Strange metals and black holes

Northwestern University
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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
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1. Quantum entanglement
2. Quantum matter *with* quasiparticles
3. Quantum matter *without* quasiparticles
4. Black holes
5. The SYK model and black holes

1. Quantum entanglement

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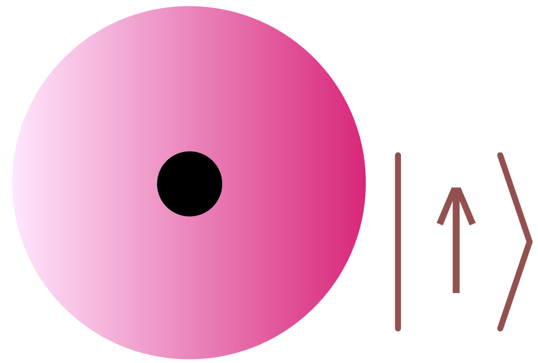
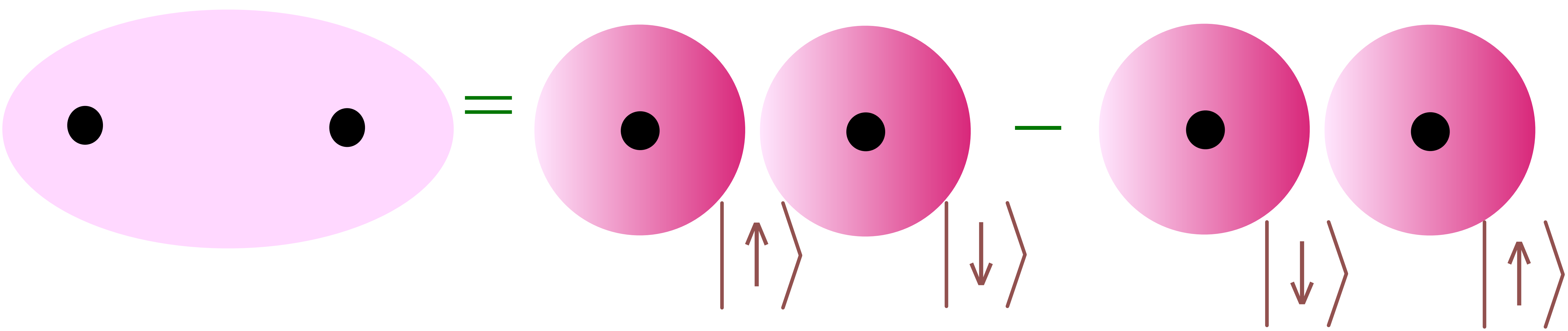
<u>Quantum Entanglement</u>: quantum superposition with more than one particle

Hydrogen atom:

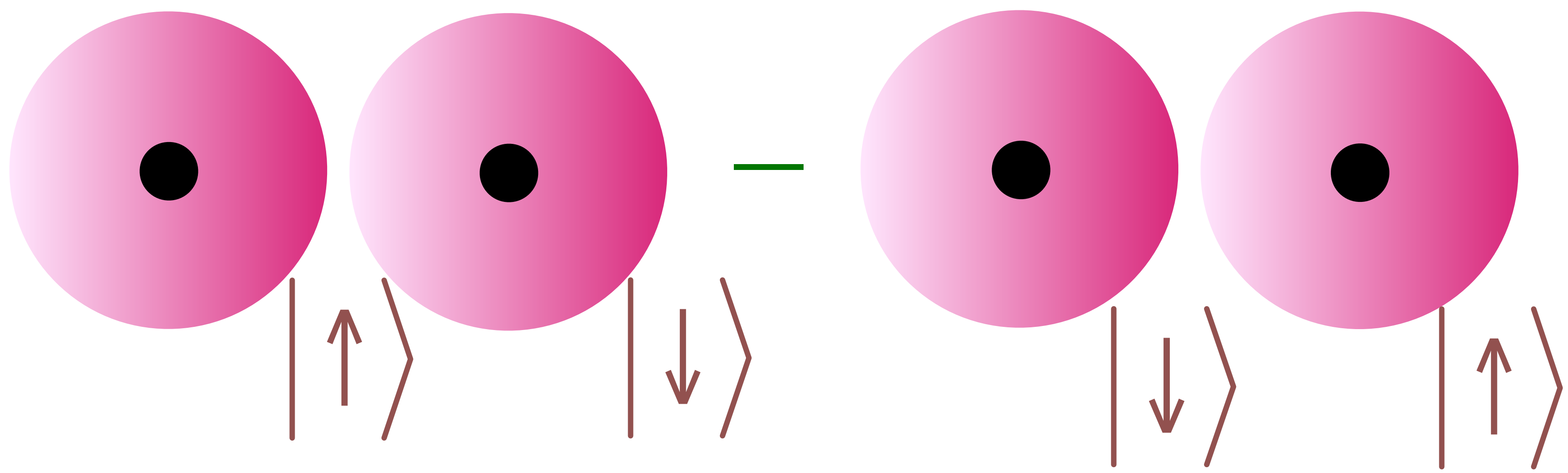


Hydrogen molecule:

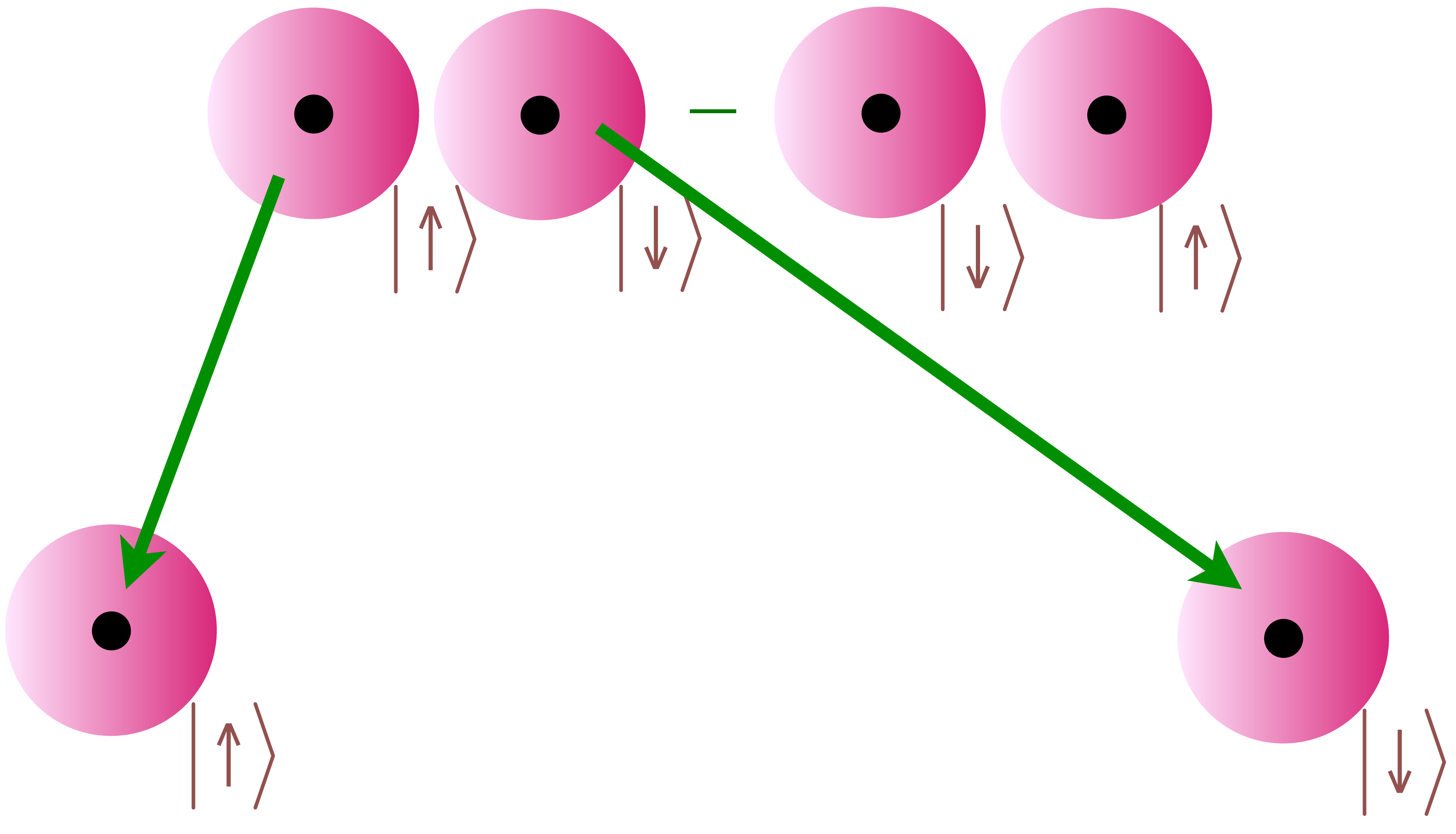


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



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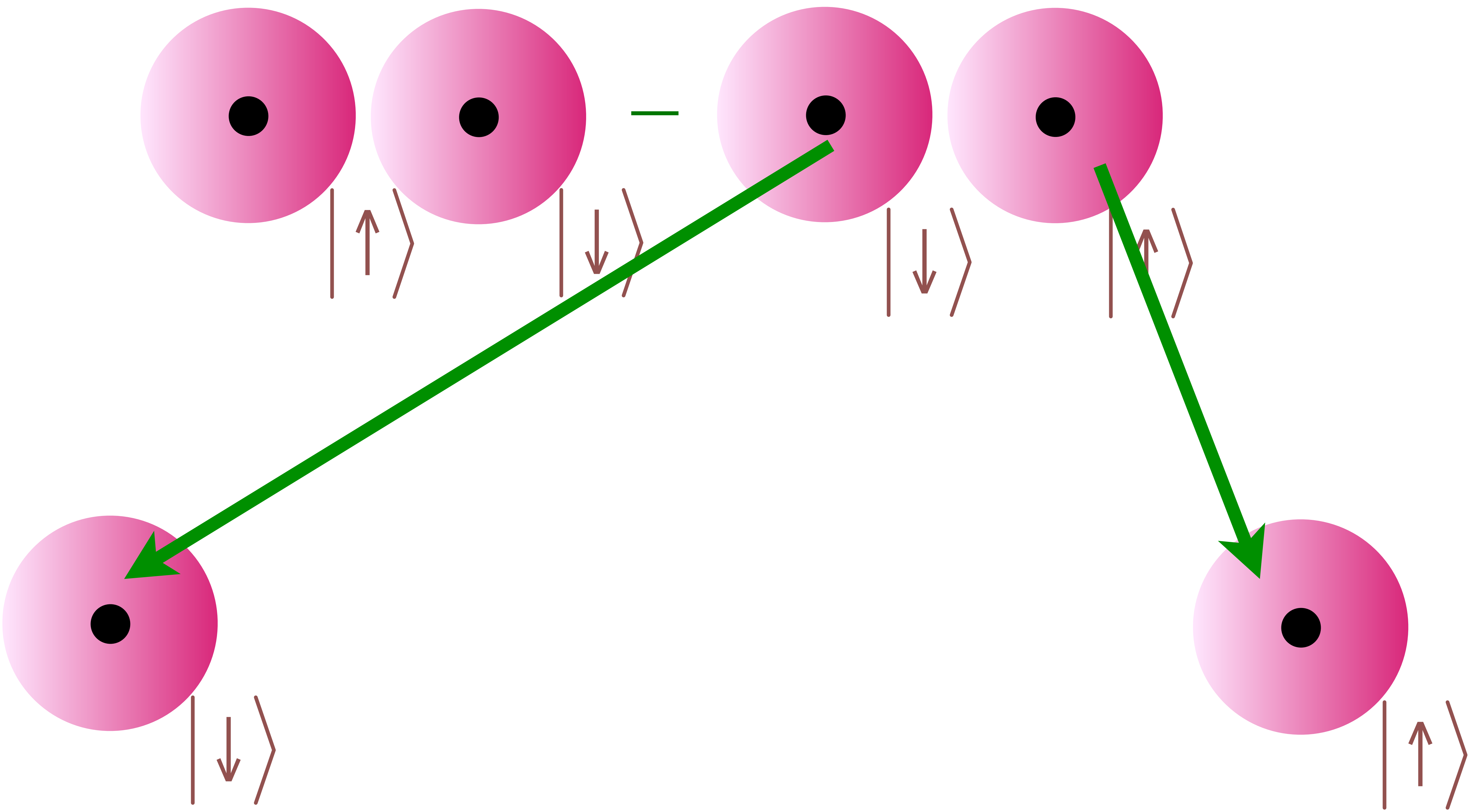
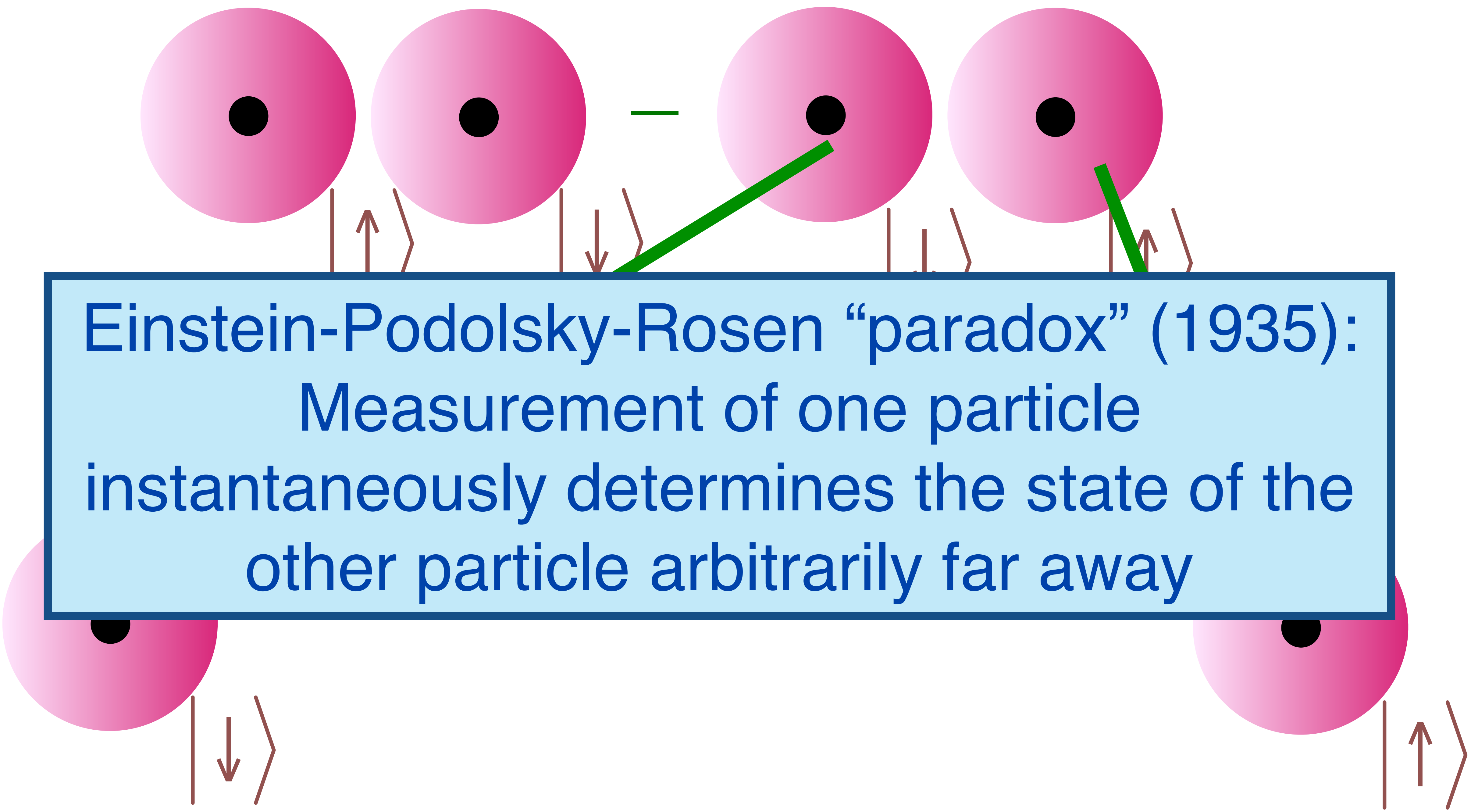


Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen “paradox” (1935):
Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

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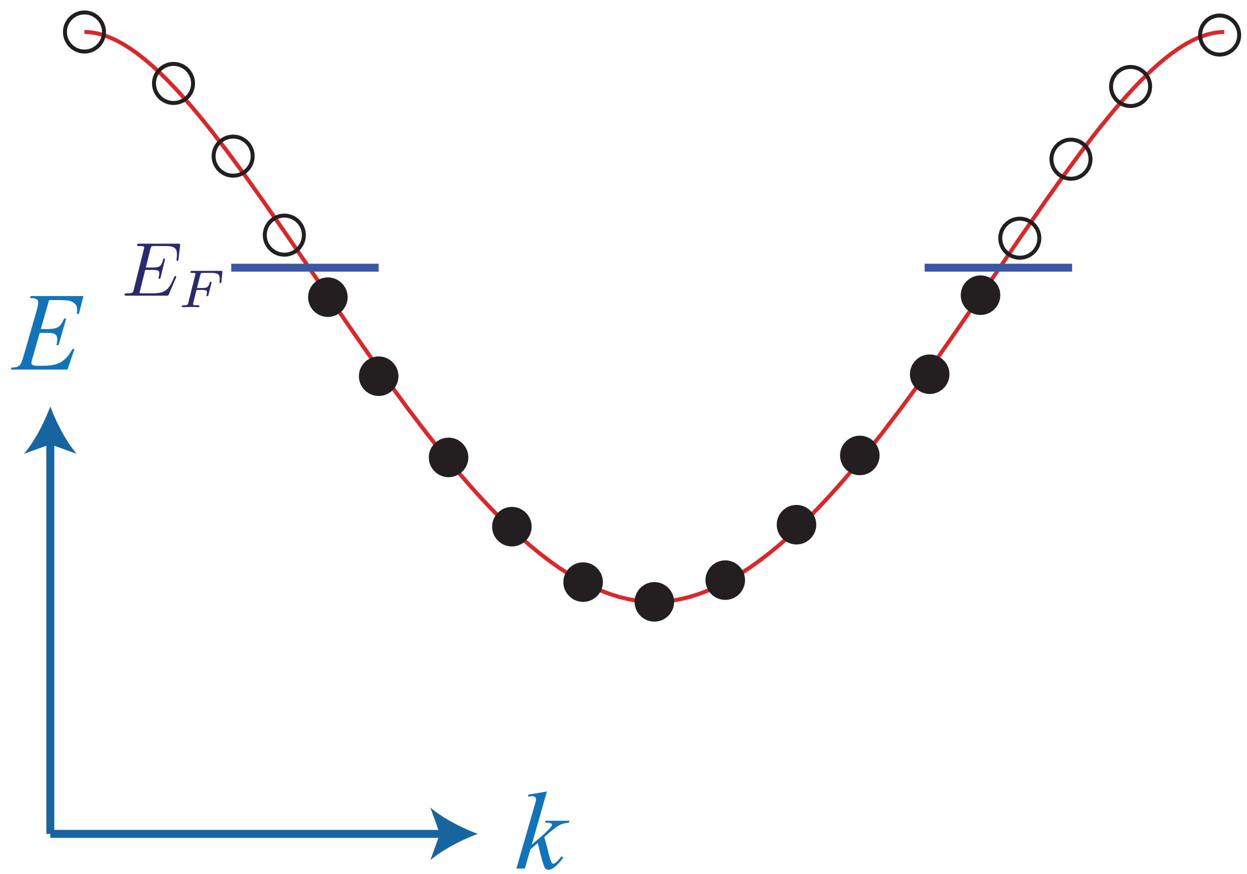
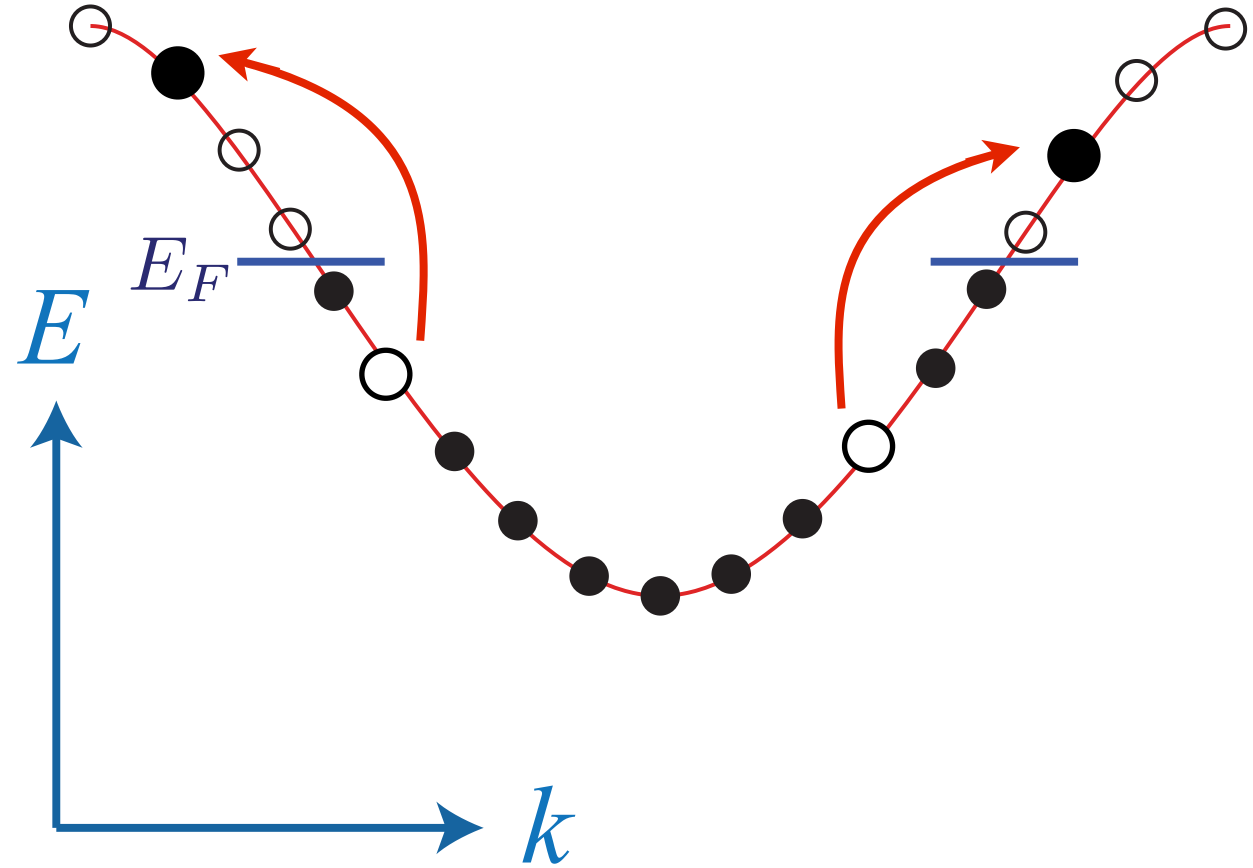
Theory of (ordinary) metals

