Quantum mechanics without particles

Institute Lecture,
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PHYSICS
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Outline

1. Key ideas from quantum mechanics
2. Many-particle systems: the concept of a quasiparticle
3. Quantum phases of a magnetic insulator

   Quantum critical point without quasiparticles
4. Connections to string theory
5. Non-zero temperatures and black holes
6. The high temperature superconductors

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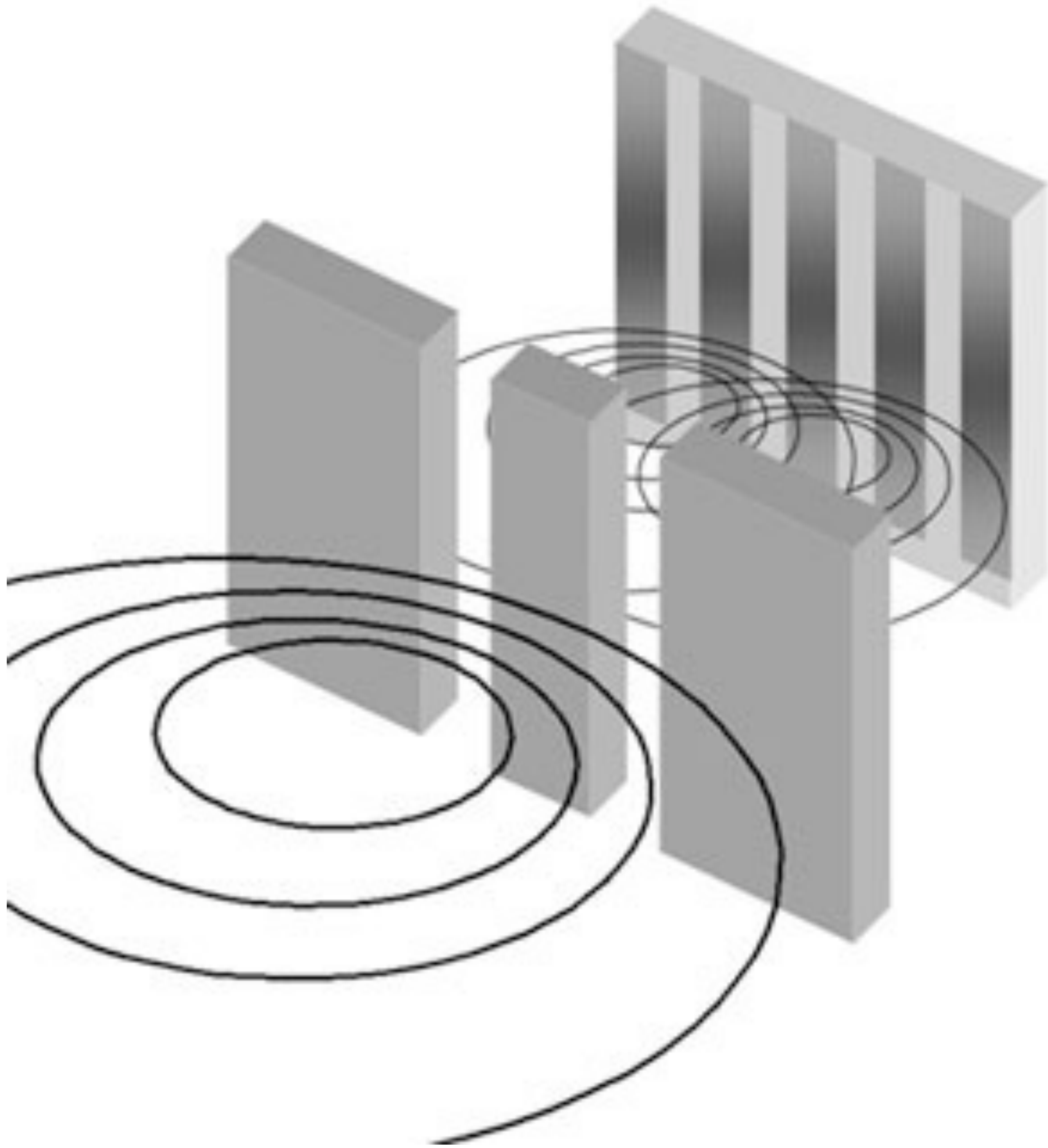
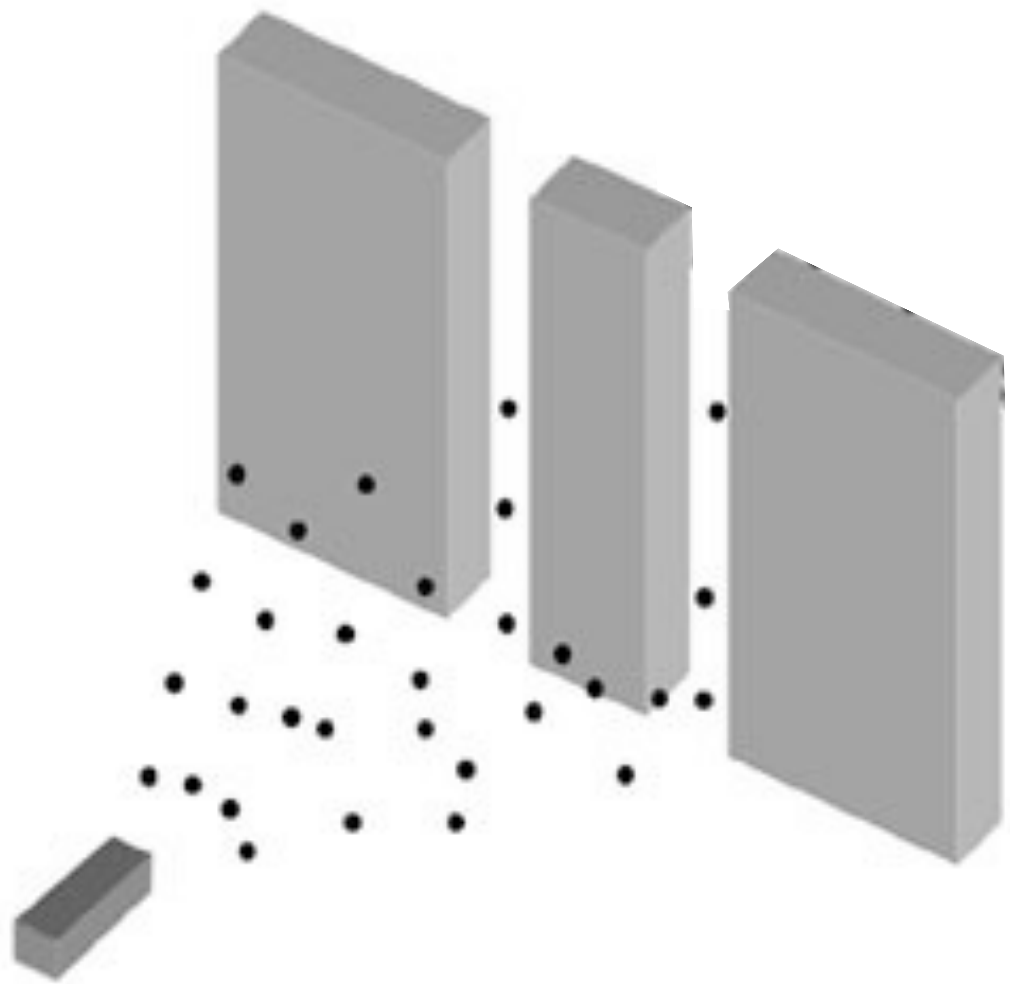
The double slit experiment



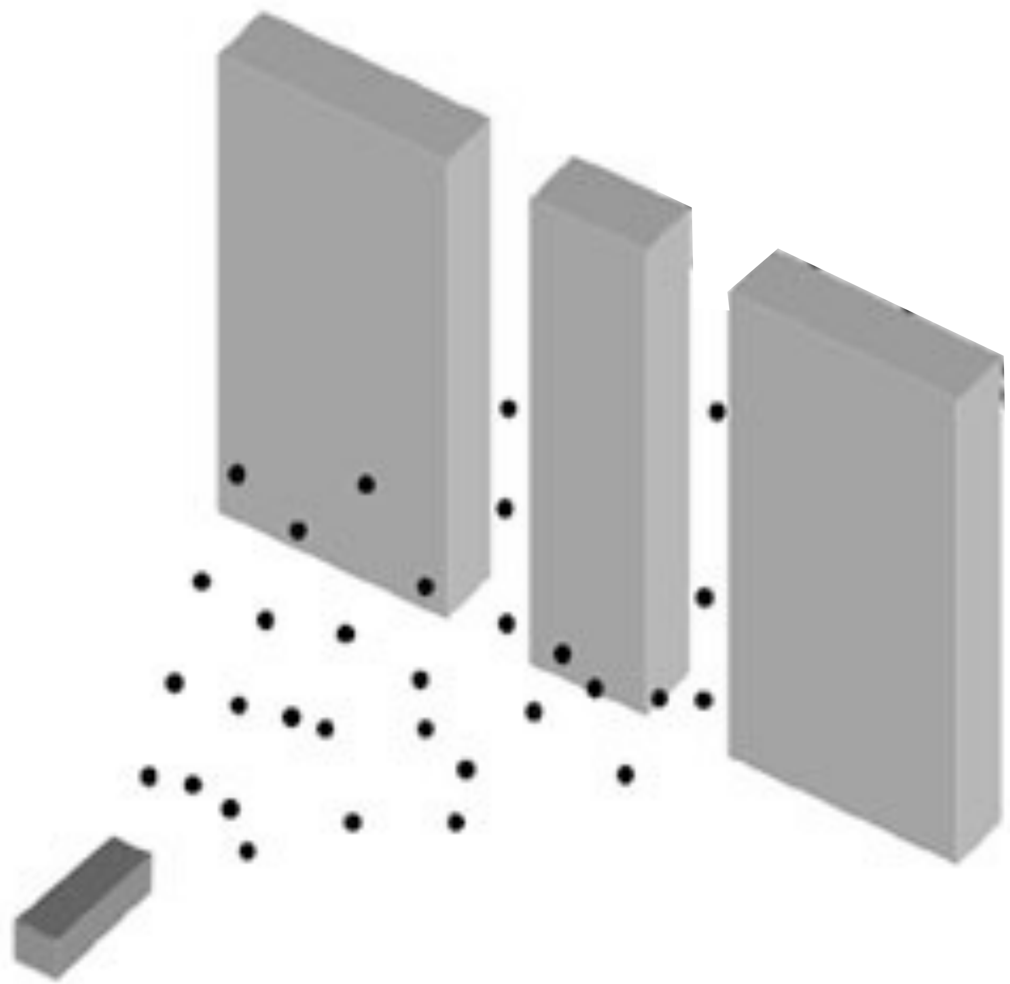
Interference of water waves

The double slit experiment



Send electrons through the slits

The double slit experiment



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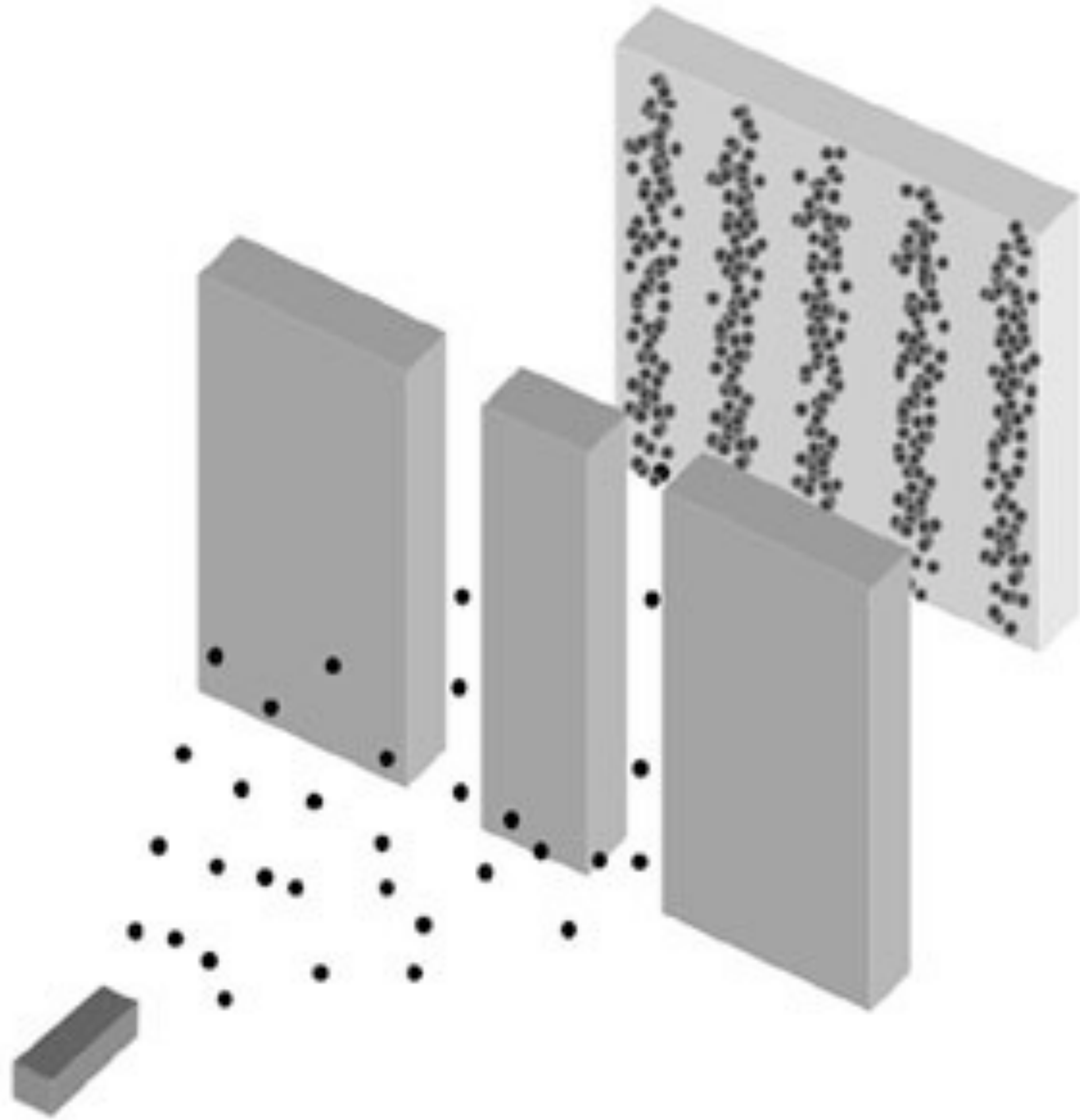
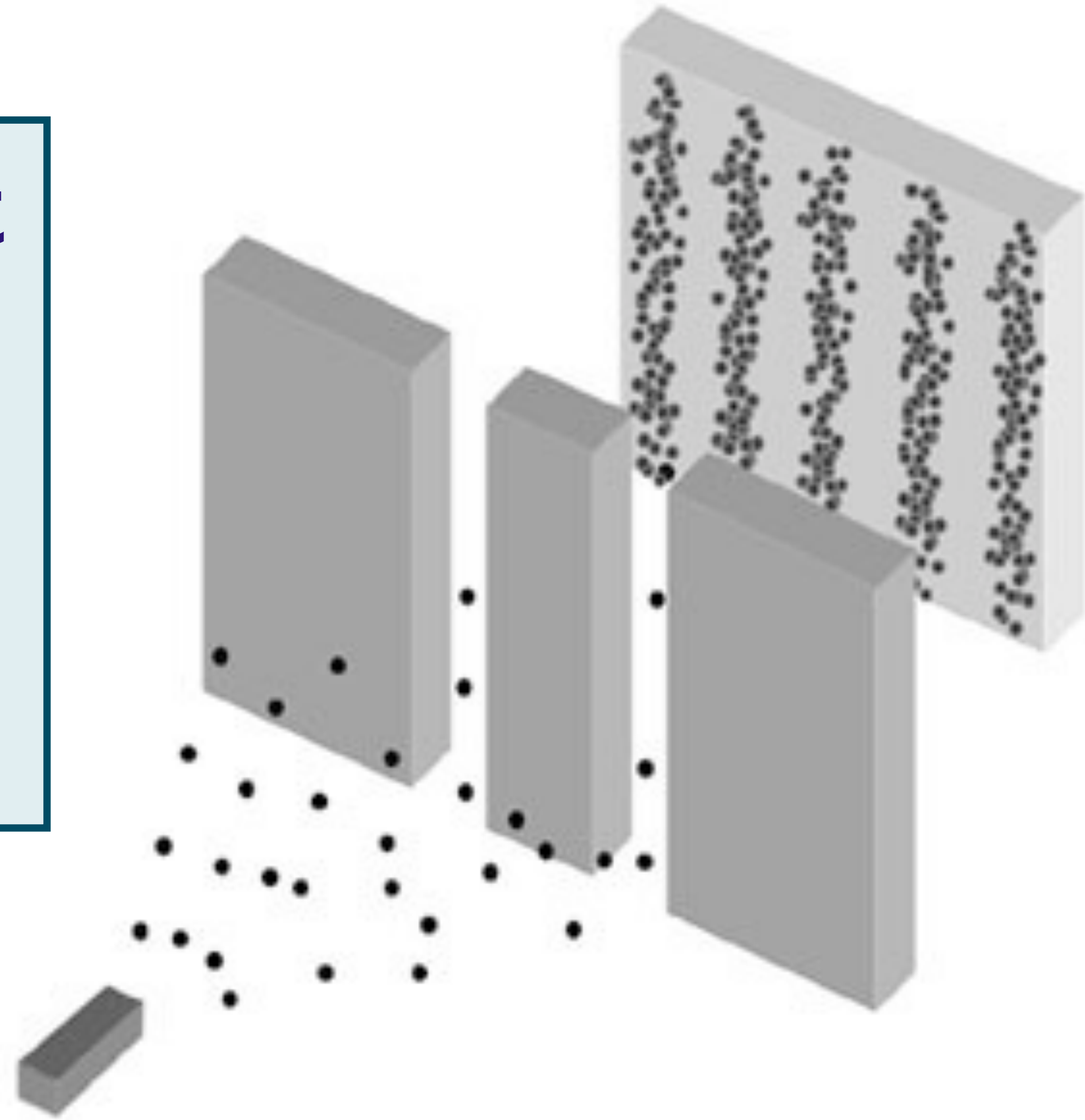


Interference of electrons

The double slit experiment

Which slit does an electron pass through ?



Interference of electrons

The double slit experiment

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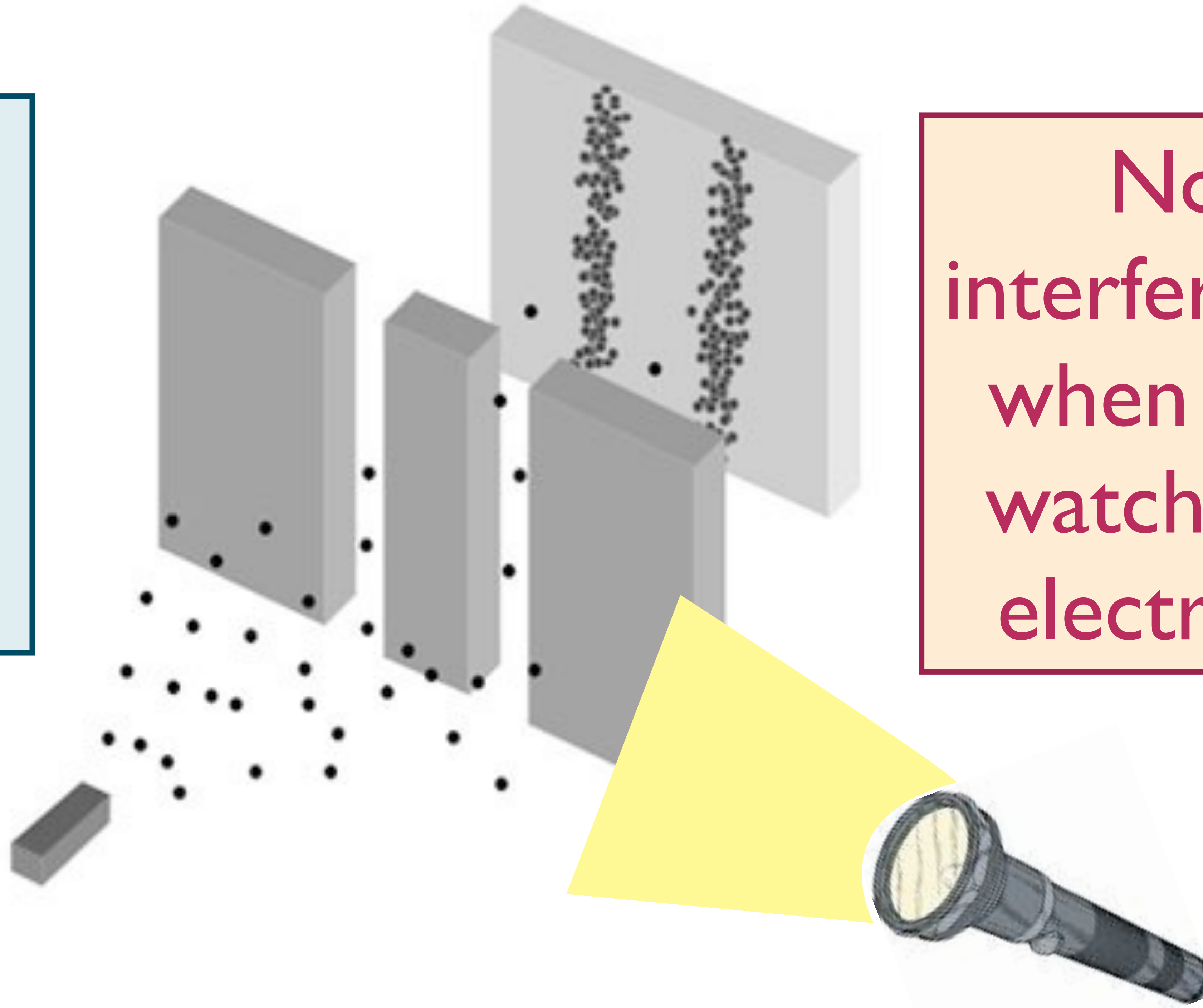
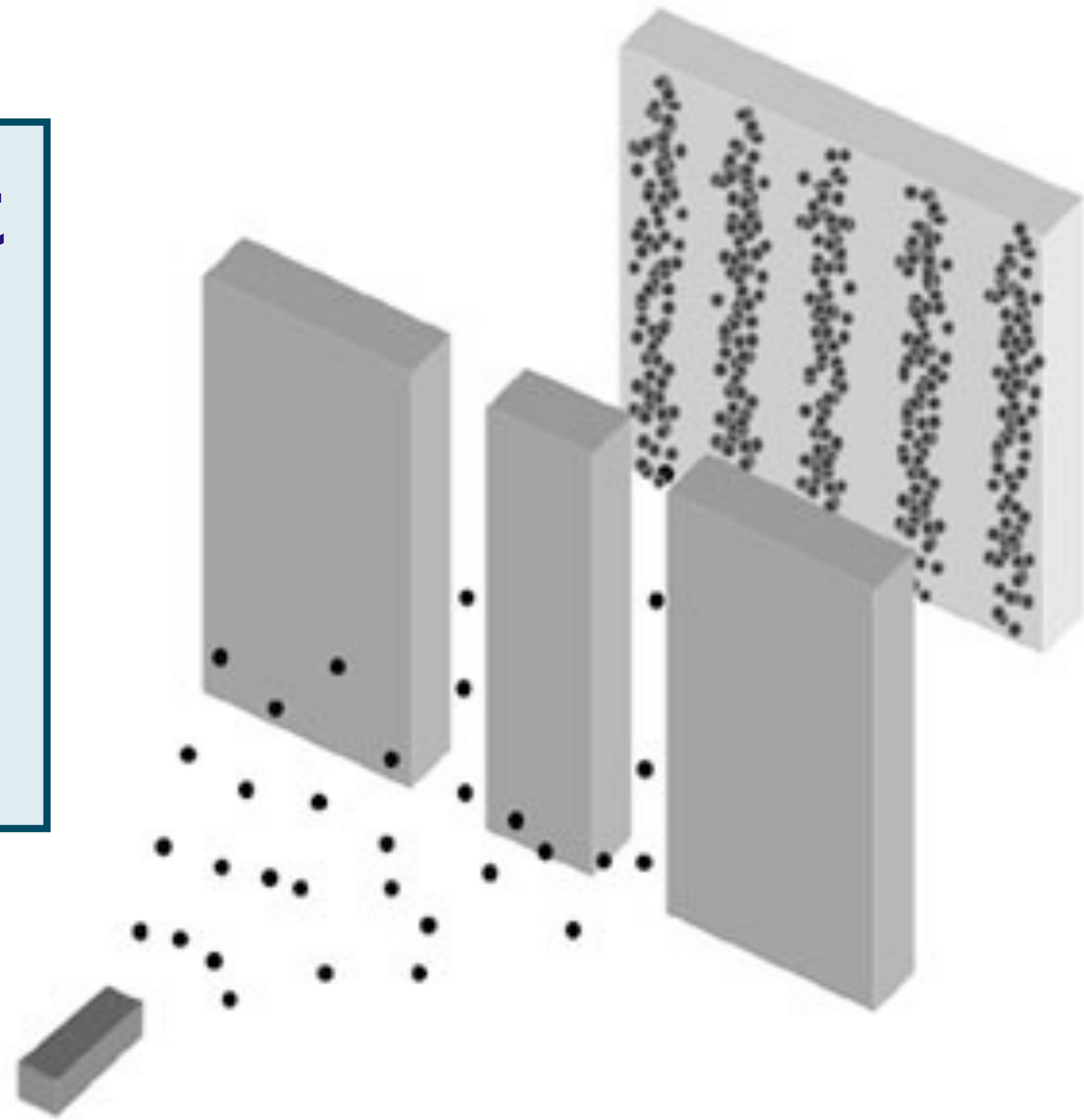


No interference when you watch the electrons

Interference of electrons

The double slit experiment

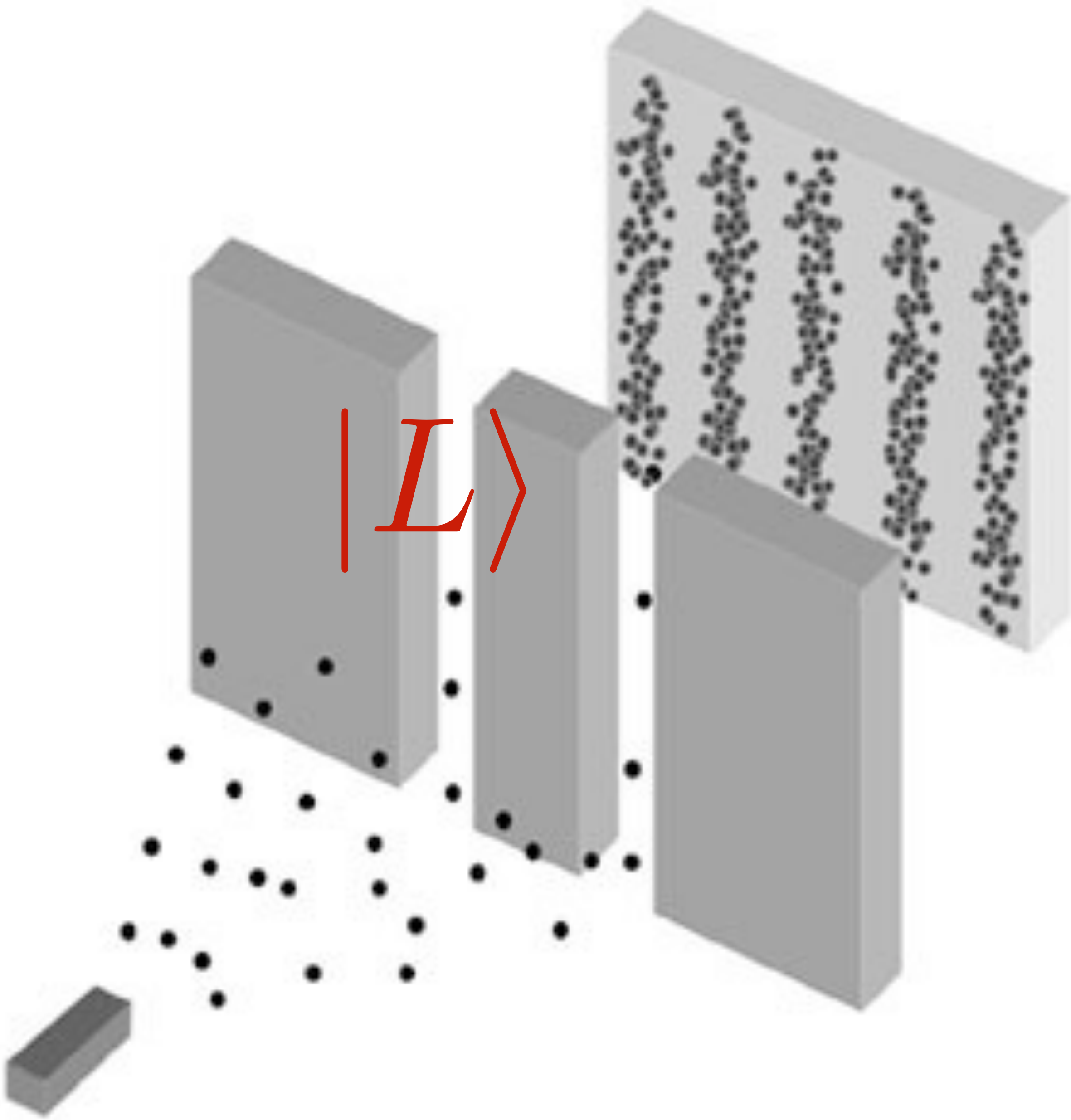
Which slit does an electron pass through ?



Each electron passes through both slits !

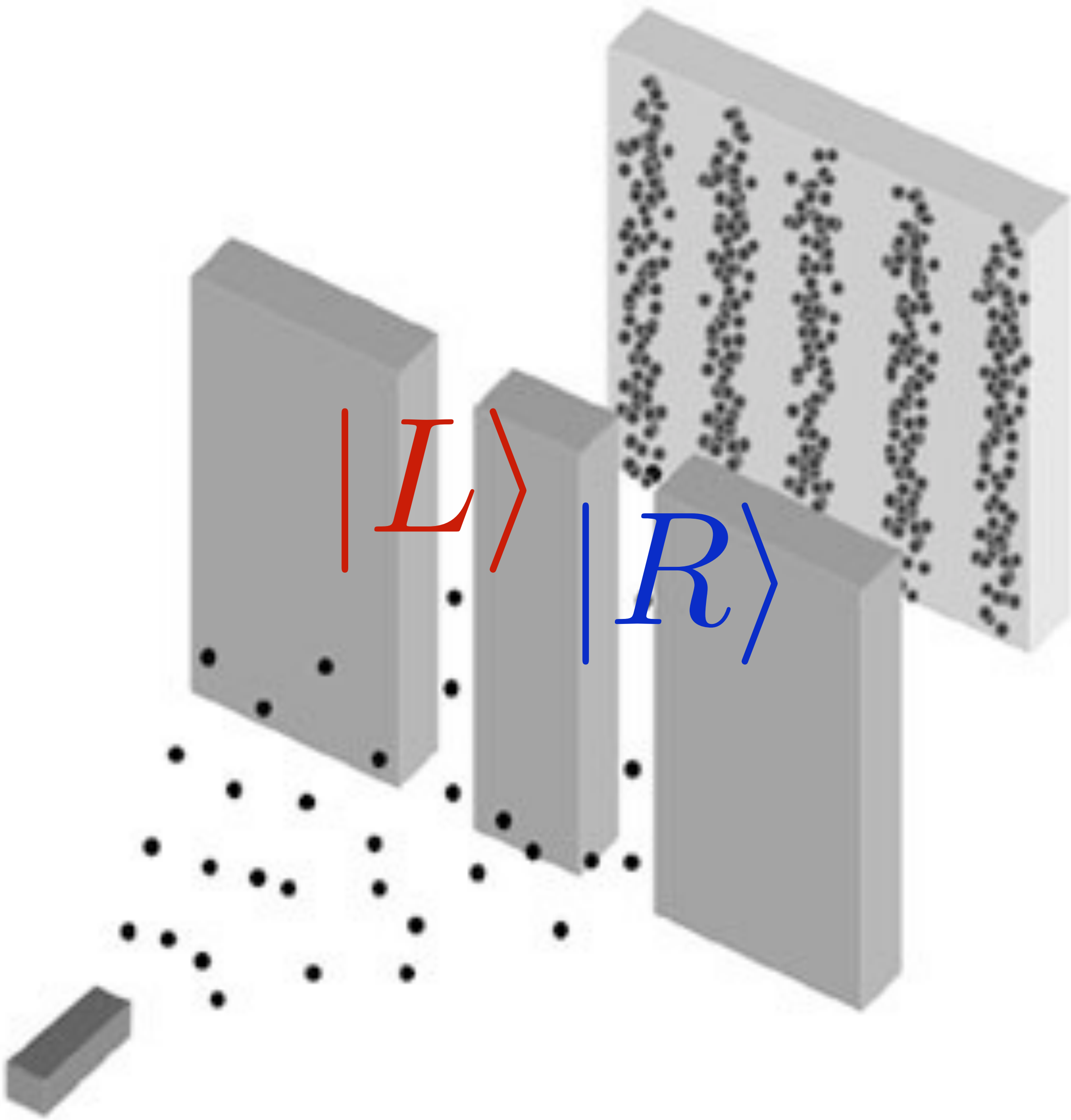
Interference of electrons

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

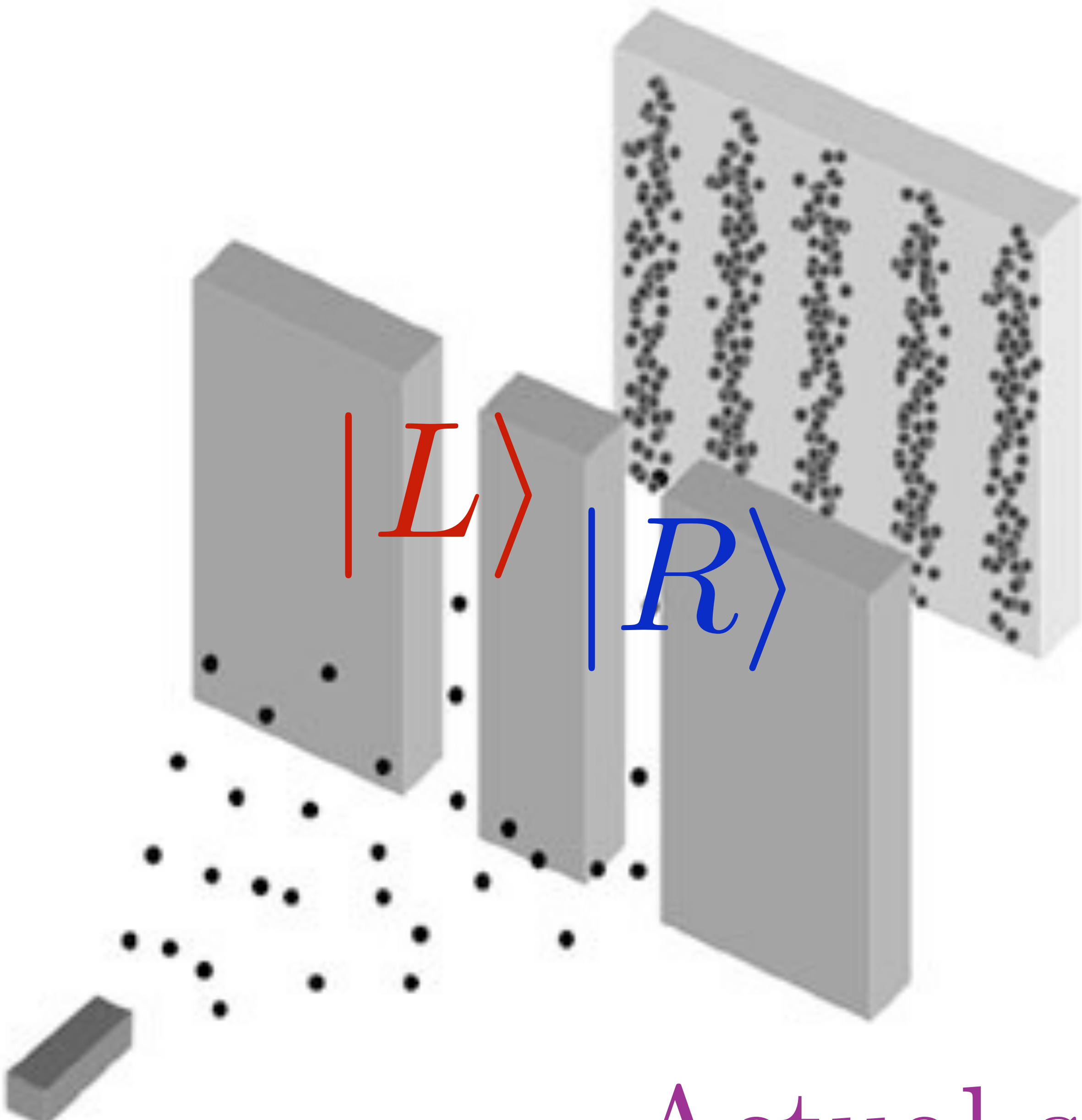
The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of the electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

