Non-Abelian dualities and quantum criticality in square lattice anitferromagnets

Field theory in condensed matter: a symposium in honor of Nick Read, Yale, April 12, 2019

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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Nick's miracle years: 1988-1990

PHYSICAL REVIEW B VOLUME 40, NUMBER 10 1 OCTOBER 1989

Statistics of the excitations of the resonating-valence-bond state

N. Read and B. Chakraborty
Section of Applied Physics, Becton Center, Yale University, P.O. Box 2157, Yale Station, New Haven, Connecticut 06520
(Received 18 July 1988; revised manuscript received 15 February 1989)

By extending recent proposals for choosing the phases in resonating-valence-bond ground states to the case of excited states, we show for these wave functions that the hole excitations are charged, spinless fermions and the spin excitations are neutral, spin-$\frac{1}{2}$ bosons. We also show that for a system with periodic boundary conditions, all states are at least fourfold degenerate.

VOLUME 62, NUMBER 1 PHYSICAL REVIEW LETTERS 2 JANUARY 1989

Order Parameter and Ginzburg-Landau Theory for the Fractional Quantum Hall Effect

N. Read
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, and Section of Applied Physics, Yale University, New Haven, Connecticut 06520[a]
(Received 15 August 1988)

A new order parameter with a novel broken symmetry is proposed for the fractional quantum Hall effect, with the Laughlin state as the mean-field ground state. The classical Ginzburg-Landau theory of Girvin is derived microscopically from this starting point and exhibits all the phenomenology of the fractional quantum Hall effect.

Nick's miracle years: 1988-1990

VOLUME 62, NUMBER 14 PHYSICAL REVIEW LETTERS 3 APRIL 1989

Valence-Bond and Spin-Peierls Ground States of Low-Dimensional Quantum Antiferromagnets

N. Read and Subir Sachdev

Center for Theoretical Physics, P.O. Box 6666, and Section of Applied Physics, P.O. Box 2157, Yale University, New Haven, Connecticut 06511

(Received 23 December 1988)

The large-N limit of a nearest-neighbor SU(N) antiferromagnet on a bipartite lattice exhibits in dimensions $d \geq 2$ a zero-temperature phase transition between a Néel-ordered state and a resonanting-valence-bond state. Here it is shown in $d=1,2$ that topological effects produce spin-Peierls or valence-bond-solid order in the non-Néel phase with a ground-state degeneracy which varies periodically with "spin" for fixed N, with periodicity given by the coordination number of the lattice. Thus a non-Néel phase of the spin-$\frac{1}{2}$ Heisenberg model on a square lattice would be a spin-Peierls state with a fourfold degeneracy due to broken lattice rotational symmetry.

VOLUME 66, NUMBER 13 PHYSICAL REVIEW LETTERS 1 APRIL 1991

Large-N Expansion for Frustrated Quantum Antiferromagnets

N. Read and Subir Sachdev

Department of Applied Physics, P.O. Box 2157, and Center for Theoretical Physics, P.O. Box 6666, Yale University, New Haven, Connecticut 06520

(Received 31 August 1990)

A large-N expansion technique based on symplectic [Sp(N)] symmetry for frustrated magnetic systems is proposed and applied to the square-lattice quantum antiferromagnet with first-, second-, and third-neighbor antiferromagnetic coupling. In addition to disordered states similar to those in unfrustrated systems, phases with incommensurate coplanar spin correlations and unconfined bosonic spinons are found. The occurrence of "order from disorder" is discussed. Neither chirally ordered nor spin-nematic states are found.

Nick's miracle years: 1988-1990

Nuclear Physics B360 (1991) 362–396
North-Holland

NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

Gregory MOORE

Department of Physics, Yale University, New Haven, CT 06511, USA

Nicholas READ

Departments of Applied Physics and Physics, Yale University, New Haven, CT 06520, USA

Received 31 May 1990
(Revised 5 December 1990)

Applications of conformal field theory to the theory of fractional quantum Hall systems are discussed. In particular, Laughlin's wave function and its cousins are interpreted as conformal blocks in certain rational conformal field theories. Using this point of view a hamiltonian is constructed for electrons for which the ground state is known exactly and whose quasihole excitations have nonabelian statistics; we term these objects "nonabelions". It is argued that universality classes of fractional quantum Hall systems can be characterized by the quantum numbers and statistics of their excitations. The relation between the order parameter in the fractional quantum Hall effect and the chiral algebra in rational conformal field theory is stressed, and new order parameters for several states are given.

HAPPY
BIRTHDAY

NICK

and many more miracle years !

1. Neel-VBS criticality in square lattice antiferromagnets

2. Recent experimental and numerical results

3. Critical theory for onset of semion topological order

4. More non-Abelian dualities

1. Neel-VBS criticality in square lattice antiferromagnets

2. Recent experimental and numerical results

3. Critical theory for onset of semion topological order

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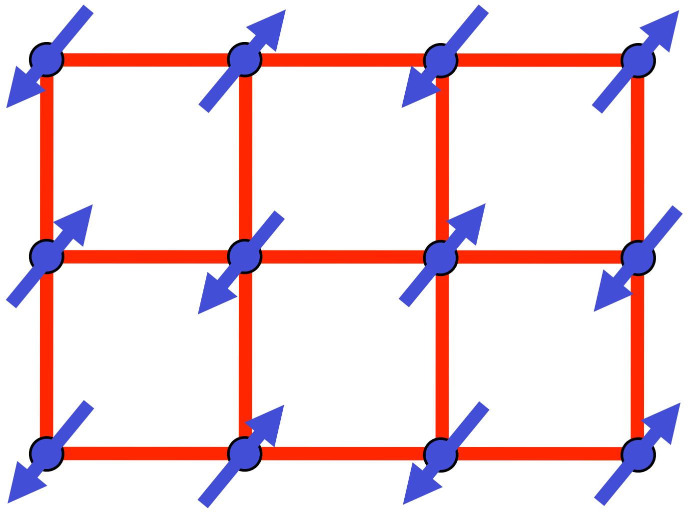
Quantum criticality in a frustrated square lattice antiferromagnet

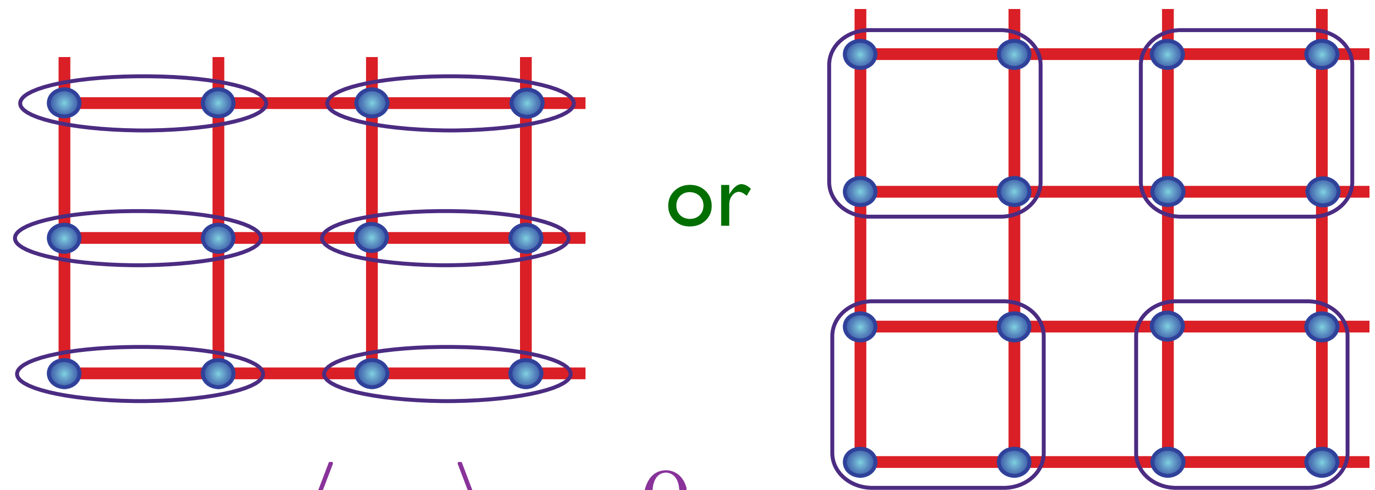
N. Read and S. Sachdev, PRL **62**, 1694 (1989)



$\langle z_\alpha \rangle \neq 0$

Néel state

$\vec{N} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$



$\langle z_\alpha \rangle = 0$

Valence bond solid (VBS) state, V_x, V_y
with a nearly gapless, emergent "photon"

s_c s

Critical $\mathbb{CP}^1$ theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A non-Abelian duality

Critical U(1) gauge (a_μ) theory of $N_b = 2$ relativistic bosons
is dual to
SU(2) gauge (A_μ) theory of $N_f = 2$ Dirac fermions.

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f \right]$$

Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

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$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s |z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f \right]$$

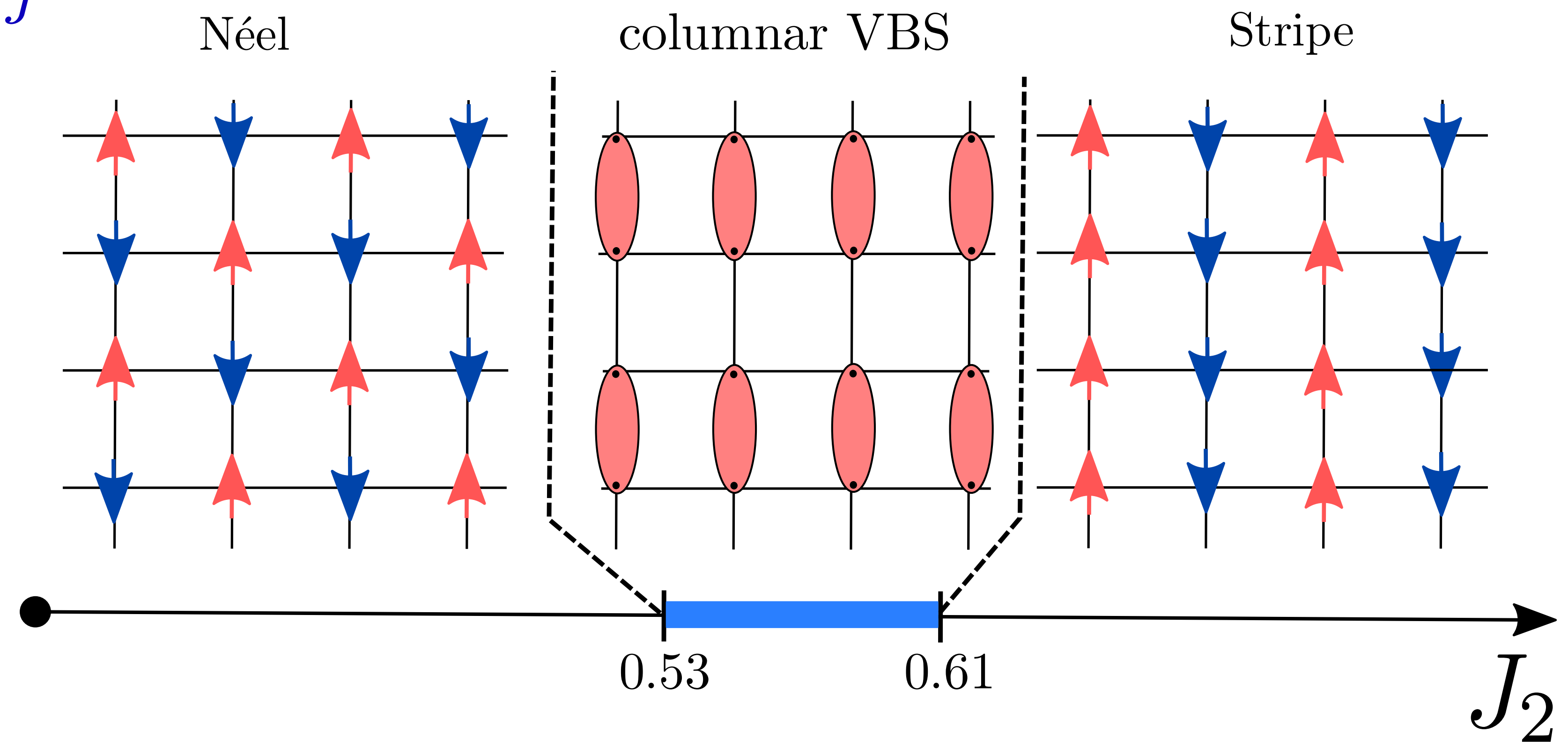
The fermion theory has a SO(5) global flavor symmetry, and the gauge-invariant fermion bilinears form a SO(5) vector which transforms as the Néel and VBS order parameters!

$$(N_x, N_y, N_z, V_x, V_y)$$

Akihiro Tanaka and Xiao Hu, PRL. **95**, 036402 (2005).
T. Senthil and M.P.A. Fisher, PRB **74**, 064405 (2006)
Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



$U(1)$-symmetric infinite projected entangled-pair states study of the spin-1/2 square J_1-J_2 Heisenberg model

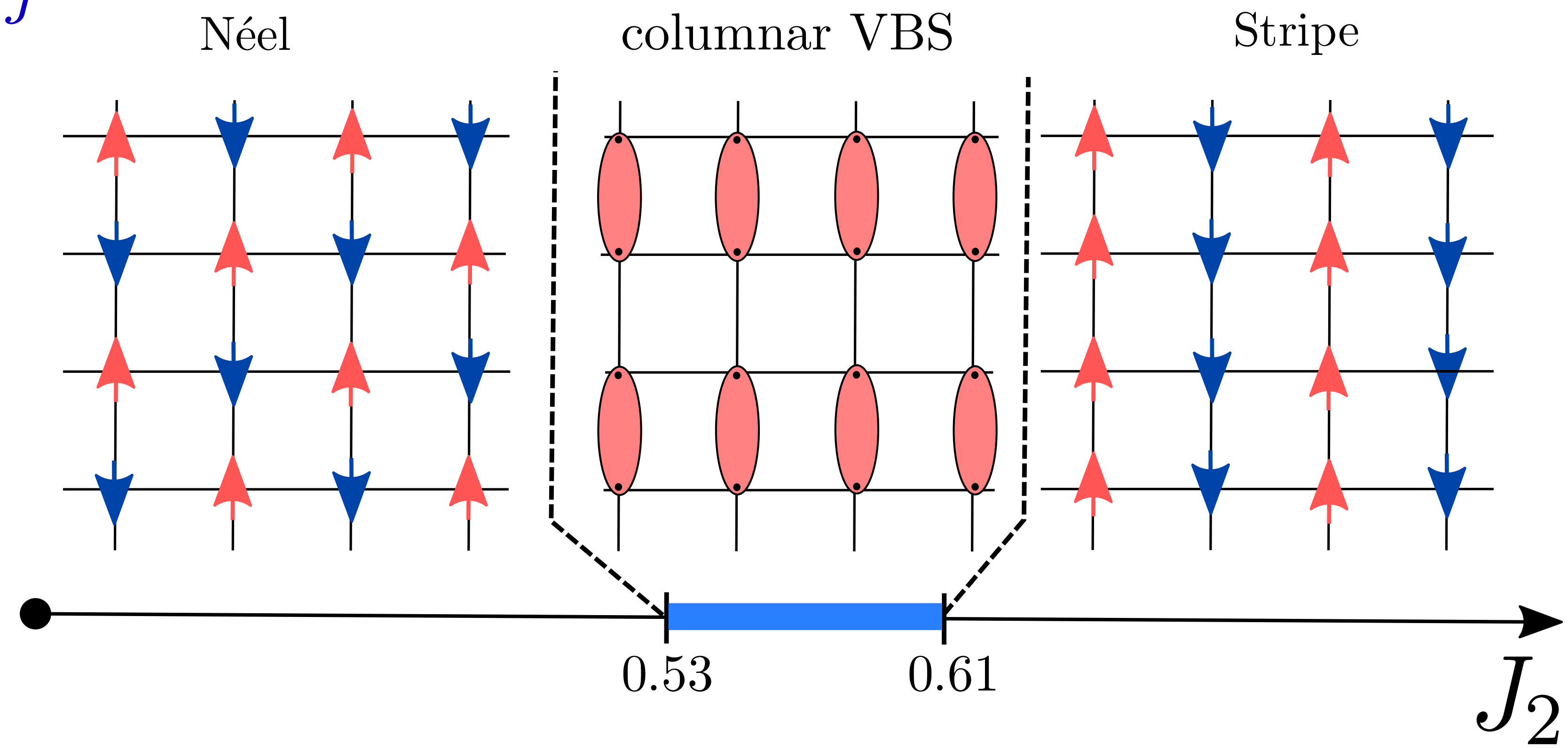
PHYSICAL REVIEW B **97**, 174408 (2018)

R. Haghshenas and D. N. Sheng

By studying the finite-D scaling of the magnetically order parameter, we find a Néel phase for $J_2/J_1 < 0.53$. For $0.53 < J_2/J_1 < 0.61$, a nonmagnetic columnar valence bond solid (VBS) state is established as observed by the pattern of local bond energy. The divergent behavior of correlation length $\xi \sim D^{1.2}$ and vanishing order parameters are consistent with a deconfined Néel-to-VBS transition at $J_2^{c1}/J_1 = 0.530(5)$, where estimated critical anomalous exponents are $\eta_s \sim 0.6$ and $\eta_d \sim 1.9$ for spin and dimer correlations, respectively.

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



Critical Level Crossings and Gapless Spin Liquid in the Square-Lattice Spin-1/2 J_1-J_2 Heisenberg Antiferromagnet

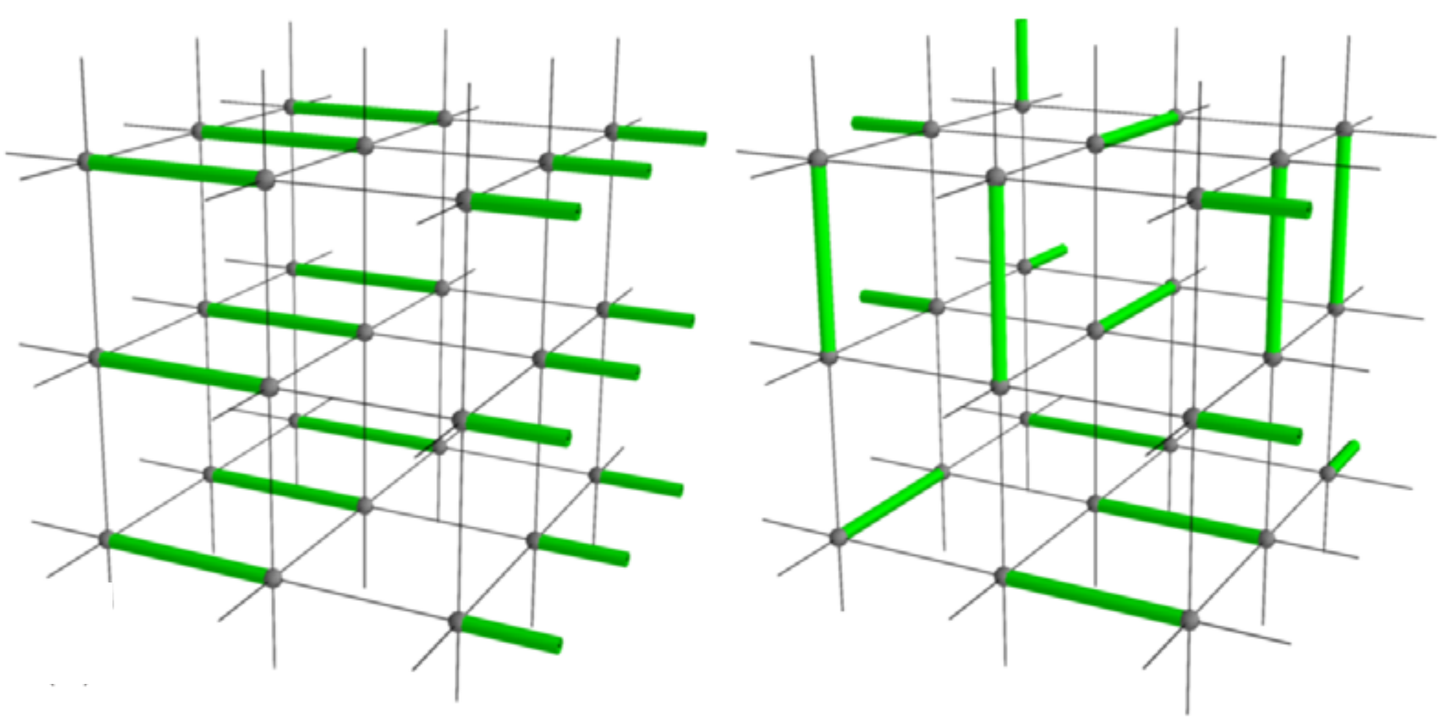
PHYSICAL REVIEW LETTERS **121,** 107202 (2018)

Ling Wang[1,*] and Anders W. Sandvik[2,1,3,†]

The lowest singlet-triplet and singlet-quintuplet crossings converge rapidly (with corrections $\propto L^{-2}$) to different g values, and we argue that these correspond to ground-state transitions between the Néel antiferromagnet and a gapless spin liquid, at $g_{c1} \approx 0.46$, and between the spin liquid and a valence-bond solid at $g_{c2} \approx 0.52$.

