The disordered Hubbard model: from Si:P to the high temperature superconductors

Subir Sachdev
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Workshop on 2D Quantum Metamaterials
NIST, Gaithersburg, MD

PHYSICS
HARVARD

1. Disordered Hubbard model for Si:P

2. Solvable disordered models:

   (A) Random matrix model of a `quantum dot'

   Metal with quasiparticles

   (B) SYK model of a `quantum dot'

   Metal without quasiparticles

3. High temperature superconductivity and strange metals.

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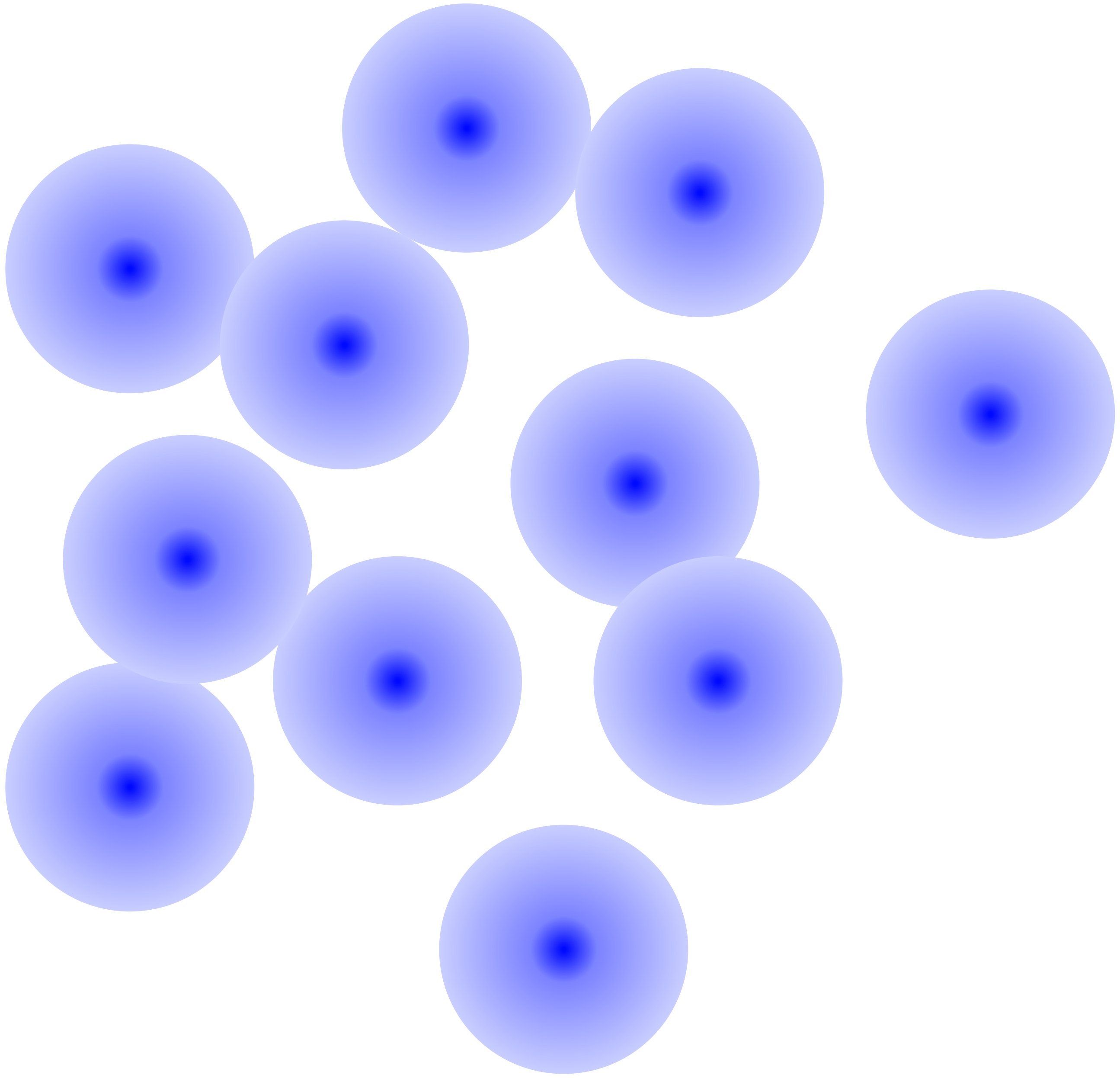
Disordered Hubbard model for Si:P

$$H = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha}$$
$$- \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$
$$+ U \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$



$$t_{ij} \sim \exp(-|\mathbf{r}_i - \mathbf{r}_j|/a_0)$$

- Electrons in doped silicon appear to separate into two components: localized spin moments and itinerant electrons

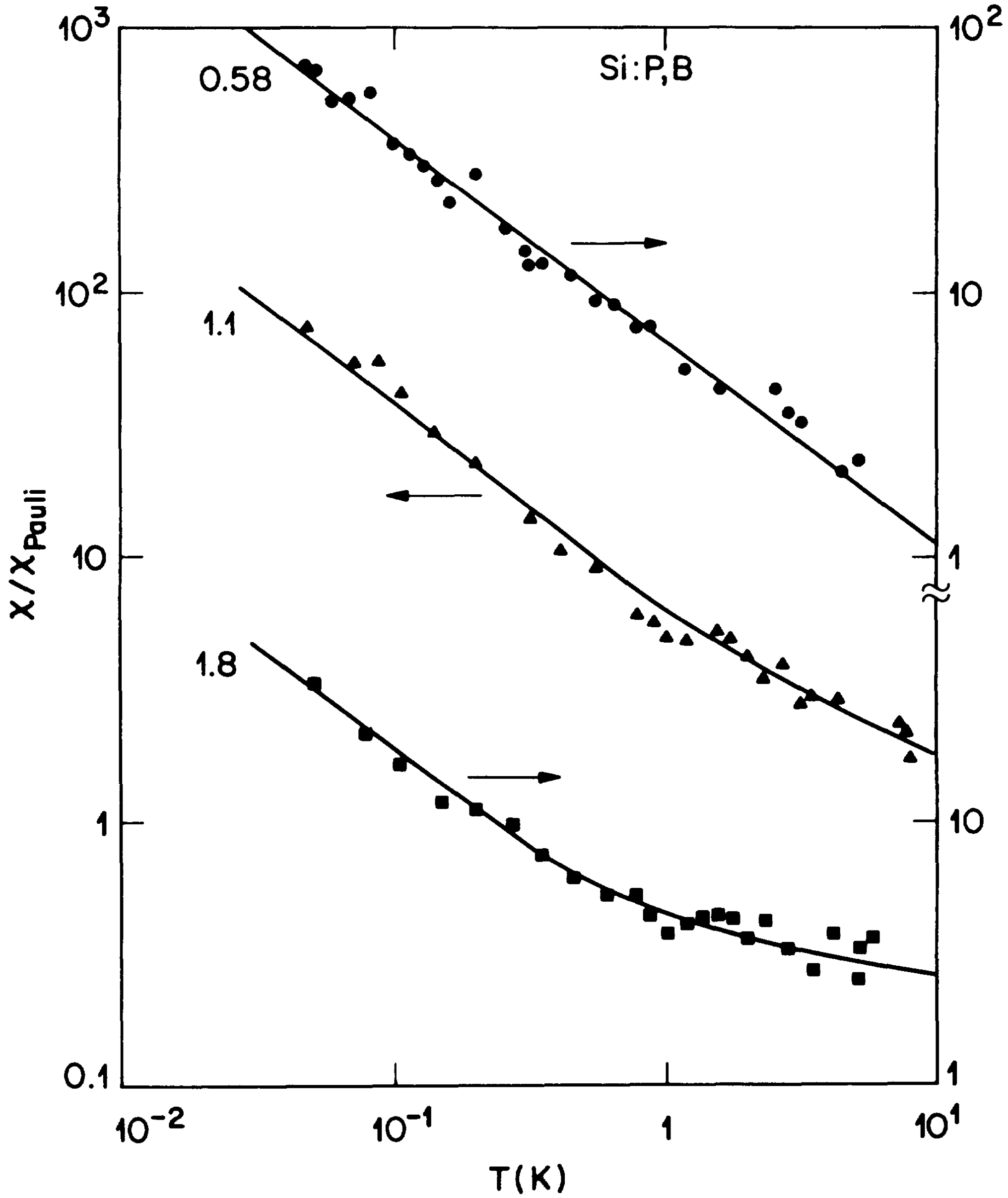
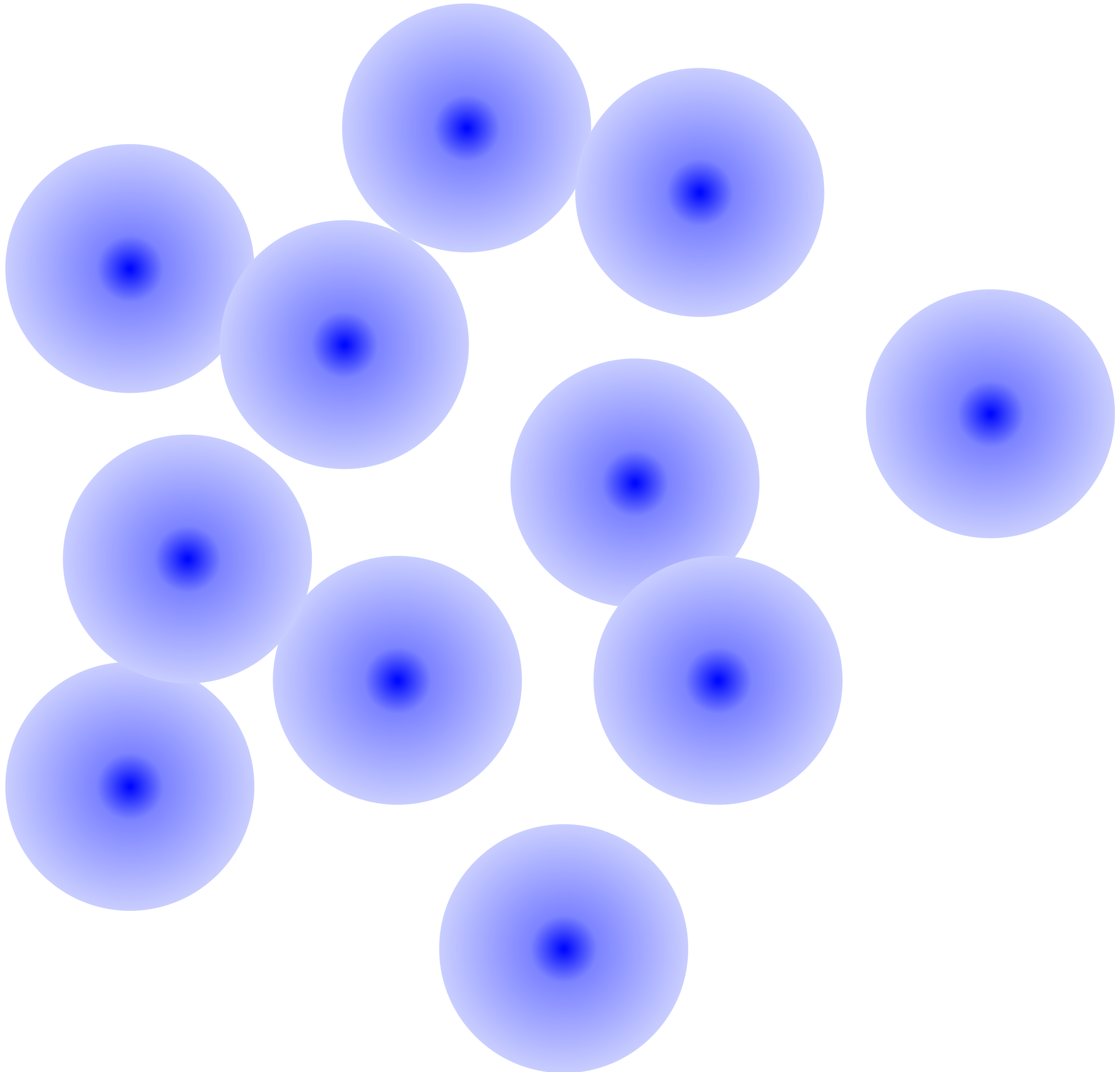


FIG. 1. Temperature dependence of normalized susceptibility χ/χ_{Pauli} of three Si:P,B samples with different normalized electron densities, n/n_c =0.58, 1.1, and 1.8. Solid lines through data are a guide to the eye.

M. J. Hirsch, D.F. Holcomb, R.N. Bhatt, and M.A. Paalanen
PRL **68**, 1418 (1992)



M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)
A.C. Potter, M. Barkeshli, J. McGreevy, T. Senthil, PRL **109**, 077205 (2012)

Theory of local moment formation in Si:P

Choose an effective Hamiltonian of the form

$$H_{\text{eff}} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} - \sum_i \gamma_i c^\dagger_{i\alpha} c_{i\alpha} + \frac{1}{2}\sum_i \vec{h}_i \cdot c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

where $\vec{h}_i$ and γ_i are variational parameters. Taking the density matrix defined by H_{eff} as the trial density matrix, and minimizing the free energy, we obtain an expansion in power of $\vec{h}_i$

$$F_{\text{eff}}[\vec{h}_i] = F_0 + \frac{1}{4}\sum_{i,j,k} \chi_{ij}(\delta_{jk} - U\chi_{jk})(\vec{h}_i \cdot \vec{h}_k) + \ldots$$

where

$$\chi_{ij} \equiv -\sum_{\lambda,\rho} \Psi_\lambda(i)\Psi^*_\rho(i)\Psi^*_\lambda(j)\Psi_\rho(j) \frac{f(\epsilon_\lambda) - f(\epsilon_\rho)}{\epsilon_\lambda - \epsilon_\rho}$$

where Ψ_λ and ϵ_λ are the eigenfunctions and eigenvalues of H_{eff} at $\vec{h}_i = 0$.

M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)

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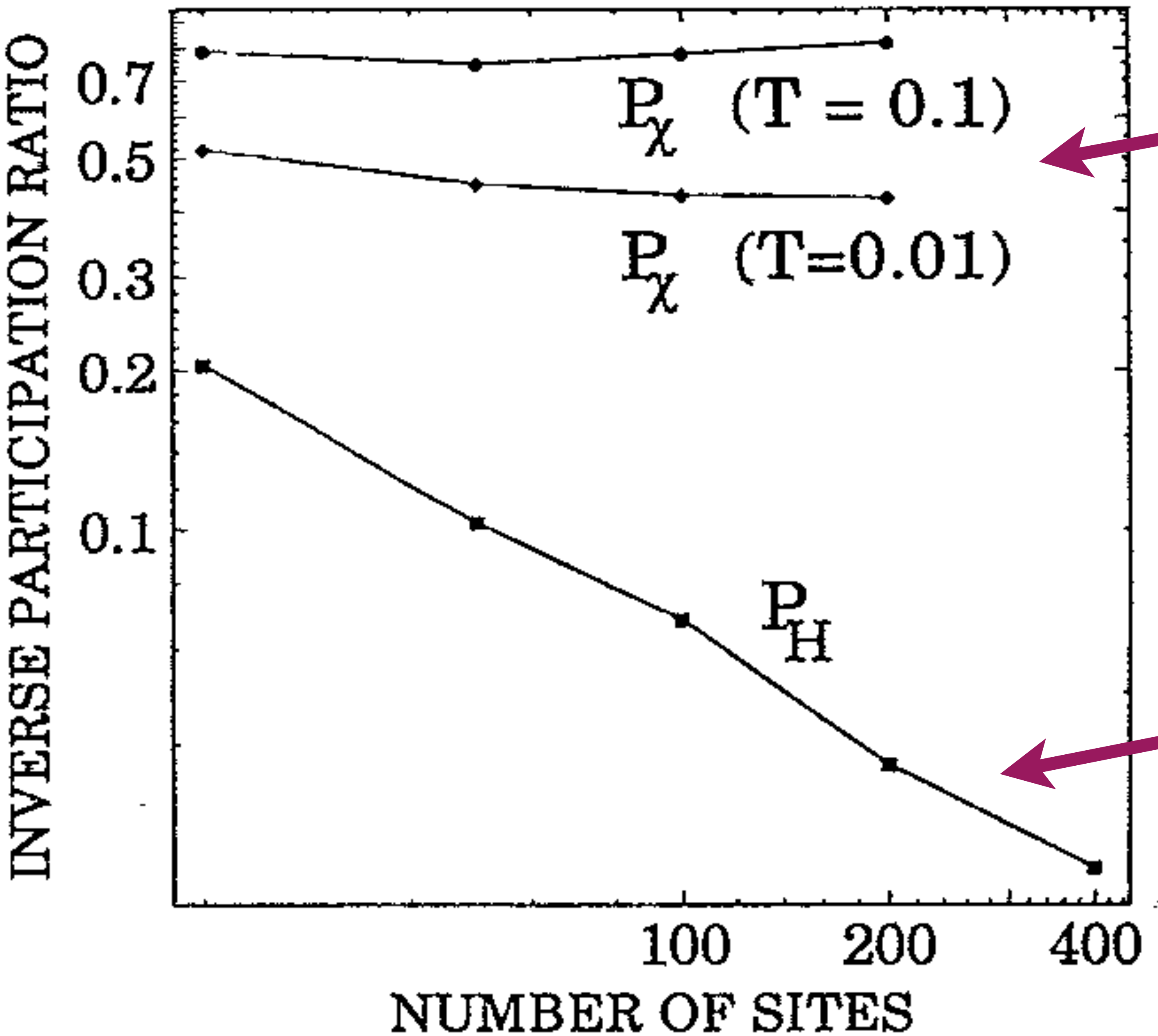
$$F_{\text{eff}}[\vec{h_i}] = F_0 + \frac{1}{4} \sum_{i,j,k} \chi_{ij} (\delta_{jk} - U \chi_{jk}) (\vec{h}_i \cdot \vec{h}_k) + \ldots$$

We numerically diagonalize χ_{ij}, and choose eigenmodes with eigenvalues $\Lambda_i > 1/U$: these are the eigenmodes corresponding to local moment instabilities.

