Quantum criticality in condensed matter: field theory vs. gauge-gravity duality

Northeastern University, May 1, 2013

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Talk online at sachdev.physics.harvard.edu

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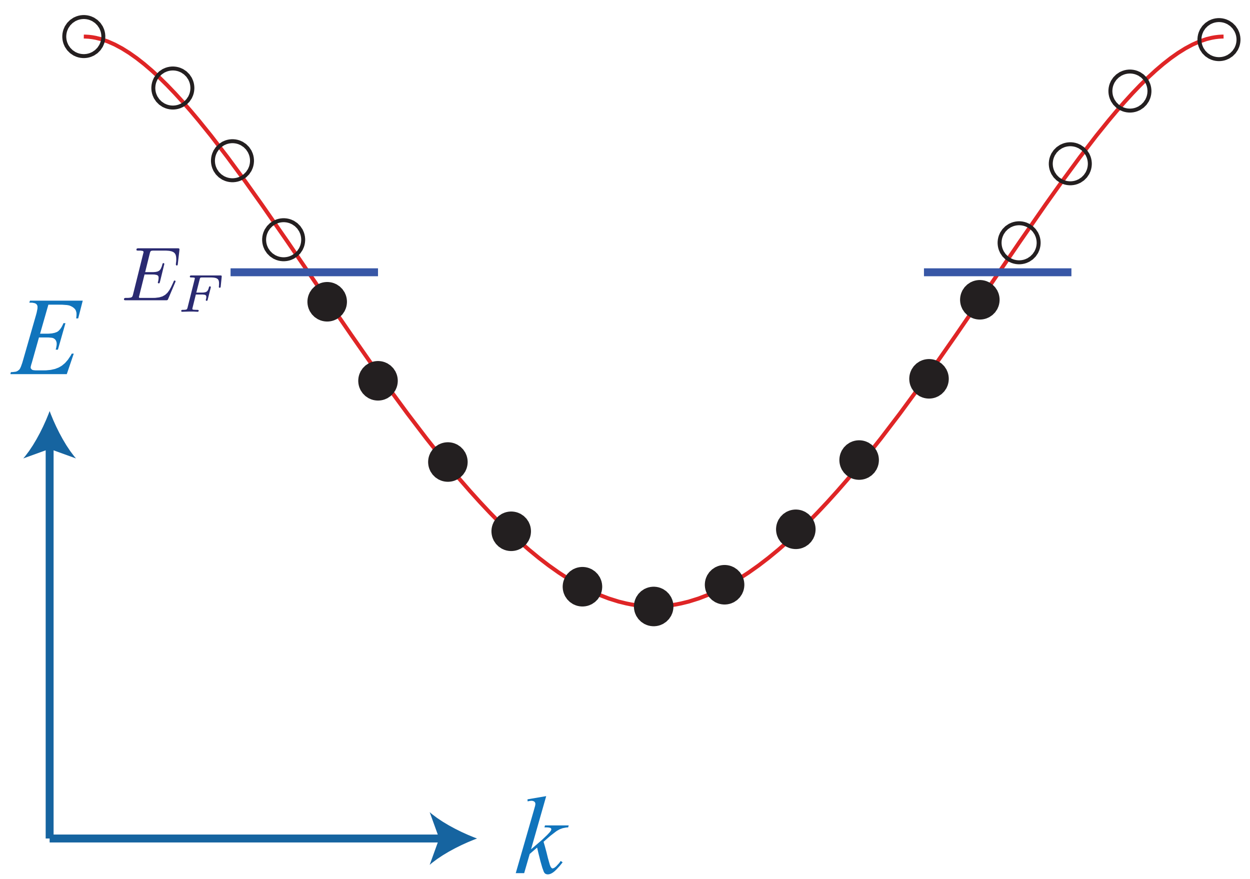
Sommerfeld-Pauli-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

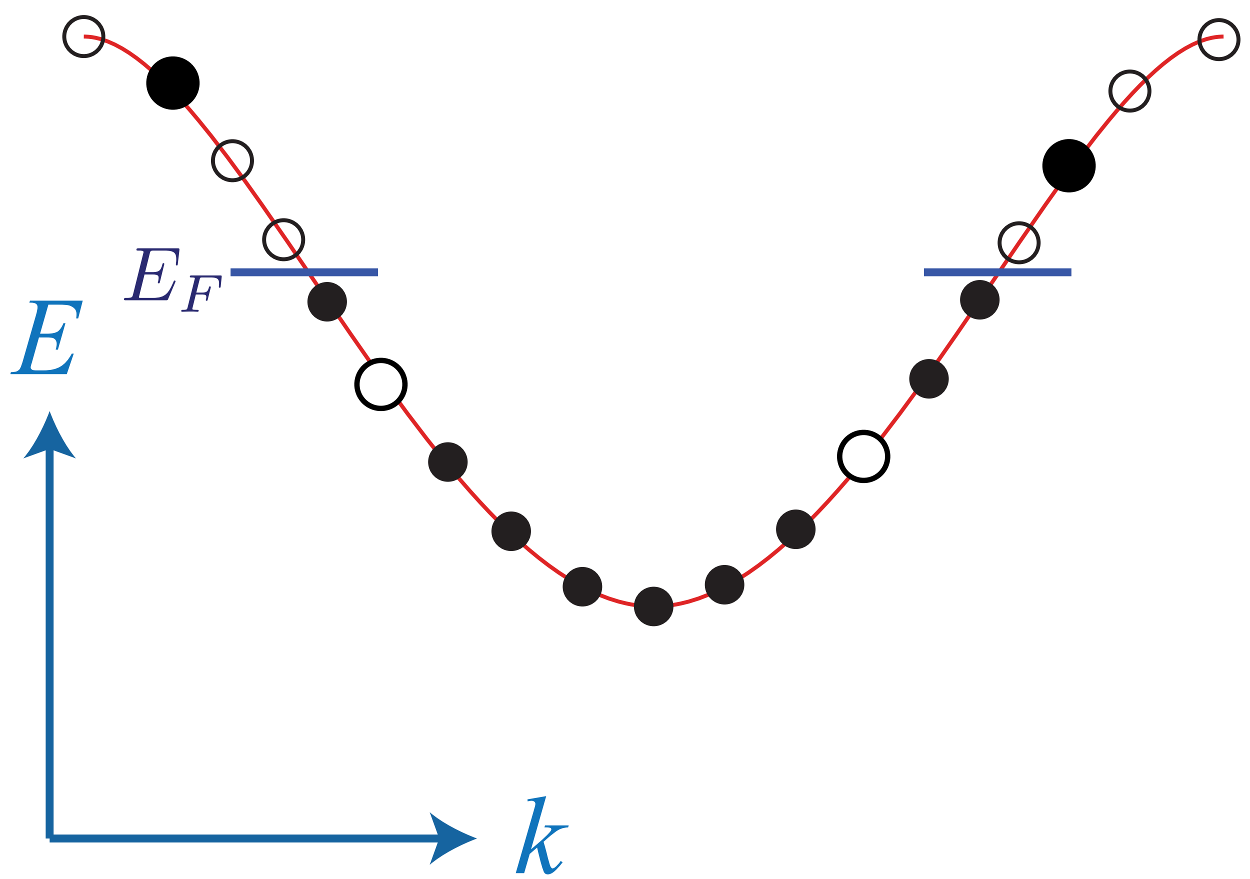
Metals



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

and no quasiparticles

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Quantum criticality and conformal field theories

2. Gauge-gravity duality
   Black-hole horizons and quasi-normal modes

3. Strange metals:
   What lies beyond the horizon ?

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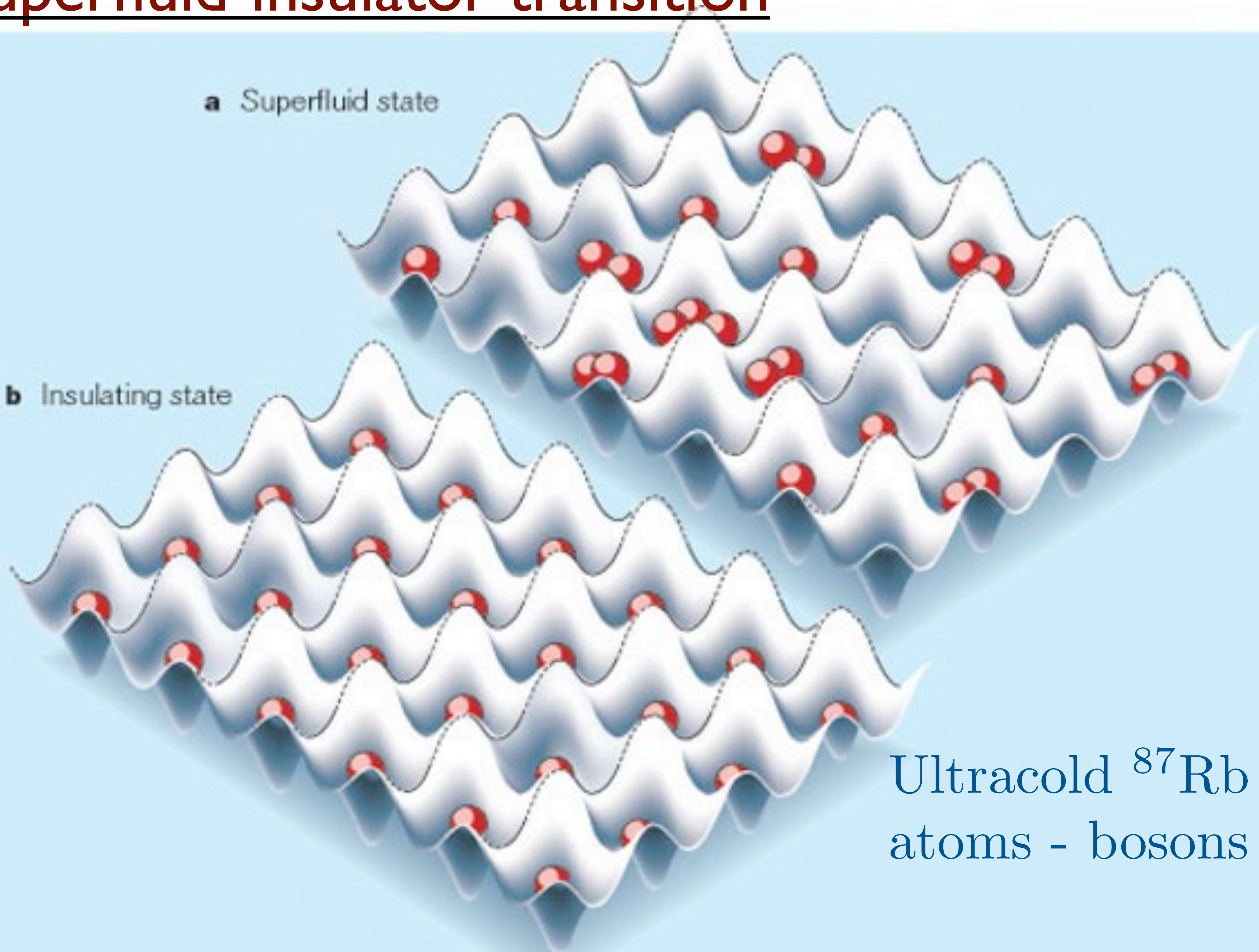
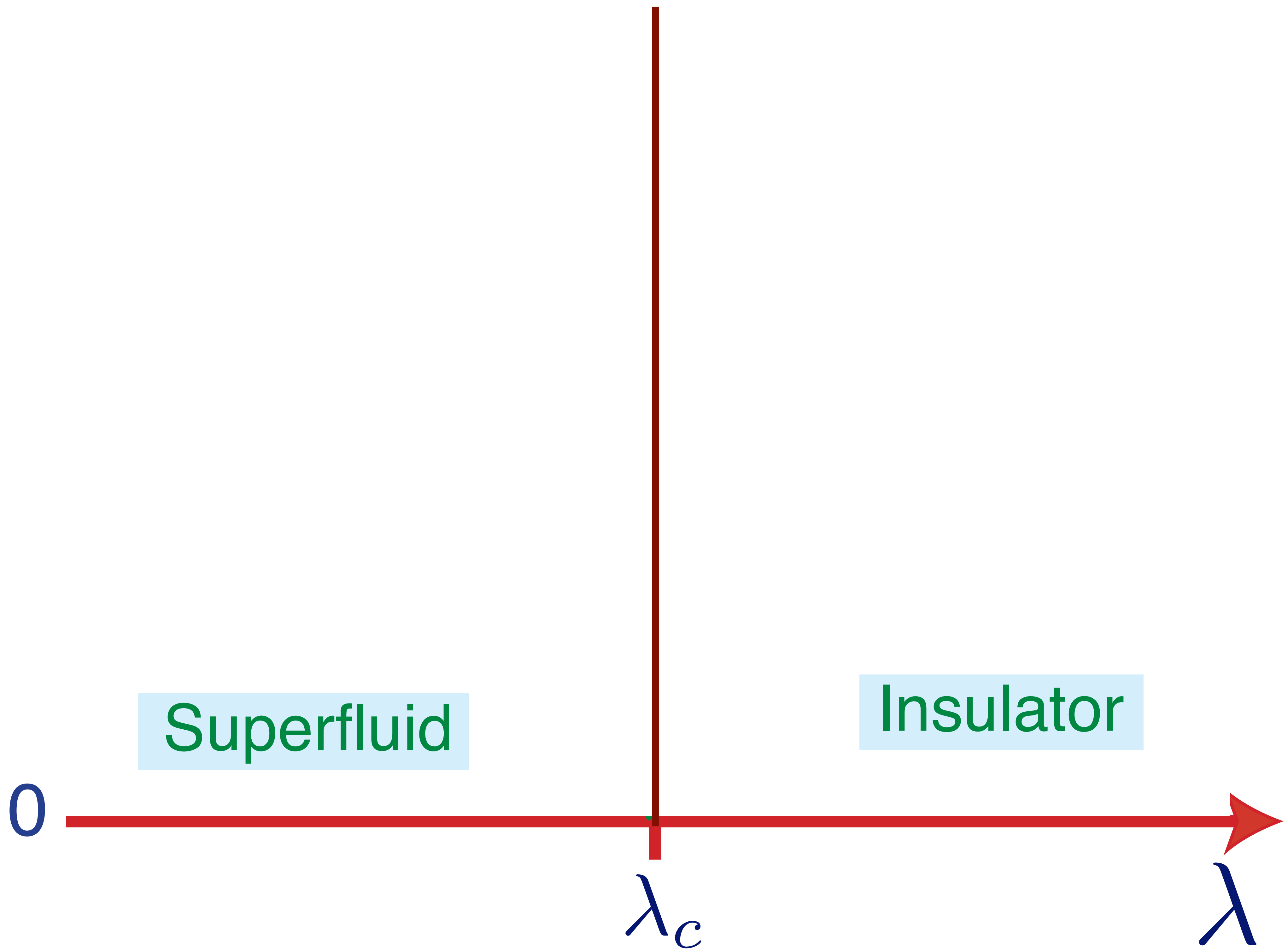
Superfluid-insulator transition

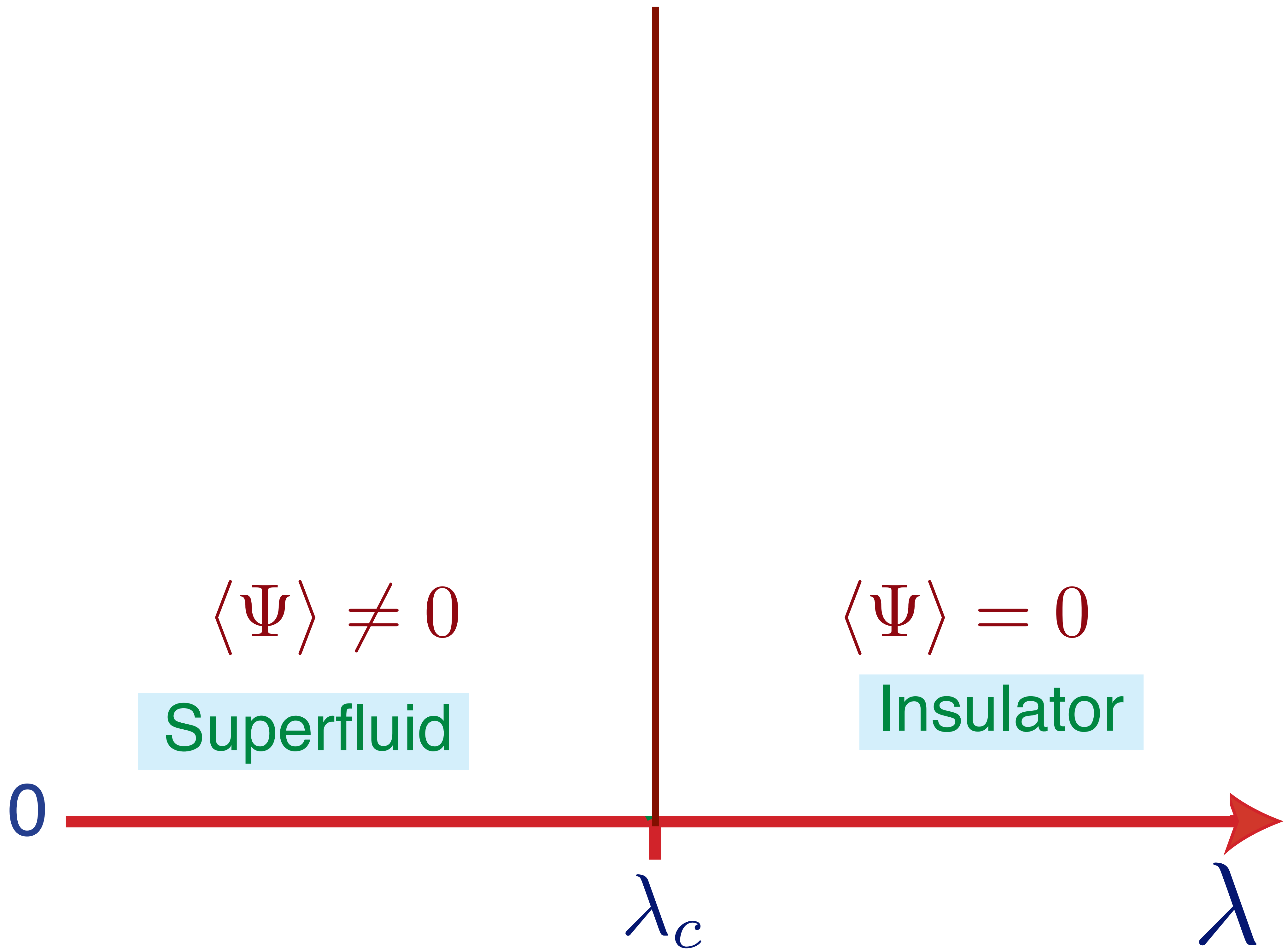


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

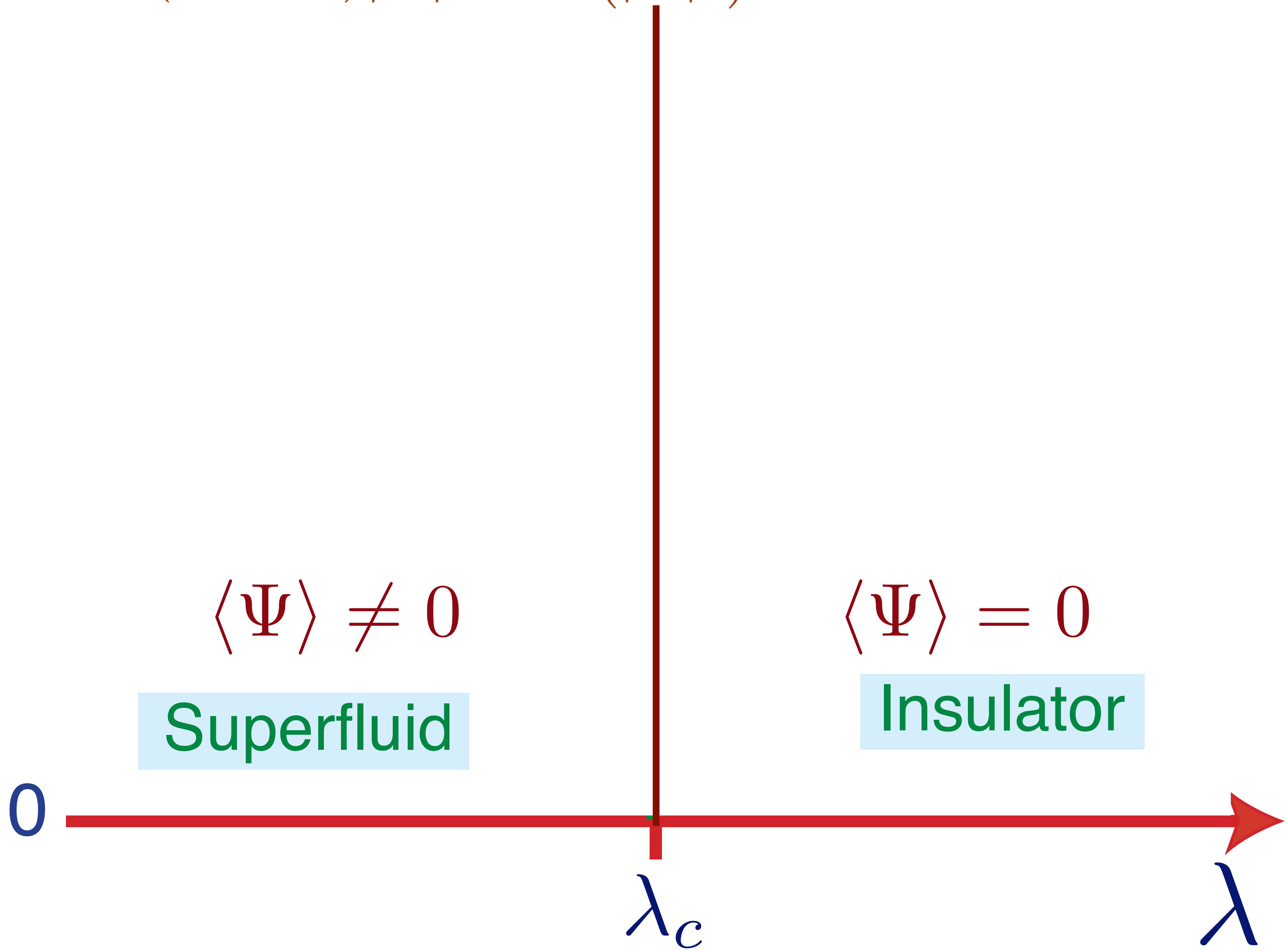

Superfluid
Insulator
0
λ_c
λ

$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



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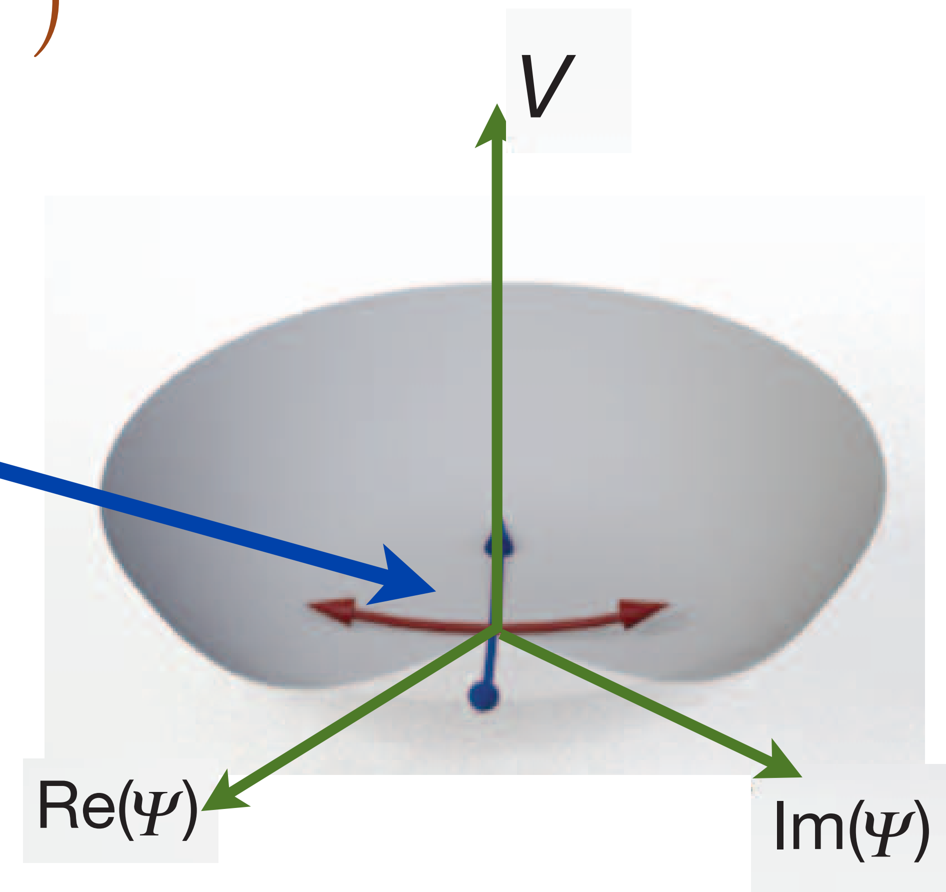
Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



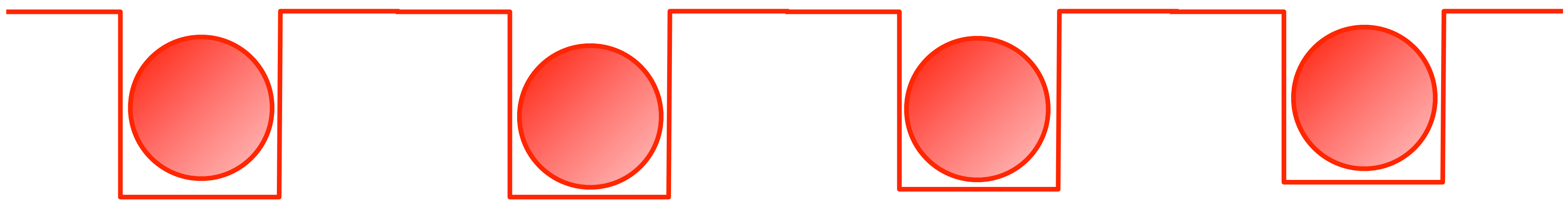
$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

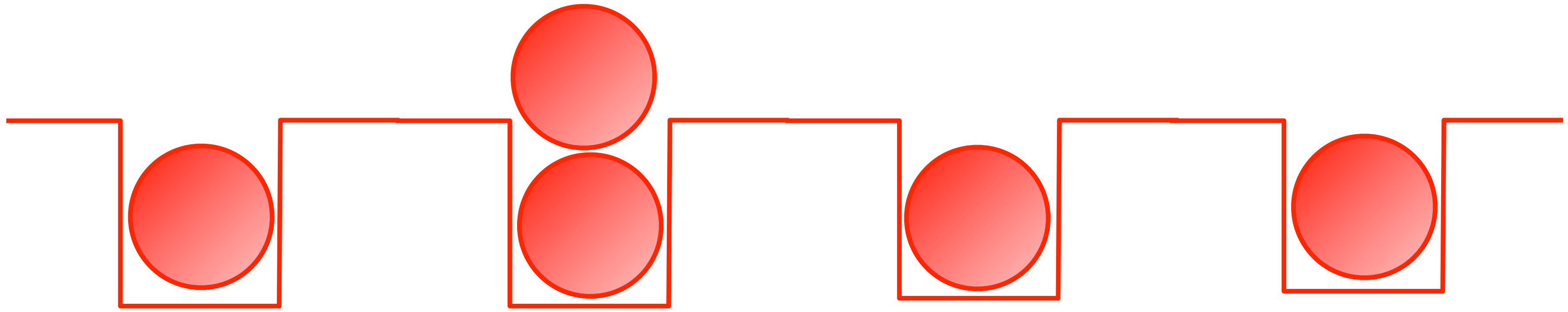
Insulator

0 λ_c λ



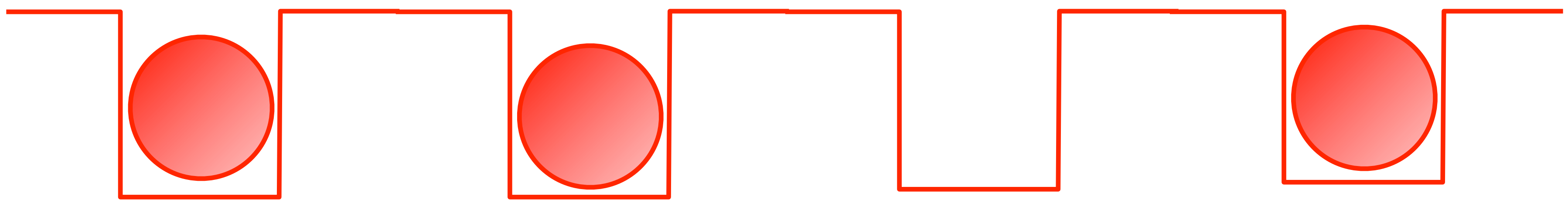
Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



$$\text{Particles} \sim \Psi^\dagger$$

Excitations of the insulator:



Holes $\sim \Psi$

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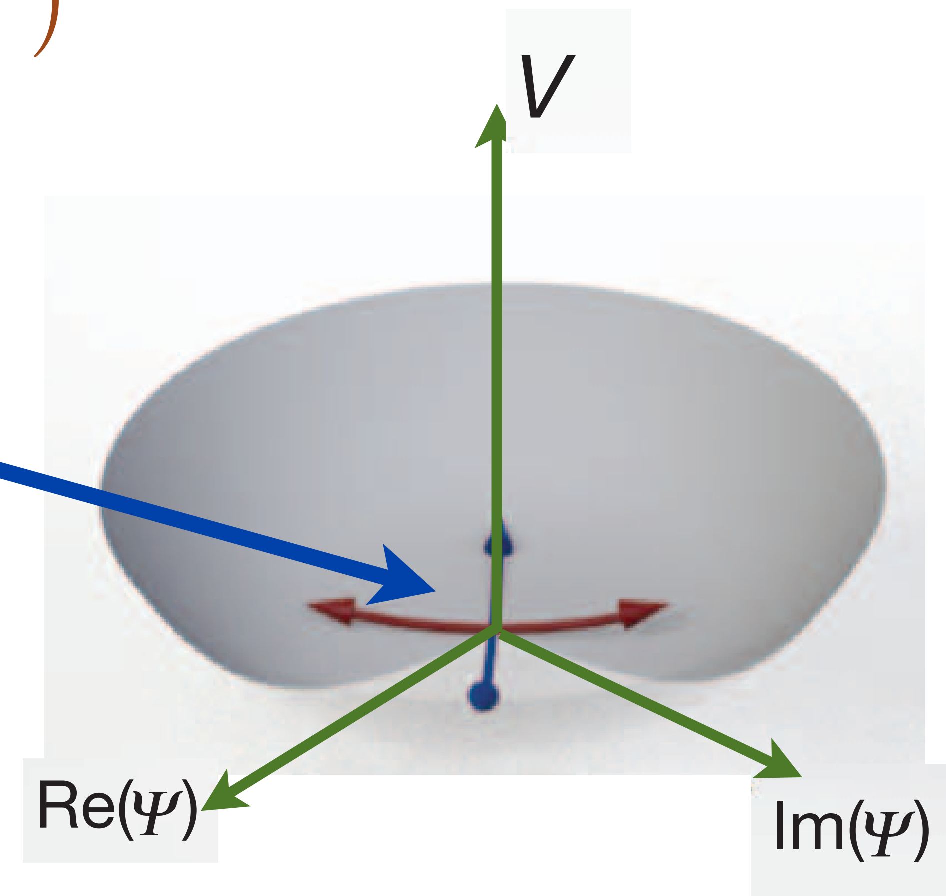

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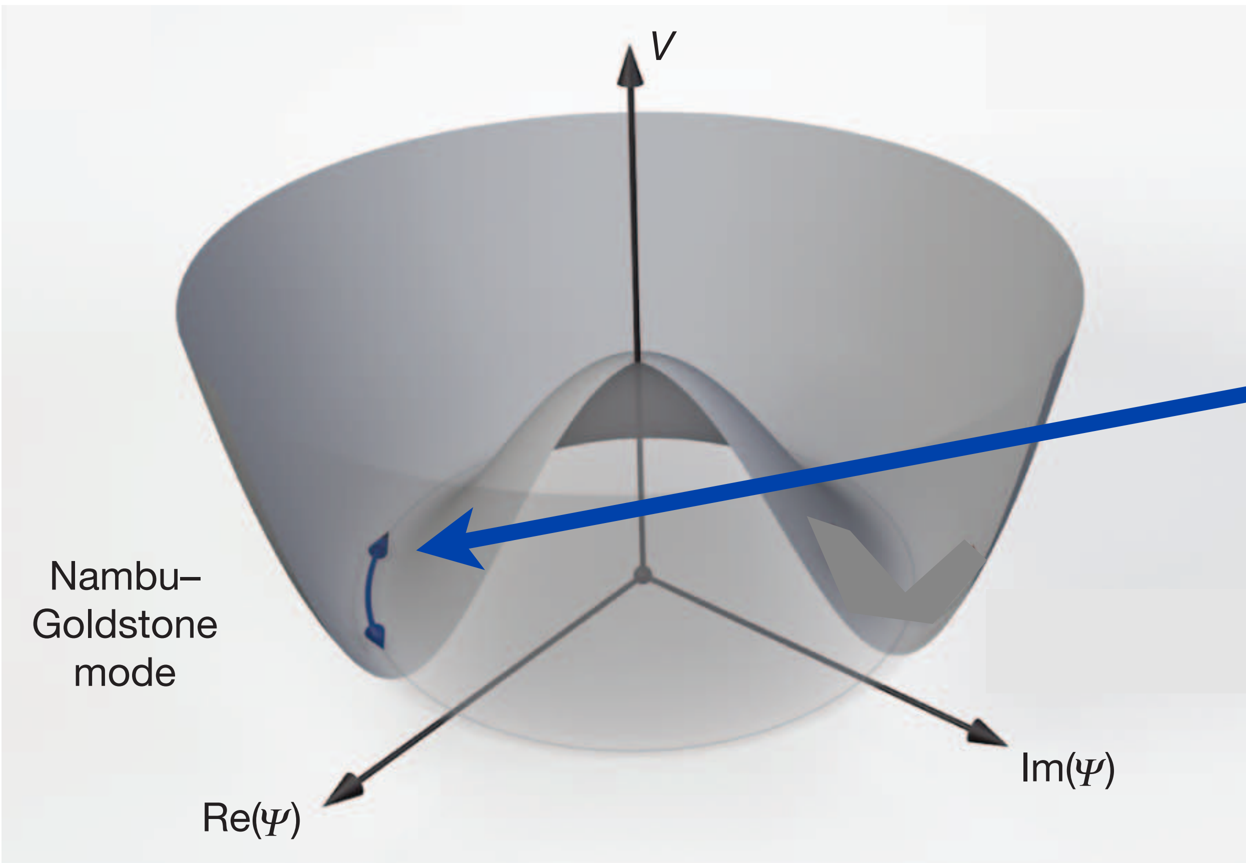

Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0 $\quad \lambda_c \quad \lambda$

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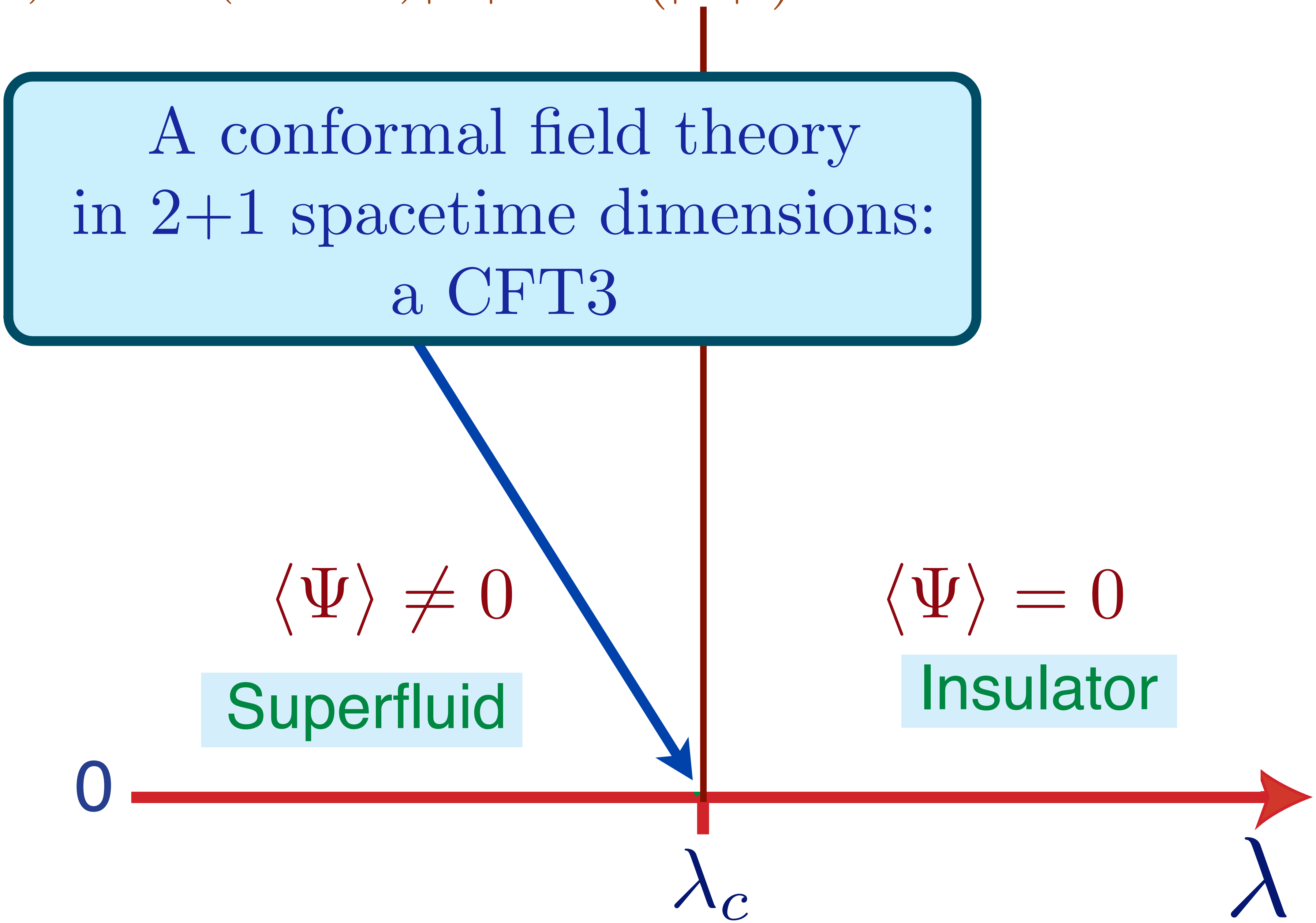

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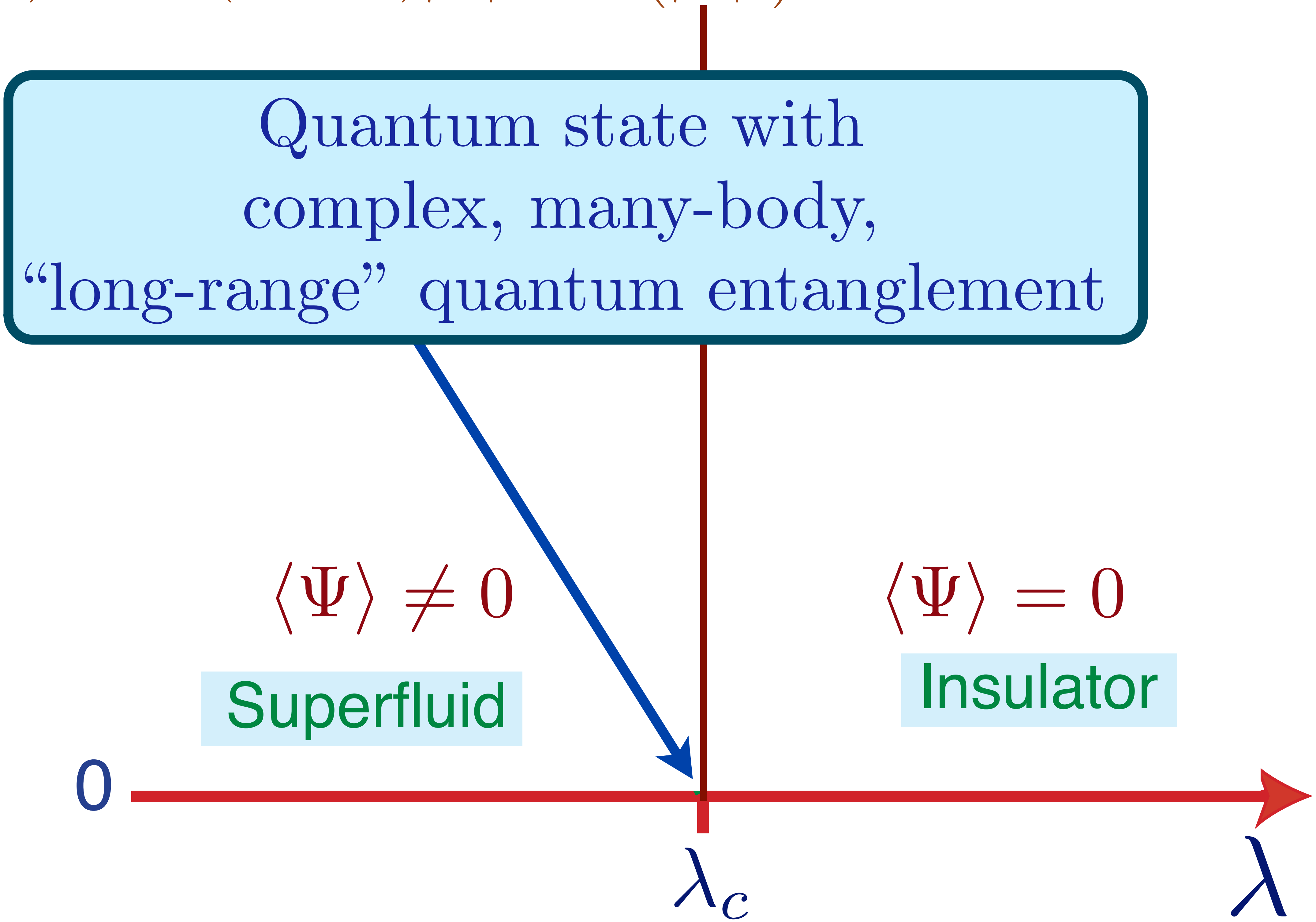

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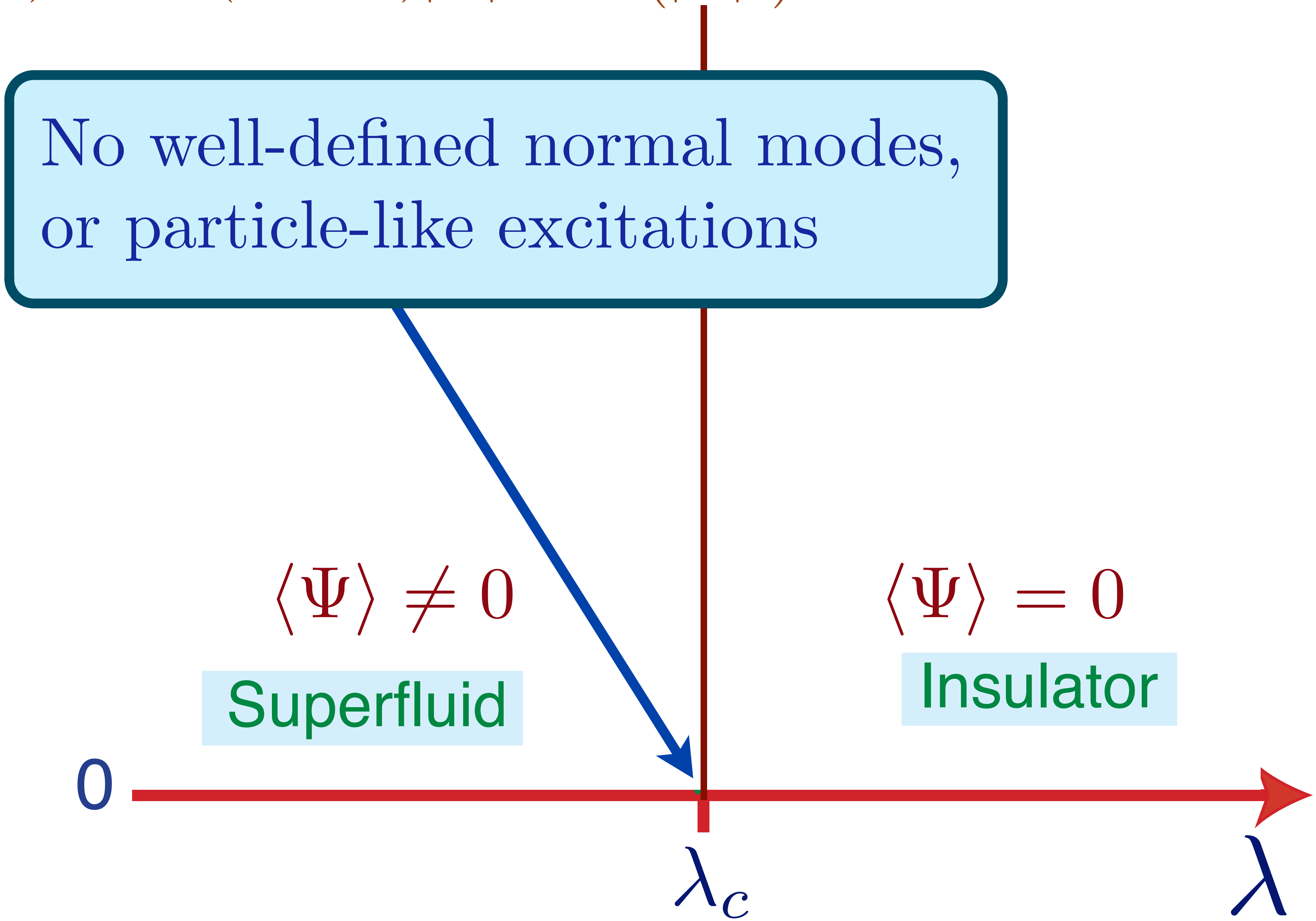

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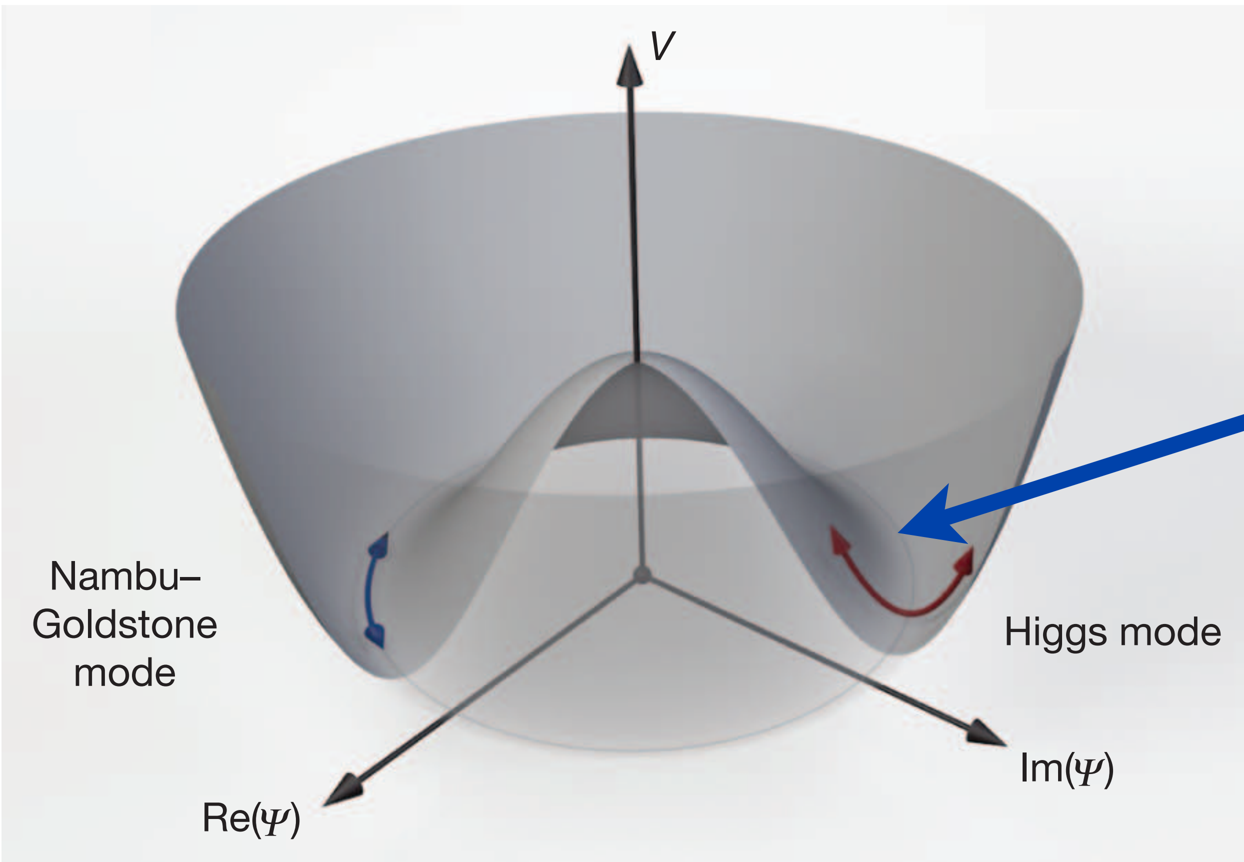


Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$\langle \Psi \rangle \neq 0$

Superfluid

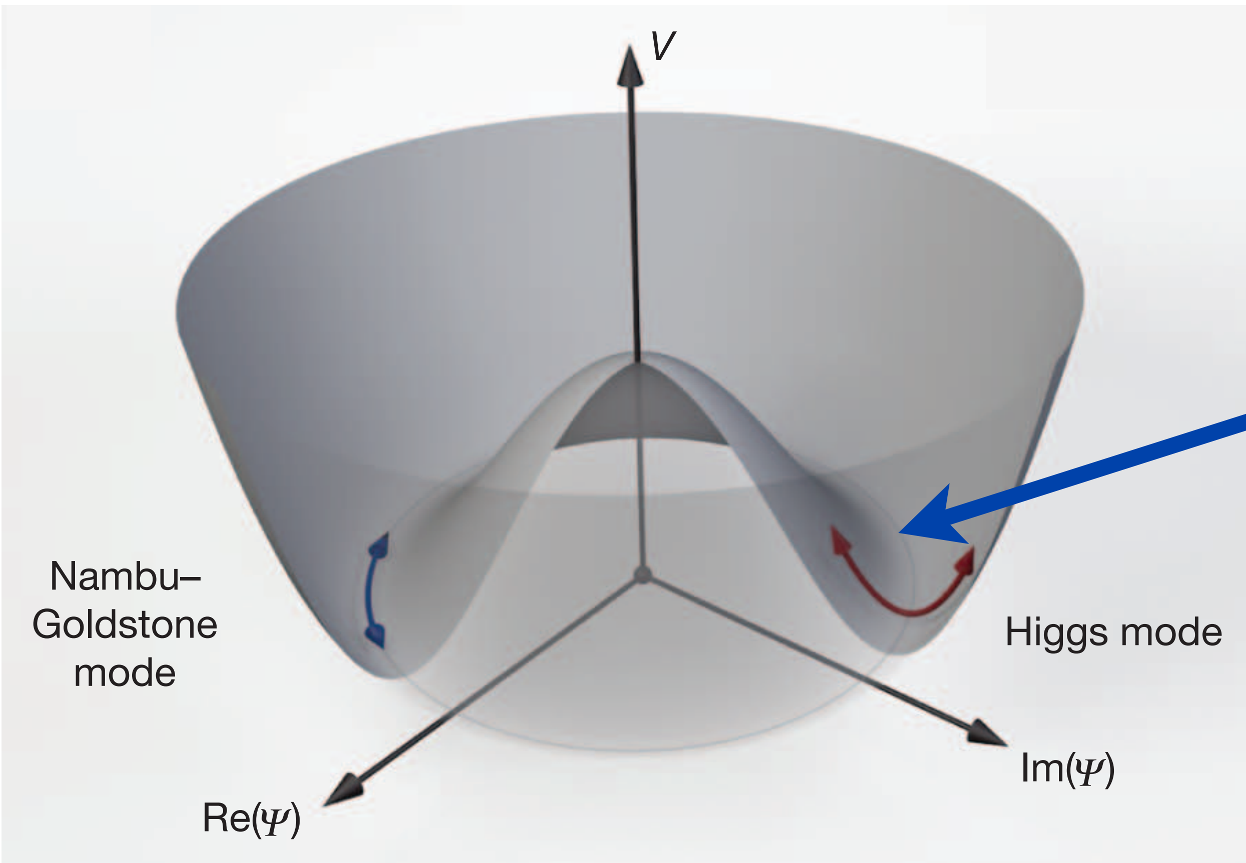
$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



Despite rapid decay,
there is a well-defined
Higgs "quasi-normal mode".
This is associated with a pole
in the lower-half
of the complex frequency plane.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0 λ_c λ

D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

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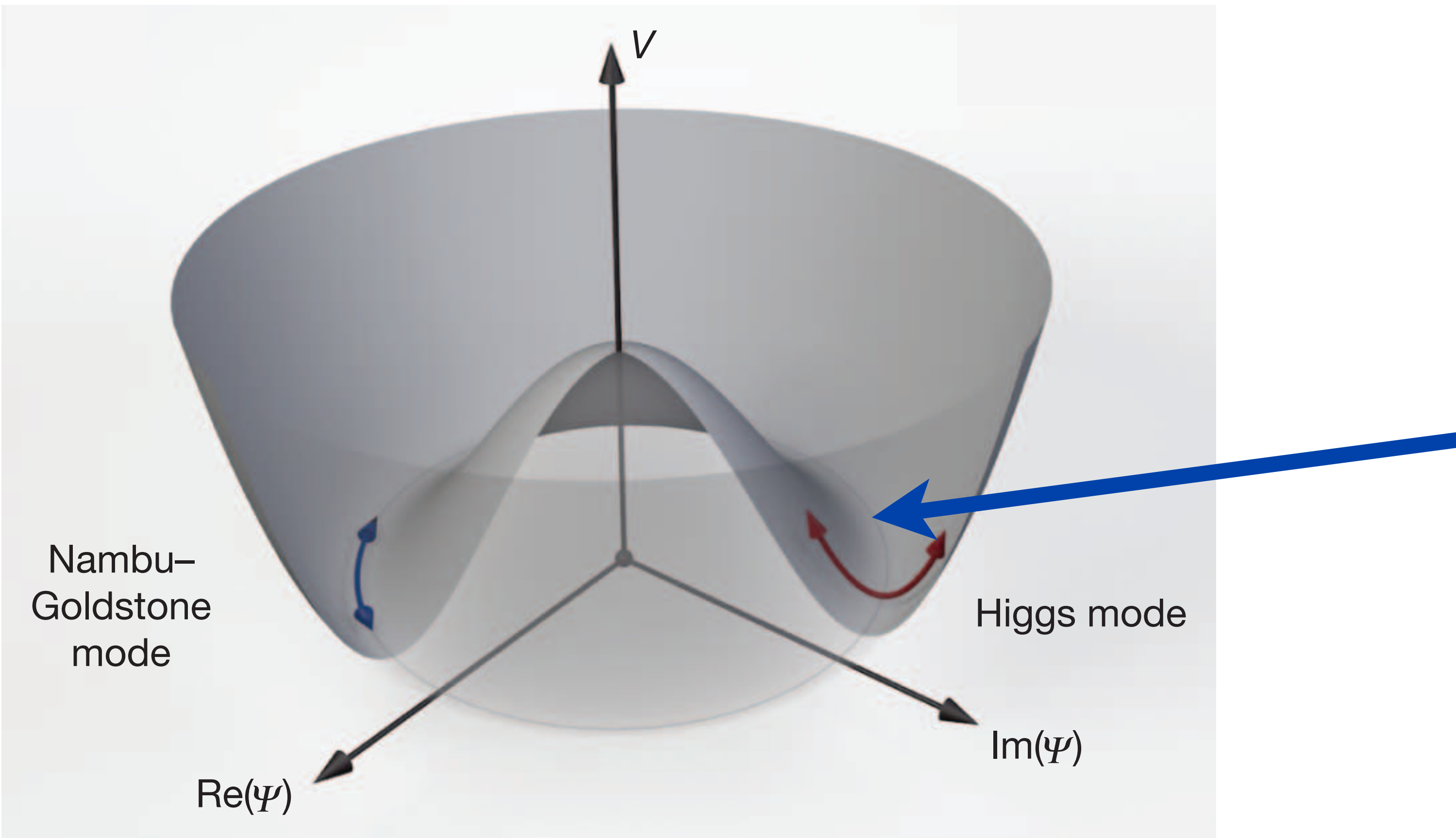
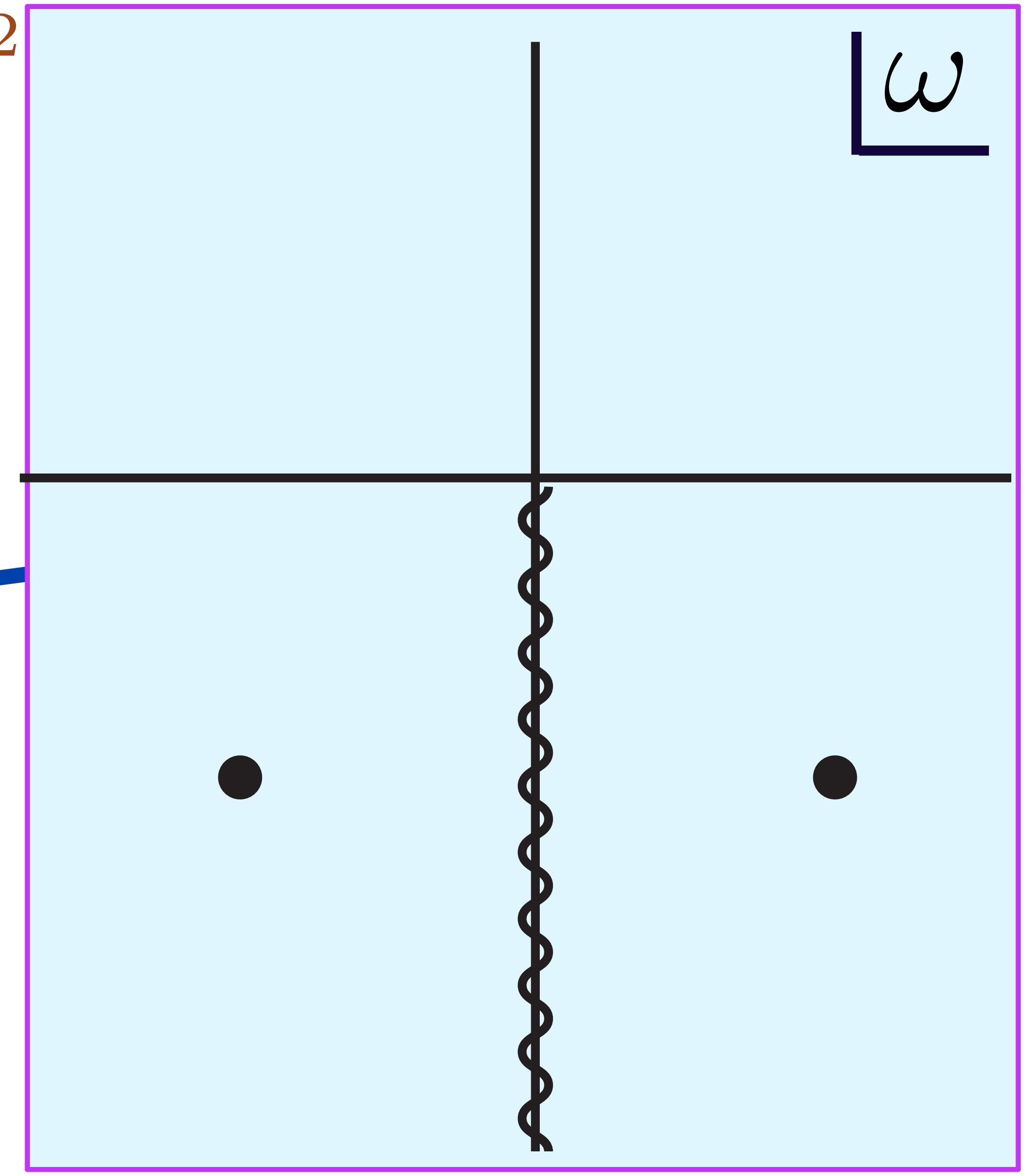




D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

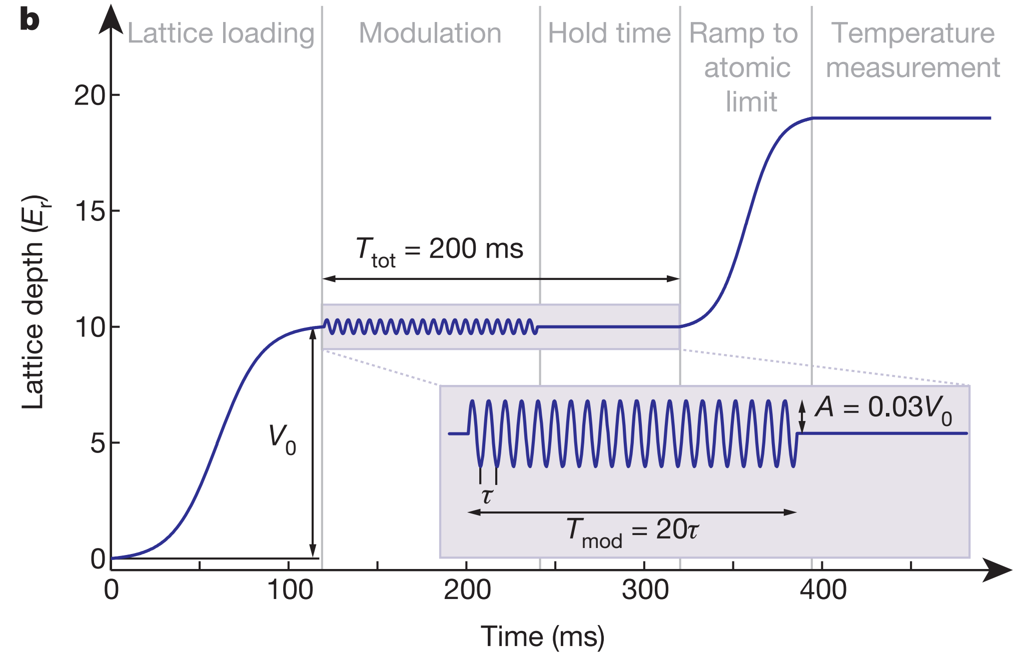
The Higgs quasi-normal mode is at the frequency

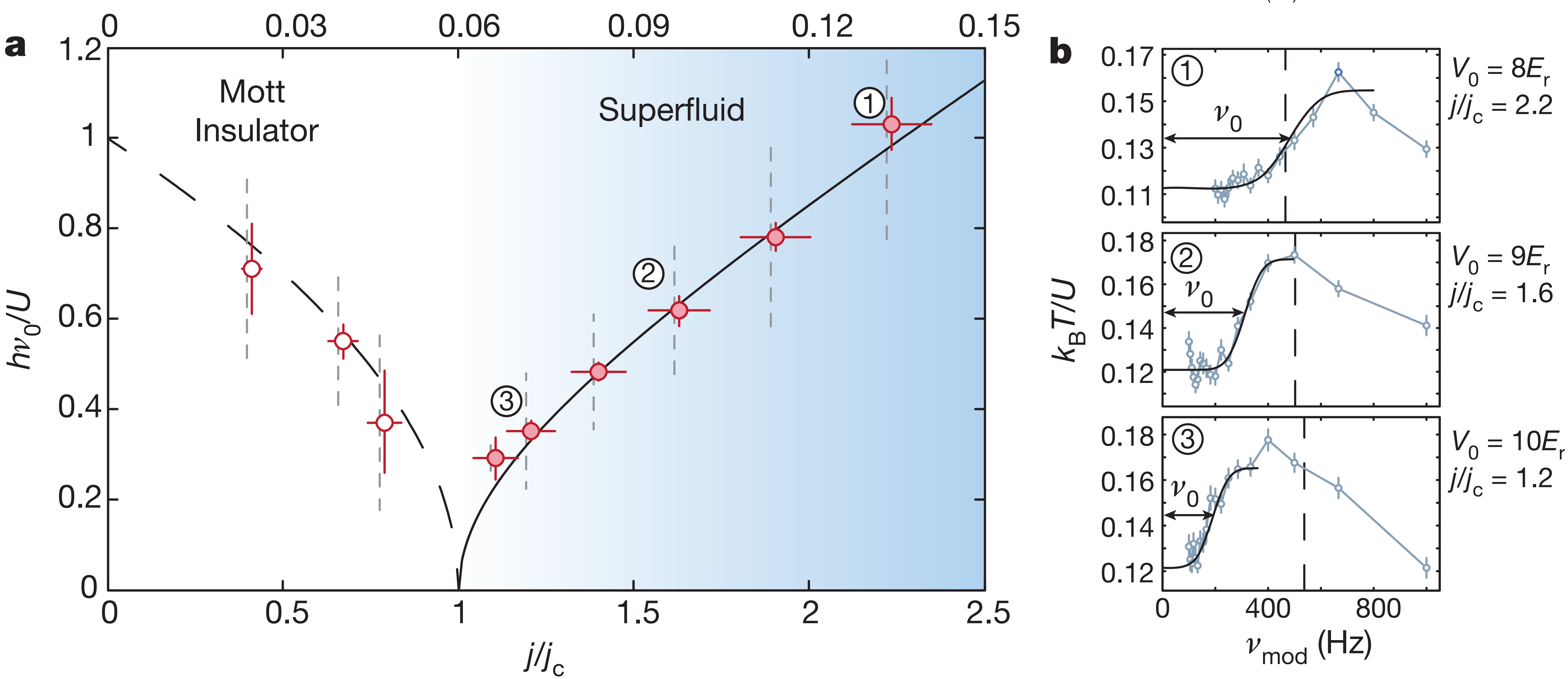
$$\frac{\omega_{\rm pole}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4 + \sqrt{2}\log\left(3 - 2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\,i \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the "paramagnetic" state with the same value of $|\lambda - \lambda_c|$, and $N = 2$ is the number of vector components of Ψ. The universal answer is a consequence of the strong interactions in the CFT3

