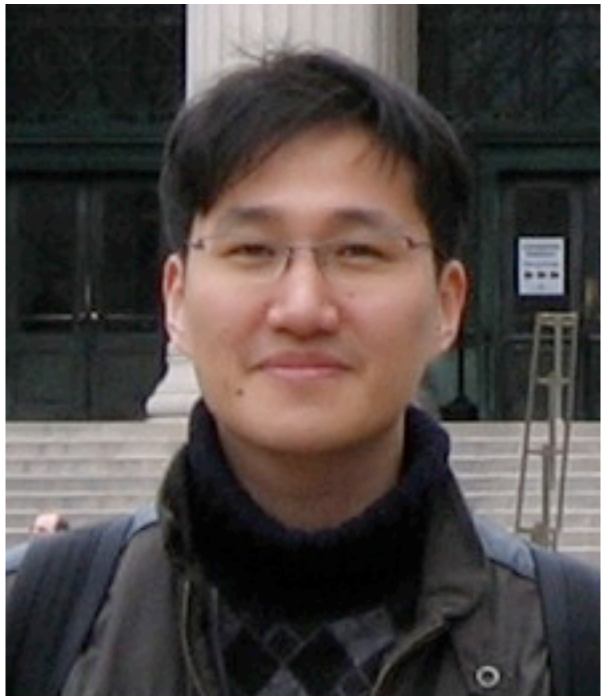
# The phase diagram of the cuprates and the quantum phase transitions of metals in two dimensions

Talk online: sachdev.physics.harvard.edu

Max Metlitski, Harvard



Eun Gook Moon, Harvard

Frederik Denef, Harvard
Lars Fritz, Cologne
Victor Galitski, Maryland
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Markus Muller, Trieste
Jorg Schmalian, Iowa

PHYSICS
HARVARD

# Outline

1. Graphene

*`Topological' Fermi surface transition*

2. The cuprate superconductors

*Fluctuating spin density waves, and pairing by gauge fluctuations*

# Outline

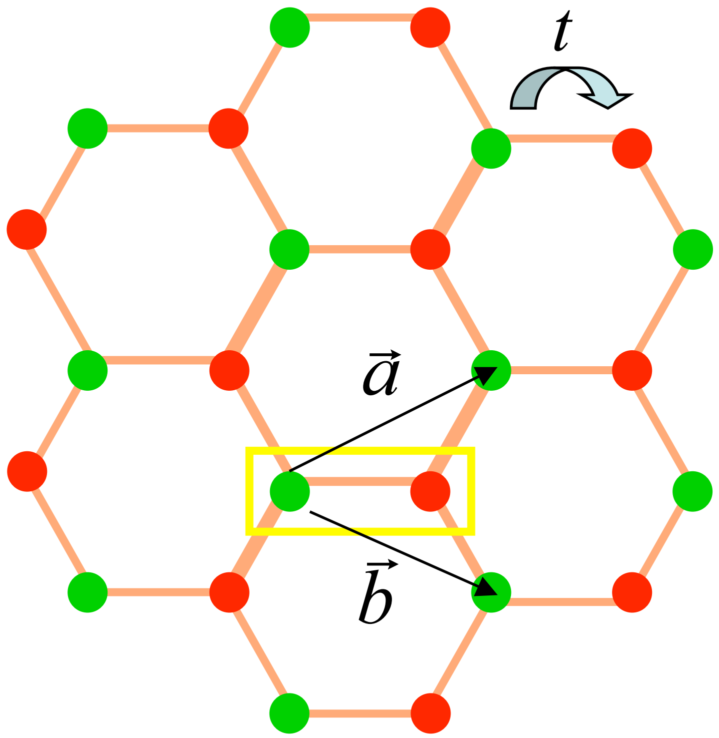
1. Graphene

   *`Topological' Fermi surface transition*

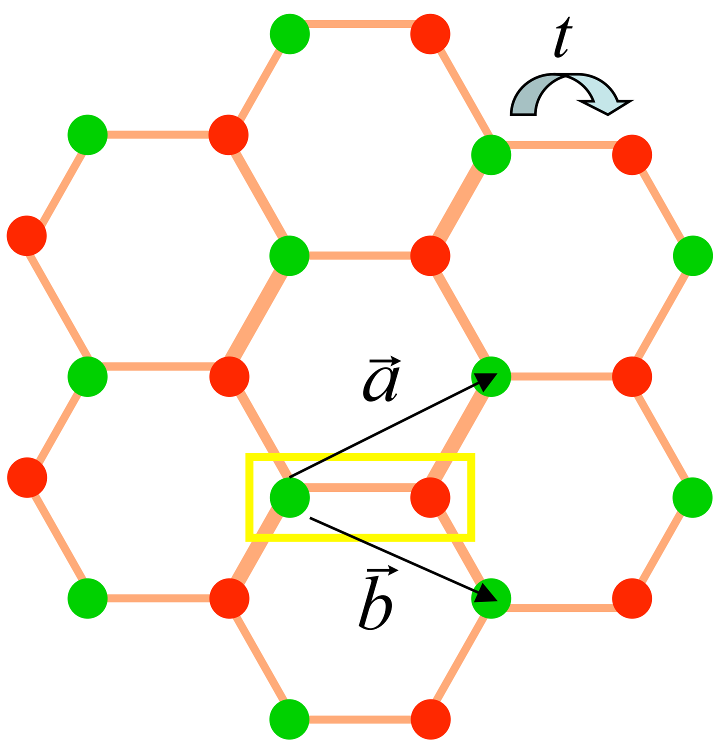
2. The cuprate superconductors

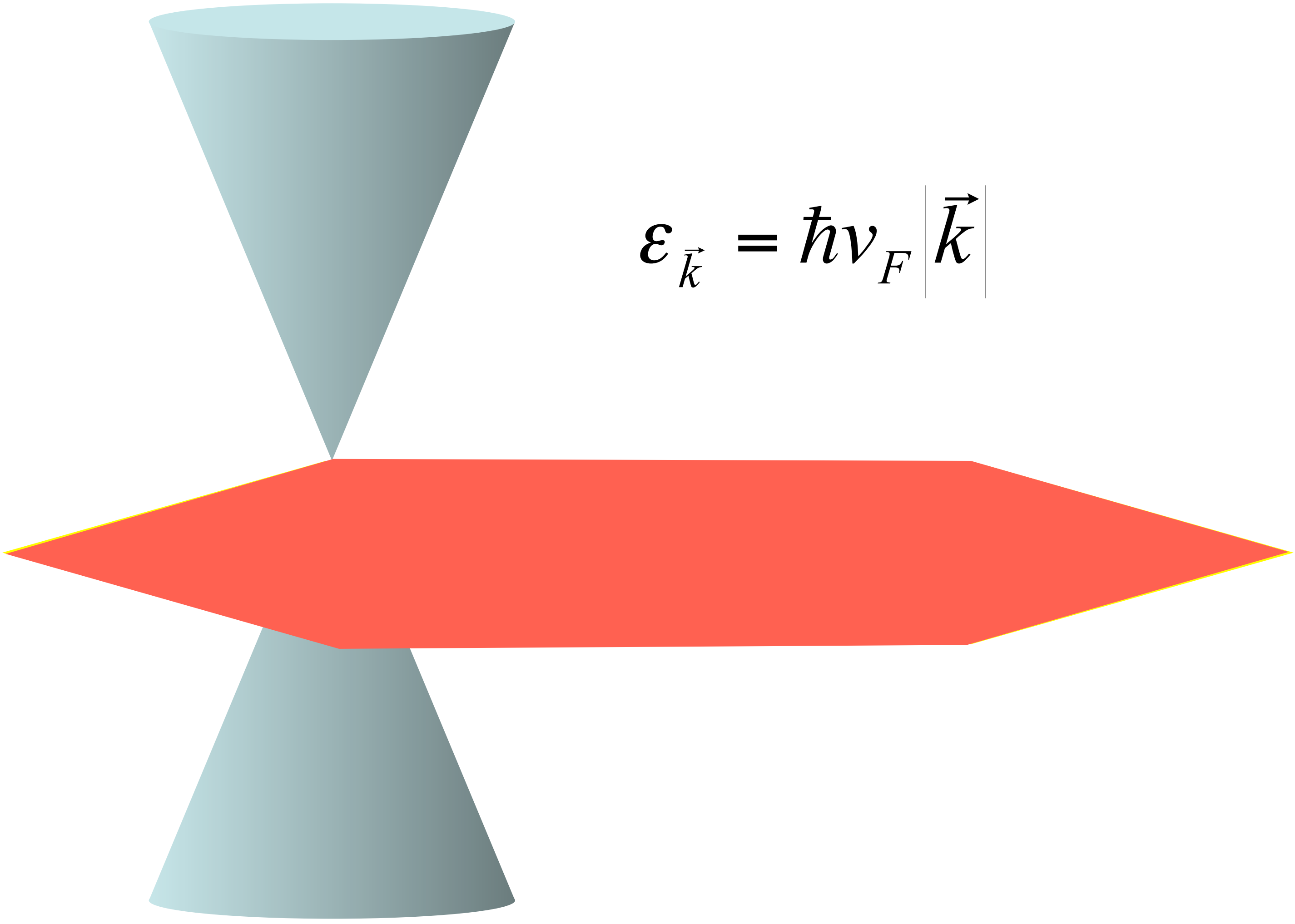
   *Fluctuating spin density waves, and pairing by gauge fluctuations*

# Graphene

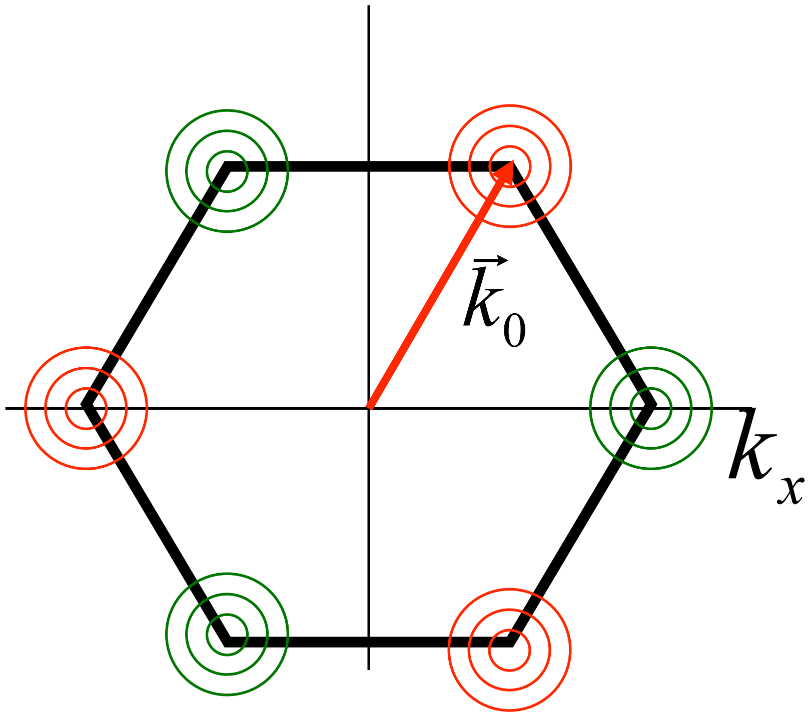


# Graphene



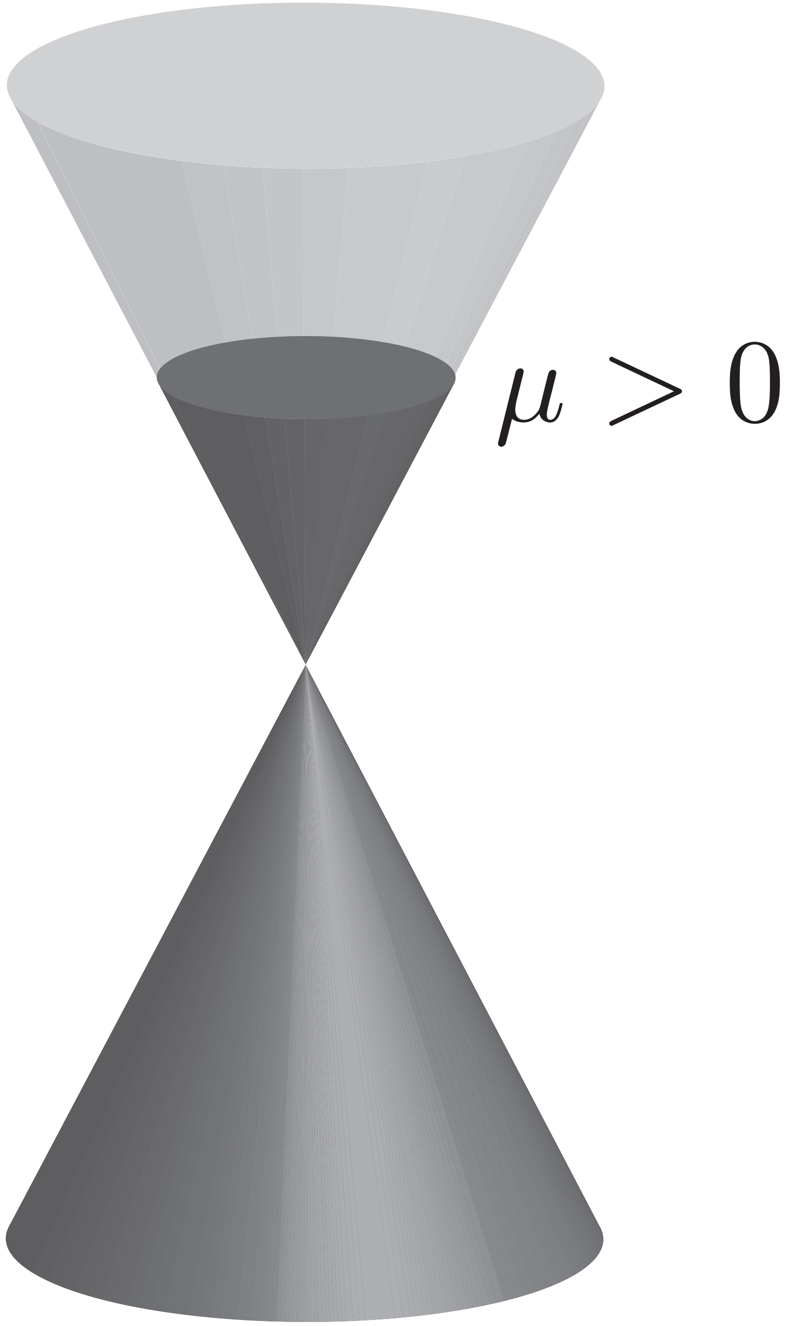


$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$



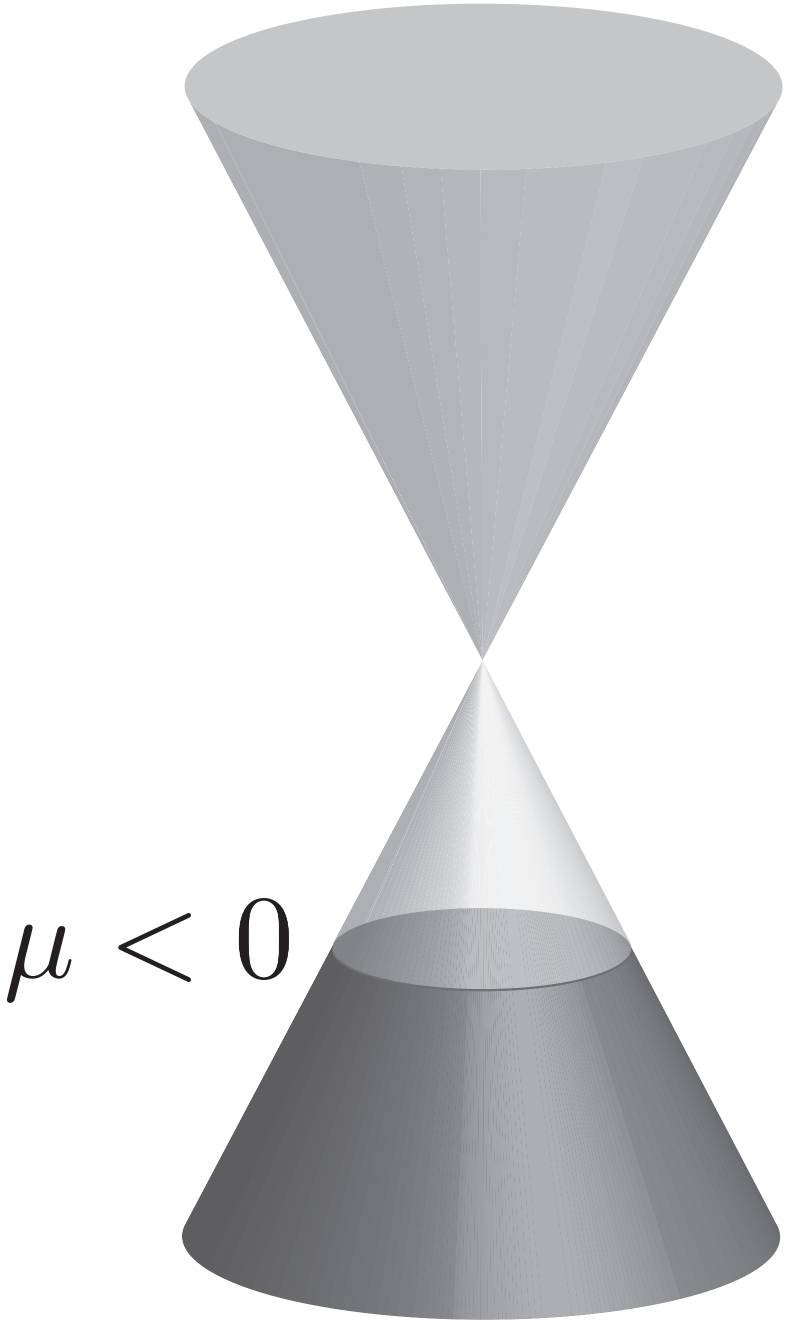
Conical Dirac dispersion

# Quantum phase transition in graphene tuned by a gate voltage

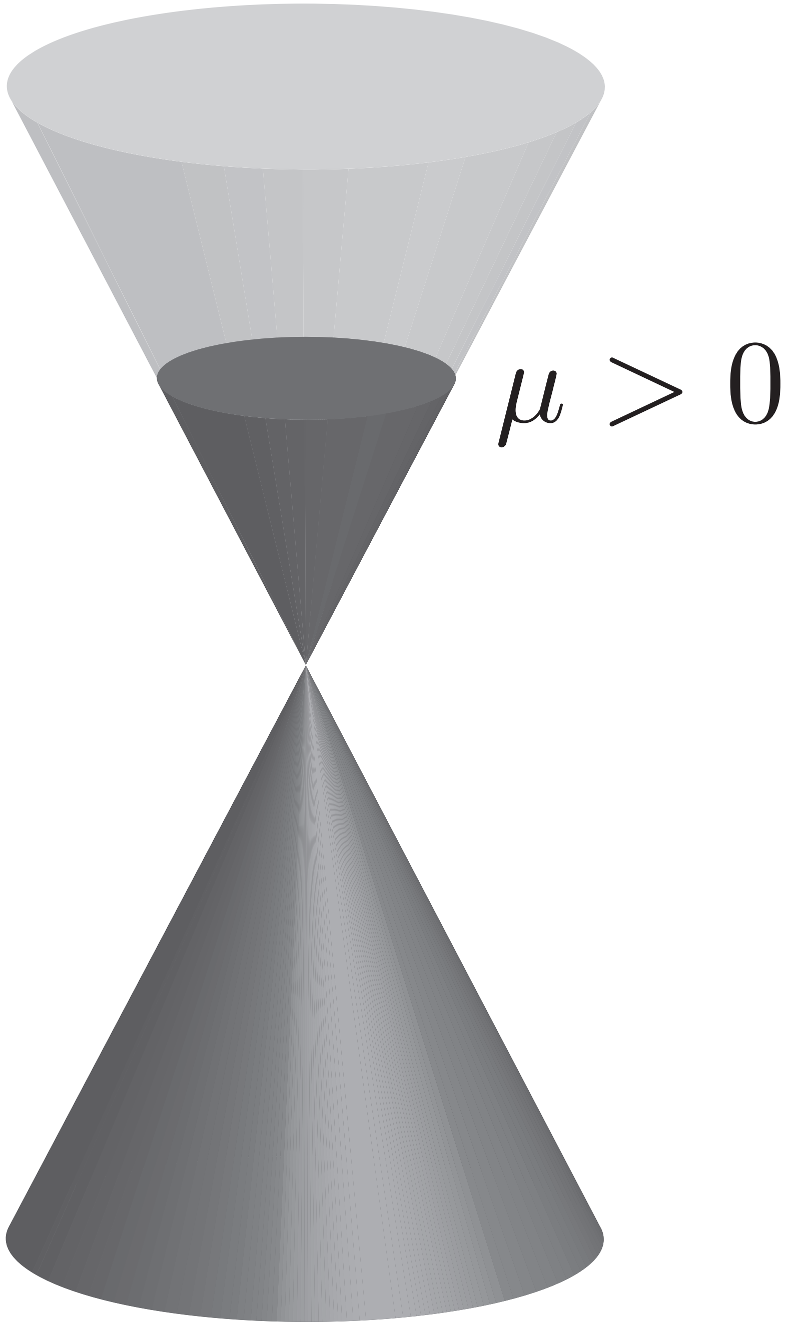


Electron
Fermi surface

# Quantum phase transition in graphene tuned by a gate voltage

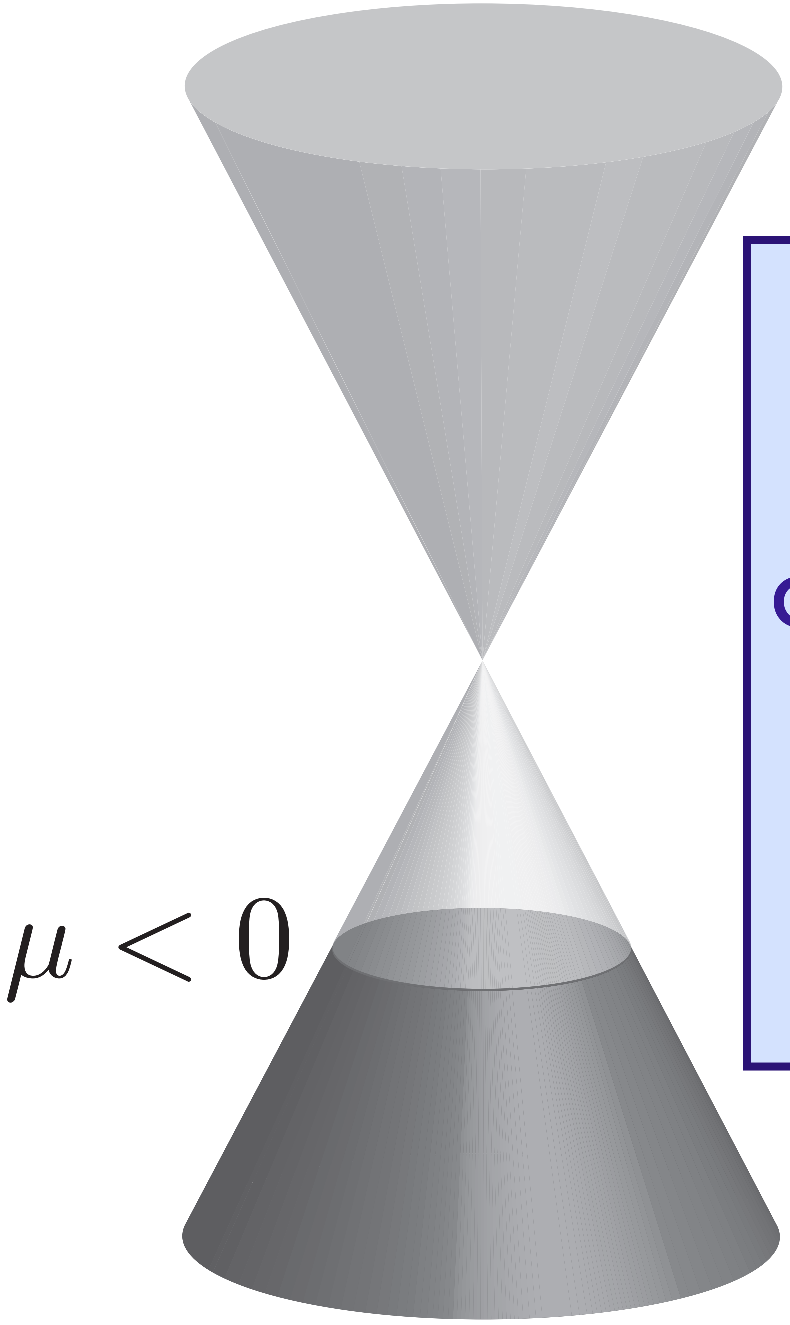


Hole
Fermi surface



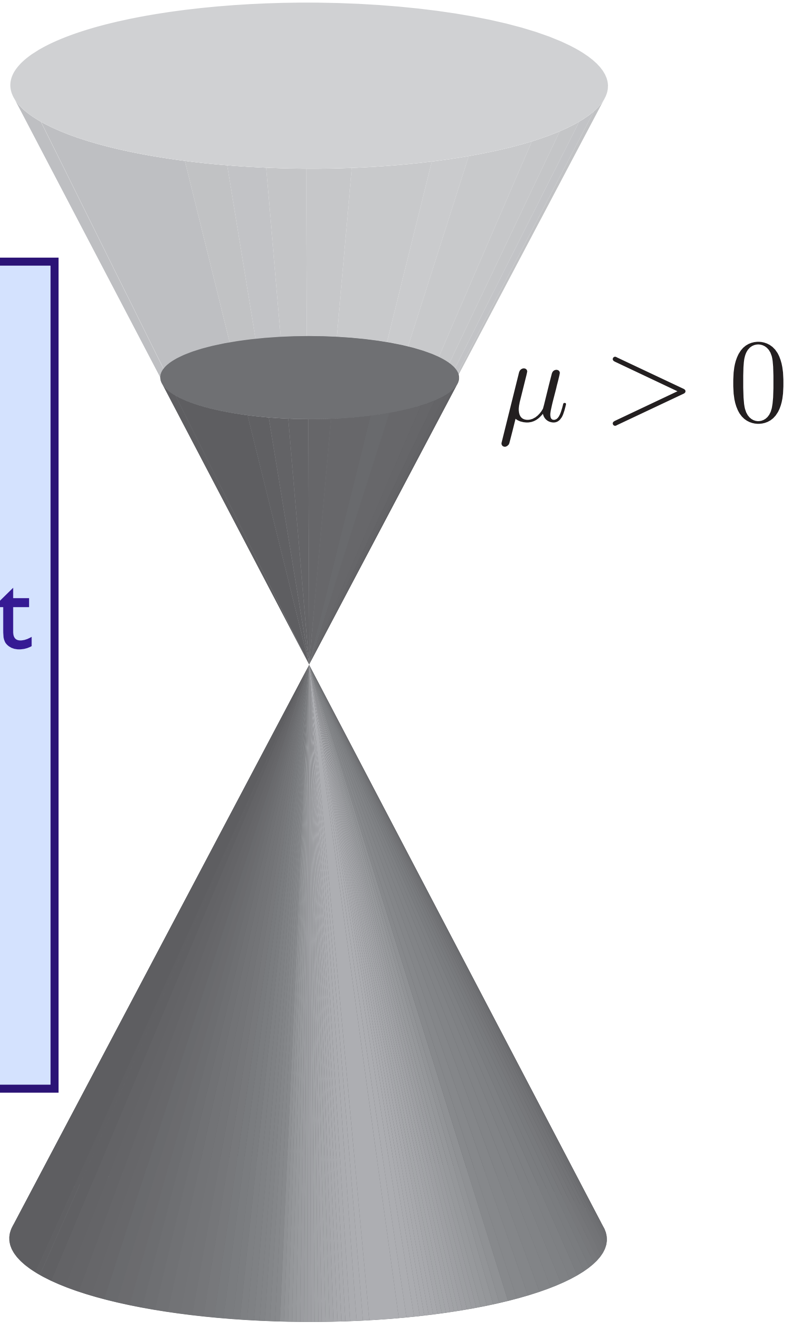
Electron
Fermi surface

# Quantum phase transition in graphene tuned by a gate voltage



Hole
Fermi surface

There must be an intermediate quantum critical point where the Fermi surfaces reduce to a Dirac point