M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)

Theory of local moment formation in Si:P



Magnetic excitations are localized (eigenmodes of the spin susceptibility)

Charged fermionic excitations are extended (eigenmodes of the electron Hamiltonian)

M. Milovanovic, S. Sachdev and R.N. Bhatt, PRL **63**, 82 (1989)

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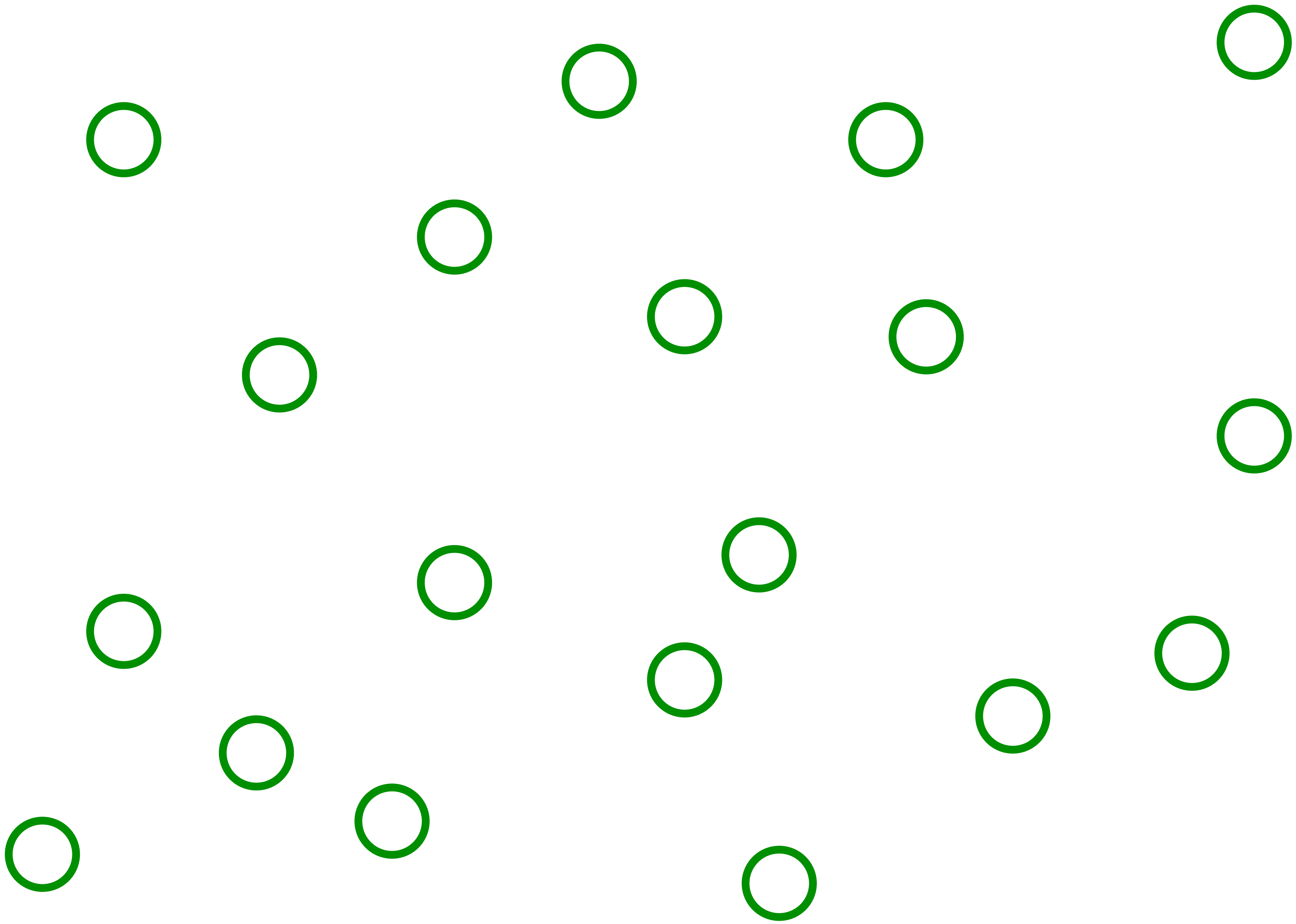
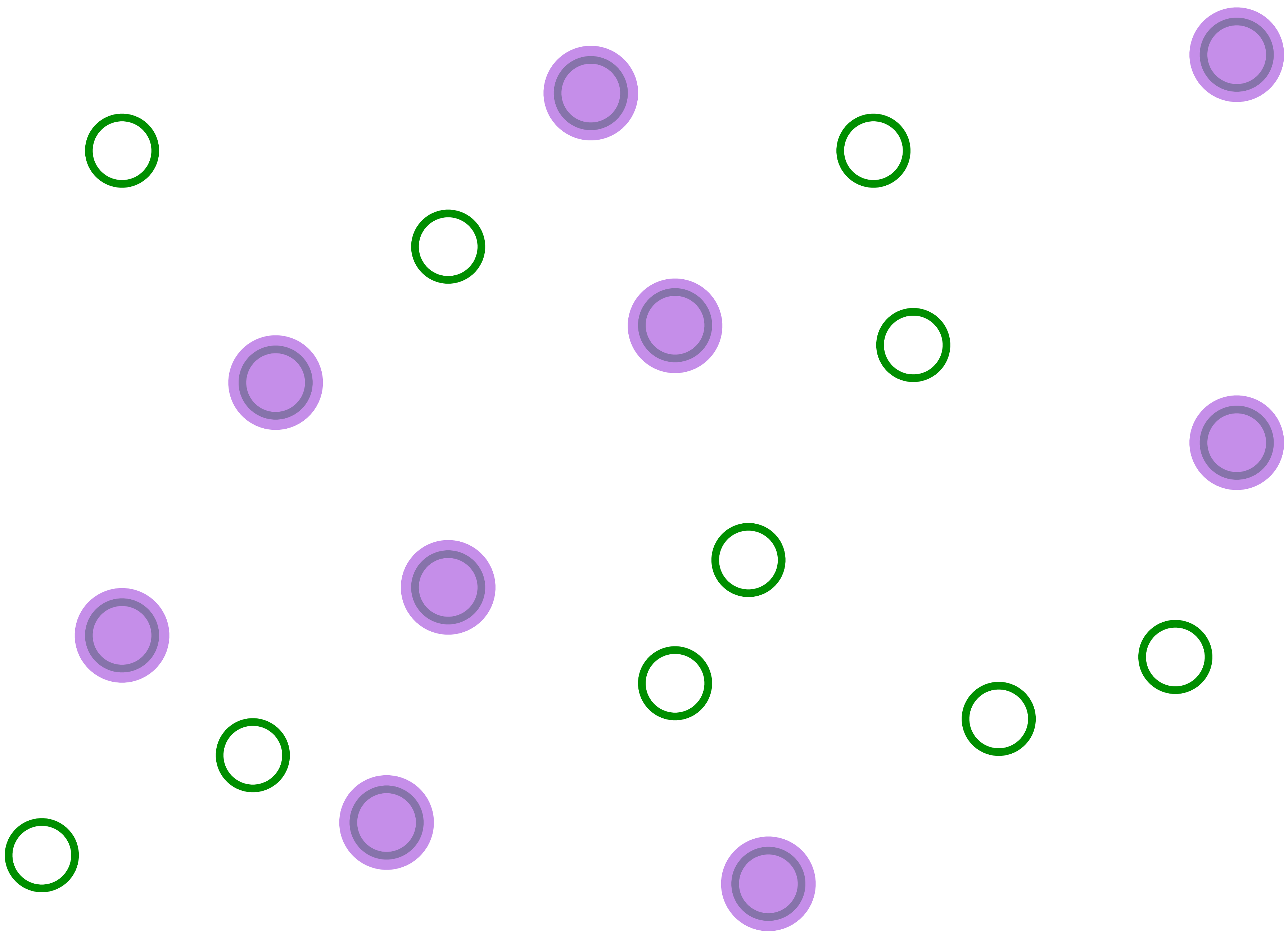
3. High temperature superconductivity and strange metals.

A simple model of a metal with quasiparticles



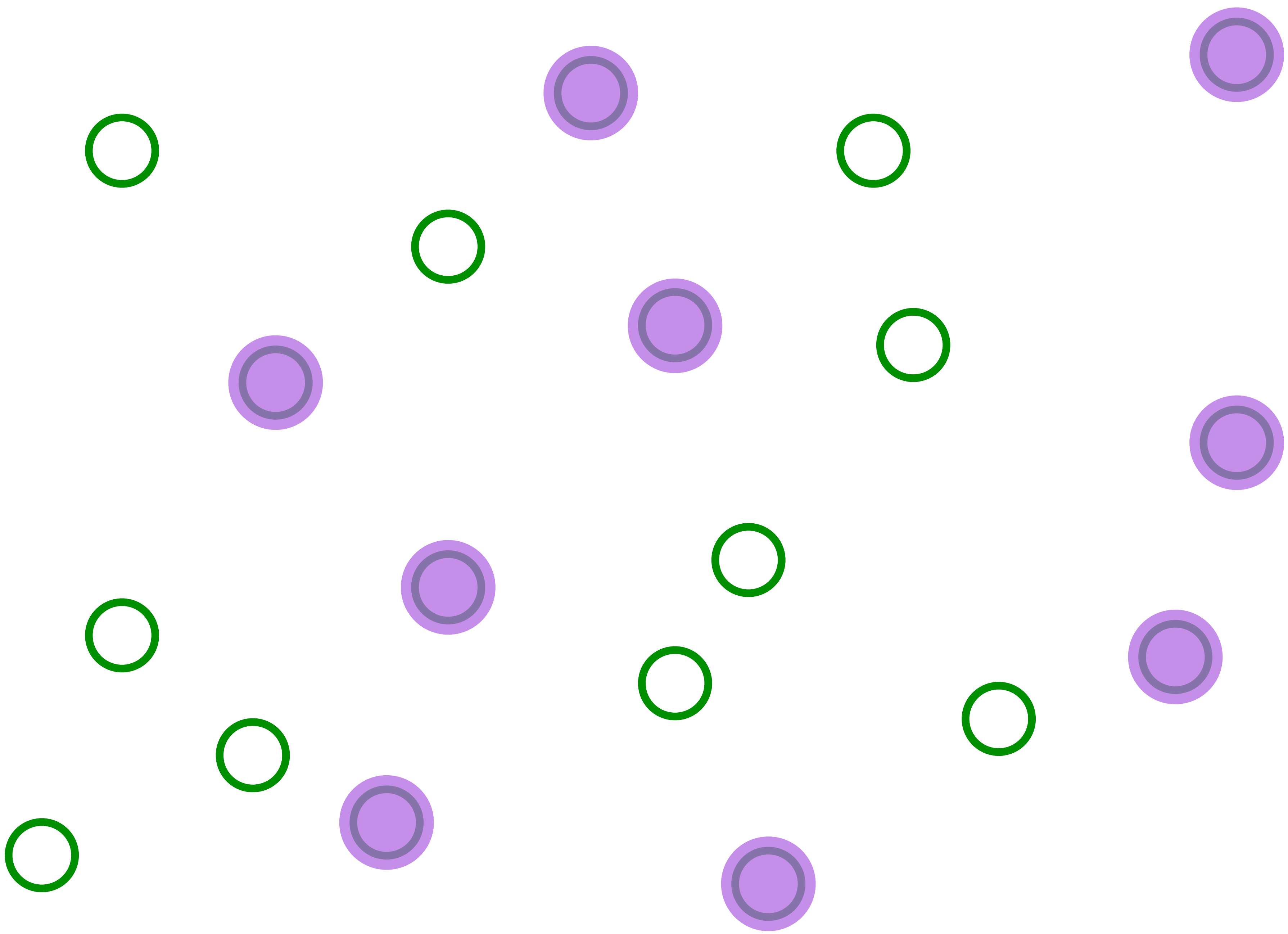
Pick a set of random positions

A simple model of a metal with quasiparticles



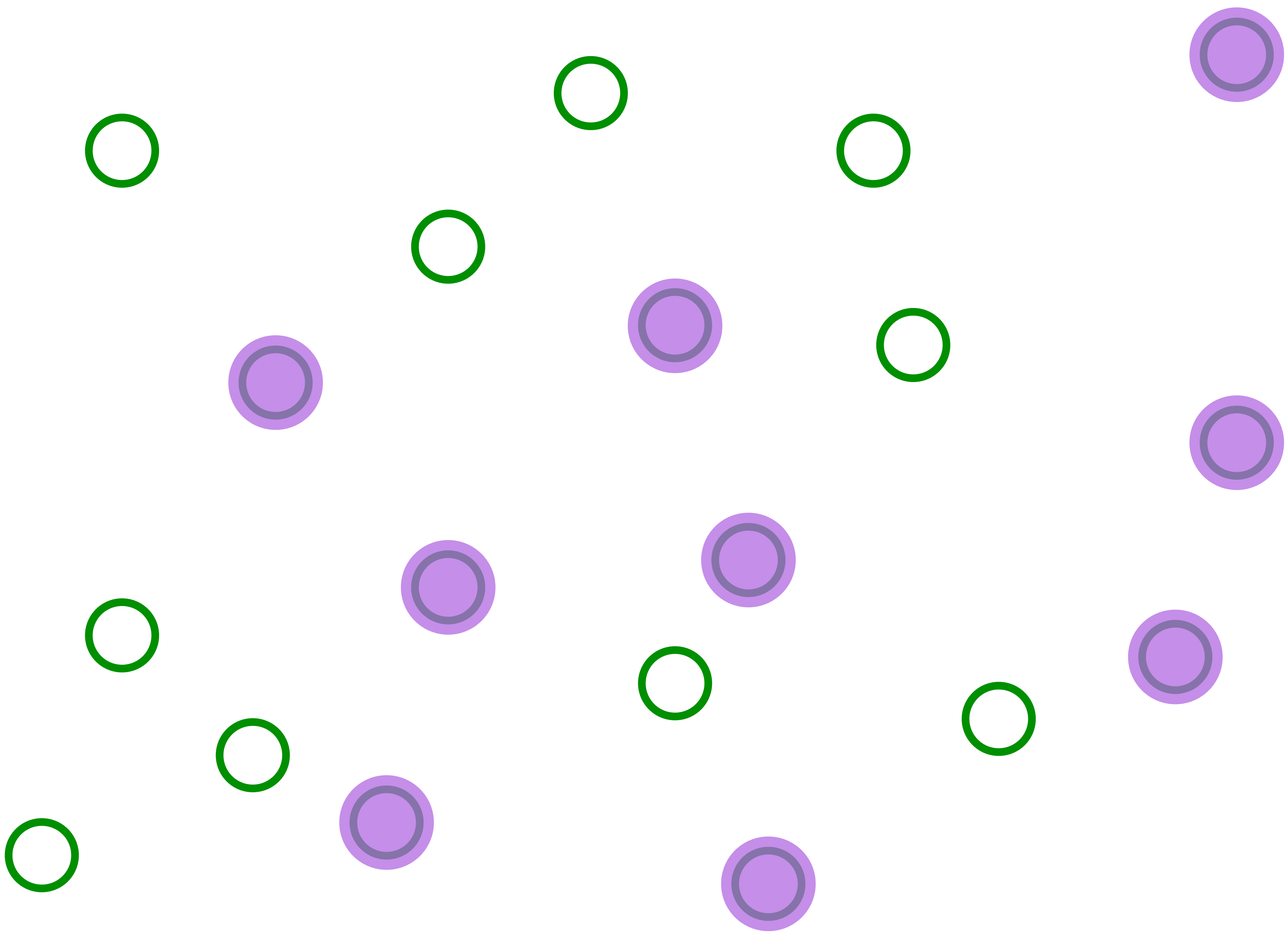
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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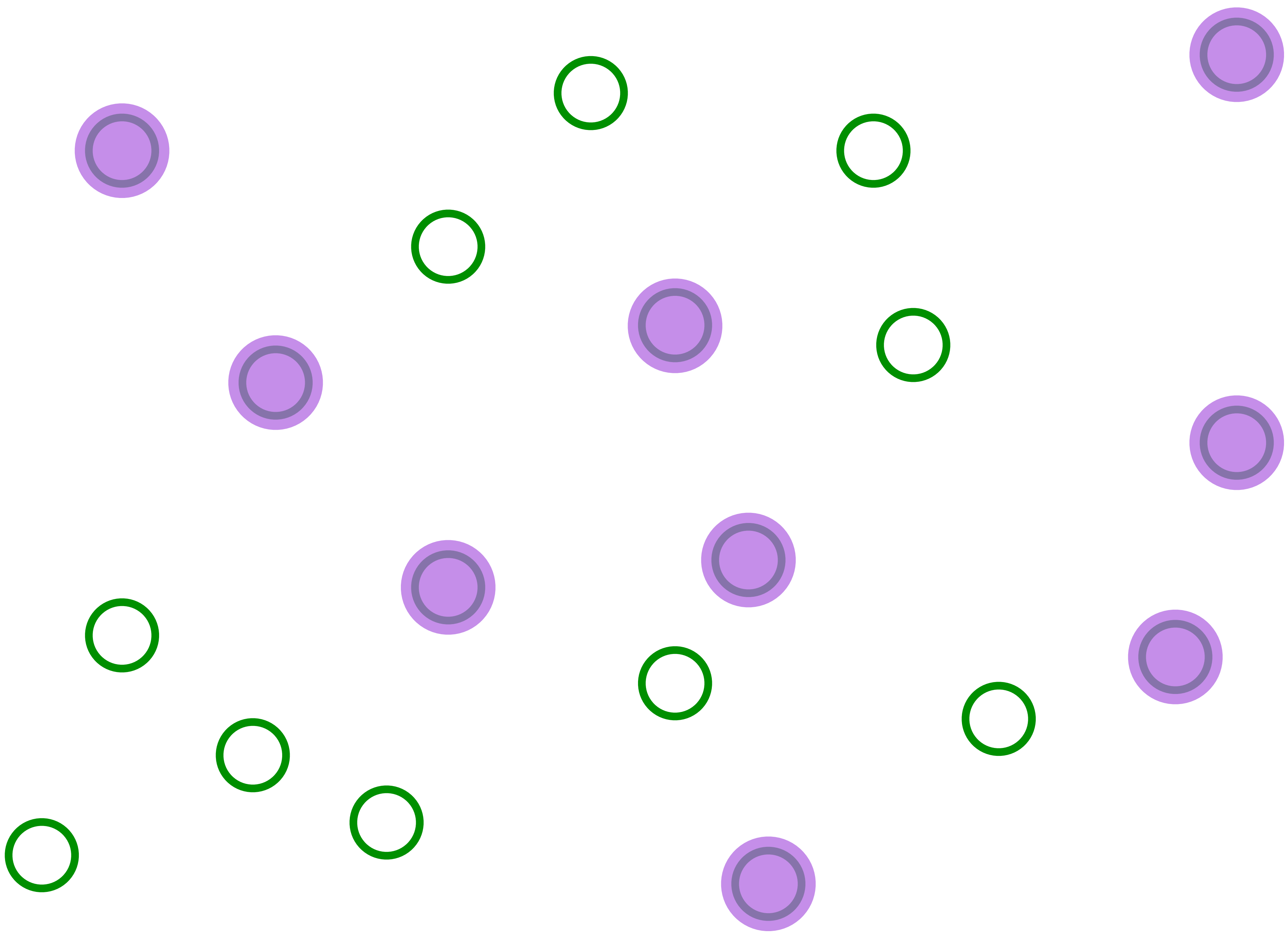

