# A non-Fermi liquid: Quantum criticality of metals near the Pomeranchuk instability

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PHYSICS
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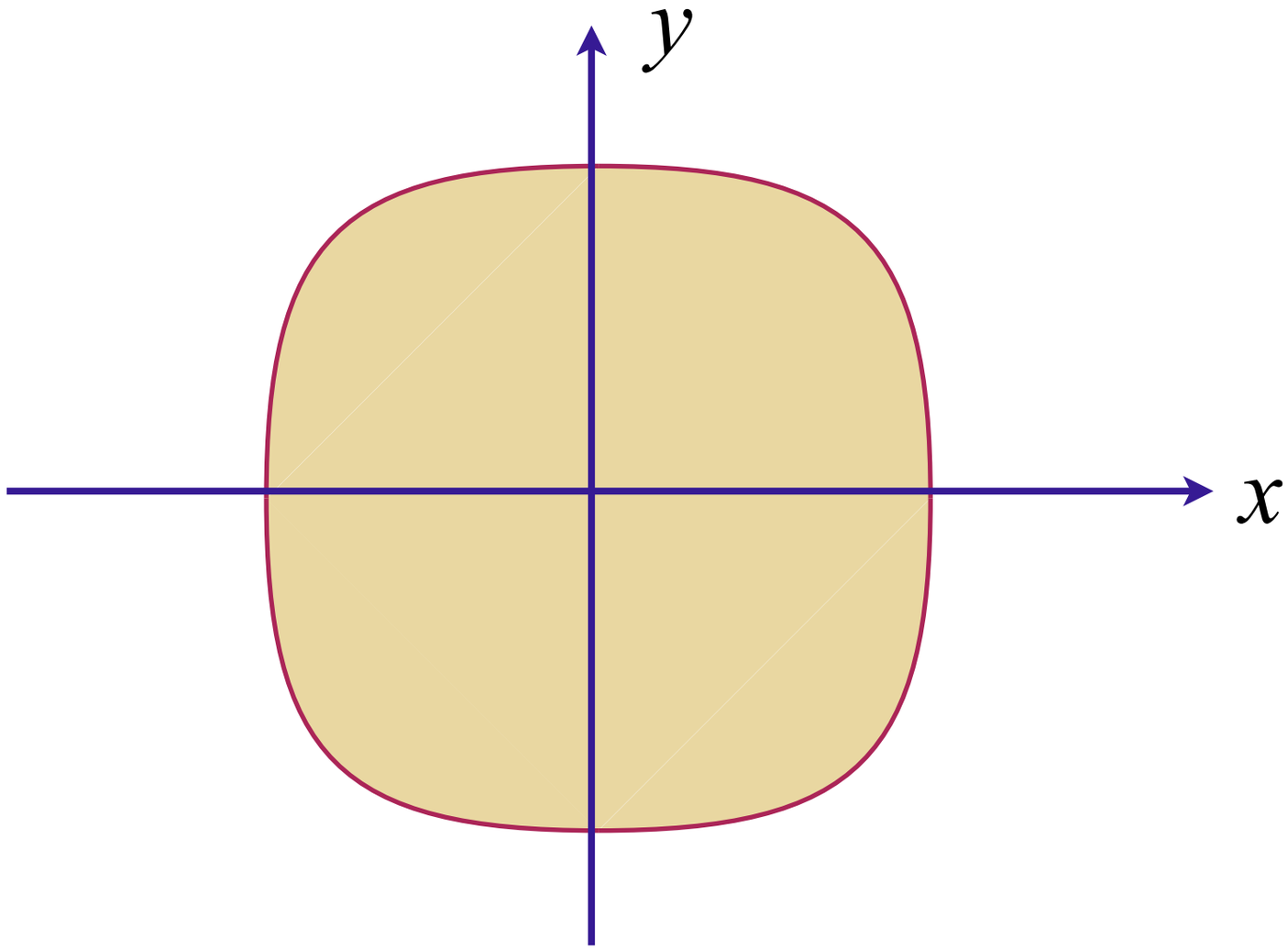
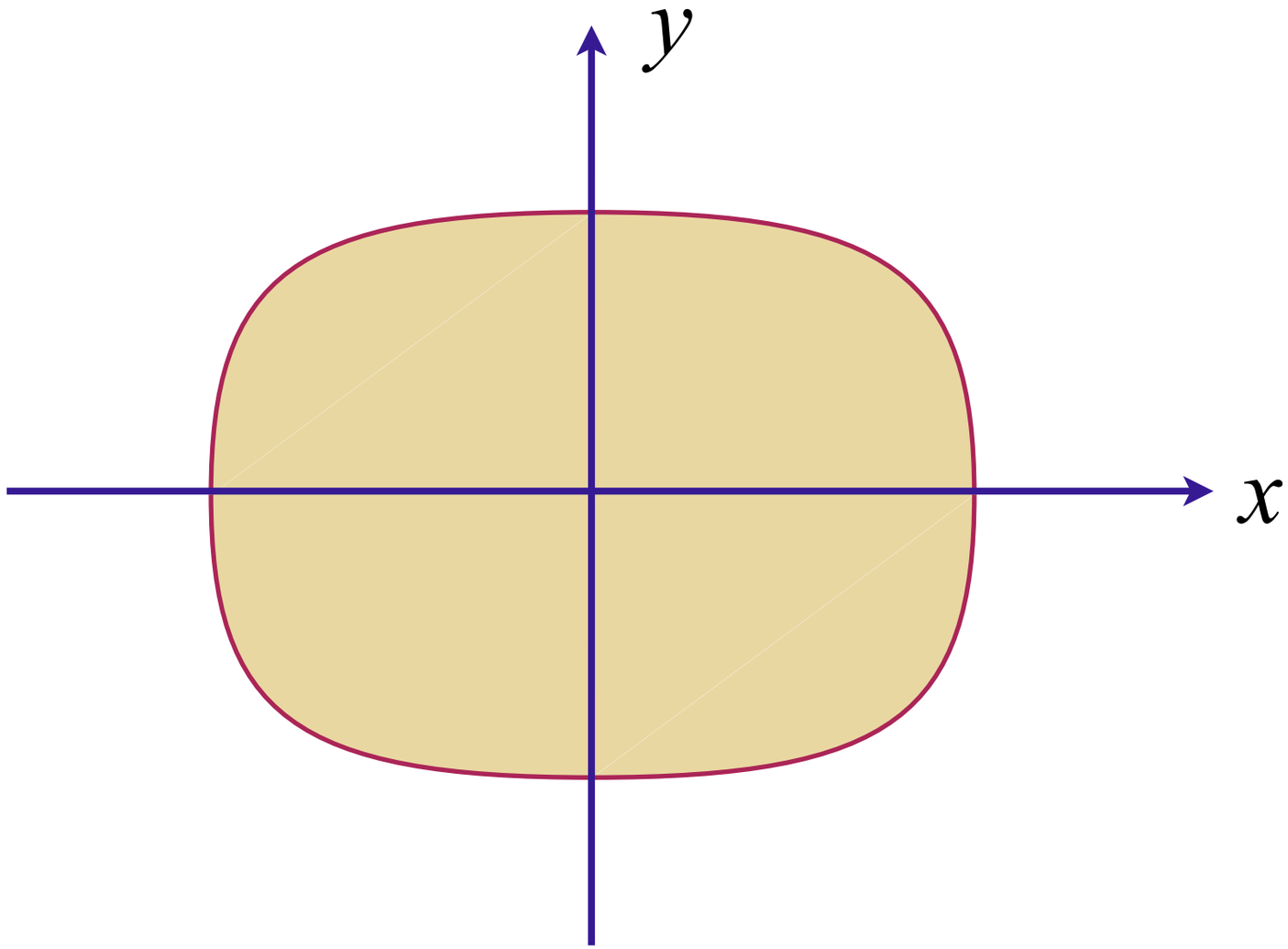
# Quantum criticality of Ising-nematic ordering