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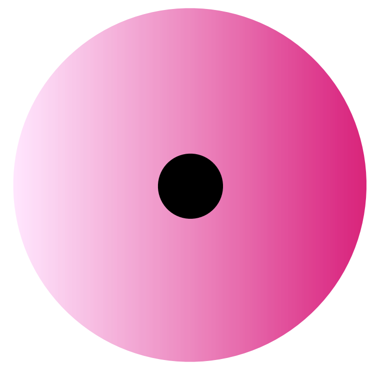
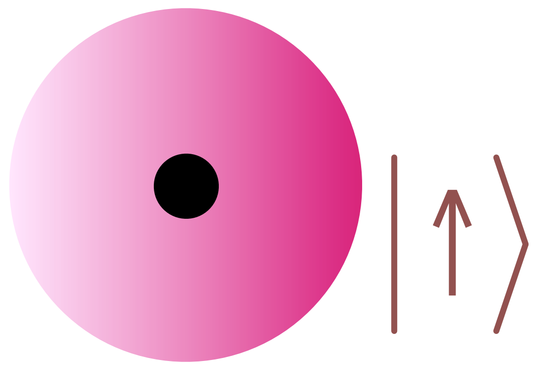
Hydrogen atom:



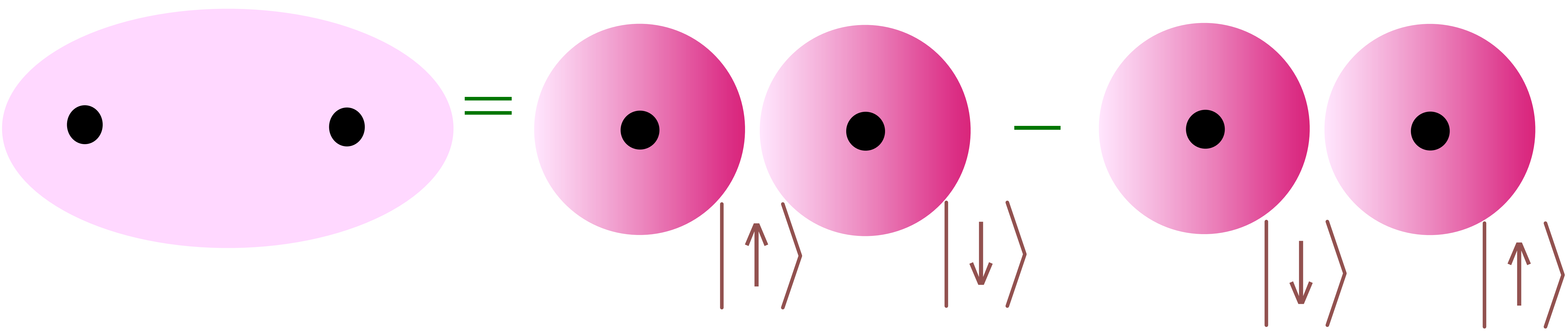
$|\uparrow\rangle$

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

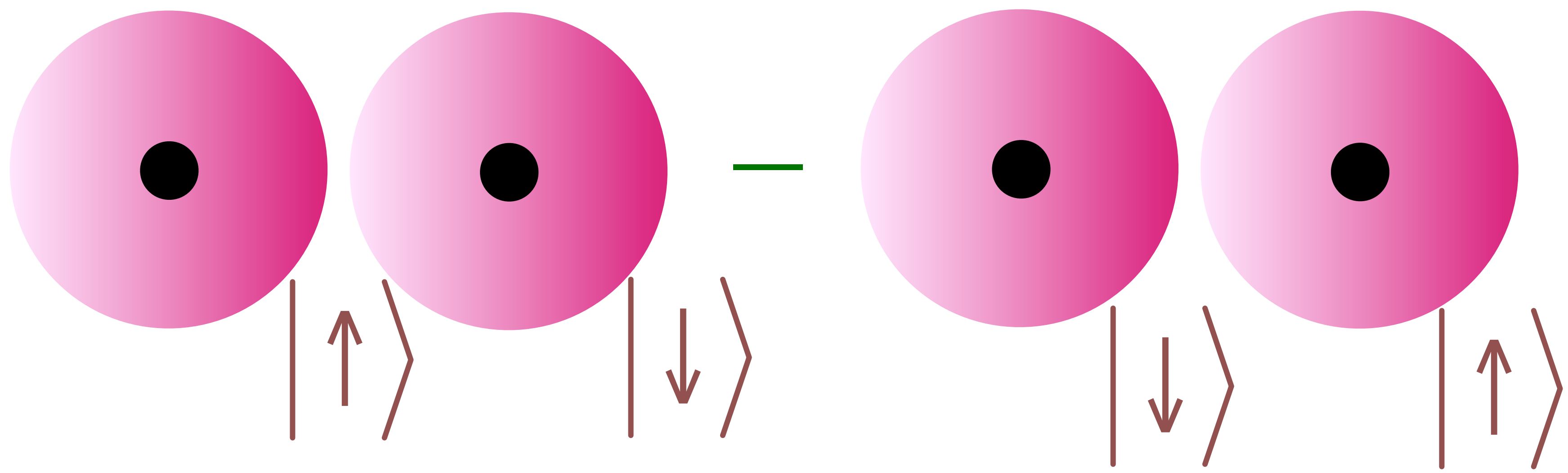


Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



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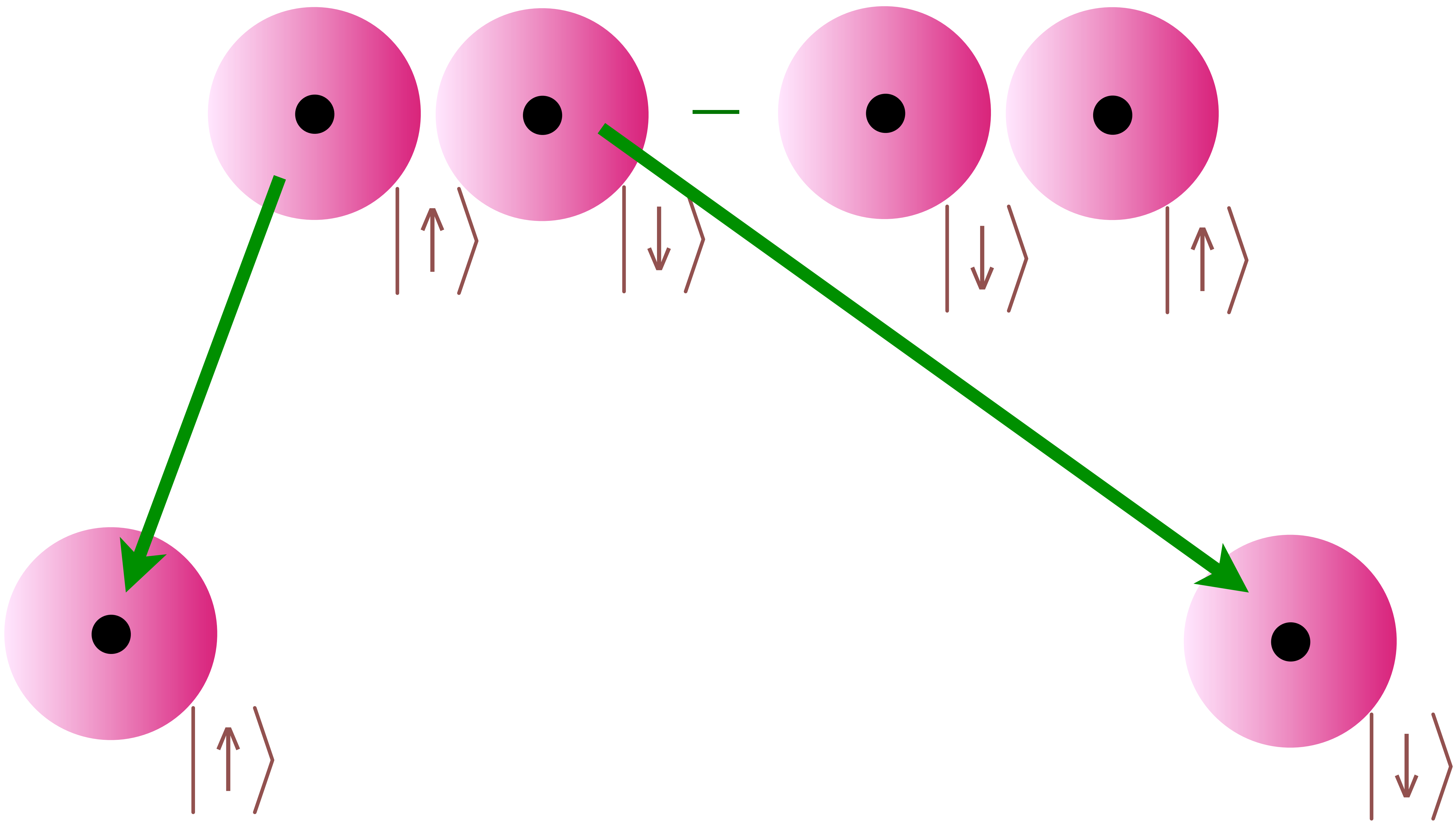

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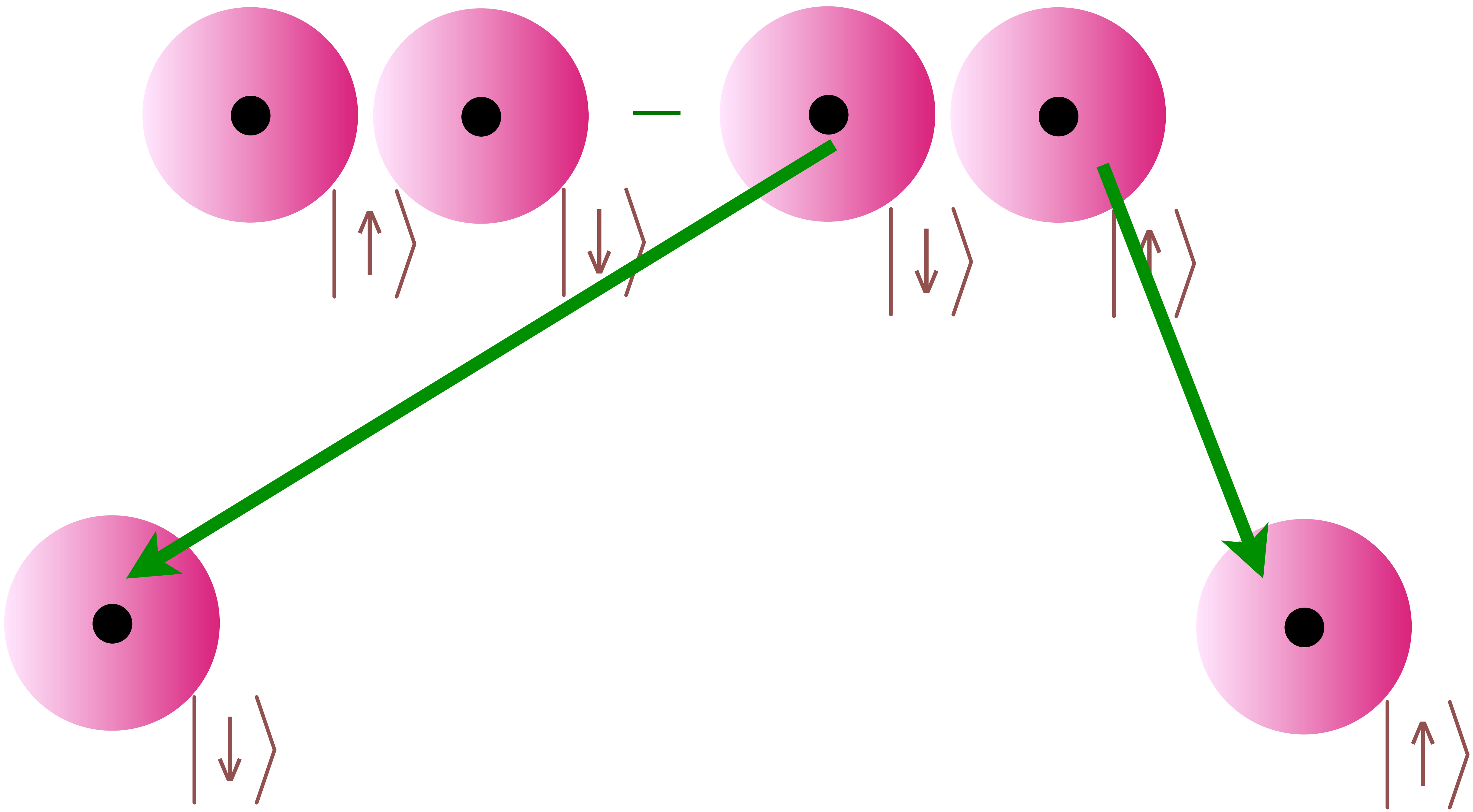

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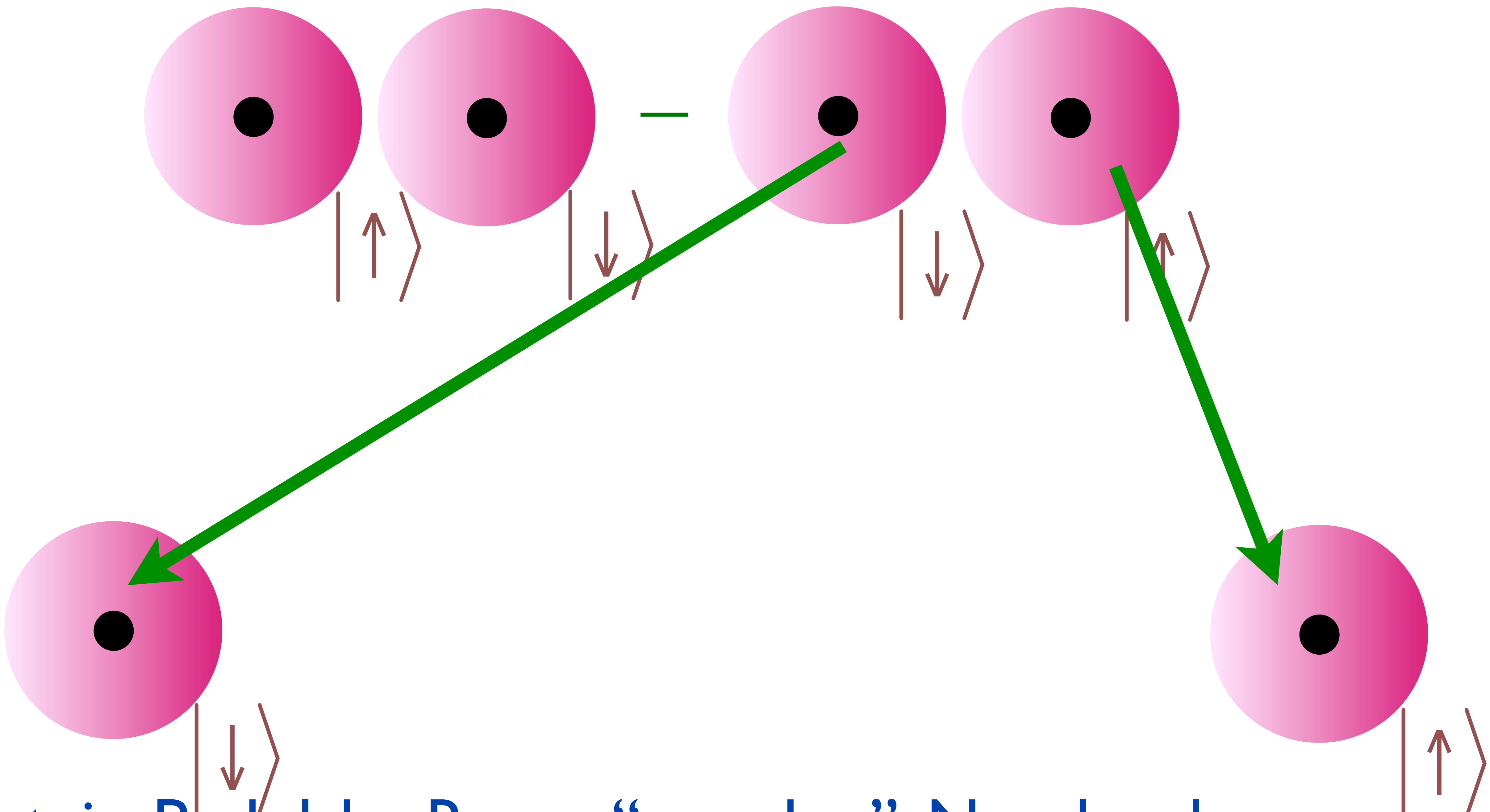

Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

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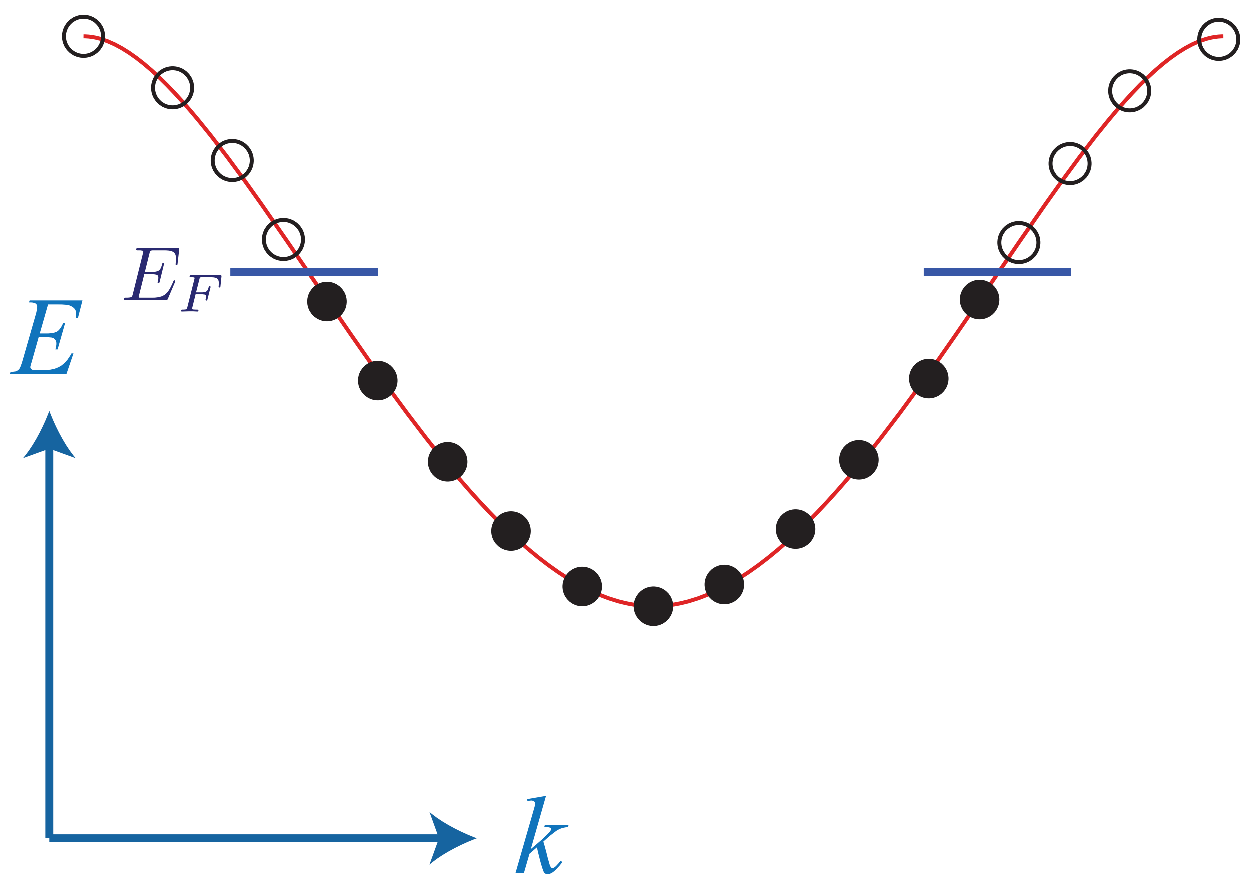
Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

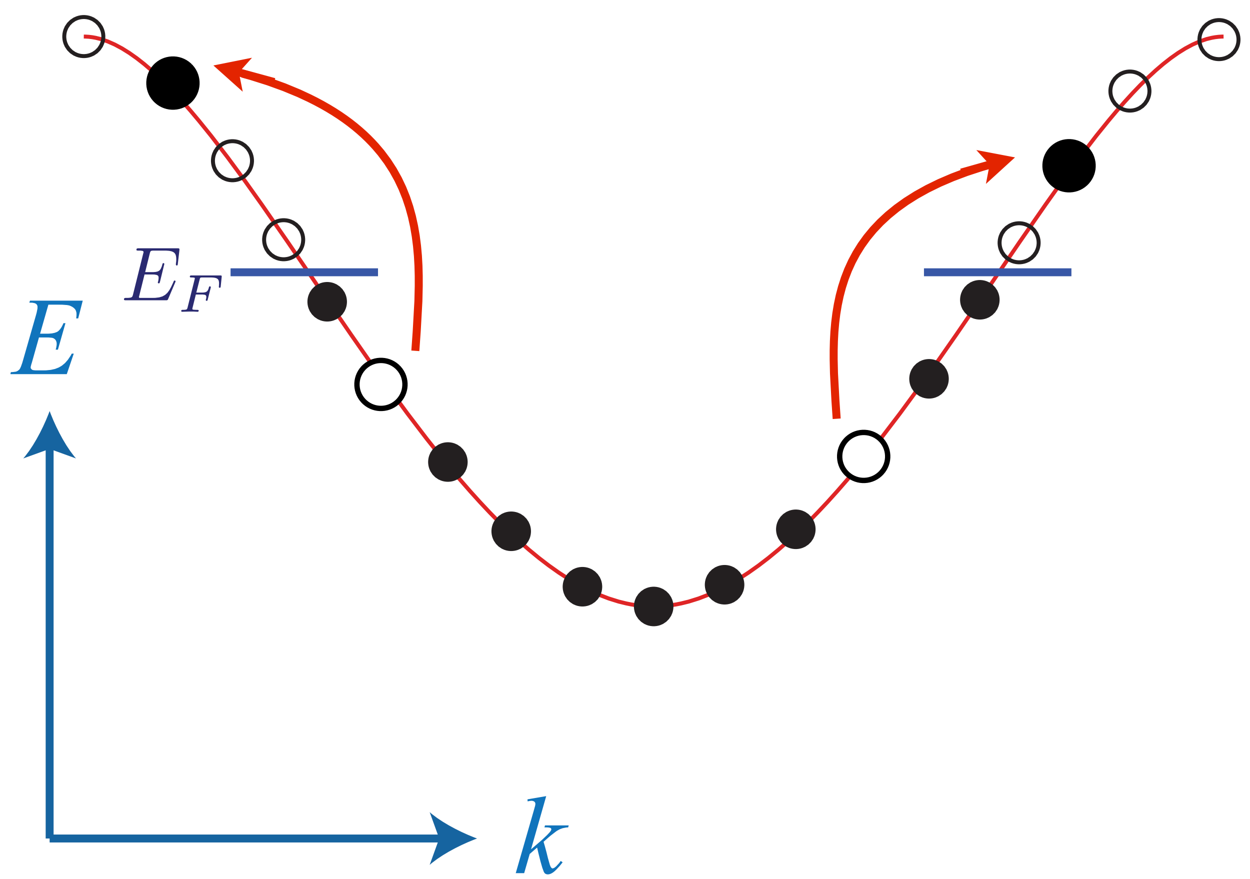
Metals



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



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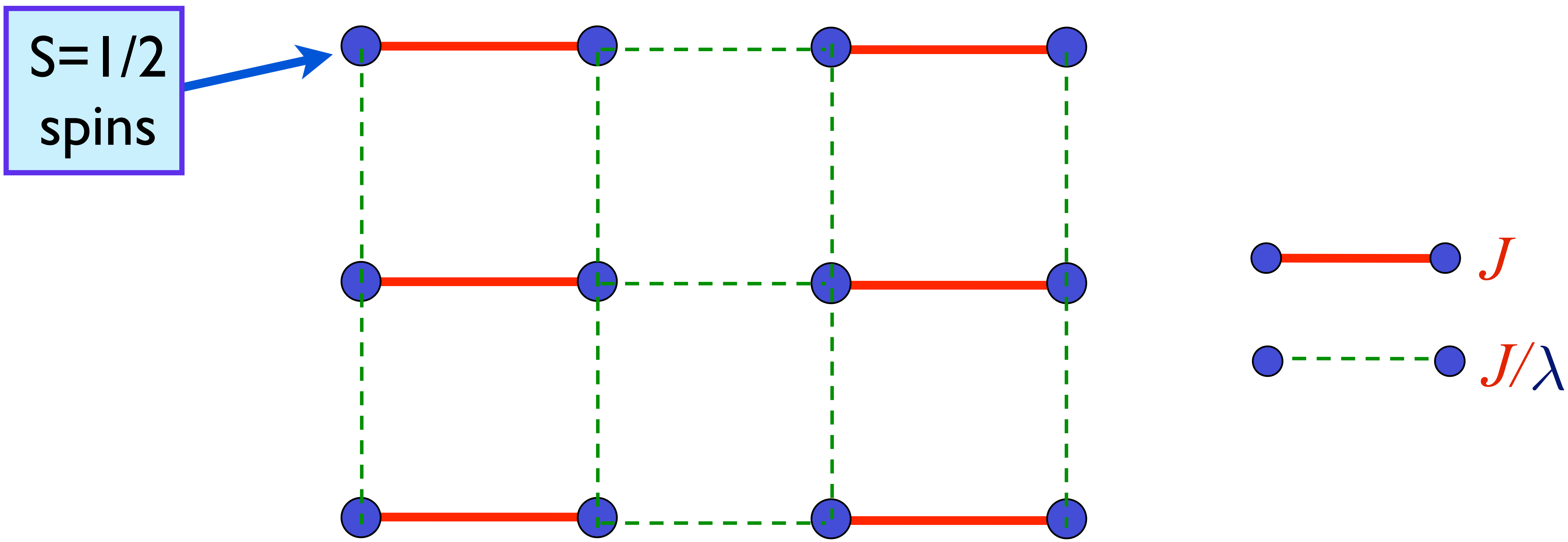
Spinning electrons localized on a square lattice

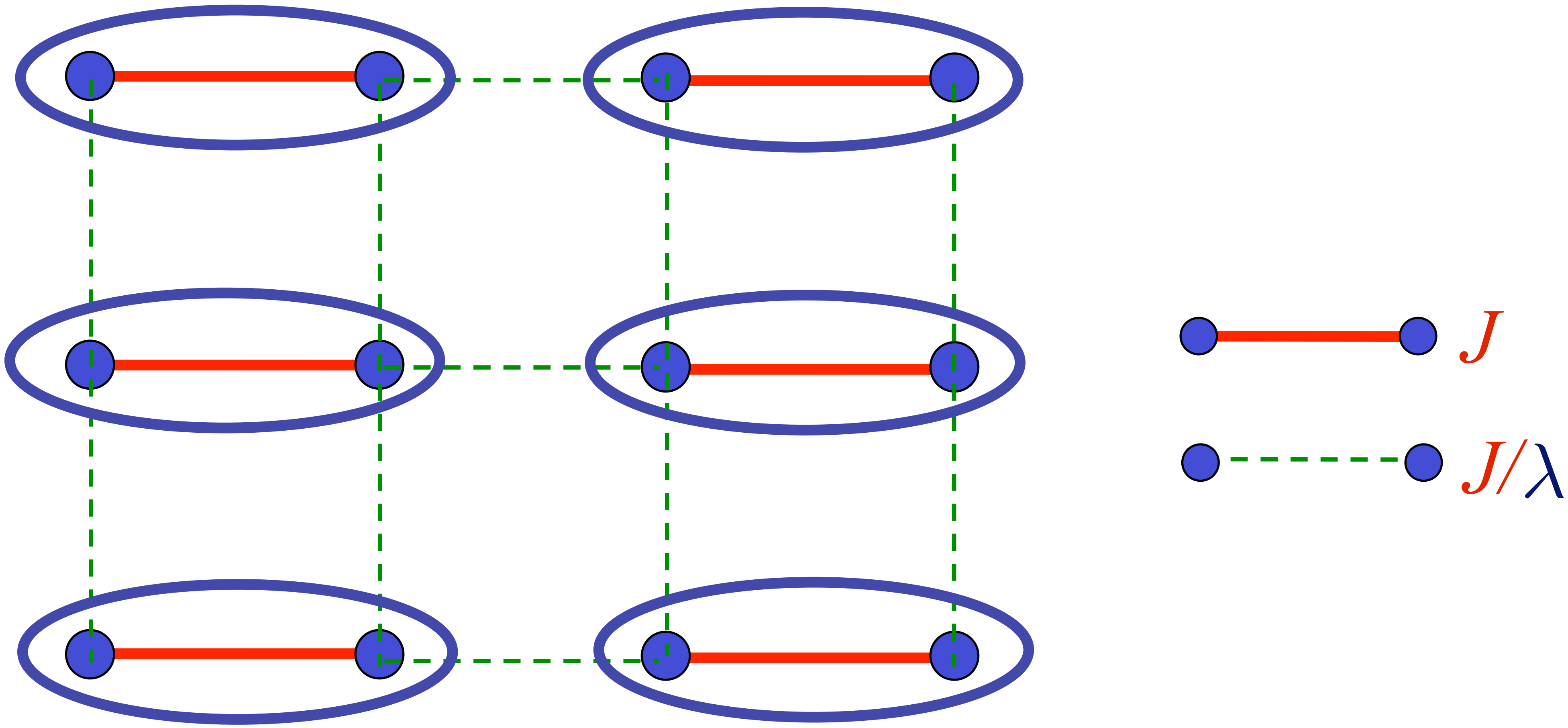
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

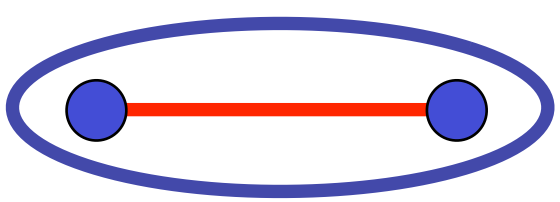


Examine ground state as a function of λ

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$





$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

Spinning electrons localized on a square lattice

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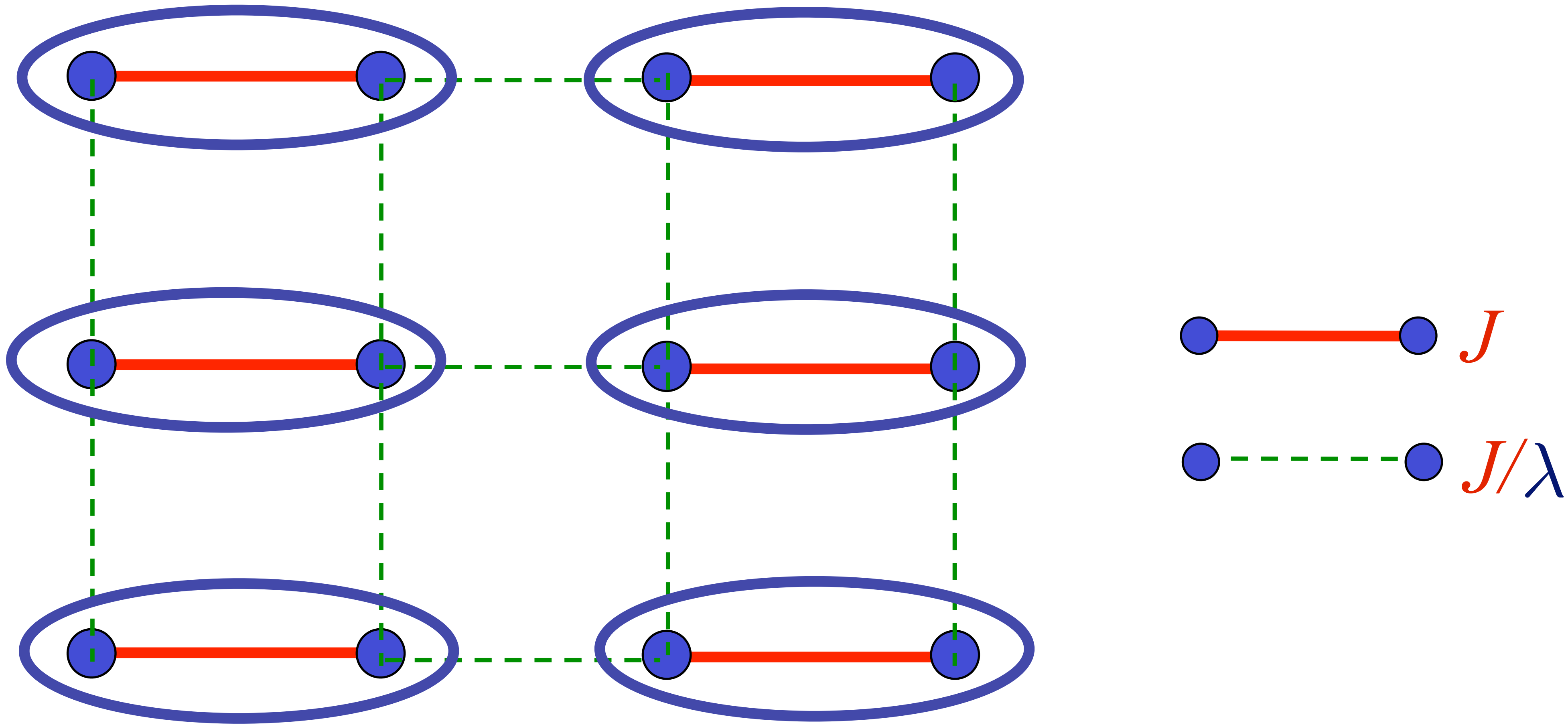
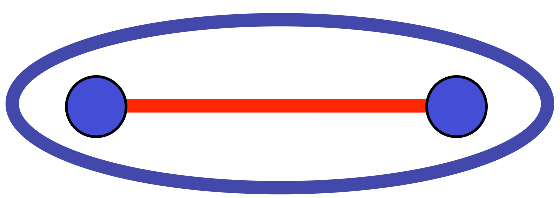


