Quantum matter without quasiparticles: random fermion models, black holes, graphene, and non-Fermi liquids

Workshop on Non-equilibrium Physics and Holography
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Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

- Quantum criticality near the superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Non-Fermi liquids in two spatial dimensions

Note: Most states with long-range entanglement, like the fractional quantum Hall states, do have quasiparticles

Local thermal equilibration or phase coherence time, τ_φ:

- As $T \to 0$, there is an *lower bound* on τ_φ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
  e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- The time over which a many-body quantum system becomes chaotic is given by $\tau_L = 1/\lambda_L$, where λ_L is the "Lyapunov exponent" determining memory of initial conditions (the "butterfly effect"):

$$D(t) = \langle W(t)V(0)W(t)V(0)\rangle \sim c_0 - \epsilon\, c_1 e^{\lambda_L t},$$

  where we make a (system-dependent) choice to arrange $\epsilon \ll 1$. As $T \to 0$, this Lyapunov time is argued to obey the lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

- Theories holographically dual to Einstein gravity have the shortest possible $\tau_L = \hbar/(2\pi k_B T)$

A.I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)
S. H. Shenker and D. Stanford, arXiv:1306.0622
J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos

Quantum matter without quasiparticles:

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- Graphene
- Non-Fermi liquids - Ising-nematic criticality of a two-dimensional metal

Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

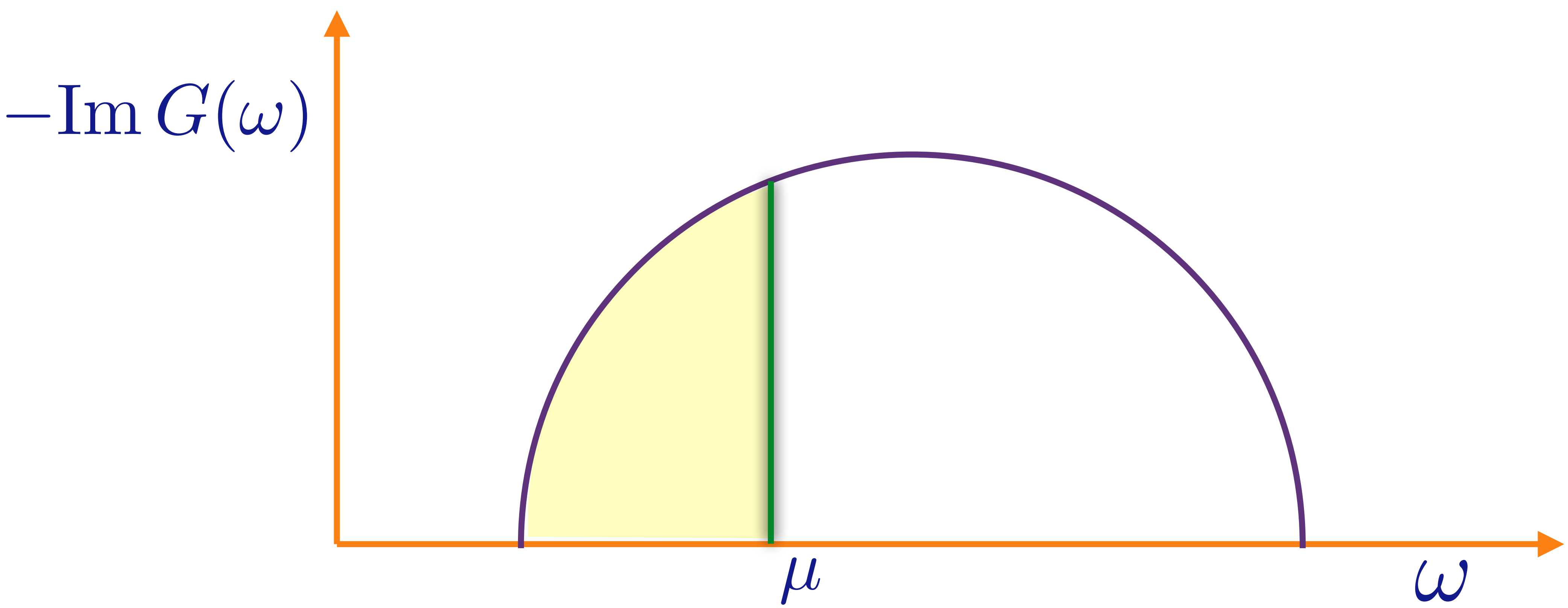
Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\frac{1}{\tau_\alpha} = \pi J^2 \rho_0^3 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta)$$
$$= \frac{\pi^3 J^2 \rho_0^3}{4} T^2$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

SY model without quasiparticles

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} \hat{S}_{i,\alpha\beta} \hat{S}_{j,\beta\alpha}$$

$$= \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c_{i\alpha}^{\dagger} c_{i\beta} c_{j\beta}^{\dagger} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c_{j\beta}^{\dagger} + c_{j\beta}^{\dagger} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = \mathcal{Q}$$

Generalization of the classical Sherrington-Kirkpatrick model to quantum SU(M) spins.

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$

$N \to \infty$ at $M = 2$ yields spin-glass ground state.

$N \to \infty$ and then $M \to \infty$ yields critical strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

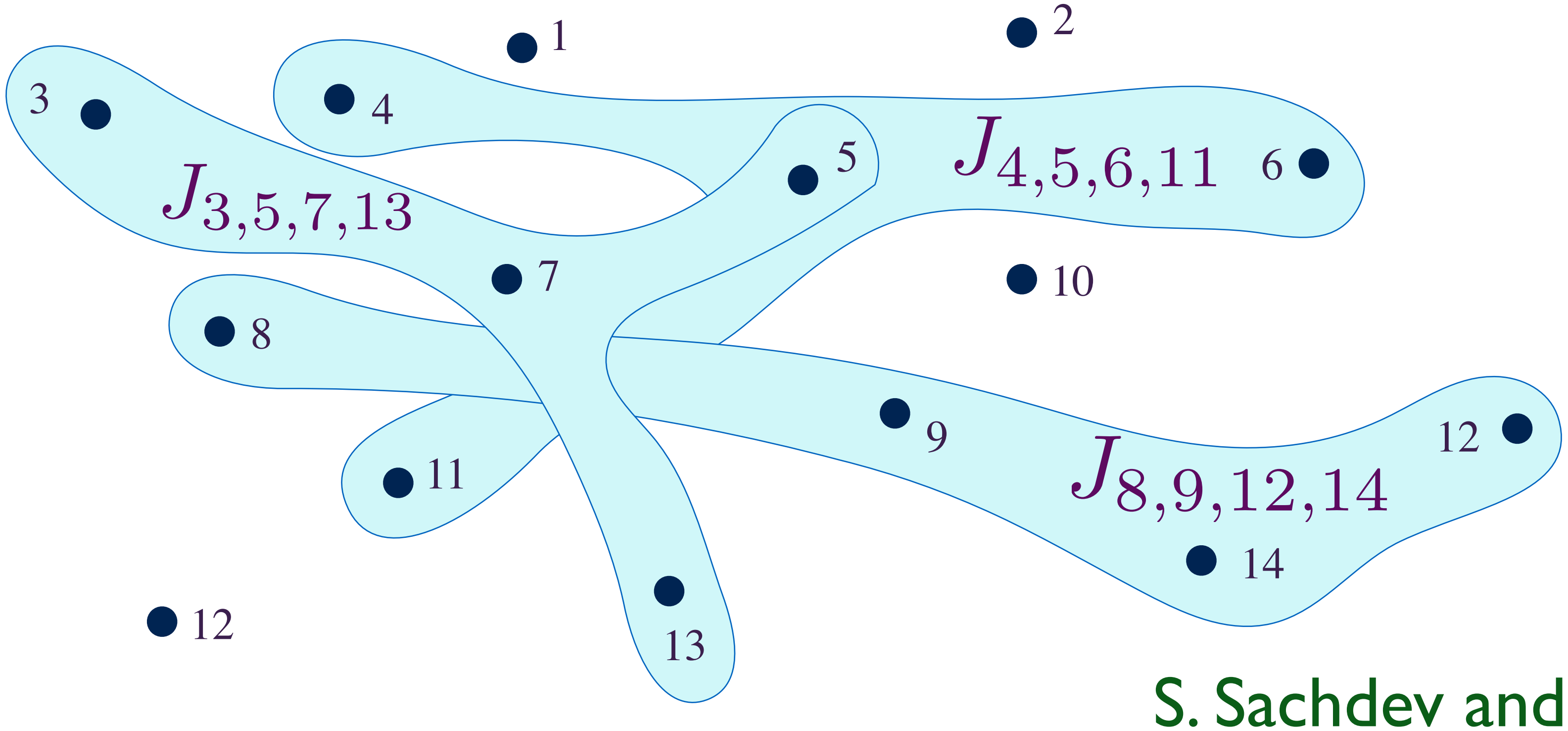
SYK model without quasiparticles

To obtain a non-Fermi liquid, we set $t_{ij} = 0$:

$$H_{\rm SYK} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$H_{\rm SYK}$ is similar, and has identical properties, to the SY model.



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
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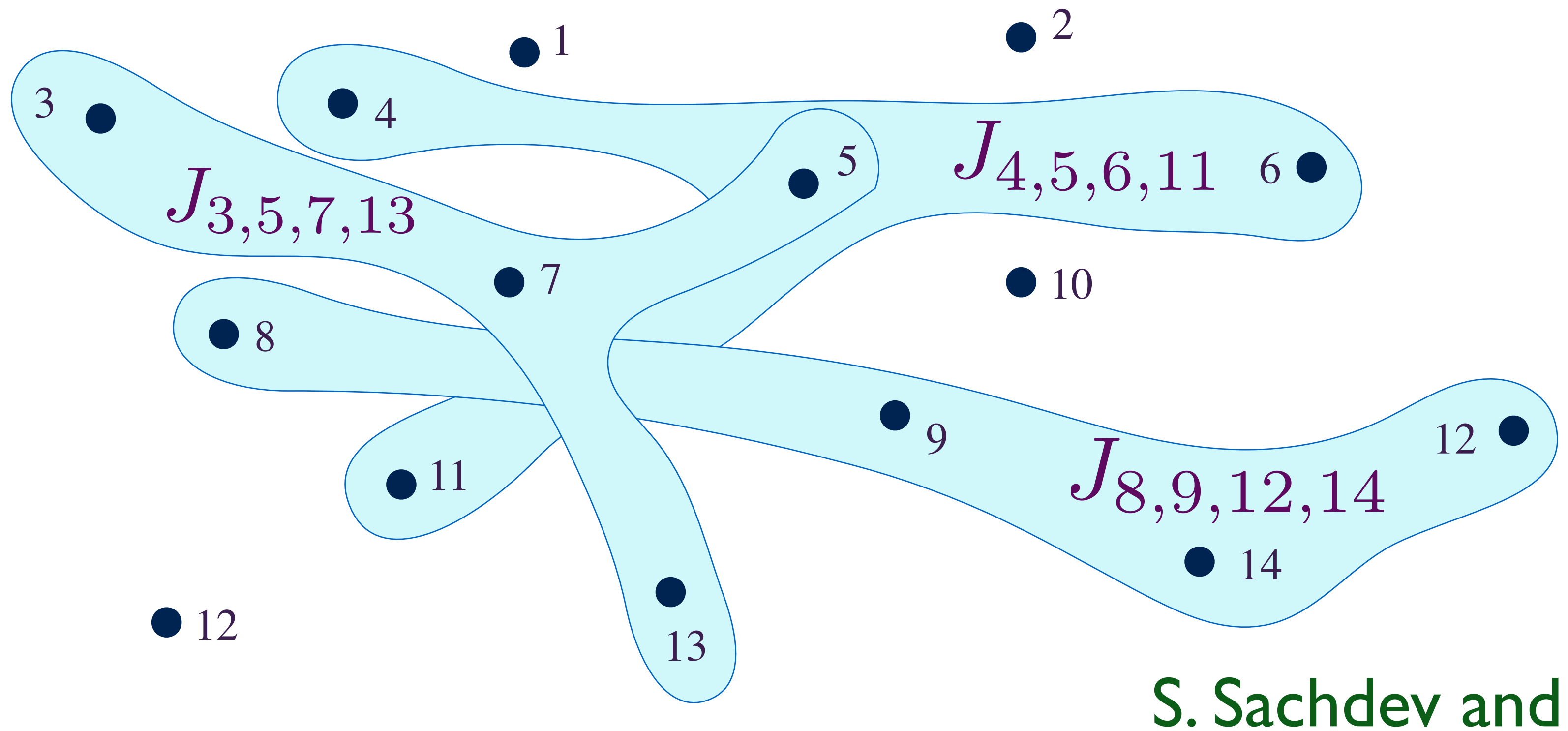


A fermion can move only by entangling with another fermion: the Hamiltonian has "nothing but entanglement".

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model without quasiparticles

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

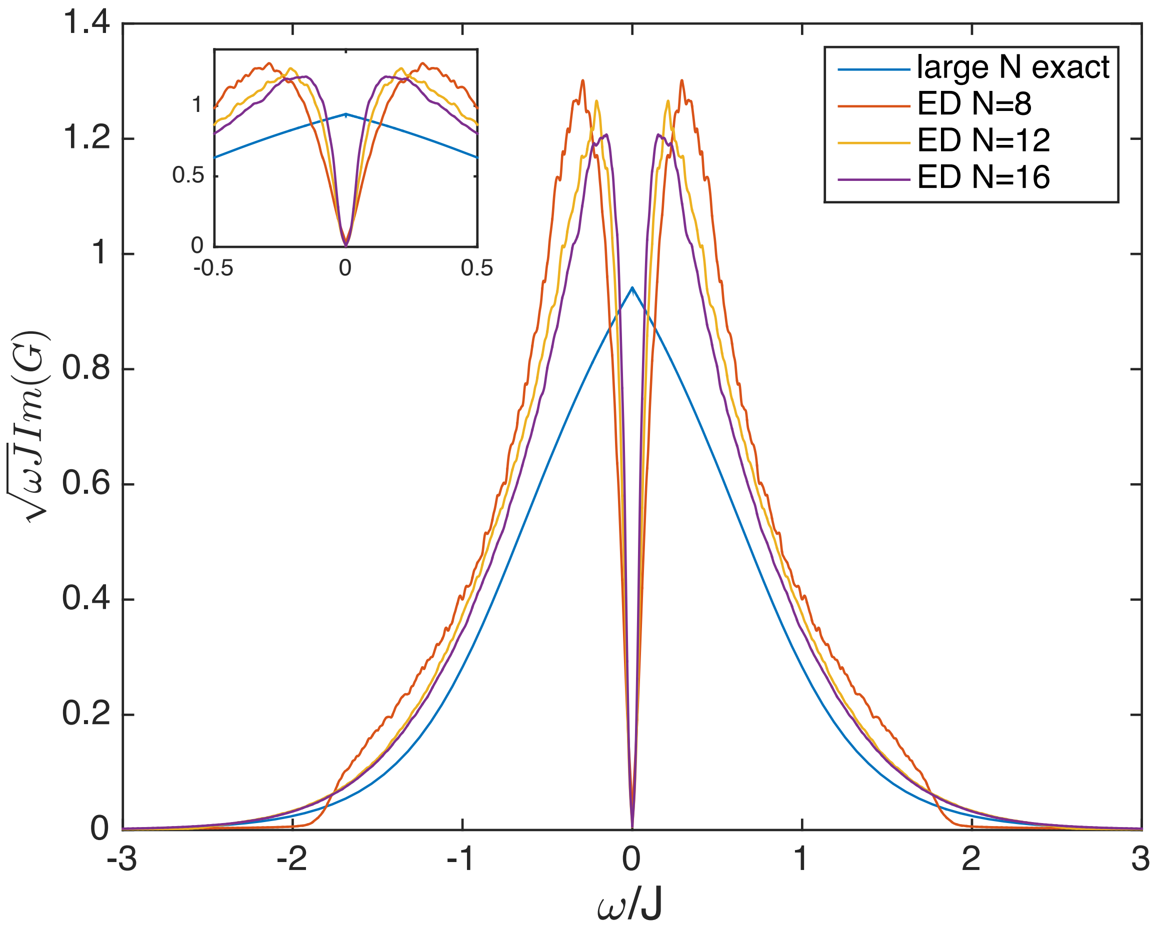
Low frequency analysis shows that the solutions must be gapless and obey

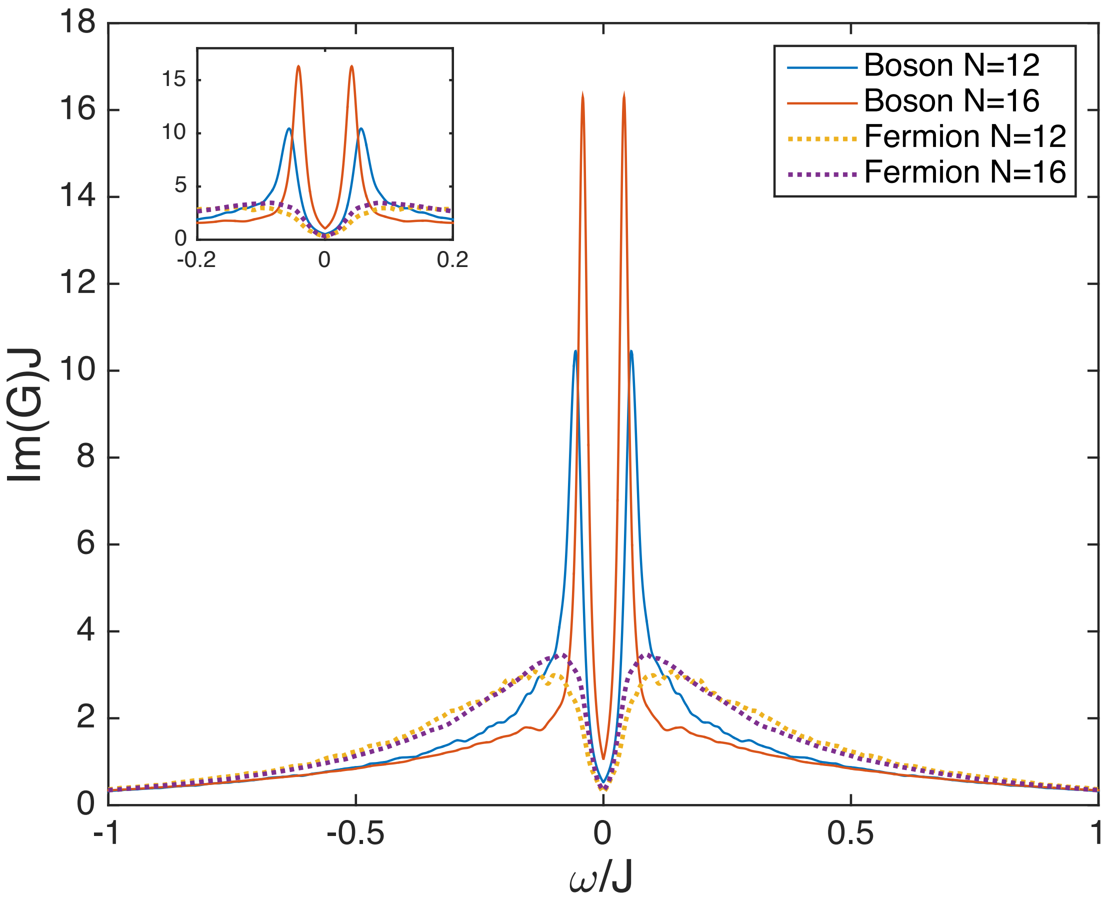
$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

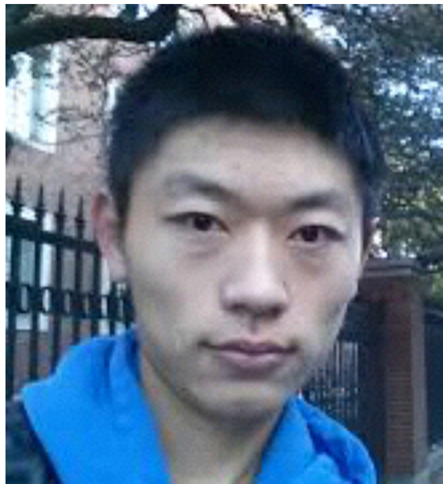
SYK model without quasiparticles





Large N solution of equations for G and Σ agree well with exact diagonalization of the finite N Hamiltonian.

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.



W. Fu and S. Sachdev, arXiv: 1603.05246

SYK model without quasiparticles

Local fermion density of states

$$\rho(\omega) = -\mathrm{Im}\, G(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

While $\mathcal{E}$ determines the *low* energy spectrum, it is determined by the *total* fermion density $\mathcal{Q}$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

Analog of the relationship between $\mathcal{Q}$ and k_F in a Fermi liquid.

