# Quantum matter without quasiparticles

Frontiers in Many Body Physics:
Memorial for Lev Petrovich Gor'kov
National High Magnetic Field Laboratory, Tallahassee

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Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Ubiquitous

"Strange",

"Bad",

or "Incoherent",

metal has a resistivity, $\rho$, which obeys

$$\rho \sim T,$$

and

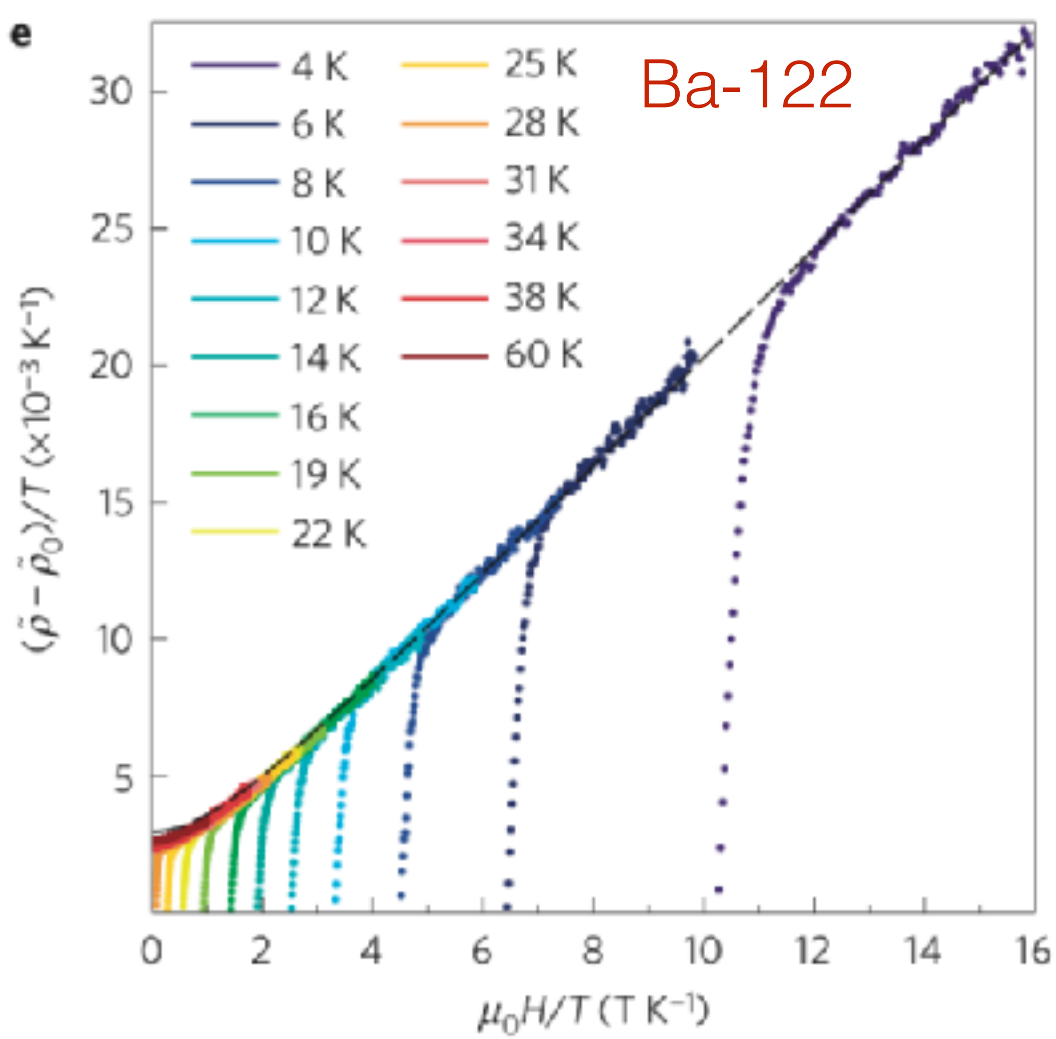
in some cases $\rho \gg h/e^2$

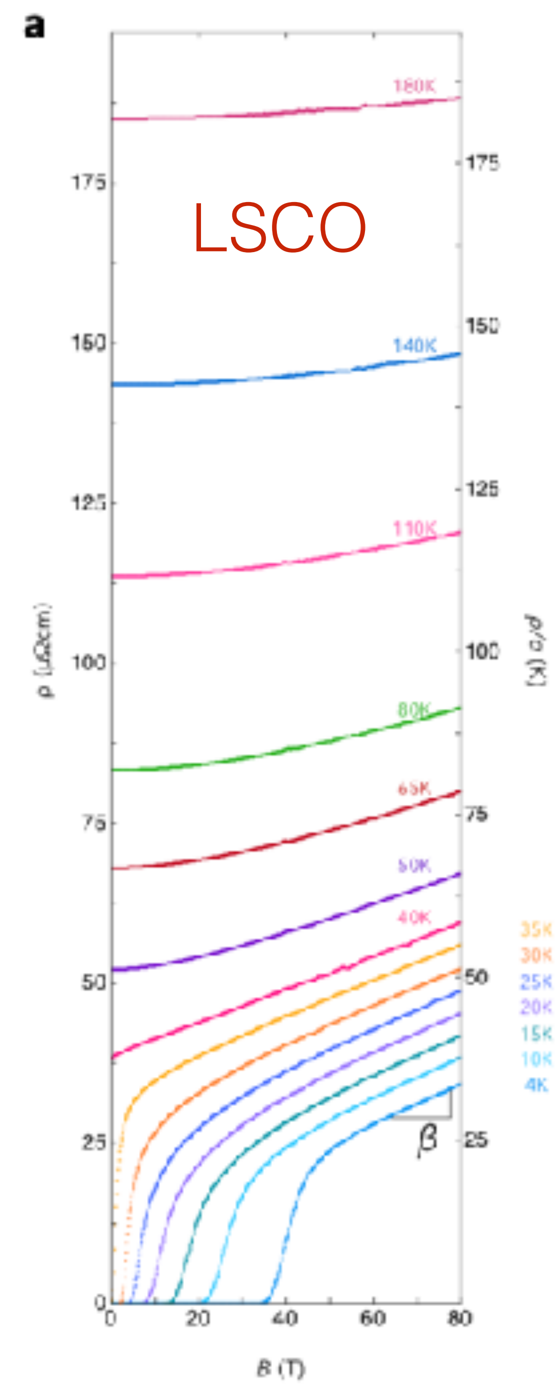
(in two dimensions),

where $h/e^2$ is the quantum unit of resistance.

# Strange metals just got stranger…

B-linear magnetoresistance!?



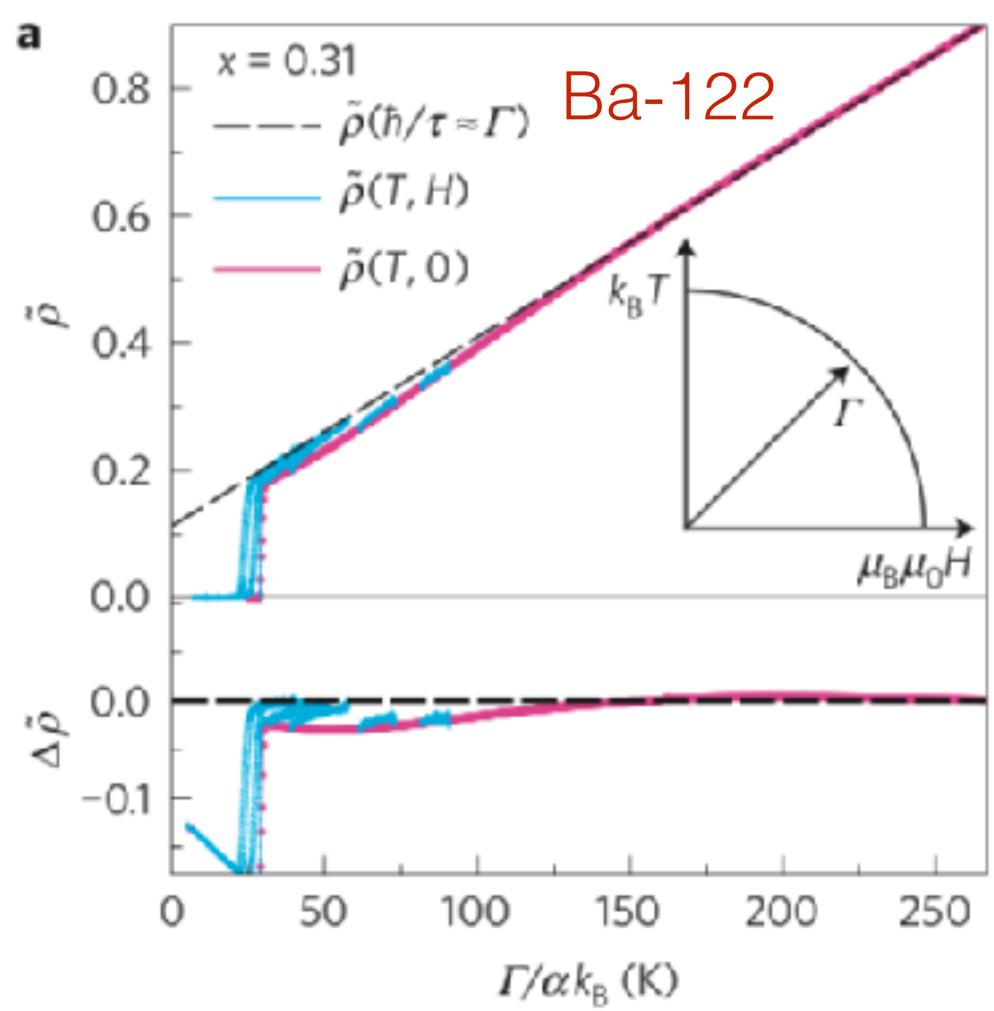




I. M. Hayes et. al., Nat. Phys. 2016

P. Giraldo-Gallo et. al., arXiv:1705.05806

# Strange metals just got stranger…

Scaling between B and T !?




$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

I. M. Hayes et. al., Nat. Phys. 2016

## Theories of metallic states without quasiparticles without disorder

- Breakdown of quasiparticles arises from long-wavelength coupling of electrons to some bosonic collective mode. In all cases this can be written in terms of a continuum theory with a conserved momentum. **The critical theory has zero resistance, even though the electron quasiparticles do not exist.**

- Need to add irrelevant (umklapp) effects to obtain a non-zero resistivity, but this not yield a large linear-in-$T$ resistivity.

## Theories of metallic states without quasiparticles in the presence of disorder

- Well-known perturbative theory of disordered metals has 2 classes of known fixed points, the insulator at strong disorder, and the metal at weak disorder. The latter state has long-lived, extended quasiparticle excitations (which are not plane waves).

- **Needed: a metallic fixed point at intermediate disorder and strong interactions without quasiparticle excitations.** Although disorder is present, it largely self-averages at long scales.

