Strange metals and black holes

Michigan State University, East Lansing
February 6, 2020

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Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates

Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates

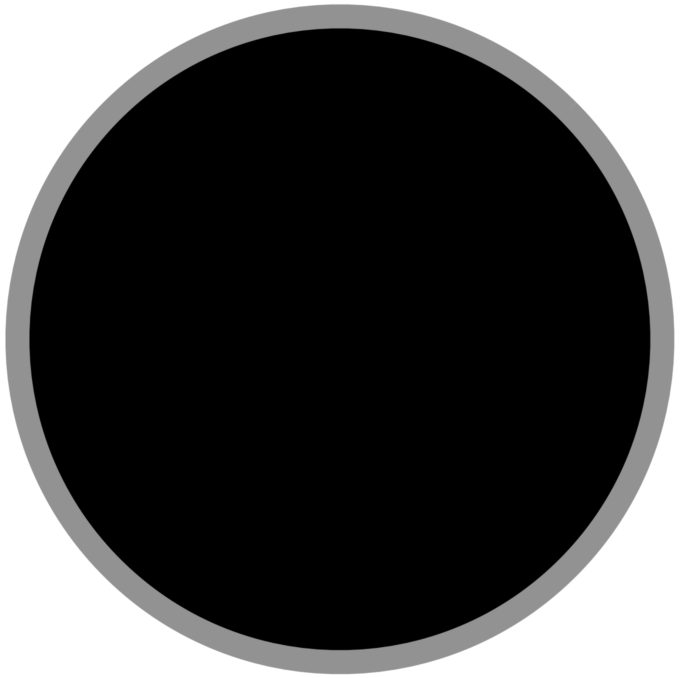
The holographic connection between strange metals and black holes

Black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.



$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

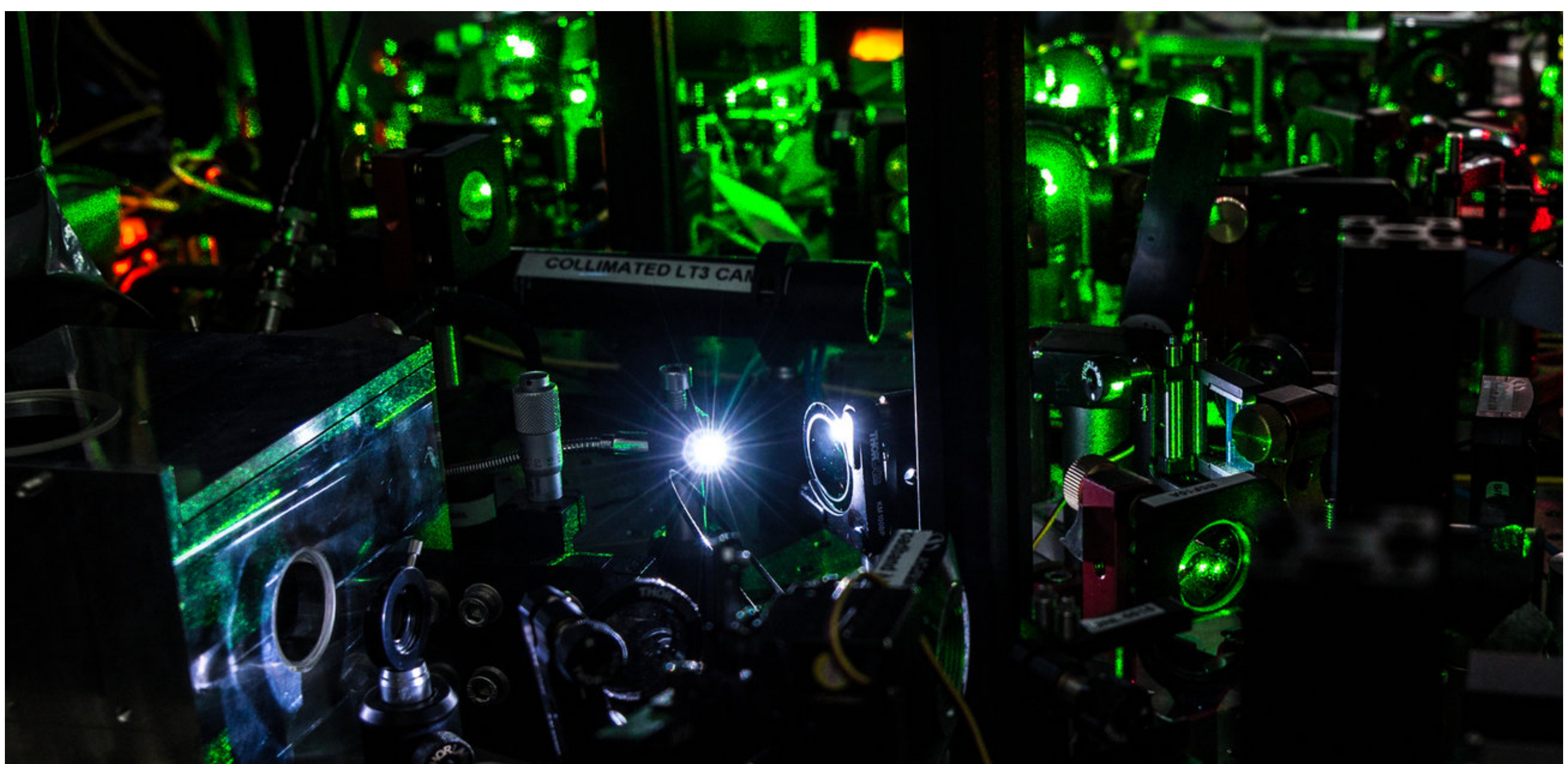
G Newton's constant, c velocity of light, M mass of black hole

The New York Times

Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By **JOHN MARKOFF** OCT. 21, 2015

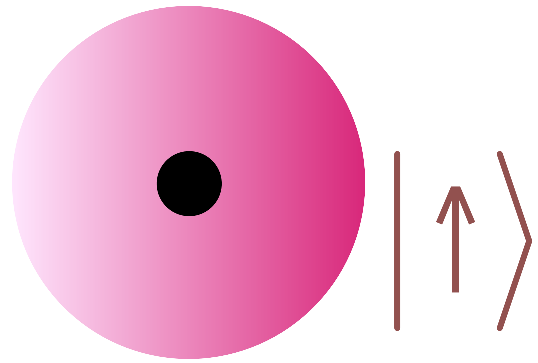
In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other's behavior.



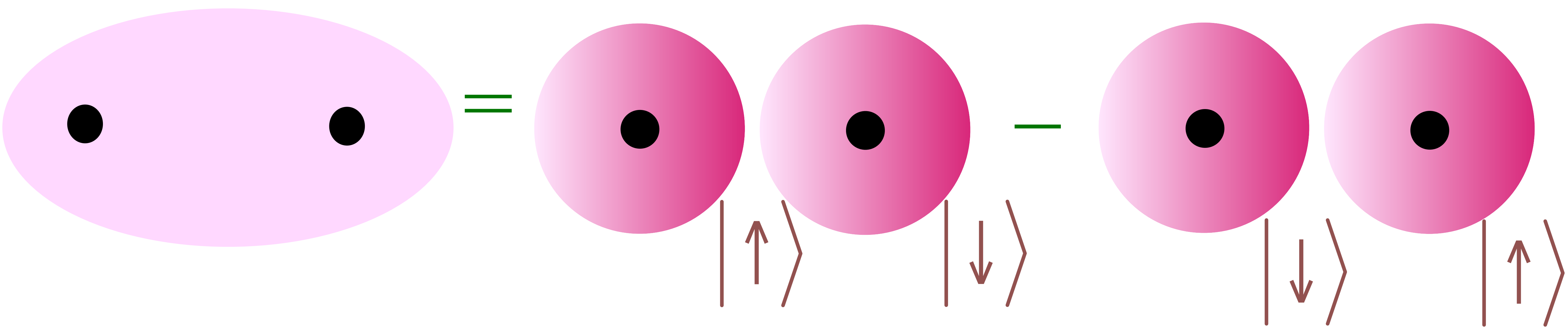
Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

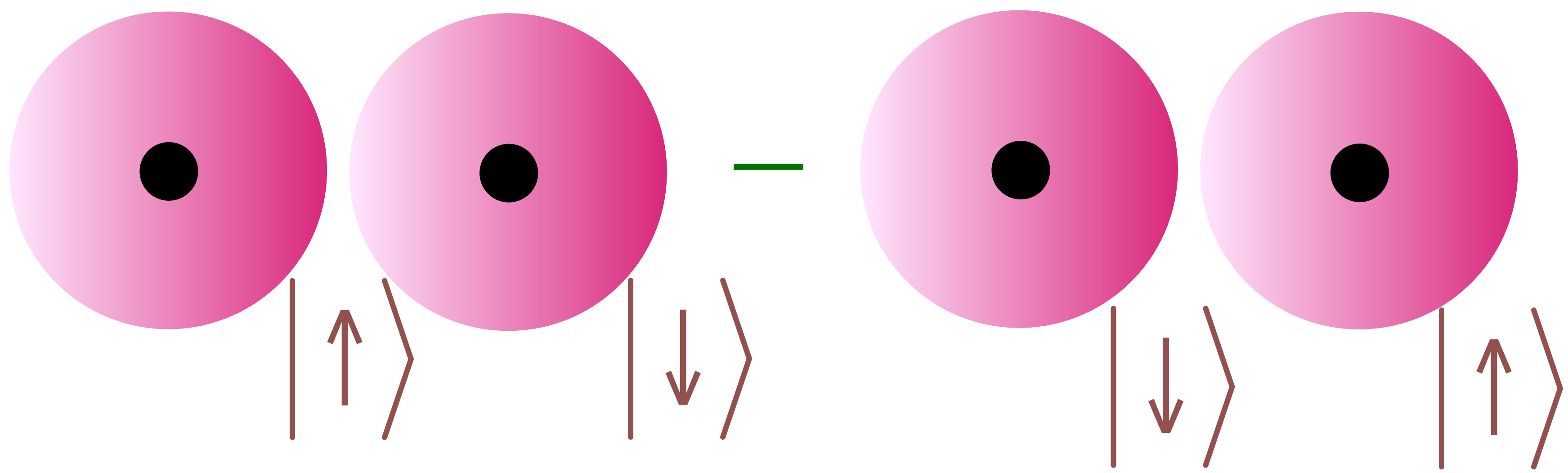


Hydrogen molecule:

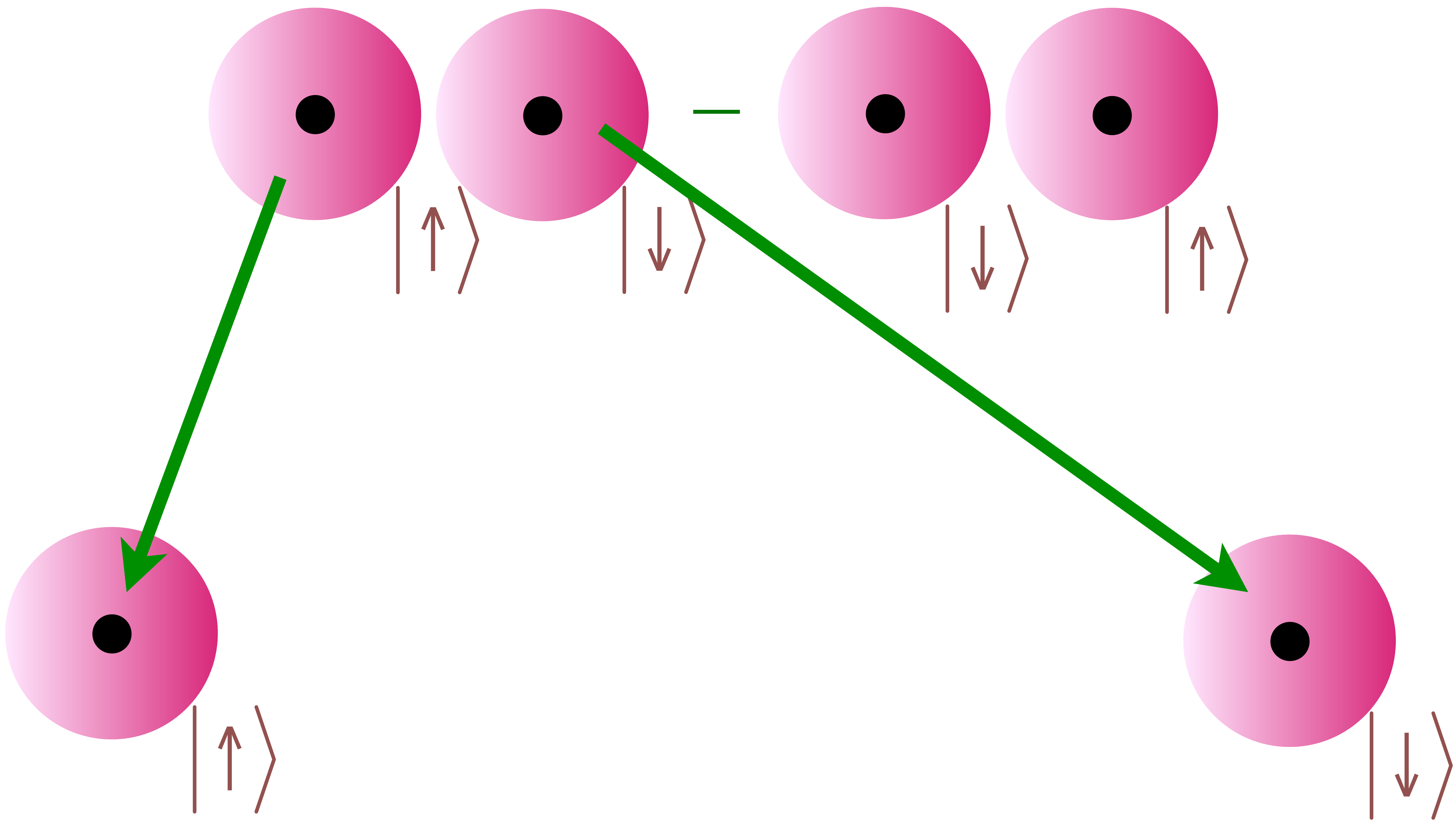


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

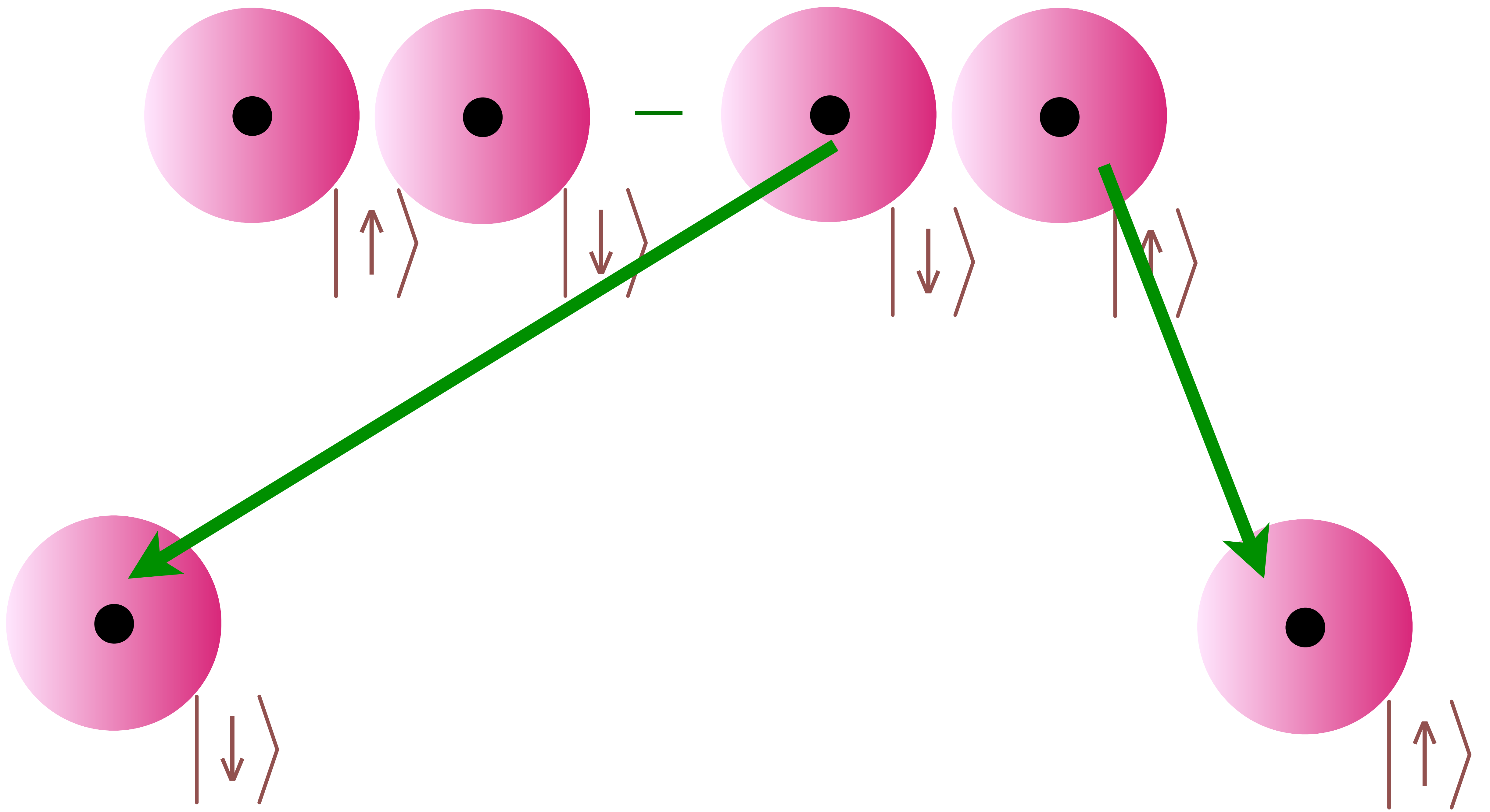
Quantum Entanglement: quantum superposition with more than one particle



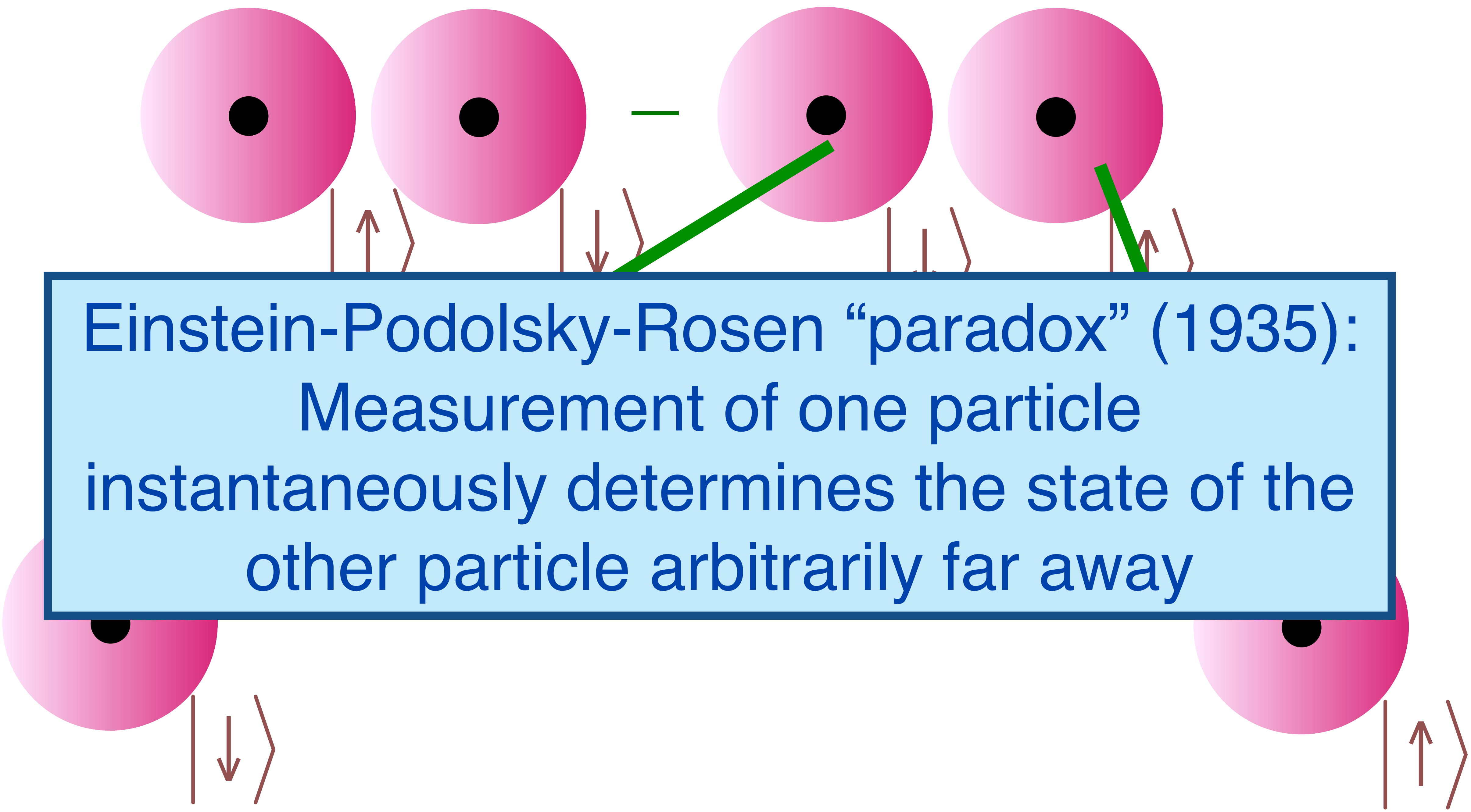
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle

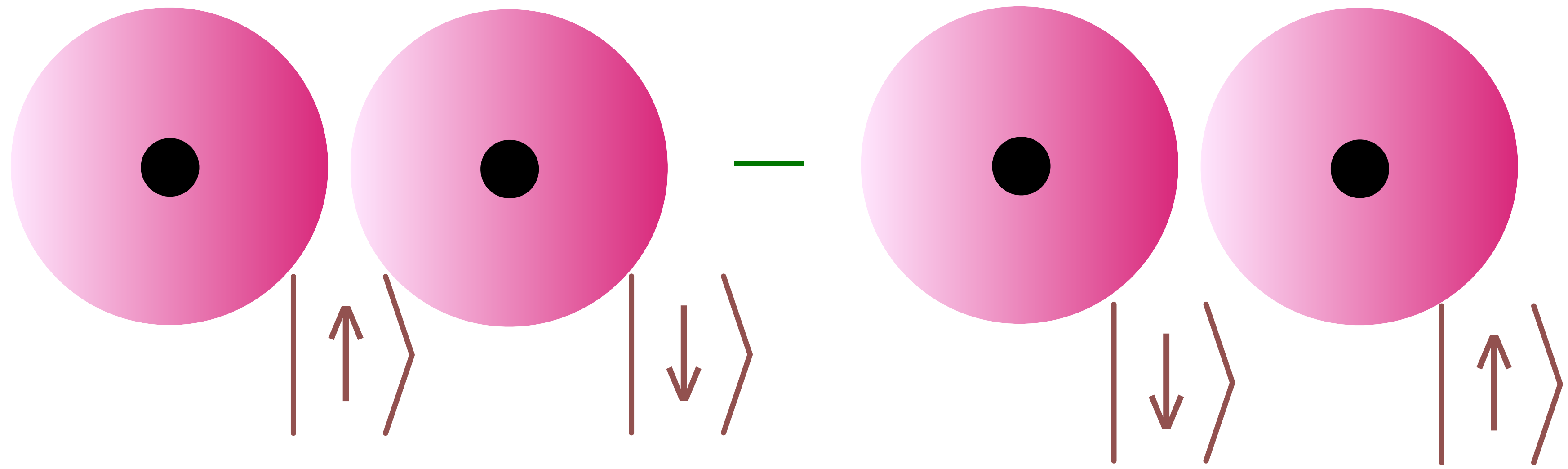


Quantum Entanglement: quantum superposition with more than one particle

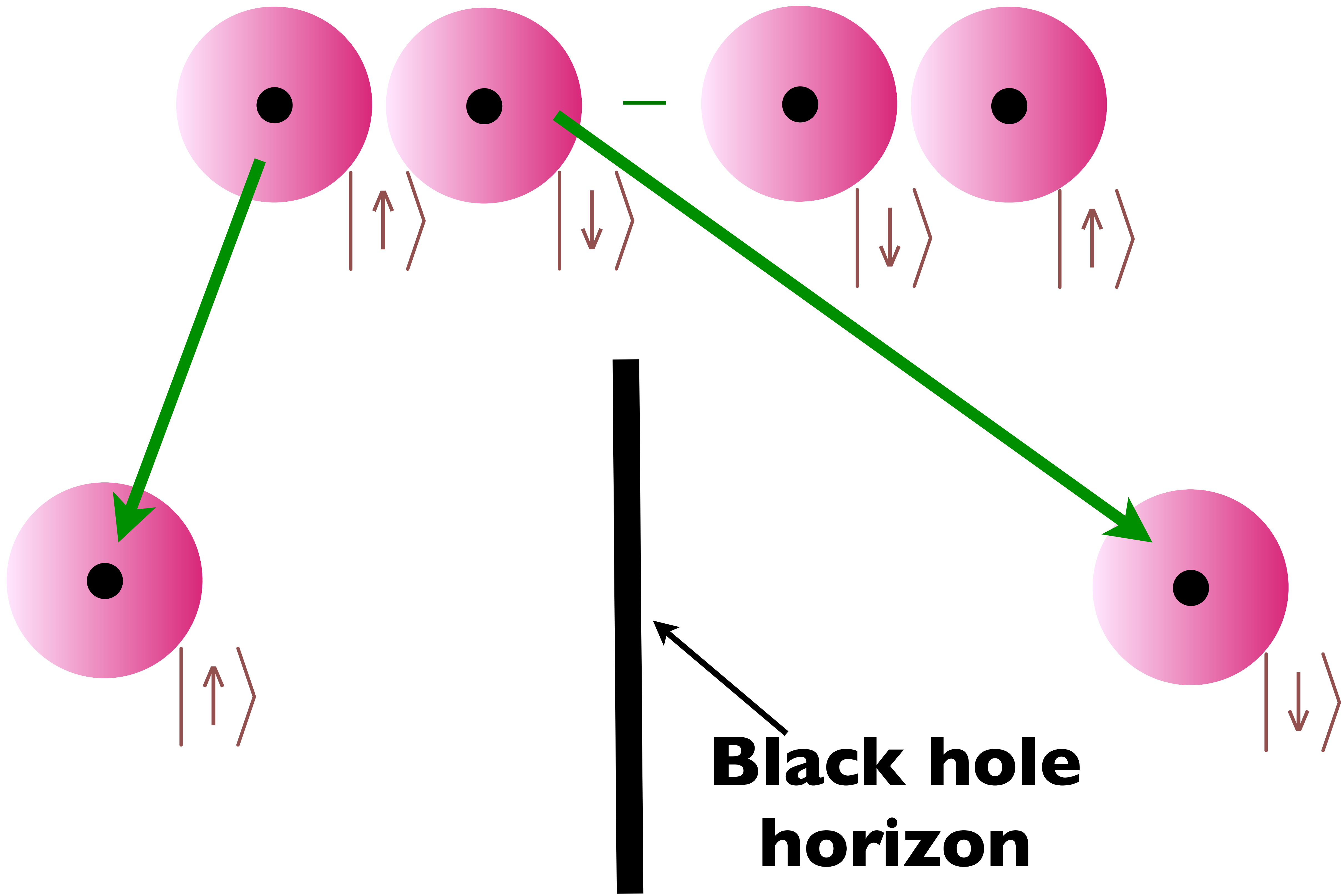


Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

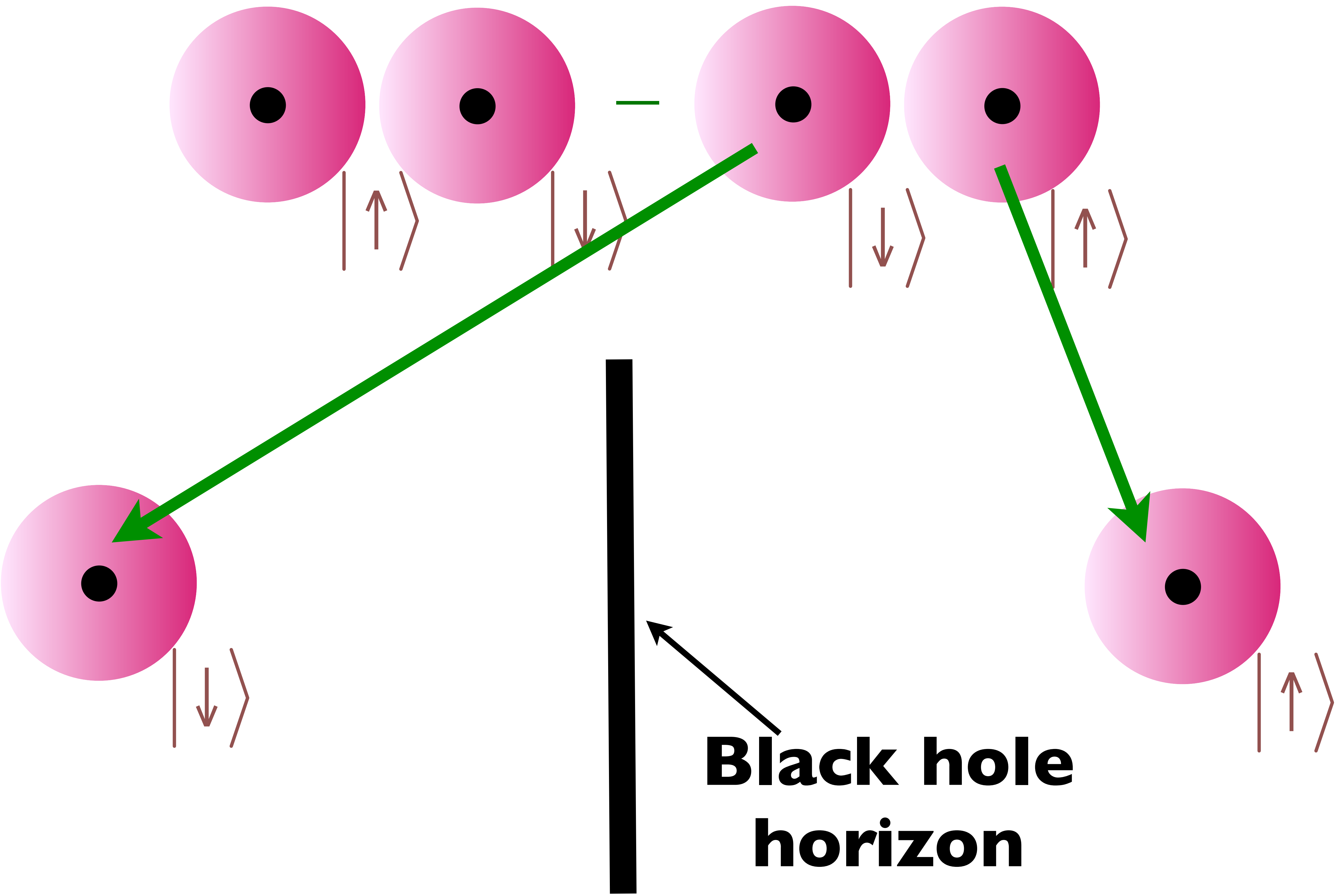
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

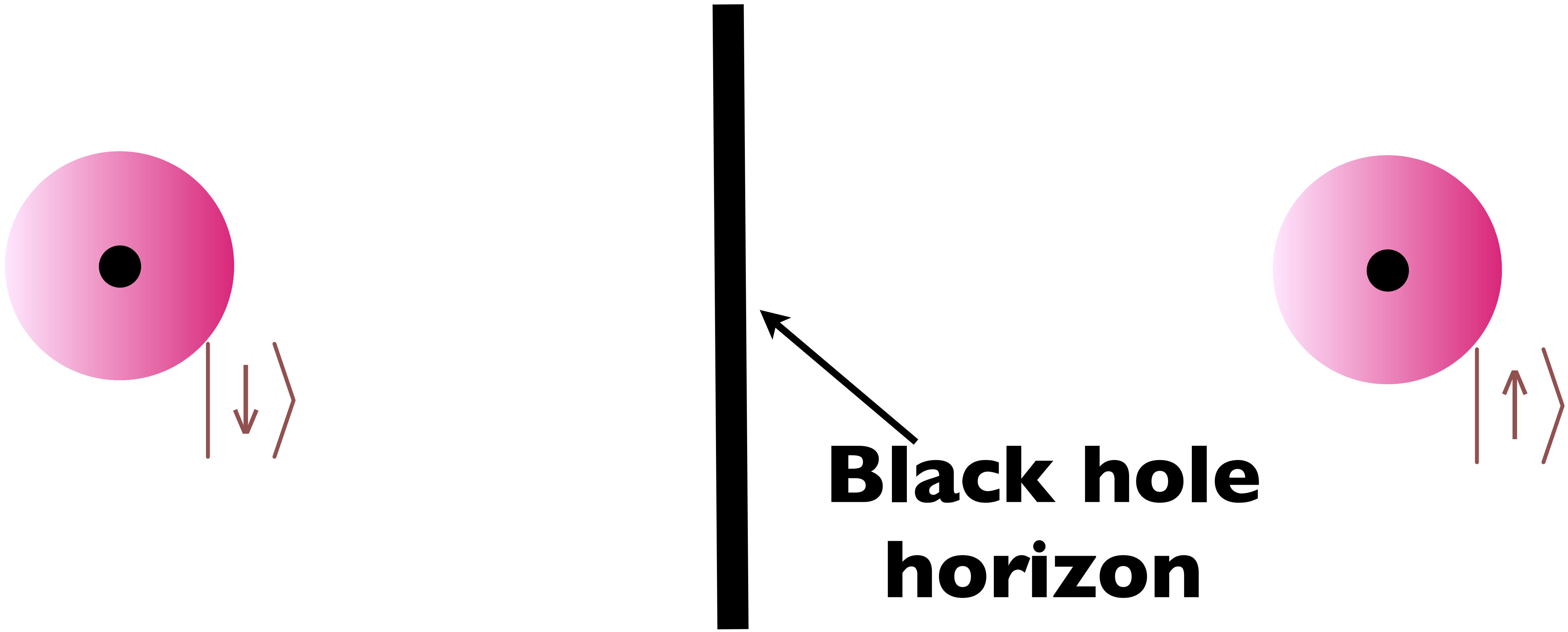


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

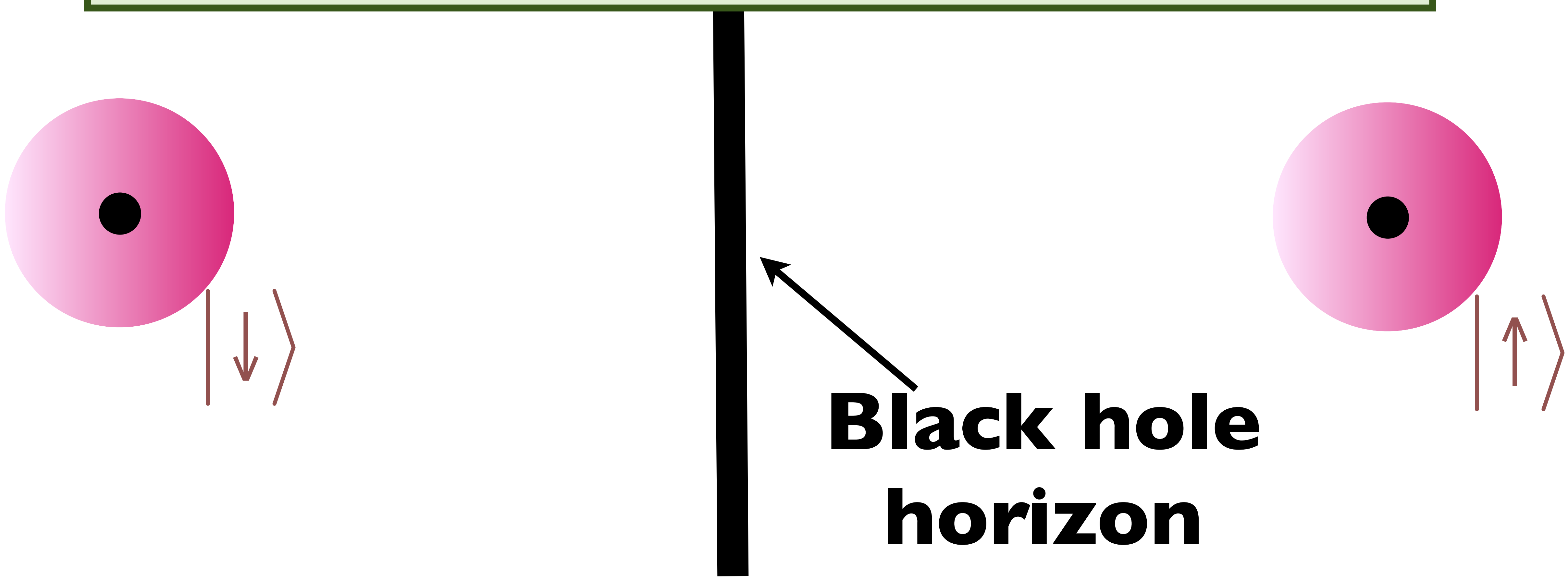
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

(because to an outside observer, the state of the electron inside the black hole is an unknown)

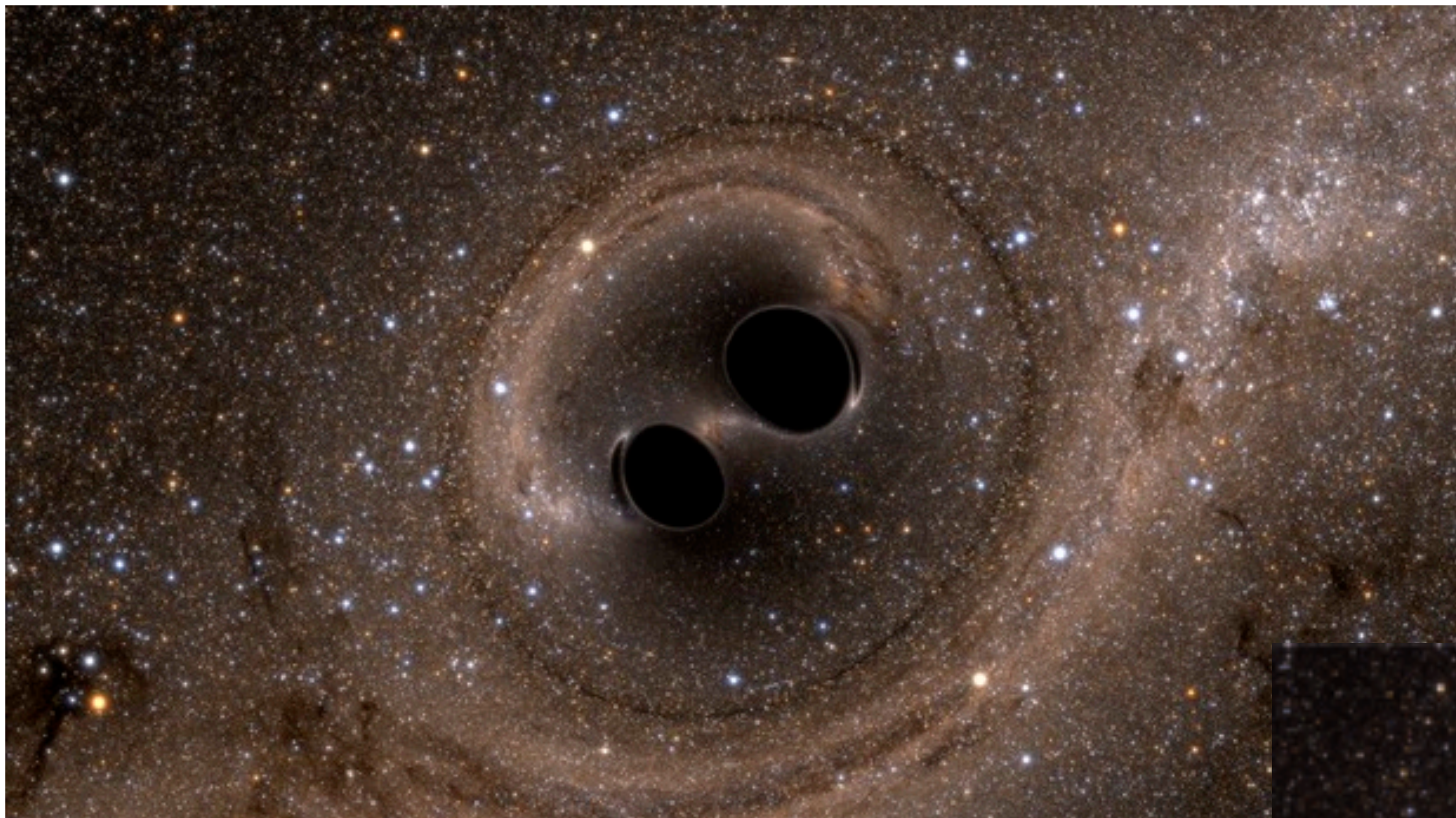


Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



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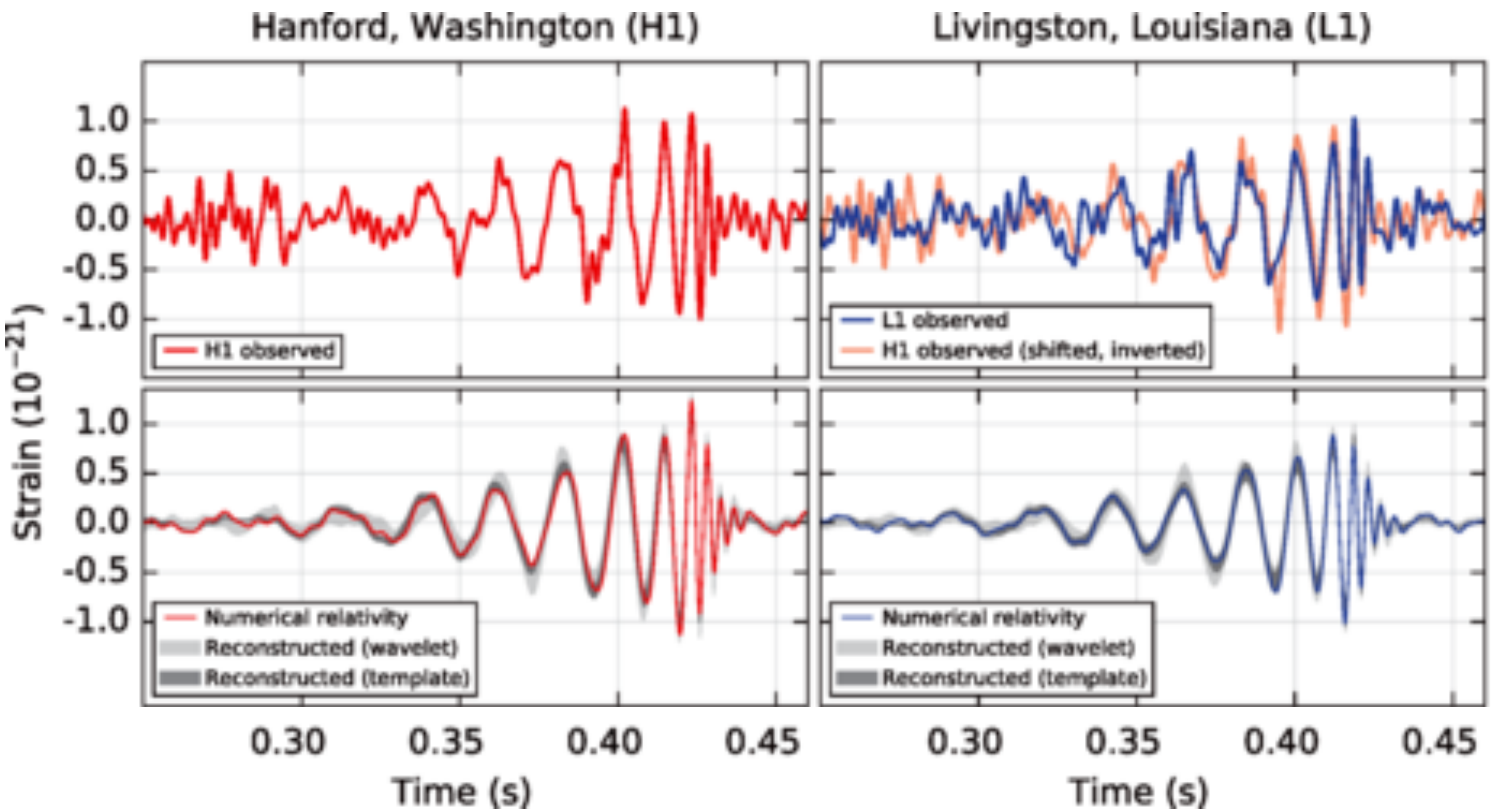
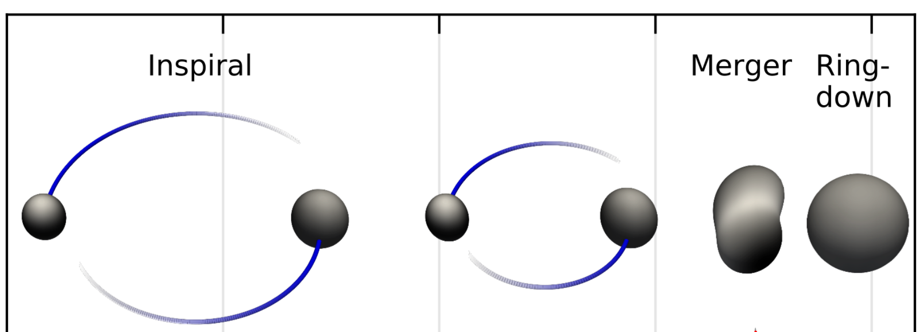




LIGO
September 14, 2015

- The ring-down time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals

$$\boxed{\frac{\hbar}{k_B T_H},}$$ $\hbar$ Planck's constant, k_B Boltzmann's constant