Electron
Fermi surface

# Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, $\psi_\sigma$, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left( \partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r') \end{aligned}$$

# Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, $\psi_\sigma$, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

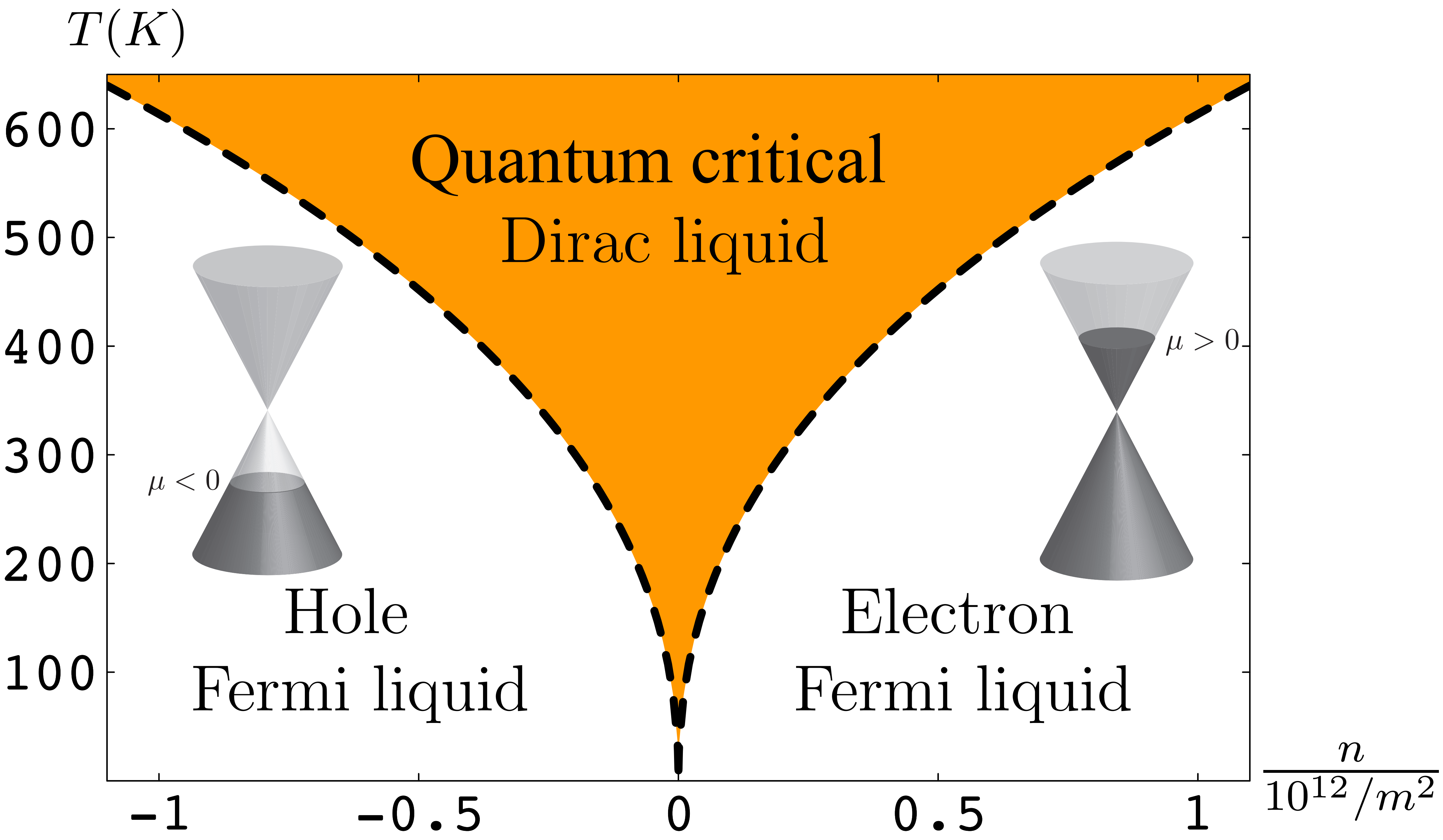
$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left( \partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r') \end{aligned}$$

Dimensionless "fine-structure" constant $\alpha = e^2/(\hbar v_F)$.
RG flow of $\alpha$:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

**Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with** $\alpha \sim 1/\ln(\text{scale})$

# Quantum phase transition in graphene



# Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, $\tau_R$

$$\boxed{\tau_R \gtrsim \frac{\hbar}{k_B T}}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

# Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

## Electrical conductivity

$$\boxed{\sigma = \frac{4e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

# Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

## Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times \left[\text{Universal constant } \mathcal{O}(1)\ \right]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

# Quantum critical transport in graphene

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

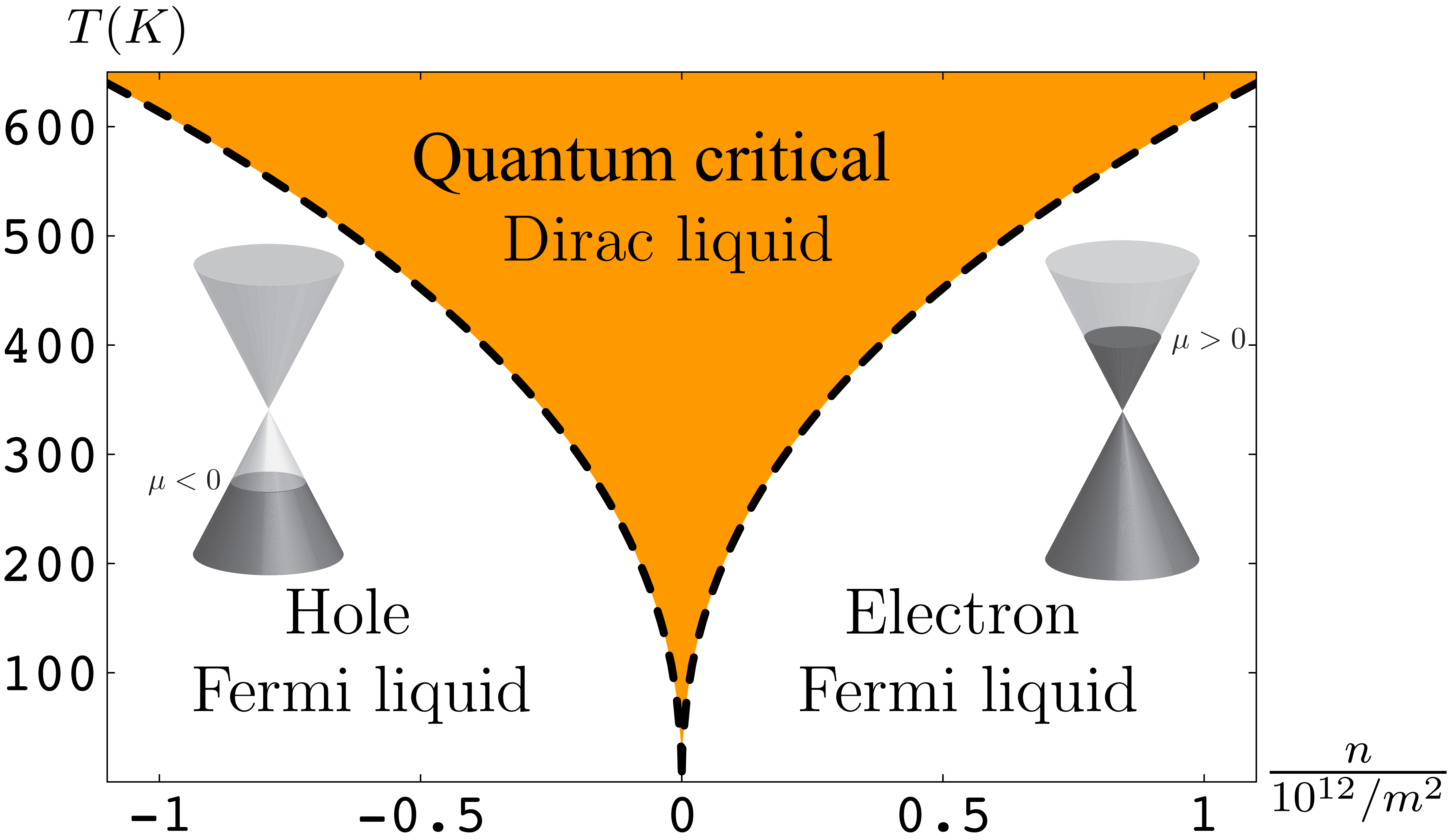
$$\frac{\eta}{s} = \frac{\hbar}{k_B \alpha^2(T)} \times 0.130$$

where the "fine structure constant" is

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

L. Fritz, J. Schmalian, M. Müller and S. Sachdev, *Physical Review B* **78**, 085416 (2008)
M. Müller, J. Schmalian, and L. Fritz, *Physical Review Letters* **103**, 025301 (2009)

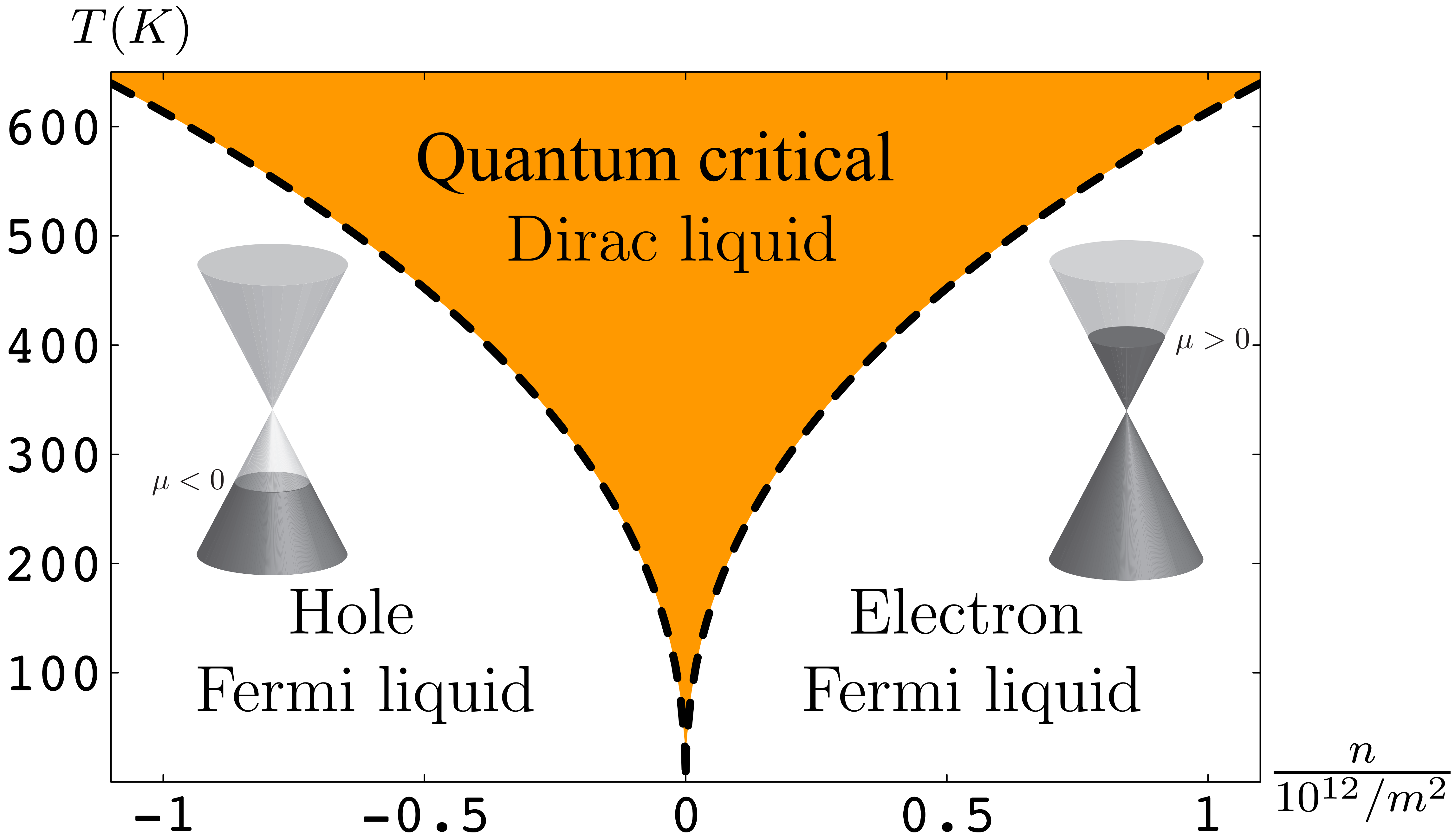
# Previously unsolved: general quantum critical transport theory for arbitrary $\mu$, applied magnetic field $B$, and small impurity density, and general $\omega/T$.



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Previously unsolved: general quantum critical transport theory for arbitrary $\mu$, applied magnetic field $B$, and small impurity density, and general $\omega/T$.

$\Rightarrow$ maps onto quasinormal modes of a Reissner-Nordstorm black hole in AdS$_4$.



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

# Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

# Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The **same** results were later obtained from the equations of generalized relativistic magnetohydrodynamics, *and* from a solution of the quantum Boltzmann equation.

So the results apply to experiments on graphene, the cuprates, *and* to the dynamics of black holes.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

# Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

As a simple example, in zero magnetic field, we can write the electrical conductivity as

$$\sigma = \sigma_Q + \frac{e^{*2}\rho^2 v^2}{\varepsilon + P}\pi\delta(\omega)$$

where $\sigma_Q$ is the universal conductivity of the CFT, $\rho$ is the charge density, $\varepsilon$ is the energy density and $P$ is the pressure.

# Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The same quantities also determine a "Wiedemann-Franz"-like relation for thermal conductivity, $\kappa$ at $B = 0$

$$\kappa = \sigma_Q \left( \frac{k_B^2 T}{e^{*2}} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 .$$

At $B \neq 0$ and $\rho = 0$ we have a "Wiedemann-Franz" relation for "vortices"

$$\kappa = \frac{1}{\sigma_Q} k_B^2 T \left( \frac{v(\varepsilon + P)}{k_B T B} \right)^2 .$$

# Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

A second example: In an applied magnetic field $B$, the dynamic transport co-efficients exhibit a **hydrodynamic cyclotron resonance** at a frequency $\omega_c$

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and damping constant $\gamma$

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}.$$

The same constants determine the **quasinormal frequency** of the Reissner-Nordstrom black hole.