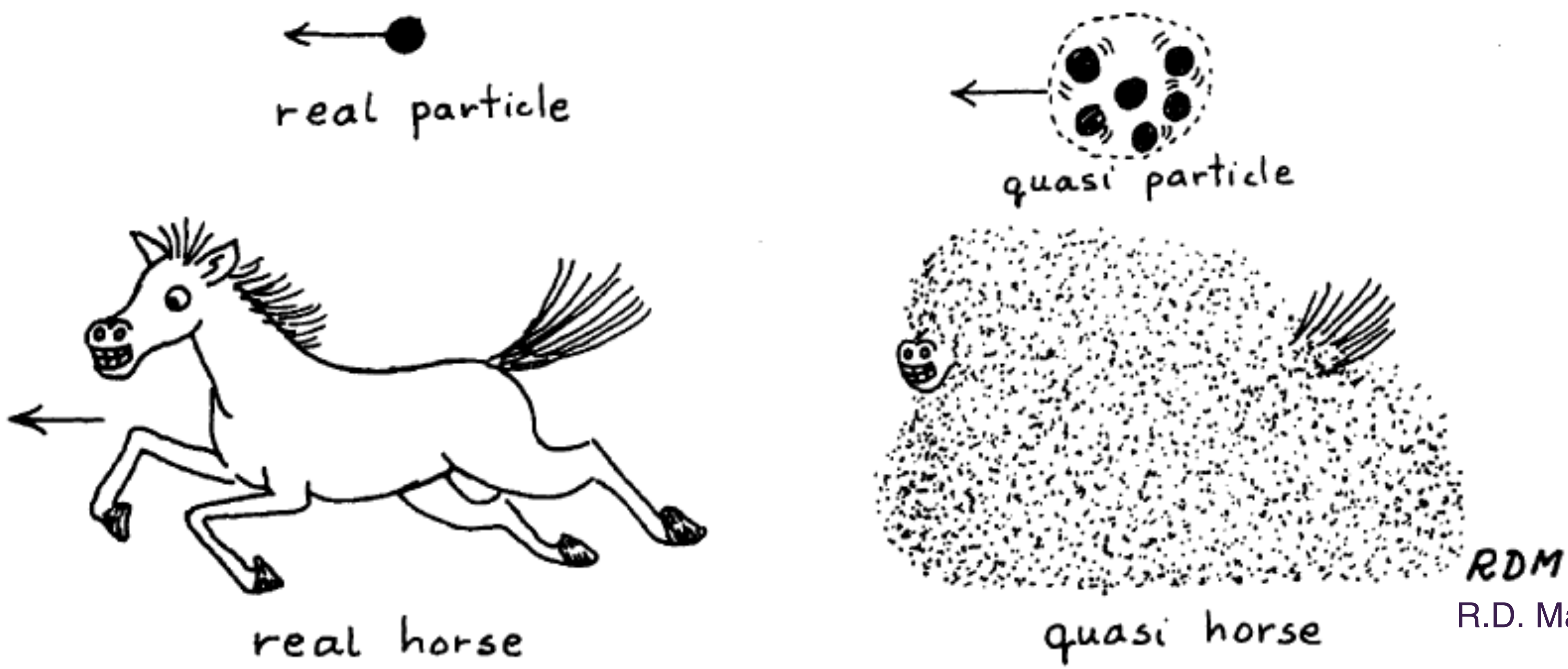


Theory of (ordinary) metals



Quasiparticles:

A quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

Quantum matter <u>with</u> quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

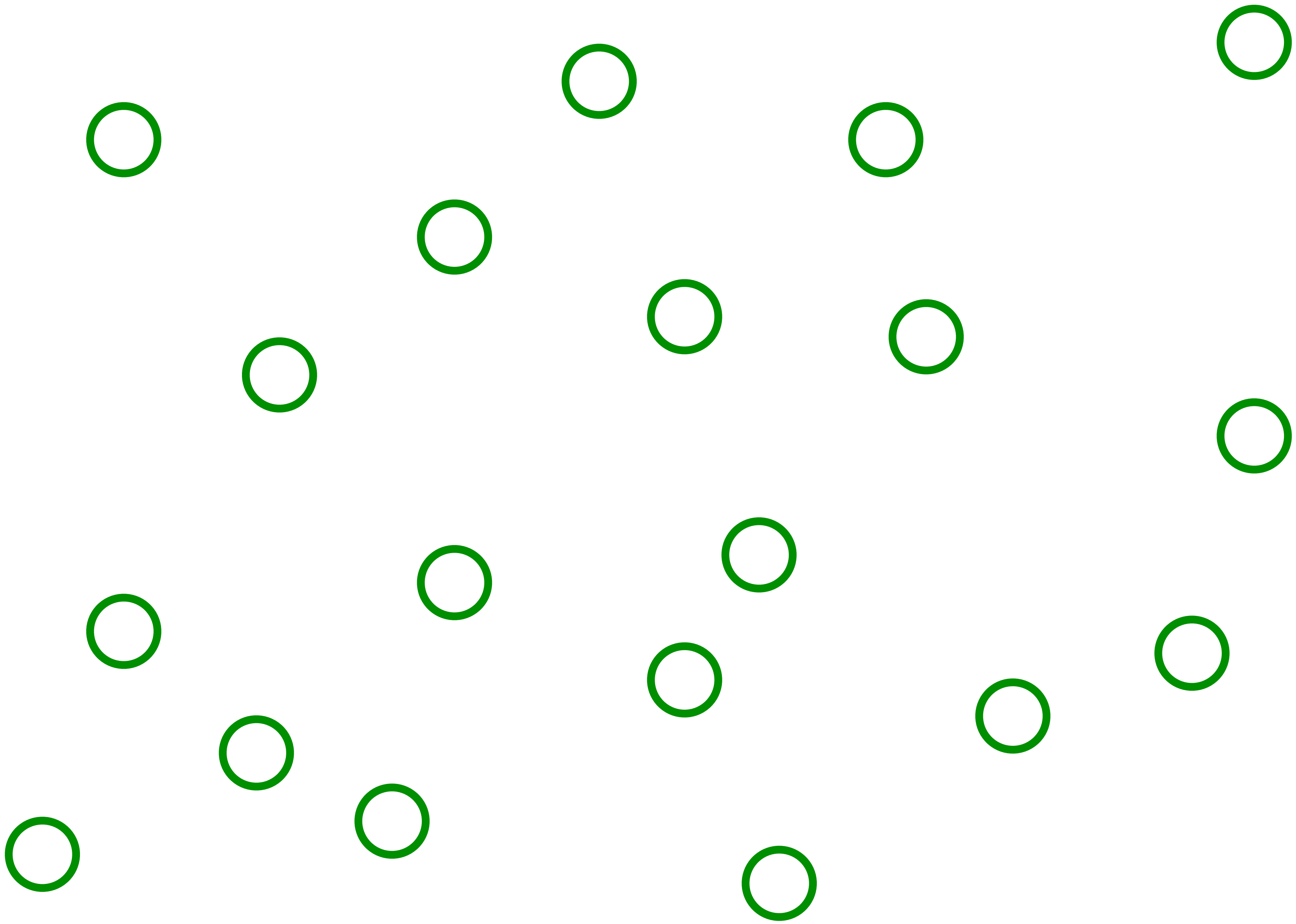
Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

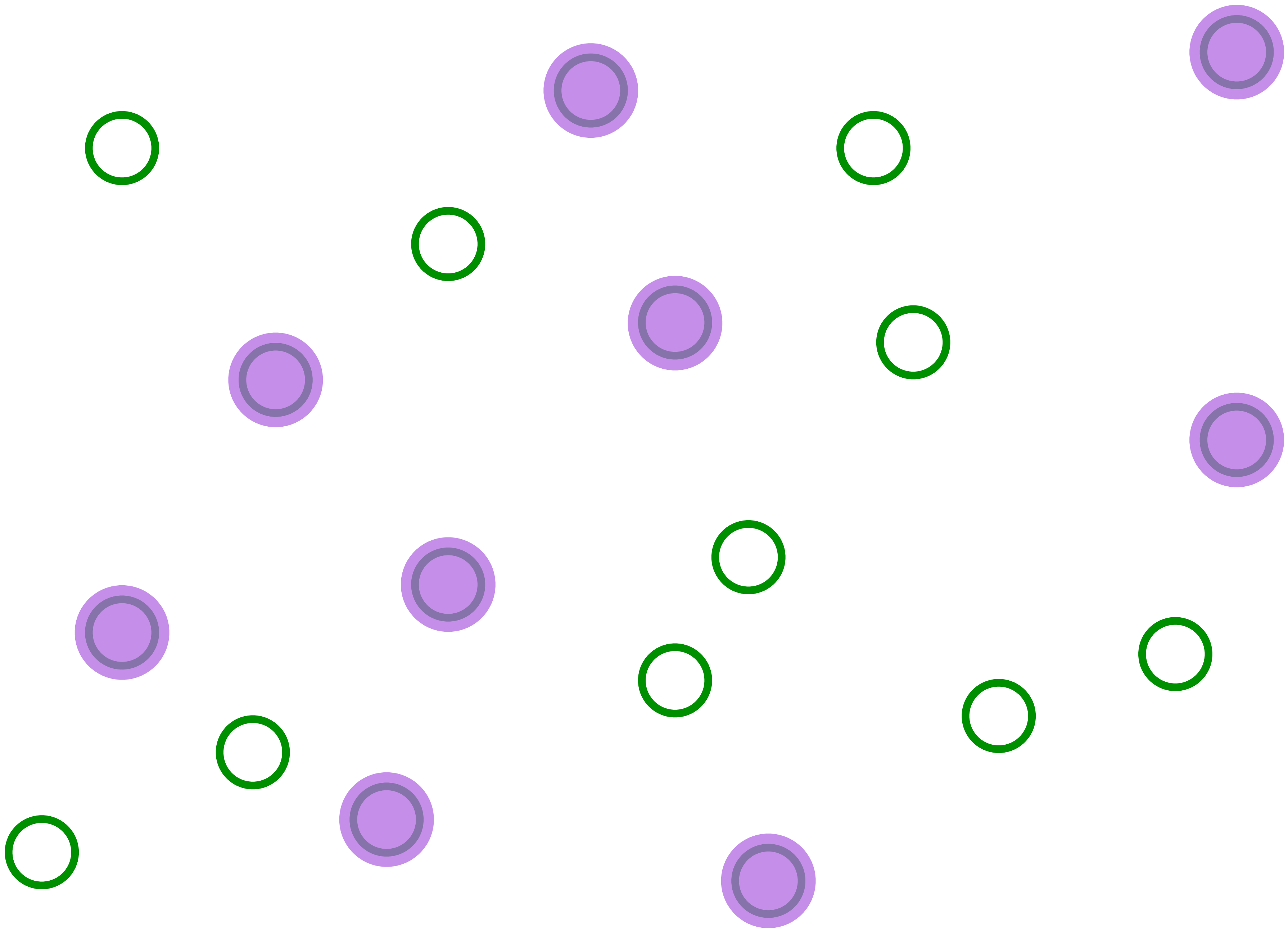
where E_F is the Fermi energy.

A simple model of a metal with quasiparticles



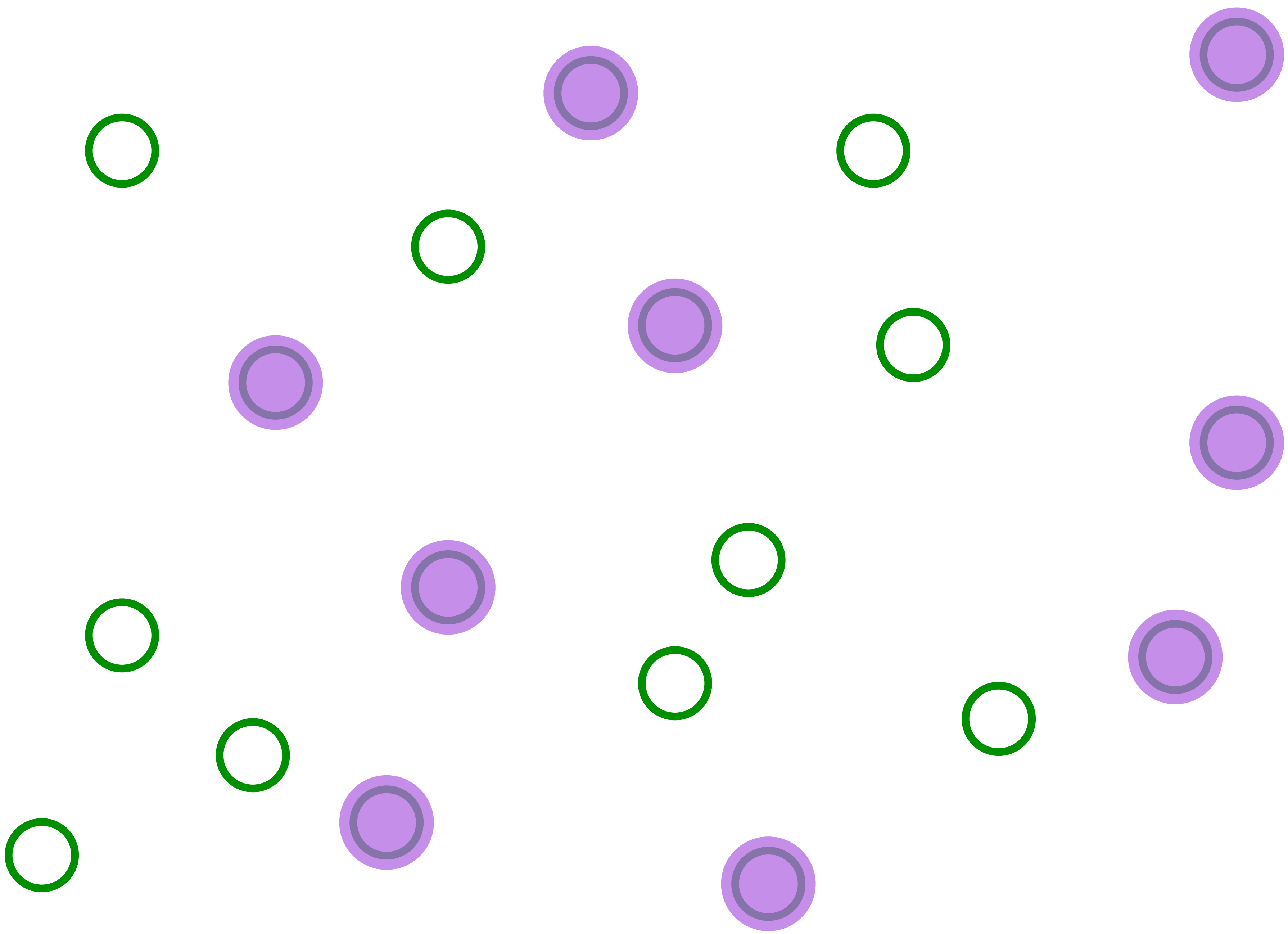
Pick a set of random positions

A simple model of a metal with quasiparticles



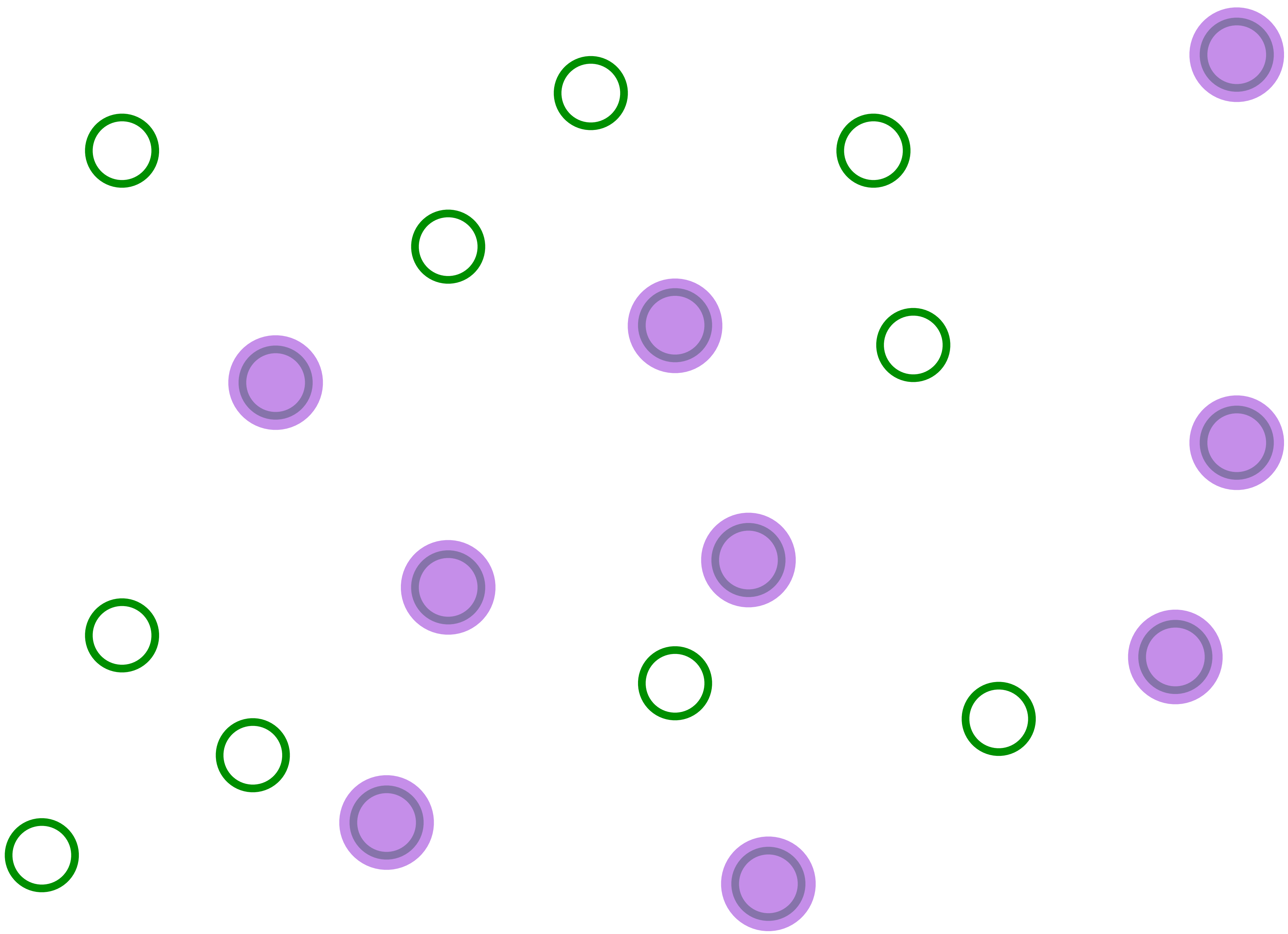
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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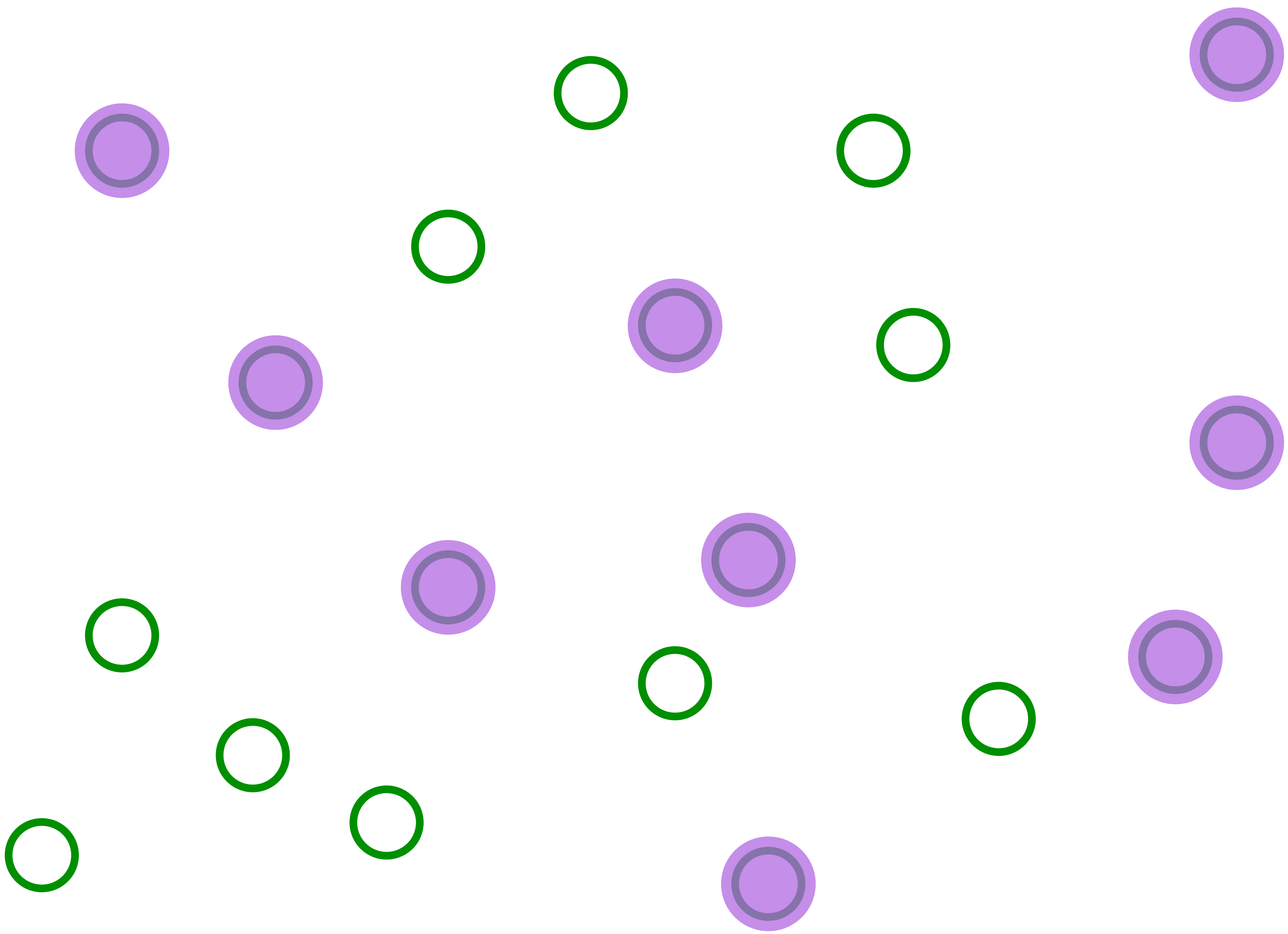