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

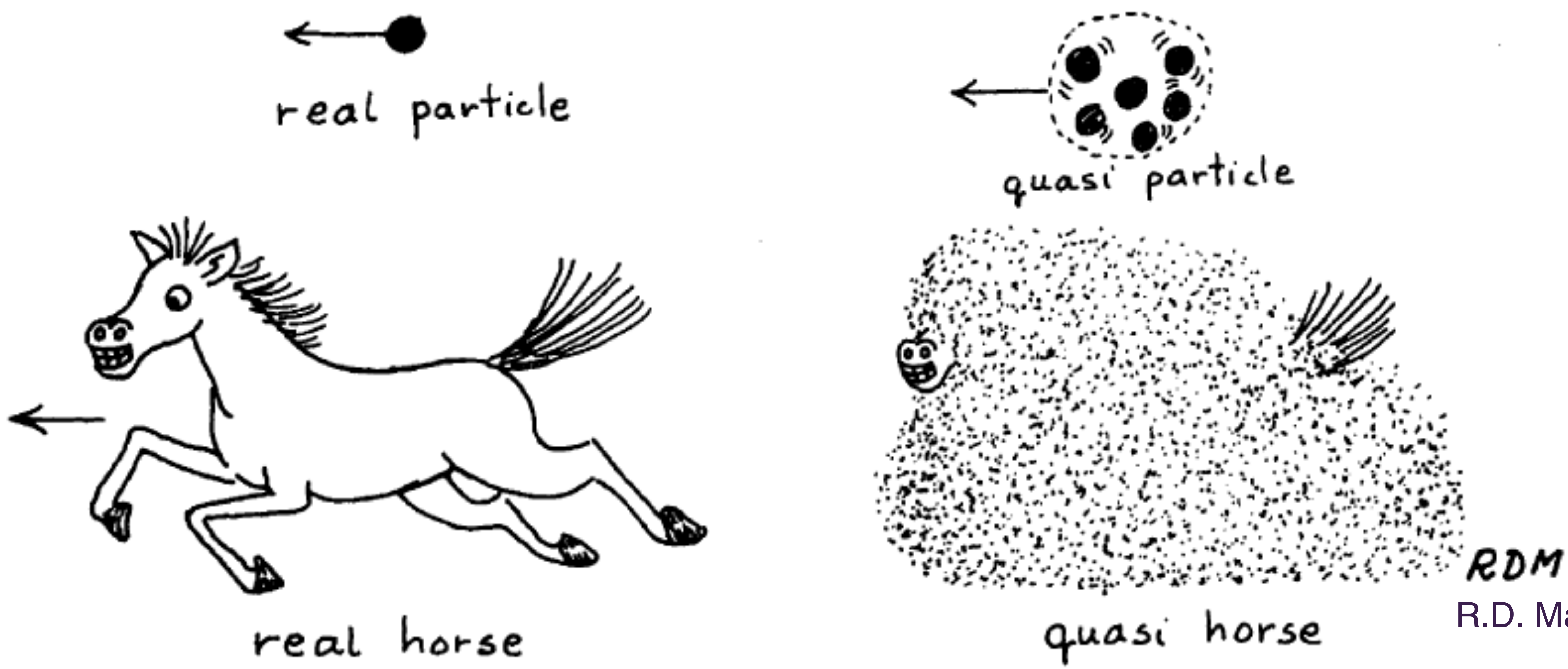
Black holes

Metals, ordinary and strange

Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

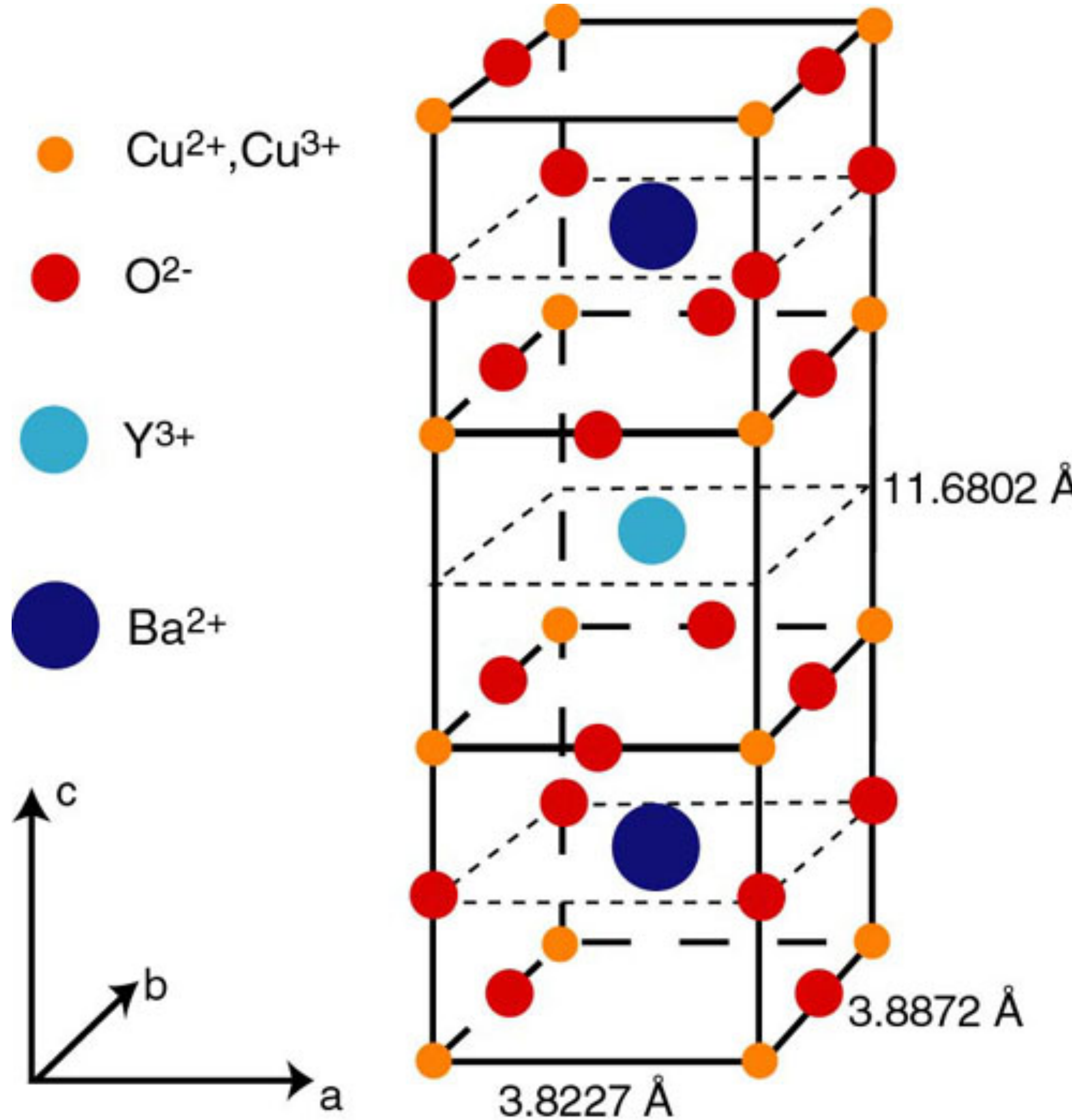


R.D. Mattuck

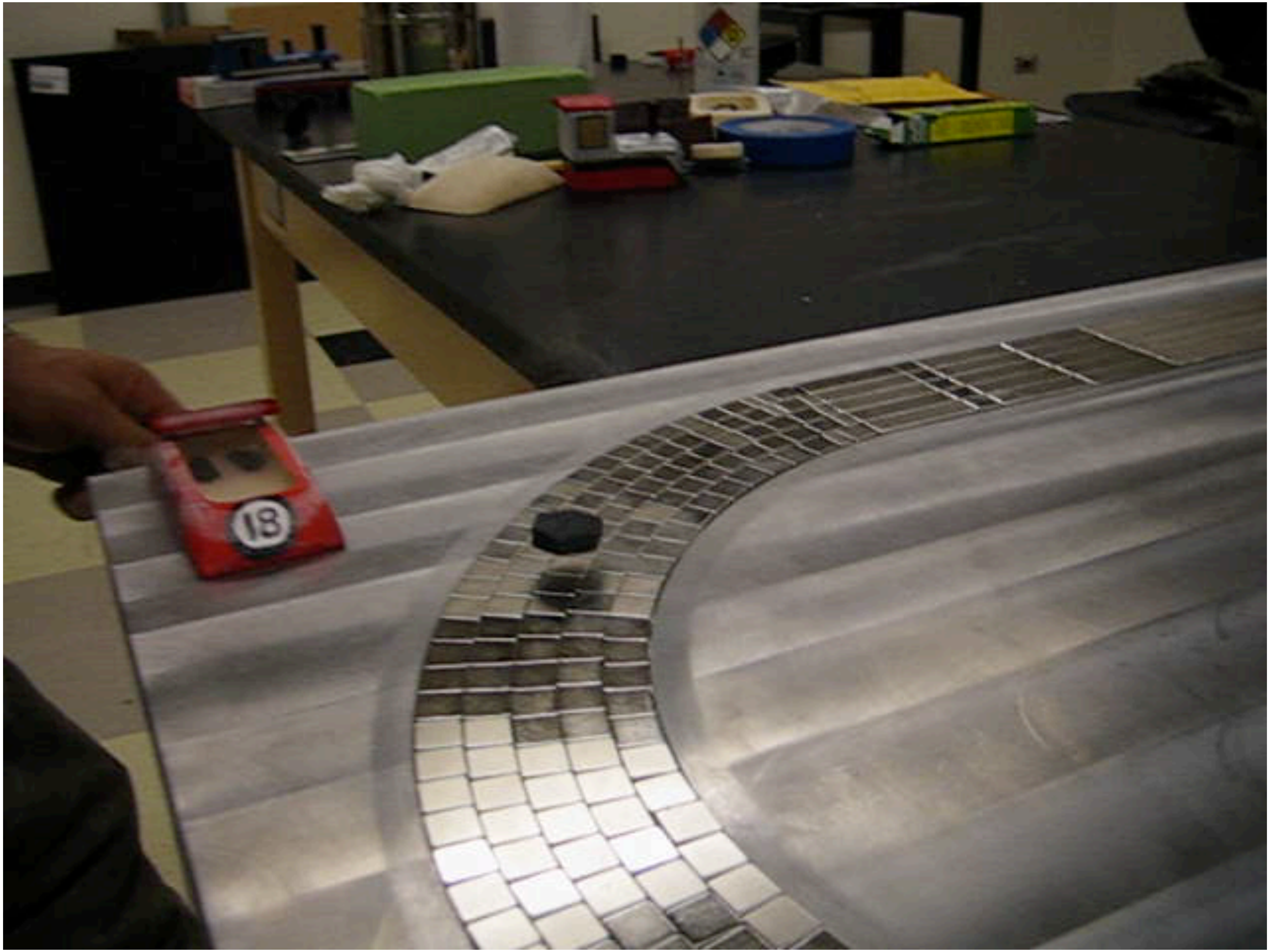
Current flow with quasiparticles

- The resistivity, ρ, of a metal from the flow of quasi-particles is
$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau}$$
where m^* is the effective mass of a quasipartcle, n is the density of electrons, e is the charge of an electron, and τ is a quasiparticle scattering time. The theory of ordinary metals implies that as the temperature $T \to 0$
$$\tau \sim \frac{1}{T^2} \gg \frac{\hbar}{k_B T}$$

High temperature superconductors



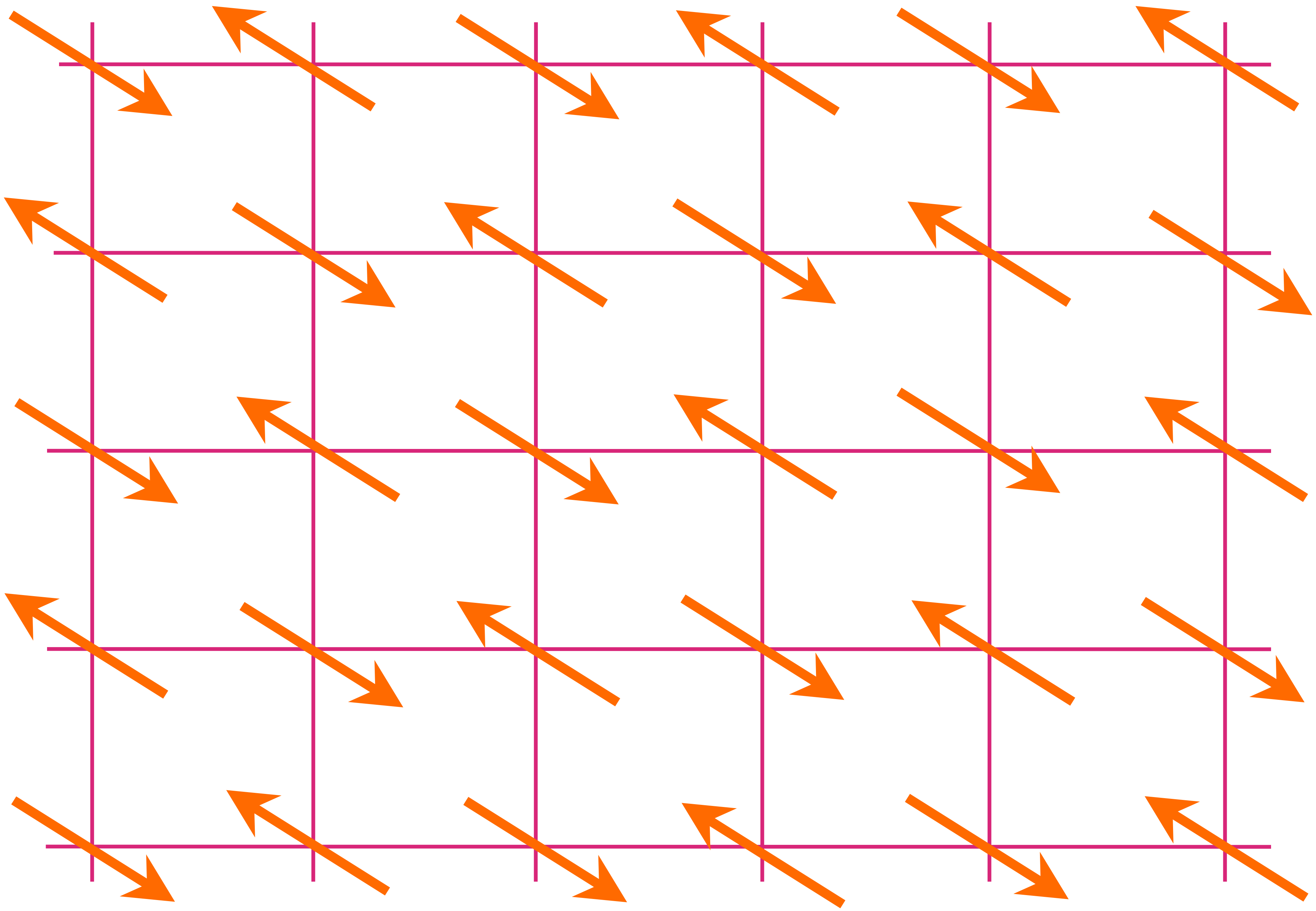
$YBa_2Cu_3O_{6+x}$

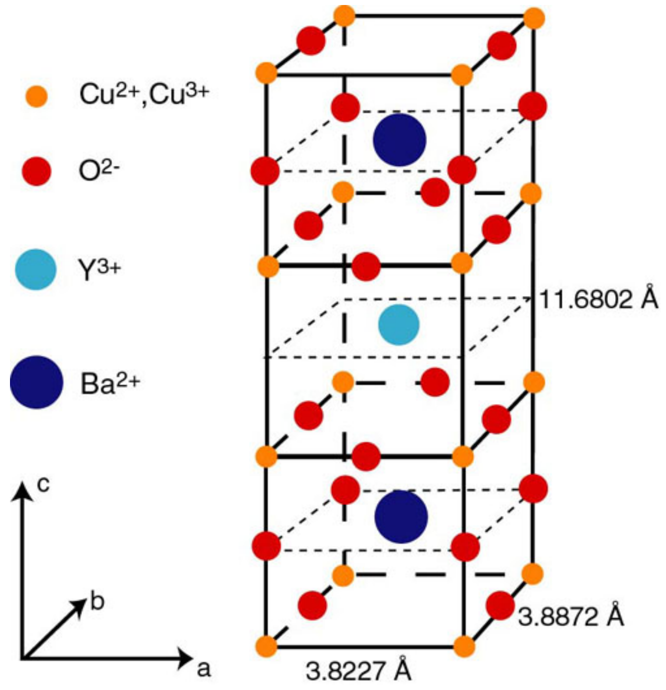


Nd-Fe-B magnets, YBaCuO superconductor

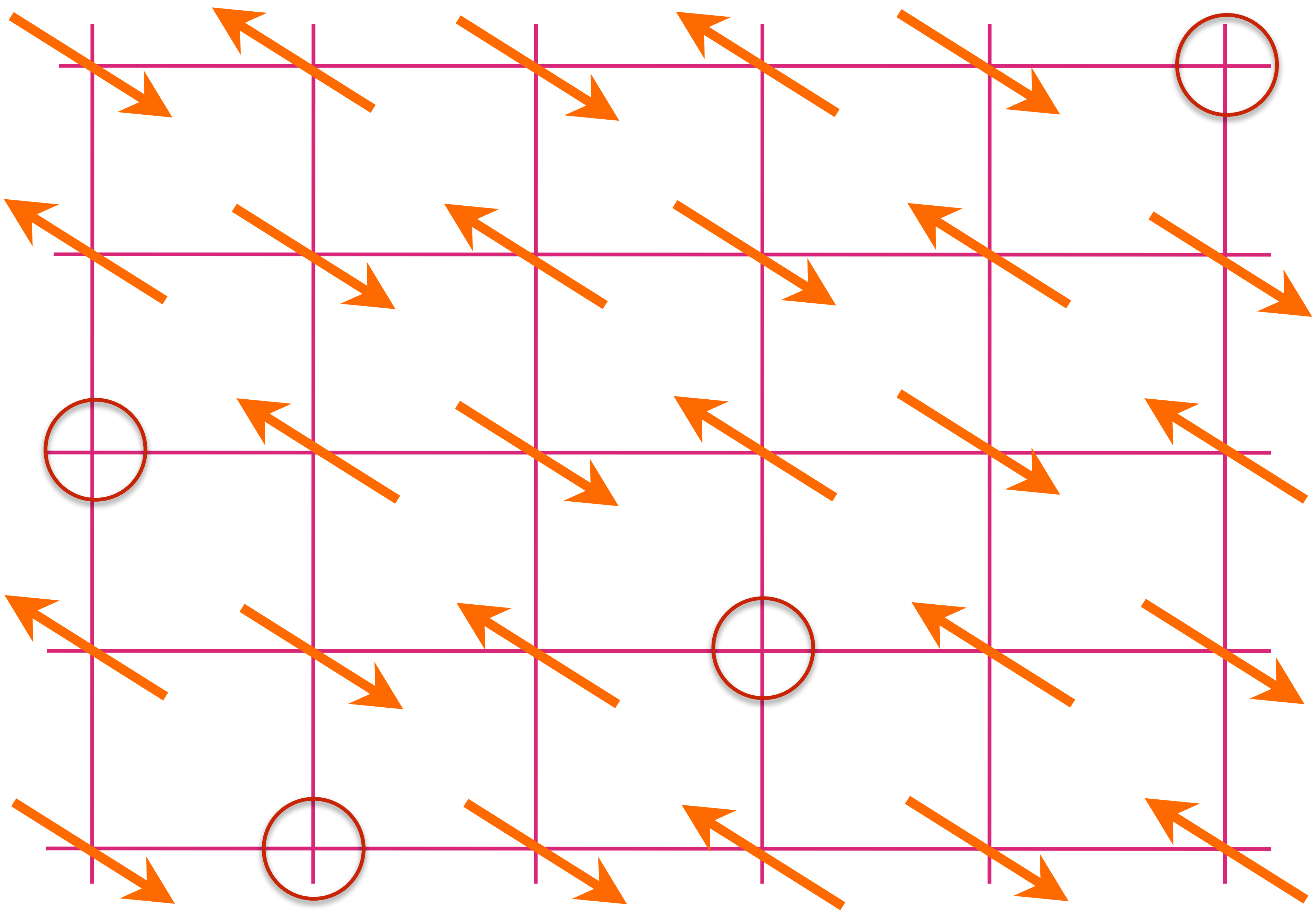
Julian Hetel and Nandini Trivedi, Ohio State University

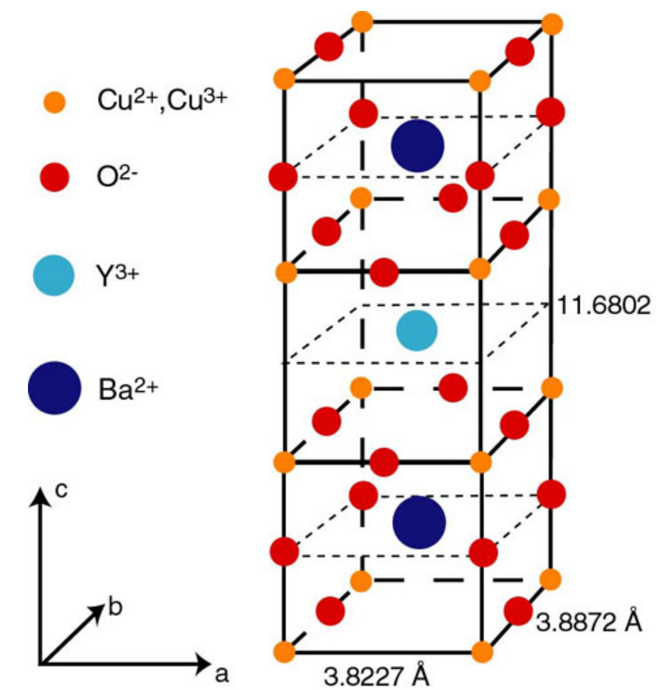
Insulating antiferromagnet

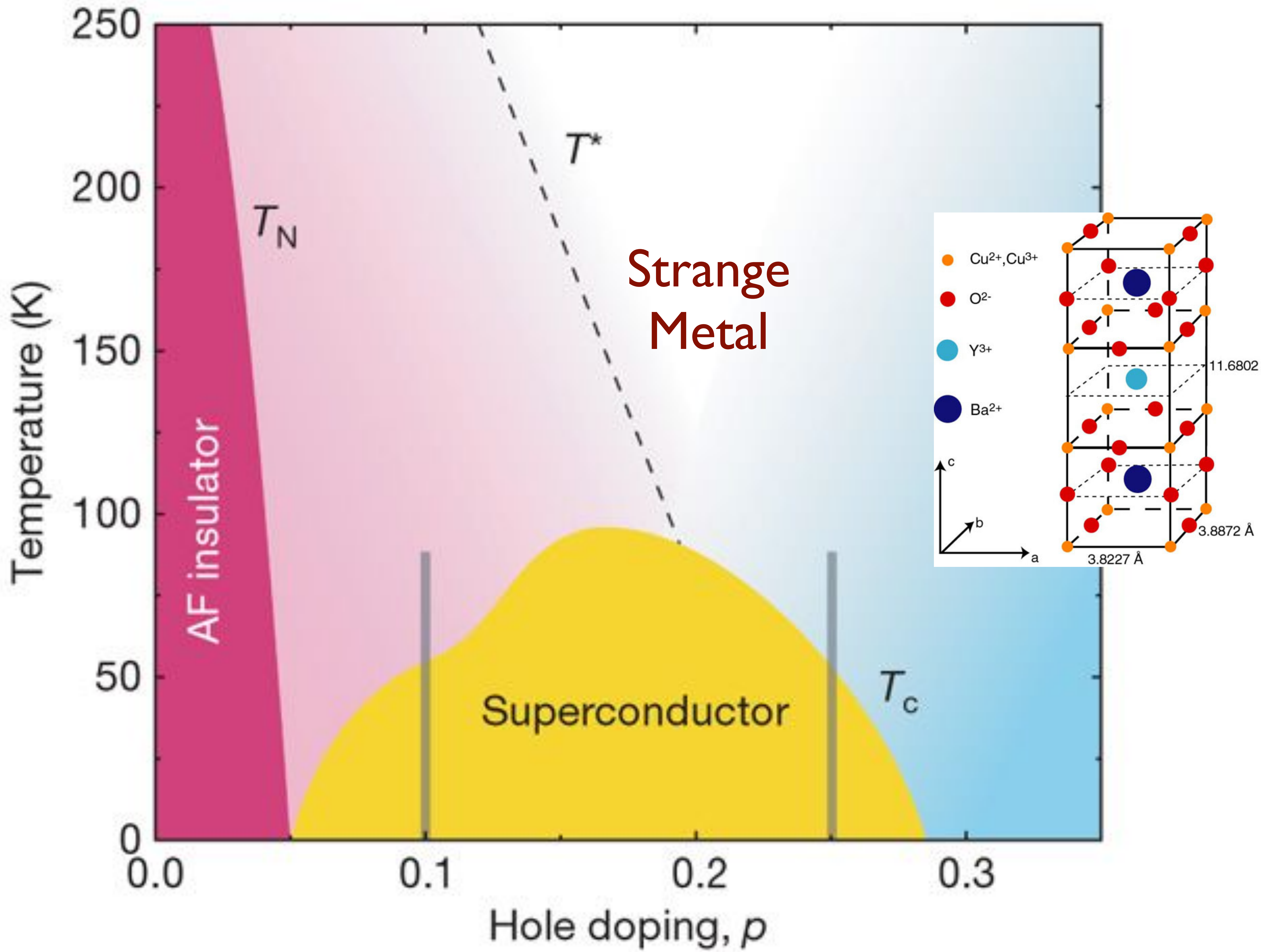




Antiferromagnet doped with hole density p







250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau}$$

with a universal scattering rate

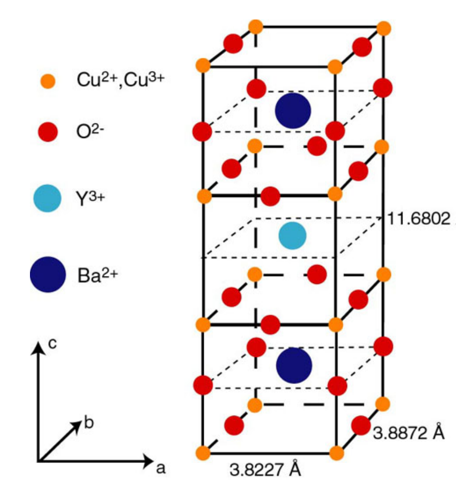
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}\,,$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

