From the Hubbard model to high temperature superconductivity

S. Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS

HARVARD

Posters

- **Max Metlitski:** Entanglement near strongly interacting quantum critical points in two and higher dimensions

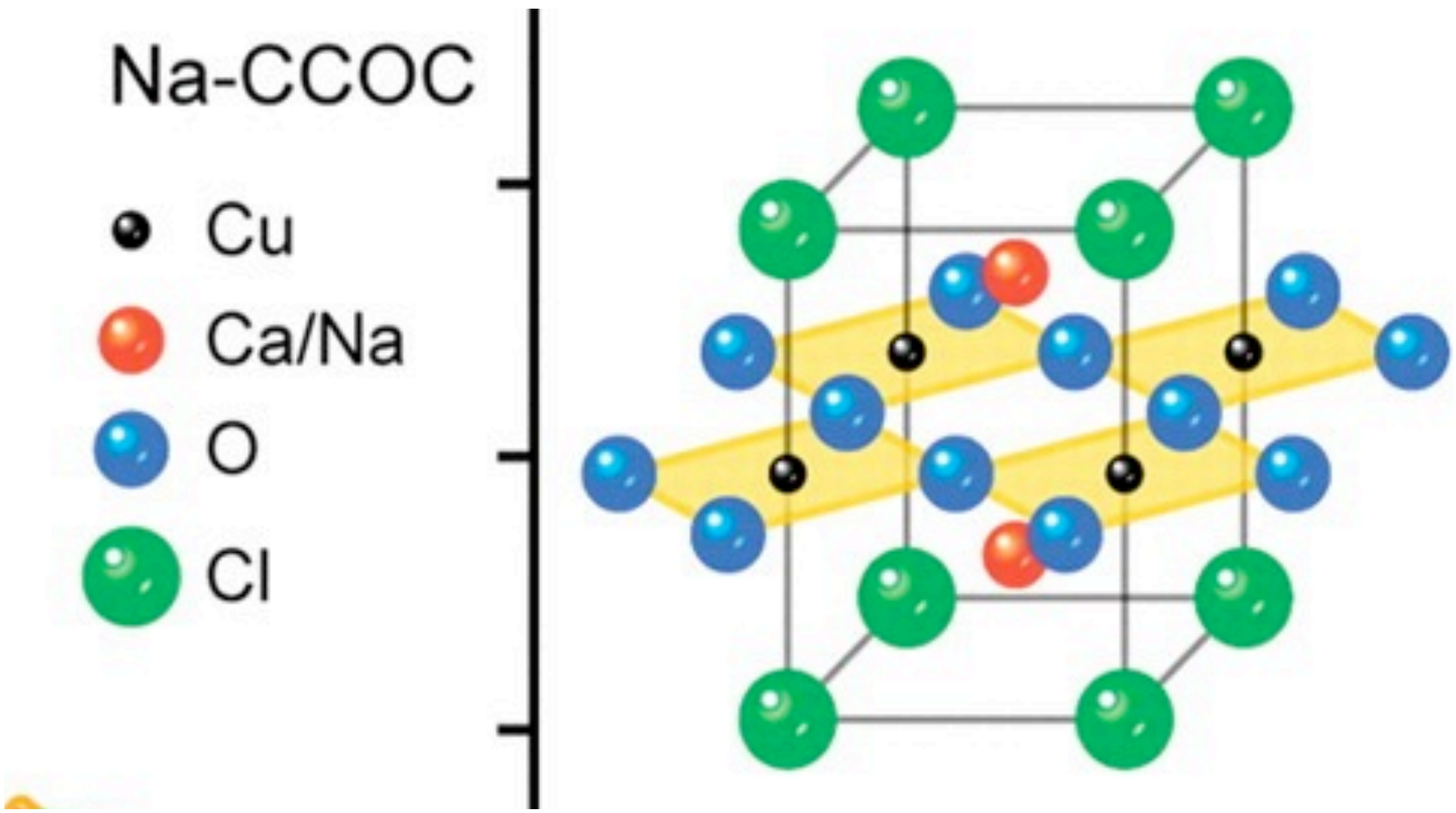
- **Yang Qi:** Frustrated antiferromagnetism and spin liquids in organic insulators

OFFICE OF SCIENTIFIC RESEARCH · AIR FORCE · UNITED STATES AIR FORCE

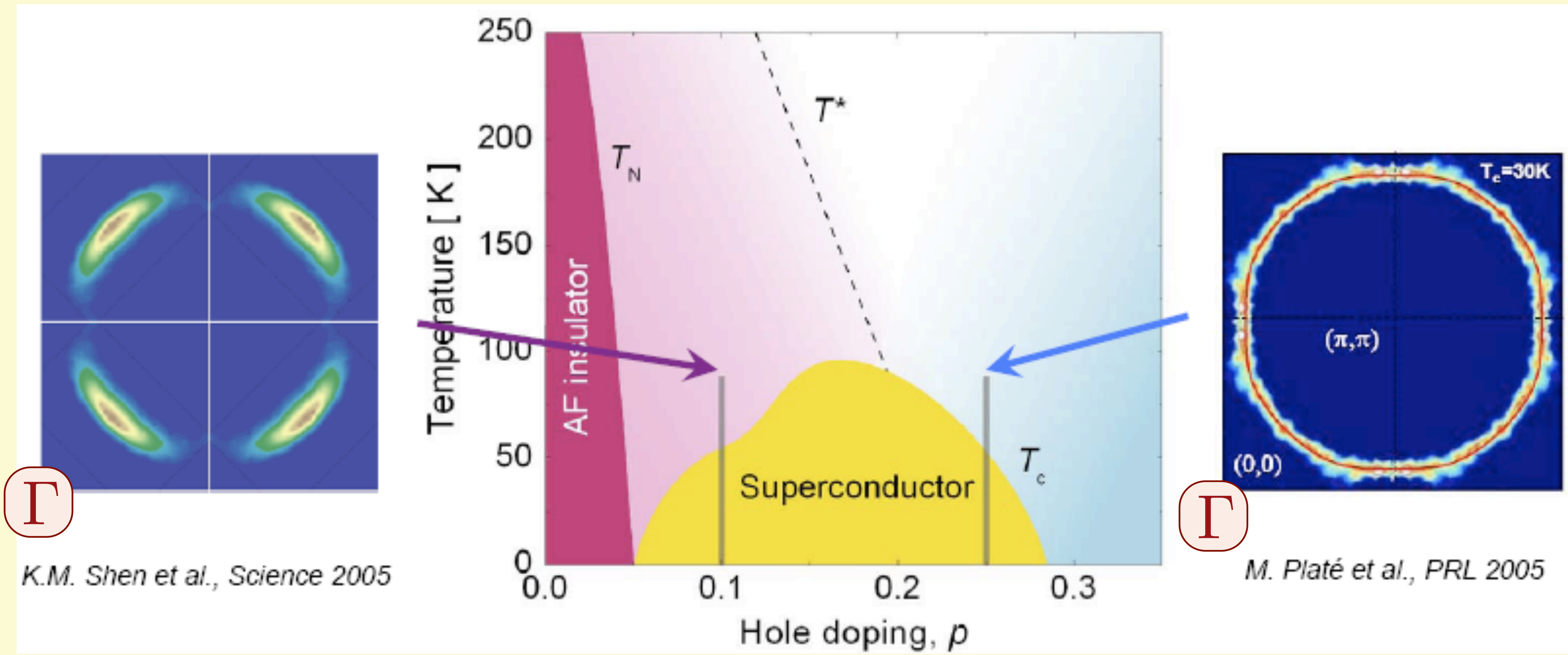
PHYSICS
VE RI TAS
HARVARD

The cuprate superconductors

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

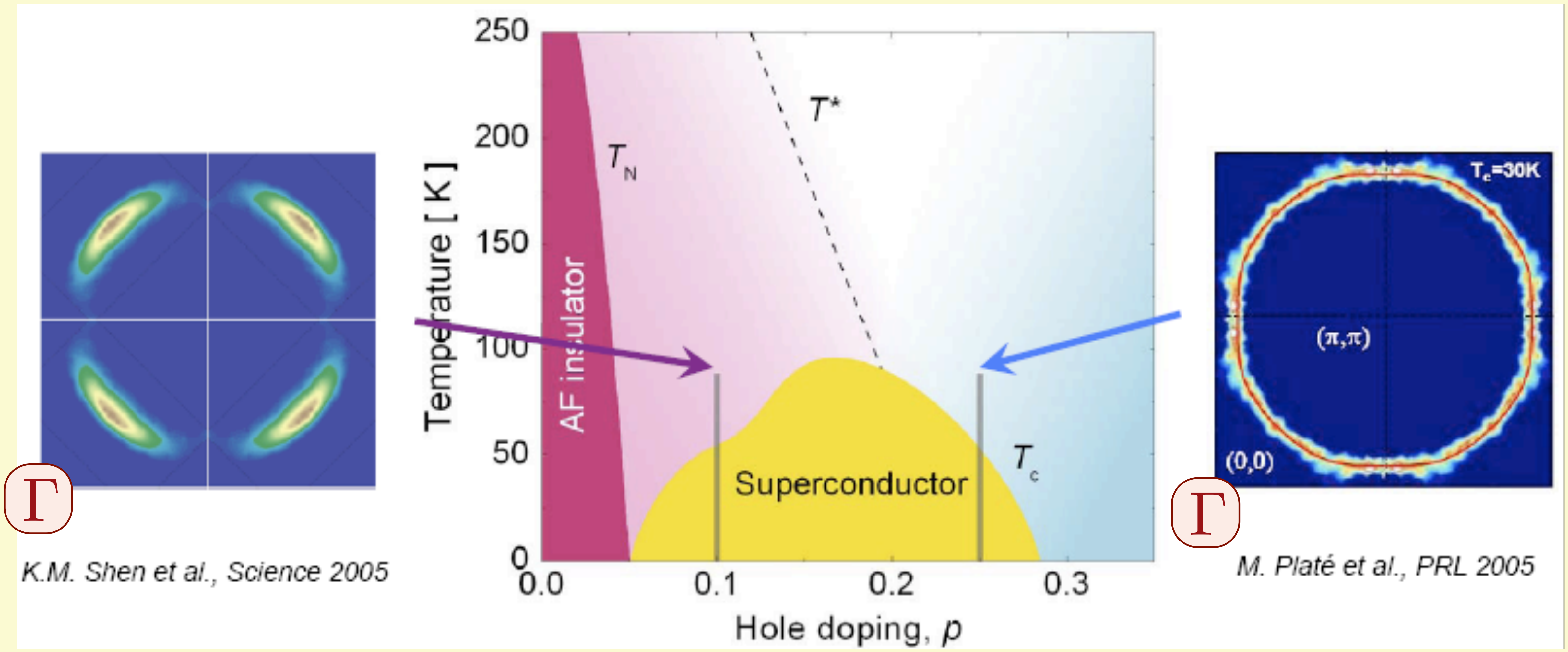
Large hole Fermi surface

Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface

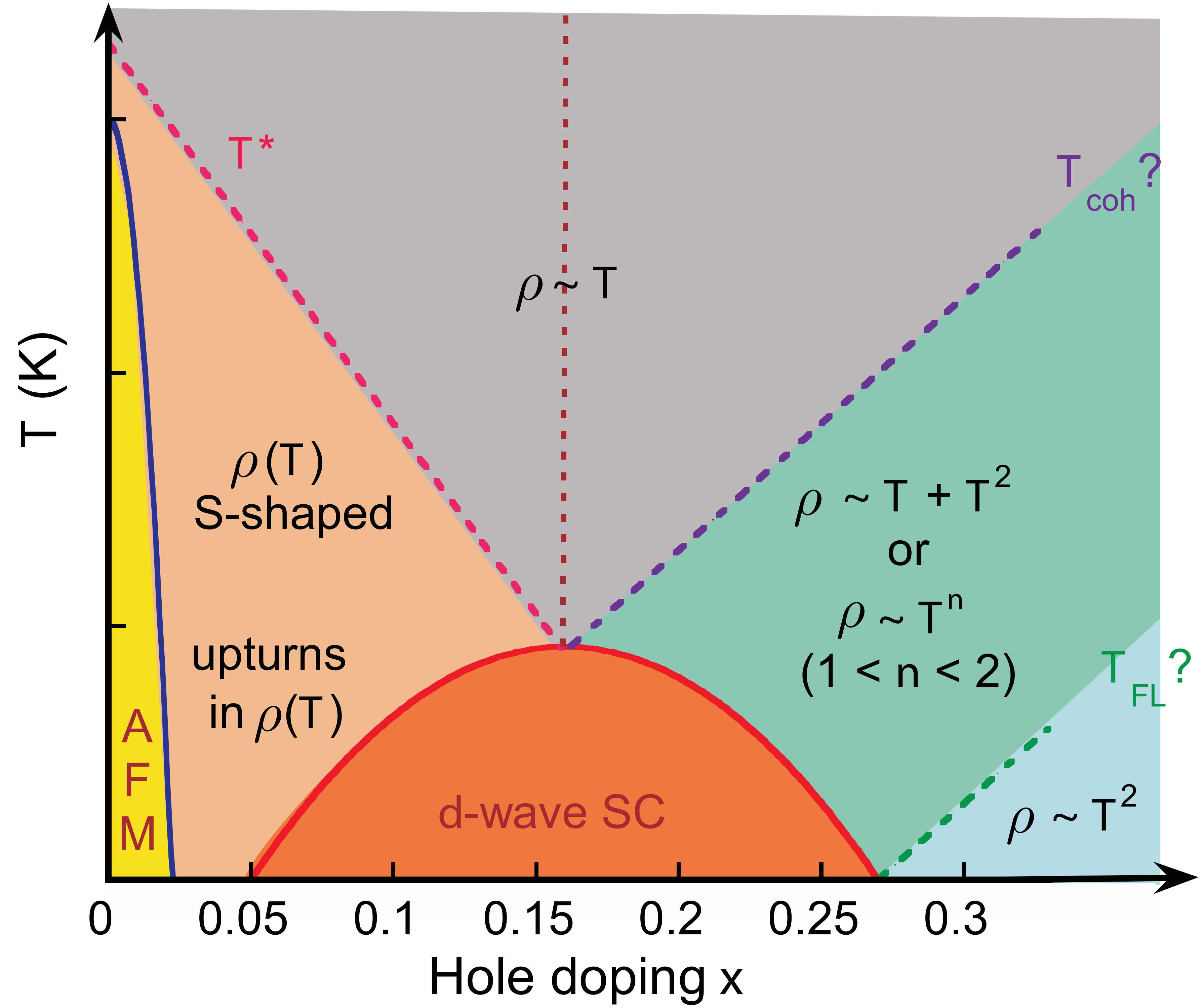
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

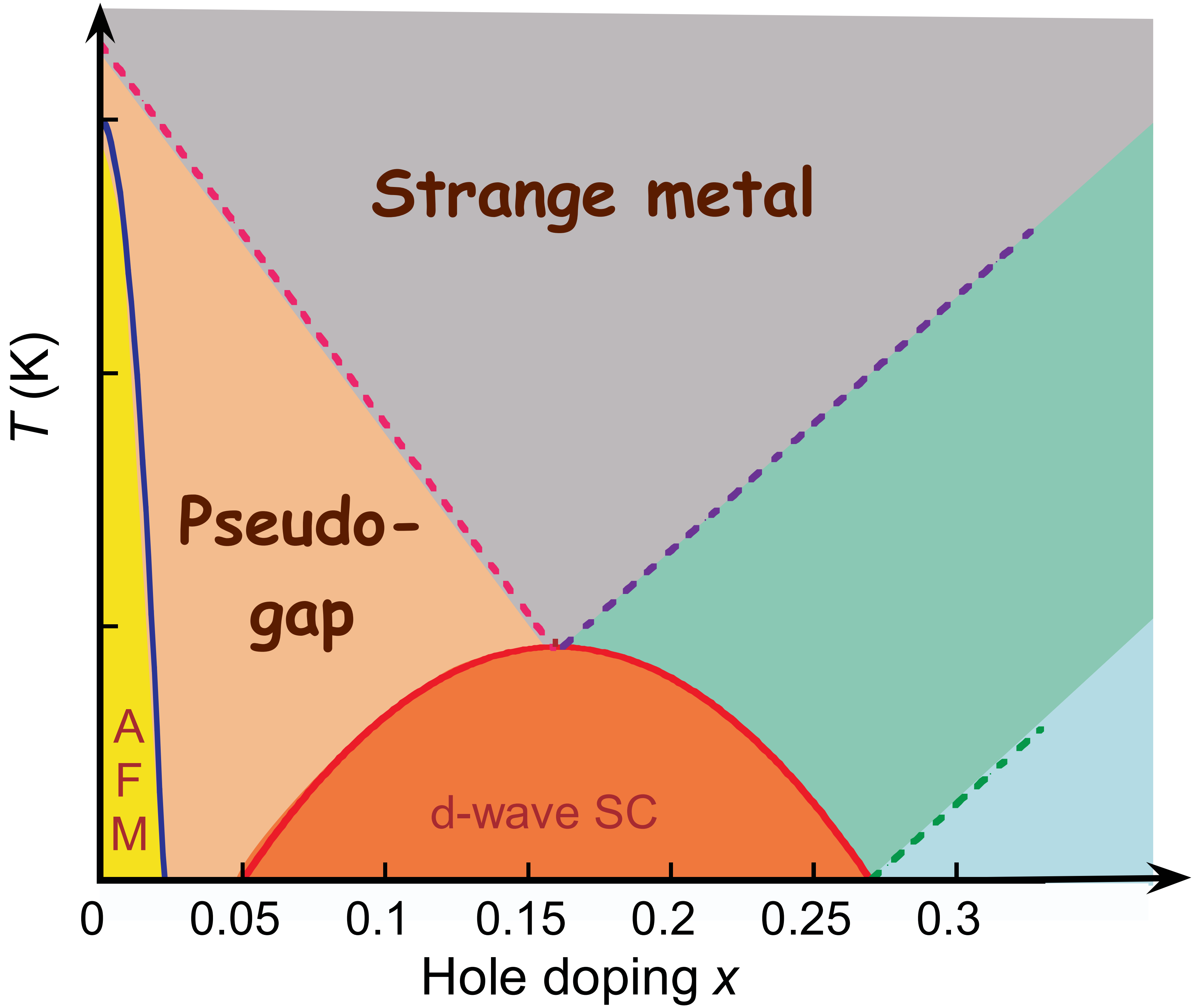
Large hole Fermi surface

Crossovers in transport properties of hole-doped cuprates

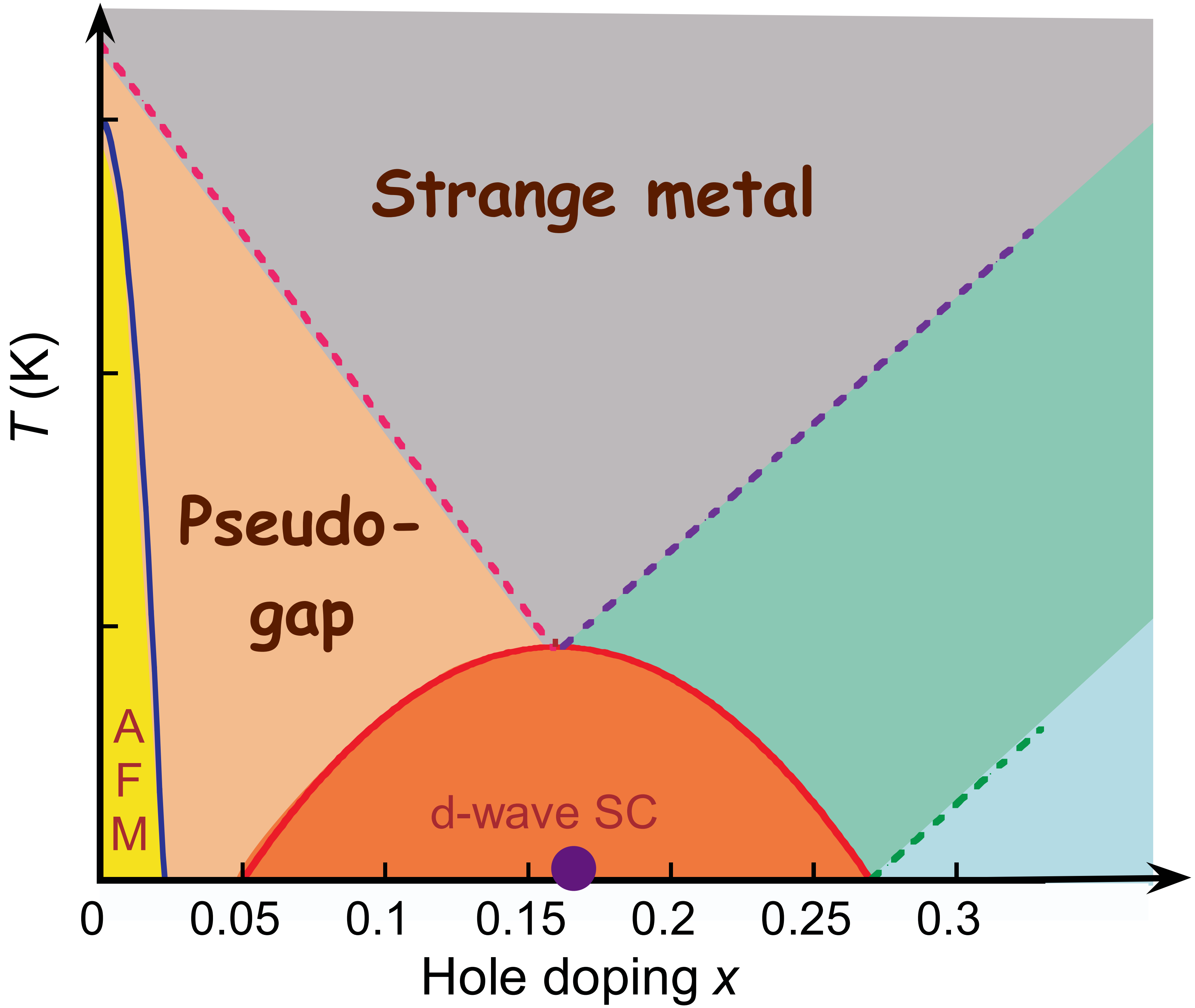


N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



Crossovers in transport properties of hole-doped cuprates



Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Questions

Questions

- How is antiferromagnetism lost with increasing doping ?

Questions

- How is antiferromagnetism lost with increasing doping ?

- How does the Fermi surface evolve with loss of antiferromagnetism ?

Questions

- How is antiferromagnetism lost with increasing doping ?
- How does the Fermi surface evolve with loss of antiferromagnetism ?
- Do antiferromagnetic fluctuations induce d-wave superconductivity ?

Questions

- How is antiferromagnetism lost with increasing doping ?

- How does the Fermi surface evolve with loss of antiferromagnetism ?

- Do antiferromagnetic fluctuations induce d-wave superconductivity ?

- How does attraction between antiferromagnetism and superconductivity turn into competition ?

Questions

- How is antiferromagnetism lost with increasing doping ?
- How does the Fermi surface evolve with loss of antiferromagnetism ?
- Do antiferromagnetic fluctuations induce d-wave superconductivity ?
- How does attraction between antiferromagnetism and superconductivity turn into competition ?
- Is there a broken symmetry in the pseudo-gap regime ?

Questions

- How is antiferromagnetism lost with increasing doping ?
- How does the Fermi surface evolve with loss of antiferromagnetism ?
- Do antiferromagnetic fluctuations induce d-wave superconductivity ?
- How does attraction between antiferromagnetism and superconductivity turn into competition ?
- Is there a broken symmetry in the pseudo-gap regime ?
- What is the role of nematic/valence-bond-solid/stripe order ?

Questions

- How is antiferromagnetism lost with increasing doping ?
- How does the Fermi surface evolve with loss of antiferromagnetism ?
- Do antiferromagnetic fluctuations induce d-wave superconductivity ?
- How does attraction between antiferromagnetism and superconductivity turn into competition ?
- Is there a broken symmetry in the pseudo-gap regime ?
- What is the role of nematic/valence-bond-solid/stripe order ?
- What is the physics of the strange metal ?

Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface

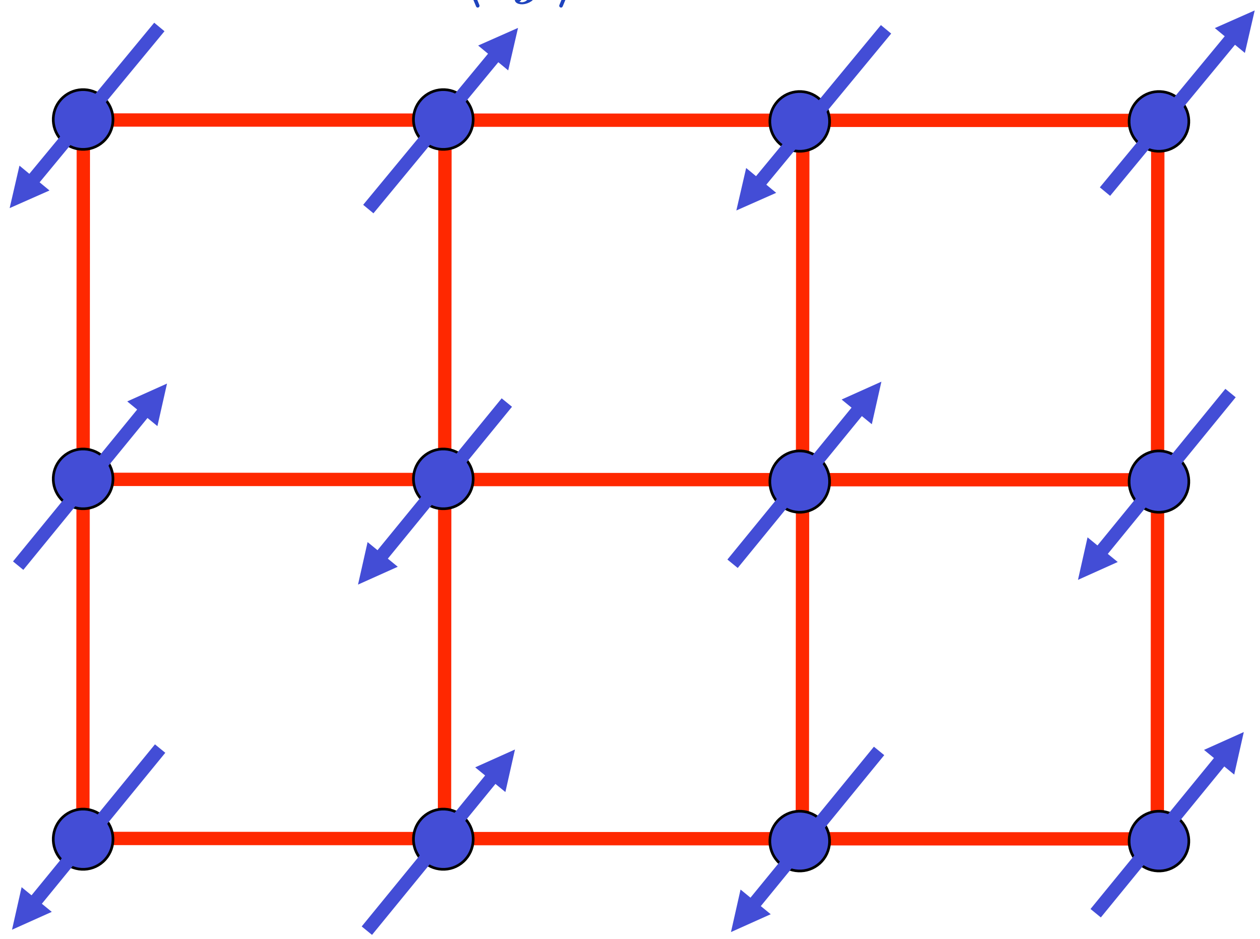
Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

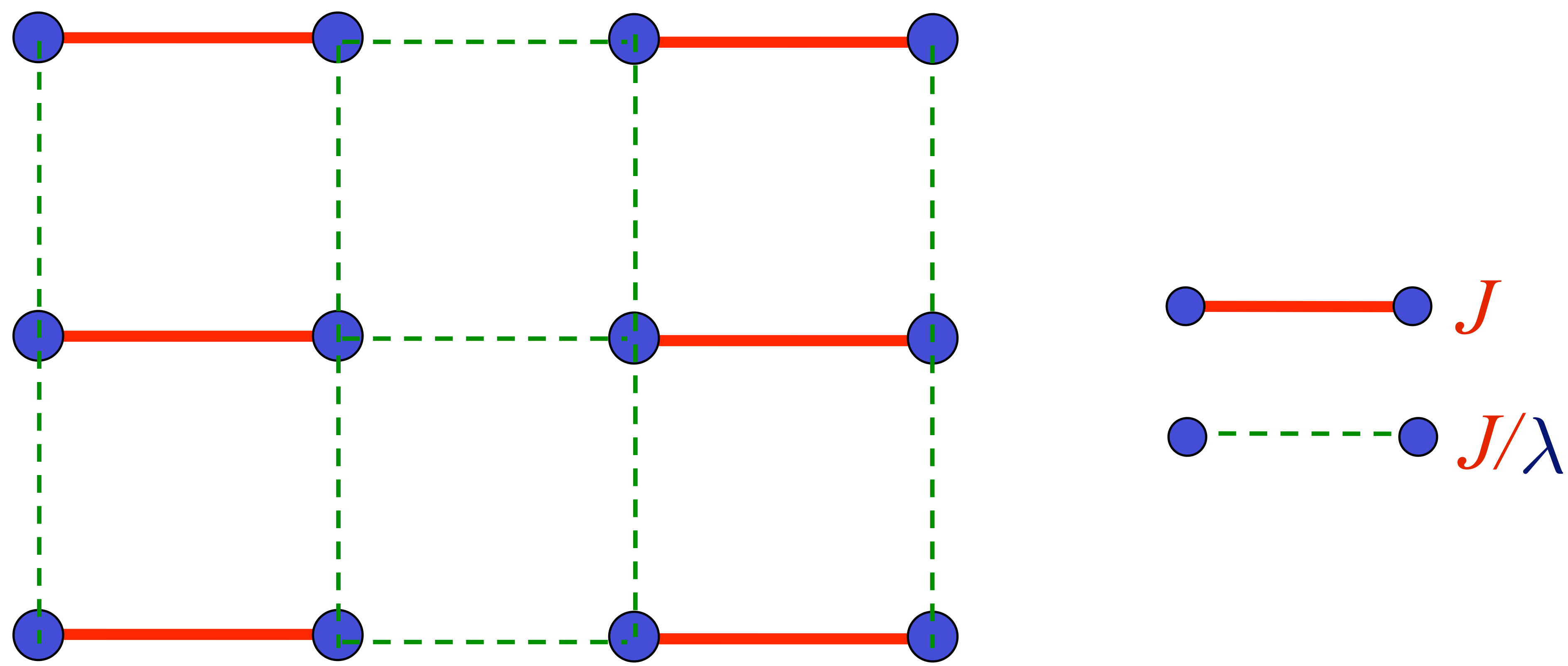
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