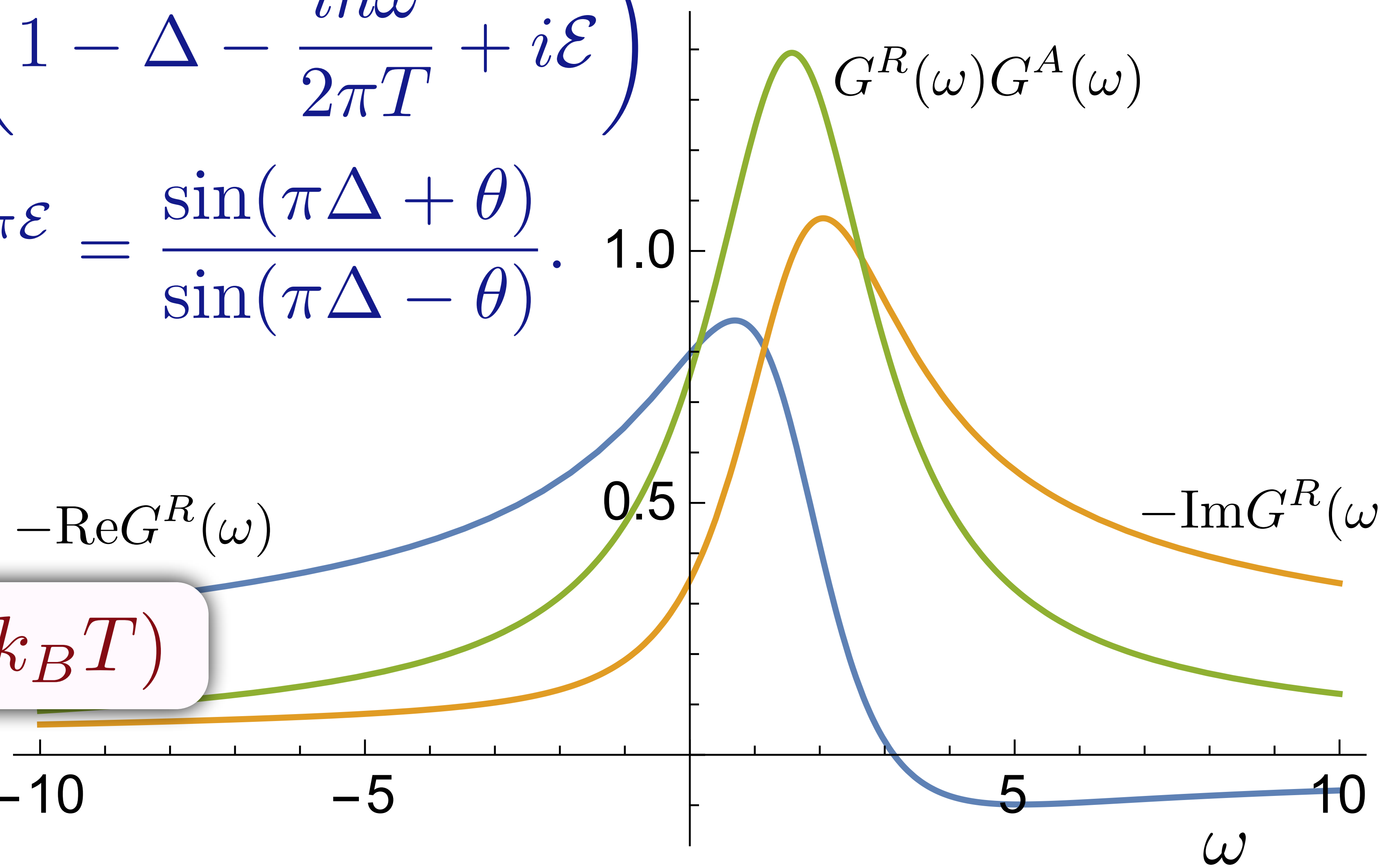
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

SYK model without quasiparticles

At non-zero temperature, T, the Green's function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}.$



Note $G(\omega) \equiv f(\hbar\omega/k_B T)$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

SYK model without quasiparticles

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$. This is *not* due to an extensive degeneracy, but due to an energy level spacing $\sim e^{-aN}$ in the entire many-body spectrum all the way down to the ground state. At low T we write

$$\mathcal{S}(T \to 0) = \mathcal{S}_0 + \gamma T + \ldots$$

where the specific heat is $\mathcal{C} = \gamma T$, and $\mathcal{S}_0$ obeys

$$\frac{d\mathcal{S}_0}{d\mathcal{Q}} = 2\pi\mathcal{E},$$

with $\mathcal{E}$ the same spectral asymmetry parameter.

Note that $\mathcal{S}_0$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure.

A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)
J. Maldacena and D. Stanford, arXiv:1604.07818

SYK model without quasiparticles

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right]$$
$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2)\left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

N. Read, S. Sachdev, and J. Ye, PRB **52**, 384 (1995)

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

SYK model without quasiparticles

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK model without quasiparticles

However the effective action must vanish for SL(2,R) transformations because G_s, Σ_s are invariant under it. In this manner we obtain the effective action as a Schwarzian

$$N S_{\text{eff}} = -\frac{N\gamma}{4\pi^2} \int d\tau \, \{f, \tau\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2 ,$$

where the specific heat, $\mathcal{C} = \gamma T$.

The Schwarzian effective action implies that the SYK model *saturates* the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK model without quasiparticles

The Schwarzian describes fluctuations of the energy operator with scaling dimension $h = 2$.
Apart from the energy operator associated with the Schwarzian, there are an infinite number of other scalar operators with irrational scaling dimensions given by the roots of

$$\tan\left(\frac{\pi(2h-1)}{4}\right) = \frac{1-2h}{3}$$

$$\Rightarrow \quad h = 3.77354\ldots,\ 5.67946\ldots,\ 7.63197\ldots,\ 9.60396\ldots,\ldots$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

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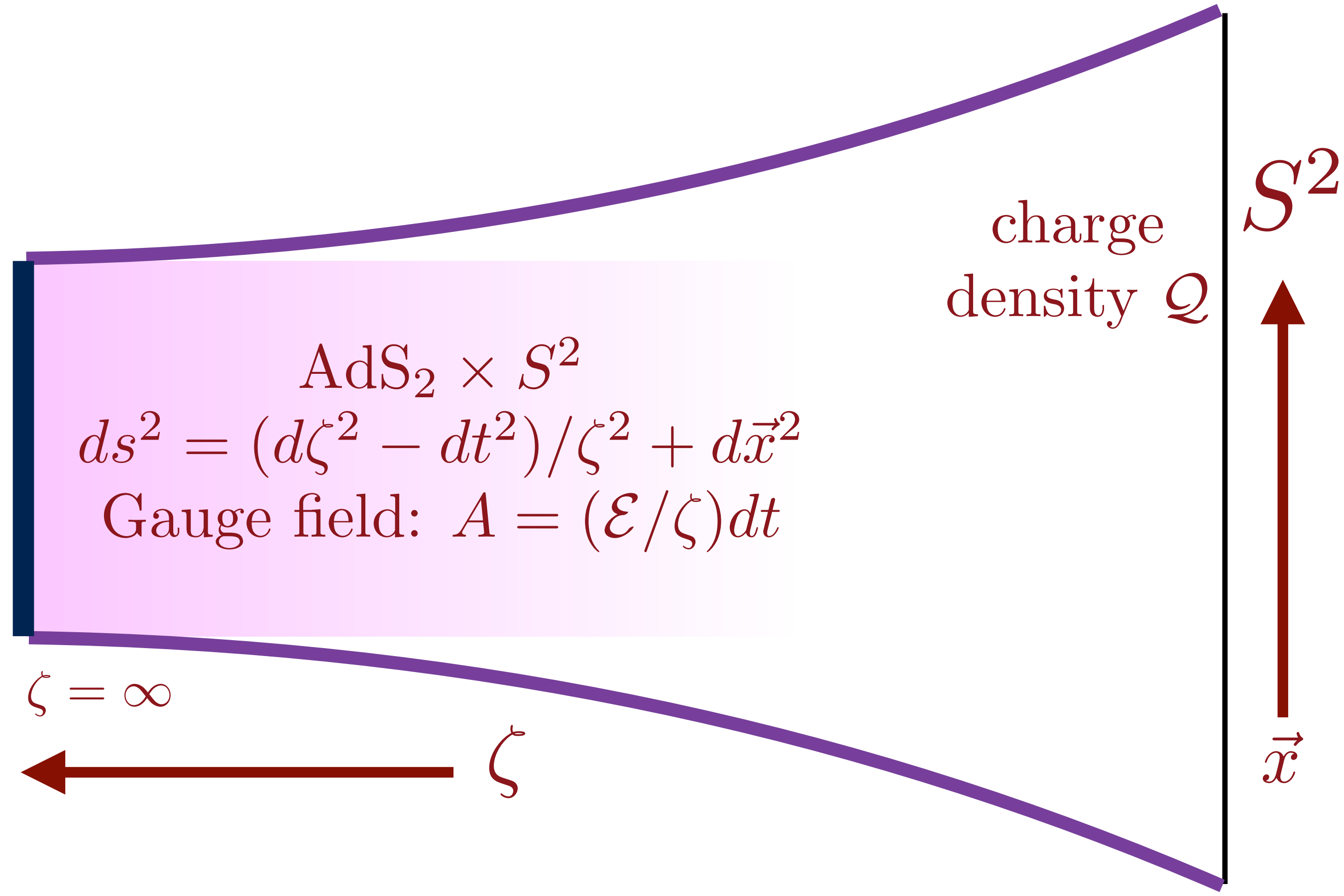
SYK and AdS_2



PHYSICAL REVIEW LETTERS 105, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

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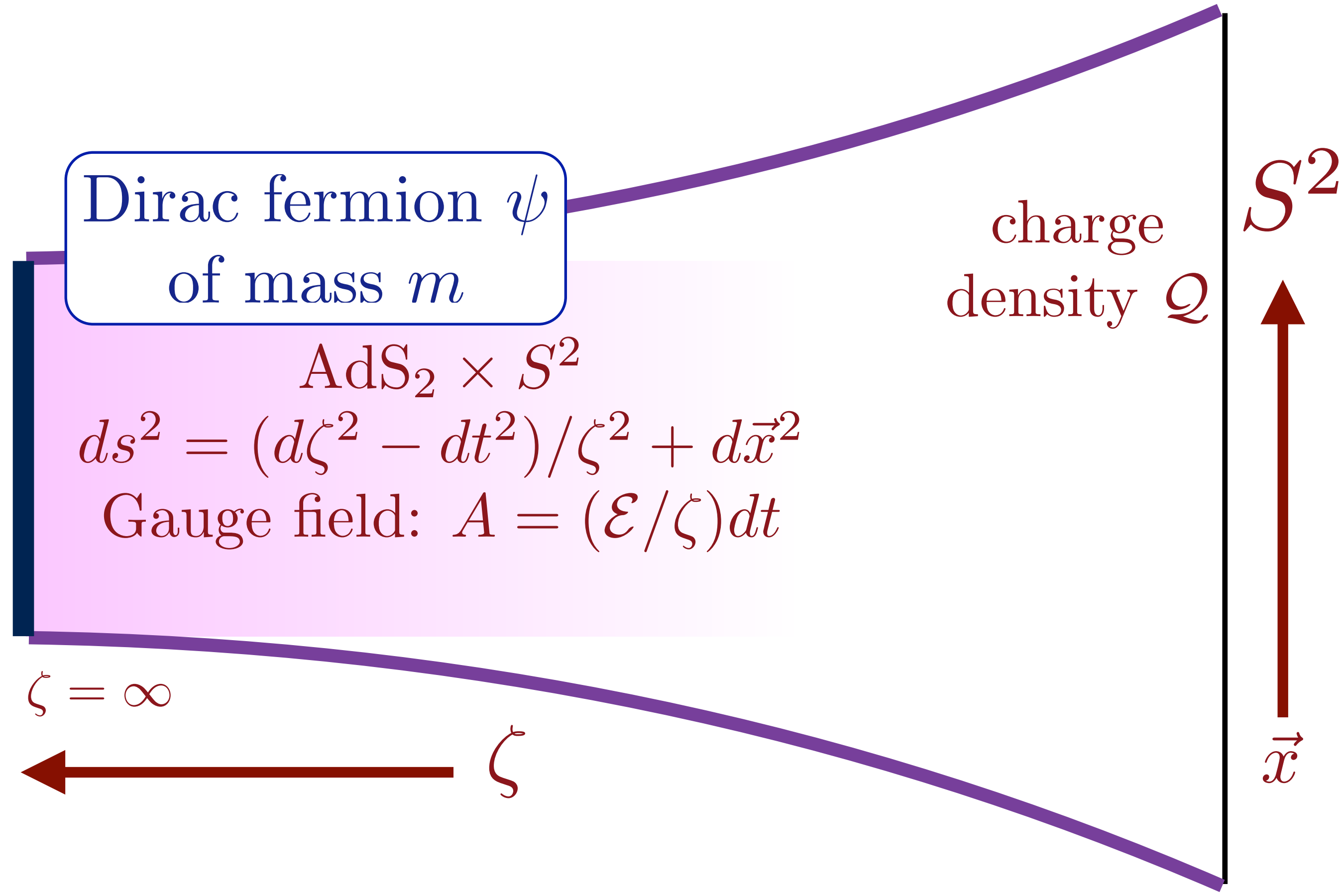
(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $\mathrm{AdS}_2 \times \mathrm{R}^2$ physics of Reissner-Nordström black holes.

SYK and AdS_2



AdS_2 boundary Green's function of ψ at $T = 0$

$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-(1-2\Delta)} & , \omega < 0. \end{cases}$$

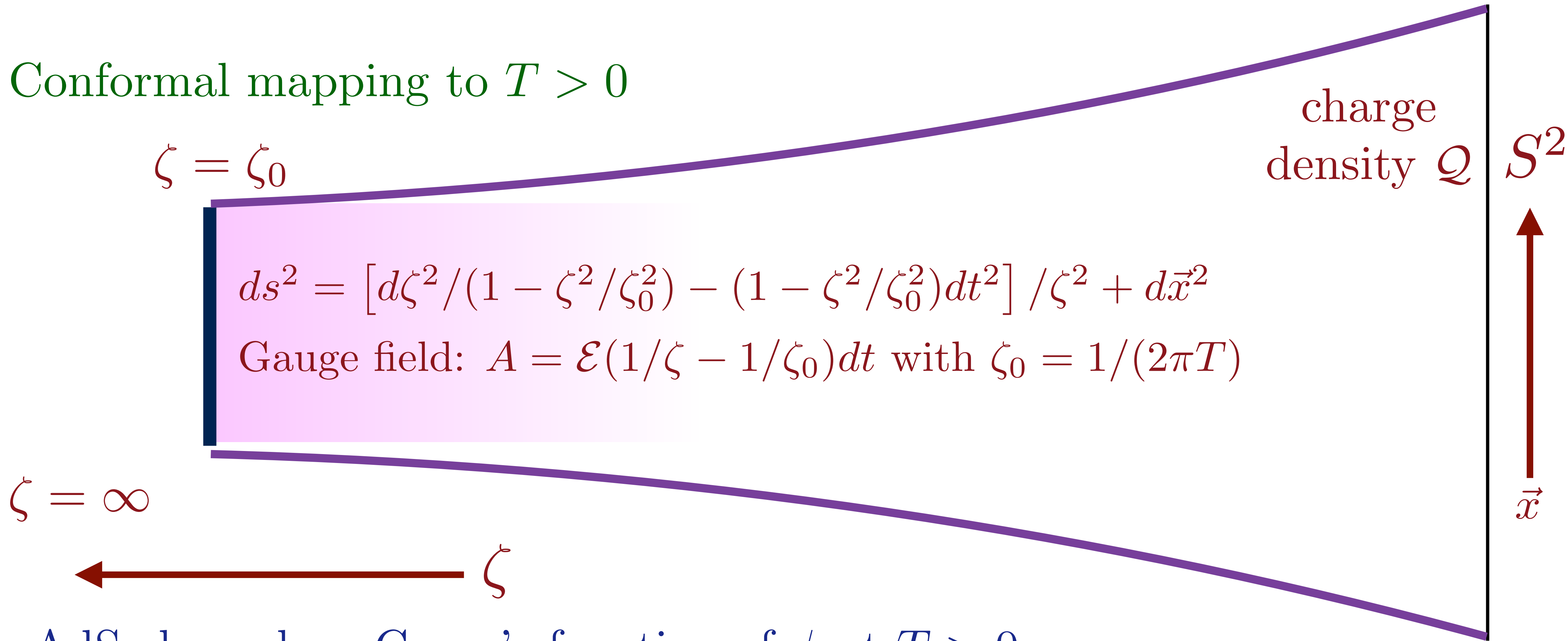
where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

SYK and AdS_2

Conformal mapping to $T > 0$



AdS_2 boundary Green's function of ψ at $T > 0$ is fully determined by $\mathcal{E}$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

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SYK and AdS_2

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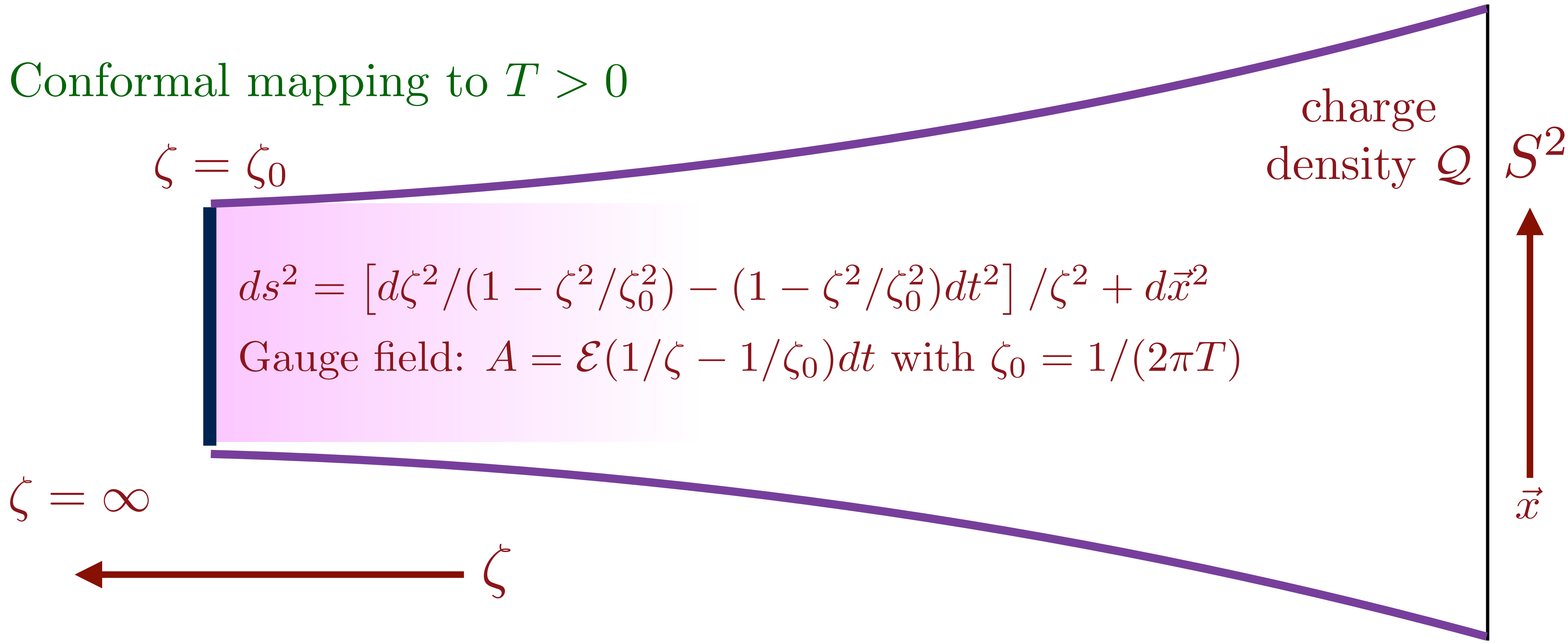


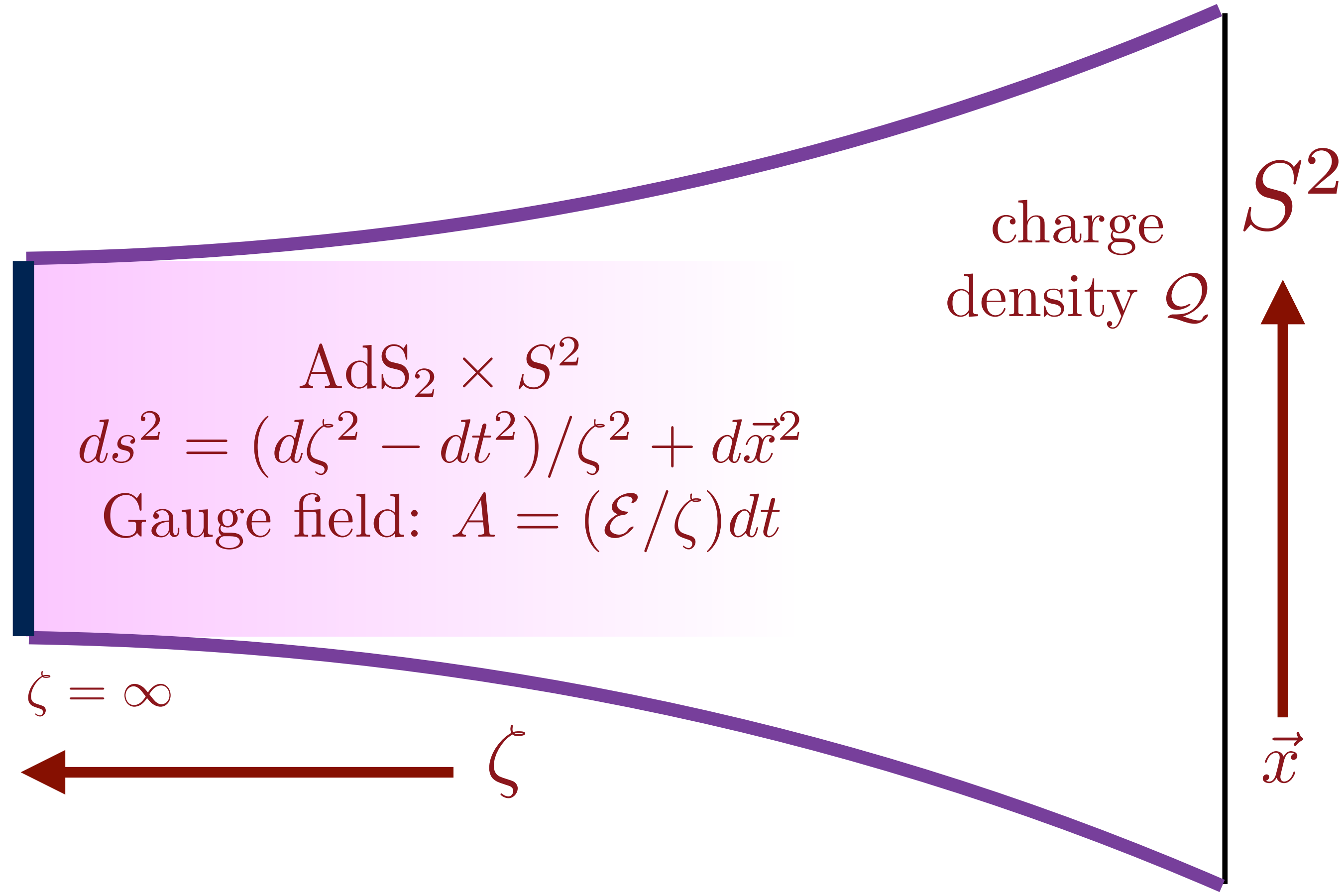
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{BH}$.
- Using Gauss's Law, it can be shown that $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen
hep-th/0506177
S. Sachdev
PRX 5, 041025 (2015)

Also obeyed by Wald entropy in higher-derivative gravity.

