The chiral clock model, duality, and Kibble-Zurek dynamics of ultracold Rydberg atoms

Harvard University
October 5, 2018

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Most slides by Seth Whitsitt

Talk online: sachdev.physics.harvard.edu

PHYSICS
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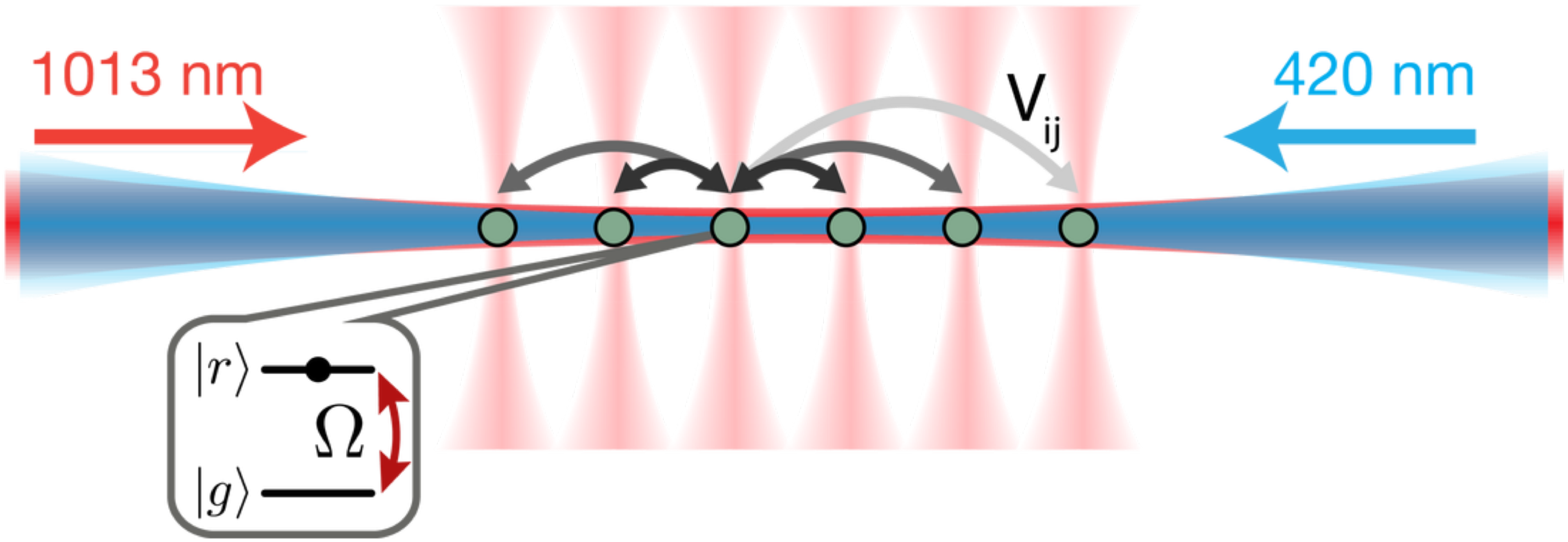
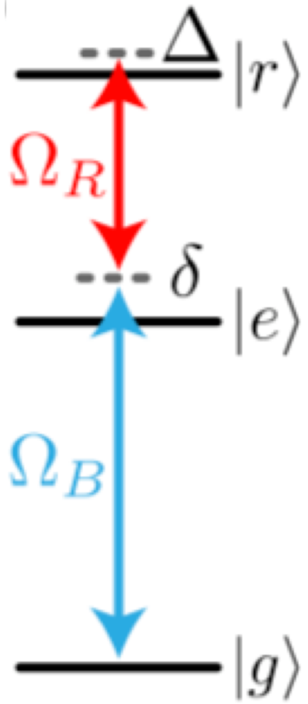
QPTs in a Rydberg quantum simulator



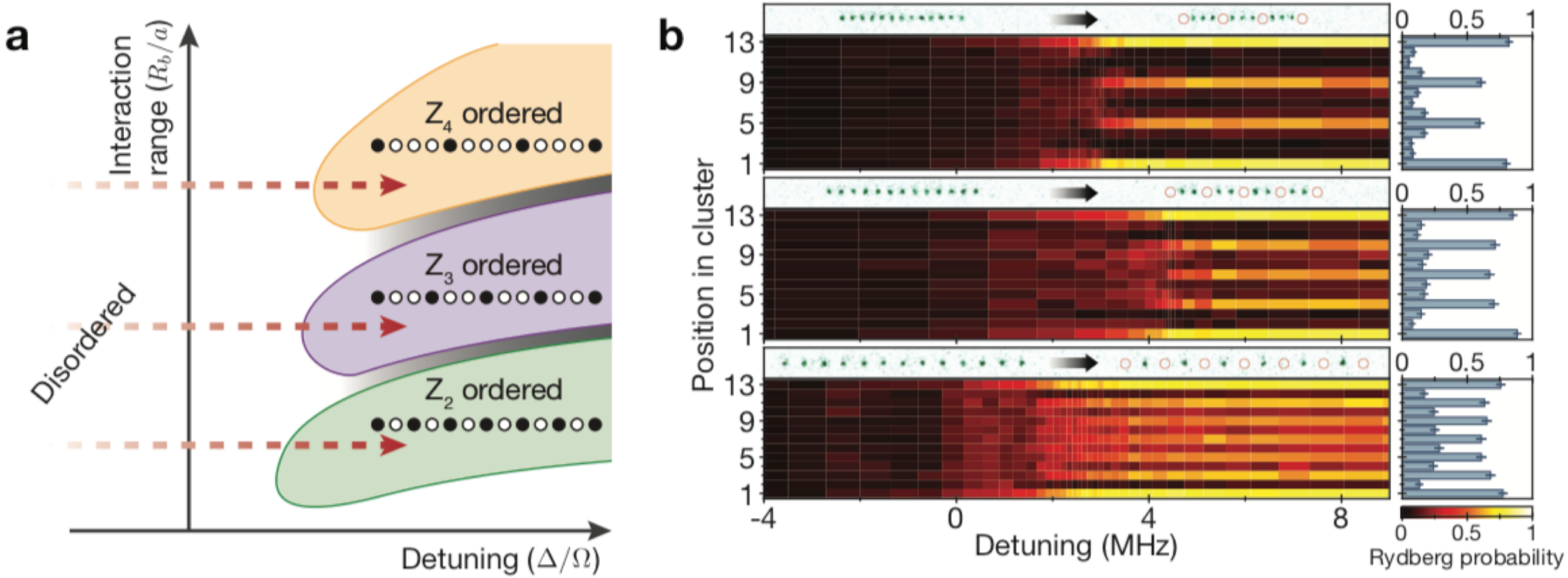


$$H_{\text{Ryd}} = \sum_{i=1}^{N} \frac{\Omega}{2}\Big(|g\rangle\langle r| + |r\rangle\langle g|\Big)_i - \Delta \sum_{i=1}^{N} |r\rangle\langle r|_i + \sum_{i<j} V_{|i-j|}\Big(|r\rangle\langle r|_i \otimes |r\rangle\langle r|_j\Big)$$

