SU(2) gauge theory for intertwined orders and hole pockets in the cuprates

Ohio State University
Columbus
March 9, 2025

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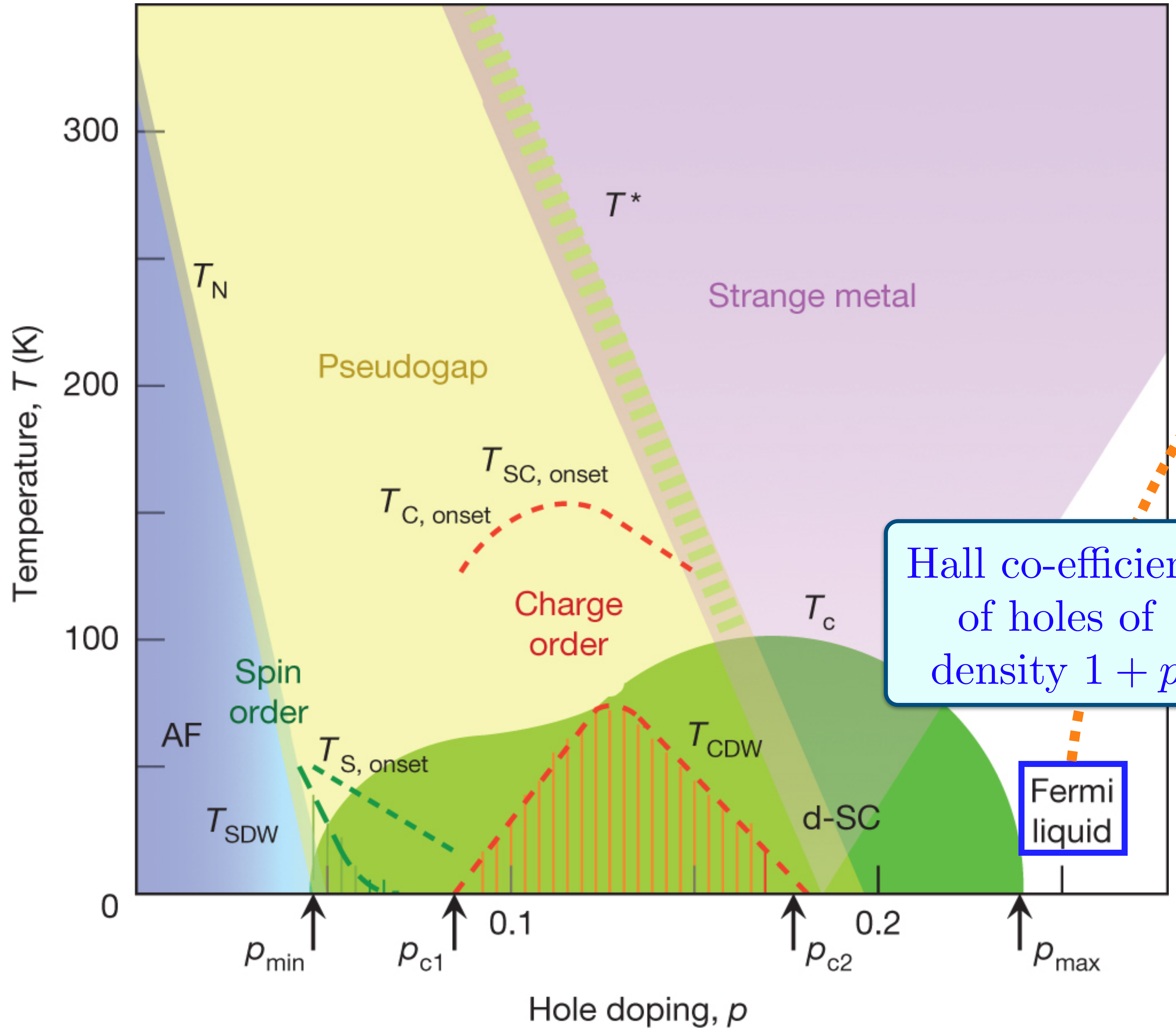
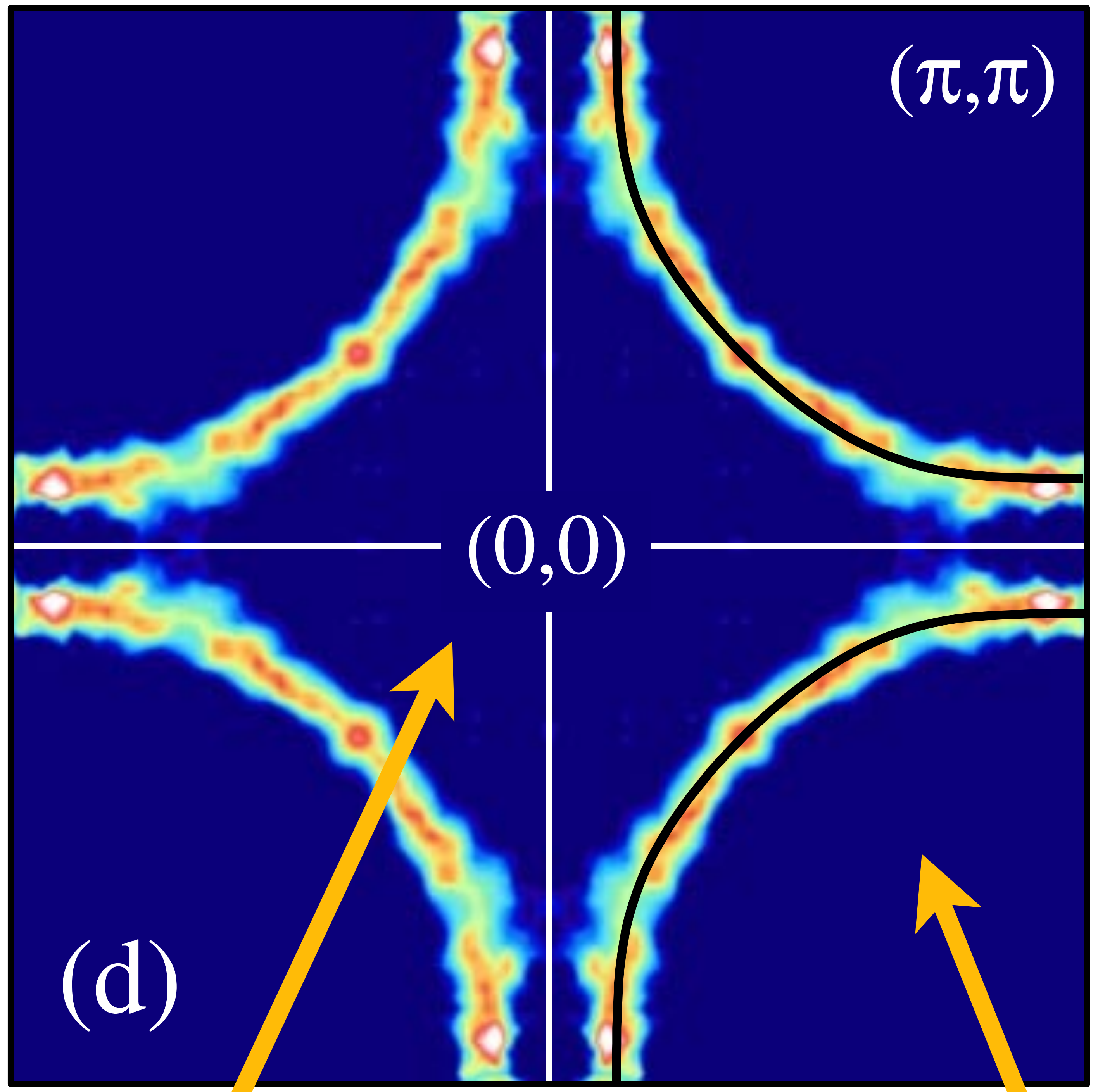
PHYSICS
VE RI TAS
HARVARD

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



M. Platé *et al.*, PRL **95**, 077001 (2005)



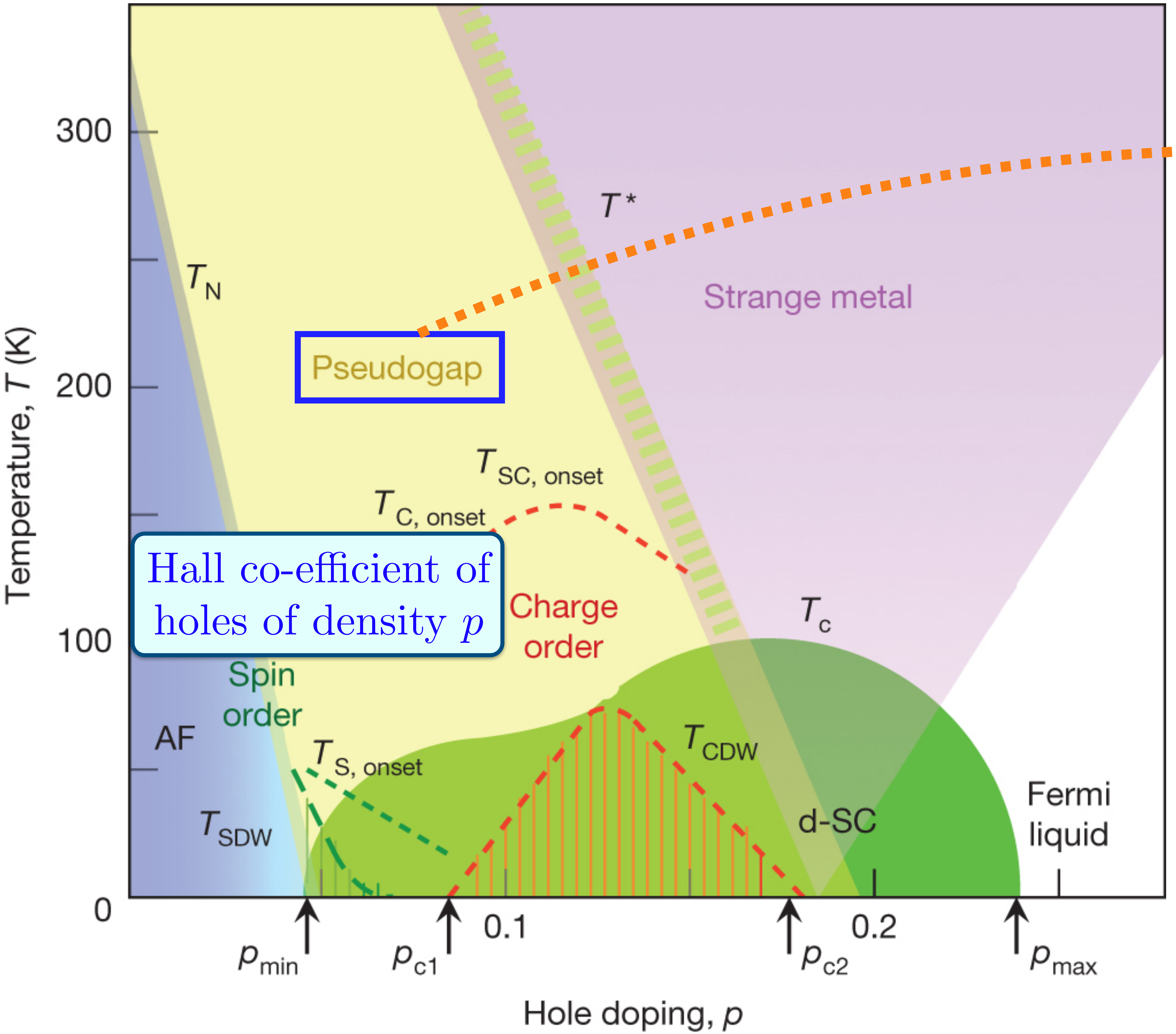
Hall co-efficient of holes of density $1 + p$

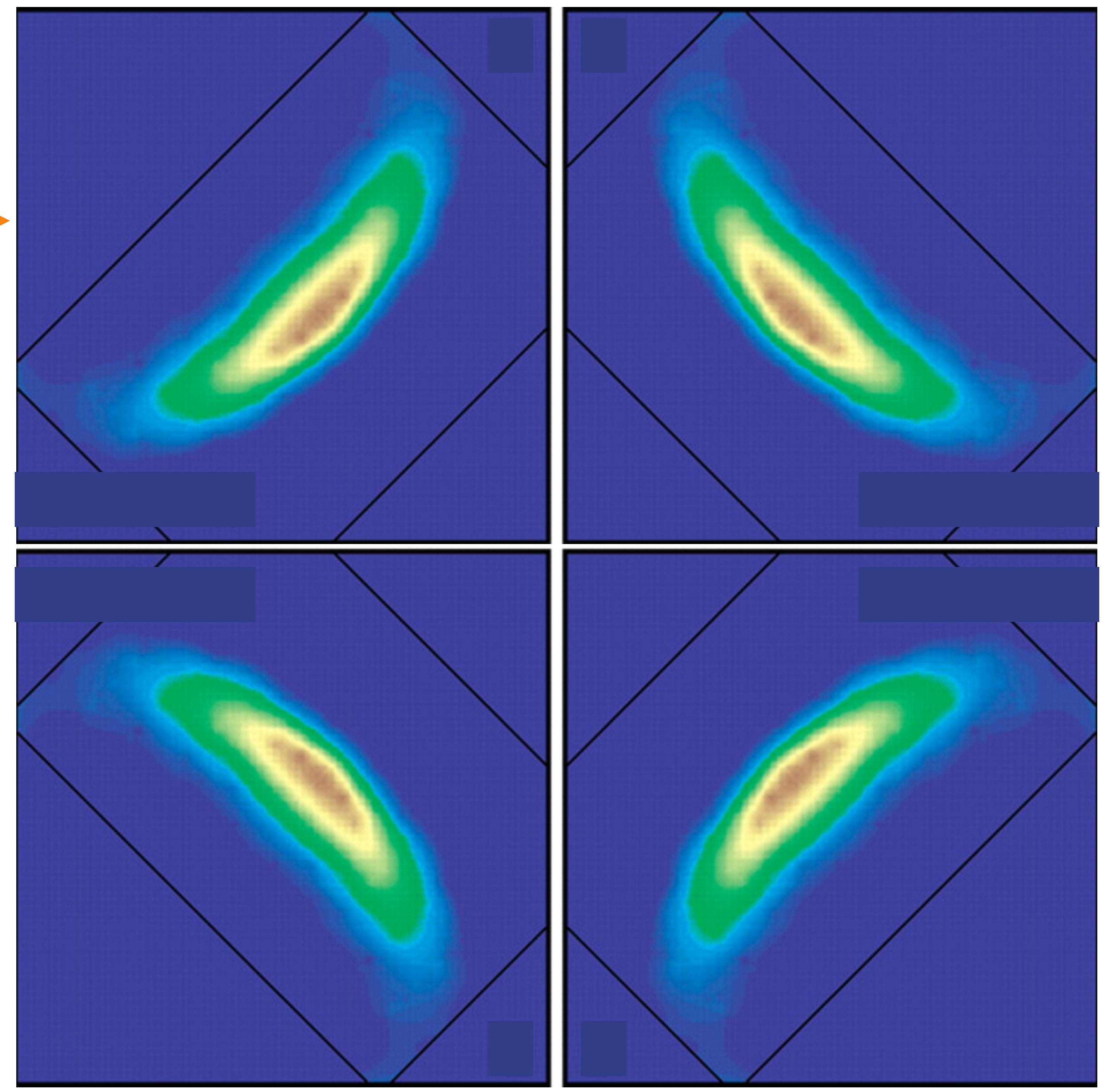
1-p electrons

1+p holes

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

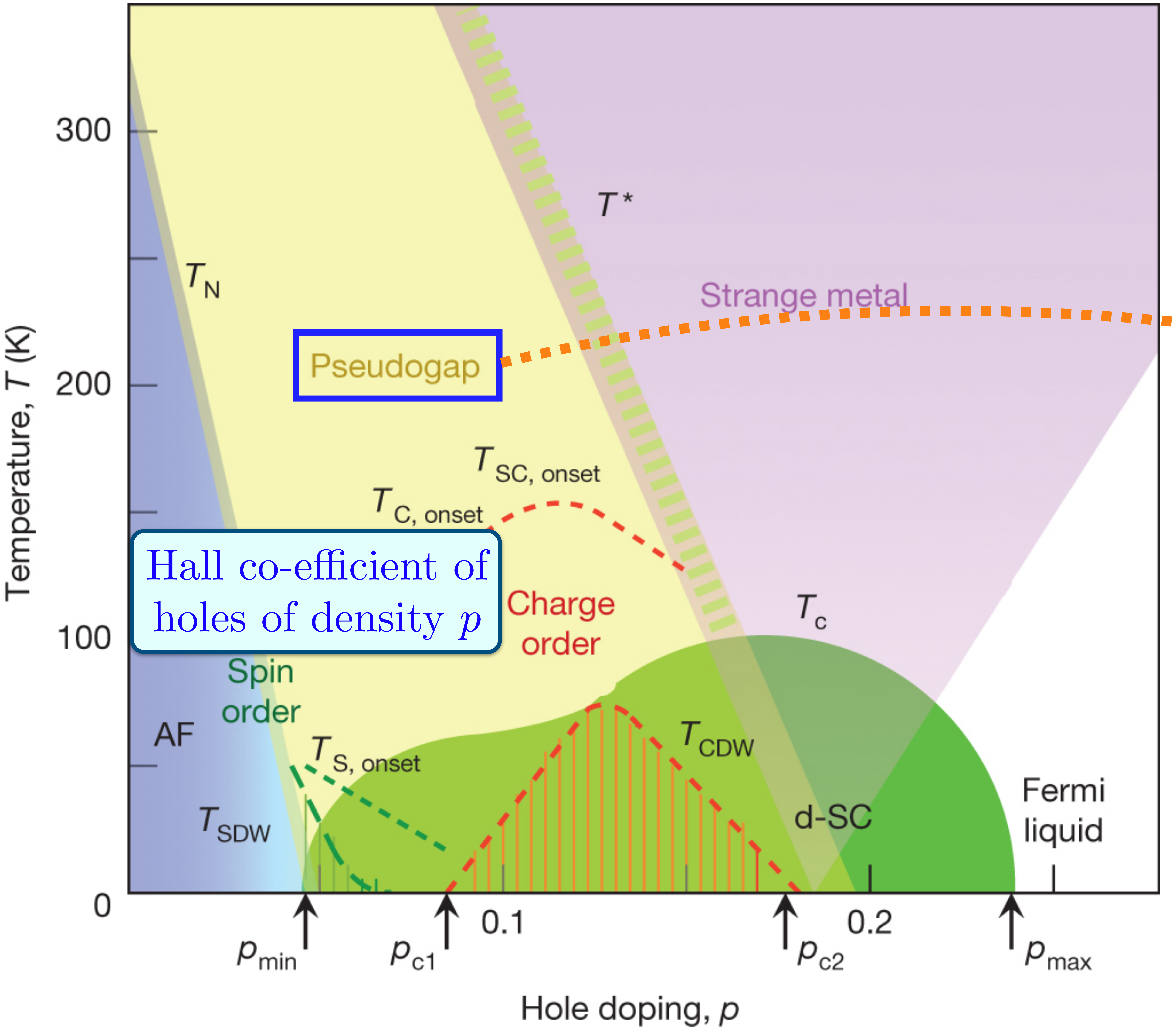
Kyle M. Shen *et al.*, Science **307**, 901 (2005)



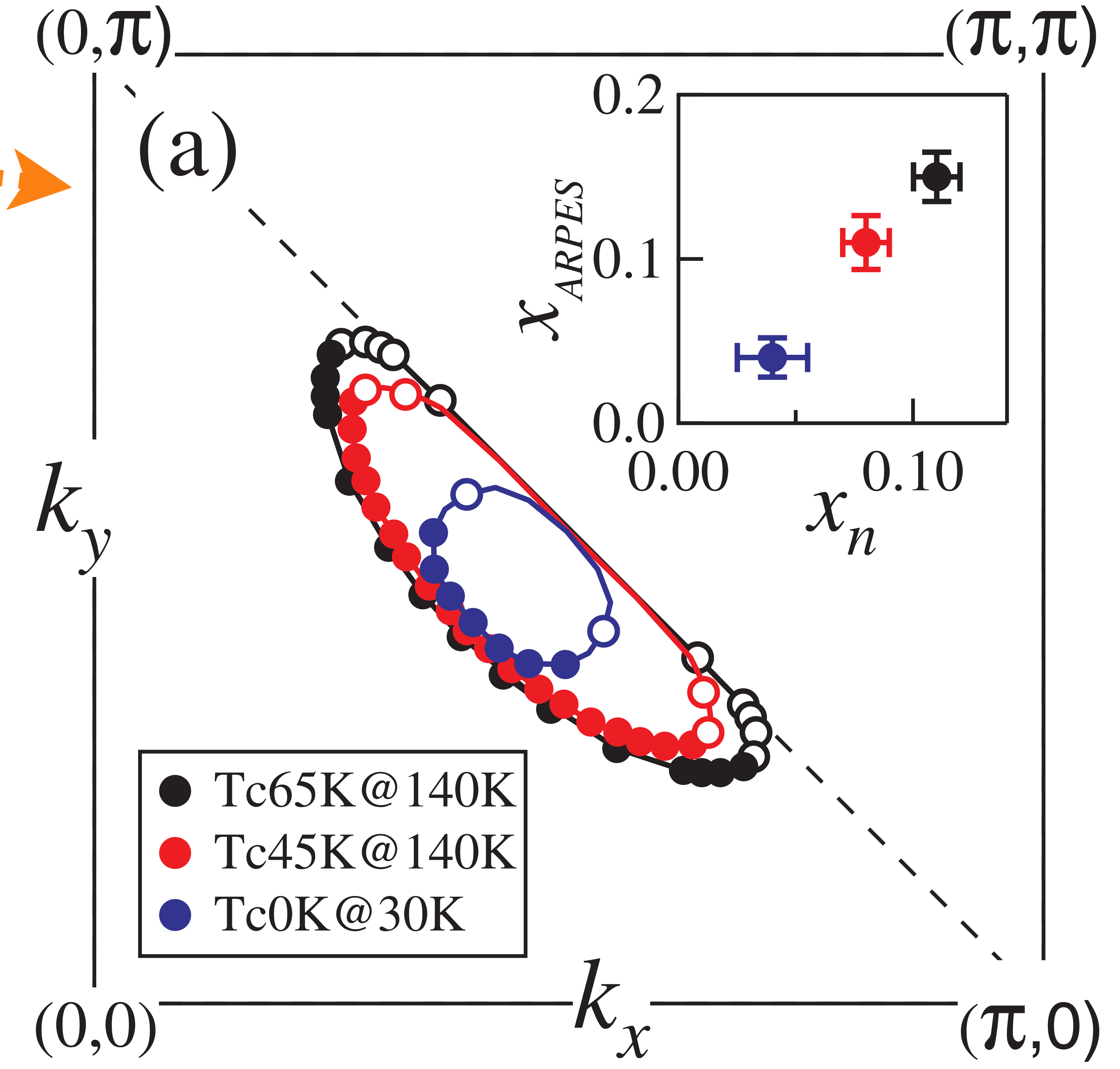


\`Fermi arcs' ?

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

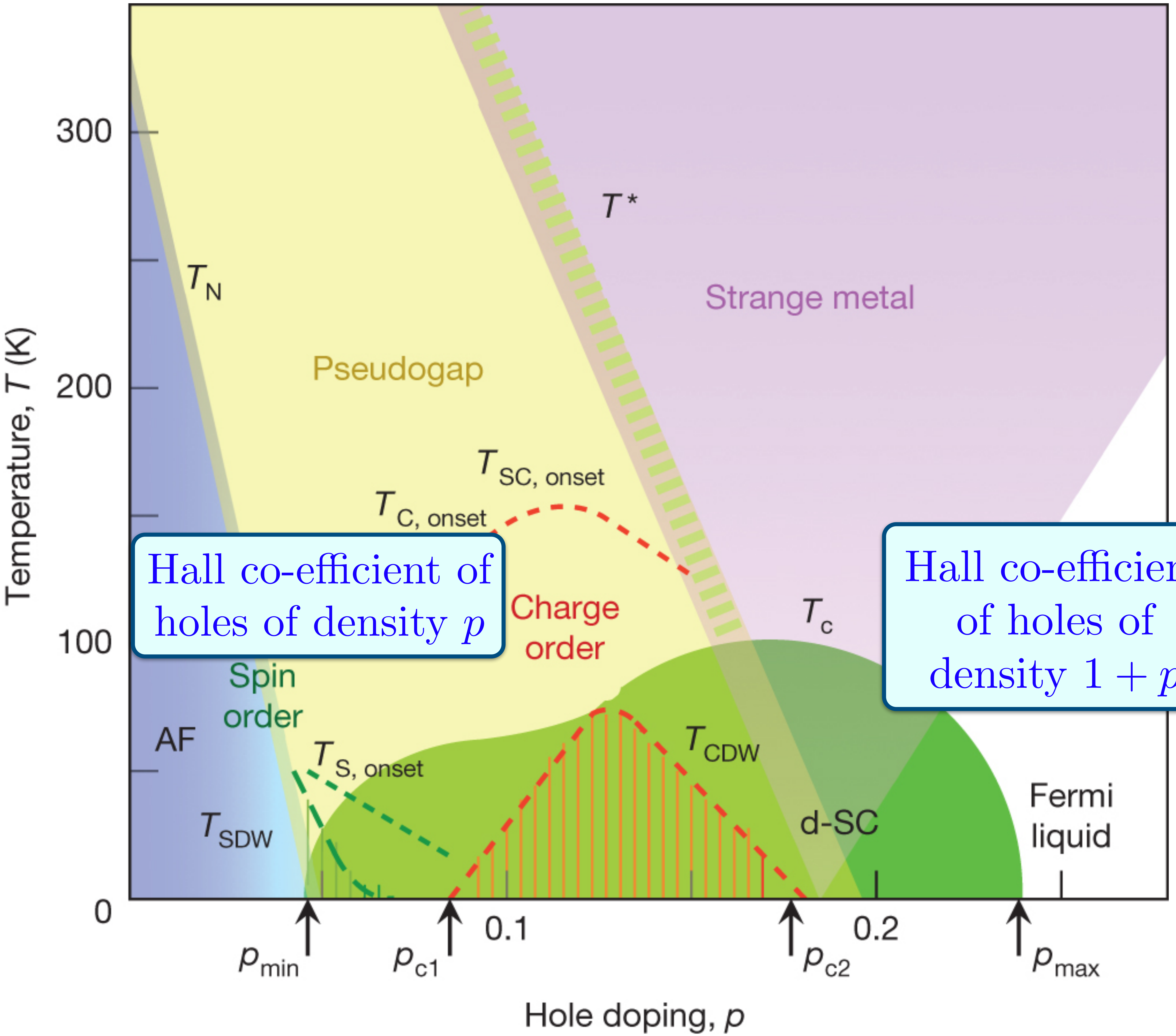


Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



Or hole pockets ?

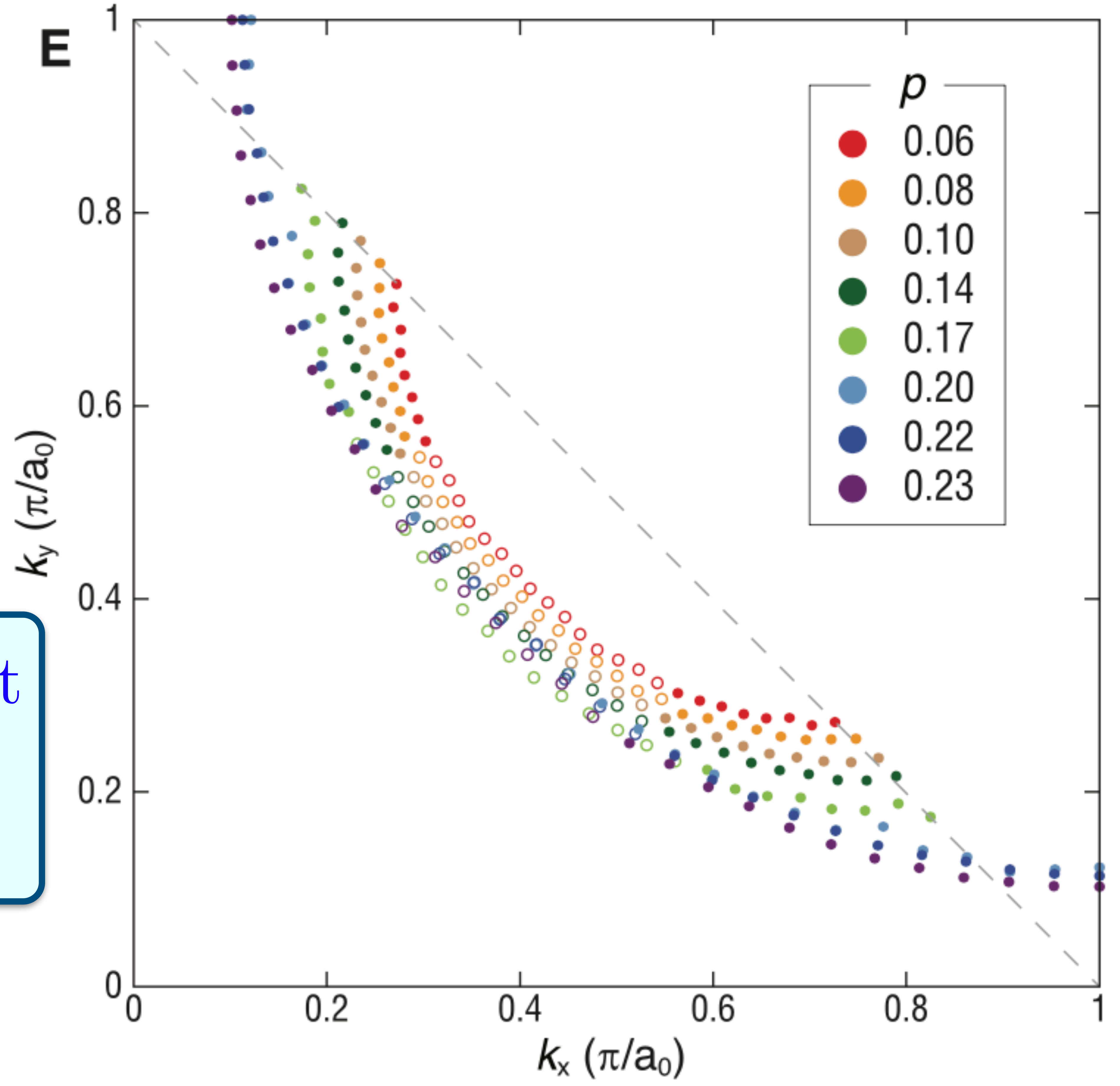
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Hall co-efficient of holes of density p

Hall co-efficient of holes of density $1 + p$

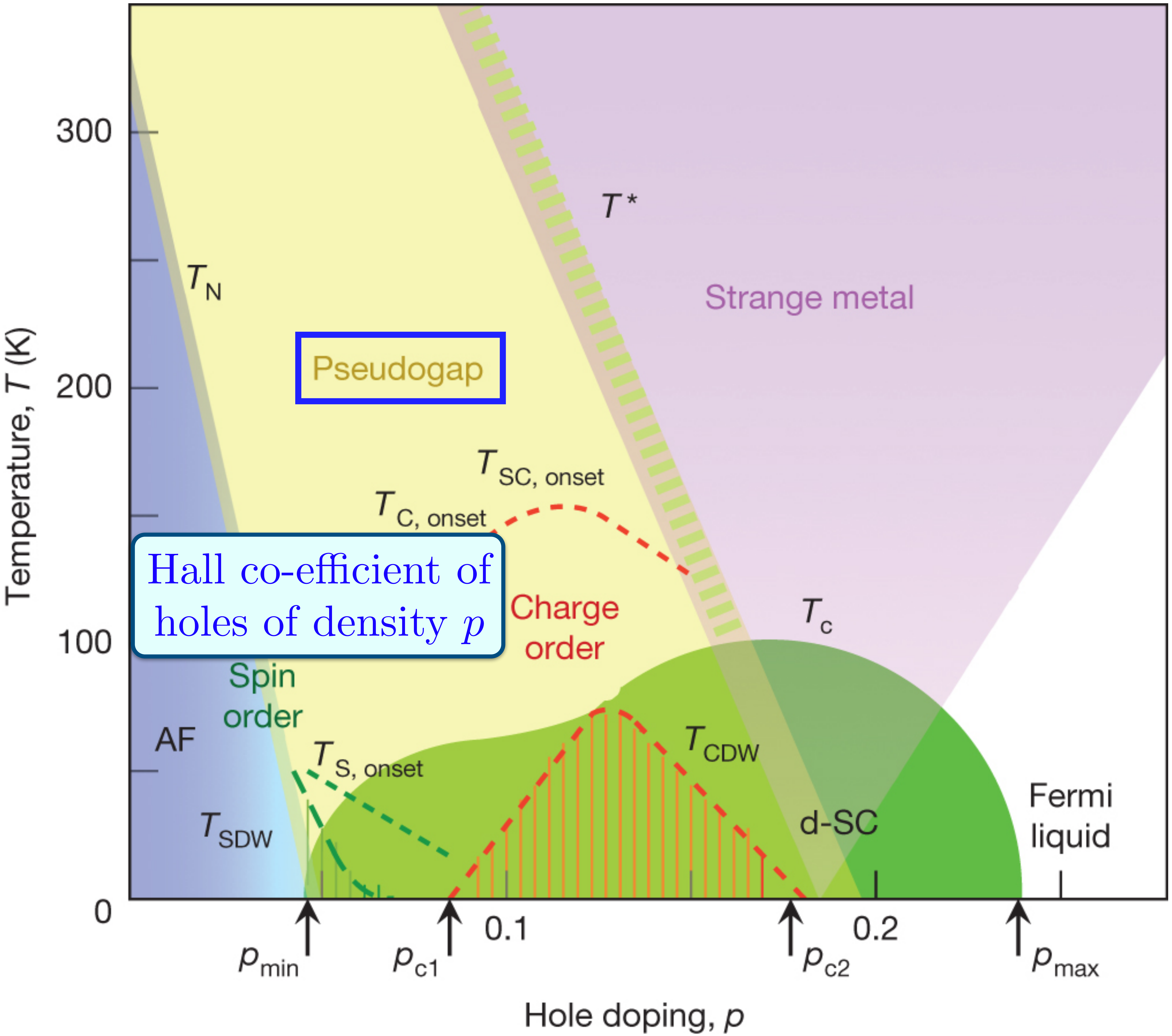
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E. -A. Kim, J. C. Davis, *Science* **344**, 612 (2014)