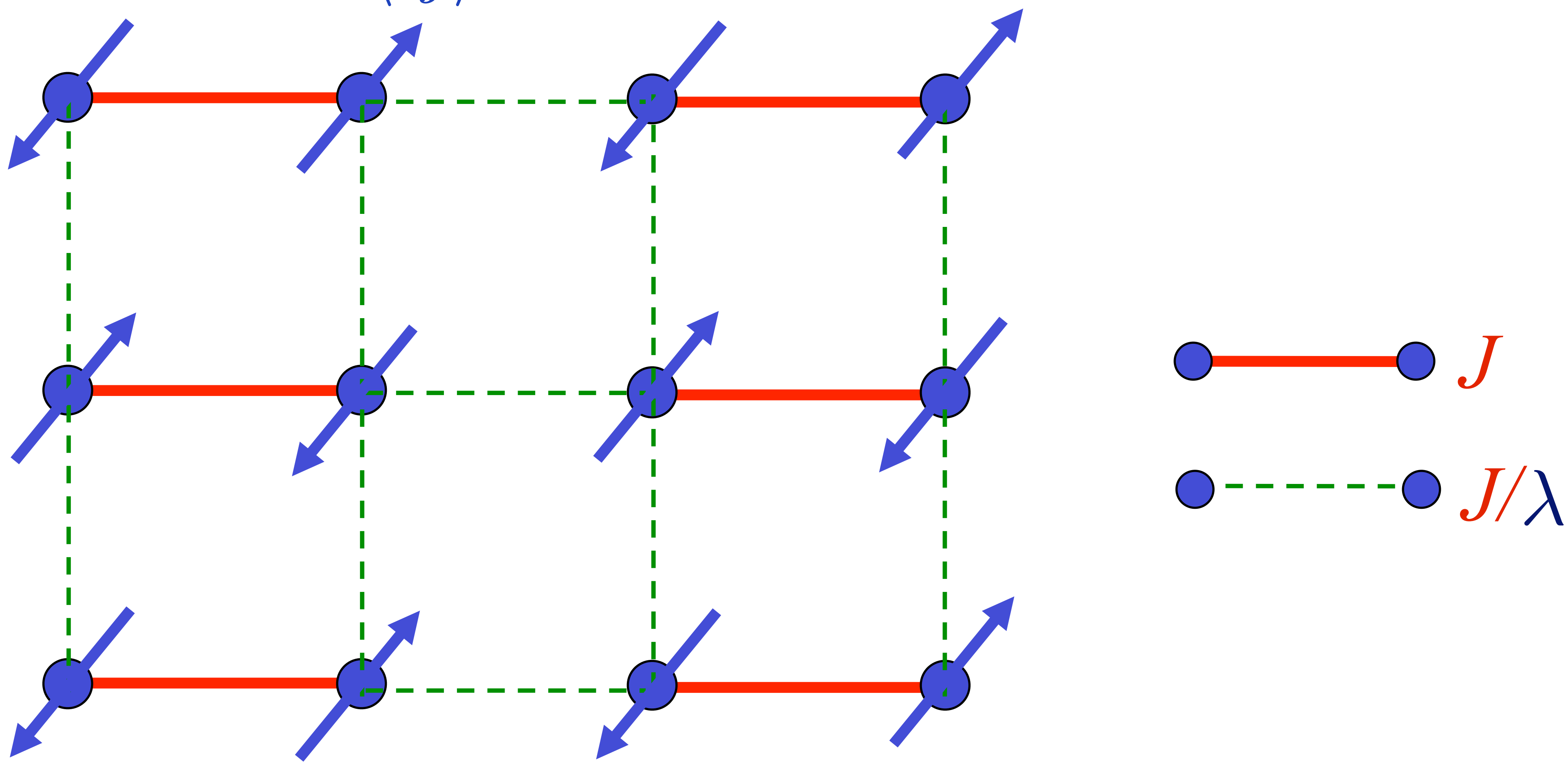


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

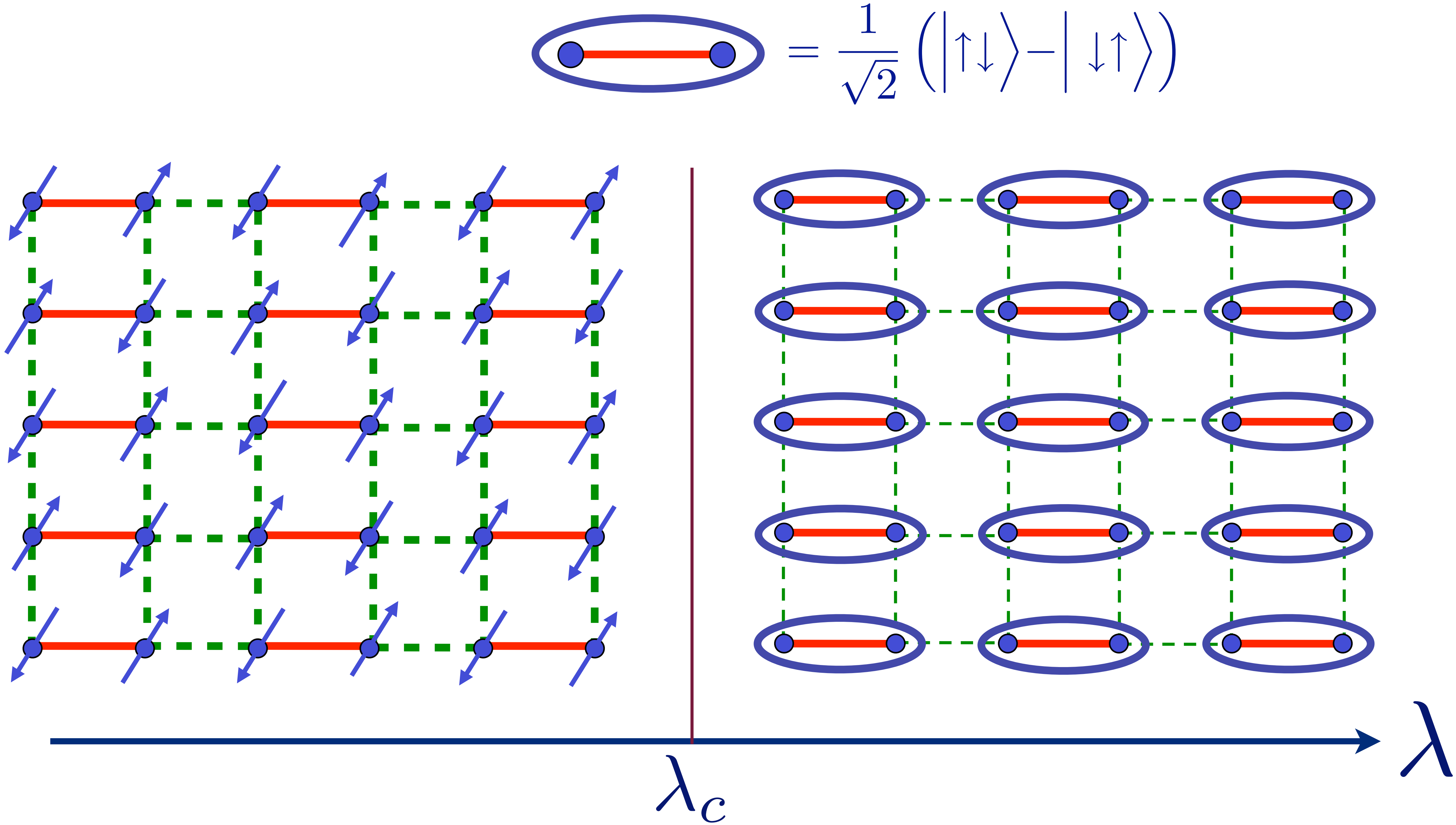
Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

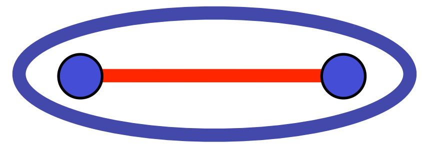
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ



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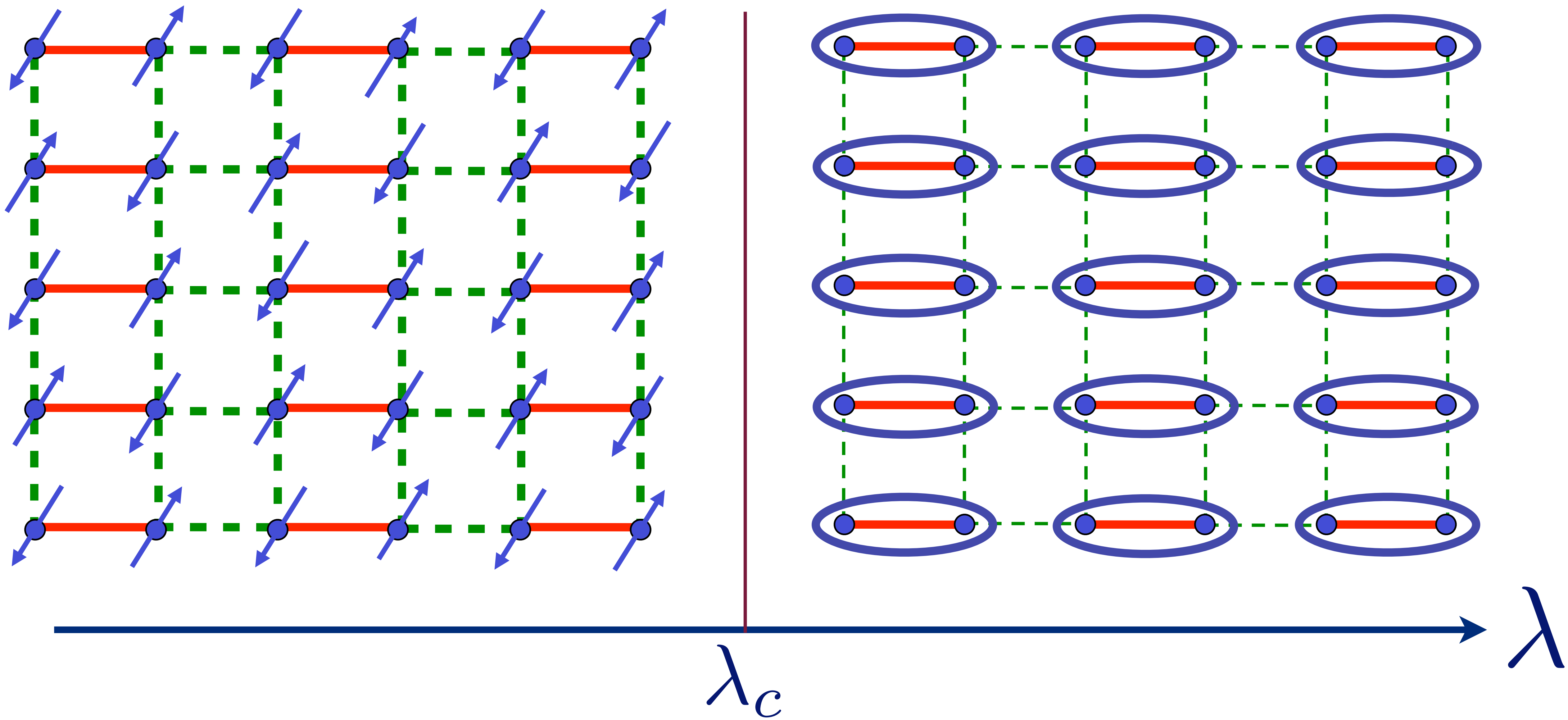
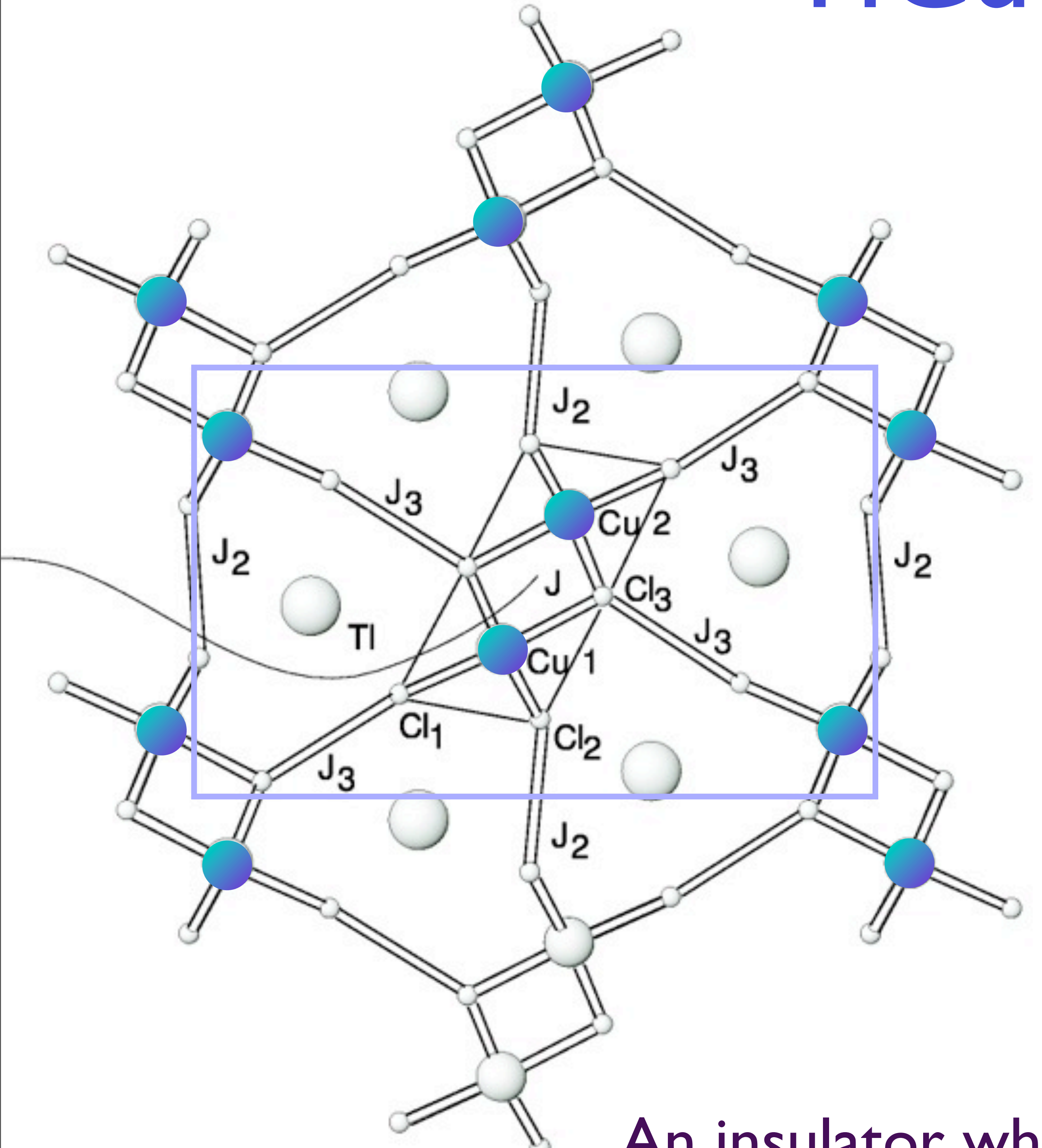
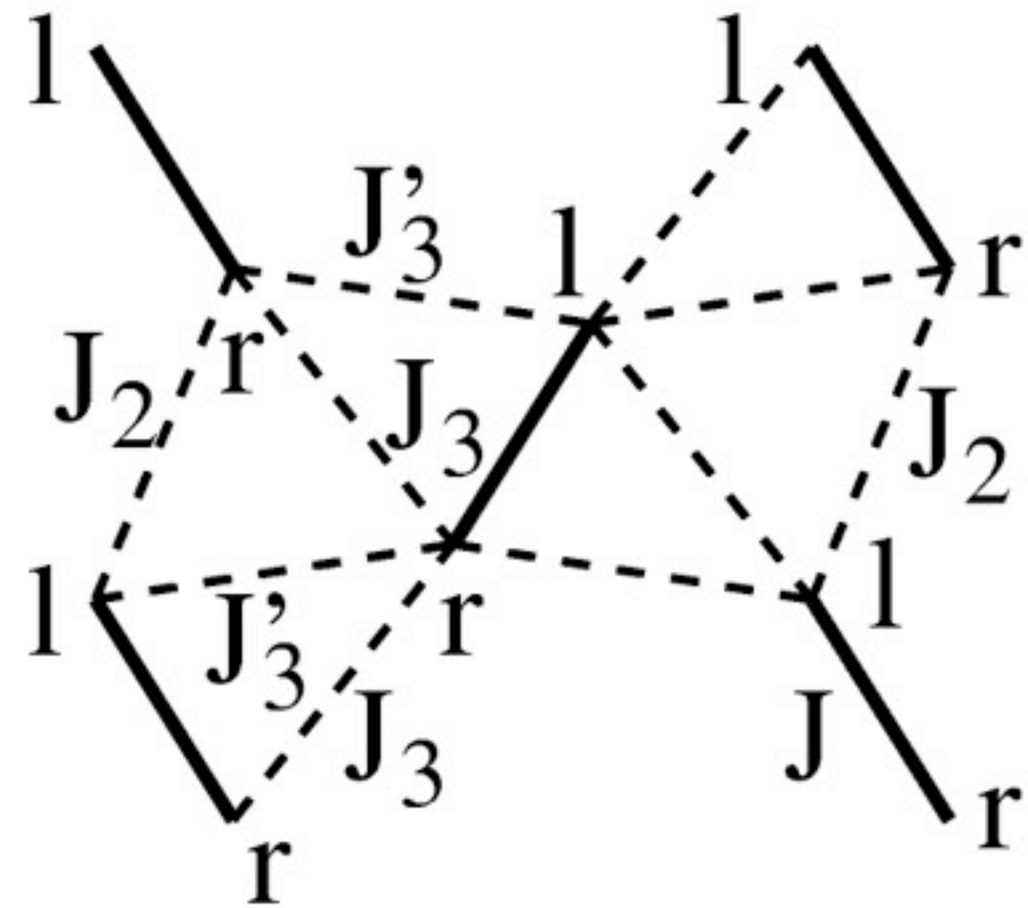


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

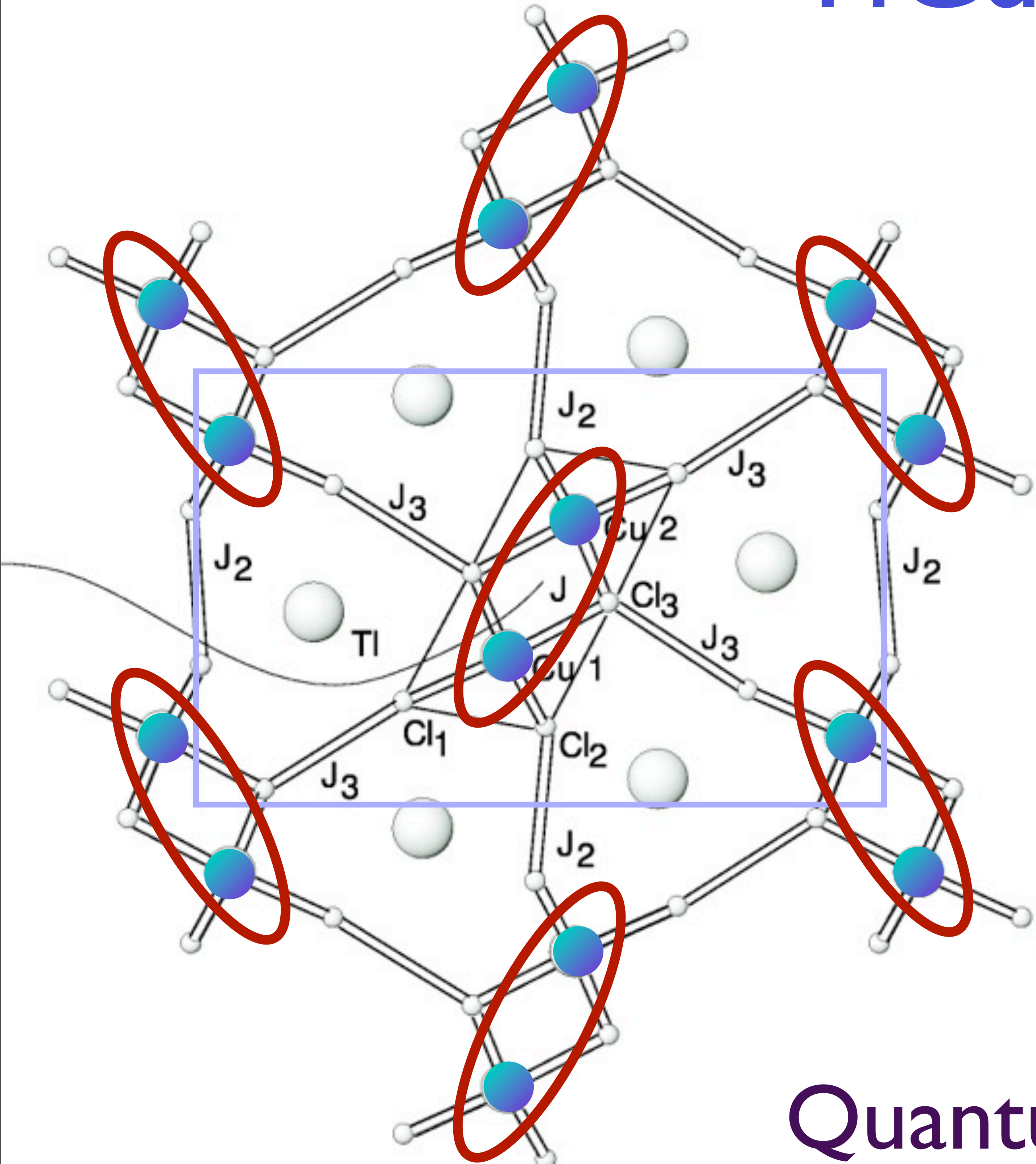
$TlCuCl_3$

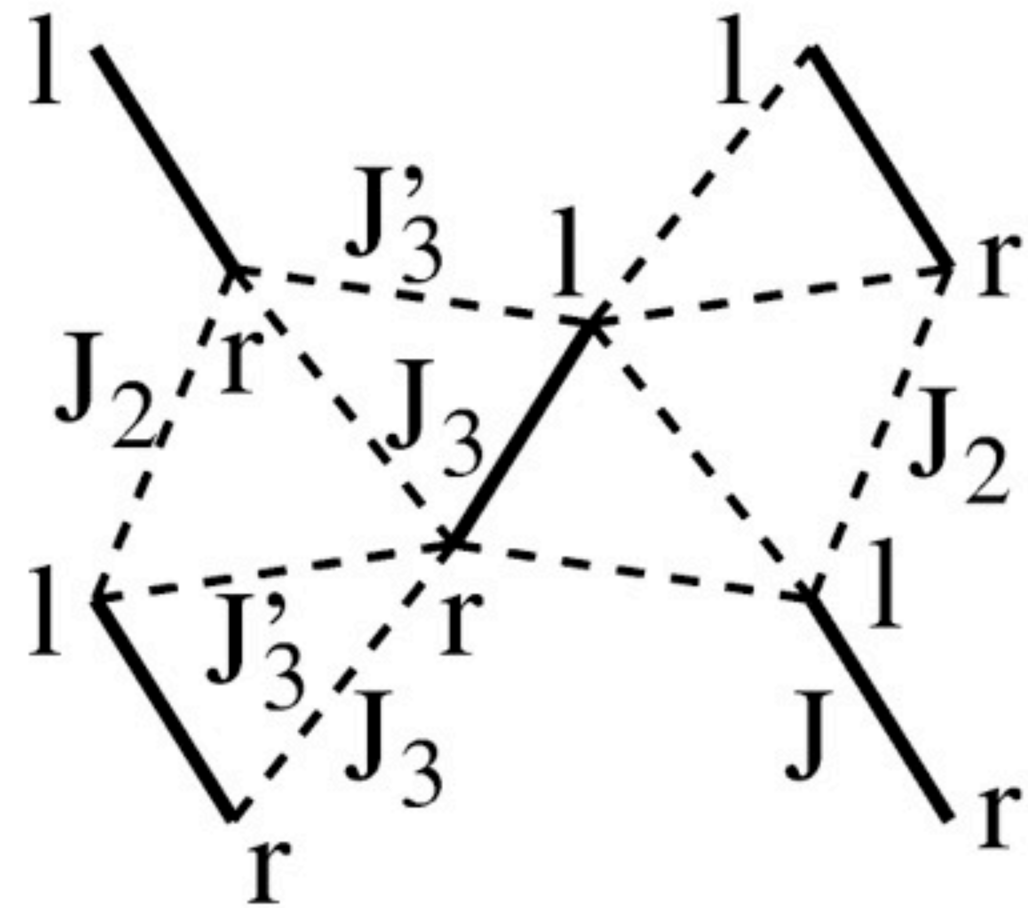




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

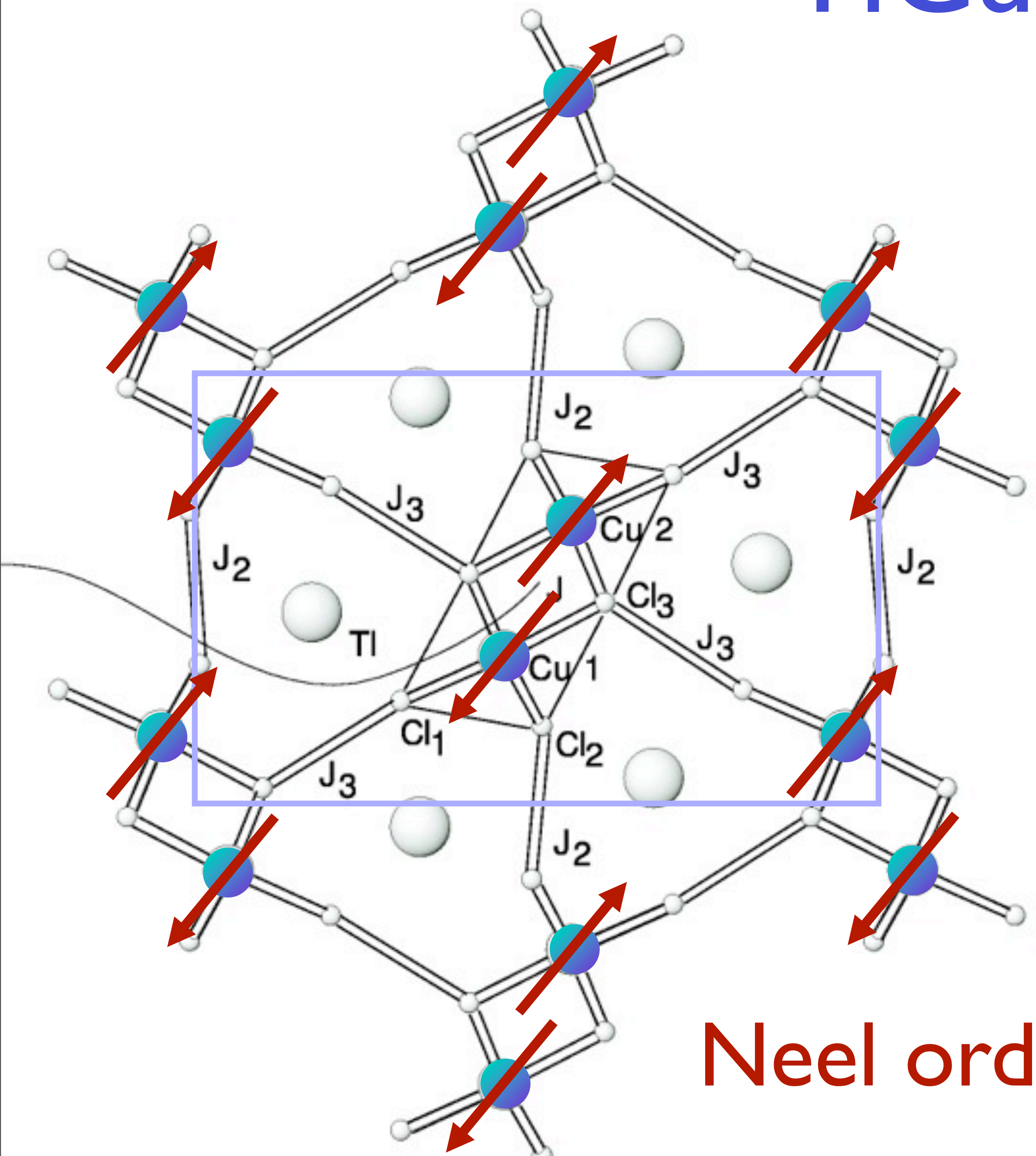
$TlCuCl_3$

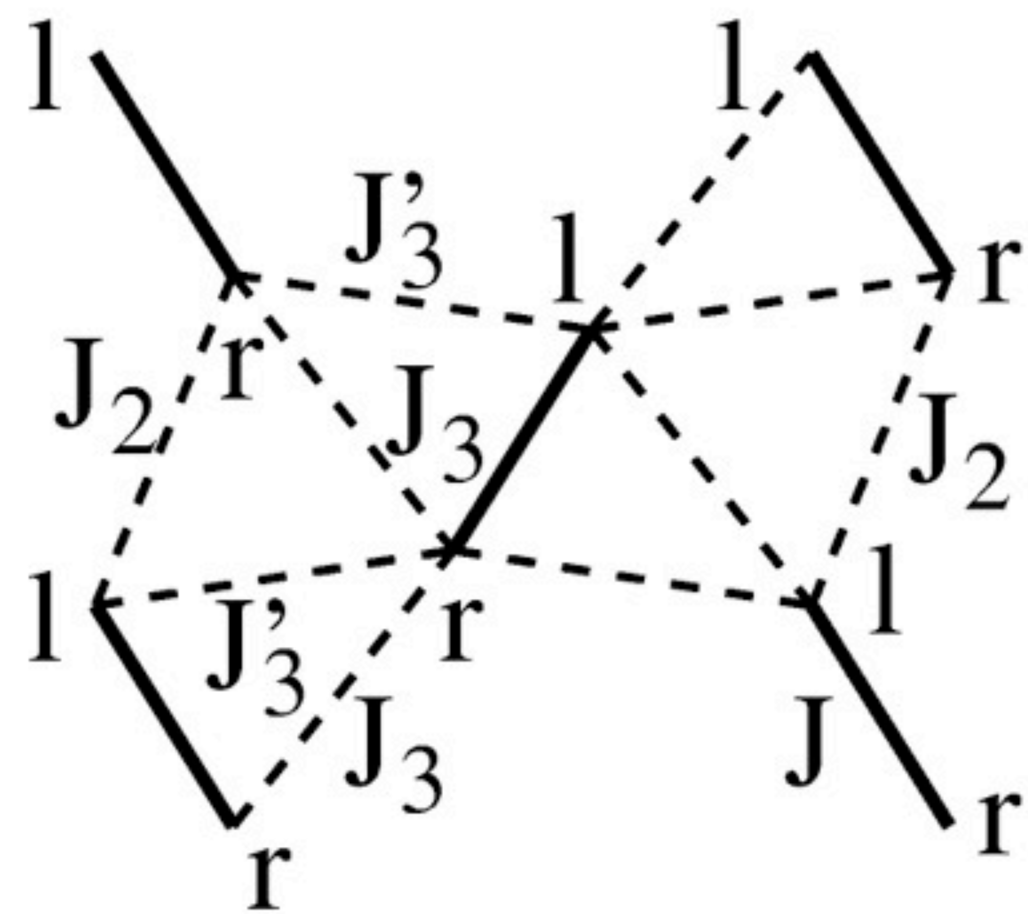




Quantum paramagnet at ambient pressure

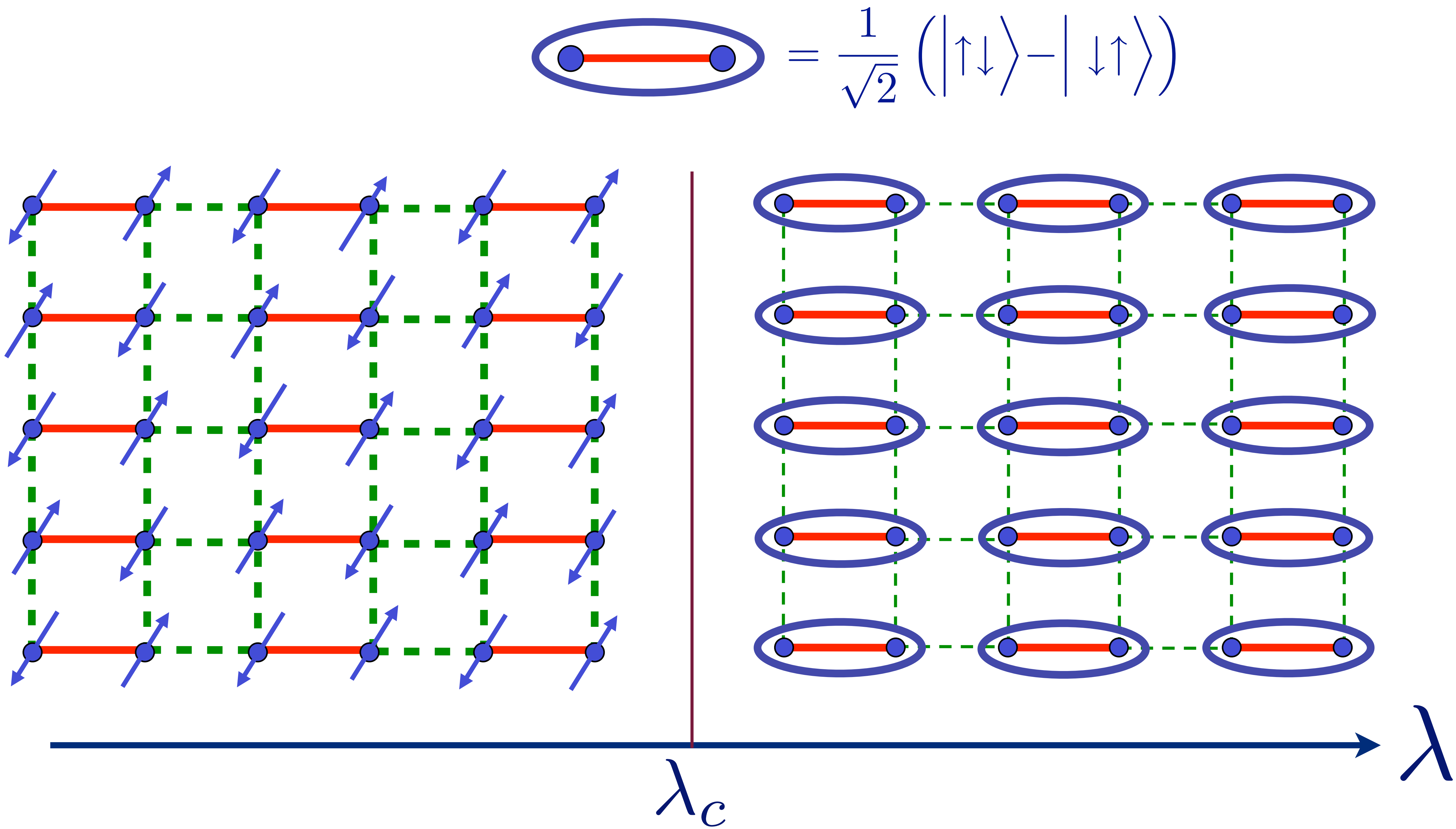
$TlCuCl_3$





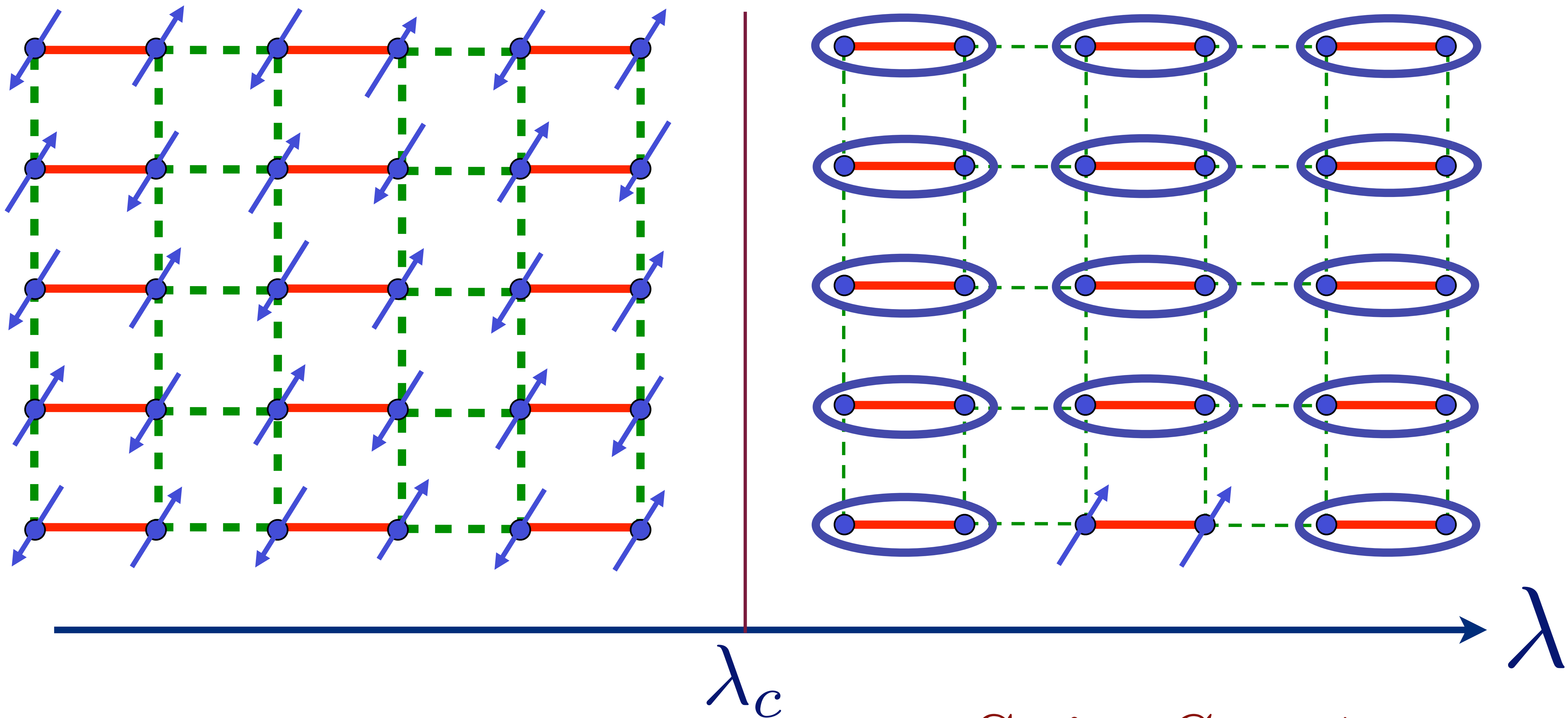
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).