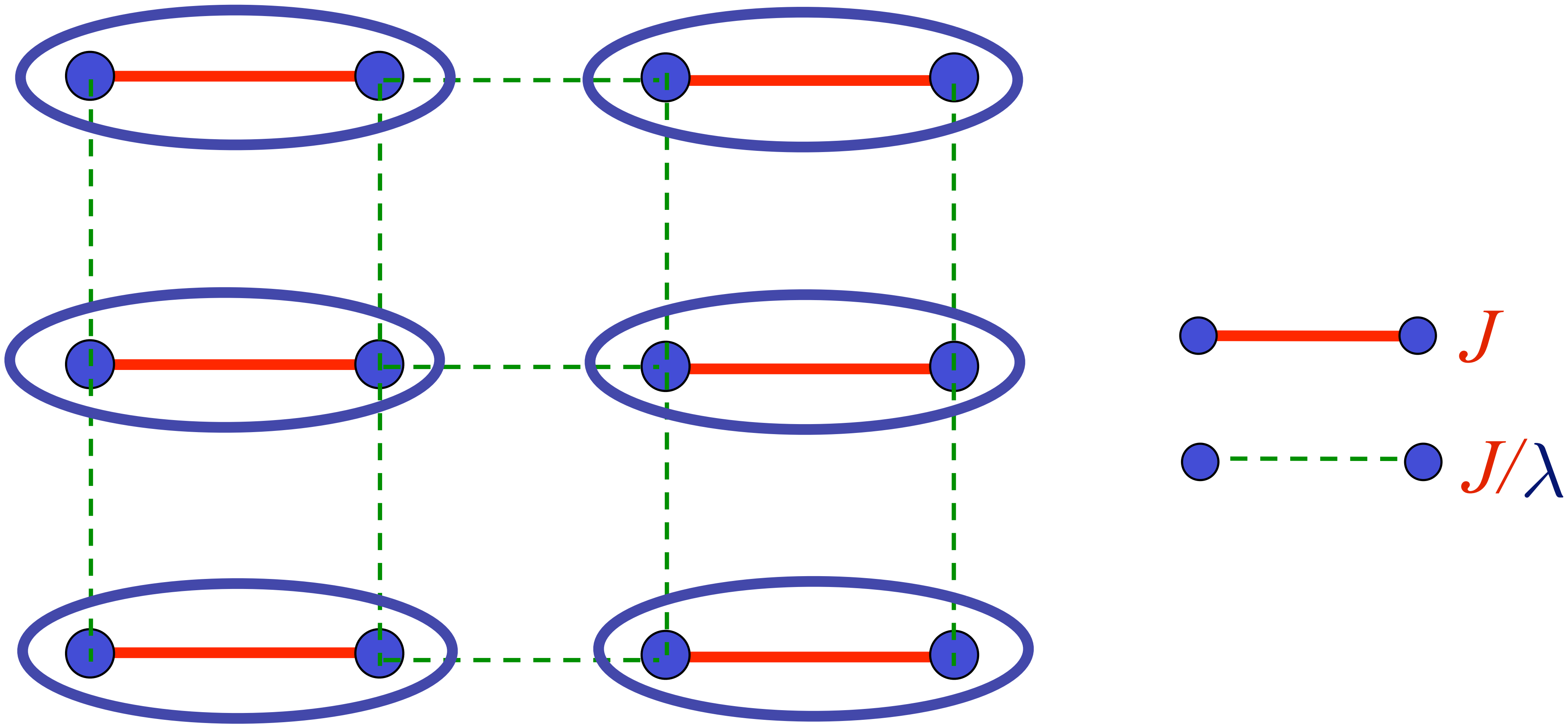
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



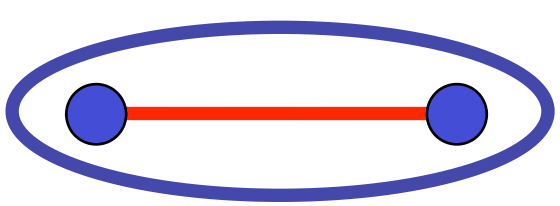
Weaken some bonds to induce spin entanglement in a new quantum phase

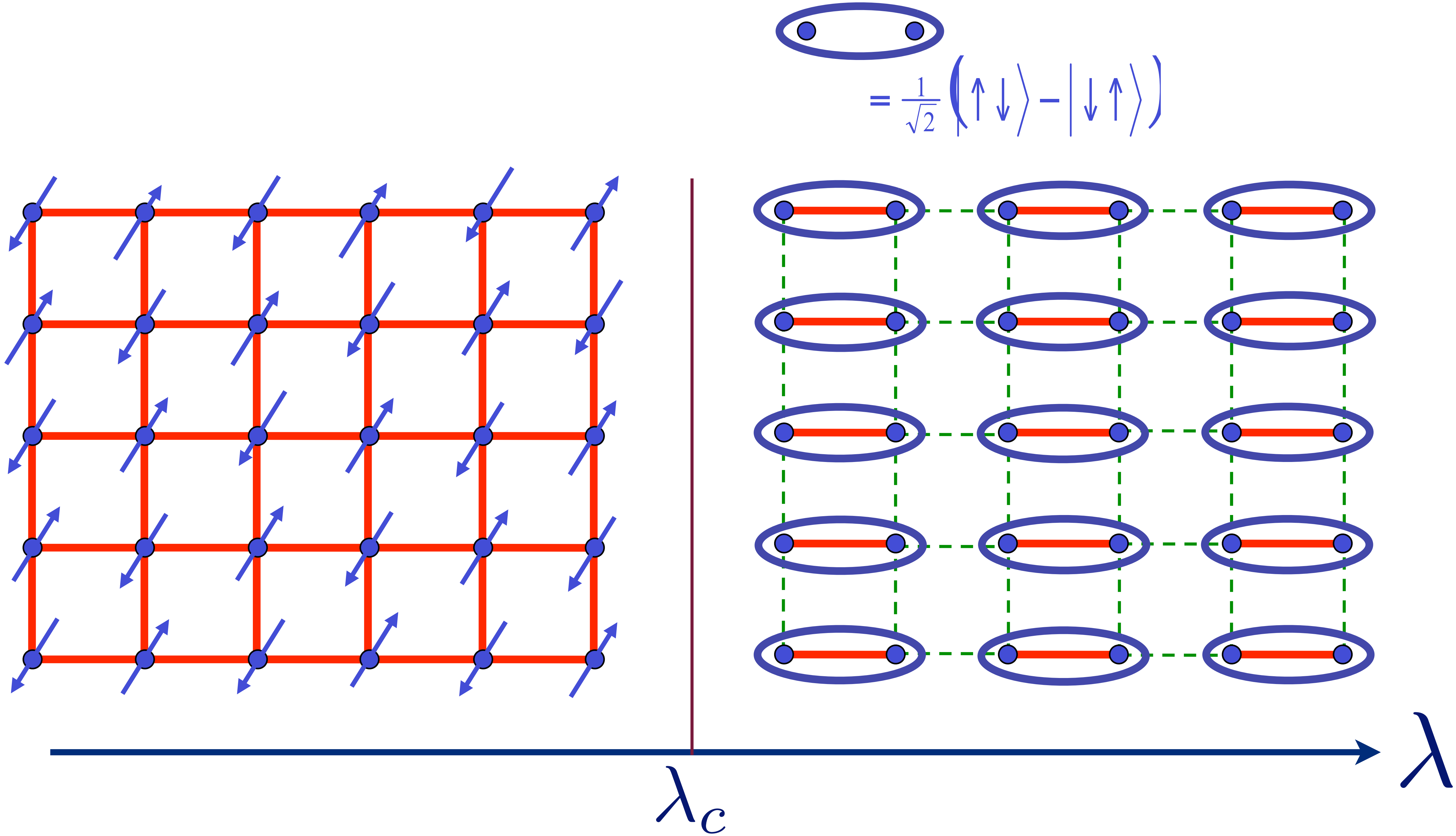
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

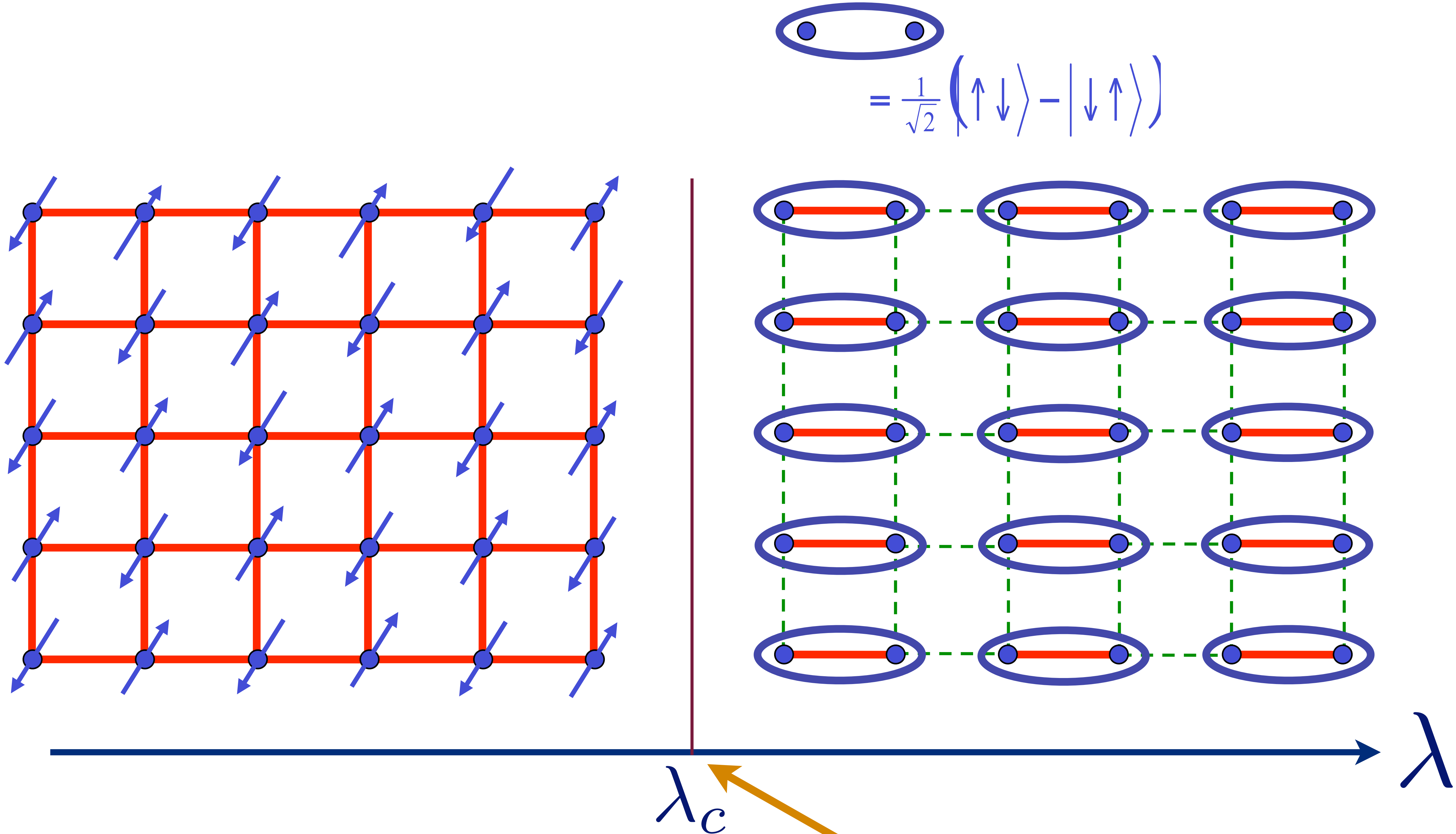


Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

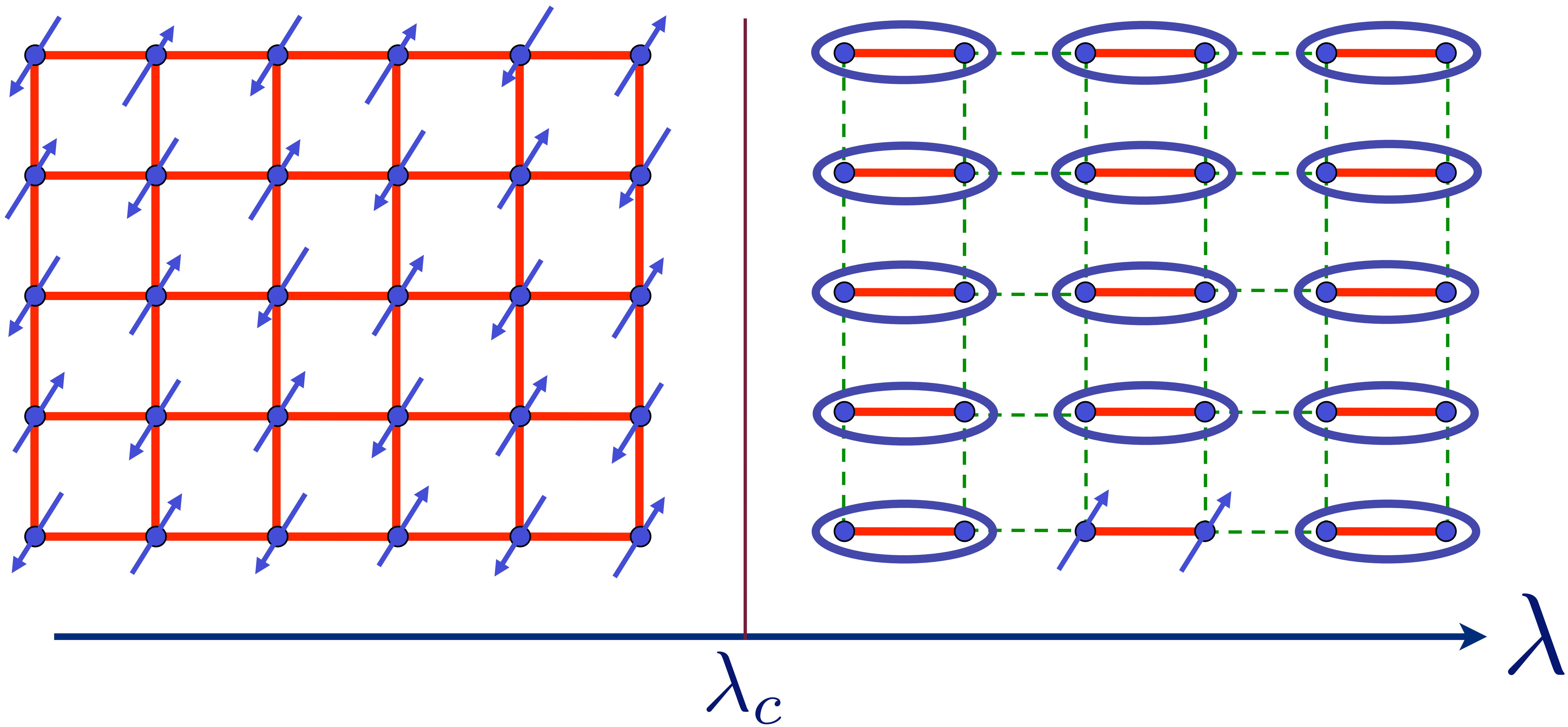


Pressure in $TlCuCl_3$

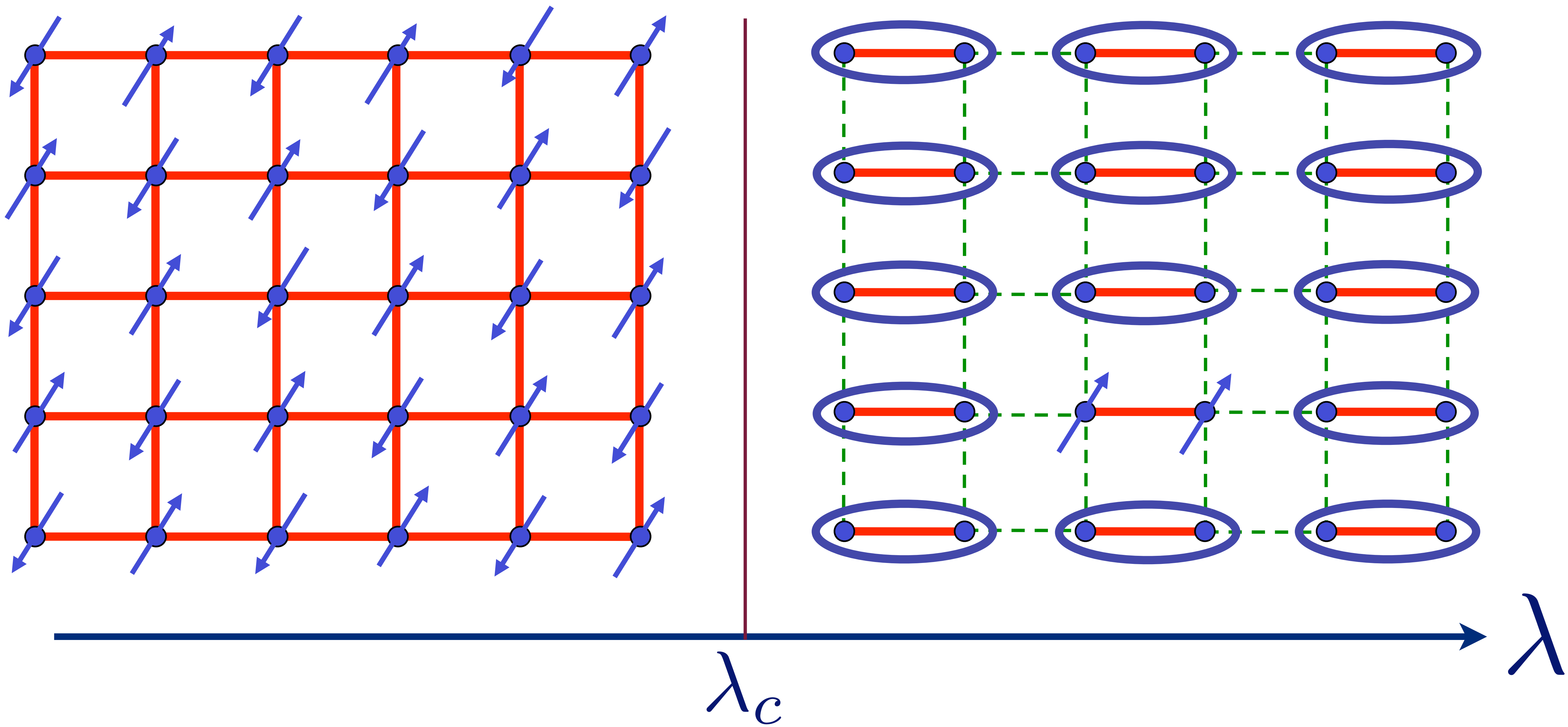


Quantum critical point with non-local entanglement in spin wavefunction

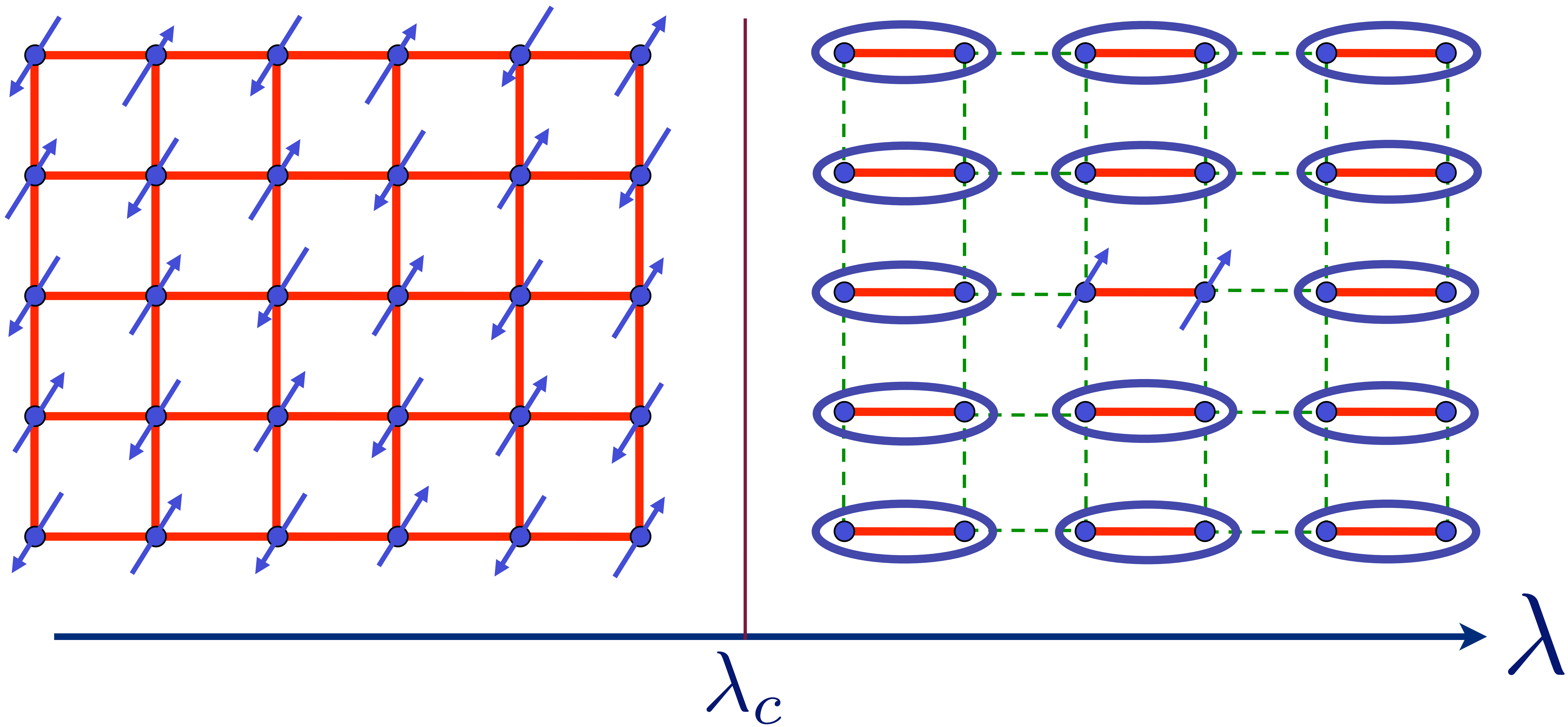
Excitation spectrum in the paramagnetic phase



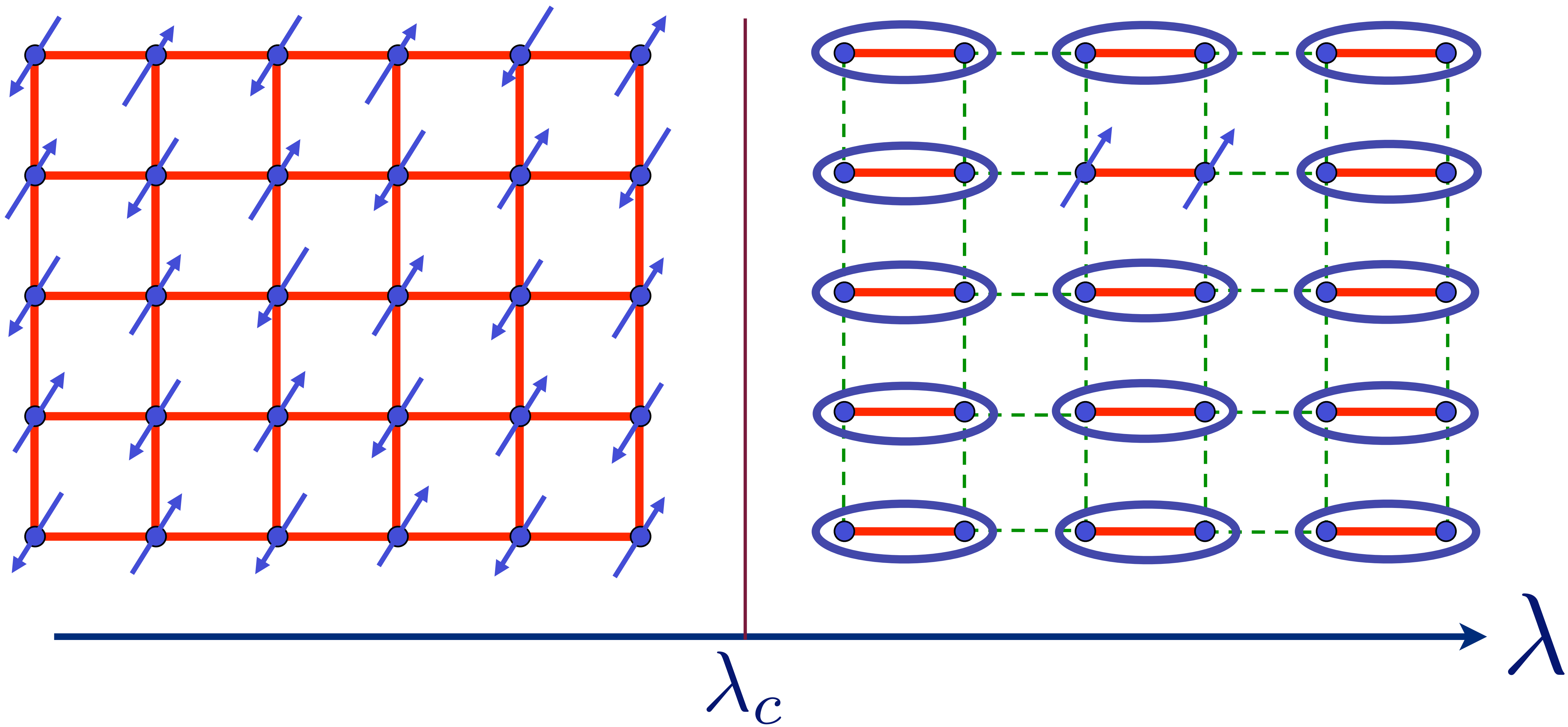
Excitation spectrum in the paramagnetic phase



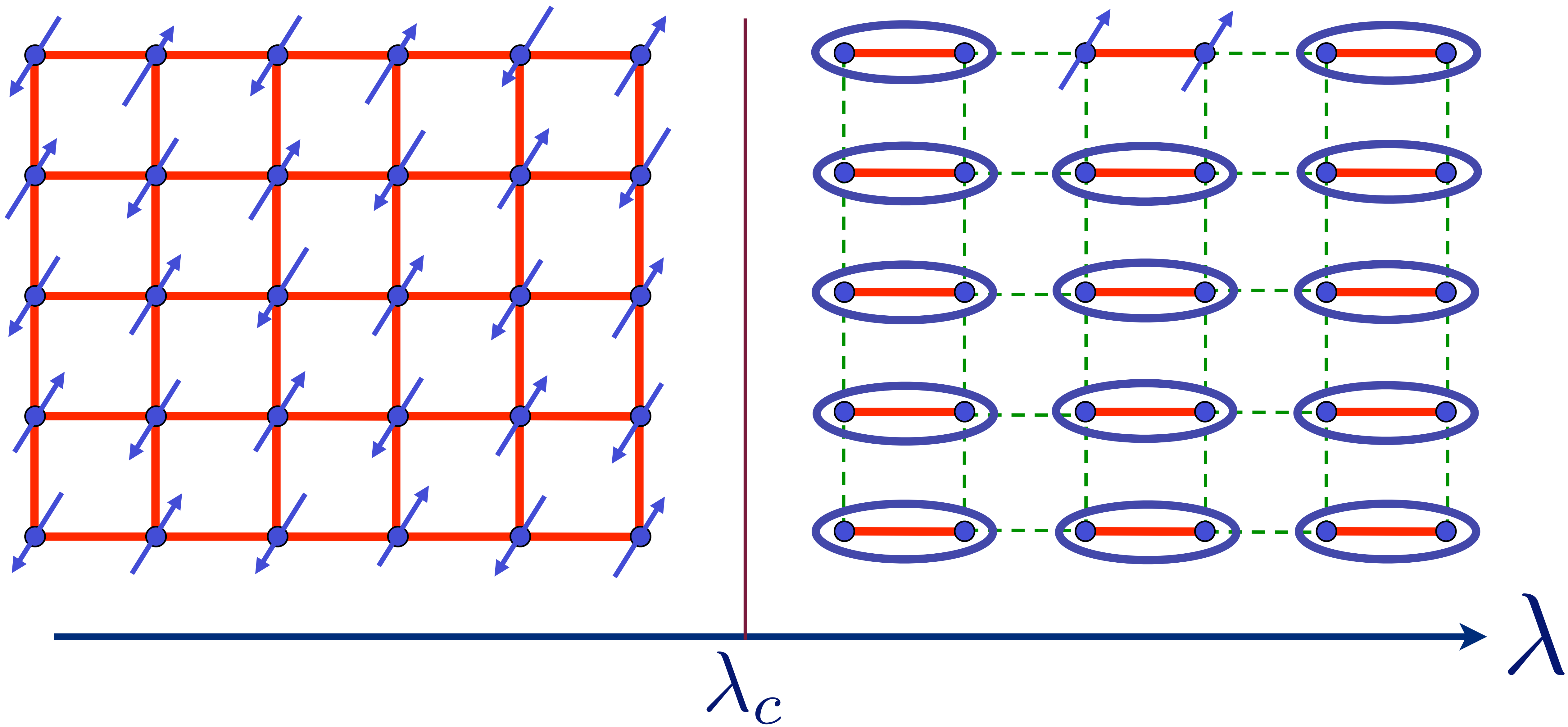
Excitation spectrum in the paramagnetic phase



