

The SYK model

Theory Winter School
National High Magnetic Field Laboratory, Tallahassee

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Talk online: sachdev.physics.harvard.edu



Quantum matter with quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are 'fractions' of an electron)

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

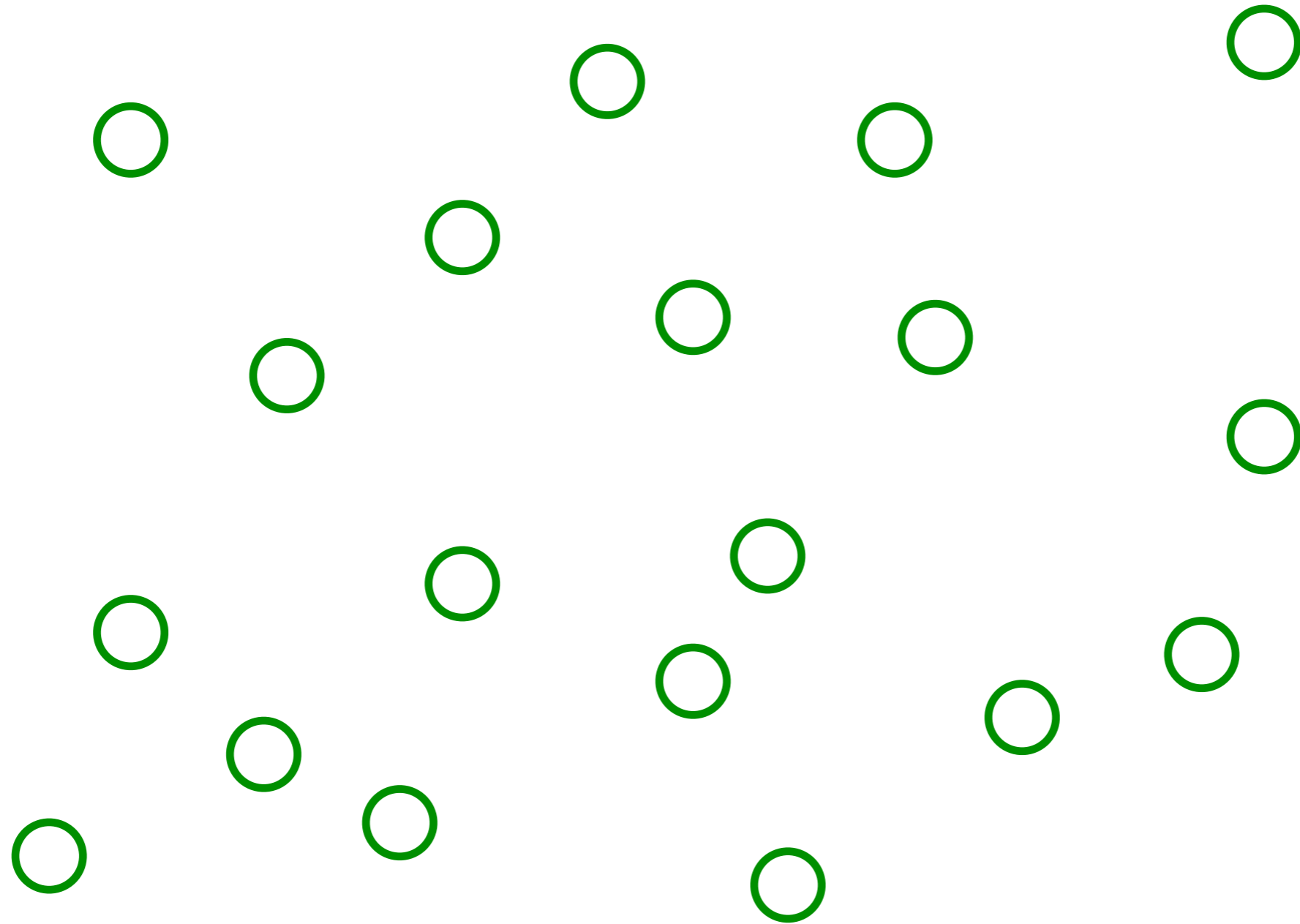
Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\text{eq}} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \rightarrow 0,$$

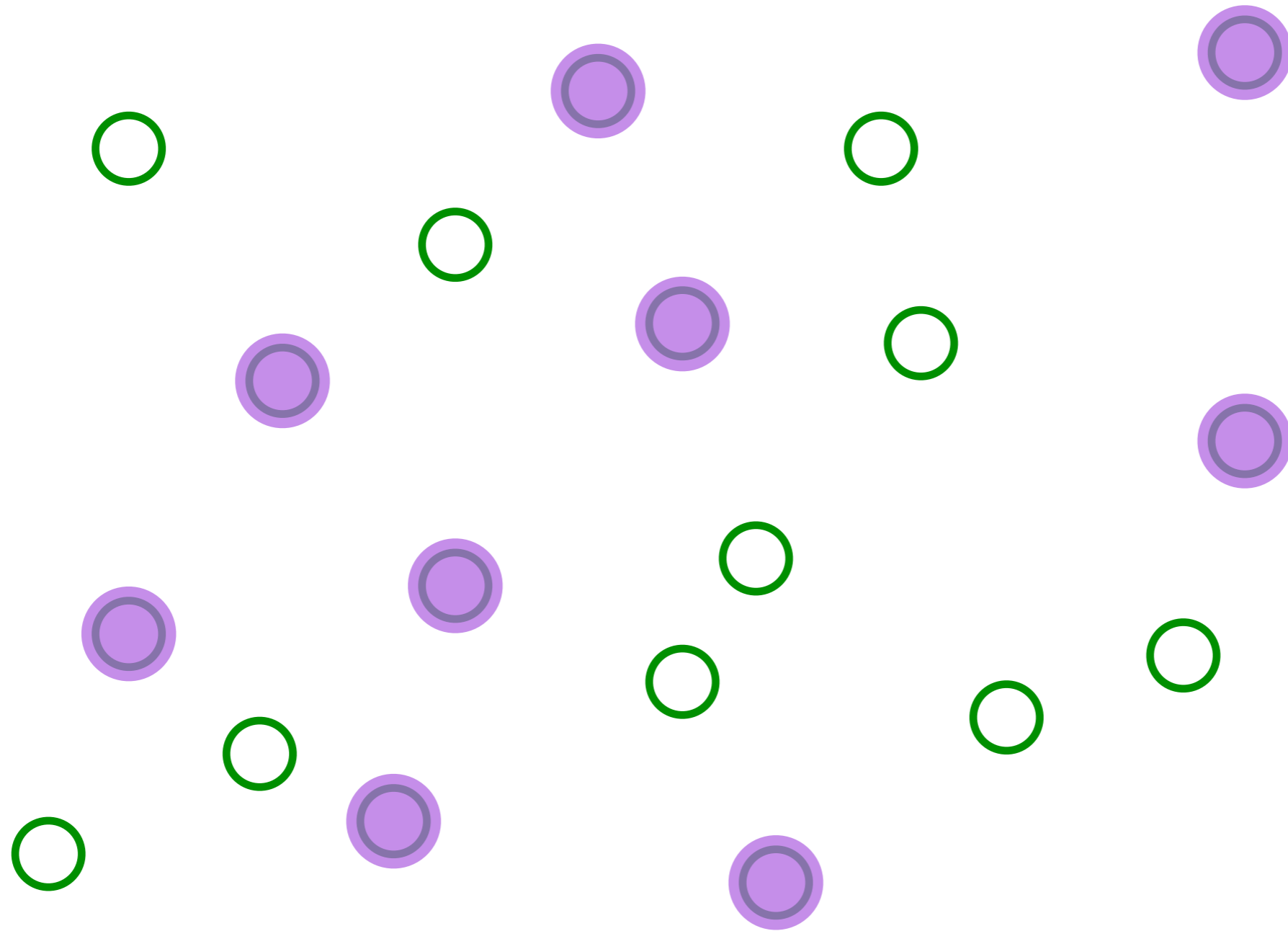
where E_F is the Fermi energy.

