1. Review of Fermi liquid theory

Topological argument for the Luttinger theorem

2. Fractionalized Fermi liquid

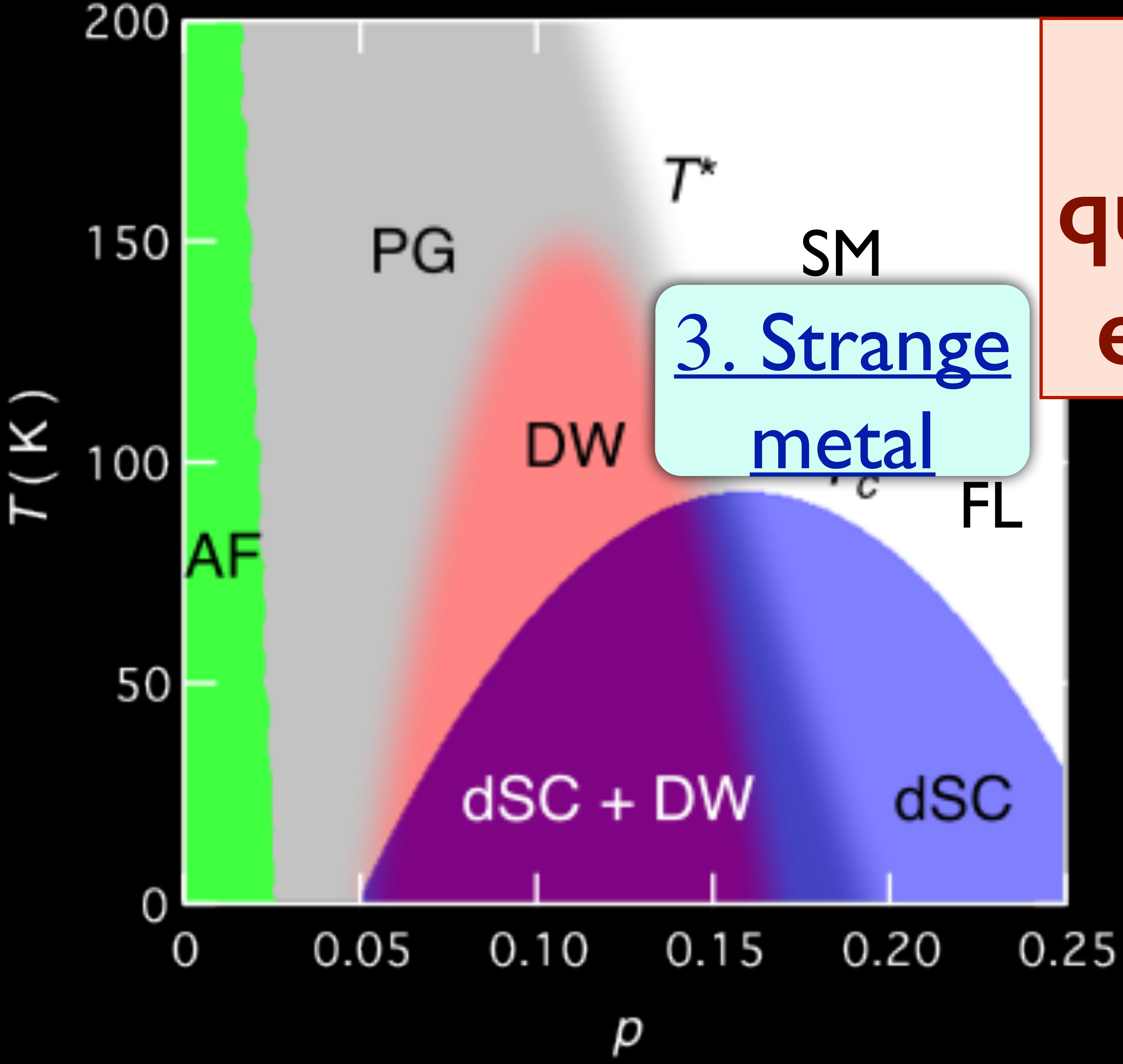
A Fermi liquid co-existing with topological order for the pseudogap metal

3. Quantum matter without quasiparticles

(A) A mean-field model of a non-Fermi liquid, and charged black holes

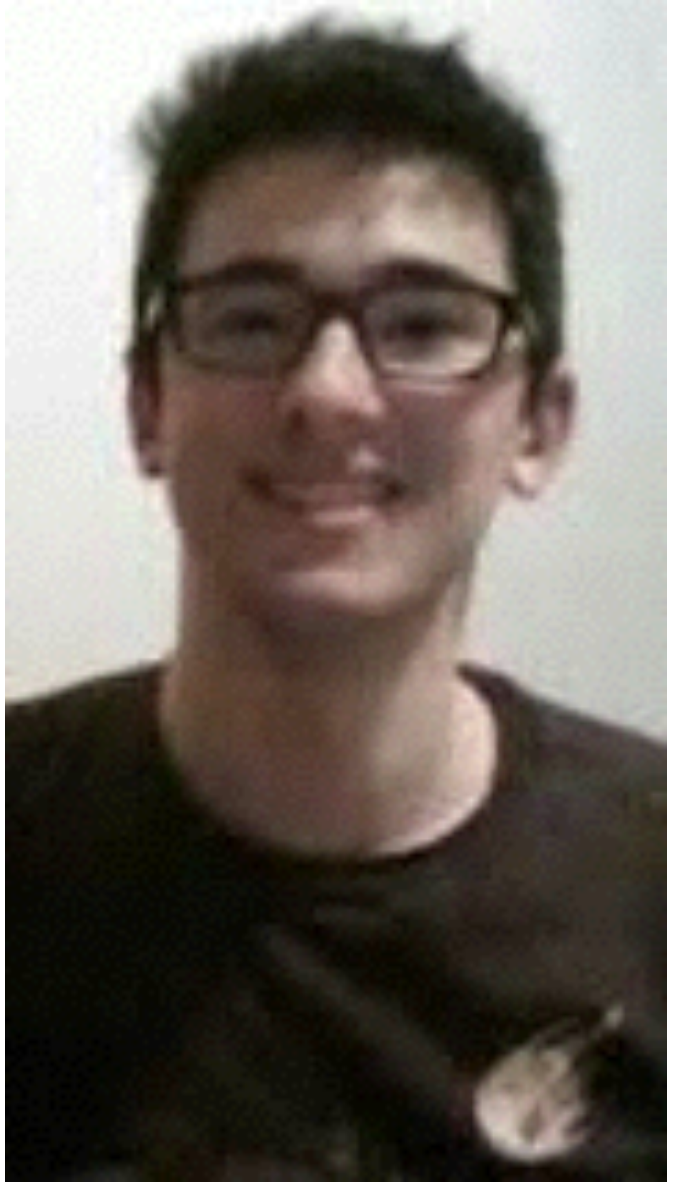
(B) Field theory of a non-Fermi liquid (Ising-nematic quantum critical point)

(C) Theory of transport in strange metals: application to the (less) strange metal in graphene



3. Strange metal

No quasiparticle excitations



Some slides and research by
Andrew Lucas

Transport in strange metals

1. Electron and phonons: Bloch vs. Peierls
2. Definitions and main results
3. Relativistic hydrodynamics
4. Memory functions
5. Holography
6. Application to experiments on graphene

Transport in strange metals

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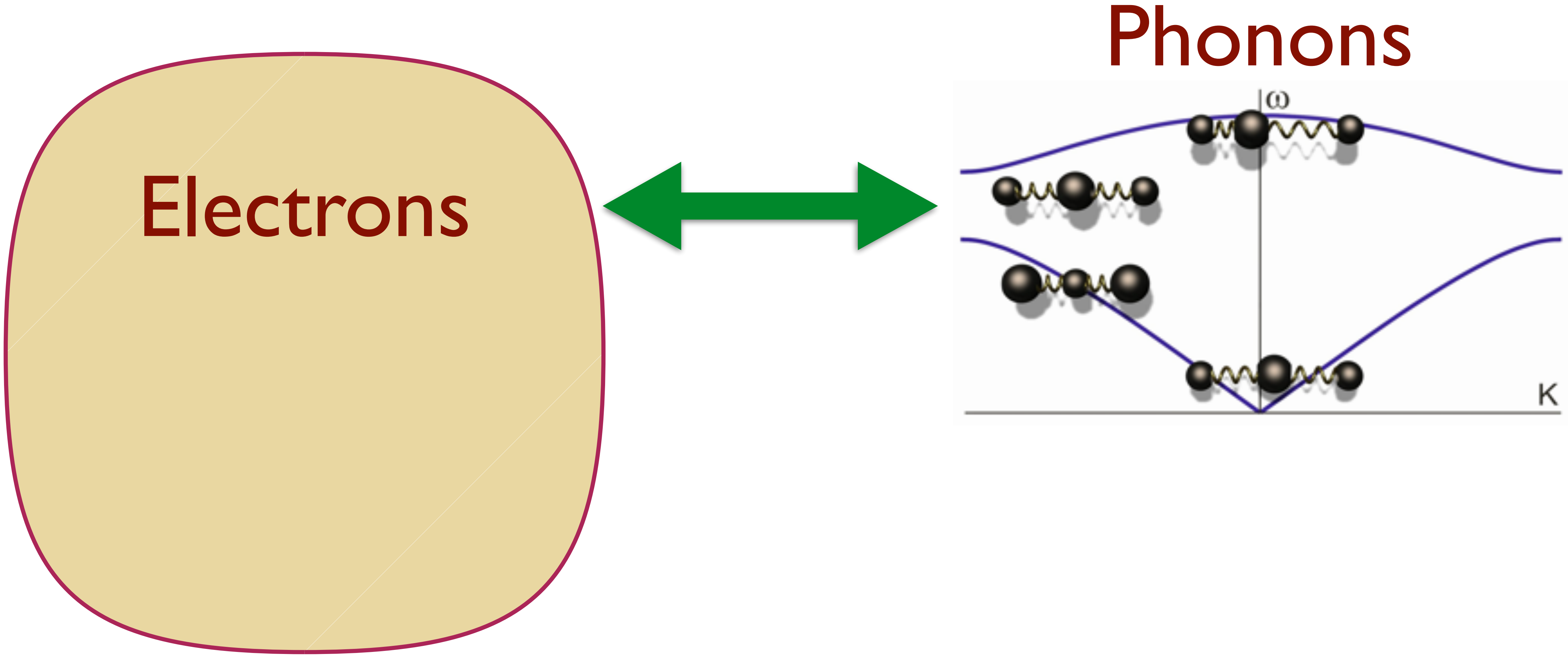
Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to Bloch's law (1930) : a resistivity $\rho(T) \sim T^5$.



Electrons

Phonons

Quasiparticle transport in metals:

- Compute the scattering ra
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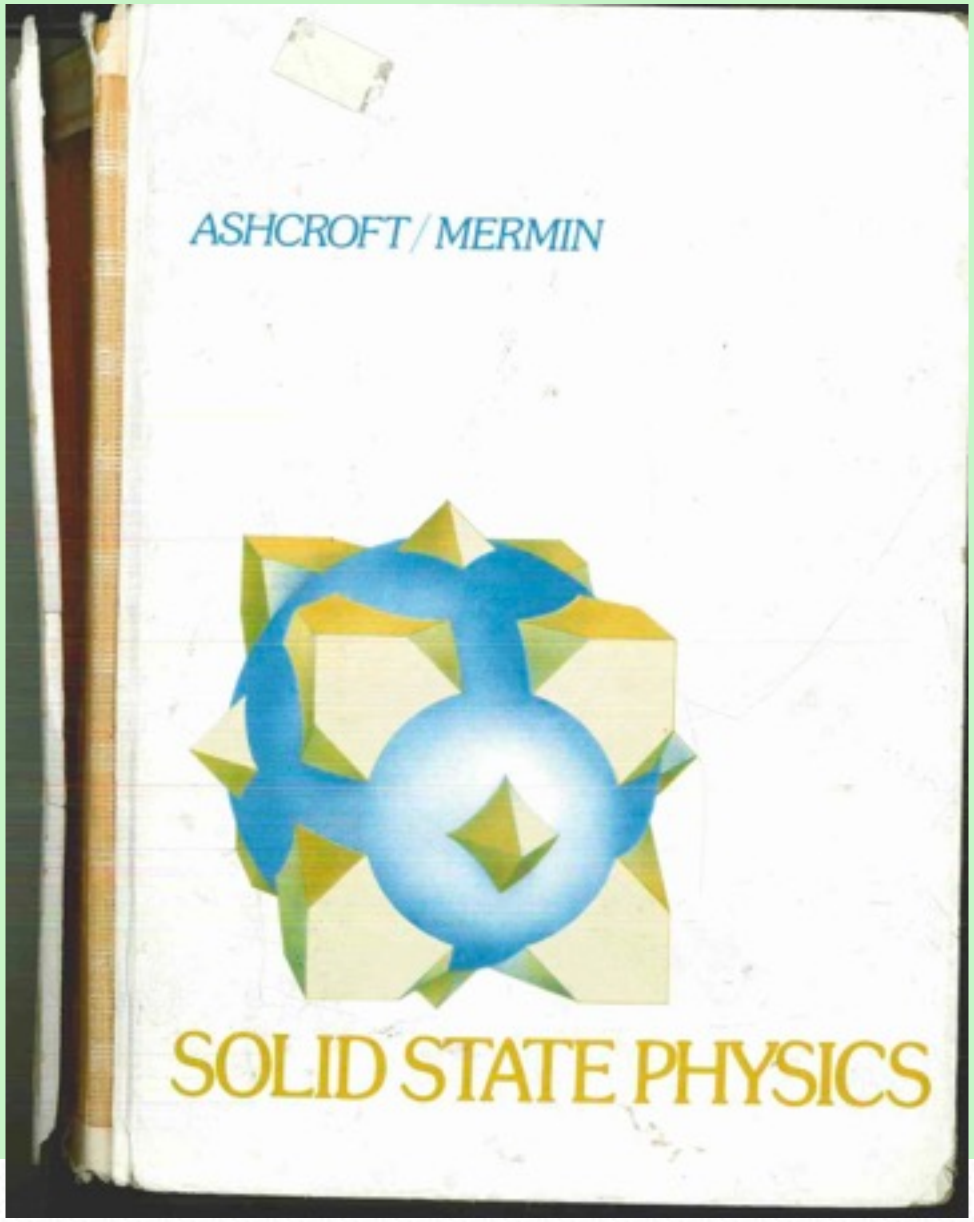
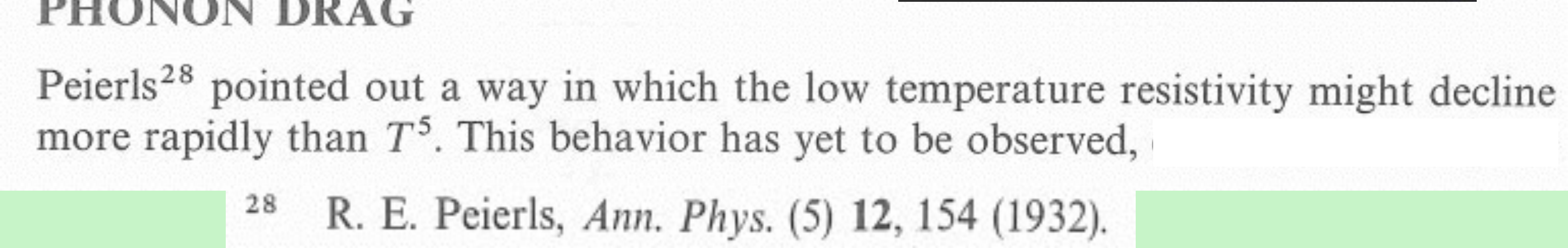
resistivity $\rho(T) \sim T^5$.