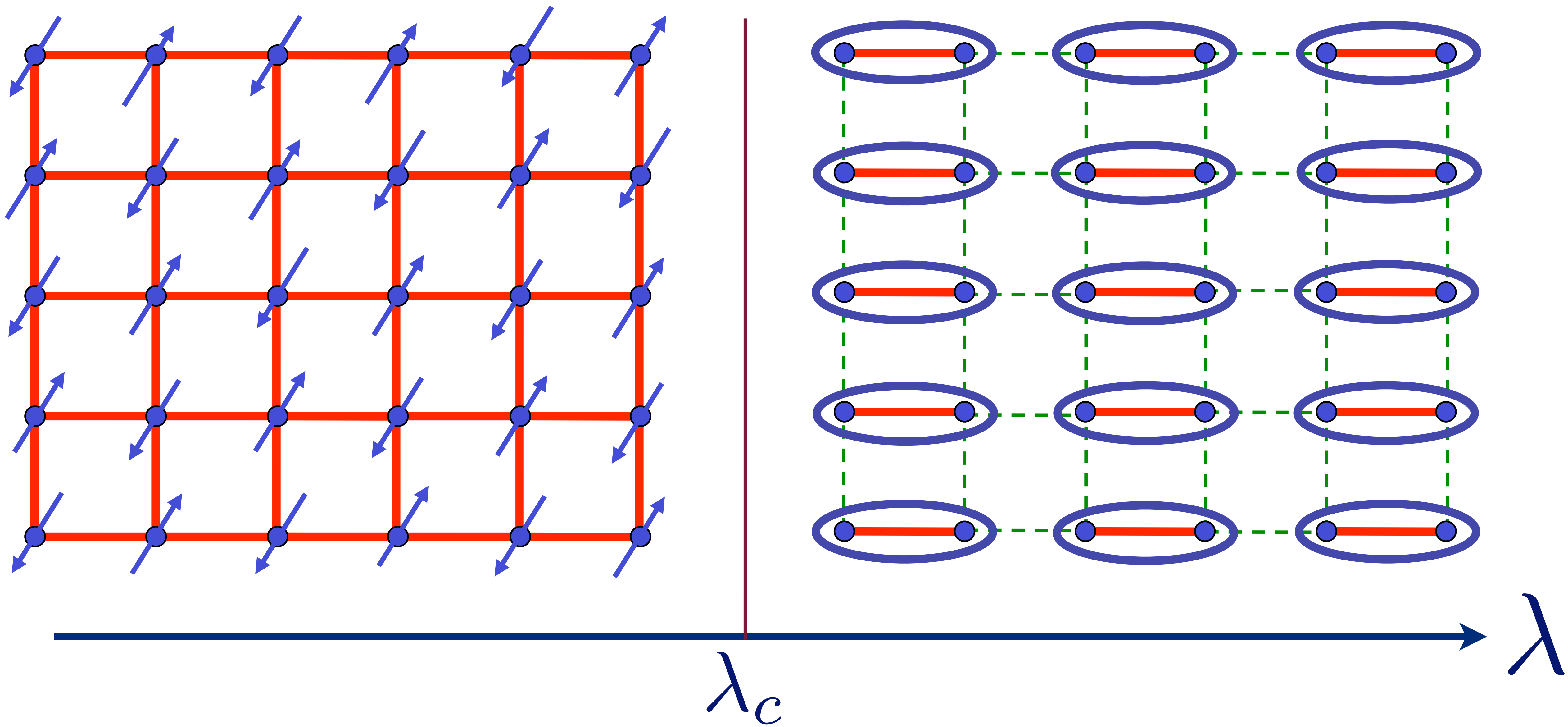
Excitation spectrum in the paramagnetic phase



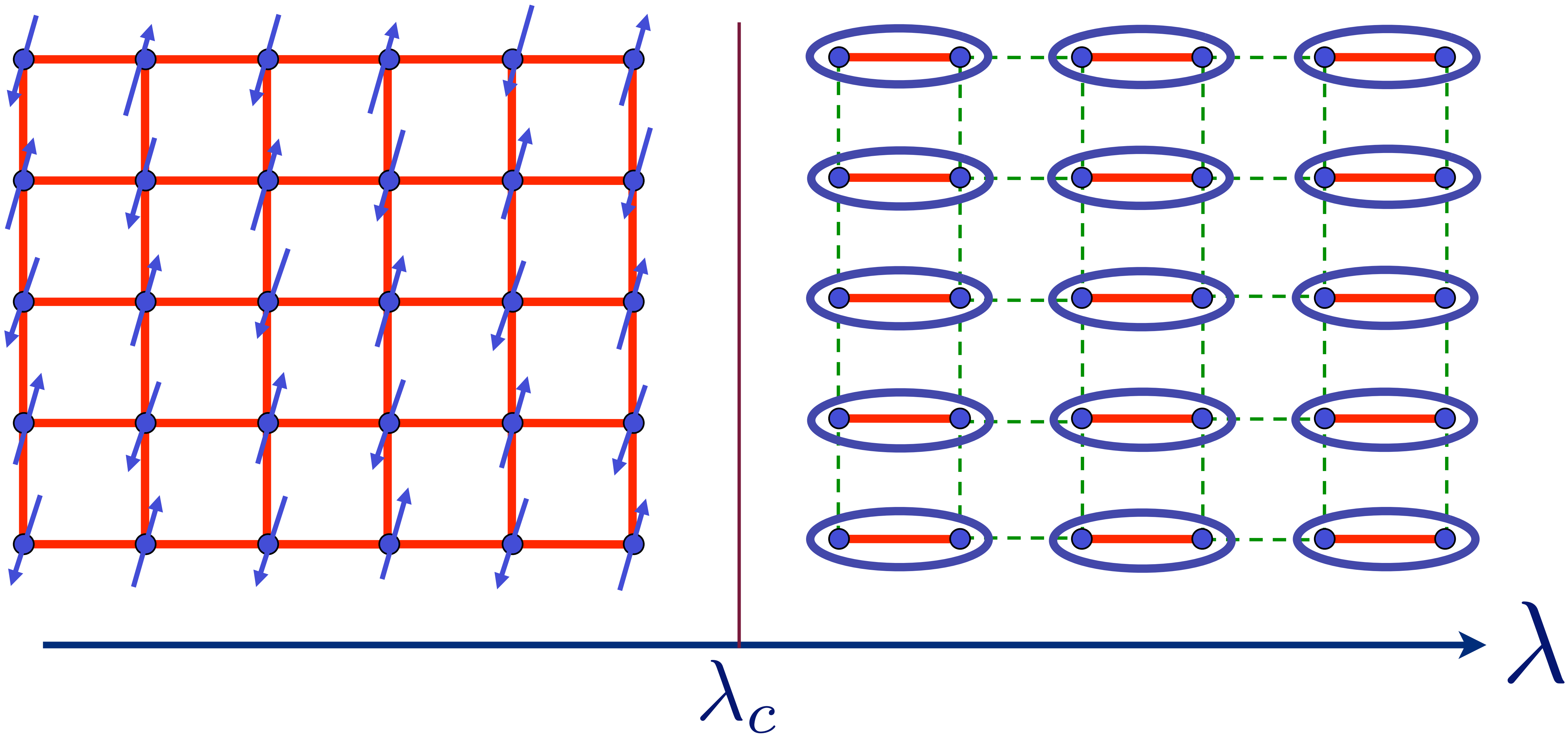
Excitation spectrum in the paramagnetic phase



Excitation spectrum in the Néel phase

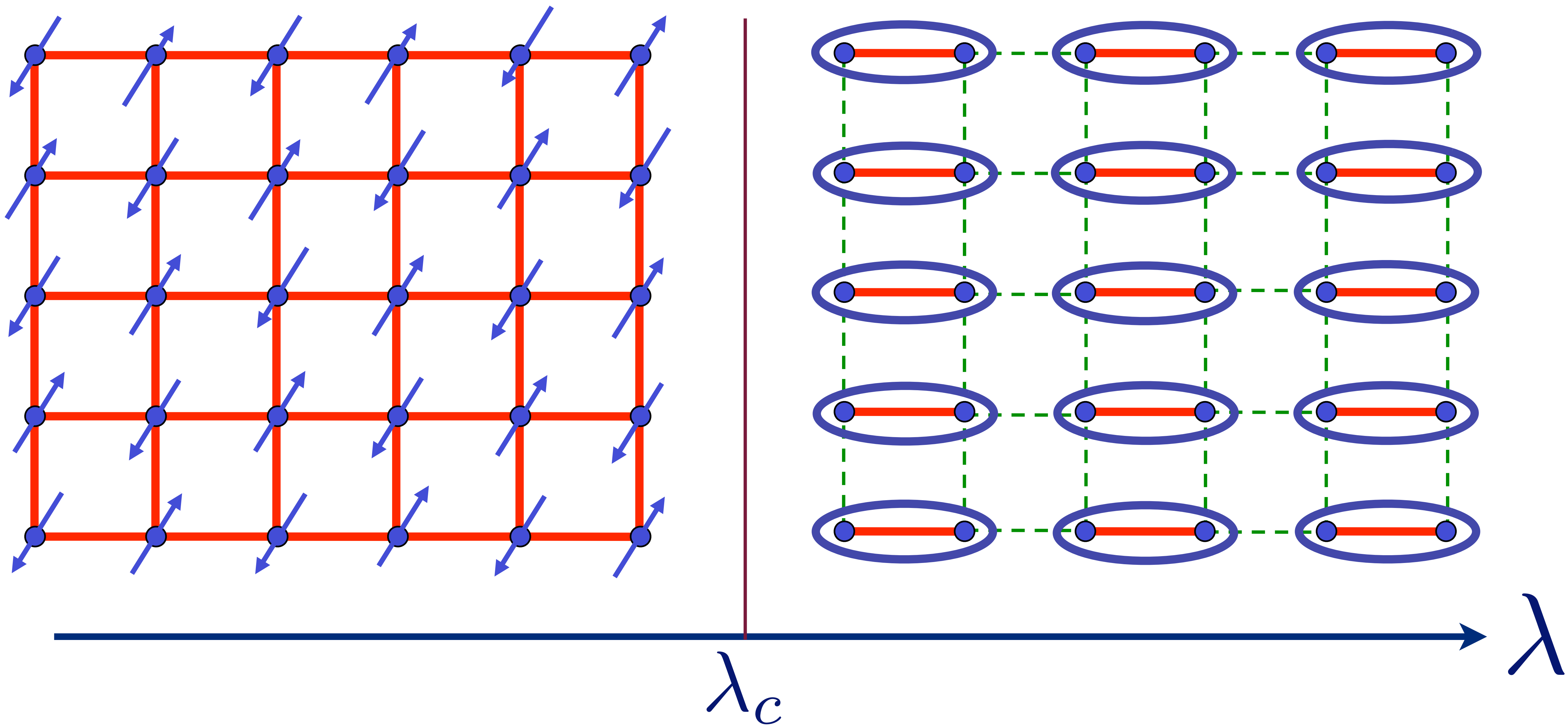


Excitation spectrum in the Néel phase



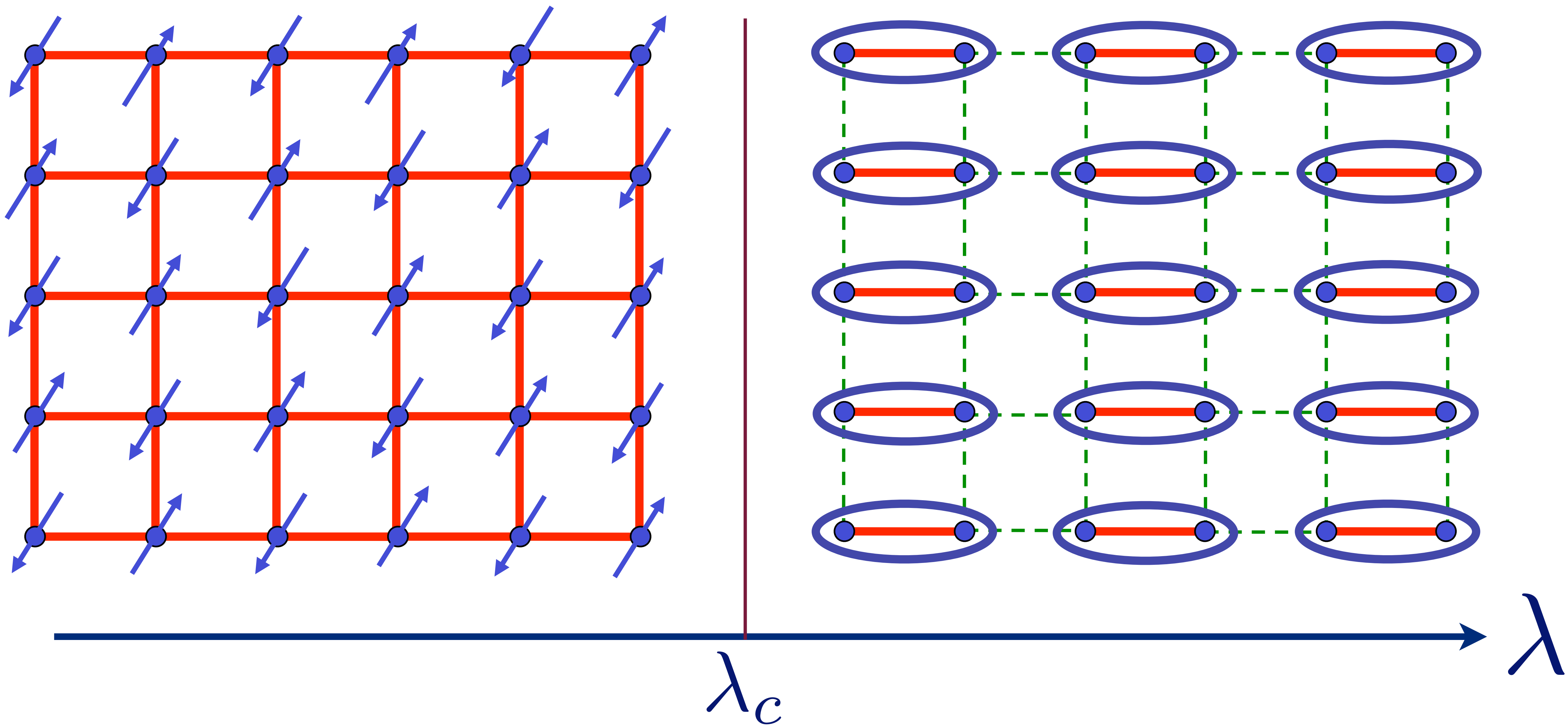
Spin waves

Excitation spectrum in the Néel phase



Spin waves

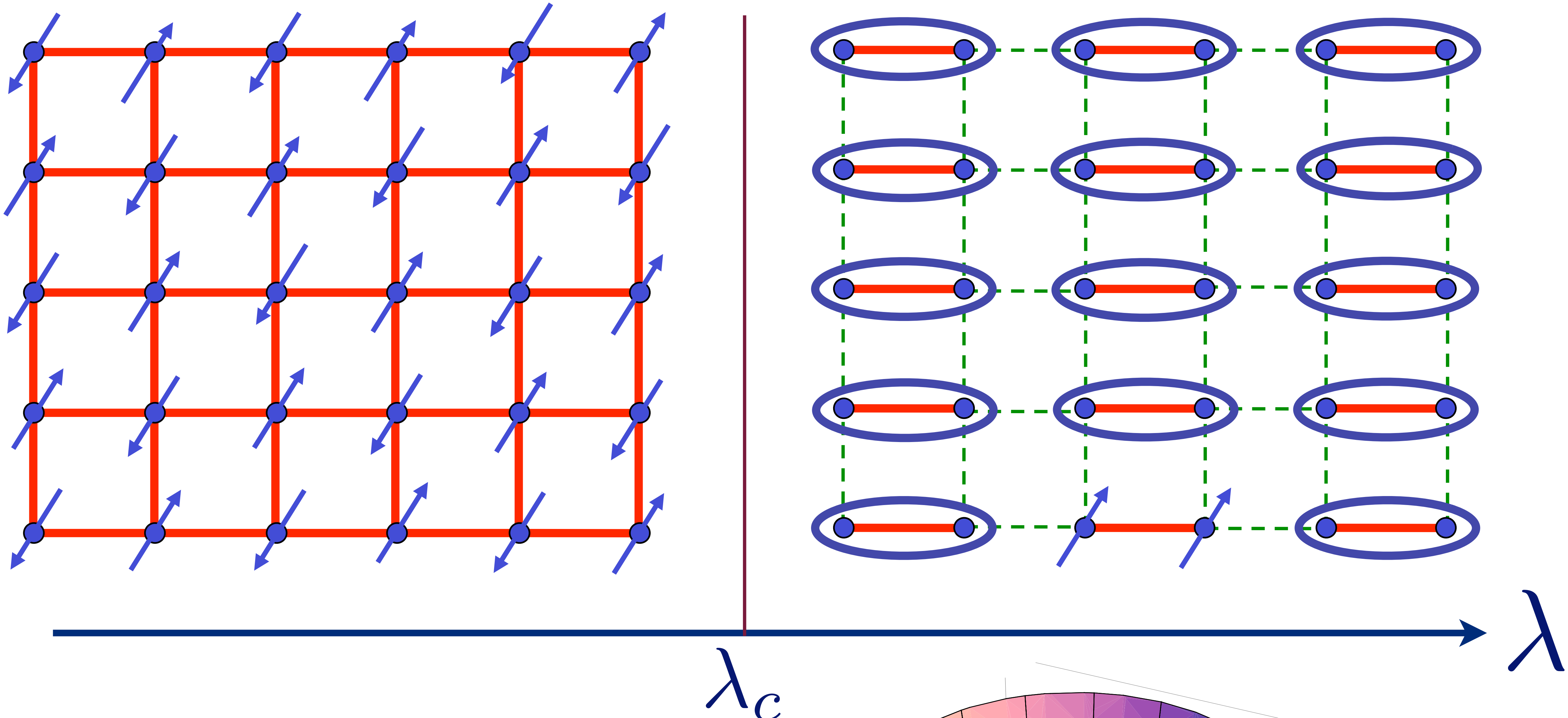
Description using Landau-Ginzburg field theory



O(3) order parameter $\vec{\varphi}$

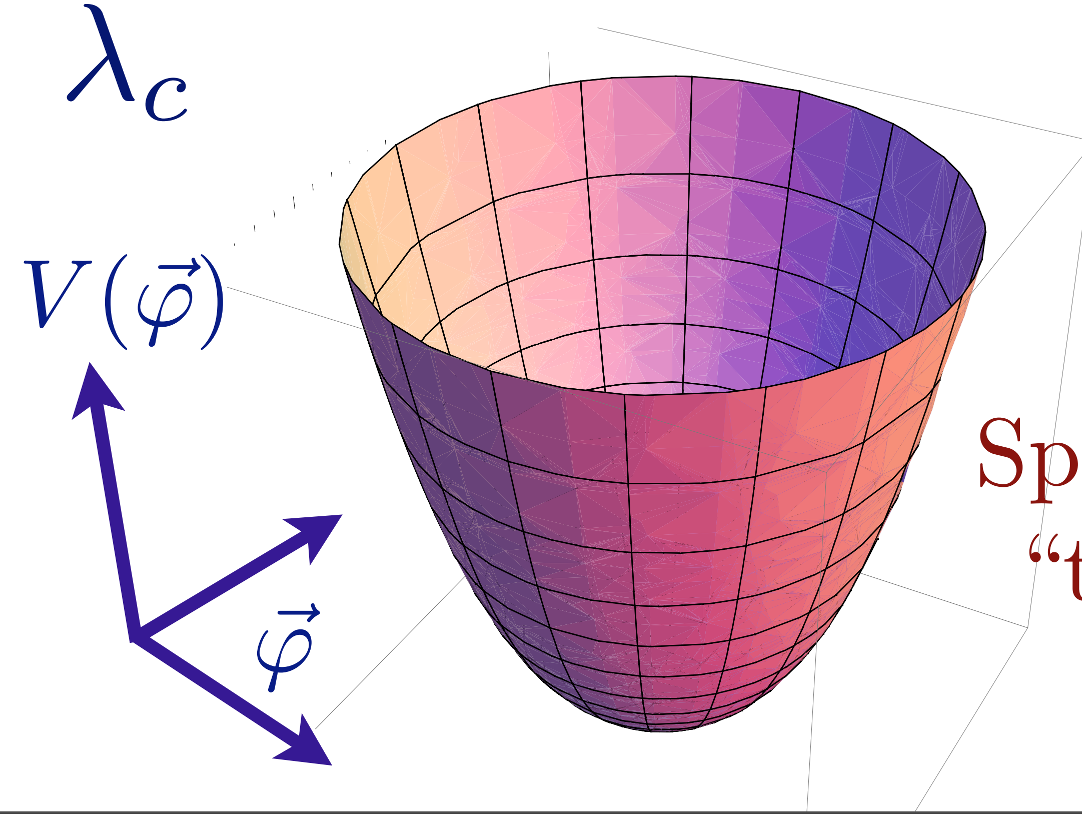
$$\mathcal{S} = \int d^2r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u \left(\vec{\varphi}^2\right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



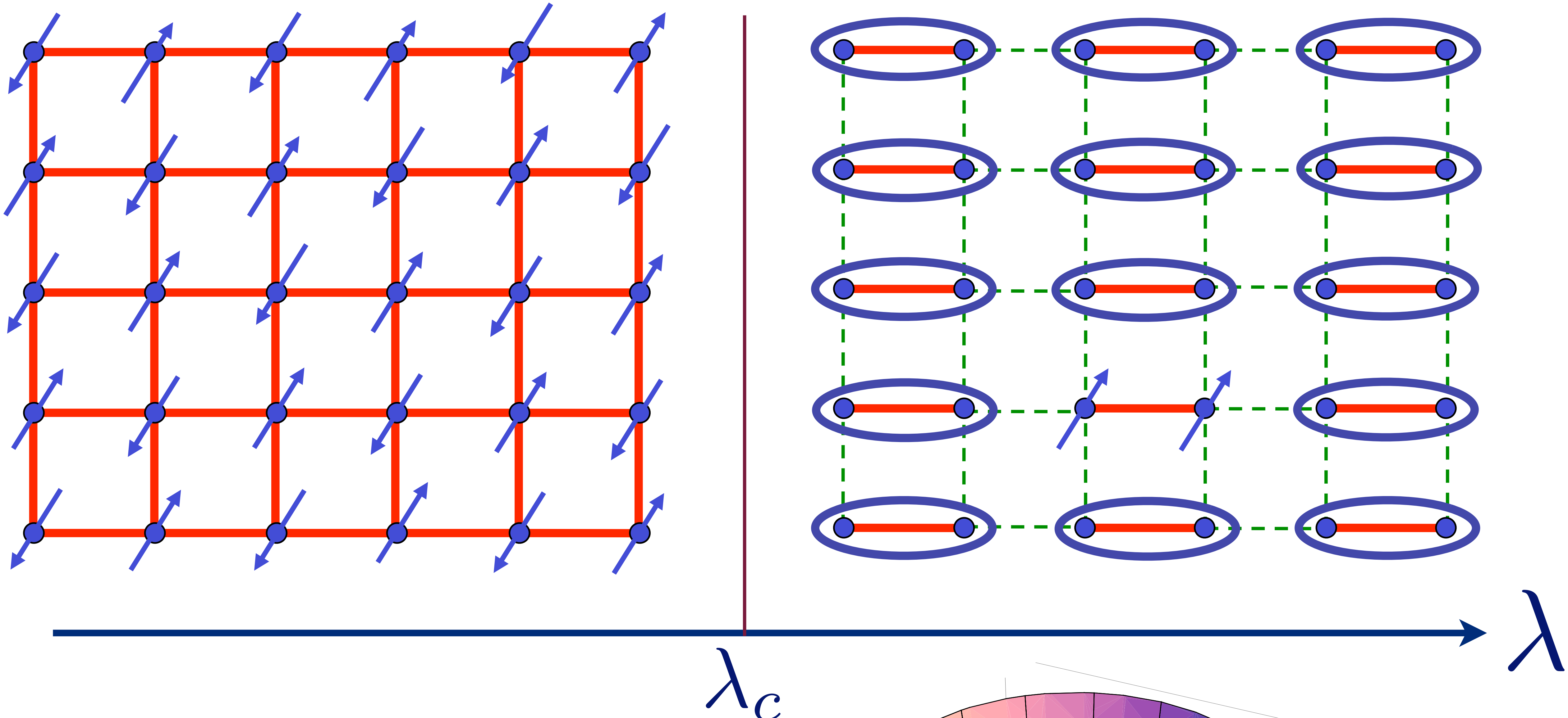
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



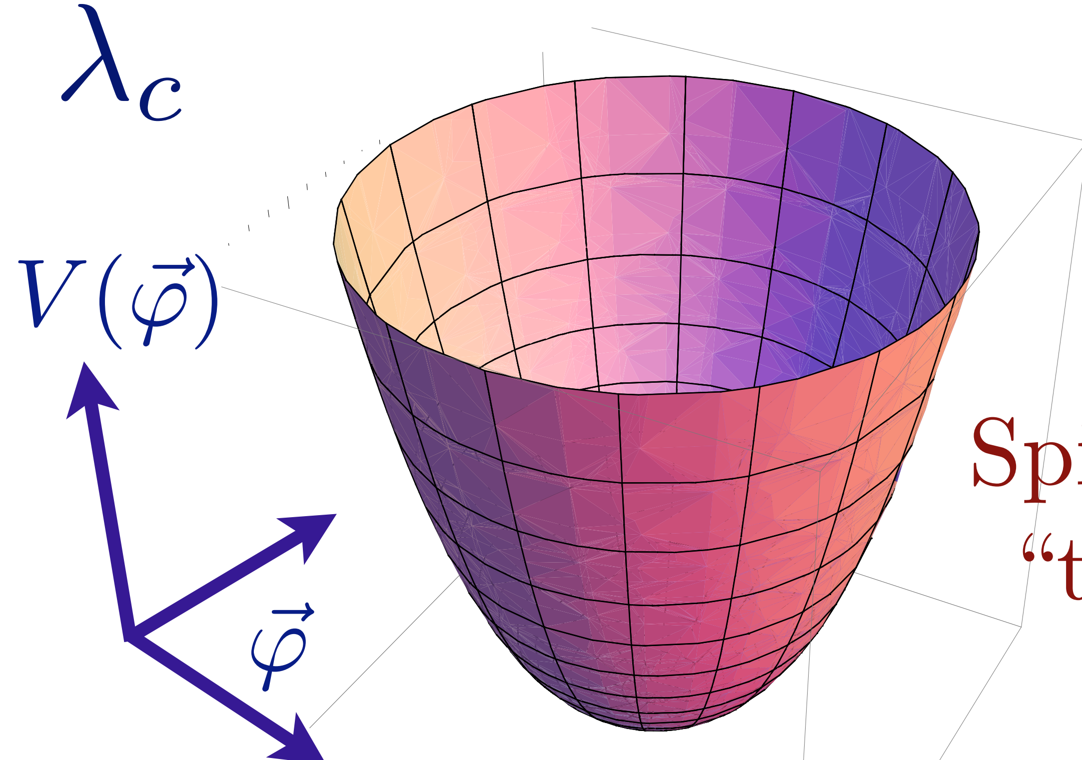
Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



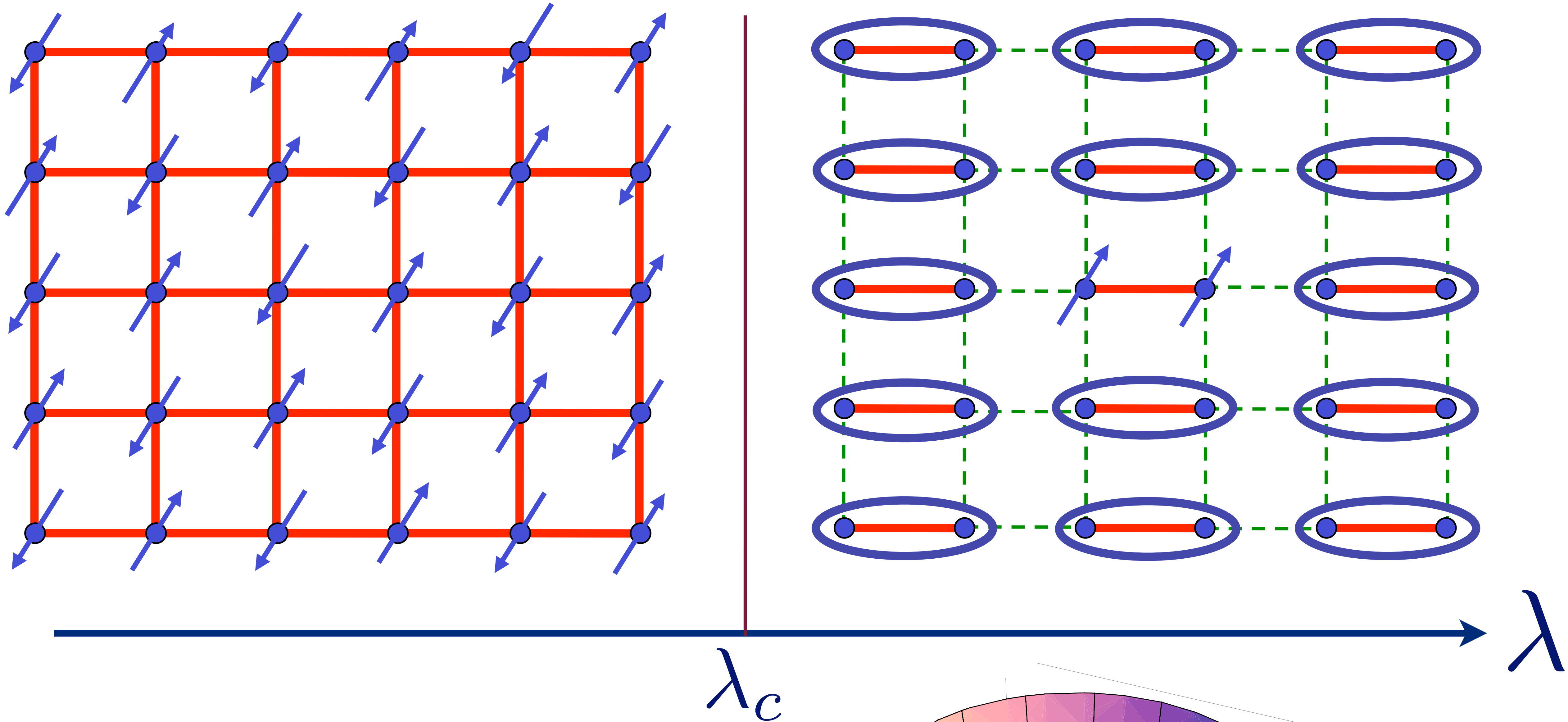
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