A simple model of a metal with quasiparticles



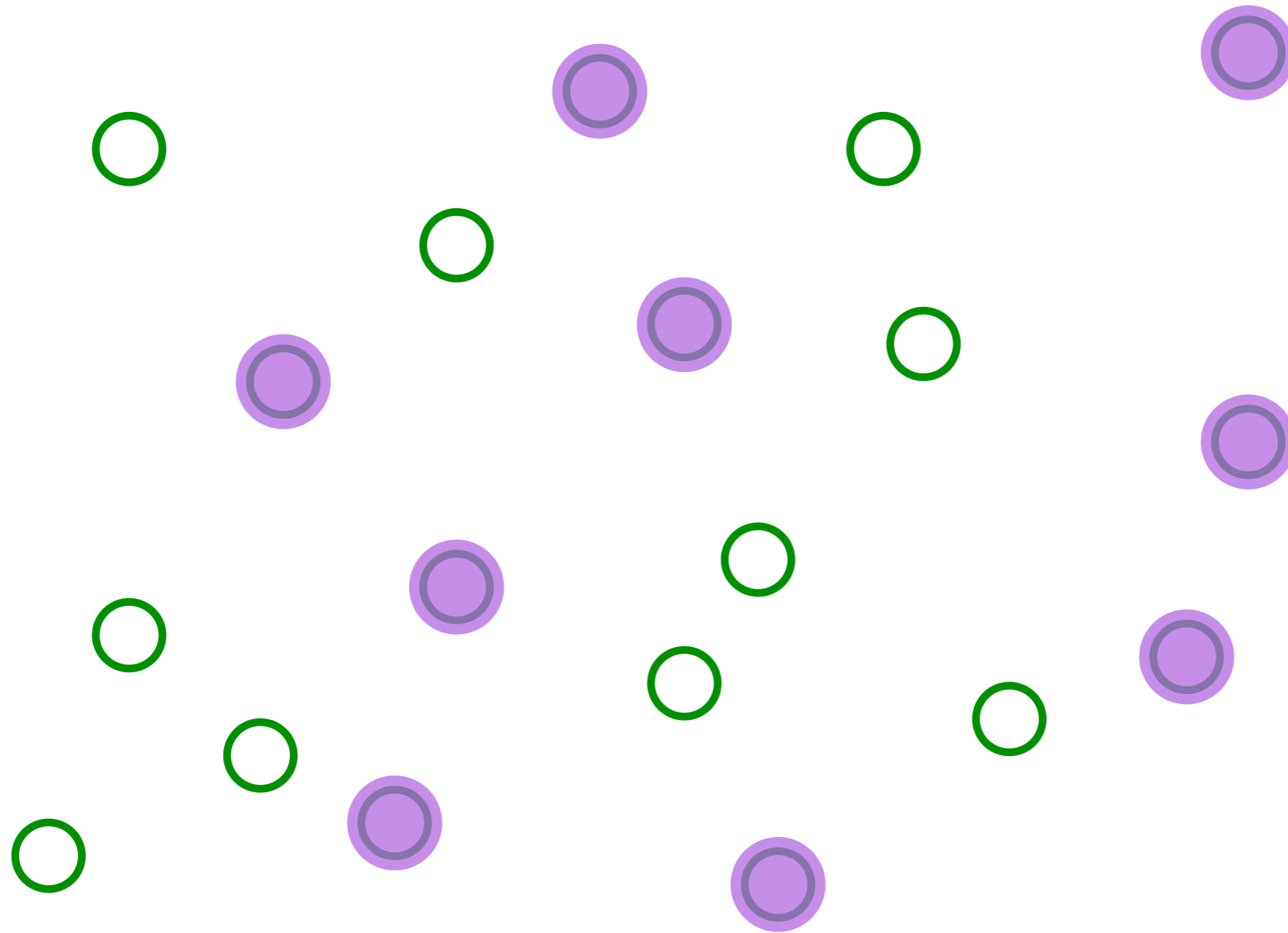
Pick a set of random positions

A simple model of a metal with quasiparticles



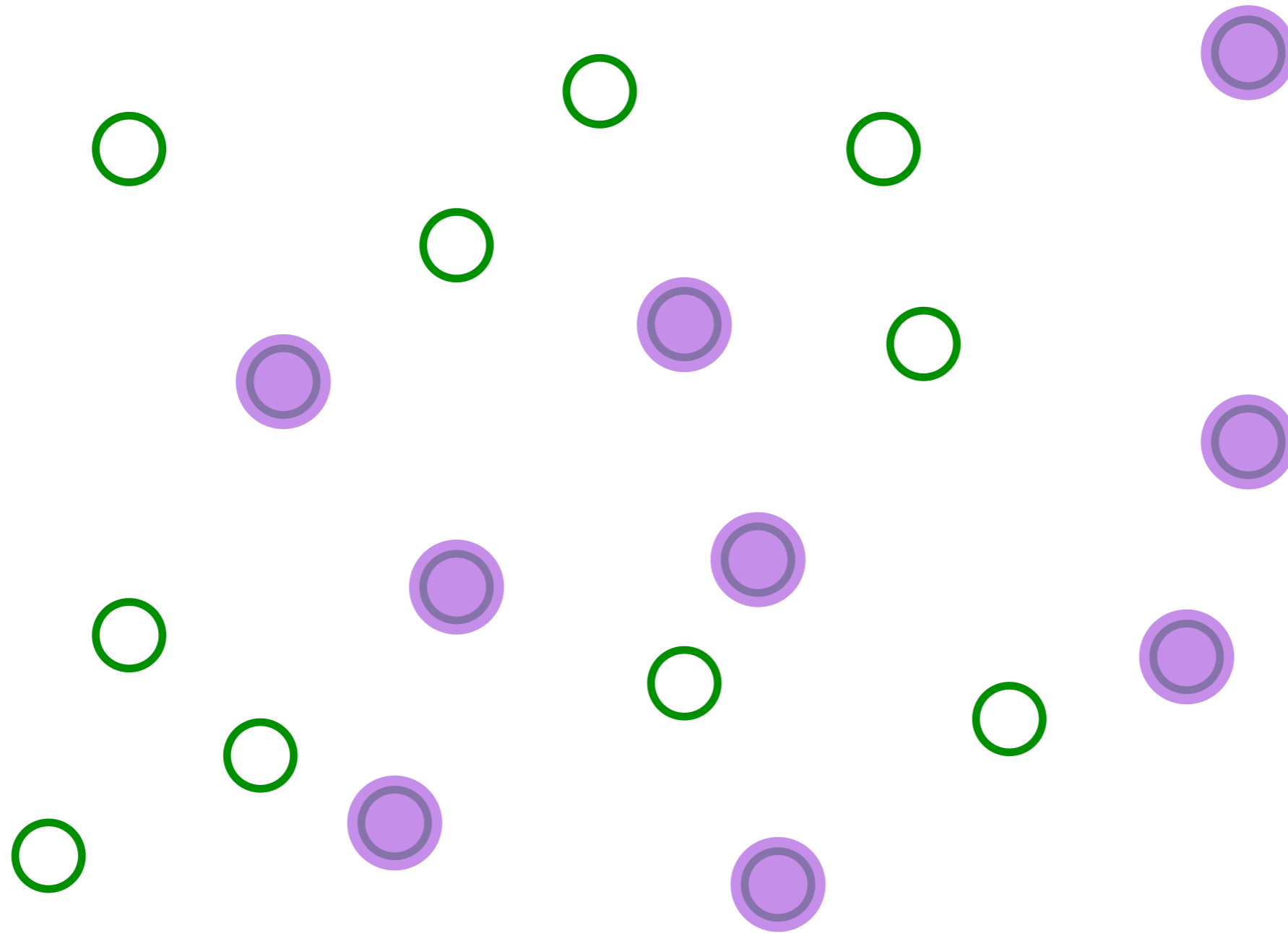
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



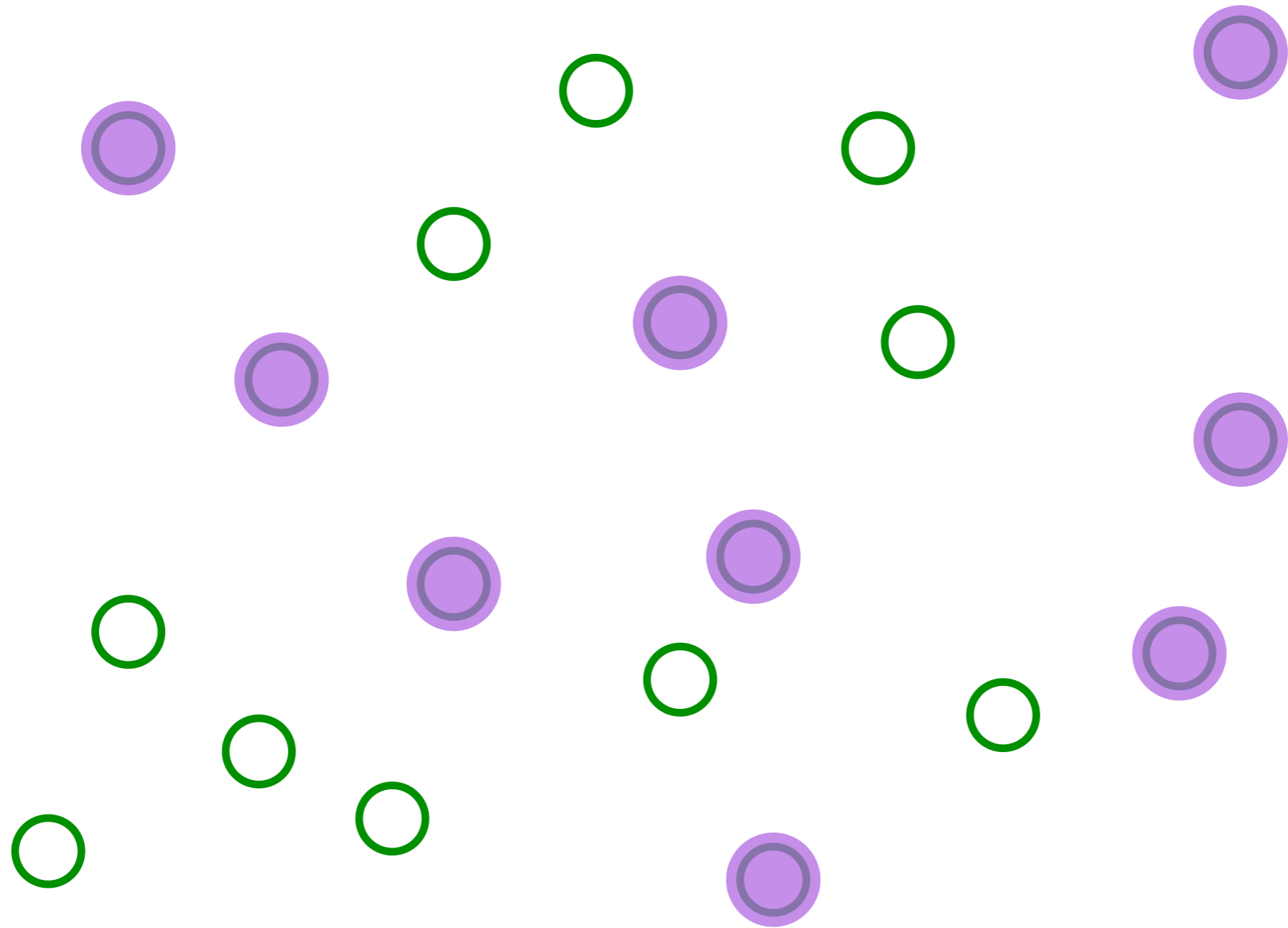
Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



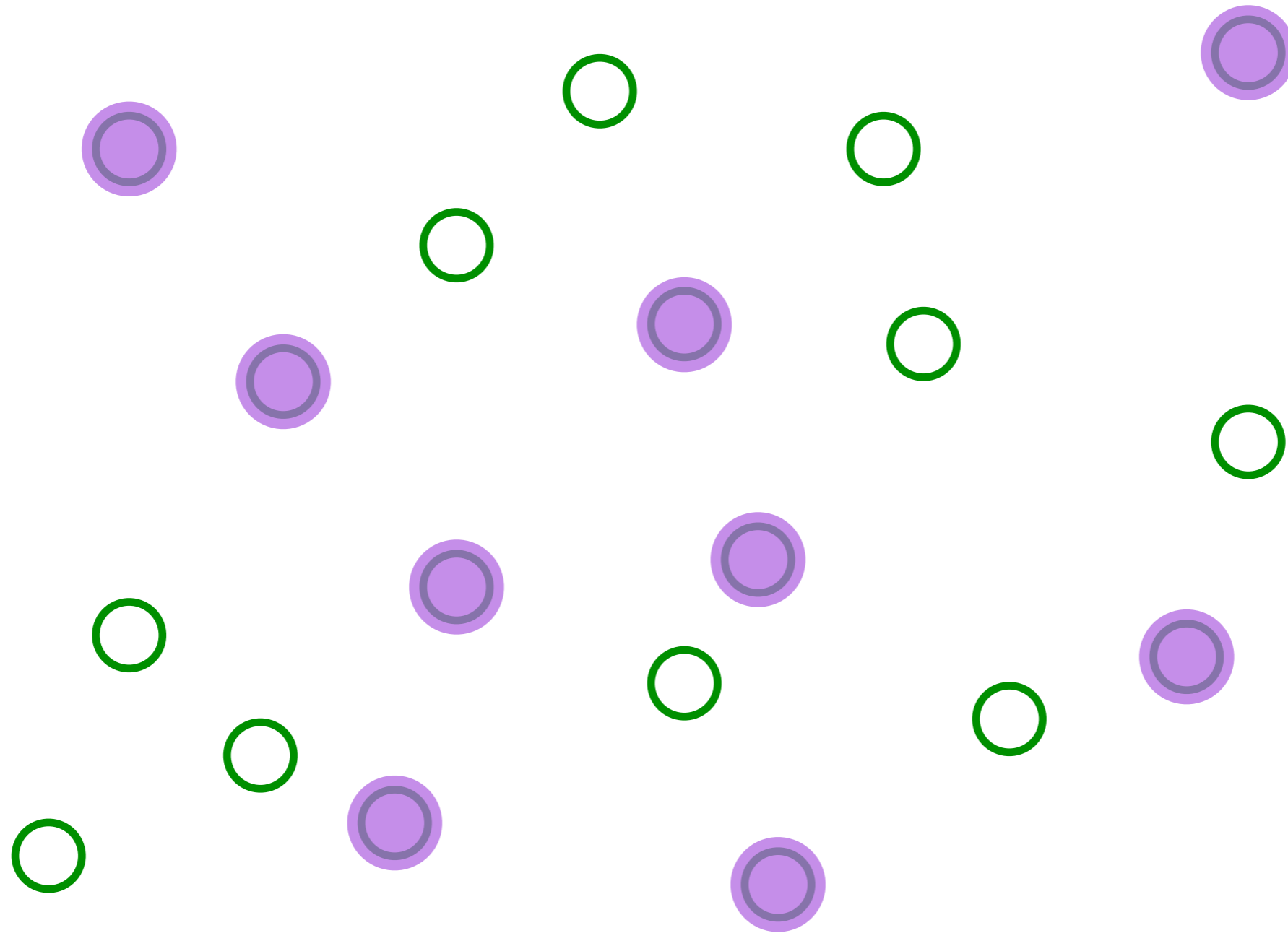
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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \dots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