Electrons move one-by-one randomly

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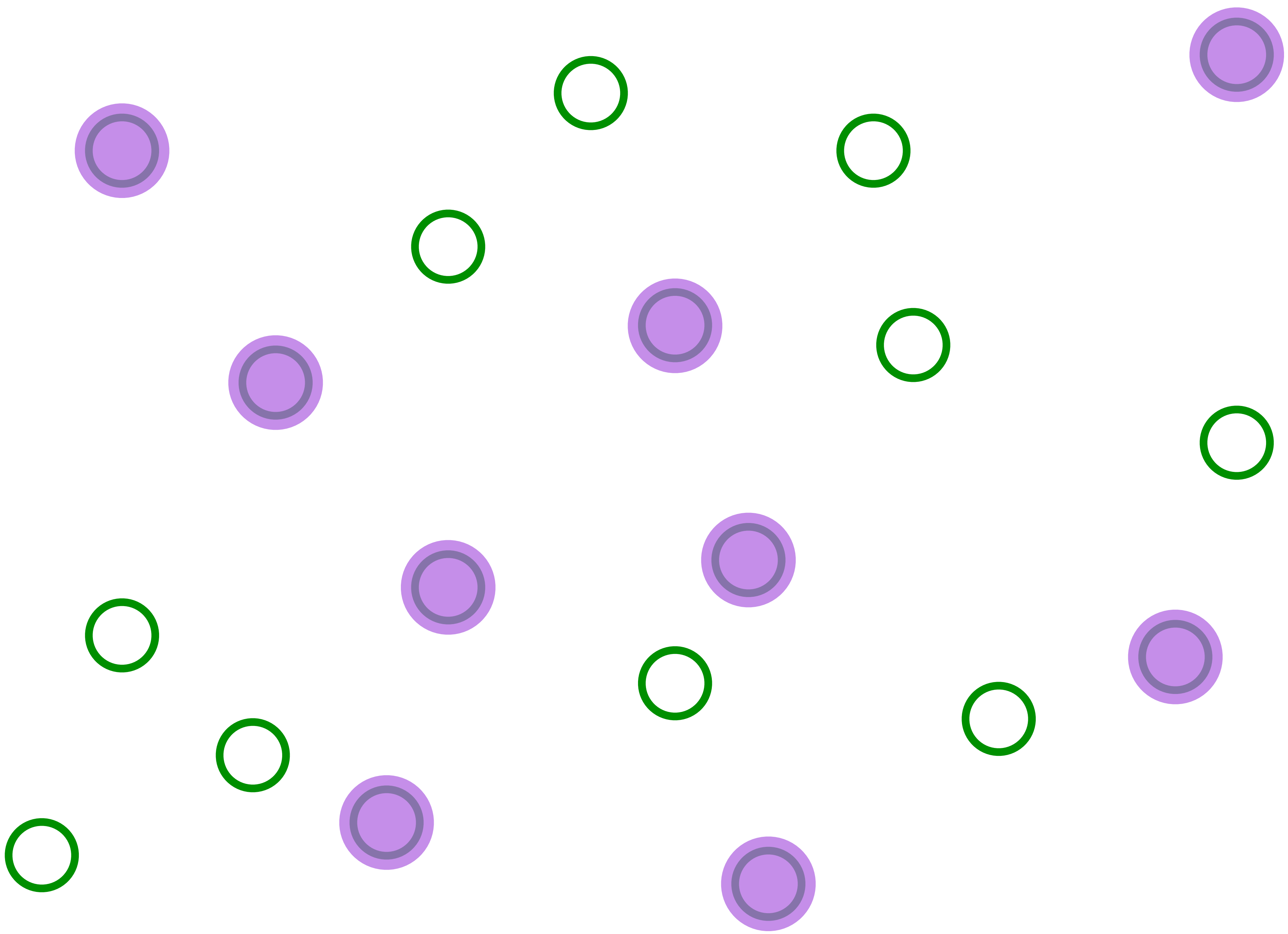


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

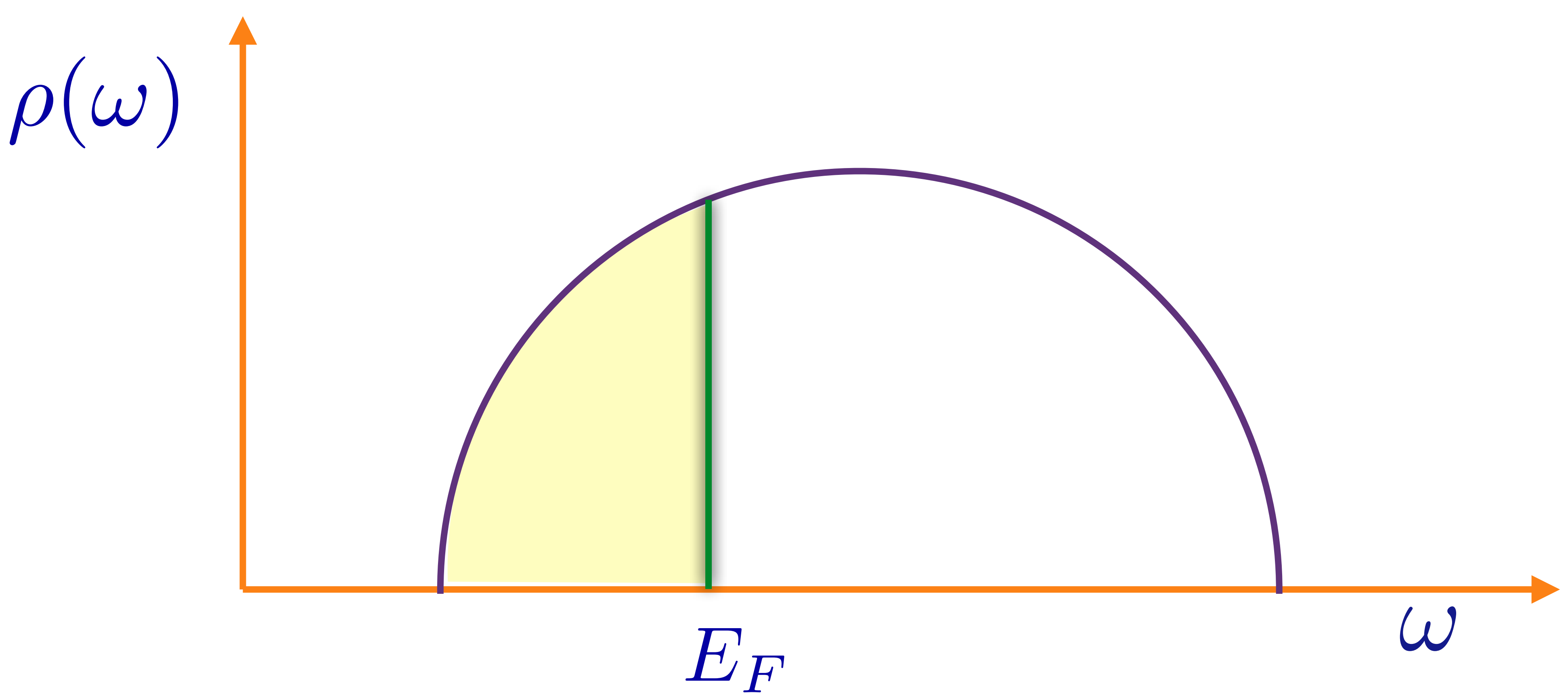
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\frac{1}{\tau_\alpha} = \pi U^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta)$$
$$= \frac{\pi^3 U^2 \rho_0^2}{4} T^2$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

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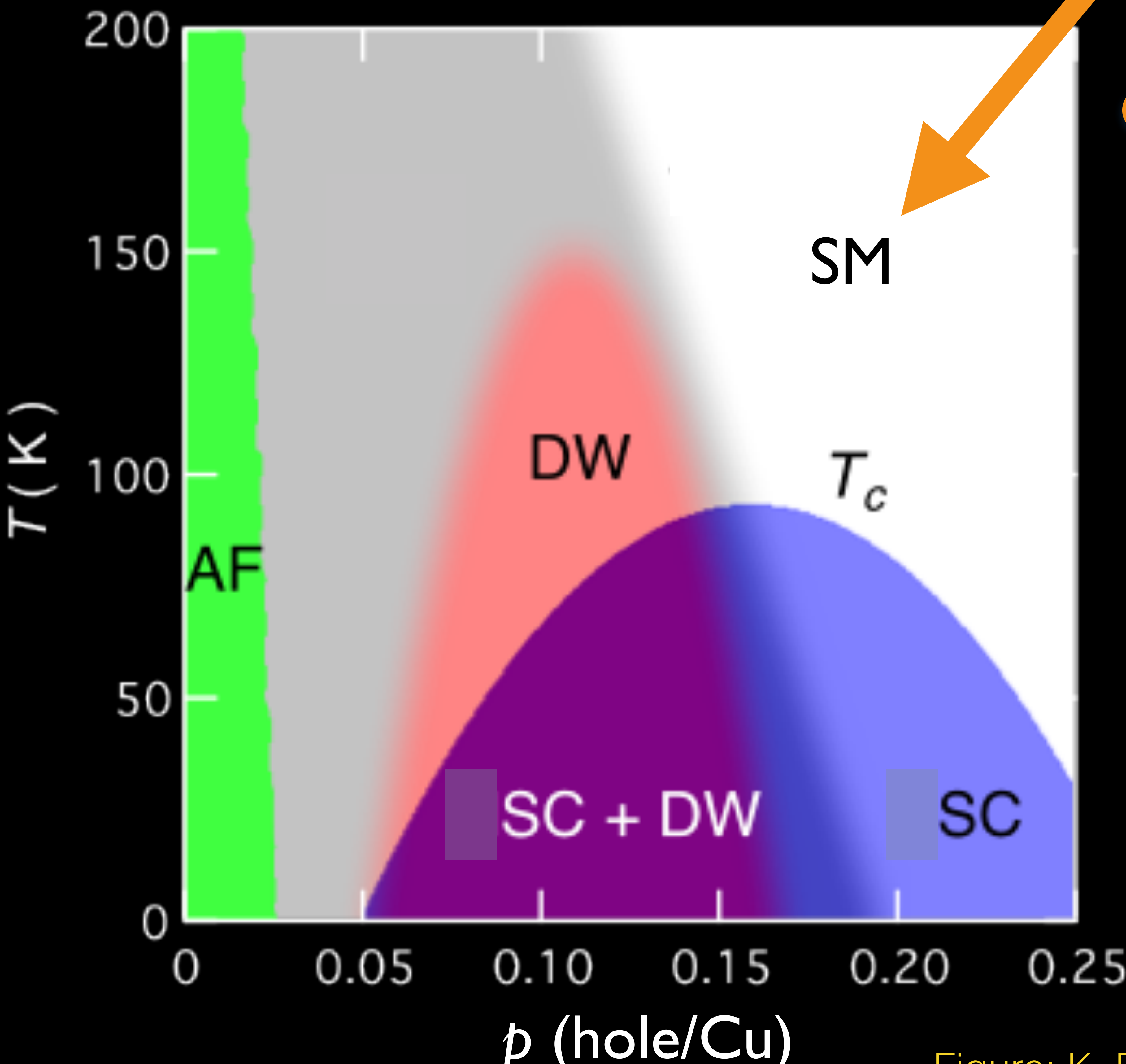
Quantum matter without quasiparticles

Strange metal

Entangled electrons lead to "strange" temperature dependence of resistivity and other properties



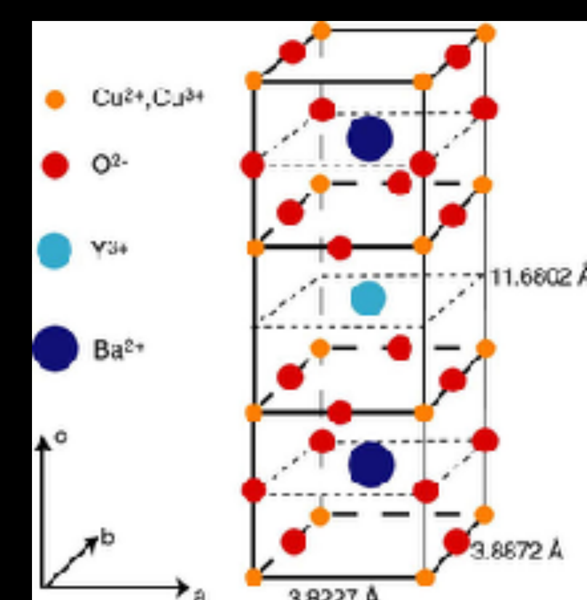


Figure: K. Fujita and J. C. Seamus Davis

"Strange",

"Bad",

or "Incoherent",

metal has a resistivity, ρ, which obeys

$$\rho \sim T,$$

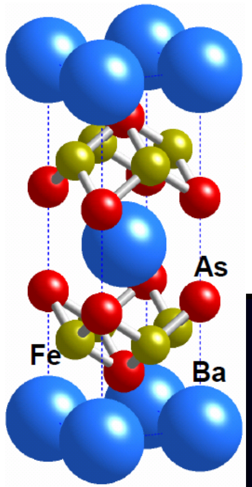
and

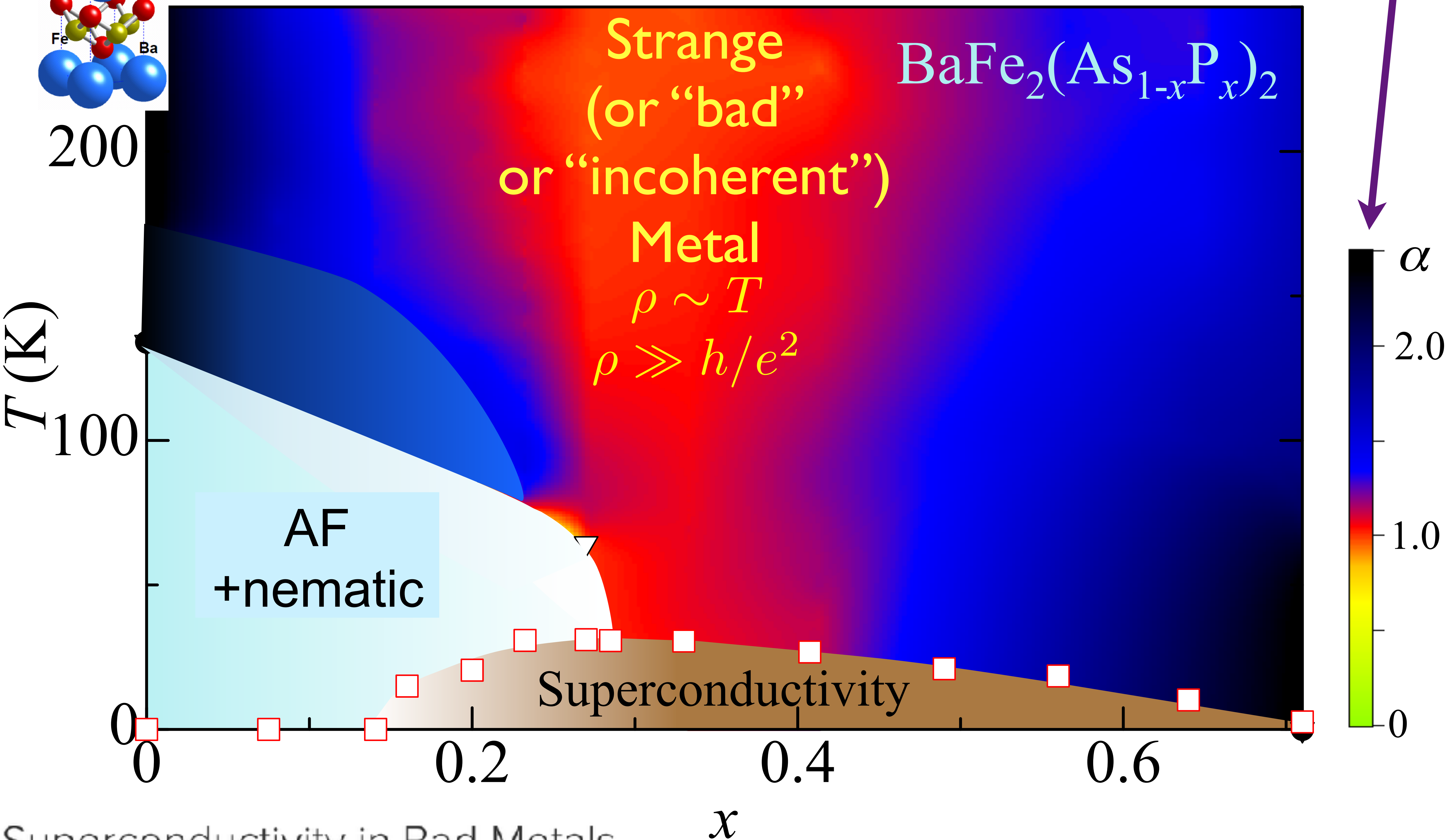
$$\rho \gg h/e^2$$

(in two dimensions),
where h/e^2 is the quantum unit of resistance.

Quantum matter without quasiparticles

Resistivity $\sim \rho_0 + AT^\alpha$



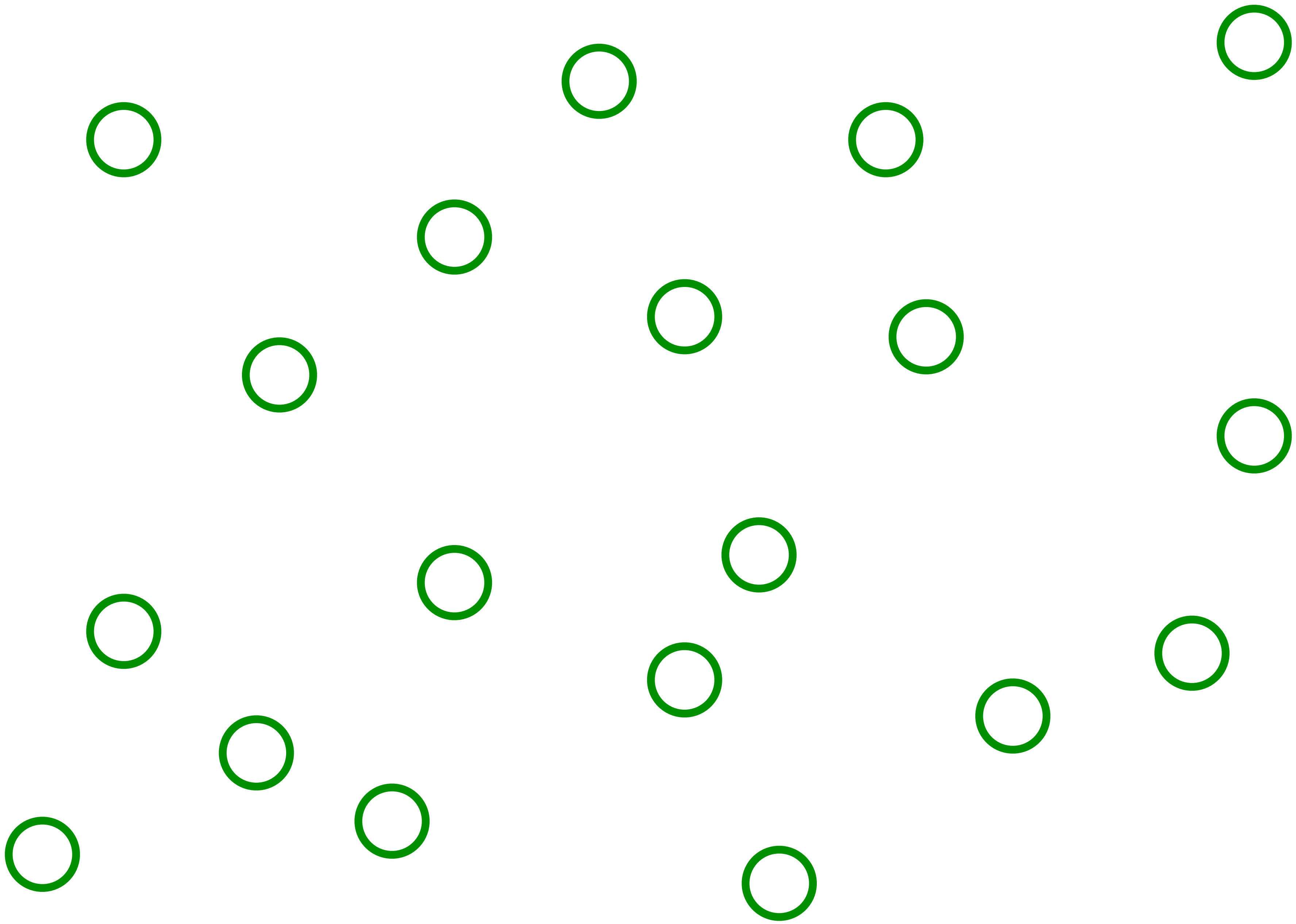


Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995

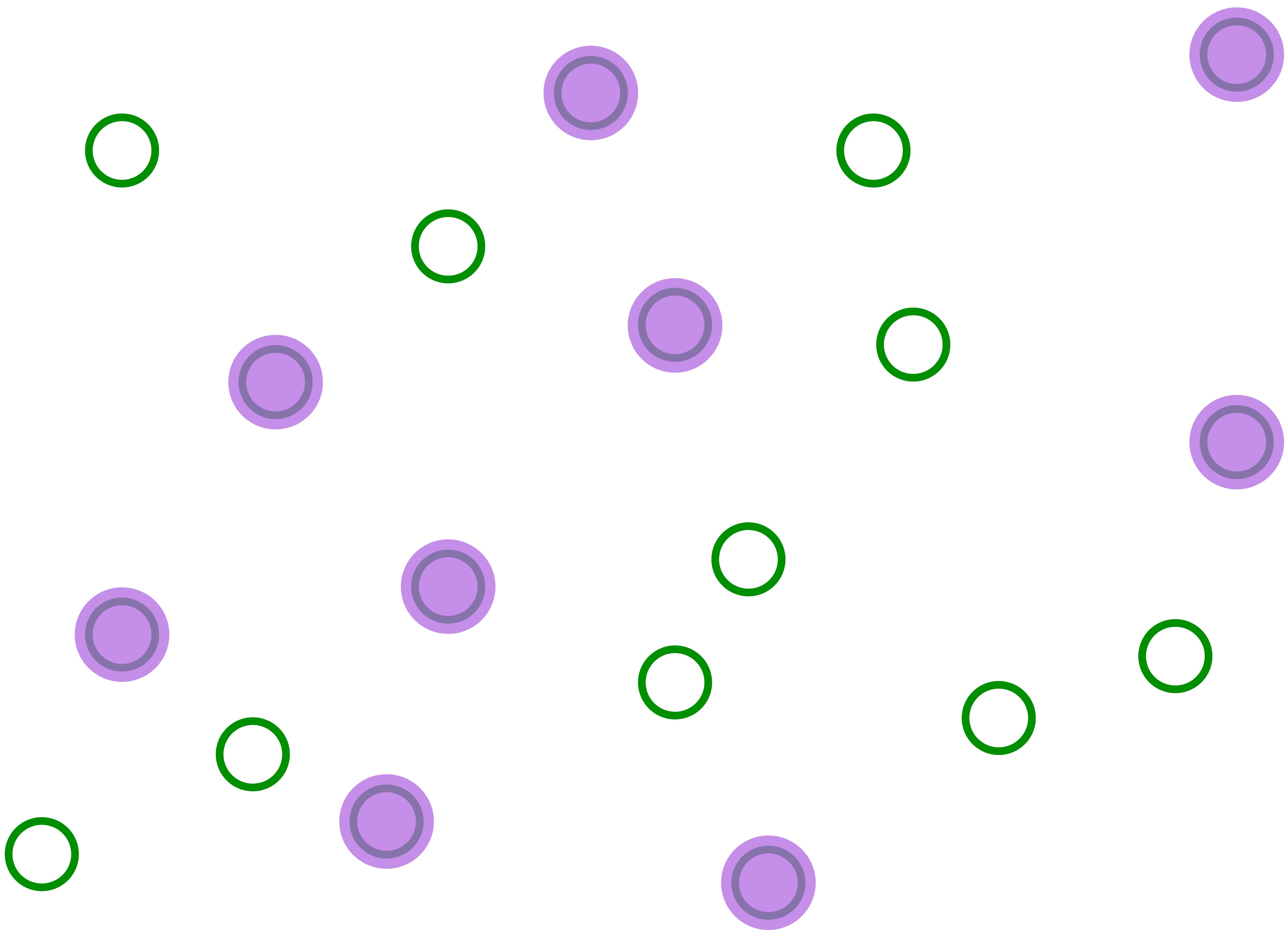
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* **81**, 184519 (2010)