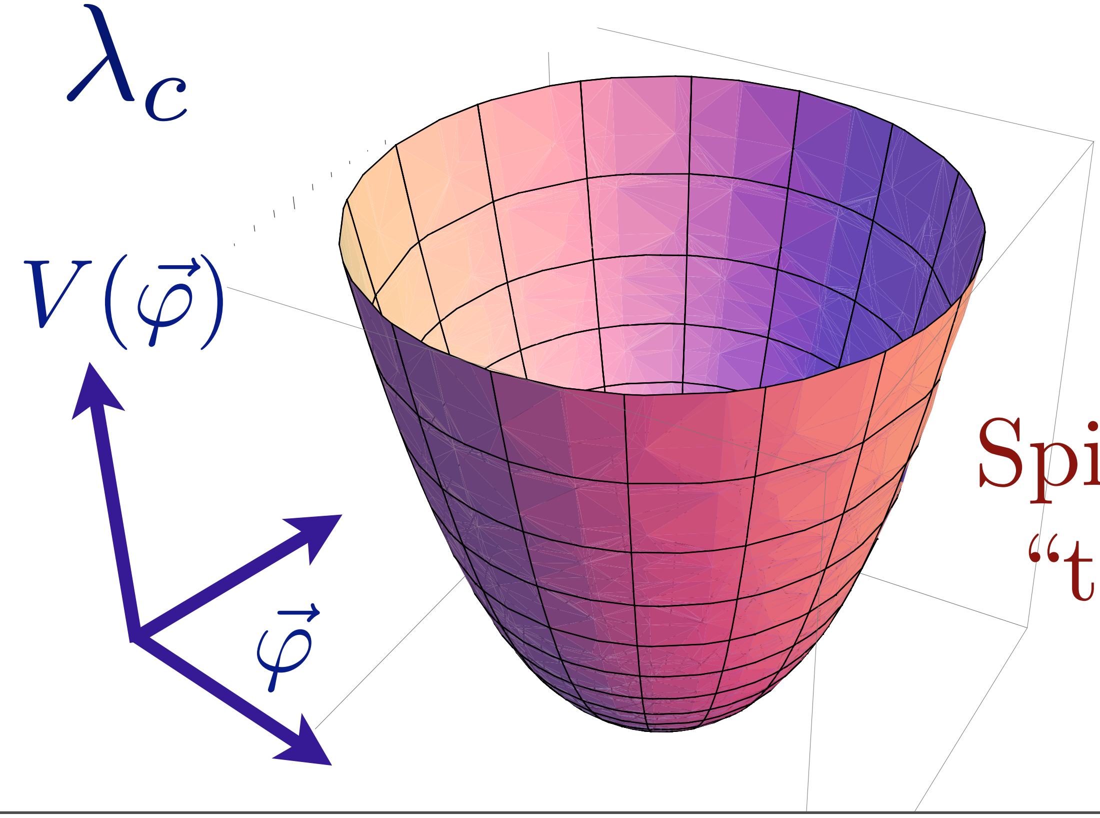
Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



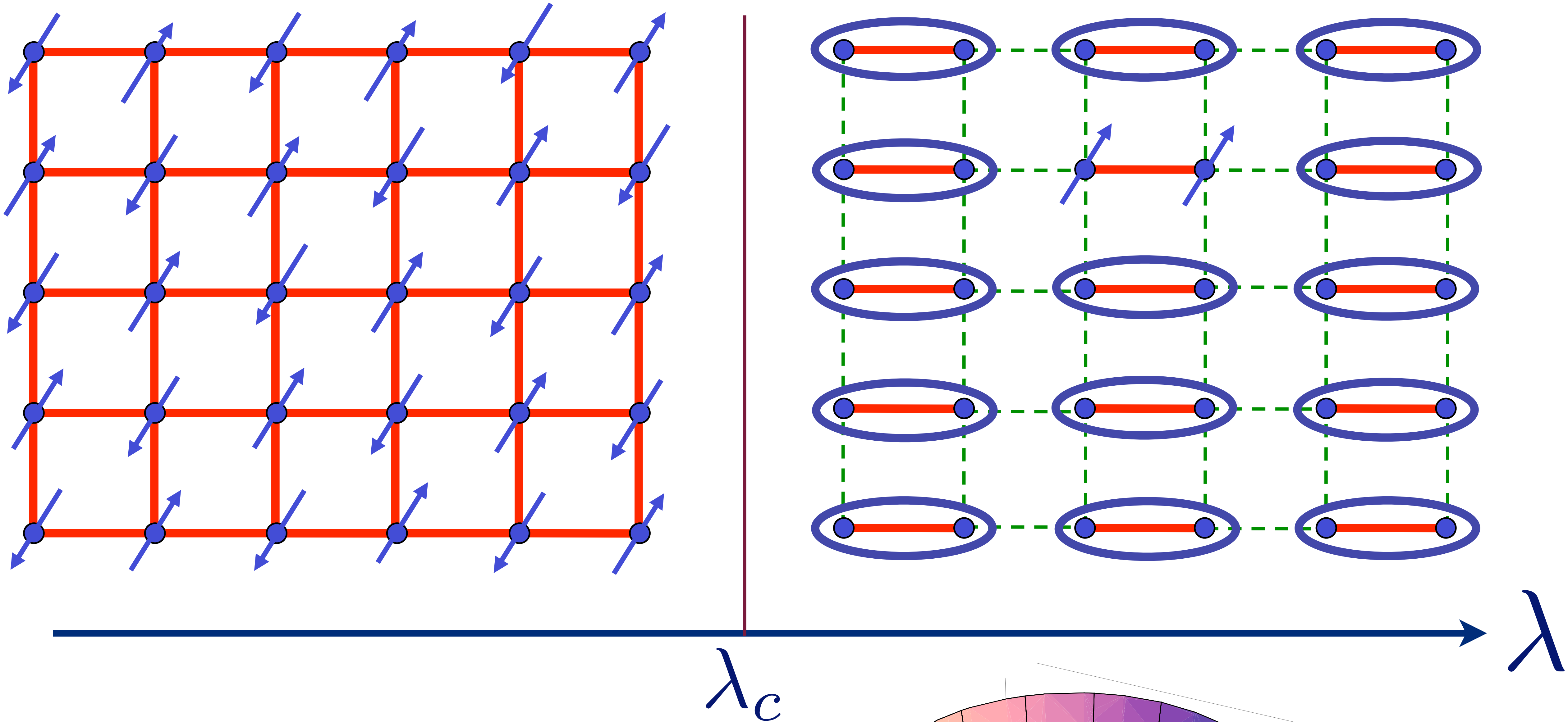
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



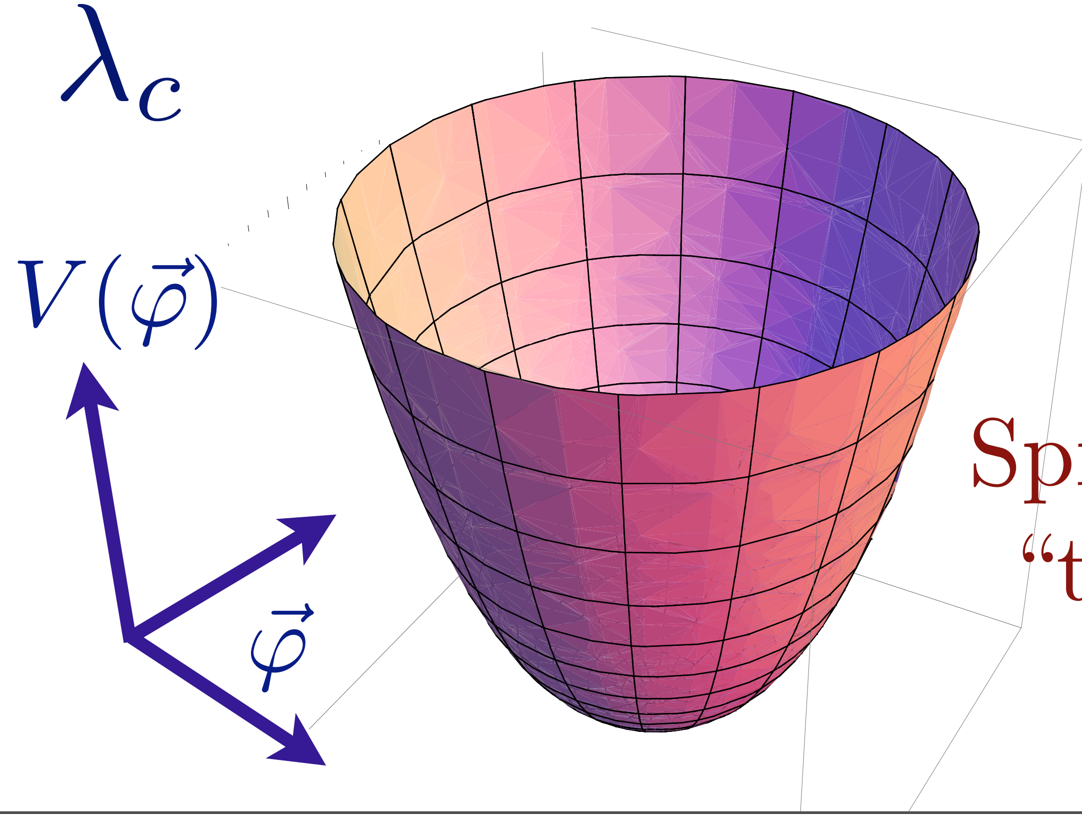
Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



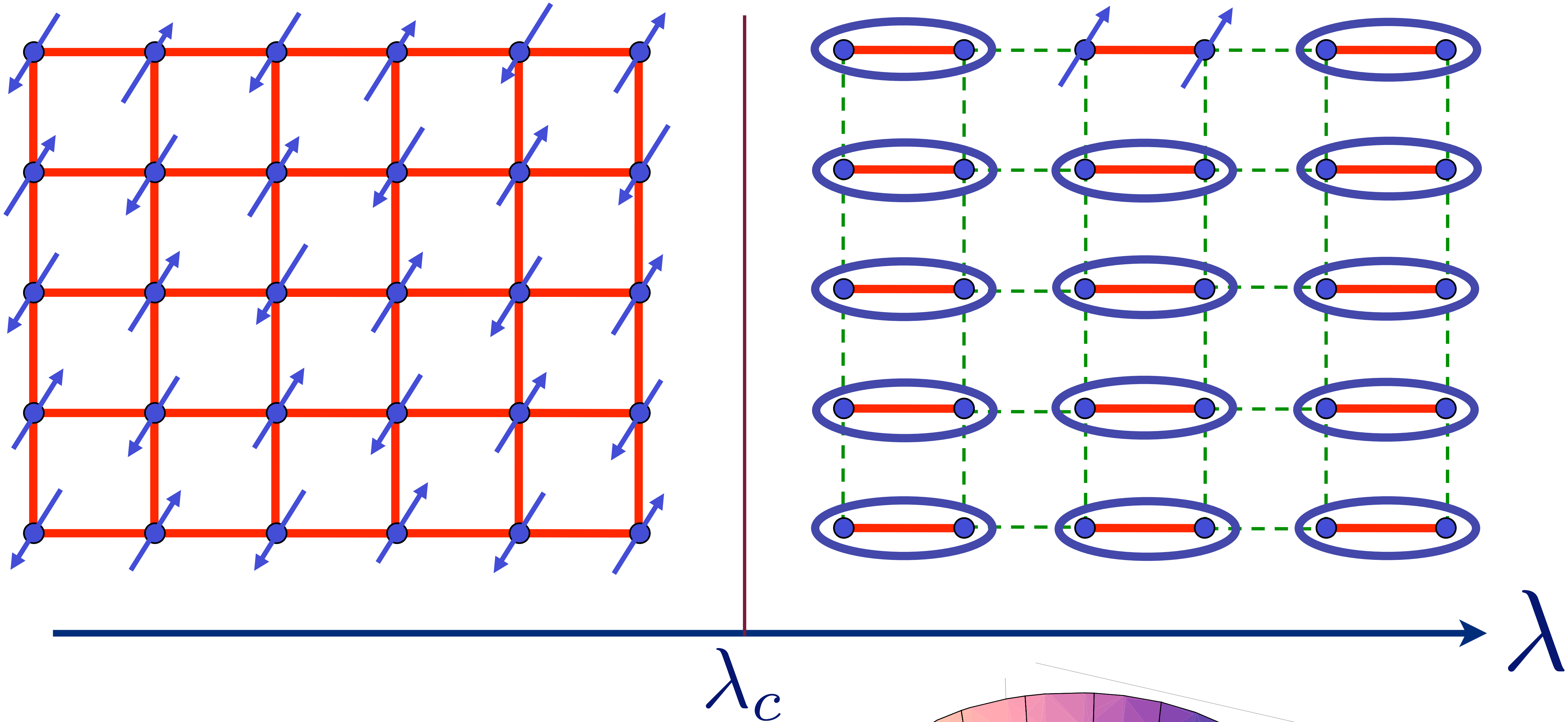
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



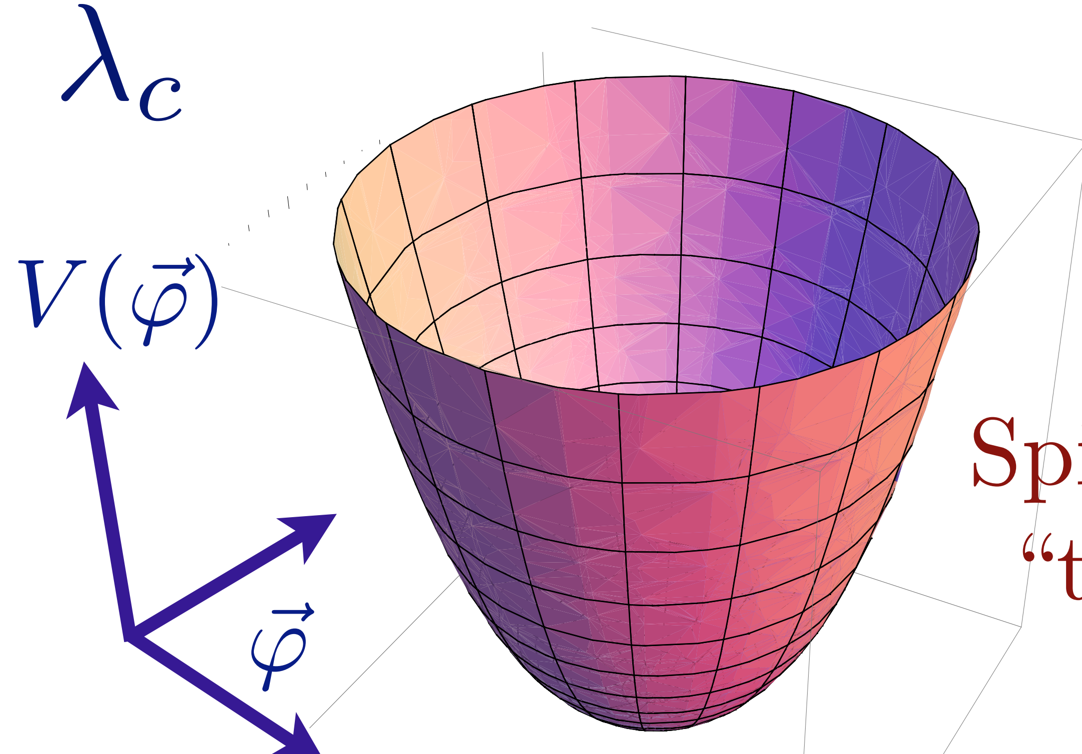
Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



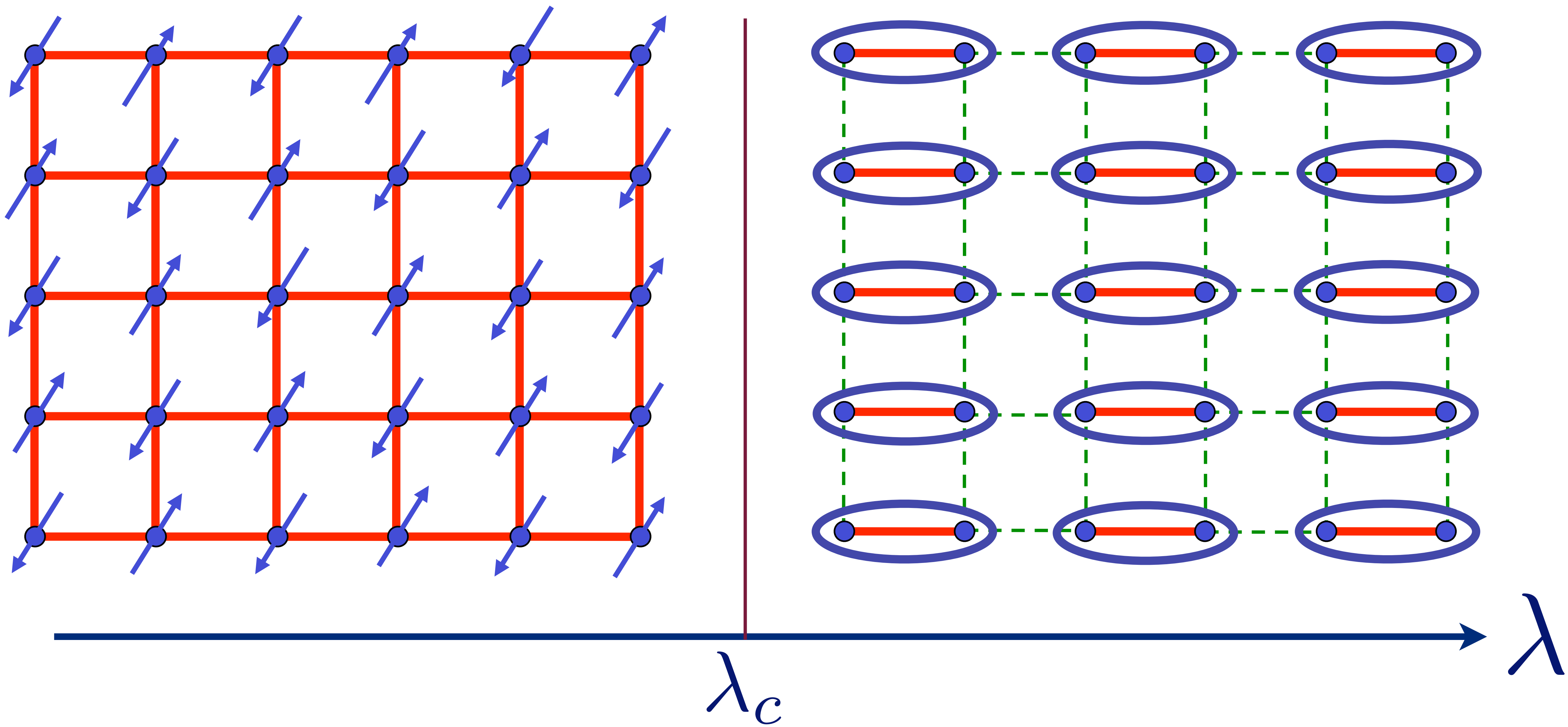
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$

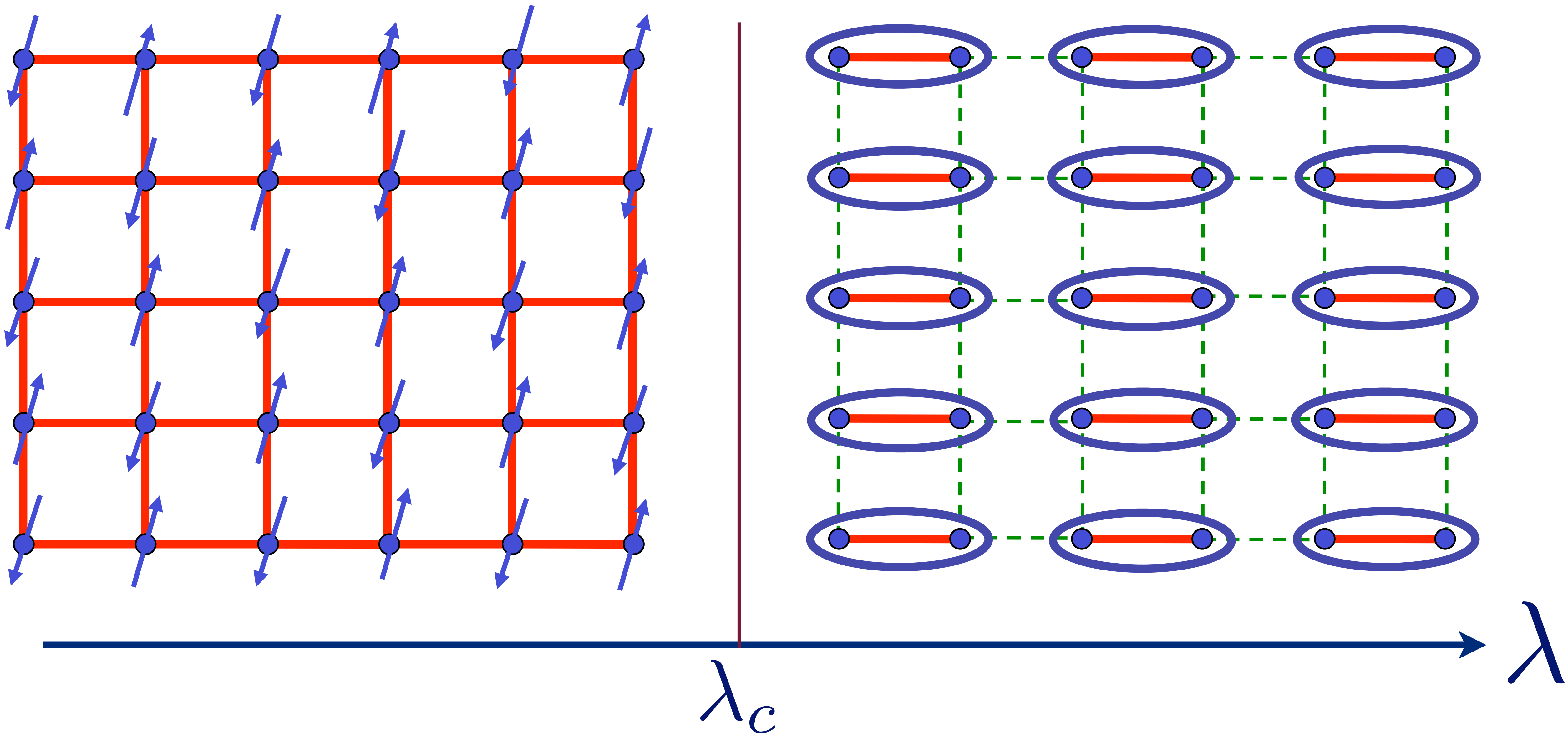


Spin $S = 1$
"triplon"

Excitation spectrum in the Néel phase

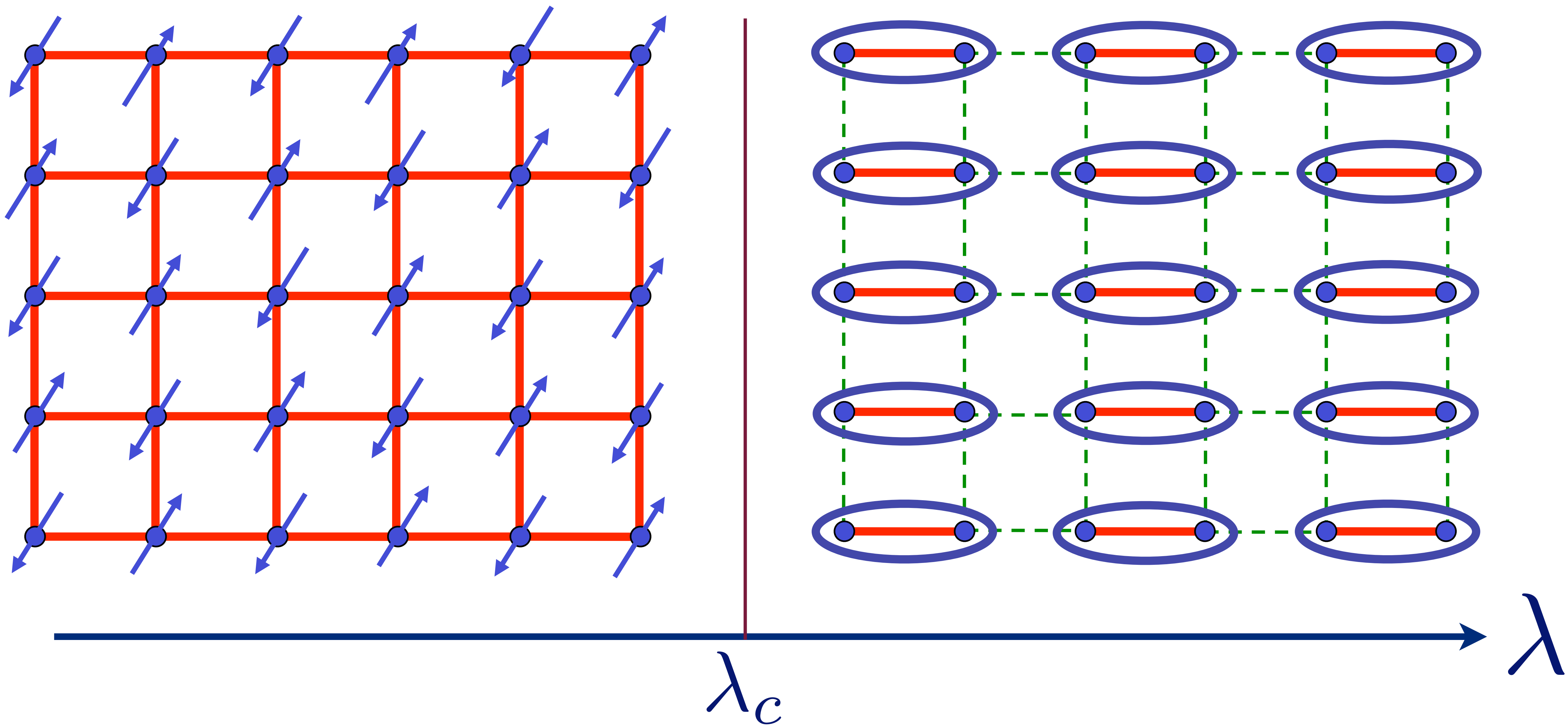


Excitation spectrum in the Néel phase



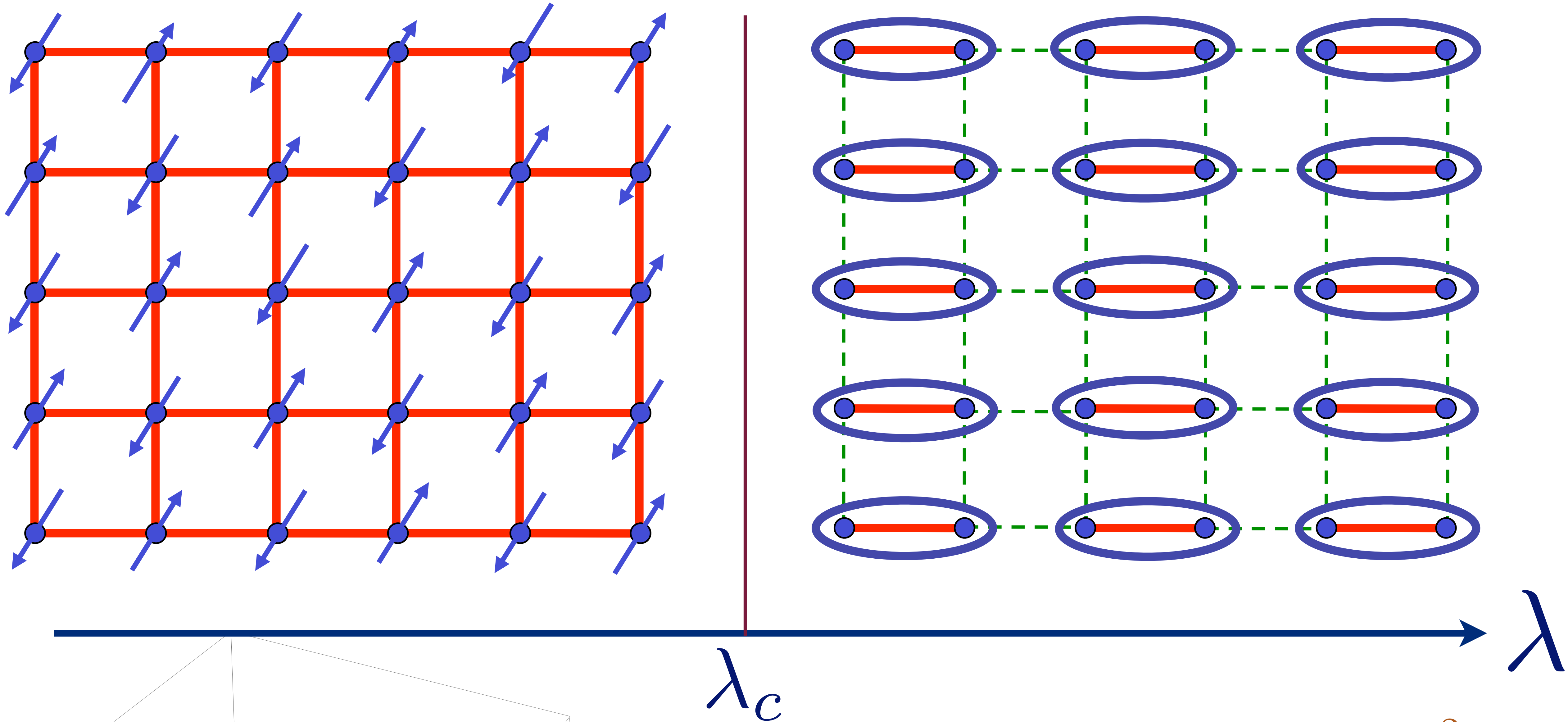
Spin waves

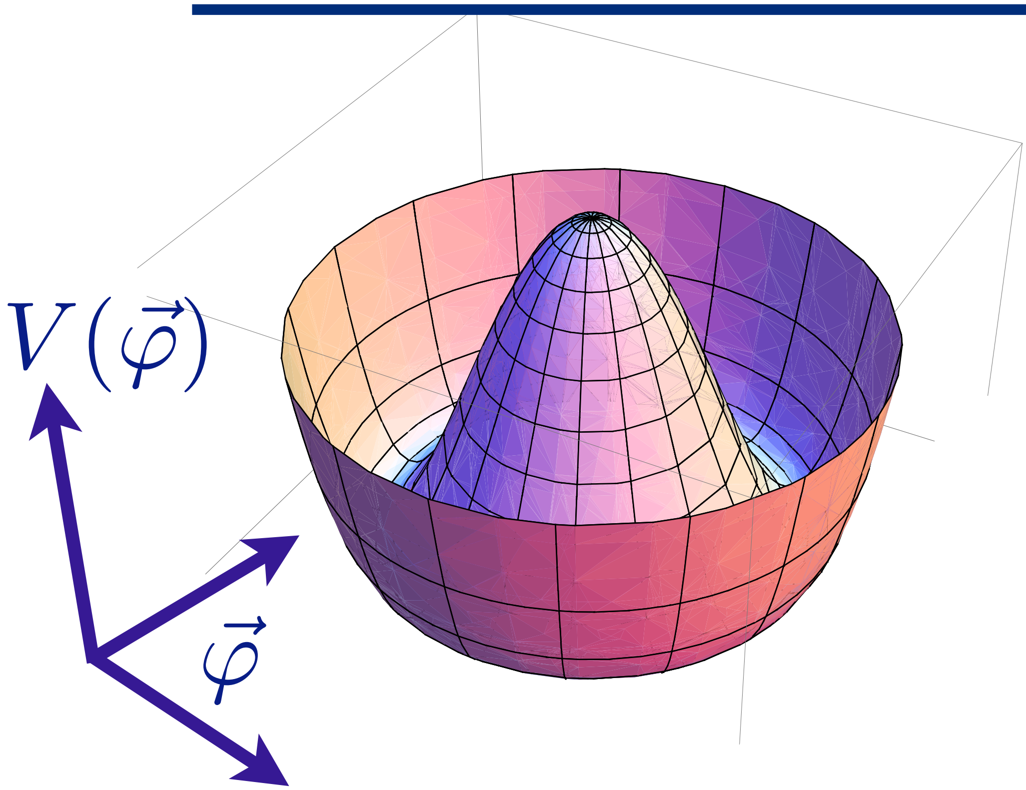
Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase

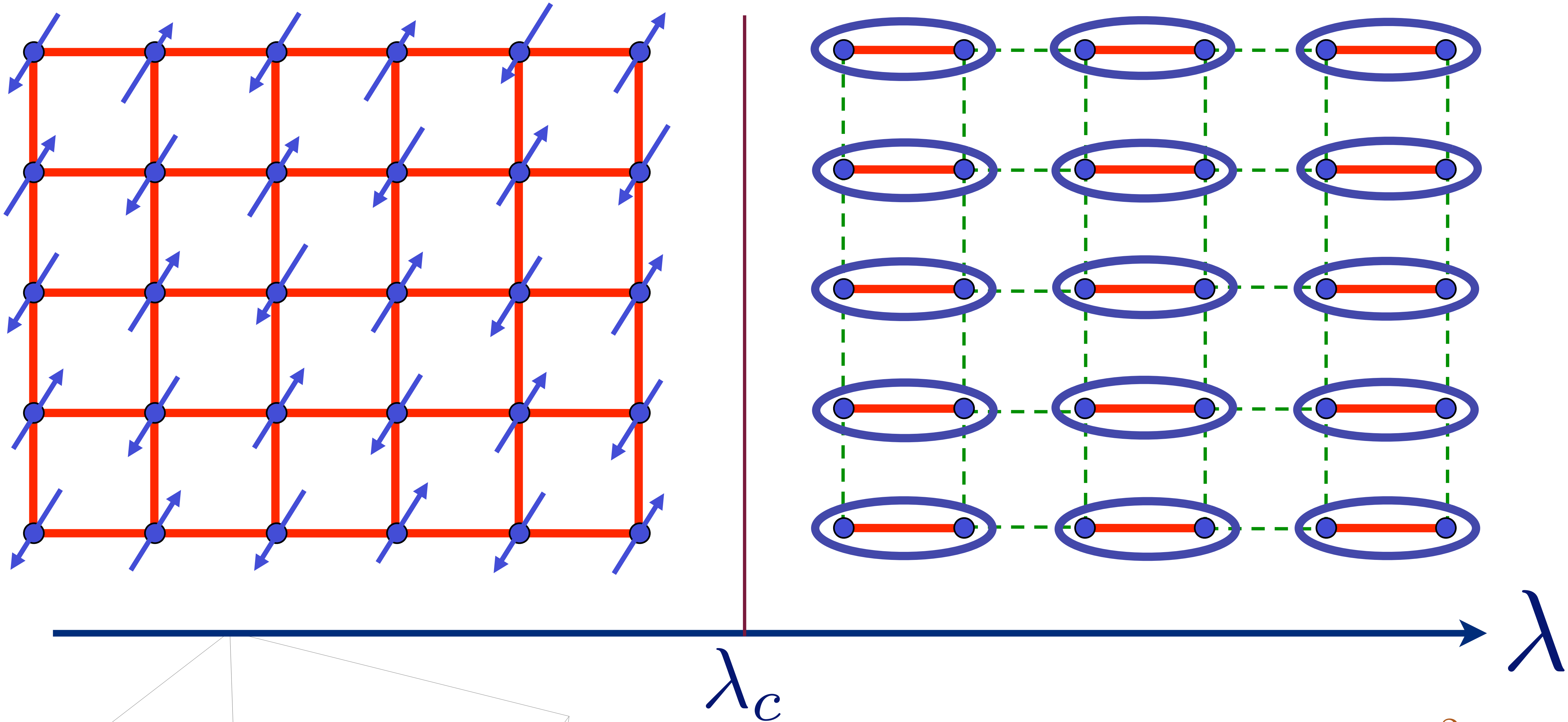


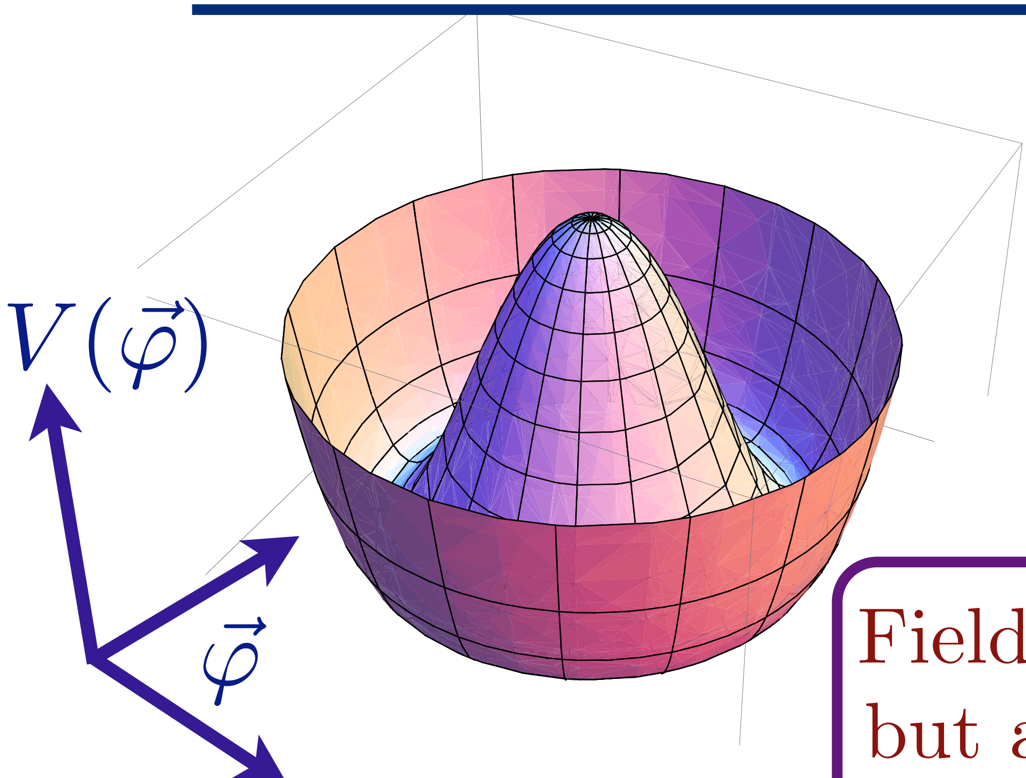


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

Excitation spectrum in the Néel phase



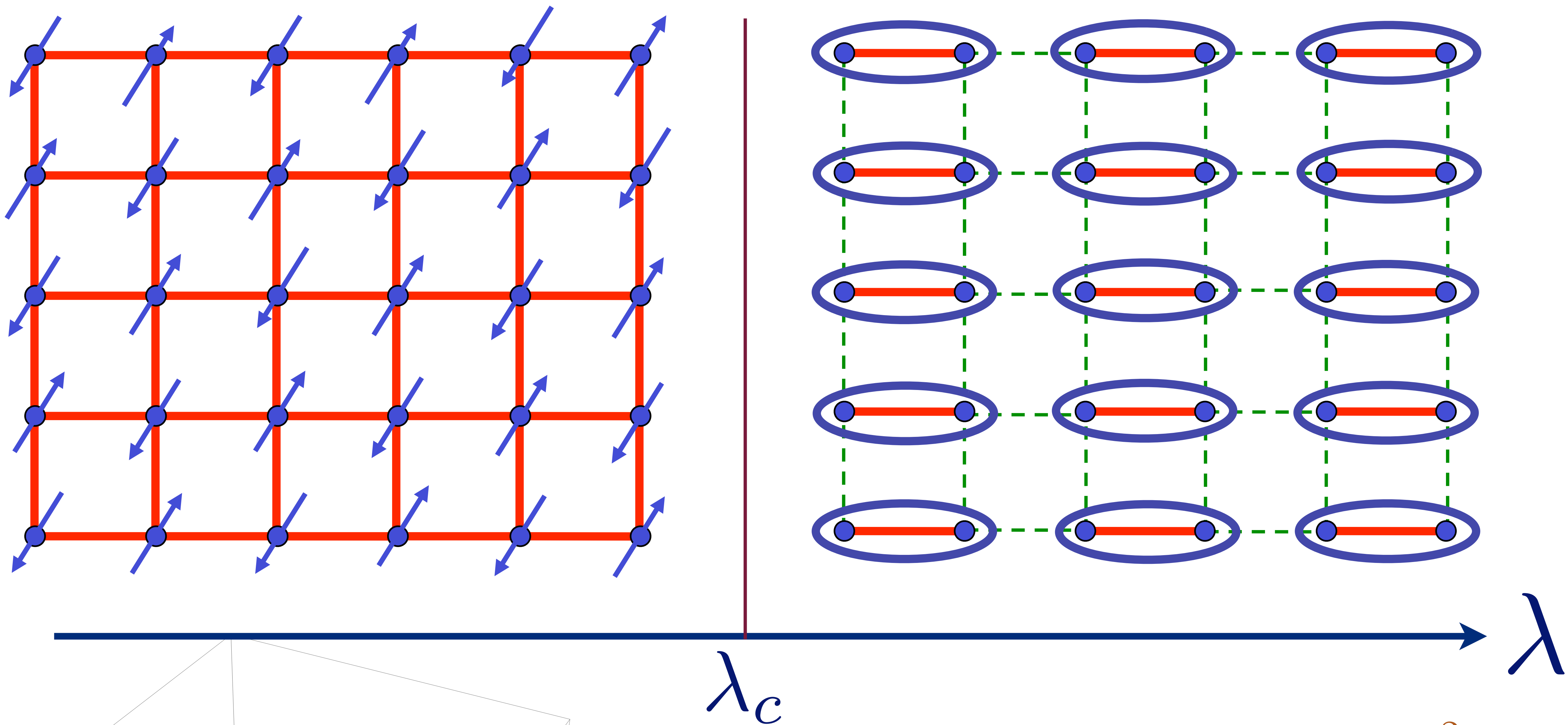


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

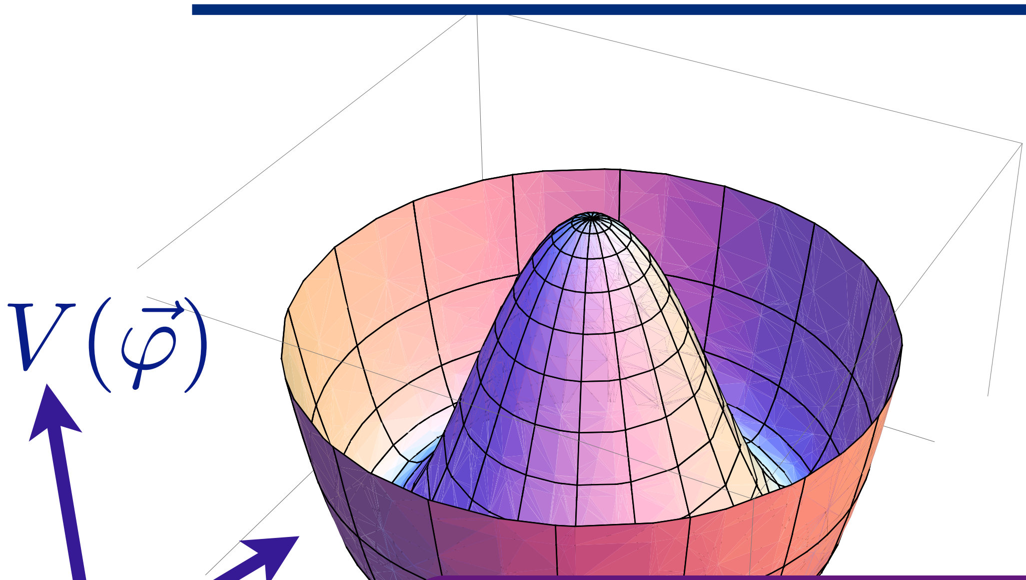
Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle

Excitation spectrum in the Néel phase



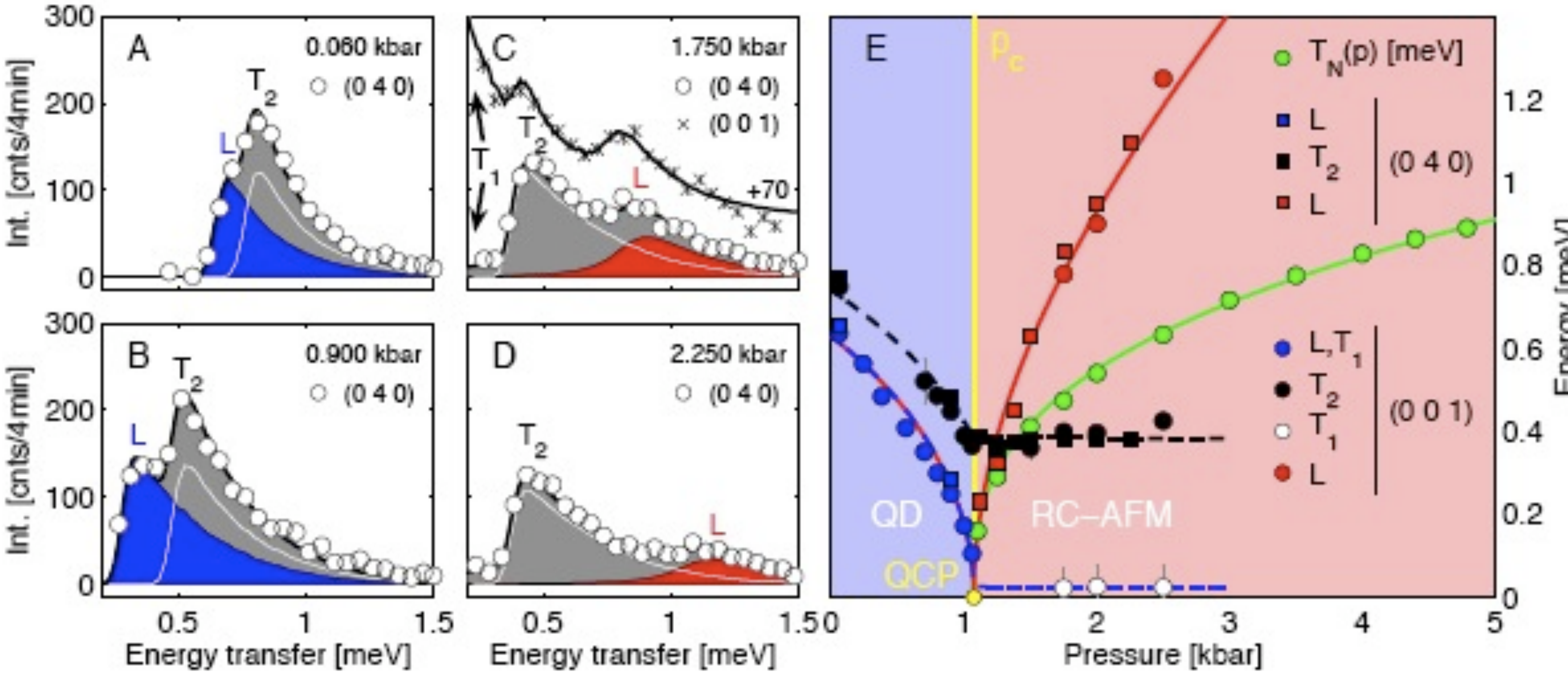
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$



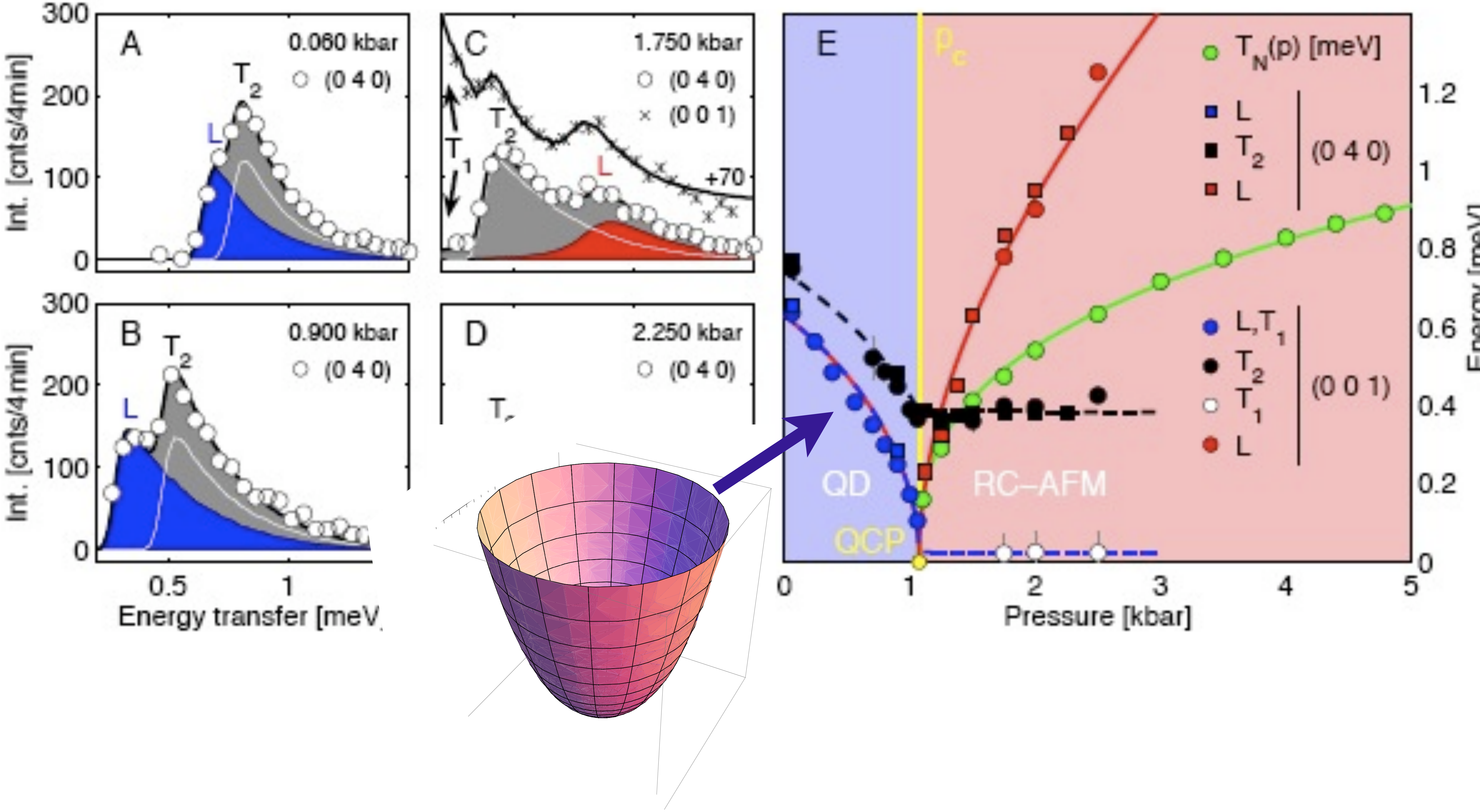
Field theory yields spin waves ("Goldstone" modes)
but also an additional longitudinal Higgs-Englert-Brout particle

$TlCuCl_3$ with varying pressure



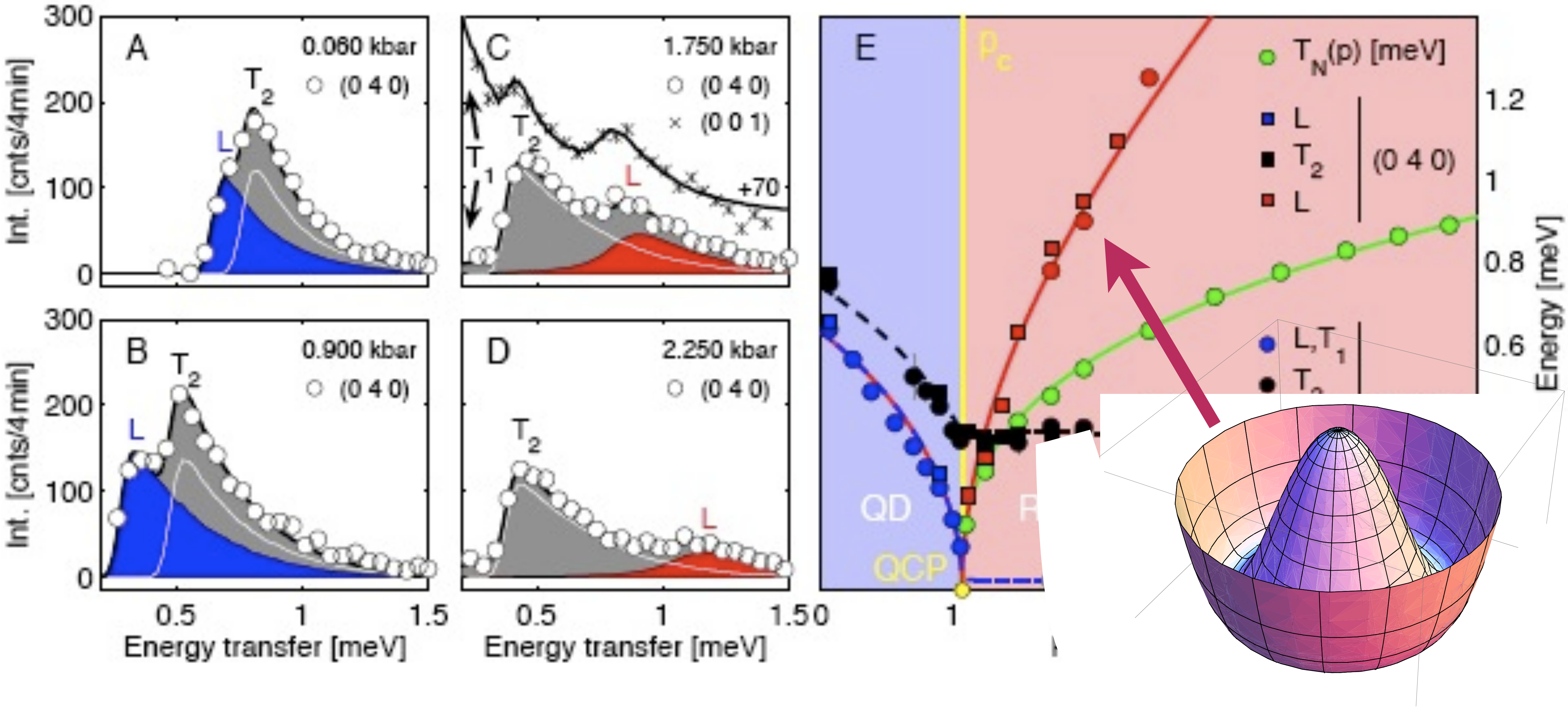
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

$TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

$TlCuCl_3$ with varying pressure

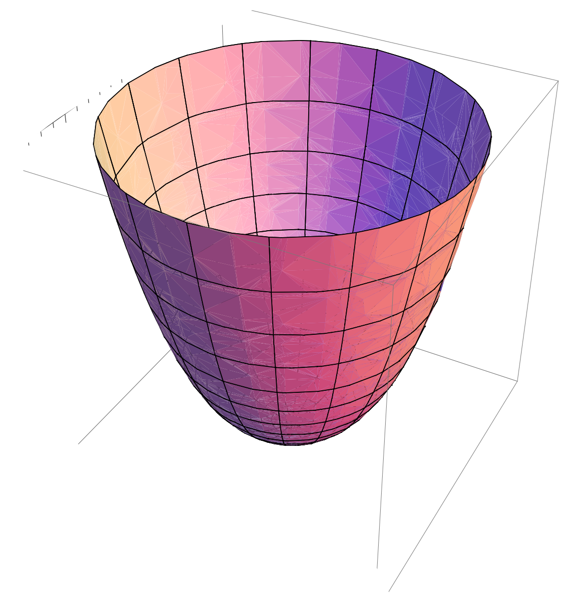


Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$



Expand about $\vec{\varphi} = 0$:

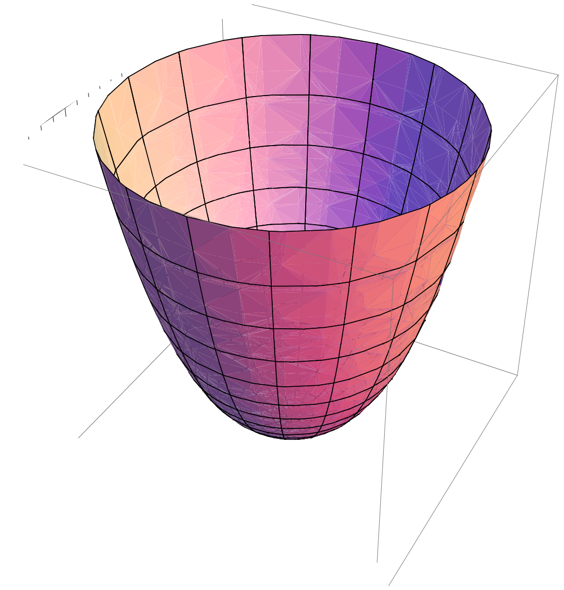
$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$

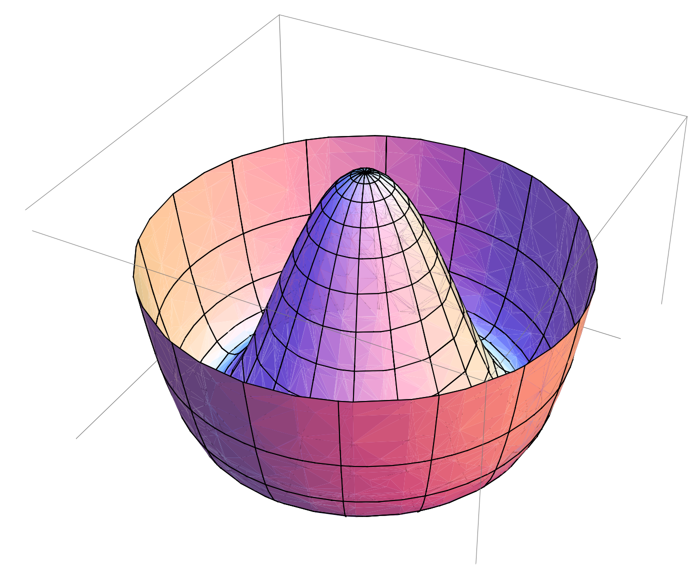


Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$



Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:

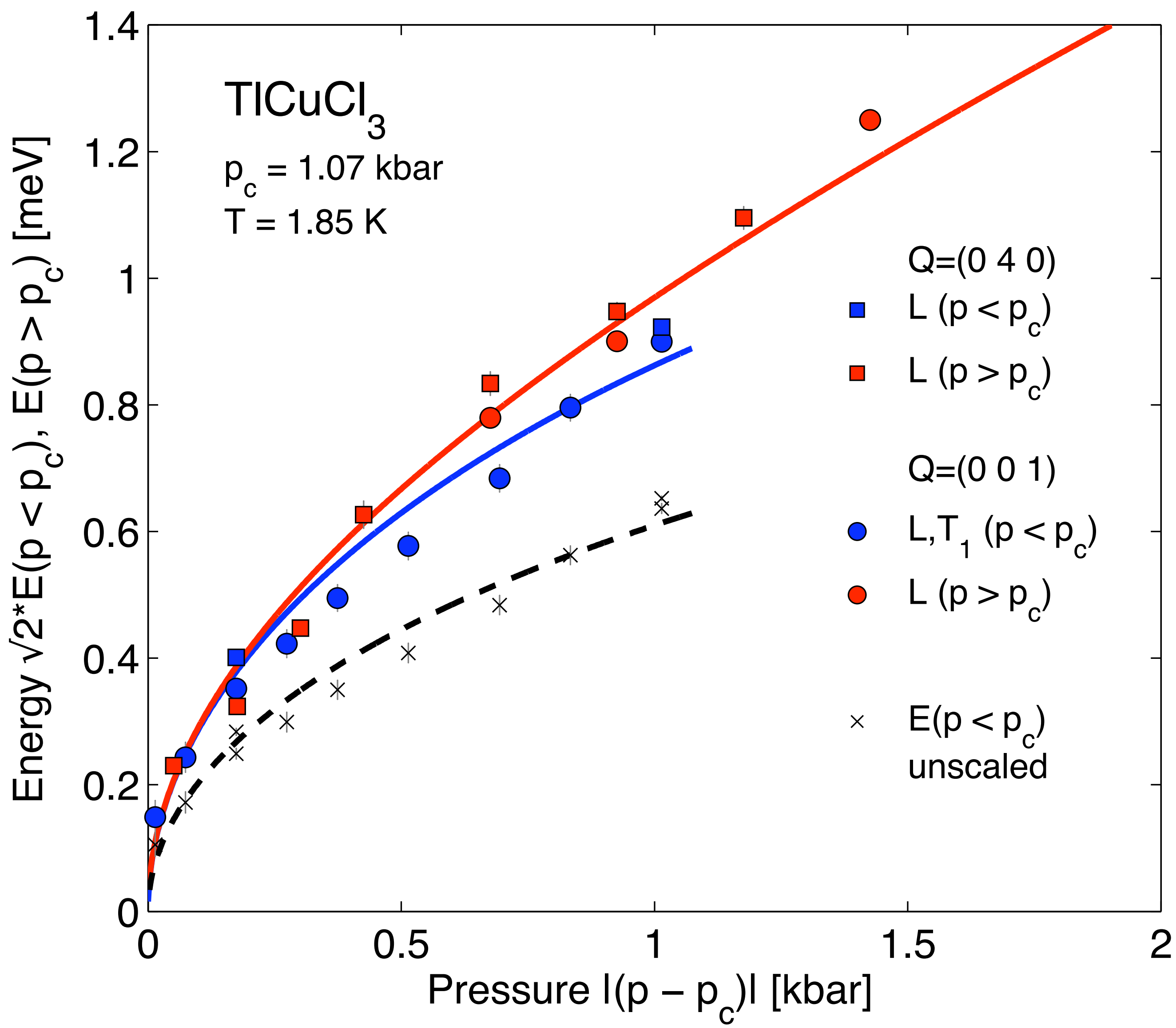
$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs-Englert-Brout particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

Prediction of quantum field theory

$$\frac{\text{Energy of Higgs-Englert-Brout particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

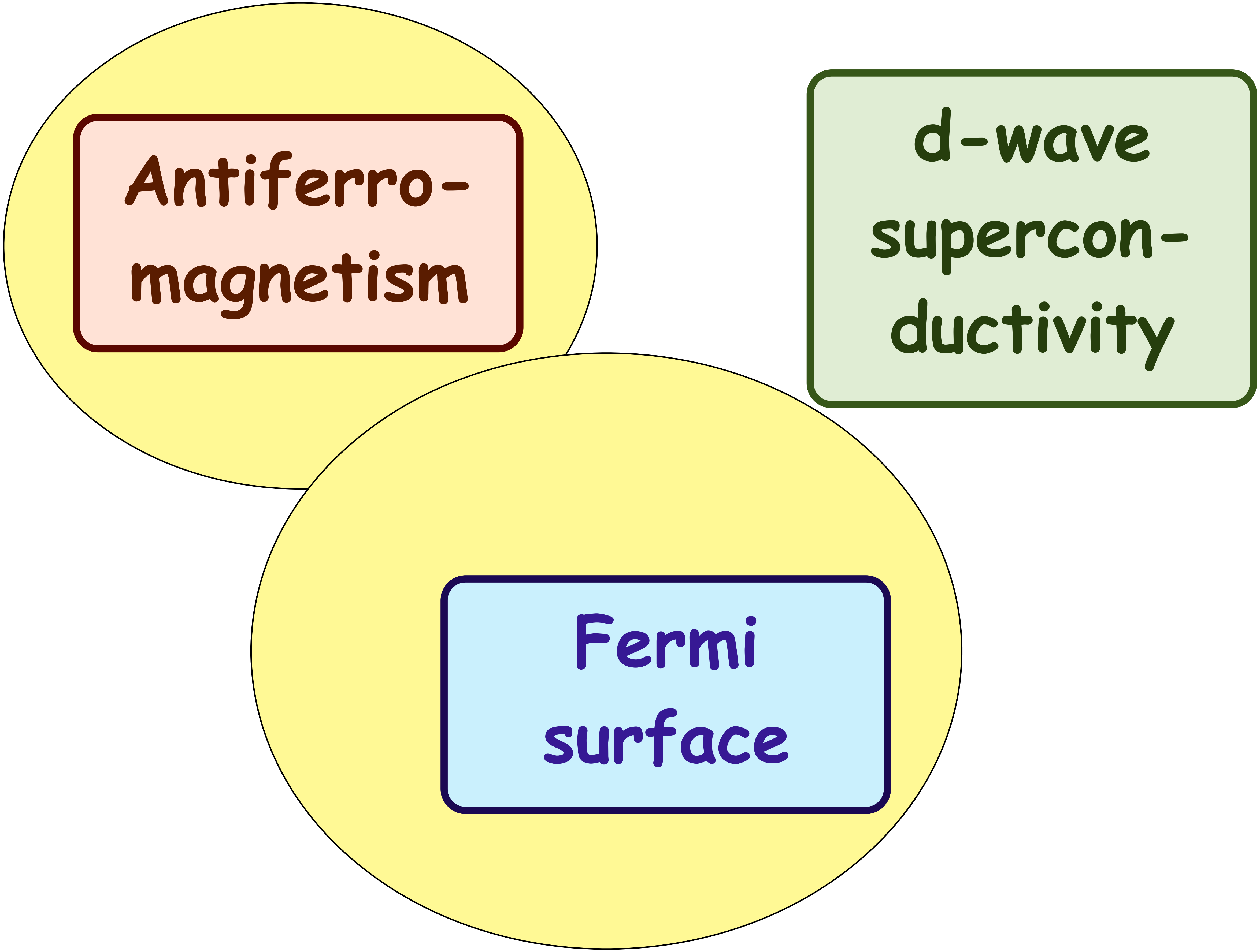


Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

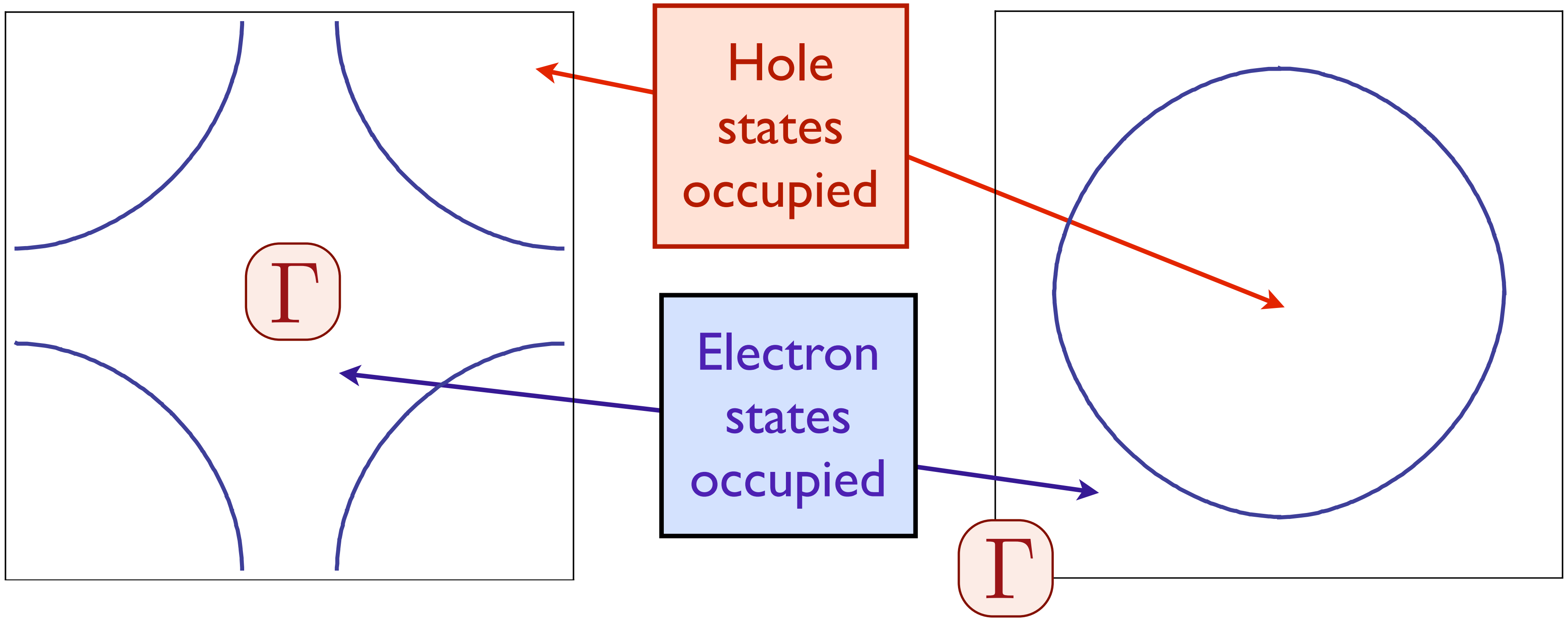
Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface


Antiferro-
magnetism
d-wave
supercon-
ductivity
Fermi
surface

"Large" Fermi surfaces in cuprates



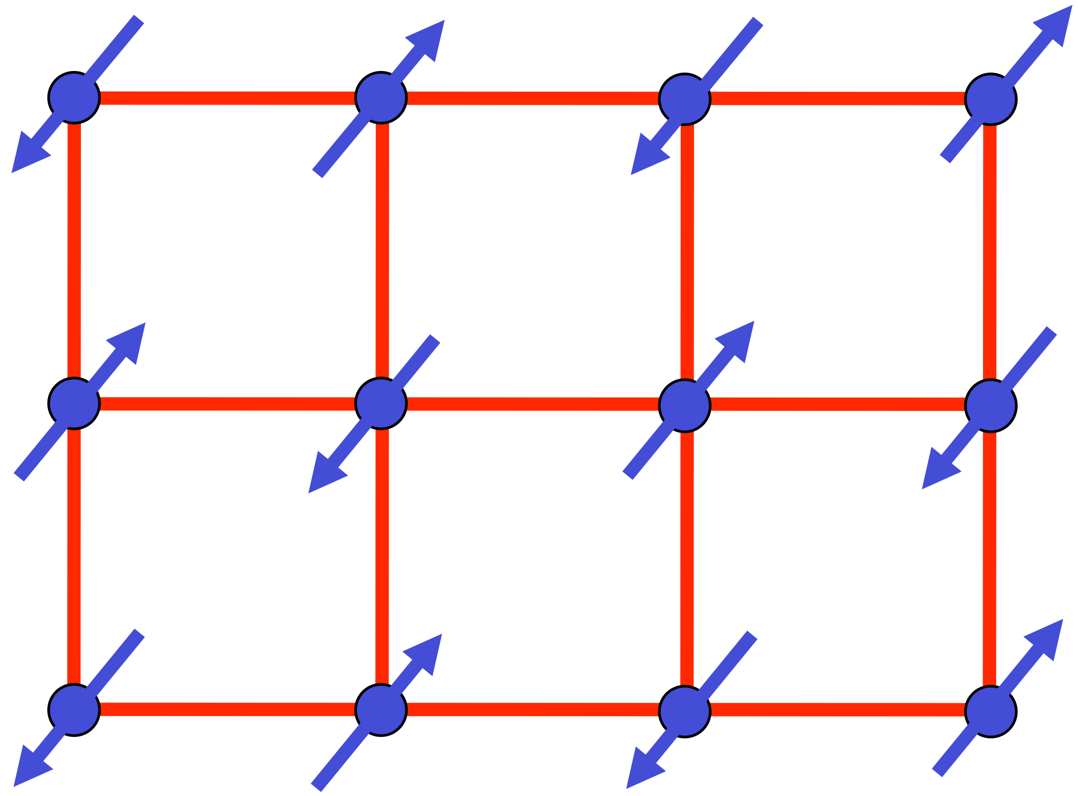
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$$

Spin density wave theory

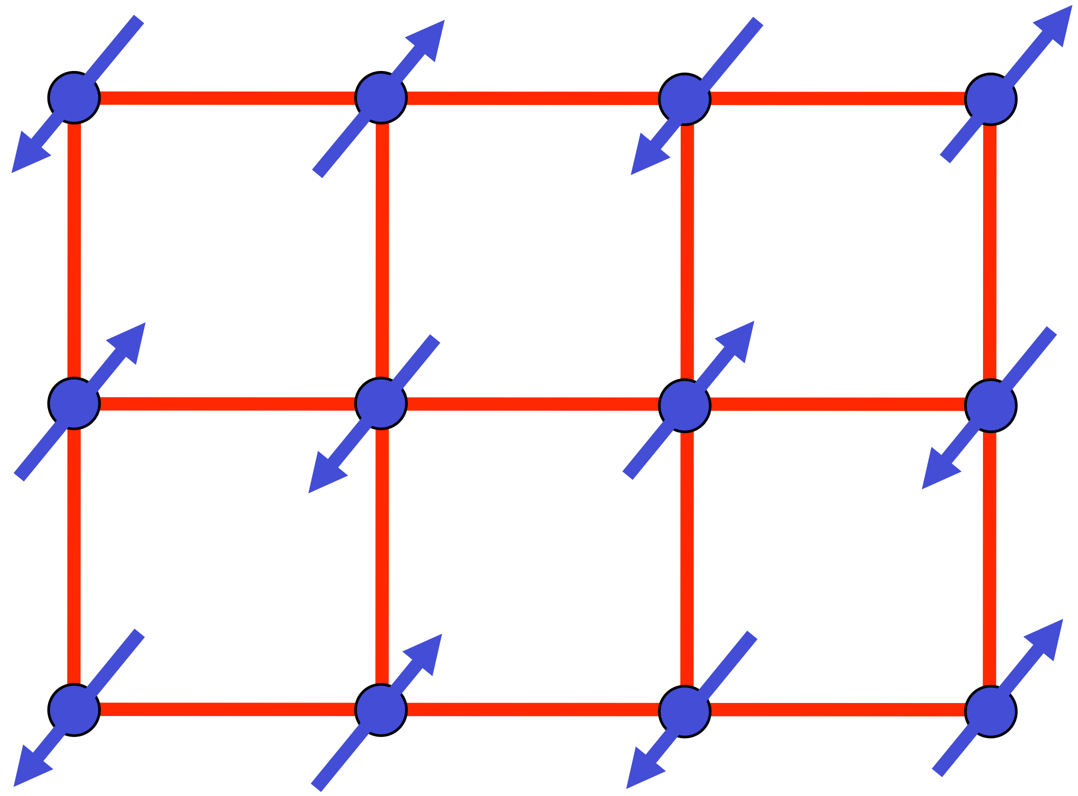


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and $\mathbf{K}$ is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



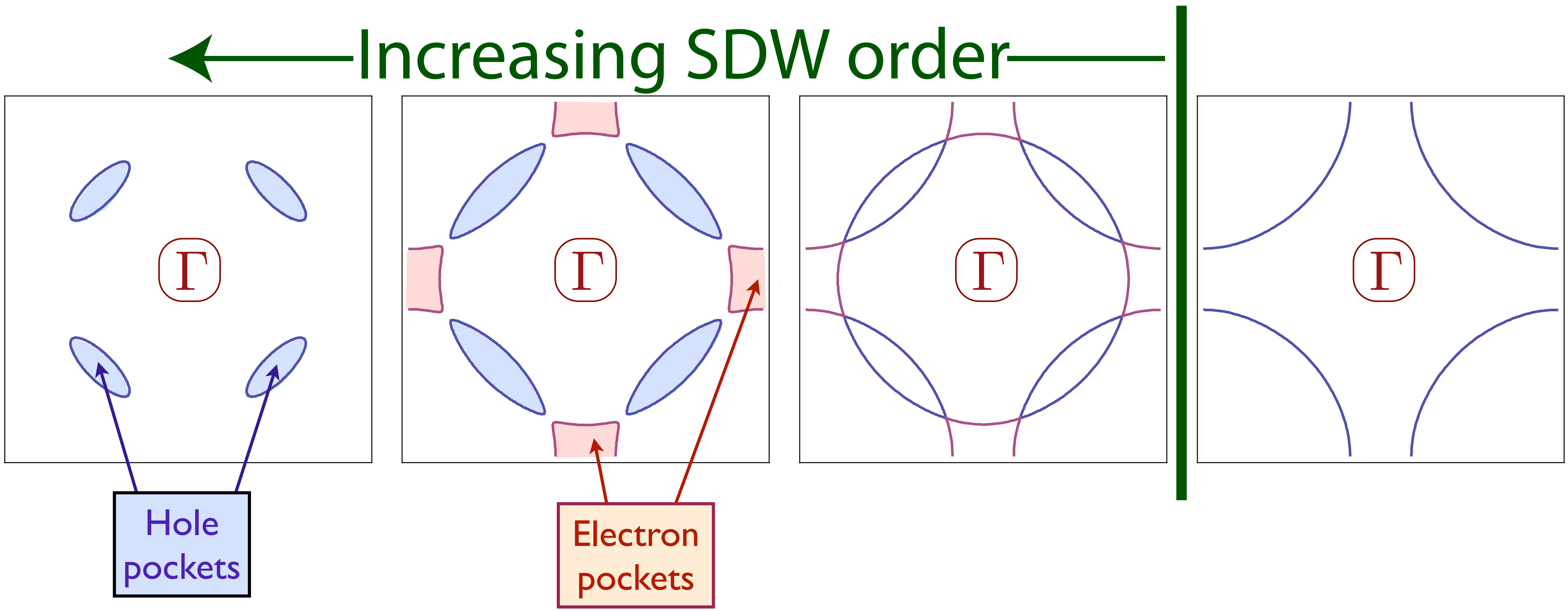
Spin density wave Hamiltonian

$$H_{\mathrm{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

Diagonalize $H_0 + H_{\mathrm{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

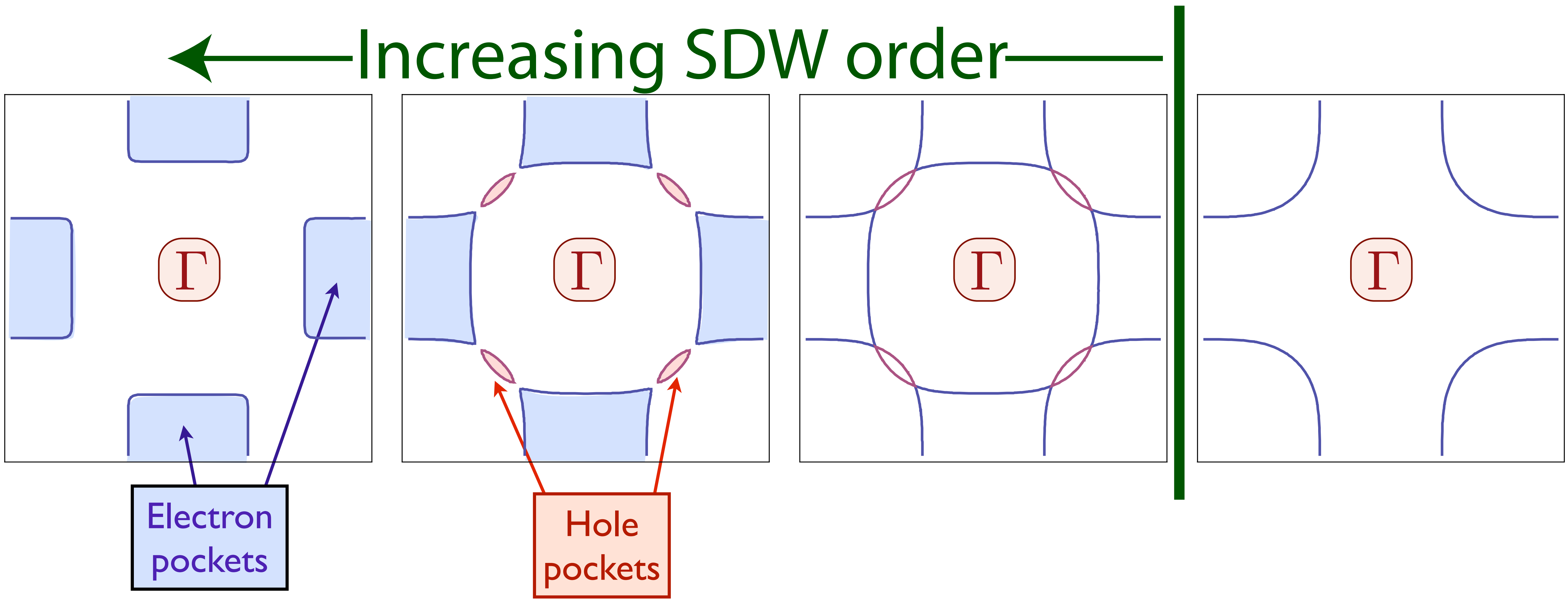
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).
D. Senechal and A.-M. S. Tremblay, *Phys. Rev. Lett.* **92**, 126401 (2004)

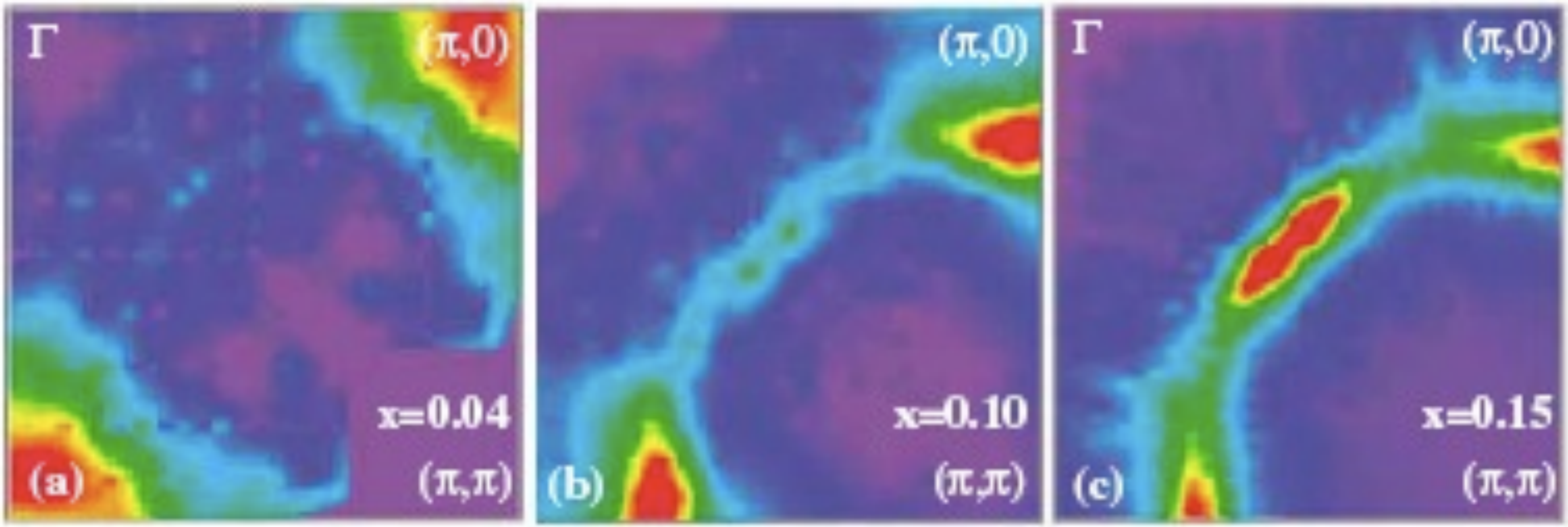
Electron-doped cuprates



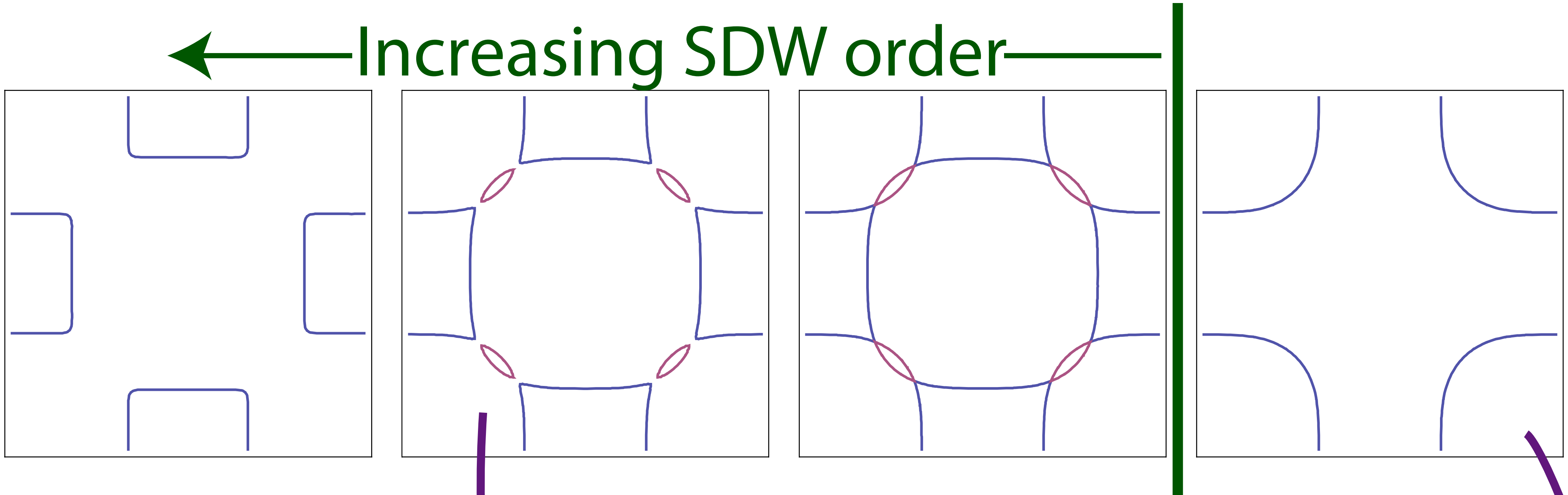
Large Fermi surface breaks up into electron and hole pockets

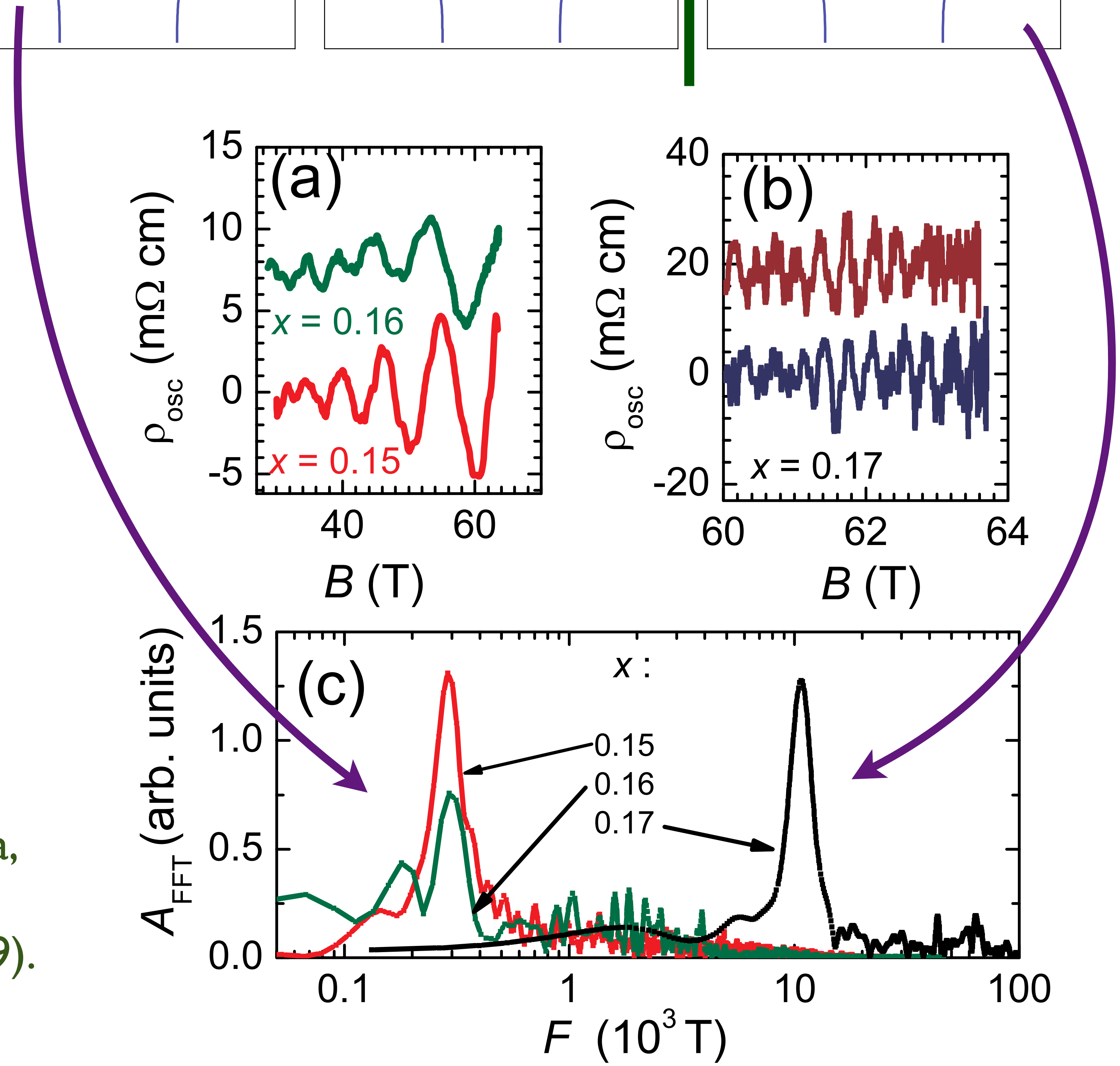
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).
D. Senechal and A.-M. S. Tremblay, *Phys. Rev. Lett.* **92**, 126401 (2004)

Photoemission in NCCO



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).