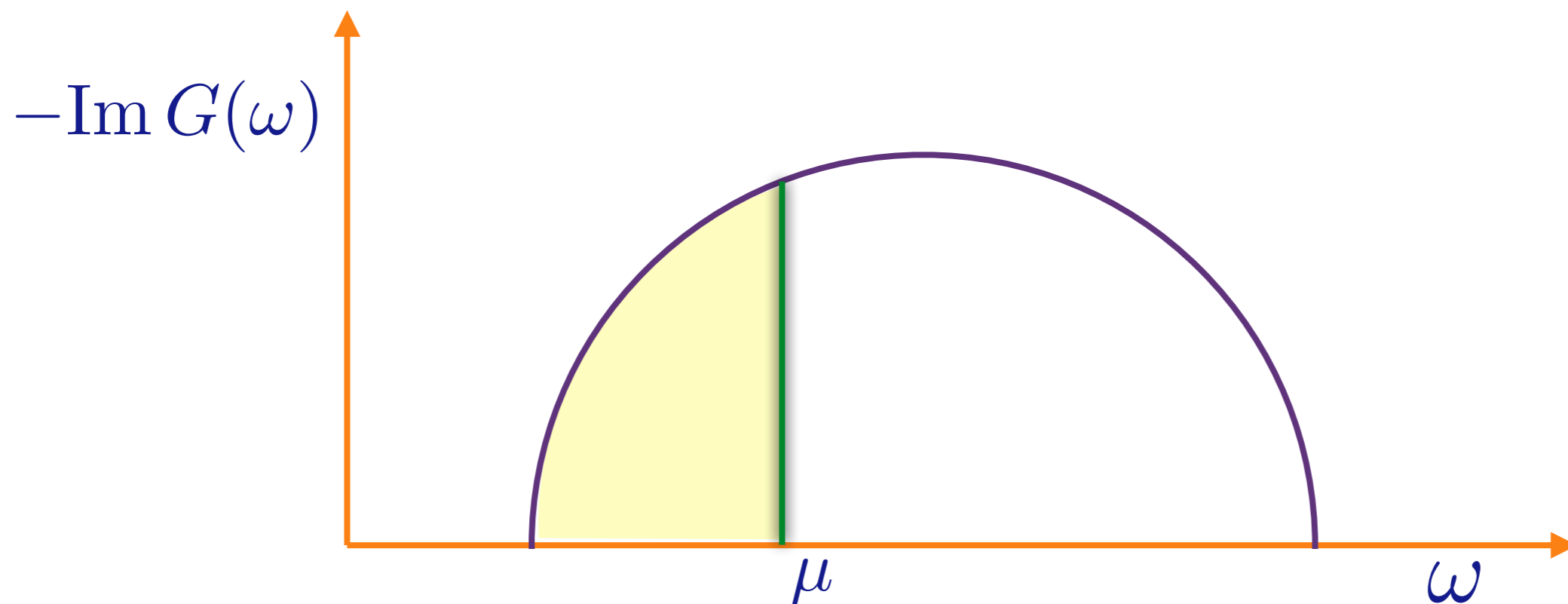
**Fermions occupying the eigenstates of a
 $N \times N$ random matrix**

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = Q.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l$$

$J_{ij;kl}$ are independent random variables with $\overline{J_{ij;kl}} = 0$ and $|\overline{J_{ij;kl}}|^2 = J^2$. We compute the lifetime of a quasiparticle, τ_α , in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamiltonian with energy E_α . By Fermi's Golden rule, for E_α at the Fermi energy

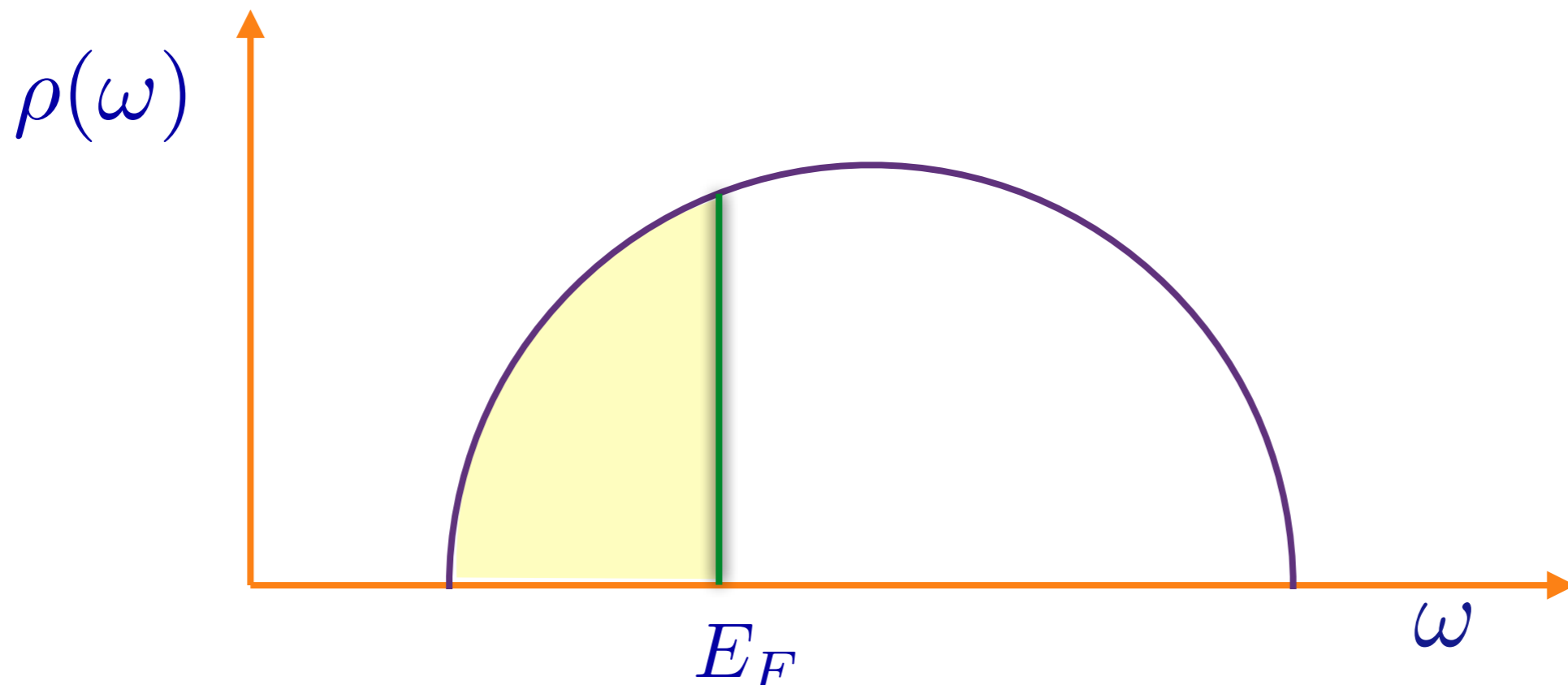
$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta) \\ &= \frac{\pi^3 J^2 \rho_0^2}{4} T^2 \end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

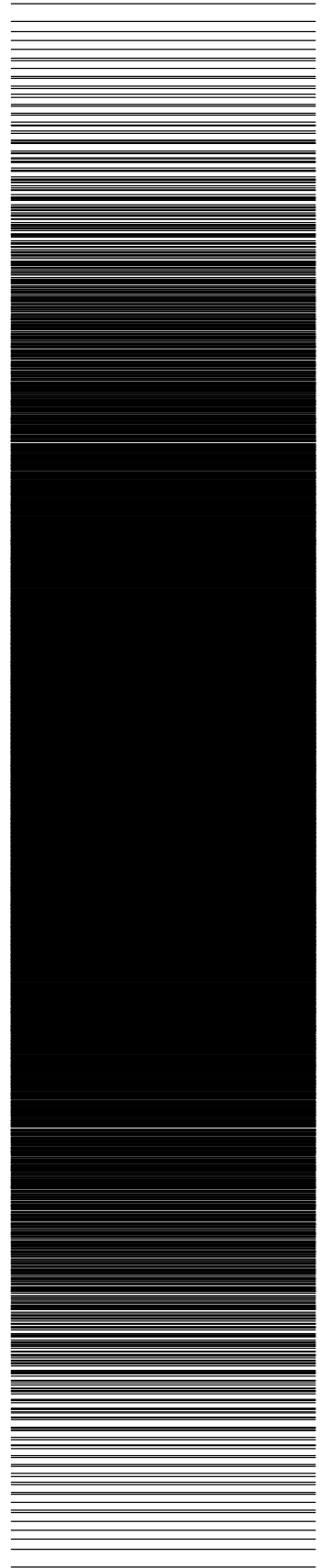
Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix t_{ij}/\sqrt{N} . The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F . The density of states is $\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