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

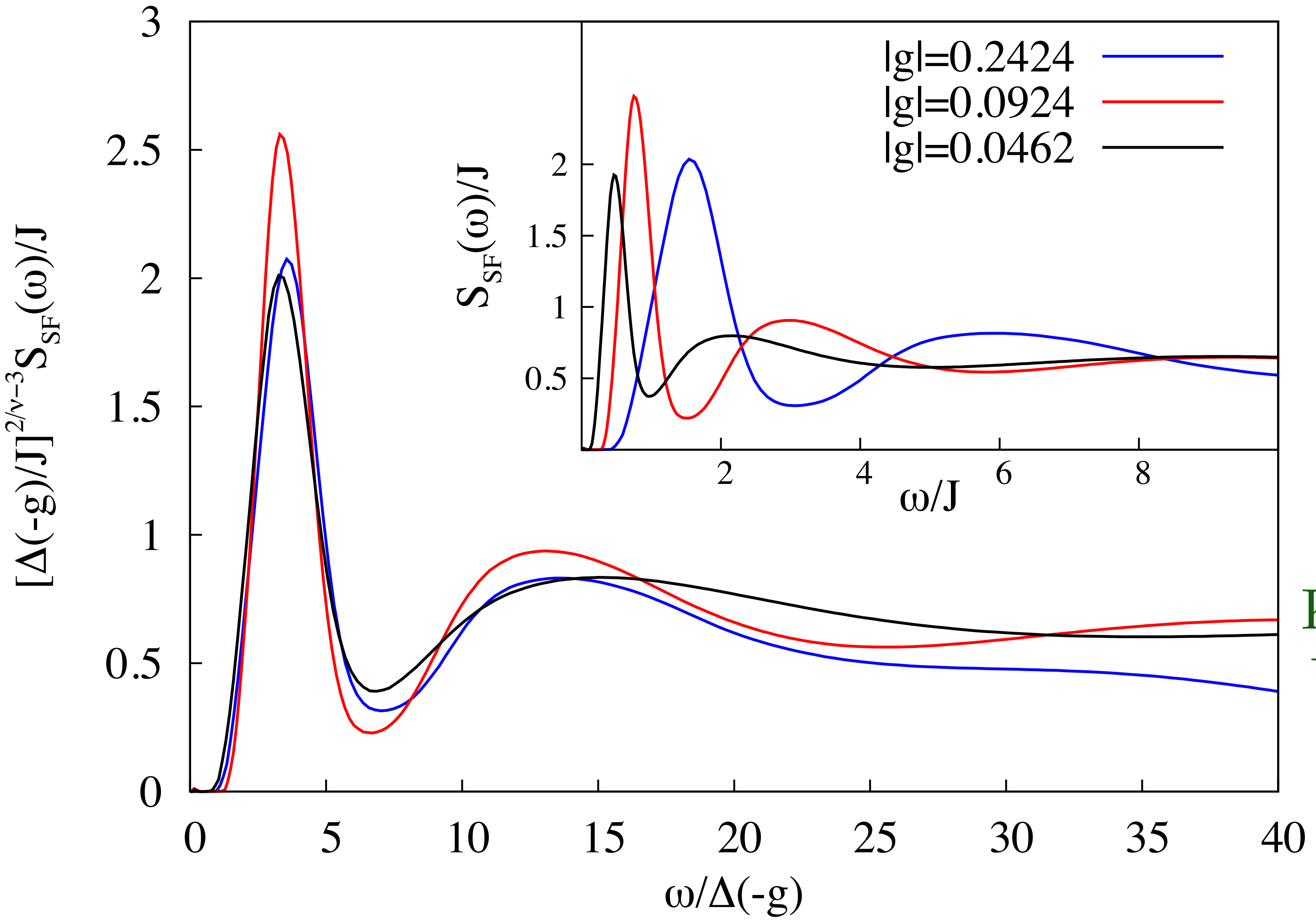
Response to modulation of lattice depth scales as expected from the LHP pole





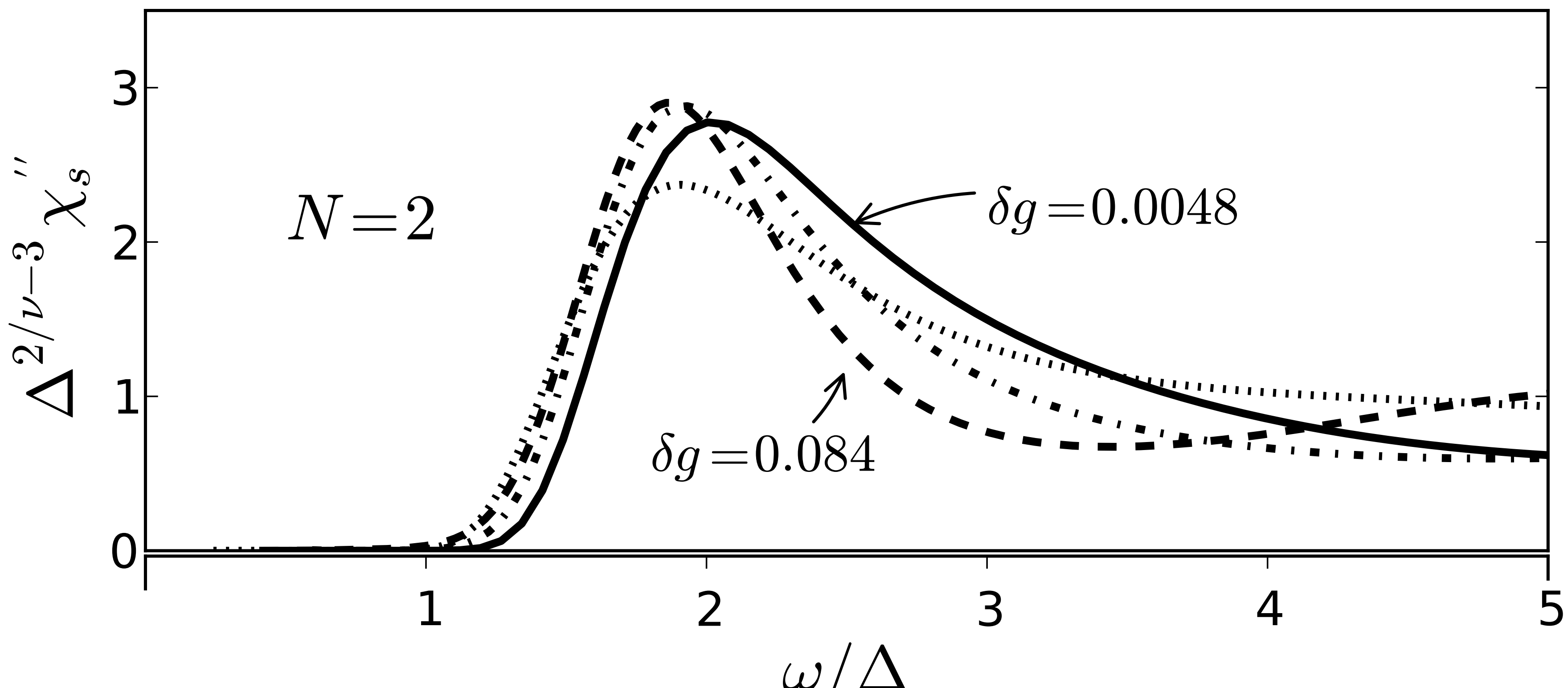
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode in quantum Monte Carlo



Scaling of spectral response functions predicted in D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, and Nikolay Prokof'ev, arXiv:1301.3139



Snir Gazit, Daniel Podolsky, and Assa Auerbach, arXiv:1212.3759

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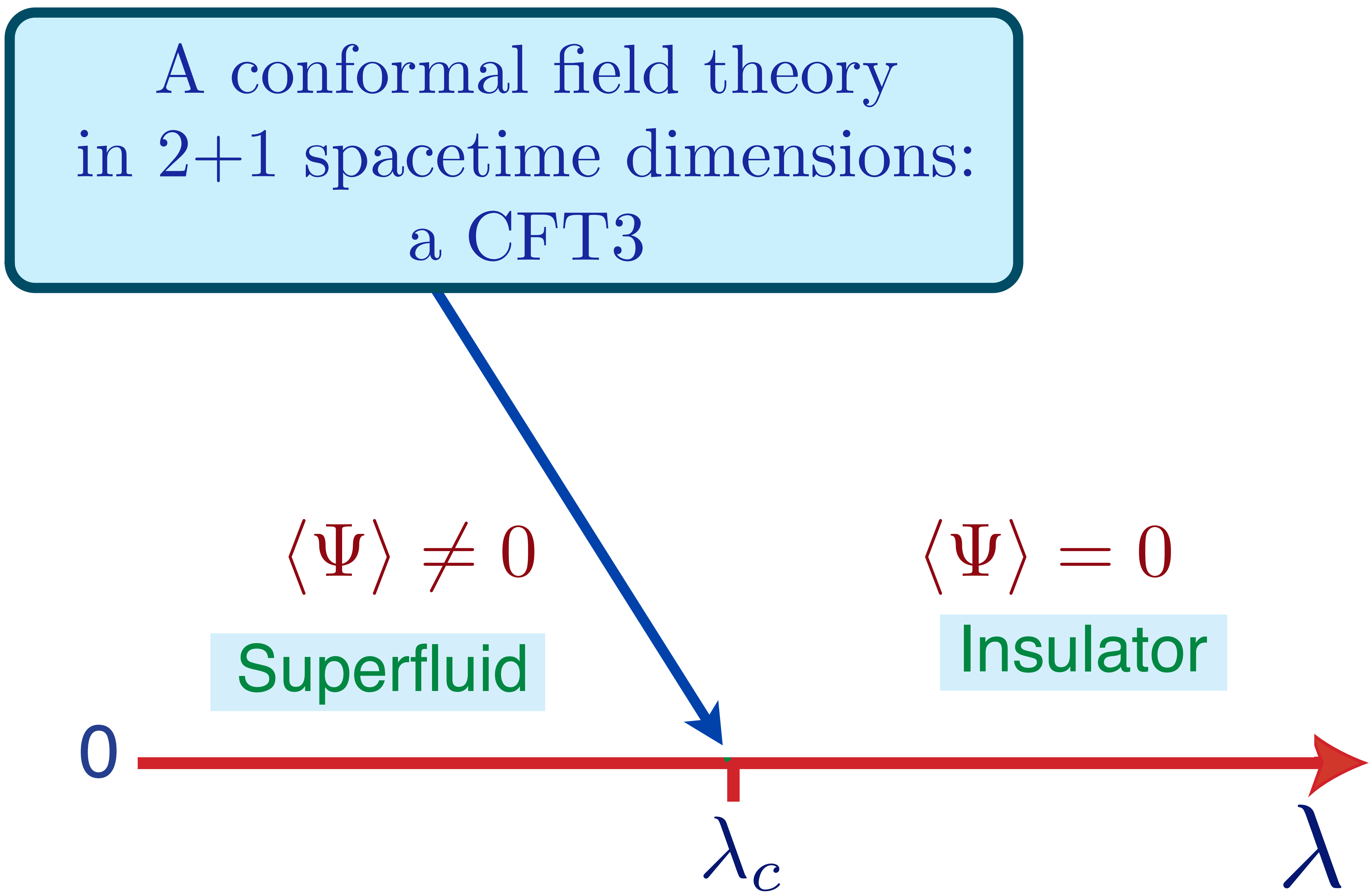
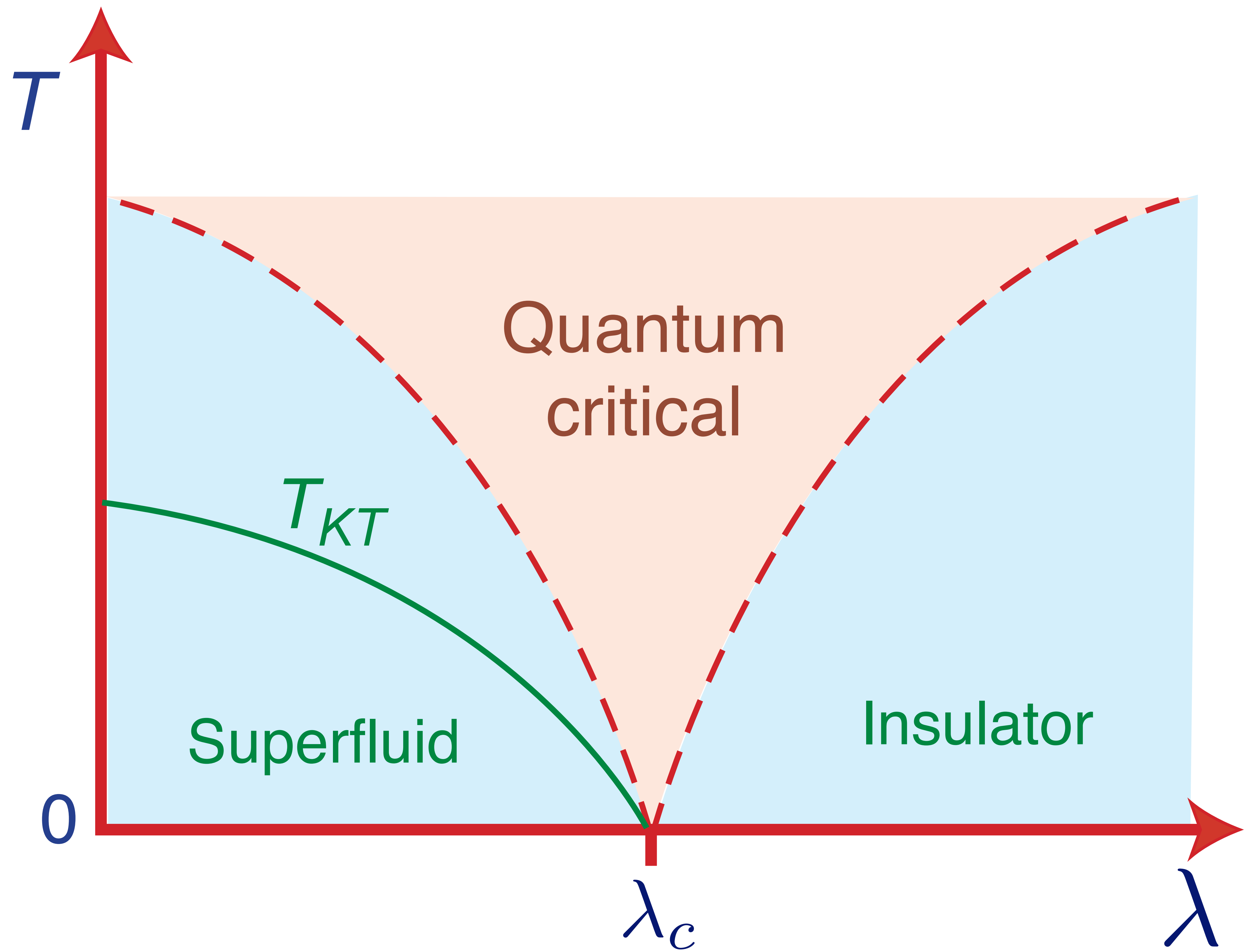
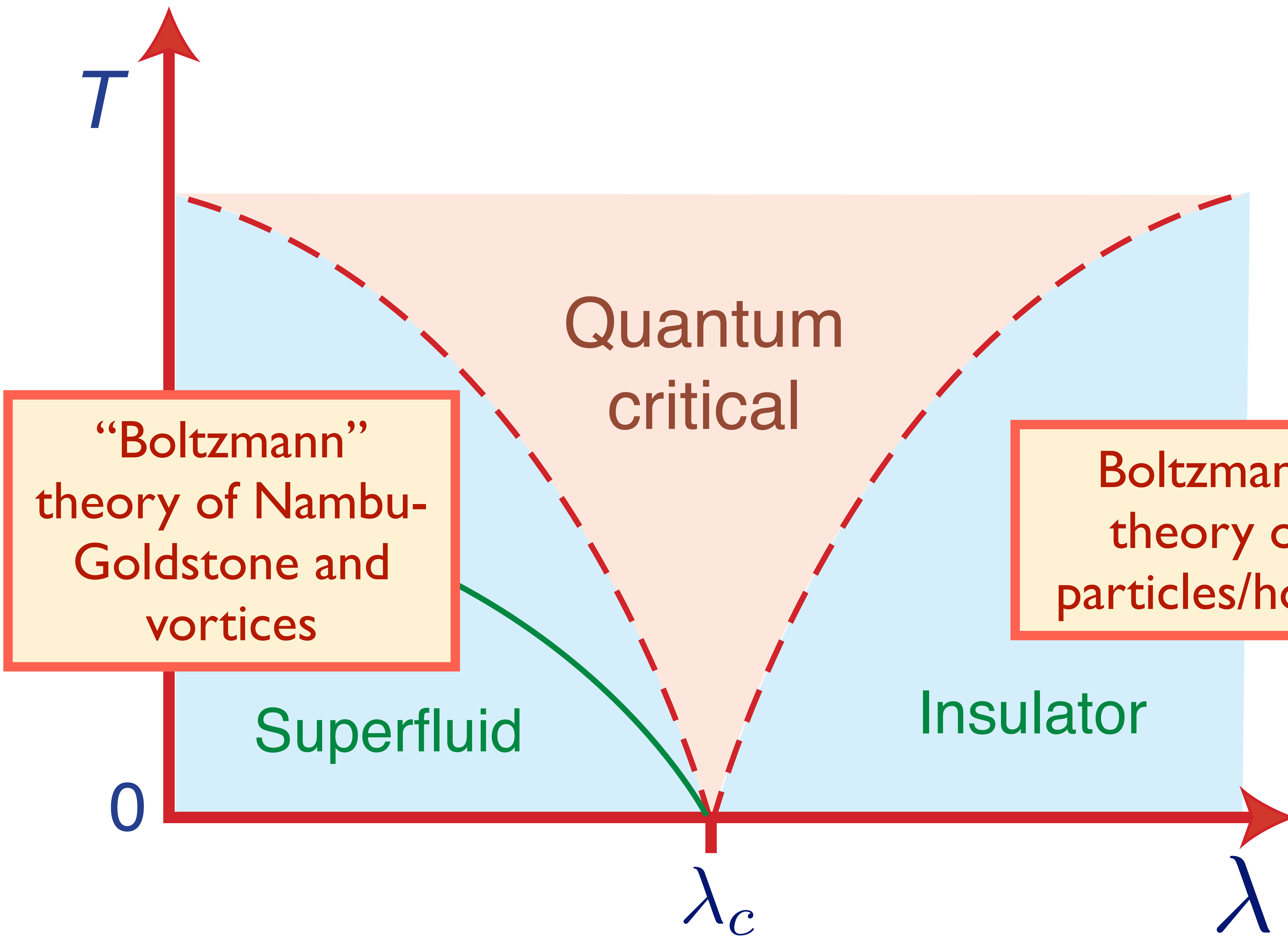
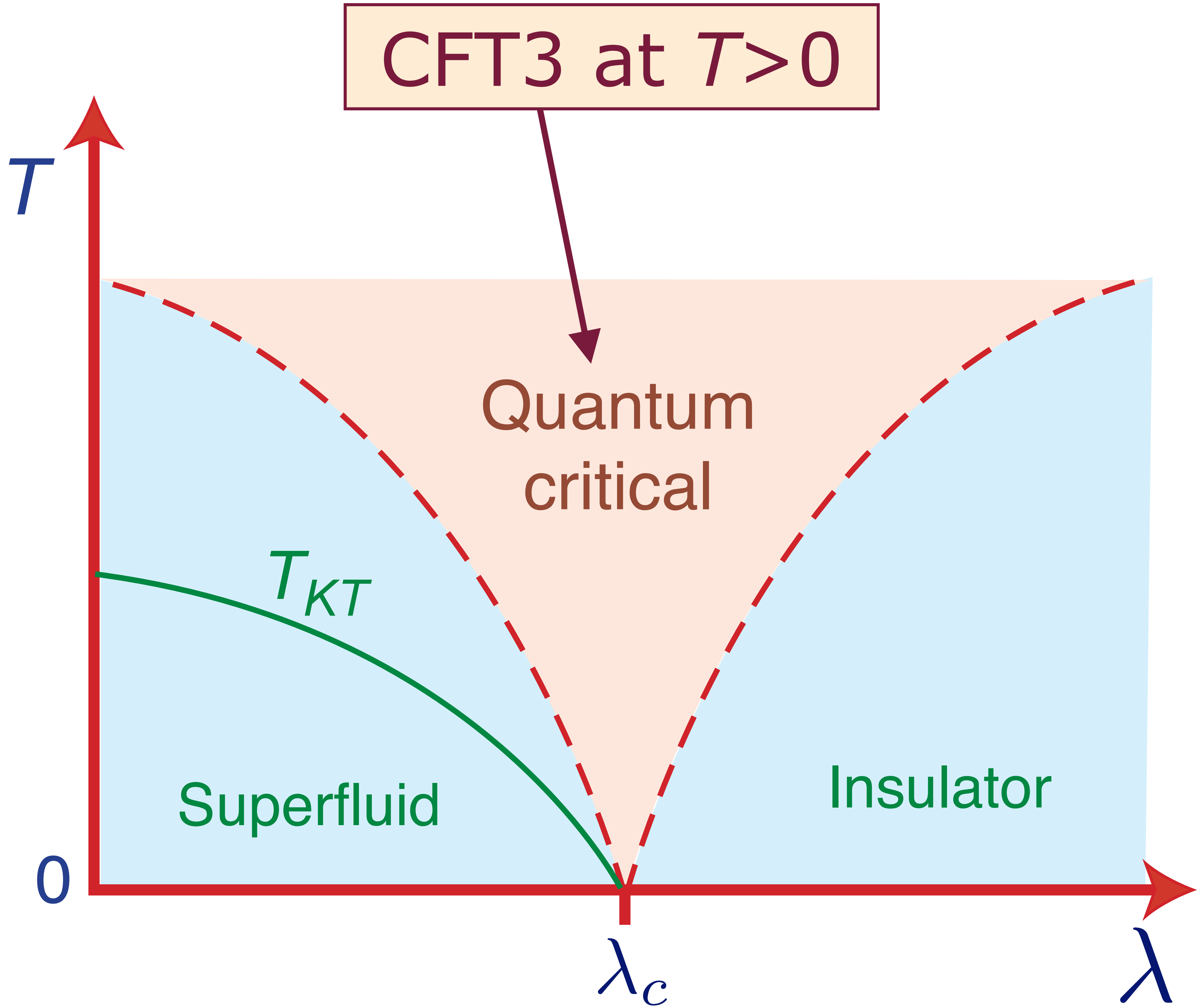




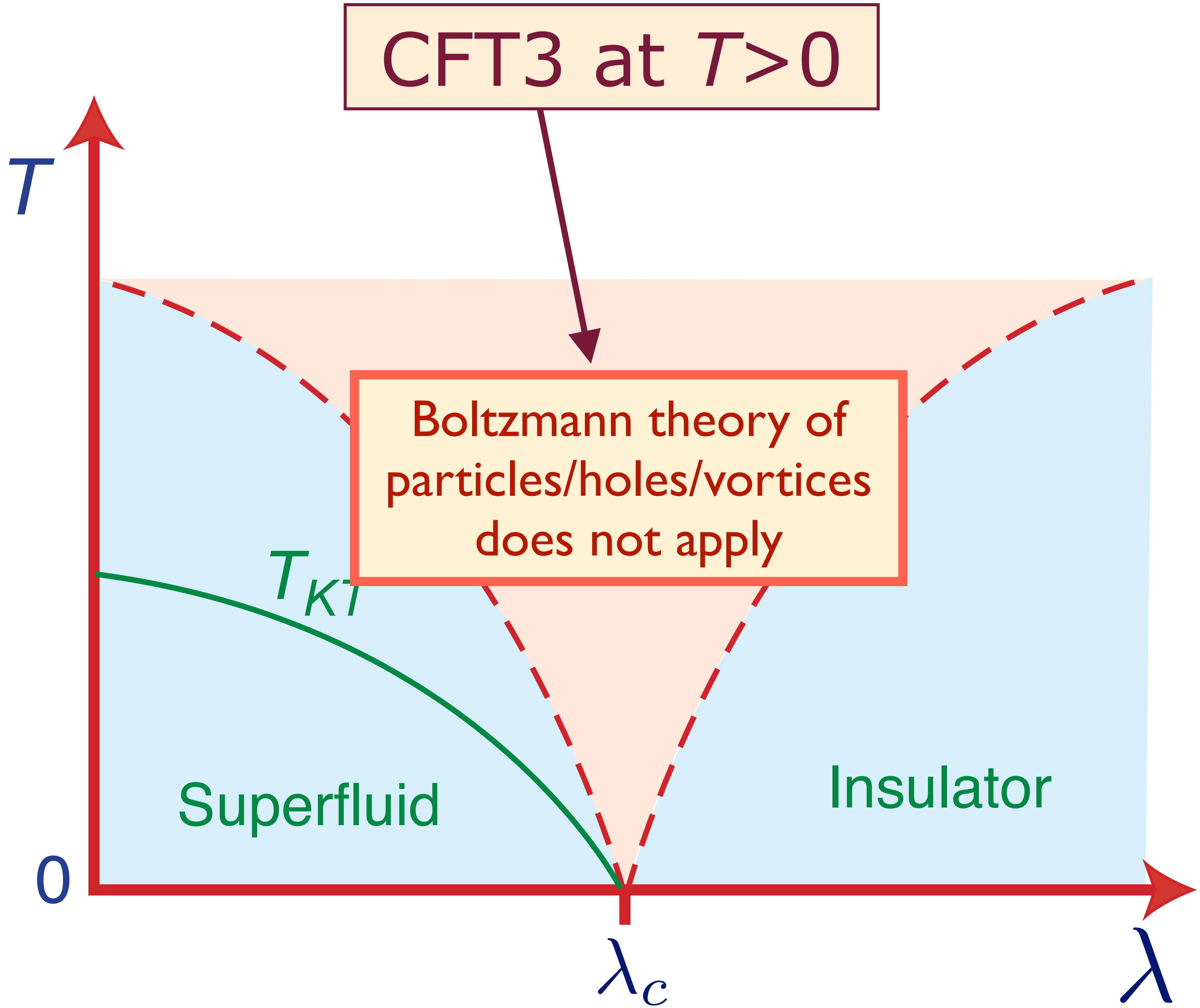
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ

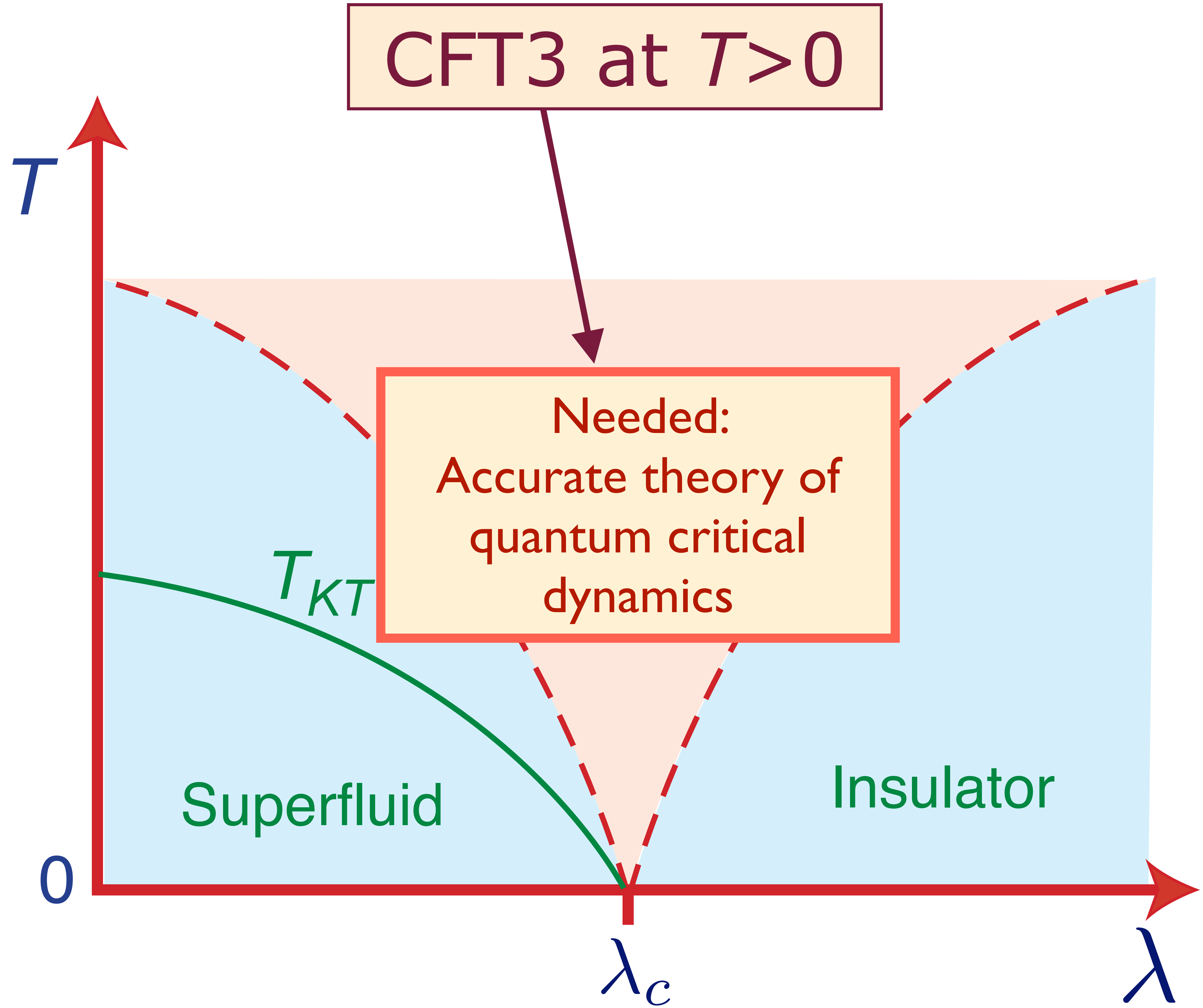


T
0
Quantum critical
"Boltzmann" theory of Nambu-Goldstone and vortices
Boltzmann theory of particles/holes
Superfluid
Insulator
λ_c
λ

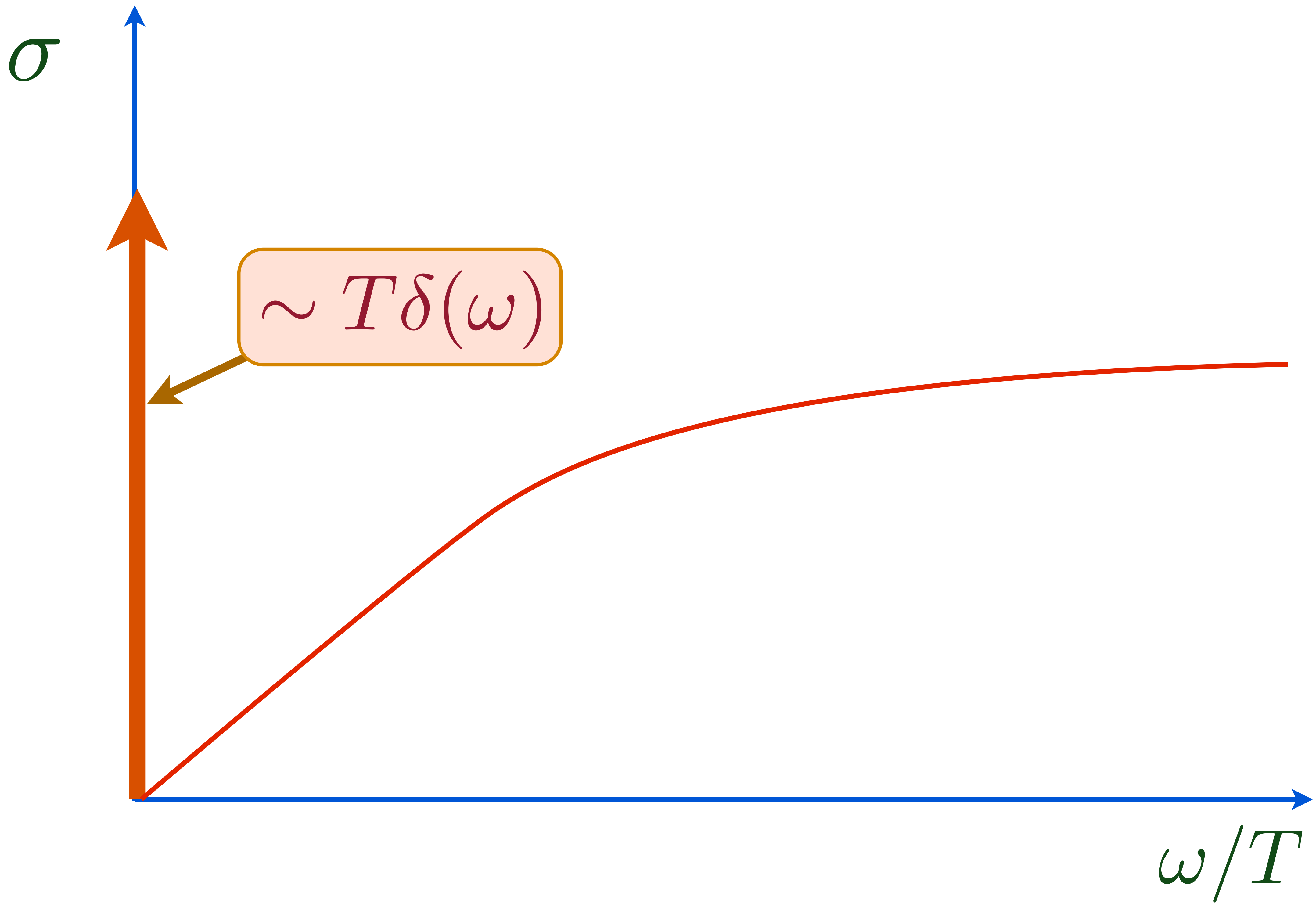


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
λ_c
λ


CFT3 at T>0
T
Boltzmann theory of
particles/holes/vortices
does not apply
T_{KT}
Superfluid
Insulator
0
λ_c
λ

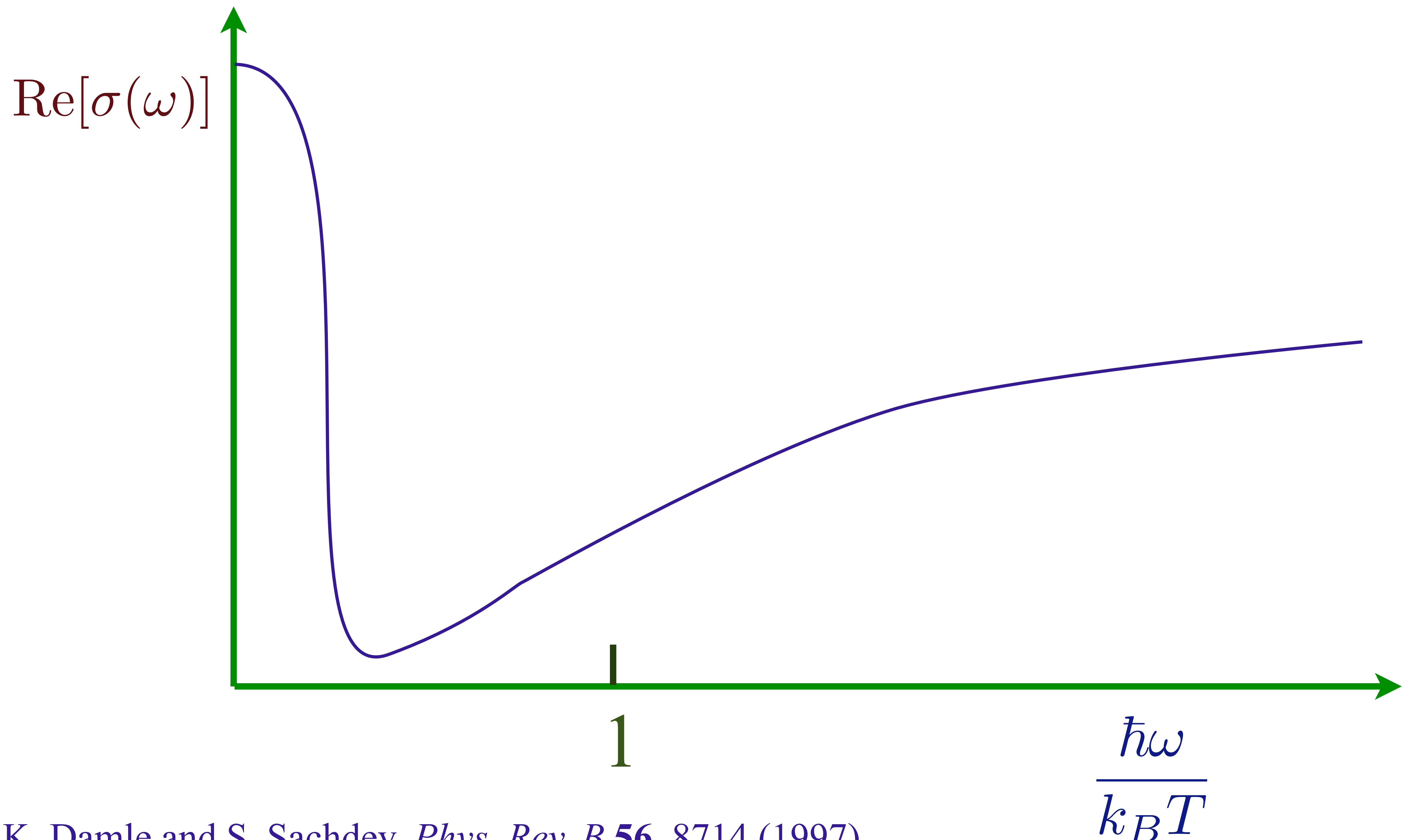

CFT3 at $T>0$
T
Needed:
Accurate theory of
quantum critical
dynamics
T_{KT}
Superfluid
Insulator
0
λ_c
λ