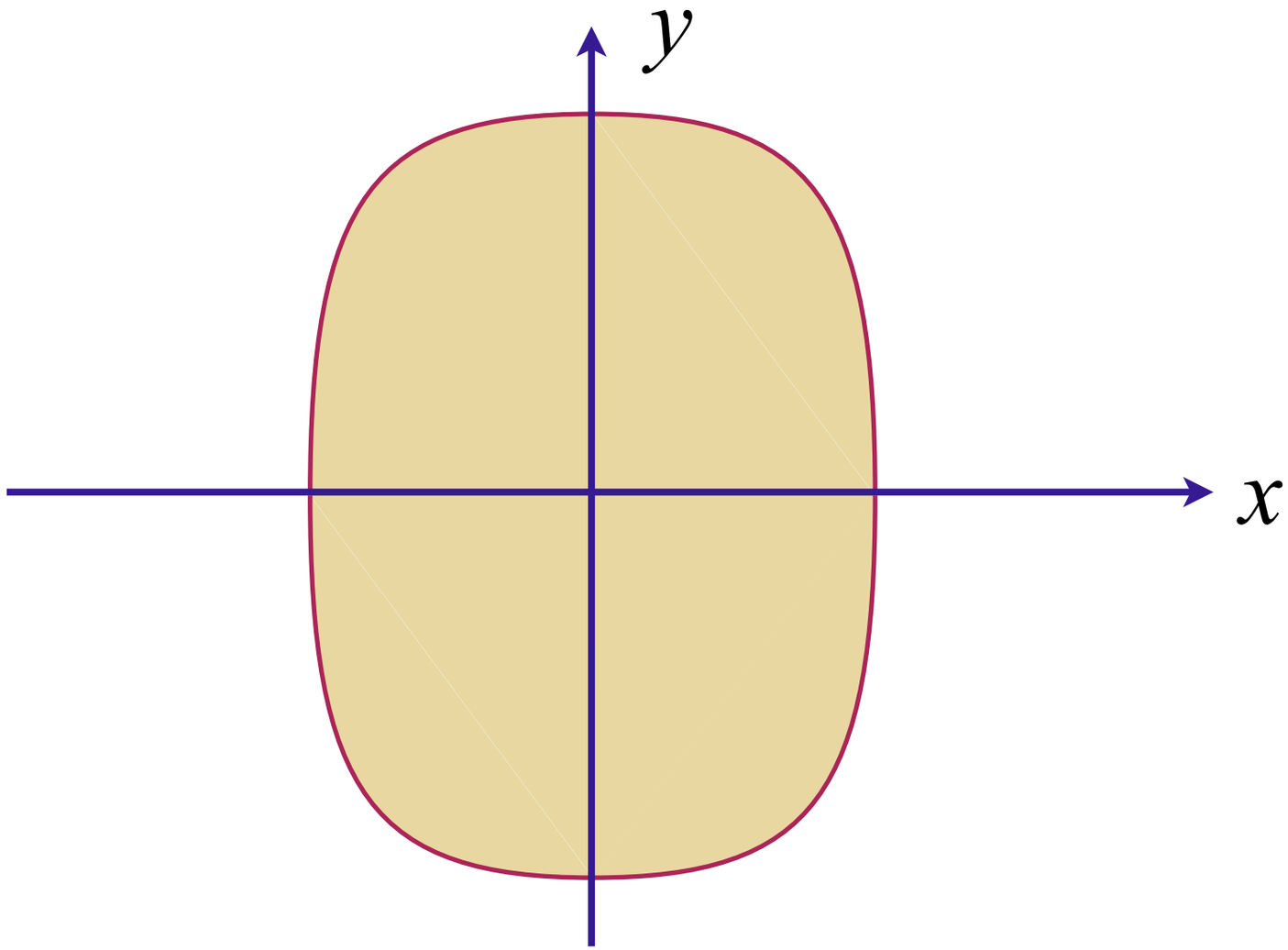

Fermi surface with full square lattice symmetry

# Quantum criticality of Ising-nematic ordering




Spontaneous elongation along $x$ direction:

# Quantum criticality of Ising-nematic ordering




Spontaneous elongation along $y$ direction:

Ising-nematic order parameter

$$\phi \sim \int d^2k \, (\cos k_x - \cos k_y) \, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