A simple model of a metal with quasiparticles



Many-body
level spacing
 $\sim 2^{-N}$

Quasiparticle
excitations with
spacing $\sim 1/N$

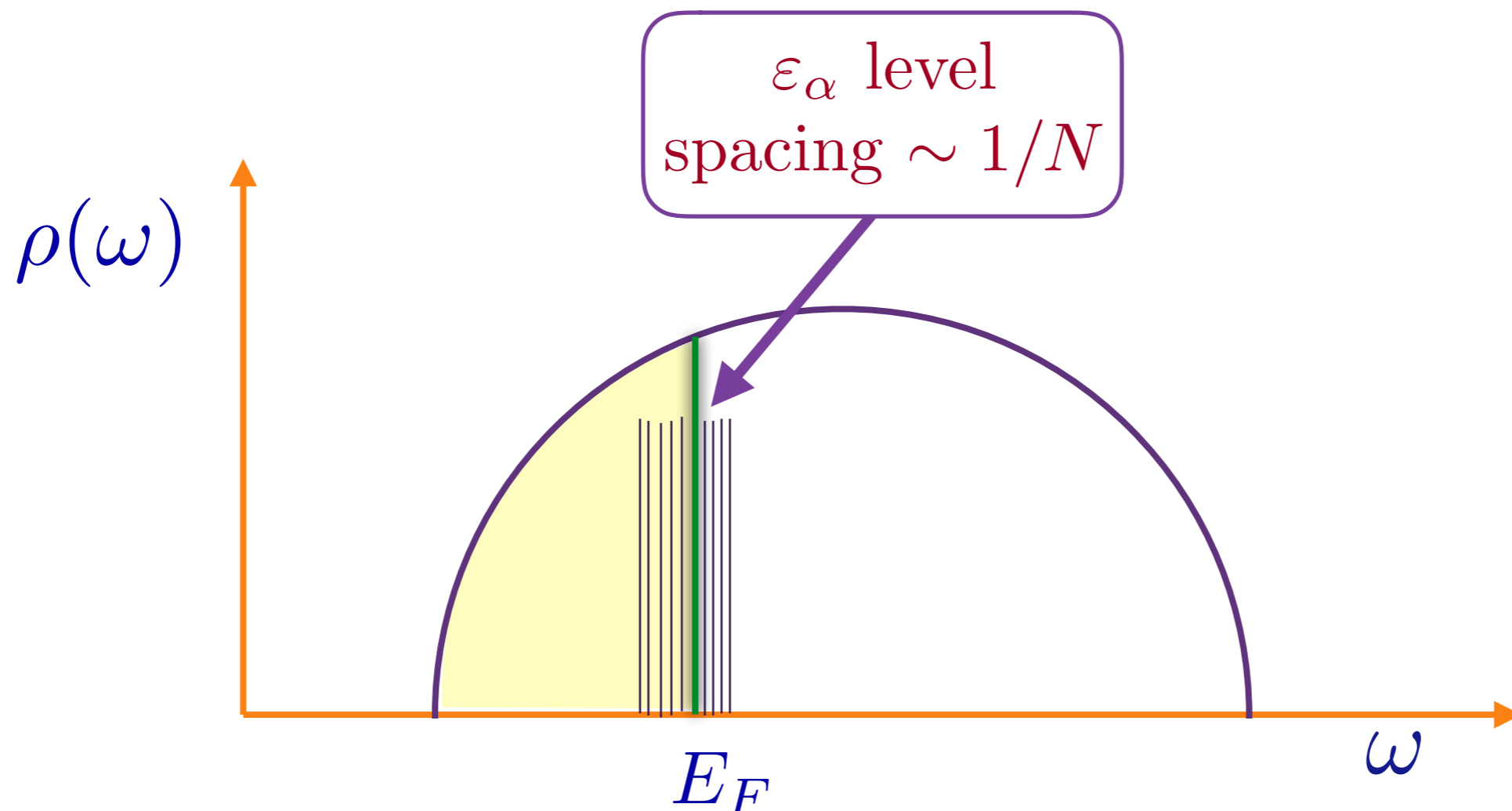
There are 2^N many
body levels with energy

$$E = \sum_{\alpha=1}^N n_{\alpha} \varepsilon_{\alpha},$$

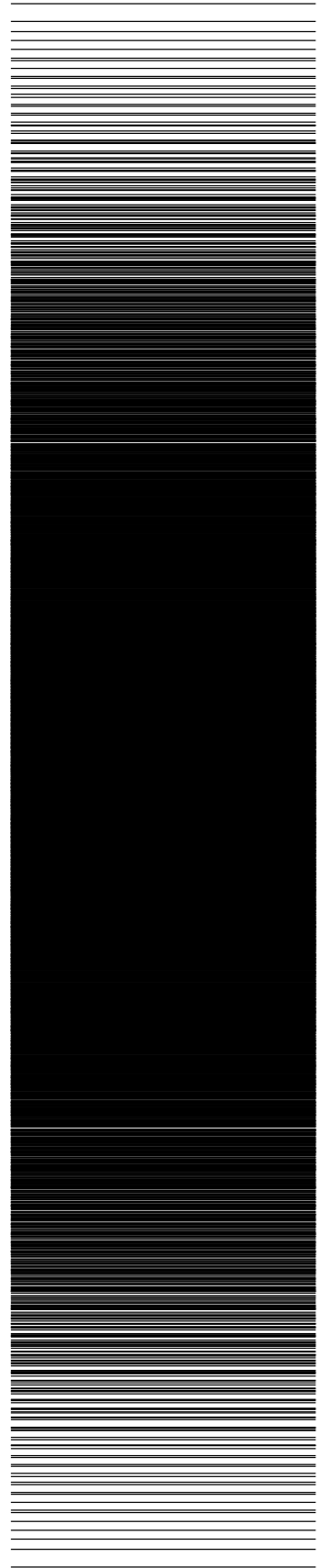
where $n_{\alpha} = 0, 1$. Shown
are all values of E for a
single cluster of size
 $N = 12$. The ε_{α} have a
level spacing $\sim 1/N$.

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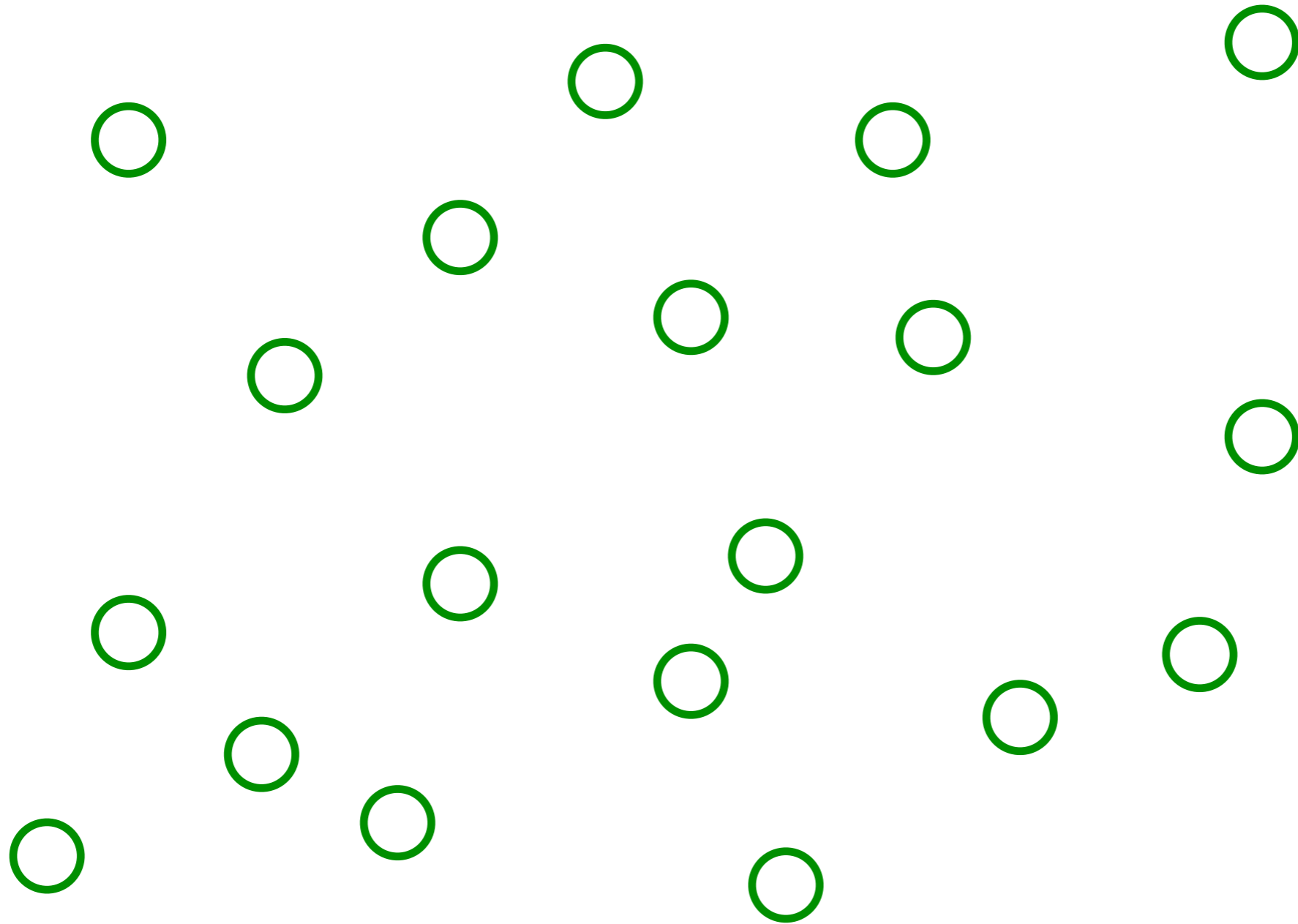
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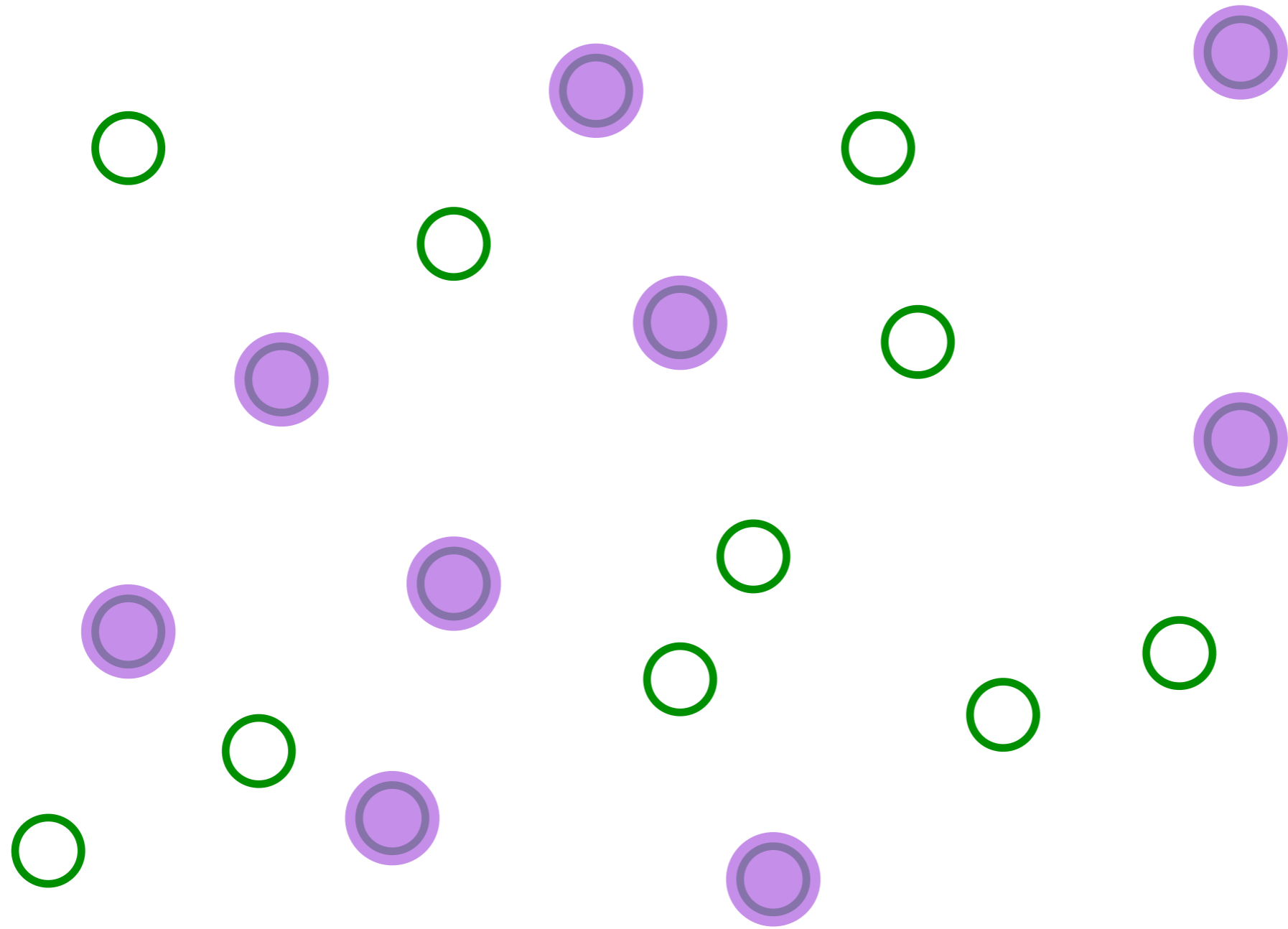
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The Sachdev-Ye-Kitaev (SYK) model