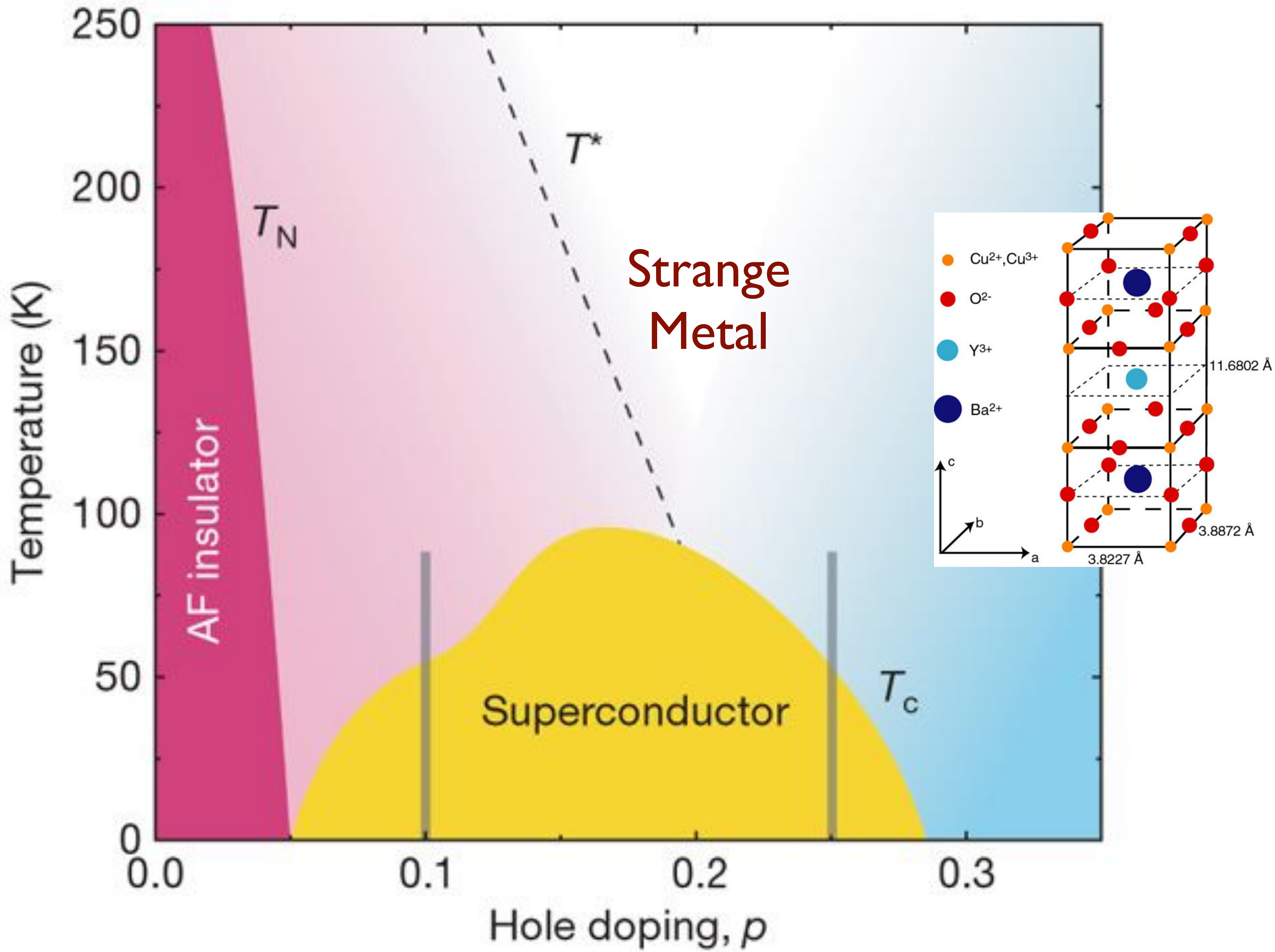


A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates


250
200
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50
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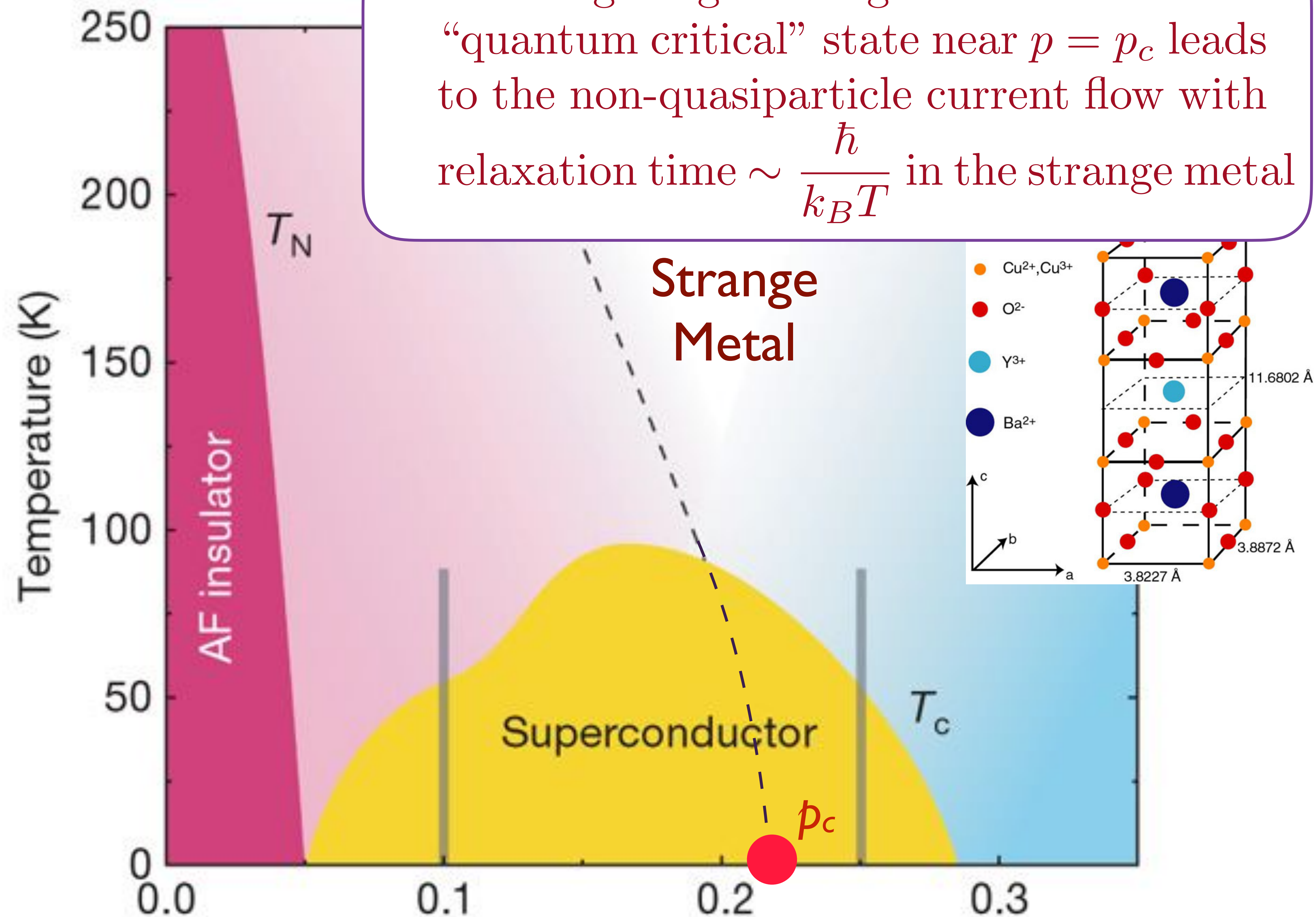
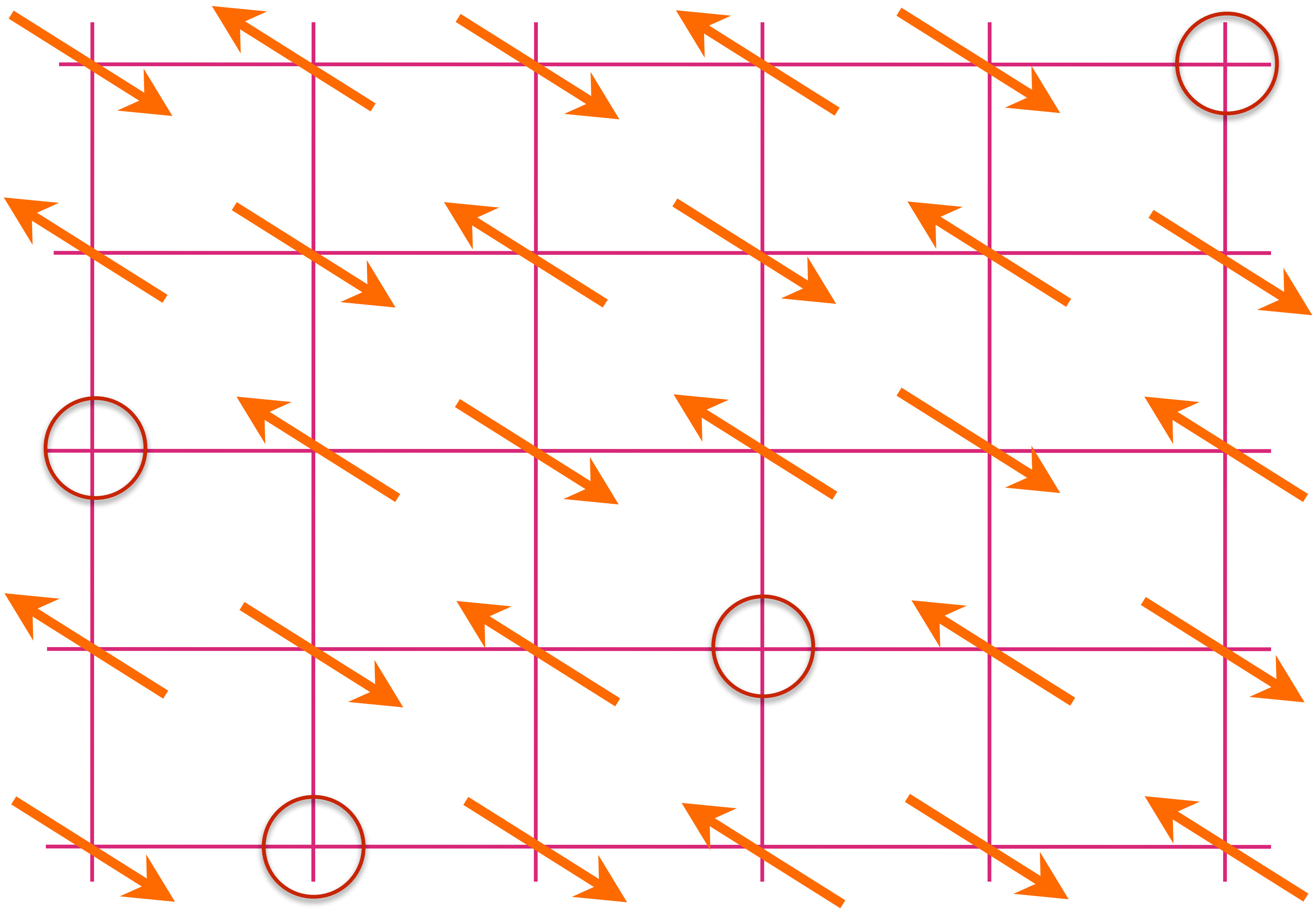
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

- The long-range entanglement in the "quantum critical" state near $p = p_c$ leads to the non-quasiparticle current flow with relaxation time $\sim \dfrac{\hbar}{k_B T}$ in the strange metal



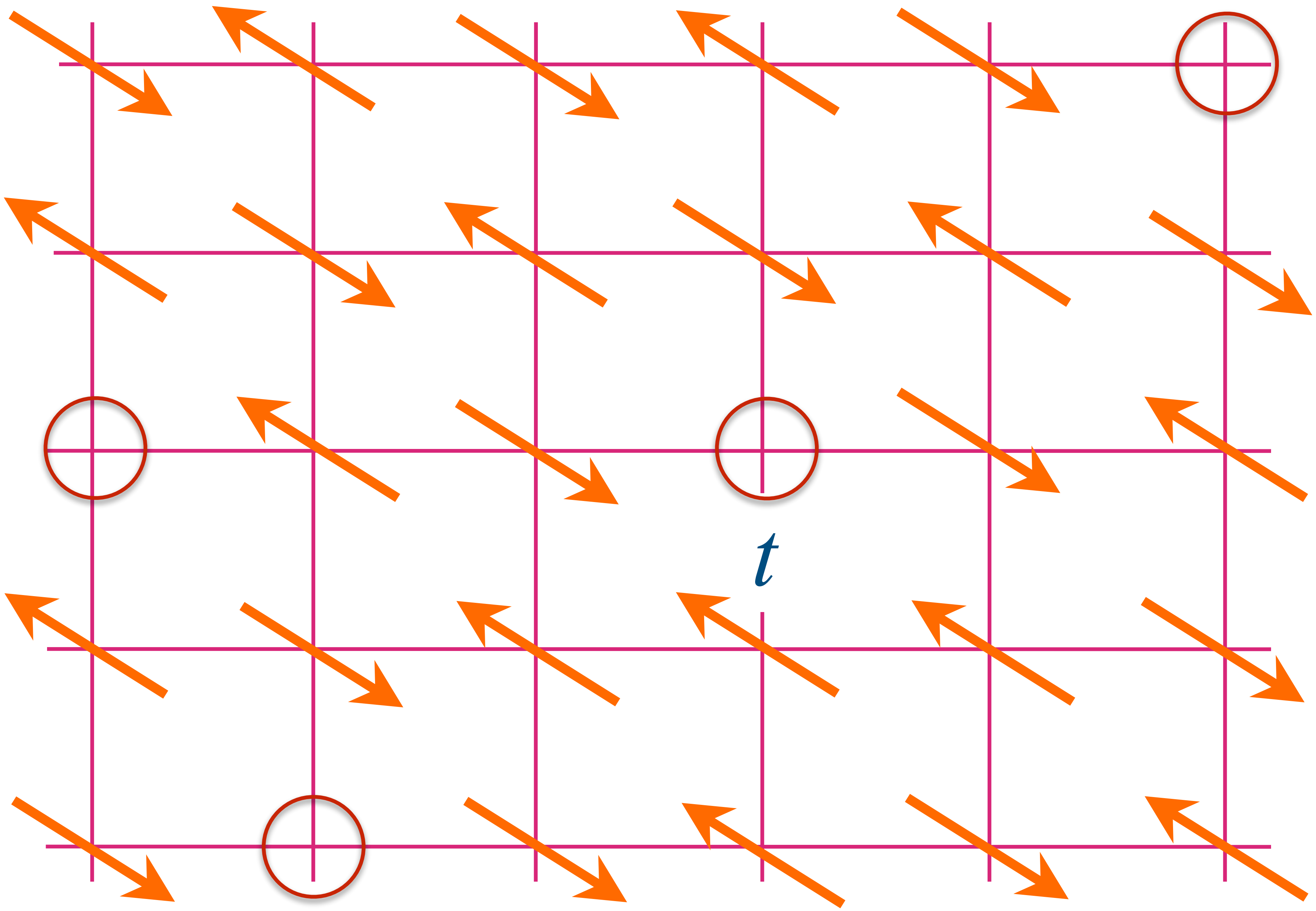
Is there a quantum phase transition at a critical $p=p_c$?

Real-space view at small p



p mobile holes in a background of fluctuating spins

Real-space view at small p



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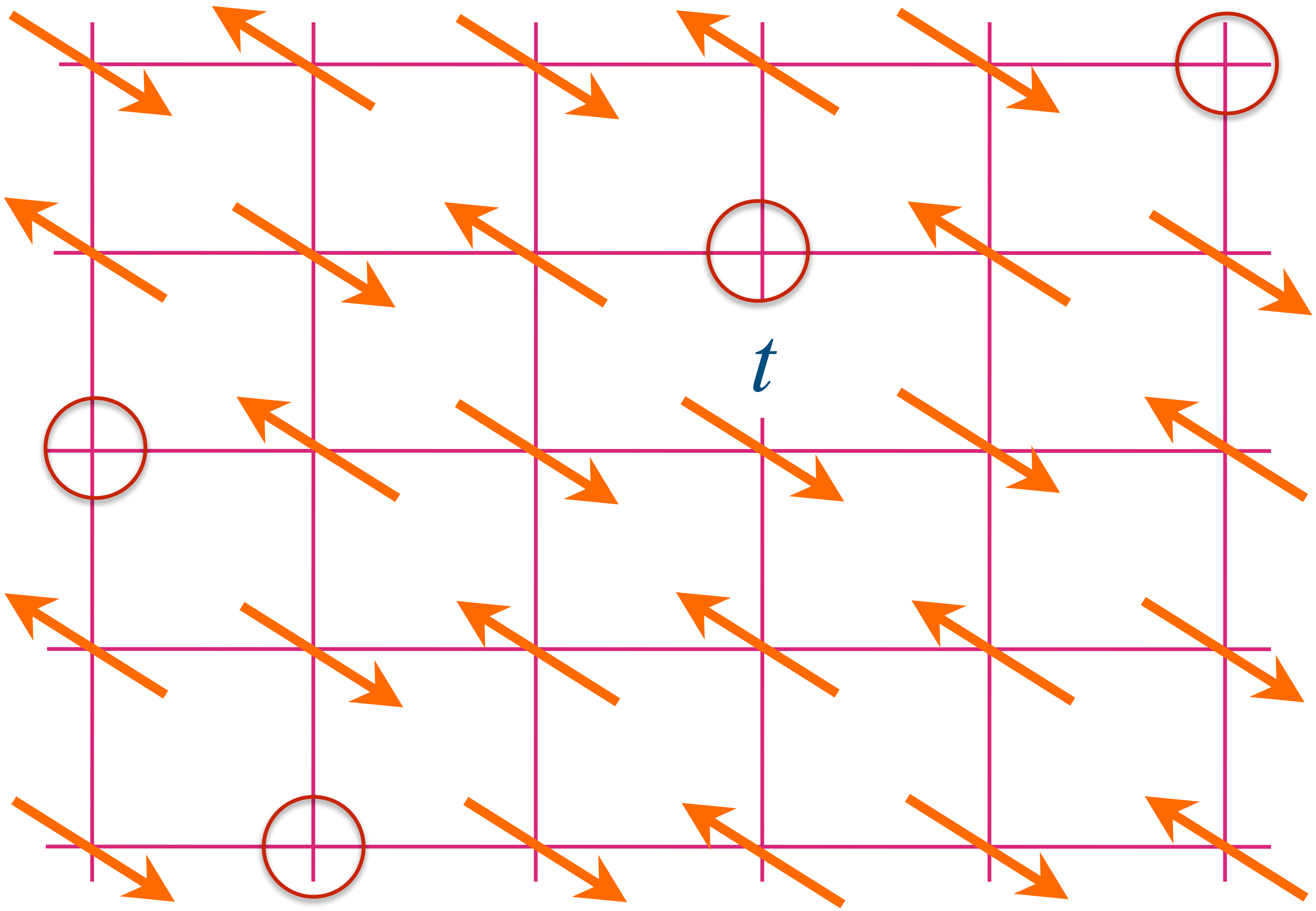
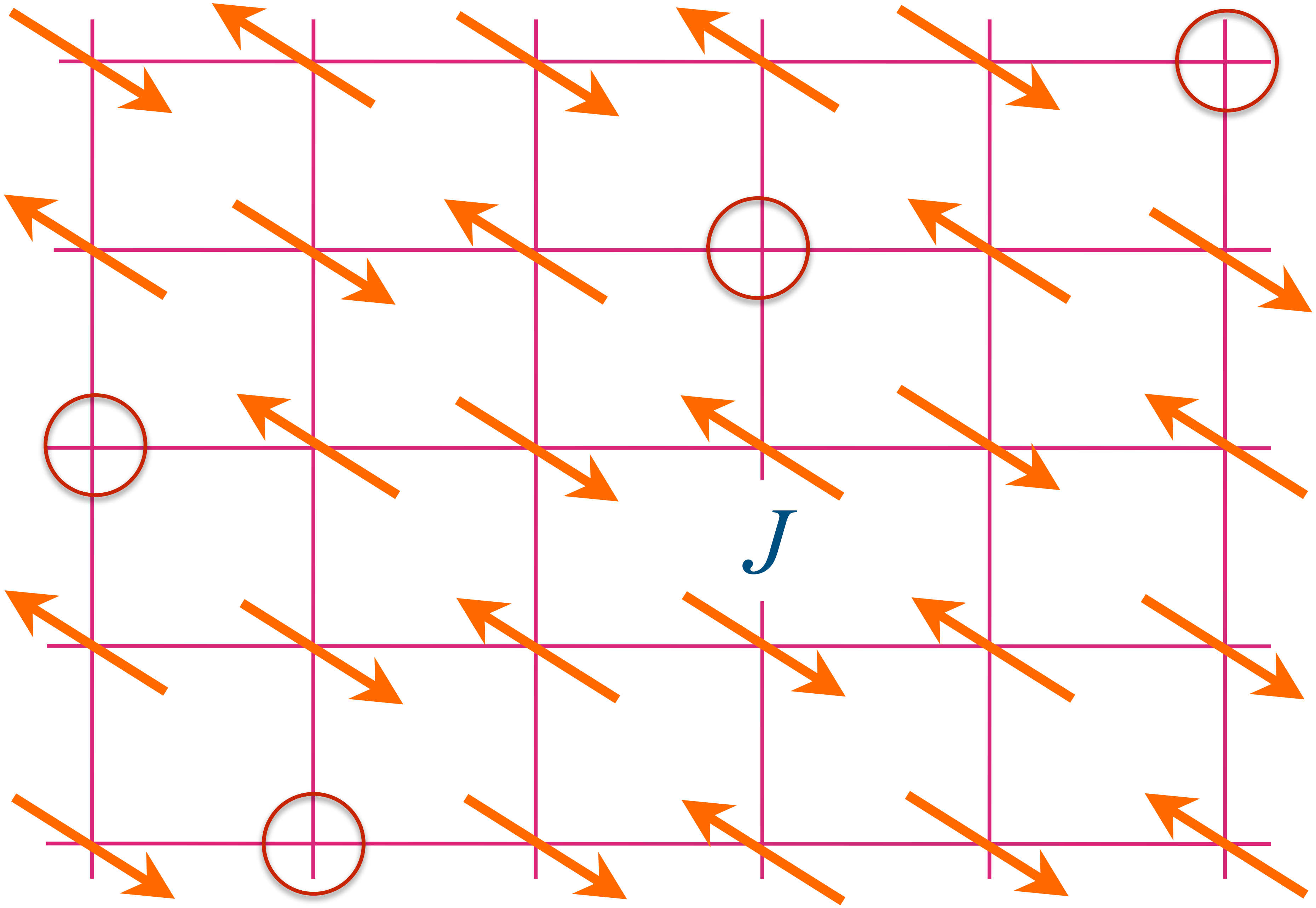


