Progress on the physics of the underdoped cuprates

International Institute of Physics
Natal, Brazil
April 11, 2013

Subir Sachdev

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Max Metlitski

Erez Berg

Rolando La Placa

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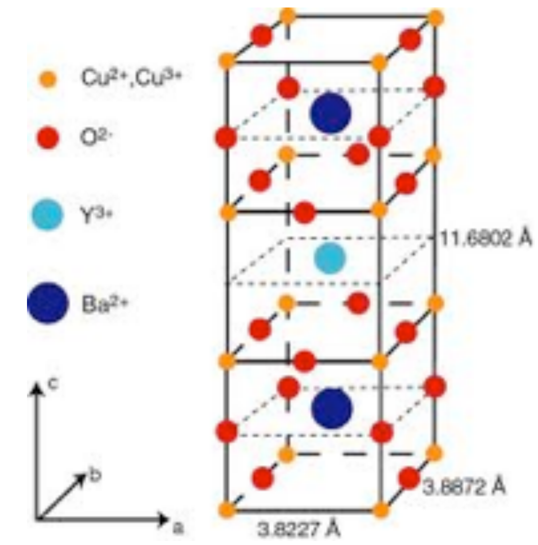
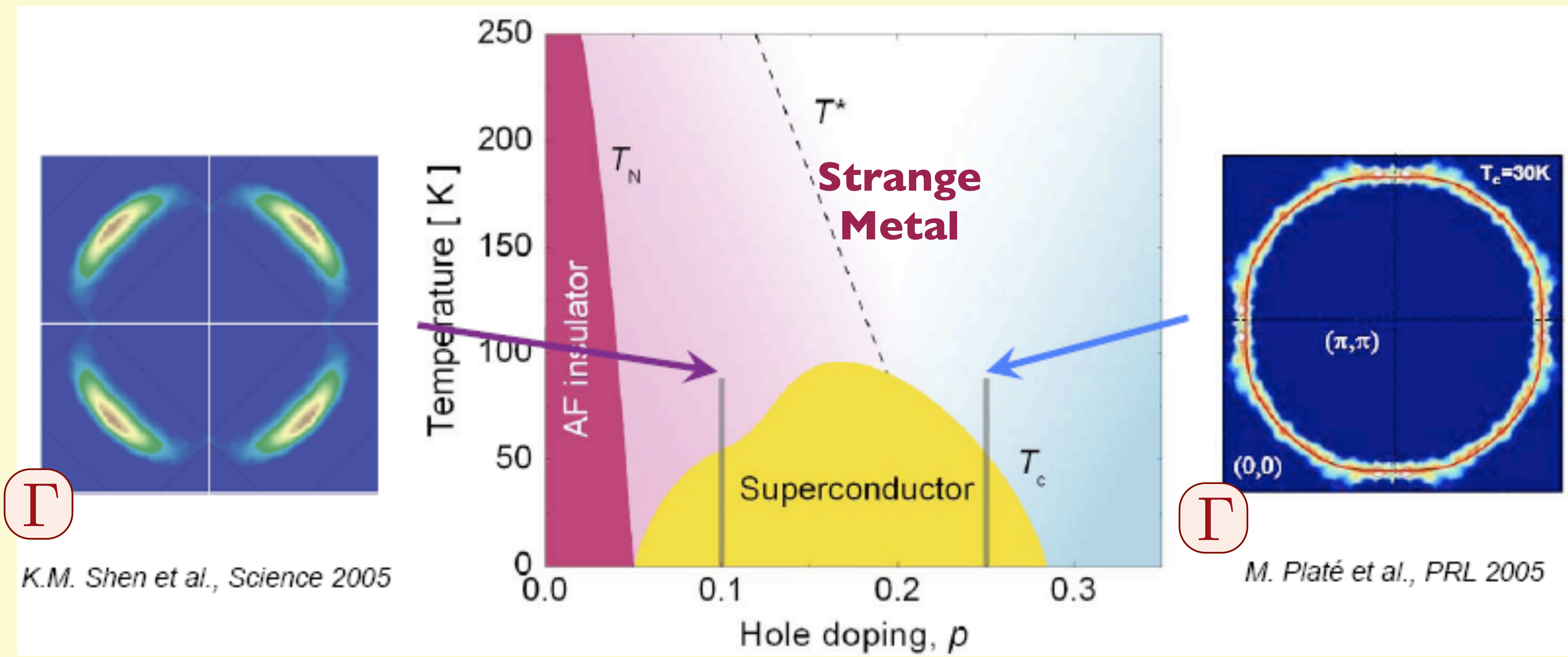




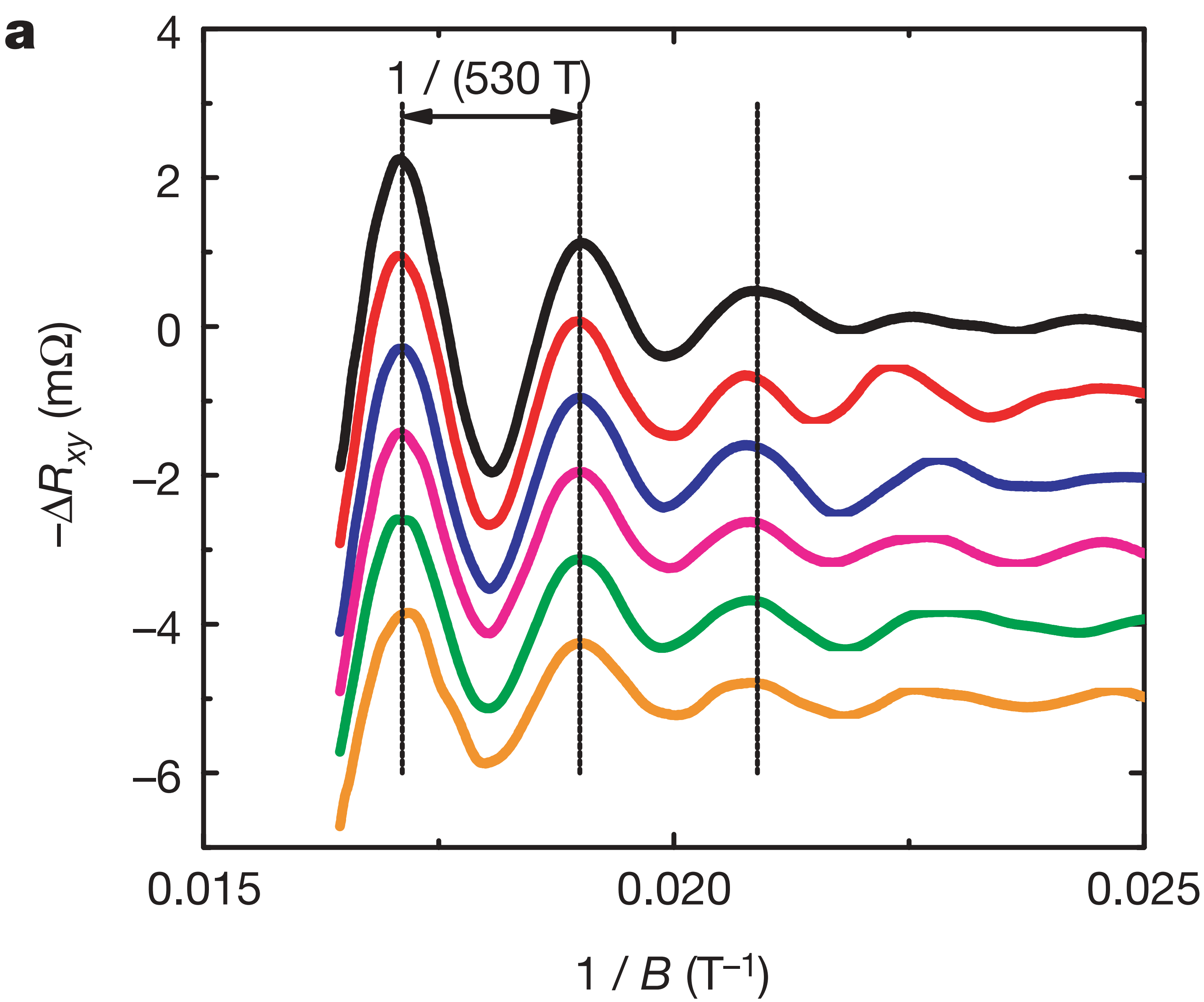
Smaller hole Fermi-pockets

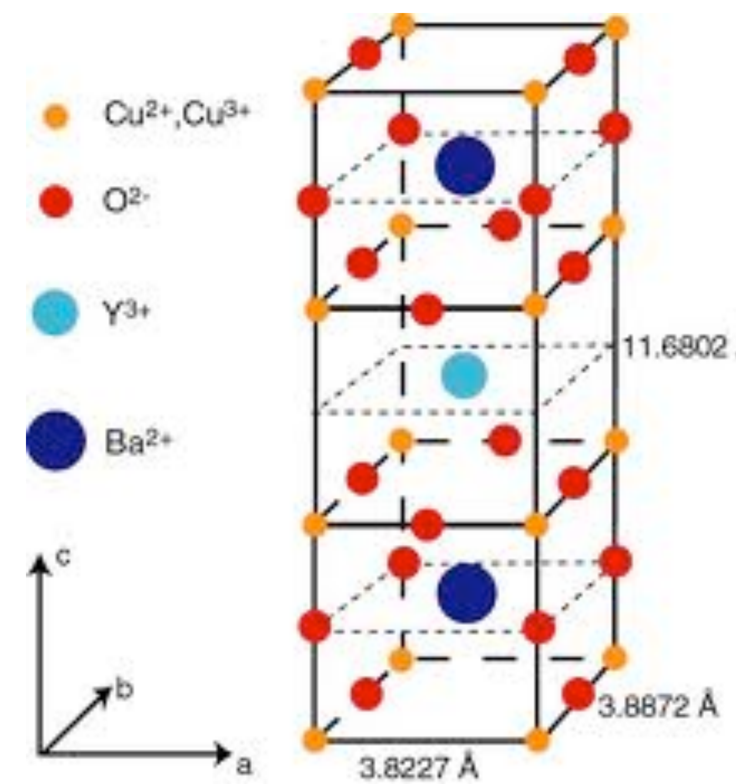
Large hole Fermi surface

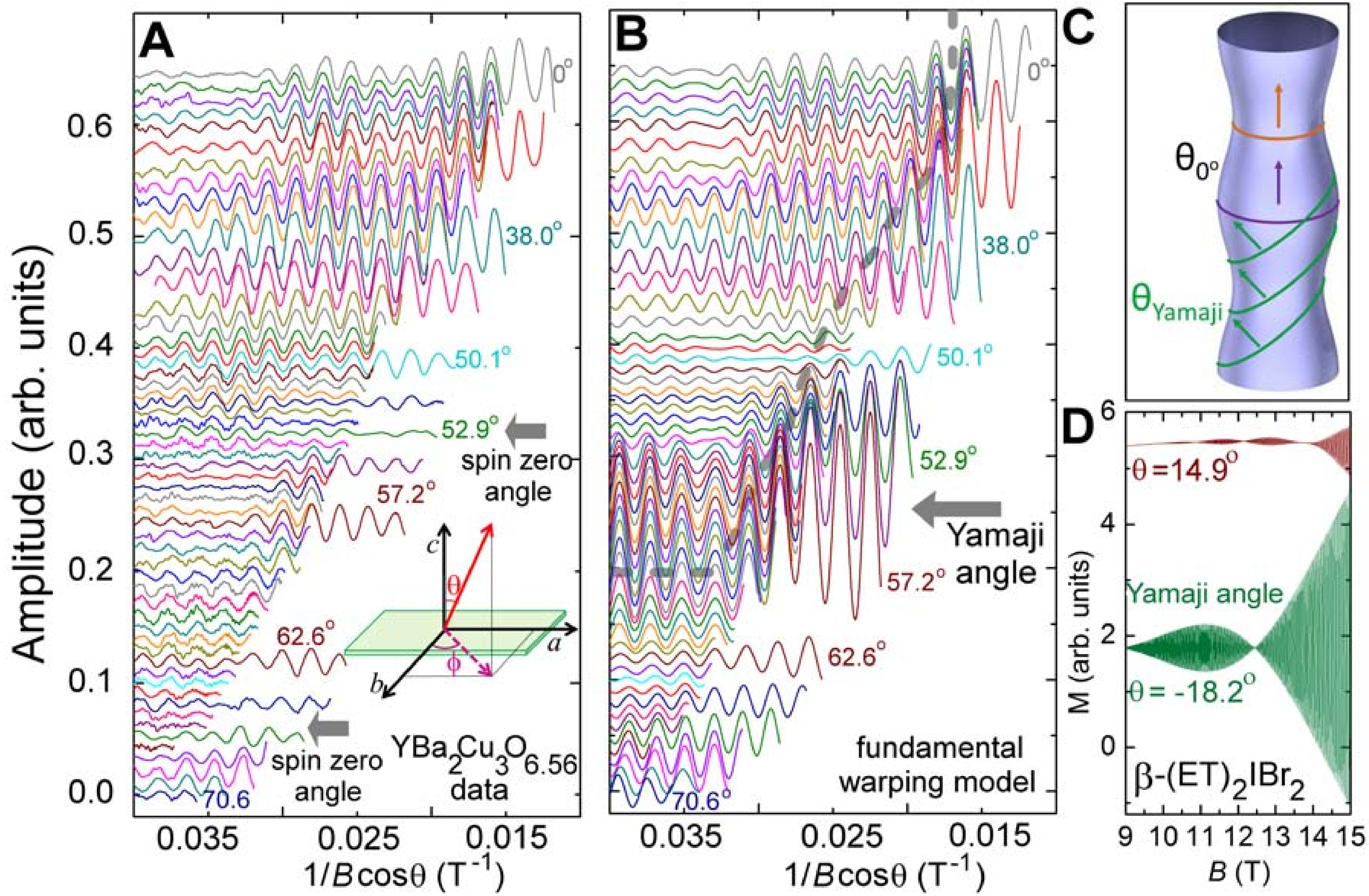
Quantum oscillations and the Fermi surface in an underdoped high-T_c superconductor

Nicolas Doiron-Leyraud[1], Cyril Proust[2], David LeBoeuf[1], Julien Levallois[2], Jean-Baptiste Bonnemaison[1], Ruixing Liang[3,4], D. A. Bonn[3,4], W. N. Hardy[3,4] & Louis Taillefer[1,4]

Nature **447**, 565 (2007)







Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

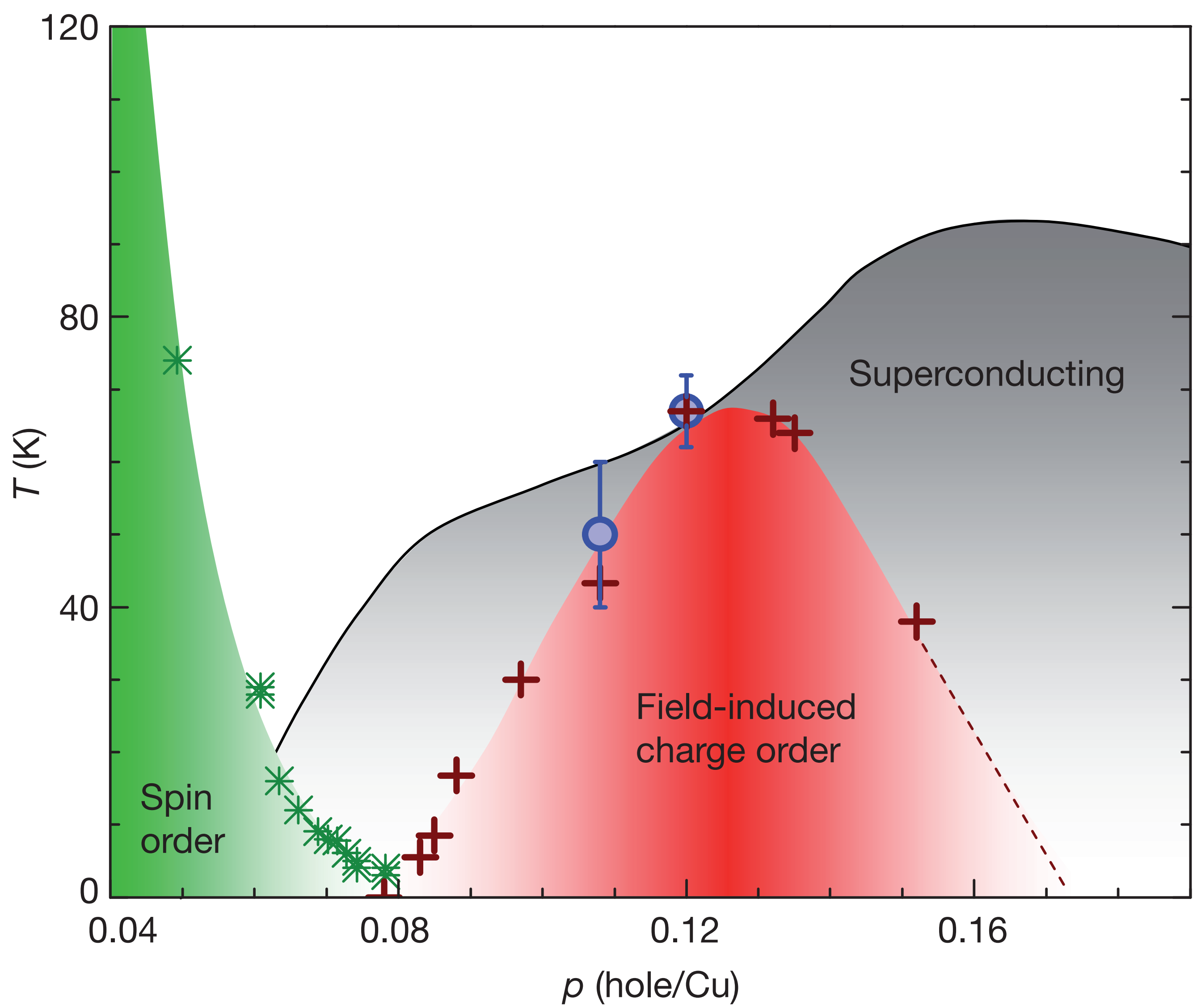
APS March meeting 2013 B2.00004

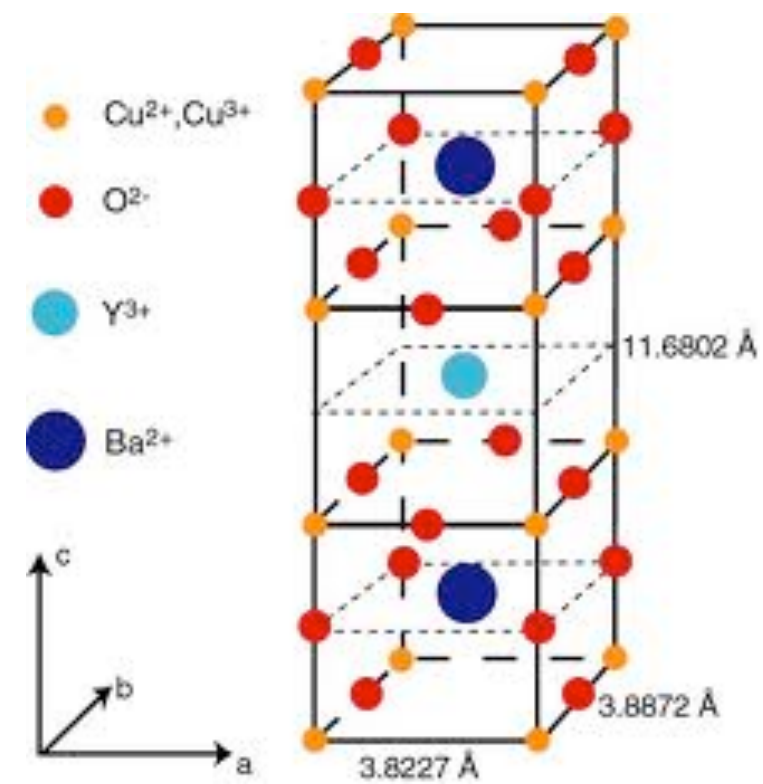
Suchitra E. Sebastian,[1*] N. Harrison,[2] F. F. Balakirev,[2] M. M. Altarawneh,[2,3]
Ruixing Liang,[4,5] D. A. Bonn,[4,5] W. N. Hardy,[4,5] G. G. Lonzarich,[1]

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191

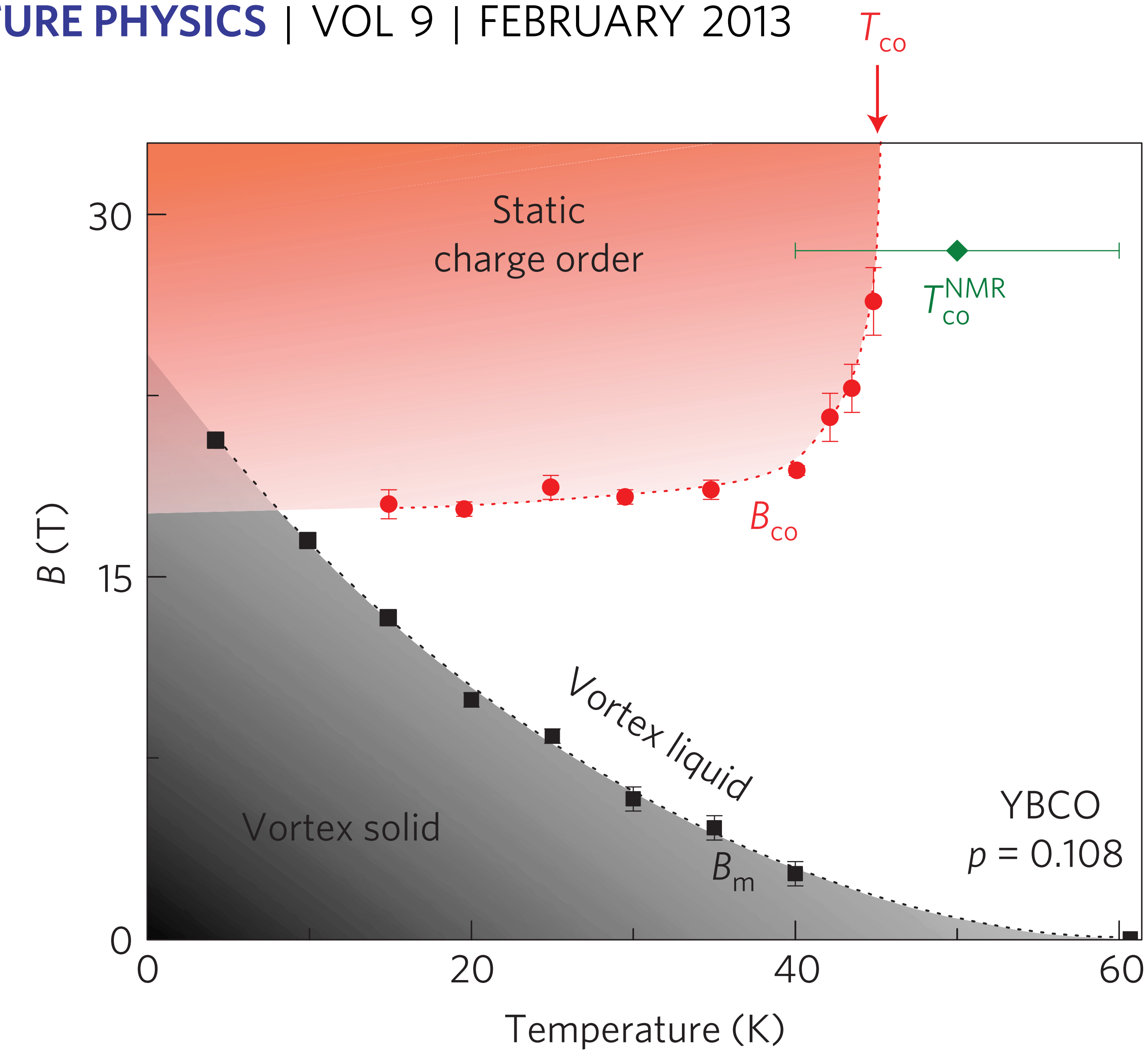




Thermodynamic phase diagram of static charge order in underdoped $YBa_2Cu_3O_y$

David LeBoeuf[1]*, S. Krämer[2], W. N. Hardy[3,4], Ruixing Liang[3,4], D. A. Bonn[3,4] and Cyril Proust[1,4]*

NATURE PHYSICS | VOL 9 | FEBRUARY 2013

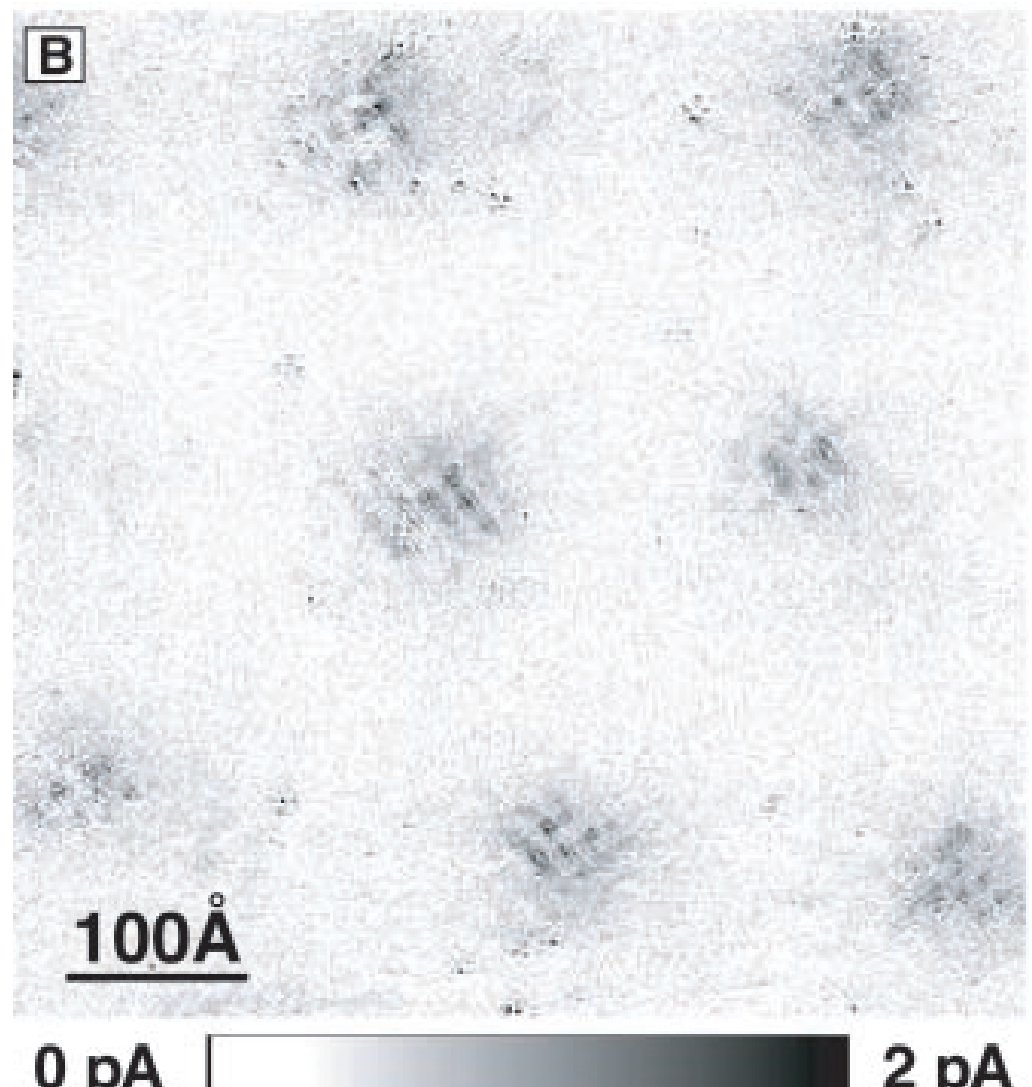


The comparison of different acoustic modes indicates that the charge modulation is biaxial, which differs from a uniaxial stripe charge order.

A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman,[1] E. W. Hudson,[1,2*] K. M. Lang,[1] V. Madhavan,[1]
H. Eisaki,[3†] S. Uchida,[3] J. C. Davis[1,2‡]

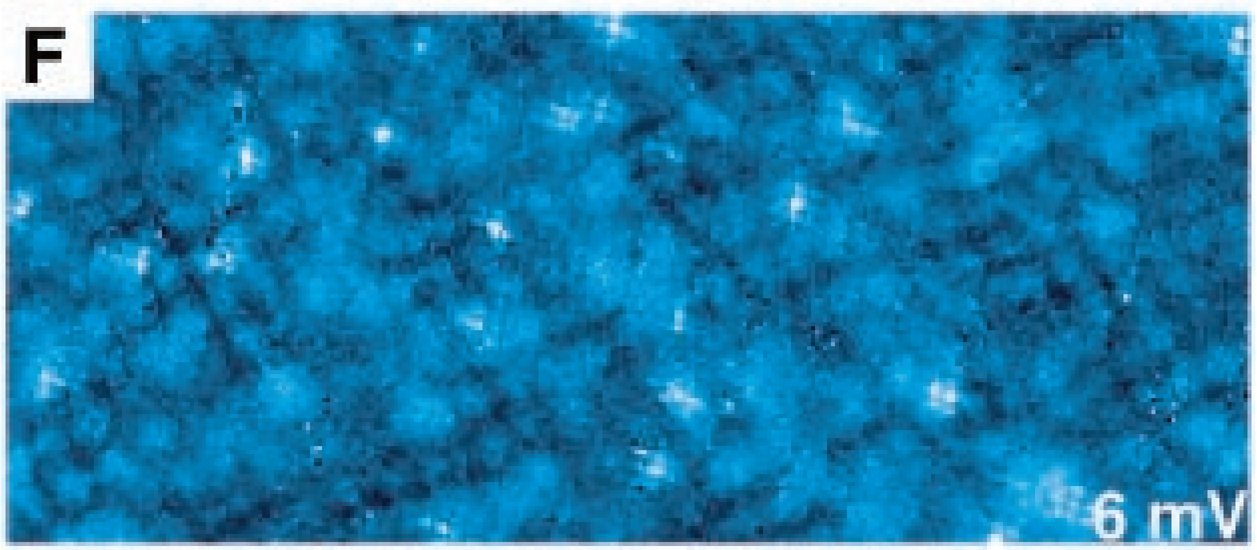
SCIENCE VOL 295 18 JANUARY 2002

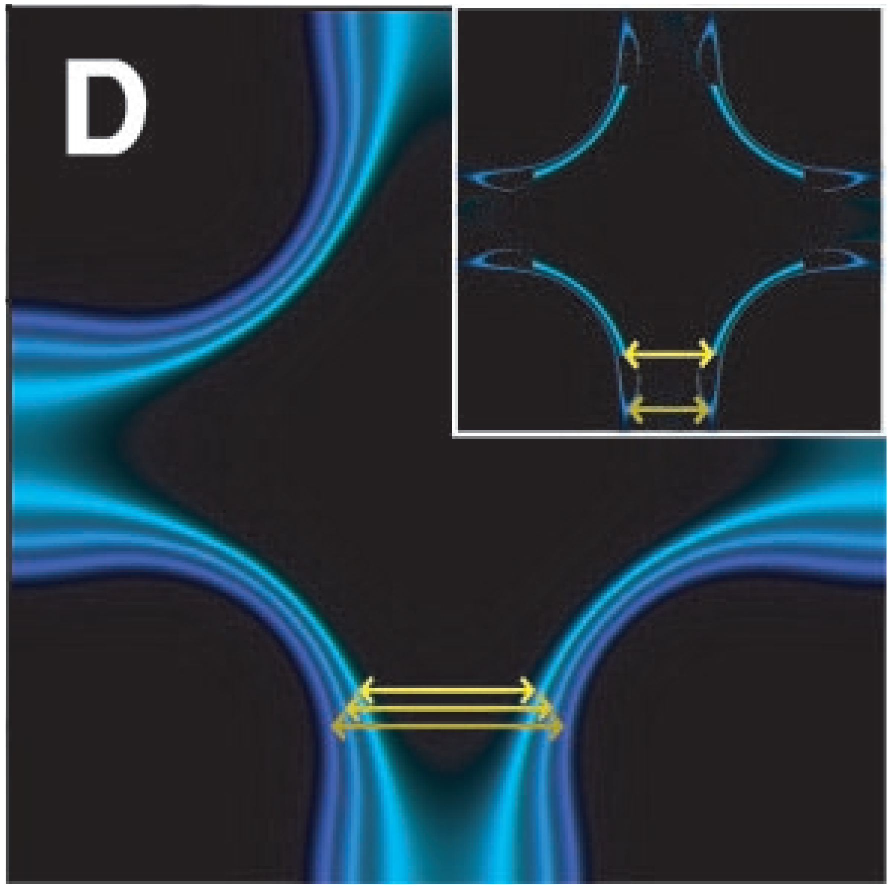


Local Ordering in the Pseudogap State of the High-T_c Superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$

Michael Vershinin,[1]* Shashank Misra,[1]* S. Ono,[2] Y. Abe,[2]† Yoichi Ando,[2] Ali Yazdani[1]‡§