However, this ignores "phonon drag"



PHONON DRAG

Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than T^5. This behavior has yet to be observed,

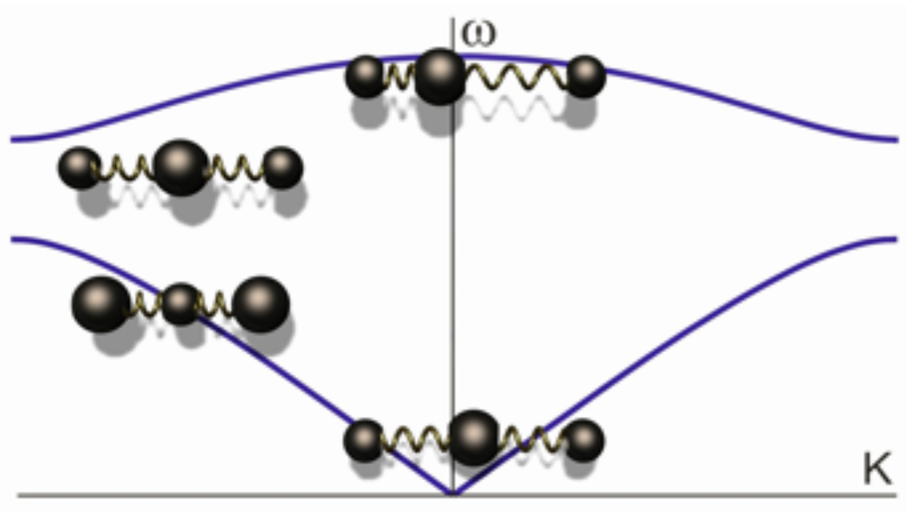
[28] R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).

Rates of Momentum Flow

Electrons

SLOW

Phonons



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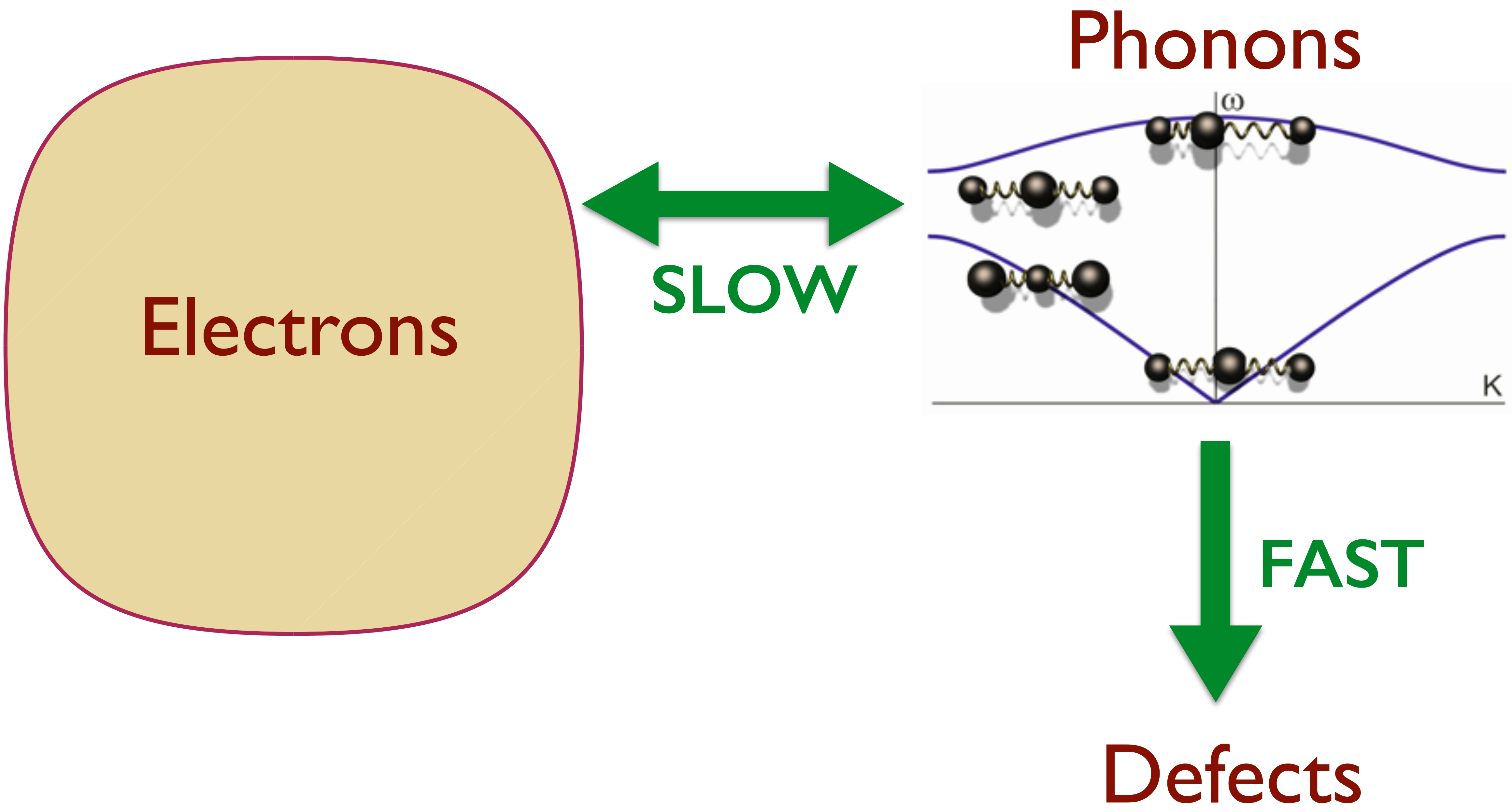


Electrons

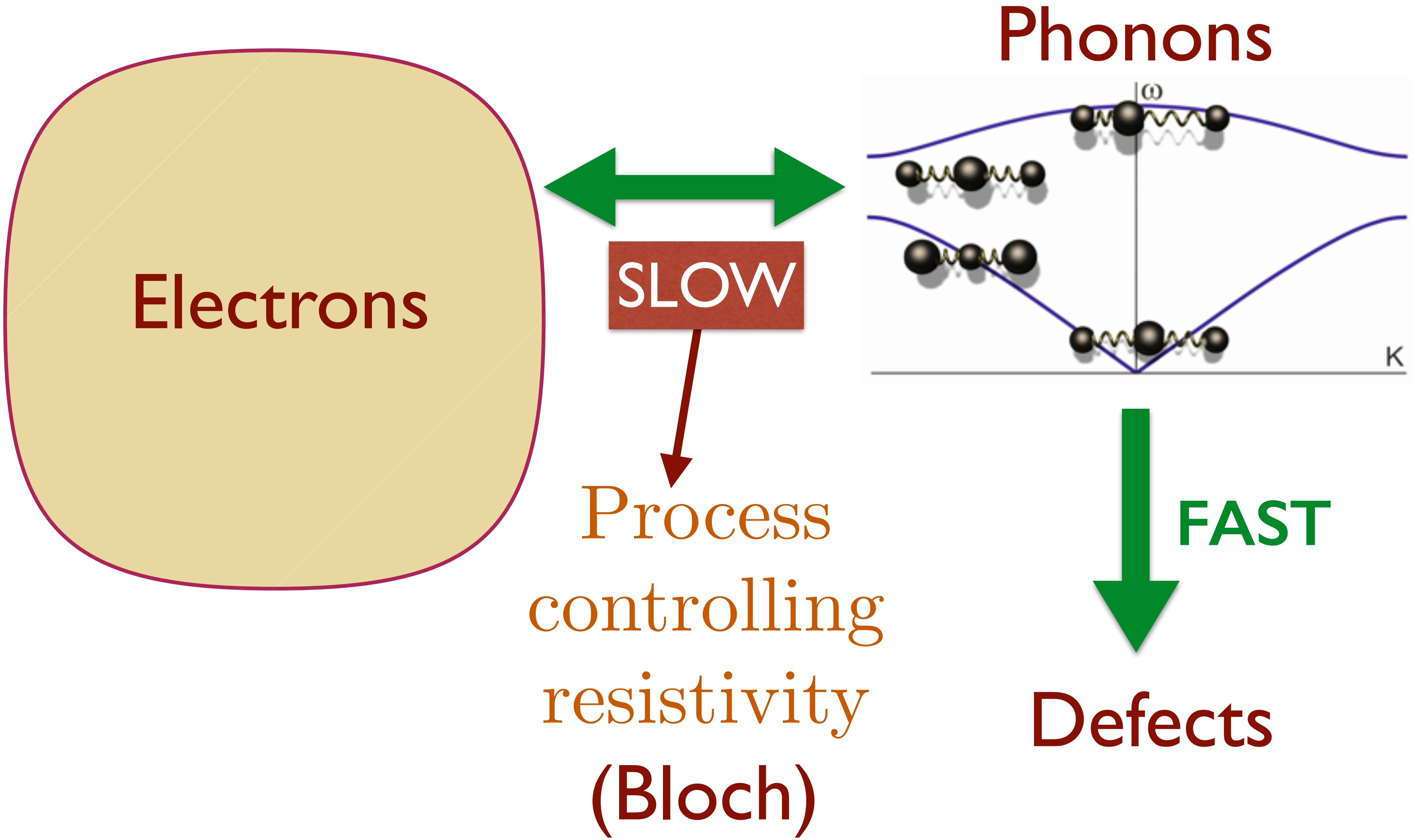
SLOW

Phonons

FAST

Defects

Rates of Momentum Flow



Electrons

Phonons

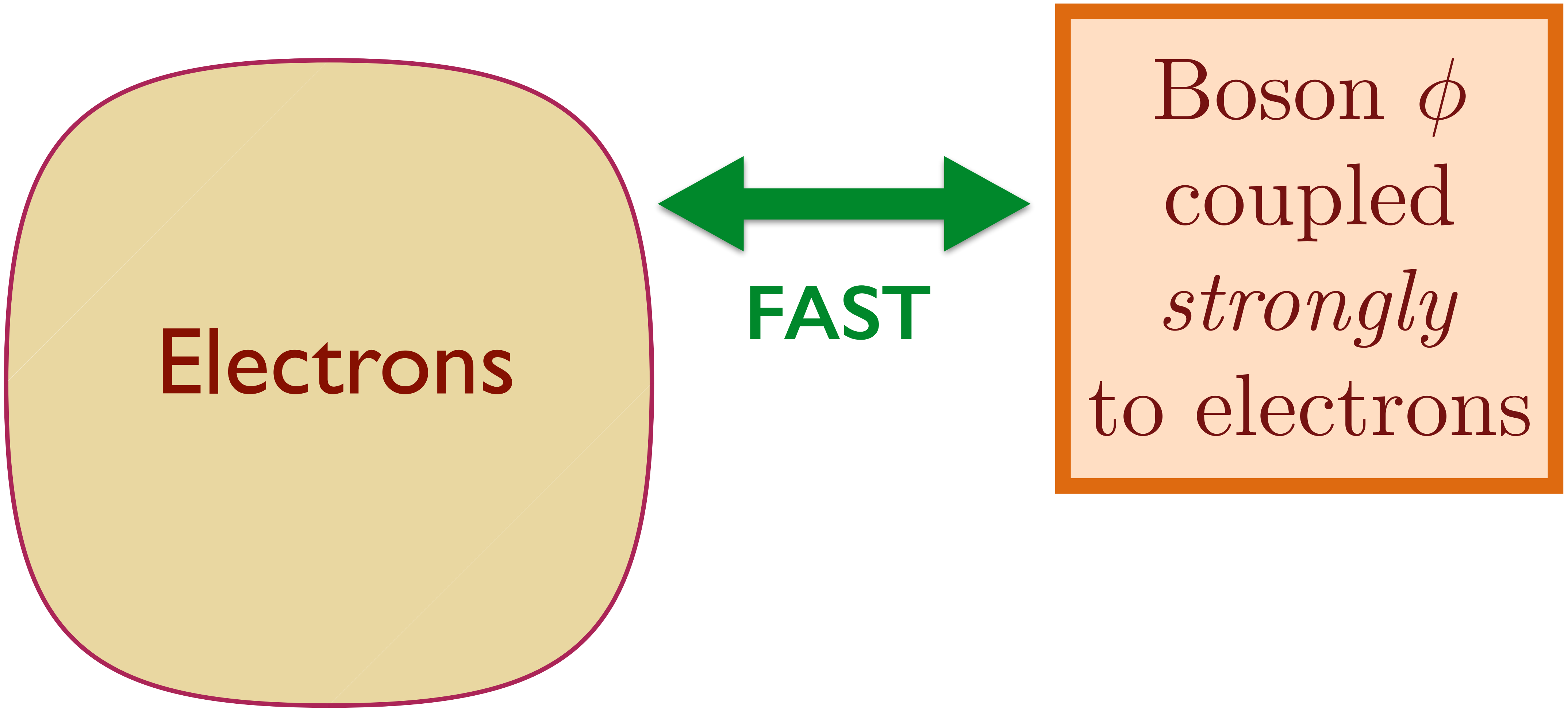
SLOW

Process controlling resistivity (Bloch)