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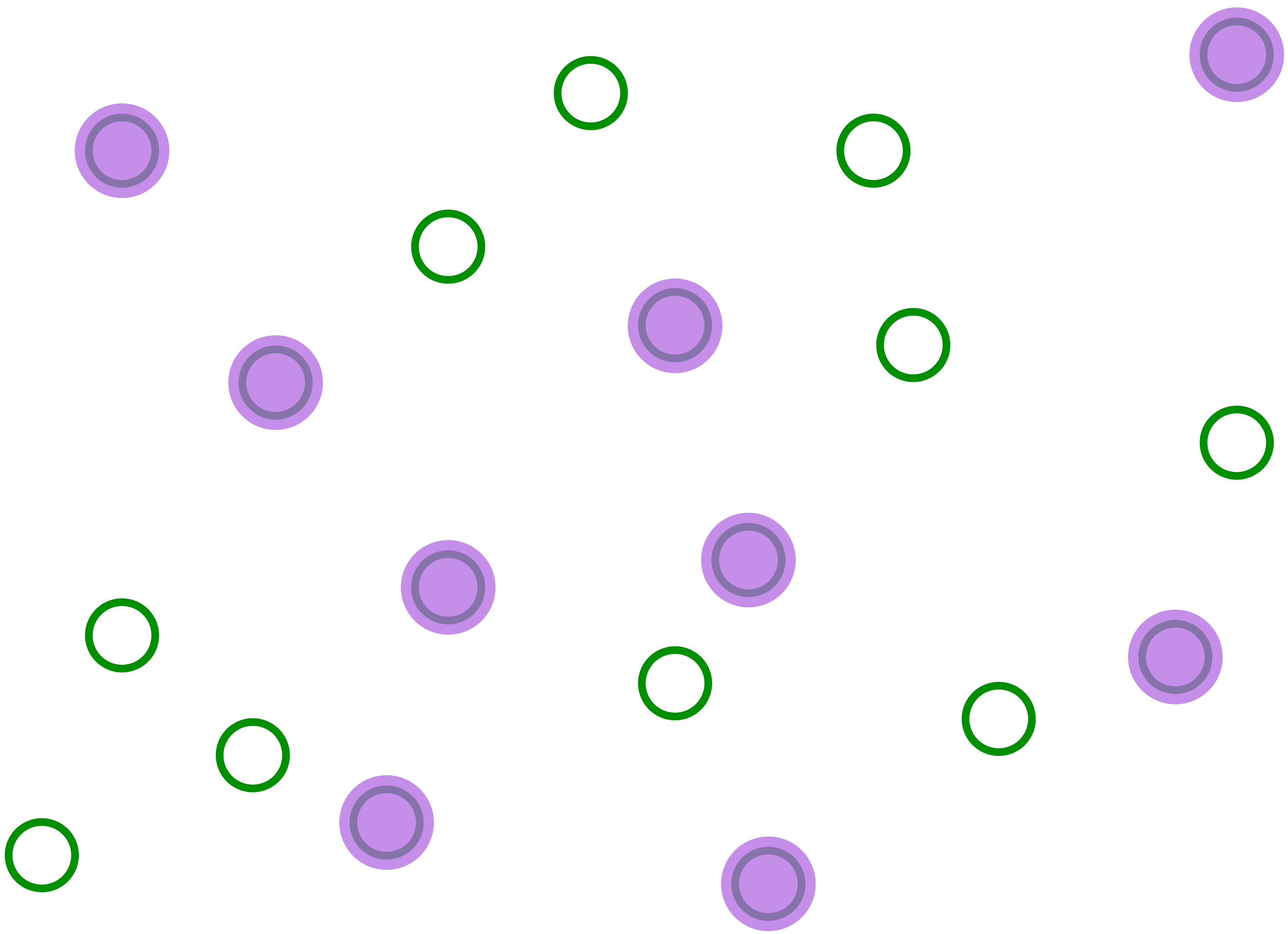


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

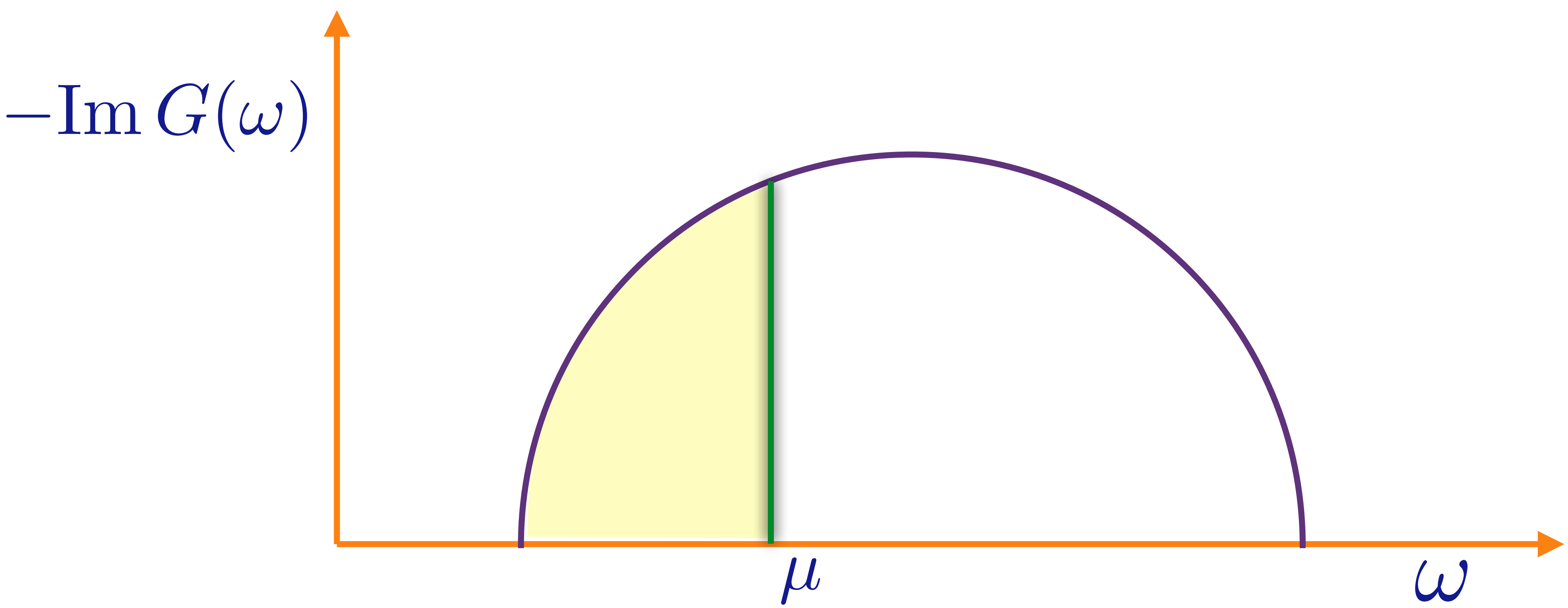
Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2 \end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

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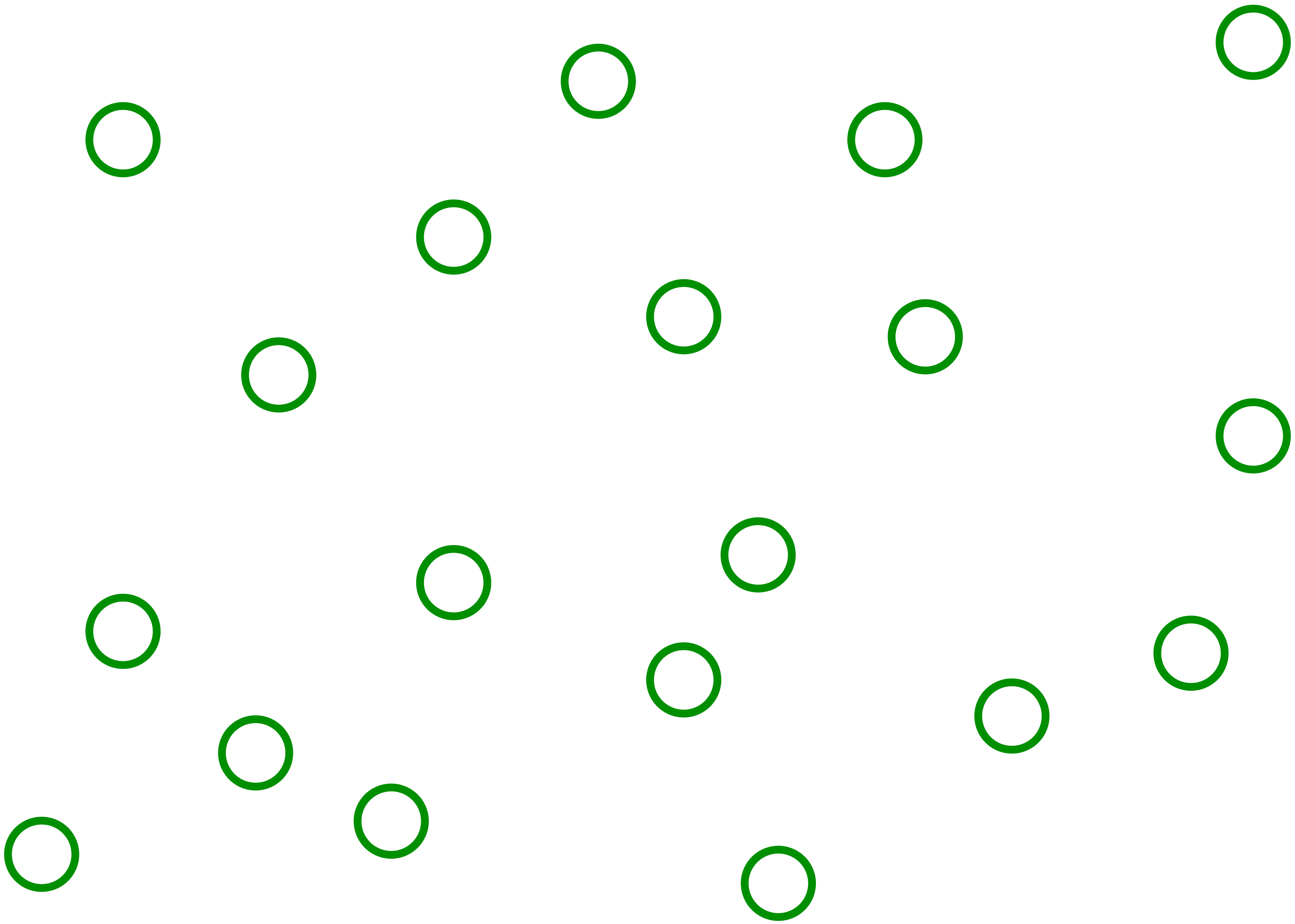
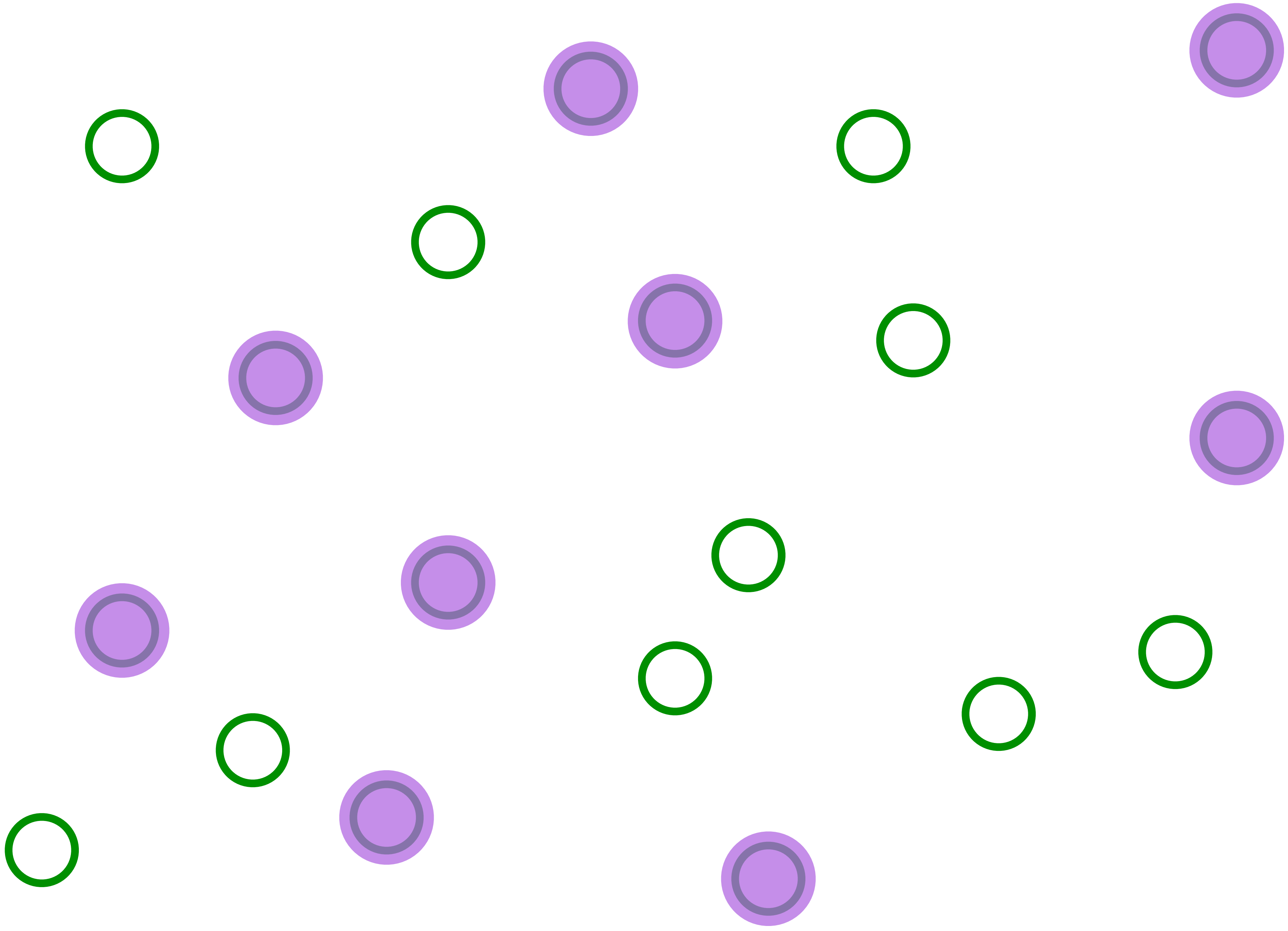
The Sachdev-Ye-Kitaev (SYK) model