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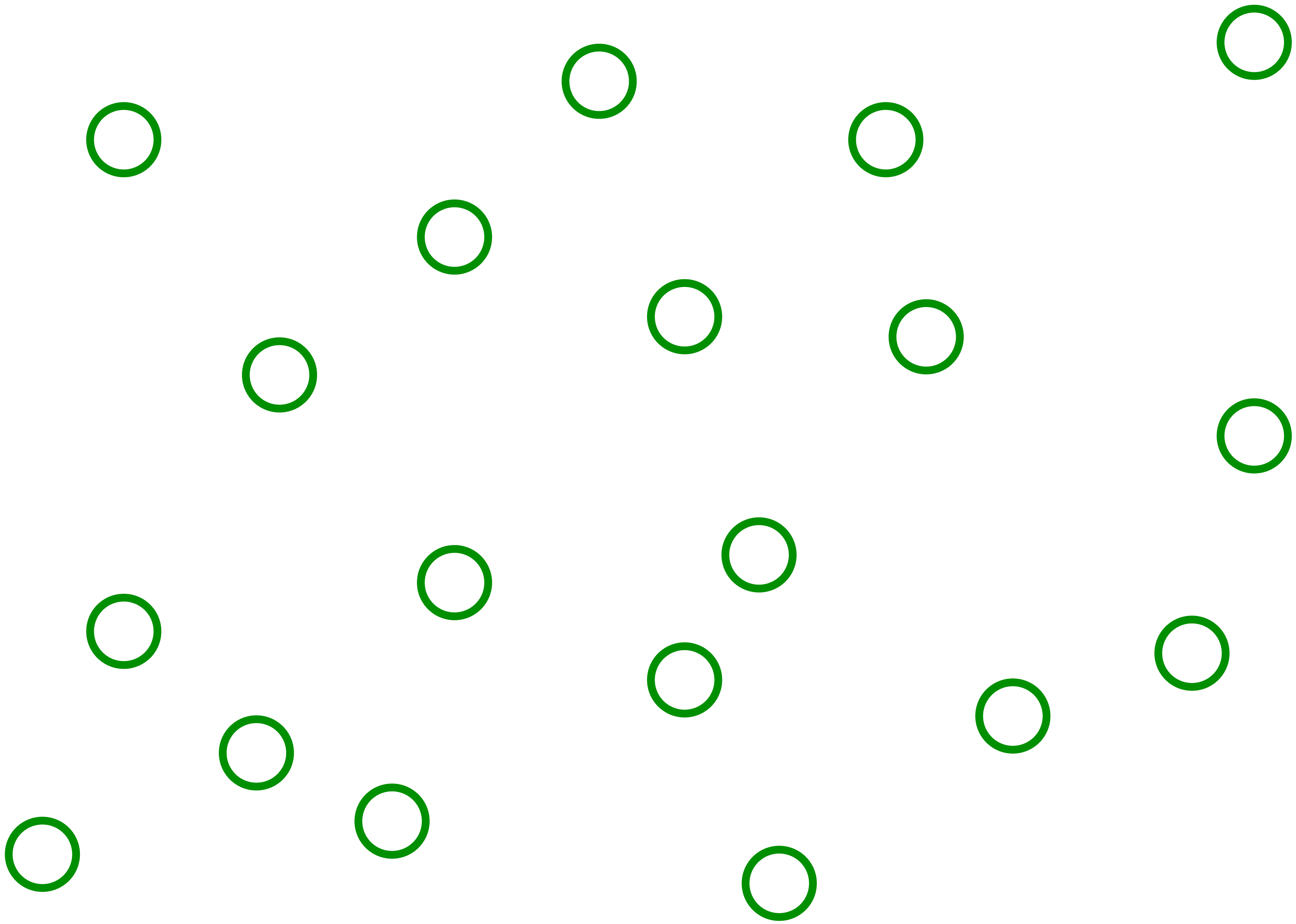
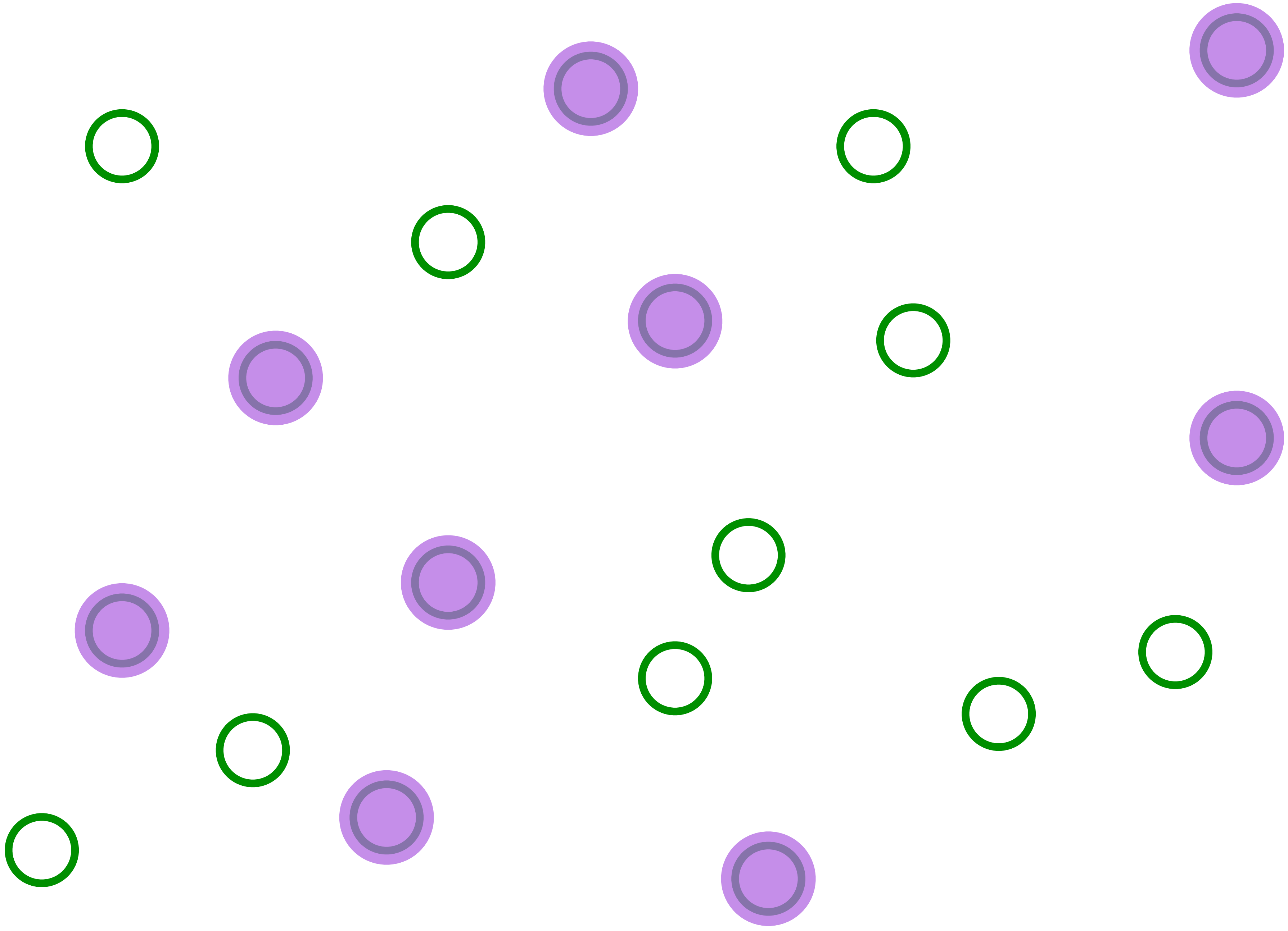
- **SYK models**