The Sachdev-Ye-Kitaev (SYK) model



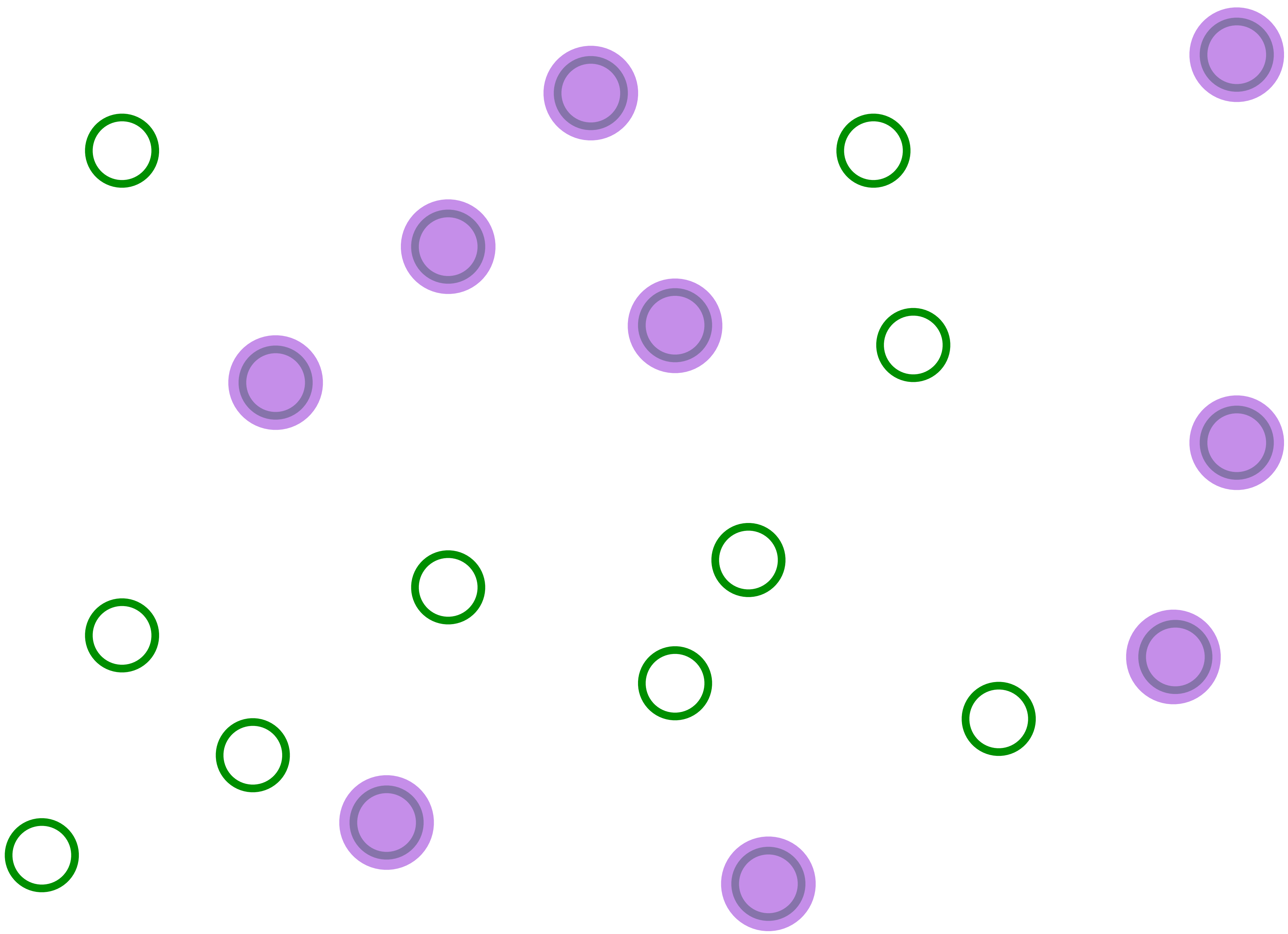
Pick a set of random positions

The SYK model



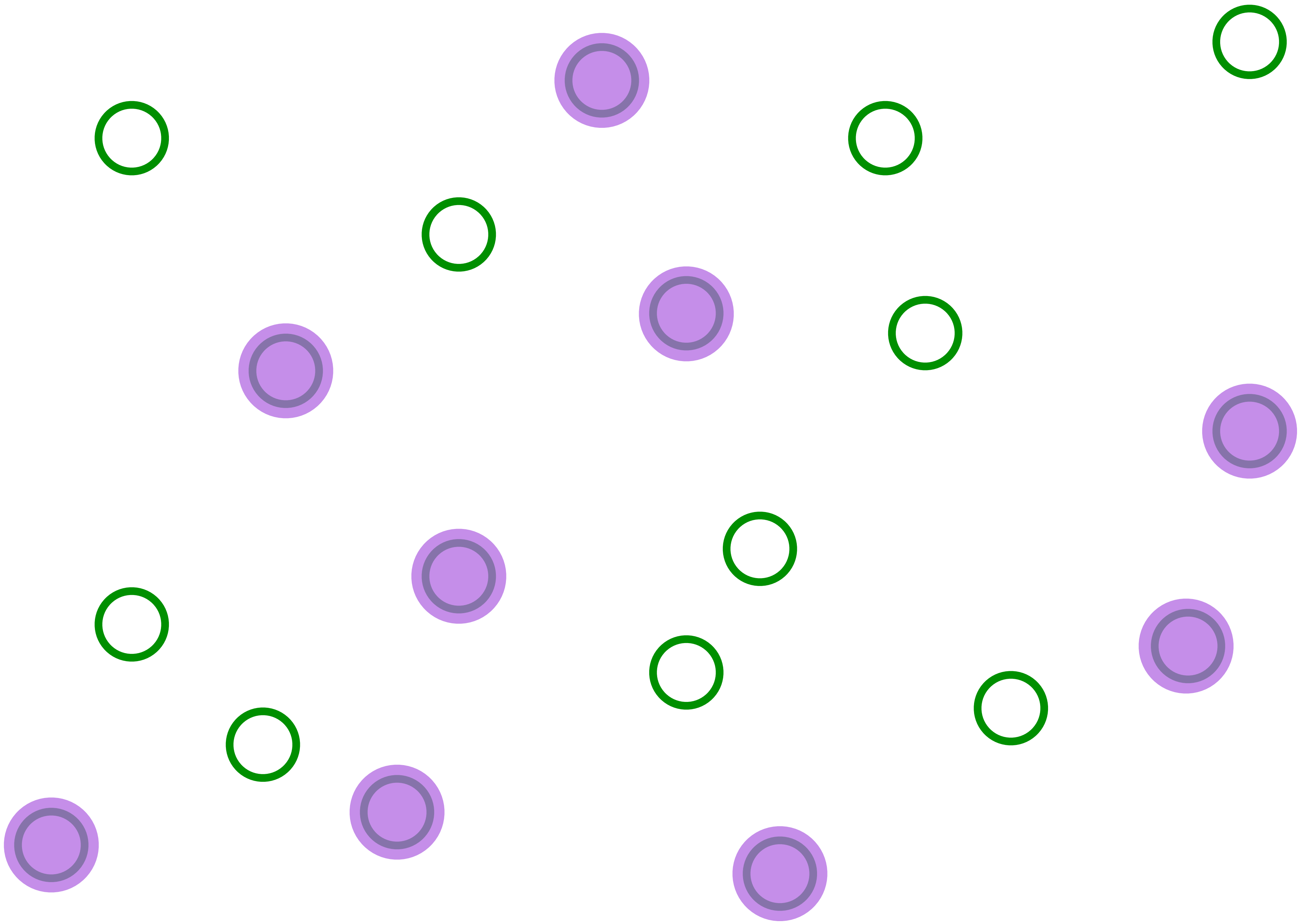
Place electrons randomly on some sites

The SYK model



Entangle electrons pairwise randomly

The SYK model



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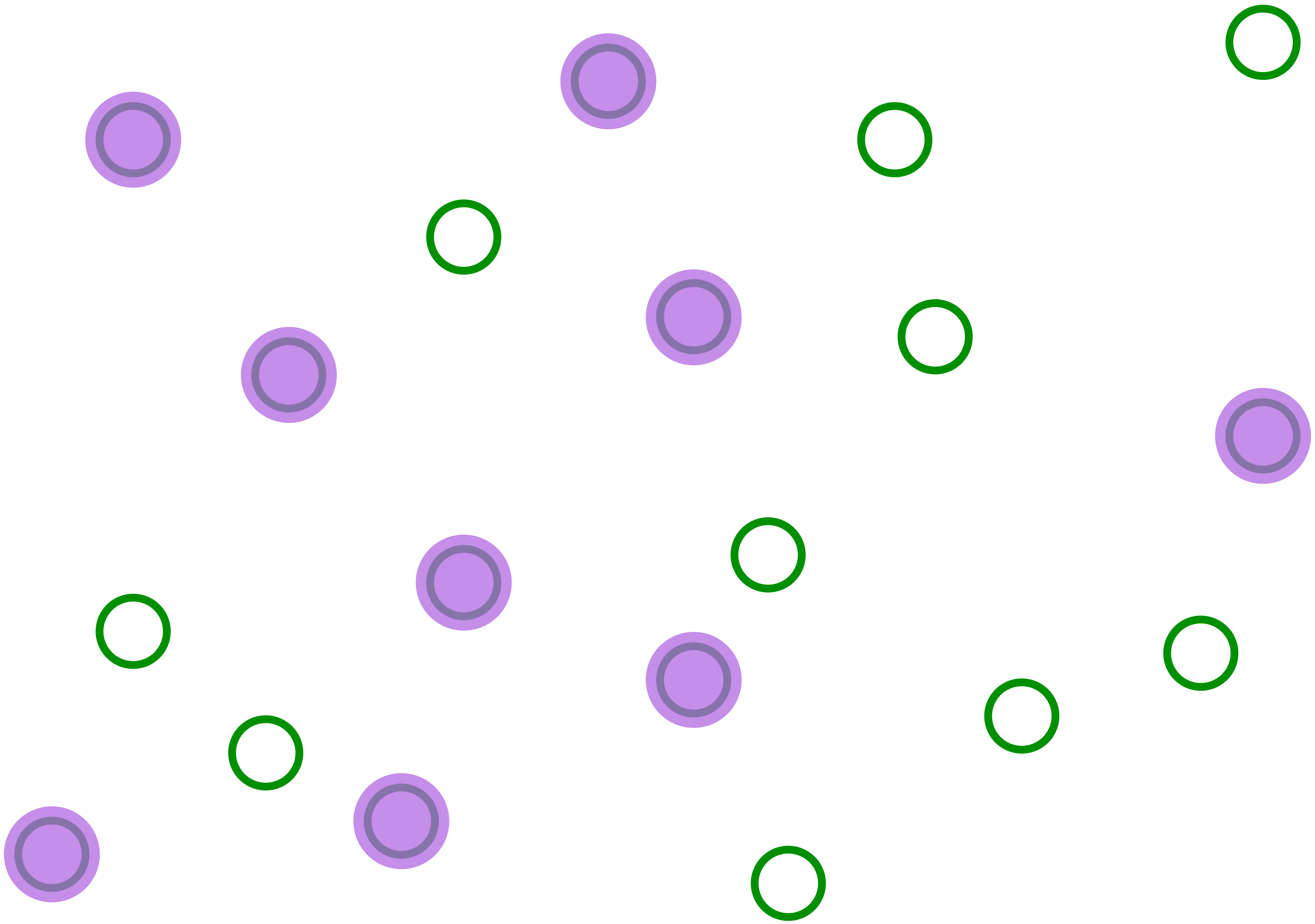

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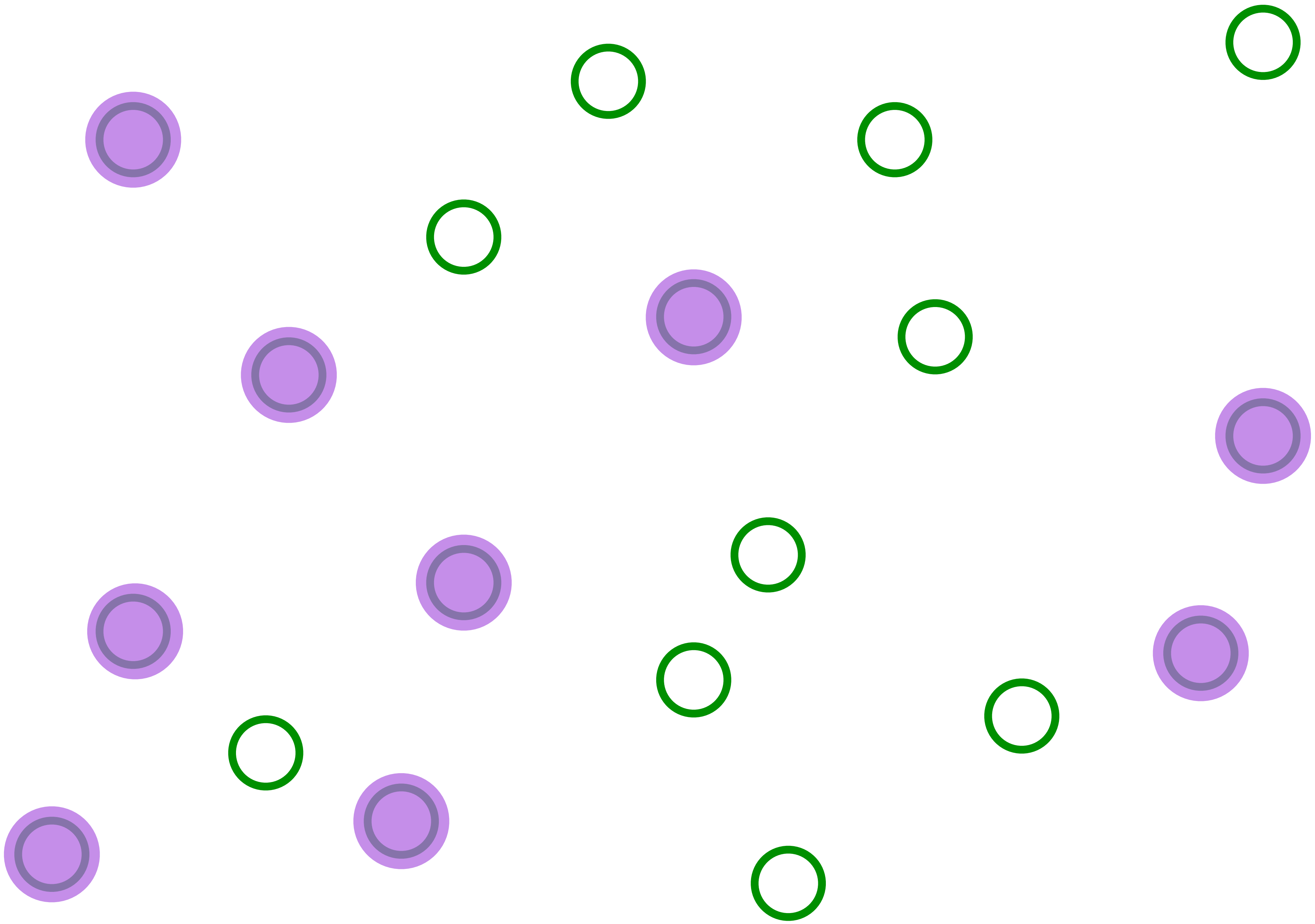

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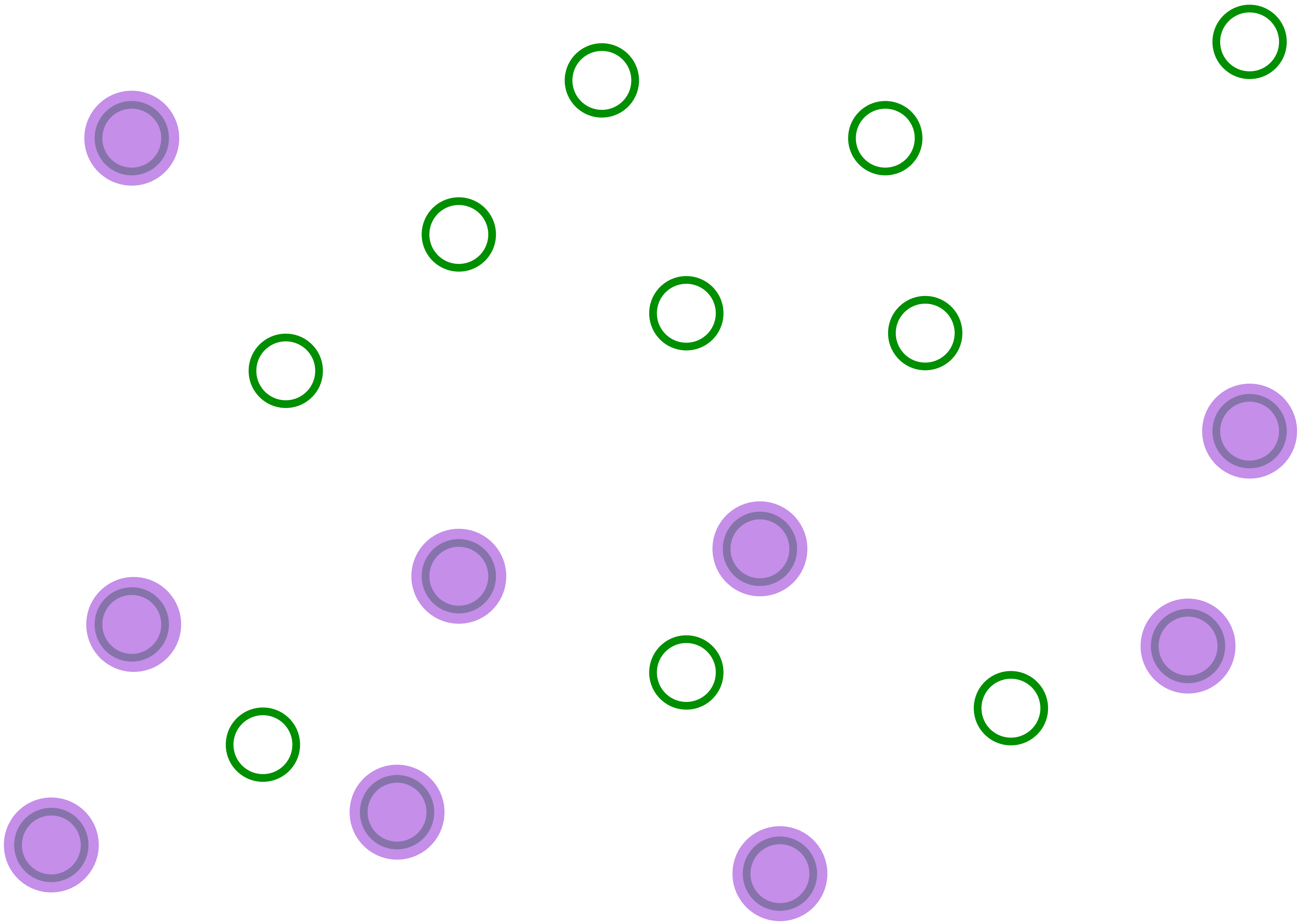

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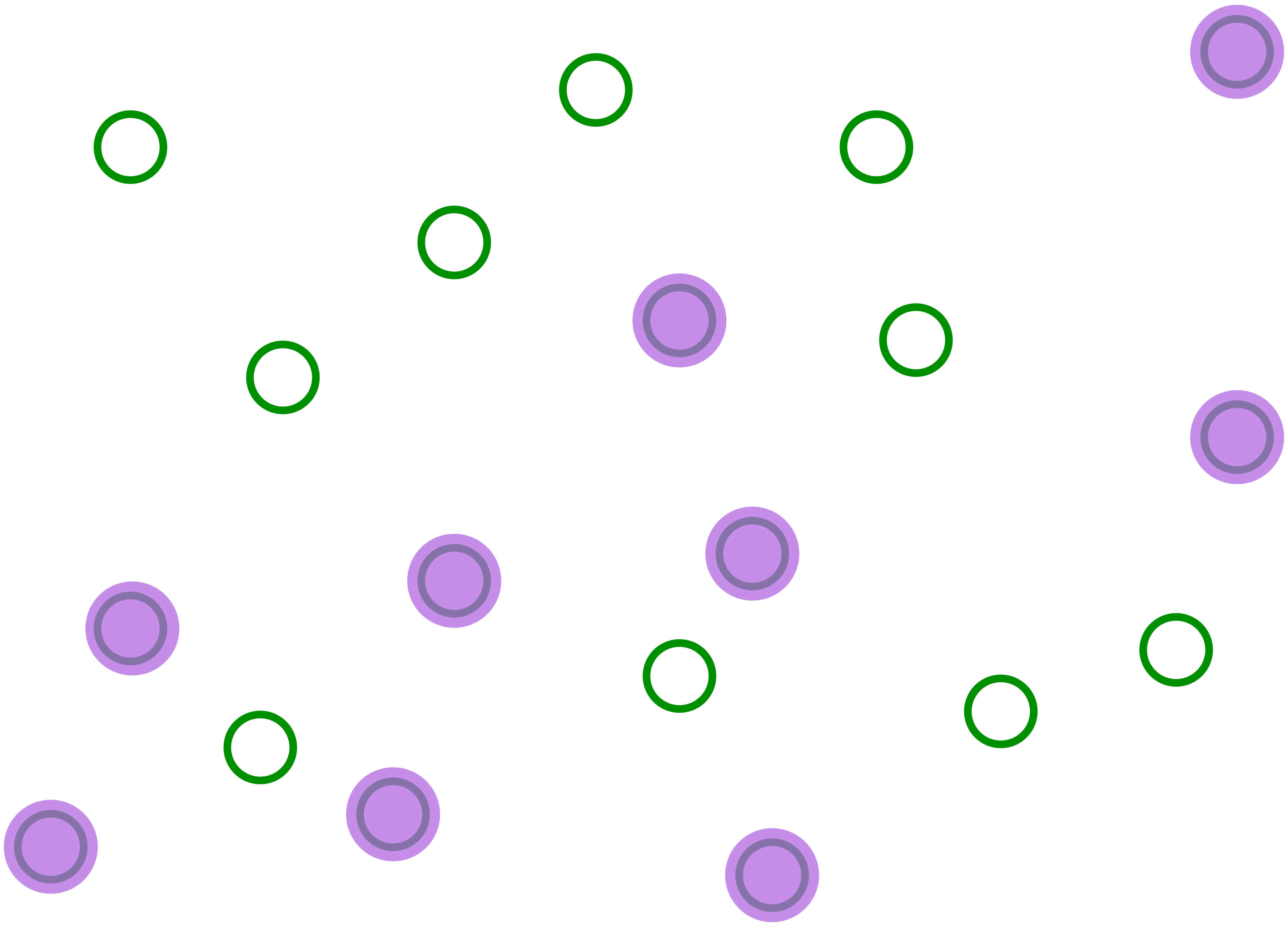

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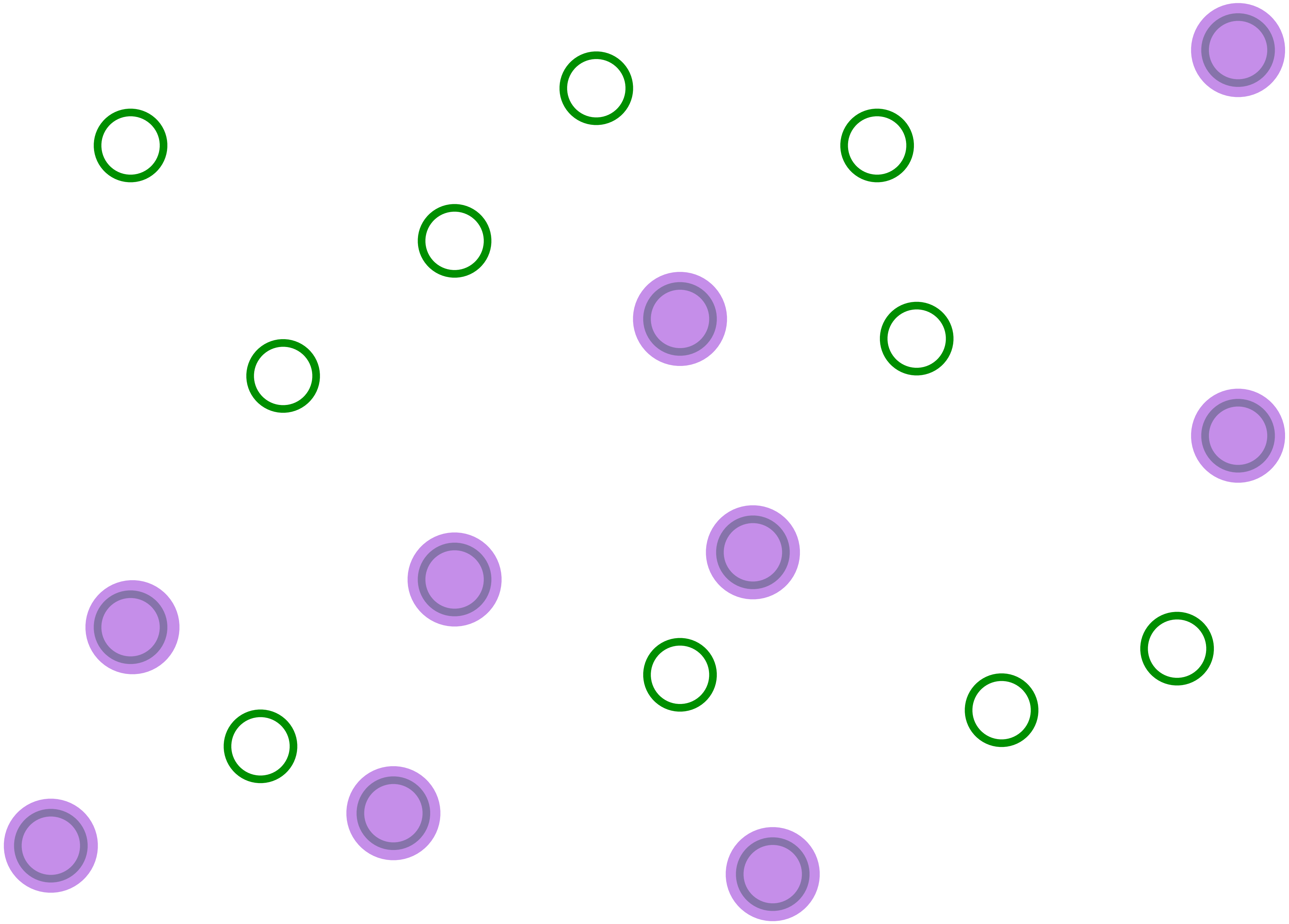


This describes both a strange metal and a black hole!