Emergent SO(5) Symmetry at the Columnar Ordering Transition in the Classical Cubic Dimer Model

"Studying linear system sizes up to L=96, we find that this symmetry applies with an excellent precision, consistently improving with system size over this range. It is remarkable that SO(5) emerges in a system as basic as the cubic dimer model, with only simple discrete degrees of freedom. Our results are important evidence for the generality of the SO(5) symmetry that has been proposed for the noncompact CP^1 field theory. We describe an interpretation for these results in terms of a consistent hypothesis for the renormalization-group flow structure, allowing for the possibility that SO(5) may ultimately be a near-symmetry rather than exact."



G.J. Sreejith, Stephen Powell, and Adam Nahum
PRL **122**, 080601 (2019)

Stephen Powell and John T. Chalker,
PRB **80**, 134413 (2009)

1. Neel-VBS criticality in square lattice antiferromagnets

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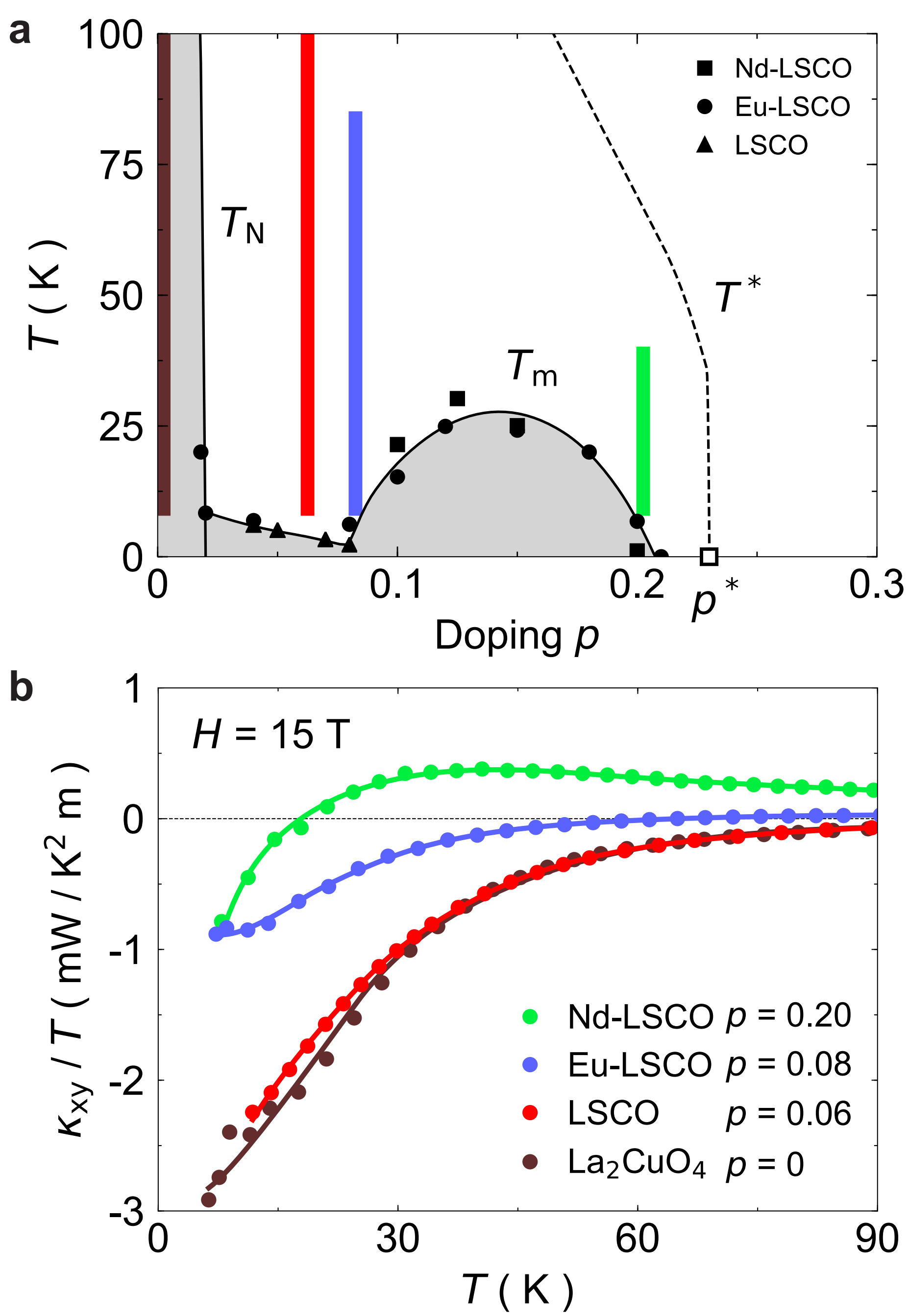
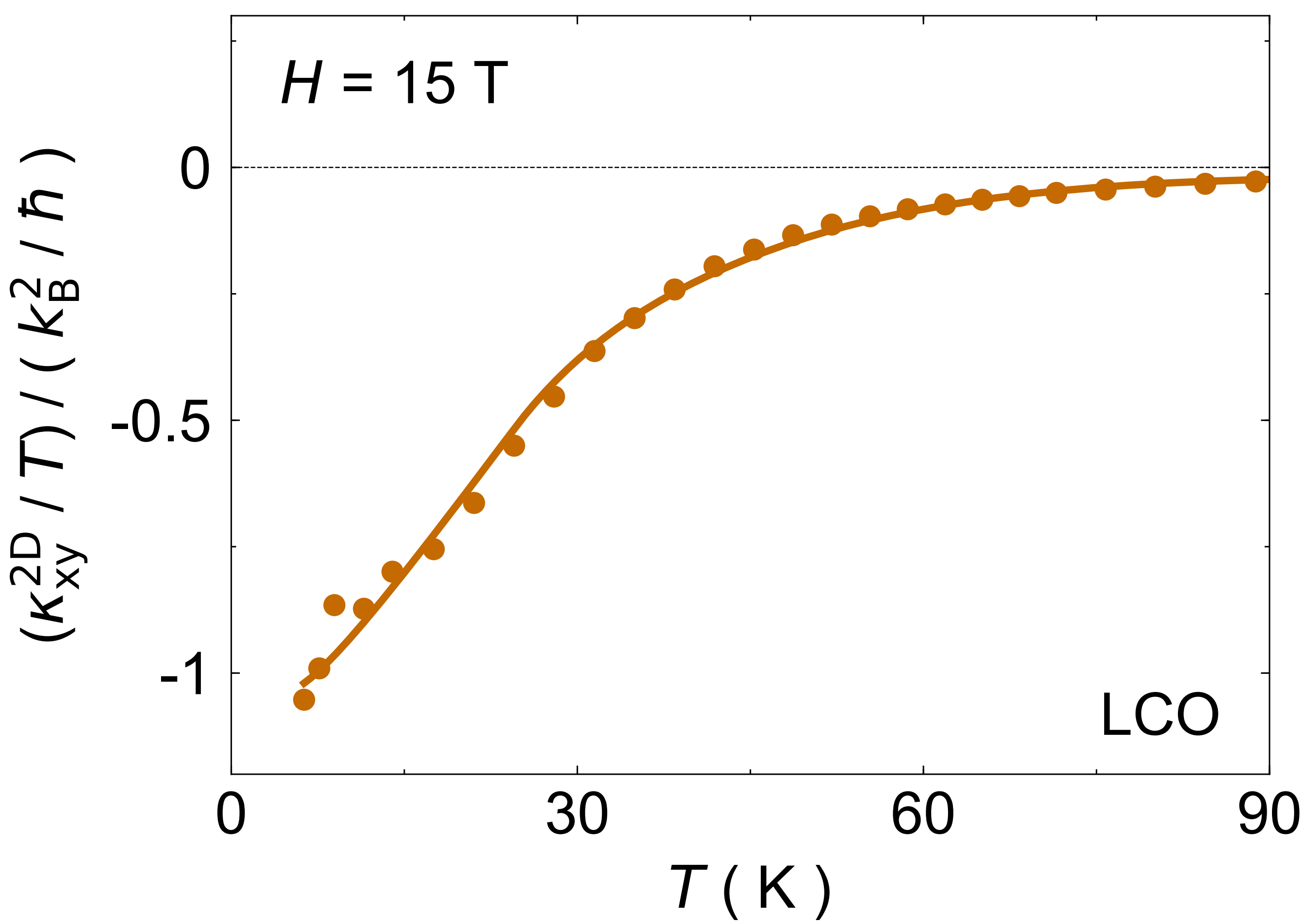
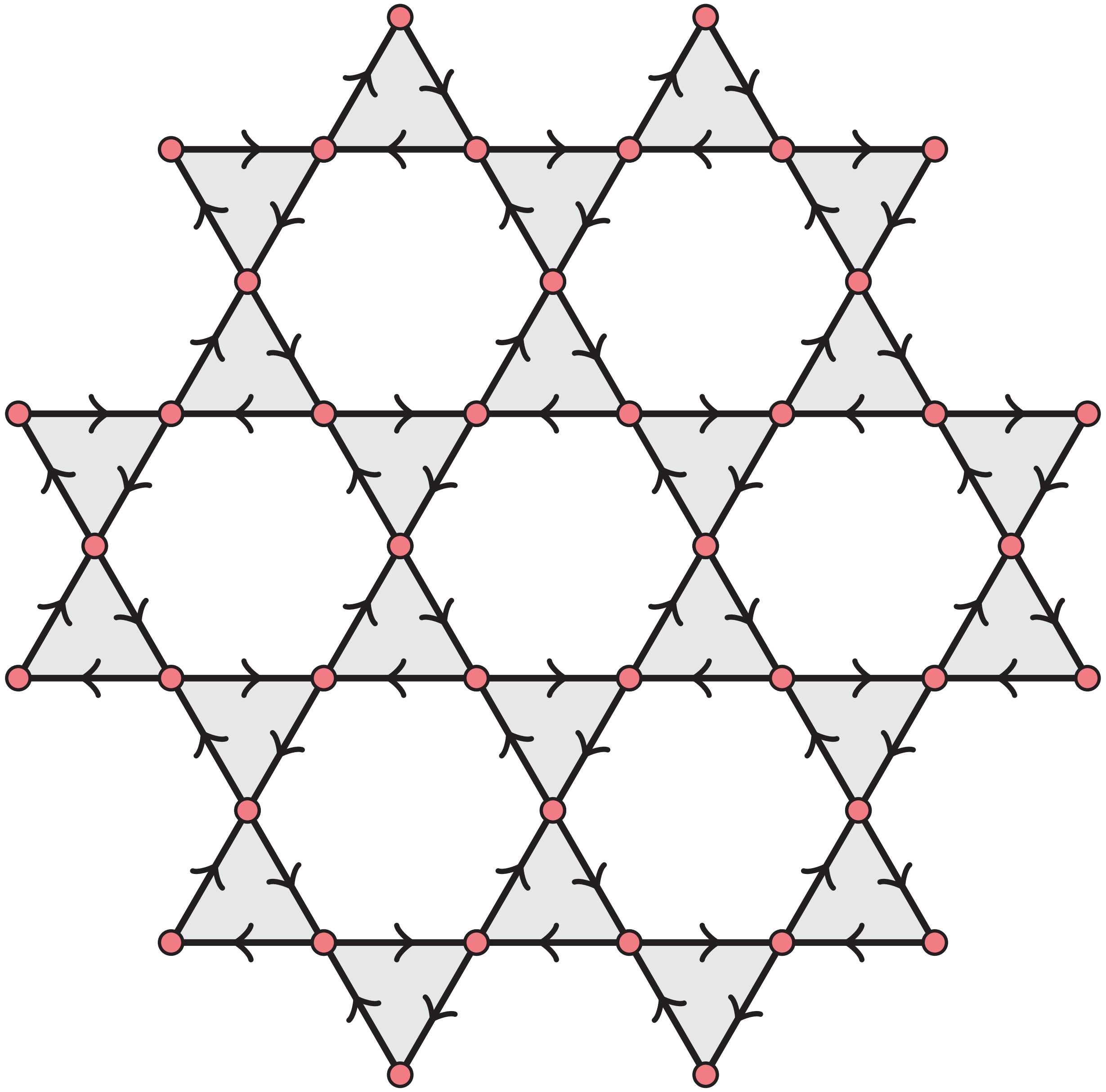
Giant thermal Hall conductivity from neutral excitations in the pseudogap phase of cuprates

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko, M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama, H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer, arXiv:1901.03104







$$H = H_1 + H_\chi$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

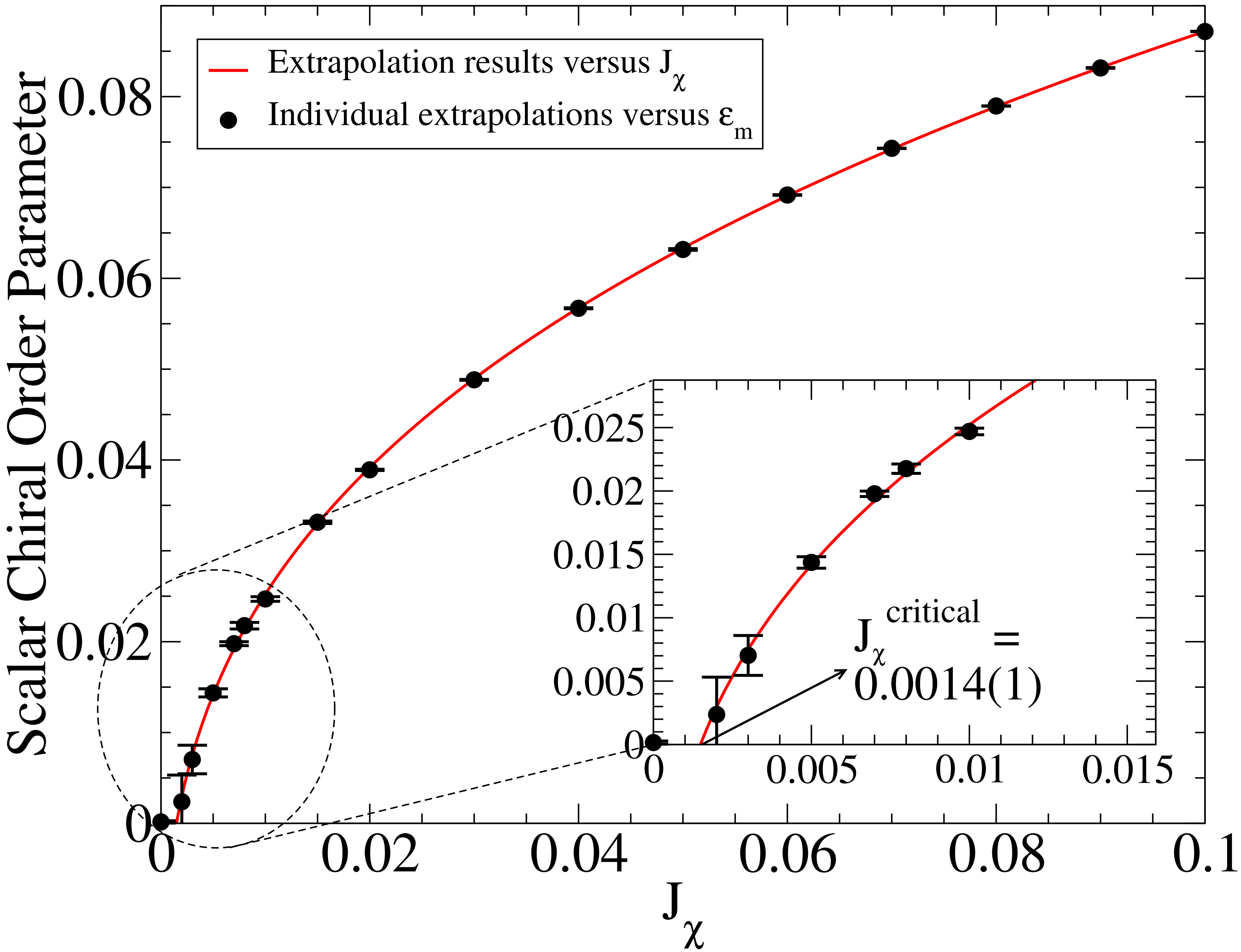
$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

B. Bauer, L. Cincio, B.P. Keller, M. Dolfi, G. Vidal, S. Trebst and A.W.W. Ludwig,
Nature Communications **5**, 5137 (2014)

Semion topological order,
i.e. the Kalmeyer-Laughlin chiral spin liquid,
appears for $J_\chi/J > 0.01$.

R. Haghshenas, Shou-Shu Gong, and D.N. Sheng, arXiv:1812.11436

Triangular lattice antiferromagnet

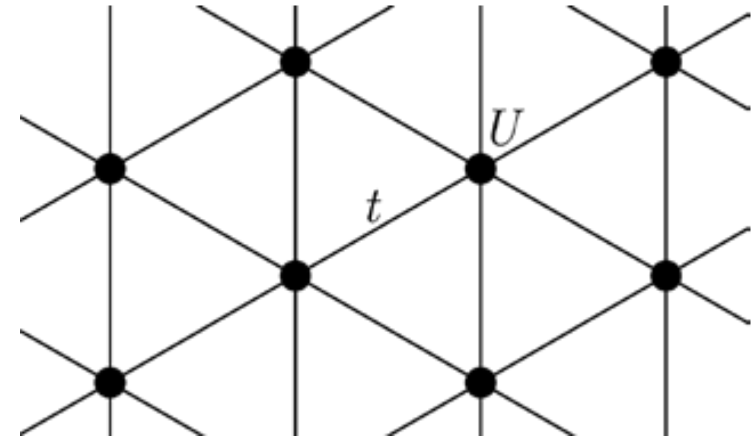


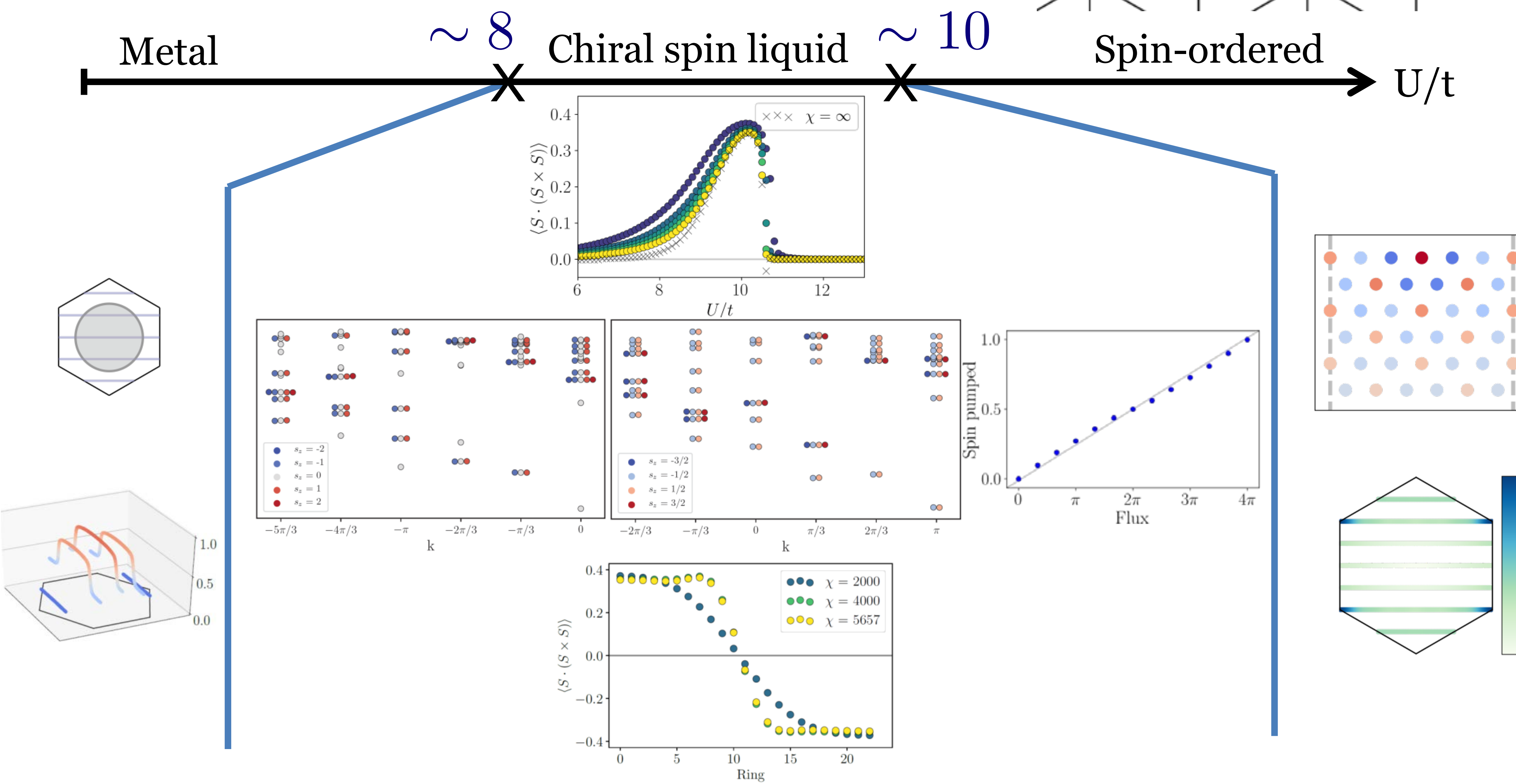
$J_2/J_1 = 1/8$; critical $J_\chi = 0.0014$