p mobile holes in a background of fluctuating spins

Real-space view at small p



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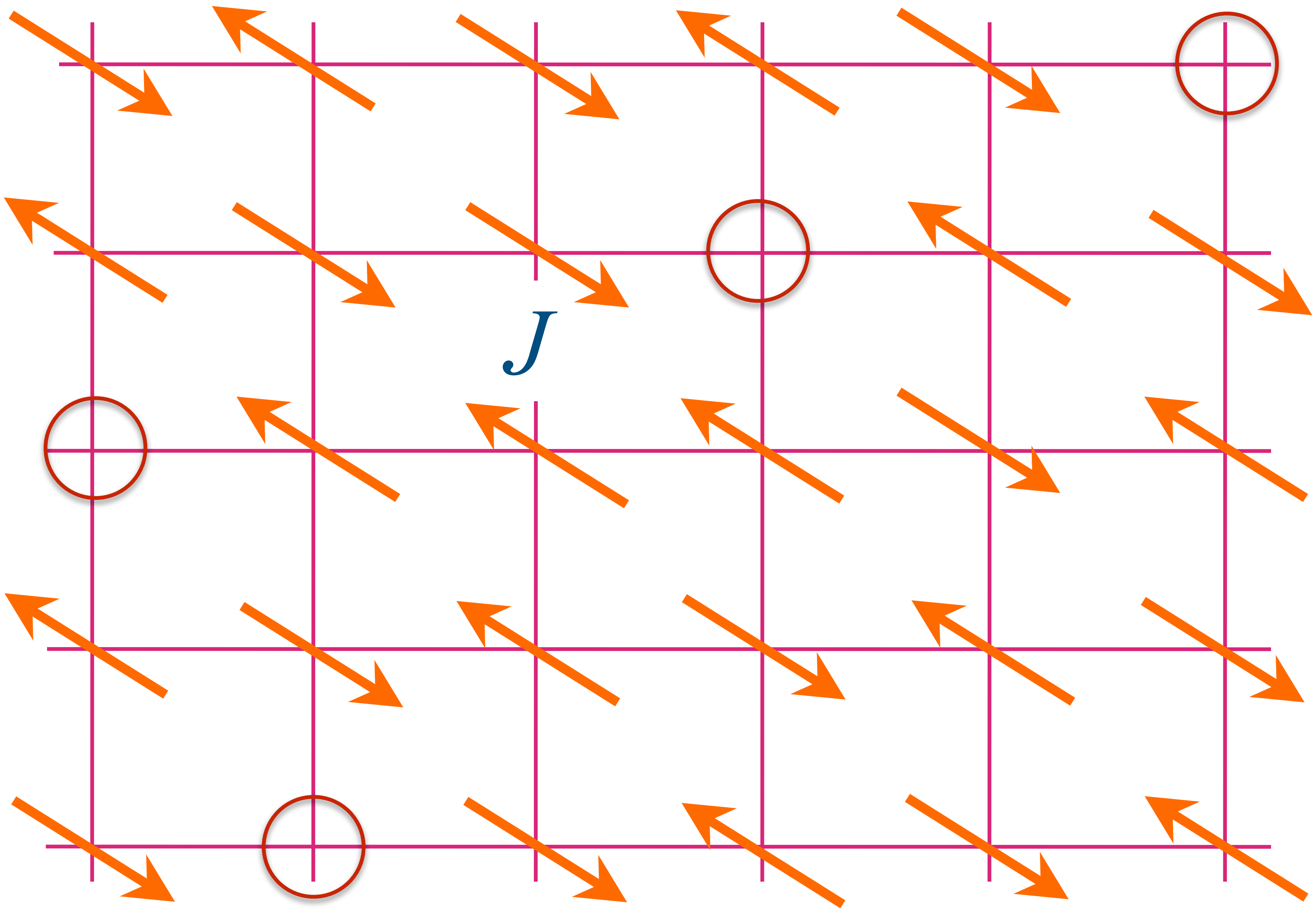
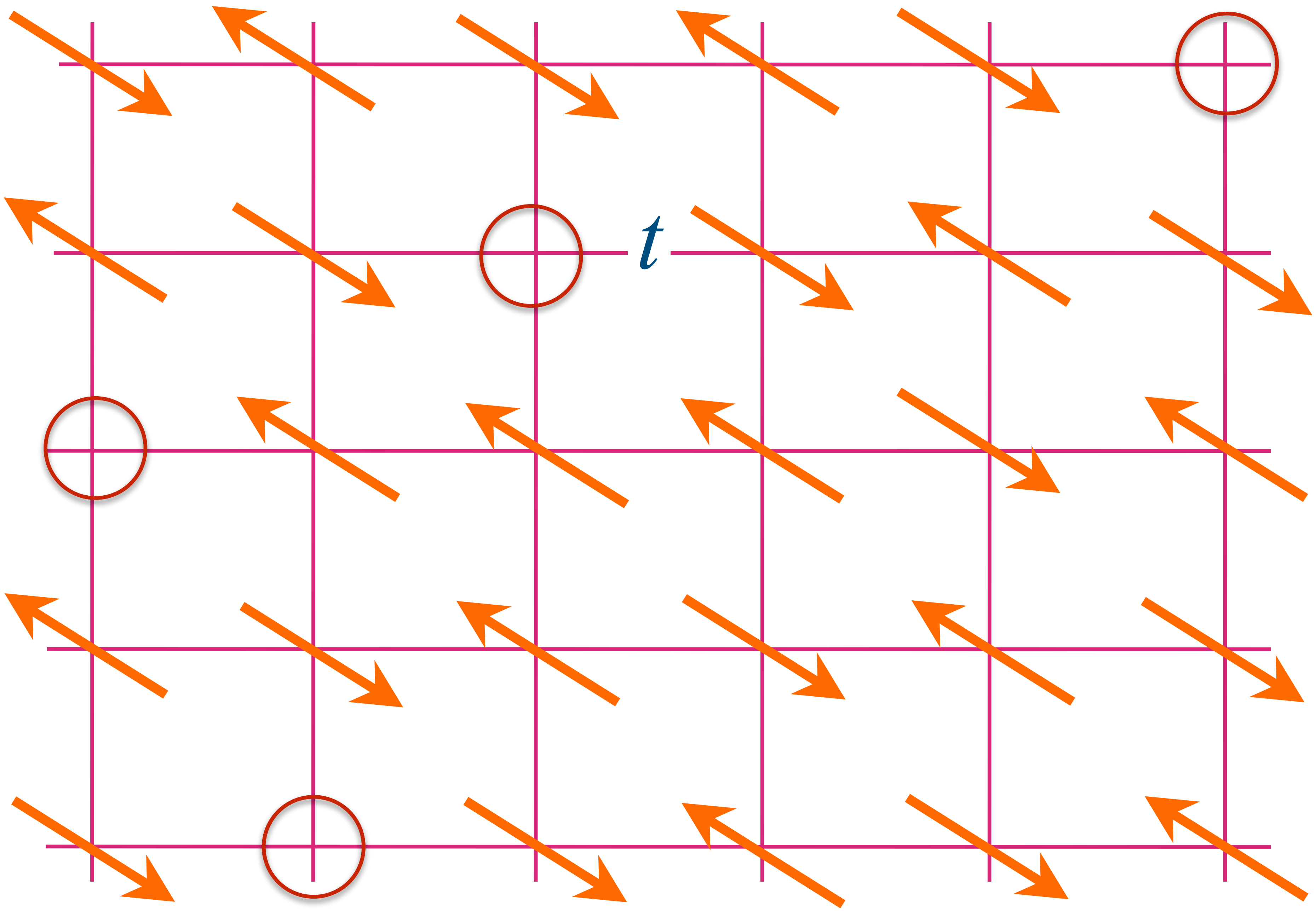
Real-space view at small p



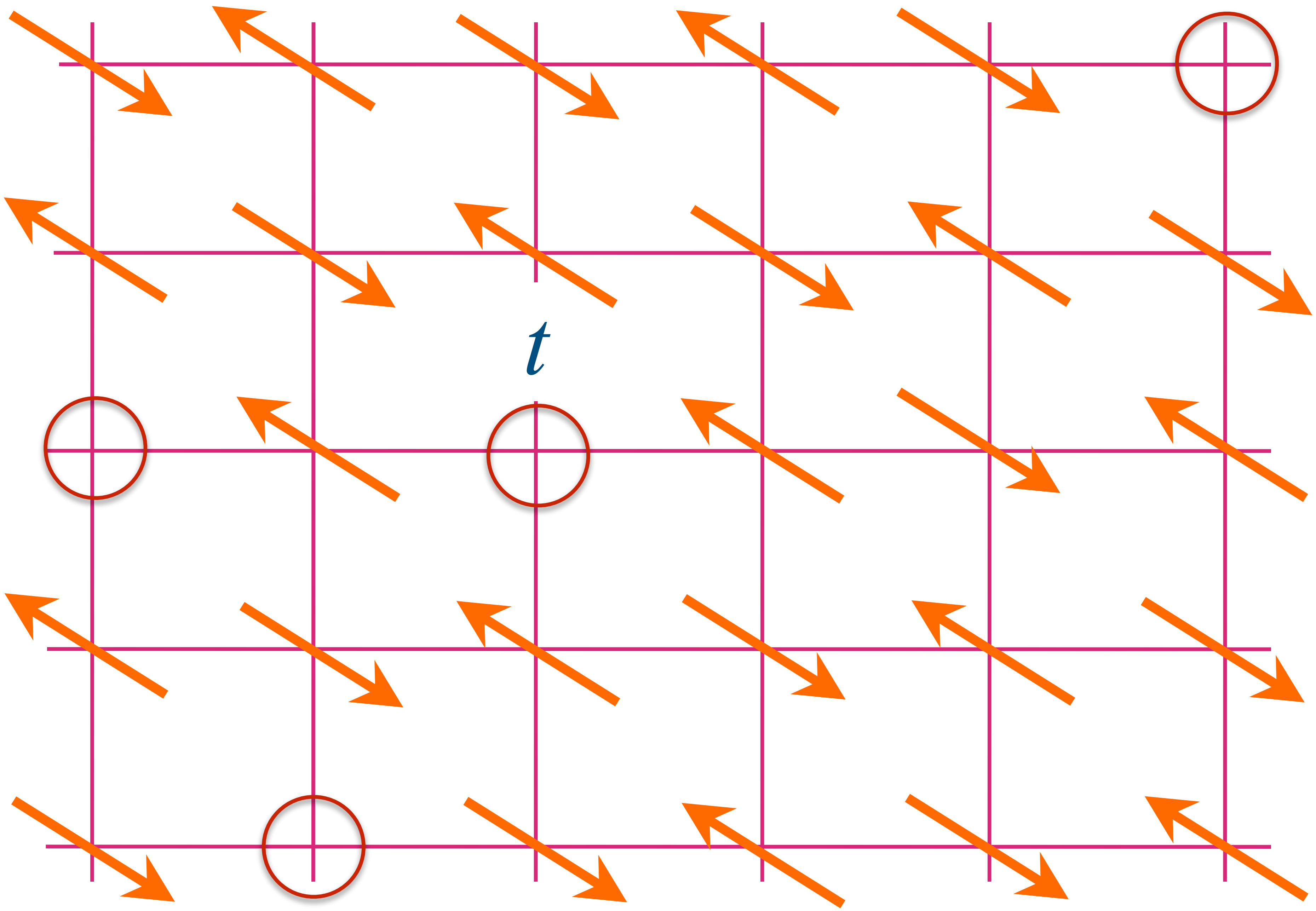
p mobile holes in a background of fluctuating spins

Real-space view at small p



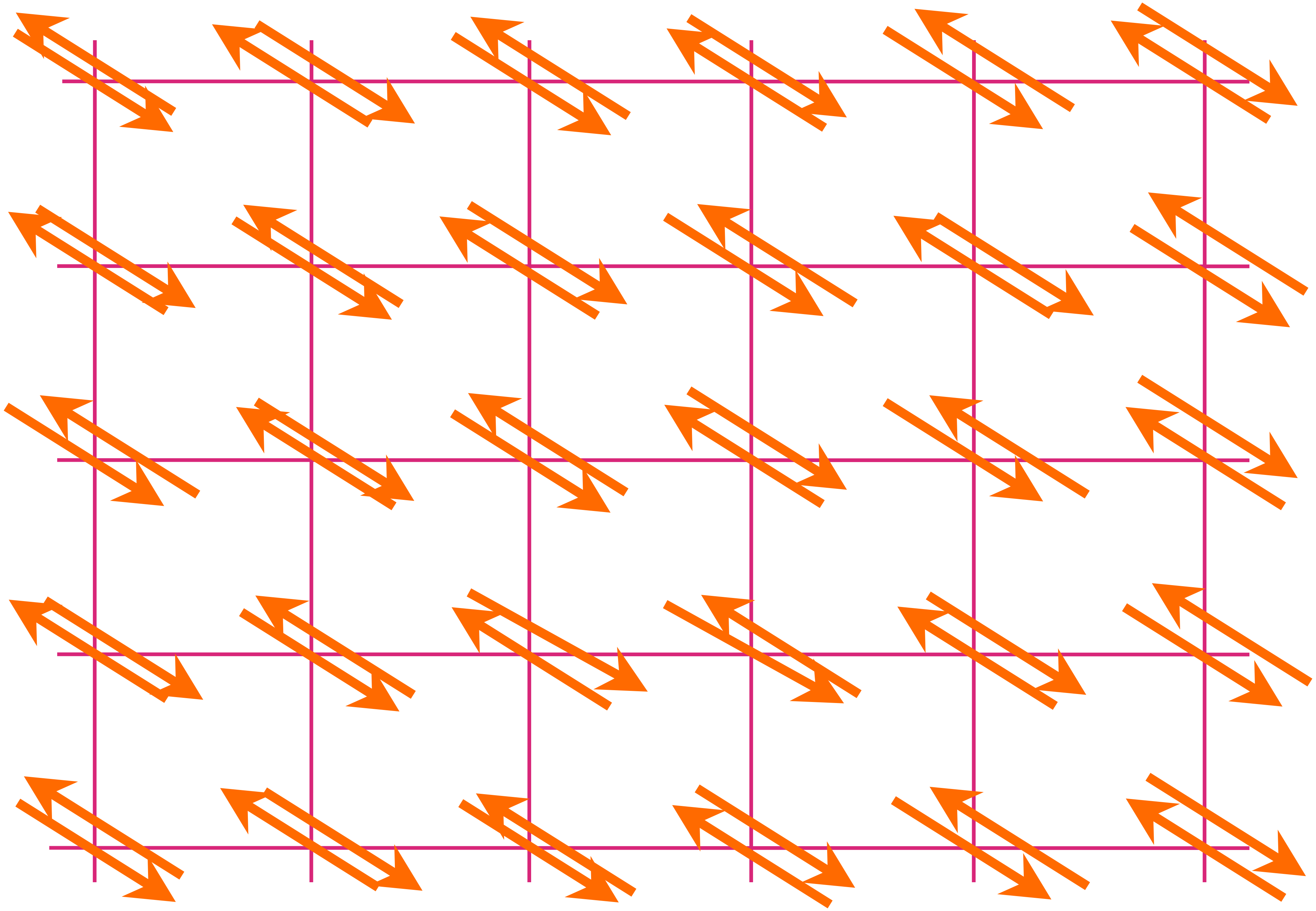
p mobile holes in a background of fluctuating spins