# The Sachdev-Ye-Kitaev (SYK) model



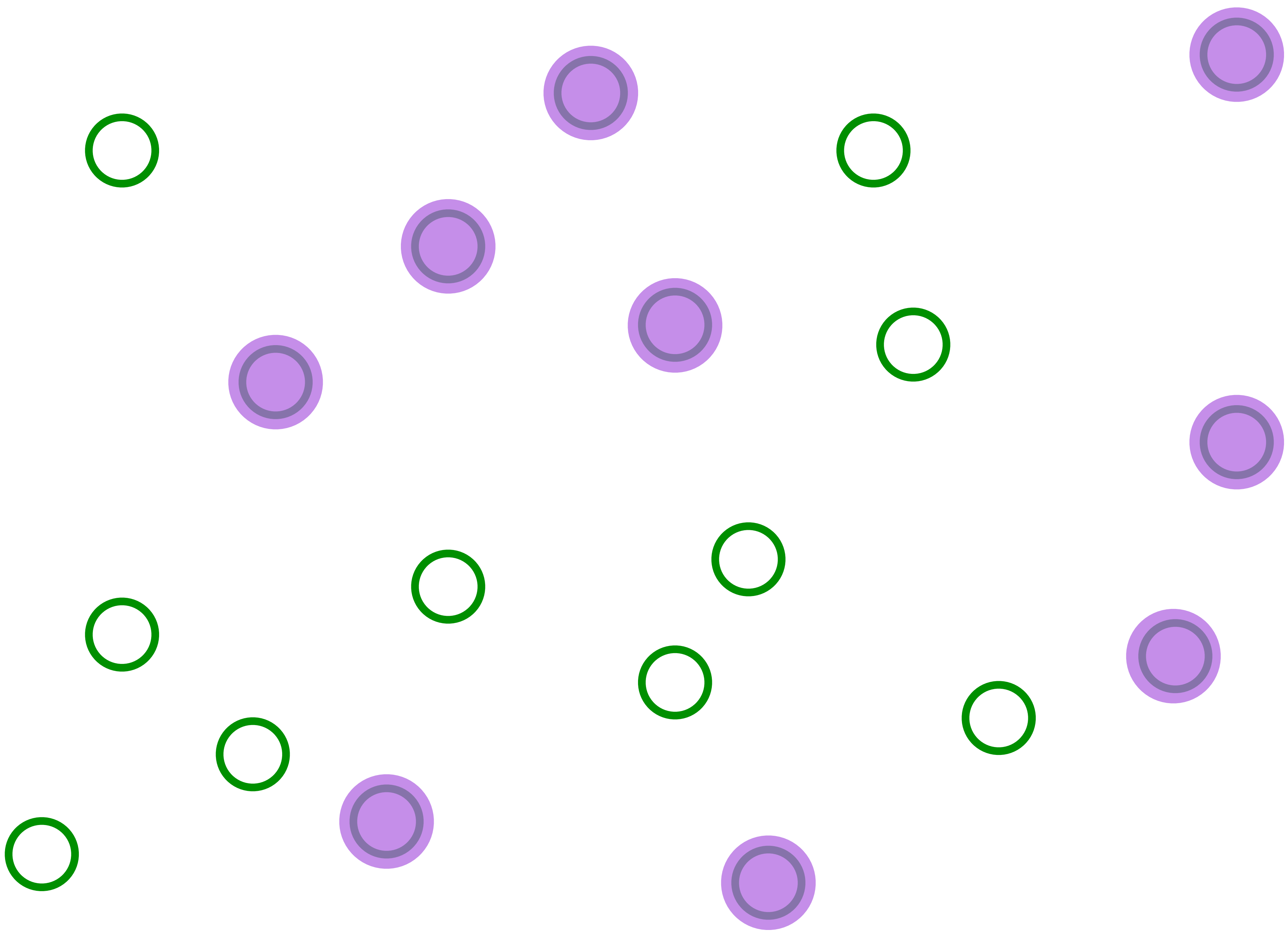
Pick a set of random positions

# The SYK model



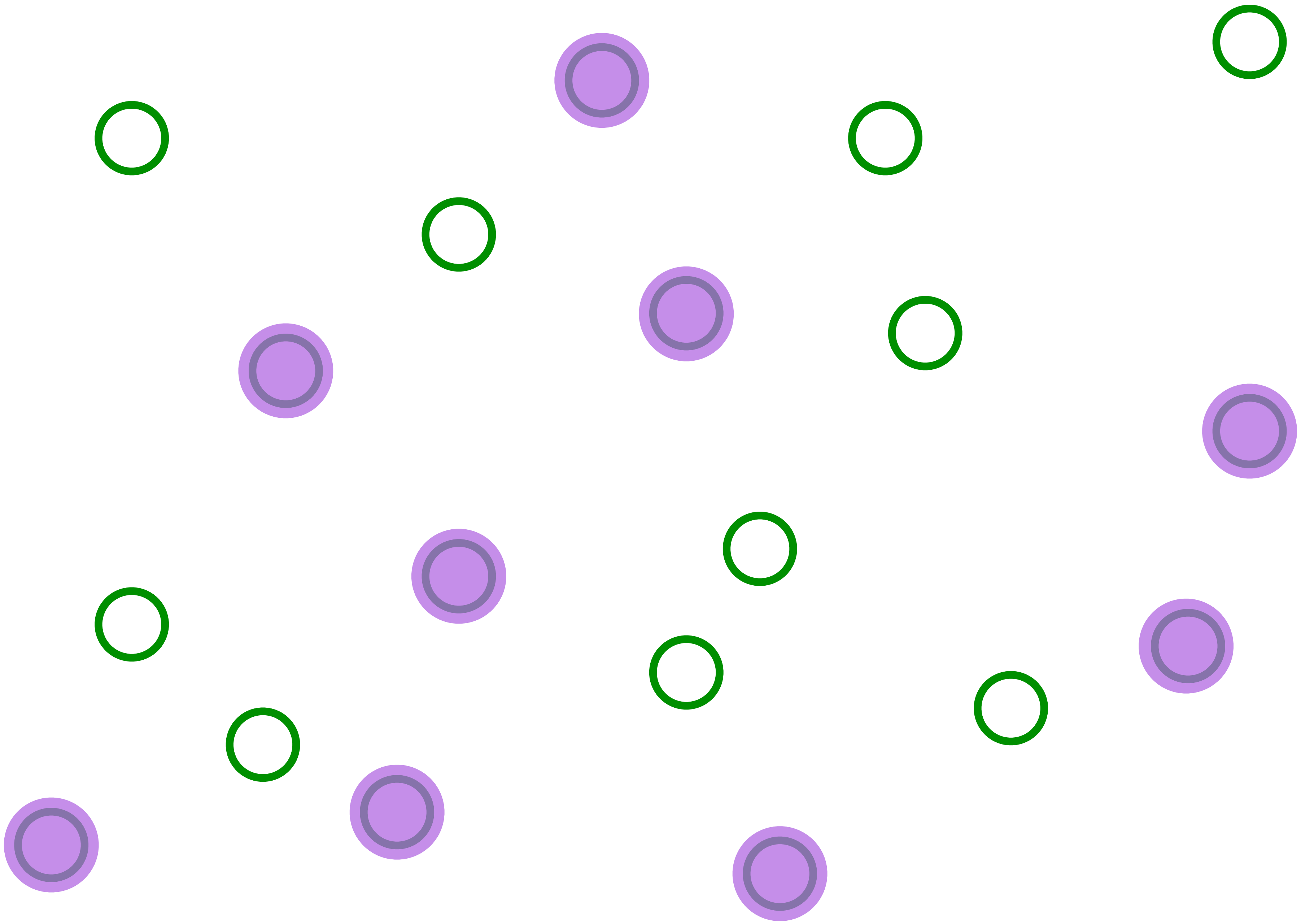
Place electrons randomly on some sites

# The SYK model



Entangle electrons pairwise randomly

# The SYK model



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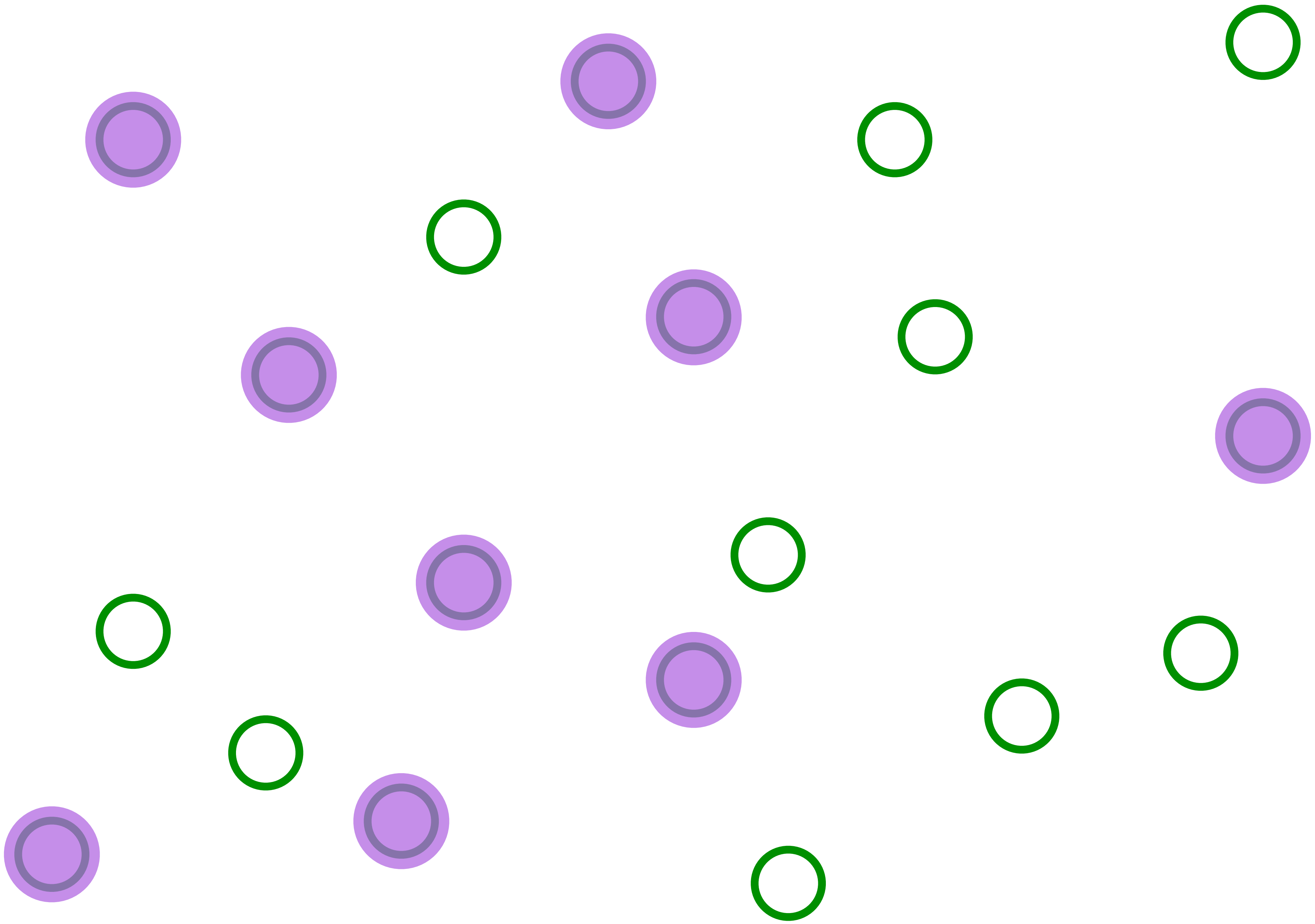

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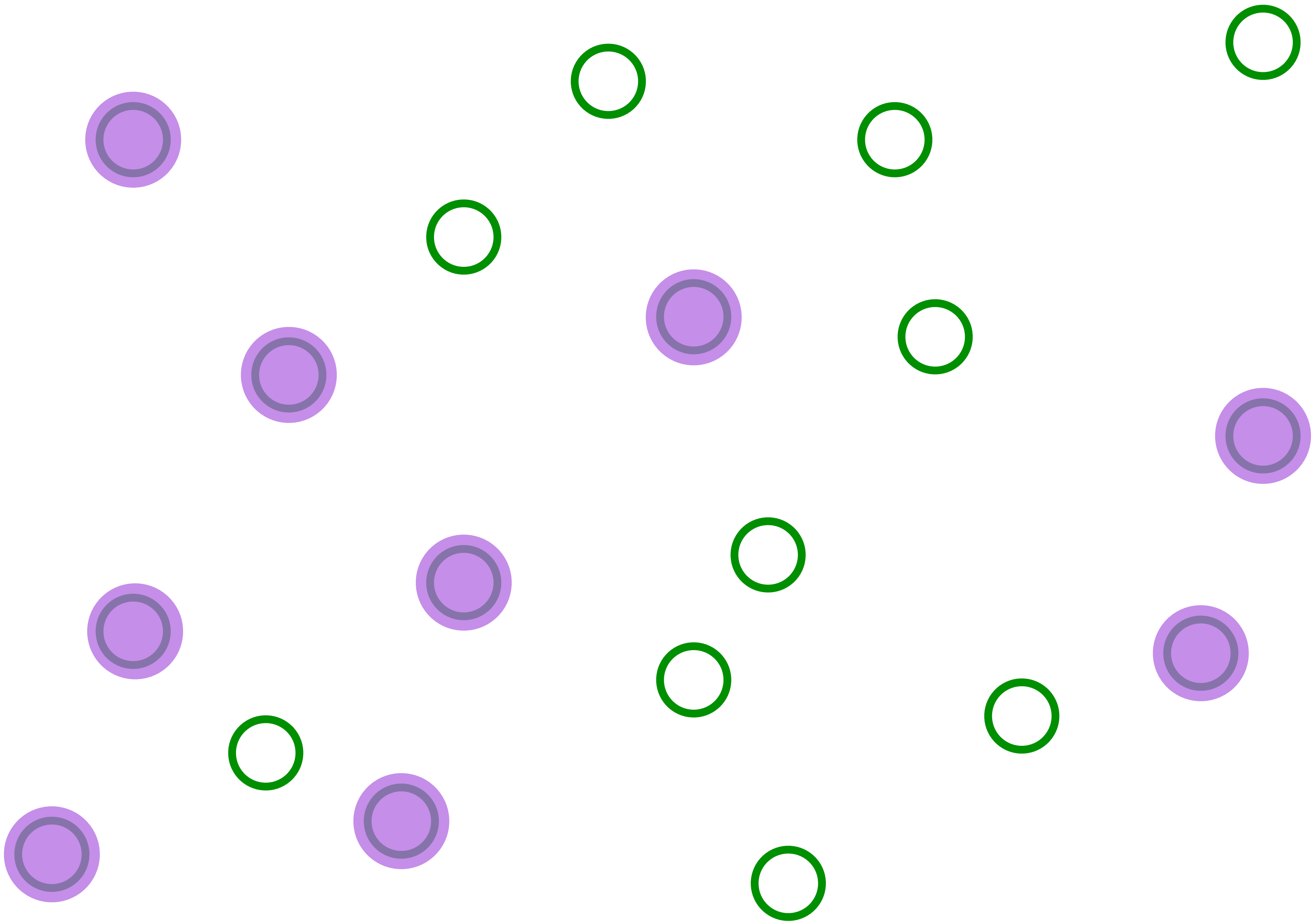