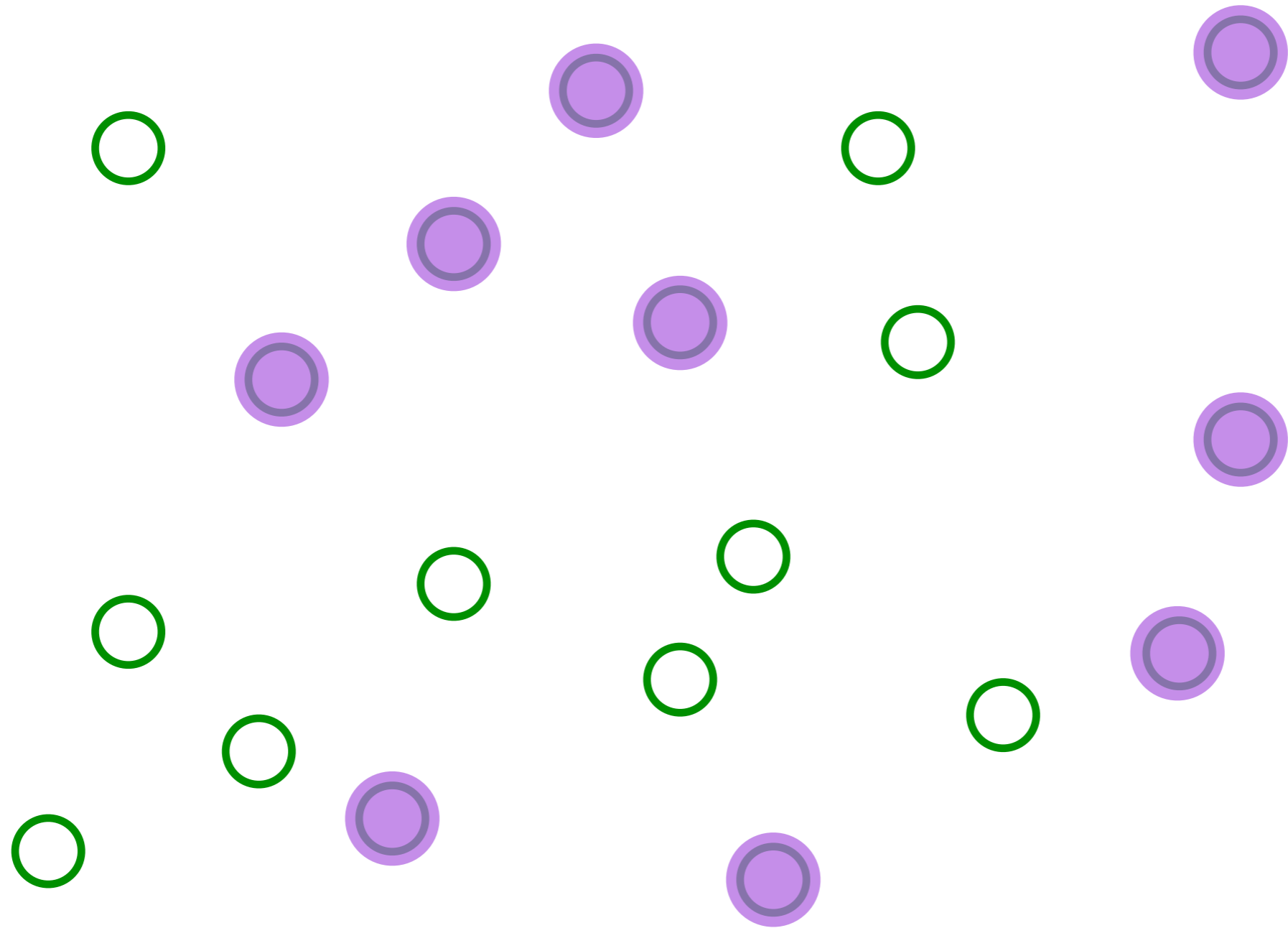
Pick a set of random positions

The SYK model



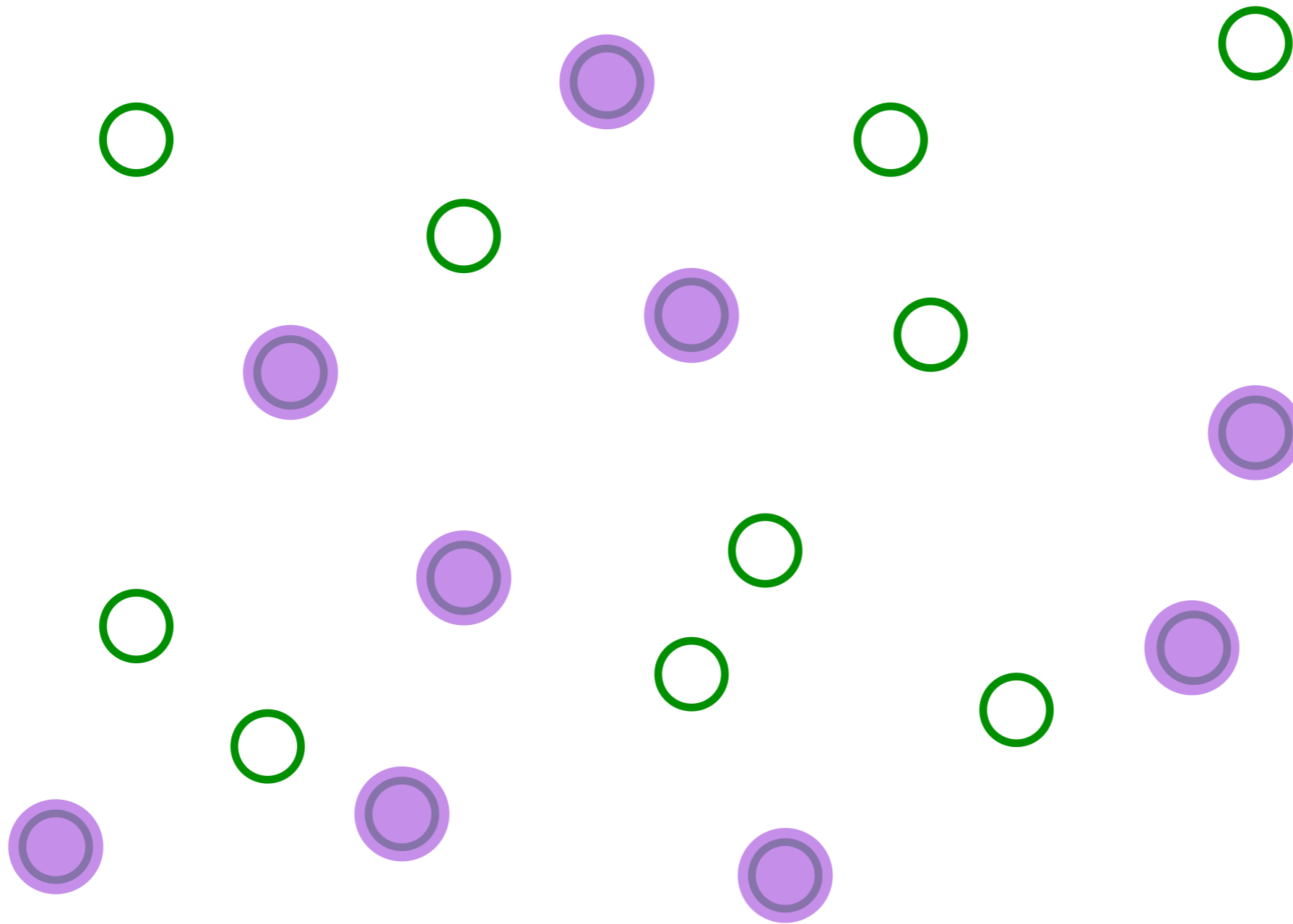
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The SYK model