$Nd_{2-x}Ce_xCuO_4$

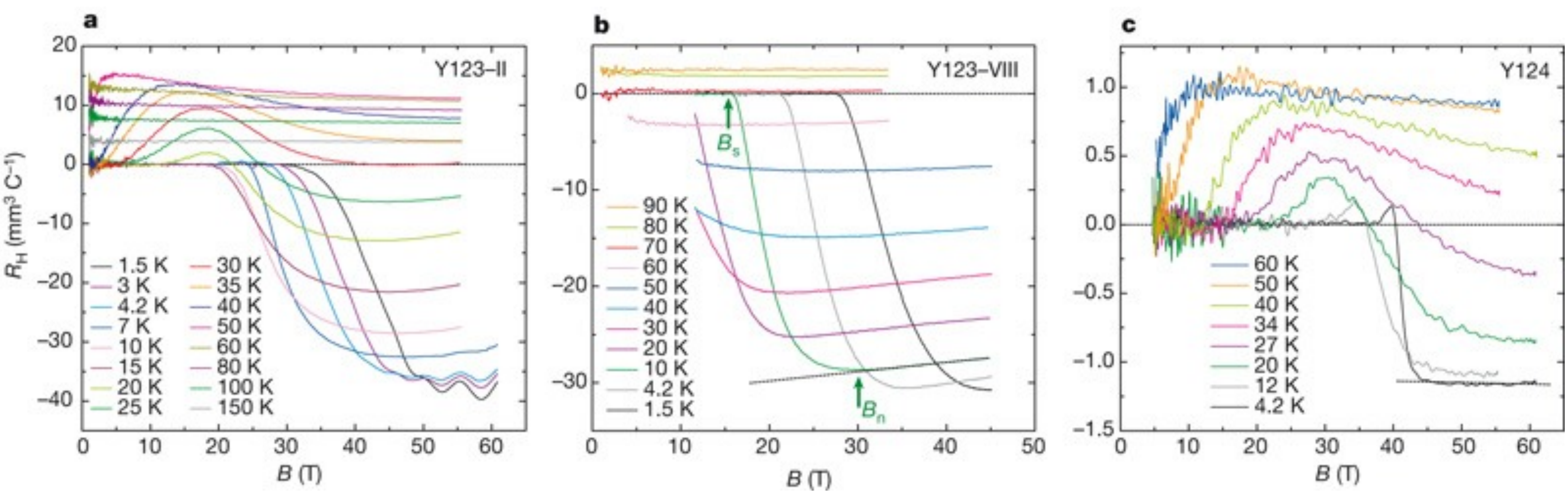
T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

Quantum oscillations

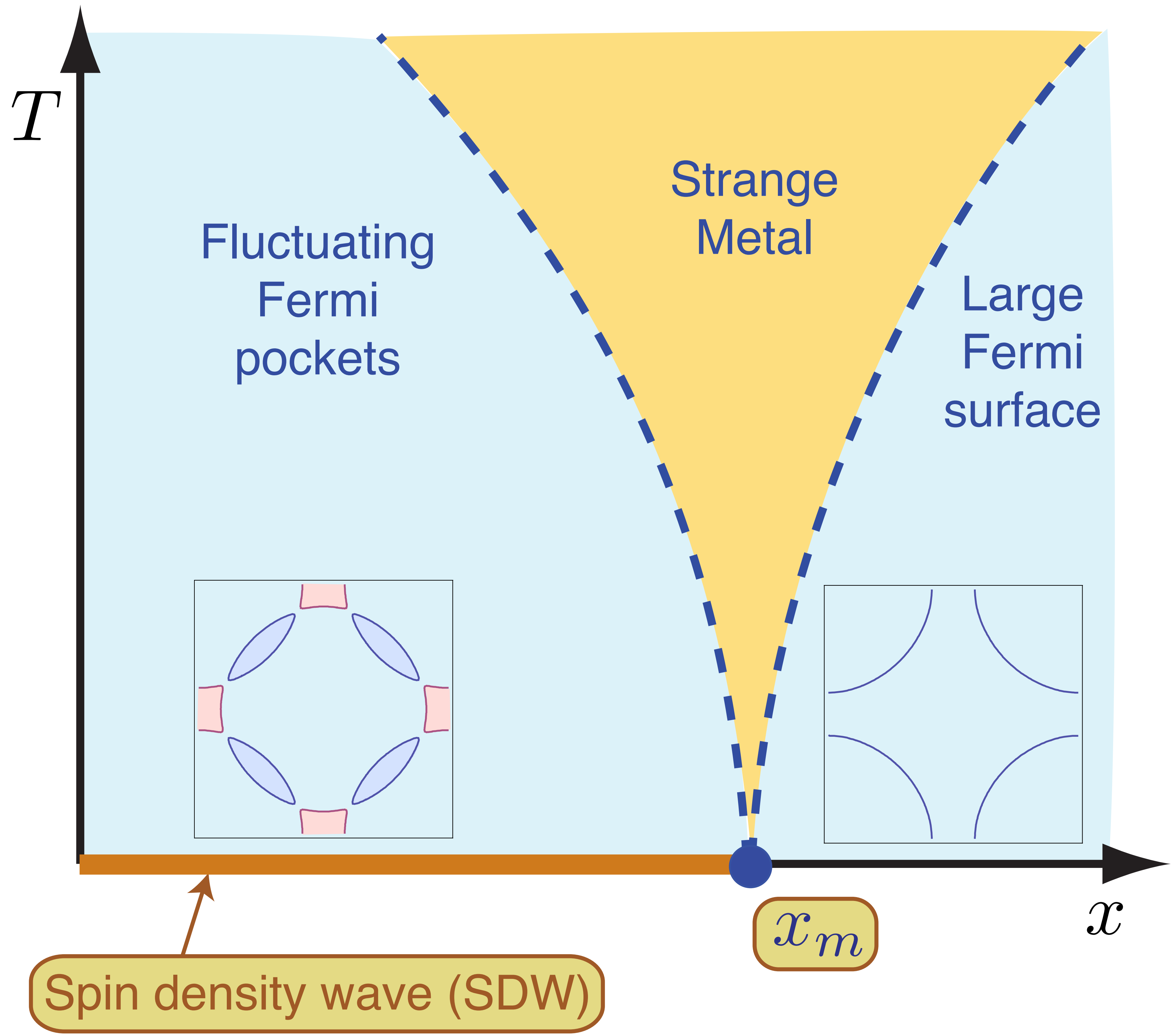
Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

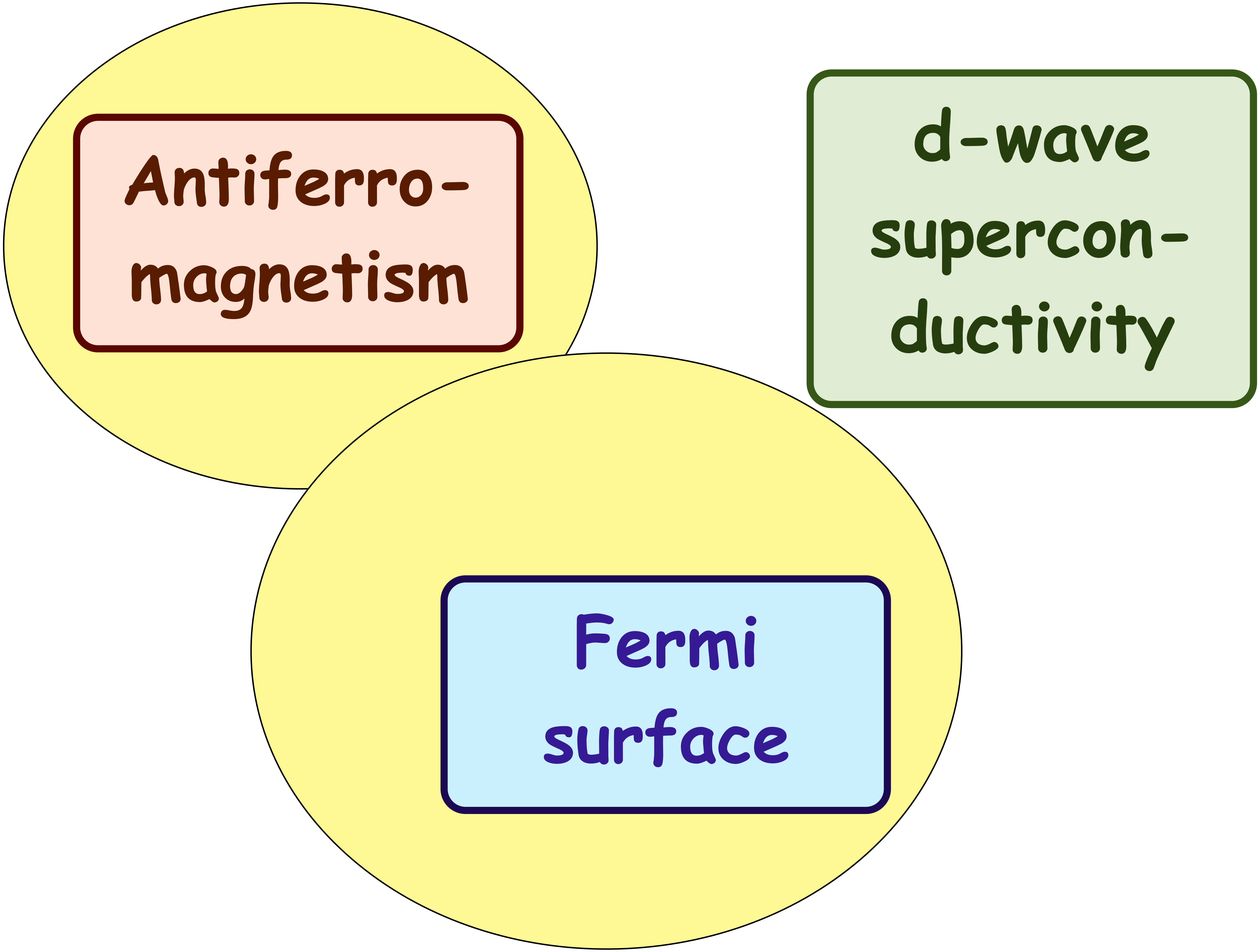
Nature **450**, 533 (2007)



Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

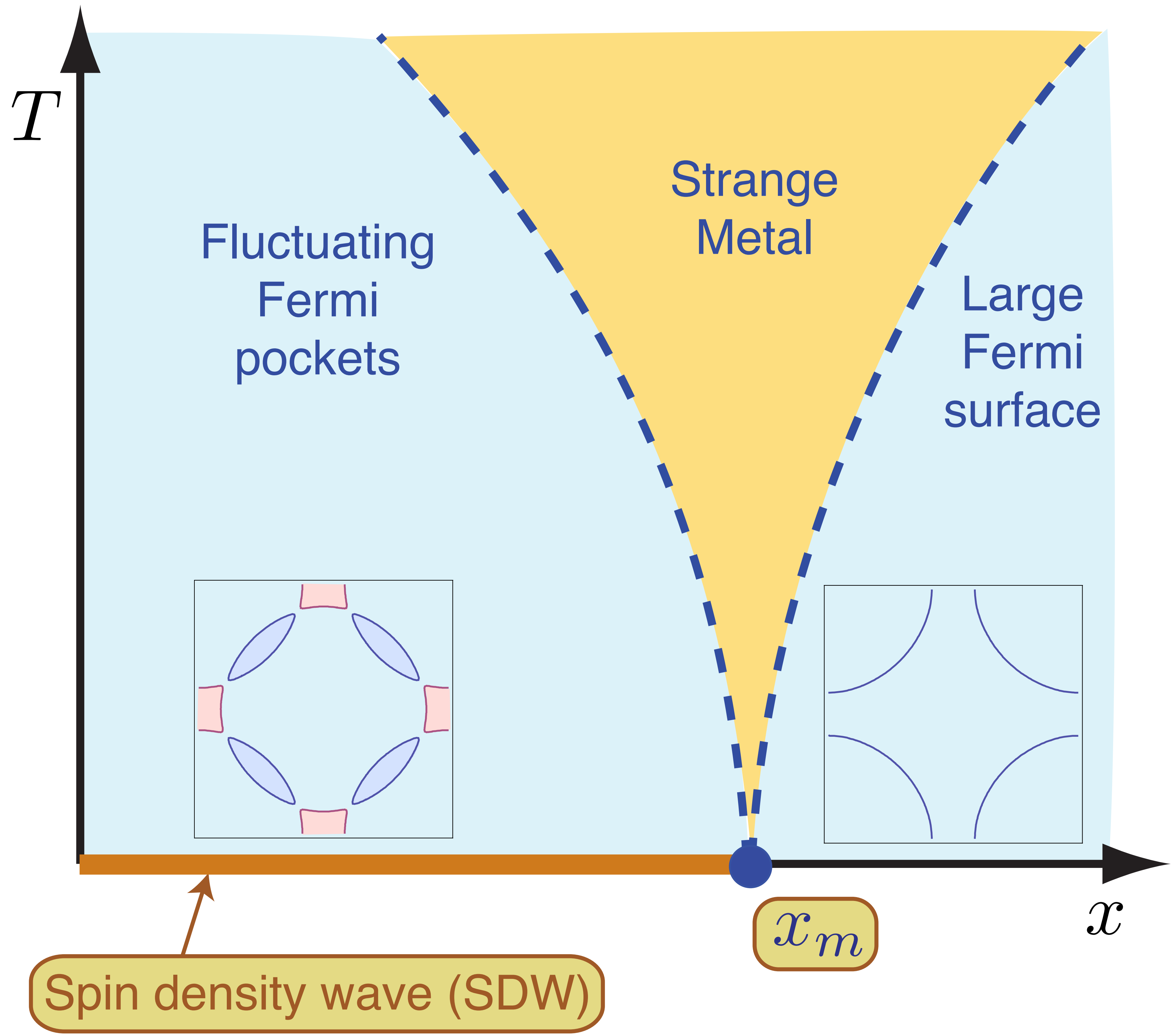

Antiferro-
magnetism
d-wave
supercon-
ductivity
Fermi
surface

Antiferro-
magnetism

d-wave
supercon-
ductivity

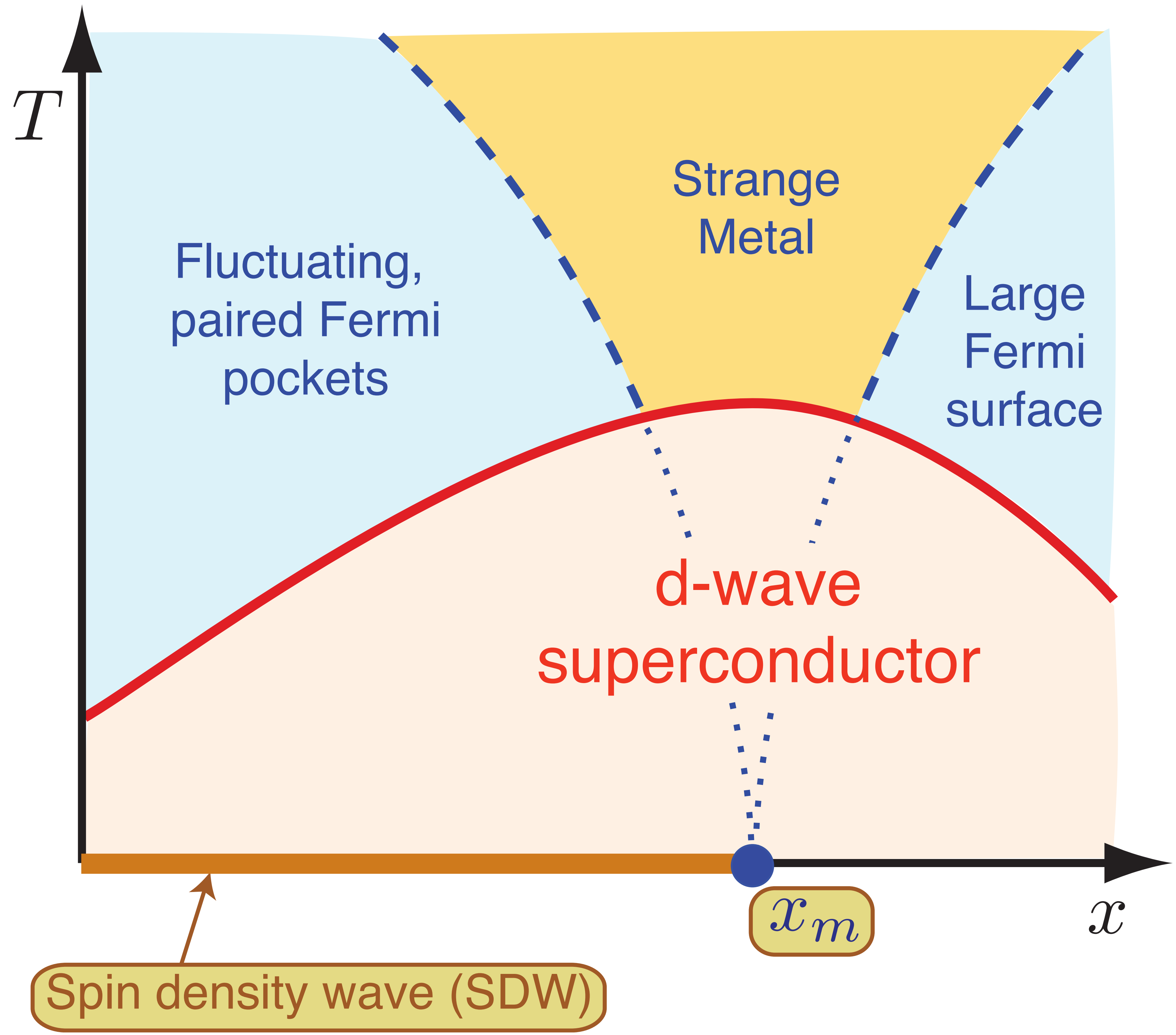
Fermi
surface

Theory of quantum criticality in the cuprates



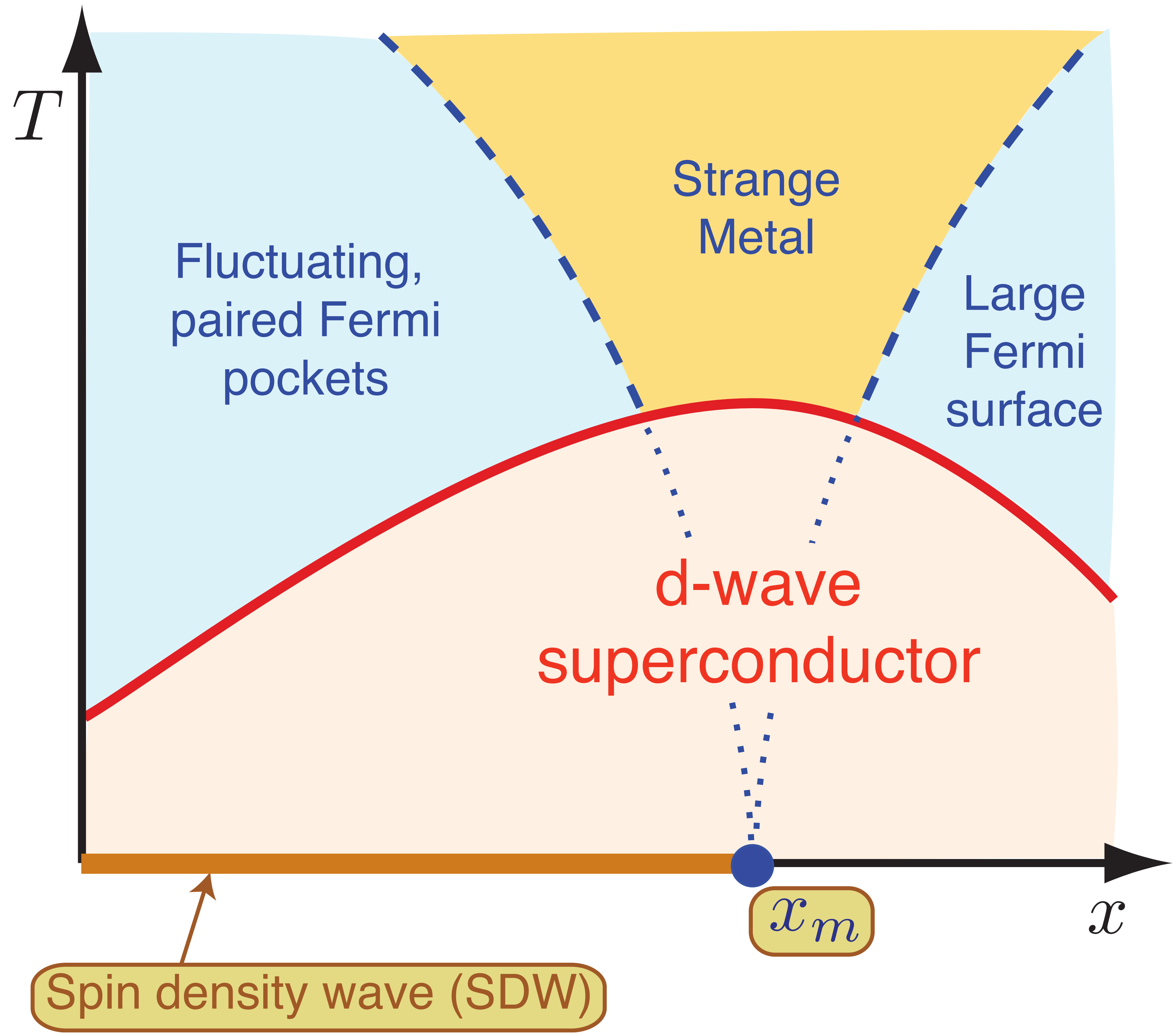
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



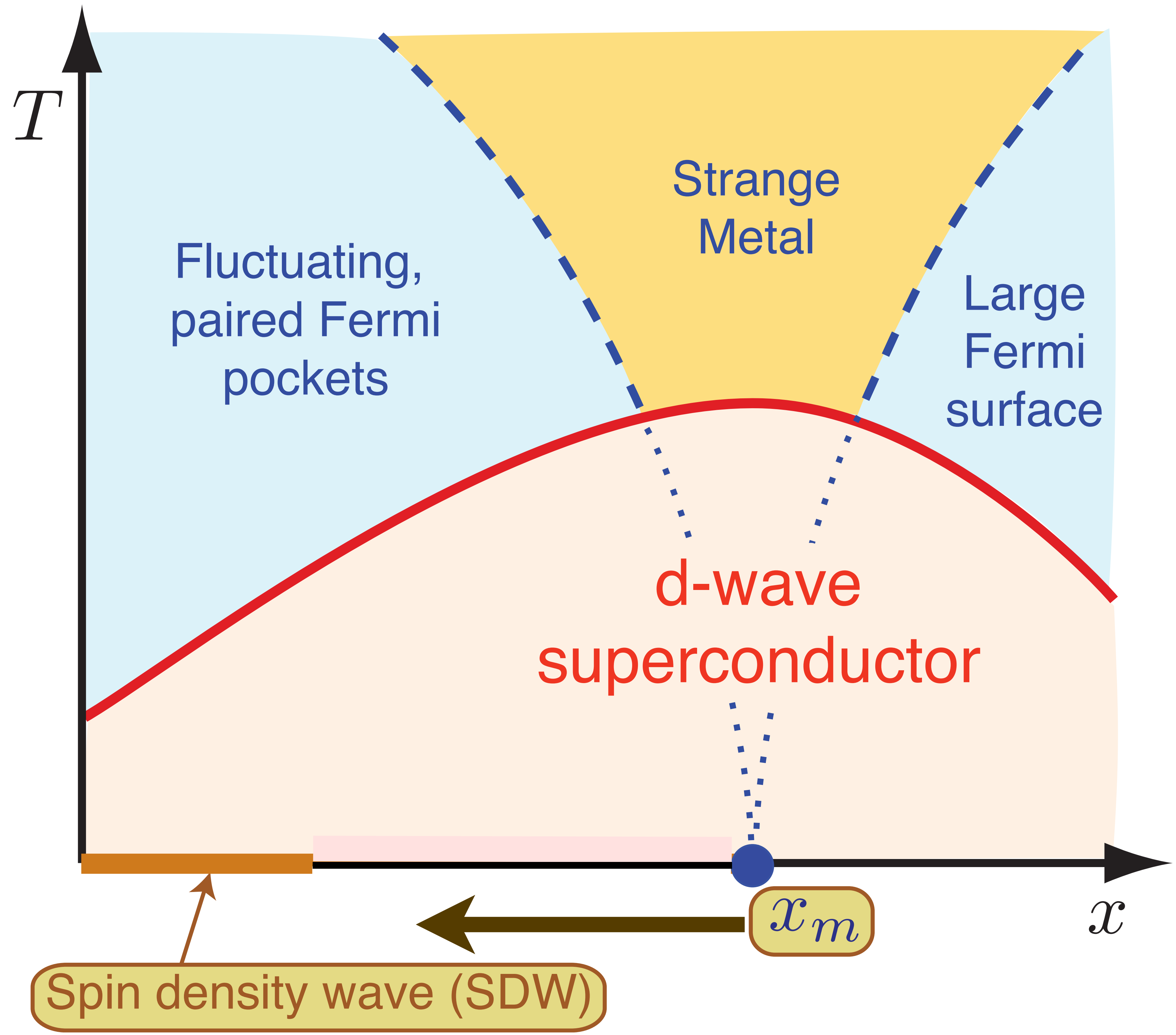
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



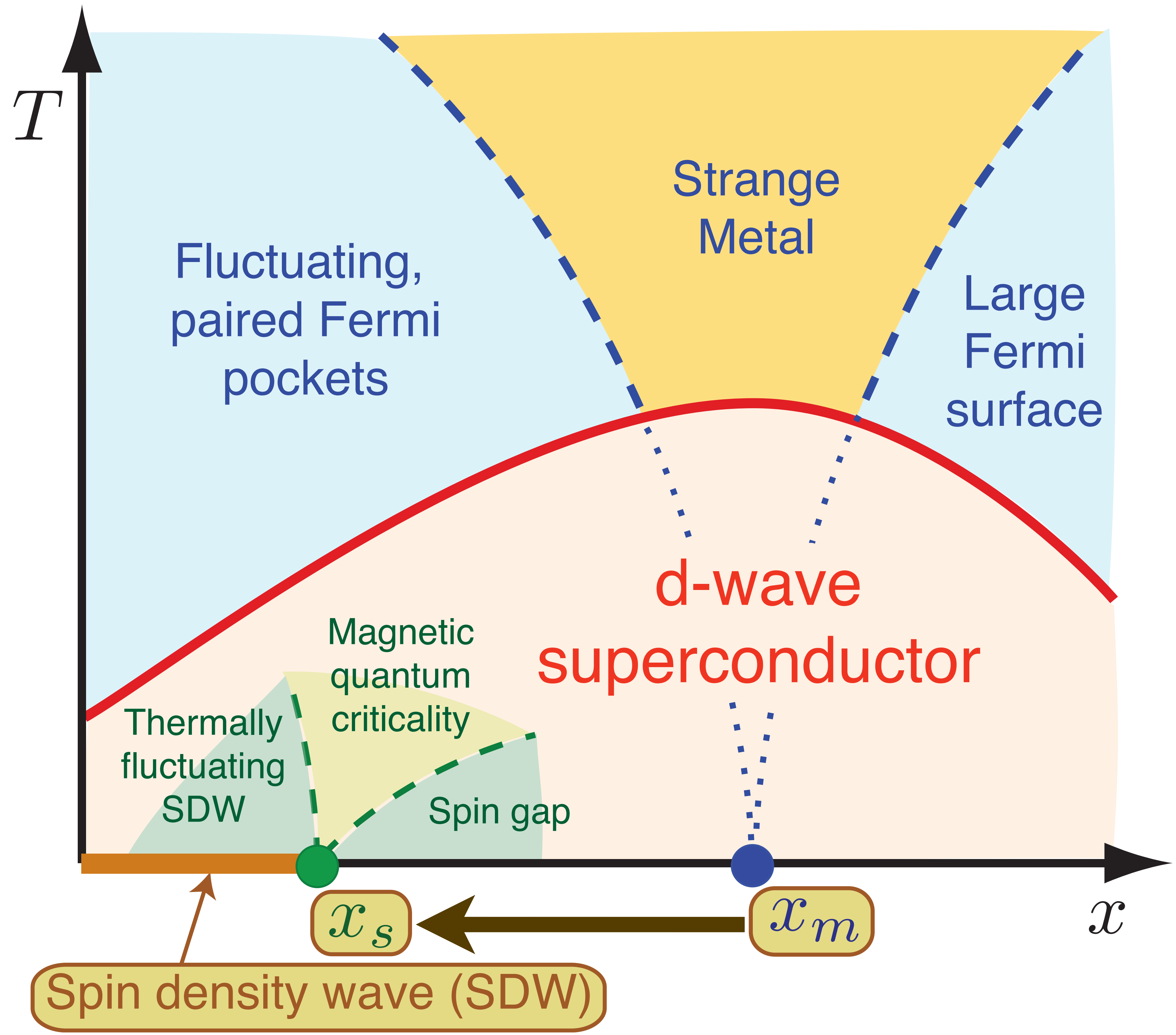
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



