Quantum spin liquids and the phase diagram of the cuprates

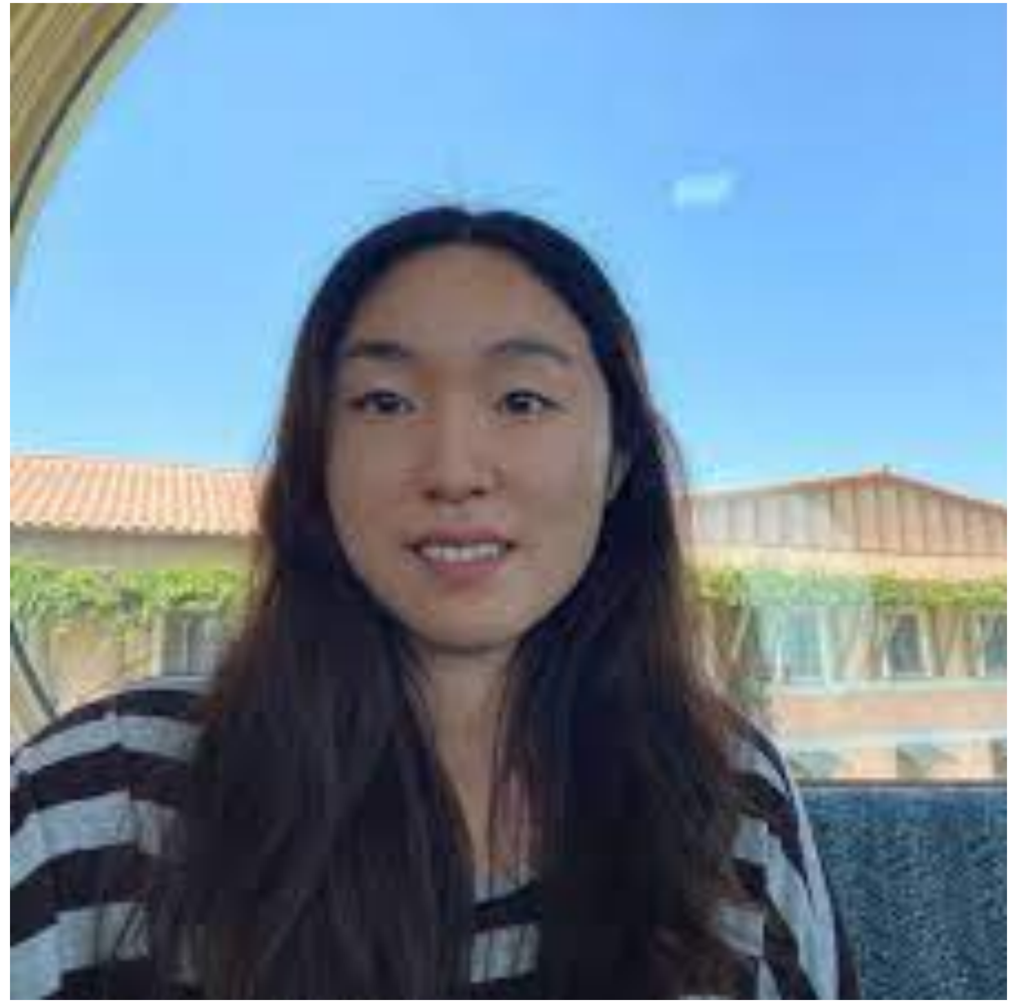
University of Oxford
July 11, 2023
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. S., PNAS **120**, e2302701120 (2023)
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.S., arXiv:2211.10452

Talk online: sachdev.physics.harvard.edu

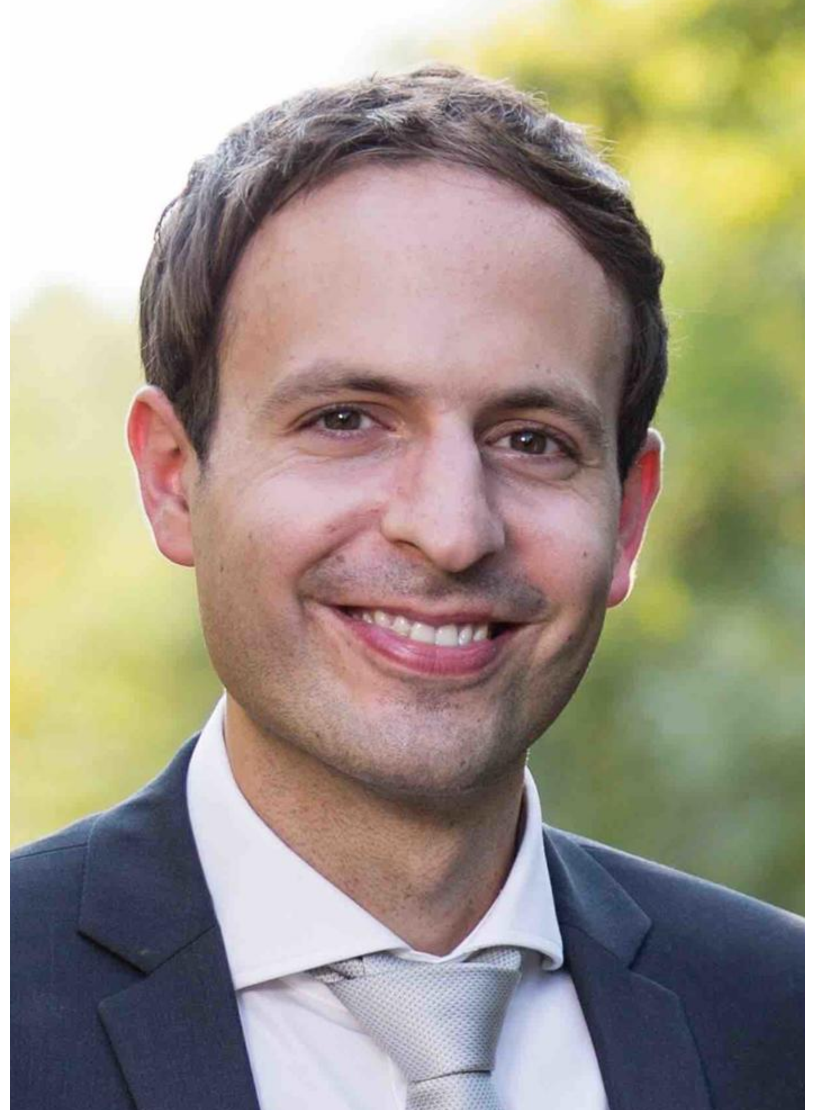
PHYSICS
HARVARD

Maine Christos

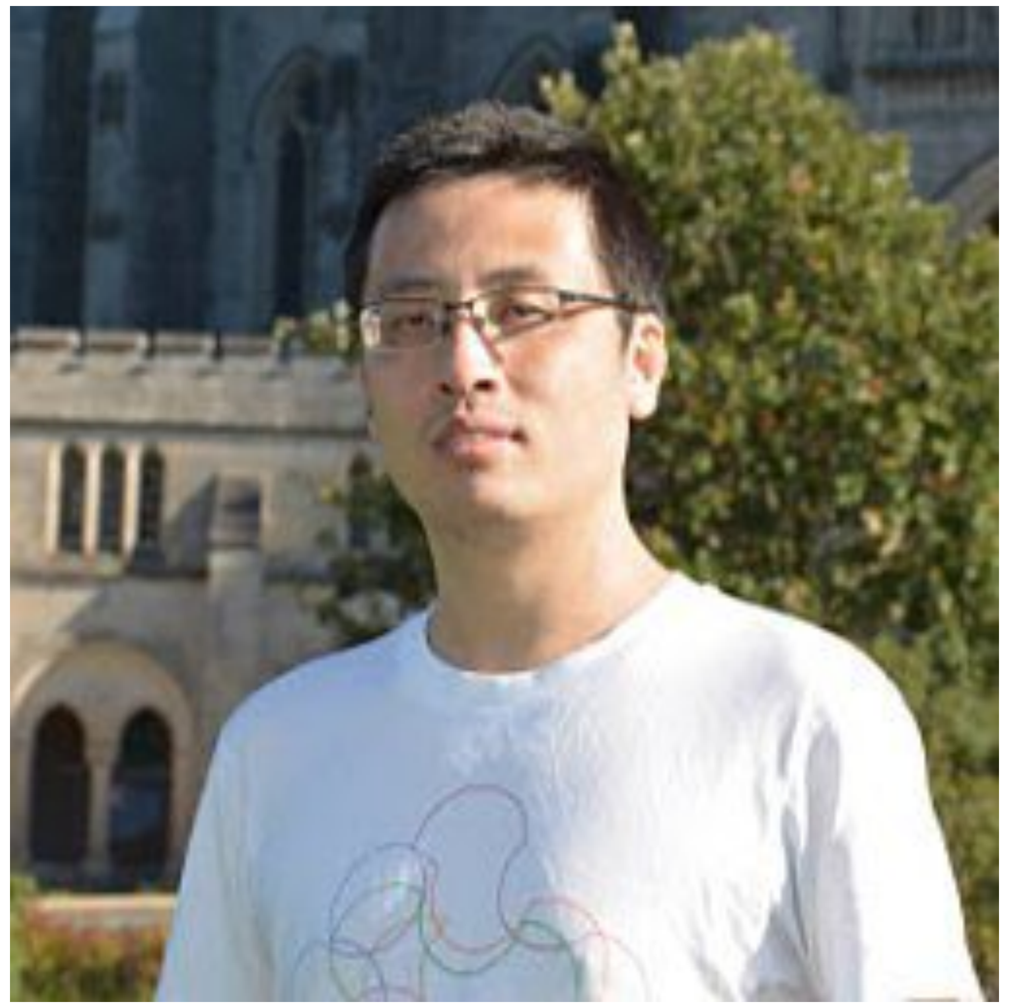


Zhu-Xi Luo
→GA Tech

Henry
Shackleton



Mathias Scheurer
Innsbruck→Stuttgart



Ya-Hui Zhang
Johns Hopkins

Alexander
Nikolaenko

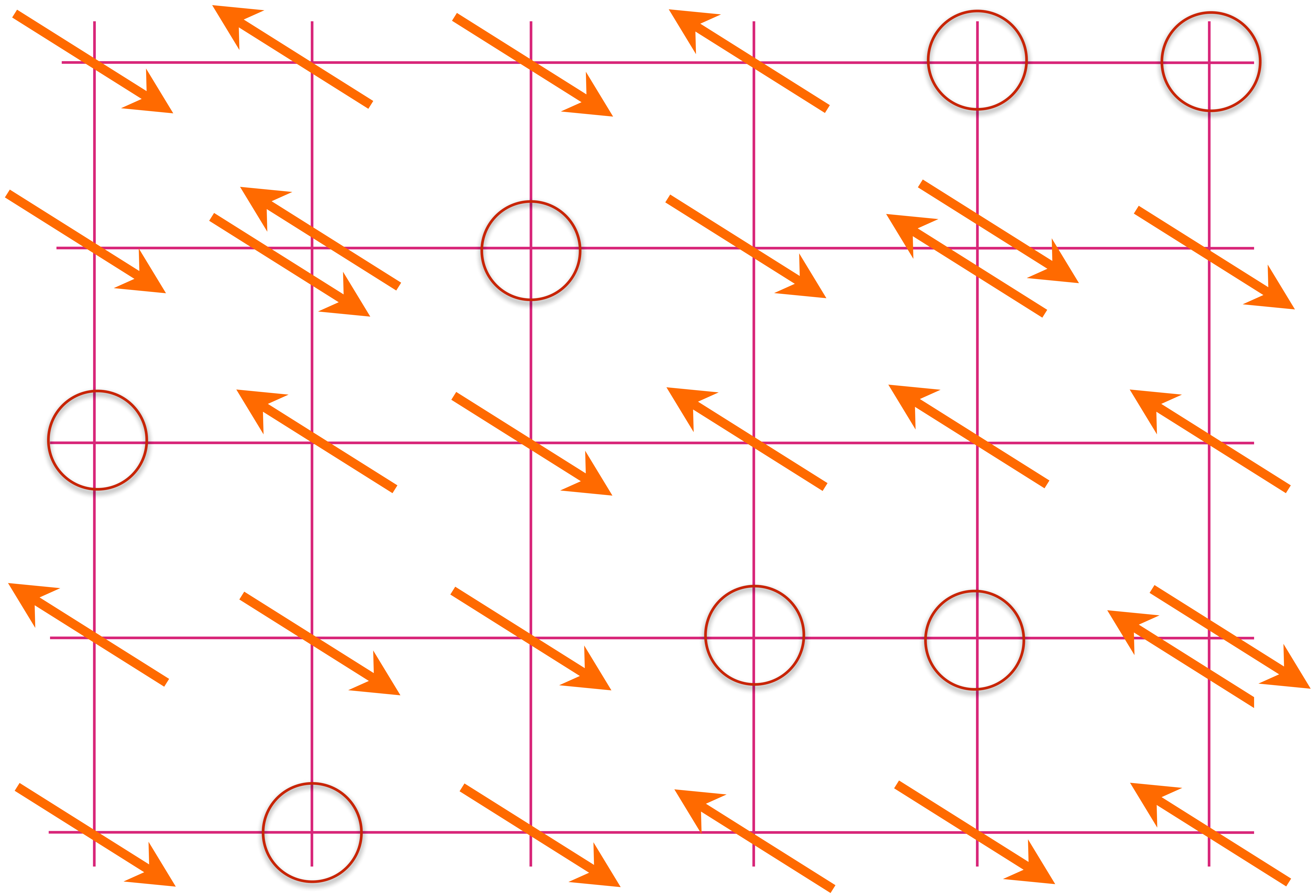
Darshan Joshi
TIFR Hyderabad

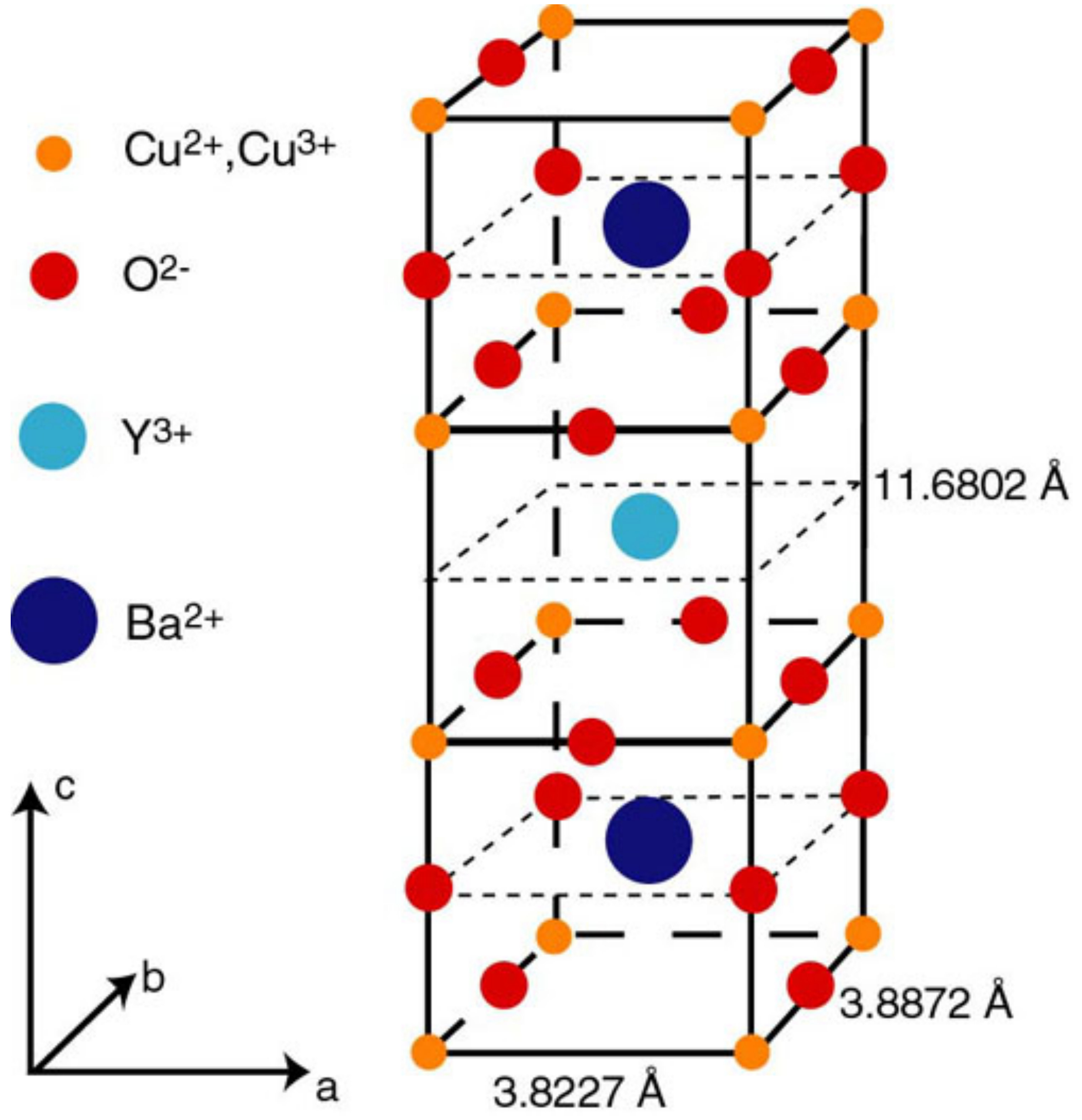
Jonas von Milczewski

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

Square lattice Hubbard model with electron density $1 - p$.

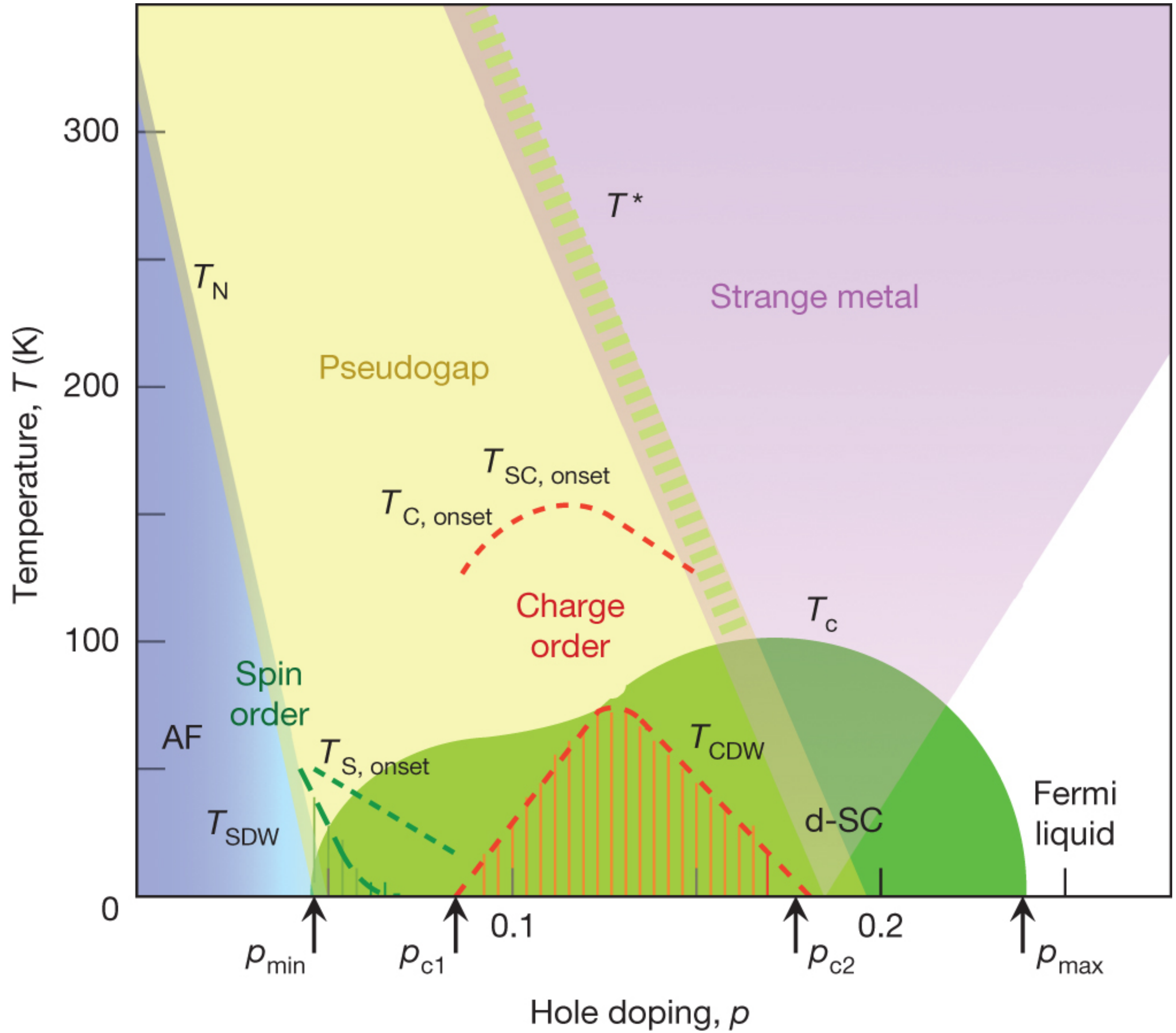
$$\mathcal{H}_{\text{Hubbard}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \ldots$$

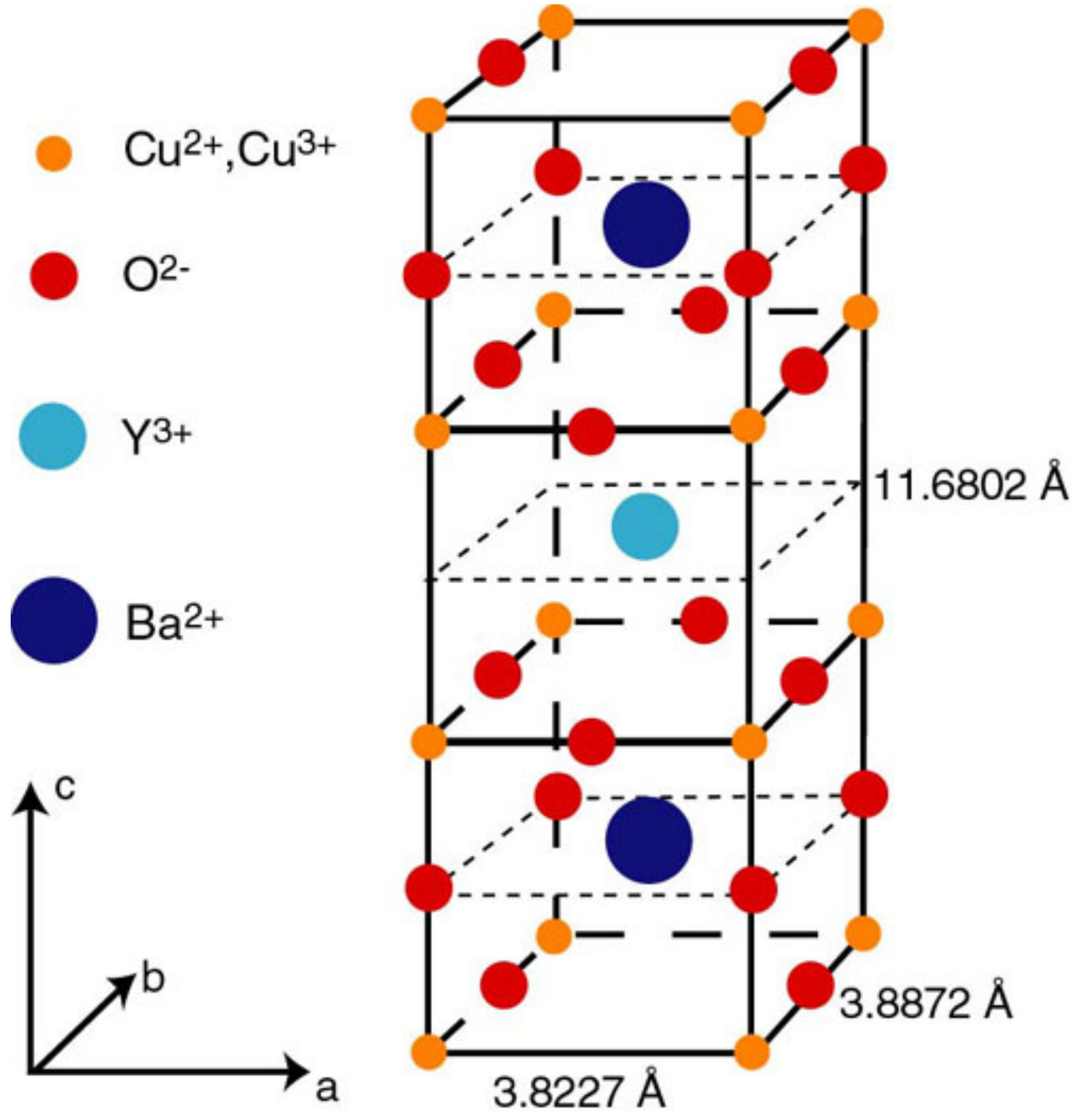




$YBa_2Cu_3O_{6+x}$

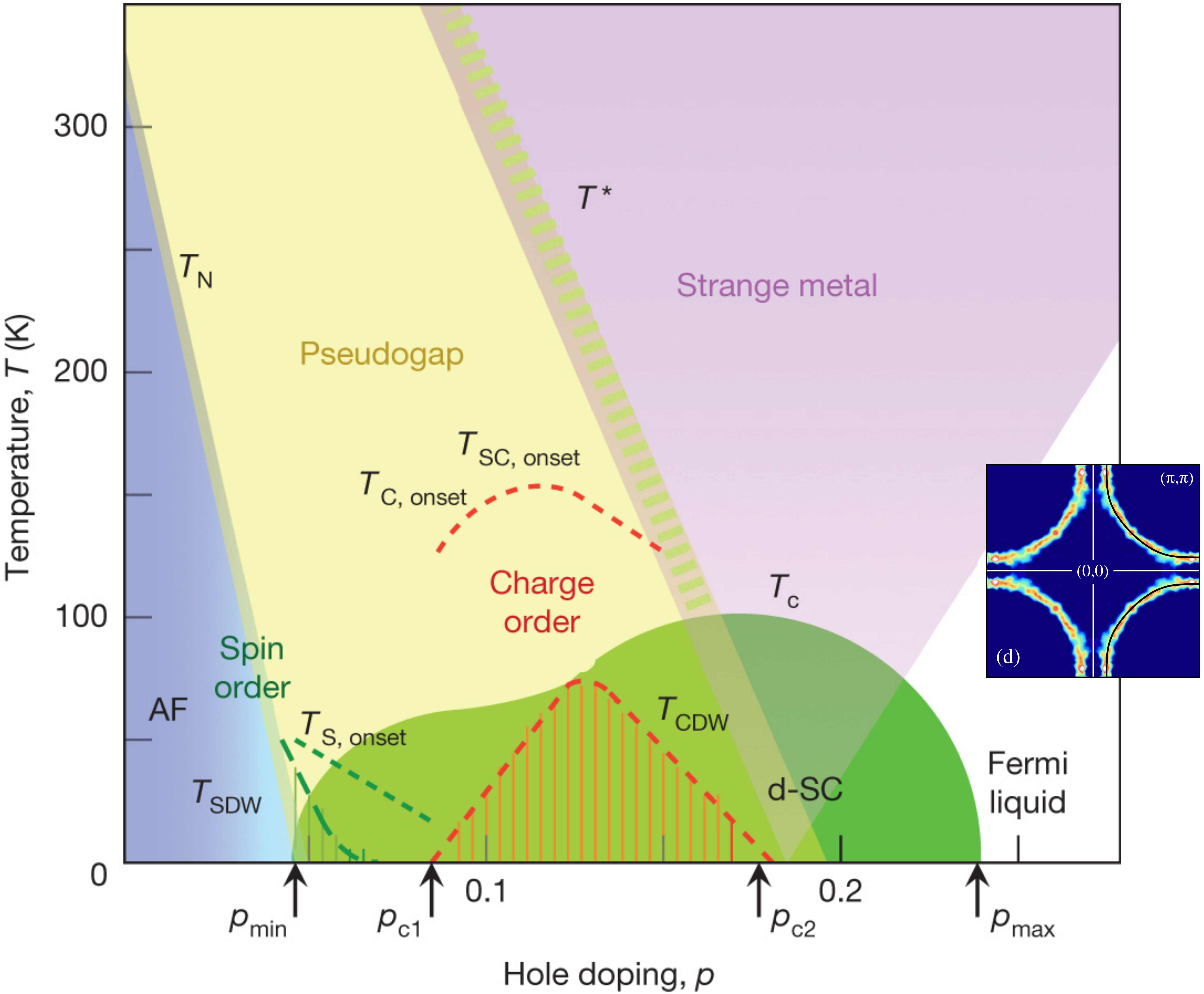
B Keimer *et al.* *Nature* **518**, 179-186 (2015)





$YBa_2Cu_3O_{6+x}$

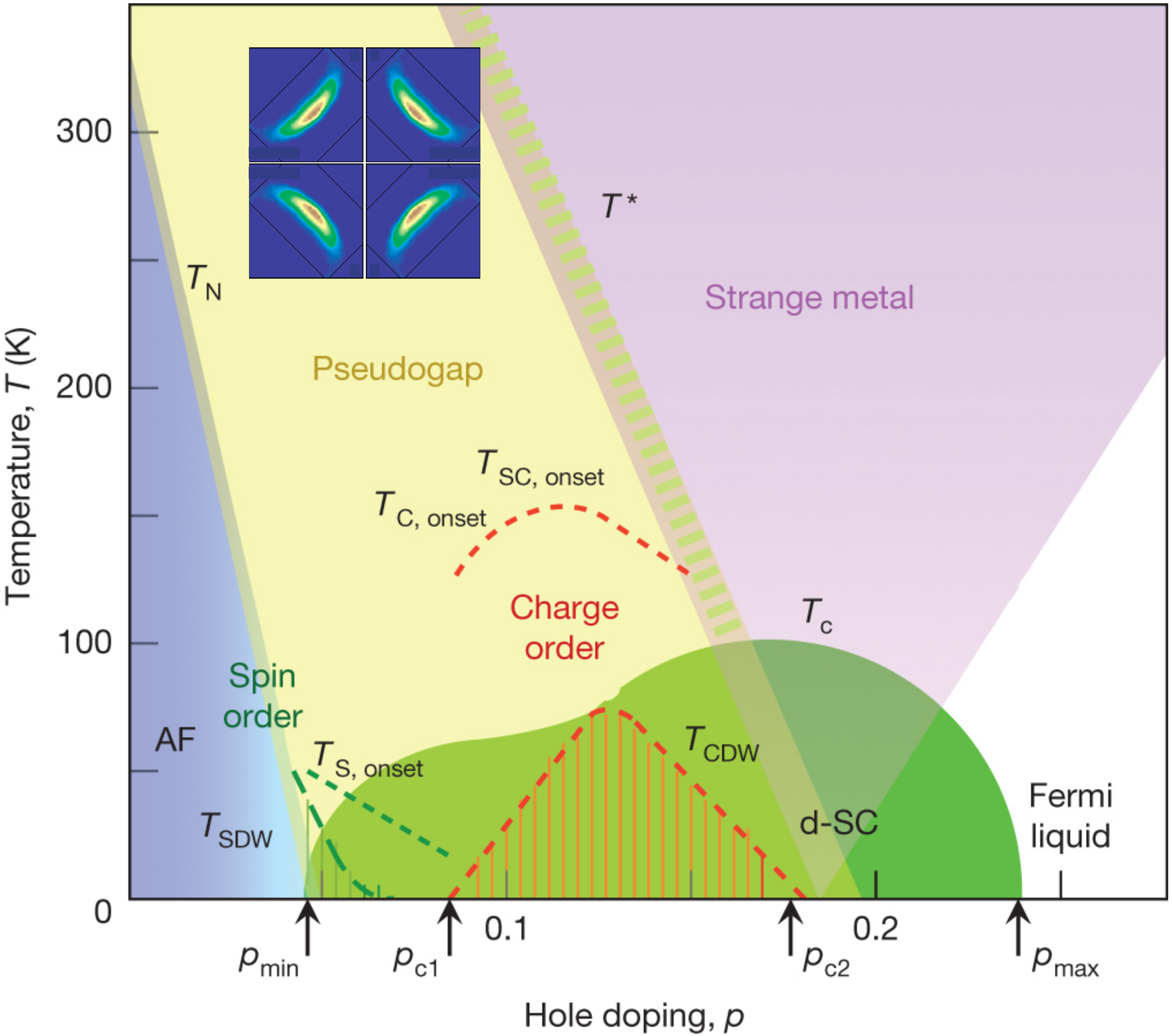
B Keimer *et al.* *Nature* **518**, 179-186 (2015)



Fermi liquid in the overdoped metal

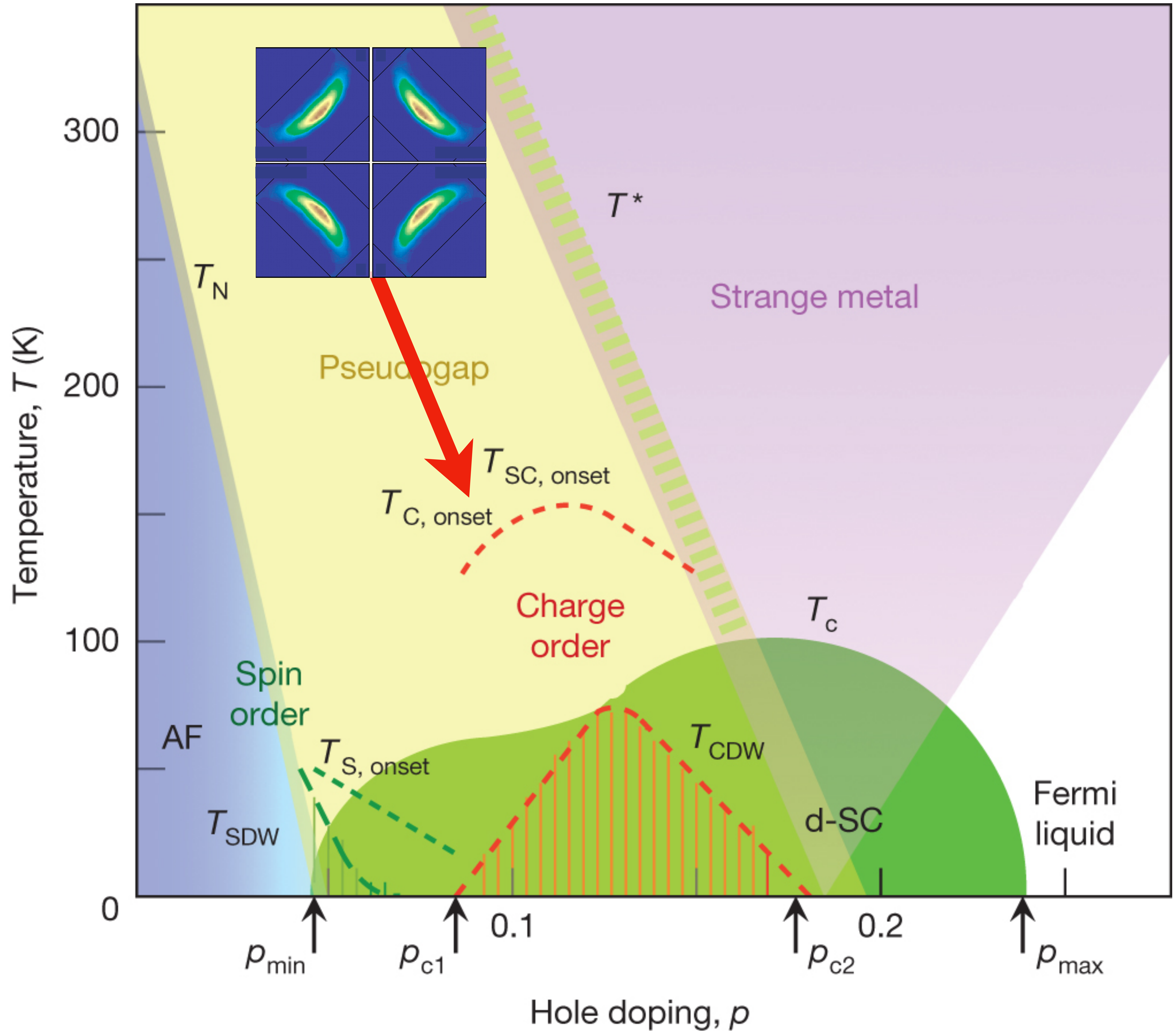
Volume enclosed by the Fermi surface $= 1 - p$

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



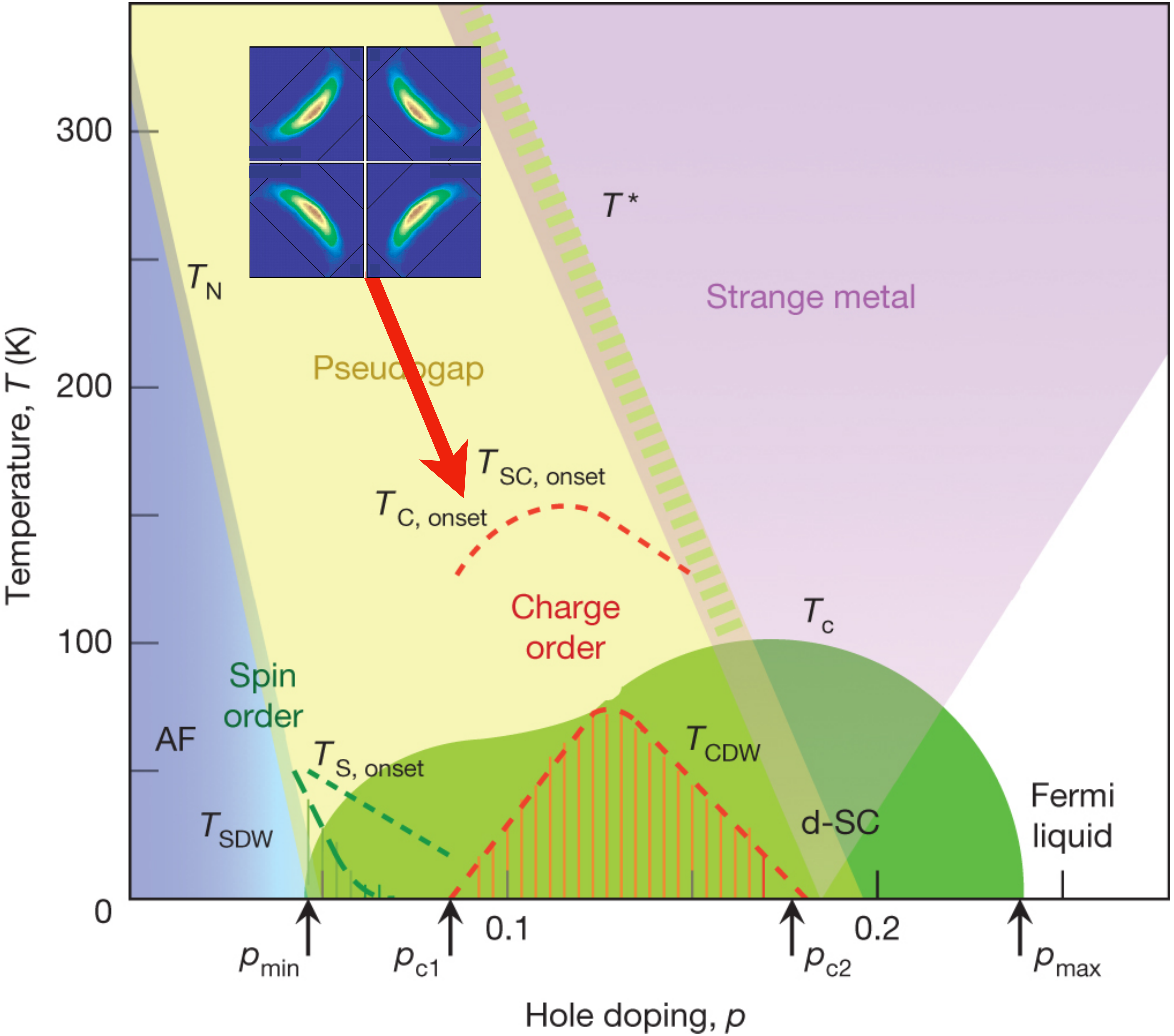
Theory for
"pseudogap metal"
with "Fermi arcs"?

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



Needed: a theory for the onset of charge order and *d*-wave superconductivity from the pseudogap metal.

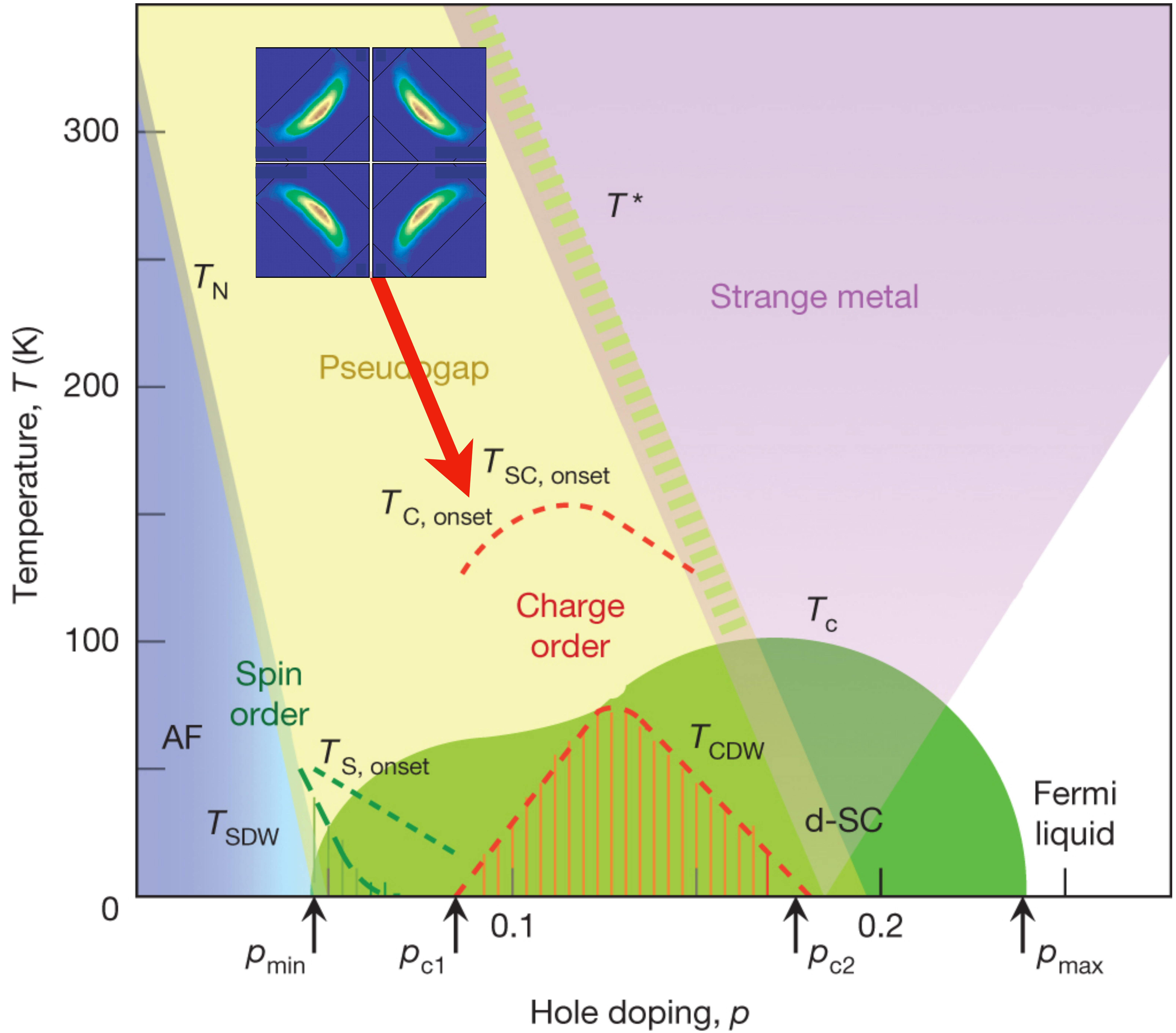
B Keimer *et al.* *Nature* **518**, 179-186 (2015)



Needed: a theory for the onset of charge order and d-wave superconductivity from the pseudogap metal.

Why are T_c and $T_{\rm CDW}$ about the same?

B Keimer *et al.* *Nature* **518**, 179-186 (2015)

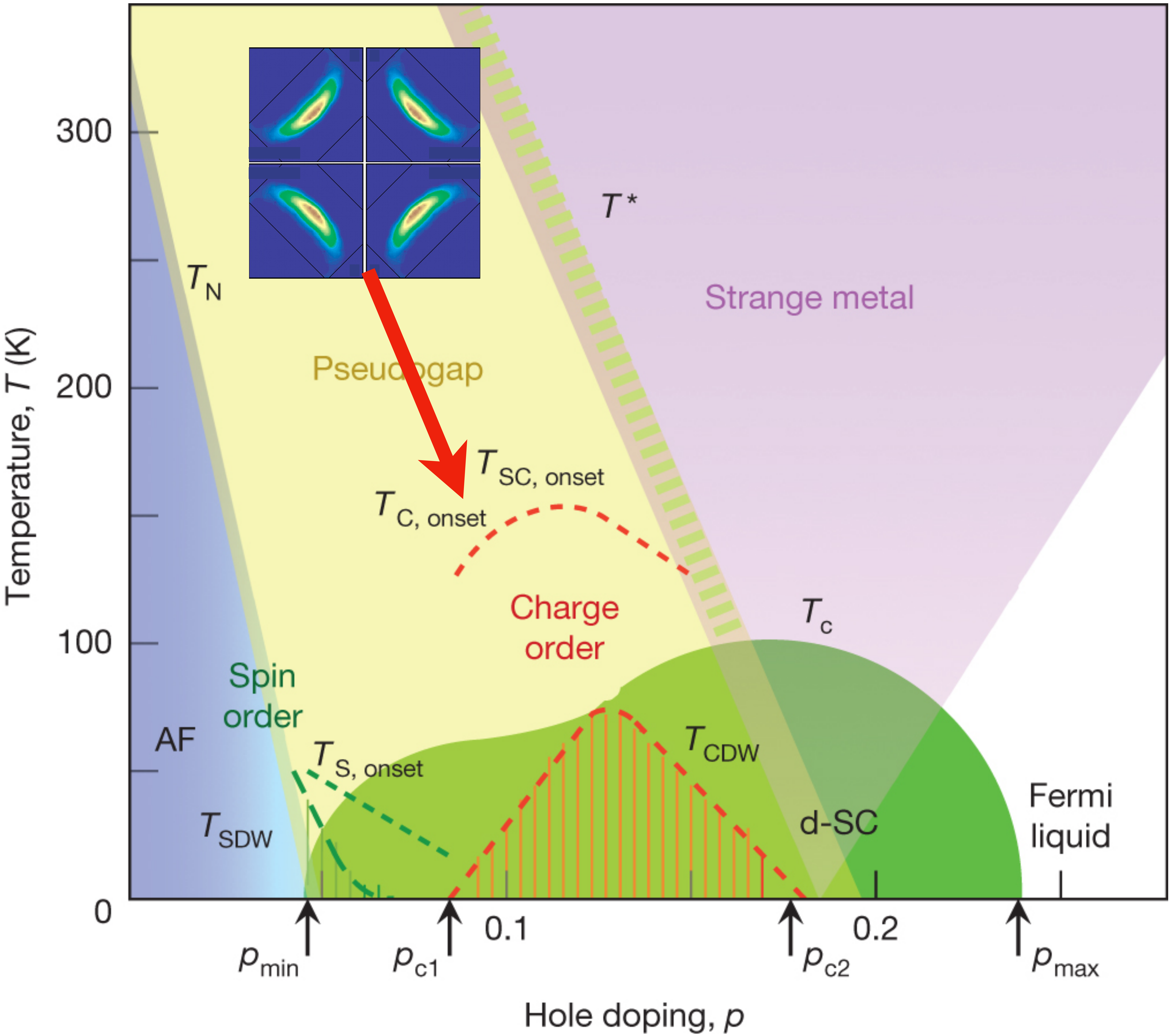


Needed: a theory for the onset of charge order and *d*-wave superconductivity from the pseudogap metal.

Why are T_c and $T_{\rm CDW}$ about the same?

Should we think of the pseudogap as a metal with "fluctuating" charge and/or superconducting order?

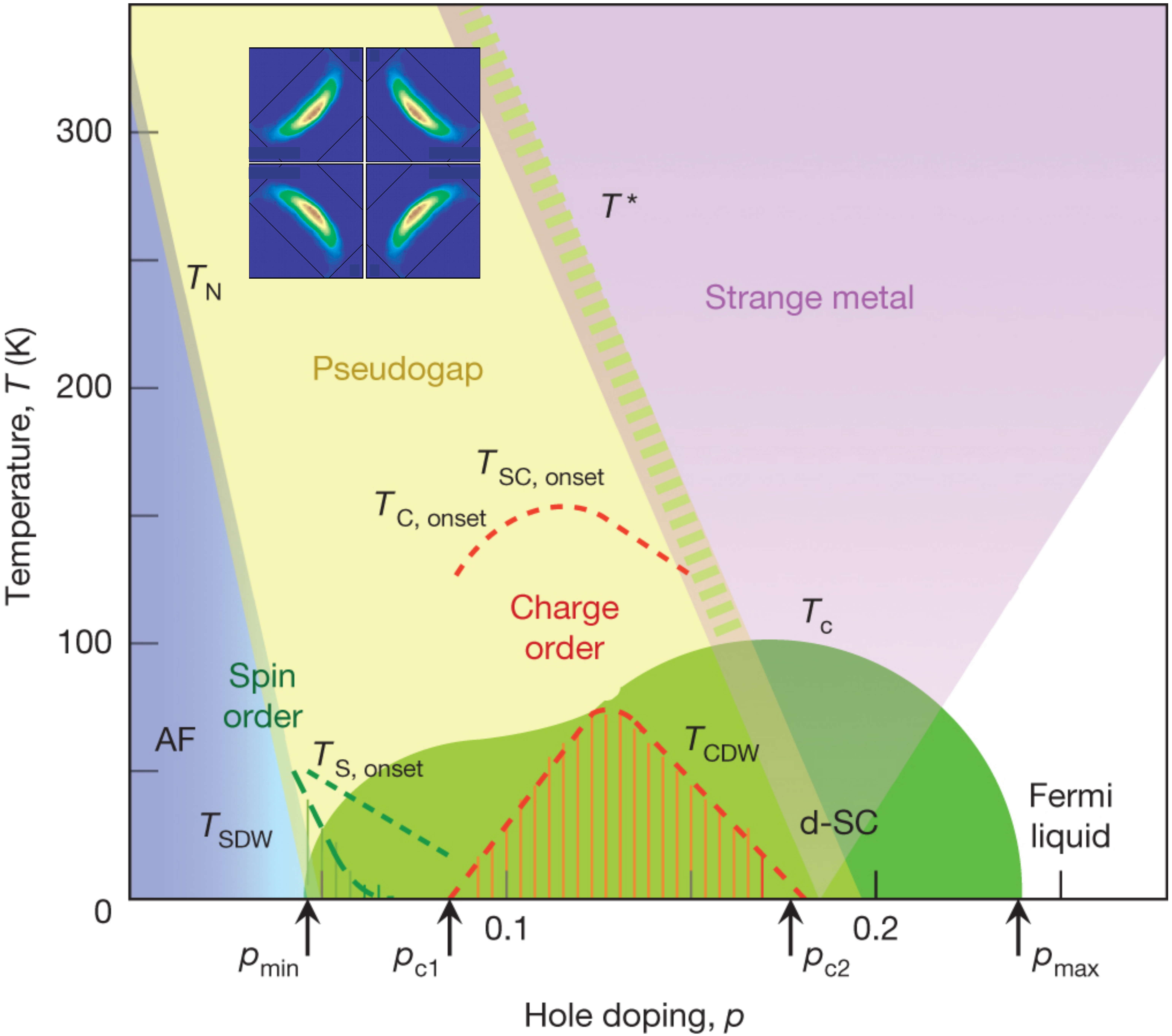
B Keimer *et al.* *Nature* **518**, 179-186 (2015)

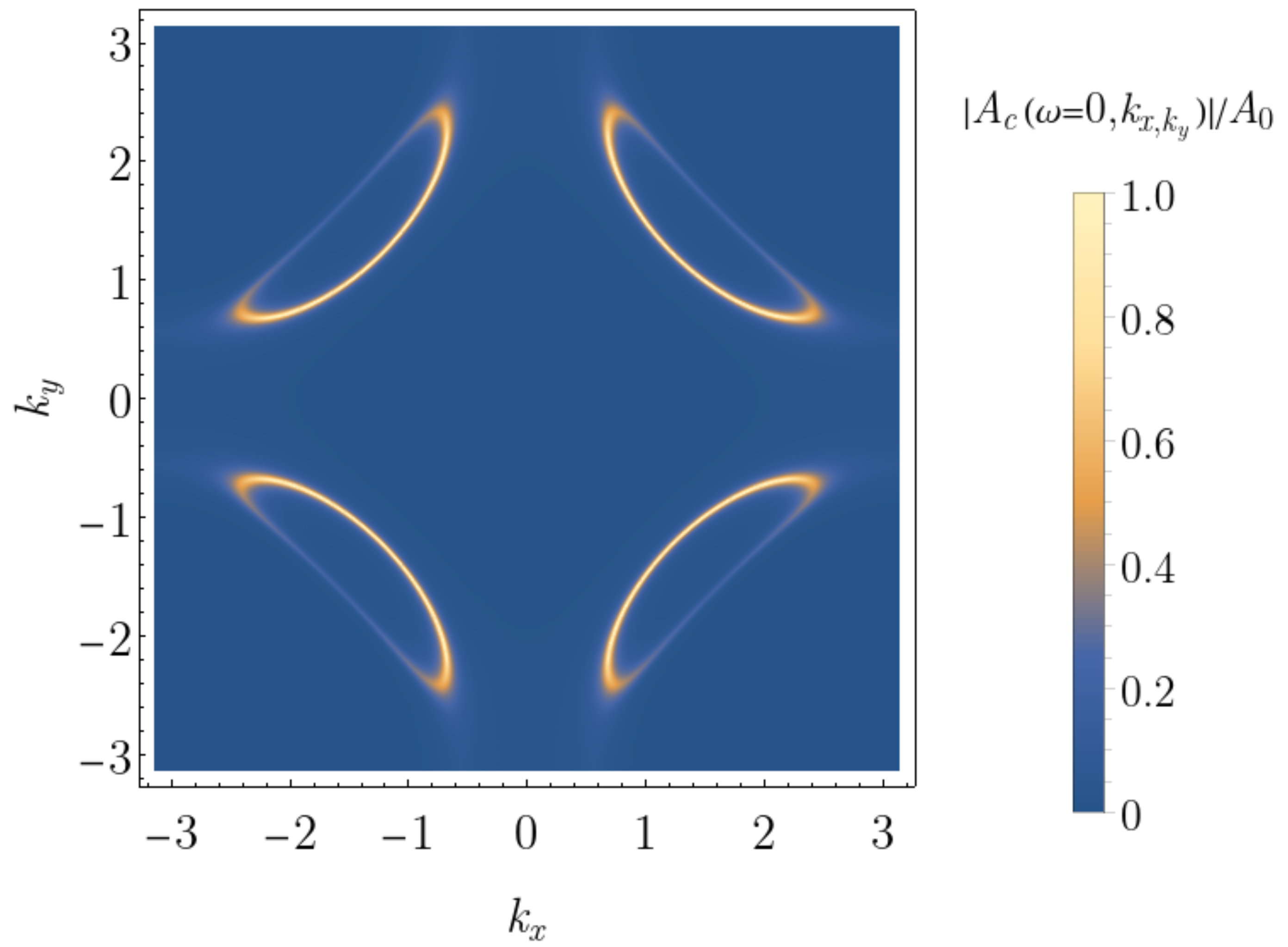


Use the pseudogap metal in place of the Fermi liquid as the 'parent' to *conventional* *d*-wave superconductor, charge density wave, spin density wave, pair density wave

. . .