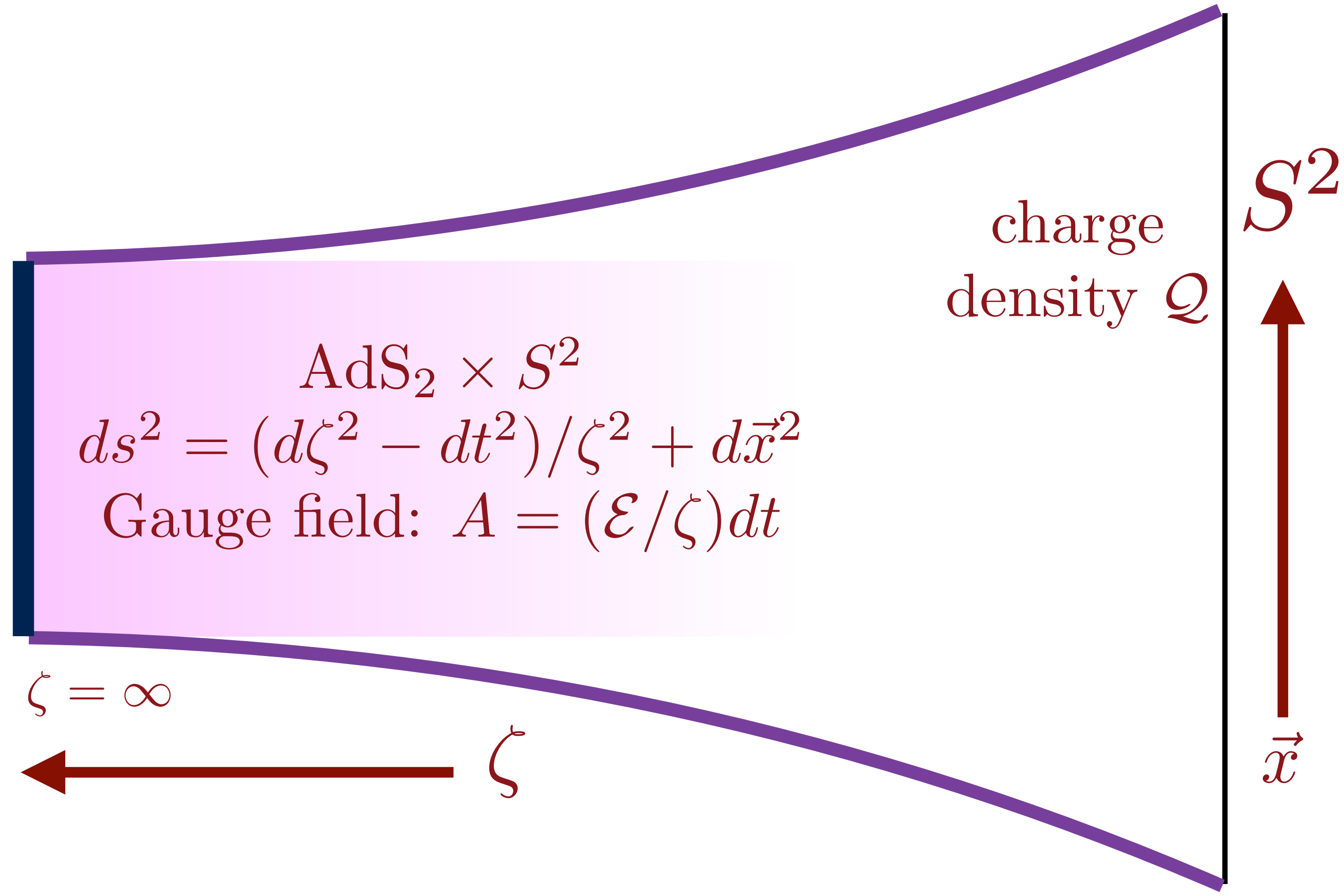
SYK and AdS_2



The same Schwarzian effective action describes low energy fluctuations on the boundary theory of gravity theories with AdS_2 near-horizon geometries (including the AdS-Reissner-Nordstrom solution of Einstein-Maxwell theory in 4 space-time dimensions). And the co-efficient of the Schwarzian, $N\gamma/4\pi^2$, determines the specific heat $\mathcal{C} = \gamma T$.

A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108

SYK and AdS_2



The Schwarzian effective action implies that both the SYK model and the AdS_2 theories *saturate* the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$

A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108

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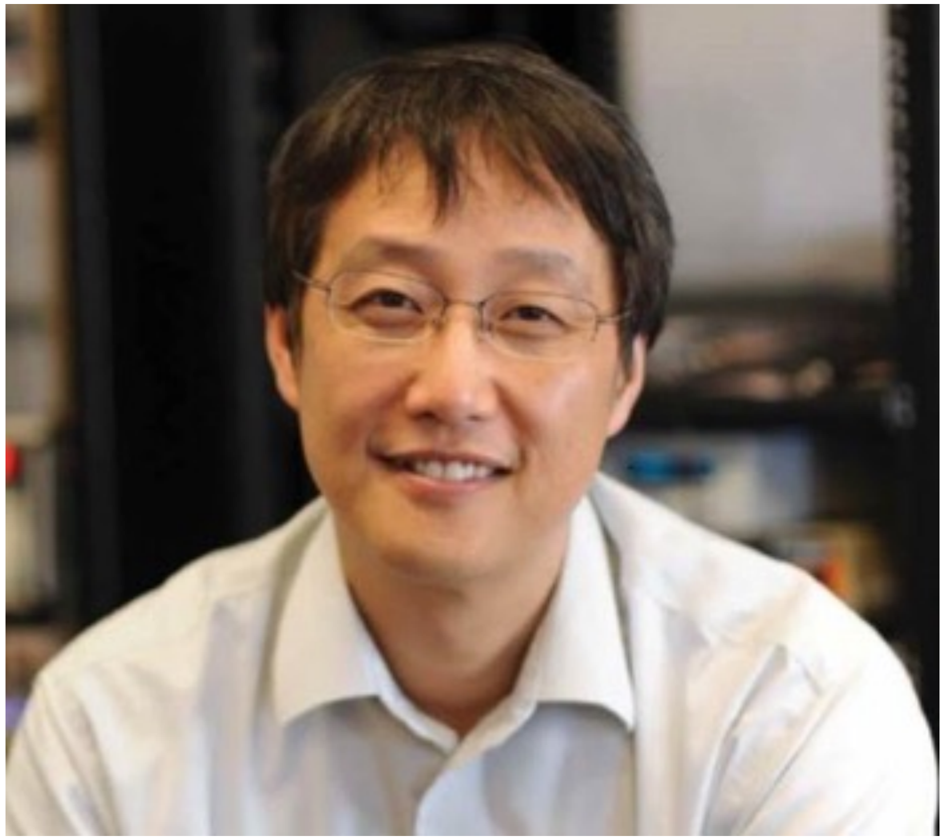
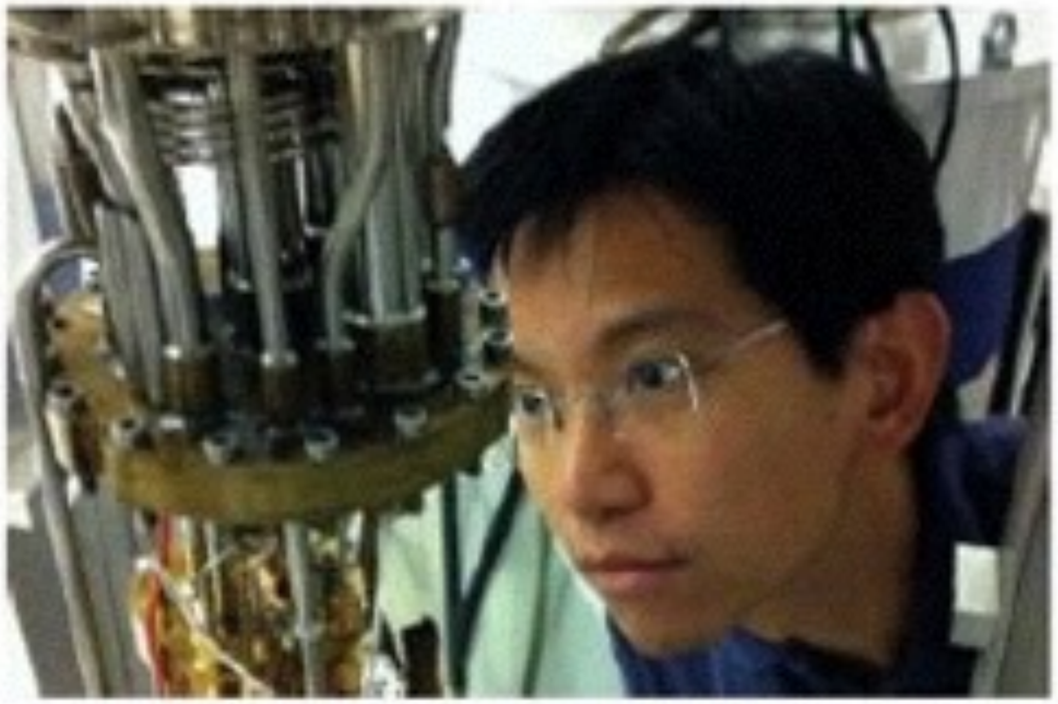
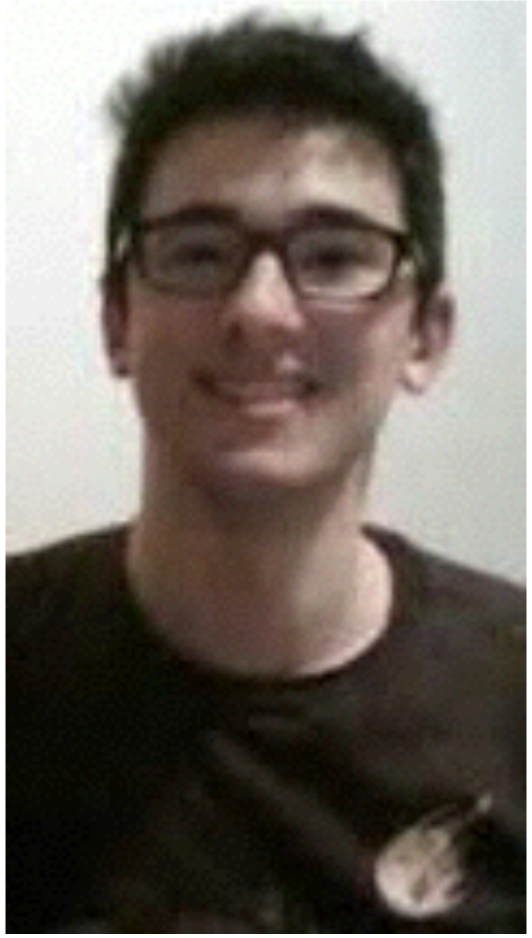


