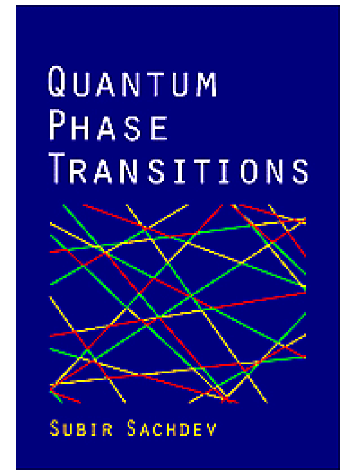
Magnetic phases and critical points of insulators and superconductors

Colloquium article:
Reviews of Modern Physics, **75**, 913 (2003).



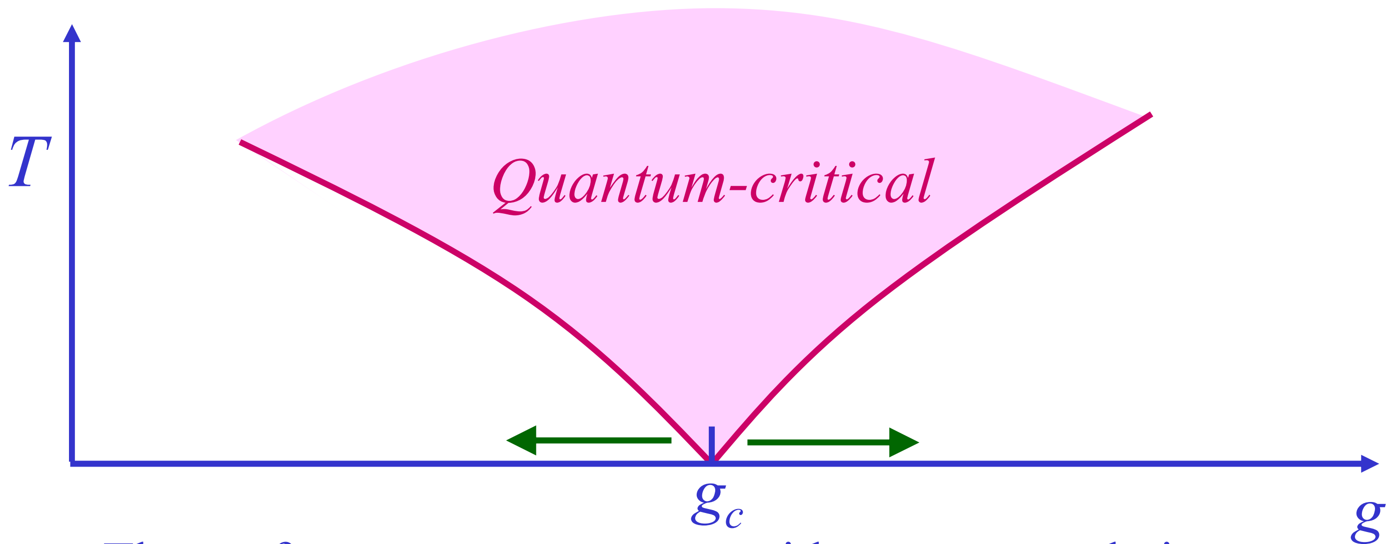
Quantum Phase Transitions
Cambridge University Press

Talks online:
Google Sachdev

What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter g

Why study quantum phase transitions ?



- Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

- Critical point is a novel state of matter without quasiparticle excitations

- Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

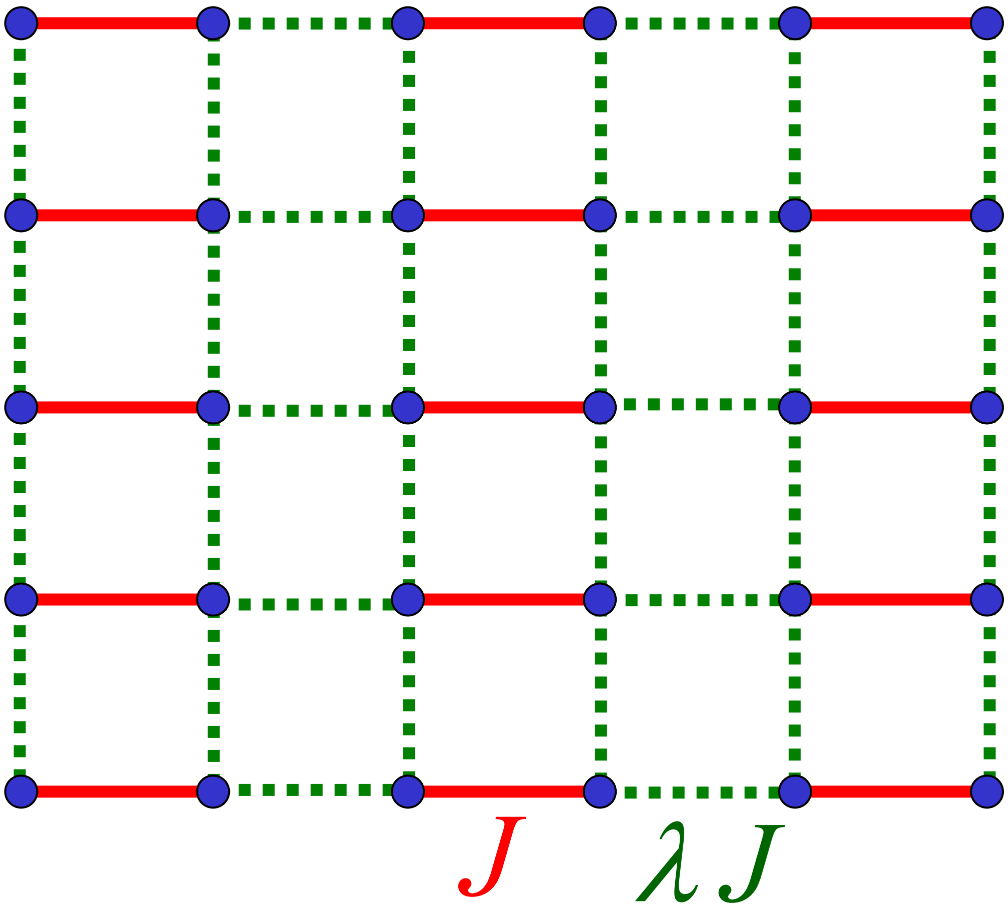
(A) Insulators

Coupled dimer antiferromagnet

Coupled Dimer Antiferromagnet

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev*. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev*. B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



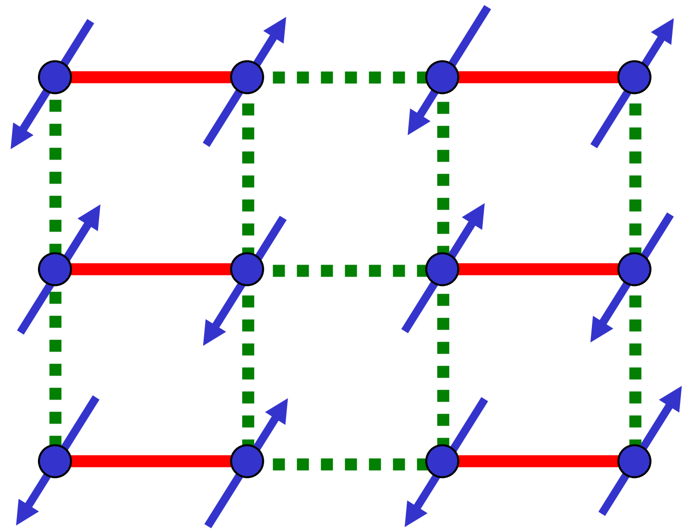
$$H = \sum_{<ij>} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



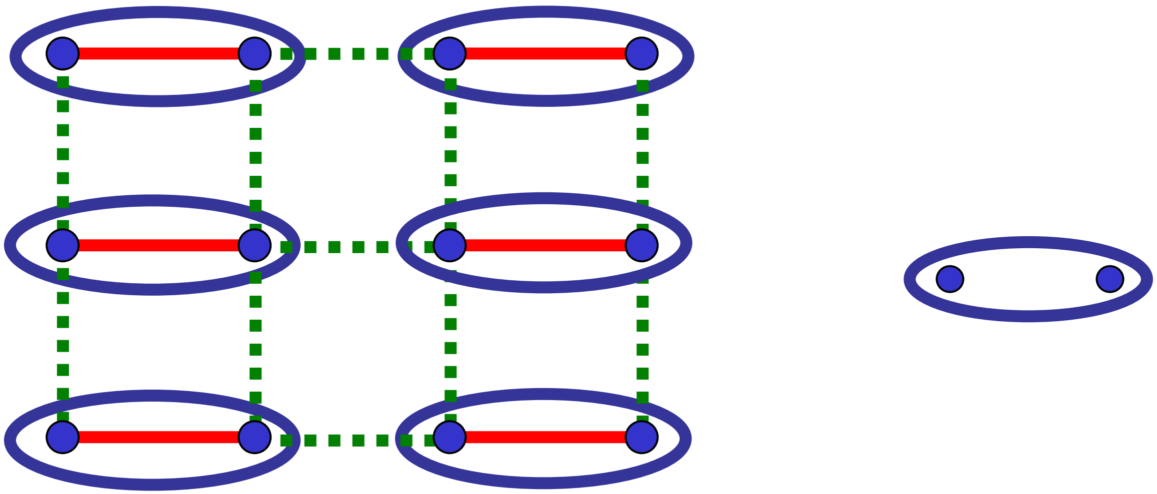
Ground state has long-range magnetic (Neel or spin density wave) order