Also

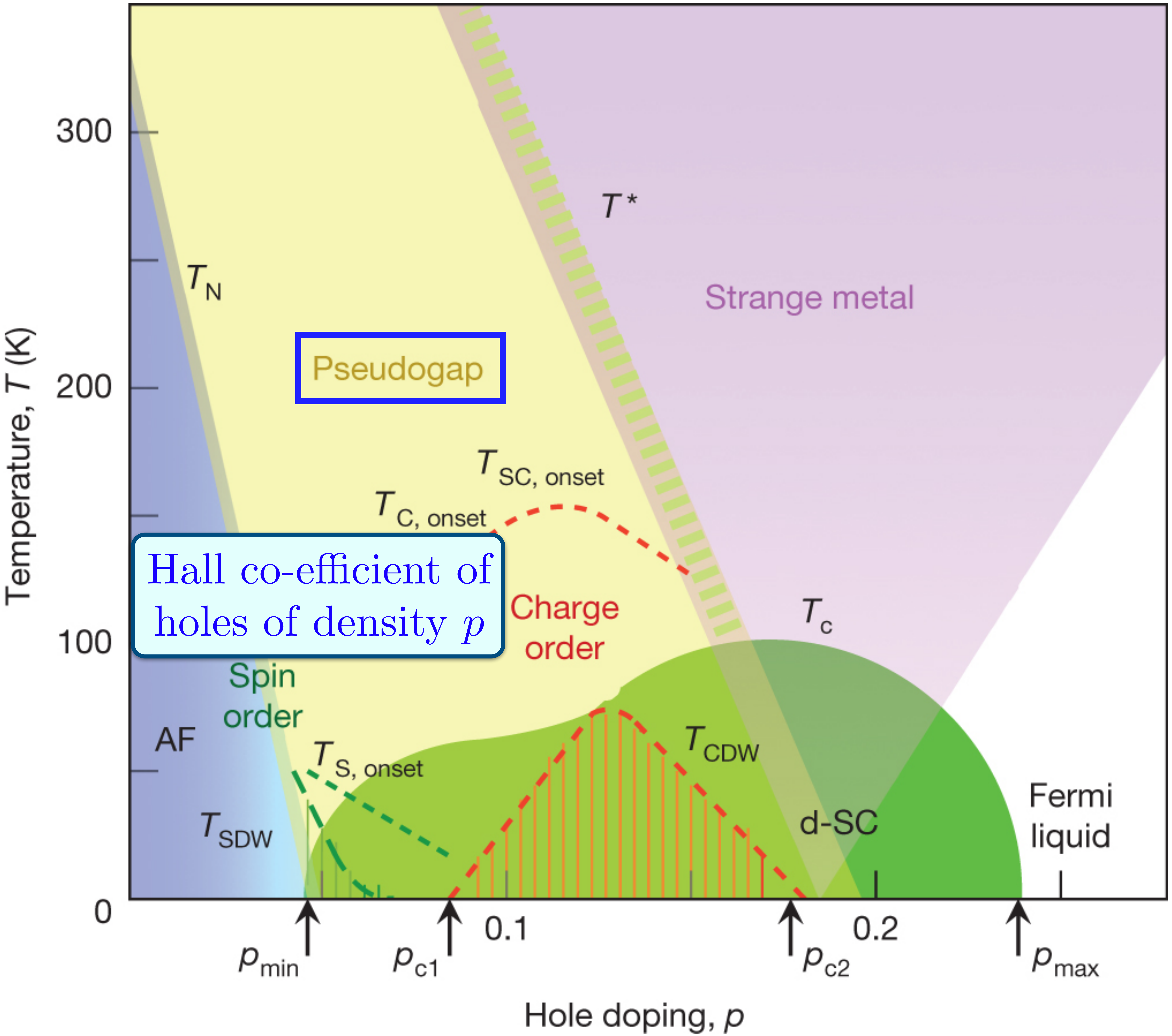
Yang He, Yi Yin, M. Zech, Anjan Soumyanarayanan, M. M. Yee, Tess Williams, M. C. Boyer, Kamalesh Chatterjee, W. D. Wise, I. Zeljkovic, Takeshi Kondo, T. Takeuchi, H. Ikuta, Peter Mistark, Robert S. Markiewicz, Arun Bansil, Subir Sachdev, E. W. Hudson, Jennifer. E. Hoffman, *Science* **344**, 608 (2014)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



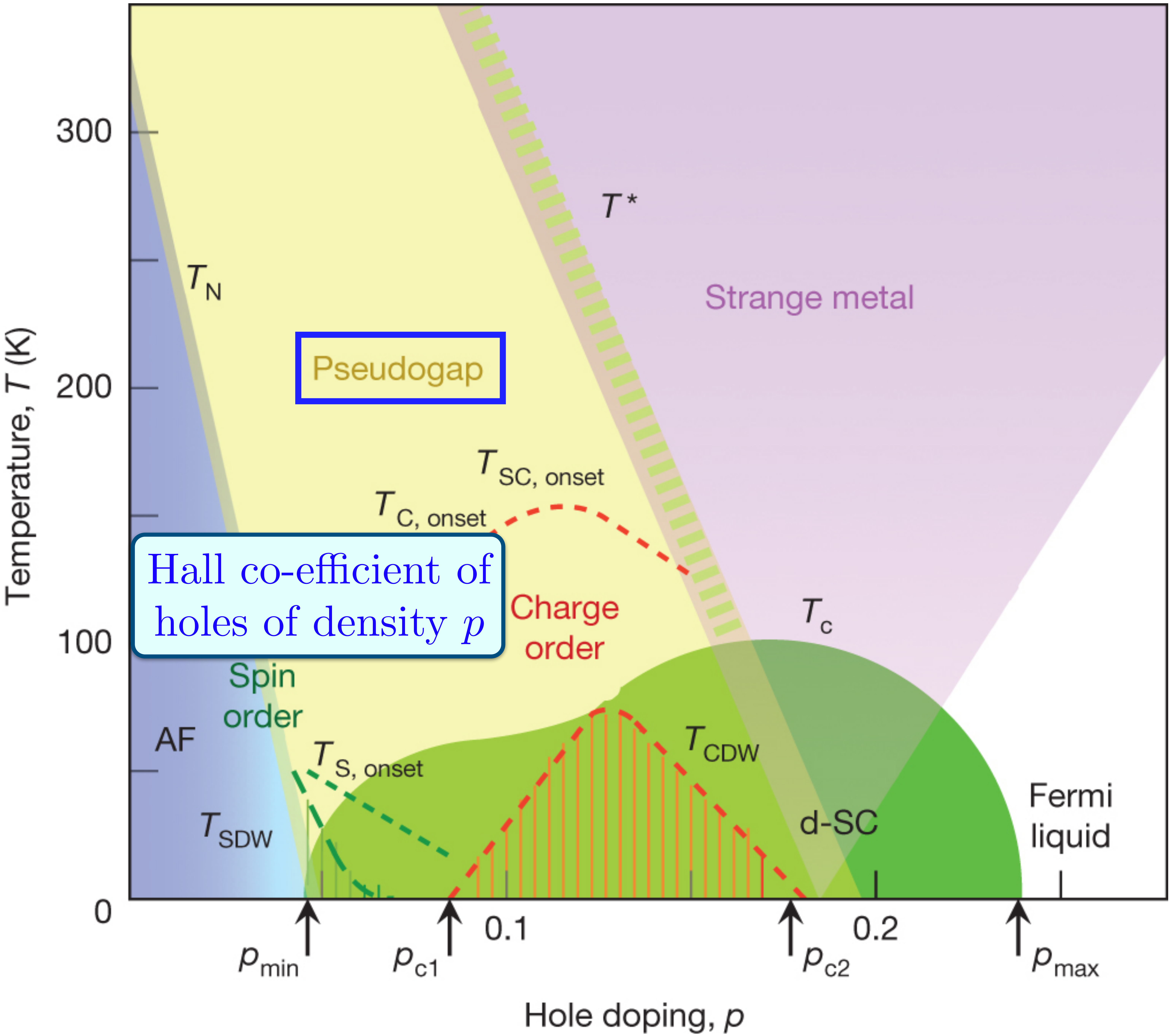
Many theories with fluctuating and intertwined AFM, d-SC and charge orders.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



I argue that a better starting point is a novel quantum ground state with no broken symmetry: the Fractionalized Fermi liquid (FL*)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



I argue that a better starting point is a novel quantum ground state with no broken symmetry: the Fractionalized Fermi liquid (FL*)

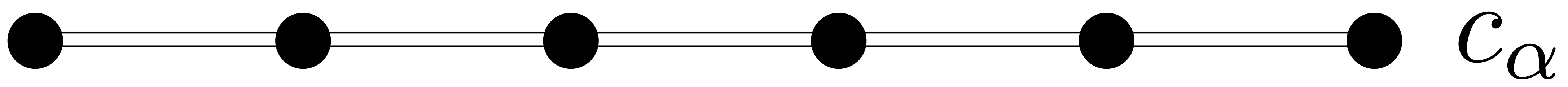
Recent angle-dependent magnetoresistance (ADMR) experiments support hole pockets with coherent interlayer tunneling of quasiparticles.

Fang et al., *Nature Physics* **18**, 558 (2022)
Chan et al., *Nature Physics* **21**, 1753 (2025)

Ancilla Layer Model (ALM) of FL* and FL phases in Hubbard-type models

Ancilla Layer Model of the Hubbard model

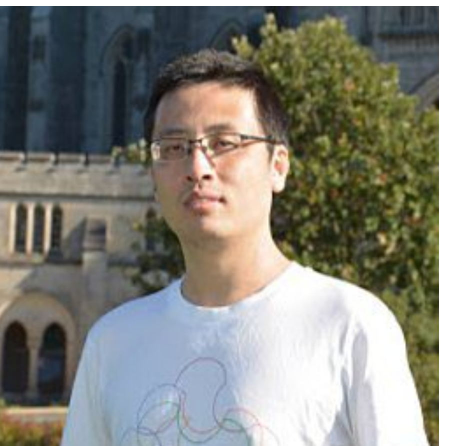
(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

$$\mathcal{H}_{\text{Hubbard}} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + U \sum_{\boldsymbol{i}} (c^\dagger_{\boldsymbol{i}\uparrow} c_{\boldsymbol{i}\uparrow})(c^\dagger_{\boldsymbol{i}\downarrow} c_{\boldsymbol{i}\downarrow})$$

Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

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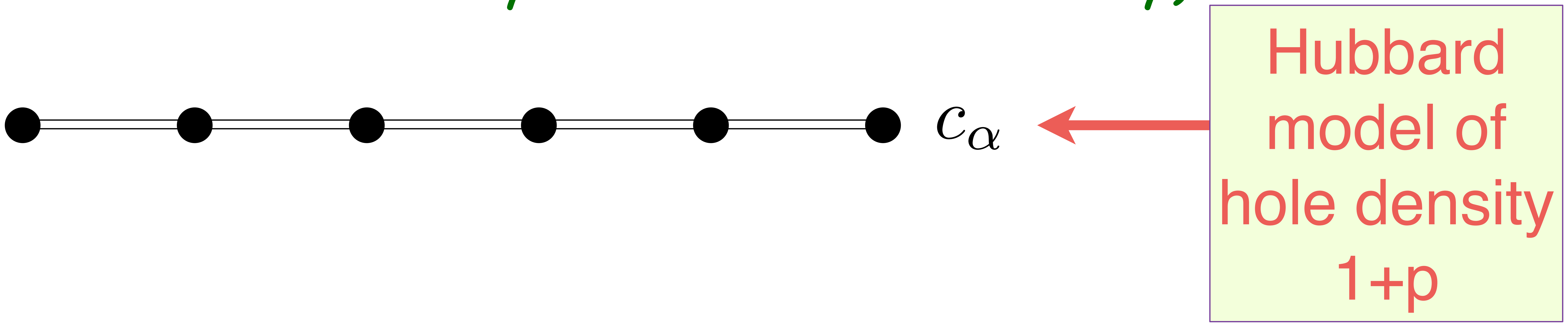
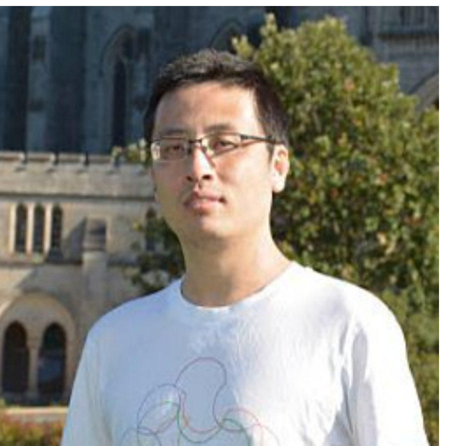


$$
\mathcal{H}_{\rm Hubbard} = - \sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}^2 + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c_{\boldsymbol{i}\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} \right] .
$$

$$
\boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \quad \Rightarrow \quad \text{Paramagnon}
$$

Ya-Hui Zhang



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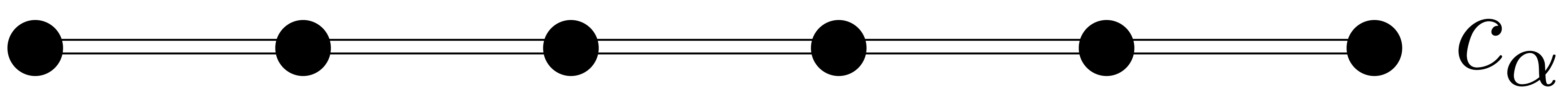

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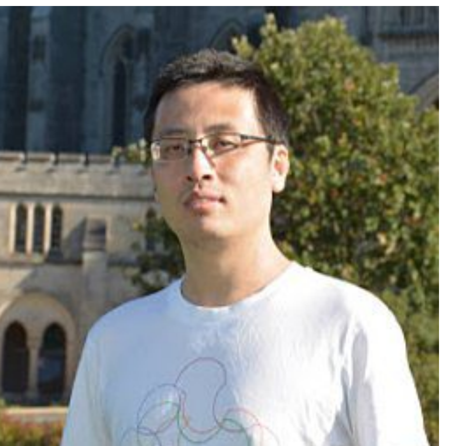
$$\boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \quad \Rightarrow \quad \text{Paramagnon}$$

Ya-Hui Zhang



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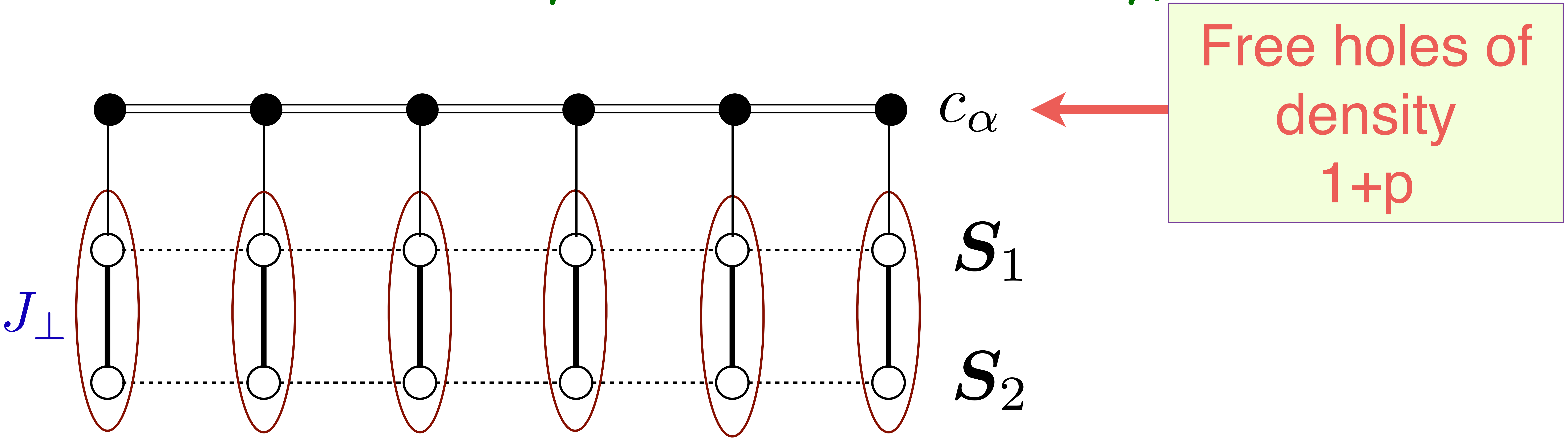

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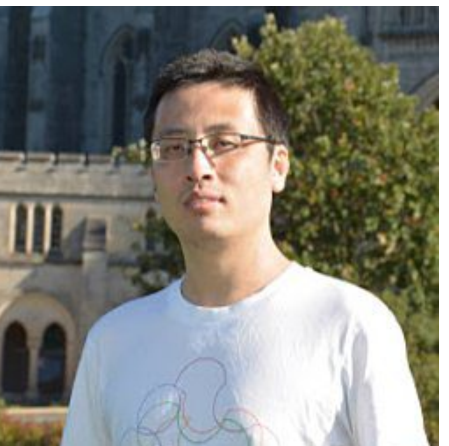
$$3\ \boldsymbol{\mathcal{P}}\ \text{oscillators' states:}\ |0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$$

$$\boldsymbol{S}_{1,2}\ \text{ancilla qubits states :}\ (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang



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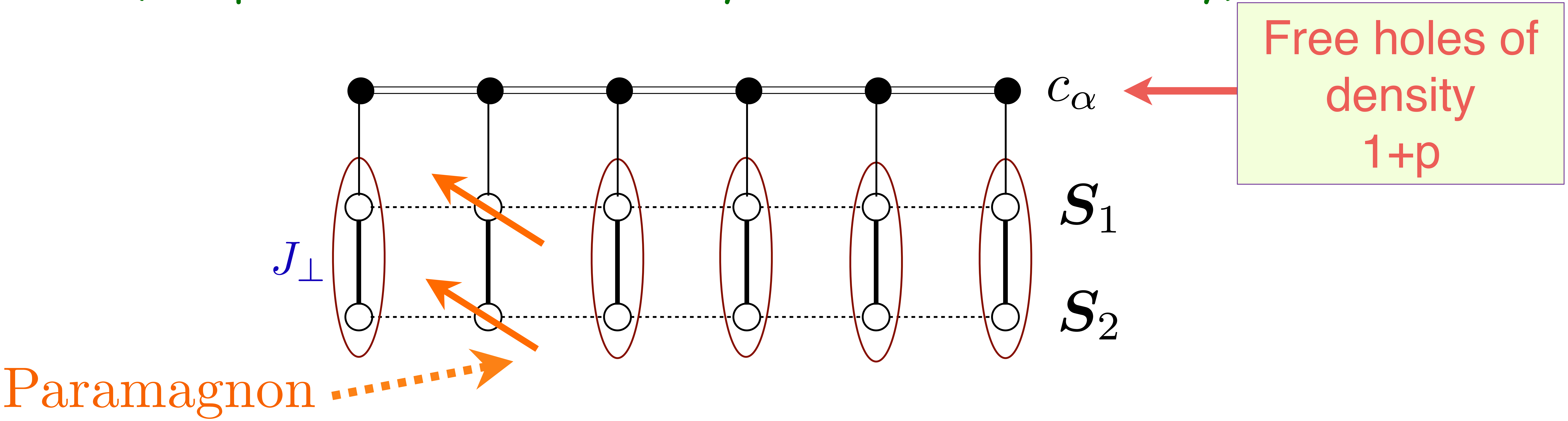

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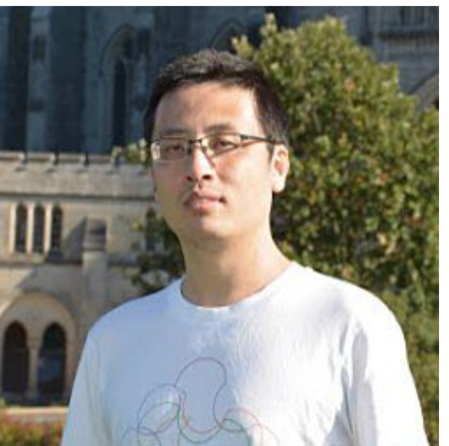

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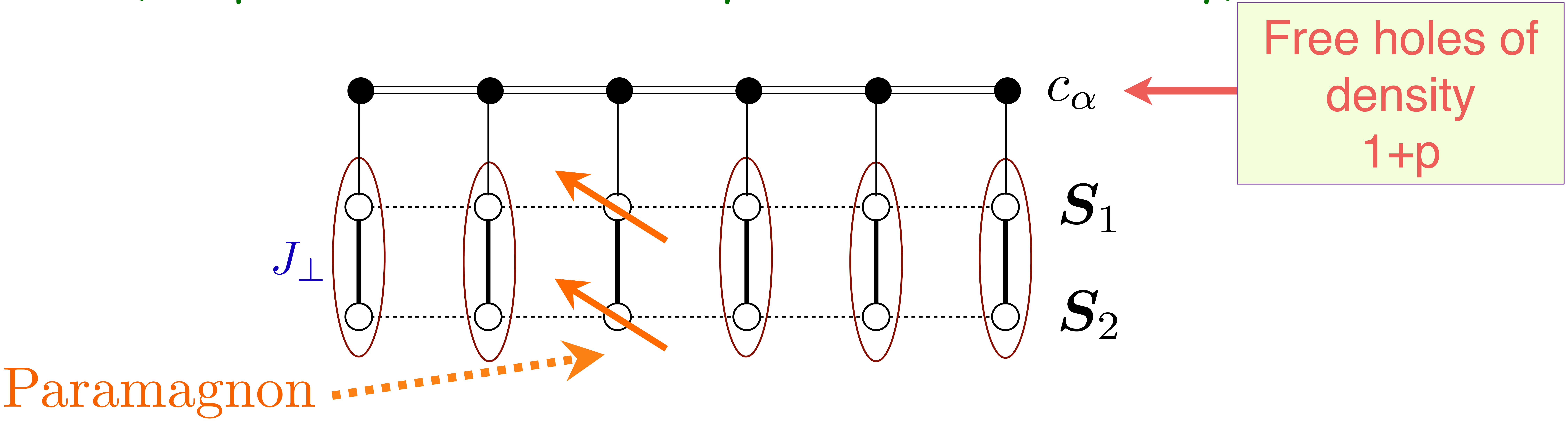

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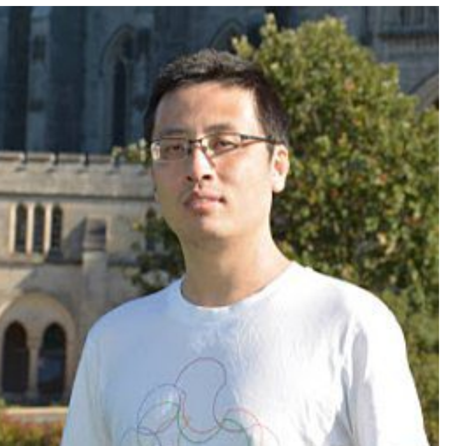

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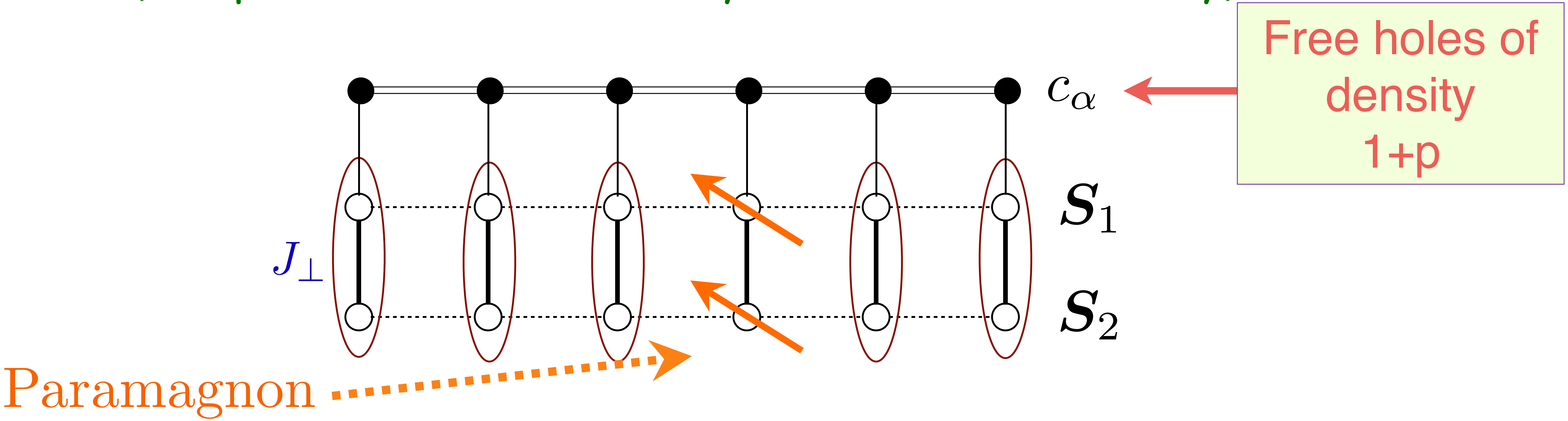

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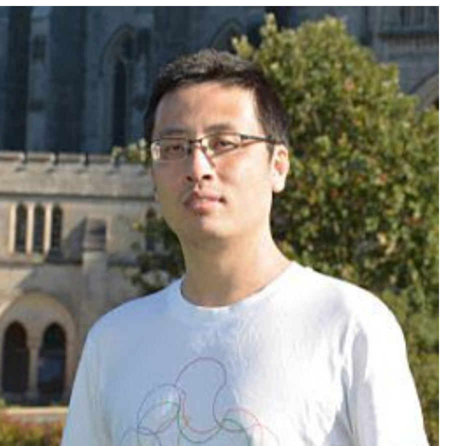

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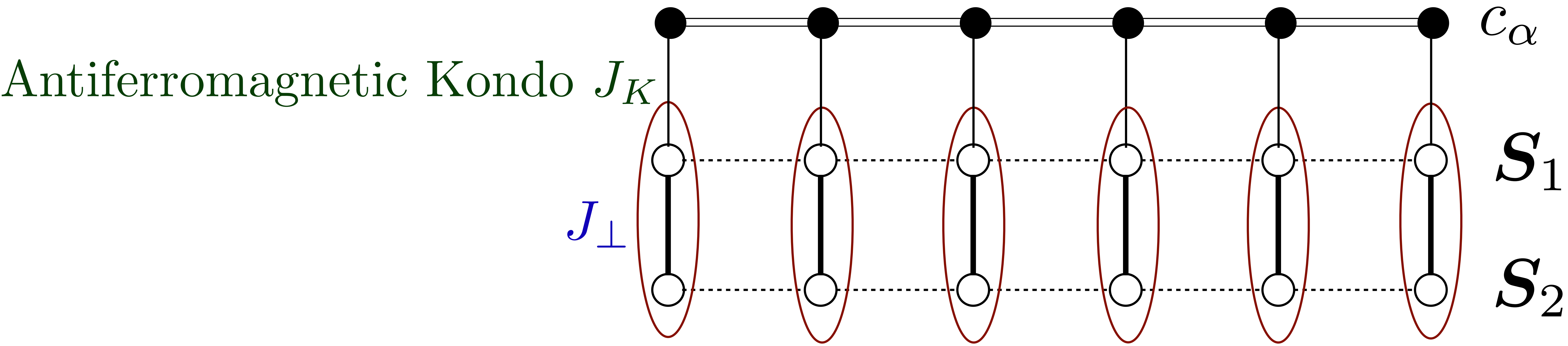
Ancilla Layer Model of the Hubbard model

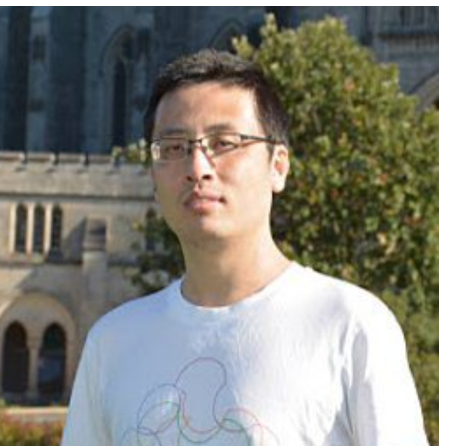
(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



$$\mathcal{H}_{\rm ALM} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \frac{J_K}{2} \boldsymbol{S}_{1i} \cdot c^\dagger_{\boldsymbol{i}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\boldsymbol{i}\beta} + J_\perp \sum_{\boldsymbol{i}} \boldsymbol{S}_{1\boldsymbol{i}} \cdot \boldsymbol{S}_{2\boldsymbol{i}} \,.$$

Ya-Hui Zhang



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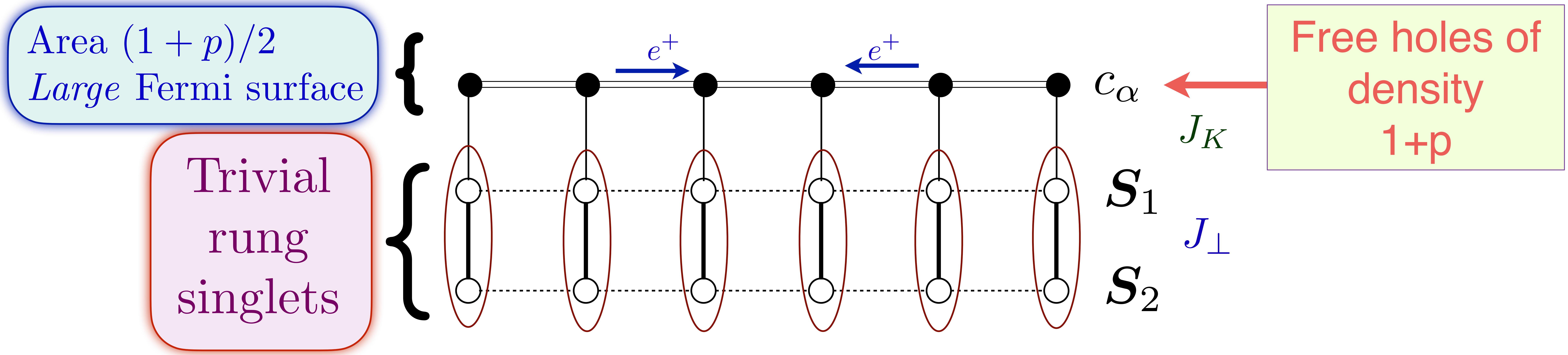
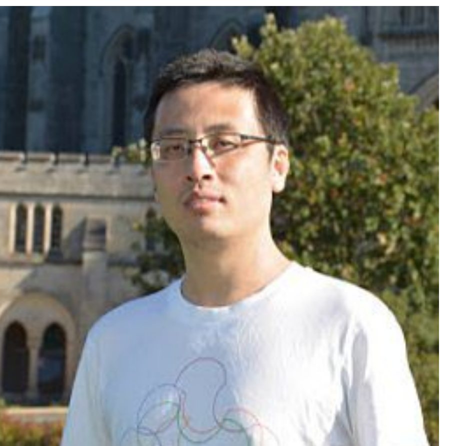
ALM of FL of Hubbard model



$$J_\perp \gg J_K$$

Ya-Hui Zhang



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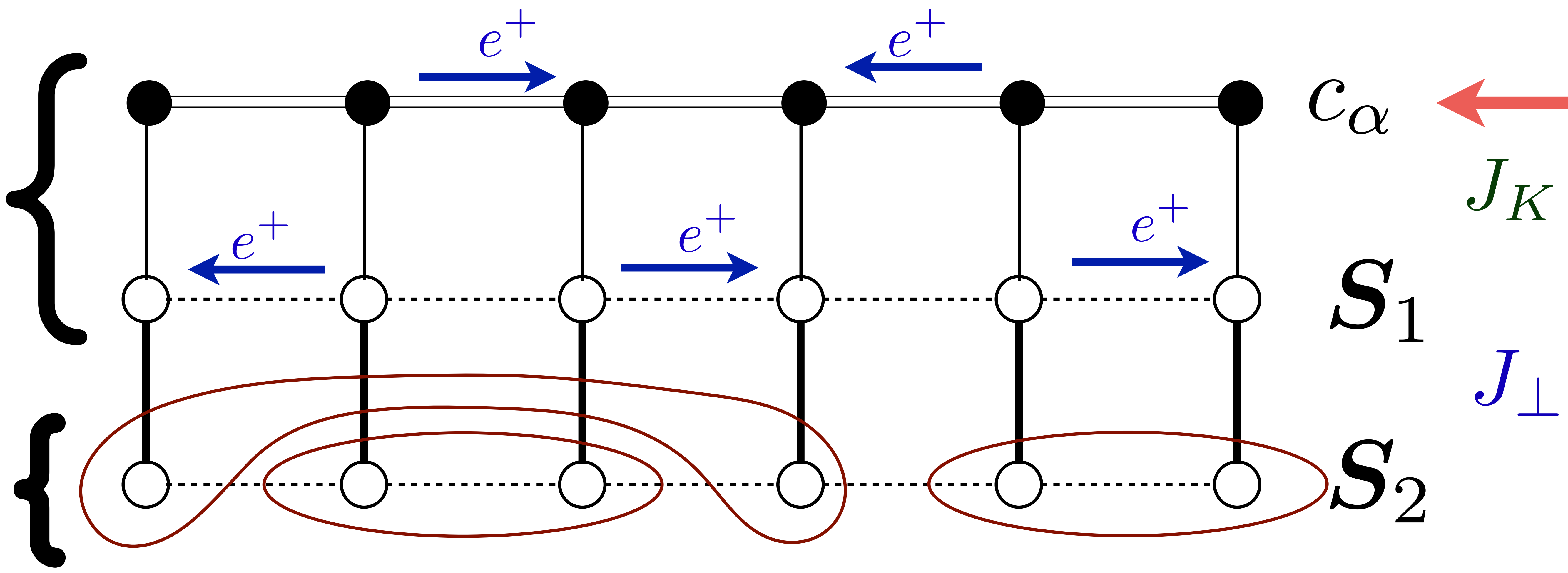
ALM of FL* of Hubbard model

Kondo lattice heavy Fermi liquid.
Area $(1 + p + 1)/2 = p/2 \pmod 1$.
Small Fermi surface!