$$V_{|i-j|} \sim \frac{1}{|r_i - r_j|^6}$$

Bernien et. al., Nature **551,** 579, (2017)
Keesling et. al., arXiv:1809.05540

QPTs in a Rydberg quantum simulator



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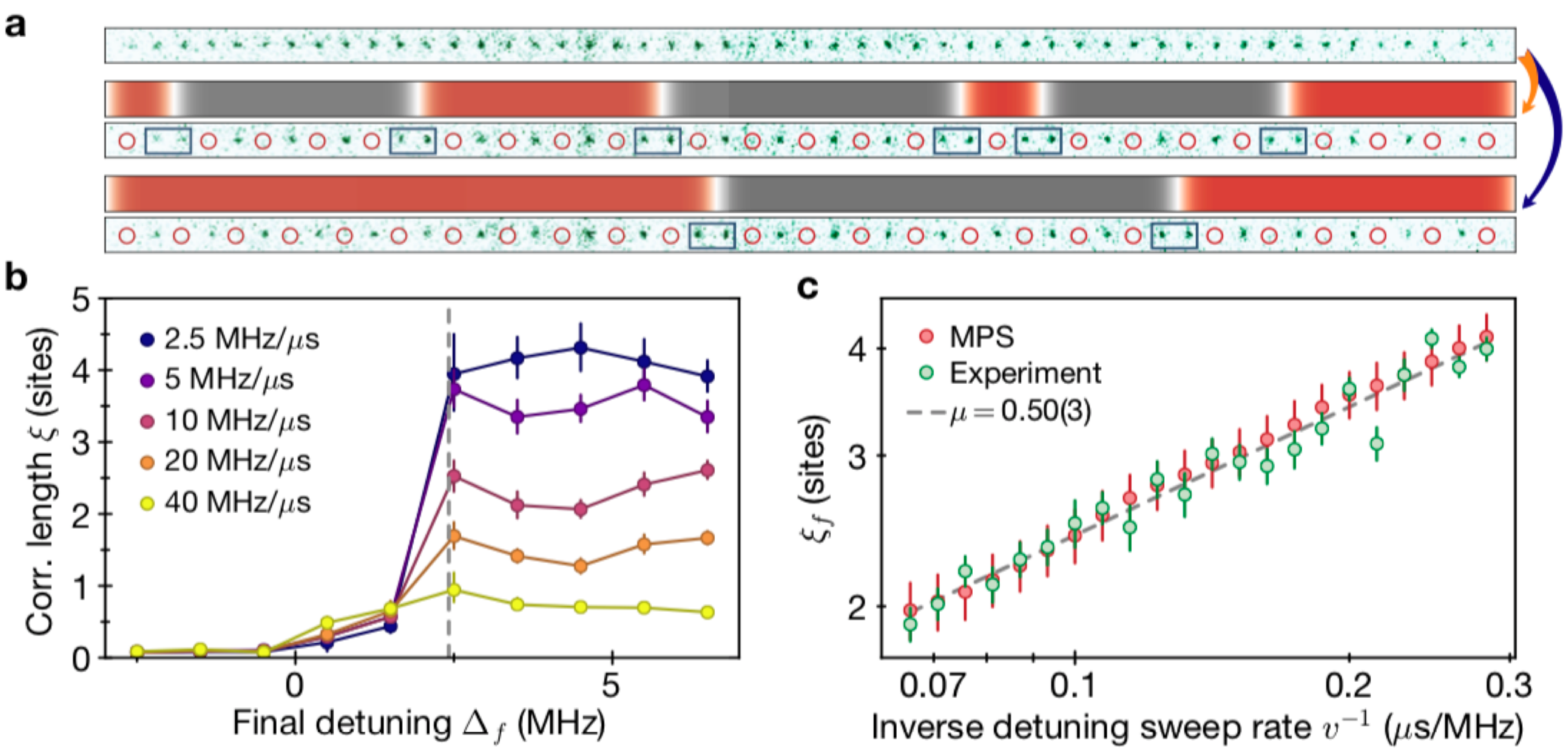
QPTs in a Rydberg quantum simulator

Universal critical dynamics: quantum Kibble-Zurek mechanism



Tune through transition at rate v:

$$\Delta(t) = \Delta_c + vt$$

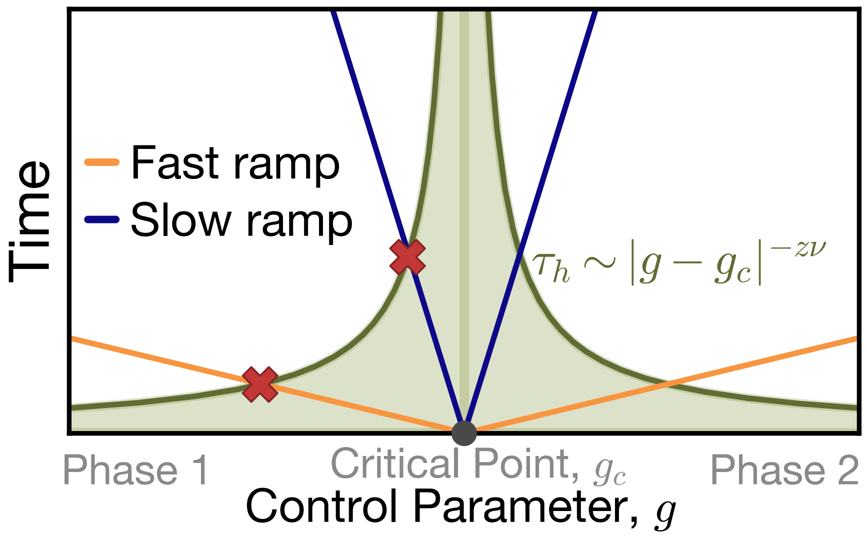
Correlation length saturates!

$$\xi \sim v^{-\nu/(1+\nu z)}$$

Experimental probe of critical exponents

Probing quantum critical dynamics on a programmable Rydberg simulator

Alexander Keesling, Ahmed Omran, Harry Levine, Hannes Bernien, Hannes Pichler, Soonwon Choi, Rhine Samajdar, Sylvain Schwartz, Pietro Silvi, Subir Sachdev, Peter Zoller, Manuel Endres, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, arXiv:1809.05540