$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves (***magnons***) $\varepsilon_p = \sqrt{c_x{}^2 p_x{}^2 + c_y{}^2 p_y{}^2}$

$\boxed{\lambda \text{ close to } 0}$

Weakly coupled dimers

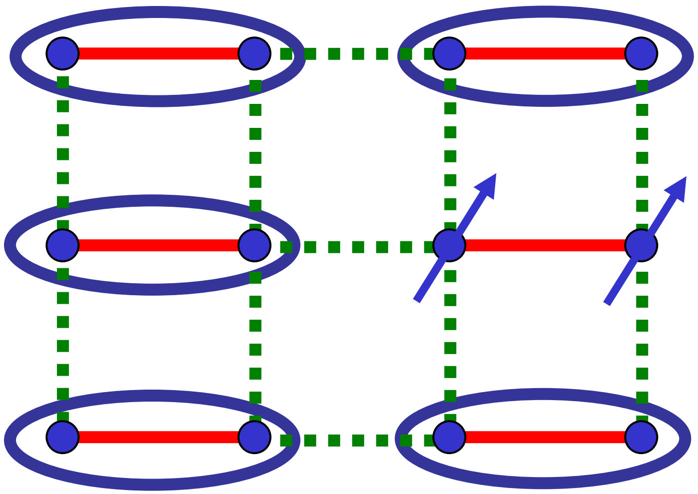


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Paramagnetic ground state $\left\langle \vec{S}_i \right\rangle = 0$

λ close to 0

Weakly coupled dimers



$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Excitation: S=1 ***triplon*** (***exciton***, spin collective mode)

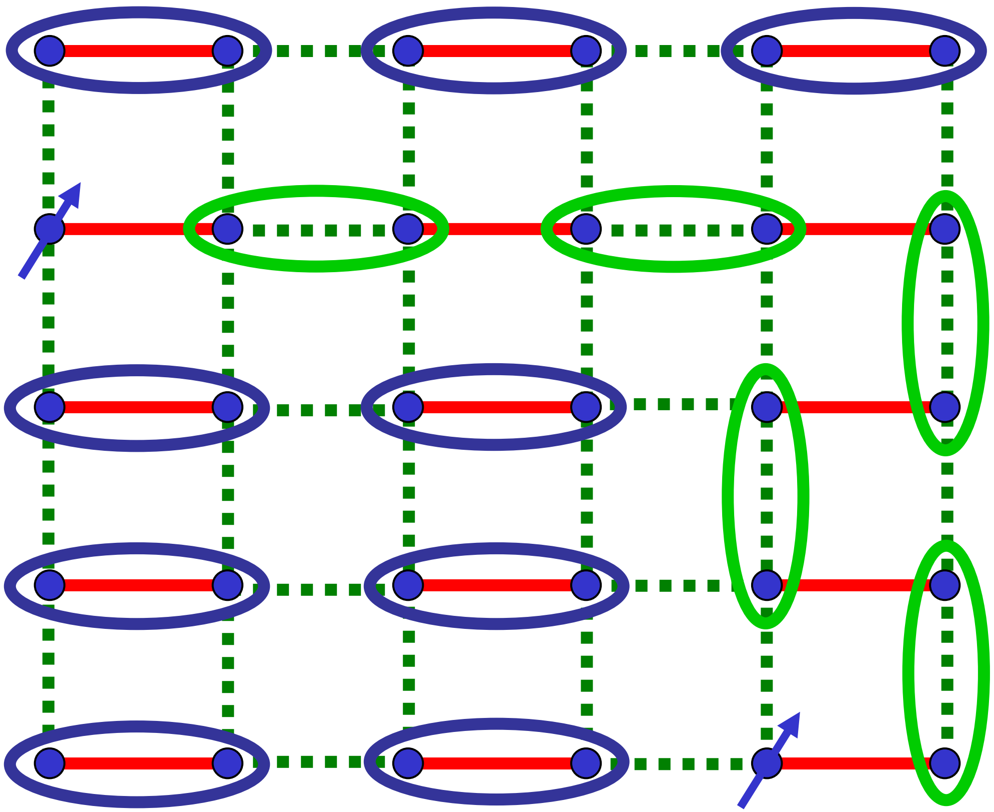
Energy dispersion away from antiferromagnetic wavevector $\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$

$\Delta \rightarrow$ spin gap

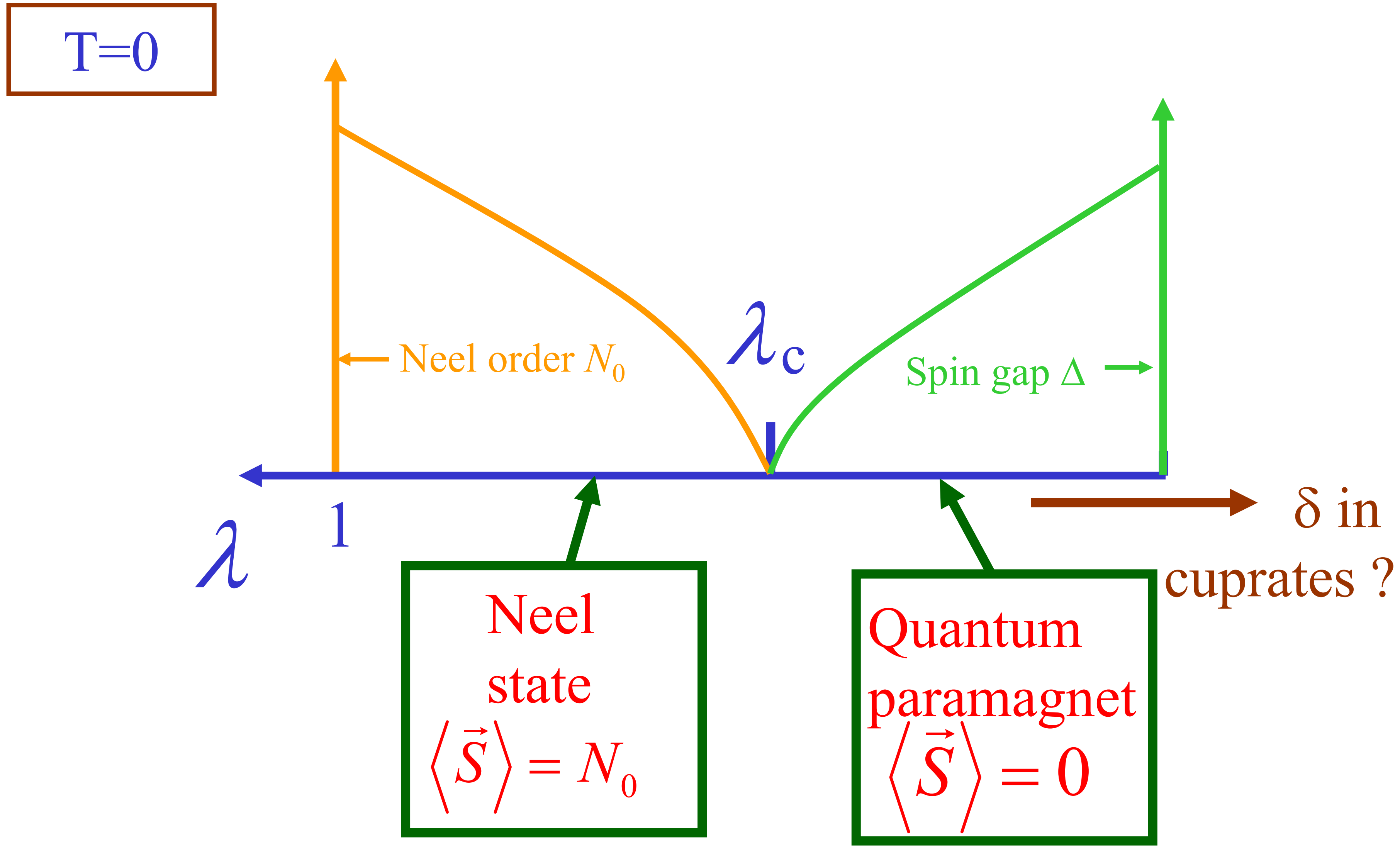
λ close to 0

Weakly coupled dimers

$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$



S=1/2 ***spinons*** are <u>*confined*</u> by a linear potential into a *S*=1 ***triplon***