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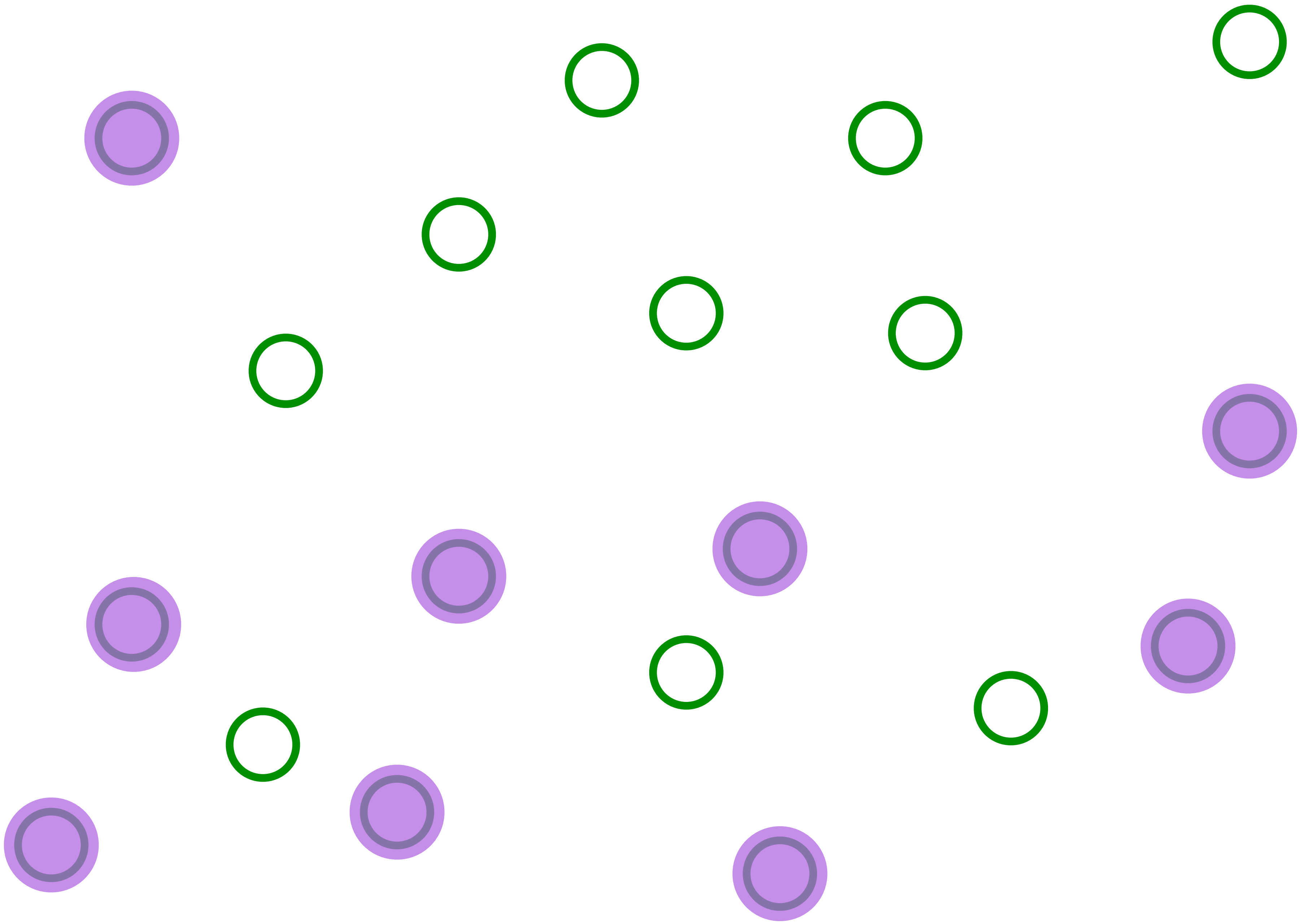

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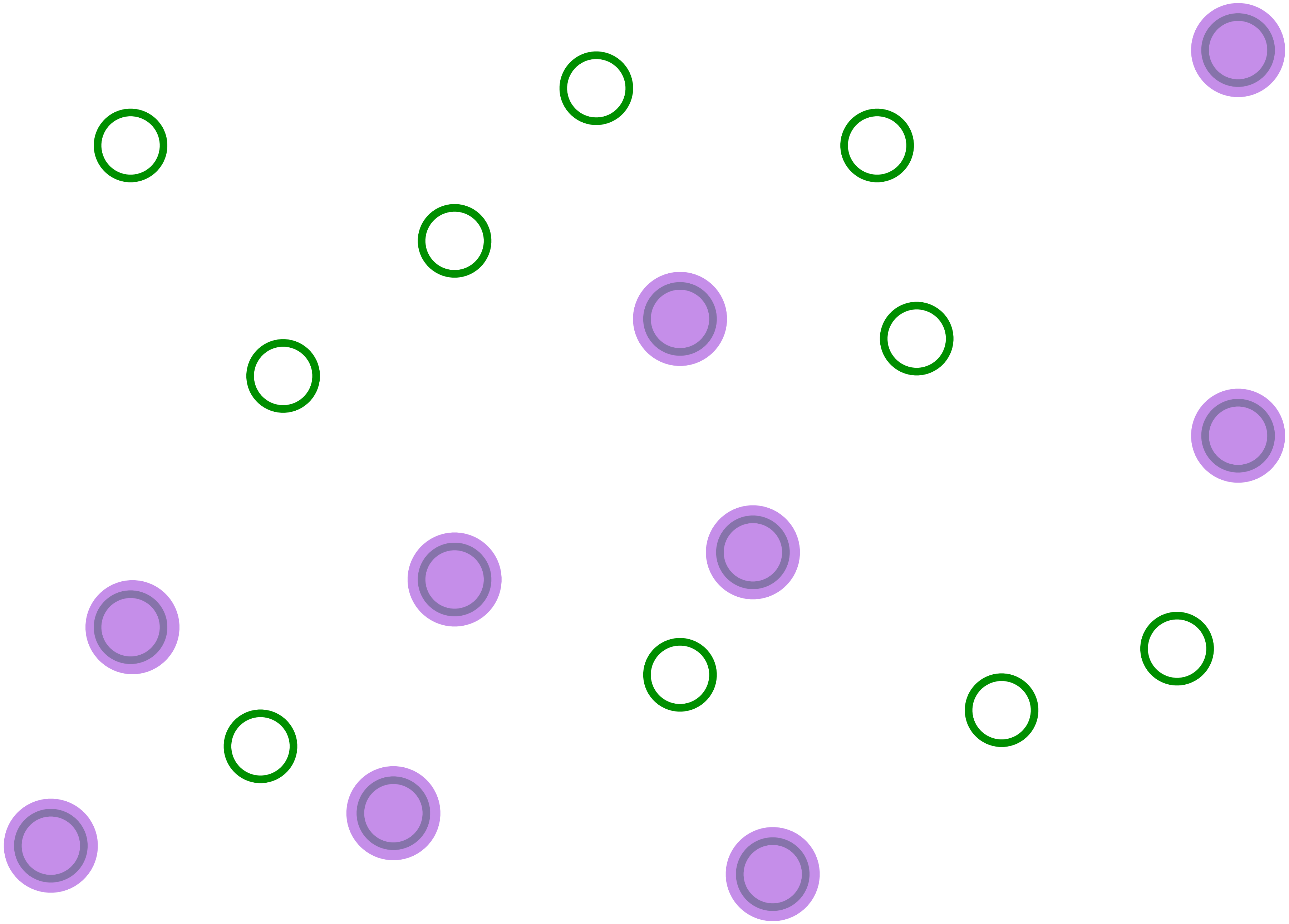

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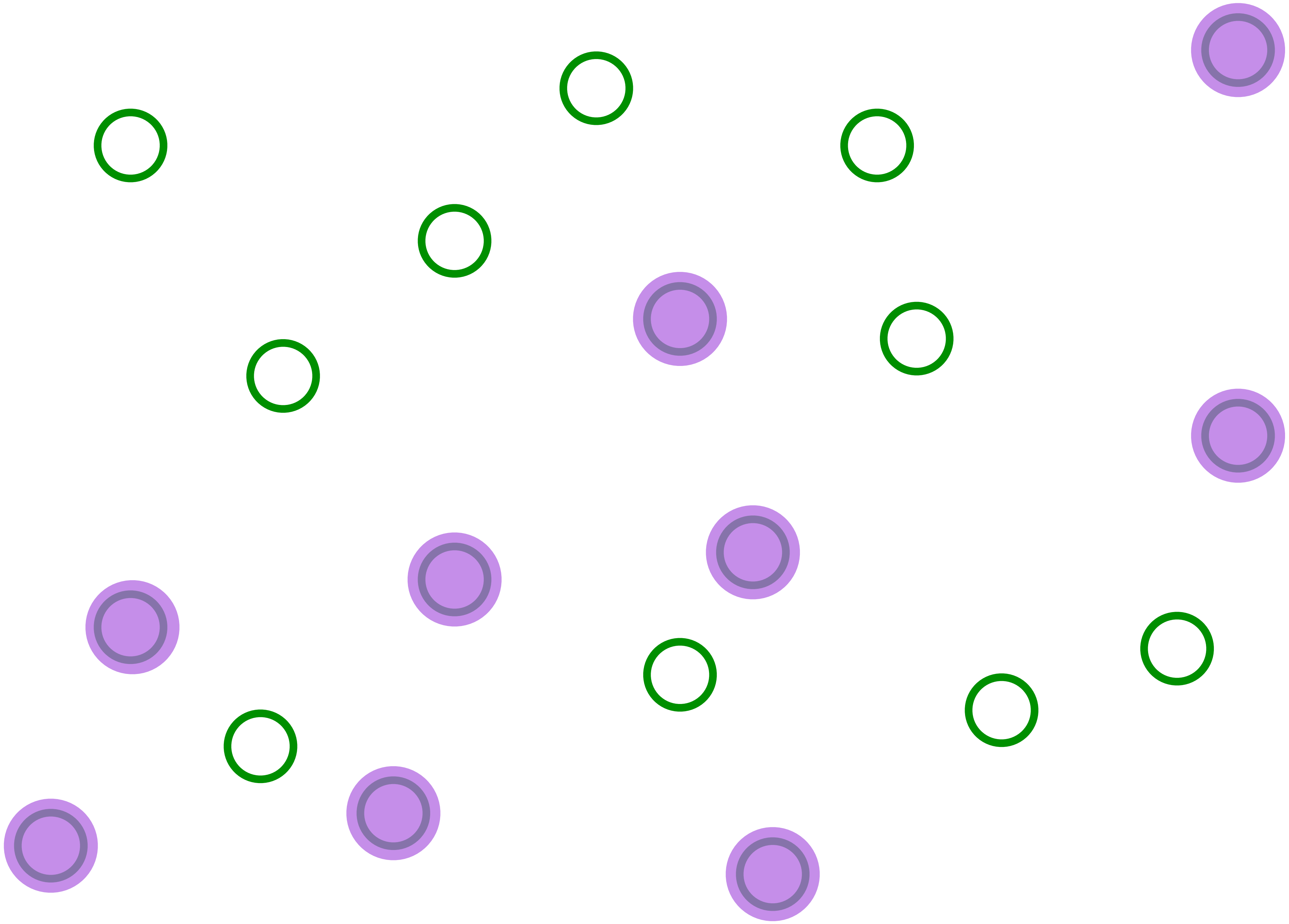


This describes both a strange metal and a black hole!

# The SYK model

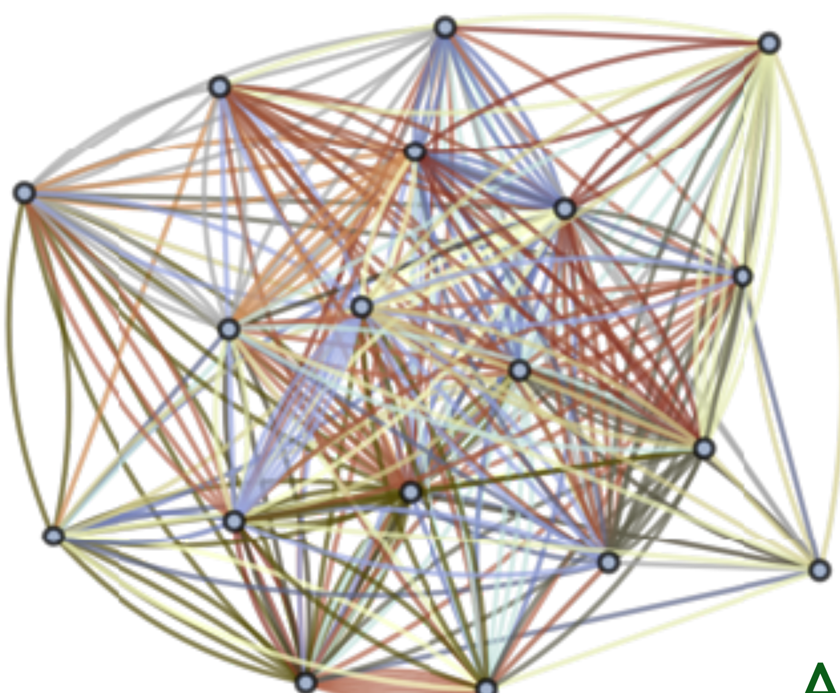
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large $N$ limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# The SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

# The SYK model

- $T = 0$ fermion Green's function is singular:

$$G(\tau) \sim \frac{1}{\sqrt{\tau}} \quad \text{at large } \tau.$$

$\Big($Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau\Big)$

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(Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

- $T > 0$ Green's function has conformal invariance, and is 'dephased' at the characteristic scale $\sim k_B T/\hbar$, which is independent of $U$.

$$G \sim e^{-2\pi \mathcal{E} T \tau} \left( \frac{T}{\sin(\pi k_B T \tau/\hbar)} \right)^{1/2}$$

$\mathcal{E}$ measures particle-hole asymmetry.