B Keimer *et al.* *Nature* **518**, 179-186 (2015)





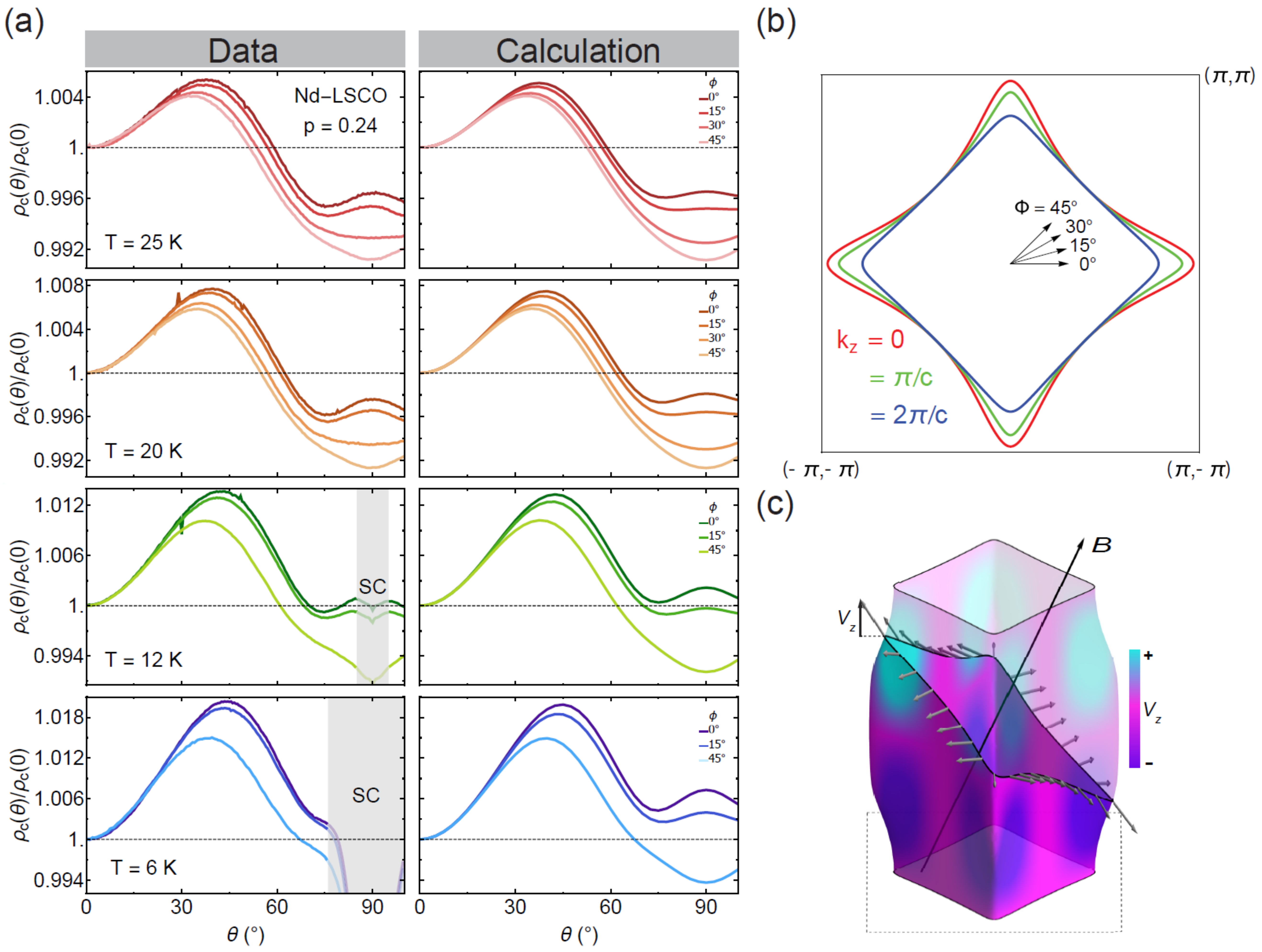
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

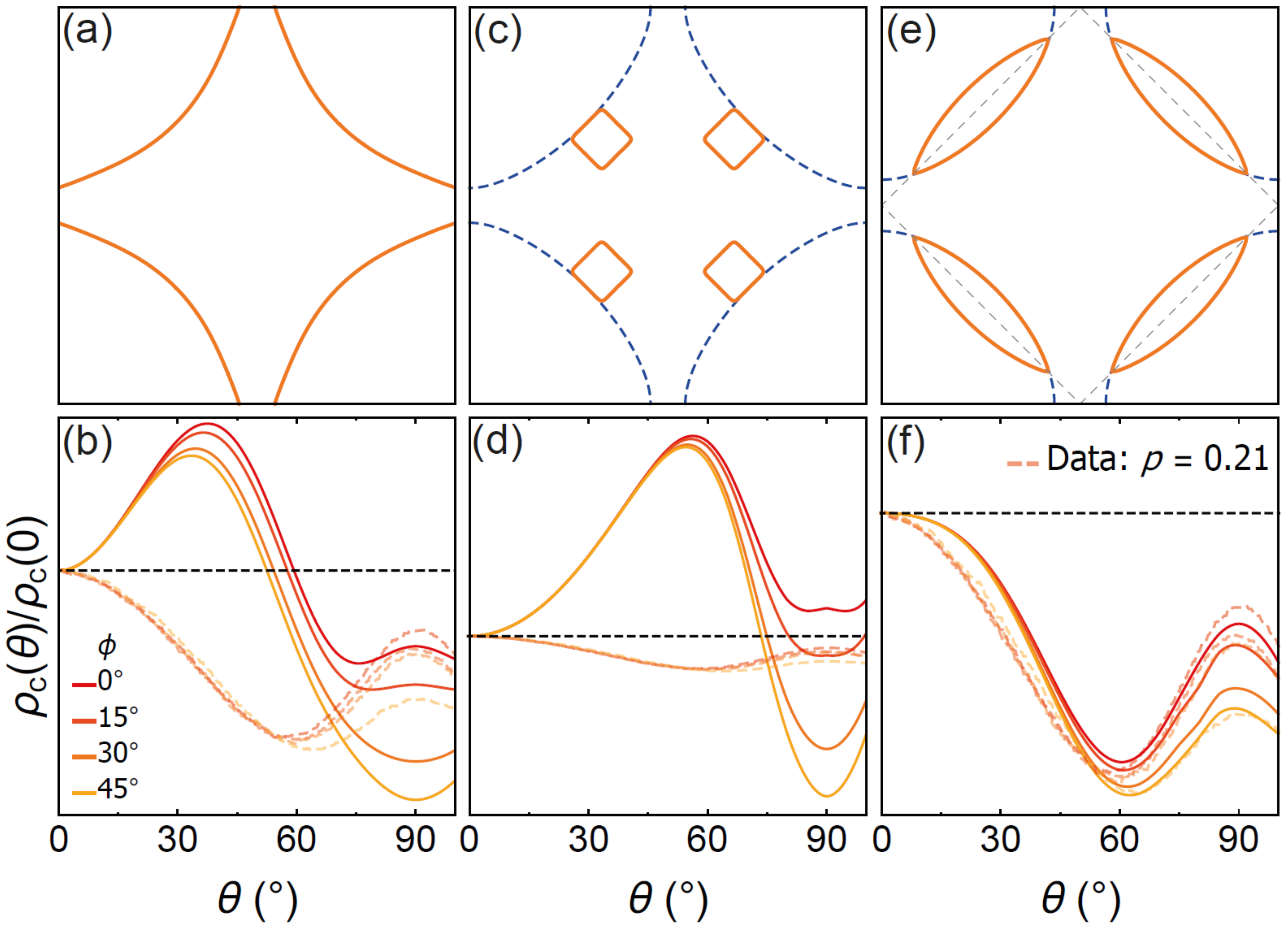
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, PRB **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, PRL **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, PRB **106**, 045109 (2022).

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics **18**, 558 (2022)

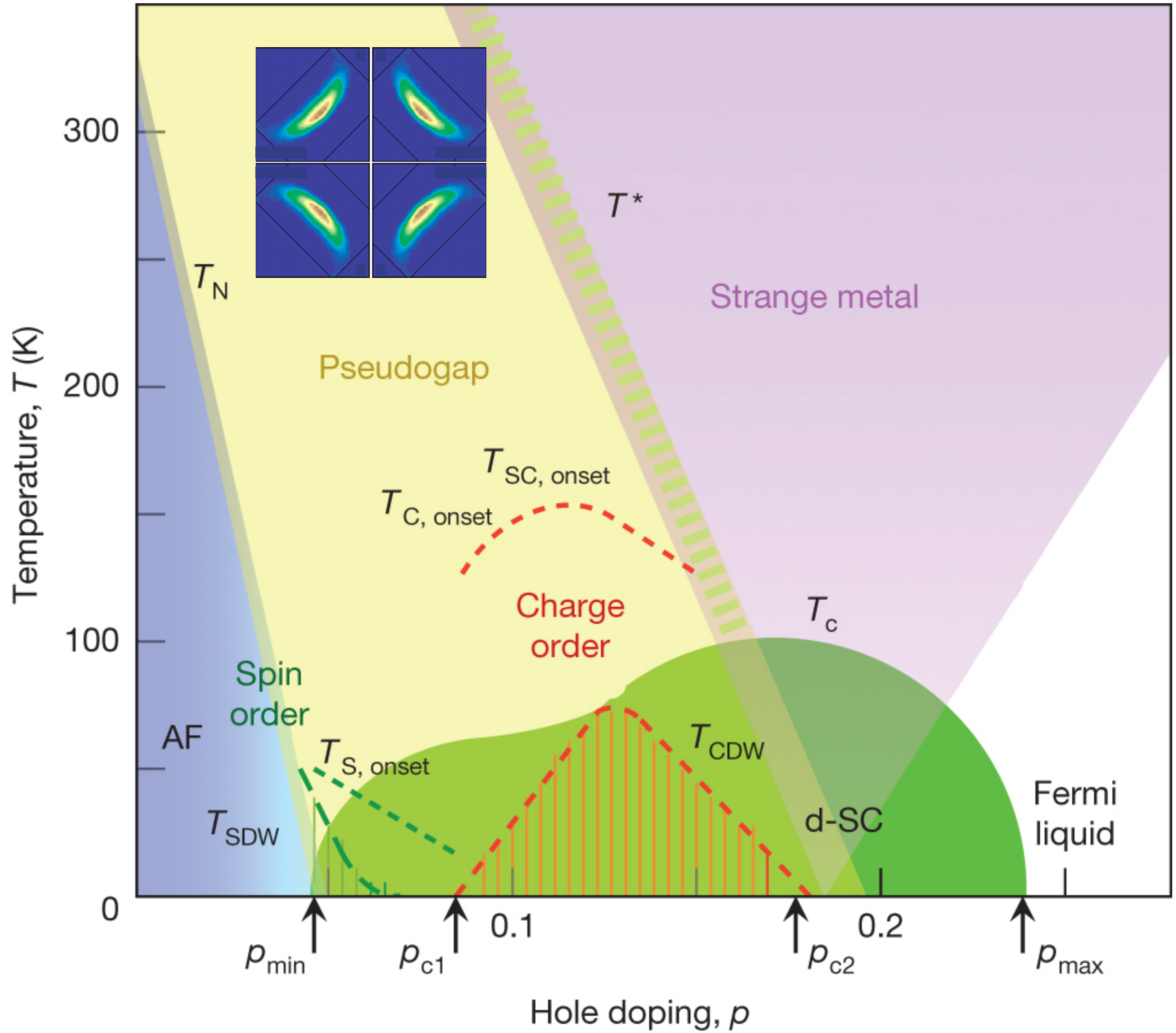


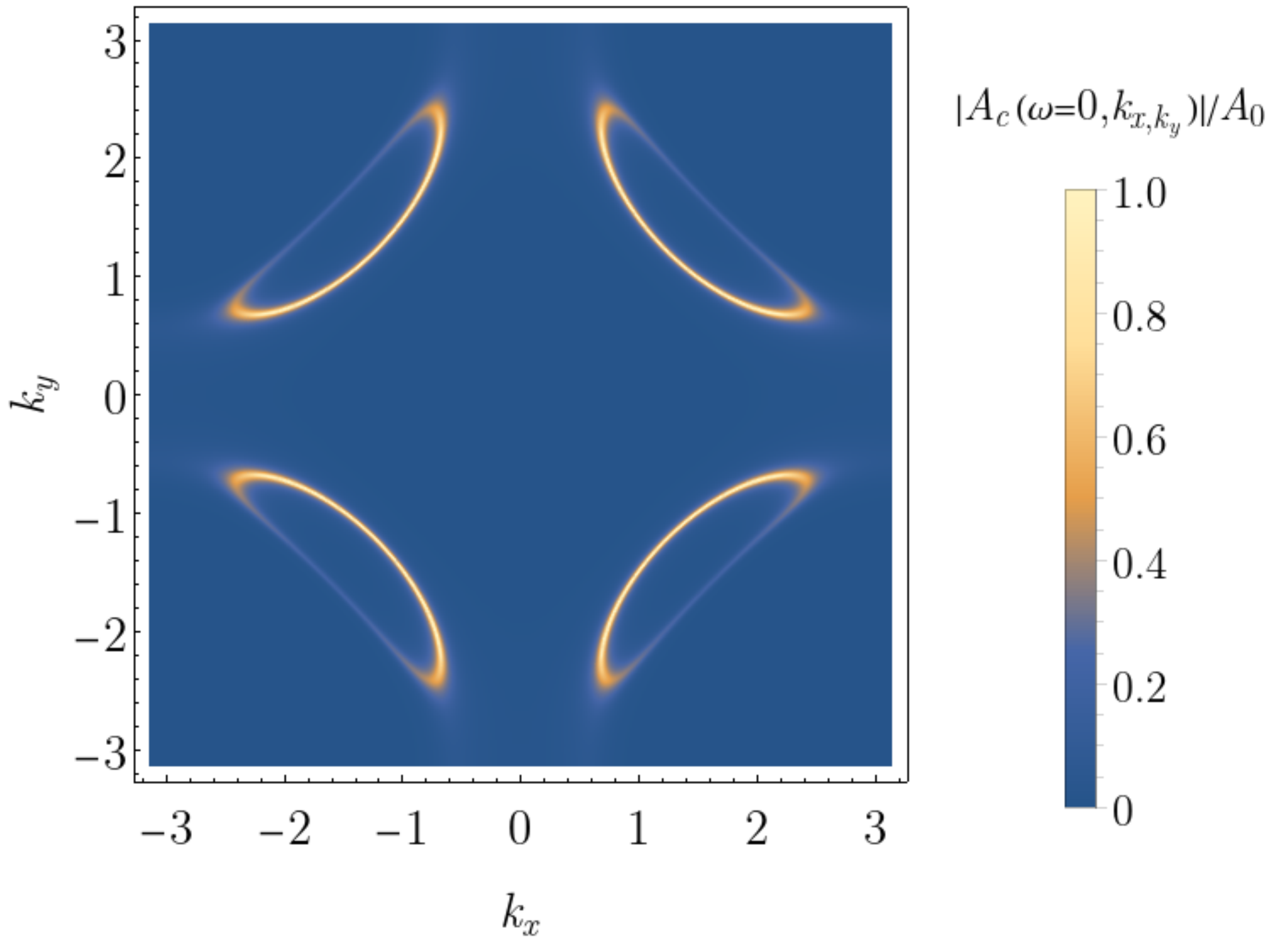
$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



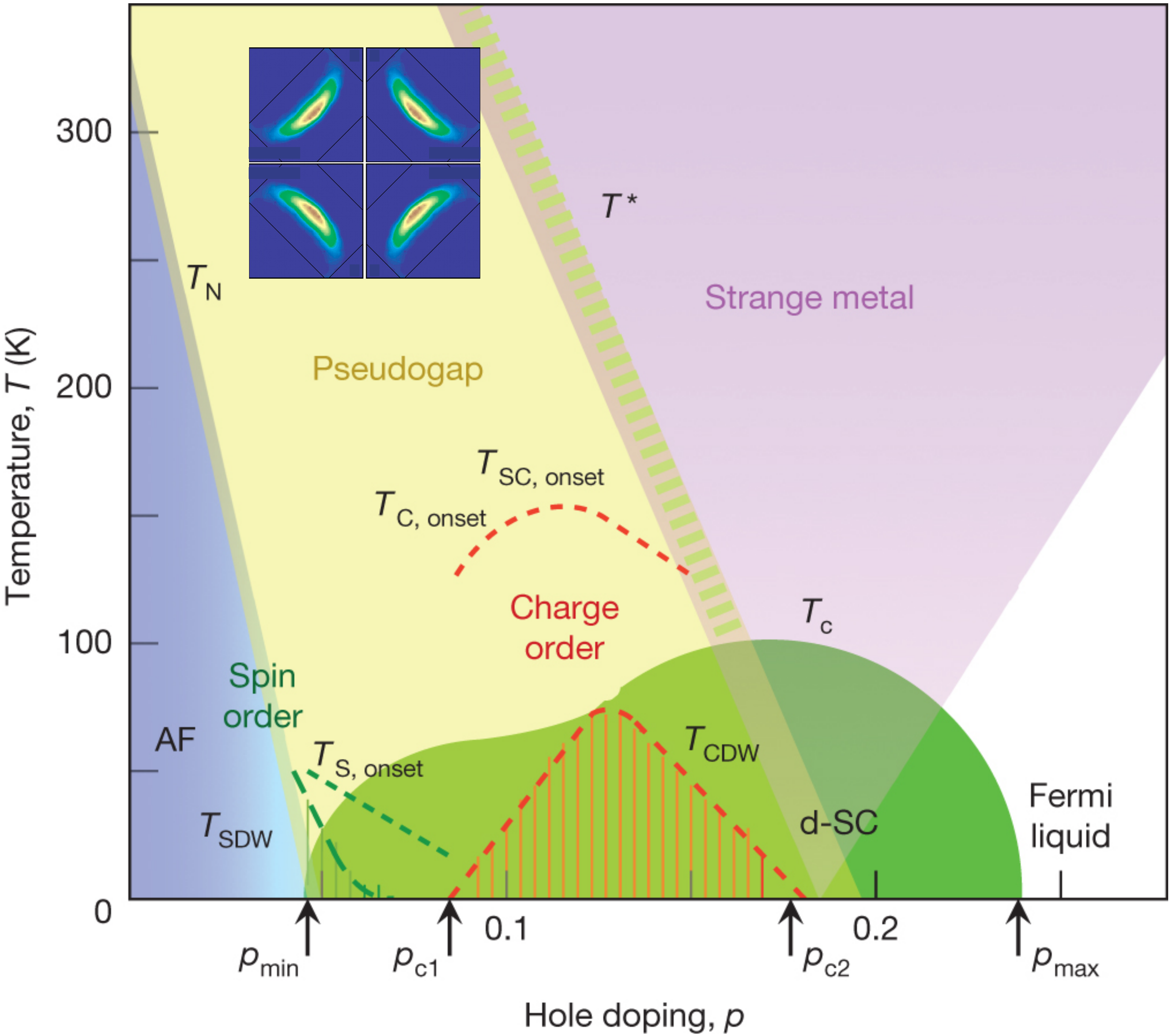


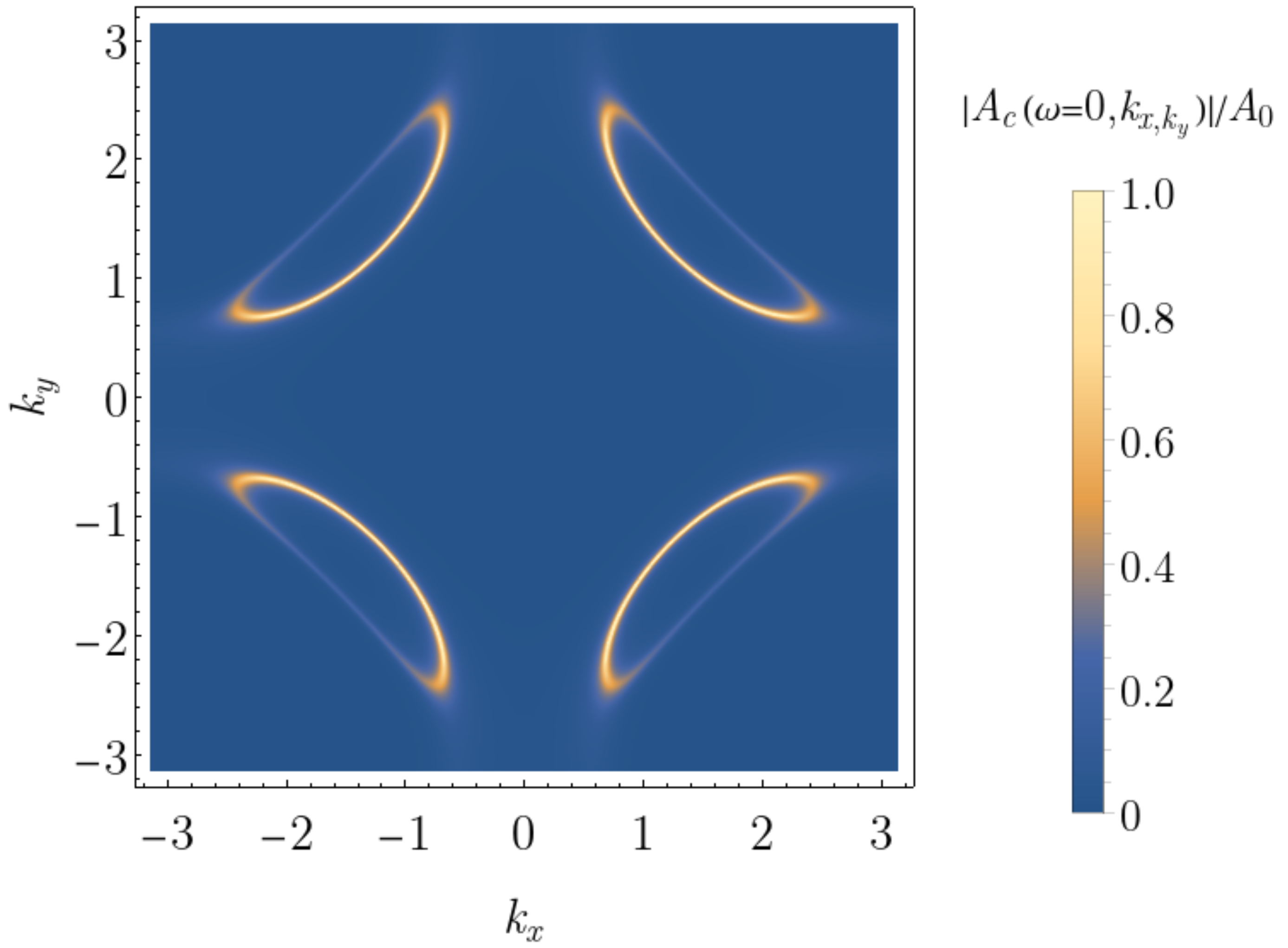
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, PRB **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, PRL **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, PRB **106**, 045109 (2022).

B Keimer *et al.* *Nature* **518**, 179-186 (2015)





E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

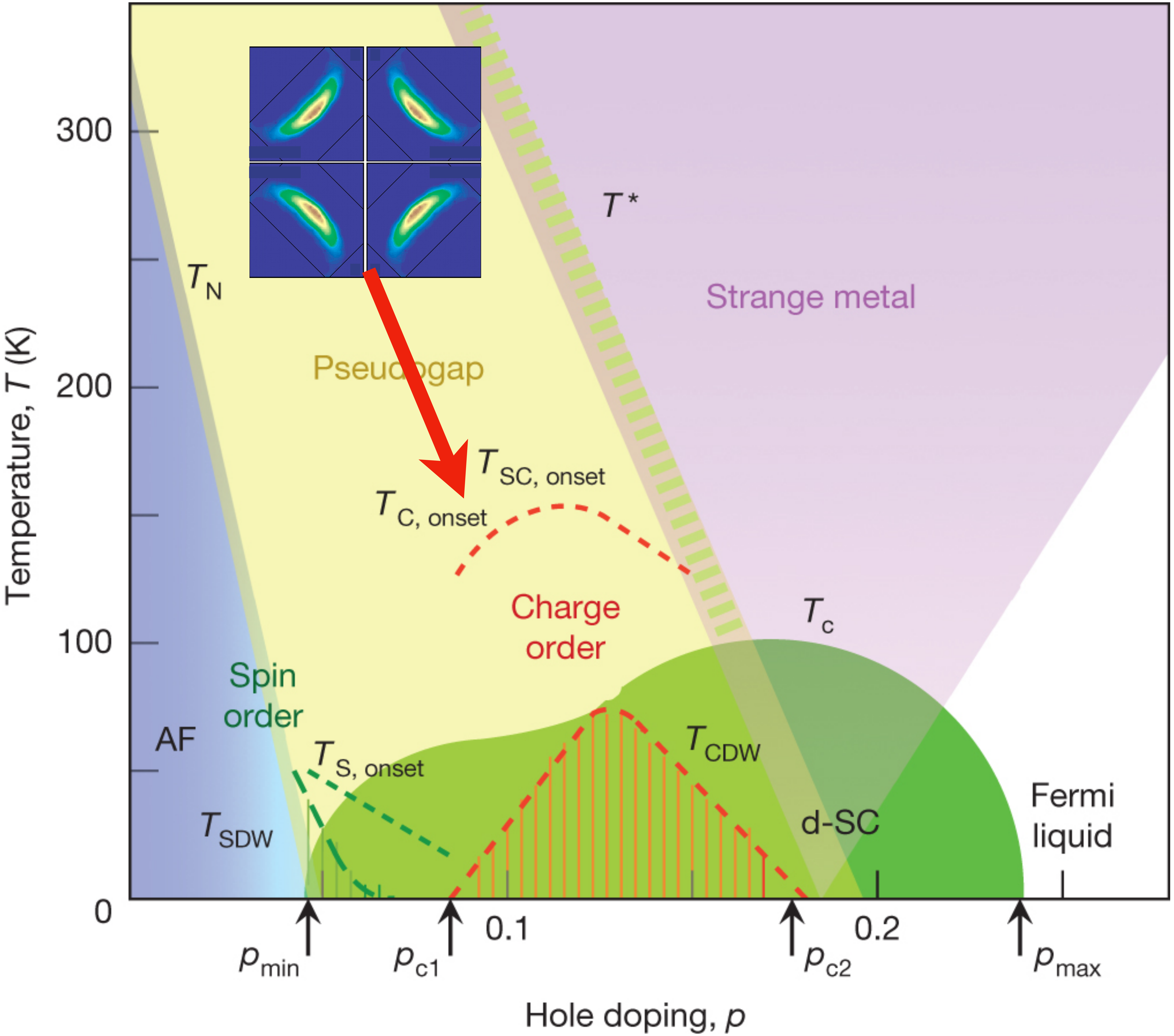
$+$

'spectator' square lattice spin liquid *at half-filling.*

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1 + p)$.

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

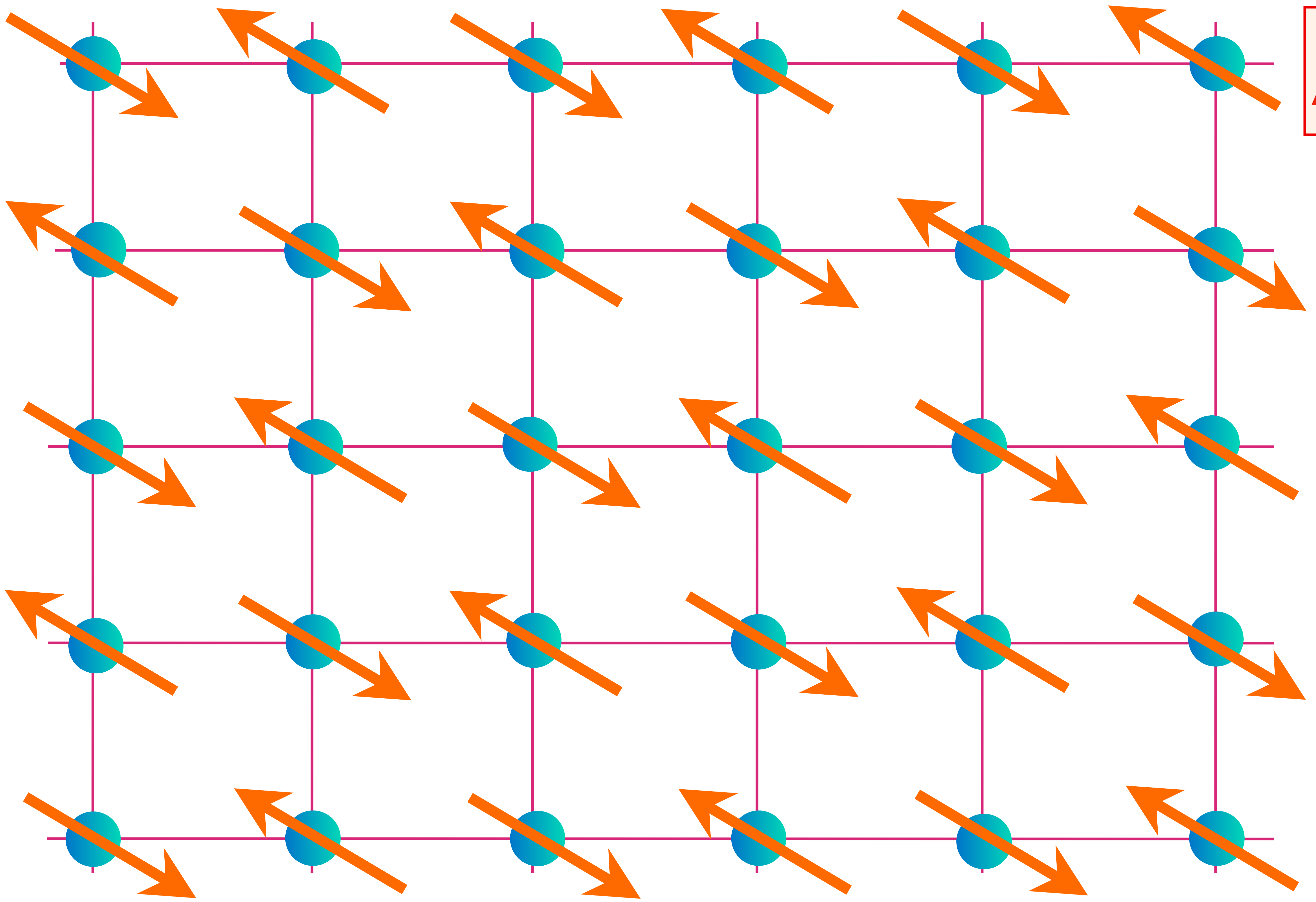
B Keimer *et al.* *Nature* **518**, 179-186 (2015)



The onset of conventional order is a *confinement transition* for the emergent gauge theory describing the fractionalized excitations of the spin liquid in the FL* state.

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

The dance of electrons on Cu atoms in YBCO

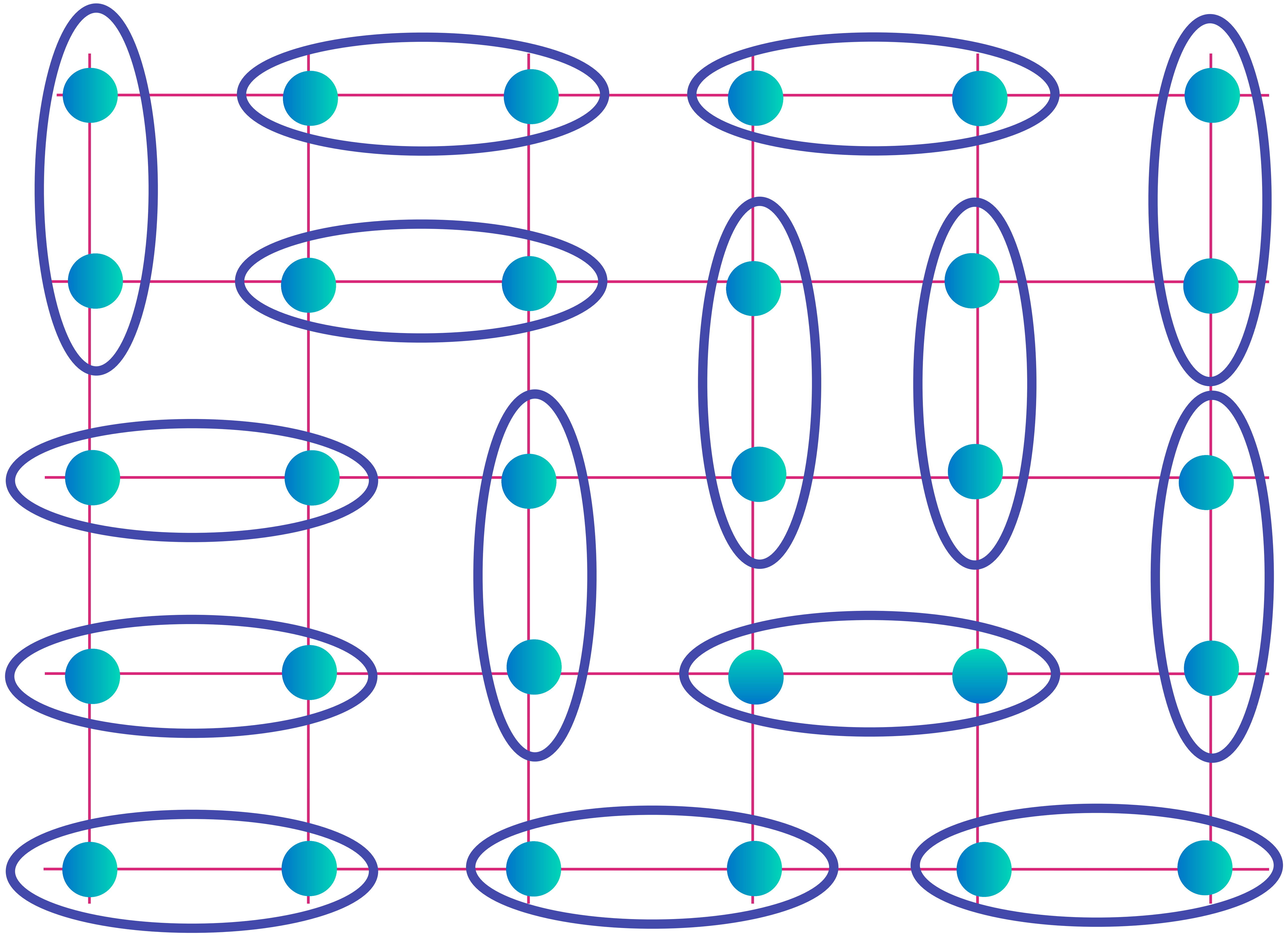


Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

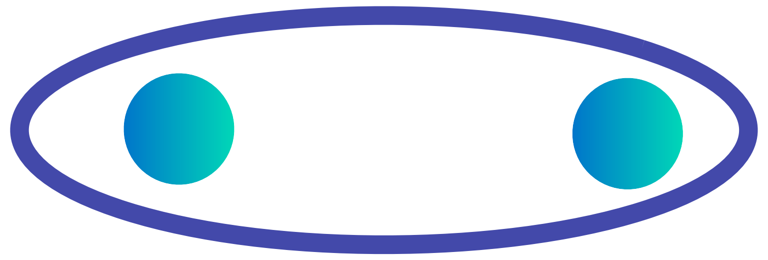
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)



Spin liquid

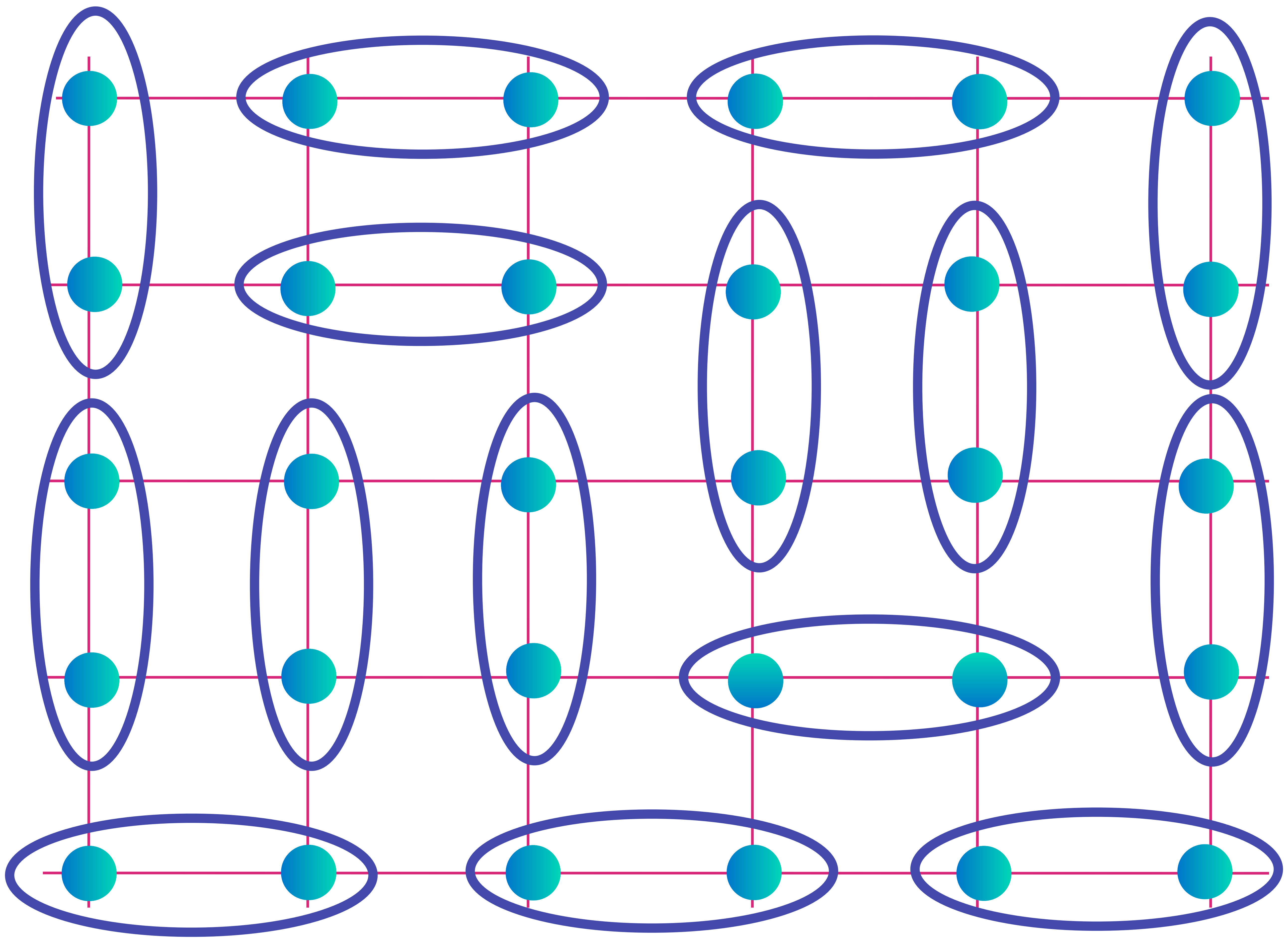
Electrons form entangled pairs, and the pairs entangle across the entire sample

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

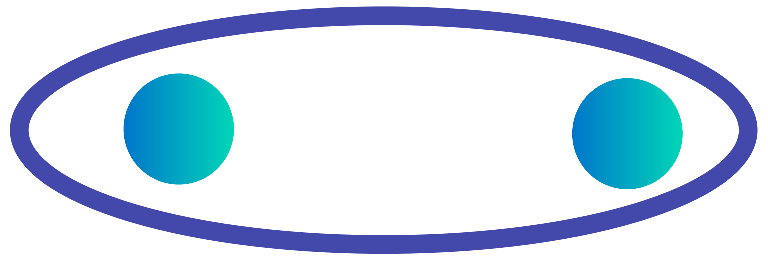
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample

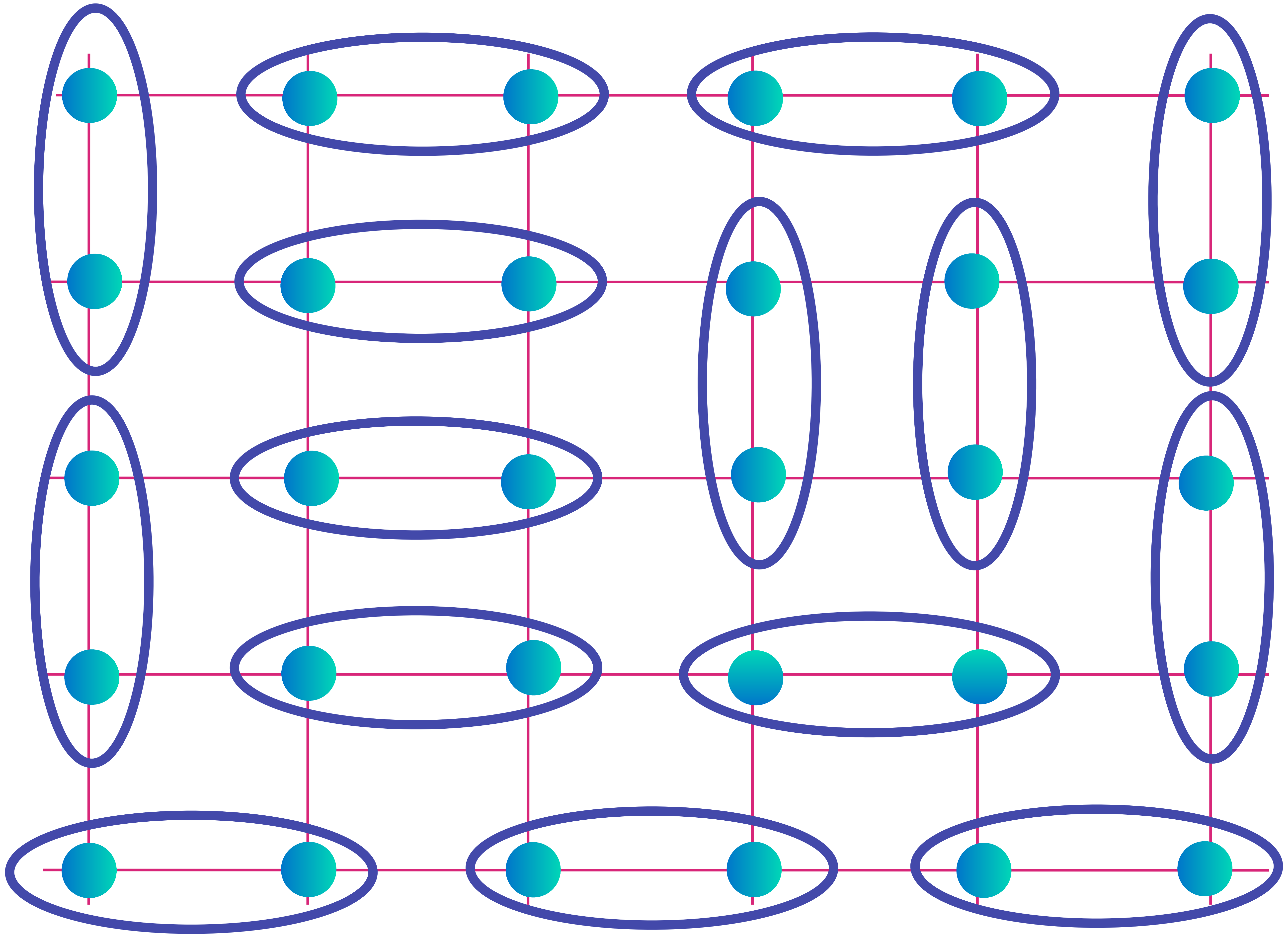


$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

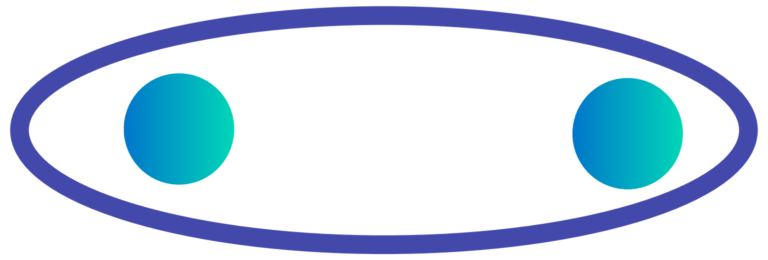
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample

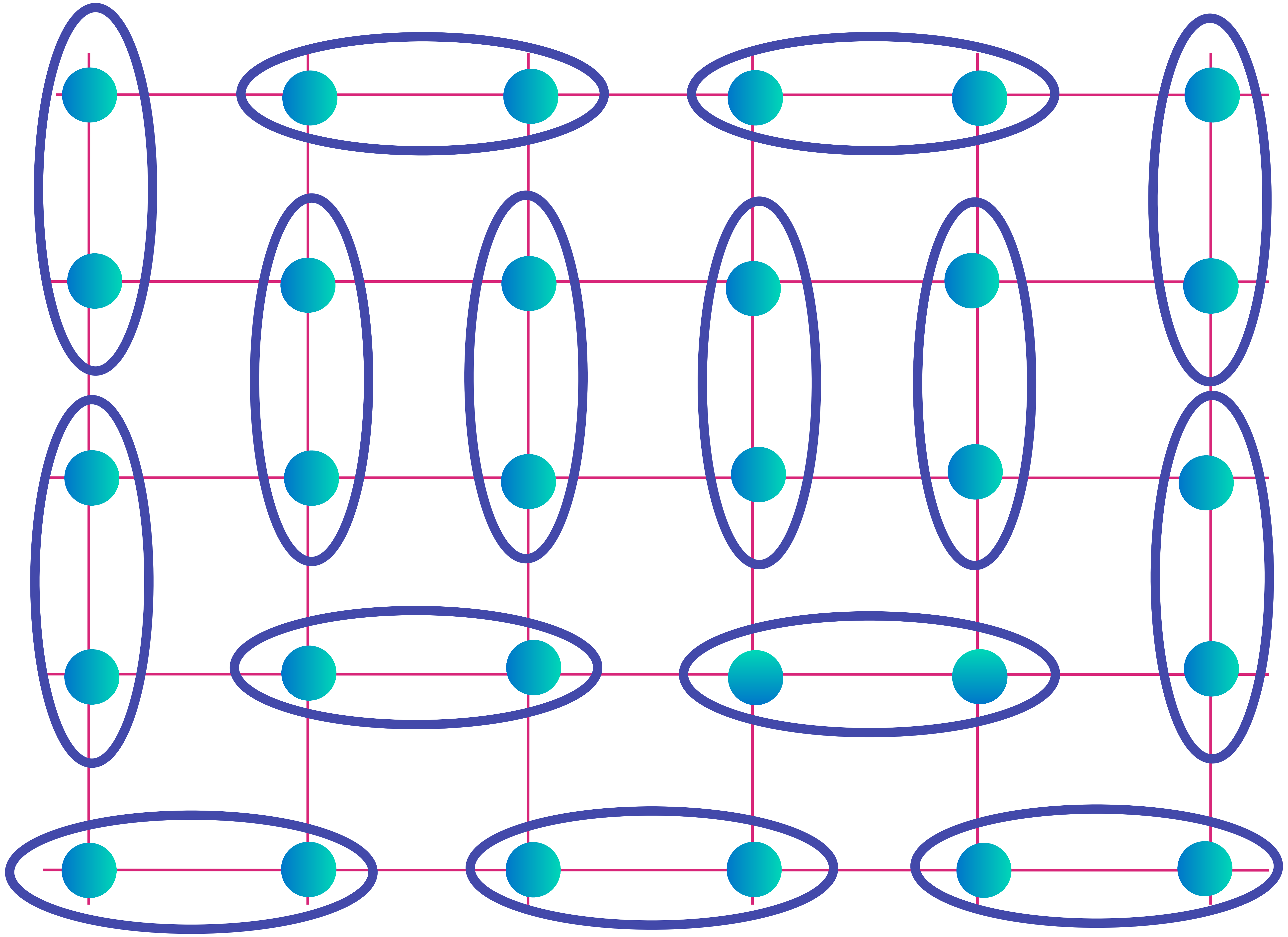


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

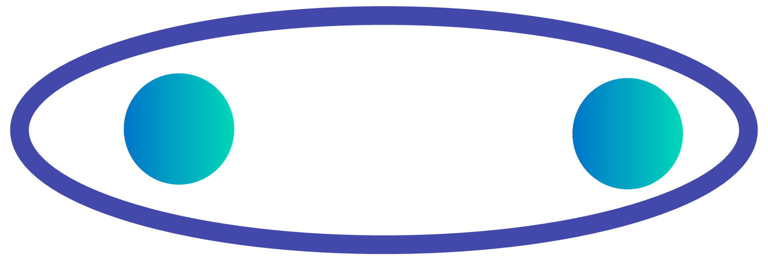
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample

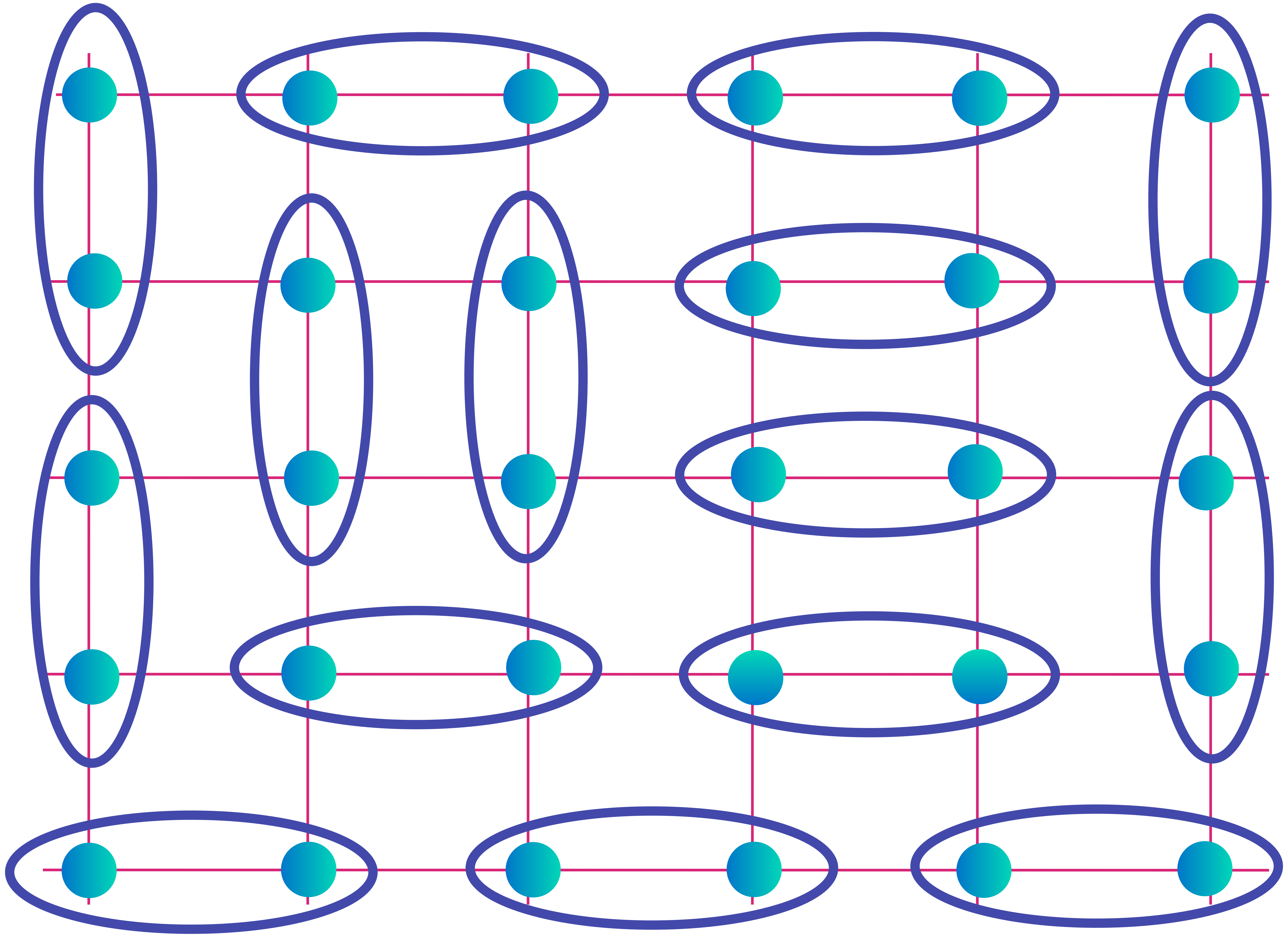


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

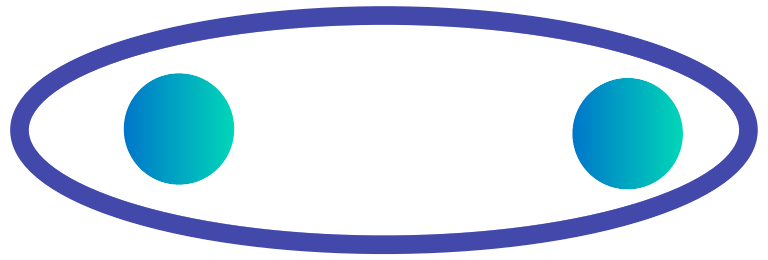
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid

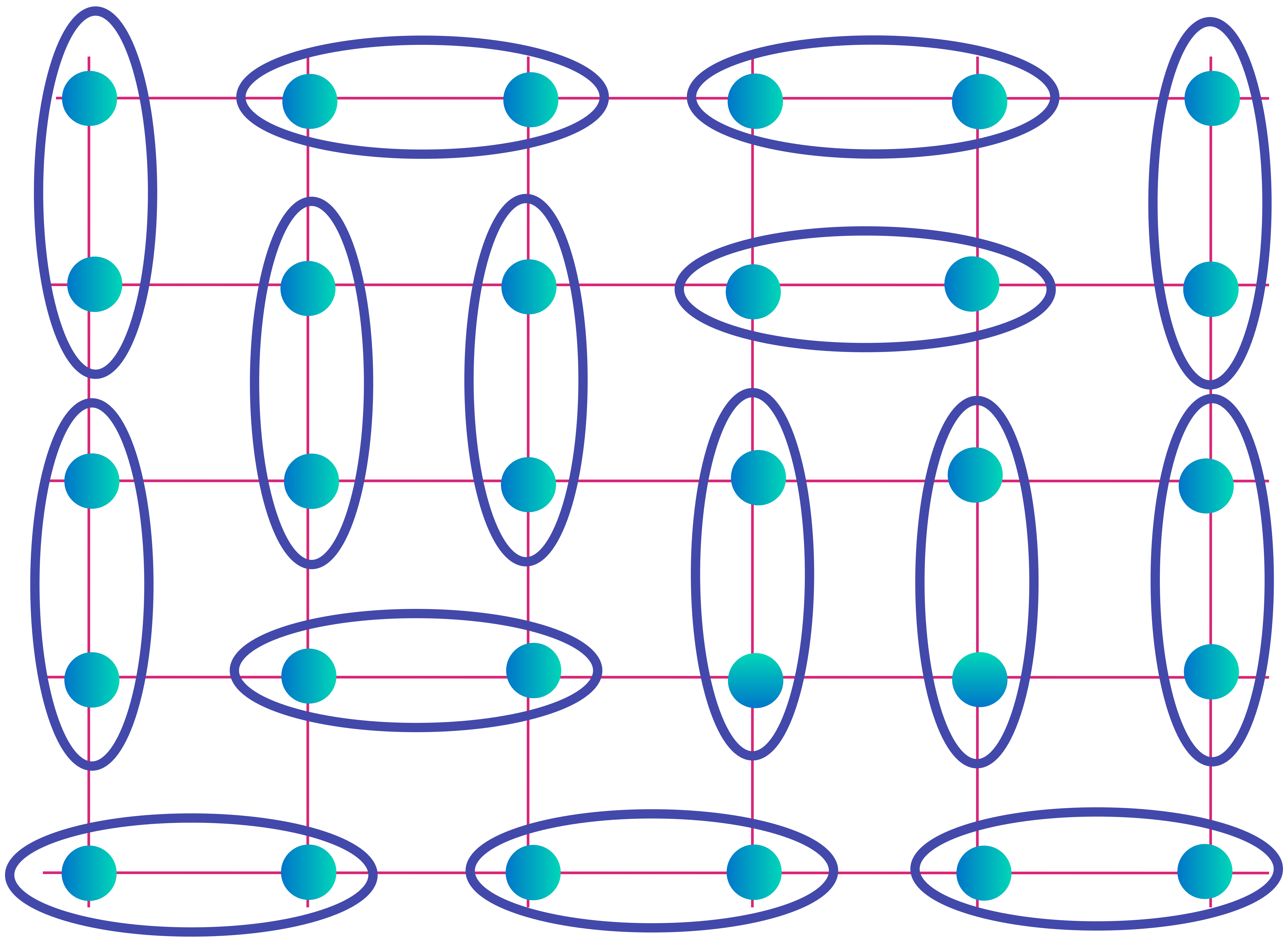


Electrons form entangled pairs, and the pairs entangle across the entire sample

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

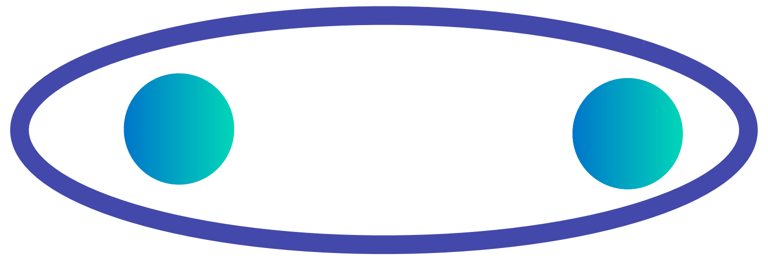
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)



Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample

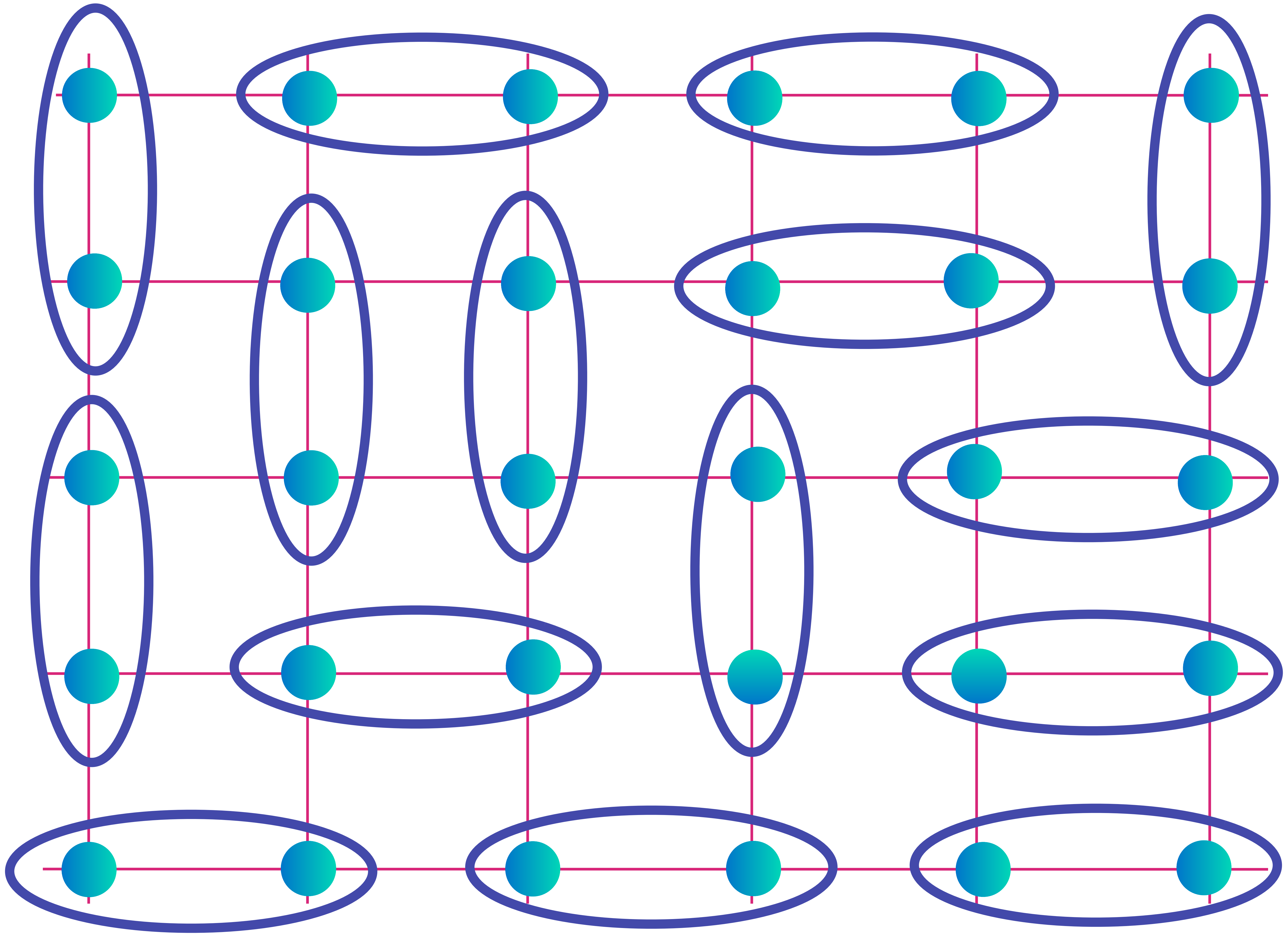


$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

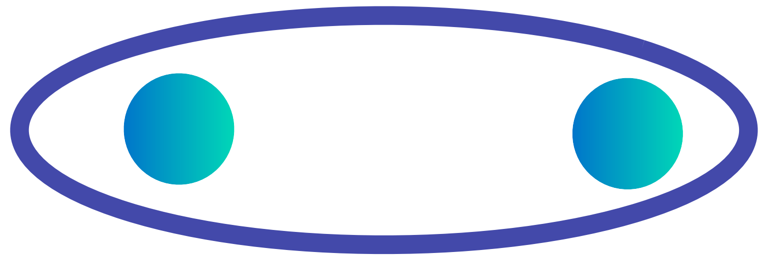
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid

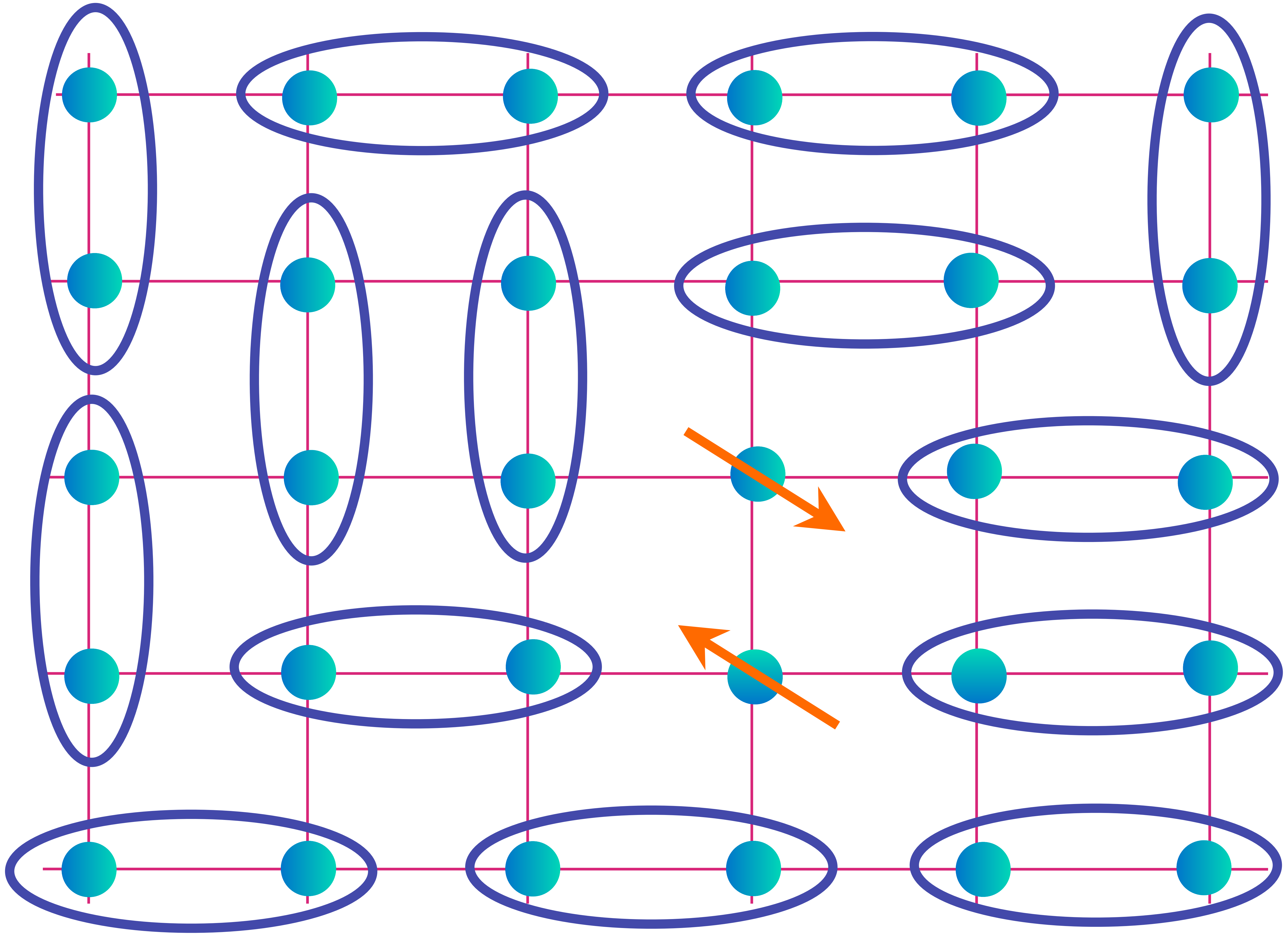


Electrons form entangled pairs, and the pairs entangle across the entire sample



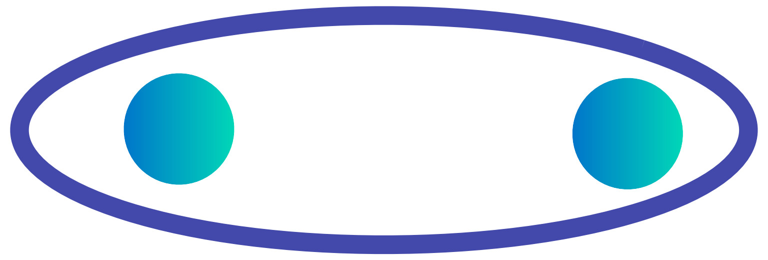
$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

The dance of electrons on Cu atoms in YBCO

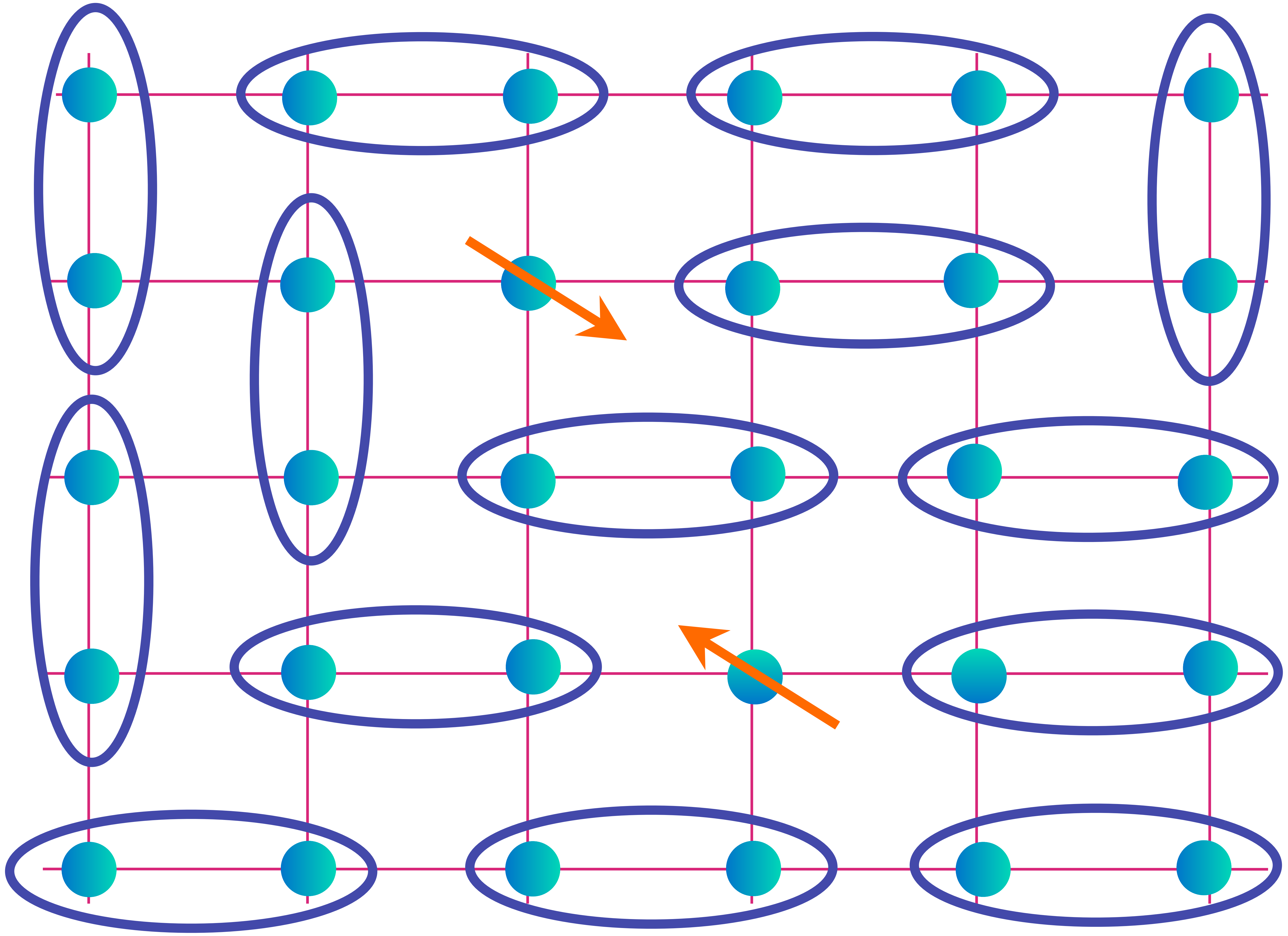


Spin liquid

Fractionalized
spinon
excitations
with spin S=1/2
and charge 0.

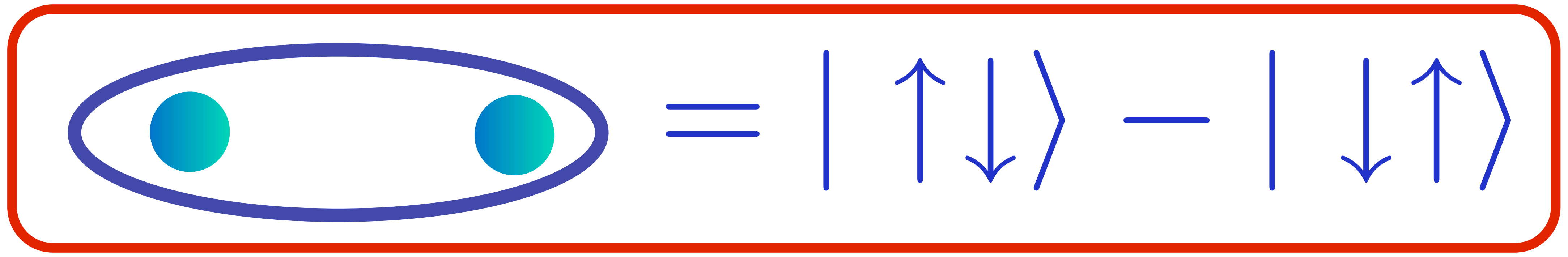
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

The dance of electrons on Cu atoms in YBCO

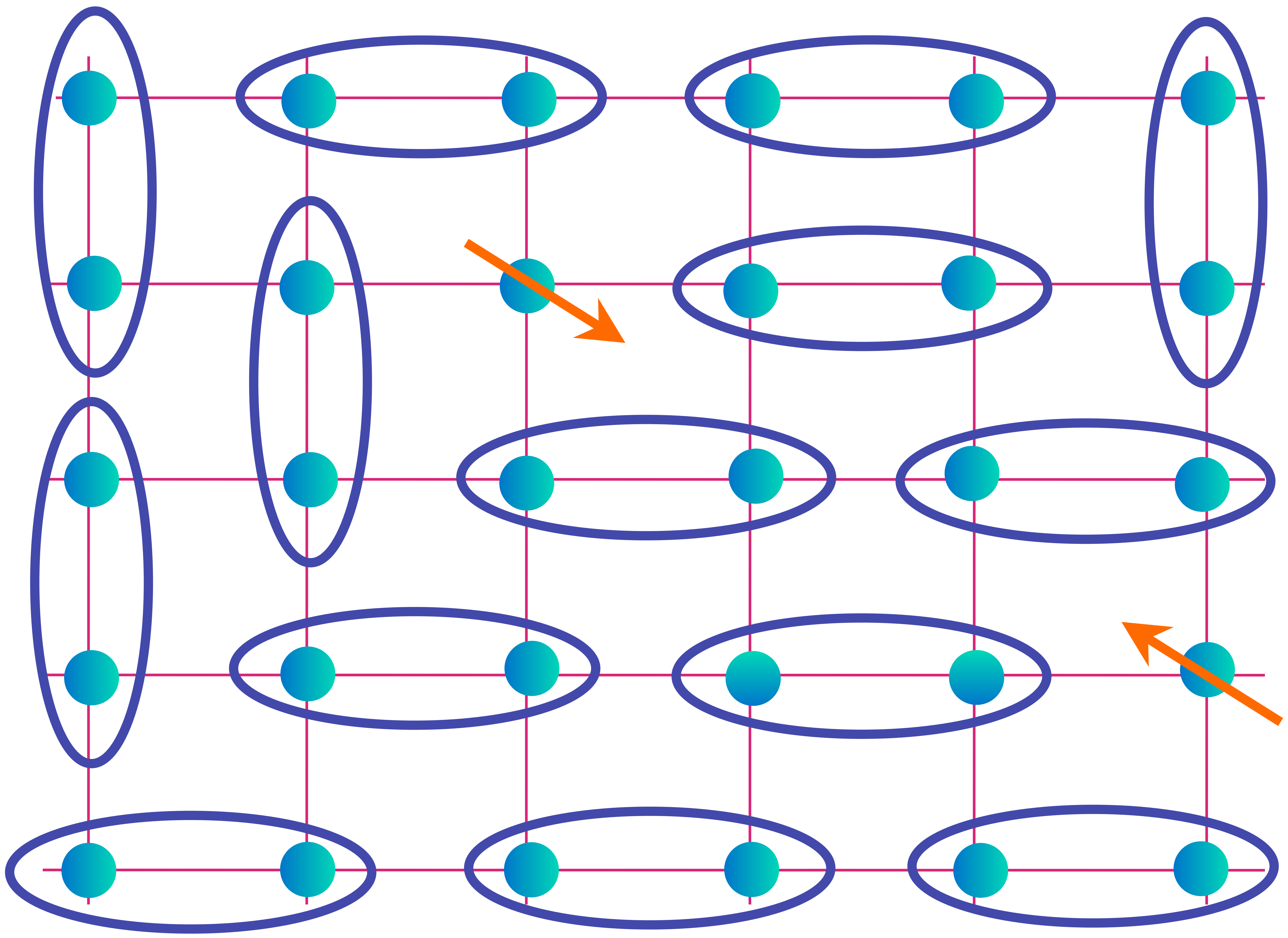


Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

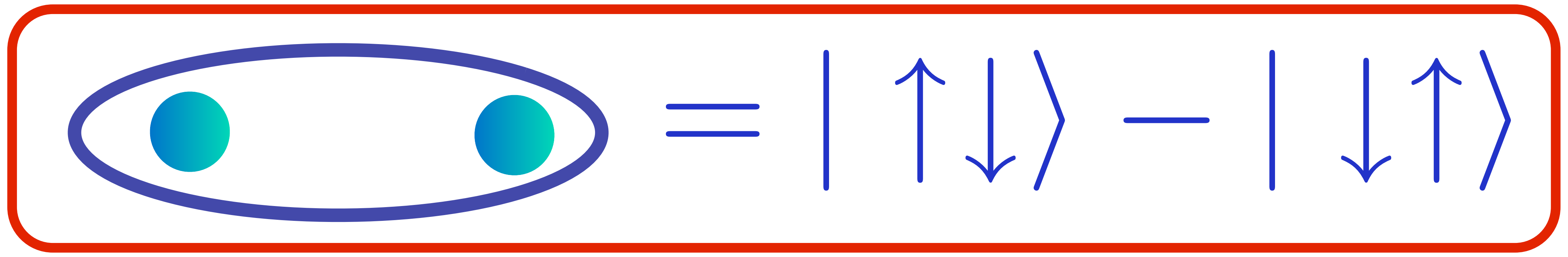


The dance of electrons on Cu atoms in YBCO

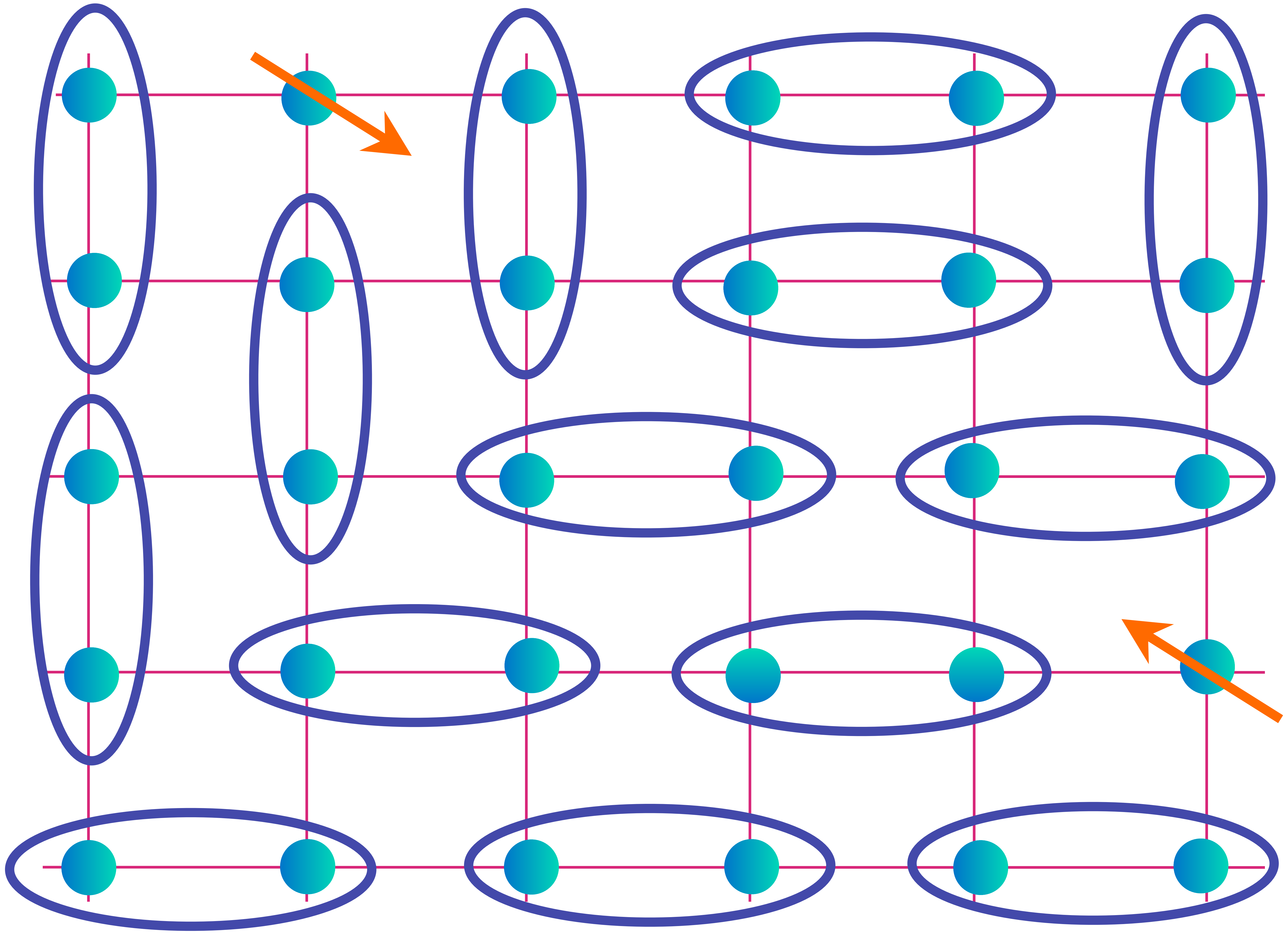


Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

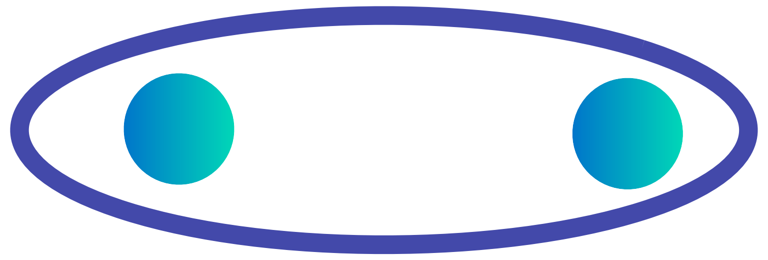


The dance of electrons on Cu atoms in YBCO



Spin liquid

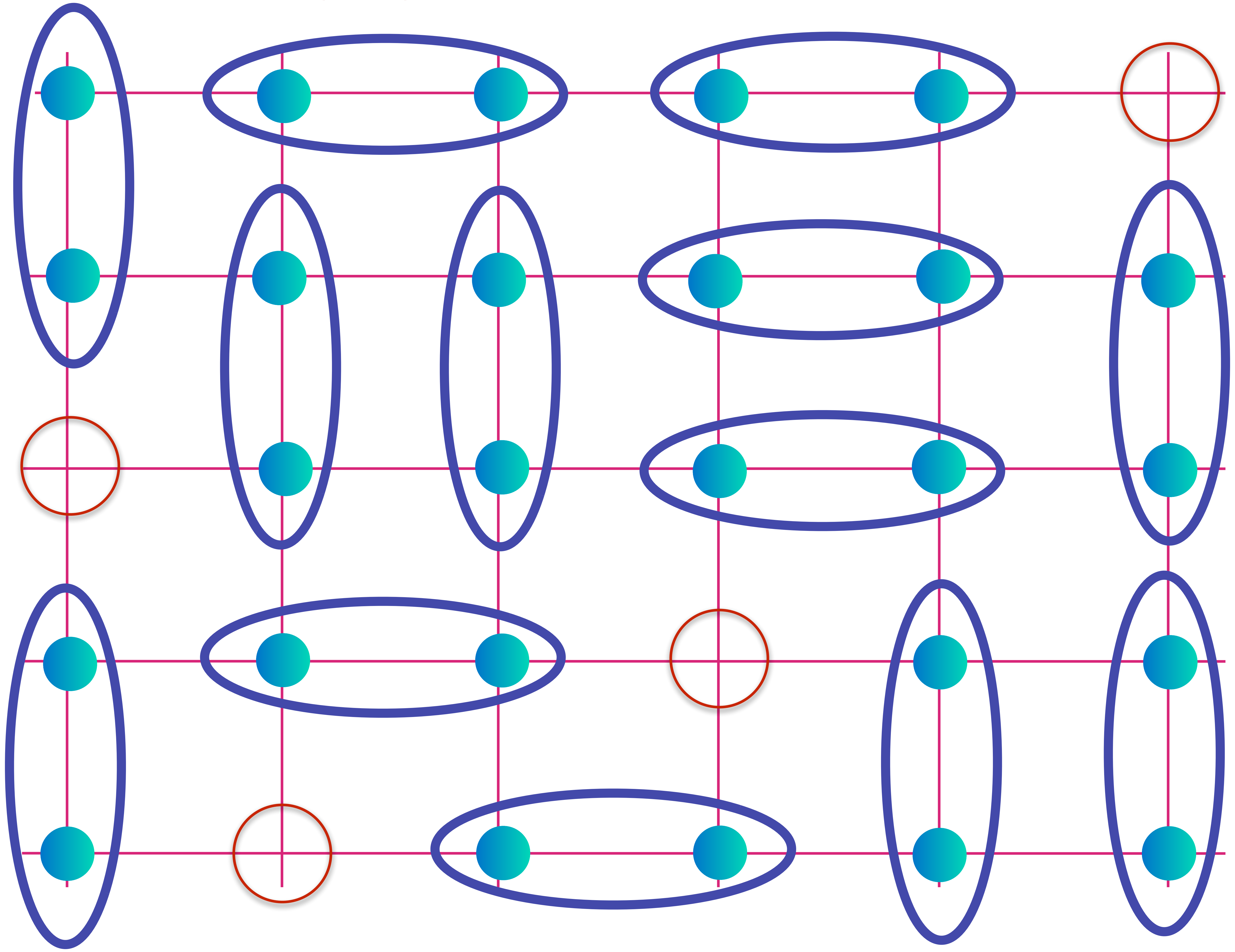
Fractionalized spinon excitations with spin S=1/2 and charge 0.

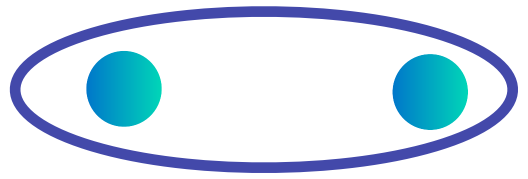


$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Holons

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



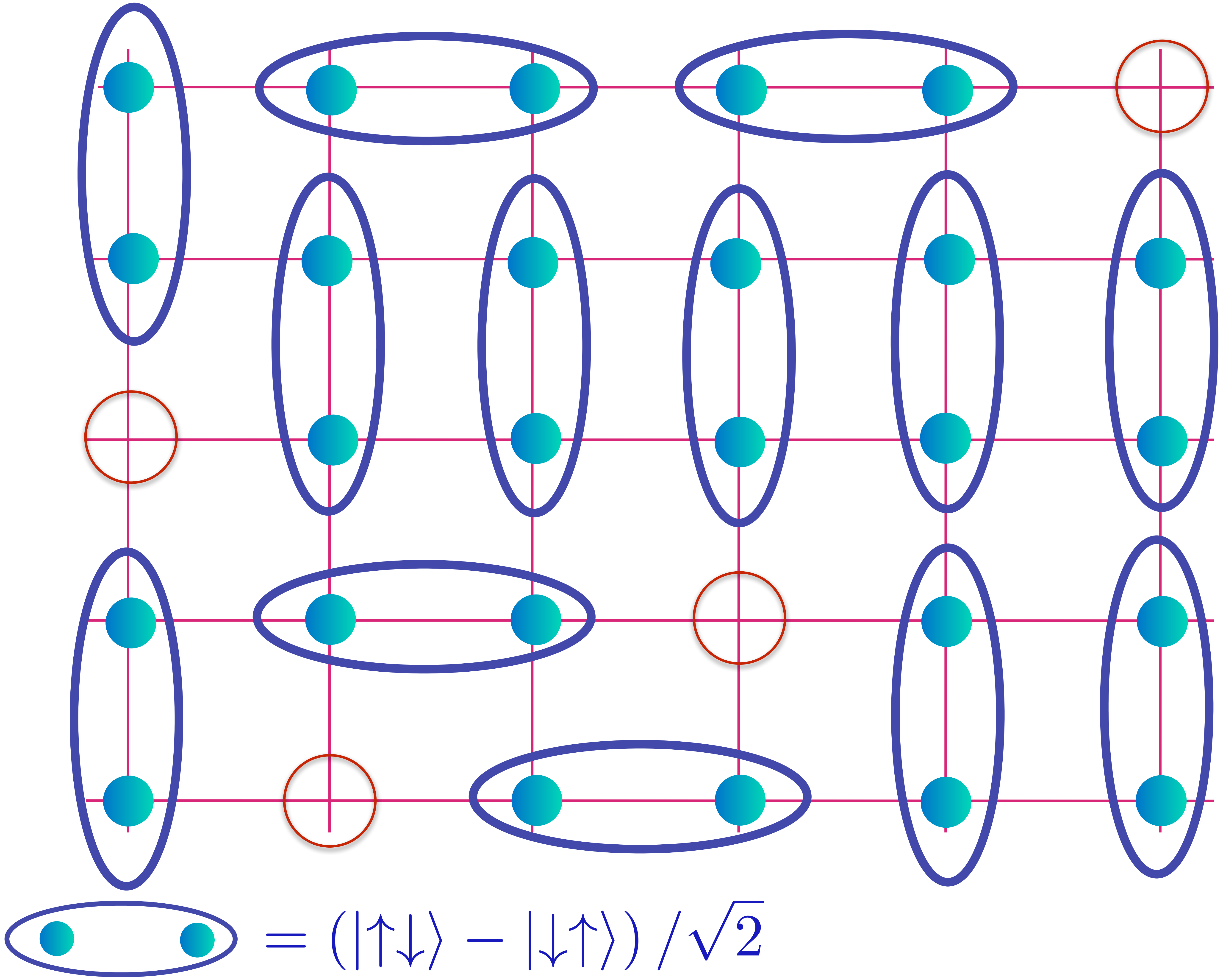
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.

Holons

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



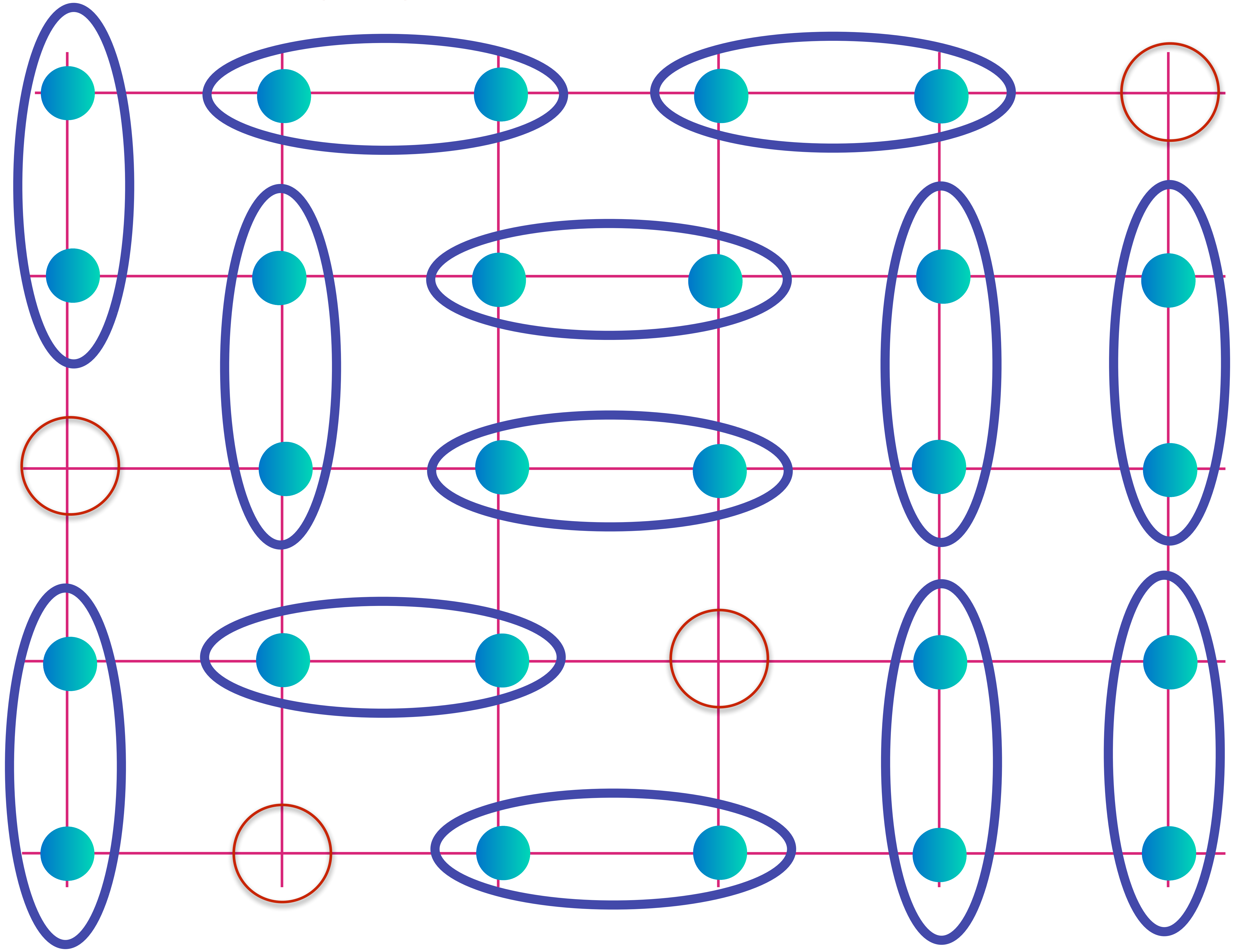
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

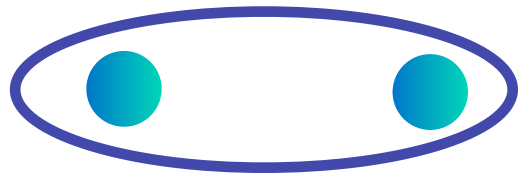
Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.

Holons

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



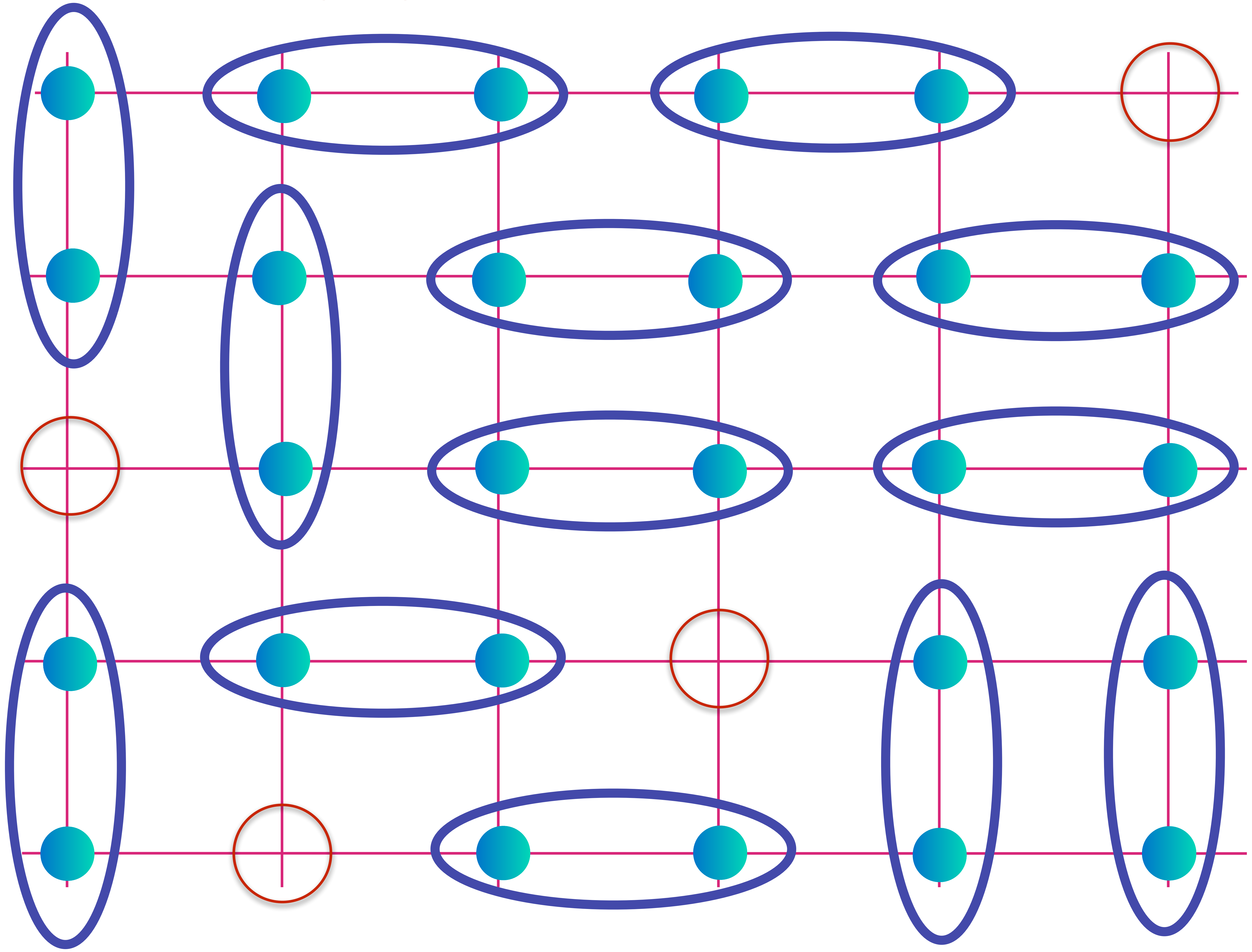
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.

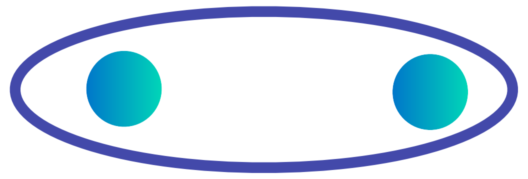
Holons

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



Spin liquid

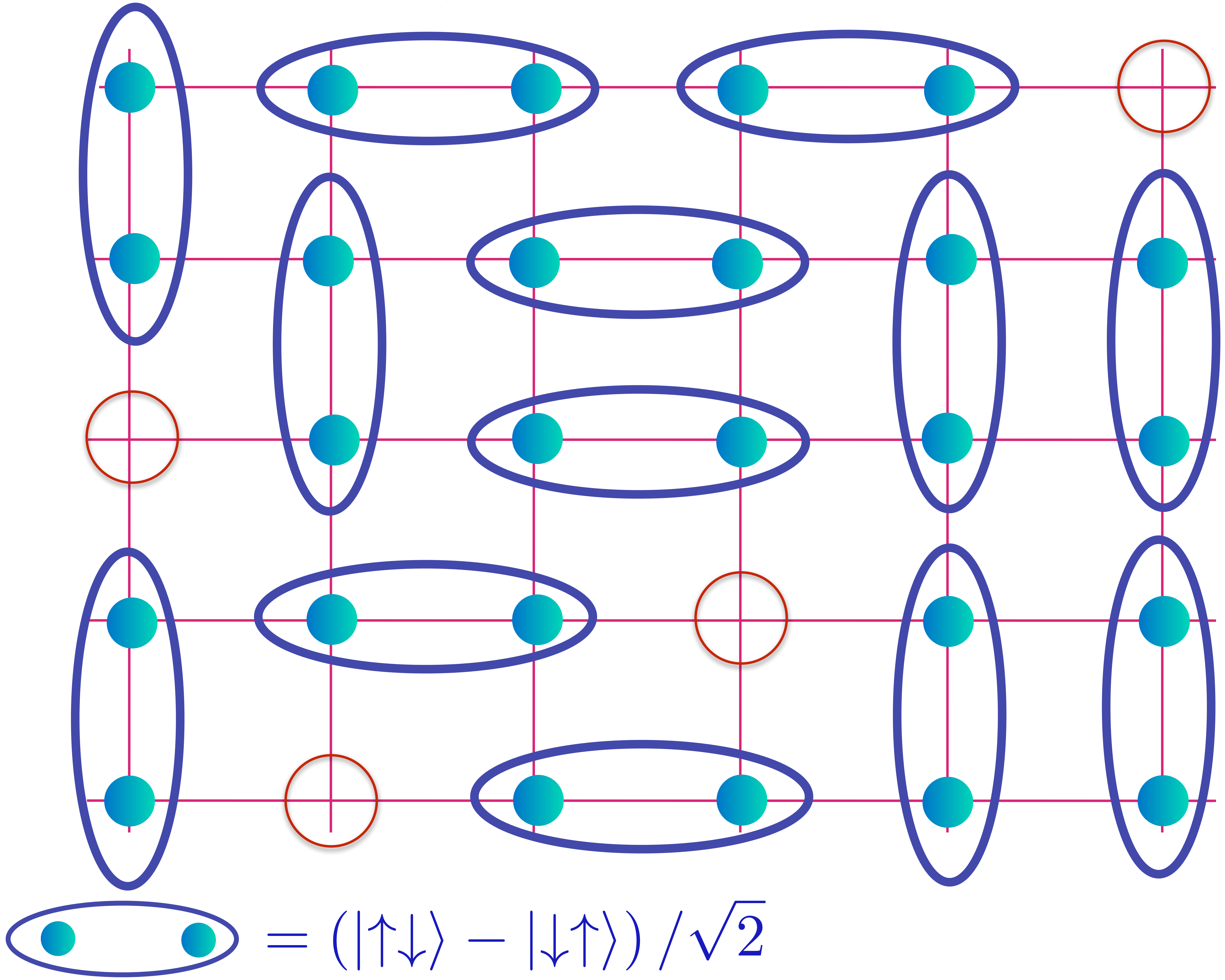
Fractionalized holon excitations with spin S=0 and charge +e.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$$

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

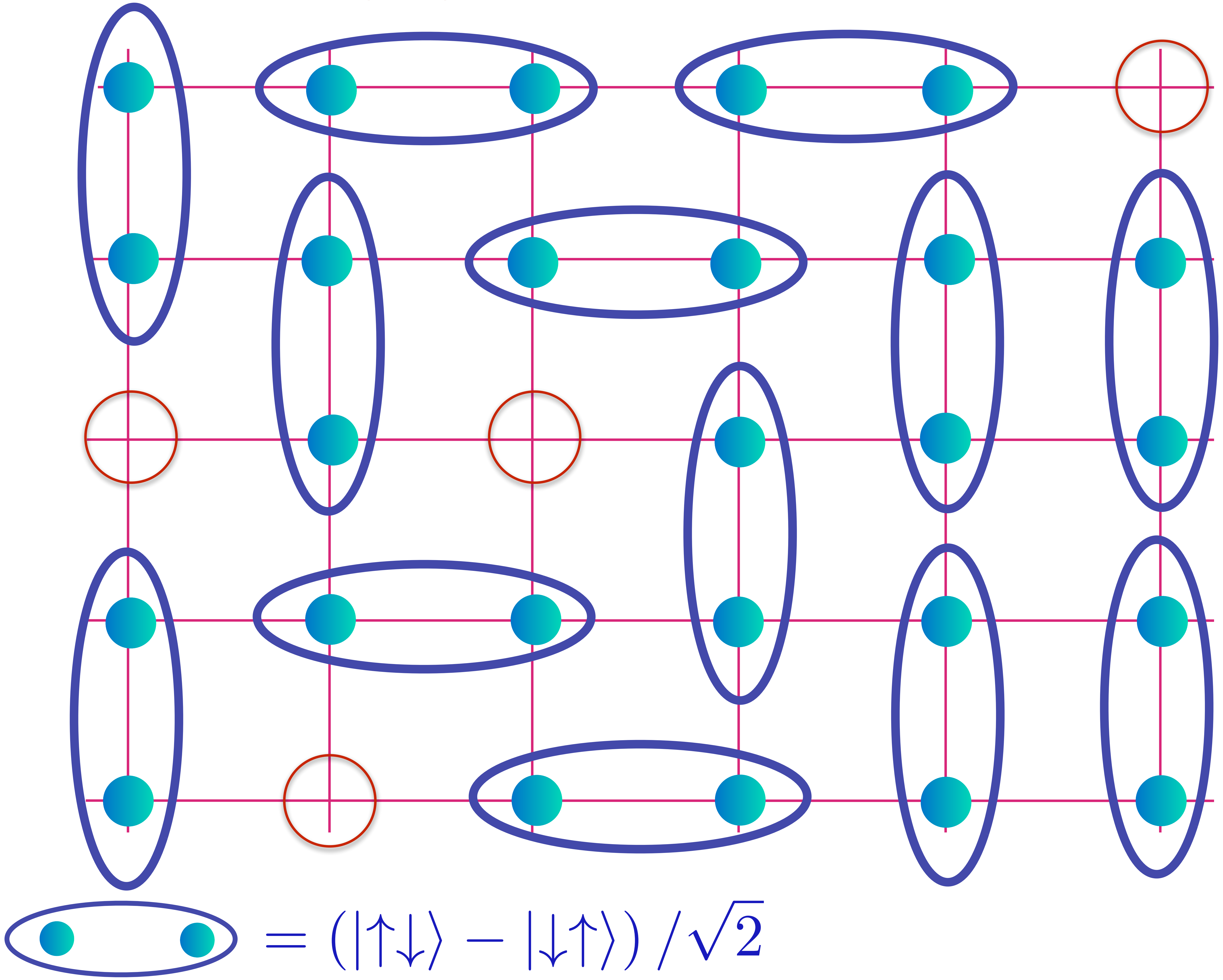


$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

If each holon is a fermion, we obtain a Fermi surface of holons of size p

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

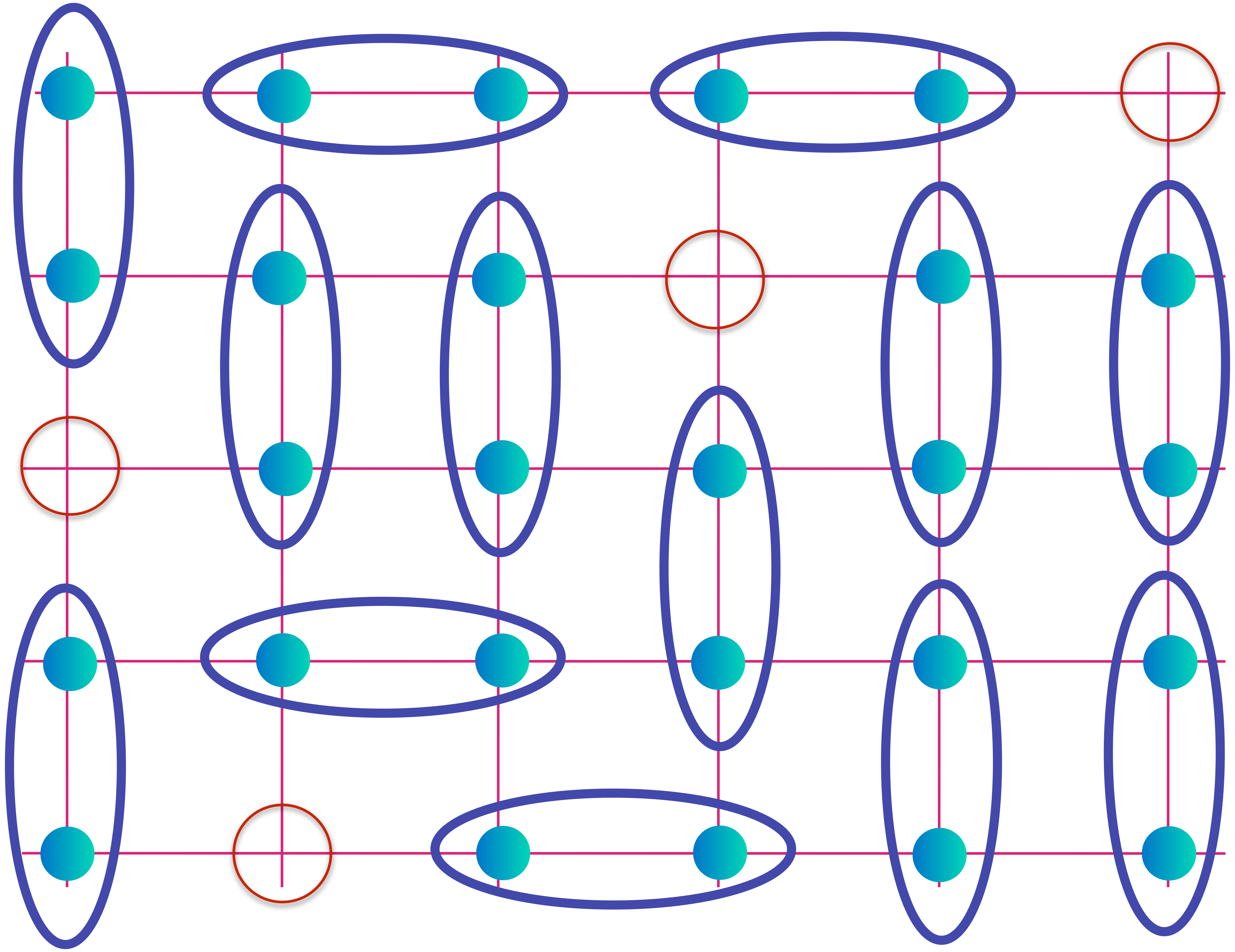


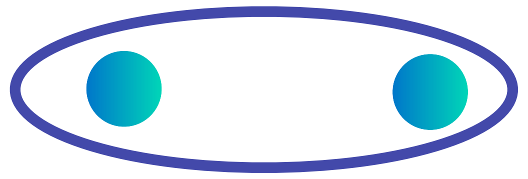
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

If each holon is
a fermion,
we obtain
a Fermi surface
of holons
of size p

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



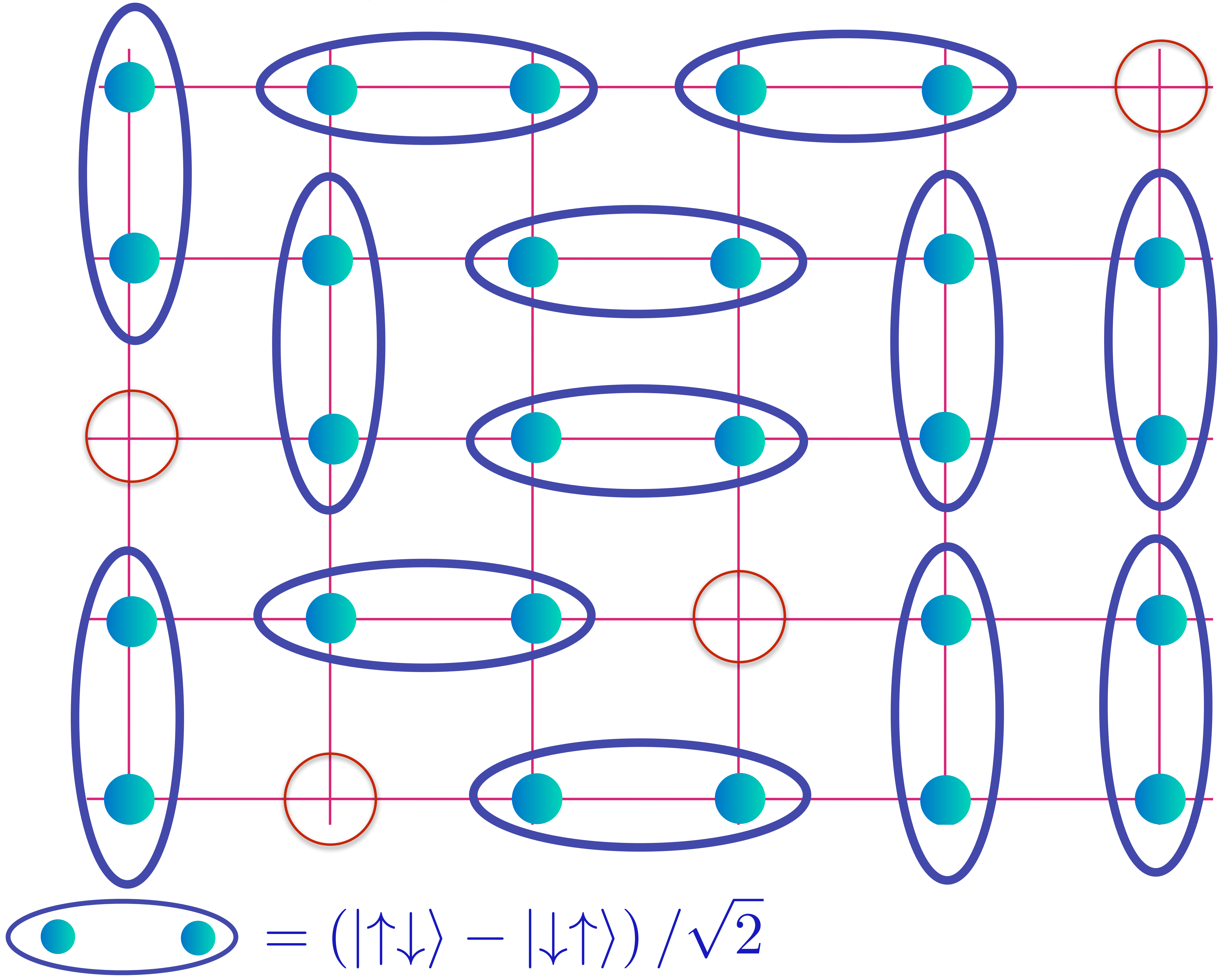
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