Your favorite spin liquid

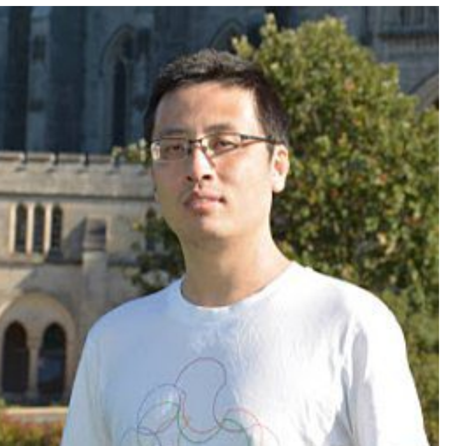


Free holes of density 1+p

$$J_K \gg J_\perp$$

$$\text{FL* Pseudogap metal} = \begin{array}{c} \text{Kondo Lattice FL of } c_\alpha \text{ and } \boldsymbol{S}_1 \\ \oplus \\ \text{Spin Liquid of } \boldsymbol{S}_2 \end{array}$$

Ya-Hui Zhang



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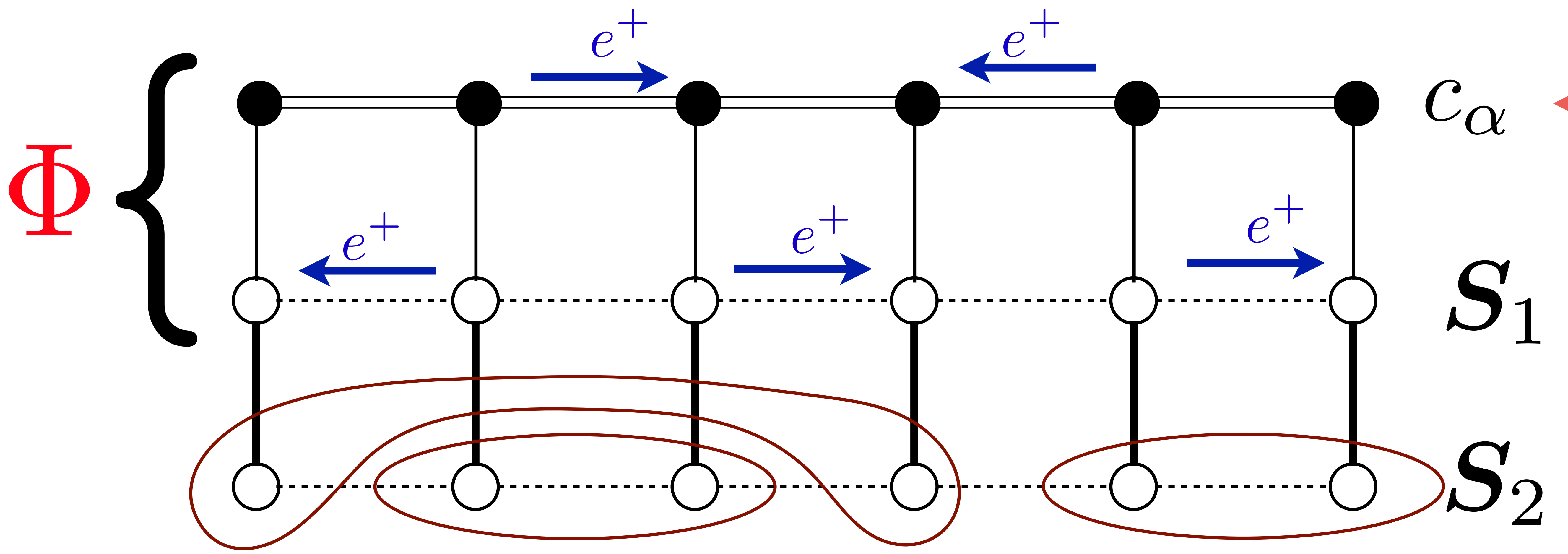
ALM of FL* of Hubbard model

Higgs field Φ determines the pseudogap.
In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.



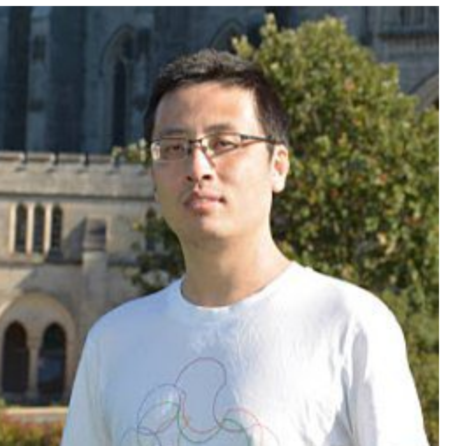
Free holes of density 1+p

$$J_K \gg J_\perp$$

$$H_{\text{Kondo lattice}} = \sum_{i,j} \left[-t_{ij} c^\dagger_{i\sigma} c_{j\sigma} - t_{1,ij} f^\dagger_{1i\sigma} f_{1j\sigma} \right] - \sum_i \Phi \left(c^\dagger_{i\sigma} f_{1i\sigma} + f^\dagger_{1i\sigma} c_{i\sigma} \right)$$

Heavy Fermi liquid of electrons c, f_1
$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

Ya-Hui Zhang



E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

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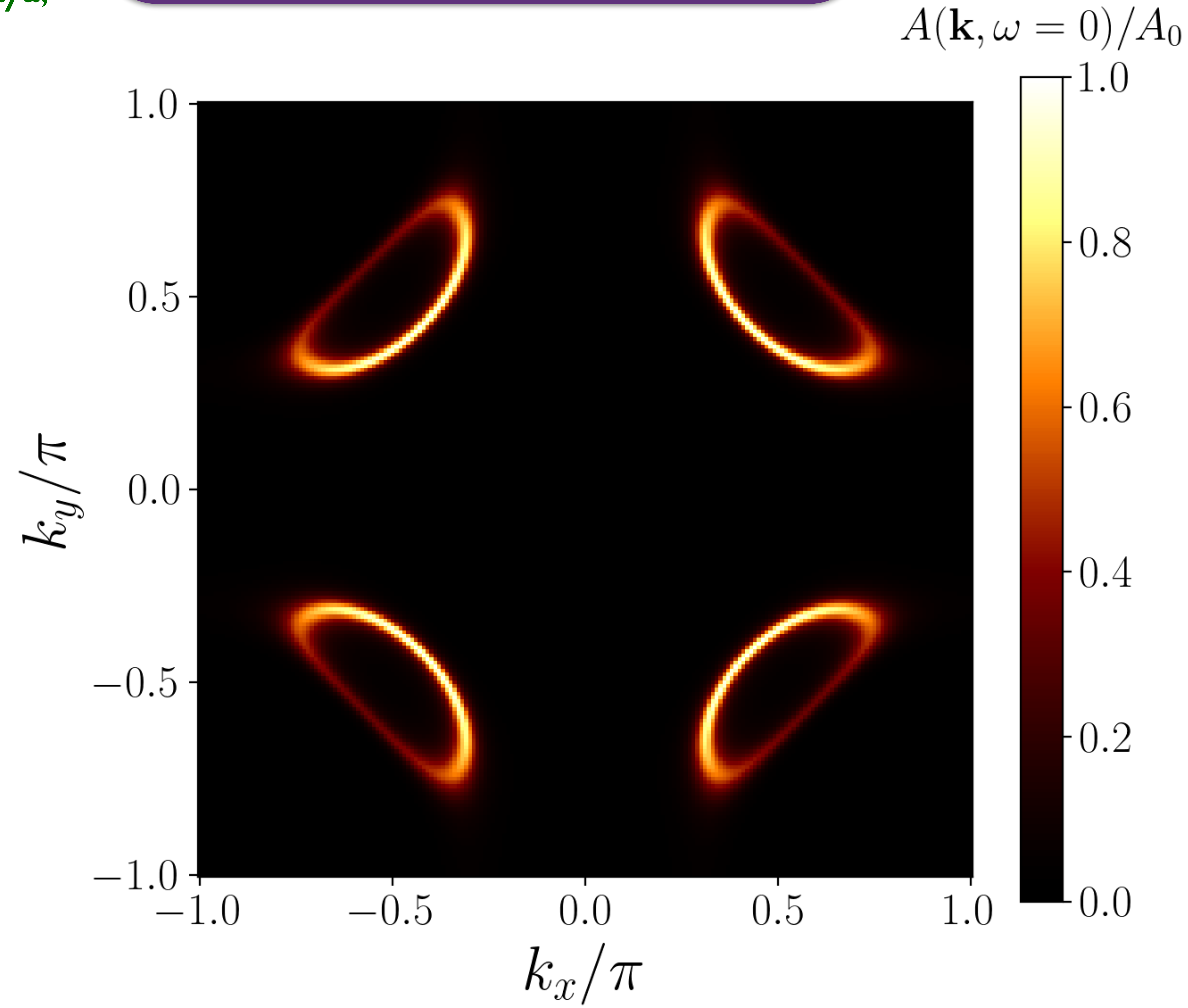
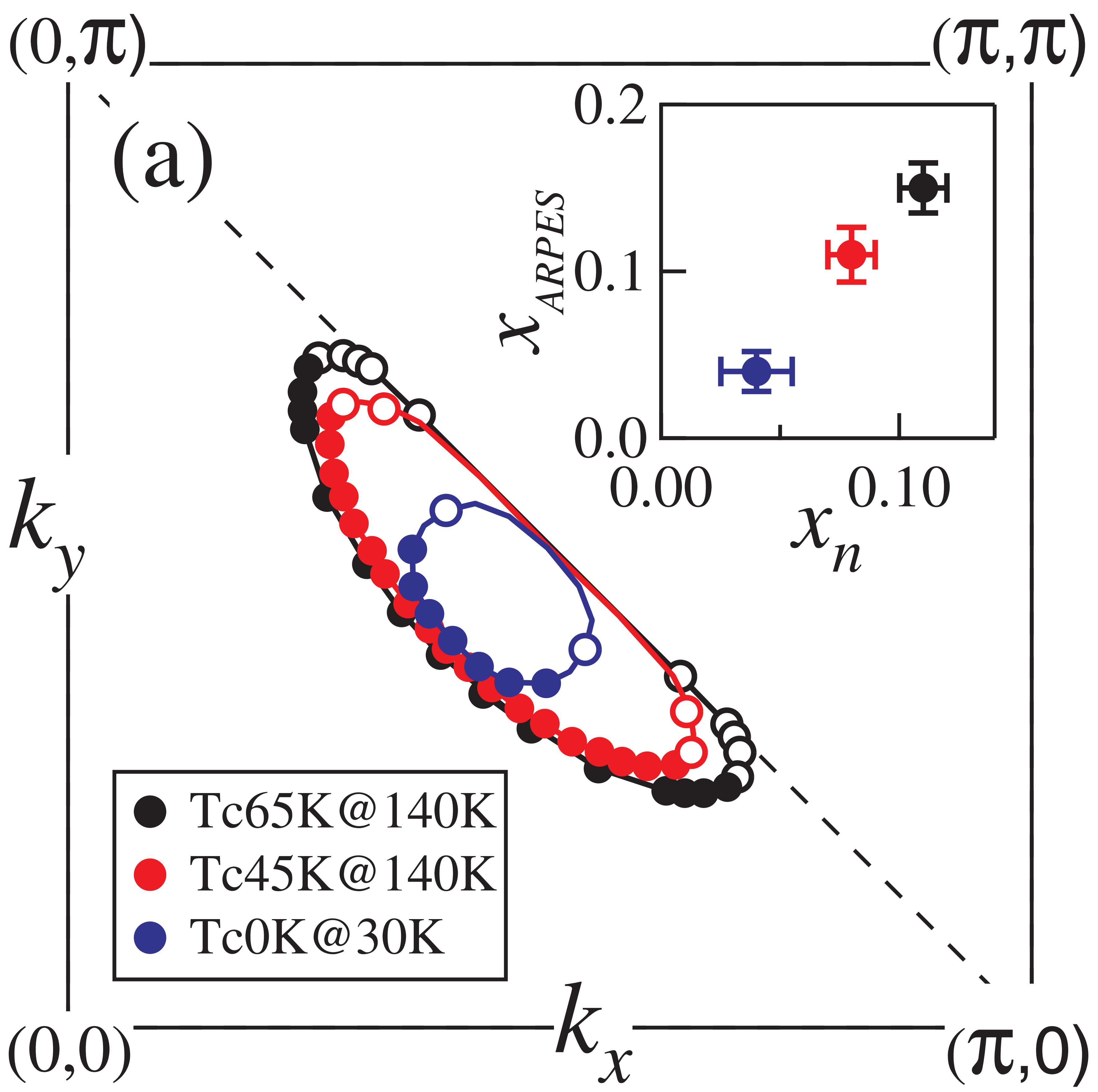
Ancilla Layer Model of FL*



Decoupled Kondo lattice and spin liquid

Photoemission expts



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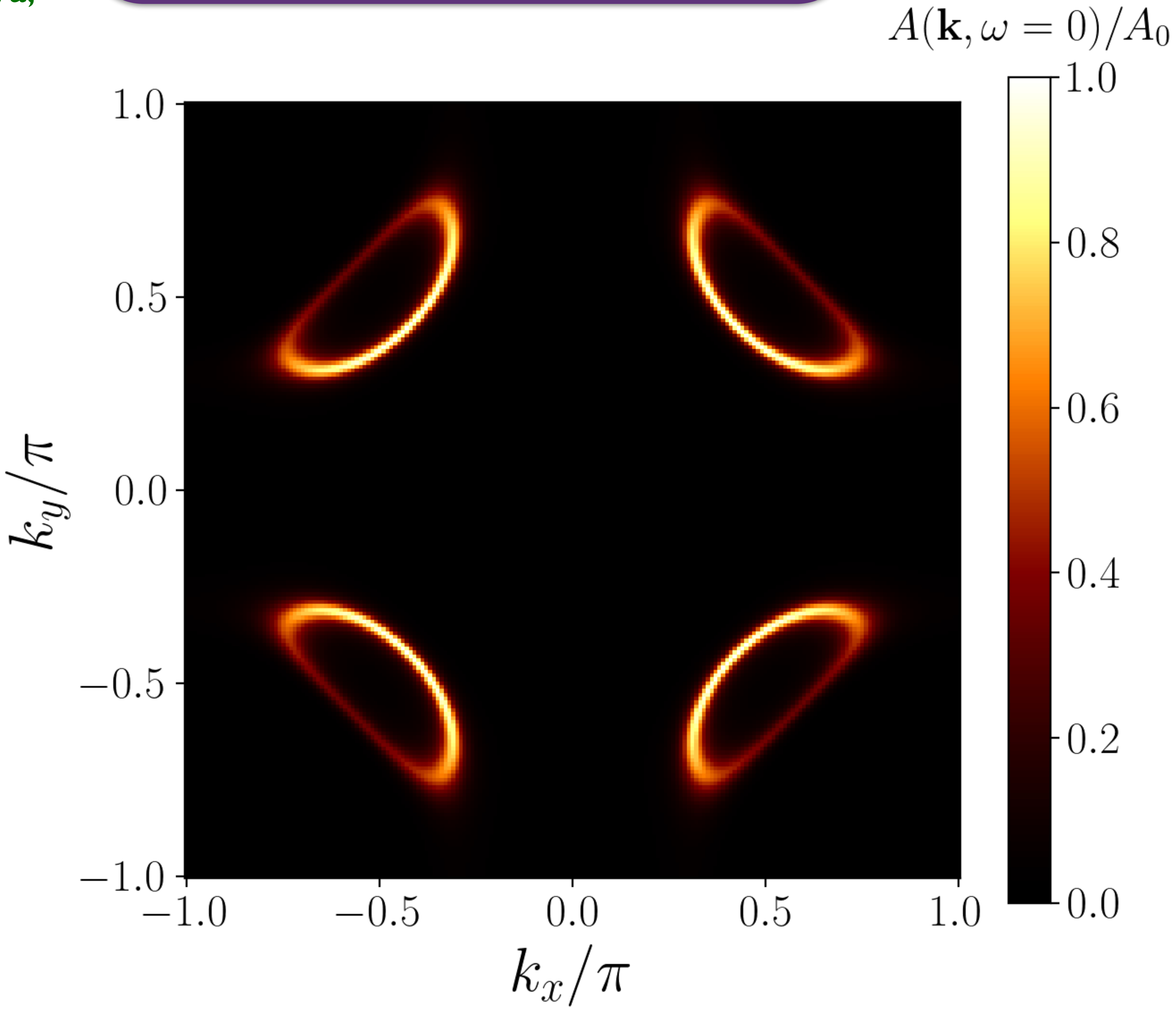
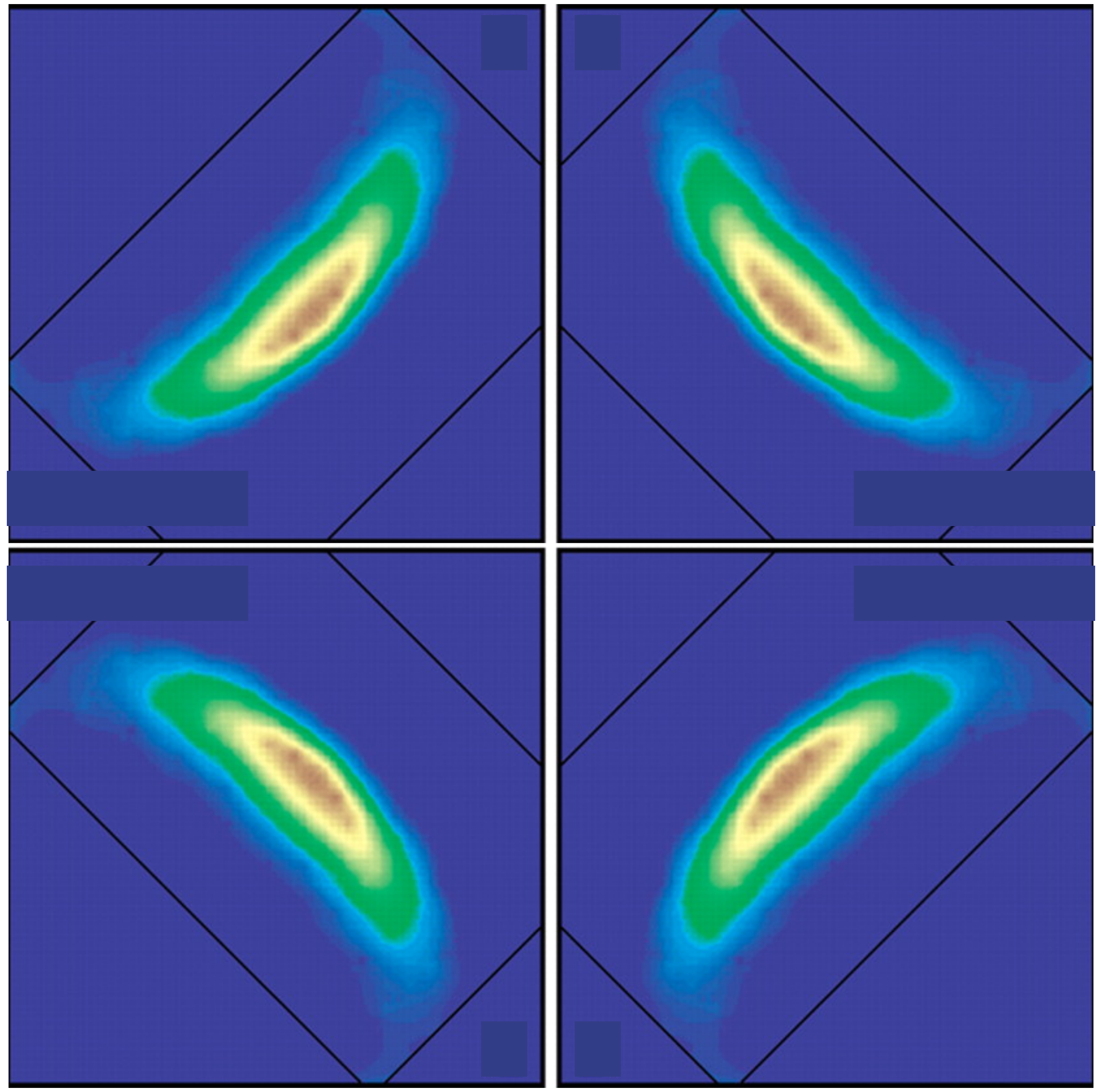


Photoemission expts



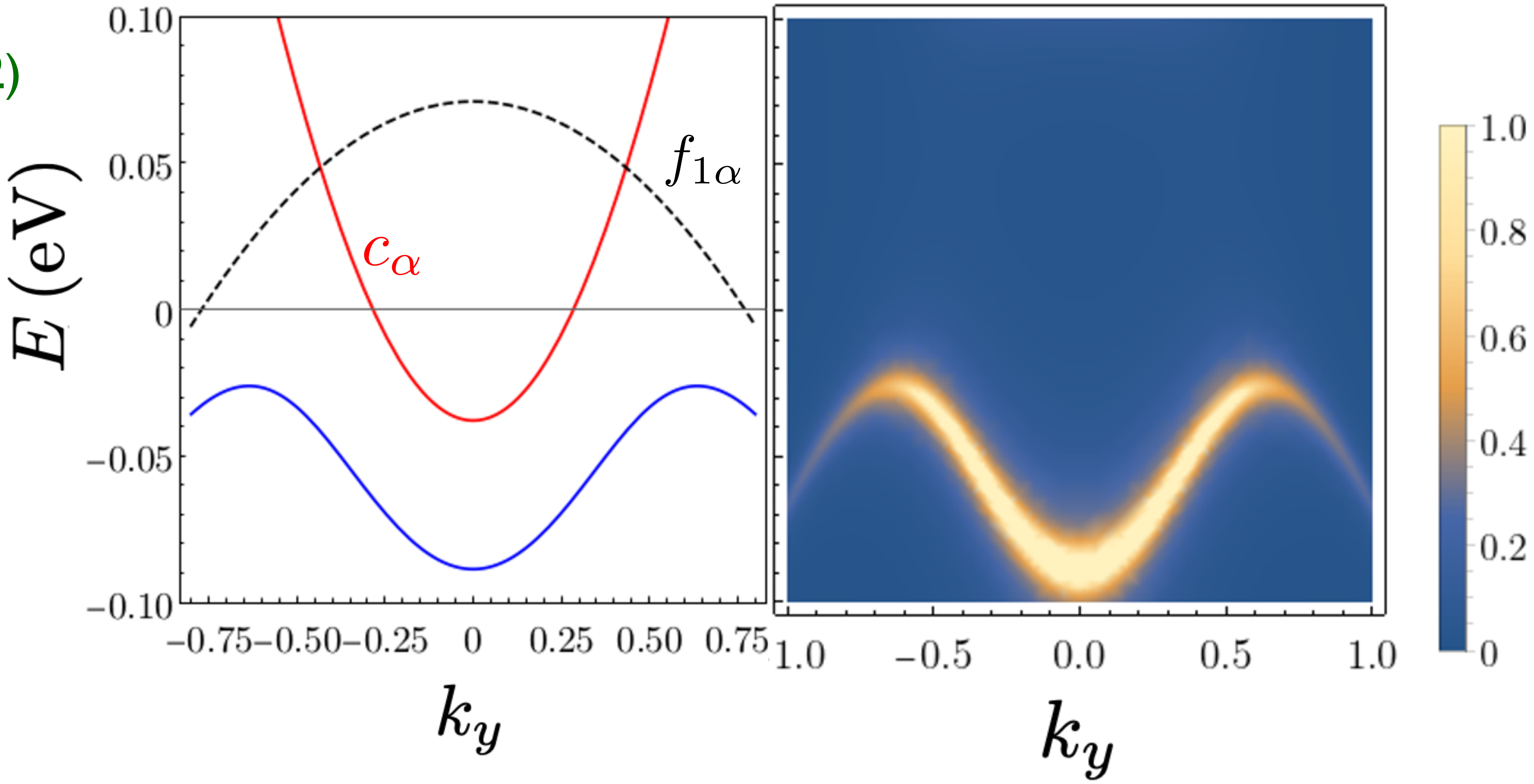
Kyle M. Shen *et al.*, Science **307**, 901 (2005)

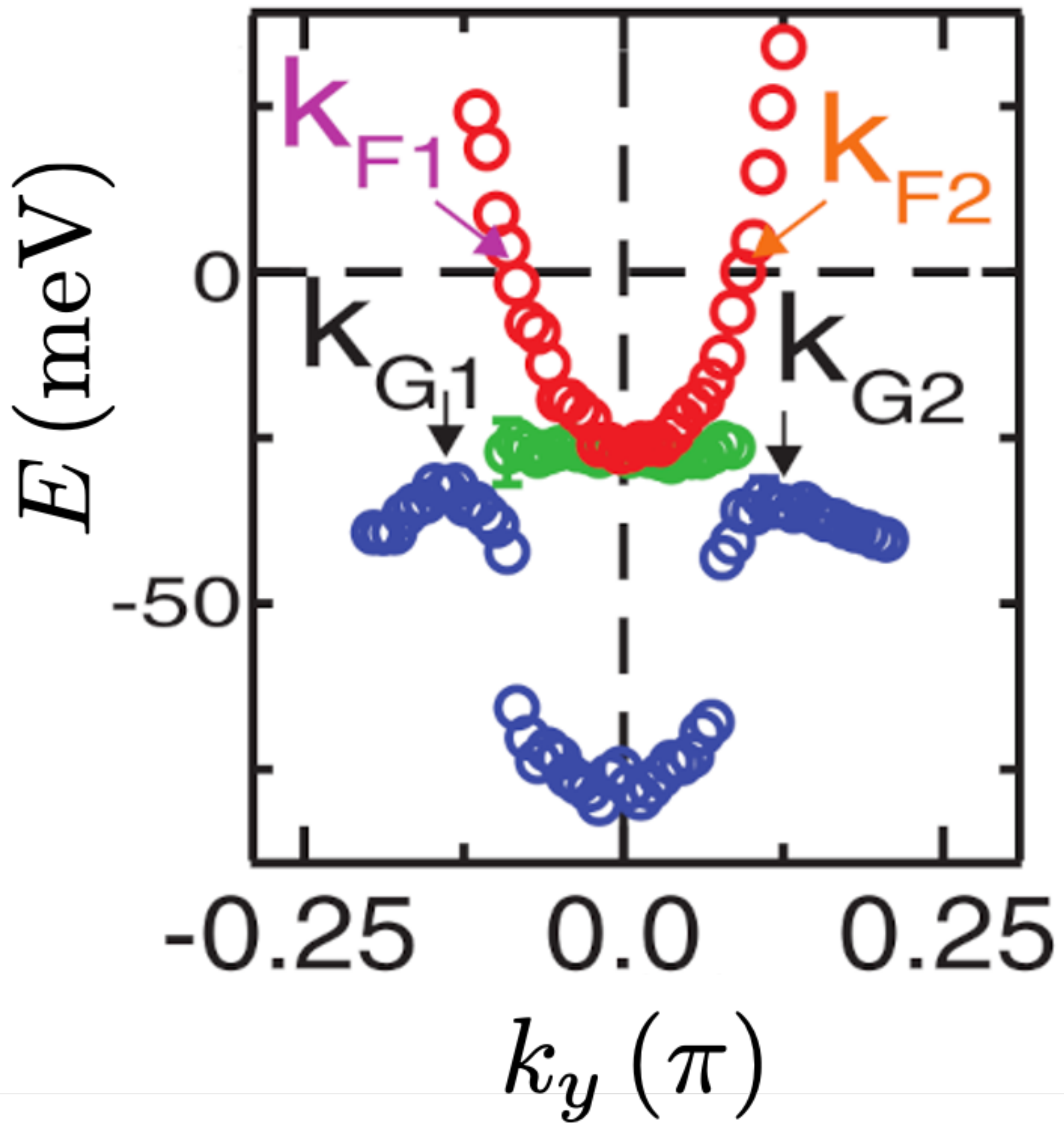
Decoupled Kondo lattice and spin liquid

E. Mascot,
A. Nikolaenko,
M. Tikhanovskaya,
Ya-Hui Zhang,
D. K. Morr,
and S. S.,
PRB **105**,
075146 (2022)

Ancilla Layer Model of FL*

Photoemission expts





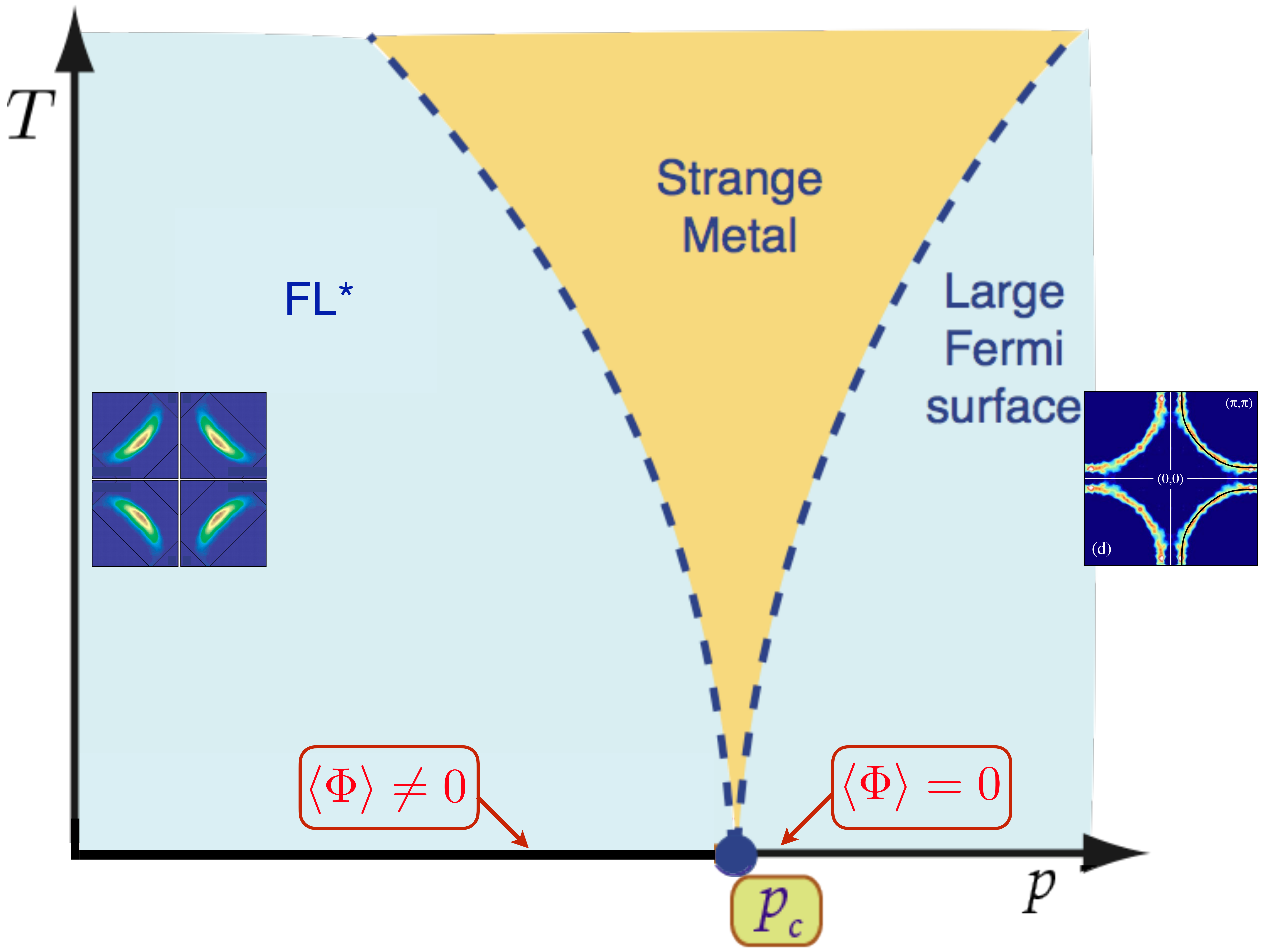
Bi2201

Decoupled Kondo lattice and spin liquid

Shift in k_F also related to
Hybridization of Fermi surfaces of size $1+p$ and 1:
(SDW theory has 2 Fermi surfaces of size $1+p$)