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
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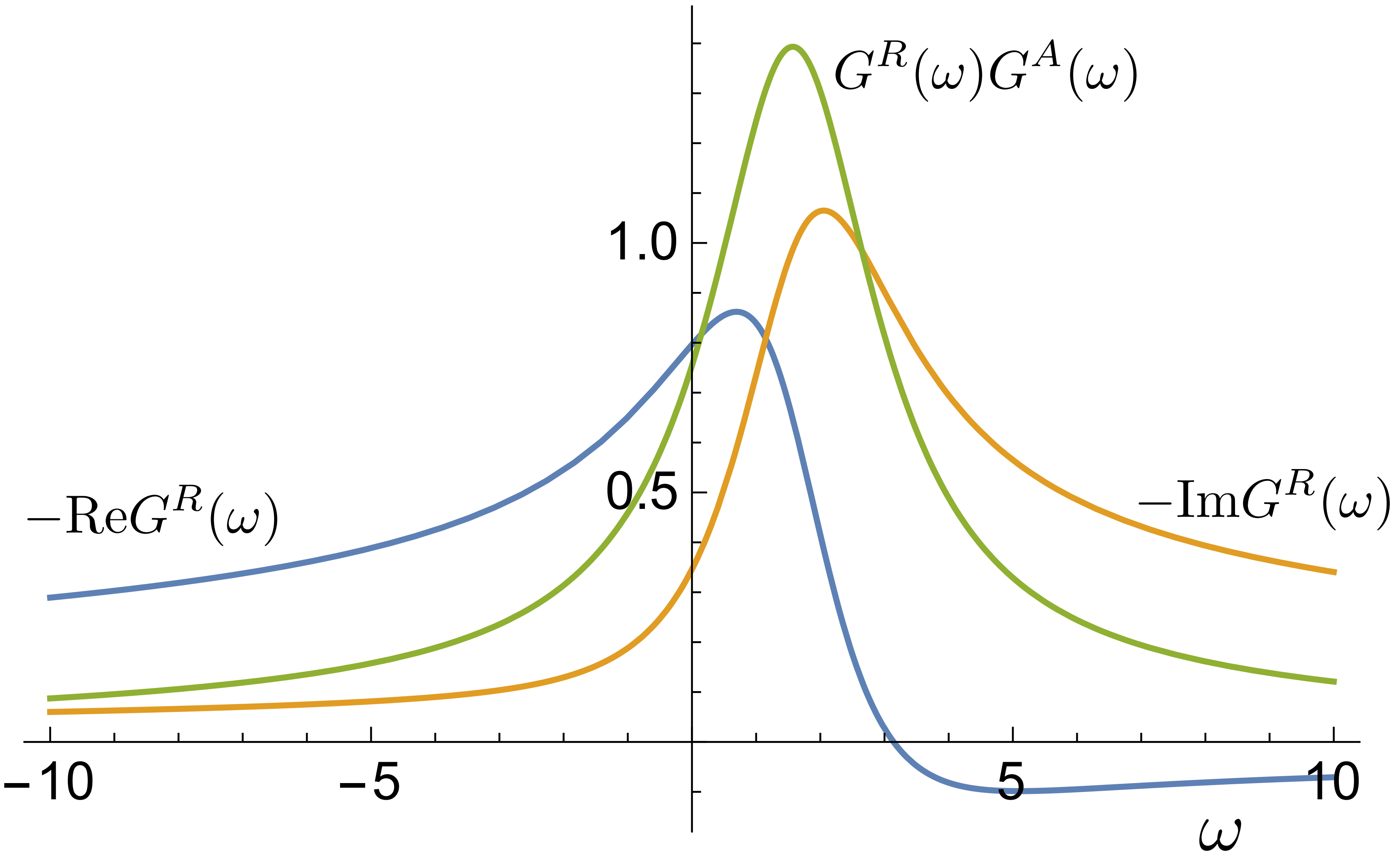
$\mathcal{E}$ measures particle-hole asymmetry.

- The last property indicates $\tau_{\text{eq}} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

# The SYK model




Green's functions away from half-filling

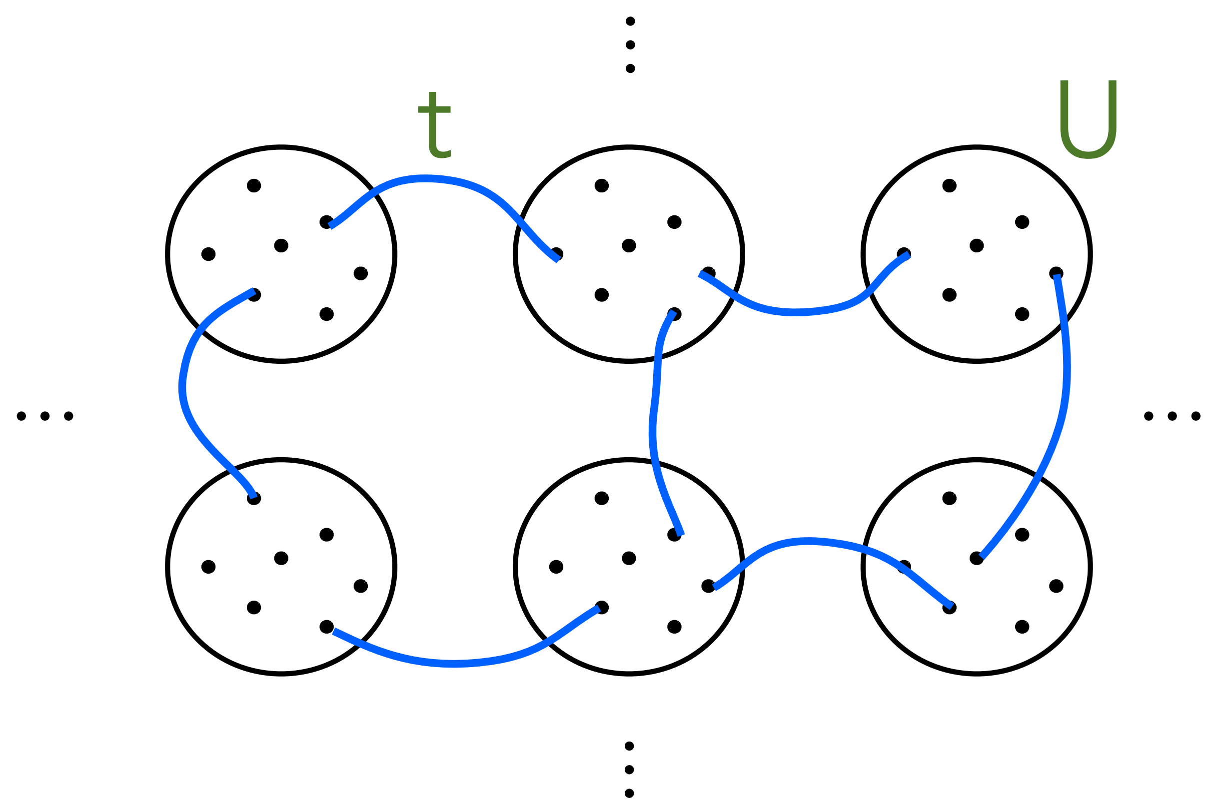
So the Green's functions display thermal 'damping' at a scale set by $T$ alone, which is independent of $U$.

arXiv:1705.00117

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents




$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

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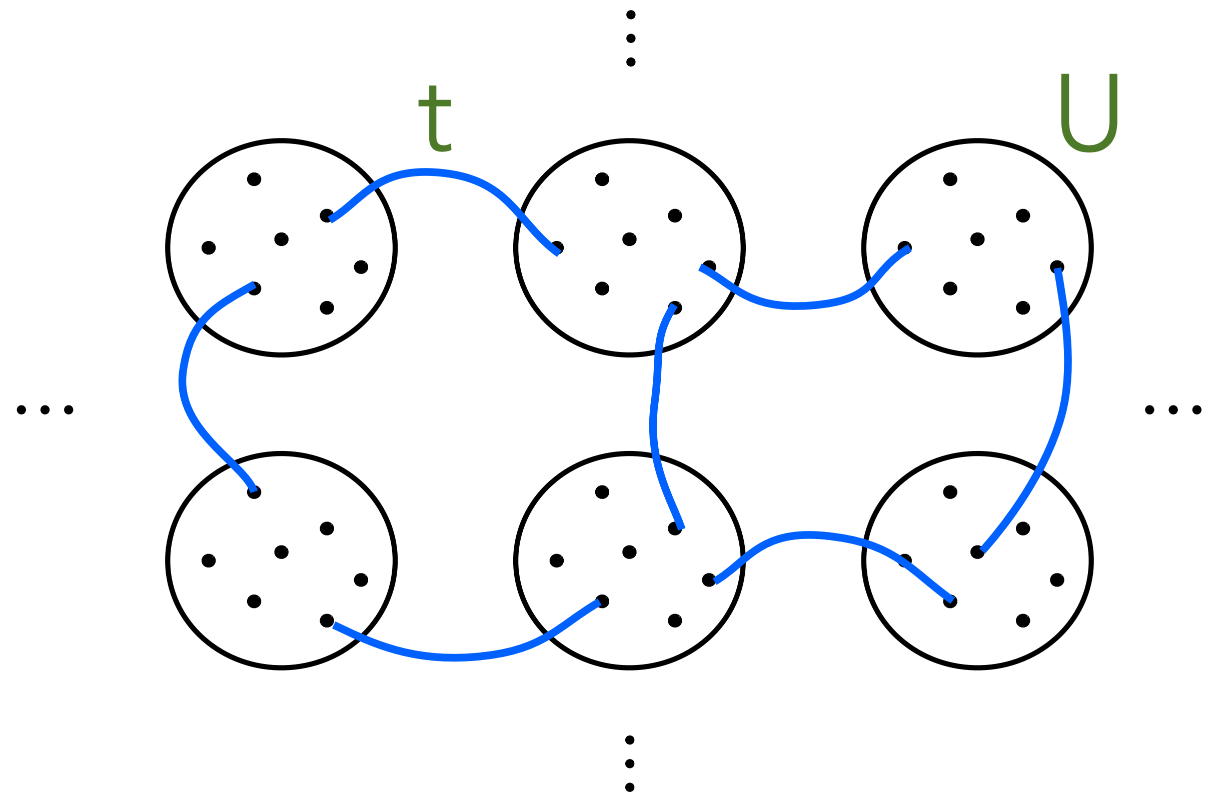


## Large *N* equations

$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$

$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau),$$

arXiv:1705.00117

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Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

- There is a low coherence scale $E_c \sim t_0^2/U$, with SYK criticality at $T > E_c$ and heavy Fermi liquid behavior for $T < E_c$.

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- There is a low coherence scale $E_c \sim t_0^2/U$, with SYK criticality at $T > E_c$ and heavy Fermi liquid behavior for $T < E_c$.