If each holon is a fermion, we obtain a Fermi surface of holons of size p

Cannot explain pseudogap metal

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

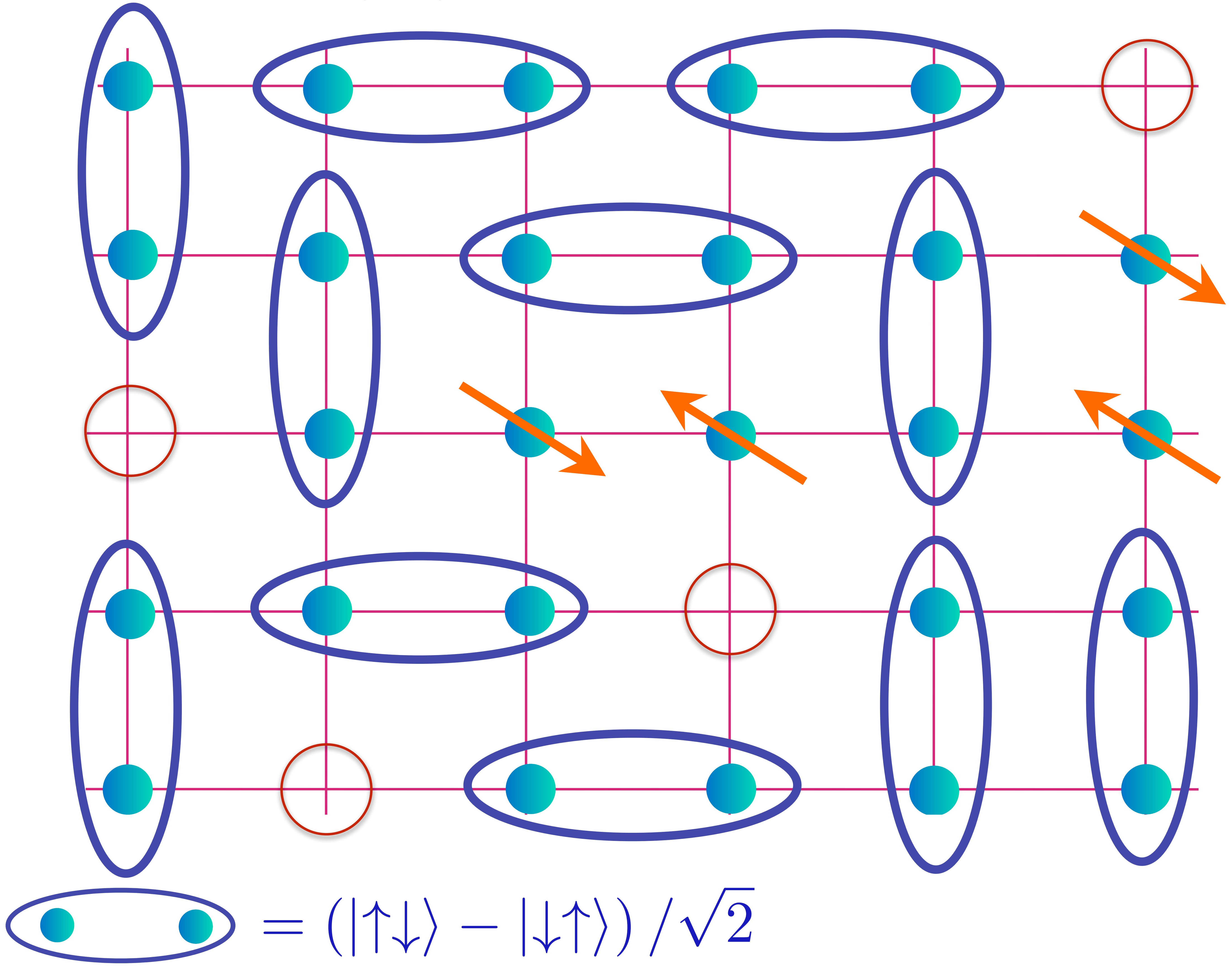


If each holon is a fermion, we obtain a Fermi surface of holons of size p

Cannot explain pseudogap metal

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

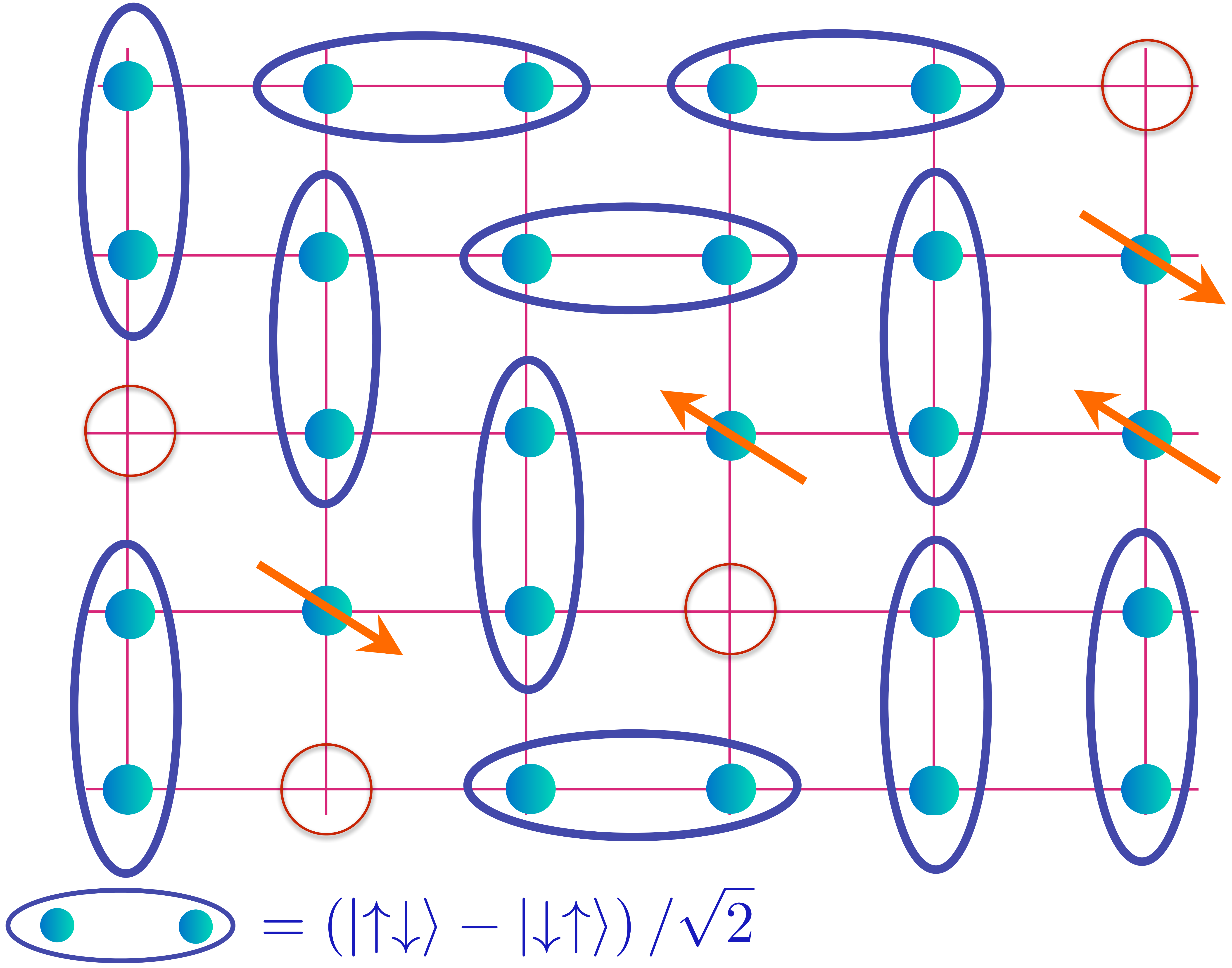


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid
with density p of
spinless, charge +e
"holons"
and
charge 0, spin-1/2
"spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

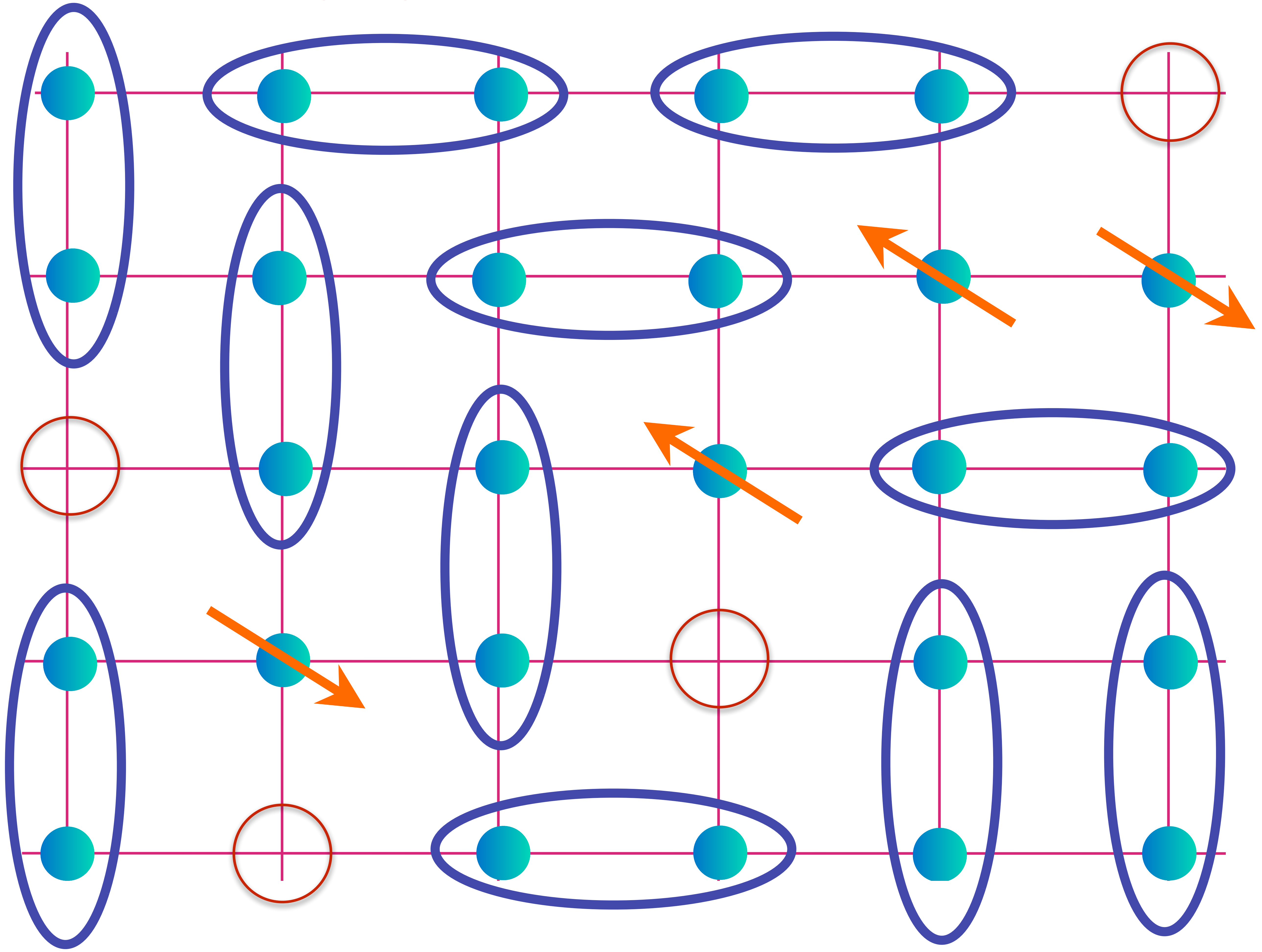


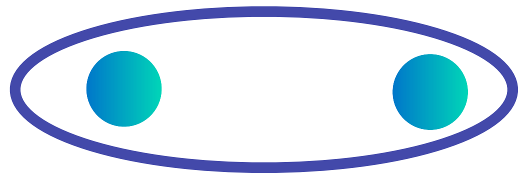
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid
with density p of
spinless, charge +e
"holons"
and
charge 0, spin-1/2
"spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



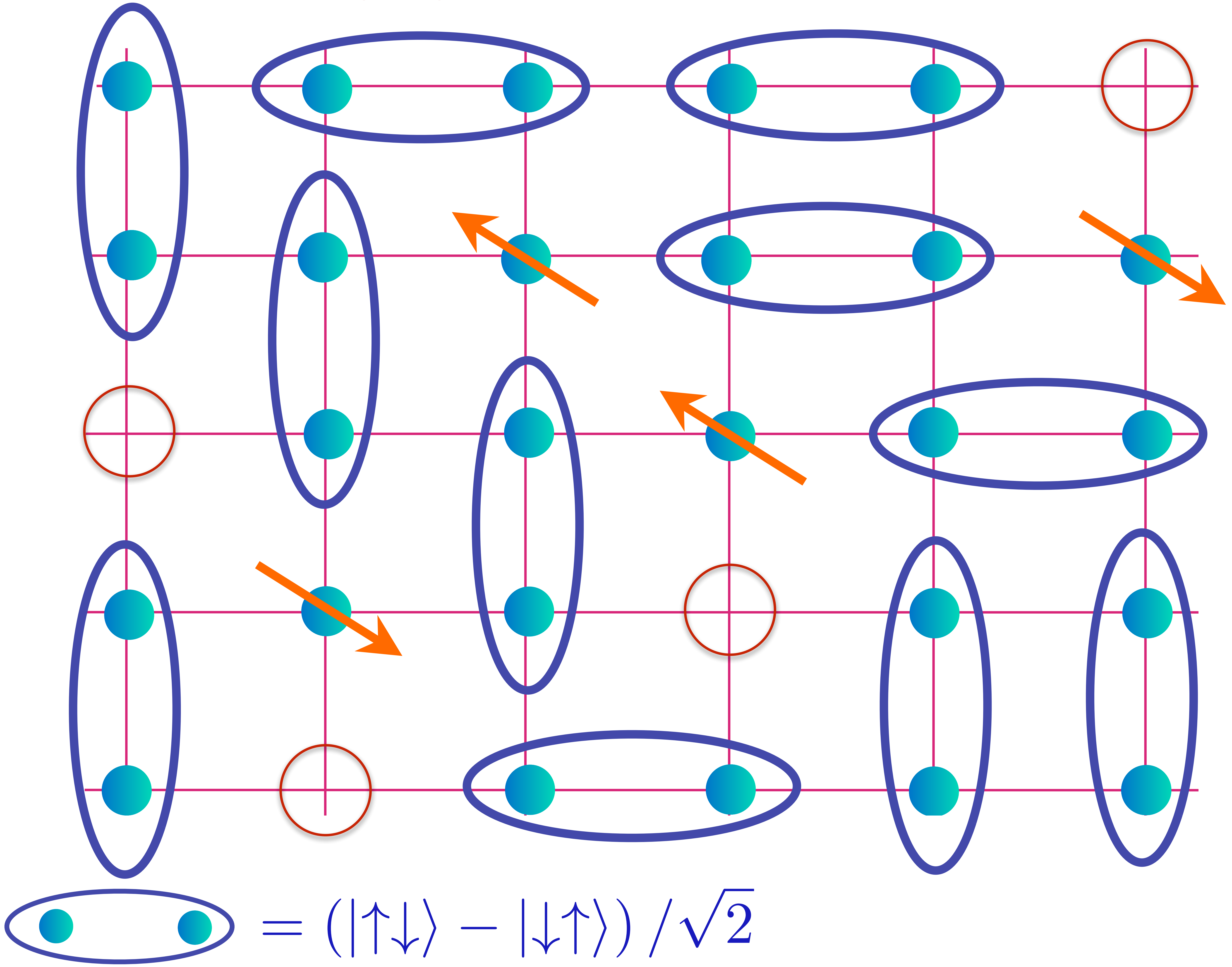


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

Spin liquid
with density p of
spinless, charge +e
"holons"
and
charge 0, spin-1/2
"spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

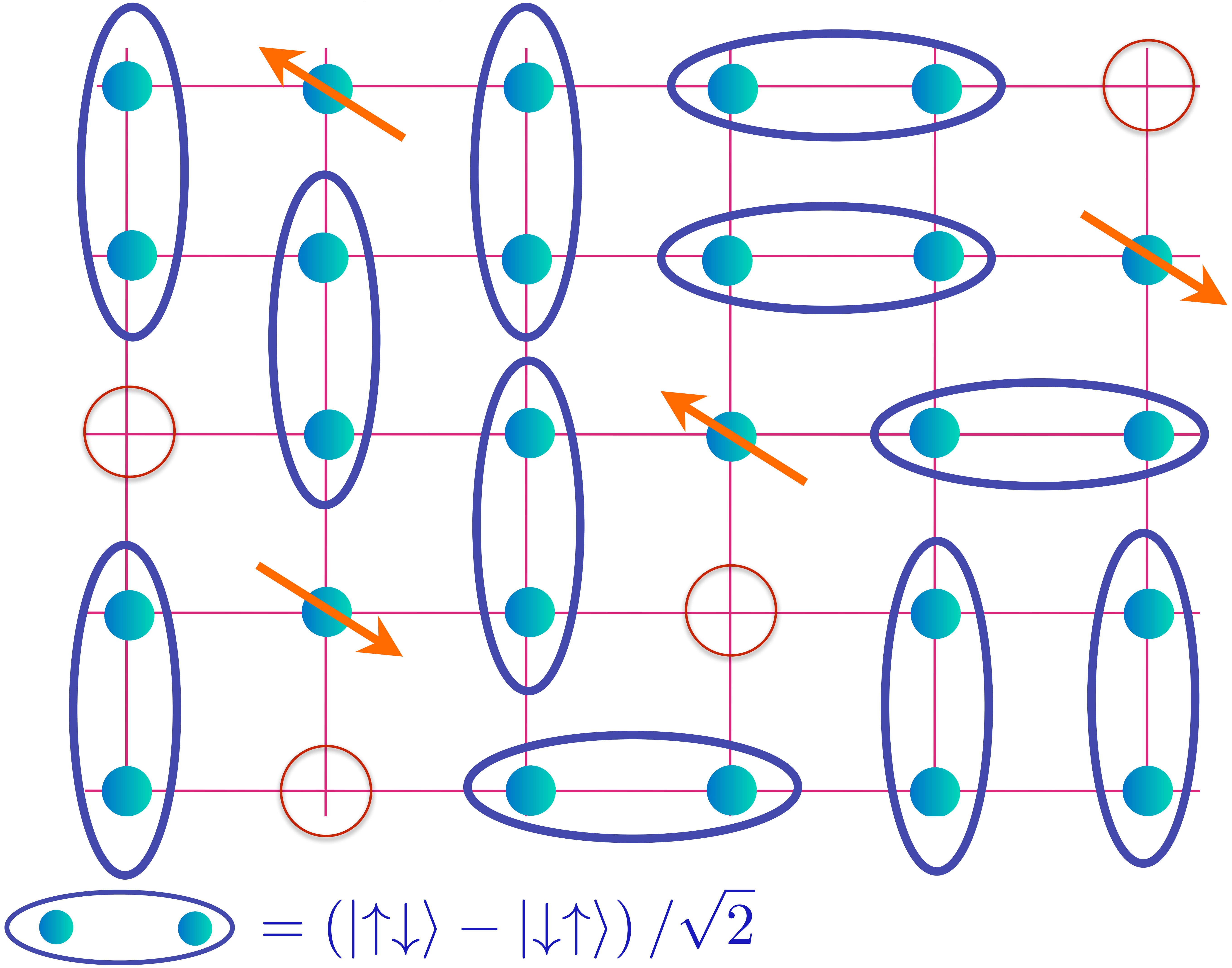


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

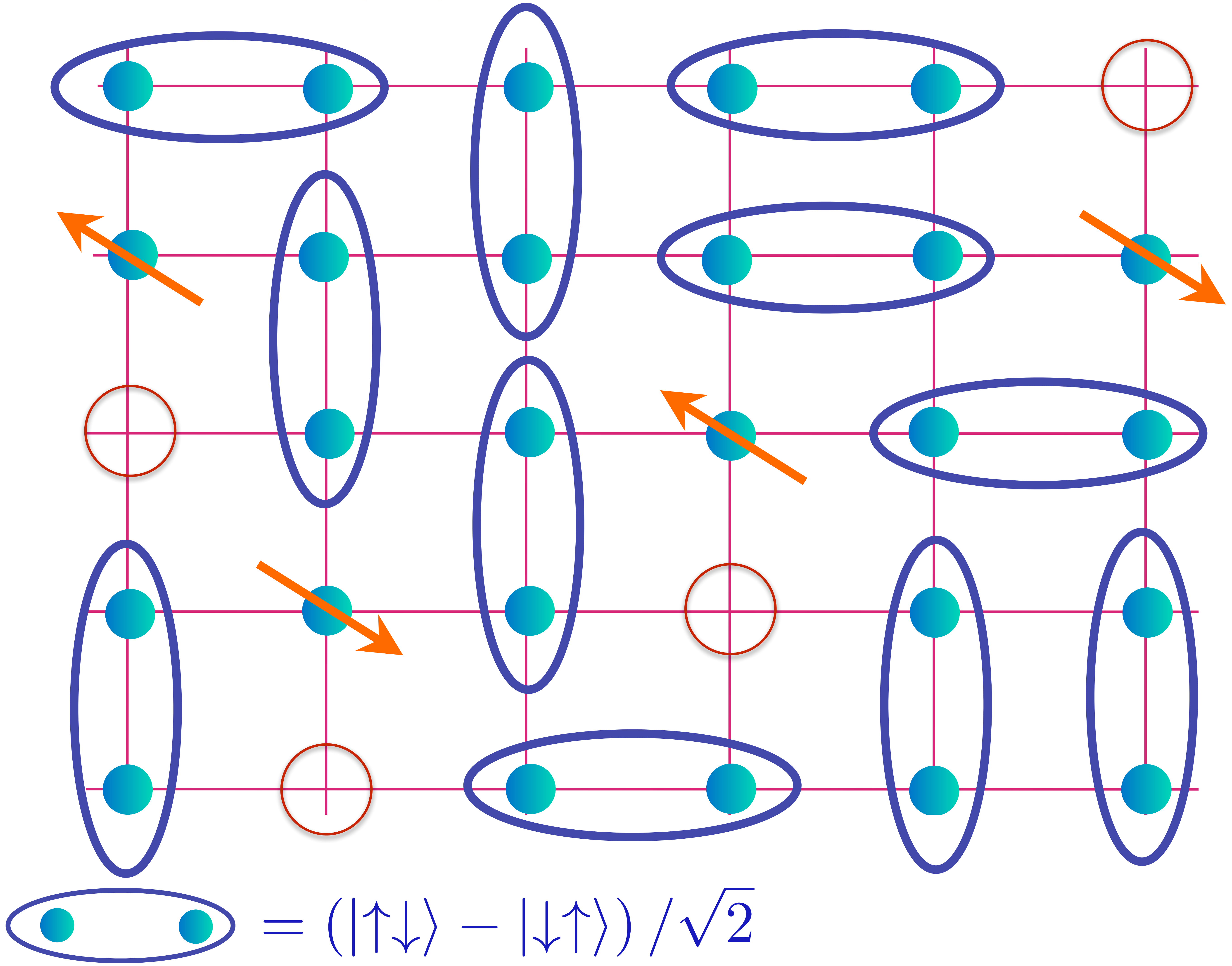


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid
with density p of
spinless, charge +e
"holons"
and
charge 0, spin-1/2
"spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

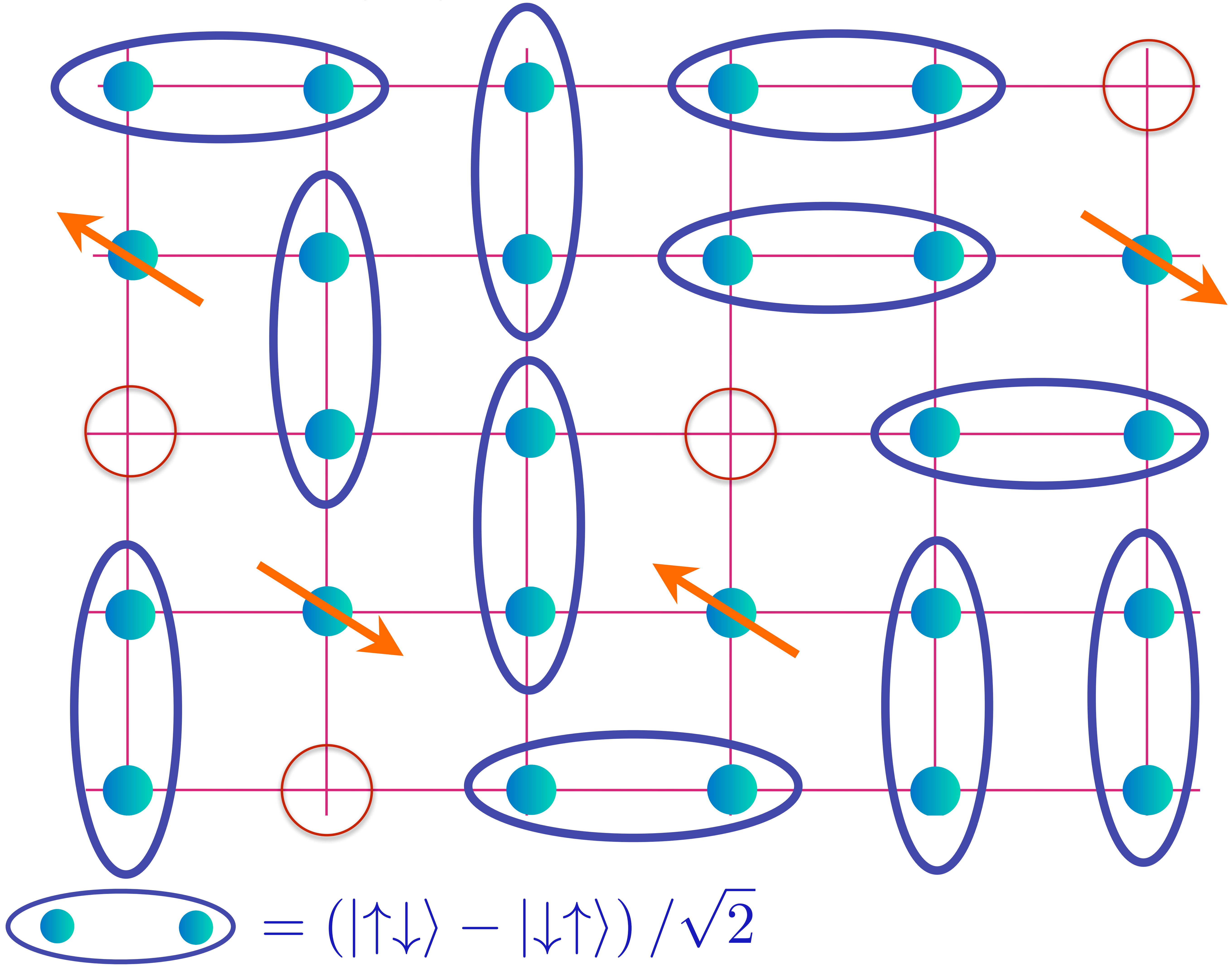


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid
with density p of
spinless, charge +e
"holons"
and
charge 0, spin-1/2
"spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

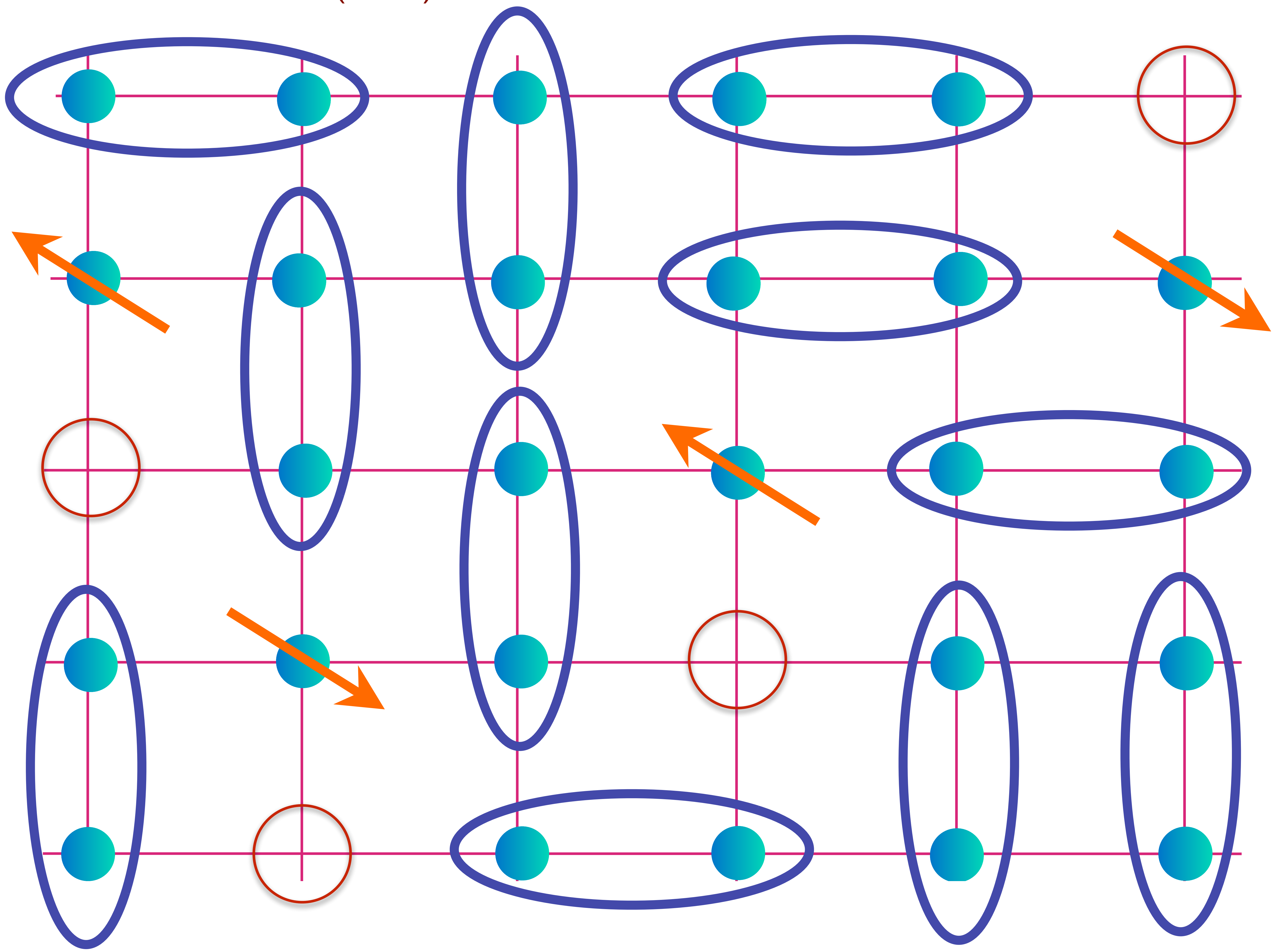


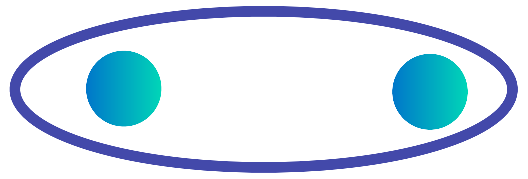
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid
with density p of
spinless, charge +e
"holons"
and
charge 0, spin-1/2
"spinons".

Holon metal

G. Baskaran, Z. Zou, P.W. Anderson, Solid State Comm. **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



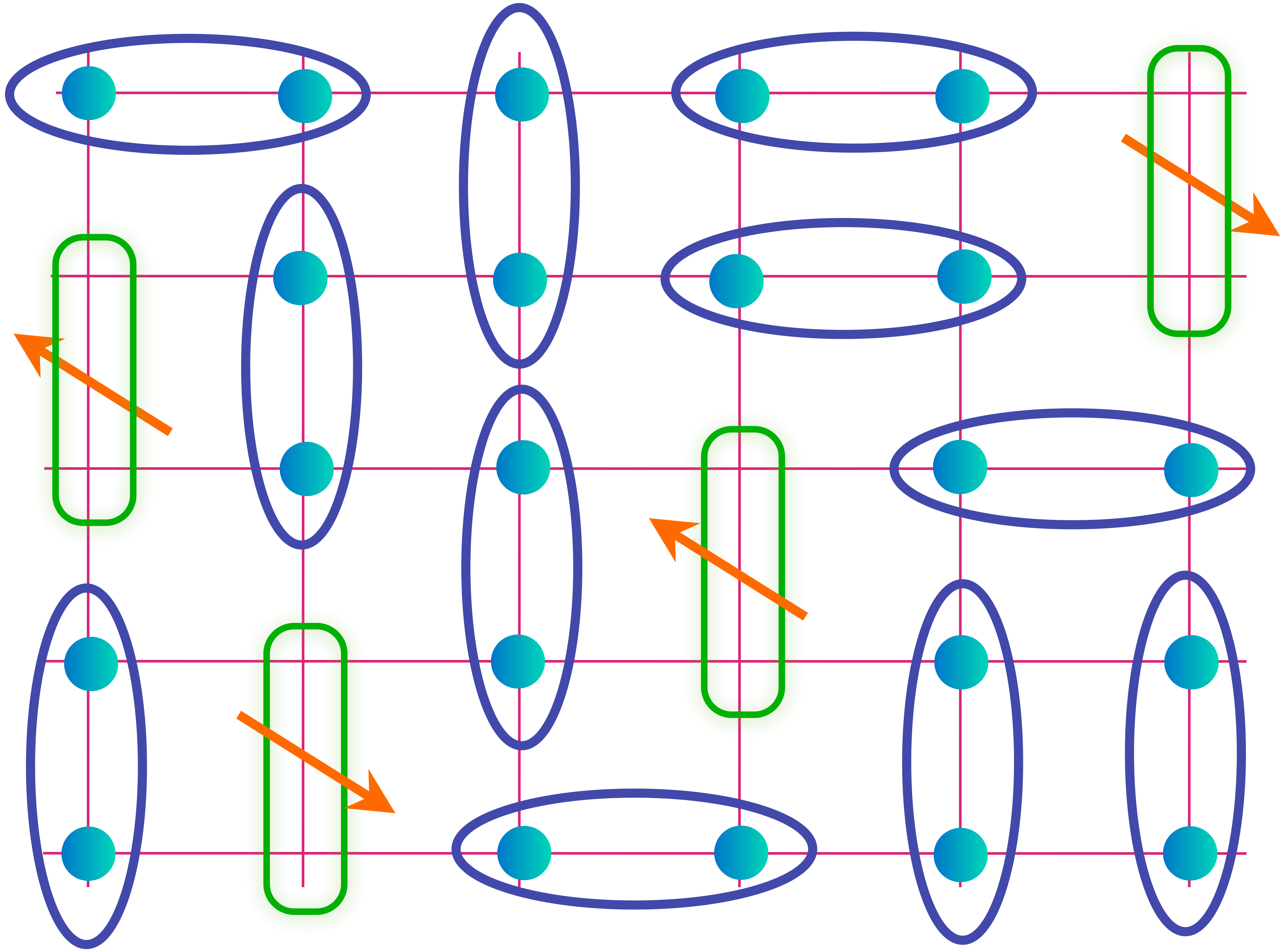


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Spin liquid
with density p of
spinless, charge +e
“holons”
and
charge 0, spin-1/2
“spinons”.

FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

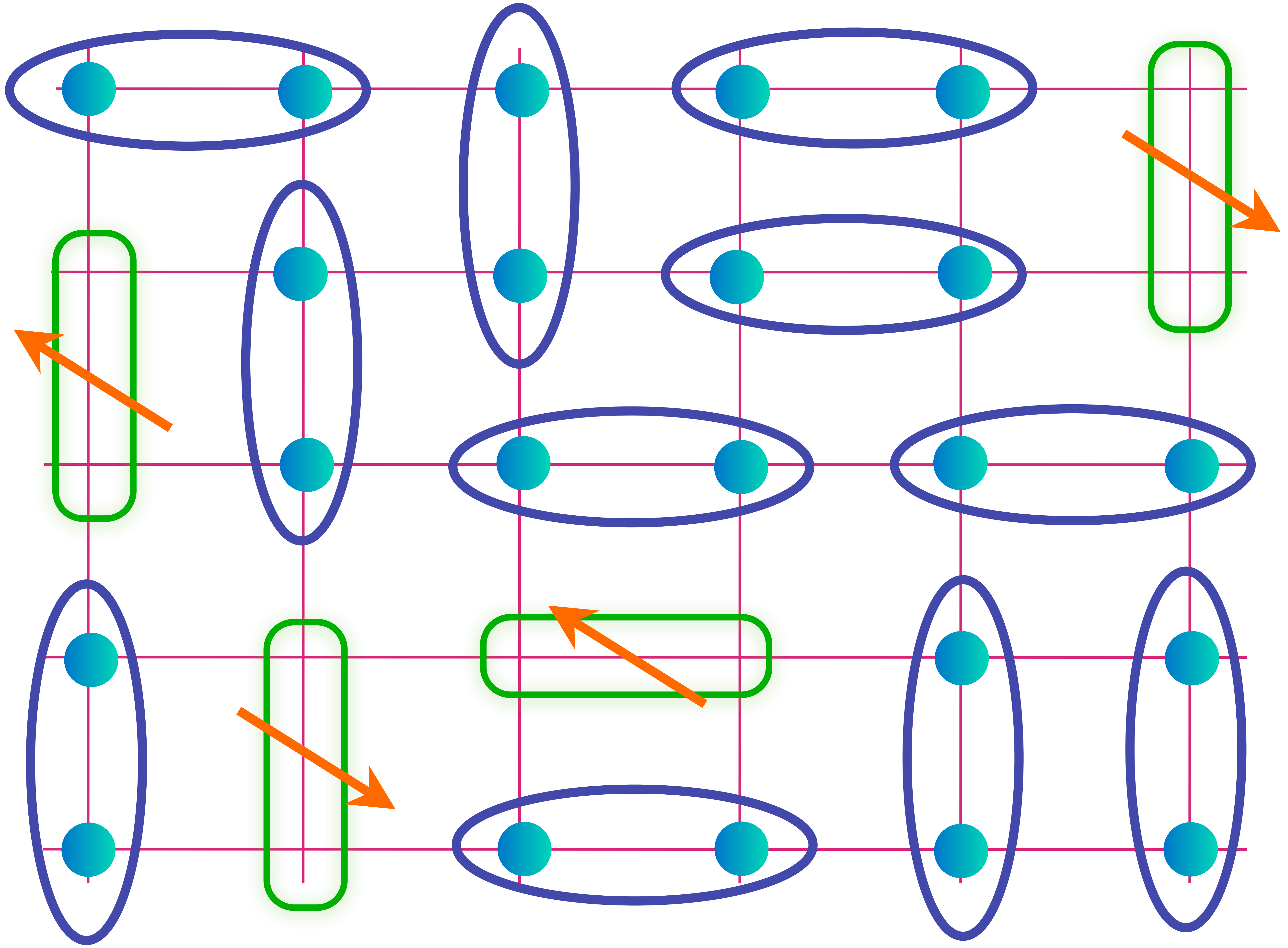
$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

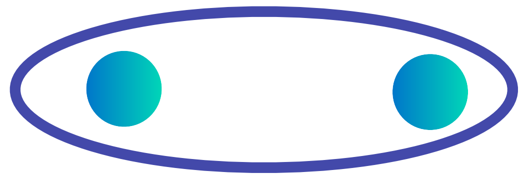
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

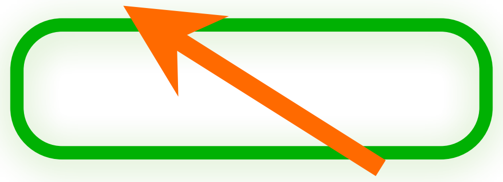
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

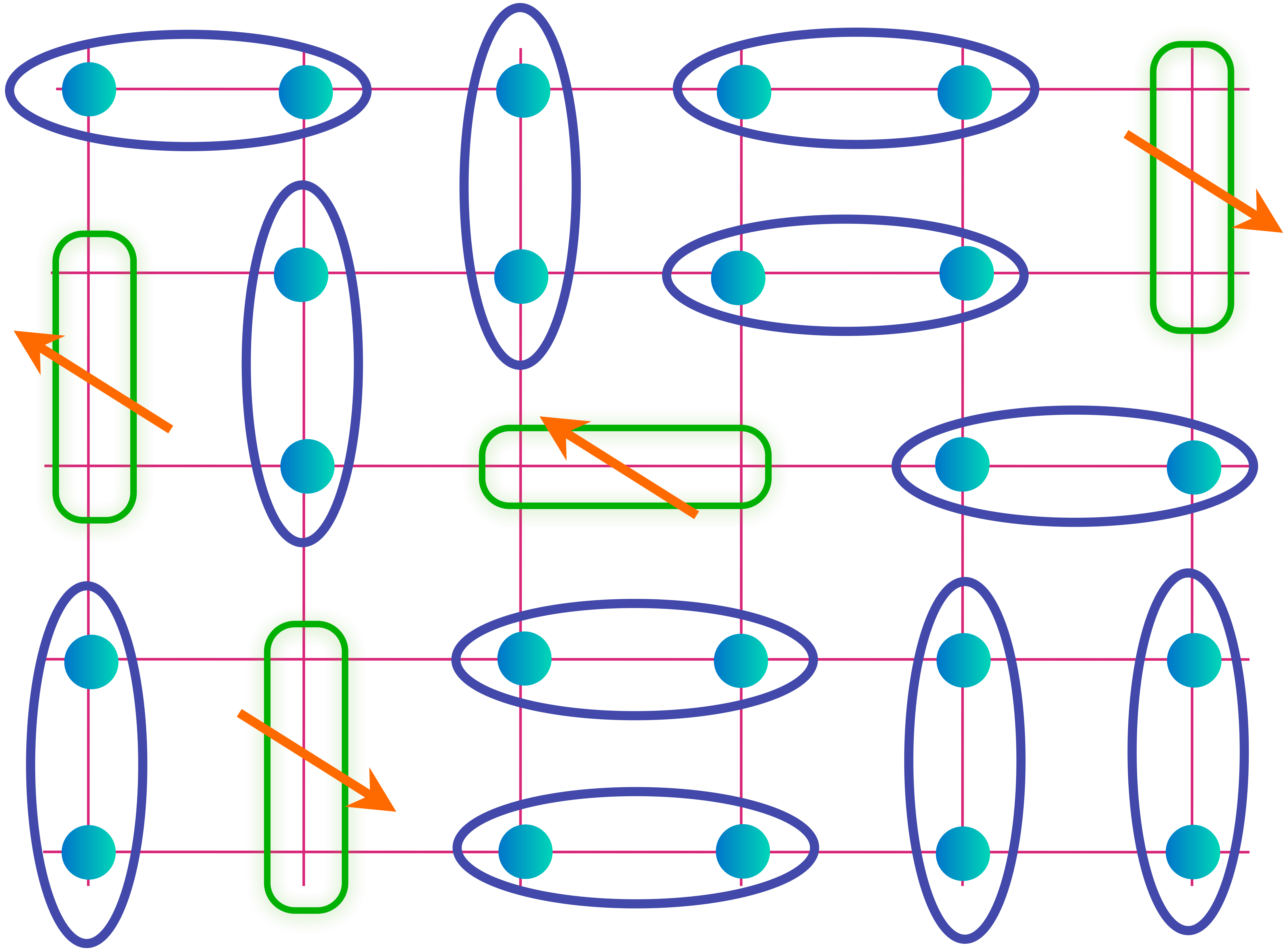


$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

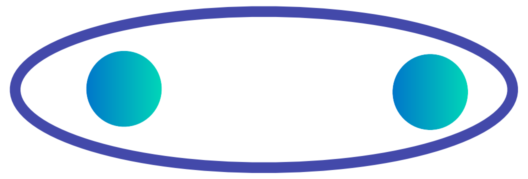
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

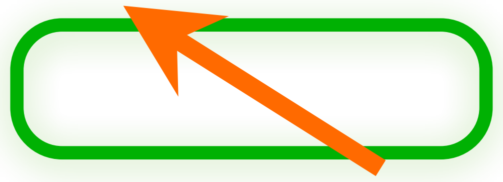
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

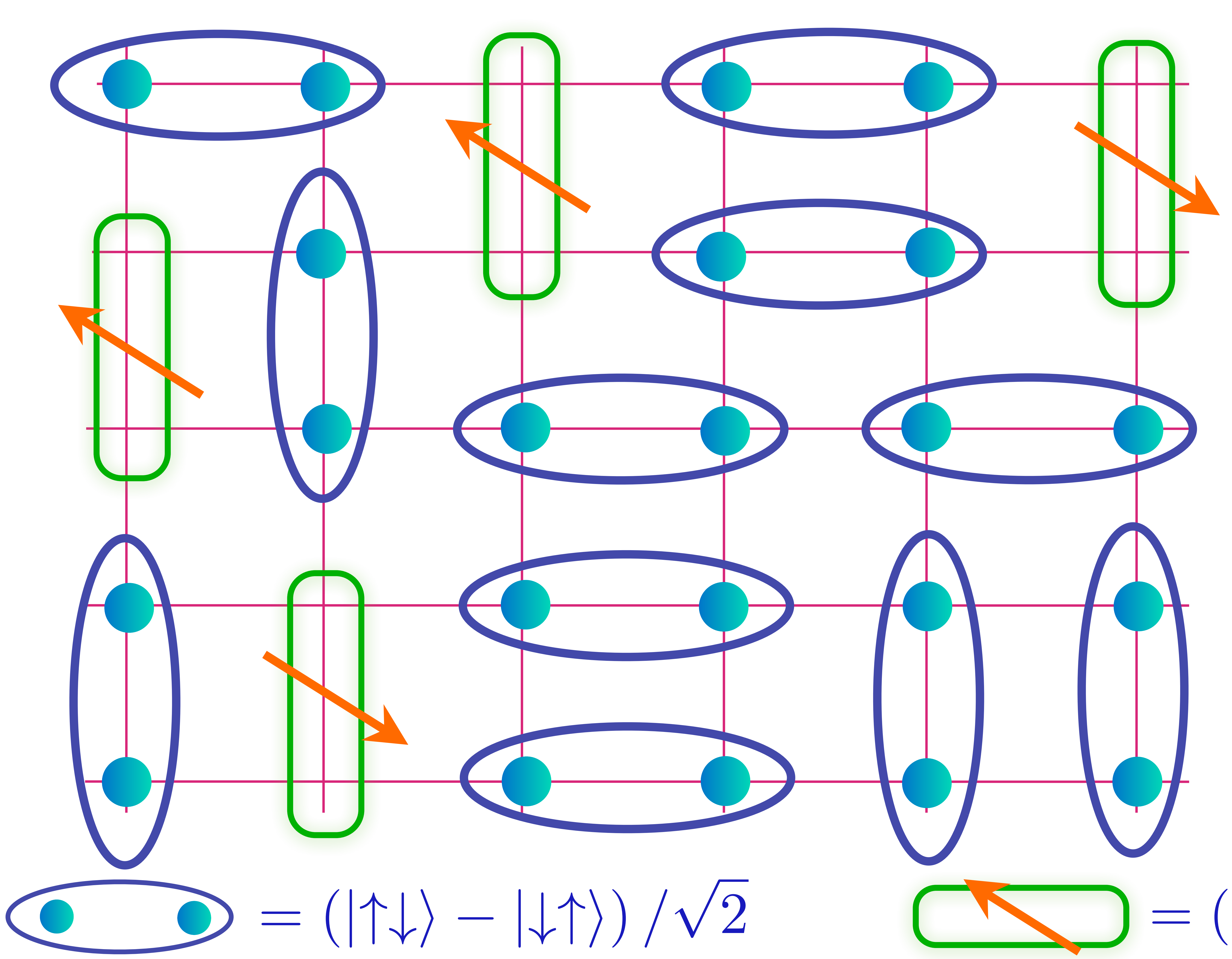
 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a *one-band* model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

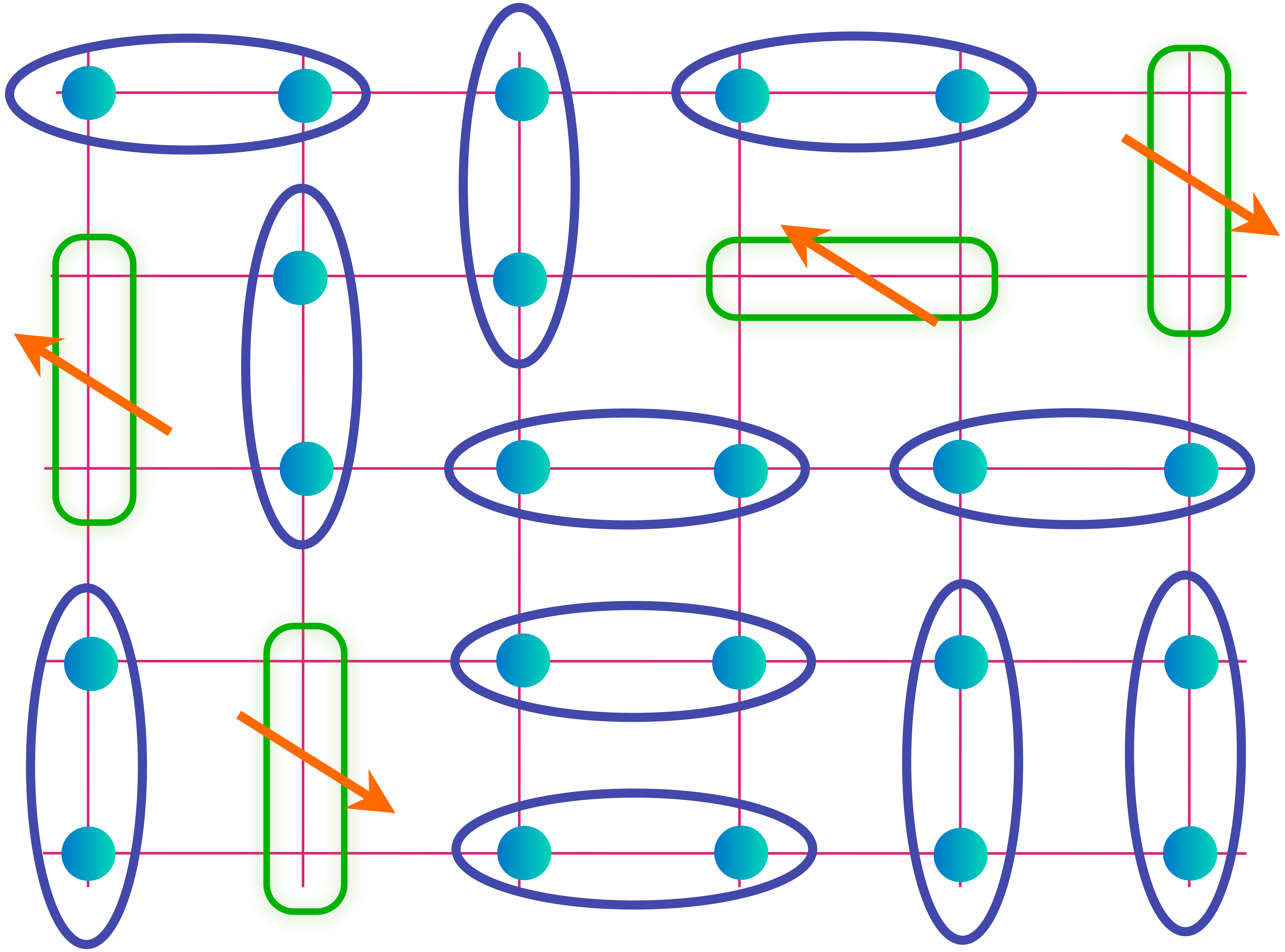
$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a *one-band* model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



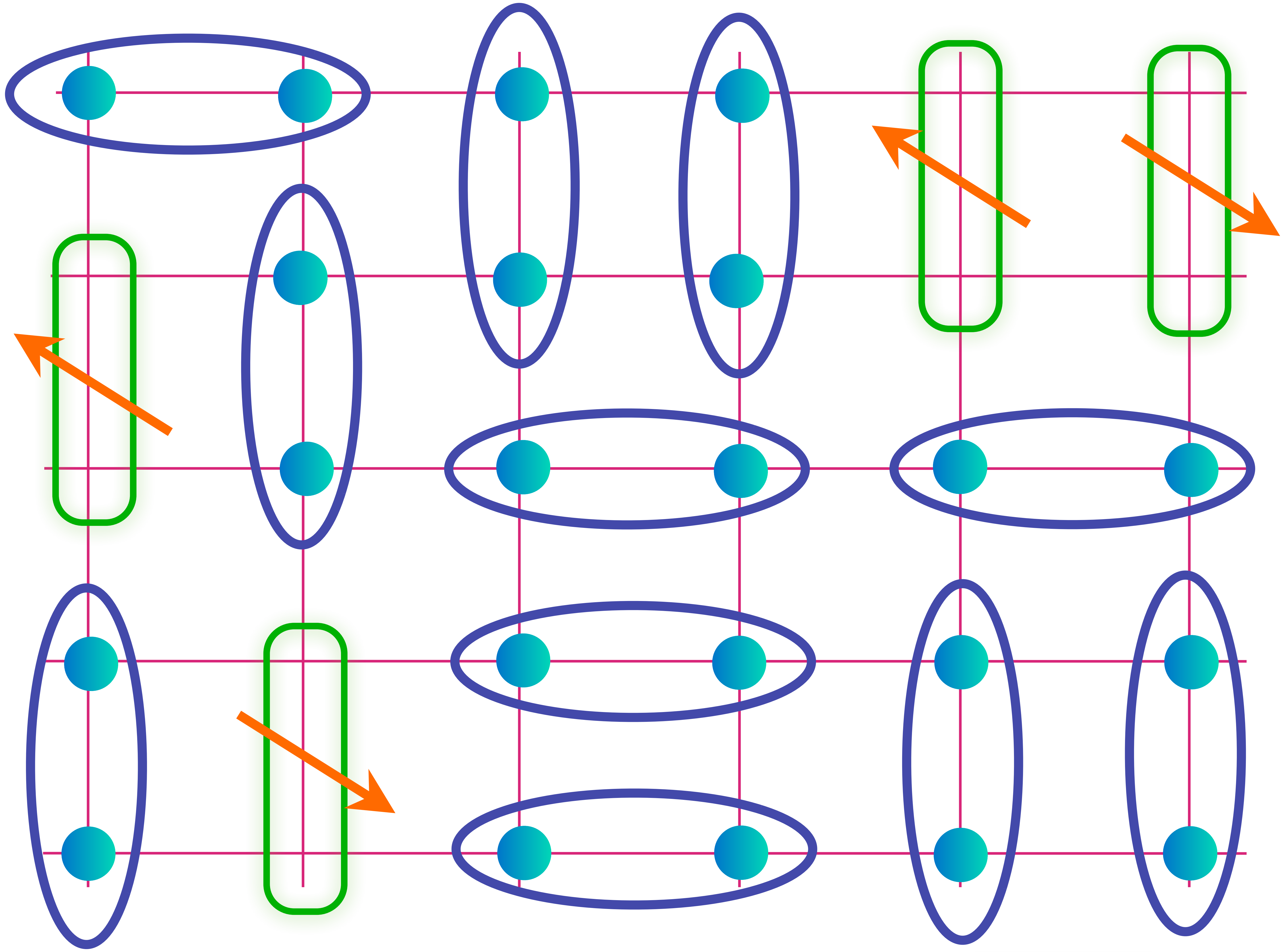
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

(blue oval) $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ (green box) $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

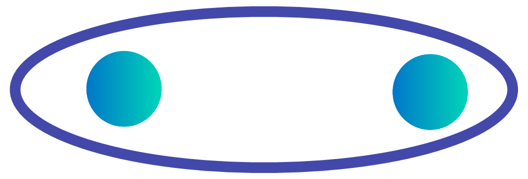
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

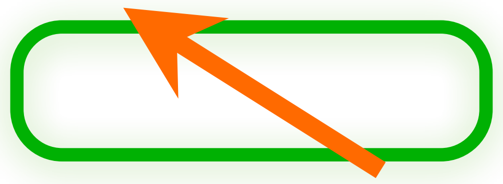
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