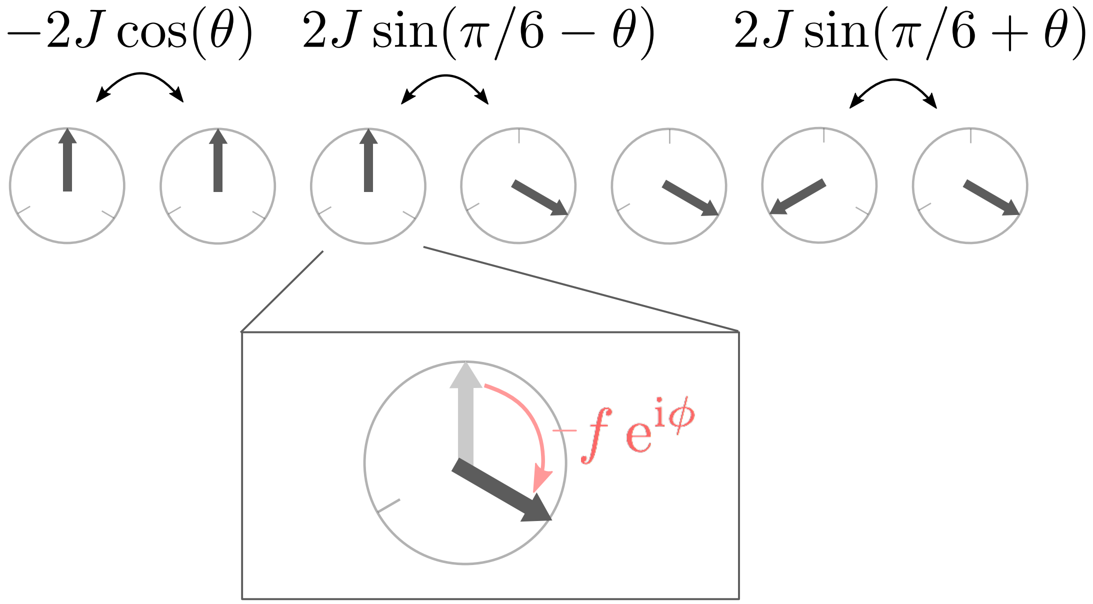
Chiral clock model

$$H_{\mathrm{CCM}} = -\,f \sum_{j=1}^{M} \left(\tau_j e^{i\phi} + \tau_j^\dagger e^{-i\phi} \right) - J \sum_{j=1}^{M-1} \left(\sigma_j \sigma_{j+1}^\dagger e^{i\theta} + \sigma_j^\dagger \sigma_{j+1} e^{-i\theta} \right)$$

$$\tau^N = \sigma^N = \mathbb{I}, \quad \tau^{-1}\,\sigma\,\tau = e^{2\pi i/N}\,\sigma$$

Simple lattice model describing $\mathbb{Z}_N$ symmetry breaking.
For $N > 2$ we may introduce *chiral* angles θ, ϕ.

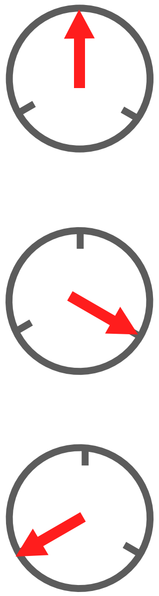
Ostlund, PRB **24**, 398 (1981).
Huse and ME Fisher, PRL **49**, 793 (1982).



Claim: Rydberg crystal transitions described by CCM with $\phi = 0$

Chiral clock model

$$\begin{aligned} H_{\mathrm{CCM}} = & - f \sum_{j=1}^{M} \left(\tau_j + \tau_j^{\dagger} \right) \\ & - J \sum_{j=1}^{M-1} \left(\sigma_j \sigma_{j+1}^{\dagger} e^{i\theta} + \sigma_j^{\dagger} \sigma_{j+1} e^{-i\theta} \right) \end{aligned}$$



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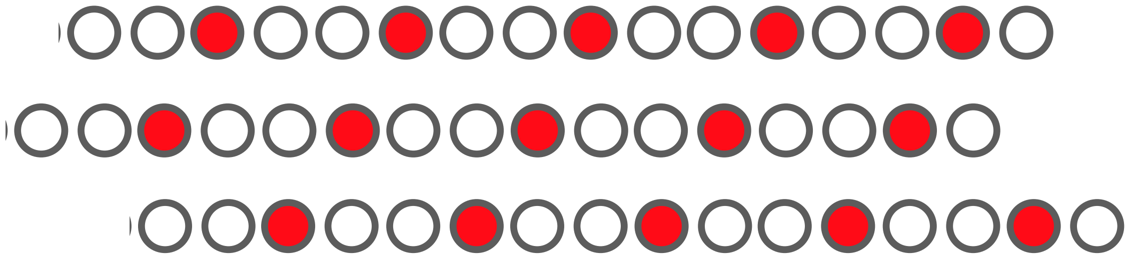
$\mathbb{Z}_3$ ordered



Fendley, Sengupta, Sachdev, PRB **69**, 075106 (2004).
Samajdar et. al., PRA **98**, 023614 (2018).