$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ
λ_c

Quasiparticles in the paramagnetic phase



Spin $S = 1$
"triplon"

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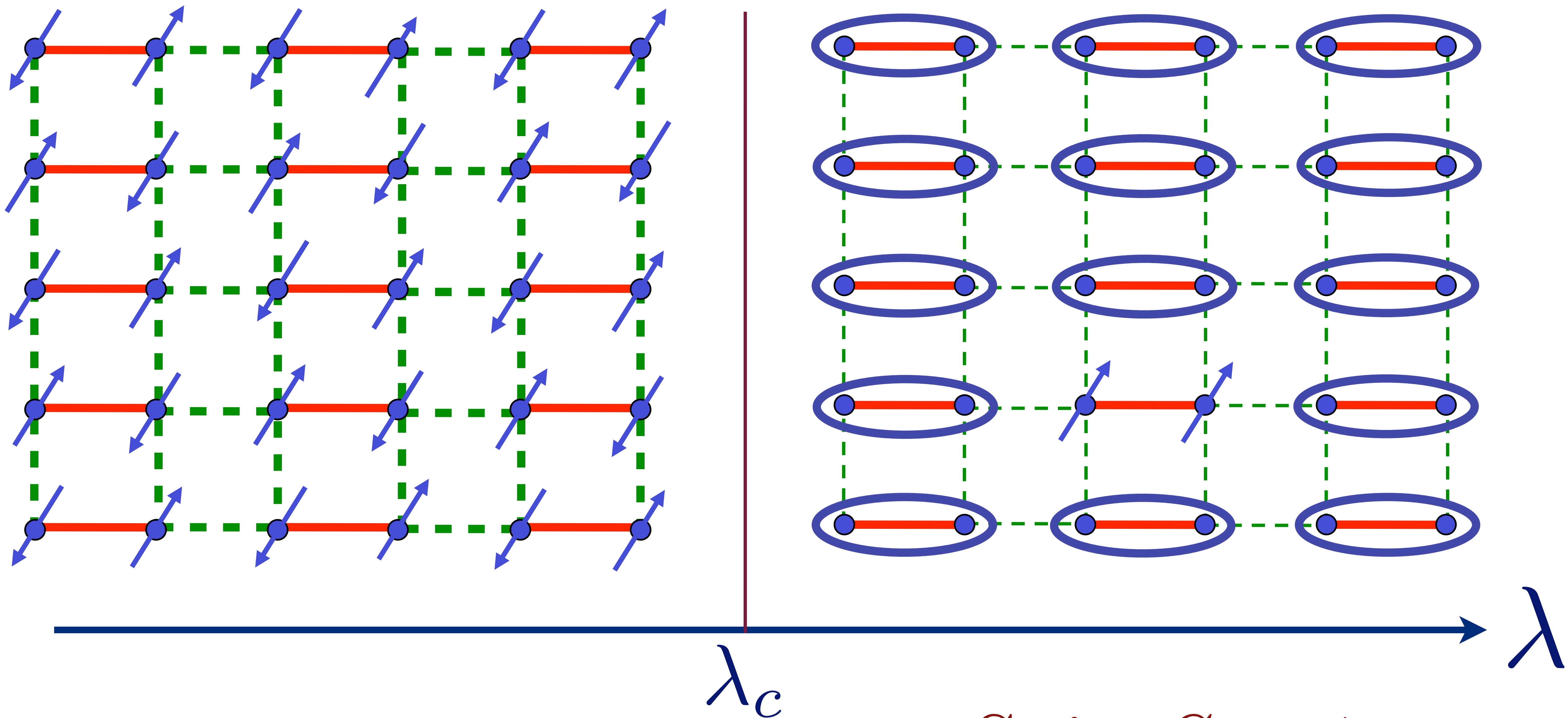

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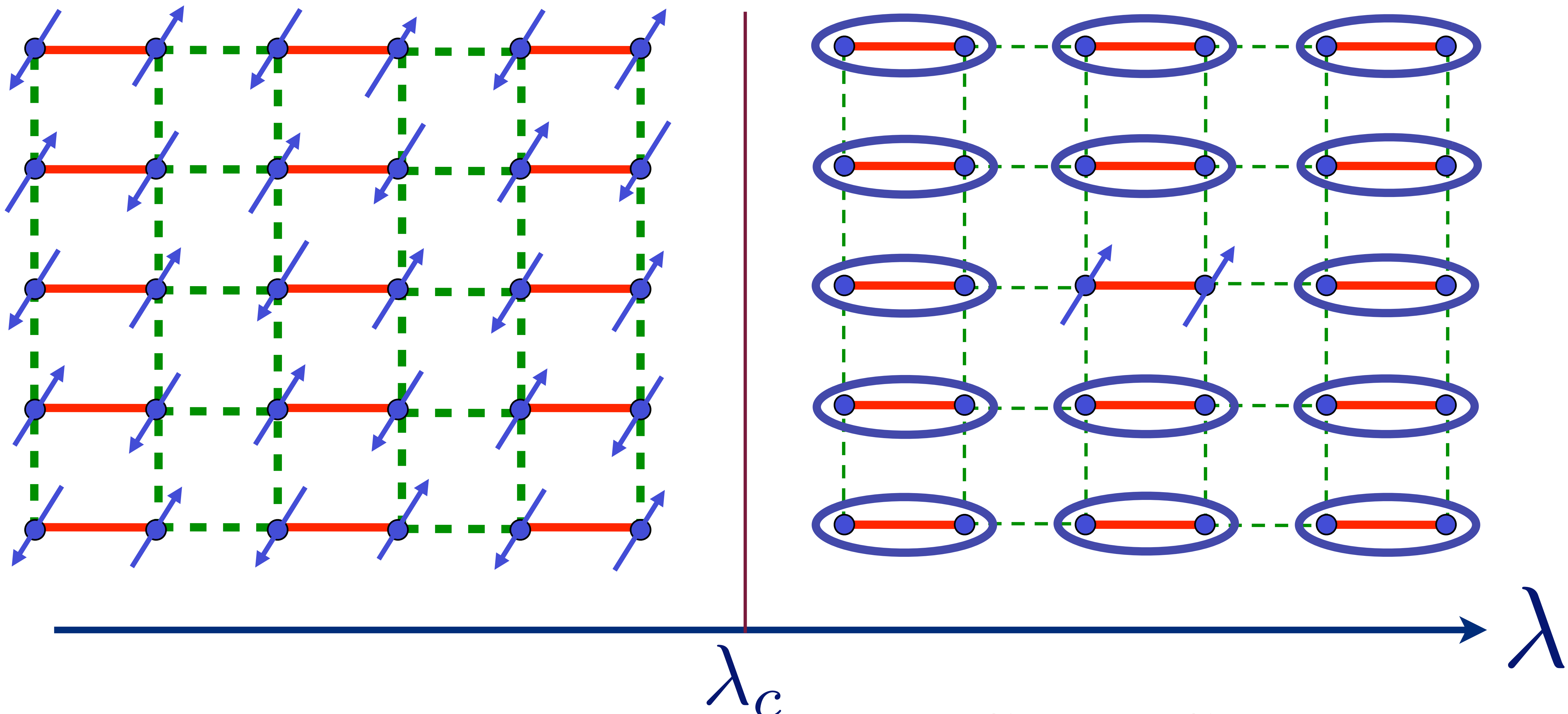

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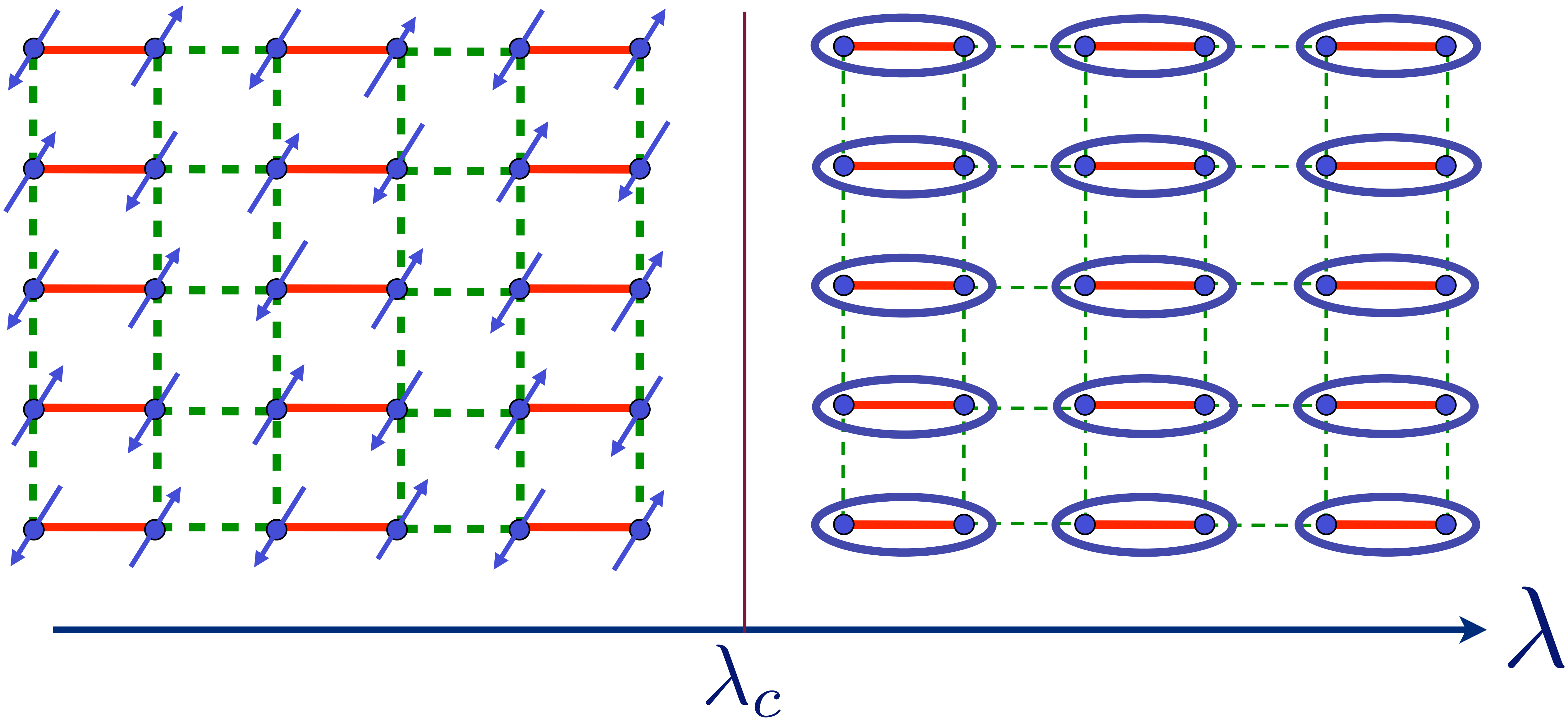
Quasiparticles in the Néel phase



Spin waves

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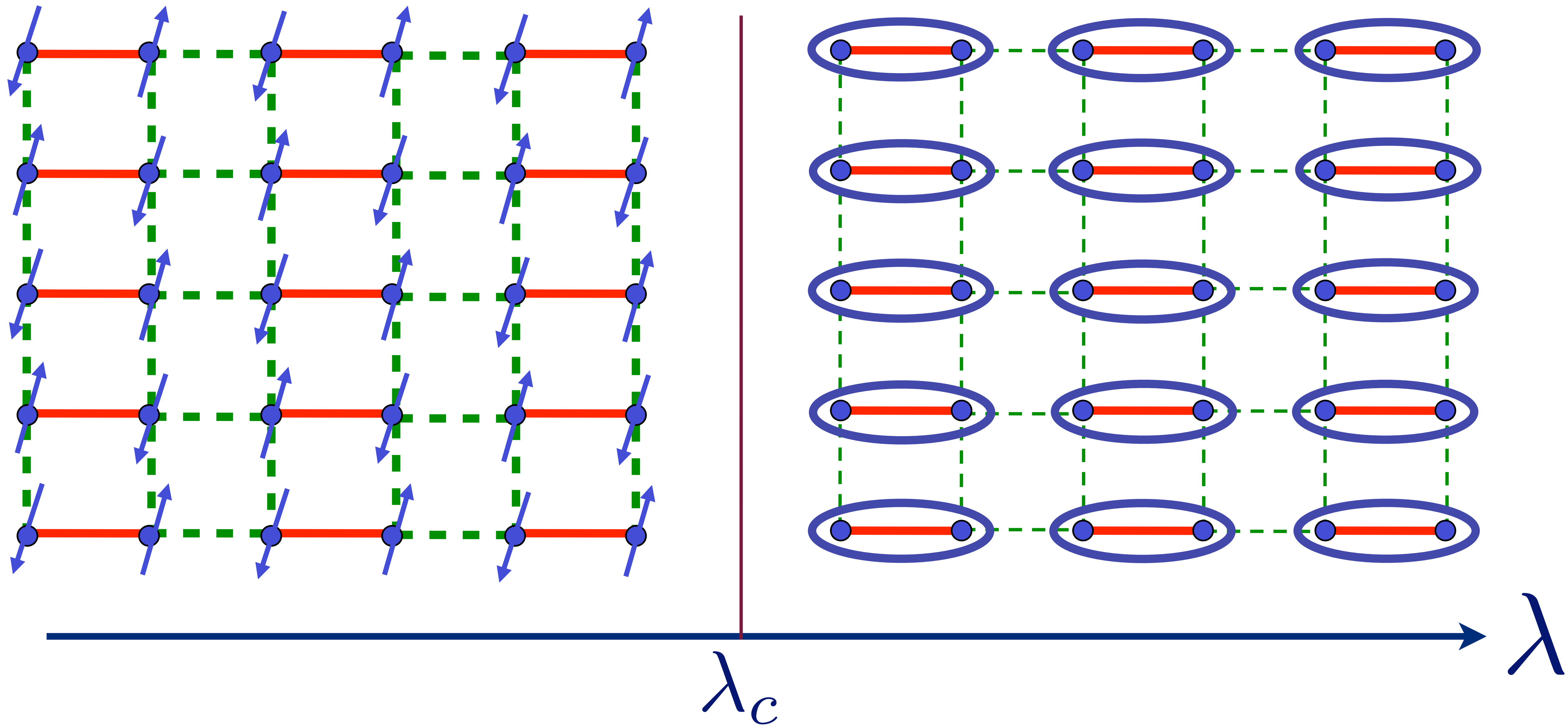

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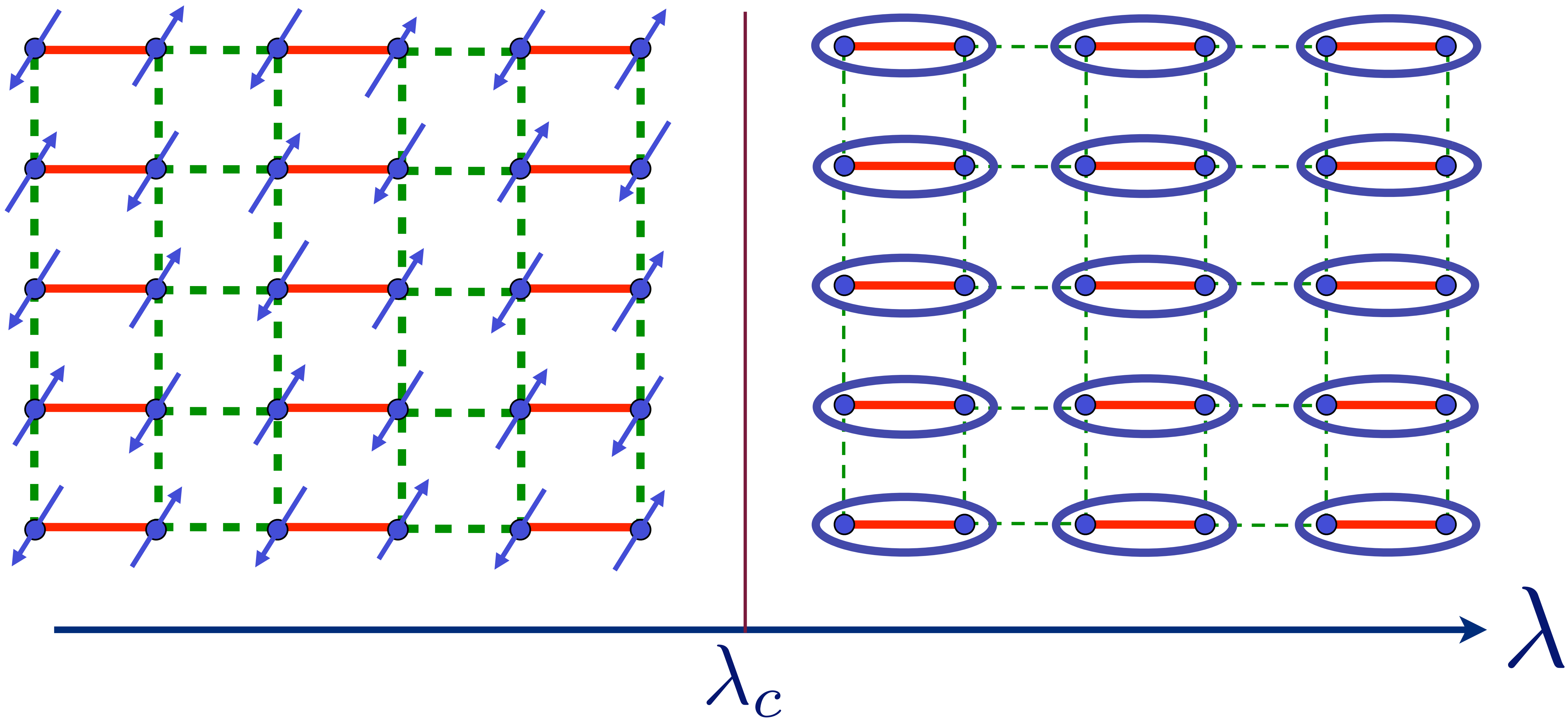
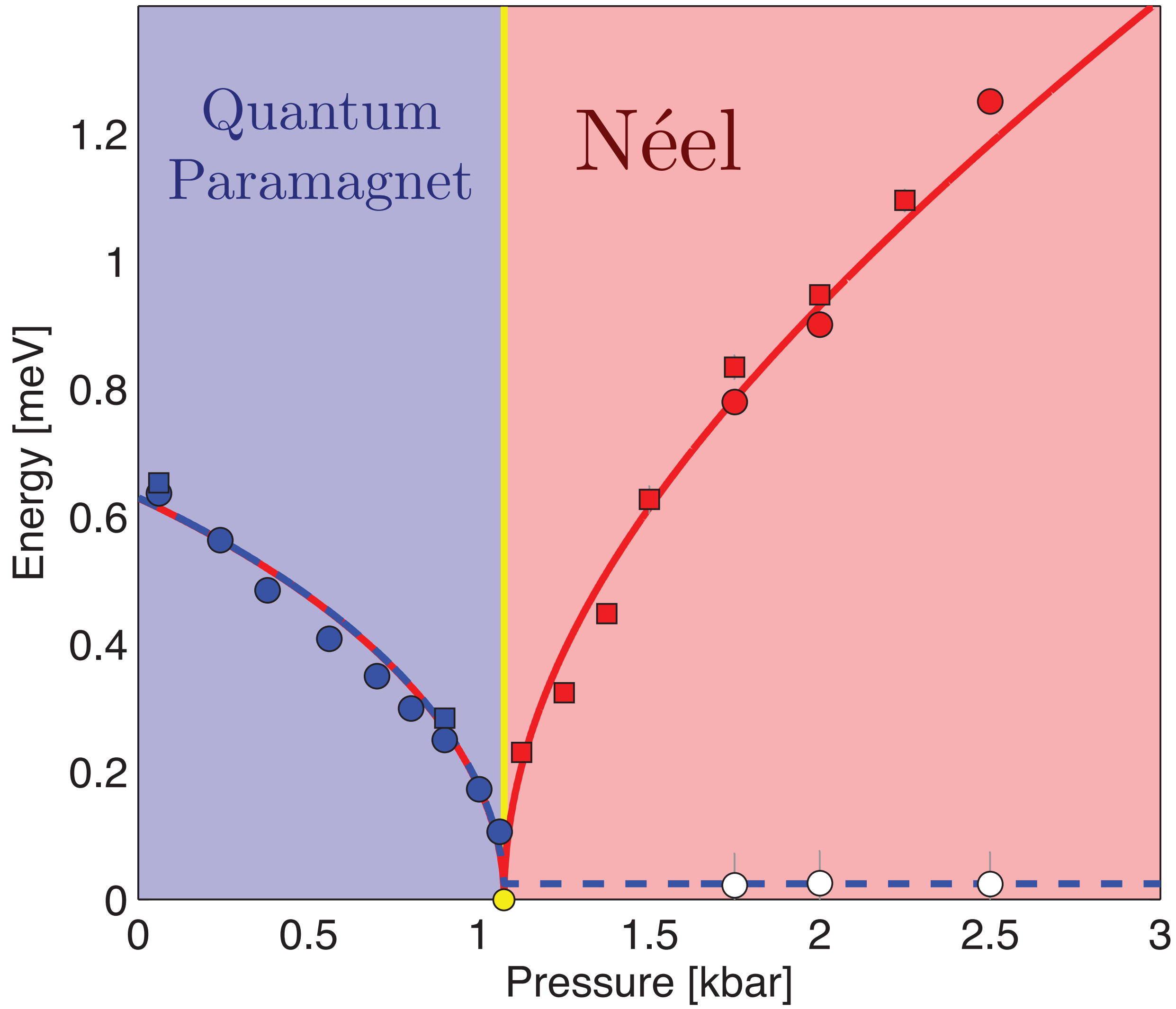


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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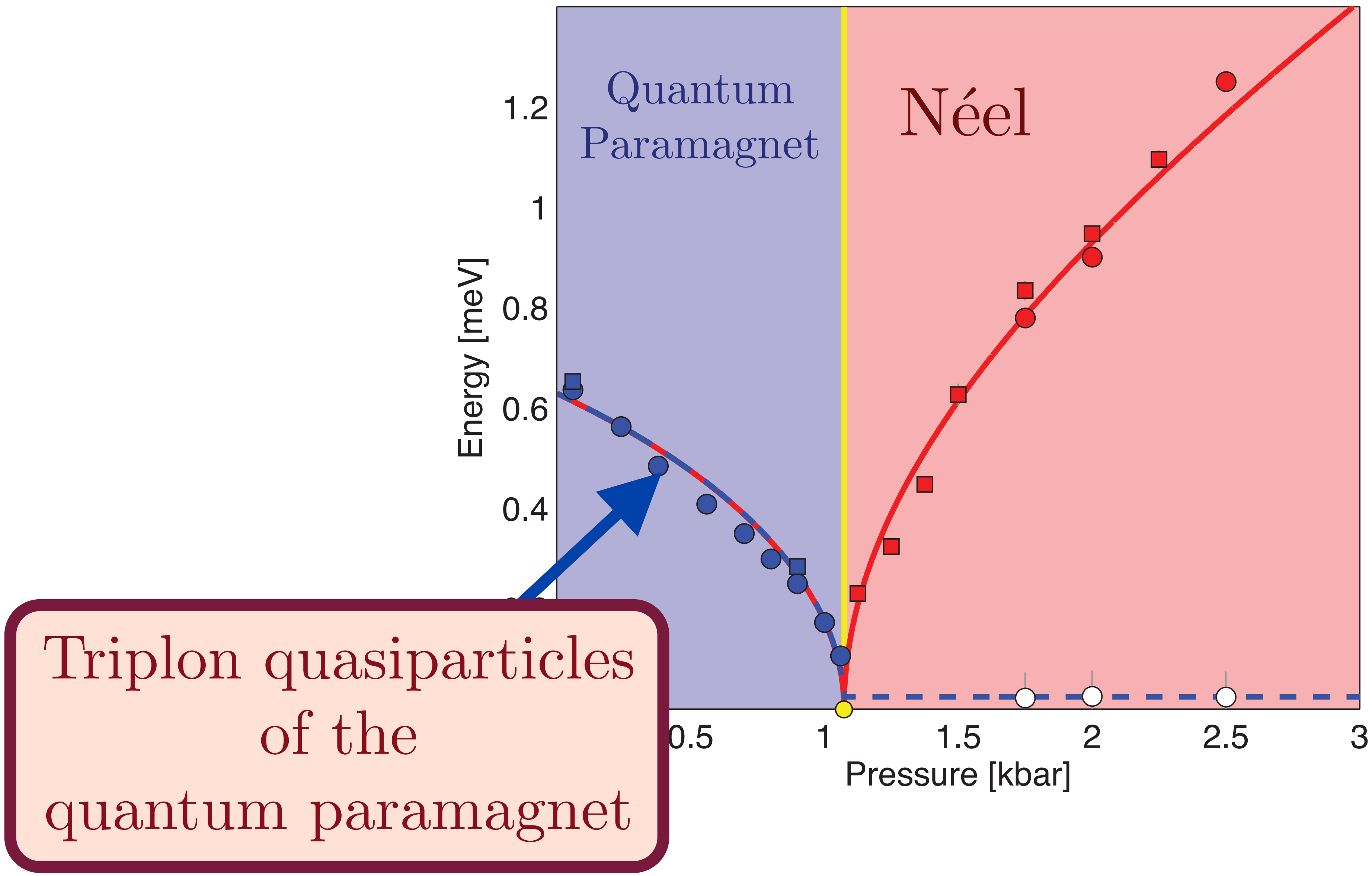

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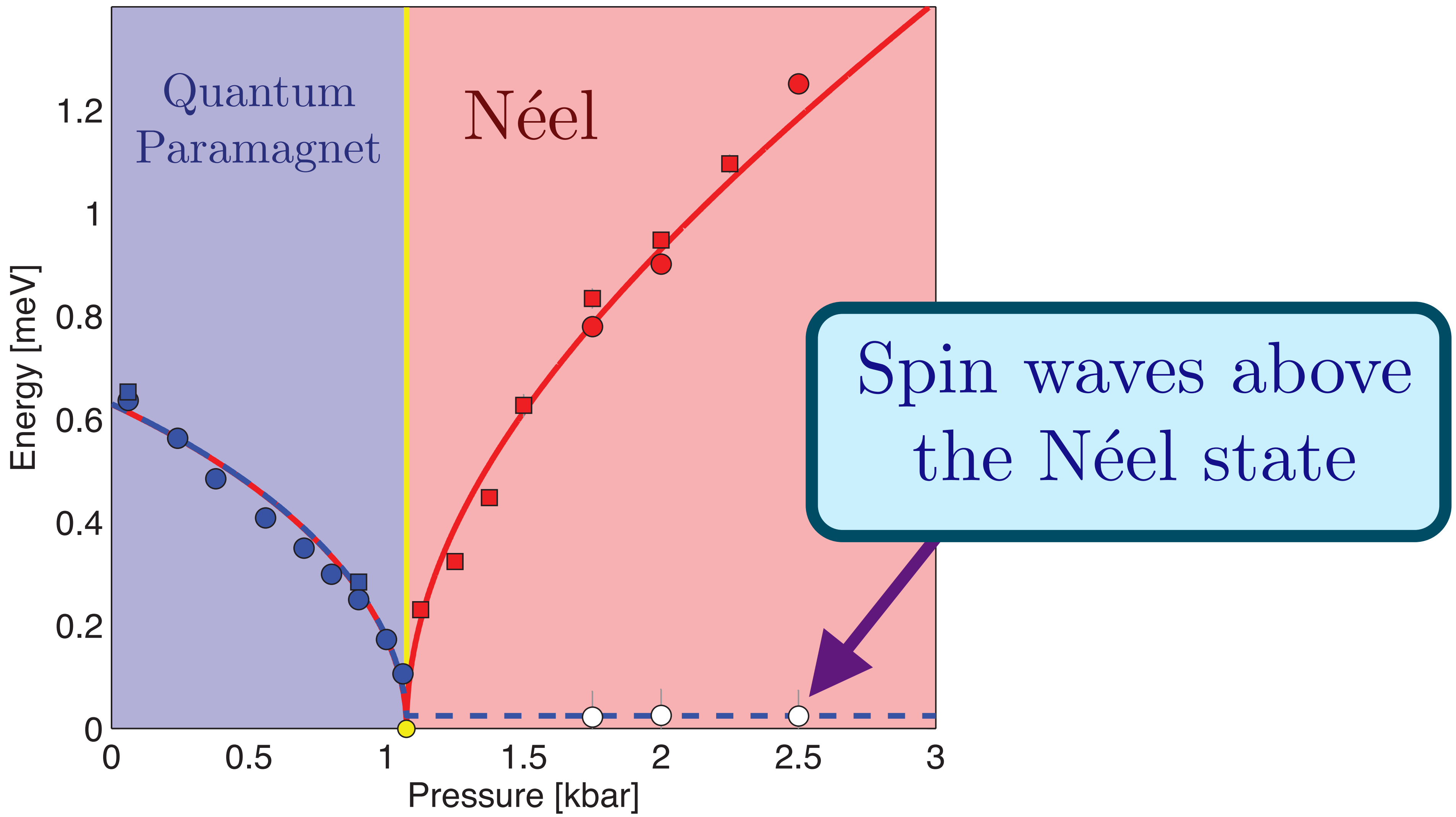

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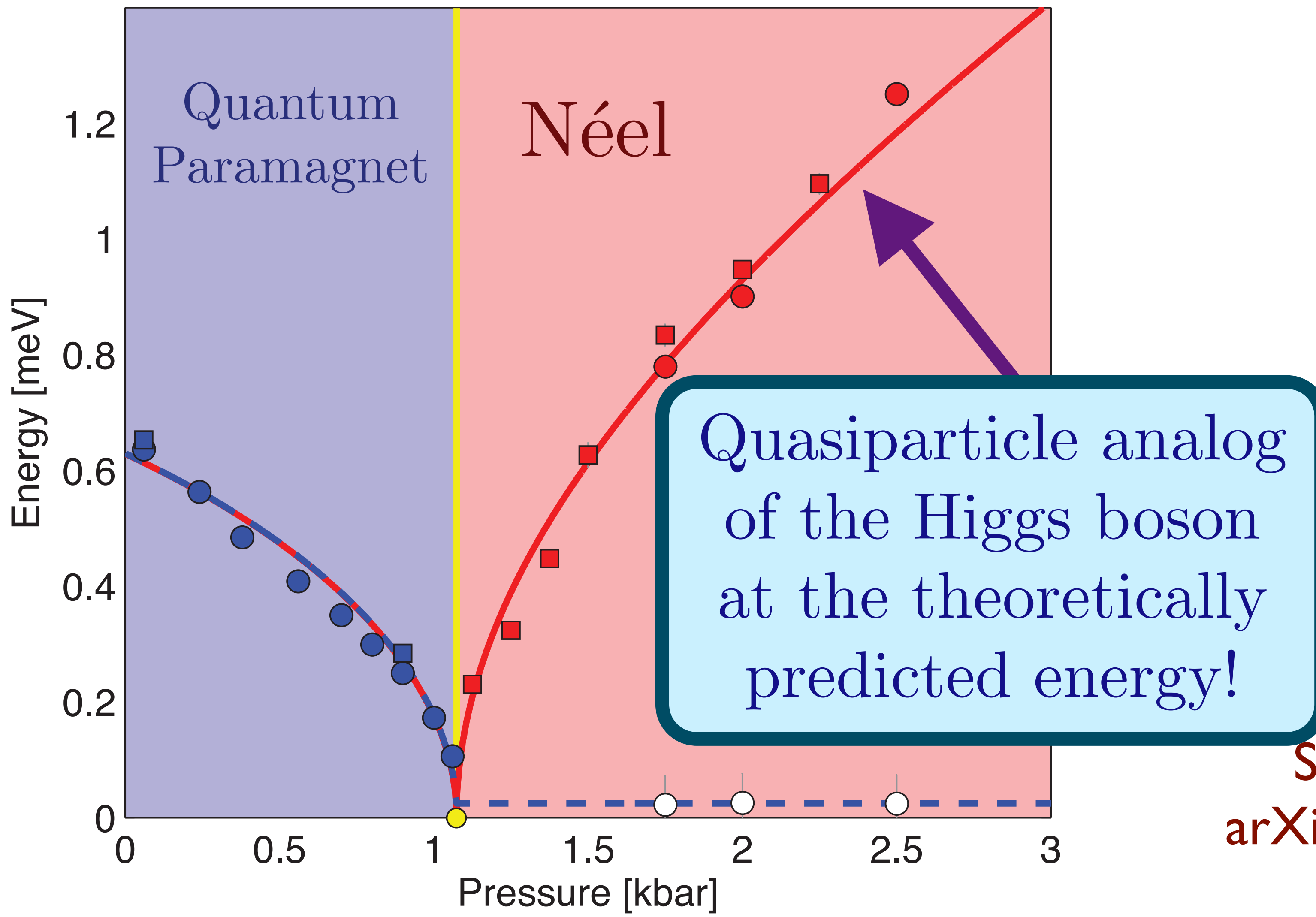