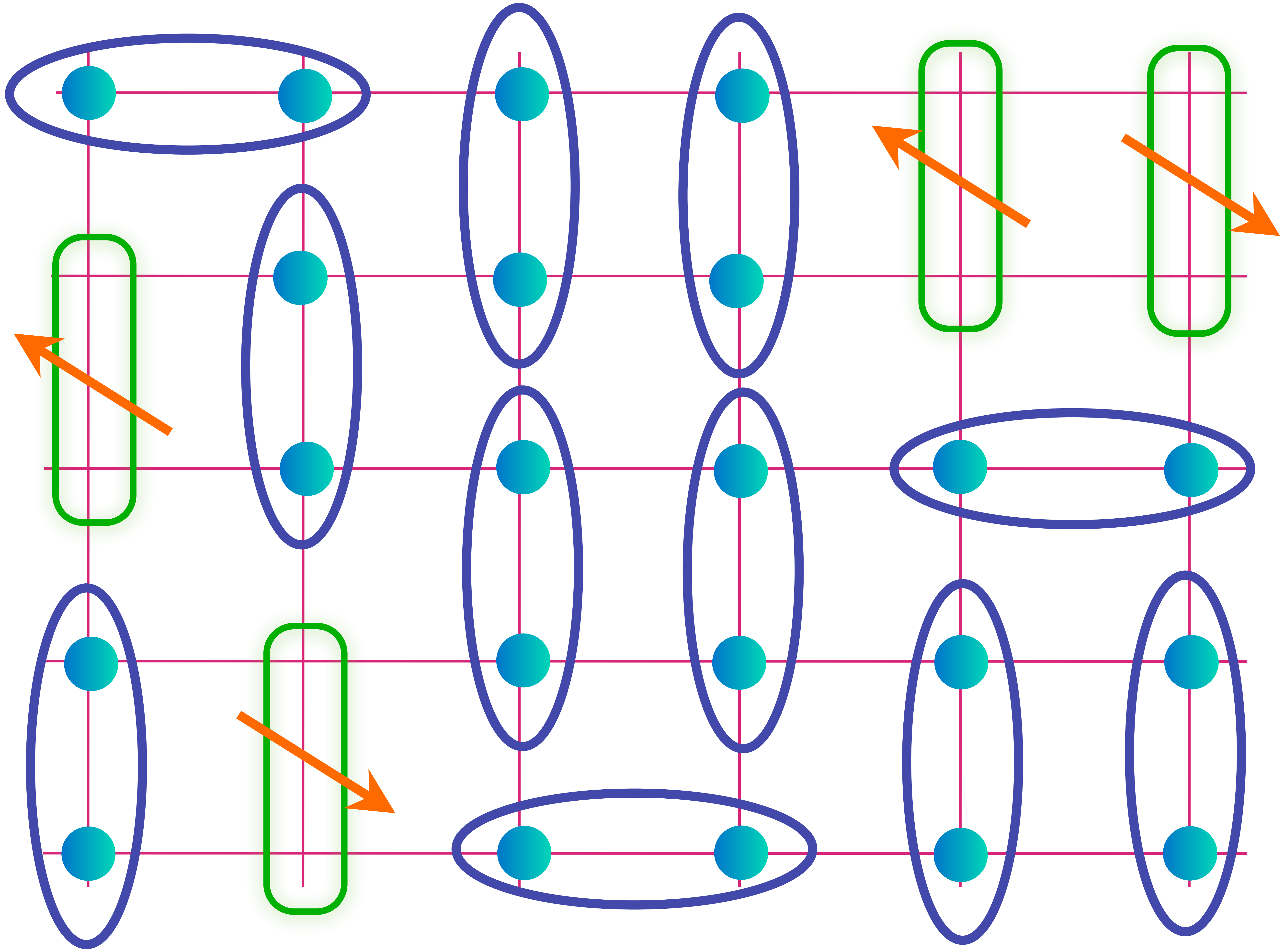
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

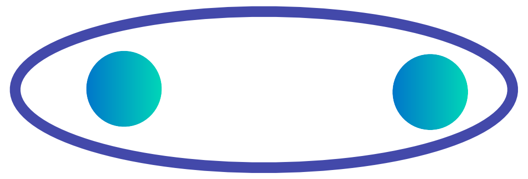
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

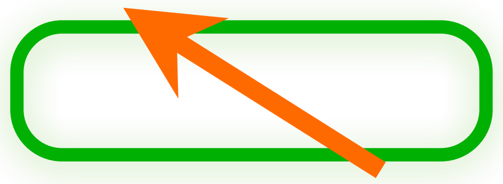
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

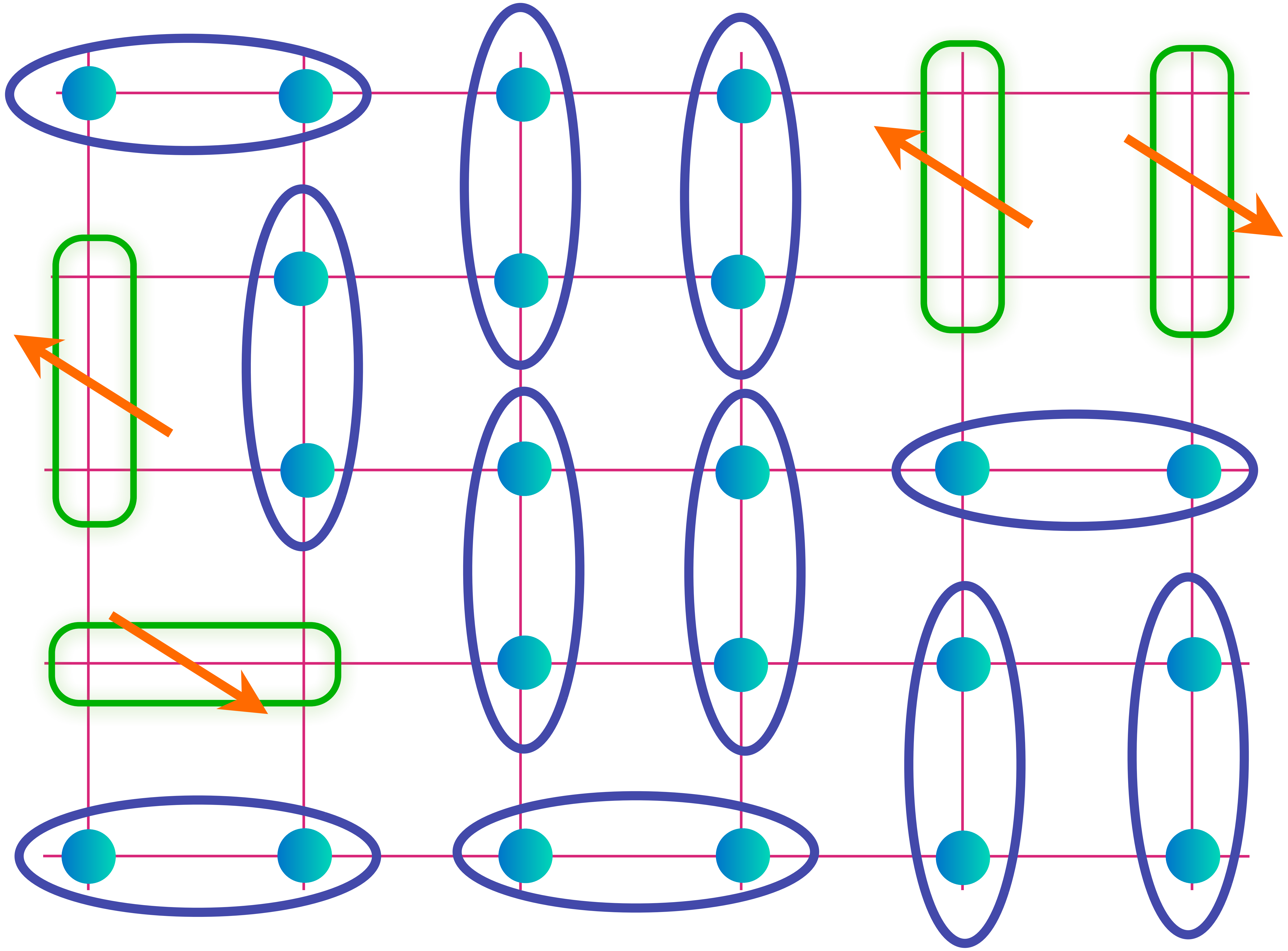
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

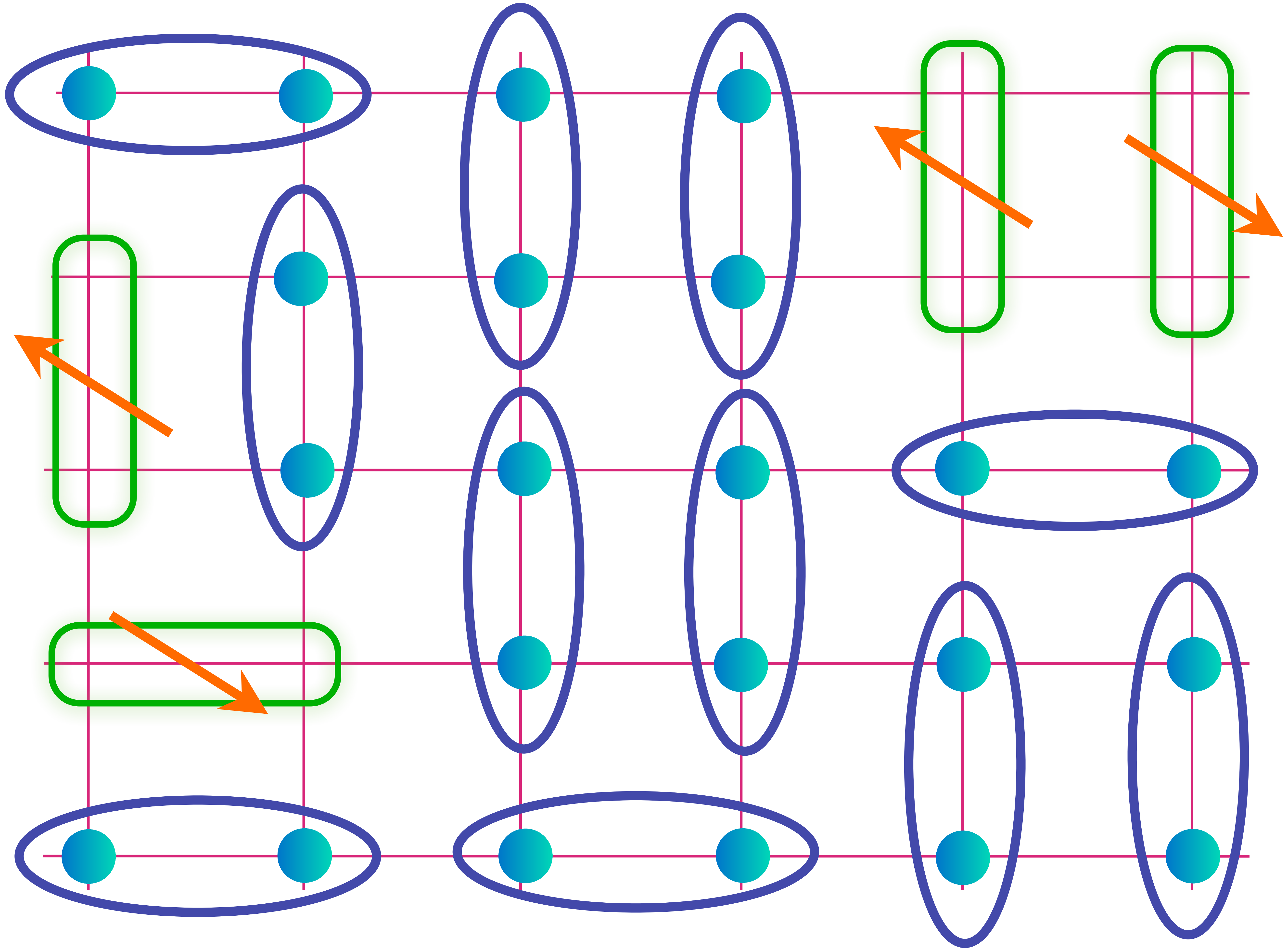
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$$

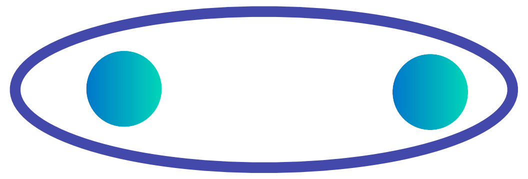
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

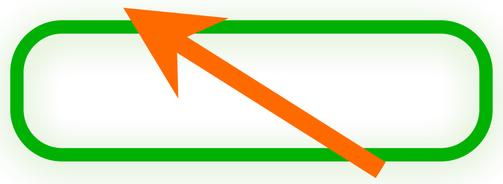
FL* in a *one-band* model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity

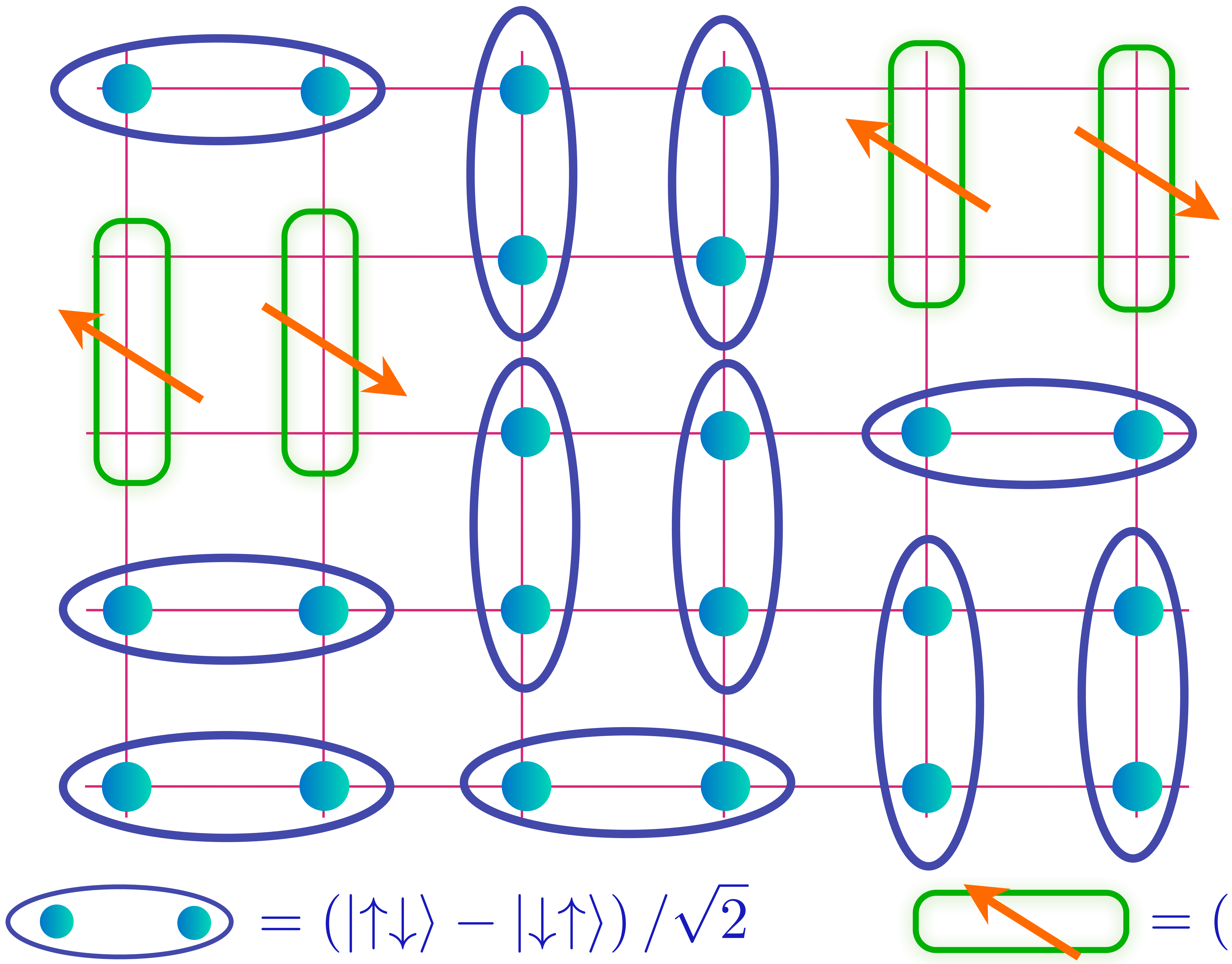
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity

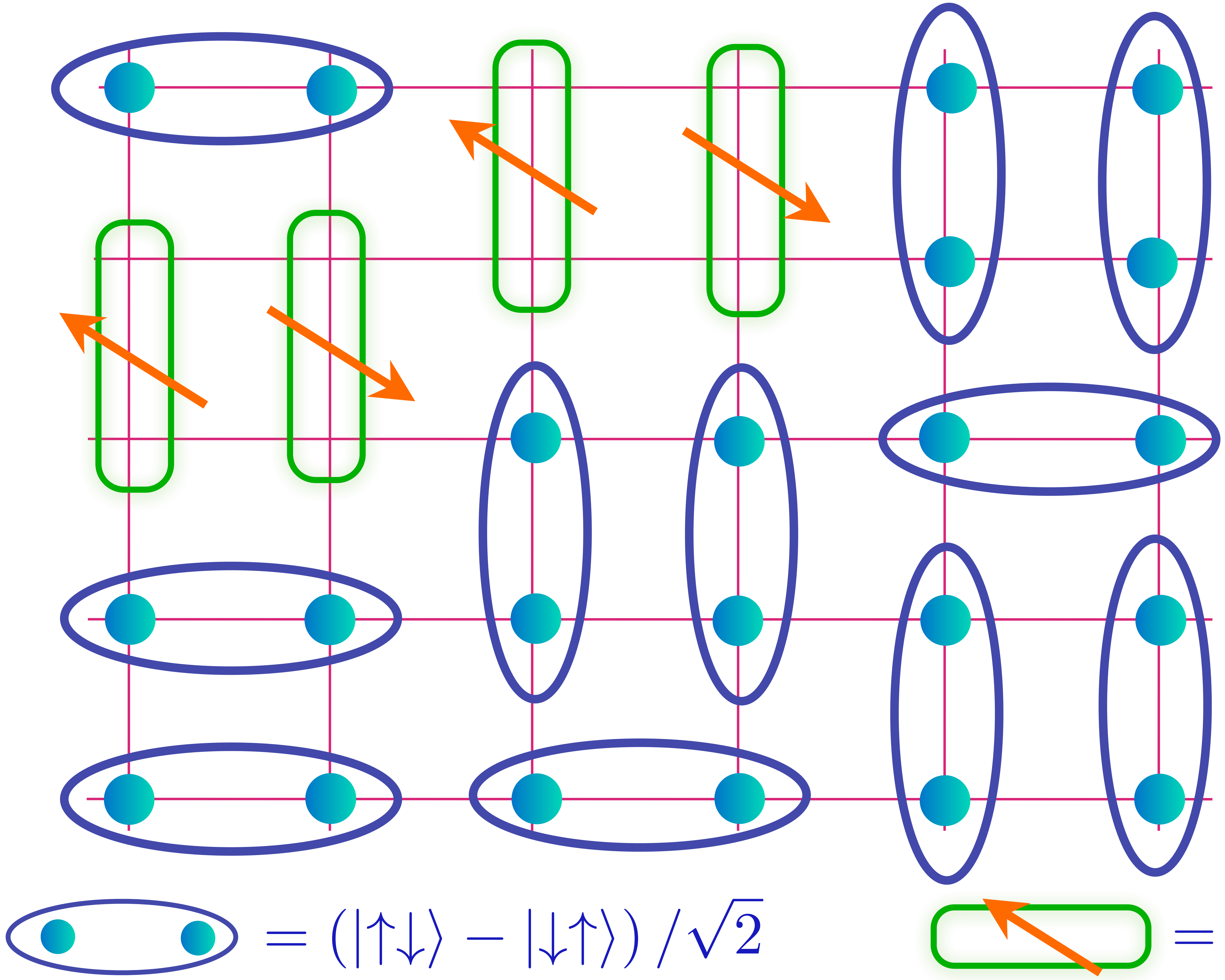
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity

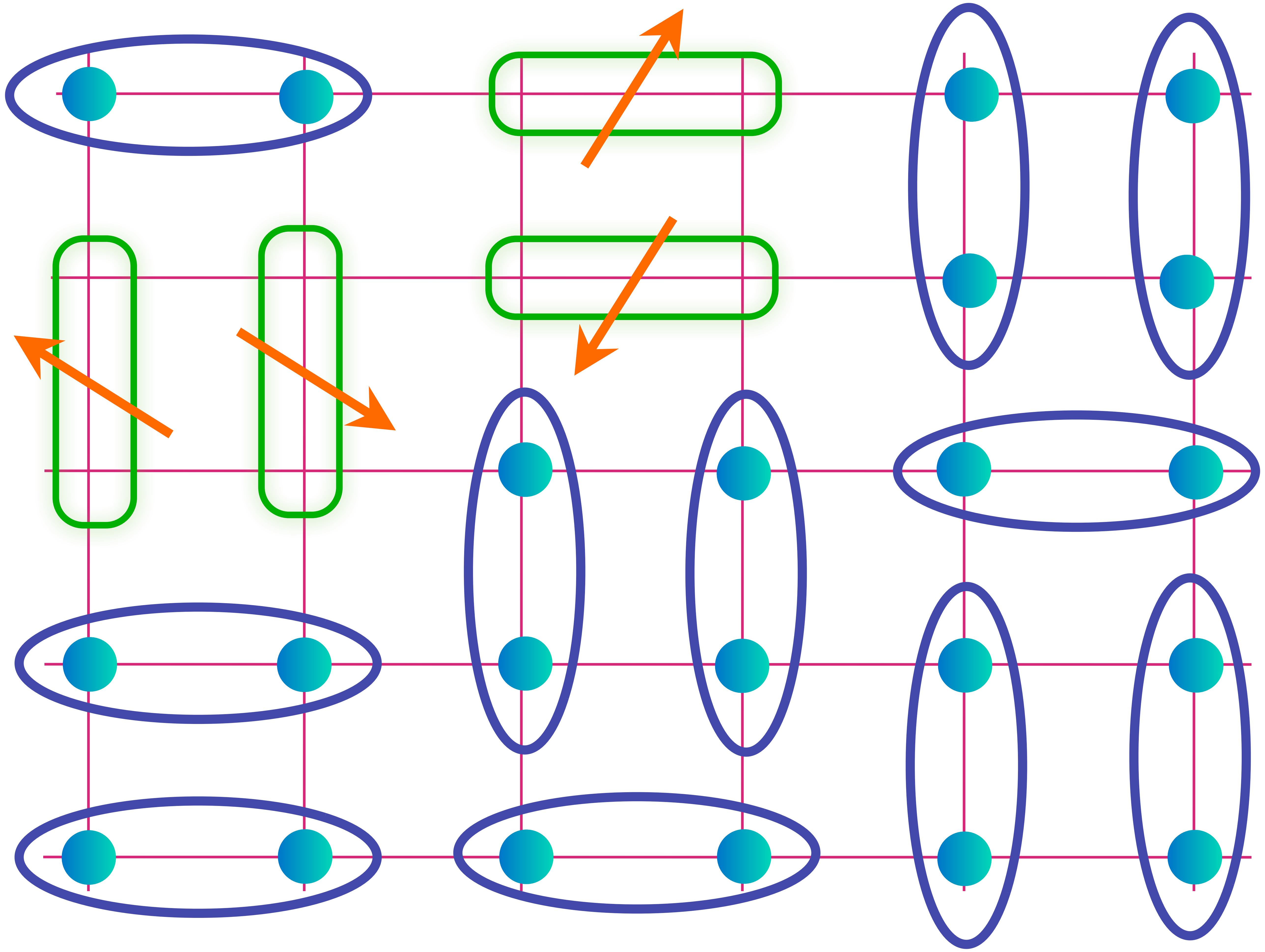
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

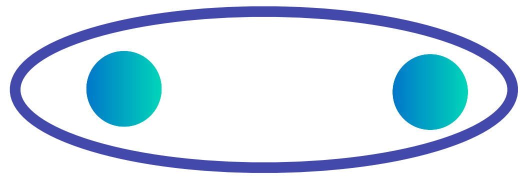
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

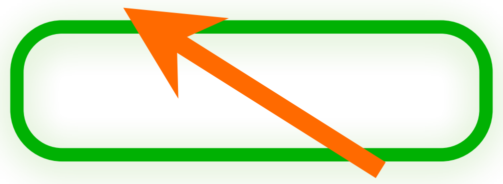
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity

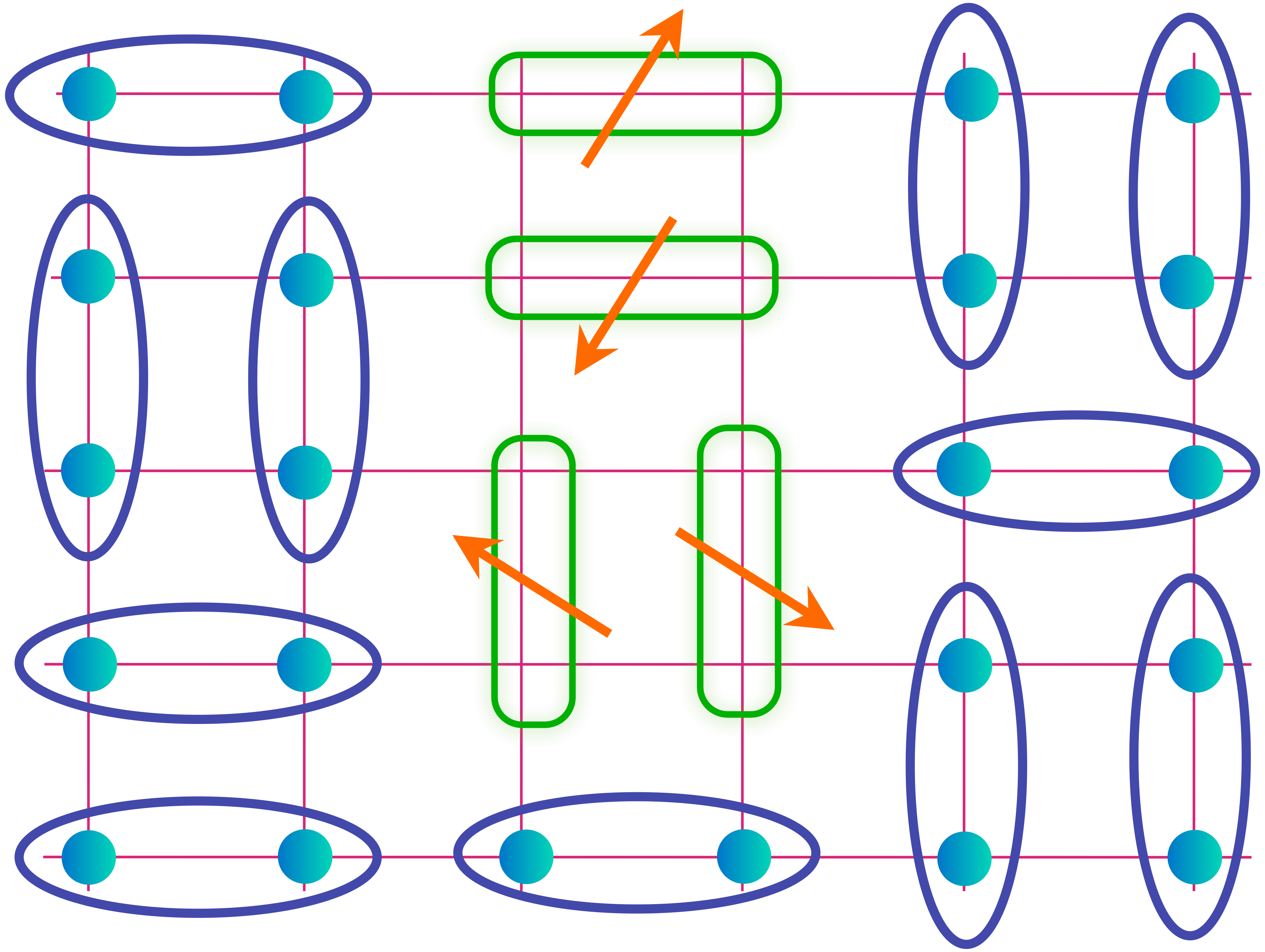
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

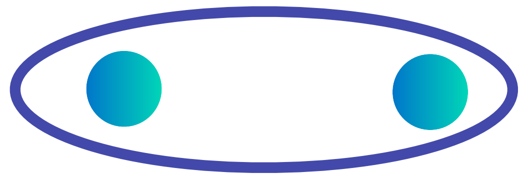
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

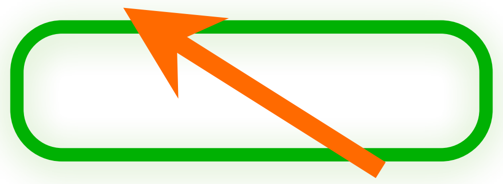
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

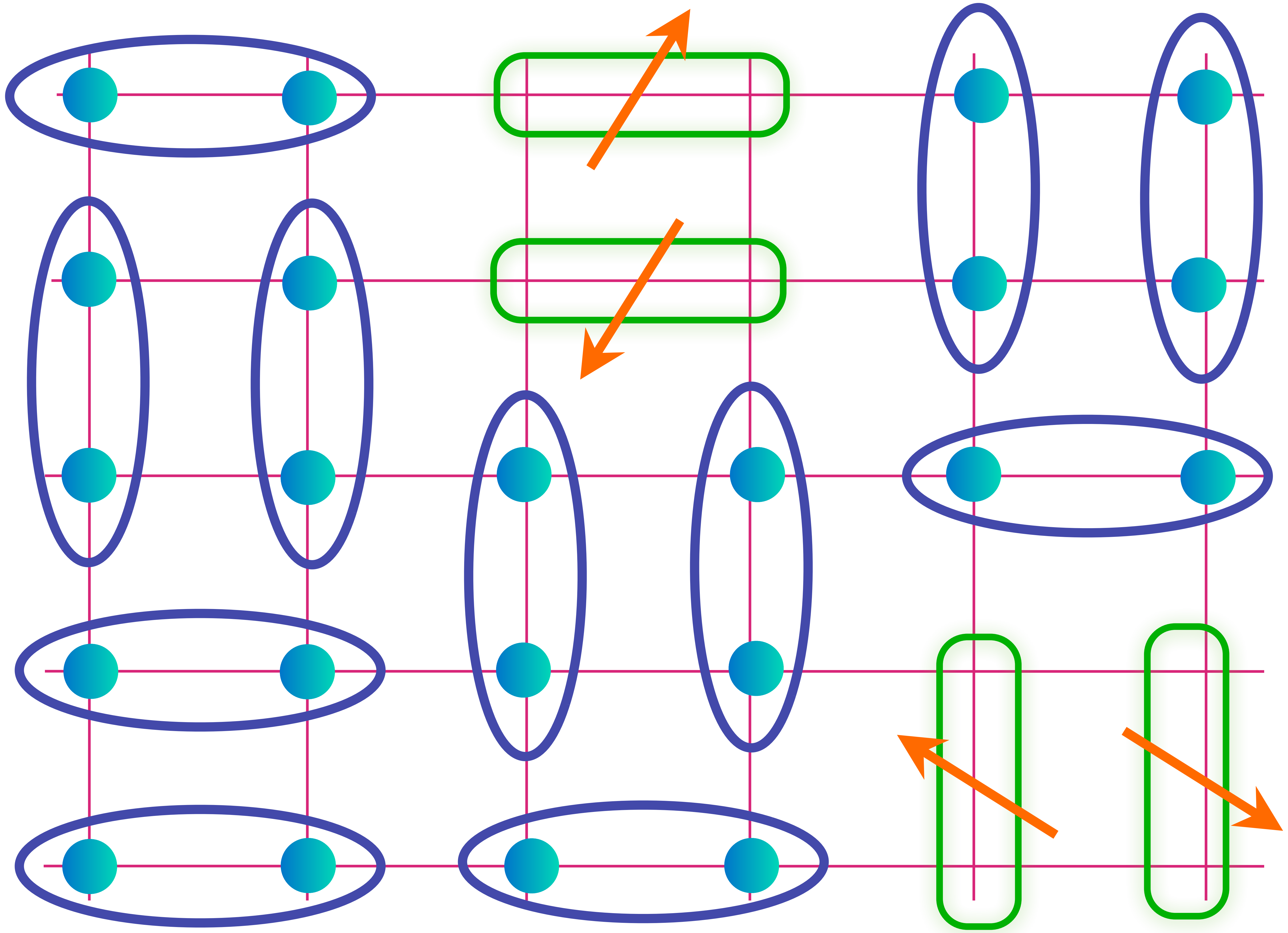


$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

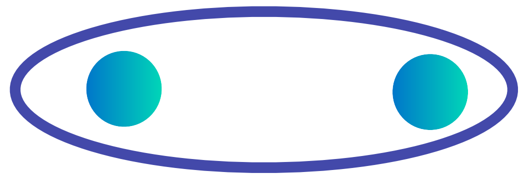
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

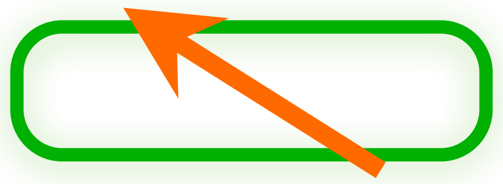
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

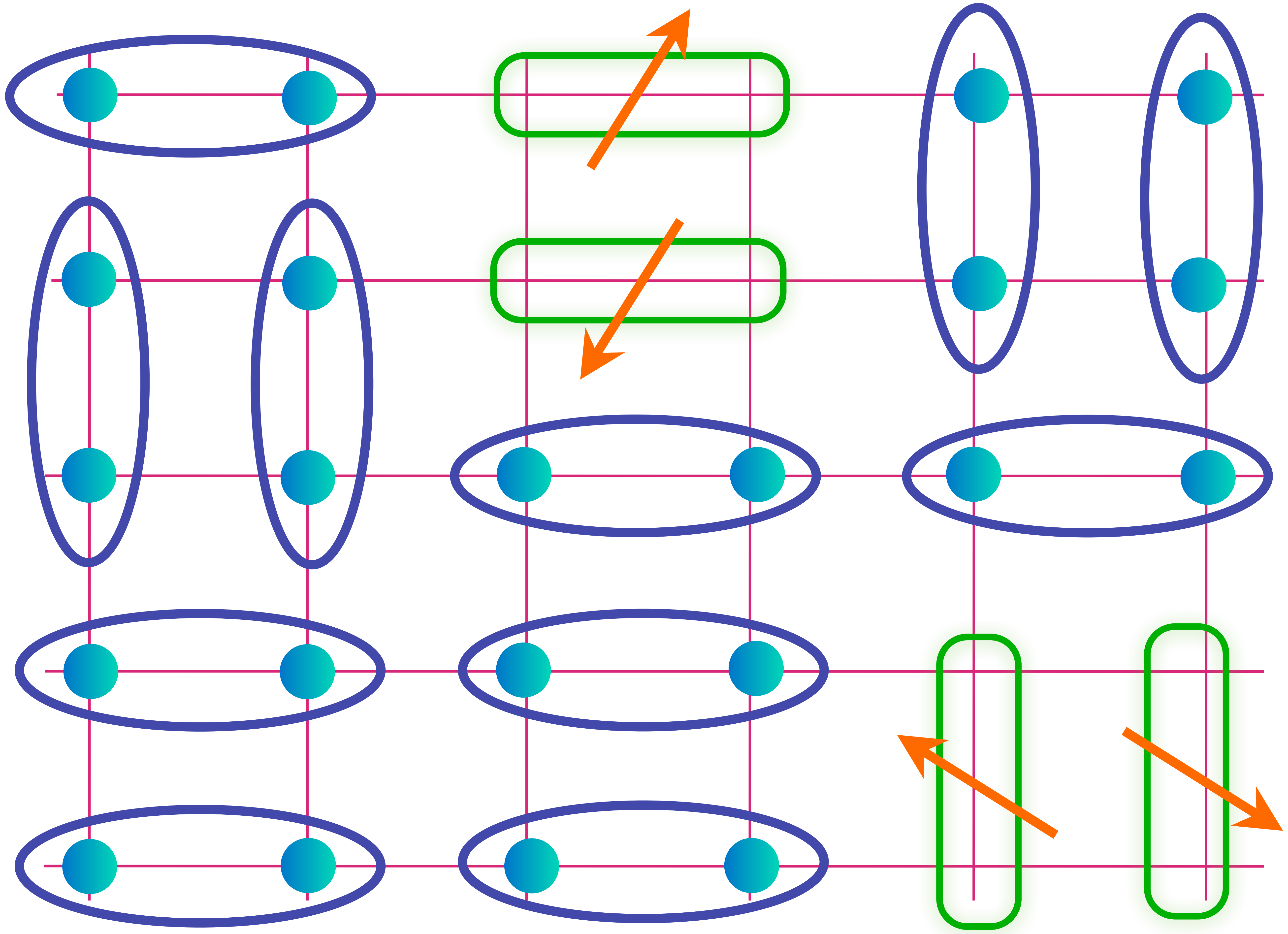


$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle) / \sqrt{2}$

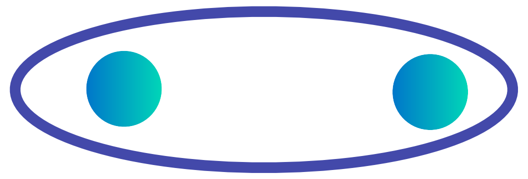
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

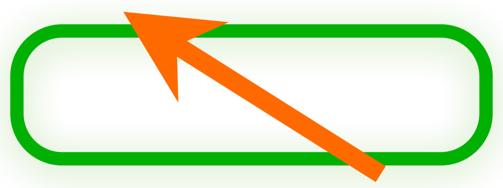
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

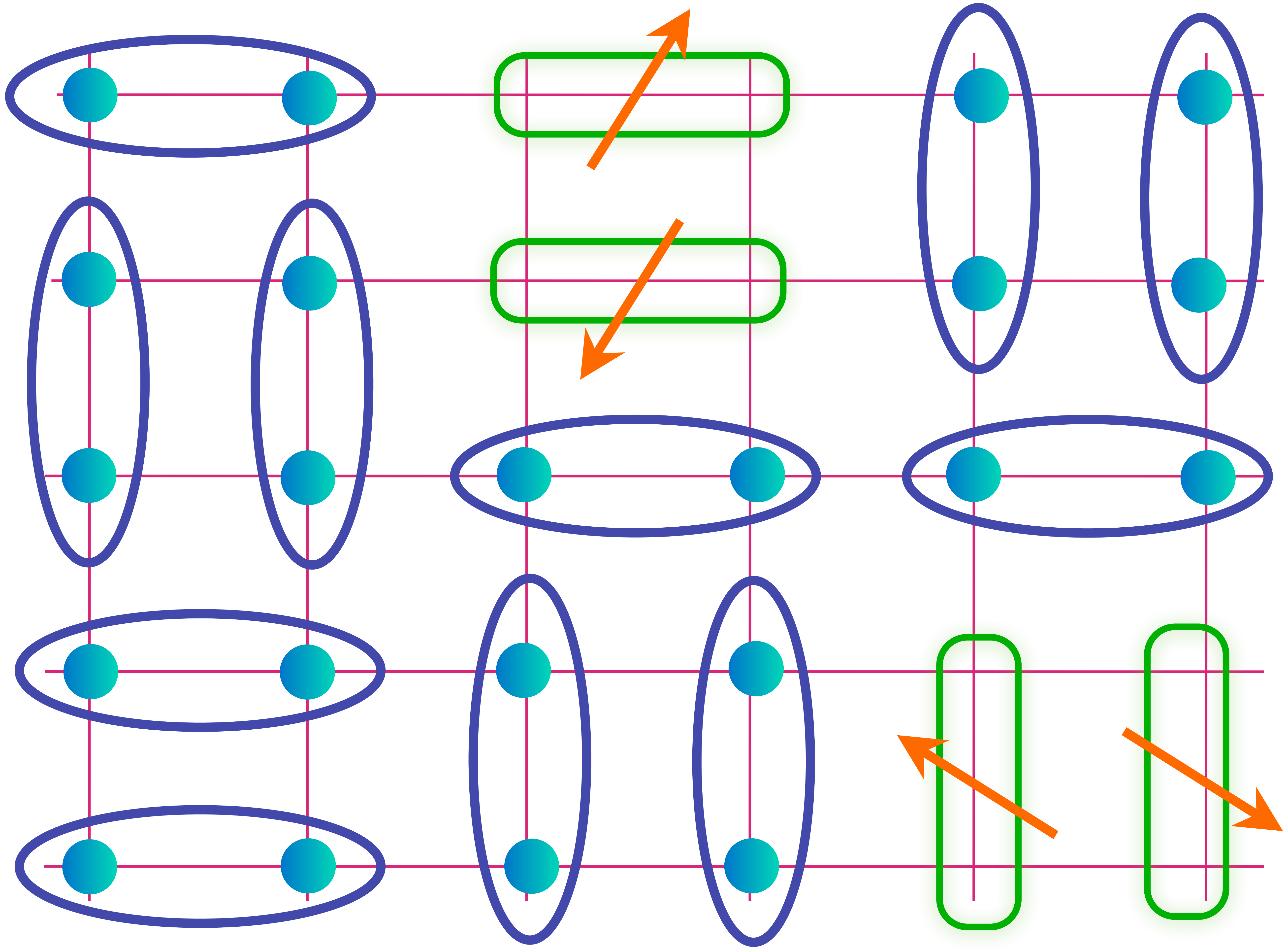


$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

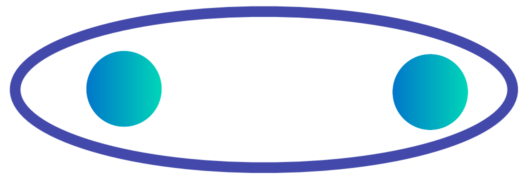
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

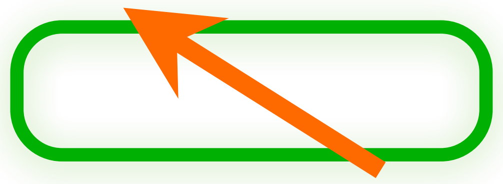
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Electrons can pair, leading to d-wave supercon-ductivity

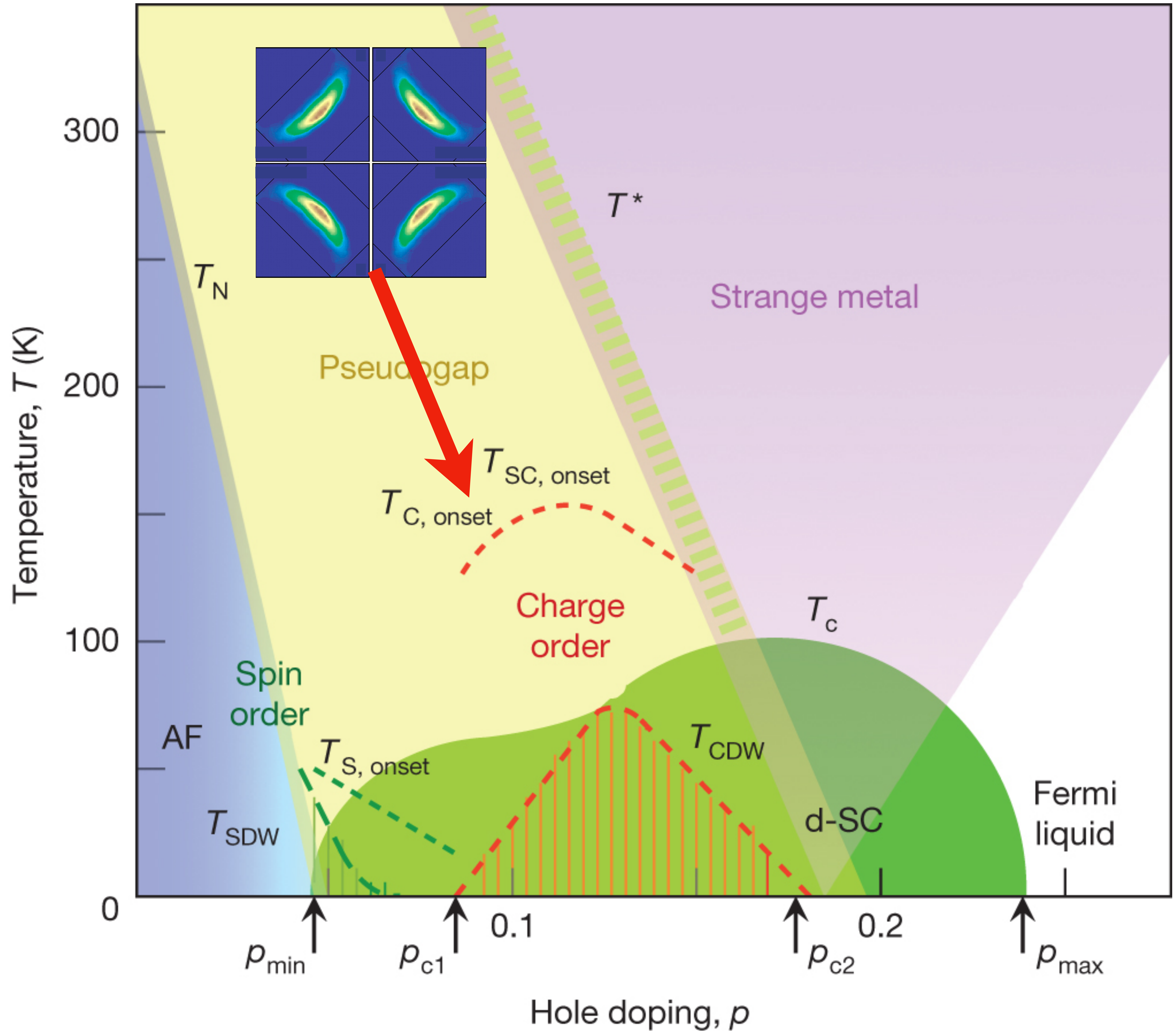
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

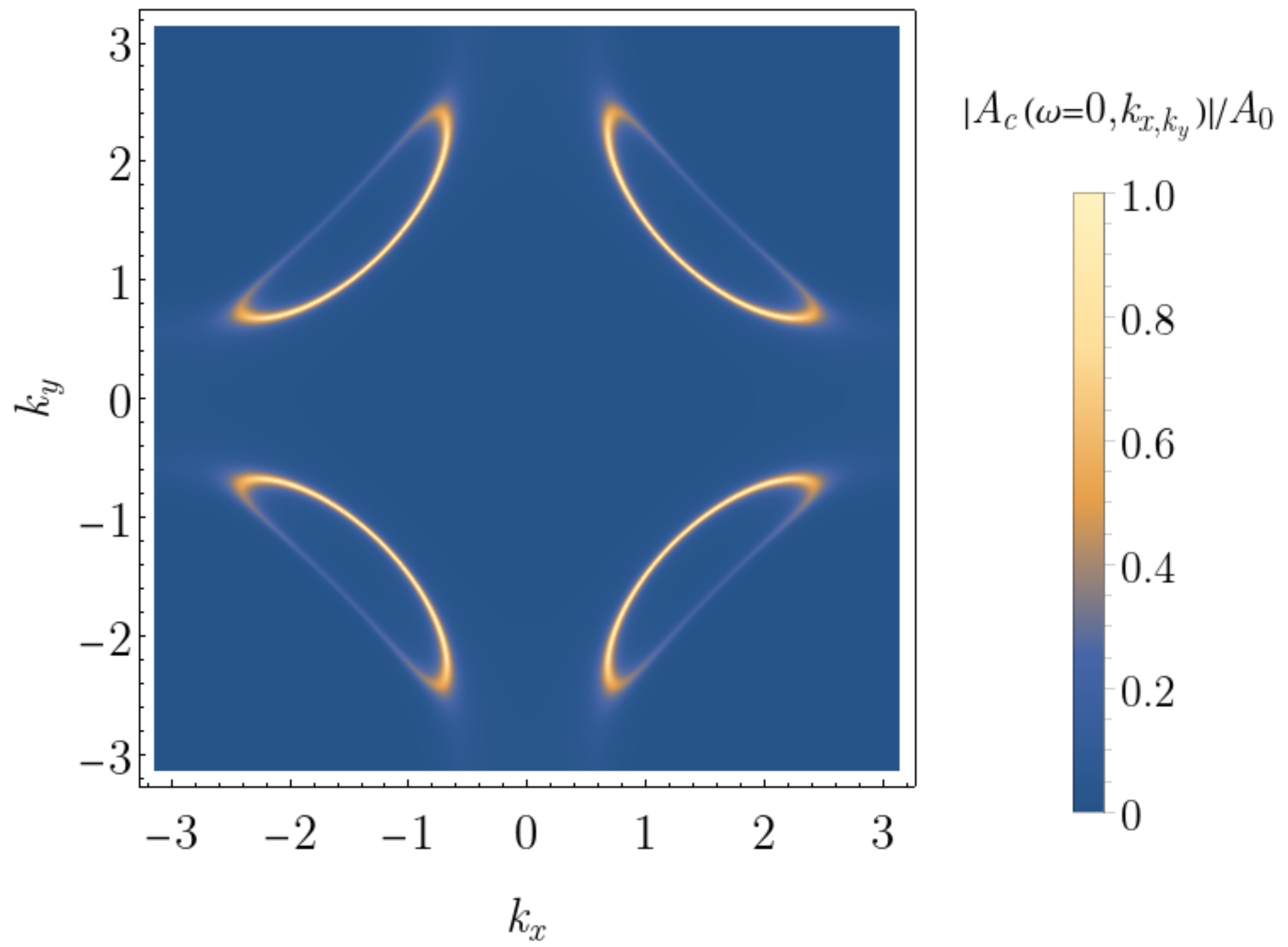
 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

B Keimer *et al.* *Nature* **518**, 179-186 (2015)

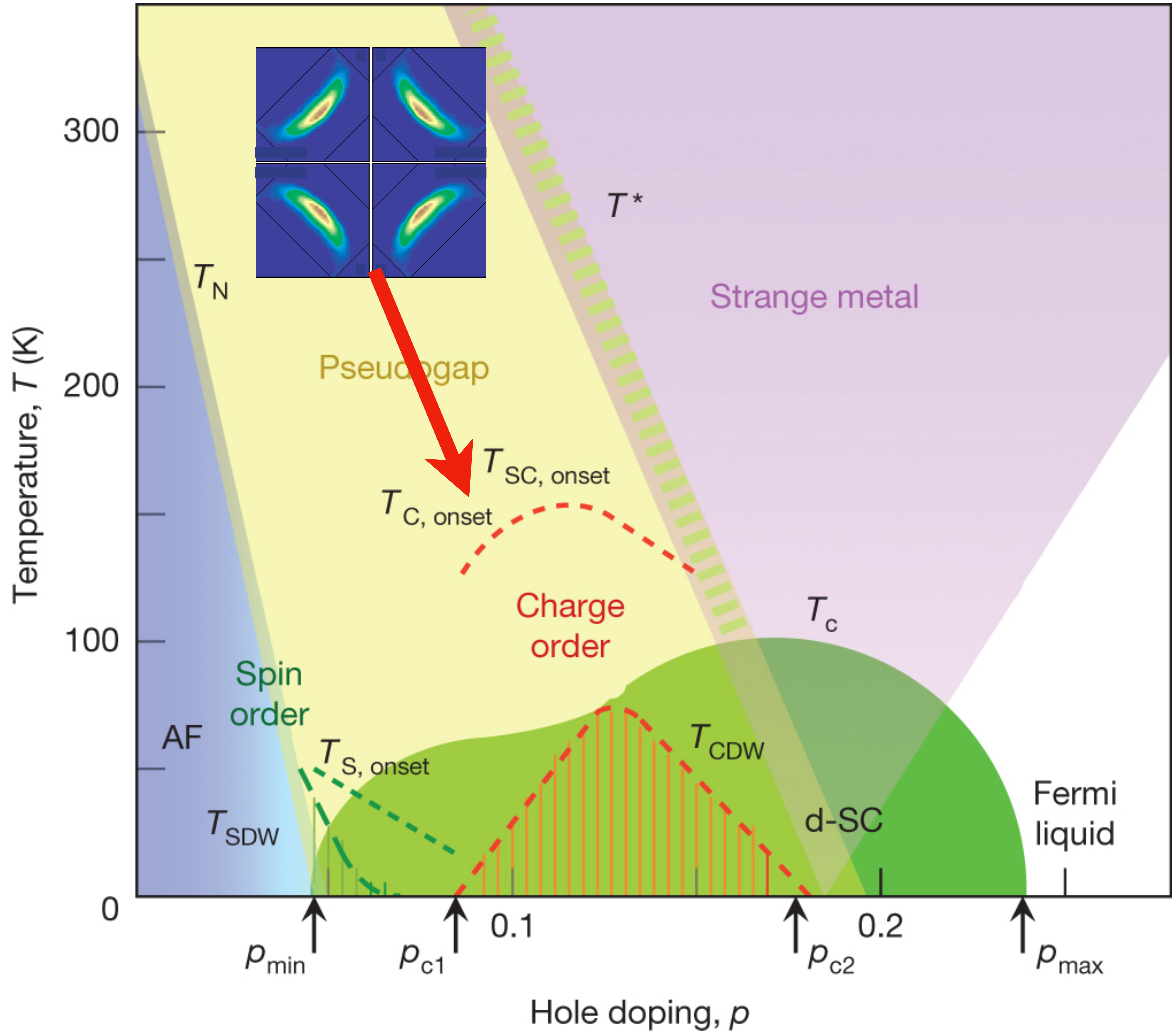


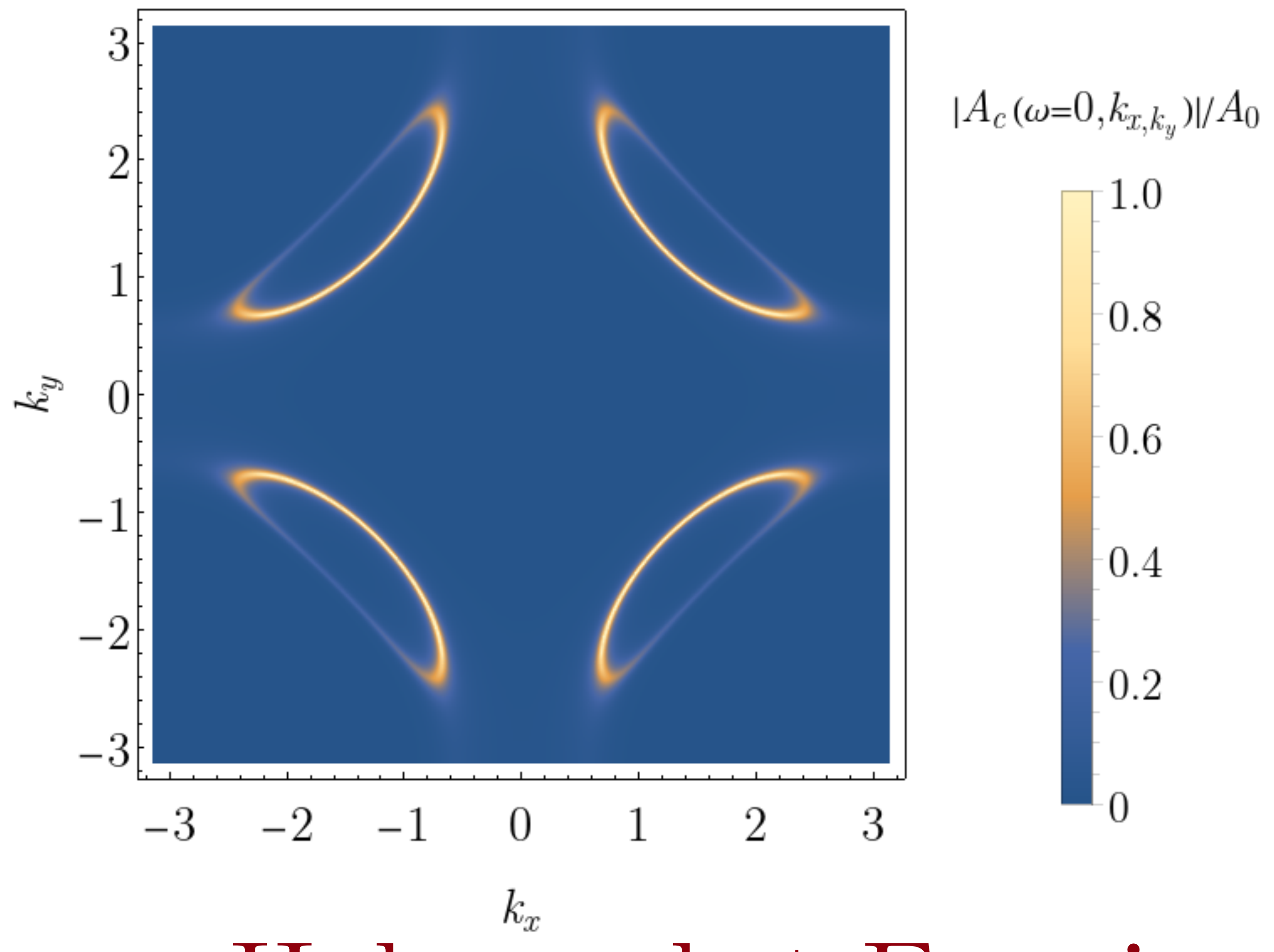


E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Hole pocket Fermi surfaces
of size p with
charge e, spin-1/2 quasiparticles
$+$
'spectator'
square lattice spin liquid
at half-filling.