The SYK model

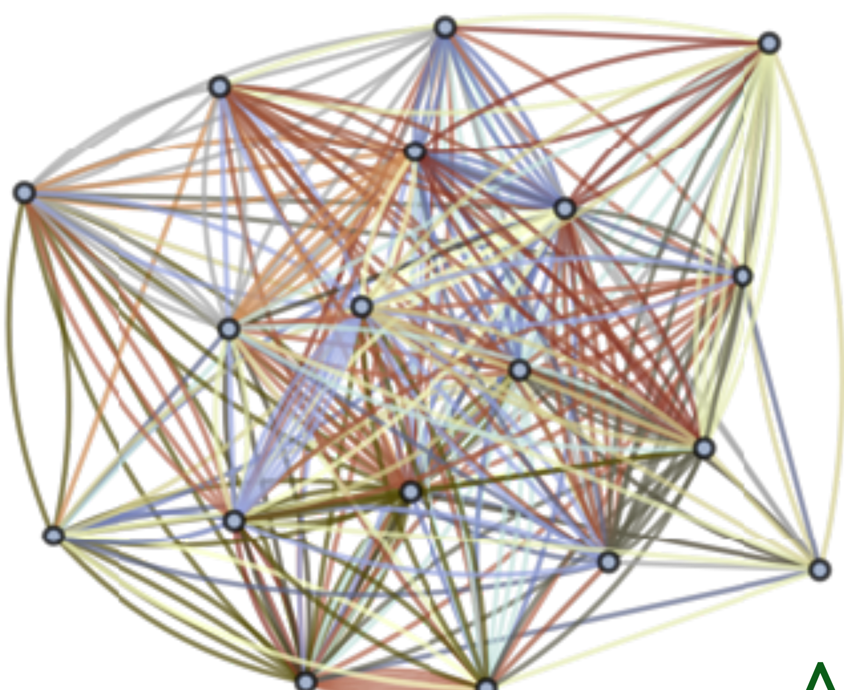
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

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- $T > 0$ Green's function has conformal invariance: $G \sim (T/\sin(\pi k_B T \tau/\hbar))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)

- The last property indicates $\tau_{\rm eq} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

Quantum matter without quasiparticles:

- If there are no quasiparticles, then

$$E \neq \sum_{\alpha} n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \dots$$

- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T}$$

S. Sachdev,
Quantum Phase Transitions,
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$$E \neq \sum_{\alpha} n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T}$$

- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

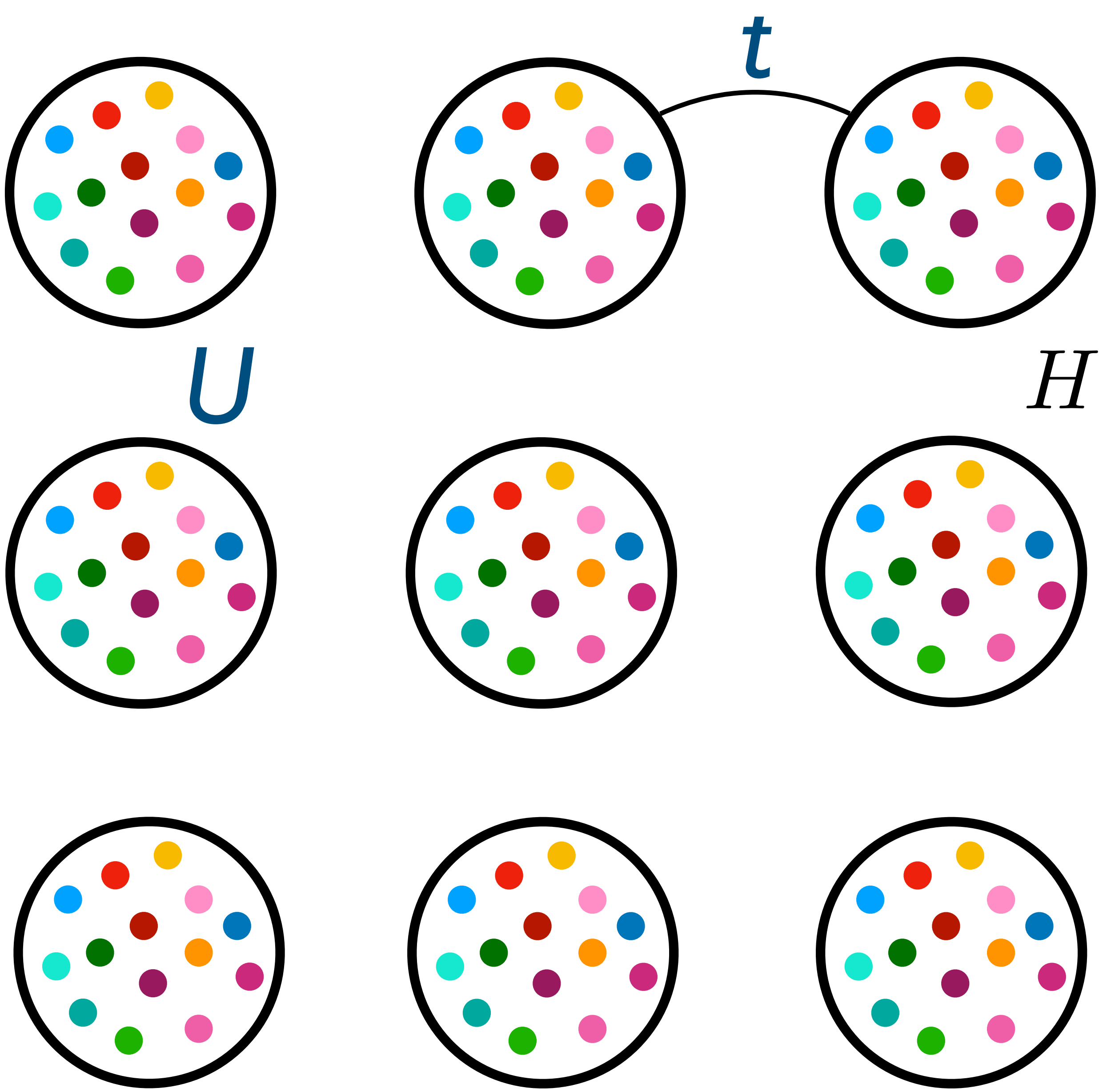
$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \,.$$

S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

SYK building blocks for a strange metal

`SYK islands' of electrons with random hopping between them.



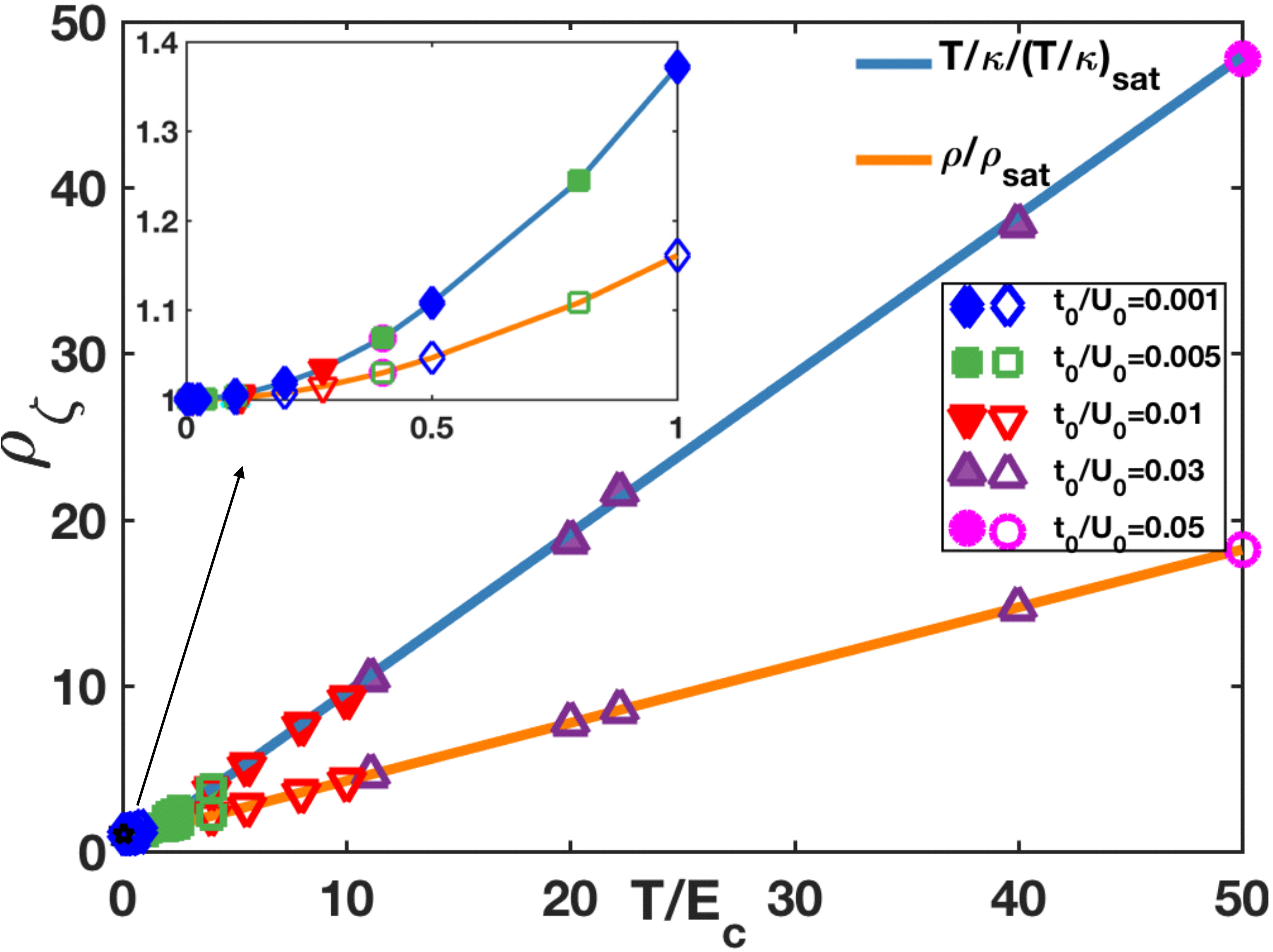
$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

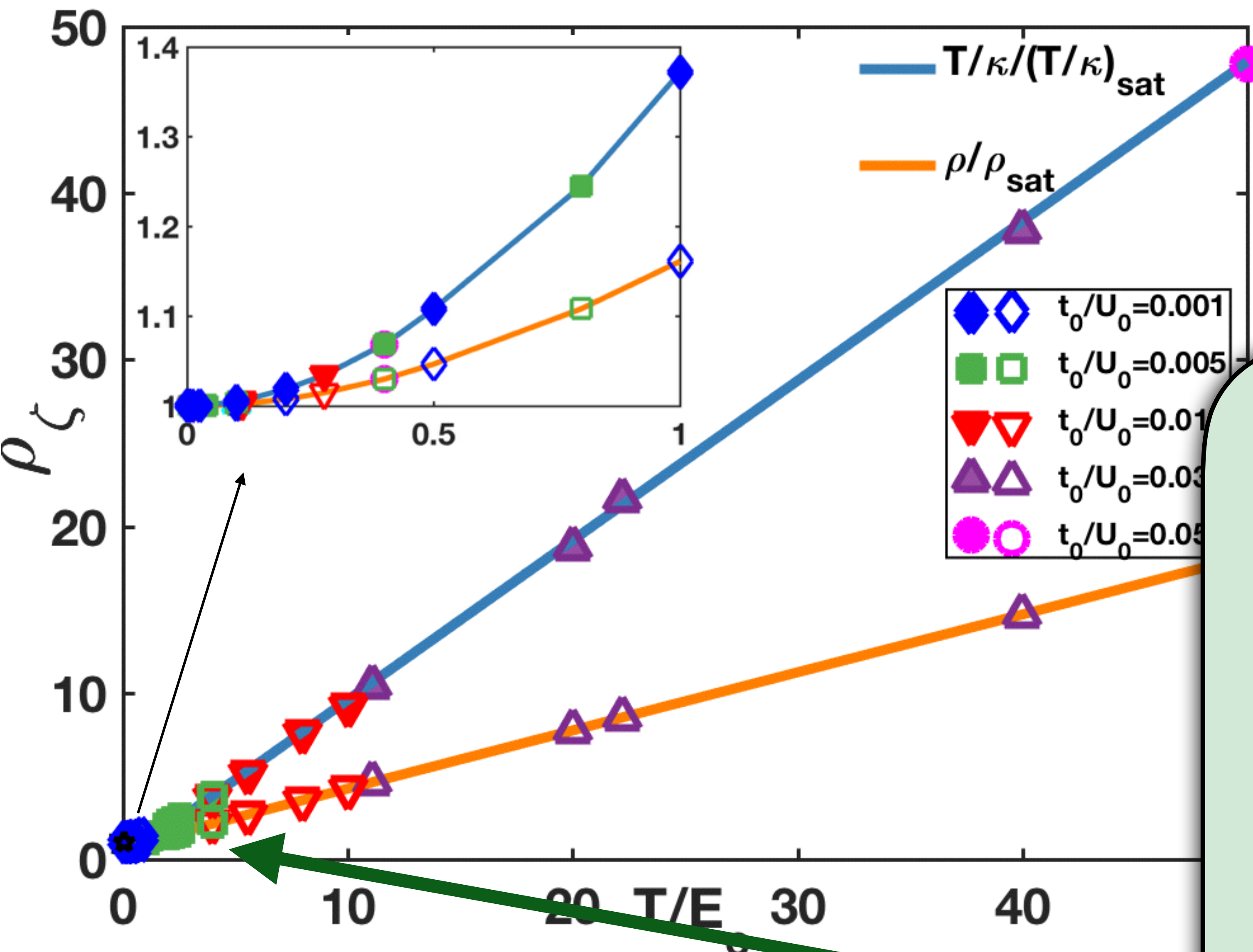
Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

For $T < E_c$, the resistivity, ρ, and entropy density, s, are

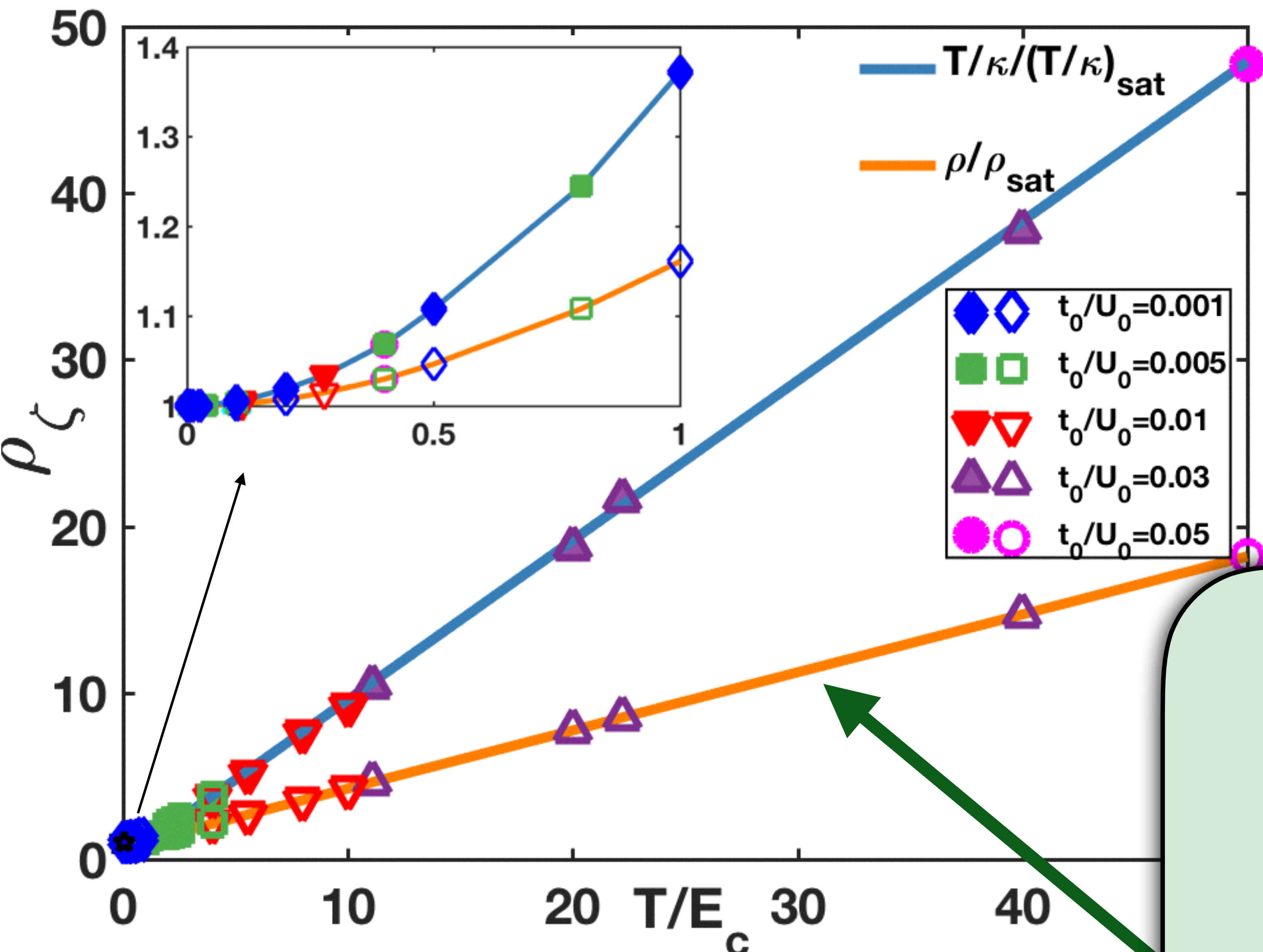
$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low ‘coherence’ scale

$$E_c \sim \frac{t_0^2}{U}$$



For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) , \quad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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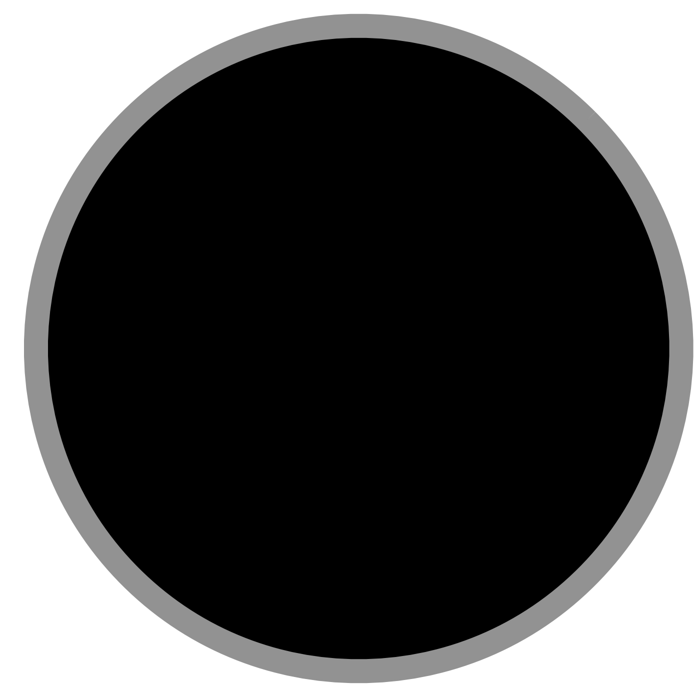
Black Holes

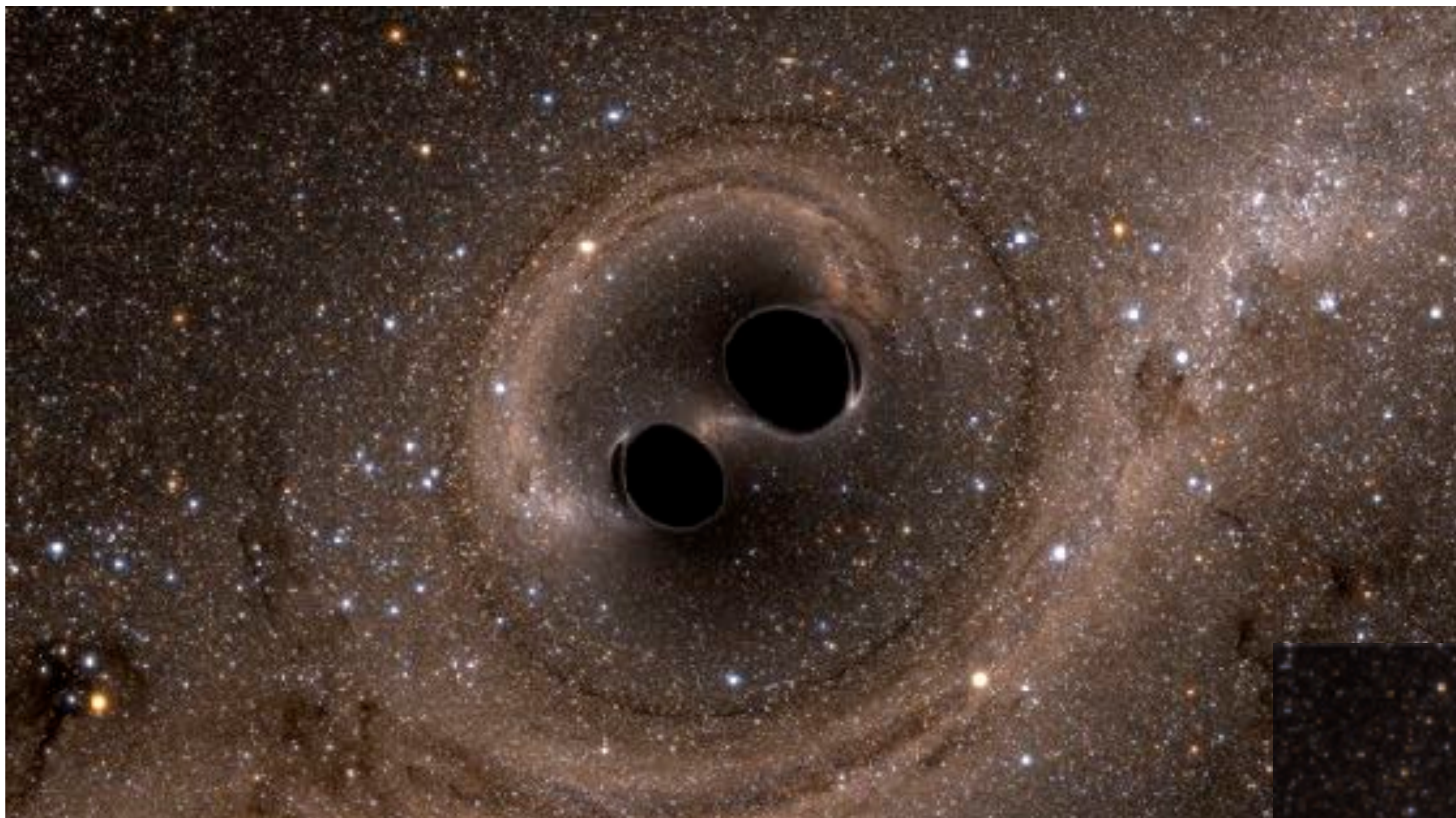
Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.