- From the Kubo formula, we have the conductivity

$$\mathrm{Re}[\sigma(\omega)] \propto t_0^2 \int d\Omega \frac{f(\omega+\Omega) - f(\Omega)}{\omega} A(\Omega) A(\omega+\Omega)$$

where $A(\omega) = \mathrm{Im}[G^R(\omega)]$.
At $T > E_c$, using $A(\omega) \sim \omega^{-1/2} F(\omega/T)$, this yields the bad metal behavior

$$\sigma \sim \frac{e^2}{h} \frac{t_0^2}{U} \frac{1}{T} \quad ; \quad \rho \sim \left(\frac{h}{e^2}\right) \frac{T}{E_c}$$

arXiv:1712.05026

Title: Magnetotransport in a model of a disordered strange metal

Authors: Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev

Aavishkar Patel

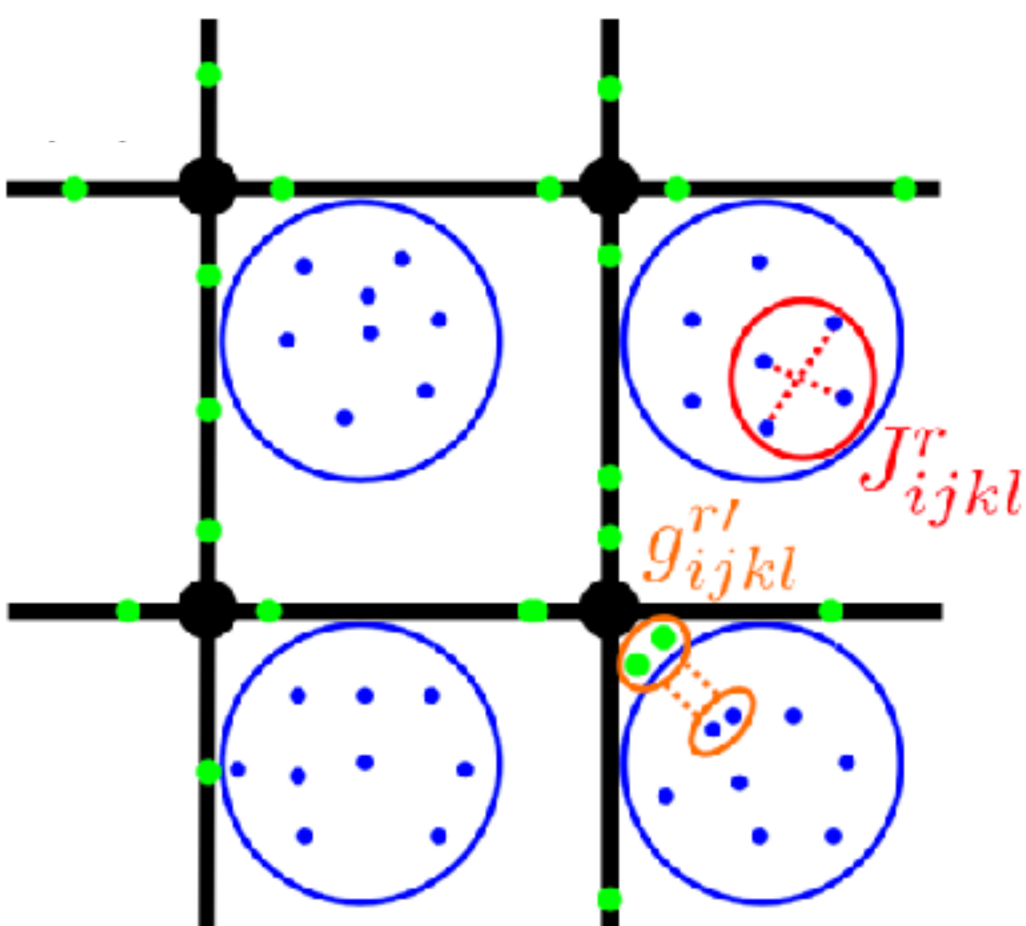
# Infecting a Fermi liquid and making it SYK

Mobile electrons ($c$, green) interacting with SYK quantum dots ($f$, blue) with exchange interactions.

This yields the first model agreeing with magnetotransport in strange metals !

$$H = -t \sum_{\langle rr' \rangle;\ i=1}^{M} (c_{ri}^\dagger c_{r'i} + \text{h.c.}) - \mu_c \sum_{r;\ i=1}^{M} c_{ri}^\dagger c_{ri} - \mu \sum_{r;\ i=1}^{N} f_{ri}^\dagger f_{ri}$$

$$+ \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^{r} f_{ri}^\dagger f_{rj} c_{rk}^\dagger c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J_{ijkl}^{r} f_{ri}^\dagger f_{rj}^\dagger f_{rk} f_{rl}.$$




A. A. Patel, J. McGreevy, D. P. Arovas and S. Sachdev, arXiv:1712.05026

Similar results in non-random models by Y. Werman, D. Chowdhury, T. Senthil, and E. Berg, to appear

# Infecting a Fermi liquid and making it SYK

$$\Sigma(\tau-\tau') = -J^2 G^2(\tau-\tau')G(\tau'-\tau) - \frac{M}{N} g^2 G(\tau-\tau')G^c(\tau-\tau')G^c(\tau'-\tau),$$

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)},$$

(*f* electrons)

$$\Sigma^c(\tau-\tau') = -g^2 G^c(\tau-\tau')G(\tau-\tau')G(\tau'-\tau),$$

$$G^c(i\omega_n) = \sum_k \frac{1}{i\omega_n - \epsilon_k + \mu_c - \Sigma^c(i\omega_n)}.$$

(*c* electrons)

Exactly solvable in the large *N*,*M* limits!

- Low-*T* phase: *c* electrons form a Marginal Fermi-liquid (MFL), *f* electrons are local SYK models

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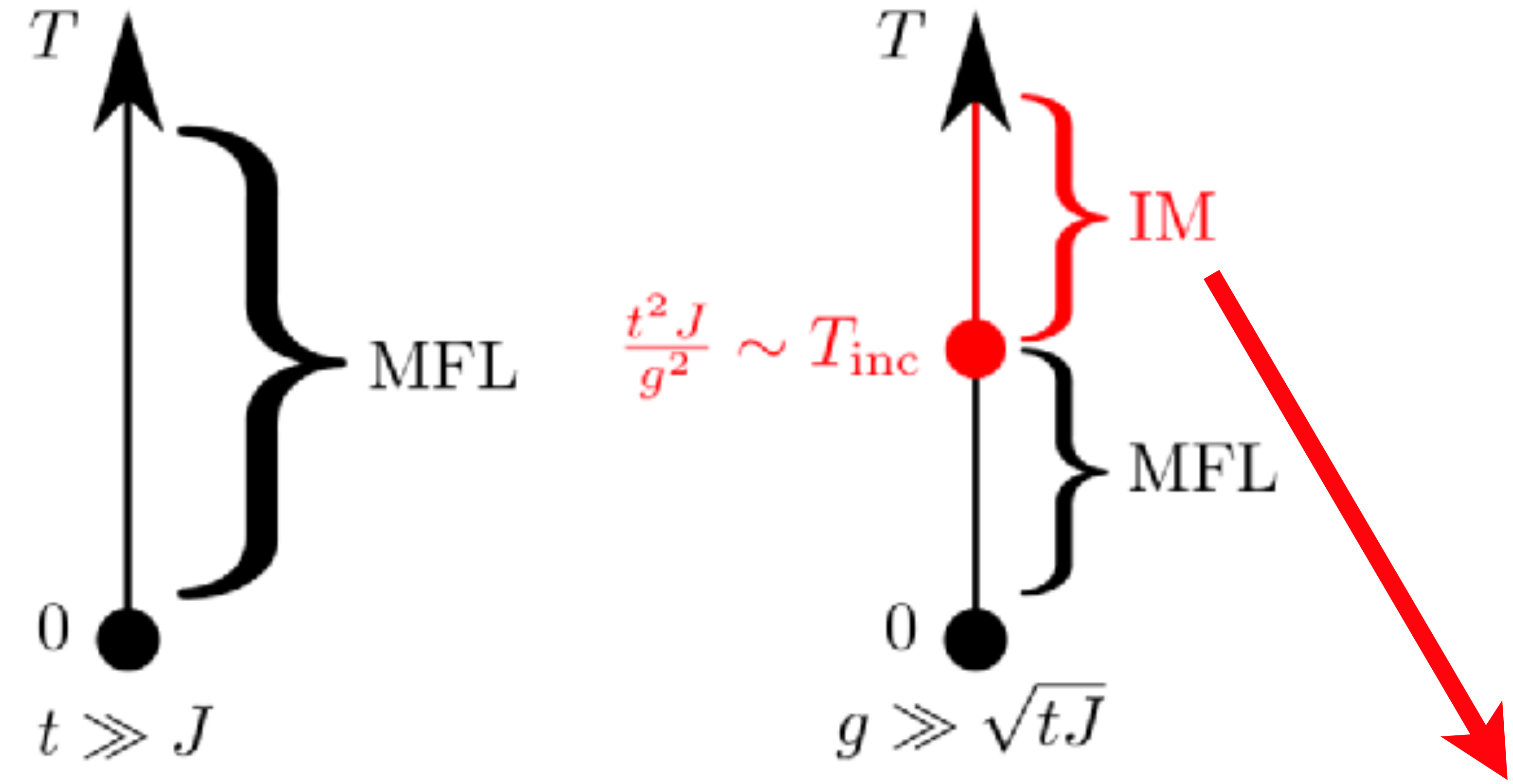
# Infecting a Fermi liquid and making it SYK



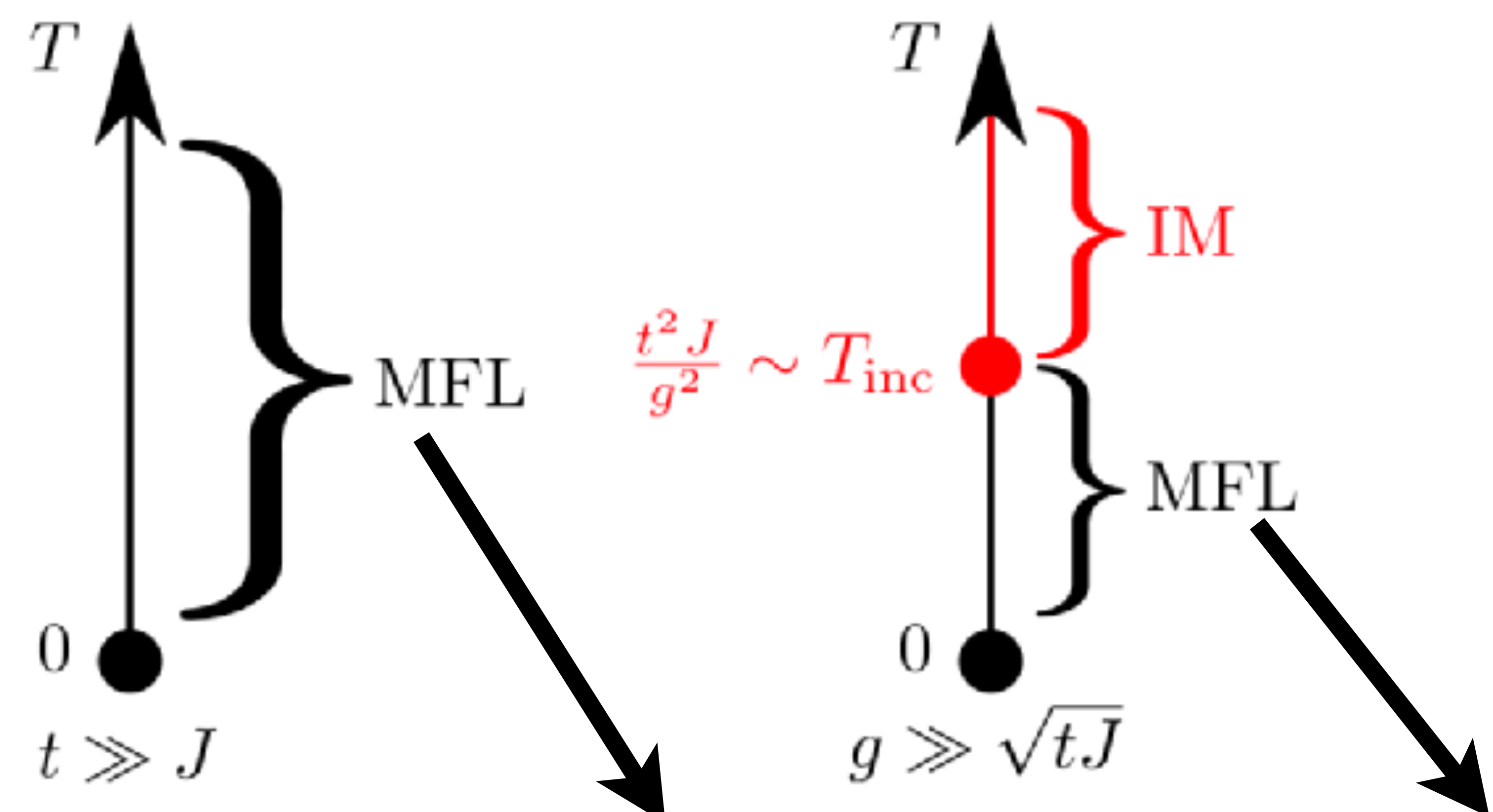

- High-$T$ phase: $c$ electrons form an "incoherent metal" (IM), with local Green's function, and no notion of momentum; $f$ electrons remain local SYK models