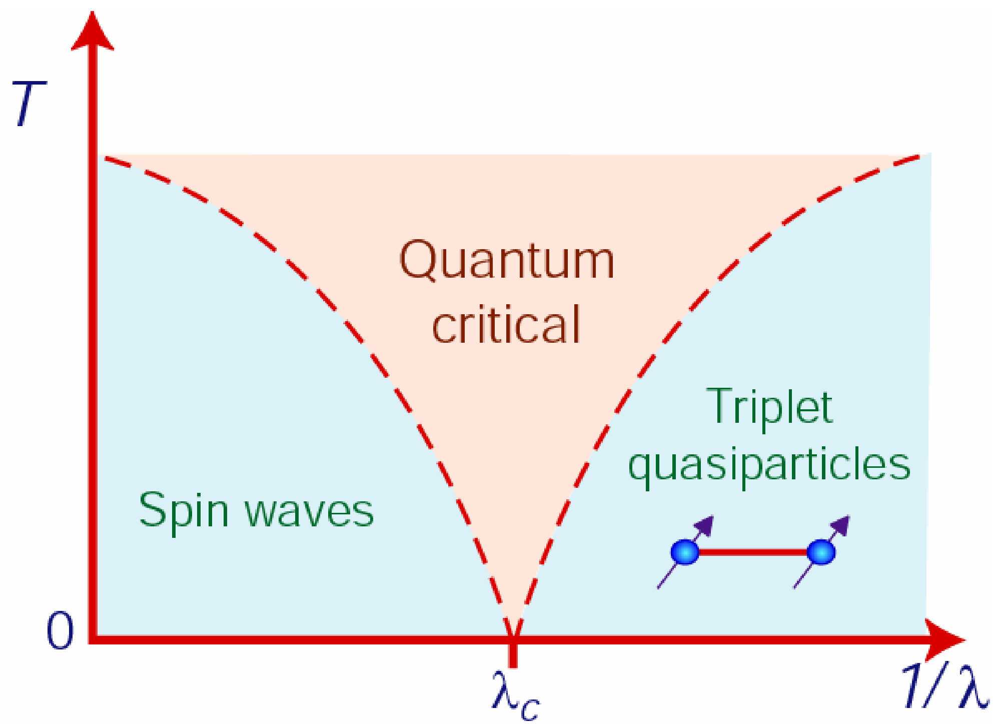
T=0
Neel order N_0
λ_c
Spin gap Δ
λ
1
δ in cuprates ?
Neel state $\langle \vec{S} \rangle = N_0$
Quantum paramagnet $\langle \vec{S} \rangle = 0$

Field theory for quantum criticality

λ close to λ_c : use "soft spin" field

$$S_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 + \left(\lambda_c - \lambda \right) \phi_\alpha^2 \right) + \frac{u}{4!} \left(\phi_\alpha^2 \right)^2 \right]$$

$\phi_\alpha \longrightarrow$ 3-component antiferromagnetic order parameter



Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega/k_B T$

$$\chi(\omega, T) = T^\eta g\left(\hbar\omega/k_B T\right)$$

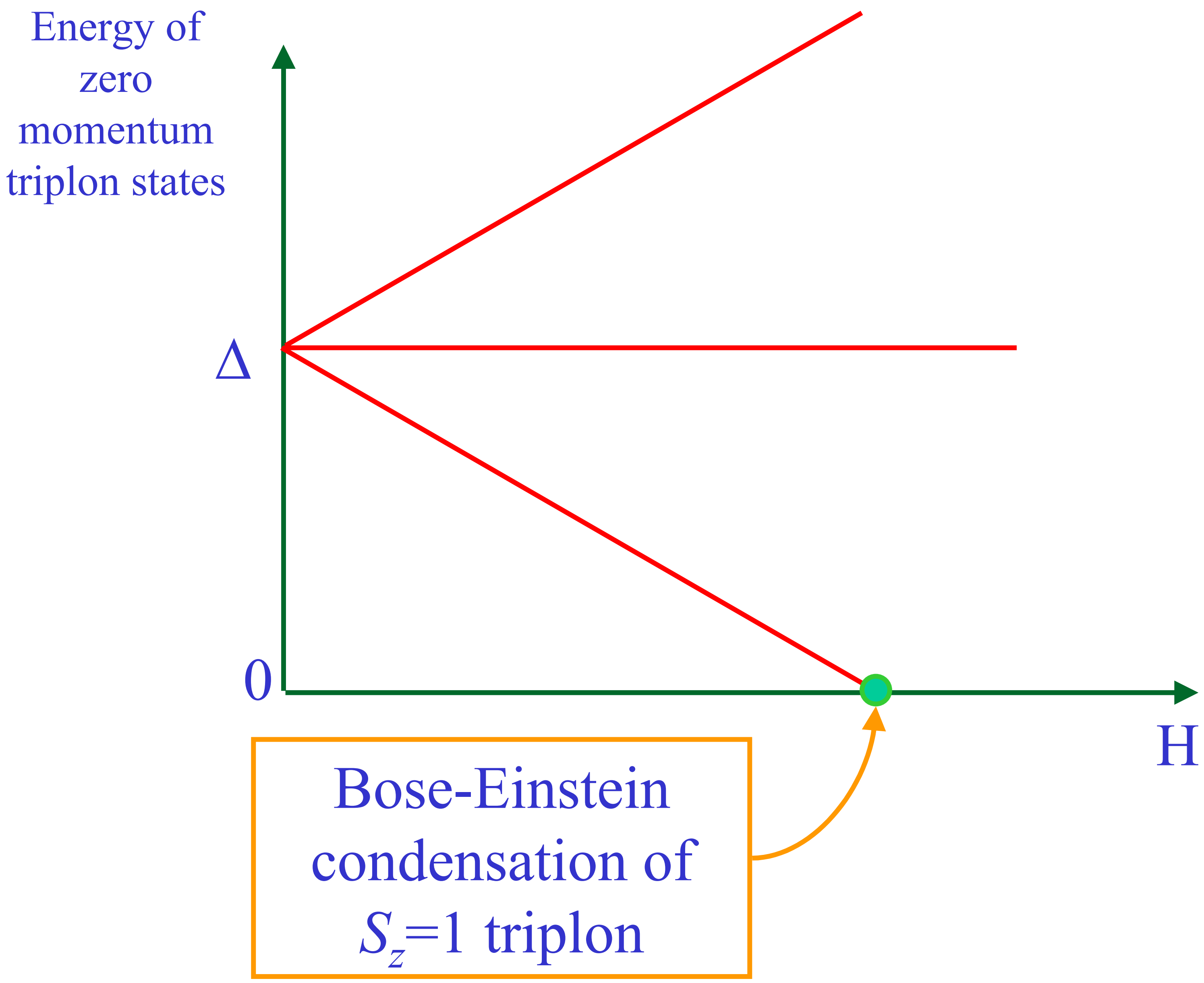
Triplon scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling u

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

(A) Insulators

Coupled dimer antiferromagnet: effect of a magnetic field.

Effect of a field on paramagnet

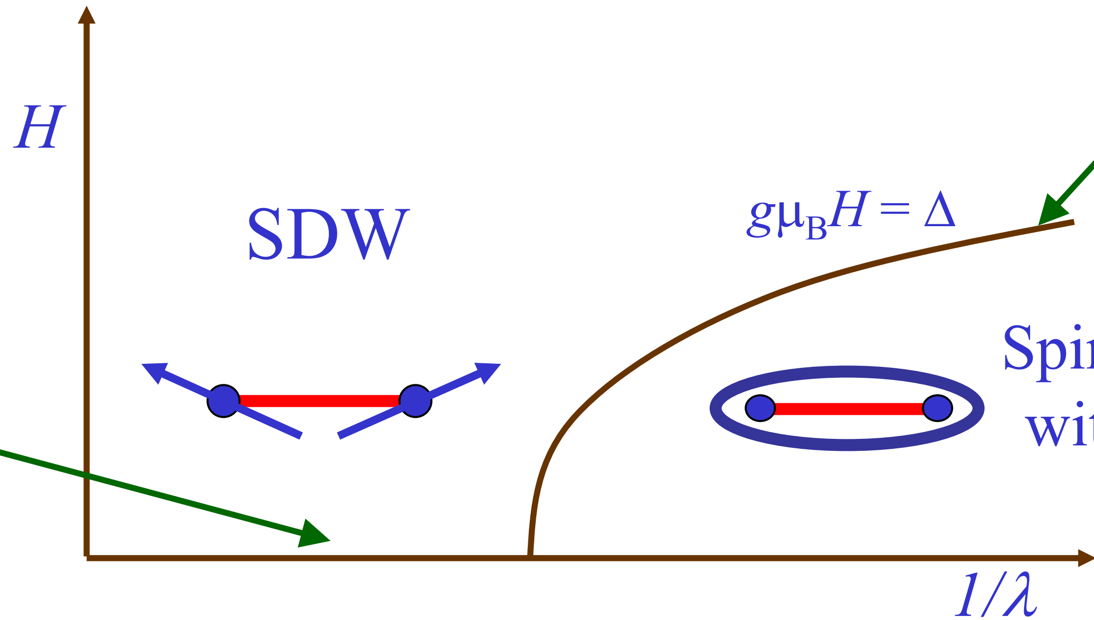


Phase diagram in a magnetic field.

