Quantum spin liquids: from Rydberg atoms to the high temperature superconductors

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Max Planck Institute of Quantum Optics

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Talk online: sachdev.physics.harvard.edu

DEPARTMENT OF ENERGY · UNITED STATES OF AMERICA

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1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

   Probing topological spin liquids

3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

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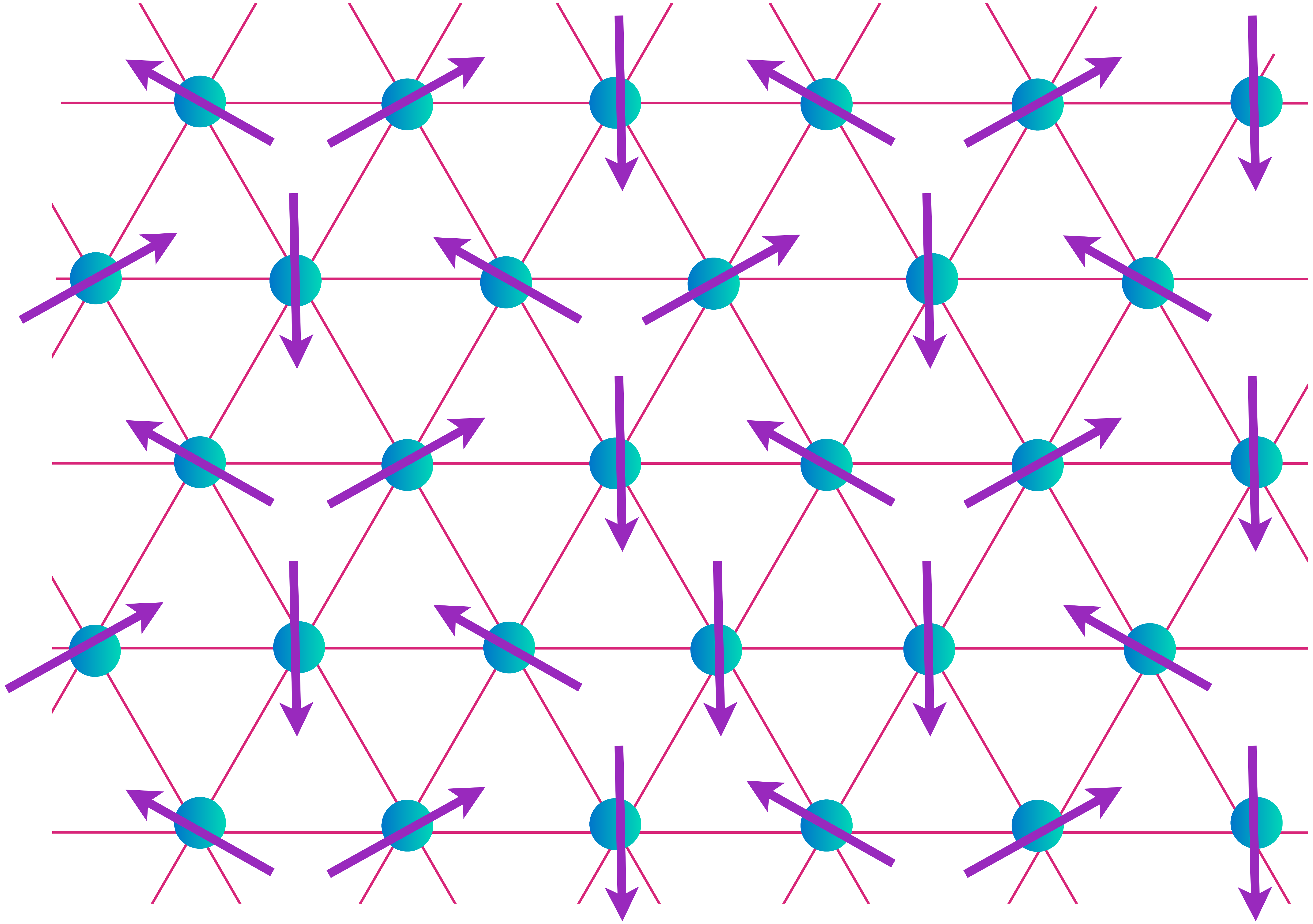
Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

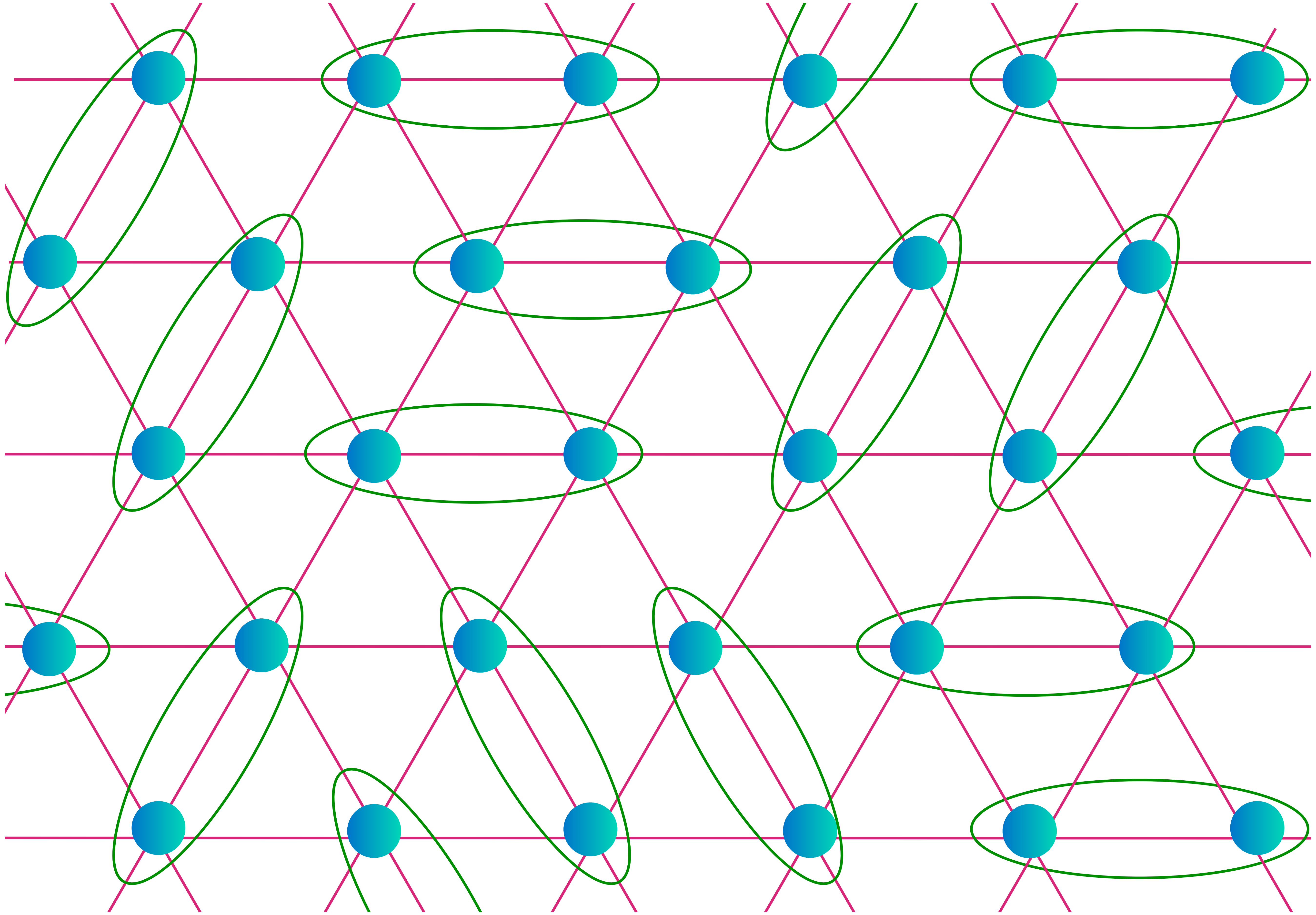
Mapping of bosons and spins

Boson B		Spin $\vec{S}$
$B^\dagger B \leq 1$		$\vec{S}^2 = 3/4$
States $\|0\rangle$, $B^\dagger\|0\rangle$		States $\|\uparrow\rangle$ $\|\downarrow\rangle$
Operators		Operators
$B^\dagger$	$\longleftrightarrow$	$S_+ = S_x + iS_y$
B	$\longleftrightarrow$	$S_- = S_x - iS_y$
$B^\dagger B - \frac{1}{2}$	$\longleftrightarrow$	S_z

Spin liquid: resonating valence bonds

Bosons at half-filling,
or a spin model with S=1/2 per unit cell

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(B_1^\dagger - B_2^\dagger \right) |0\rangle$$



$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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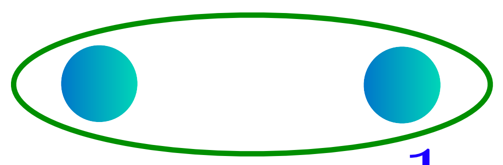

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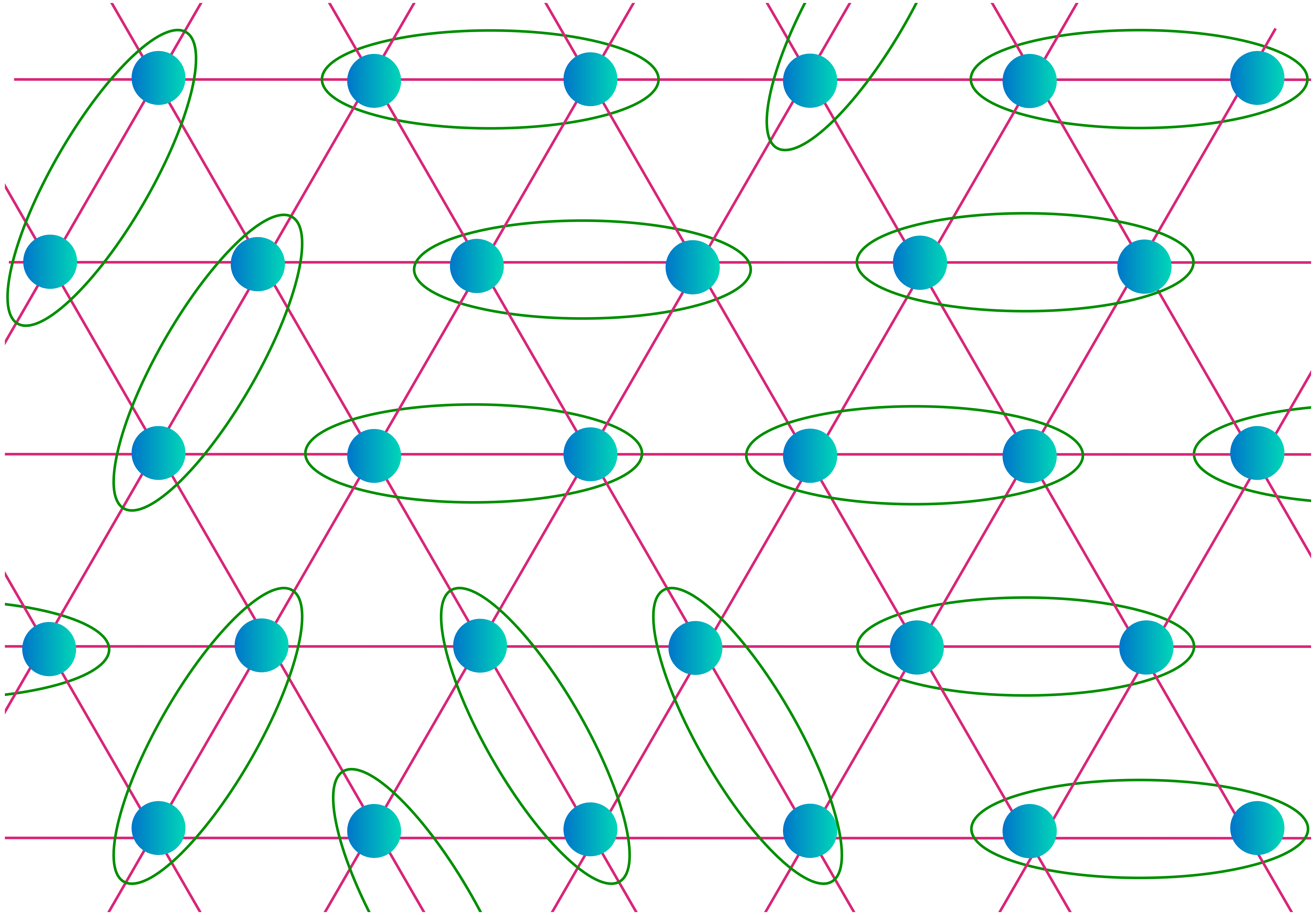

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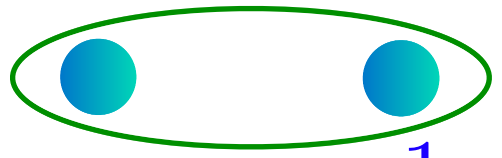

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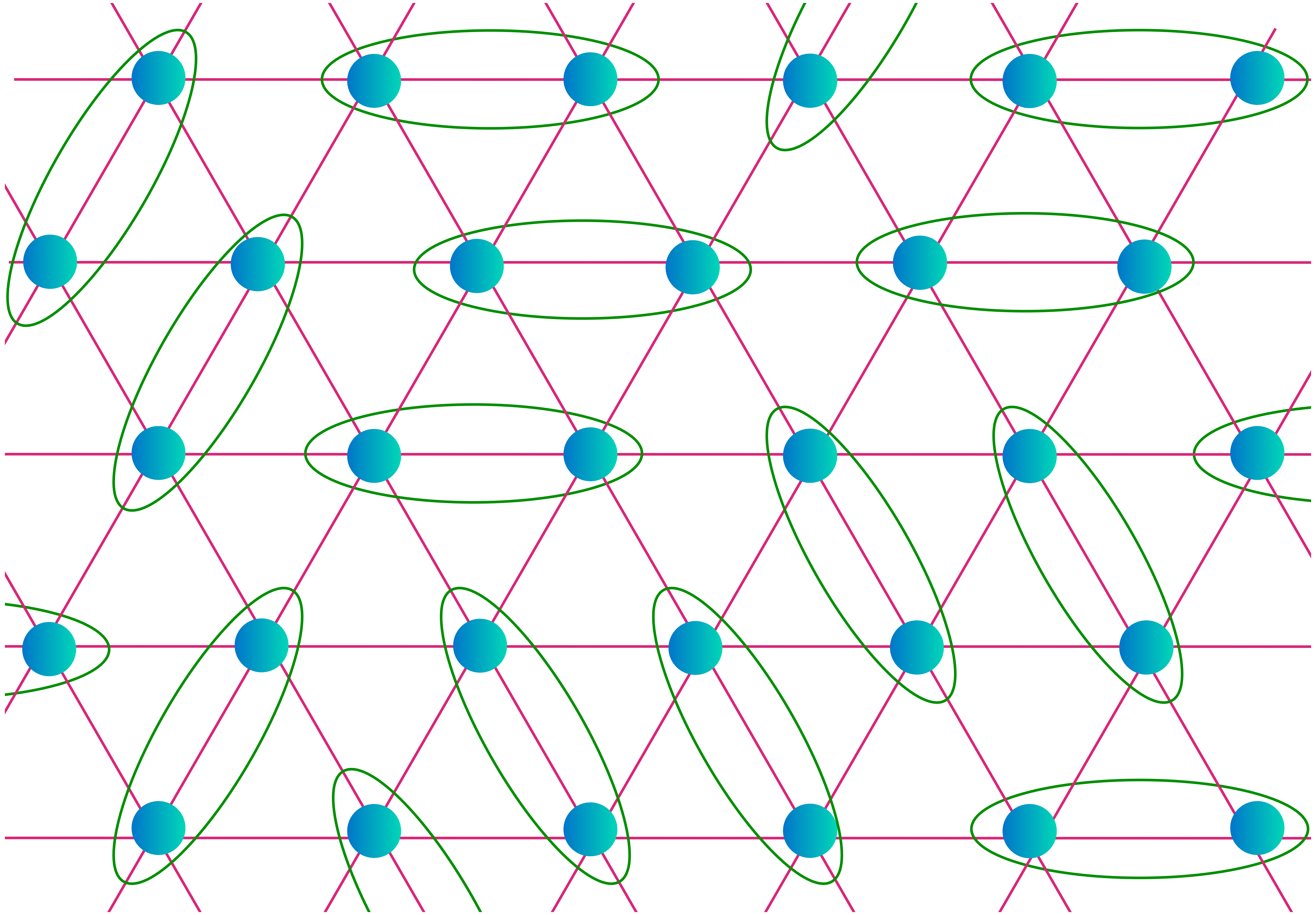

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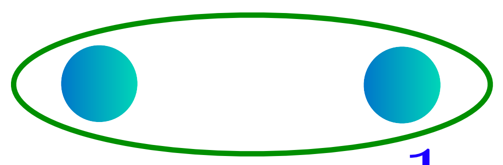

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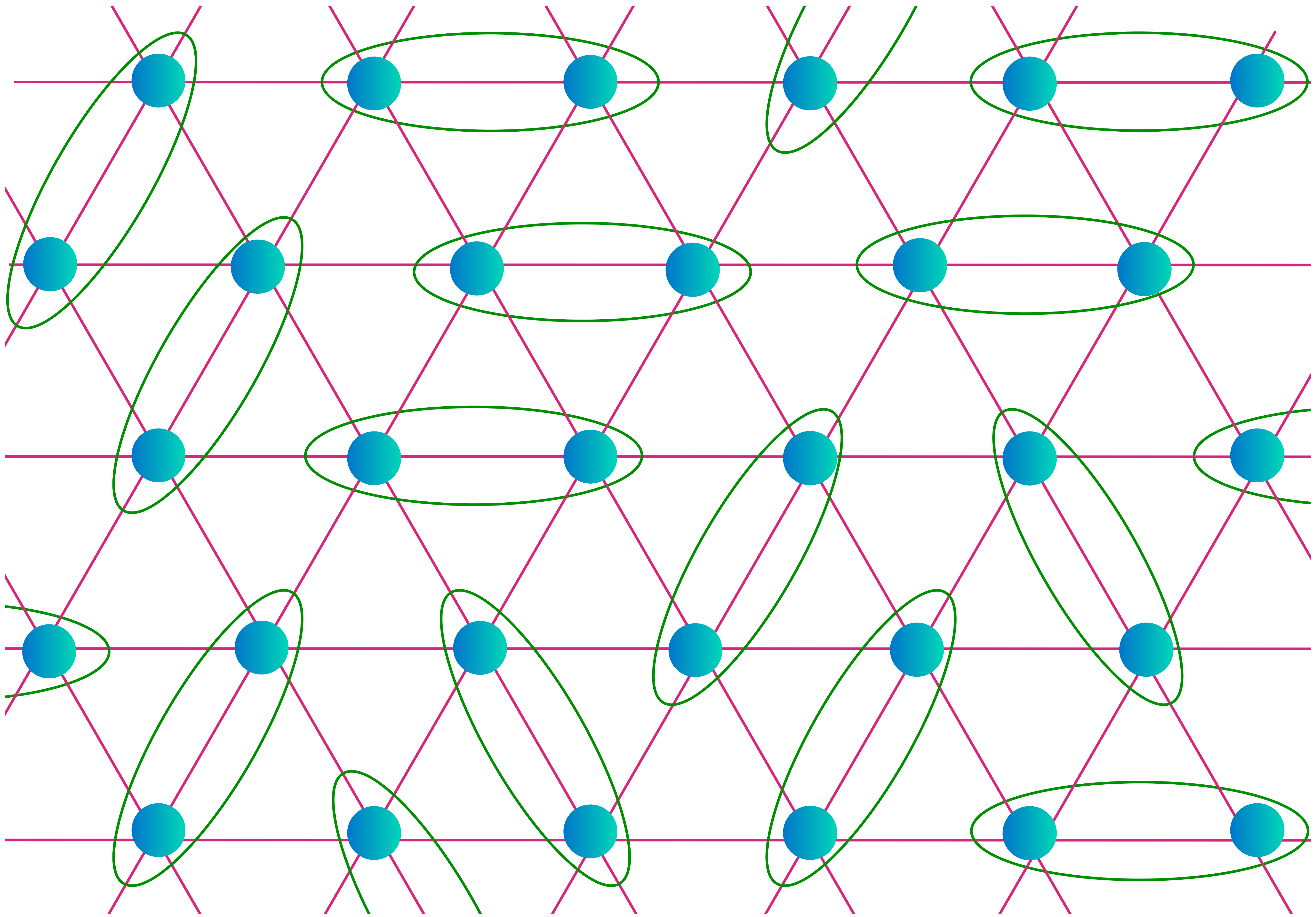

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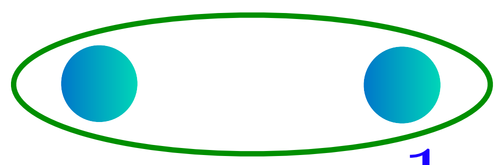

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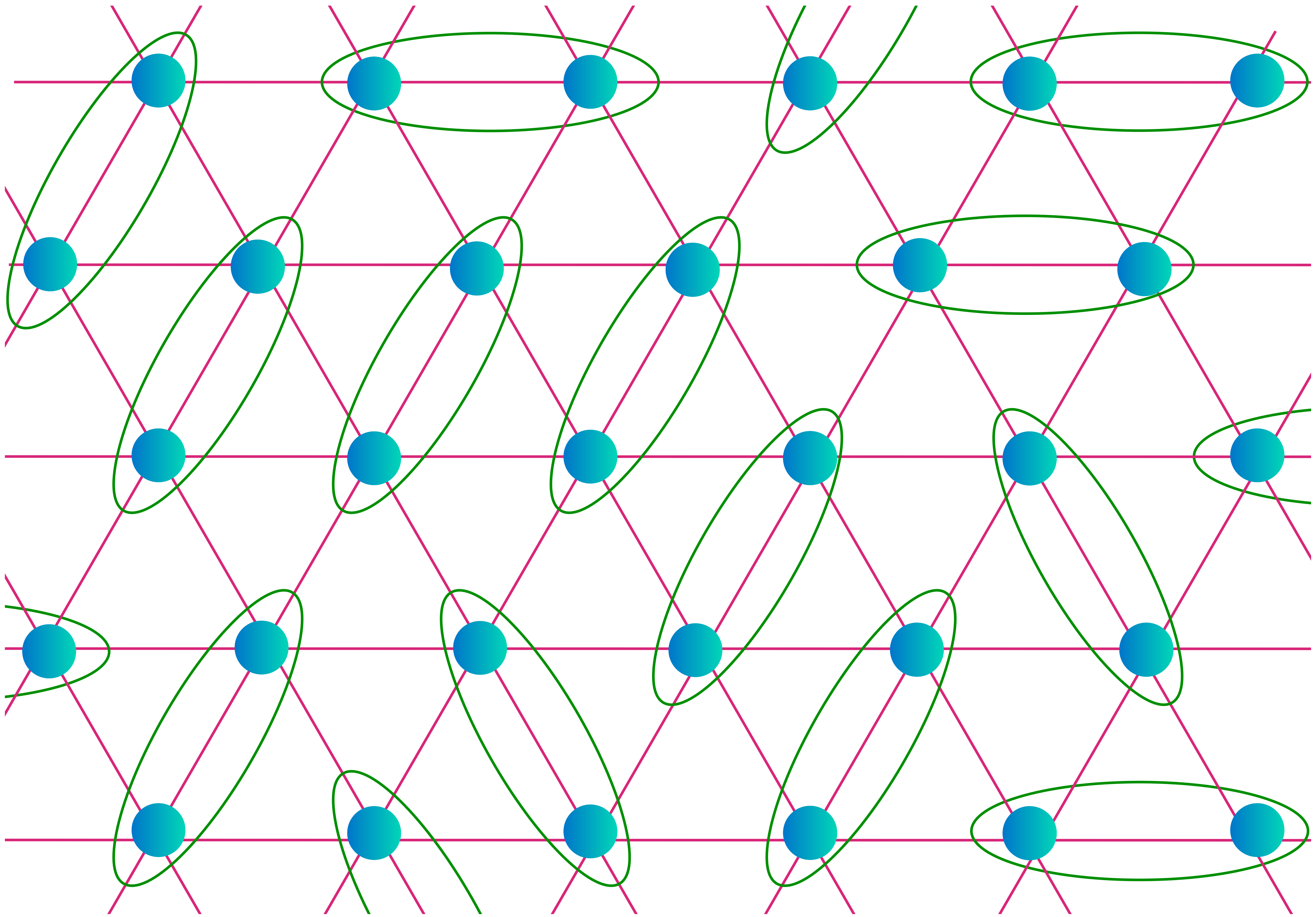

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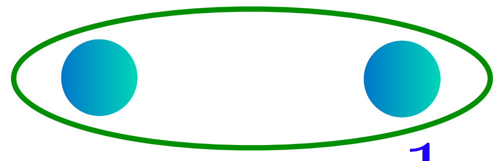

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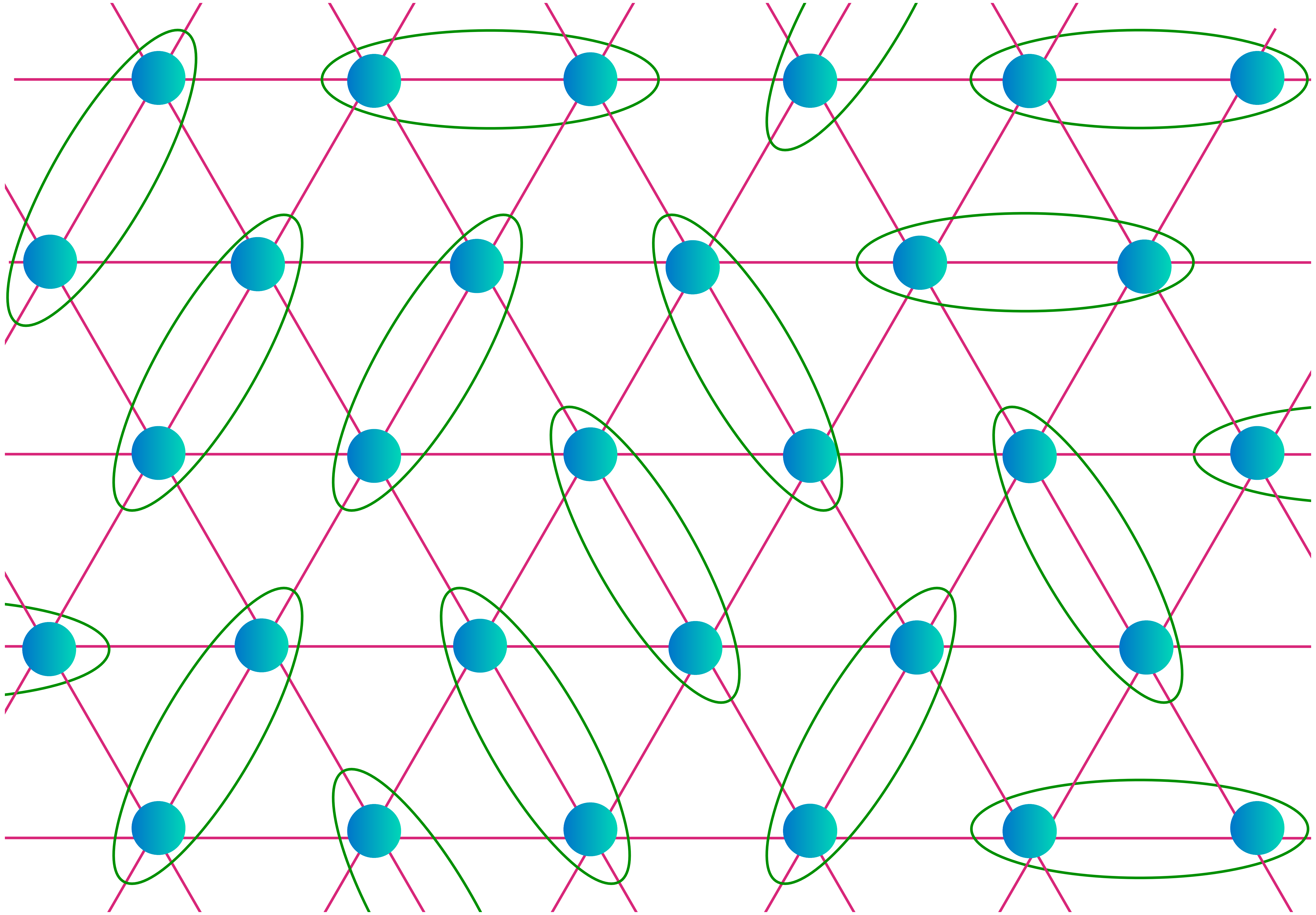

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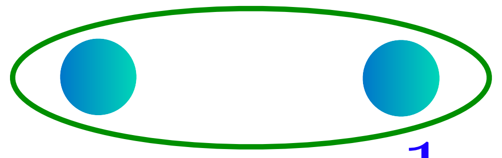
RVB: Z_2 spin liquid

Excitations with boson number 1/2



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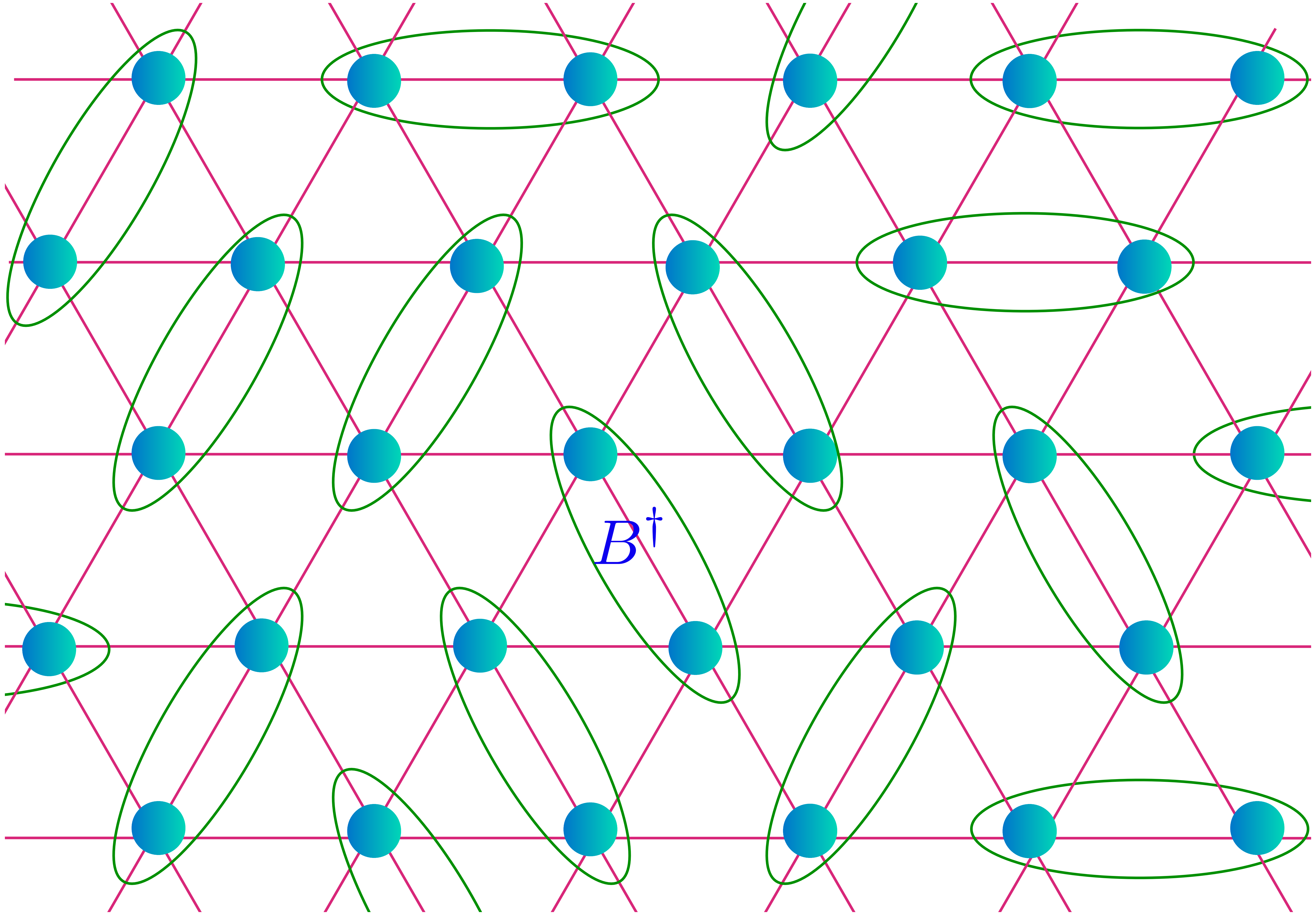
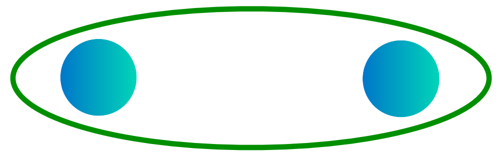
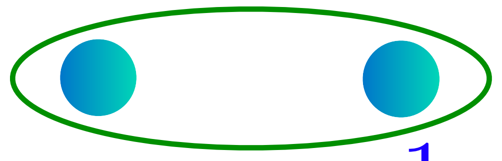


$B_2^\dagger$ 

$$= \frac{1}{\sqrt{2}} B_1^\dagger B_2^\dagger |0\rangle = \frac{1}{\sqrt{2}} |\uparrow\uparrow\rangle$$

RVB: Z_2 spin liquid

Excitations with boson number 1/2
a "spinon"