SCIENCE VOL 303 26 MARCH 2004

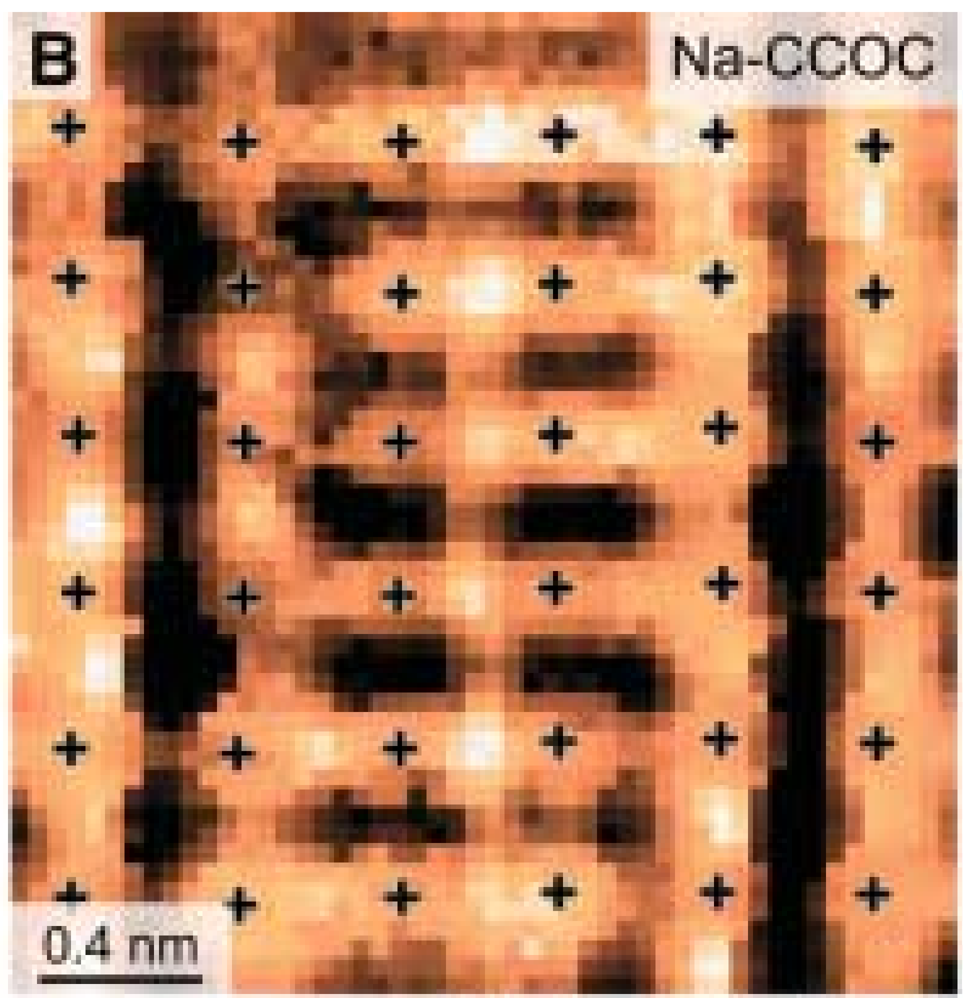




An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,[1] C. Taylor,[1] K. Fujita,[1,2] A. Schmidt,[1] C. Lupien,[3] T. Hanaguri,[4] M. Azuma,[5] M. Takano,[5] H. Eisaki,[6] H. Takagi,[2,4] S. Uchida,[2,7] J. C. Davis[1,8]*

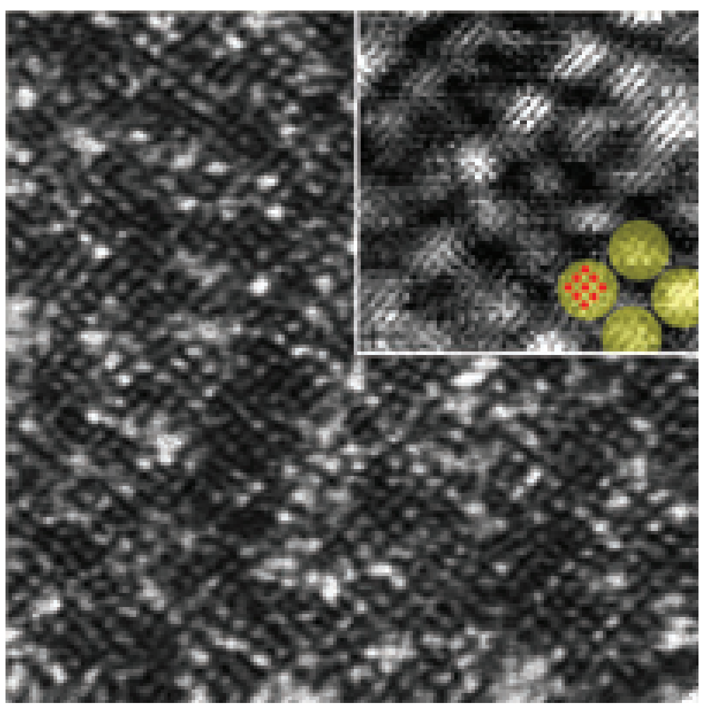
9 MARCH 2007 VOL 315 SCIENCE

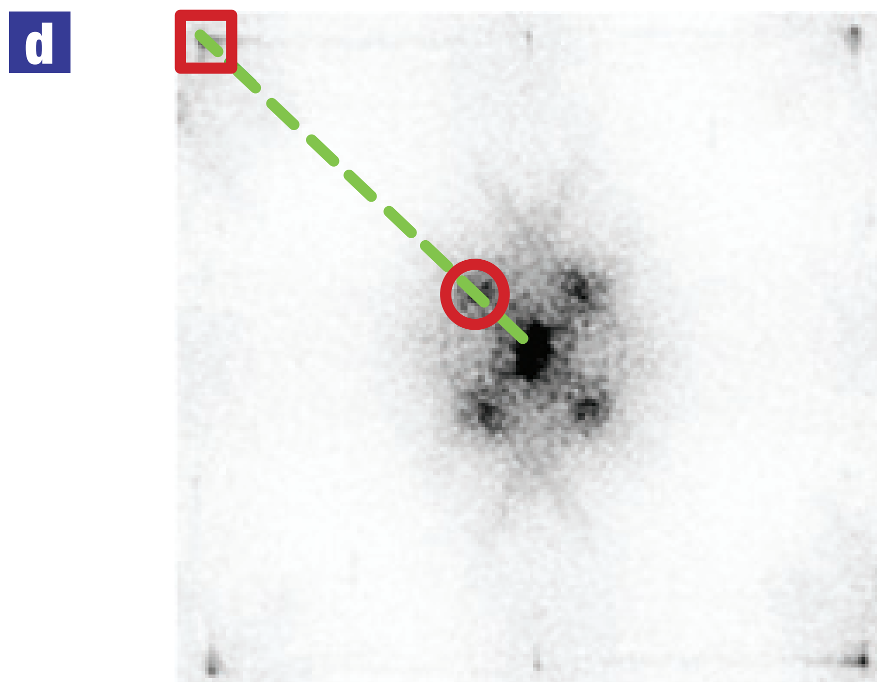


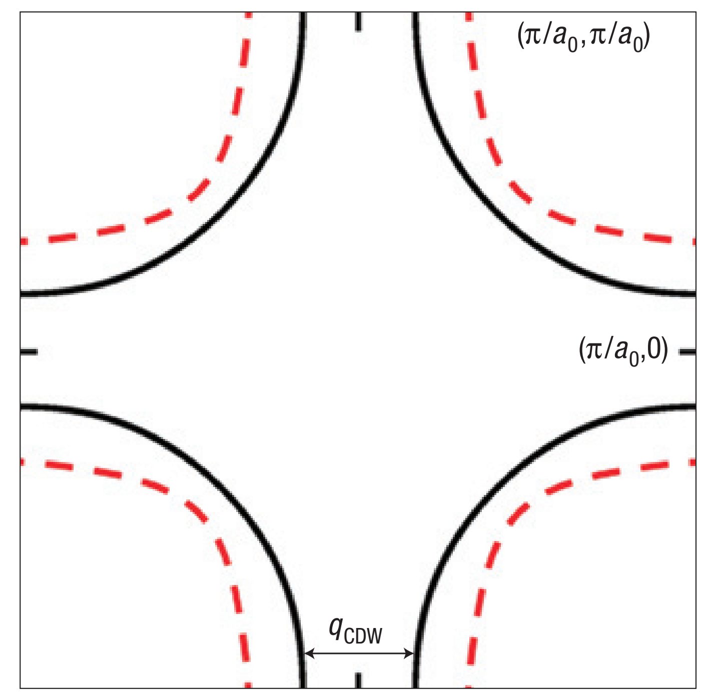
Charge-density-wave origin of cuprate checkerboard visualized by scanning tunnelling microscopy

W. D. WISE[1], M. C. BOYER[1], KAMALESH CHATTERJEE[1], TAKESHI KONDO[1,2*], T. TAKEUCHI[2,3], H. IKUTA[2], YAYU WANG[1*] AND E. W. HUDSON[1†]

nature physics | VOL 4 | SEPTEMBER 2008 |







Long-Range Incommensurate Charge Fluctuations in $(Y,Nd)Ba_2Cu_3O_{6+x}$

G. Ghiringhelli,[1*] M. Le Tacon,[2] M. Minola,[1] S. Blanco-Canosa,[2] C. Mazzoli,[1] N. B. Brookes,[3] G. M. De Luca,[4] A. Frano,[2,5] D. G. Hawthorn,[6] F. He,[7] T. Loew,[2] M. Moretti Sala,[3] D. C. Peets,[2] M. Salluzzo,[4] E. Schierle,[5] R. Sutarto,[7,8] G. A. Sawatzky,[8] E. Weschke,[5] B. Keimer,[2*] L. Braicovich[1]

SCIENCE VOL 337 17 AUGUST 2012

resonant soft x-ray scattering

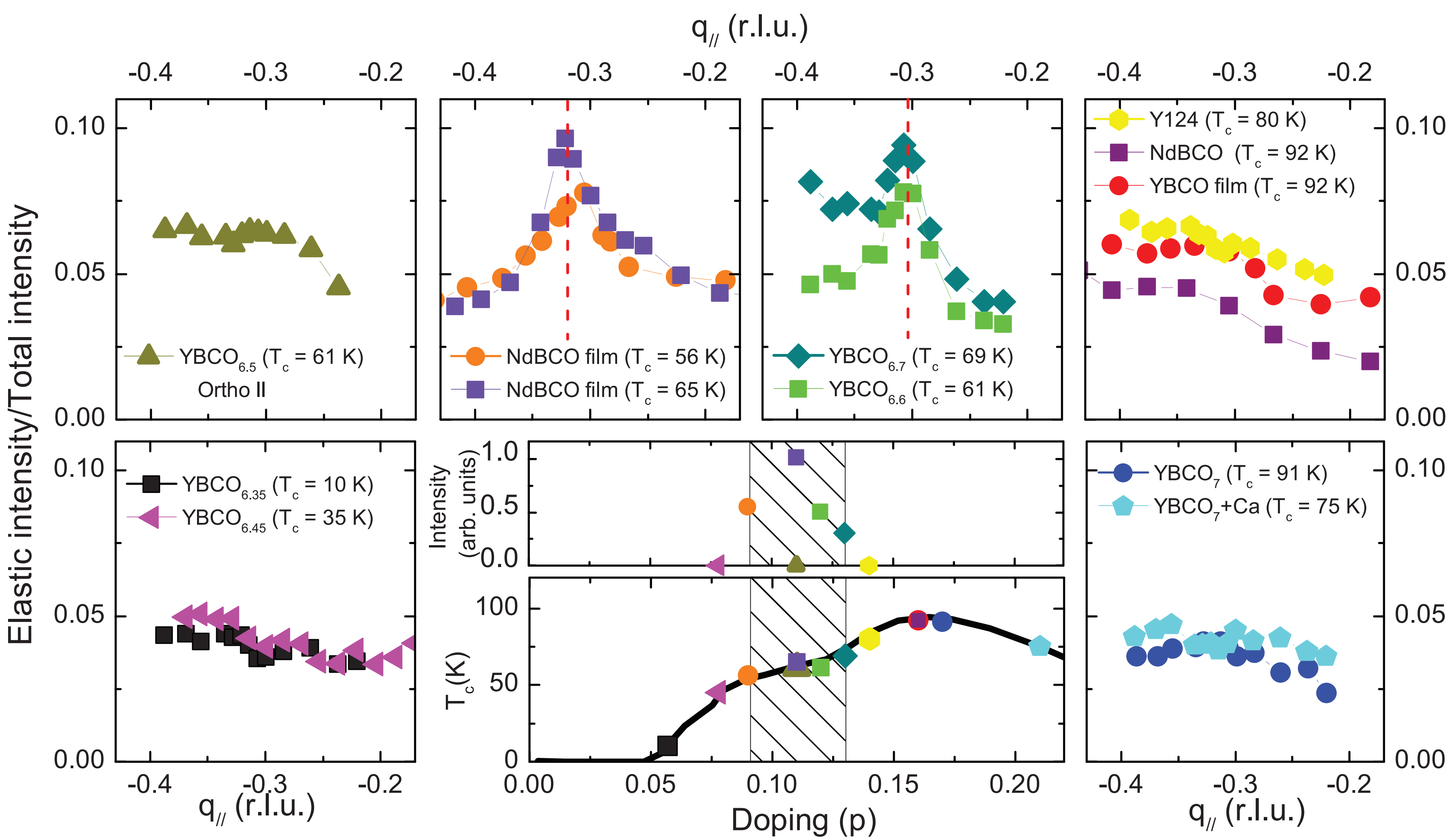
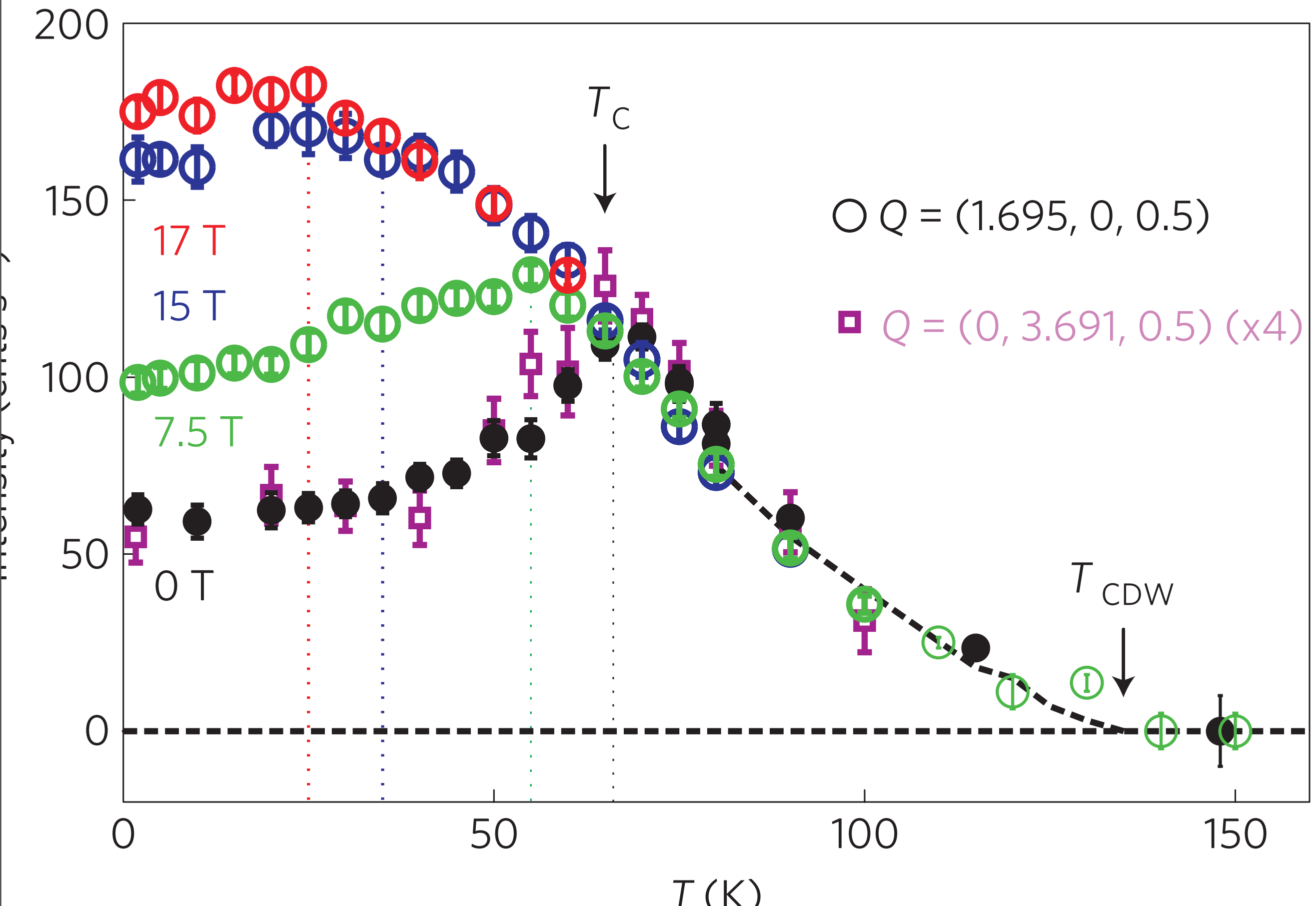


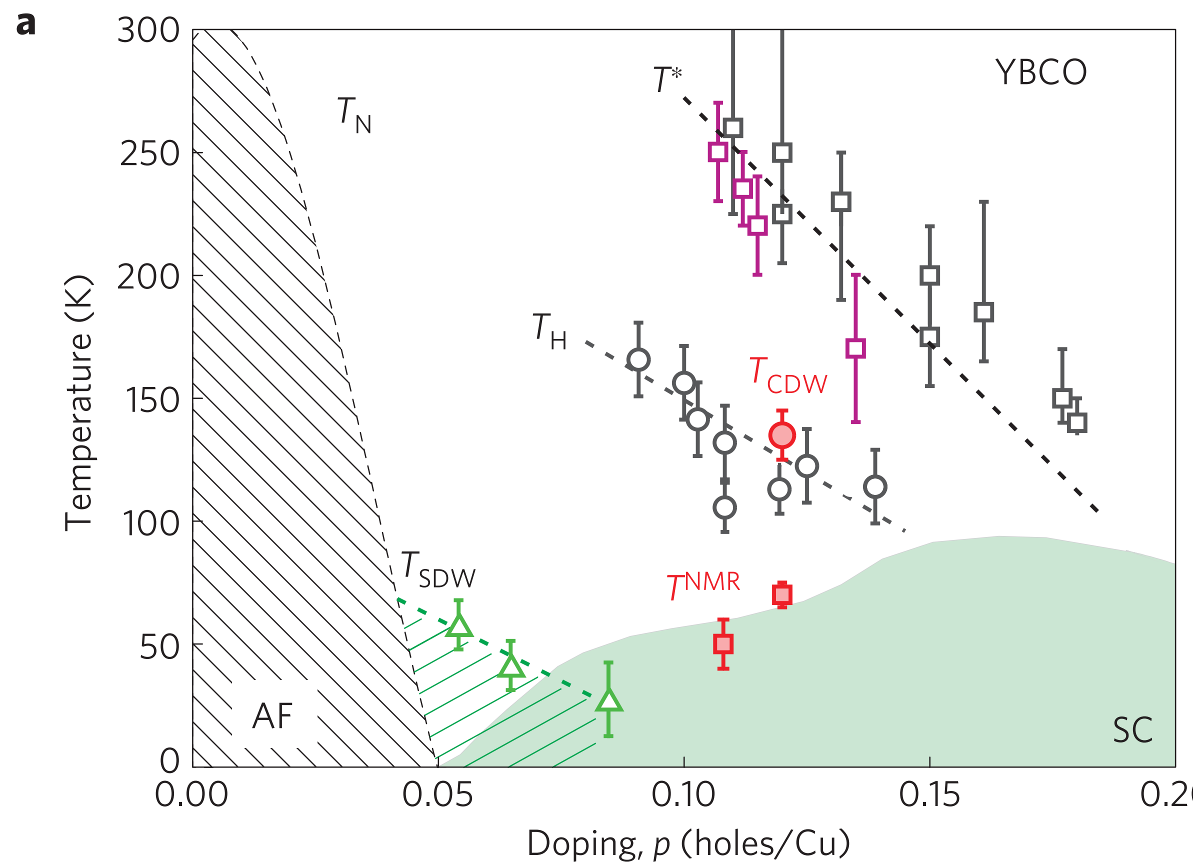
Fig. 3. Dependence of the CDW signal at 15 K on the hole doping level p. The CDW signal is present in several $YBa_2Cu_3O_{6+x}$ and $Nd_{1+y}Ba_{2-y}Cu_3O_7$ samples, but only for $0.09 \leq p \leq 0.13$. In this doping range (shaded in the central panel), the T_c-versus-p relation exhibits a plateau. The CDW peak position does not change with p outside of the experimental error, but its intensity is maximum at $p \simeq 0.11$.

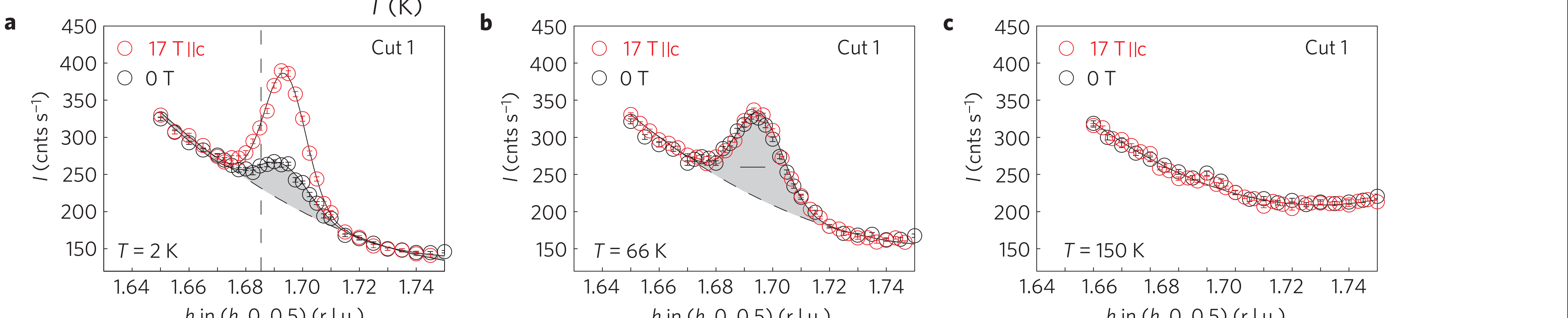
NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]



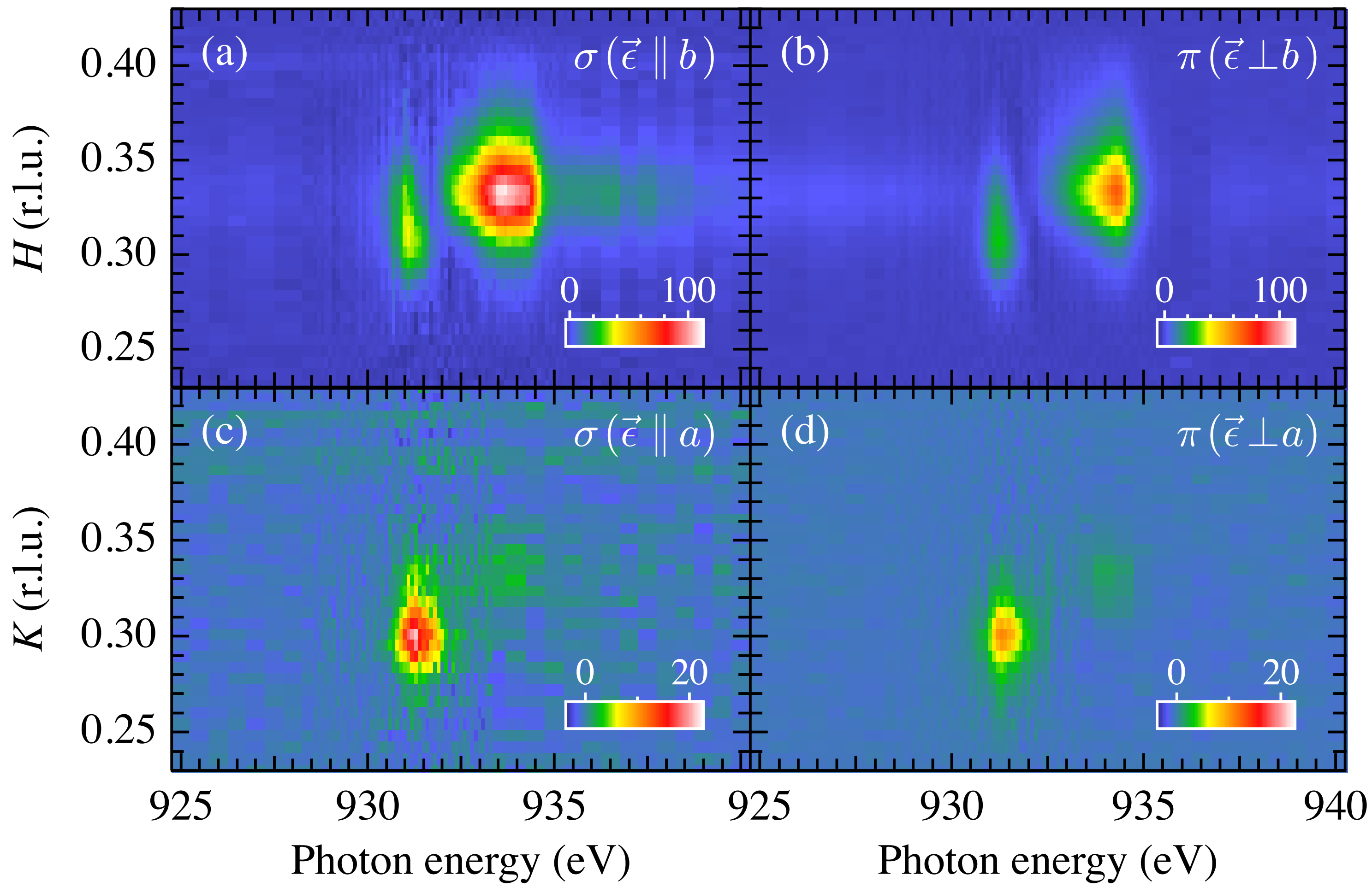




Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

PRL **109,** 167001 (2012)

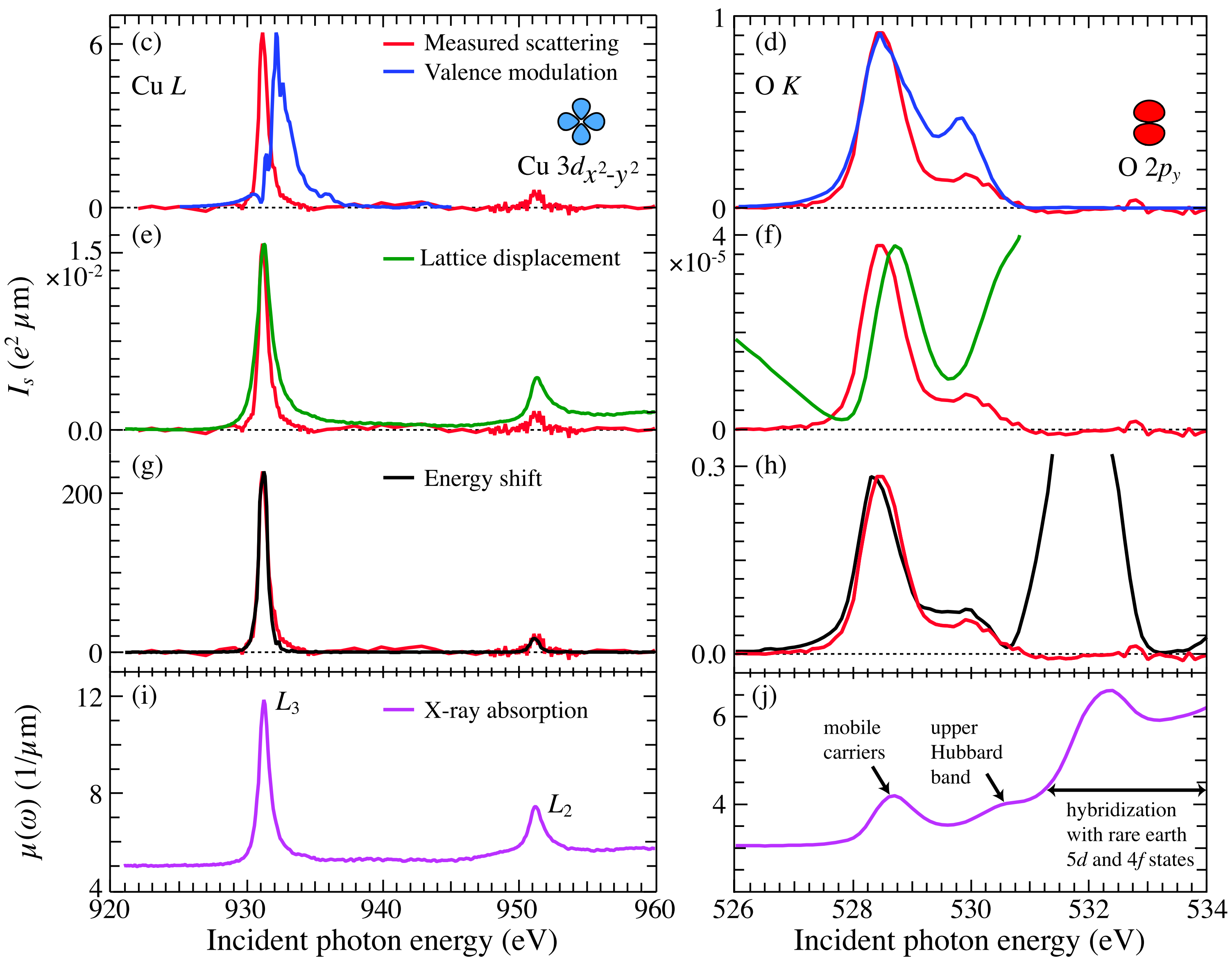


Moreover, the energy dependence of the CDW order in the planes is shown to result from a spatial modulation of energies of the Cu $2p$ to $3d_{x^2-y^2}$ transition, similar to stripe-ordered 214 cuprates.