$$G^c(\tau) = -\frac{C_c}{\sqrt{1+e^{-4\pi\mathcal{E}_c}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E}_c T\tau},$$

$$G(\tau) = -\frac{C}{\sqrt{1+e^{-4\pi\mathcal{E}}}}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} e^{-2\pi\mathcal{E} T\tau}, \quad 0 \le \tau < \beta$$

Similar results in non-random models by Y. Werman, D. Chowdhury, T. Senthil, and E. Berg, to appear

# Infecting a Fermi liquid and making it SYK




- Low-$T$ phase: $c$ electrons form a Marginal Fermi-liquid (MFL), $f$ electrons are local SYK models

$$\Sigma^c(i\omega_n) = \frac{ig^2\nu(0)T}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\left(\frac{\omega_n}{T}\ln\left(\frac{2\pi T e^{\gamma_E - 1}}{J}\right) + \frac{\omega_n}{T}\psi\left(\frac{\omega_n}{2\pi T}\right) + \pi\right),$$

$$\Sigma^c(i\omega_n) \to \frac{ig^2\nu(0)}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}}\omega_n \ln\left(\frac{|\omega_n| e^{\gamma_E - 1}}{J}\right), \quad |\omega_n| \gg T \quad (\nu(0) \sim 1/t)$$

Similar results in non-random models by Y. Werman, D. Chowdhury, T. Senthil, and E. Berg, to appear

# Linear-in-*T* resistivity

*Both* the MFL and the IM are not translationally-invariant and have linear-in-*T* resistivities!

$$\sigma_0^{\mathrm{MFL}} = 0.120251 \times M T^{-1} J \times \left( \frac{v_F^2}{g^2} \right) \cosh^{1/2}(2\pi\mathcal{E}). \quad (v_F \sim t)$$

$$\sigma_0^{\mathrm{IM}} = (\pi^{1/2}/8) \times M T^{-1} J \times \left( \frac{\Lambda}{\nu(0) g^2} \right) \frac{\cosh^{1/2}(2\pi\mathcal{E})}{\cosh(2\pi\mathcal{E}_c)}.$$

[Can be obtained straightforwardly from Kubo formula in the large-*N,M* limits]

The IM is also a “Bad metal” with $\sigma_0^{\mathrm{IM}} \ll 1$

Similar results in non-random models by Y. Werman, D. Chowdhury, T. Senthil, and E. Berg, to appear

# Magnetotransport: Marginal-Fermi liquid

- Thanks to large *N,M*, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields…

$$(1 - \partial_\omega \mathrm{Re}[\Sigma^c_R(\omega)])\partial_t \delta n(t,k,\omega) + v_F \hat{k} \cdot \mathbf{E}(t)\; n'_f(\omega) + v_F(\hat{k} \times \mathcal{B}\hat{z}) \cdot \nabla_k \delta n(t,k,\omega) = 2\delta n(t,k,\omega)\mathrm{Im}[\Sigma^c_R(\omega)],$$

$(\mathcal{B} = eBa^2/\hbar)$ (i.e. flux per unit cell)

$$\sigma_L^{\mathrm{MFL}} = M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{-\mathrm{Im}[\Sigma^c_R(E_1)]}{\mathrm{Im}[\Sigma^c_R(E_1)]^2 + (v_F/(2k_F))^2\mathcal{B}^2},$$

$$\sigma_H^{\mathrm{MFL}} = -M\frac{v_F^2\nu(0)}{16T}\int_{-\infty}^{\infty}\frac{dE_1}{2\pi}\mathrm{sech}^2\left(\frac{E_1}{2T}\right)\frac{(v_F/(2k_F))\mathcal{B}}{\mathrm{Im}[\Sigma^c_R(E_1)]^2 + (v_F/(2k_F))^2\mathcal{B}^2}.$$

$$\sigma_L^{\mathrm{MFL}} \sim T^{-1}s_L((v_F/k_F)(\mathcal{B}/T)), \quad \sigma_H^{\mathrm{MFL}} \sim -\mathcal{B}T^{-2}s_H((v_F/k_F)(\mathcal{B}/T)).$$

$$s_{L,H}(x \to \infty) \propto 1/x^2, \quad s_{L,H}(x \to 0) \propto x^0.$$

Scaling between magnetic field and temperature in **orbital** magnetotransport!

# Magnetotransport with mesoscopic homogeneity

- No macroscopic momentum, so equations describing charge transport are just

$$\nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla \Phi(\mathbf{x}).$$


A. M. Dykhne, "Anomalous plasma resistance in a strong magnetic field," Journal of Experimental and Theoretical Physics **32**, 348 (1971).

D. Stroud, "Generalized effective-medium approach to the conductivity of an inhomogeneous material," Phys. Rev. B **12**, 3368 (1975).

M. M. Parish and P. B. Littlewood, "Non-saturating magnetoresistance in heavily disordered semiconductors," Nature **426**, 162 (2003), cond-mat/0312020 .

M. M. Parish and P. B. Littlewood, "Classical magnetotransport of inhomogeneous conductors," Phys. Rev. B **72**, 094417 (2005), cond-mat/0508229 .

V. Guttal and D. Stroud, "Model for a macroscopically disordered conductor with an exactly linear high-field magnetoresistance," Phys. Rev. B **71**, 201304 (2005), cond-mat/0502162 .

J. C. W. Song, G. Refael, and P. A. Lee, "Linear magnetoresistance in metals: Guiding center diffusion in a smooth random potential," Phys. Rev. B **92**, 180204 (2015), arXiv:1507.04730 [cond-mat.mes-hall] .

N. Ramakrishnan, Y. T. Lai, S. Lara, M. M. Parish, and S. Adam, "Equivalence of Effective Medium and Random Resistor Network models for disorder-induced unsaturating linear magnetoresistance," ArXiv e-prints (2017), arXiv:1703.05478 [cond-mat.mes-hall] .

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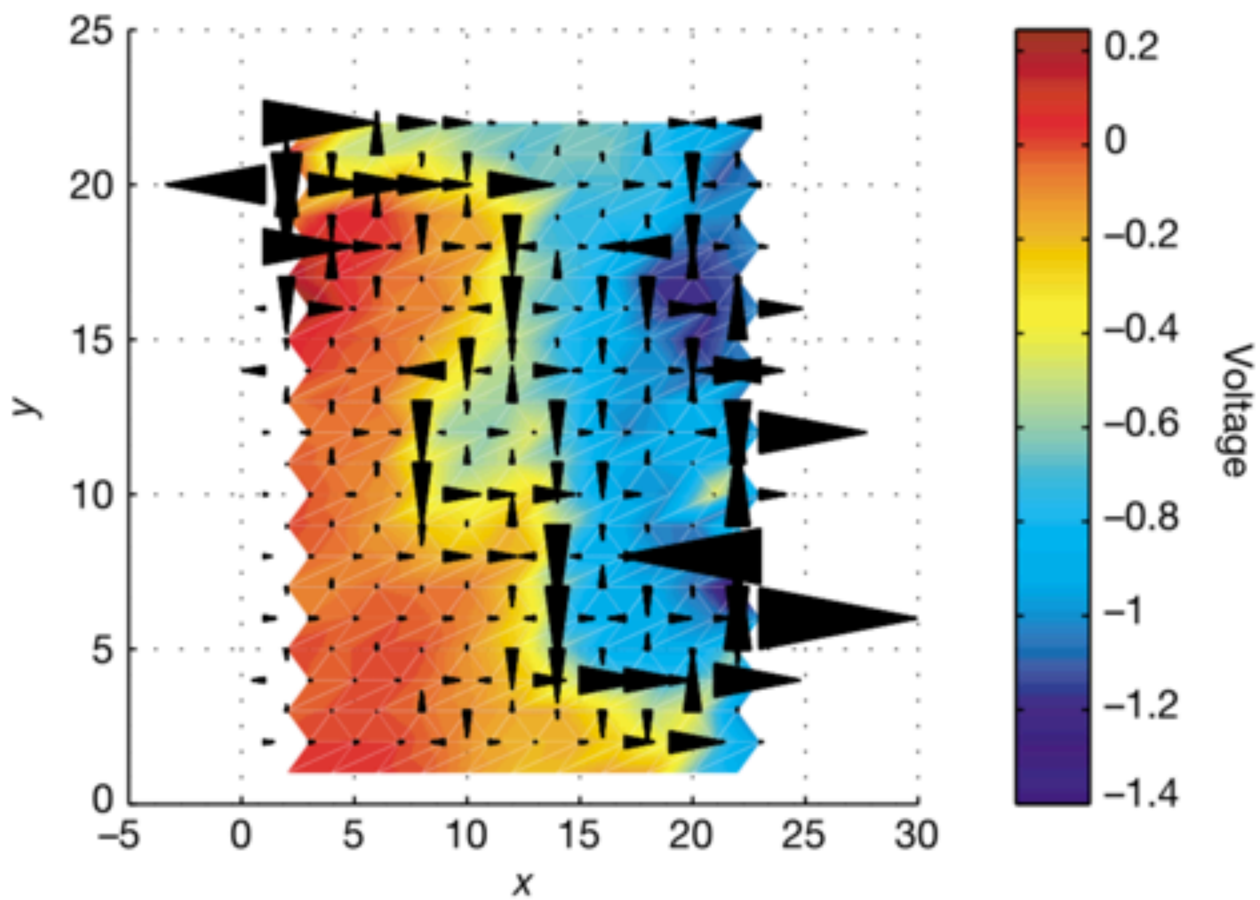
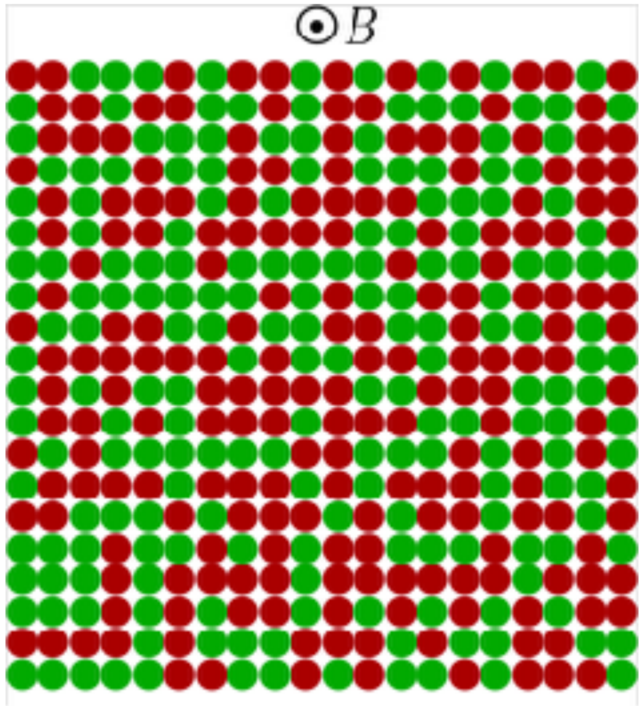



**Figure 3** Visualization of currents and voltages at large magnetic field in a 10 × 10 random network of disks with radii 1 (arbitrary units), where the potential difference $U = -1$ V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in *H*.