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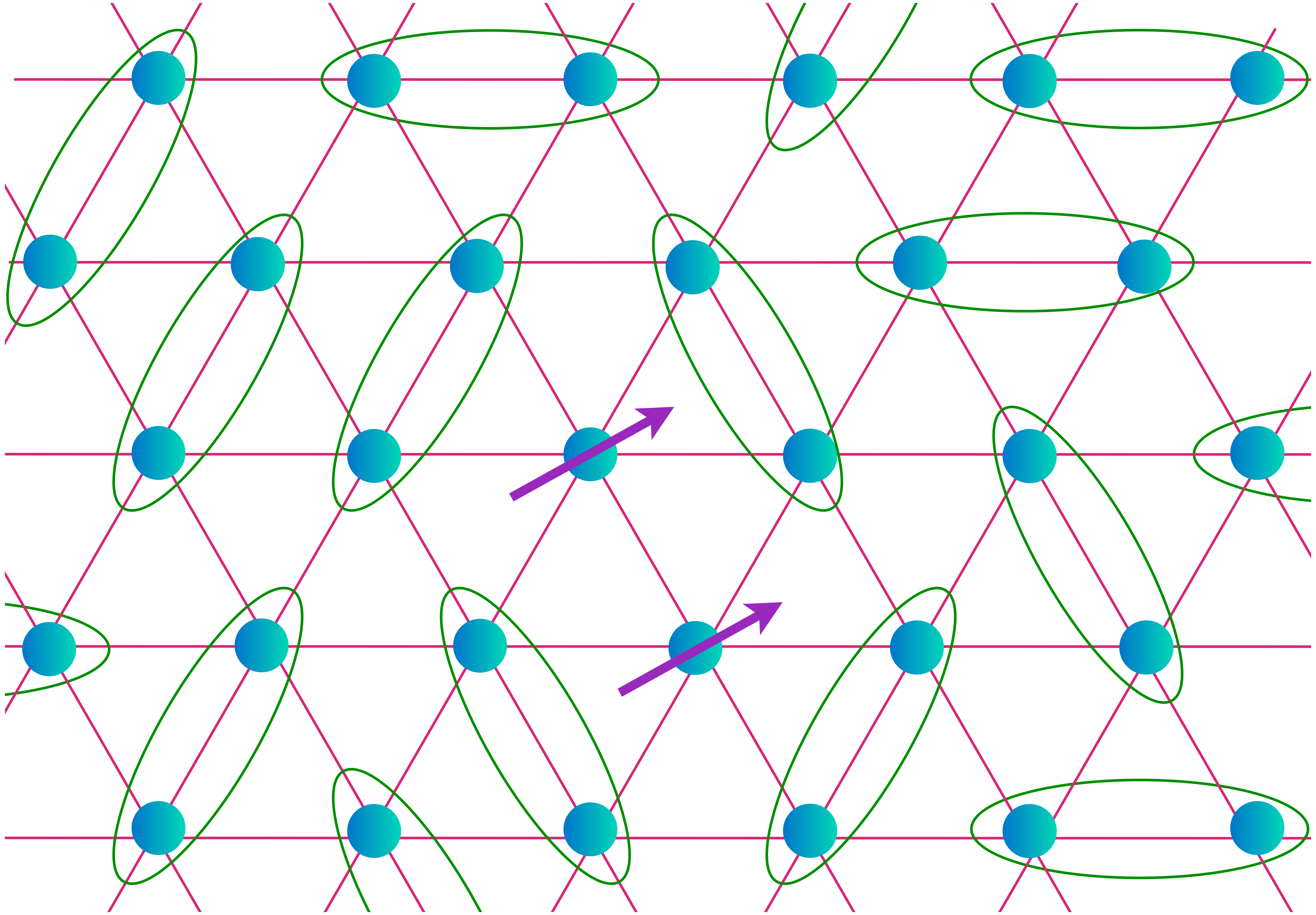

- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

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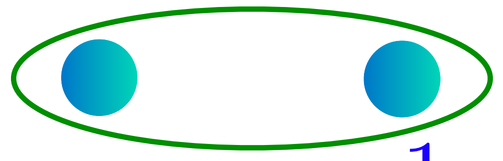

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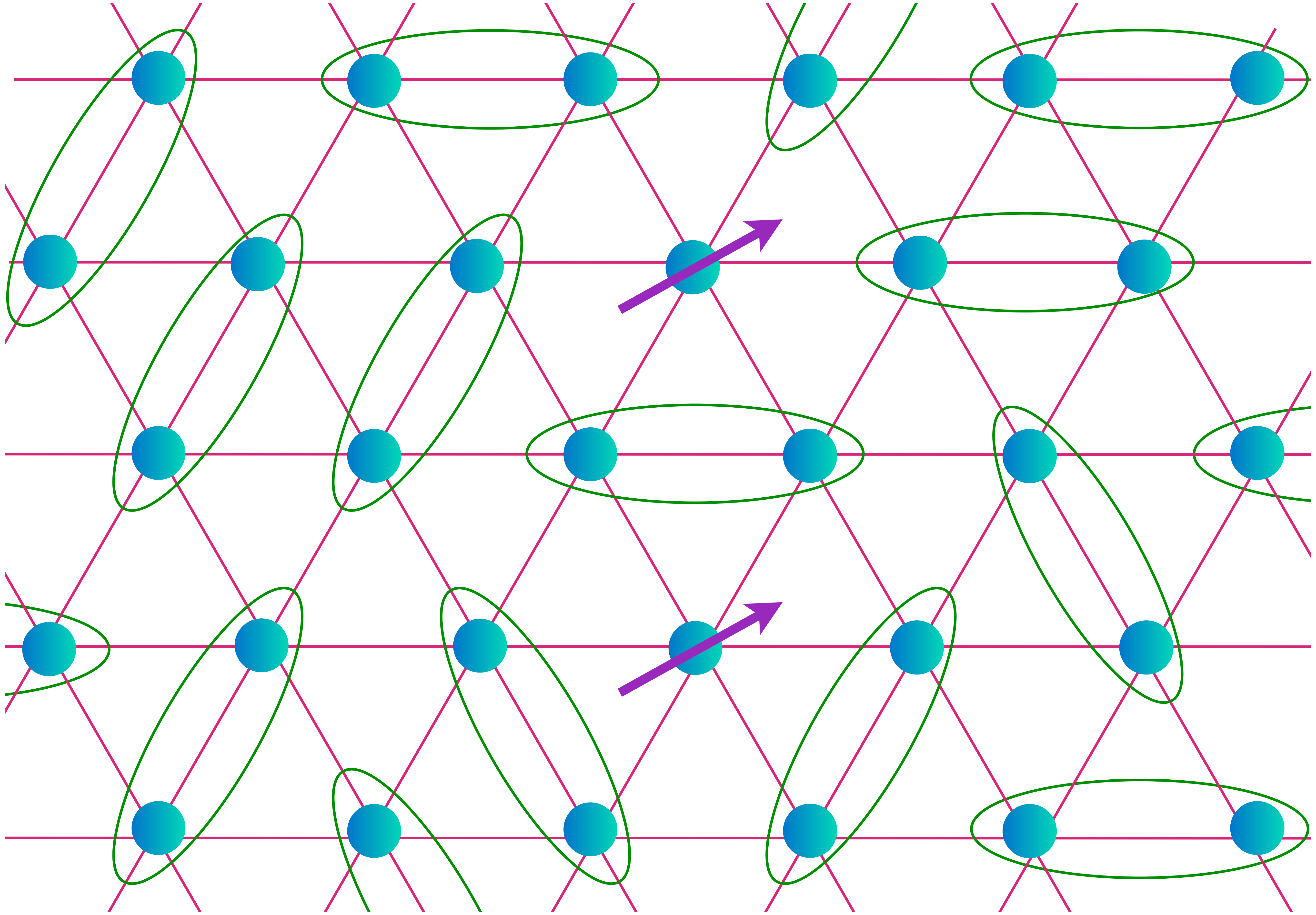

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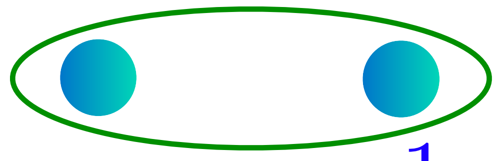

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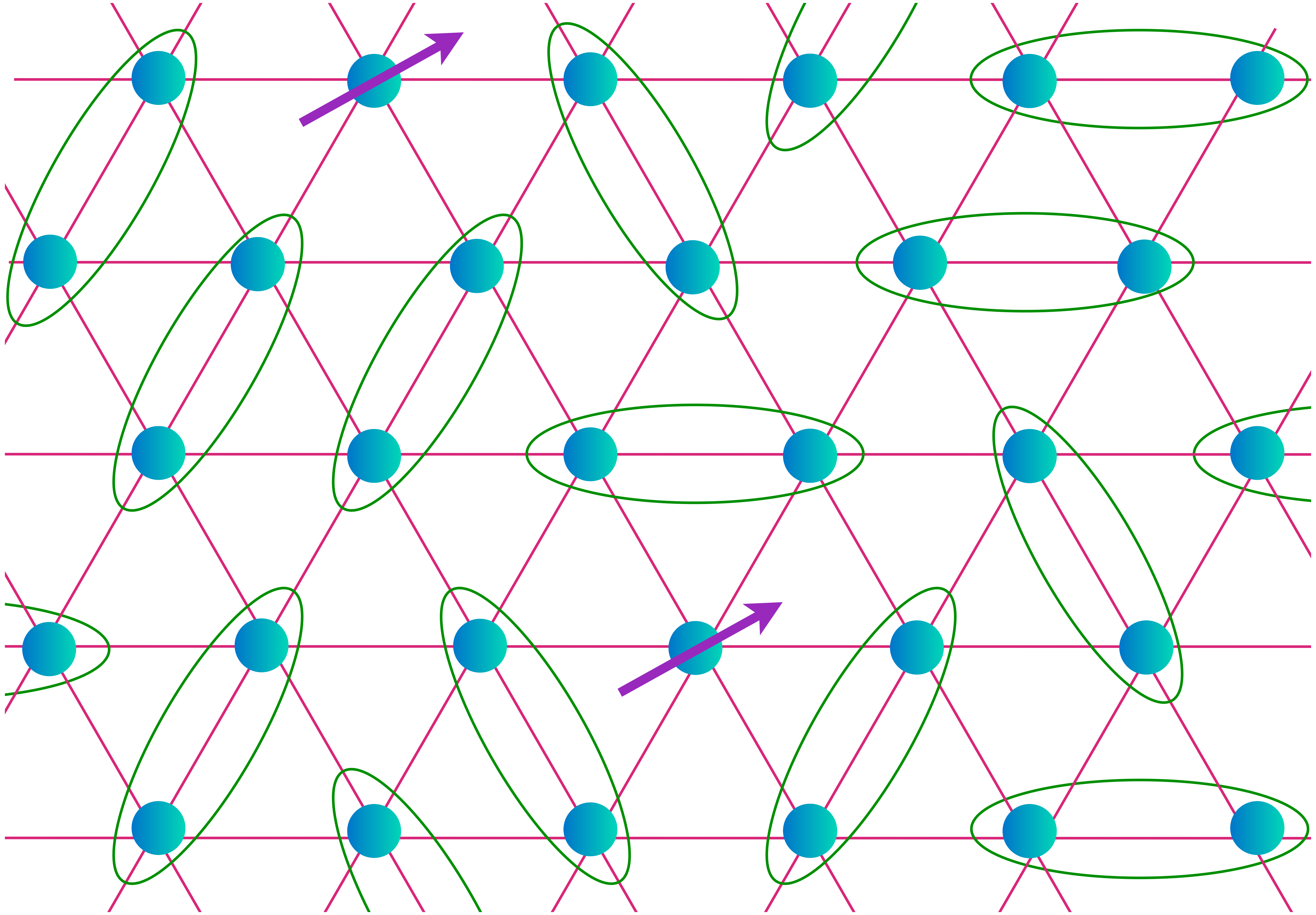

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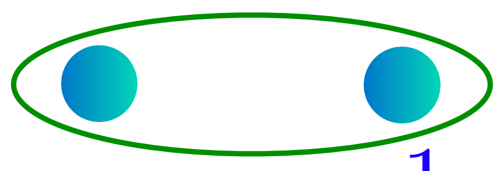

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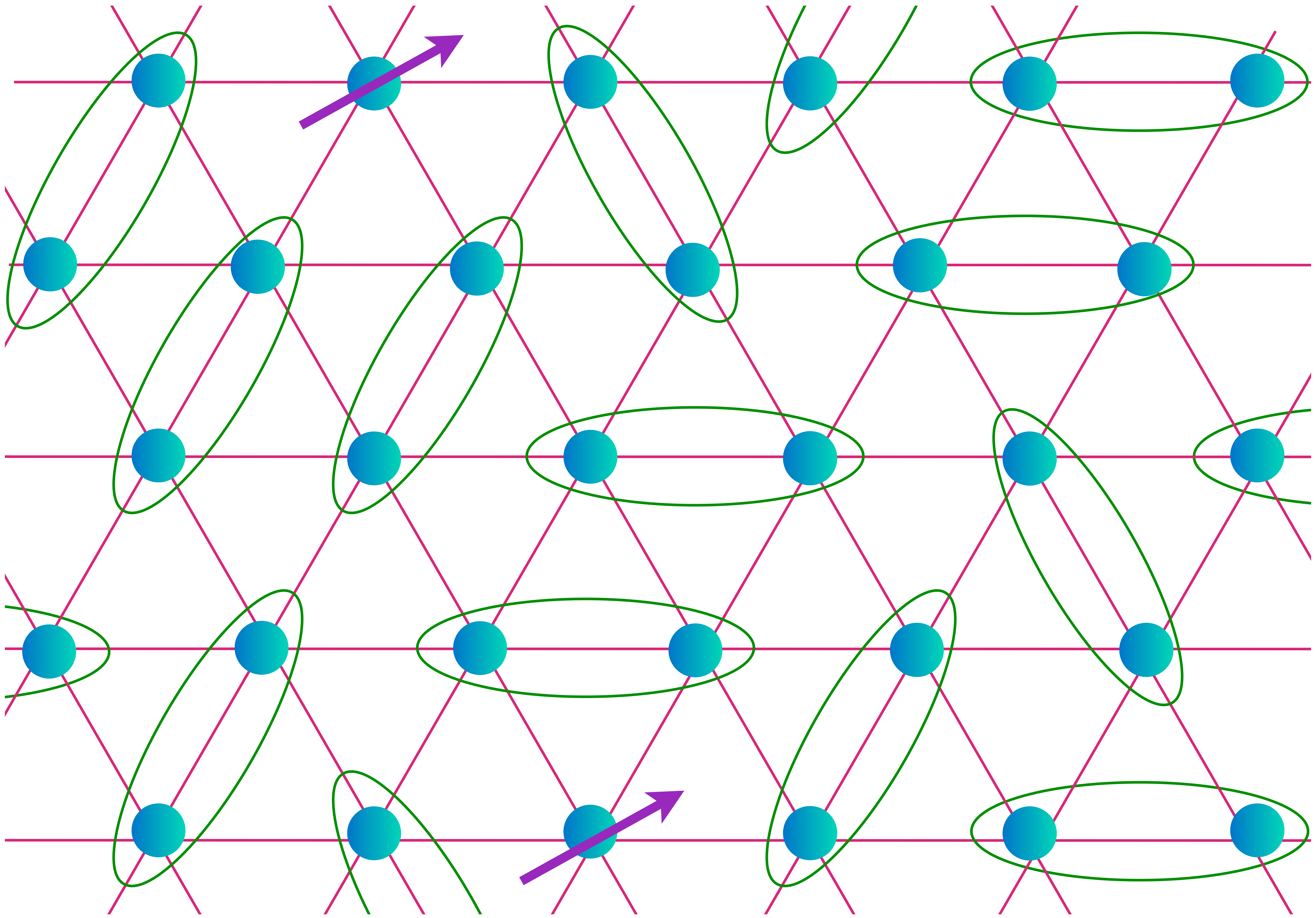

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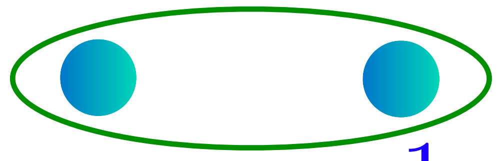
Excitations with boson number 0
a vison (*m* particle)



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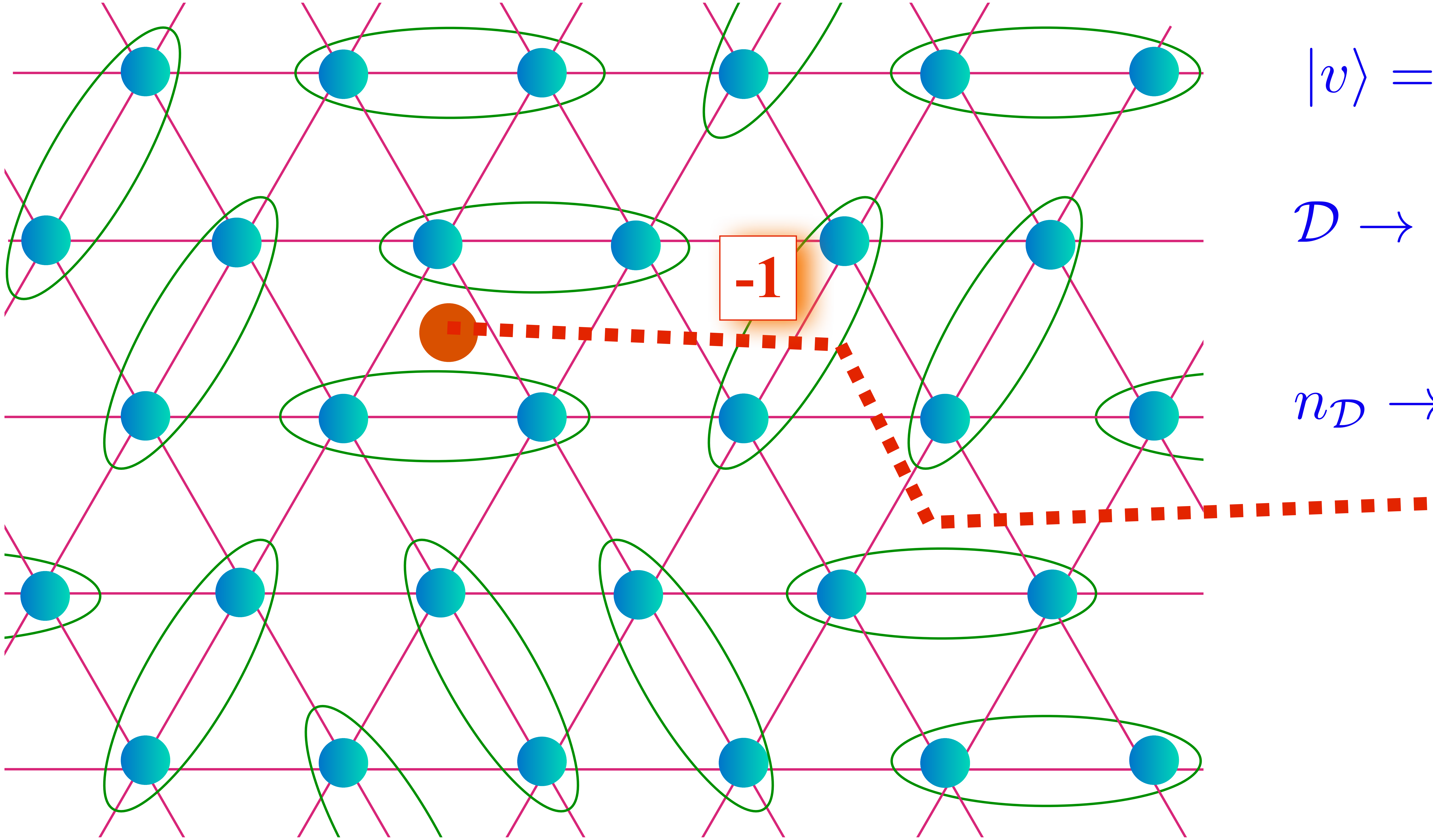

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$\mathcal{D} \rightarrow$ dimer covering of lattice

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N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

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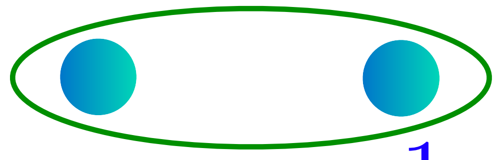

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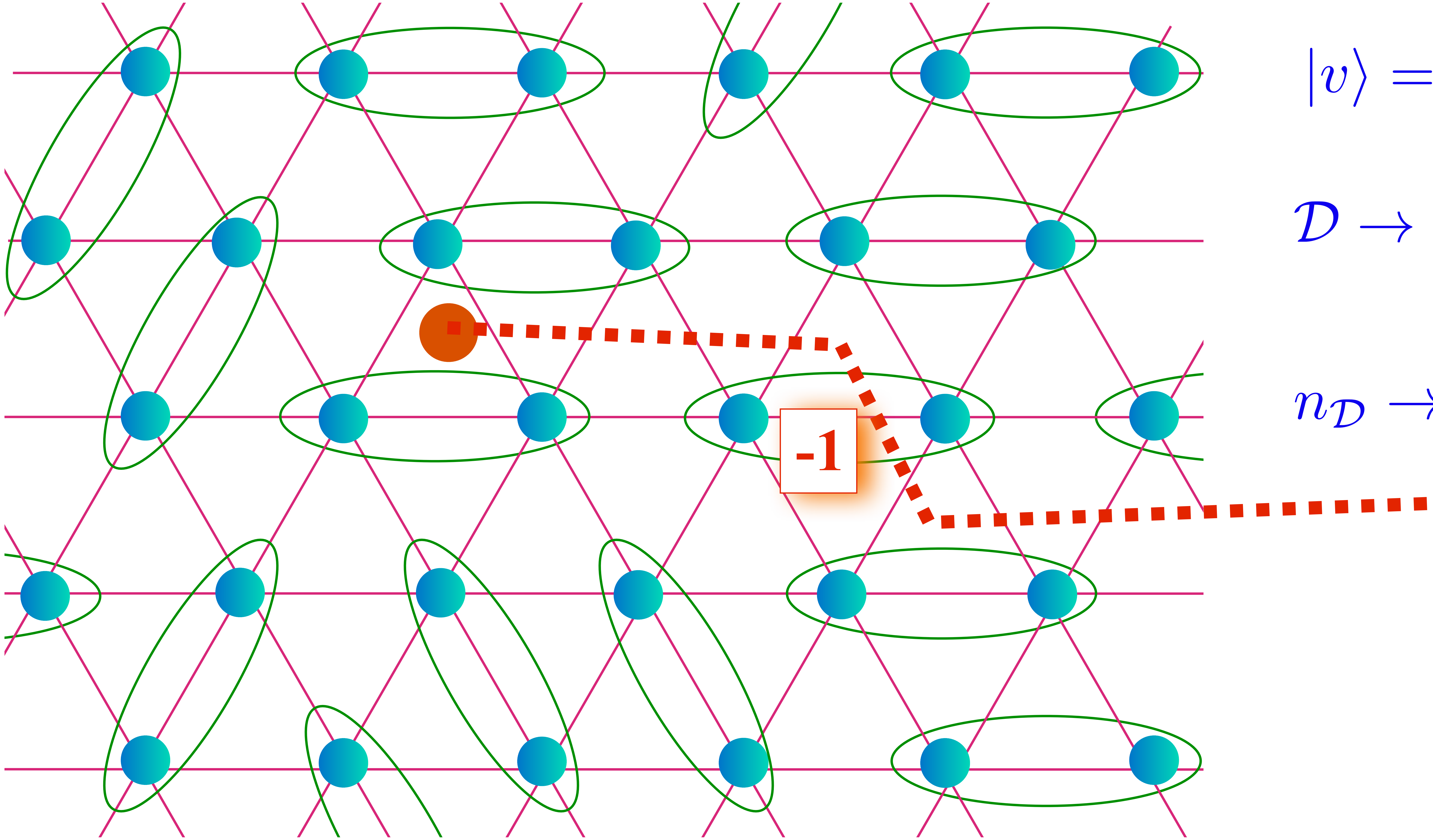

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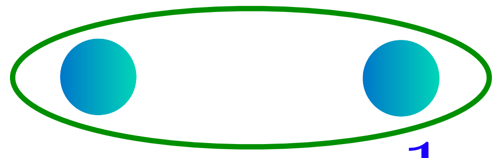

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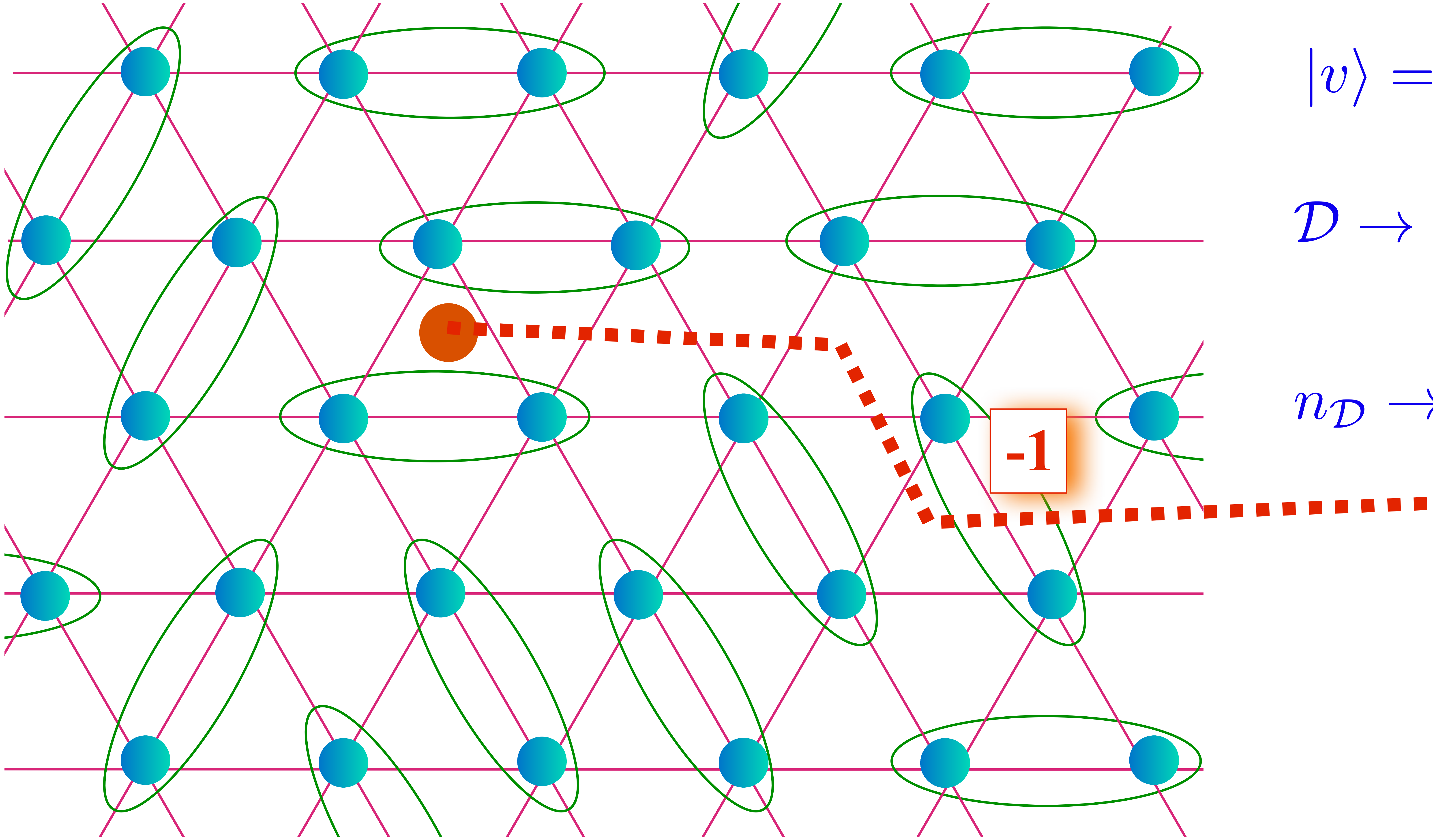

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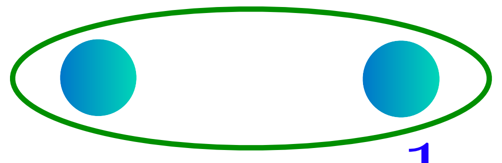

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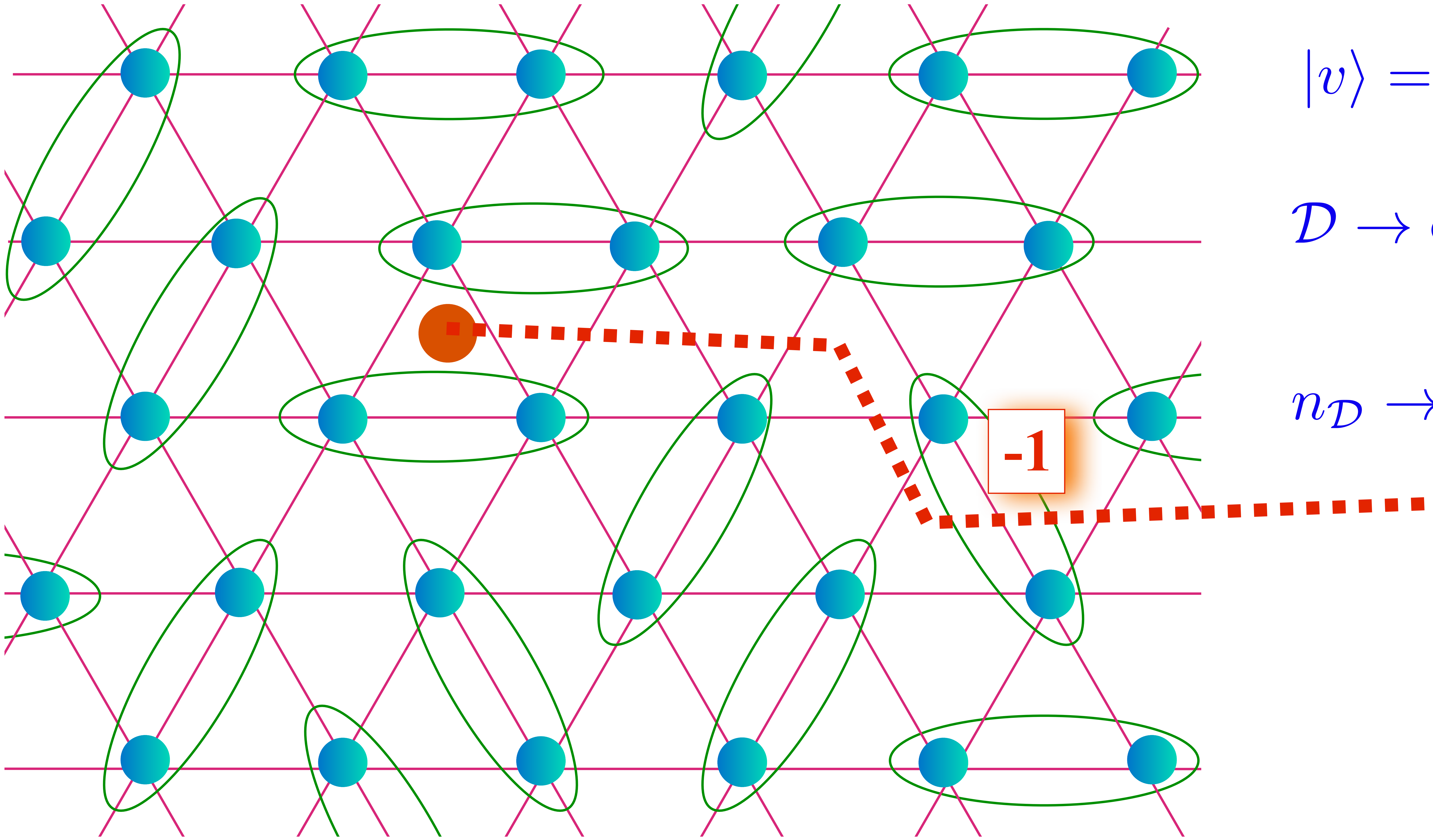

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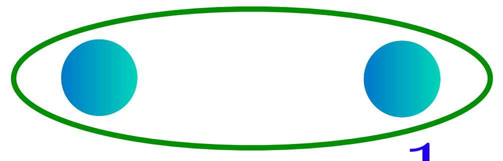

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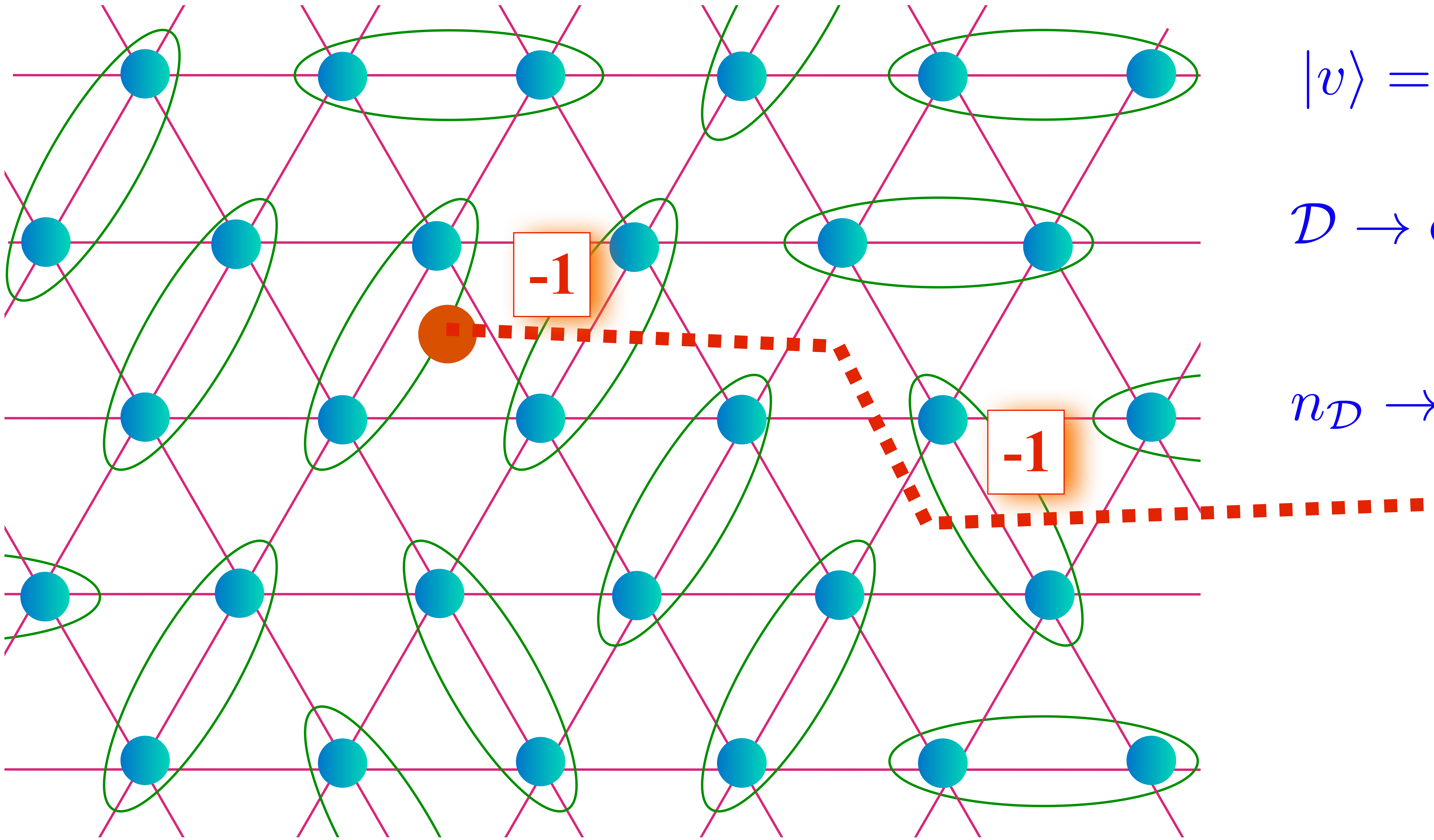