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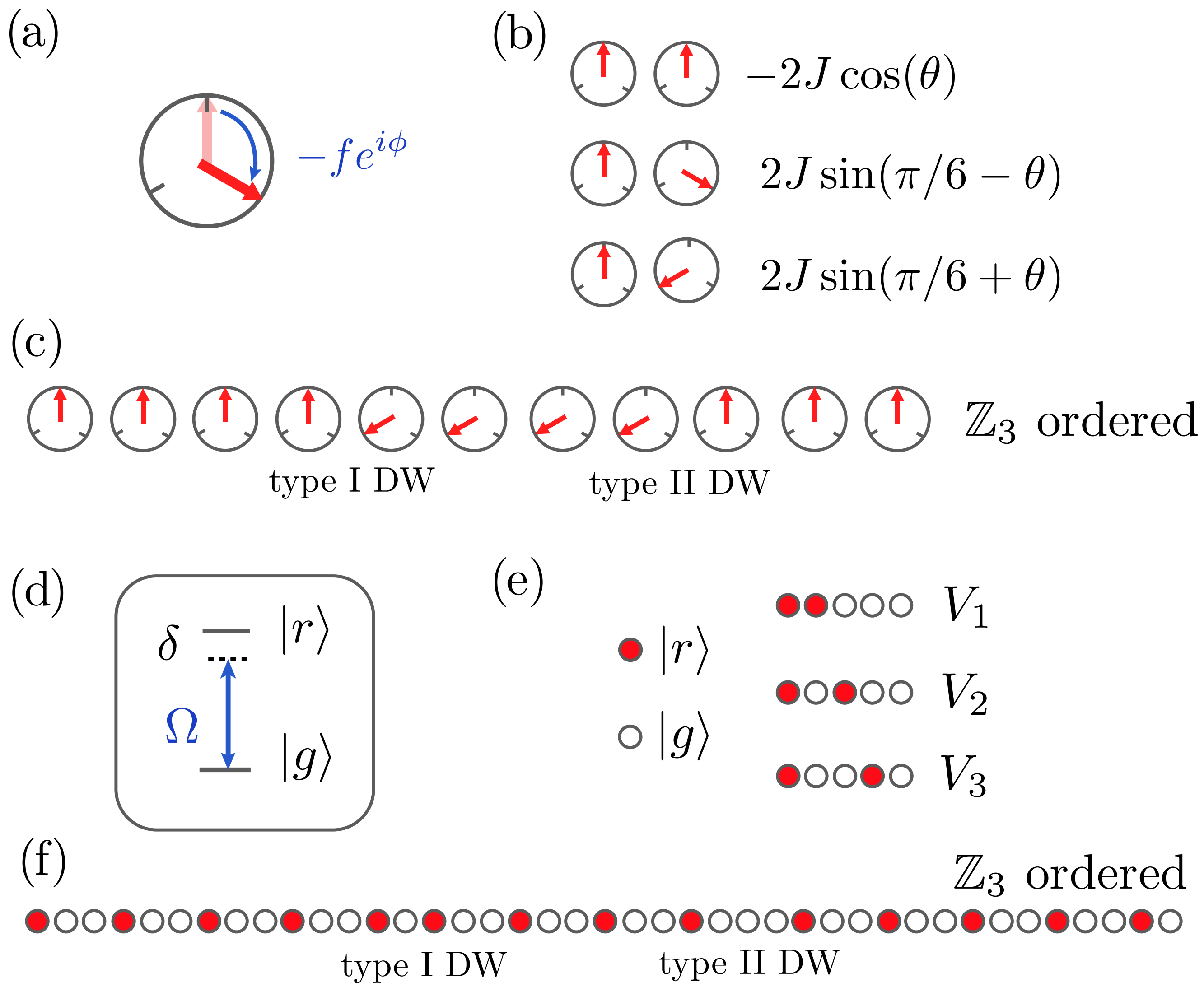

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Chiral clock model

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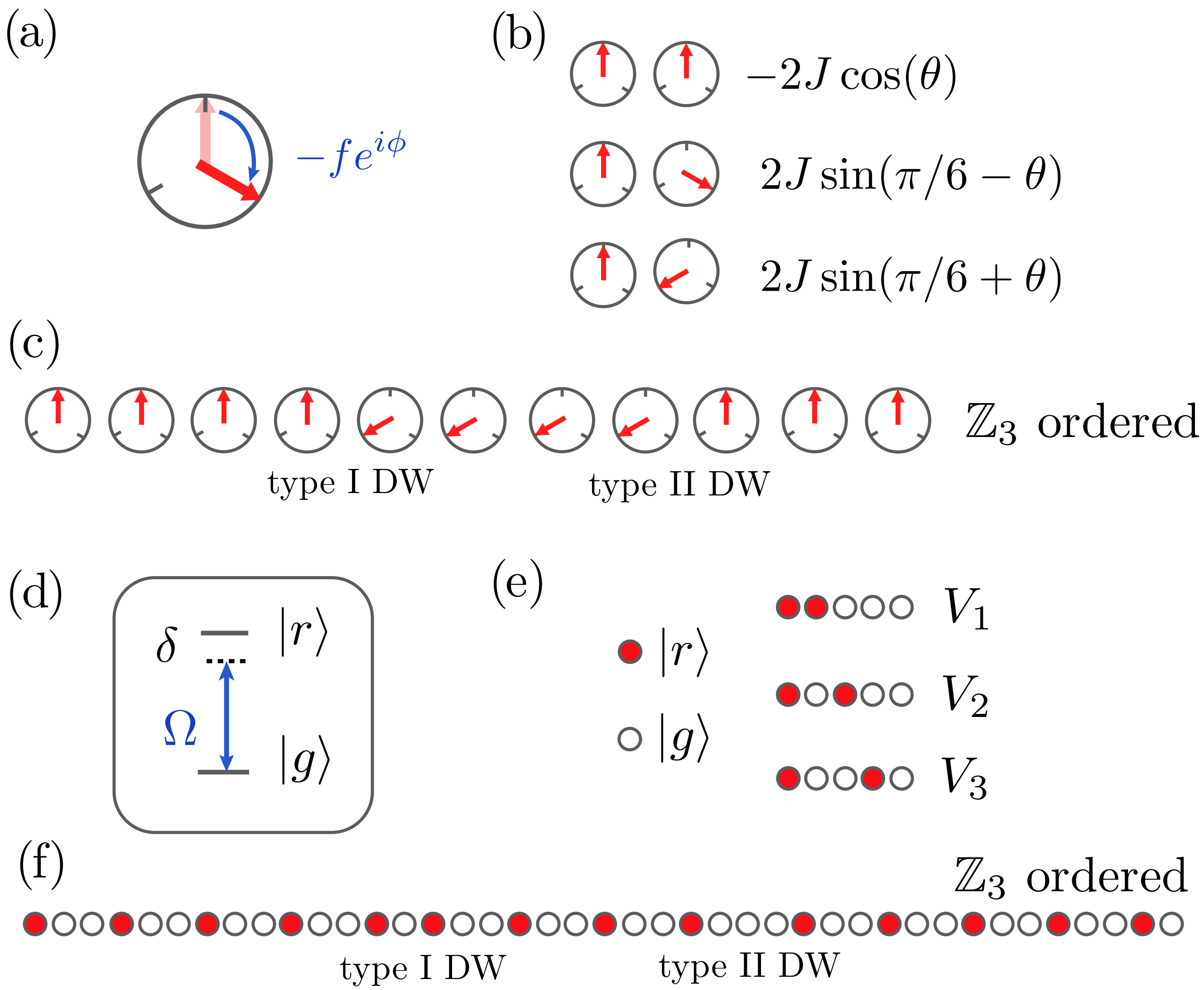
Parity symmetry of Rydberg system maps to parity + conjugation:

$$\sigma_j \to \sigma_{M-j}^\dagger$$

$$\tau_j \to \tau_{M-j}^\dagger$$

Fendley, Sengupta, Sachdev, PRB **69**, 075106 (2004).
Samajdar et. al., PRA **98**, 023614 (2018).