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

From FL* and FL to the d-wave superconductor

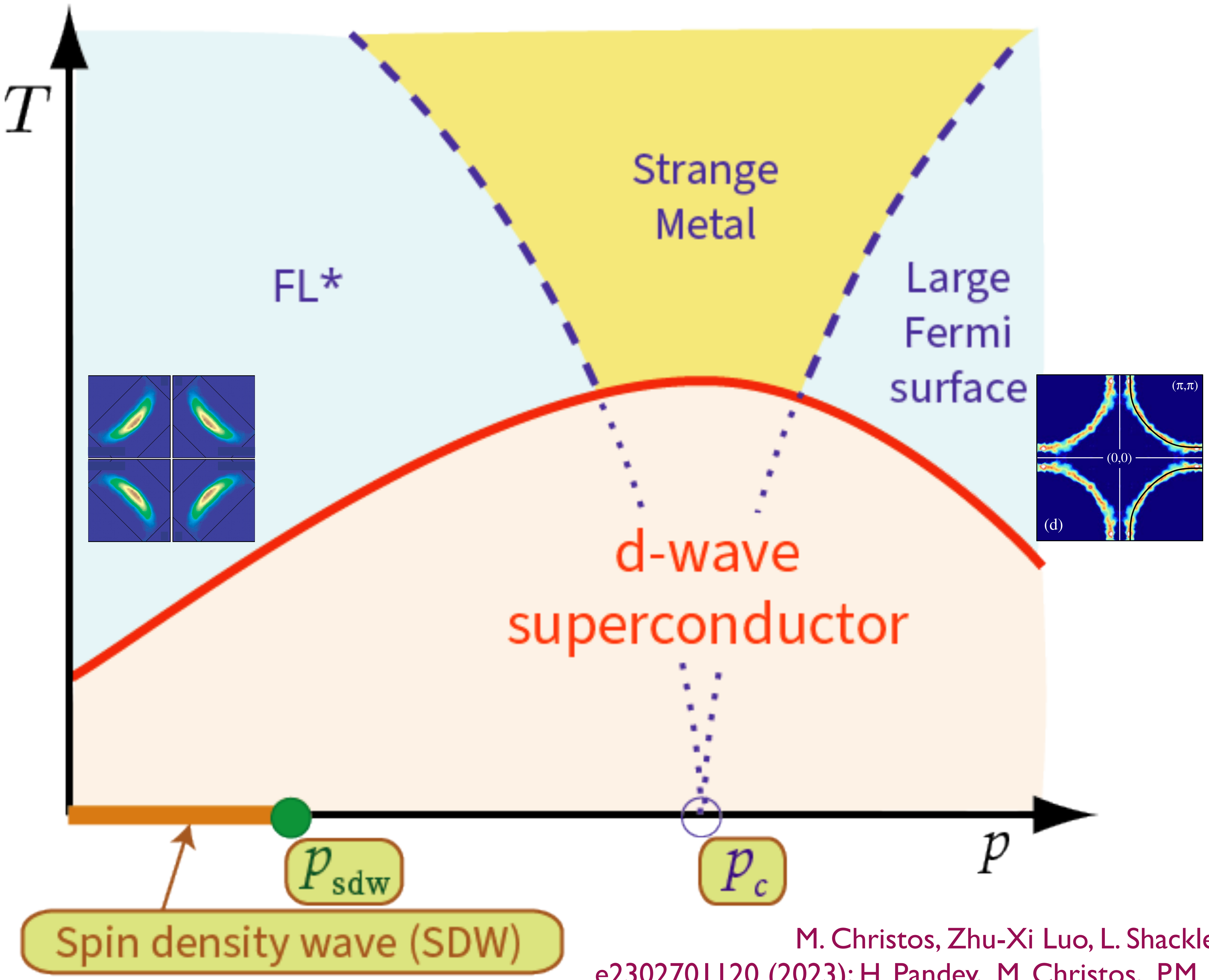


Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

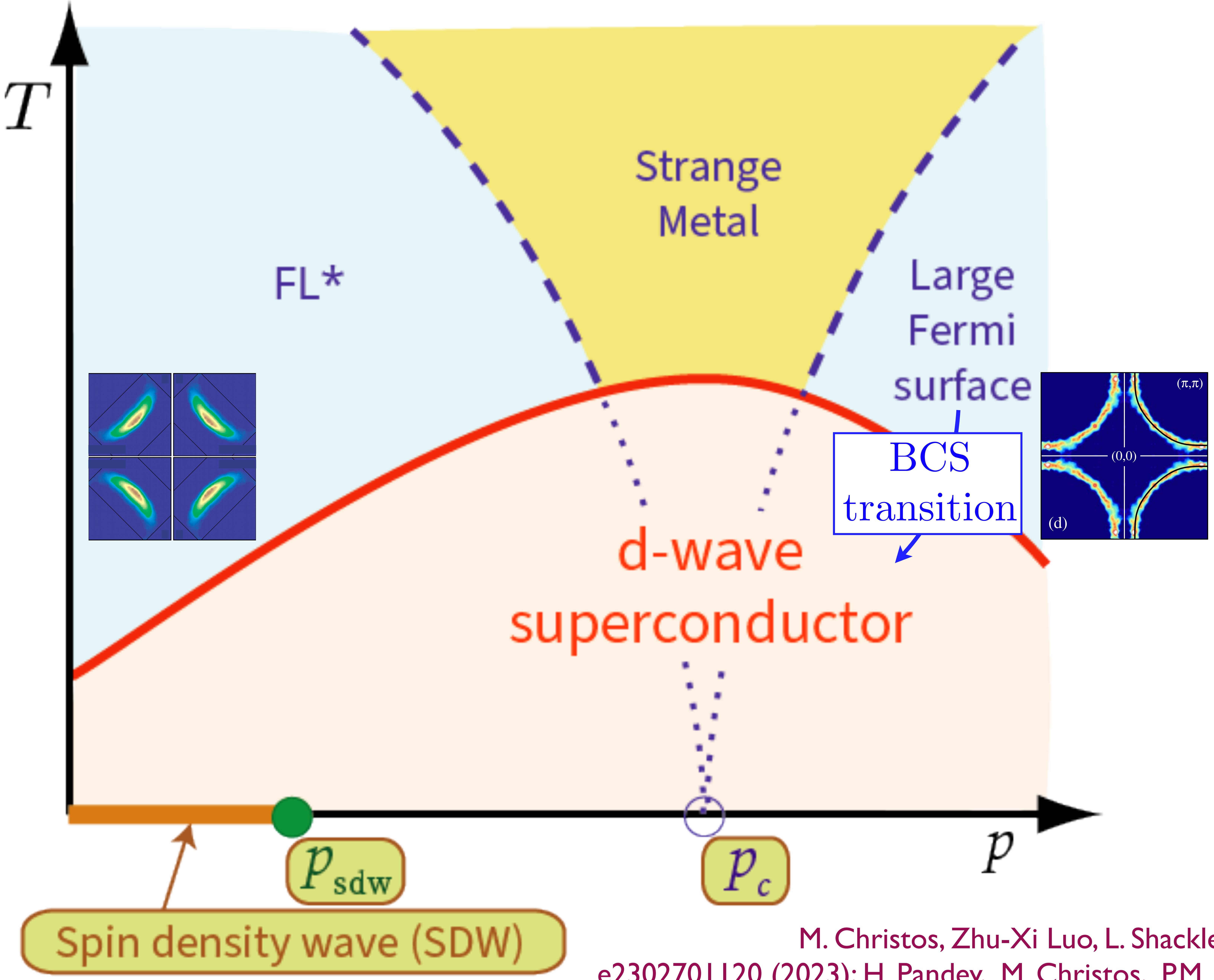
S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



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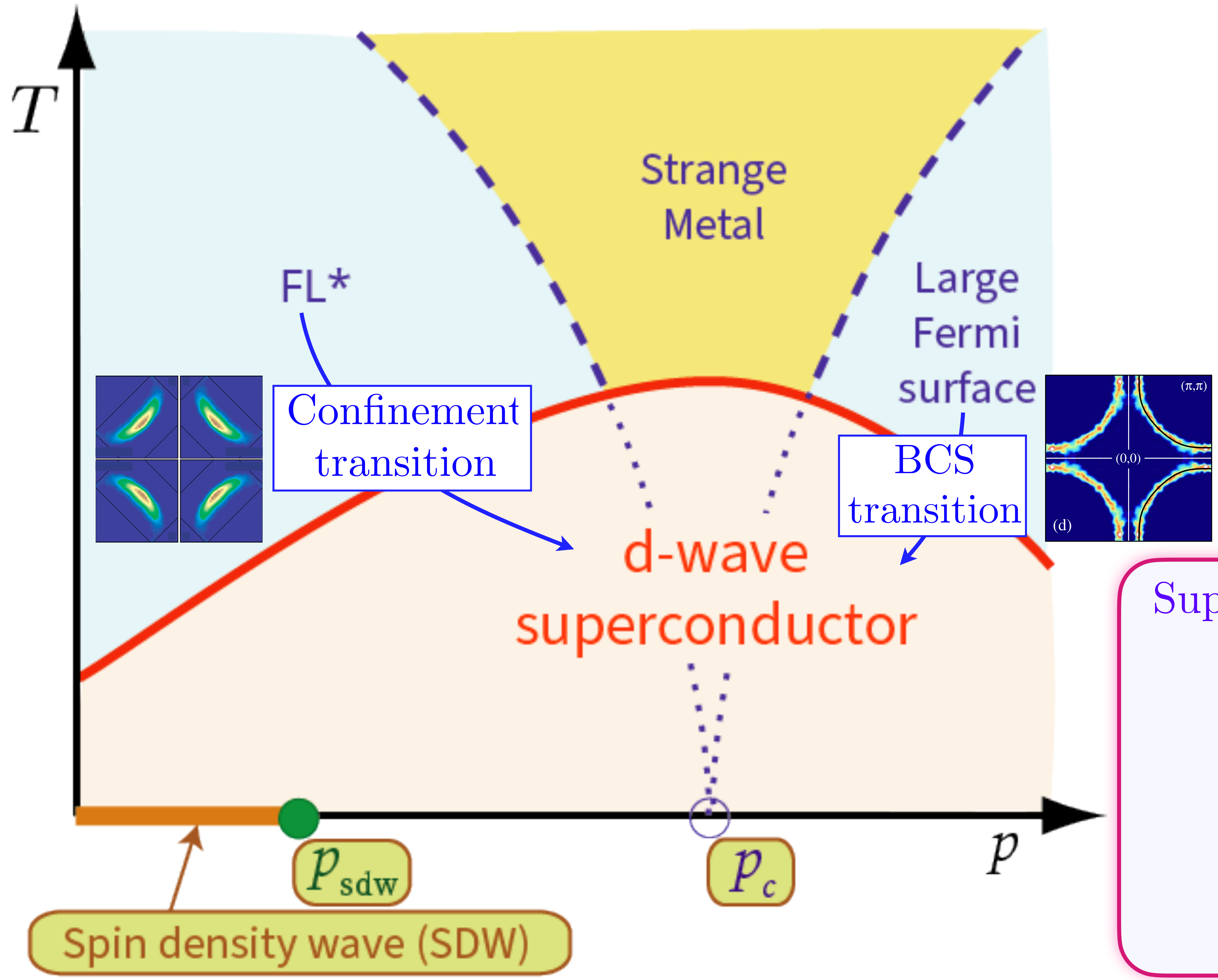
M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

Superconductor obtained from FL*:

- d-wave pairing with 4 nodal quasiparticles with $v_F \gg v_\Delta$.
- Vortex cores don't have Wang-Macdonald peak, and show charge order.

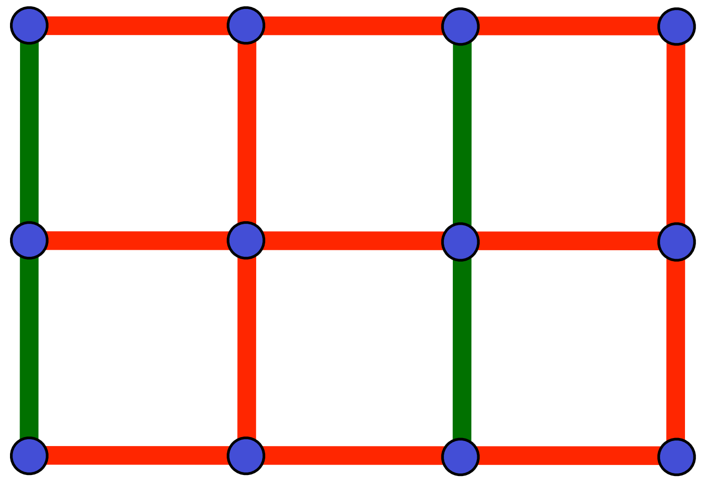
Critical quantum spin liquid on the square lattice

S=1/2 square lattice

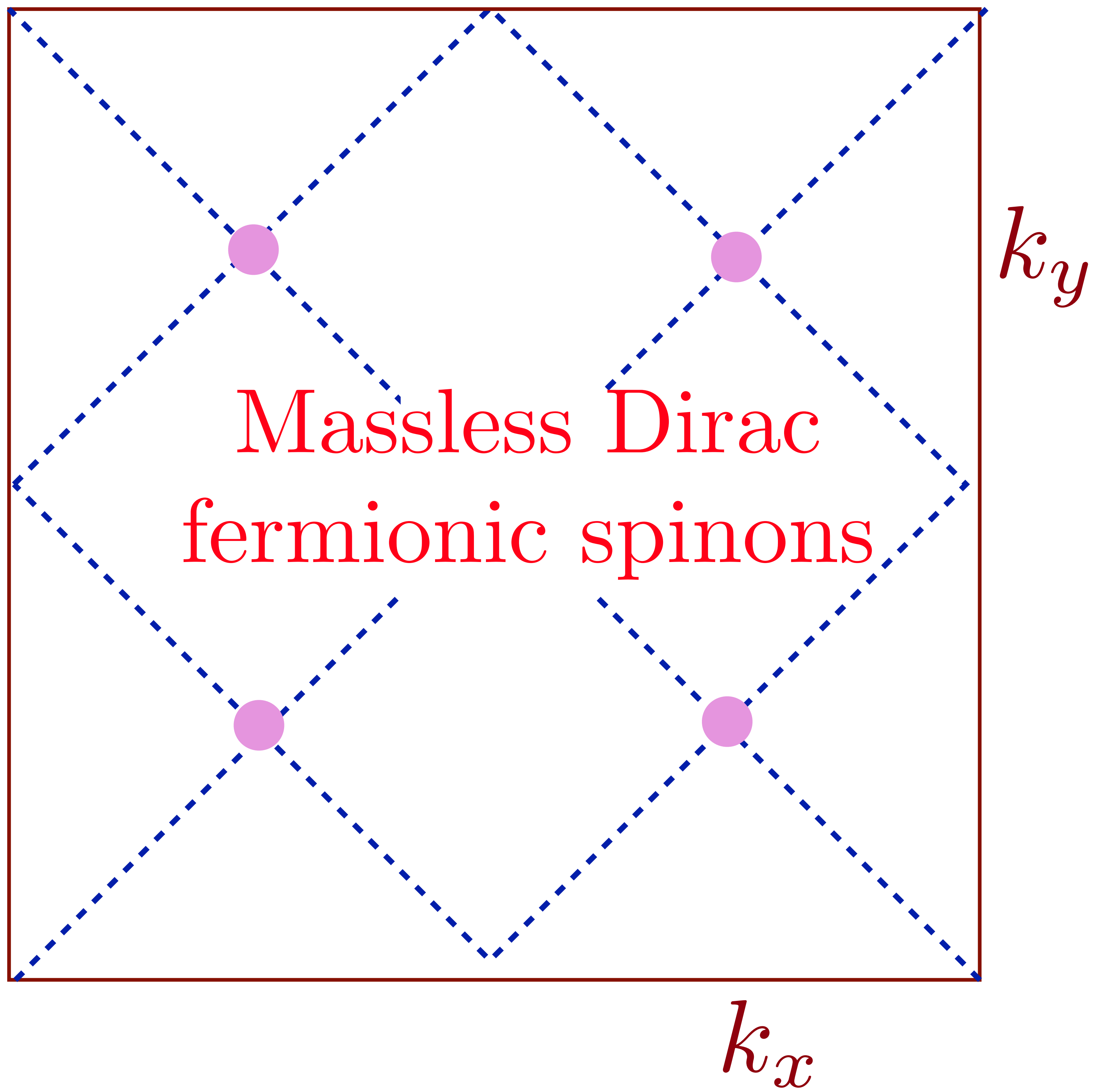
Represent spins in terms of
$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(f_{\boldsymbol{i}\alpha}^\dagger f_{\boldsymbol{j}\alpha} - f_{\boldsymbol{j}\alpha}^\dagger f_{\boldsymbol{i}\alpha} \right)$$



$e_{\boldsymbol{ij}} = 1$
$e_{\boldsymbol{ij}} = -1$



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I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

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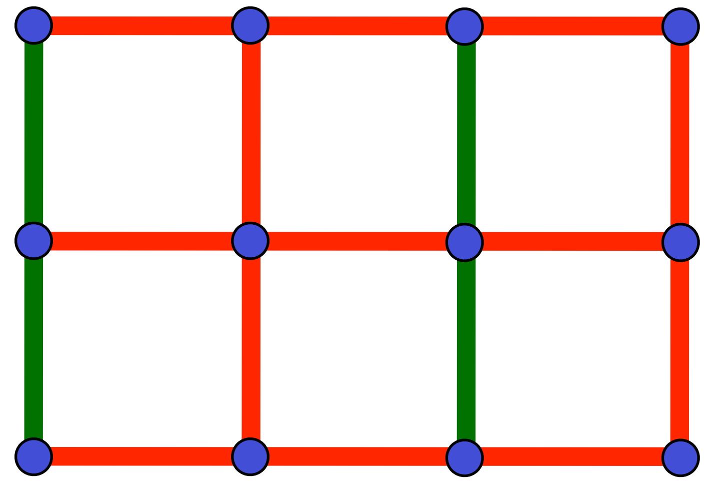
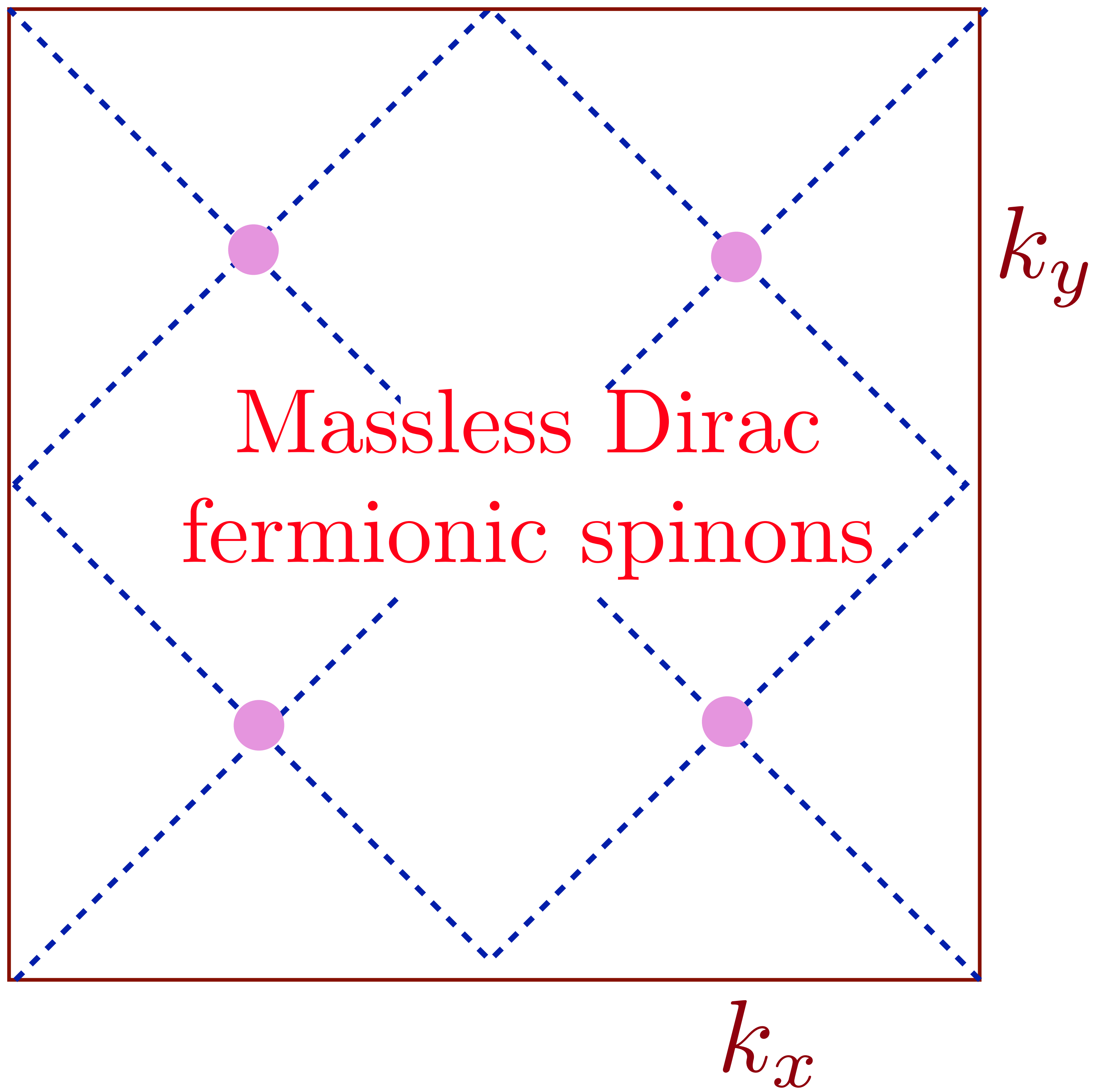


$e_{ij} = 1$

$e_{ij} = -1$

$$\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.$$



J_2/J_1

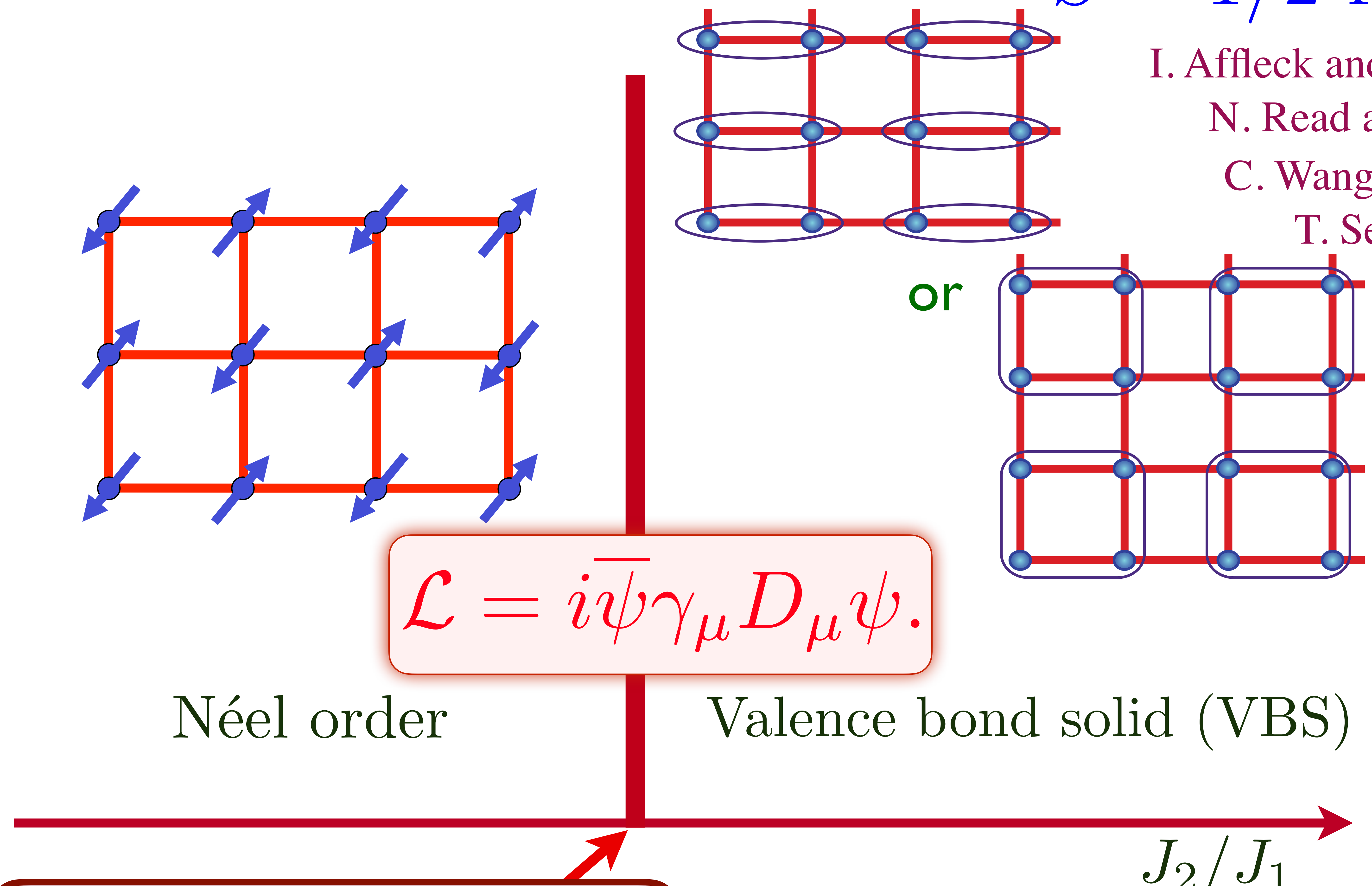
$N_f = 2$ SU(2) QCD

S=1/2 square lattice

Represent spins in terms of

$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

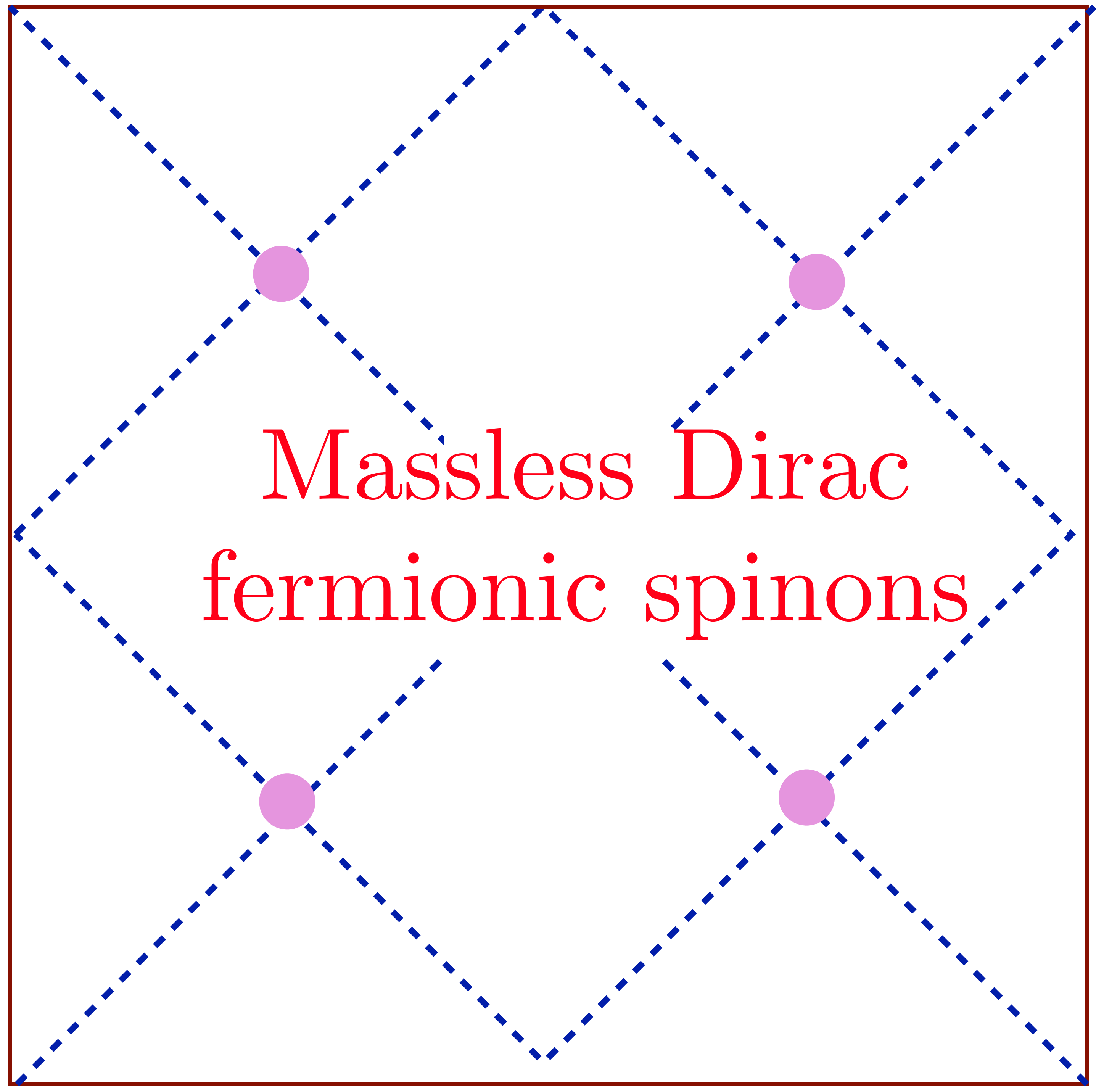
I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)
N. Read and S. Sachdev, PRL **62**, 1694 (1989)
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)



Critical spin liquid without quasiparticles?

$N_f = 2$ SU(2) QCD

Confining instability to Néel and VBS, as in $\mathbb{CP}^1$ theory of Read+SS



S=1/2 square lattice

Bosonic spinons:
$\mathbb{CP}^1$ U(1) gauge theory

N. Read and S. Sachdev, PRL **62**, 1694 (1989)

Nearly-critical S=1/2 square lattice antiferromagnet without quasiparticles

SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

Obtained from a saddle-point of fermionic spinons moving in π-flux.