$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



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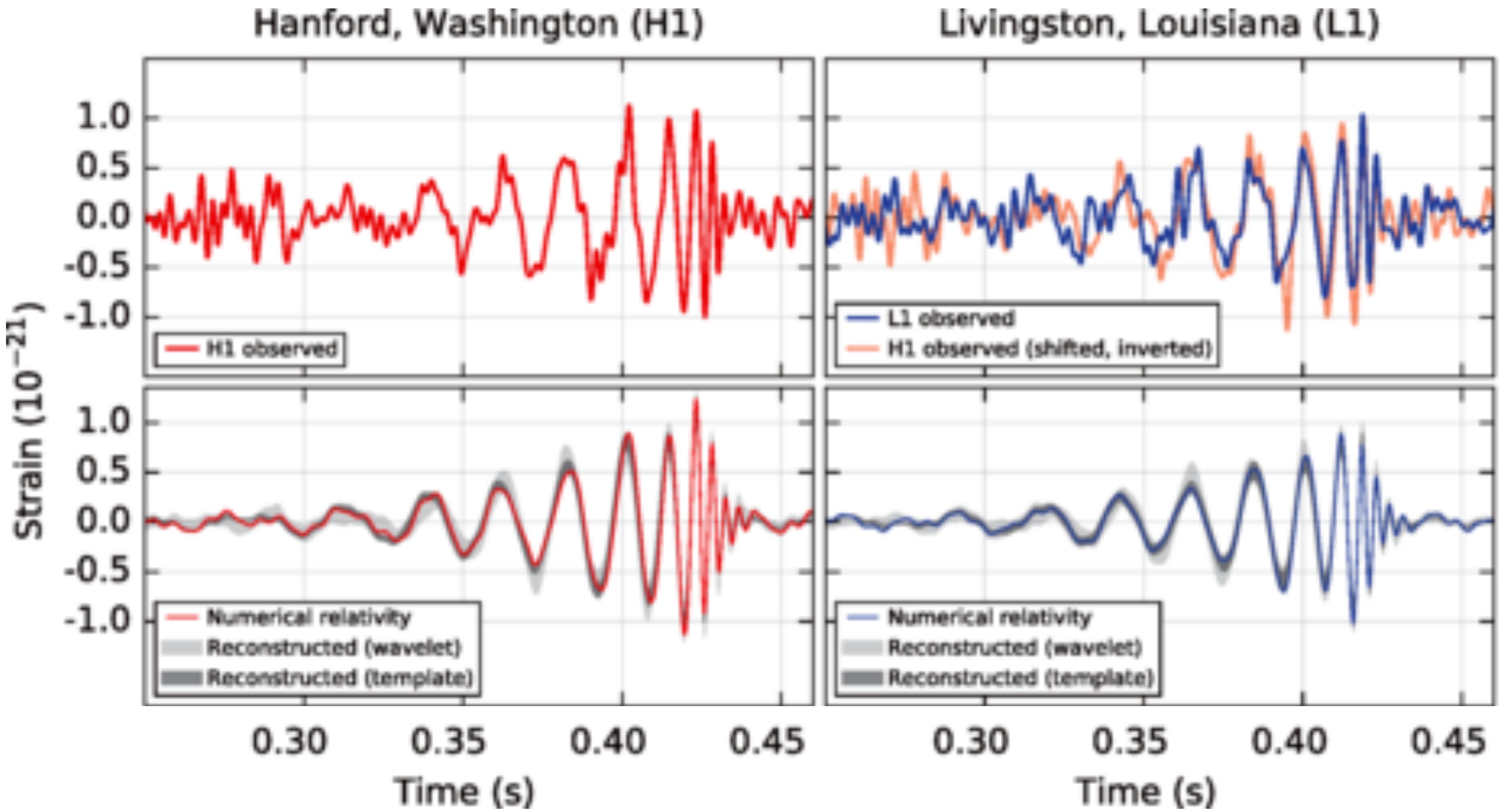
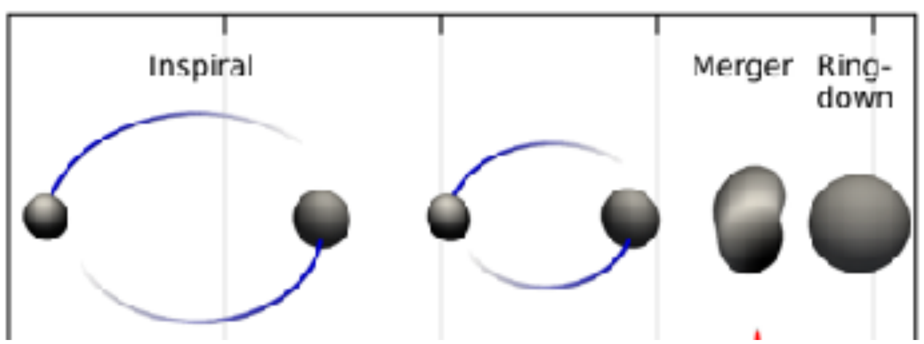
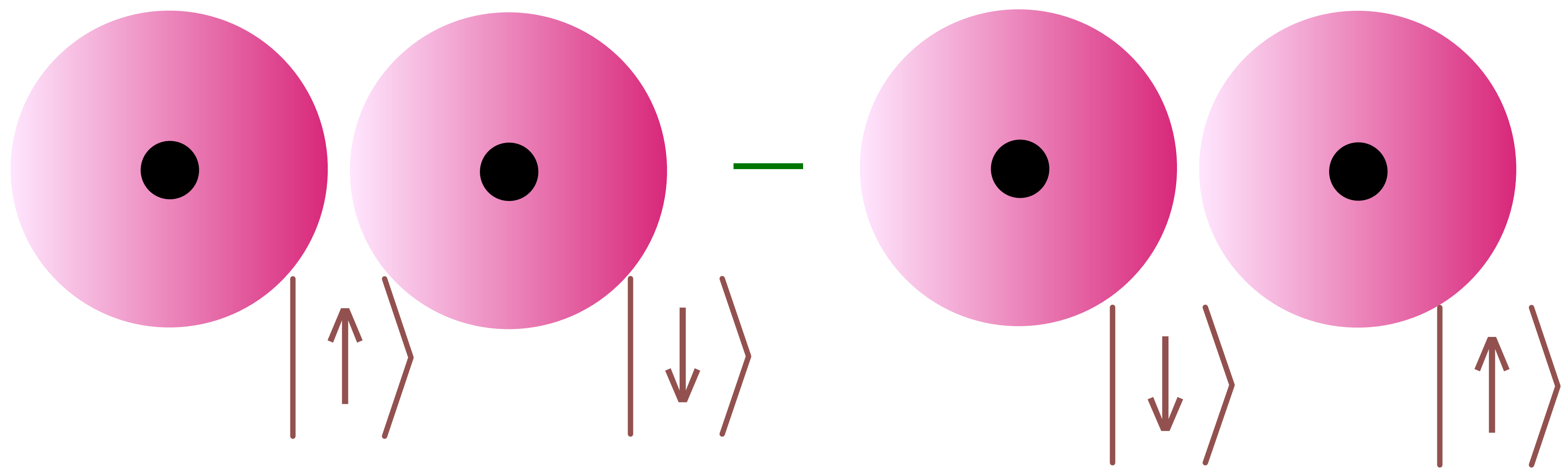




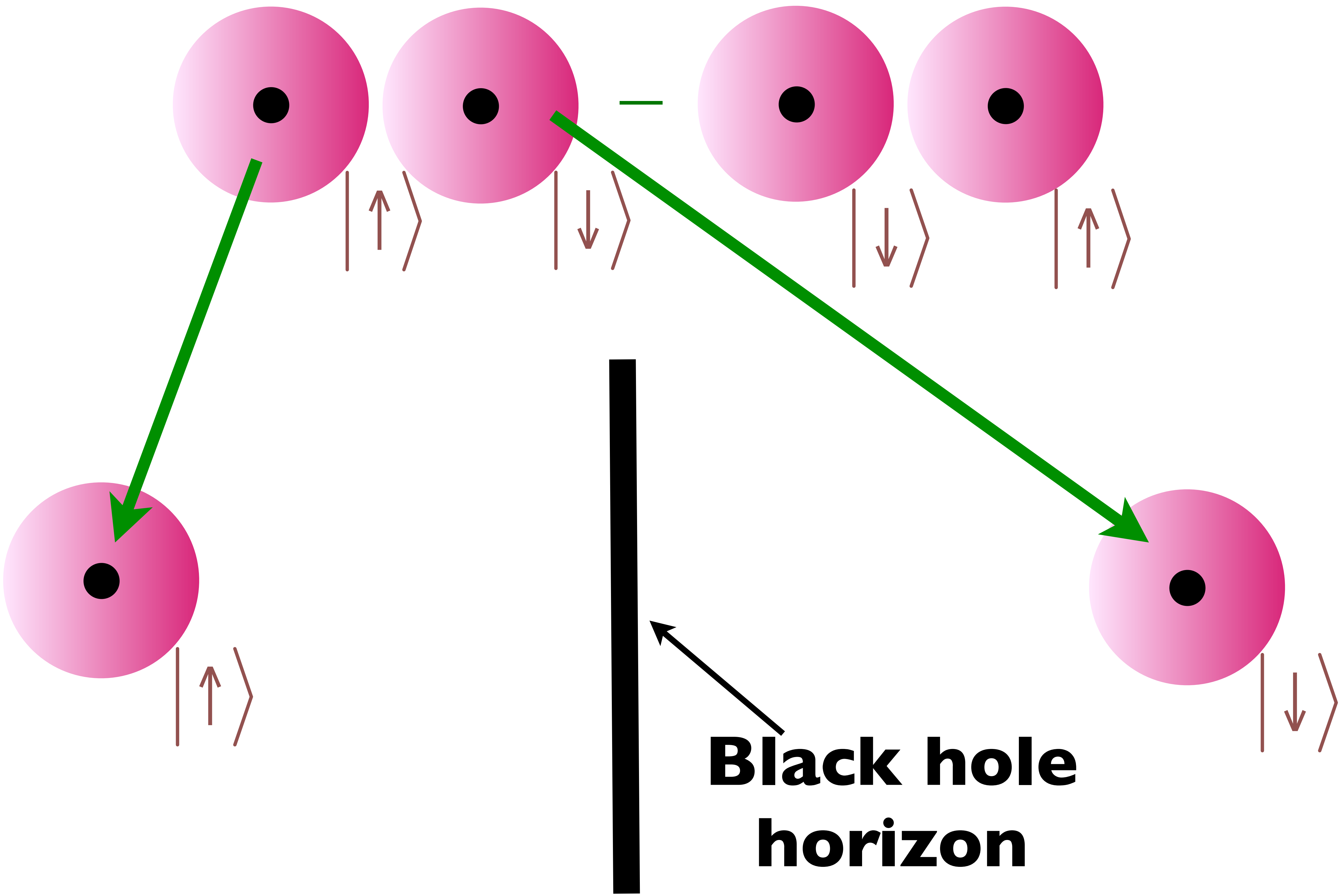
LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds.

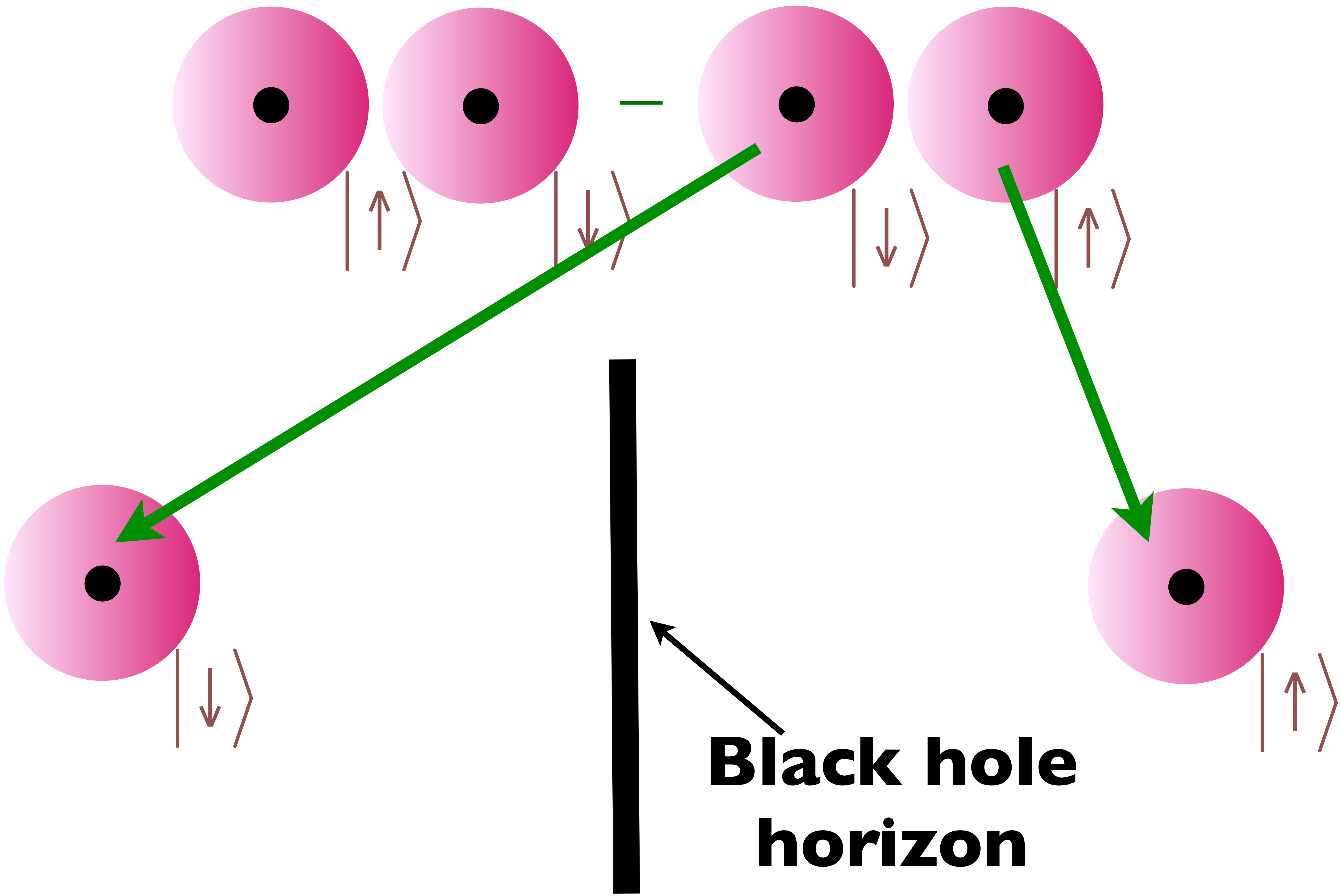
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

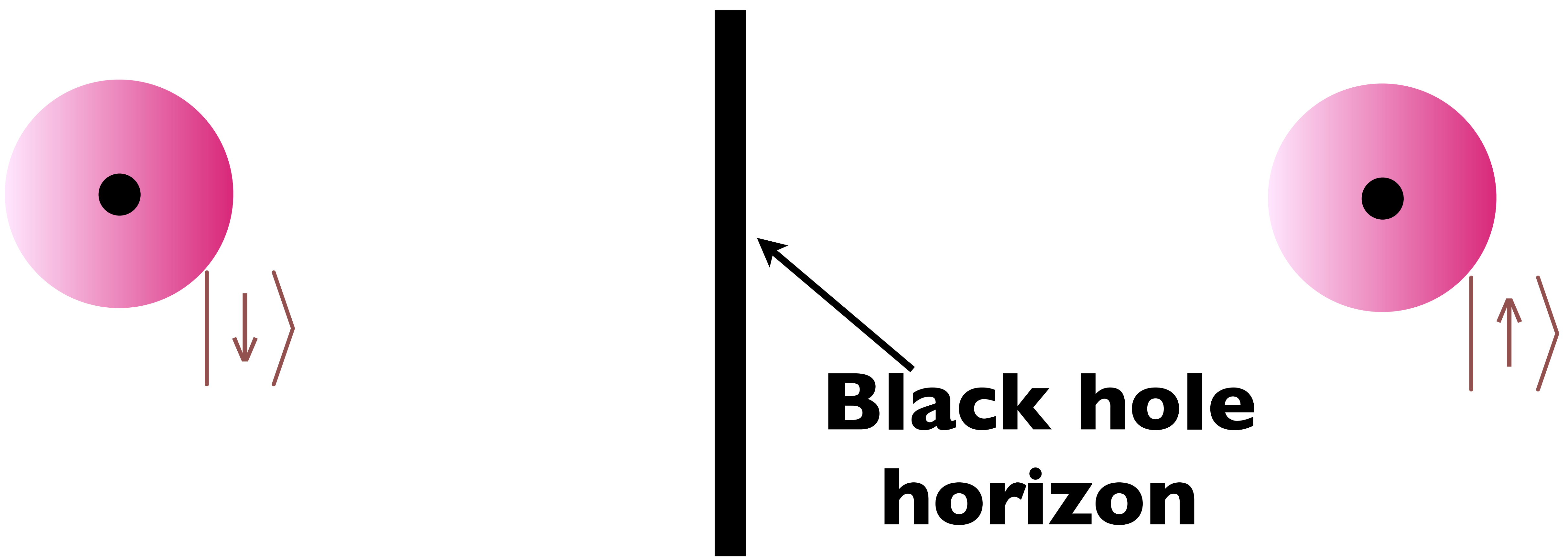


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

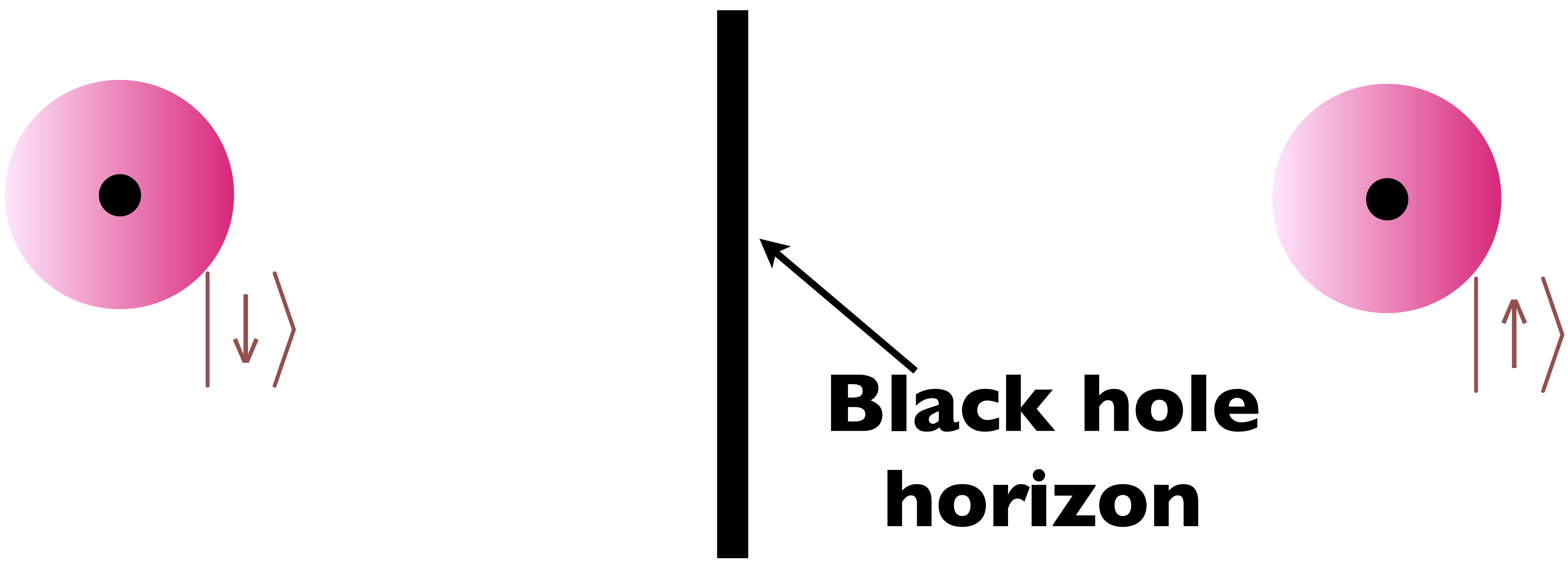
There is long-range quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