# Outline

1. Graphene

   *`Topological' Fermi surface transition*

2. The cuprate superconductors

   *Fluctuating spin density waves, and pairing by gauge fluctuations*

# Outline

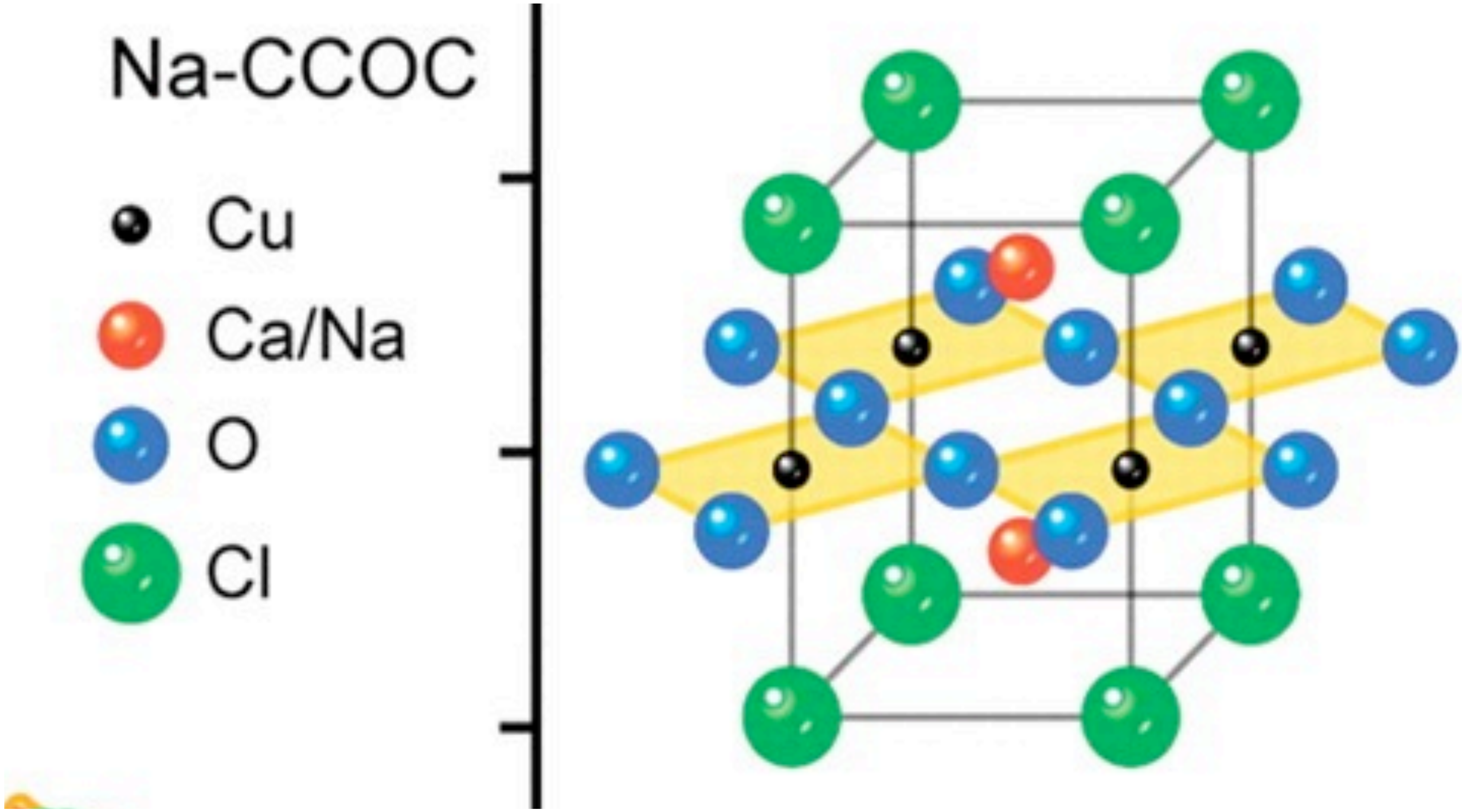
1. Graphene

   *`Topological' Fermi surface transition*

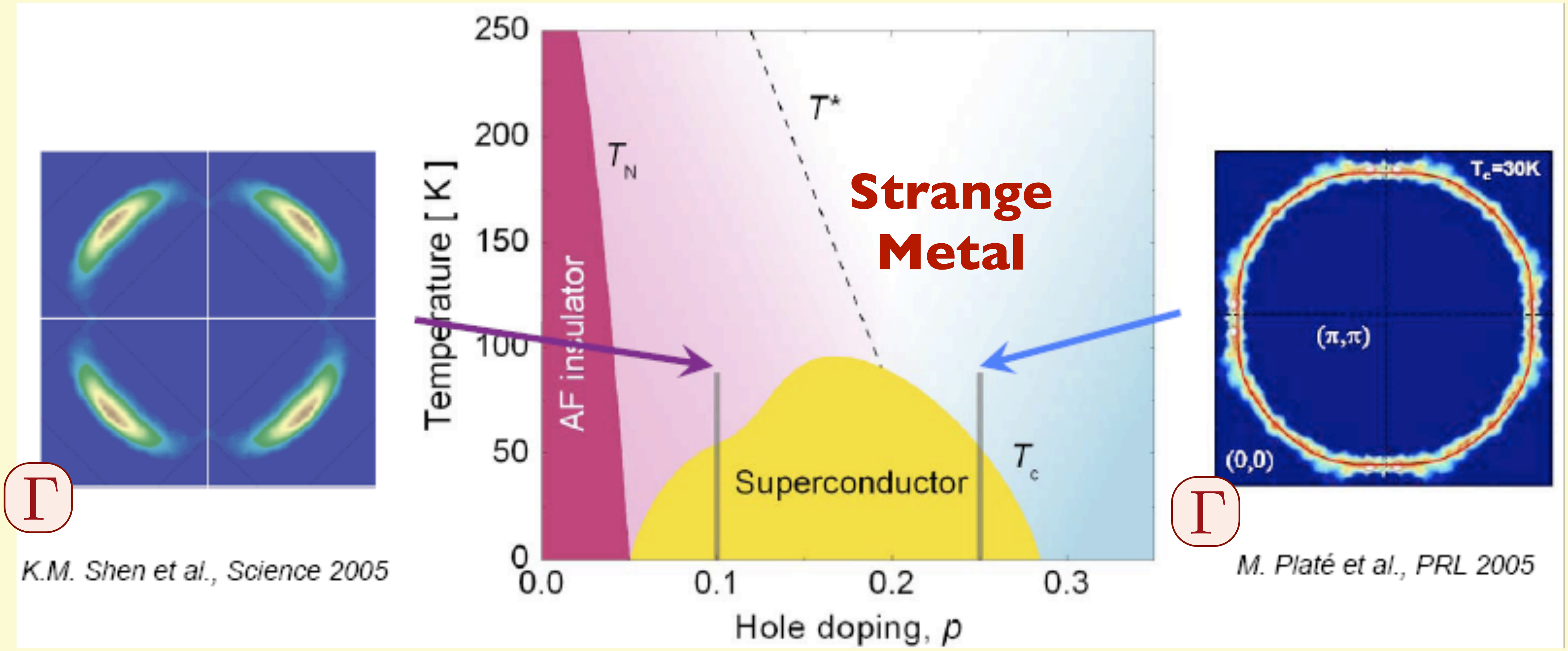
2. The cuprate superconductors

   *Fluctuating spin density waves, and pairing by gauge fluctuations*

# The cuprate superconductors



# Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



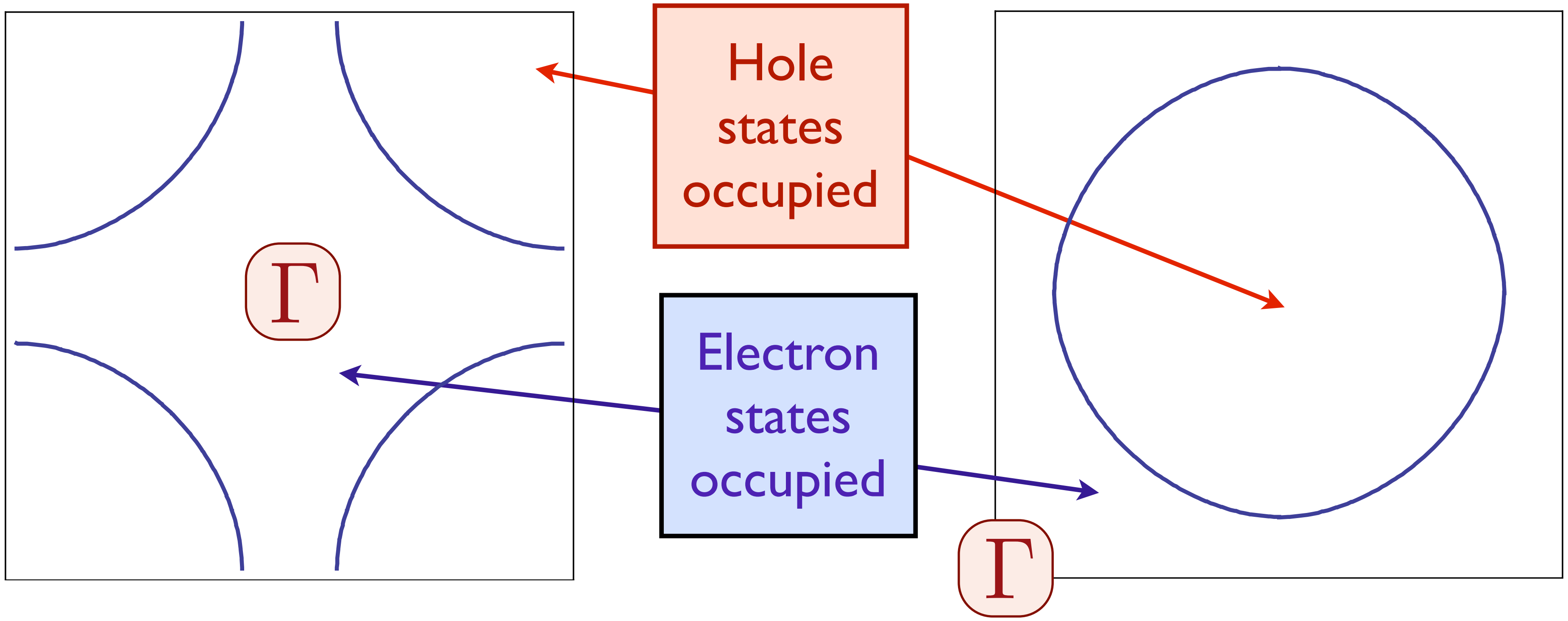
*K.M. Shen et al., Science 2005*

*M. Platé et al., PRL 2005*

Smaller hole Fermi-pockets

Large hole Fermi surface

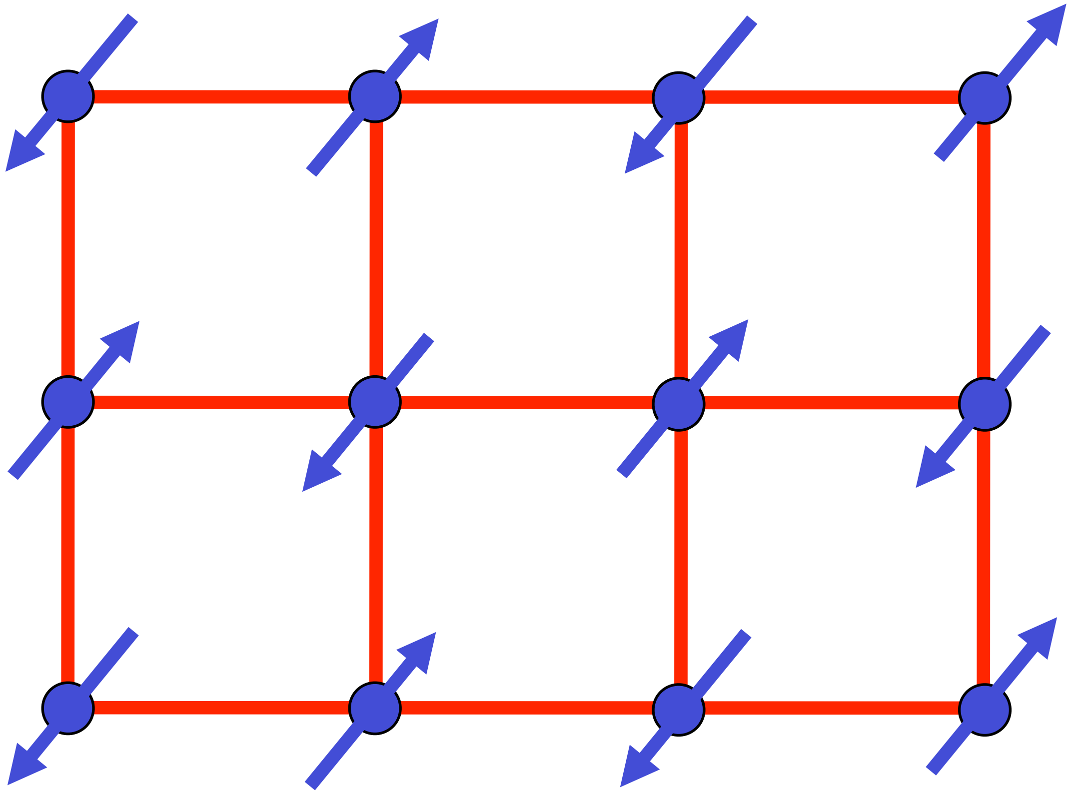
# Fermi surface+antiferromagnetism



Hole states occupied

Electron states occupied

+

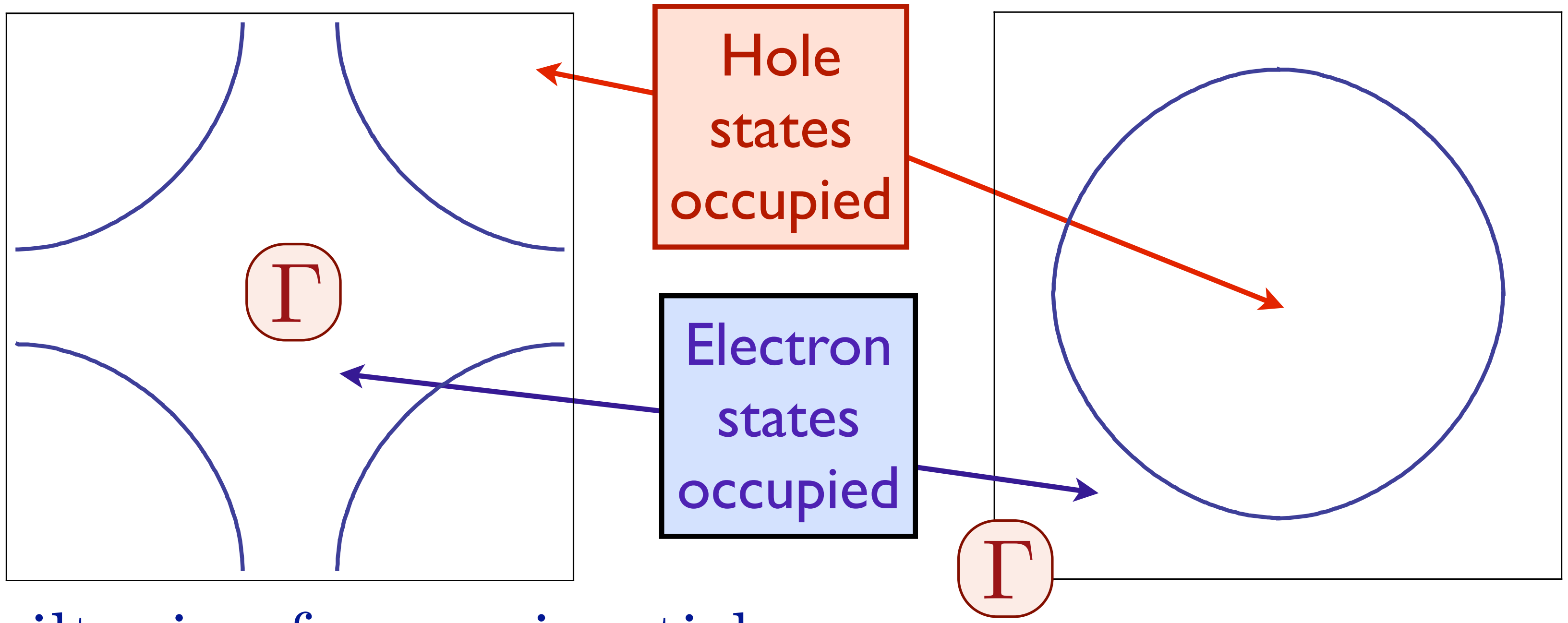


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

# Fermi surfaces in electron- and hole-doped cuprates



Hole states occupied

Electron states occupied

Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with $t_{ij}$ non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

# Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

$$H_{\mathrm{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

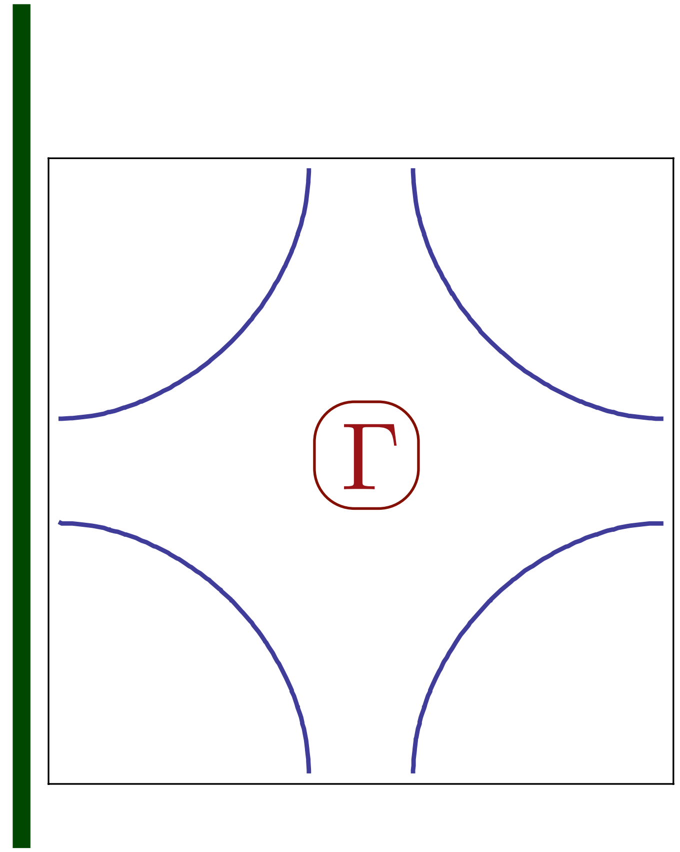
where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\mathrm{sdw}}$ for $\vec{\varphi} \propto (0, 0, 1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

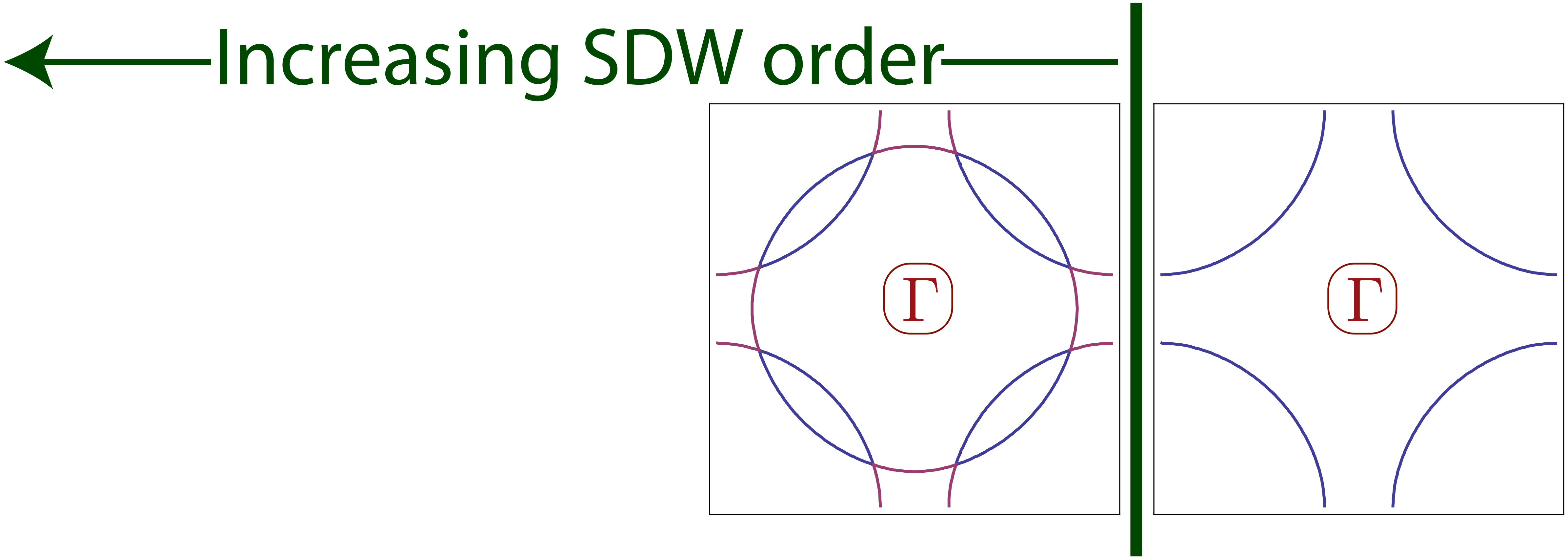
# Hole-doped cuprates

Increasing SDW order



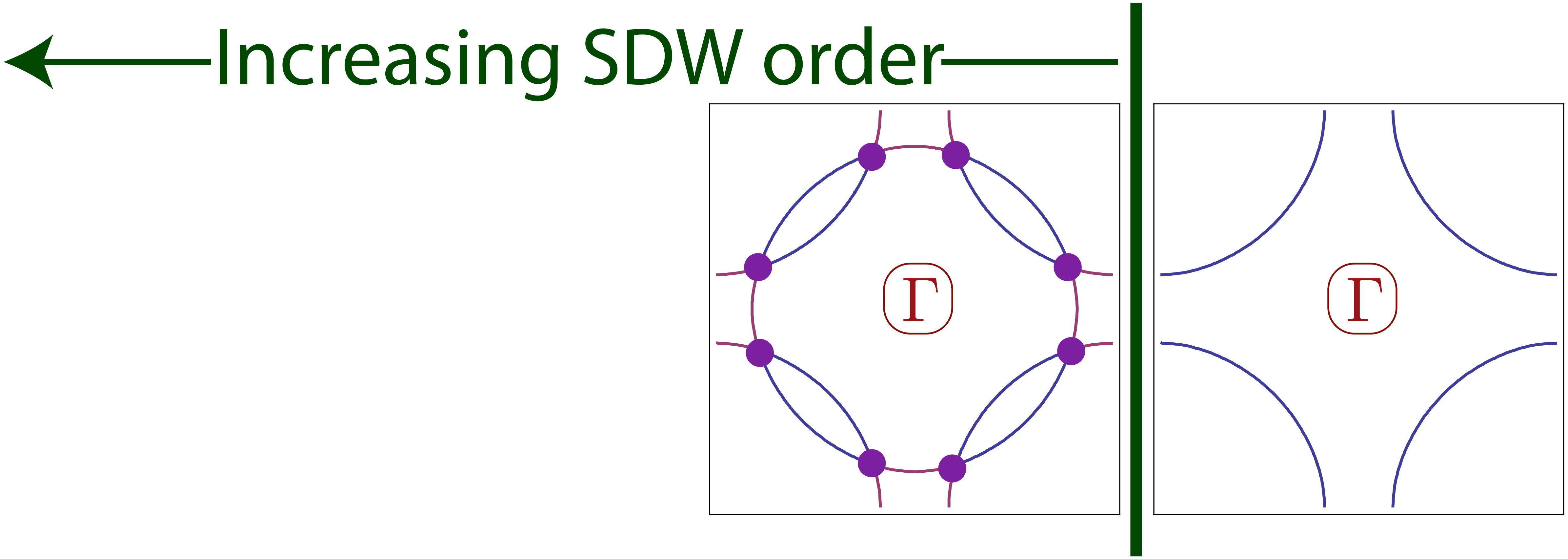
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Hole-doped cuprates


Increasing SDW order

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

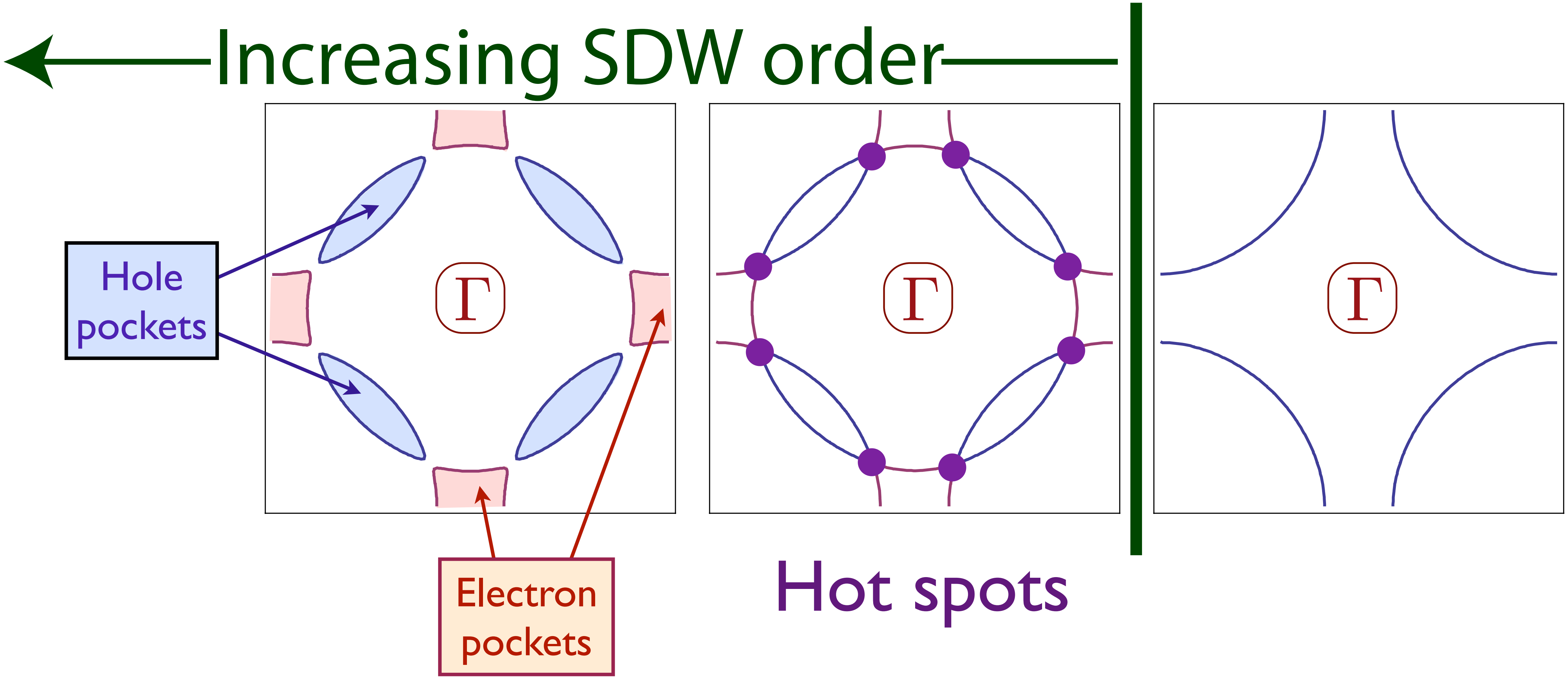
# Hole-doped cuprates



Hot spots

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Hole-doped cuprates



Increasing SDW order

Hole pockets

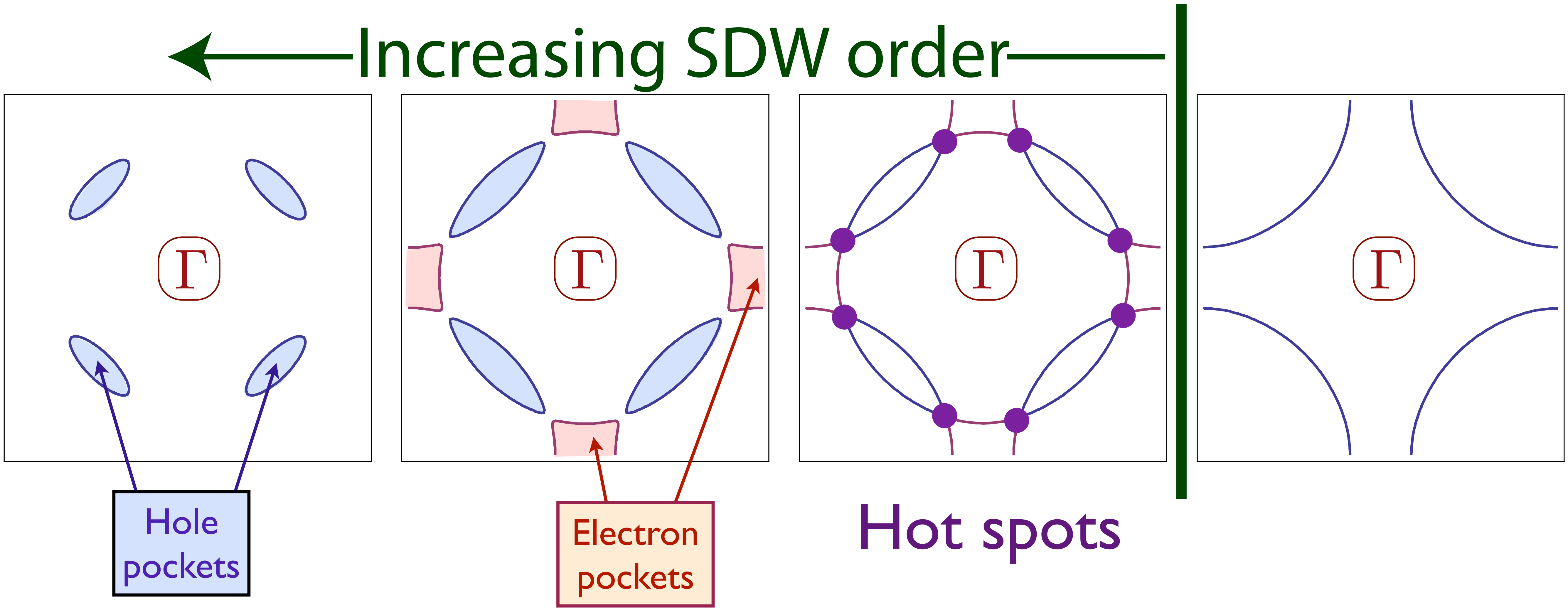
Electron pockets

Hot spots

Fermi surface breaks up at hot spots into electron and hole "pockets"

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Hole-doped cuprates



Increasing SDW order

Hole pockets

Electron pockets

Hot spots

Fermi surface breaks up at hot spots
into electron and hole “pockets”

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Evidence for small Fermi pockets

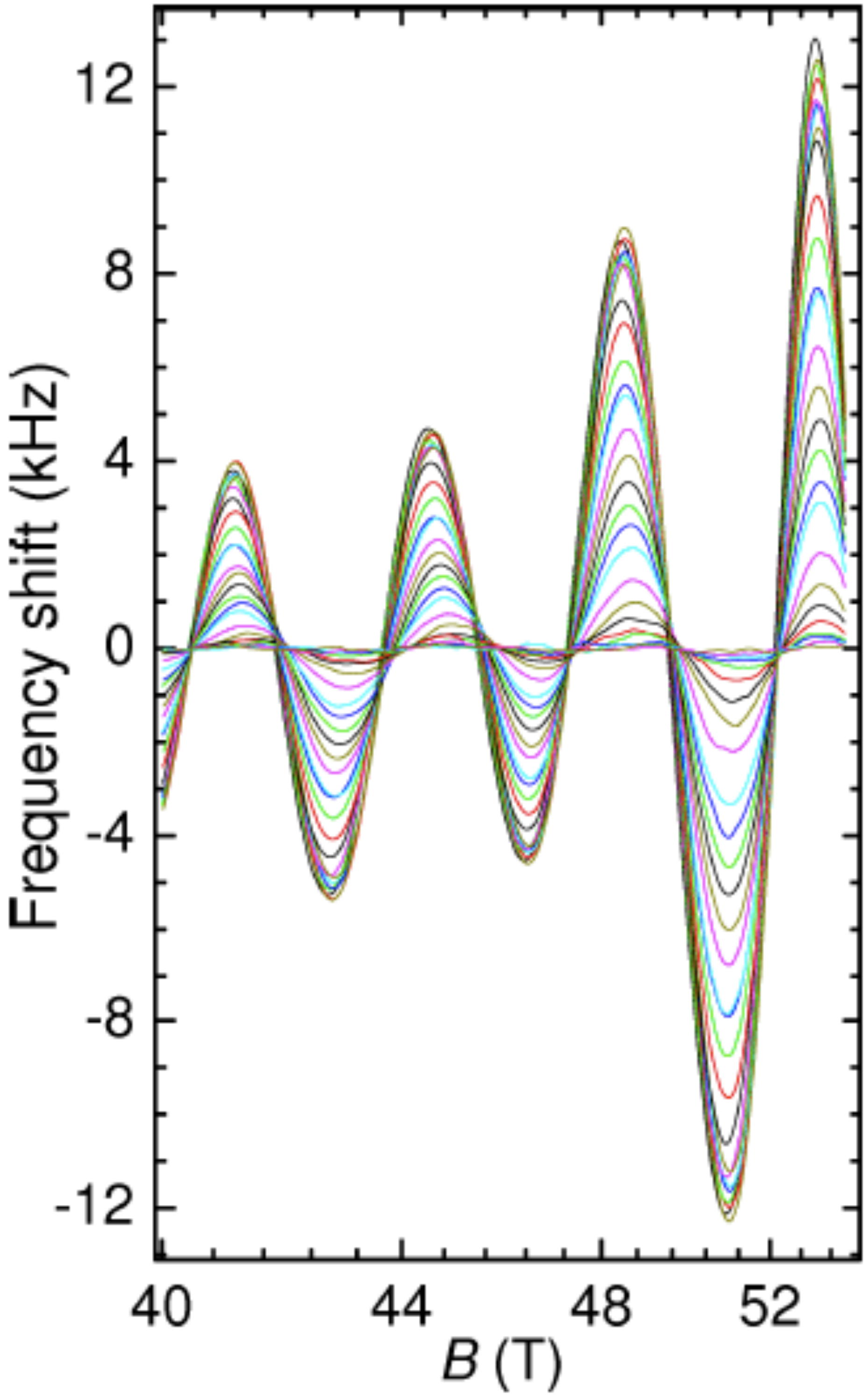


FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between $T$ = 1 and 18 K. The actual $T$ values are provided in Fig. 3.