Real-space view at small p



p mobile holes in a background of fluctuating spins

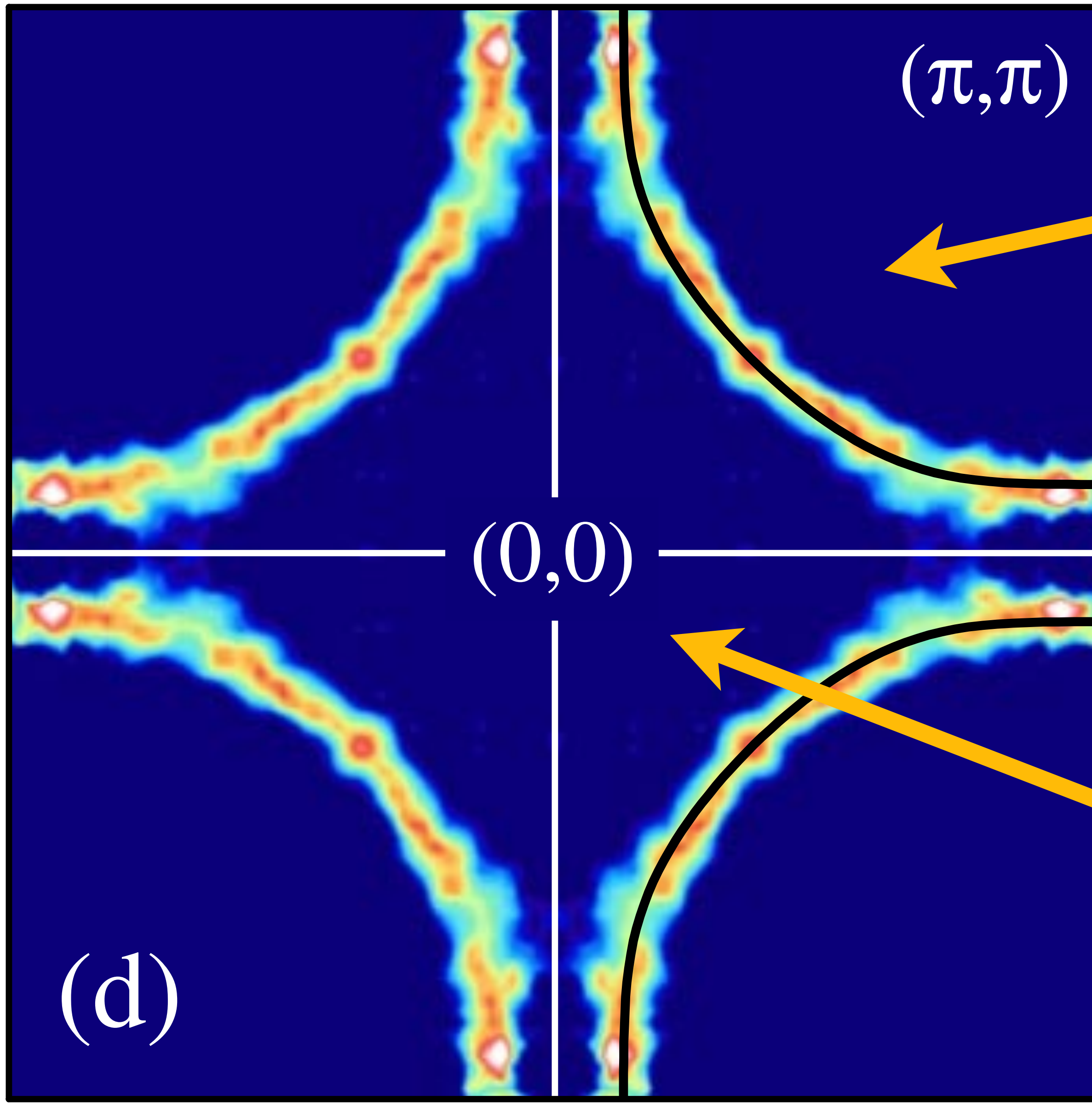
Momentum-space view at large p



Filled Band

$1+p$ mobile holes in a filled band

Momentum-space view at large p



$1+p$ holes

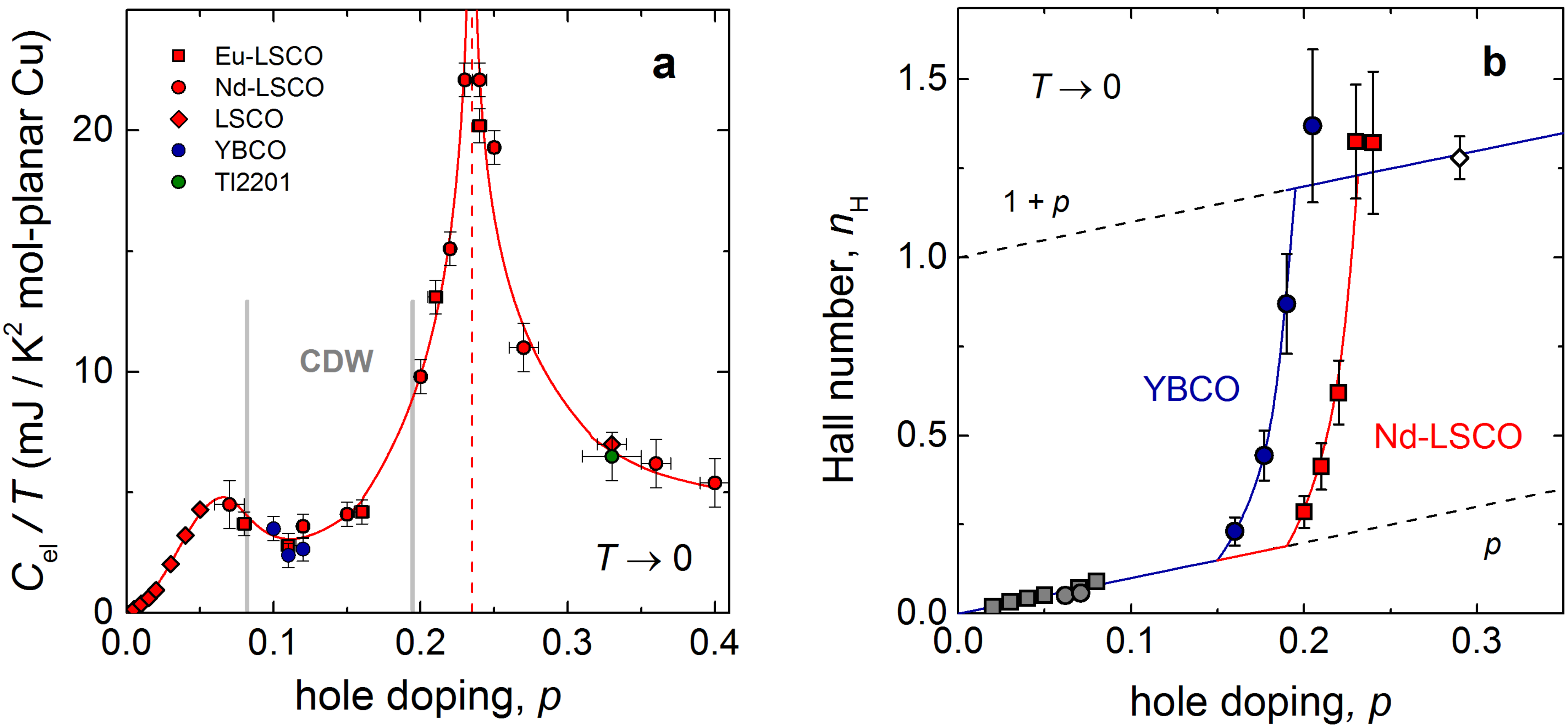
1-p electrons

$1+p$ mobile holes in a filled band

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

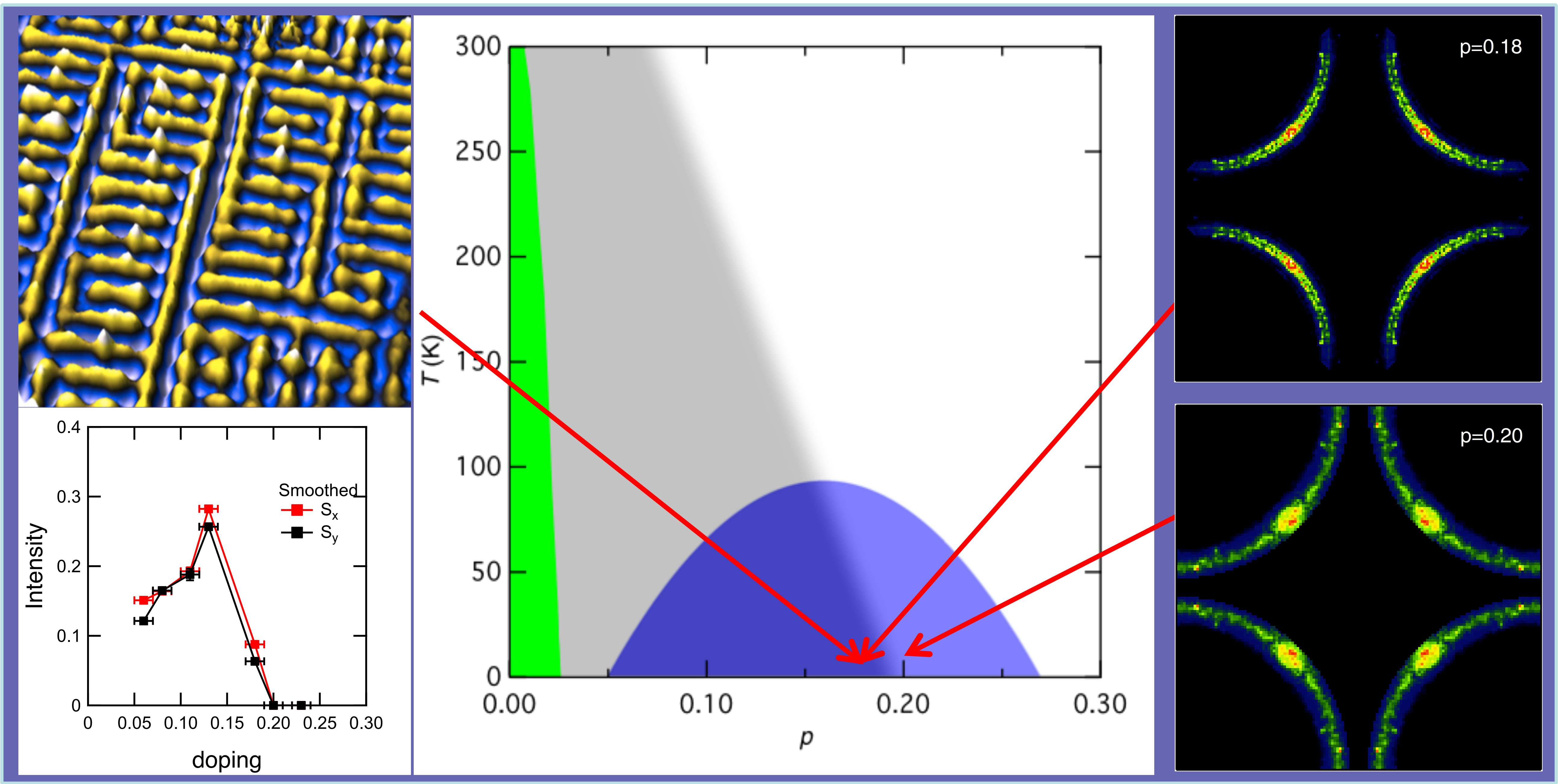
Cyril Proust and Louis Taillefer, arXiv:1807.0507



Hole doped cuprates

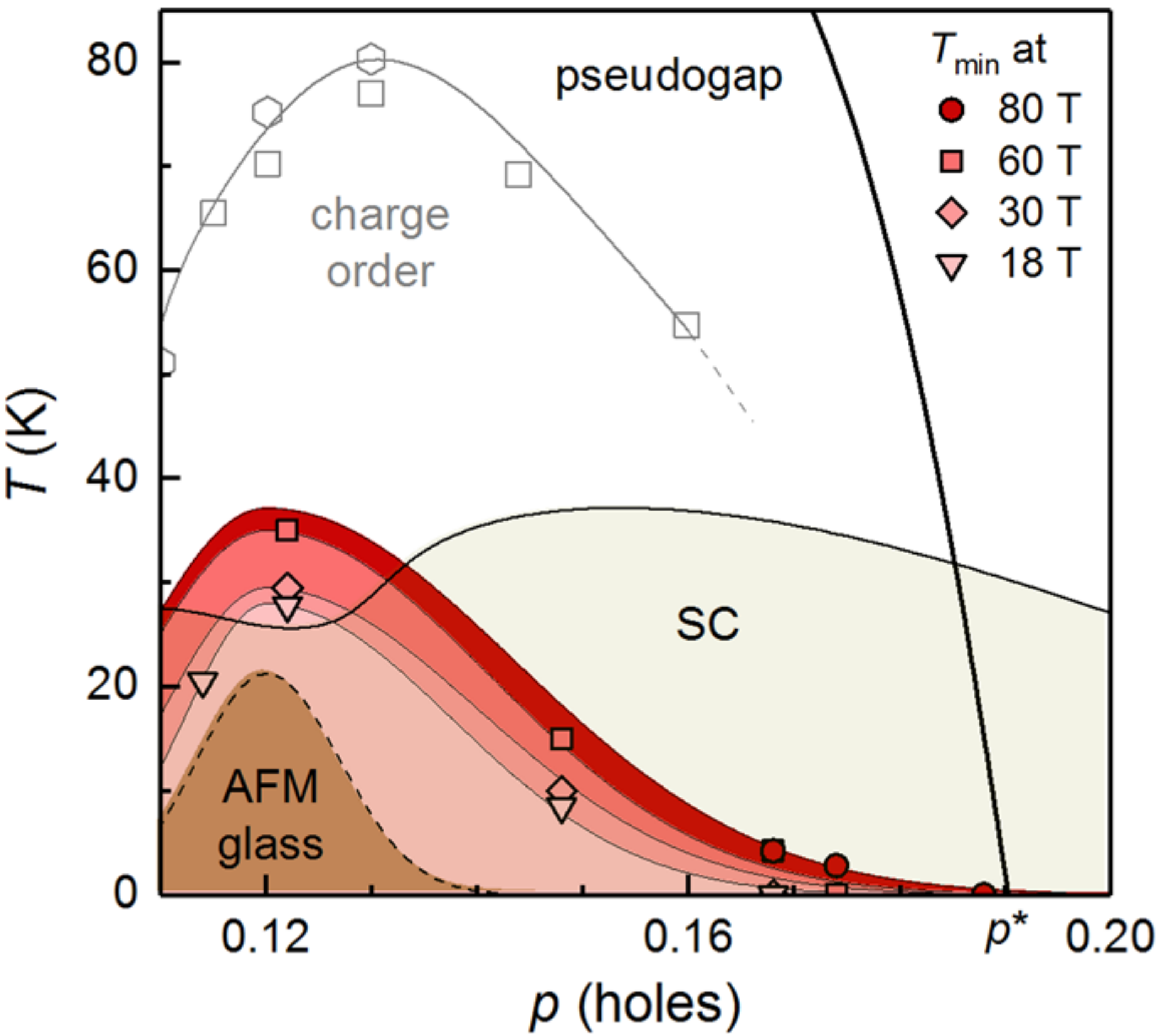
Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

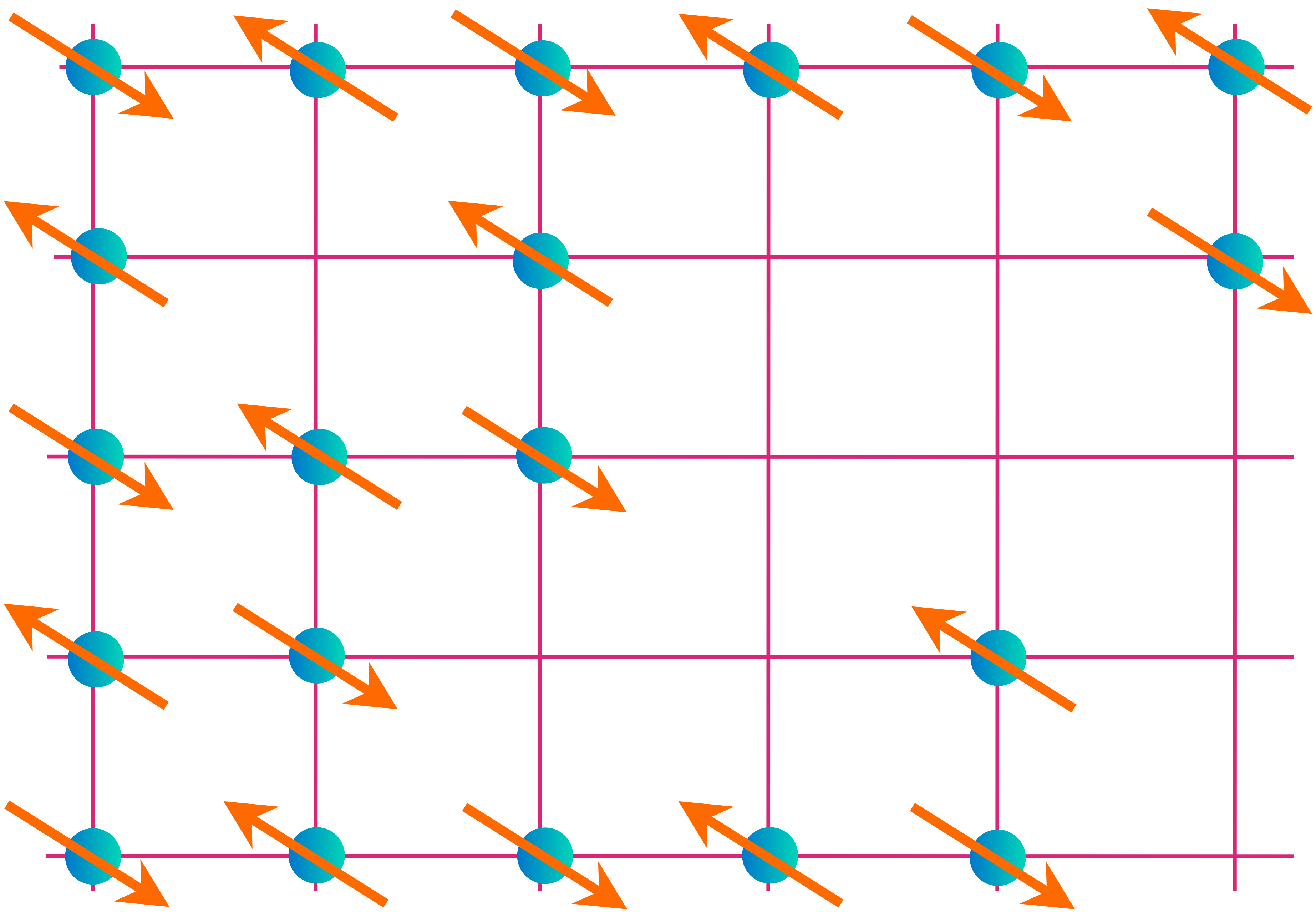
Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



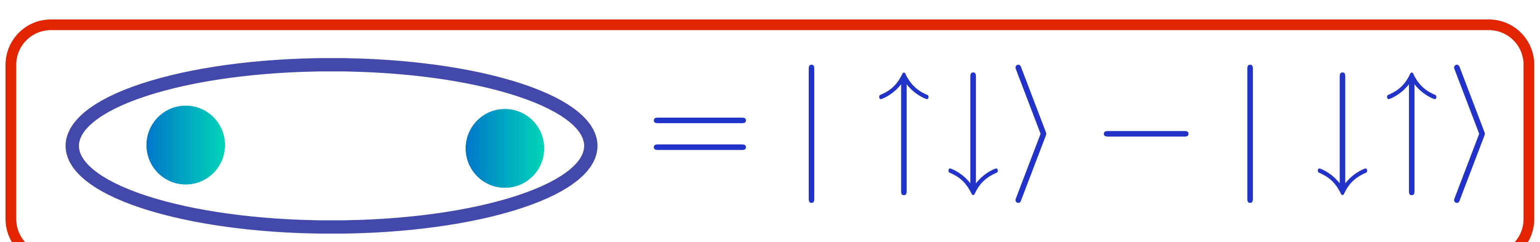
arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B{=}0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

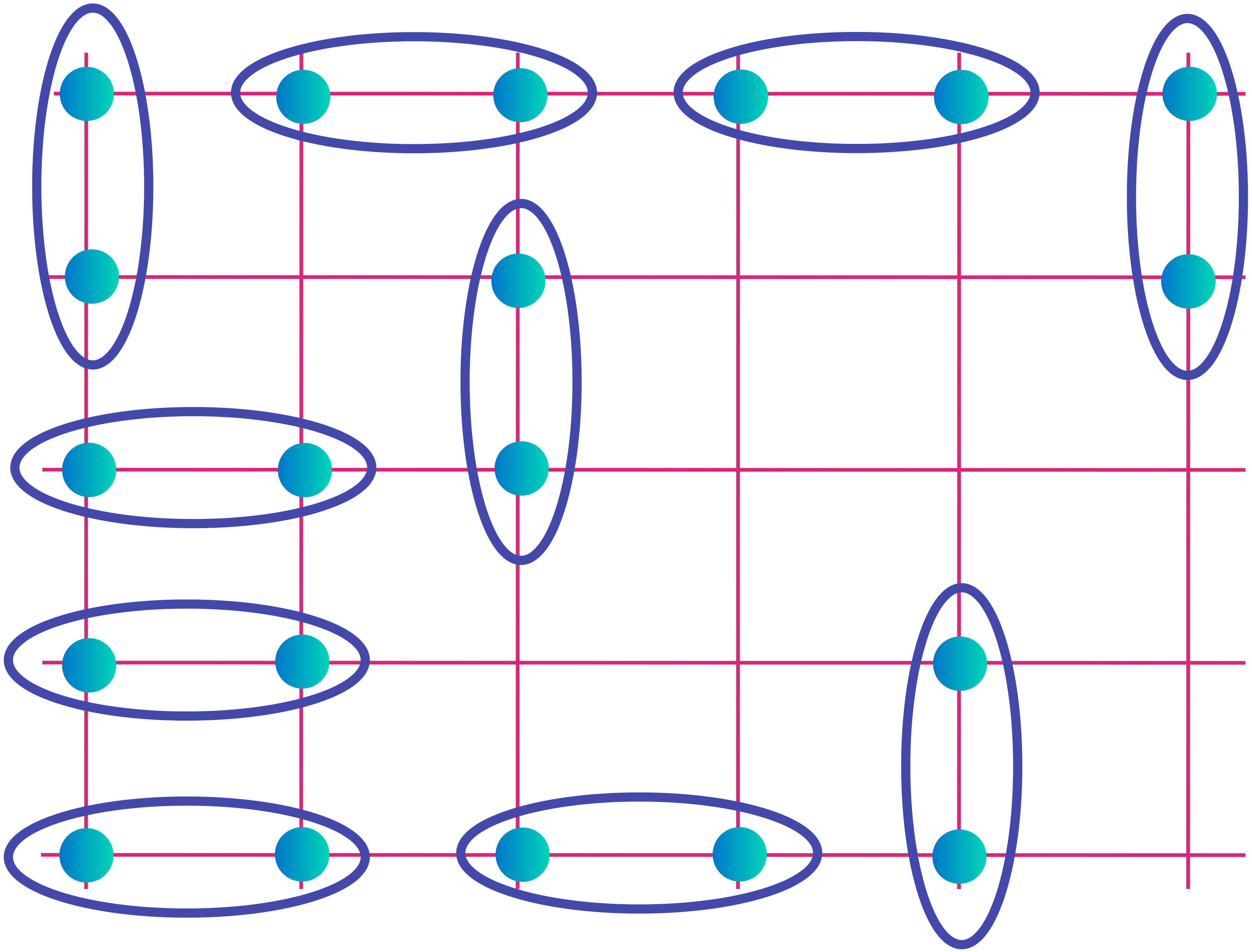
Square lattice of Cu sites at $p=p_c$



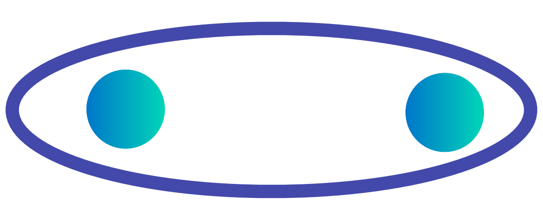
Remove fraction p electrons



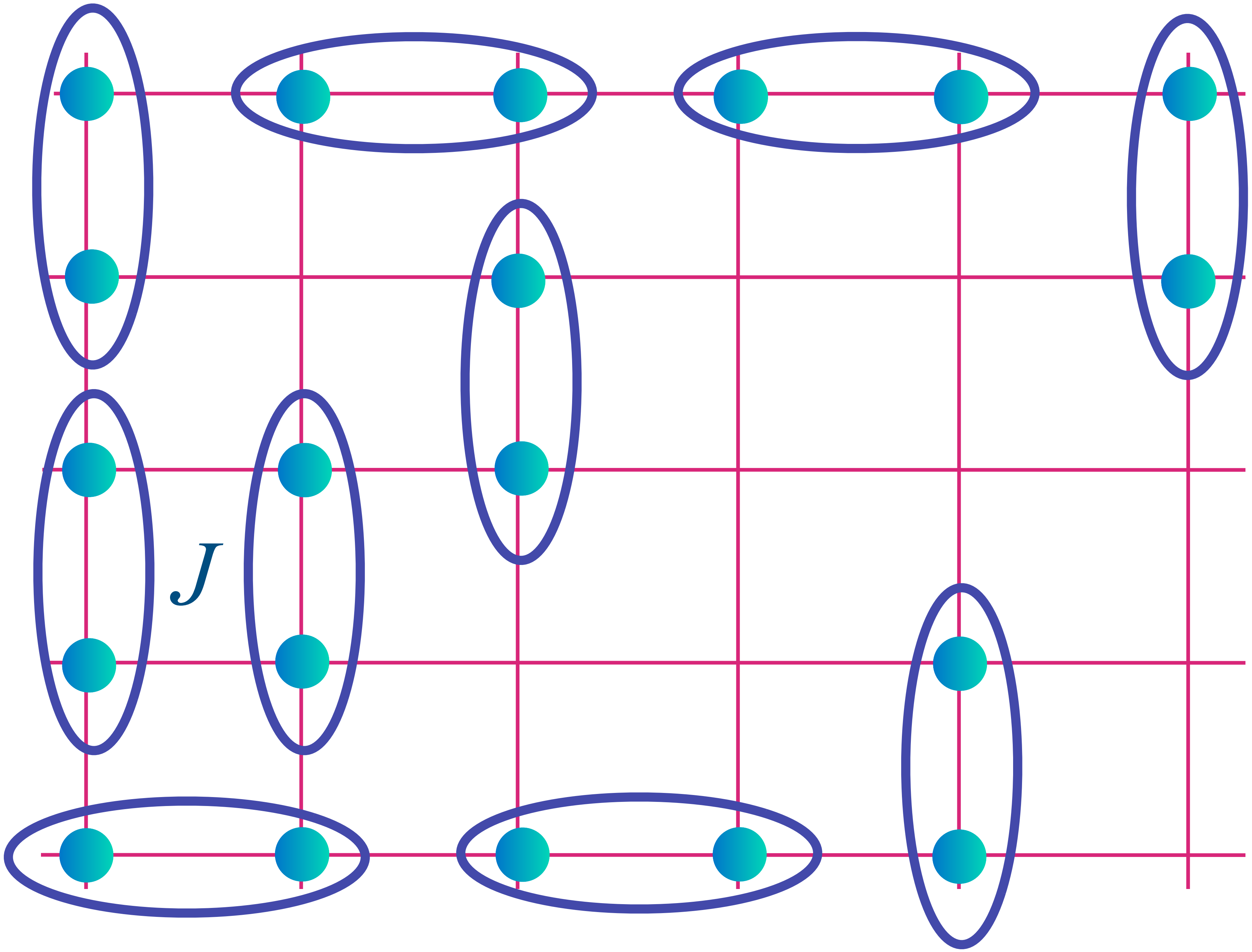
Square lattice of Cu sites at $p = p_c$



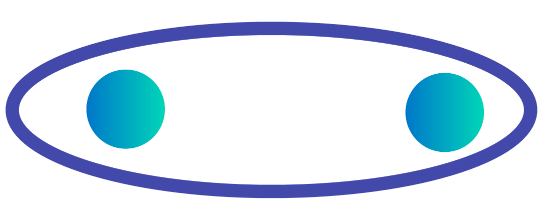
Electrons entangle in ("Cooper") pairs into chemical bonds