FAST

Defects

Rates of Momentum Flow

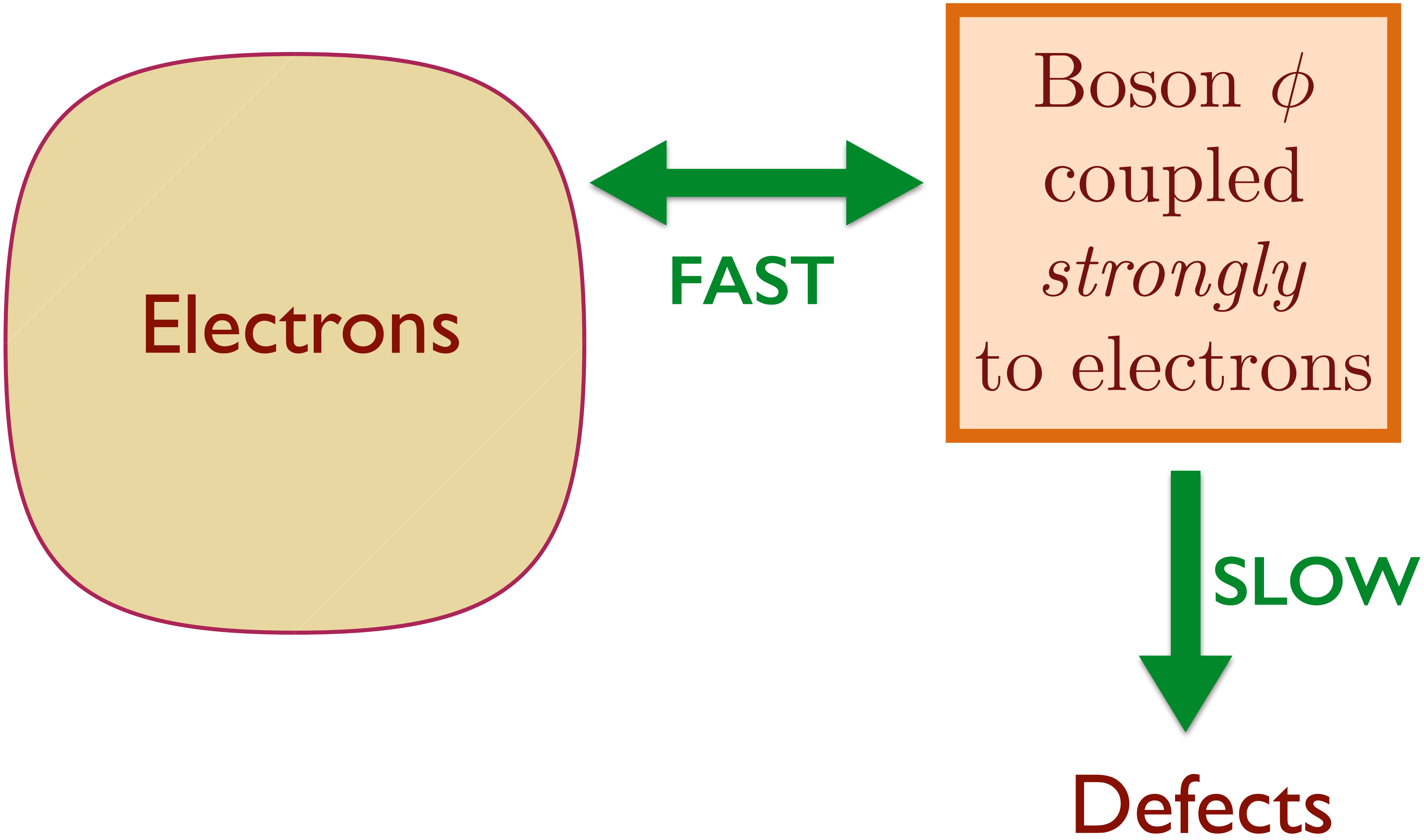


Electrons

FAST

Boson ϕ coupled *strongly* to electrons

Rates of Momentum Flow



Electrons

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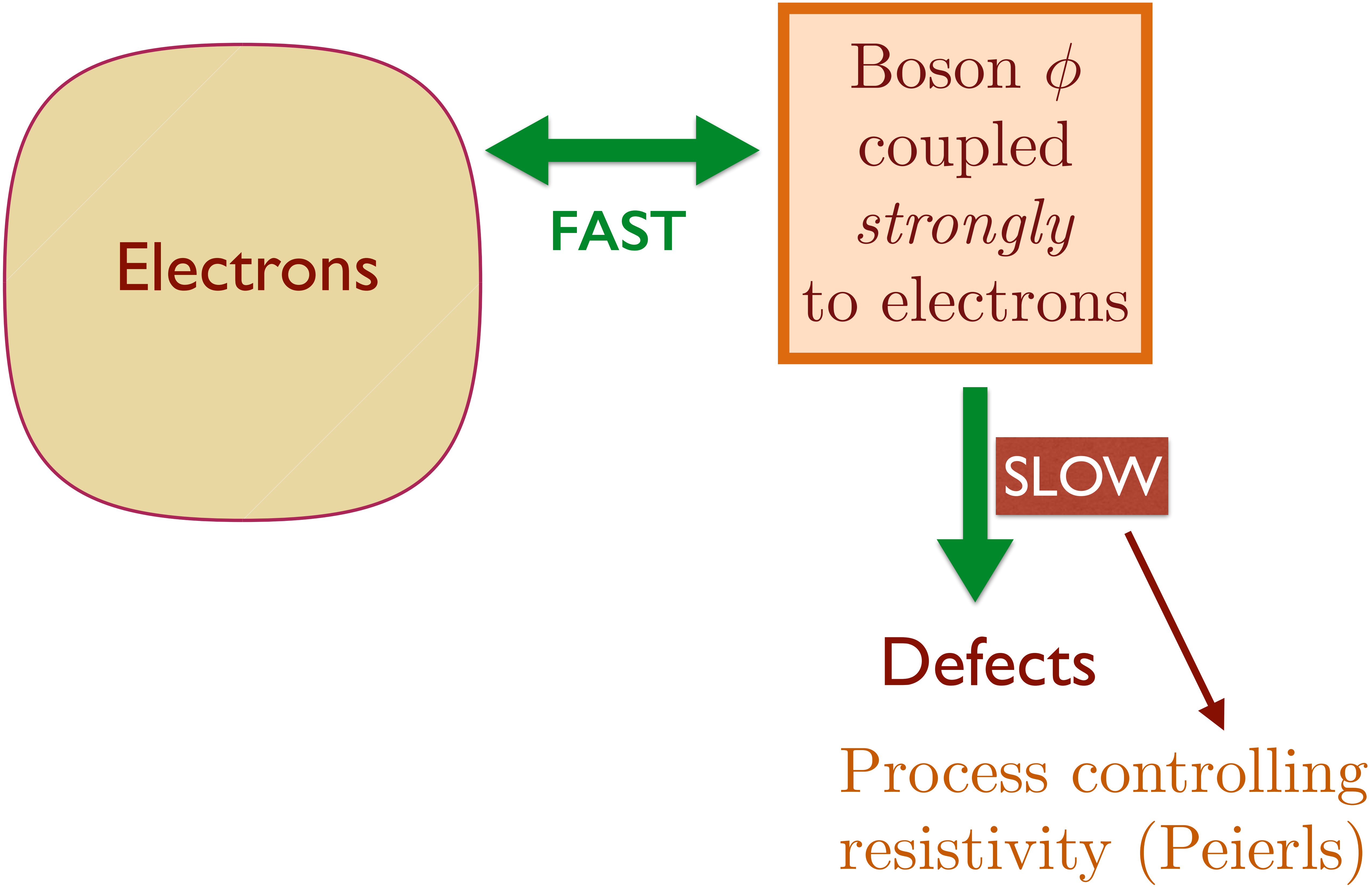
Rates of Momentum Flow



Electrons

FAST

Boson ϕ coupled *strongly* to electrons

SLOW

Defects

Process controlling resistivity (Peierls)

Transport in strange metals

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Fermi "Liquids"

- old metallic theory: nearly free streaming quasiparticles (Fermi "liquid")
- typical time scales:

$$t_{\mathrm{ee}} \gg t_{\mathrm{imp}}$$

- (quantum) kinetic description of transport:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = -\mathcal{C}[f]$$

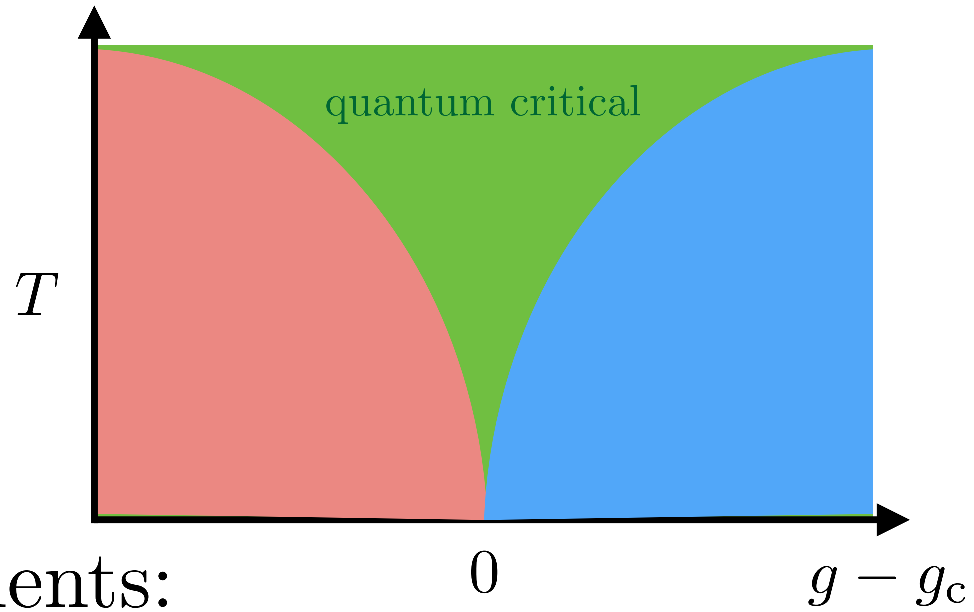
  and a nearly universal prediction:

$$\sigma \sim T^{-2}$$

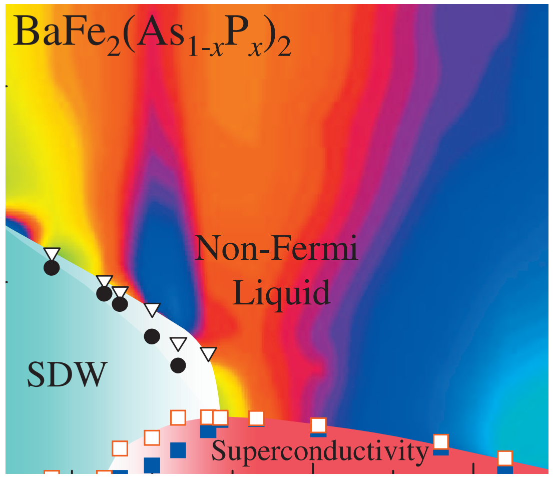
- but $\sigma \sim T^{-2}$ violated in many experiments...