S. N. Saadatmand and I. P. McCulloch, PRB **96**, 075117 (2017)

Hubbard model:

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(c^{\dagger}_{i\sigma} c_{j\sigma} + \mathrm{h.c.} \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$





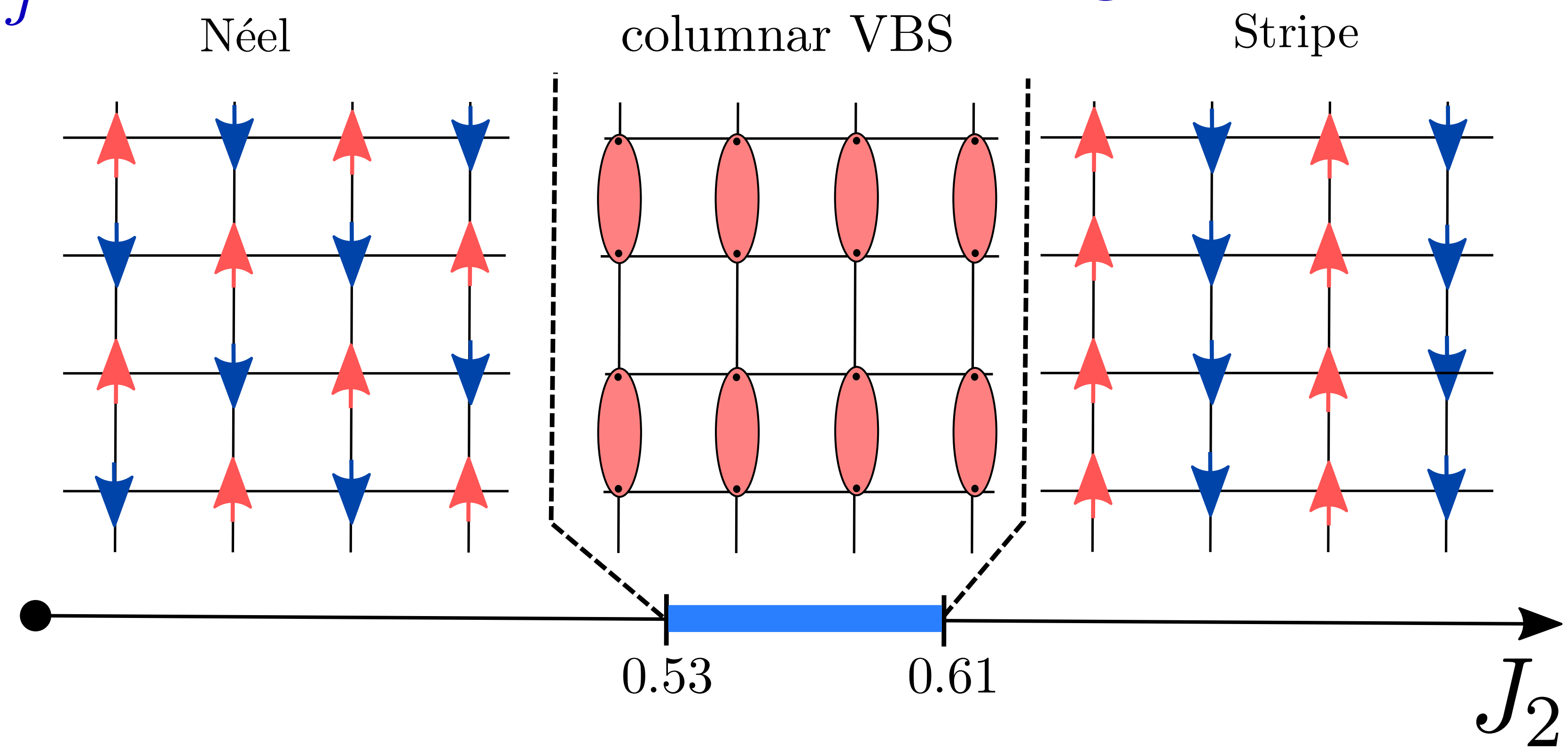
A. Szasz, J. Motruk, M. P. Zaletel, and J. E. Moore, arXiv: 1808.00463

(from slides by Aaron Szasz)

Square lattice antiferromagnet

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



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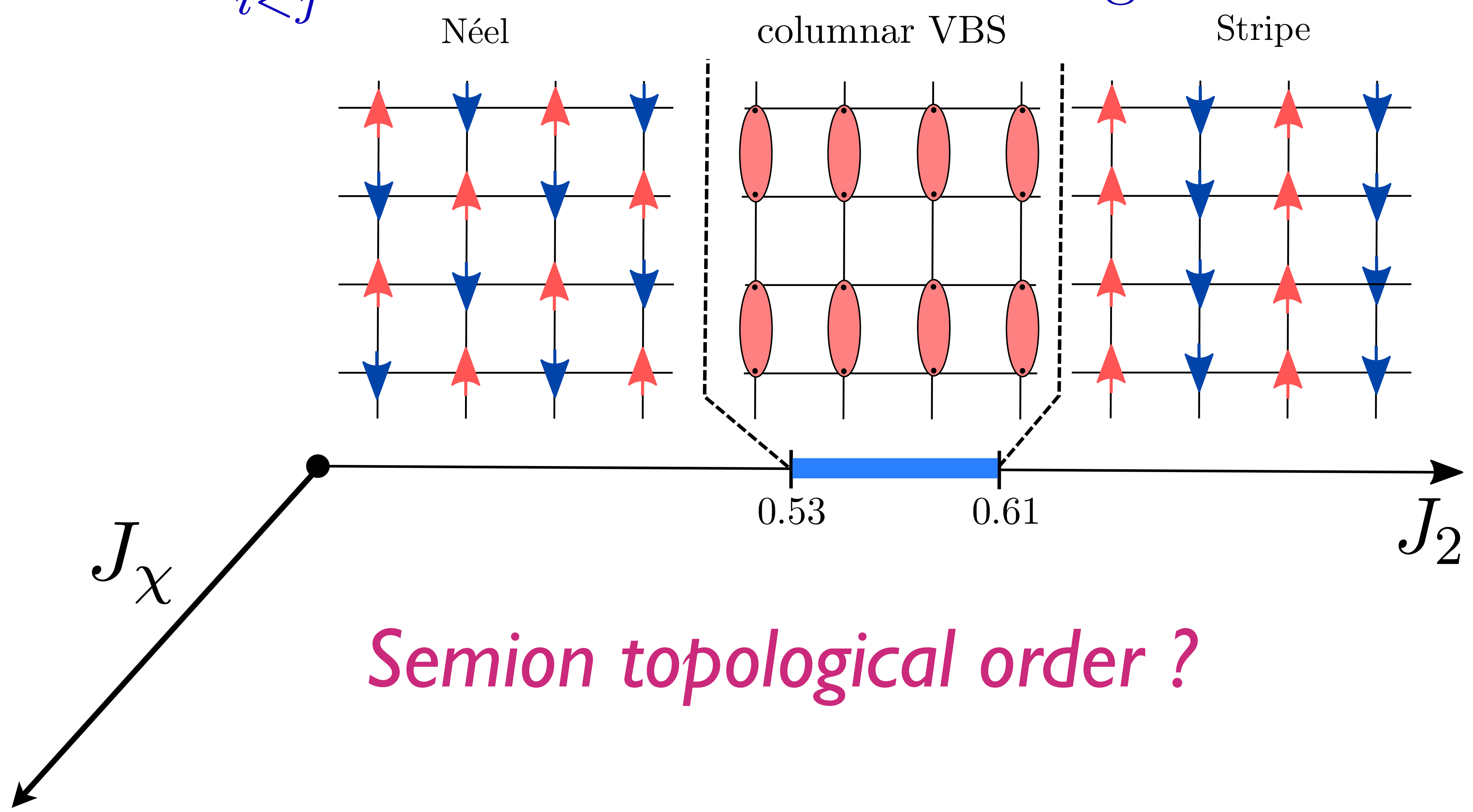
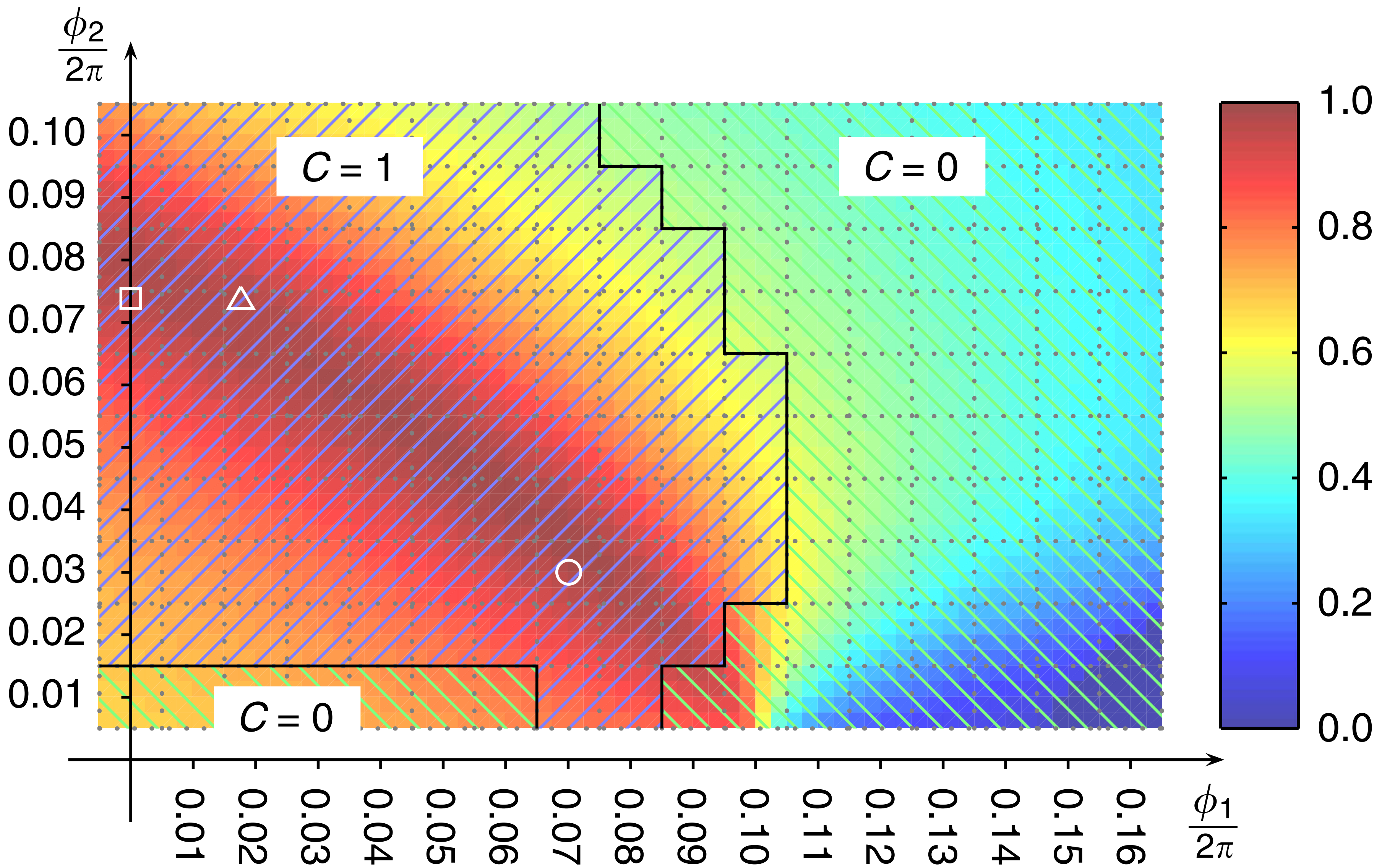


Semion topological order ?

$$H = H_1 + H_\chi$$

$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

Square lattice antiferromagnet



$$J_1 = 2J\cos(\phi_1)\cos(\phi_2)$$
$$J_2 = 2J\sin(\phi_1)\cos(\phi_2)$$
$$J_\chi = 4J\sin(\phi_2)$$

Anne E.B. Nielsen, German Sierra, and J. Ignacio Cirac,
Nature Communications **4**, 2864 (2013)

Giant thermal Hall conductivity from neutral excitations in the pseudogap phase of cuprates

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko,
M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama,
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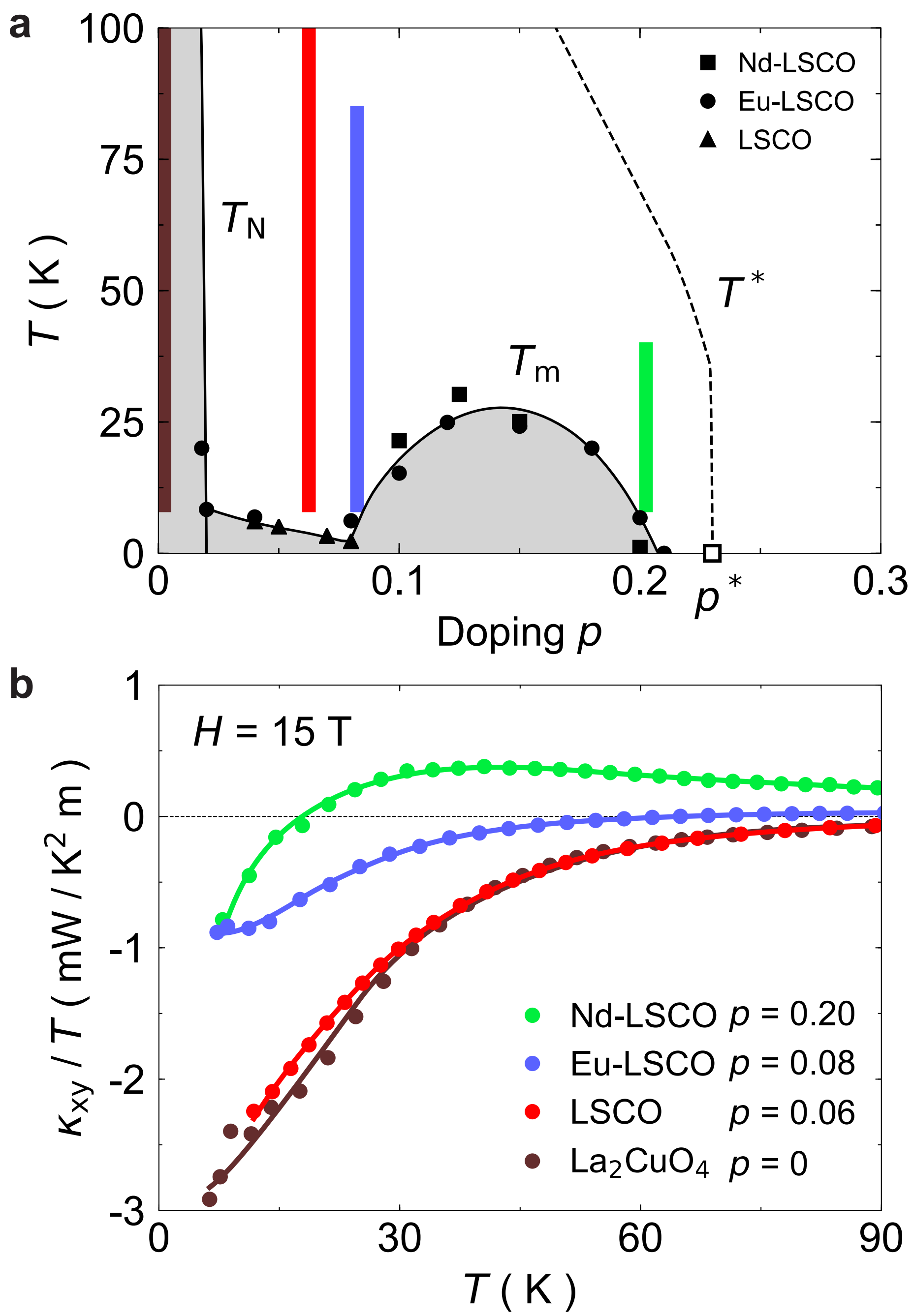
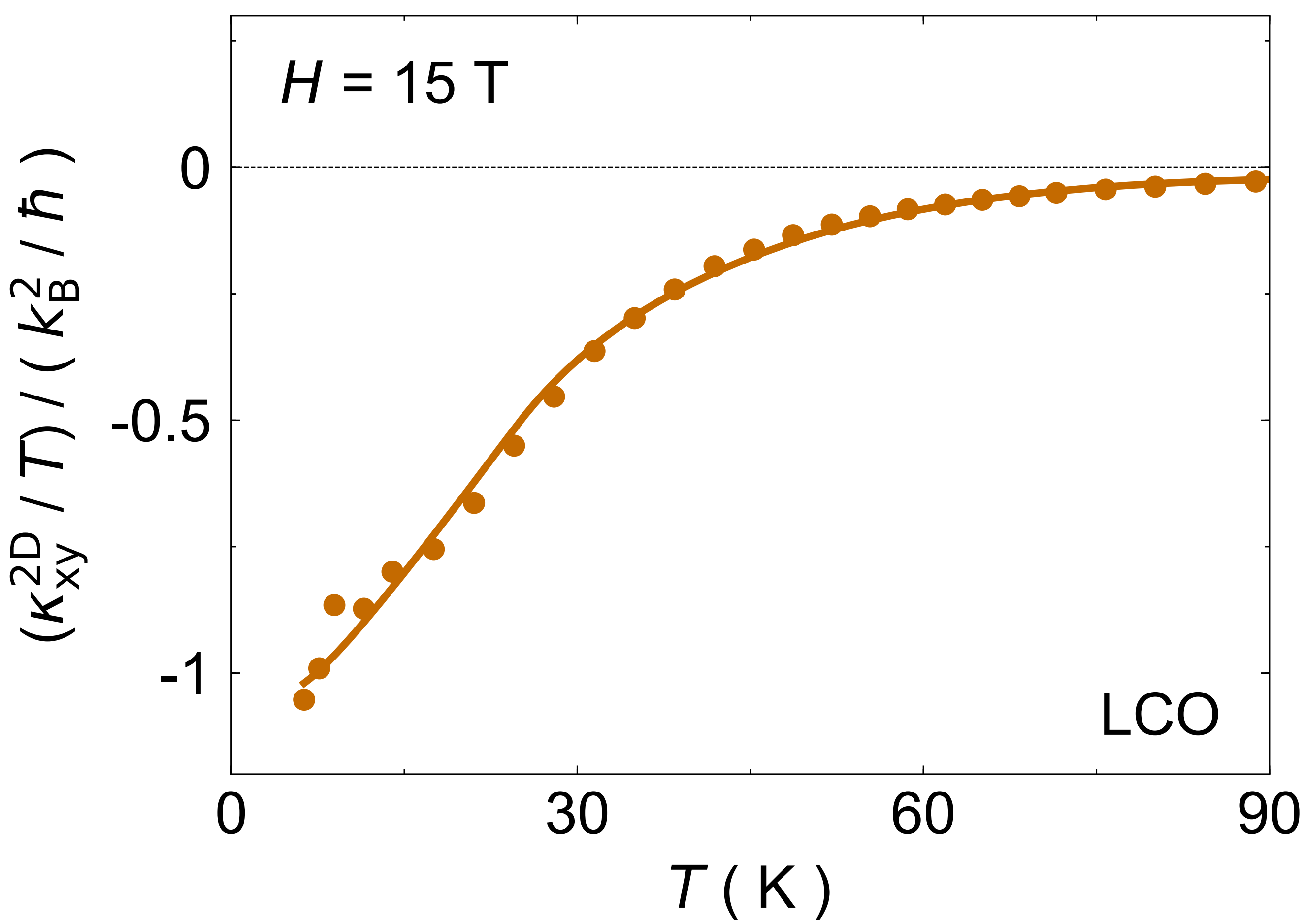
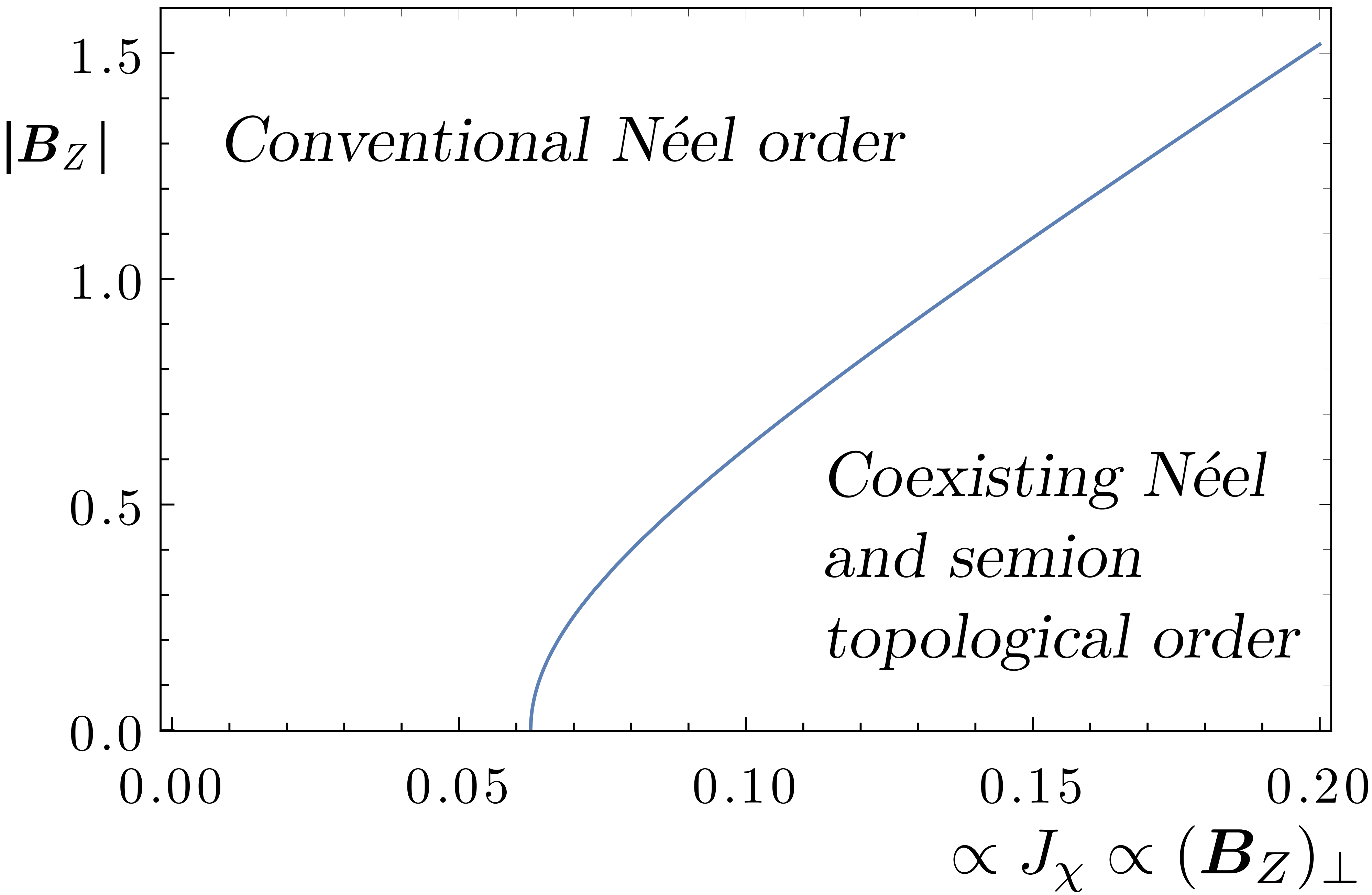