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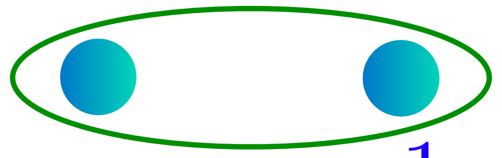

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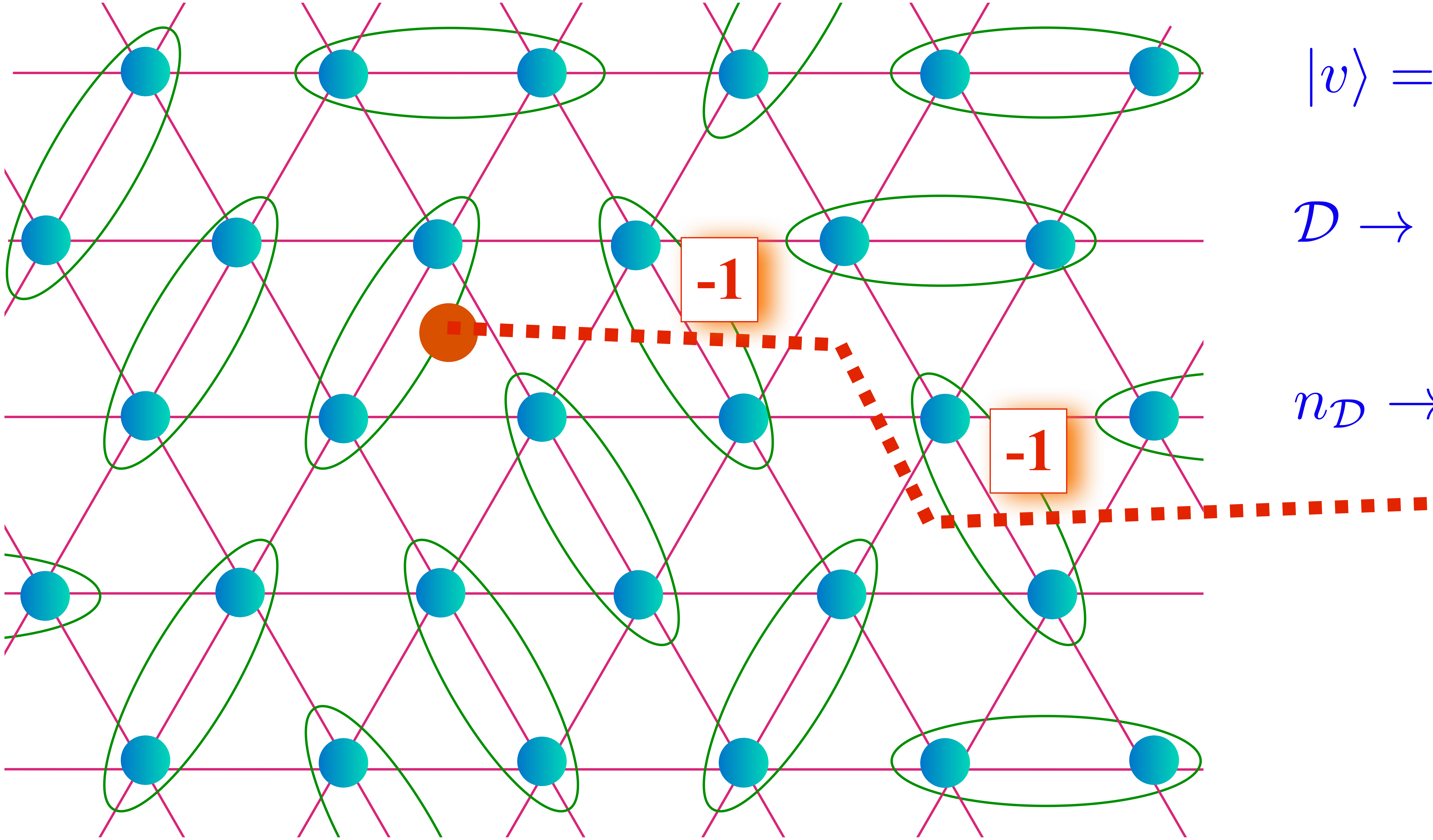

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RVB: Z_2 spin liquid

Read and Sachdev (1990); Wen (1991)

The simplest stable spin liquid (which need not break time-reversal) is the deconfined phase of a $\mathbb{Z}_2$ gauge theory. There are 'spinon' excitations which carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry π $\mathbb{Z}_2$ magnetic flux.

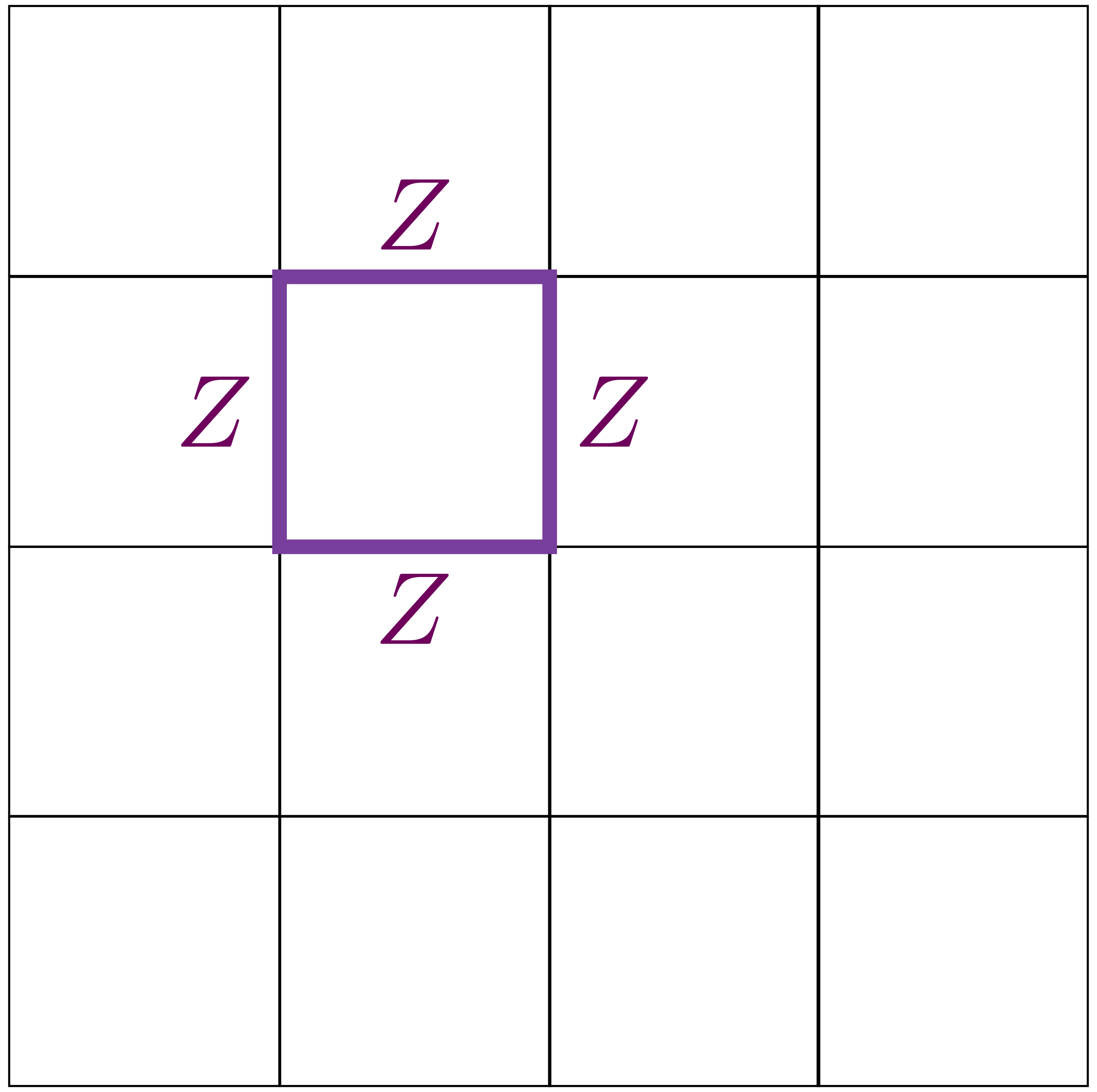
Anyon	e (spinon)	ϵ (spinon)	m (vison)
Boson number	$1/2$	$1/2$	0
Self-statistics	boson	fermion	boson

Any pair of e, ϵ, m are mutual semions.

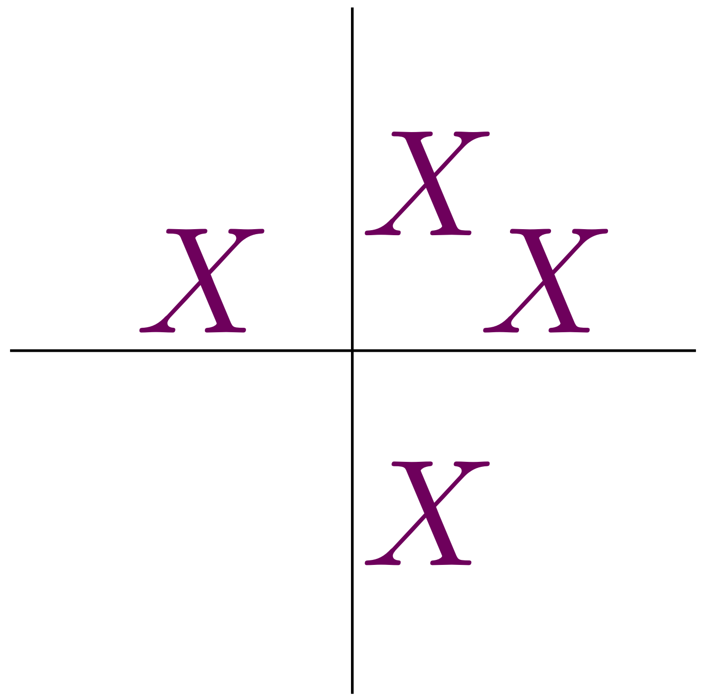
These anyons are 'topological': they cannot be created individually by any local operator, and their existence implies a four-fold ground state degeneracy on a large torus.

Pure Z_2 gauge theory: only describes vison sector

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$



$G_i =$



$$[\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0$$

$$G_i = (-1)^{2S} \quad \text{for spin } S \text{ antiferromagnets}$$

R. Jalabert and S. Sachdev, Physical Review B **44**, 686 (1991)

S. Sachdev and M. Vojta, Journal of the Physical Society of Japan **69,** Suppl. B, 1 (2000); cond-mat/9910231

Z_2 gauge theory with matter: describes visons and spinons

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$

$$- J \sum_{\ell \in (i,j)} \tau_i^z Z_\ell \tau_j^z - h \sum_i \tau_\ell^x$$

$$G_i = \tau_i^x \prod_{\ell \in i} X_\ell \quad , \quad [\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0$$

Now we choose $G_i = 1$ and $\text{sgn}(h) = (-1)^{2S}$.

The τ_i^x operator creates a $\mathbb{Z}_2$ electric charge – a 'spinon' which has mutual semionic statistics with a vison.

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Quantum phases of Rydberg atoms on a kagome lattice,
Rhine Samajdar, Wen Wei Ho, Hannes Pichler, M. D. Lukin, and S. S.,
Proceedings of the National Academy of Sciences **118**, e2015785118 (2021); arXiv:2011.12295

Emergent Z_2 gauge theories and topological excitations in Rydberg atom arrays,
Rhine Samajdar, Darshan G. Joshi, Yanting Teng, and S. S., arXiv:2204.00632

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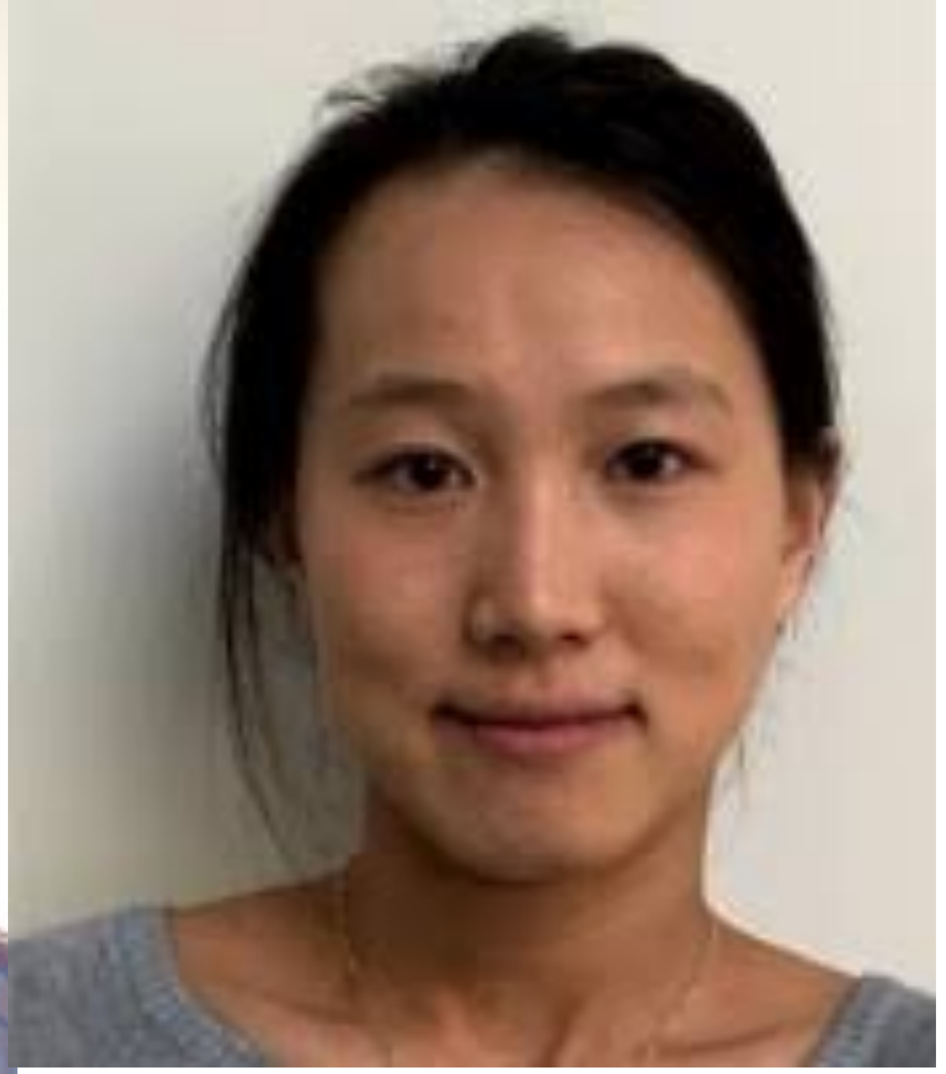
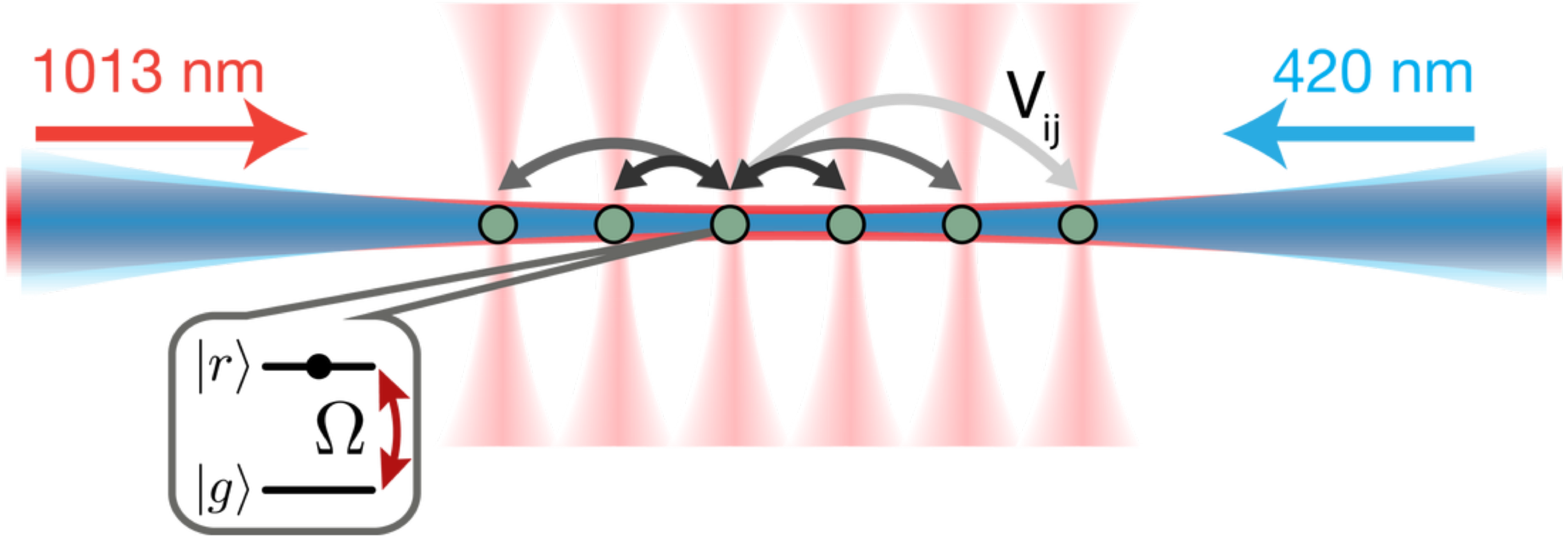
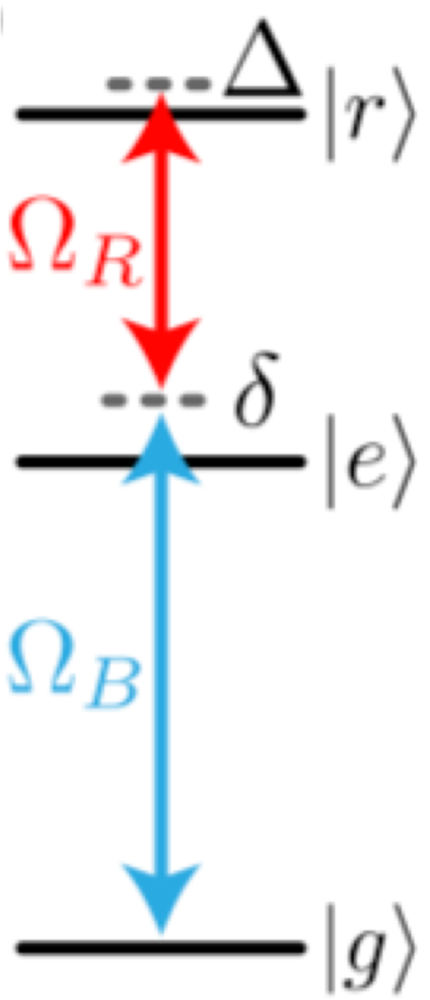


Yanting Teng

QPTs in a Rydberg quantum simulator





$|g\rangle \equiv |0\rangle$

$|r\rangle \equiv b^\dagger |0\rangle$

$$\mathcal{H} = \sum_\ell \left[\frac{\Omega}{2} \left(b_\ell + b_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

$$V_{|\ell-\ell'|} \sim \frac{1}{|\ell - \ell'|^6}$$

$n_\ell \equiv b_\ell^\dagger b_\ell$

$n_\ell = 0, 1$ 'hard core' bosons

FSS model

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

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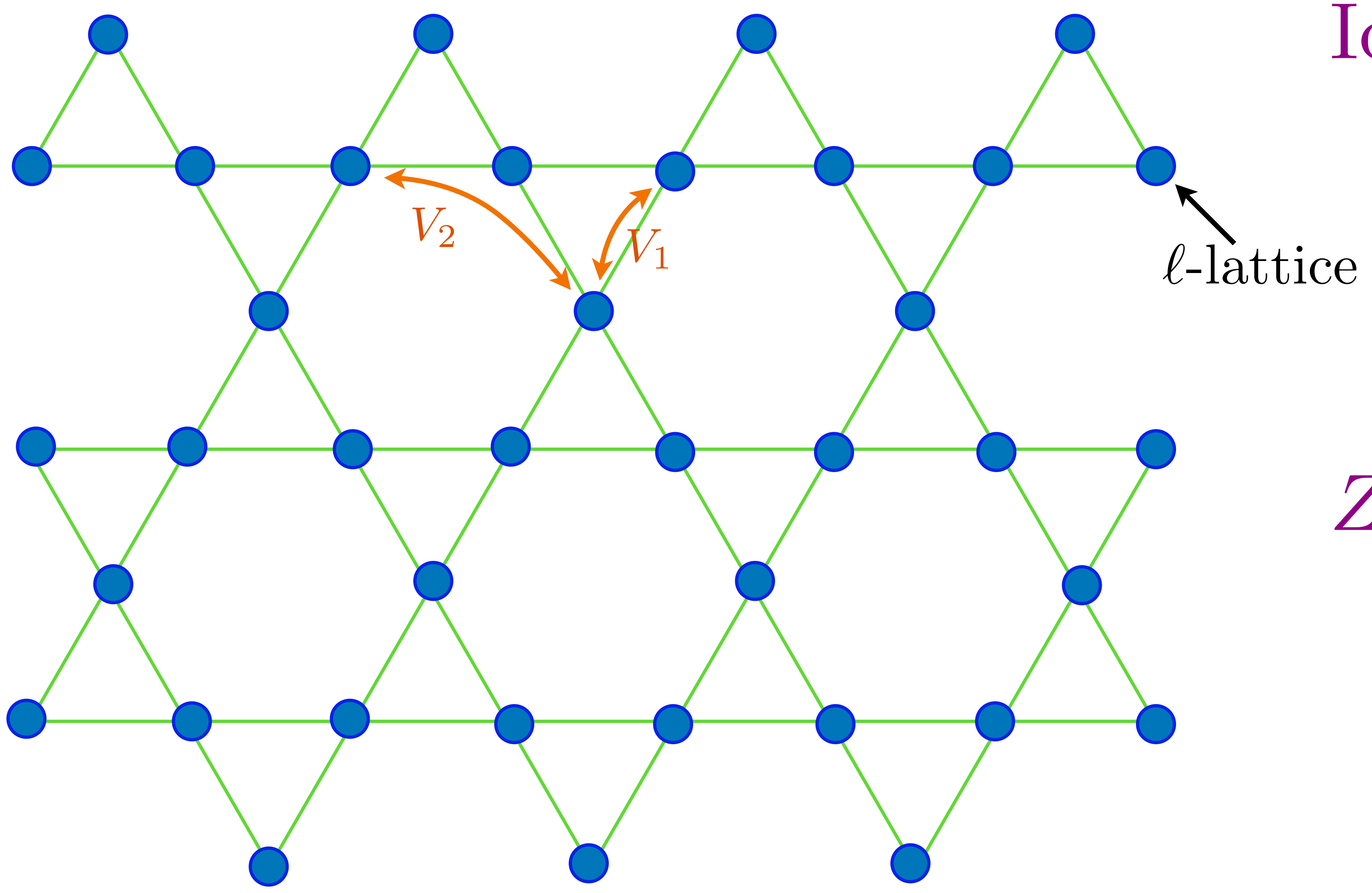

Identify hard core bosons with a qubit X, Y, Z

$$b_\ell + b_\ell^\dagger \quad \Leftrightarrow \quad Z_\ell$$

$$n_\ell \quad \Leftrightarrow \quad (1 - X_\ell)/2$$

Z will become the $\mathbb{Z}_2$ gauge field

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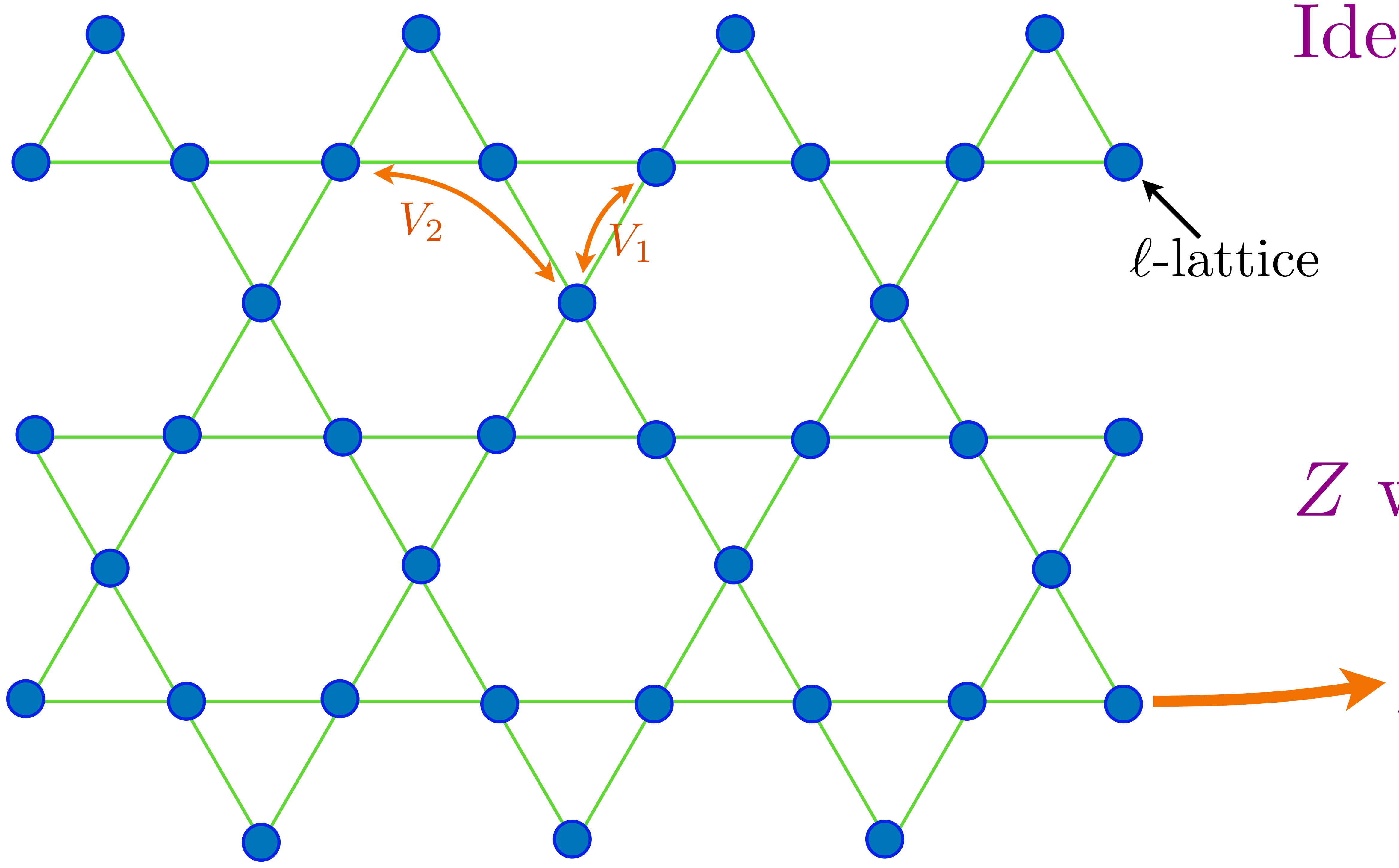
$$\mathcal{H} = \sum_{\ell} \left[\frac{\Omega}{2} Z_\ell + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$



Z_ℓ

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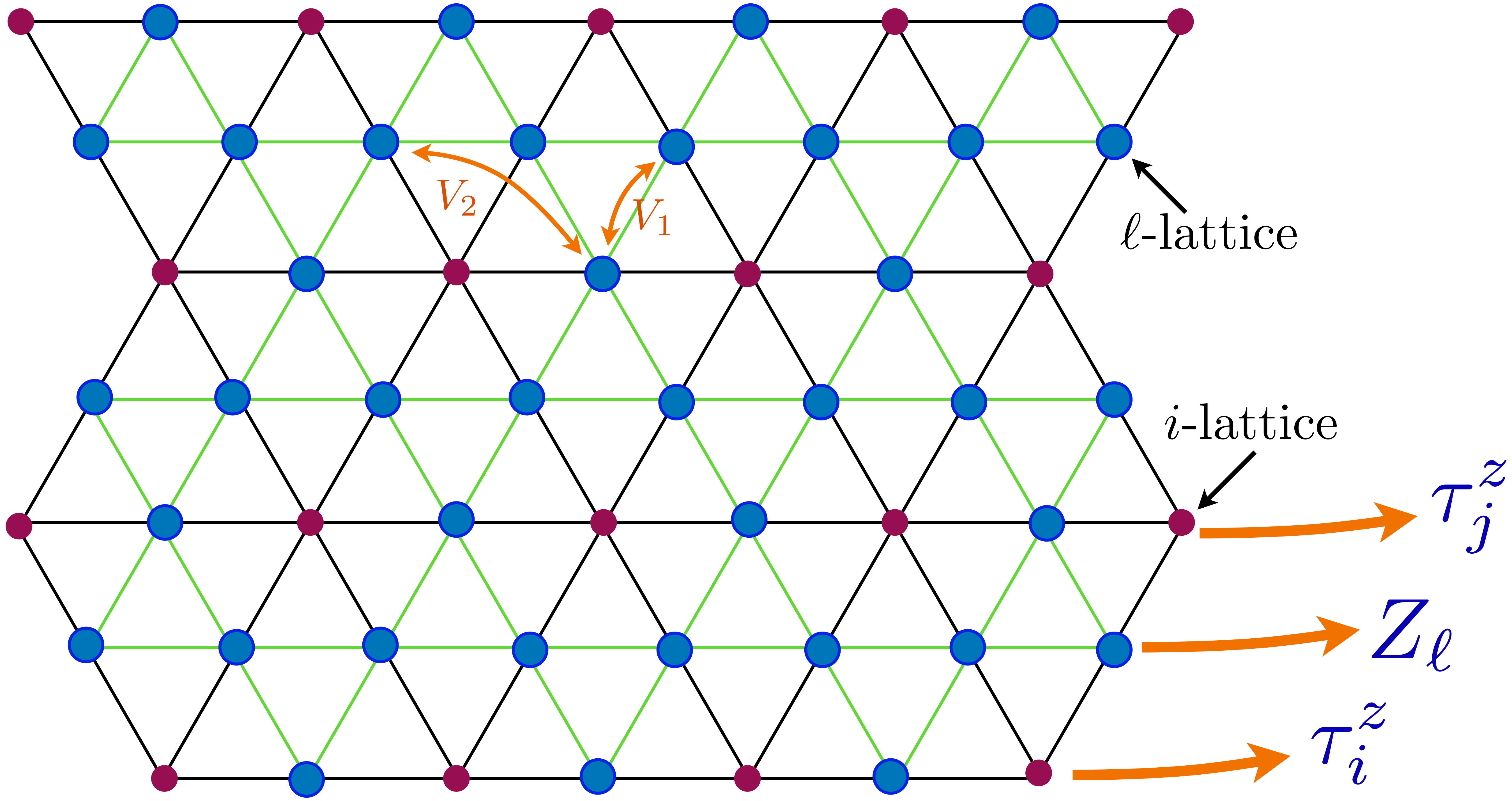
$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[\frac{\Omega}{2} \tau_i^z \, Z_\ell \, \tau_j^z + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$

Introduce $\mathbb{Z}_2$ matter fields on 'i sites'. Gauge invariance: $\tau_i^z \to \varrho_i \tau_i^z$, $Z_{ij} \to \varrho_i Z_{ij} \varrho_j$, $\tau_i^x \to \tau_i^x$, $X_\ell \to X_\ell$, $\varrho_i = \pm 1$. Gauss law constraint: $G_i = \tau_i^x \prod_{\ell \in i} X_\ell = 1$.