- Current path length increases linearly with *B* at large *B* due to local Hall effect, which causes the global resistance to increase linearly with *B* at large *B*.

M.M. Parish and P.B. Littlewood, Nature **426**, 162 (2003)

# Solvable toy model: two-component disorder




- Two types of domains *a,b* with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.

- Effective medium equations can be solved exactly

$$\left(\mathbb{I}+\frac{\sigma^a-\sigma^e}{2\sigma^e_L}\right)^{-1}\cdot(\sigma^a-\sigma^e)+\left(\mathbb{I}+\frac{\sigma^b-\sigma^e}{2\sigma^e_L}\right)^{-1}\cdot(\sigma^b-\sigma^e)=0.$$

$$\rho^e_L\equiv\frac{\sigma^e_L}{\sigma^{e2}_L+\sigma^{e2}_H}=\frac{\sqrt{(\mathcal{B}/m)^2\left(\gamma_a\sigma^{\mathrm{MFL}}_{0a}-\gamma_b\sigma^{\mathrm{MFL}}_{0b}\right)^2+\gamma_a^2\gamma_b^2\left(\sigma^{\mathrm{MFL}}_{0a}+\sigma^{\mathrm{MFL}}_{0b}\right)^2}}{\gamma_a\gamma_b(\sigma^{\mathrm{MFL}}_{0a}\sigma^{\mathrm{MFL}}_{0b})^{1/2}\left(\sigma^{\mathrm{MFL}}_{0a}+\sigma^{\mathrm{MFL}}_{0b}\right)},$$

$$\rho^e_H\equiv-\frac{\sigma^e_H/\mathcal{B}}{\sigma^{e2}_L+\sigma^{e2}_H}=\frac{\gamma_a+\gamma_b}{m\gamma_a\gamma_b\left(\sigma^{\mathrm{MFL}}_{0a}+\sigma^{\mathrm{MFL}}_{0b}\right)}.\quad(m=k_F/v_F\sim 1/t)$$

$\gamma_{a,b}\sim T$ (i.e. effective transport scattering rates)

$$\rho^e_L\sim\sqrt{c_1T^2+c_2B^2}$$

Scaling between *B* and *T* at microscopic orbital level has been transferred to global MR!

# *Magnetotransport in strange metals*

- Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-$T$ resistance, with a magnetoresistance which scales as $B \sim T$.

# *Magnetotransport in strange metals*

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- Higher temperatures lead to an incoherent metal with a local Green's function and a linear-in-$T$ resistance, but negligible magnetoresistance.

- This simple two-component model describes a new state of matter which is realized by electrons in the presence of strong interactions and disorder.

- Can such a model be realized as a fixed-point of a generic theory of strongly-interacting electrons in the presence of disorder?

- Can we start from a single-band Hubbard model with disorder, and end up with such two-band fixed point, with emergent local conservation laws?

- Electrons in doped silicon appear to separate into two components: localized spin moments and itinerant electrons

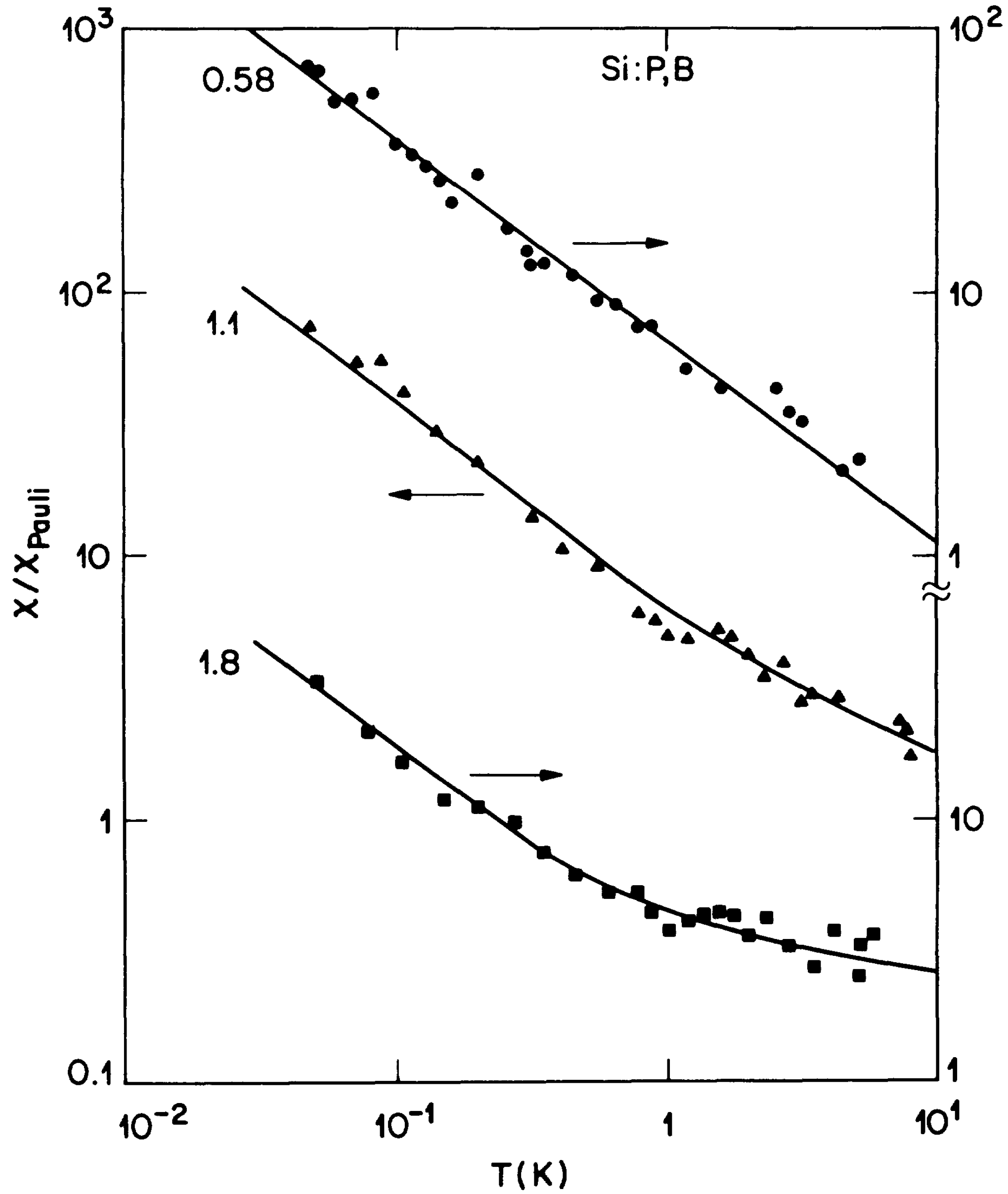


FIG. 1. Temperature dependence of normalized susceptibility $\chi/\chi_{Pauli}$ of three Si:P,B samples with different normalized electron densities, $n/n_c$ =0.58, 1.1, and 1.8. Solid lines through data are a guide to the eye.

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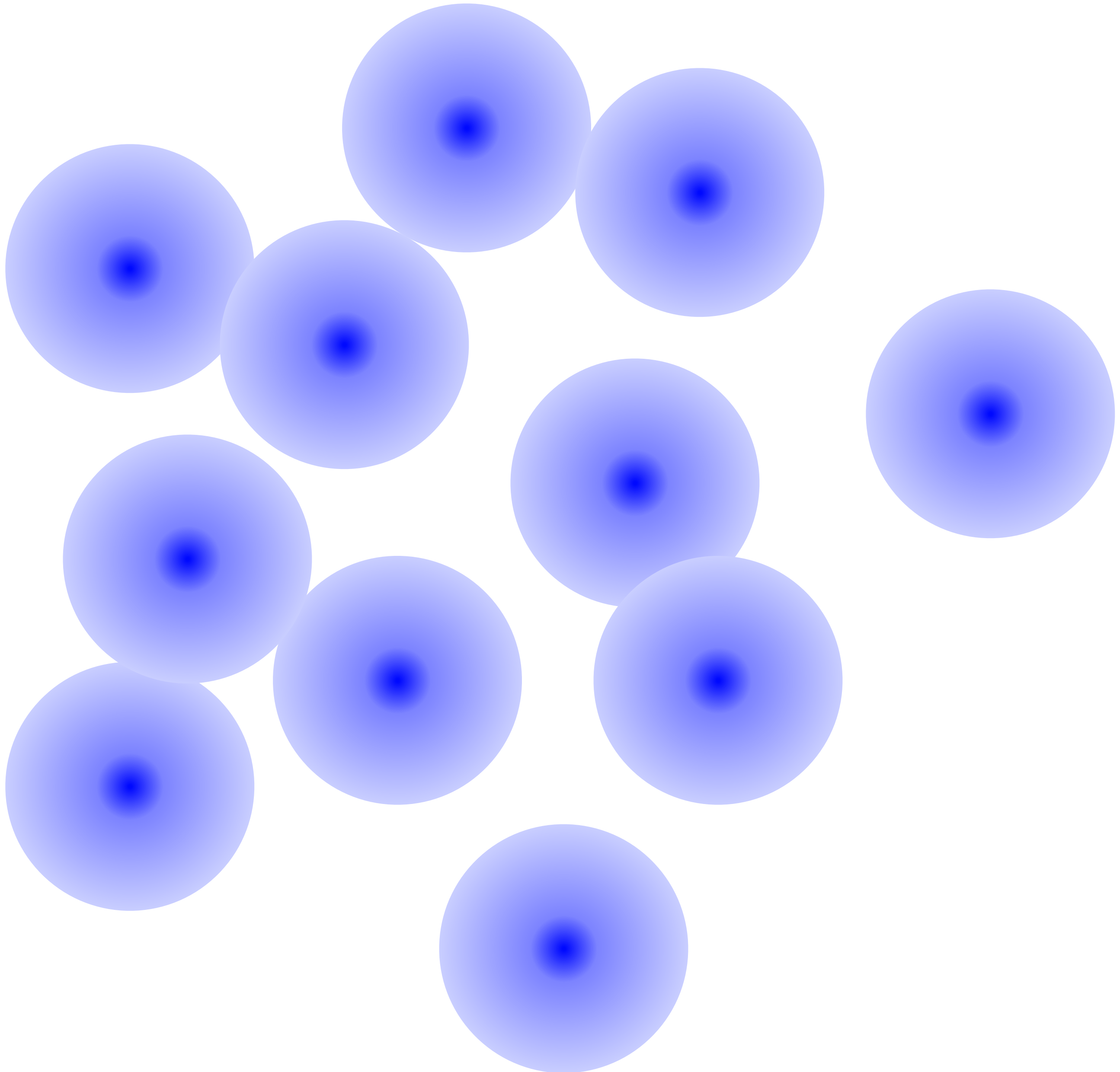

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