Hydrodynamics at Quantum Criticality

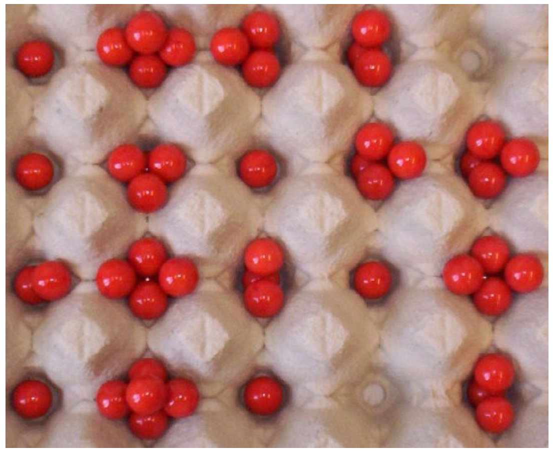
$$t_{\mathrm{ee}} \sim \frac{\hbar}{k_{\mathrm{B}} T} \sim 10^{-13}\ \mathrm{s} \qquad (T = 100\ \mathrm{K})$$



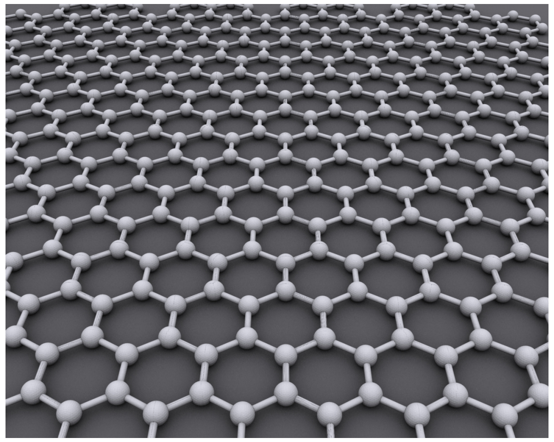
- quantum criticality in experiments:



strange metal

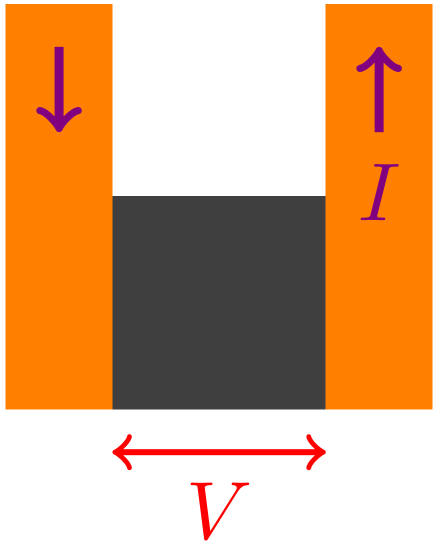


superfluid-insulator (cold atoms)



graphene

- hydrodynamic behavior of electrons in very clean system with $v_{\mathrm{F}} t_{\mathrm{ee}} \ll l_{\mathrm{imp}}$; $v_{\mathrm{F}} t_{\mathrm{ee}} \sim 100$ nm (graphene)



$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, measure thermoelectric transport:

$$\begin{pmatrix} \delta J_i \\ \delta Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & \alpha_{ij} \\ T\bar{\alpha}_{ij} & \bar{\kappa}_{ij} \end{pmatrix} \begin{pmatrix} \delta E_j \\ -\partial_j \delta T \equiv T\delta\zeta_j \end{pmatrix}.$$

- $\sigma =$ easy experiment; related to QFT correlators:

$$\sigma_{ij}(\omega) = \frac{\mathrm{i}}{\omega}\langle J_i(-\omega) J_j(\omega)\rangle, \quad \text{etc.}$$

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)$$

with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x, P_x}$, and $\mathcal{M} \equiv \chi_{P_x, P_x}$.

Obtained in hydrodynamics, holography, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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In theories which are relativistic at high energies (including graphene), $T\alpha_Q(\omega) = -\mu\sigma_Q(\omega)$, $T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega)$, $\mathcal{M} = T\mathcal{S} + \mu\mathcal{Q} = \mathcal{H}$ the enthalpy density, and $\mathcal{Q} = n$ the electron density

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$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \sigma_Q(\omega)$$

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$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \bar{\kappa}_Q(\omega)$$

Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$\begin{aligned} H &= H_0 - \int d^d x \, h(x) \, \mathcal{O}(x) . \\ \frac{\mathcal{M}}{\tau} &= \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\operatorname{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \text{higher orders in } h \end{aligned}$$

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$$\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} B\mathcal{Q}.$$

Electrical and thermal magnetotransport in a magnetic field *B* with no additional parameters

(assuming σ_Q is field-independent)

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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Electrical and thermal magnetotransport with no additional parameters
(assuming σ_Q is field-independent)

Blake and Donos: With $\sigma_Q \sim 1/T$ and $\tau \sim 1/T^2$, we obtain $\sigma_{xx} \sim 1/T$ and $\tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2$, in agreement with data on cuprates (Ong, PRL 1991); such data cannot be explained in a quasiparticle model.

M. Blake and A. Donos, PRL **114**, 021601 (2015)

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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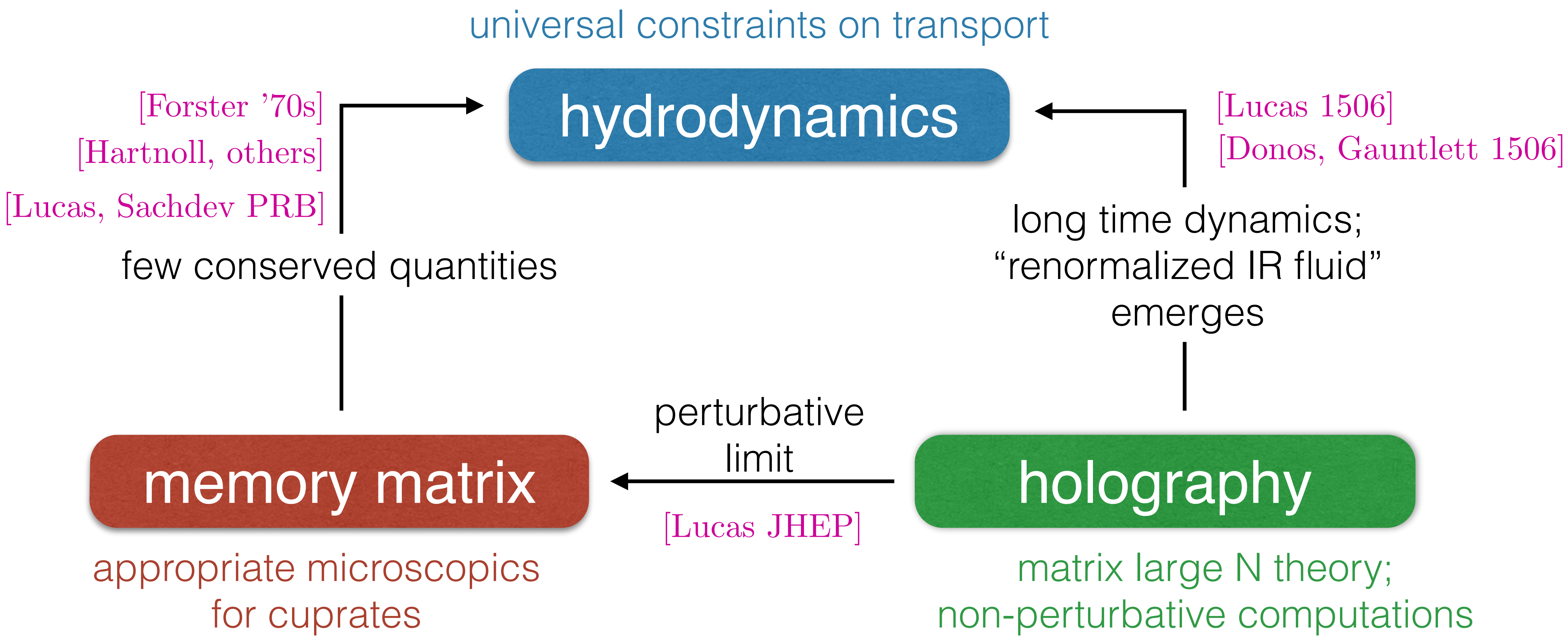

universal constraints on transport

hydrodynamics

[Forster ’70s]
[Hartnoll, others]
[Lucas, Sachdev PRB]

few conserved quantities

[Lucas 1506]
[Donos, Gauntlett 1506]

long time dynamics;
“renormalized IR fluid”
emerges

memory matrix

appropriate microscopics
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matrix large N theory;
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figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

Beyond Perturbation Theory

Summary

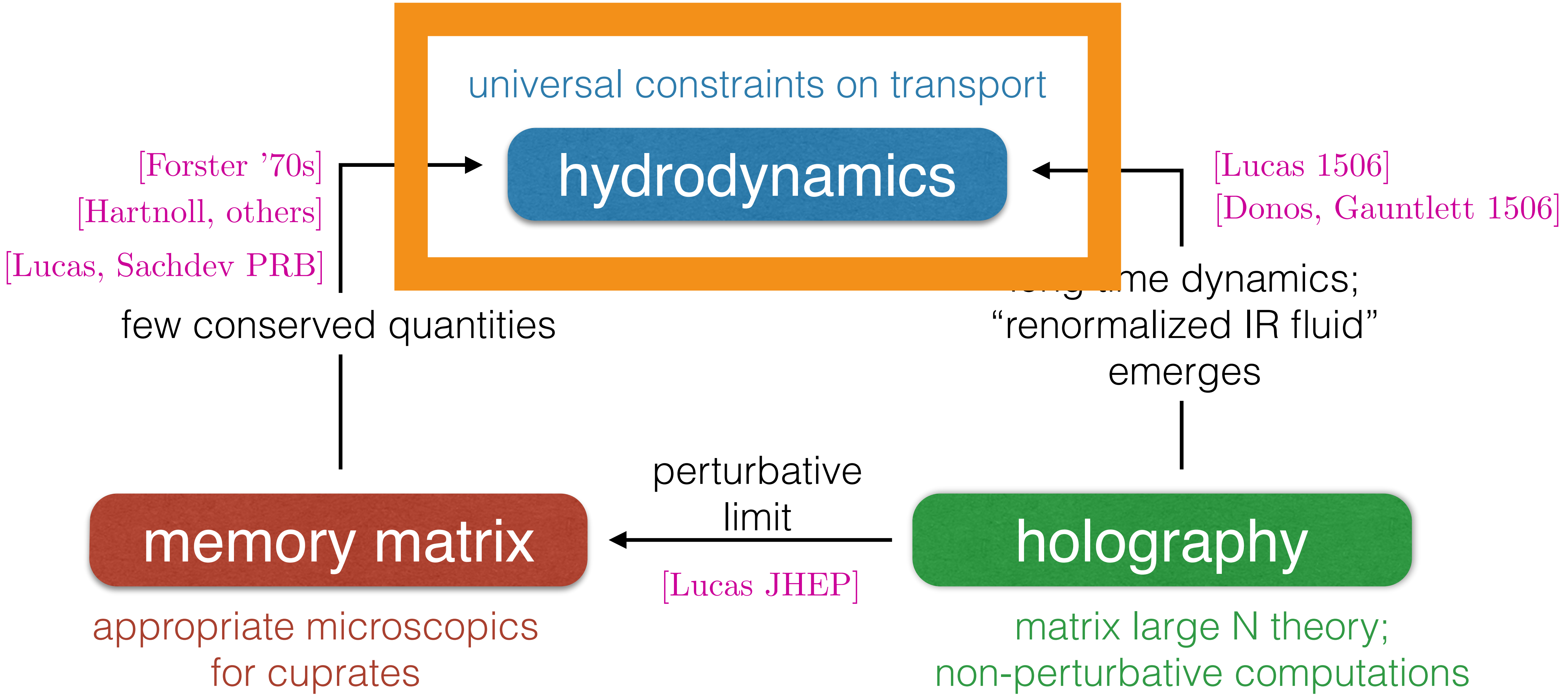

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figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

- hydrodynamics when $l \gg l_{\rm ee}$, $t \gg t_{\rm ee}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\rm ext}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, J^μ in perturbative parameter $l_{\rm ee}\partial_\mu$:

$$\begin{aligned} T^{\mu\nu} &= P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu - 2\mathcal{P}^{\mu\rho}\mathcal{P}^{\nu\sigma}\eta\partial_{(\rho}u_{\sigma)} - \mathcal{P}^{\mu\nu}\left(\zeta - \frac{2\eta}{d}\right)\partial_\rho u^\rho + \cdots, \\ J^\mu &= \mathcal{Q}u^\mu - \sigma_{\rm Q}\mathcal{P}^{\mu\rho}\left(\partial_\rho\mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\rm ext}_{\rho\nu}\right) + \cdots, \\ \mathcal{P}^{\mu\nu} &\equiv \eta^{\mu\nu} + u^\mu u^\nu, \\ Q^i &= J^i - \mu T^{ti} \end{aligned}$$

- Determines $\mathcal{M}$, α_Q, and $\overline{\kappa}_Q$ in terms of σ_Q and P.