Philip Kim

Jesse Crossno

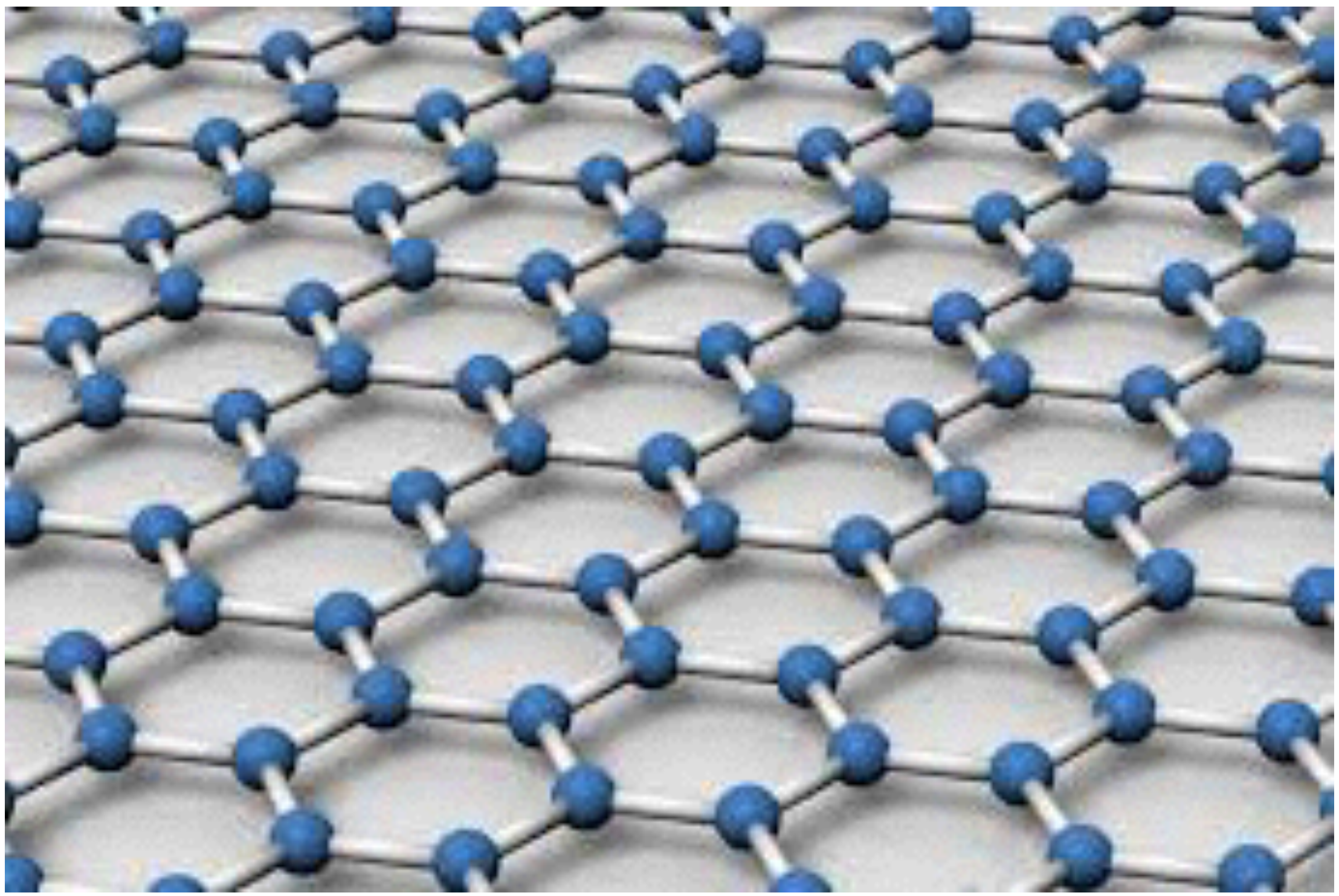


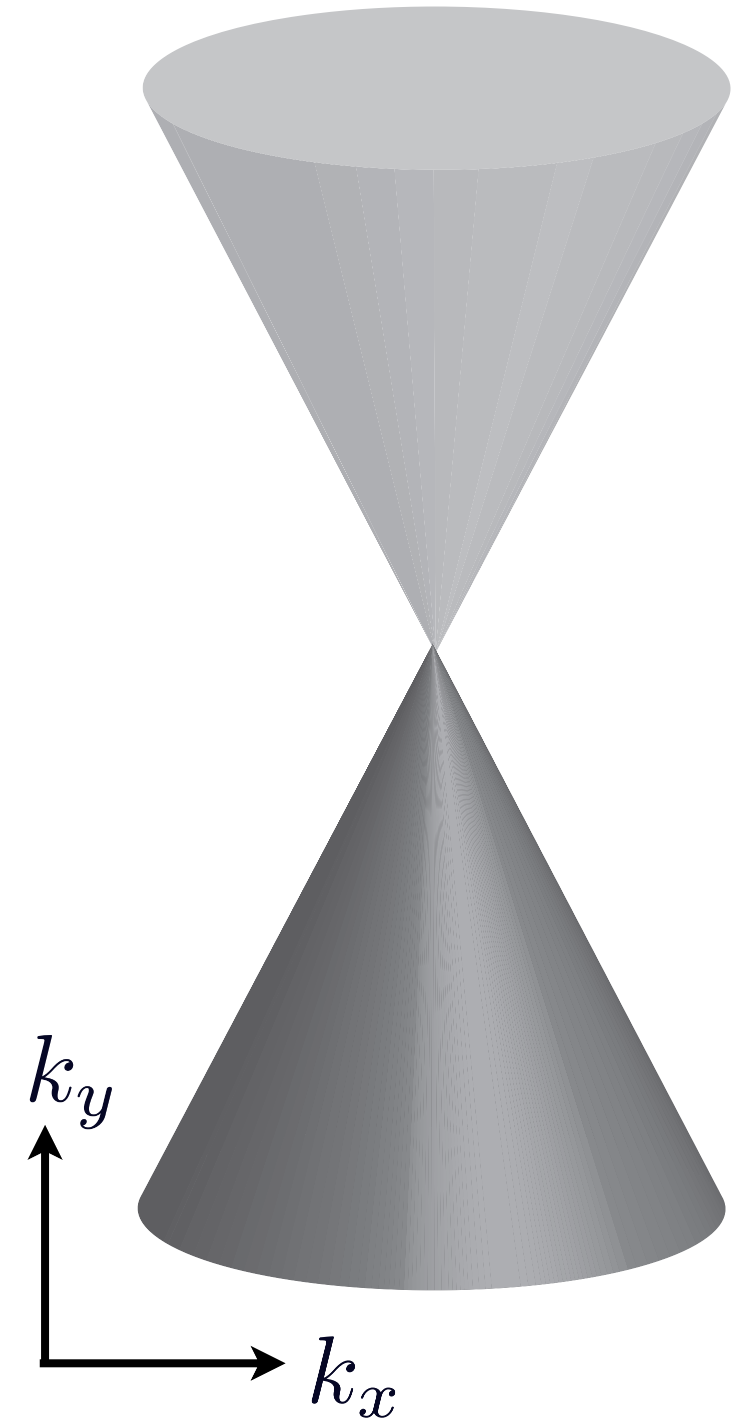
Kin Chung Fong



Andrew Lucas

Graphene

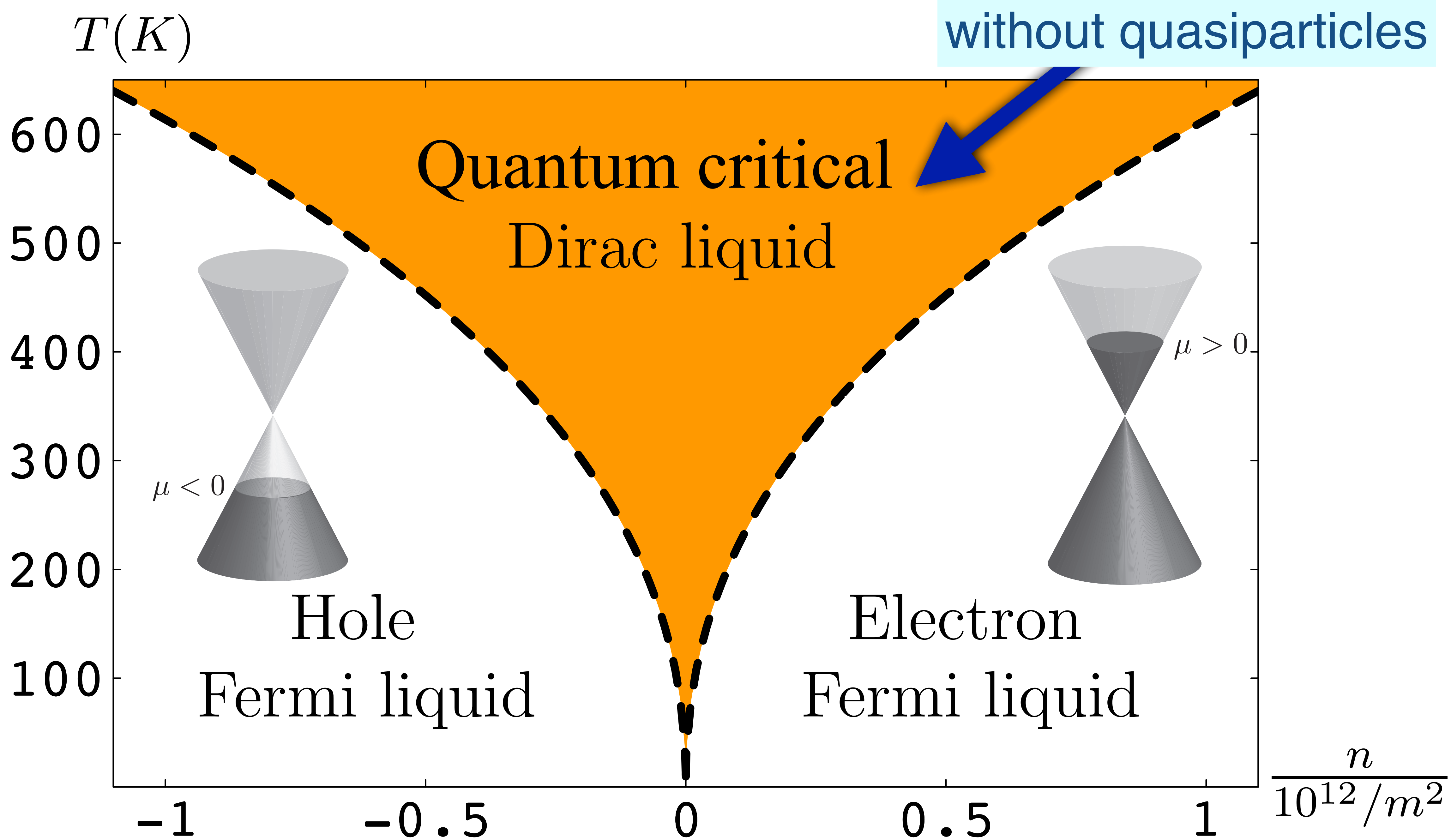




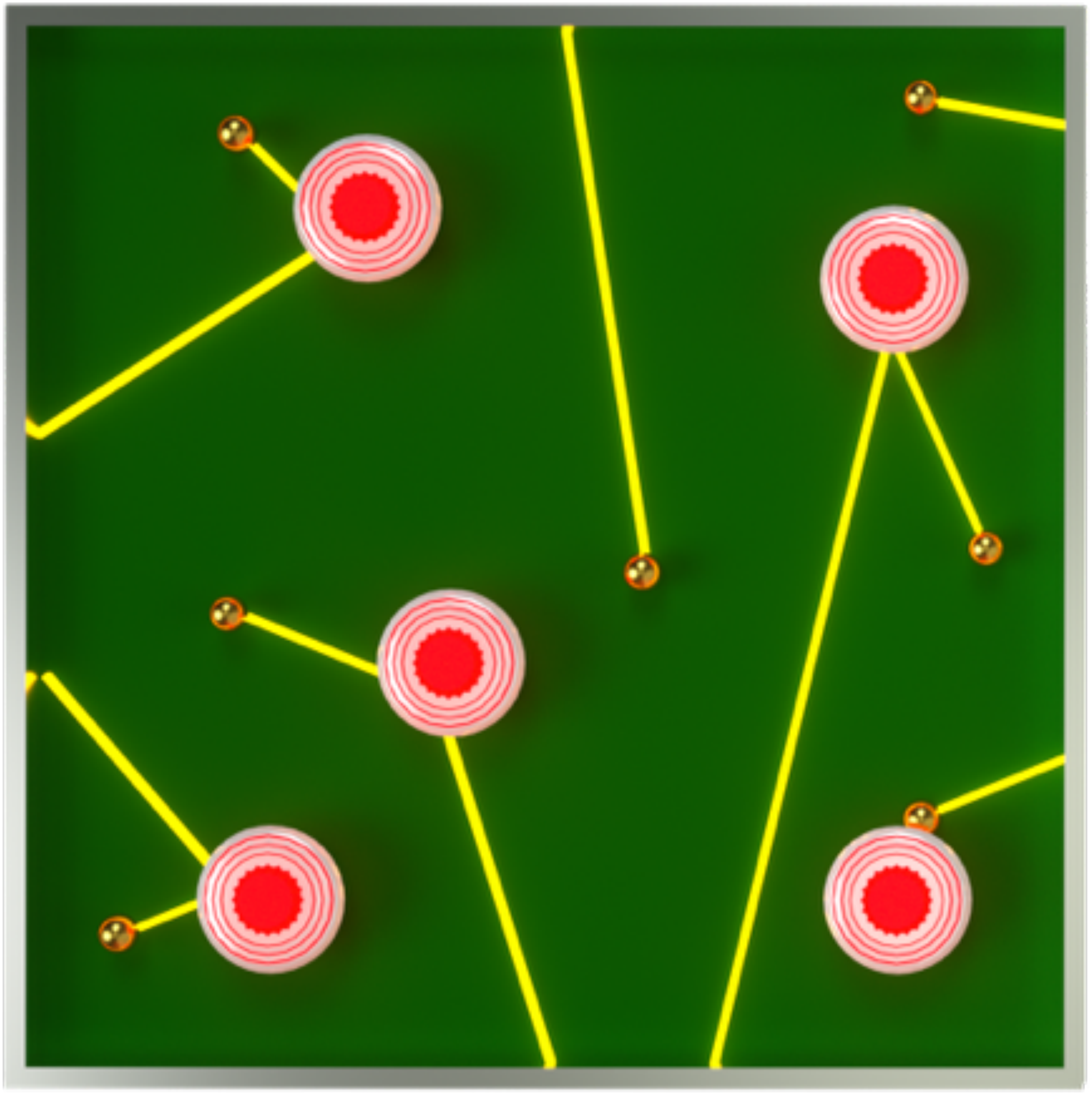
Same "Hubbard" model as for ultracold atoms, but for electrons on the honeycomb lattice

Graphene

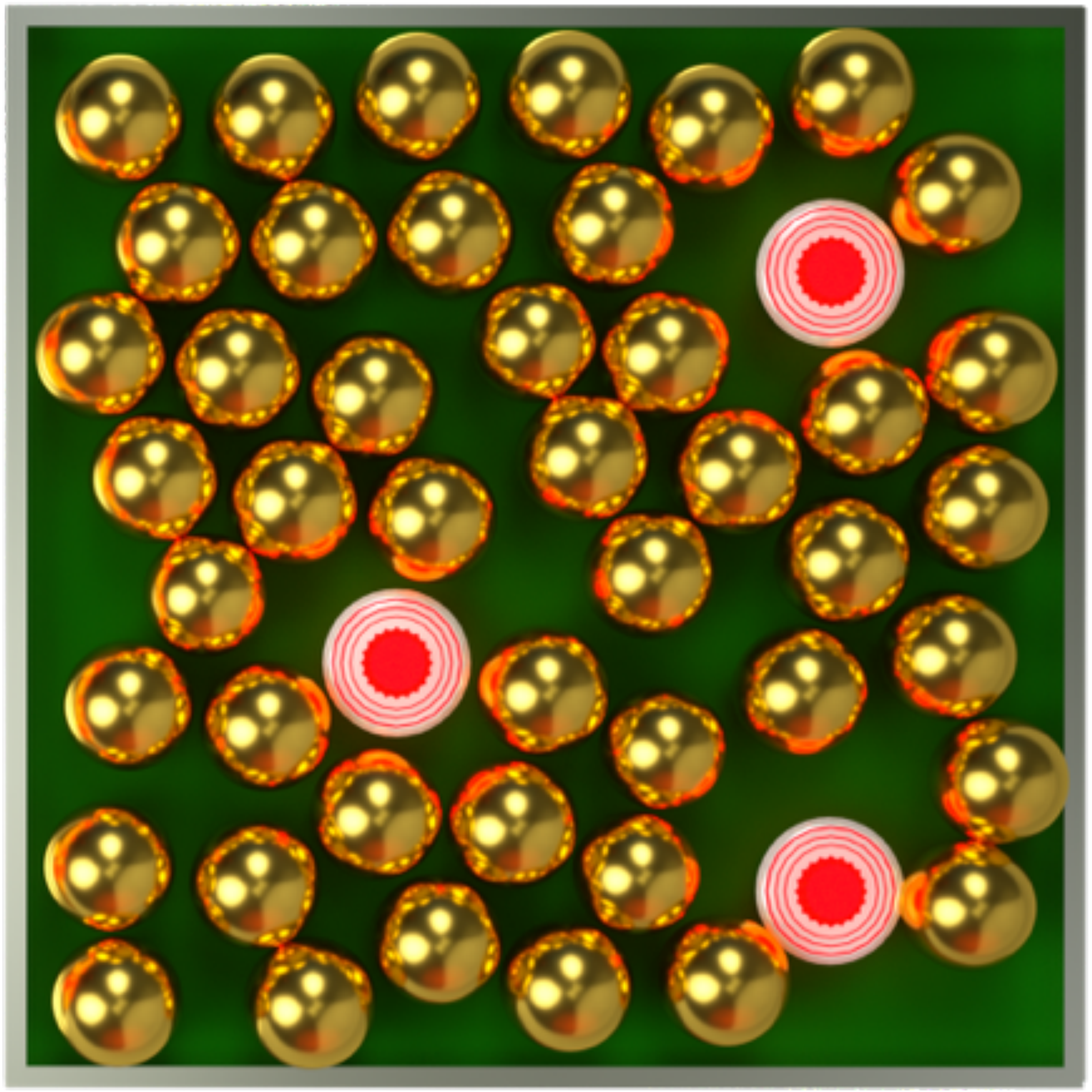
Predicted "strange metal" without quasiparticles



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

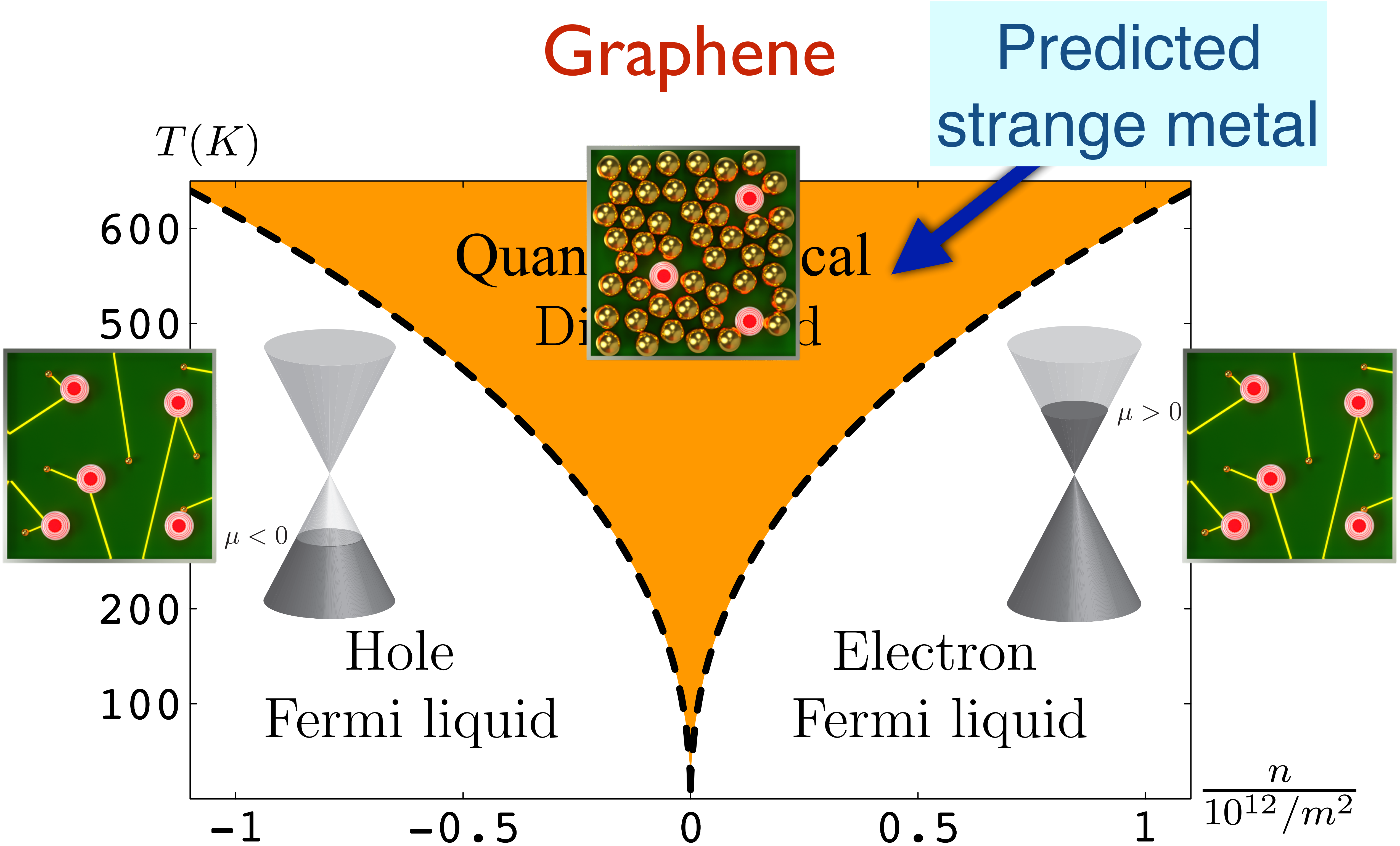


Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events



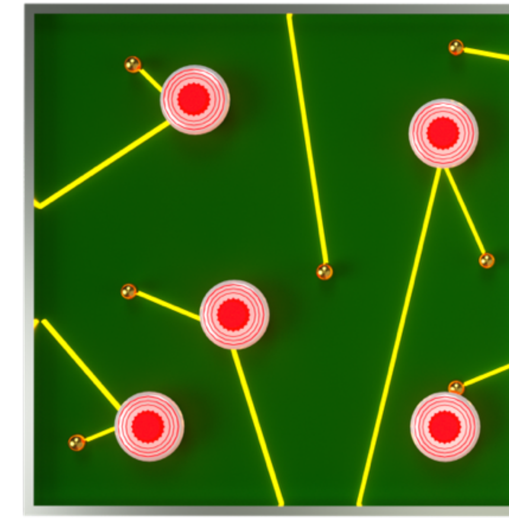
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron "liquid" then "flows" around impurities

Graphene



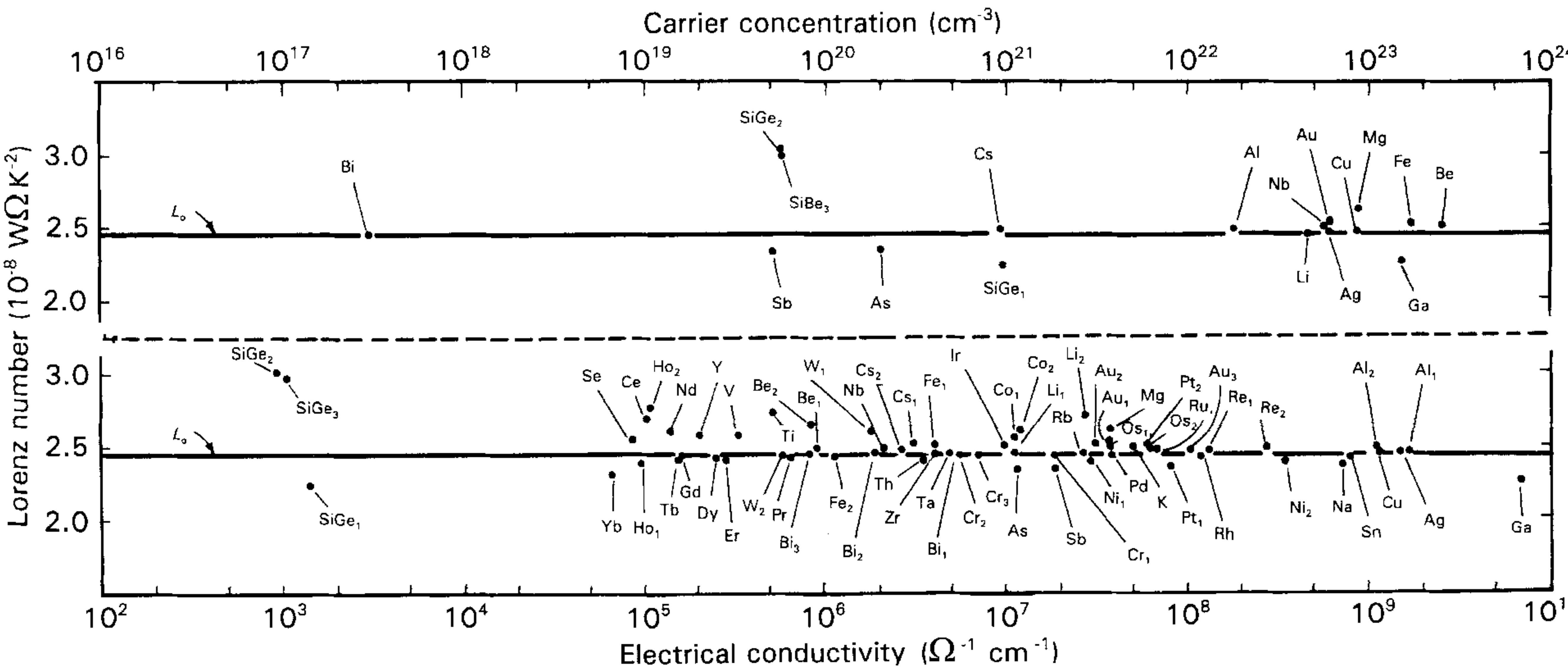
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal and electrical conductivity with quasiparticles



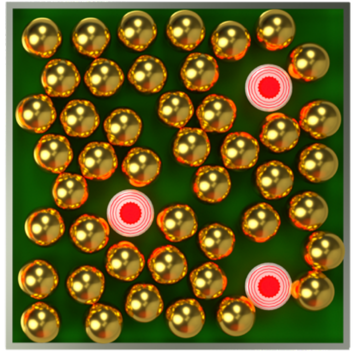
- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Transport in Strange Metals



- hydrodynamics when $l \gg l_{\rm ee}$, $t \gg t_{\rm ee}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\rm ext}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, J^μ in perturbative parameter $l_{\rm ee}\partial_\mu$:

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu$$

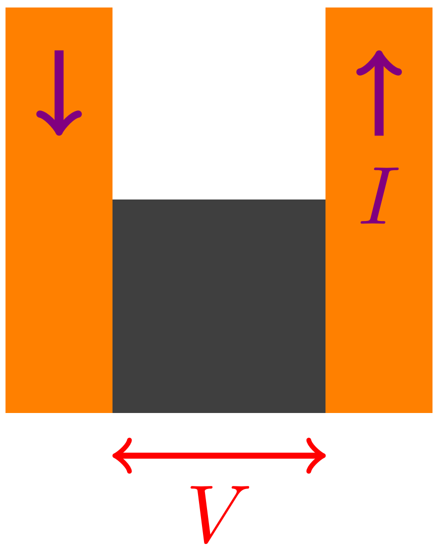
$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\rm Q}\mathcal{P}^{\mu\rho}\left(\partial_\rho \mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\rm ext}_{\rho\nu}\right) + \cdots,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

- New (and only) transport co-efficient, σ_Q: "quantum critical" conductivity at $\mathcal{Q} = 0$.

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)



$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

- σ = easy experiment; related to QFT correlators:

$$\sigma_{ij}(\omega) = \frac{\mathrm{i}}{\omega}\langle J_i(-\omega) J_j(\omega)\rangle, \quad \text{etc.}$$

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \pi\delta(\omega) + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \pi\delta(\omega) + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \pi\delta(\omega) + \bar{\kappa}_Q(\omega)$$

with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

In theories which are relativistic at high energies (including graphene), $T\alpha_Q(\omega) = -\mu\sigma_Q(\omega)$, $T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega)$, $\mathcal{M} = T\mathcal{S} + \mu\mathcal{Q} = \mathcal{H}$ the enthalpy density, and $\mathcal{Q} = n$ the electron density

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Translational symmetry breaking

Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\rm imp}} + \ldots$$

"Memory function" methods yield an explicit expression for $\tau_{\rm imp}$:

$$\frac{\mathcal{M}}{\tau_{\rm imp}} = \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\mathrm{Im}\left(G^{\rm R}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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Obtained in hydrodynamics, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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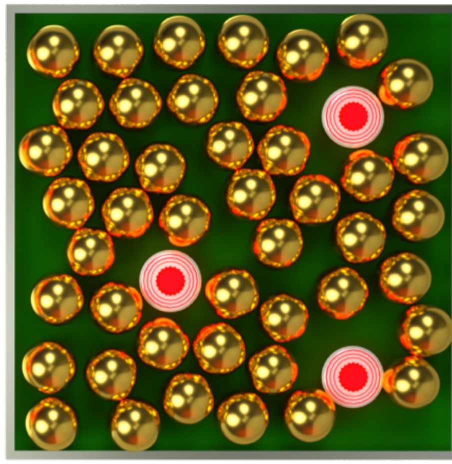
Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \bar{\kappa}_Q(\omega)$$



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S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Prediction for transport in the graphene strange metal

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield for the Lorentz ratio $L = \kappa/(T\sigma)$

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

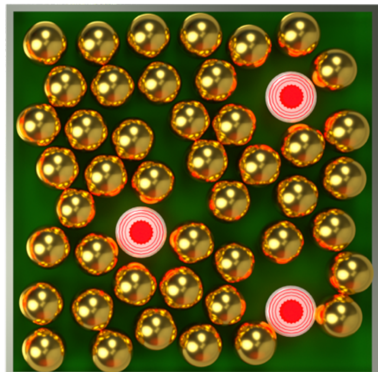
$$L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, τ_{imp} is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

- For $\mathcal{Q} = 0$, as $\tau_{\text{imp}} \to \infty$, σ remains finite, while $\kappa \to \infty$, and so $L \to \infty$.
- For $\mathcal{Q} \neq 0$, as $\tau_{\text{imp}} \to \infty$, $\sigma \to \infty$, while κ remains finite, and so $L \to 0$.