Resonant X-Ray Scattering Measurements of a Spatial Modulation of the Cu 3*d* and O 2*p* Energies in Stripe-Ordered Cuprate Superconductors

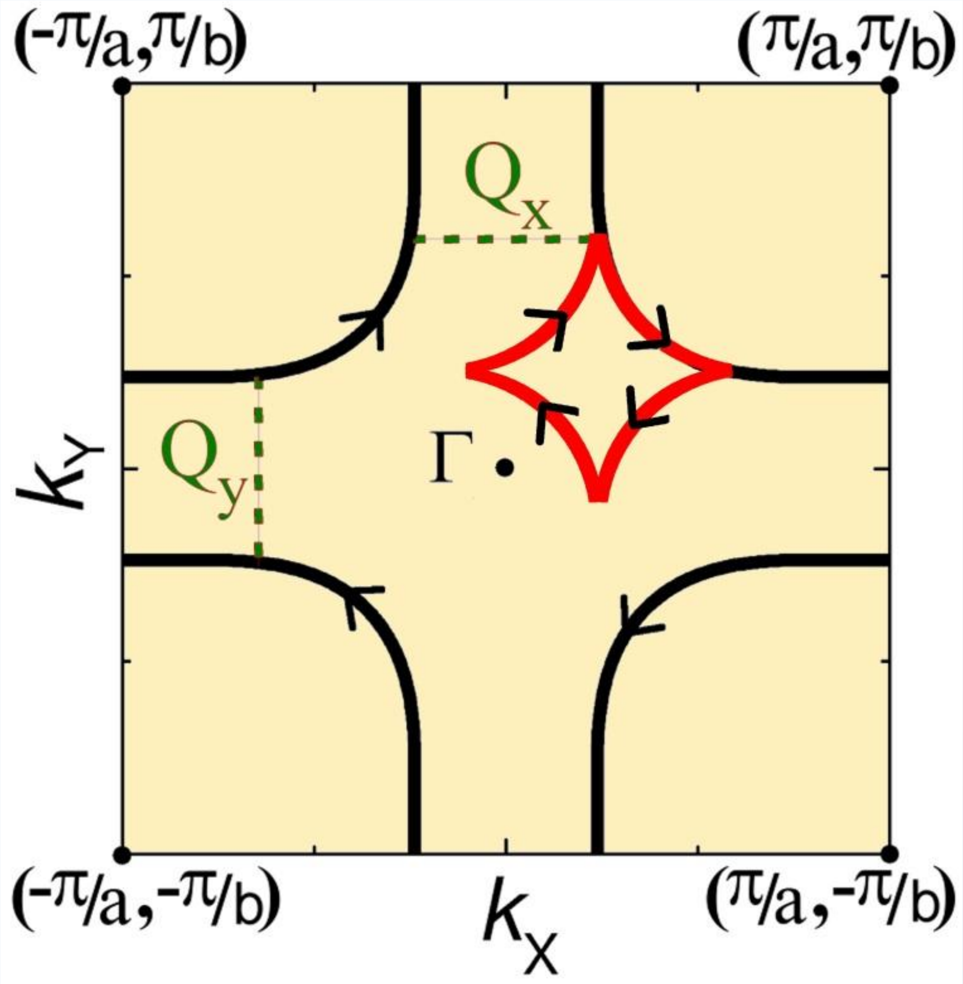
A. J. Achkar,[1] F. He,[2] R. Sutarto,[3] J. Geck,[4] H. Zhang,[5] Y.-J. Kim,[5] and D. G. Hawthorn[1]

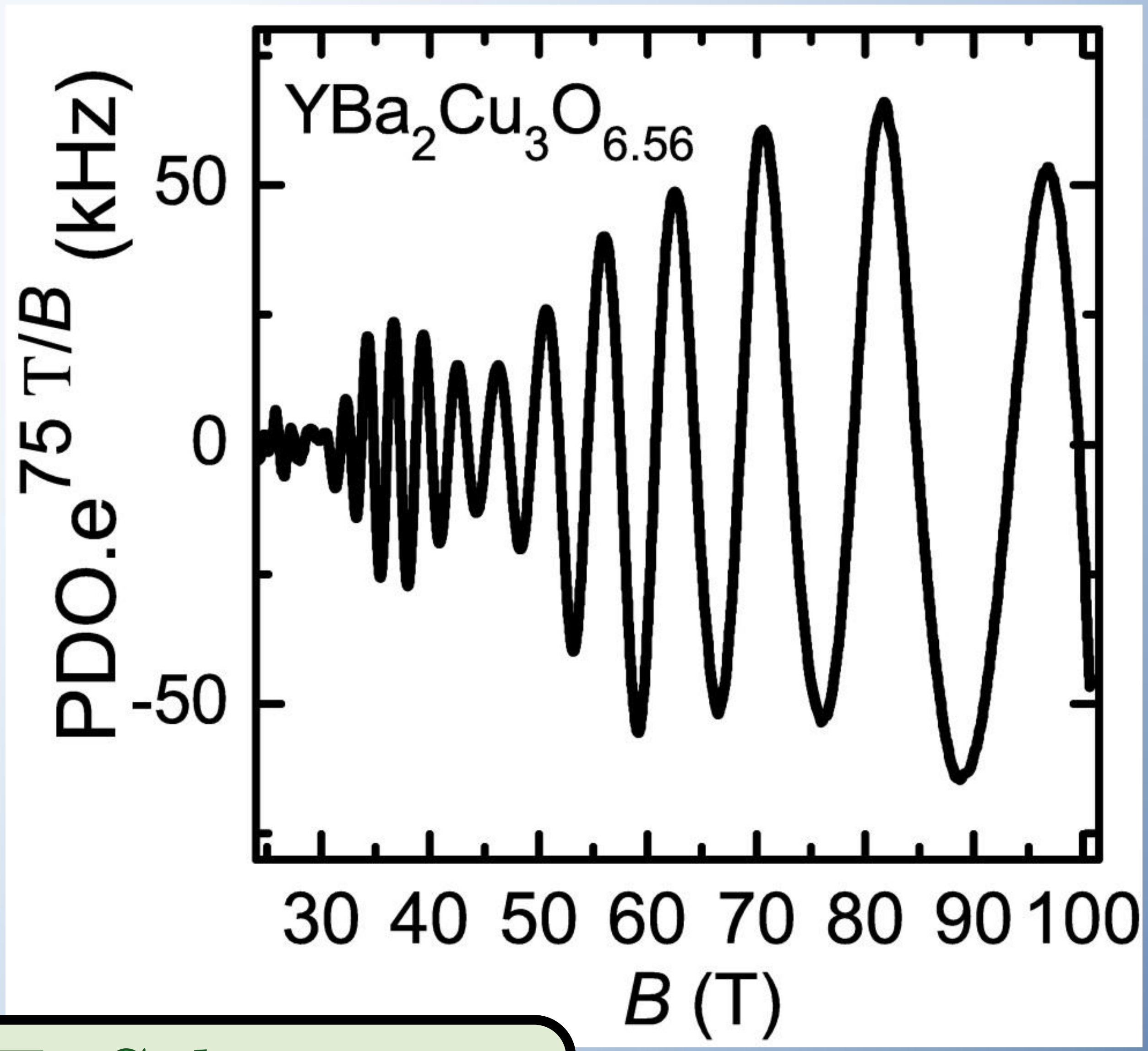
PRL **110,** 017001 (2013)



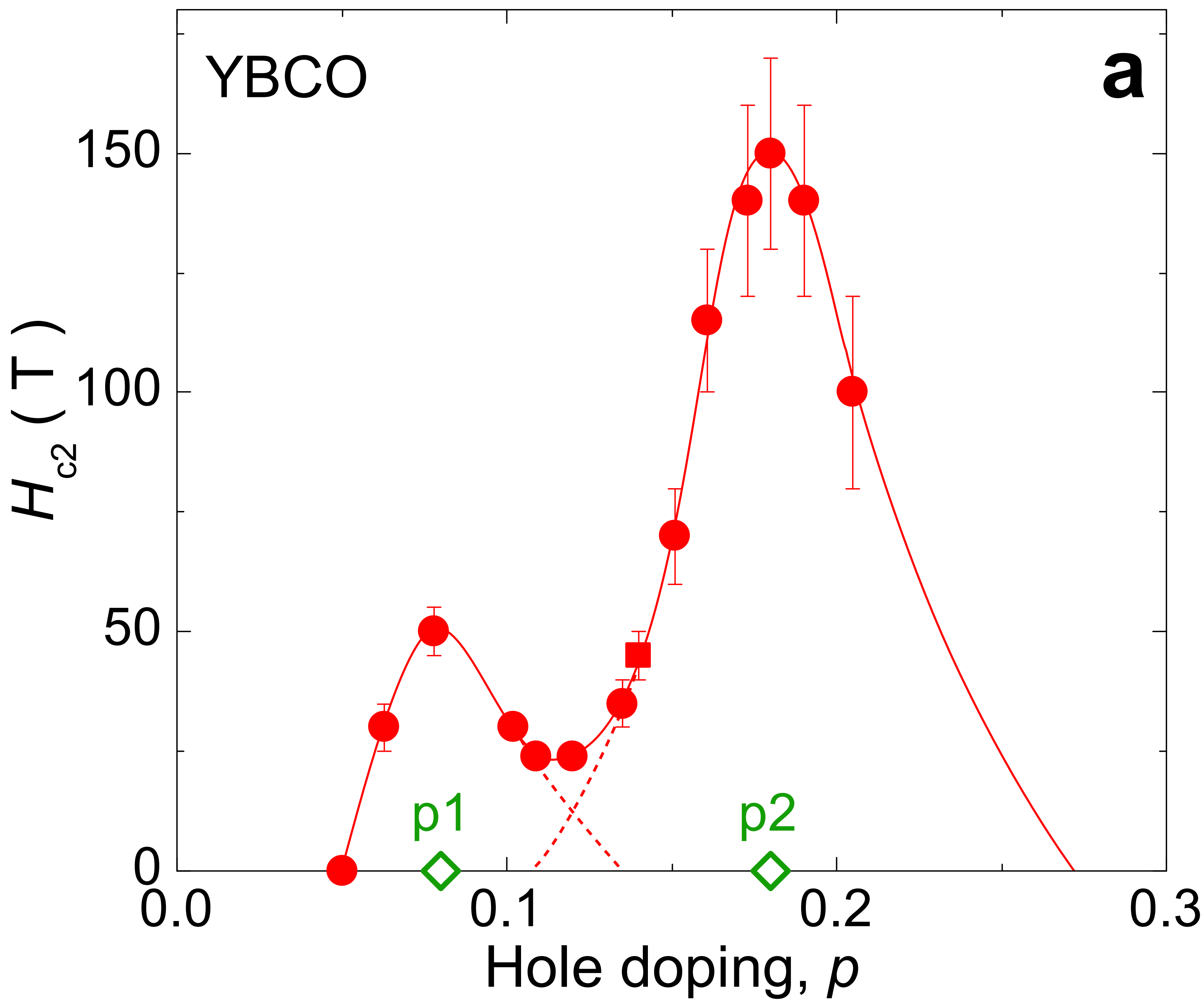
These energy shifts are interpreted as a spatial modulation of the electronic structure and may point to a valence-bond-solid interpretation of the stripe phase.

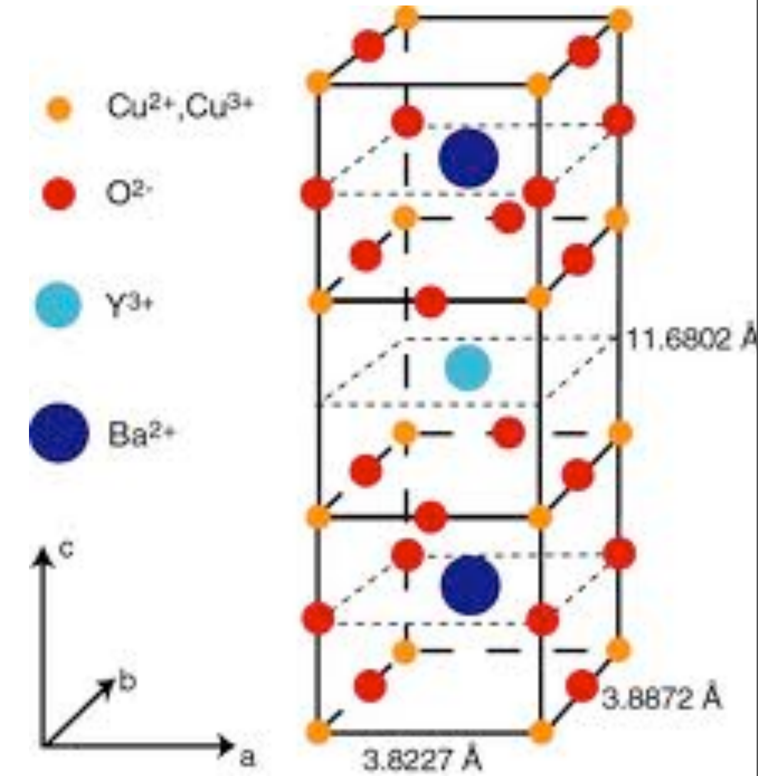
Do we finally have a resolution to the low energy electronic structure of underdoped YBCO?





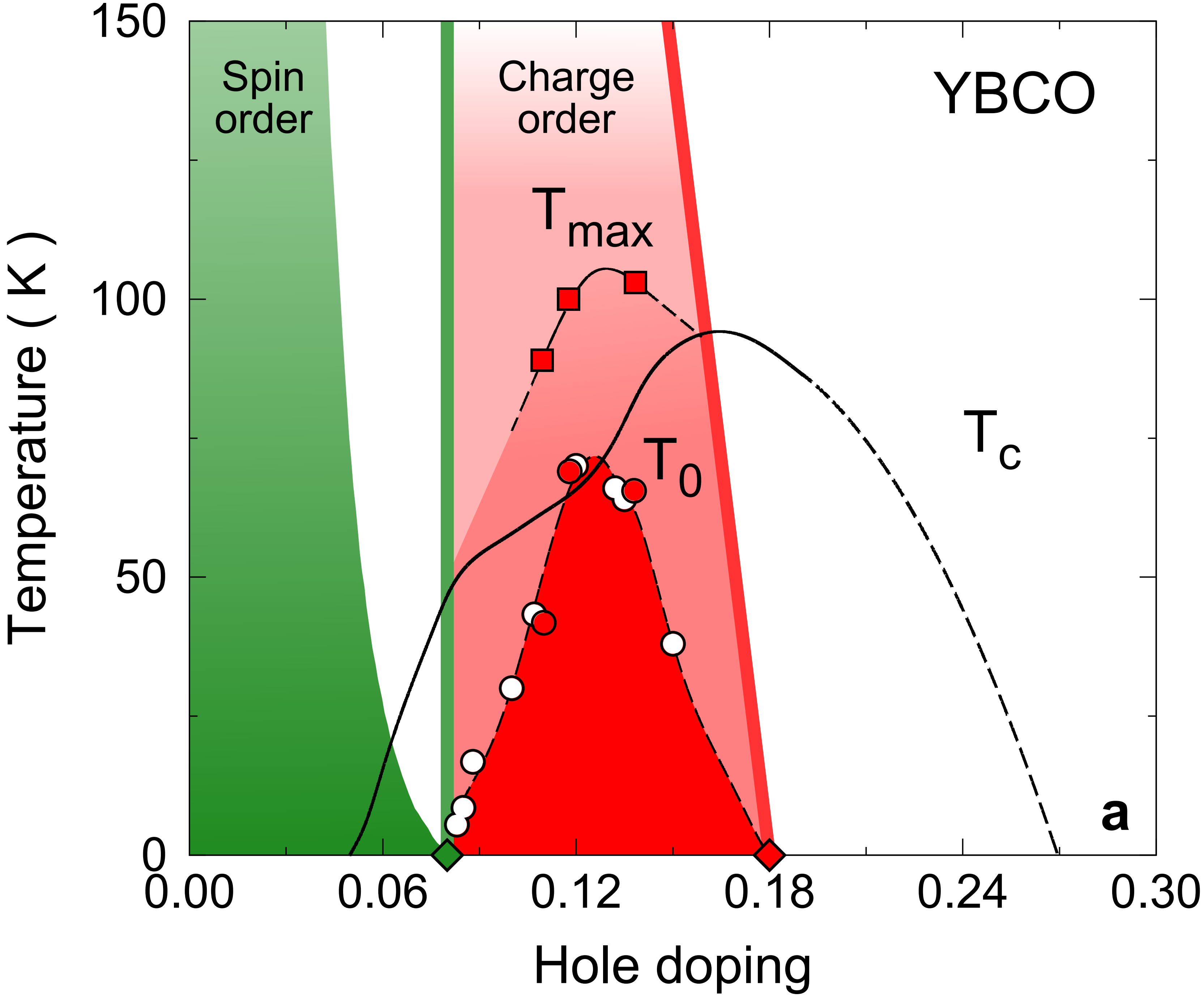
N. Harrison and S. E. Sebastian
Phys. Rev. Lett. **106**, 226402 (2011).

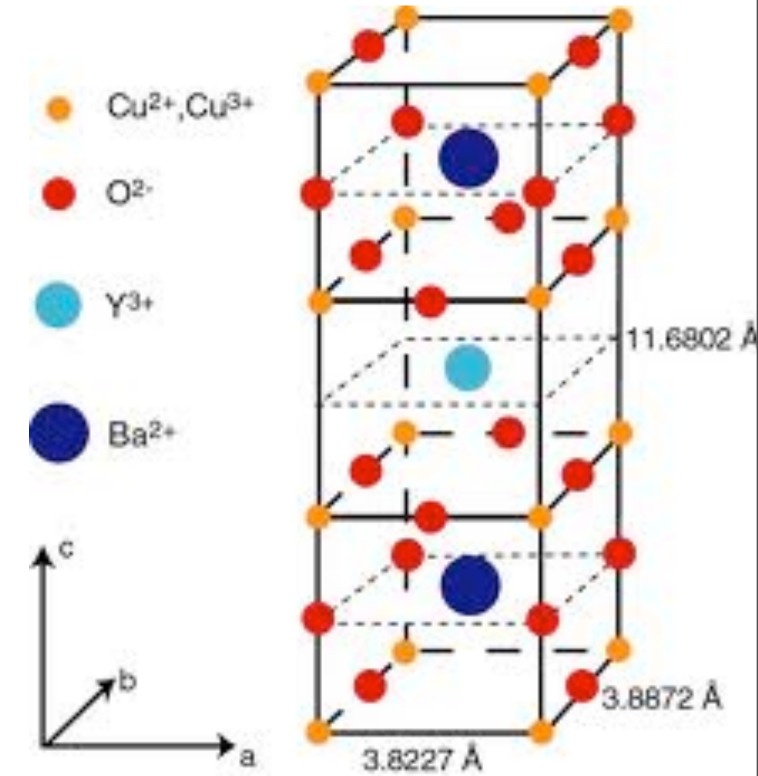




APS March meeting 2013 B36.00002

G. Grissonnanche, O. Cyr-Choinière, F. Laliberté, S. René de Cotret, A. Juneau-Fecteau, S. Dufour-Beauséjour, M.-È. Delage, D. LeBoeuf, J. Chang, B. J. Ramshaw, D. A. Bonn, W. N. Hardy, R. Liang, S. Adachi, N. E. Hussey, B. Vignolle, C. Proust, M. Sutherland, S. Krämer, J.-H. Park, D. Graf, N. Doiron-Leyraud & Louis Taillefer





APS March meeting 2013 B36.00002

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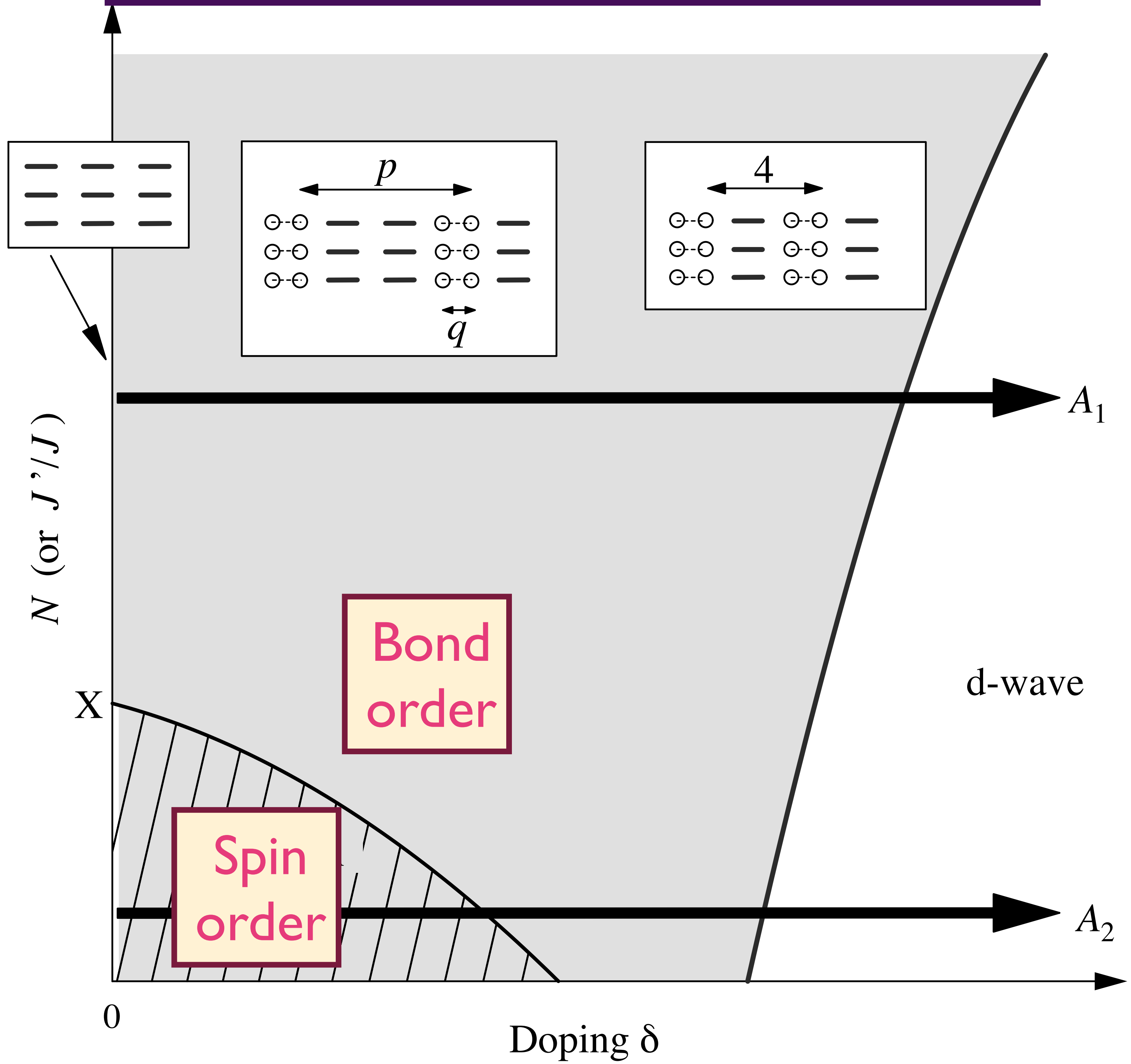
Low T phase diagram of a doped antiferromagnet



M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991)

Main results

- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

- The pseudospin partner of *d*-wave superconductivity is an incommensurate *d*-wave bond order

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)
K. B. Efetov, H. Meier, and C. Pépin, arXiv:1210.3276.
S. Sachdev and R. La Placa, arXiv:1303.2114

Outline

1. Stability of metal in Hartree-Fock-BCS theory

2. Emergent pseudospin symmetry in low energy theory of metal with antiferromagnetic interactions

3. Quantum Monte Carlo without the sign problem

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1. Stability of metal in Hartree-Fock-BCS theory

2. Emergent pseudospin symmetry in low energy theory of metal with antiferromagnetic interactions

3. Quantum Monte Carlo without the sign problem

$$H_{tJ} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Optimize the free energy w.r.t. a mean field Hamiltonian which allows for spin-singlet superconductivity (pairing order $\Delta_S(\mathbf{k})$) and spin-singlet "charge" order ($\Delta_{\mathbf{Q}}(\mathbf{k})$):

$$\begin{aligned} H_{MF} &= -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{\mathbf{k}} \Delta_S(\mathbf{k}) \epsilon_{\alpha\beta} c_{\mathbf{k},\alpha} c_{-\mathbf{k},\beta} + \text{H.c.} \\ &\quad + \sum_{\mathbf{k},\mathbf{Q}} \Delta_{\mathbf{Q}}(\mathbf{k}) c^\dagger_{\mathbf{k}+\mathbf{Q}/2,\alpha} c_{\mathbf{k}-\mathbf{Q}/2,\alpha} \end{aligned}$$

$$H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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In real space, the "charge" order $\Delta_{\mathbf{Q}}(\mathbf{k})$ corresponds to a modulation in local and non-local "density" variables:

$$\left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle \sim \left[\sum_{\mathbf{k}} \Delta_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i+\mathbf{r}_j)/2}$$

We obtain purely "bond" order with no modulation in the site density, $\left\langle c_{i\alpha}^\dagger c_{i\alpha} \right\rangle$, if $\sum_{\mathbf{k}} \Delta_{\mathbf{Q}}(\mathbf{k}) = 0$.

$$H_{tJ} = -\sum_{i,j} t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Optimize the free energy w.r.t. a mean field Hamiltonian which allows for spin-singlet superconductivity (pairing order $\Delta_S(\mathbf{k})$) and spin-singlet "charge" order ($\Delta_{\mathbf{Q}}(\mathbf{k})$):

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Expanding the free energy in powers of the order parameters we obtain

$$\begin{aligned} F = F_0 &+ \sum_{\mathbf{k}} \Delta^*_S(\mathbf{k}) \mathcal{M}_S(\mathbf{k},\mathbf{k}') \Delta_S(\mathbf{k}') \\ &+ \sum_{\mathbf{k},\mathbf{Q}} \Delta^*_{\mathbf{Q}}(\mathbf{k}) \mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}') \Delta_{\mathbf{Q}}(\mathbf{k}') \end{aligned}$$

Due to pseudospin symmetry, the kernels have identical forms:

$$\mathcal{M}_S(\mathbf{k},\mathbf{k}') = \Pi_S(\mathbf{k})\left[\delta_{\mathbf{k},\mathbf{k}'} - \frac{3}{V}J(\mathbf{k}-\mathbf{k}')\Pi_S(\mathbf{k}')\right]$$
$$\mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}') = \Pi_{\mathbf{Q}}(\mathbf{k})\left[\delta_{\mathbf{k},\mathbf{k}'} - \frac{3}{V}J(\mathbf{k}-\mathbf{k}')\Pi_{\mathbf{Q}}(\mathbf{k}')\right]$$

The pseudospin symmetry is broken only by the differing forms of the polarizabilities of fermions with dispersion $\varepsilon(\mathbf{k})$.

$$\Pi_S(\mathbf{k}) = \frac{1-2f(\varepsilon(\mathbf{k}))}{2\varepsilon(\mathbf{k})}$$
$$\Pi_{\mathbf{Q}}(\mathbf{k}) = \frac{f(\varepsilon(\mathbf{k}+\mathbf{Q}/2)) - f(\varepsilon(\mathbf{k}-\mathbf{Q}/2))}{\varepsilon(\mathbf{k}-\mathbf{Q}/2) - \varepsilon(\mathbf{k}+\mathbf{Q}/2)},$$

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We diagonalized $J(\mathbf{k}-\mathbf{k}')\Pi_S(\mathbf{k}')$ and $J(\mathbf{k}-\mathbf{k}')\Pi_{\mathbf{Q}}(\mathbf{k}')$ and obtained the lowest eigenvalues λ_S and $\lambda_{\mathbf{Q}}$ and the corresponding right eigenvectors $\Delta_S(\mathbf{k})$ and $\Delta_{\mathbf{Q}}(\mathbf{k})$.

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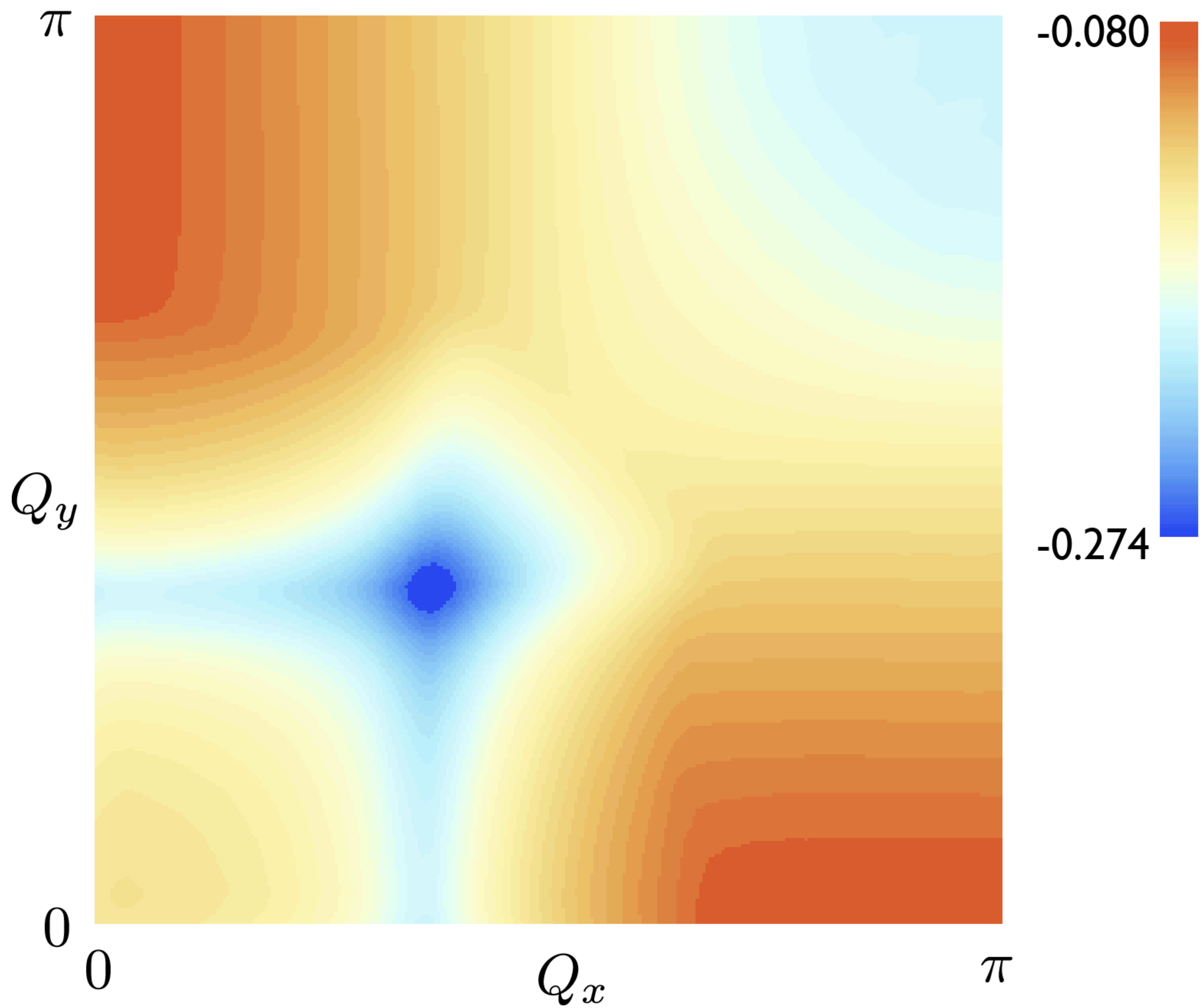
$$
\begin{aligned}
\mathcal{M}_S(\mathbf{k},\mathbf{k}') &= \Pi_S(\mathbf{k})\left[\delta_{\mathbf{k},\mathbf{k}'} - \frac{3}{V} J(\mathbf{k}-\mathbf{k}')\Pi_S(\mathbf{k}')\right] \\
\mathcal{M}_{\mathbf{Q}}(\mathbf{k},\mathbf{k}') &= \Pi_{\mathbf{Q}}(\mathbf{k})\left[\delta_{\mathbf{k},\mathbf{k}'} - \frac{3}{V} J(\mathbf{k}-\mathbf{k}')\Pi_{\mathbf{Q}}(\mathbf{k}')\right]
\end{aligned}
$$

The pseudospin symmetry is broken only by the differing forms of the polarizabilities of fermions with dispersion $\varepsilon(\mathbf{k})$.

$$
\begin{aligned}
\Pi_S(\mathbf{k}) &= \frac{1-2f(\varepsilon(\mathbf{k}))}{2\varepsilon(\mathbf{k})} \\
\Pi_{\mathbf{Q}}(\mathbf{k}) &= \frac{f(\varepsilon(\mathbf{k}+\mathbf{Q}/2)) - f(\varepsilon(\mathbf{k}-\mathbf{Q}/2))}{\varepsilon(\mathbf{k}-\mathbf{Q}/2) - \varepsilon(\mathbf{k}+\mathbf{Q}/2)},
\end{aligned}
$$

We diagonalized $J(\mathbf{k}-\mathbf{k}')\Pi_S(\mathbf{k}')$ and $J(\mathbf{k}-\mathbf{k}')\Pi_{\mathbf{Q}}(\mathbf{k}')$ and obtained the lowest eigenvalues λ_S and $\lambda_{\mathbf{Q}}$ and the corresponding right eigenvectors $\Delta_S(\mathbf{k})$ and $\Delta_{\mathbf{Q}}(\mathbf{k})$.
The smallest eigenvalue was always λ_S with with eigenvector $\Delta_S(k) = \Delta_0(\cos k_x - \cos k_y) + \ldots$.



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, arXiv:1303.2114

$$\Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \sum_{\gamma} c_{\boldsymbol{Q},\gamma}\, \psi_\gamma(\boldsymbol{k})$$

γ	$\psi_\gamma(\boldsymbol{k})$	$\boldsymbol{Q} =$ (1.15,1.15)	$\boldsymbol{Q} =$ (1.15, 0)	$\boldsymbol{Q} =$ (0,0)	$\boldsymbol{Q} =$ (π,π)	$\Delta_S(\boldsymbol{k})$
s	1	0	-0.231	0	0	0
s'	$\cos k_x + \cos k_y$	0	0.044	0	0	0
s''	$\cos(2k_x) + \cos(2k_y)$	0	-0.046	0	0	0
d	$\cos k_x - \cos k_y$	0.993	0.963	0.997	0	0.997
d'	$\cos(2k_x) - \cos(2k_y)$	- 0.069	-0.067	-0.057	0	-0.056
d''	$2 \sin k_x \sin k_y$	0	0	0	0	0
p_x	$\sqrt{2} \sin k_x$	0	0	0	0.706	0
p_y	$\sqrt{2} \sin k_y$	0	0	0	-0.706	0
g	$(\cos k_x - \cos k_y) \times \sqrt{8} \sin k_x \sin k_y$	-0.009	0	0	0	0

Charge-ordering eigenvector

S. Sachdev and R. La Placa, arXiv:1303.2114

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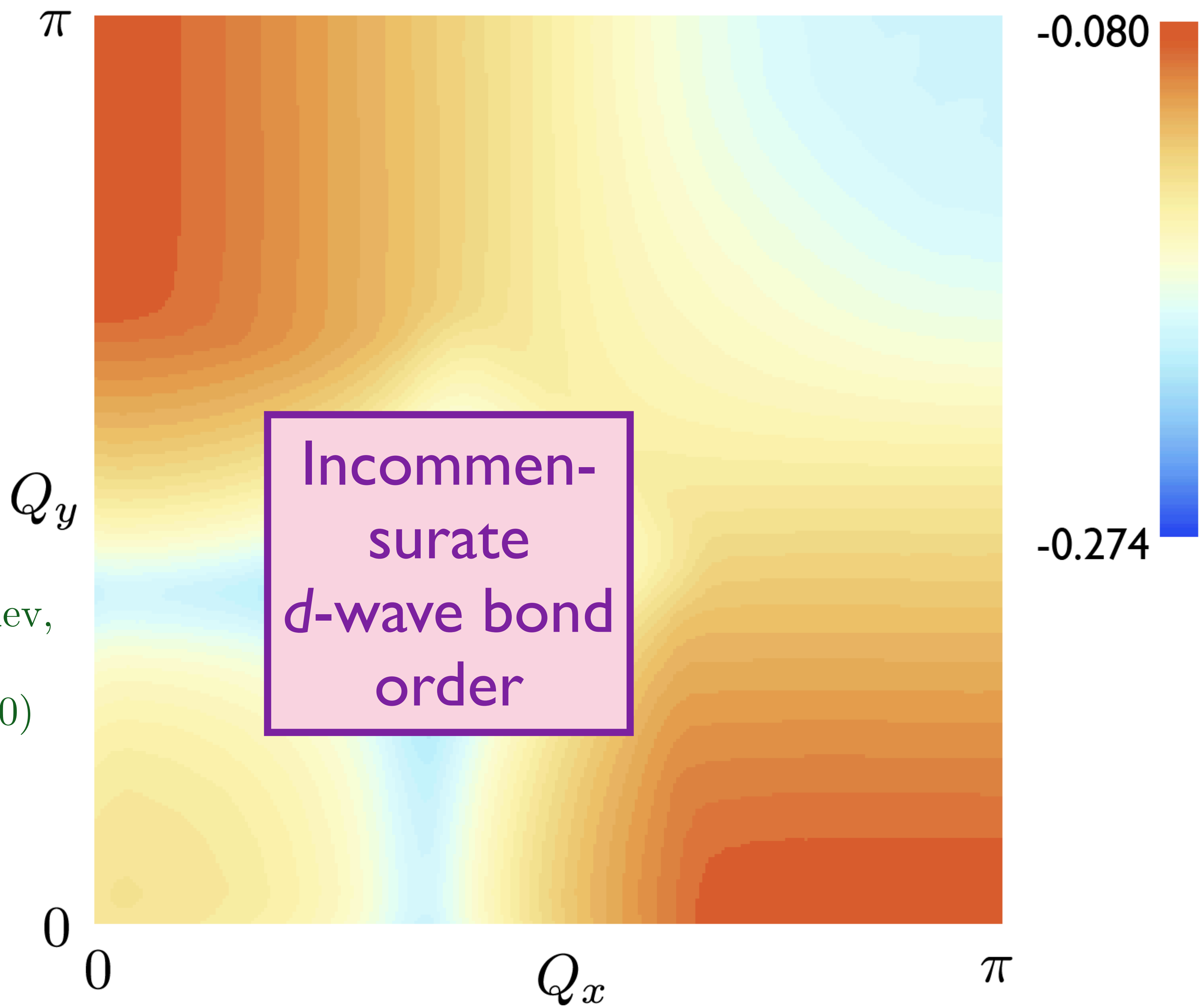
S. Sachdev and R. La Placa, arXiv:1303.2114