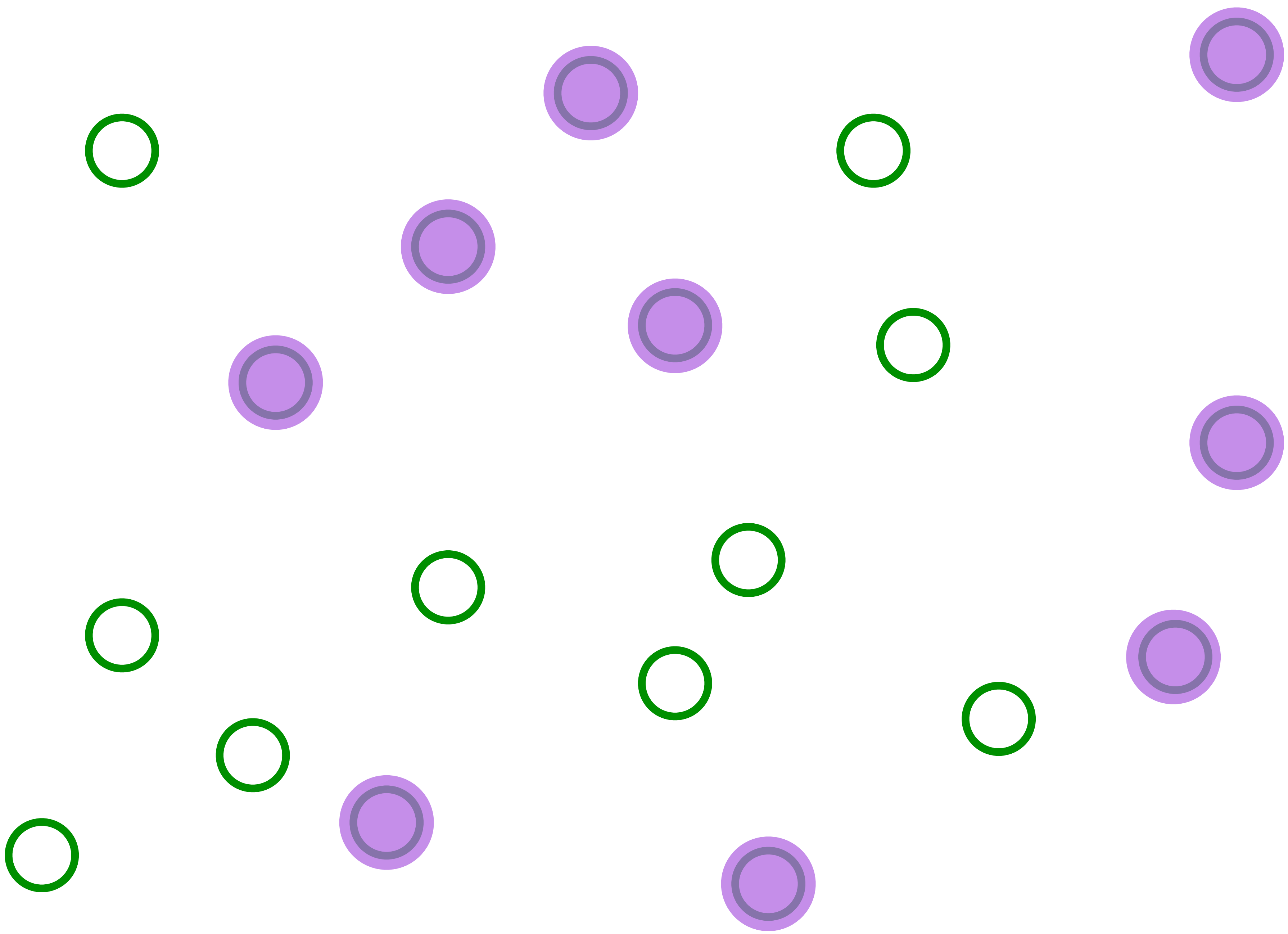
Pick a set of random positions

The SYK model



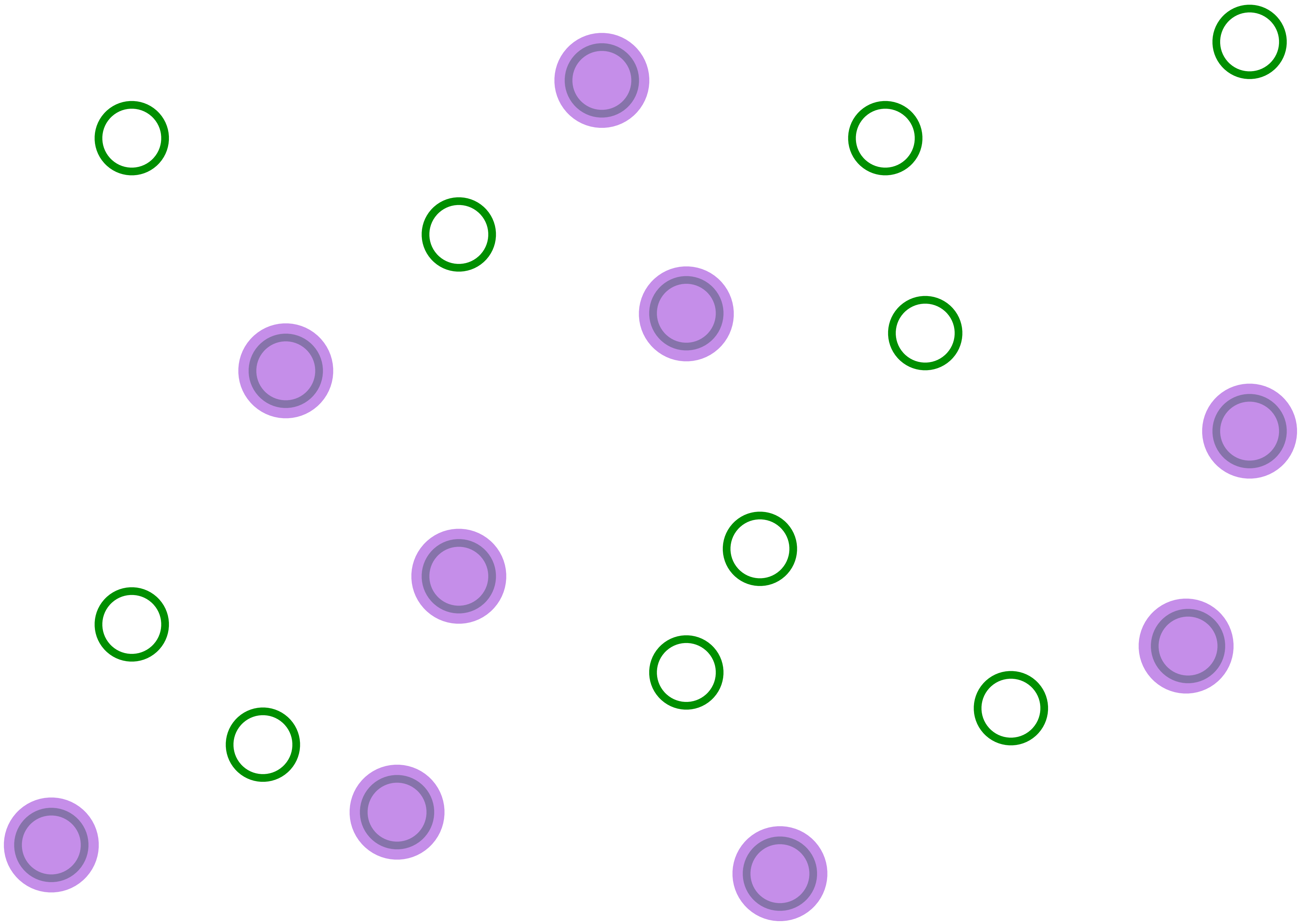
Place electrons randomly on some sites

The SYK model



Entangle electrons pairwise randomly

The SYK model



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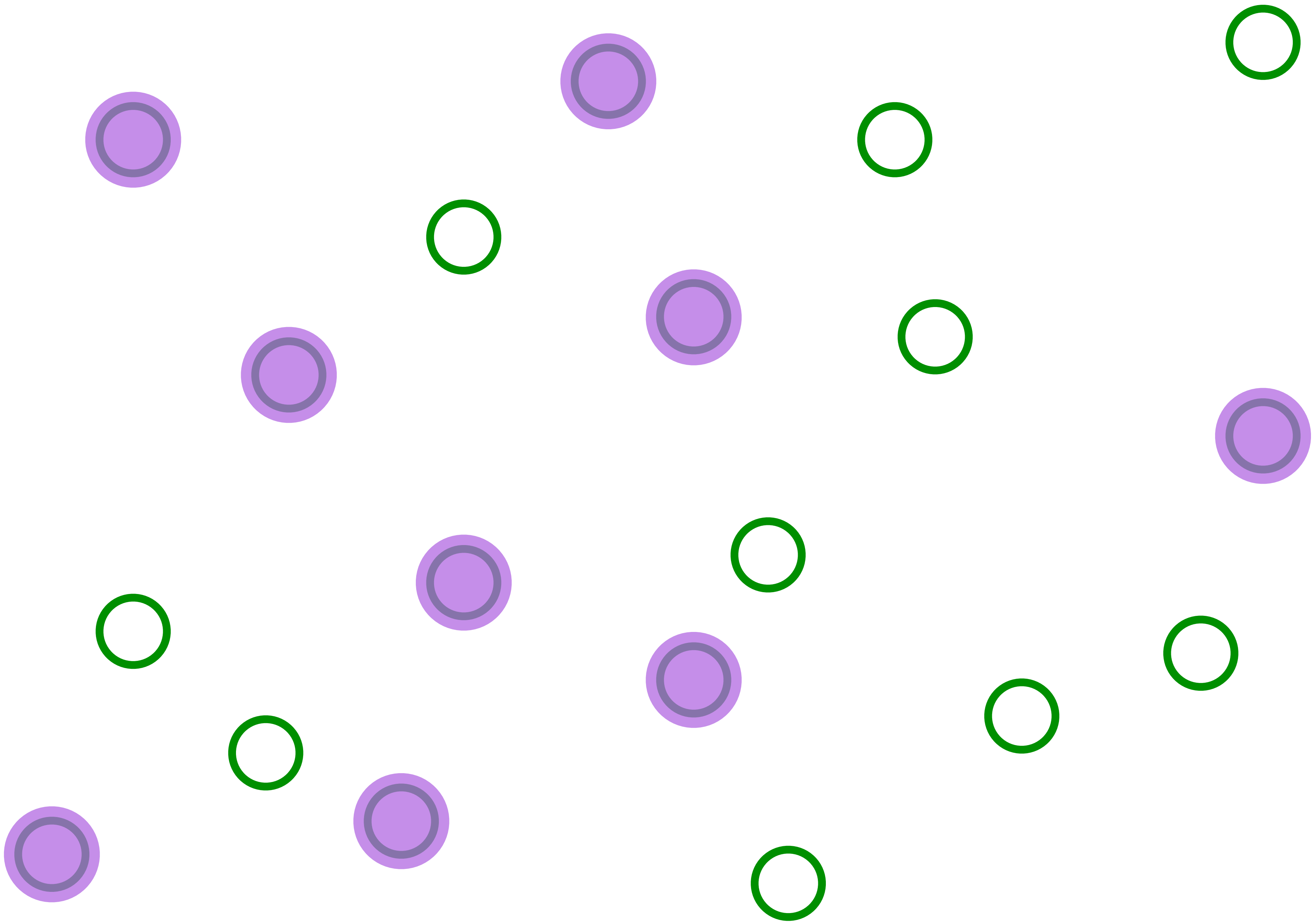

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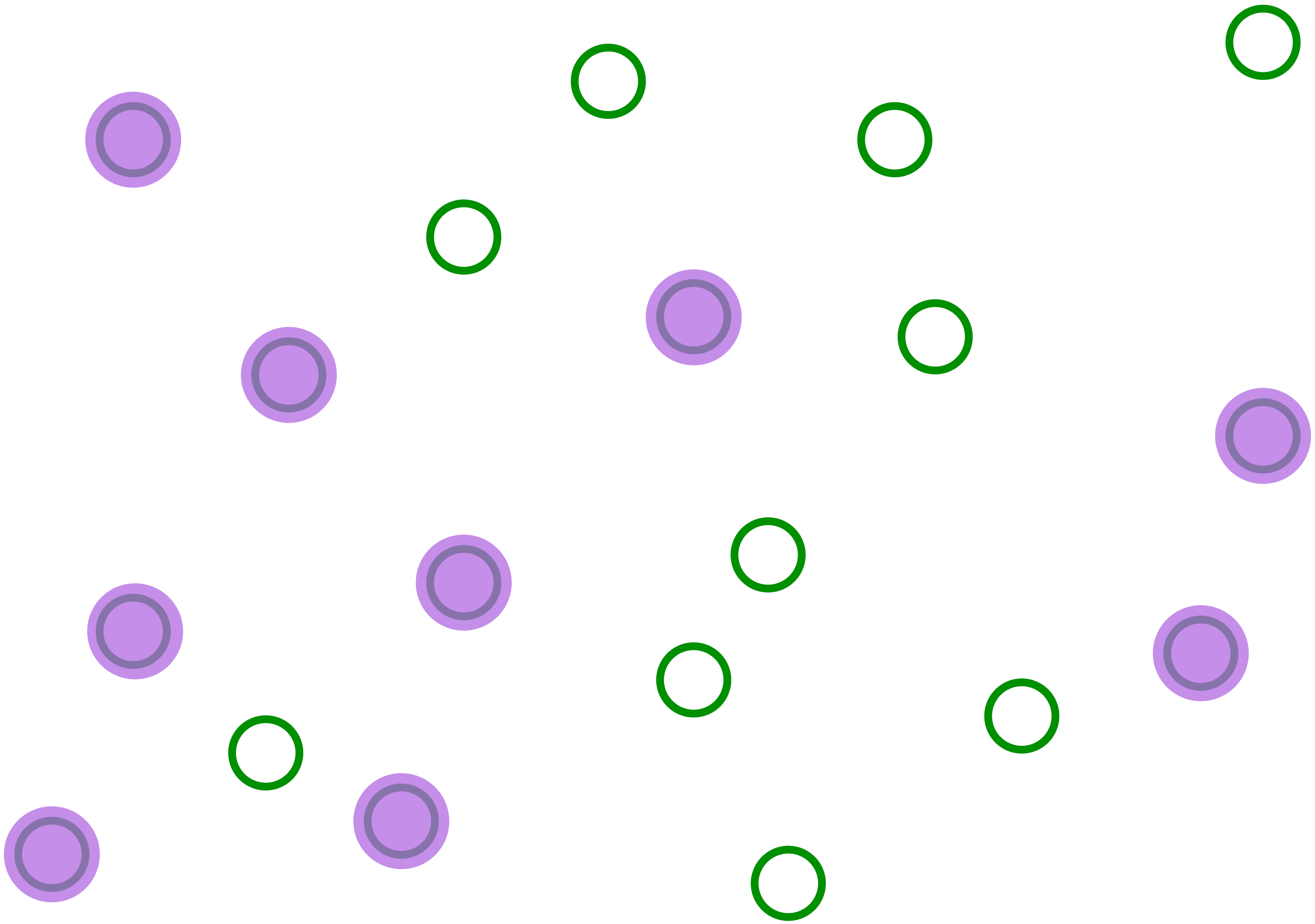

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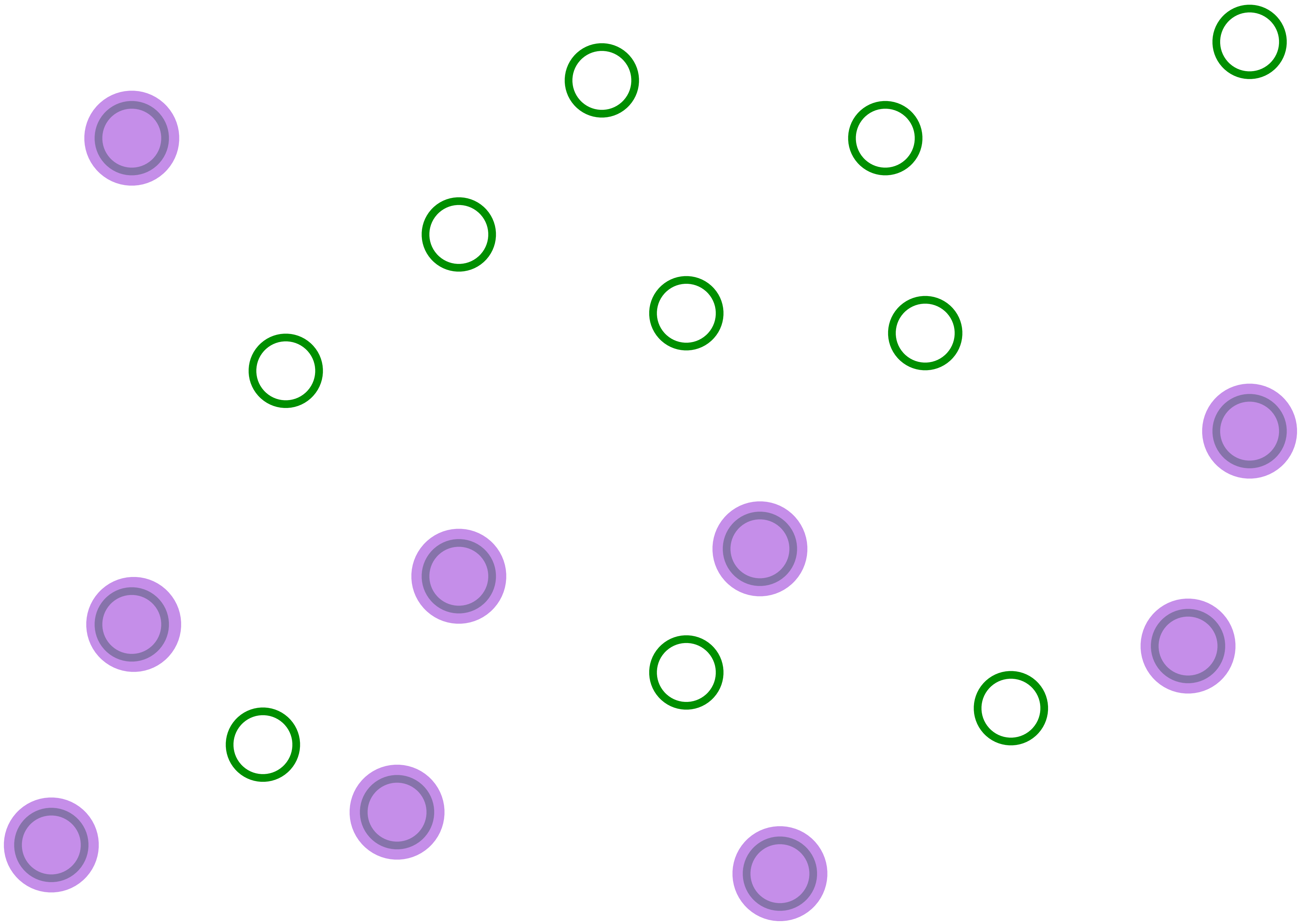

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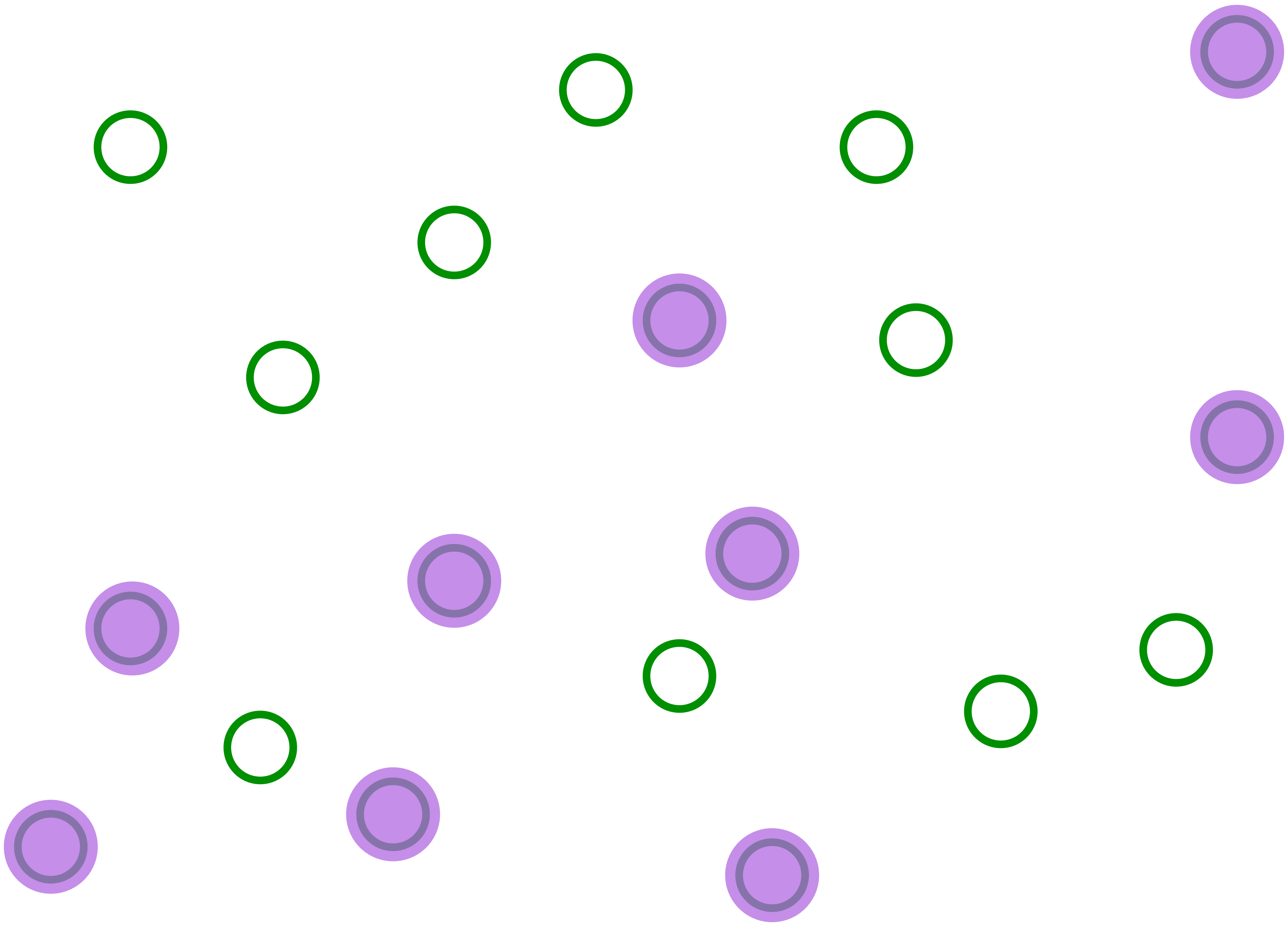