Electrical transport in a free CFT3 for *T* > *0*



Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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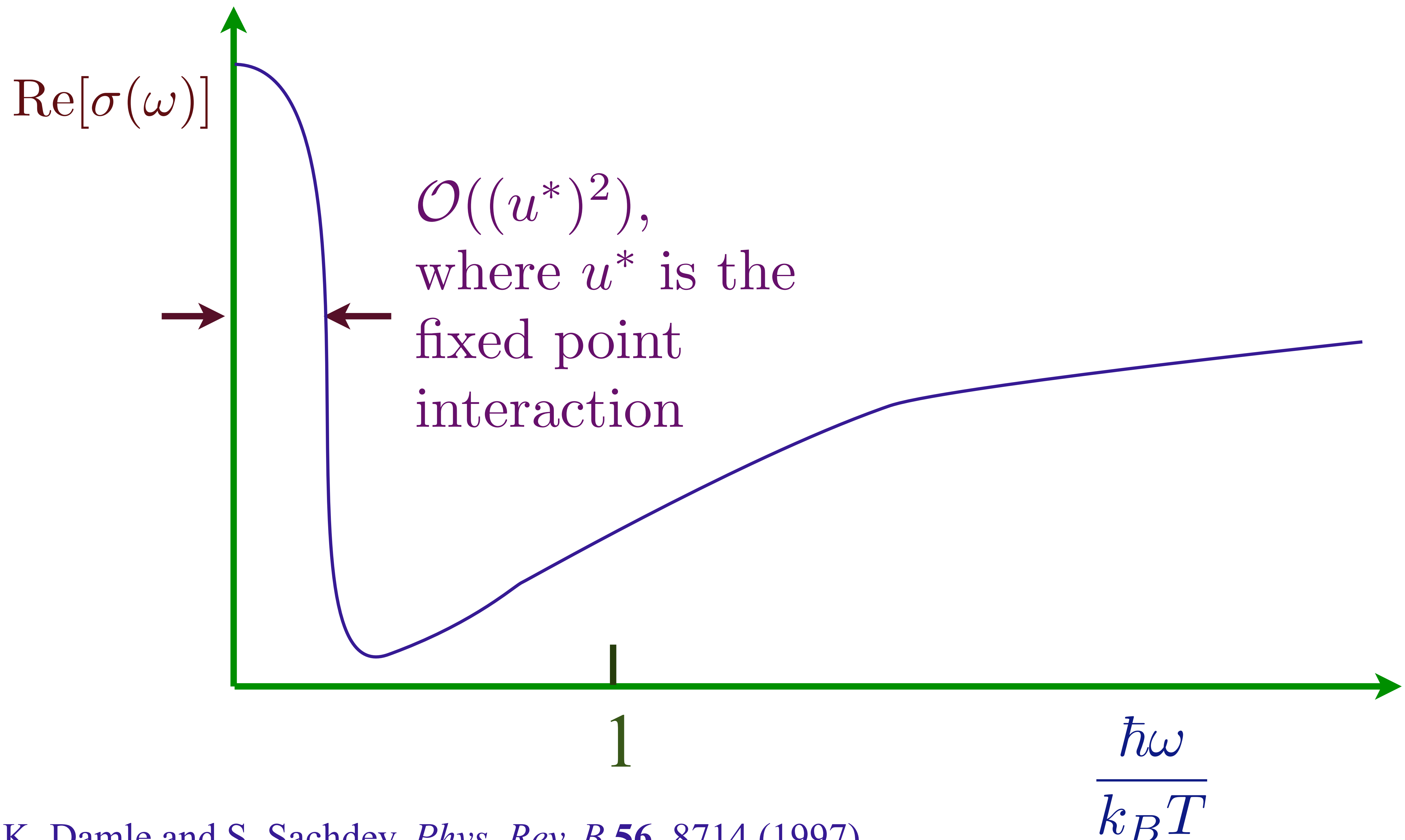

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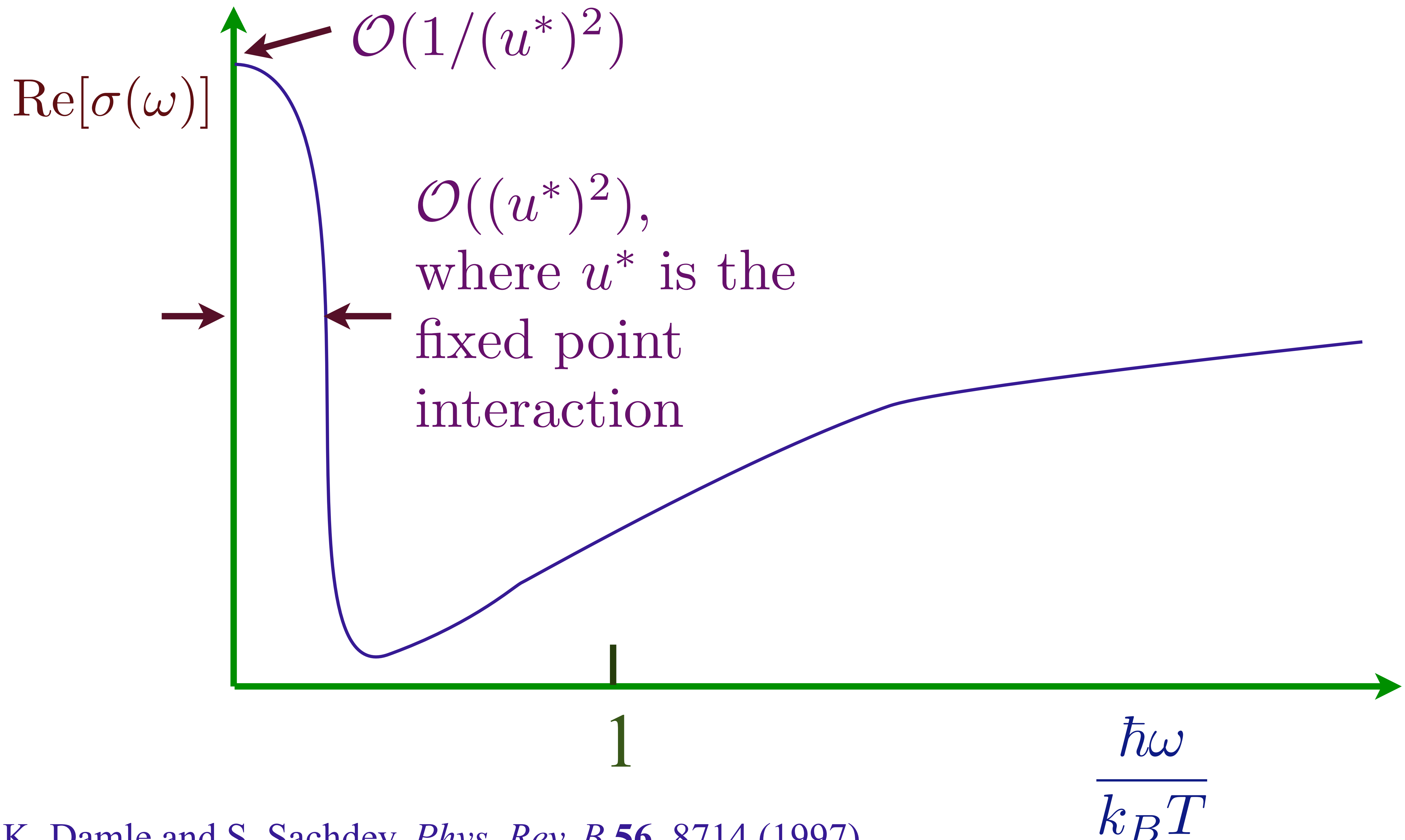

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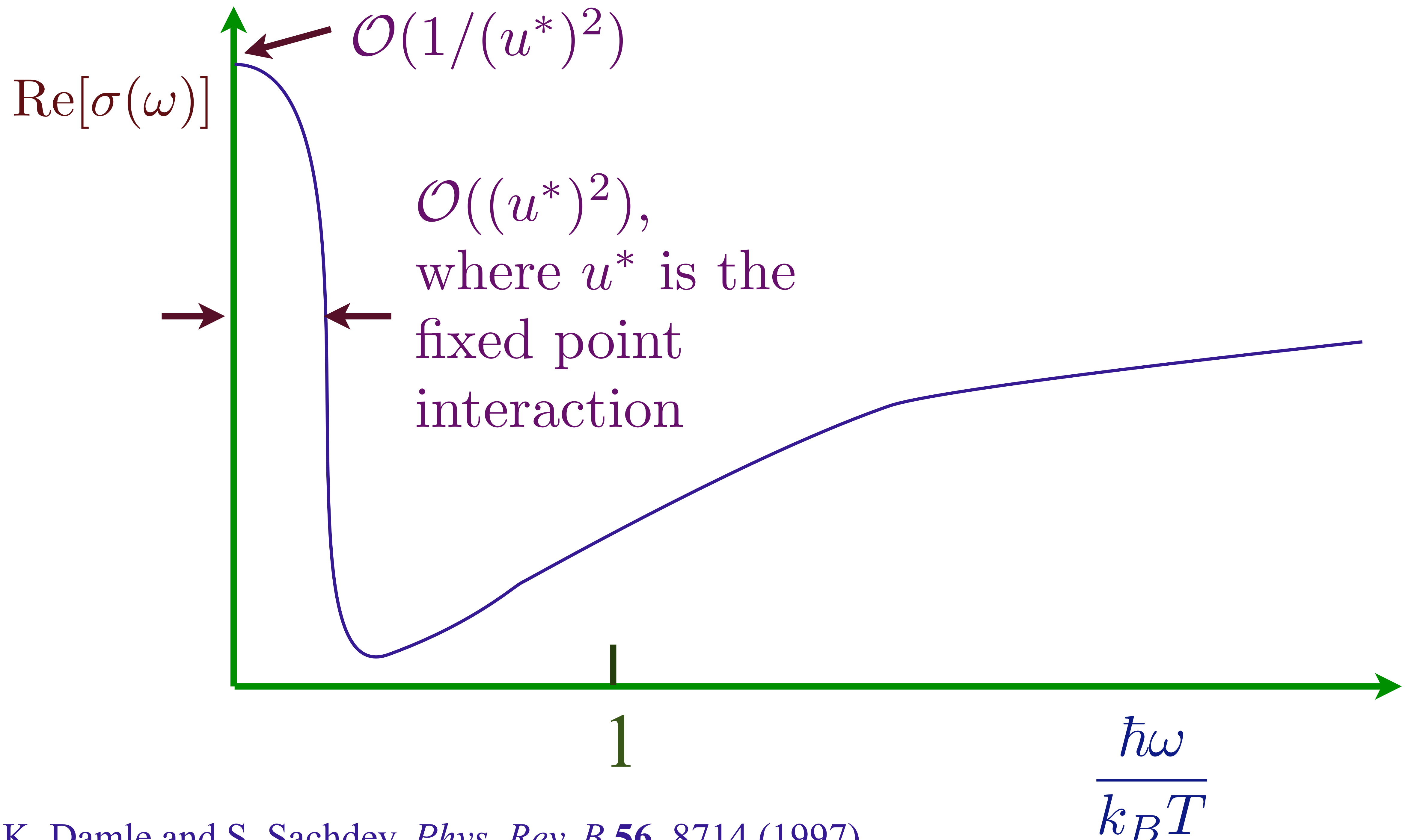

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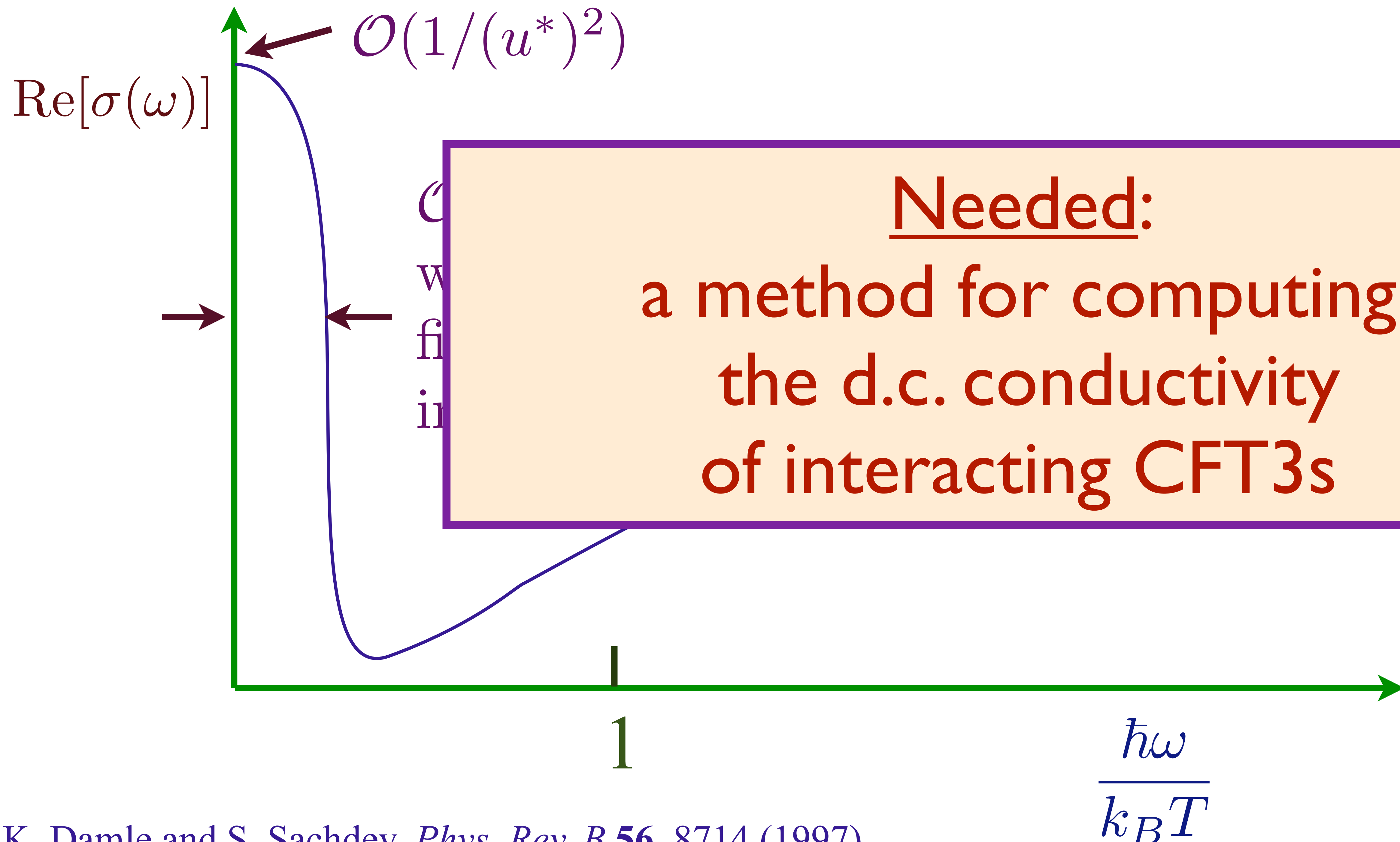

Needed:
a method for computing
the d.c. conductivity
of interacting CFT3s

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible *local* equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in the holographic theory.
(Analogs of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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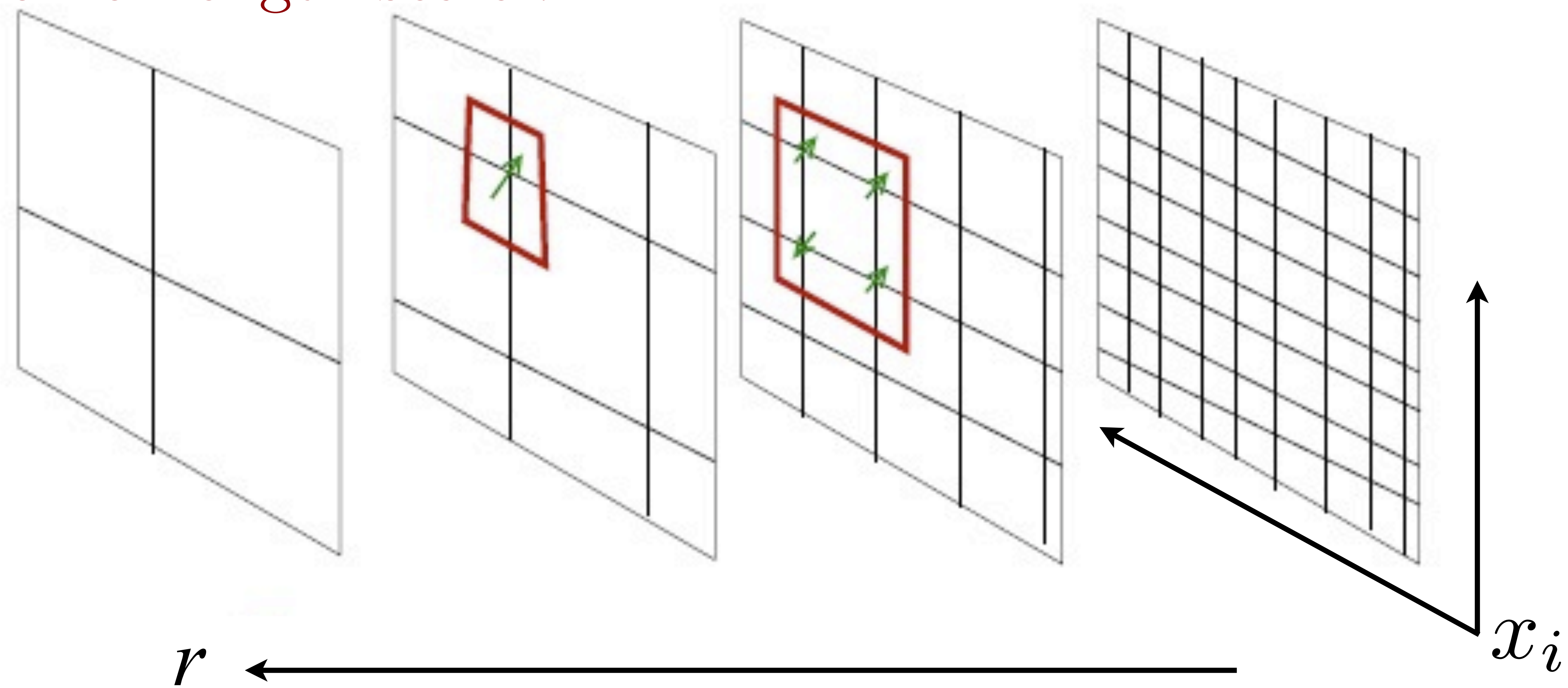
Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale r



J. McGreevy, arXiv0909.0518

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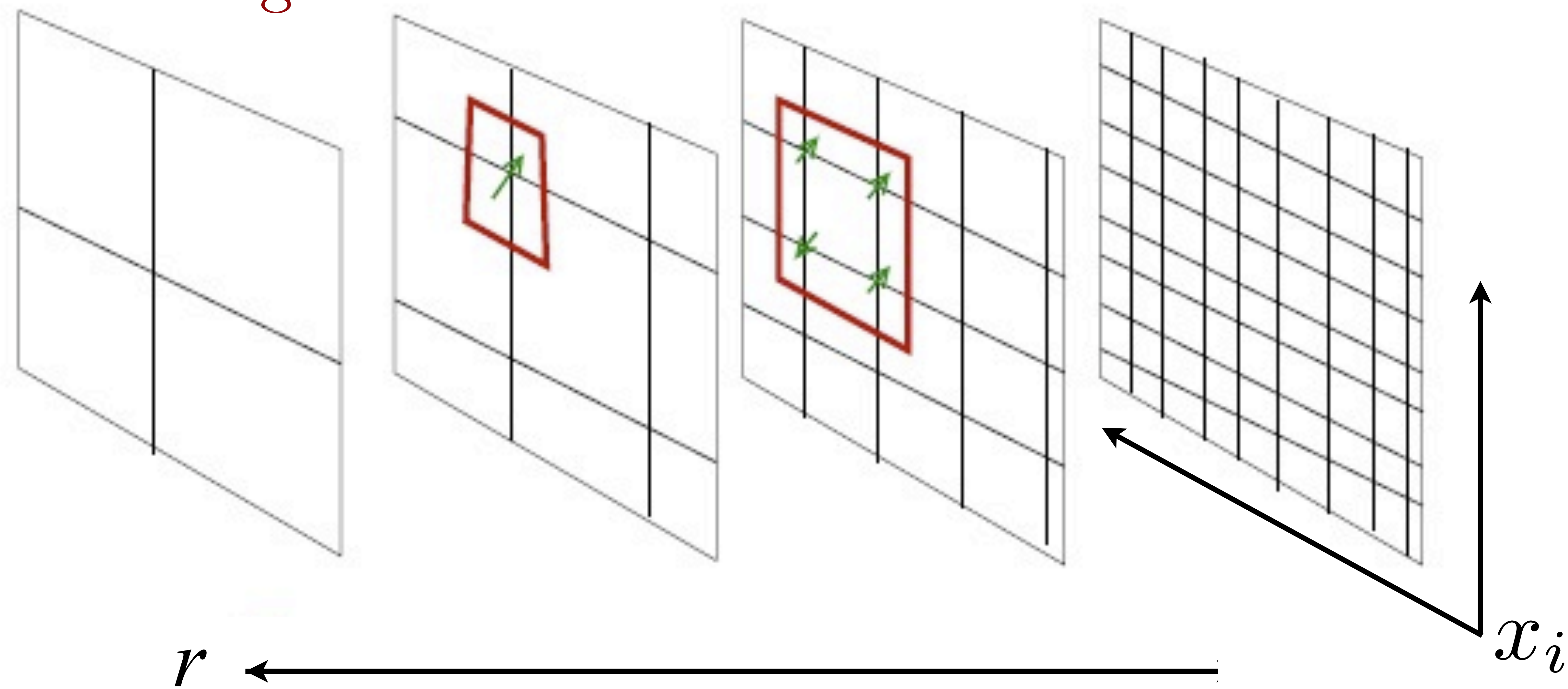
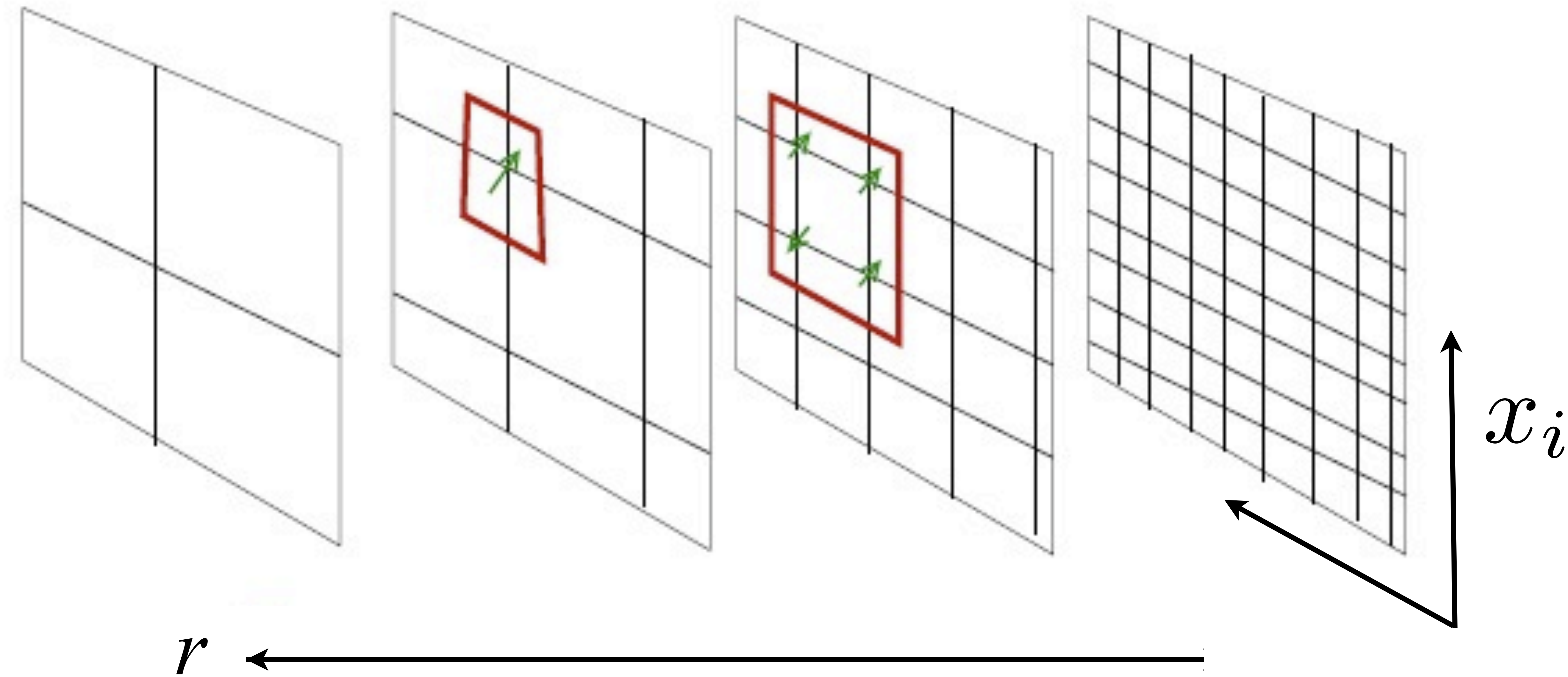


Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

J. McGreevy, arXiv0909.0518

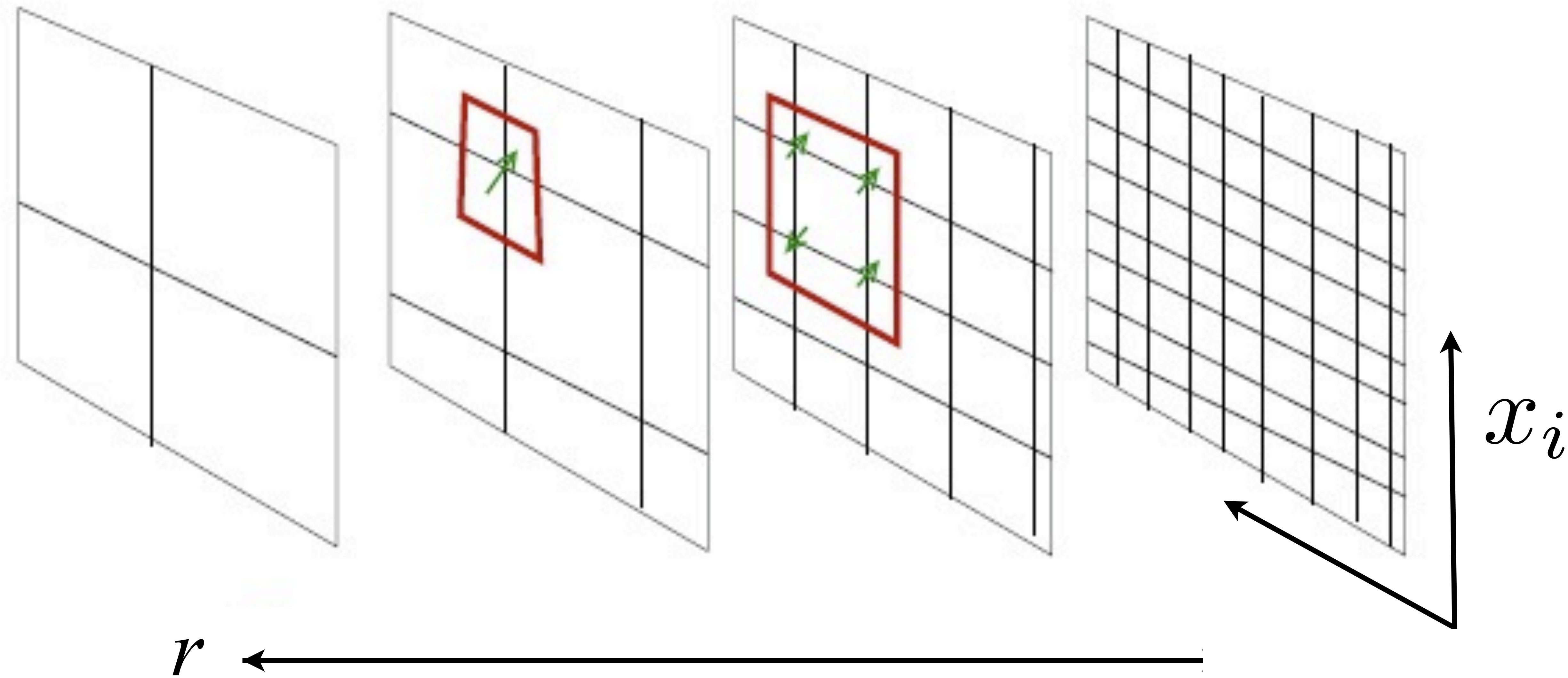
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

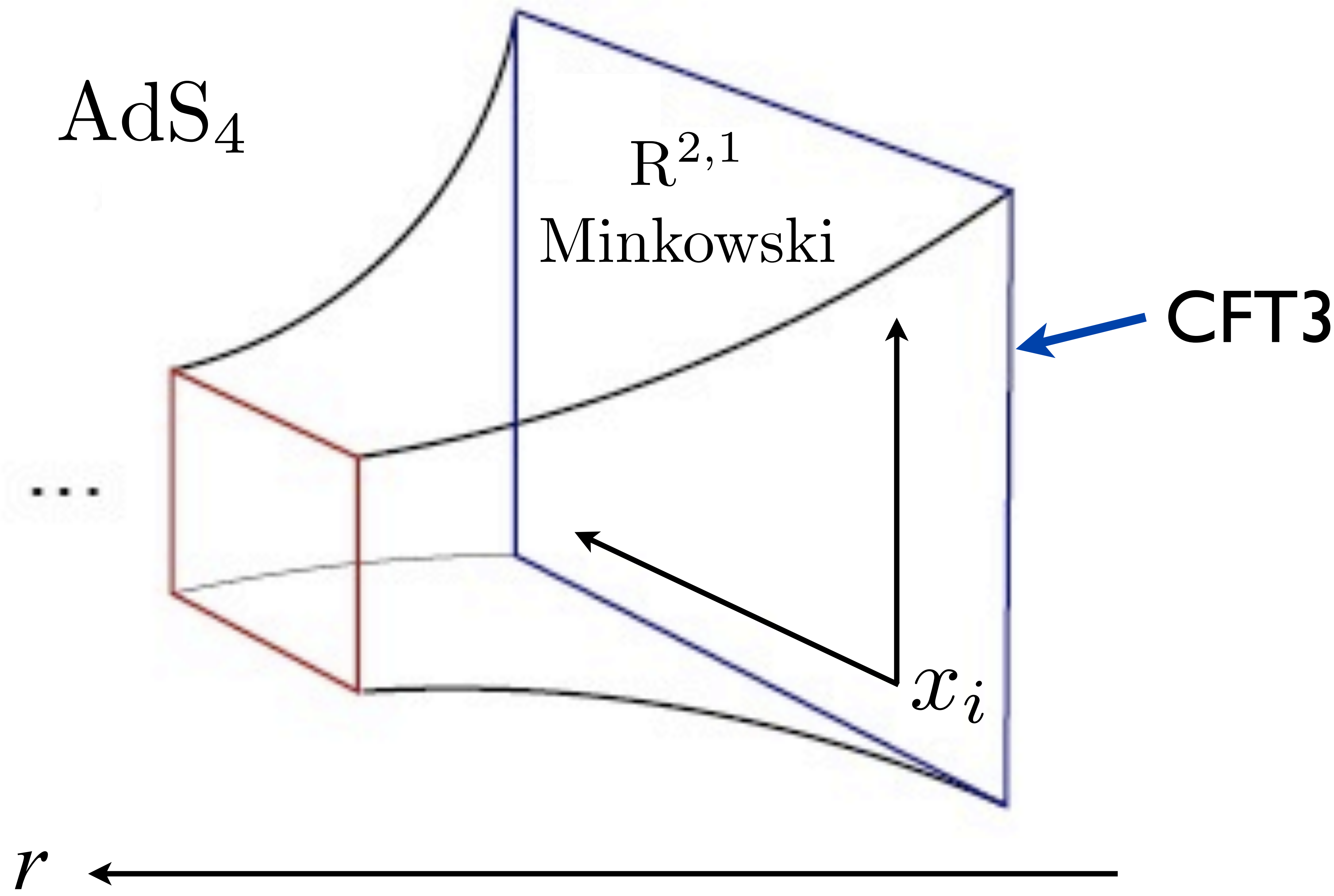


This gives the unique metric

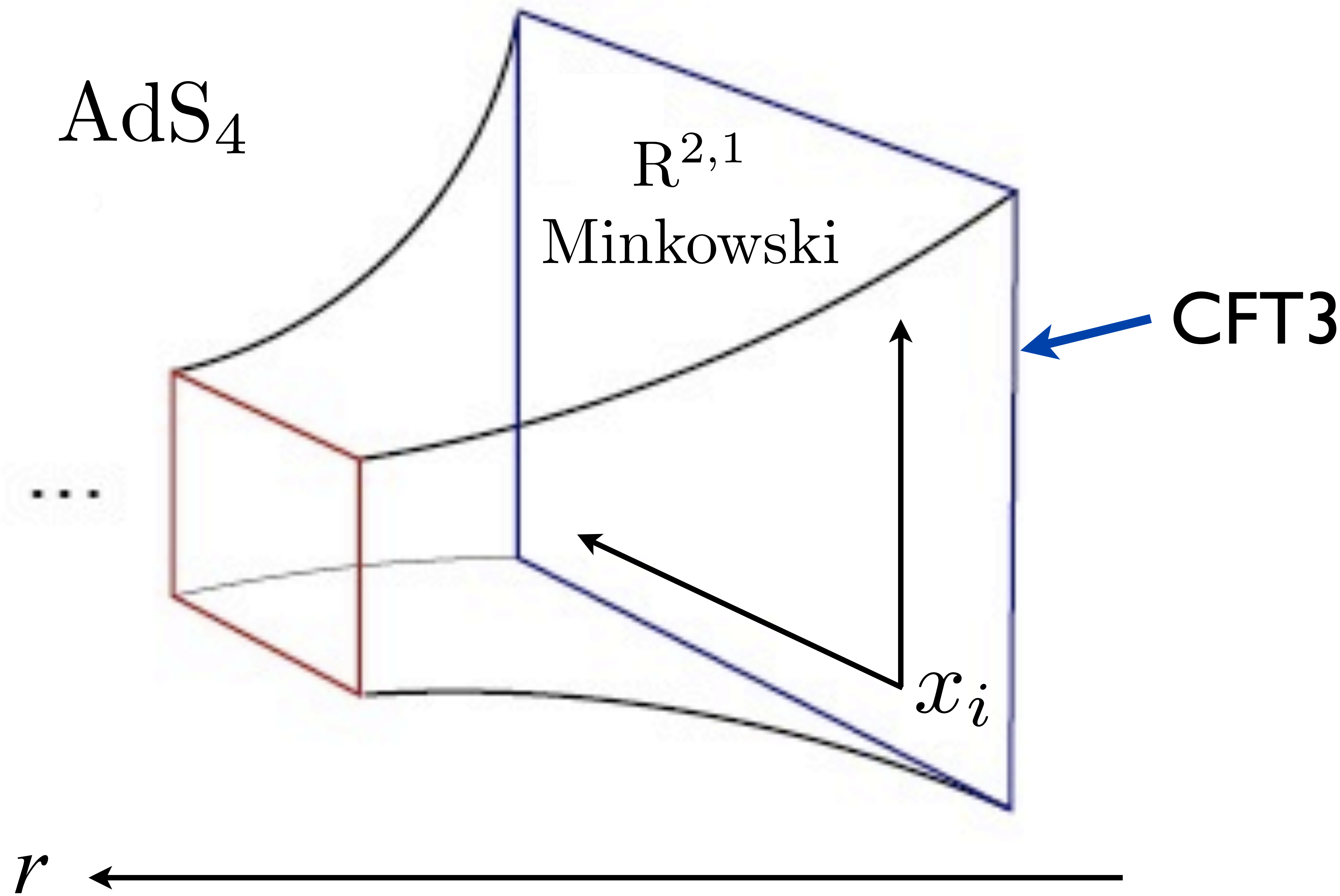
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

AdS/CFT correspondence



AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence

For every primary operator $O(\boldsymbol{x})$ in the CFT, there is a corresponding field $\phi(\boldsymbol{x}, r)$ in the bulk (gravitational) theory. For a scalar operator $O(\boldsymbol{x})$ of dimension Δ, the correlators of the boundary and bulk theories are related by

$$\langle O(\boldsymbol{x}_1) \ldots O(\boldsymbol{x}_n) \rangle_{\mathrm{CFT}} = Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(\boldsymbol{x}_1, r_1) \ldots \phi(\boldsymbol{x}_n, r_n) \rangle_{\mathrm{bulk}}$$

where the "wave function renormalization" factor $Z = (2\Delta - D)$.