[Hartnoll, Kovtun, Müller, Sachdev, *Physical Review* **B76** 144502 (2007)]

Generalized Drude (HKMS) Model

mean field treatment of translational symmetry breaking:

[Hartnoll, Kovtun, Müller, Sachdev, *Physical Review* **B76** 144502 (2007)]

$$\partial_\mu T^{\mu i} = -\frac{T^{it}}{\tau} + \mathcal{Q}E_i.$$

"constitutive relations":

$$\begin{aligned} T^{it} &= \mathcal{M}v^i \\ J^i &= \sigma_{\mathrm{Q}} E^i + \mathcal{Q}v^i: \end{aligned}$$

$$\sigma(\omega) = \sigma_{\mathrm{Q}} + \frac{\mathcal{Q}^2\tau}{\mathcal{M}(1-\mathrm{i}\omega\tau)}.$$

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Hydrodynamic equations yield *B* dependence of magnetotransport with no additional parameters

(assuming σ_Q is field-independent)

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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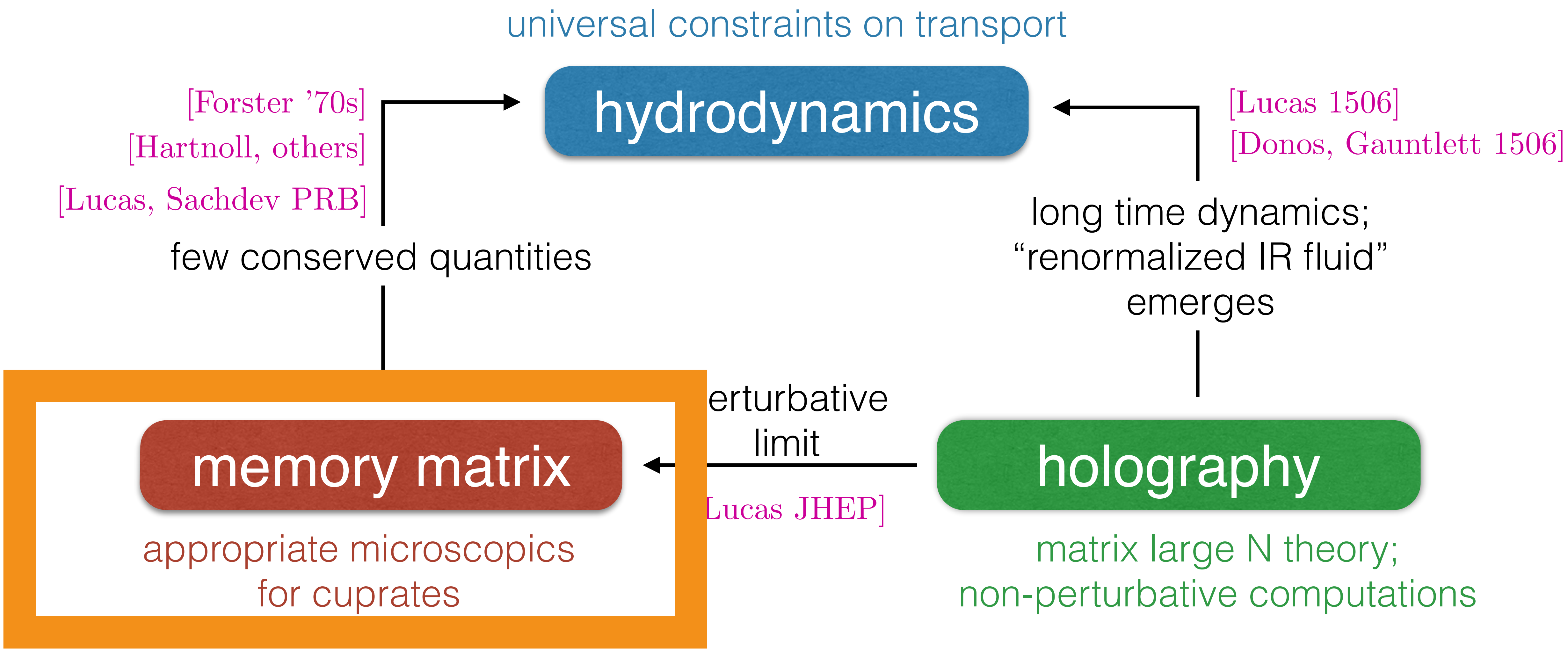

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figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

Memory Functions

Operator Formalism

- σ parametrically *big* in perturbation theory!
- memory functions: keep small subset of operators A:

$$\sigma_{AB} = \chi_{AC}\mathfrak{m}^{-1}_{CD}\chi_{DB}, \quad \mathfrak{m} = \text{small!}$$

- $\mathfrak{m}$ takes the generic form:

$$\mathfrak{m} = M + N - \mathrm{i}\omega\chi$$

neglected DOFs

included DOFs

[Zwanzig, *Journal of Chemical Physics* **33** 1338 (1960)]

[Mori, *Progress of Theoretical Physics* **34** 399 (1965)]

Memory Functions

Momentum Relaxation Time

start with H_0 translation invariant, isotropic, deform to

$$H = H_0 - \int \mathrm{d}^d\mathbf{x}\; h(\mathbf{x})\mathcal{O}(\mathbf{x}).$$

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$$\dot{P}_i = \mathrm{i}[H, P_i] = -\int \mathrm{d}^d\mathbf{x}\; h(\mathbf{x})(\partial_i\mathcal{O}).$$

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$$\sigma = \frac{\chi_{JP}^2}{M_{PP} - \mathrm{i}\omega\chi_{PP}} = \mathcal{Q}^2\left[-\mathrm{i}\omega\mathcal{M} + \sum_{\mathbf{k}} \frac{k^2}{d} \lim_{\omega\to 0} \frac{\mathrm{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\mathbf{k},\omega)\right)}{\omega}\right]^{-1}$$

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if $\langle\mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{x}')\rangle \sim |\mathbf{x}-\mathbf{x}'|^{-2\Delta}$: h random field:

$$\sigma \sim T^{-2(1+\Delta-z)/z}$$

[Lucas, Sachdev, Schalm, *Physical Review* **D89** 066018 (2014)]

Perturbative Hydrodynamic Derivation

intuitive derivation from hydrodynamics:

[Lucas, `arXiv:1506.02662`]

$$-\mathrm{i}\omega\mathcal{M}\delta\bar{v}_i = \mathcal{Q}\delta E_i + \int\frac{\mathrm{d}^d\mathbf{x}}{V_d}\delta\mathcal{O}\partial_i h, \qquad \delta J_i \approx \mathcal{Q}\delta\bar{v}_i,$$

$$\mathcal{O}_0(\mathbf{k}) + \delta\mathcal{O}(\mathbf{k}) = G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\mathbf{k}, -\mathbf{k}\cdot\delta\bar{\mathbf{v}})h(\mathbf{k})$$

Taylor expand, combine equations and recover τ above!

Memory Functions

Diffusion in the Memory Matrix

- σ_{Q} from memory matrix:
  Green's function in a fluid takes the form:

$$G^{\mathrm{R}}_{nn}(\mathbf{k},\omega) = \frac{\sigma_{\mathrm{Q}} k^2}{Dk^2 - \mathrm{i}\omega} + \text{non-singular}$$

$$M_{\partial_x n \partial_x n} = \sigma_{\mathrm{Q}} k^2 + \mathrm{i}\omega\chi_{nn} + \cdots$$

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$$M_{\partial_x n \partial_x n} = \sigma_{\mathrm{Q}} k^2 + \mathrm{i}\omega\chi_{nn} + \cdots$$

- effects of magnetic field:

$$\dot{P}_i = B\epsilon_{ij}J_j + \cdots; \qquad N_{P_y P_x} = B\chi_{J_x P_x} = B\mathcal{Q}, \text{ etc.}$$

[Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

Application: the Hall Angle

- combining together we reproduce HKMS

$$\sigma_{xx} = \frac{(\tau^{-1} - \mathrm{i}\omega)\mathcal{M}\sigma_{\mathrm{Q}} + \mathcal{Q}^2 + B^2\sigma_{\mathrm{Q}}^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - \mathrm{i}\omega)\mathcal{M} + B^2\sigma_{\mathrm{Q}})^2}\mathcal{M}\left(\frac{1}{\tau} - \mathrm{i}\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau^{-1} - \mathrm{i}\omega)\mathcal{M}\sigma_{\mathrm{Q}} + \mathcal{Q}^2 + B^2\sigma_{\mathrm{Q}}^2}{\mathcal{Q}^2 B^2 + ((\tau^{-1} - \mathrm{i}\omega)\mathcal{M} + B^2\sigma_{\mathrm{Q}})^2} B\mathcal{Q}.$$

- old puzzle: $\sigma_{xx}(B=0) \sim 1/T$, $\sigma_{xy}(B \to 0) \sim B/T^3$.

  [Chien, Wang, Ong, *Physical Review Letters* **67** 2088 (1991)]
- this is consistent if

  [Blake, Donos, *Physical Review Letters* **114** 021601 (2015)]

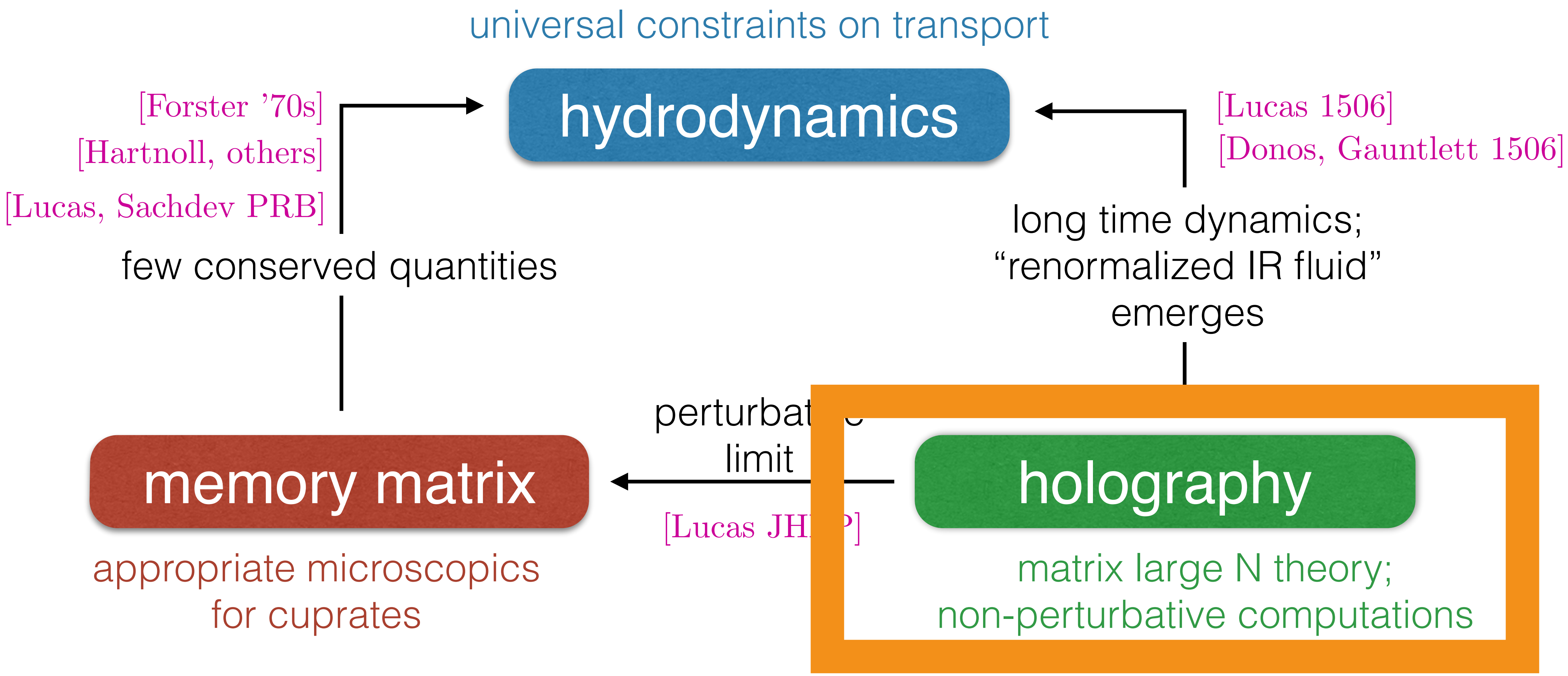
$$\frac{\mathcal{Q}^2\tau}{\mathcal{M}} \ll \sigma_{\mathrm{Q}} \sim \frac{1}{T}, \qquad \tau \sim \frac{1}{T^2}.$$

Transport in strange metals

1. Electron and phonons: Bloch vs. Peierls
2. Definitions and main results
3. Relativistic hydrodynamics
4. Memory functions
5. Holography
6. Application to experiments on graphene

Beyond Perturbation Theory

Summary



universal constraints on transport

hydrodynamics

[Forster '70s]
[Hartnoll, others]
[Lucas, Sachdev PRB]

few conserved quantities

[Lucas 1506]
[Donos, Gauntlett 1506]

long time dynamics;
"renormalized IR fluid"
emerges

memory matrix

appropriate microscopics
for cuprates

perturbat e
limit

[Lucas JH P]