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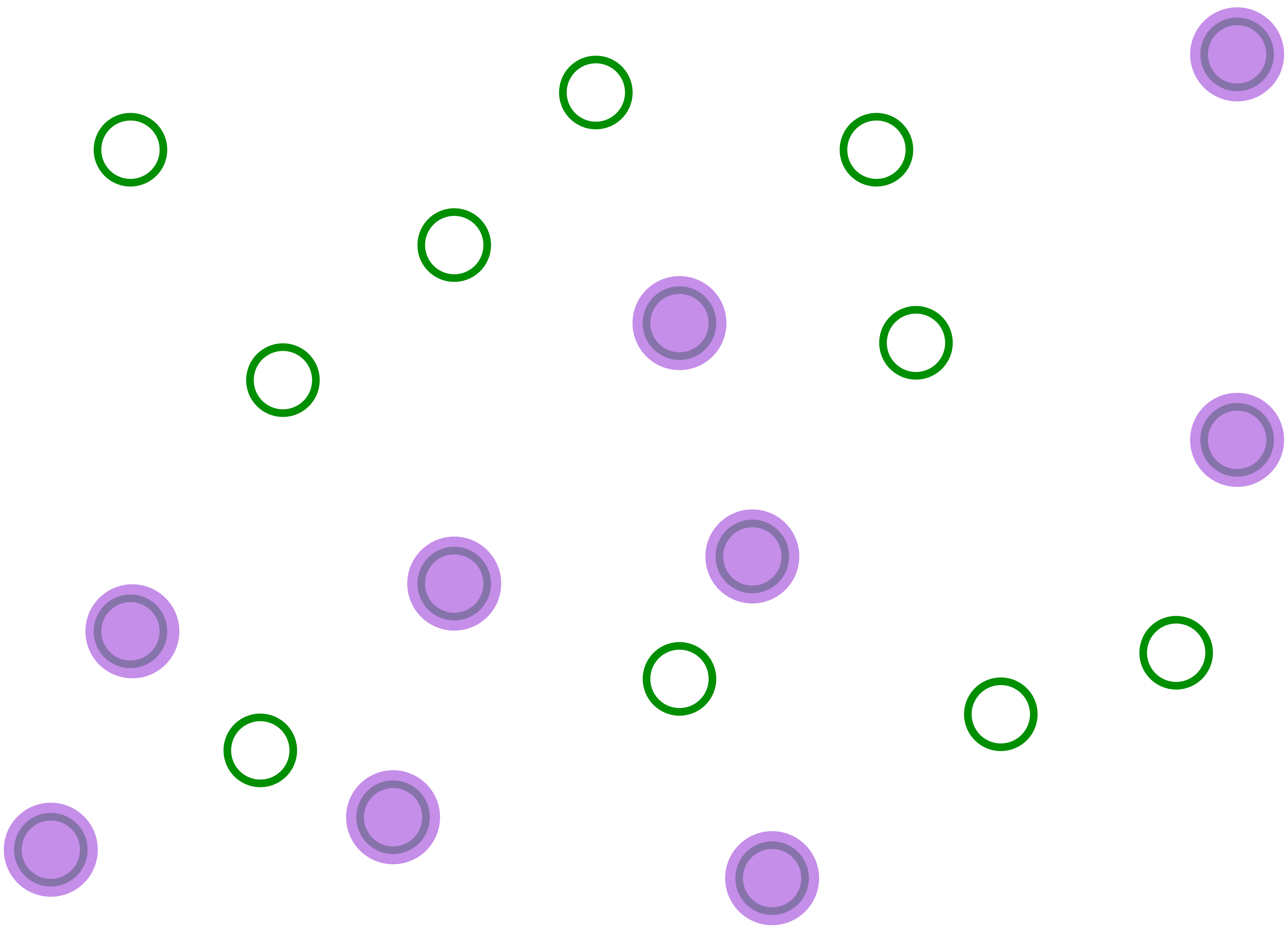

This describes both a strange metal and a black hole!

The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

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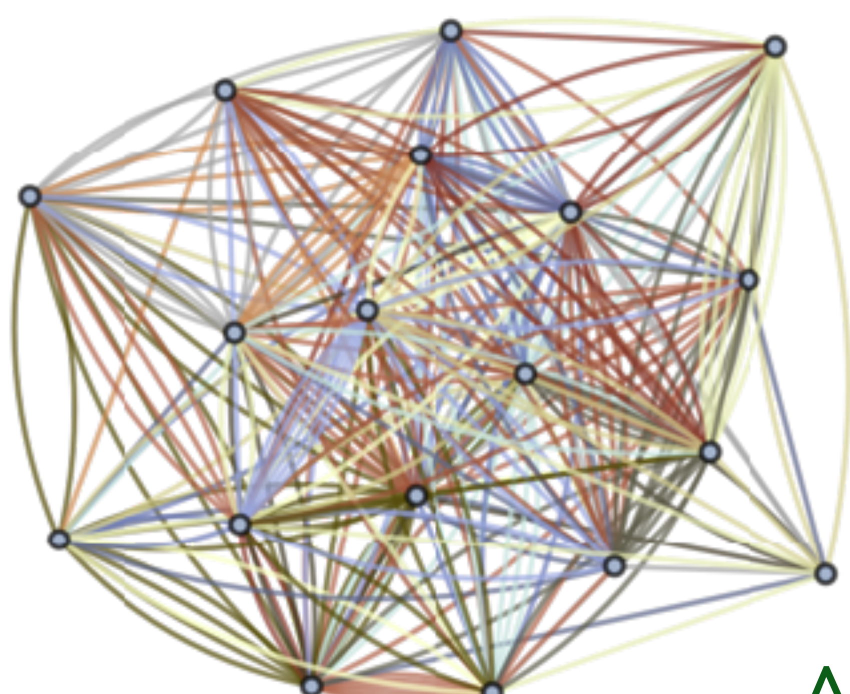
$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

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$$G(\tau = 0^-) = \mathcal{Q}.$$

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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

where $A = e^{-i\pi/4}(\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The SYK model

- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)

- The last property indicates $\tau_{\text{eq}} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

Quantum matter <u>without</u> quasiparticles:

- If there are no quasiparticles, then

$$E \neq \sum_{\alpha} n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

Quantum matter without quasiparticles:

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- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T}$$

S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

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- If there are no quasiparticles, then

$$\tau_{\rm eq} = \# \frac{\hbar}{k_B T}$$

- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} .$$

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

  - In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
  - This bound rules out quantum systems with *e.g.* $\tau_{\rm eq} \sim \hbar/(J k_B T)^{1/2}$.
  - There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

SYK models and black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.

- Black holes relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.

Black holes

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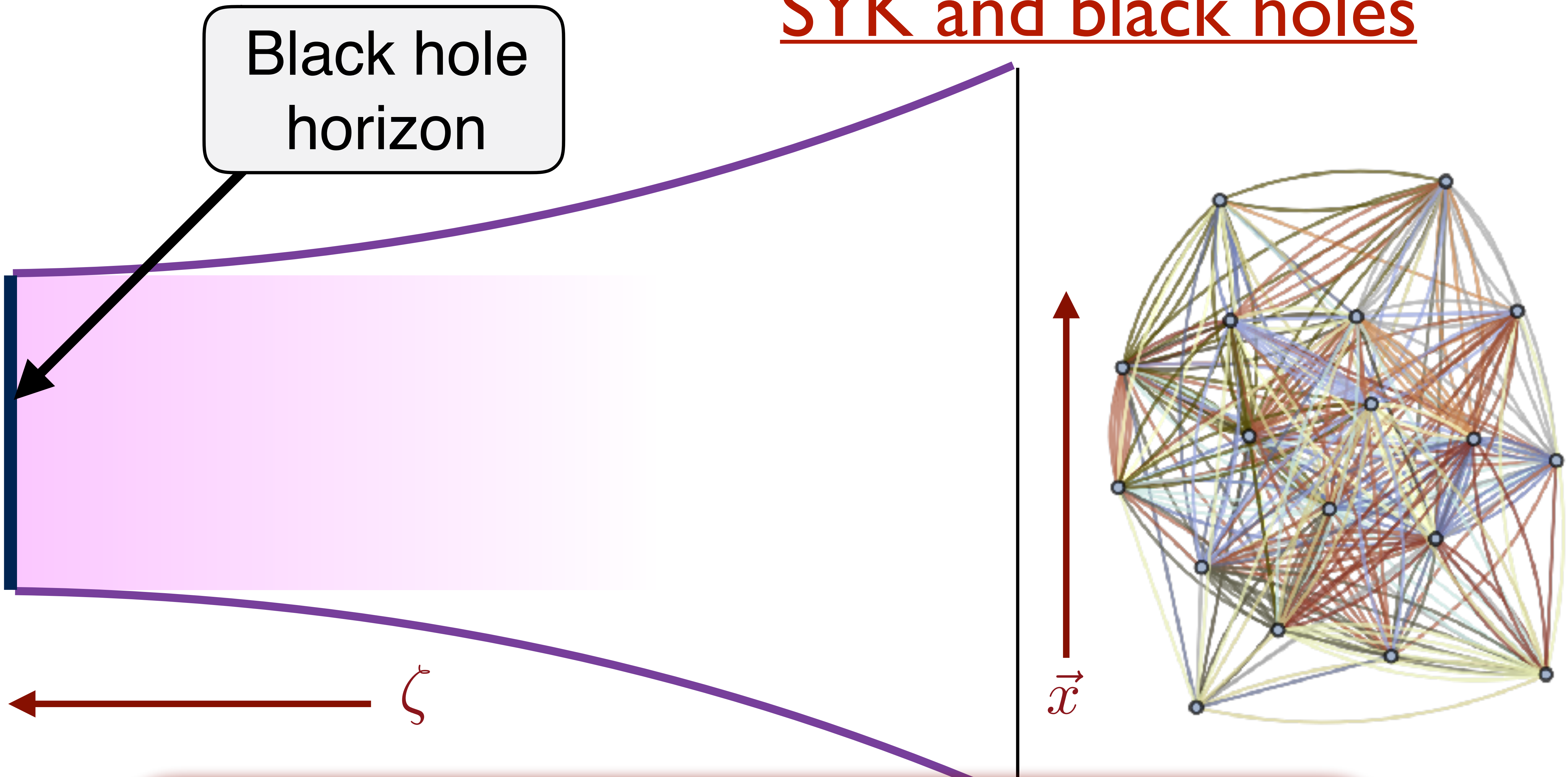
- Black holes relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.
- The entropy of black holes is proportional to their surface area: the SYK model (in 0+1 dimensions) maps to a black hole in 1+1 dimensions.

Black holes

SYK and black holes



Black holes with a near-horizon AdS_2 geometry (described by quantum gravity in 1+1 spacetime dimensions) match the properties of the 0+1 dimensional SYK model in the previous slide: Ns_0 is the Bekenstein-Hawking entropy

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

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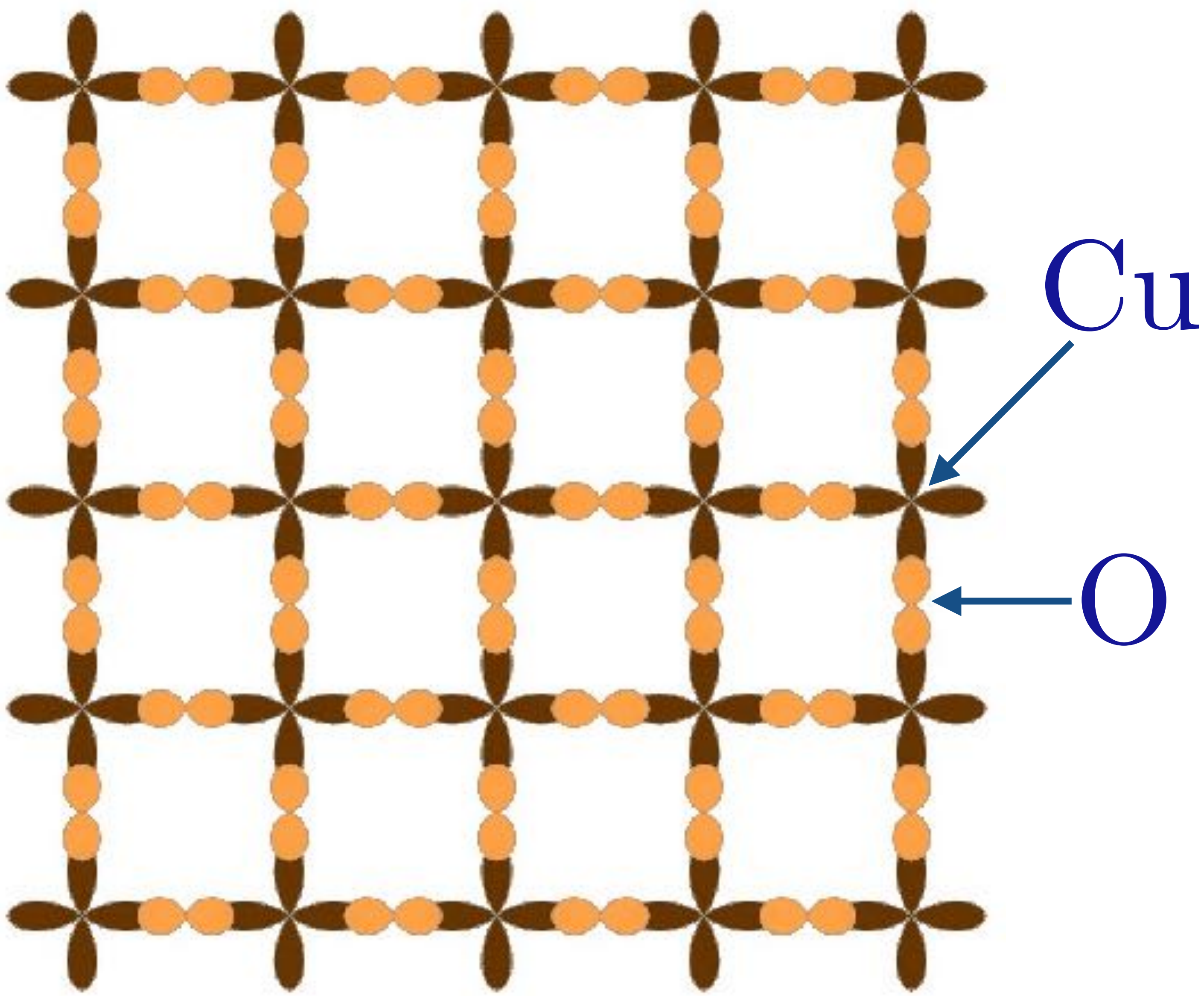
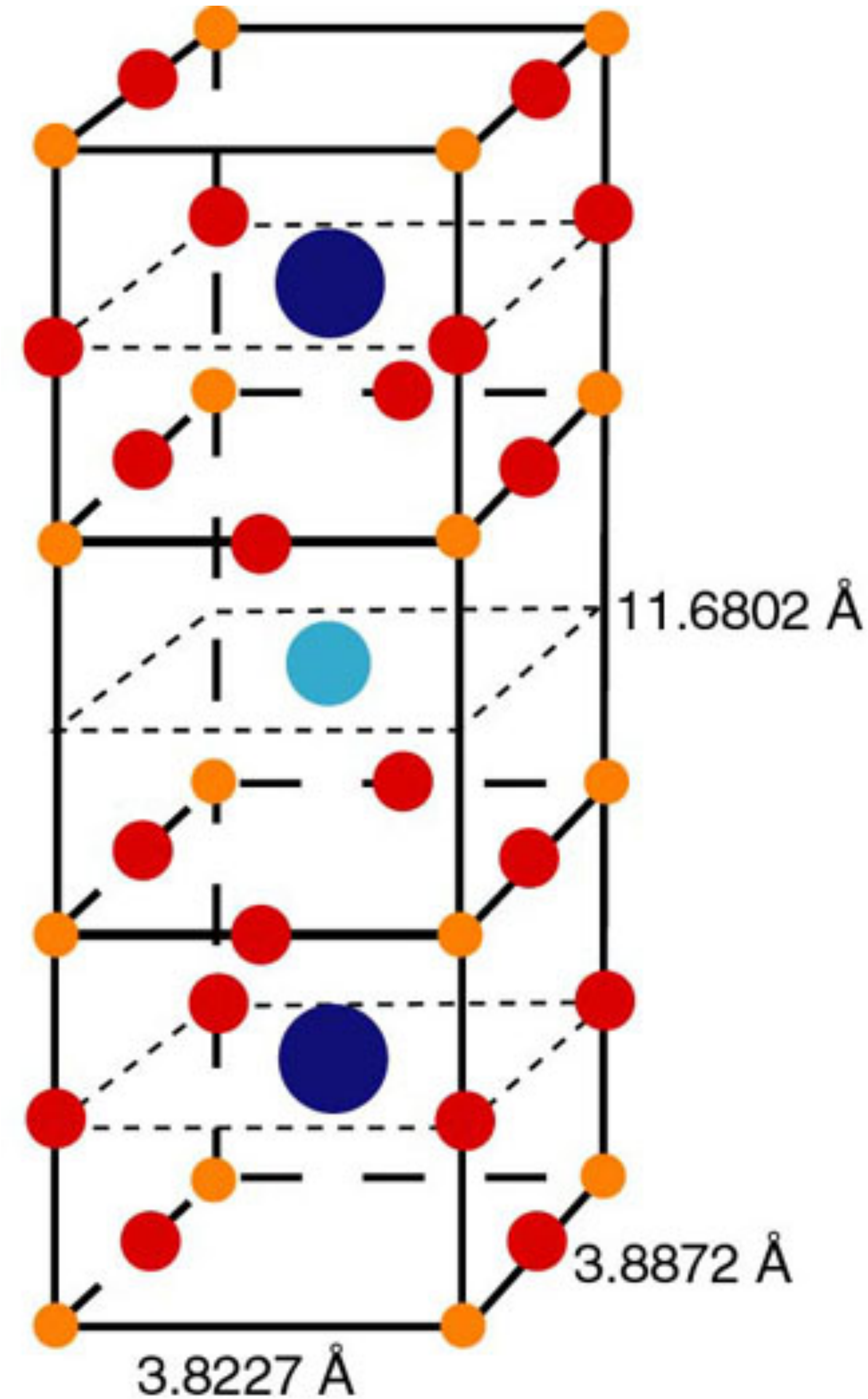
High temperature superconductors