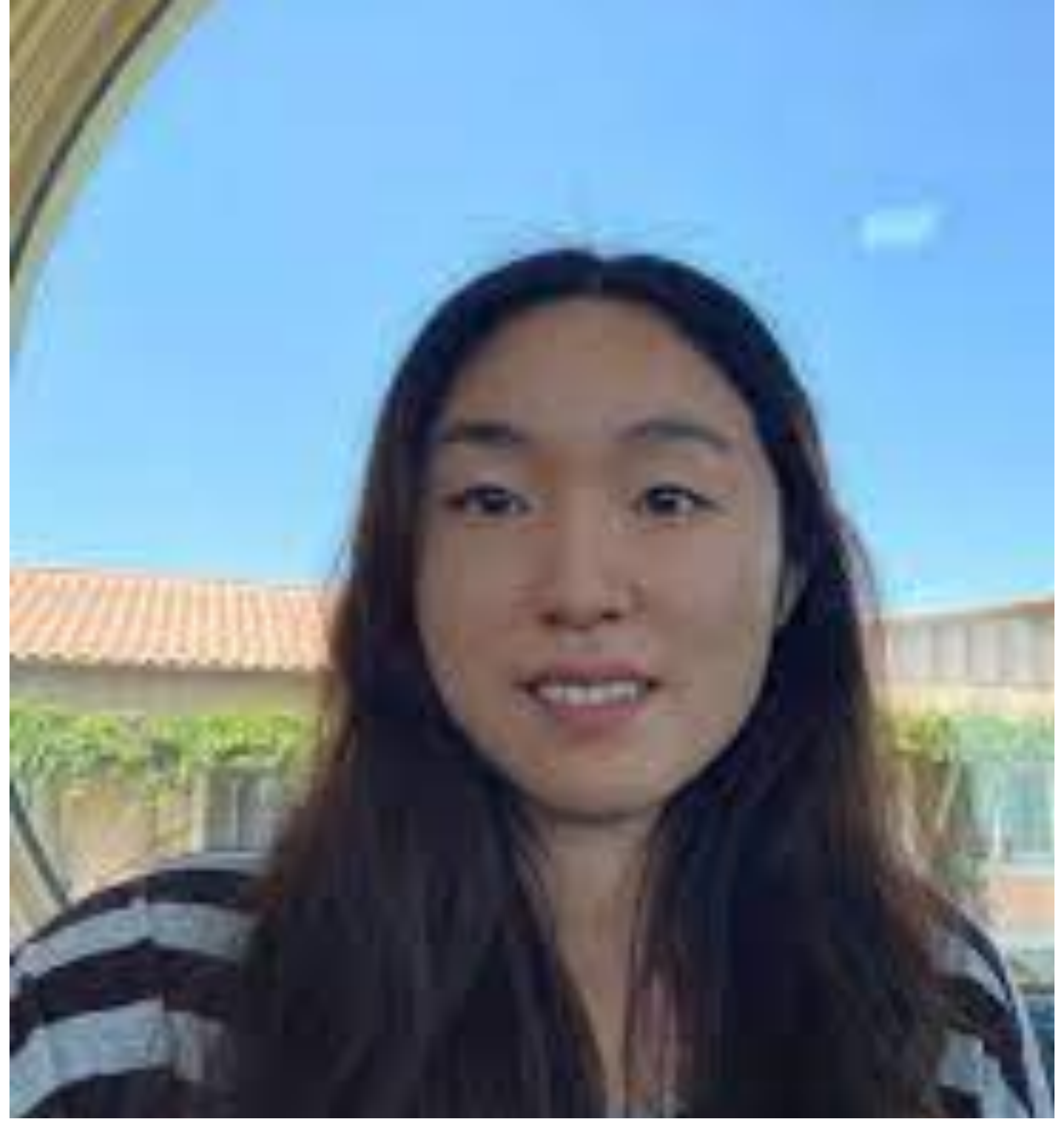
SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

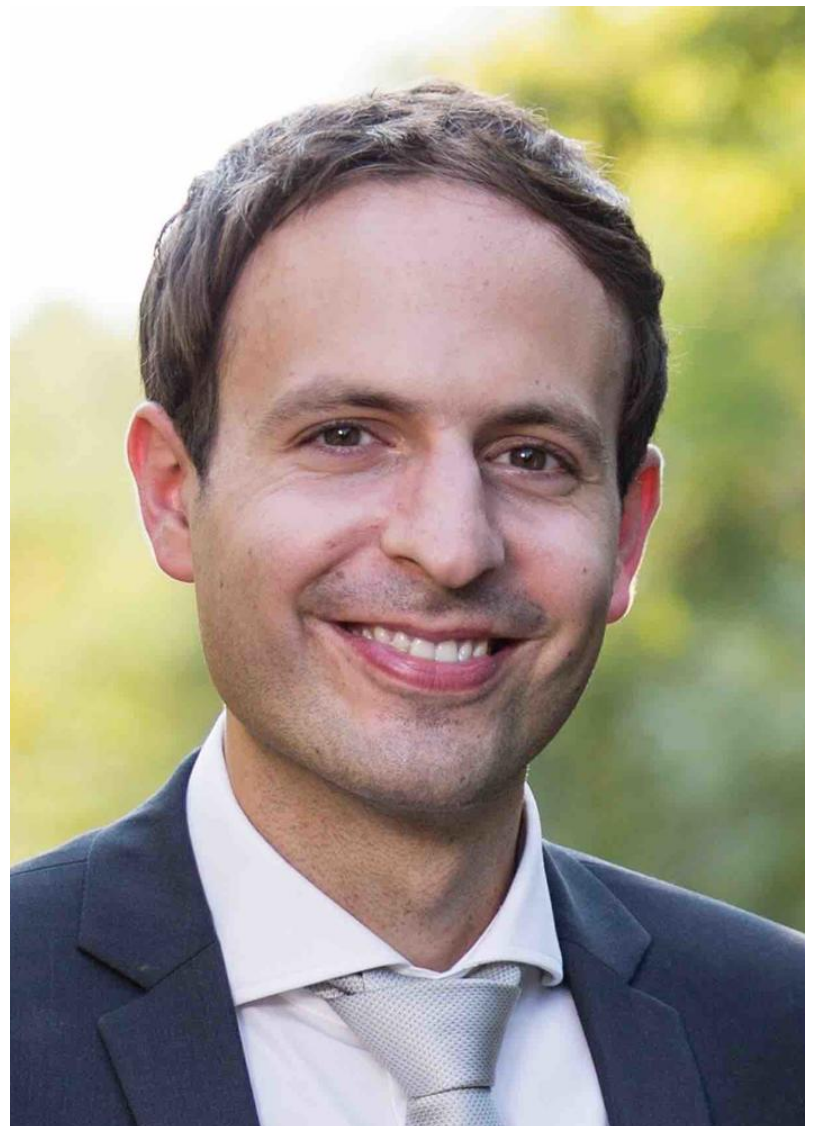
Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

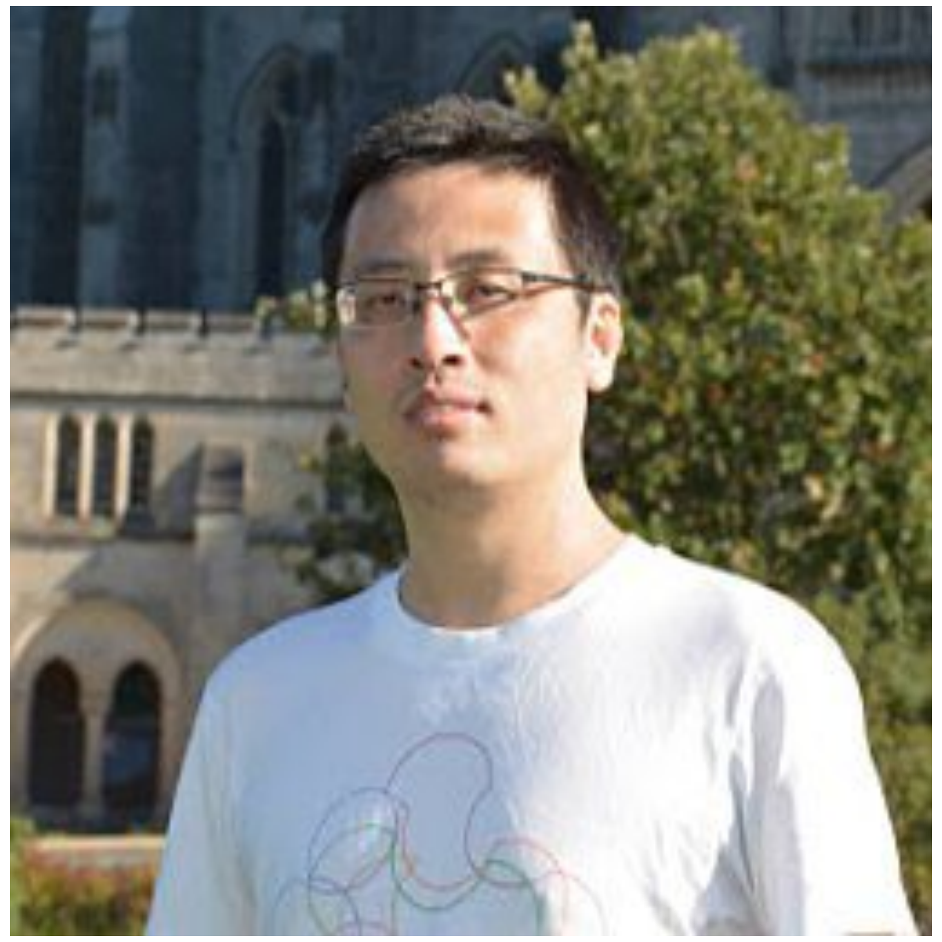
A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017); Zheng Zhou, Liangdong Hu, Wei Zhu, Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester N. Su, PRL **132**, 111601 (2024). B.-B. Chen, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL **132**, 246503 (2024); J. Takahashi, H. Shao, B. Zhao, W. Guo, A. W. Sandvik, arXiv:2405.06607.

SU(2) gauge theory for underdoped cuprates


Maine Christos


Zhu-Xi Luo


Mathias Scheurer


Ya-Hui Zhang


Leyna Shackleton

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)
M. Christos, L. Shackleton, S.S., and Zhu-Xi Luo, Physical Review Research **6,** 033018 (2024)

Ancilla Layer Model of FL*

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

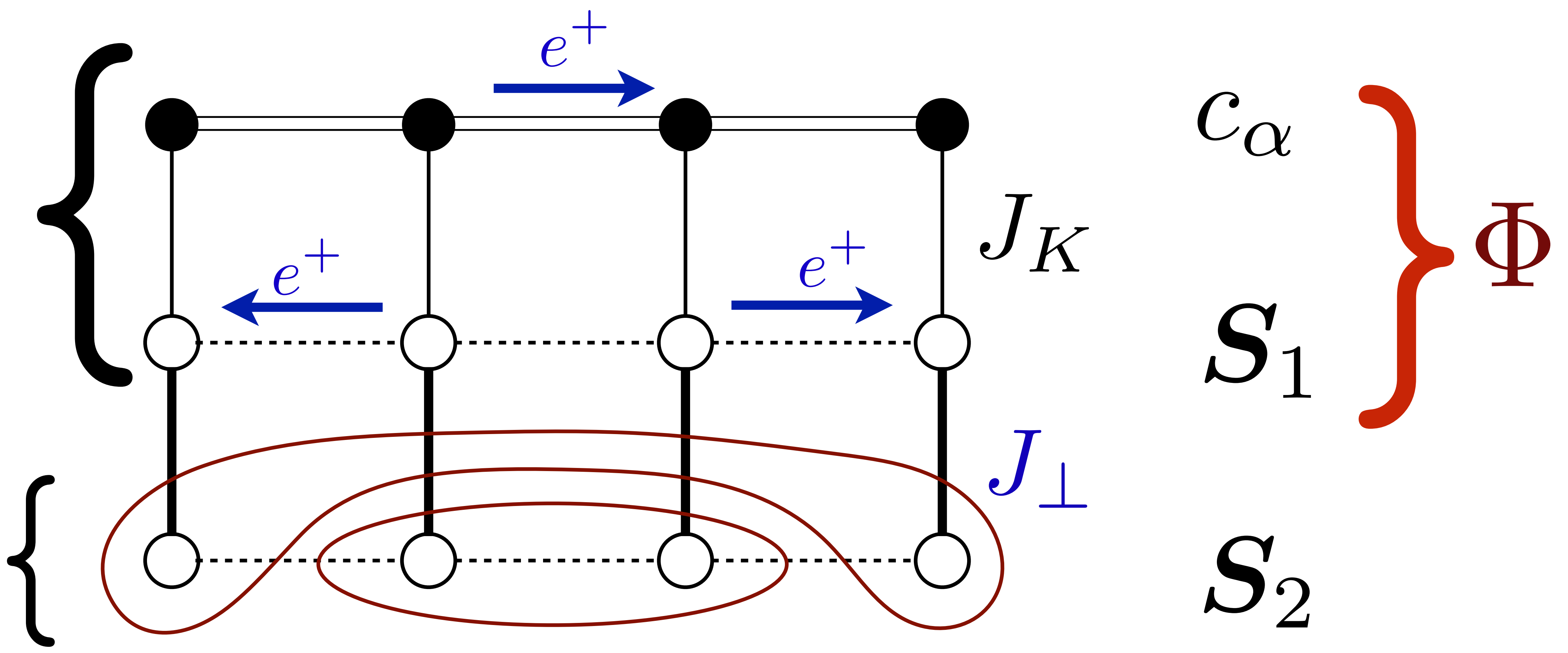
Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

Your favorite spin liquid



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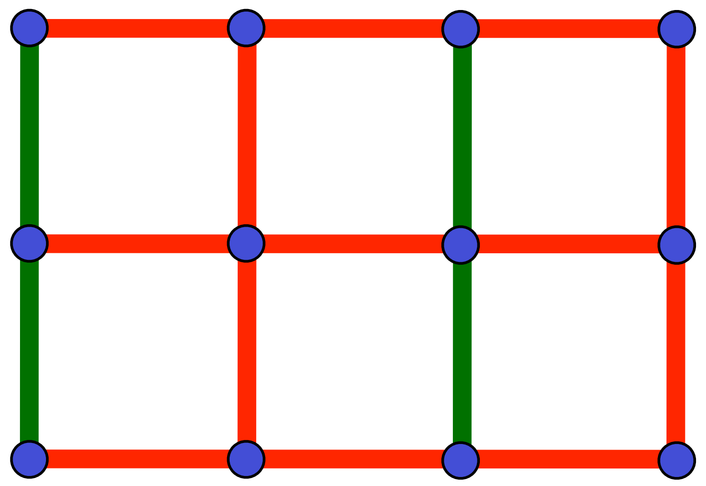
Heavy Fermi liquid of electrons c, f_1
$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) + \mathcal{E}_{YM}[U] \; ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

π-flux $\boldsymbol{S}_2$ spin liquid.
$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

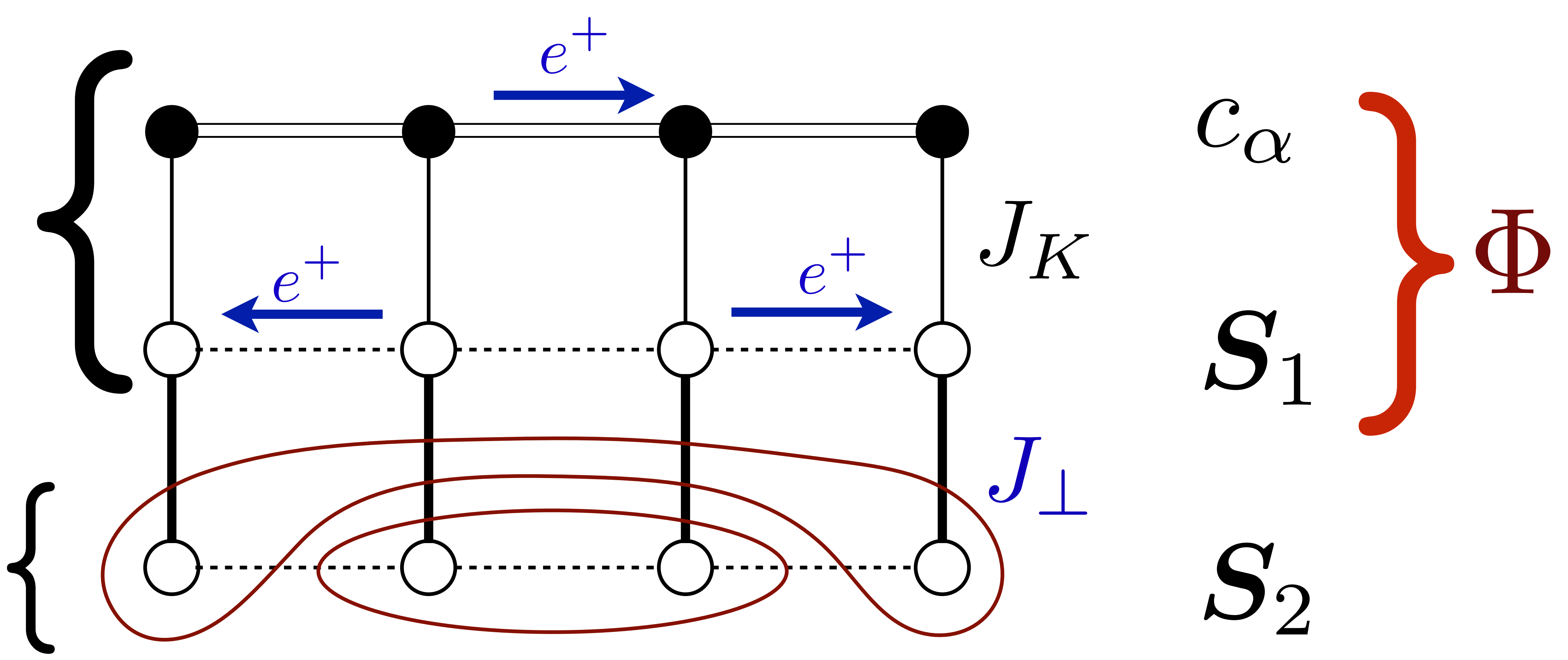


$e_{\boldsymbol{ij}} = 1$
$e_{\boldsymbol{ij}} = -1$

Fermionic spinons f moving in π-flux and an emergent SU(2) gauge field U

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi surface!

π-flux spin liquid



Ancilla Layer Model of FL*

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Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

$$+\mathcal{E}_{YM}[U]$$

π-flux $\boldsymbol{S}_2$ spin liquid.

$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

$$H_{\text{coupling}} = \sum_{\boldsymbol{i}} \left(B^*_{1\boldsymbol{i}} f^\dagger_{1\boldsymbol{i}\alpha} f_{\boldsymbol{i}\alpha} + B^*_{2\boldsymbol{i}} \varepsilon_{\alpha\beta} f^\dagger_{1\boldsymbol{i}\alpha} f^\dagger_{\boldsymbol{i}\beta} + \text{H.c.} \right)$$

Couple Kondo lattice and spin liquid by charge e, SU(2) fundamental Higgs boson B

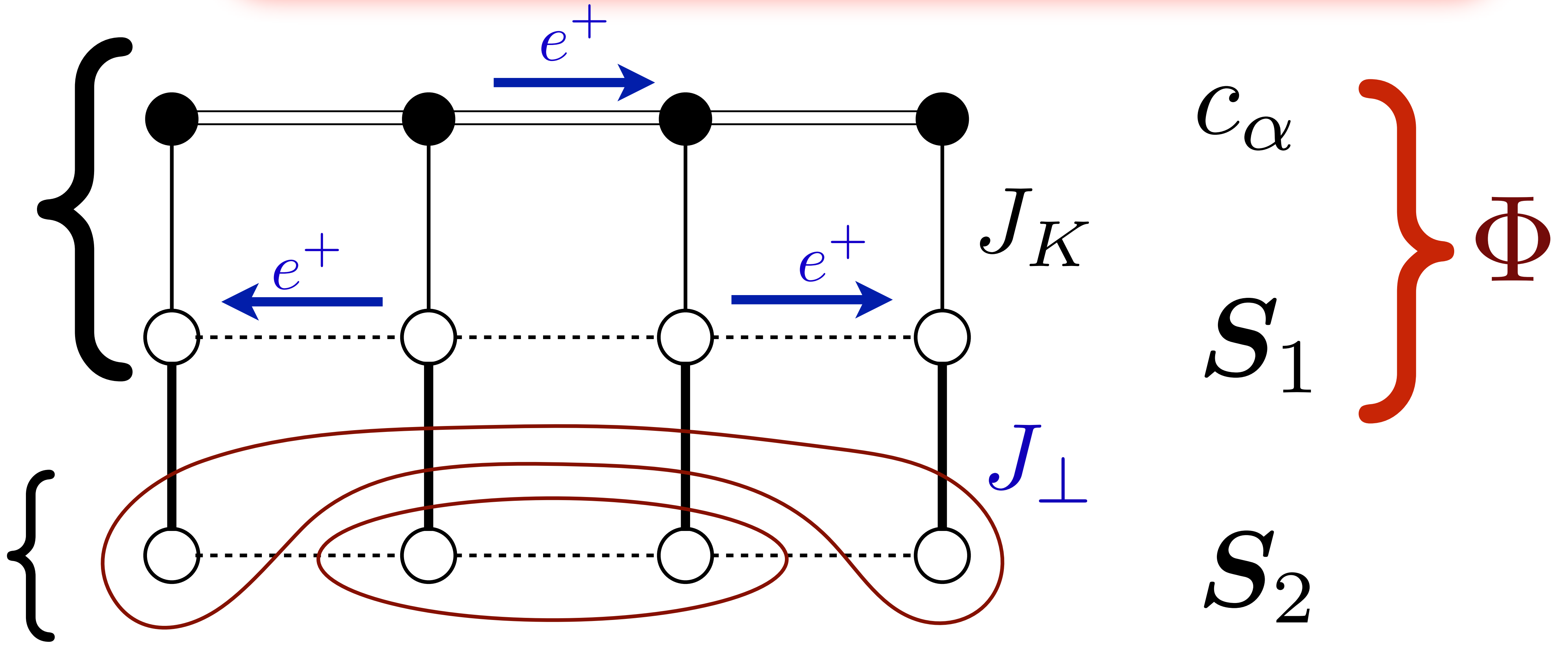
$$V_{\text{Higgs}} = \mathcal{E}_2[B, U] + \mathcal{E}_4[B, U]$$

V_{Higgs} dictated by symmetry of spin liquid

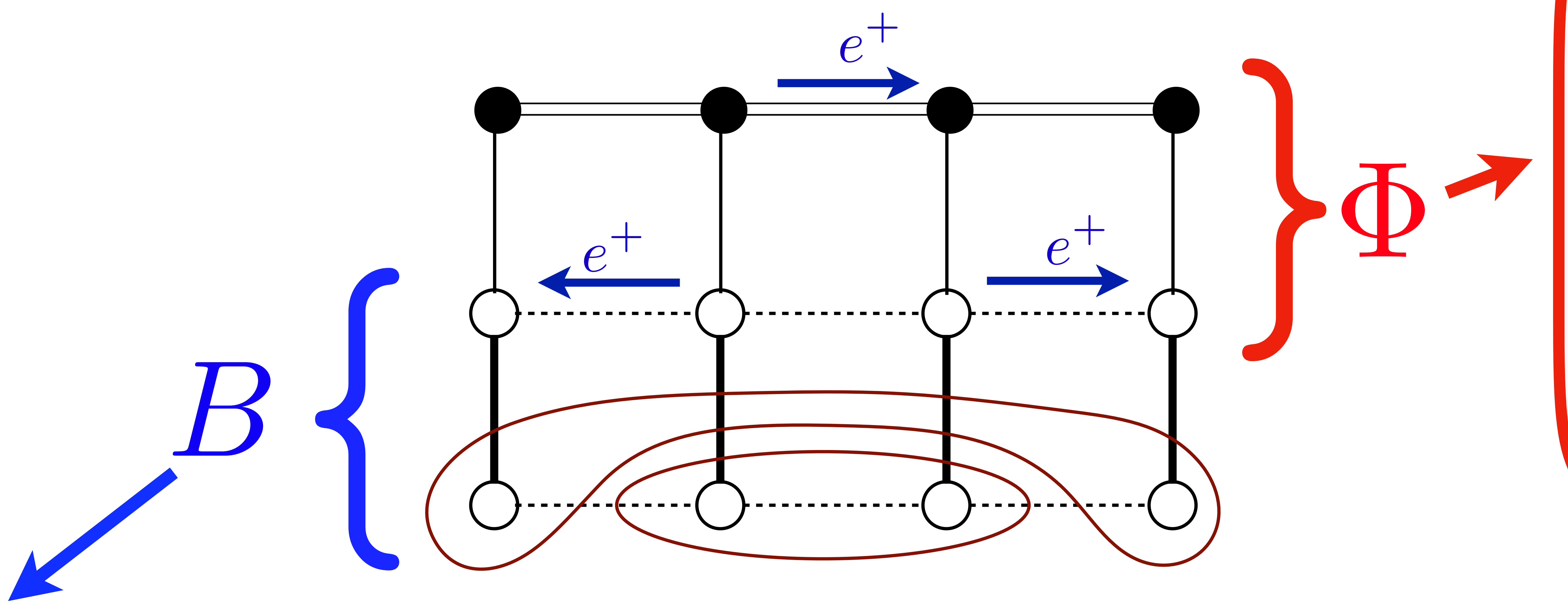
Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi surface!

B

π-flux spin liquid



Ancilla Layer Model of FL*

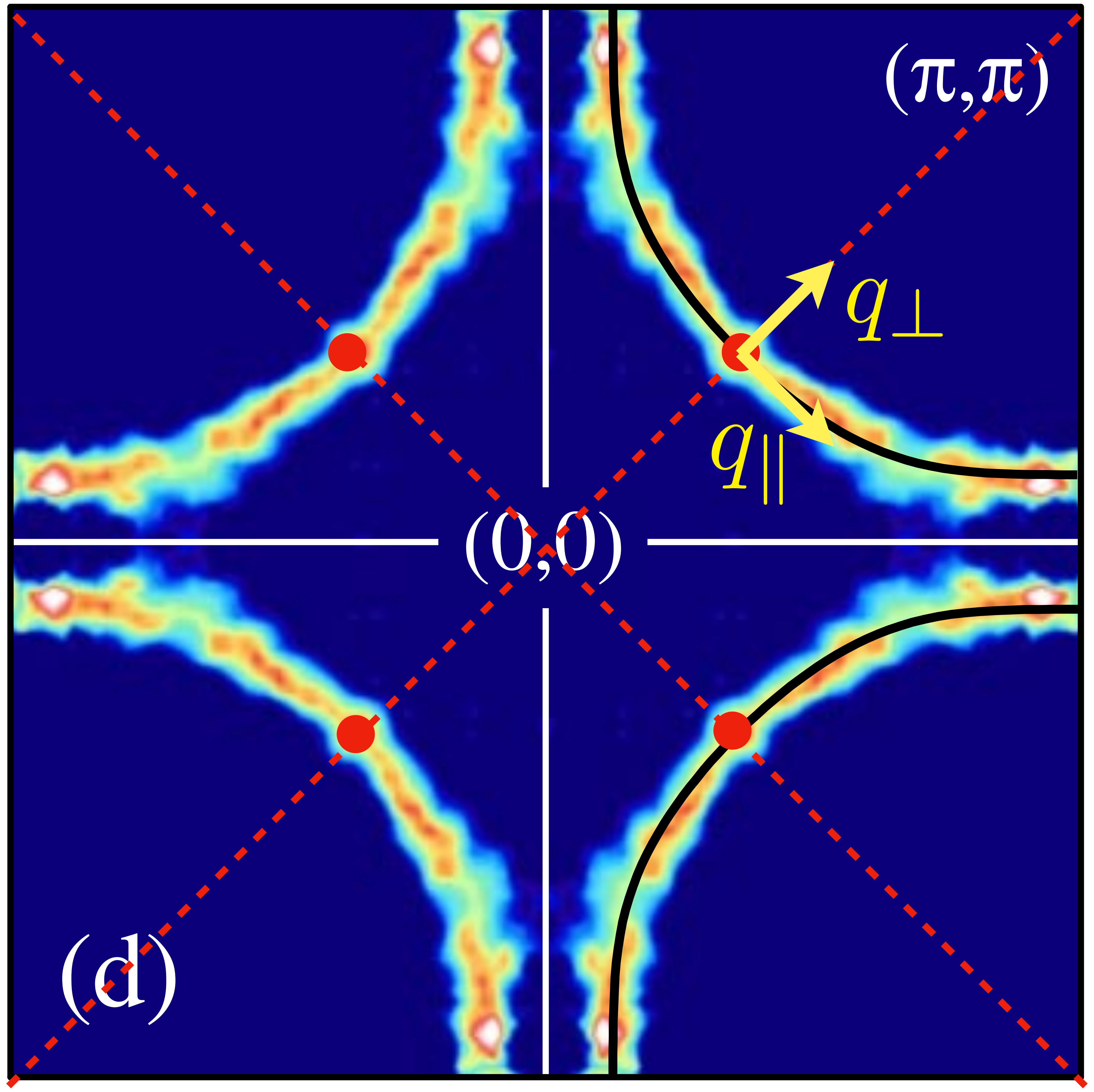


Higgs field Φ determines the pseudogap.
In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.
- Yukawa coupling between c_α, f_α and B.
- B is a **fractionalized order parameter**, whose composites describe numerous superconducting and charge order parameters!

From FL* to dSC

FL→dSC



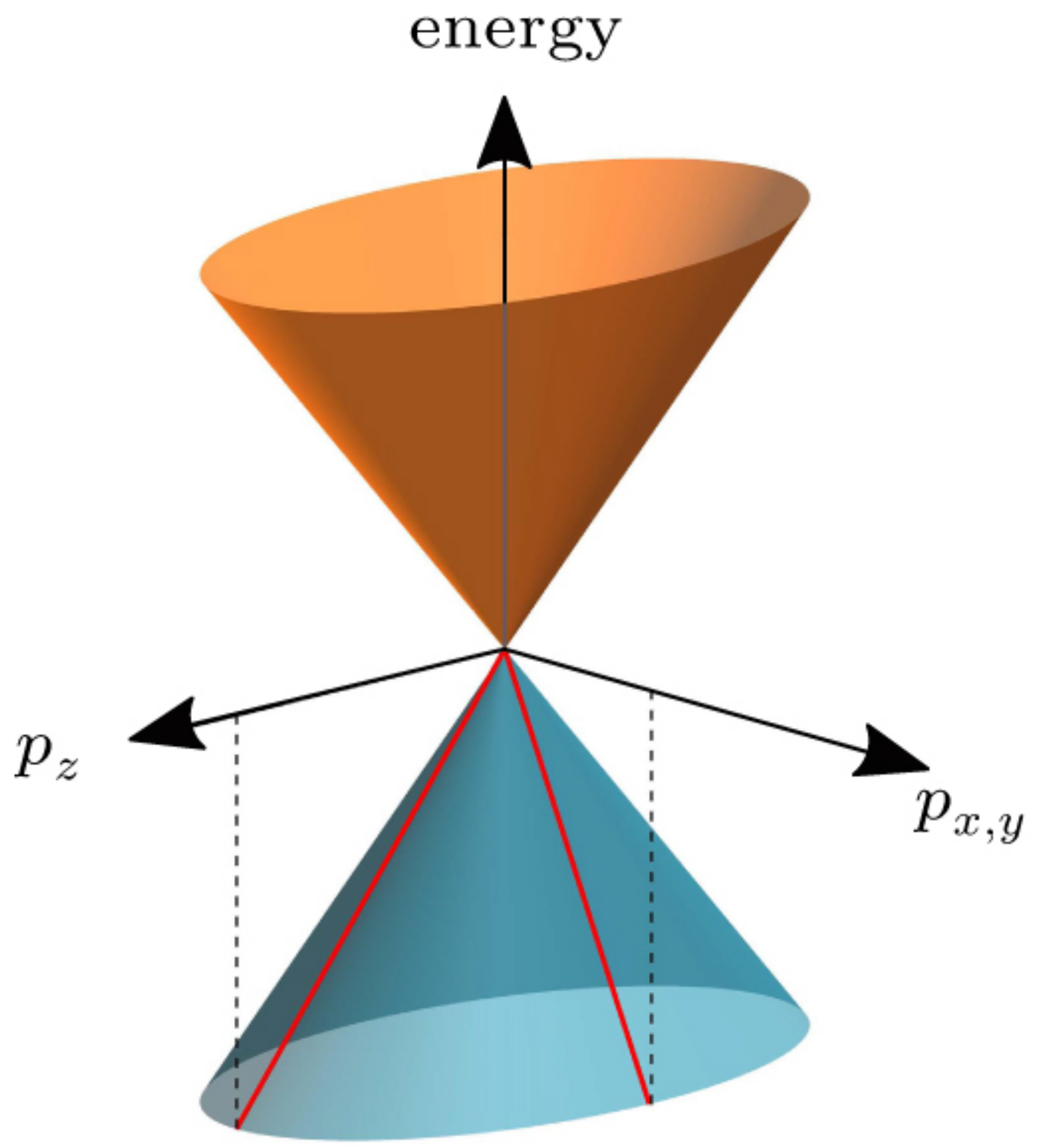
BCS/Bogoliubov quasiparticles
in a d-wave superconductor

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

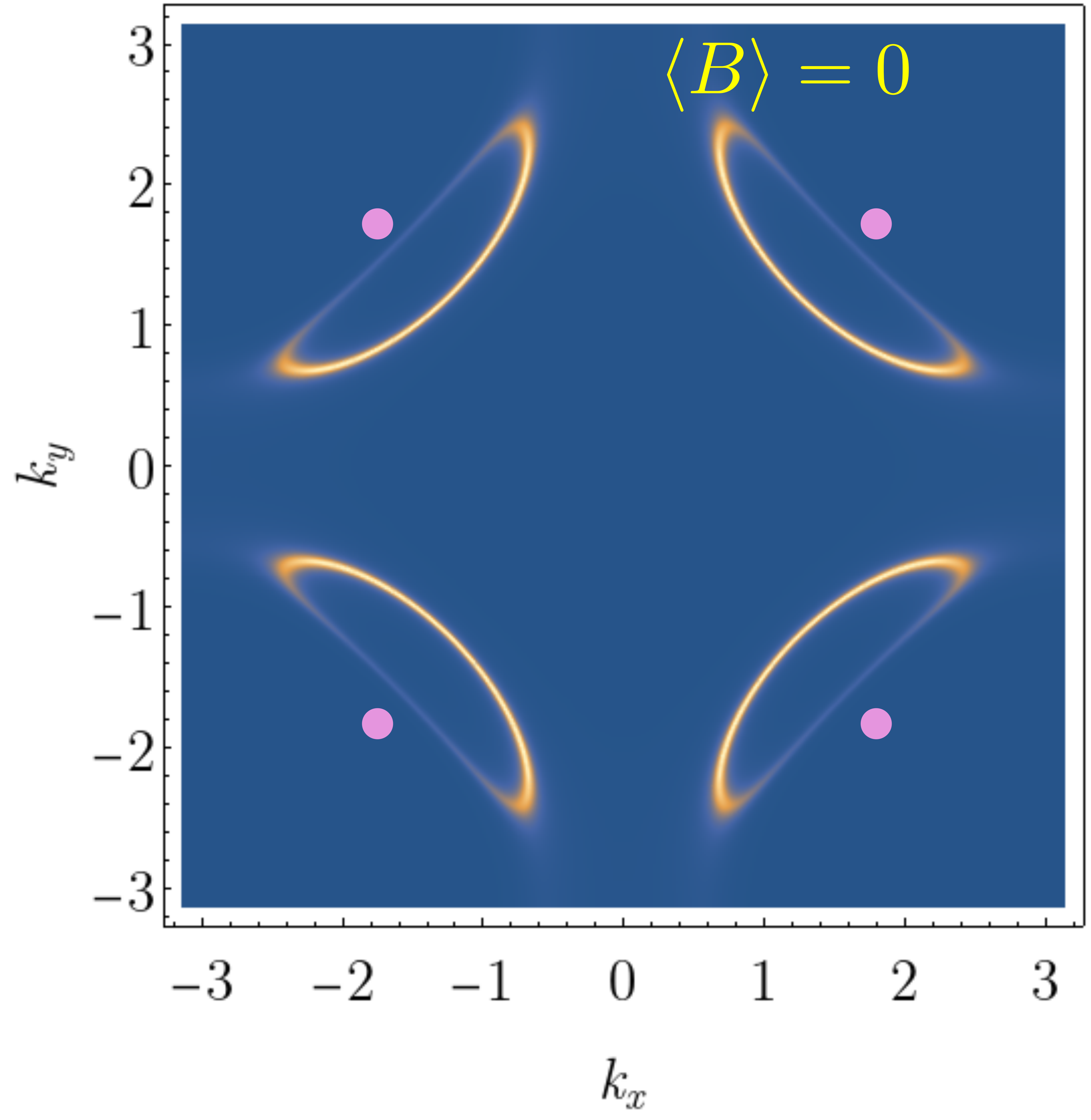
4 nodal points where

$$E_{\boldsymbol{k}_0+\boldsymbol{q}} = \left(v_F^2 q_\perp^2 + v_\Delta^2 q_\|^2\right)^{1/2}$$

with $v_F \gg v_\Delta$.