Elastic scattering intensity

$$I[H] = I[0] + a\left(\frac{H}{J}\right)^2$$

$$H_c \sim \sqrt{\lambda_c - \lambda}$$



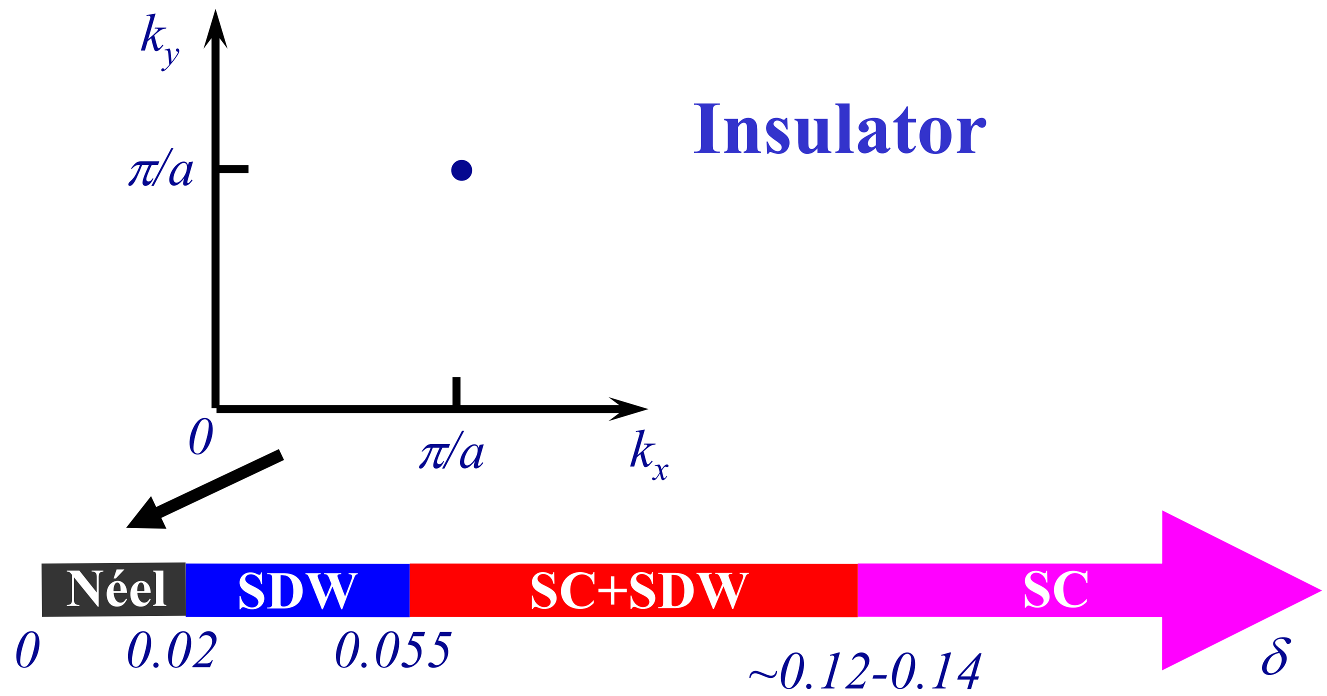
1 Tesla = 0.116 meV

(B) Superconductors

Magnetic transitions in a superconductor:
effect of a magnetic field.

Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO



(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al., Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

Interplay of SDW and SC order in the cuprates

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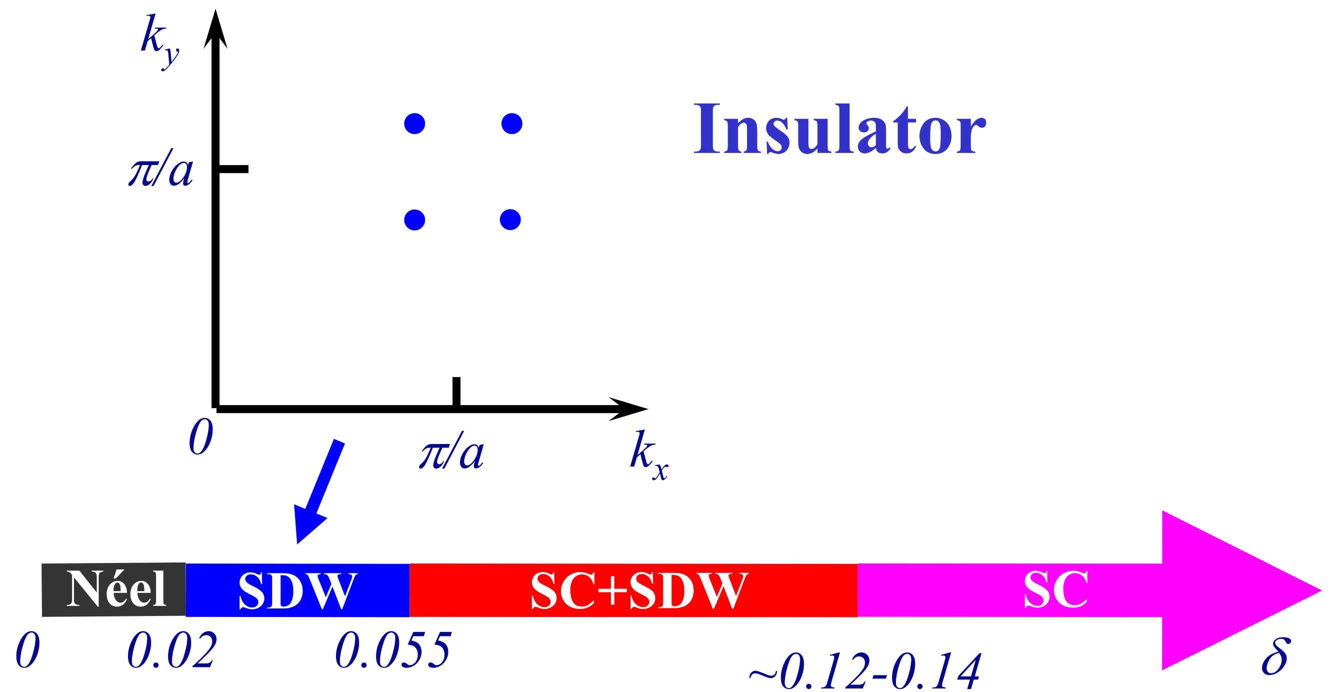

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Interplay of SDW and SC order in the cuprates

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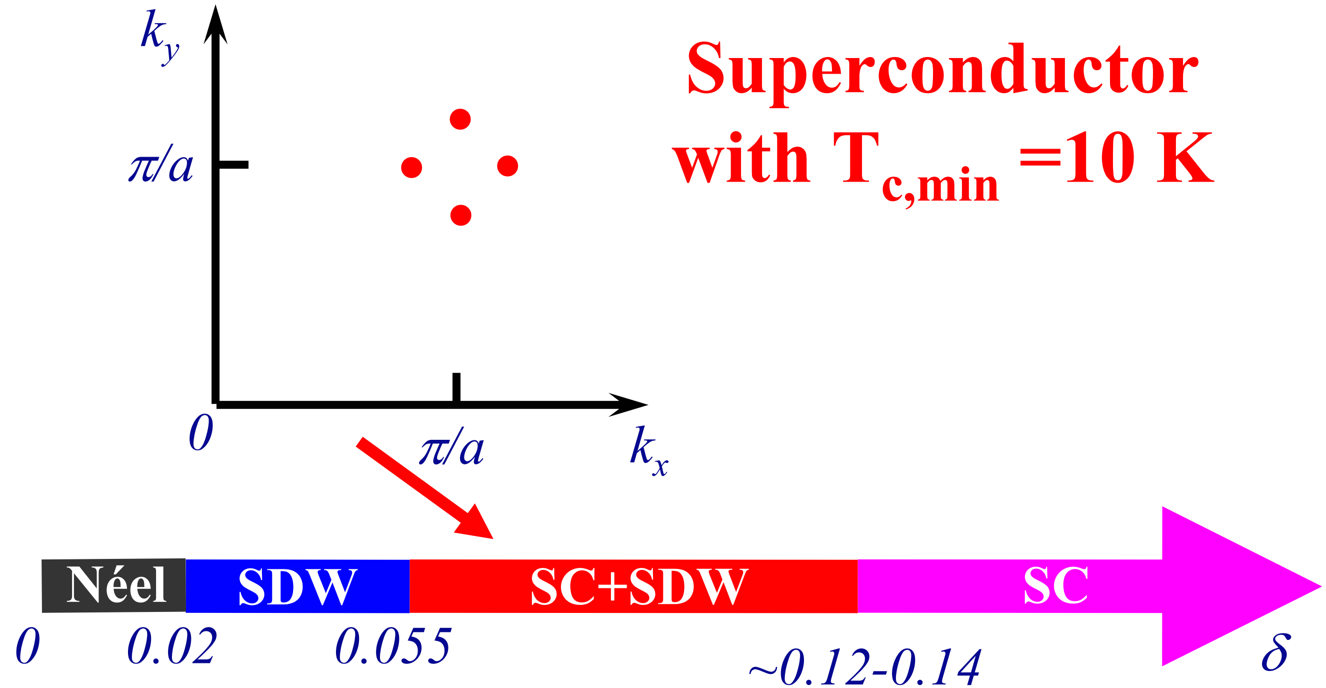