R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021)

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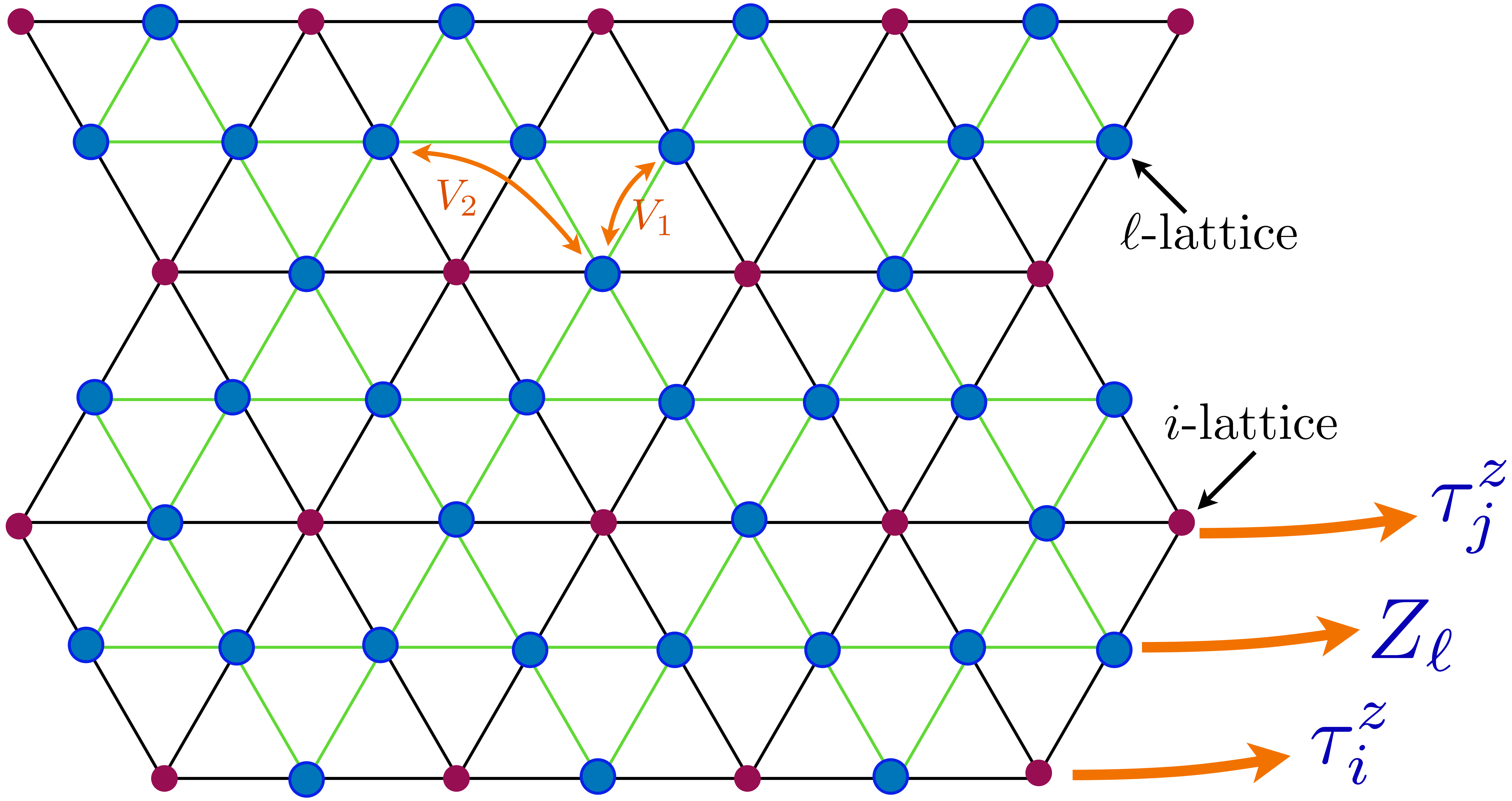
The K_{loop} terms are generated in a large V expansion: 'resonance' between Rydberg states can stabilize a phase with deconfined $\mathbb{Z}_2$ gauge charges *i.e.* a $\mathbb{Z}_2$ spin liquid



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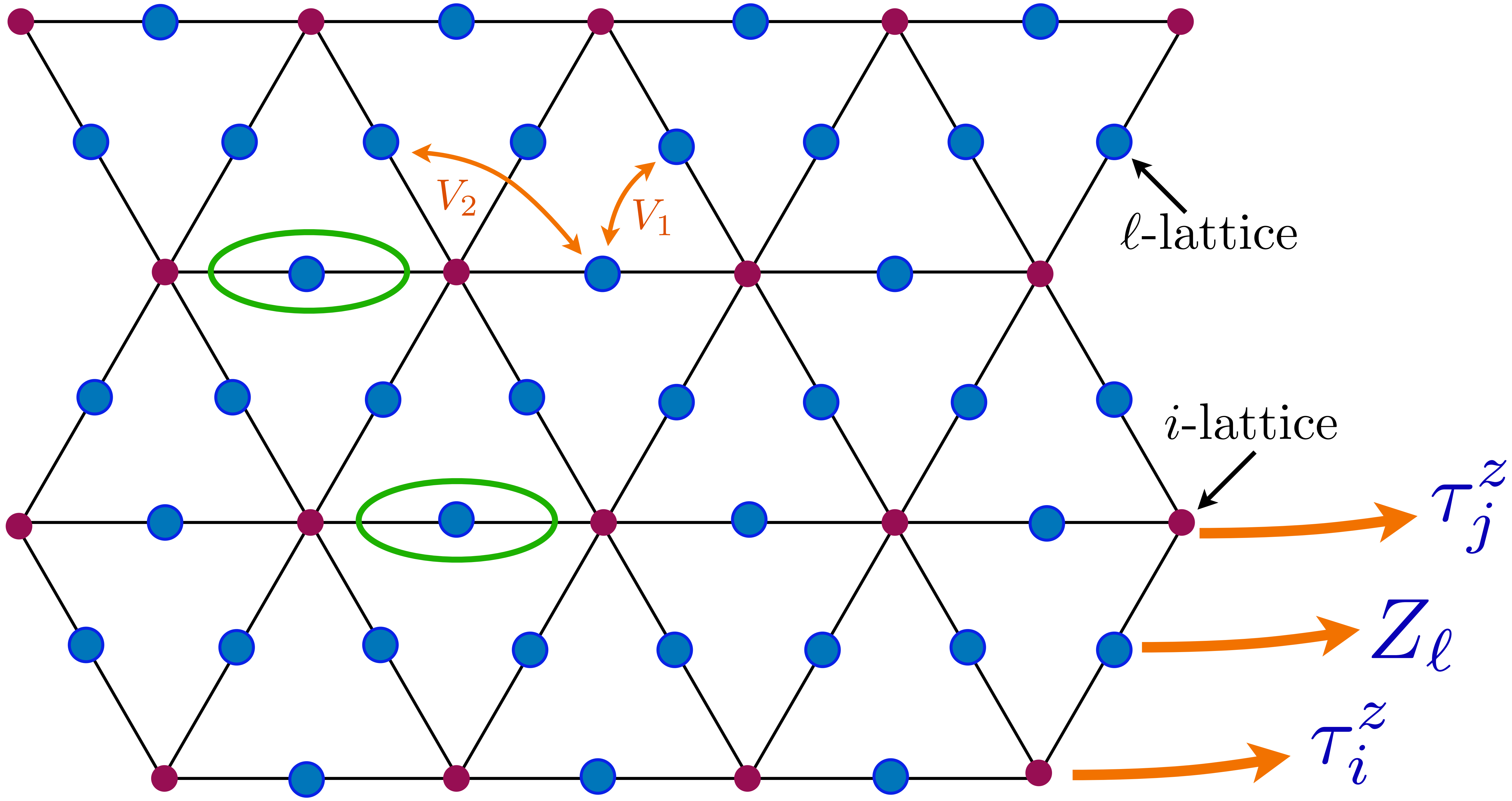

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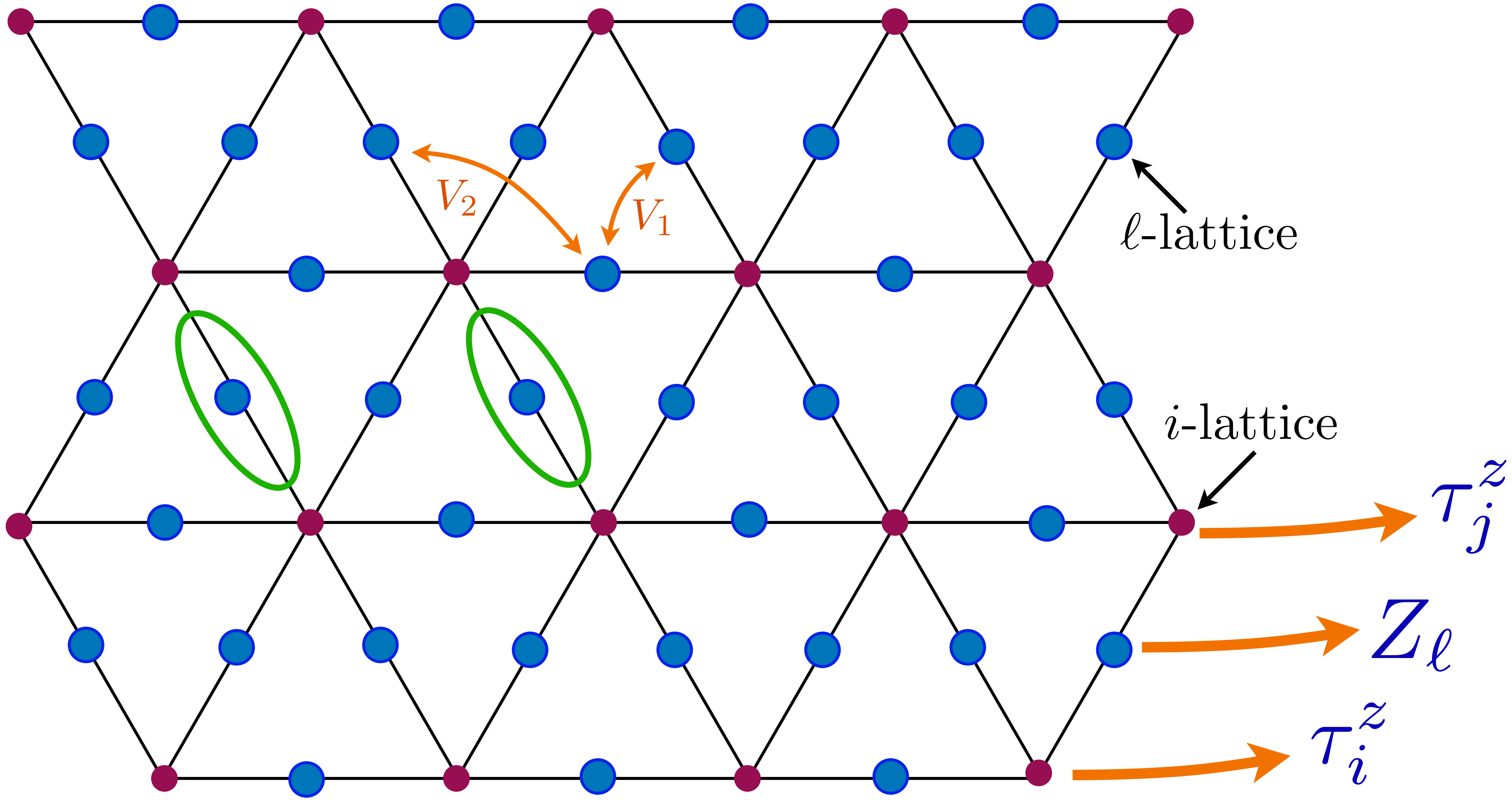

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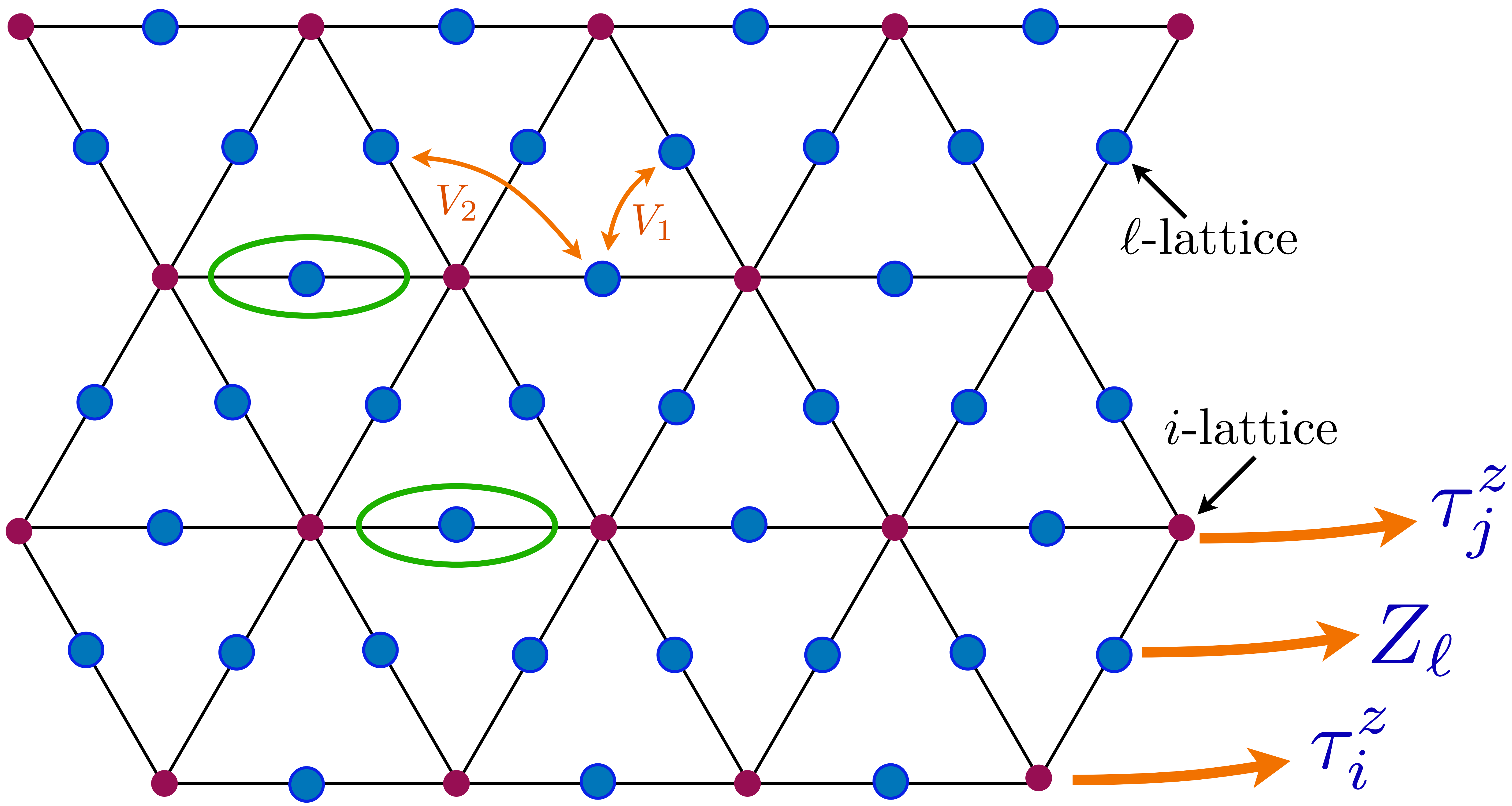

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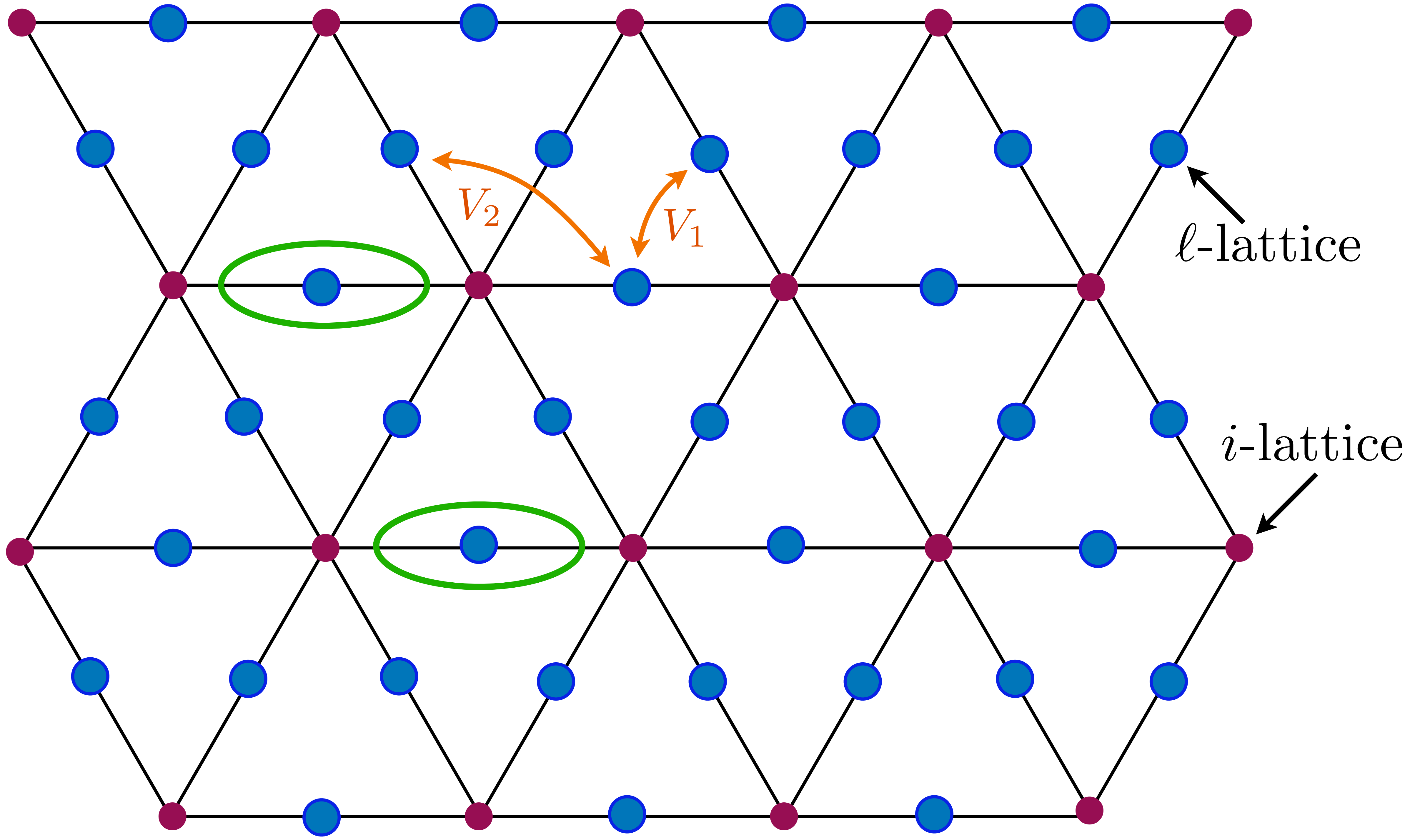

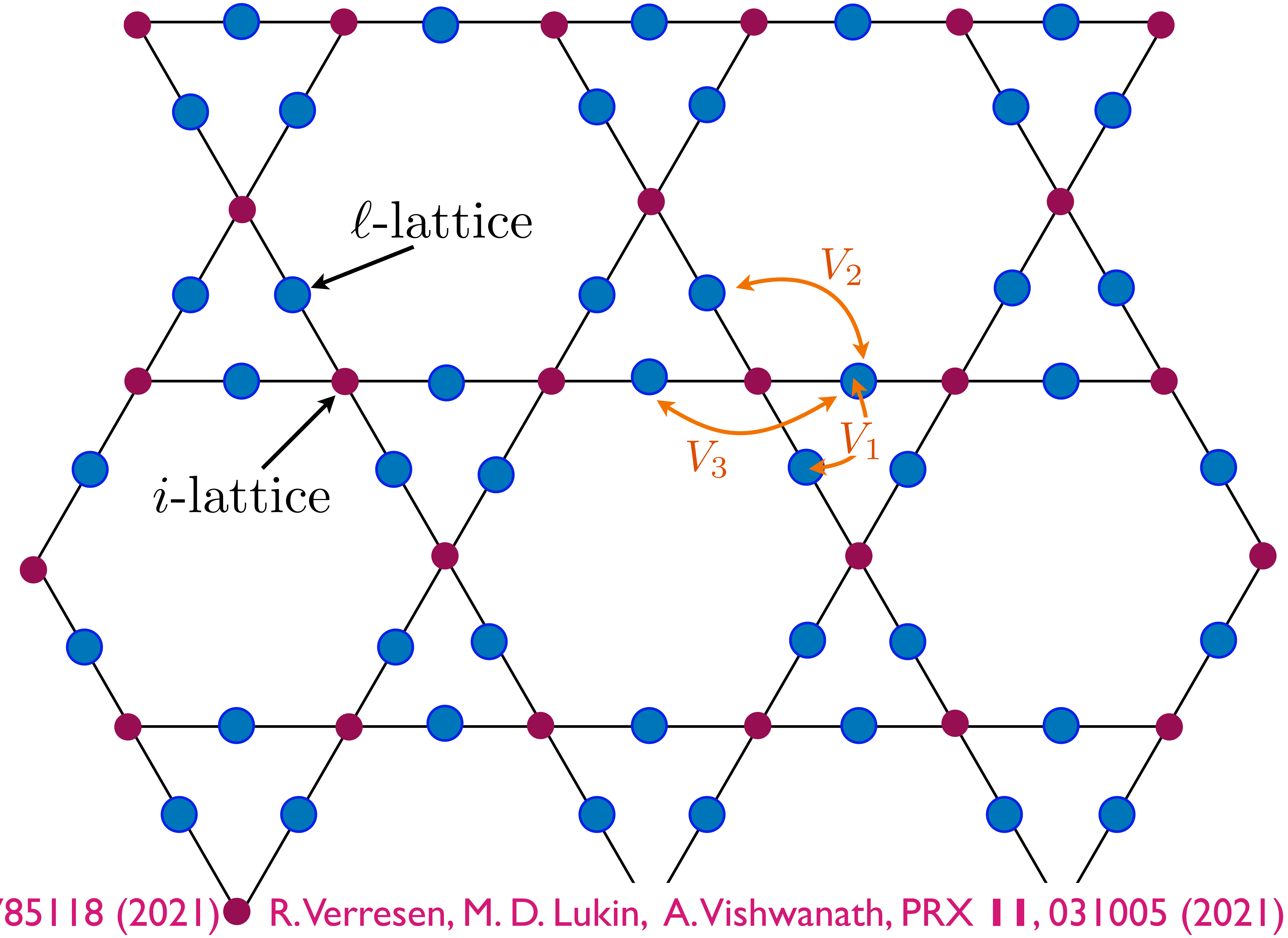

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021) R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

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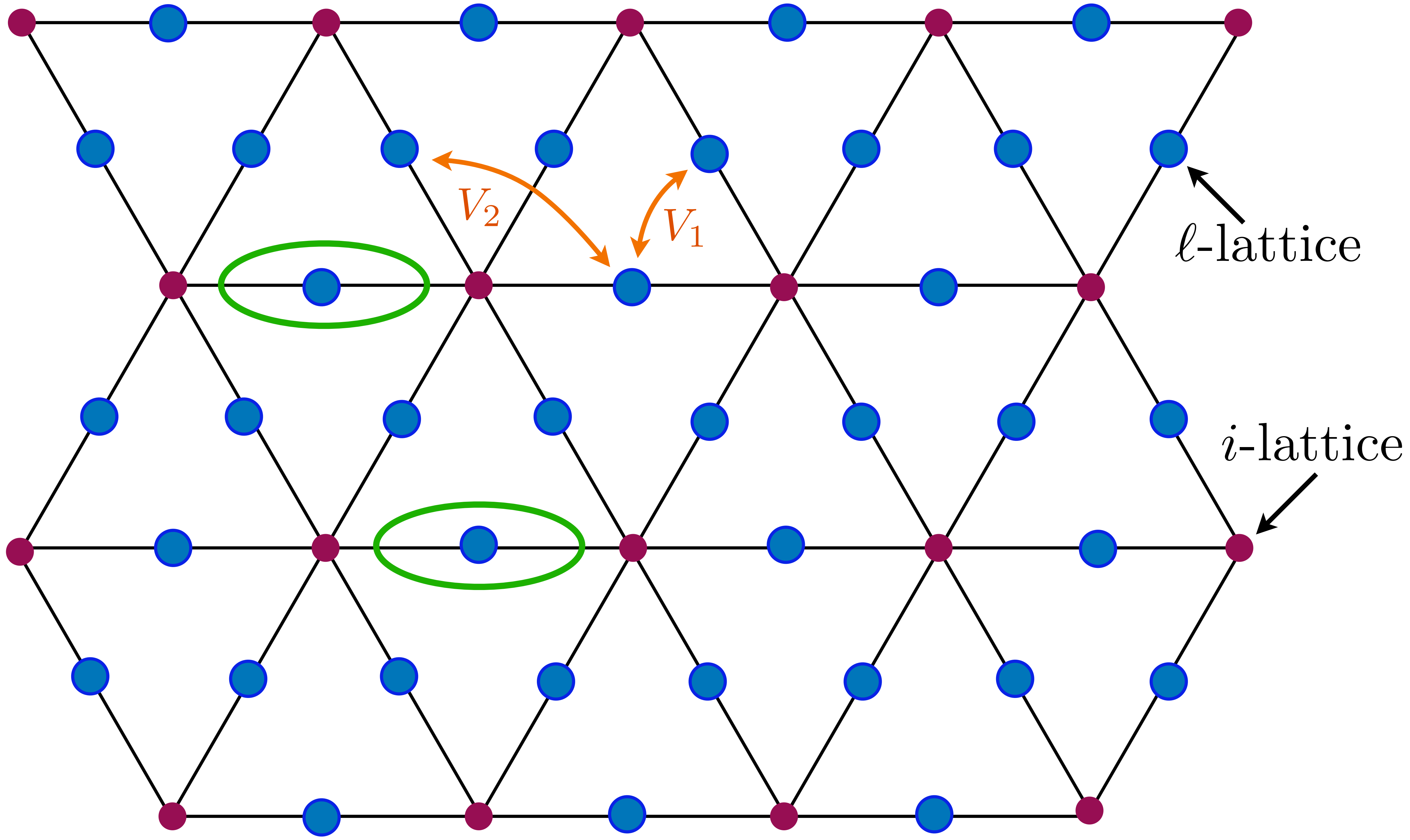

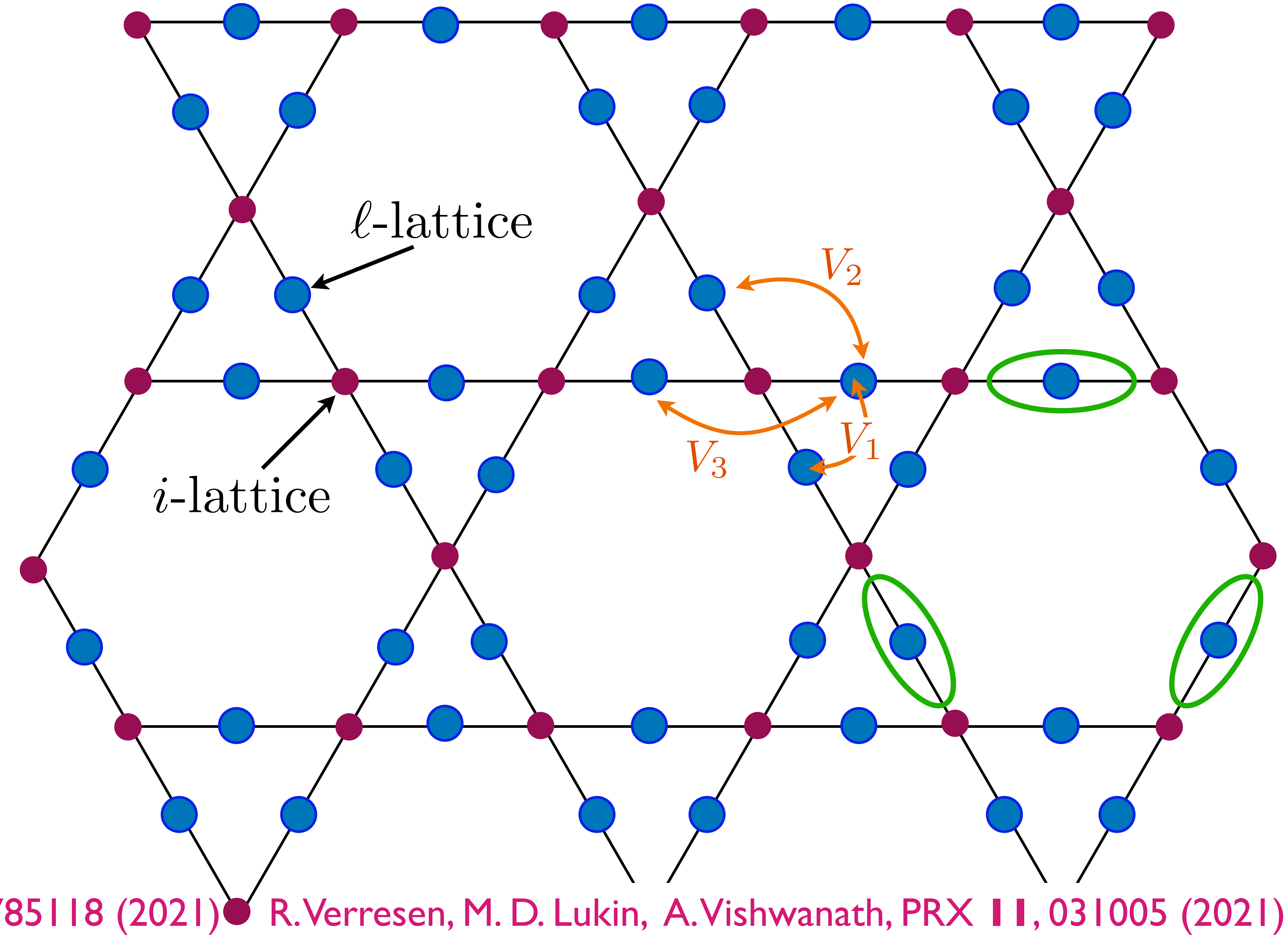

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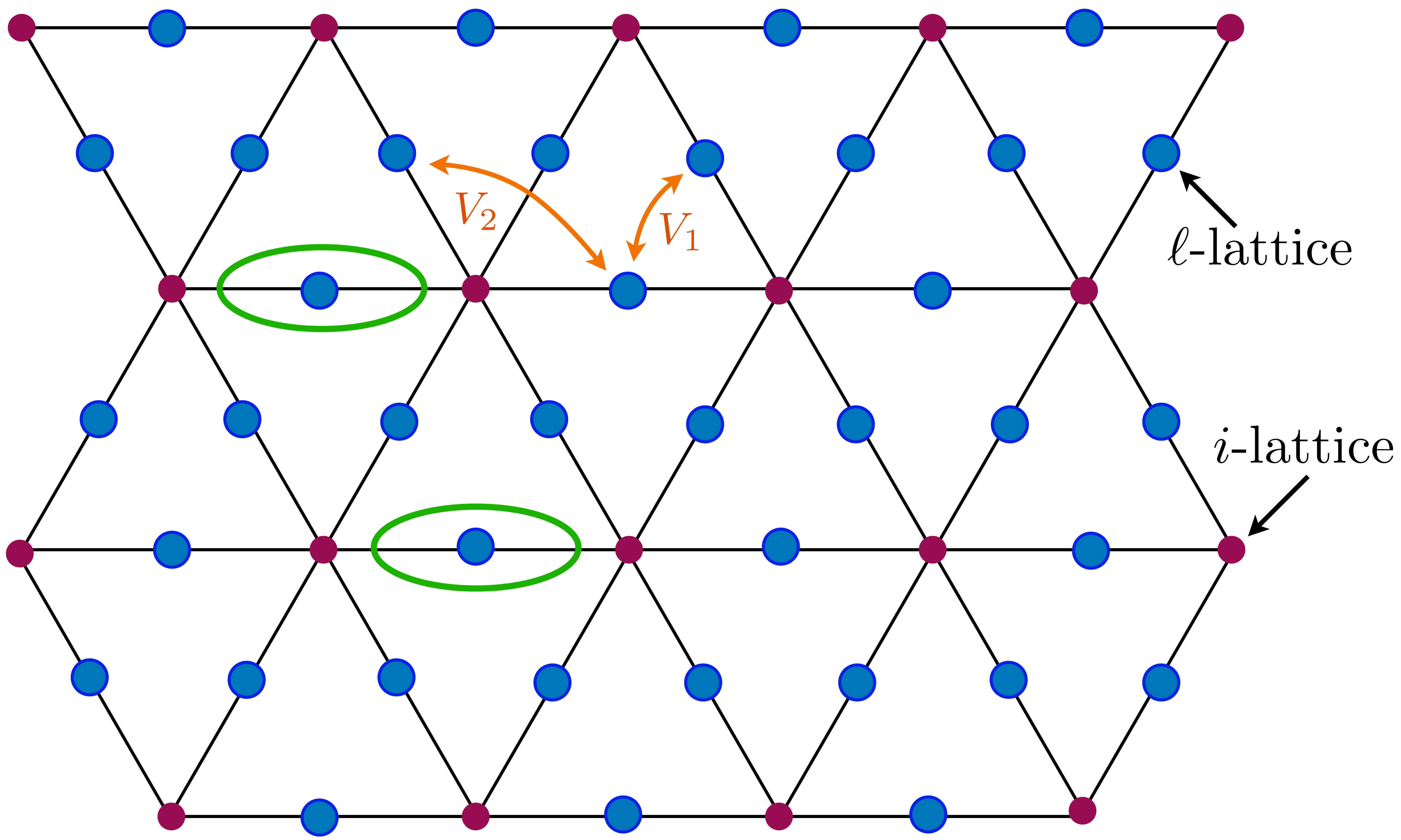