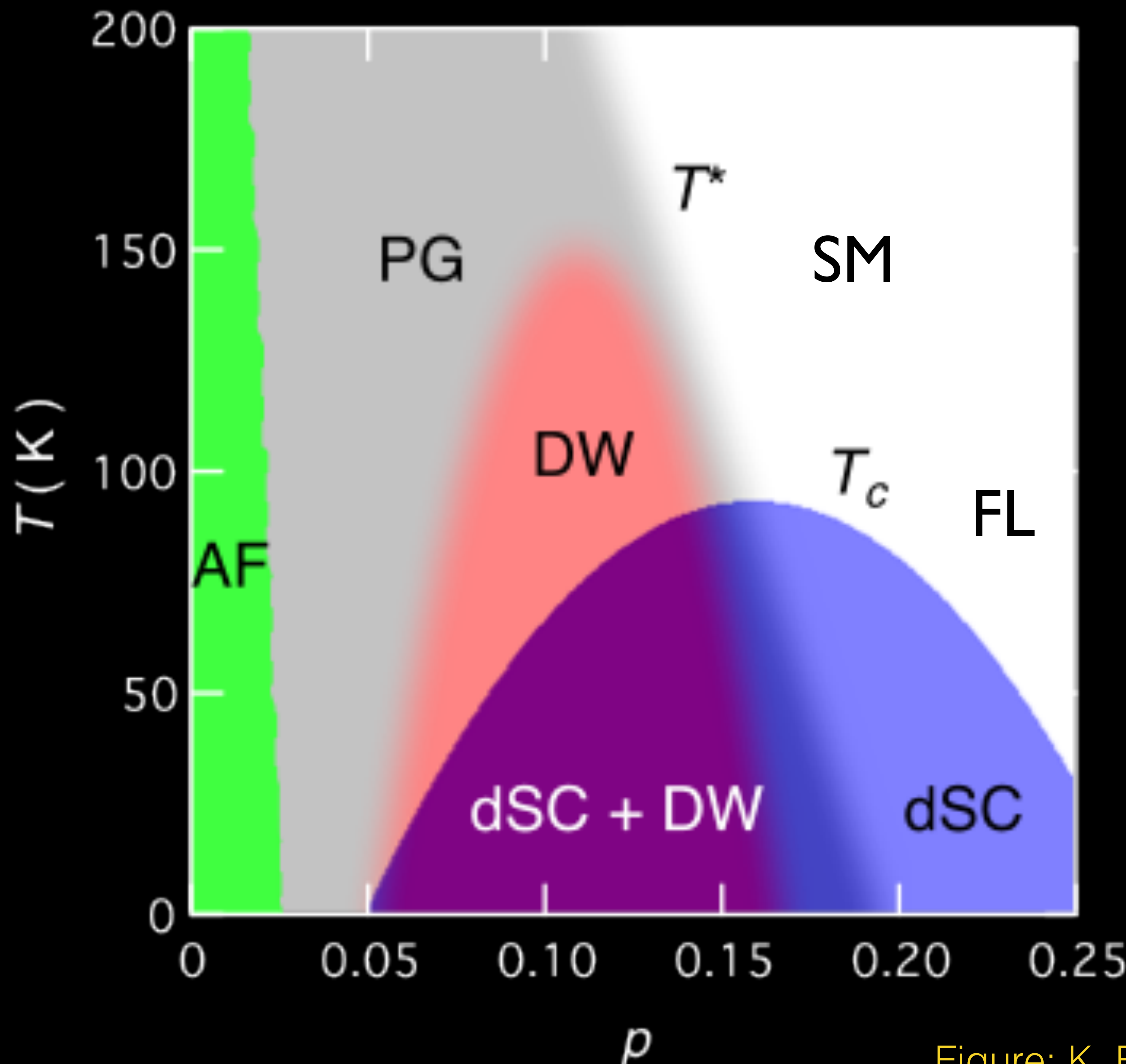


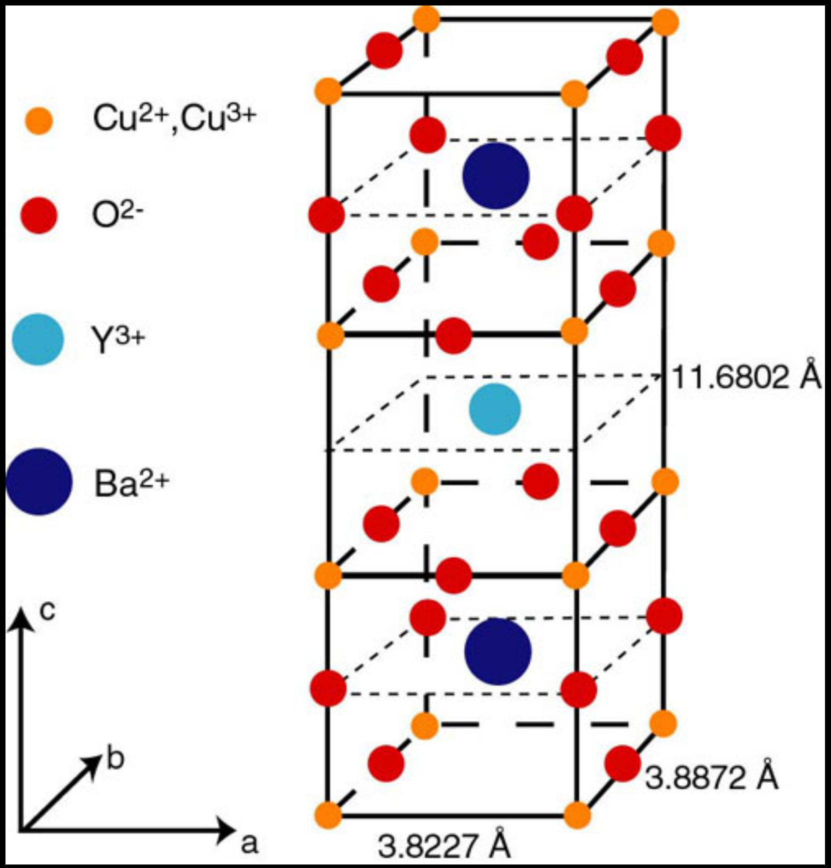
CuO_2 plane

Described by a Hubbard model on the Cu sites



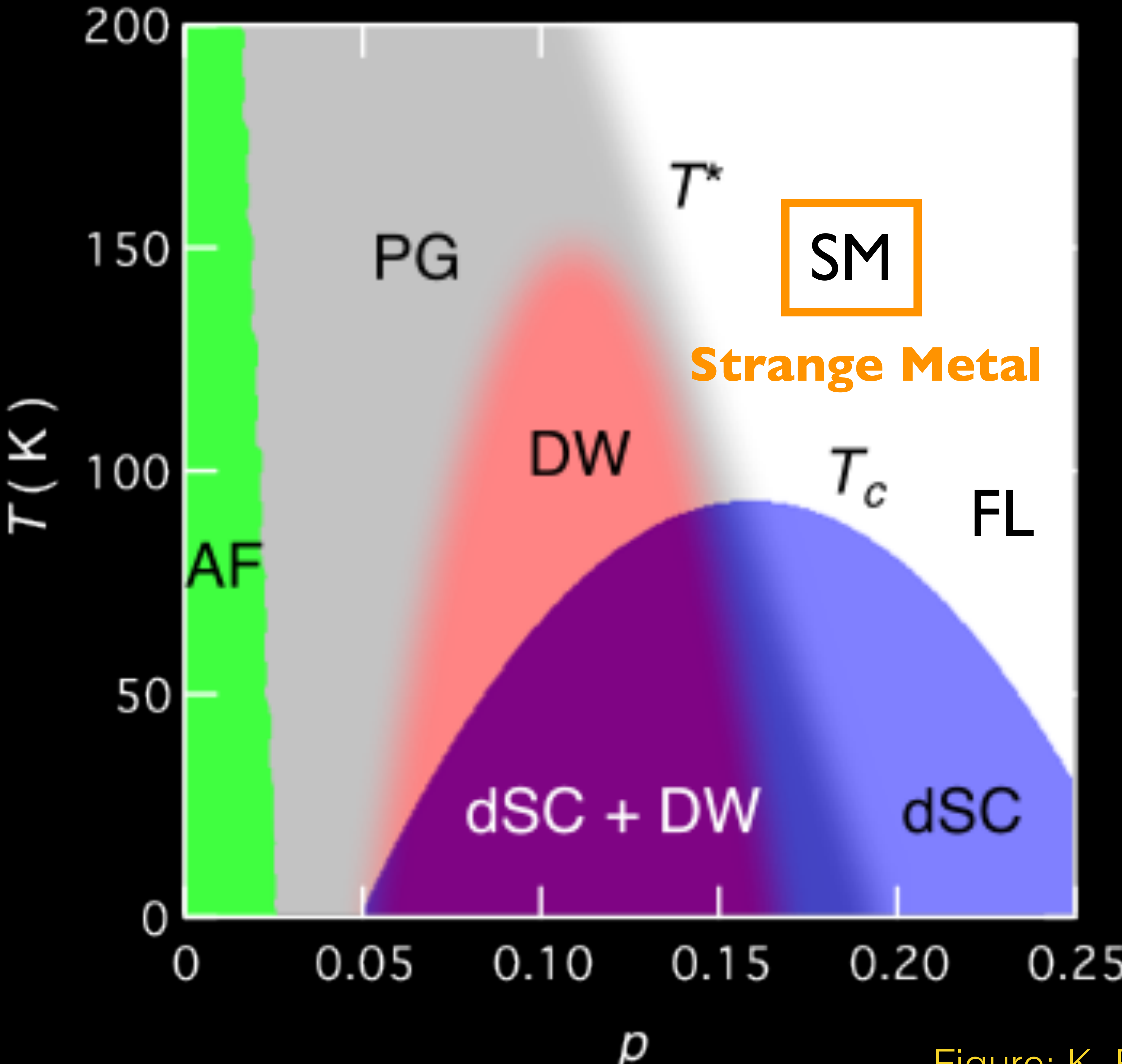
$YBa_2Cu_3O_{6+x}$

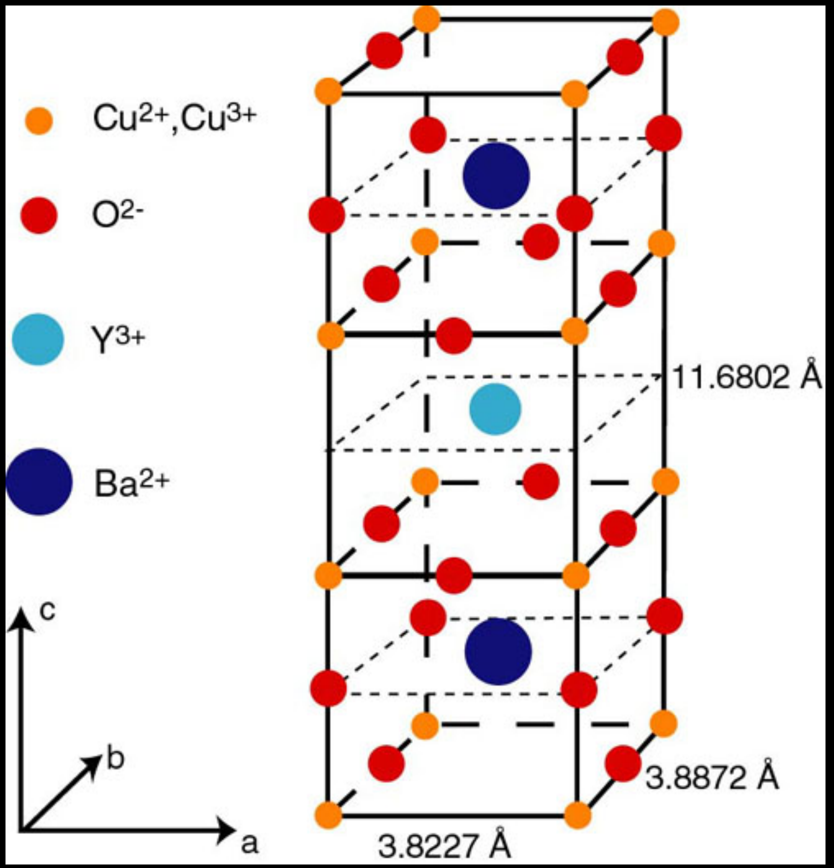




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

Ubiquitous

"Strange",

"Bad",

"Incoherent",

or "Ultra-quantum"

metal has a resistivity, ρ, which obeys

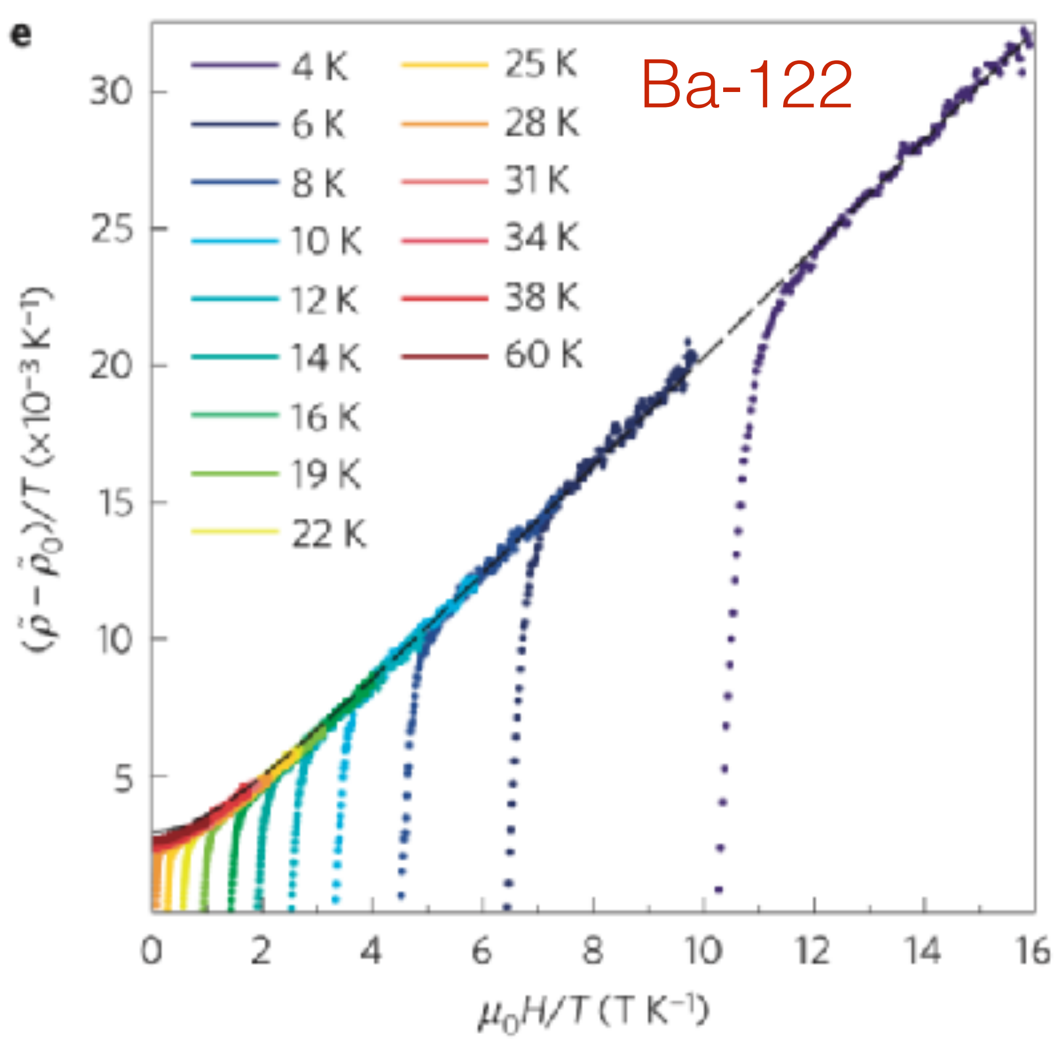
$$\rho \sim T,$$

and

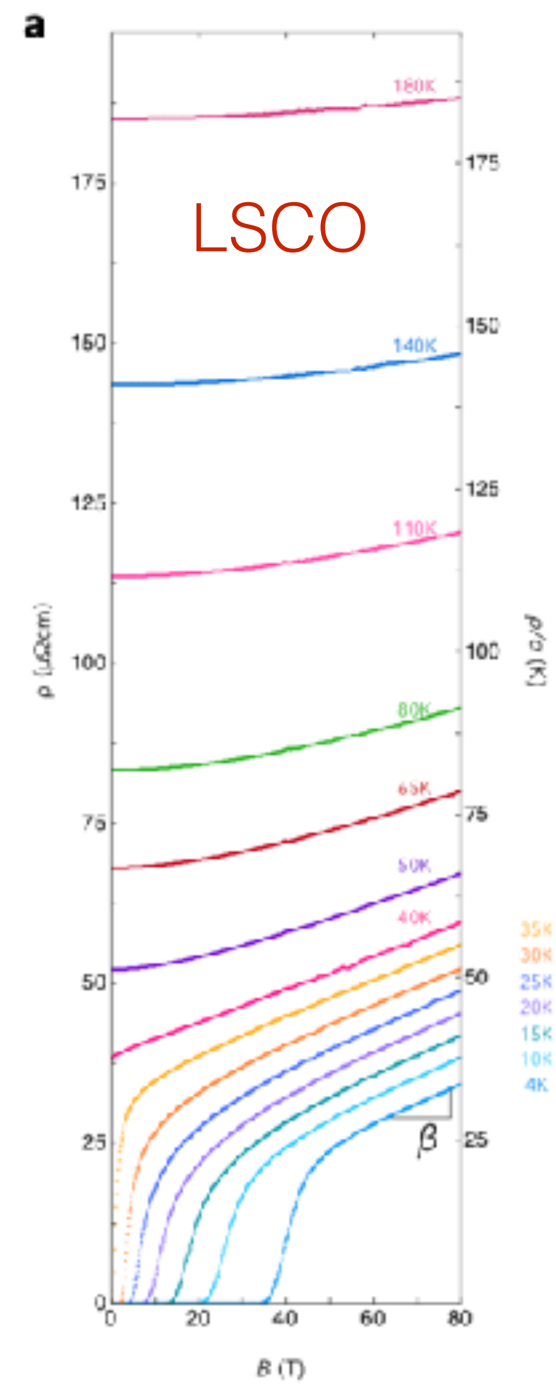
in some cases $\rho \gg h/e^2$
(in two dimensions),
where h/e^2 is the quantum unit of resistance.

Ultra-quantum metals just got stranger…

B-linear magnetoresistance!?