AdS/CFT correspondence

For a U(1) conserved current J_μ of the CFT, the corresponding bulk operator is a U(1) *gauge* field A_μ. With a Maxwell action for the gauge field

$$\mathcal{S}_M = \frac{1}{4g_M^2} \int d^{D+1}x \sqrt{g} F_{ab} F^{ab}$$

we have the bulk-boundary correspondence

$$\langle J_\mu(\boldsymbol{x}_1) \ldots J_\nu(\boldsymbol{x}_n) \rangle_{\text{CFT}} = (Z g_M^{-2})^n \lim_{r \to 0} r_1^{2-D} \ldots r_n^{2-D} \langle A_\mu(\boldsymbol{x}_1, r_1) \ldots A_\nu(\boldsymbol{x}_n, r_n) \rangle_{\text{bulk}}$$

with $Z = D - 2$.

AdS/CFT correspondence

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\boldsymbol{x}_1)\dots T_{\rho\sigma}(\boldsymbol{x}_n)\rangle_{\rm CFT} =$$
$$\left(\frac{ZL^2}{\kappa^2}\right)^n \lim_{r\to 0} r_1^{-D}\dots r_n^{-D}\,\langle \chi_{\mu\nu}(\boldsymbol{x}_1,r_1)\dots\chi_{\rho\sigma}(\boldsymbol{x}_n,r_n)\rangle_{\rm bulk}\,,$$

with $Z = D$.

AdS/CFT correspondence

So the minimal bulk theory for a CFT with a conserved U(1) current is the *Einstein-Maxwell* theory with a cosmological constant

$$\mathcal{S} = \frac{1}{4g_M^2}\int d^4x\,\sqrt{g}F_{ab}F^{ab} + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2, which are related to the conductivity $\sigma(\omega) = \mathcal{K}$ and the central charge of the CFT.

AdS/CFT correspondence

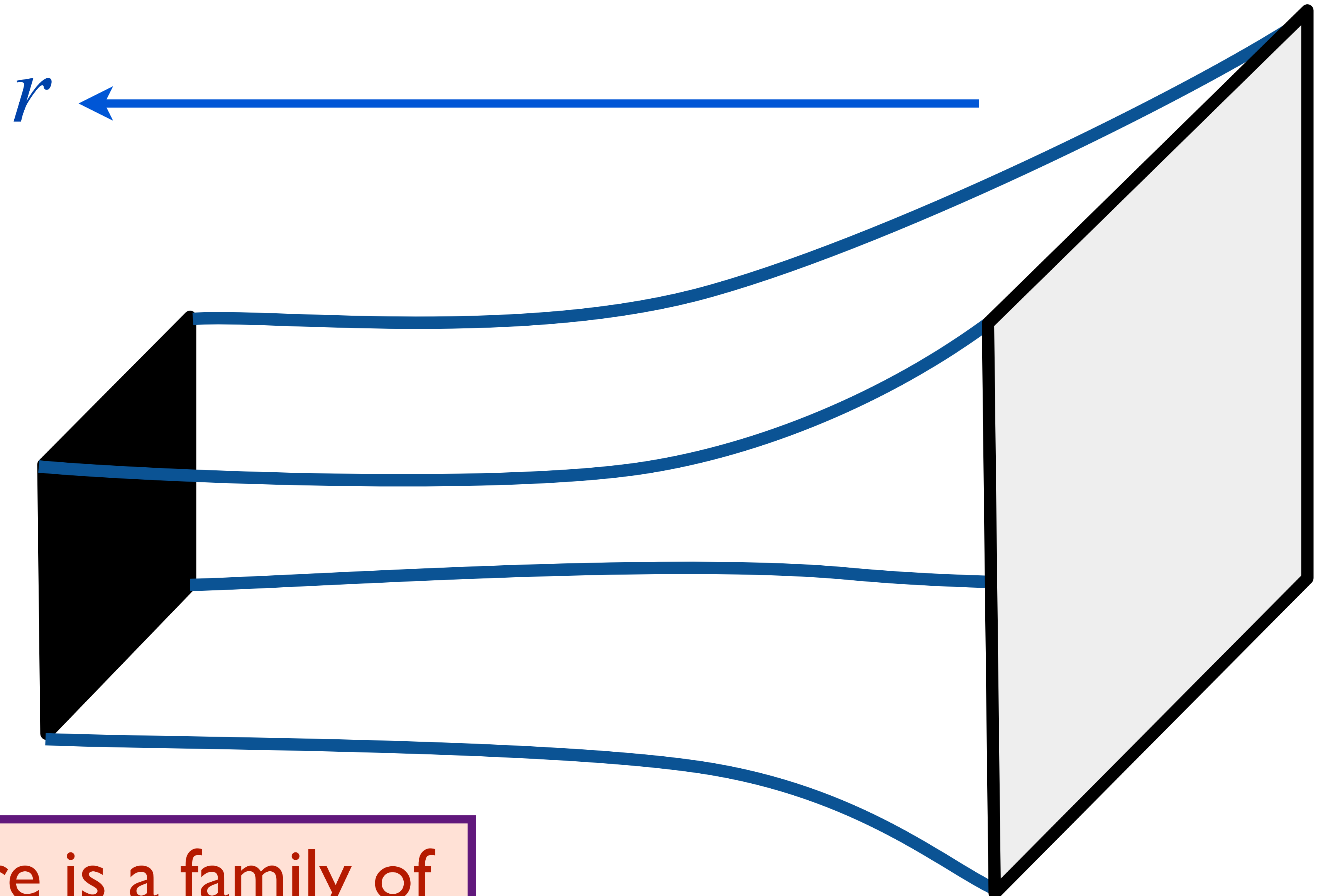
This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right]$$
$$+ \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

where C_{abcd} is the Weyl tensor. The parameter γ can be related to 3-point correlators of J_μ and $T_{\mu\nu}$. Both boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247

Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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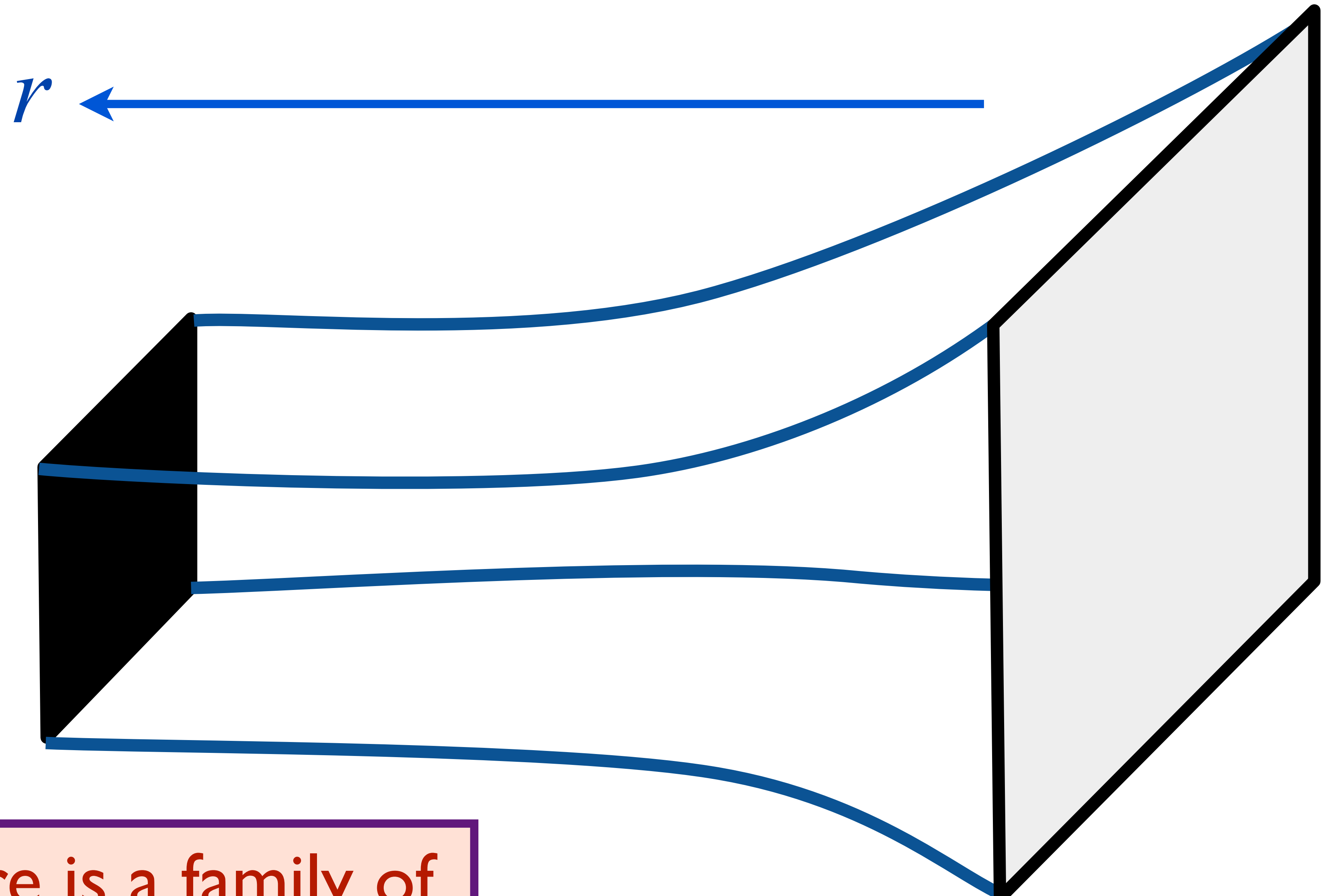
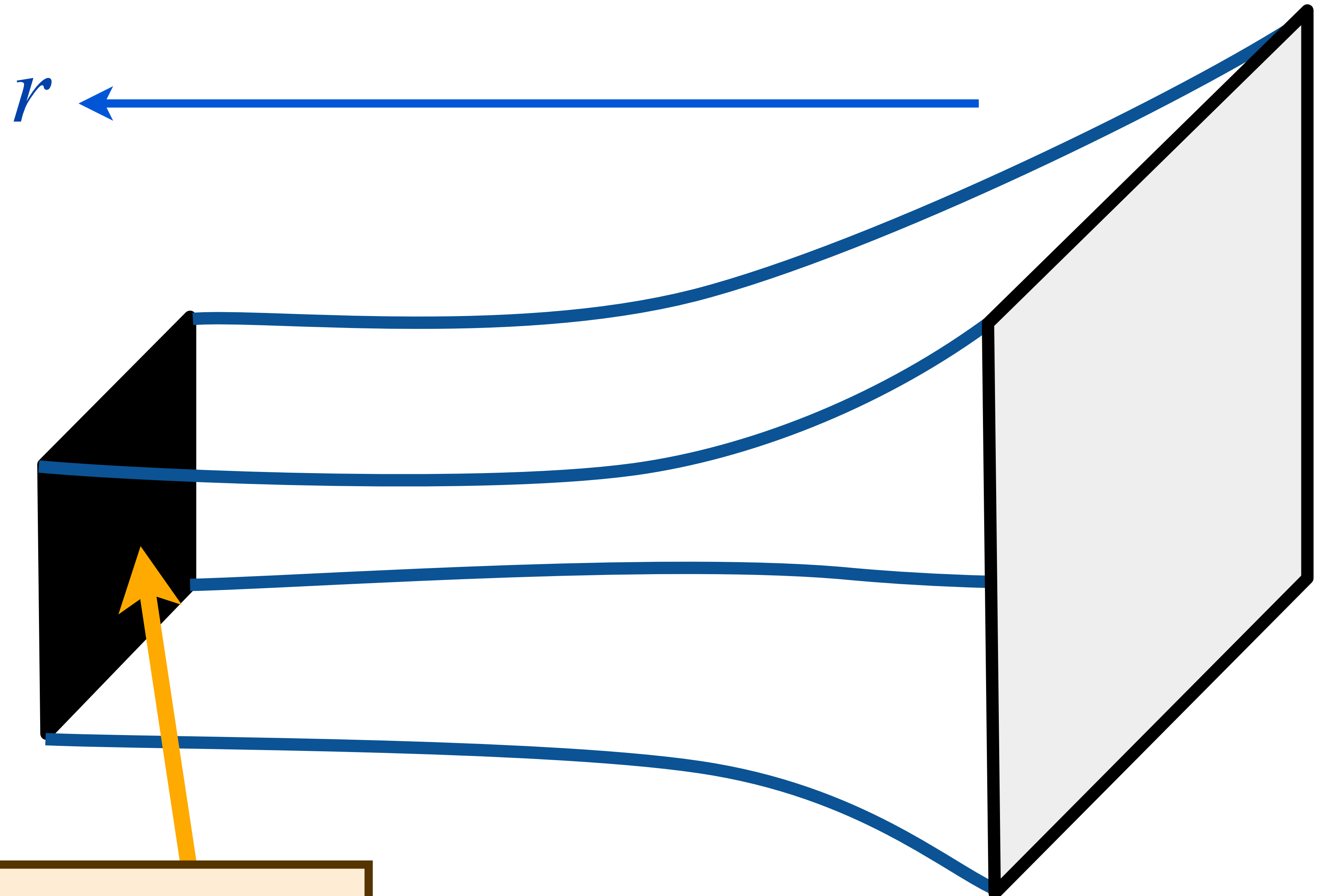


There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

Gauge-gravity duality at non-zero temperatures



A "horizon", similar to the surface of a black hole at $r = R$!

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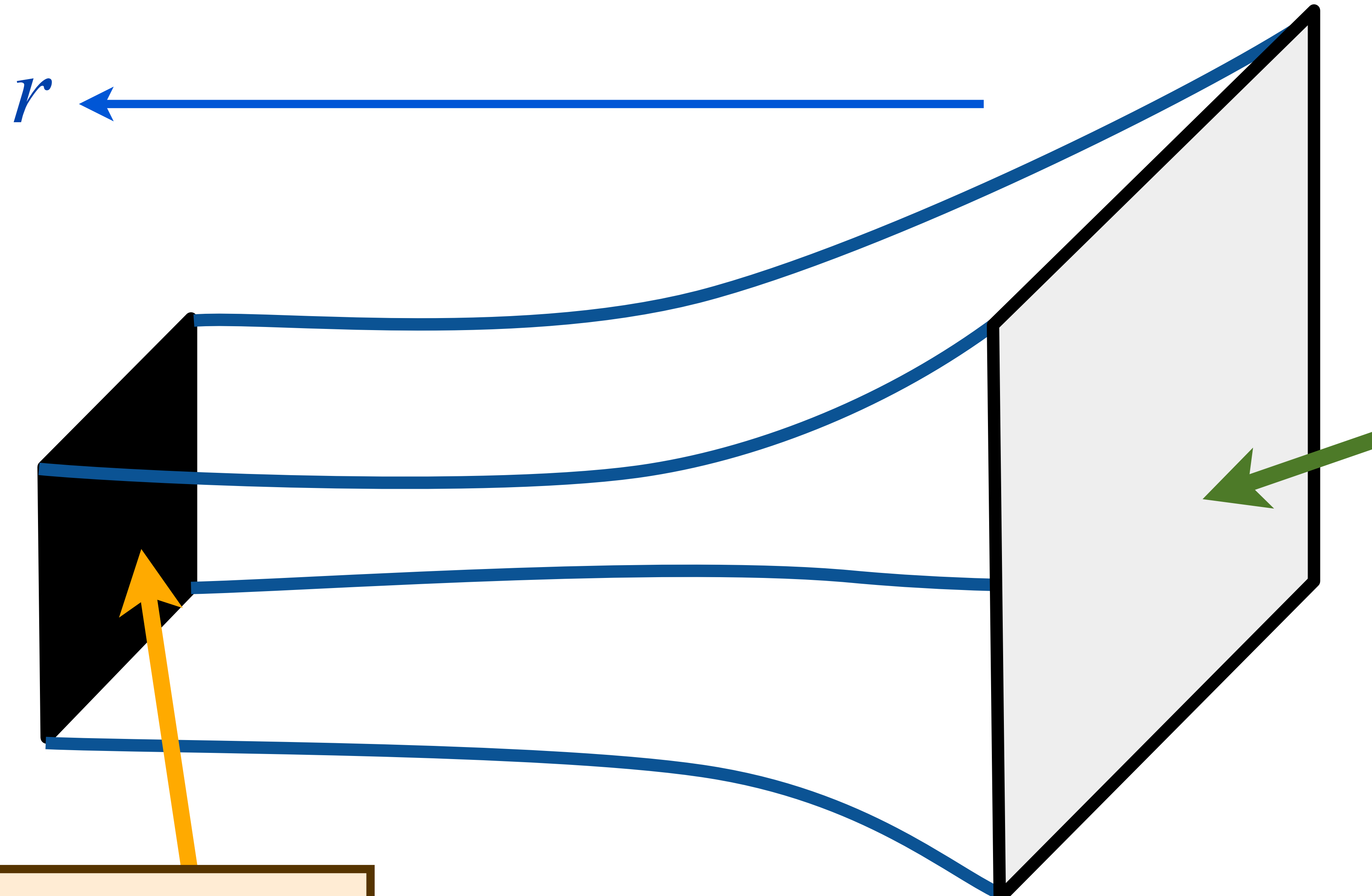

A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R}.$$

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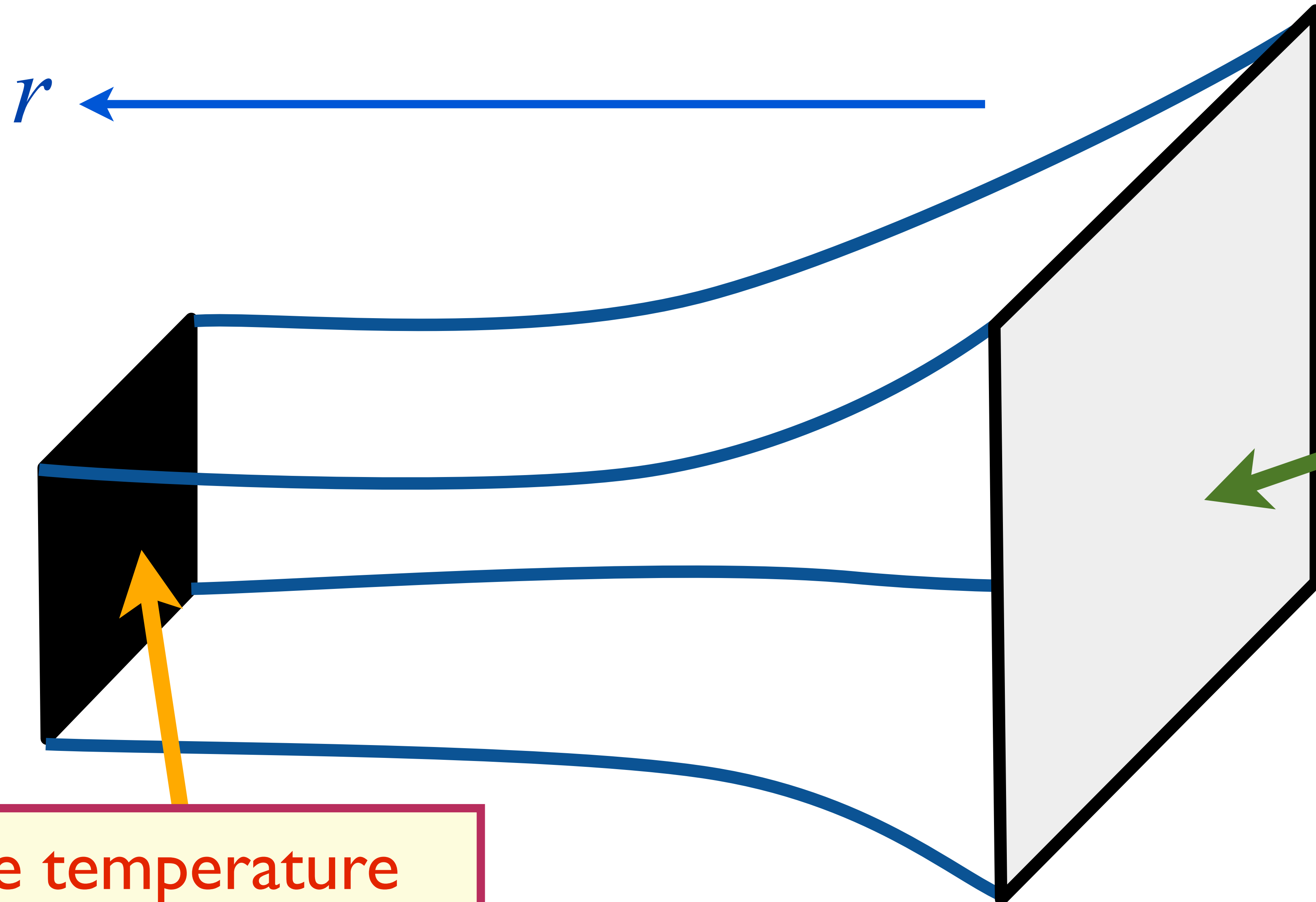

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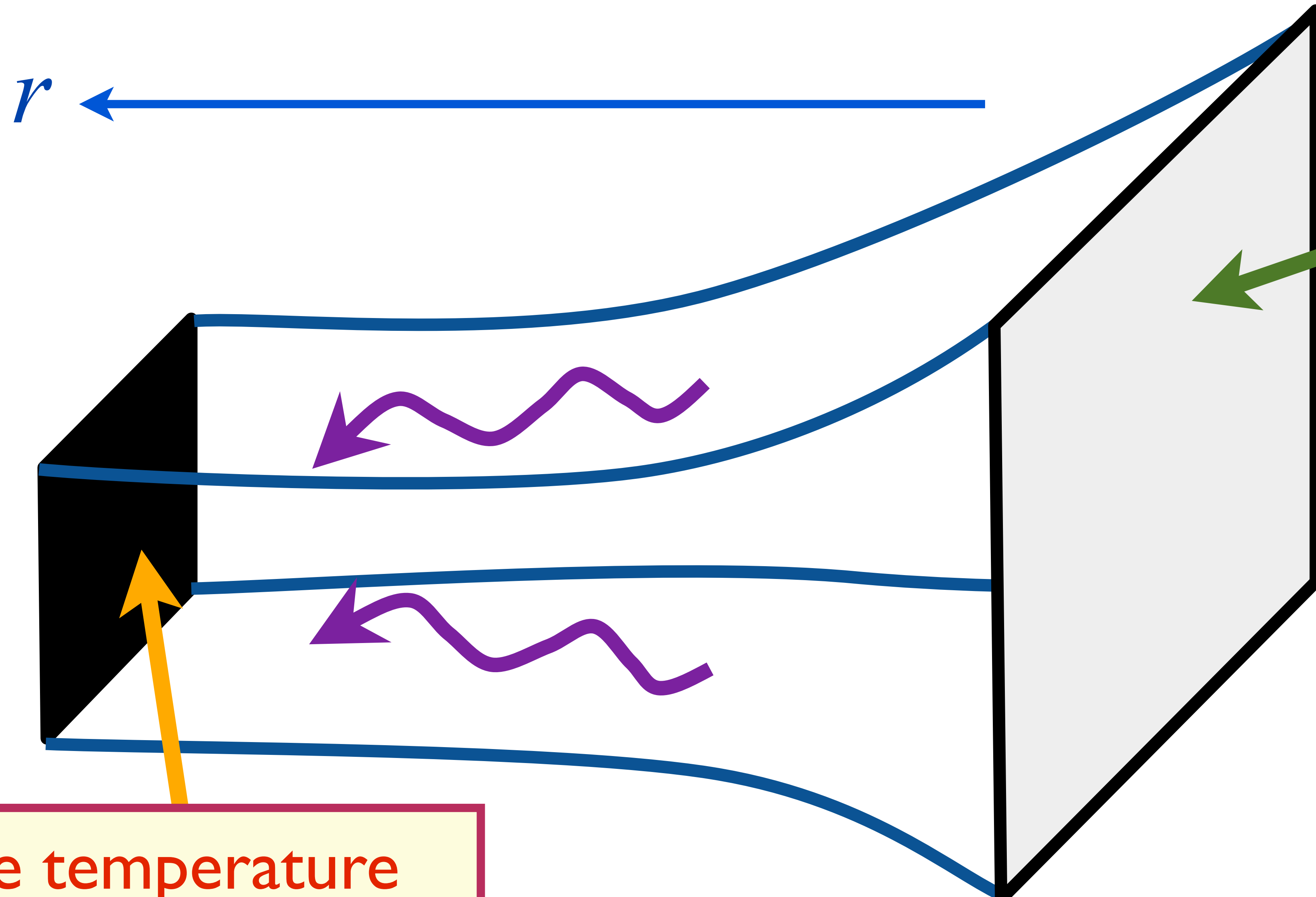


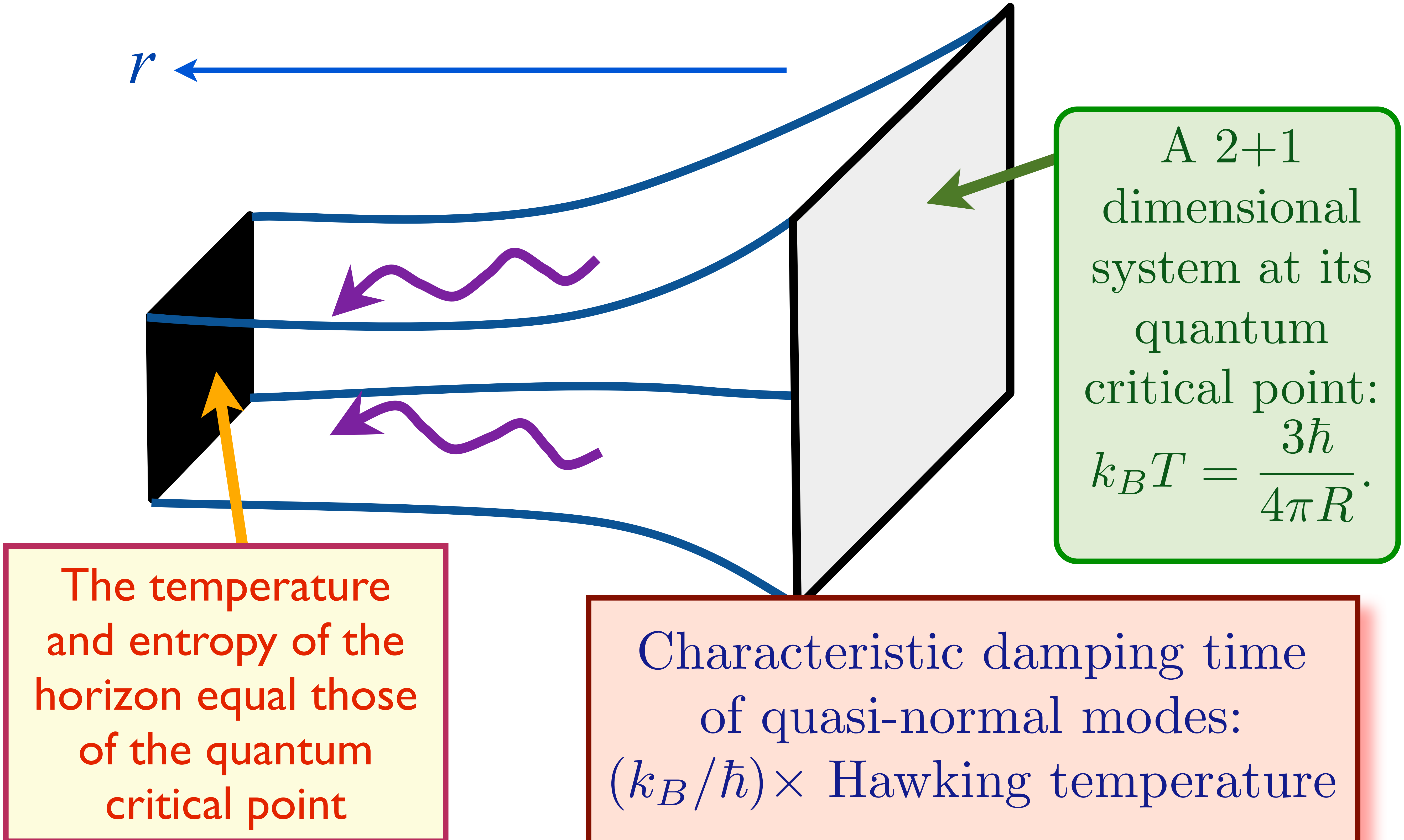
A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R}.$$

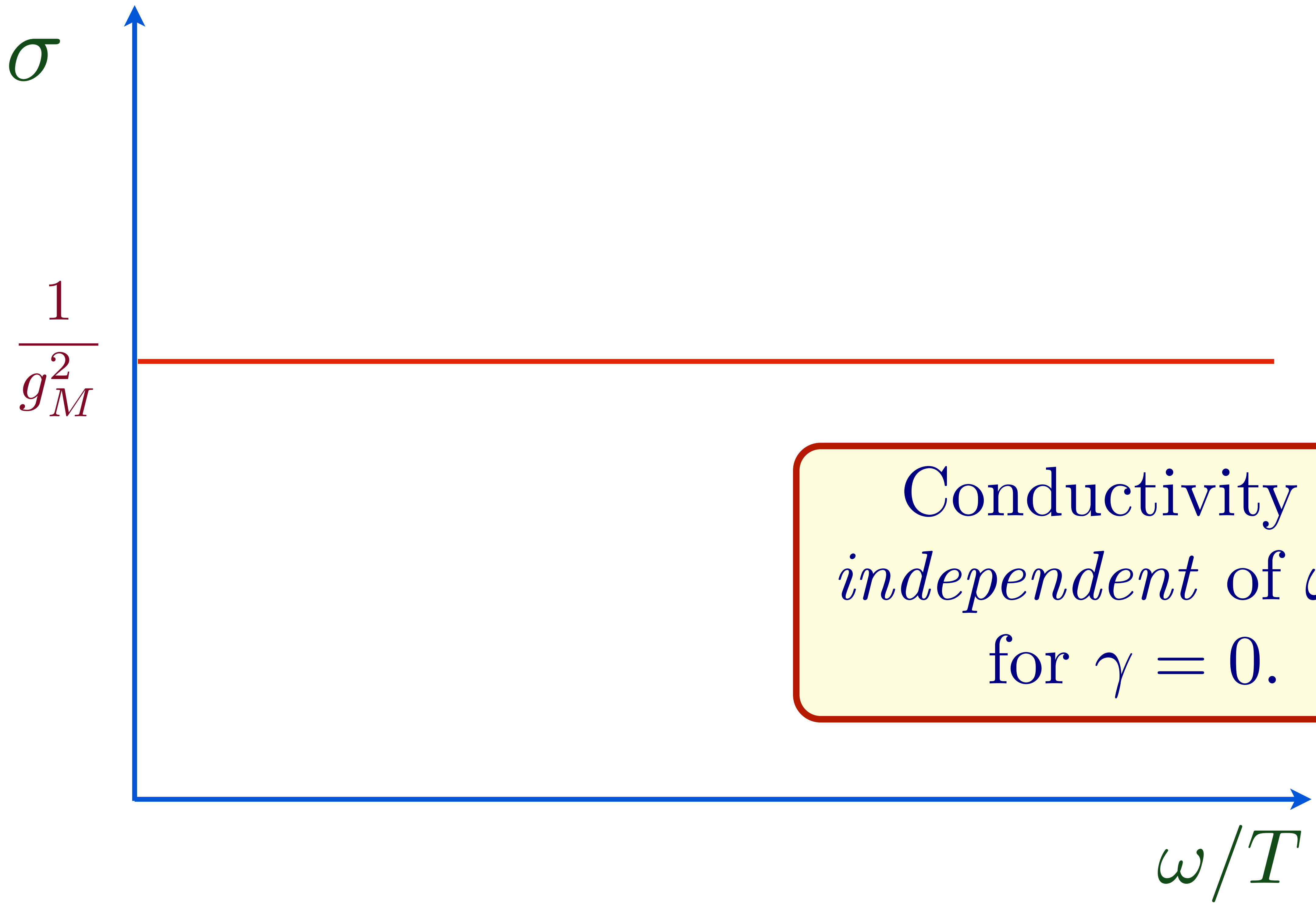
The temperature and entropy of the horizon equal those of the quantum critical point

Quasi-normal modes of quantum criticality = waves falling into black hole

Gauge-gravity duality at non-zero temperatures



AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*



Conductivity is *independent* of ω/T for $\gamma = 0$.