S. Sachdev, arXiv:0901.4103

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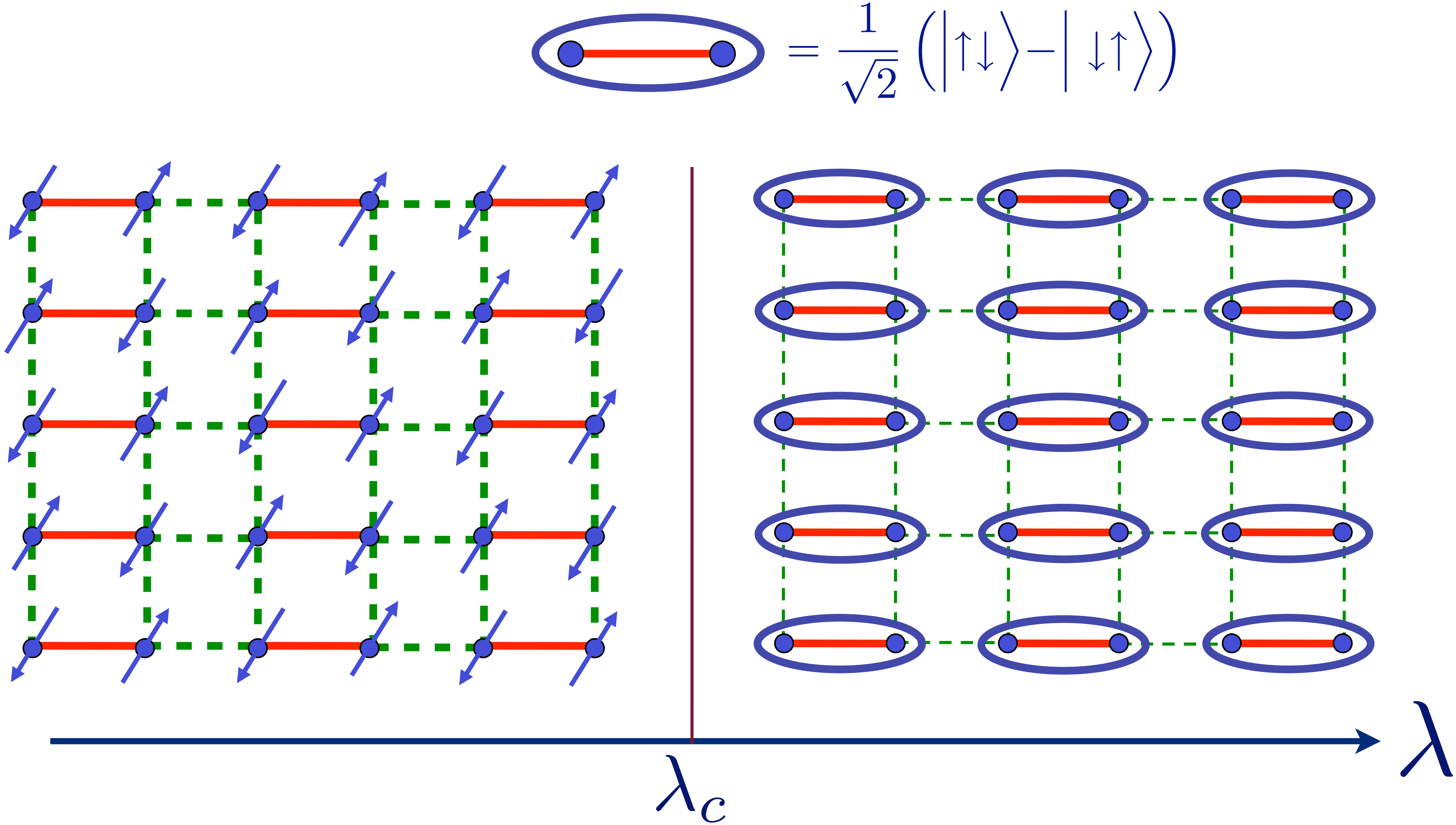
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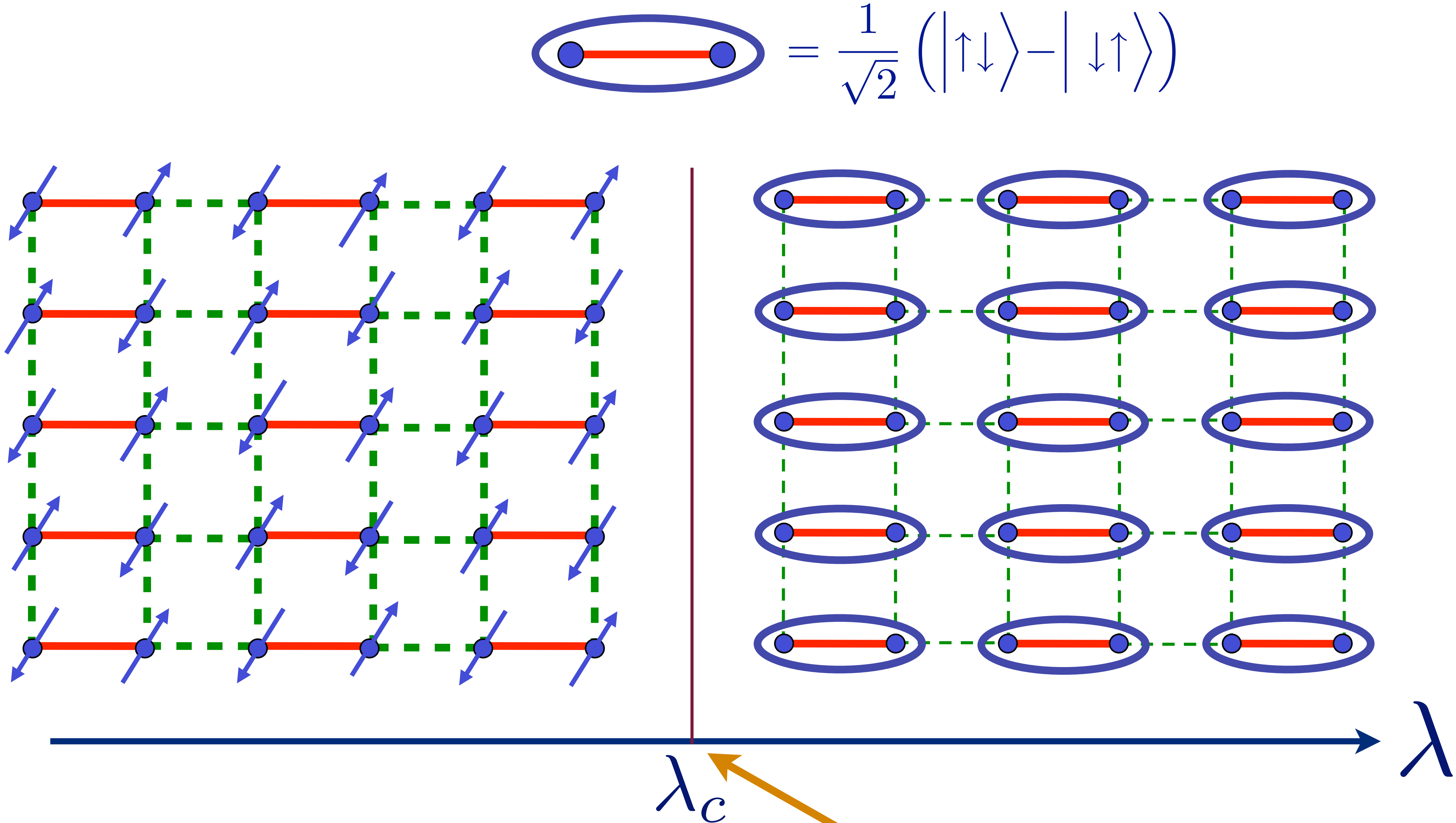
λ_c
λ



Quantum critical point with non-local entanglement in spin wavefunction

A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

Characteristics of quantum critical point

- Long-range entanglement

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- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

Characteristics of quantum critical point

- Long-range entanglement
- No quasiparticles - no simple description of excitations.
- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
- The theory of the critical point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a **CFT3**

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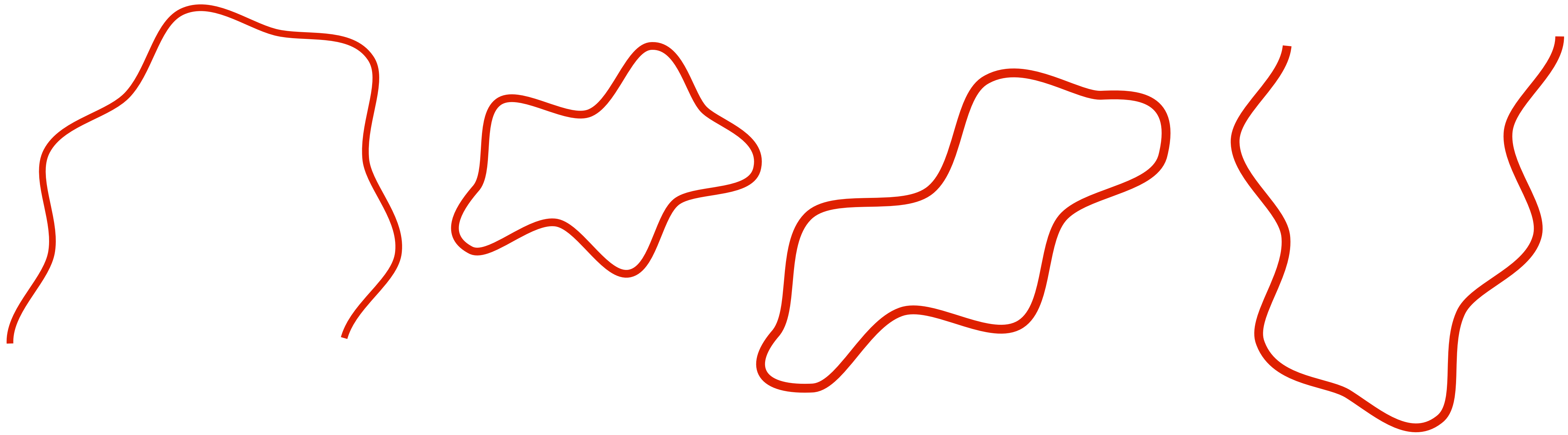
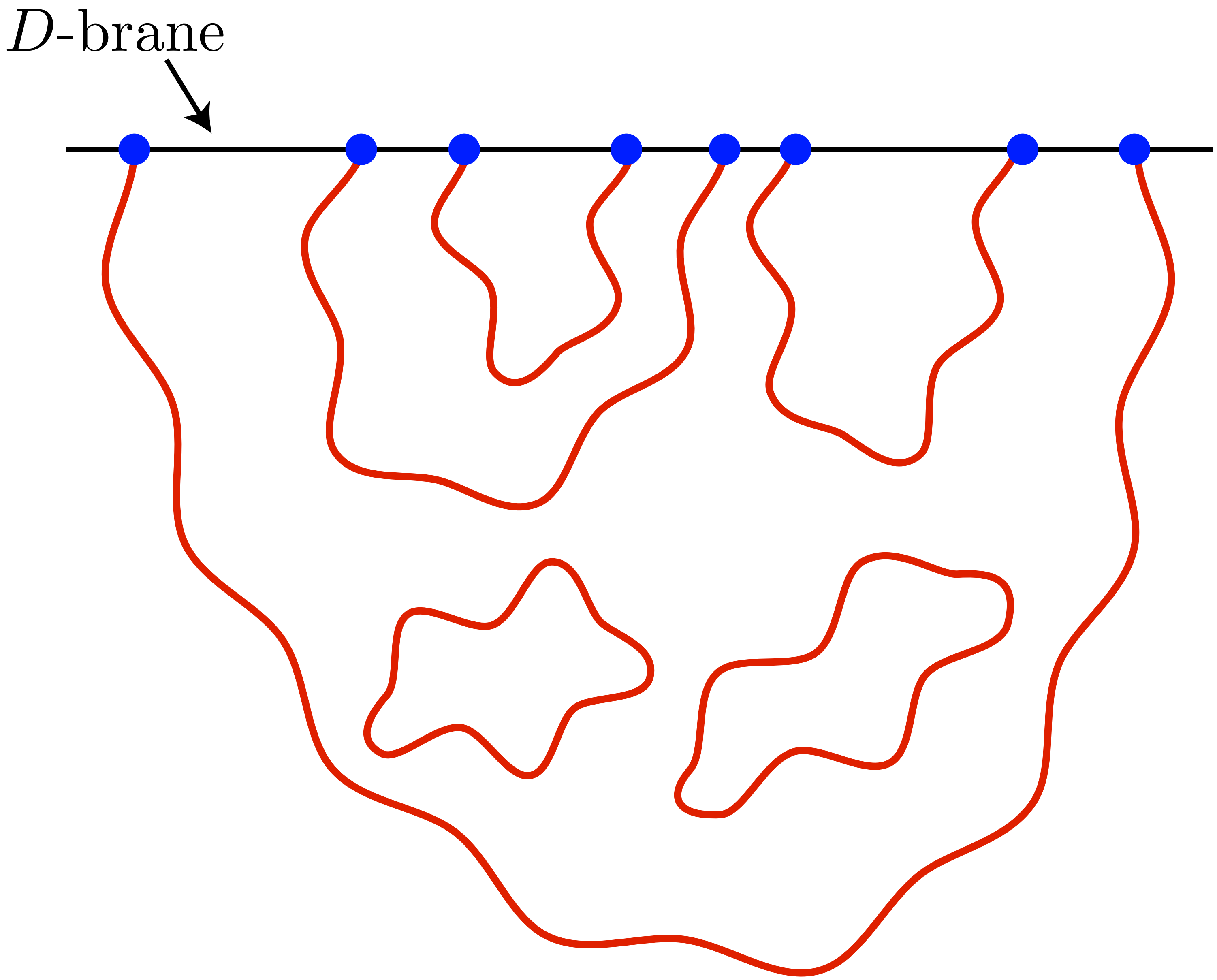
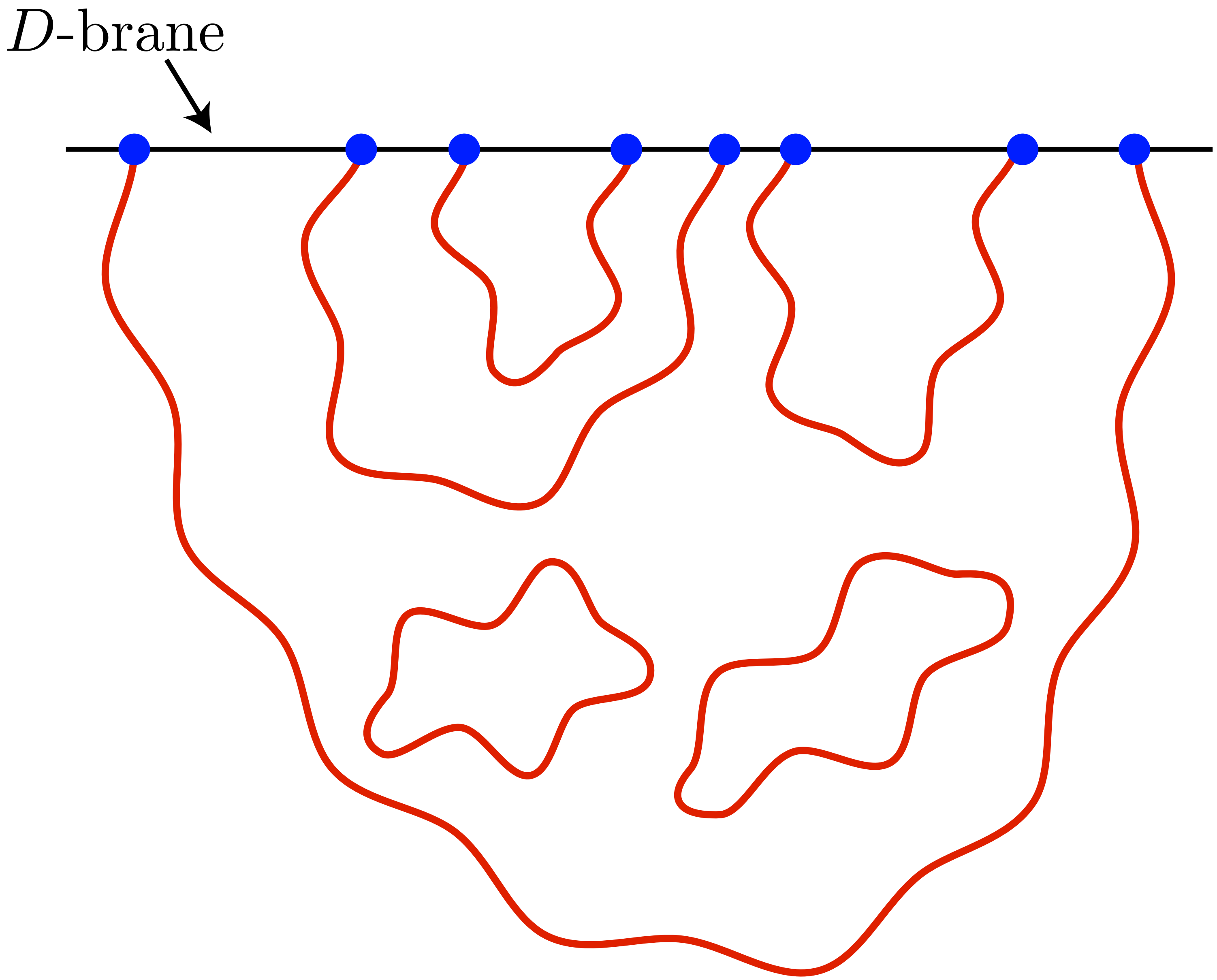
String theory



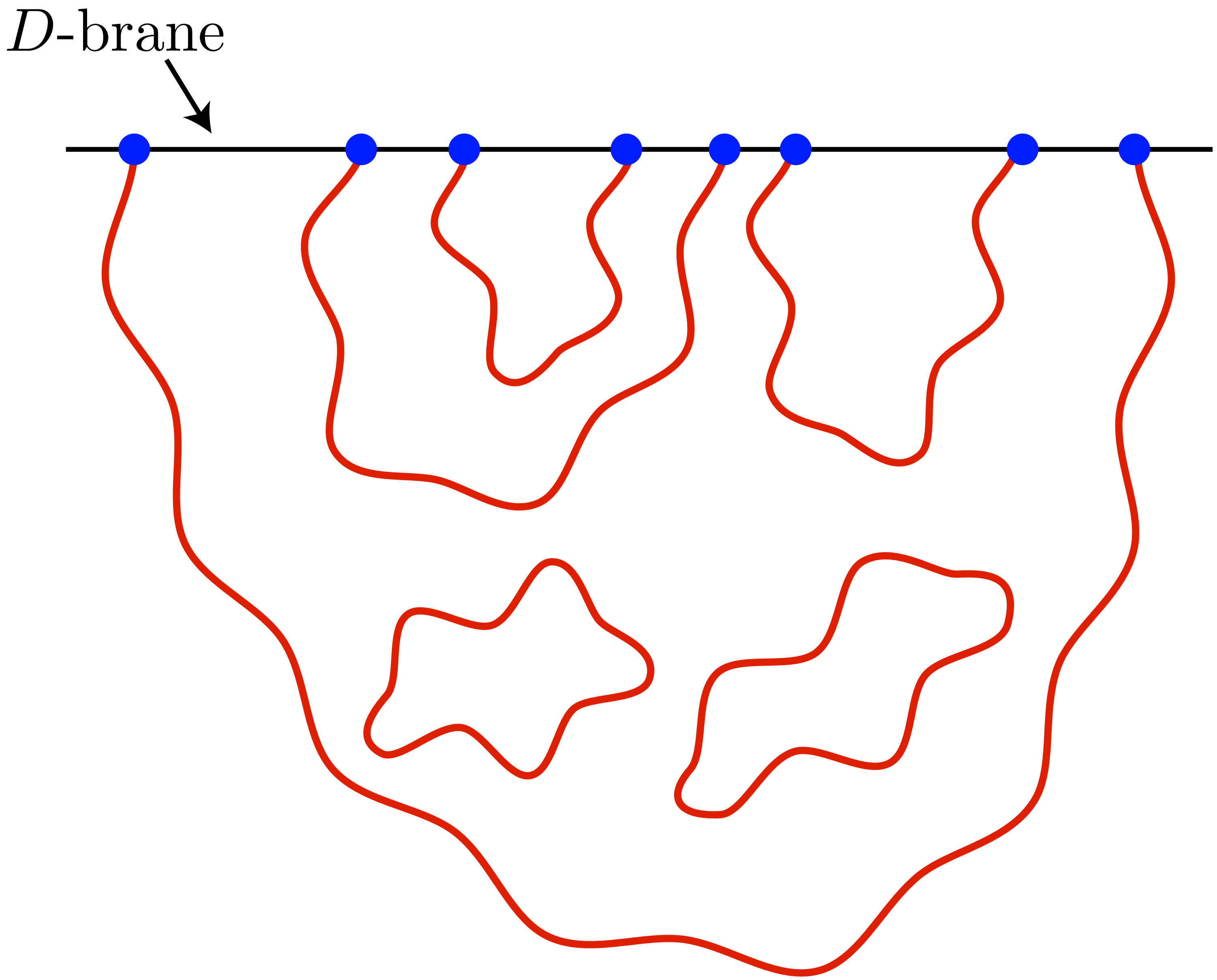
- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks . . .



- A D-brane is a d-dimensional surface on which strings can end.



- A *D*-brane is a *d*-dimensional surface on which strings can end.
- The low-energy theory on a *D*-brane has no gravity, similar to theories of entangled electrons of interest to us.



- A D-brane is a d-dimensional surface on which strings can end.

- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.

- In $d = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

Outline

1. Key ideas from quantum mechanics
2. Many-particle systems: the concept of a quasiparticle
3. Quantum phases of a magnetic insulator
   Quantum critical point without quasiparticles
4. Connections to string theory
5. Non-zero temperatures and black holes
6. The high temperature superconductors

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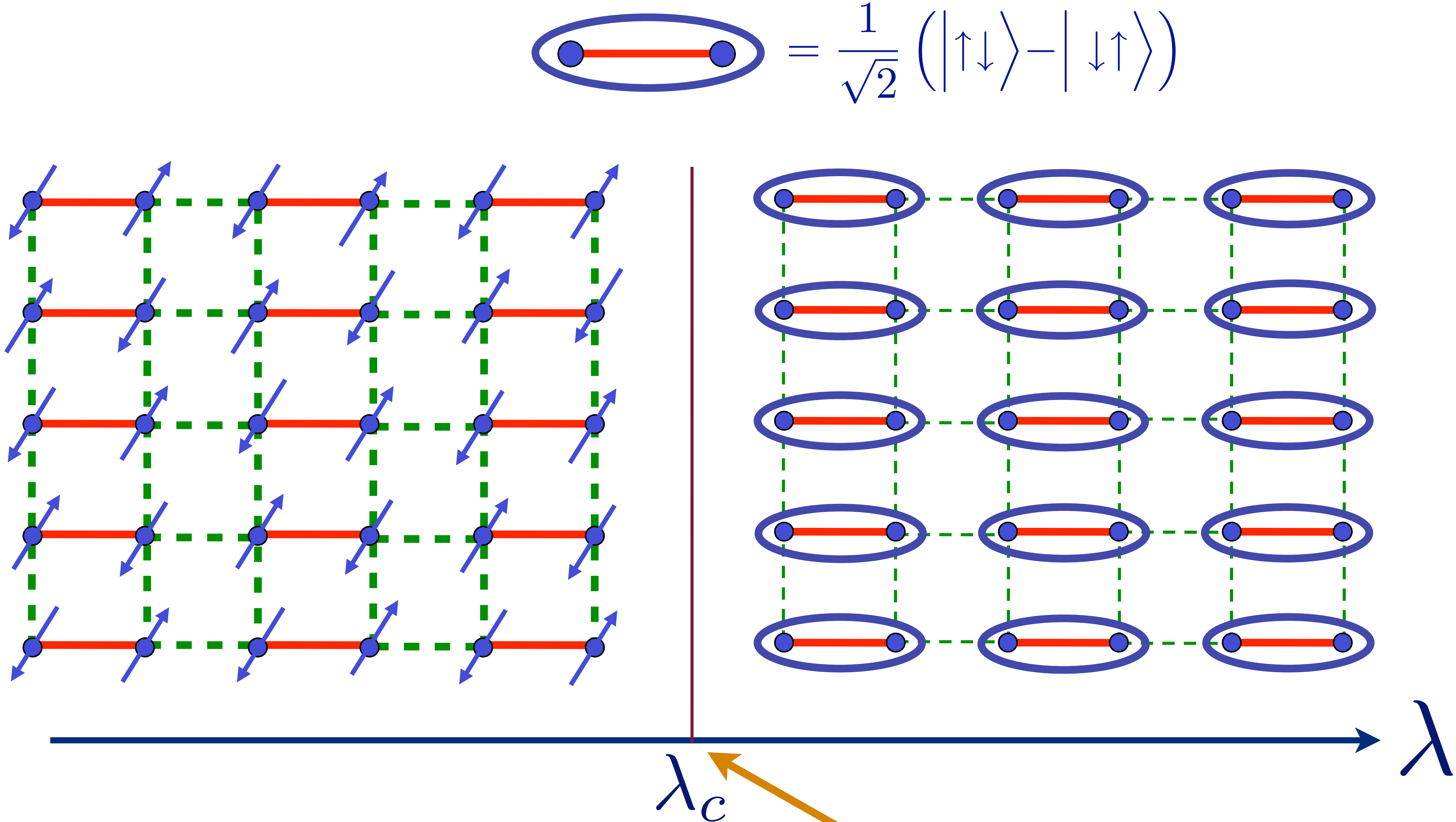
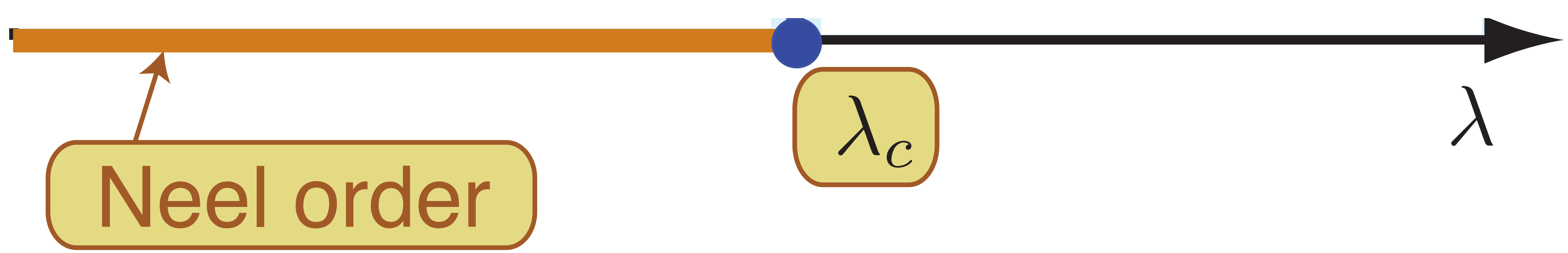
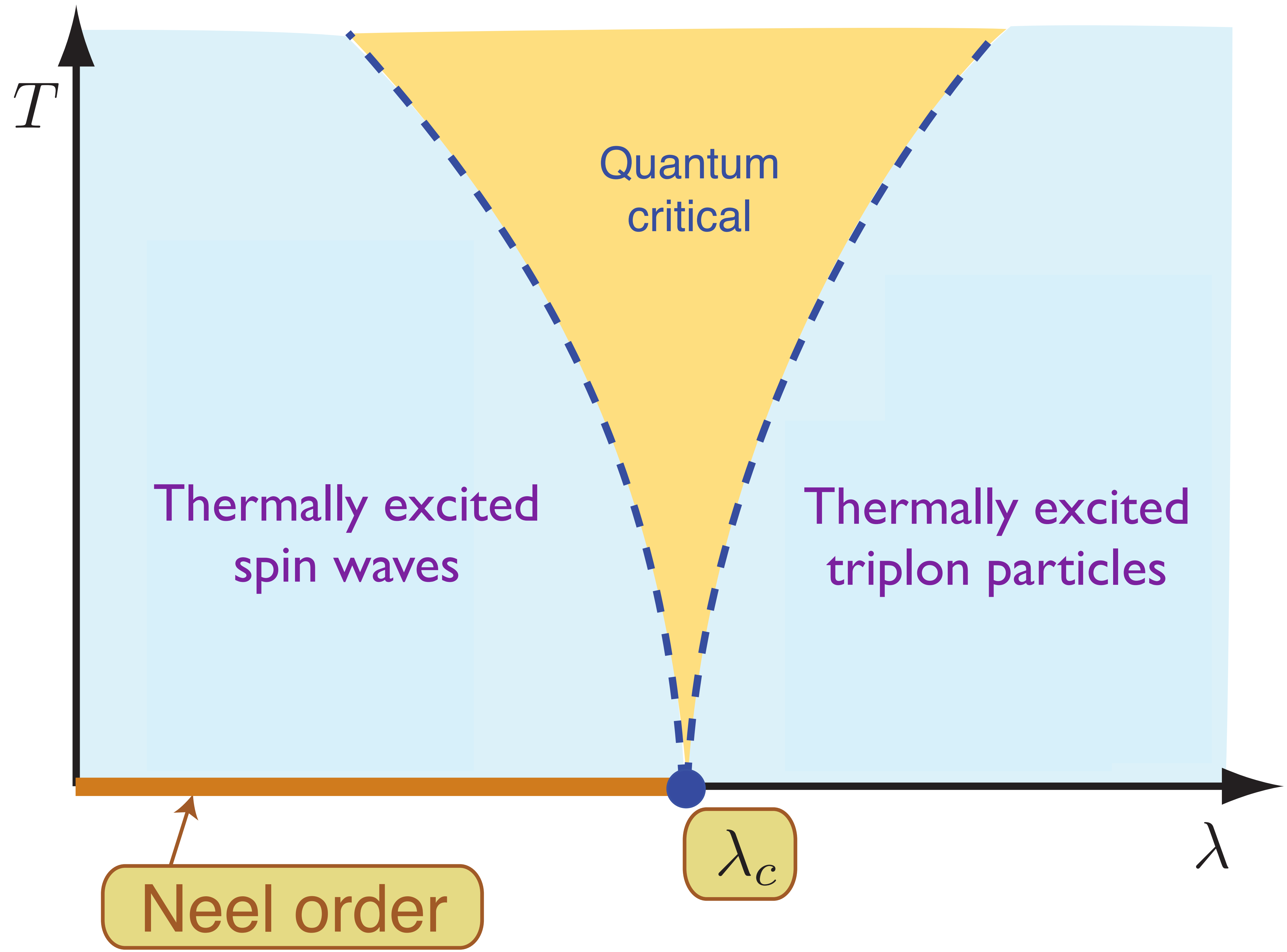


$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Quantum critical point with non-local entanglement in spin wavefunction


Neel order
λ_c
λ

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



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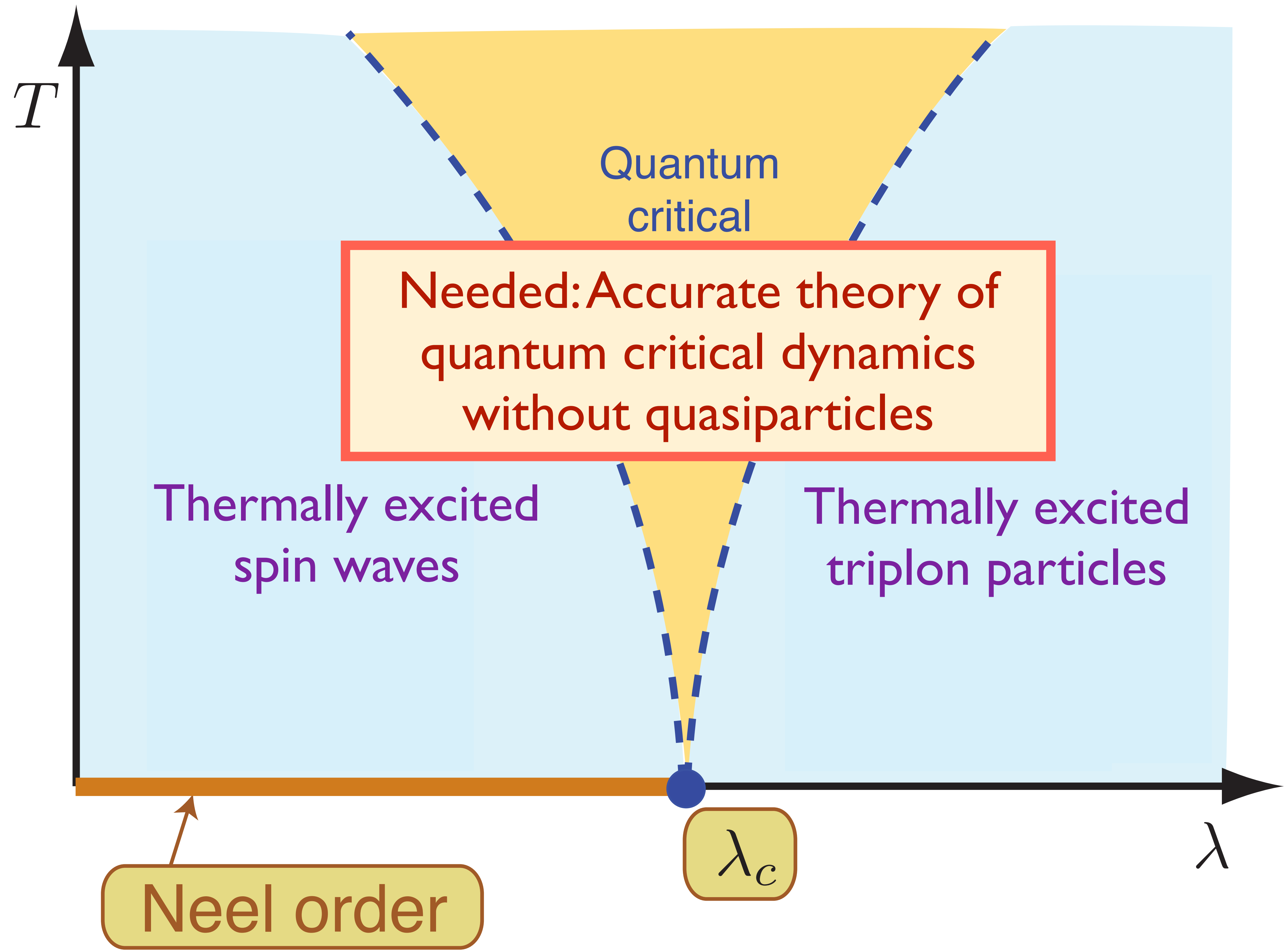
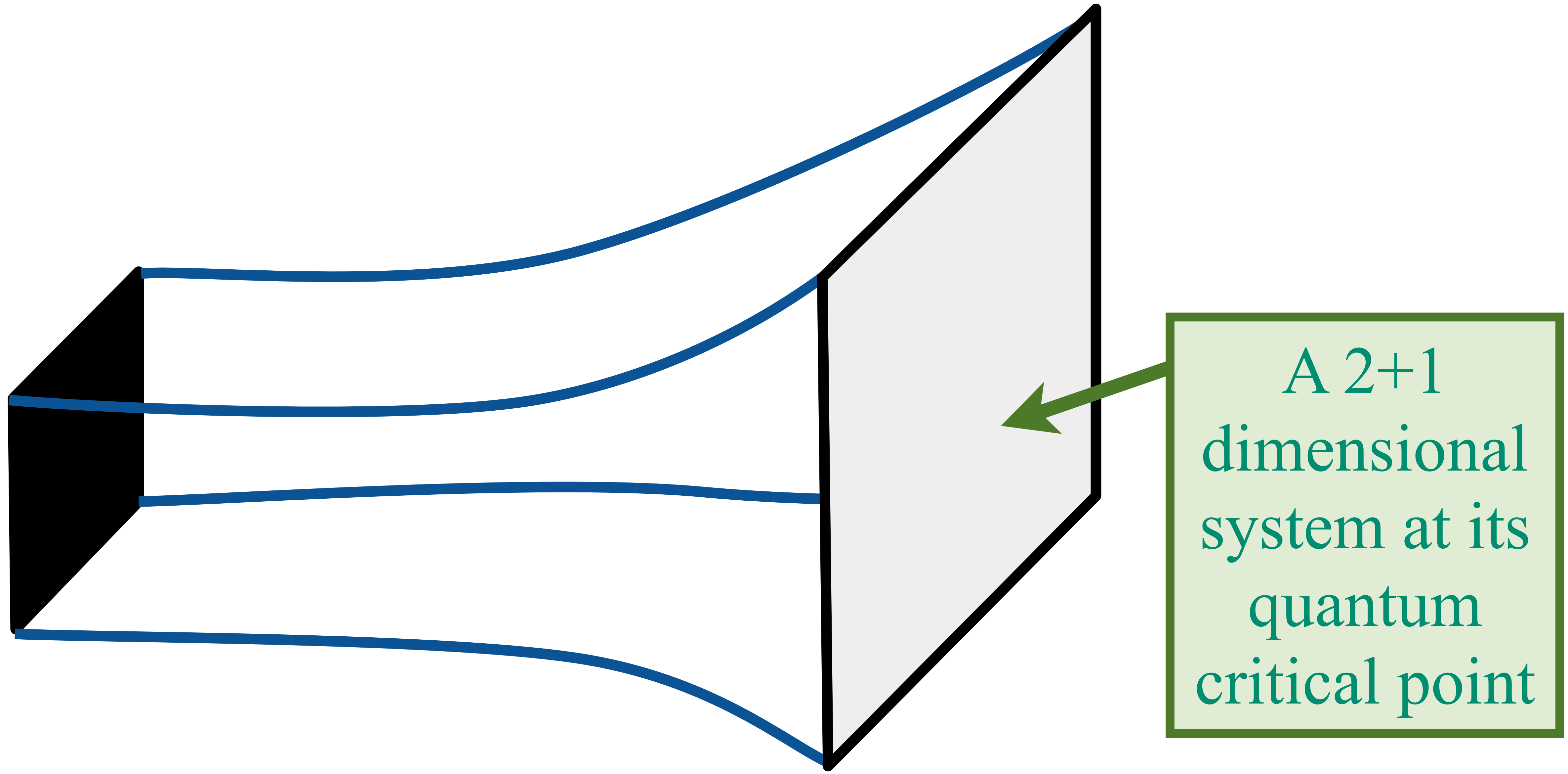