Chiral clock model

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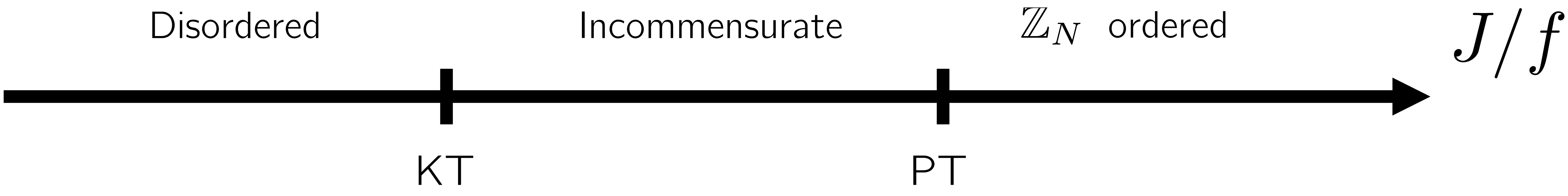
For $N \gtrsim 4$, there is always an intermediate incommensurate phase.



Disordered

Incommensurate

$\mathbb{Z}_N$ ordered

J/f

KT

PT

Chiral clock model

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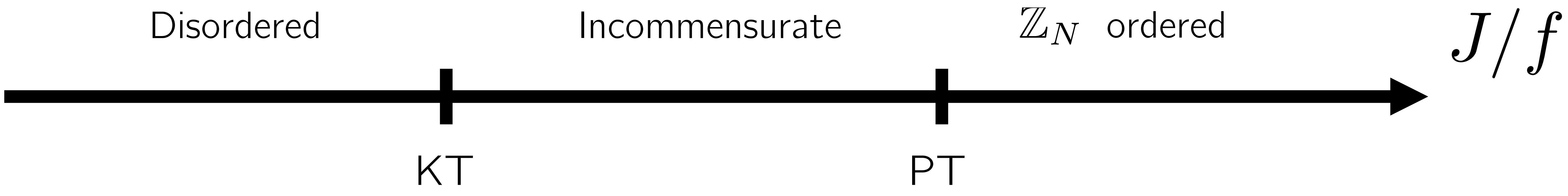

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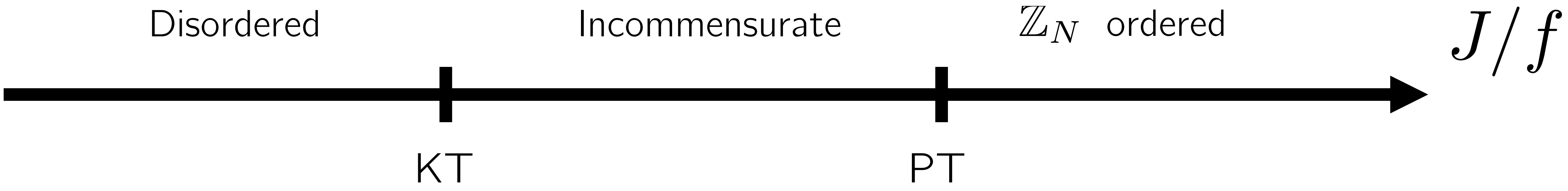

Disordered

Incommensurate

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J/f

KT

PT

Early work on the $N = 3$ case was controversial, with theoretical and numerical evidence for either an IC phase or a direct transition.

IC:

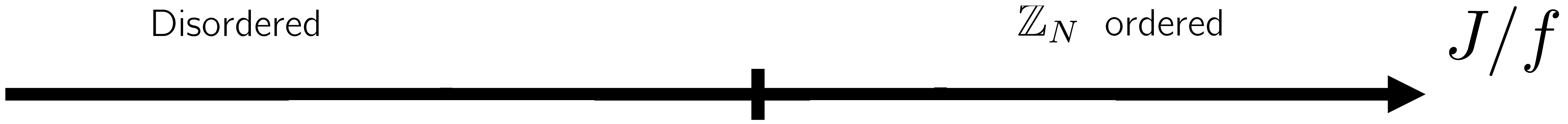
Haldane, Bak, Bohr, PRB **28**, 2743 (1983).
Schulz, PRB **28**, 2746 (1983)
Von Gehlen and Rittenberg, Nucl. Phys. B, 473 (1984)

Direct:

Ostlund, PRB **24**, 398 (1981).
Huse and ME Fisher, PRL **49**, 793 (1982).
Huse, PRB **24**, 5180 (1981).
Selke and Yeomans, Z. Phys. B **46**, 311 (1982).
Howes, Kadanoff, den Nijs, Nucl. Phys. B **215**, 169 (1983)
Huse, Szpilka, Fisher, Physica A **121**, 363 (1983).

Chiral clock model

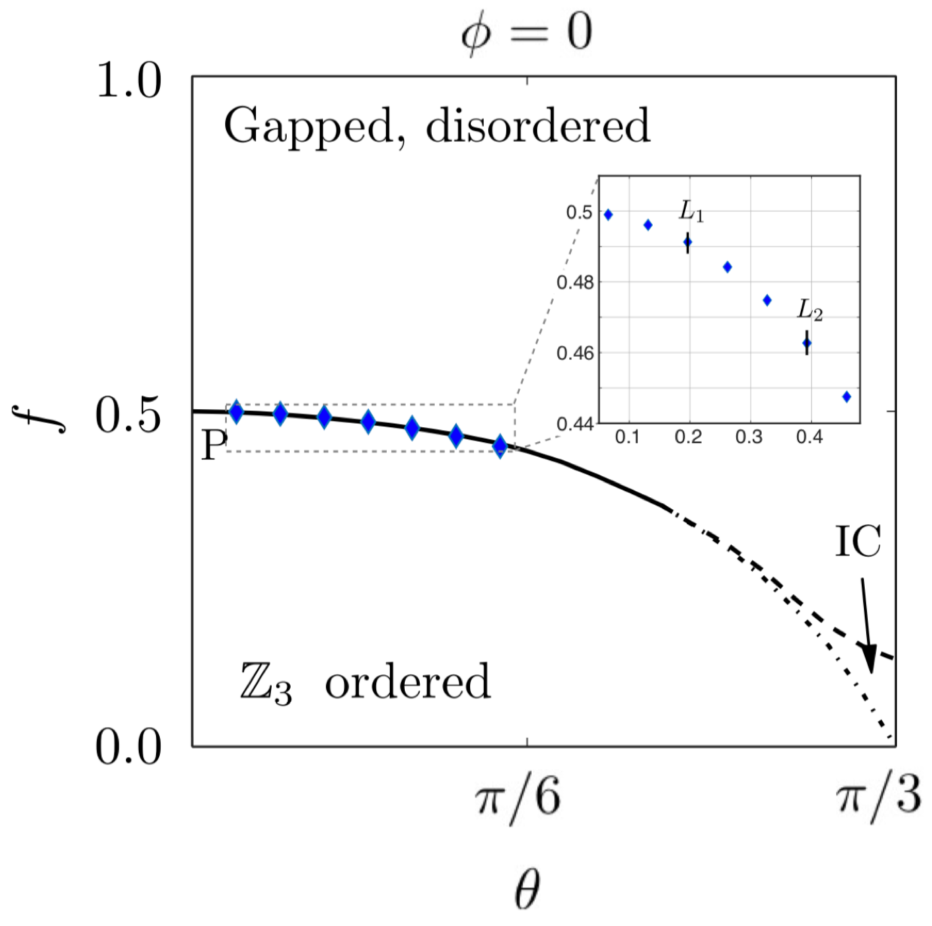
$$H_{\rm CCM} = -f\sum_{j=1}^{M}\left(\tau_j + \tau_j^\dagger\right) - J\sum_{j=1}^{M-1}\left(\sigma_j\sigma_{j+1}^\dagger e^{i\theta} + \sigma_j^\dagger\sigma_{j+1}e^{-i\theta}\right)$$