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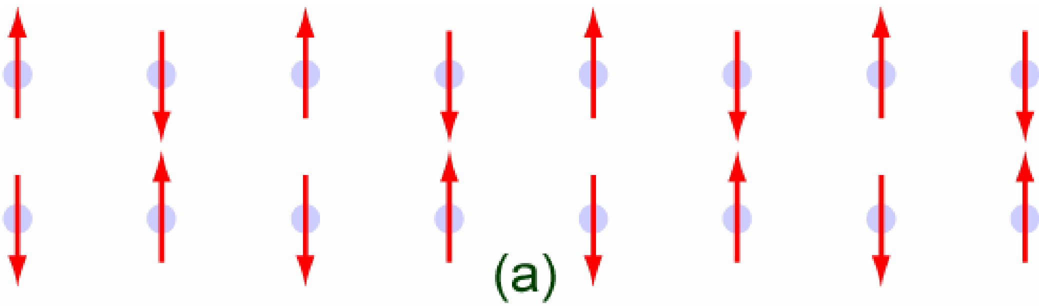
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

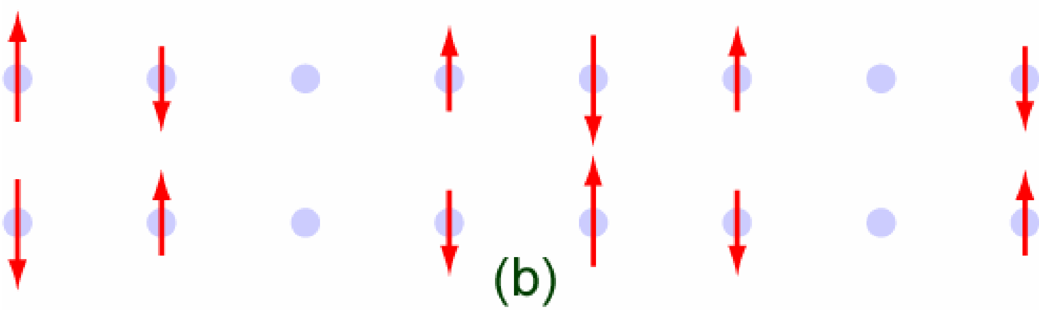
Collinear magnetic (spin density wave) order

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K}.\vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K}.\vec{r}_j\right)$$

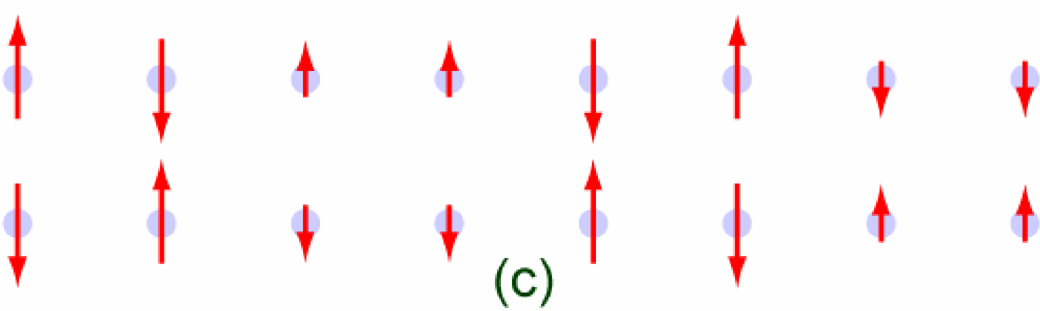
Collinear spins



$$\vec{K} = (\pi,\pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$



$$\vec{K} = (3\pi/4,\pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$

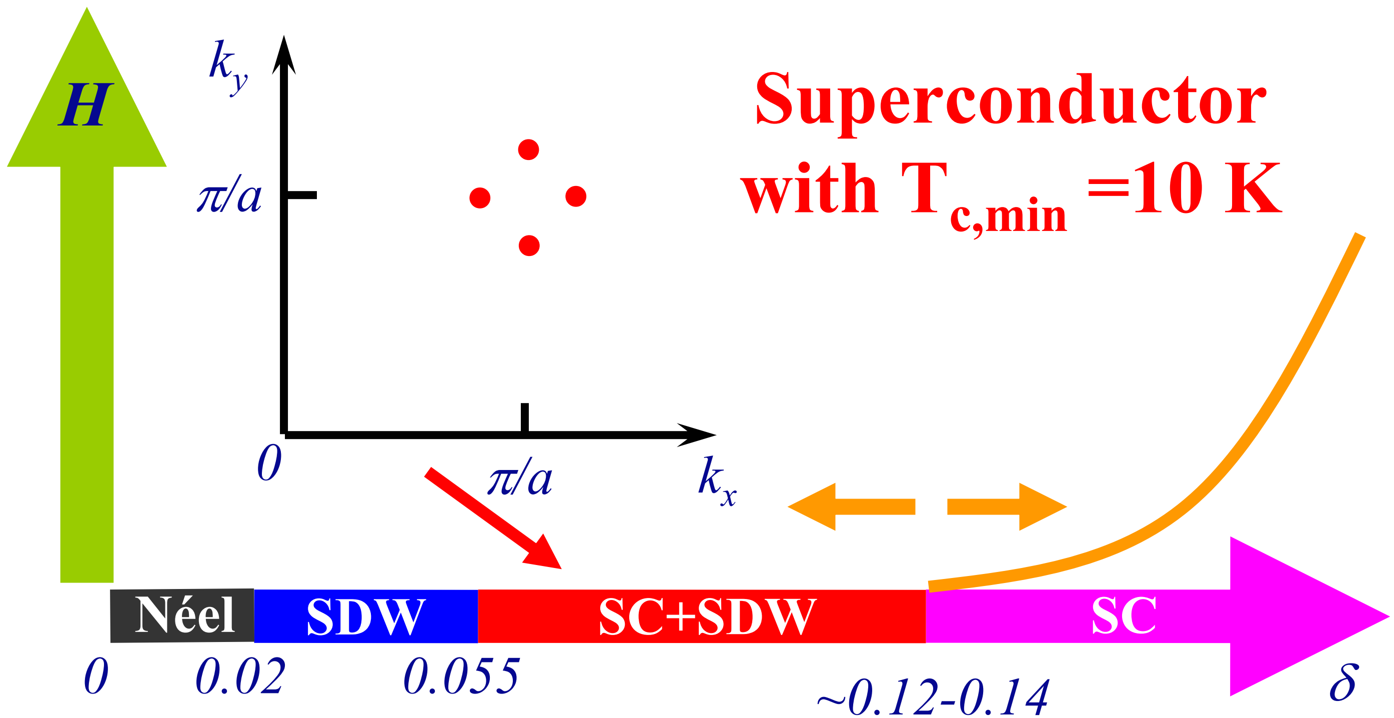


$$\vec{K} = (3\pi/4,\pi) \;\; ;$$

$$\boldsymbol{N}_2 = \left(\sqrt{2}-1\right)\boldsymbol{N}_1$$

Interplay of SDW and SC order in the cuprates

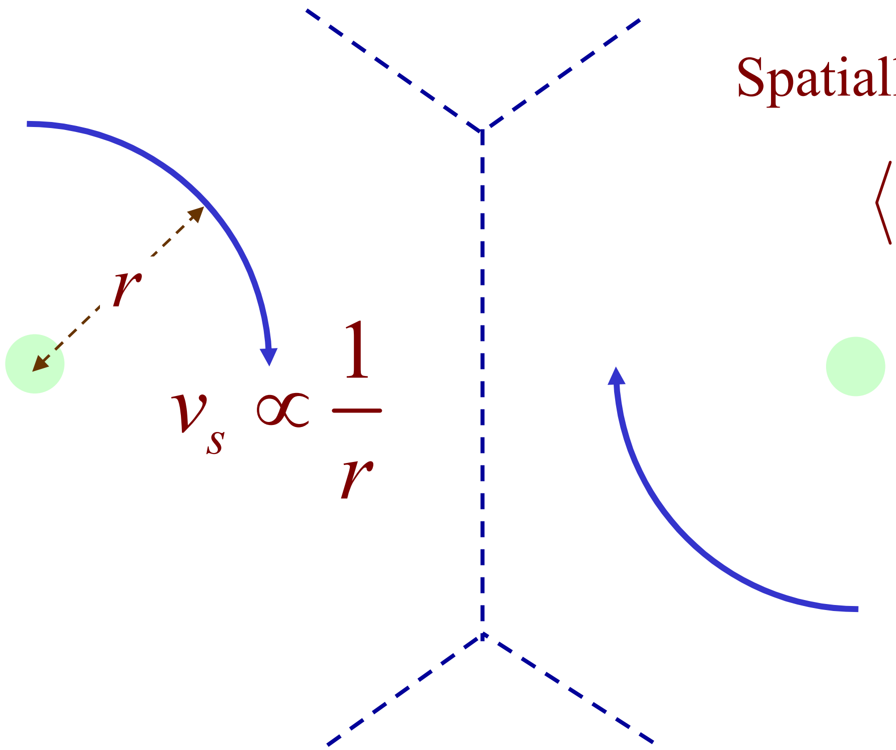
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field