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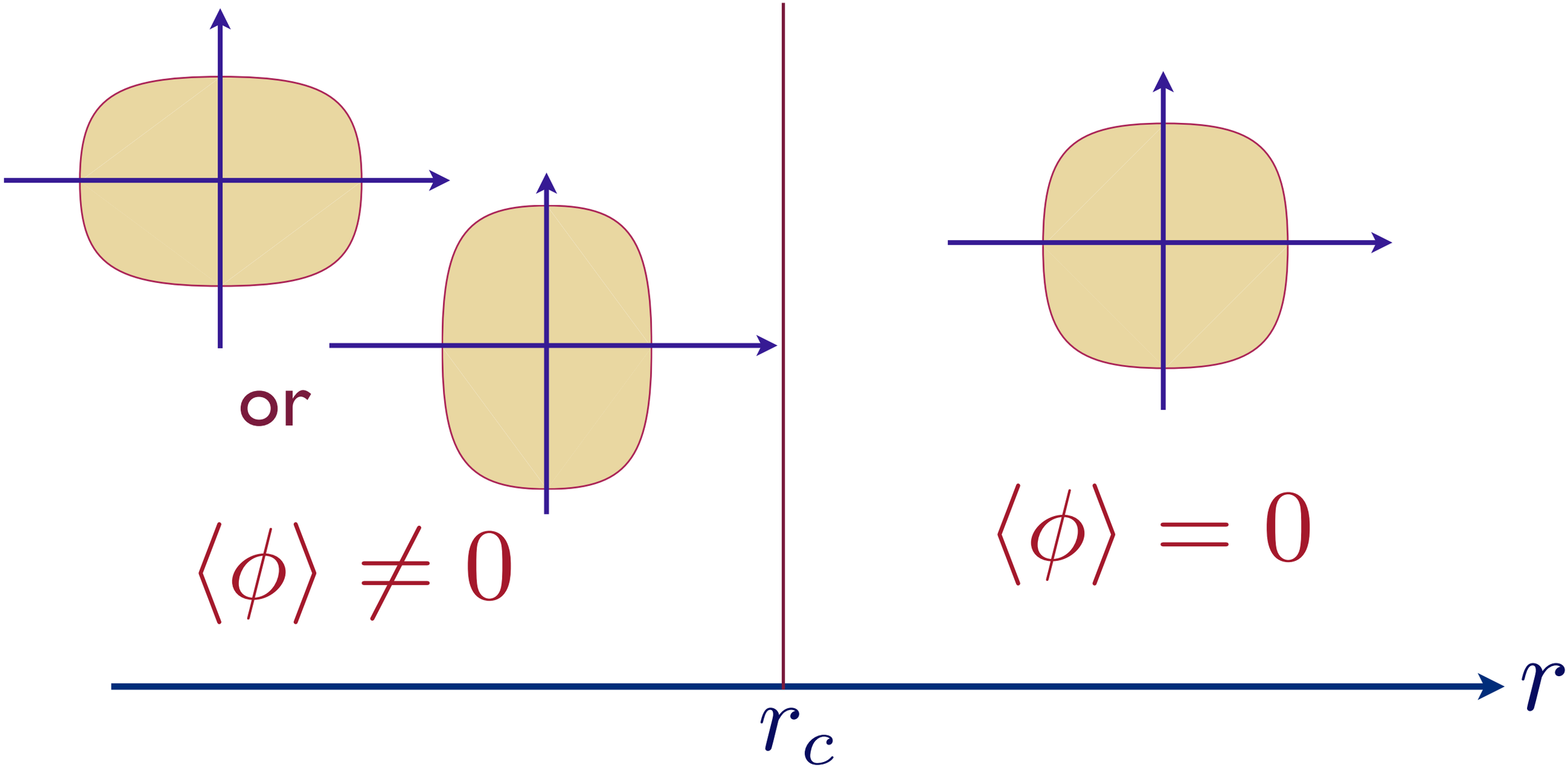
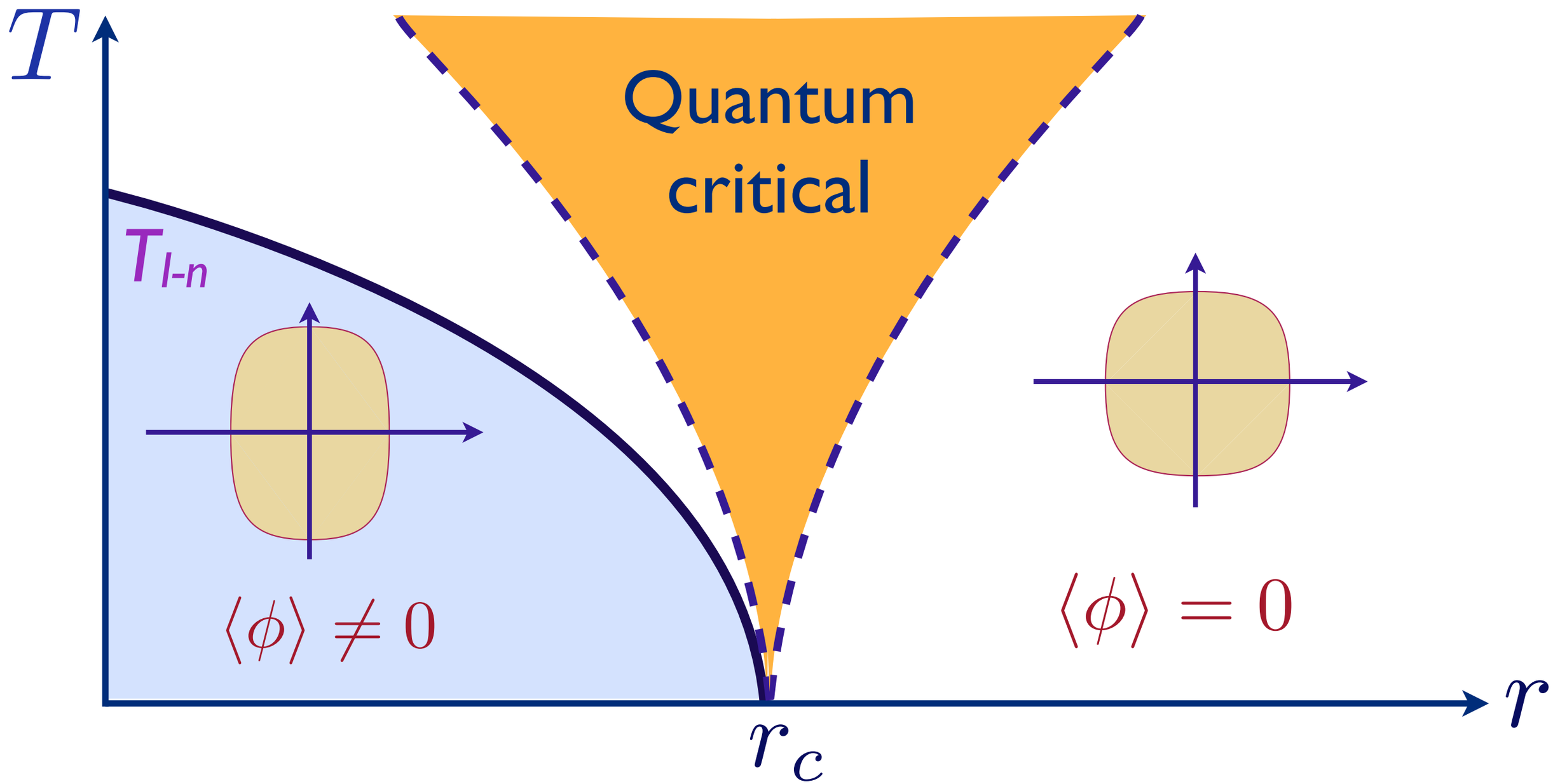



Pomeranchuk instability as a function of coupling $r$

# Quantum criticality of Ising-nematic ordering




Phase diagram as a function of $T$ and $r$

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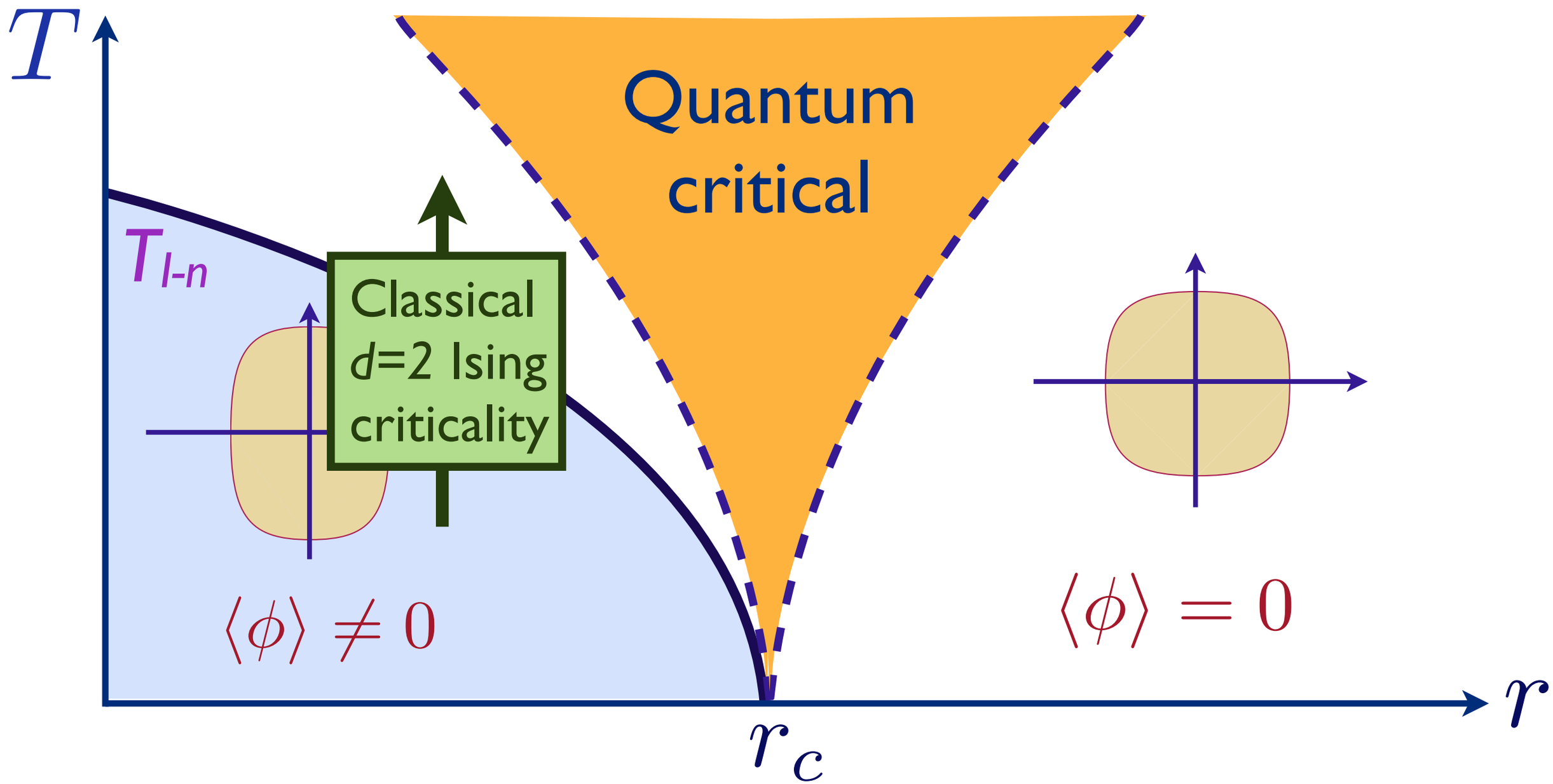