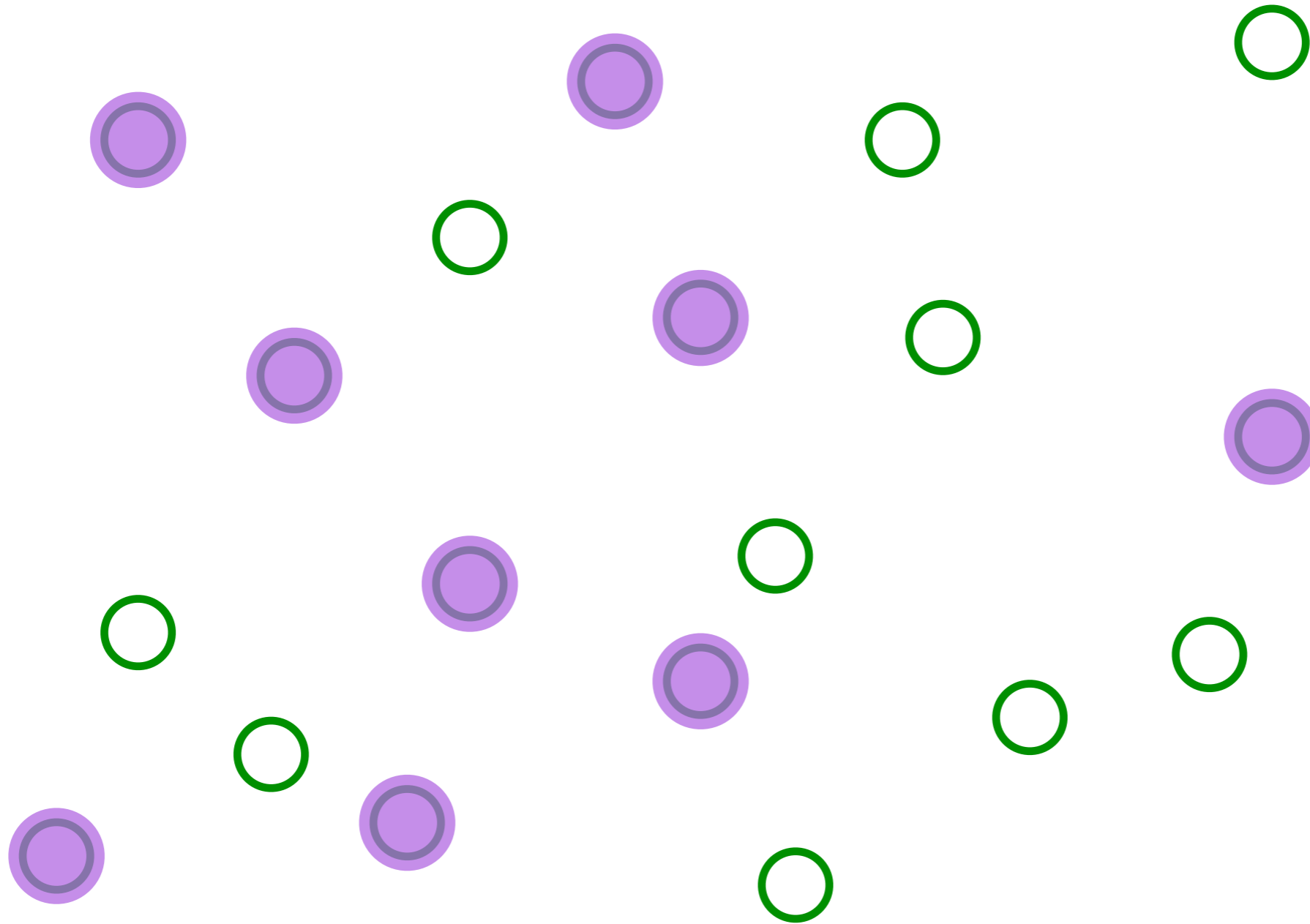
Entangle electrons pairwise randomly

The SYK model



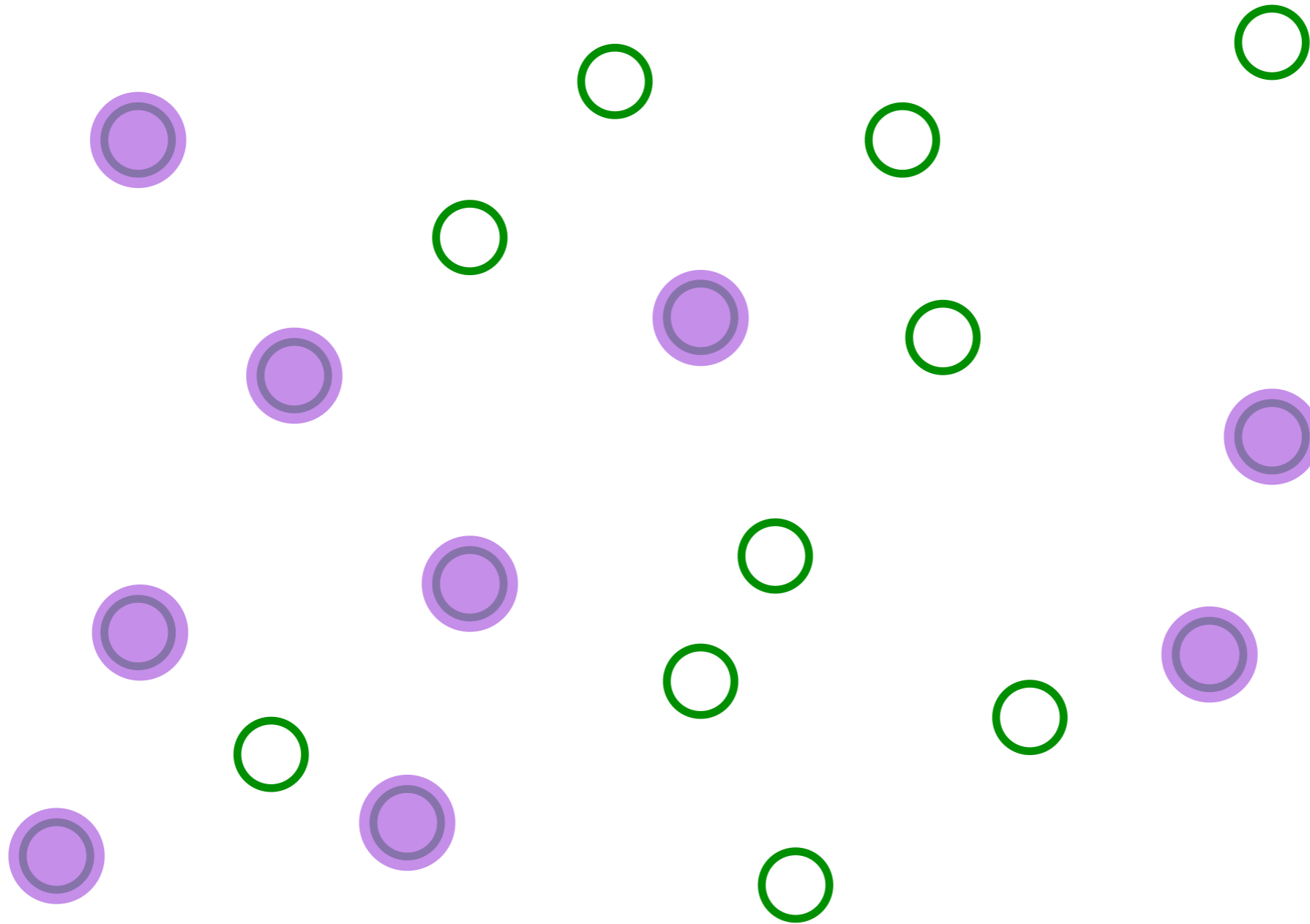
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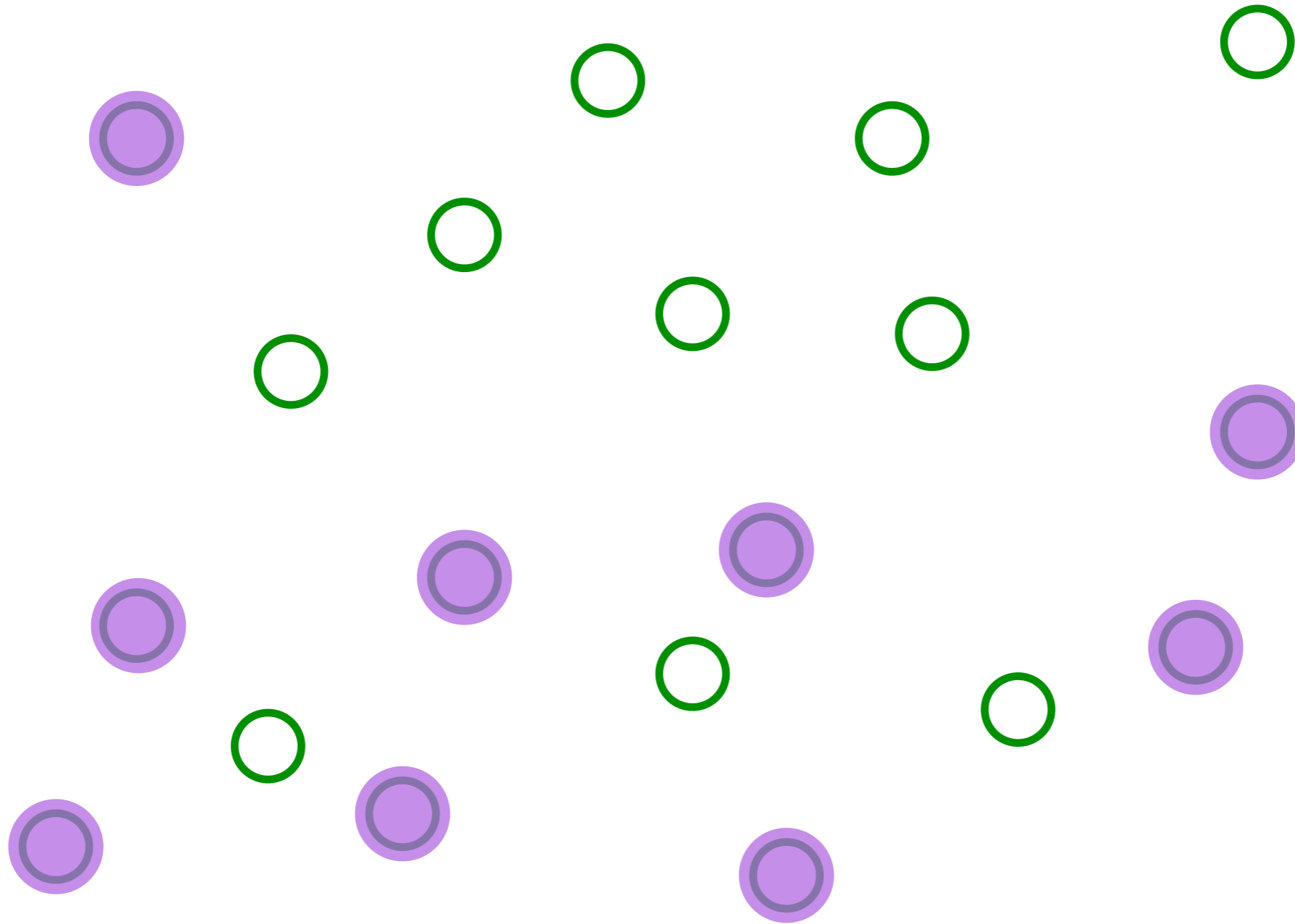
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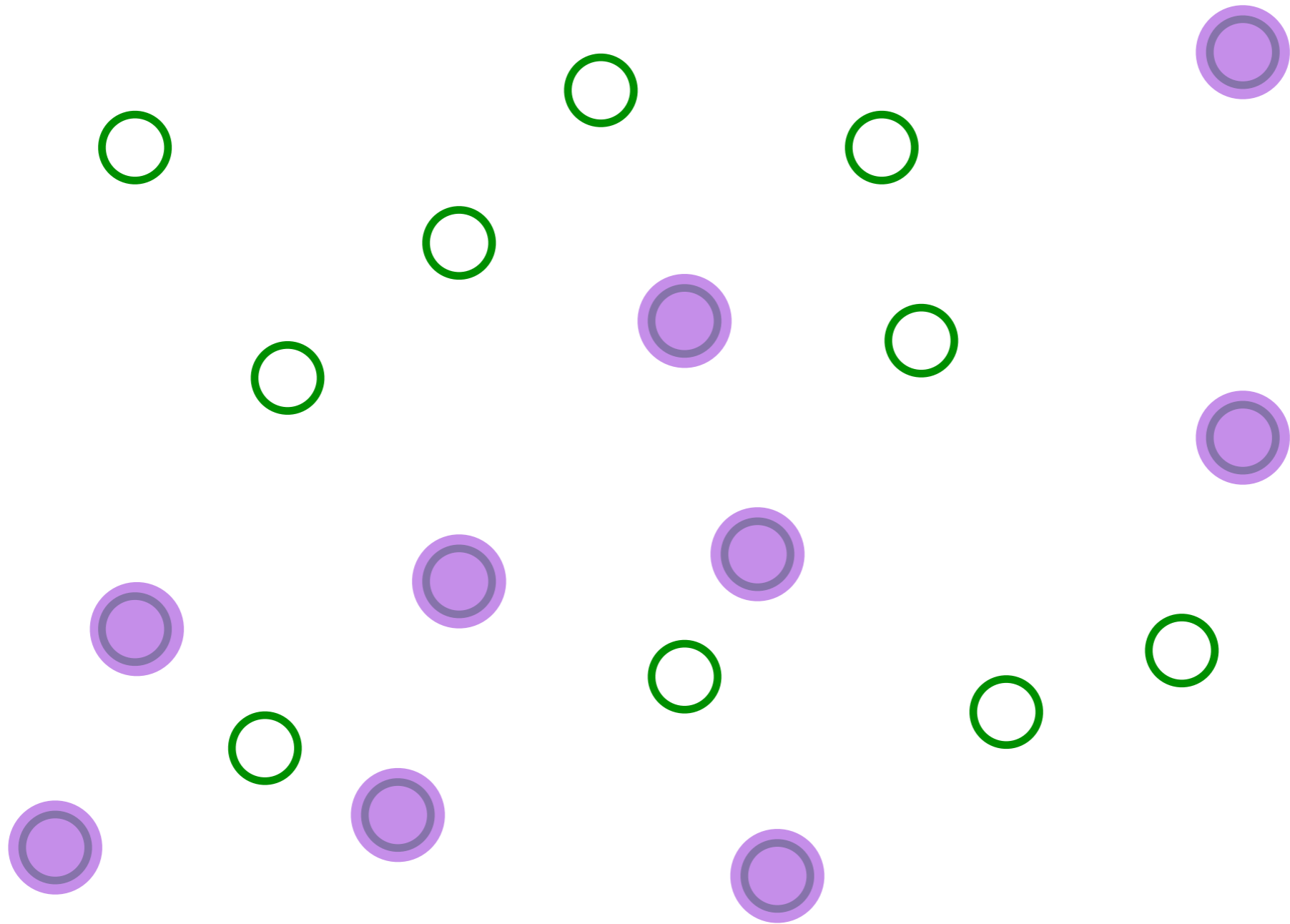
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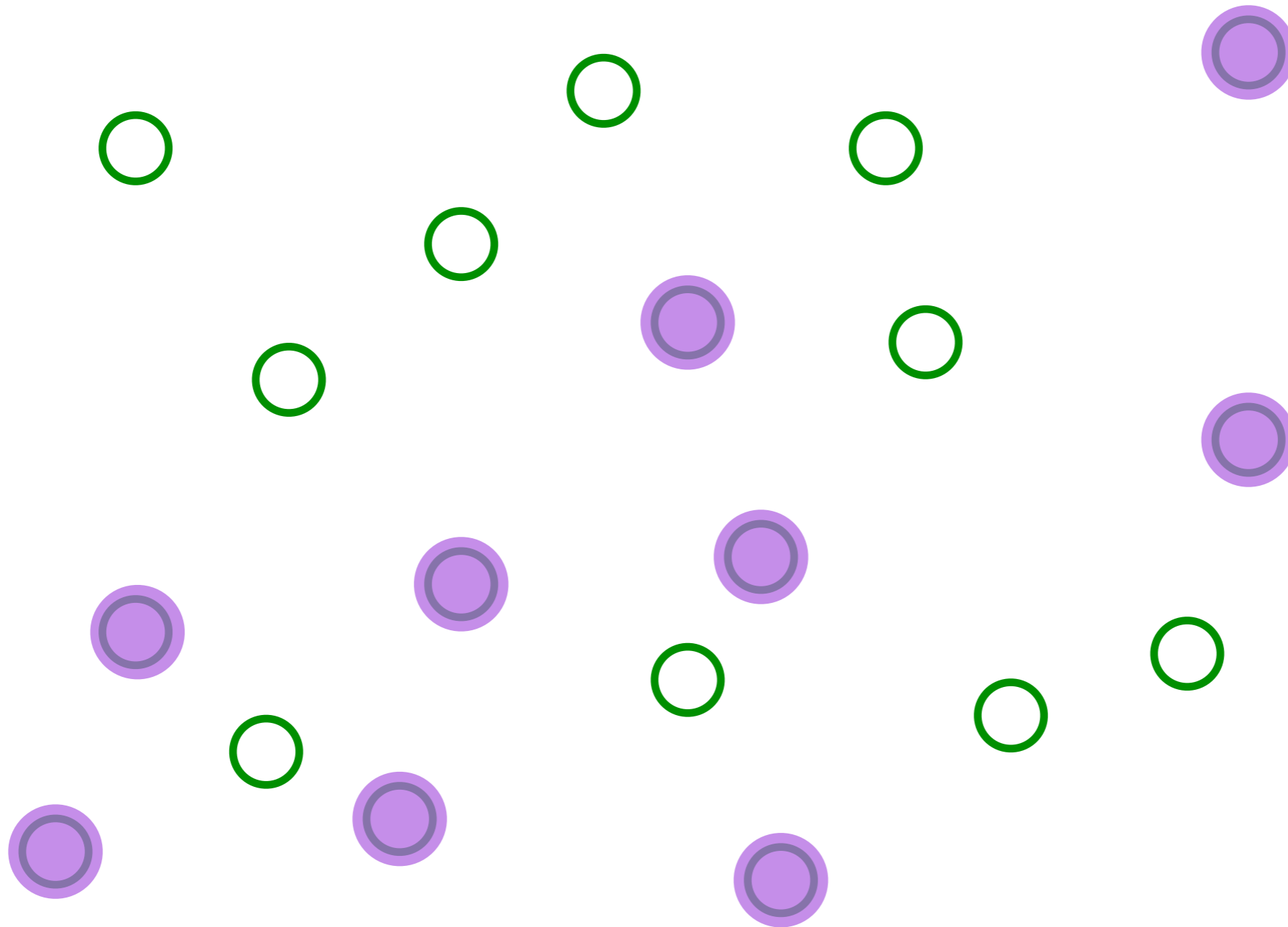
Entangle electrons pairwise randomly