Disordered

$\mathbb{Z}_N$ ordered

J/f

Recent DMRG studies for $\mathbb{Z}_3$ give very strong evidence for direct transition!



Direct transition appears to be in a new universality class.

“The only known example of a strongly coupled generic transition between gapped states in 1+1D with $z \neq 1$“

Zhuang, Changlani, Tubman, Hughes, PRB **92**, 035154 (2015).
Samajdar et. al., PRA **98**, 023614 (2018).
Chepiga and Mila, arXiv:1808.08990

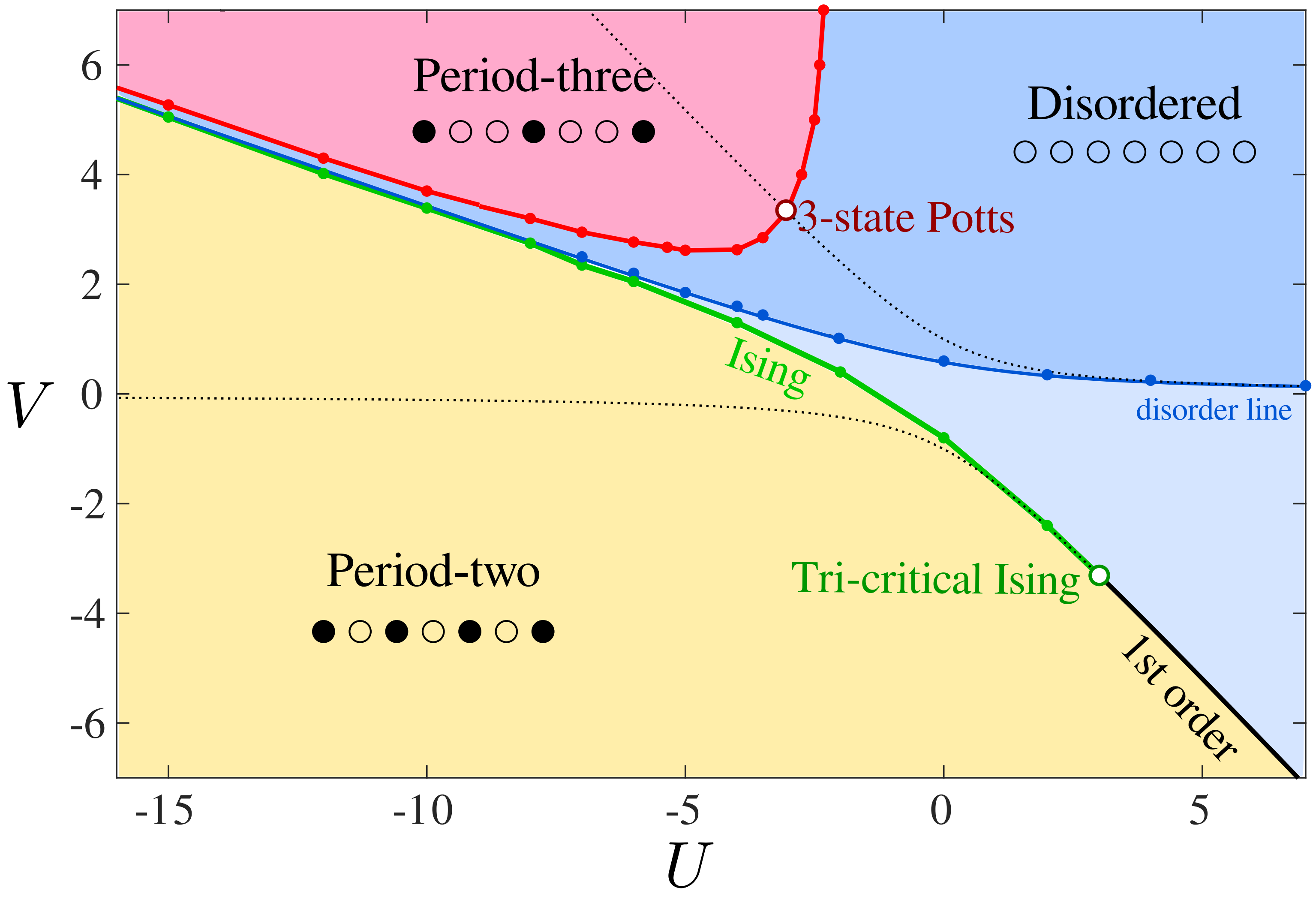
Floating phase versus chiral transition in a 1D hard-boson model

Natalia Chepiga[1] and Frédéric Mila[2]

We investigate the nature of the phase transition between the period-three charge-density wave and the disordered phase of a hard-boson model proposed in the context of cold-atom experiments. Building on a density-matrix renormalization group algorithm that takes full advantage of the hard-boson constraints, we study systems with up to 9’000 sites and calculate the correlation length and the wave-vector of the incommensurate short-range correlations with unprecedented accuracy. We provide strong numerical evidence that there is an intermediate floating phase far enough from the integrable Potts point, while in its vicinity, our numerical data are consistent with a unique transition in the Huse-Fisher chiral universality class.

$$H = \sum_j \Big[-w(d_j^\dagger + d_j) + U n_j + V n_{j-1} n_{j+1} \Big] ,$$

arXiv:1808.08990



QFT for chiral clock model

What is the critical field theory? First try: write the most general theory for order parameter with appropriate symmetries.

$$\Phi \to e^{2\pi i/N}\,\Phi \qquad\qquad \Phi(x,\tau) \to \Phi^*(-x,\tau)$$

$$\mathcal{S}_\Phi = \int dx\,d\tau \Big[|\partial_\tau \Phi|^2 + |\partial_x \Phi|^2 + i\alpha_x \Phi^* \partial_x \Phi + s|\Phi|^2 \\ + u|\Phi|^4 + \lambda\left(\Phi^N + (\Phi^*)^N\right) + \cdots \Big]$$