B Keimer *et al.* *Nature* **518**, 179-186 (2015)





E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

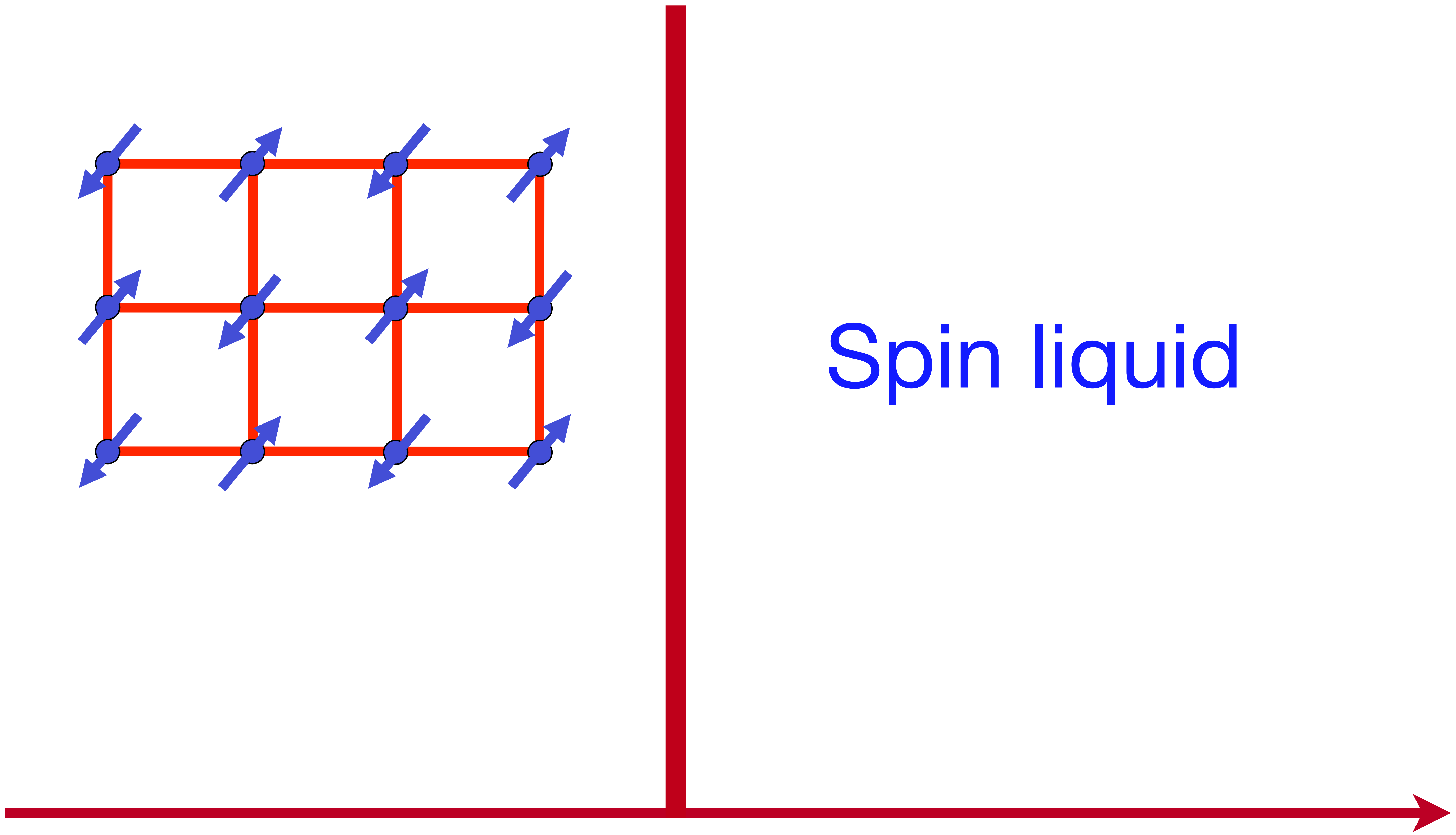
+

'spectator' square lattice spin liquid *at half-filling.*

But which spin liquid?

A spin liquid which is ultimately IR-unstable to confinement

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

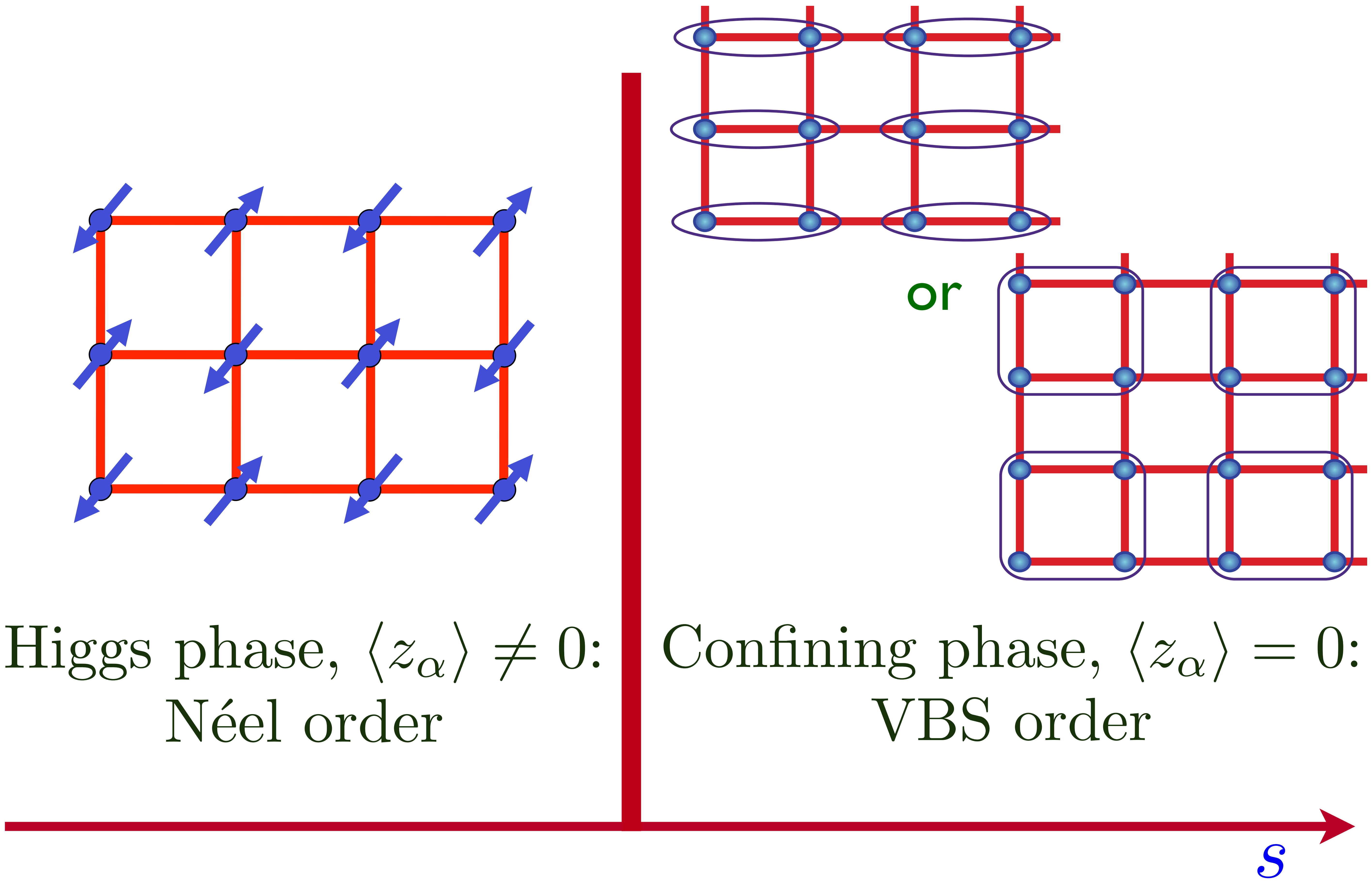
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \, , \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, PRB **38**, 316 (1988)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

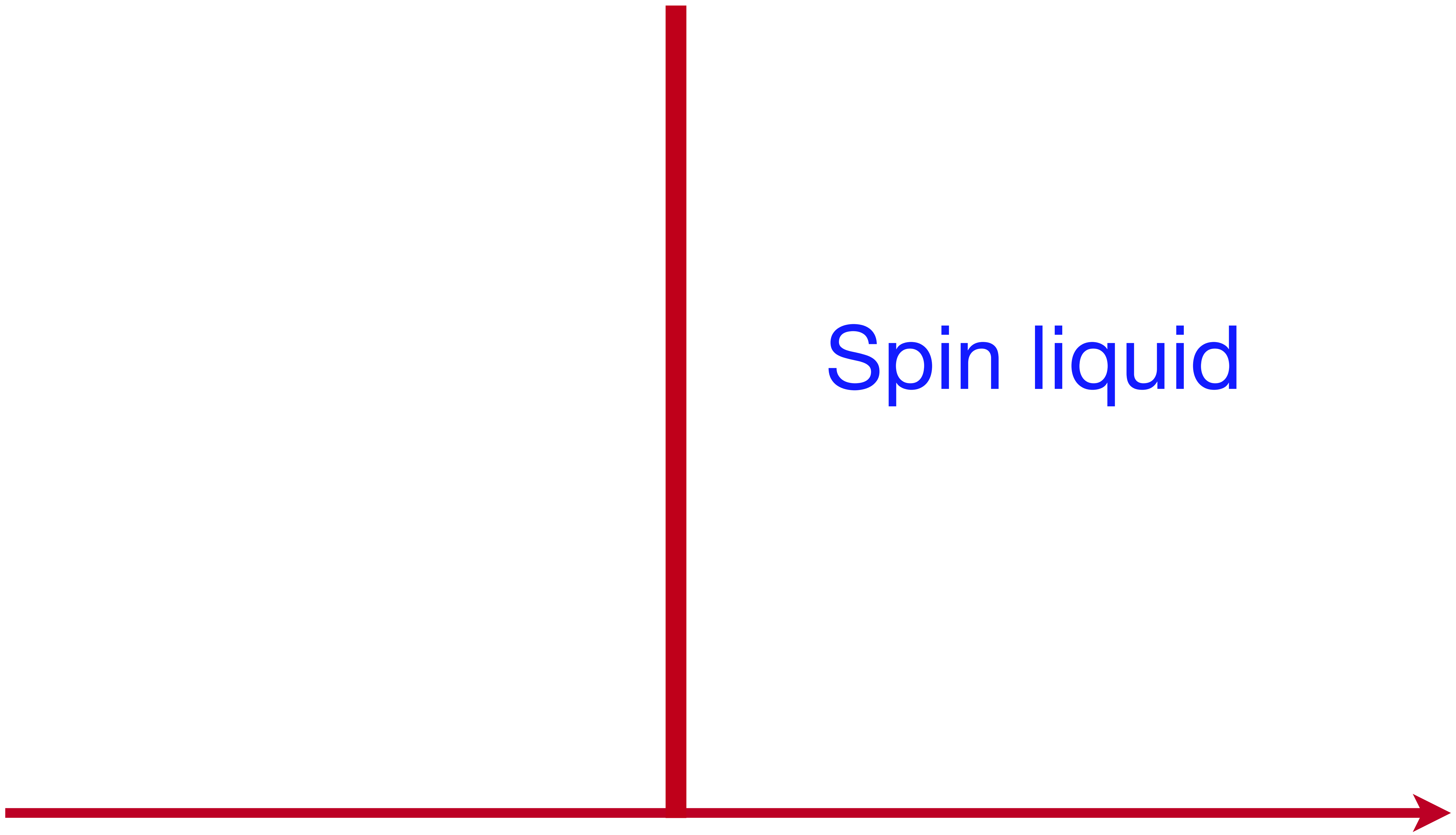
Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. S., PRL **62**, 1694 (1989)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

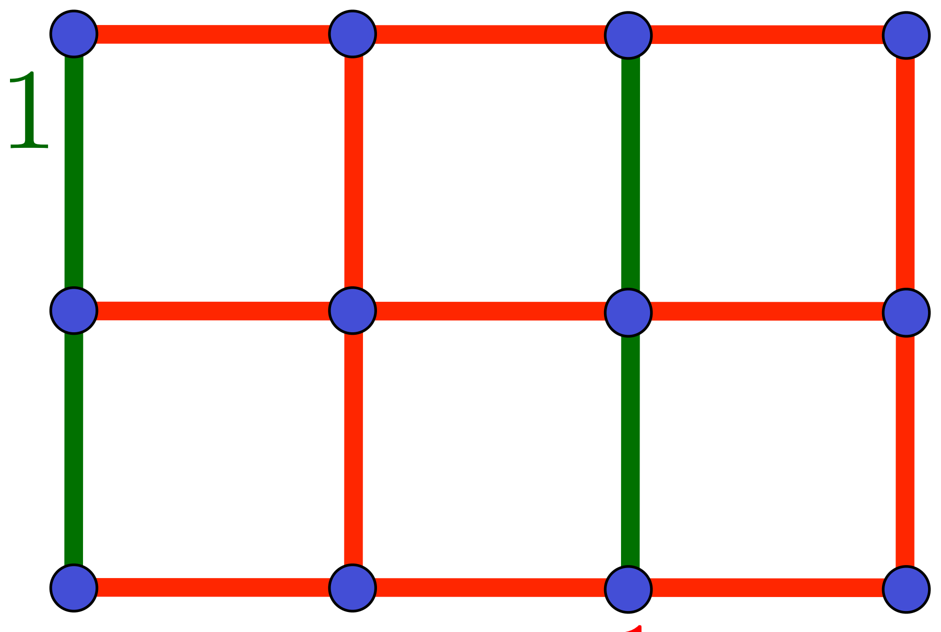
Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

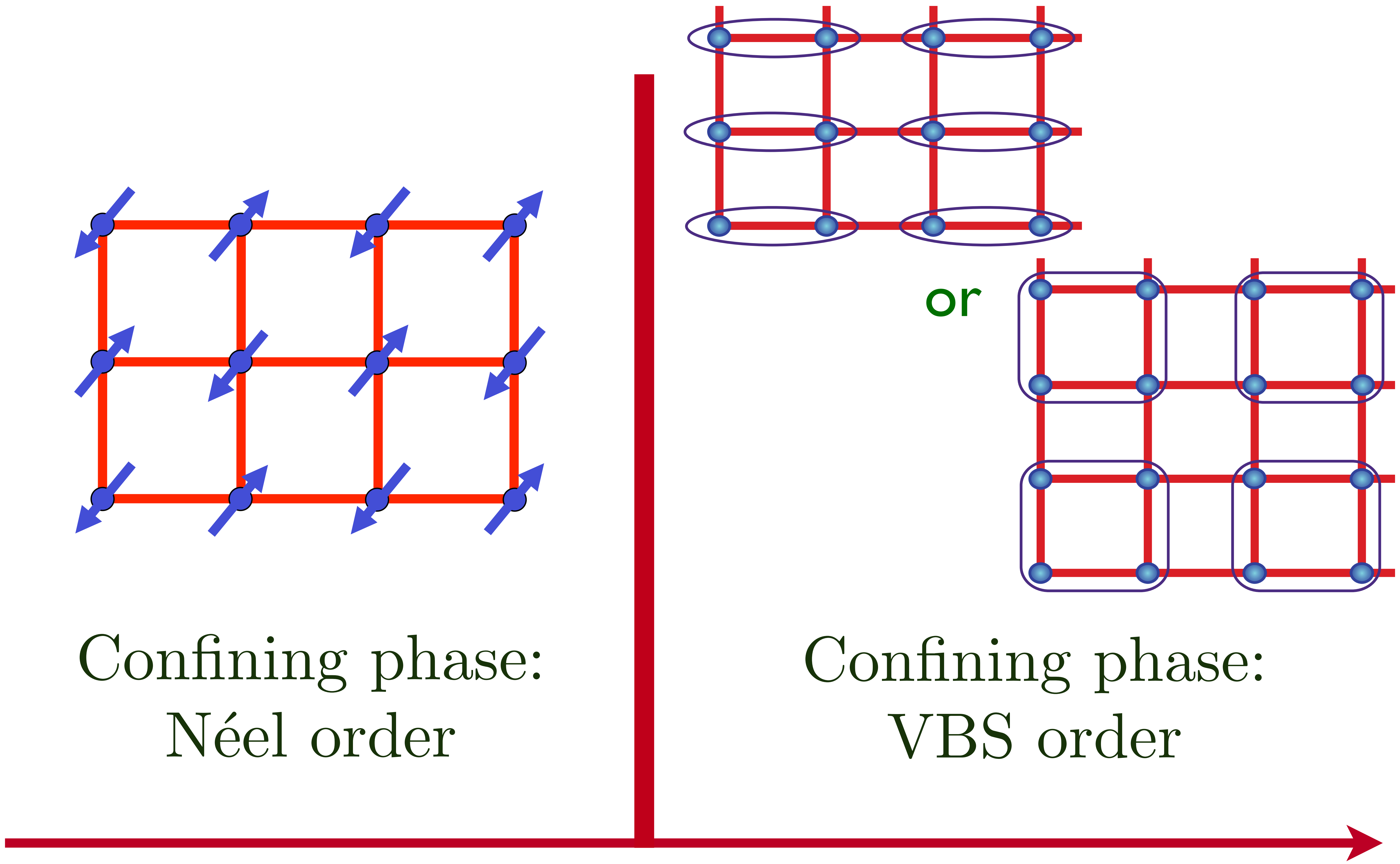
$e_{ij} = -1$



$e_{ij} = 1$

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) \quad , \quad \varepsilon_{\mathbf{k}} = 2J\sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

Insulating S=1/2 antiferromagnet



$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$$

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

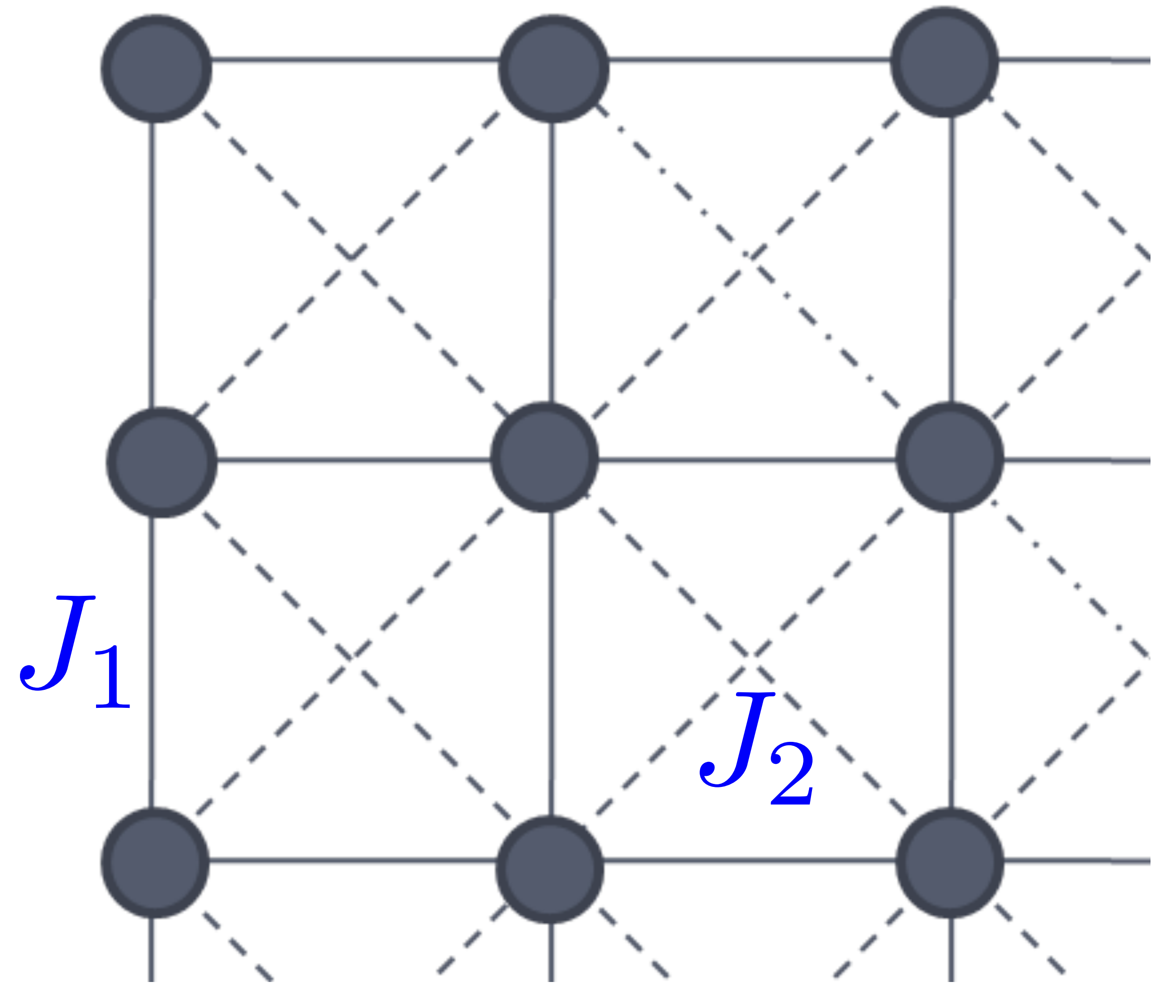
$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

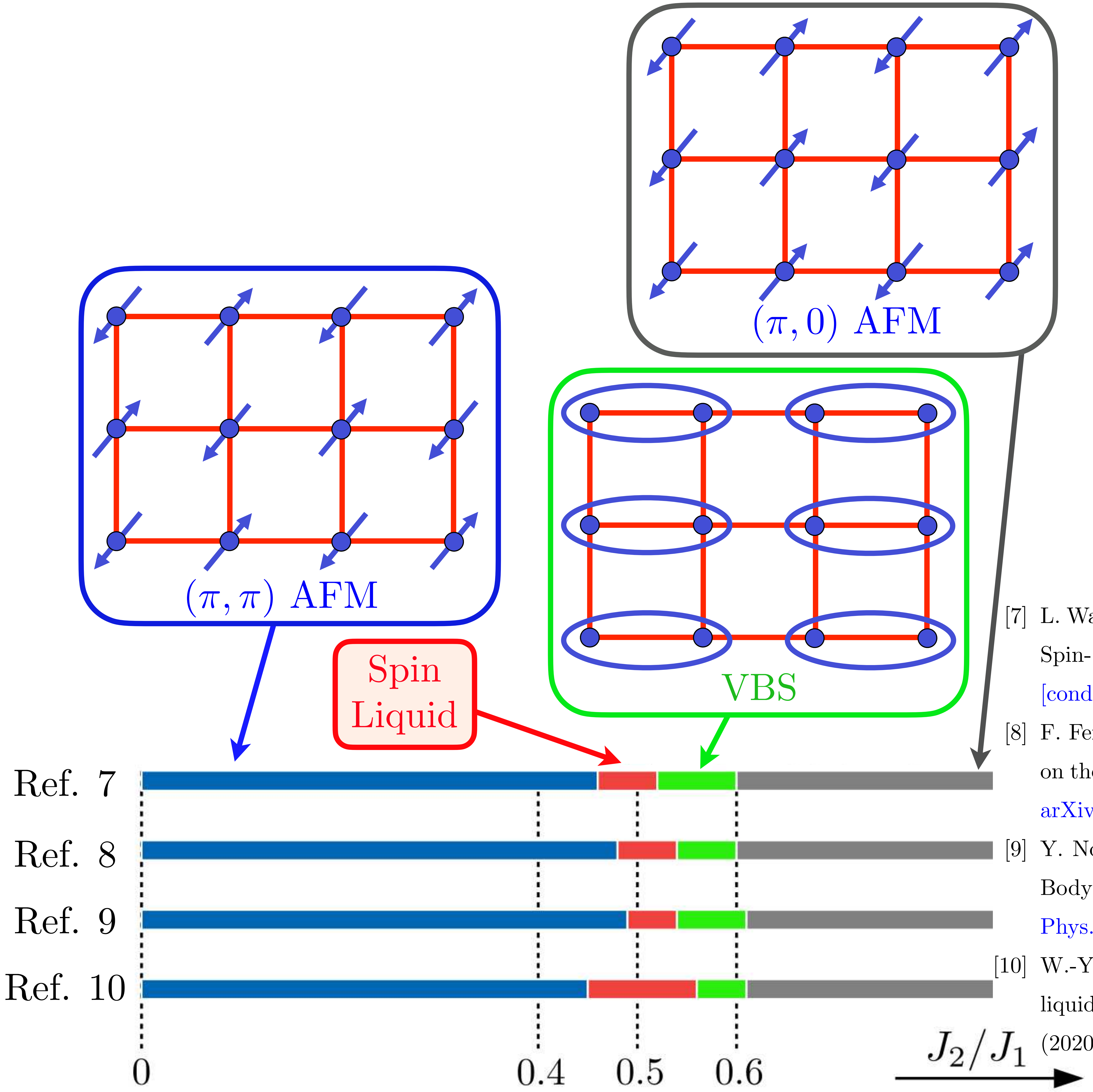
π-flux mean-field theory with gapless spinons at 2 Dirac points. Low energy theory of $N_f = 2$ Dirac fermions Ψ_s coupled to an emergent $\mathrm{SU}(2)_N$ gauge field. Confining order parameters are Néel and VBS states, with a global $\mathrm{SO}(5)_f$ symmetry!

Dual to $\mathbb{CP}^1$ U(1) gauge theory.

A. Tanaka and X. Hu, PRL **95**, 036402 (2005); T. Senthil and M.P.A. Fisher PRB **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Physical Review X **7**, 031051 (2017)

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





[7] L. Wang and A. W. Sandvik, "Critical Level Crossings and Gapless Spin Liquid in the Square-Lattice Spin-1 /2 J_1-J_2 Heisenberg Antiferromagnet," Phys. Rev. Lett. **121**, 107202 (2018), arXiv:1702.08197 [cond-mat.str-el].

[8] F. Ferrari and F. Becca, "Gapless spin liquid and valence-bond solid in the J_1-J_2 Heisenberg model on the square lattice: Insights from singlet and triplet excitations," Phys. Rev. B **102**, 014417 (2020), arXiv:2005.12941 [cond-mat.str-el].

[9] Y. Nomura and M. Imada, "Dirac-Type Nodal Spin Liquid Revealed by Refined Quantum Many-Body Solver Using Neural-Network Wave Function, Correlation Ratio, and Level Spectroscopy," Phys. Rev. X **11**, 031034 (2021), arXiv:2005.14142 [cond-mat.str-el].

[10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, "Gapless quantum spin liquid and global phase diagram of the spin-1/2 J_1-J_2 square antiferromagnetic Heisenberg model," (2020), arXiv:2009.01821 [cond-mat.str-el].

[Submitted on 28 Jun 2023]

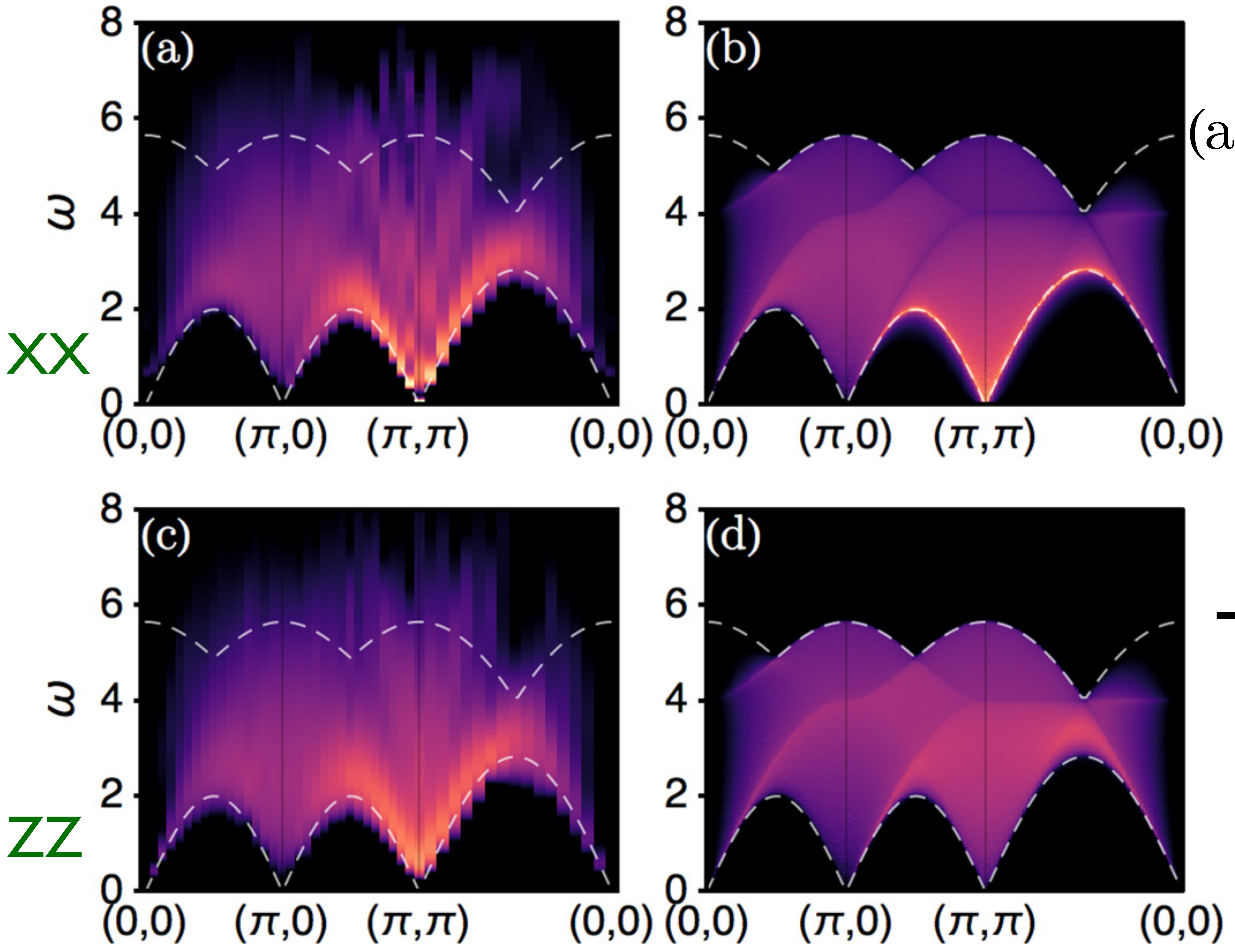
The SO(5) Deconfined Phase Transition under the Fuzzy Sphere Microscope: Approximate Conformal Symmetry, Pseudo-Criticality, and Operator Spectrum

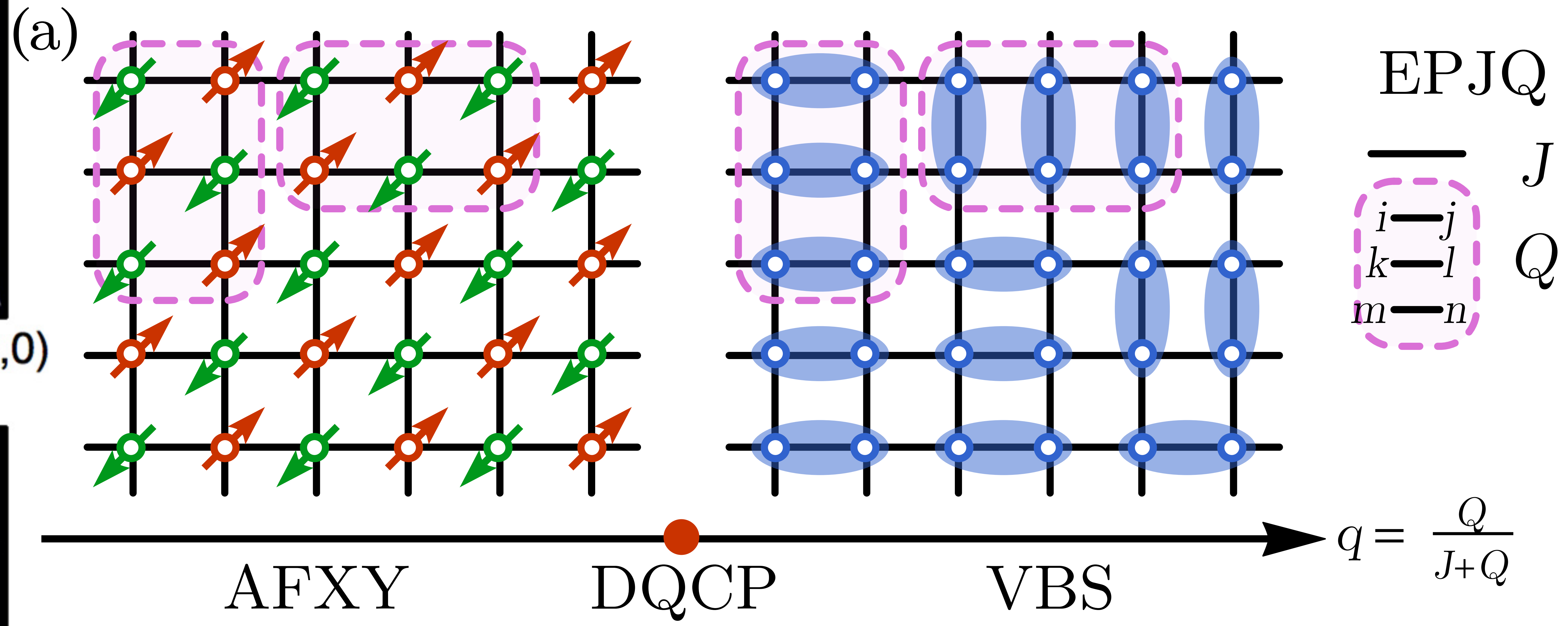
Zheng Zhou, Liangdong Hu, W. Zhu, Yin-Chen He

The deconfined quantum critical point (DQCP) is an example of phase transitions beyond the Landau symmetry breaking paradigm that attracts wide interest. However, its nature has not been settled after decades of study. In this paper, we apply the recently proposed fuzzy sphere regularization to study the SO(5) non-linear sigma model (NLσM) with a topological Wess-Zumino-Witten term, which serves as a dual description of the DQCP with an exact SO(5) symmetry. We demonstrate that the fuzzy sphere functions as a powerful microscope, magnifying and revealing a wealth of crucial information about the DQCP, ultimately paving the way towards its final answer. In particular, through exact diagonalization, we provide clear evidence that the DQCP exhibits approximate conformal symmetry. The evidence includes the existence of a conserved SO(5) symmetry

QMC

Free fermion spinons in π-flux



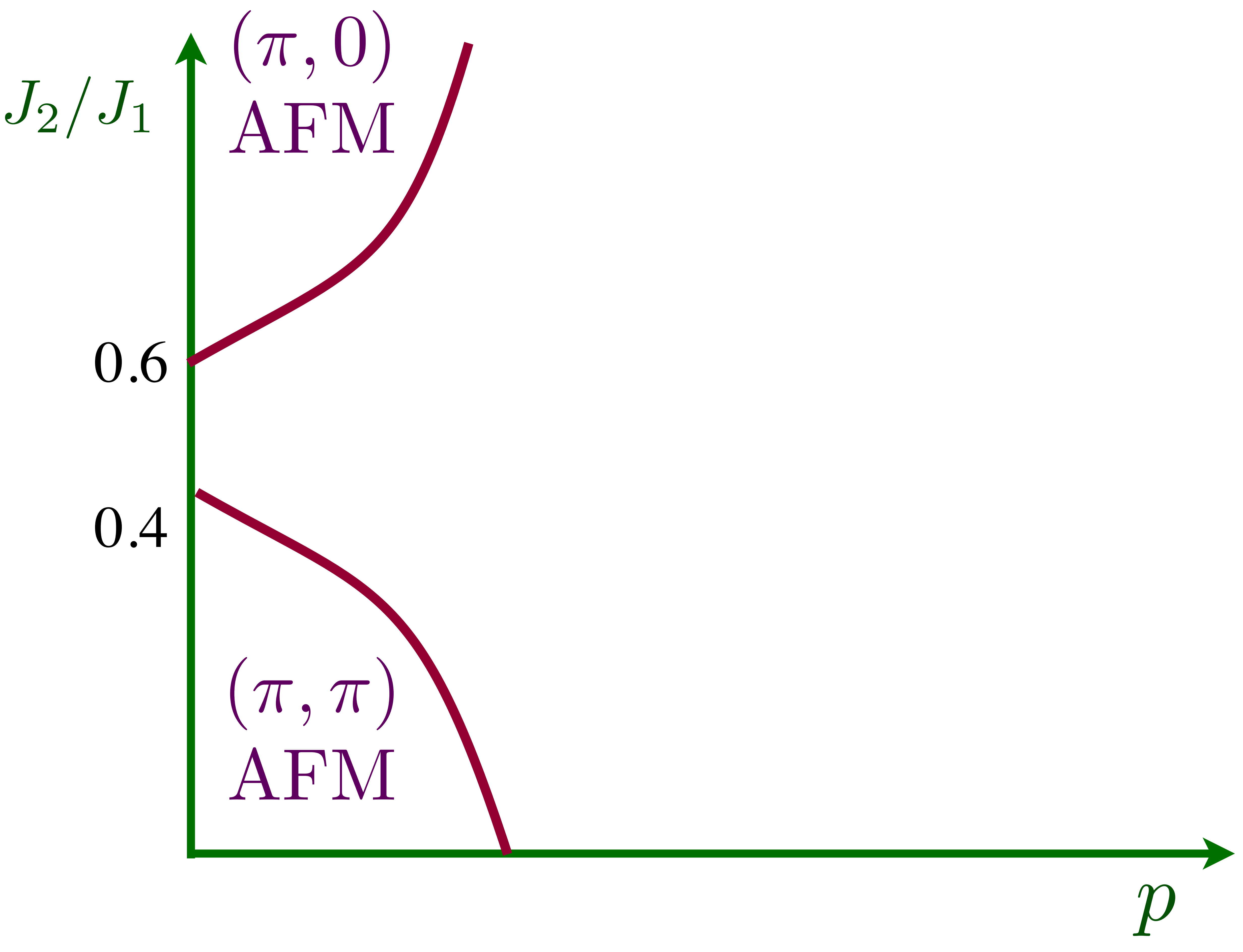


Nvsen Ma, Guang-Yu Sun, Yi-Zhuang You, Cenke Xu, Ashvin Vishwanath, Anders W. Sandvik, and Zi Yang Meng, PRB **98**, 174421 (2018)

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

PHYSICAL REVIEW LETTERS **127**, 097002 (2021)



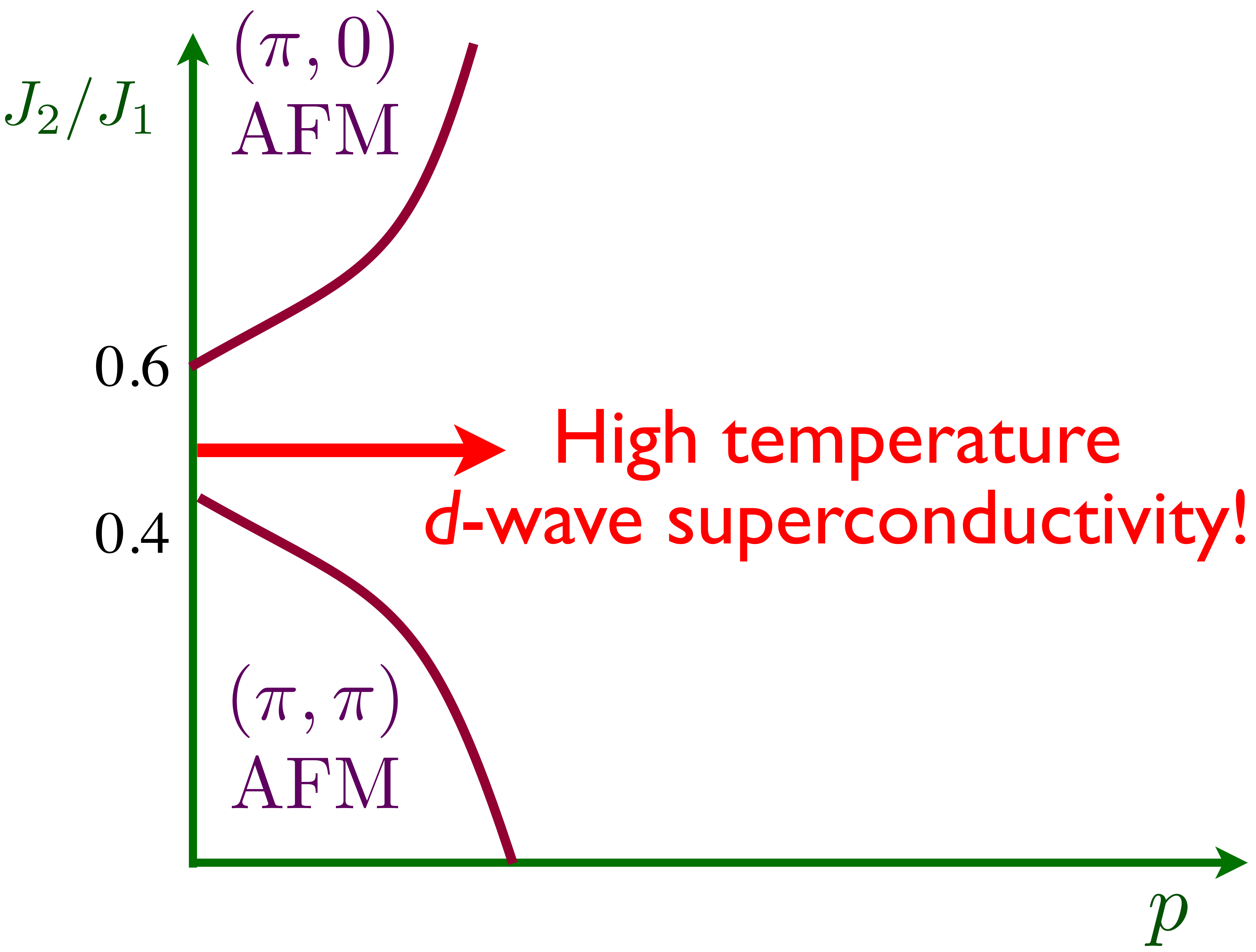
Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

PHYSICAL REVIEW LETTERS **127**, 097002 (2021)

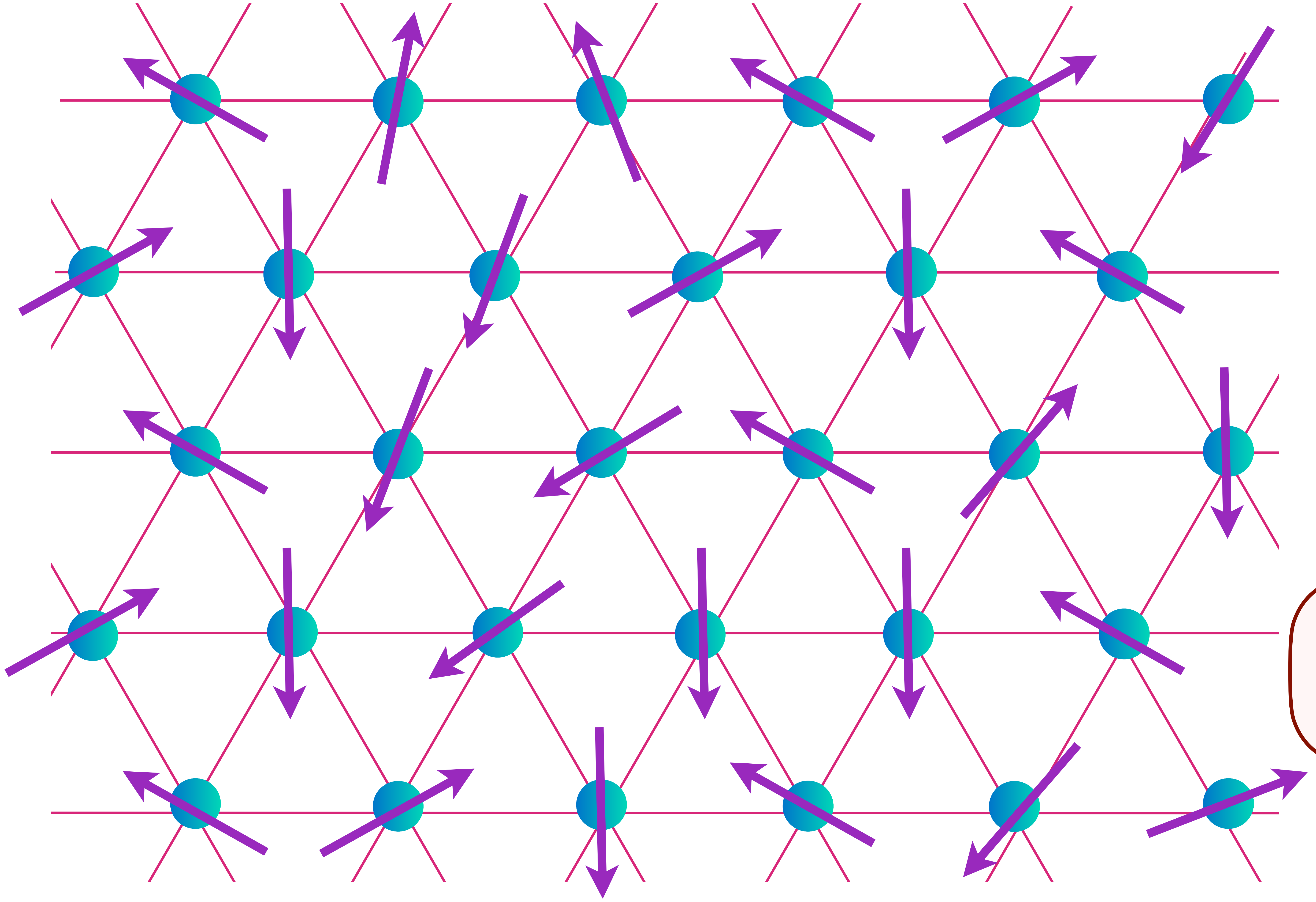


Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

Kondo lattice



Kondo exchange J_K

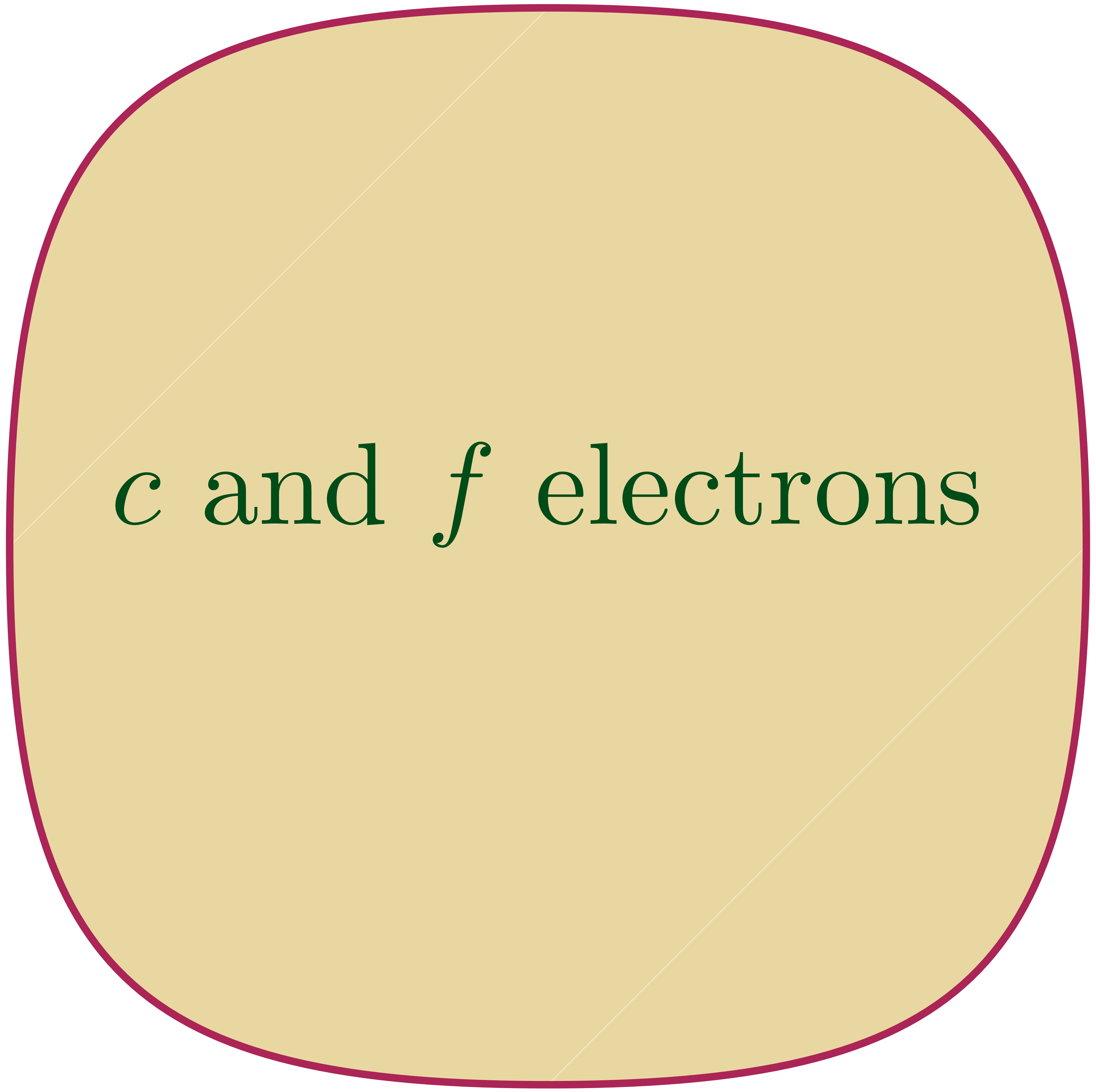
c electrons

Density of the electrons per unit cell $= 1 + p$

$$J_K \boldsymbol{S}_{fi} \cdot c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{i\beta}$$

f electrons

Kondo lattice: Heavy Fermi Liquid phase



Kondo
exchange
J_K

Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= 1 + p$.
Luttinger volume “large” Fermi surface.

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

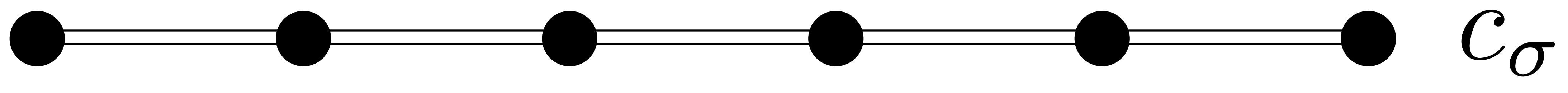
Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

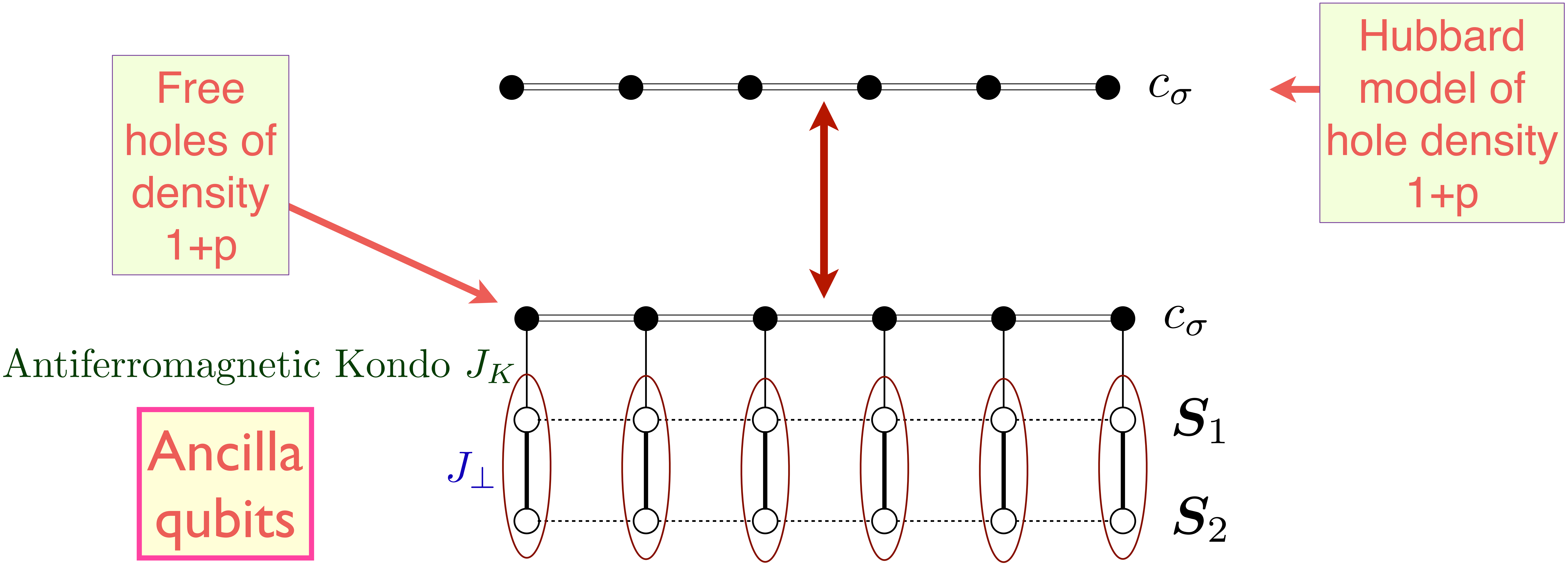
$\boldsymbol{\Phi}_i$ is the creation/annihilation operator for charge 0, spin $S = 1$ particle.