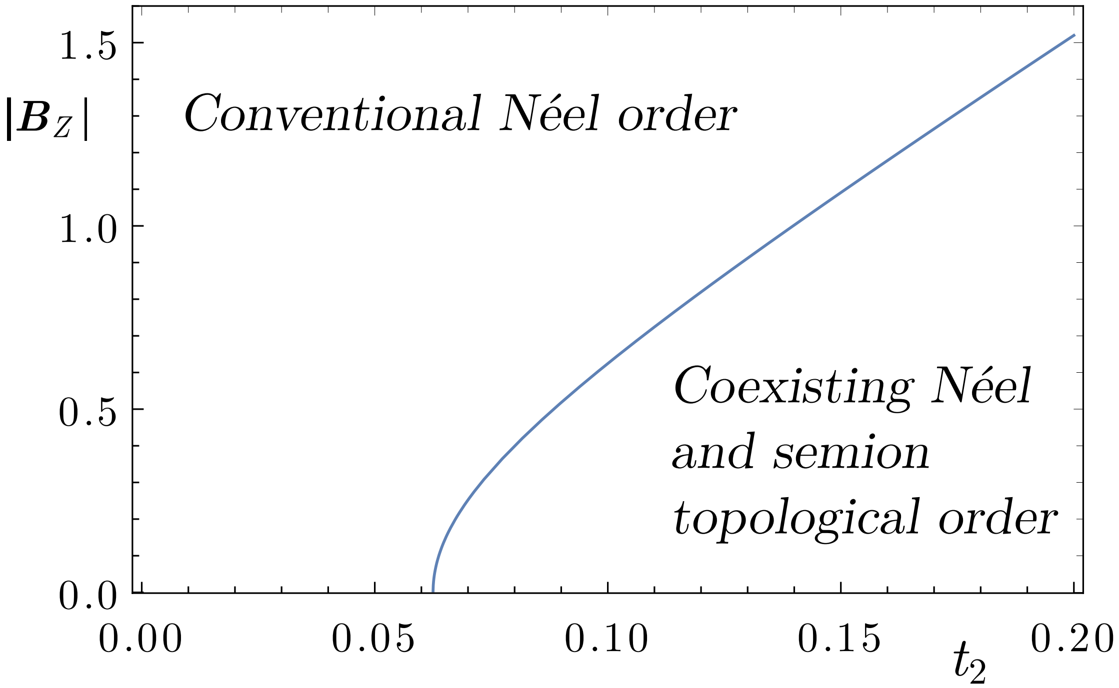


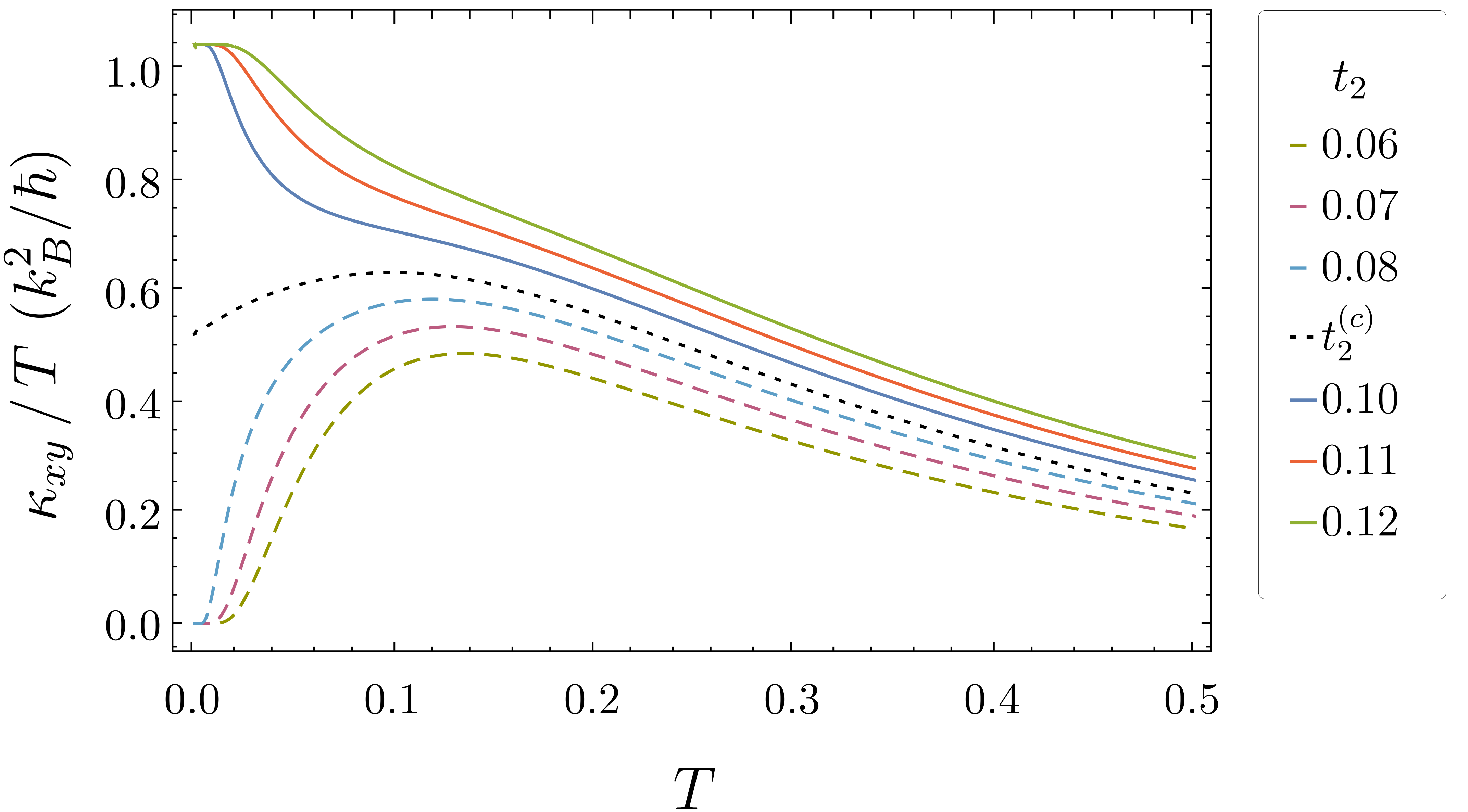
$$H = H_1 + H_B \qquad H_B = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots \qquad - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \, .$$



S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992





S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

1. Neel-VBS criticality in square lattice antiferromagnets

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Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):

Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = F_i R_{ci}$$

$$F_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow}^\dagger \\ f_{i\downarrow} & f_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad R_{ci} = \begin{pmatrix} b_{i1} & b_{i2} \\ -b_{i2}^* & b_{i1}^* \end{pmatrix}$$

F are fermionic spinons, R_c is a SU(2) rotation. Pseudospin rotations are *right* multiplication of R_c, while *left* multiplication is an emergent SU(2) gauge symmetry:

$$F \to FU, \quad R_c \to U^\dagger R_c \, .$$

The F spinons move in a π-flux background, while the $b_{1,2}$ are gapped chargons. The low energy theory is a SU(2) gauge theory of $N_f = 2$ Dirac fermions, f

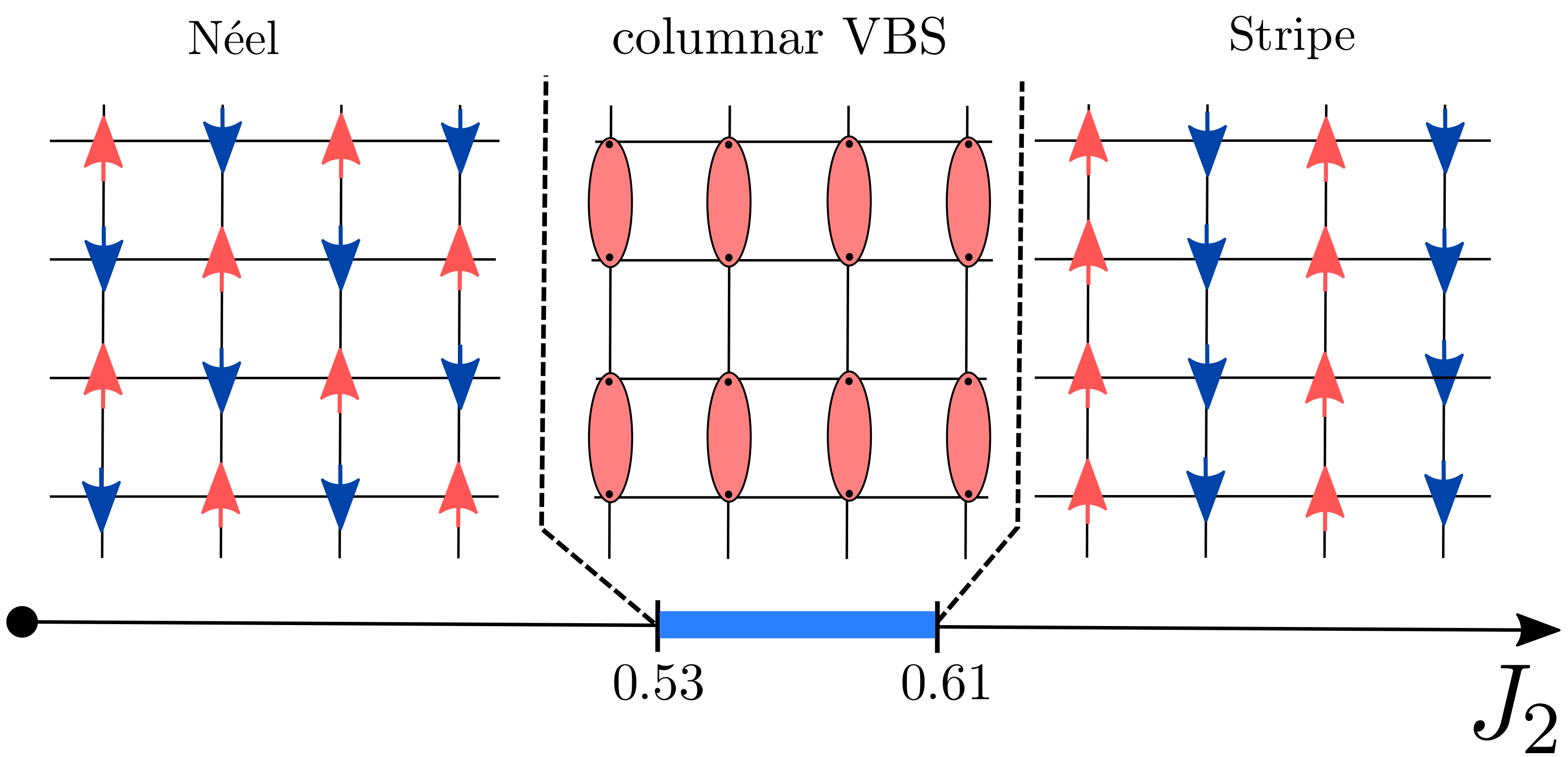
Neel-VBS deconfined criticality

M. Scheurer and S. Sachdev, PRB **98**, 235126 (2018)

Square lattice antiferromagnet

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f \right]$$

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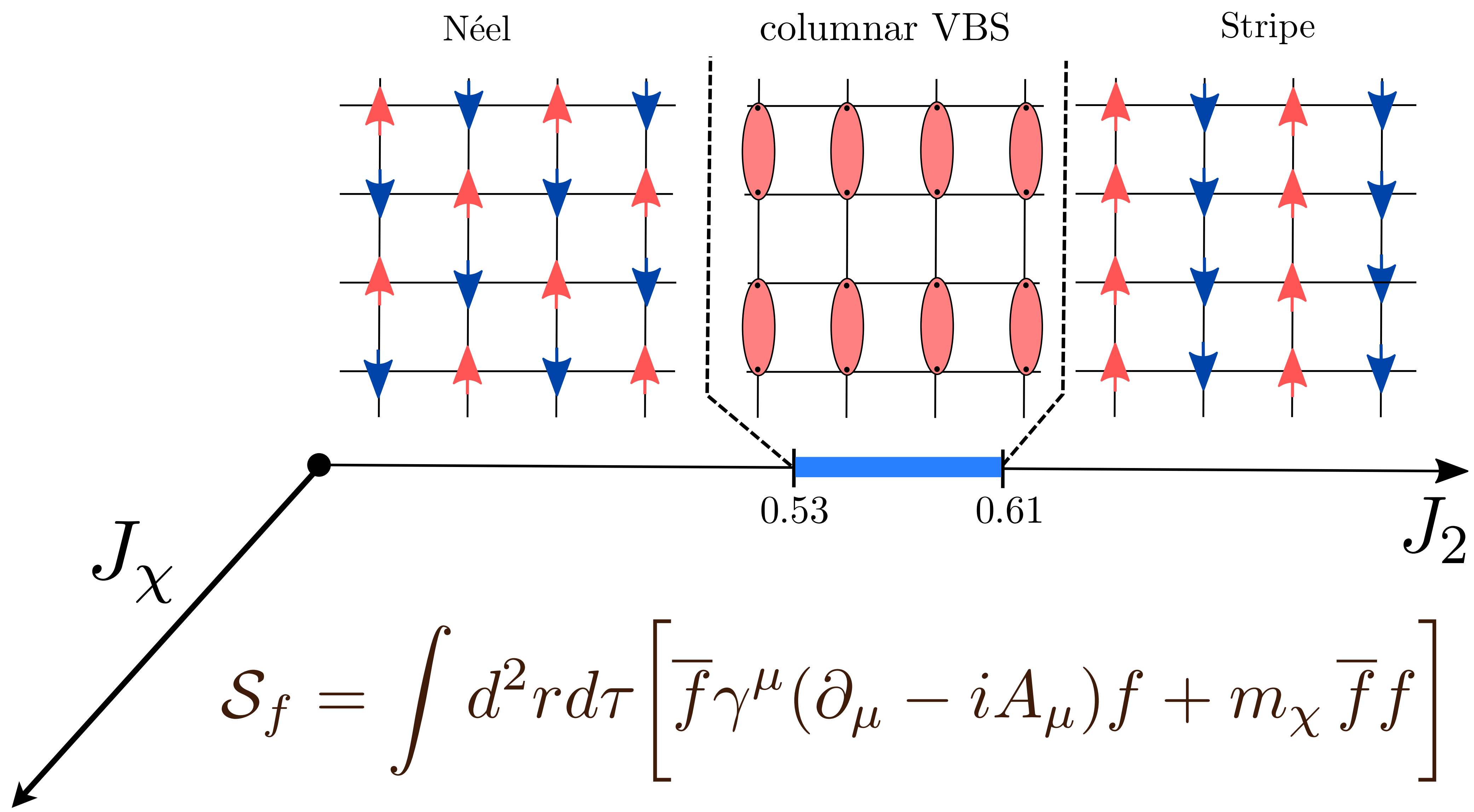