String theory at non-zero temperatures



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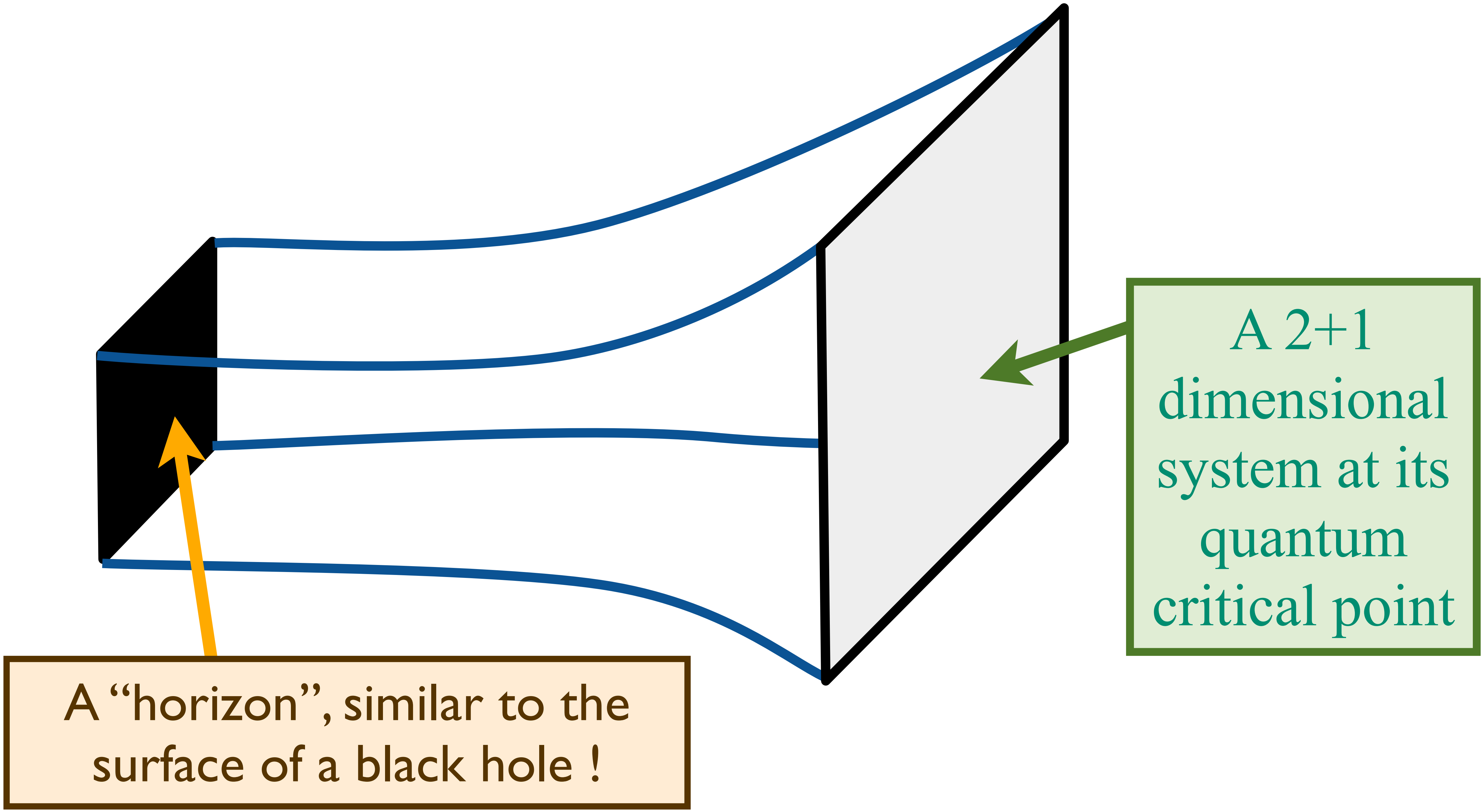
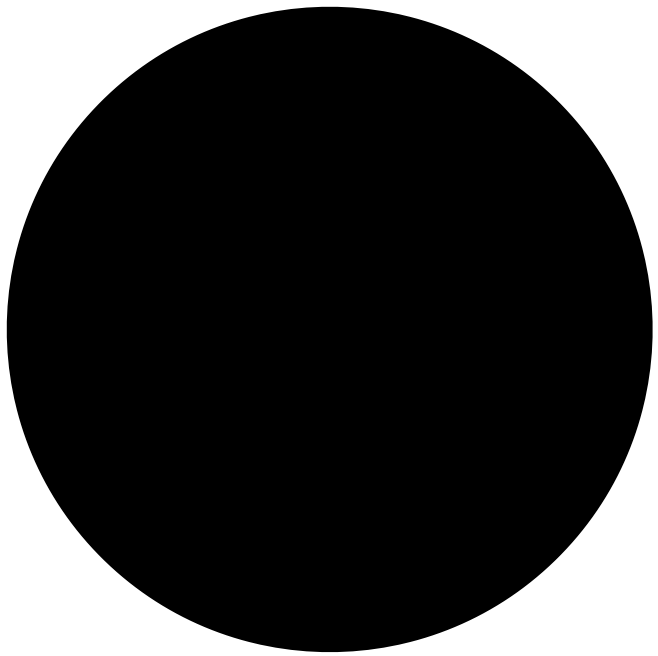


Black Holes

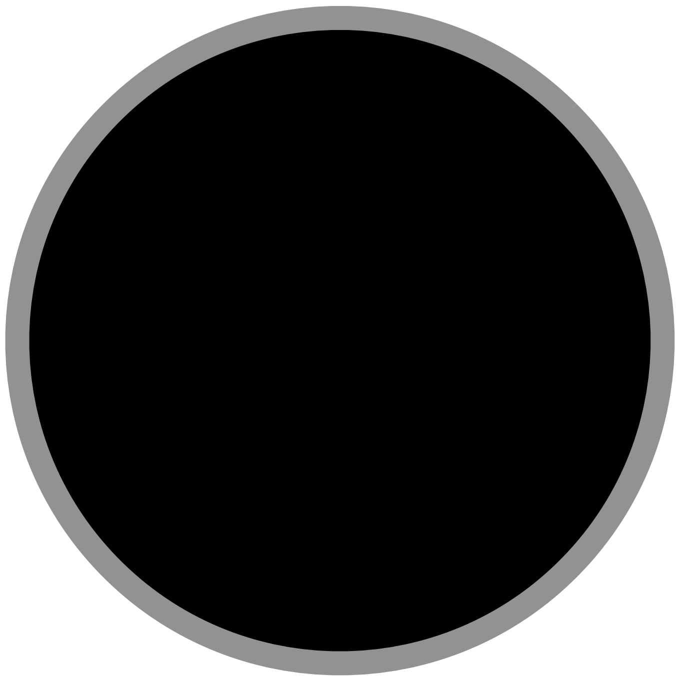
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

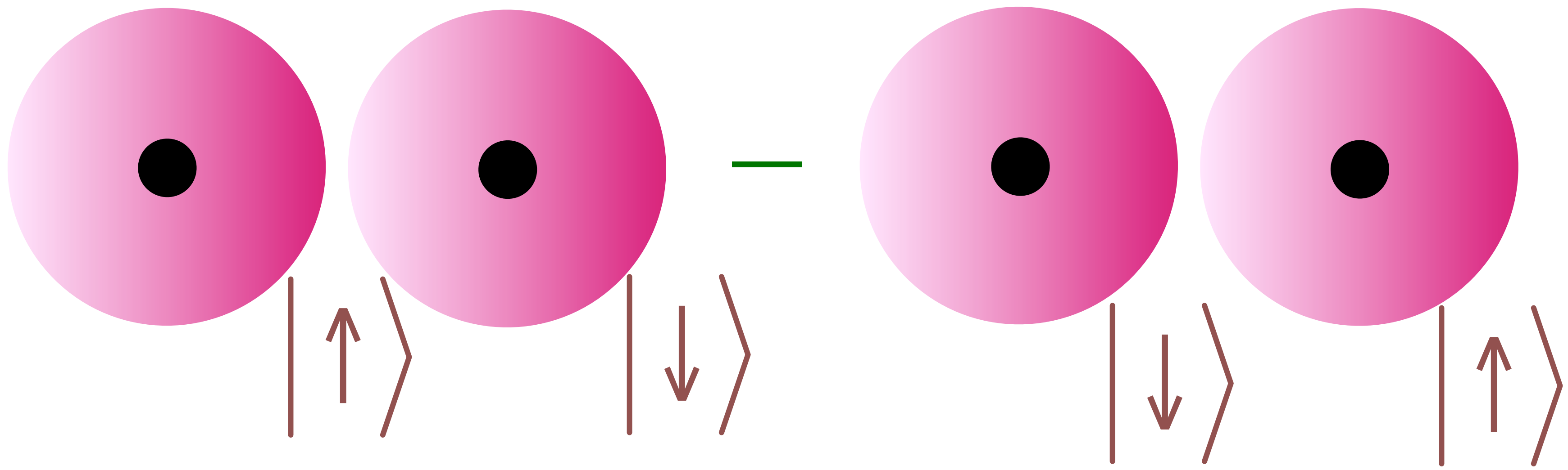


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

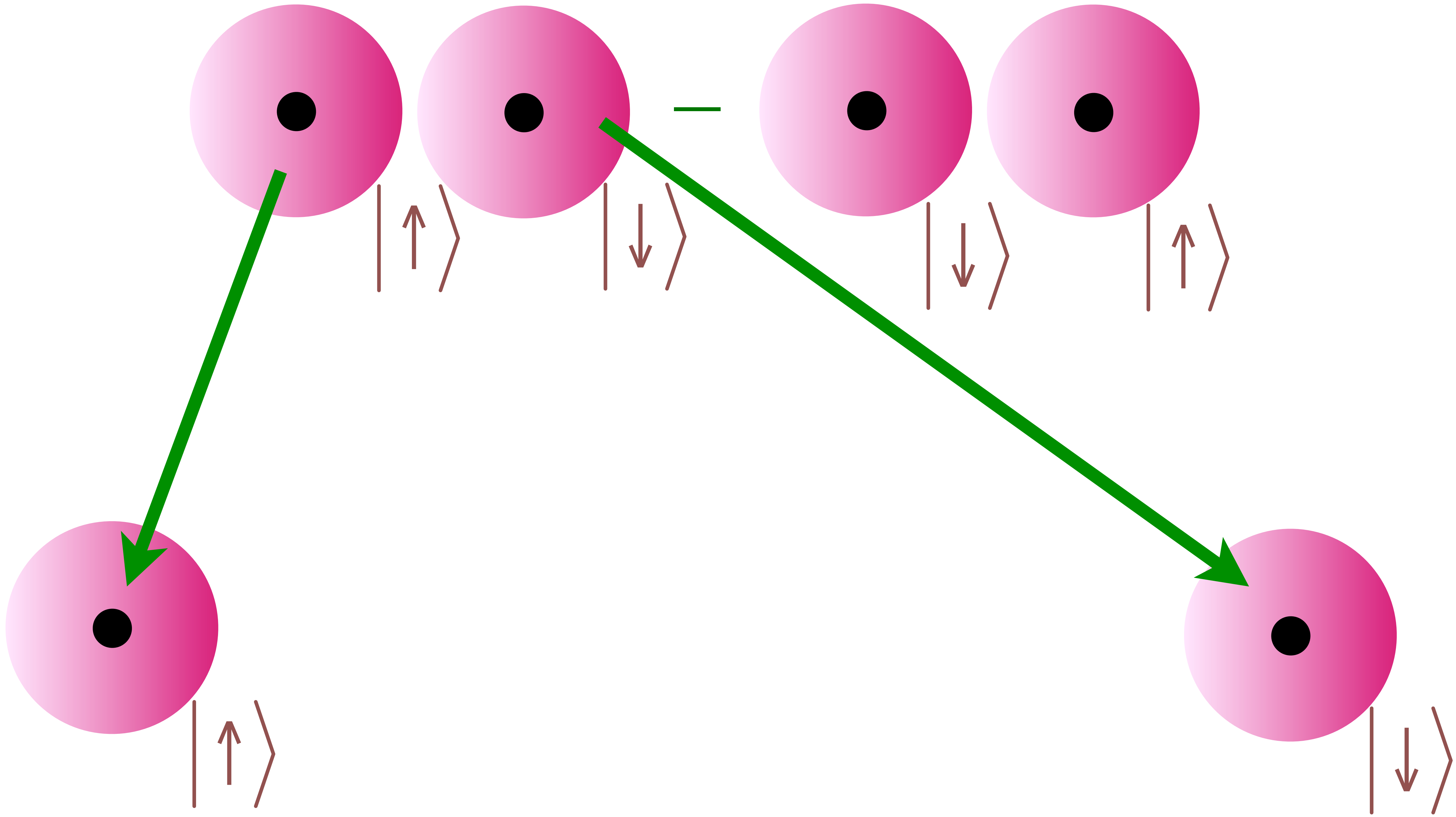
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



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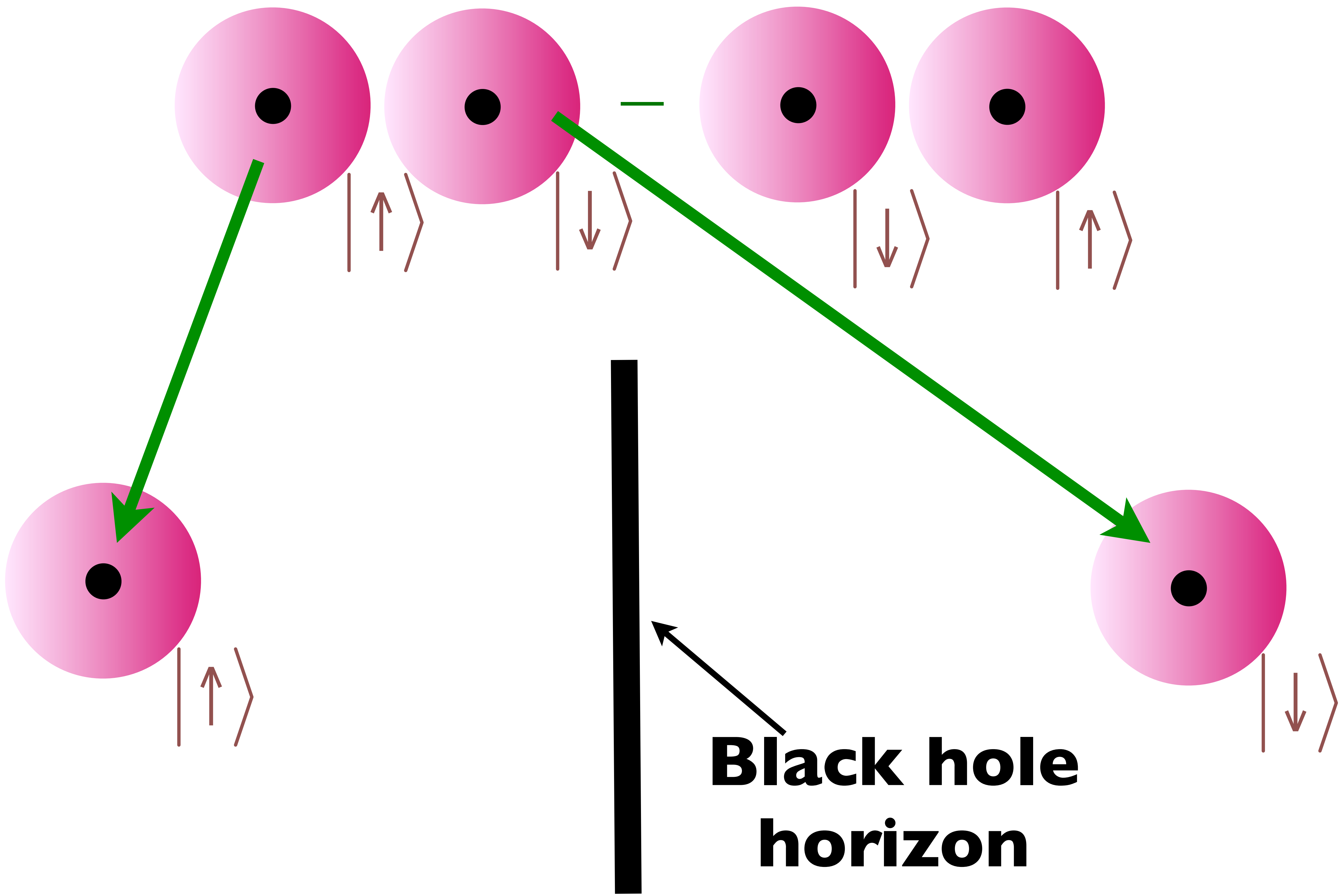

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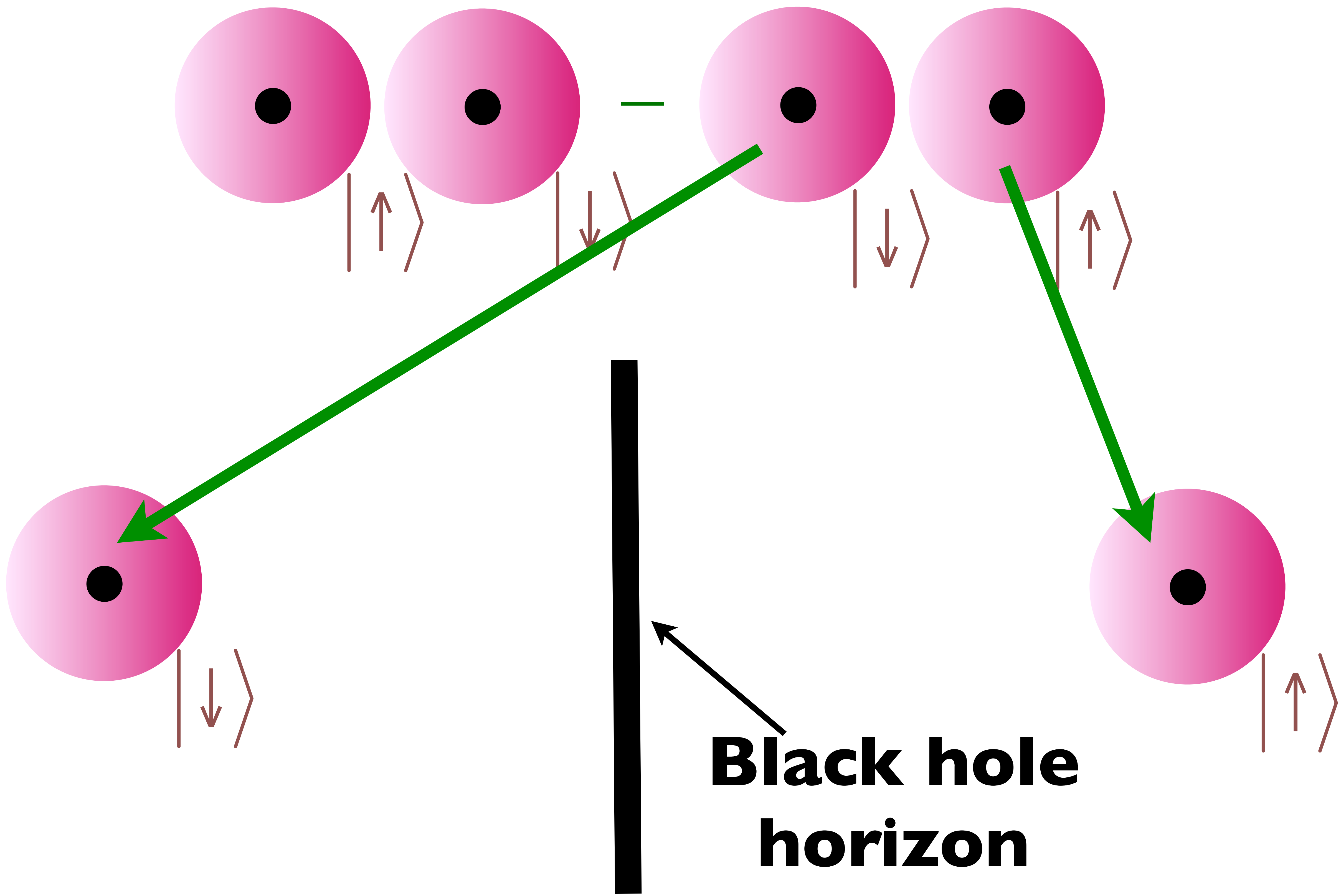
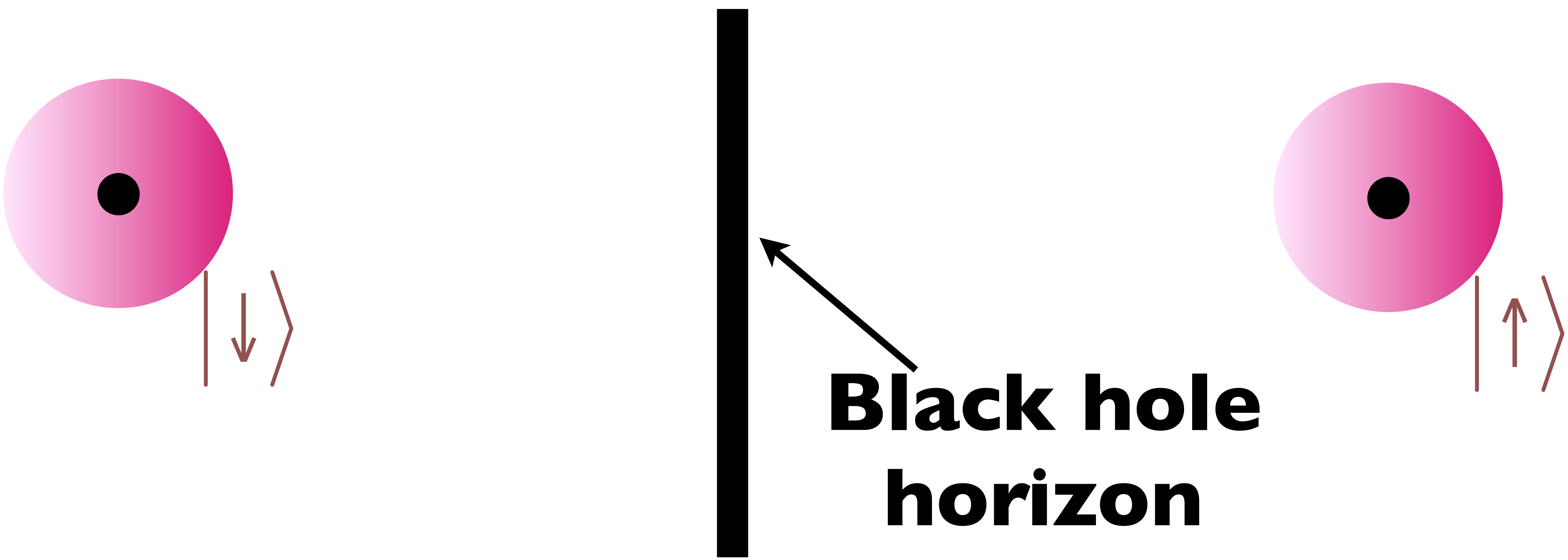


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



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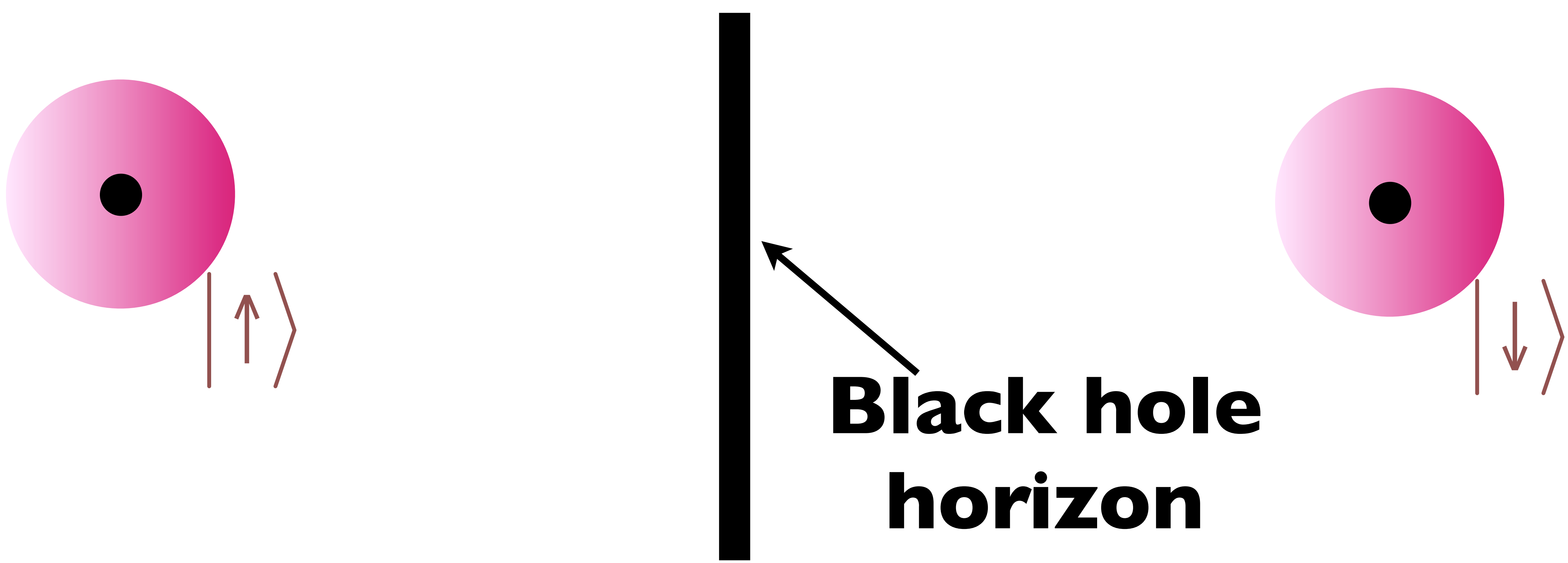
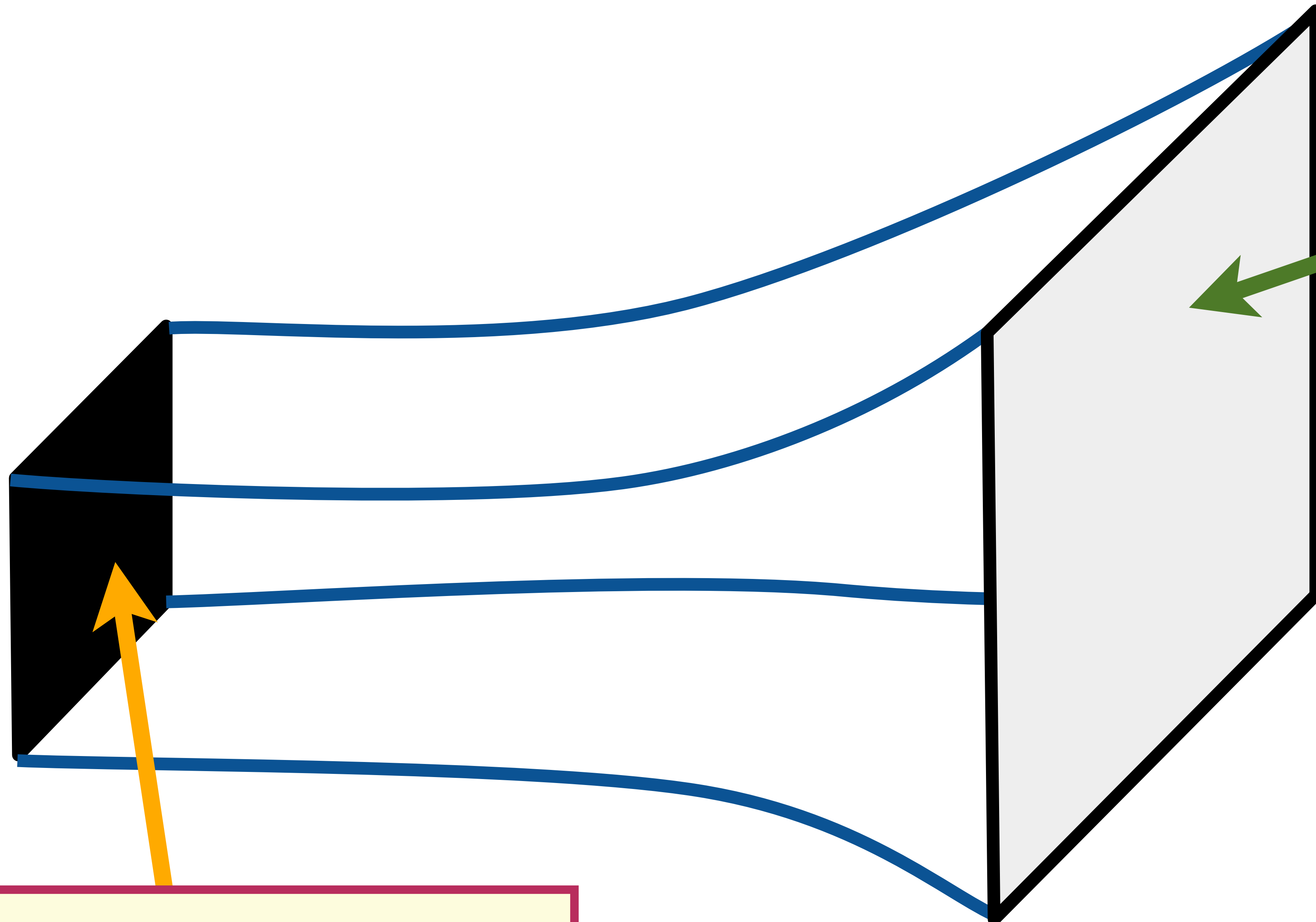


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

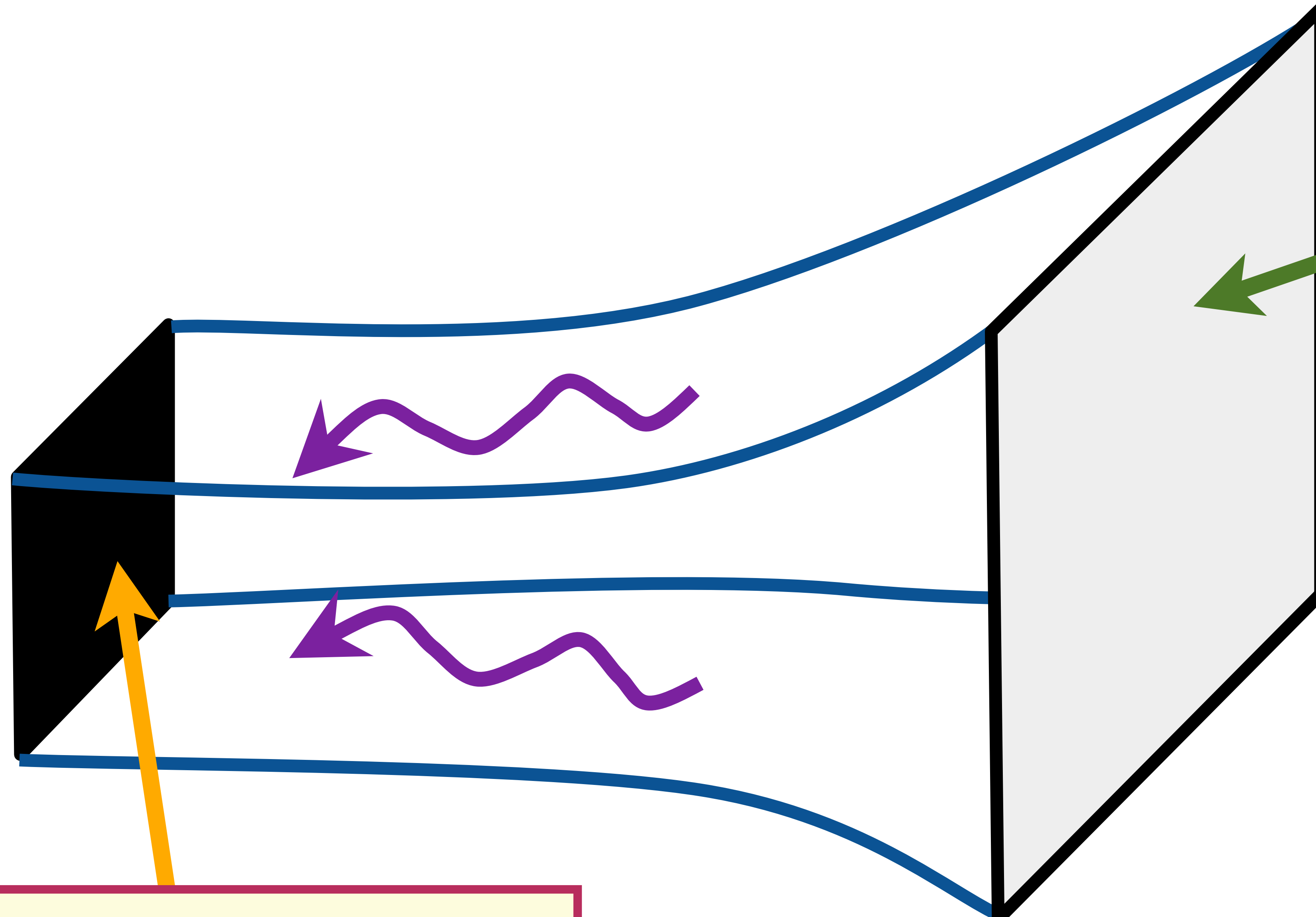
String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

A “horizon”, whose temperature and entropy equal those of the quantum critical point