**Fermi liquid behaviour in an underdoped high Tc superconductor**

Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D. A. Bonn,
W. N. Hardy, and G. G. Lonzarich

arXiv:0912.3022

# Start from the "spin-fermion" model

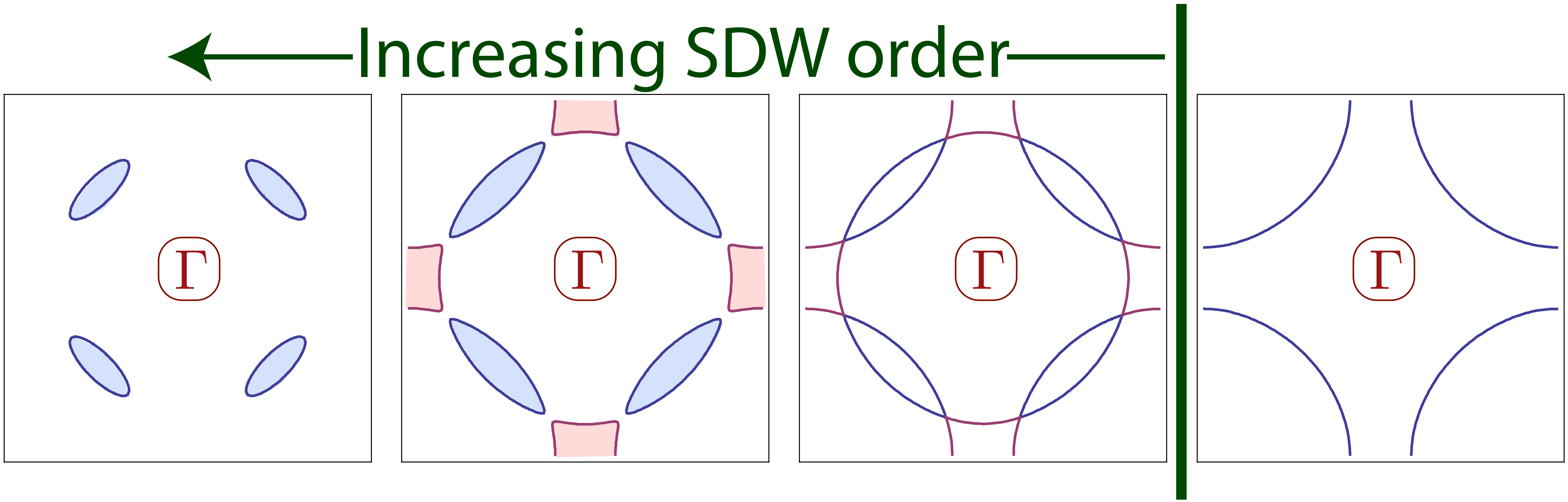
$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left( \frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

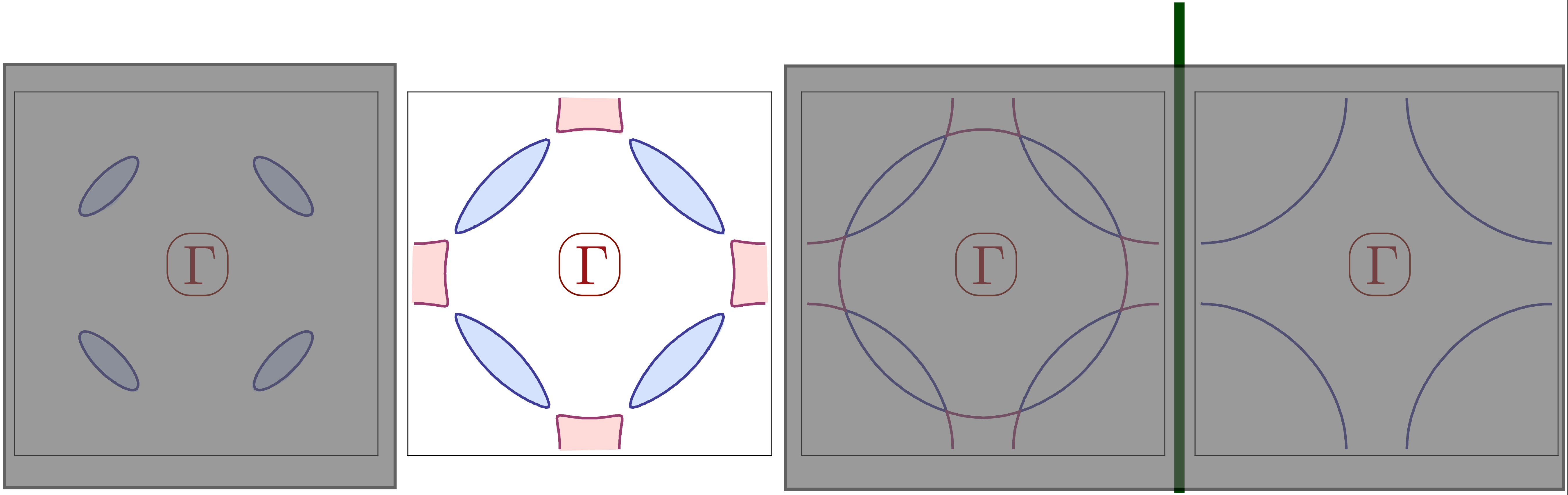
$$+ \int d\tau d^2 r \left[ \frac{1}{2} \left( \boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left( \partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

# Spin density wave theory in hole-doped cuprates



Increasing SDW order

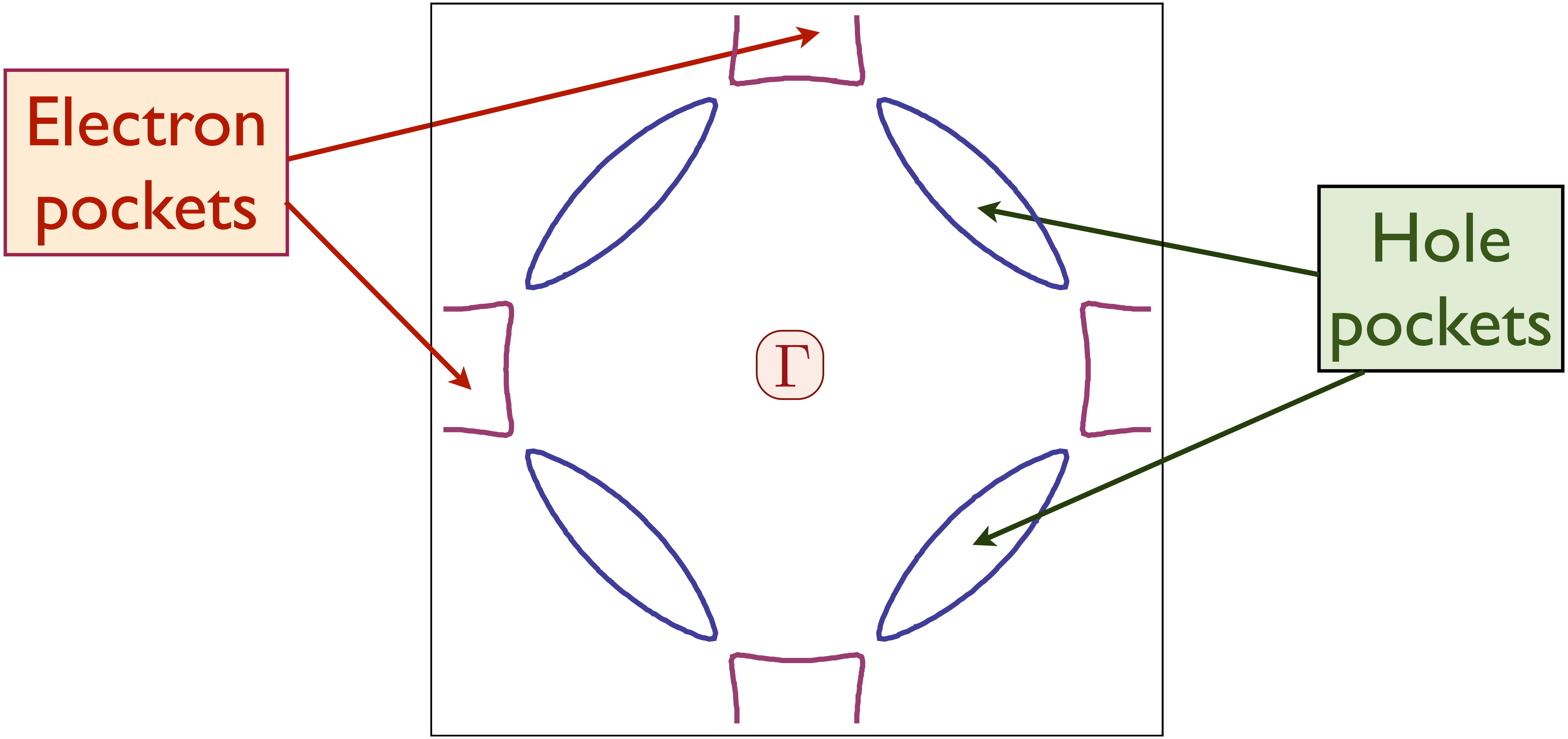
# Fermi pockets in hole-doped cuprates



Begin with SDW ordered state, and focus on fluctuations in the *orientation* of $\vec{\varphi}$, by using a unit-length bosonic spinor $z_\alpha$

$$\vec{\varphi} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$$

# Charge carriers in the lightly-doped cuprates with Neel order



Electron pockets

Hole pockets

# Spin density wave theory for electrons near $(0,\pi)$ and $(\pi,0)$

Let us write $c_{(0,\pi)\alpha} = c_{1\alpha}$, $c_{(\pi,0)\alpha} = c_{2\alpha}$ and $\varepsilon_{(0,\pi)} = \varepsilon_{(\pi,0)} = \varepsilon_0$. Then the Hamiltonian for $\vec{\varphi} = (0,0,\varphi)$ with $\varphi > 0$ is

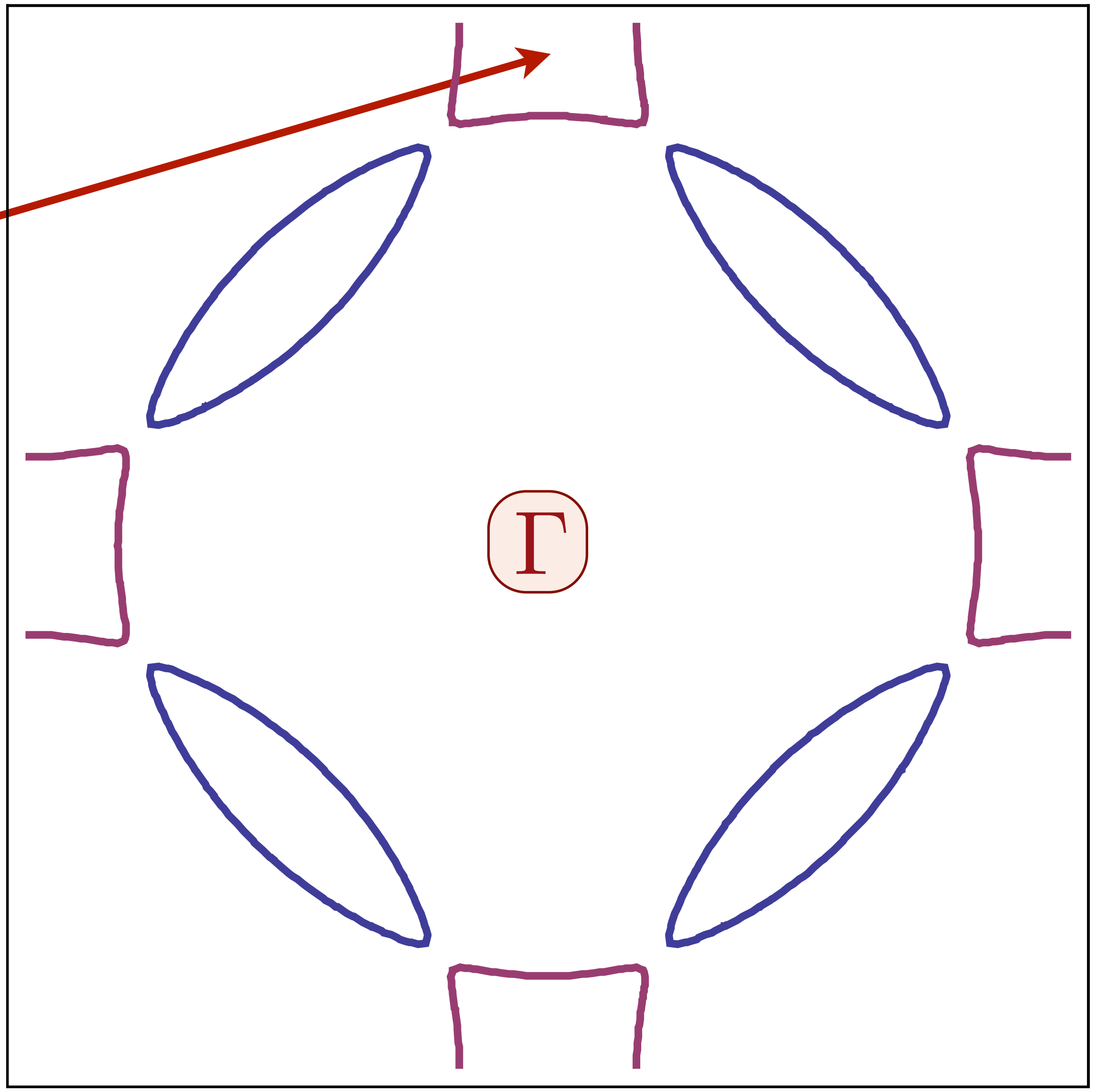
$$\begin{aligned} H_0 + H_{\text{sdw}} &= \varepsilon_0 \left( c_{1\alpha}^\dagger c_{1\alpha} + c_{2\alpha}^\dagger c_{2\alpha} \right) \\ &\quad - \varphi \left( c_{1\uparrow}^\dagger c_{2\uparrow} - c_{1\downarrow}^\dagger c_{2\downarrow} + c_{2\uparrow}^\dagger c_{1\uparrow} - c_{2\downarrow}^\dagger c_{1\downarrow} \right) \end{aligned}$$

We diagonalize this by writing

$$H_0 + H_{\text{sdw}} = (\varepsilon_0 - \varphi) \left( g_+^\dagger g_+ + g_-^\dagger g_- \right) + (\varepsilon_0 + \varphi) \left( h_+^\dagger h_+ + h_-^\dagger h_- \right)$$

where

$$\begin{aligned} c_{1\uparrow} &= (g_+ + h_+)/\sqrt{2} \\ c_{2\uparrow} &= (g_+ - h_+)/\sqrt{2} \\ c_{1\downarrow} &= (g_- + h_-)/\sqrt{2} \\ c_{2\downarrow} &= (-g_- + h_-)/\sqrt{2} \end{aligned}$$



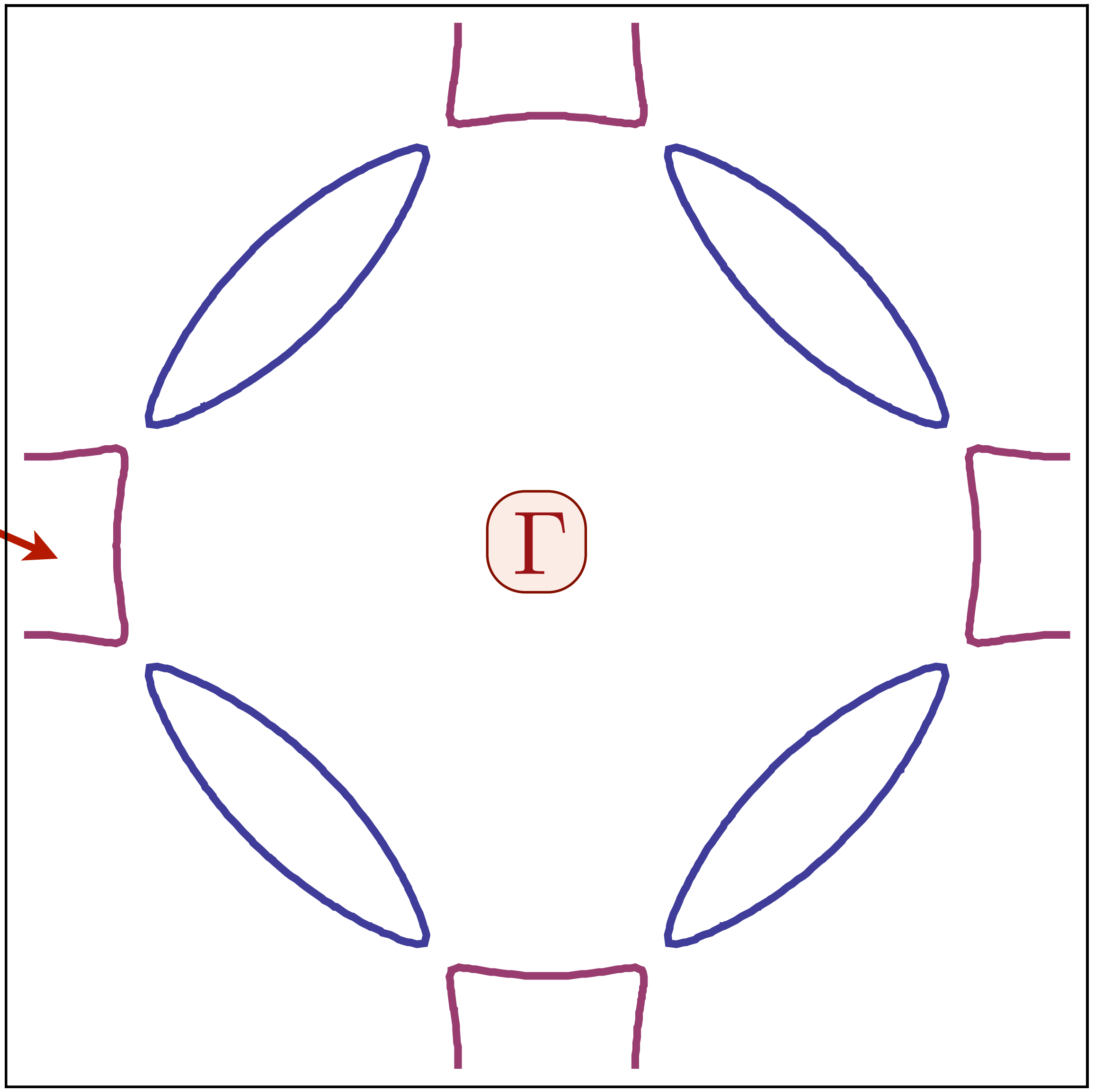
Electron operator
$c_{1\alpha}$

For a uniform SDW order with $\vec{\varphi} = (0, 0, \varphi)$, write

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \begin{pmatrix} g_{+} \\ g_{-} \end{pmatrix}$$

**Electron operator**

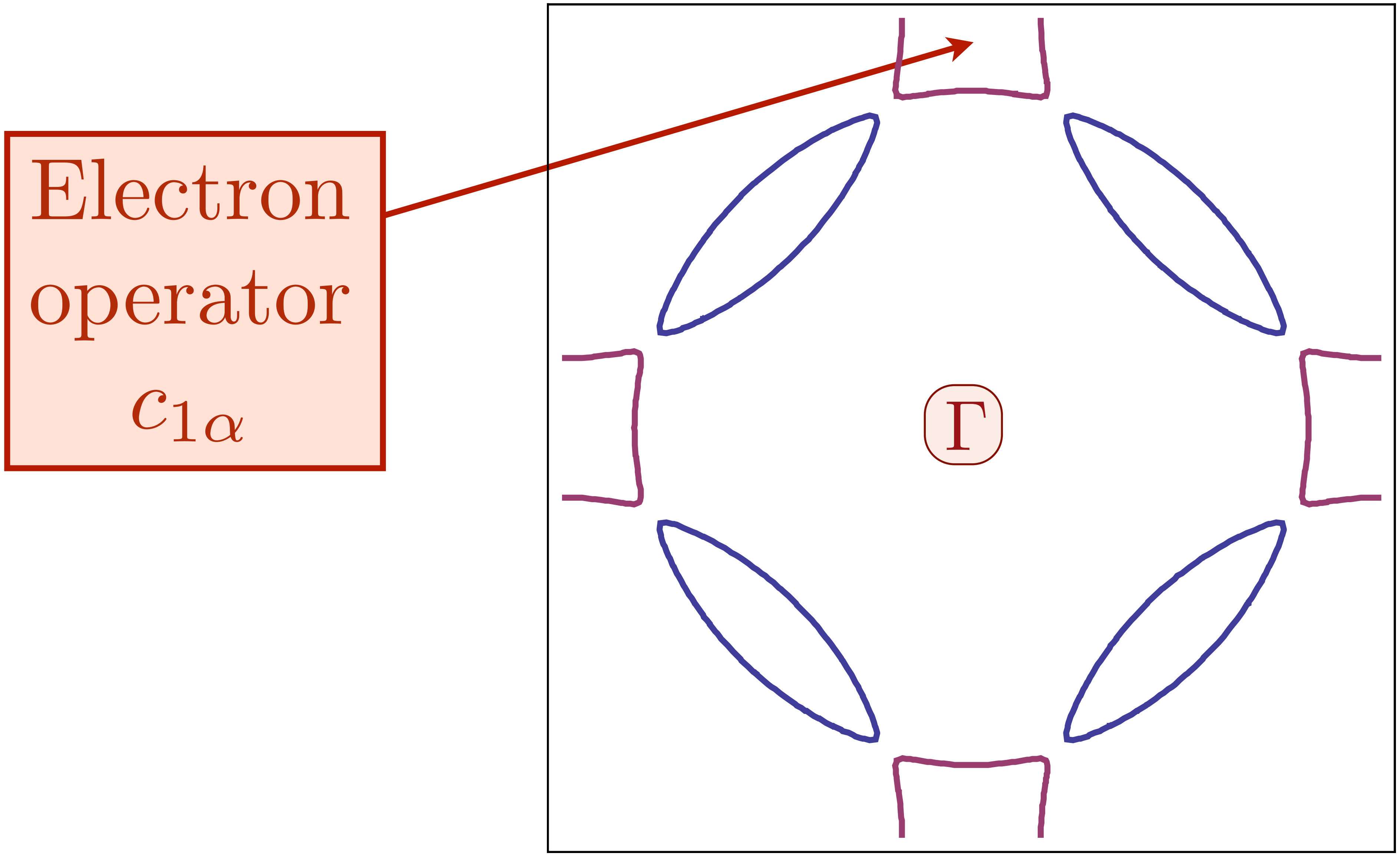
$c_{2\alpha}$



SDW theory also specifies electrons
at second pocket for $\vec{\varphi} = (0, 0, \varphi)$

,

$$\begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

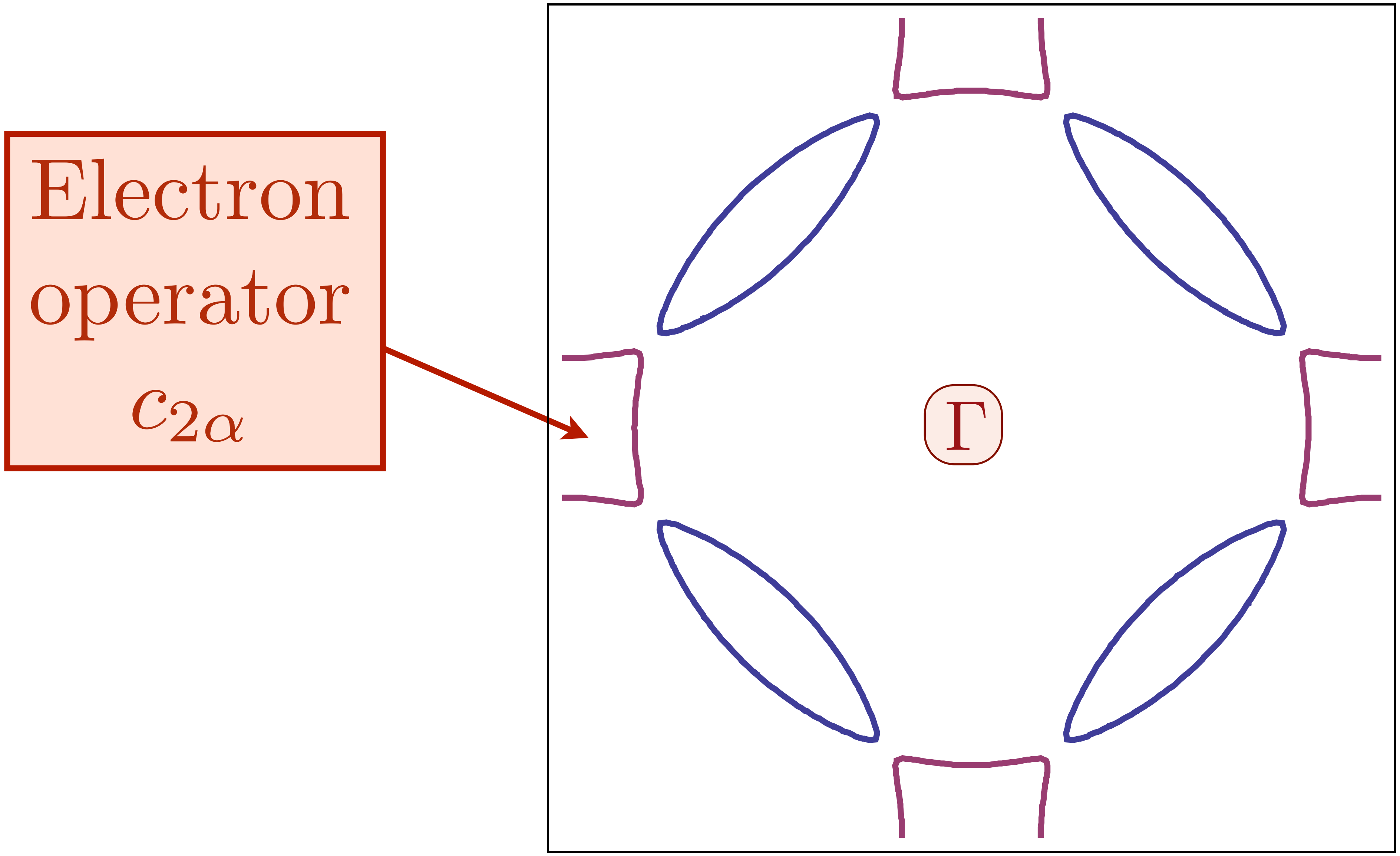


Electron operator $c_{1\alpha}$

For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\left( \begin{array}{c} c_{1\uparrow} \\ c_{1\downarrow} \end{array} \right) = \mathcal{R}_z \left( \begin{array}{c} g_+ \\ g_- \end{array} \right) \quad ; \quad \mathcal{R}_z \equiv \left( \begin{array}{cc} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{array} \right).$$

So $g_\pm$ are the "up/down" electron operators
in a rotating reference frame defined by the local SDW order



Electron operator

$c_{2\alpha}$

For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

Same SU(2) matrix also rotates electrons in second pocket.

# Fluctuating pocket theory for electrons near $(0,\pi)$ and $(\pi,0)$

Summarizing, in the low energy theory, the $c_{1,2\alpha}$ are expressed in terms of the $g_\pm$ fermions and the $z_\alpha$ by

$$\begin{aligned} c_{1\uparrow} &= z_\uparrow g_+ - z_\downarrow^* g_- \\ c_{2\uparrow} &= z_\uparrow g_+ + z_\downarrow^* g_- \\ c_{1\downarrow} &= z_\downarrow g_+ + z_\uparrow^* g_- \\ c_{2\downarrow} &= z_\downarrow g_+ - z_\uparrow^* g_- \end{aligned}$$

Note that this invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\phi} z_\alpha \quad ; \quad g_+ \rightarrow e^{-i\phi} g_+ \quad ; \quad g_- \rightarrow e^{i\phi} g_- ,$$

which must be obeyed by the effective action for $z_\alpha$ and $g_\pm$.

# Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \quad \text{or} \quad \hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

the theory is invariant under

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \;\; ; \;\; \psi_+ \rightarrow e^{-i\theta} \psi_+ \;\; ; \;\; \psi_- \rightarrow e^{i\theta} \psi_{.-}$$

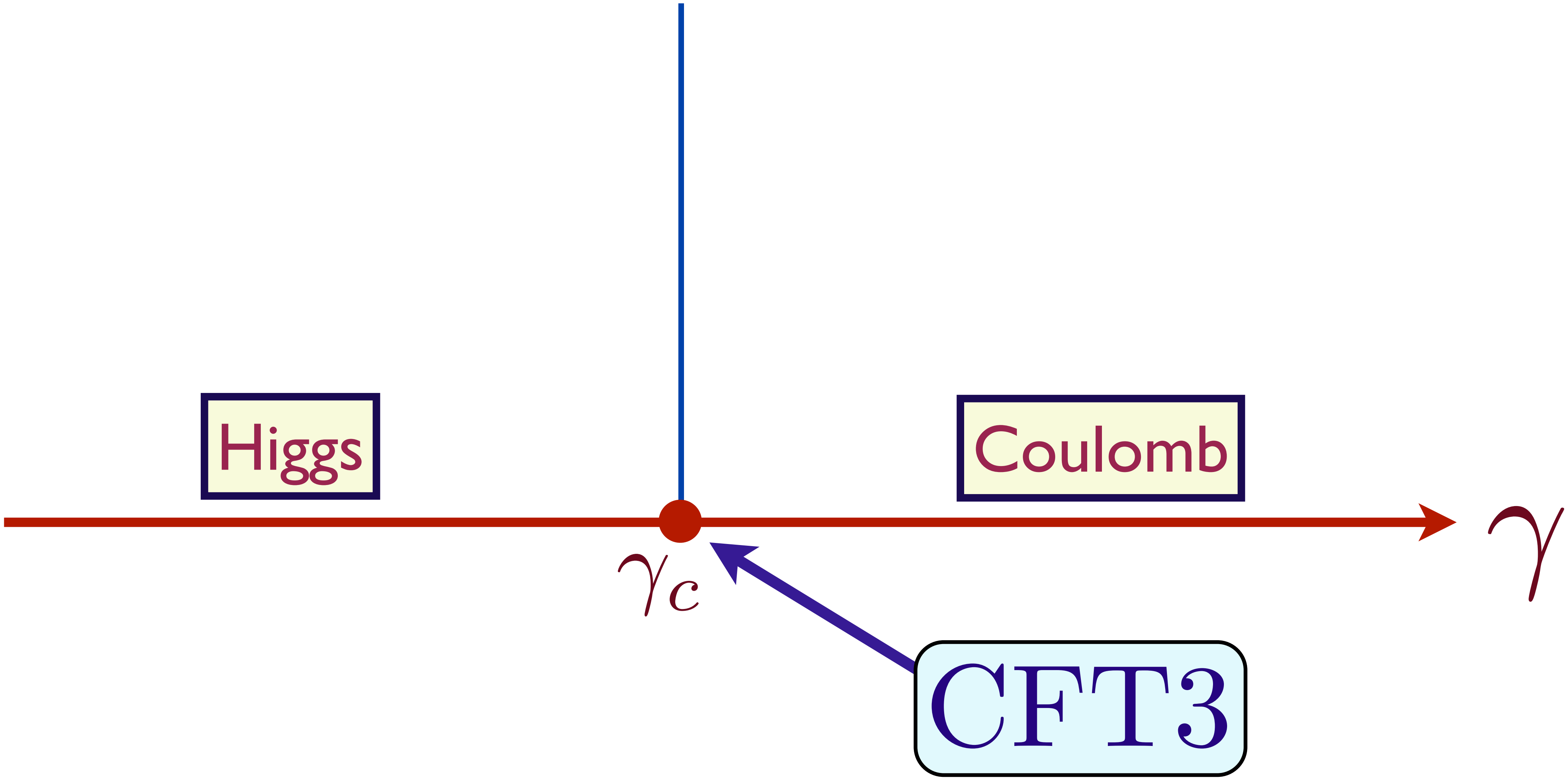
We obtain a U(1) gauge theory of

- bosonic neutral spinons $z_\alpha$;
- spinless, charged fermions $\psi_\pm$
  with small 'pocket' Fermi surfaces;
- an emergent U(1) gauge field $A_\mu$.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Phys. Rev. B* **80**, 155129 (2009).

- Begin with a CFT3: the CP$^1$ model.

$$\mathcal{L}_z = \frac{1}{\gamma}|(\partial_\mu - iA_\mu)z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$



Higgs

Coulomb

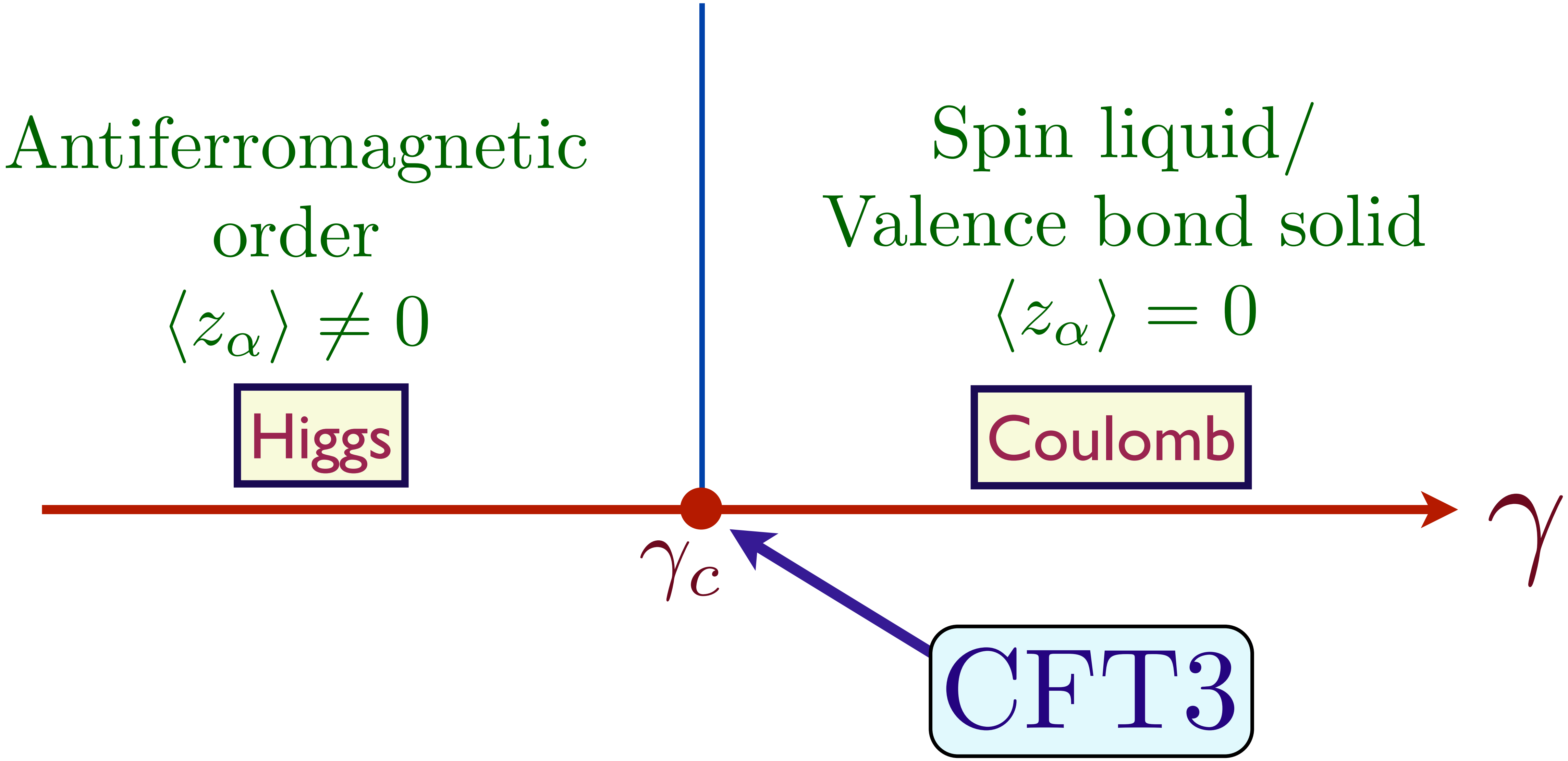
$\gamma_c$

$\gamma$

CFT3

- Begin with a CFT3: the CP$^1$ model.

$$\mathcal{L}_z = \frac{1}{\gamma} |(\partial_\mu - i A_\mu) z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$



Antiferromagnetic order

$\langle z_\alpha \rangle \neq 0$

Higgs

Spin liquid/ Valence bond solid

$\langle z_\alpha \rangle = 0$

Coulomb

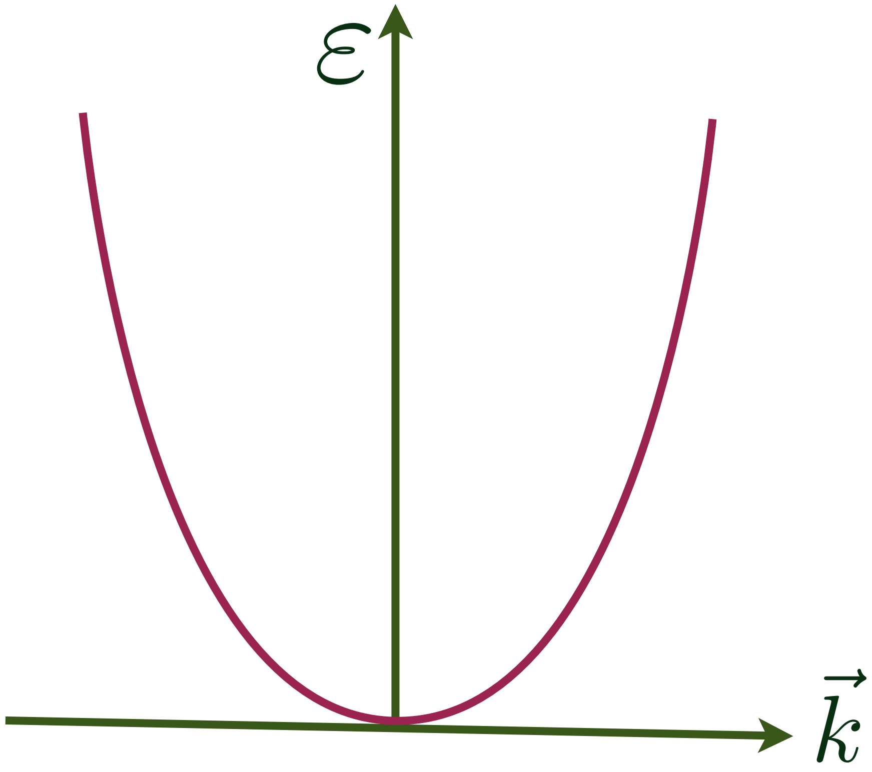
$\gamma_c$

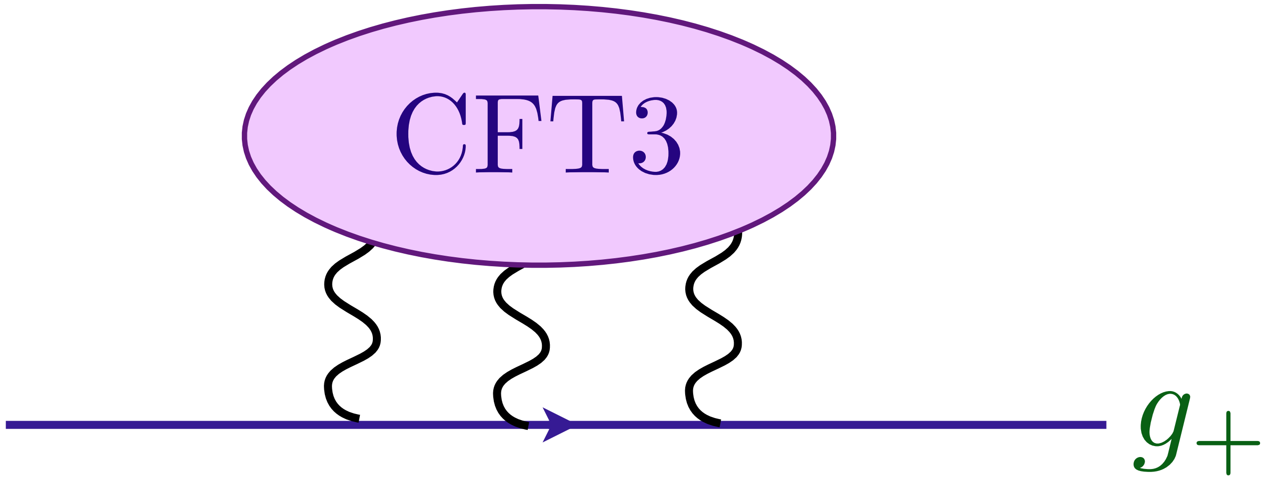
$\gamma$

CFT3

- Begin with a CFT3: the CP$^1$ model.
- Add "probe" non-relativistic fermions, $g_+$ and $g_-$, with opposite gauge charges

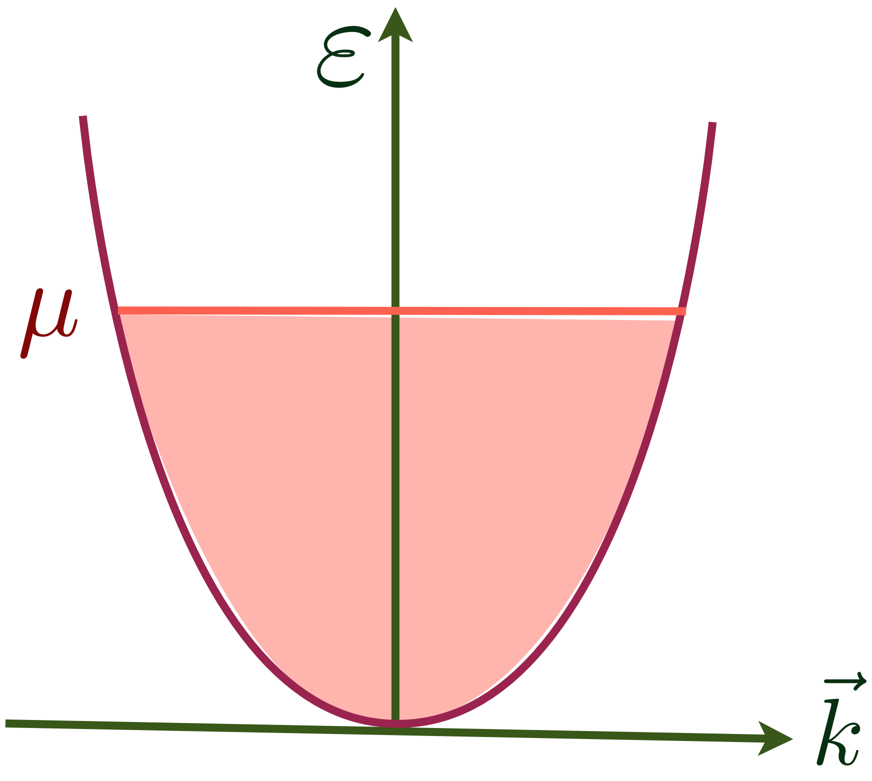
$$\mathcal{L}_f = g_+^\dagger \left( \frac{\partial}{\partial\tau} - iA_\tau - \frac{1}{2m}\left(\vec{\nabla} - i\vec{A}\right)^2 \right) g_+ + g_-^\dagger \left( \frac{\partial}{\partial\tau} + iA_\tau - \frac{1}{2m}\left(\vec{\nabla} + i\vec{A}\right)^2 \right) g_-$$

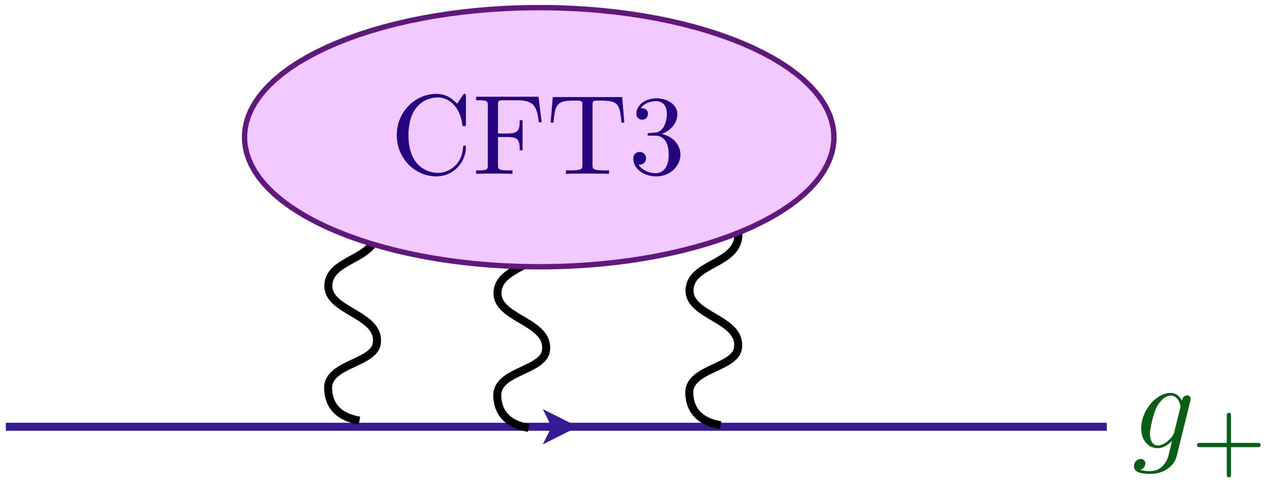




- Begin with a CFT3: the CP$^1$ model.
- Add "probe" non-relativistic fermions, $g_+$ and $g_-$, with opposite gauge charges
- Turn on fermion chemical potential:

$$\begin{aligned}\mathcal{L}_f &= g_+^\dagger \left( \frac{\partial}{\partial \tau} - i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} - i \vec{A} \right)^2 \right) g_+ \\ &\quad + g_-^\dagger \left( \frac{\partial}{\partial \tau} + i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} + i \vec{A} \right)^2 \right) g_-\end{aligned}$$





# Complete theory

$$\mathcal{L} = \mathcal{L}_z + \mathcal{L}_f$$

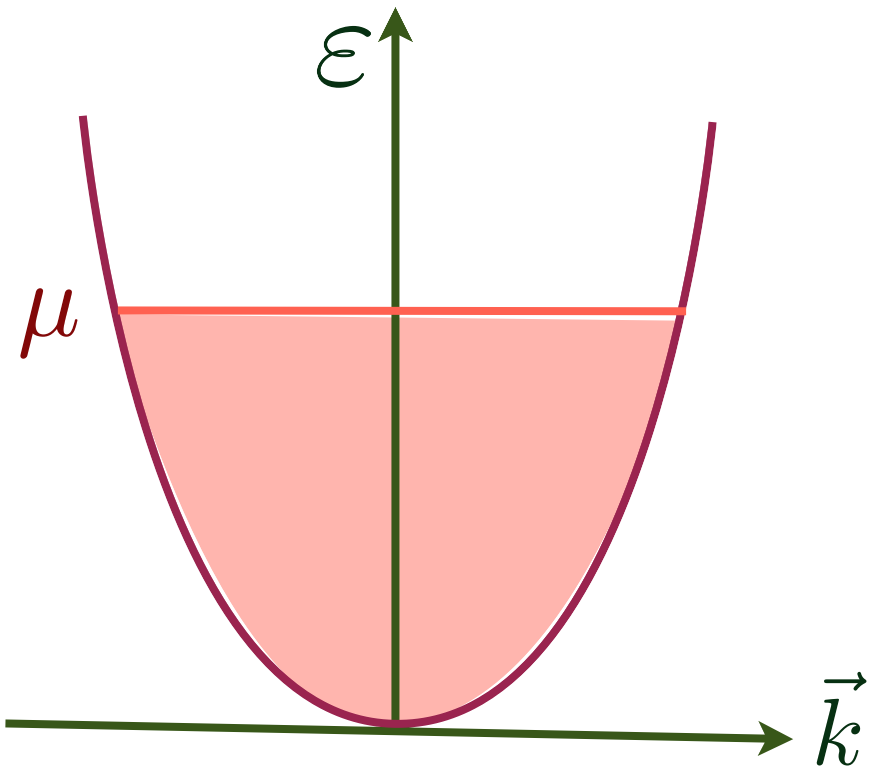
$$\mathcal{L}_z = \frac{1}{\gamma} \left| (\partial_\mu - i A_\mu) z_\alpha \right|^2 \quad ; \quad |z_\alpha|^2 = 1$$

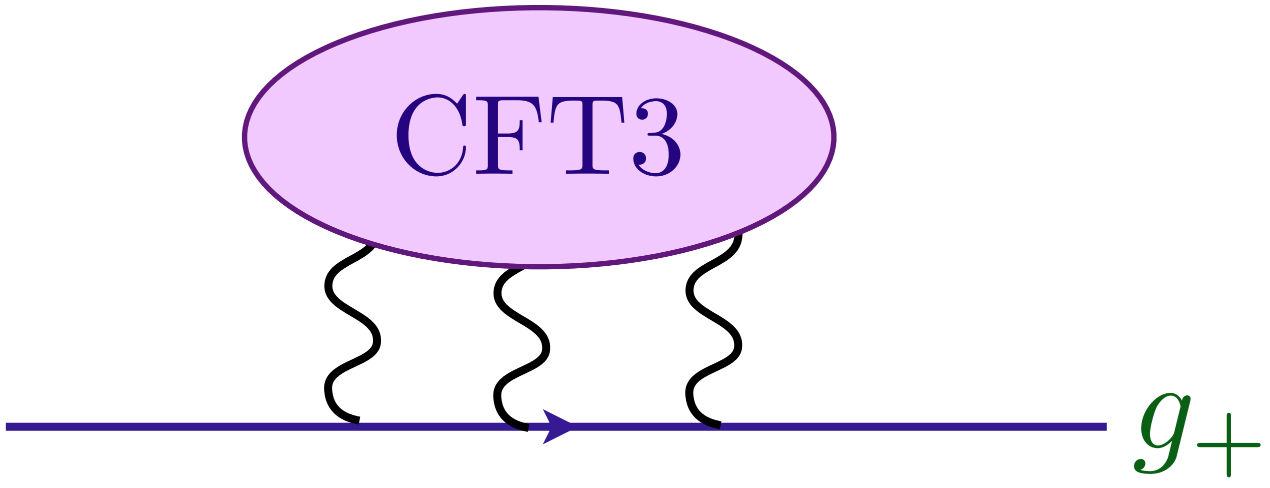
$$\begin{aligned} \mathcal{L}_f \quad = \quad & g_+^\dagger \left( \frac{\partial}{\partial \tau} - i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} - i \vec{A} \right)^2 \right) g_+ \\ & + g_-^\dagger \left( \frac{\partial}{\partial \tau} + i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} + i \vec{A} \right)^2 \right) g_- \end{aligned}$$

V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

- Begin with a CFT3: the CP$^1$ model.
- Add "probe" non-relativistic fermions, $g_+$ and $g_-$, with opposite gauge charges
- Turn on fermion chemical potential:

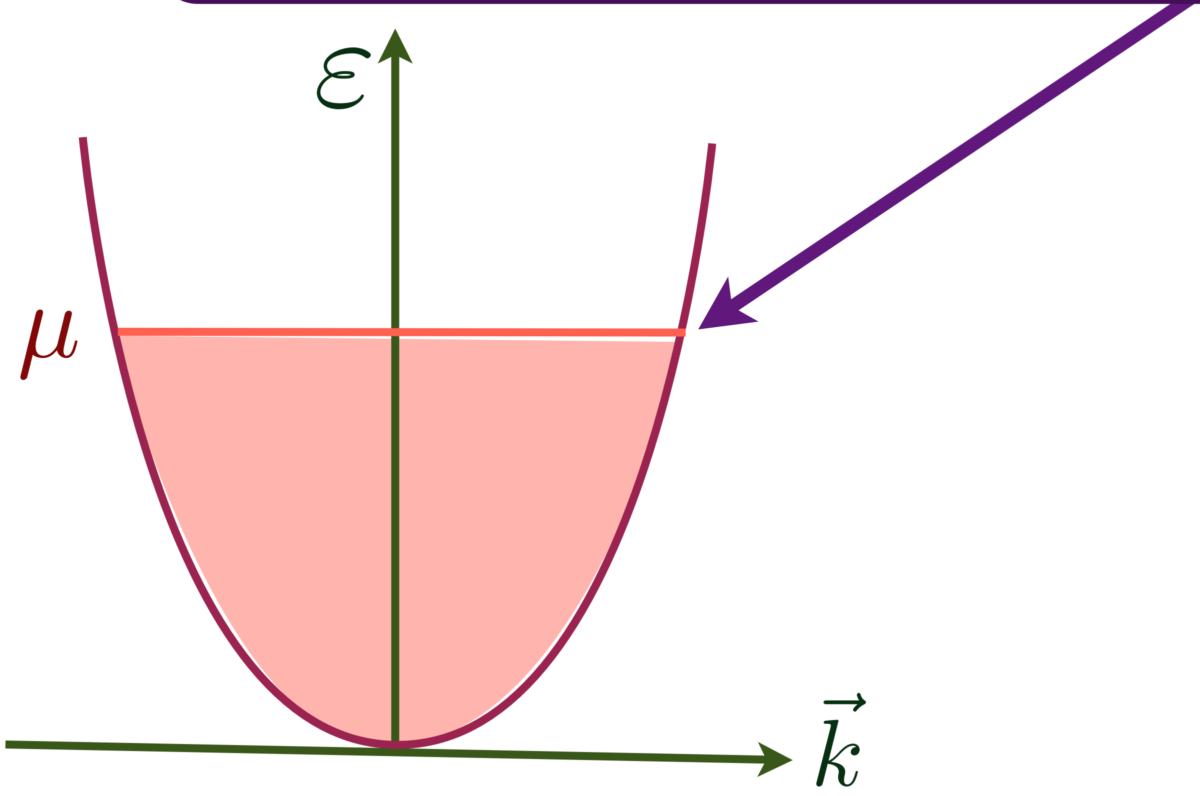
$$\begin{aligned}\mathcal{L}_f &= g_+^\dagger \left( \frac{\partial}{\partial \tau} - i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} - i \vec{A} \right)^2 \right) g_+ \\ &\quad + g_-^\dagger \left( \frac{\partial}{\partial \tau} + i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} + i \vec{A} \right)^2 \right) g_-\end{aligned}$$