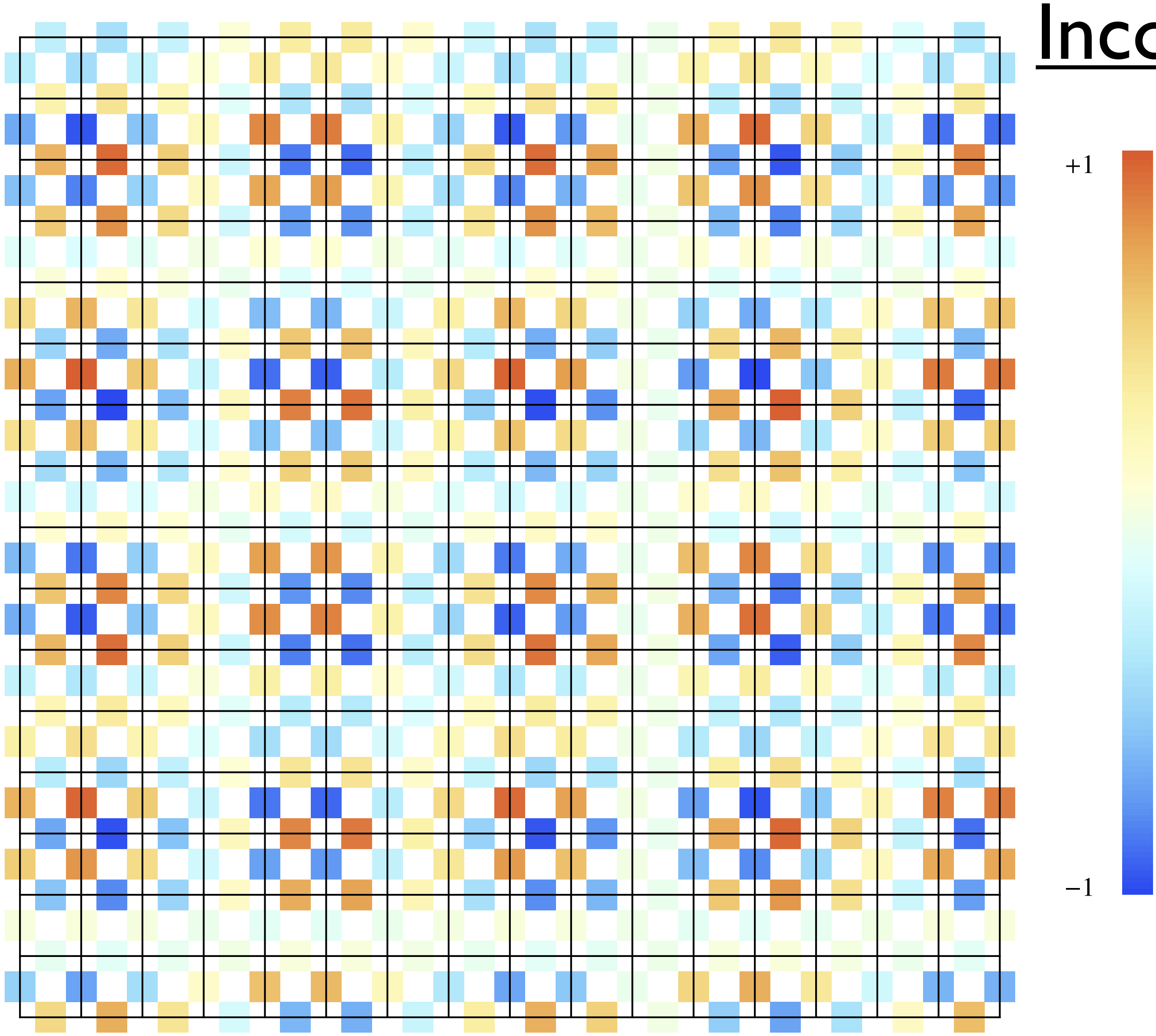


M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010)

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Incommensurate *d*-wave bond order



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c_{\mathbf{r}\alpha}^{\dagger} c_{\mathbf{s}\alpha} \right\rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle$$

where $\mathbf{Q}$ extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Note $\left\langle c_{\mathbf{r}\alpha}^{\dagger} c_{\mathbf{s}\alpha} \right\rangle$ is non-zero *only* when $\mathbf{r}, \mathbf{s}$ are nearest neighbors.

$$\Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \sum_{\gamma} c_{\boldsymbol{Q},\gamma}\,\psi_\gamma(\boldsymbol{k})$$

γ	$\psi_\gamma(\boldsymbol{k})$	$\boldsymbol{Q}$ = (1.15,1.15)	$\boldsymbol{Q}$ = (1.15, 0)	$\boldsymbol{Q}$ = (0,0)	$\boldsymbol{Q}$ = (π,π)	$\Delta_S(\boldsymbol{k})$
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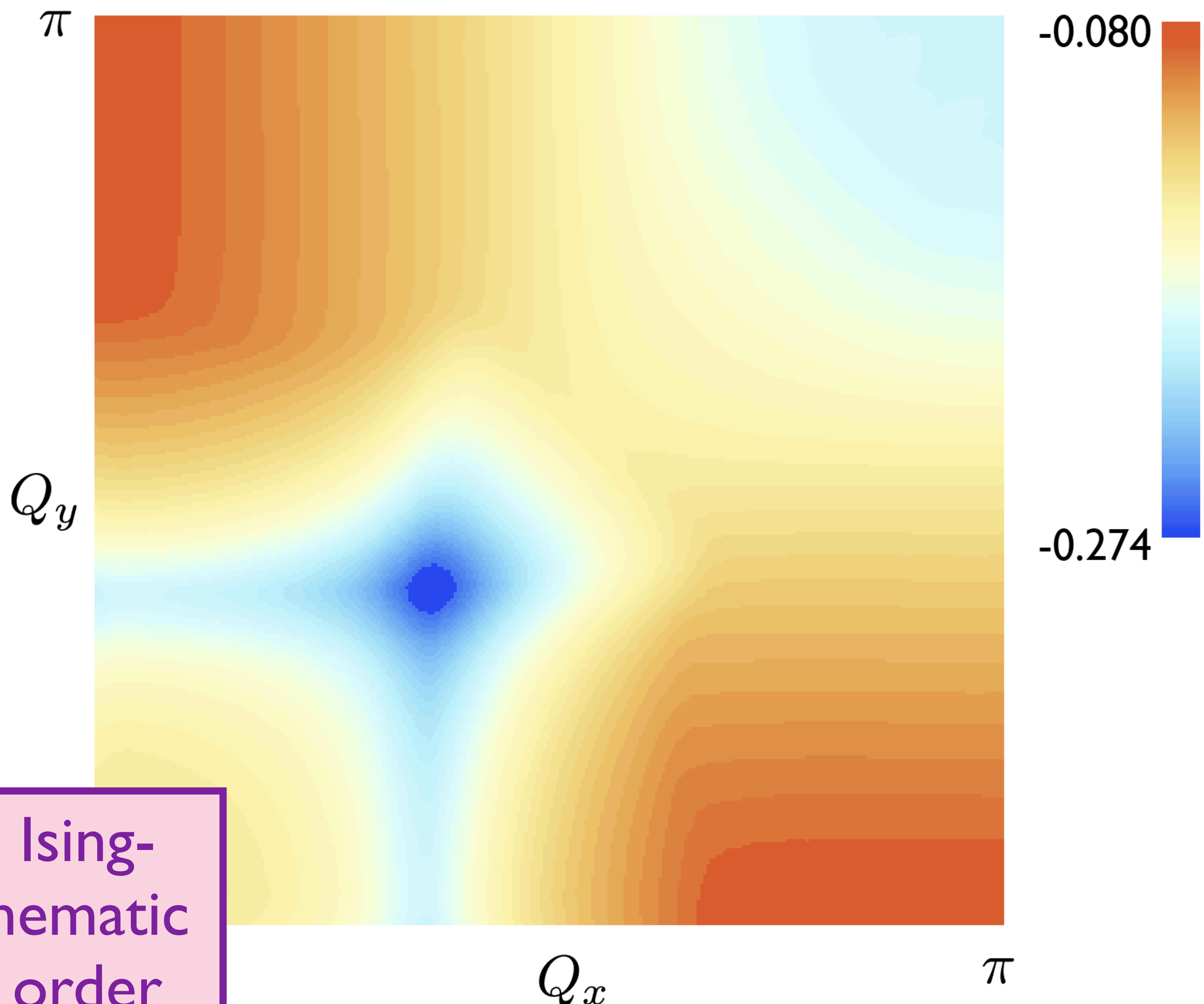
Charge-ordering eigenvector

S. Sachdev and R. La Placa, arXiv:1303.2114

S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature **393**, 550 (1998).
H. Yamase and H. Kohno, J. Phys. Soc. Jpn. **69**, 2151 (2000).
C. J. Halboth and W. Metzner, Phys. Rev. Lett. **85**, 5162 (2000).



Ising-nematic order

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, arXiv:1303.2114

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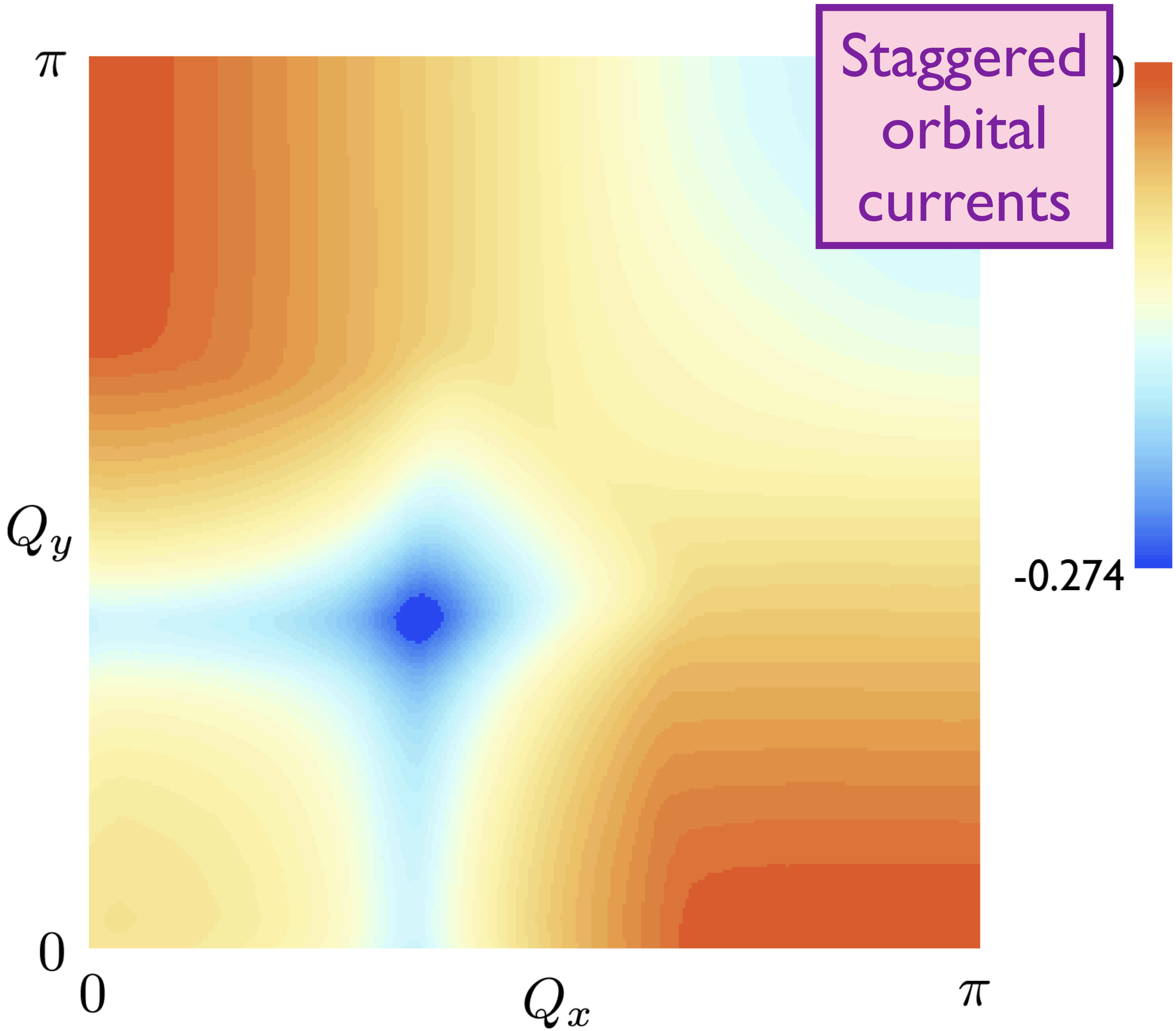

S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001). Also the related staggered-flux liquid of X.-G. Wen and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996).

Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, arXiv:1303.2114

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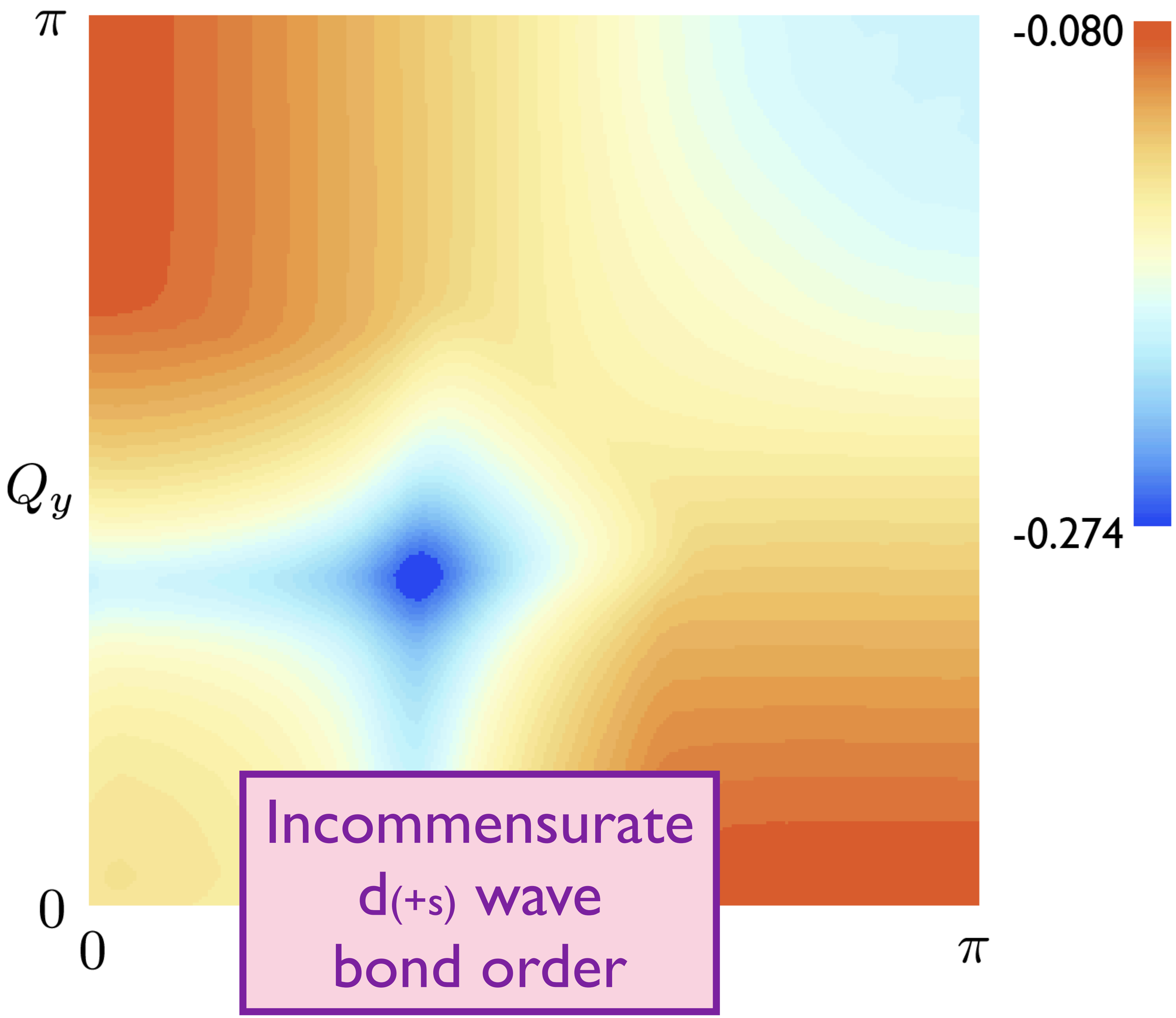

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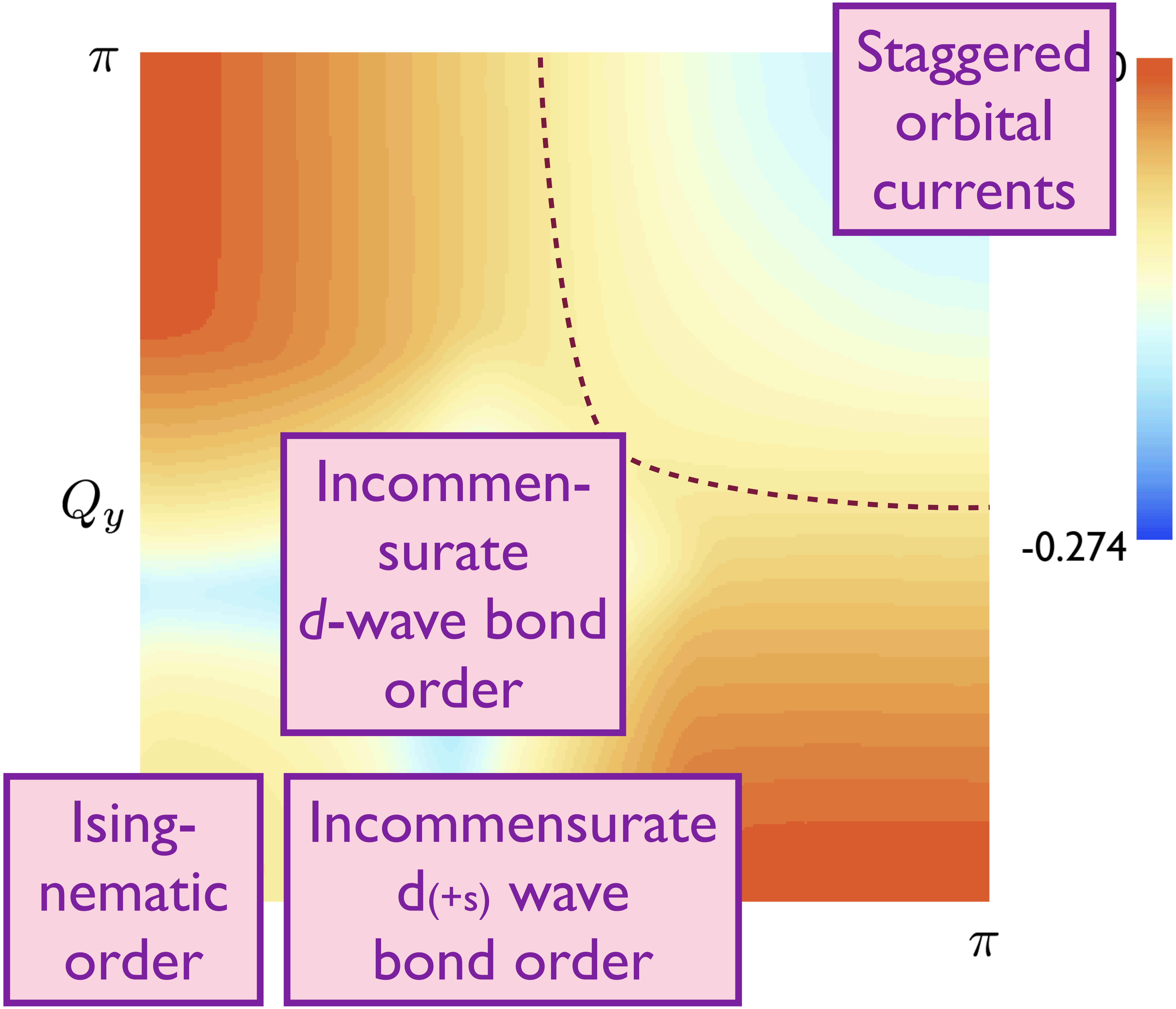
S. Sachdev and R. La Placa, arXiv:1303.2114



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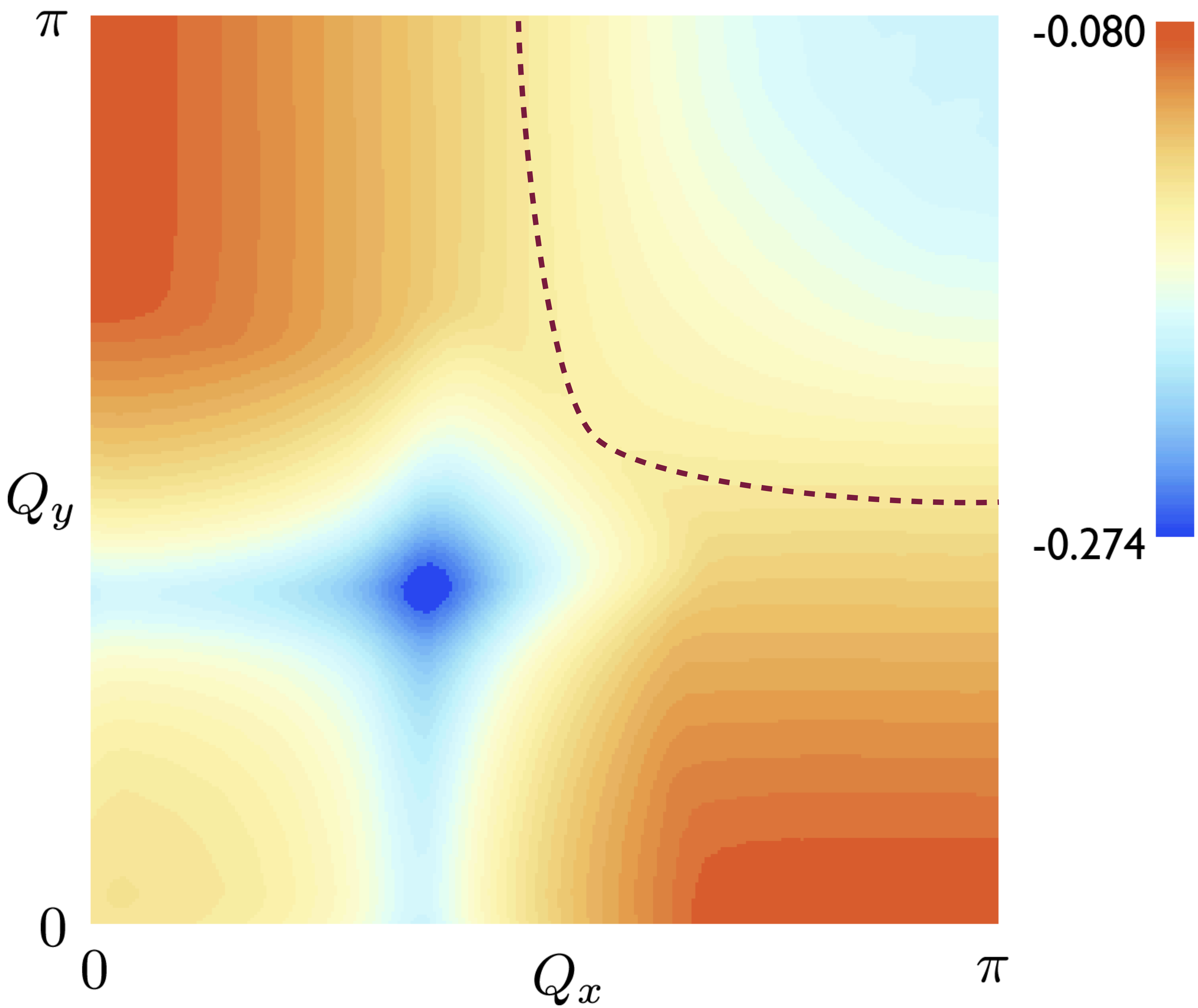

What determines the $\mathbf{Q}$ at which $\lambda_{\mathbf{Q}}$ is a minimum ?

S. Sachdev and R. La Placa, arXiv:1303.2114

Outline

1. Stability of metal in Hartree-Fock-BCS theory

2. Emergent pseudospin symmetry in low energy theory of metal with antiferromagnetic interactions

3. Quantum Monte Carlo without the sign problem

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Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\,\vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi^\dagger_{j\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{i\delta} \right)$$

which is invariant under the SU(2) pseudospin transformations

$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

This pseudospin symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
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This pseudospin symmetry is important in classifying spin liquid ground states of H_J. It is fully broken by the electron hopping t_{ij} but does have remnant consequences in doped spin liquid states.

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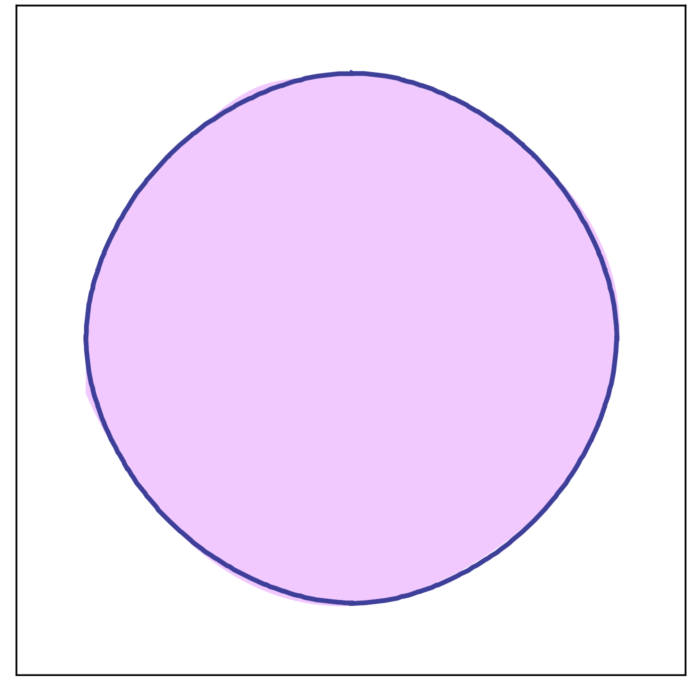
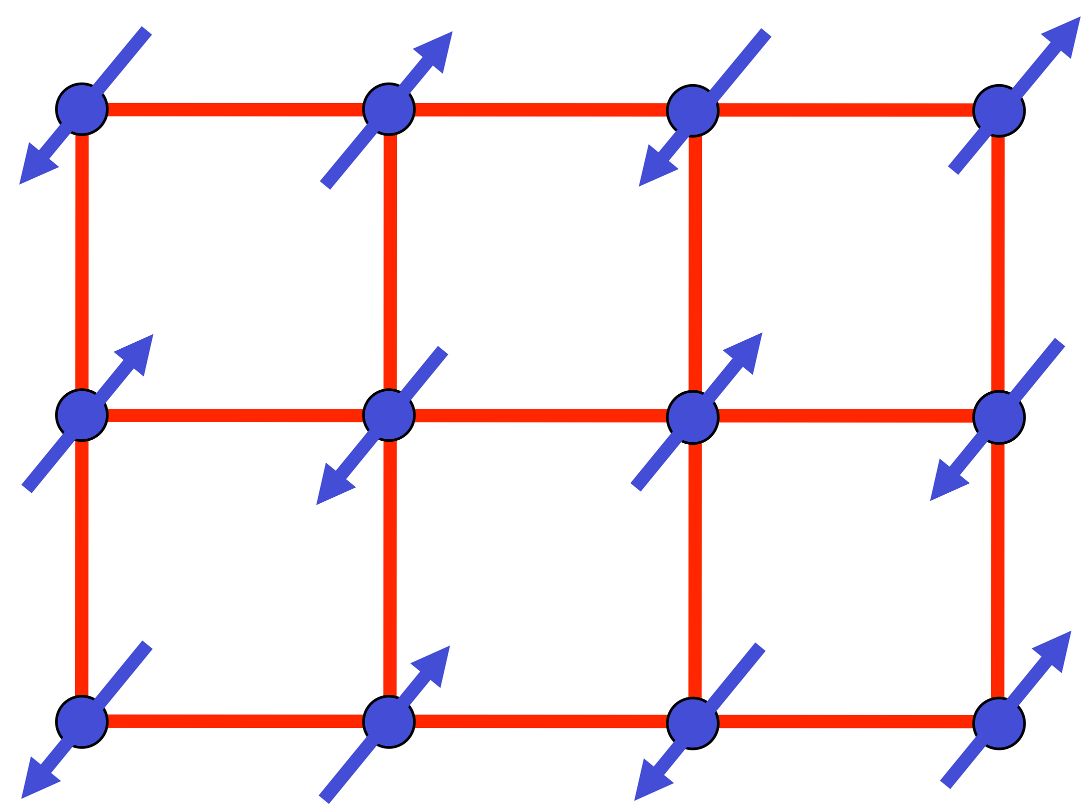
We will start with the Néel state, and find important consequences of the pseudospin symmetry in metals with antiferromagnetic correlations.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



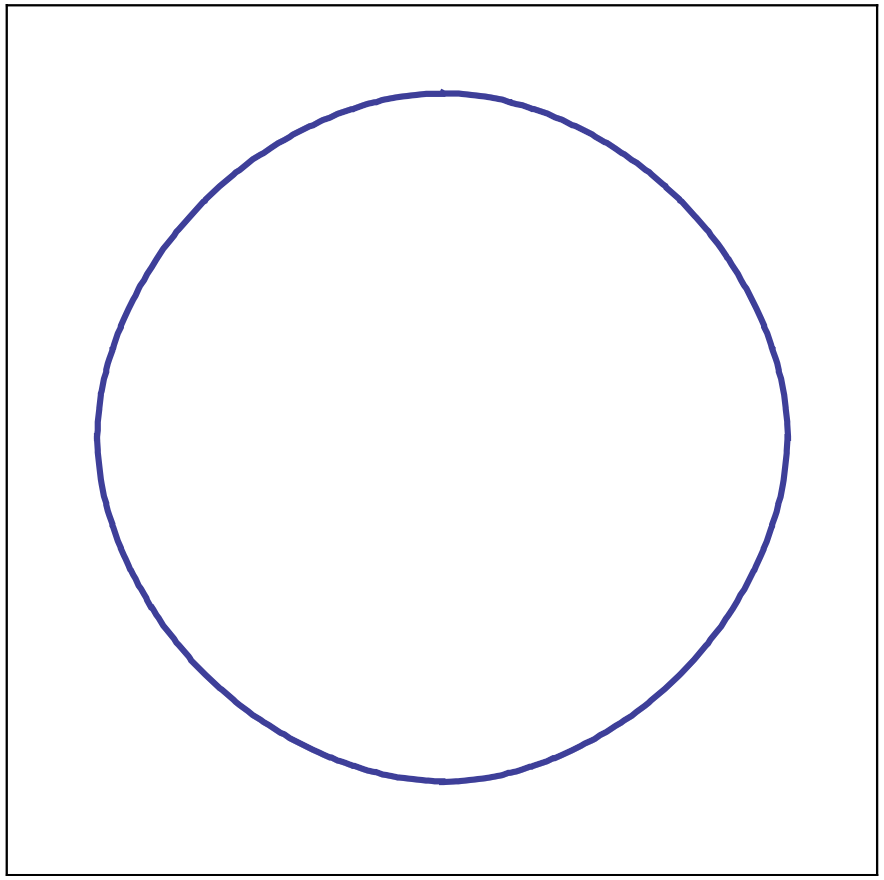


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

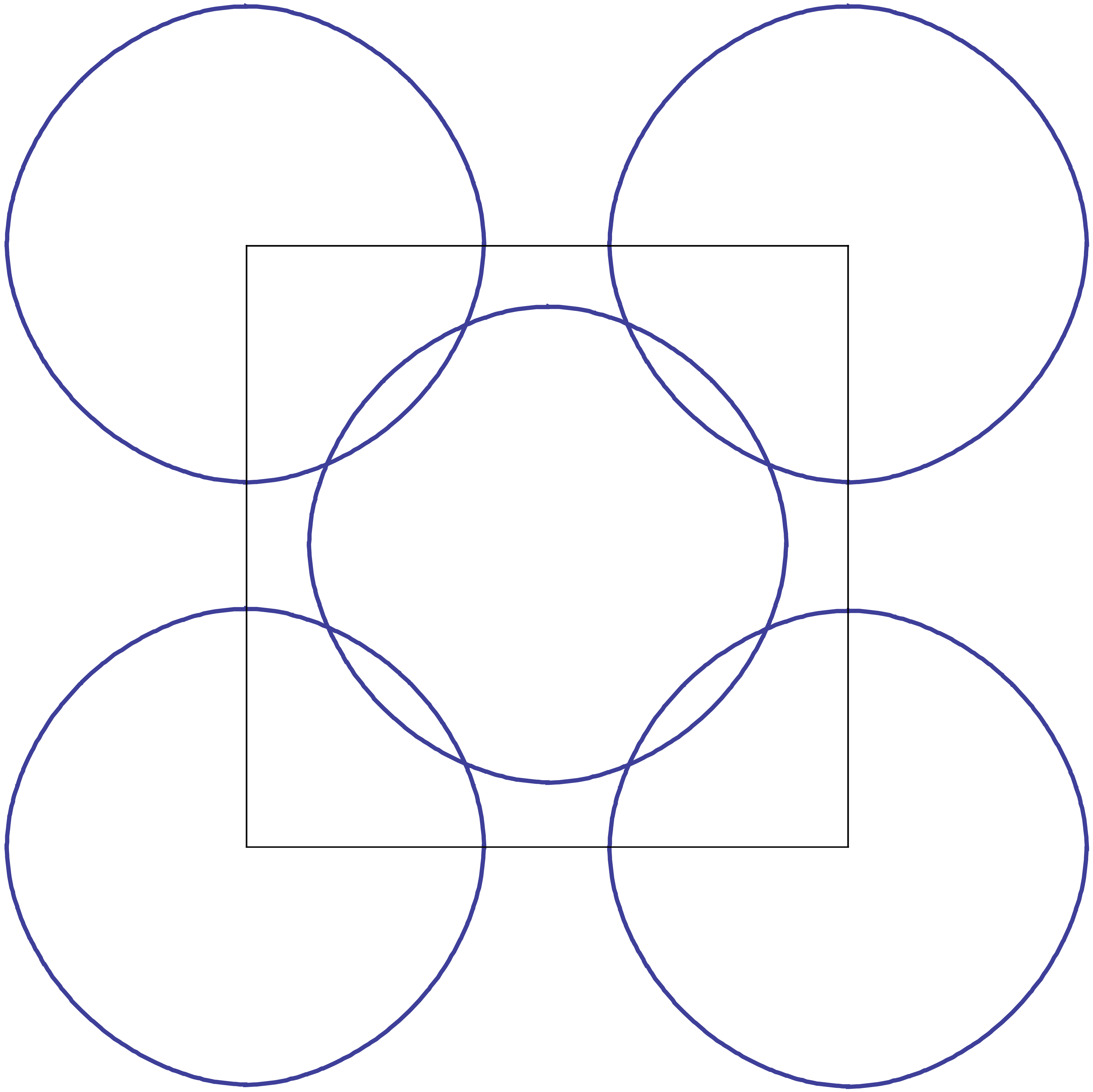
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