FL*

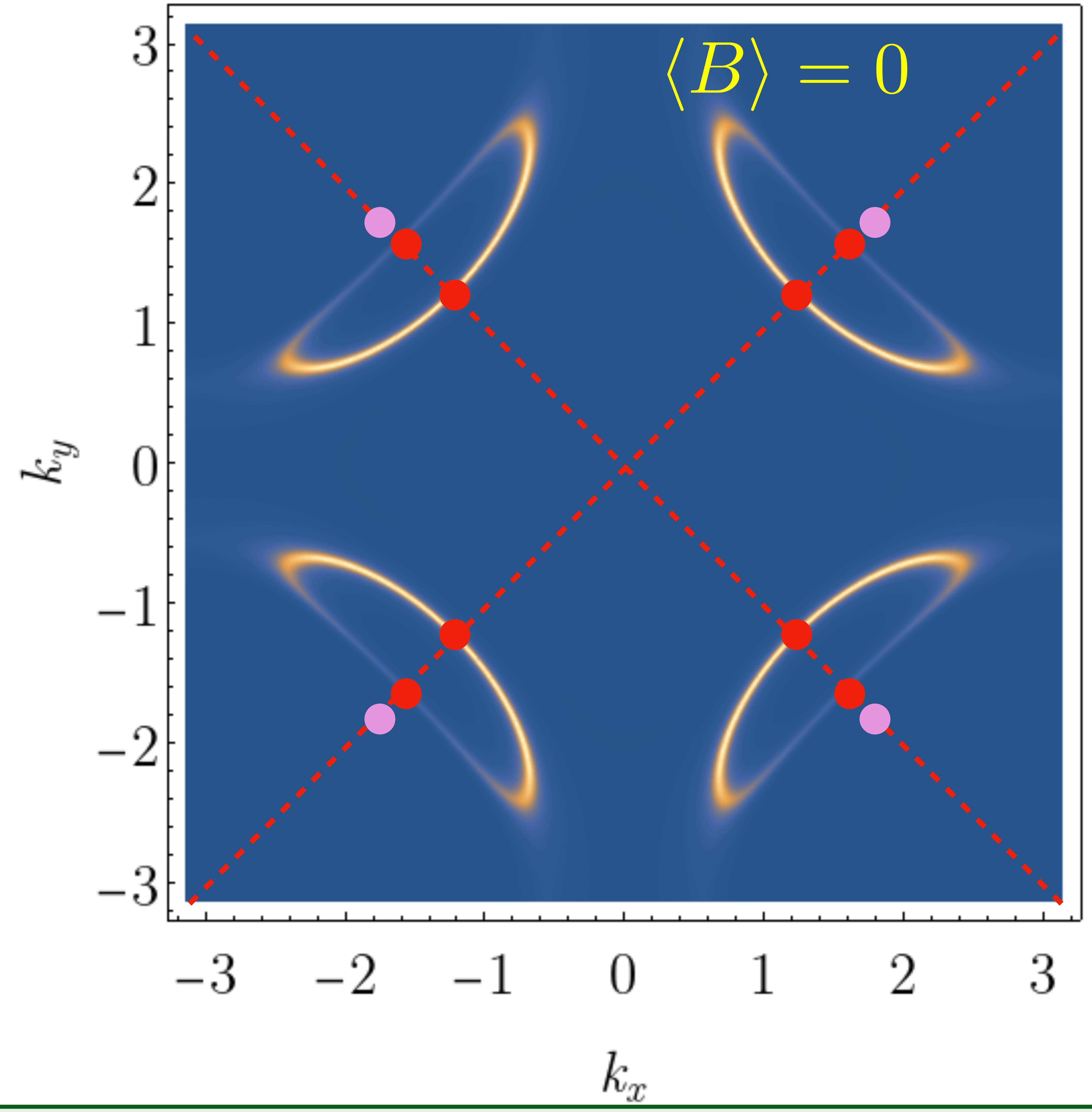


FL* $\Rightarrow$ d-SC:

Cooper pairing of the Fermi surface?

FL* has 4 electron-like pockets
and 4 nodal spinons
of the π-flux spin liquid

FL*→d-SC*



FL* ⇒ d-SC:

Cooper pairing of the Fermi surface?

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

No!

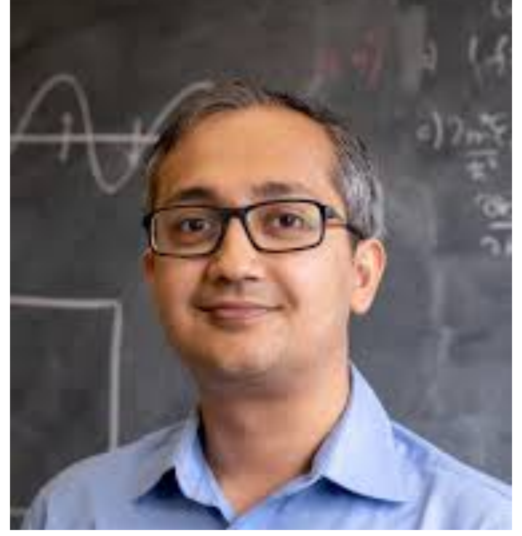
Leads to 8 nodal points of Bogoliubov quasiparticles and 4 nodal spinons of π-flux spin liquid.

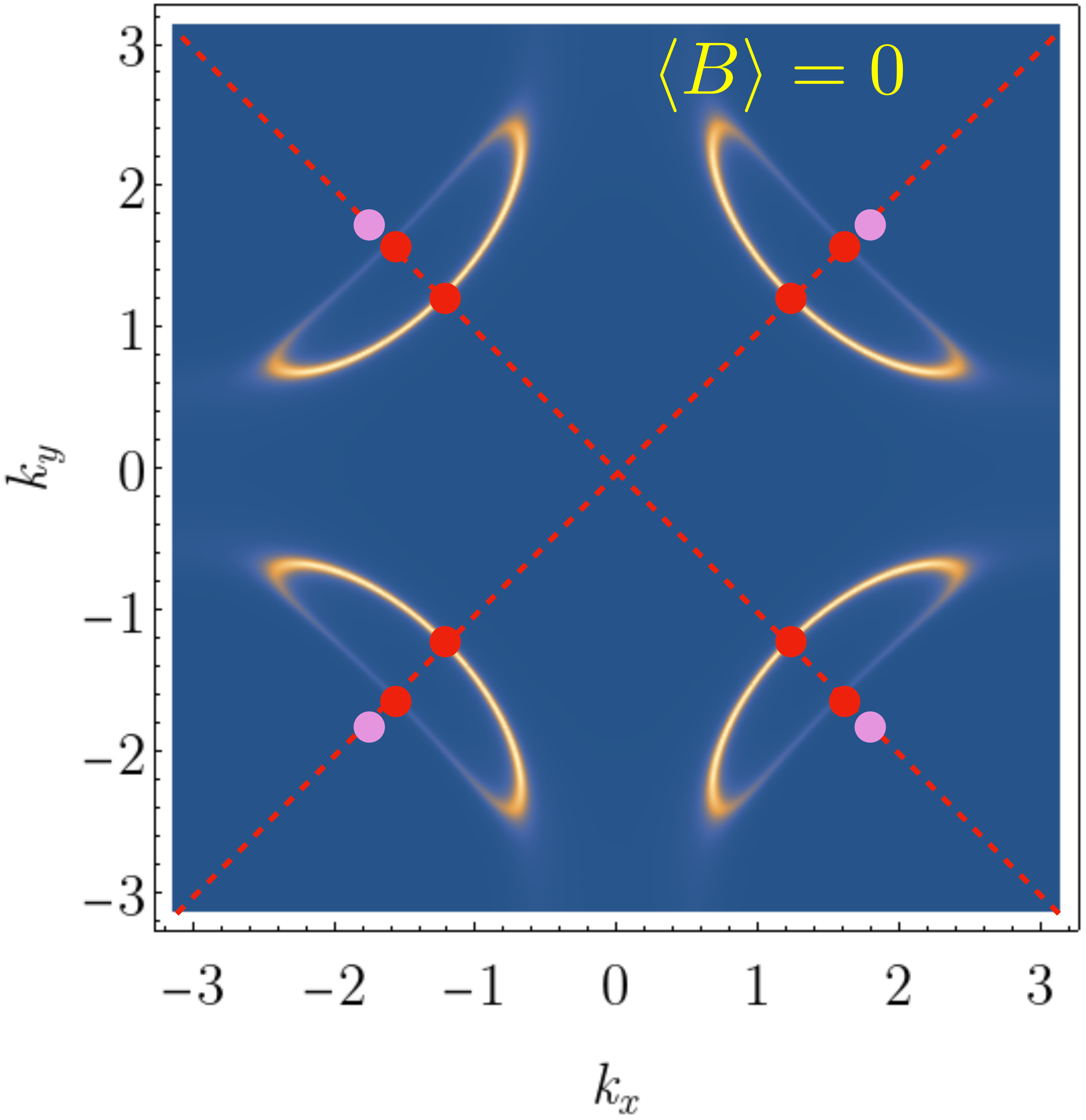
FL* ⇒ d-SC*

BCS mechanism applied to FL* pseudogap leads to non-BCS superconductor!

FL*→d-SC

Shubhayu Chatterjee and S. S., PRB **94**, 205117 (2016)
Maine Christos and S.S., npj Quantum Materials **9**, 4 (2024)

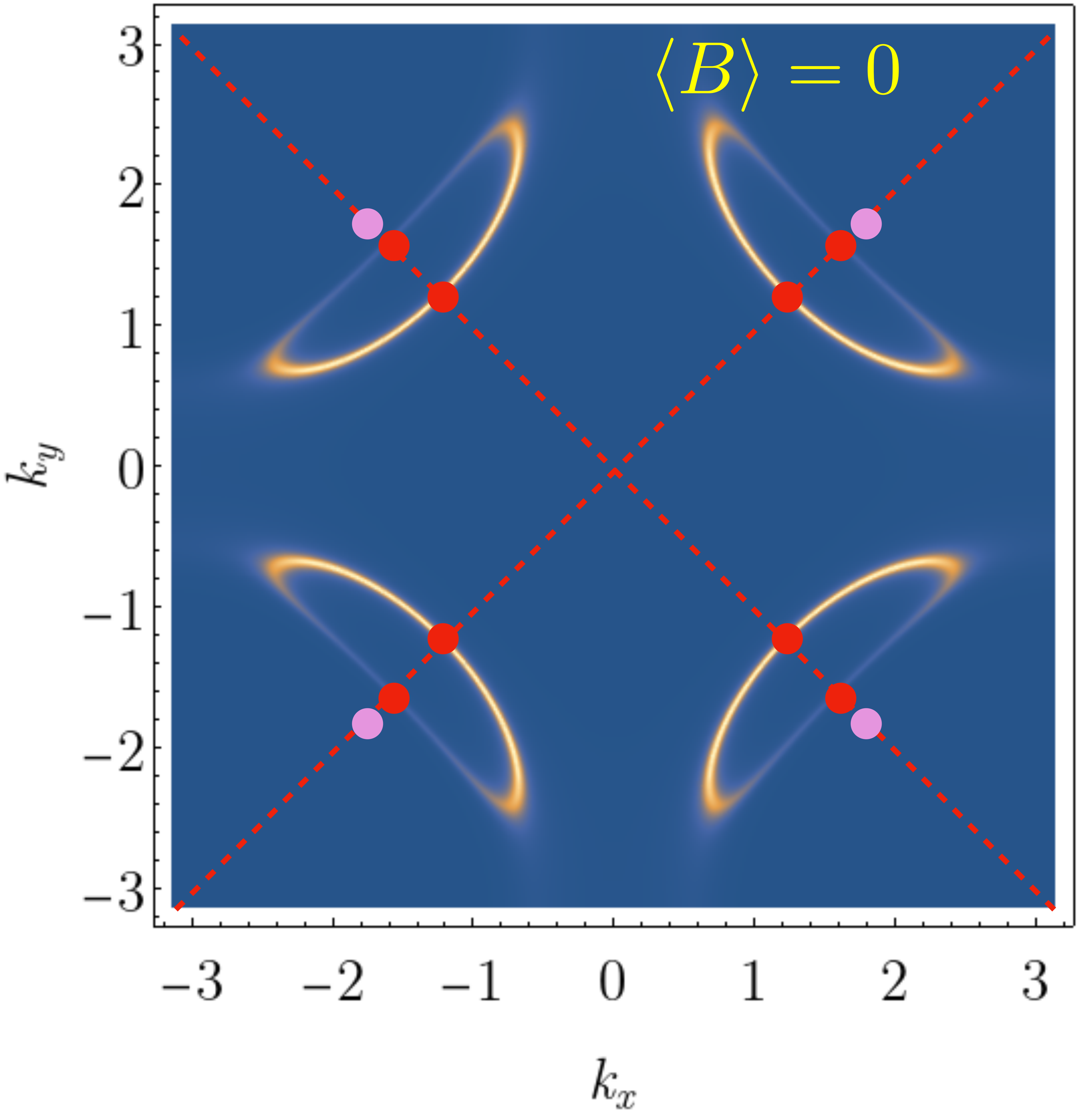




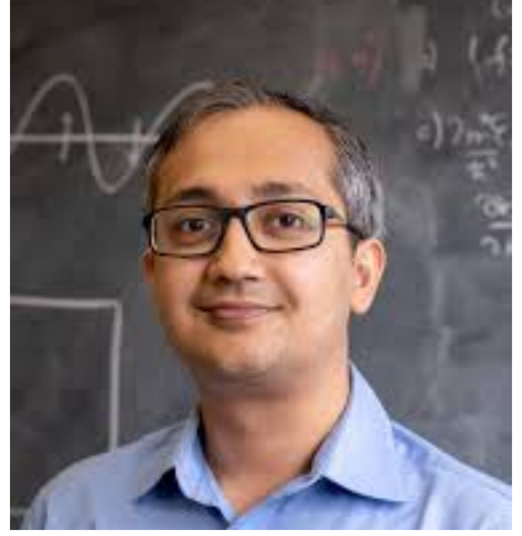
Alternative route to d-wave superconductivity:

Use the pre-existing pairing of the underlying spin liquid and confine the spin liquid!

FL*→d-SC



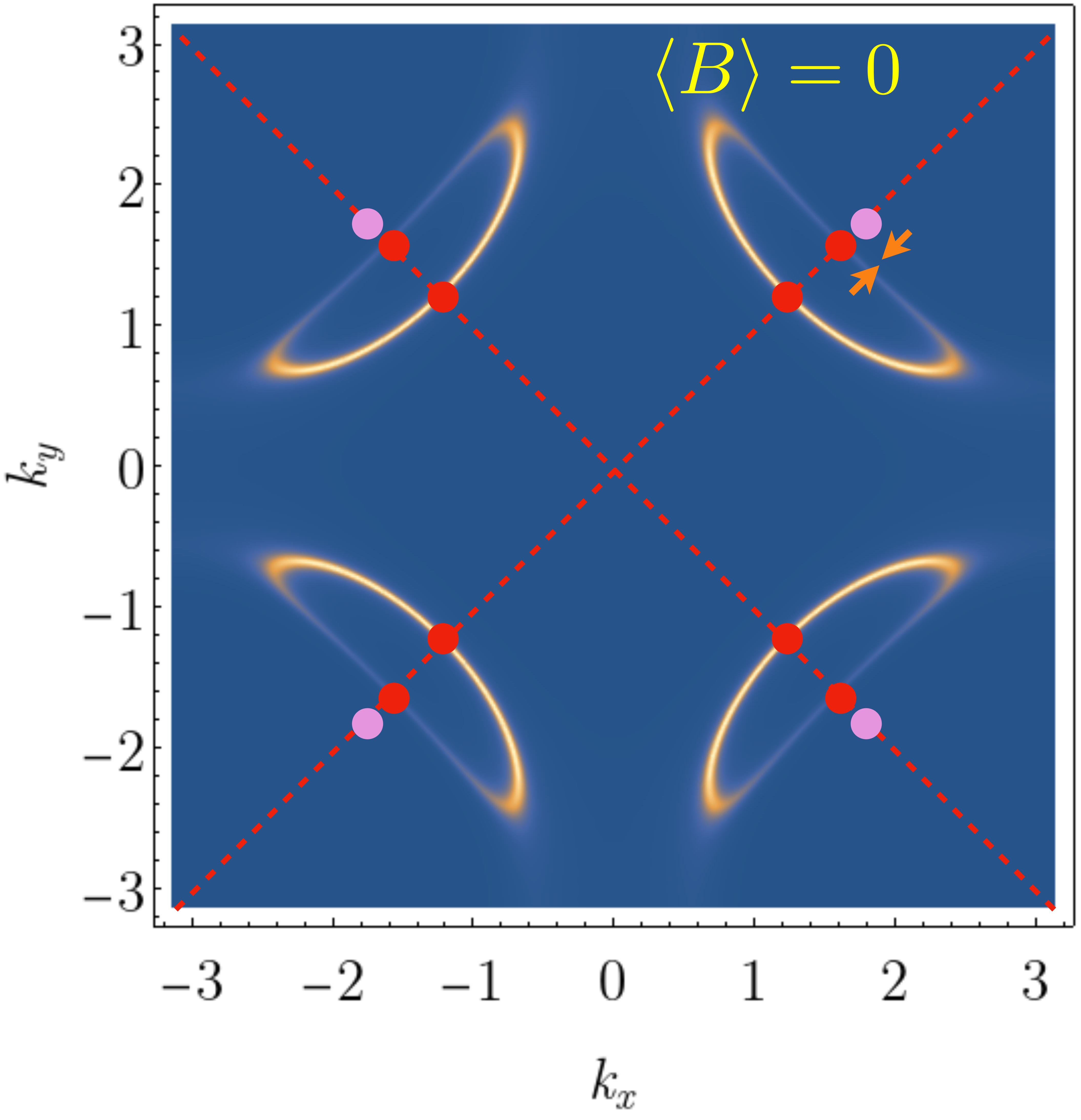
Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)



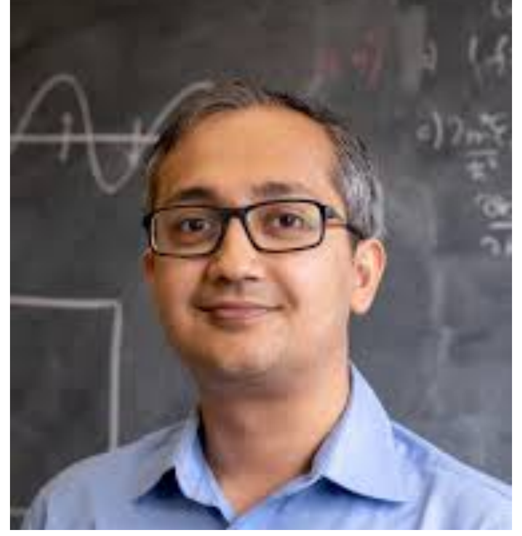
Alternative route to d-wave superconductivity:

Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

FL*→d-SC



Shubhayu Chatterjee and S. S., PRB **94**, 205117 (2016)
Maine Christos and S.S., npj Quantum Materials **9**, 4 (2024)



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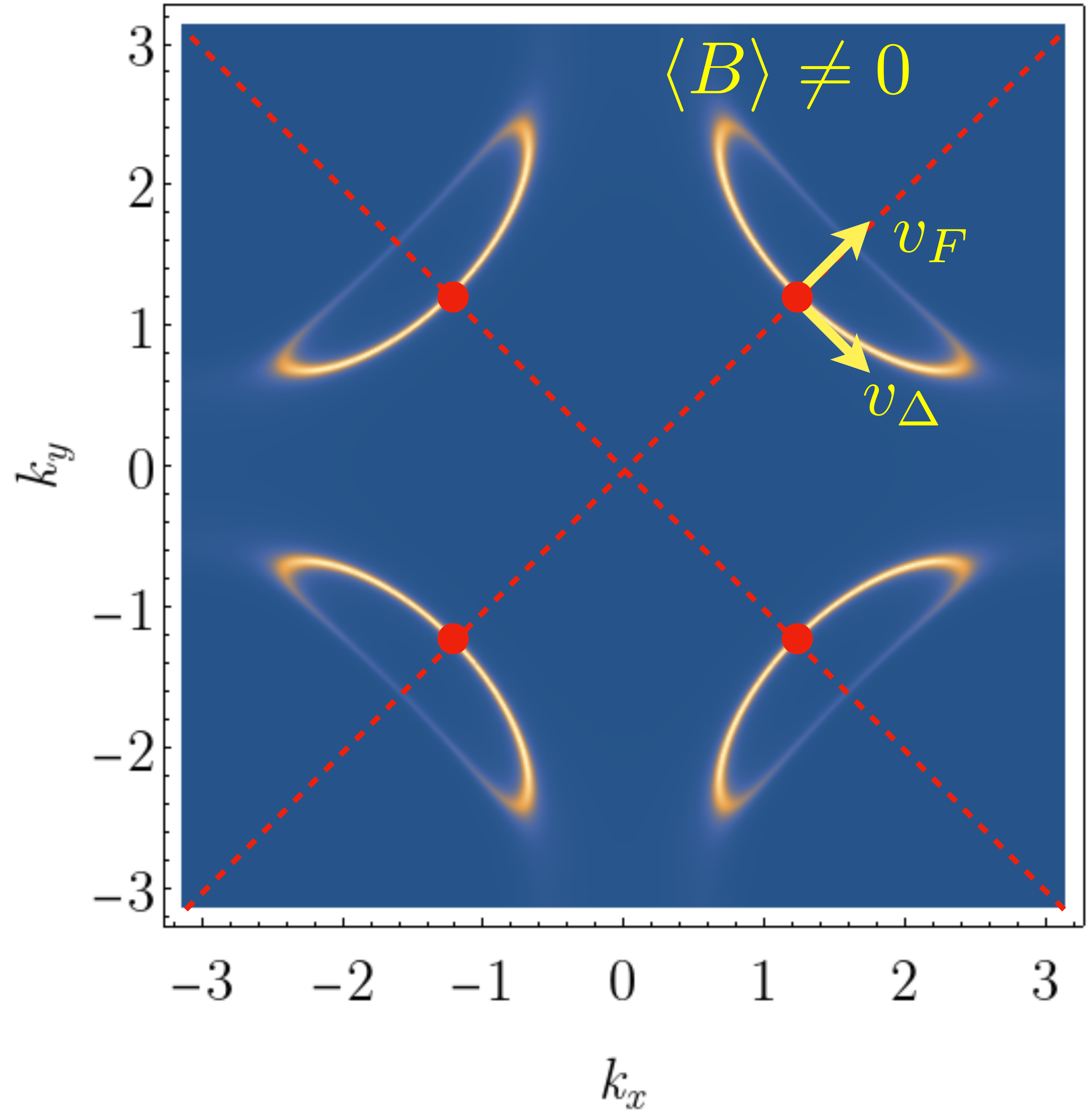
FL*→d-SC



Shubhayu Chatterjee and S. S.,
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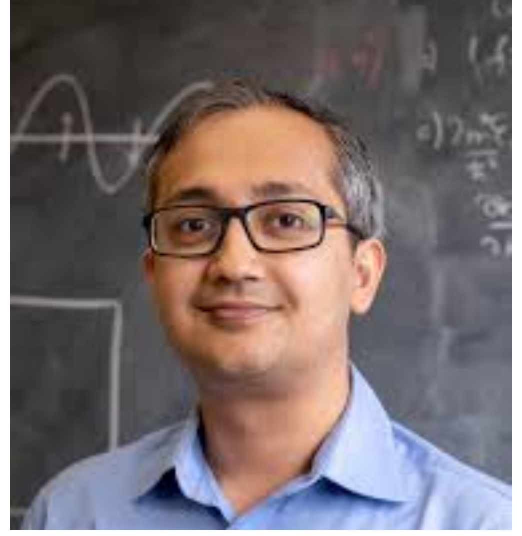
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)



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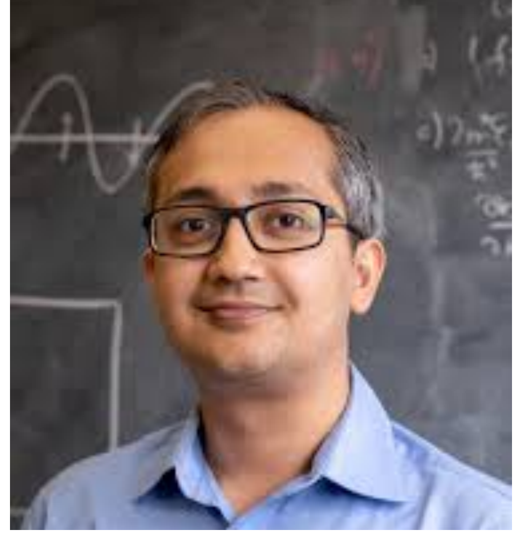
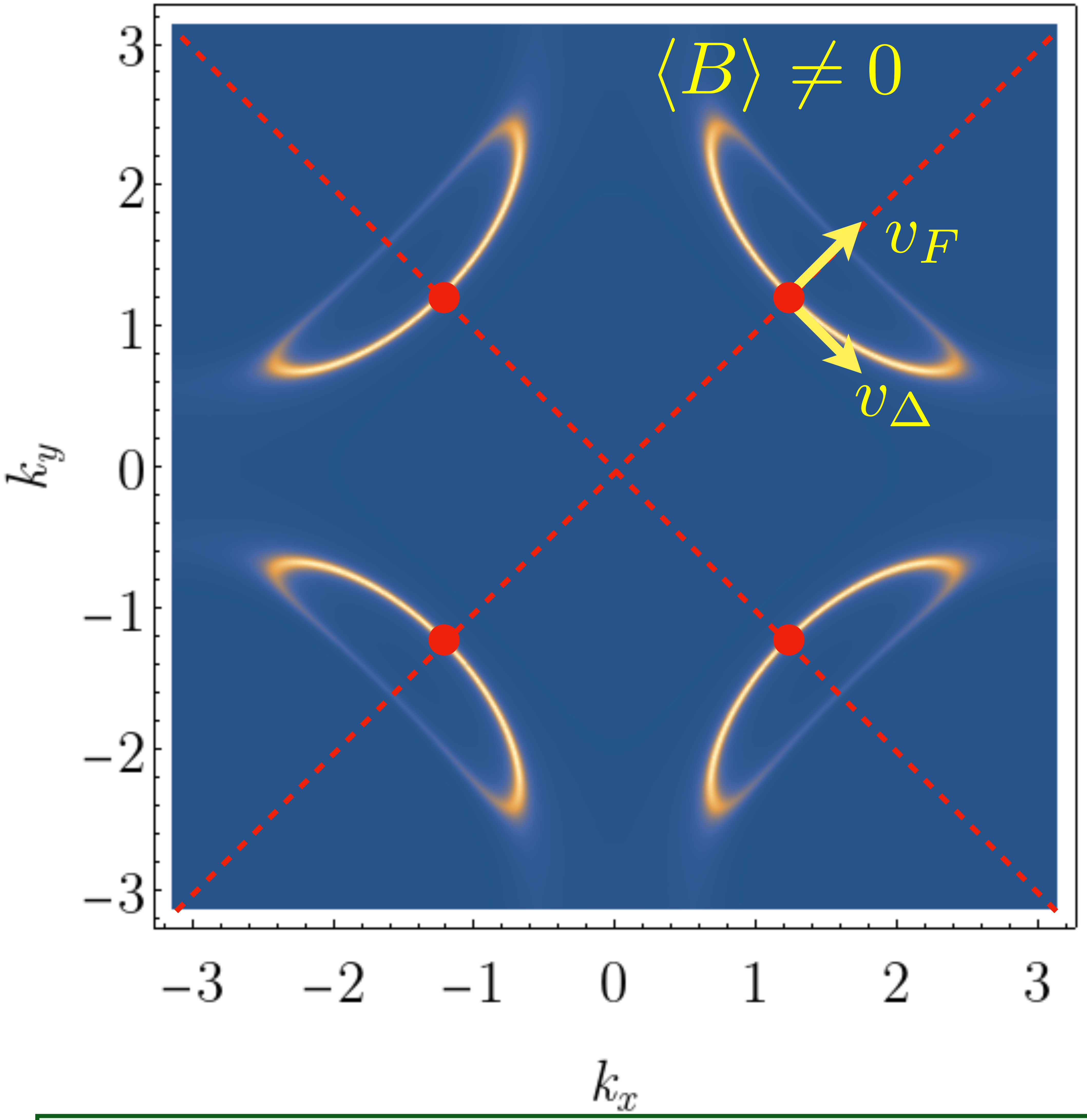
Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

Resolves difficulty in early theories of d-SC from spin liquids (Zhang, Gros,Rice, Shiba (1988); Kotliar, Liu (1988)), in which spinons turned into Bogoliubov quasiparticles with $v_F \approx v_\Delta$.