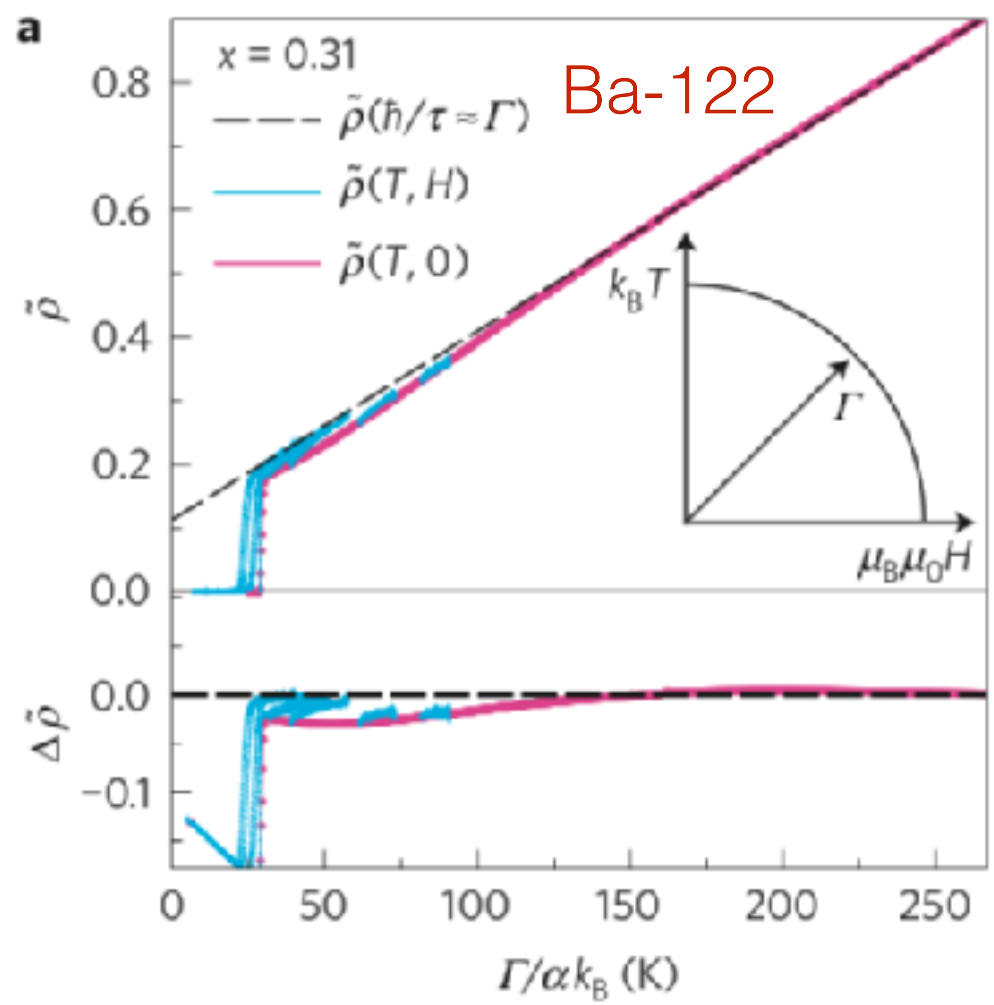
I. M. Hayes et. al., Nat. Phys. 2016



P. Giraldo-Gallo et. al., arXiv:1705.05806

Ultra-quantum metals just got stranger…

Scaling between B and T !?

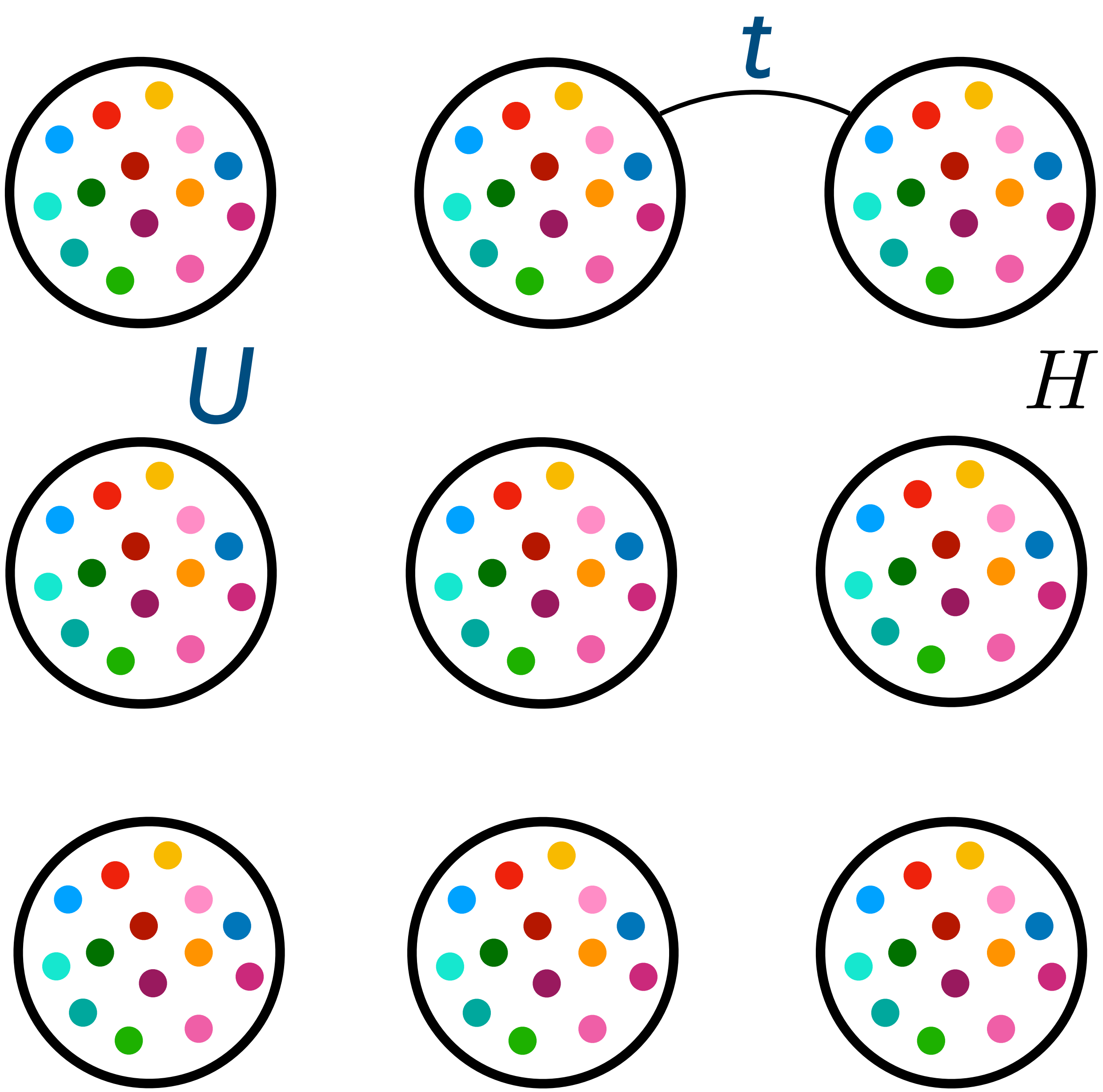


$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

I. M. Hayes et. al., Nat. Phys. 2016

SYK building blocks for a strange metal

SYK quantum dots of electrons with
random hopping between them.



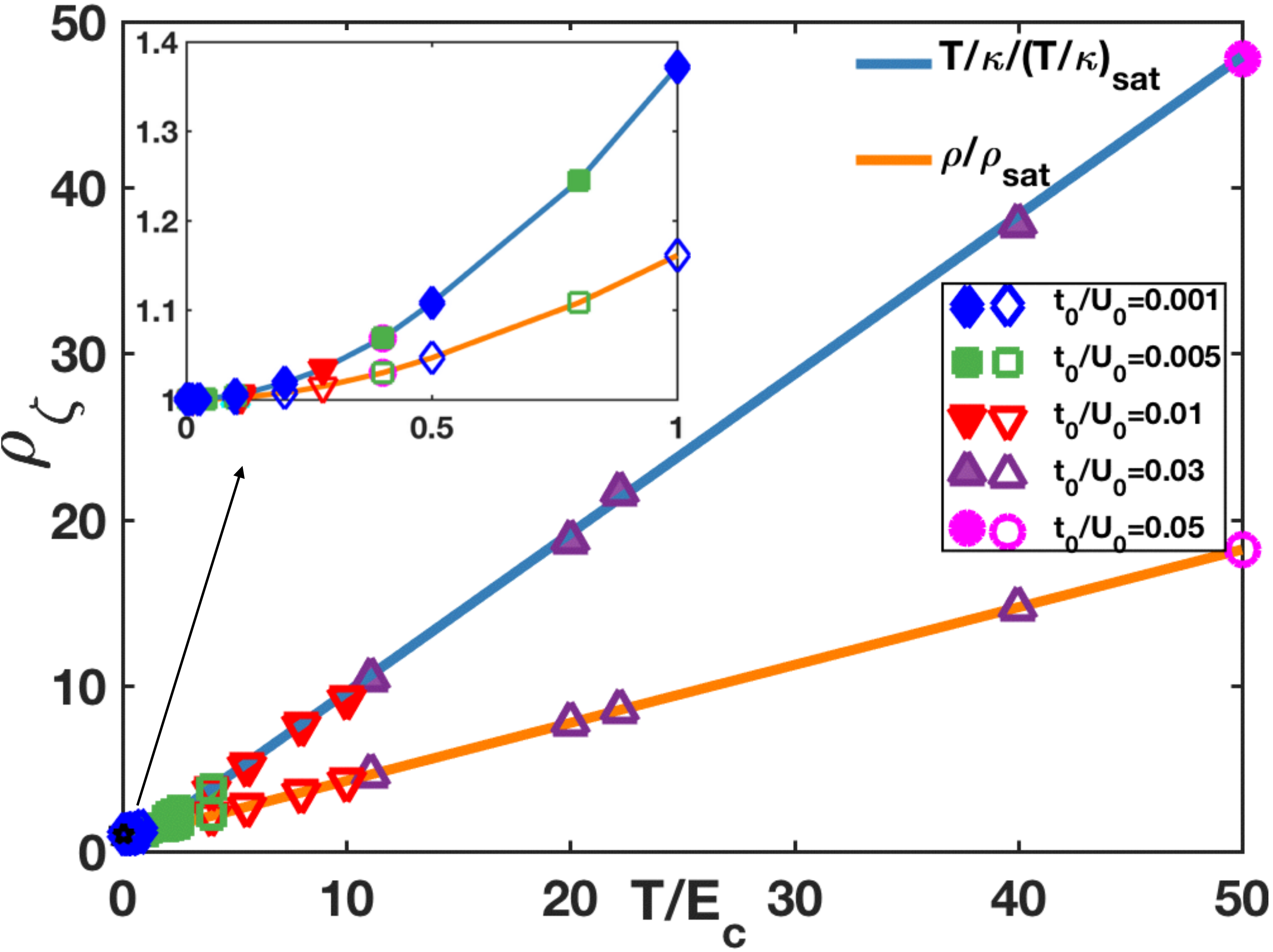
$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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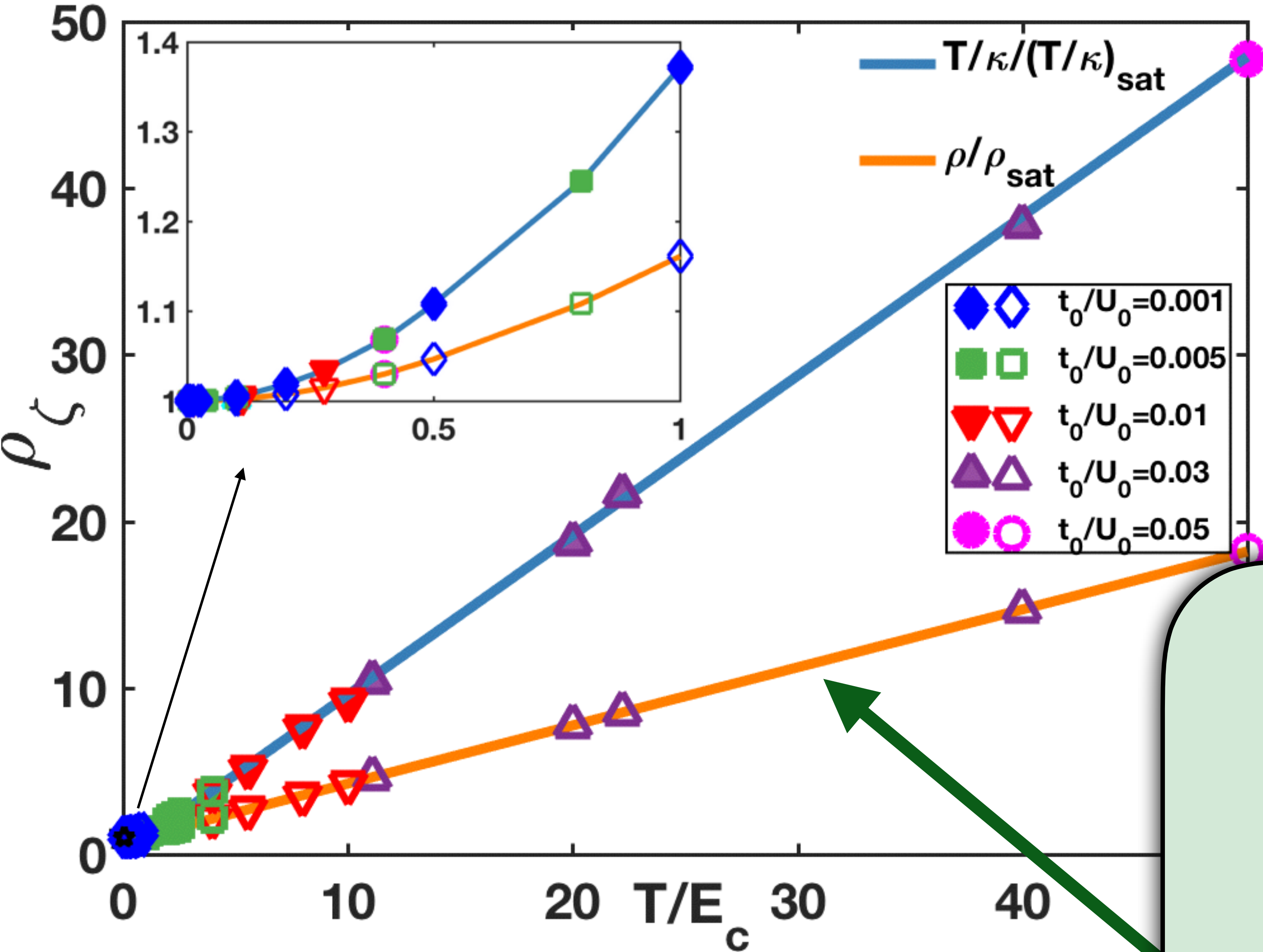
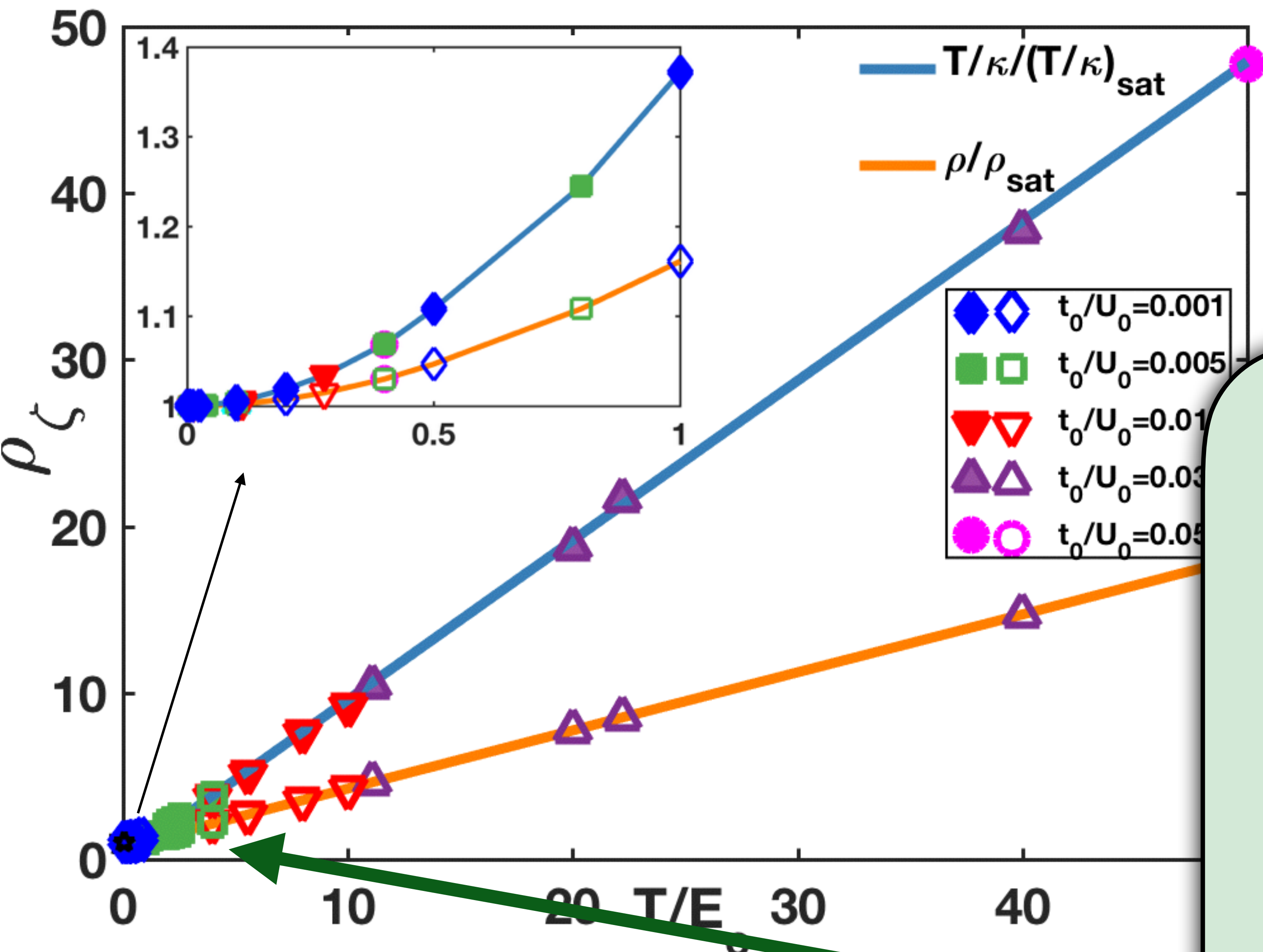


For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \quad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