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites at $p=p_c$

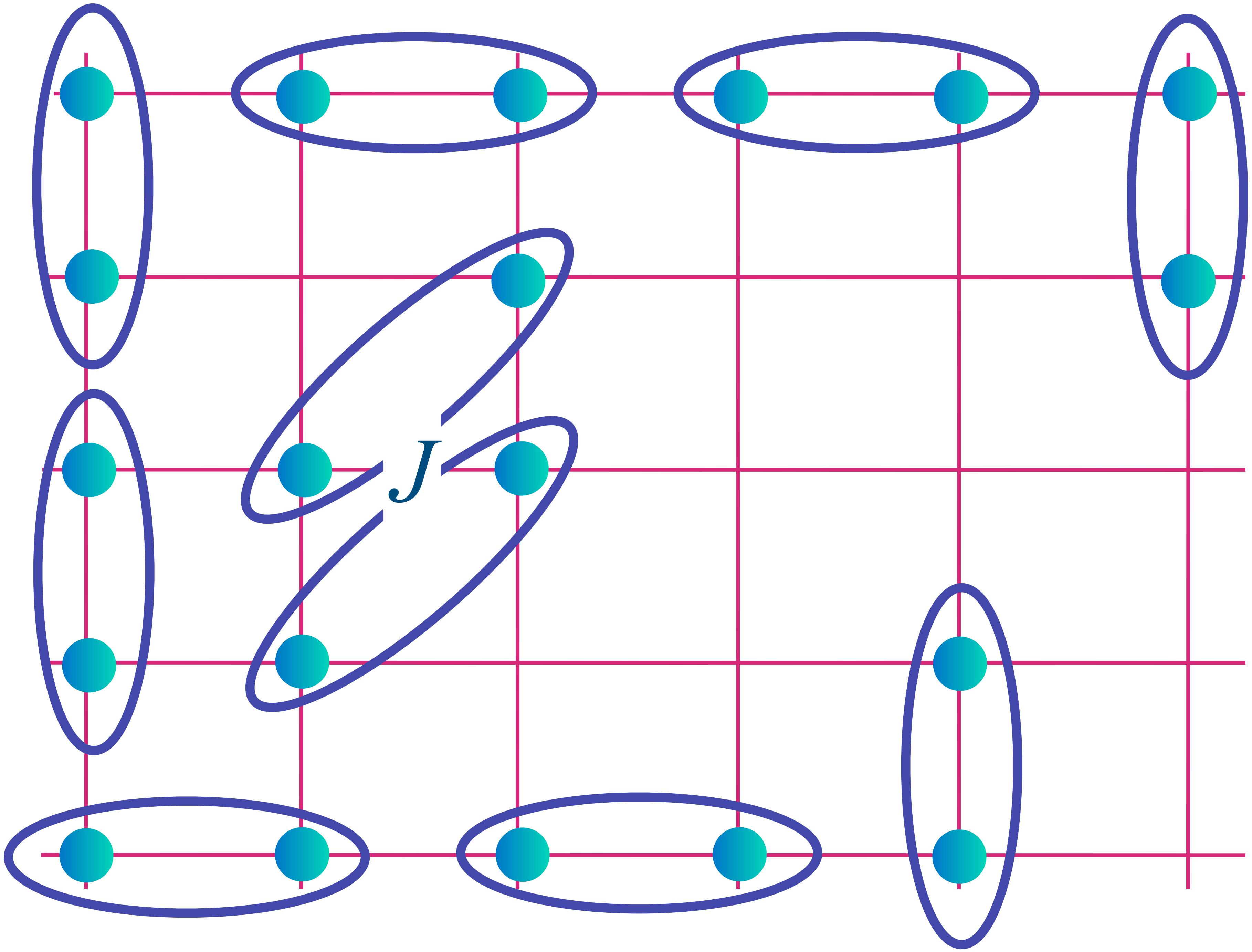


Electrons <u>entangle</u> “en masse” by exchanging partners, and there is <u>long-range quantum entanglement</u>

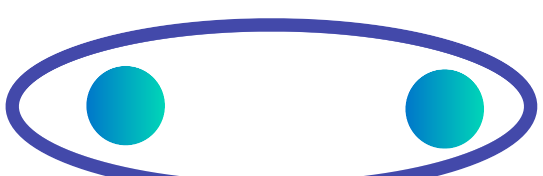


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites at $p=p_c$

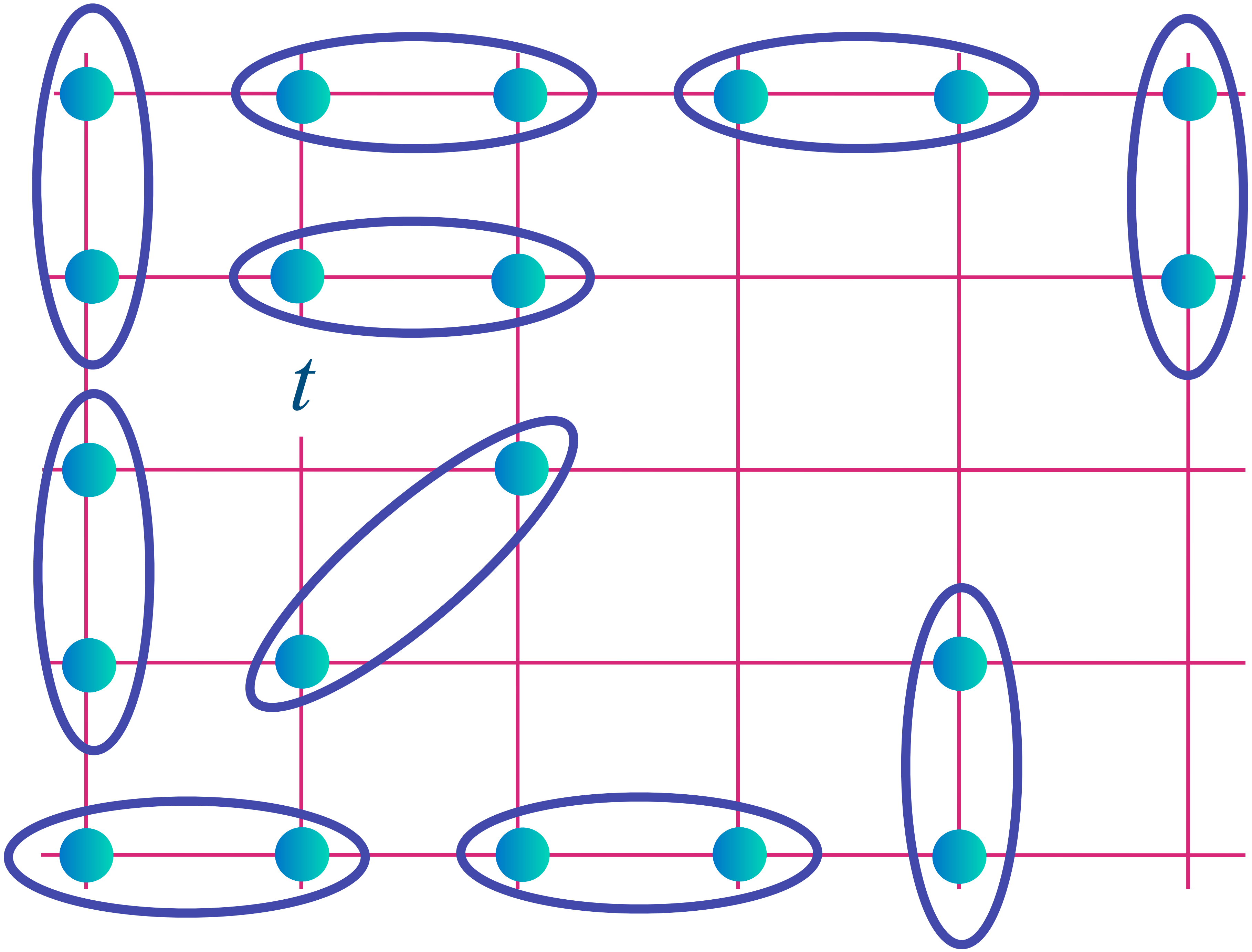


Electrons entangle "en masse" by exchanging partners, and there is long-range quantum entanglement

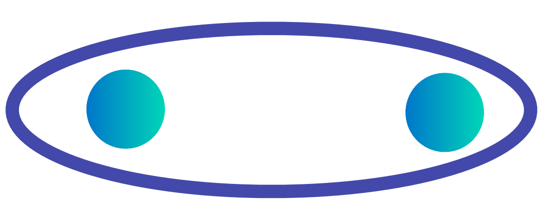


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites at $p=p_c$



Electrons <u>entangle</u> "en masse" by exchanging partners, and there is <u>long-range quantum entanglement</u>



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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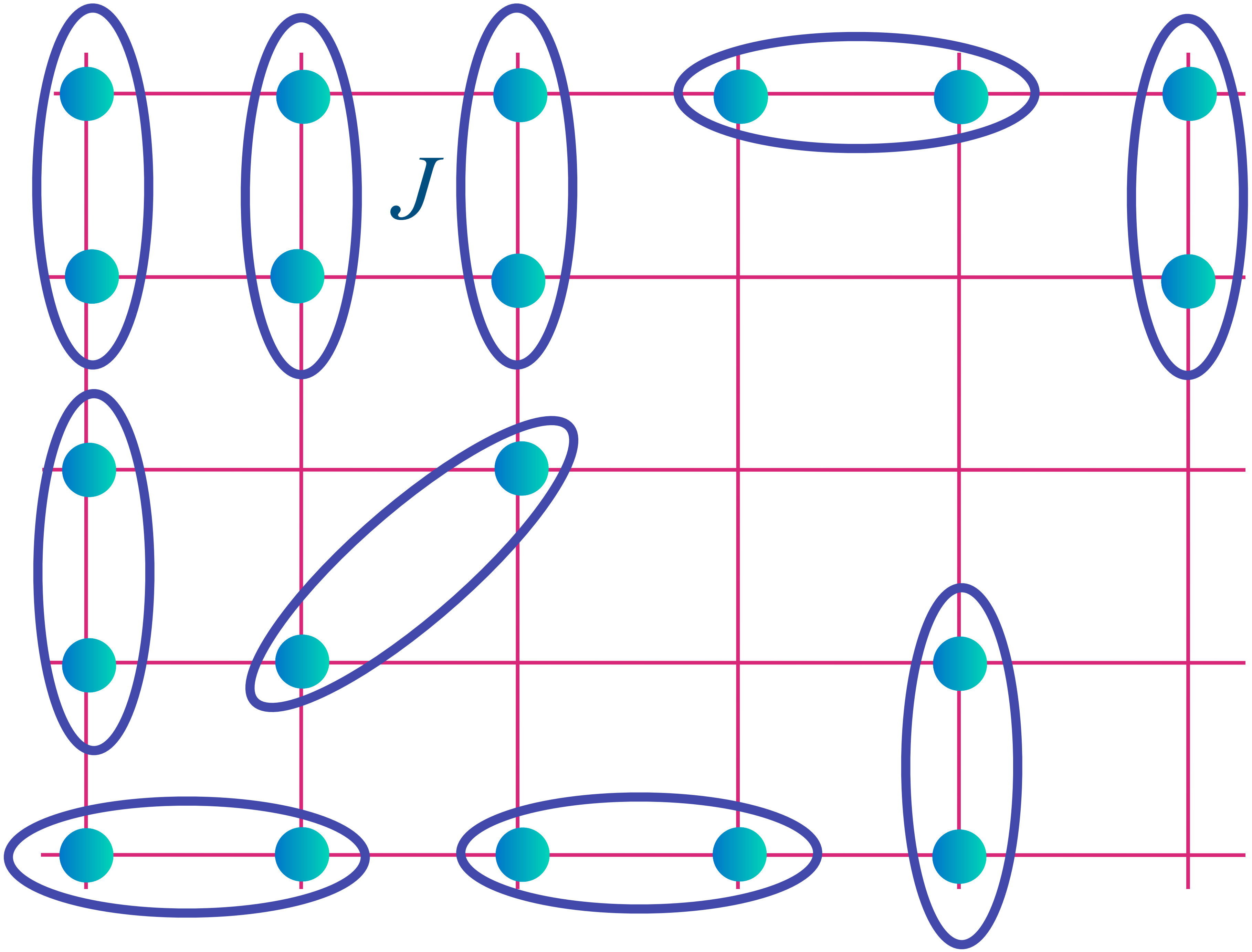

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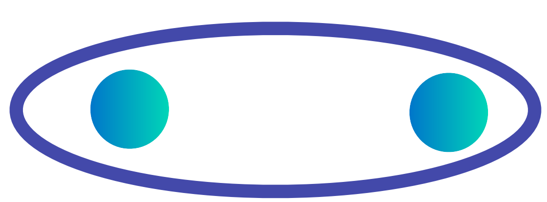

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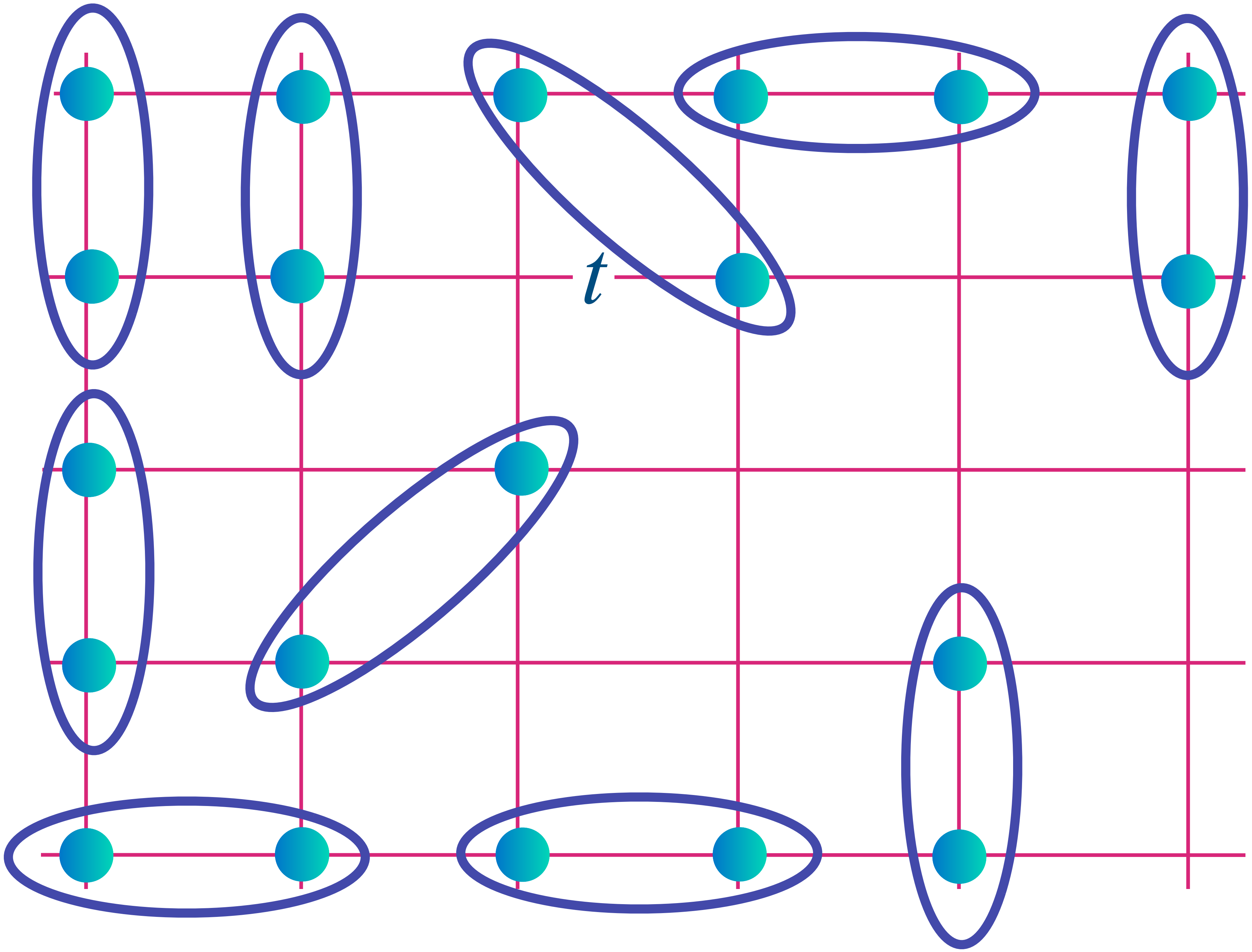

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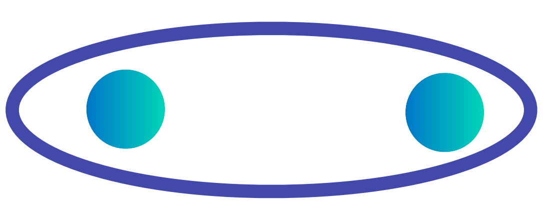
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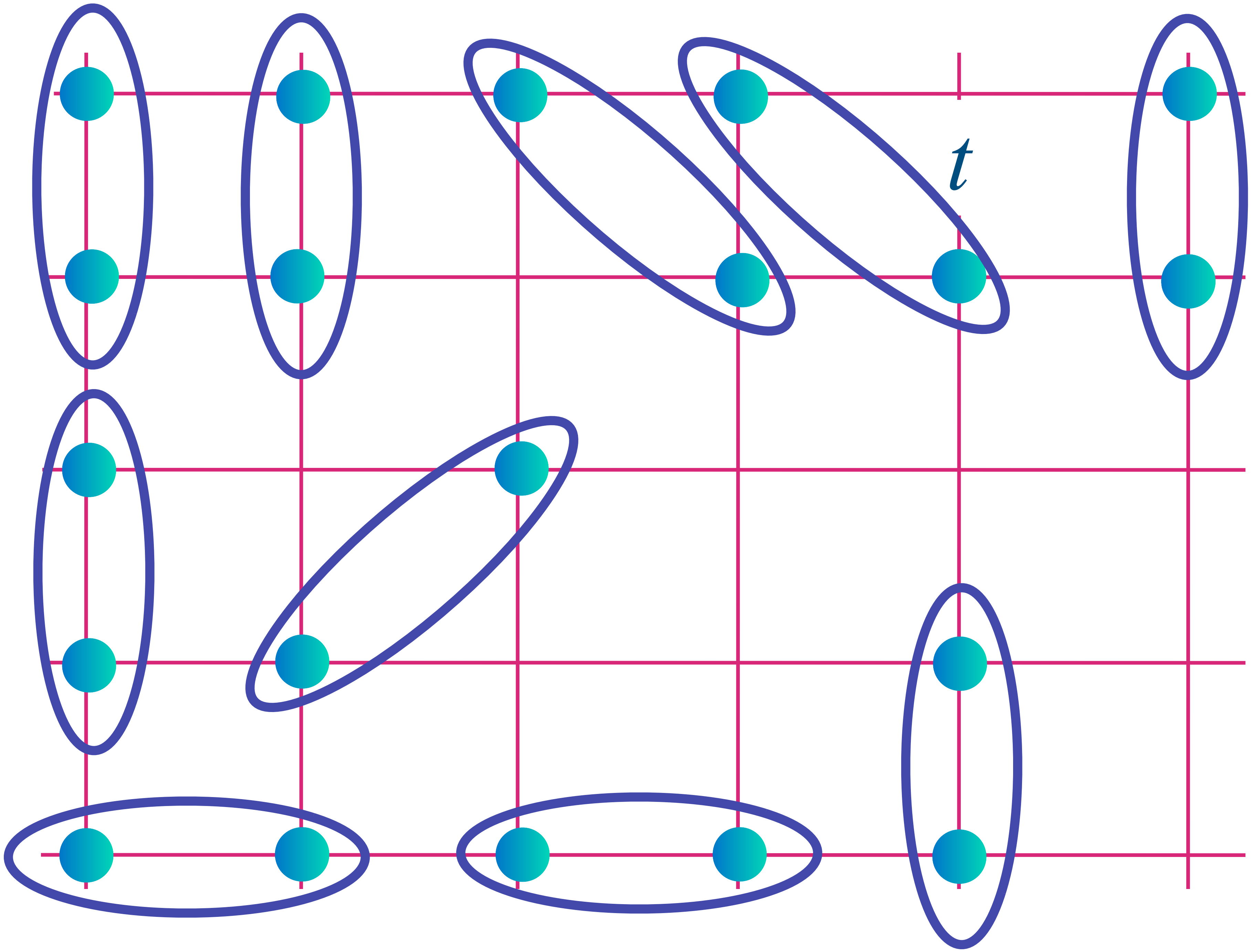

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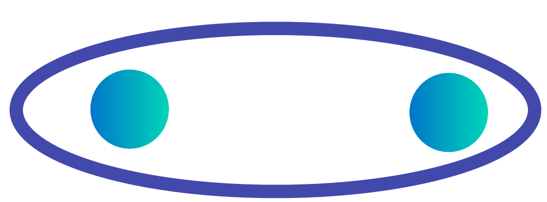

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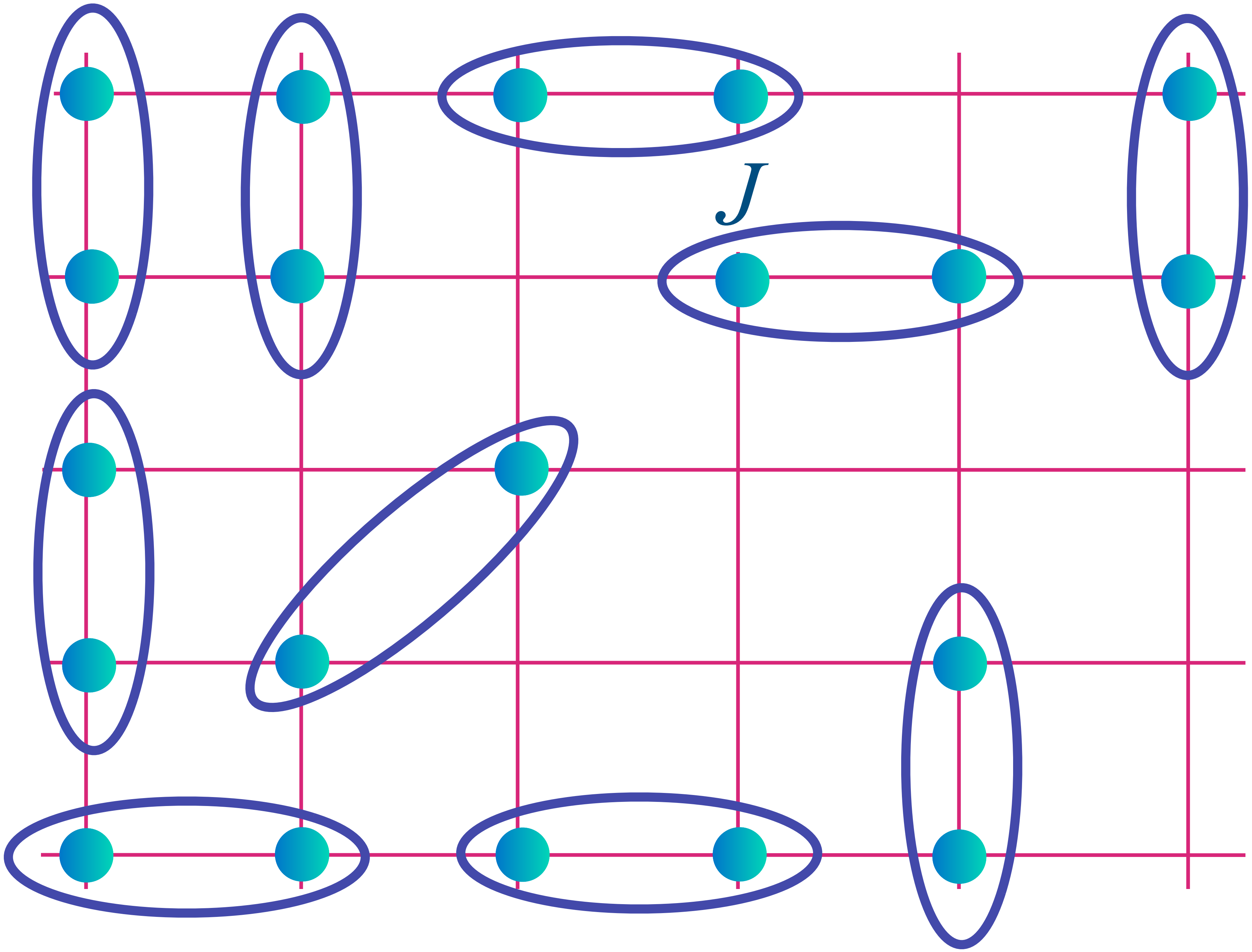

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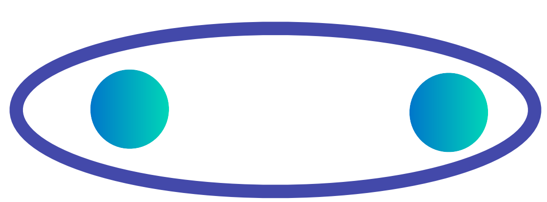