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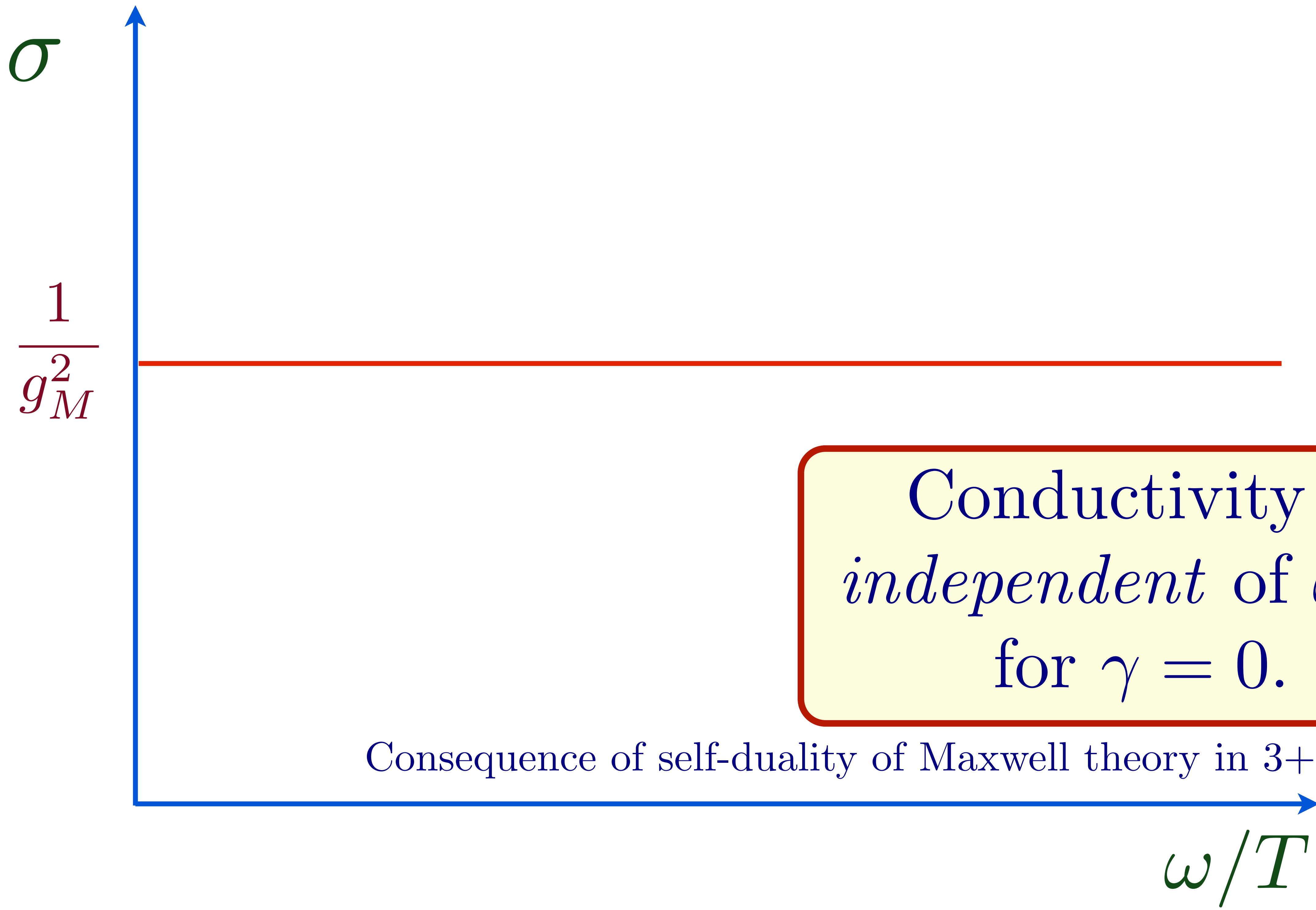
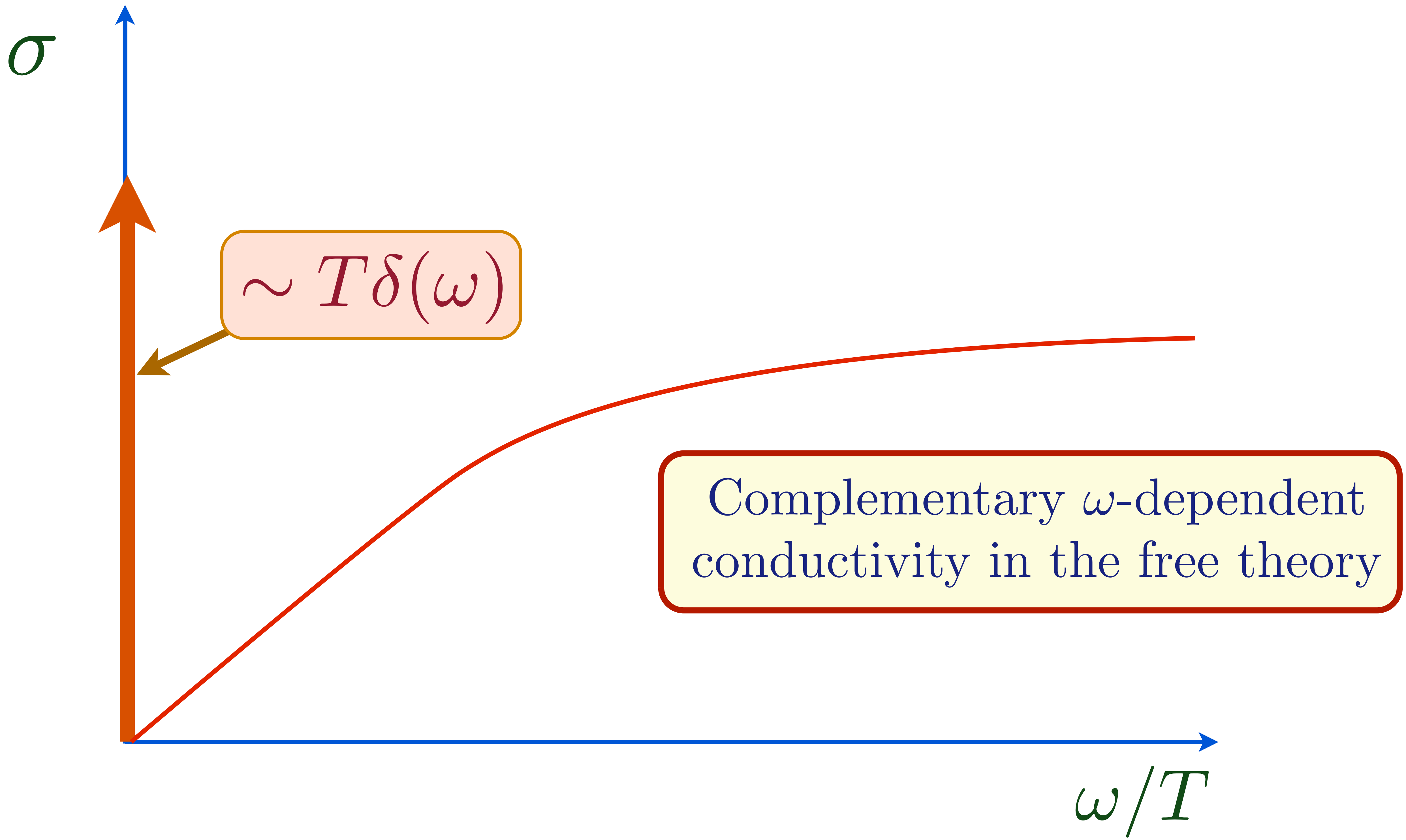


Conductivity is *independent* of ω/T for $\gamma = 0$.

Consequence of self-duality of Maxwell theory in 3+1 dimensions

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

Electrical transport in a free CFT3 for *T > 0*



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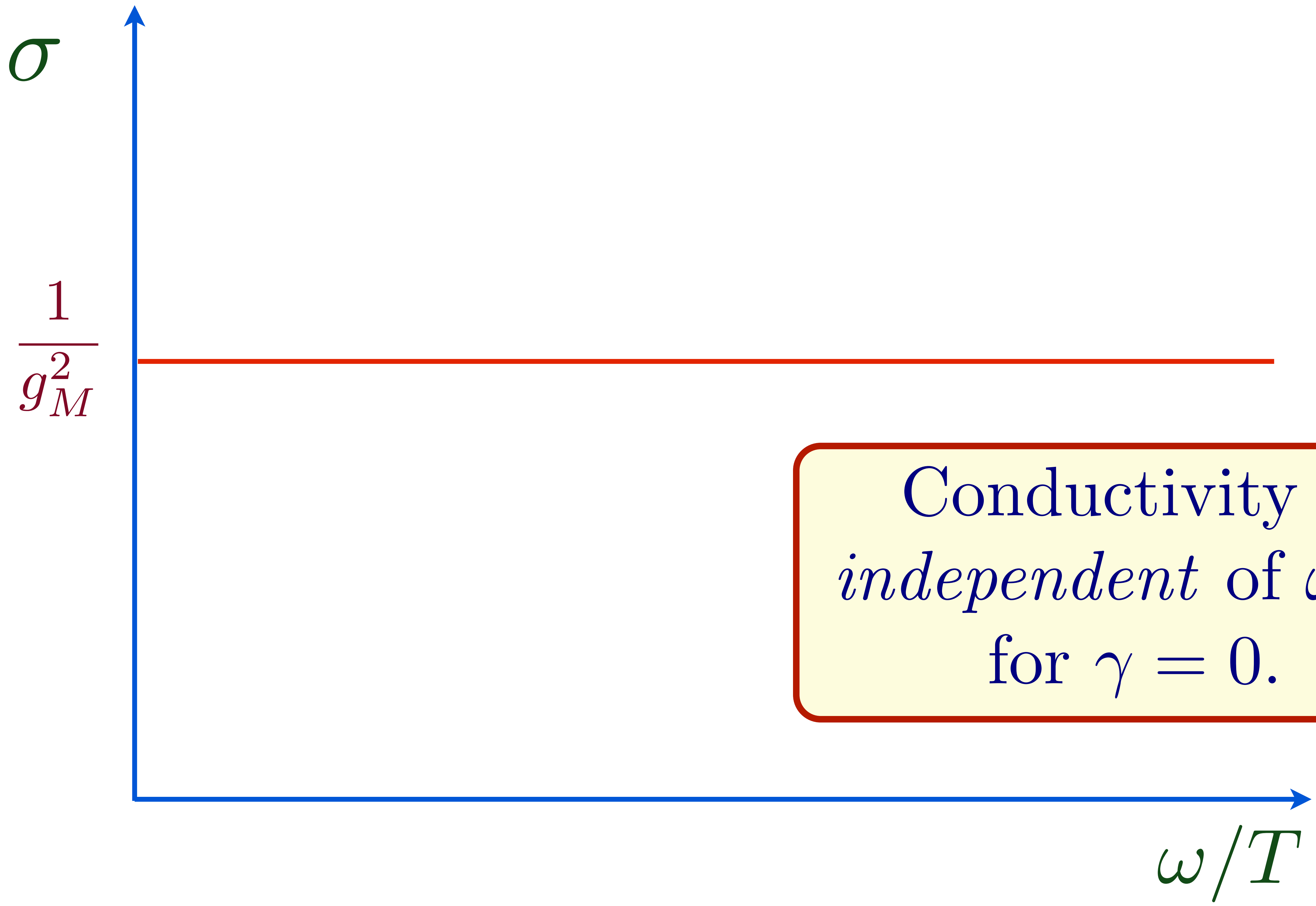

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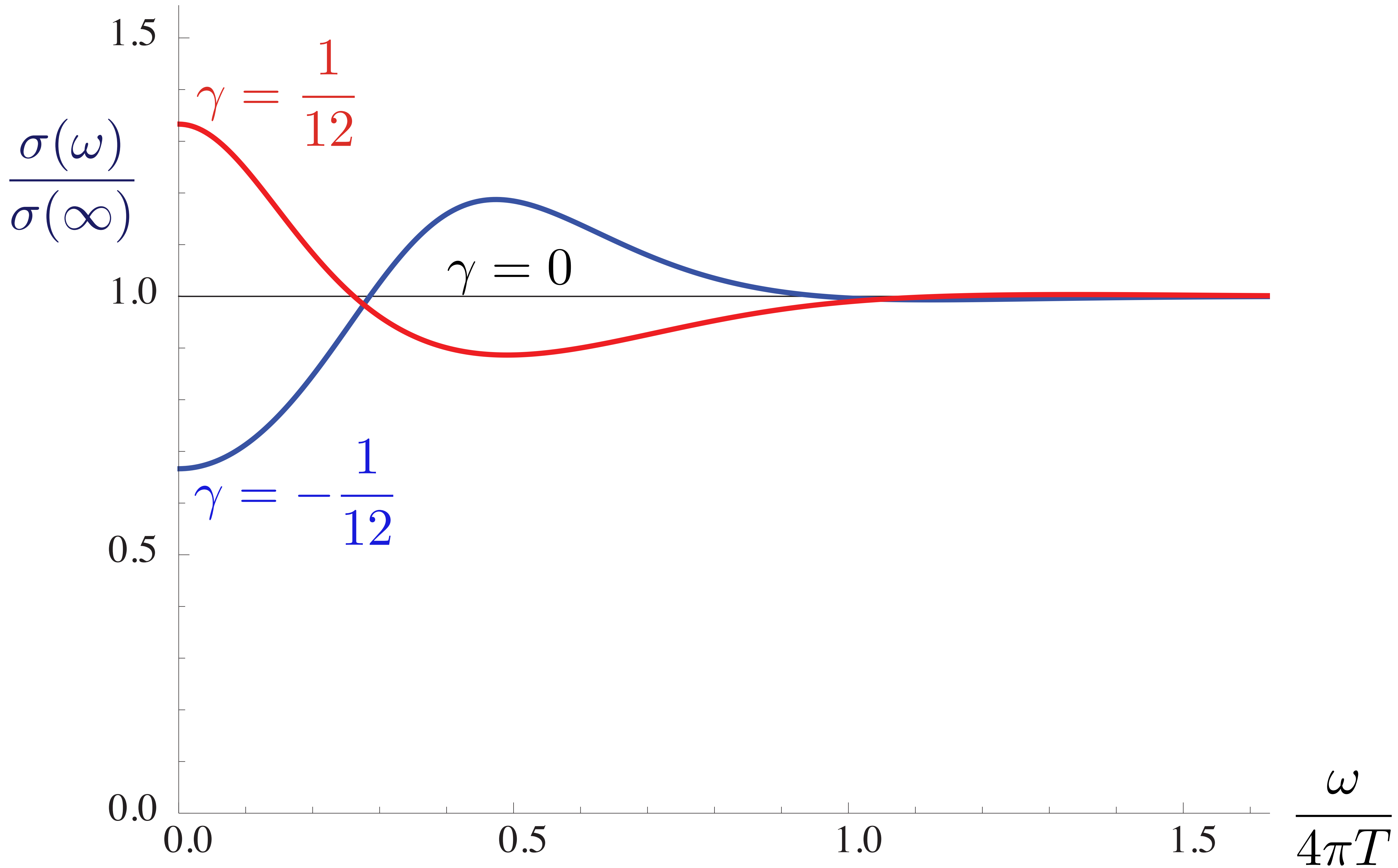
AdS4 theory of "nearly perfect fluids"



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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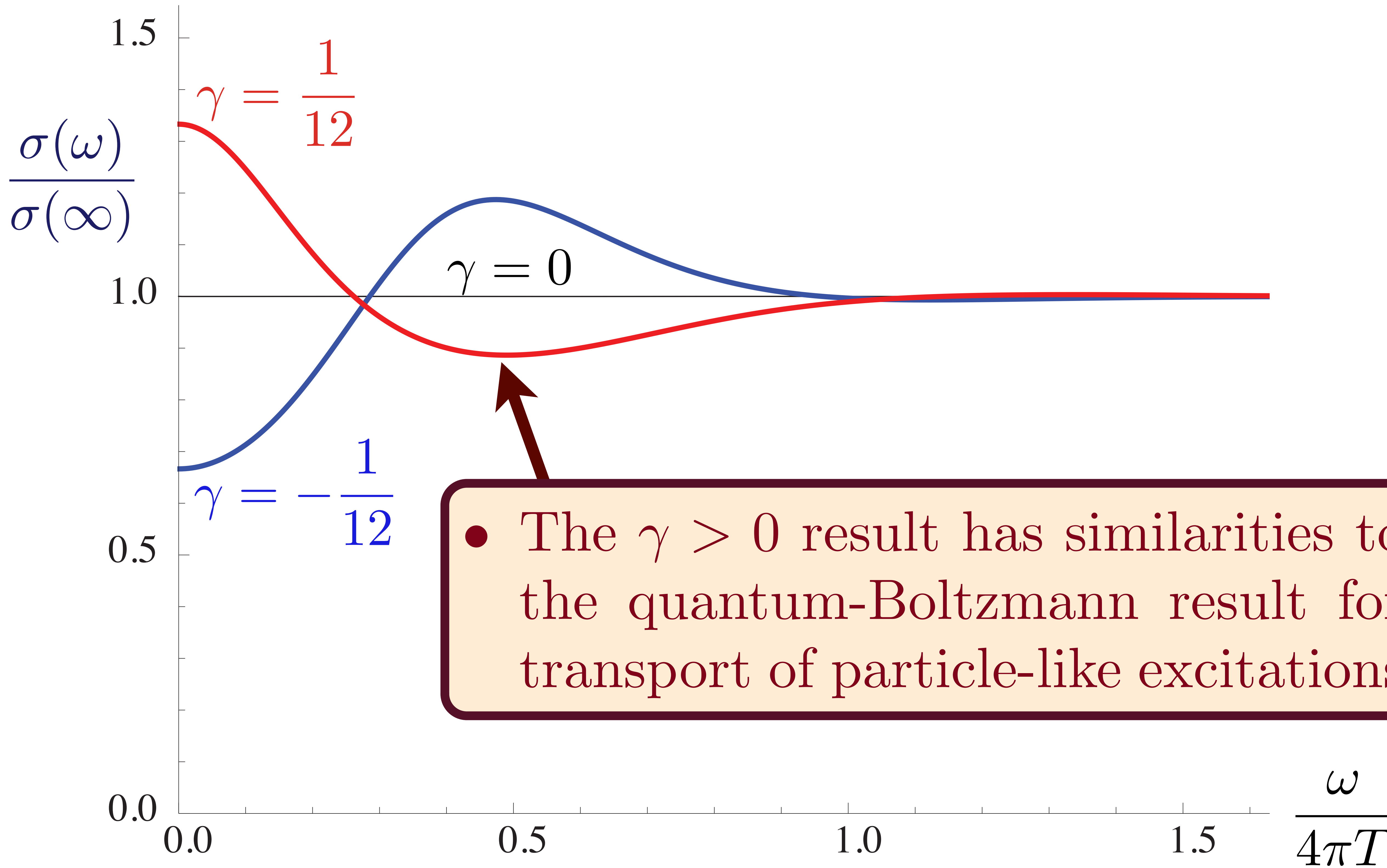
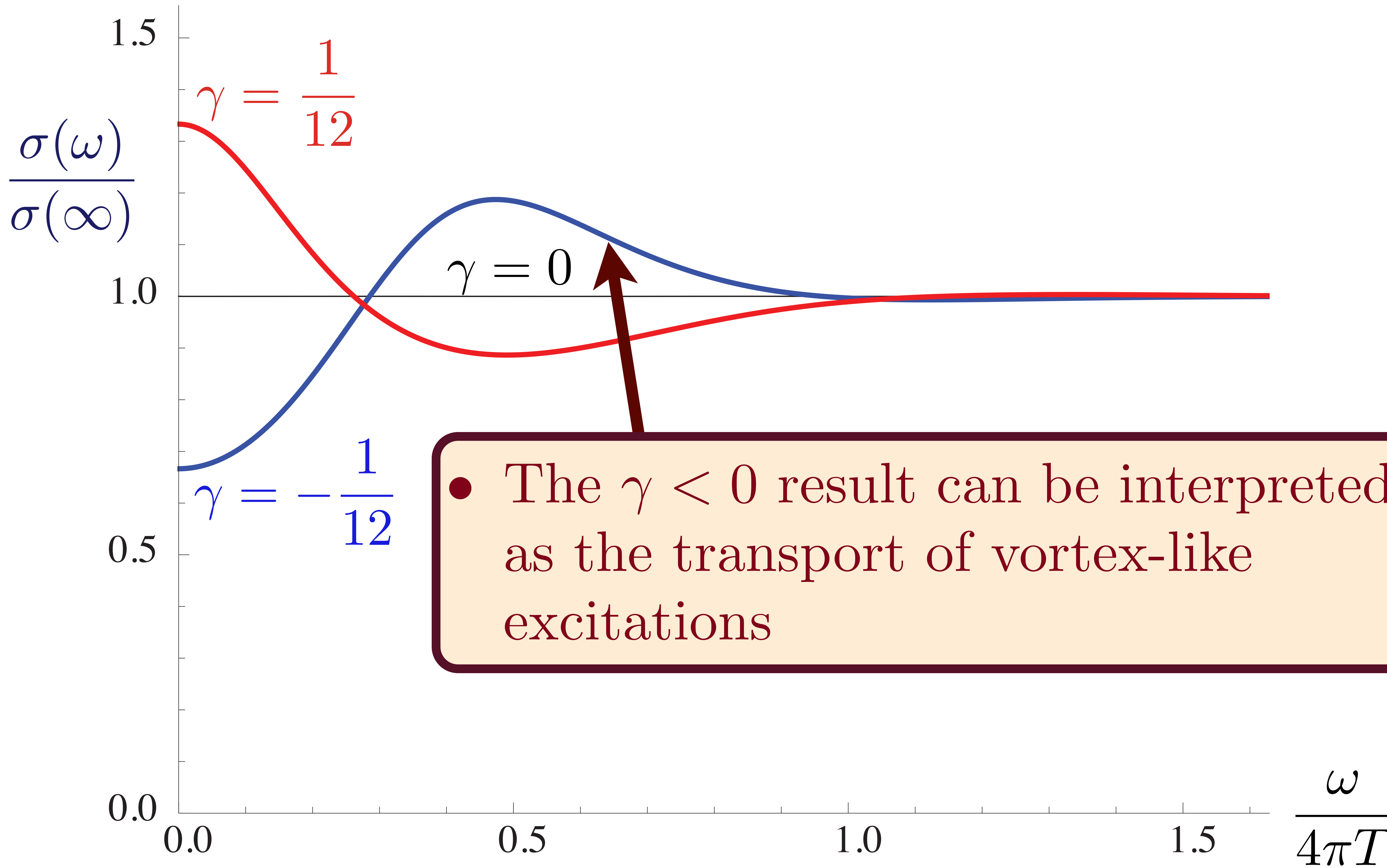


- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of "nearly perfect fluids"



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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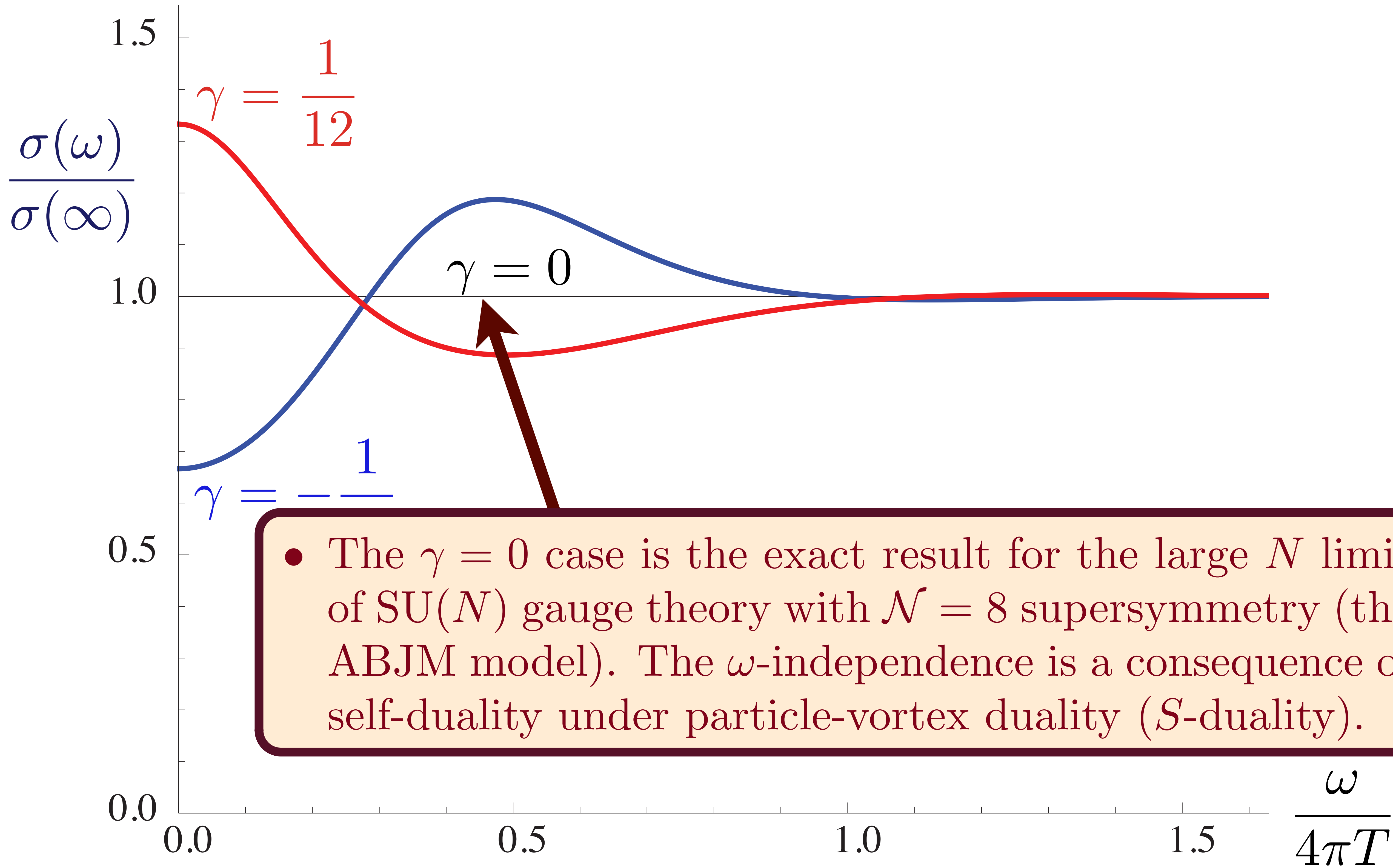

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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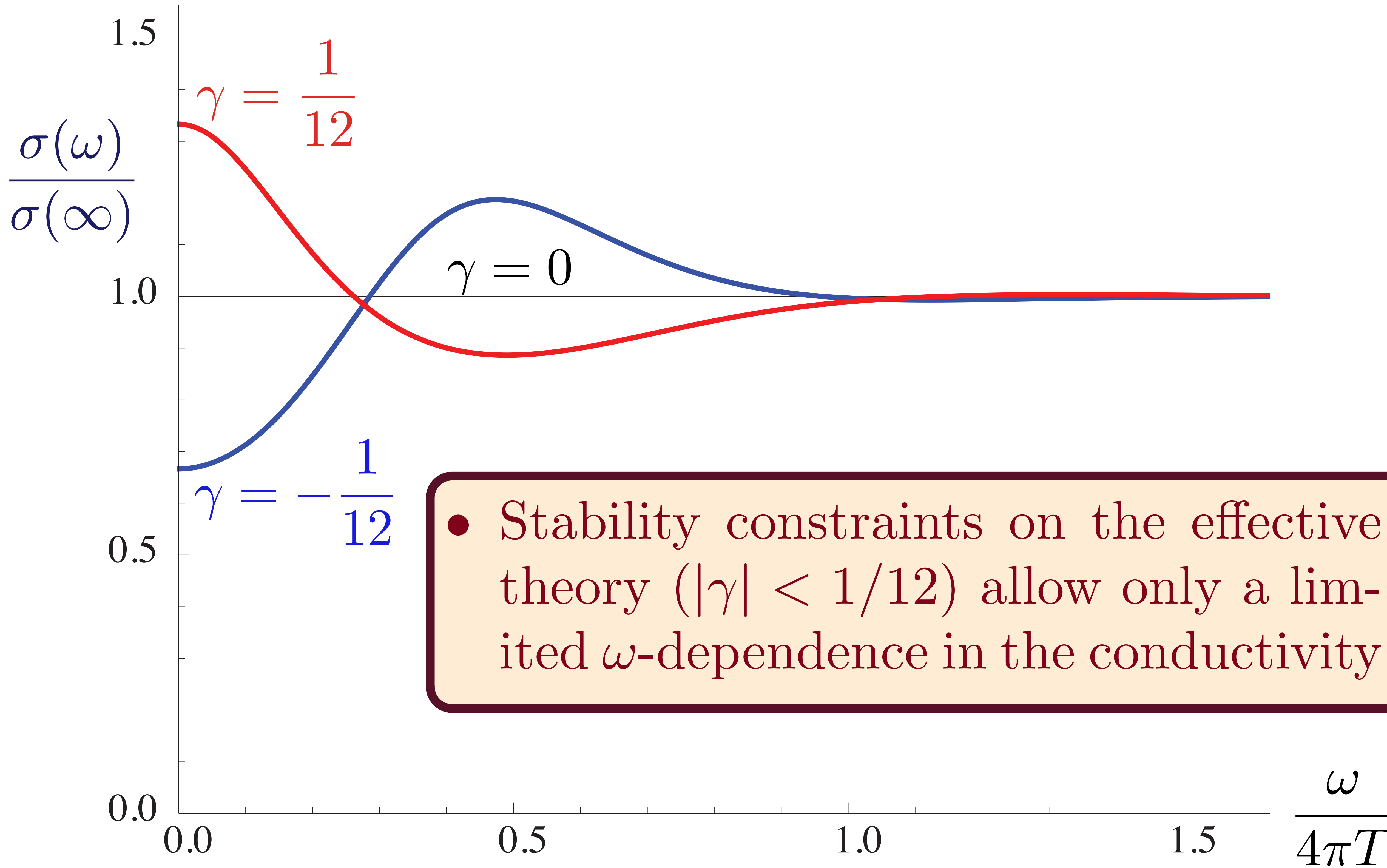
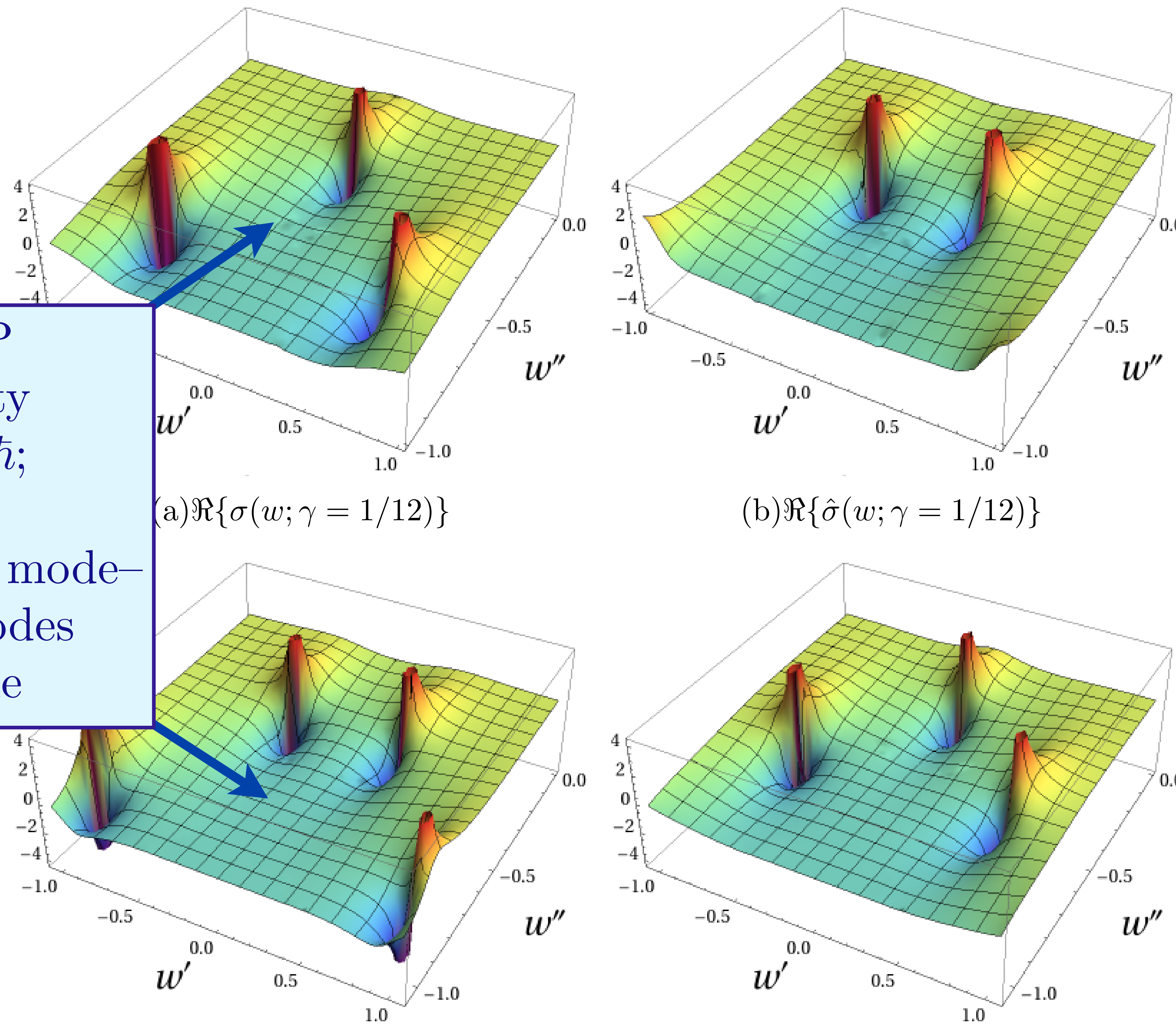


- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality



Poles in LHP
of conductivity
at $\omega \sim k_B T/\hbar$;
analog of
Higgs quasinormal mode–
quasinormal modes
of black brane

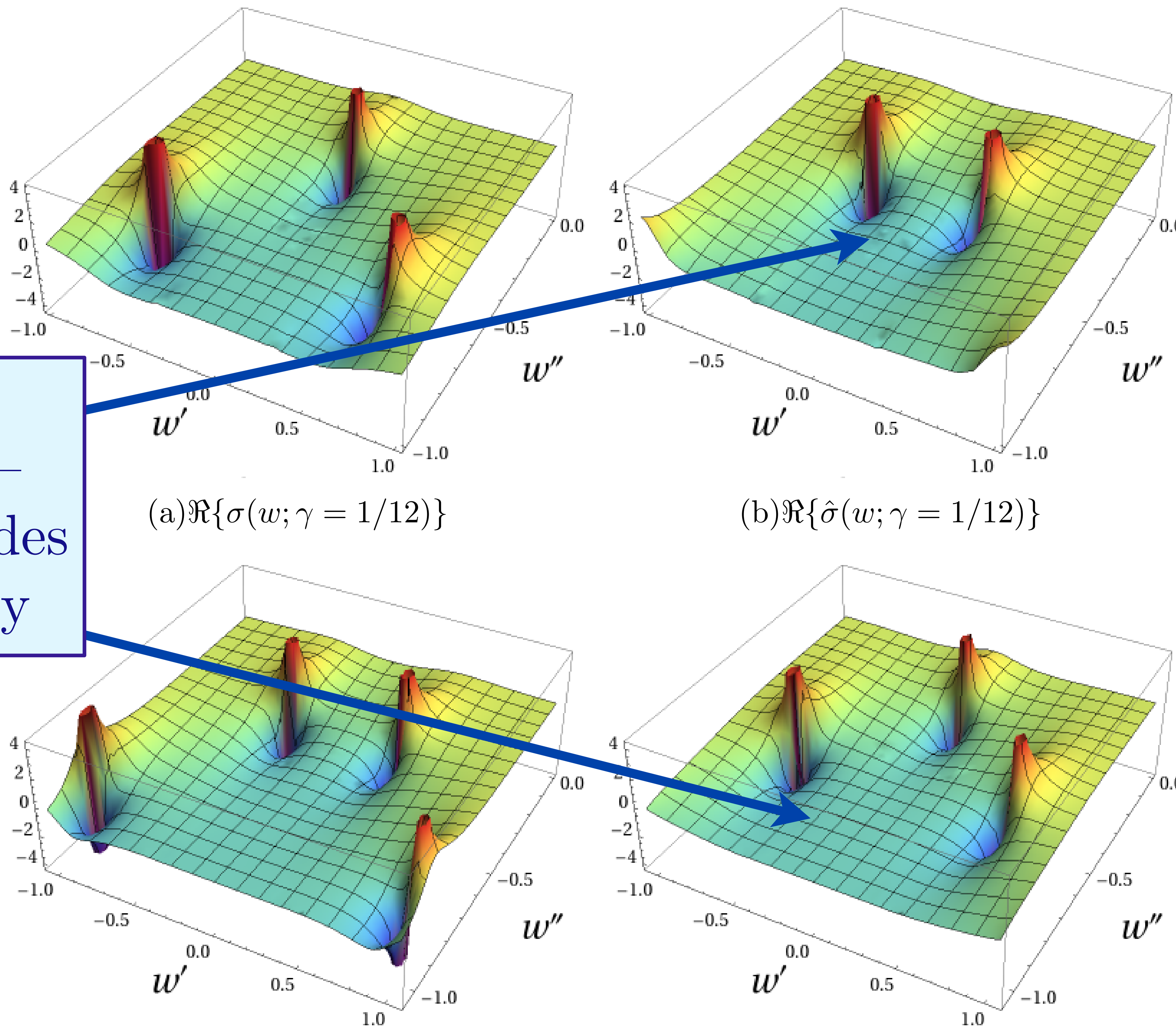
(a)$\Re\{\sigma(w;\gamma=1/12)\}$ (b)$\Re\{\hat\sigma(w;\gamma=1/12)\}$

(c)$\Re\{\sigma(w;\gamma=-1/12)\}$ (d)$\Re\{\hat\sigma(w;\gamma=-1/12)\}$

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

Poles in LHP of resistivity — quasinormal modes of S-dual theory



(a) $\Re\{\sigma(w;\gamma = 1/12)\}$ (b) $\Re\{\hat{\sigma}(w;\gamma = 1/12)\}$

(c) $\Re\{\sigma(w;\gamma = -1/12)\}$ (d) $\Re\{\hat{\sigma}(w;\gamma = -1/12)\}$

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of "nearly perfect fluids"

The holographic solutions for the conductivity satisfy two sum rules, valid for all CFT3s. (W. Witzack-Krempa and S. Sachdev, Phys. Rev. B **86**, 235115 (2012))

$$\int_0^\infty d\omega \mathrm{Re}\left[\sigma(\omega) - \sigma(\infty)\right] = 0$$

$$\int_0^\infty d\omega \mathrm{Re}\left[\frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)}\right] = 0$$

The second rule follows from the existence of a EM-dual CFT3.

Boltzmann theory chooses a "particle" basis: this satisfies only *one* sum rule but not the other.

Holographic theory satisfies both sum rules.

AdS$_4$ theory of quantum criticality

PRL **95,** 180603 (2005) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2005

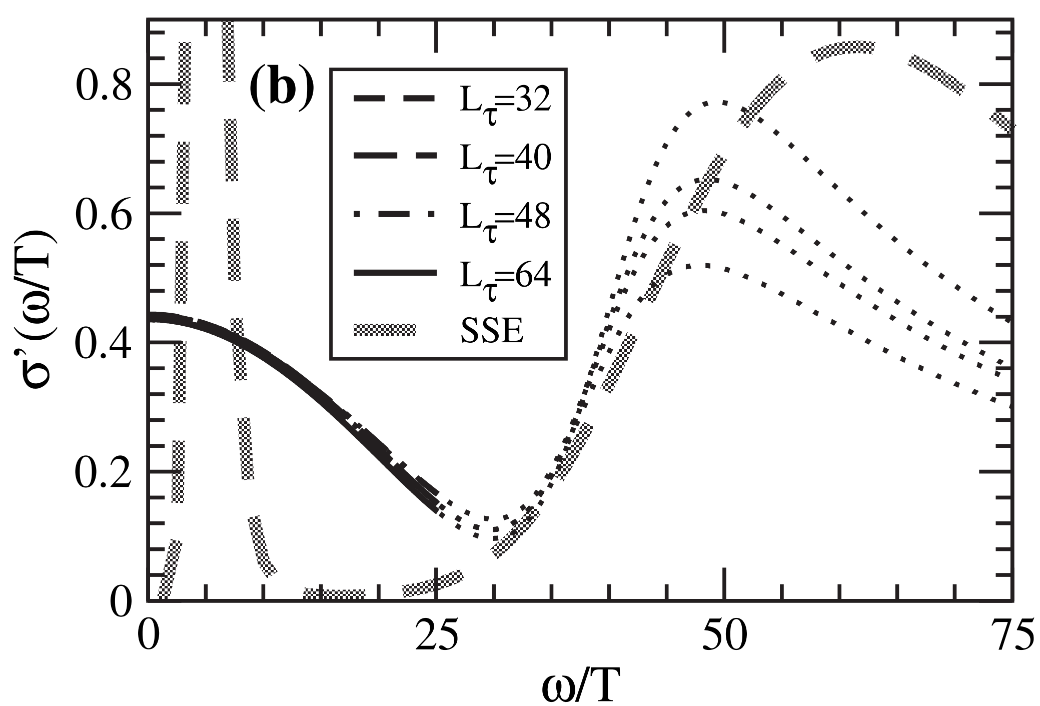
Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

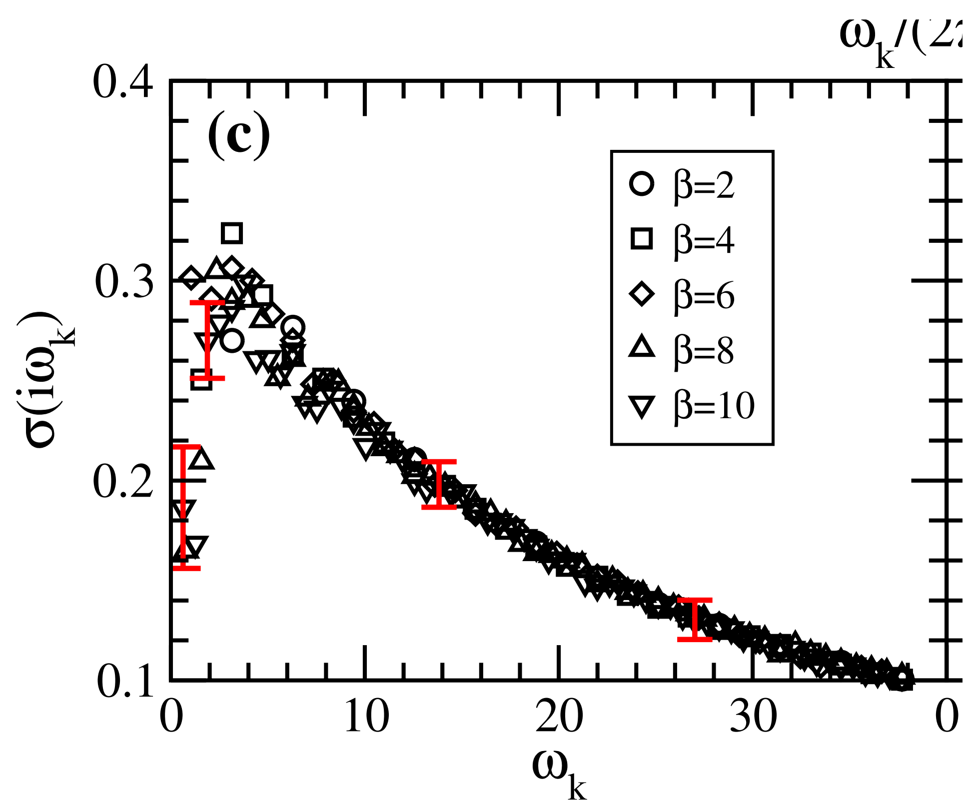
Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature T. We find clear evidence for *deviations* from ω_k scaling of the conductivity towards ω_k/T scaling at low Matsubara frequencies ω_k. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with ω/T at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.





AdS$_4$ theory of quantum criticality

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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, arXiv:1302.0847

Traditional CMT

- Identify quasiparticles and their dispersions

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- Compute scattering matrix elements of quasiparticles (or of collective modes)

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- Start with strongly interacting CFT without particle- or wave-like excitations

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- Solve Einsten-Maxwell-... equations, allowing for a horizon at non-zero temperatures.

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Quantum criticality and conformal field theories

2. Gauge-gravity duality
   Black-hole horizons and quasi-normal modes

3. Strange metals:
   What lies beyond the horizon ?

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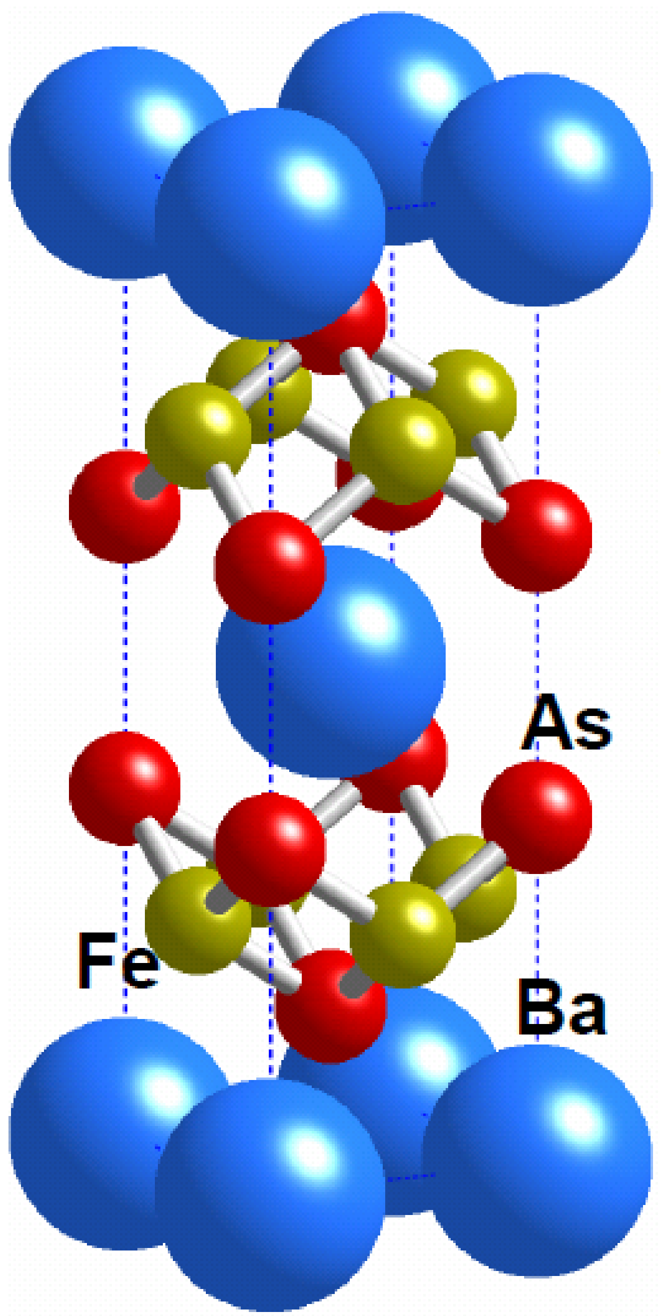
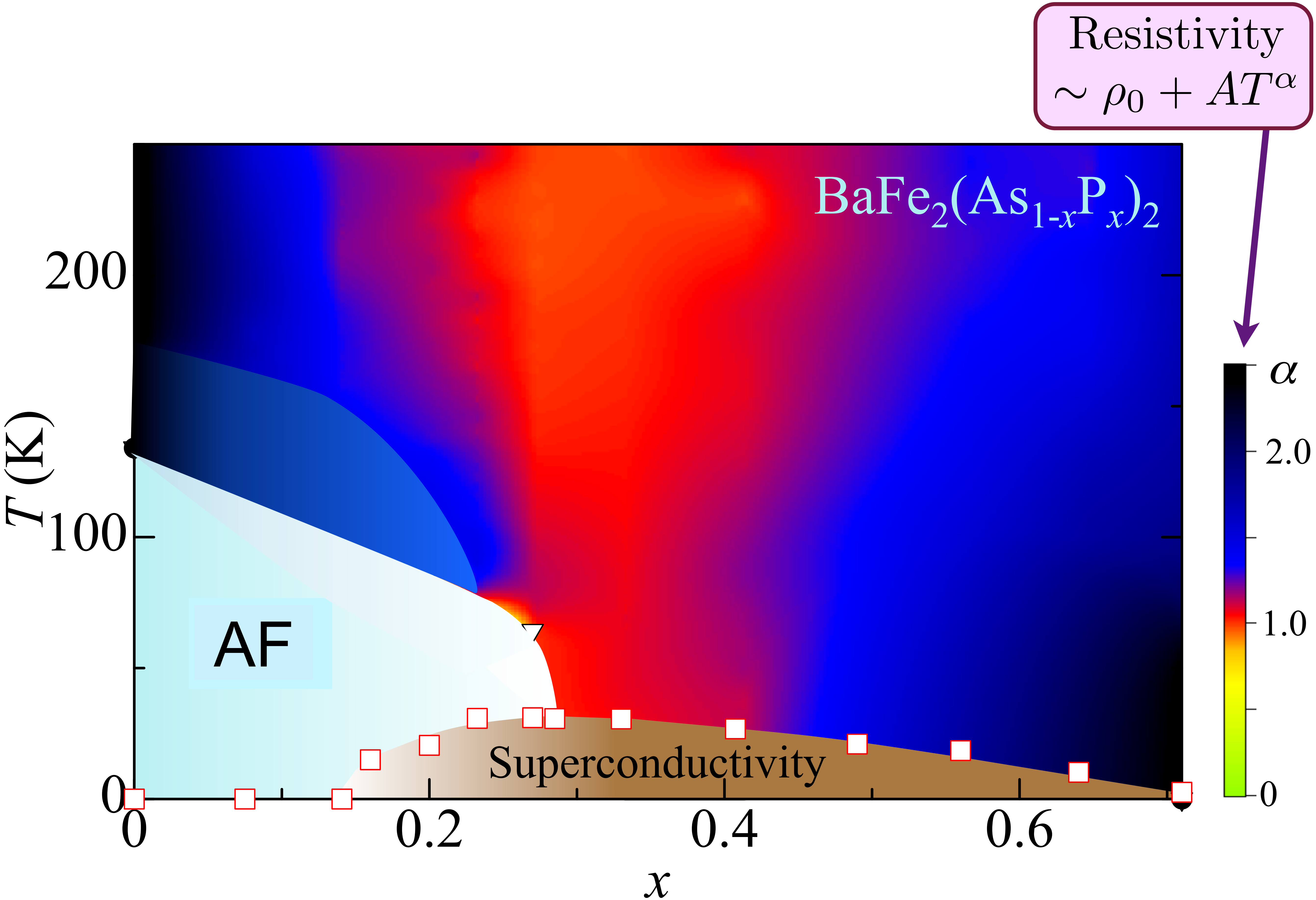
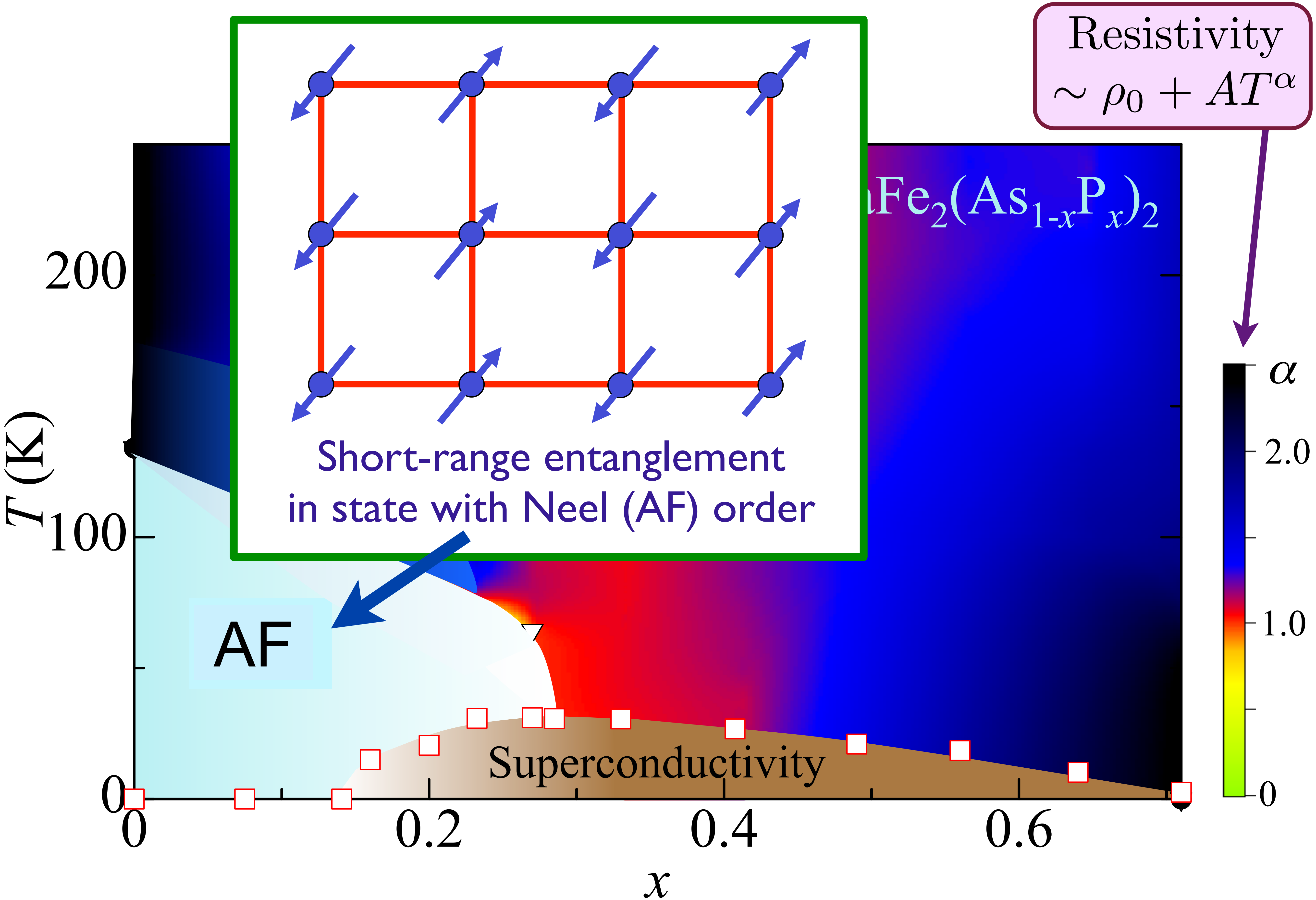
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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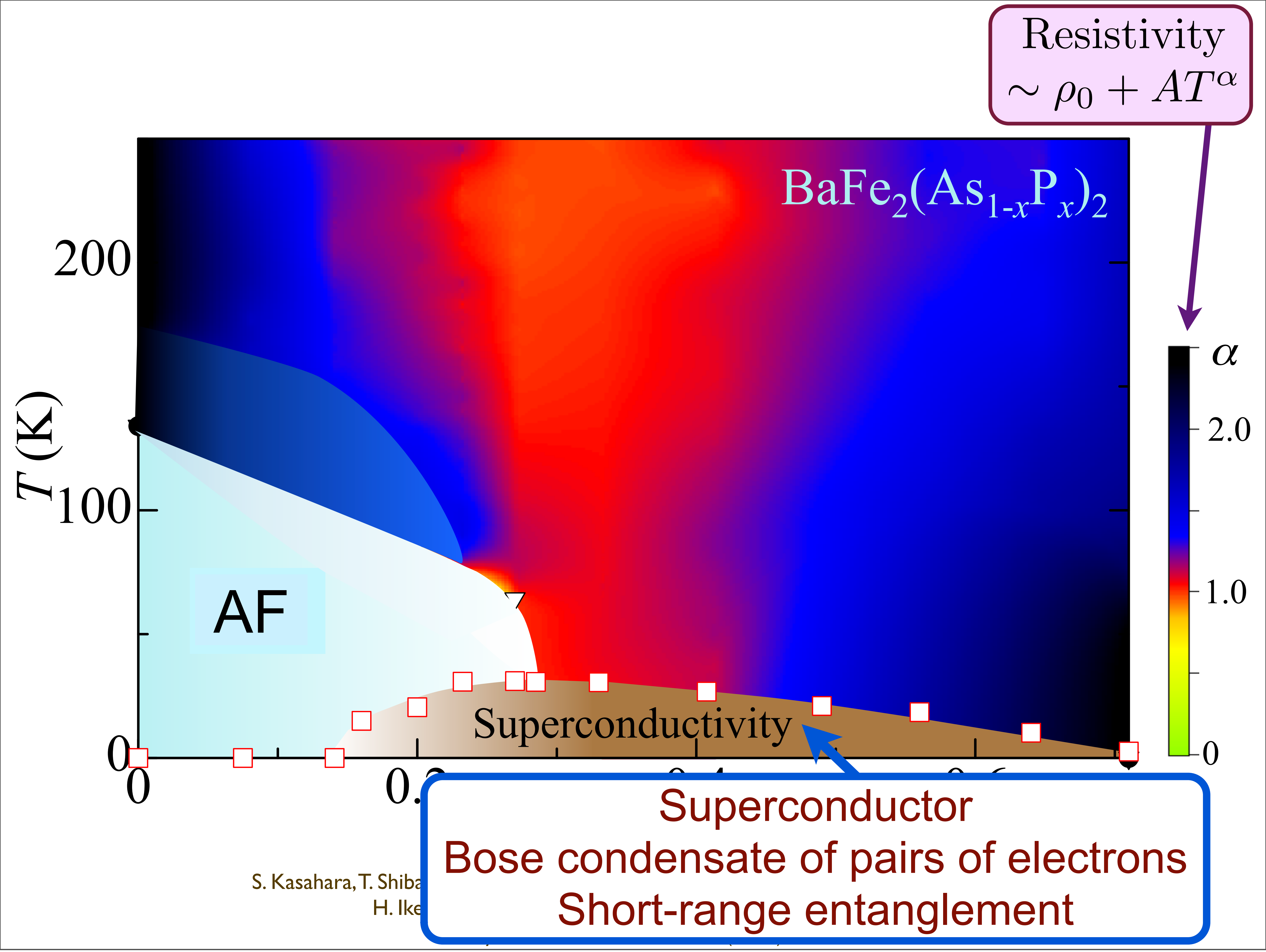
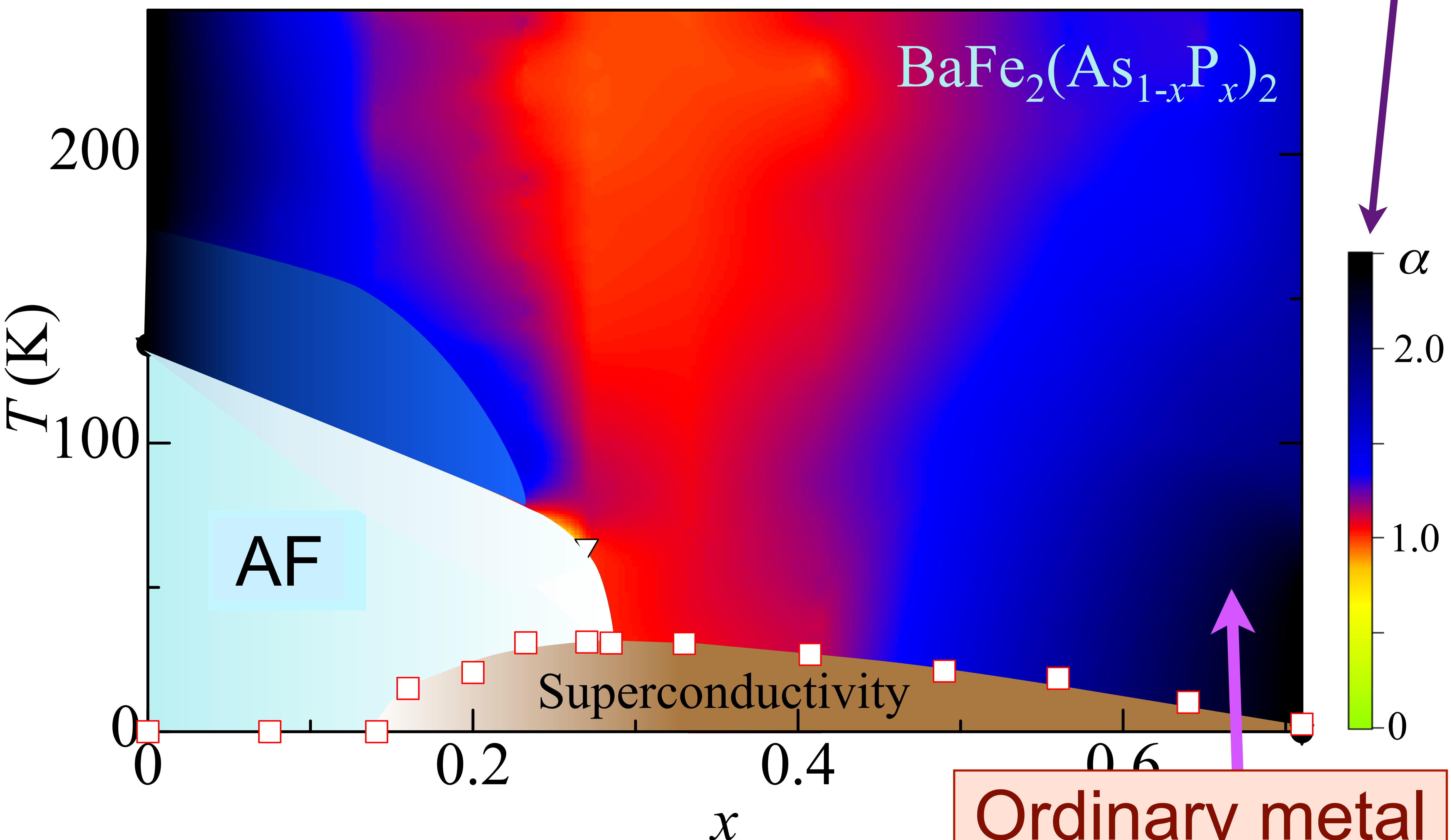
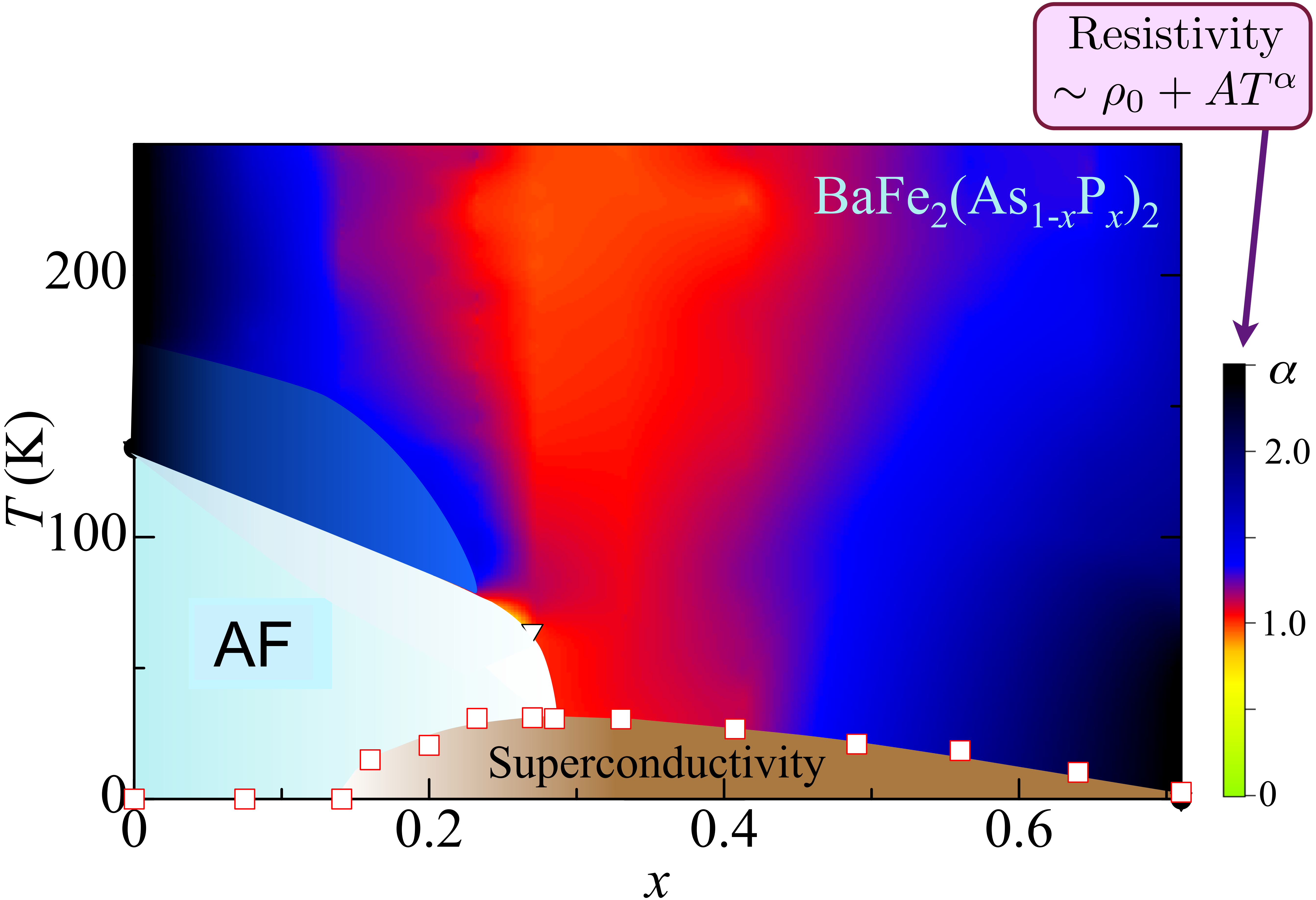

Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
200
100
0
T (K)
AF
Superconductivity
α
2.0
1.0
0
0
Superconductor
Bose condensate of pairs of electrons
Short-range entanglement
S. Kasahara, T. Shiba
H. Ike

Resistivity
$\sim \rho_0 + AT^{\alpha}$



Ordinary metal
(Fermi liquid)

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
Physical Review B **81**, 184519 (2010)



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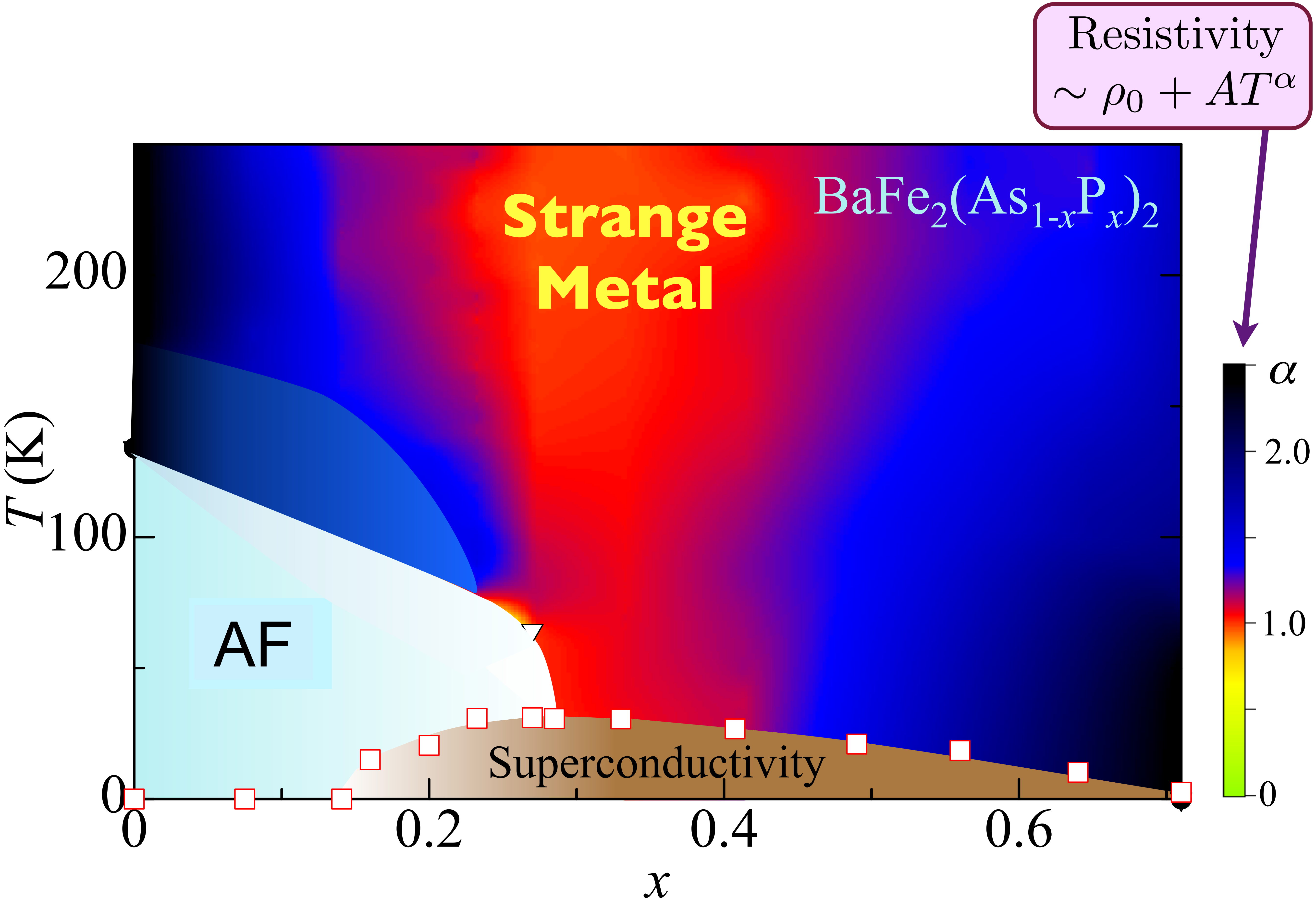

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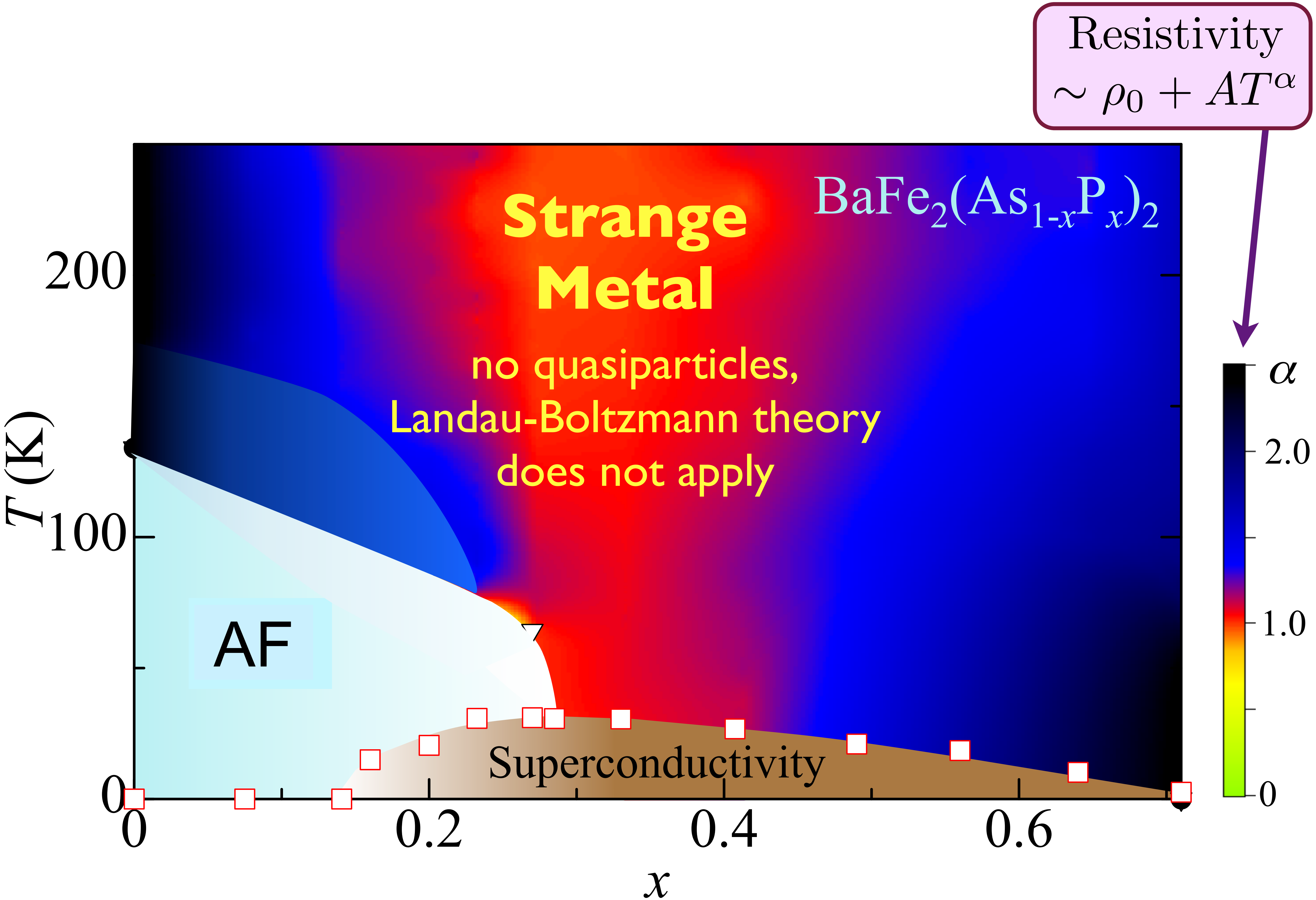

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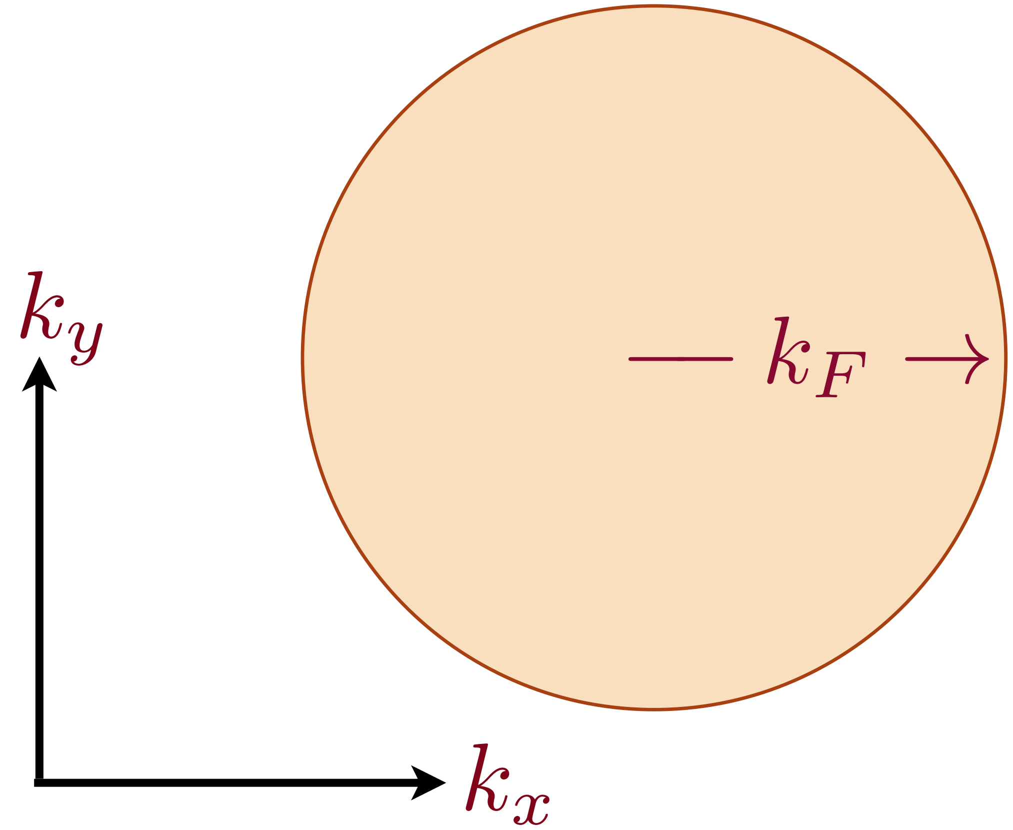
The Metal



Electrons (fermions) occupy states inside a Fermi "surface" (circle) of radius k_F which is determined by the density of electrons, $\mathcal{Q}$.

A Strange Metal

Can bosons form a metal ?

A Strange Metal

O. I. Motrunich and M. P.A. Fisher, *Phys. Rev.* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Phys. Rev.* D **84**, 026001 (2011)
S. Sachdev, arXiv:1209.1637

Can bosons form a metal ?

Yes, if each boson, b, *fractionalizes* into 2 fermions ('quarks')
$b = f_1 f_2$!

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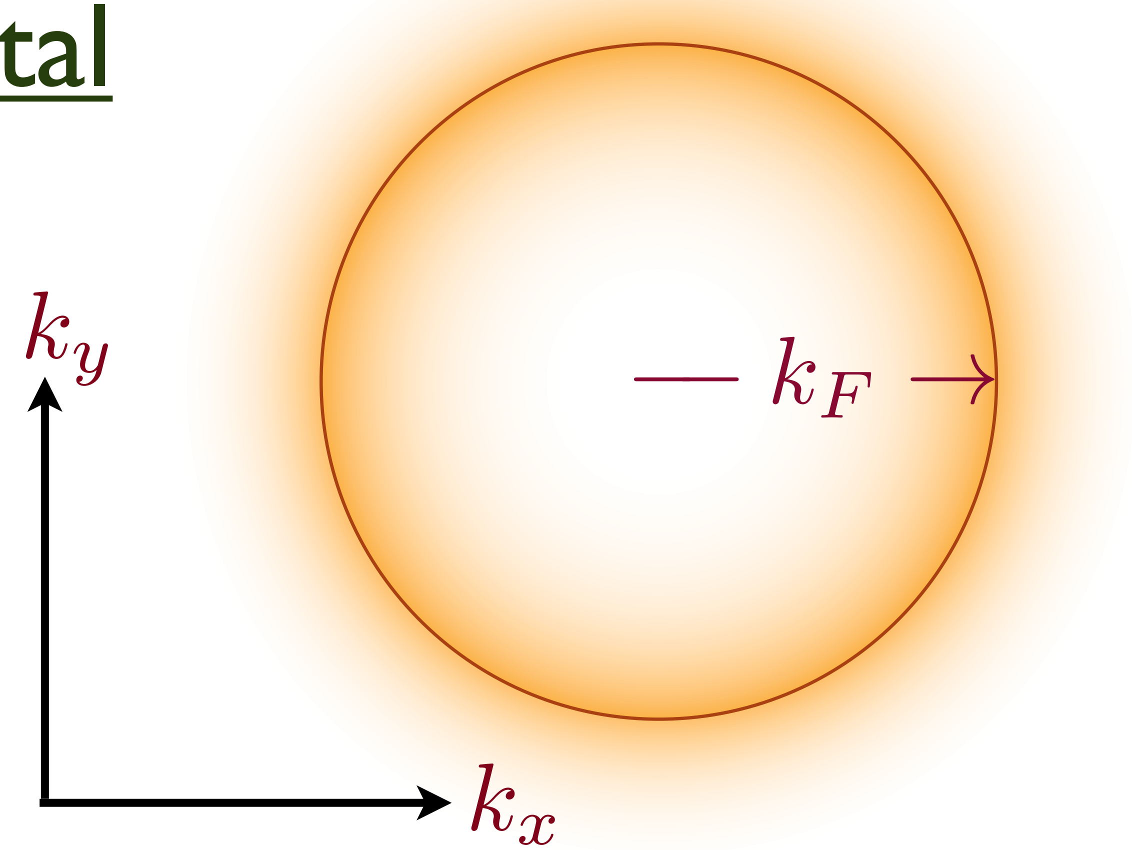
- Each quark is charged under an *emergent* gauge force, which encapsulates the entanglement in the ground state.

A Strange Metal



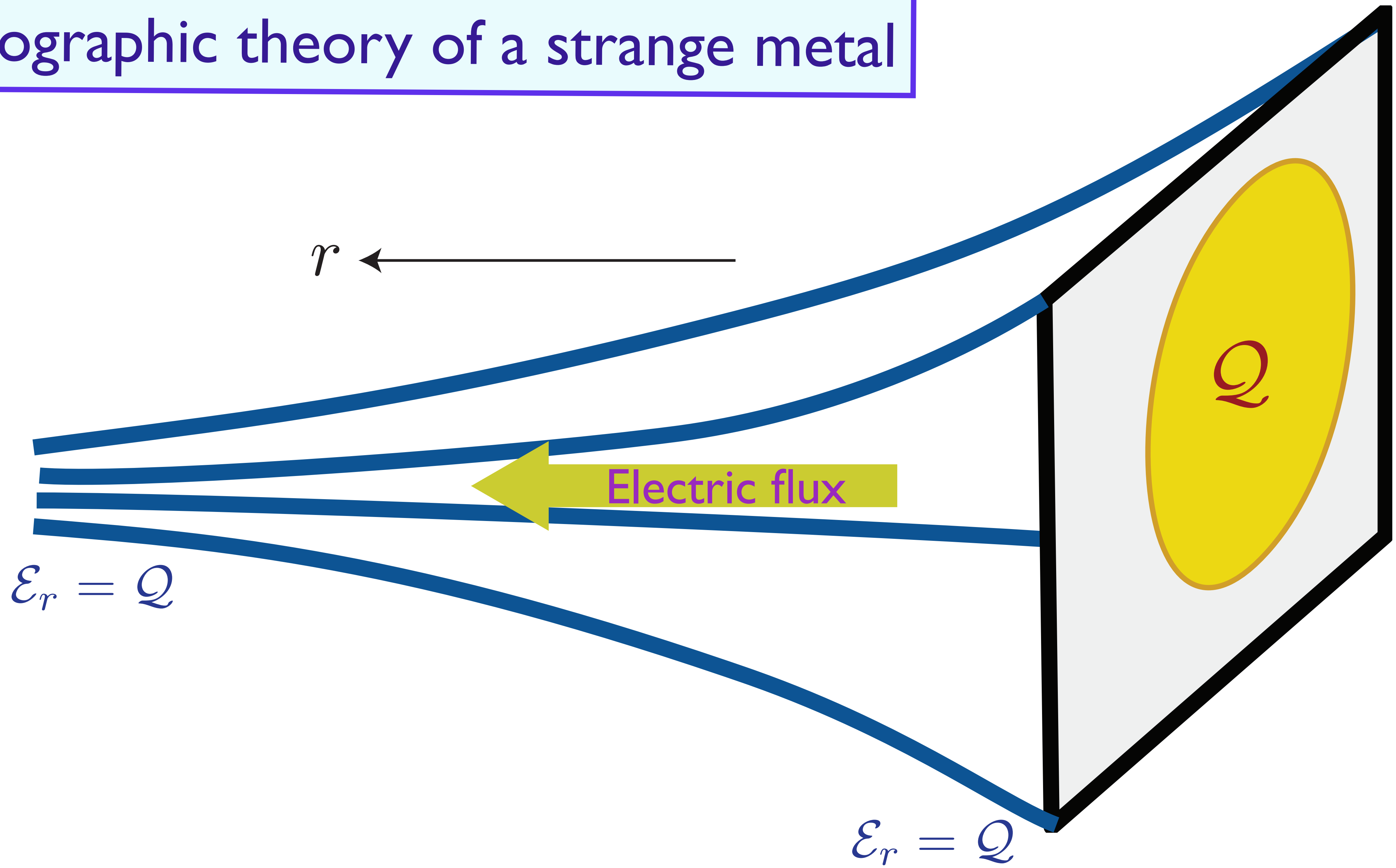
Can bosons form a metal ?

Yes, if each boson, b, *fractionalizes* into 2 fermions ('quarks') $b = f_1 f_2$!

- Each quark is charged under an *emergent* gauge force, which encapsulates the entanglement in the ground state.

- The quarks have "hidden" Fermi surfaces of radius k_F.

Holographic theory of a strange metal



The density of particles $\mathcal{Q}$ creates an electric flux $\mathcal{E}_r$ which modifies the metric of the emergent spacetime.

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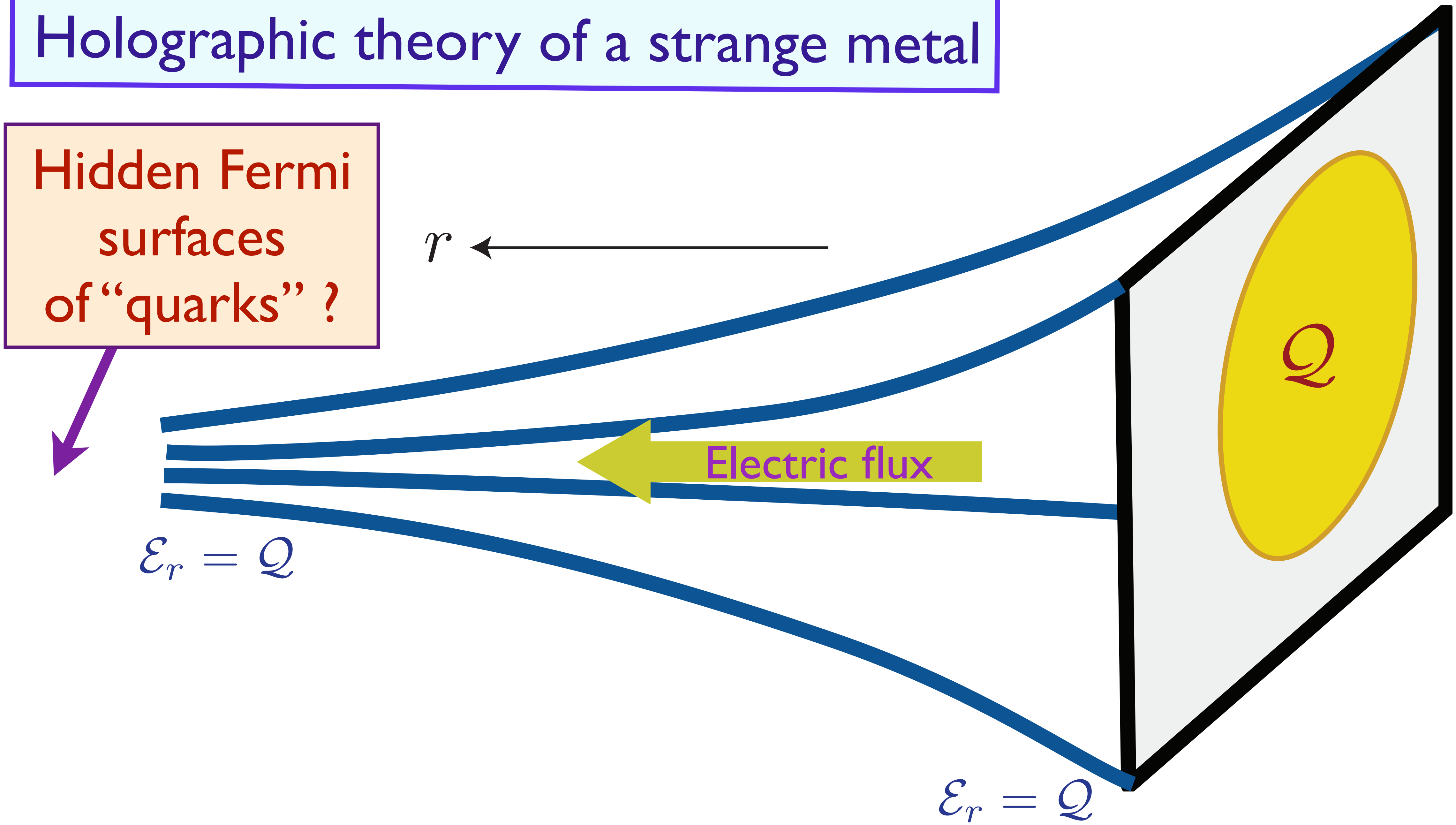

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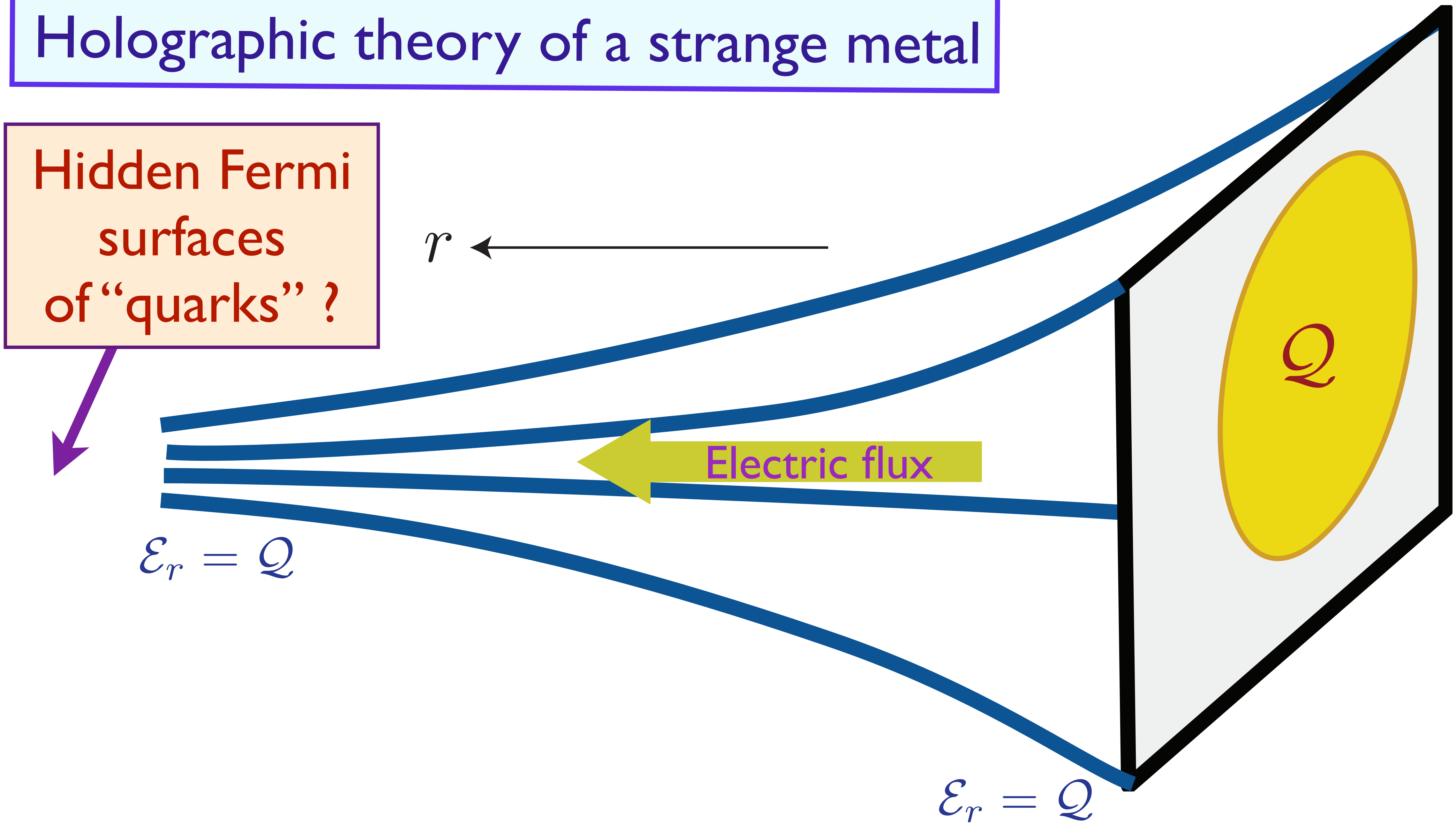
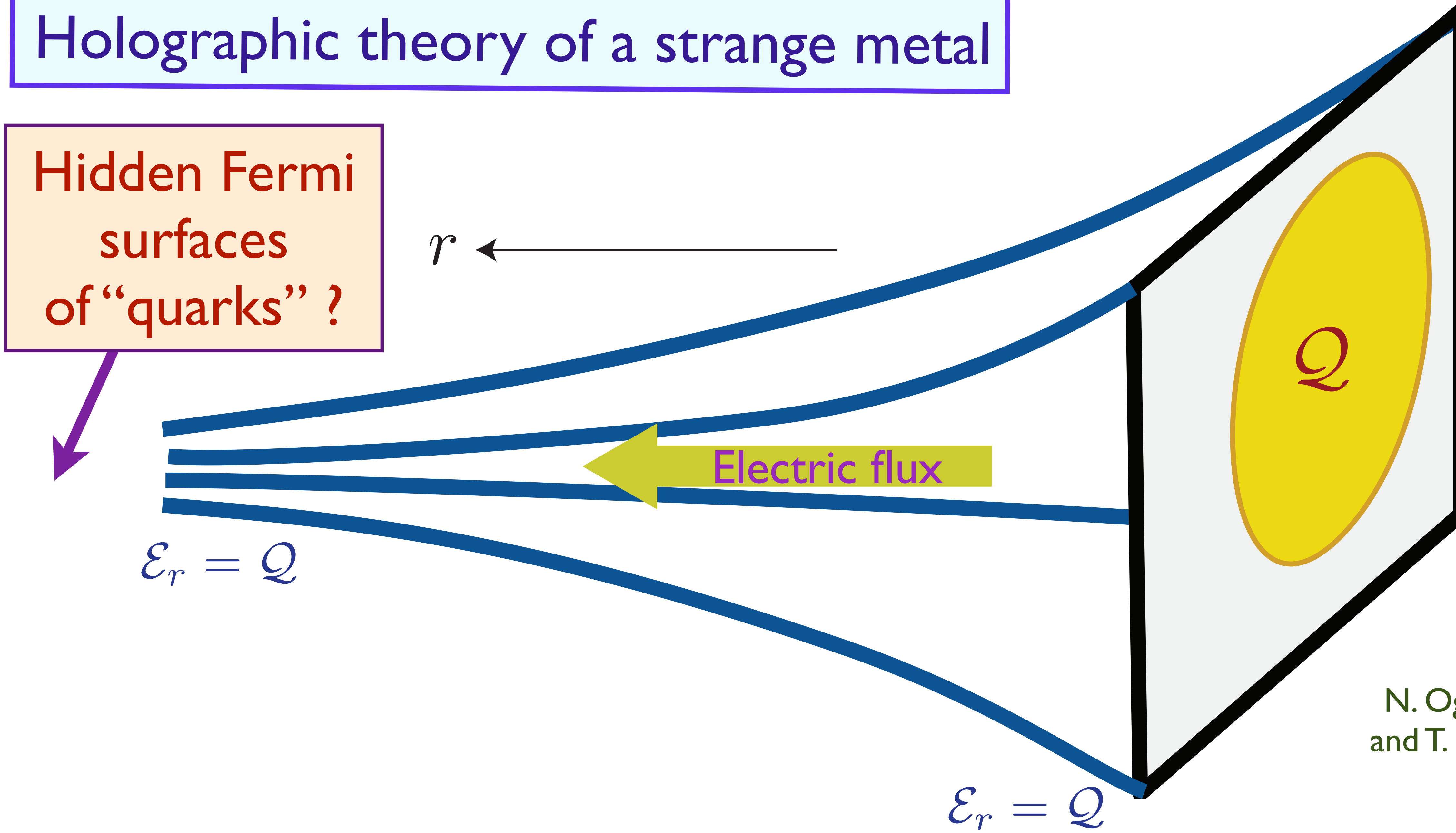


The general metric transforms under rescaling as

$$x_i \to \zeta \, x_i, \quad t \to \zeta^z \, t, \quad ds \to \zeta^{\theta/d} \, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Holographic theory of a strange metal



N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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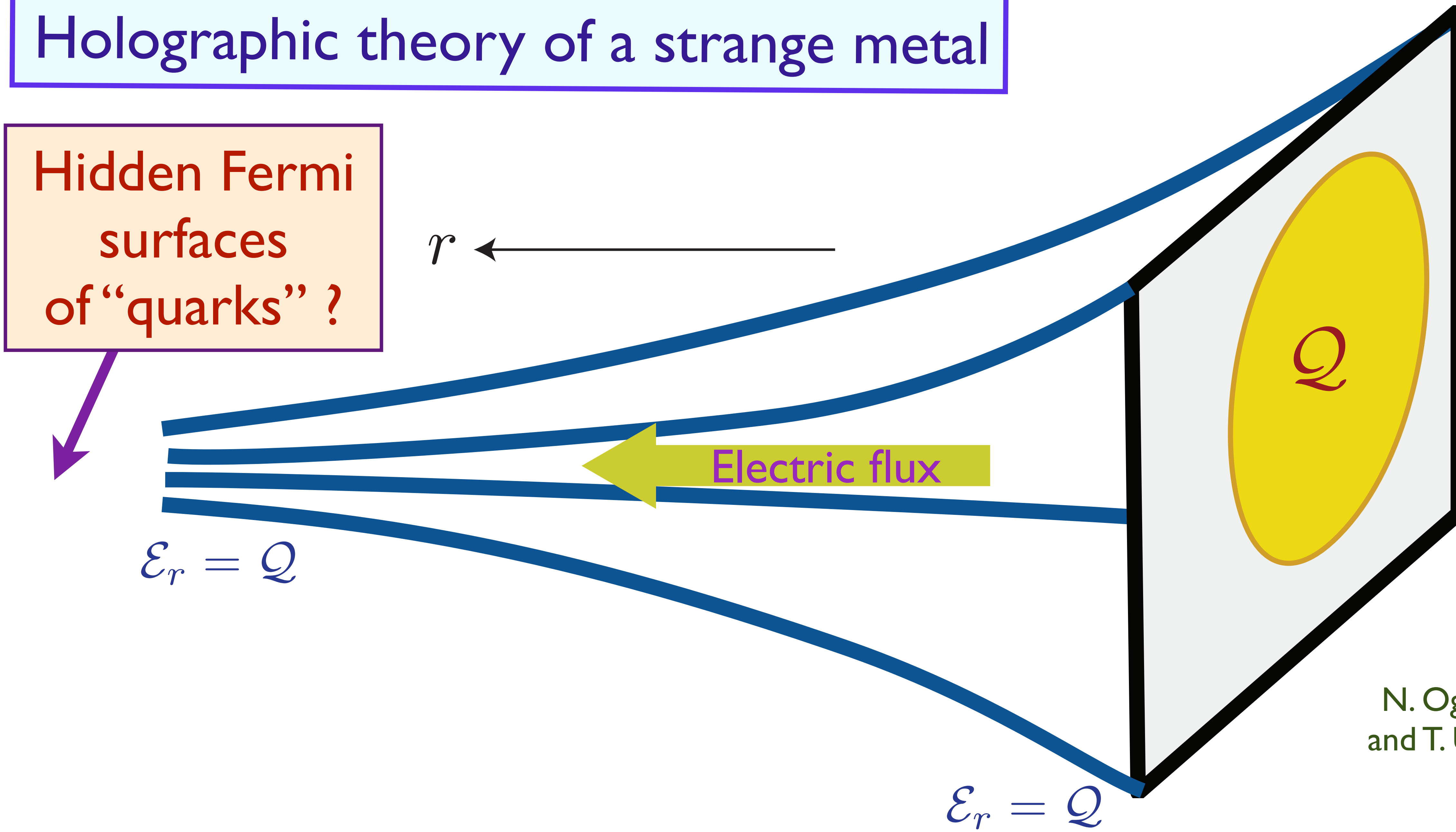
The value $\theta = d - 1$ reproduces *all* the essential characteristics of the entropy and entanglement entropy of a strange metal.

Holographic theory of a strange metal



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The null-energy condition of gravity yields $z \geq 1 + \theta/d$. In $d = 2$, this leads to $z \geq 3/2$. Field theory on strange metal yields $z = 3/2$ to 3 loops!

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

<u>Conclusions</u>

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

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- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of "strange metals"