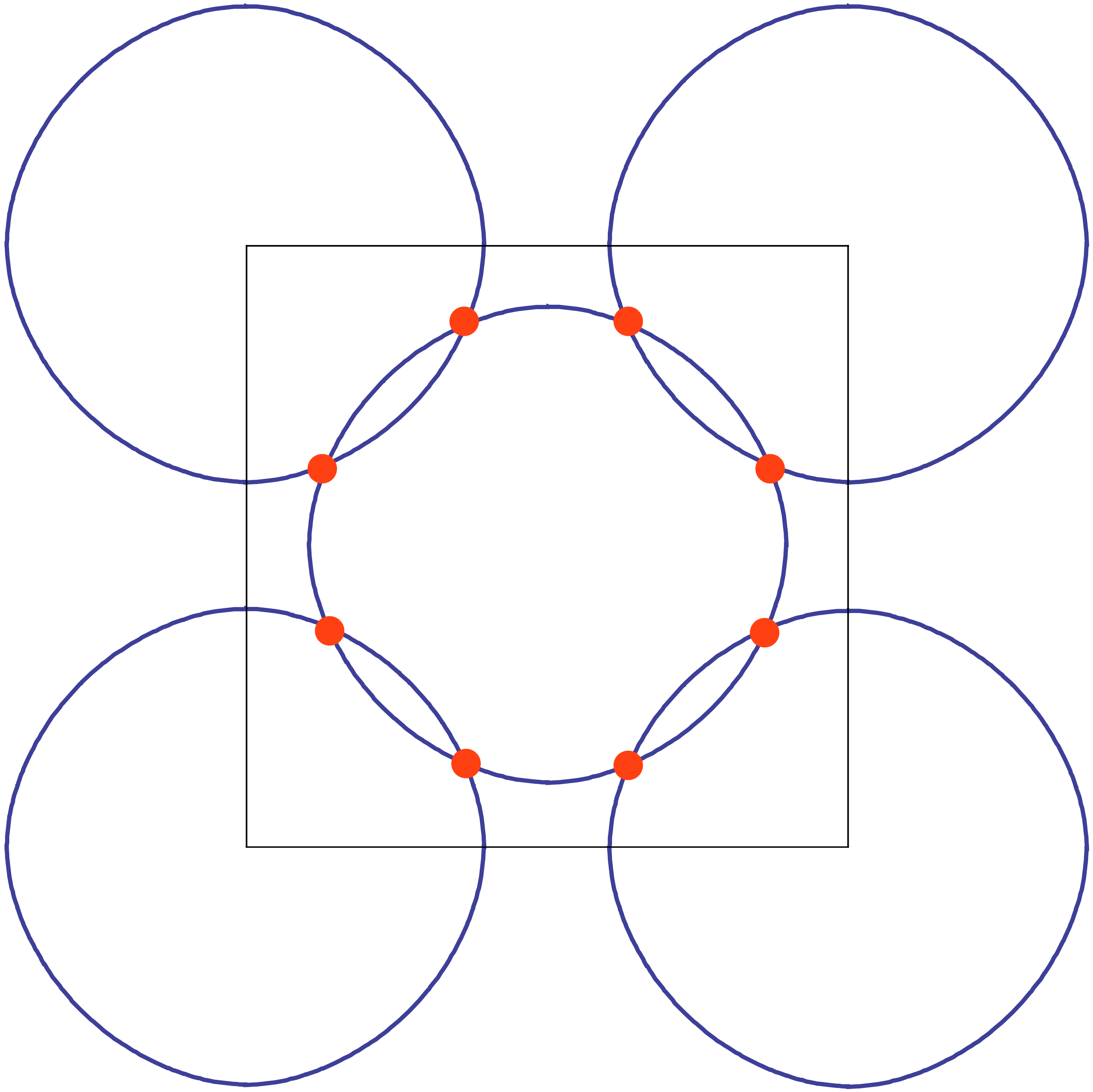
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



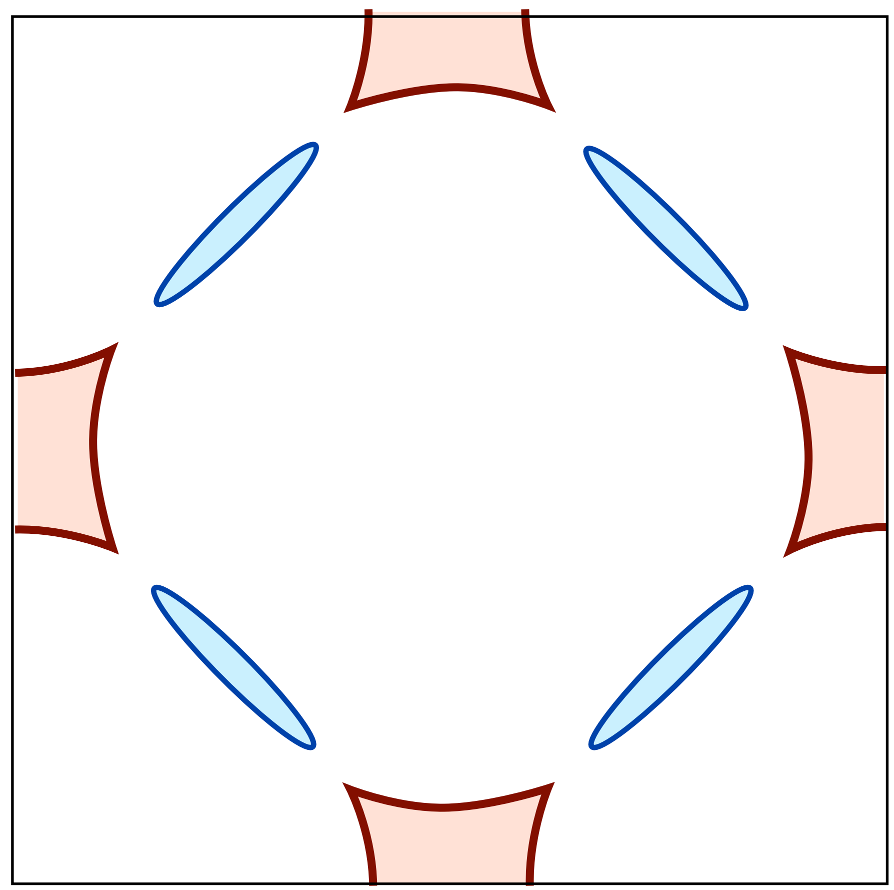
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



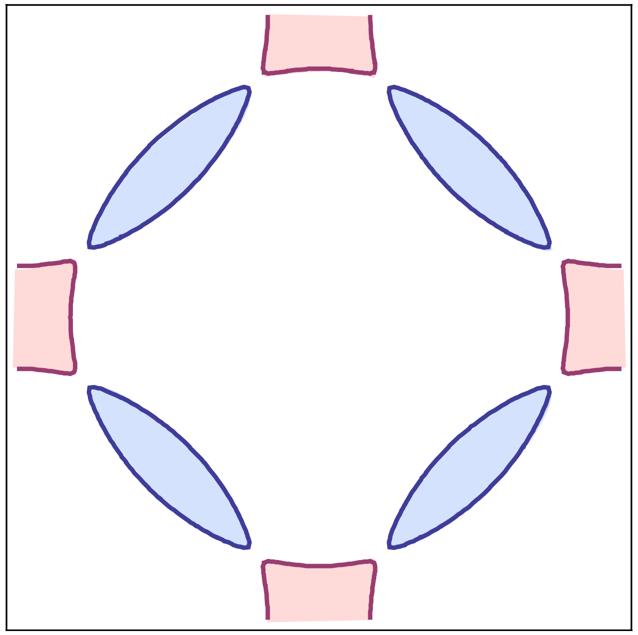
“Hot” spots

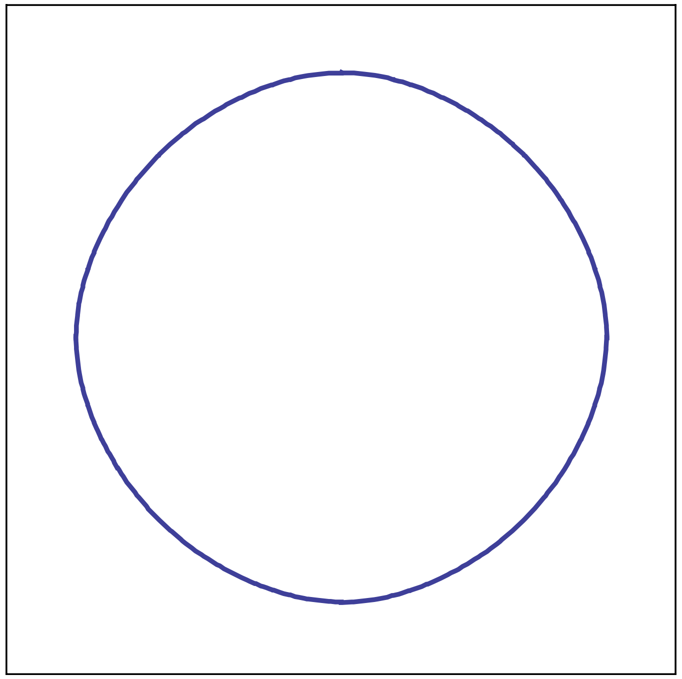
Fermi surface+antiferromagnetism



Electron and hole pockets in
antiferromagnetic phase
with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism





$\langle \vec{\varphi} \rangle \neq 0$

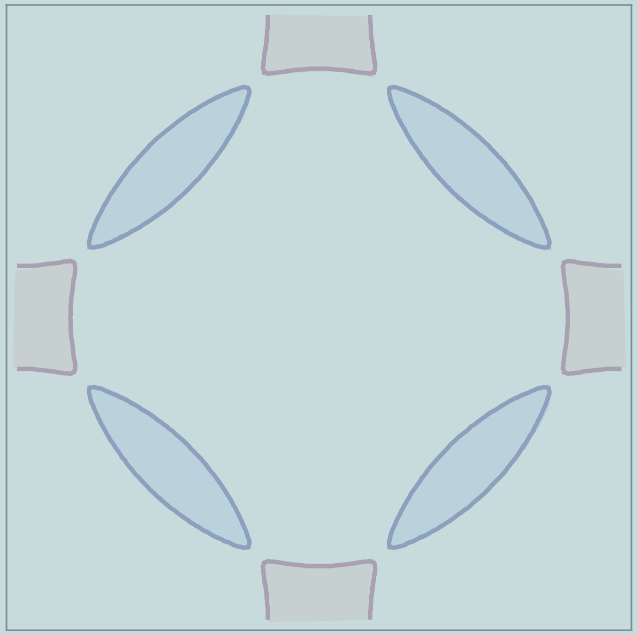
$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

r

Fermi surface+antiferromagnetism



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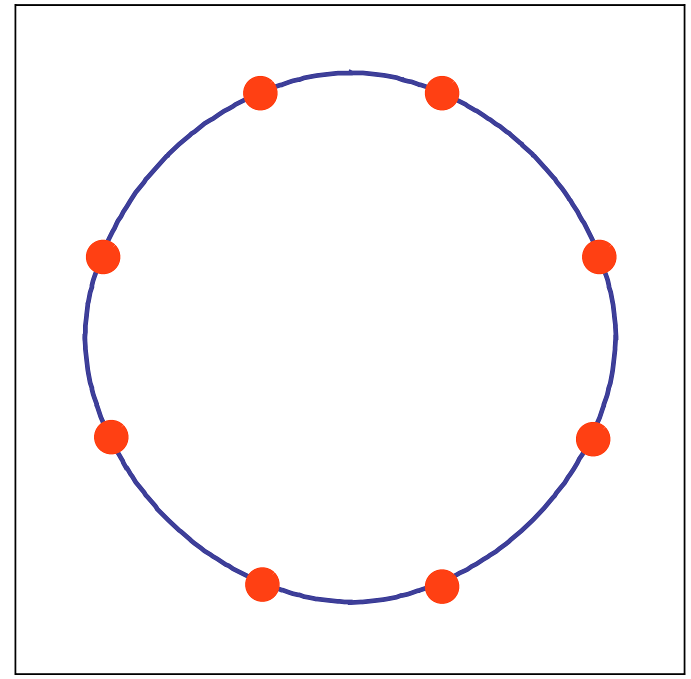

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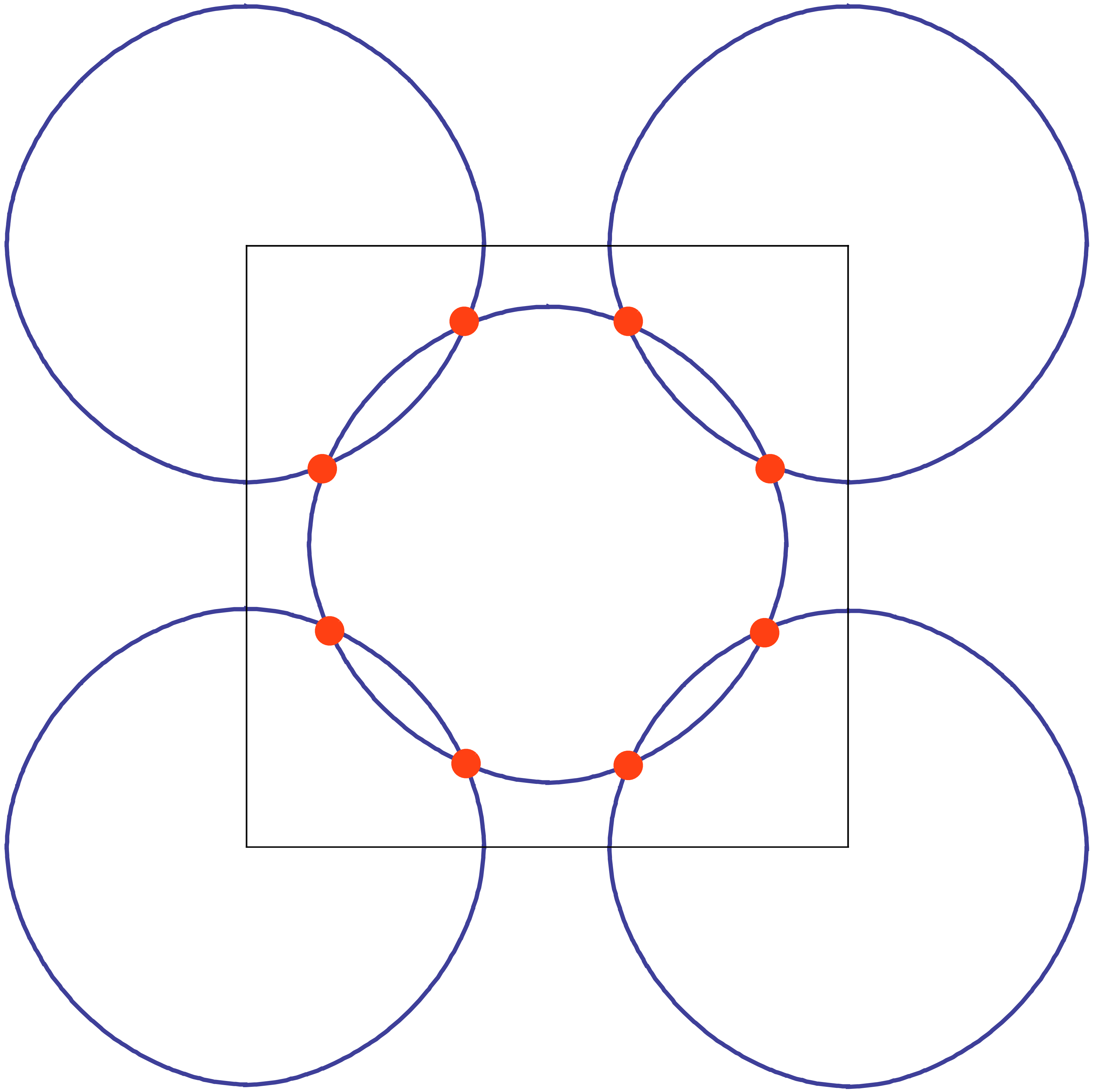
Metal with "large" Fermi surface

Focus of this talk

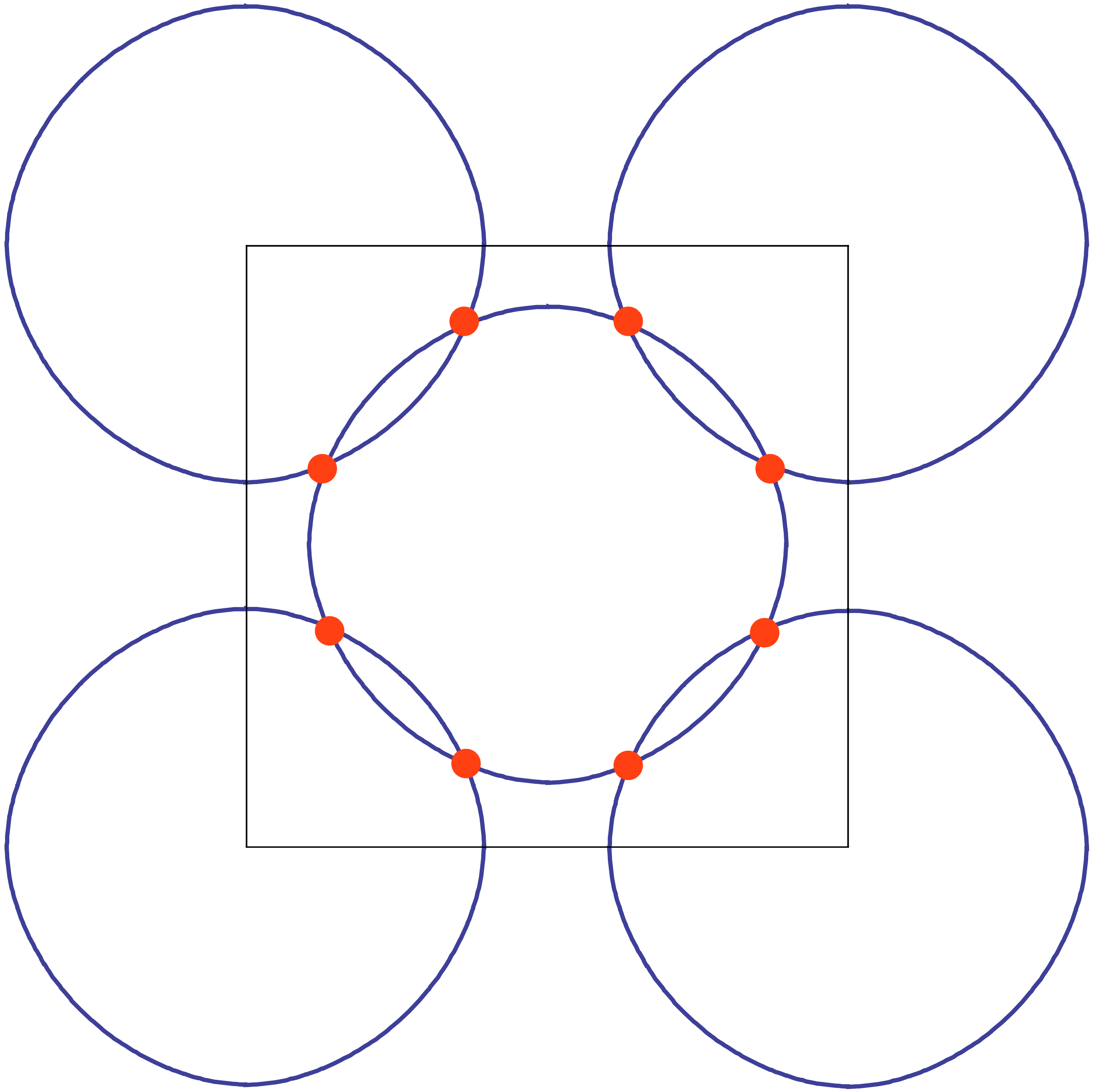
r

Fermi surface+antiferromagnetism



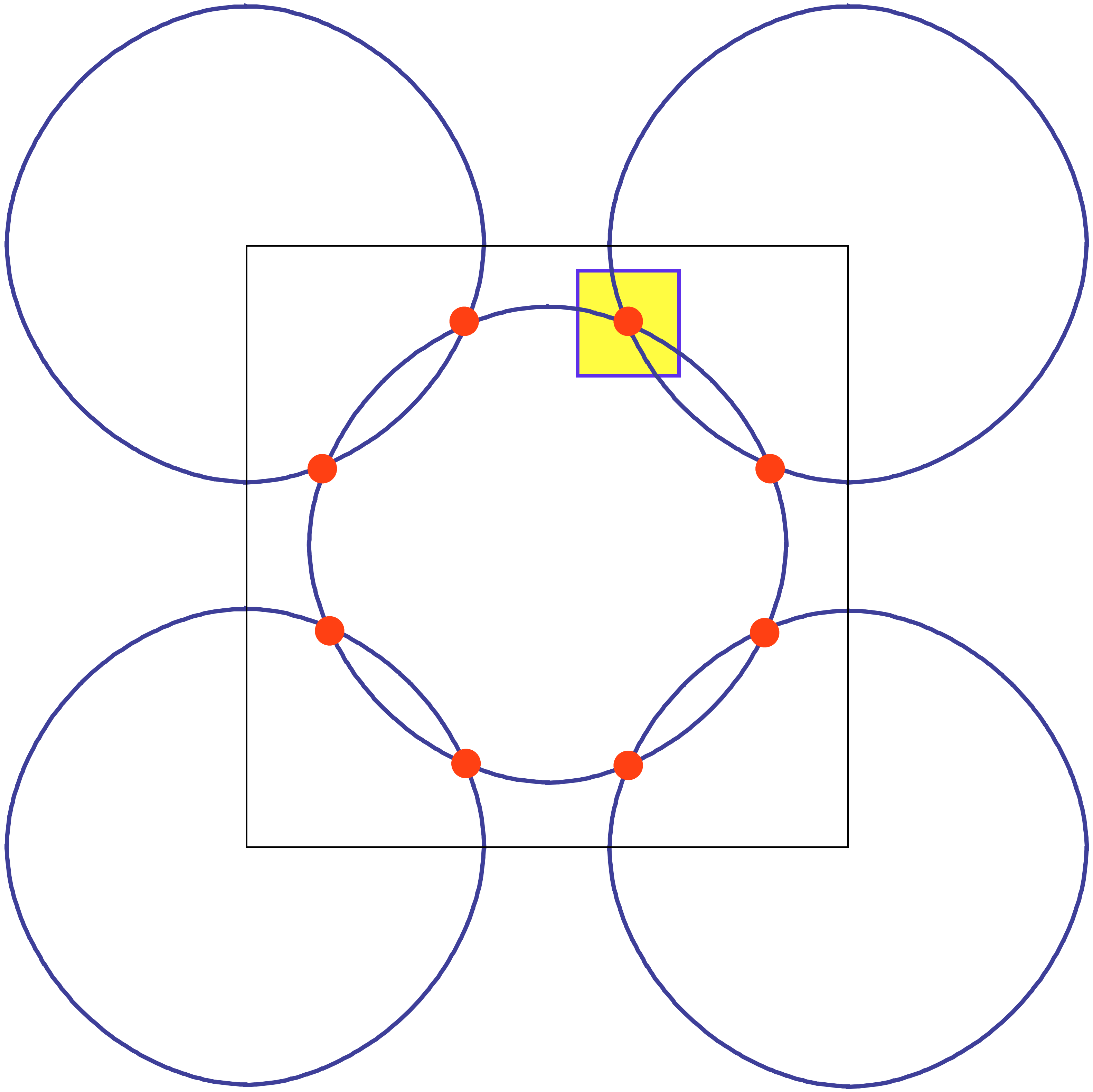
"Hot" spots

Fermi surface+antiferromagnetism



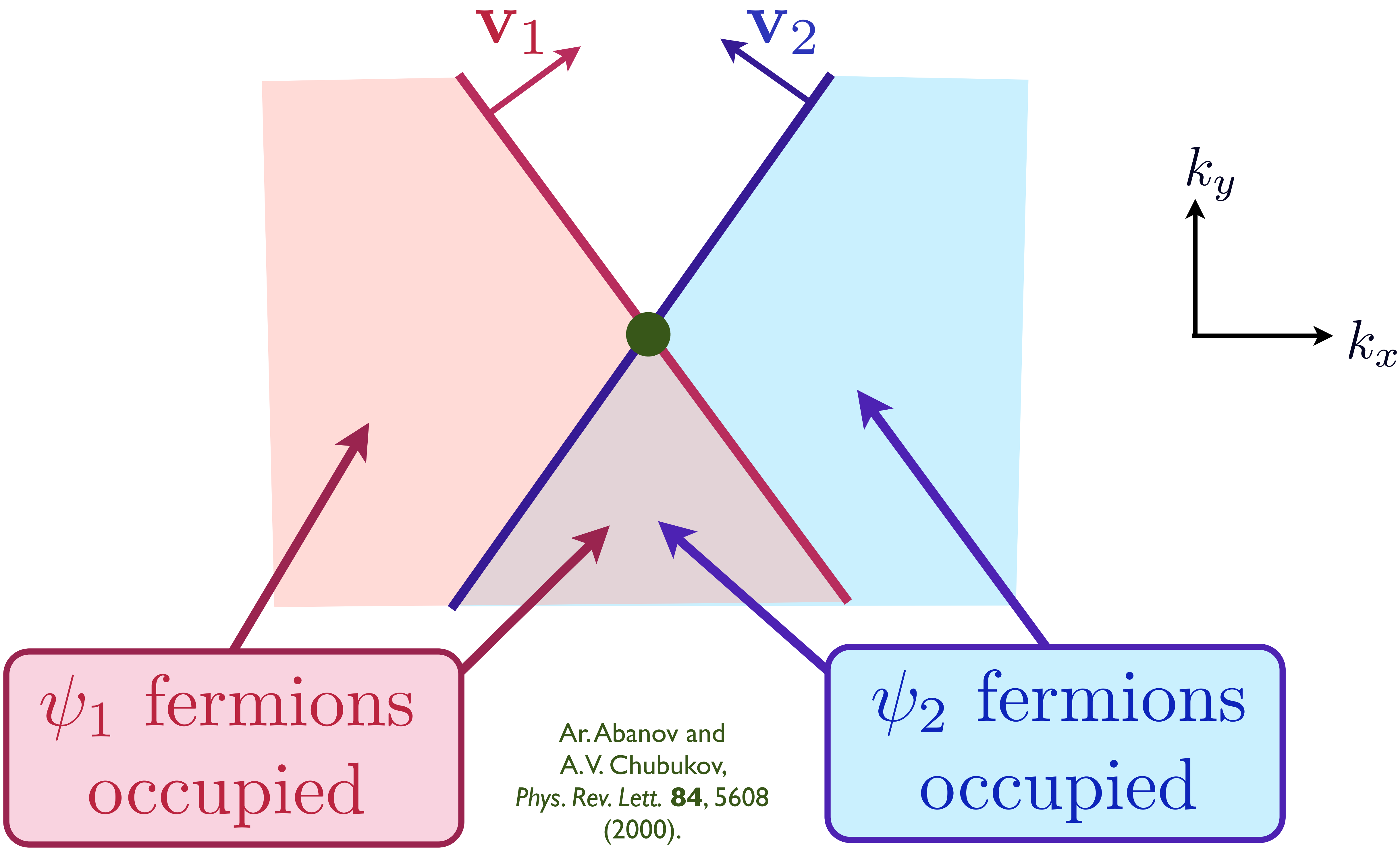
Low energy theory for critical point near hot spots

Fermi surface+antiferromagnetism

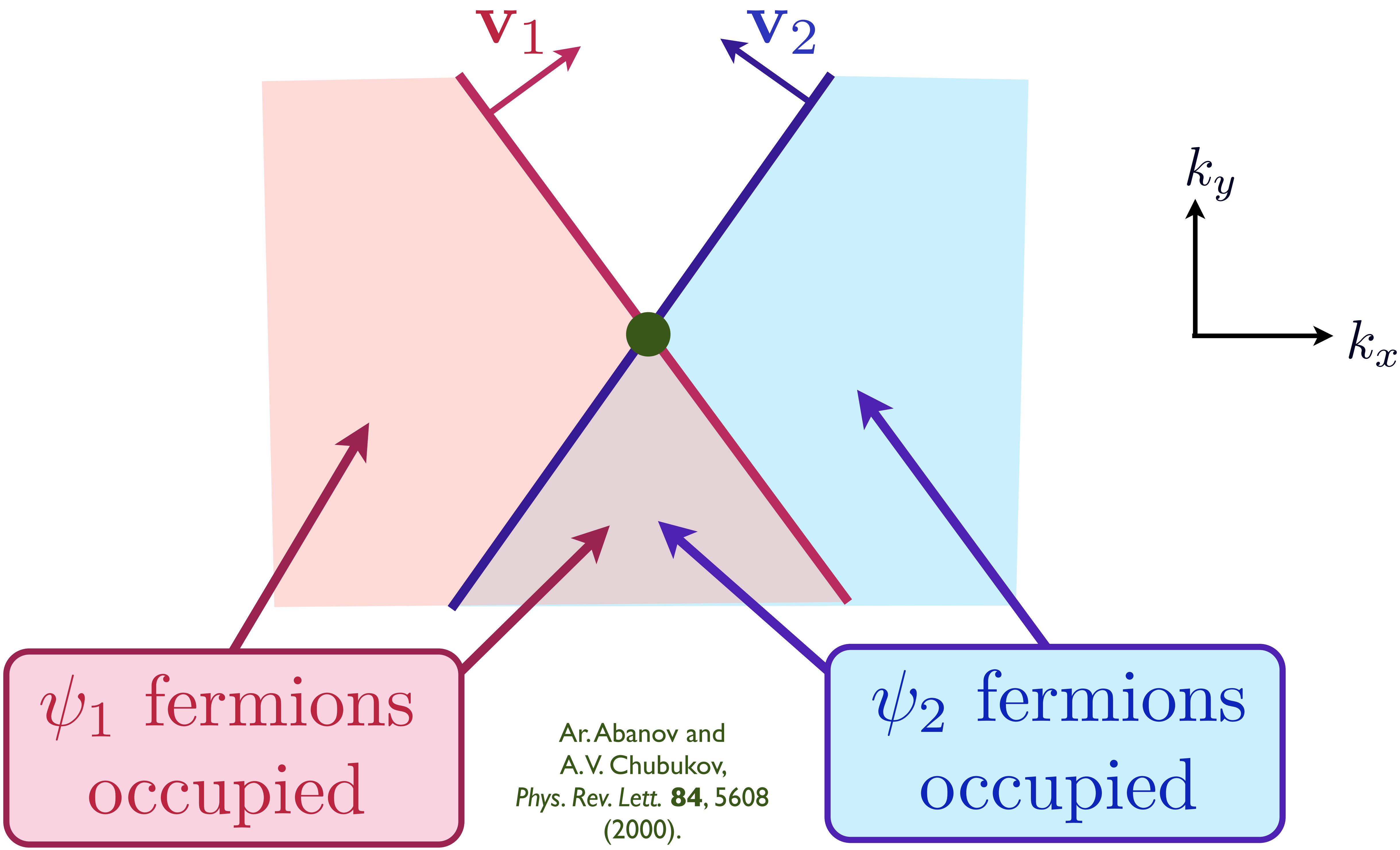


Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ

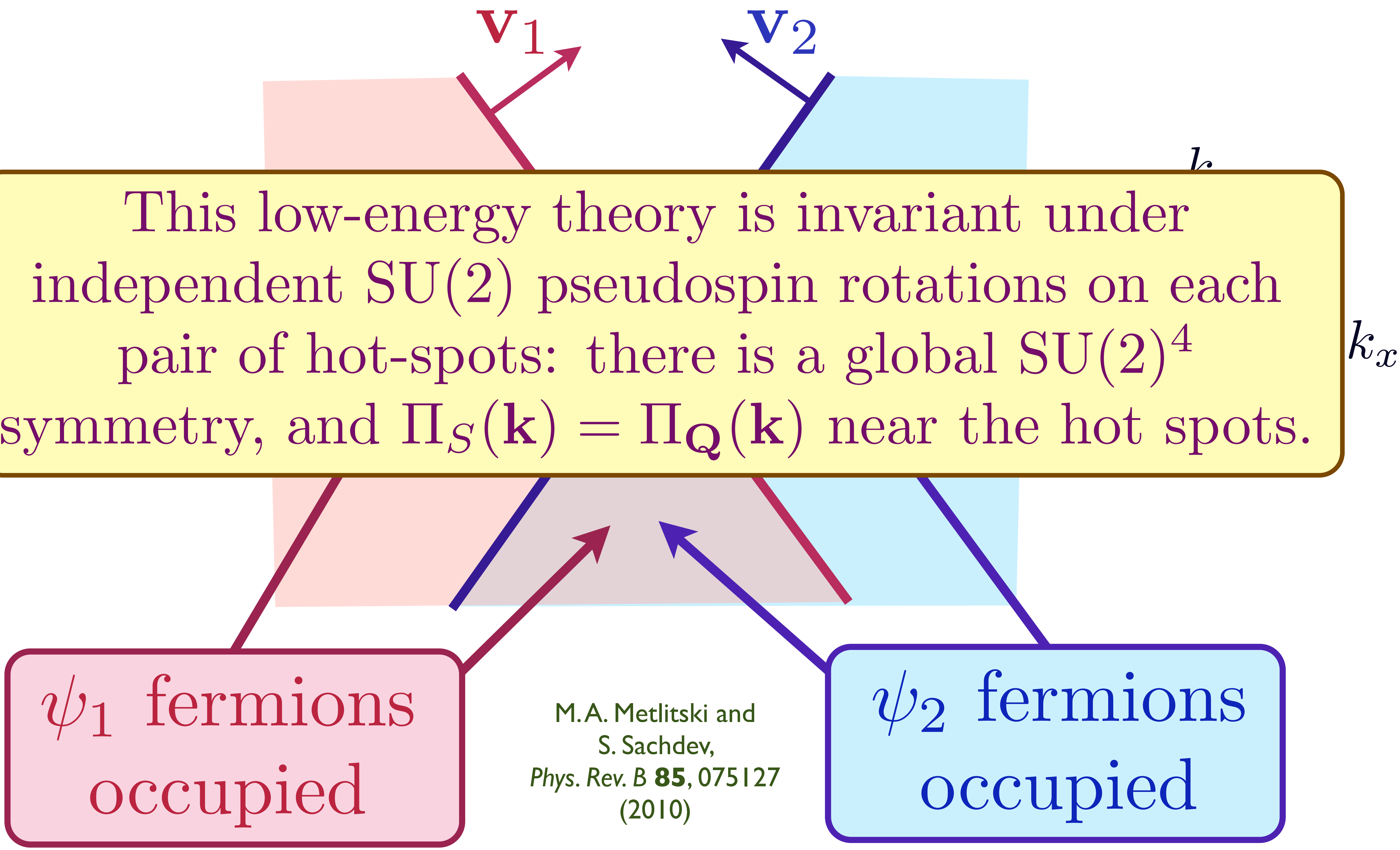


$$
\begin{aligned}
\mathcal{S} \;=\; & \int d^2r\,d\tau \Big[\psi^\dagger_{1\alpha} \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi^\dagger_{2\alpha} \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha} \\
& + \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2} \vec{\varphi}^{\,2} + \frac{u}{4} \vec{\varphi}^{\,4} - \lambda \vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha} \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi^\dagger_{2\alpha} \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \Big]
\end{aligned}
$$



Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **84**, 5608 (2000).