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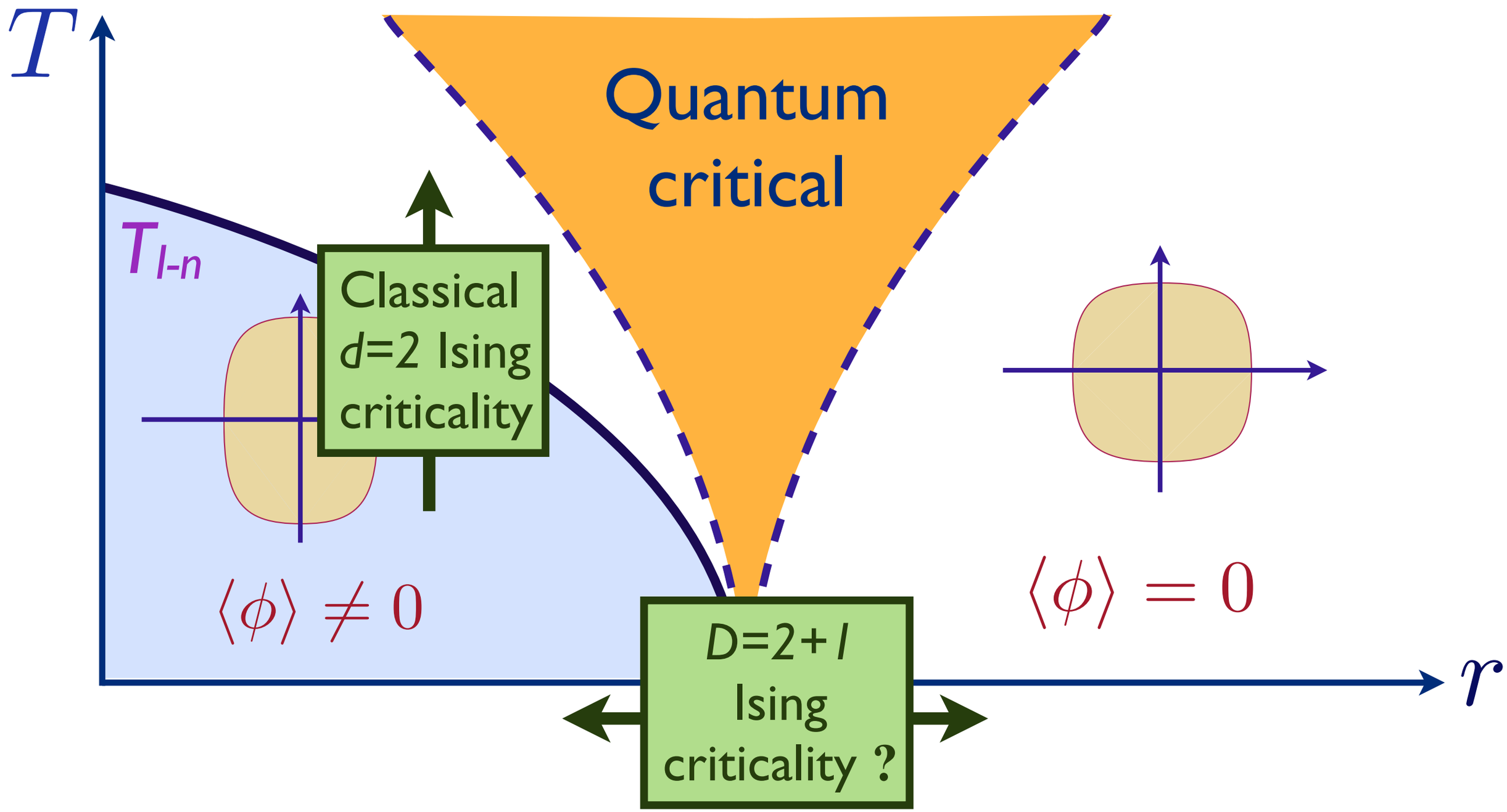