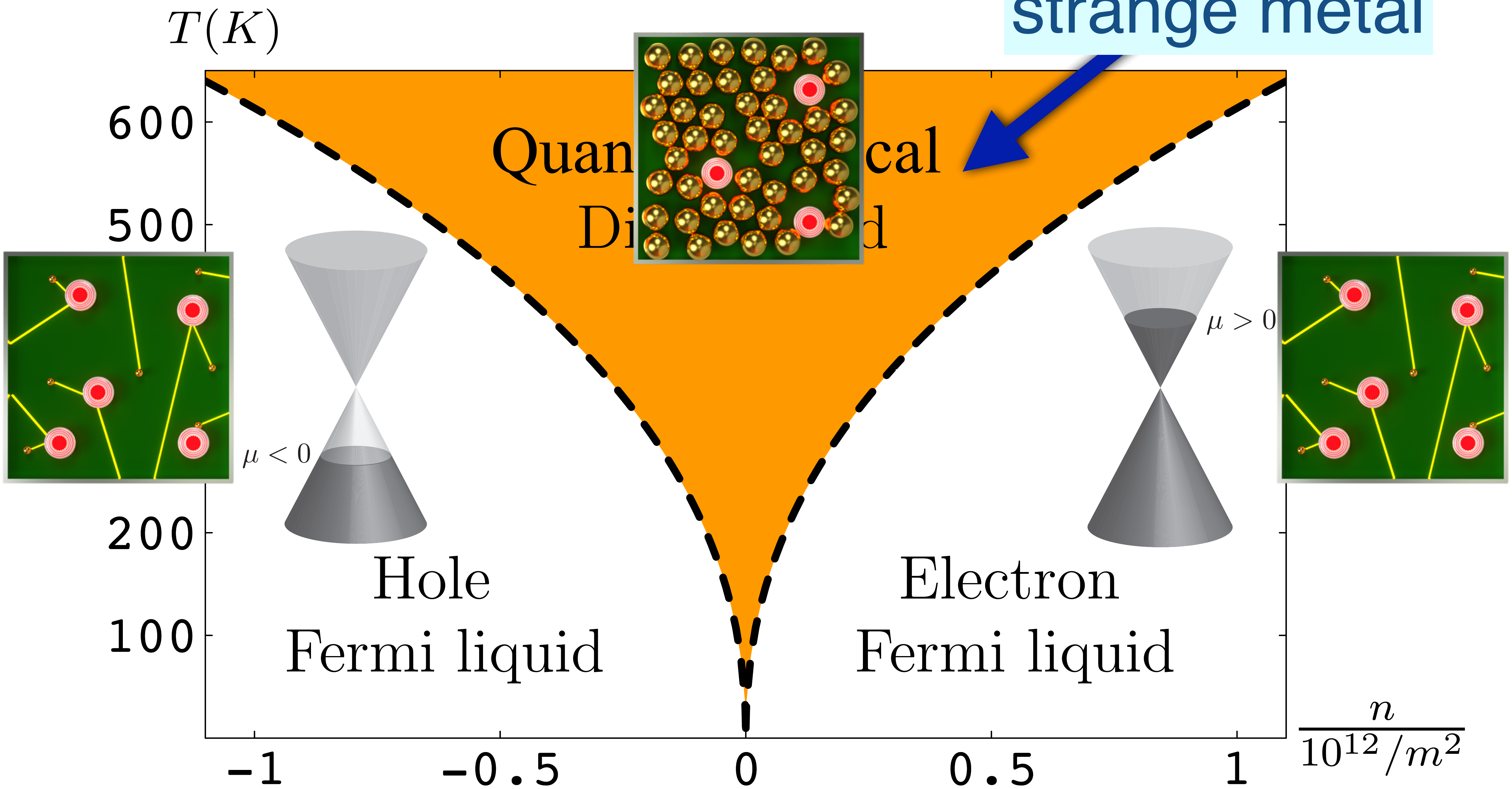
<u>Prediction</u>: L diverges as $1/\mathcal{Q}^4$ near $\mathcal{Q} = 0$ in clean graphene.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)



Graphene

Predicted strange metal



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
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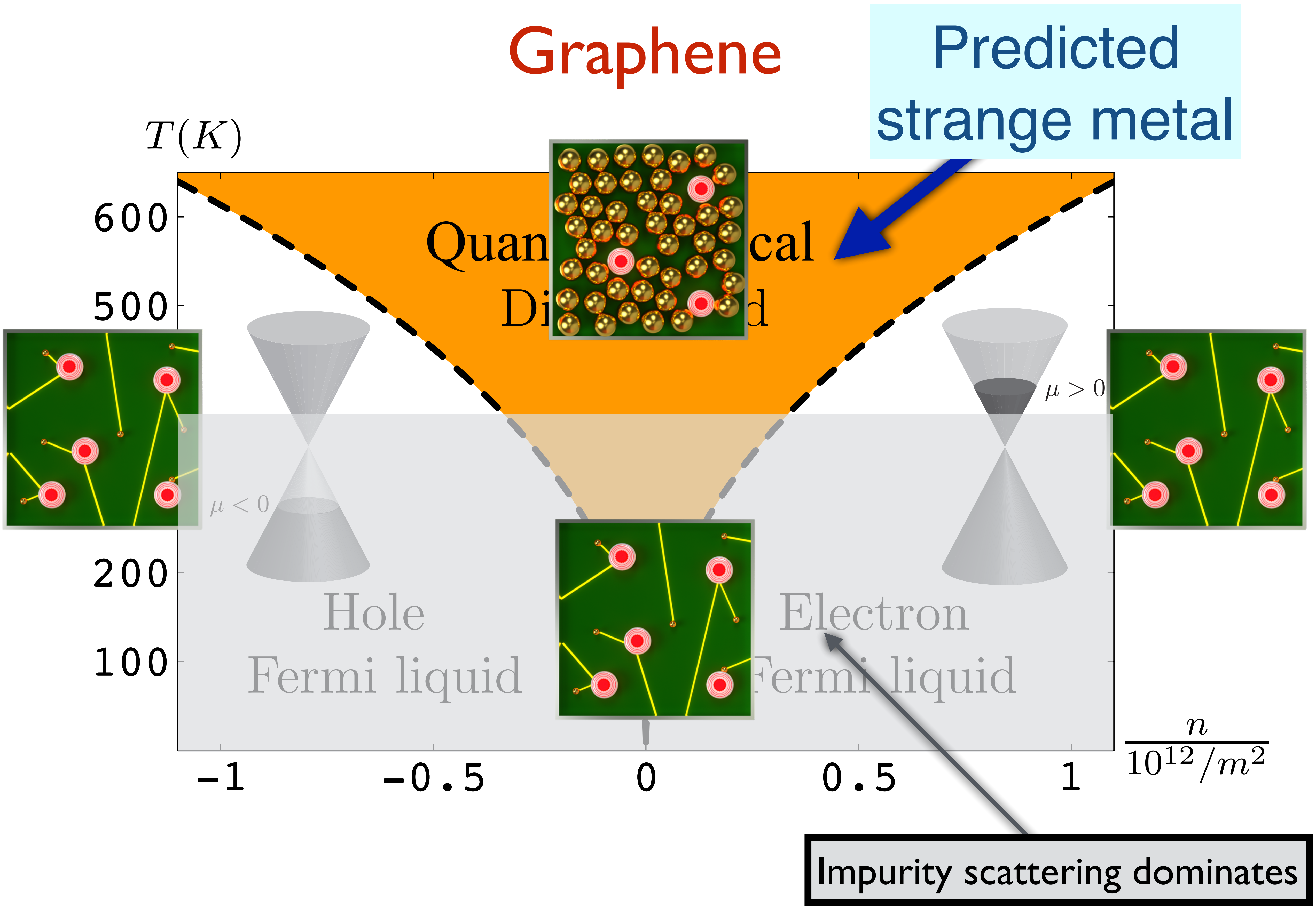
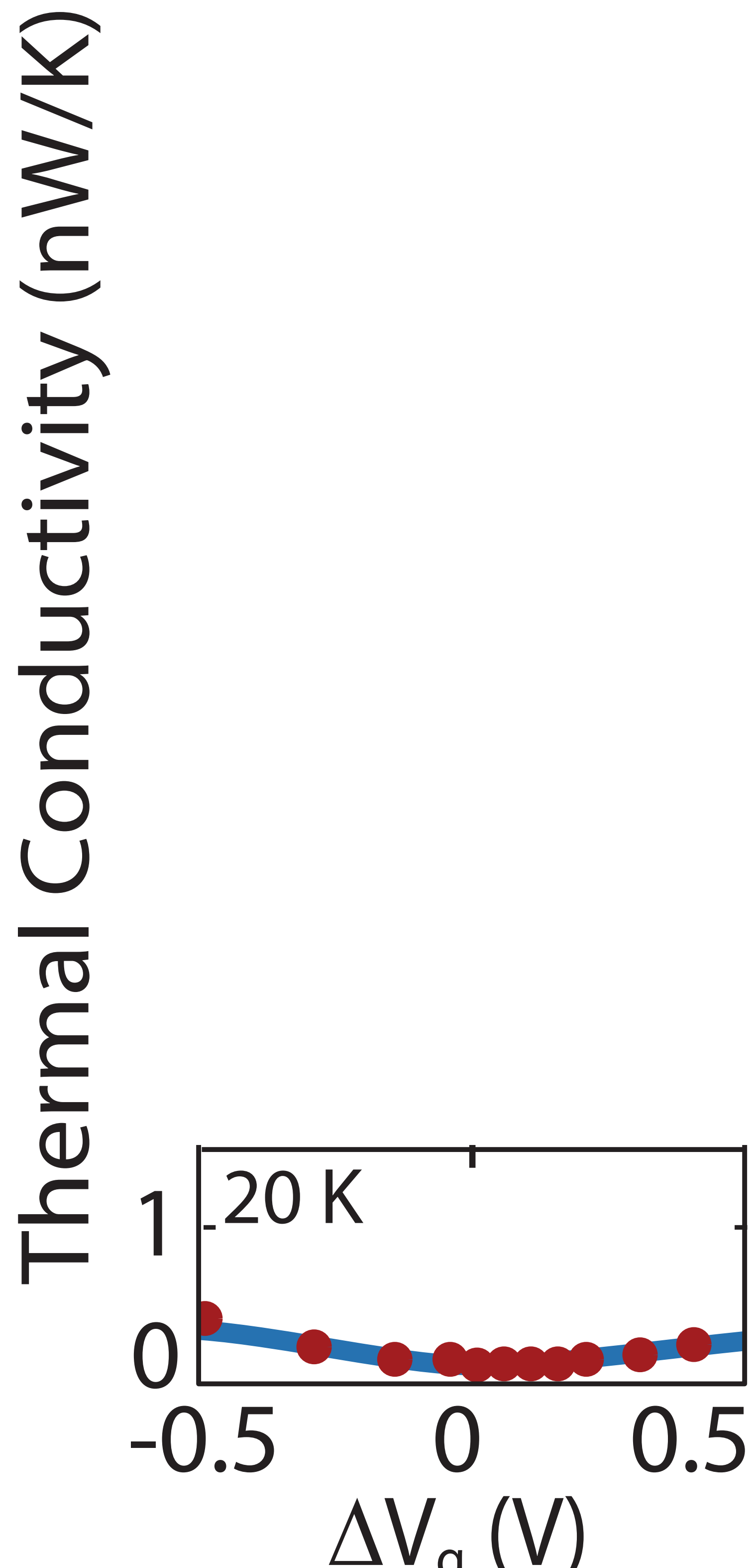
Graphene



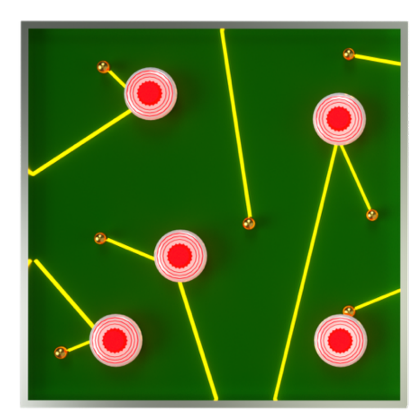
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

J. Crossno et al., Science **351**, 1058 (2016)

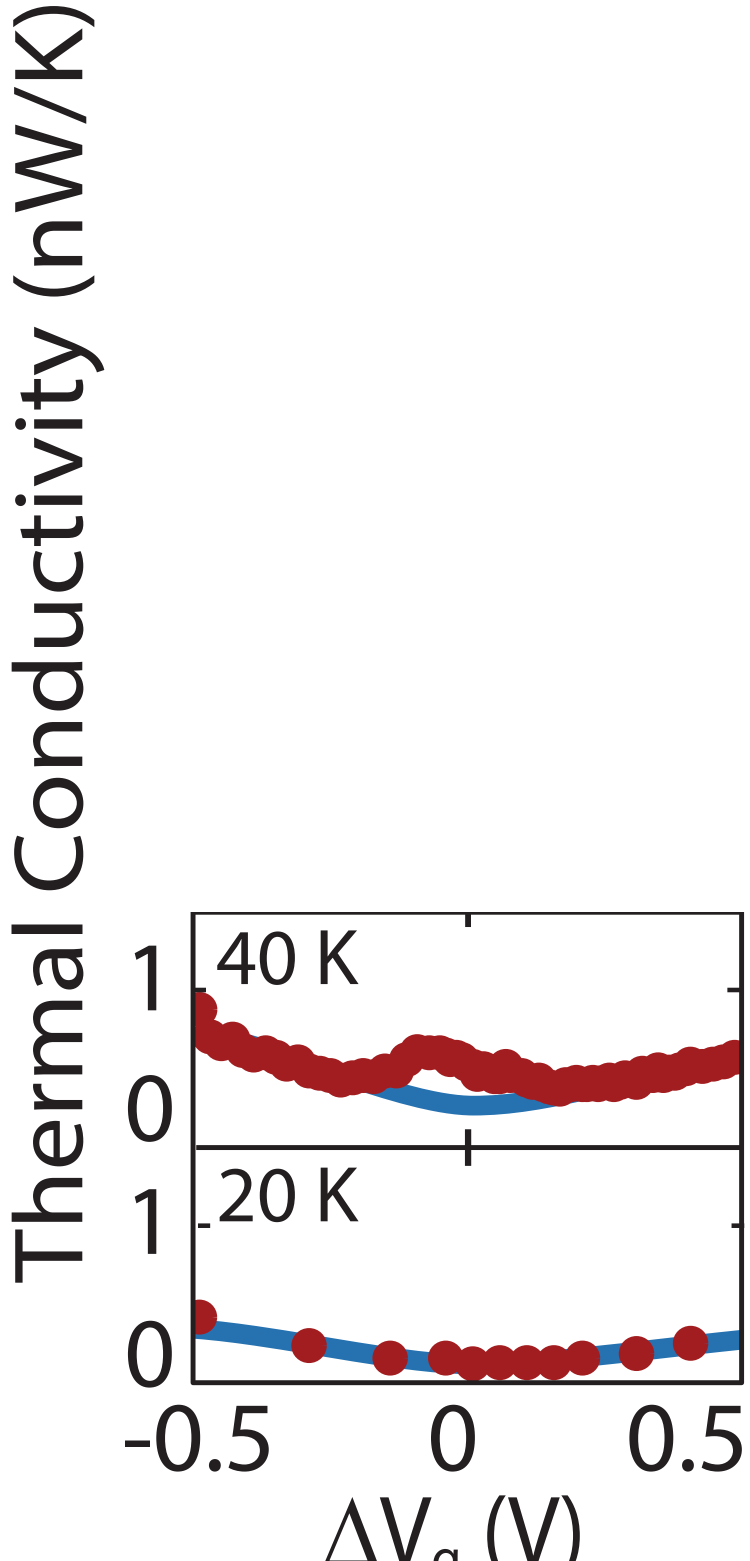


Red dots: data

Blue line: value for $L = L_0$



J. Crossno et al., Science **351**, 1058 (2016)



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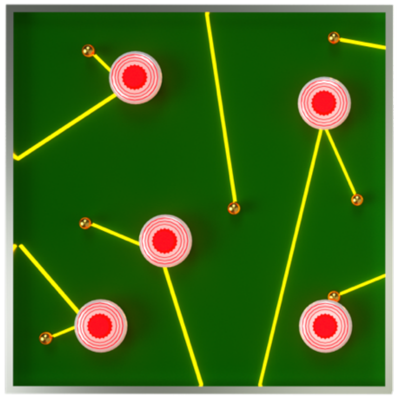
Blue line: value for $L = L_0$



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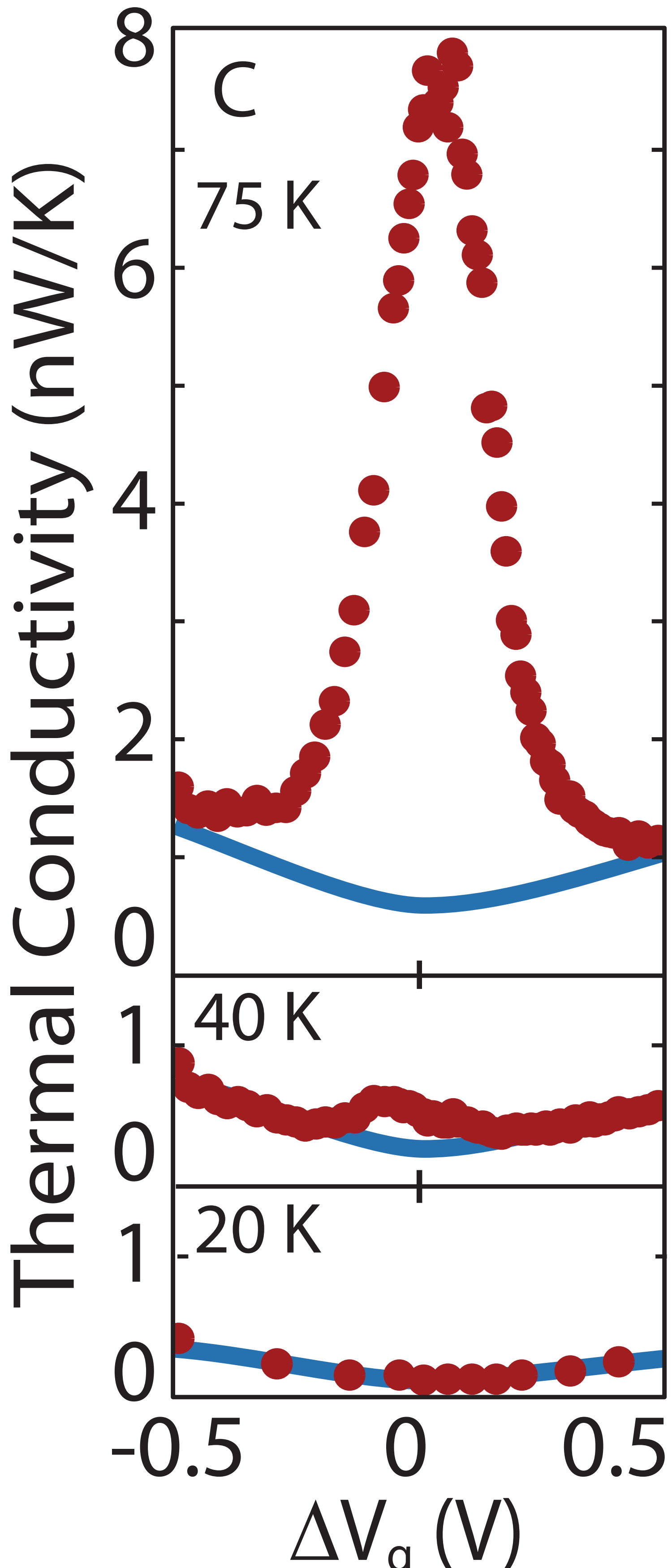

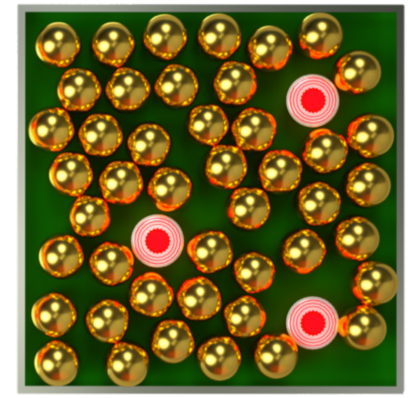

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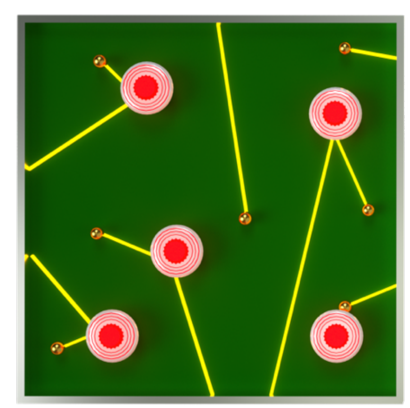
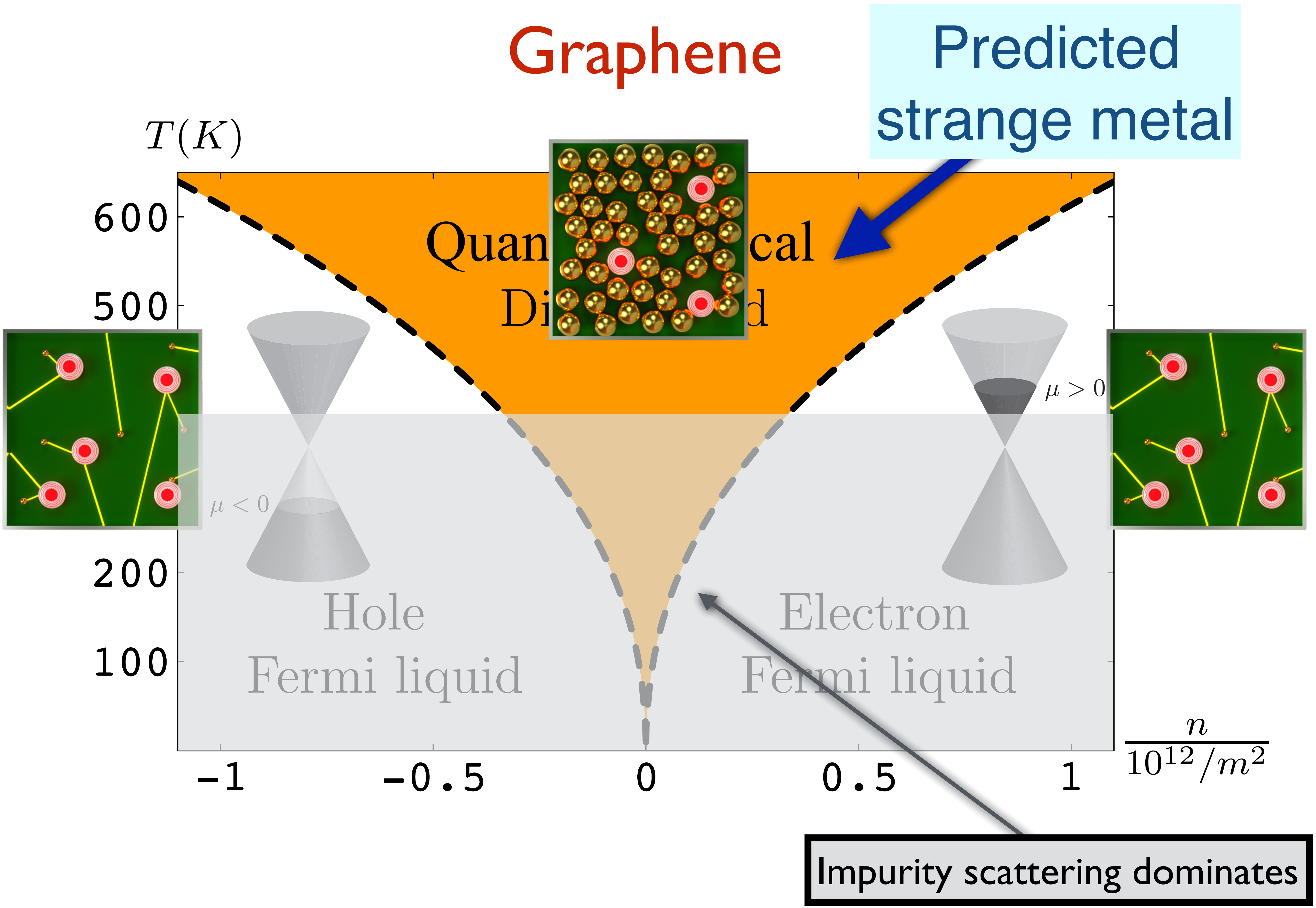