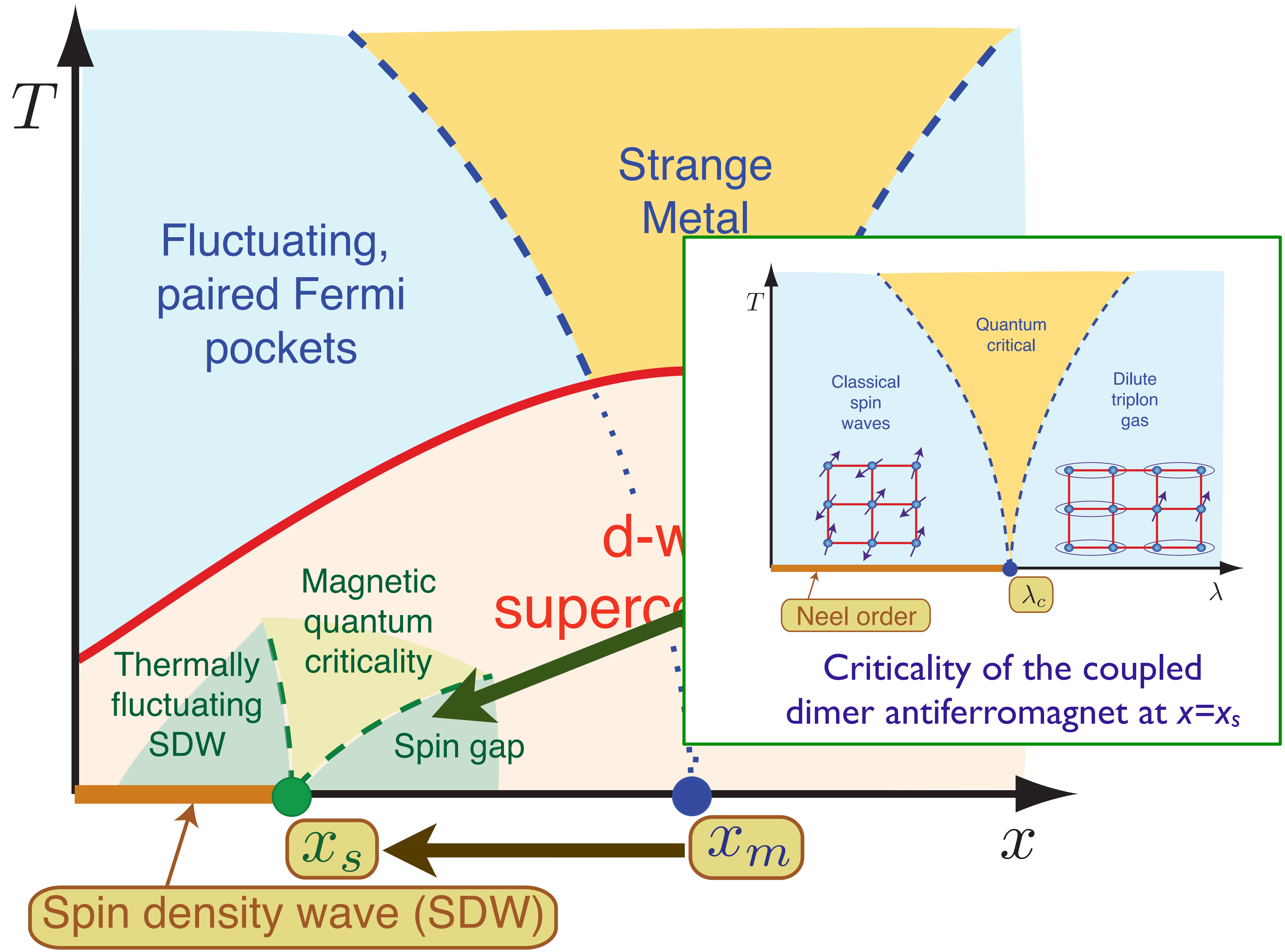
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



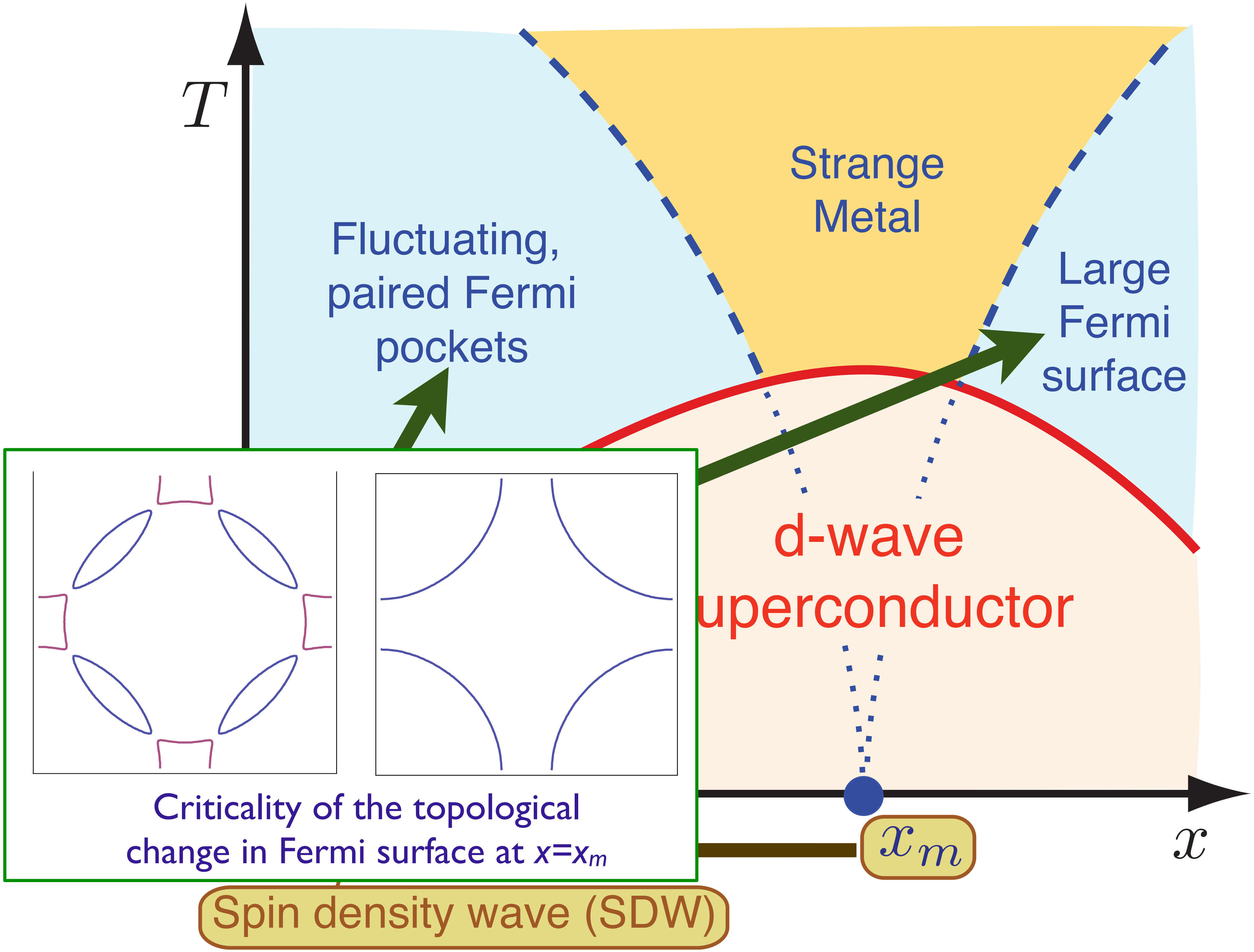
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



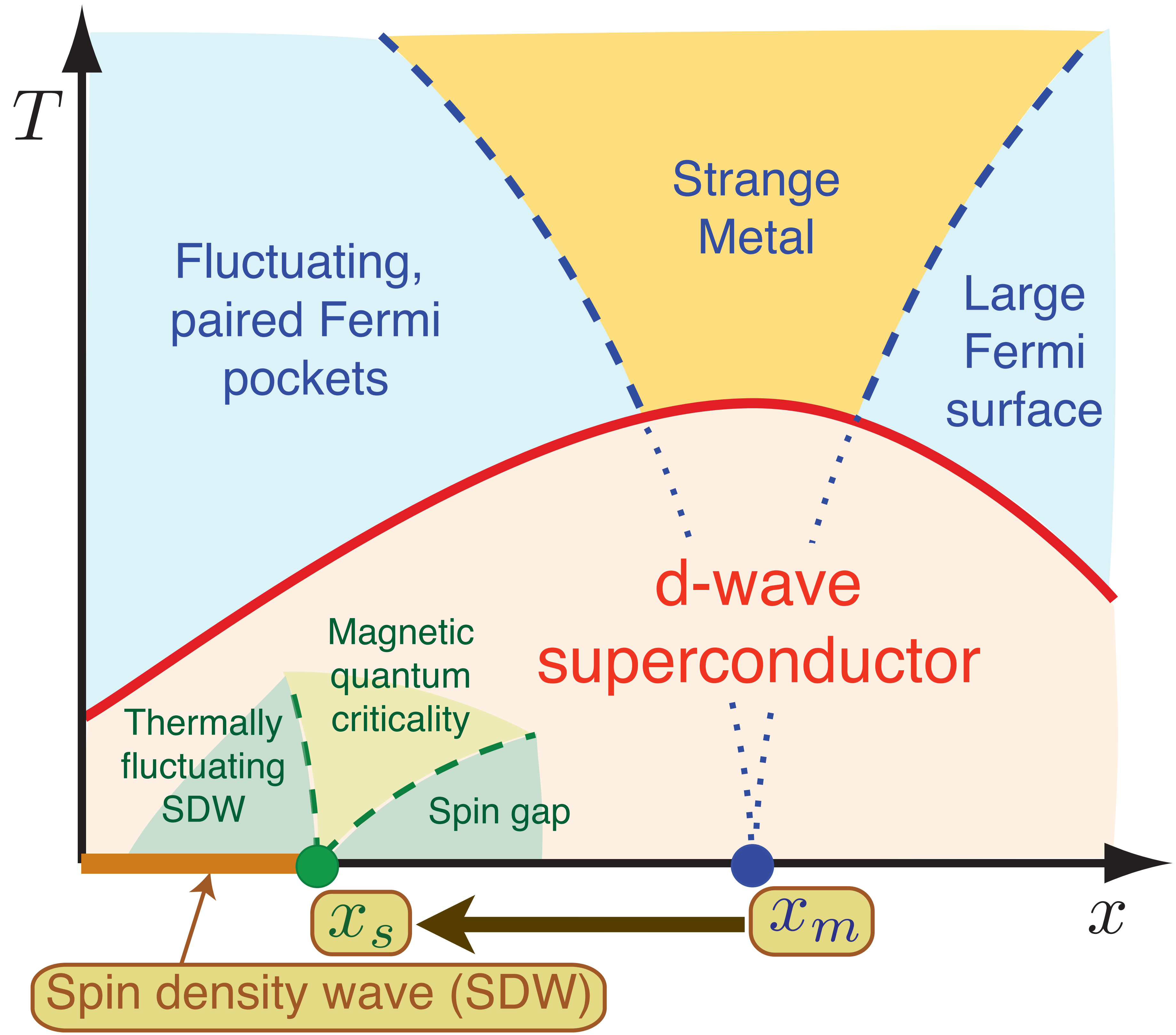
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

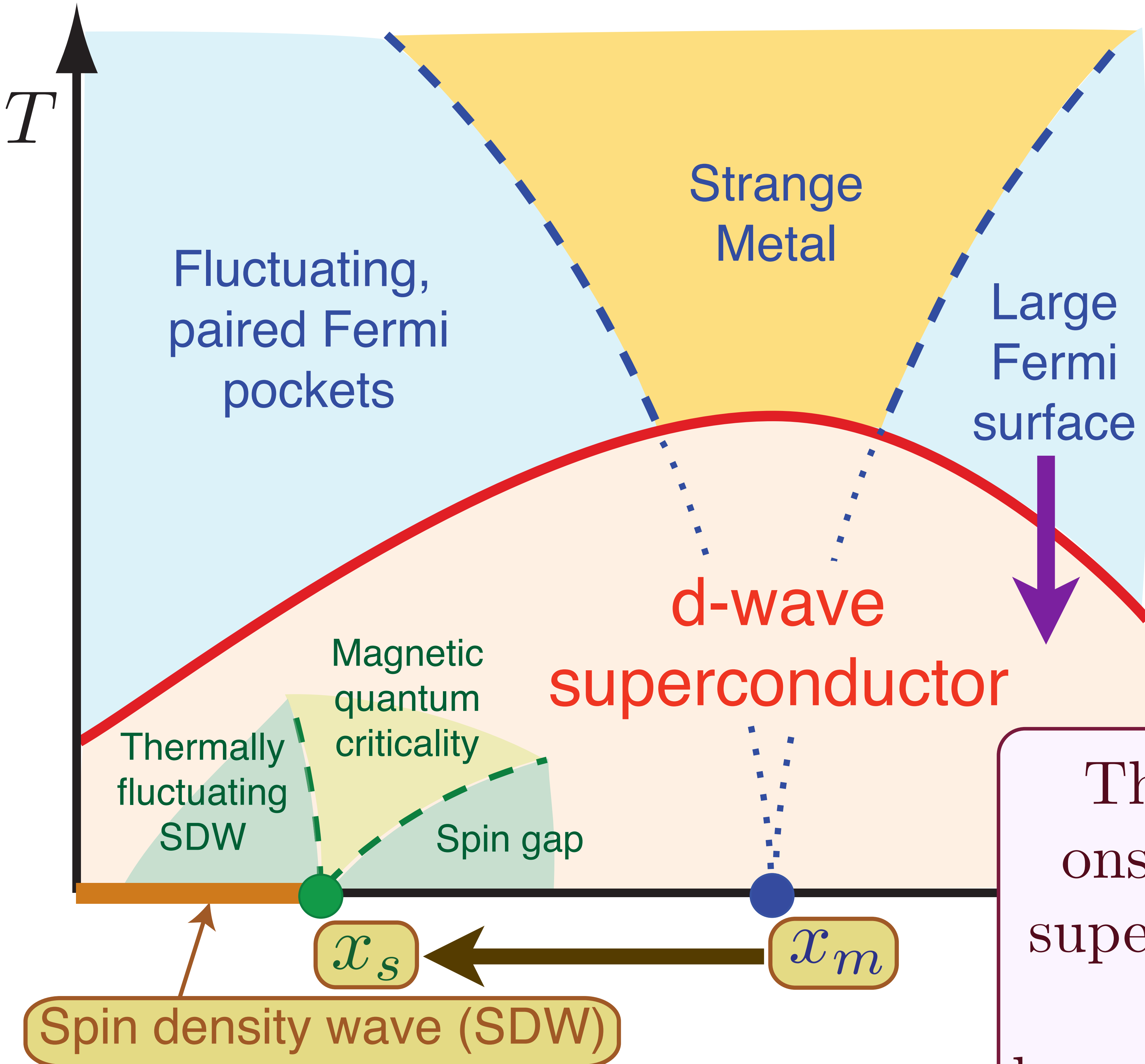


Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

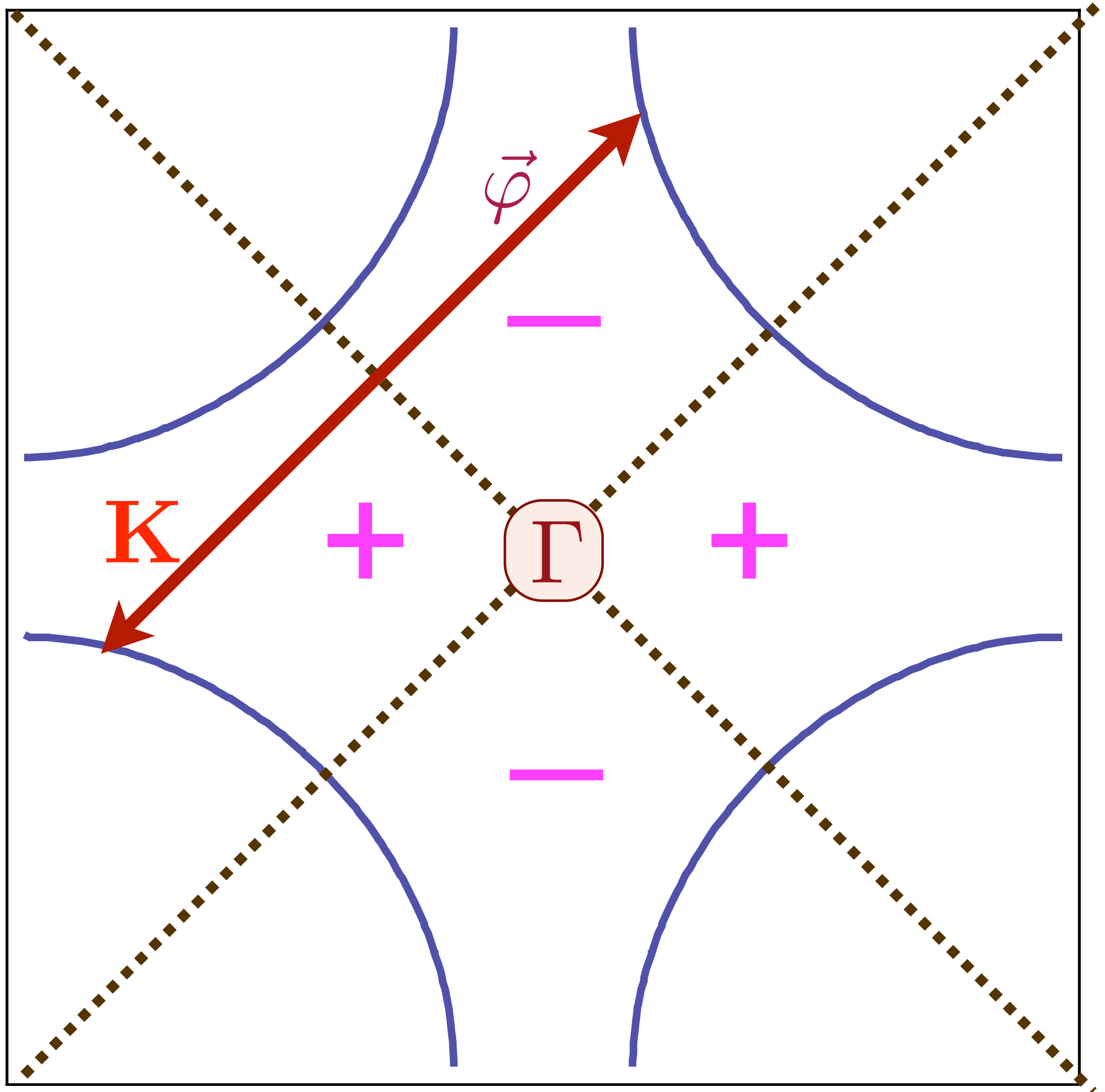


Theory of quantum criticality in the cuprates



Theory of the onset of d-wave superconductivity from a large Fermi surface

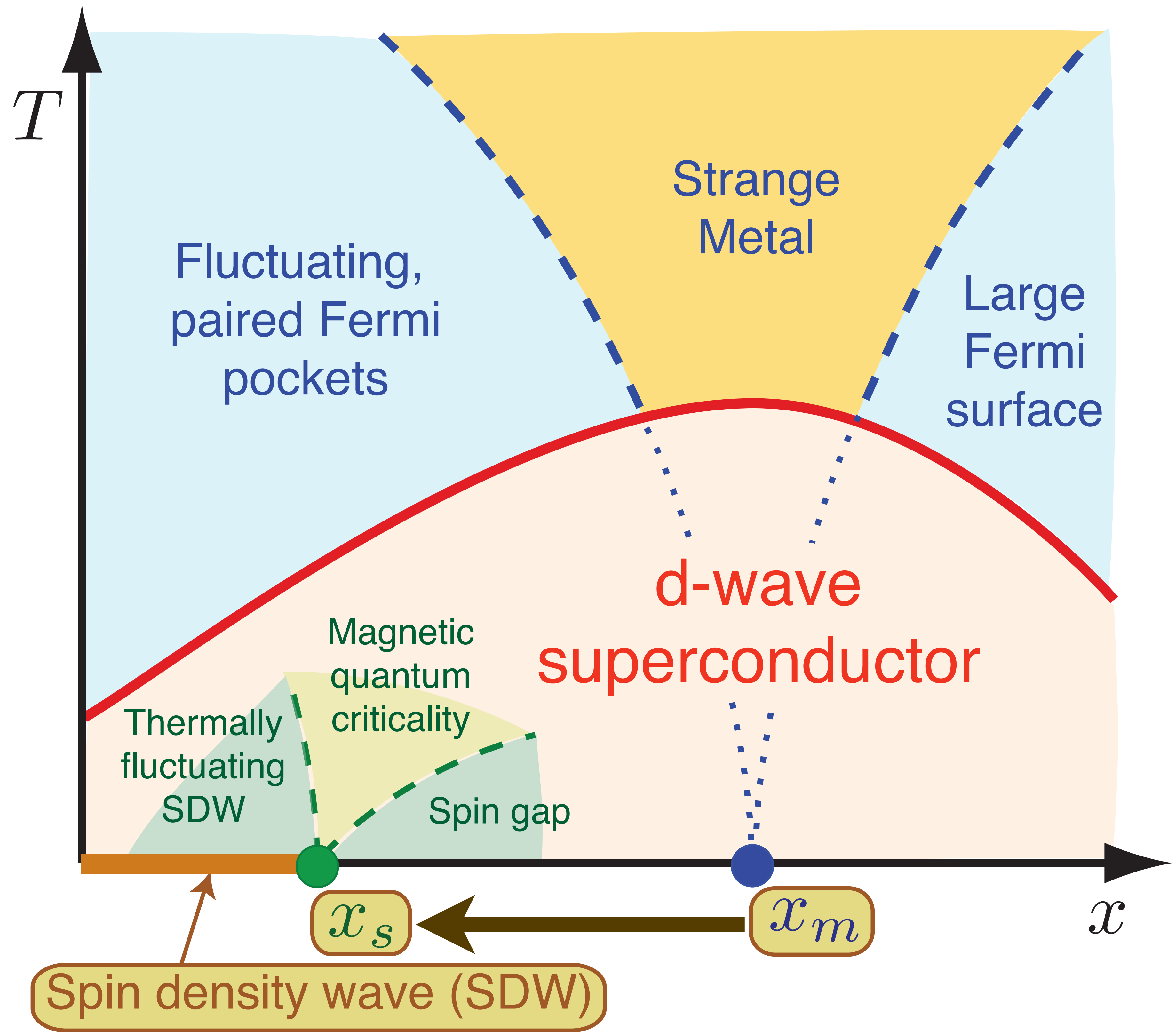
d-wave pairing of the large Fermi surface



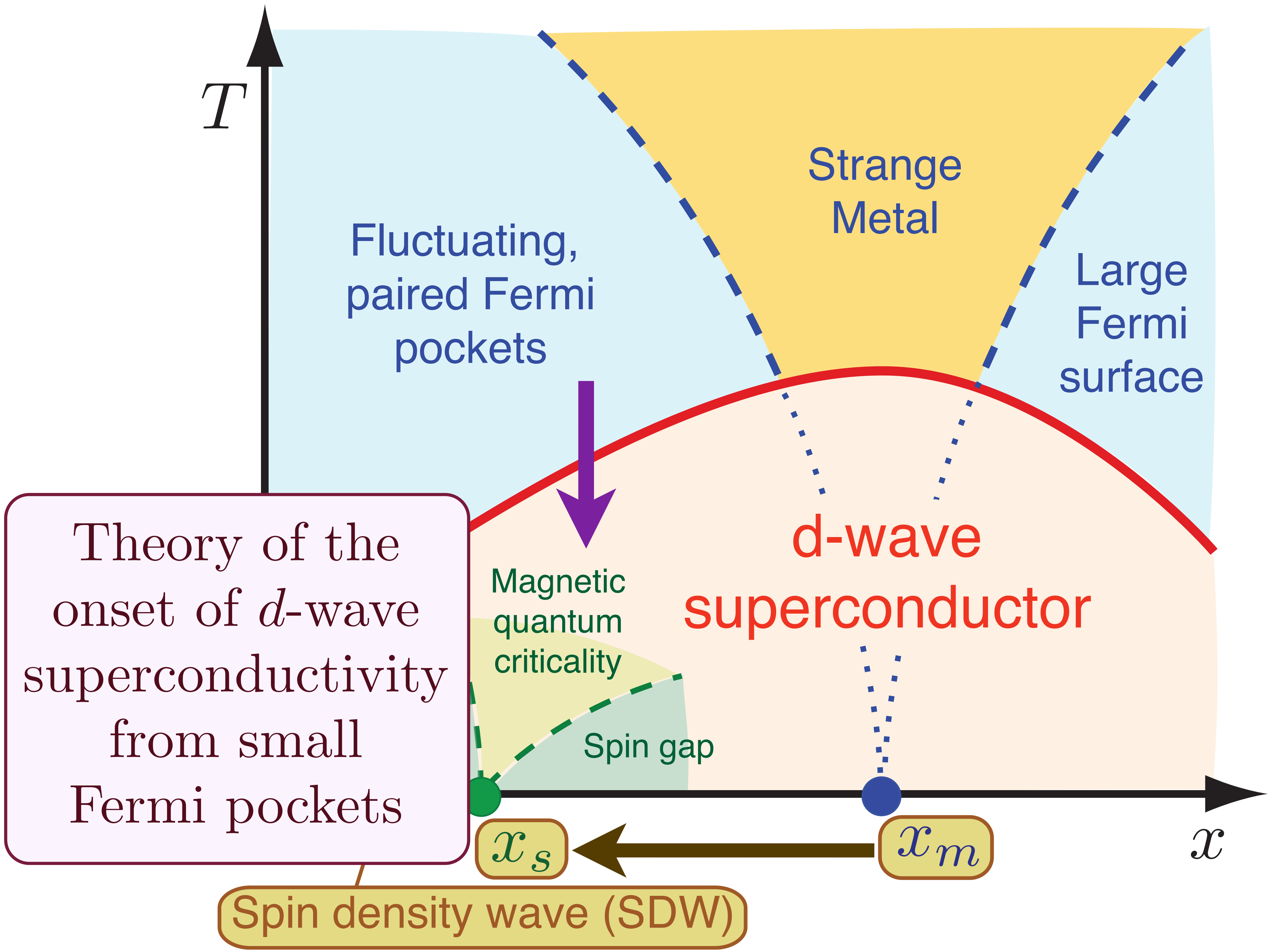
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

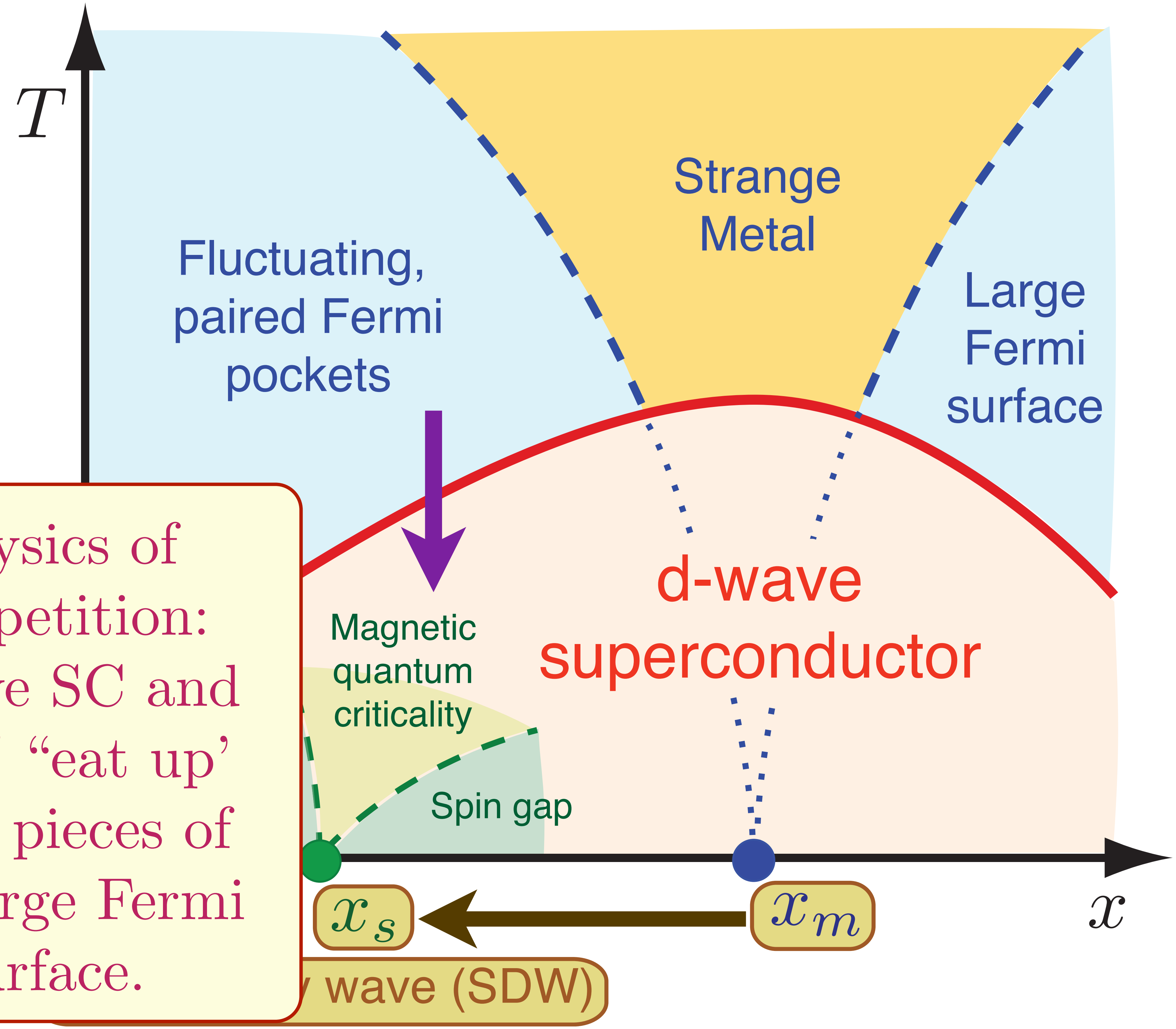
Theory of quantum criticality in the cuprates



Theory of quantum criticality in the cuprates



Theory of quantum criticality in the cuprates



Physics of competition: d-wave SC and SDW "eat up' same pieces of the large Fermi surface.

Questions

- How is antiferromagnetism lost with increasing doping ?
- How does the Fermi surface evolve with loss of antiferromagnetism ?
- Do antiferromagnetic fluctuations induce d-wave superconductivity ?
- How does attraction between antiferromagnetism and superconductivity turn into competition ?
- Is there a broken symmetry in the pseudo-gap regime ?
- What is the role of nematic/valence-bond-solid/stripe order ?
- What is the physics of the strange metal ?

Posters

- **Max Metlitski:** Entanglement near strongly interacting quantum critical points in two and higher dimensions

- **Yang Qi:** Frustrated antiferromagnetism and spin liquids in organic insulators

PHYSICS

HARVARD