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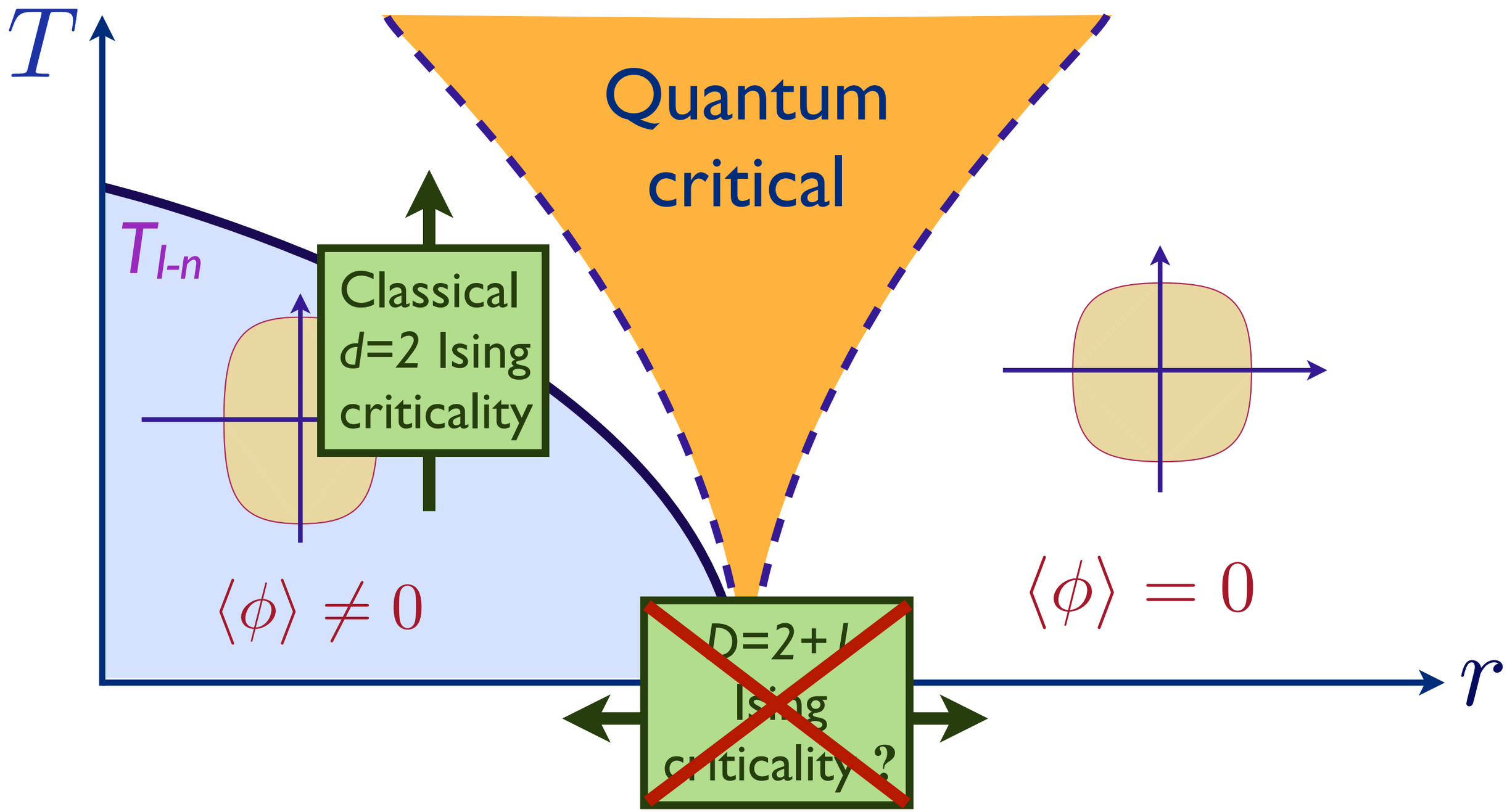


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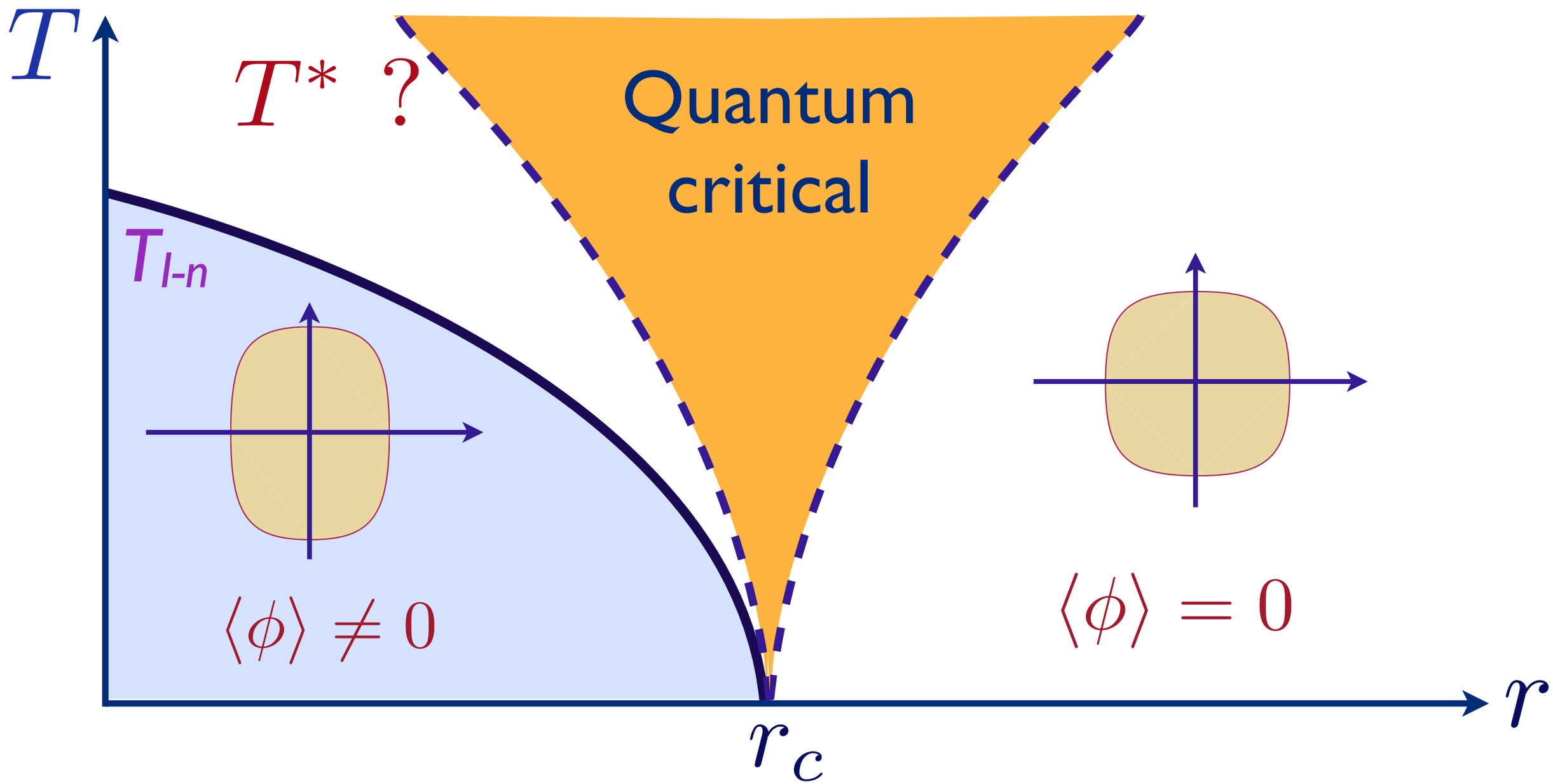