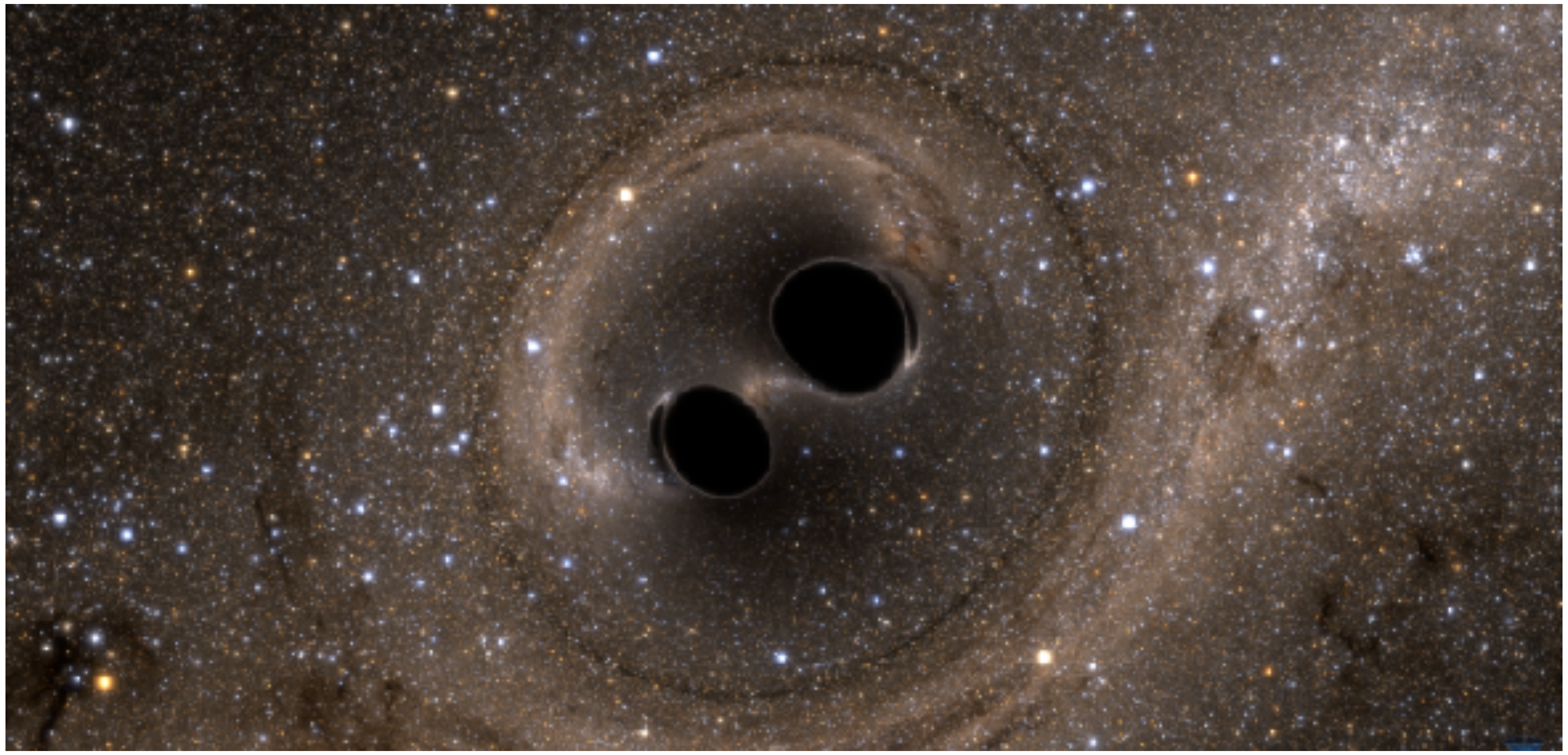
Hawking used this to show that black hole horizons have an entropy and a temperature

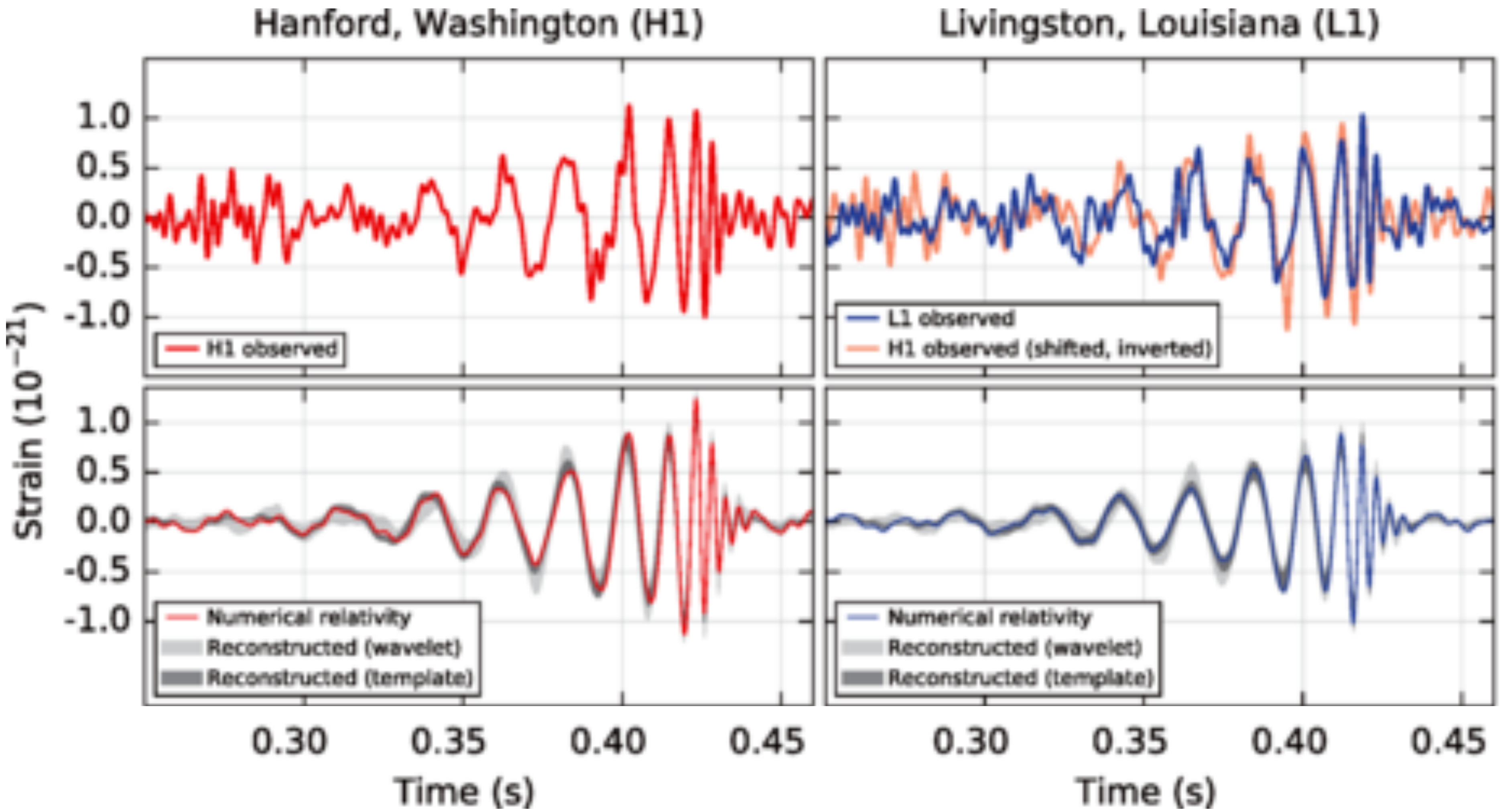
(because to an outside observer, the state of the electron inside the black hole is an unknown)

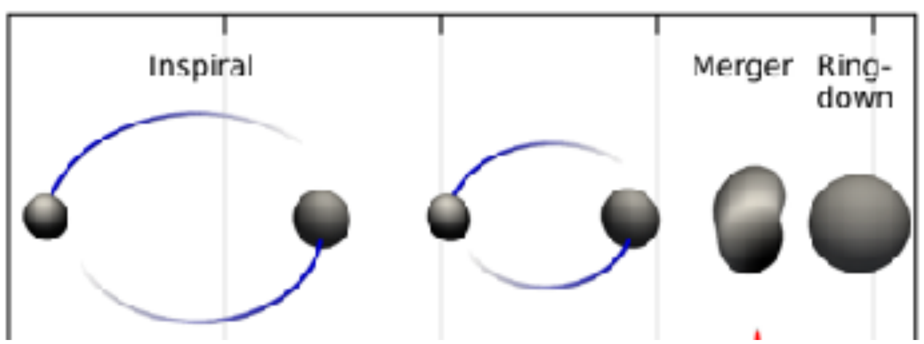


Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.

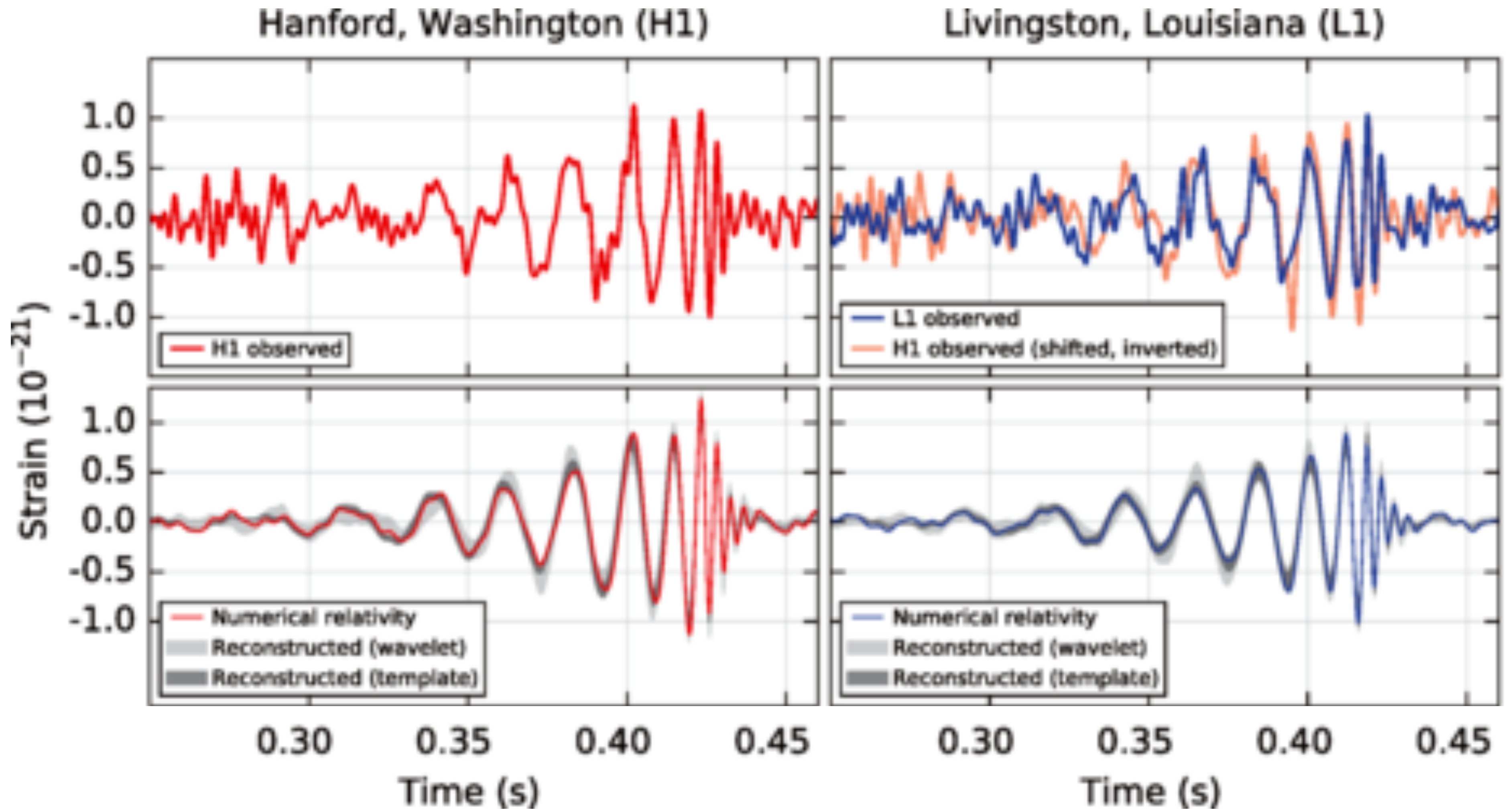


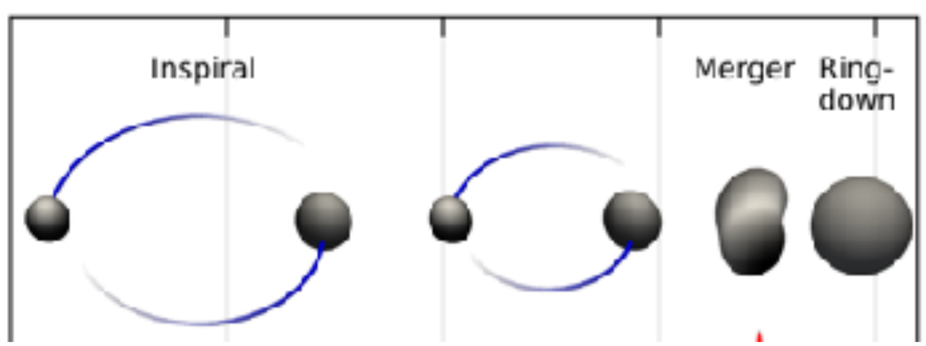




LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds.



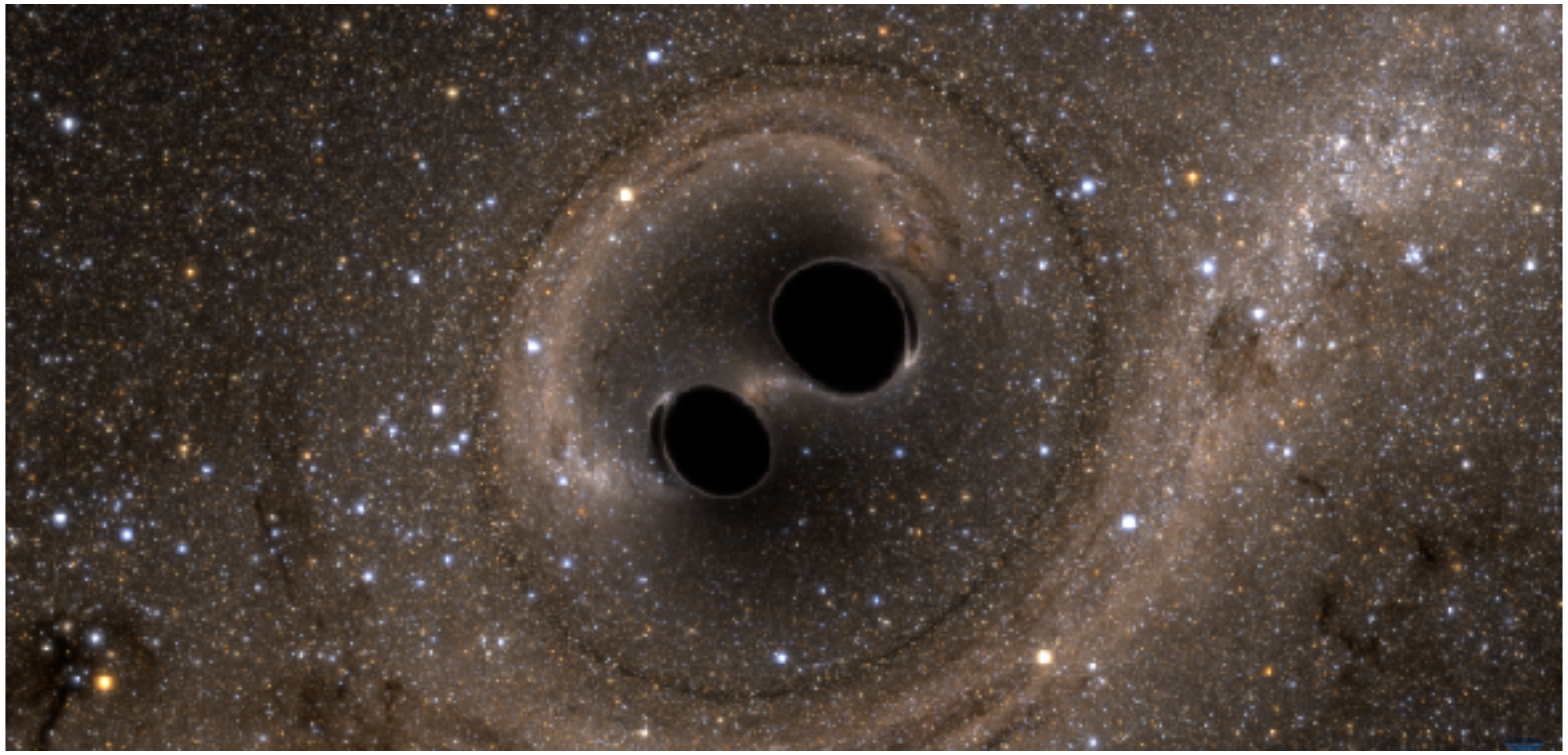


LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\boxed{\dfrac{\hbar}{k_B T_H}}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

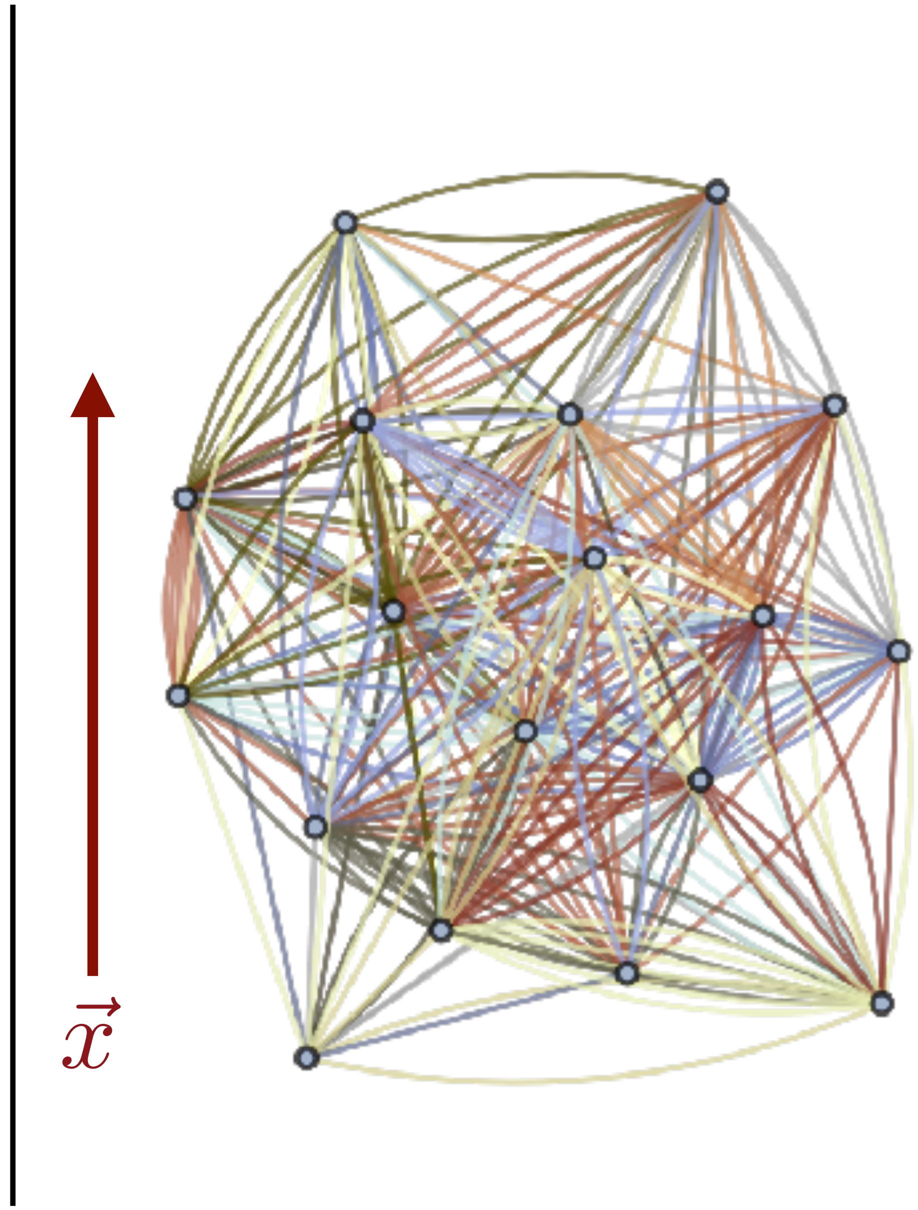
Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.

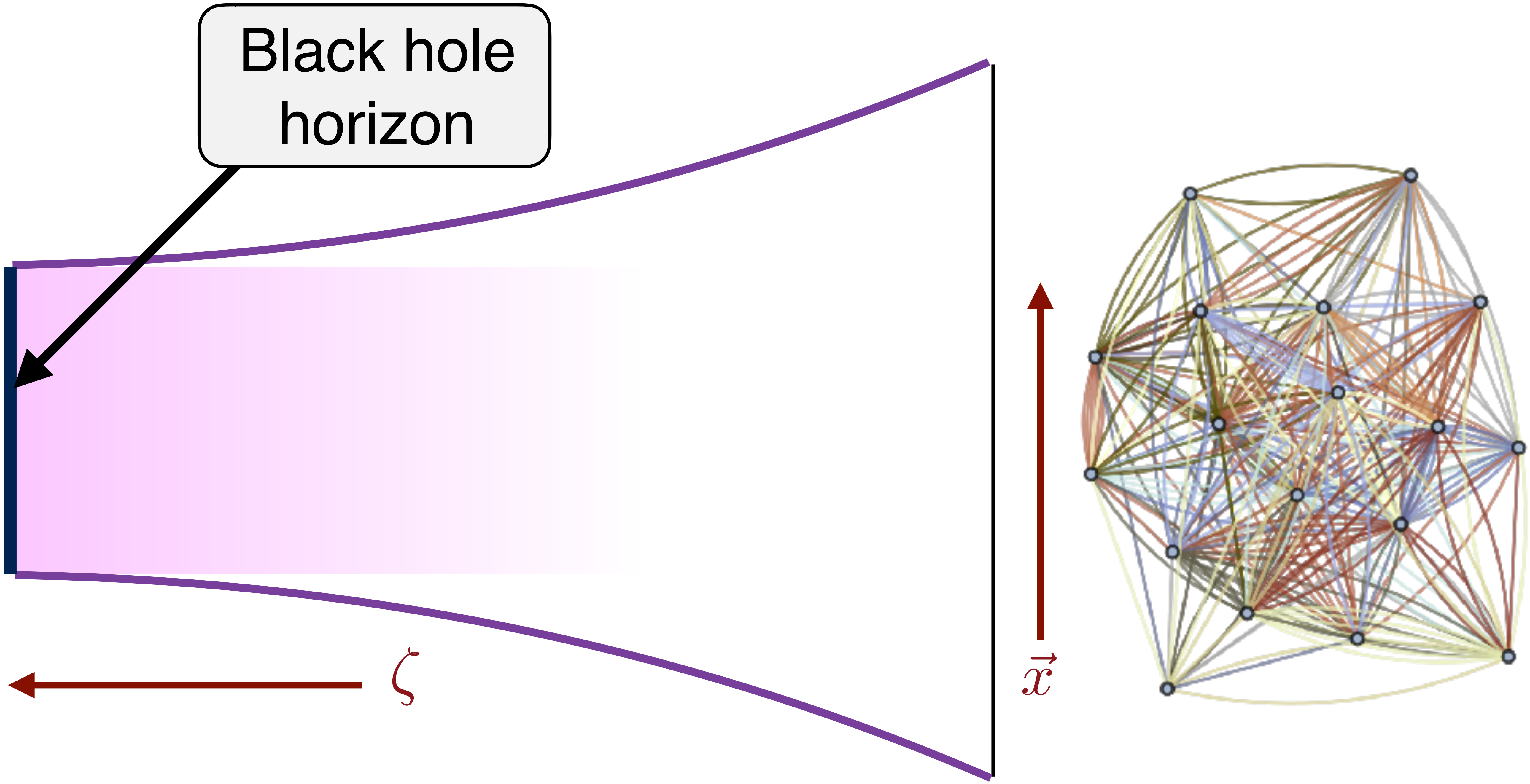


1. Quantum entanglement
2. Quantum matter *with* quasiparticles
3. Quantum matter *without* quasiparticles
4. Black holes
5. The SYK model and black holes