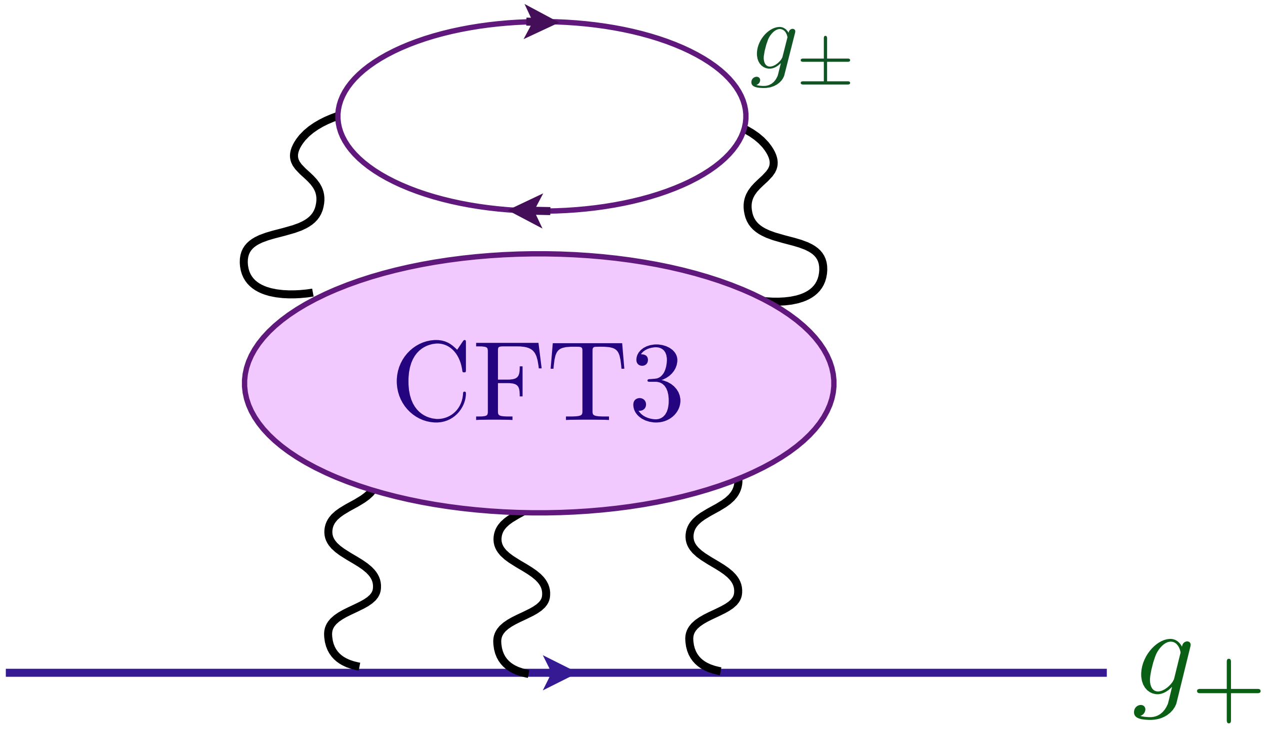




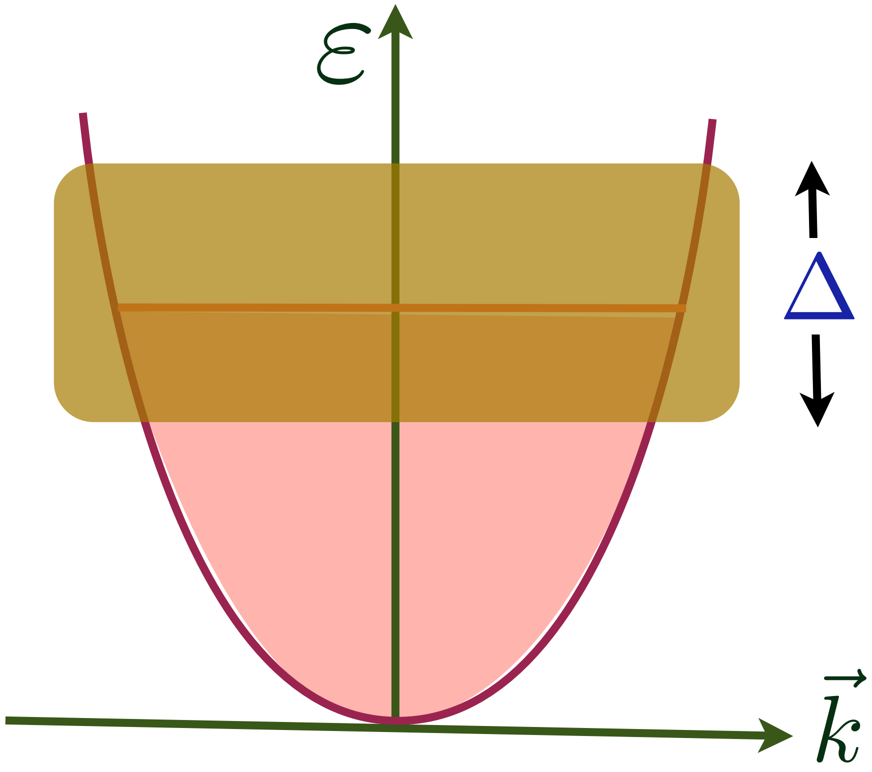
- Begin with a CFT3: the $\mathrm{CP}^1$ model.
- Add "probe" non-relativistic fermions, $g_+$ and $g_-$, with opposite gauge charges
- Turn on fermion chemical potential: leads to a marginal Fermi liquid of $g_\pm$ (not electrons)

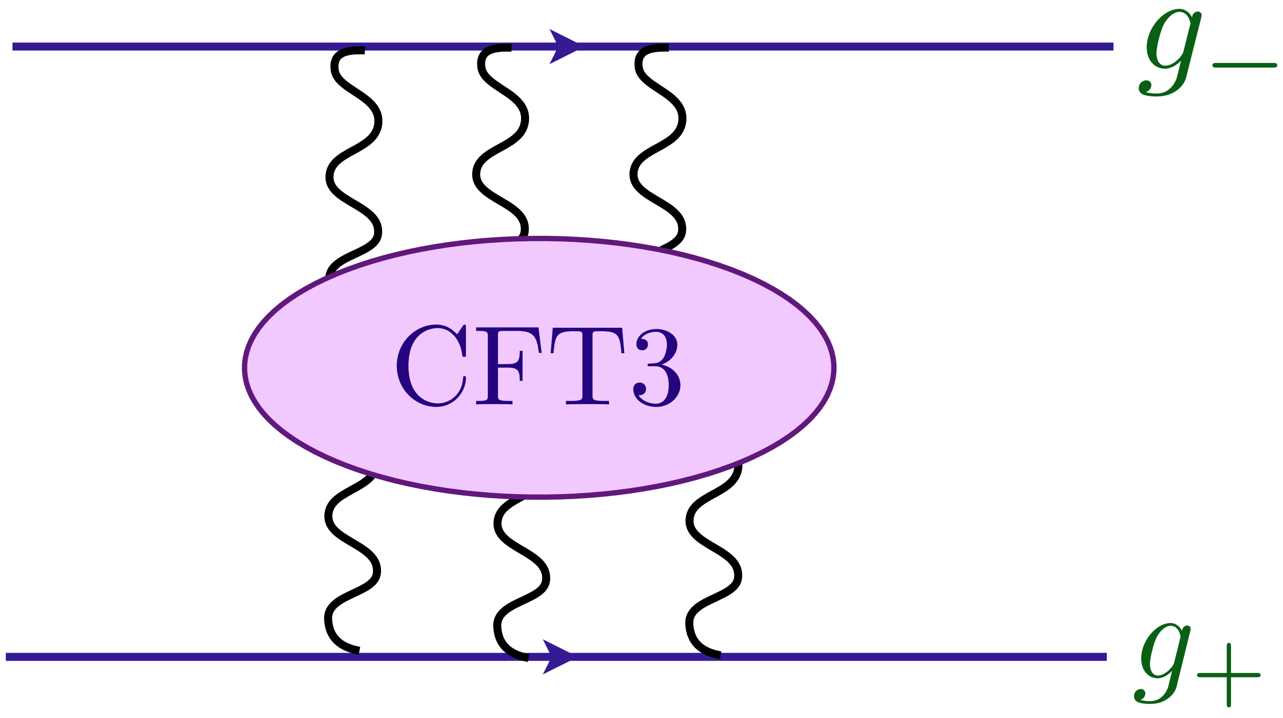
$$G(\vec{k},\omega) = \frac{1}{\omega - v_F(|\vec{k}| - k_F) + c\,\omega[\ln(|\omega|) + i\pi\,\mathrm{sgn}(\omega)]}$$





- Begin with a CFT3: the $\mathrm{CP}^1$ model.
- Add "probe" non-relativistic fermions, $g_+$ and $g_-$, with opposite gauge charges
- Turn on fermion chemical potential: leads to a marginal Fermi liquid of $g_\pm$ (not electrons)
- Low $T$ state is a superconductor with $\langle g_+ g_- \rangle = \Delta \neq 0$





Theory has *many* similarities to holographic superconductors (Gubser, Hartnoll, Herzog, Horowitz) solved via the AdS/CFT correspondence, which (presumably) describe SYM3 theories in which gluinos pair via exchange of gluons into color singlets, and then Bose condense:

- Fermi surfaces with non-Fermi singularities in spectral functions
- Cooper pairs which are gauge neutral
- Are obtained after doping a CFT3 with finite density of a conserved global charge
- Fermion and current spectral functions in superconducting and normal states have many similarities to cuprates

# Why is the pairing $d$-wave ?

## Fluctuating pocket theory for electrons near $(0,\pi)$ and $(\pi,0)$

Summarizing, in the low energy theory, the $c_{1,2\alpha}$ are expressed in terms of the $g_\pm$ fermions and the $z_\alpha$ by

$$\begin{aligned} c_{1\uparrow} &= z_\uparrow g_+ - z_\downarrow^* g_- \\ c_{2\uparrow} &= z_\uparrow g_+ + z_\downarrow^* g_- \\ c_{1\downarrow} &= z_\downarrow g_+ + z_\uparrow^* g_- \\ c_{2\downarrow} &= z_\downarrow g_+ - z_\uparrow^* g_- \end{aligned}$$

Note that this invariant under the U(1) gauge transformation

$$z_\alpha \to e^{i\phi} z_\alpha \quad ; \quad g_+ \to e^{-i\phi} g_+ \quad ; \quad g_- \to e^{i\phi} g_-,$$

which must be obeyed by the effective action for $z_\alpha$ and $g_\pm$.

# Why is the pairing $d$-wave ?

## Fluctuating pocket theory for electrons near $(0,\pi)$ and $(\pi,0)$

Attractive gauge forces lead to simple $s$-wave pairing of the $g_\pm$

$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

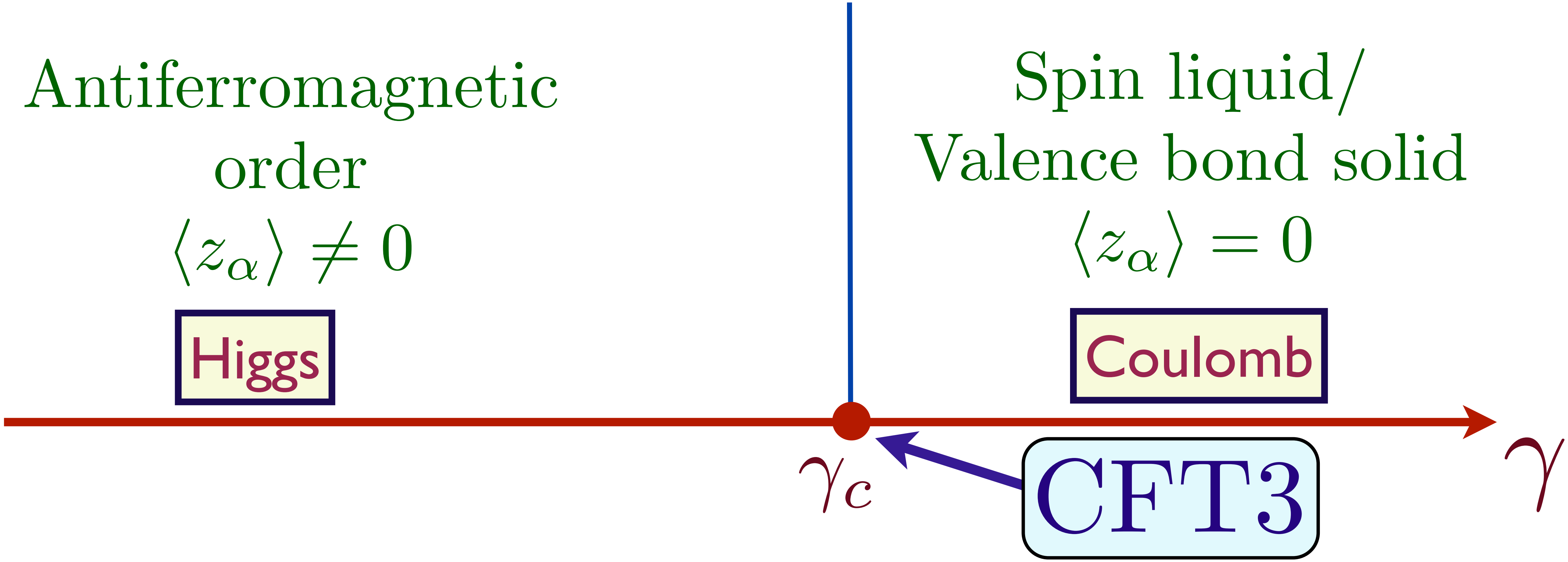
$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

$i.e.$ $d$-wave pairing !

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

# T=0 Phase diagram

$$\mathcal{L}_z = \frac{1}{\gamma}|(\partial_\mu - iA_\mu)z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$



Antiferromagnetic order

$\langle z_\alpha \rangle \neq 0$

**Higgs**

Spin liquid/ Valence bond solid

$\langle z_\alpha \rangle = 0$

**Coulomb**

$\gamma_c$

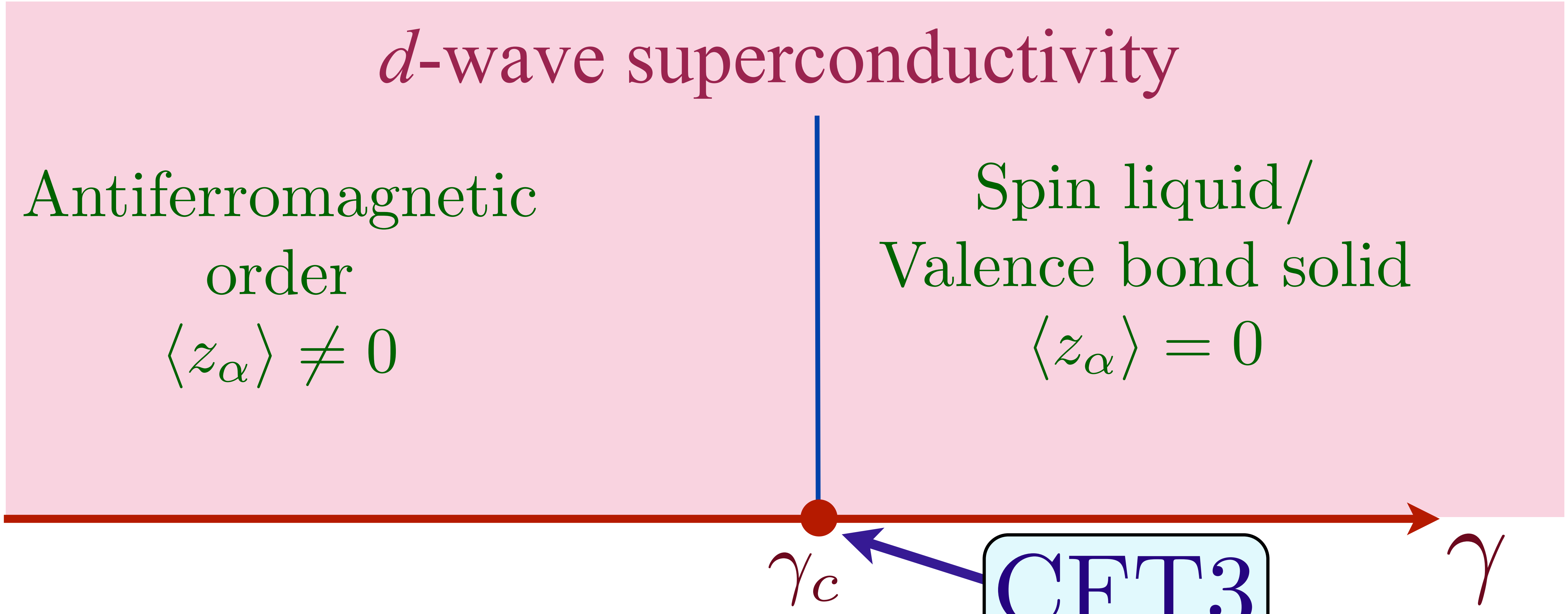
CFT3

$\gamma$

# T=0 Phase diagram

$$\mathcal{L}_z = \frac{1}{\gamma} |(\partial_\mu - i A_\mu) z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$

$$\begin{aligned} \mathcal{L}_f \quad = \quad & g_+^\dagger \left( \frac{\partial}{\partial \tau} - i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} - i \vec{A} \right)^2 \right) g_+ \\ & + g_-^\dagger \left( \frac{\partial}{\partial \tau} + i A_\tau - \mu - \frac{1}{2m} \left( \vec{\nabla} + i \vec{A} \right)^2 \right) g_- \end{aligned}$$



*d*-wave superconductivity

Antiferromagnetic order

$\langle z_\alpha \rangle \neq 0$

Spin liquid/
Valence bond solid

$\langle z_\alpha \rangle = 0$

$\gamma_c$

CFT3

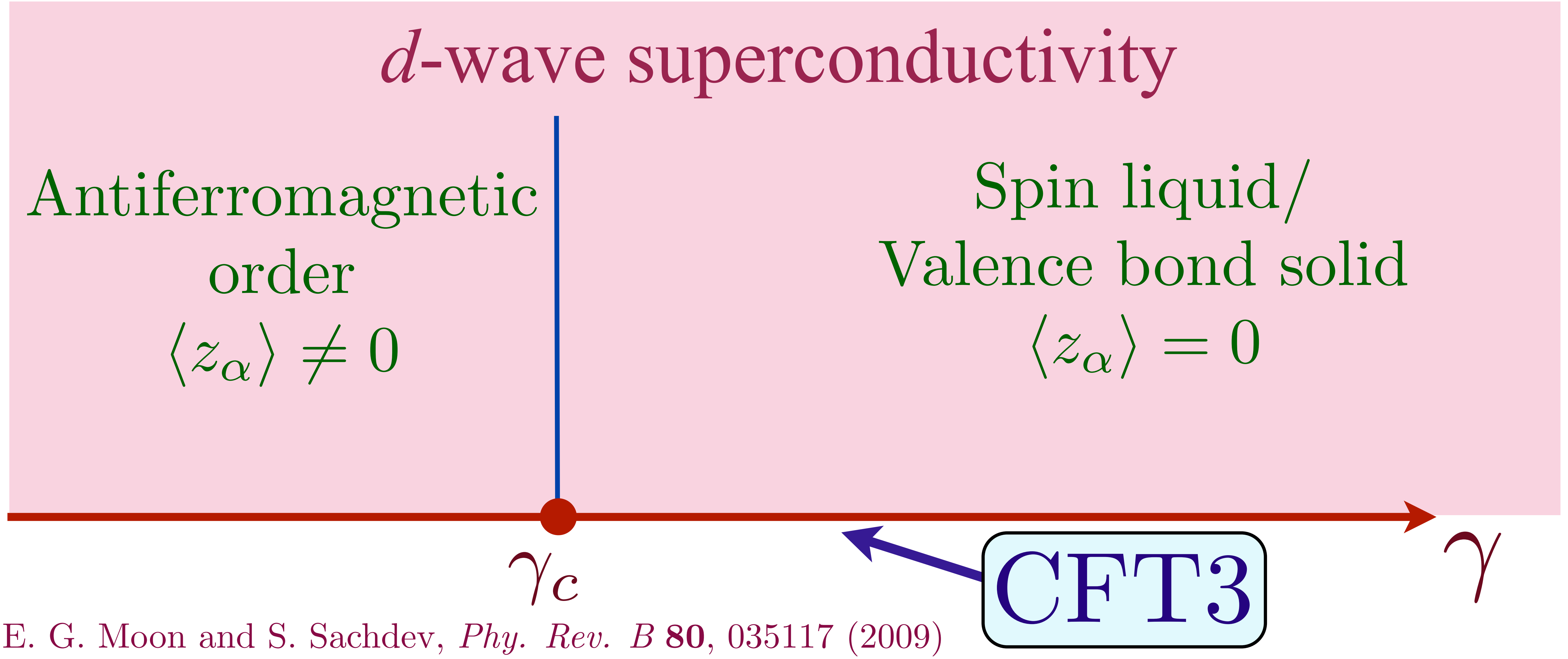
$\gamma$

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

# T=0 Phase diagram

$$\mathcal{L}_z = \frac{1}{\gamma}|(\partial_\mu - iA_\mu)z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$

$$\begin{aligned}\mathcal{L}_f \;=\; & g_+^\dagger \left( \frac{\partial}{\partial \tau} - iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} - i\vec{A}\right)^2 \right) g_+ \\ & + g_-^\dagger \left( \frac{\partial}{\partial \tau} + iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} + i\vec{A}\right)^2 \right) g_- \end{aligned}$$



*d*-wave superconductivity

Antiferromagnetic order

$\langle z_\alpha \rangle \neq 0$

Spin liquid/ Valence bond solid

$\langle z_\alpha \rangle = 0$

$\gamma_c$

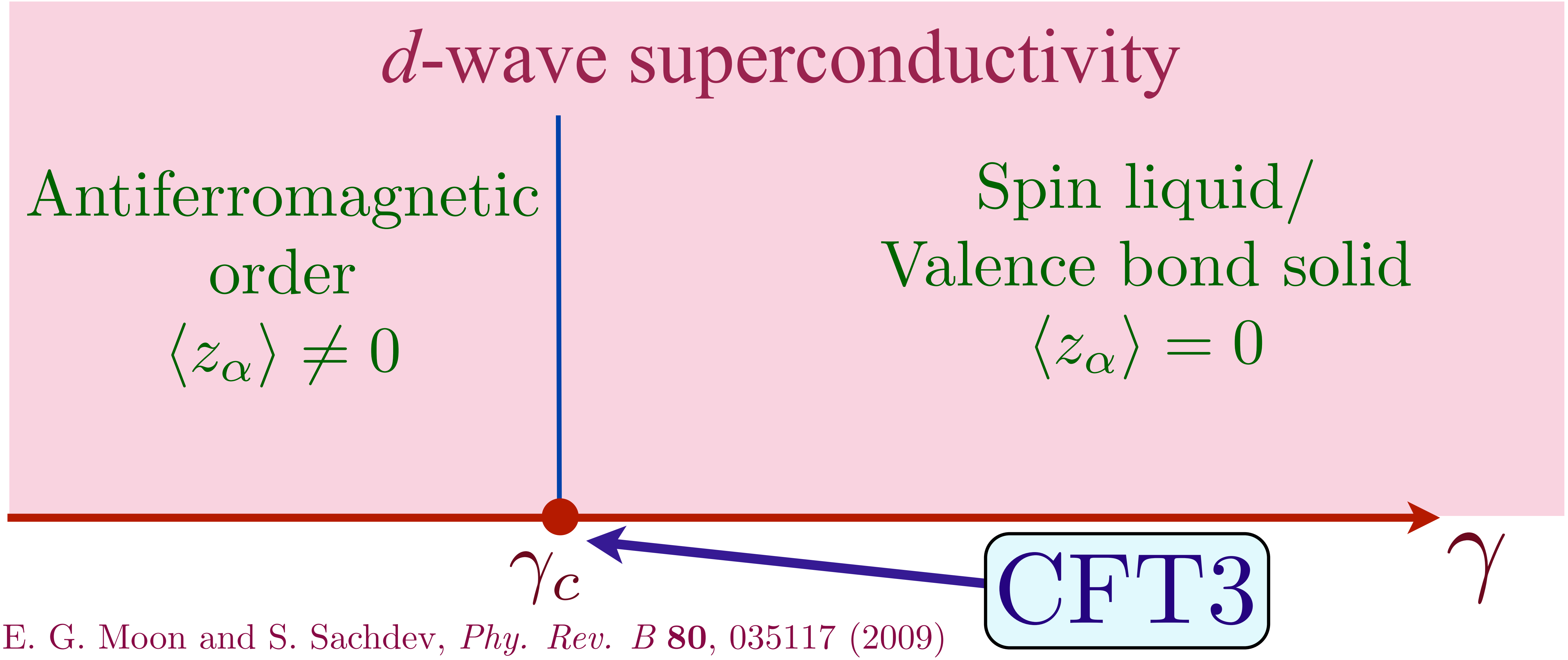
CFT3

$\gamma$

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

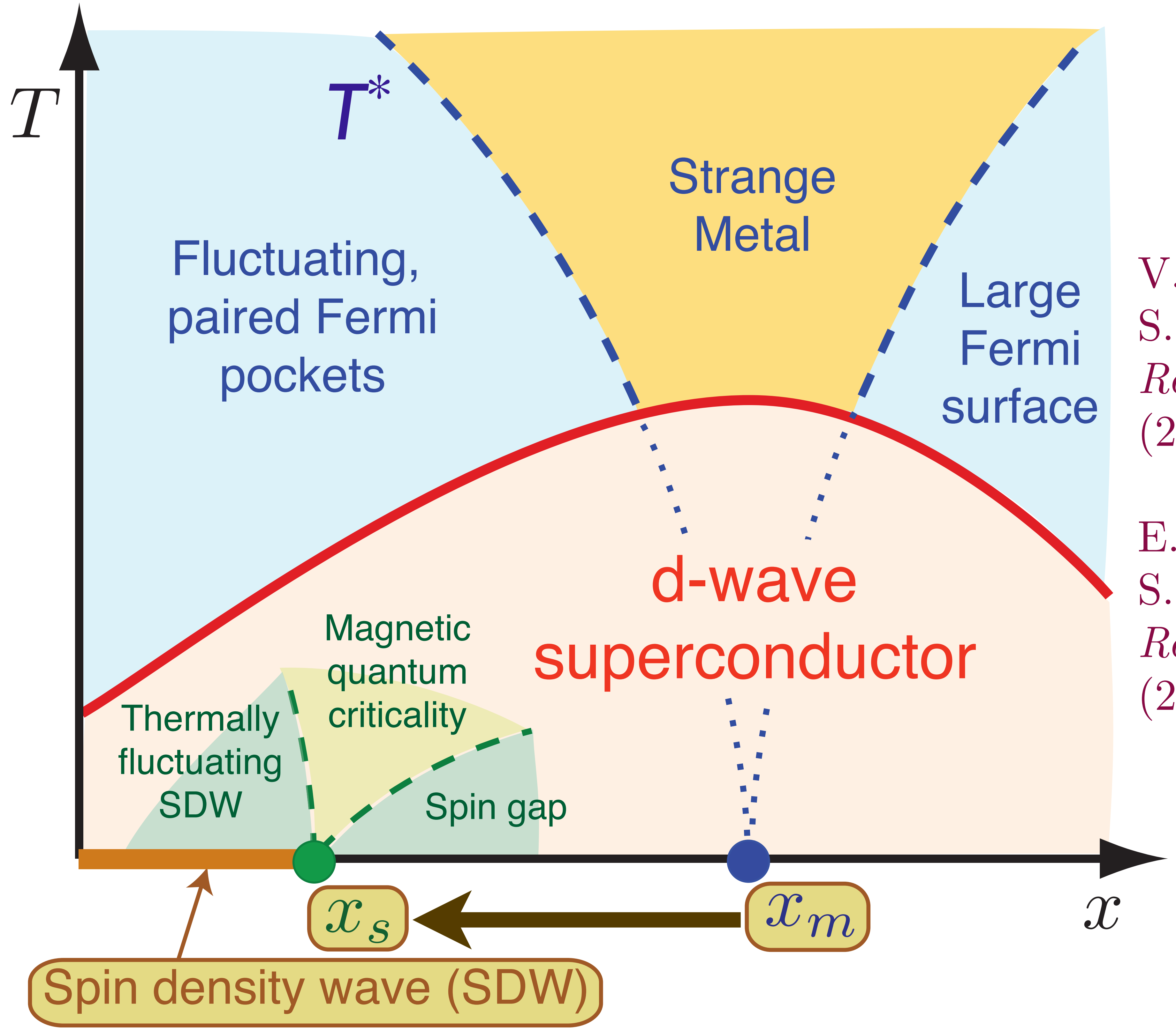
# T=0 Phase diagram

Competition between antiferromagnetism and superconductivity shrinks region of antiferromagnetic order: feedback of "probe fermions" on CFT is important