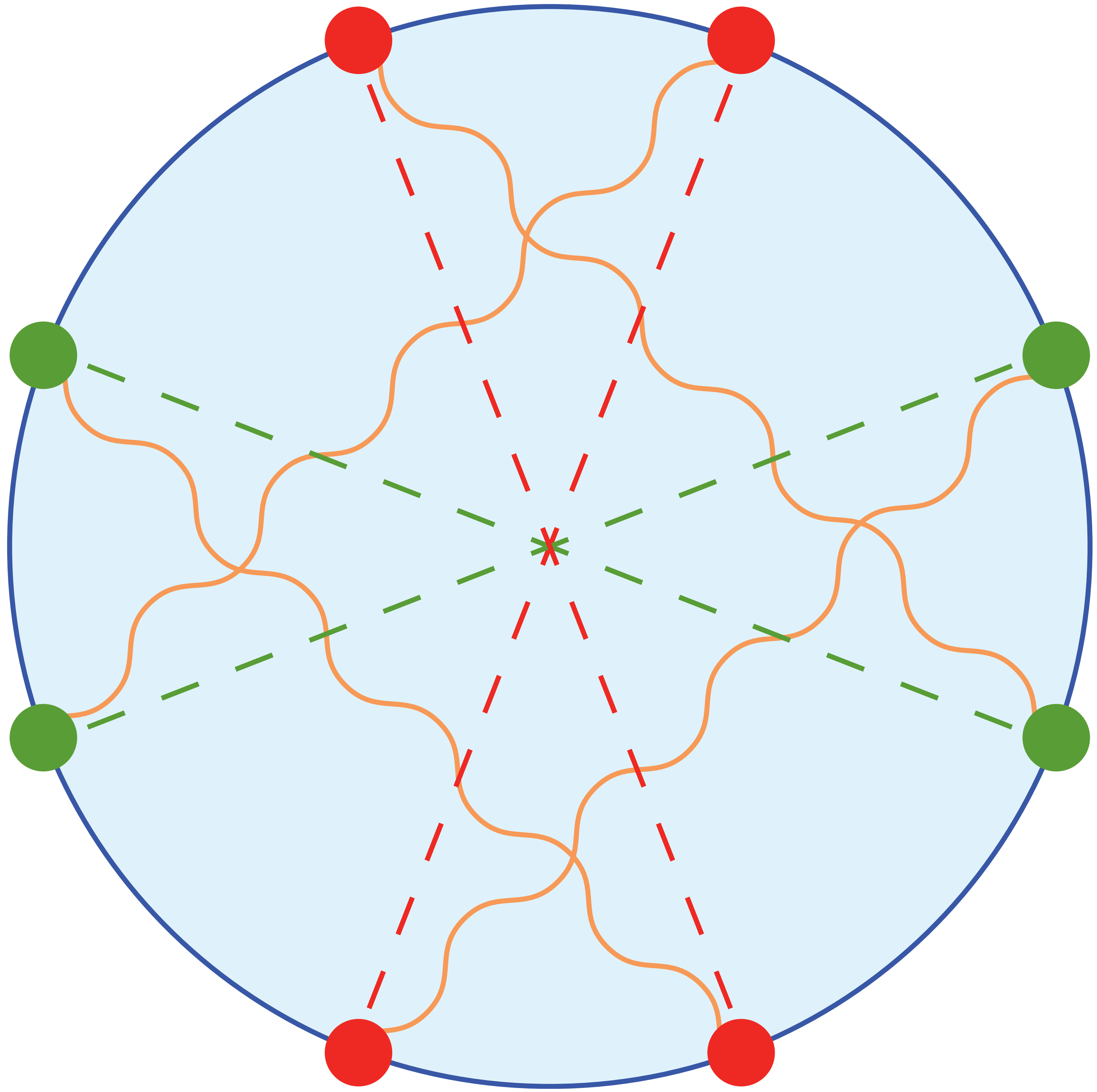
$$\mathcal{S} = \int d^2 r d\tau \Big[\psi^\dagger_{1\alpha} \left(\partial_\tau - i \mathbf{v}_1 \cdot \boldsymbol{\nabla}_r \right) \psi_{1\alpha} + \psi^\dagger_{2\alpha} \left(\partial_\tau - i \mathbf{v}_2 \cdot \boldsymbol{\nabla}_r \right) \psi_{2\alpha}$$
$$+ \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi^\dagger_{1\alpha} \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi^\dagger_{2\alpha} \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \Big]$$



This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots: there is a global $\mathrm{SU}(2)^4$ symmetry, and $\Pi_S(\mathbf{k}) = \Pi_{\mathbf{Q}}(\mathbf{k})$ near the hot spots.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

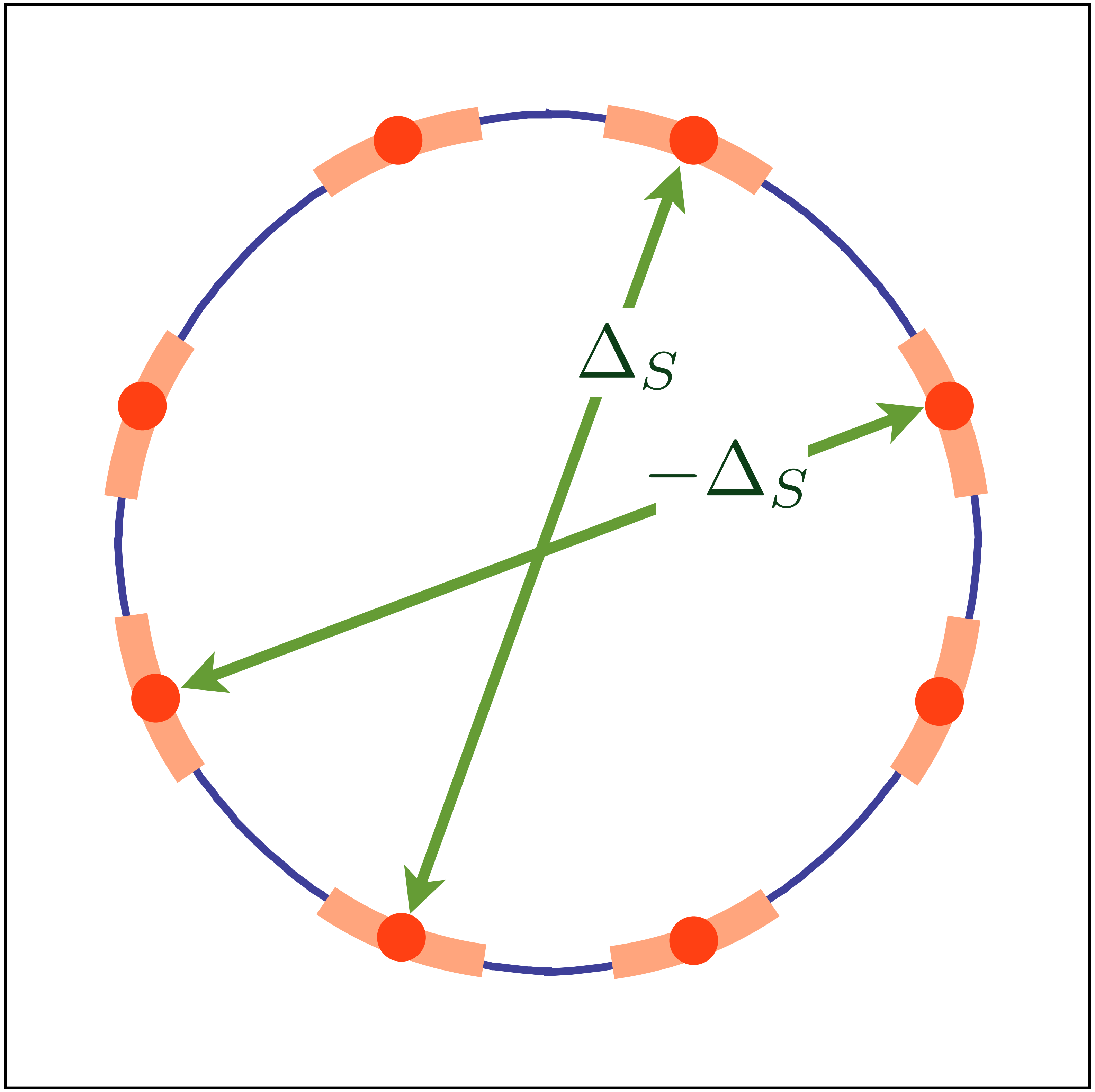
V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

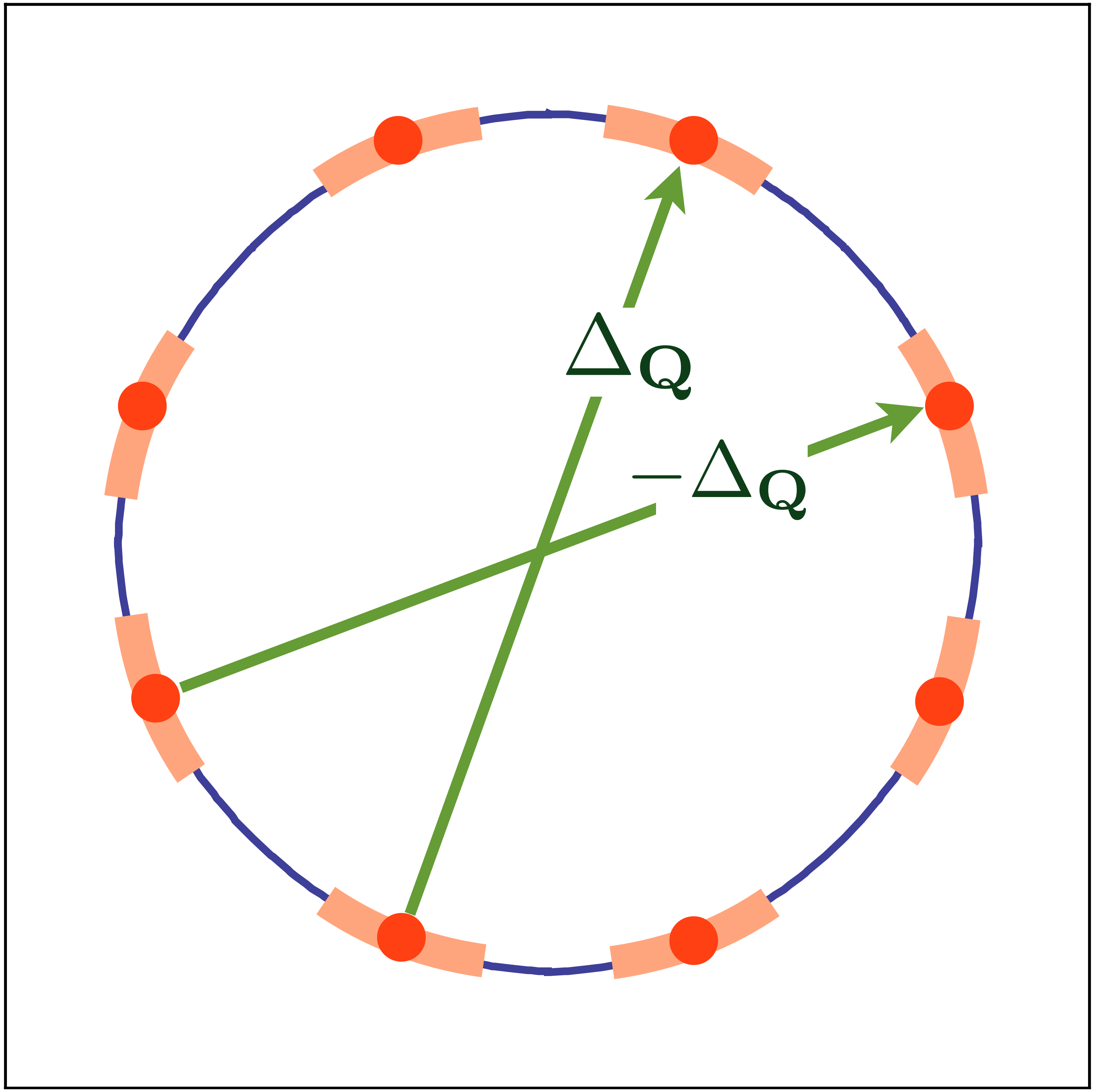
V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation on *half* the hot-spots

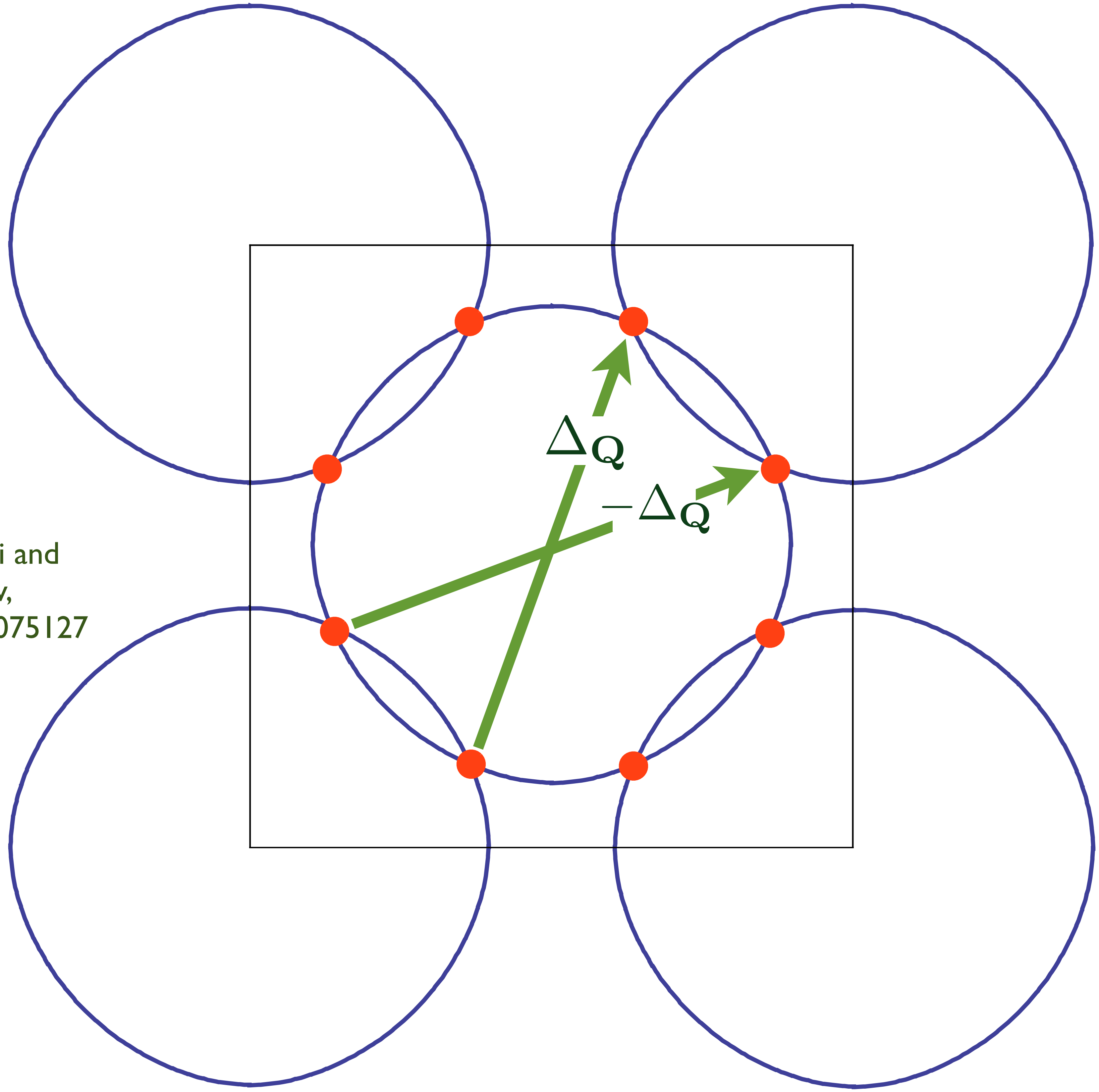


$\mathbf{Q}$ is '$2k_F$' wavevector

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Unconventional particle-hole pairing at *and near* hot spots

Incommensurate *d*-wave bond order



M. A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

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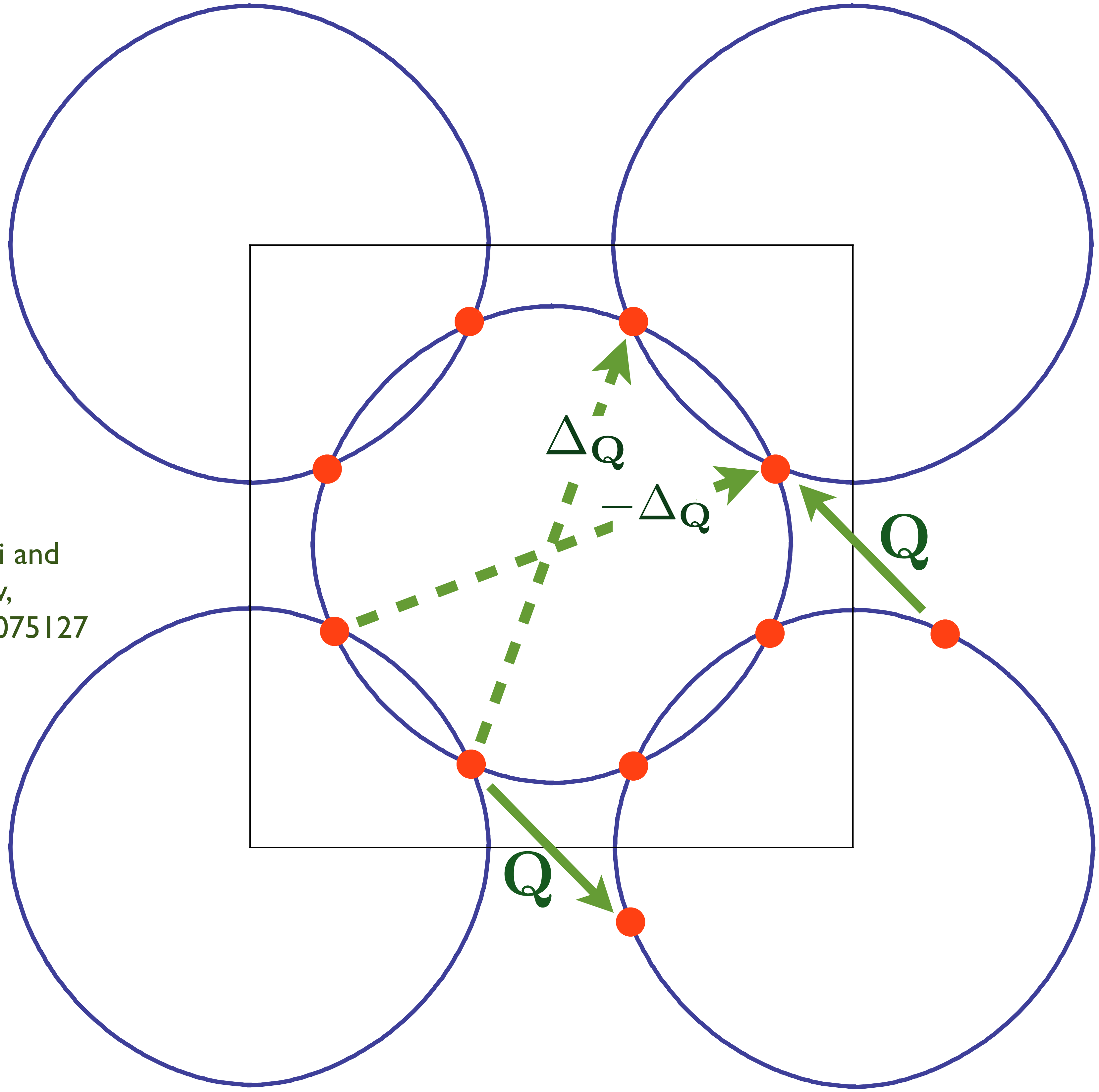
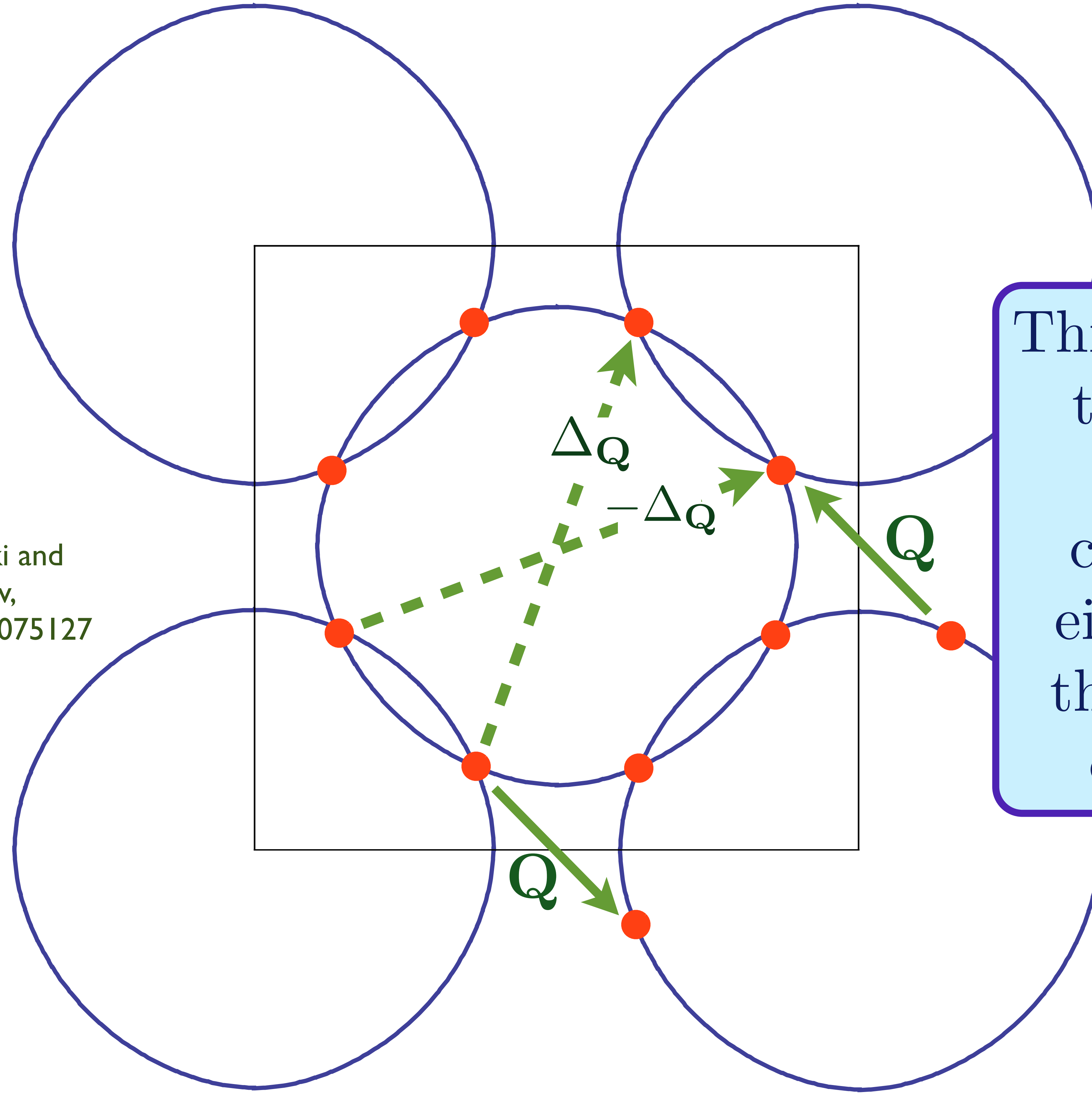


M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

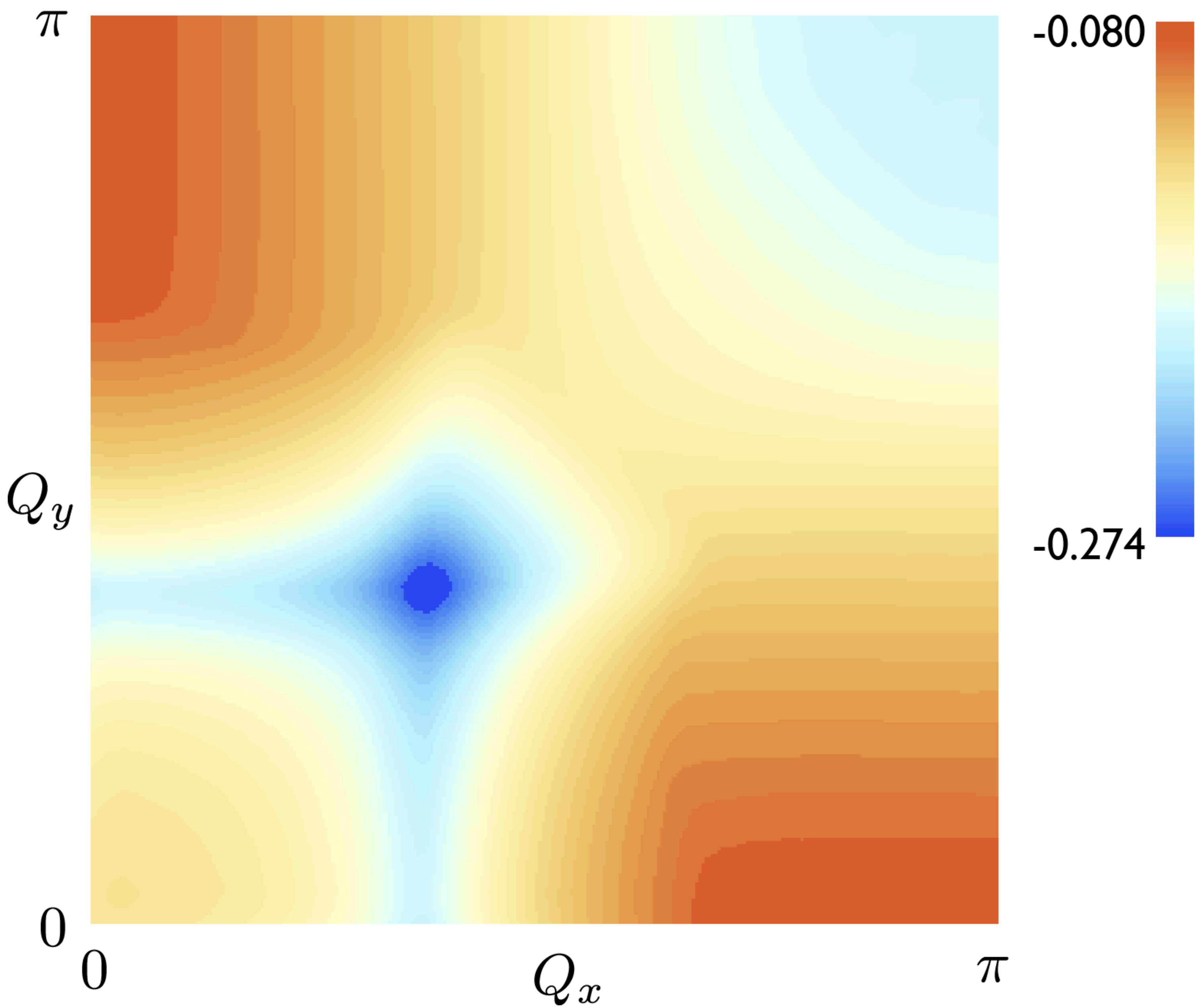
Incommensurate *d*-wave bond order



This $\mathbf{Q}$ is very close to the $\mathbf{Q}$ which minimizes the charge-ordering eigenvalue $\lambda_{\mathbf{Q}}$ in the Hartree-Fock computation !