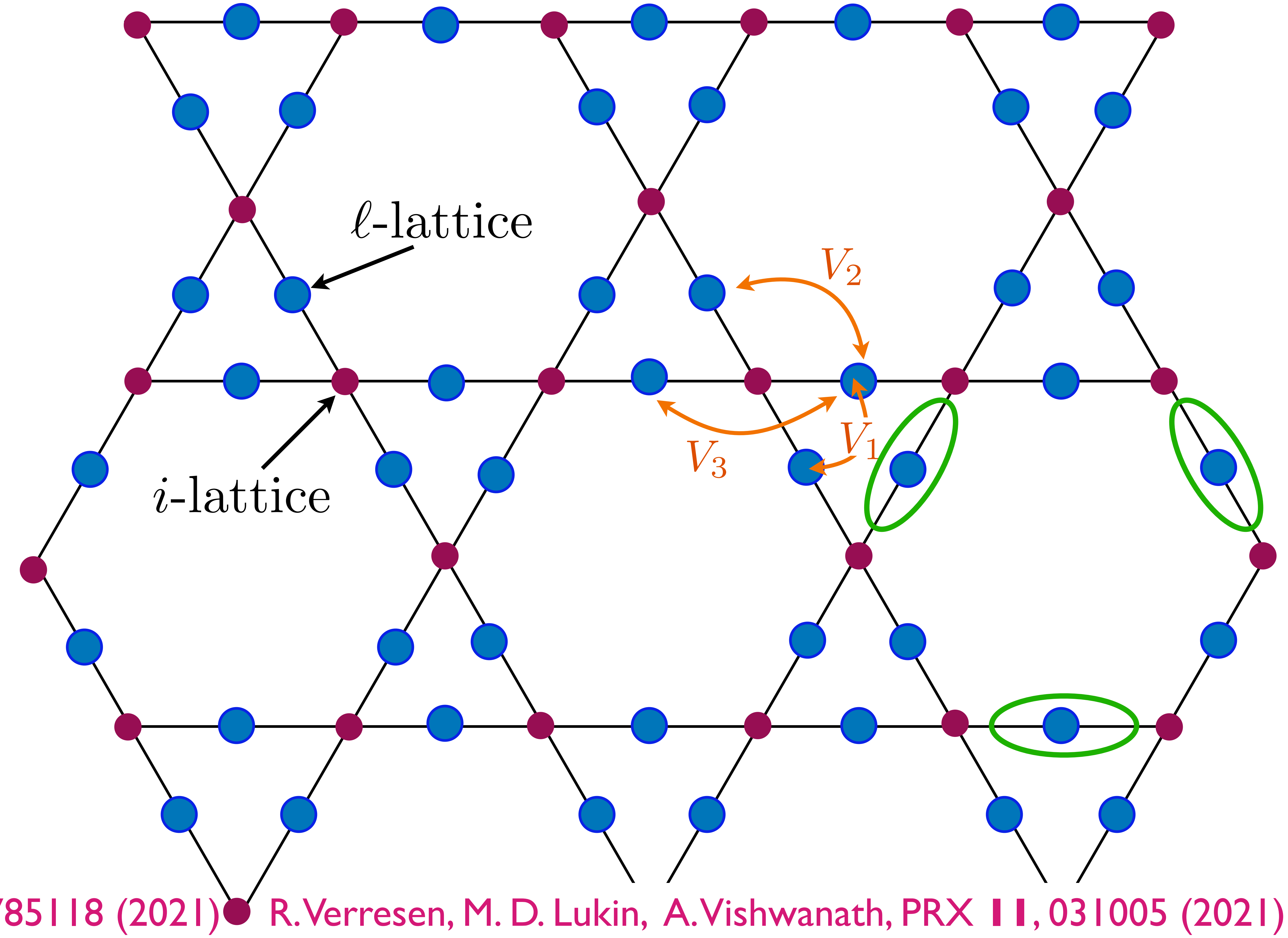

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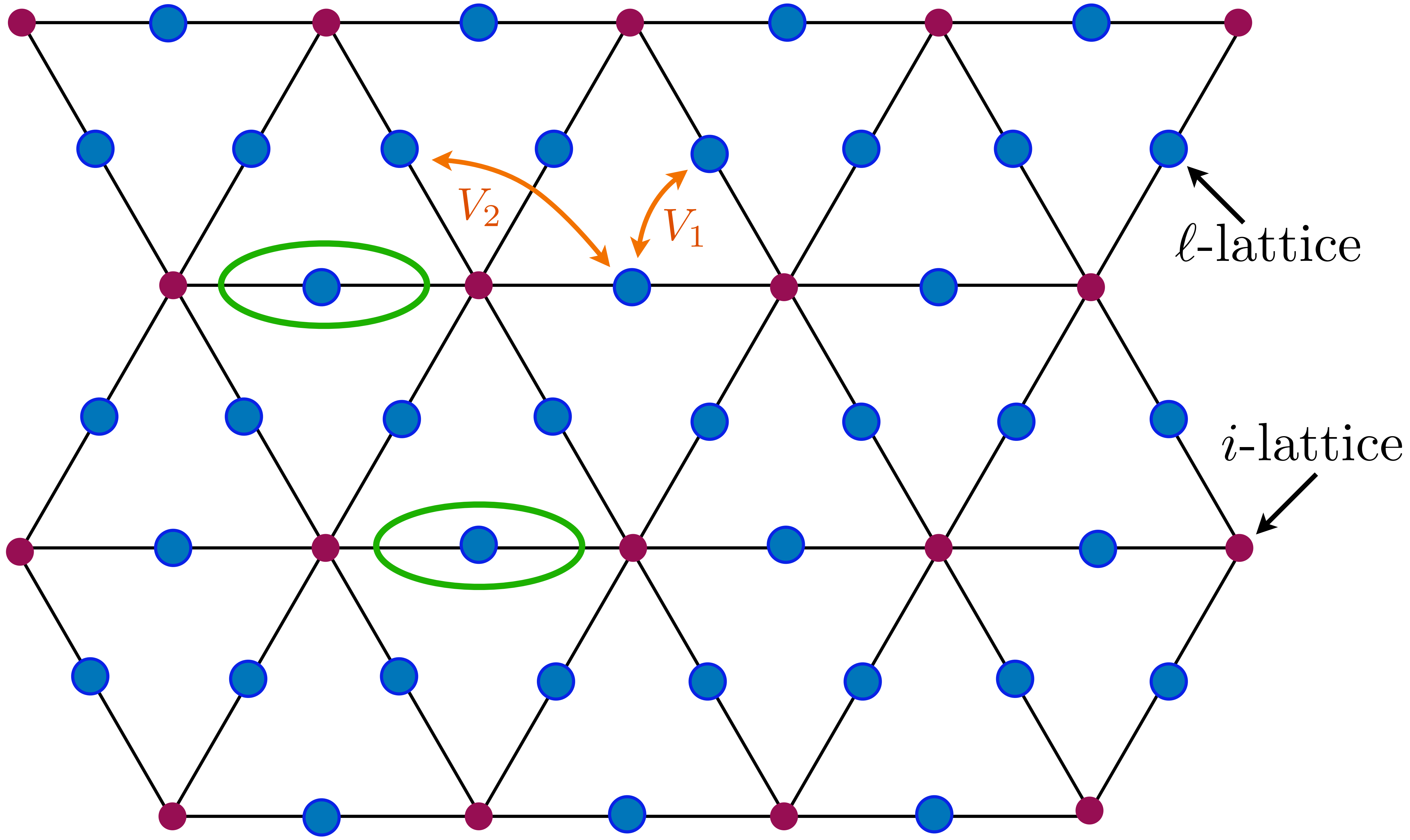

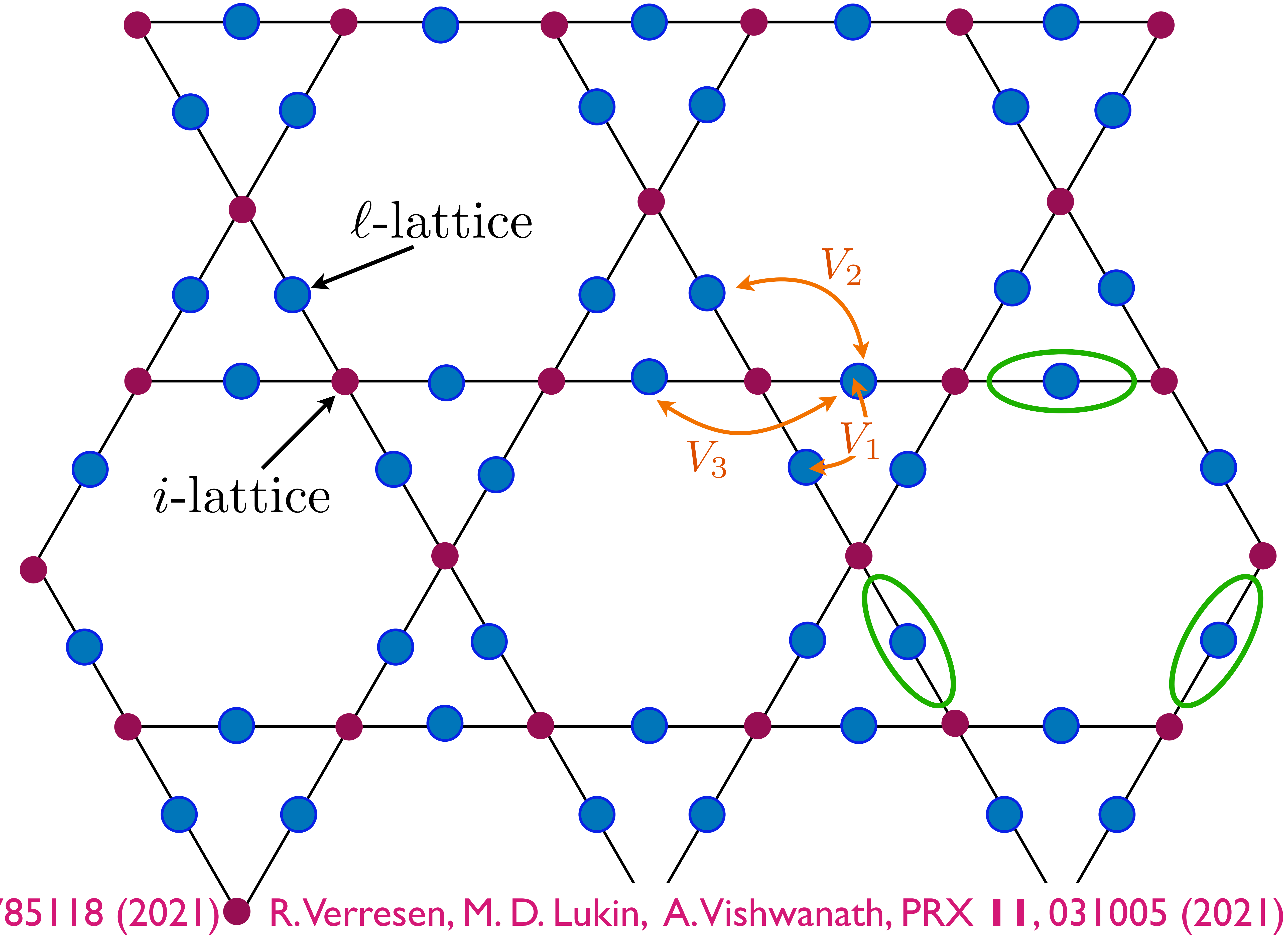

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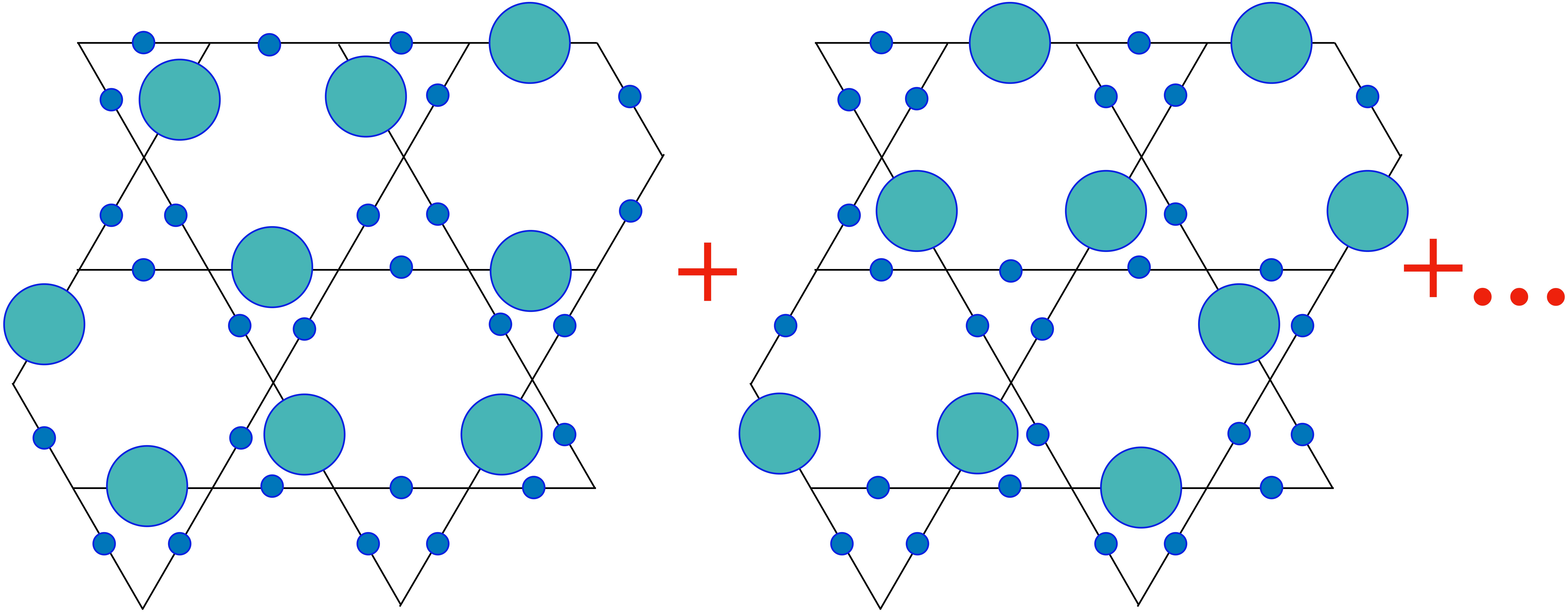
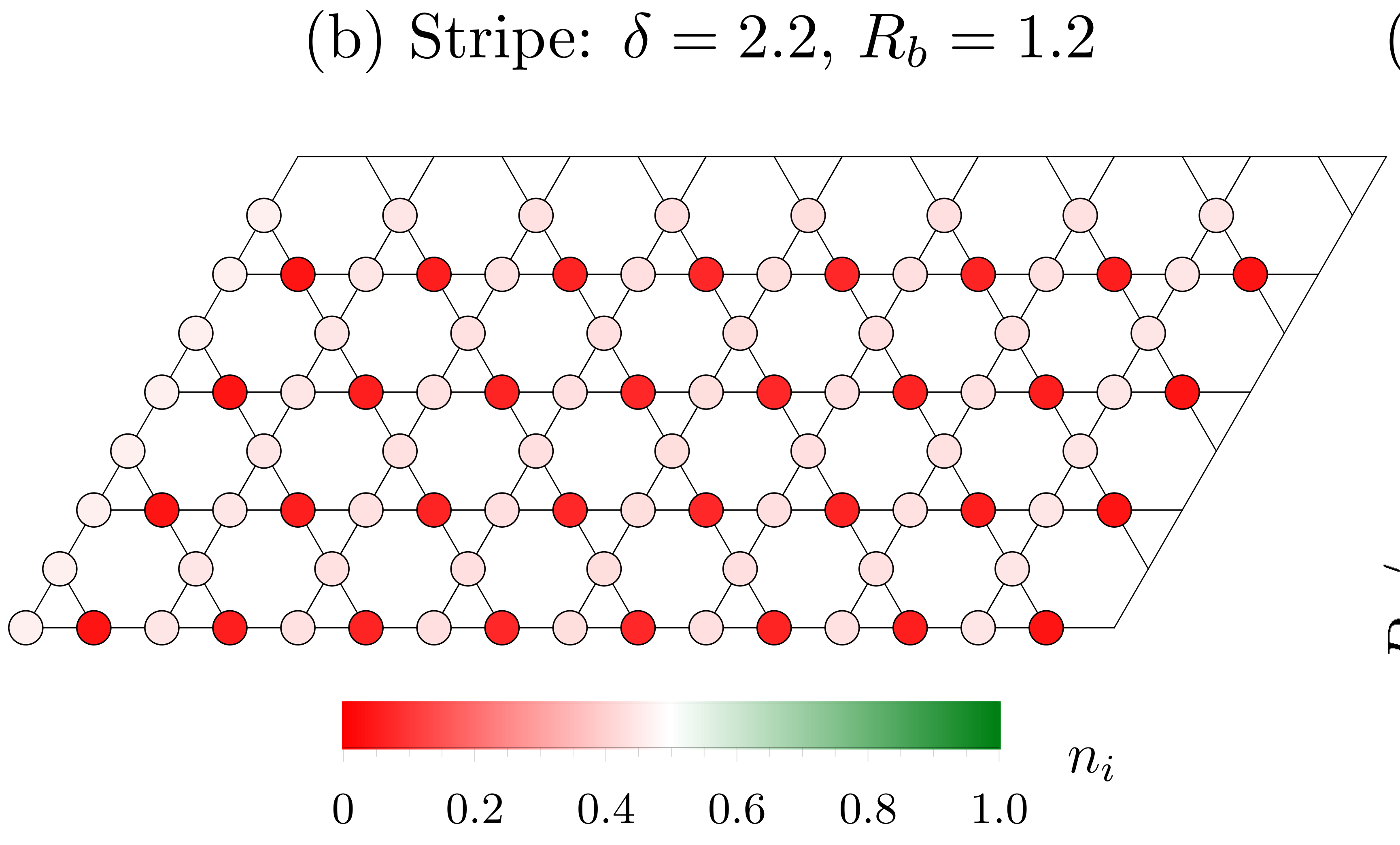
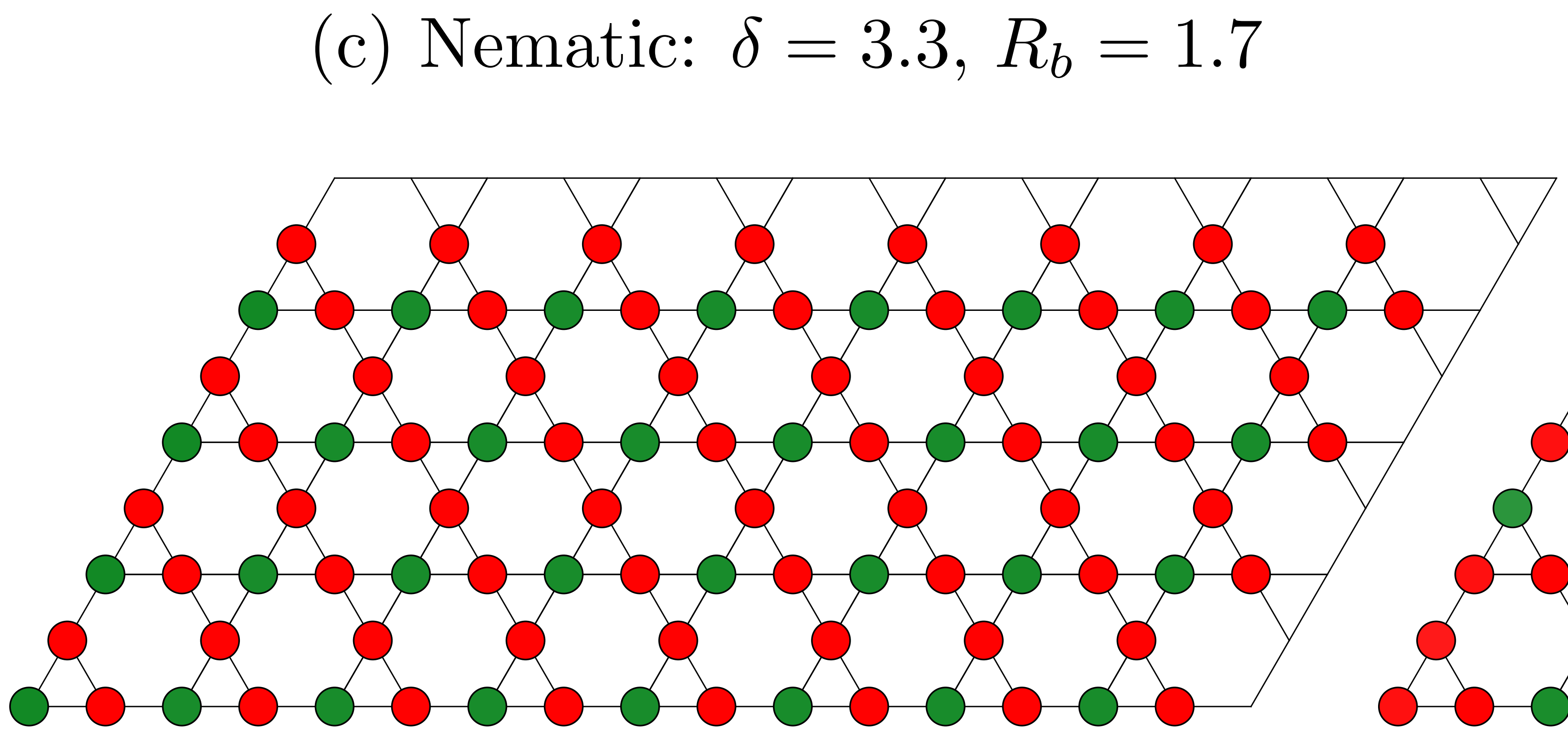
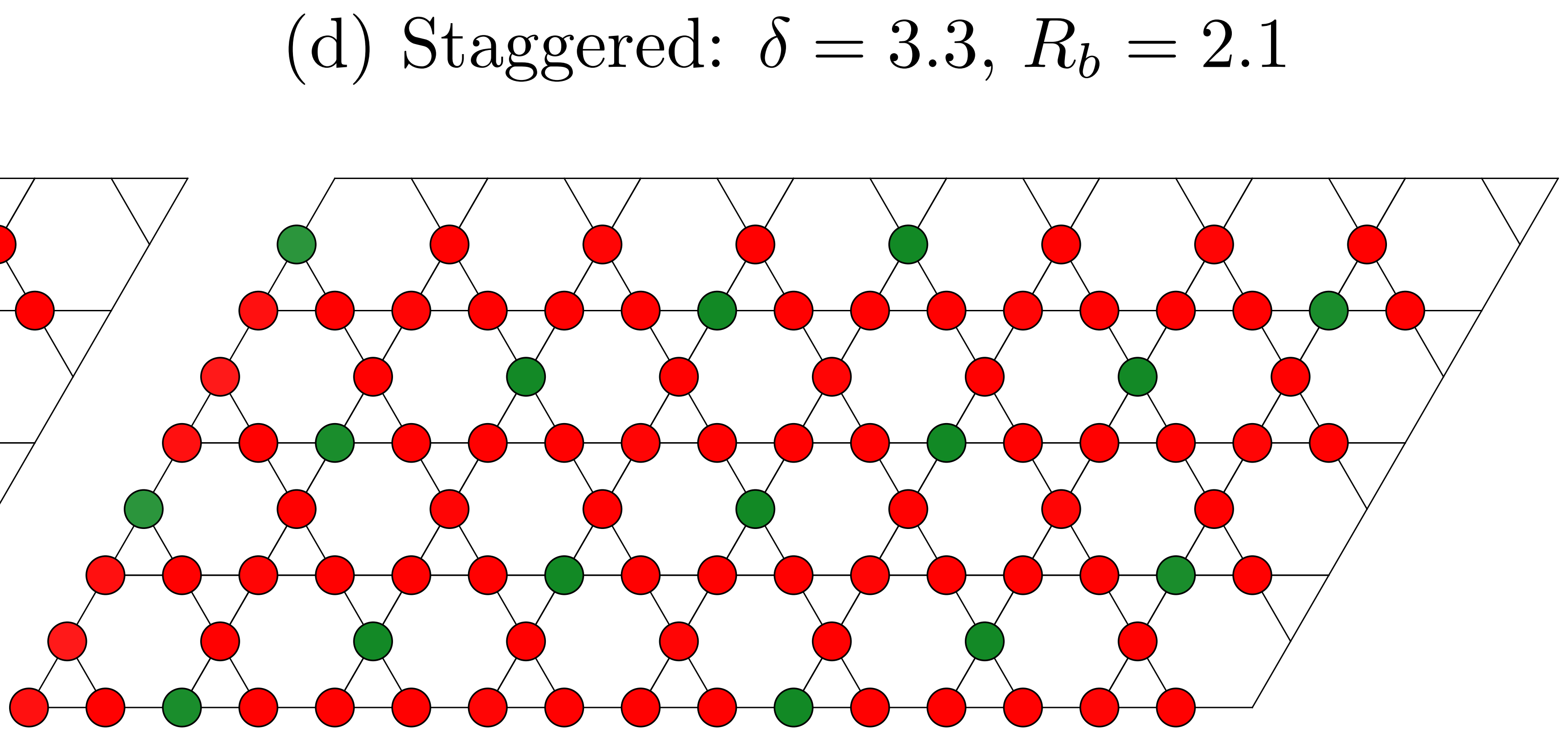
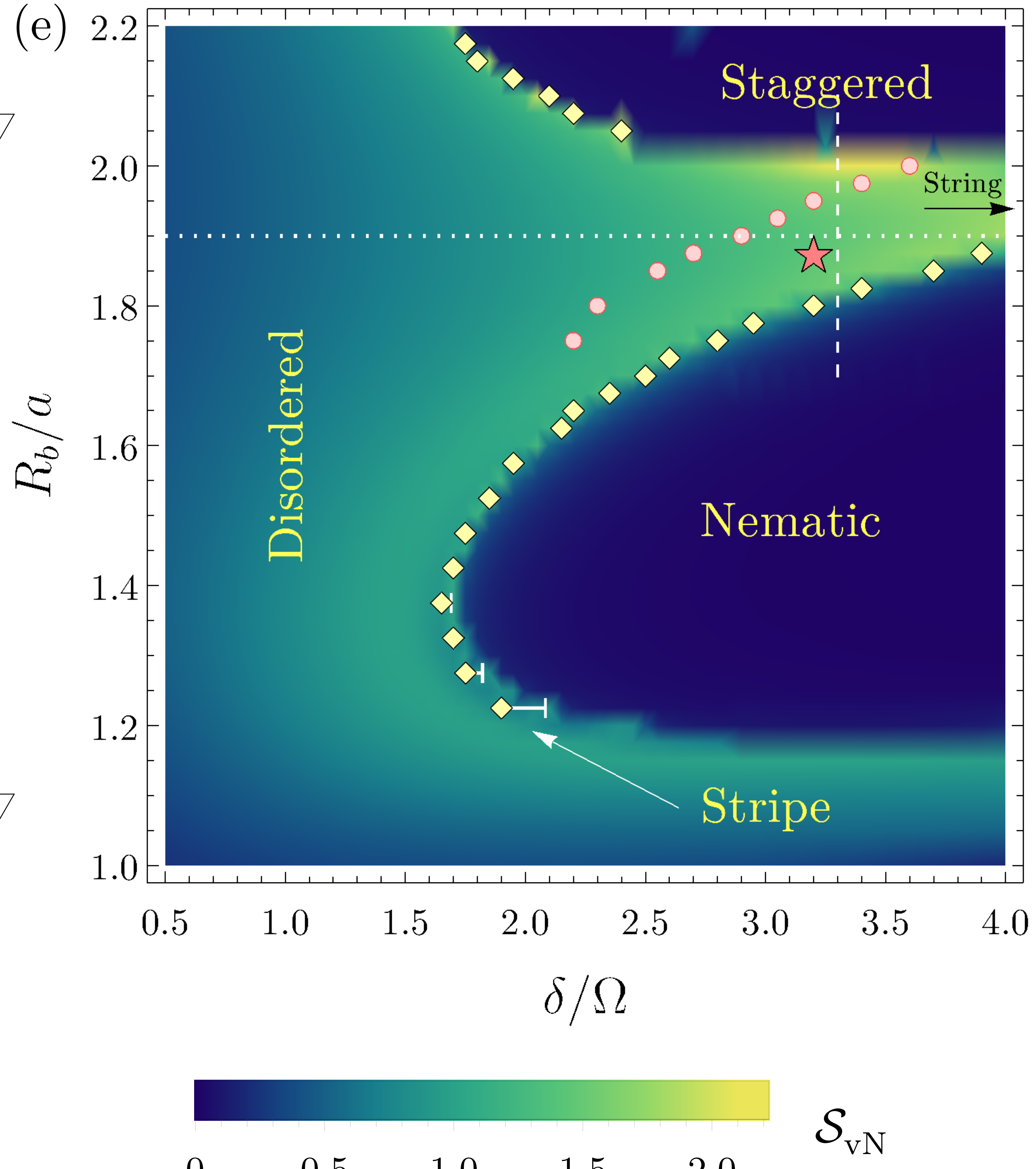


Rydberg atoms on site-kagome lattice: theory









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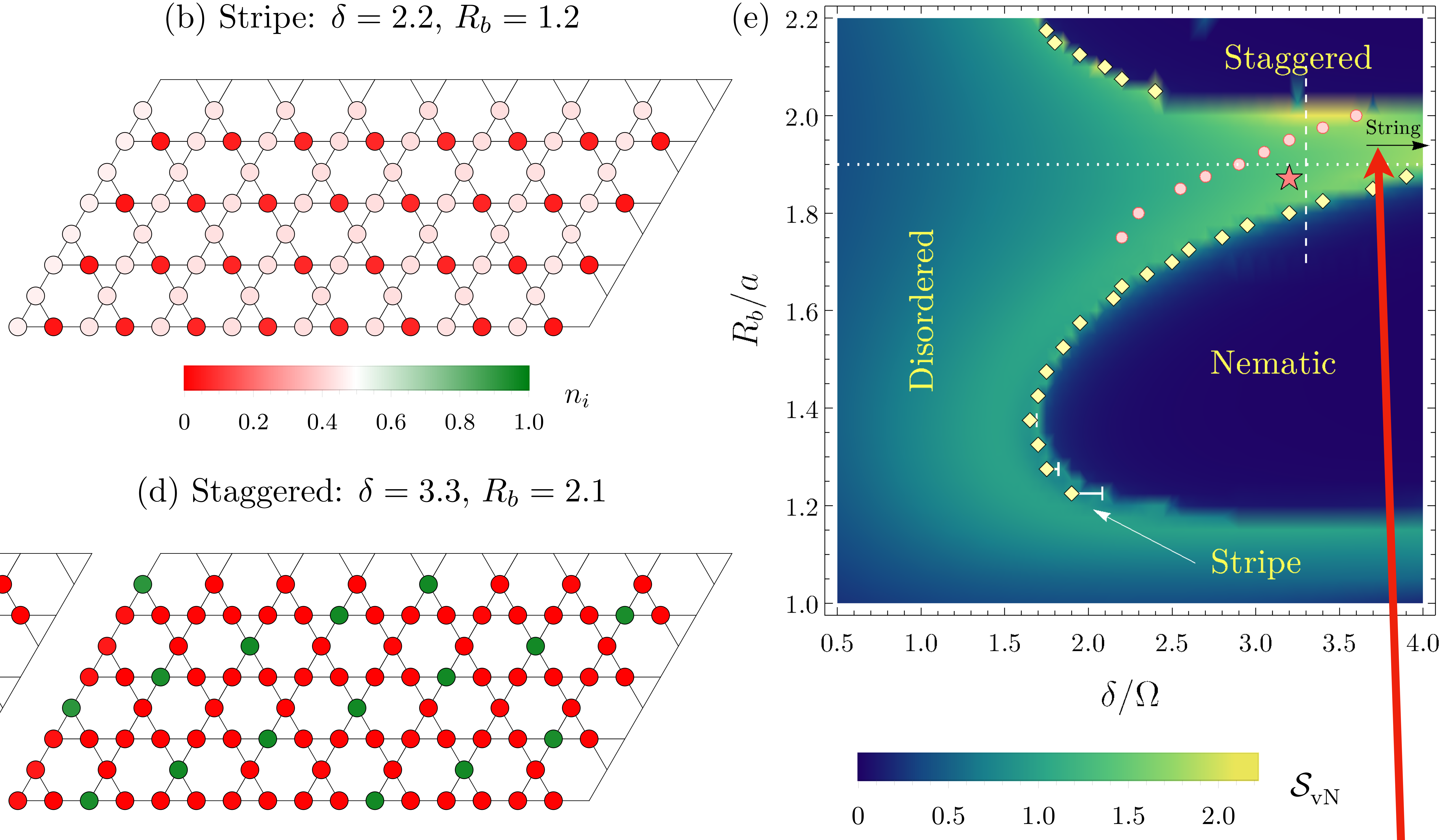
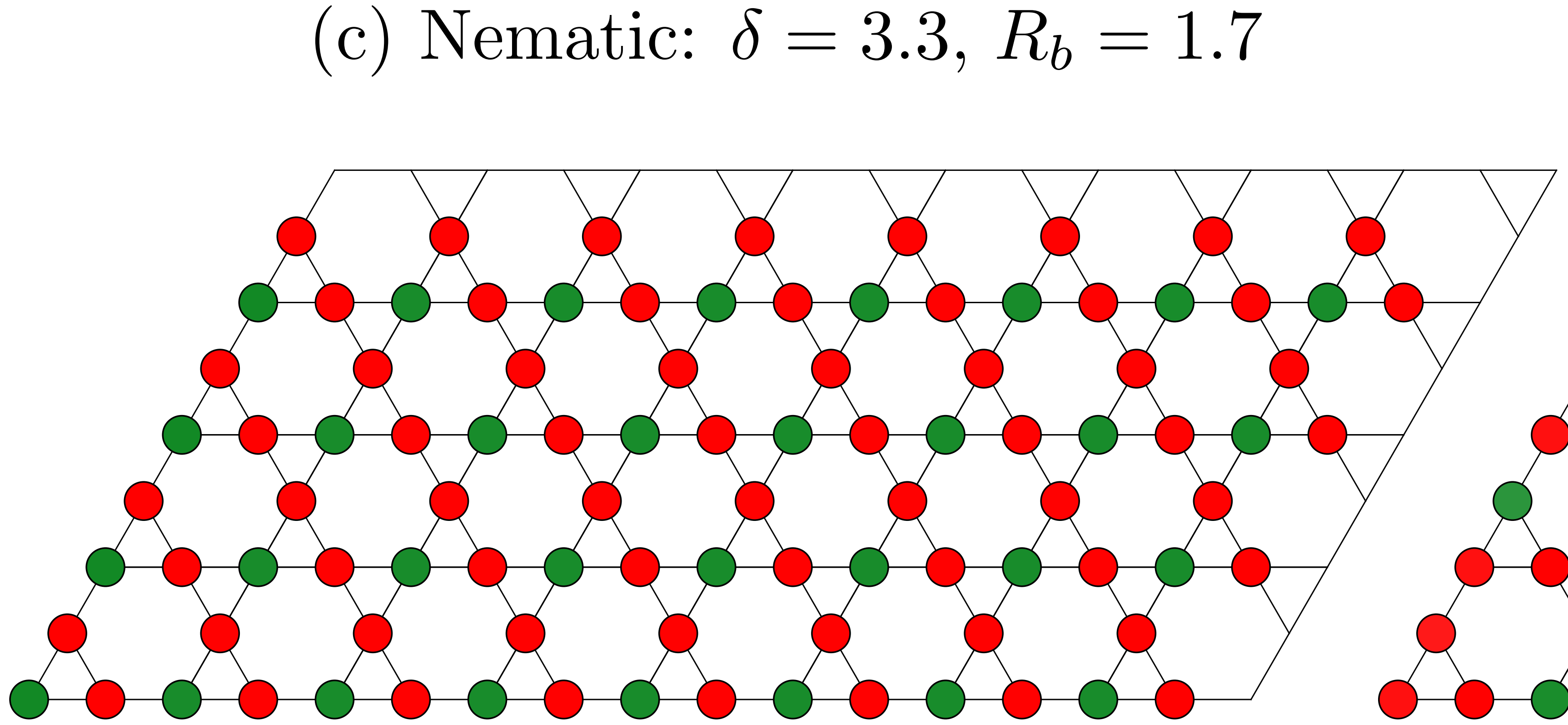
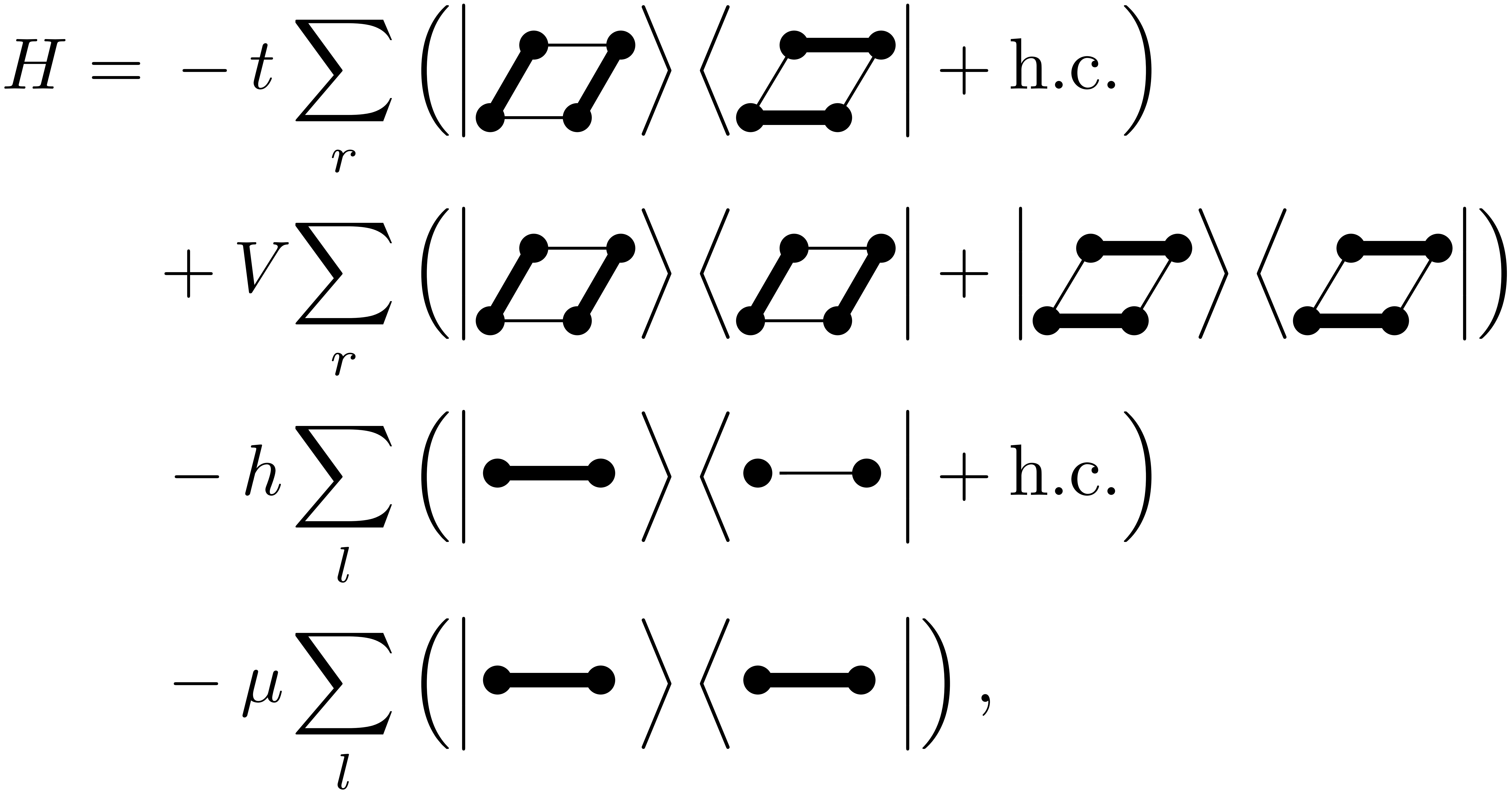




Topological spin liquid described by emergent $\mathbb{Z}_2$ gauge theory ?