$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f + m_\chi \overline{f} f \right]$$

$$H = H_1 + H_\chi$$

$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

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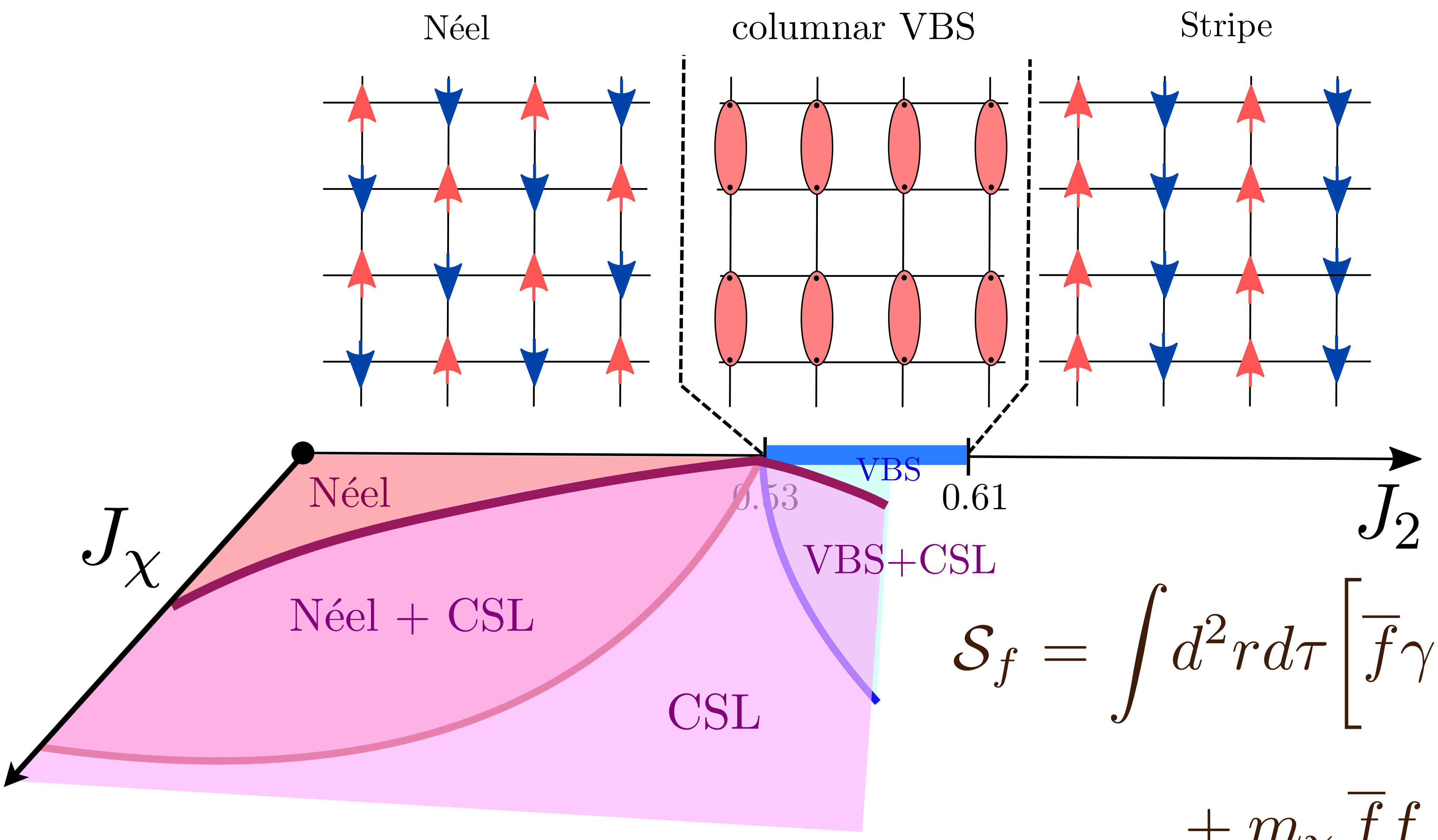

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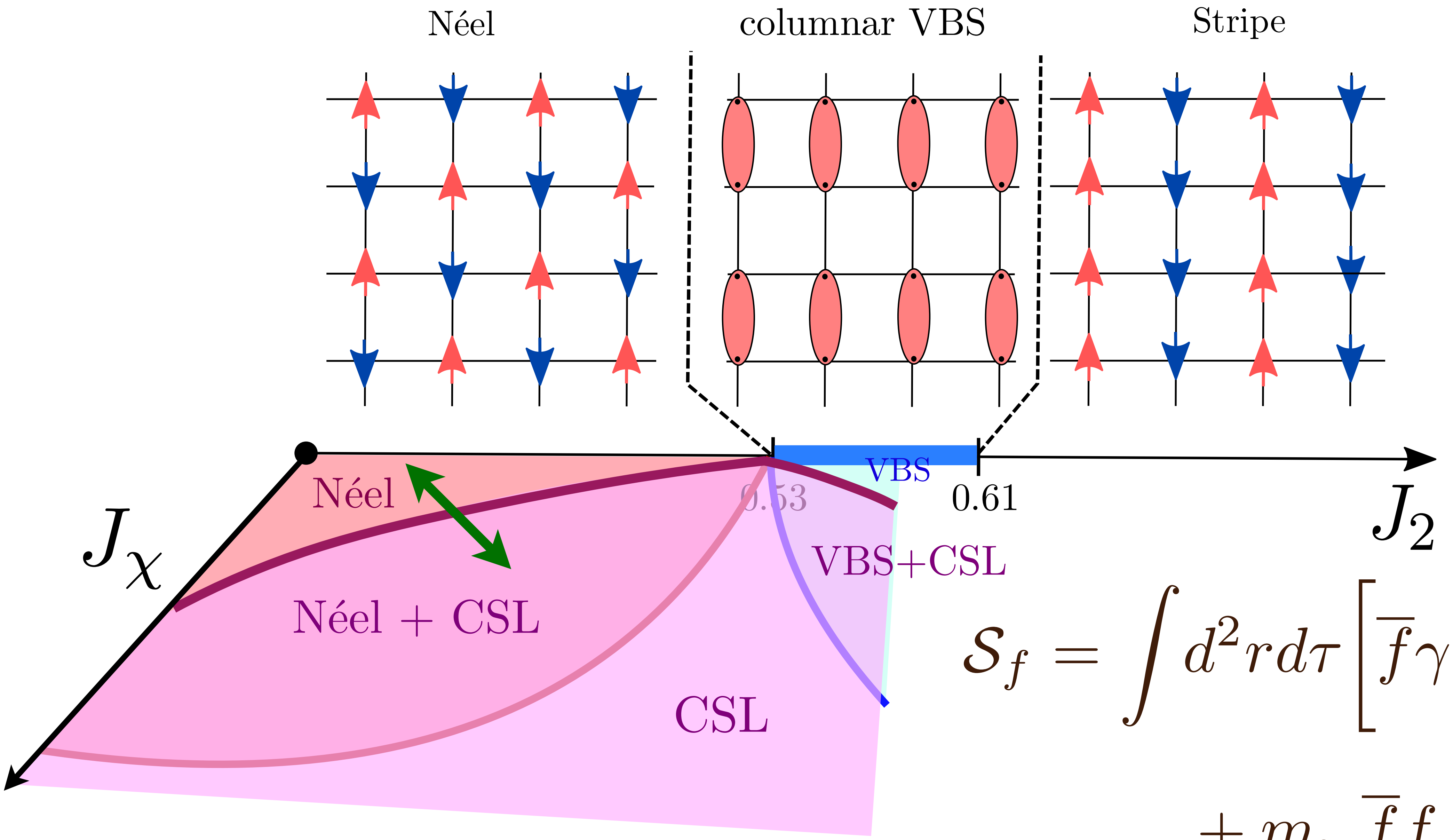

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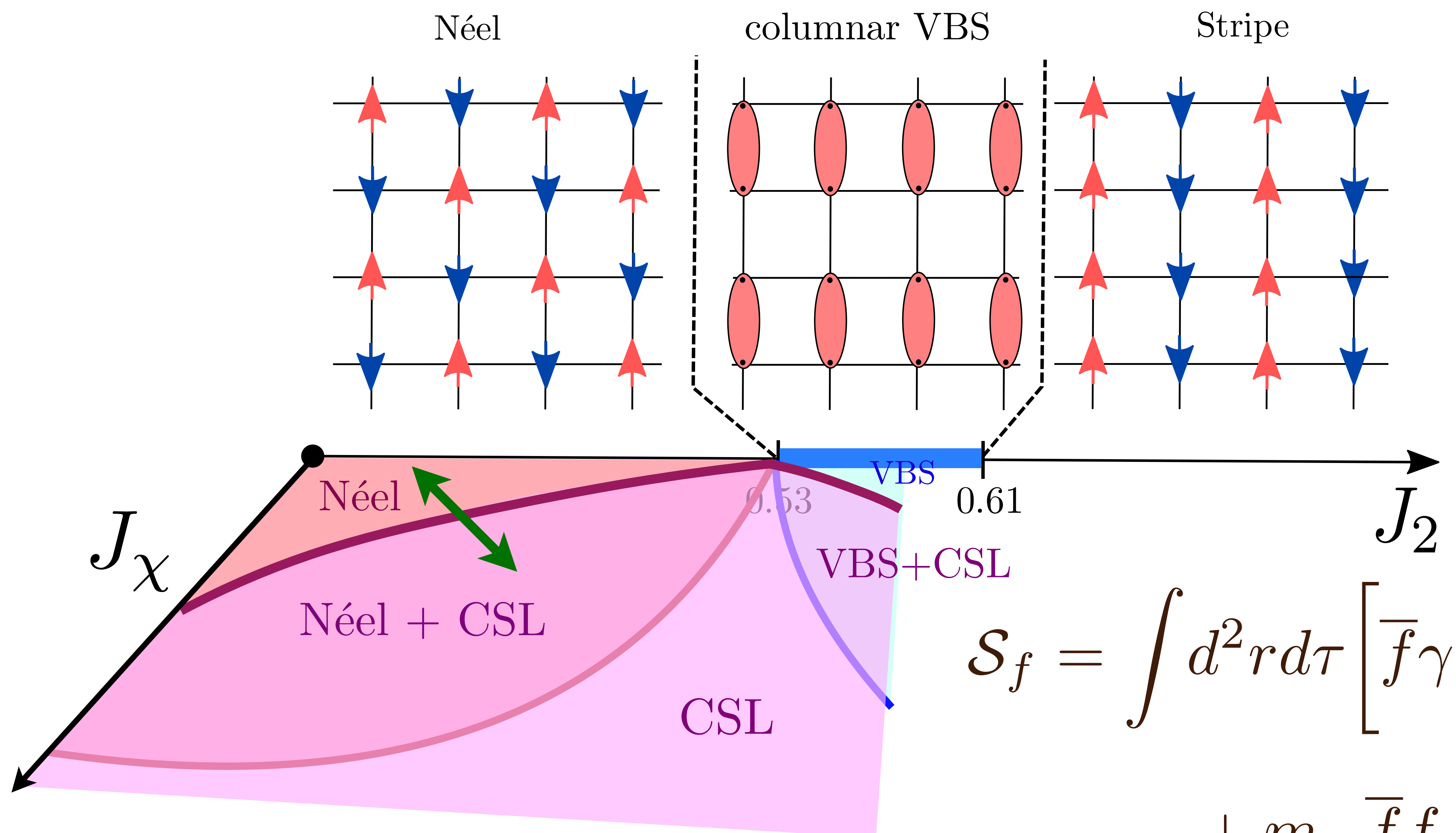
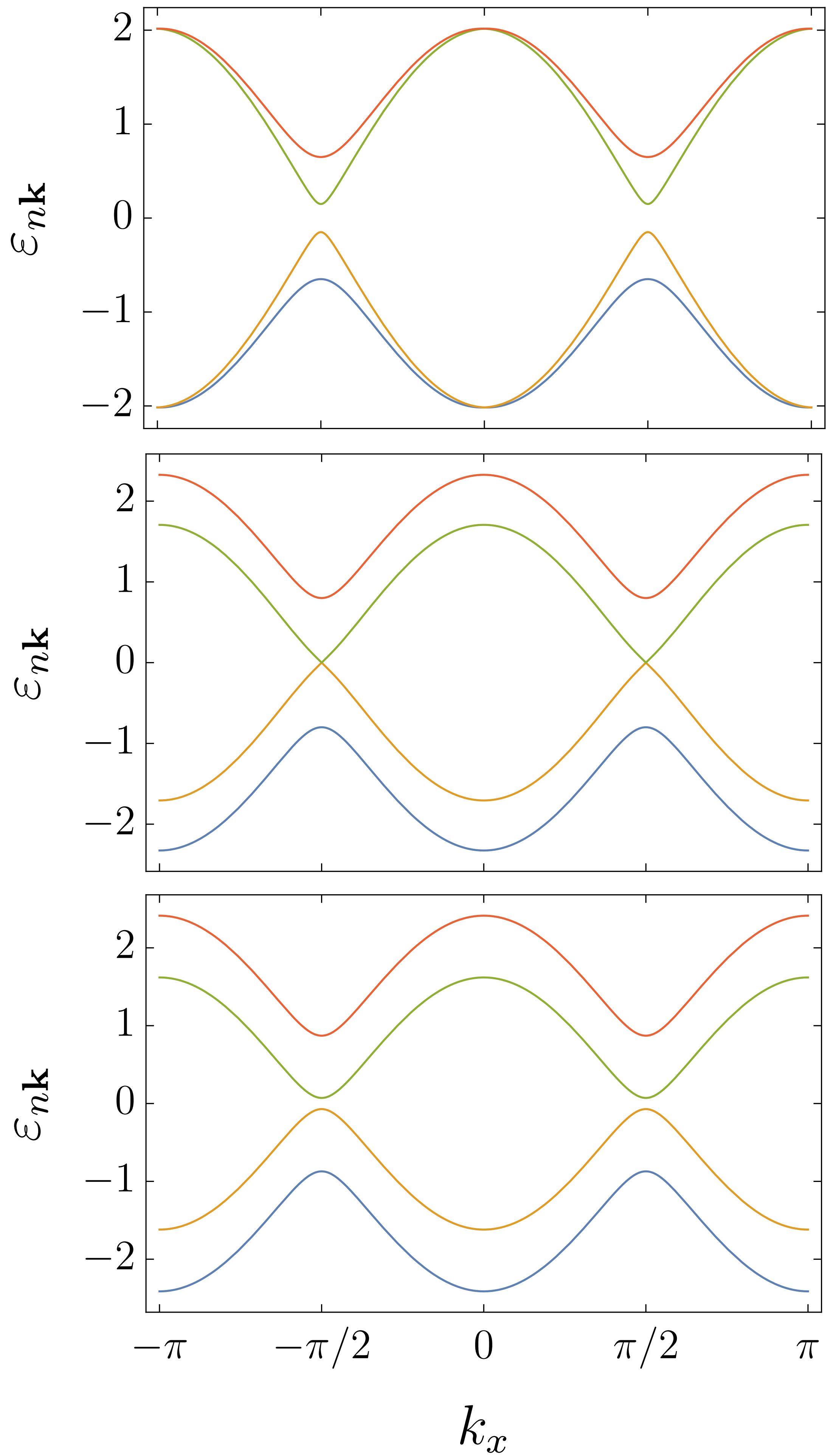


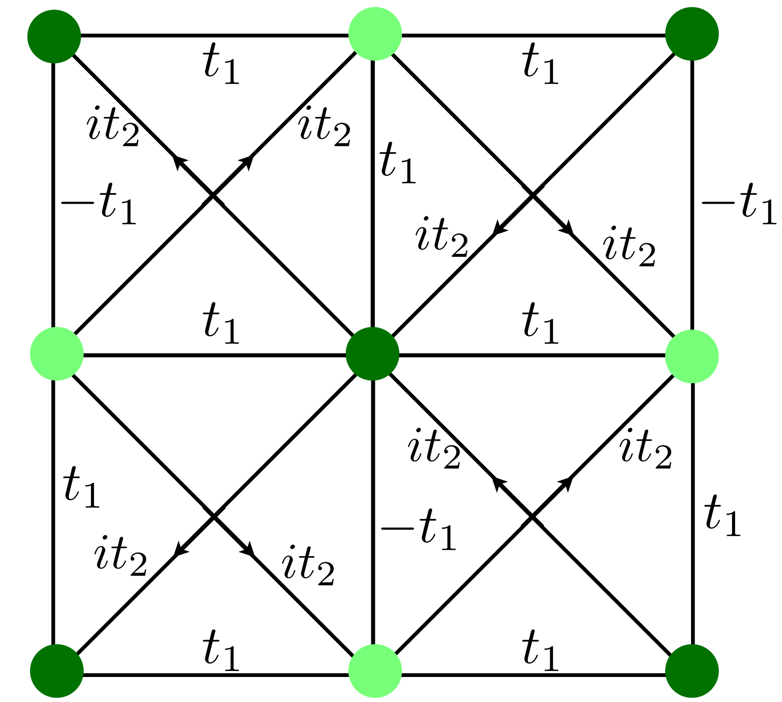
Mass from Néel order

$$\mathcal{S}_f = \int d^2 r d\tau \left[\bar{f} \gamma^\mu (\partial_\mu - i A_\mu) f + m_\chi \bar{f} f + m_N \bar{f} \Gamma f \right]$$

$$H = H_1 + H_\chi$$

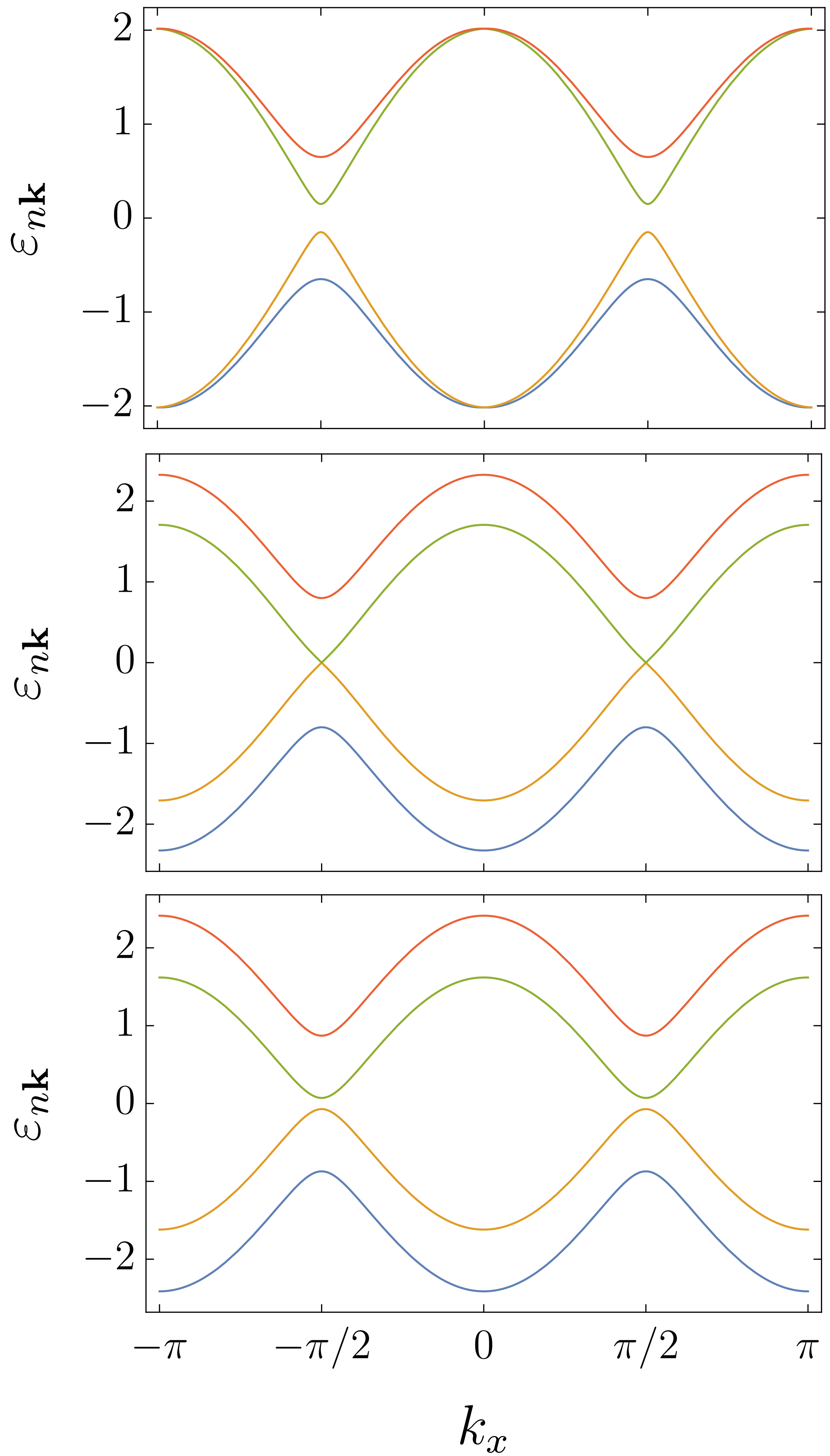
$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

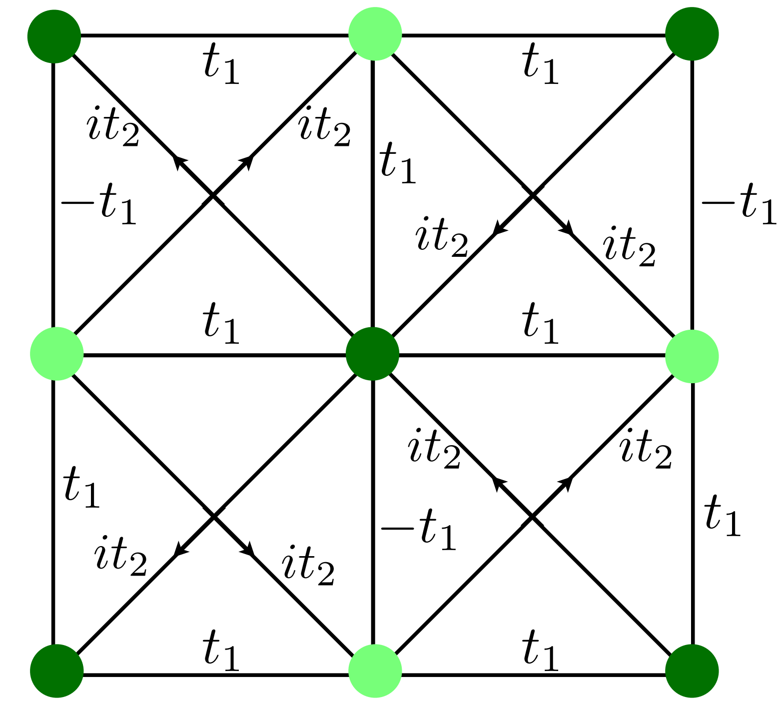




We focus on two possible mass terms for the Dirac fermions: m_χ, from the spin chirality term, and m_N induced by Néel order. There is a line in the m_χ, m_N plane where the occupied bands switch from Chern numbers $\{1, -1\}$ (the Néel state) to $\{1, 1\}$ (Néel order co-existing with semion topological order).