String theory at non-zero temperatures

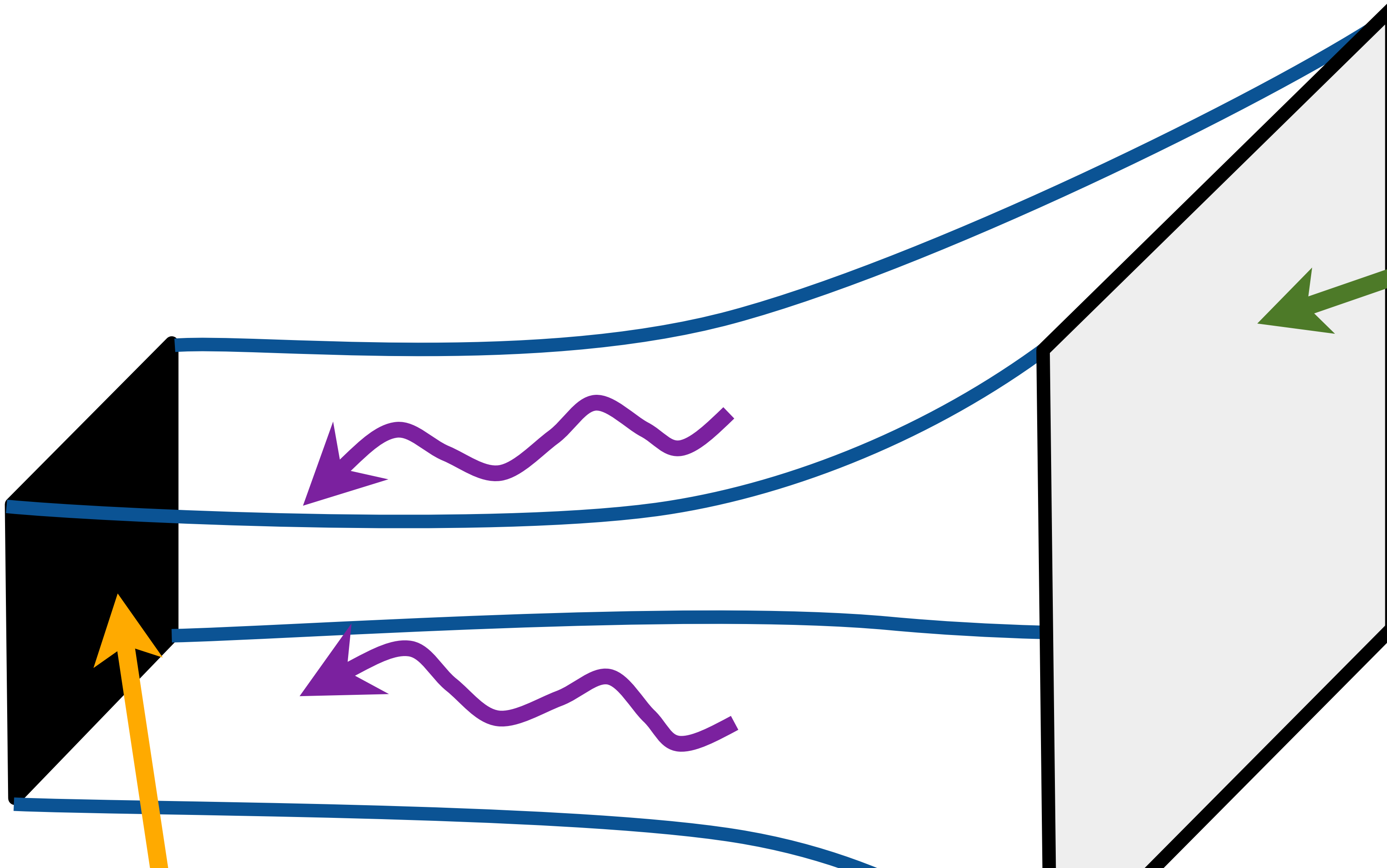


A 2+1 dimensional system at its quantum critical point

A “horizon”, whose temperature and entropy equal those of the quantum critical point

Friction of quantum criticality = waves falling into black brane

String theory at non-zero temperatures

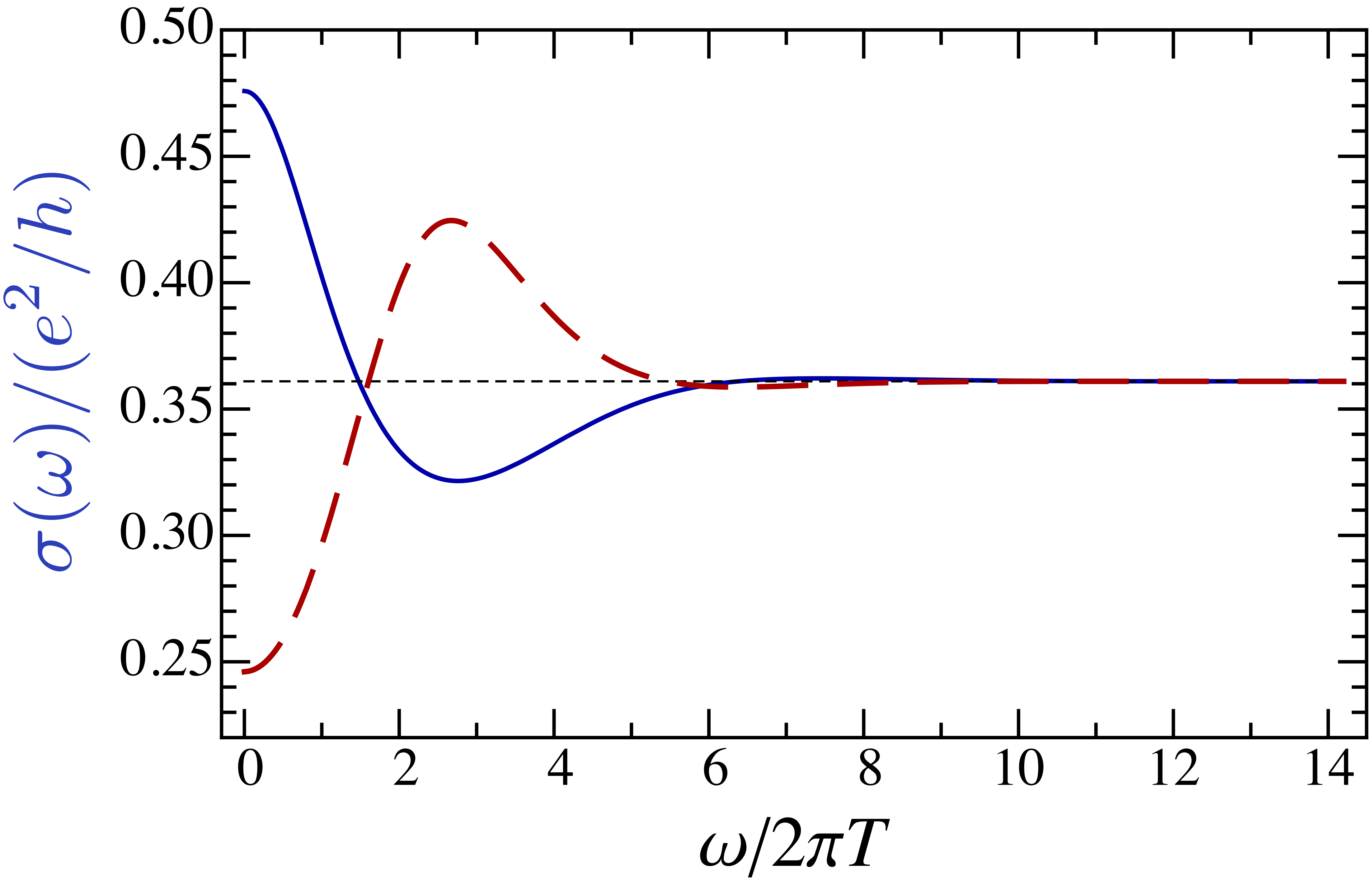


A 2+1 dimensional system at its quantum critical point

A "horizon", whose temperature and entropy equal those of the quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

Black hole predictions for quantum critical dynamics



Predictions of holographic theory,
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941
See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

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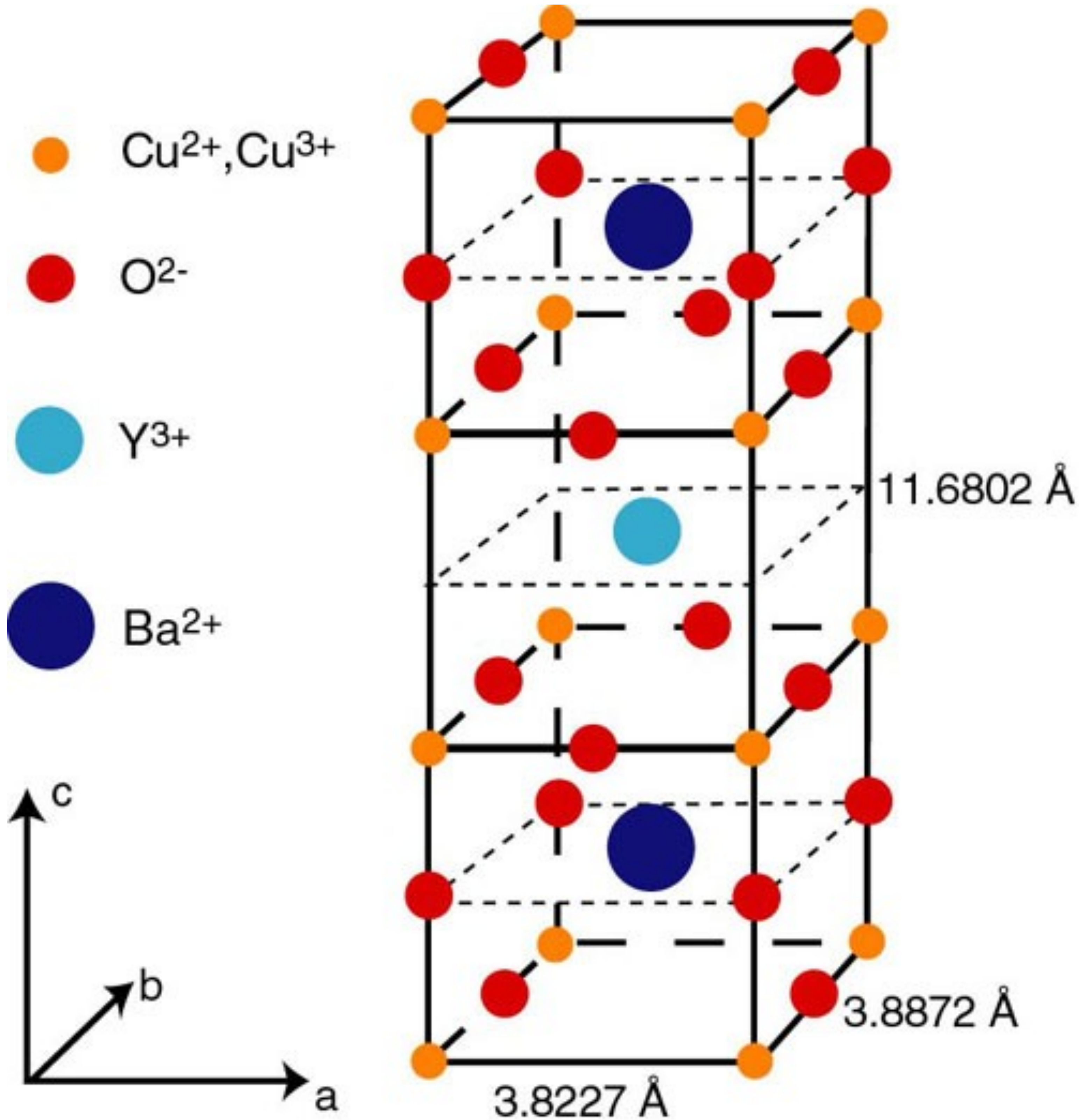
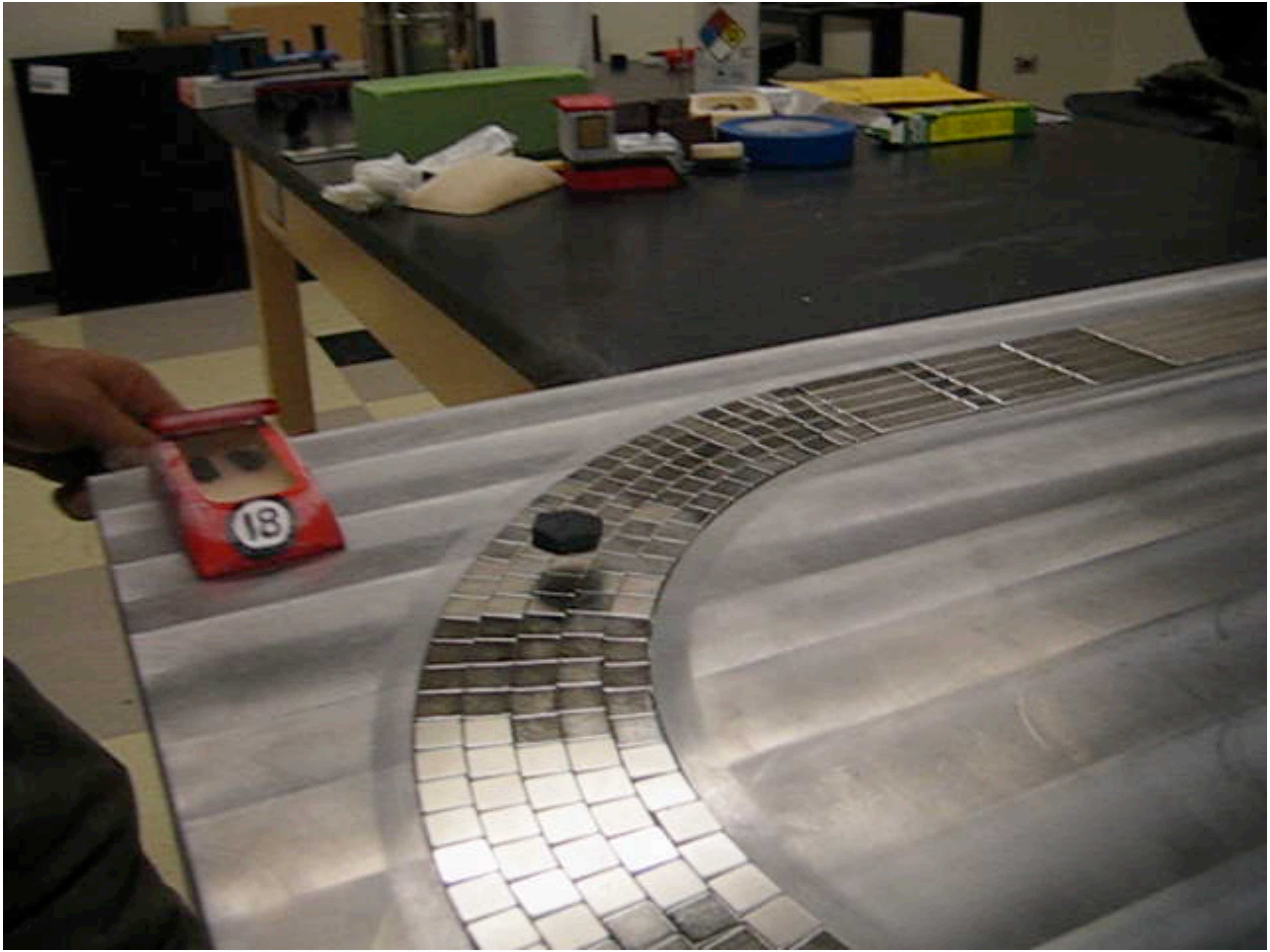


$YBa_2Cu_3O_{6+x}$

Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University



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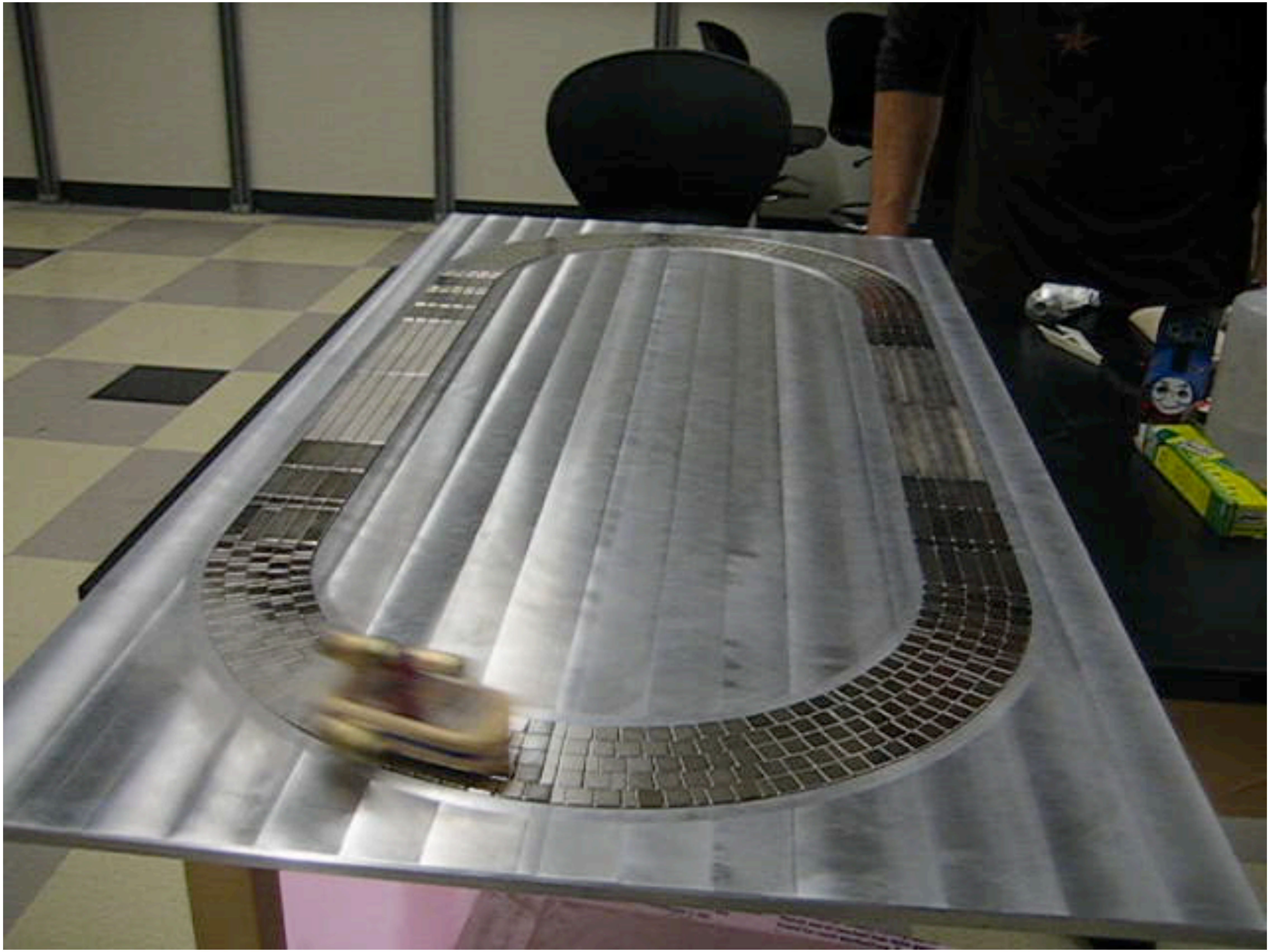
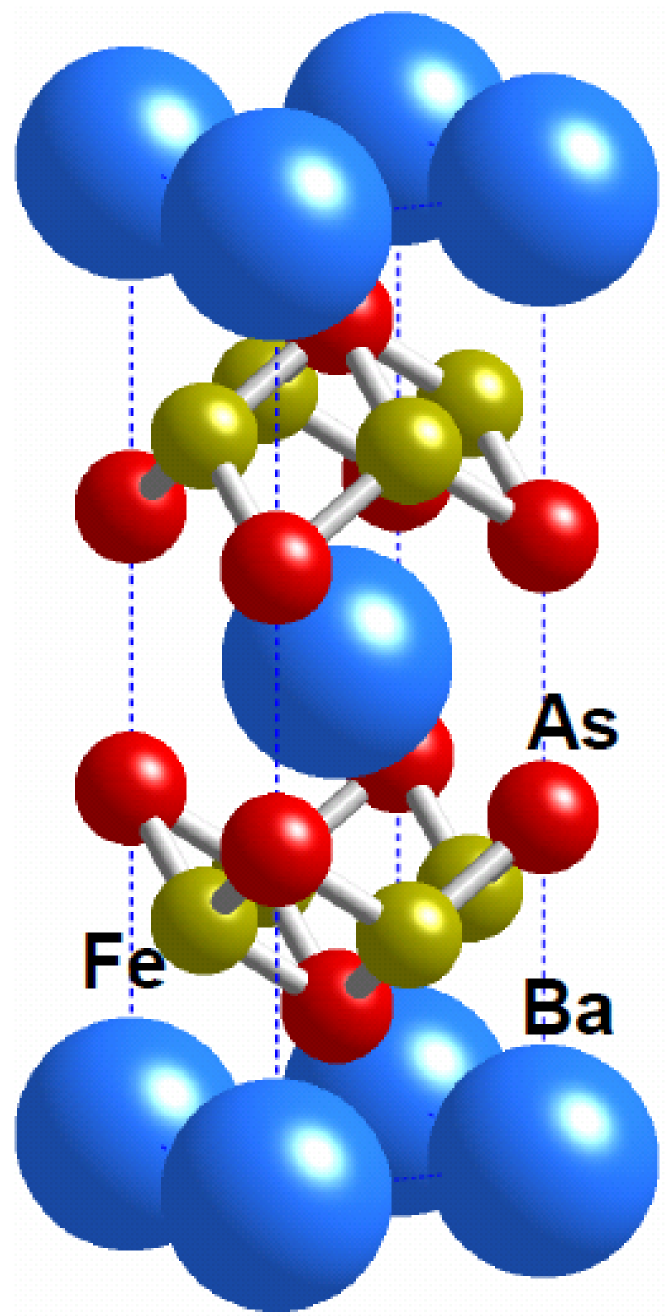


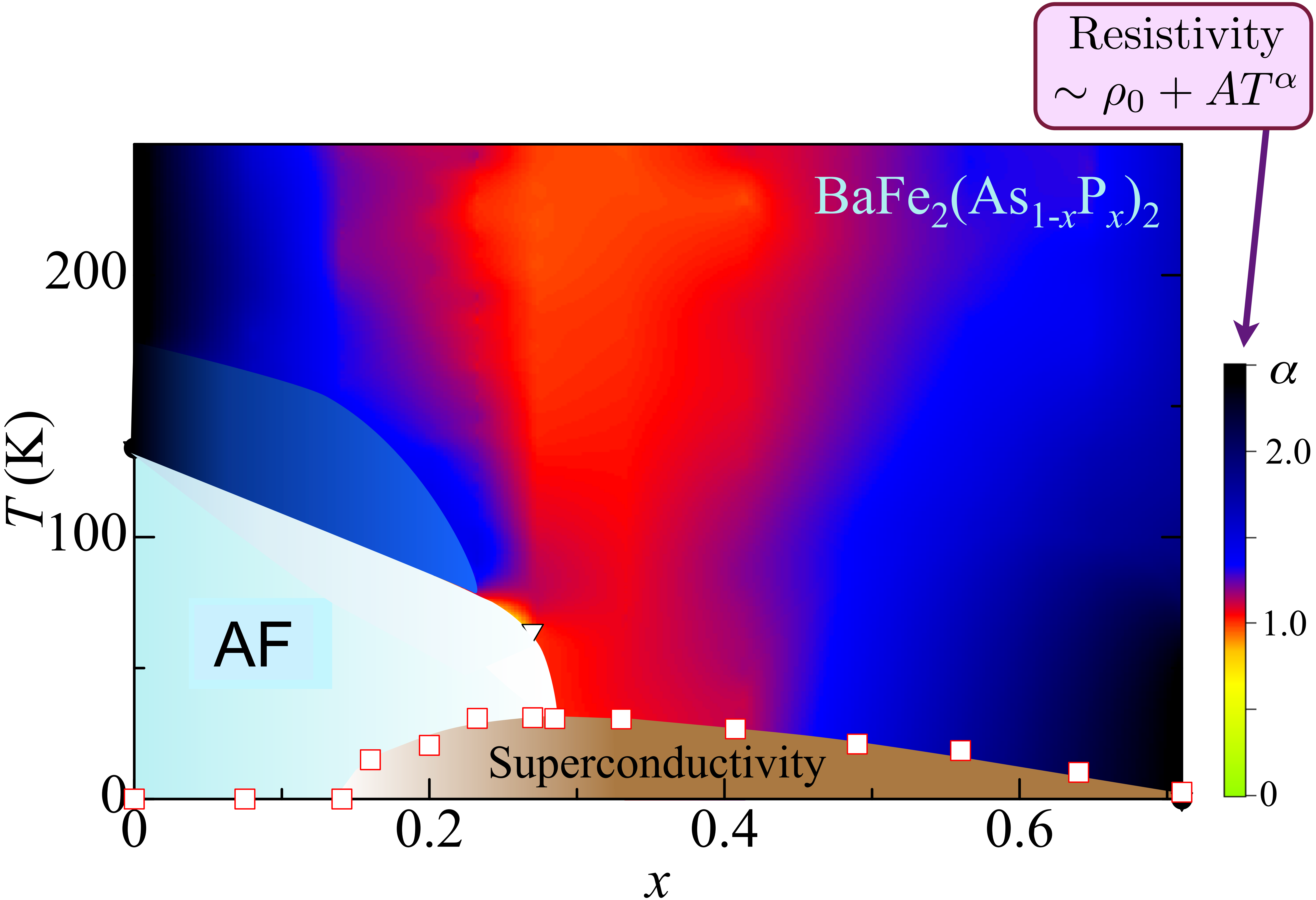
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

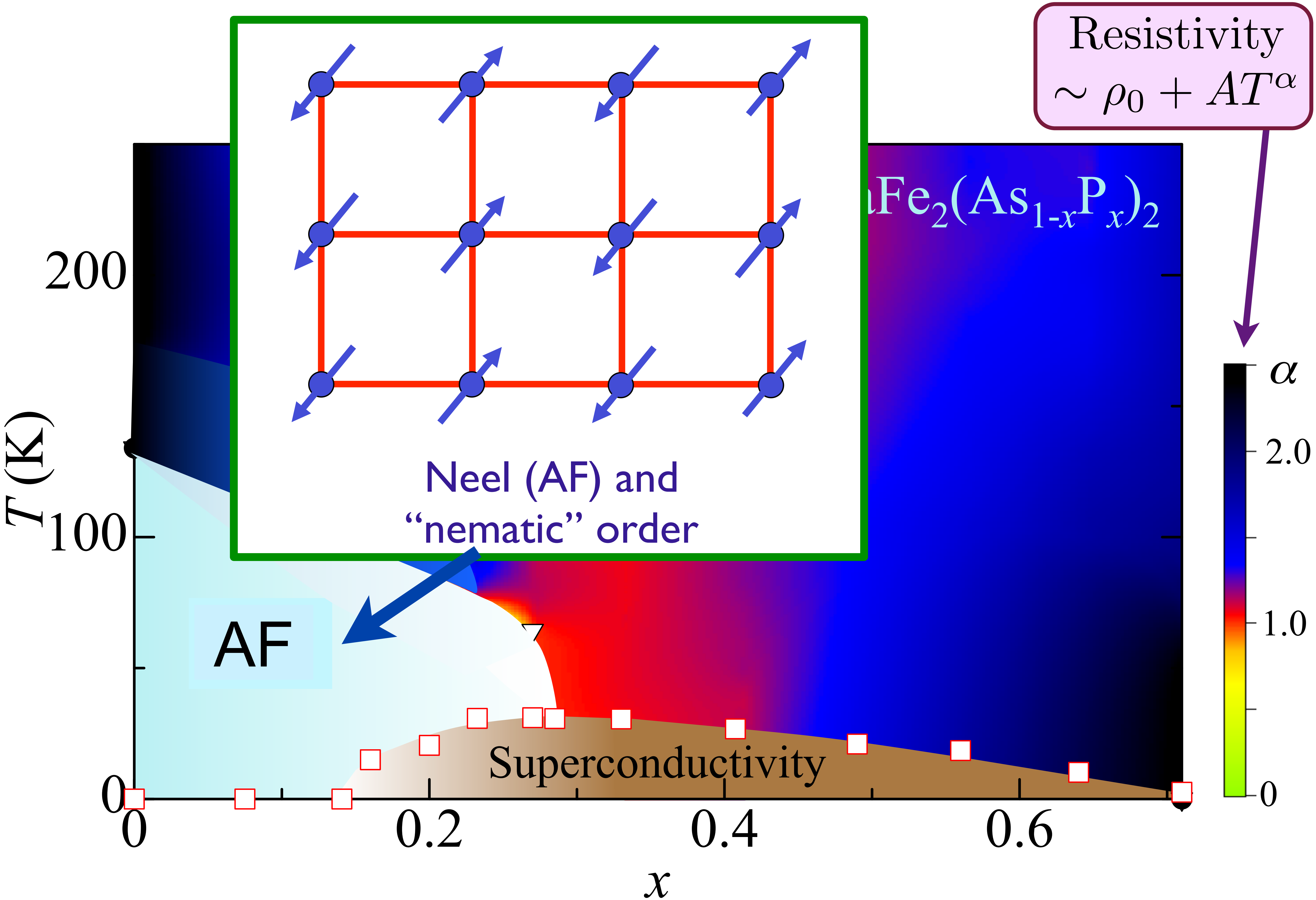
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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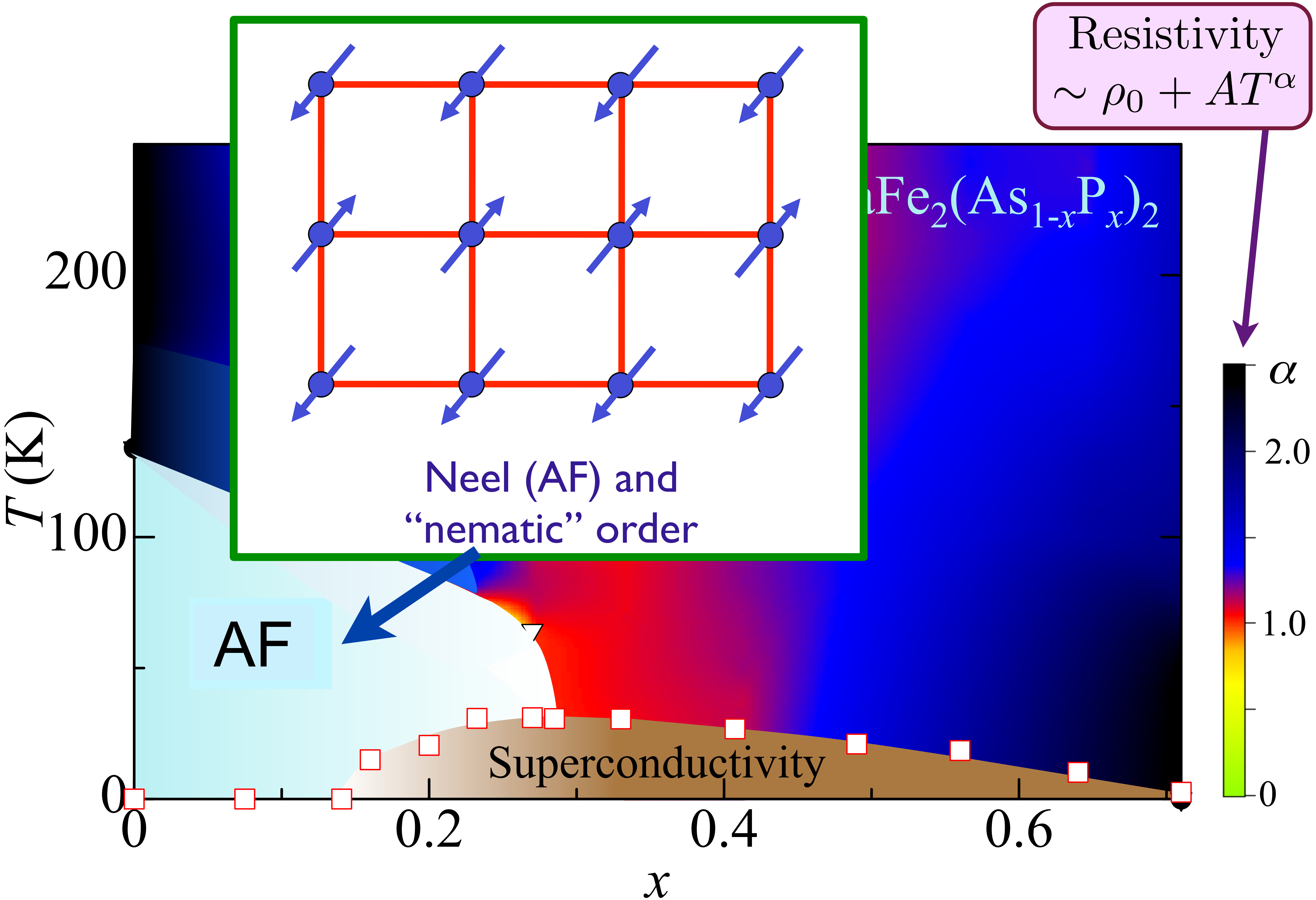