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

Triangular lattice quantum dimer model with variable dimer density



Zheng Yan, R. Samajdar, Yan-Cheng Wang, S. Sachdev, and Zi Yang Meng, arXiv:2202.11100.

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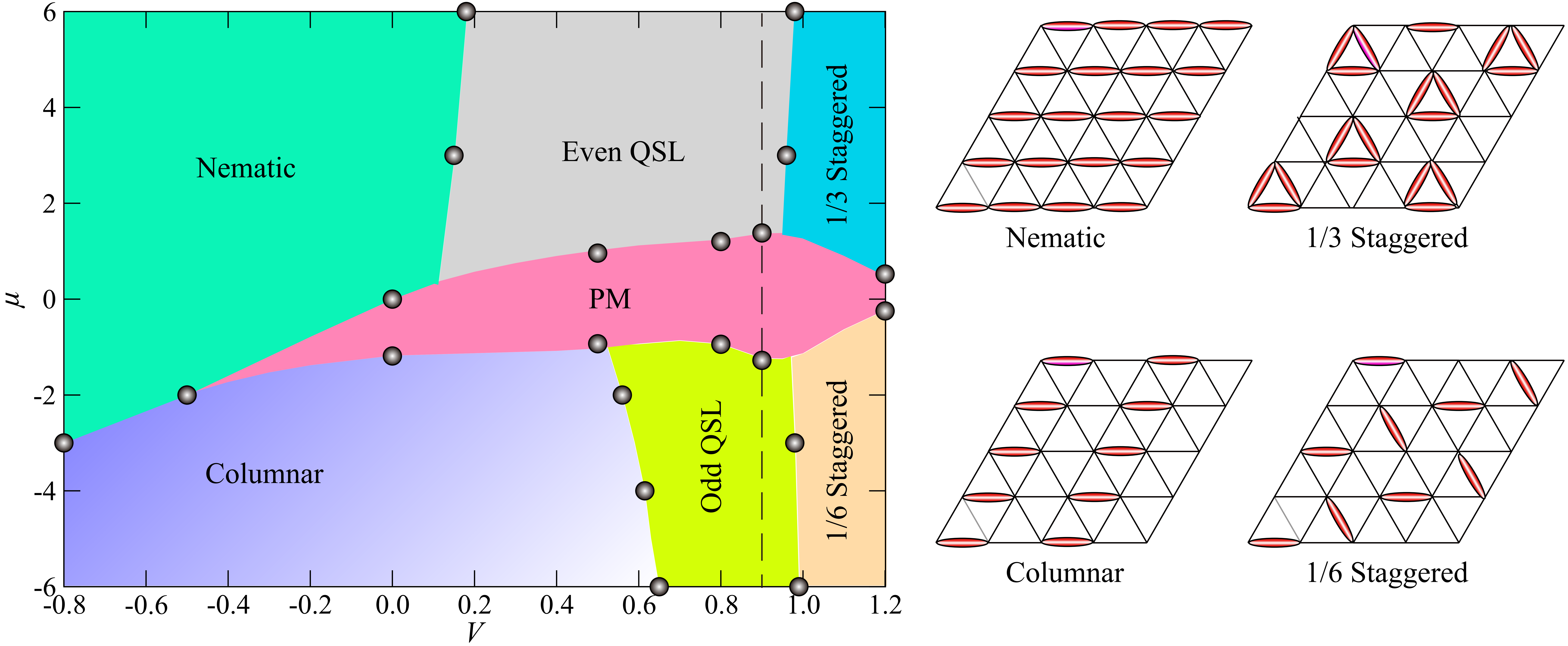

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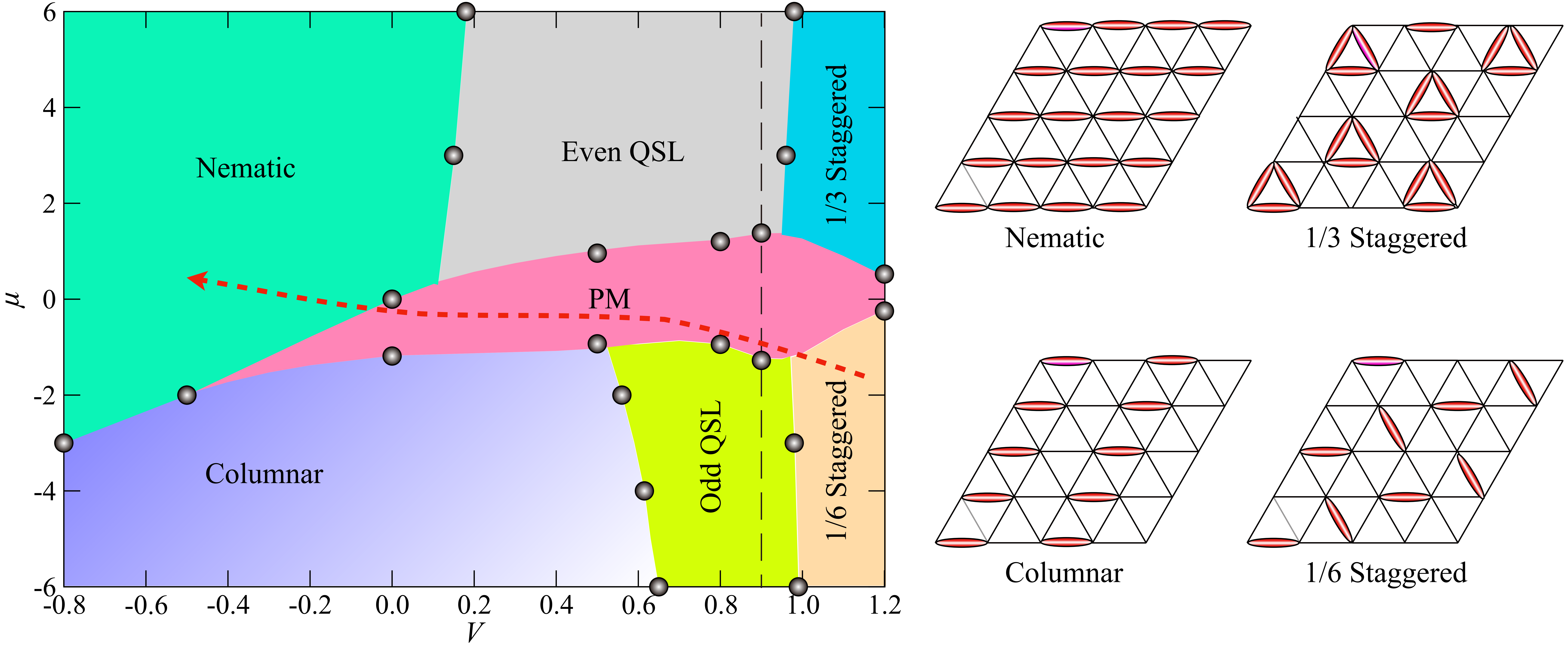

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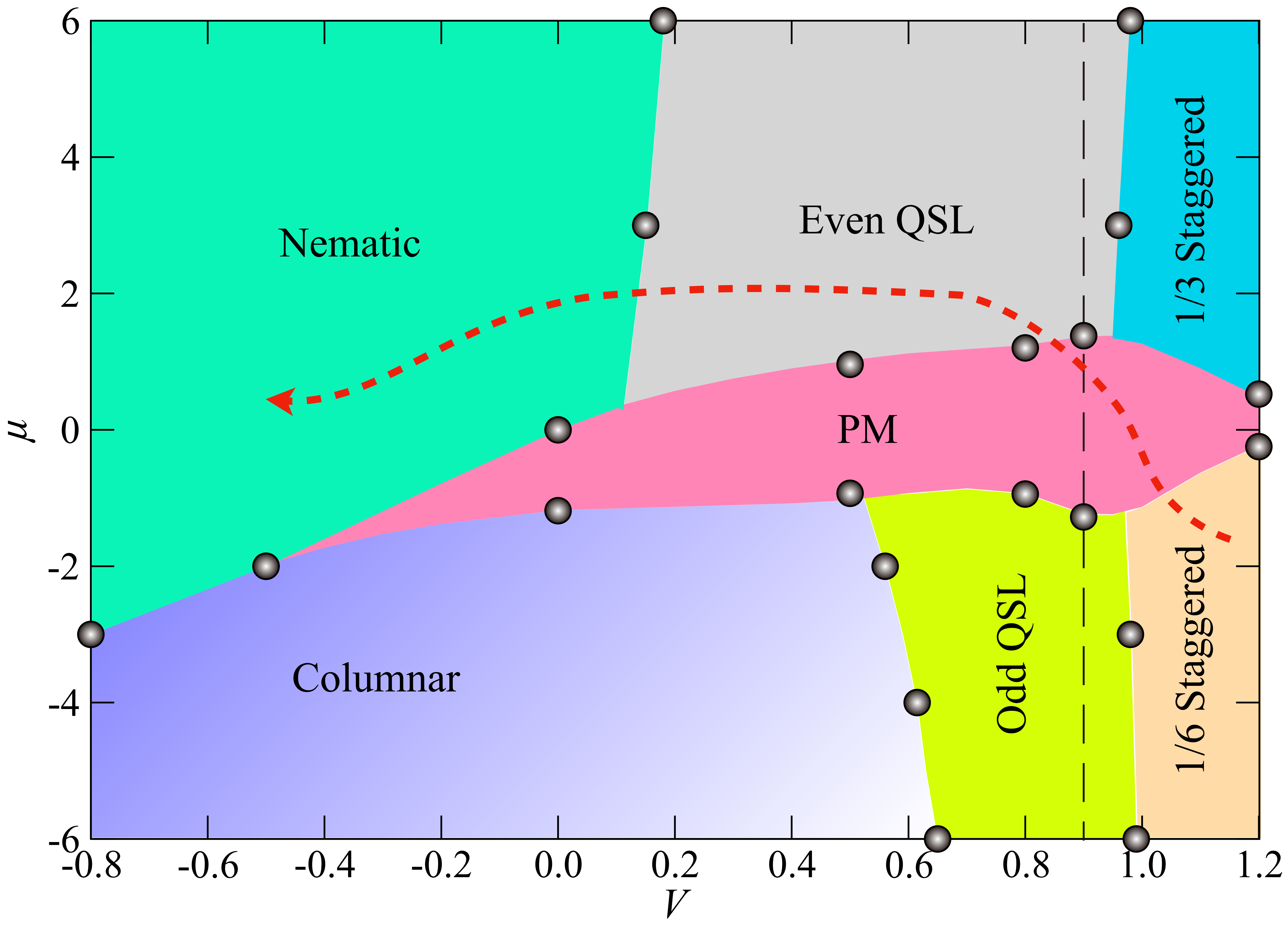

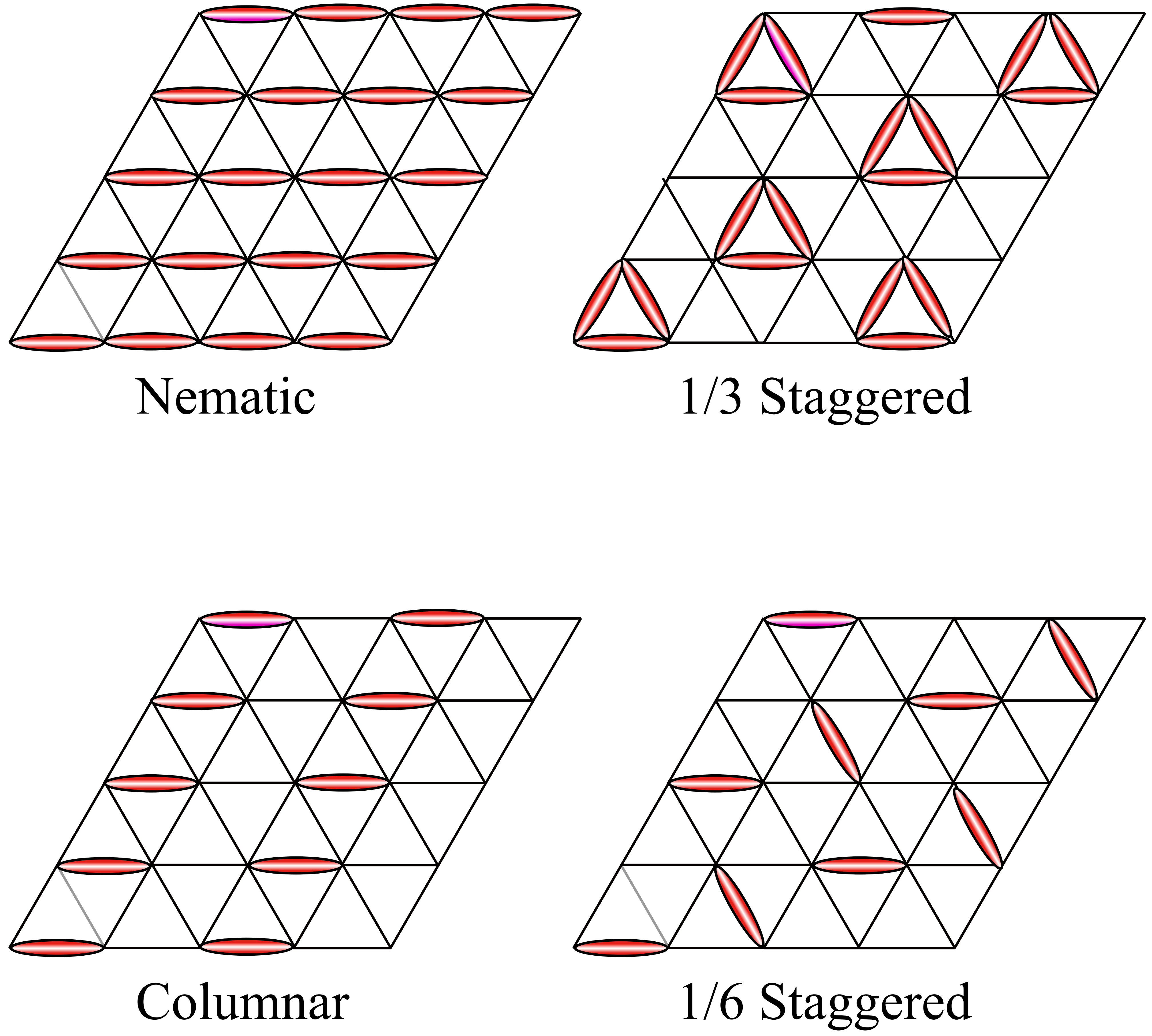

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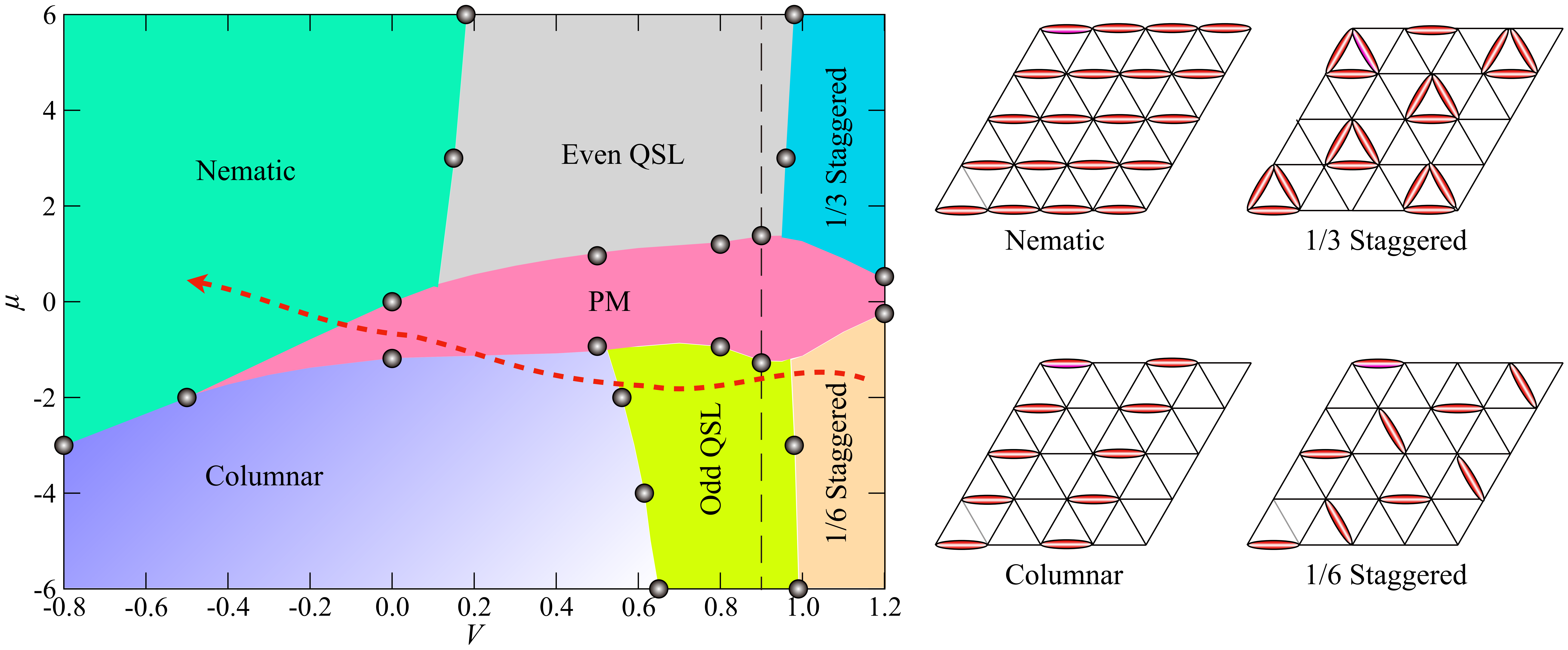

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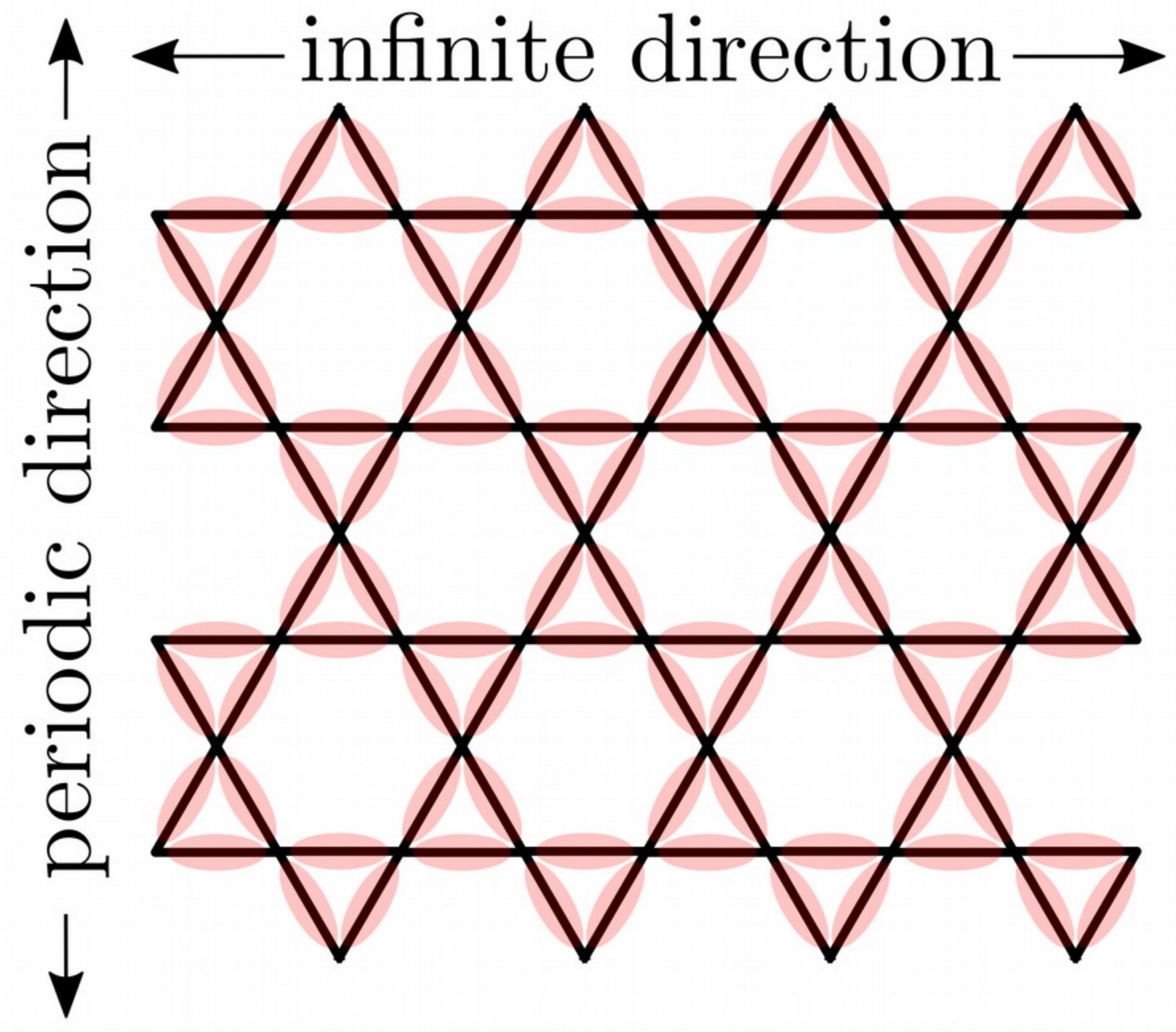
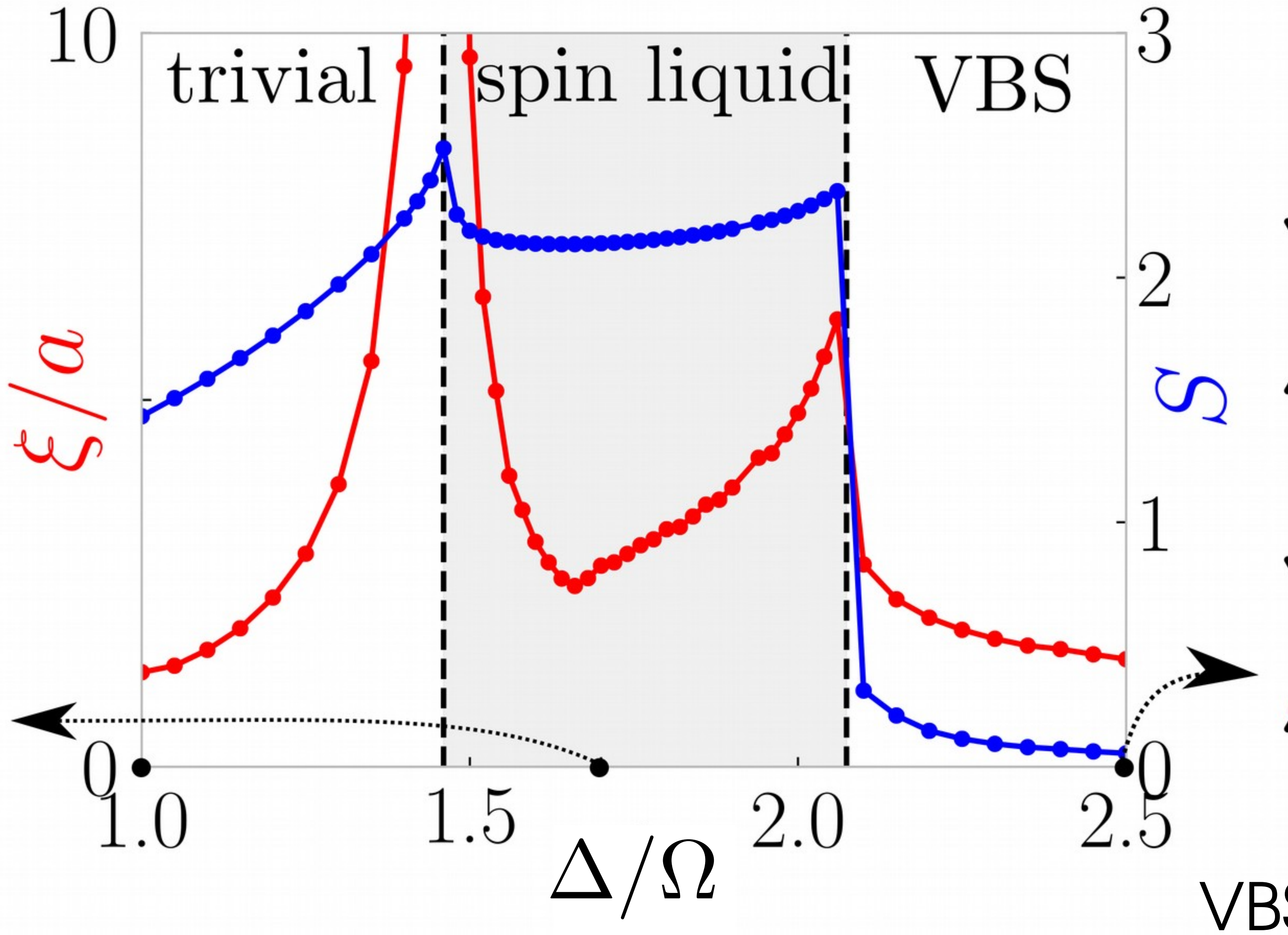
Rydberg atoms on link-kagome lattice: theory

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2}\left(b_j + b_j^\dagger\right) - \Delta n_j\right] + \sum_{i<j} V_{|i-j|} n_i n_j \,, \quad n_j \equiv b_j^\dagger b_j = 0, 1.$$

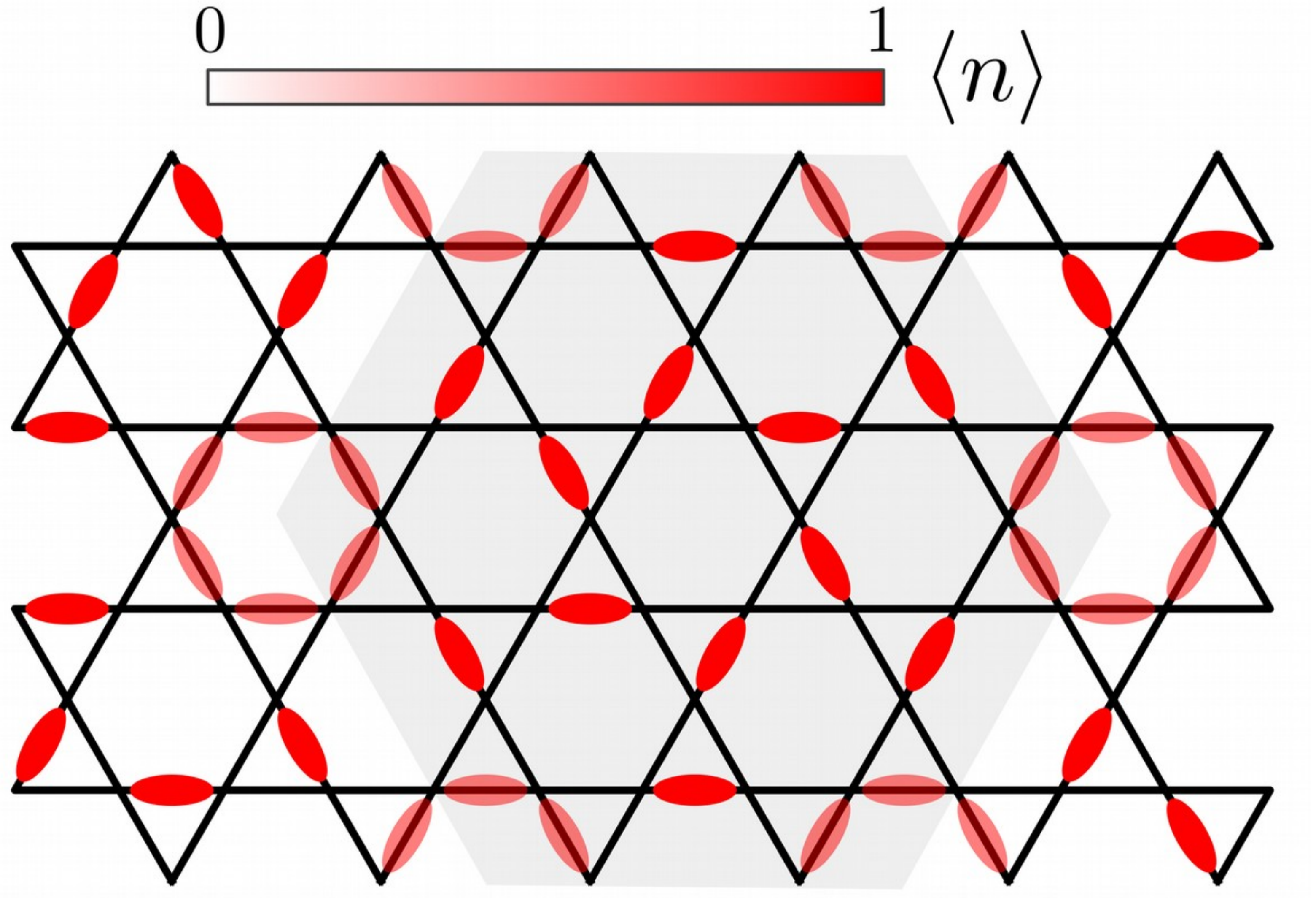
The sites j are on the links of the kagome lattice.
Examine the PXP model, $V_{\text{nearest neighbor}} = \infty$, other $V_k = 0$.



intermediate featureless phase



DMRG



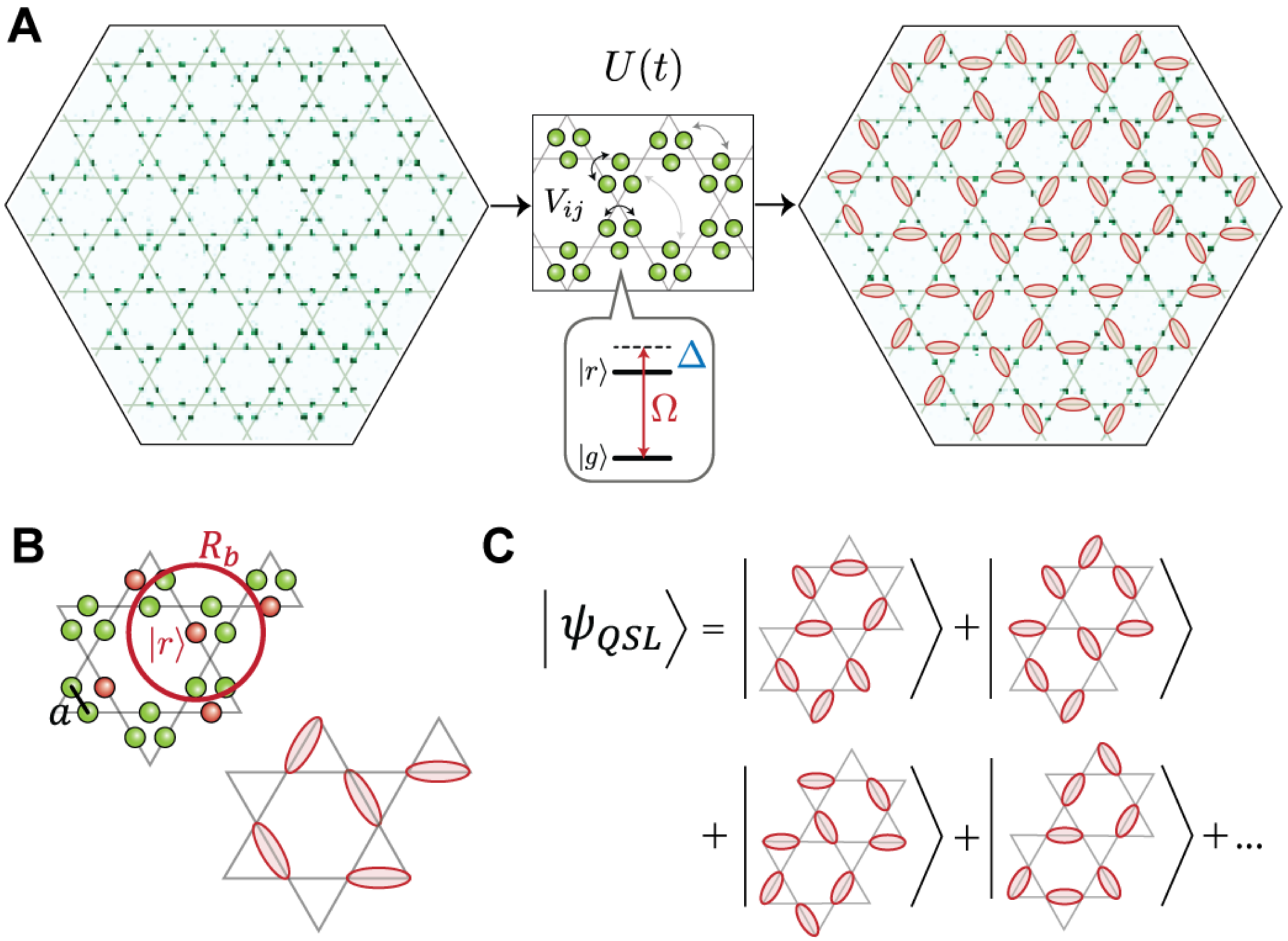
VBS pattern studied before in context of Heisenberg model on kagome lattice (Nikolic and Senthil, Singh and Huse, ...)