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



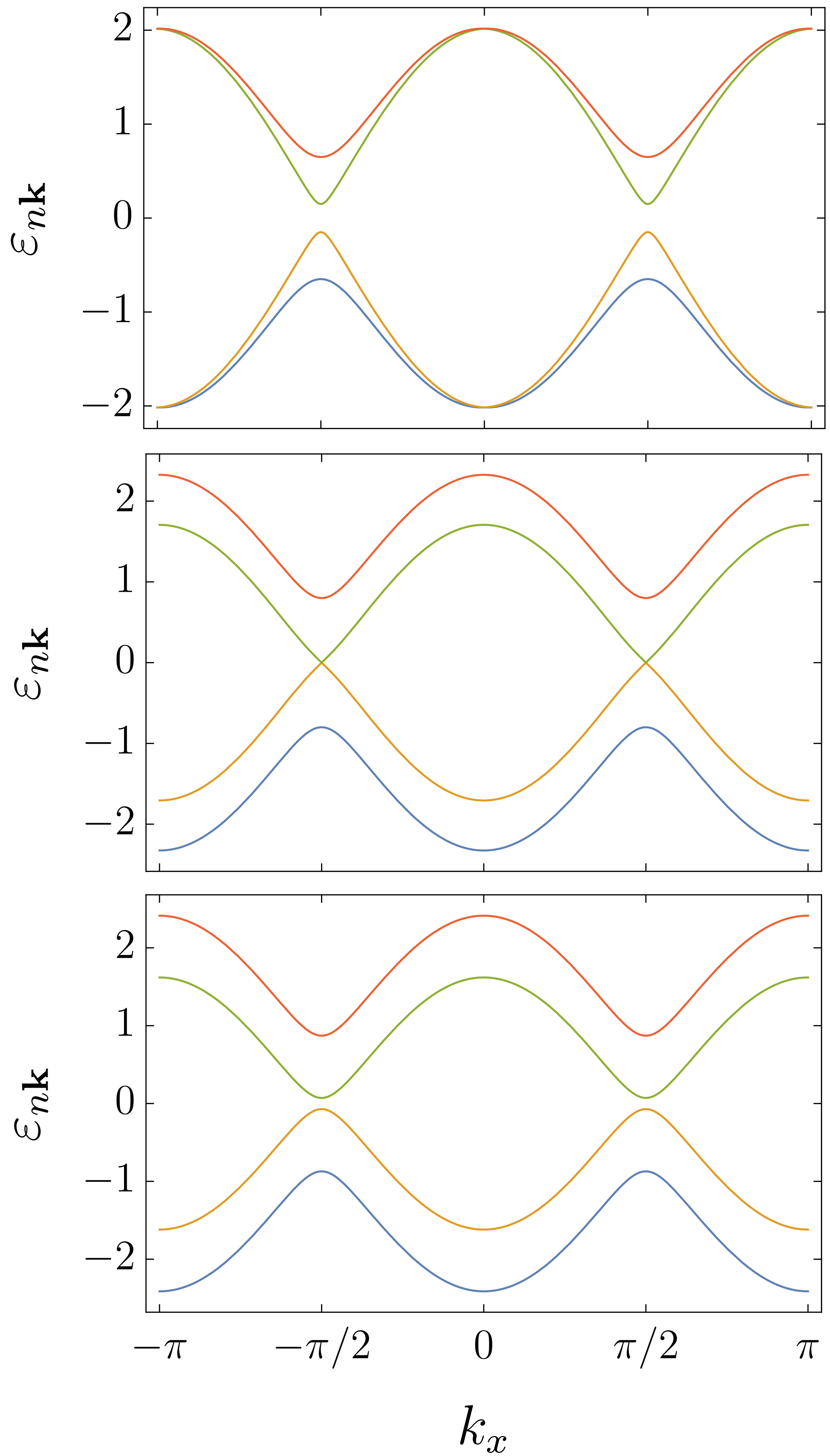


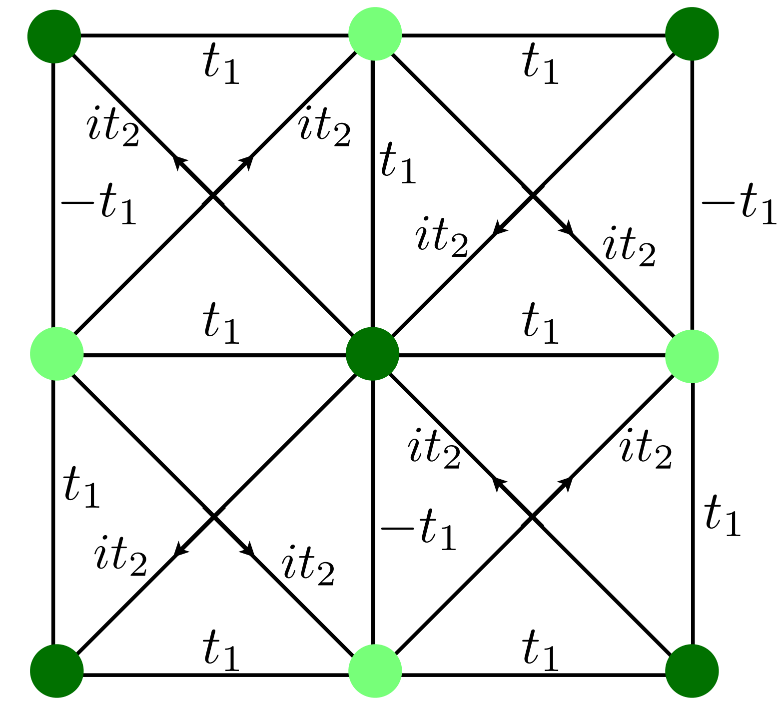
The vicinity of the critical point is described by $N_f = 1$ Dirac fermion coupled to a SU(2) gauge field A_μ at level $-1/2$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

The transition is tuned by the change in sign of m.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

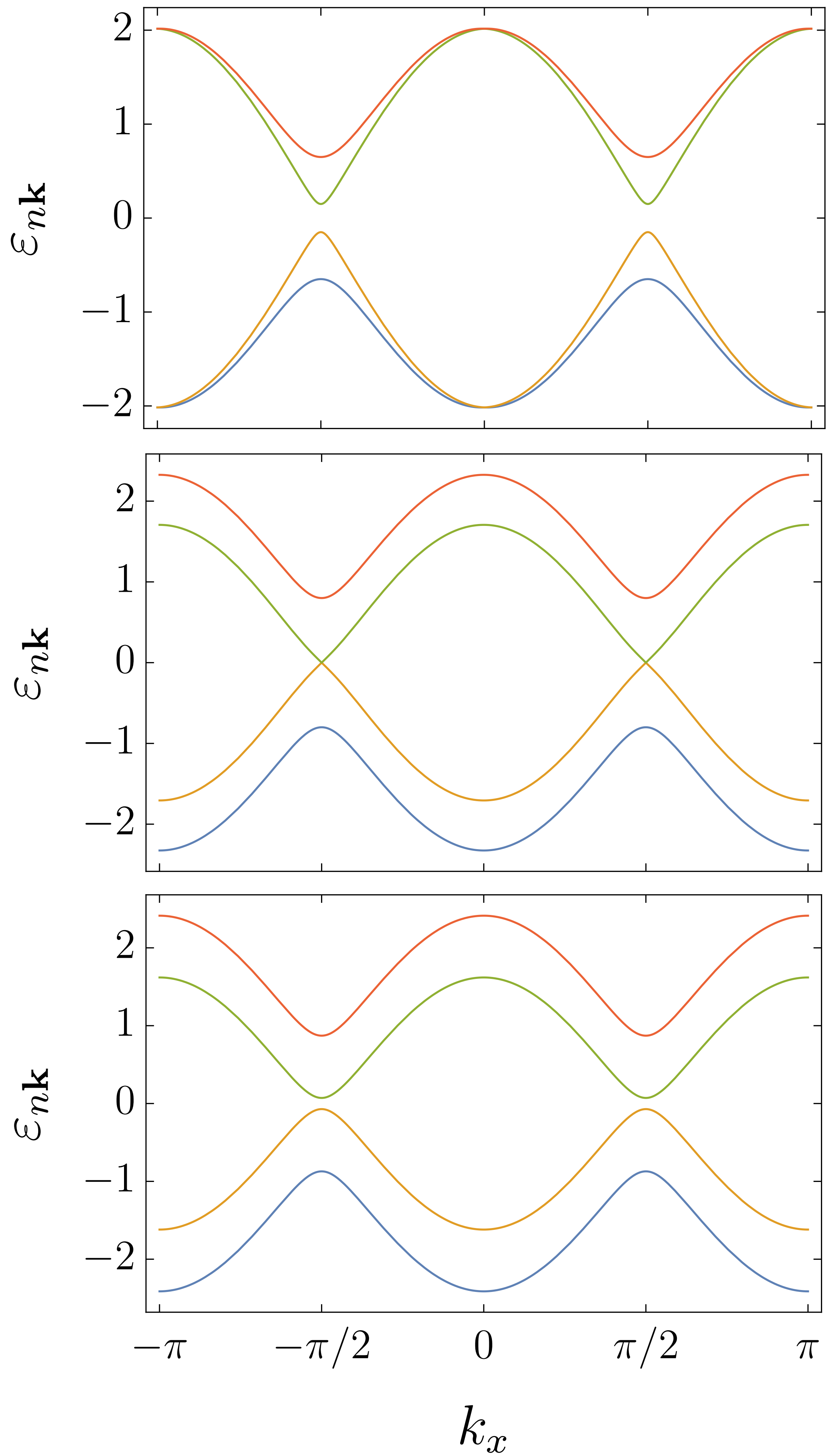


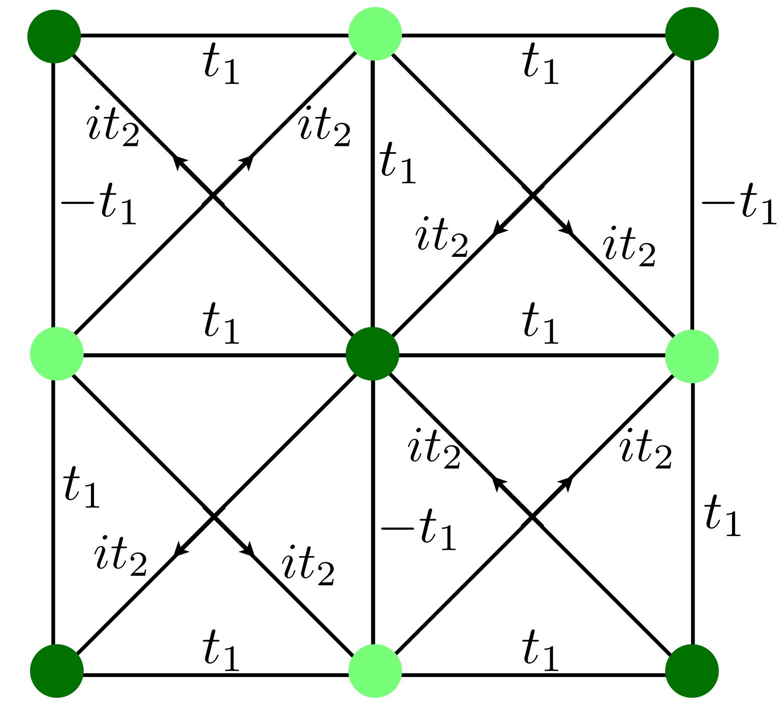


$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

When $m > 0$, we can integrate out f and there is no net CS term. The SU(2) gauge theory confines, and we obtain Néel order.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992





$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

When $m < 0$, we can integrate out f and we obtain a net CS term at level -1. The SU(2) gauge theory at level -1 describes the semion topological phase.

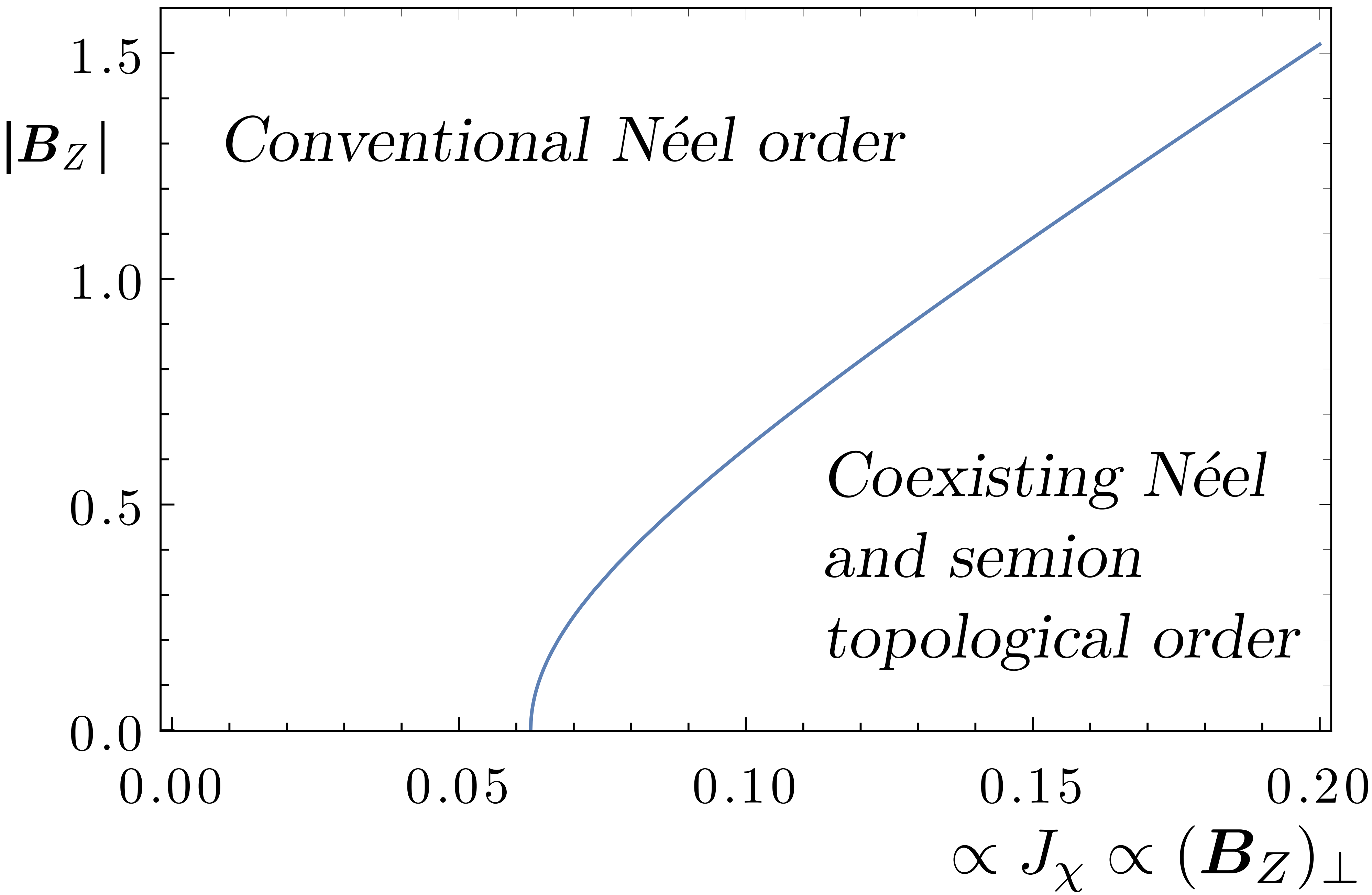
S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