In perturbation theory, the field condenses at nonzero momentum.

$$\mathcal{S}_\Phi = \int \frac{d\omega\,dk}{(2\pi)^2}\ \Phi^* \left[\omega^2 + k^2 - \alpha_x k + s\right] \Phi + \cdots$$

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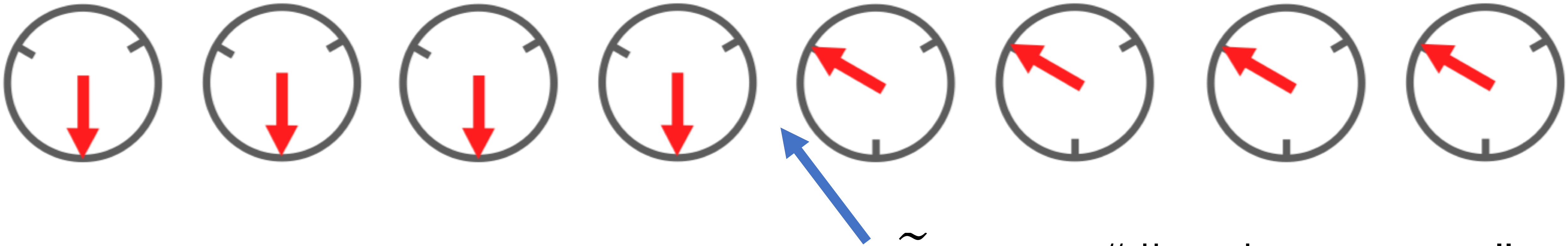
In perturbation theory, the field condenses at nonzero momentum. Can **only** describe transition to IC phase.

$$E(k) \sim (k - k_0) \qquad\qquad \langle \Phi(x)\Phi(0)\rangle \sim e^{ik_0 x}$$

Chiral clock model: duality

$$H_{\mathrm{CCM}} = -f\sum_{j=1}^{M}\left(\tau_j e^{i\phi} + \tau_j^\dagger e^{-i\phi}\right) - J\sum_{j=1}^{M-1}\left(\sigma_j\sigma_{j+1}^\dagger e^{i\theta} + \sigma_j^\dagger\sigma_{j+1} e^{-i\theta}\right)$$

$$\tilde{\tau}_{j+1/2} = \sigma_j\sigma_{j+1}^\dagger, \qquad \tilde{\sigma}_{j+1/2} = \prod_{k=1}^{j}\tau_k^\dagger$$



$\tilde{\sigma}$ a “disorder operator”

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$$\tilde{\tau}_{j+1/2} = \sigma_j \sigma_{j+1}^\dagger, \qquad \tilde{\sigma}_{j+1/2} = \prod_{k=1}^{j} \tau_k^\dagger$$

Same algebra as the original variables. Bulk Hamiltonian takes the form:

$$H_{\text{CCM}} = -J \sum_{j} \left(\tilde{\tau}_j e^{i\theta} + \tilde{\tau}_j^\dagger e^{-i\theta} \right) - f \sum_{j} \left(\tilde{\sigma}_j \tilde{\sigma}_{j+1}^\dagger e^{i\phi} + \tilde{\sigma}_j^\dagger \tilde{\sigma}_{j+1} e^{-i\phi} \right)$$

$$\phi \longleftrightarrow \theta, \qquad J \longleftrightarrow f$$

QFT for chiral clock model

$$H_{\text{CCM}} = -J \sum_j \left(\tilde{\tau}_j e^{i\theta} + \tilde{\tau}_j^\dagger e^{-i\theta} \right) - f \sum_j \left(\tilde{\sigma}_j \tilde{\sigma}_{j+1}^\dagger + \tilde{\sigma}_j^\dagger \tilde{\sigma}_{j+1} \right)$$

What is the QFT in terms of “disorder parameter” $\tilde{\sigma} \sim \Psi$?

Note $\phi = 0$, and now θ is a Berry phase in the kinetic term.

QFT for chiral clock model

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$$\mathcal{S}_B = \int d\tau \, d^d x \left[\Psi^* \partial_\tau \Psi + |\nabla \Psi|^2 + s|\Psi|^2 + \frac{u}{2} |\Psi|^4 + \frac{\lambda_0}{N!} \left(\Psi^N + \Psi^{*N} \right) \right]$$

Describes the quantum phase transition associated with the onset of a single boson condensate, $\langle \Psi \rangle$, in the presence of a background N boson condensate ($\langle \Psi^N \rangle \neq 0$, on both sides of the transition). This Bose gas transition is dual to the *direct* transition from the $\mathbb{Z}_N$ ordered phase to the disordered phase of the original chiral clock model in $d = 1$

Field theory for $\mathbb{Z}_N$ density wave ordering

$$\mathcal{S}_\Phi = \int dx\, d\tau \Big[|\partial_\tau \Phi|^2 + |\partial_x \Phi|^2 + i\alpha_x \Phi^* \partial_x \Phi + s|\Phi|^2 + u|\Phi|^4 + \lambda \left(\Phi^N + (\Phi^*)^N \right) + \cdots \Big]$$

is Kramers-Wannier dual to

$$\mathcal{S}_B = \int d\tau\, d^d x \left[\Psi^* \partial_\tau \Psi + |\nabla \Psi|^2 + s|\Psi|^2 + \frac{u}{2} |\Psi|^4 + \frac{\lambda_0}{N!} \left(\Psi^N + \Psi^{*N} \right) \right]$$