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)





Alternative route to d-wave superconductivity:

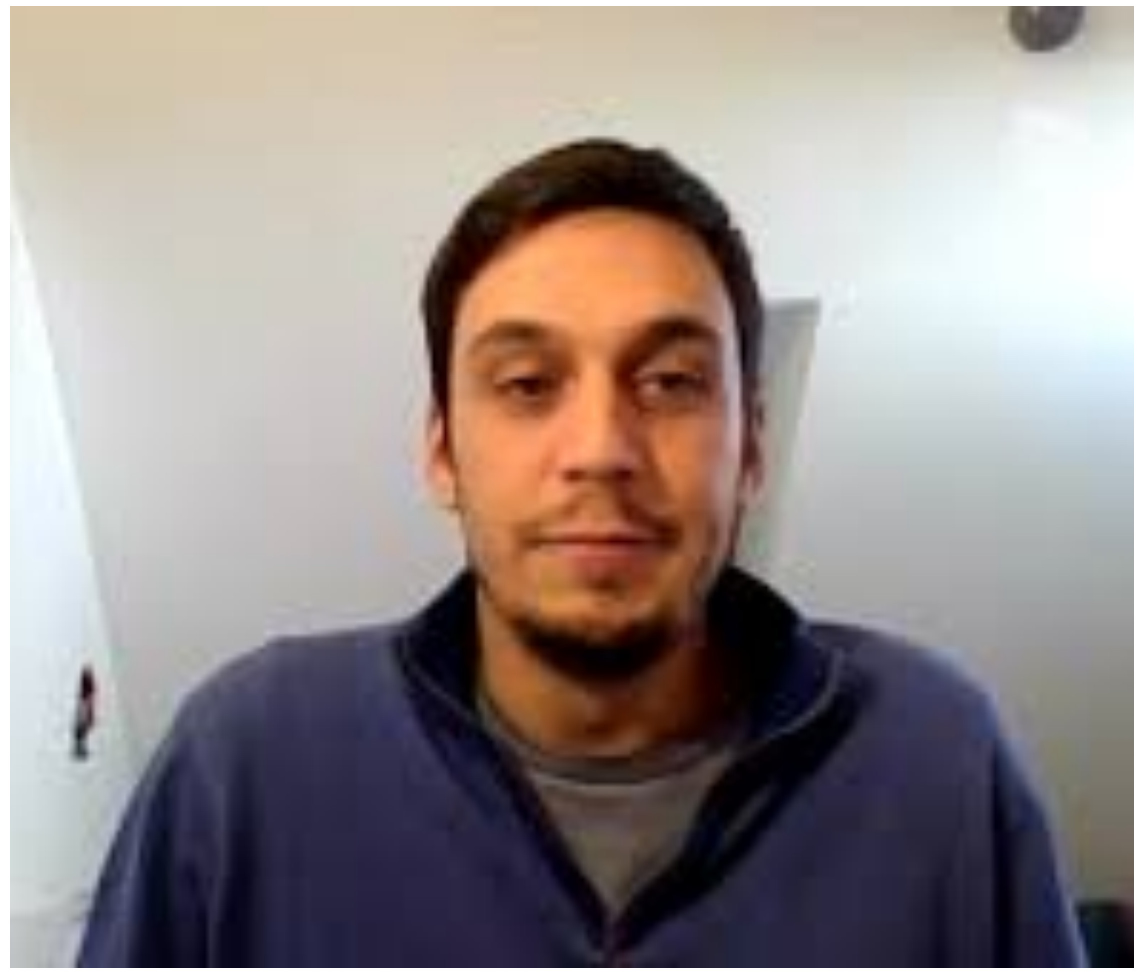
Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

Resolves difficulty in early theories of d-SC from spin liquids (Zhang, Gros,Rice, Shiba (1988); Kotliar, Liu (1988)), in which spinons turned into Bogoliubov quasiparticles with $v_F \approx v_\Delta$.

Non-BCS mechanism applied to pseudogap leads to BCS superconductor!

Intertwined orders and hole pockets

Maine Christos
Caltech



Pietro Bonetti

Thermal SU(2) lattice gauge theory of intertwined orders and and hole pockets

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

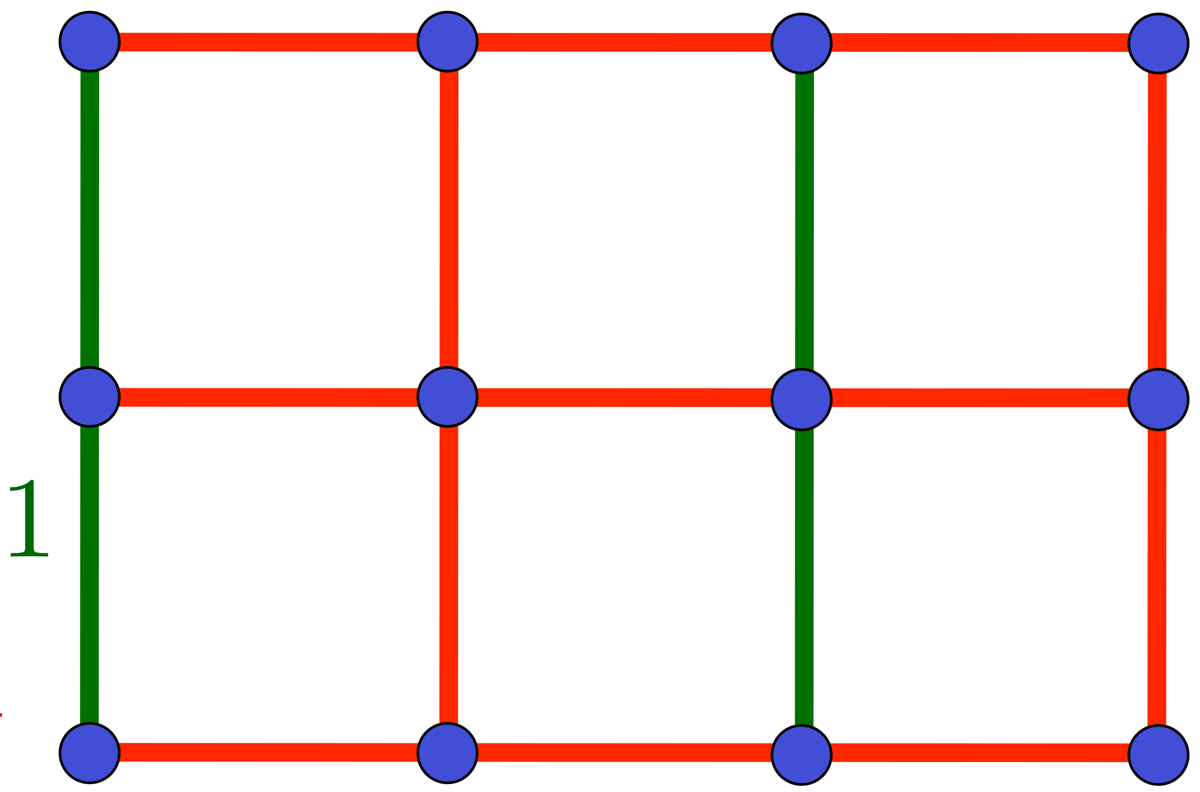
Harshit Pandey

Ravi Shanker

Sayantan Sharma

The Institute of Mathematical Sciences, Chennai

f_α and B both move in π-flux

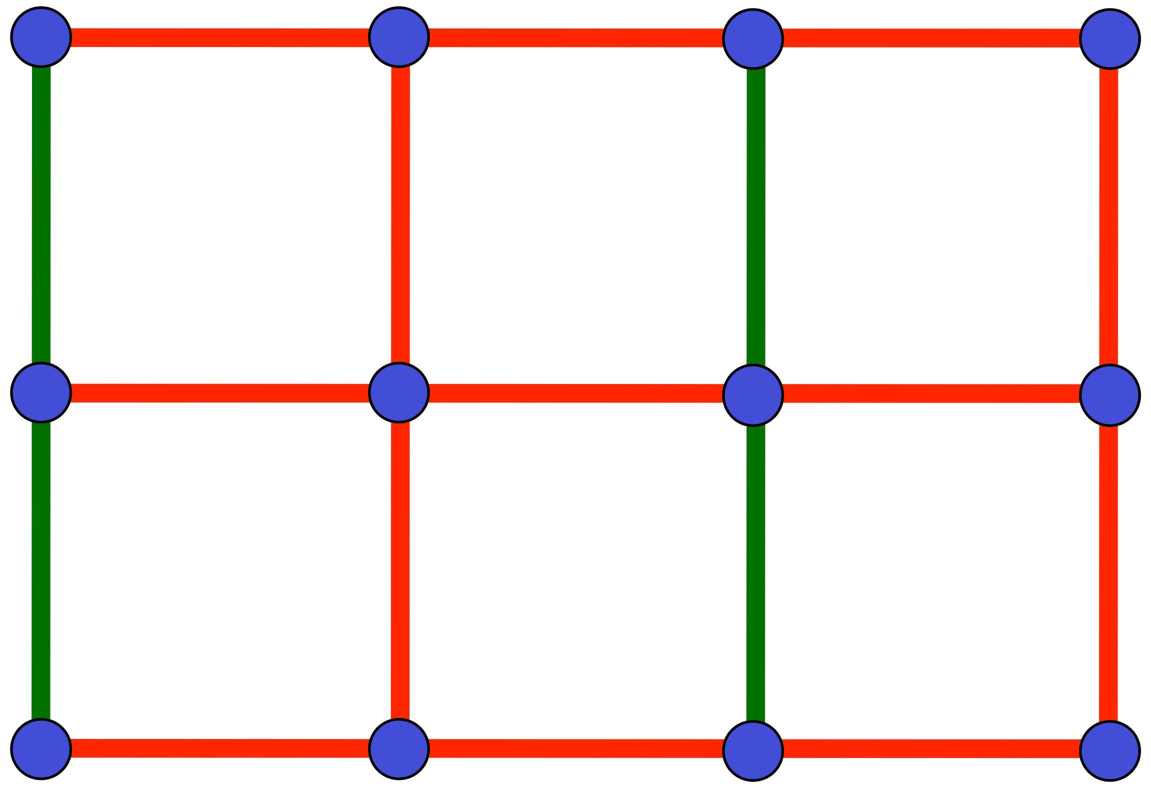


Symmetry	f_α	B_a
T_x	$(-1)^y f_\alpha$	$(-1)^y B_a$
T_y	f_α	B_a
P_x	$(-1)^x f_\alpha$	$(-1)^x B_a$
P_y	$(-1)^y f_\alpha$	$(-1)^y B_a$
P_{xy}	$(-1)^{xy} f_\alpha$	$(-1)^{xy} B_a$
$\mathcal{T}$	$(-1)^{x+y} \varepsilon_{\alpha\beta} f_\beta$	$(-1)^{x+y} B_a$

Projective transformations of the f spinons and B chargons on lattice sites $\boldsymbol{i} = (x, y)$ under the symmetries $T_x : (x, y) \to (x+1, y)$; $T_y : (x, y) \to (x, y+1)$; $P_x : (x, y) \to (-x, y)$; $P_y : (x, y) \to (x, -y)$; $P_{xy} : (x, y) \to (y, x)$; and time-reversal $\mathcal{T}$. The indices α, β refer to global SU(2) spin, while the index $a = 1, 2$ refers to gauge SU(2).

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

f_α and B both move in π-flux



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Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim$

$$\Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$$

site charge density: $\left\langle c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B^\dagger_{\boldsymbol{i}} B_{\boldsymbol{i}}$

bond density: $\left\langle c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + c^\dagger_{\boldsymbol{j}\alpha} c_{\boldsymbol{i}\alpha} \right\rangle$

$$\sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B^\dagger_{\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$$

bond current: $i \left\langle c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - c^\dagger_{\boldsymbol{j}\alpha} c_{\boldsymbol{i}\alpha} \right\rangle$

$$\sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B^\dagger_{\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Energy functional for B and U: $\mathcal{E}[B,U] = \mathcal{E}_2[B,U] + \mathcal{E}_4[B,U] + \mathcal{E}_{YM}[U]$

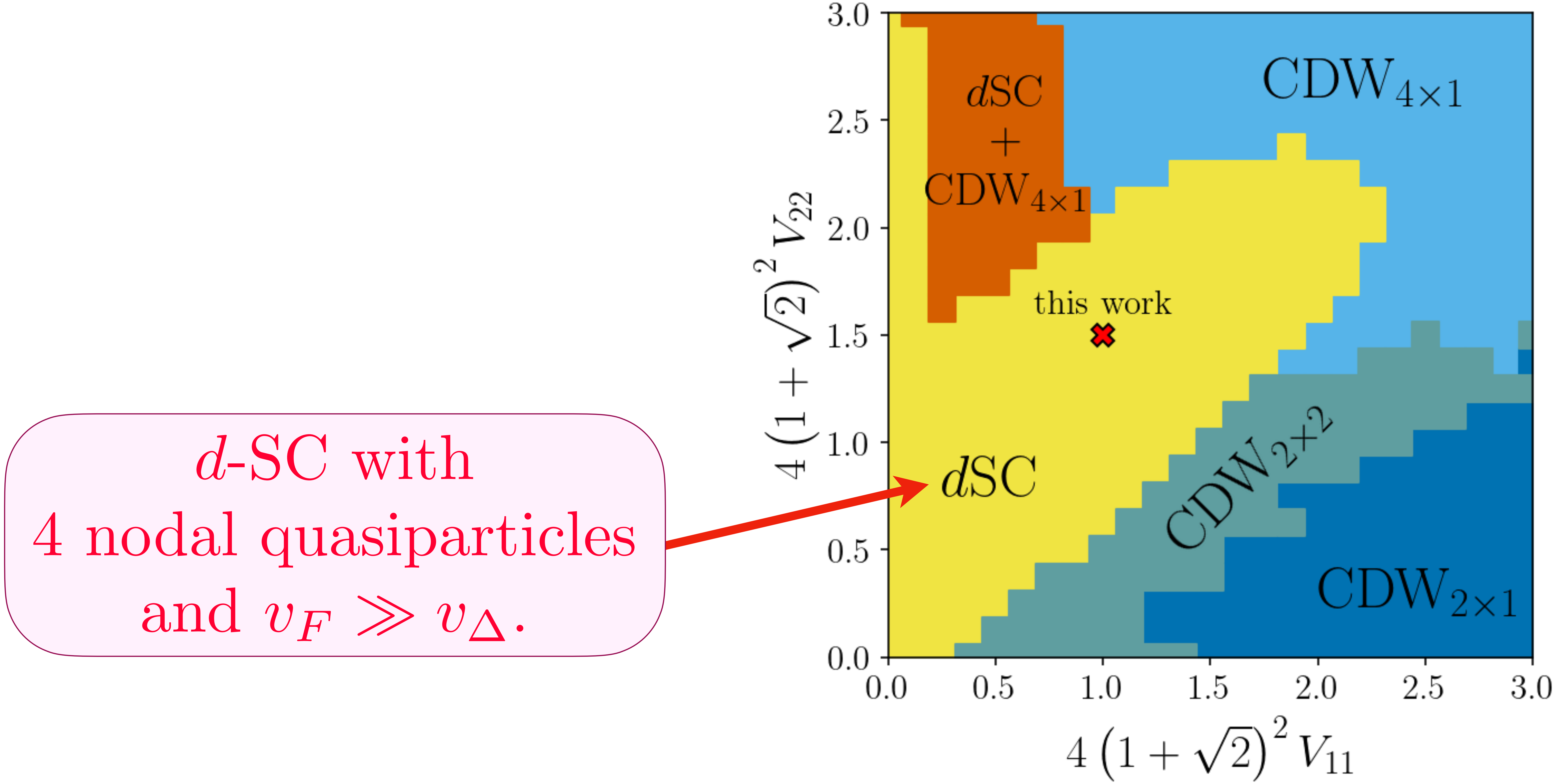
$$\mathcal{E}_2[B,U] = (r + 2\sqrt{2}w) \sum_{\boldsymbol{i}} B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}} + iw \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(B_{\boldsymbol{i}}^\dagger U_{\boldsymbol{ij}} B_{\boldsymbol{j}} - B_{\boldsymbol{j}}^\dagger U_{\boldsymbol{ji}} B_{\boldsymbol{i}} \right)$$

$$\mathcal{E}_4[B,U] = \frac{u}{2} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}} \right) + g \sum_{\langle \boldsymbol{ij} \rangle} |\Delta_{\boldsymbol{ij}}|^2 + J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2$$
$$+ V_{11} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}} \right) + V_{22} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}+2\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}-2\hat{\boldsymbol{y}}} \right)$$

$$\mathcal{E}_{YM}[U] = \kappa \sum_{\square} \left[1 - \frac{1}{2} \mathrm{ReTr} \prod_{\boldsymbol{ij} \in \square} U_{\boldsymbol{ij}} \right]$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

At $T = 0$, minimize $\mathcal{E}[B, U]$.



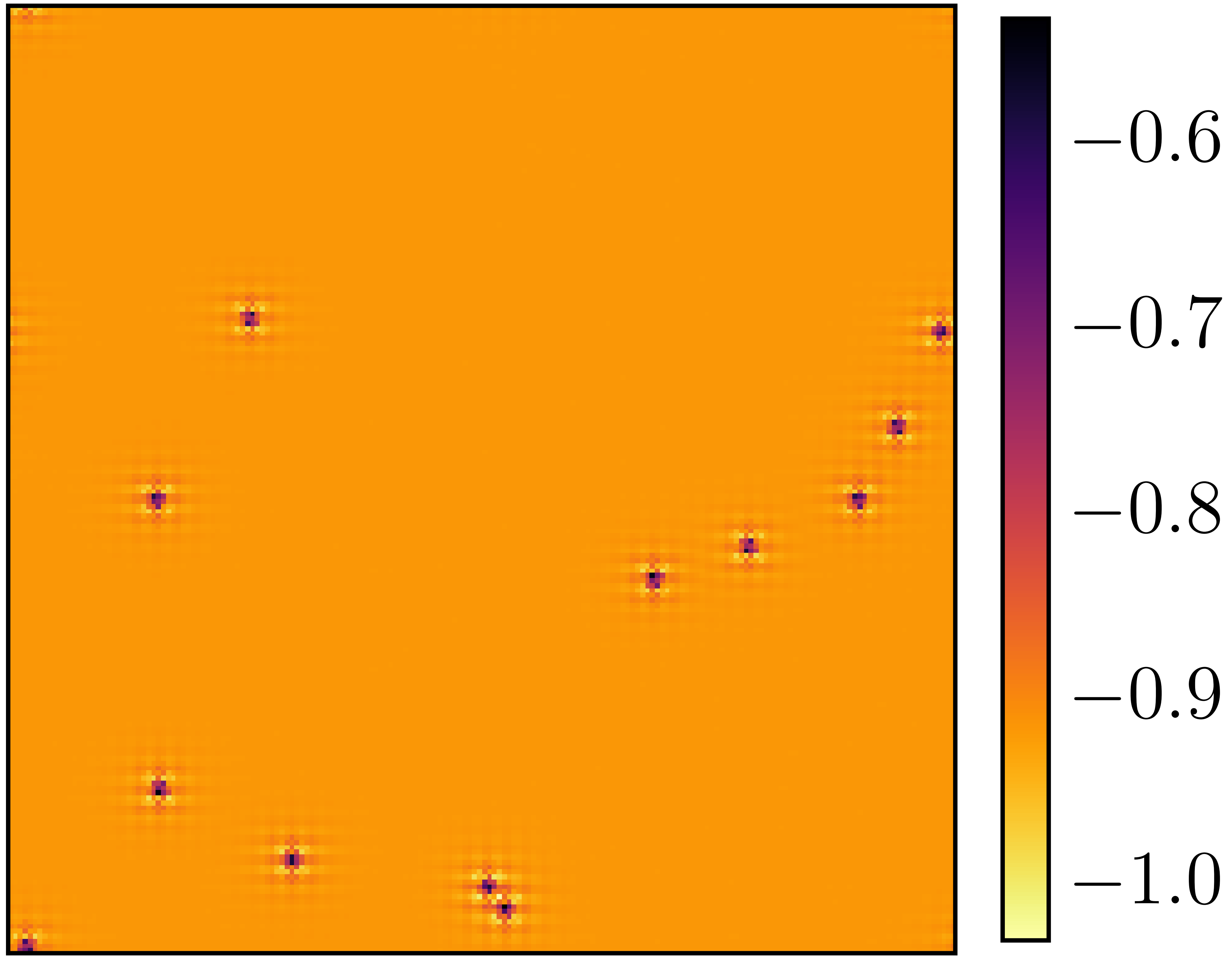
Parameters chosen so that the ground state is a d-wave superconductor, and second best state is a period-4 stripe.

Monte Carlo at a temperature T

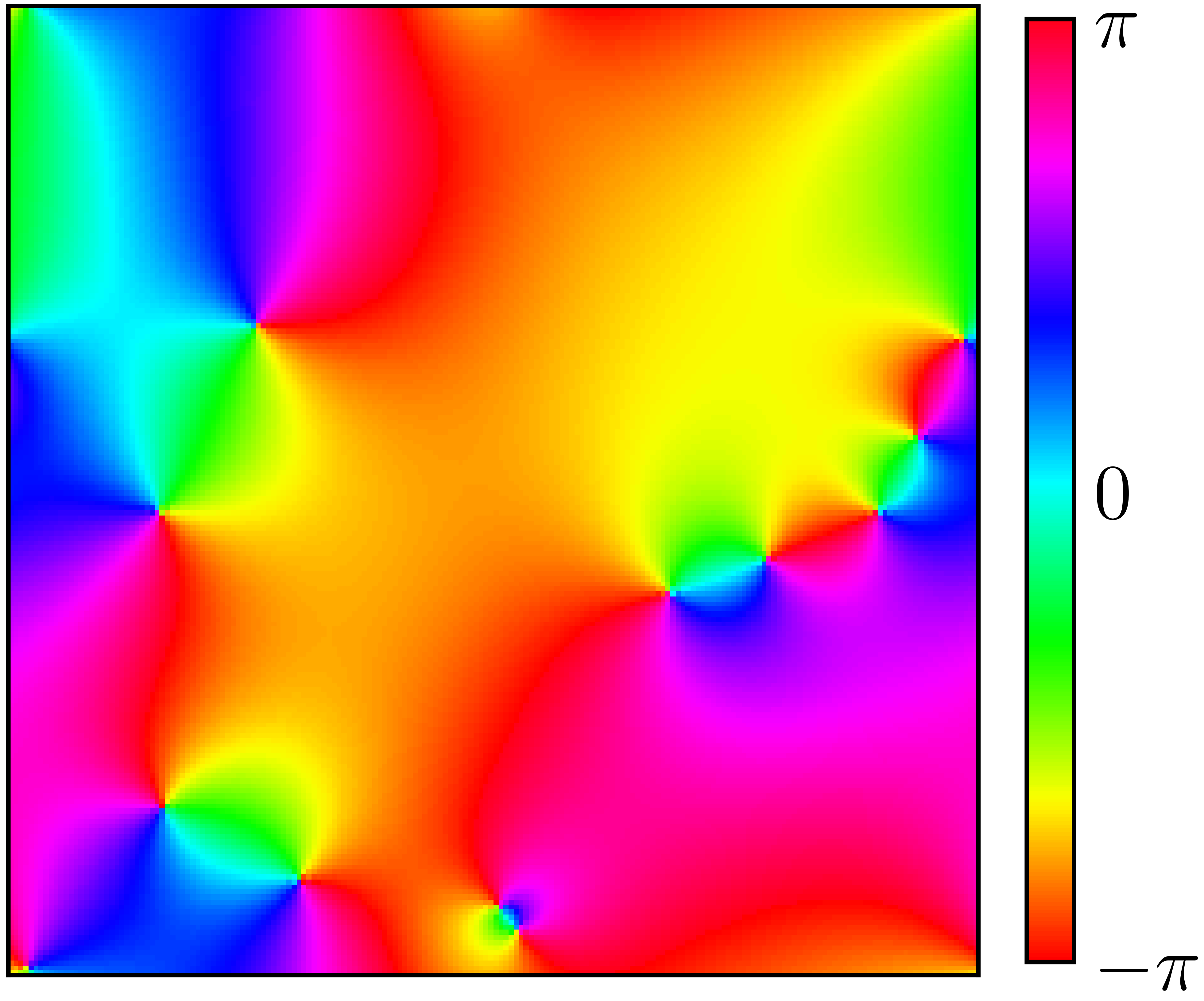
$$\mathcal{Z}_{2+0} = \int \prod_i \mathcal{D}B_{\boldsymbol{i}} \int \prod_{\langle \boldsymbol{ij} \rangle} \mathcal{D}U_{\boldsymbol{ij}} \exp\left[-\mathcal{E}[B,U]/T\right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$
- Diagonalize 3-layer fermion Hamiltonian for c, f_1, f for each snapshot of B, U, and average.