Phase diagram as a function of $T$ and $r$

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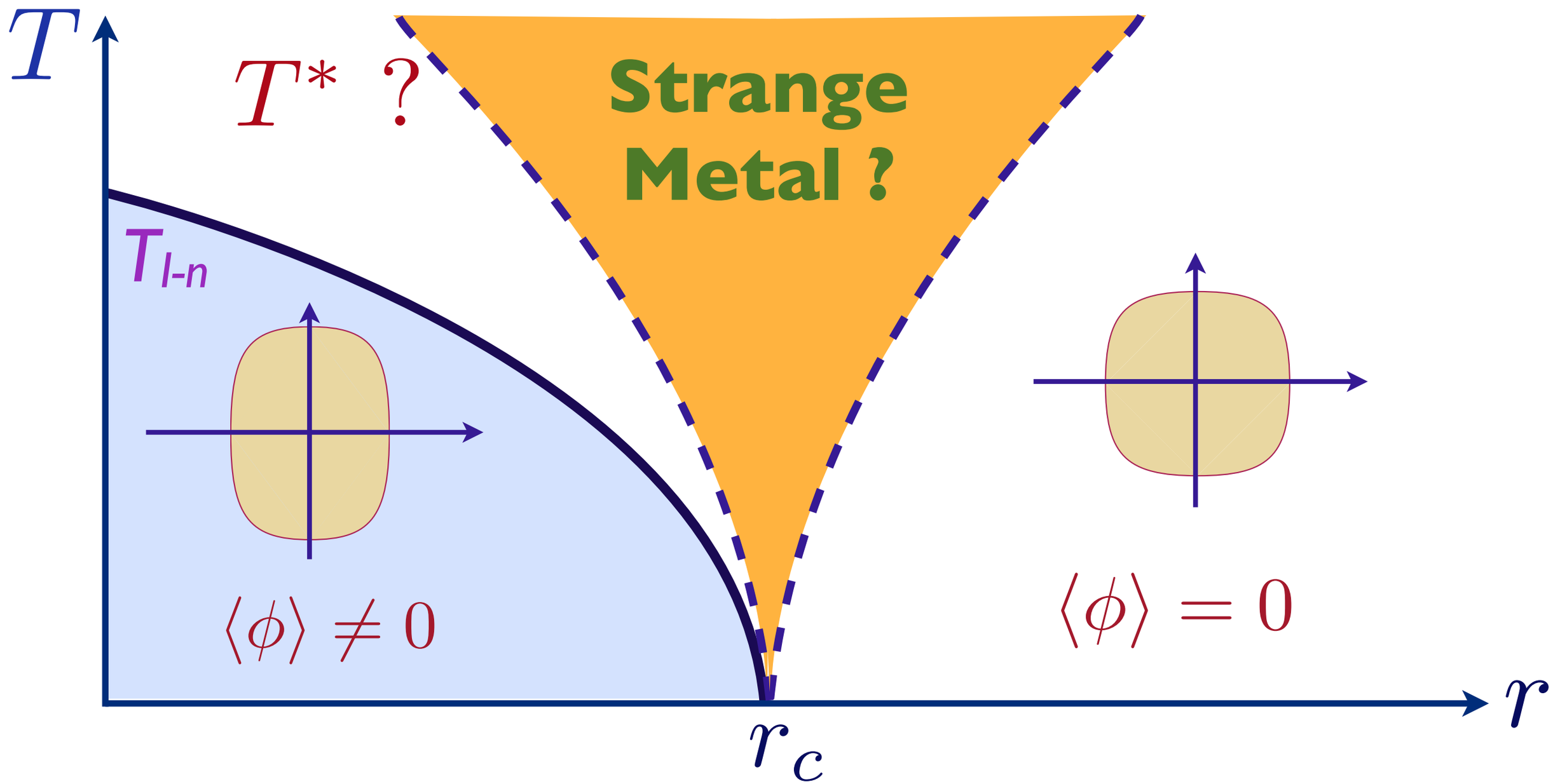


Phase diagram as a function of $T$ and $r$

# Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

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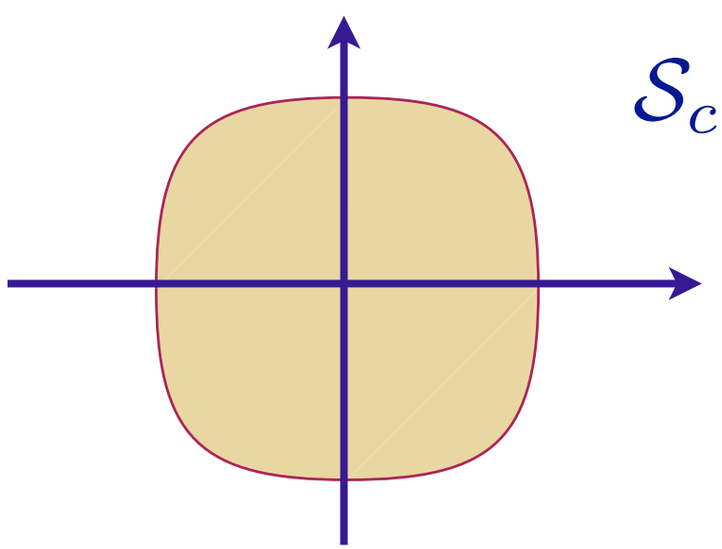
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[ \sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

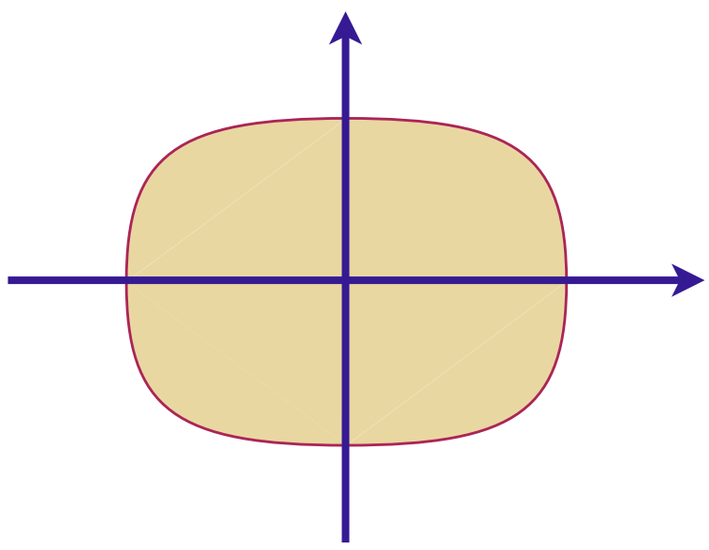
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left( \partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

# Quantum criticality of Ising-nematic ordering

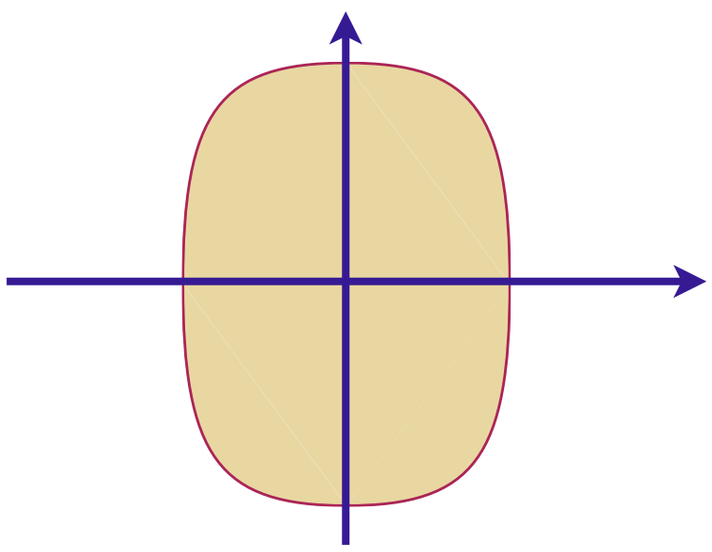
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent $\phi$



$\langle \phi \rangle > 0$



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# Quantum criticality of Ising-nematic ordering