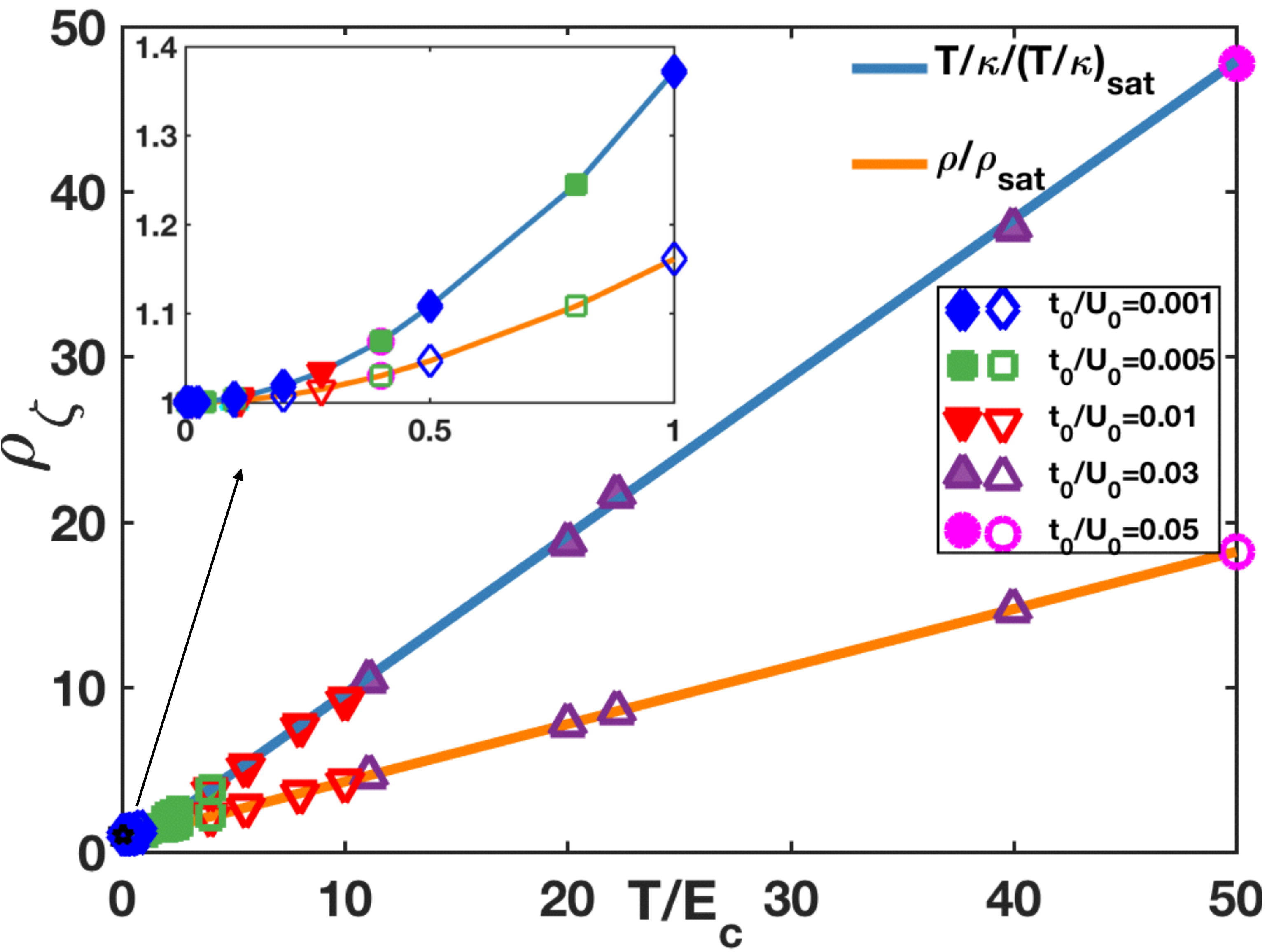
For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

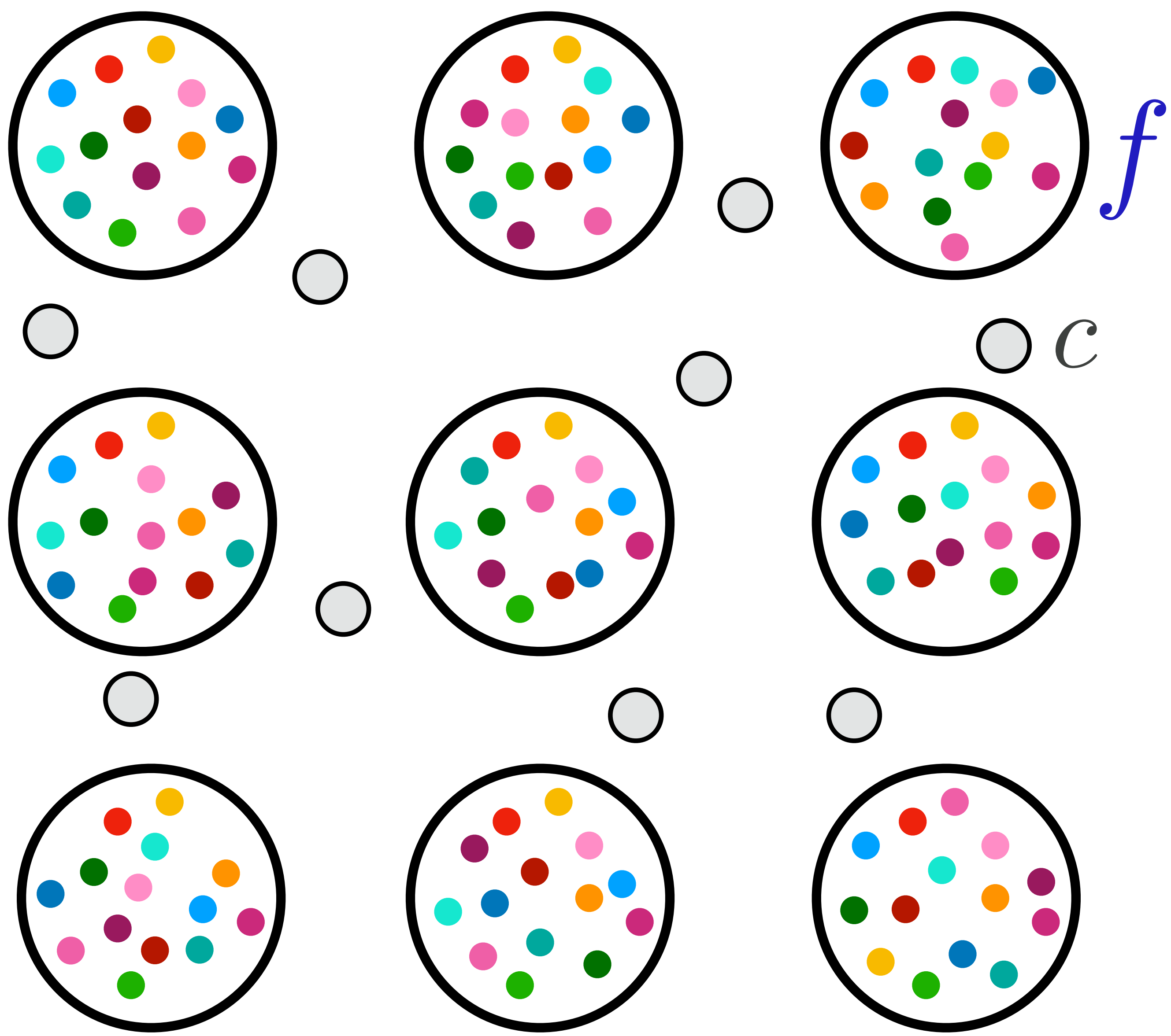
But this model exhibits negligible magnetoresistivity !

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Infecting a Fermi liquid and making it SYK

Mobile electrons (c) interacting with SYK quantum dots (f) with exchange interactions.

This yields the first model agreeing with magnetotransport in strange metals !



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

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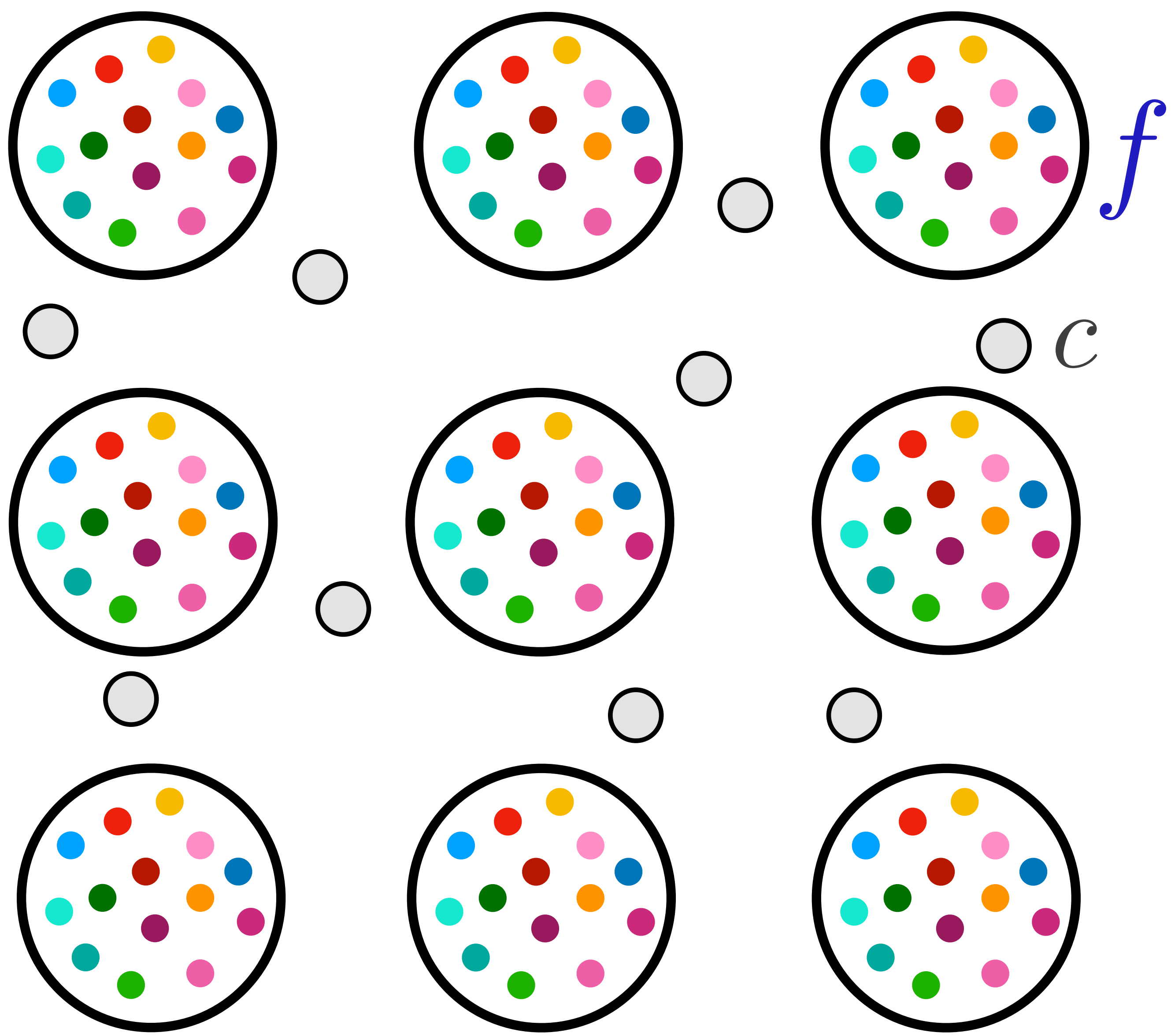


Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

Infecting a Fermi liquid and making it SYK

Mobile electrons (*c*) interacting with SYK quantum dots (*f*) with exchange interactions.

Large N solution (with or without microscopic disorder) yields a 'marginal Fermi liquid' metal, with conductivities of the form:

$$\sigma_{xx}(B,T) = \frac{1}{T}\Phi_L\left(\frac{B}{T}\right)$$
$$\sigma_{xy}(B,T) = \frac{B}{T^2}\Phi_H\left(\frac{B}{T}\right)$$

where the scaling functions interpolate as

$$\Phi_{L,H}(b \to 0) \sim \text{constant} \quad ; \quad \Phi_{L,H}(b \to \infty) \sim 1/b^2$$

This solution exhibits B/T scaling, but the magnetoresistance ρ_{xx} saturates for $B \gg T$.

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

Infecting a Fermi liquid and making it SYK

Need mesoscopic disorder to obtain linear-in-B magnetoresistance

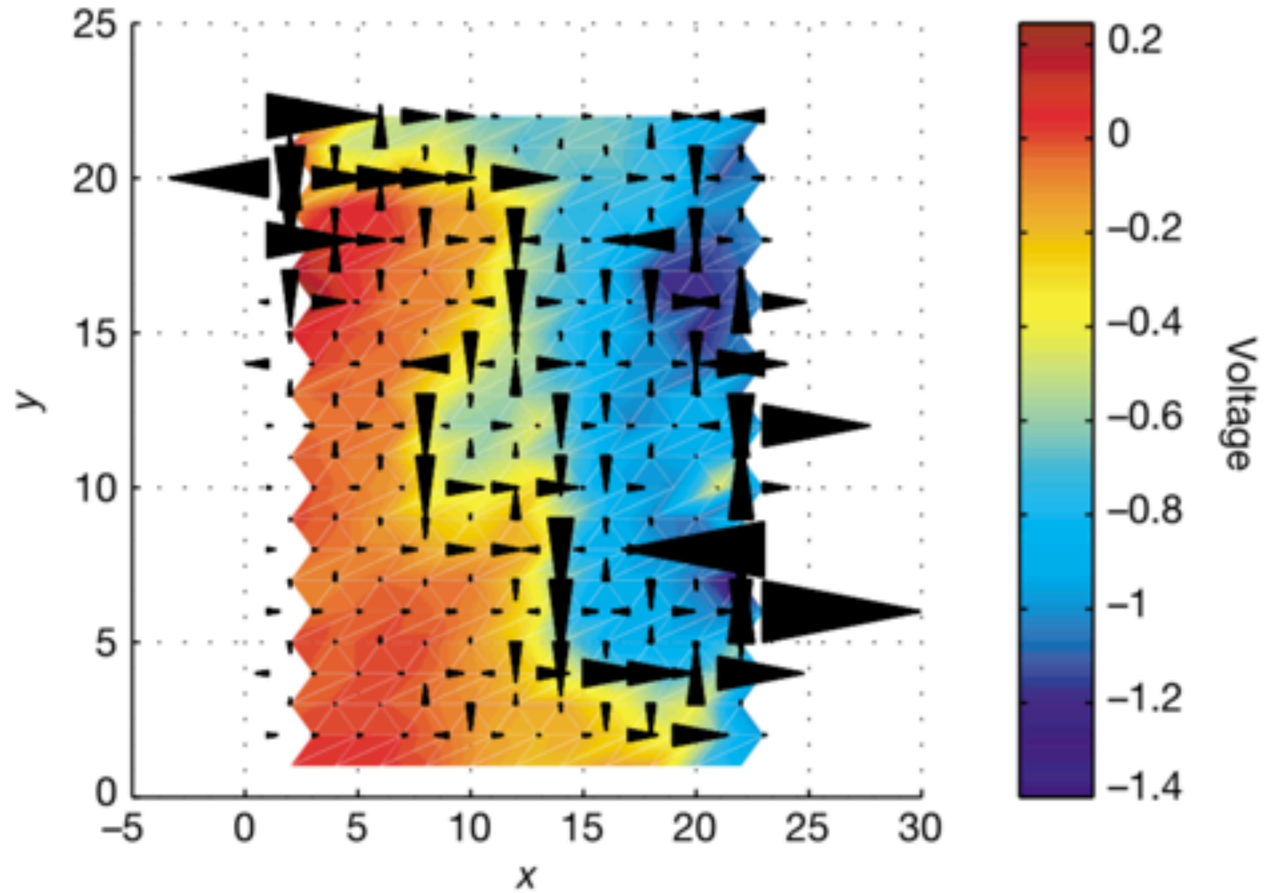
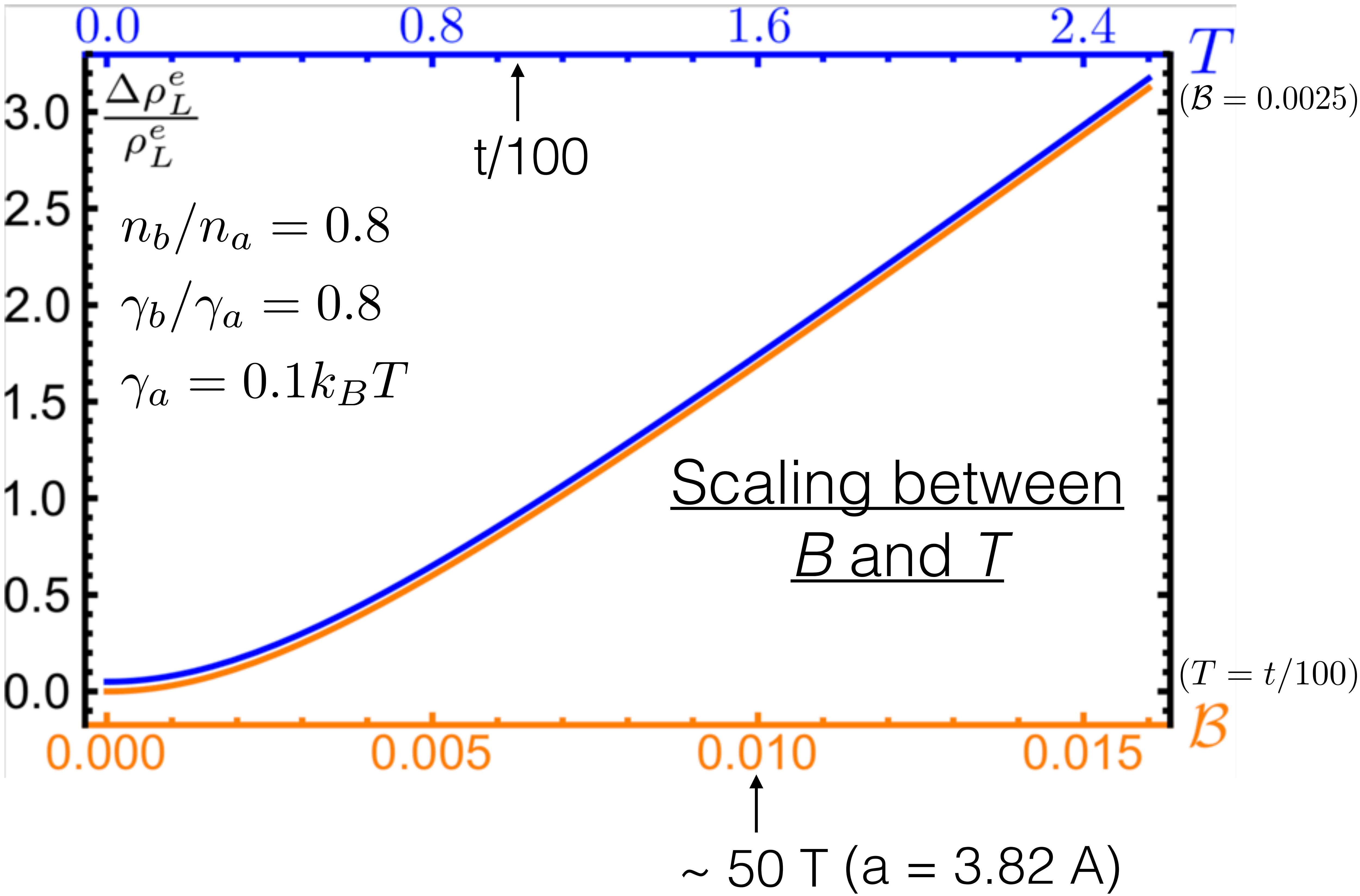


Figure 3 Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in *H*.

- Current path length increases linearly with *B* at large *B* due to local Hall effect, which causes the global resistance to increase linearly with *B* at large *B*.

Exact numerical solution of charge-transport equations in a random-resistor network. (M. M. Parish and P. Littlewood, Nature 426, 162 (2003))

Infecting a Fermi liquid and making it SYK



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, arXiv:1712.05026

- This simple two-component model describes a new state of matter which is realized by electrons in the presence of strong interactions and disorder.

- Can such a model be realized as a fixed-point of a generic theory of strongly-interacting electrons in the presence of disorder?

- Can we start from a single-band Hubbard model with (or without) disorder, and end up with such two-band fixed point, with emergent local conservation laws? Experiments on Si:P indicate an affirmative answer.