Ancilla theory of the Hubbard model



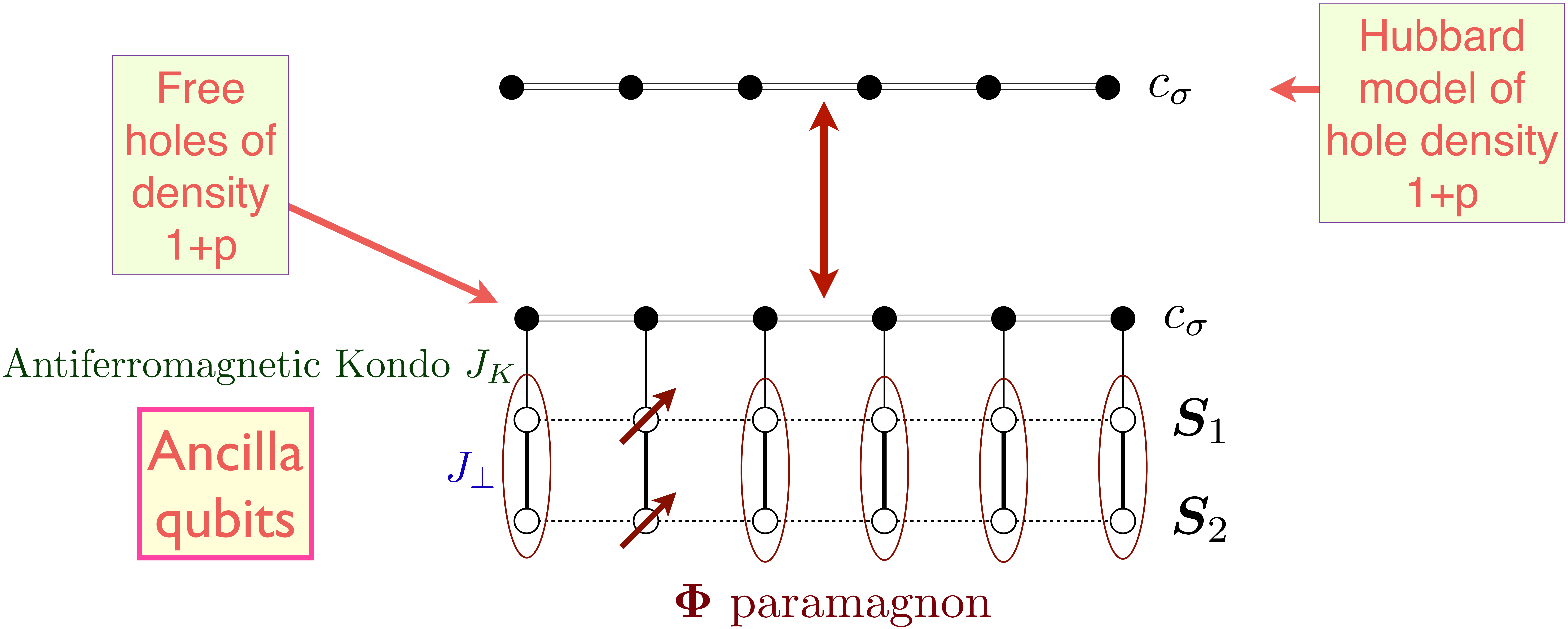
Hubbard model of hole density 1+p

Ancilla theory of the Hubbard model



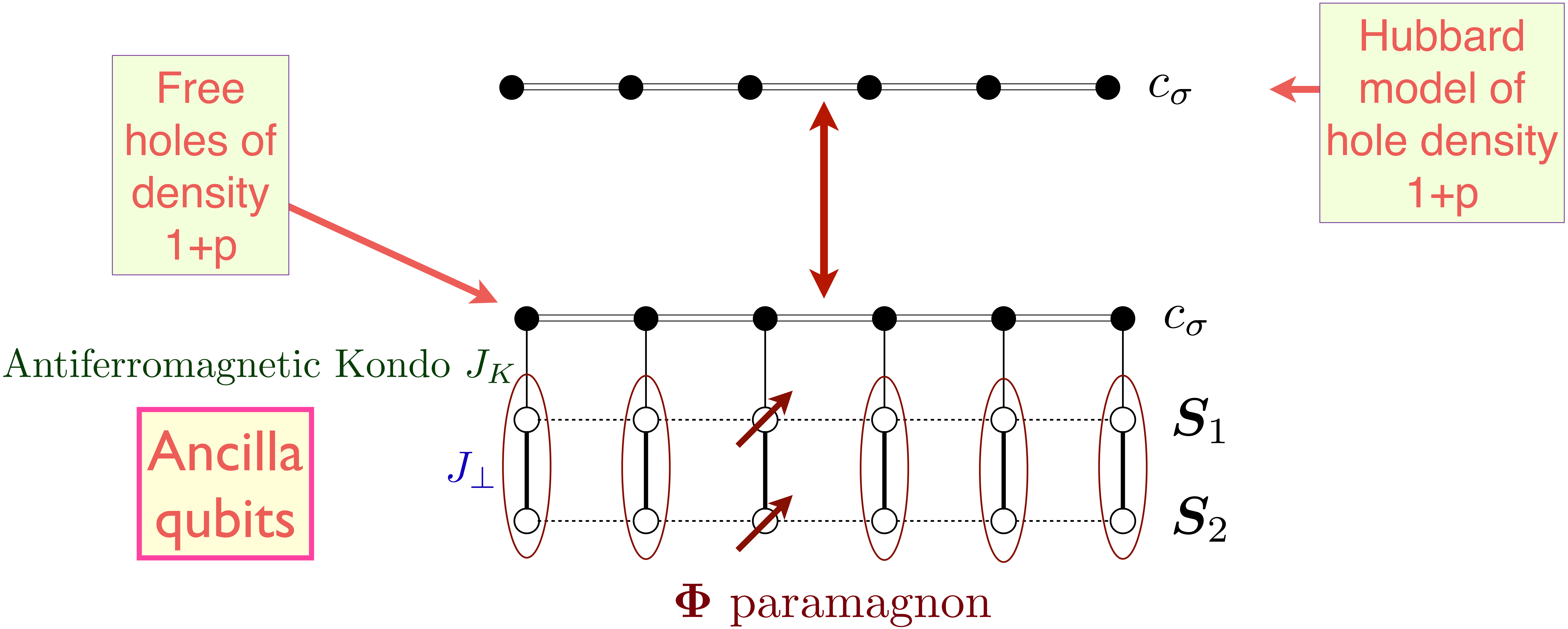
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model



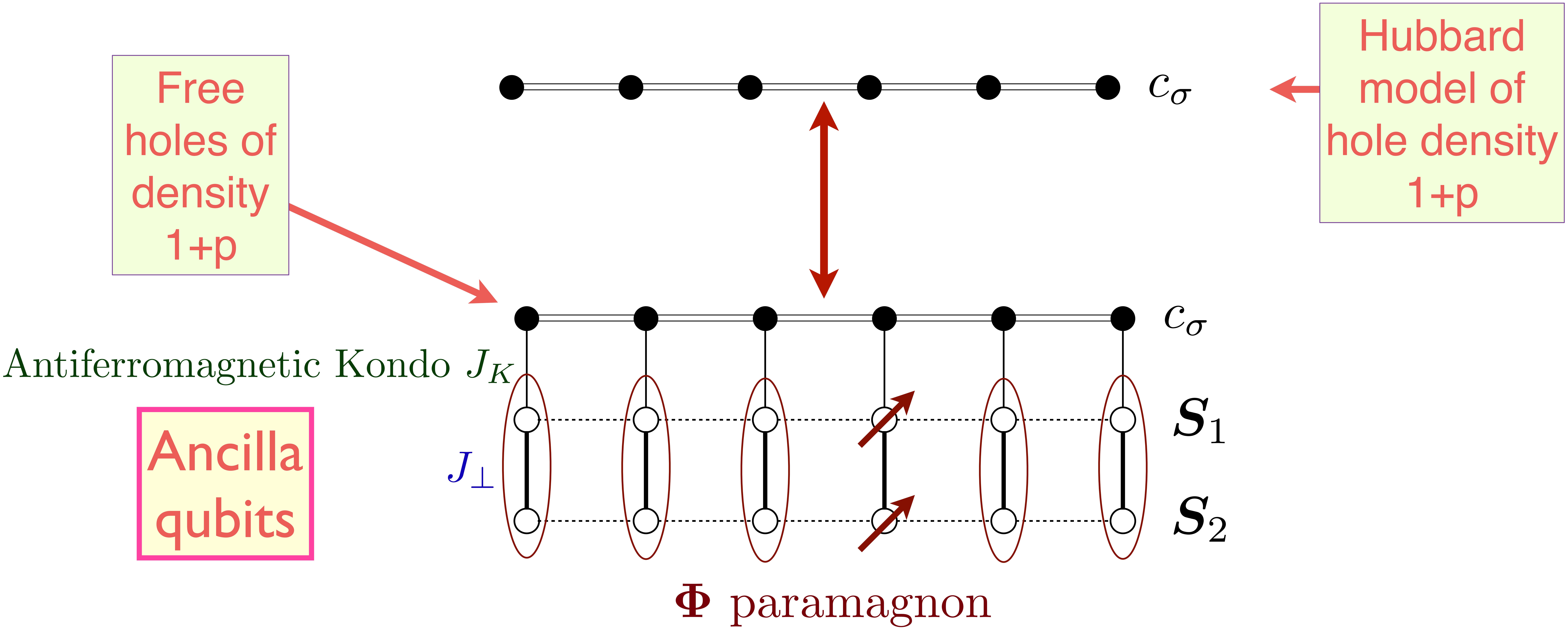
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model



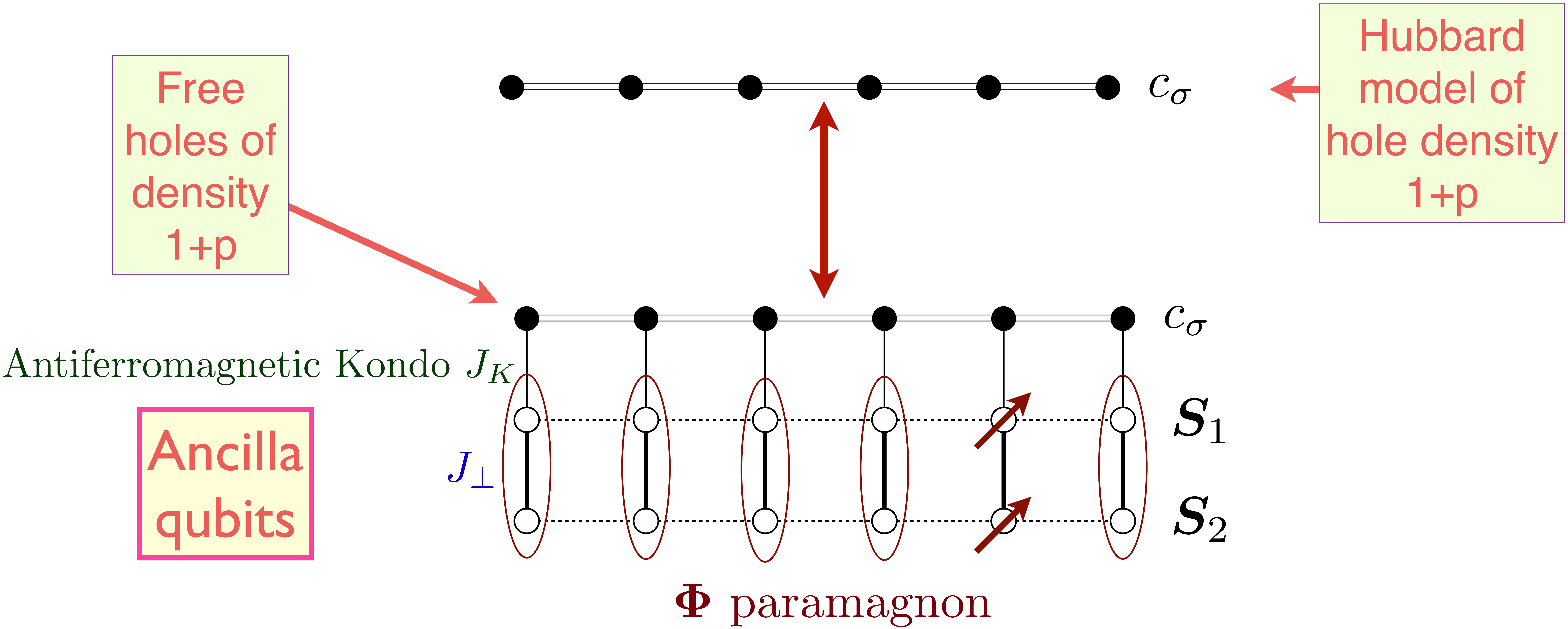
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

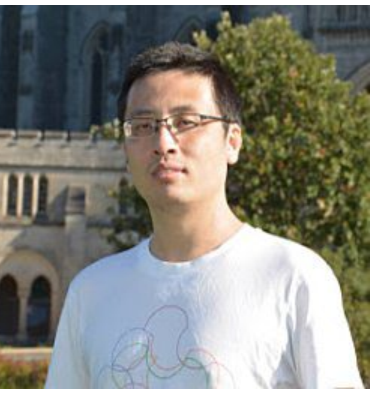
Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

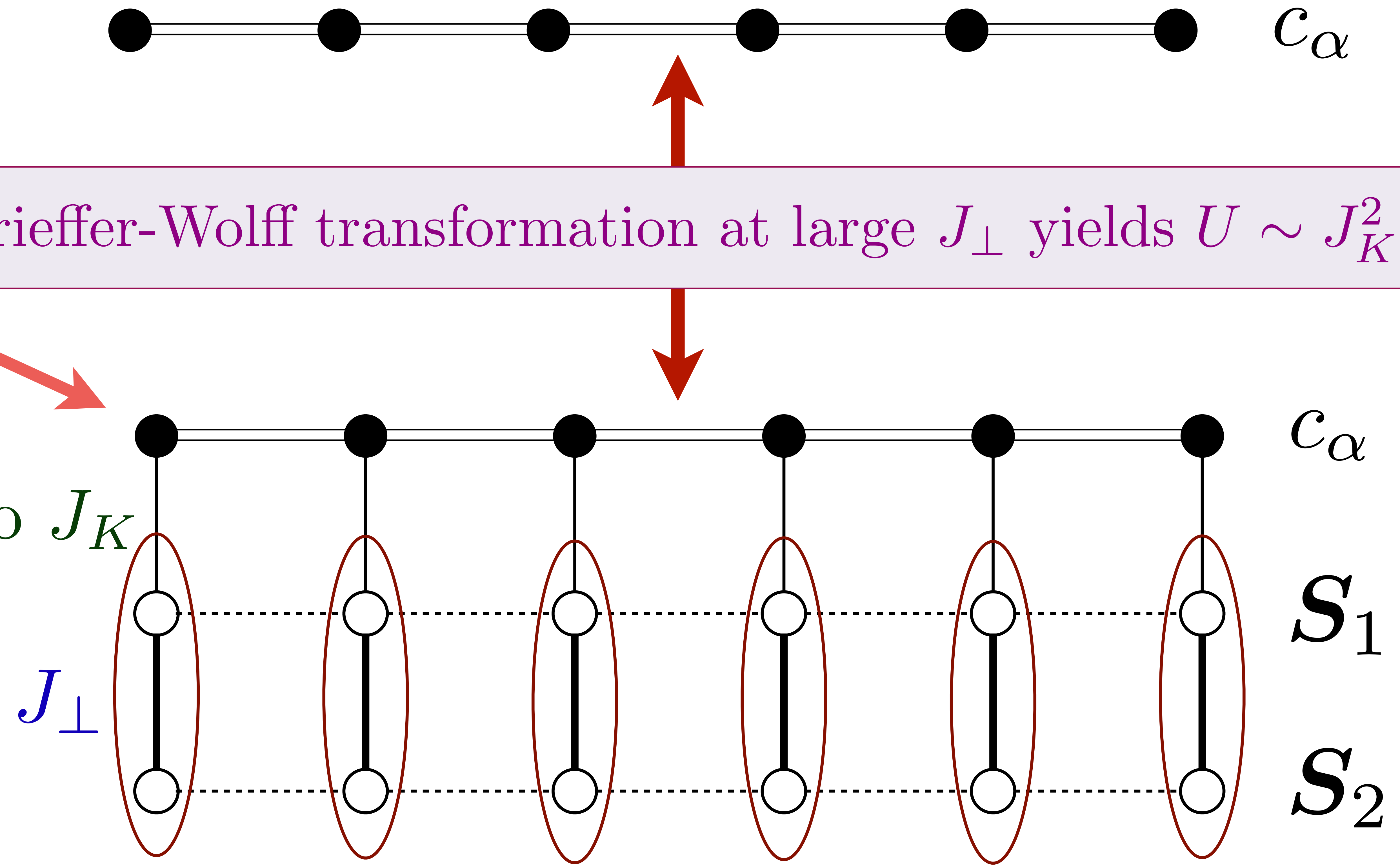
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



Ya-Hui Zhang

Free holes of density 1+p

Hubbard model of hole density 1+p



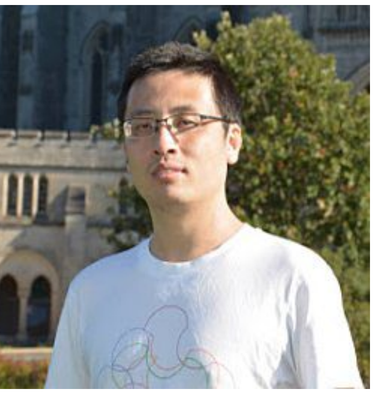
Antiferromagnetic Kondo J_K

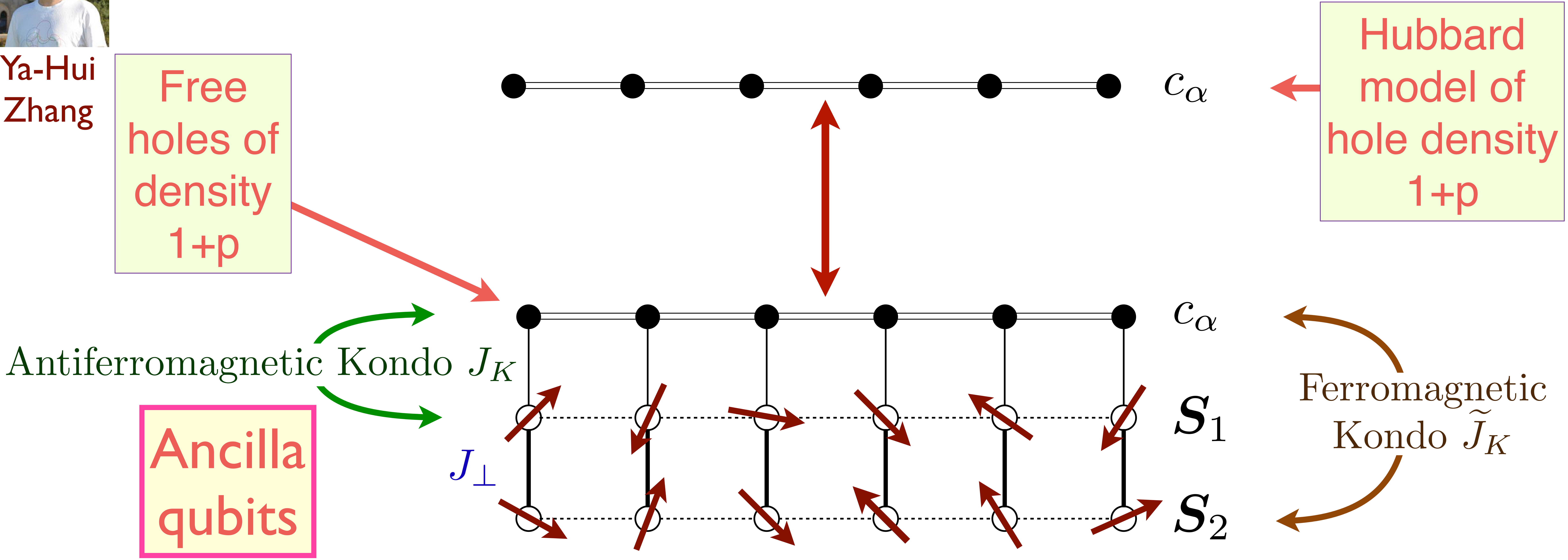
Ancilla qubits

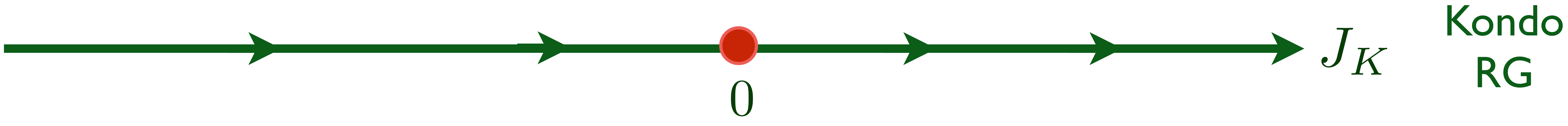
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)





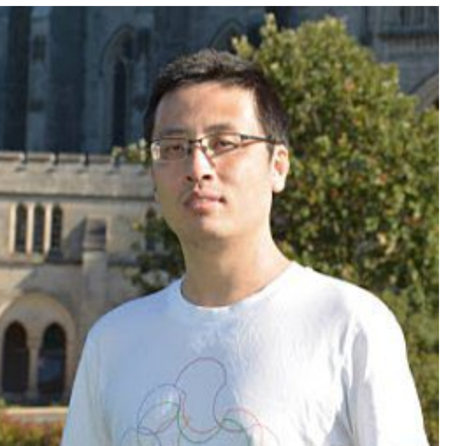


$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Ya-Hui Zhang

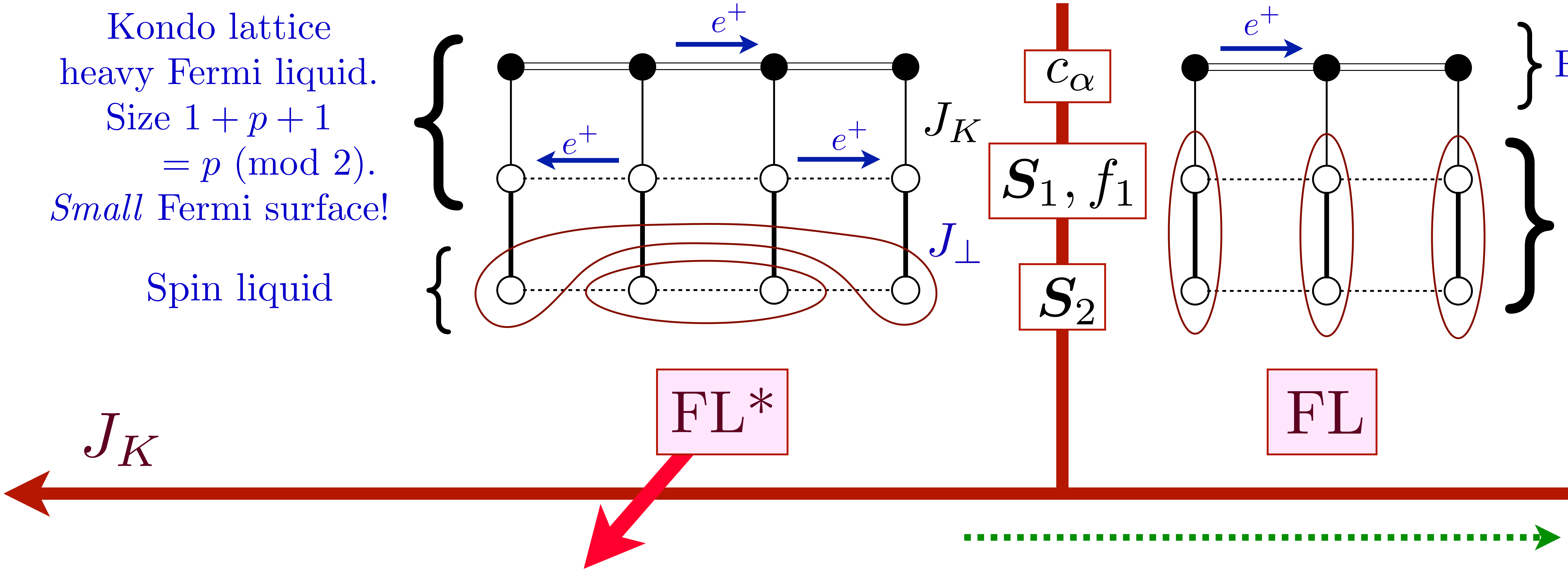


Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Kondo lattice heavy Fermi liquid. Size $1 + p + 1 = p \pmod 2$. *Small* Fermi surface!

Spin liquid



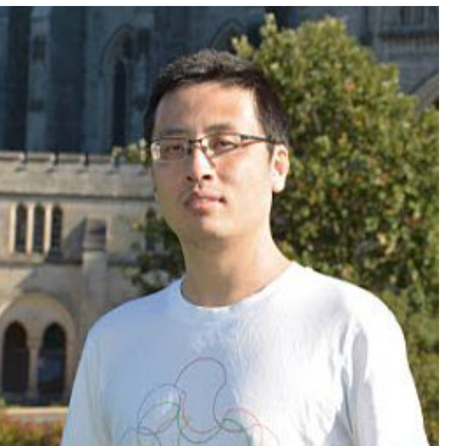
Large Fermi surface. Size: $1 + p$

Trivial insulator

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

$\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$

Ya-Hui Zhang



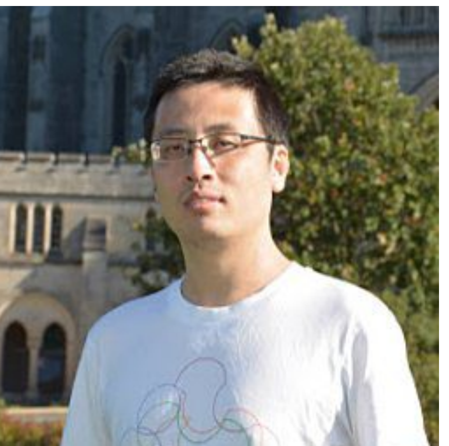
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

$\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$

Ya-Hui Zhang



Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Kondo lattice heavy Fermi liquid. Size $1 + p + 1 = p \pmod 2$. *Small* Fermi surface!

Spin liquid

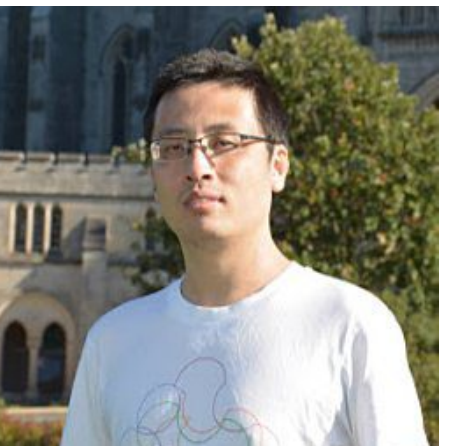
Large Fermi surface. Size: $1 + p$

Trivial insulator

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

$\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$

Ya-Hui Zhang



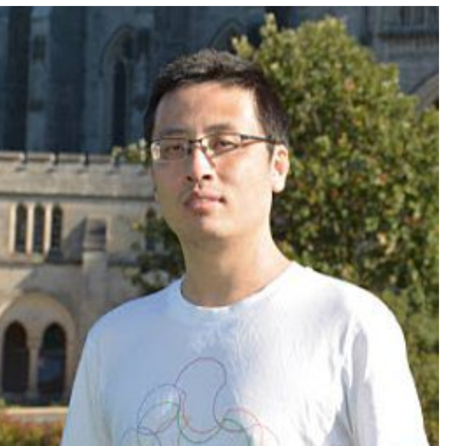
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

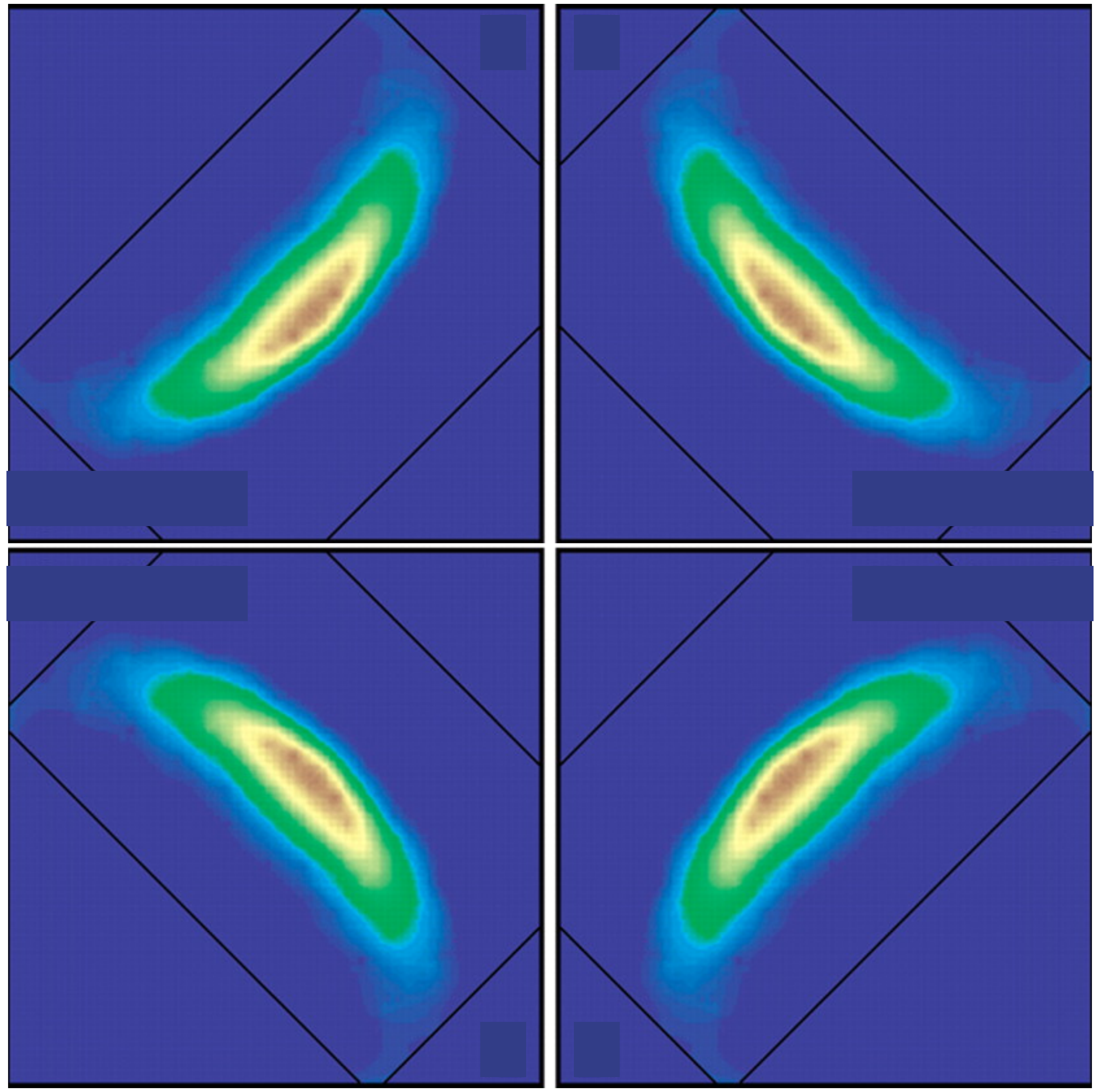
$\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$

Ya-Hui Zhang

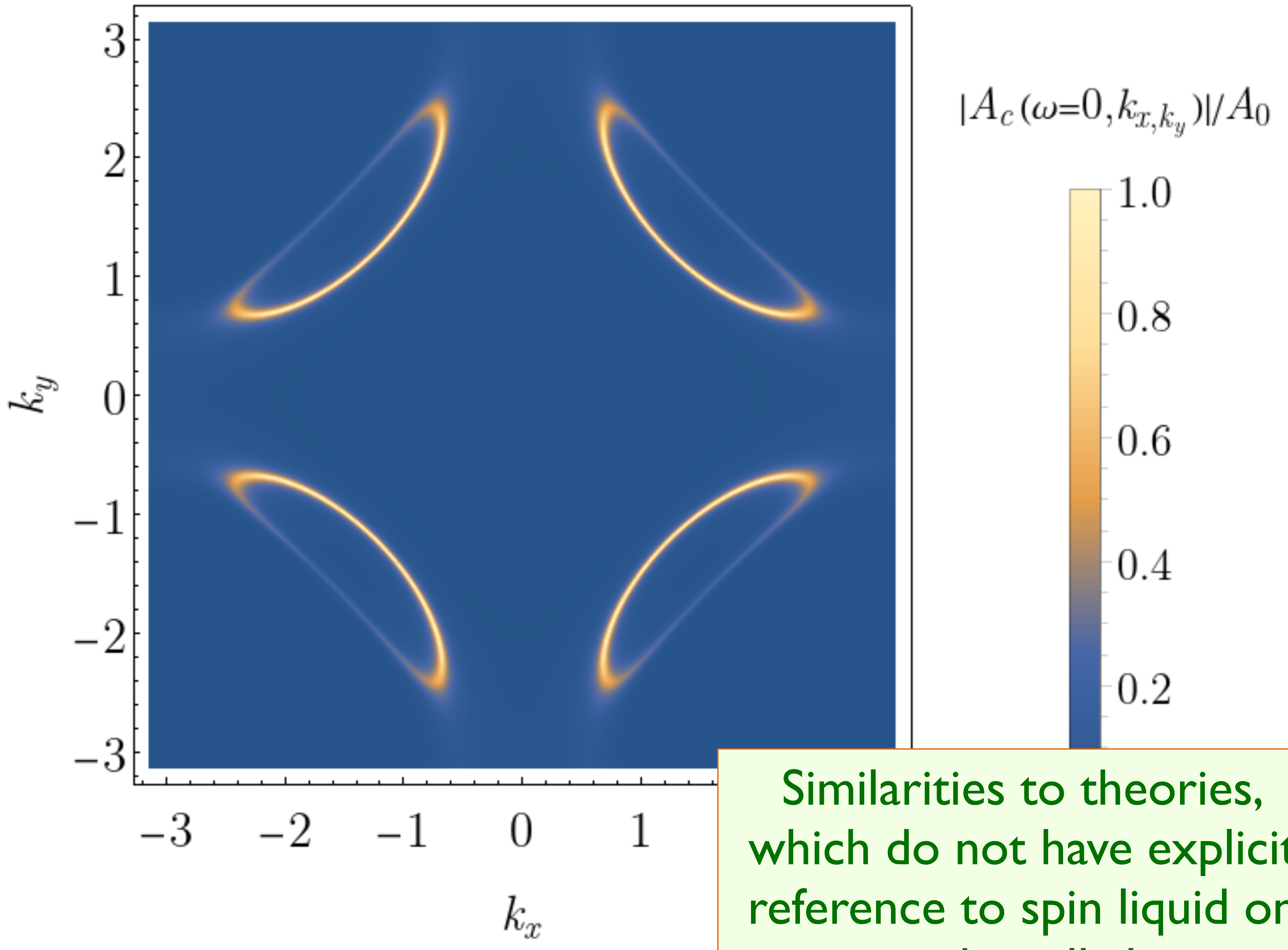


Photoemission at small p

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$



"Fermi arcs"

Similarities to theories, which do not have explicit reference to spin liquid on second ancilla layer

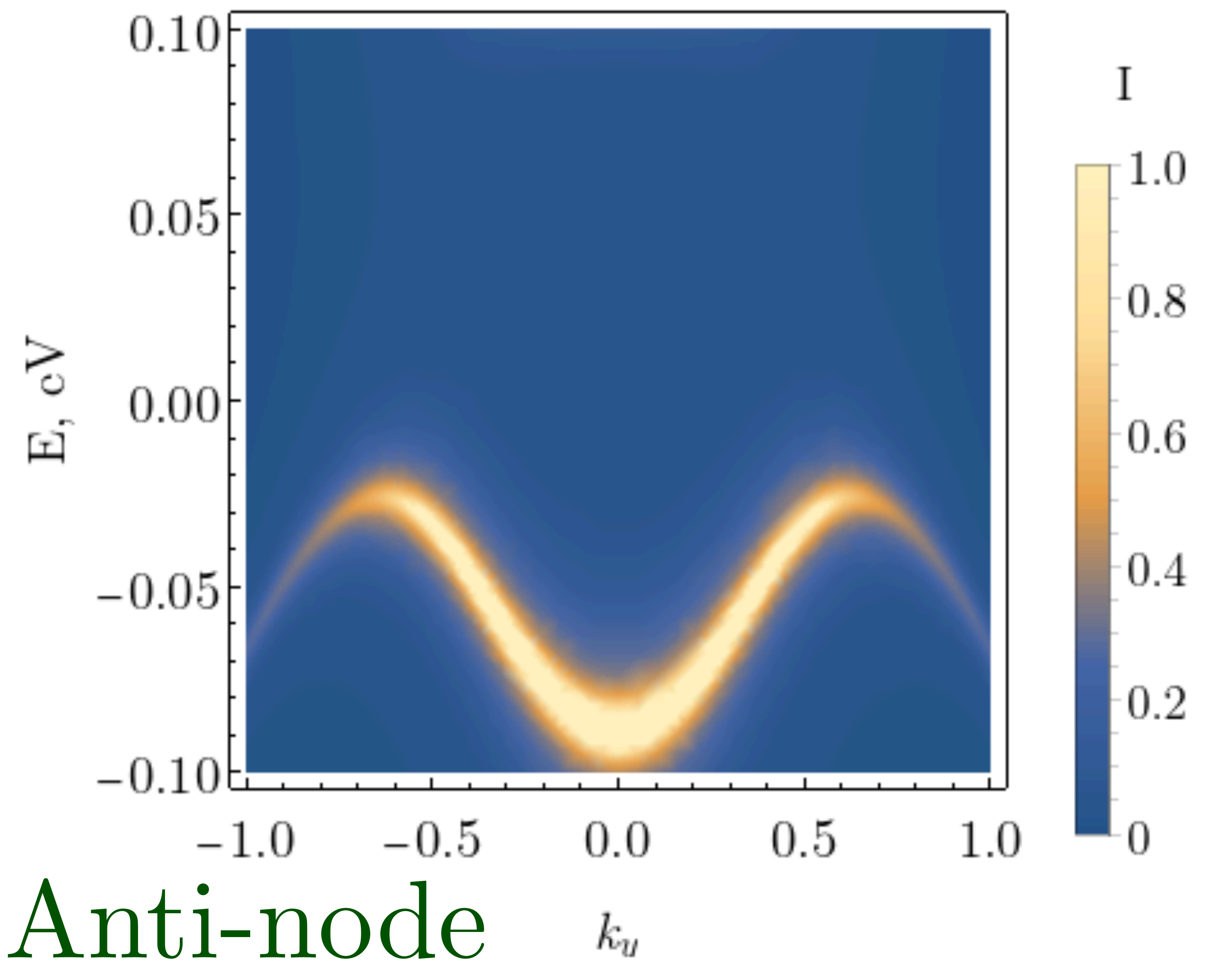
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009)

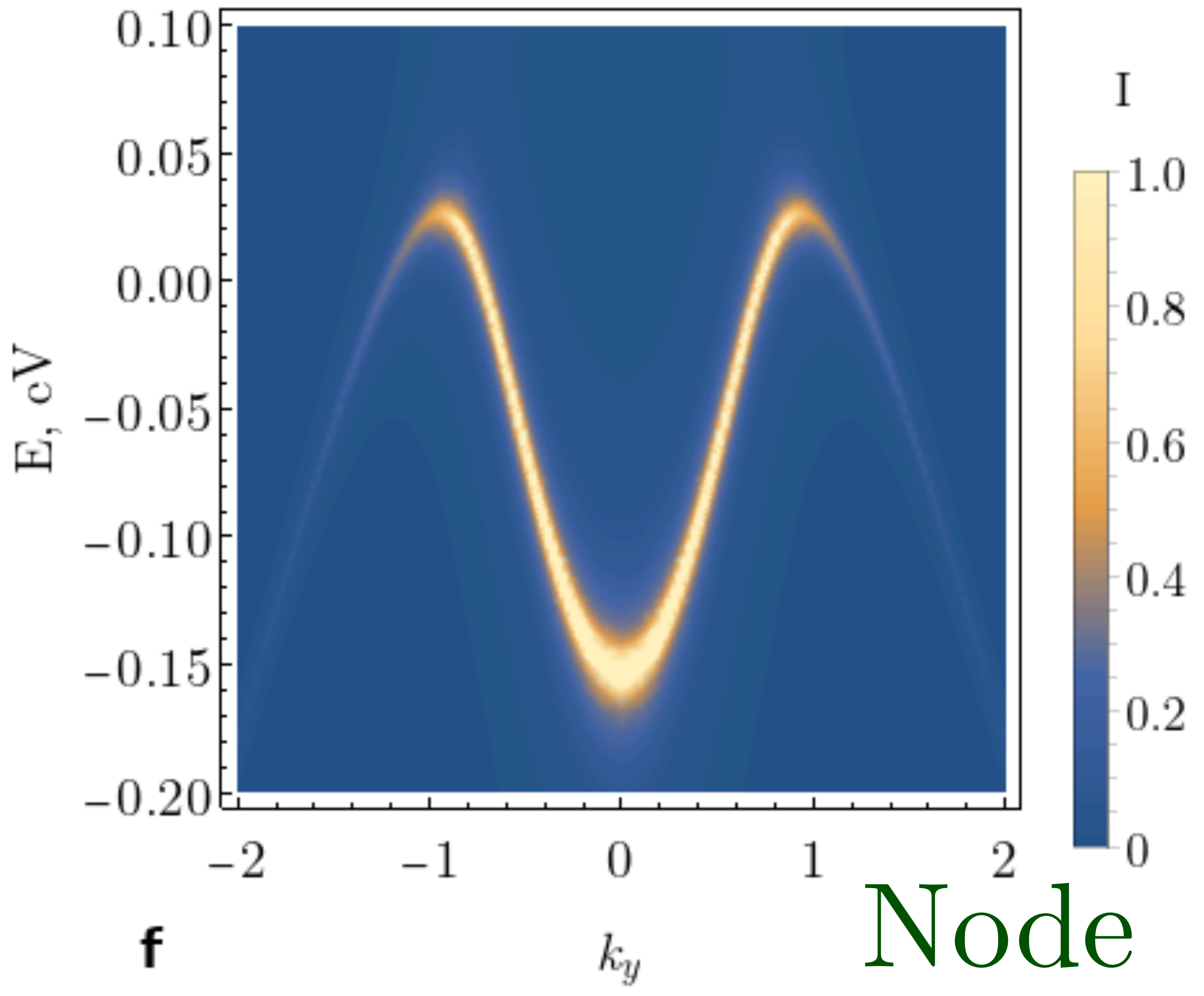
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

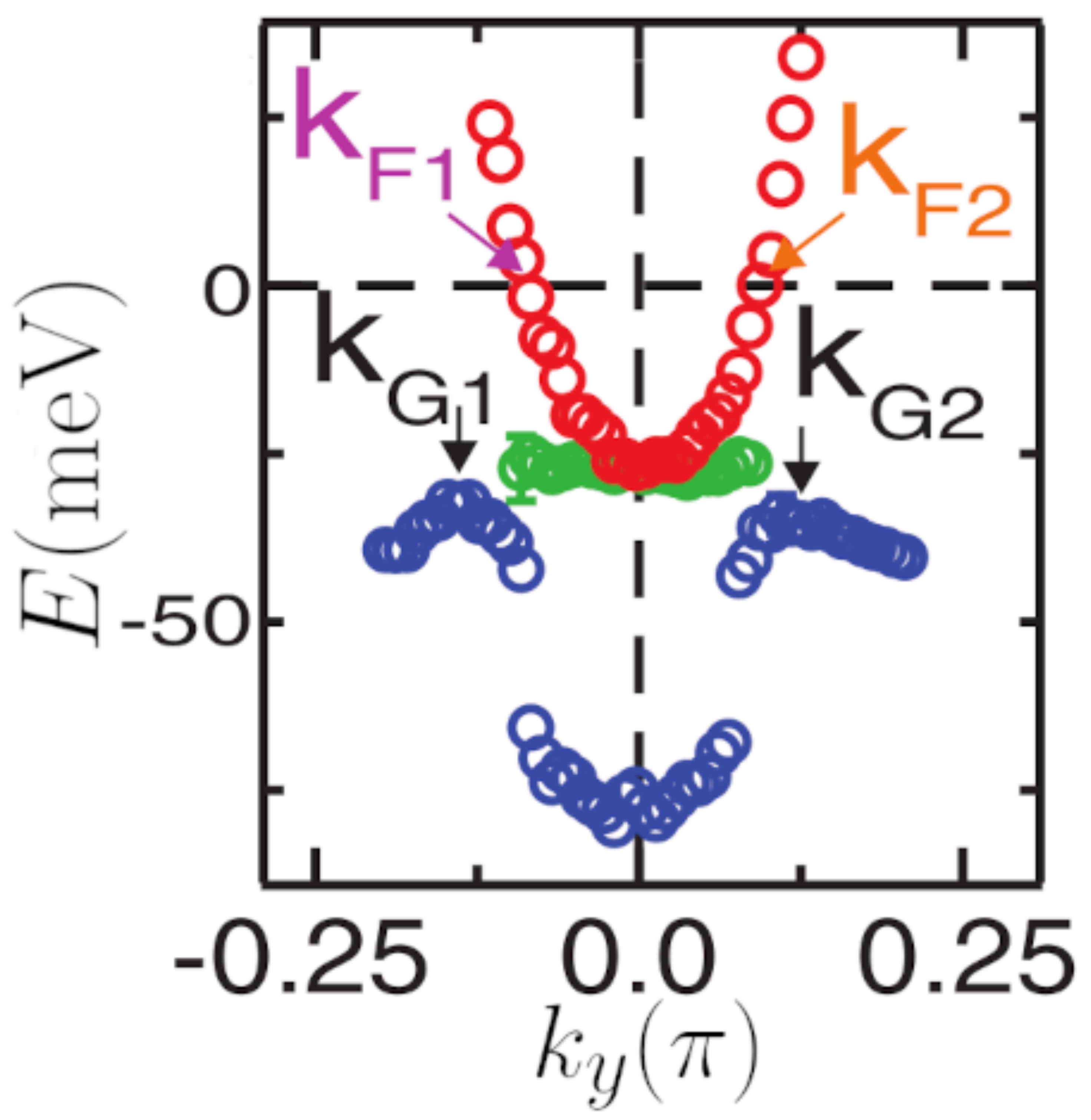
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

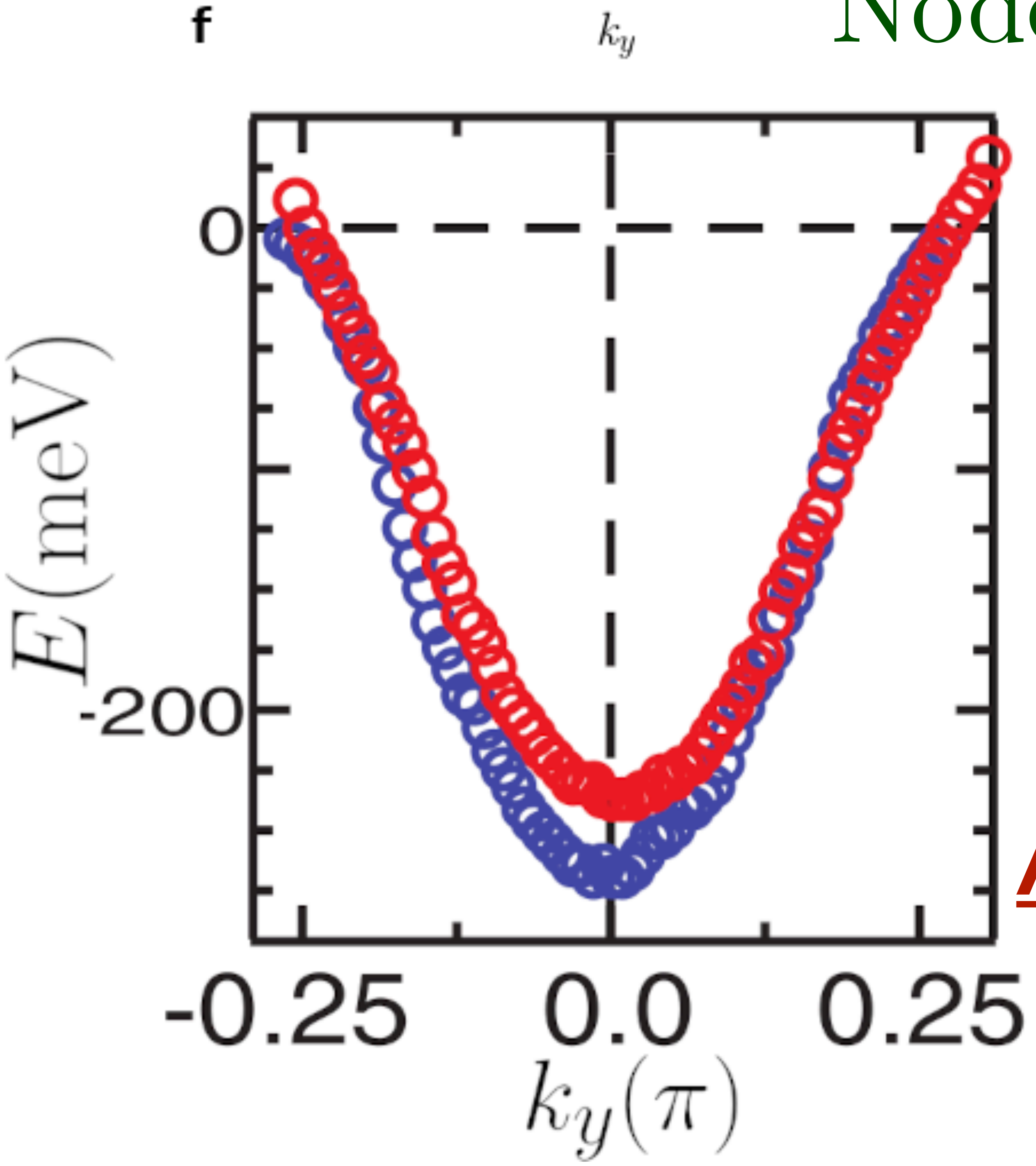
FL* in a ***one-band*** model

Second ancilla layer is needed to describe MDC and EDC









R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

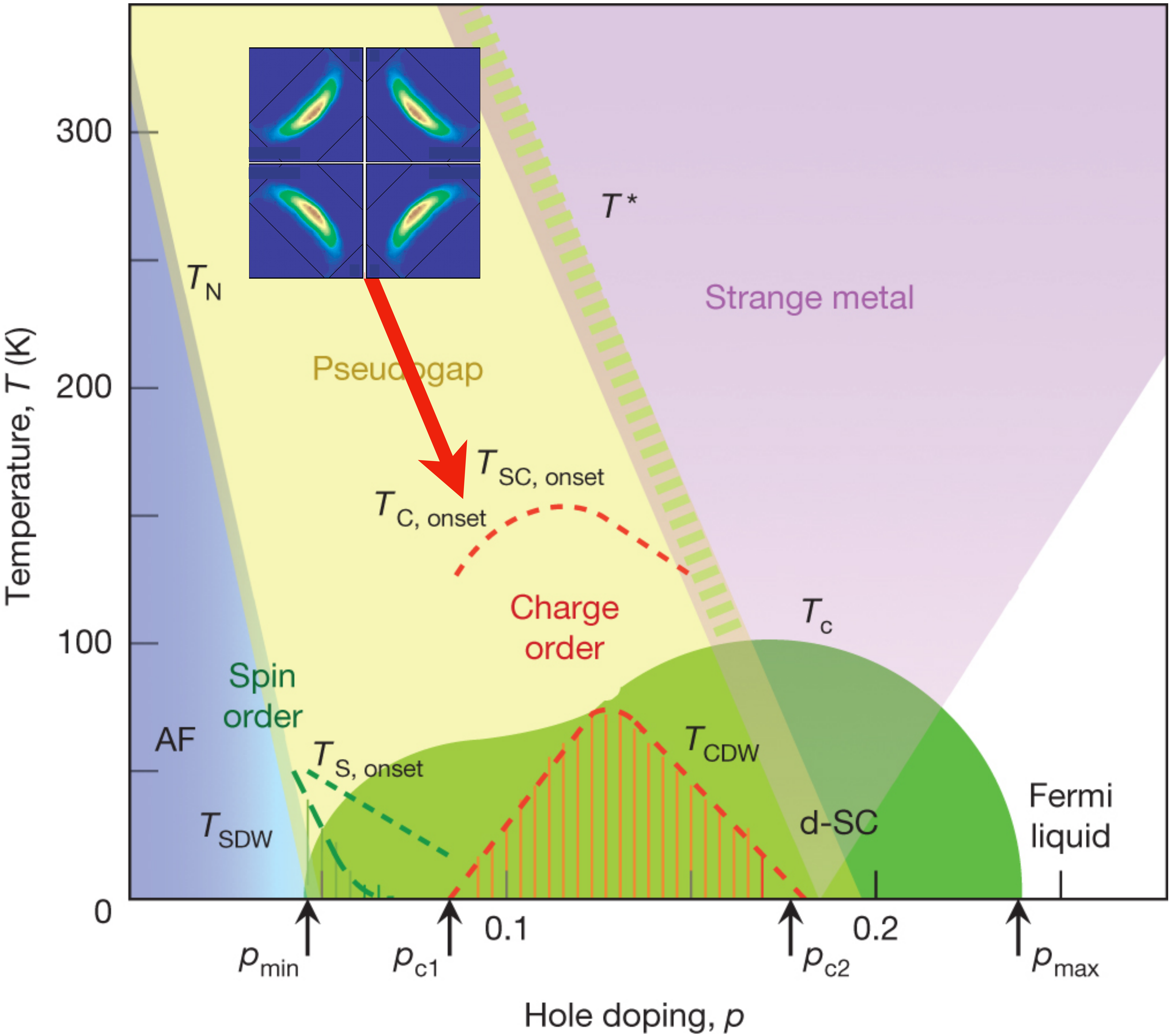
ARPES on Bi2201

Similarities to theories, which do not have explicit reference to spin liquid on second ancilla layer

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009)

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



A theory for the confinement of fractionalized excitations in the *π-flux spin liquid with fermionic spinons* dual to the *$\mathbb{CP}^1$ spin liquid with bosonic spinons* from electrically charged excitations.

Ancilla theory of the Hubbard model

Pseudogap metal: $\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$

f_α form π-flux spin liquid with $\mathrm{SU}(2)_N$ gauge field

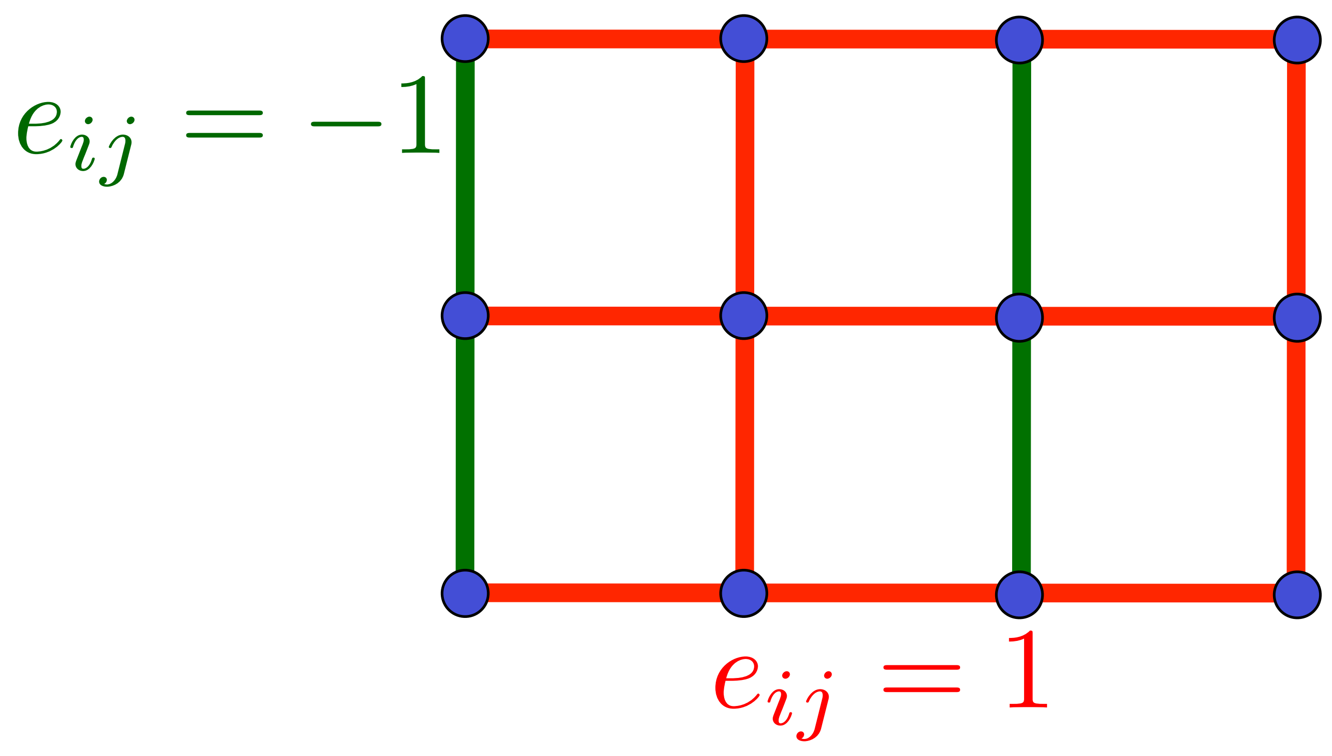
Charge e, $\mathrm{SU}(2)_N$ fundamental, Higgs boson $B \sim \begin{pmatrix} f_{1\alpha}^\dagger f_\alpha \\ \varepsilon_{\alpha\beta} f_{1\alpha}^\dagger f_\beta^\dagger \end{pmatrix}$

Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f^\dagger_{i\alpha} f_{j\alpha} - f^\dagger_{j\alpha} f_{i\alpha} \right)$$

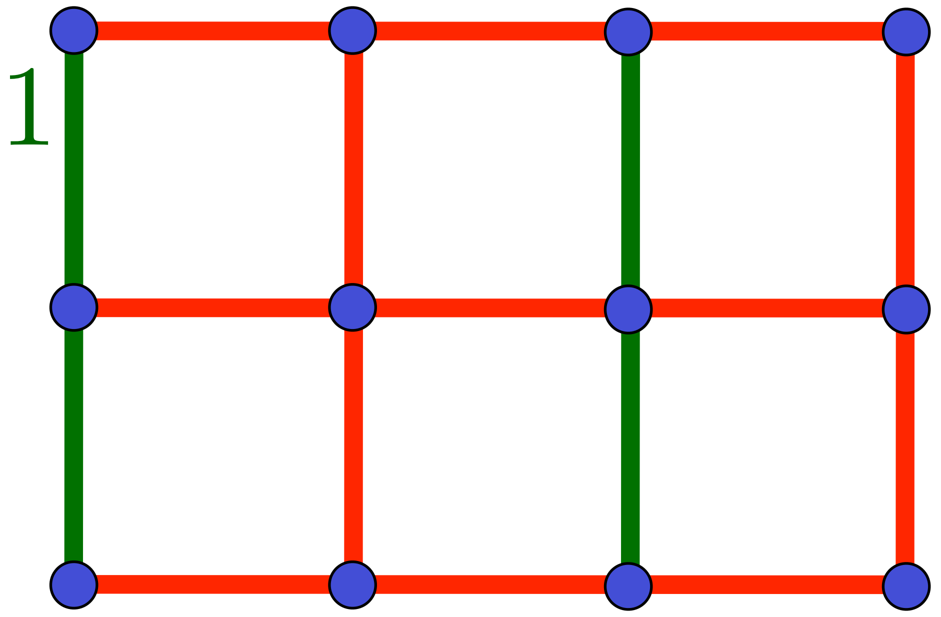


Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.