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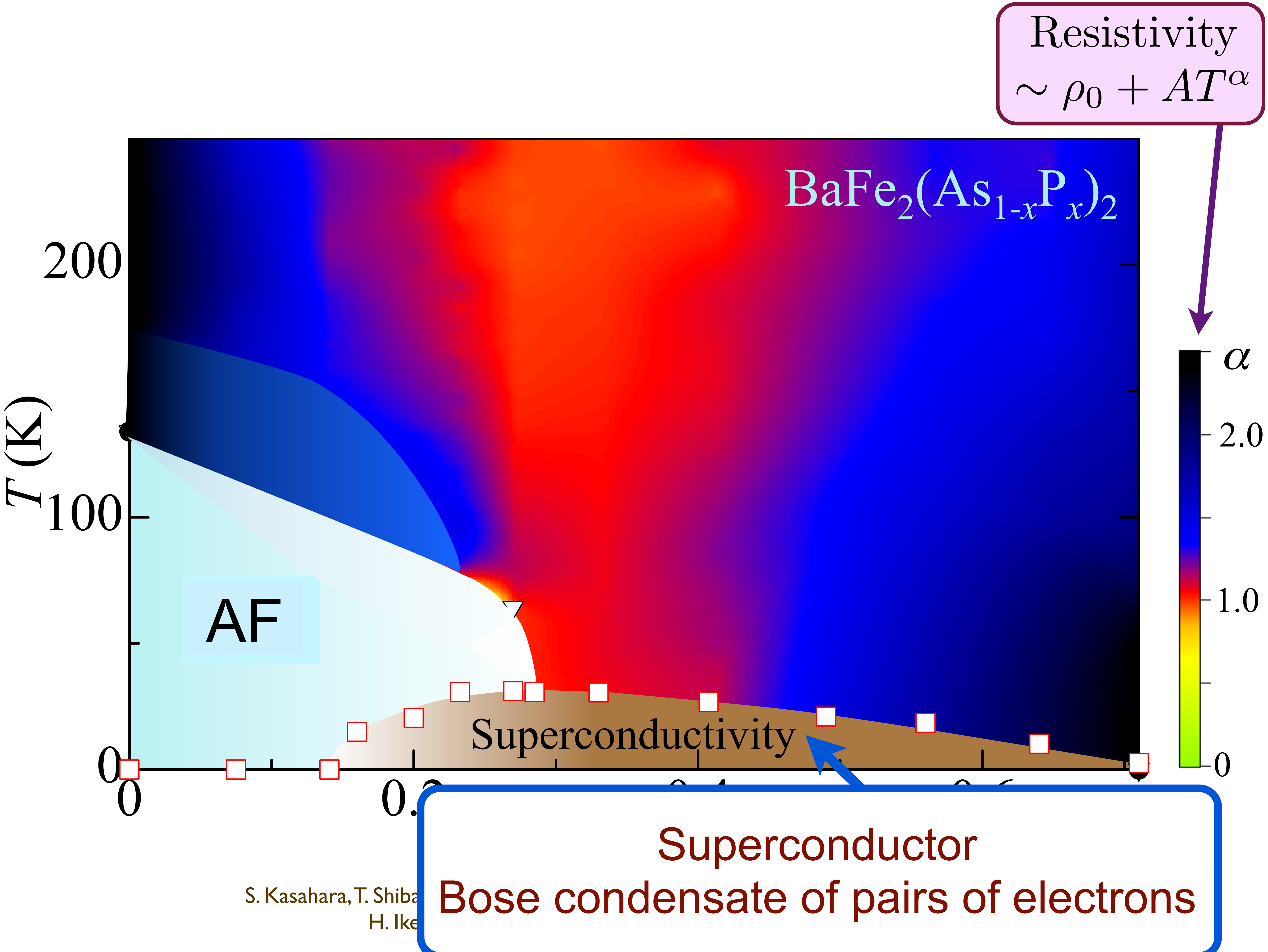
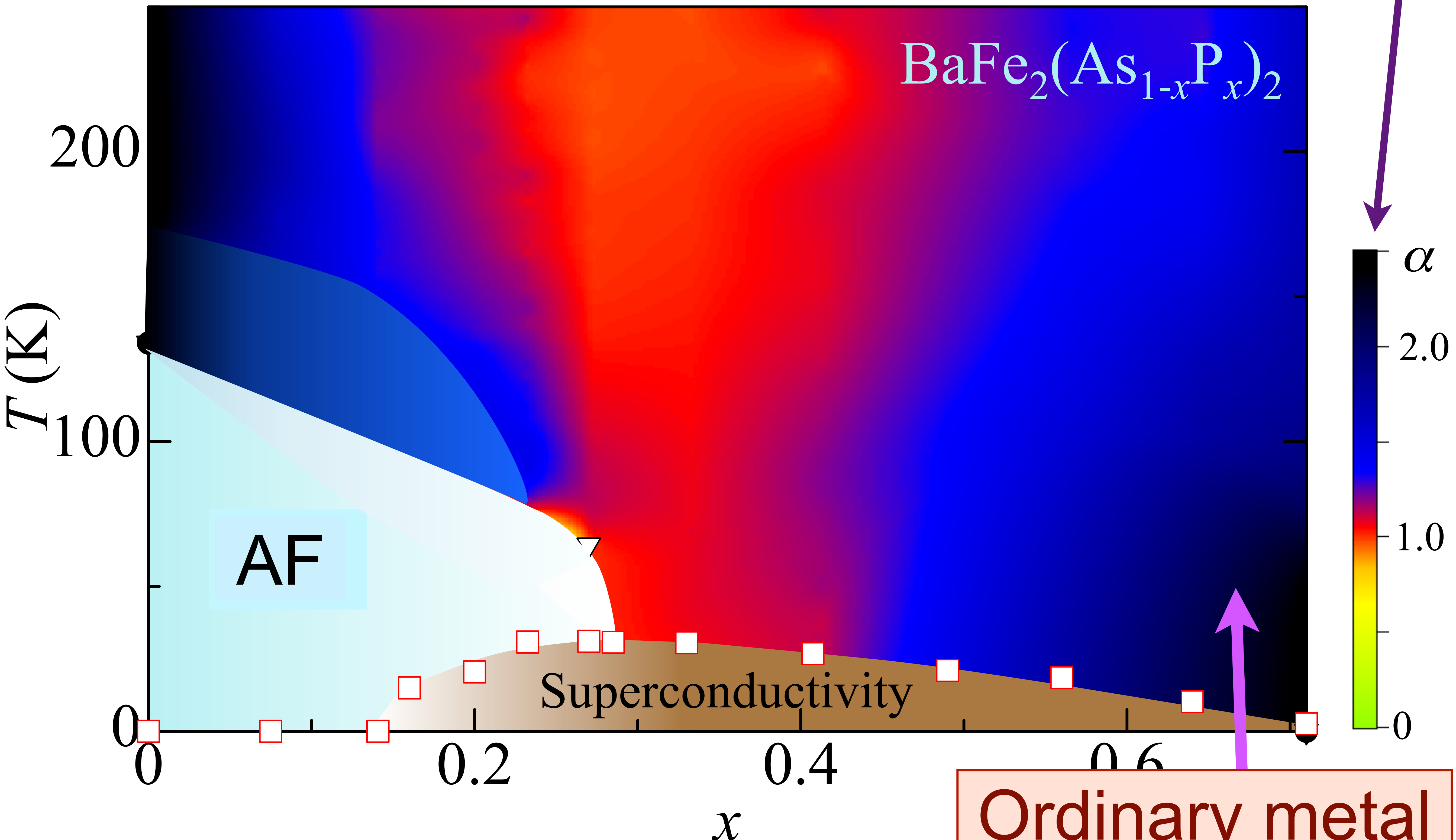


S. Kasahara, T. Shiba
H. Ike

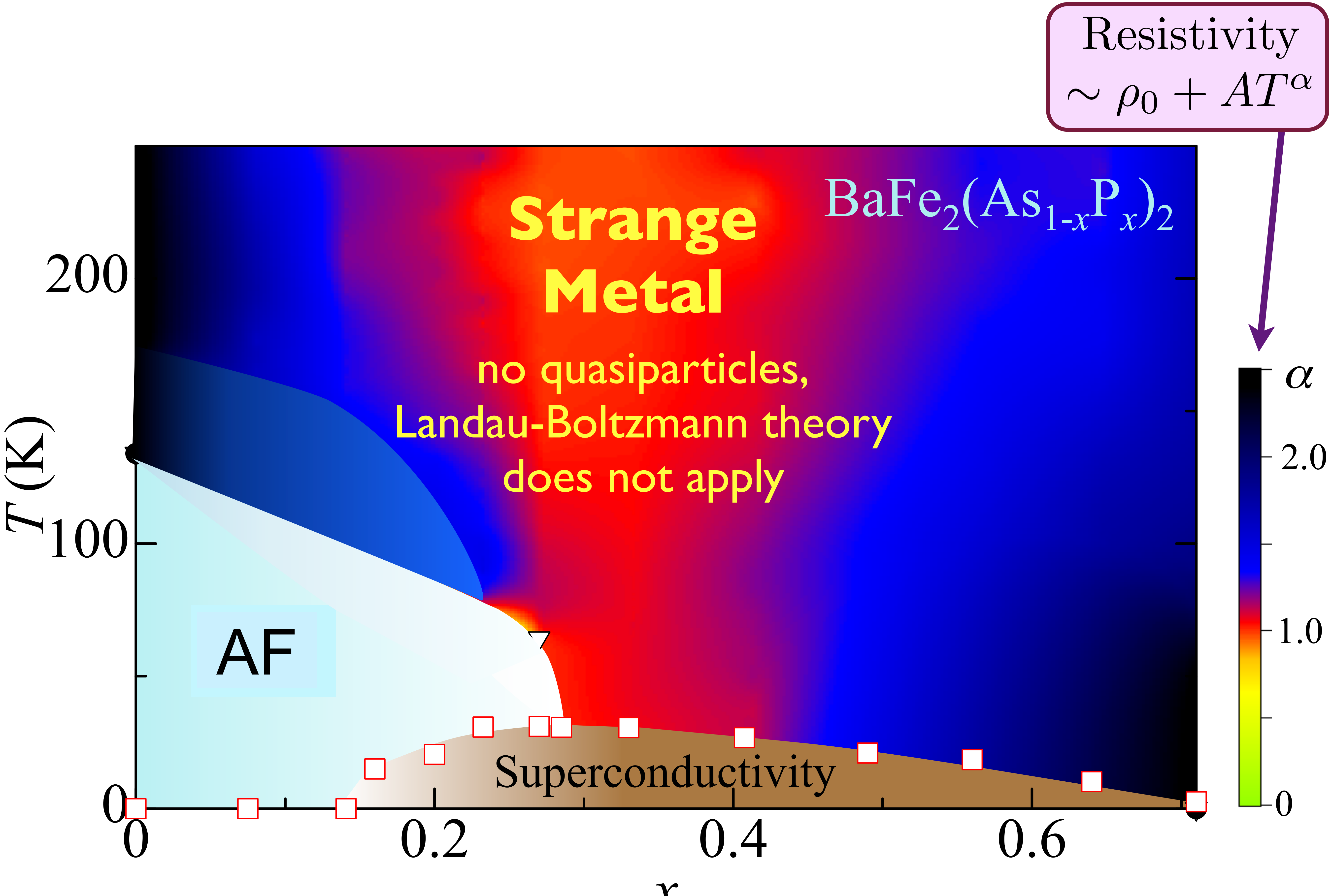
Tuesday, January 21, 14

Resistivity $\sim \rho_0 + AT^{\alpha}$



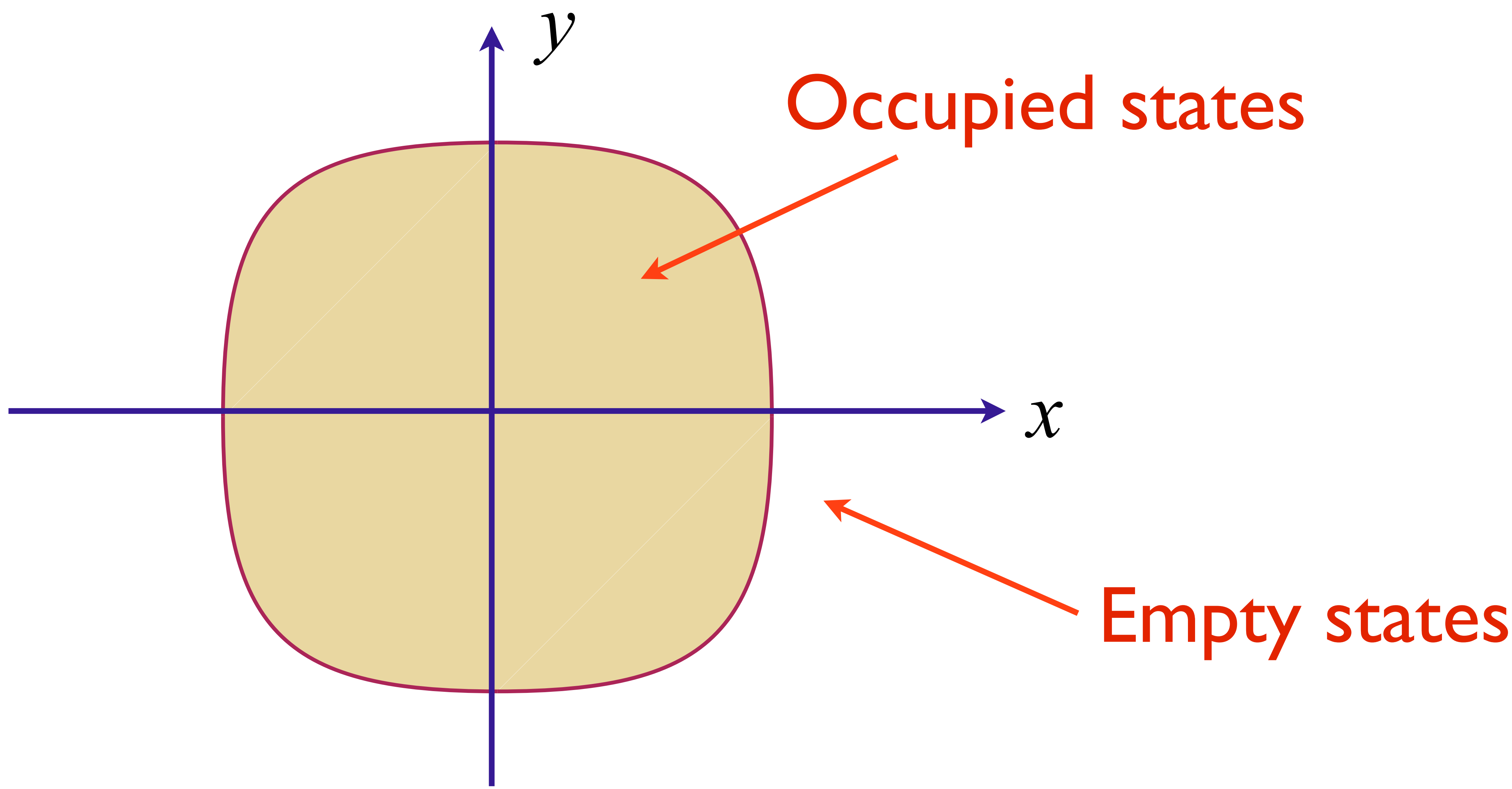
Ordinary metal
(Fermi liquid)

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. C
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
Physical Review B **81**, 184519 (2010)



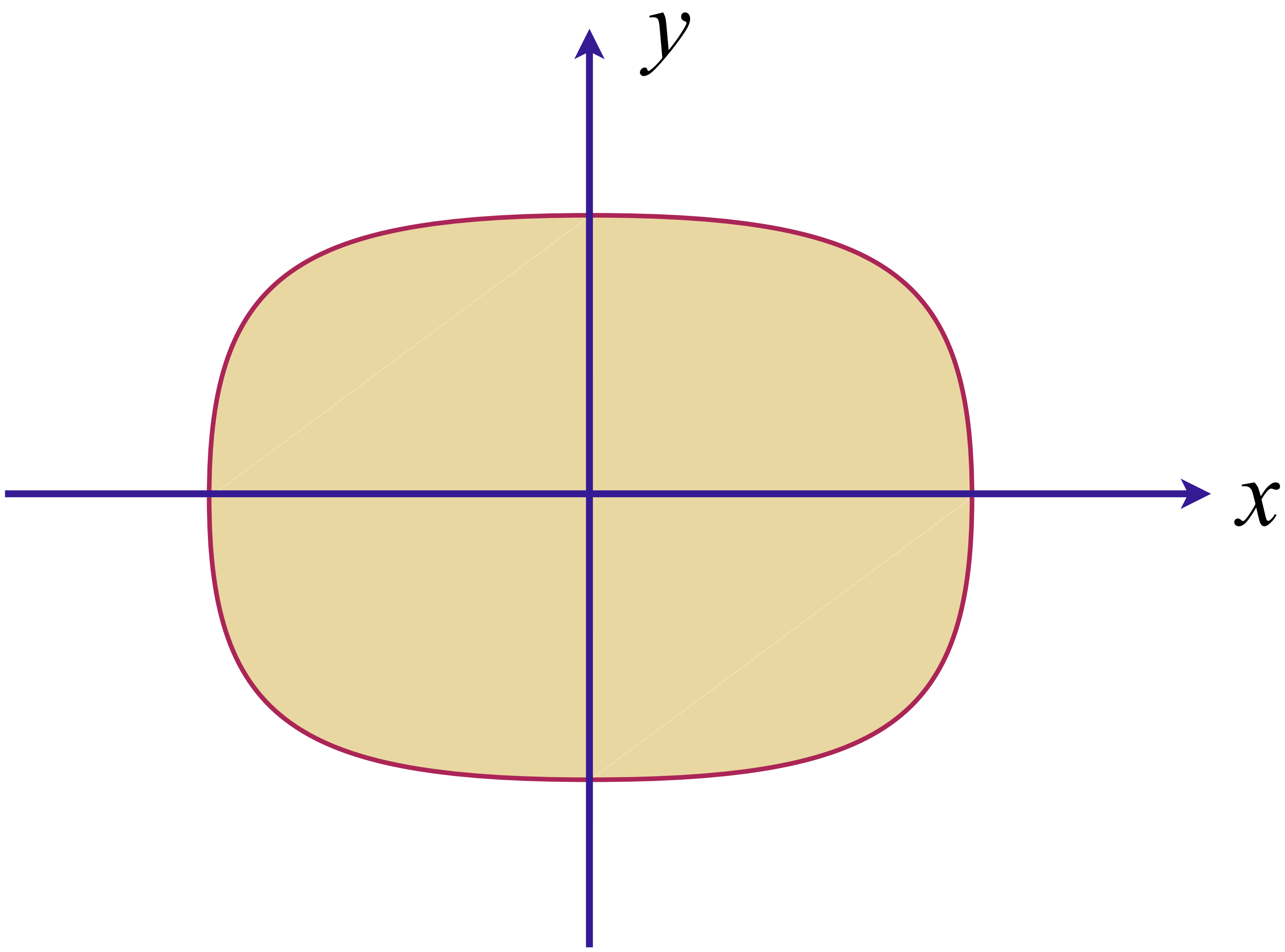
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

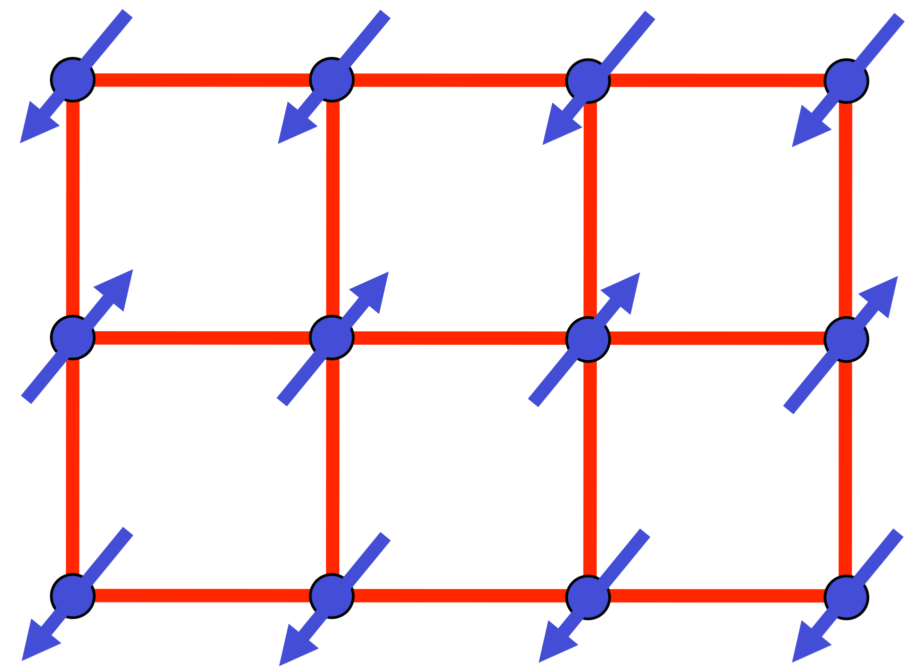
Quantum criticality of Ising-nematic ordering in a metal



A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal

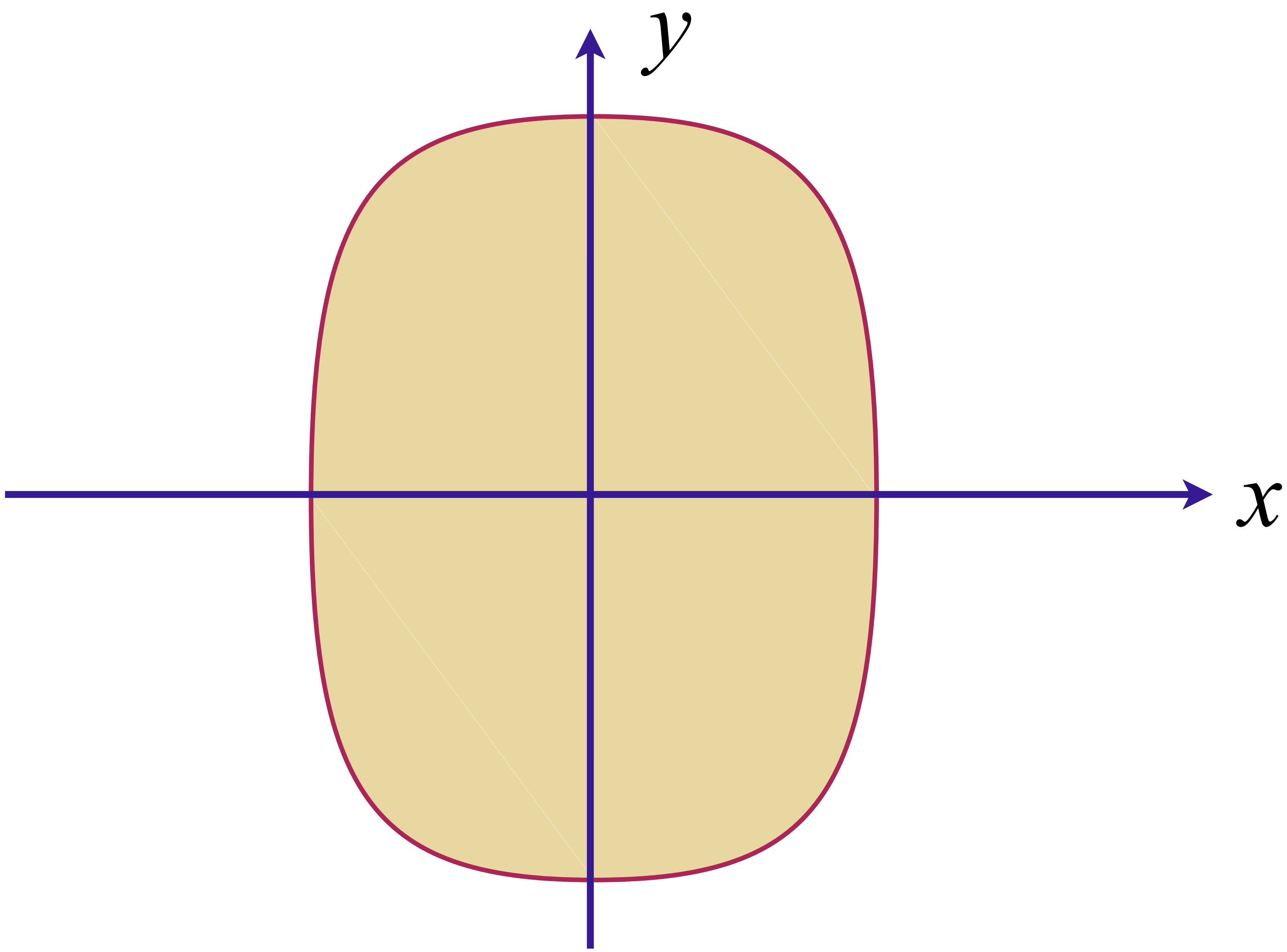


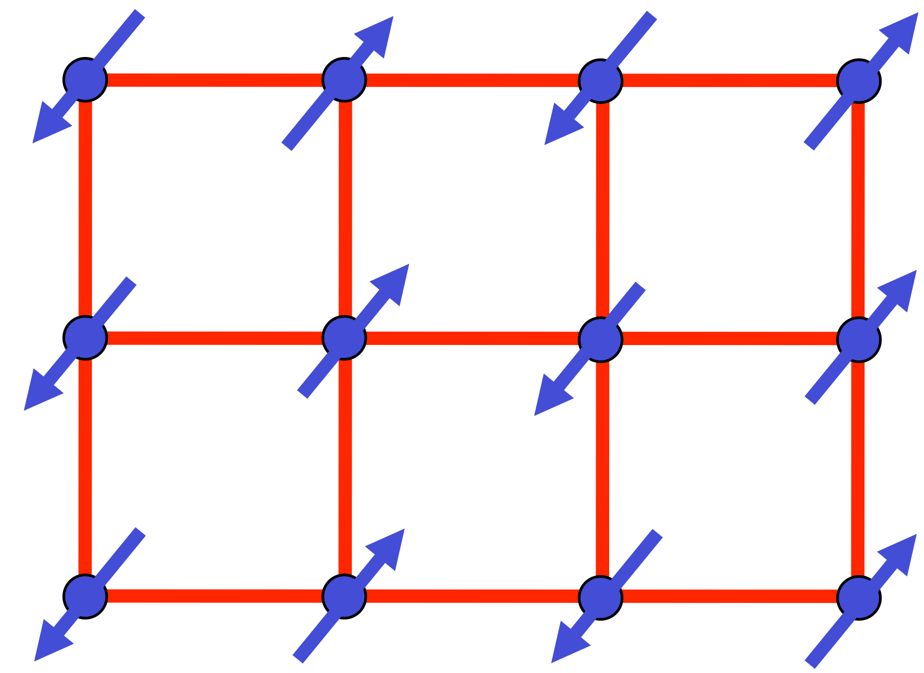


Neel (AF) and
"nematic" order

Spontaneous elongation along x direction

Quantum criticality of Ising-nematic ordering in a metal





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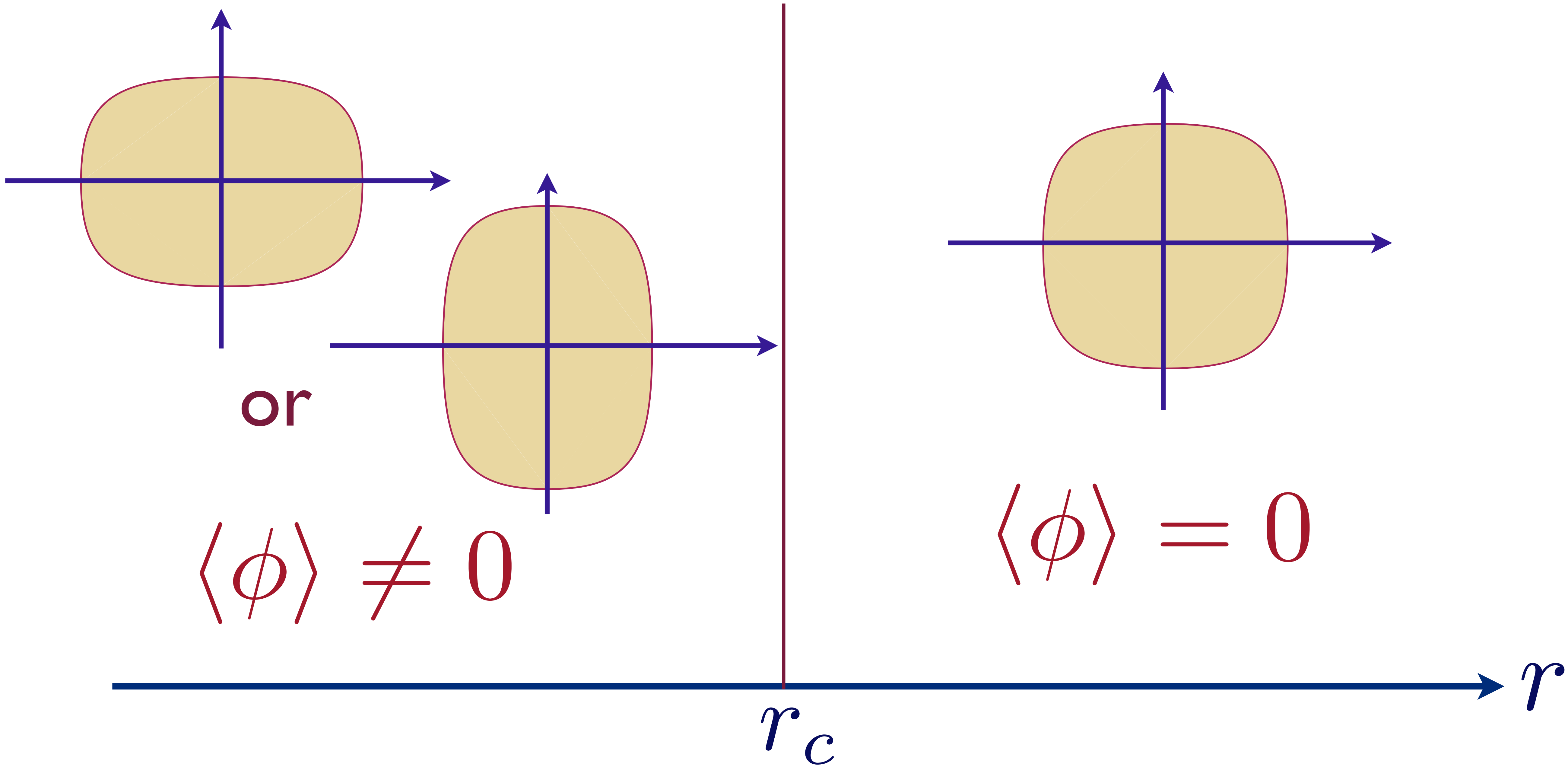
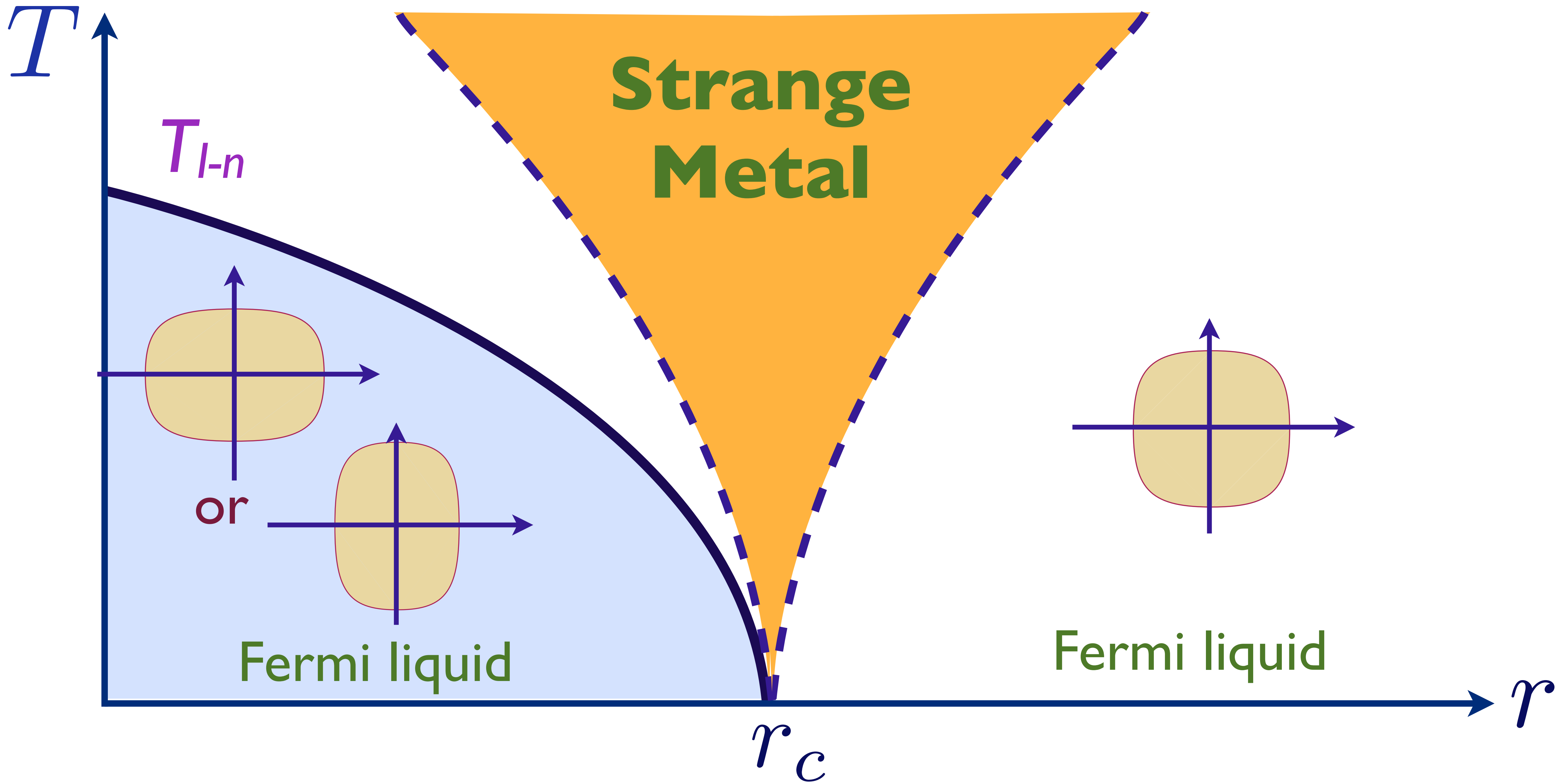


Zero temperature phase diagram as a function of some parameter r

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and r

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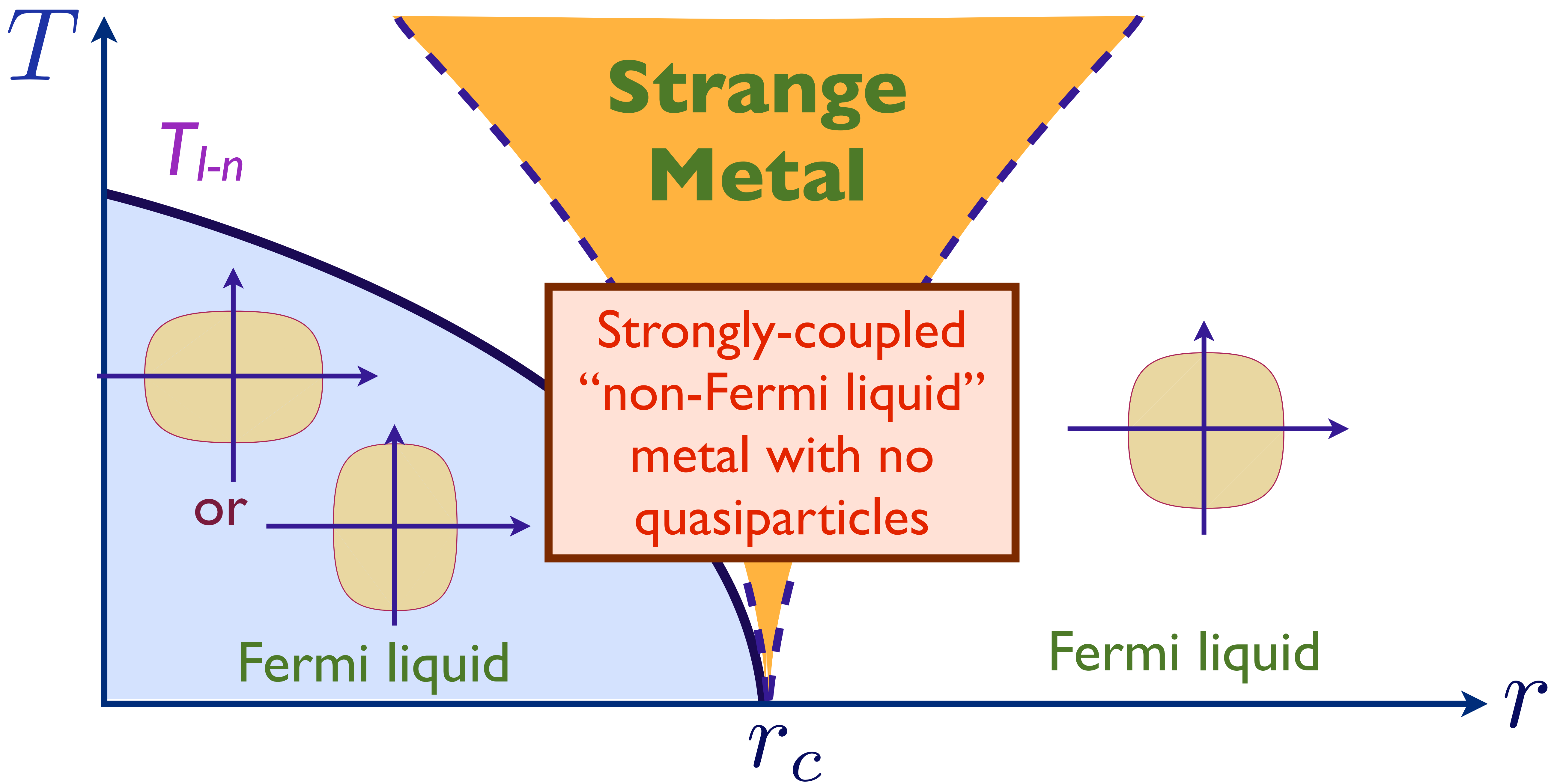

Phase diagram as a function of T and r

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- Exciting recent progress on the description of metallic states without quasiparticles, via quantum field theory and holography