R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

Rydberg atoms
on the
link-kagome lattice:
experiment



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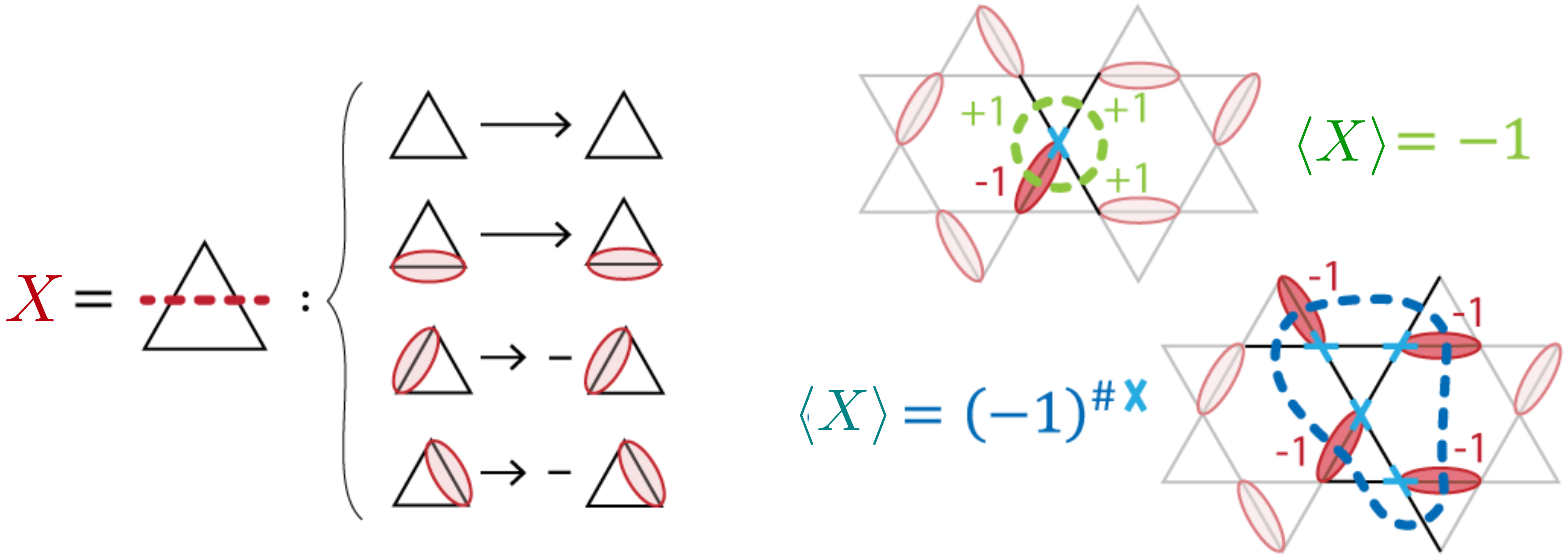
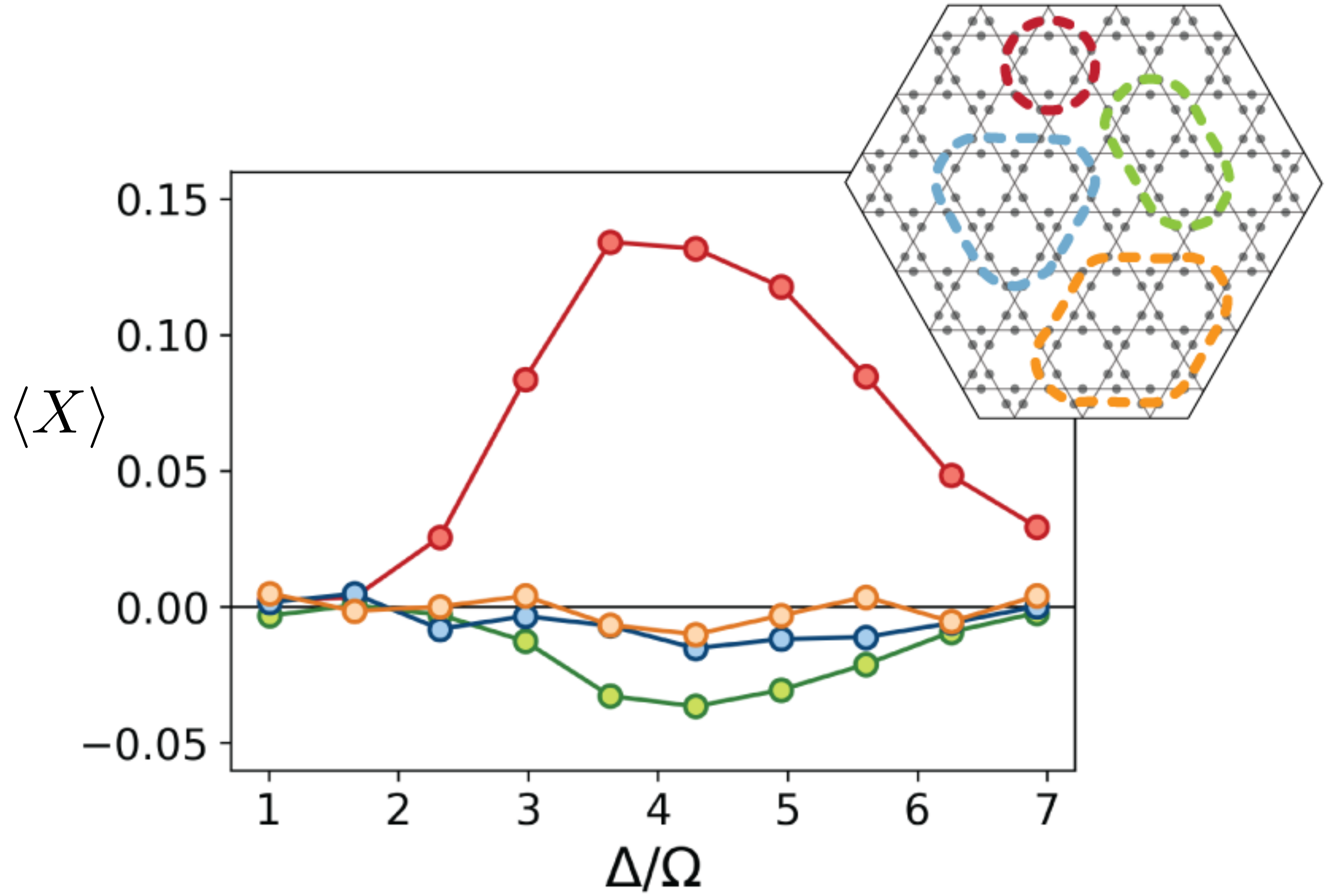


Measurement of
the topological
X operator
$= \prod_{\text{loop}} X_\ell$.
Detects close-packed dimers.



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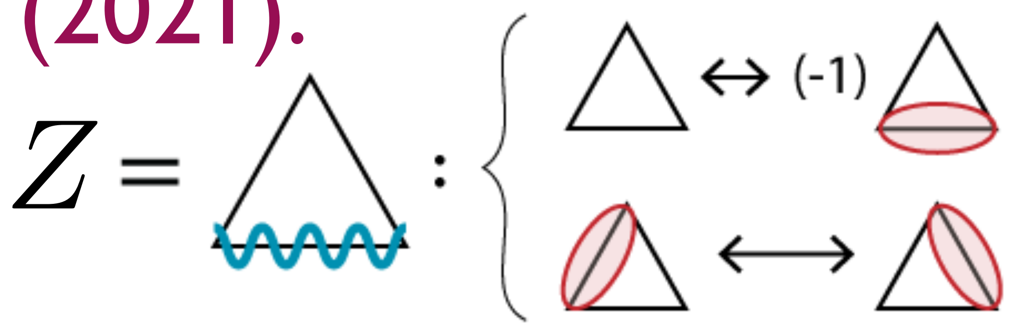

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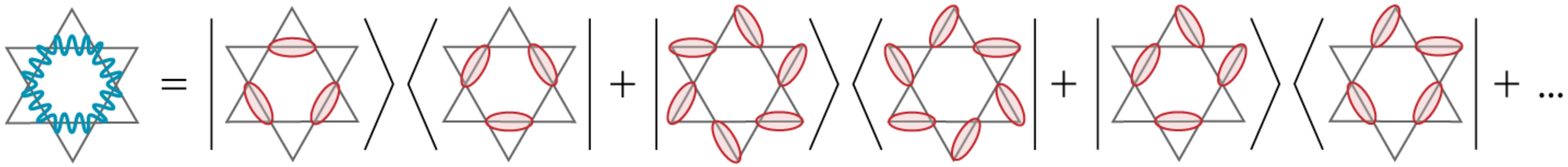
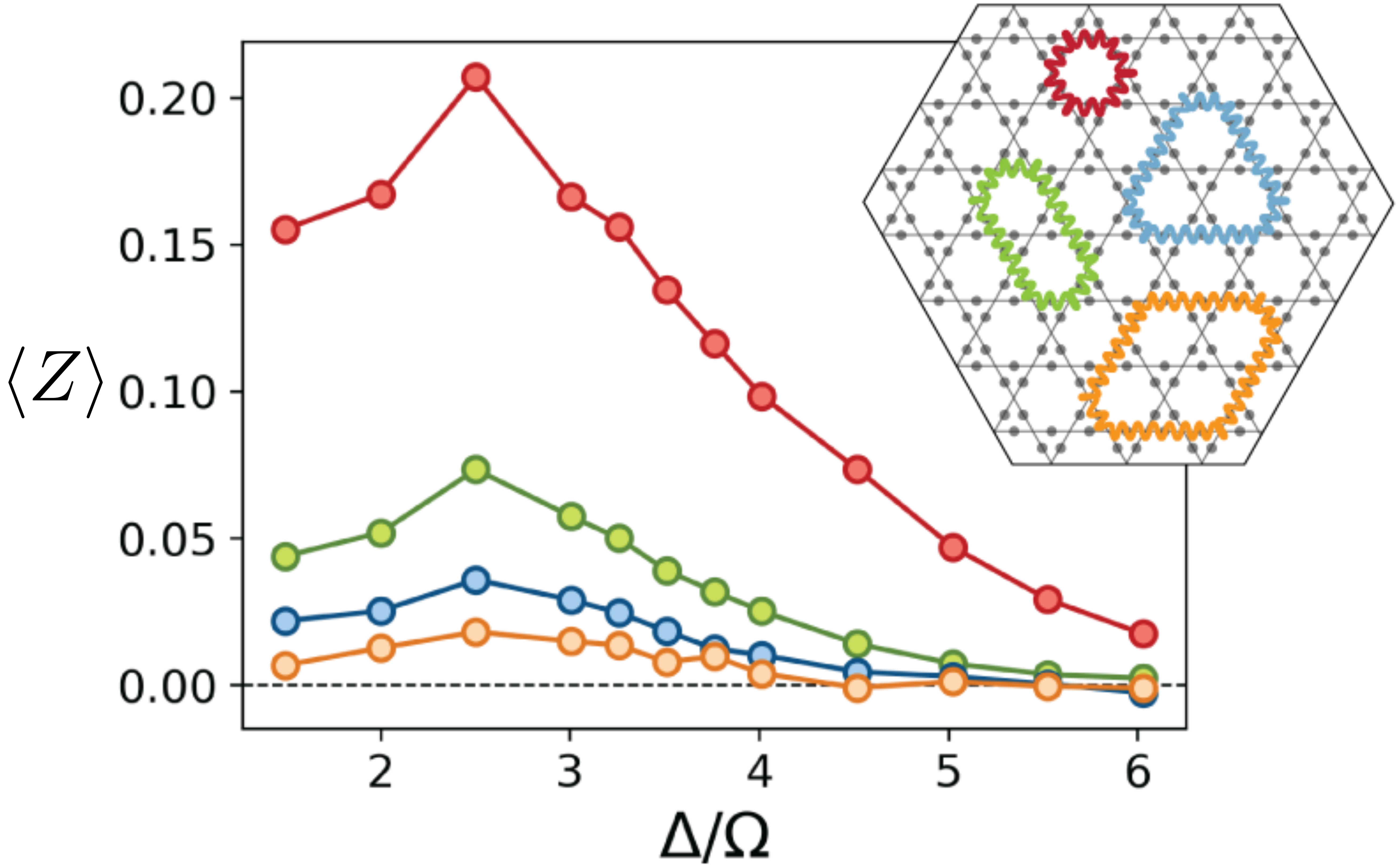
link-kagome lattice:
experiment



Measurement of
the topological
Z operator.
Detects resonance
between dimer loops.



1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

Probing topological spin liquids

3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

Yahui Zhang

Alexander Nikolaenko

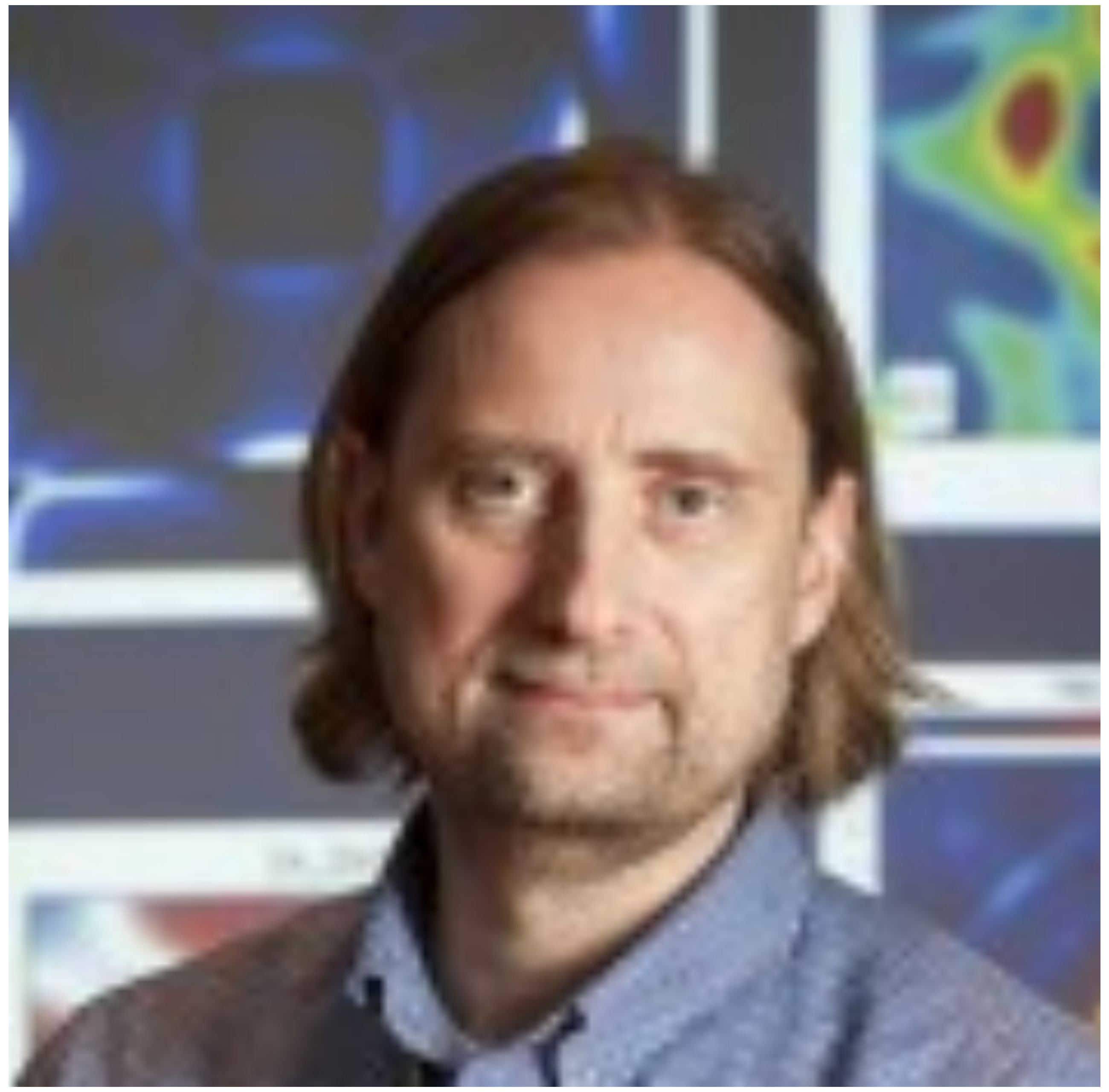
Maria Tikhanovskaya

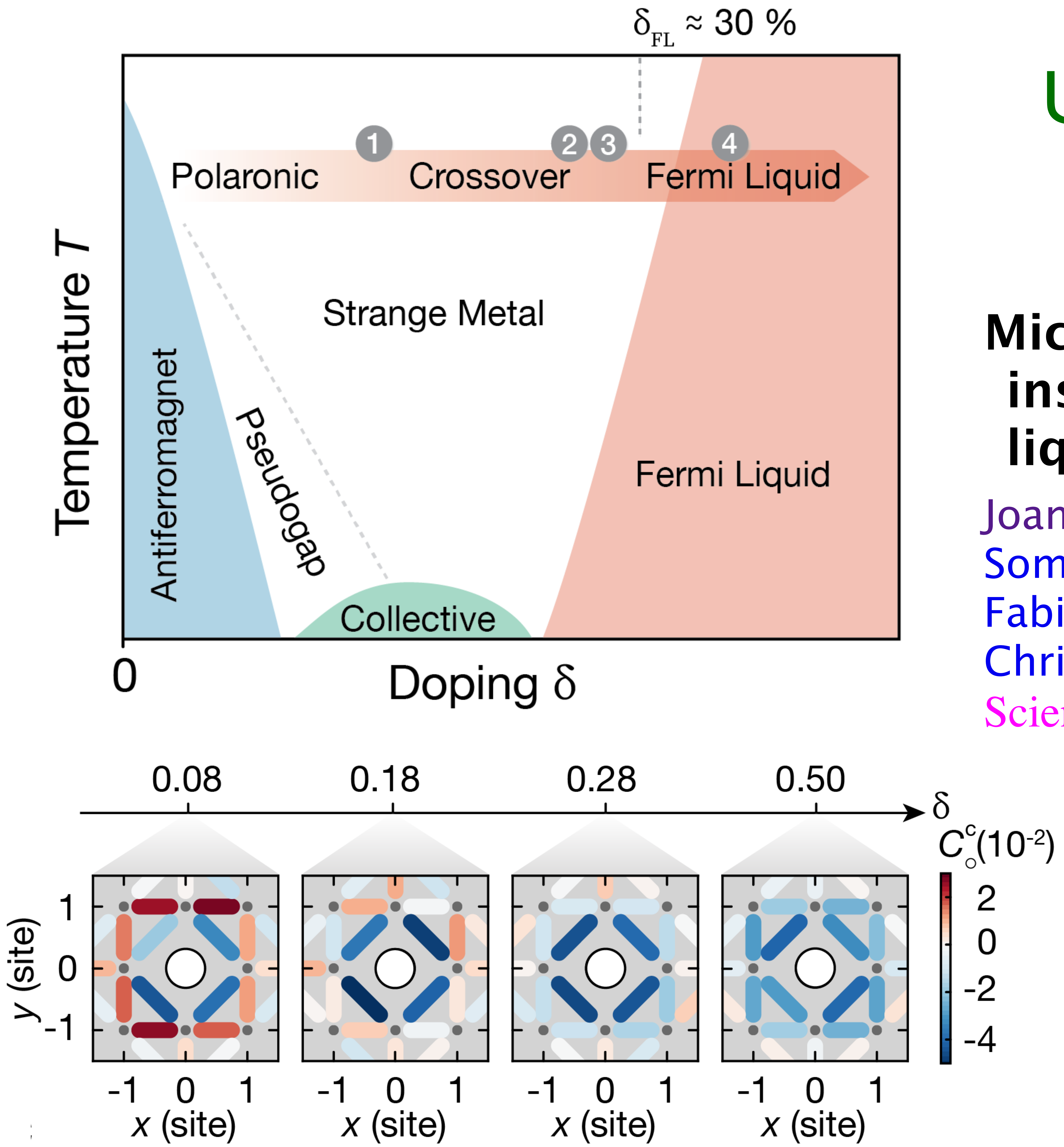
Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr



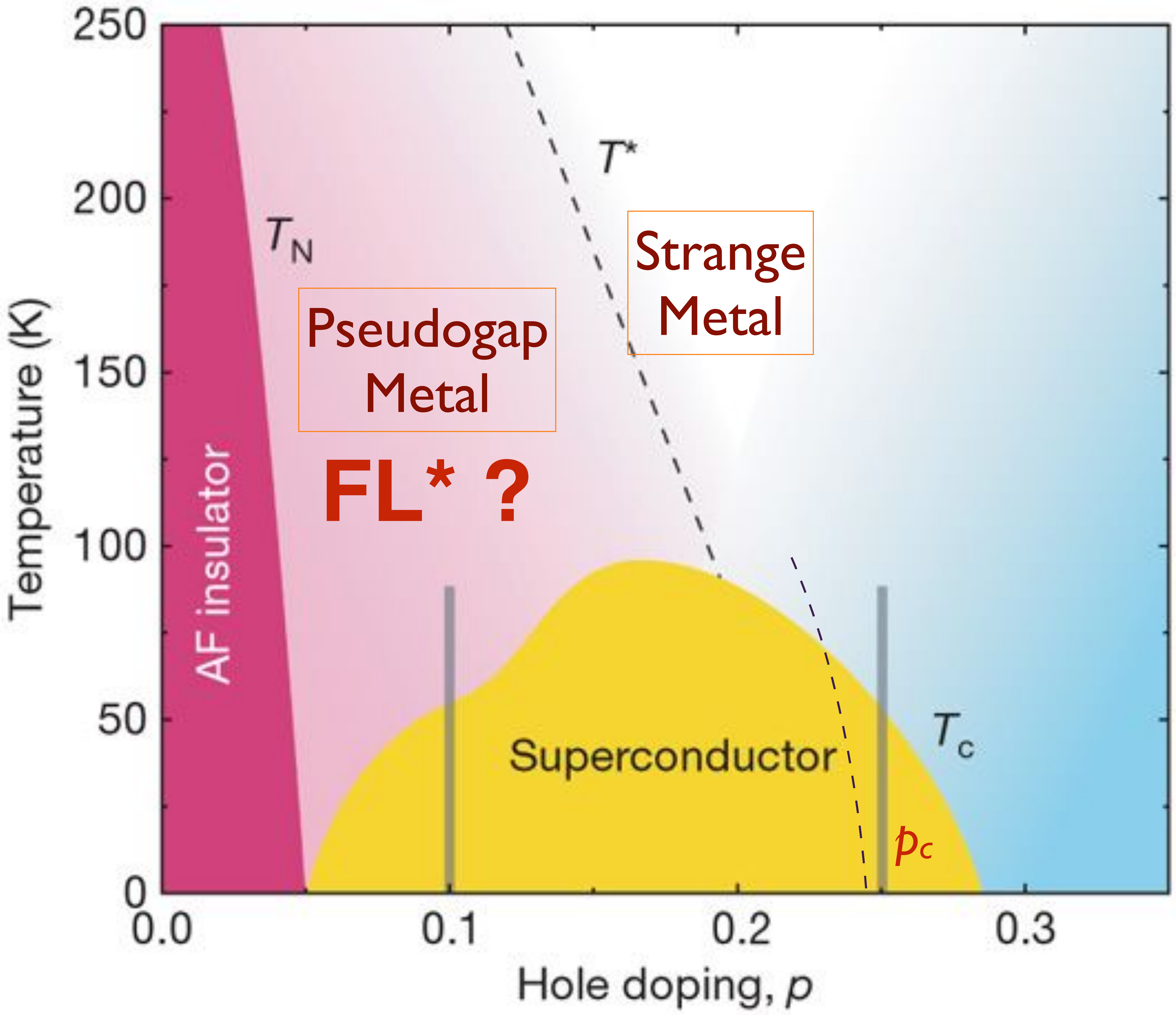


Ultracold fermionic atoms in optical lattices

Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82



Can a FL* state
in a
single-band Hubbard model
describe the
pseudogap metal
over an intermediate
temperature range,
along with a
crossover/transition to
confinement at lower
temperatures?

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

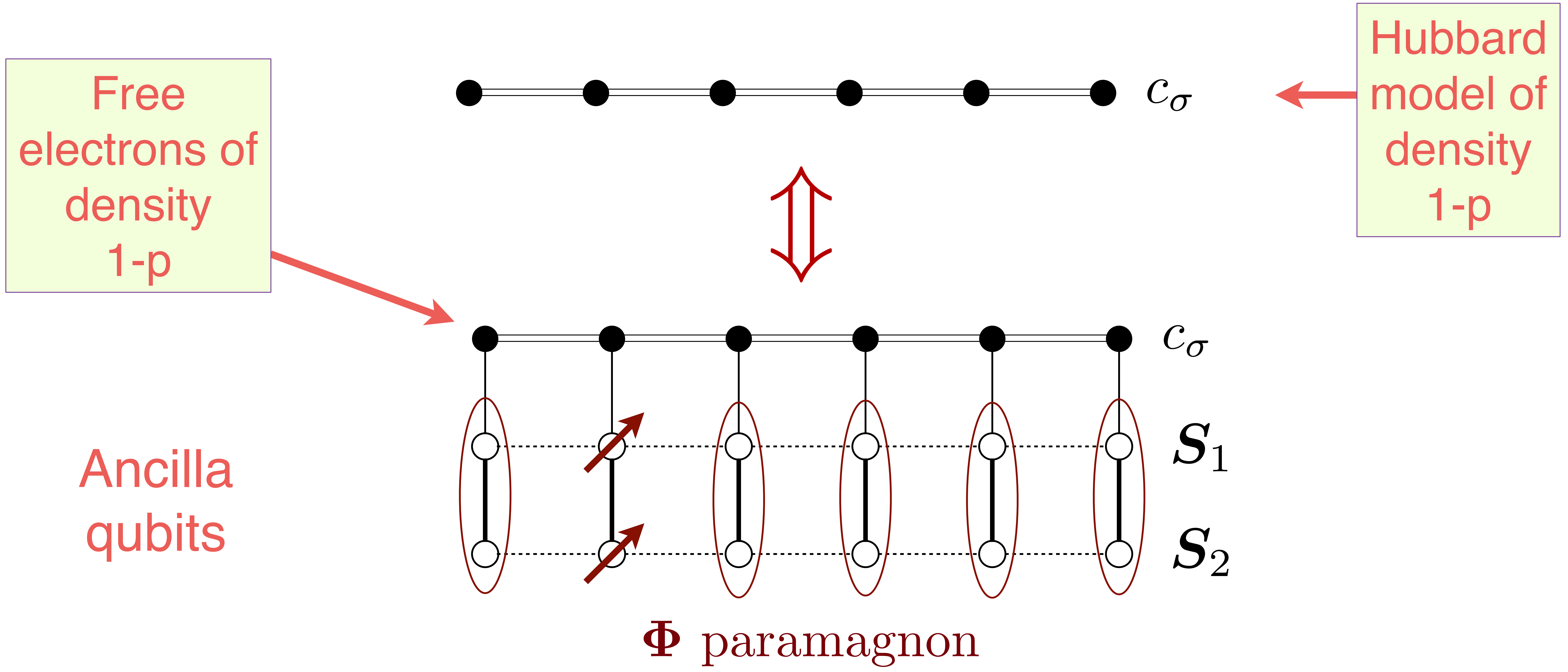
$$U\left(n_{\uparrow} - \frac{1}{2}\right)\left(n_{\downarrow} - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

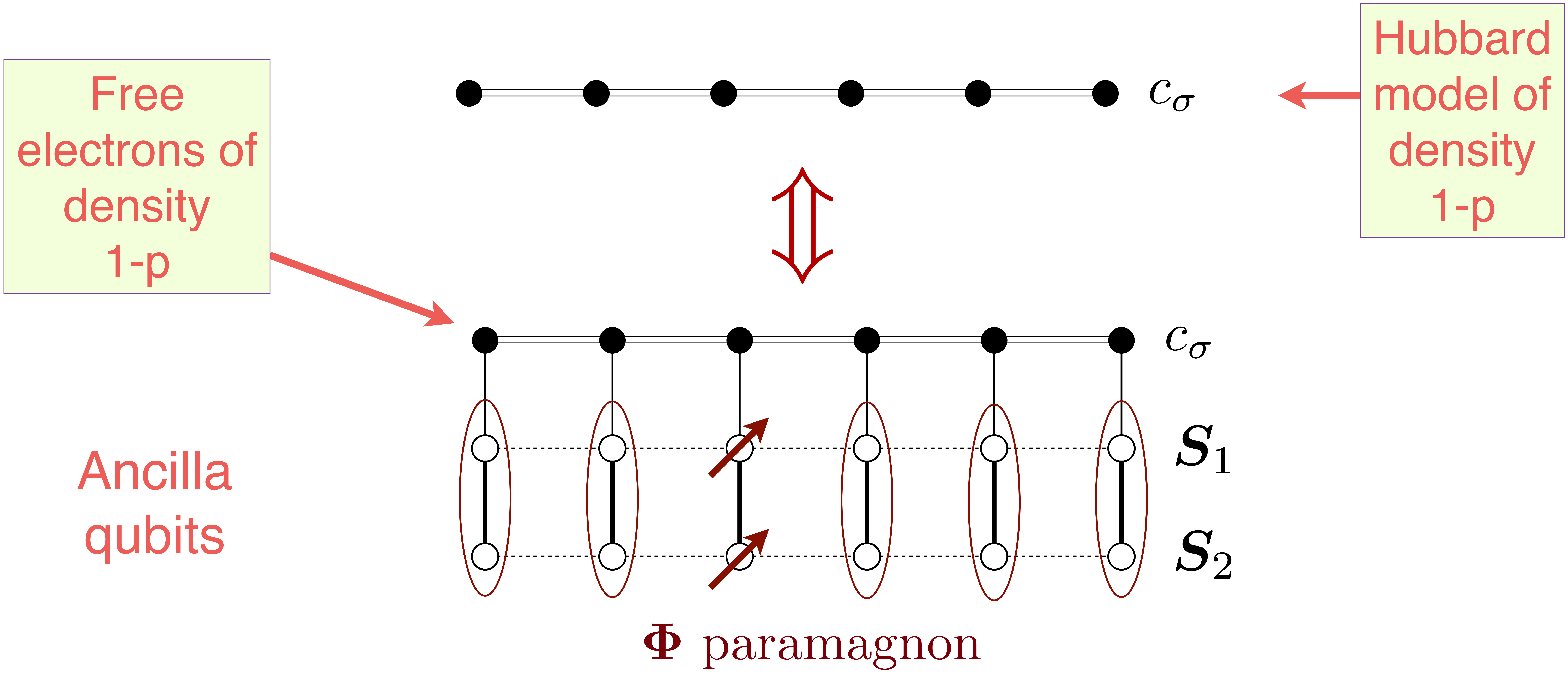
This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model