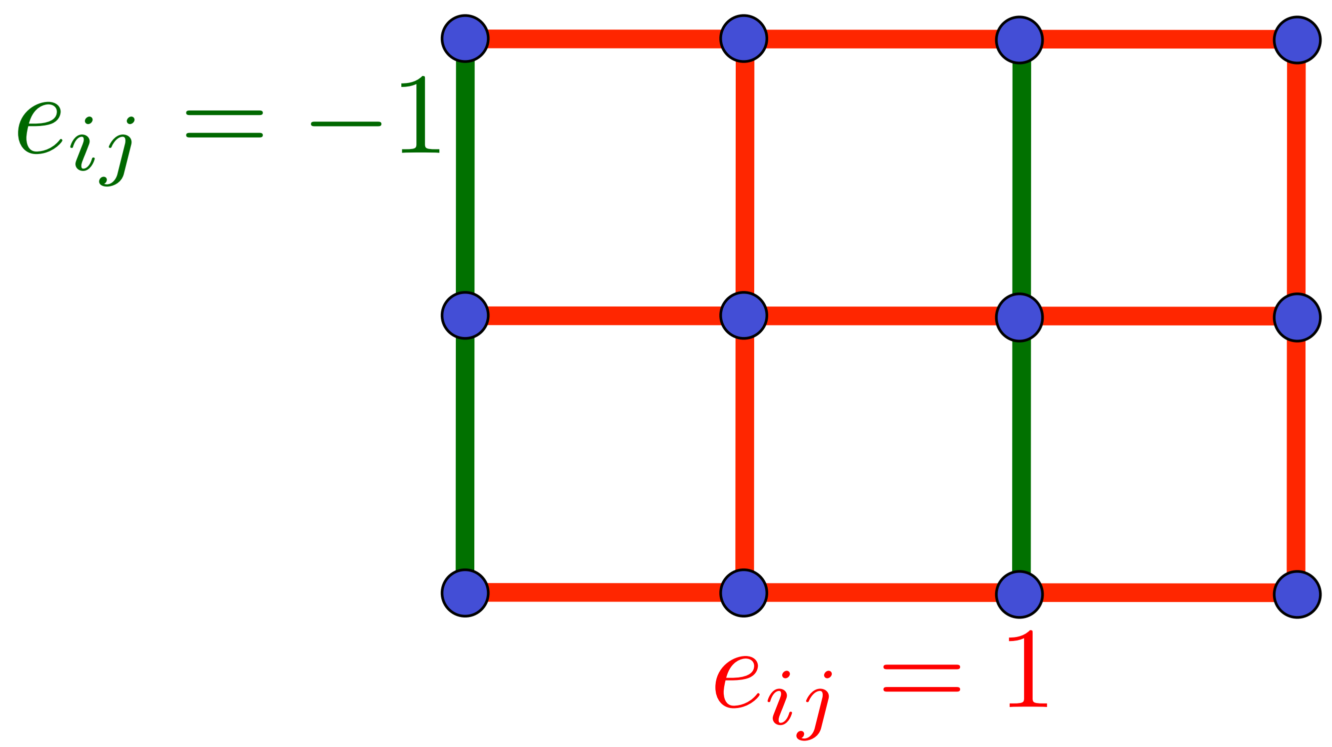


Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f^\dagger_{i\alpha} f_{j\alpha} - f^\dagger_{j\alpha} f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi^\dagger_i U_{ij} \Psi_j - \Psi^\dagger_j U_{ji} \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f^\dagger_{i\downarrow} \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



- The nearest-neighbor effective Hamiltonian for charge e, SU(2)$_N$ fundamental boson B_i is constrained by the fact that the composite of B_i and Ψ_i is an electron:

$$H_B = r \sum_i B^\dagger_i B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B^\dagger_i U_{ij} B_j - B^\dagger_j U_{ji} B_i \right) + \ldots$$

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{ij} \rangle} \left|\Delta_{\boldsymbol{ij}}\right|^2 + J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2 .$$

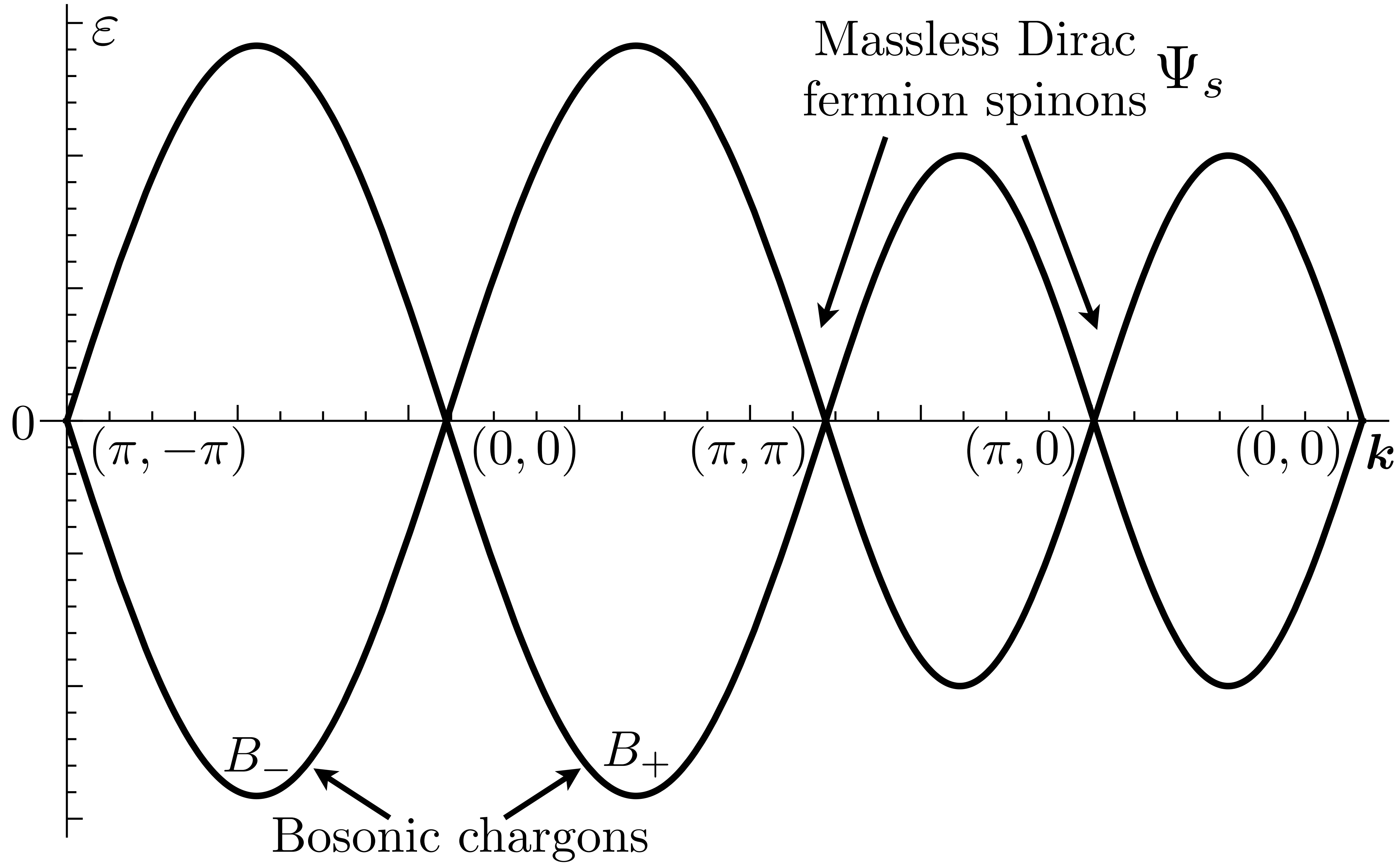
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

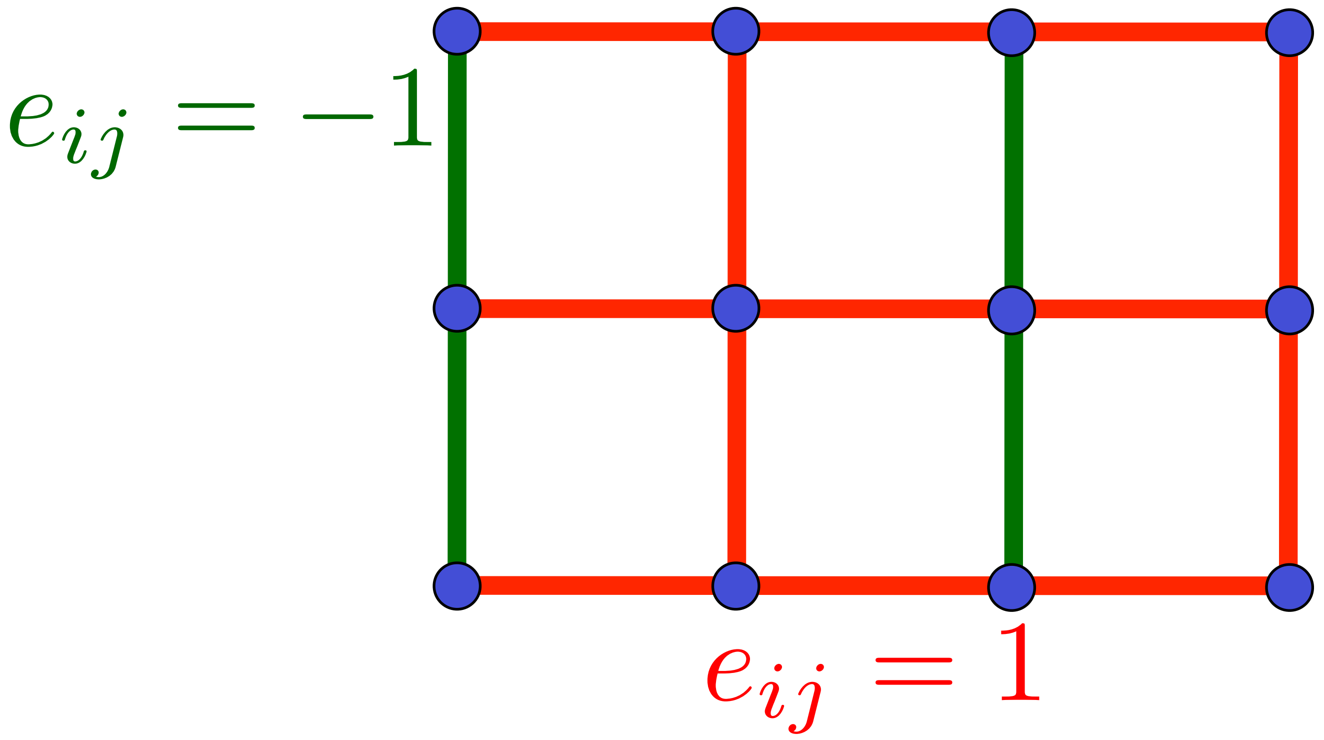
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

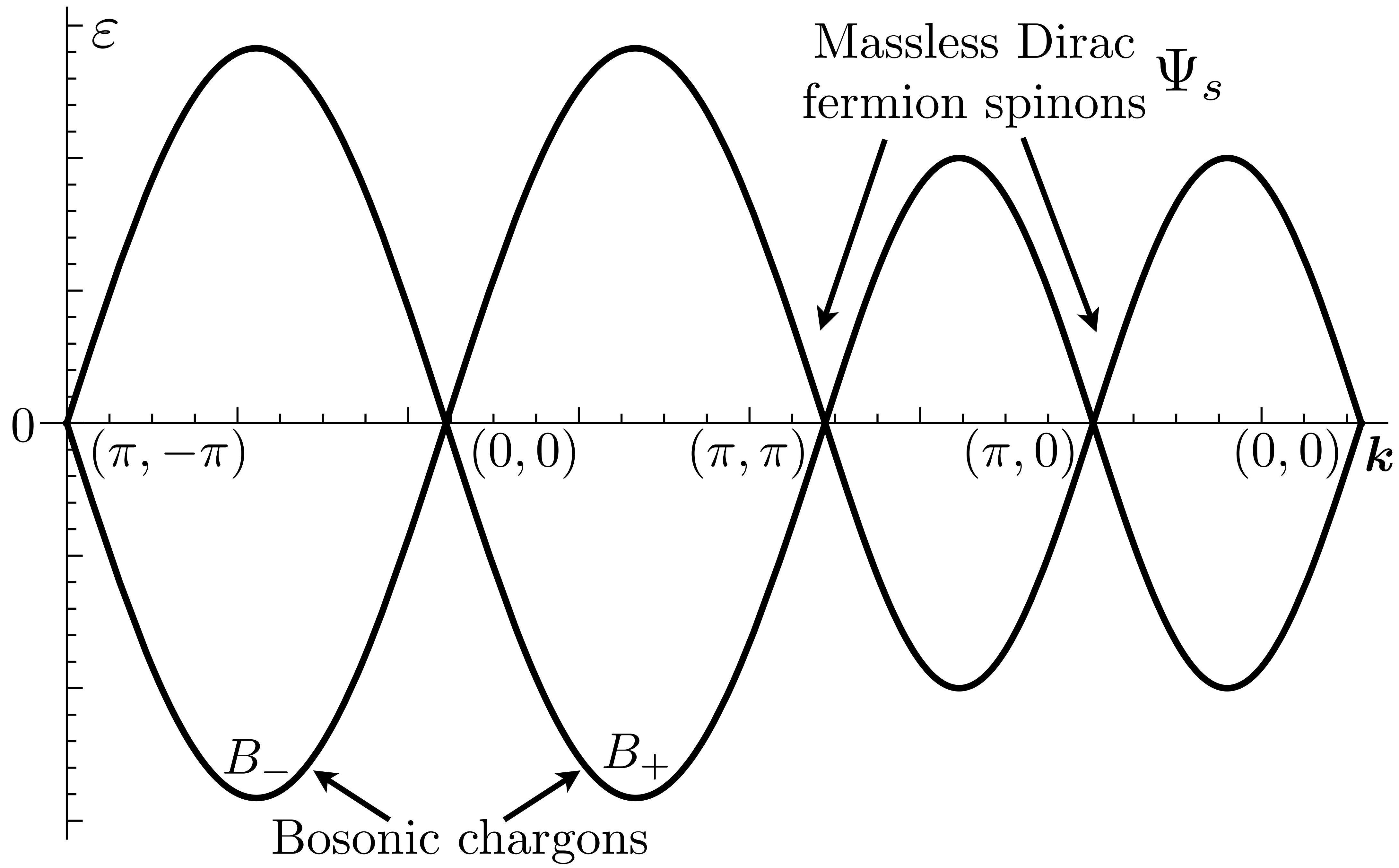
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.

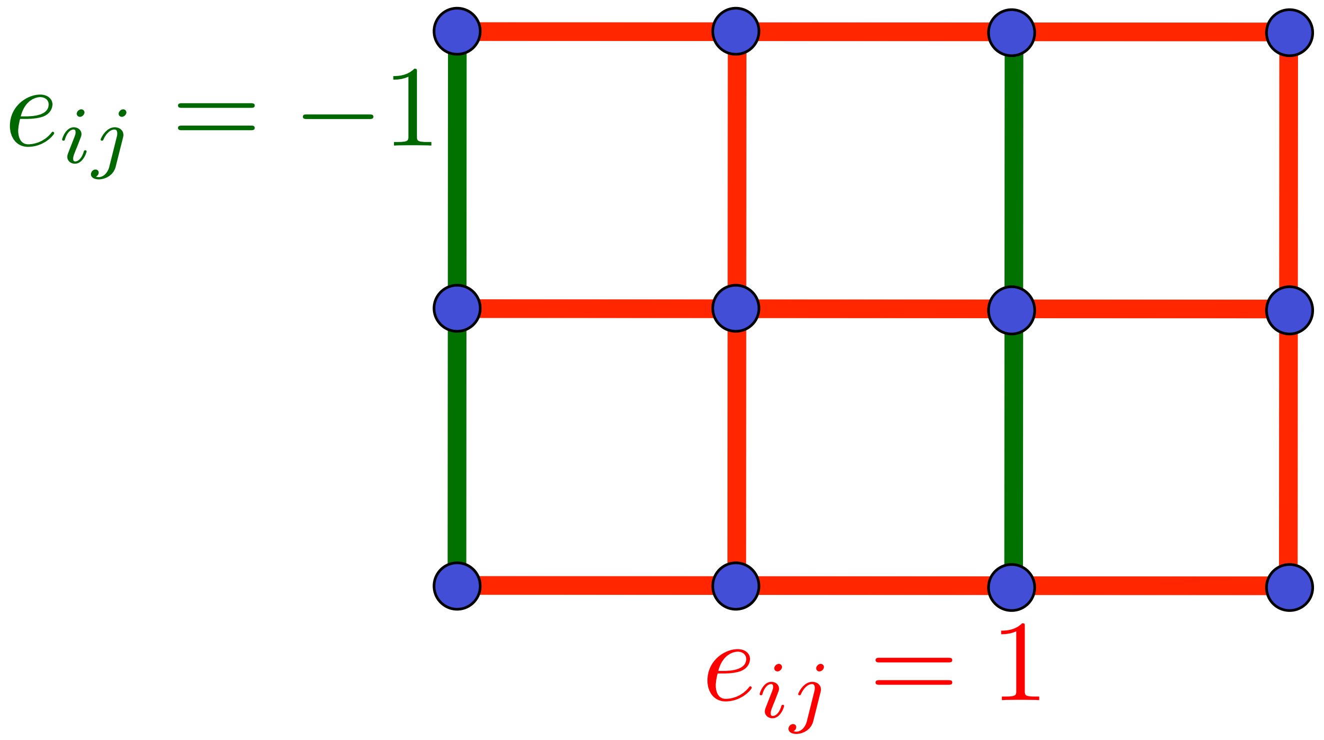
Confinement of SU(2)$_N$ gauge theory by charge fluctuations





Confinement of $SU(2)_N$ gauge theory by charge fluctuations





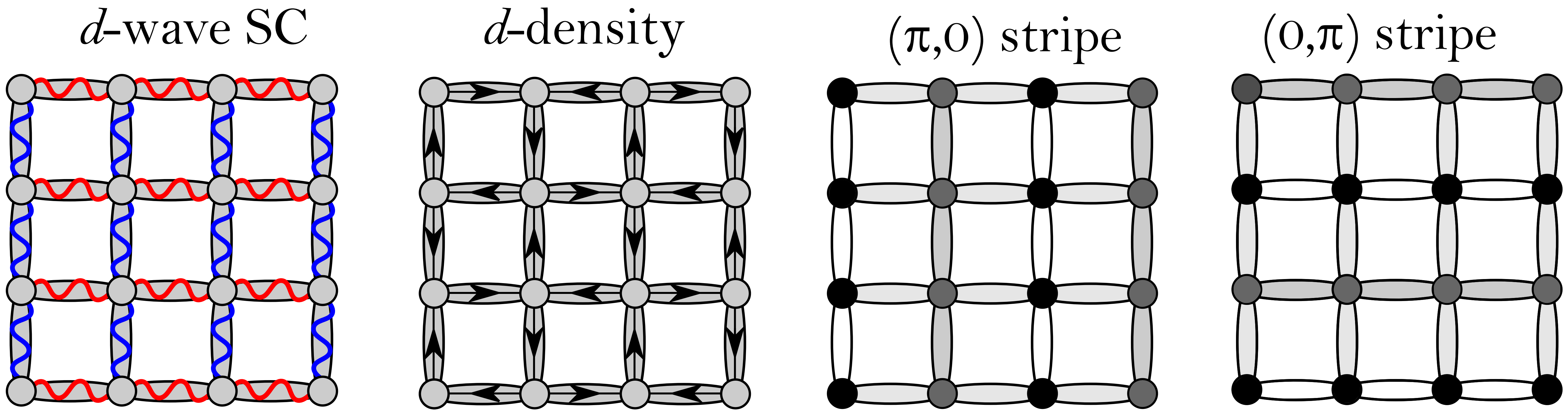
$SU(2)_N$ gauge-invariant and $SU(2)$ spin invariant order parameters of Higgs phases:

$$x\text{-CDW}: \quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-}$$

$$y\text{-CDW}: \quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+}$$

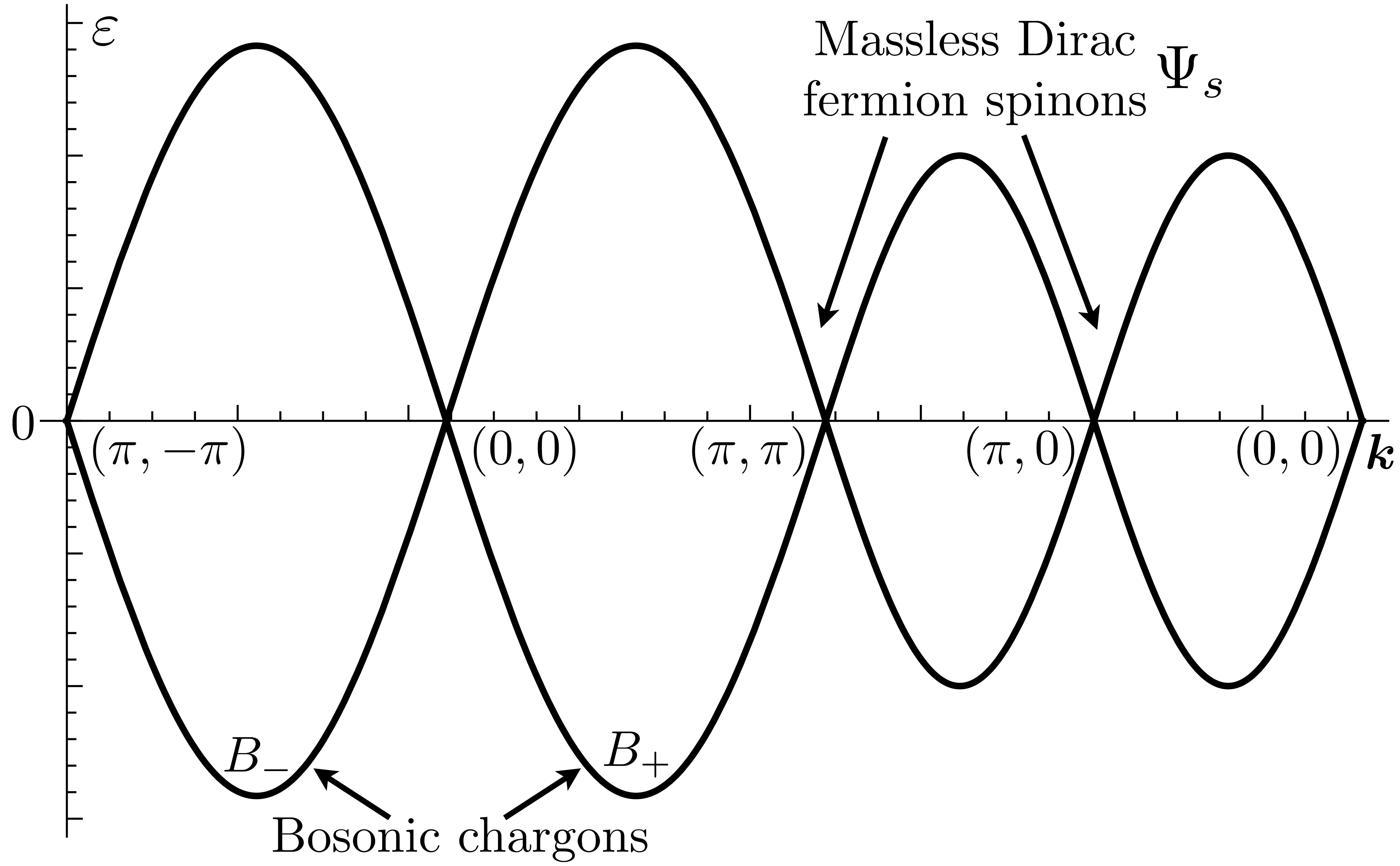
$$d\text{-density wave}: \quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right)$$

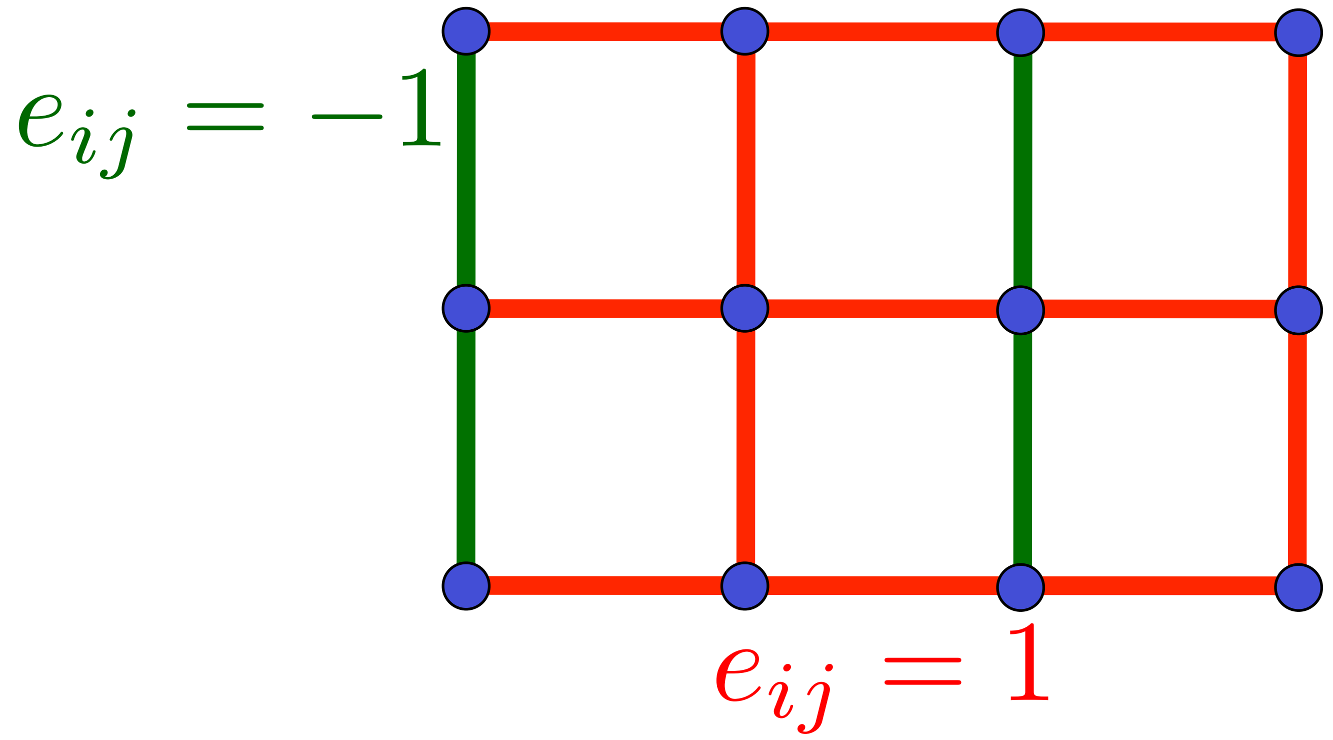
$$d\text{-wave superconductor}: \quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}$$

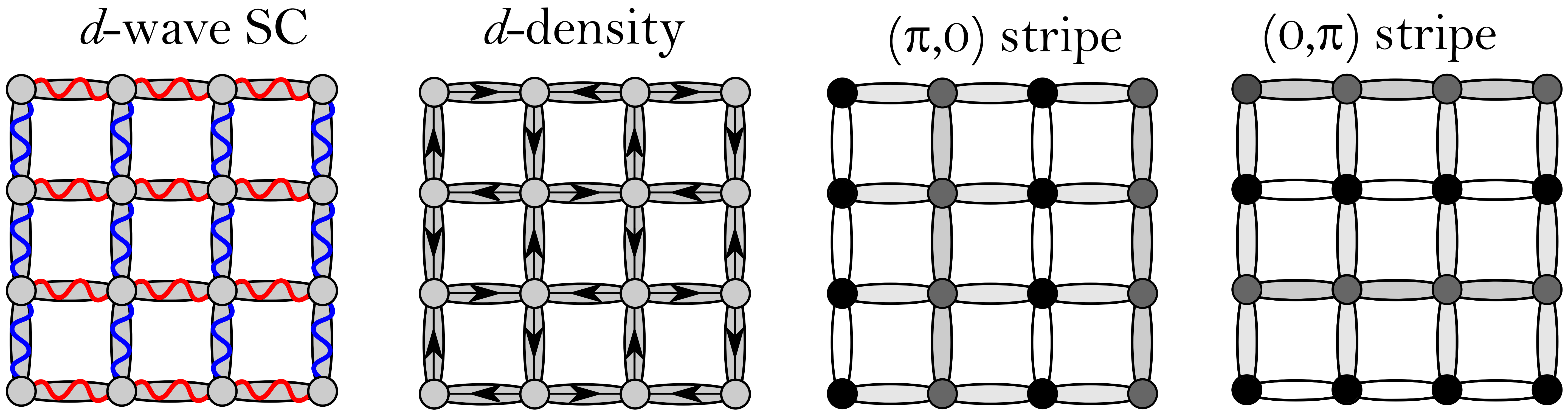


The $\mathcal{O}(B^2_{a\pm})$ terms in the energy have a $SO(5)_b$ rotation symmetry between these orders.

Confinement of SU(2)$_N$ gauge theory by charge fluctuations







The B_{av}

($a \rightarrow$ SU(2)$_N$ gauge, $v \rightarrow$ valley)

are the "square roots" of

conventional

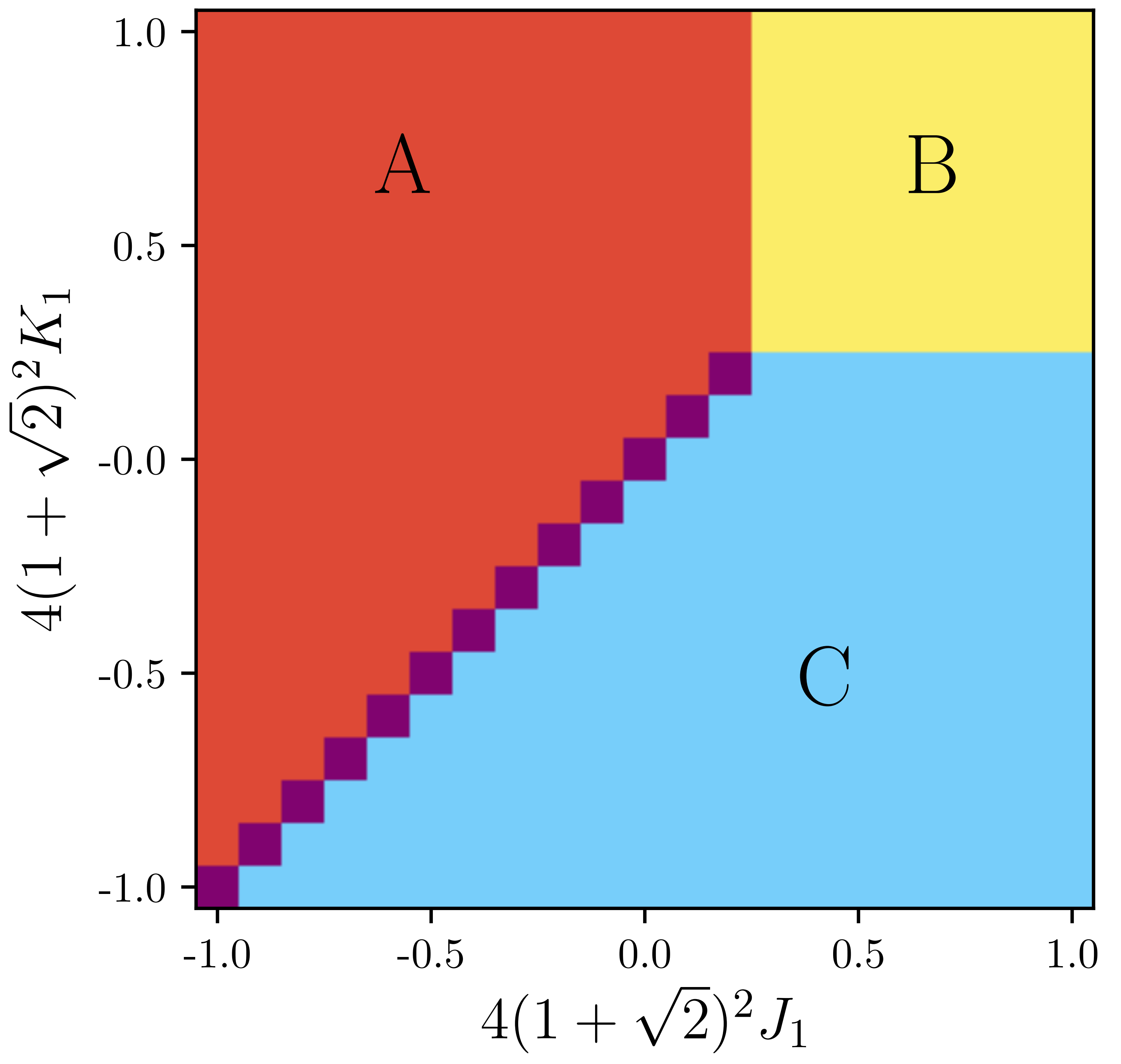
d-wave superconductor,

charge density wave,

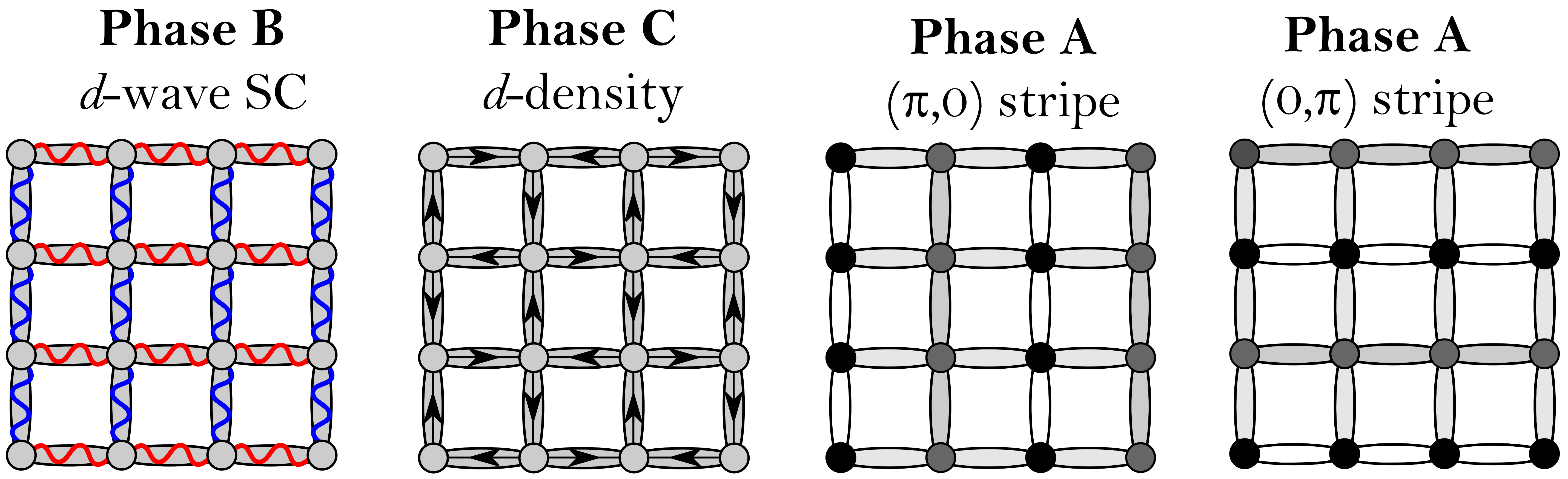
pair density wave

. . .

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

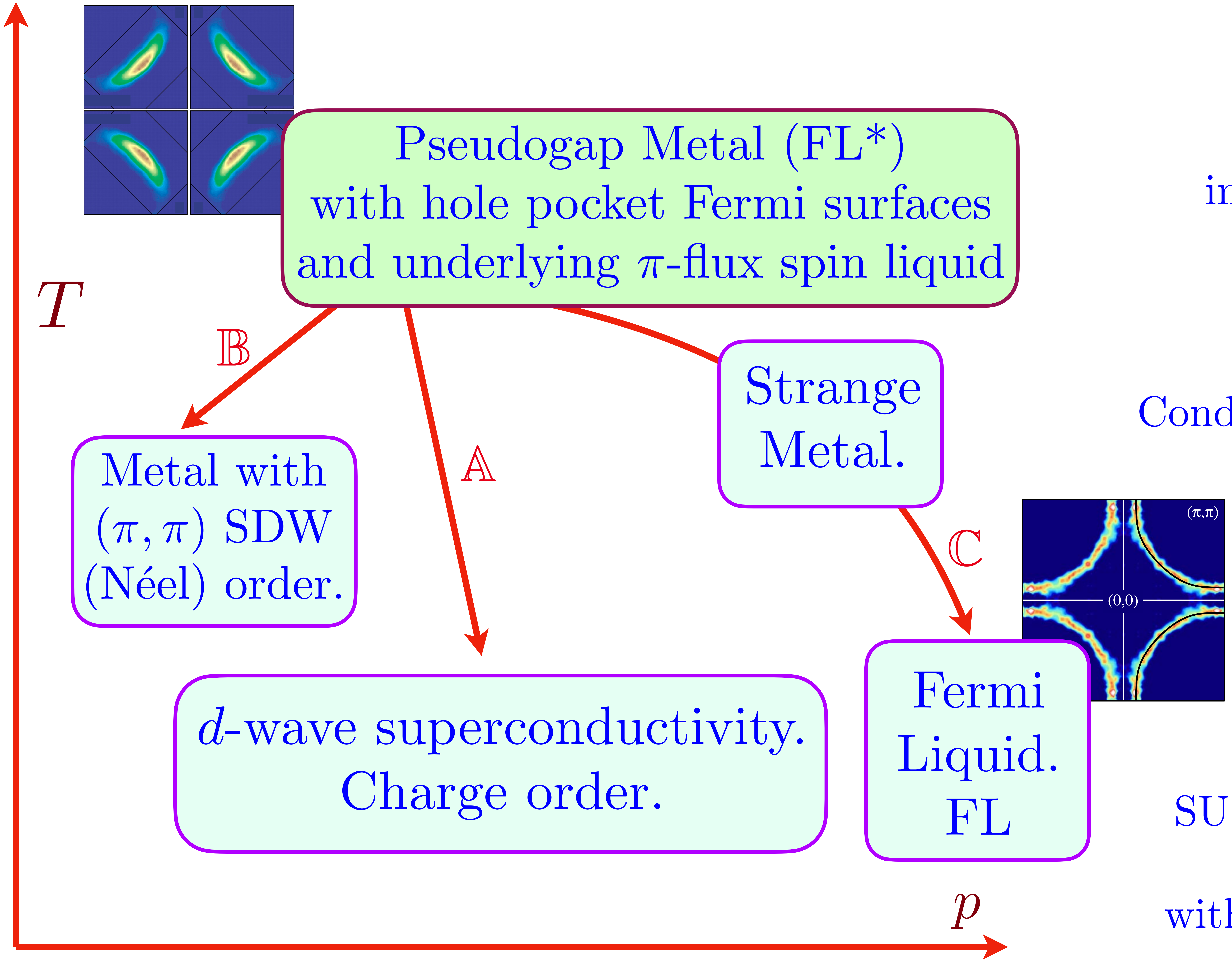


$$\langle B \rangle \neq 0$$



Global phase diagram of SU(2)$_N$ gauge theory

Global phase diagram of $SU(2)_N$ gauge theory



Arrow $\mathbb{A}$

Condensation of B
in SU$(2)_N$ gauge theory.

Arrow $\mathbb{B}$

Condensation of z_α in dual $\mathbb{CP}^1$
U(1) gauge theory.

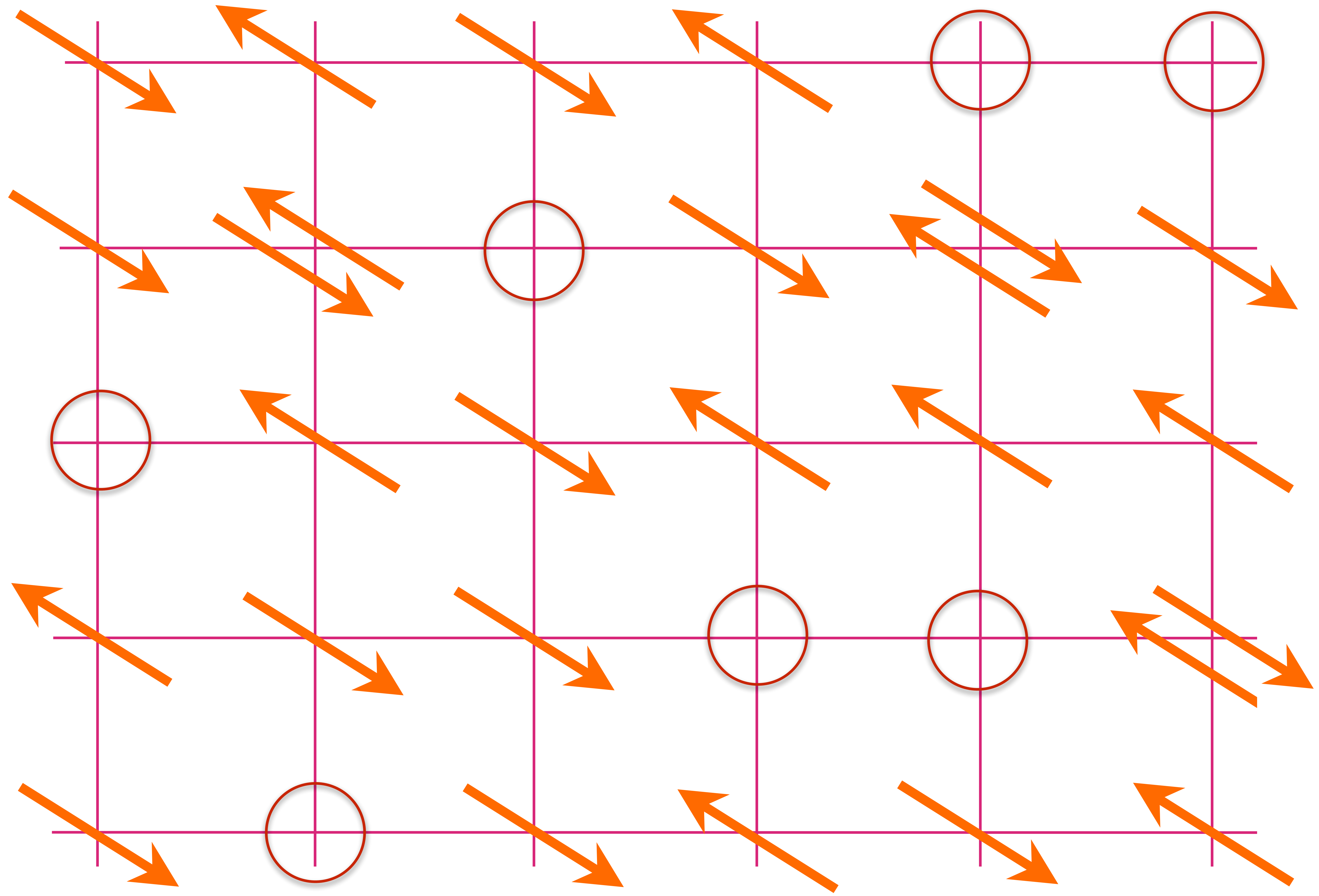
Arrow $\mathbb{C}$

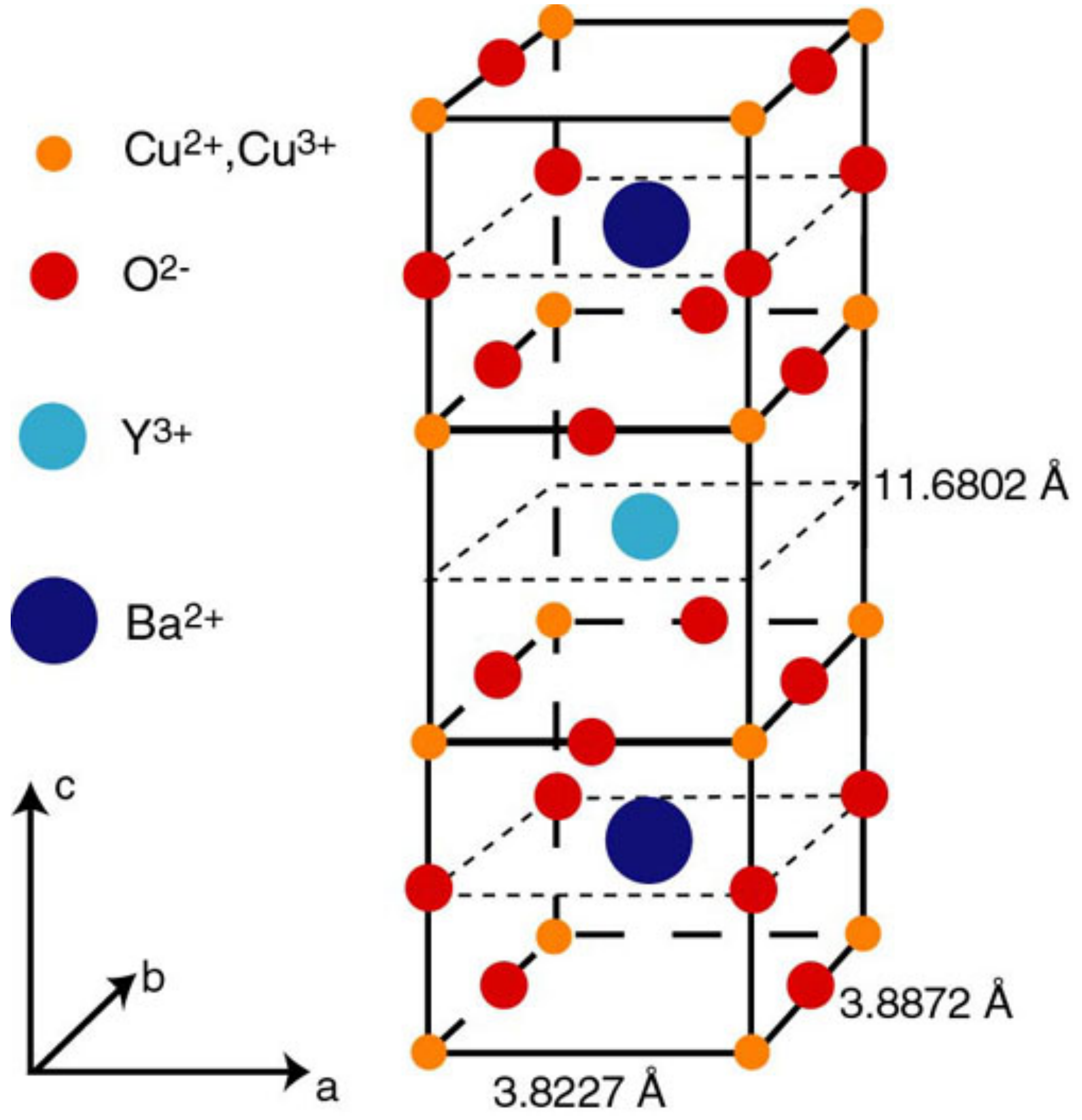
SU$(2)_s \times$U$(1)_a$ gauge theory
in ancilla model
with Higgs field $H_{\alpha a} \sim c_\alpha^\dagger f_{1a}$

1. The phase diagram of the cuprates
2. Introduction to quantum spin liquids and FL*
3. The π-flux spin liquid
4. The heavy Fermi liquid of the Kondo lattice
5. Ancilla theory of FL*
6. Confinement transitions of π-flux-FL*
7. Recap

Square lattice Hubbard model with electron density $1 - p$.

$$\mathcal{H}_{\text{Hubbard}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + \ldots$$





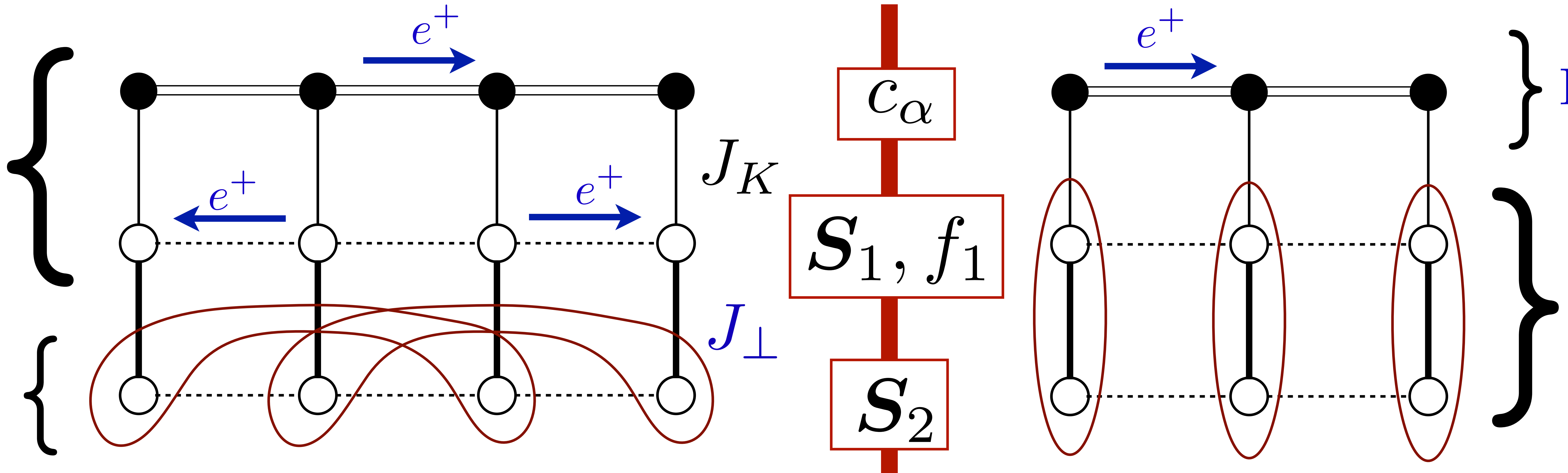
$YBa_2Cu_3O_{6+x}$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,
PRR **2**, 023172 (2020)

Kondo lattice heavy Fermi liquid. Size $1 + p + 1 = p \pmod 2$. *Small* Fermi surface!

Spin liquid



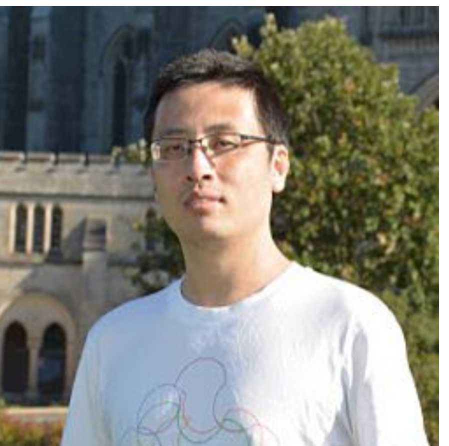
Large Fermi surface. Size: $1 + p$

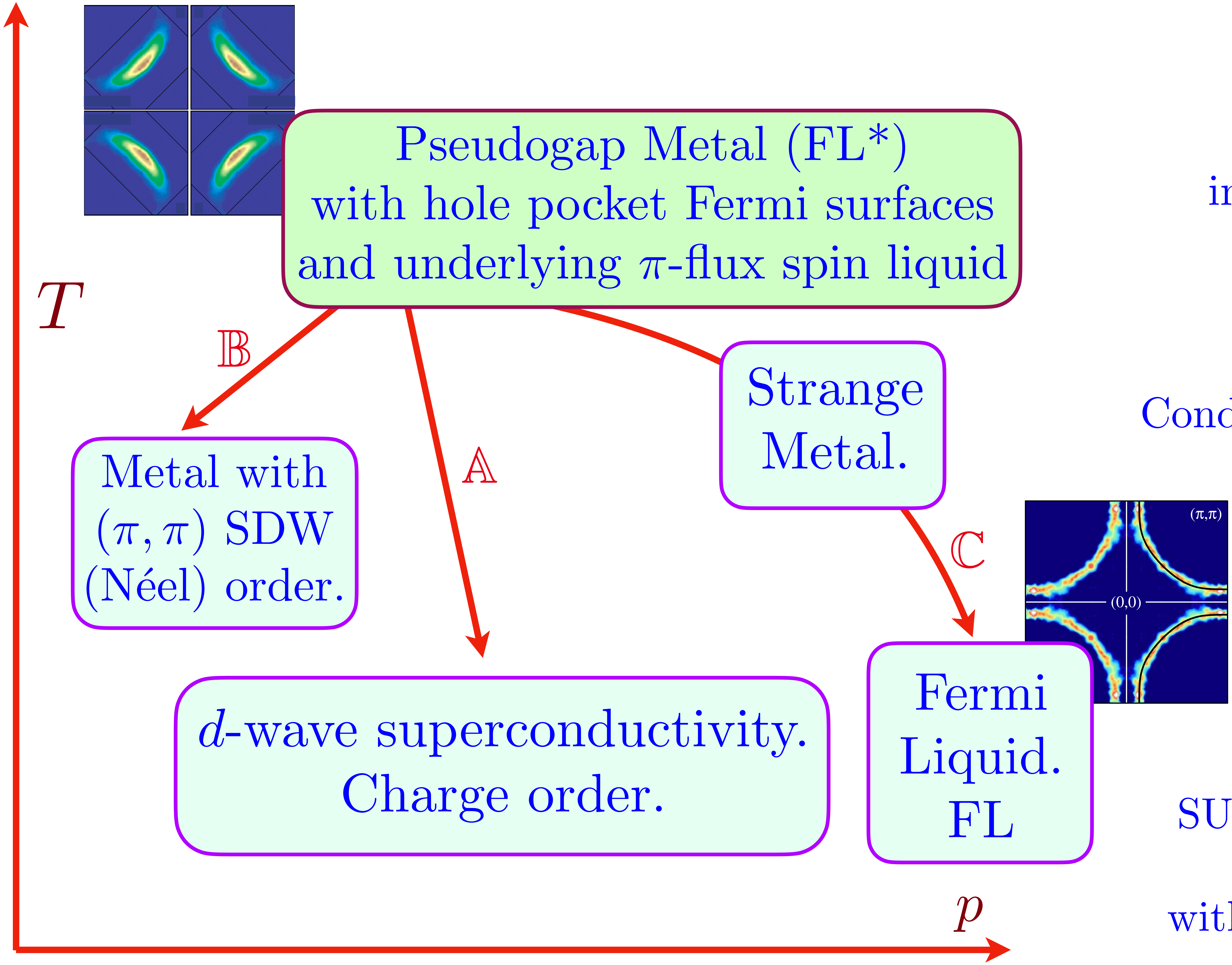
Trivial insulator

Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

$$\langle c_\alpha^\dagger f_{1\alpha} \rangle \neq 0$$

Ya-Hui Zhang





Arrow $\mathbb{A}$

Condensation of B
in SU(2)$_N$ gauge theory.

Arrow $\mathbb{B}$

Condensation of z_α in dual $\mathbb{CP}^1$
U(1) gauge theory.

Arrow $\mathbb{C}$

SU(2)$_s \times$U(1)$_a$ gauge theory
in ancilla model
with Higgs field $H_{\alpha a} \sim c_\alpha^\dagger f_{1a}$

Unified SU(2)xU(1) gauge theory of spinons, electrons and Higgs bosons:
uncanny similarities to the Salam-Weinberg-Glashow theory of weak interactions

- The electromagnetic U(1) is effectively global, because $\alpha \ll 1$.
- The fermionic spinons transform as a fundamental of gauge SU(2), with a massless Dirac spectrum

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) ,$$

where U_{ij} is the (lattice) SU(2) gauge field. The spinons are the analog of the neutrinos

- The Higgs sector has a boson B_i which is fundamental of SU(2)

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \mathcal{O}(B_i^4)$$

- The hole pockets in the nodal region of the Brillouin zone are described by electron $\bar{c}_{i\alpha}$ which have a Yukawa coupling to the spinons and the Higgs field $B_i = (B_{1i}, B_{2i})$:

$$H_Y = \sum_{ij} \bar{t}_{ij} \bar{c}_{i\alpha}^\dagger \bar{c}_{j\alpha} + i \sum_i \left(B_{1i} f_{i\alpha}^\dagger \bar{c}_{i\alpha} - B_{2i} \varepsilon_{\alpha\beta} f_{i\alpha} \bar{c}_{i\beta} \right) + \text{H.c.} \, .$$