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

# Theory of quantum criticality in the cuprates

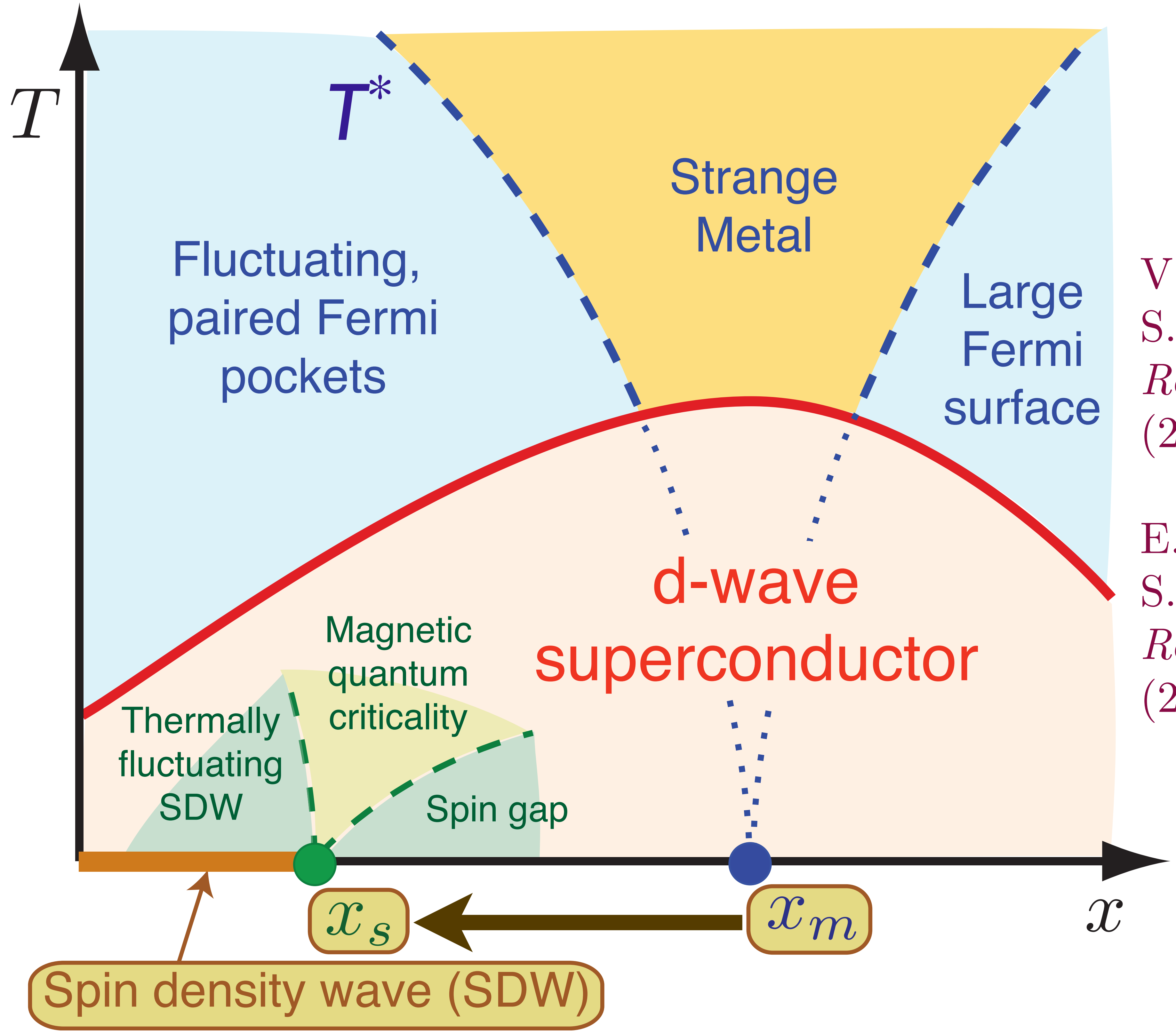


V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

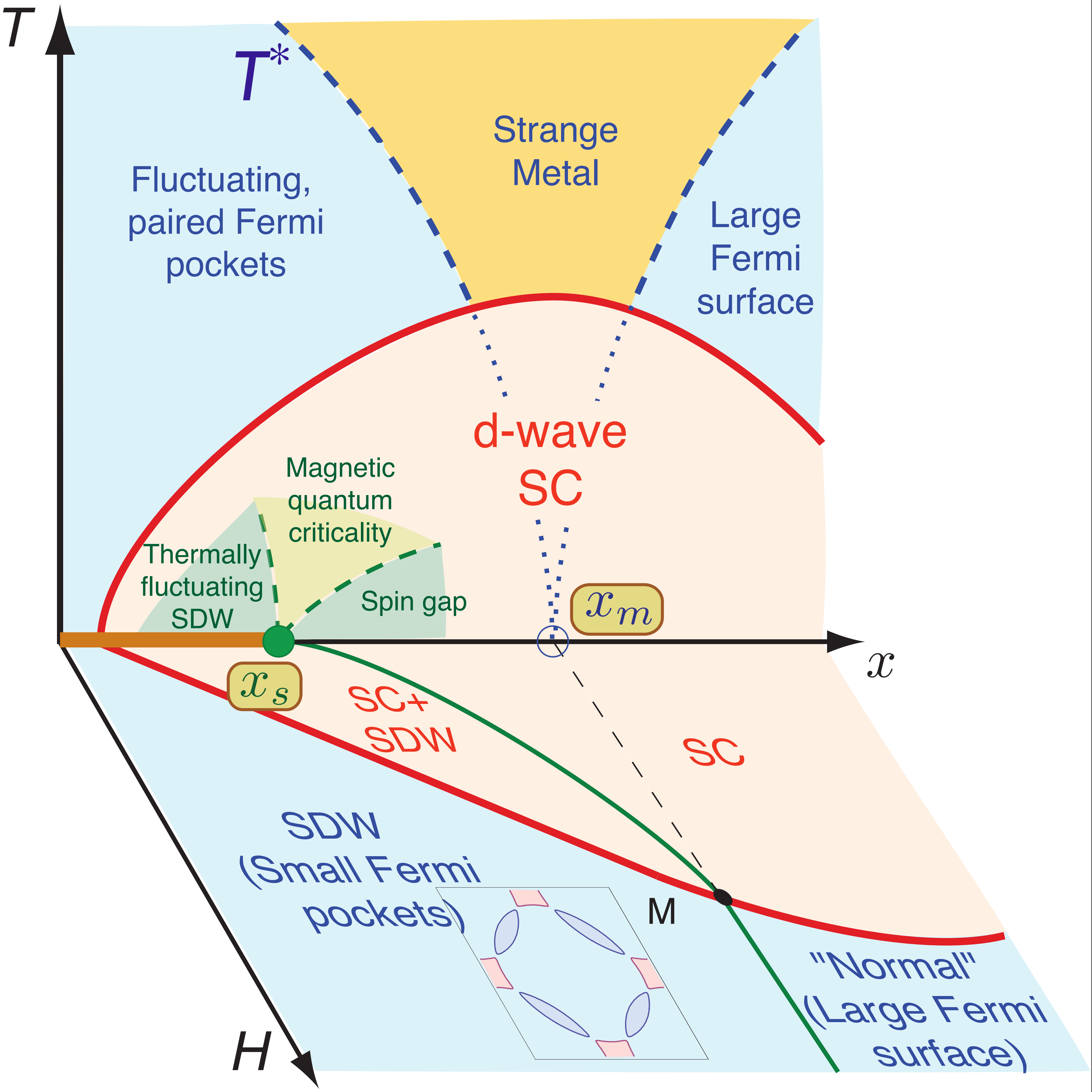
# Theory of quantum criticality in the cuprates



V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

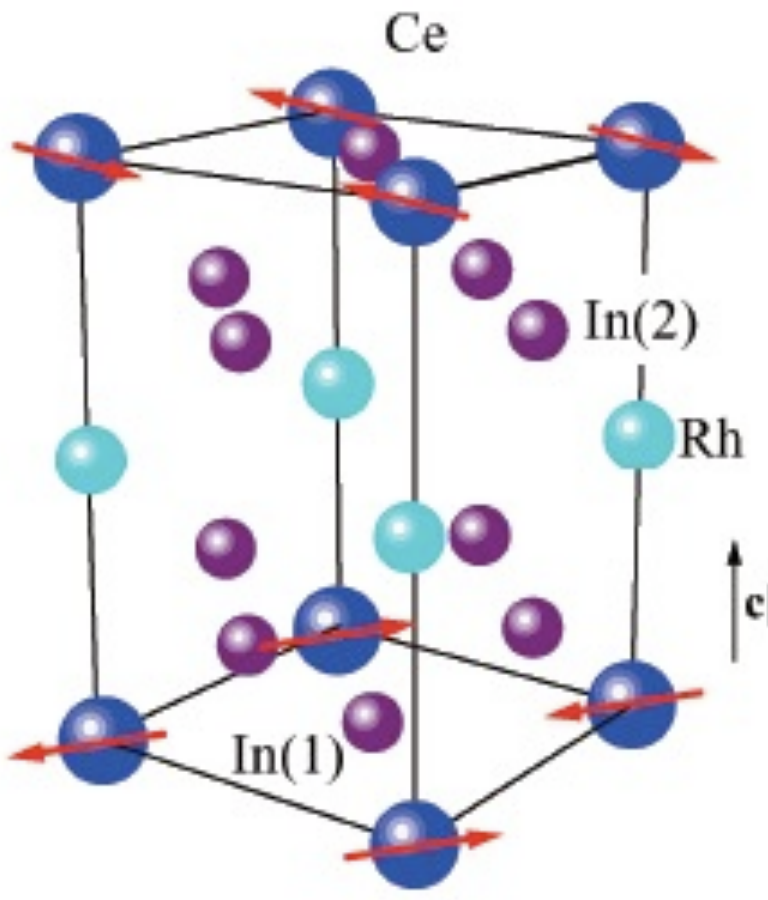
Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.

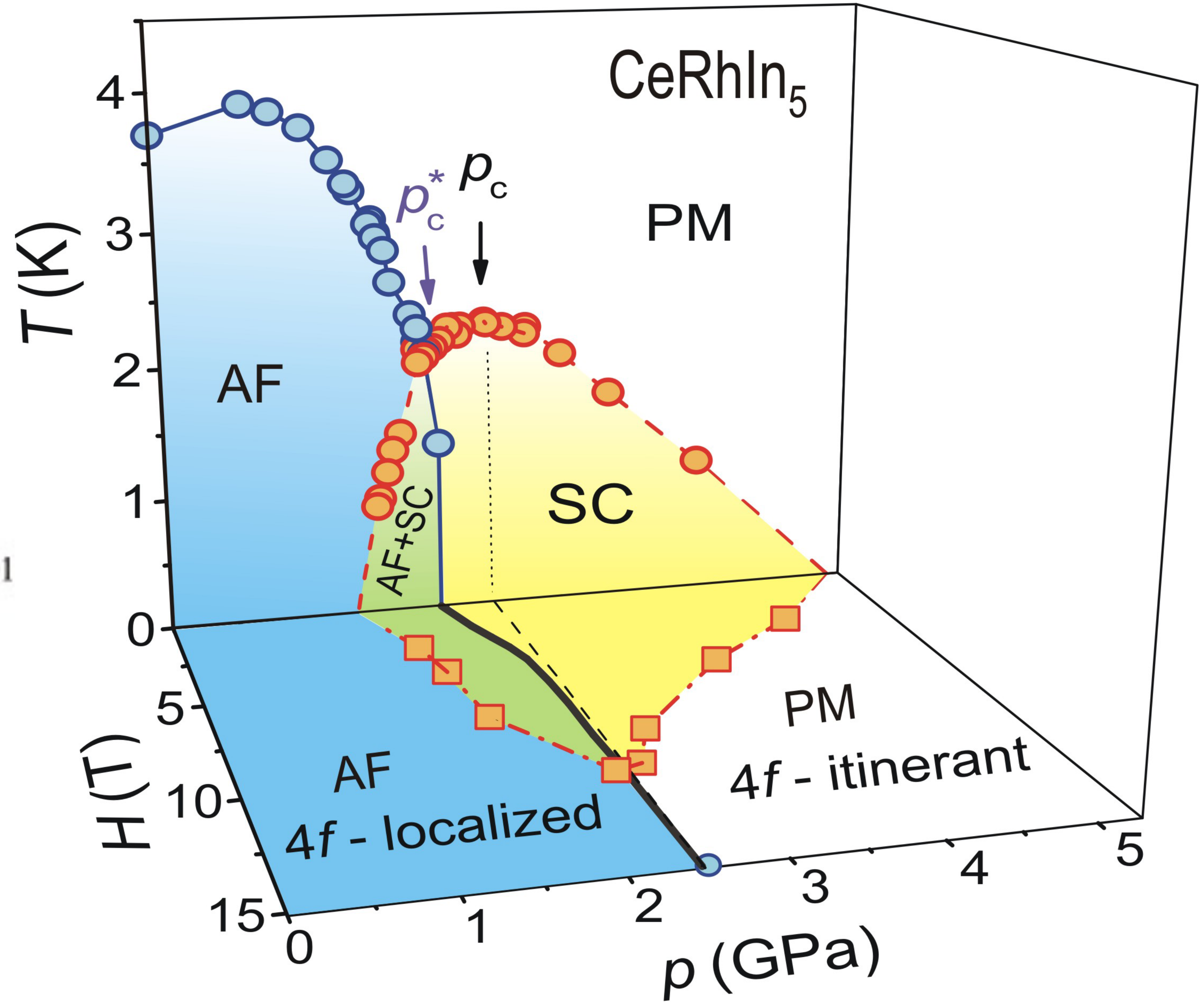


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

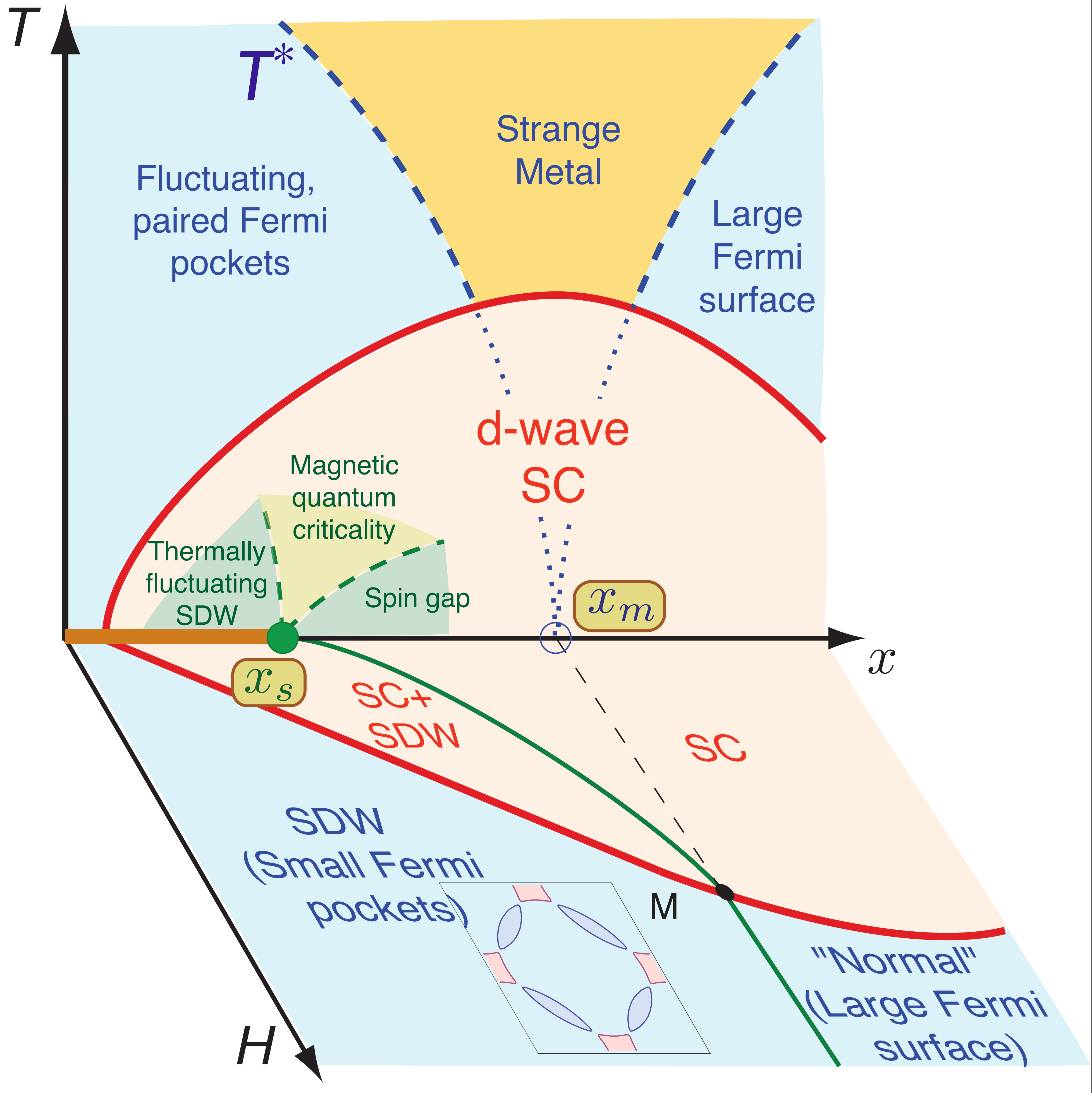
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

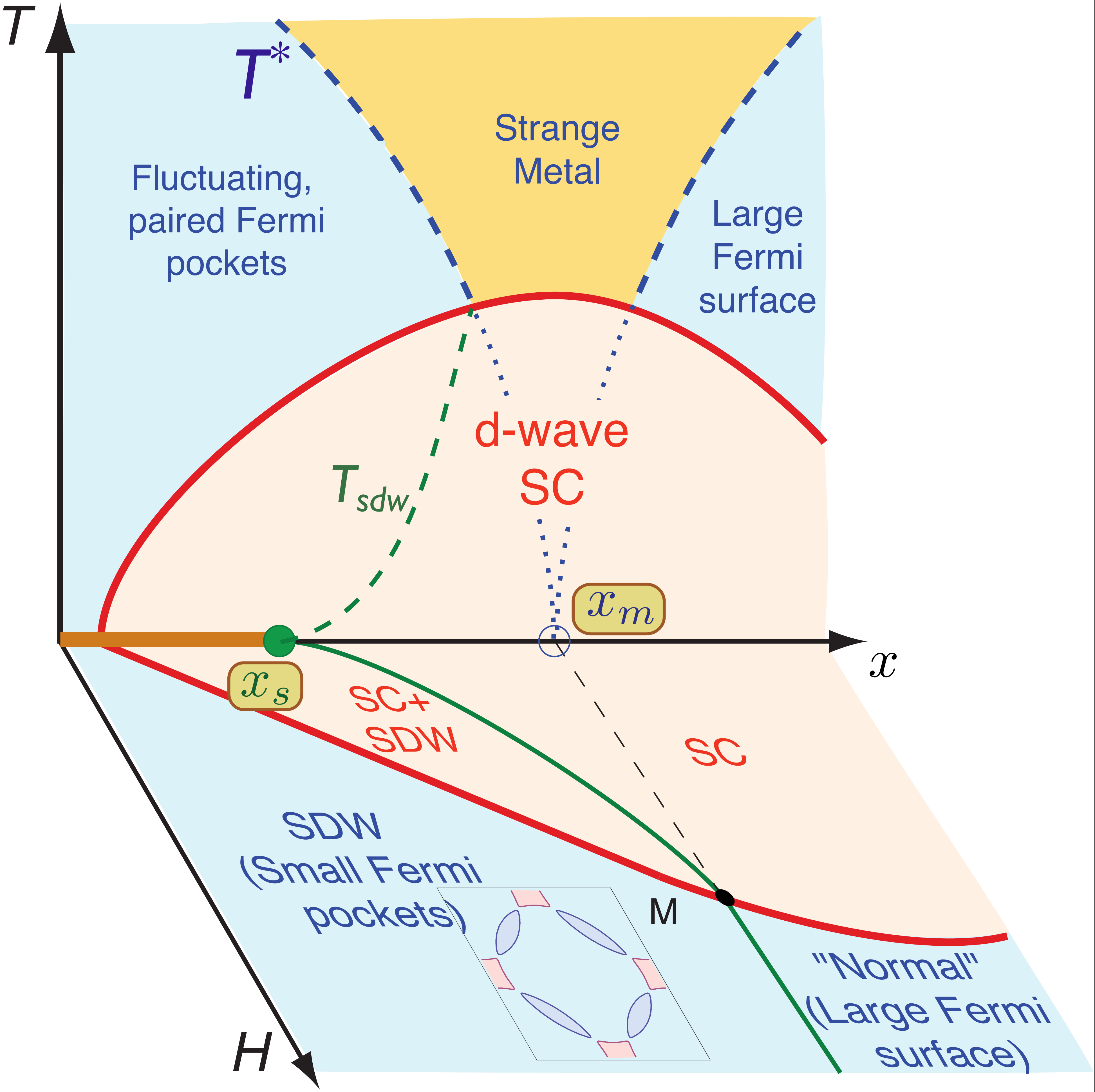
# Similar phase diagram for $CeRhIn_5$





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

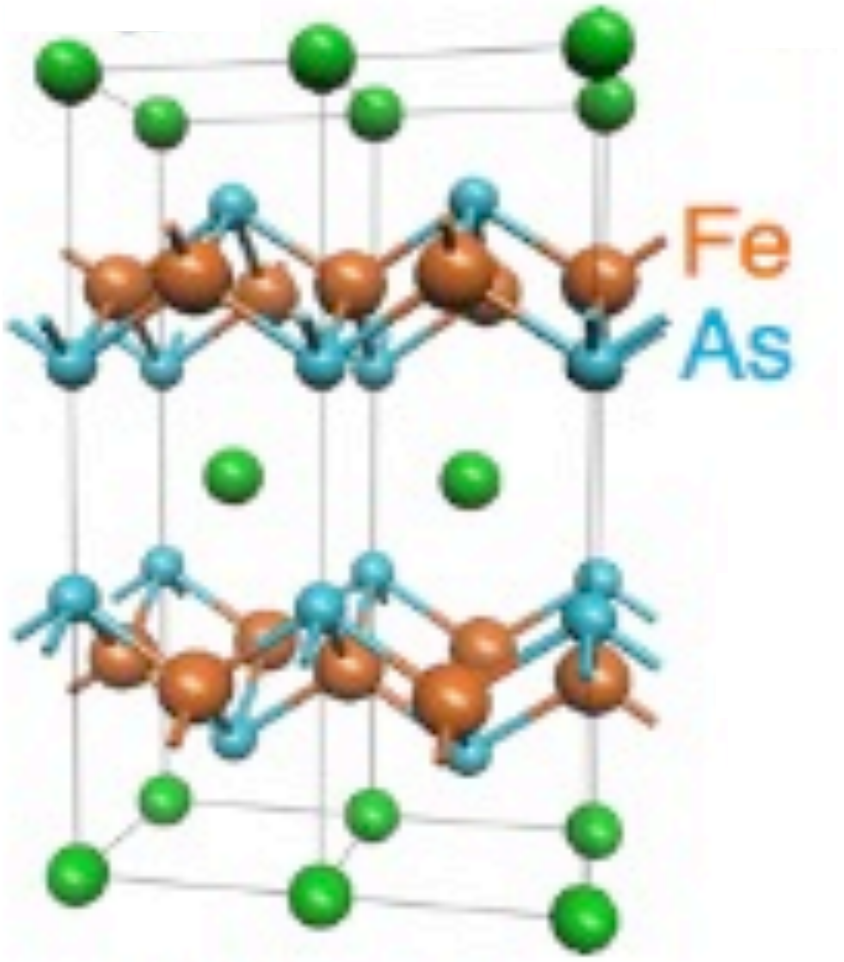

$T$
$T^*$
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
$x_m$
$x$
$x_s$
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
$H$