$$H = H_1 + H_B$$

$$H_B = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

$$- \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$



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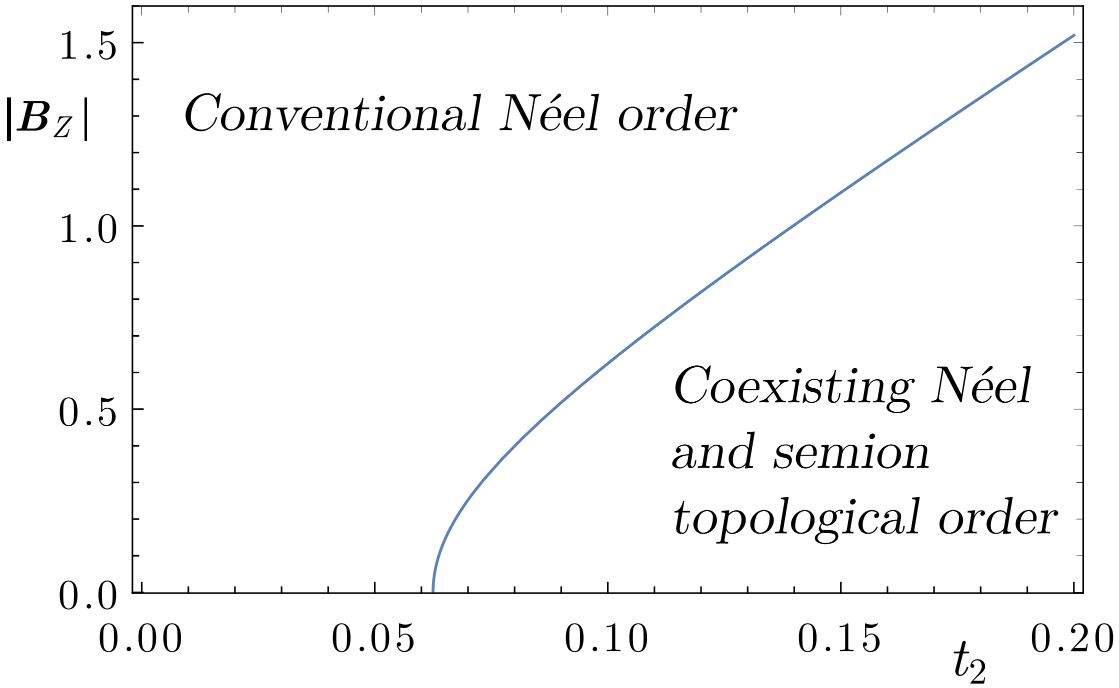
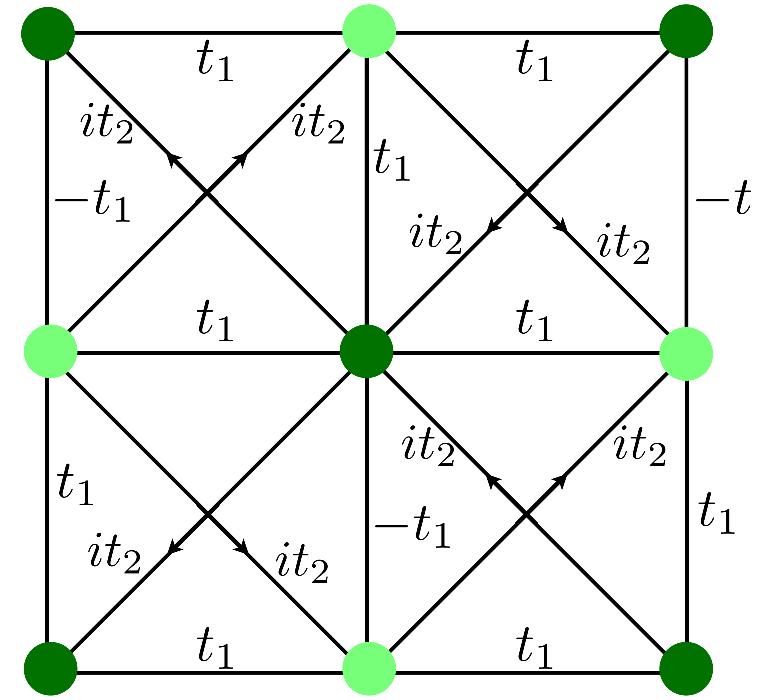
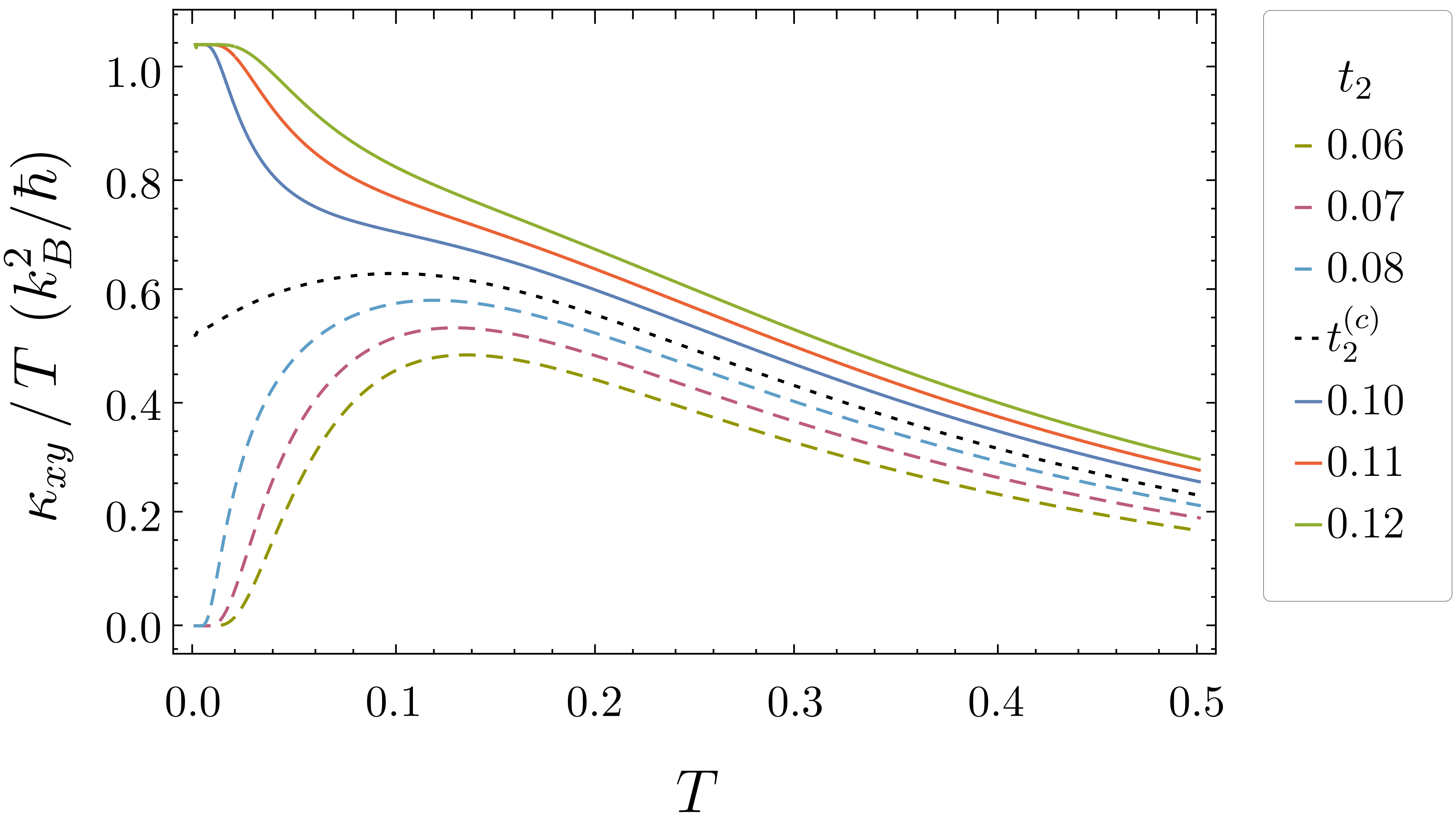






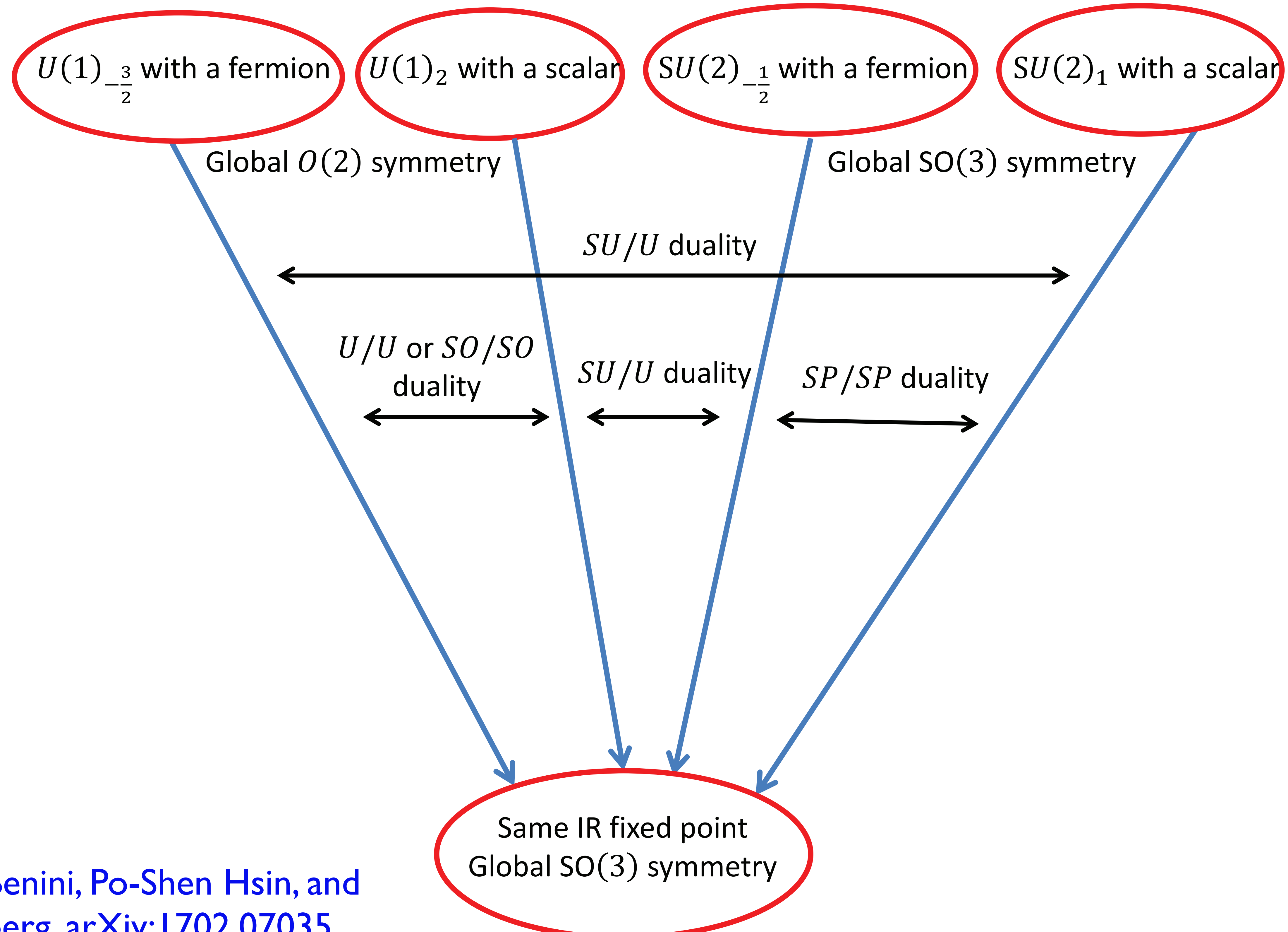
S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

1. Neel-VBS criticality in square lattice antiferromagnets

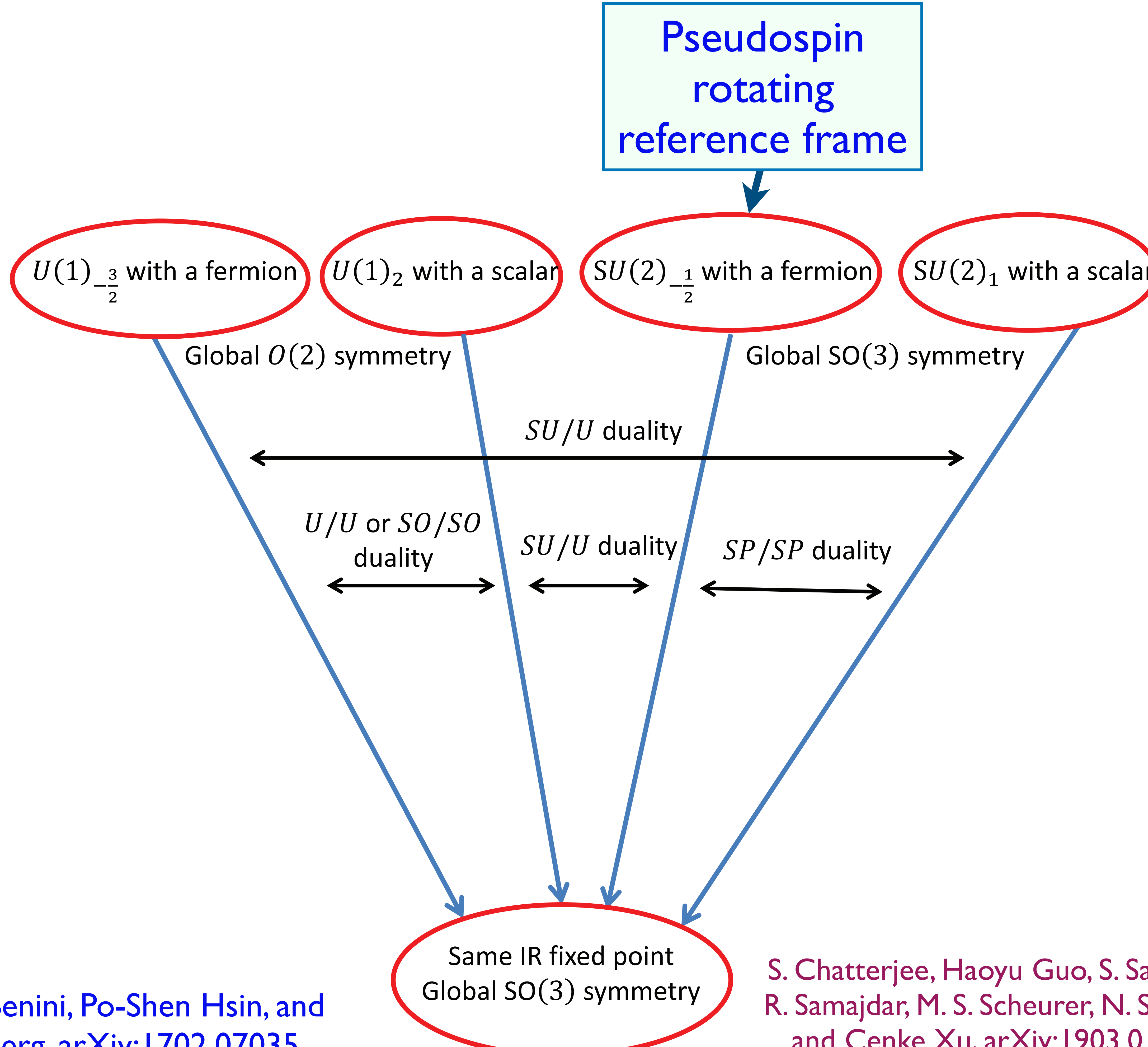
2. Recent experimental and numerical results

3. Critical theory for onset of semion topological order

4. More non-Abelian dualities



Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035



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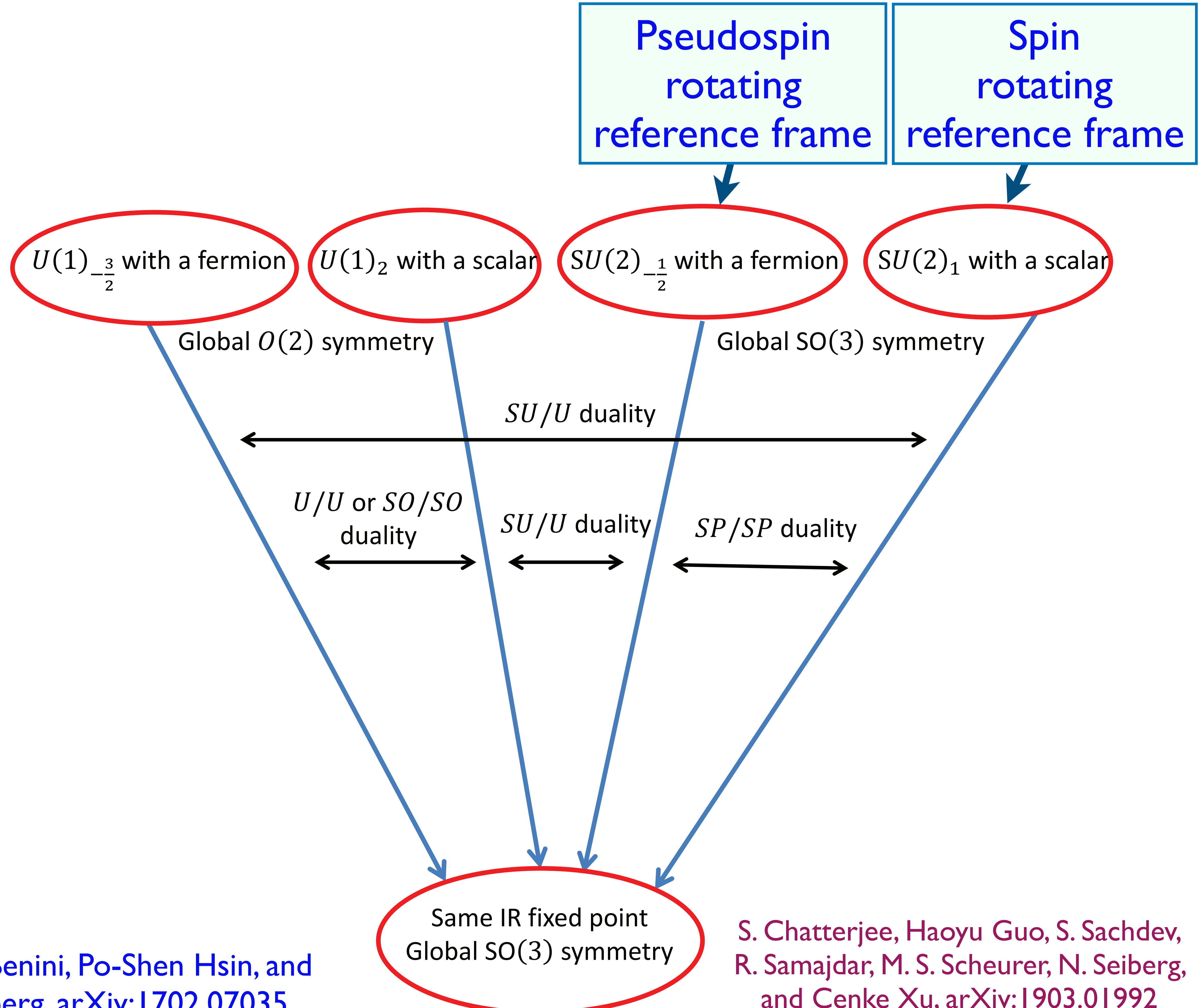

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S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):

Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = F_i R_{ci}$$

$$F_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow}^\dagger \\ f_{i\downarrow} & f_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad R_{ci} = \begin{pmatrix} b_{i1} & b_{i2} \\ -b_{i2}^* & b_{i1}^* \end{pmatrix}$$

F are fermionic spinons, R_c is a SU(2) rotation. Pseudospin rotations are *right* multiplication of R_c, while *left* multiplication is an emergent SU(2) gauge symmetry:

$$F \rightarrow FU, \quad R_c \rightarrow U^\dagger R_c \, .$$

M. Scheurer and S. Sachdev, PRB **98**, 235126 (2018)

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):

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$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = R_{si}\Psi_i$$

$$\Psi_i = \begin{pmatrix} \psi_{i+} & -\psi_{i-}^\dagger \\ \psi_{i-} & \psi_{i+}^\dagger \end{pmatrix} \quad , \quad R_{si} = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^* \\ z_{i\downarrow} & z_{i\uparrow}^* \end{pmatrix}$$

Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger \, .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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We Higgs the SU(2) down to U(1) by condensing $(-1)^i \psi_i^\dagger \sigma^z \psi_i$, and the chargons are then gapped, fully filling the lower band. The resulting low energy theory is a U(1) gauge theory for the bosonic spinons z_α.

Neel-VBS deconfined criticality

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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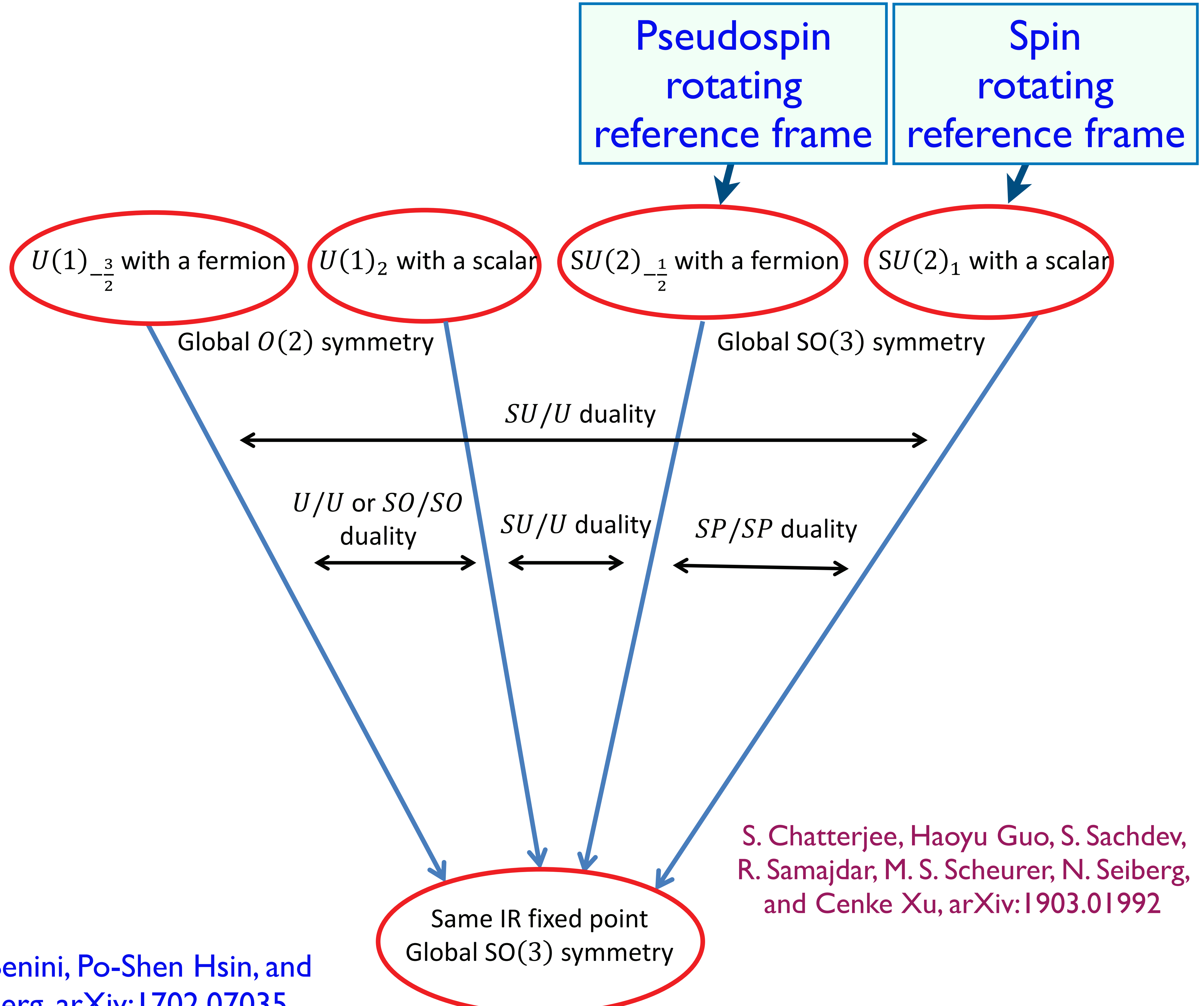
Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger .$$