The SYK model



Entangle electrons pairwise randomly

The SYK model



This describes both a strange metal and a black hole!

The SYK model

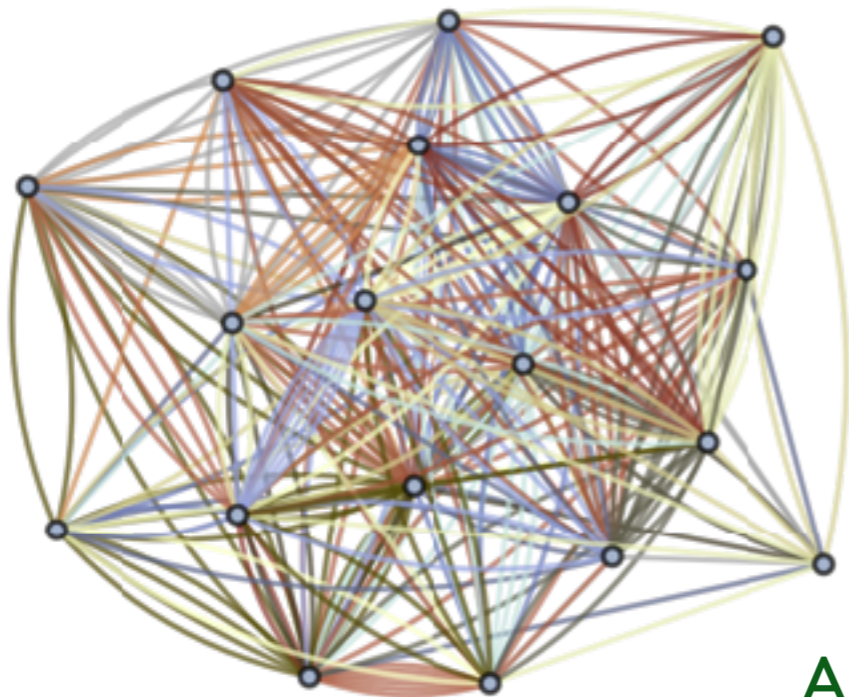
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N U_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

There are 2^N many body levels with energy E , which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \rightarrow 0$ state has an entropy $S_{GPS} = N s_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\dots$$
$$< \ln 2$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

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Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

No quasiparticles !

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

PRB **63**, 134406 (2001)

The SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
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Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

where $A = e^{-i\pi/4} (\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density Q .

The SYK model

The equations for the Green's function can also be solved at non-zero T . We “guess” the solution

$$G(\tau) = B \operatorname{sgn}(\tau) \left| \frac{\pi T}{\sin(\pi T \tau)} \right|^\rho$$

Then the self-energy is

$$\Sigma(\tau) = U^2 B^3 \operatorname{sgn}(\tau) \left| \frac{\pi T}{\sin(\pi T \tau)} \right|^{3\rho}$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

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Taking Fourier transforms, we have as a function of the Matsubara frequency ω_n

$$G(i\omega_n) = [iB\Pi(\rho)] \frac{T^{\rho-1} \Gamma\left(\frac{\rho}{2} + \frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1 - \frac{\rho}{2} + \frac{\omega_n}{2\pi T}\right)}$$

$$\Sigma_{\text{sing}}(i\omega_n) = [iU^2 B^3 \Pi(3\rho)] \frac{T^{3\rho-1} \Gamma\left(\frac{3\rho}{2} + \frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1 - \frac{3\rho}{2} + \frac{\omega_n}{2\pi T}\right)},$$

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The SYK model

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$$\Sigma_{\text{sing}}(i\omega_n) = [iU^2 B^3 \Pi(3\rho)] \frac{T^{3\rho-1} \Gamma\left(\frac{3\rho}{2} + \frac{\omega_n}{2\pi T}\right)}{\Gamma\left(1 - \frac{3\rho}{2} + \frac{\omega_n}{2\pi T}\right)},$$

where we have dropped a less-singular term in Σ , and

$$\Pi(s) \equiv \pi^{s-1} 2^s \cos\left(\frac{\pi s}{2}\right) \Gamma(1-s).$$

Now the singular part of Dyson's equation is

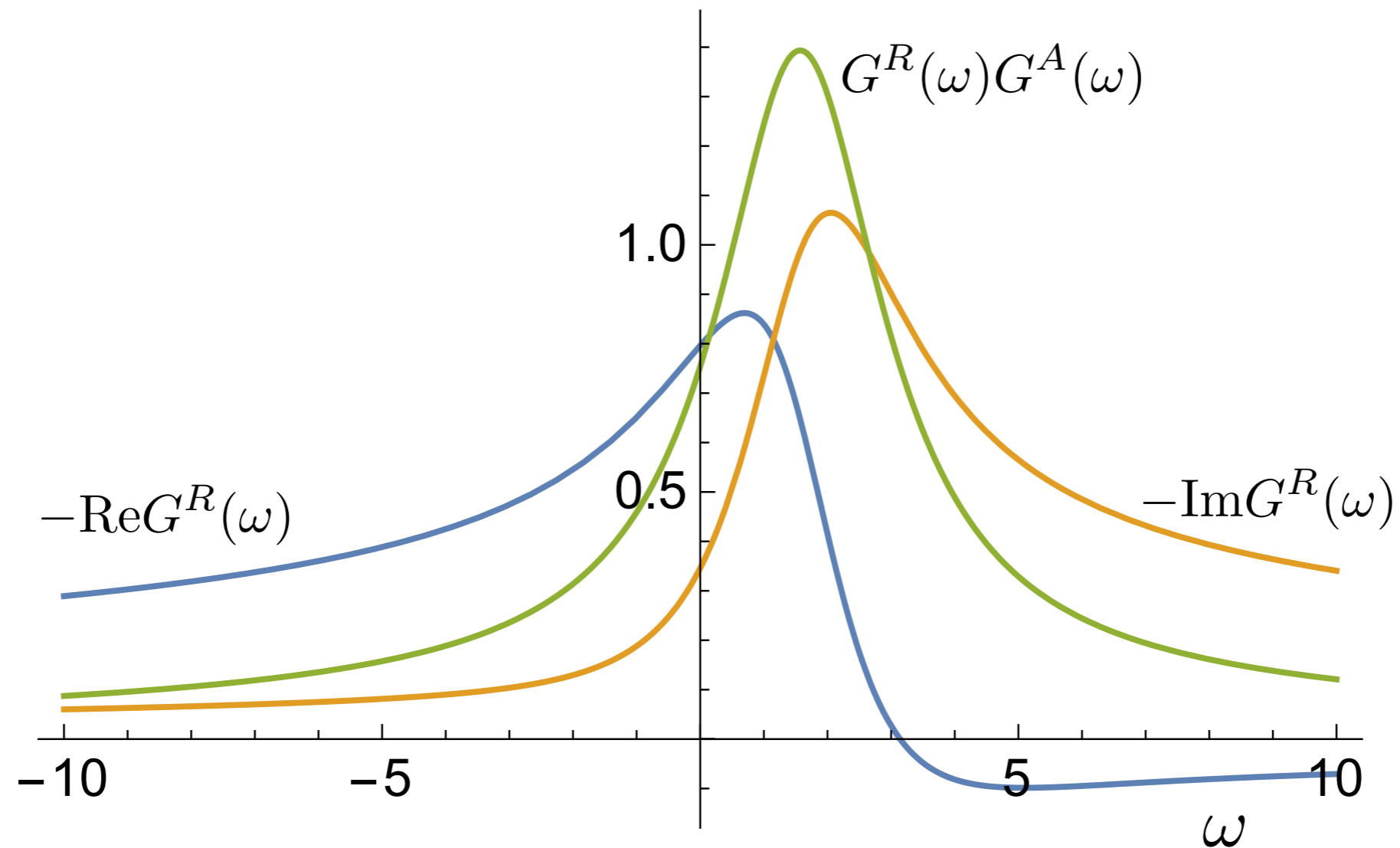
$$G(i\omega_n)\Sigma_{\text{sing}}(i\omega_n) = -1$$