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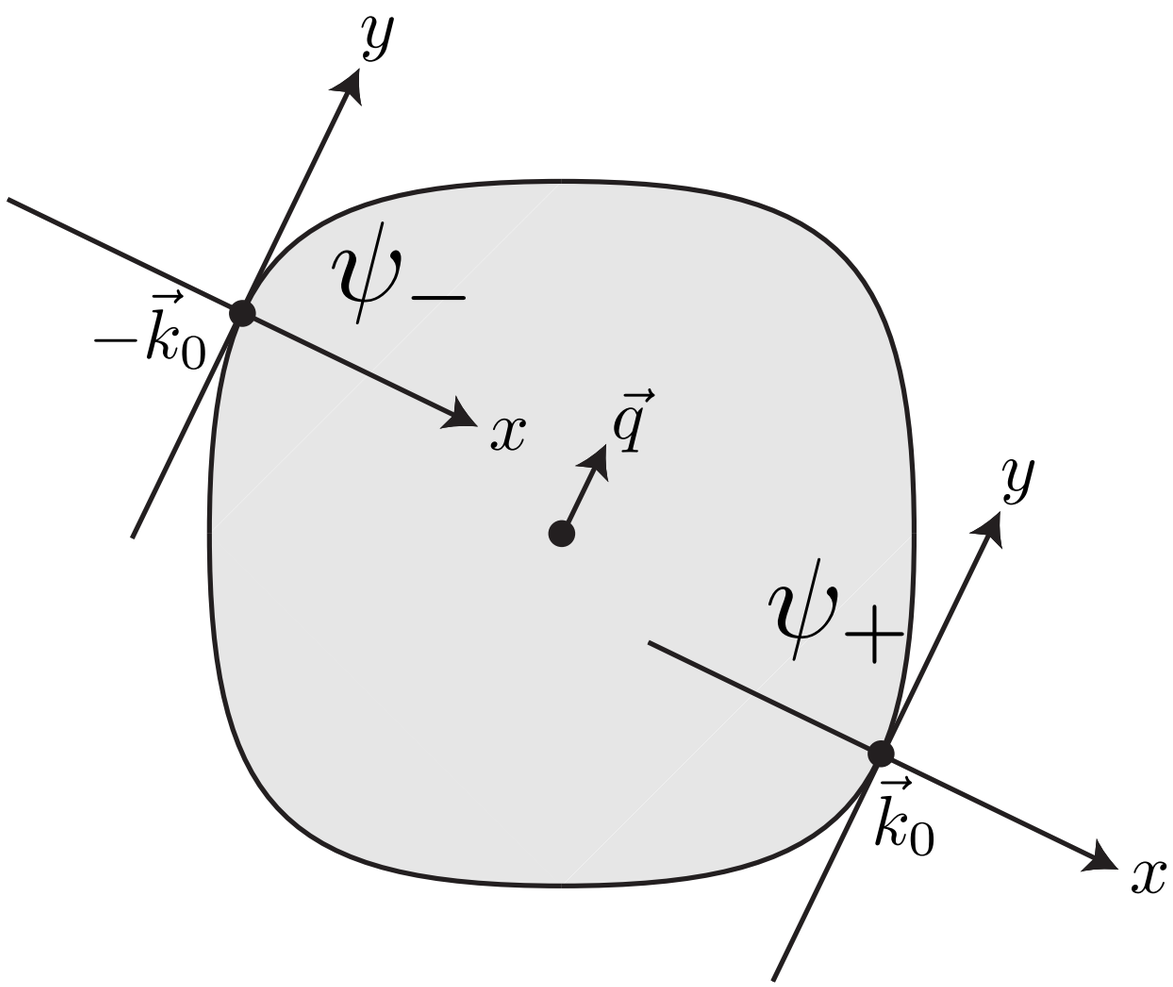


- $\phi$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

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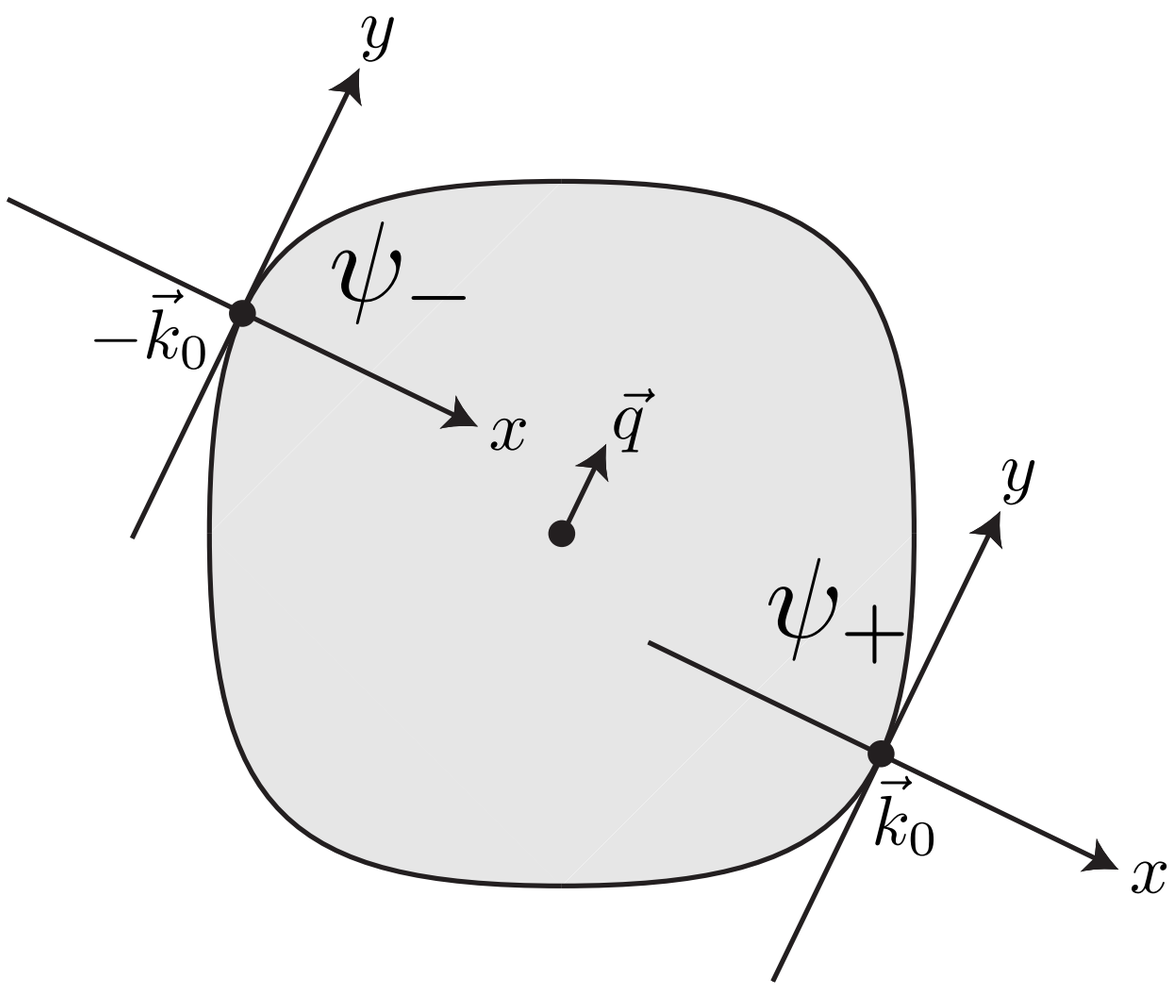


- $\phi$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson ($\phi$) kinetic energy about $\vec{q} = 0$.