The fermionic chargons Ψ fully occupy a trivial band, and are gapped. The R bosons form a bosonic SPT state connected to the O(4) model at $\theta = 2\pi$. After we gauge away the right-SU(2), the left $\mathrm{SU}(2)_1$ theory describes the chiral spin liquid. When the bosons condense, we obtain a trivial state. The critical theory is a SU(2) gauge theory at level 1 coupled to massless self-interacting scalar.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



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Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

Another non-Abelian duality

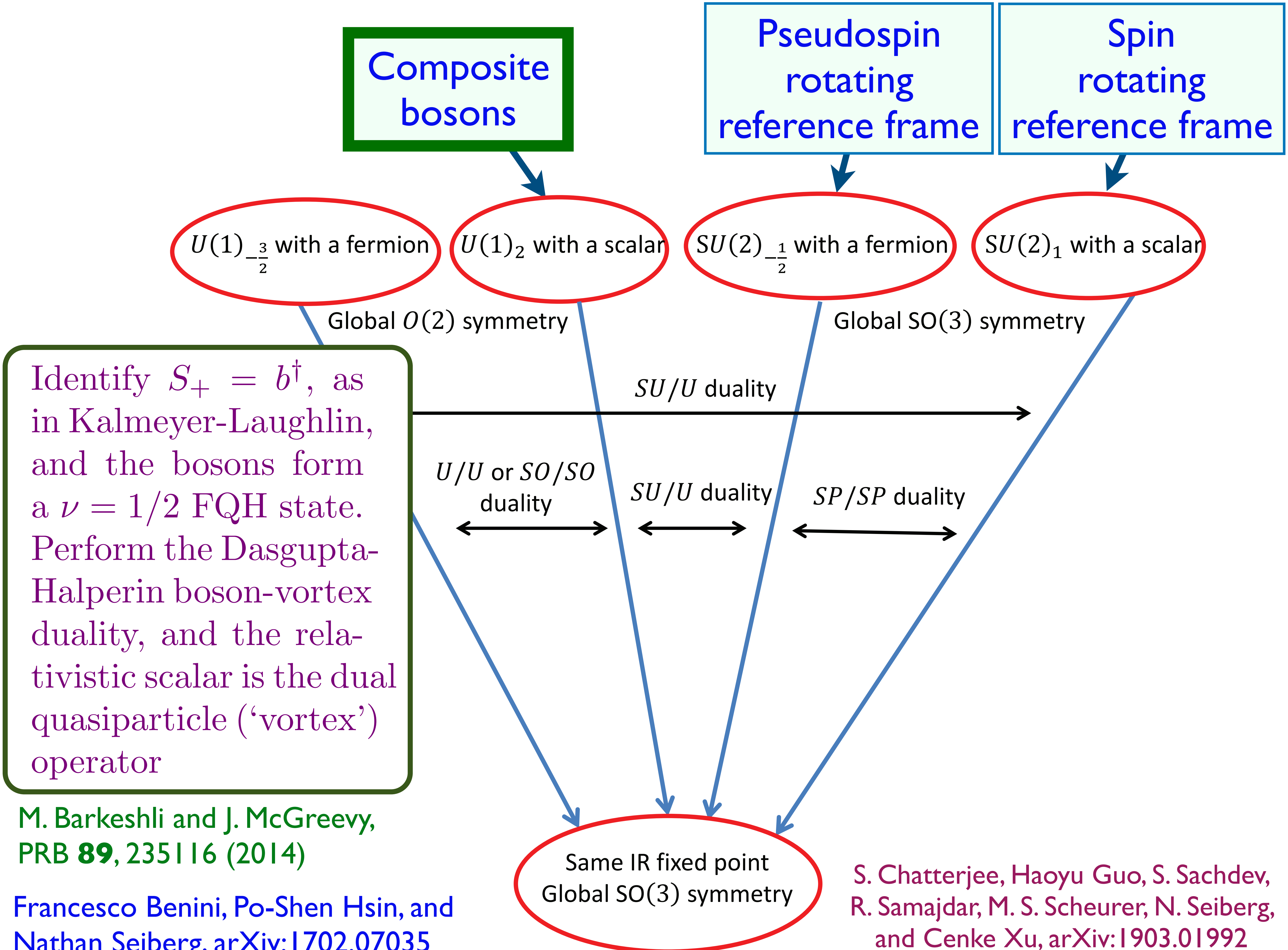
Critical SU(2) gauge theory of $N_b = 1$ relativistic bosons
at CS level 1
is dual to
SU(2) gauge theory of $N_f = 1$ Dirac fermion
at CS level -1/2.

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

Both theories have an emergent global SO(3) symmetry

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992


Composite bosons
Pseudospin rotating reference frame
Spin rotating reference frame
$U(1)_{-\frac{3}{2}}$ with a fermion
$U(1)_2$ with a scalar
$SU(2)_{-\frac{1}{2}}$ with a fermion
$SU(2)_1$ with a scalar
Global $O(2)$ symmetry
Global SO(3) symmetry
Identify $S_+ = b^\dagger$, as in Kalmeyer-Laughlin, and the bosons form a $\nu = 1/2$ FQH state. Perform the Dasgupta-Halperin boson-vortex duality, and the relativistic scalar is the dual quasiparticle ('vortex') operator
SU/U duality
U/U or SO/SO duality
SU/U duality
SP/SP duality
Same IR fixed point
Global SO(3) symmetry
M. Barkeshli and J. McGreevy, PRB **89**, 235116 (2014)
Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035
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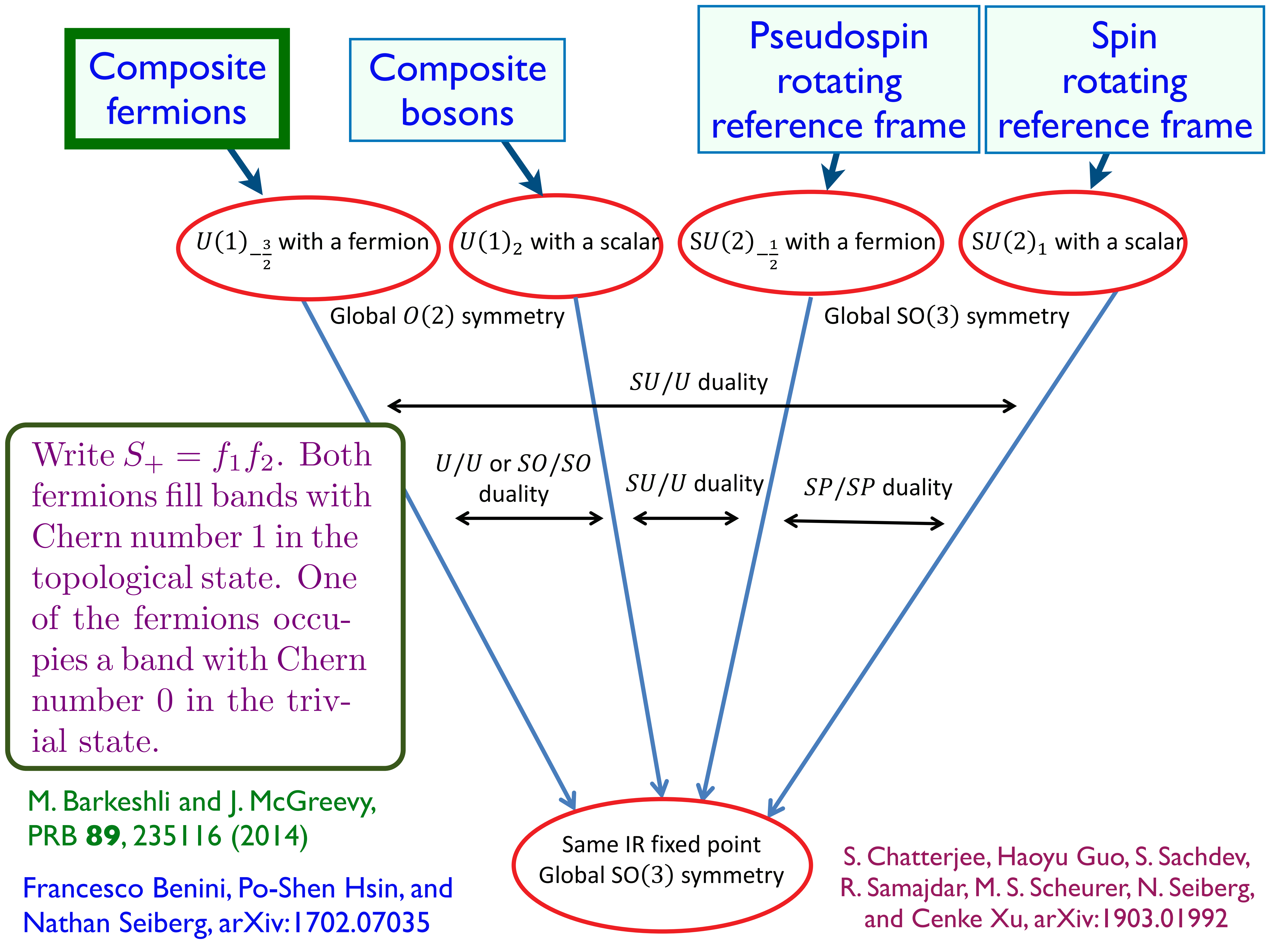

Composite fermions
Composite bosons
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Global $O(2)$ symmetry
Global SO(3) symmetry
SU/U duality
U/U or SO/SO duality
SU/U duality
SP/SP duality
Write $S_+ = f_1 f_2$. Both fermions fill bands with Chern number 1 in the topological state. One of the fermions occupies a band with Chern number 0 in the trivial state.
Same IR fixed point
Global SO(3) symmetry
M. Barkeshli and J. McGreevy, PRB **89**, 235116 (2014)
Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035
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A quadrilarity

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

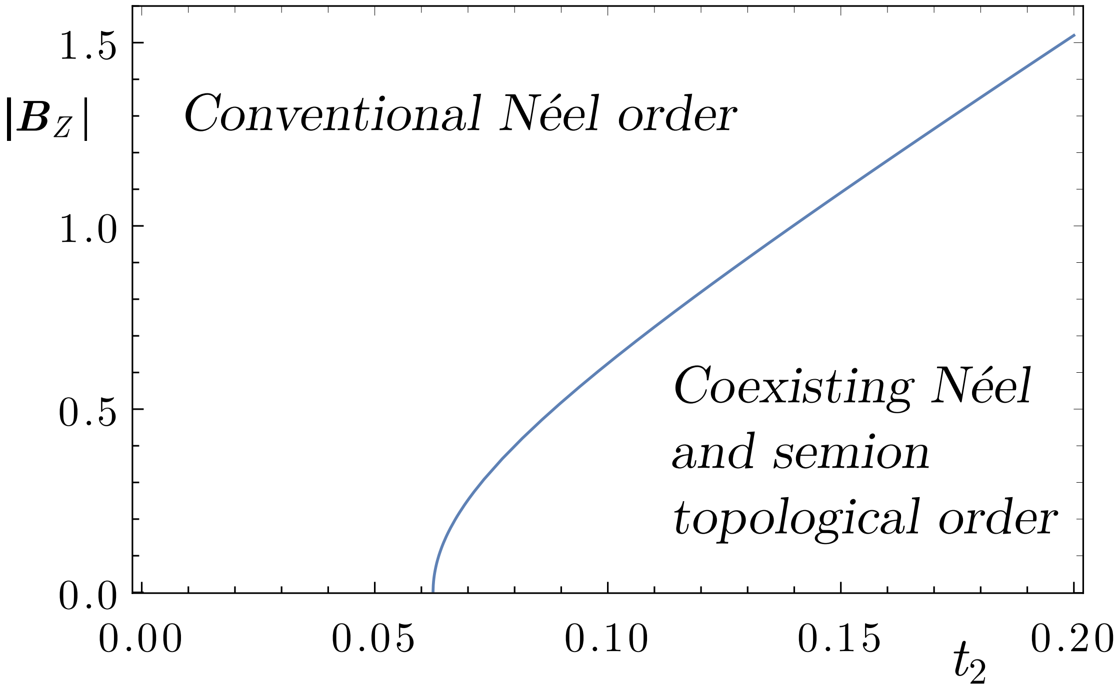
$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

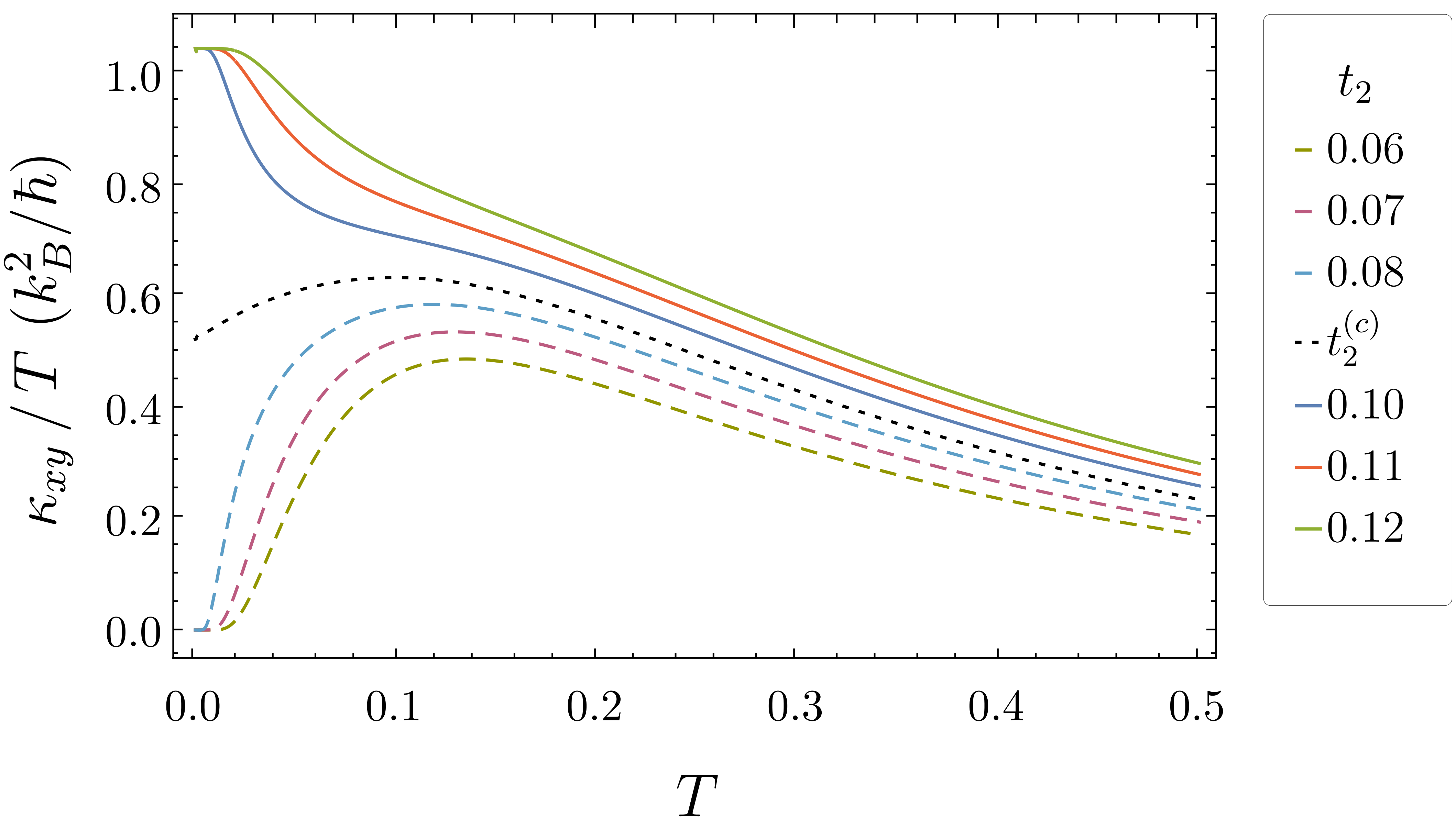
$$\mathcal{L}_\phi = |(\partial_\mu - ia_\mu)\phi|^2 + s|\phi|^2 + u(|\phi|^2)^2 + 2\mathrm{CS}[a_\mu]$$

$$\mathcal{L}_g = \overline{g}\gamma^\mu(\partial_\mu - ia_\mu)g + m\overline{g}g - \frac{3}{2}\mathrm{CS}[a_\mu]$$

Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992





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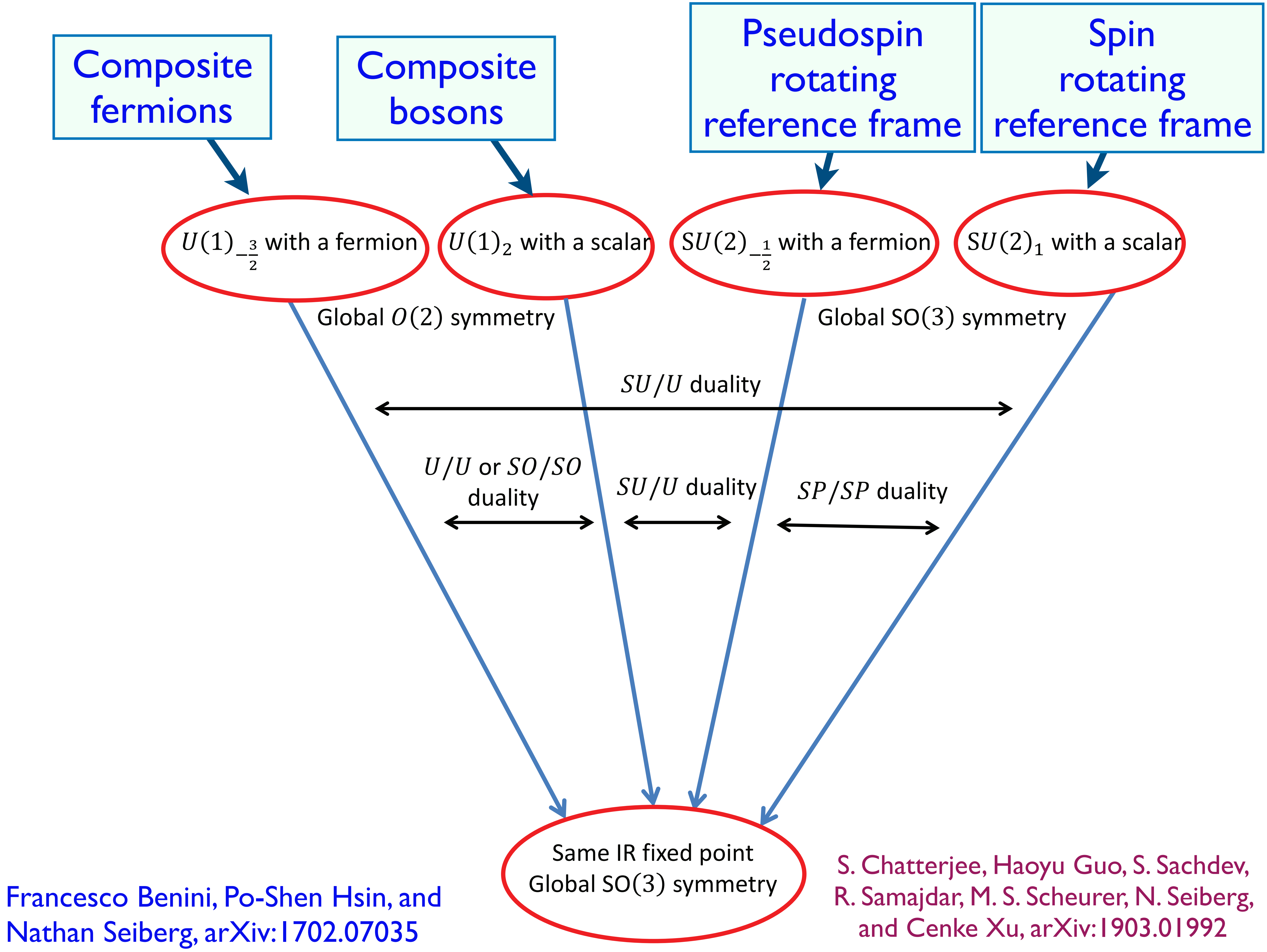

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