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

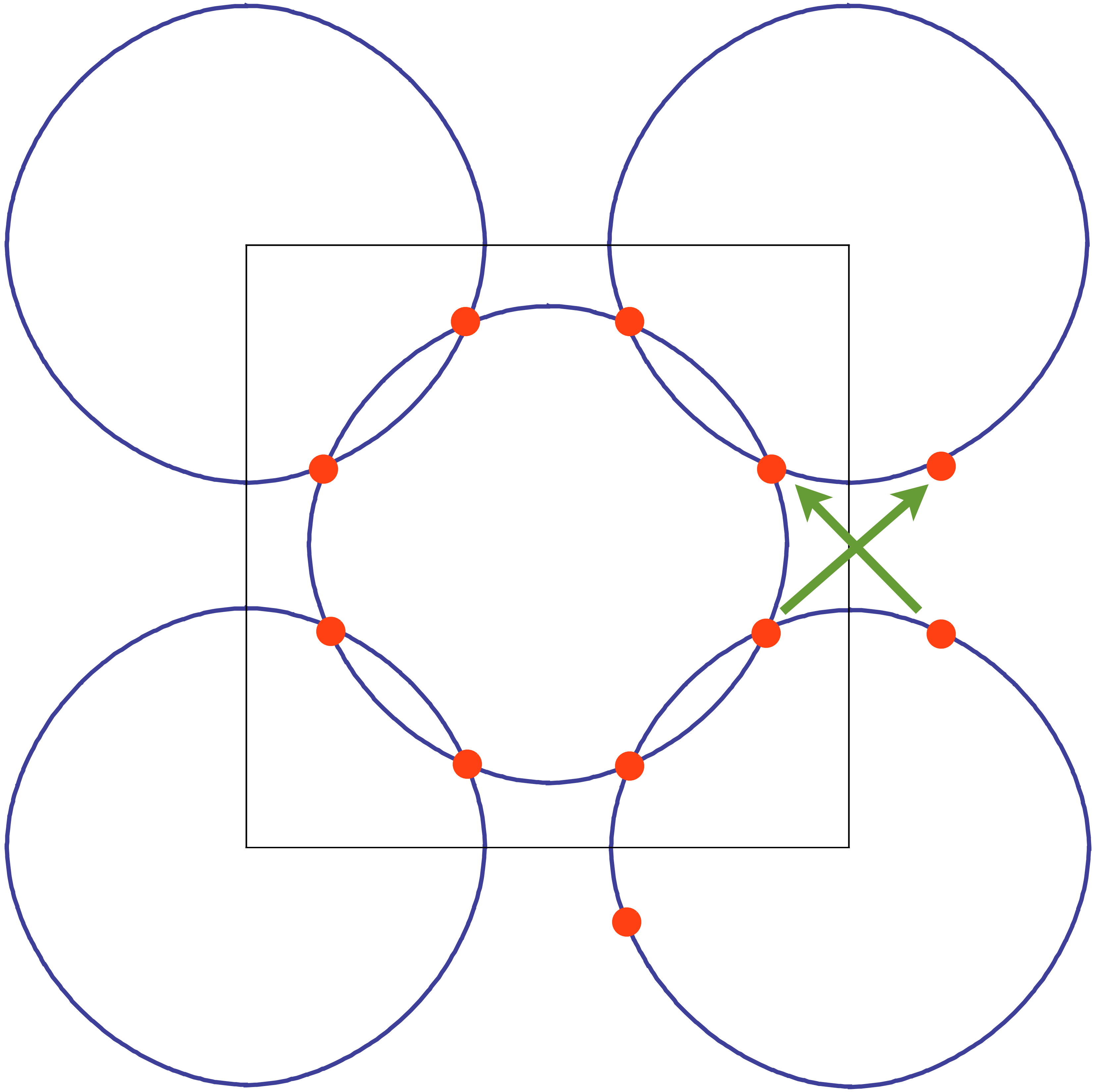
$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, arXiv:1303.2114

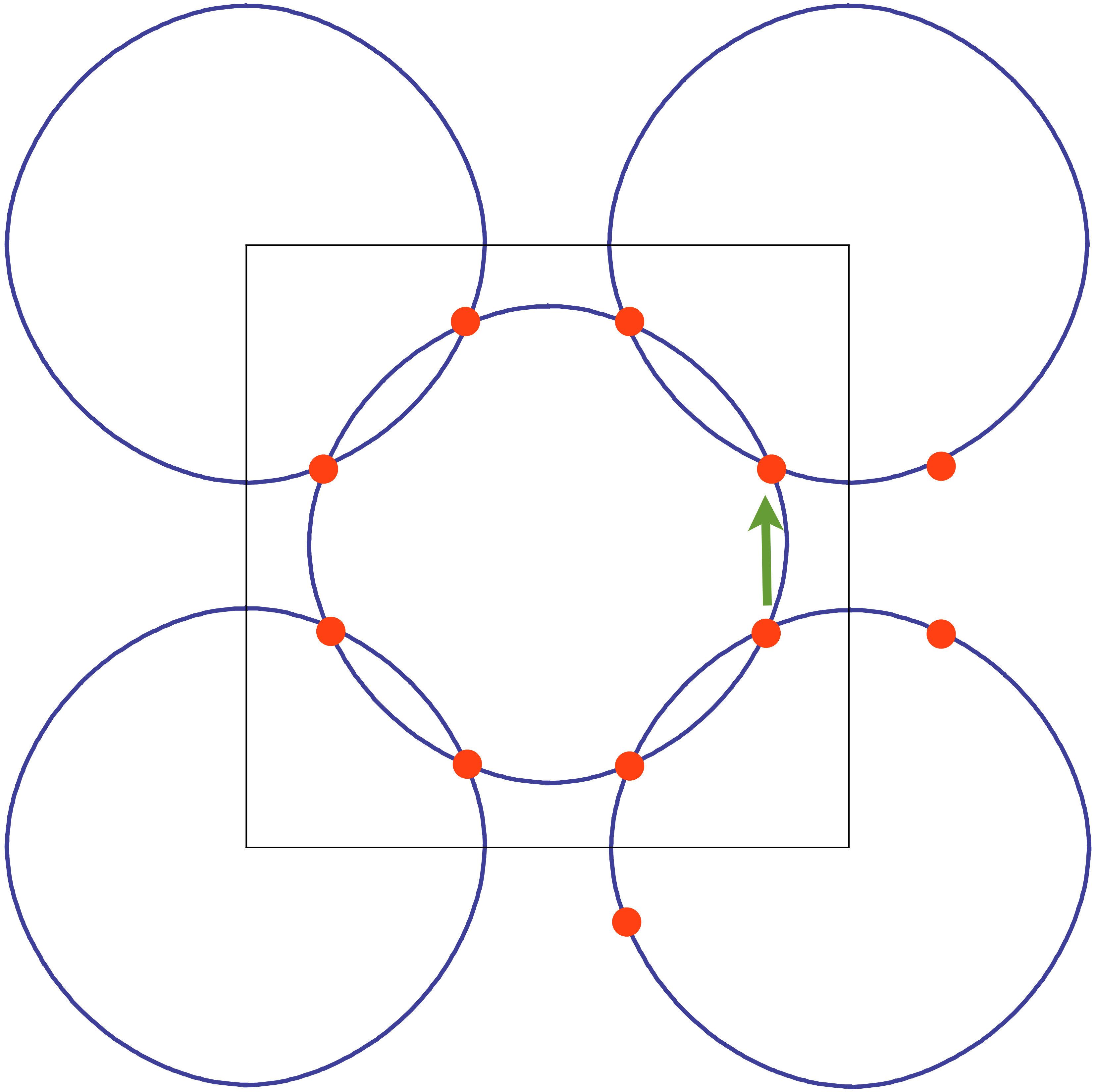
Incommensurate *d*-wave bond order



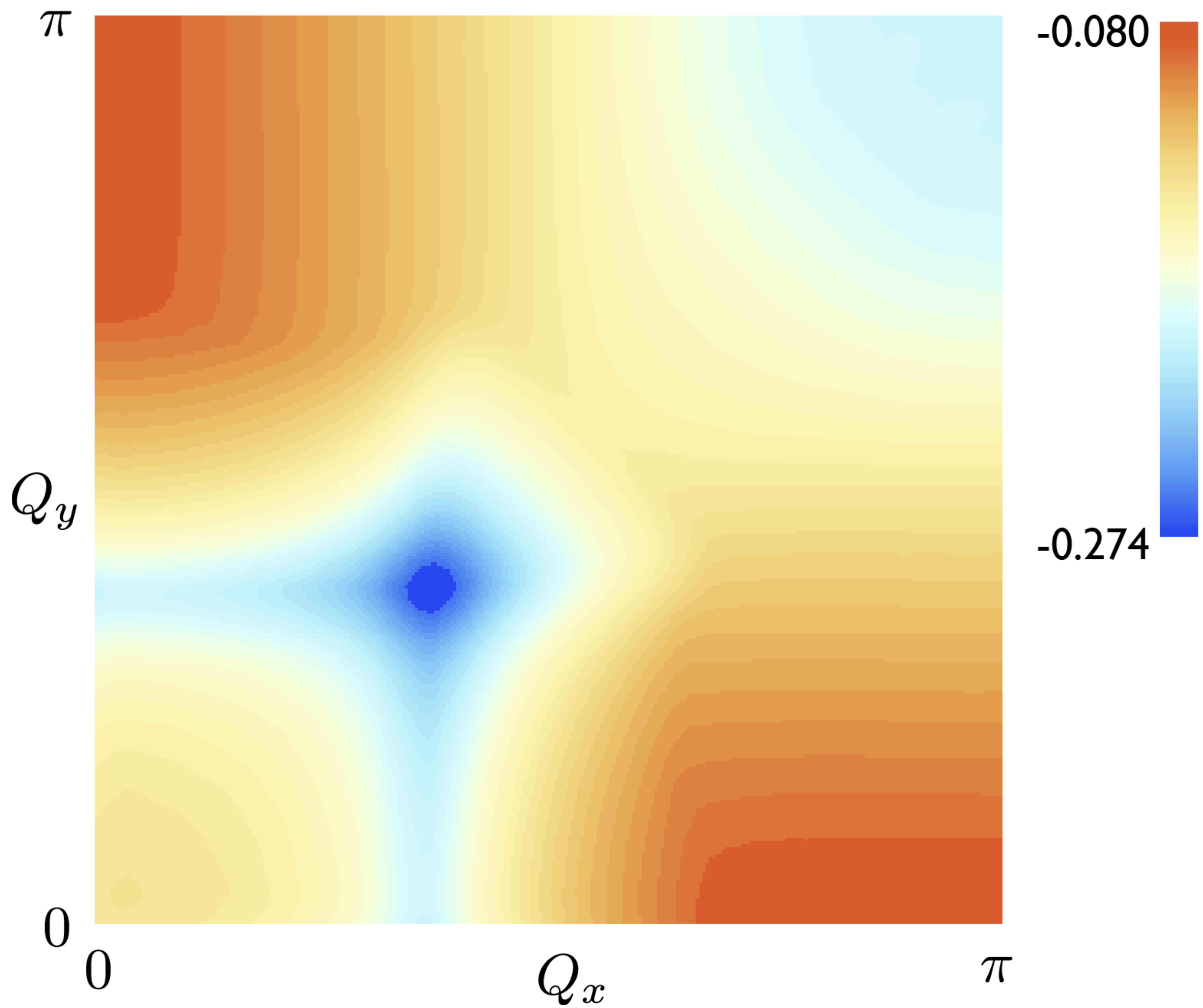
High T pseudogap: Fluctuating composite order parameter of nearly degenerate d-wave pairing and incommensurate d-wave bond order. (Approximate) SU(2) symmetry of composite order prevents long-range order $T > 0$.

K. B. Efetov, H. Meier, and C. Pepin, arXiv:1210.3276

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$



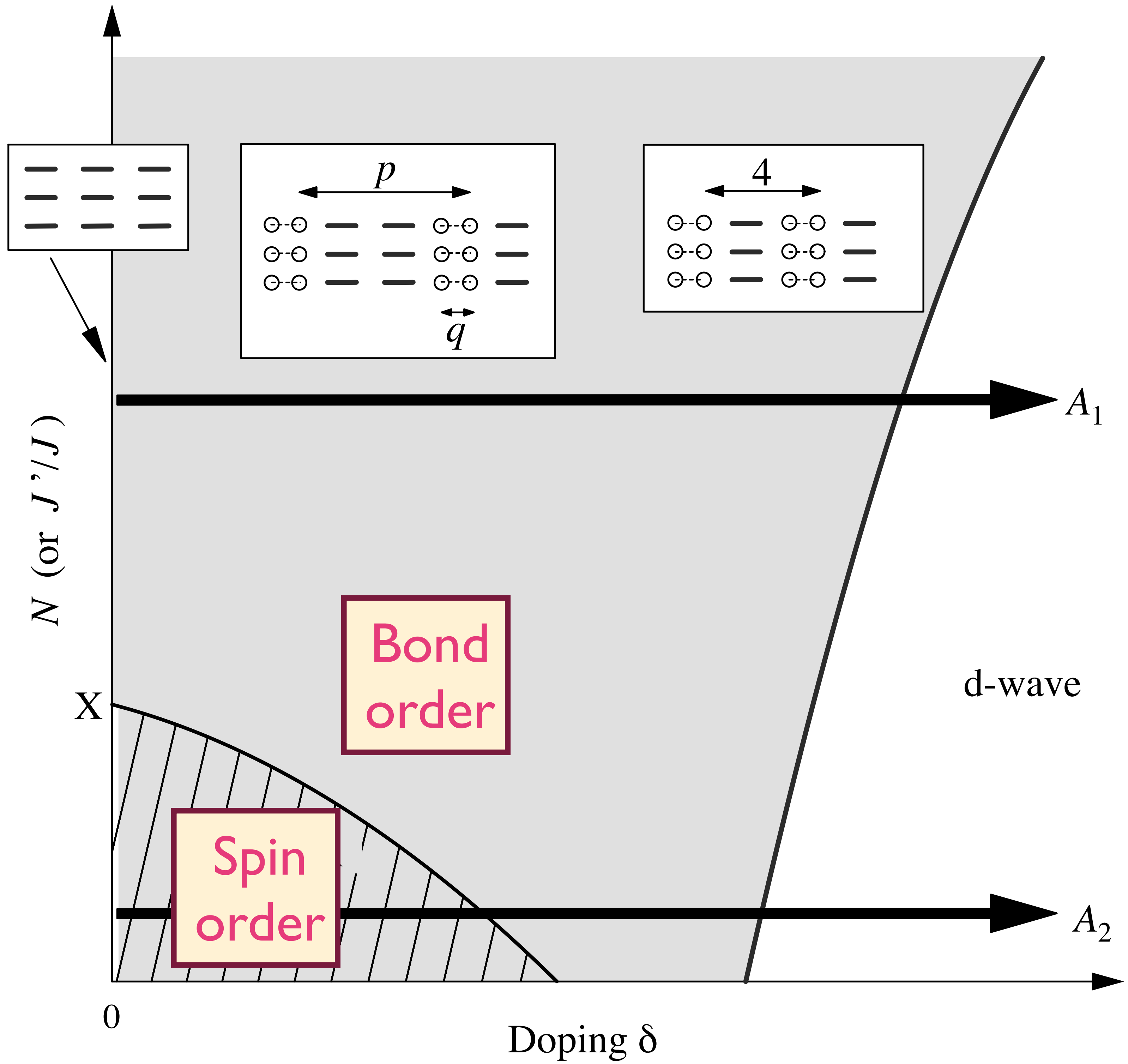
Observed
low T ordering



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Evidence bond order is along (1,0), (0,1) directions in low *T* superconducting phase



M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991)

Evidence bond order is along (1,0), (0,1) directions in low *T* superconducting phase

PHYSICAL REVIEW B **77**, 094504 (2008)

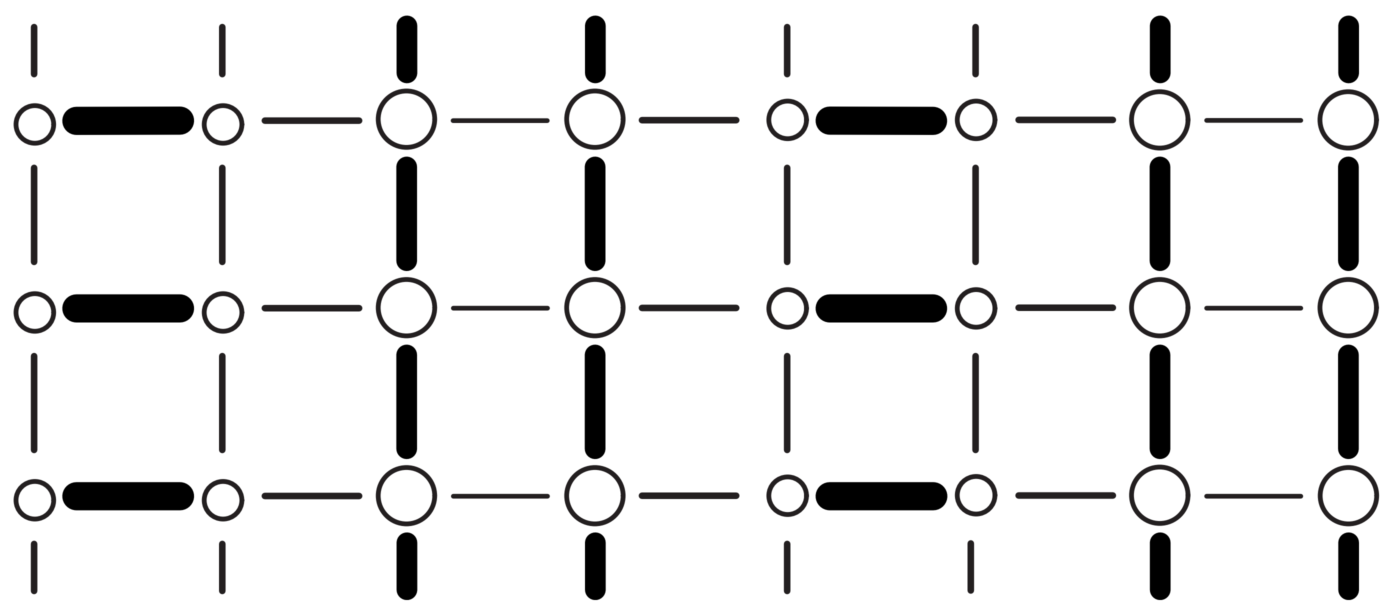
Superconducting *d*-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

Matthias Vojta and Oliver Rösch

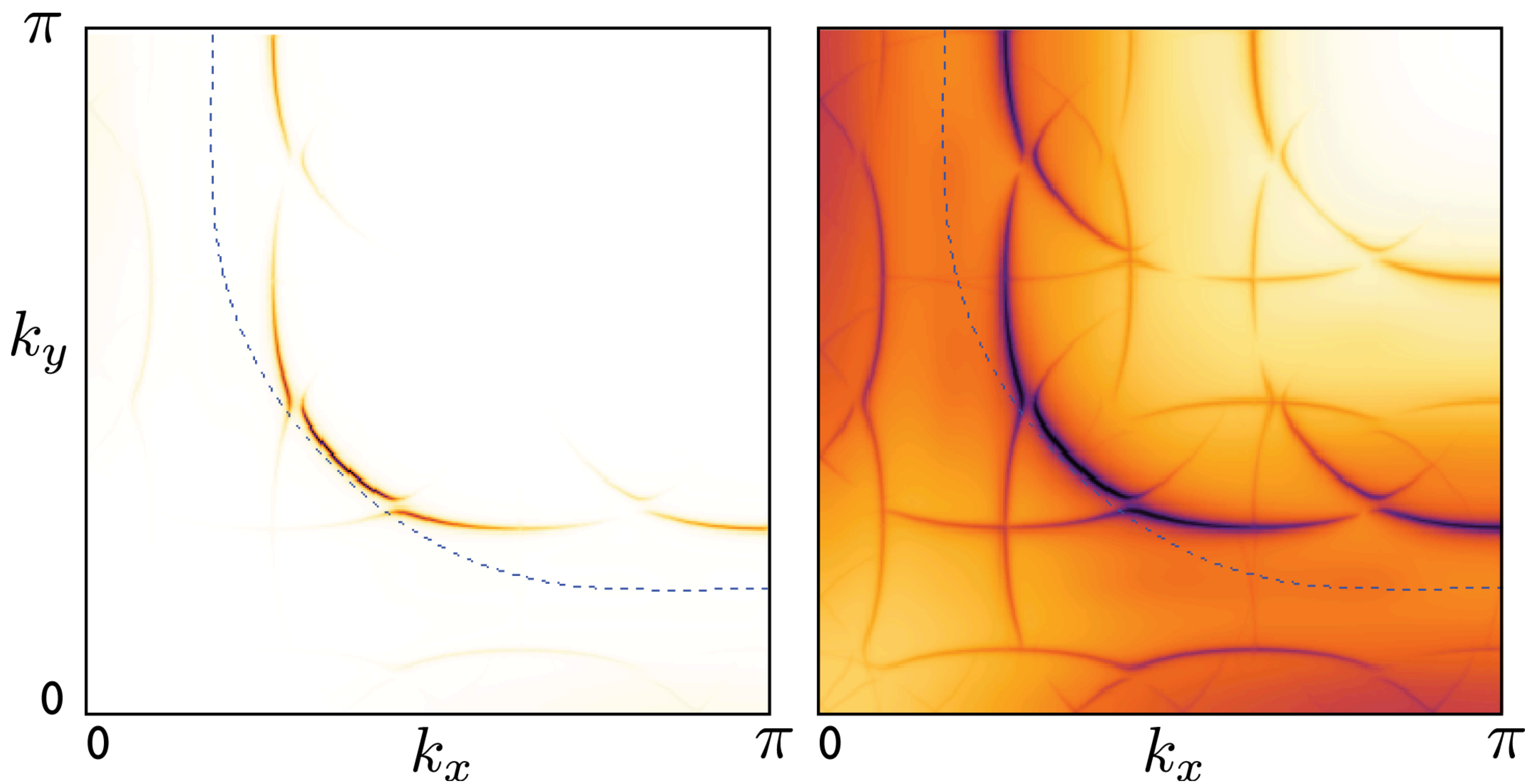
Institut für Theoretische Physik, Universität zu Köln, Zülpicher Straße 77, 50937 Köln, Germany

(Received 8 January 2008; revised manuscript received 10 January 2008; published 6 March 2008)

We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both *s*- and *d*-wave channels, with nonlocal Coulomb repulsion suppressing the *s*-wave component. The resulting bond-charge-density wave, coexisting with superconductivity, is compatible with recent photoemission and tunneling data and as well as neutron-scattering measurements, once long-range order is destroyed by slow fluctuations or glassy disorder. In particular, the real-space structure of *d*-wave stripes is consistent with the scanning-tunneling-microscopy measurements on both underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ and $Ca_{2-x}Na_xCuO_2Cl_2$ of Kohsaka *et al.* [Science **315**, 1380 (2007)].



Electron spectral function



$\mathrm{Im}G(\mathbf{k}, \omega + i\eta)$ $\quad$ $\log \mathrm{Im}G(\mathbf{k}, \omega + i\eta)$

$$\left\langle c^\dagger_{\boldsymbol{k}-\boldsymbol{Q}/2,\alpha} c_{\boldsymbol{k}+\boldsymbol{Q}/2,\alpha} \right\rangle \propto \Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \begin{cases} \Delta_s + \Delta_d(\cos k_x - \cos k_y) & , \quad \boldsymbol{Q} = (\pm Q_0, 0) \\ \Delta_s - \Delta_d(\cos k_x - \cos k_y) & , \quad \boldsymbol{Q} = (0, \pm Q_0) \end{cases}$$

with $\Delta_s/\Delta_d = -0.234.$

S. Sachdev and R. La Placa, arXiv:1303.2114

Outline

1. Stability of metal in Hartree-Fock-BCS theory

2. Emergent pseudospin symmetry in low energy theory of metal with antiferromagnetic interactions

3. Quantum Monte Carlo without the sign problem

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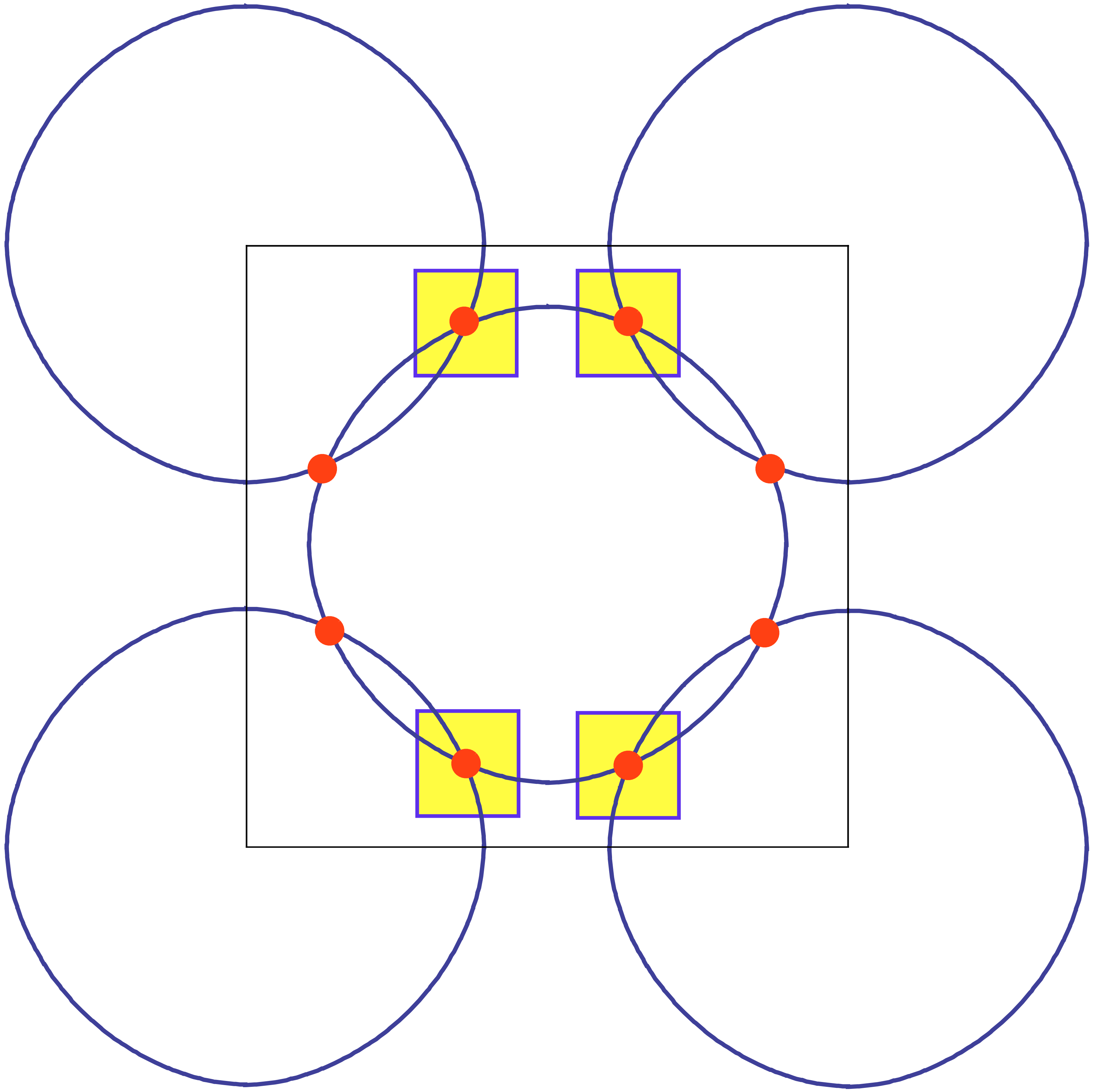
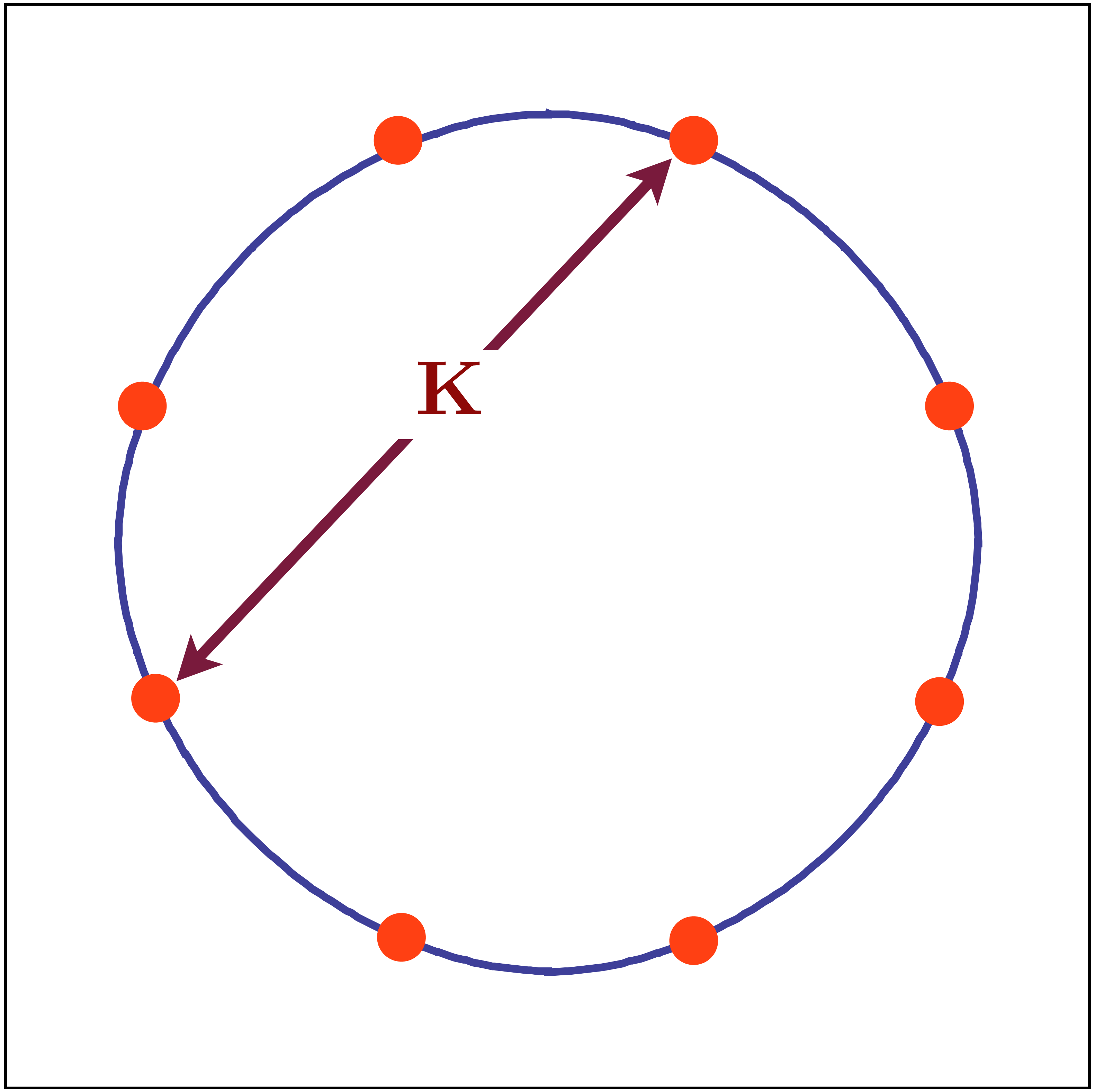


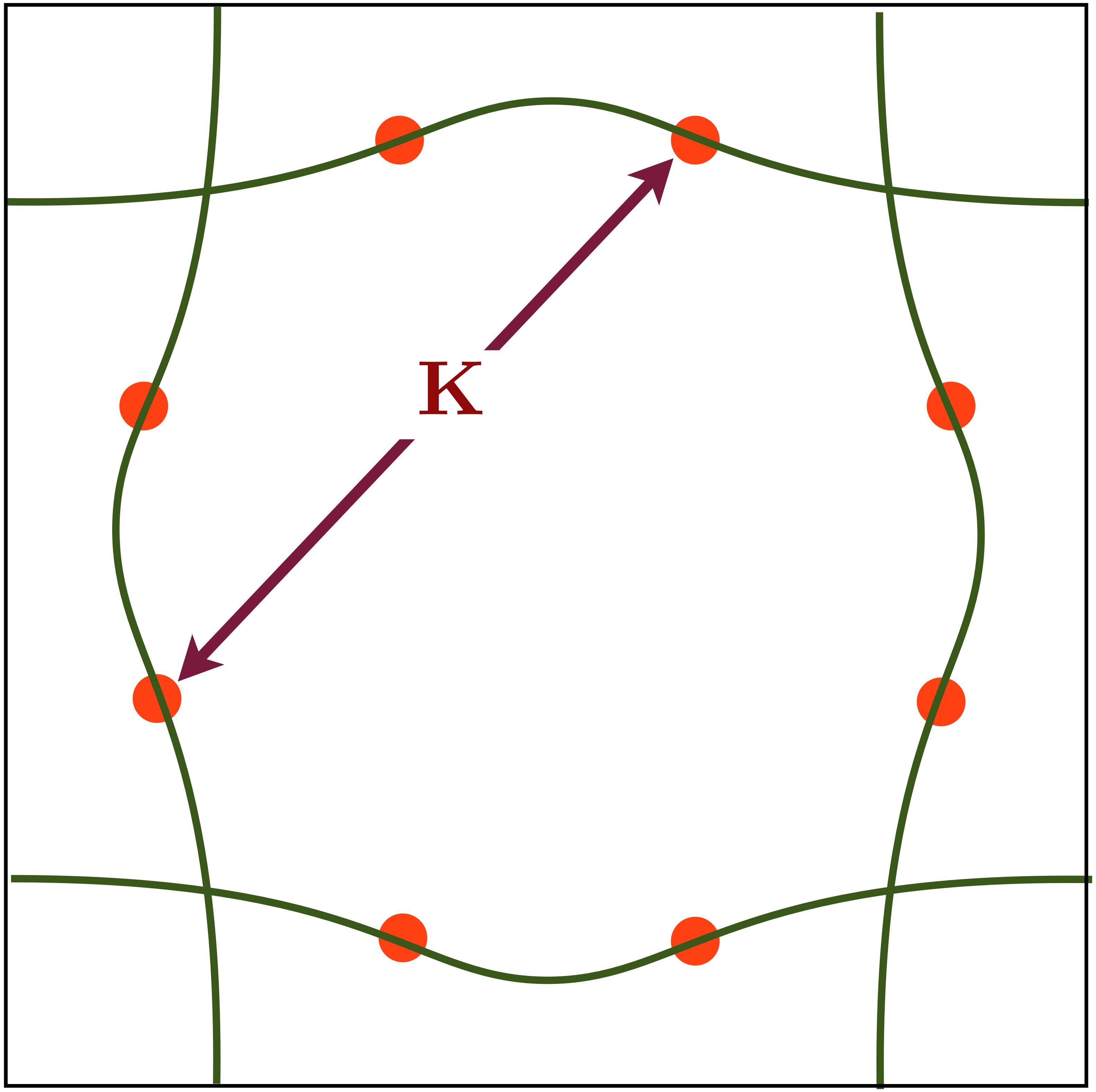
Low energy theory for critical point near hot spots

QMC for the onset of antiferromagnetism



Hot spots in a single band model

QMC for the onset of antiferromagnetism

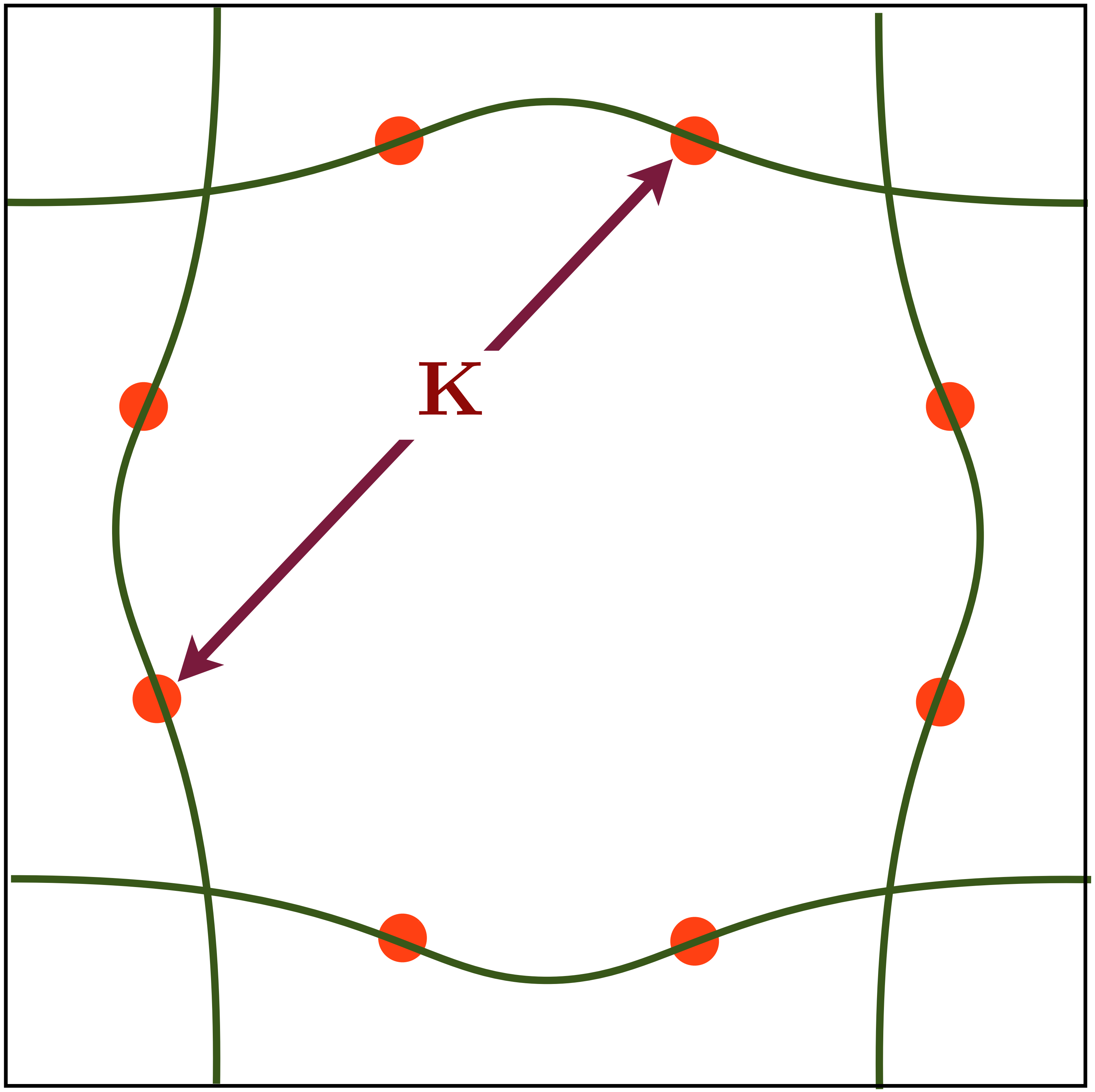


E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

Hot spots in a two band model

QMC for the onset of antiferromagnetism

Faithful realization of the *generic* universal low energy theory for the onset of antiferromagnetism.