SYK and black holes



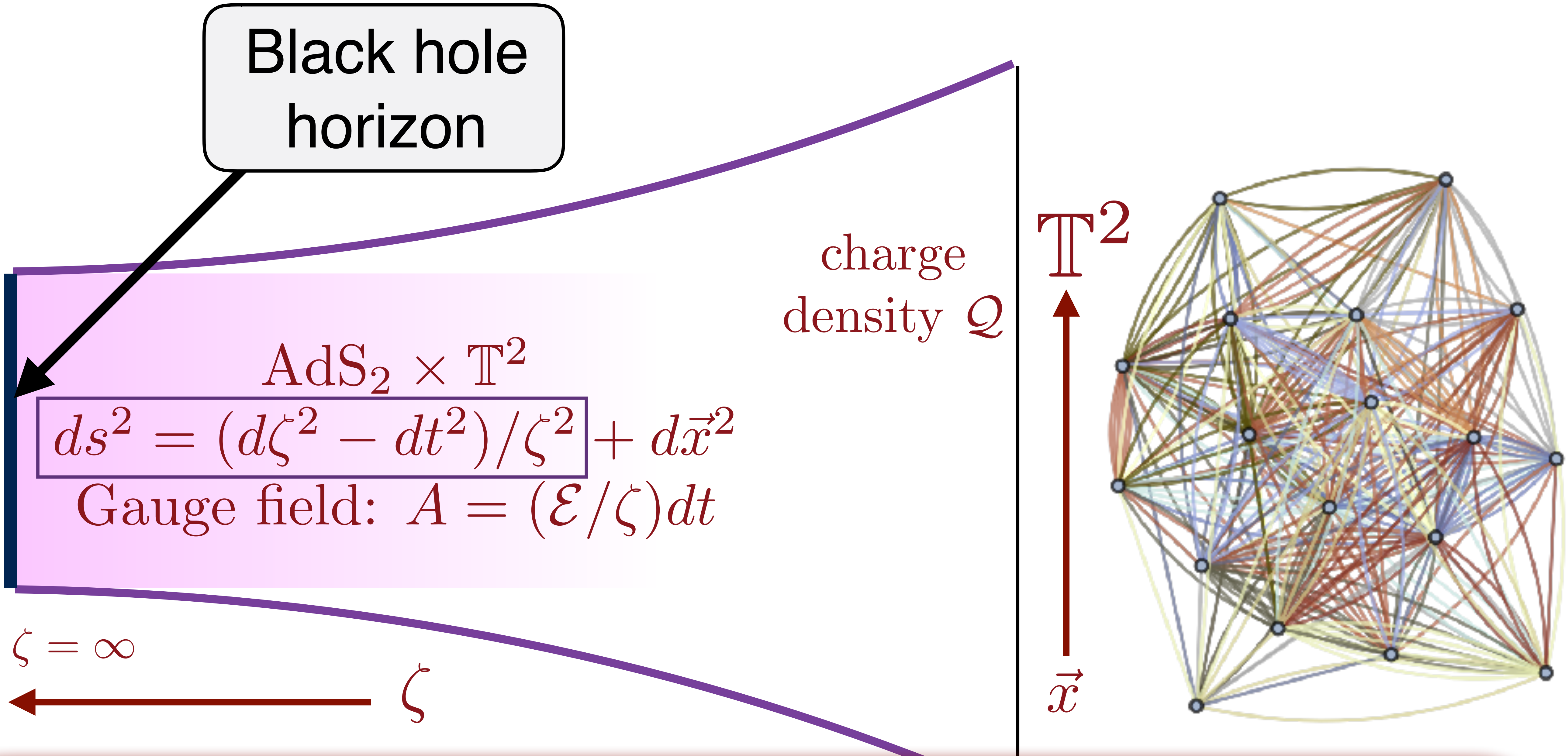
SYK and black holes



The SYK model has "dual" description
in which an extra spatial dimension, ζ, emerges.
The curvature of this "emergent" spacetime is described
by Einstein's theory of general relativity

SS 2010; A. Kitaev, 2015

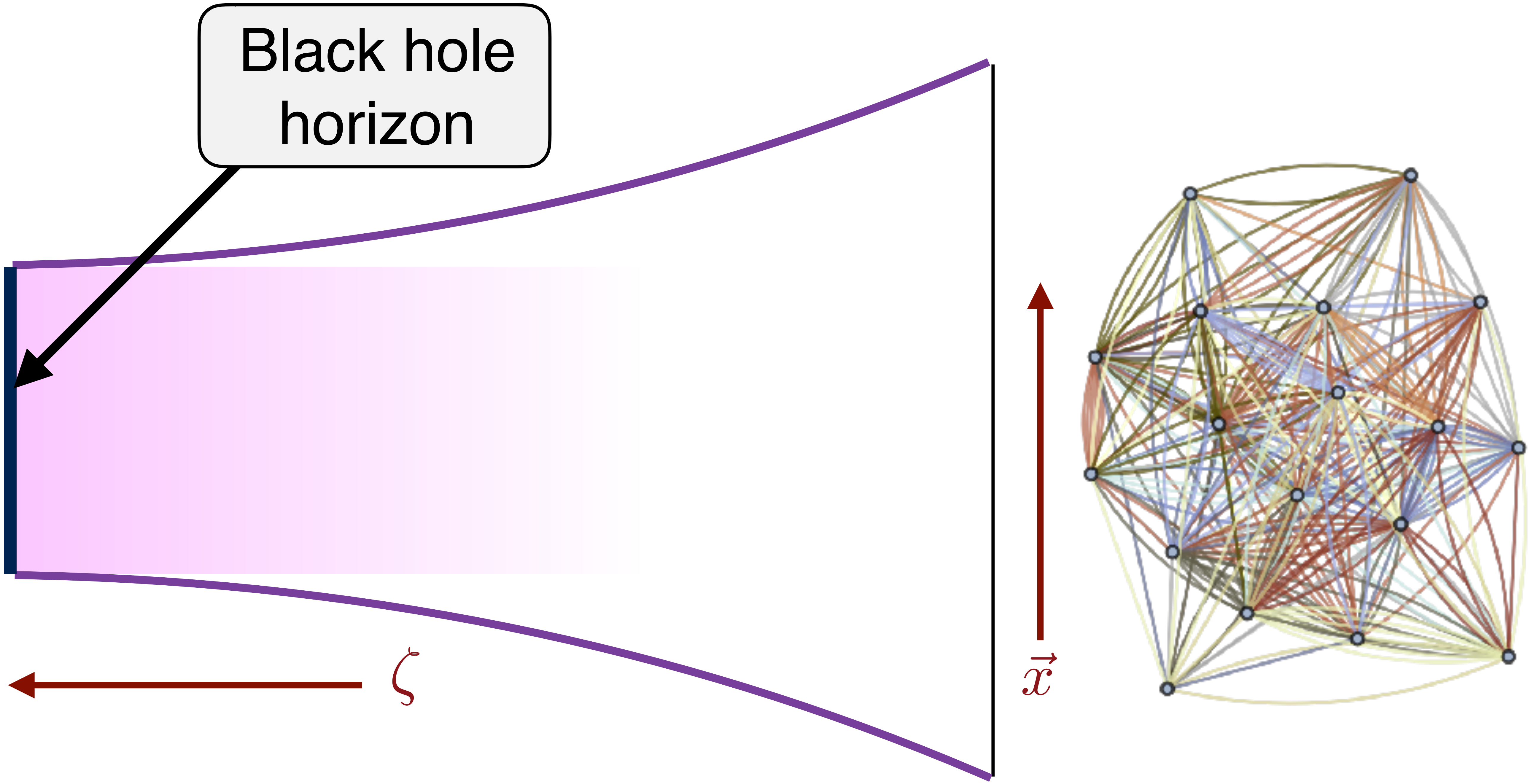
SYK and black holes



Quantum gravity on the 1+1 dimensional spacetime AdS_2 (when embedded in the $RNAdS_4$ solution of Einstein-Maxwell theory) is holographically matched to the 0+1 dimensional SYK model

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (unpublished); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

SYK and black holes

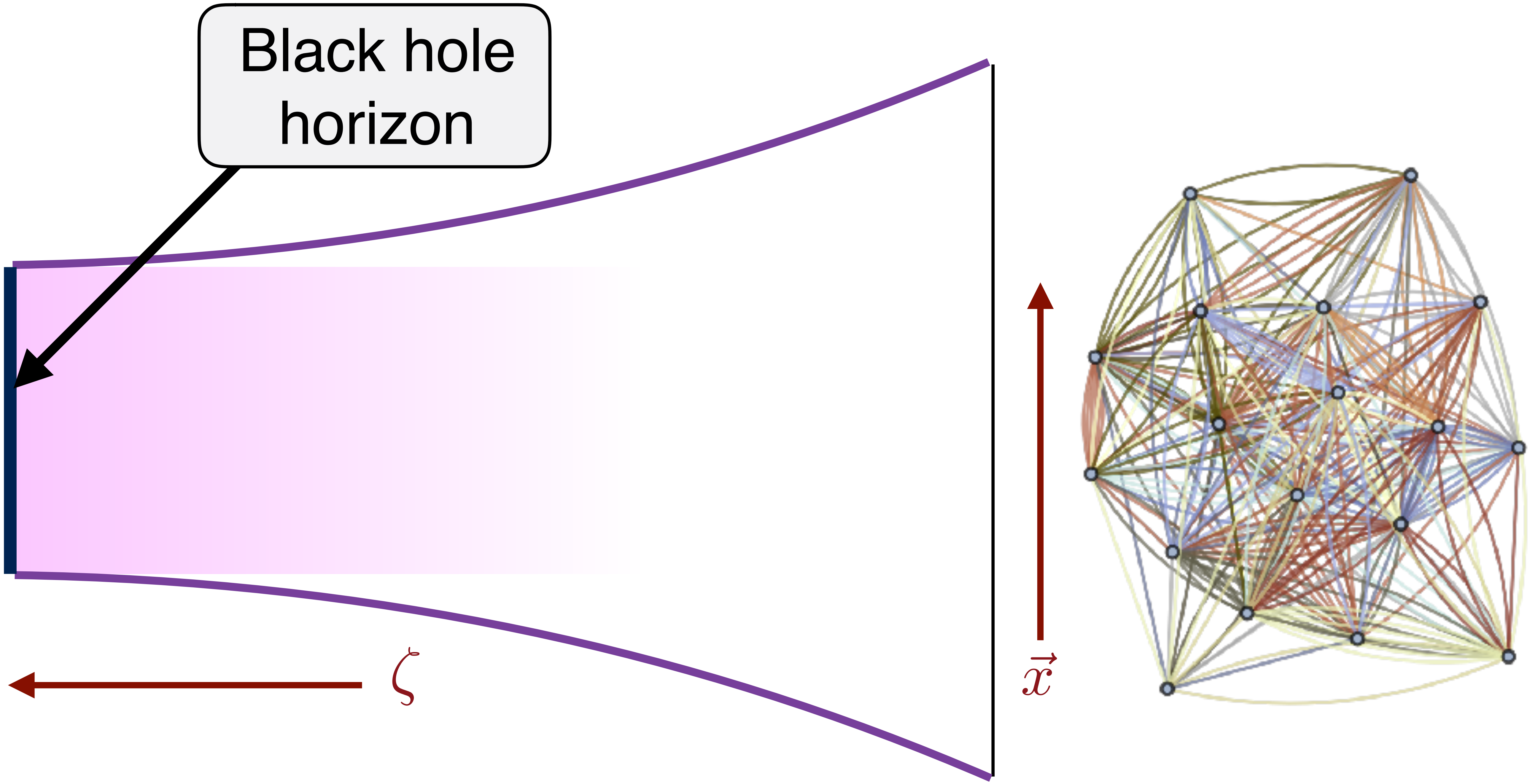


There is a black hole in the emergent spacetime at the same temperature, T_H, as the SYK model.

The duality explains:

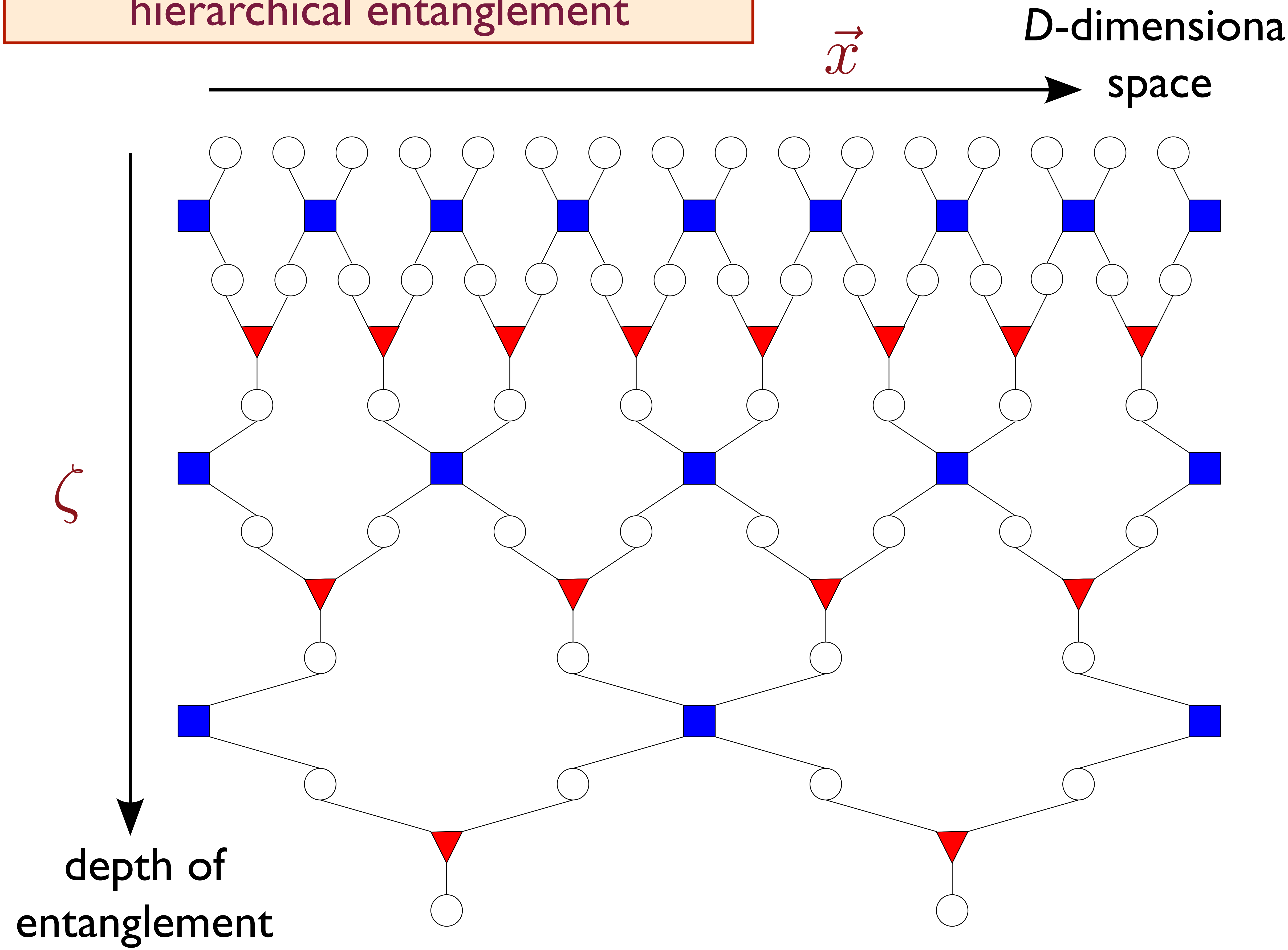
(i) why they have a common thermalization time $\hbar/(k_B T_H)$.

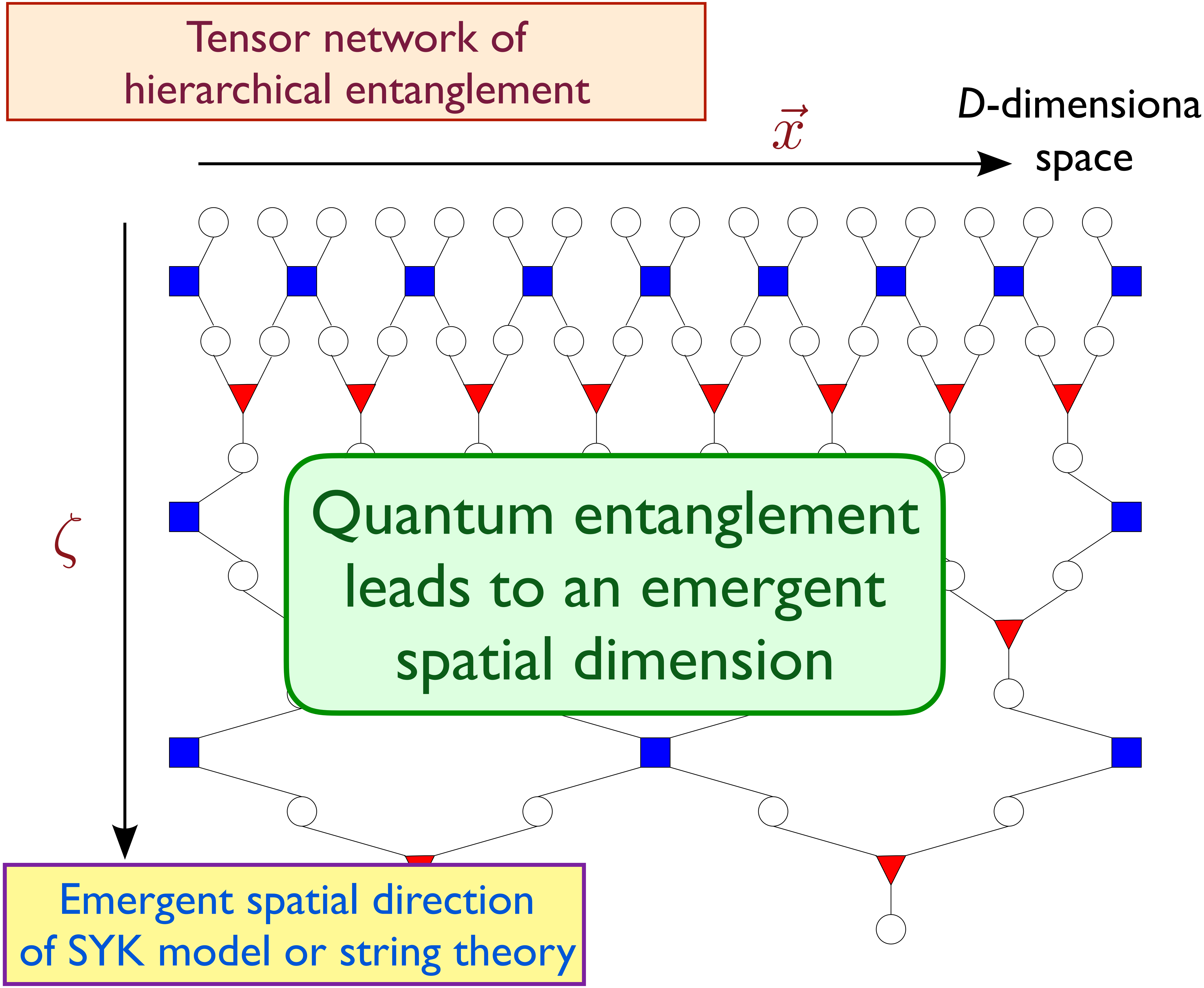
SYK and black holes



There is a black hole in the emergent spacetime
at the same temperature, T_H, as the SYK model.
The duality explains:
(ii) why the black hole entropy is proportional to its surface area.

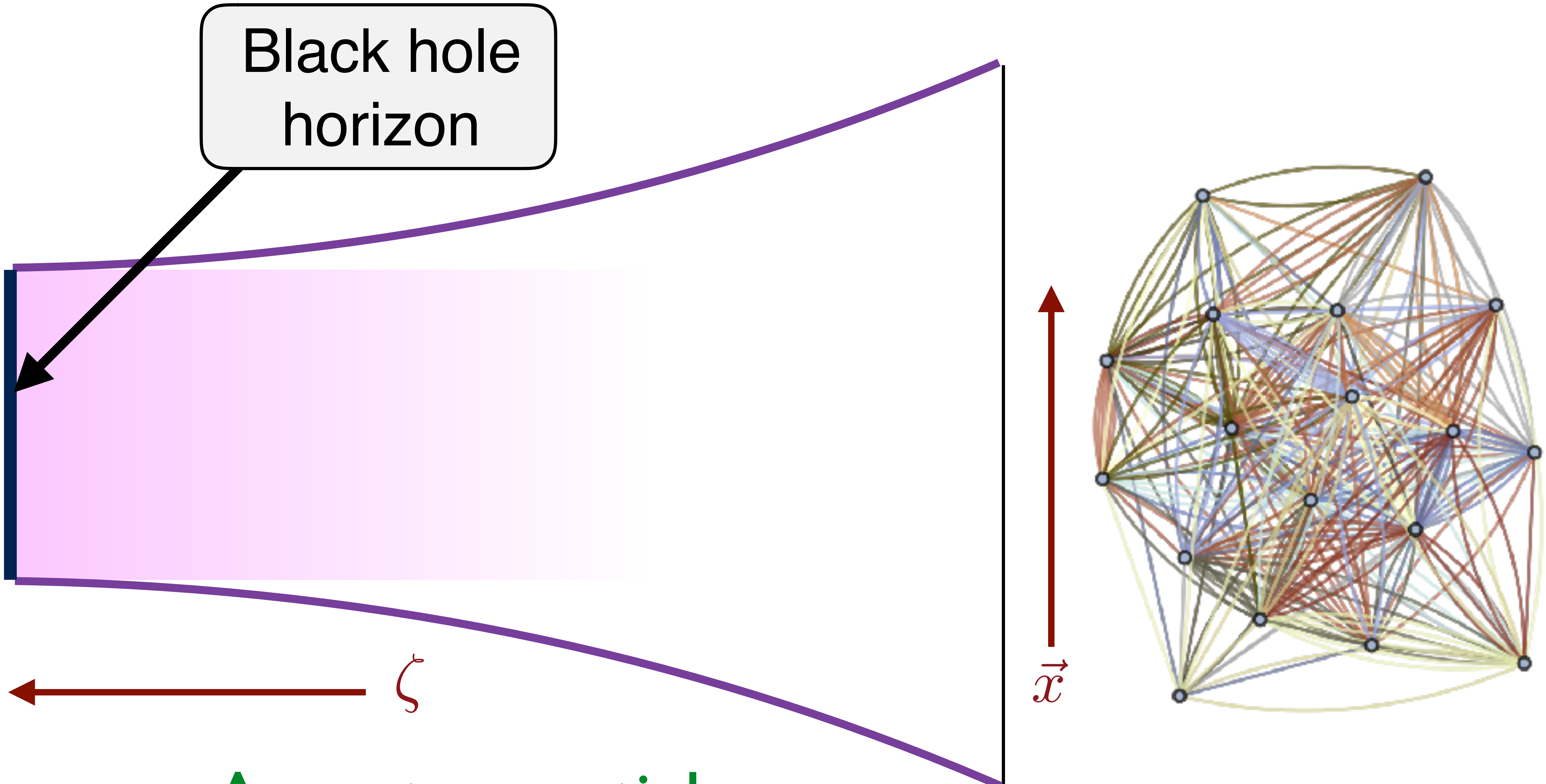
Tensor network of hierarchical entanglement





Tensor network of
hierarchical entanglement
$\vec{x}$
D-dimensional
space
ζ
Quantum entanglement
leads to an emergent
spatial dimension
Emergent spatial direction
of SYK model or string theory

SYK and black holes



An extra spatial dimension emerges from quantum entanglement!

SS 2010; A. Kitaev, 2015

Quantum matter <u>without</u> quasiparticles:

- No quasiparticle decomposition of low-lying states.
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.

Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states.
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.
- These are also characteristics of black holes in quantum gravity.