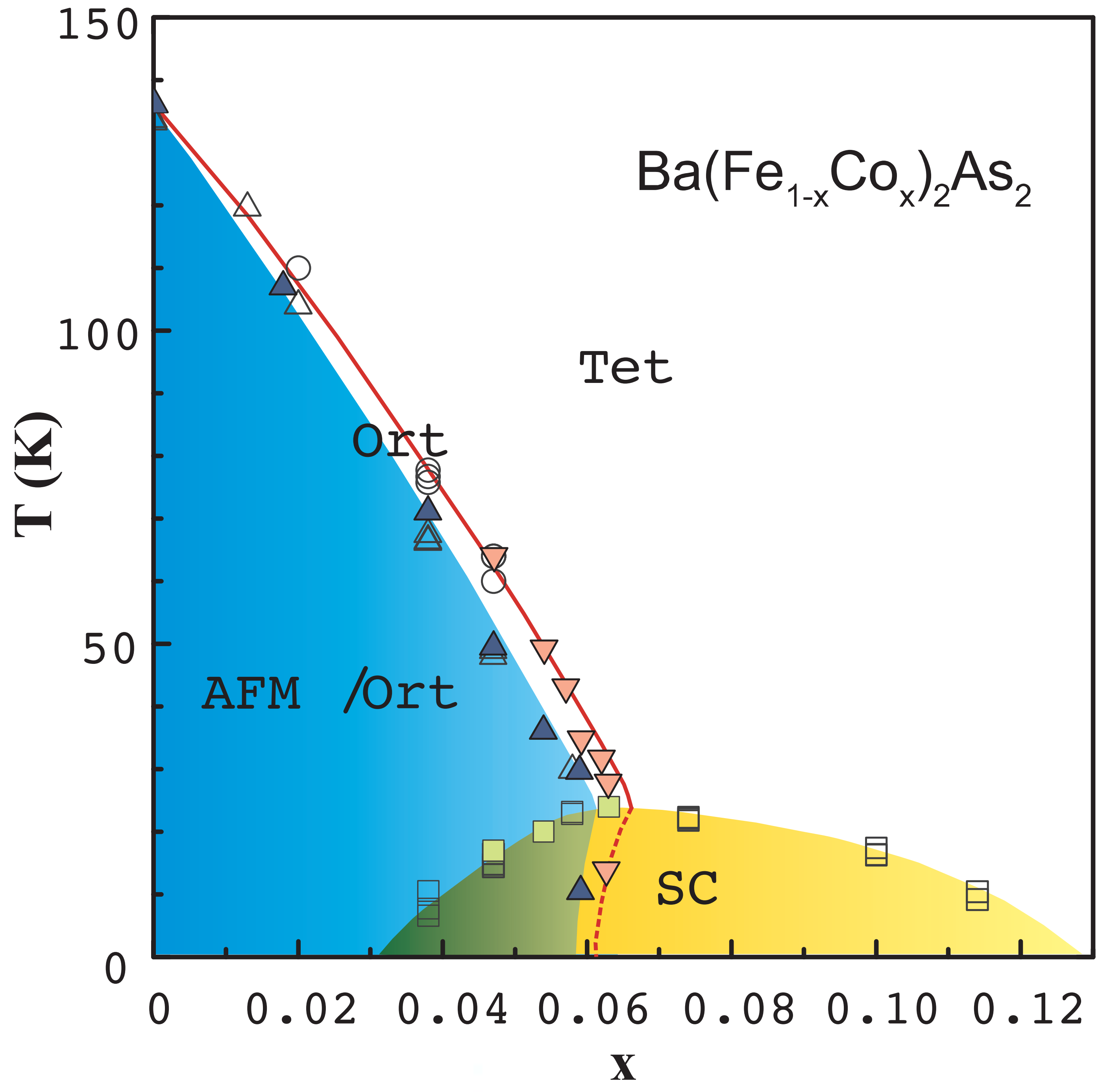
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

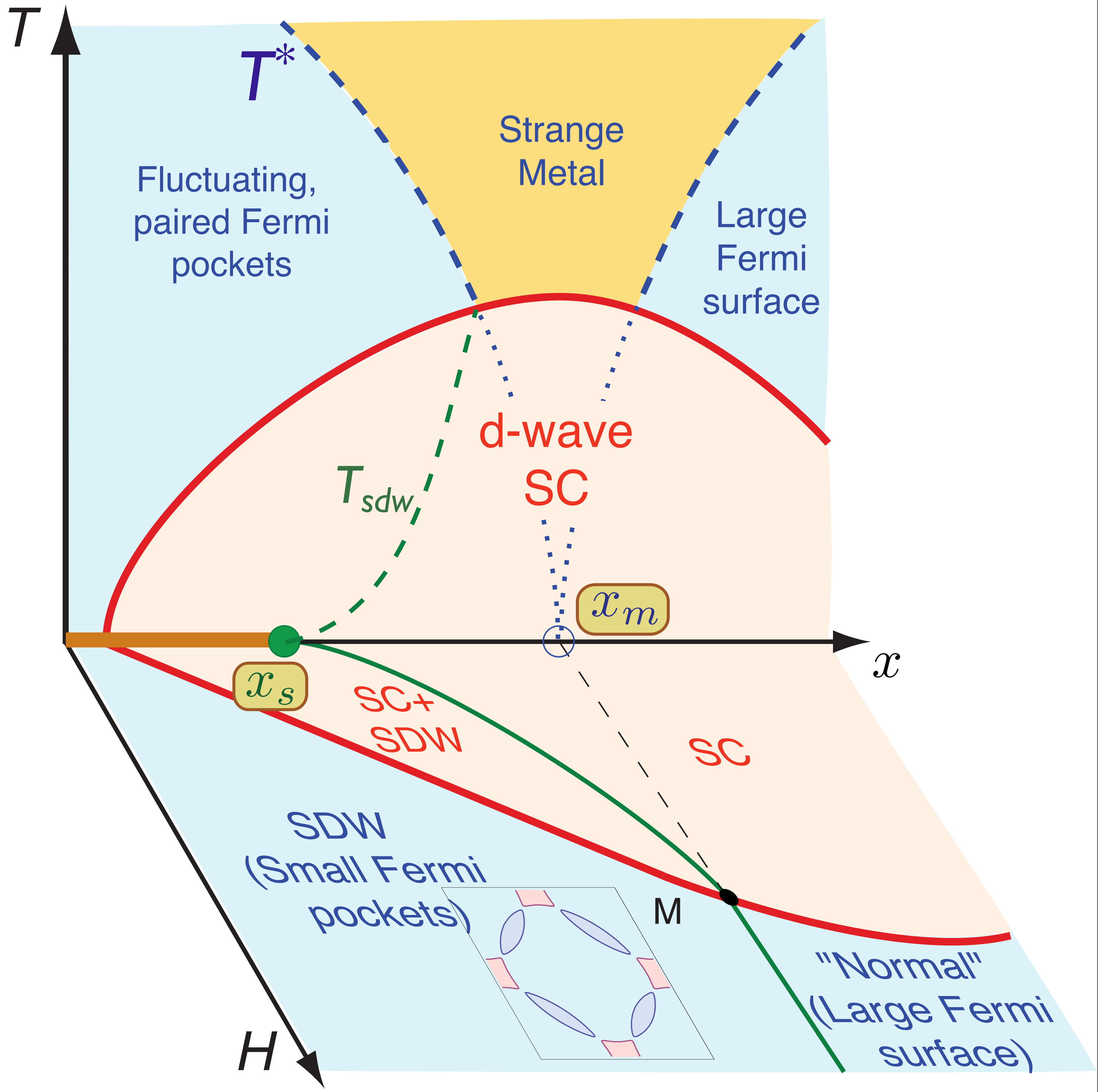
# Similar phase diagram for the pnictides



$BaFe_2As_2$
Unit cell



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, arXiv:0911.3136.

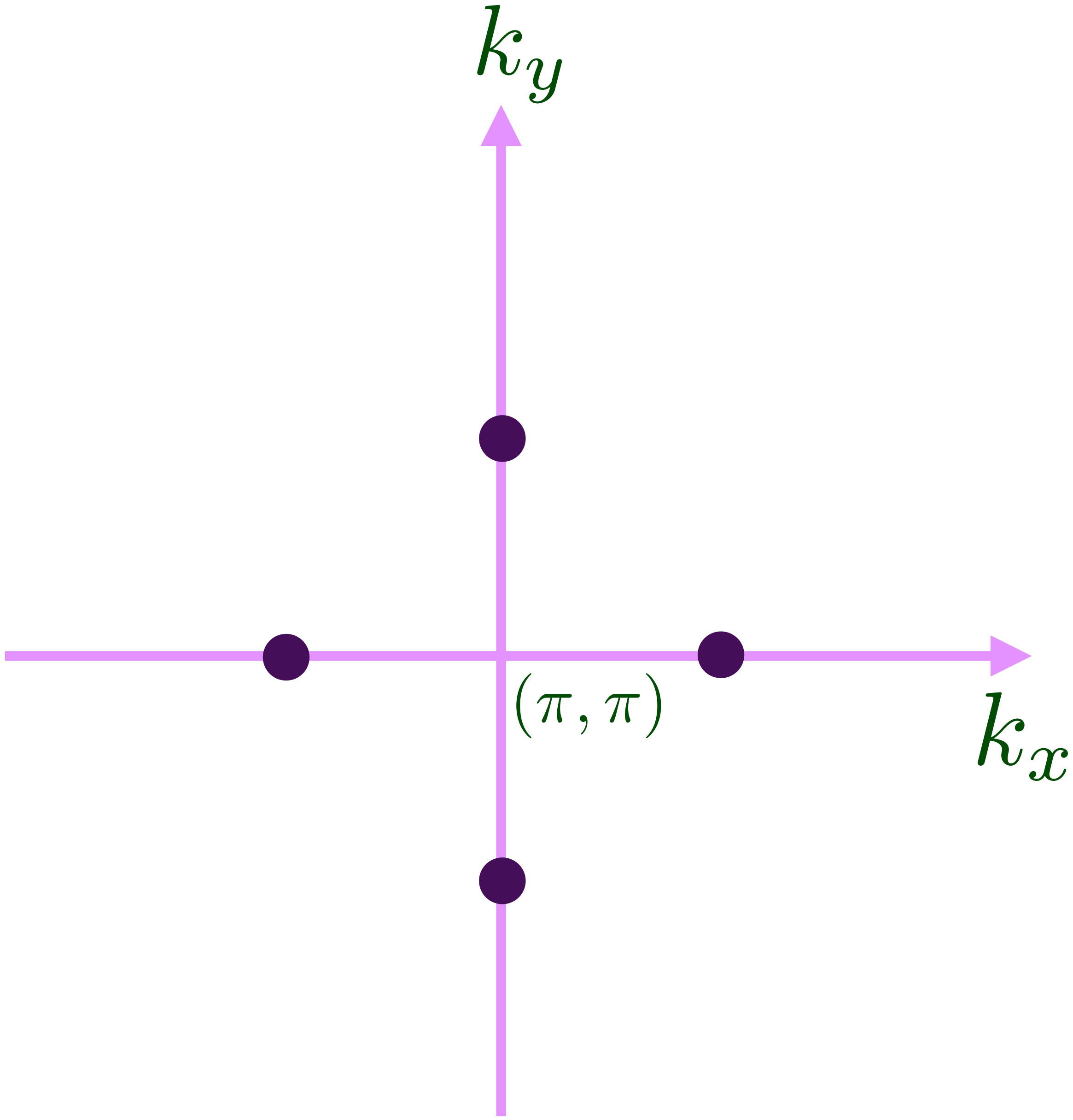

$T$
$T^*$
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
d-wave SC
$T_{sdw}$
$x_m$
$x$
$x_s$
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
$H$

# Remnants of SDW order for $x_s < x < x_m$

For incommensurate ordering, the SDW order parameter consists of 2 complex 3-component vectors $\vec{\Phi}_x$, $\vec{\Phi}_y$:

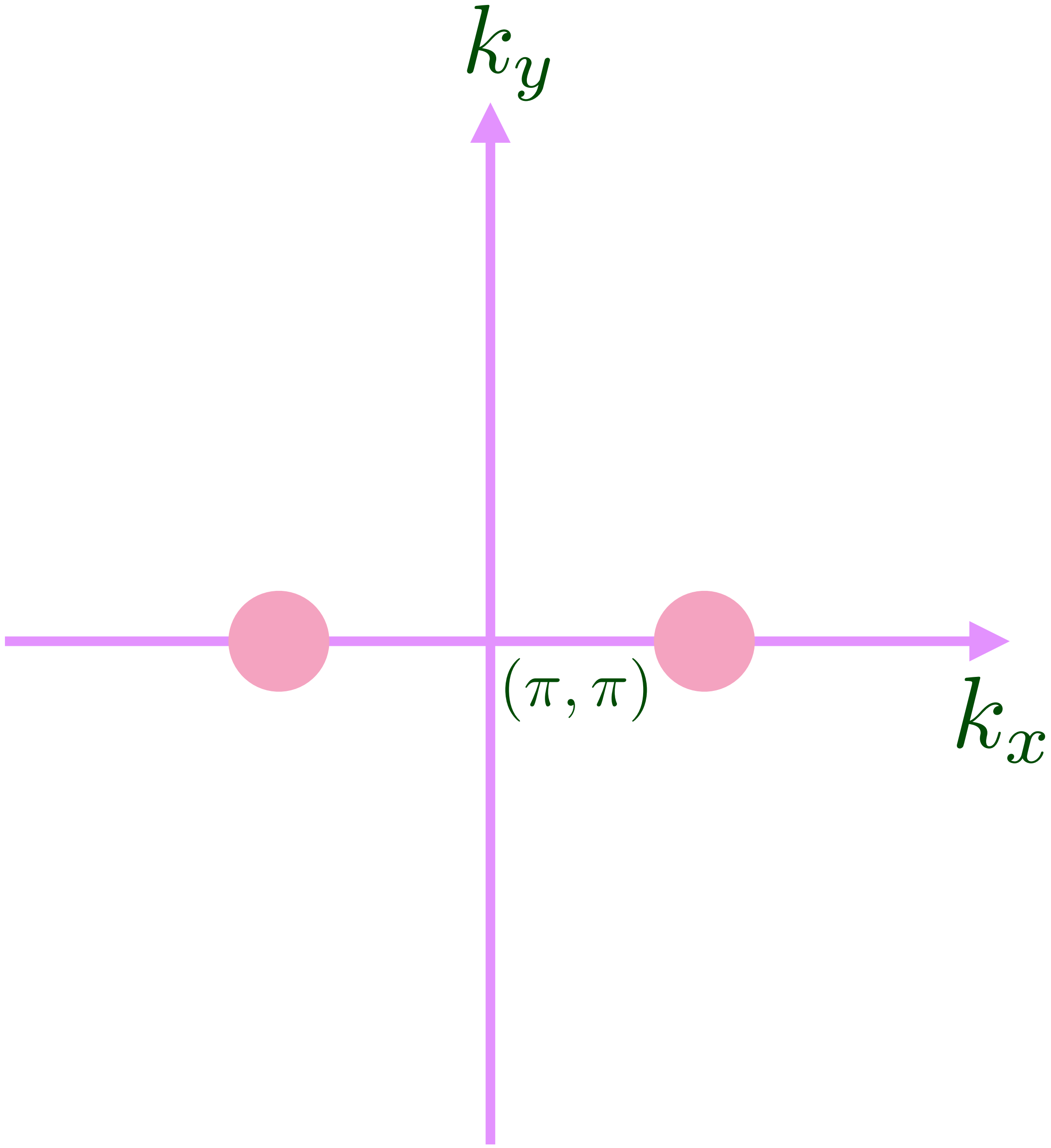
$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\Phi}_x(\mathbf{r},\tau) e^{i\mathbf{K}_x \cdot \mathbf{r}} + \vec{\Phi}_y(\mathbf{r},\tau) e^{i\mathbf{K}_y \cdot \mathbf{r}} + \text{c.c.}$$

where $\mathbf{K}_x = (\pi(1-\vartheta), \pi)$ and $\mathbf{K}_y = (\pi, \pi(1-\vartheta))$, with $\vartheta = 1/4$ near $1/8$ doping.



# Remnants of SDW order for $x_s < x < x_m$

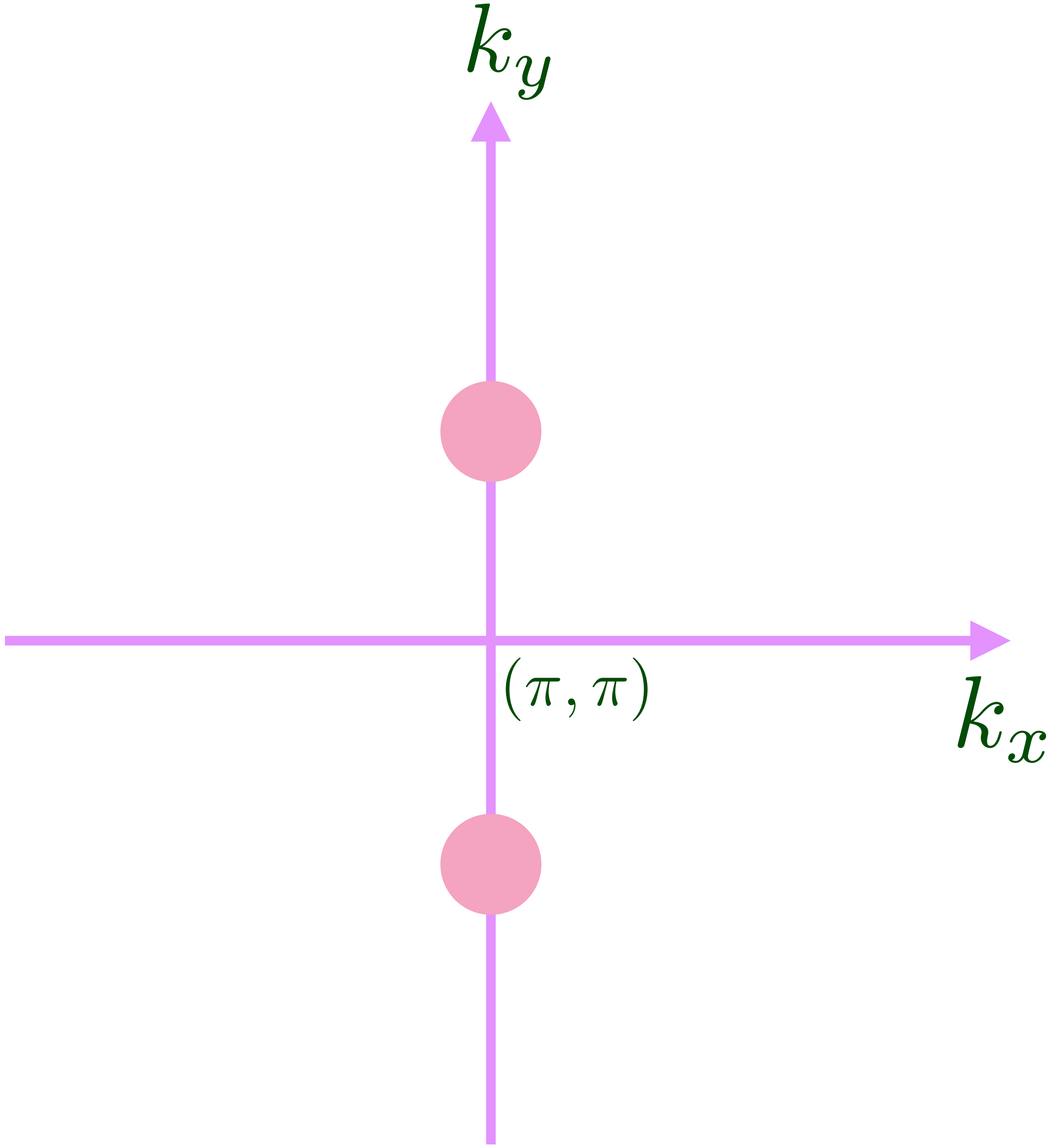
SDW correlations also Ising nematic order $\phi \propto |\Phi_x|^2 - |\Phi_y|^2$, which can be long-ranged, with SDW and VBS/CDW order all short ranged. This implies of preferential enhancement of electronic exchange/pairing energies along the $x$ or $y$ directions.



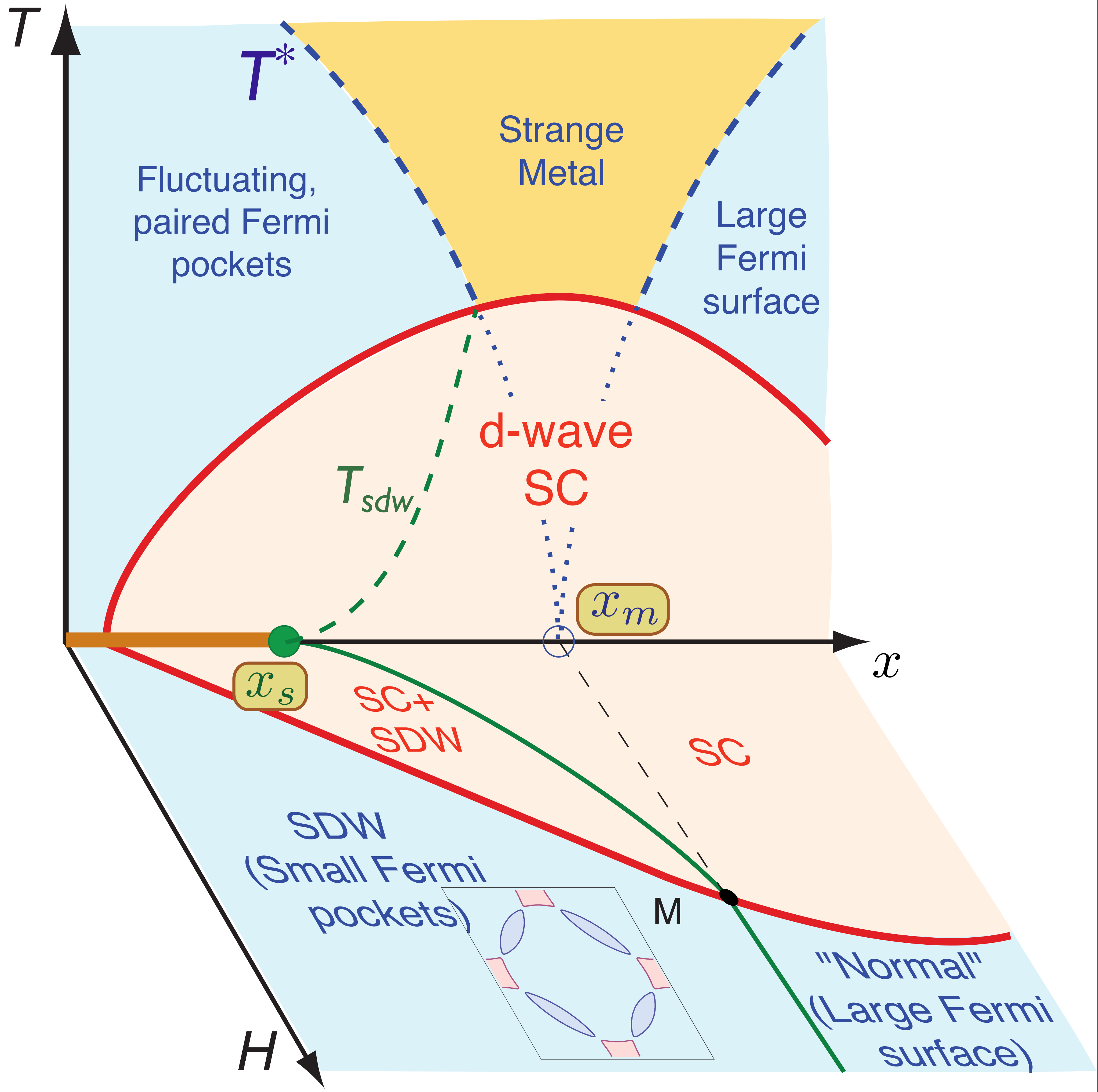
S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

# Remnants of SDW order for $x_s < x < x_m$

SDW correlations also Ising nematic order $\phi \propto |\Phi_x|^2 - |\Phi_y|^2$, which can be long-ranged, with SDW and VBS/CDW order all short ranged. This implies of preferential enhancement of electronic exchange/pairing energies along the $x$ or $y$ directions.

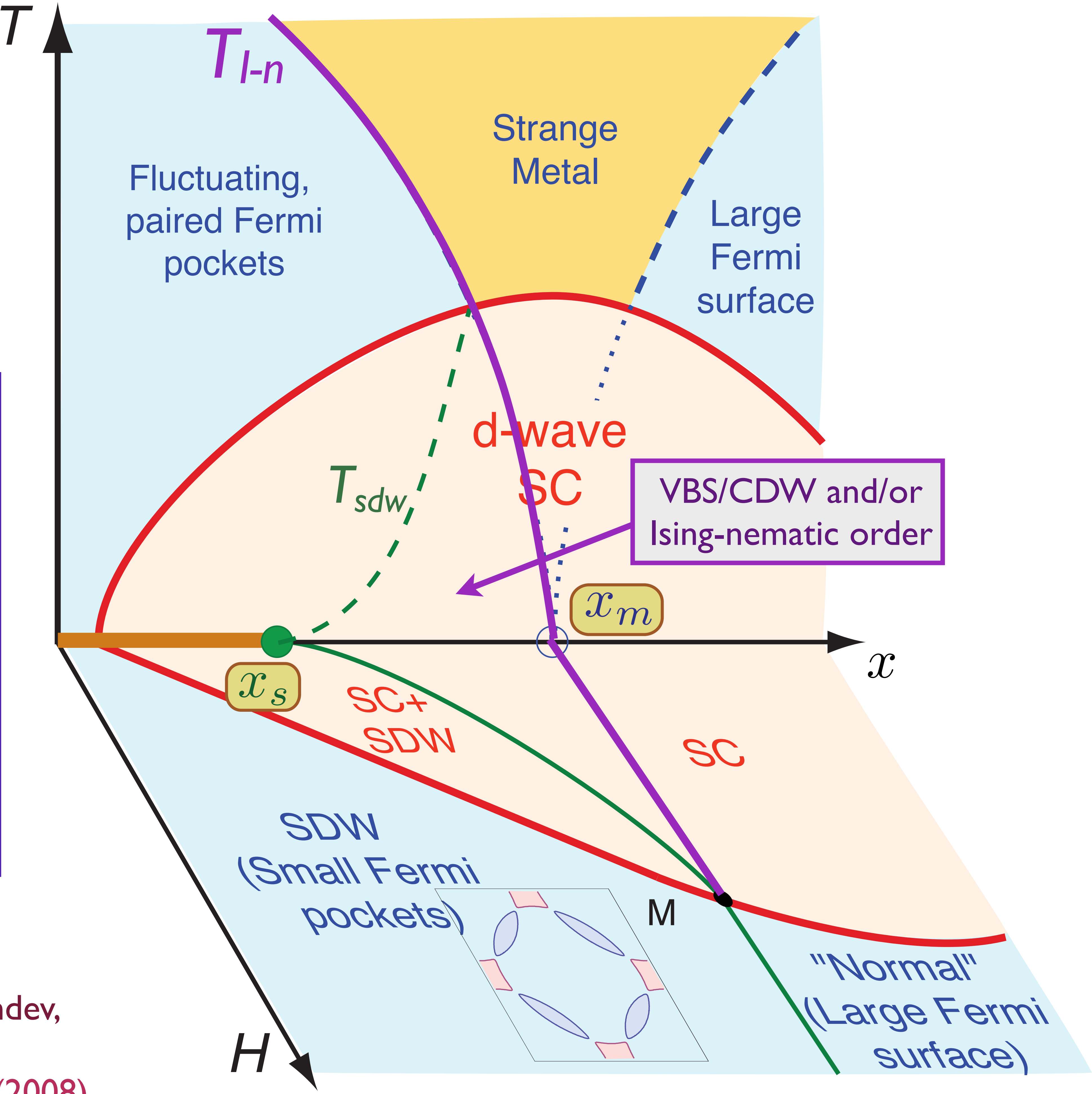


S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).



$T$
$T^*$
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
d-wave SC
$T_{sdw}$
$x_m$
$x$
$x_s$
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
$H$

Onset of superconductivity disrupts SDW order, but VBS/CDW/ Ising-nematic ordering can survive



R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

# Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

# Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density

# Conclusions

Gauge theory for pairing of Fermi pockets in a metal with fluctuating spin density wave order: Many qualitative similarities to holographic strange metals and superconductors