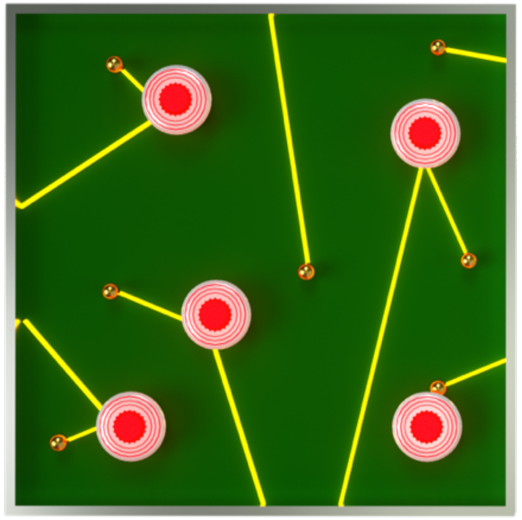
Graphene

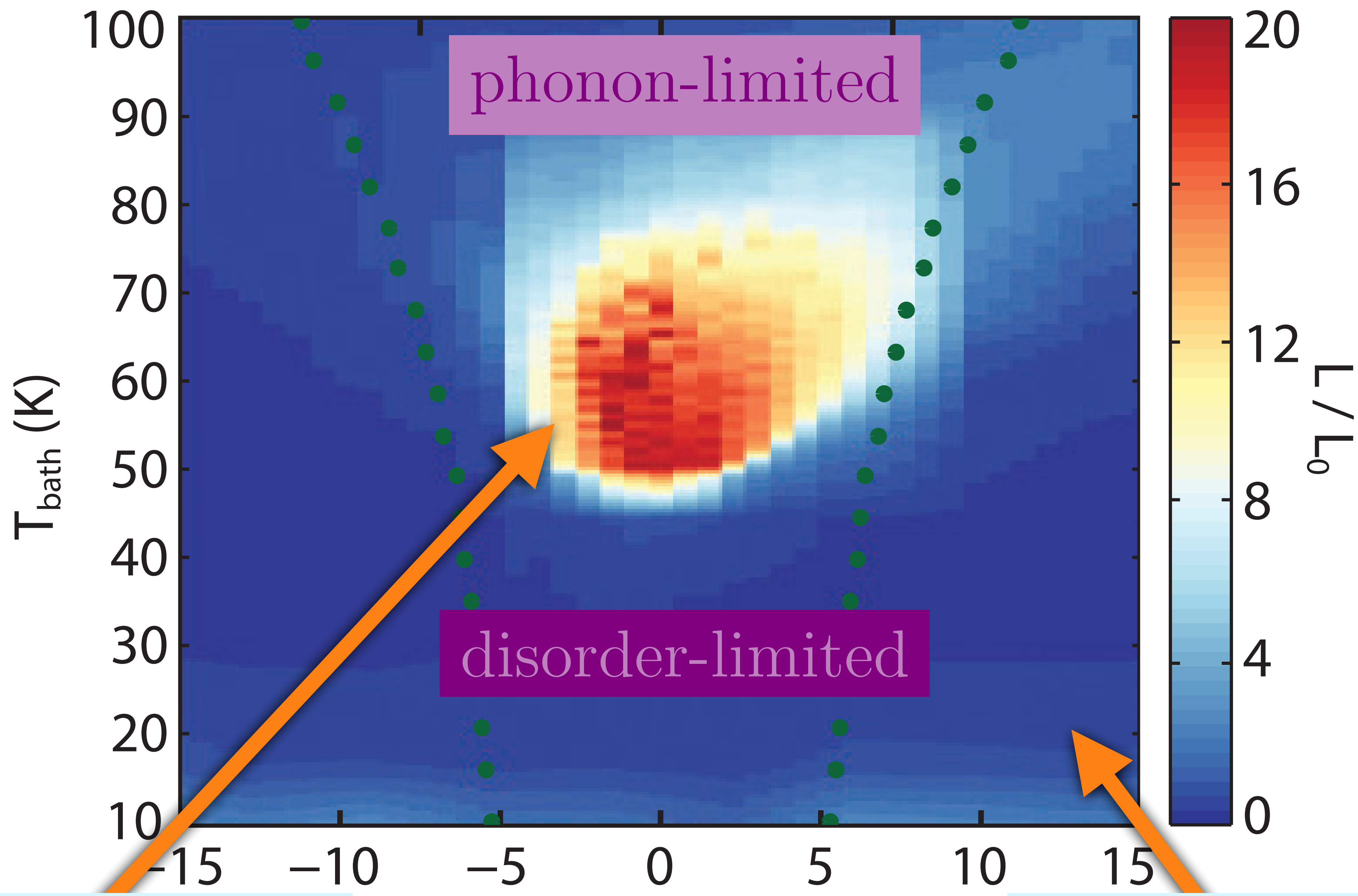


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

J. Crossno et al., Science **351**, 1058 (2016)

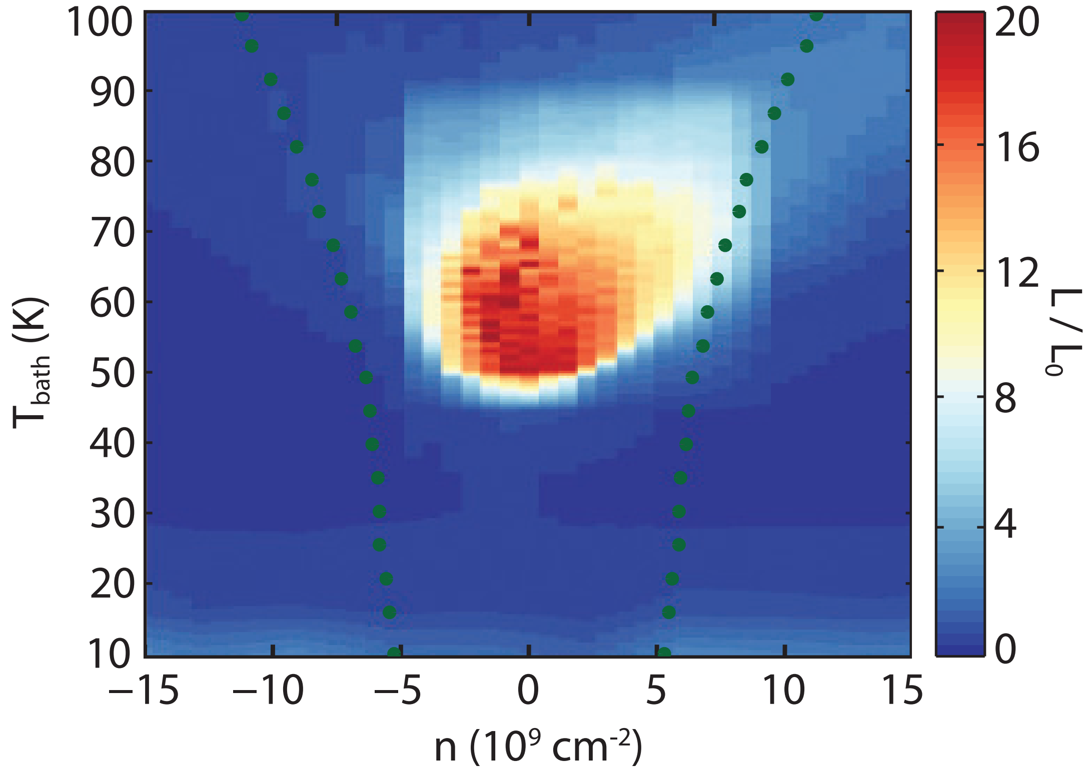
Strange metal in graphene

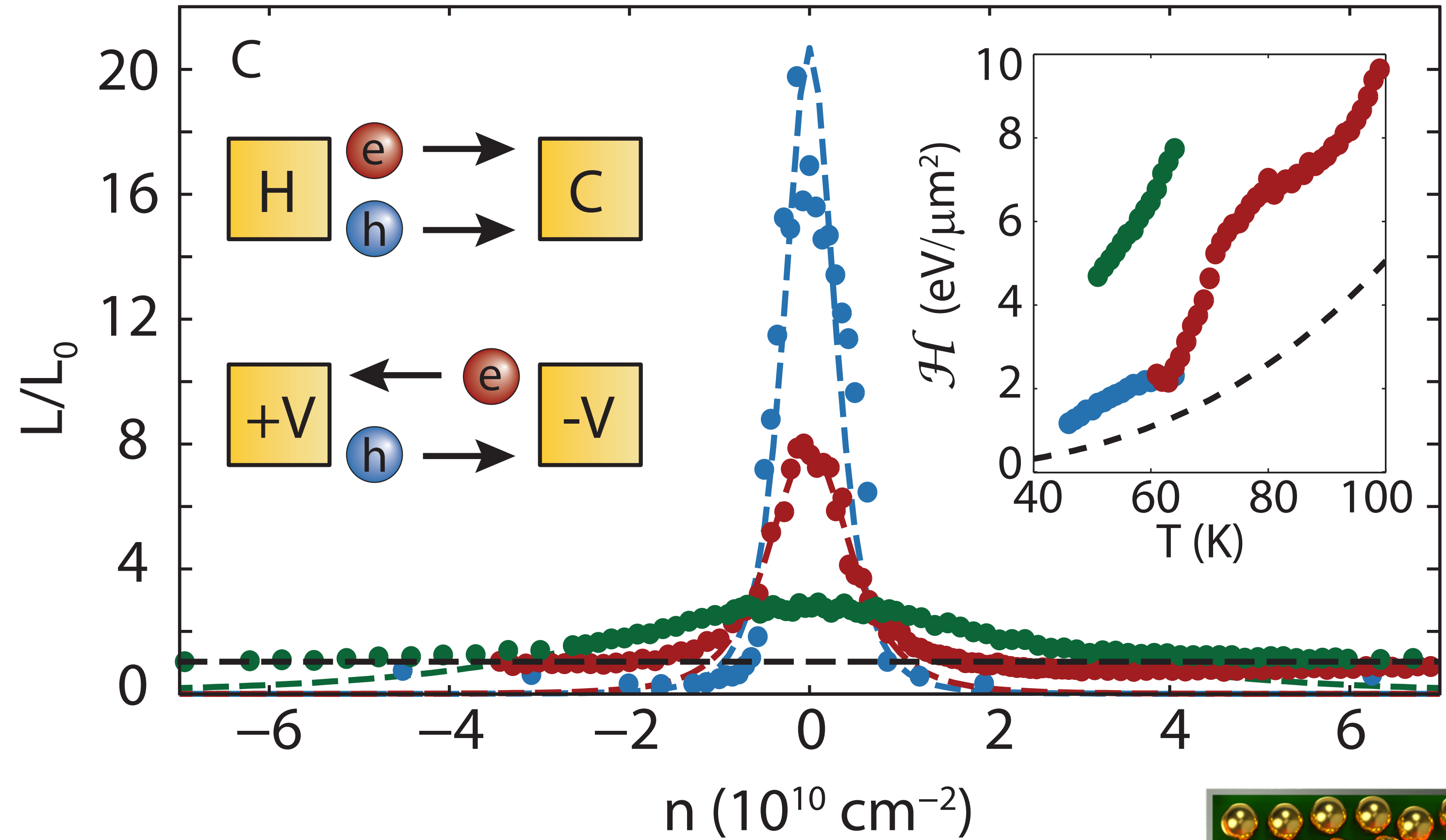


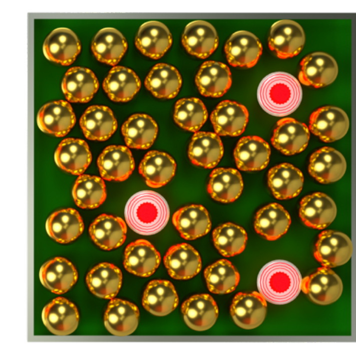


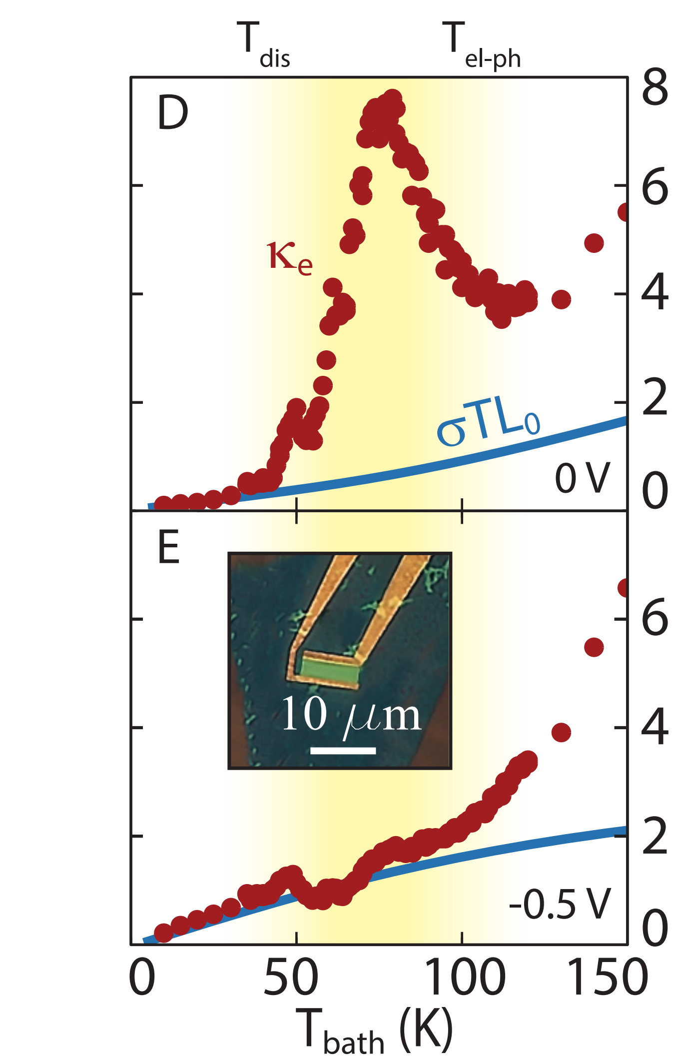
Wiedemann-Franz violated !

Wiedemann-Franz obeyed









Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

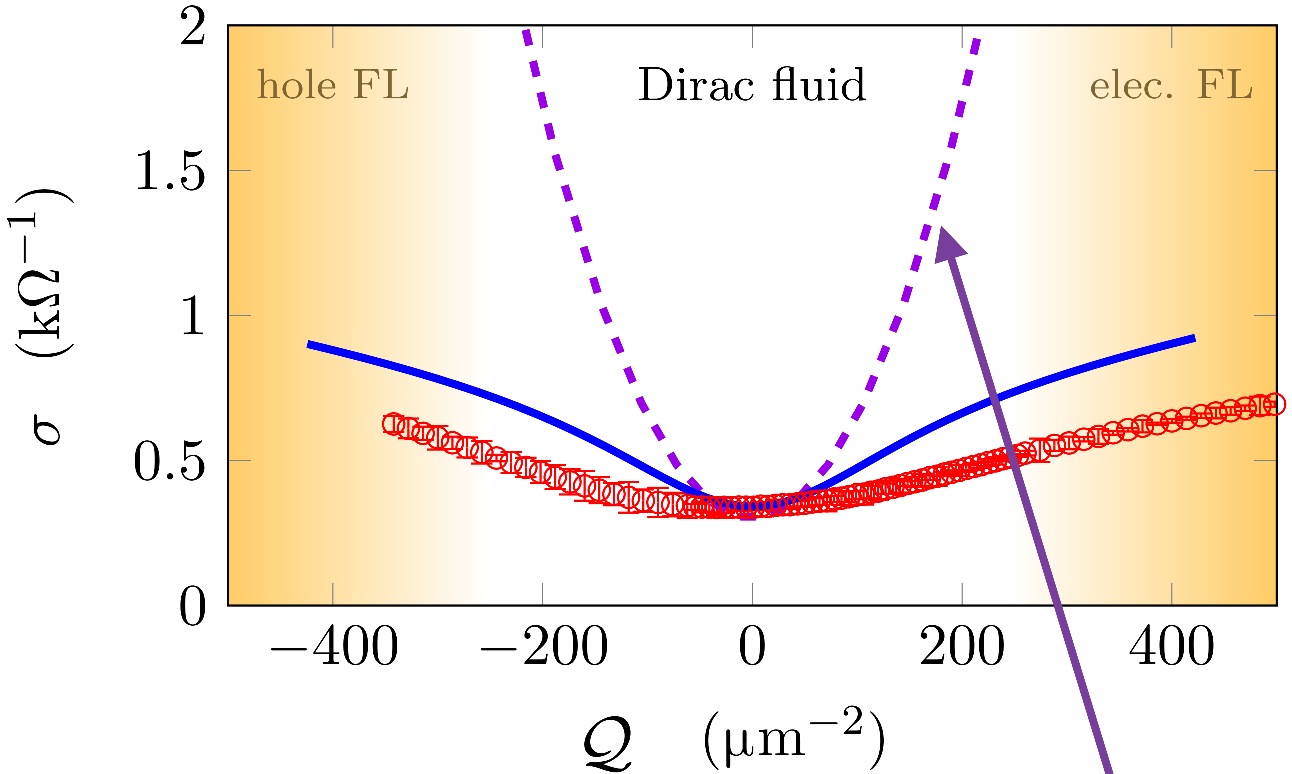
$\mathcal{Q} \to$ electron density; $\mathcal{H} \to$ enthalpy density

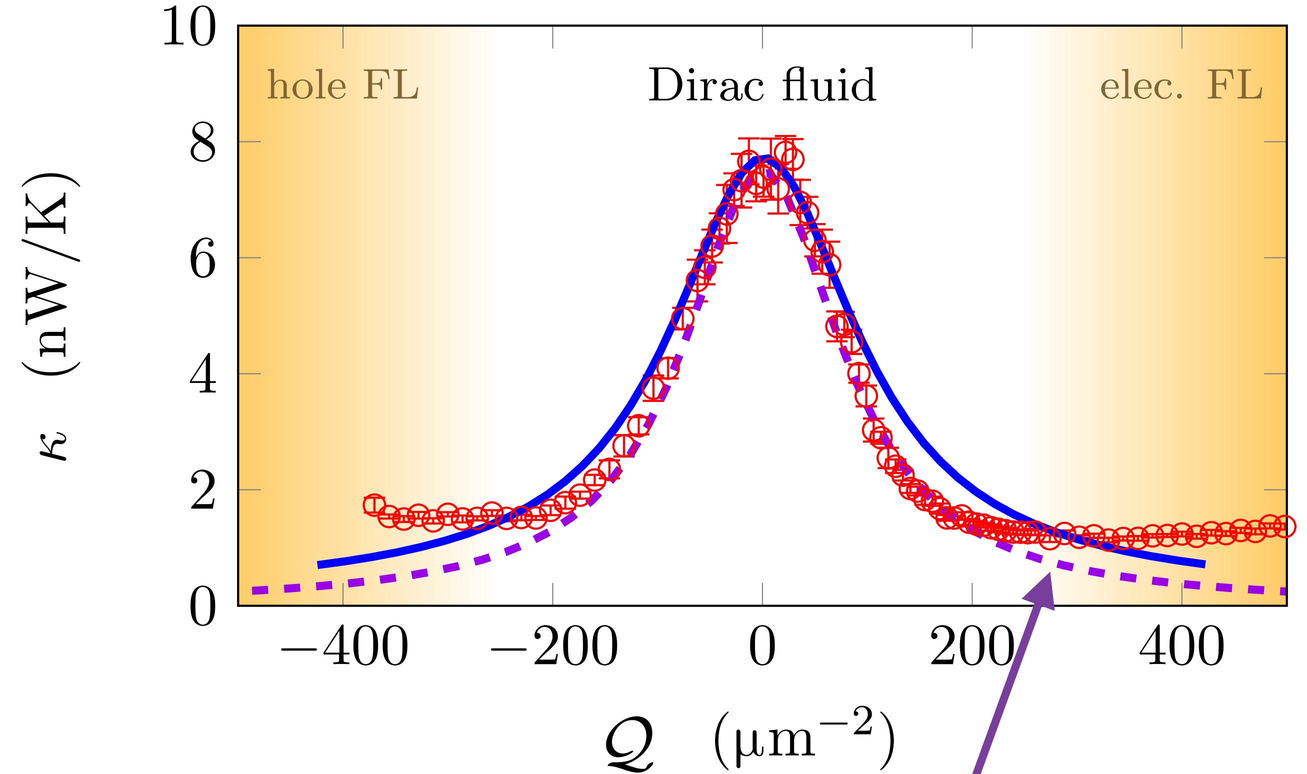
$\sigma_Q \to$ quantum critical conductivity