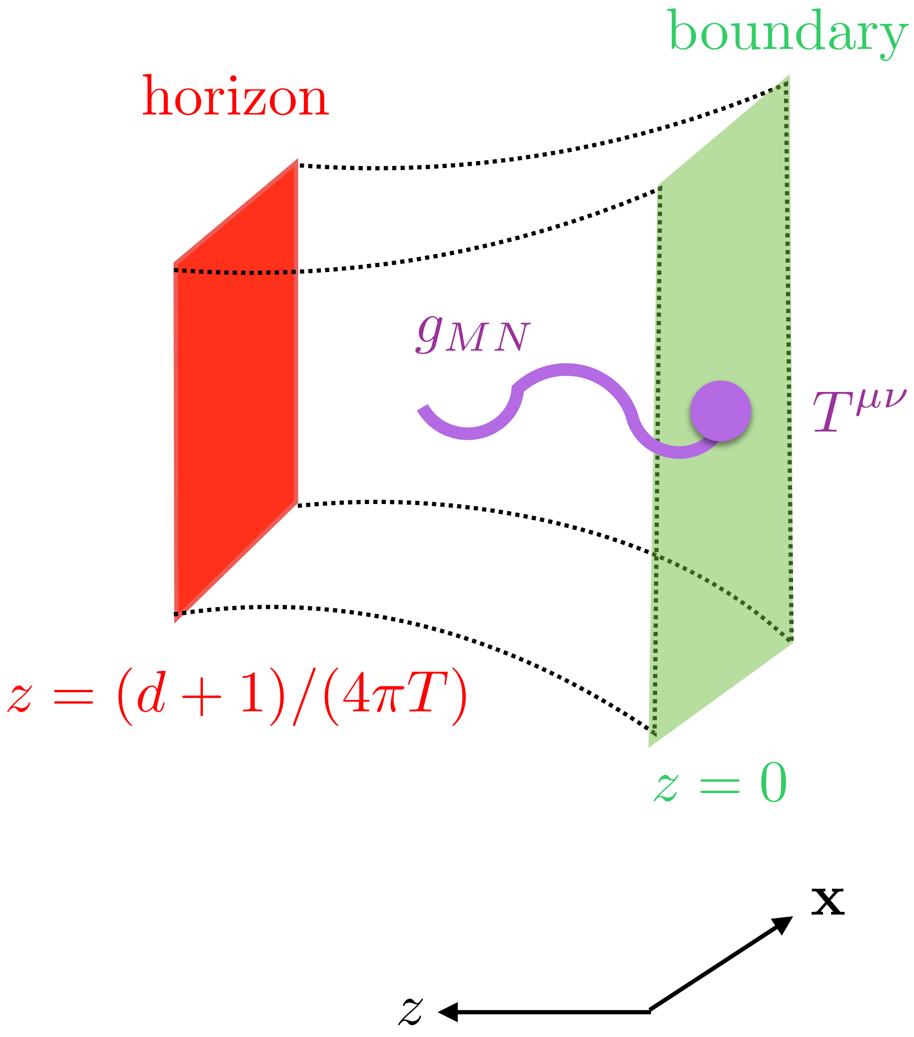
holography

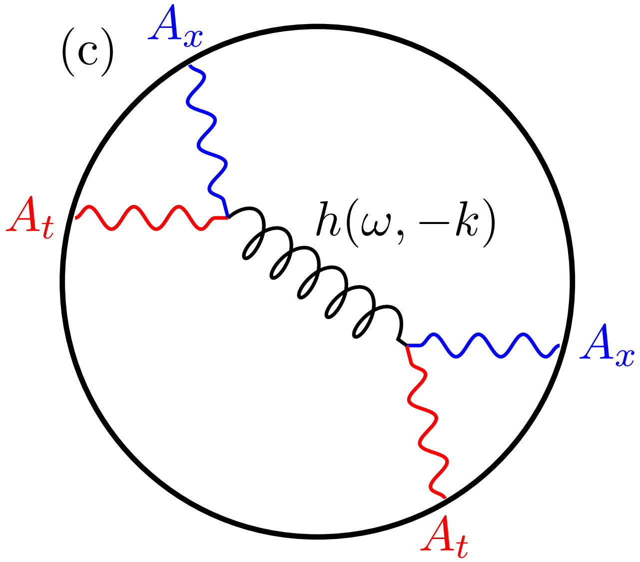
matrix large N theory;
non-perturbative computations

figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

Motivation

- conformal group $\mathrm{SO}(4,2)$ = isometry of AdS_5.
- $g_{MN} \to T^{\mu\nu}$; $A_M \to J^\mu$.
- black hole $\implies$ finite T
- finite density → charged BH
- gravity theory classical: $L^3/G_5 \sim N^2 \gg 1$:

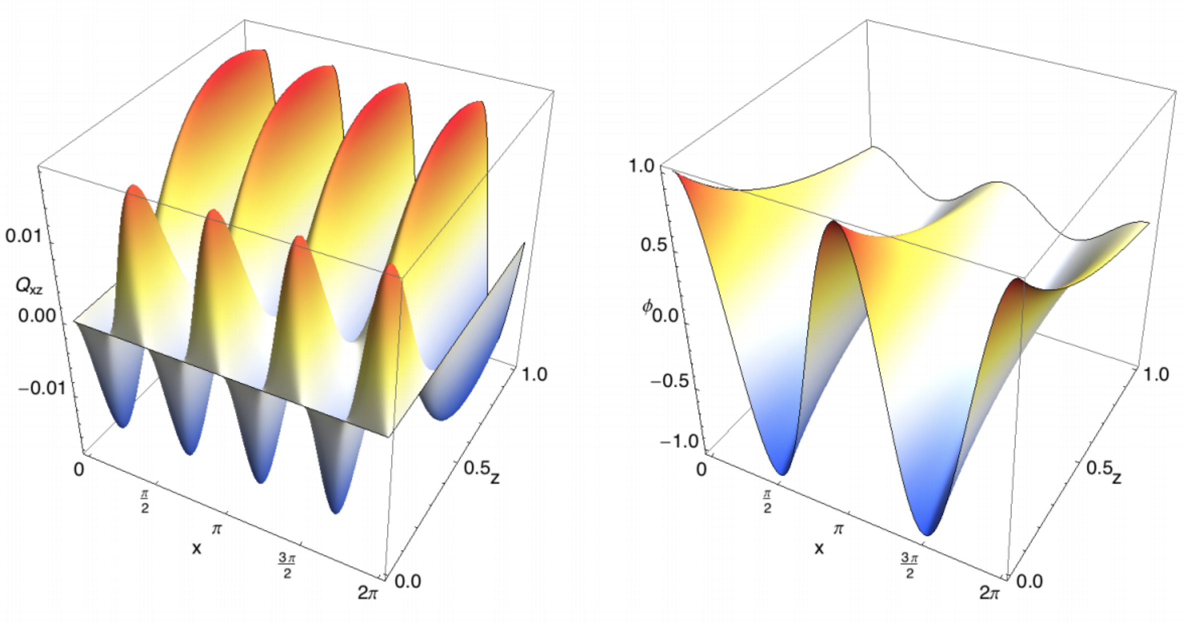




"Holographic Lattices", Disorder and Massive Gravity

how to break translational invariance in holography:

- explicitly (numerical GR): [Horowitz, Santos, Tong, *Journal of High Energy Physics* **07** 168 (**2012**)]



- massive gravity: [Vegh, arXiv:1301.0537]

$$\mathcal{L} = R - 2\Lambda - m^2 \sum \mathcal{U}[f, g].$$

- "Q-lattice": [Donos, Gauntlett, *JHEP* **04** 040 (**2014**)]

$$\mathcal{L} = R - 2\Lambda - \frac{1}{2}(\partial\phi_i)^2, \quad \phi_i = m x_i$$

“Old” Analytic Computation of dc Transport

- “lattice”/disorder $\Longrightarrow$ “massive gravity” perturbatively

  [Blake, Tong, Vegh, *Physical Review Letters* **112** 071602 (2014)]

  [Lucas, Sachdev, Schalm, *Physical Review* **D89** 066018 (2014)]

From Holography to Memory Matrices

- step 1: $\omega \ll T \implies$ bulk response to δE_i is

$$\delta(\text{bulk field}) \sim \text{constant} - \mathrm{i}\omega\tau \times \text{Galilean boost} + \cdots$$

$$\frac{\epsilon + P}{\tau} \sim \sum_{\mathbf{k}} k^2 |h(\mathbf{k})|^2 \psi(\mathbf{k}, r_{\mathrm{h}})^2$$

and so $\sigma(\omega) \approx \dfrac{Q^2\tau}{(\epsilon+P)(1-\mathrm{i}\omega\tau)}$.

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- step 2: reduction of order method: proved

$$\psi(\mathbf{k}, r_{\mathrm{h}})^2 \to \lim_{\omega \to 0} \frac{\mathrm{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\mathbf{k}, \omega)\right)}{\omega}.$$

which directly implies $\tau_{\mathrm{AdS/CFT}} = \tau_{\mathrm{mem.\ matrix}}$

[Lucas, *Journal of High Energy Physics* **03** 071 (**2015**)]

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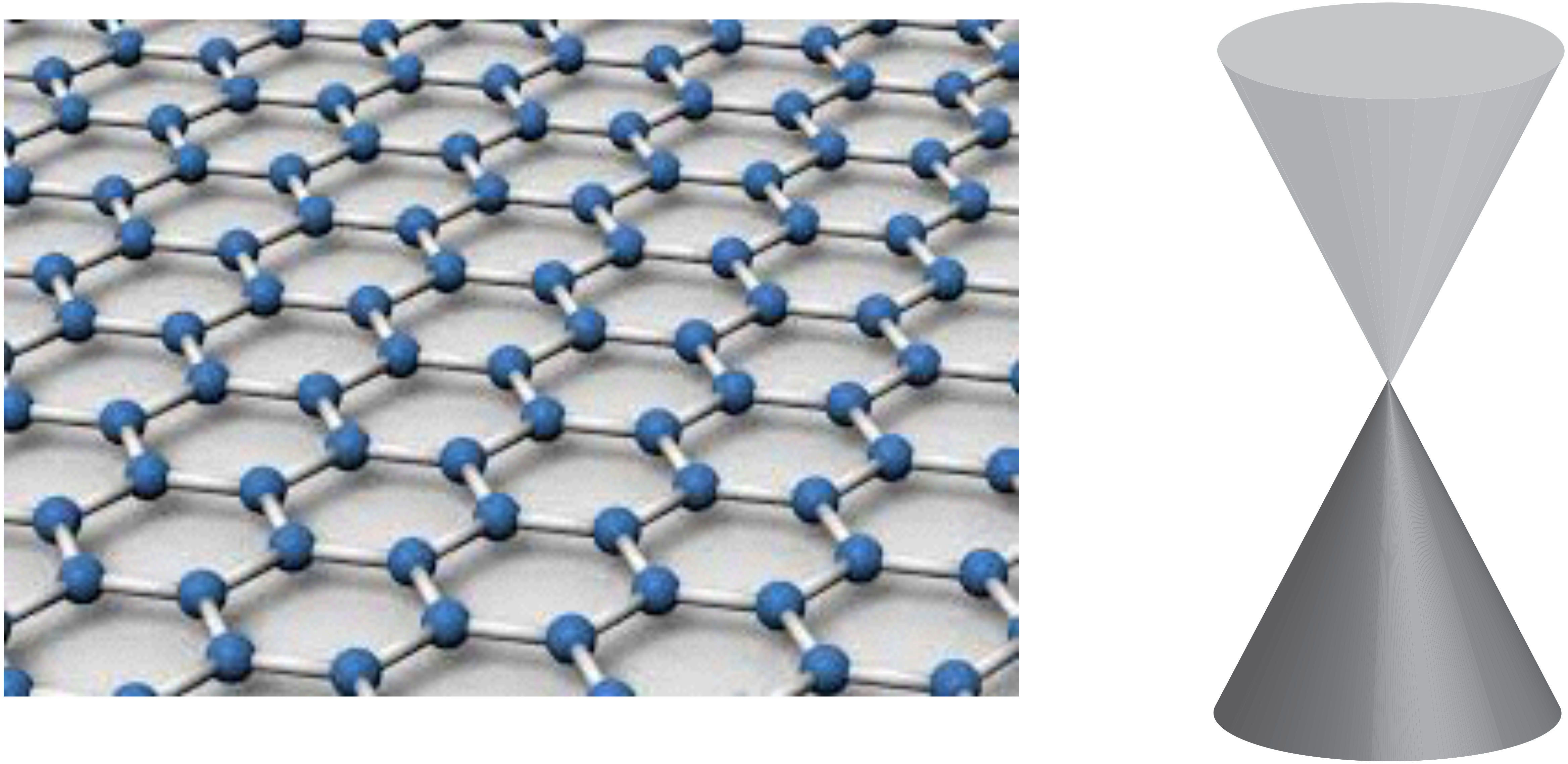
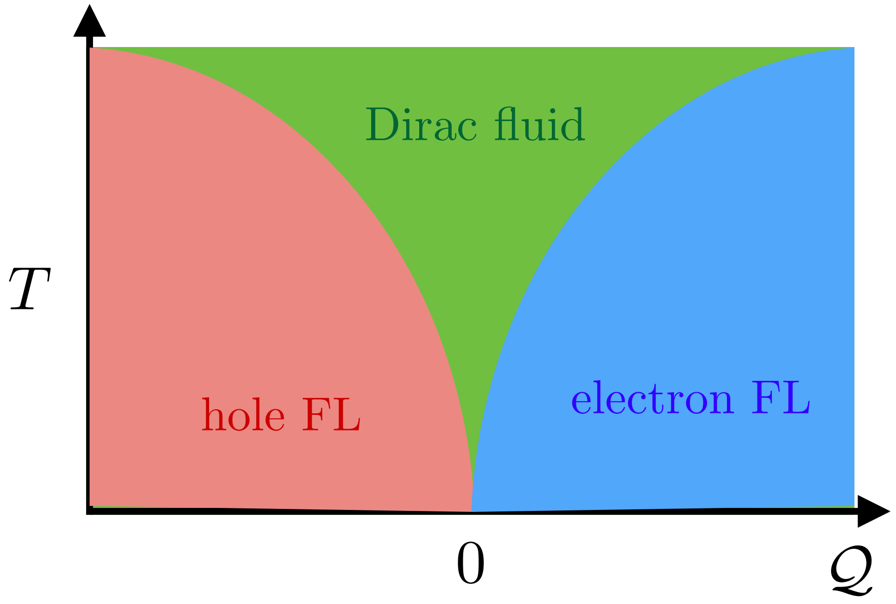


The Dirac Fluid

$$\epsilon_{a\sigma} = \hbar v_{\rm F} k$$

$$+$$

$$V_{\rm int} = \frac{\alpha_{\rm eff}}{r}$$



- marginally irrelevant $1/r$ Coulomb interactions:

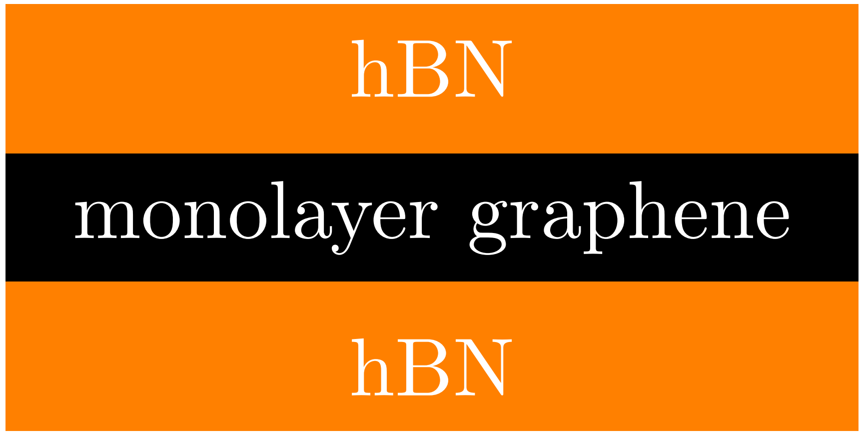
$$\alpha_{\rm eff} = \frac{\alpha_0}{1 + (\alpha_0/4)\log((10^5\ {\rm K})/T)}, \quad \alpha_0 \approx \frac{1}{137}\frac{c}{v_{\rm F}\epsilon_{\rm r}} \sim 0.5.$$

- thermo/hydro nearly that of relativistic theory
- $\alpha_{\rm eff} \sim 0.3$ at $T = 100$ K

e.g. [Sheehy, Schmalian, *Physical Review Letters* **99** 226803 (2007)]
[Müller, Fritz, Sachdev, *Physical Review* **B78** 115406 (2008)]

Graphene: an Ideal Experimental Platform

- fabricating ultra pure monolayer graphene:
  [Dean *et al*, *Nature Nanotechnology* **5** 722 (2010)]

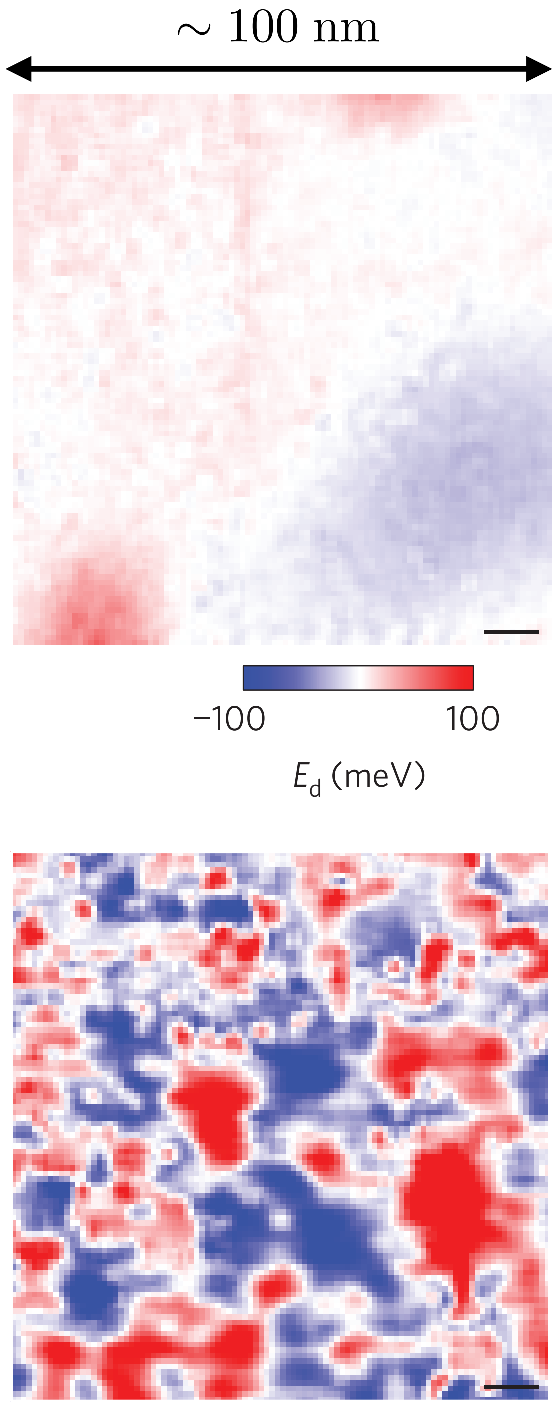


hBN

monolayer graphene

hBN

- weak disorder: charge puddles
  [Xue *et al*, *Nature Materials* **10** 282 (2011)]



hBN

$\Delta\mu \sim 30$ K

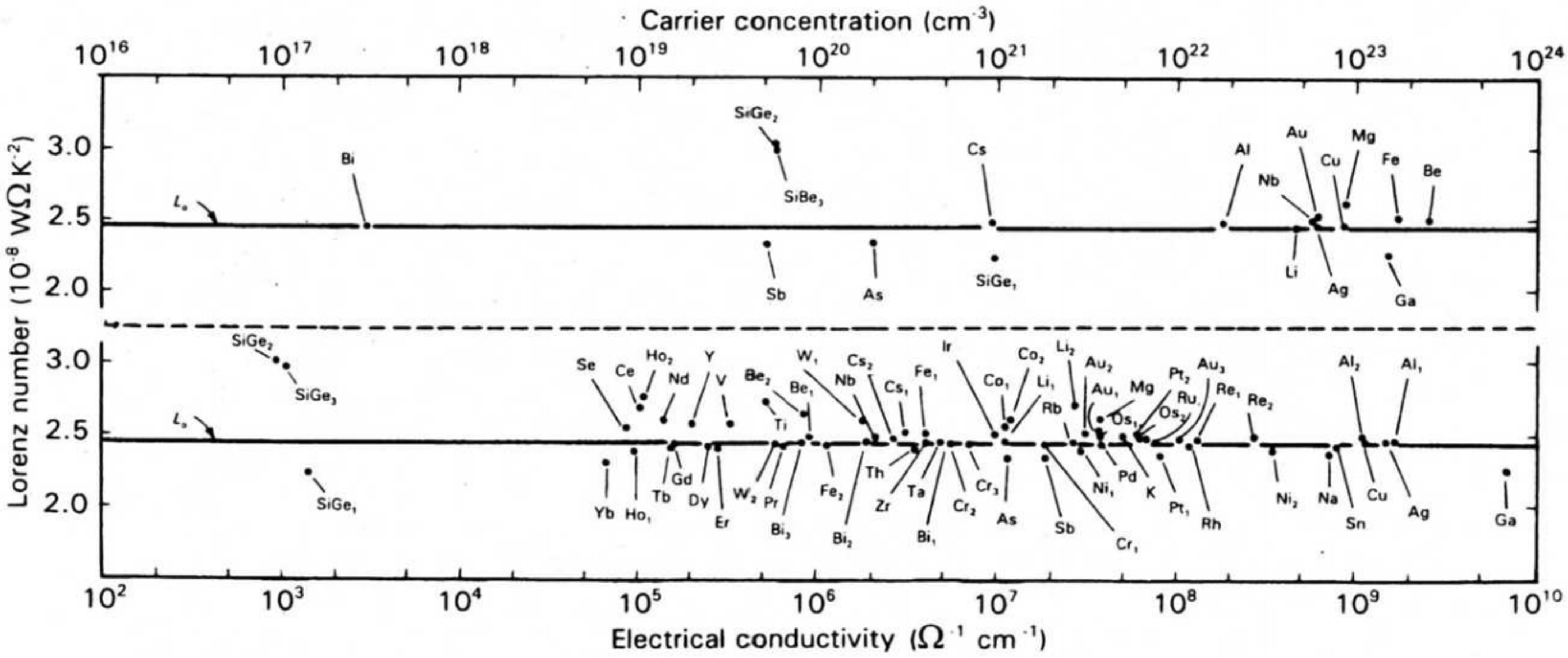
SiO_2

$\Delta\mu \sim 300$ K

Wiedemann-Franz Law

- Wiedemann-Franz law in a Fermi liquid:

$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$

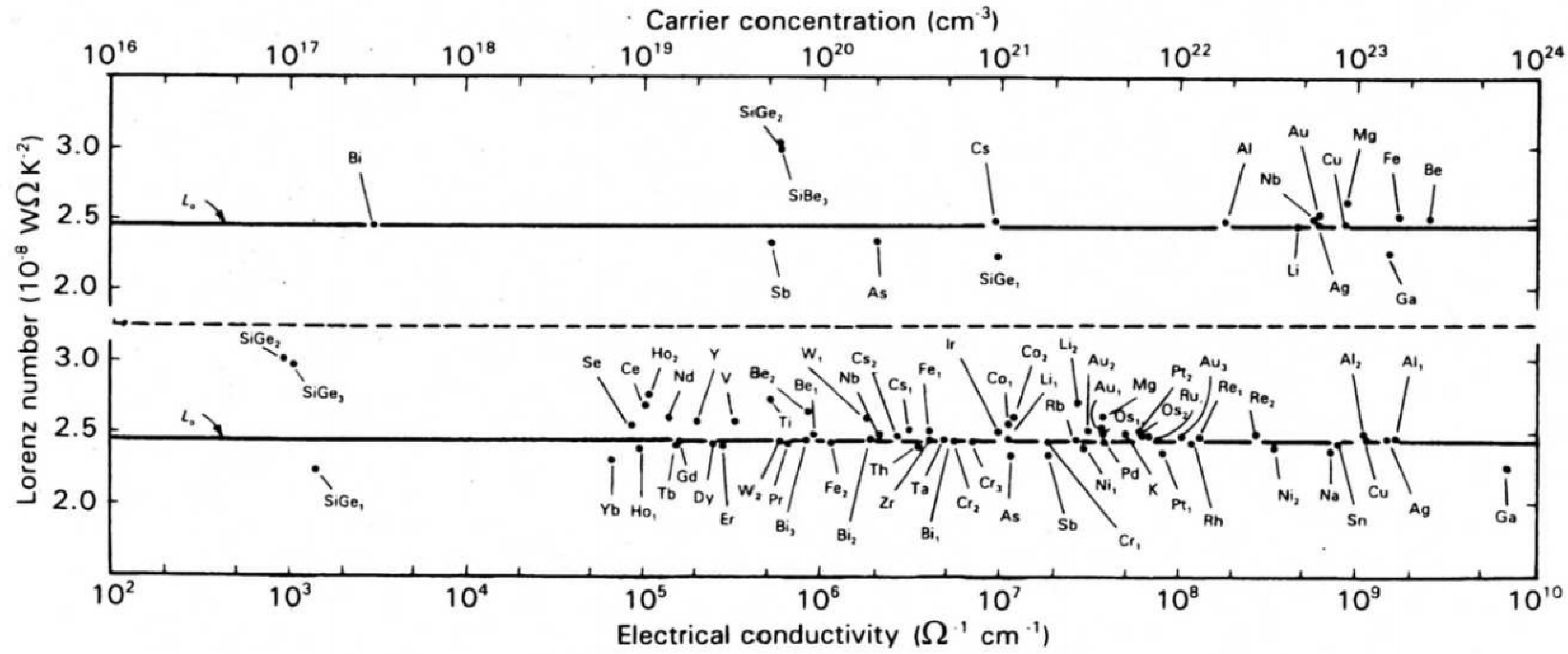


Dirac Fluid in Graphene

Wiedemann-Franz Law

- Wiedemann-Franz law in a Fermi liquid:

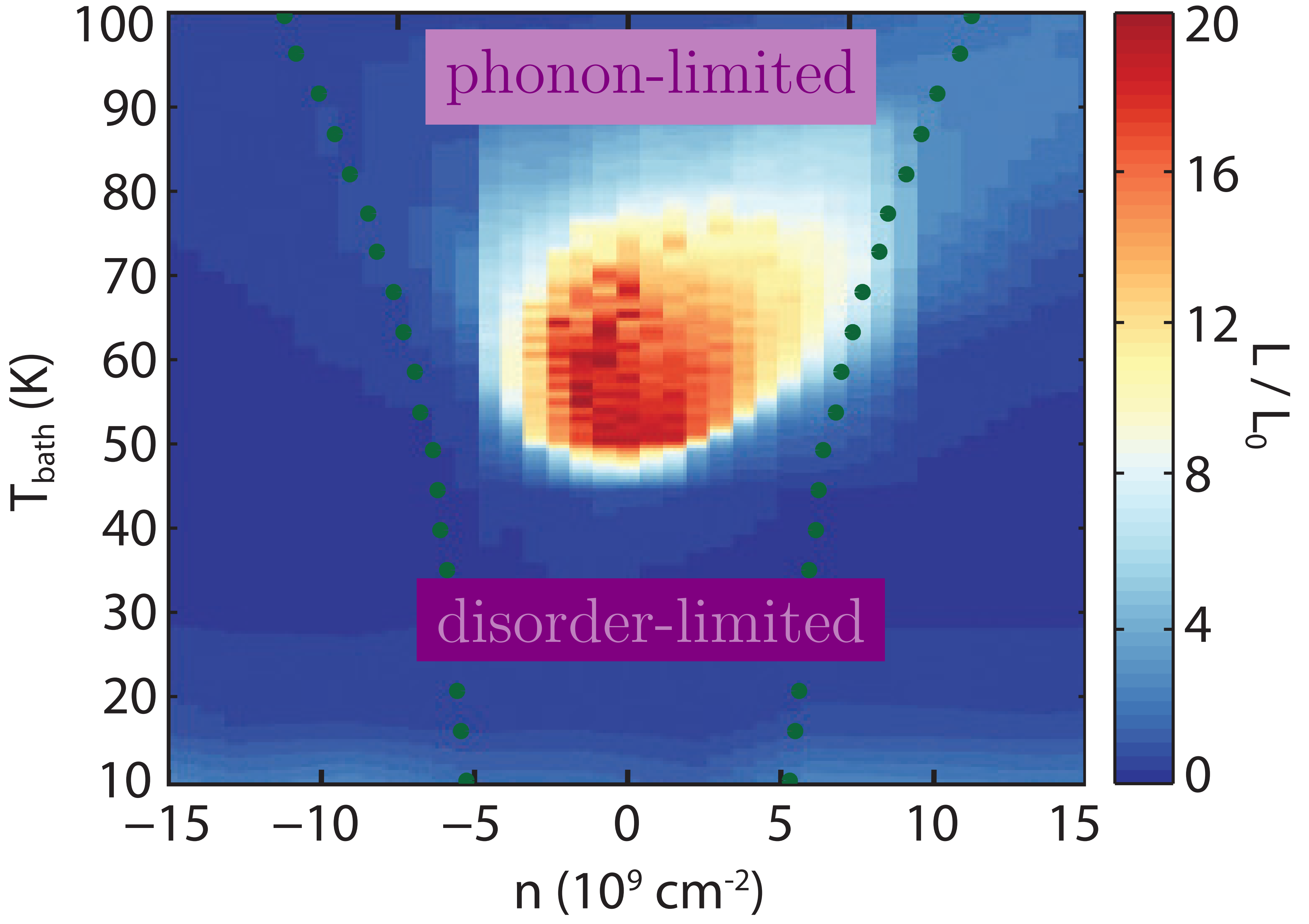
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_{\mathrm{B}}^2}{3e^2} \approx 2.45 \times 10^{-8}\,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