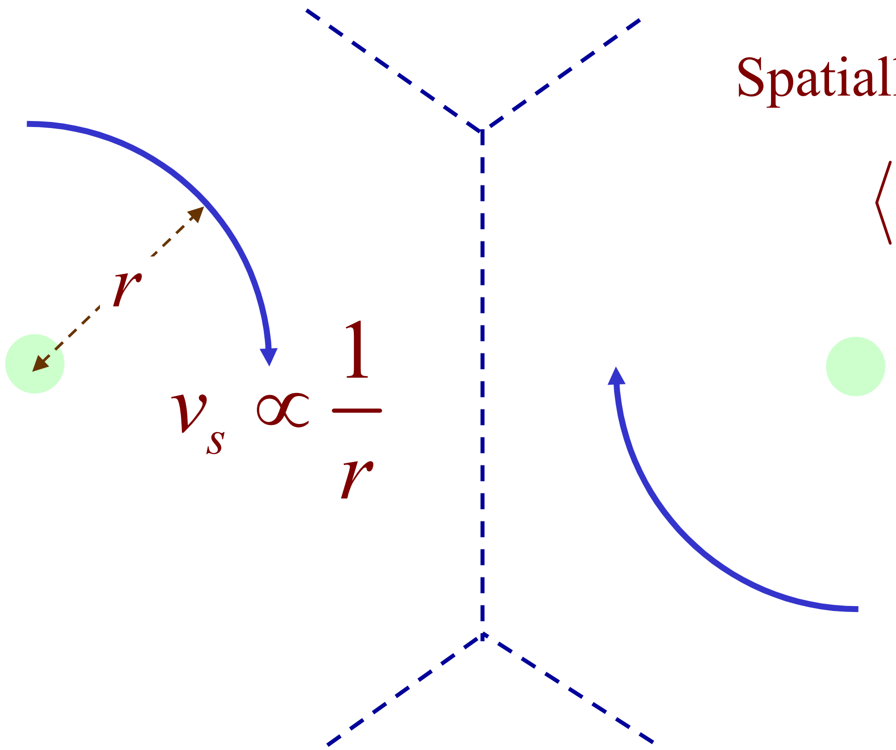
A magnetic field applied to a superconductor induces a lattice of vortices in superflow



Spatially averaged superflow kinetic energy

$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Dominant effect with coexisting superconductivity: **uniform** softening of triplon spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

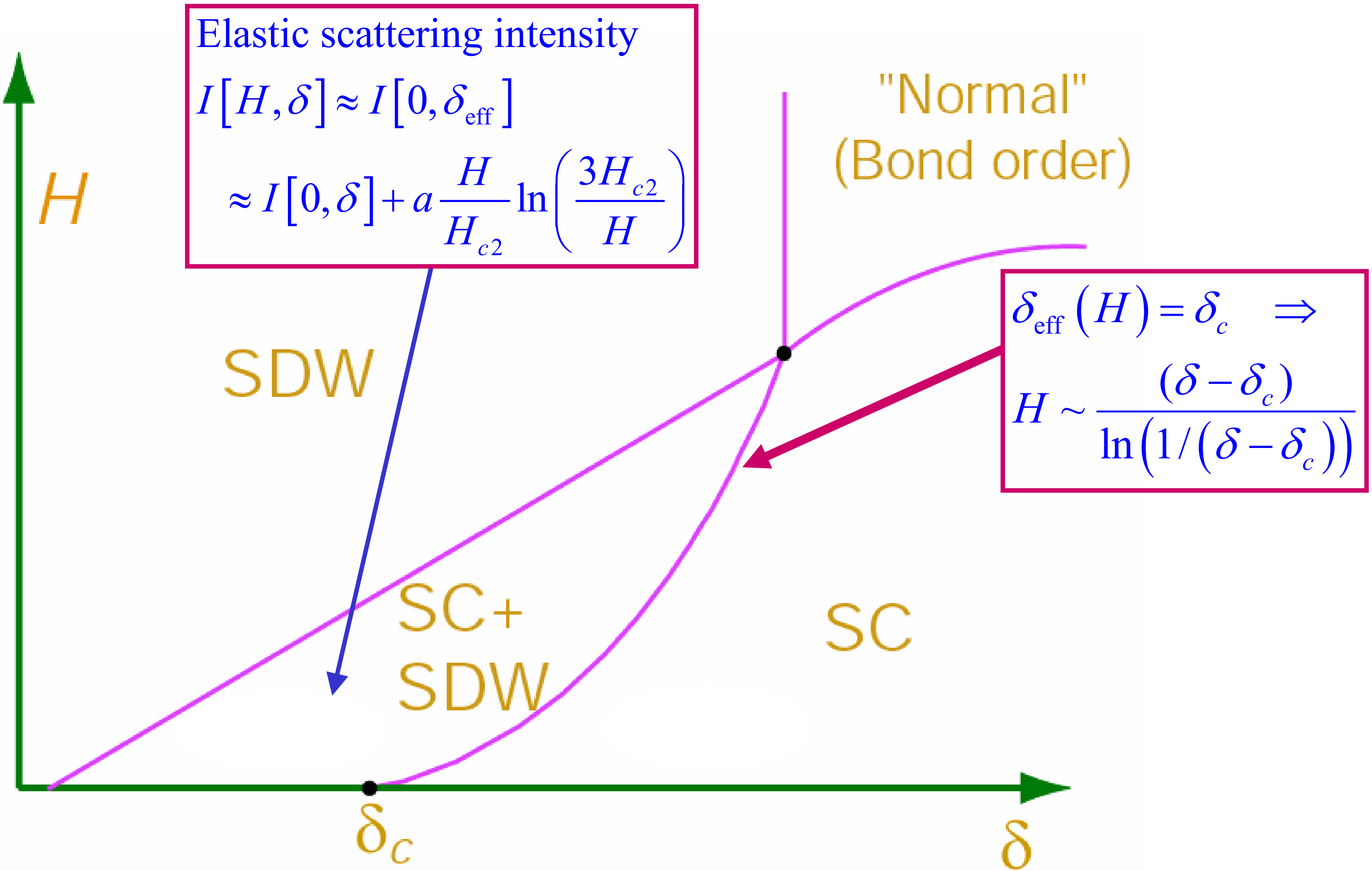
$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

The suppression of SC order appears to the SDW order as a ***uniform*** effective "doping" δ:

$$\delta_{\text{eff}}\left(H\right) = \delta - C\frac{H}{H_{c2}} \ln\left(\frac{3H_{c2}}{H}\right)$$