A. Georges and O. Parcollet
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Remarkably, the Γ functions appear with just the right arguments, so that there is a solution of the Dyson equation at $\rho = 1/2$!

So the Green's functions display thermal 'damping' at a scale set by T alone, which is independent of U .

The SYK model



Green's functions away from half-filling

So the Green's functions display thermal 'damping' at a scale set by T alone, which is independent of U .

The SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

The SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll U$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations.

The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2 \Sigma_{\text{sing}}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\text{sing}}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

The SYK model

$$\int_0^\beta d\tau_2 \Sigma(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$
$$\Sigma(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

By using $f(\sigma) = \tan(\pi T \sigma) / (\pi T)$ we can

now obtain the $T > 0$ solution from the $T = 0$ solution.

The SYK model

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reparamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\tilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the $\text{SL}(2, \mathbb{R})$ transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to $\text{SL}(2, \mathbb{R})$ by the saddle point.

SYK and AdS₂

Connections of SYK to gravity and AdS₂ horizons

- Reparameterization and gauge invariance are the ‘symmetries’ of the Einstein-Maxwell theory of gravity and electromagnetism
- $SL(2, \mathbb{R})$ is the isometry group of AdS₂.

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

Infinite-range (SYK) model without quasiparticles

After introducing replicas $a = 1 \dots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{ia}(\tau) \exp \left[- \sum_{ia} \int_0^\beta d\tau c_{ia}^\dagger \left(\frac{\partial}{\partial \tau} - \mu \right) c_{ia} - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{ia}^\dagger(\tau) c_{ib}(\tau') \right|^4 \right].$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \left[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left(G(\tau_2, \tau_1) + \frac{1}{N} \sum_i c_i(\tau_2) c_i^\dagger(\tau_1) \right) \right].$$

Infinite-range (SYK) model without quasiparticles

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$
$$S = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)]$$
$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) [G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

A. Kitaev, 2015

S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1) \mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G, \Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s , and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-NS_{\text{eff}}[f, \phi]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849;

S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;

K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

The SYK model

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-N S_{\text{eff}}[f, \phi]}.$$

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T(\tau + \epsilon(\tau))), \tau \},$$

where $f(\tau) \equiv \tau + \epsilon(\tau)$, the couplings K , γ , and \mathcal{E} can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left(\frac{g''}{g'} \right)^2.$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\text{eff}} \left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849;

S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;

K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

The SYK model

- Low energy, many-body density of states

$$\rho(E) \sim e^{N s_0} \sinh(\sqrt{2(E - E_0)N\gamma})$$

(for Majorana model)

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

D. Stanford and E. Witten, 1703.04612

A. M. Garcia-Garcia, J.J.M. Verbaarschot, 1701.06593

D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

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A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818

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- $T = 0$ fermion Green's function is incoherent: $G(\tau) \sim \tau^{-1/2}$ at large τ . (Fermi liquids with quasiparticles have the coherent: $G(\tau) \sim 1/\tau$)

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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- The last property indicates $\tau_{\text{eq}} \sim \hbar / (k_B T)$, and this has been found in a recent numerical study.

Quantum matter without quasiparticles:

- If there are no quasiparticles, then

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

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- Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \rightarrow 0$

$$\tau_{\text{eq}} > C \frac{\hbar}{k_B T}.$$

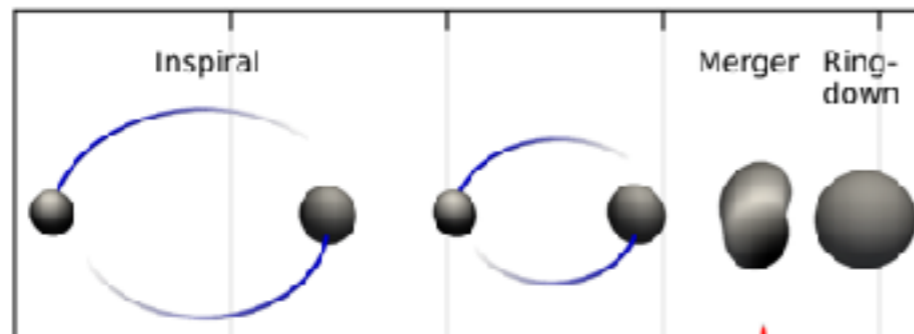
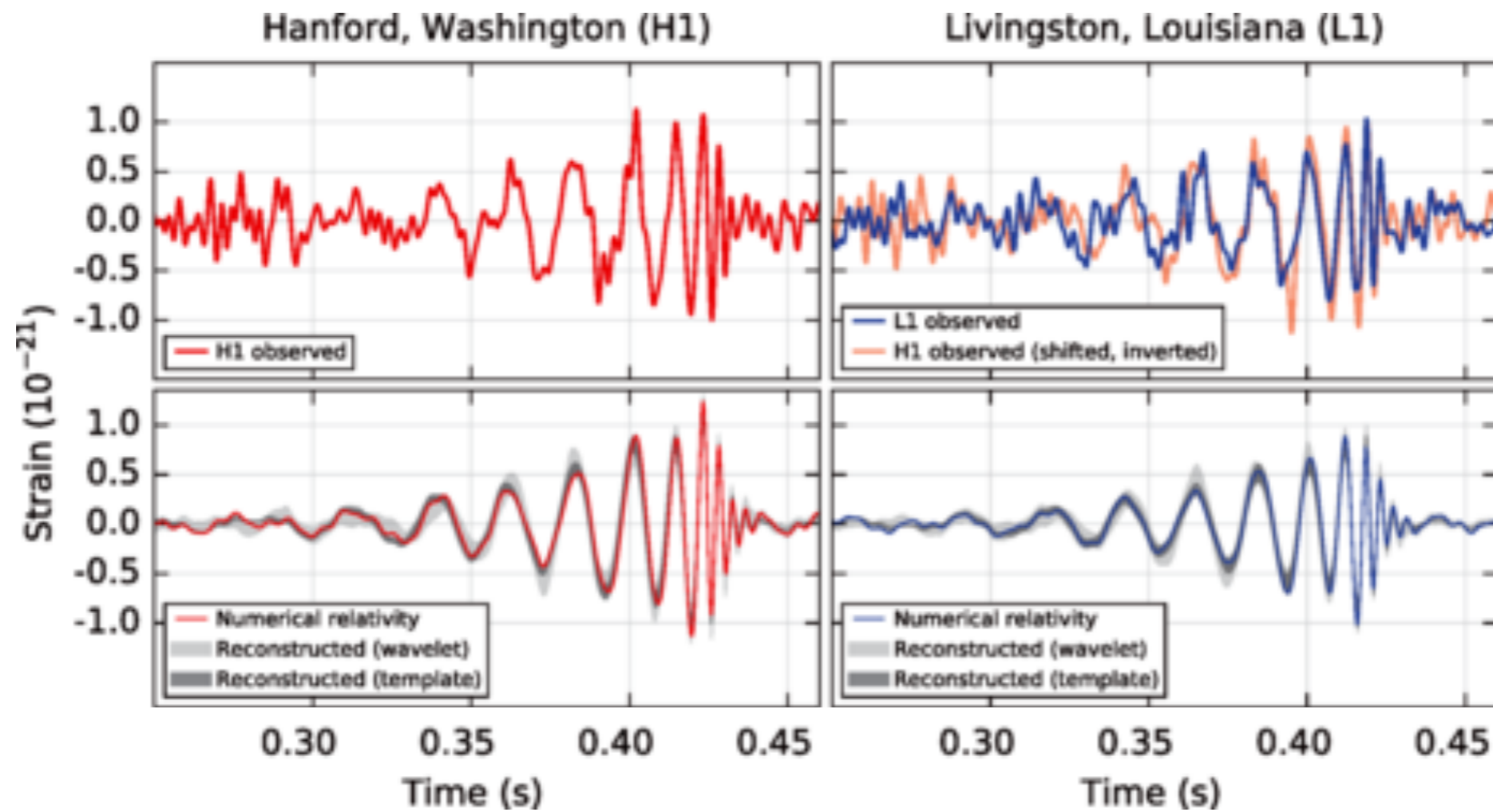
S. Sachdev,
Quantum Phase Transitions,
Cambridge (1999)

- In Fermi liquids $\tau_{\text{eq}} \sim 1/T^2$, and so the bound is obeyed as $T \rightarrow 0$.
- This bound rules out quantum systems with *e.g.* $\tau_{\text{eq}} \sim \hbar/(Jk_B T)^{1/2}$.
- There is no bound in classical mechanics ($\hbar \rightarrow 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3 / (8\pi G M k_B)$.
- The entropy is proportional to their surface area.





LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal

$$\frac{\hbar}{k_B T_H}$$

so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3 / (8\pi G M k_B)$.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar / (k_B T_H)$.



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The
has b

Black holes with a near-horizon AdS_2 geometry (described by quantum gravity in $1+1$ spacetime dimensions) match these properties of the $0+1$ dimensional SYK model:
 Ns_0 is the Bekenstein-Hawking entropy

Many-body quantum chaos

- Using holographic analogies, Shenker and Stanford introduced the “Lyapunov time”, τ_L , the time over which a generic many-body quantum system loses memory of its initial state.

S. Shenker and D. Stanford, arXiv:1306.0622

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- The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

A. Kitaev, unpublished
J. Maldacena and D. Stanford,
arXiv:1604.07818

Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states:
$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.
- These are also characteristics of black holes in quantum gravity.