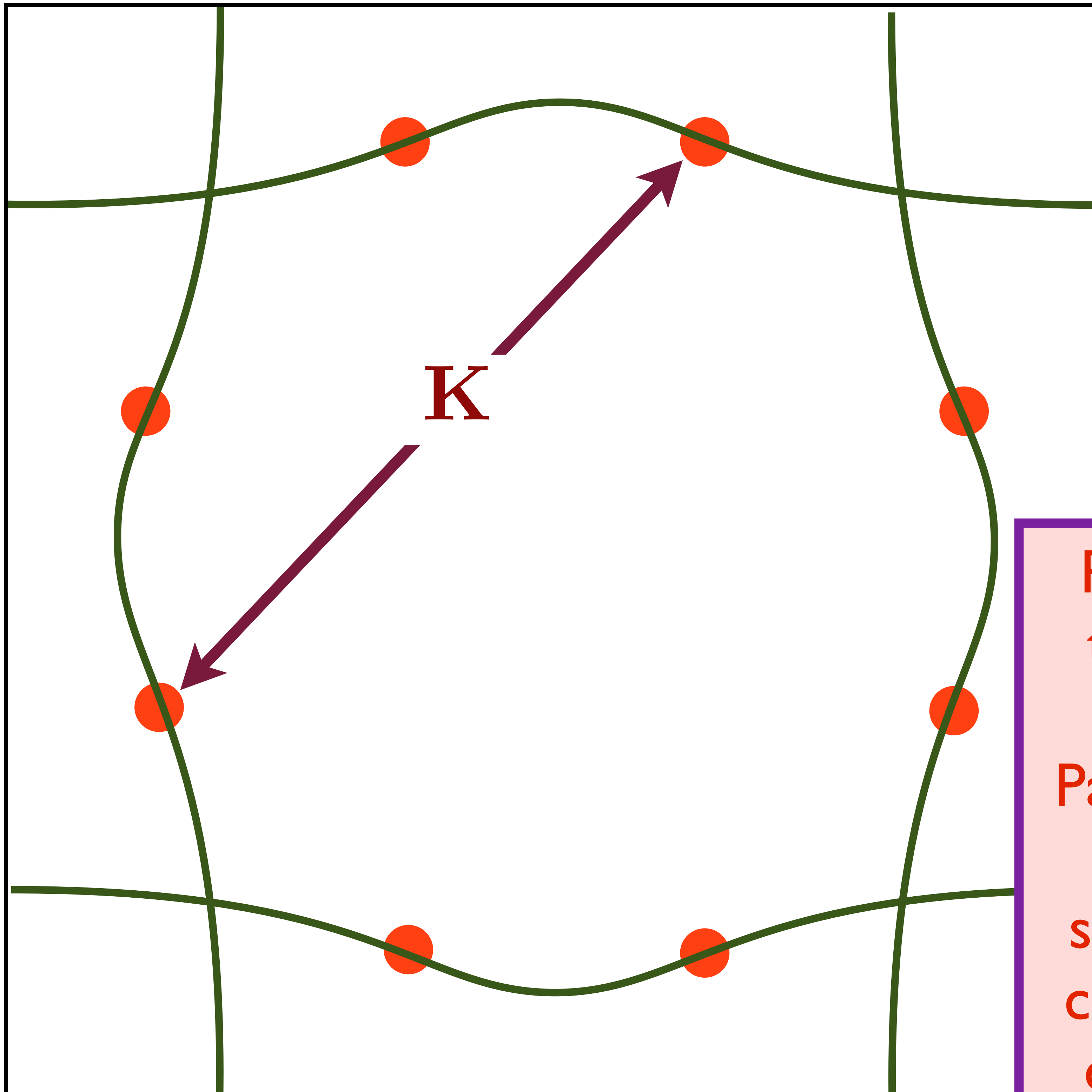


Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism

Sign problem is absent as long as ***K*** connects hotspots in distinct bands



Hot spots in a two band mode

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Requires only time-reversal symmetry. Particle-hole or point-group symmetries or commensurate densities ***not*** required !

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_{\alpha}^{(x)} \mathcal{D}c_{\alpha}^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

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E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
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E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

No sign problem !

QMC for the onset of antiferromagnetism

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E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Applies without changes to the microscopic band structure in the iron-based superconductors

QMC for the onset of antiferromagnetism

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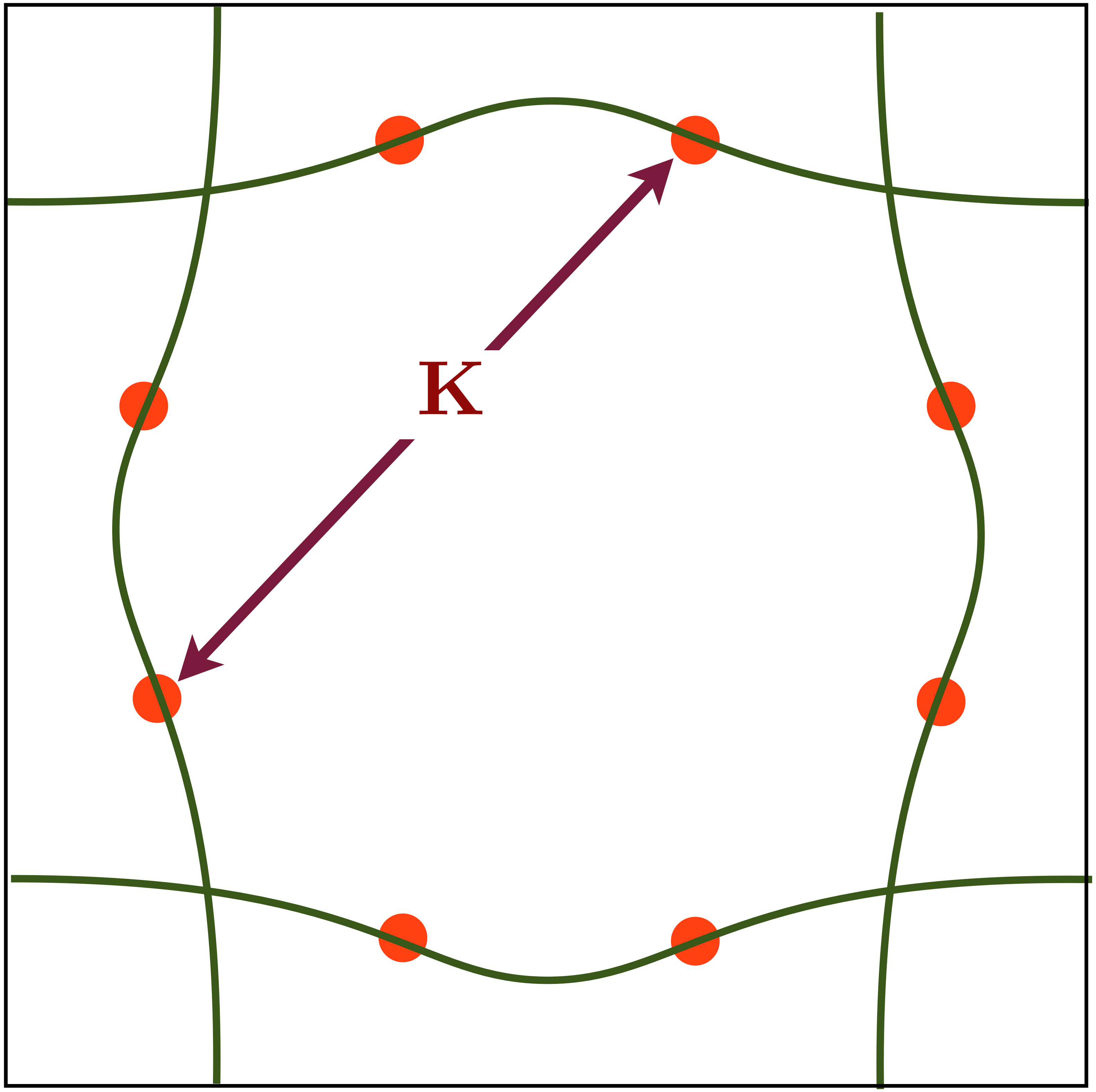
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Can integrate out $\vec{\varphi}$ to obtain an extended Hubbard model. The interactions in this model only couple electrons in separate bands.

QMC for the onset of antiferromagnetism



E. Berg,
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Science **338**, 1606
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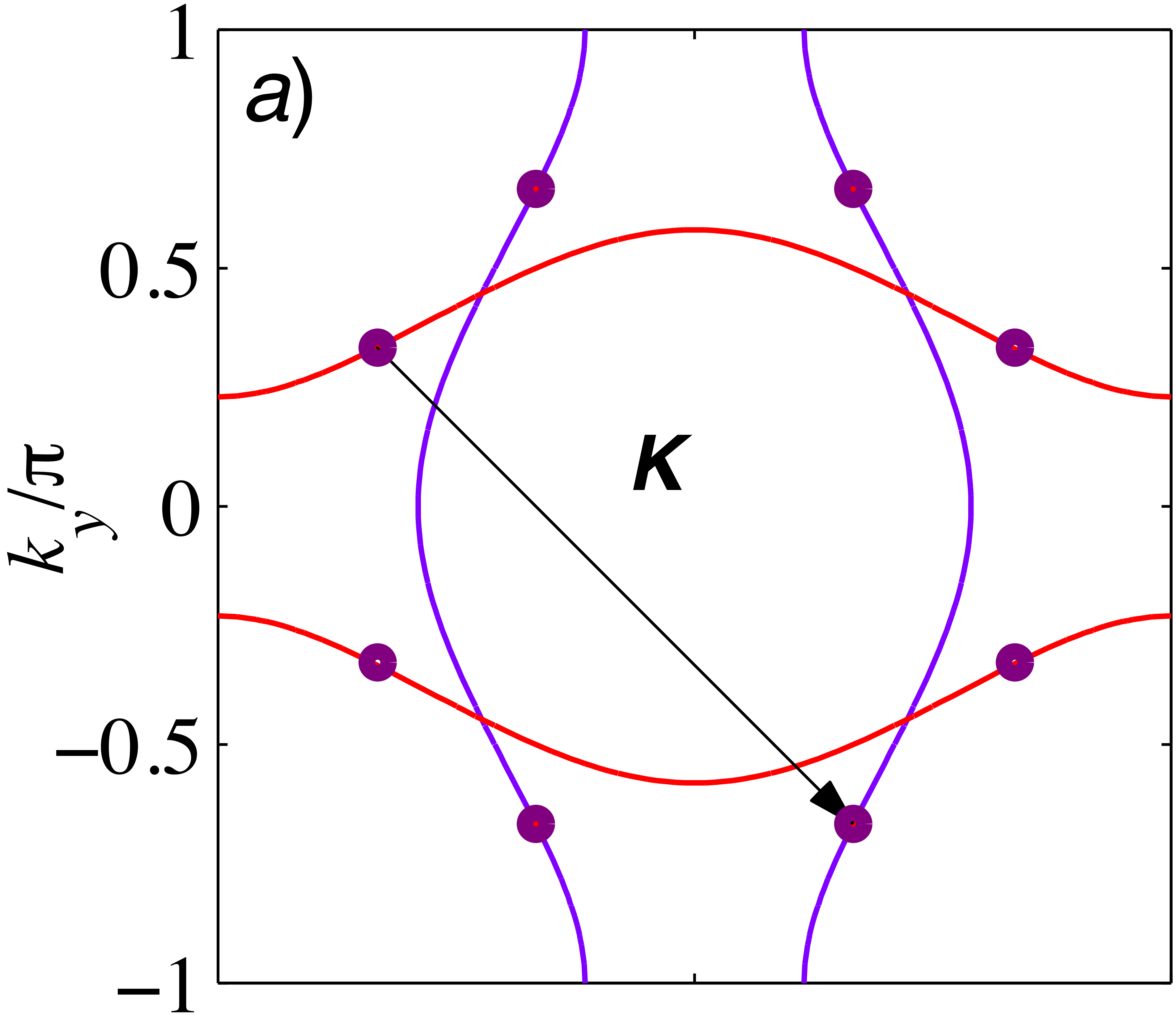
Hot spots in a two band model

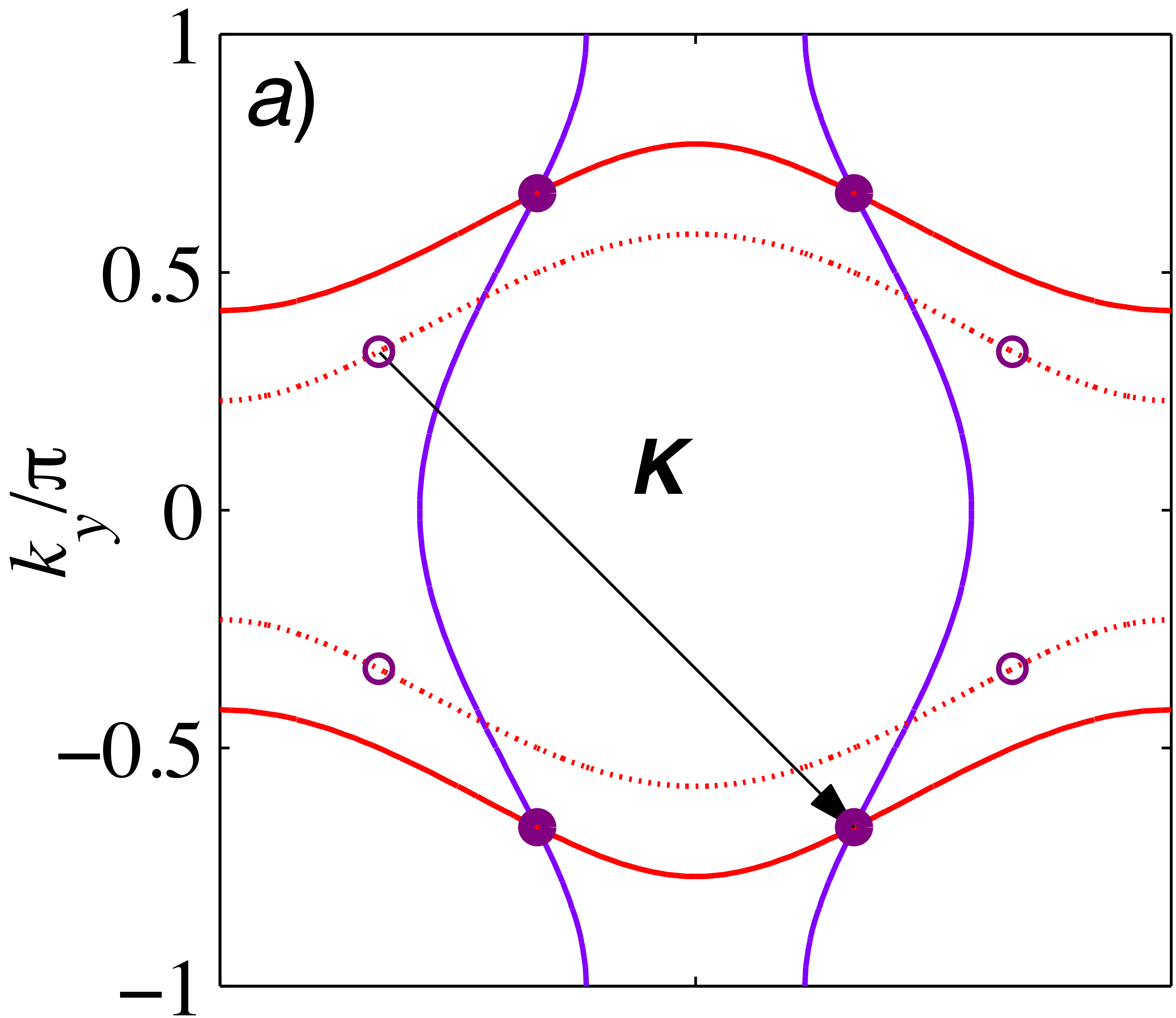
QMC for the onset of antiferromagnetism



Center Brillouin zone at (π,π,)

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism



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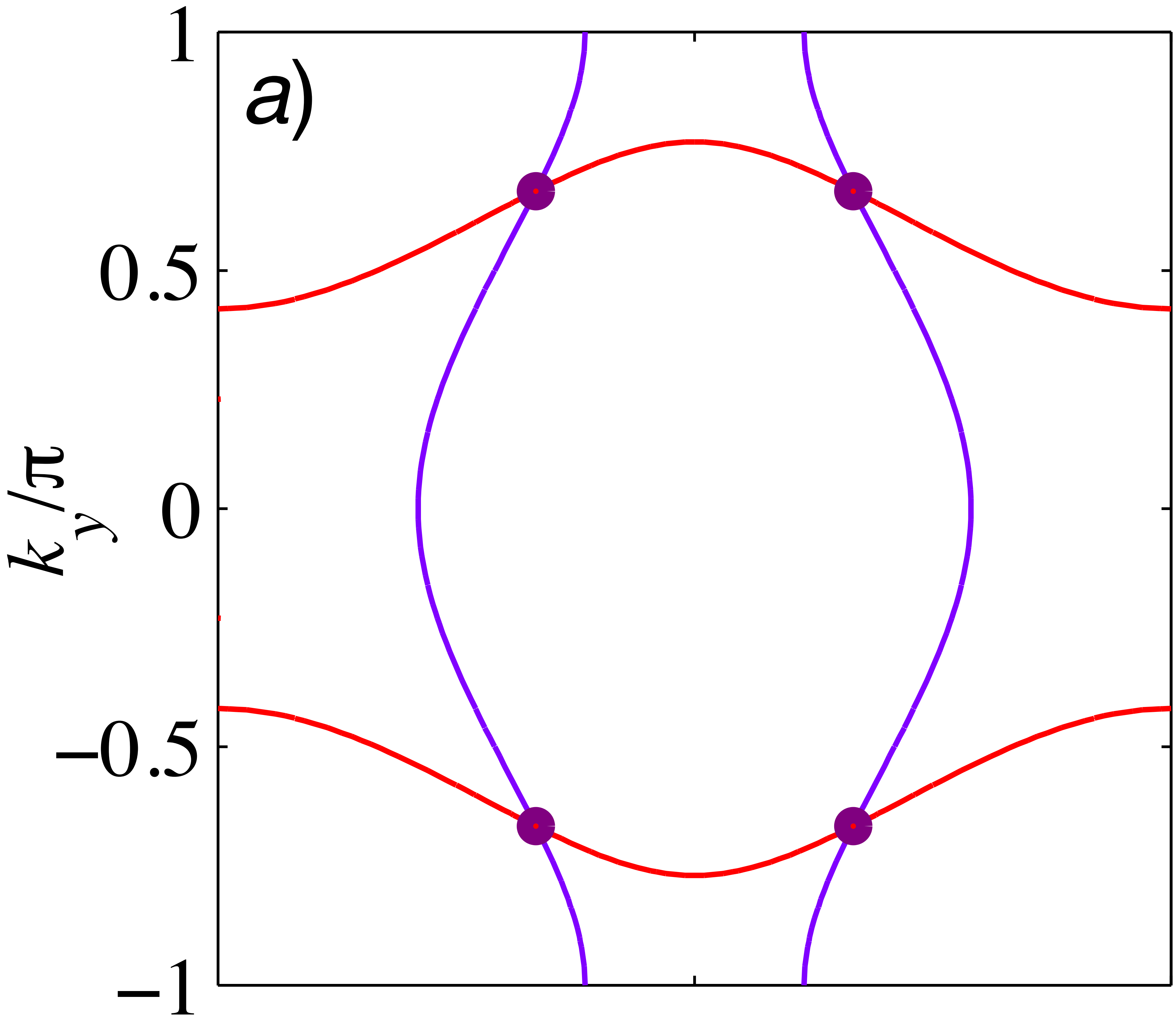
Move one of the Fermi surface by (π,π,)

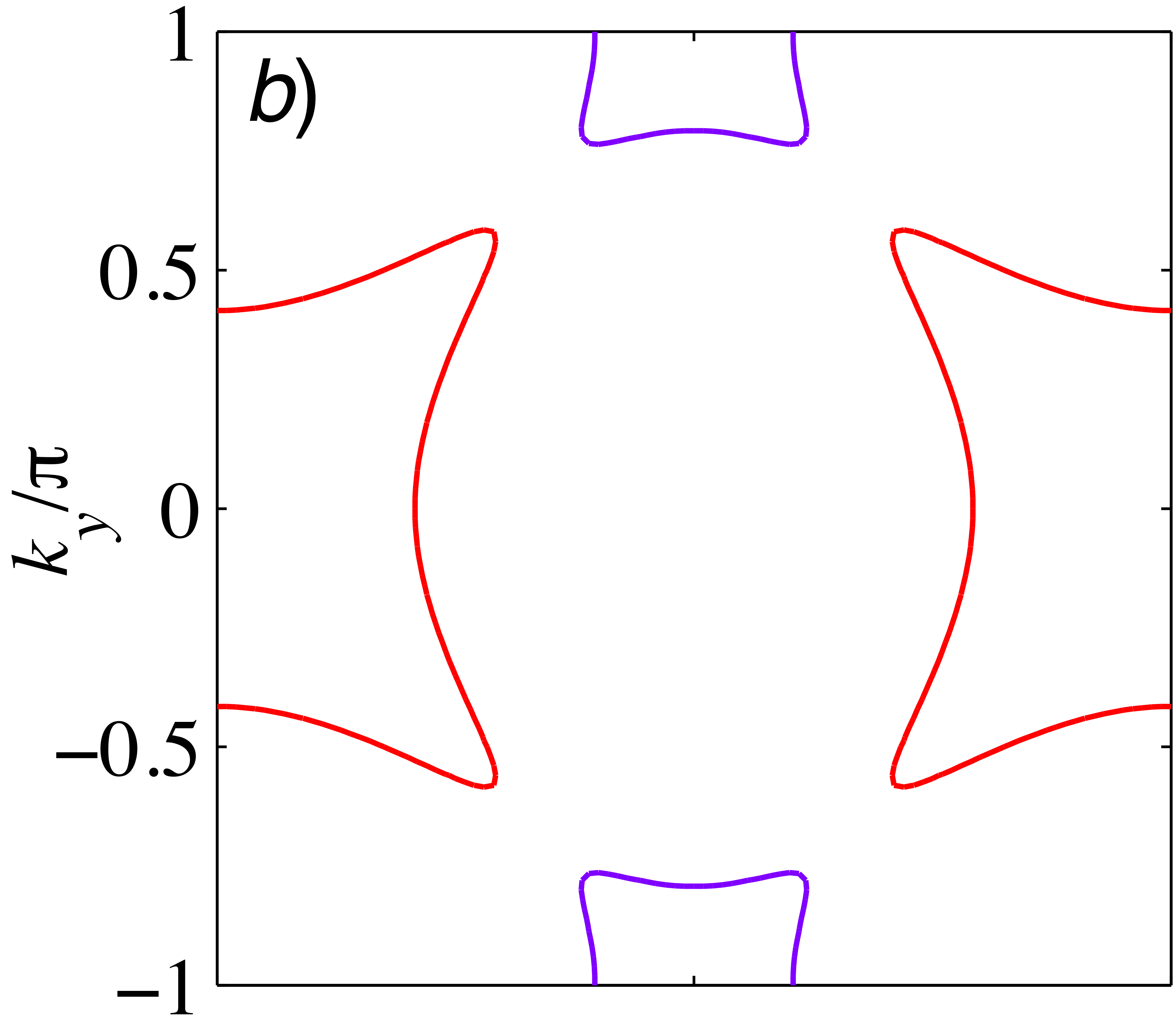
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Now hot spots are at Fermi surface intersections

QMC for the onset of antiferromagnetism



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Science **338**, 1606
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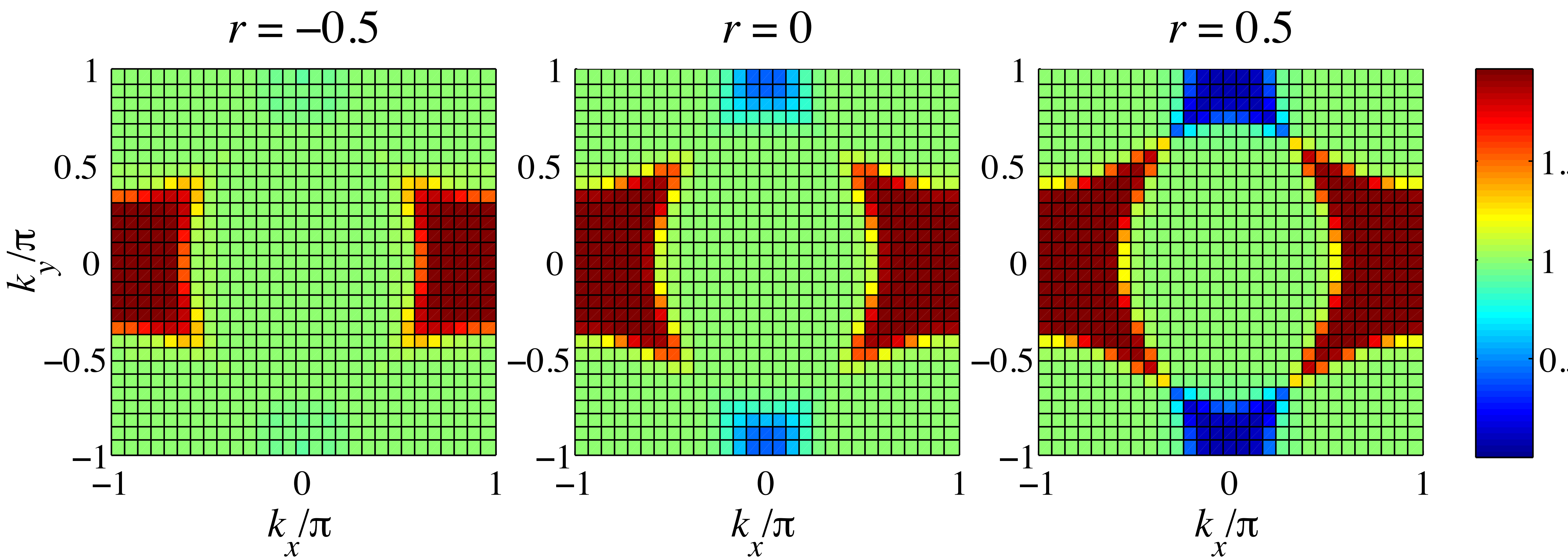
Expected Fermi surfaces in the AFM ordered phase

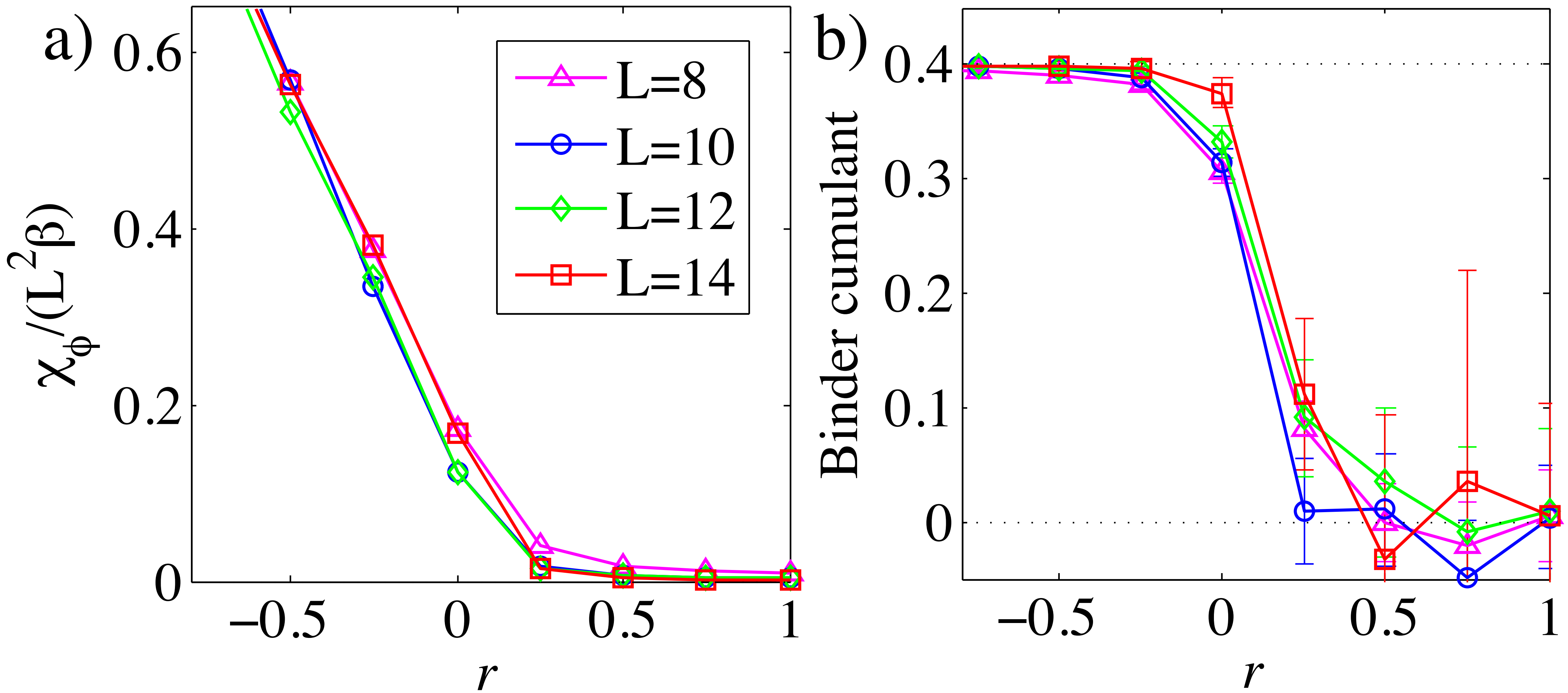
QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

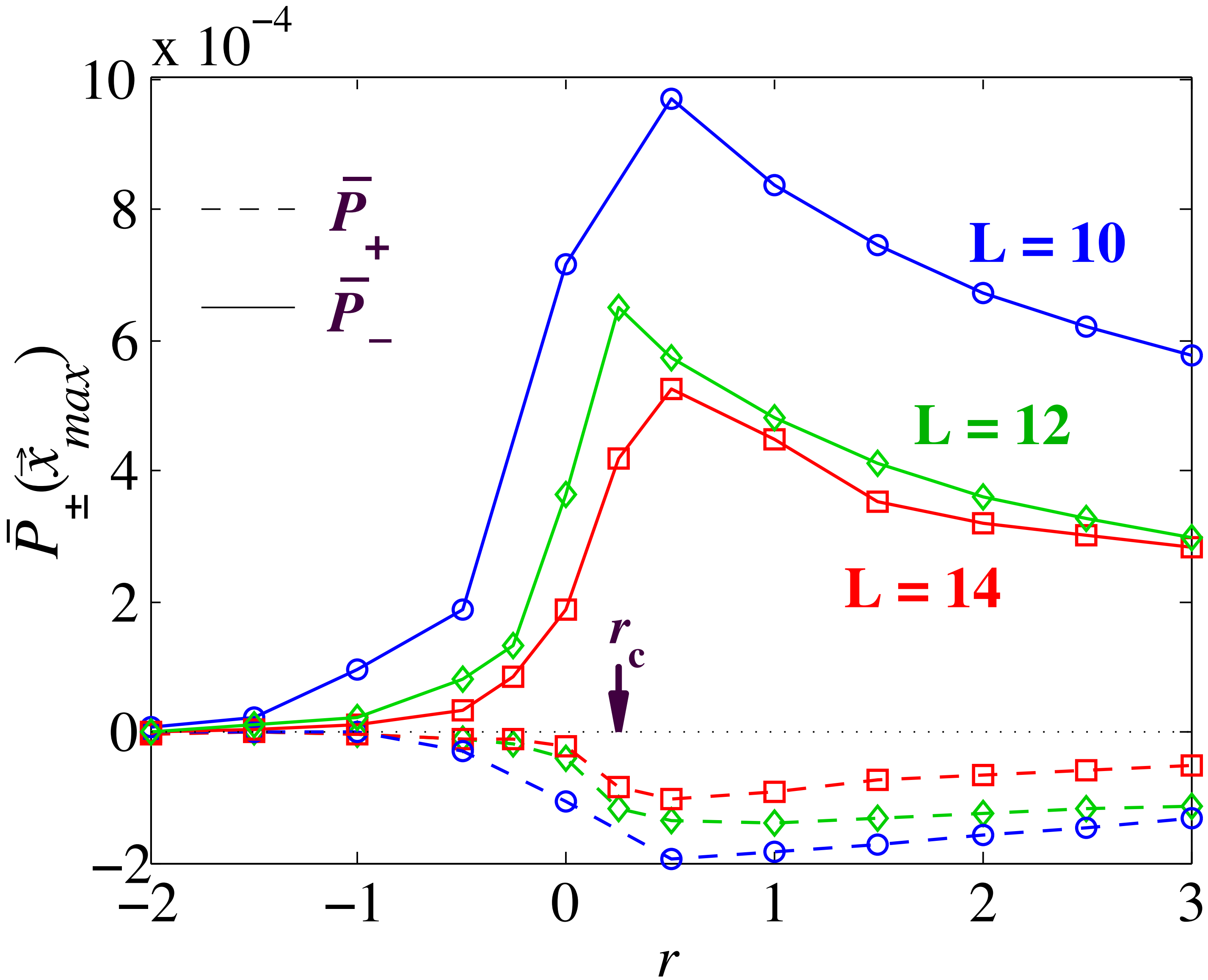
QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Conclusions

- Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to *d*-wave superconductivity, and to a charge density wave with a *d*-wave form factor.

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- Good prospects for studying competing charge orders, and non-Fermi liquid physics at non-zero temperature.