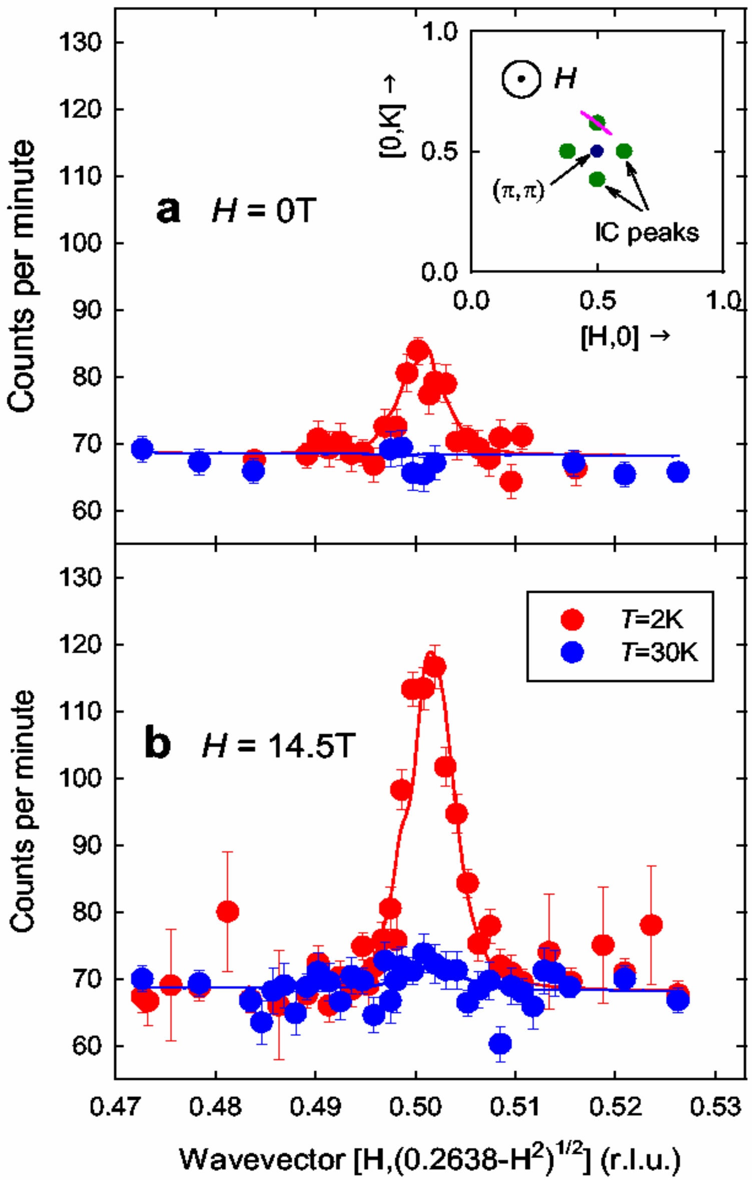
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phase diagram of SC and SDW order in a magnetic field

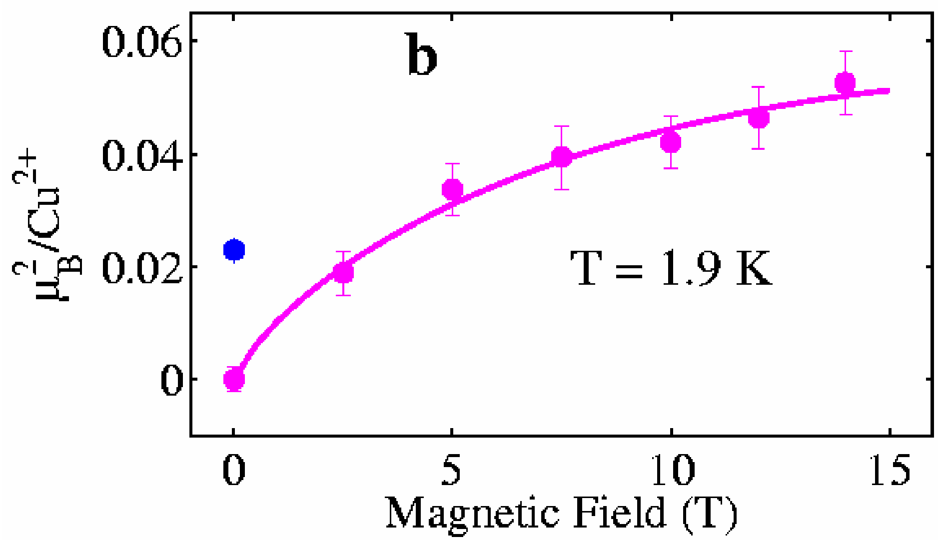


E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



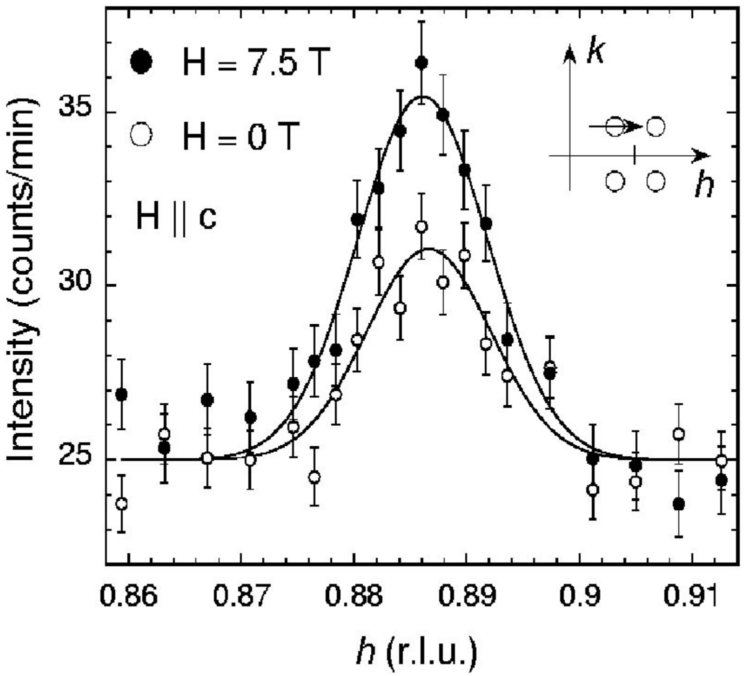
$$\text{Solid line - fit to : } I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$$

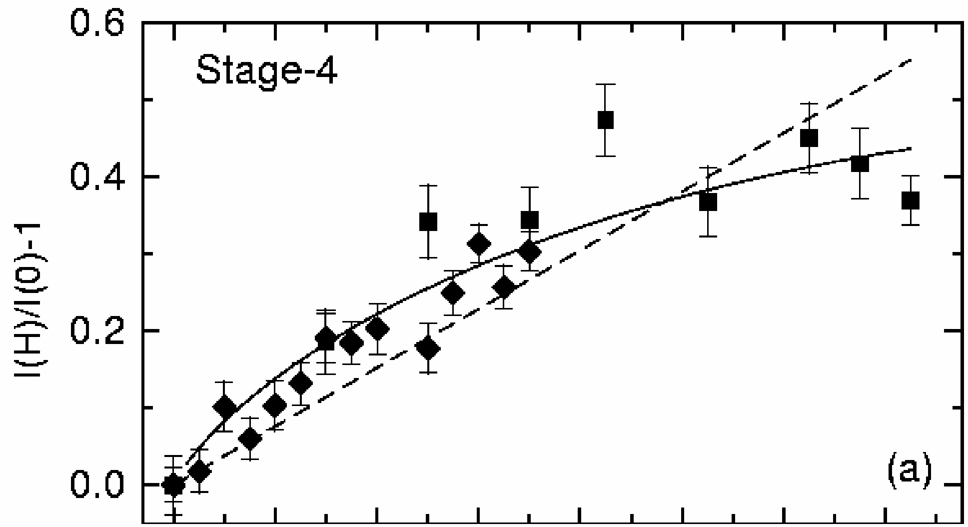
See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev*. B **62**, R14677 (2000).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+CM) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto,
K. J. Thomas, M. A. Kastner,
and R.J. Birgeneau, *Phys. Rev.* B **66**, 014528 (2002).





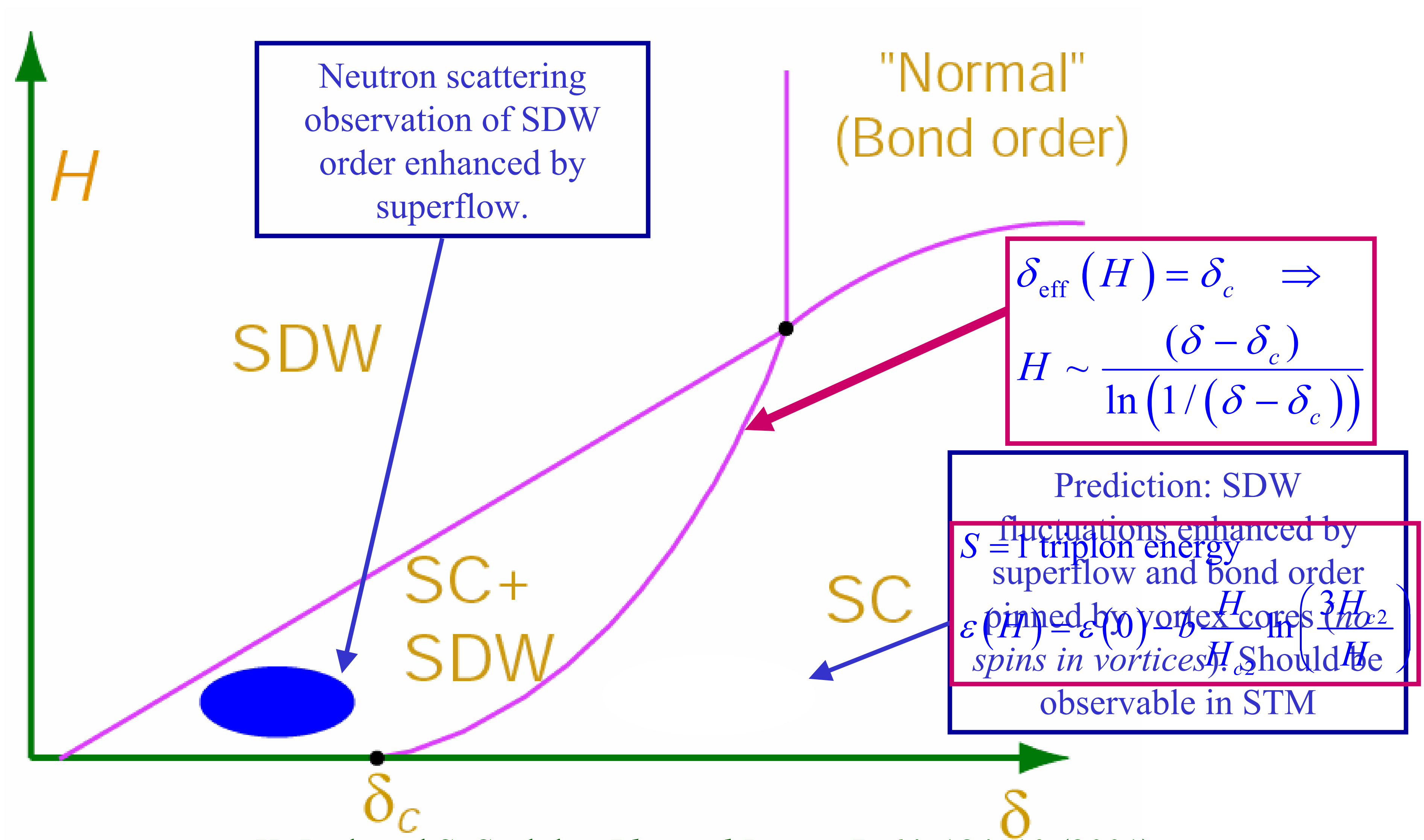
H (Tesla)

Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Phase diagram of a superconductor in a magnetic field

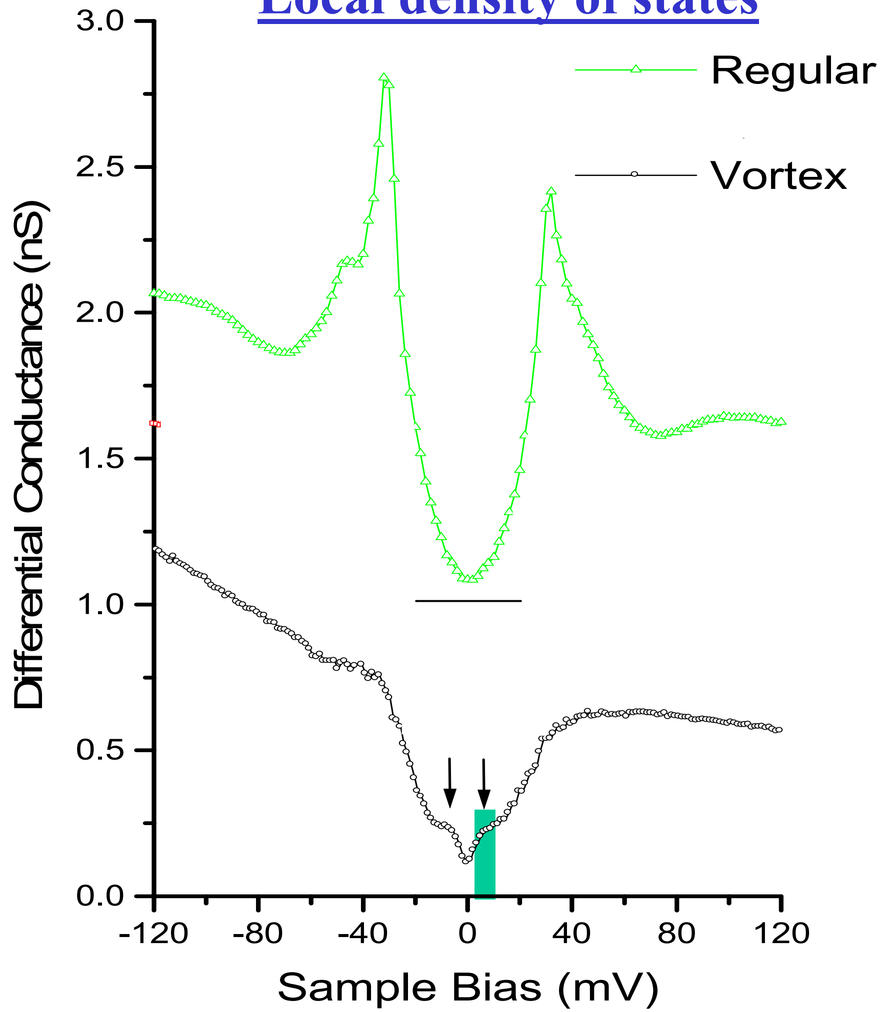


K. Park and S. Sachdev *Physical Review* B **64**, 184510 (2001);
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
Y. Zhang, E. Demler and S. Sachdev, *Physical Review* B **66**, 094501 (2002).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

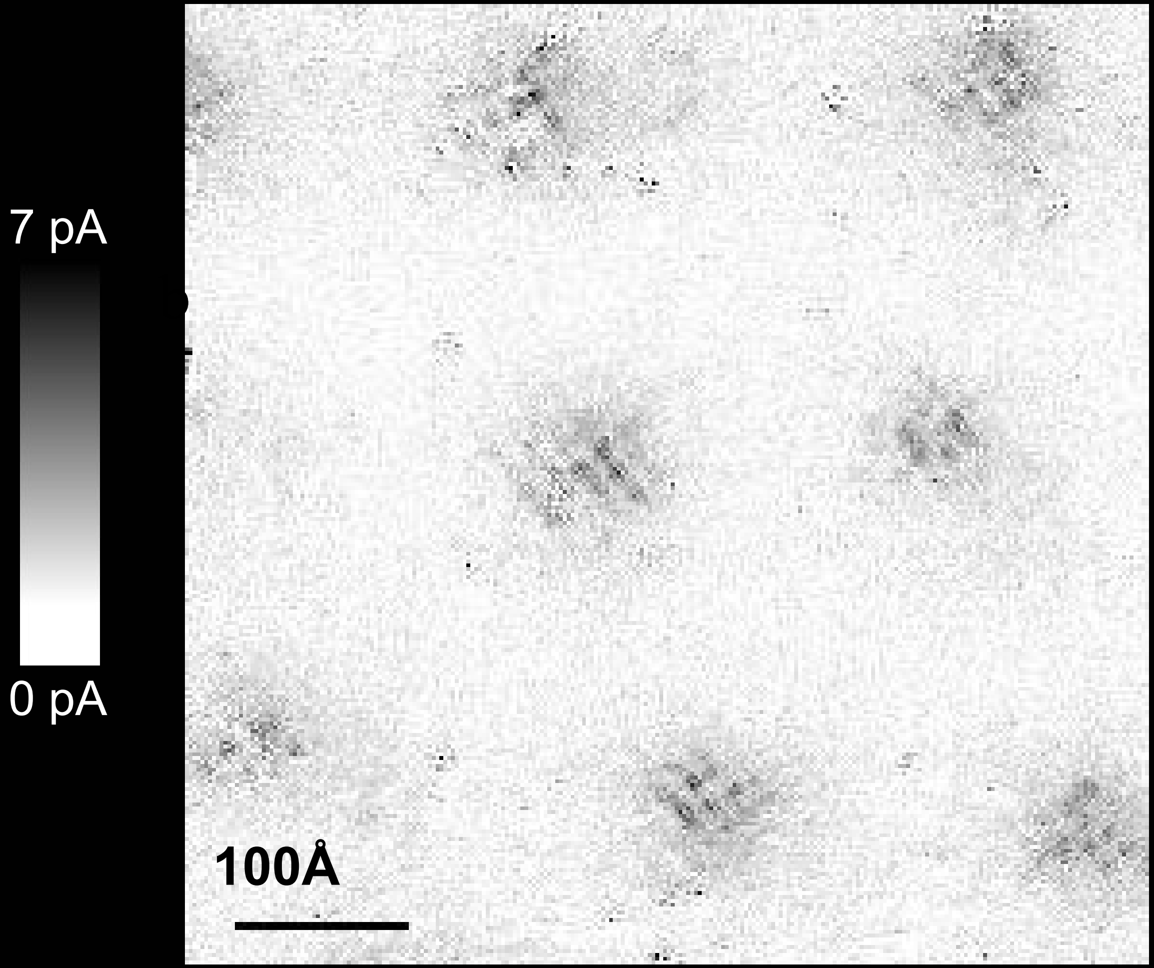


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



Our interpretation: LDOS modulations are signals of bond order of period 4 revealed in vortex halo

See also:
S. A. Kivelson, E. Fradkin, V. Oganesyan, I. P. Bindloss, J. M. Tranquada, A. Kapitulnik, and C. Howald, cond-mat/0210683.

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Conclusions

I. Introduction to magnetic quantum criticality in coupled dimer antiferromagnet.

II. Theory of quantum phase transitions provides semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.