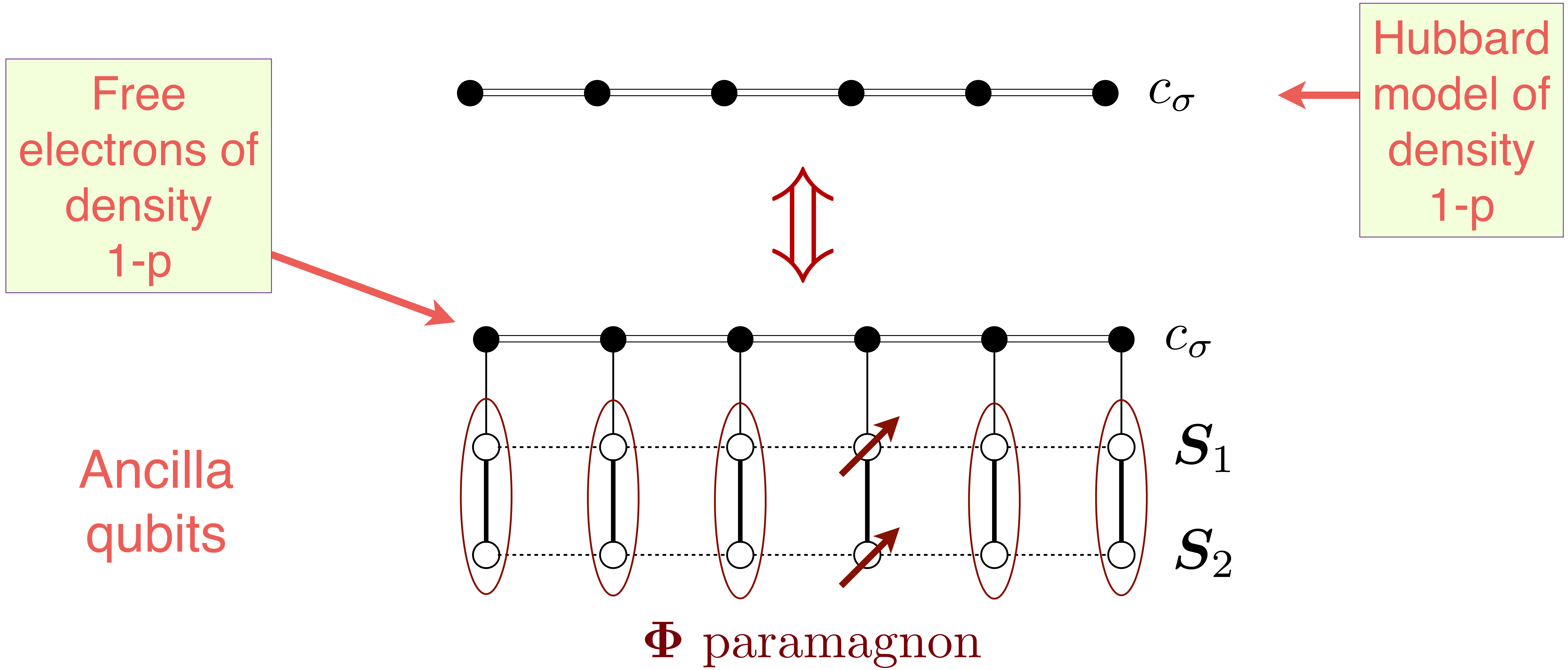
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^{\dagger} c_{\mathbf{p}\sigma} - \lambda \sum_{i} c_{i\sigma}^{\dagger} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model



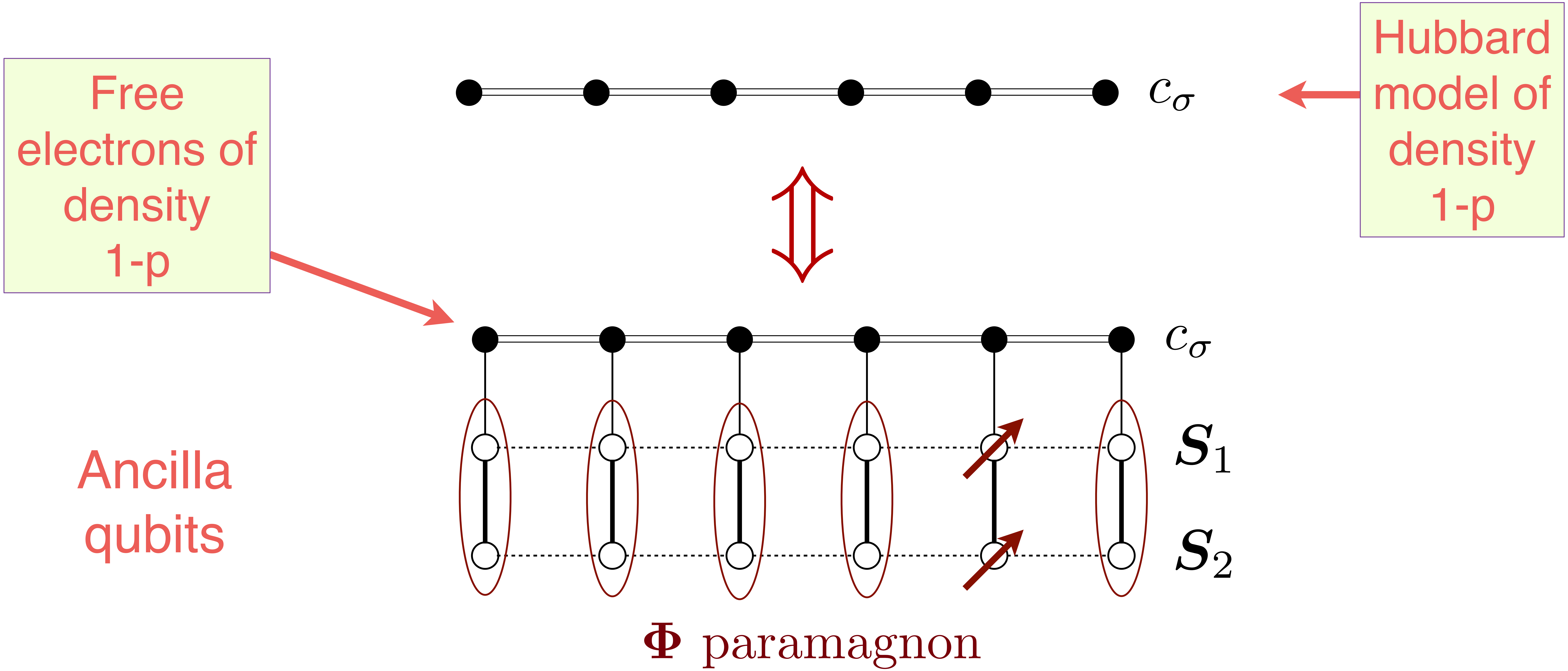
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model



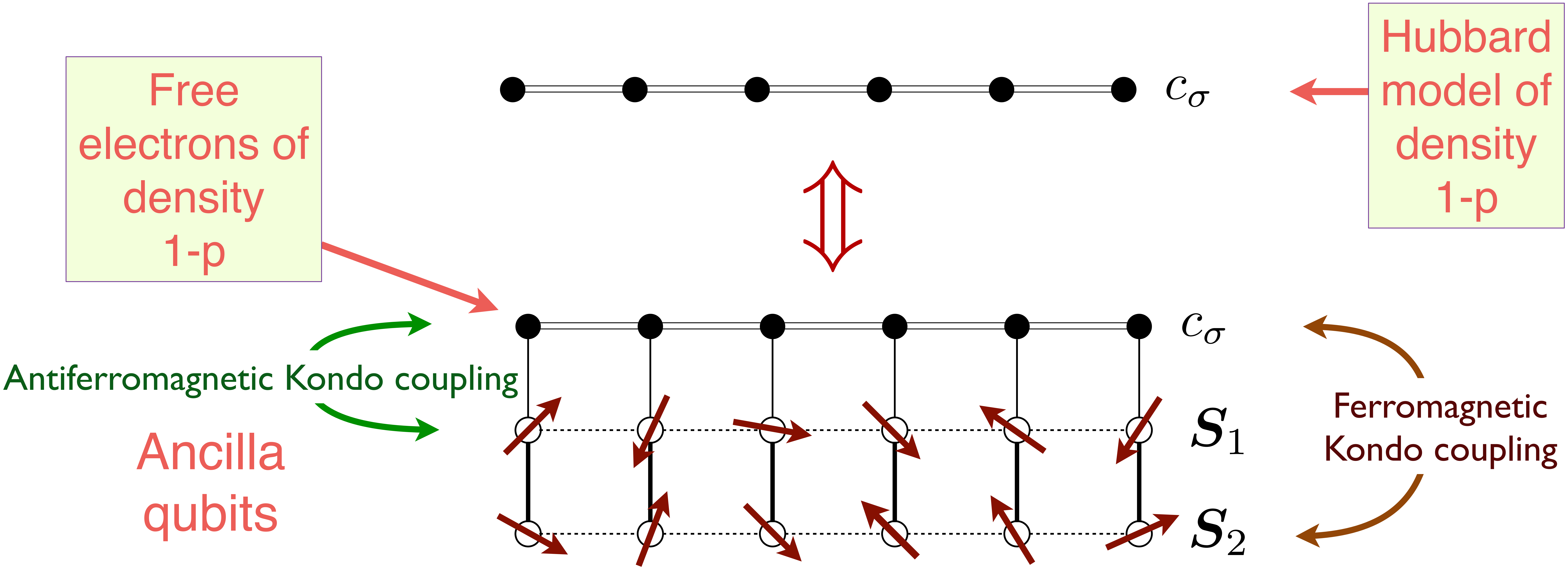
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model



$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

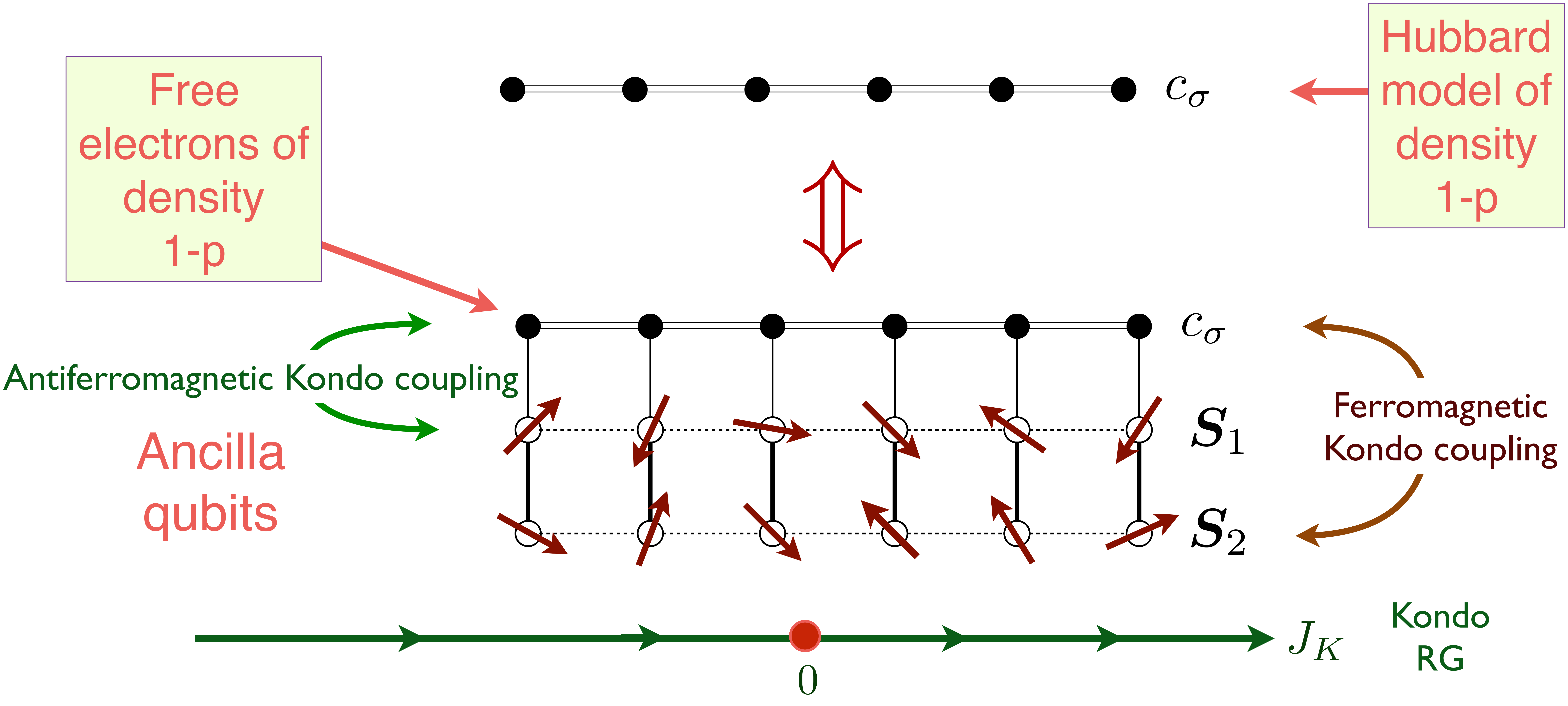
Paramagnon fractionalization theory of the Hubbard model



$$\mathbf{\Phi}_i = \frac{1}{\sqrt{3}}\left(\mathbf{S}_{2i} - \mathbf{S}_{1i}\right)$$

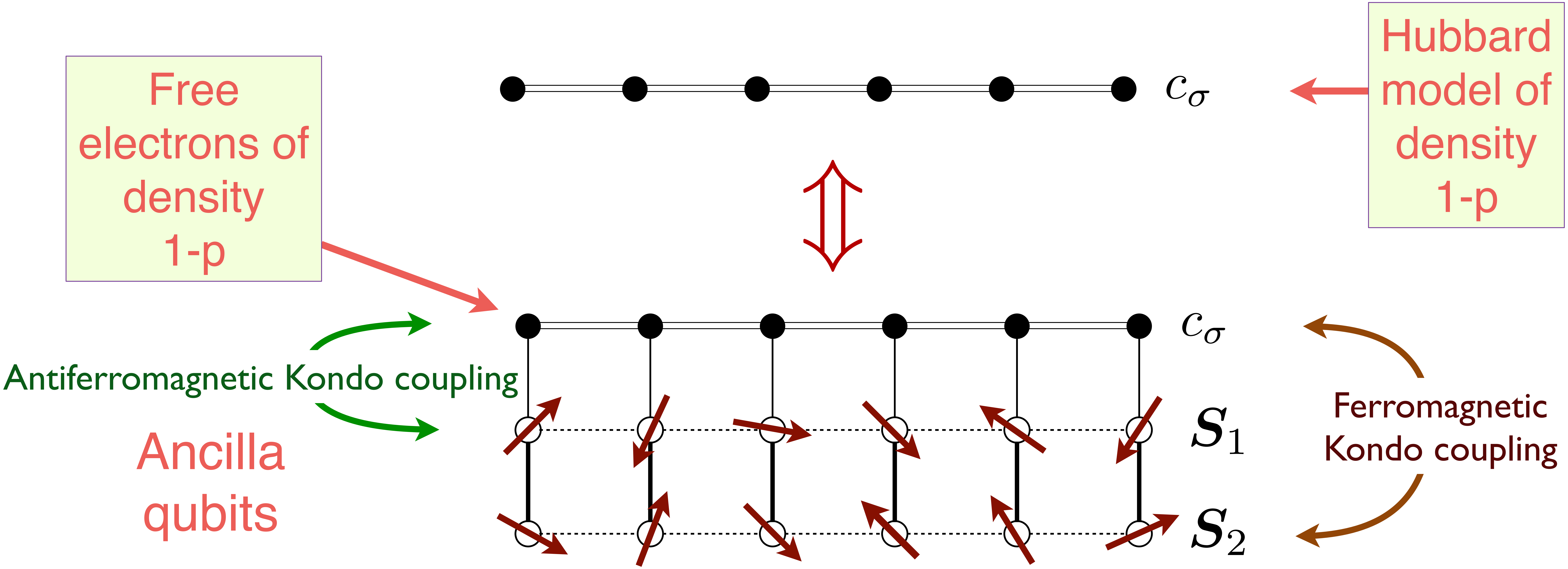
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{\Phi}_i + \ldots$$

Paramagnon fractionalization theory of the Hubbard model



$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + J_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + -\widetilde{J}_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{2i} + \ldots$$

Paramagnon fractionalization theory of the Hubbard model

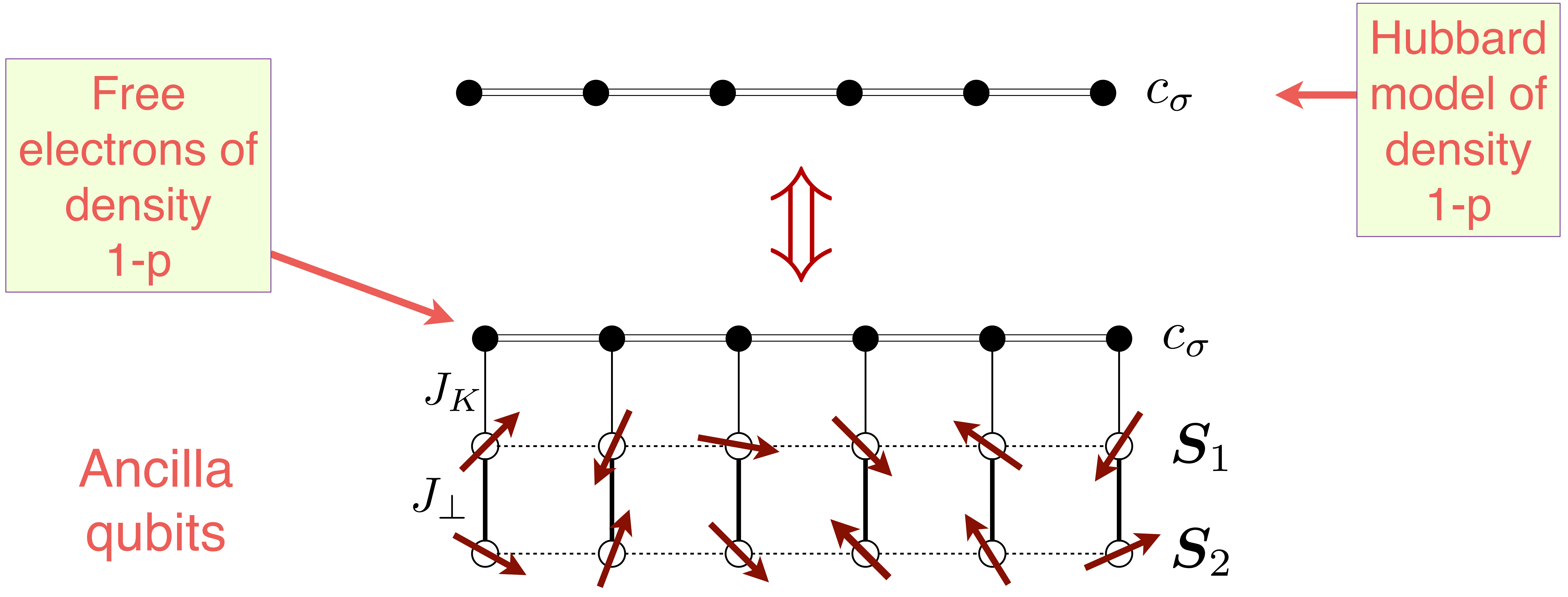


A FL* state is realized when the antiferromagnetic Kondo coupling dominates, and the c_σ and $\boldsymbol{S}_1$ form a "large" Fermi surface of hole density

$$(1+p)+1 = 2+p = p \text{ mod } 2!$$

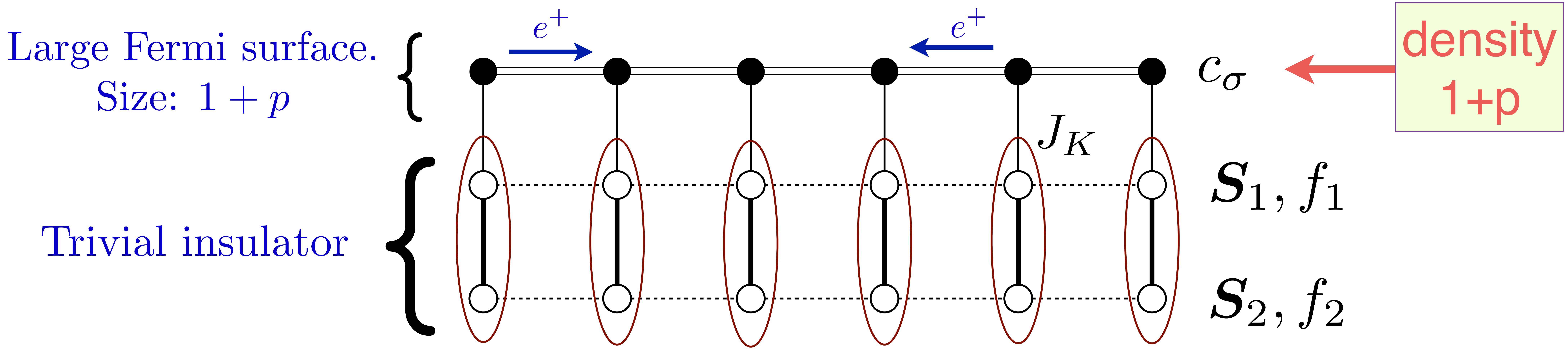
The $\boldsymbol{S}_2$ must form a decoupled spin liquid which does not break translational symmetry, to obtain a metal with a non-Luttinger volume Fermi surface.

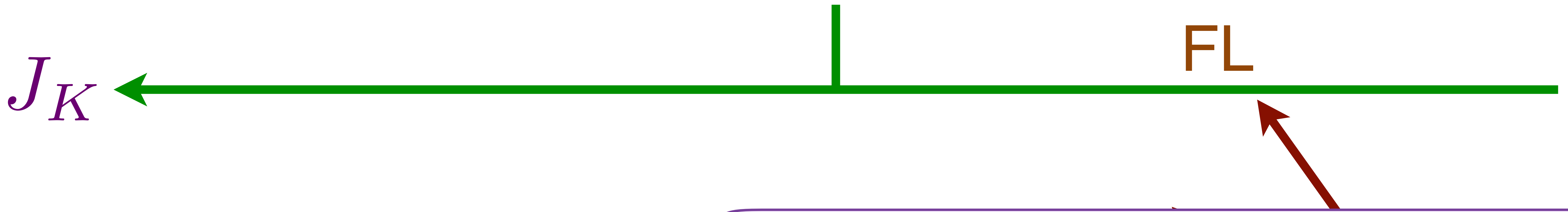
Paramagnon fractionalization theory of the Hubbard model



Related by a Schrieffer-Wolff canonical transformation with $U = \dfrac{3J_K^2}{8J_\perp} + \dfrac{3J_K^3}{16J_\perp} + \ldots$

Trial wavefunctions in the paramagnon fractionalization theory





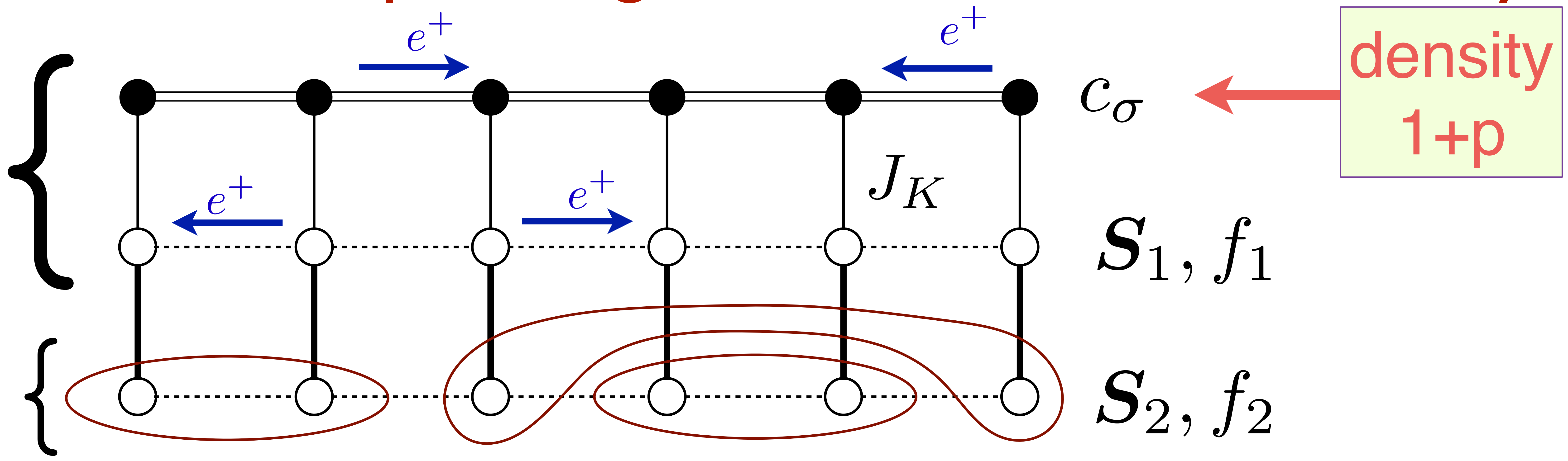
Large Fermi surface of size $1+p$

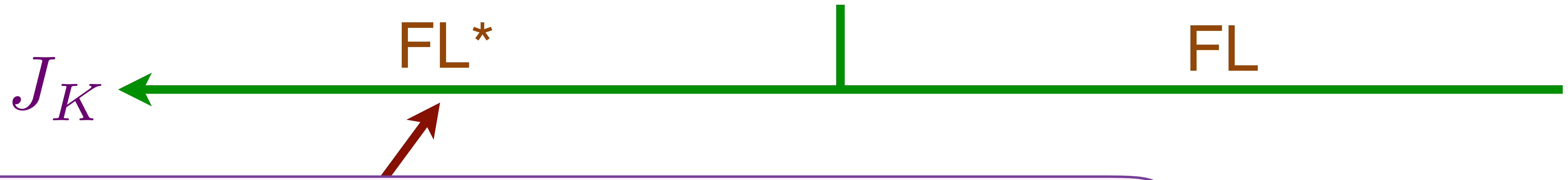
$$|\mathrm{FL}\rangle = |\text{Rung singlets of } f_1, f_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2+p \cong p$

Spin liquid





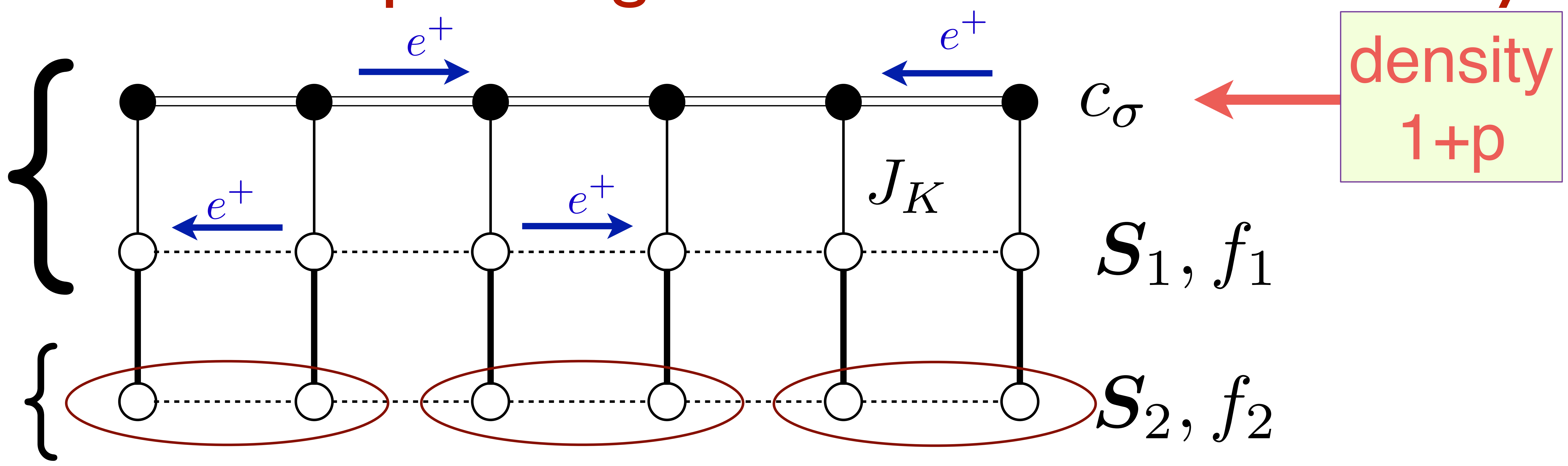
Small Fermi surface of size p

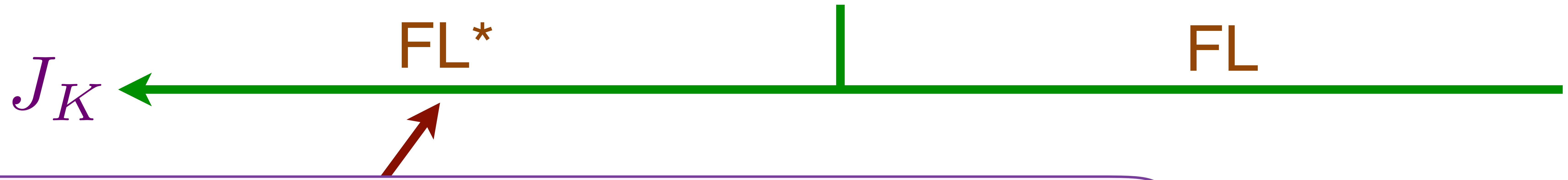
$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } f_1, f_2]$$
$$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$$
$$\otimes |\text{Slater determinant of } f_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2 + p \cong p$

Spin liquid





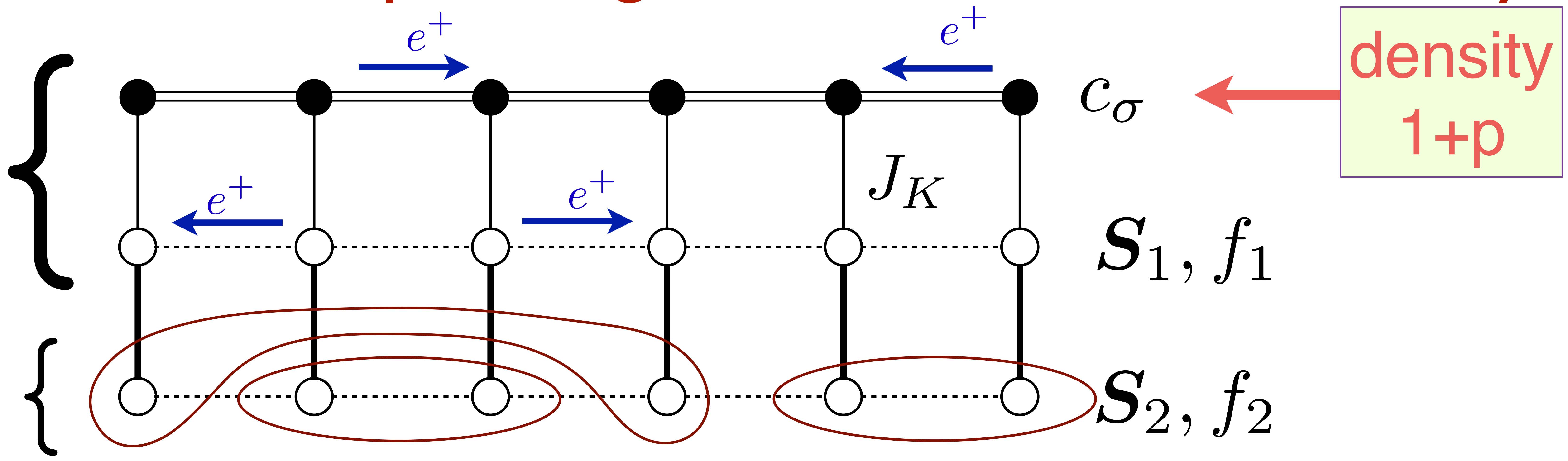
Small Fermi surface of size p

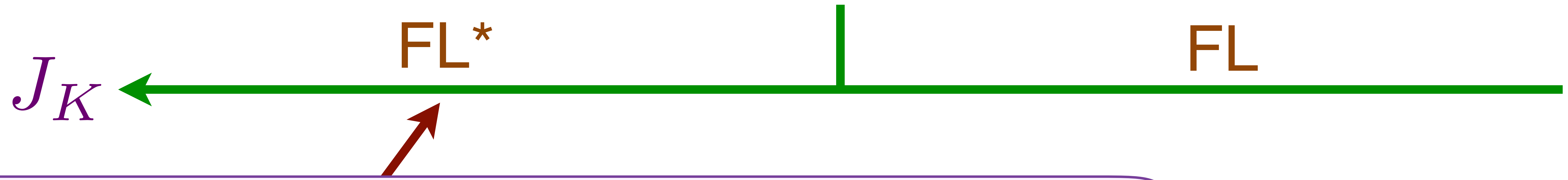
$$
\begin{aligned}
|\mathrm{FL}*\rangle = &\,[\text{Projection onto rung singlets of } f_1, f_2] \\
&\bowtie |\text{Slater determinant of } (c, f_1)\rangle \\
&\otimes |\text{Slater determinant of } f_2\rangle
\end{aligned}
$$

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2 + p \cong p$

Spin liquid





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$$\otimes |\text{Slater determinant of } f_2\rangle$$

$(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2$ gauge theory of **one-band** model

Fermion partons of ancilla spins: $\boldsymbol{S}_1 = f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$, $\boldsymbol{S}_2 = f^\dagger_{2\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{2\beta}$.
Write fermion partons as 2×2 matrices

$$\boldsymbol{f}_1 = \begin{pmatrix} f_{1\uparrow} & -f^\dagger_{1\downarrow} \\ f_{1\downarrow} & f^\dagger_{1\uparrow} \end{pmatrix} \quad , \quad \boldsymbol{f}_2 = \begin{pmatrix} f_{2\uparrow} & -f^\dagger_{2\downarrow} \\ f_{2\downarrow} & f^\dagger_{2\uparrow} \end{pmatrix}$$

P.A. Lee, N. Nagaosa, and X.-G. Wen, RMP **78**, 17 (2006)

Constraints $f^\dagger_{1\alpha} f_{1\alpha} = 1$ and $f^\dagger_{2\alpha} f_{2\alpha} = 1$ lead to:

$$\mathrm{SU}(2)_1 : \quad \boldsymbol{f}_1 \to \boldsymbol{f}_1 U_1 \quad , \quad \boldsymbol{f}_2 \to \boldsymbol{f}_2$$
$$\mathrm{SU}(2)_2 : \quad \boldsymbol{f}_1 \to \boldsymbol{f}_1 \quad , \quad \boldsymbol{f}_2 \to \boldsymbol{f}_2 U_2$$

S. Sachdev, M.A. Metlitski, Yang Qi, and Cenke Xu, PRB **80**, 155129 (2009)
S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)

Rung singlet formation $\boldsymbol{S}_1 + \boldsymbol{S}_2 \approx 0$ leads to:

$$\mathrm{SU}(2)_S : \quad \boldsymbol{f}_1 \to U_S \boldsymbol{f}_1 \quad , \quad \boldsymbol{f}_2 \to U_S \boldsymbol{f}_2$$

Summary

- Probing $\mathbb{Z}_2$ spin liquid with Rydberg atoms:
  Two-state Rydberg atoms on the kagome and ruby lattices can be written *exactly* as a $\mathbb{Z}_2$ gauge theory. Evidence for intermediate scale deconfinement of a $\mathbb{Z}_2$ gauge theory on the ruby lattice.

Summary

- Probing $\mathbb{Z}_2$ spin liquid with Rydberg atoms:
  Two-state Rydberg atoms on the kagome and ruby lattices can be written *exactly* as a $\mathbb{Z}_2$ gauge theory. Evidence for intermediate scale deconfinement of a $\mathbb{Z}_2$ gauge theory on the ruby lattice.

- Paramagnon fractionalization theory of FL* for the pseudogap metal of the cuprate high temperature superconductors:
  Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'.
  Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.
  $(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2$ theory for transition from FL* to FL.