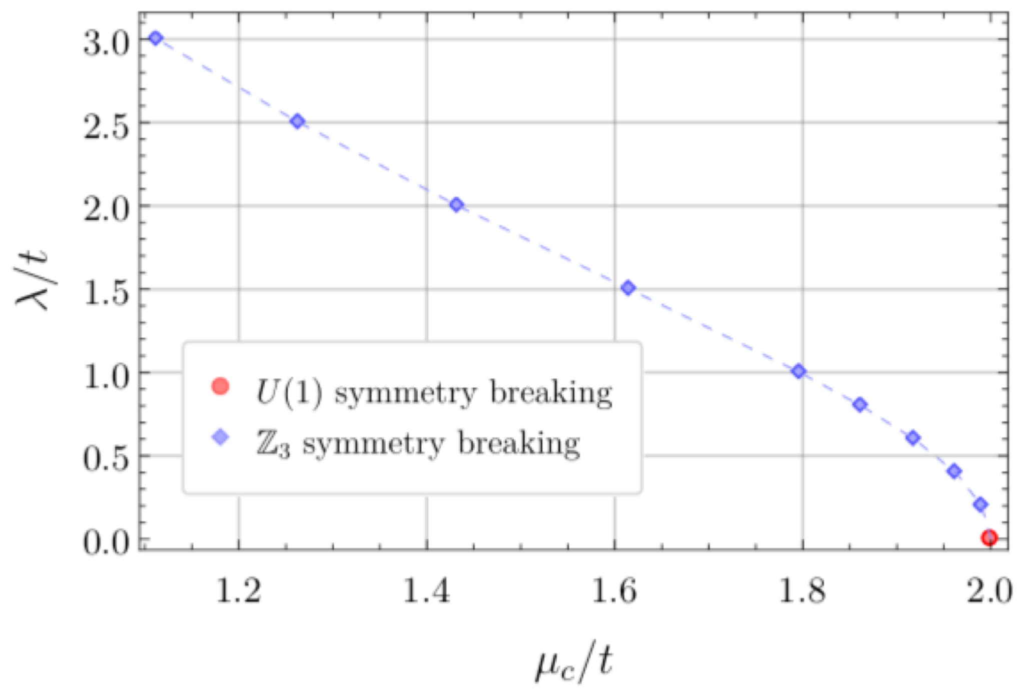
field theory for Bose condensation in the presence of a background N boson condensate

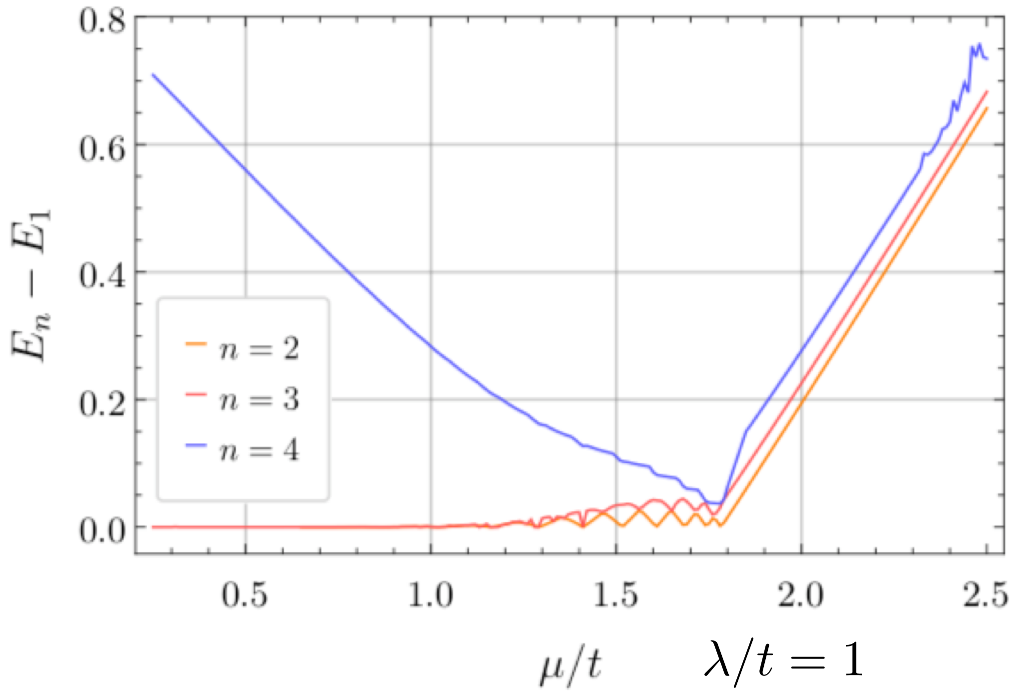
Note: this is not a Wick rotation—
there is a crucial difference in the factor of i !!

Numerics on a $\mathbb{Z}_3$ dilute Bose gas

DMRG simulations on a Bose gas transition with $\mathbb{Z}_3$ symmetry breaking

$$H = -\,t \sum_{\langle i,j \rangle} \left(b_i^\dagger\, b_j + b_i\, b_j^\dagger \right) - \mu \sum_i n_i$$
$$+\, \lambda \sum_i \left(b_i\, b_{i+1}\, b_{i+2} + b_i^\dagger\, b_{i+1}^\dagger\, b_{i+2}^\dagger \right) \; ; \quad n_i \leq 1,$$





Clear evidence for a direct transition between gapped phases, without an intermediate gapless incommensurate phase

RG for $\mathbb{Z}_N$ dilute Bose gas

$\mathbb{Z}_3$ CCM critical exponents

$$E(k) \sim k^z$$

$$\xi \sim |s - s_c|^{-\nu}$$

$$\langle \Psi(x,0)\Psi(0,0)\rangle \sim |x|^{-(d+z-2+\eta)}$$

Exponent	LO	NLO
z	1.87	1.57
ν	0.60	N/A
η	0.03	0.11

Kibble-Zurek exponent:

$$\#(\text{excitations}) \sim v^{\mu}$$

$$\mu = \frac{d\nu}{1+\nu z}$$

$$\mu \approx .28$$

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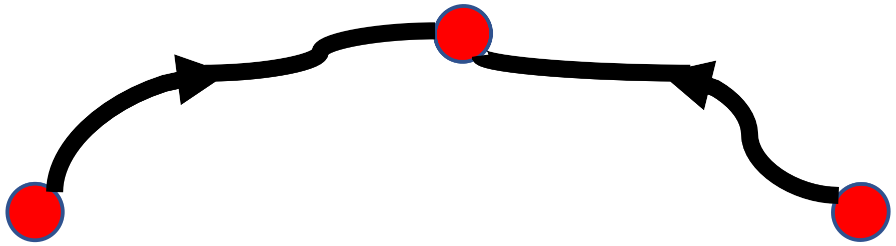
$\mathbb{Z}_3$ CCM



3-state Potts

$z = 1$

$\nu = 5/6$

$\eta = 4/15$

U(1) DBG

$z = 2$

$\nu = 1/2$

$\eta = 0$

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Numerics:

$$z \approx 1.33$$

$$\nu \approx .71$$

Samajdar et. al., PRA **98**, 023614 (2018).

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Experiments: $\mu \sim .37$

Keesling et. al., arXiv:1809.05540

Summary

- QPTs seen in Rydberg arrays enable the study of chiral clock universality class

- The ideal QFT to study this universality class is written in terms of the “disorder parameter.” A controlled expansion exists with access to a nontrivial fixed point.

- Further work: tests of our assumed RG flow?
  - E.g. compute stability of U(1) DGB directly in one dimension
  - Conformal perturbation theory around 3-state Potts model?

- Chiral clock transitions in other systems?