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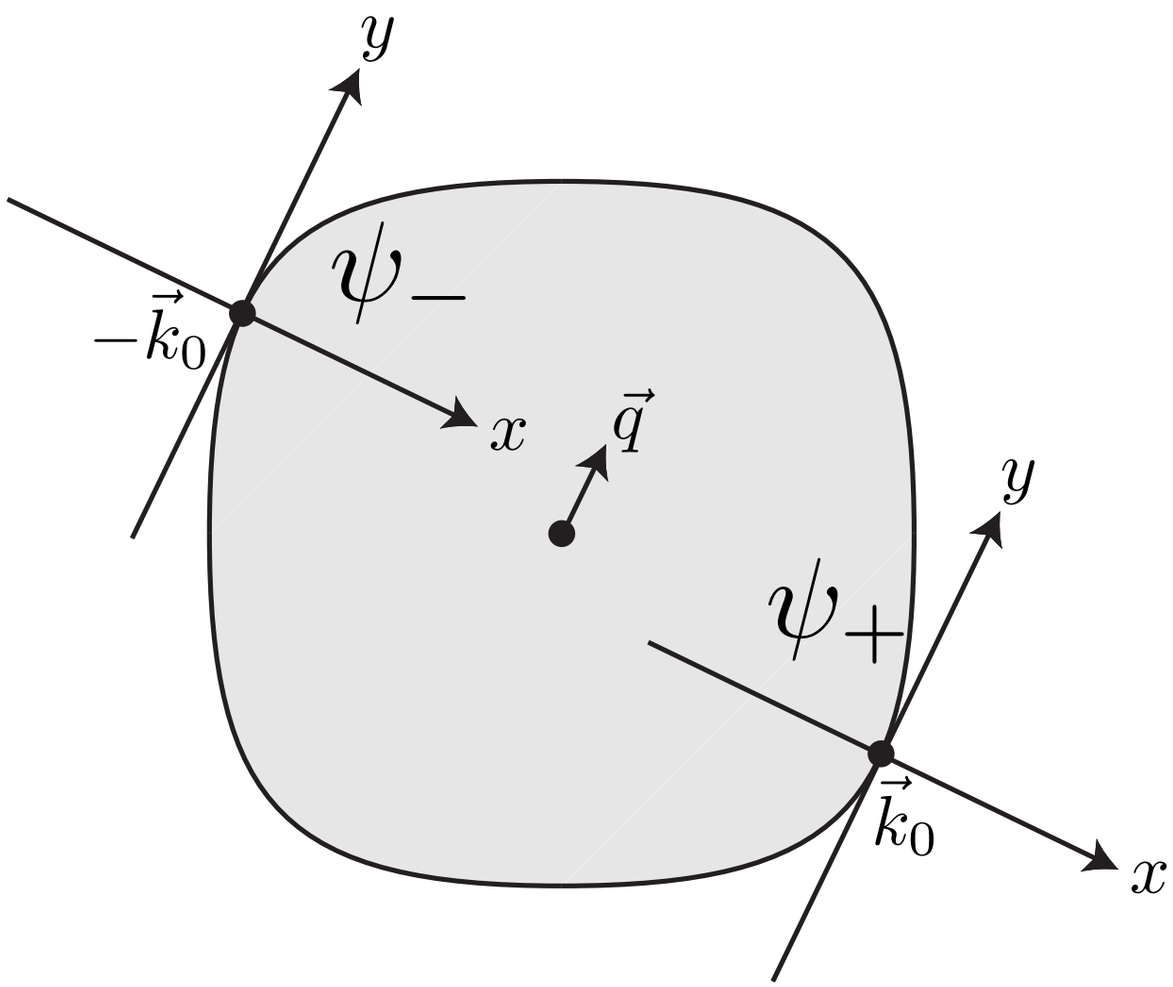



$$\mathcal{L}[\psi_\pm, \phi] =$$

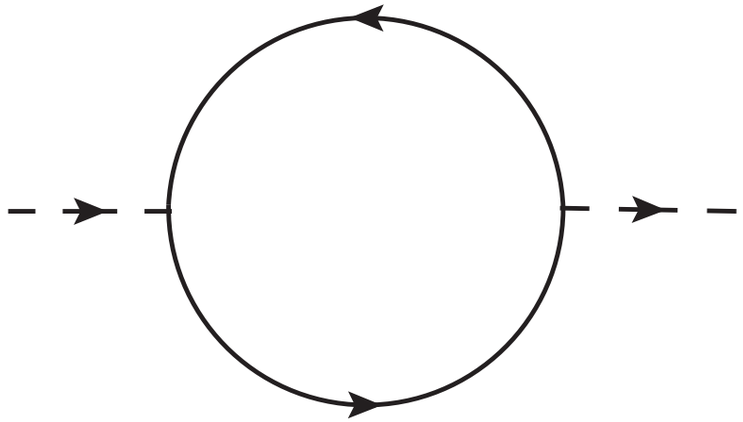
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

# Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



One loop $\phi$ self-energy with $N_f$ fermion flavors:

$$\Sigma_\phi(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]}$$

$$= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

# Quantum criticality of Ising-nematic ordering

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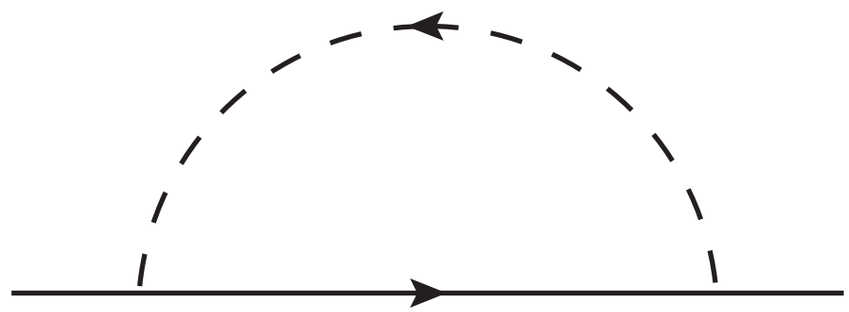

Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int \frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

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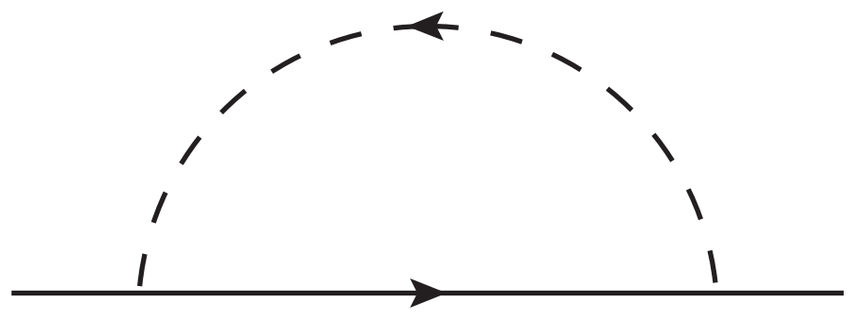

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$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

$\sim |\Omega|^{d/3}$ in dimension $d$.

# Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

Schematic form of $\phi$ and fermion Green's functions in $d$ dimensions

$$D(\vec{q},\omega) = \frac{1/N_f}{q_\perp^2 + \frac{|\omega|}{|q_\perp|}} \quad , \quad G_f(\vec{q},\omega) = \frac{1}{q_x + q_\perp^2 - i\,\mathrm{sgn}(\omega)|\omega|^{d/3}/N_f}$$

In the boson case, $q_\perp^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.
In the fermion case, $q_x \sim q_\perp^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2 \Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in $g$ is OK.
Strongly-coupled theory in $d = 2$.

# Quantum criticality of Ising-nematic ordering

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In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in $G_f^{-1}$ is irrelevant.

Strongly-coupled theory without quasiparticles.

# Quantum criticality of Ising-nematic ordering

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Simple scaling argument for $z = 3/2$.

# Quantum criticality of Ising-nematic ordering

$$\mathcal{L}_{\text{scaling}} = \psi_+^\dagger \left(-i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(+i\partial_x - \partial_y^2\right) \psi_- - g\,\phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \left(\partial_y \phi\right)^2$$

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Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\to \phi\, s^{(2z+1)/4} \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.