- in hydrodynamics one finds

$$\frac{\kappa}{\sigma T} = \frac{\mathcal{L}_{\mathrm{hydro}}}{\left(1+(\mathcal{Q}/\mathcal{Q}_0)^2\right)^2}, \qquad \mathcal{L}_{\mathrm{hydro}} \gg 1.$$

Wiedemann-Franz Law Violations in Experiment

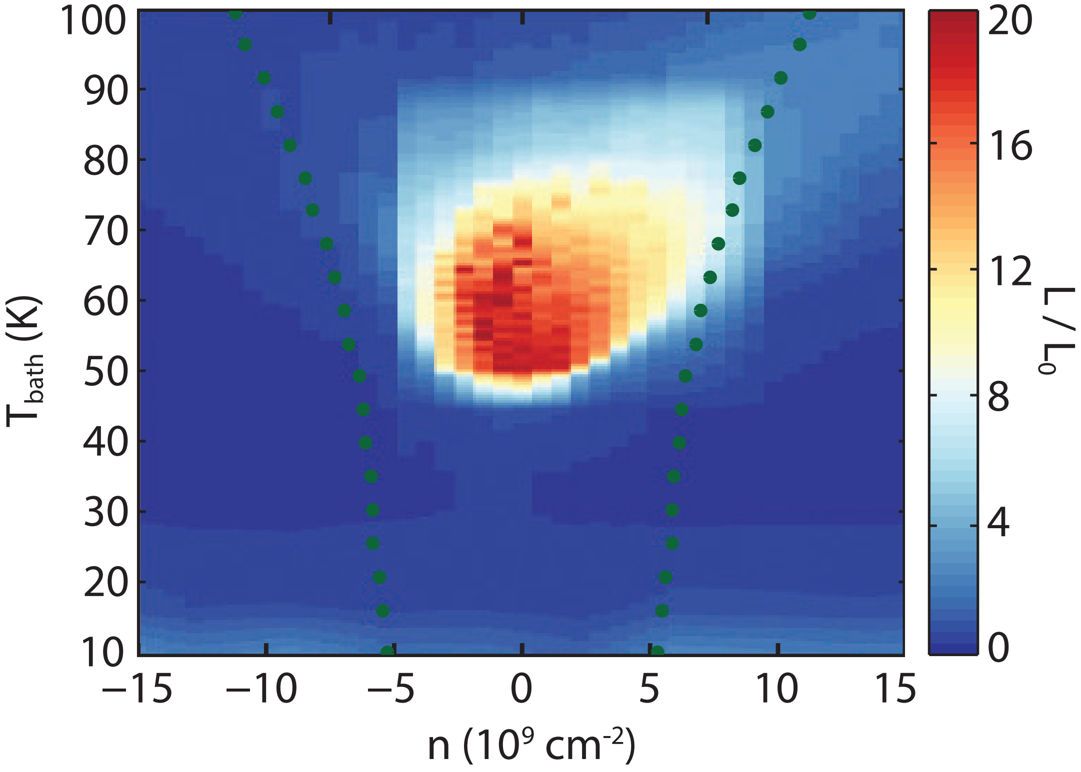


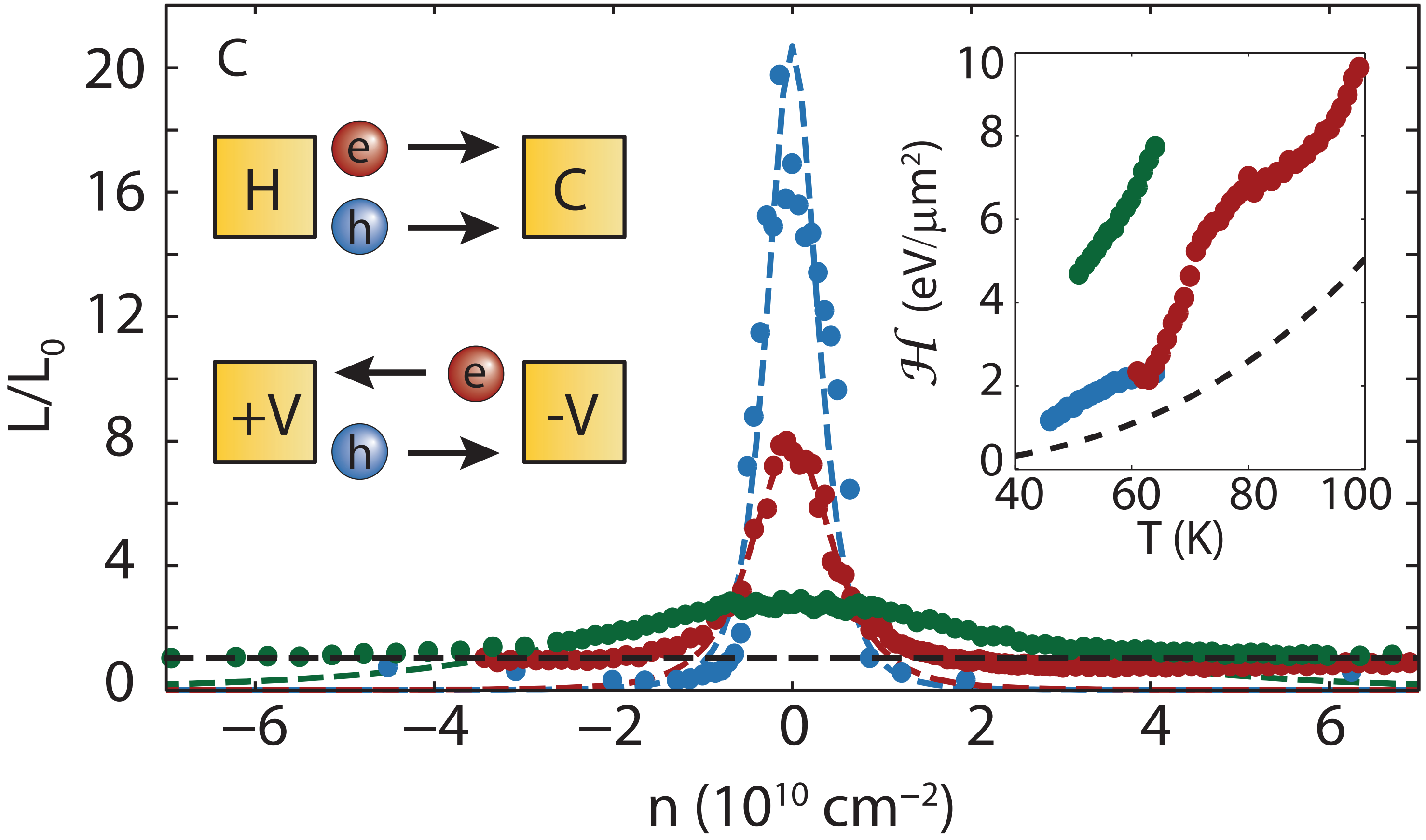
[Crossno *et al, submitted*]

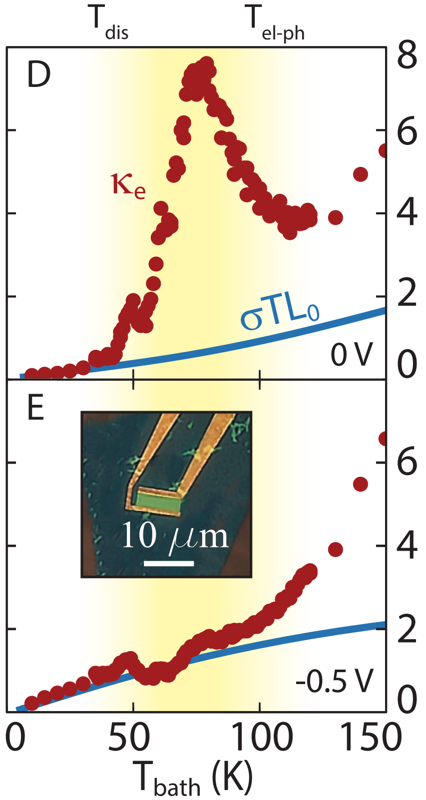
(submitted)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3]
Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]







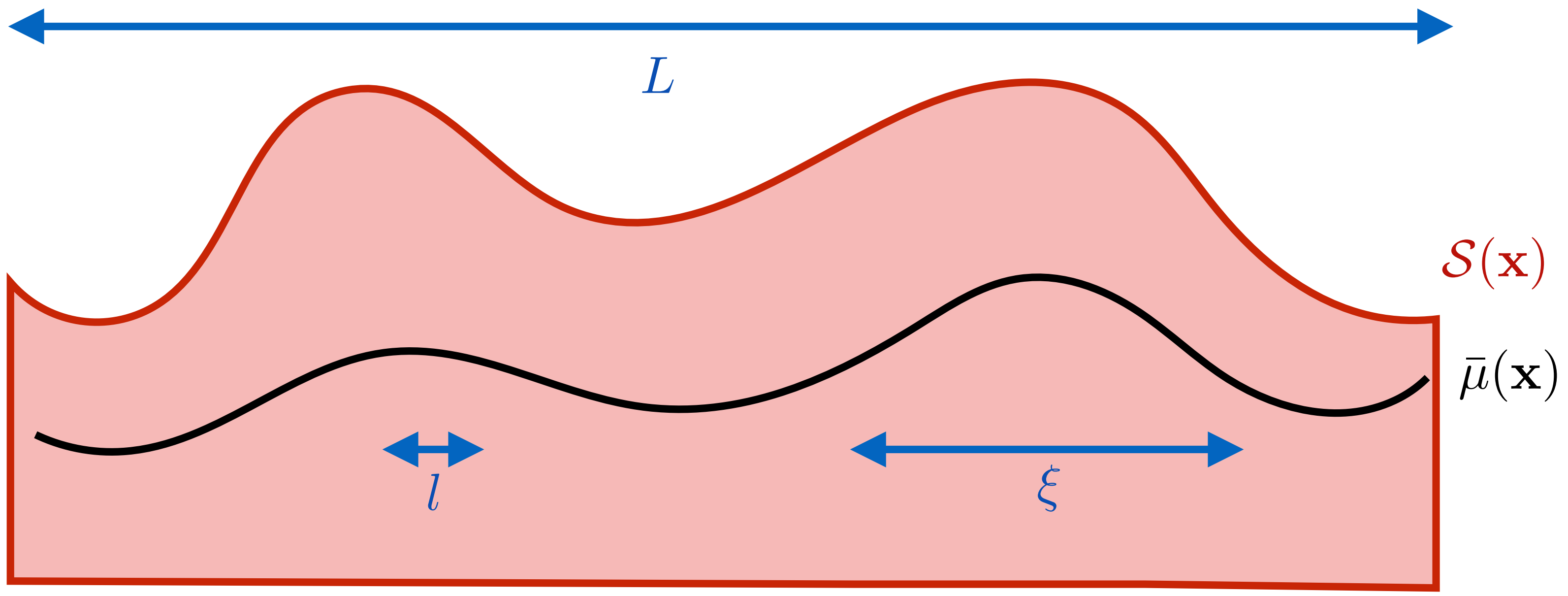
Thermal conductivity $\kappa = \bar{\kappa} - T\alpha^2/\sigma$

Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{\mathcal{H}\tau}{T^2\sigma_Q}\frac{1}{\left(1+n^2\tau/(\mathcal{H}\sigma_Q)\right)^2}$$

Beyond Perturbation Theory

Transport in a Disordered Fluid



- *non-perturbative* hydrodynamic limit: $\xi \gg l_{\mathrm{ee}}$
  [Lucas, `arXiv:1506.02662`]

$$\mathcal{Q}(\delta E_i - \partial_i \delta\mu) + \mathcal{S}(T\delta\zeta_i - \partial_i \delta T) = \partial_j \left(\eta_{ijkl}\partial_l \delta v_k\right), \text{ etc.}$$

$$\mathbb{E}\left[\delta J_i\right] = \sigma_{ij}\delta E_j + T\alpha_{ij}\delta\zeta_j, \text{ etc.}$$

This non-perturbative approach yields satisfactory quantitative agreement with experiments