$\tau_{\rm imp} \to$ momentum relaxation time from impurities

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

J. Crossno et al., Science **351**, 1058 (2016)

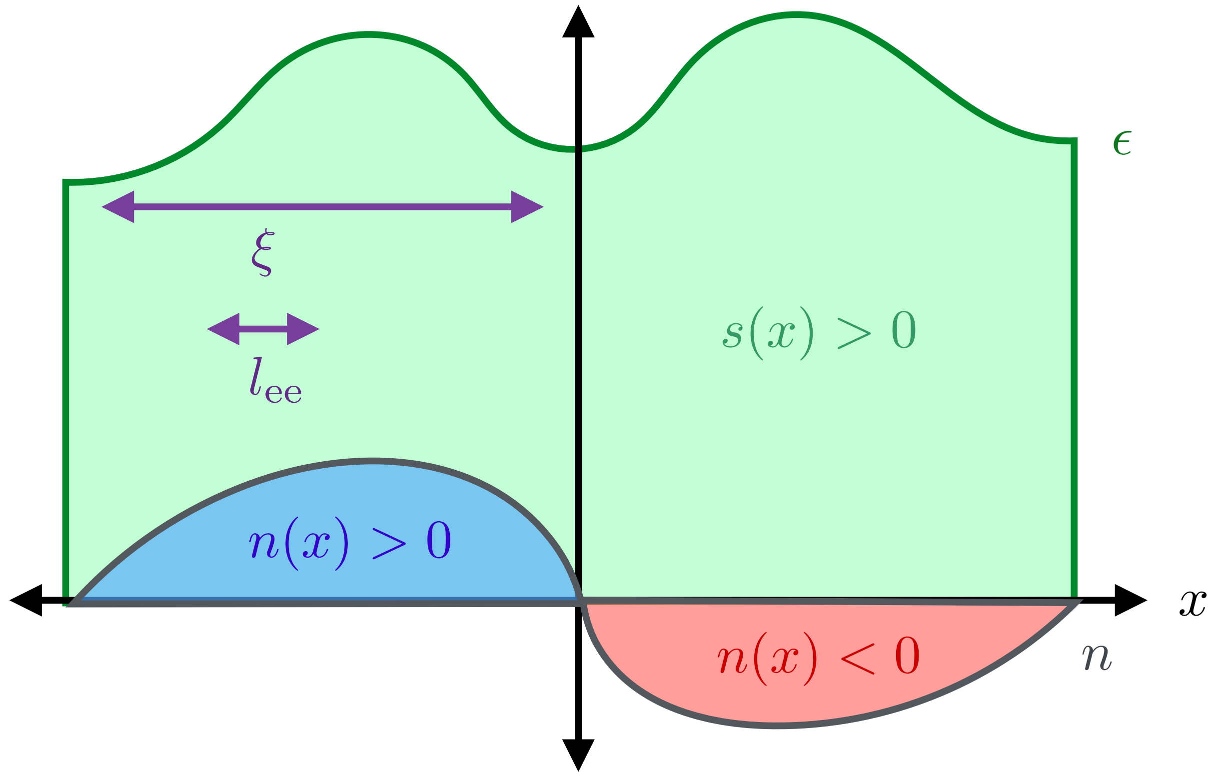




Comparison to theory with a single momentum relaxation time τ_{imp}. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)

Non-perturbative treatment of disorder

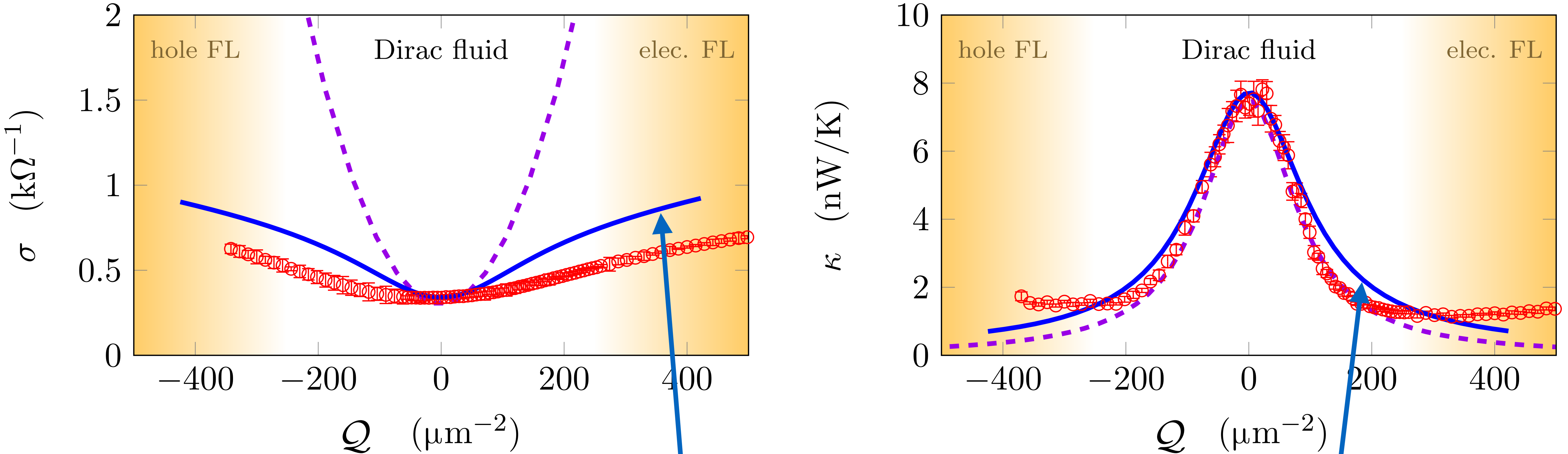


Note
$n \equiv \mathcal{Q}$

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_B T$, and so the entropy density s/k_B is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than l_{ee}, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)

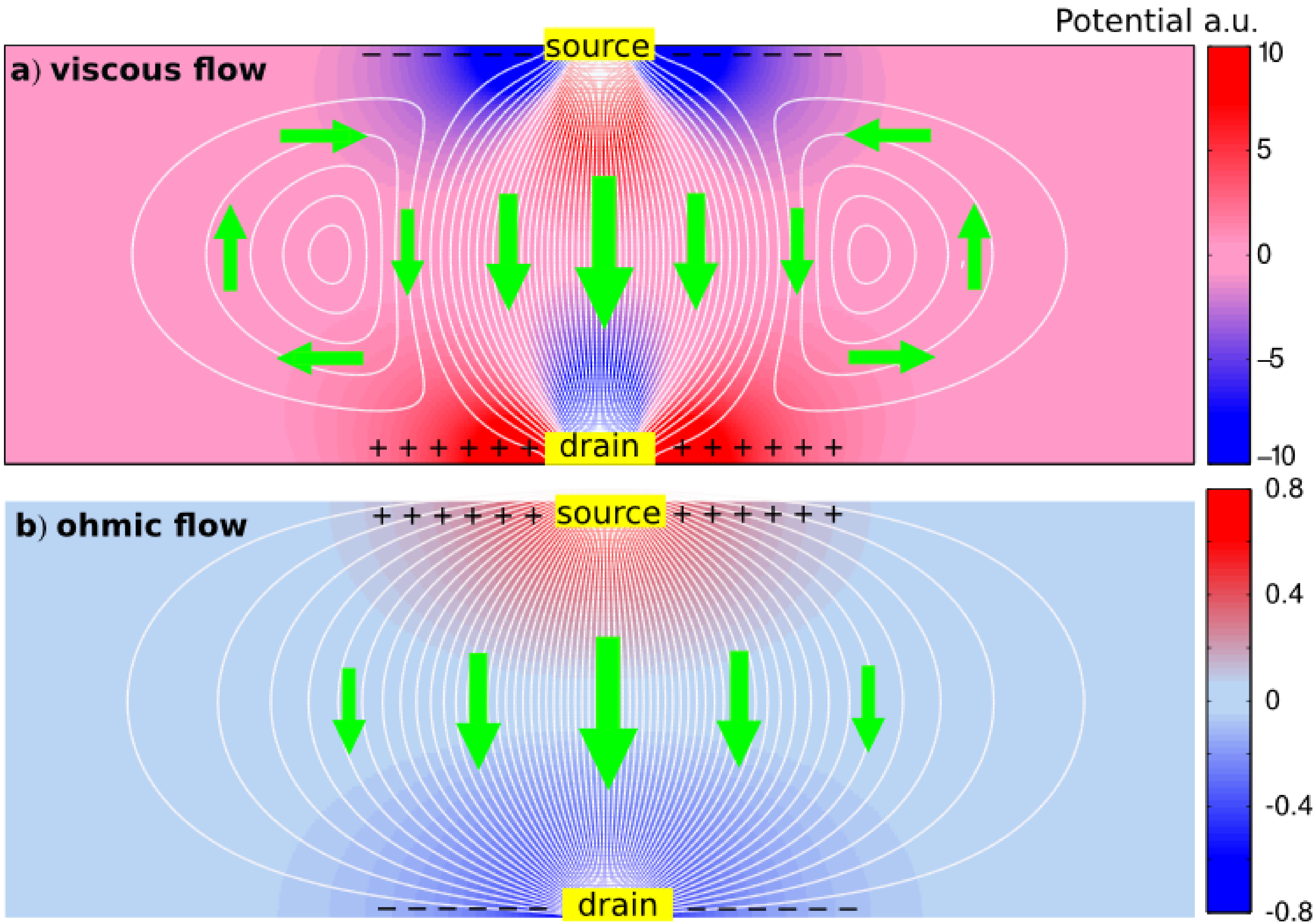


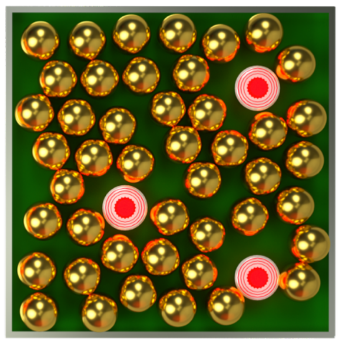
Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

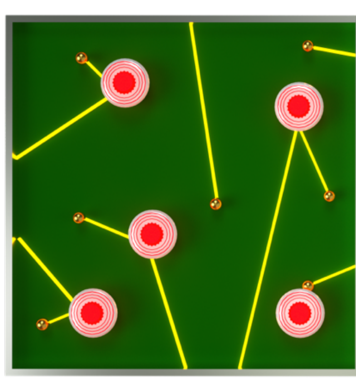
A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, PRB **93**, 075426 (2016)

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene







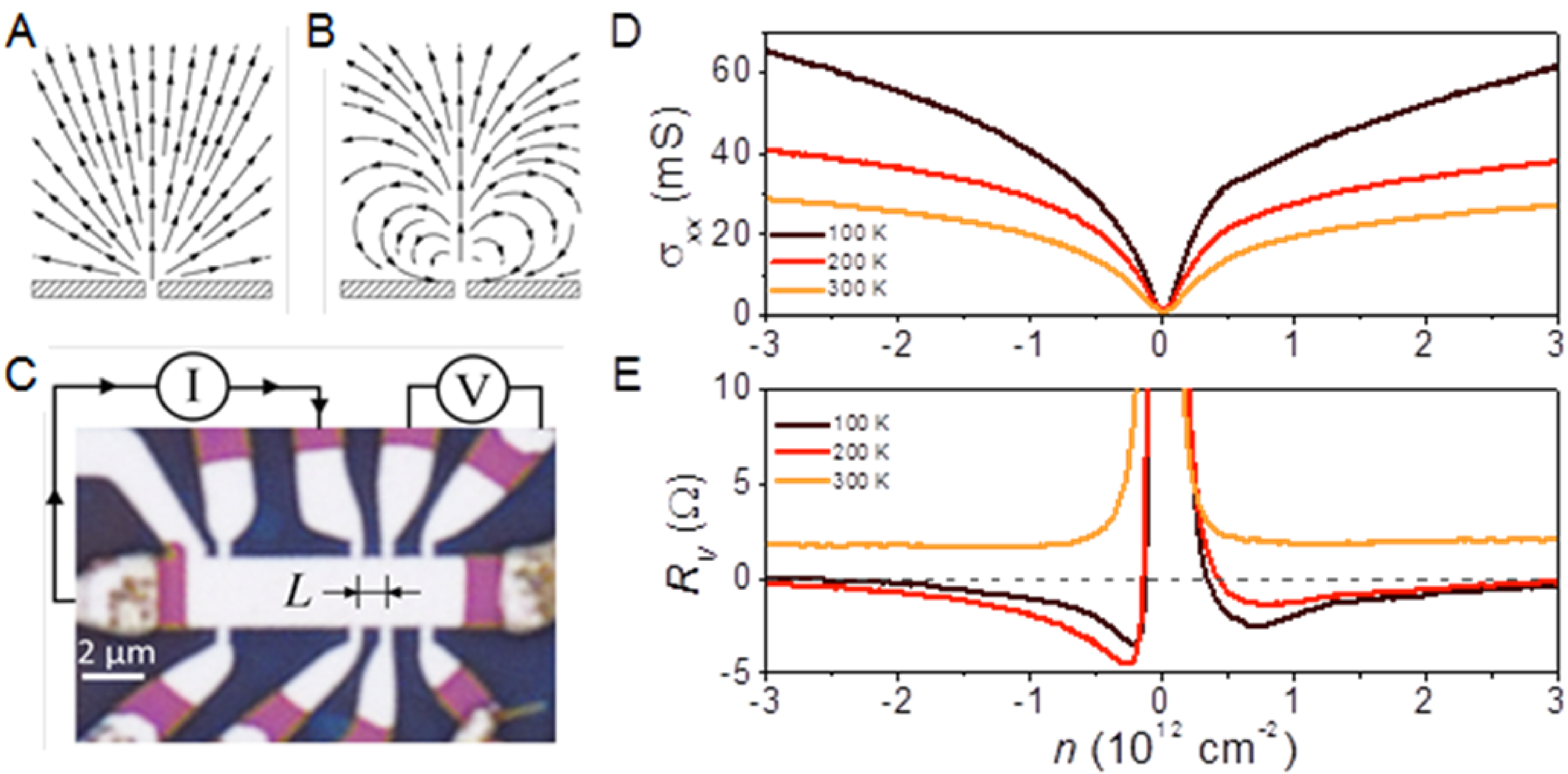
L. Levitov and G. Falkovich, *Nature Physics* **12**, 672 (2016)

Strange metal in graphene

Science **351**, 1055 (2016)

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]



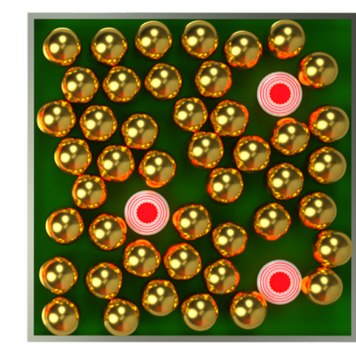
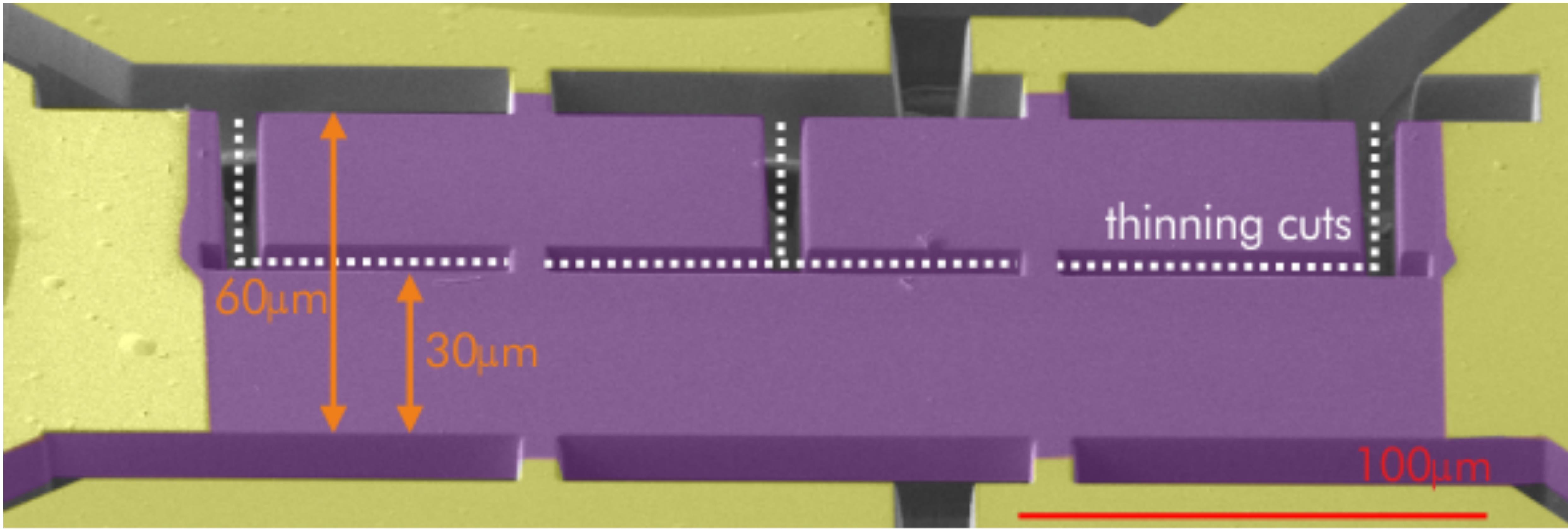


Figure 1. Viscous backflow in doped graphene. (**a,b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d,e**) Longitudinal conductivity σ_{xx} and R_V for this device as a function of n induced by applying gate voltage. $I = 0.3$ μA; $L = 1$ μm. For more detail, see Supplementary Information.

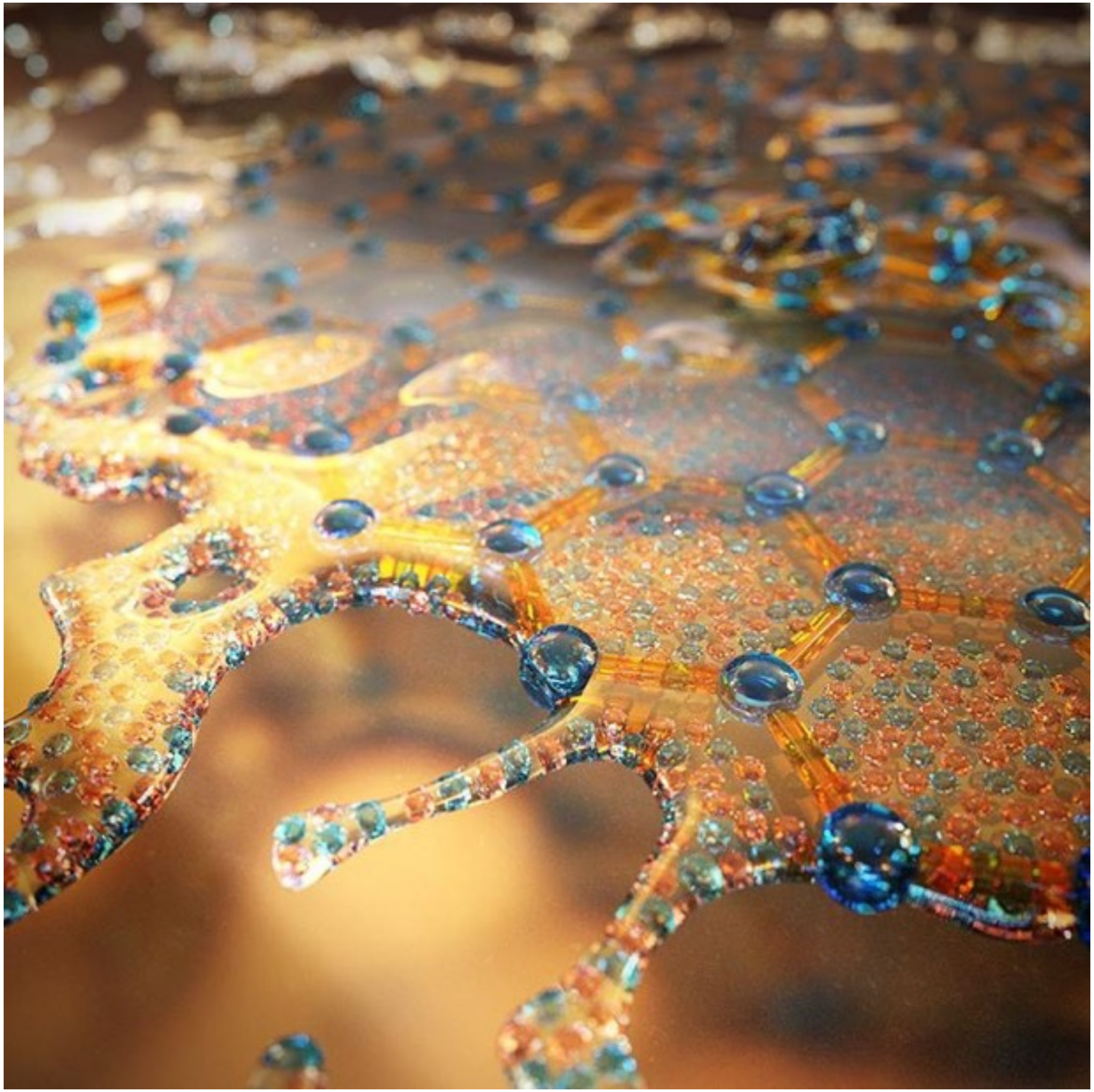
Signature of Navier-Stokes hydrodynamic flow in $PdCoO_2$



Experiment: Successively narrow the channel in factors of 2, measuring the resistance after every step.