Monte Carlo at a temperature T

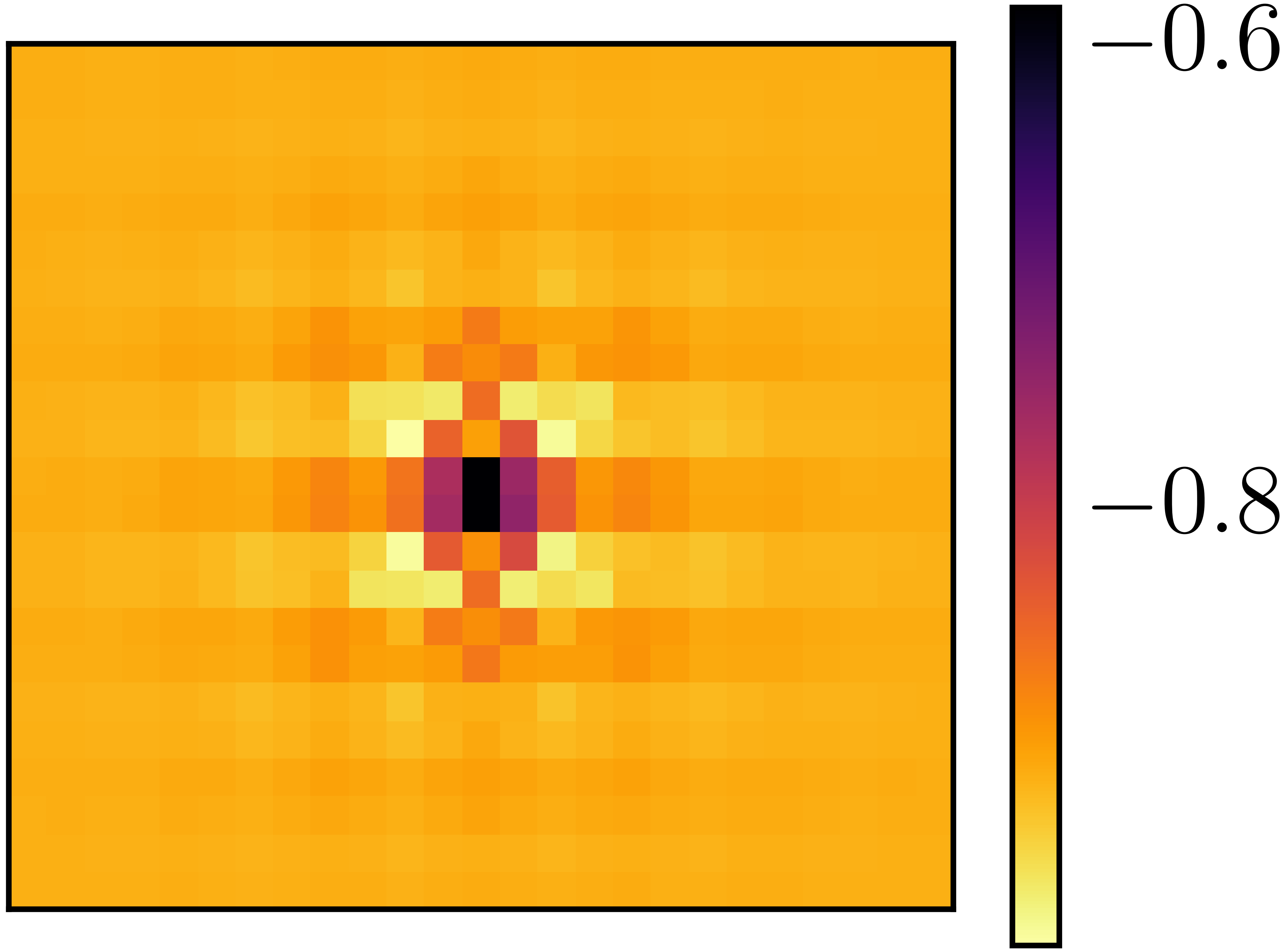


Bond density

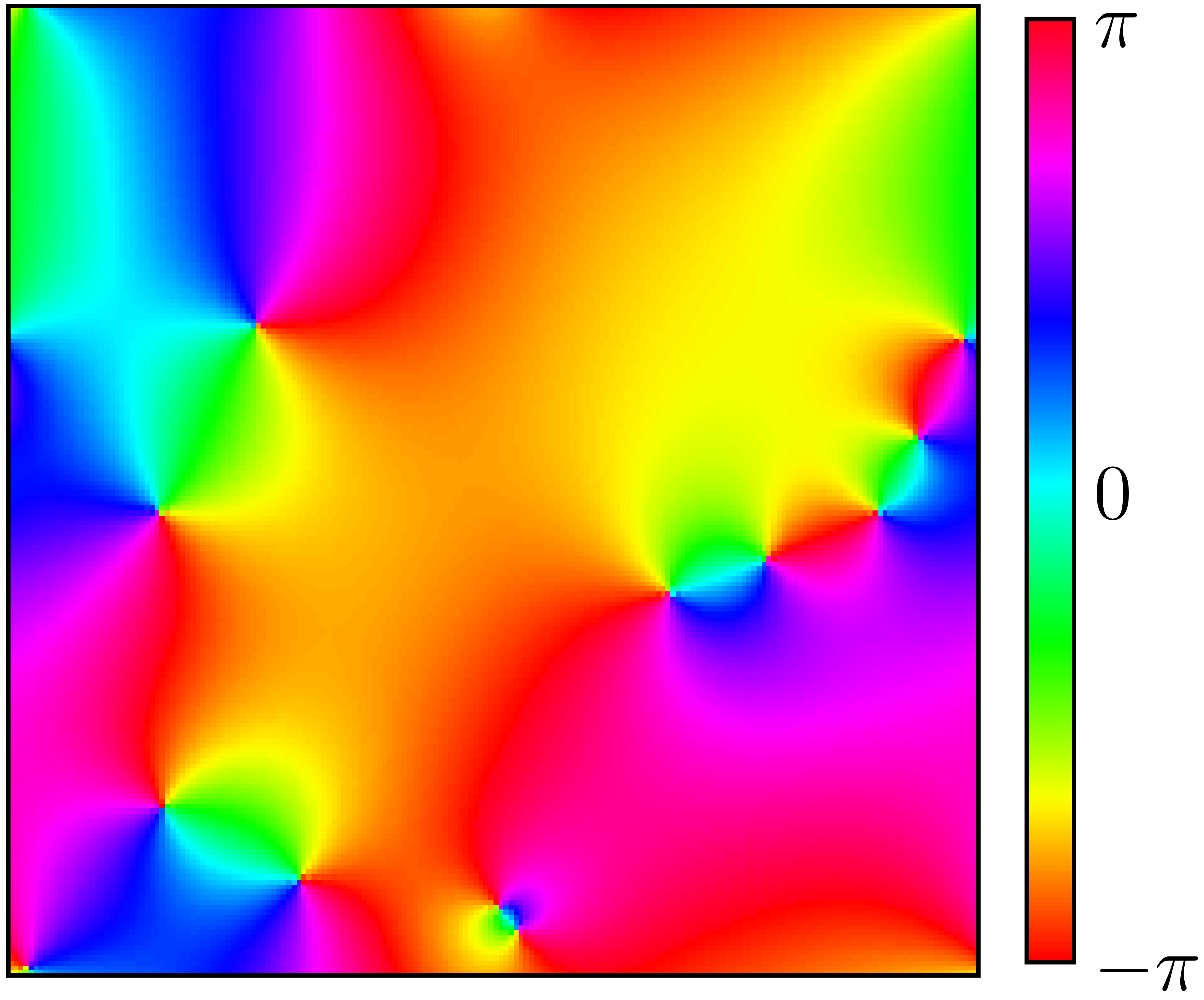


Phase of pairing amplitude

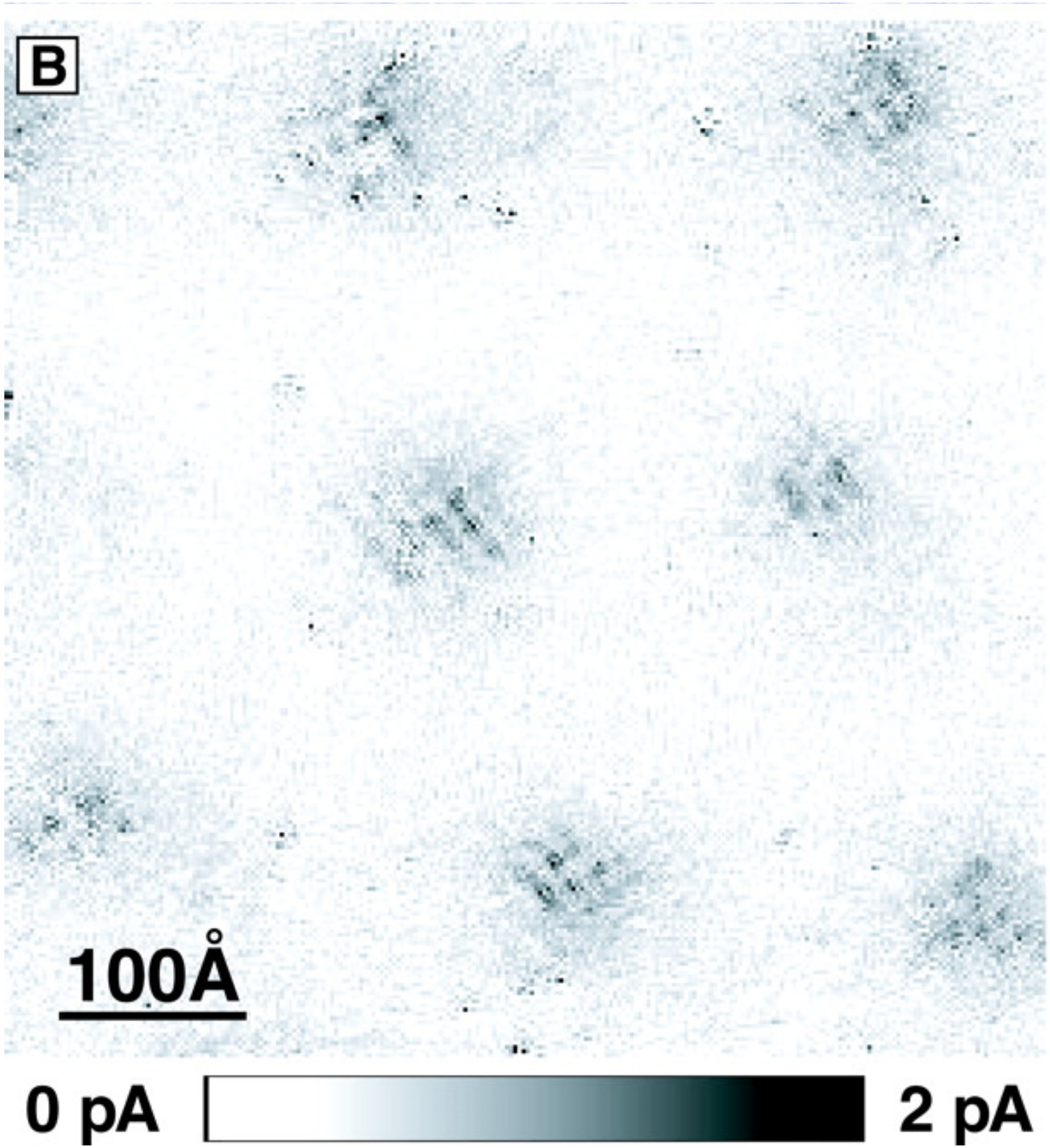
Monte Carlo at a temperature T



Bond density



Phase of pairing amplitude

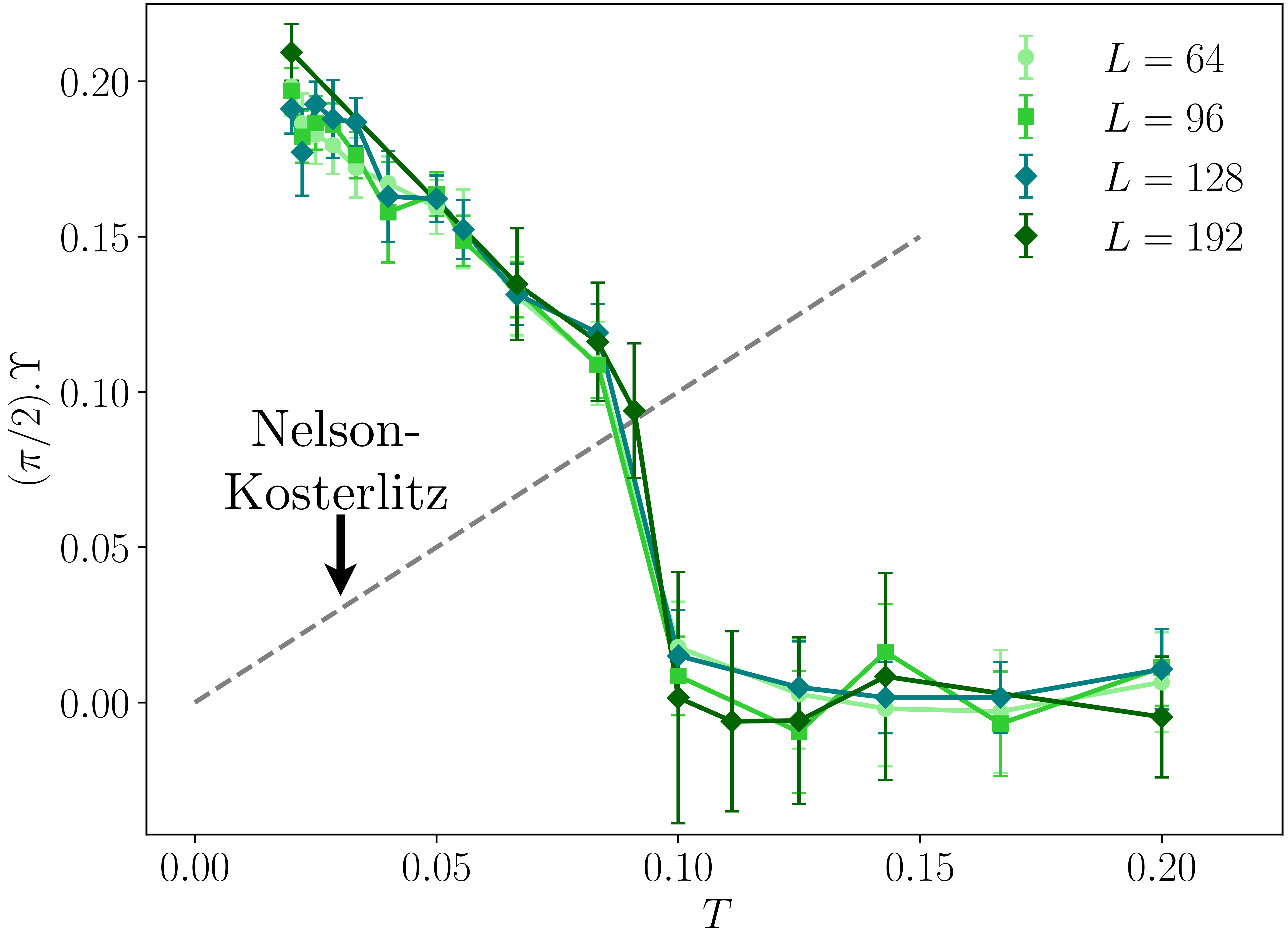


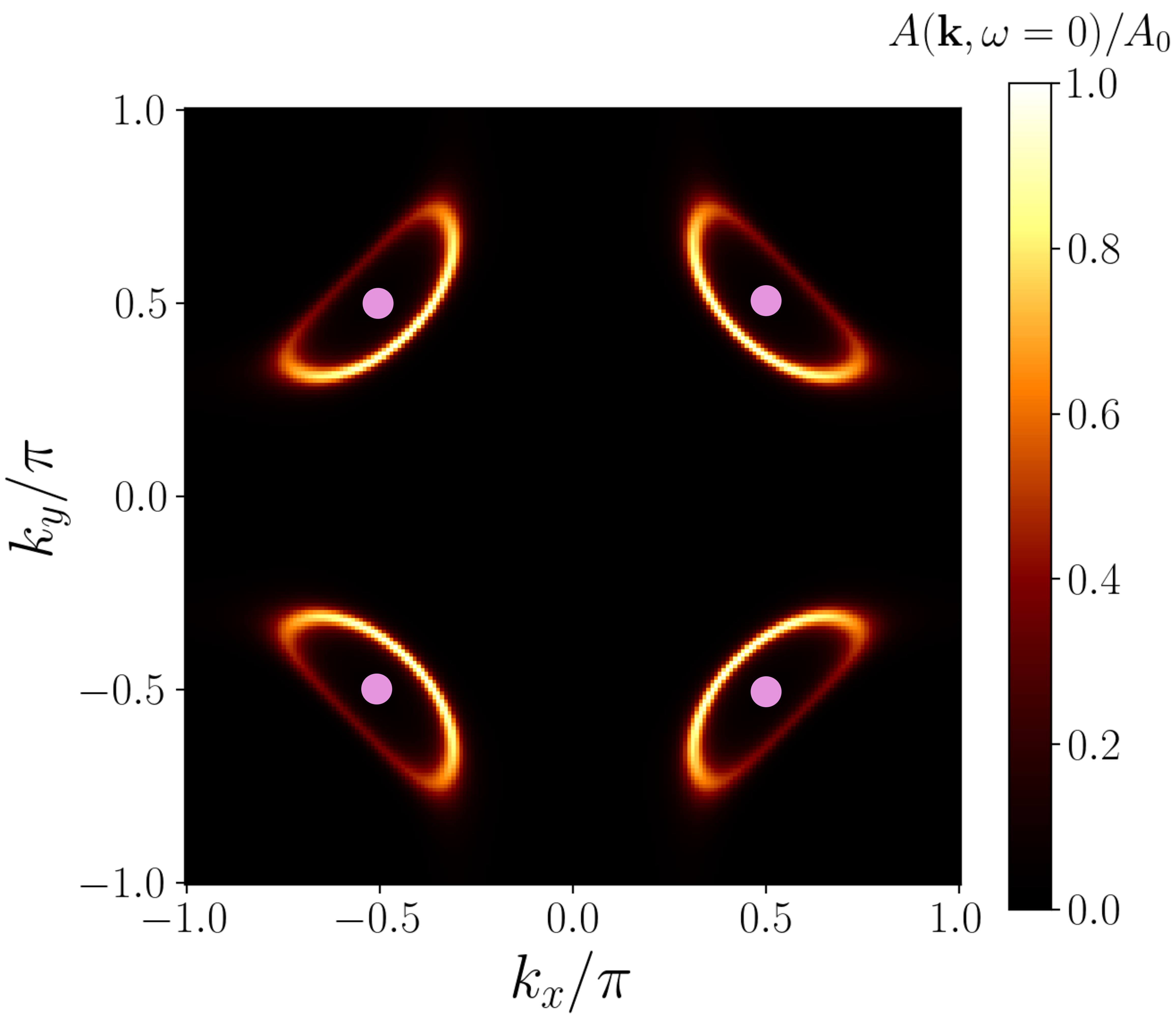
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis
Science **295**, 466 (2002)

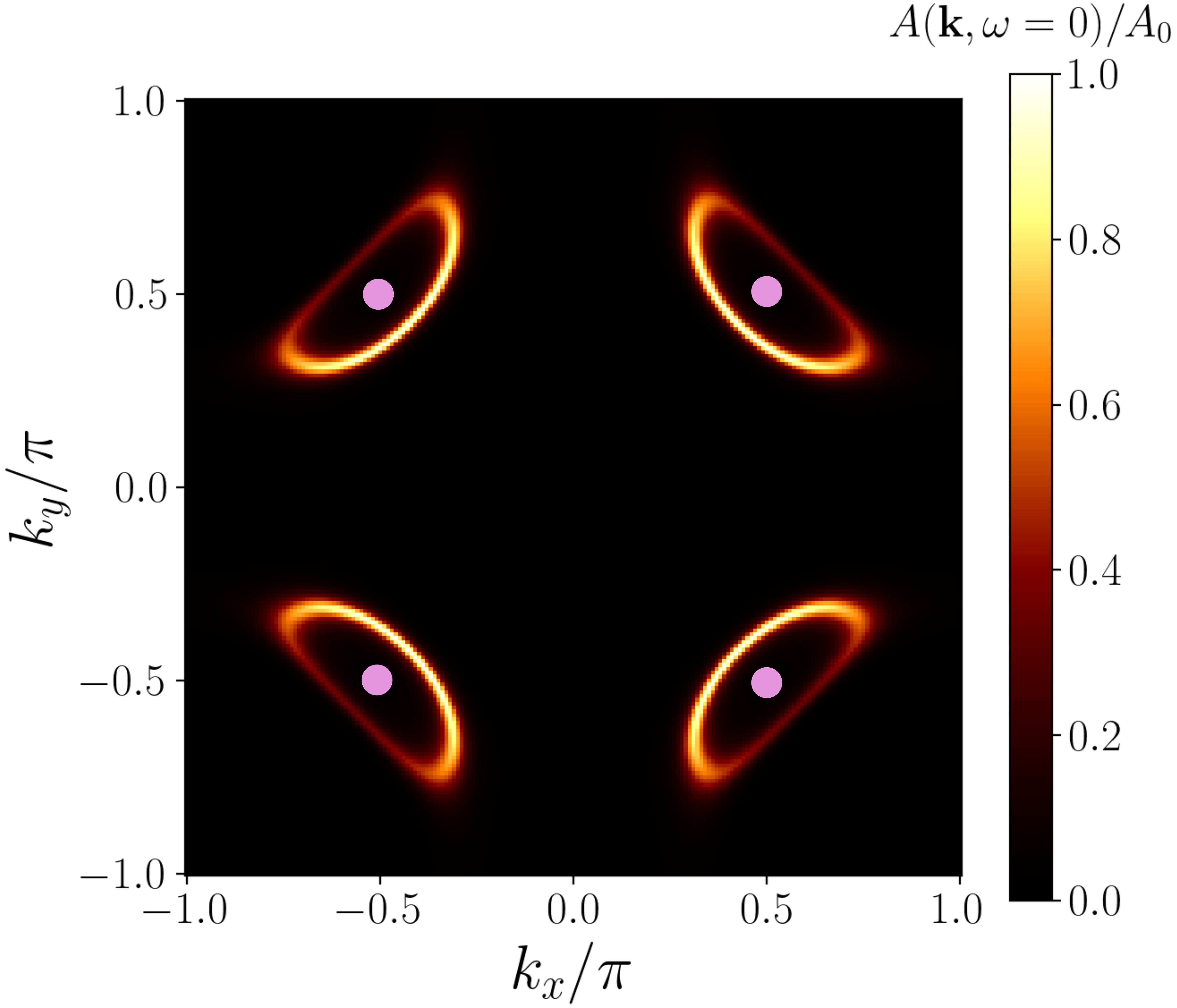
Monte Carlo at a temperature T

$\Upsilon =$
Helicity
Modulus





FL* fermionic spectrum with $B = 0$, $U = 1$
4 holes pockets of size $p/8$;
4 nodal spinons



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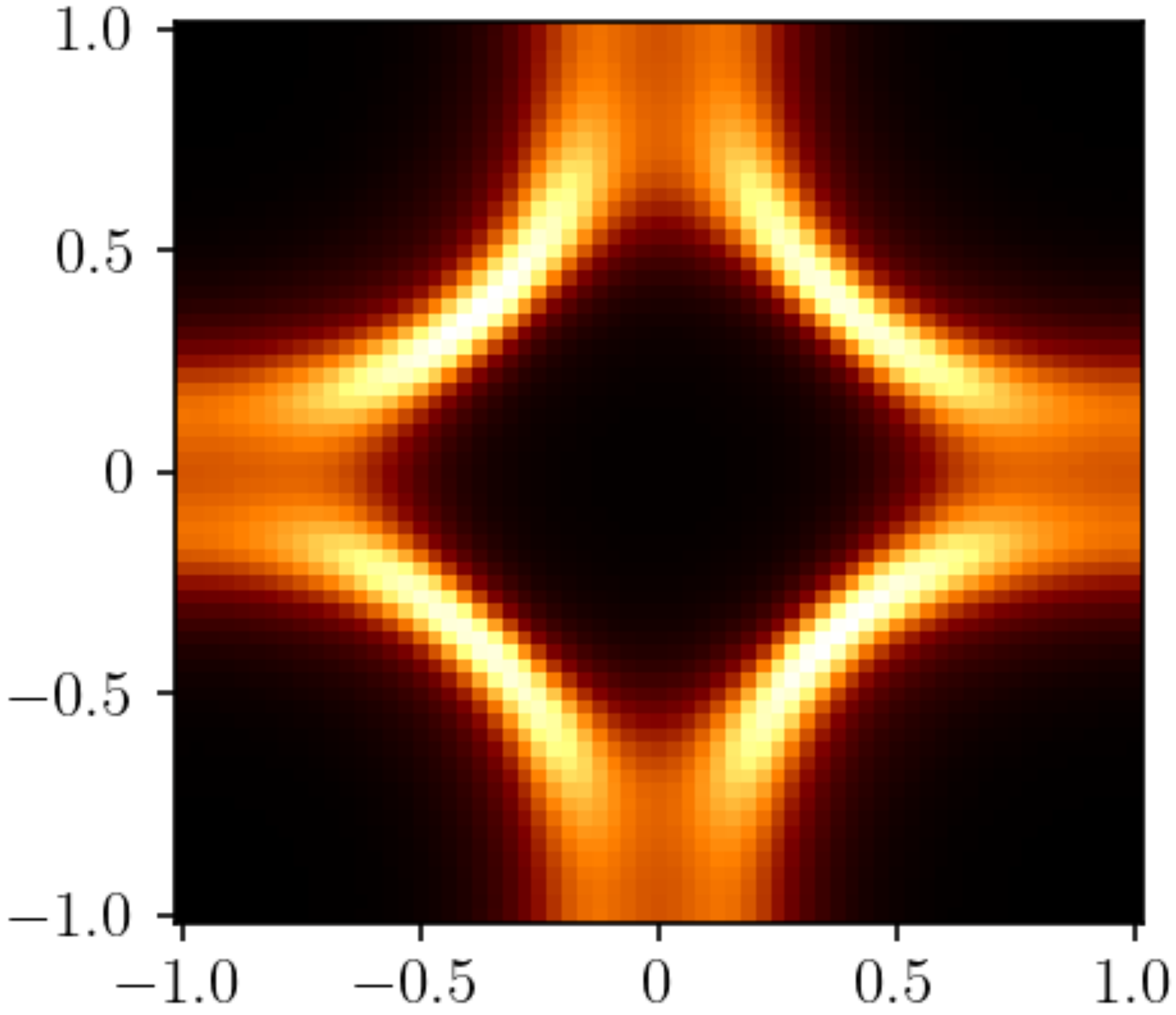
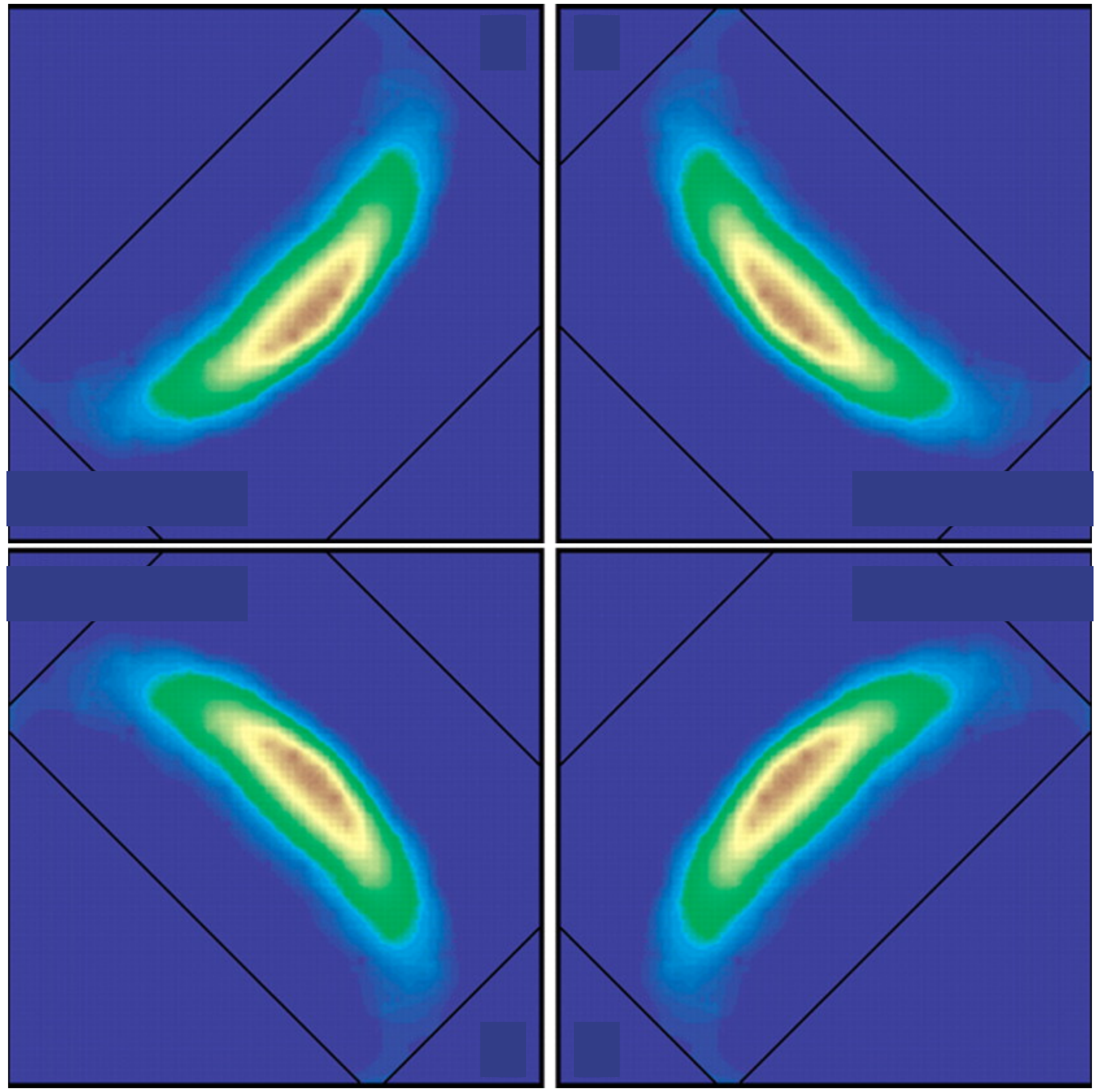
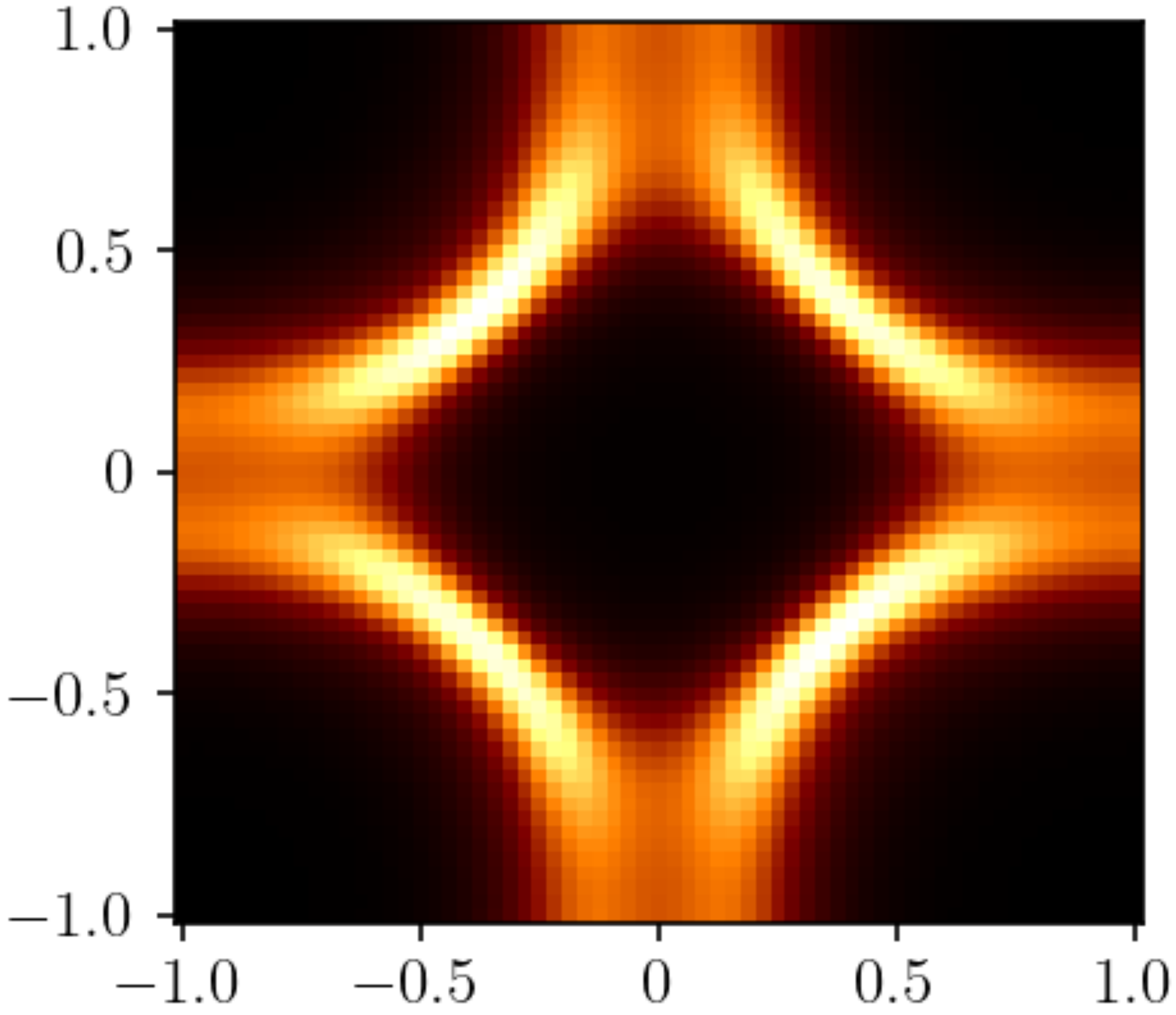


Monte Carlo at a temperature $T > T_{KT}$



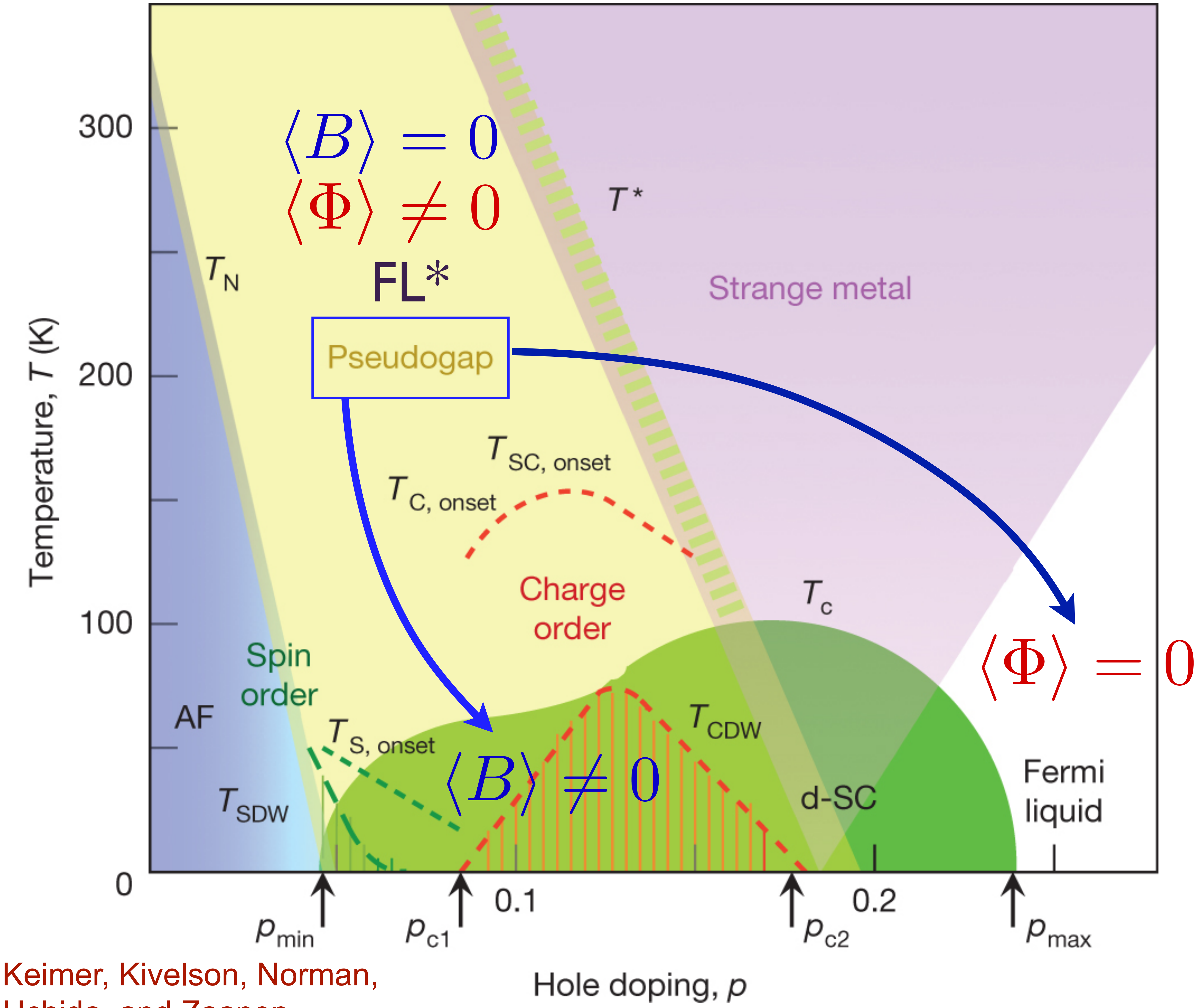
Kyle M. Shen, . . . Z.-X. Shen, Science **307**, 901 (2005)

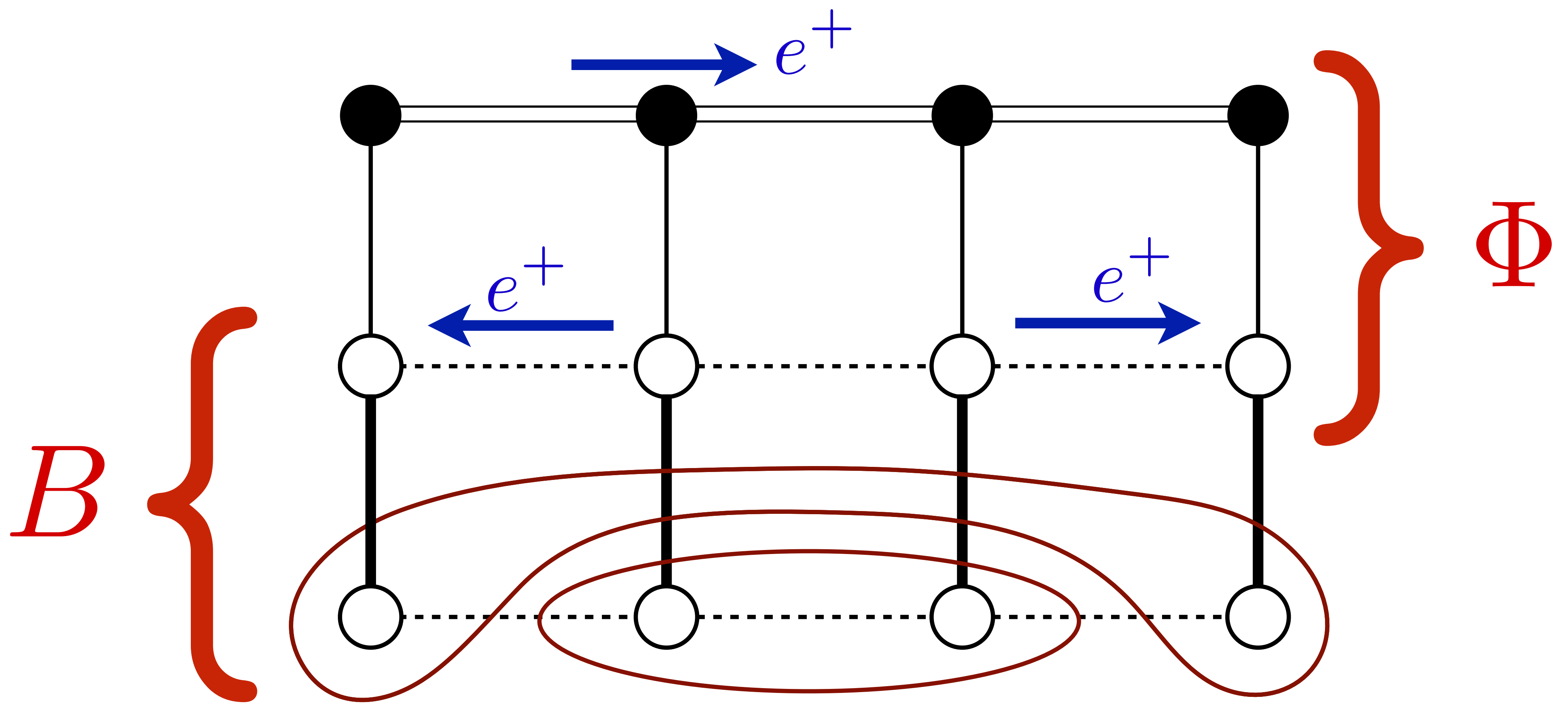
Photoemission observations



Monte Carlo at a temperature $T > T_{KT}$

From the FL* pseudogap to d-SC and FL





Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

- *Lectures on insulating and conducting quantum spin liquids*, S. Sachdev, arXiv:2512.23962
- *Fractionalized Fermi liquids and the cuprate phase diagram*, P. M. Bonetti, M. Christos, A. Nikolaenko, A.A. Patel, and S. Sachdev, arXiv:2508.20164
- *Thermal SU(2) lattice gauge theory of the cuprate pseudogap*, H. Pandey, M. Christos, P. M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, and S. Sachdev, arXiv:2507.05336