P.J.W. Moll, P. Kushwaha, N. Nandi, B. Schmidt and A.P. Mackenzie, Science ***351****, 1061 (2016)*

Graphene: "a metal that behaves like water"



Quantum matter without quasiparticles:

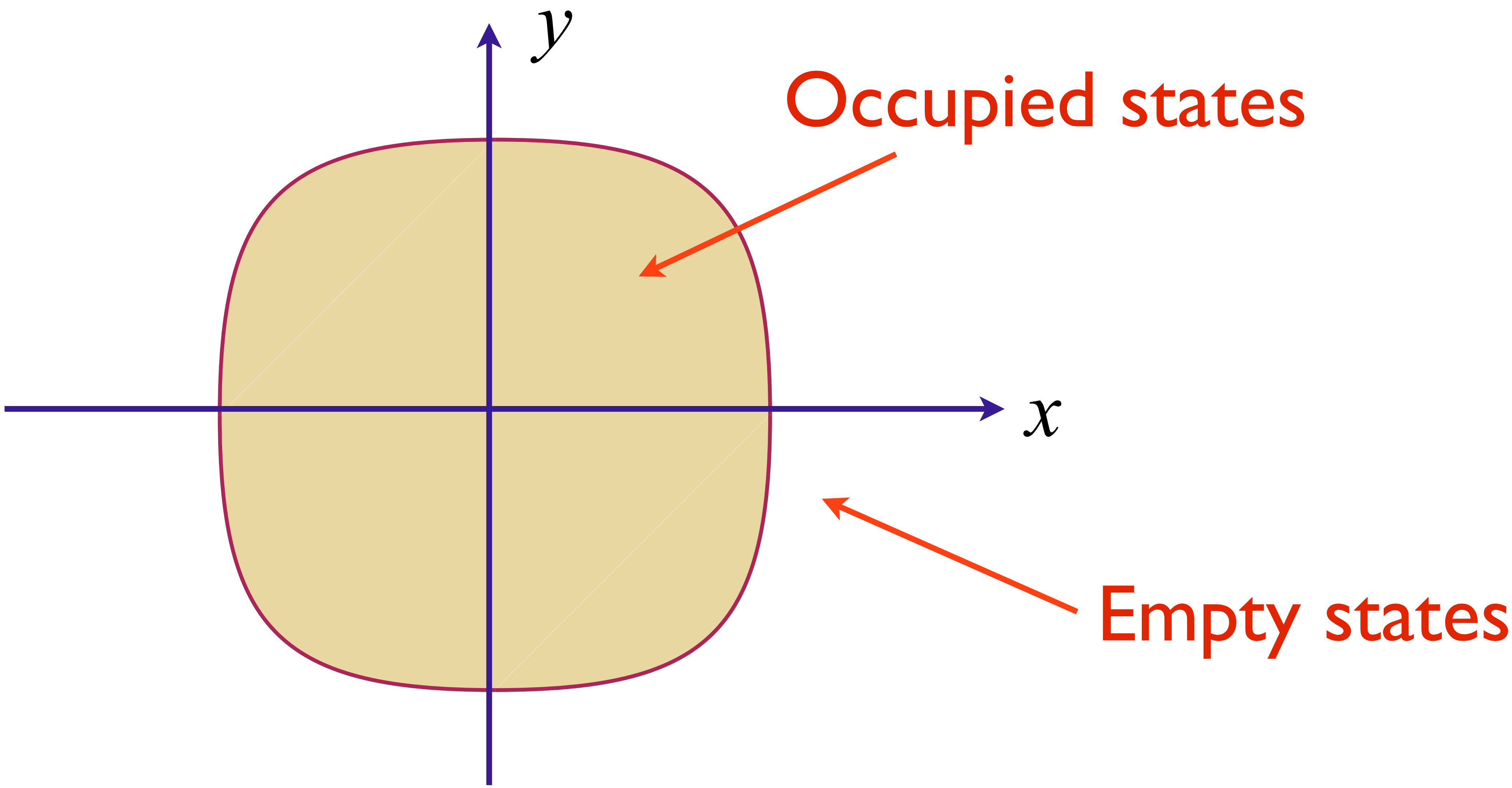
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Graphene
- Non-Fermi liquids - Ising-nematic criticality of a two-dimensional metal

Aavishkar
Patel

Andreas
Eberlein

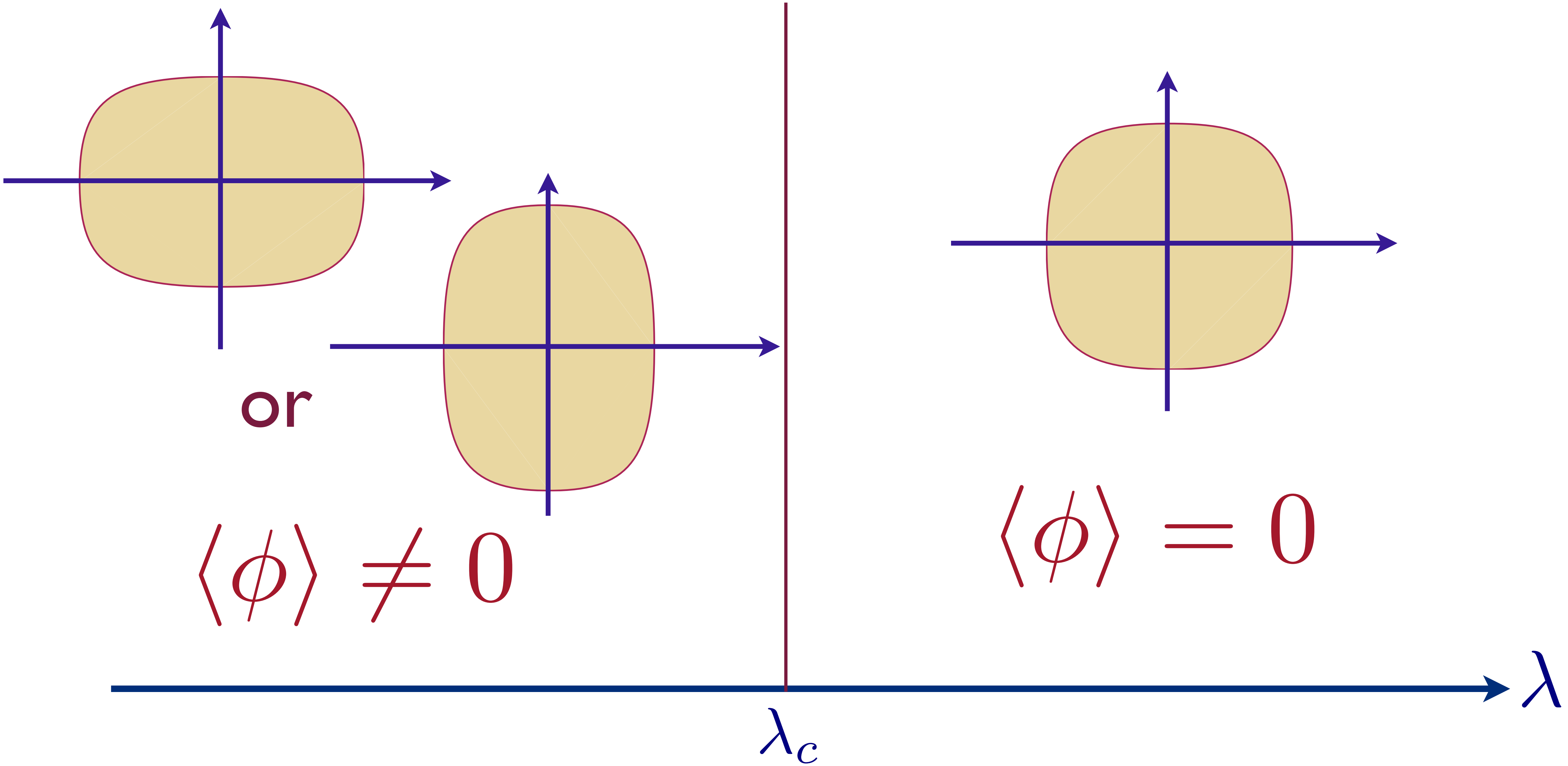
Ipsita Mandal

Quantum criticality of Ising-nematic ordering in a metal



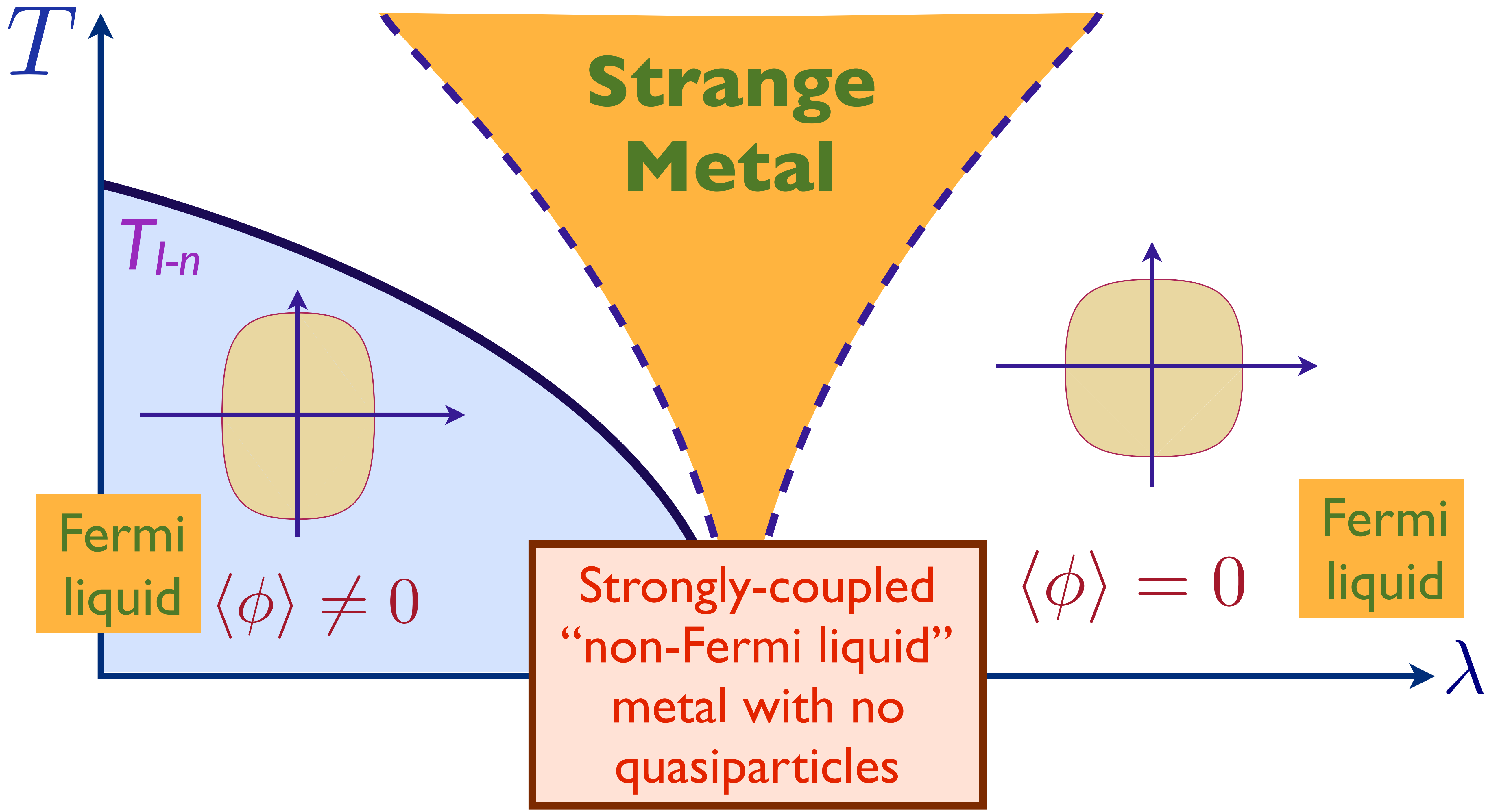
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



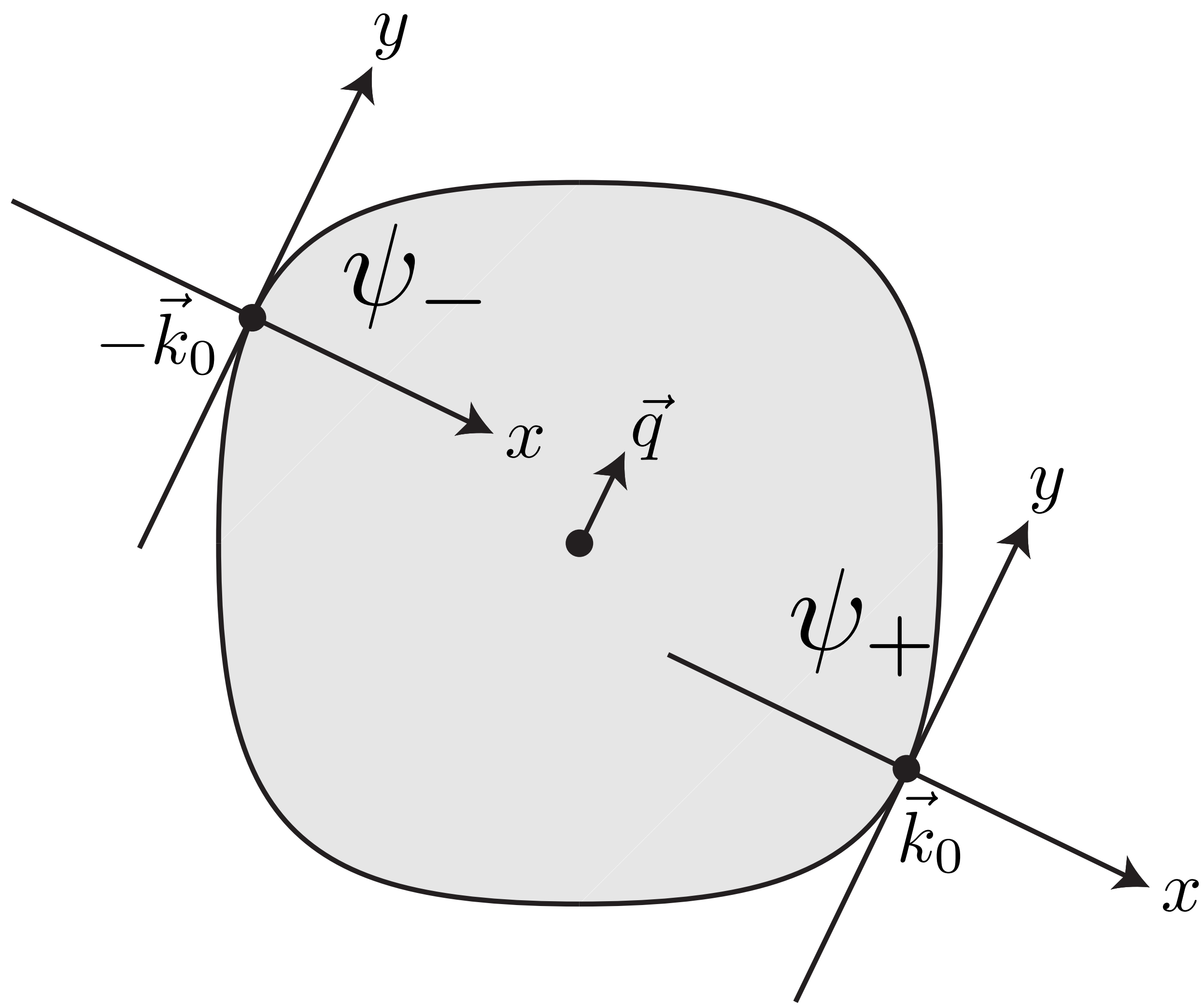
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

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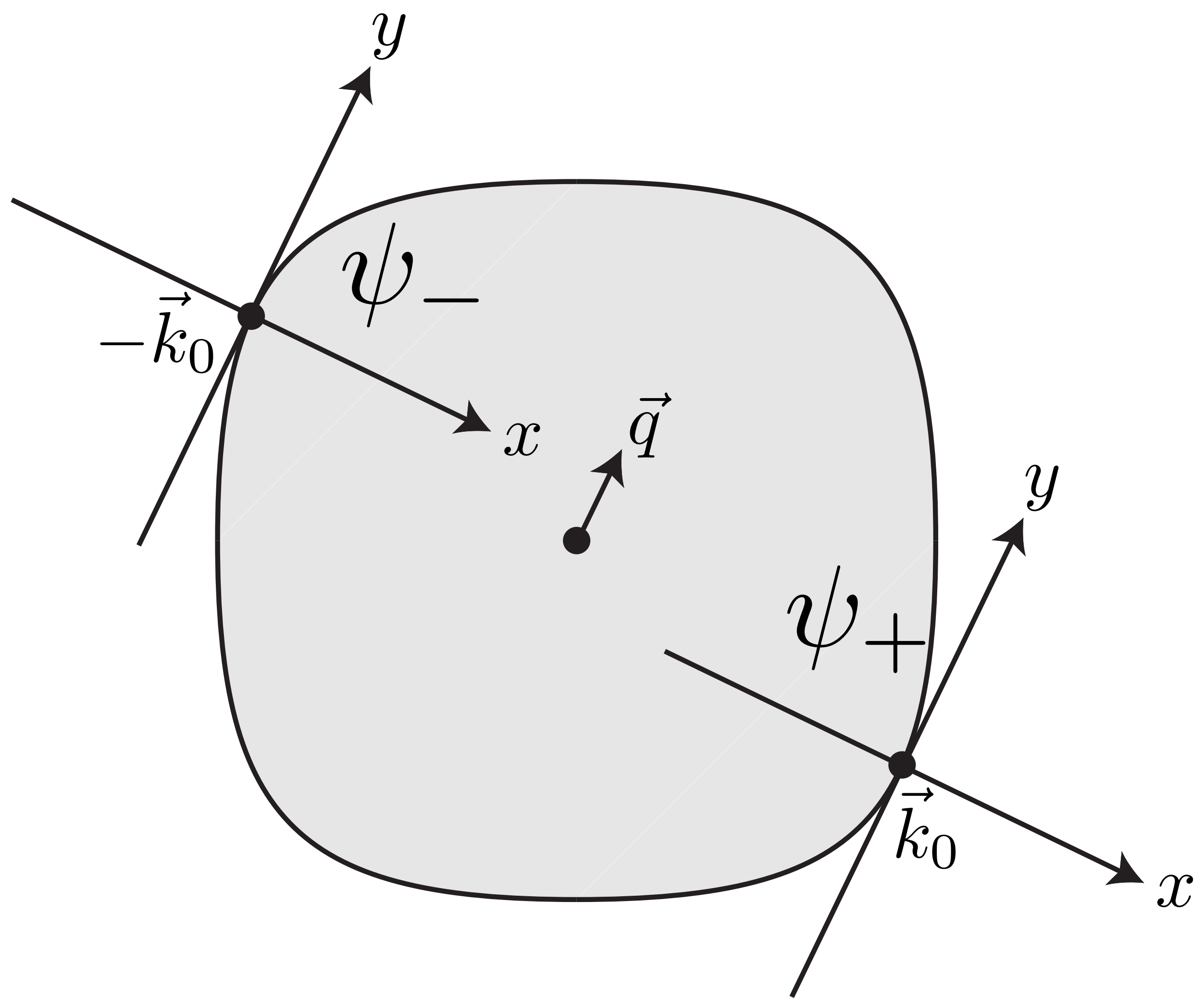

$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering in a metal

Thermodynamics and transport co-efficients in a non-Fermi liquid:
The entropy density, s, obeys

$$s \sim T^{(d-\theta)/z}$$

where $z = 3/2$ is the dynamic critical exponent for fermionic excitations dispersing normal to the Fermi surface, and $d - \theta = 1$ is the number of dimensions normal to the Fermi surface.

A RG analysis using a dimensionality expansion below $d = 5/2$ shows that the quantum critical conductivity obeys

$$\sigma_Q \sim T^{(d-\theta-2)/z}$$

A. Eberlein, I. Mandal, and S. Sachdev, arXiv:1605.00657

We also computed the shear viscosity and found

$$\eta \sim T^{(d-\theta-2)/z}$$

A. A. Patel, A. Eberlein, and S. Sachdev, arXiv:1607.03894

Note that $\eta/s \sim T^{-2/z}$ does not scale to a constant: this is a consequence of the anisotropic scaling between the directions normal and parallel to the Fermi surface.

Entangled quantum matter without quasiparticles

- No quasiparticle excitations

- Shortest possible "phase coherence" time, fastest possible local equilibration time, or fastest possible Lyapunov time towards quantum chaos, all of order $\dfrac{\hbar}{k_B T}$

- Realization in solvable SYK model, which saturates the lower bound on the Lyapunov time. Its properties have some similarities to non-rational, large central charge CFT2s.

- Remarkable match between SYK and quantum gravity of black holes with AdS_2 horizons, including a SL(2,R)-invariant Schwarzian effective action for thermal energy fluctuations.

- Experiments on graphene agree well with predictions of a theory of a nearly relativistic quantum liquid without quasiparticles.

- Non-Fermi liquids with a critical Fermi surface have a divergent η/s as $T \to 0$. This is a consequence of the spatial anisotropy in the vicinity of a Fermi surface point, and unlike existing holographic models.