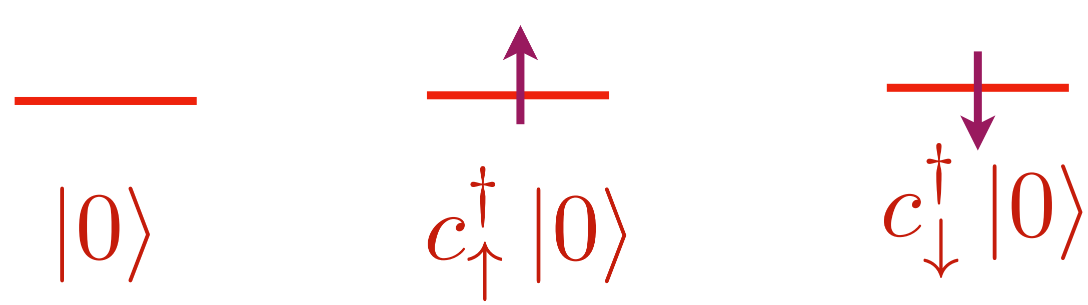
t-J model

$$H = \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

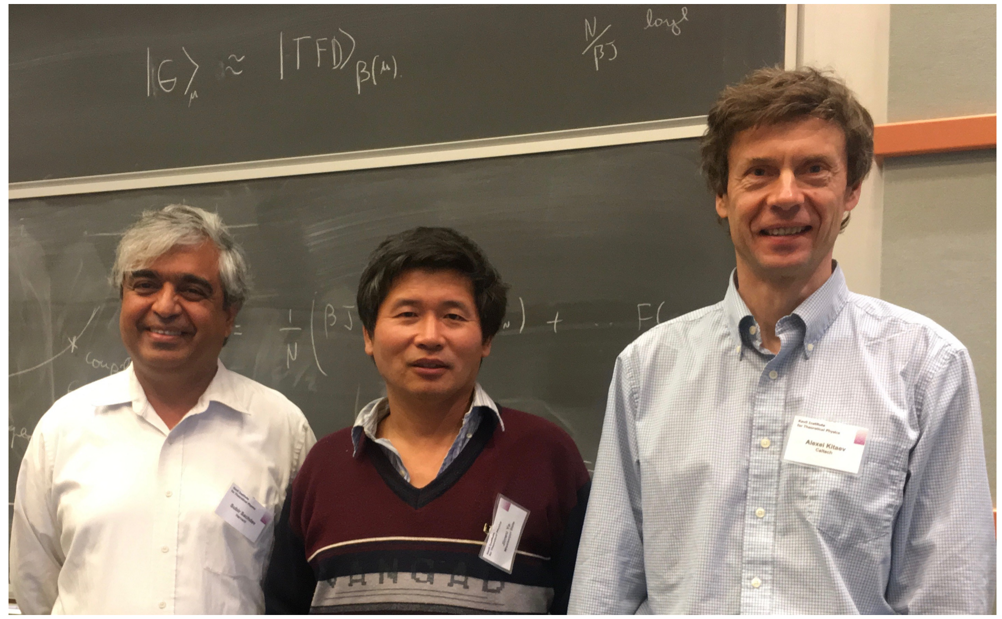
We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N}\sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

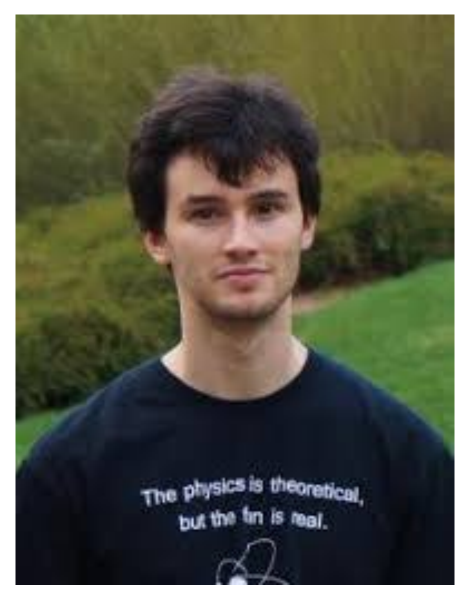


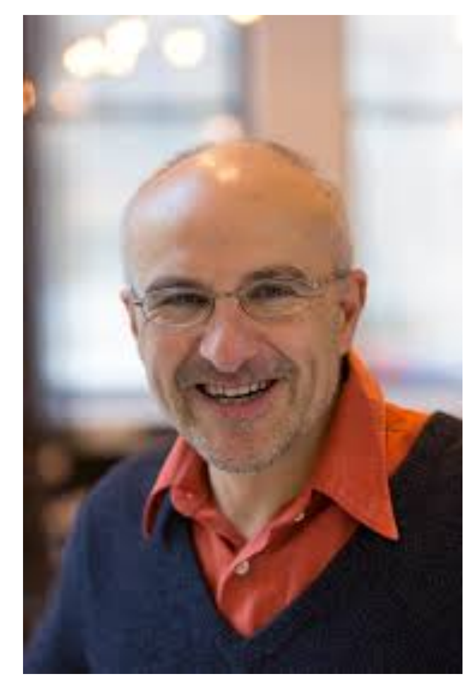
The Sachdev-Ye-Kitaev (SYK) model



S. Sachdev and J. Ye (1993); A. Kitaev (2015)

Variation described in
D. G. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges,
and S. Sachdev, arXiv:1912.08822





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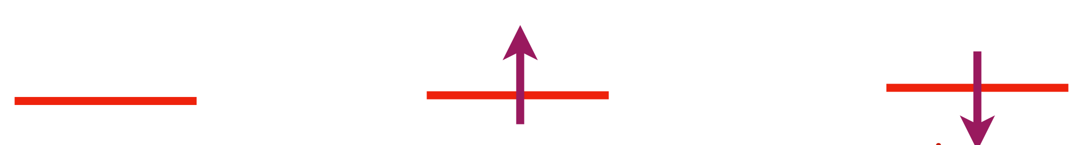

$|0\rangle$ $\quad$ $c_\uparrow^\dagger |0\rangle$ $\quad$ $c_\downarrow^\dagger |0\rangle$

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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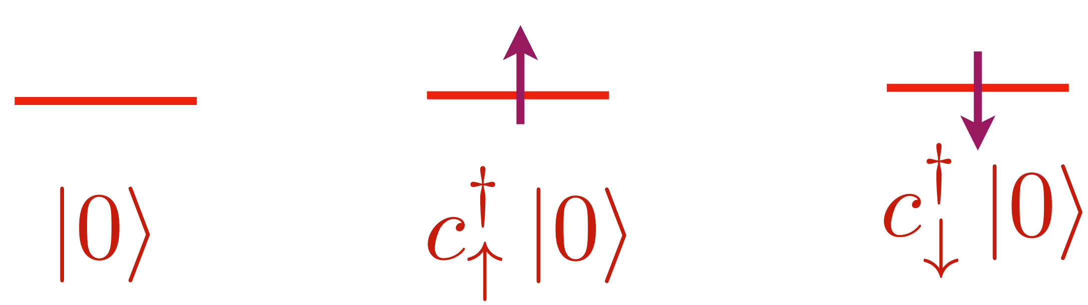
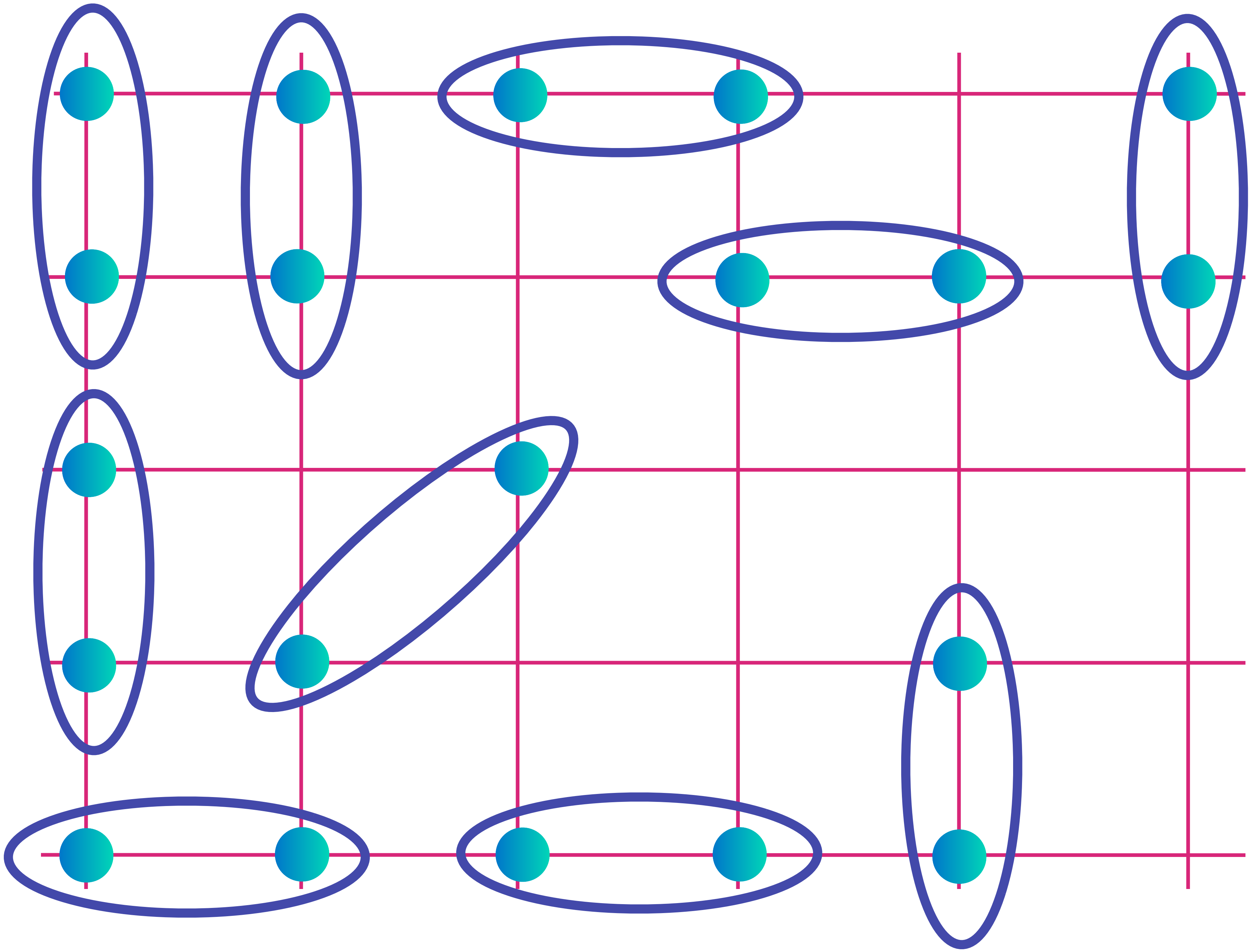
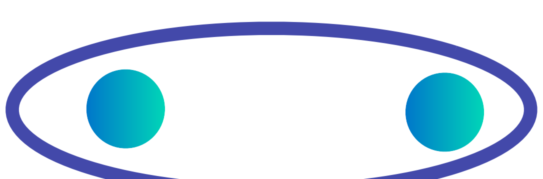
$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$

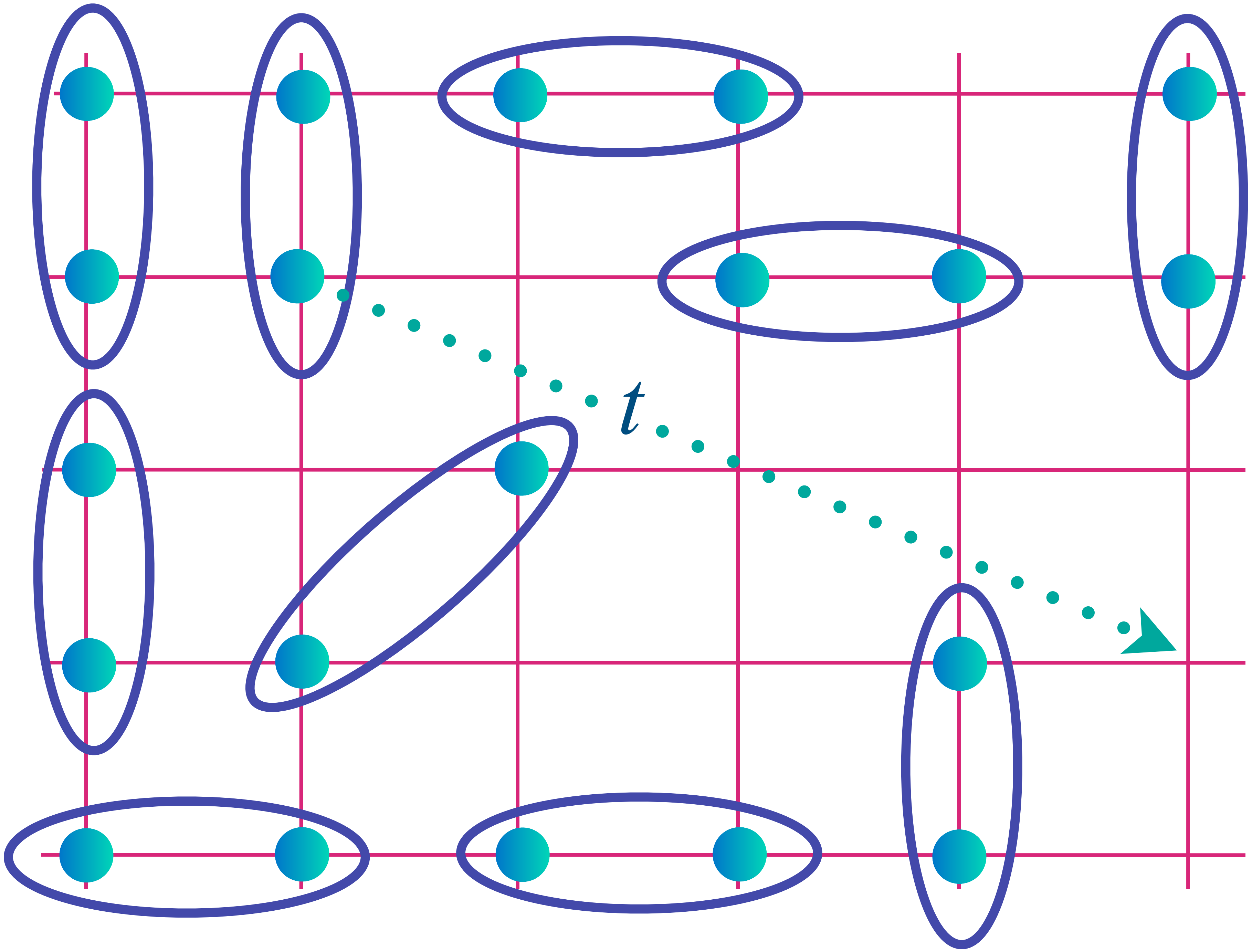




Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



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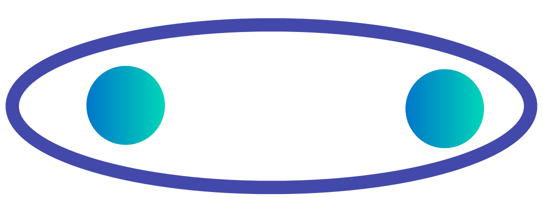

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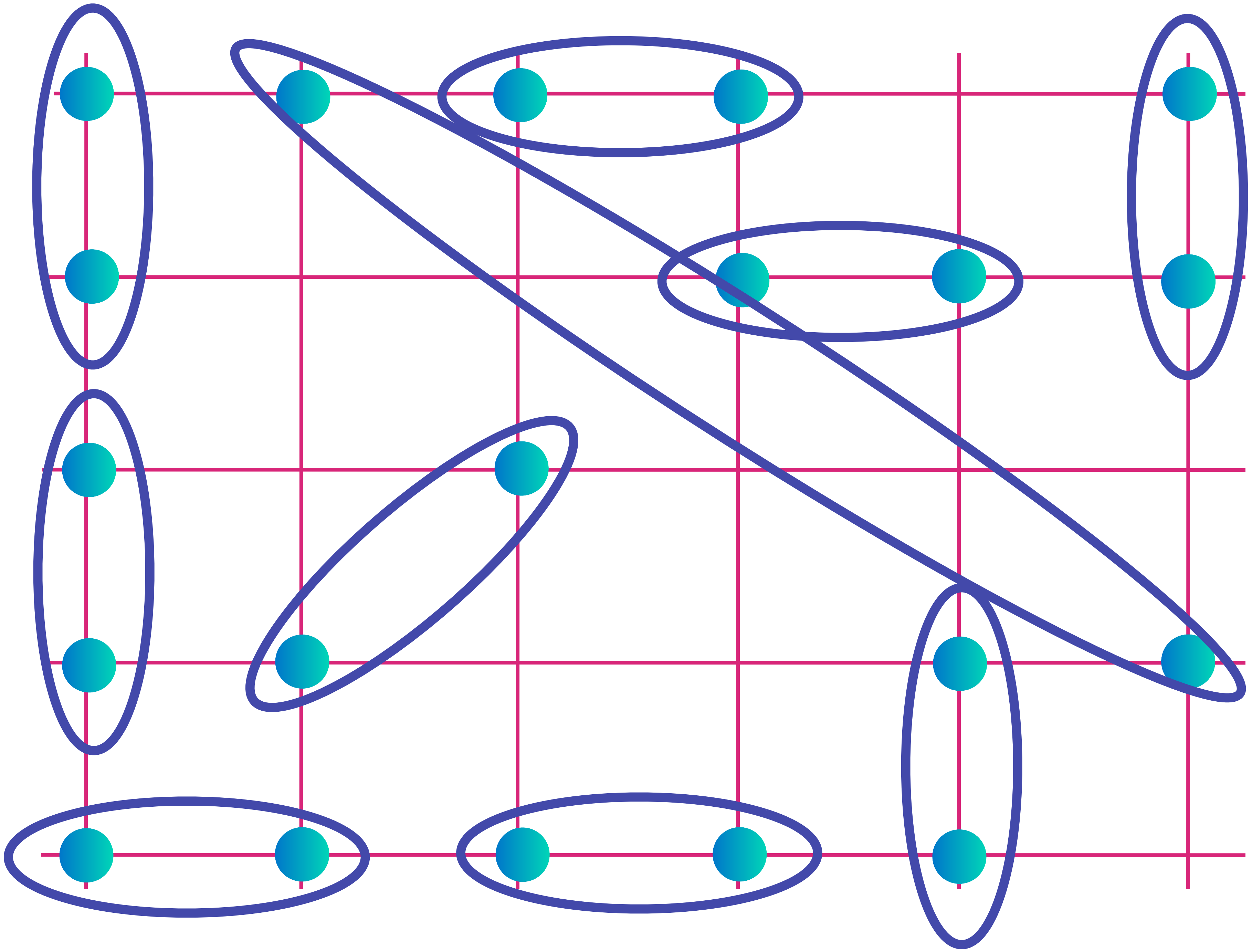

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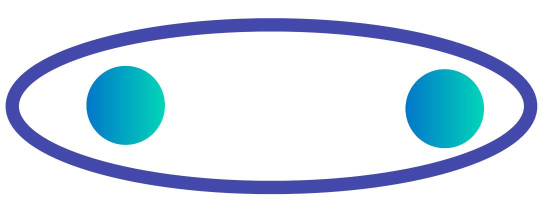
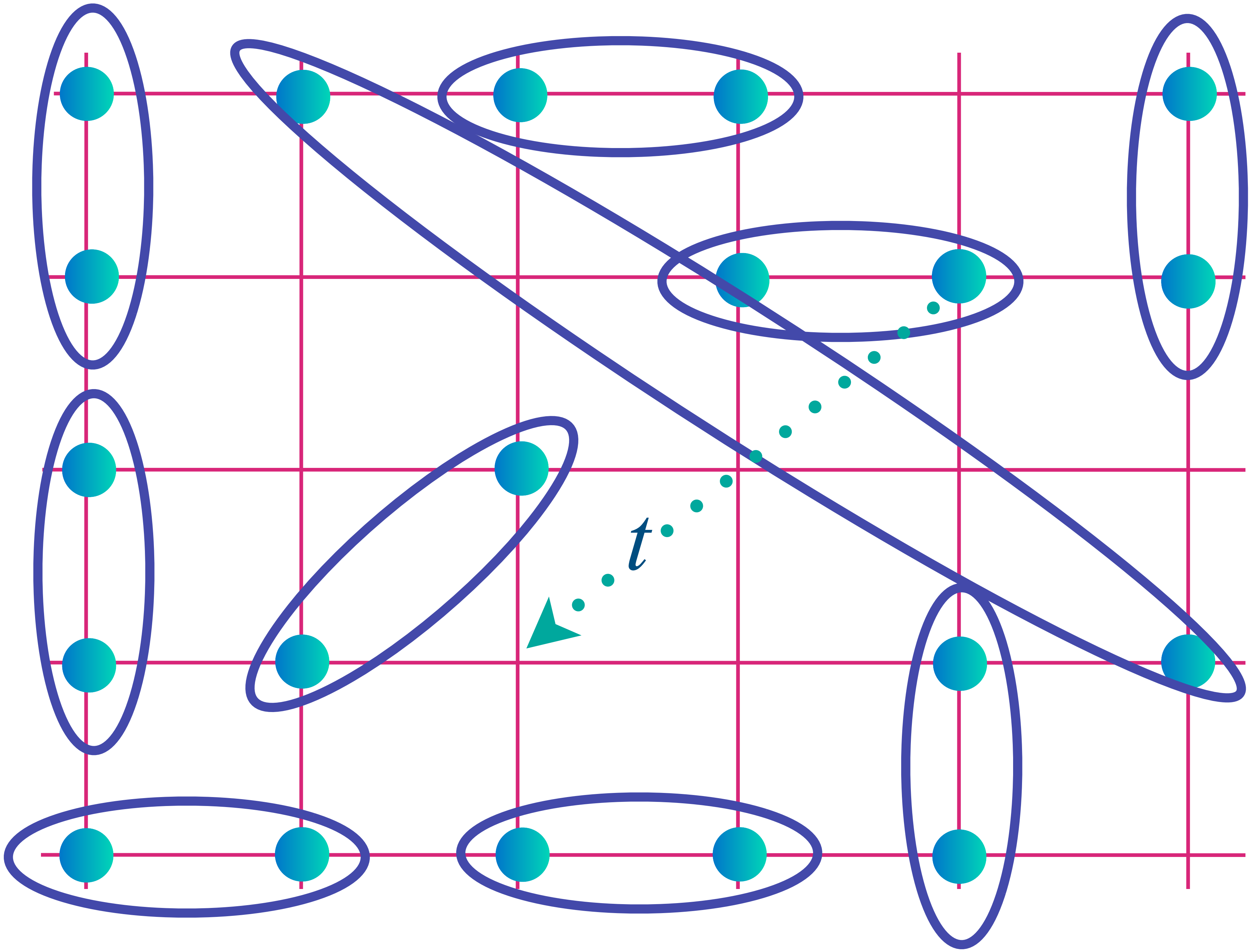
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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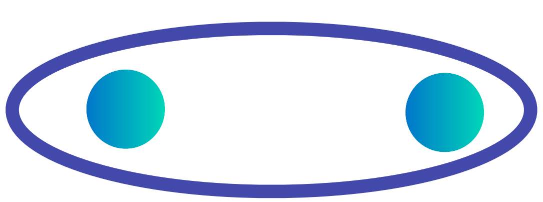

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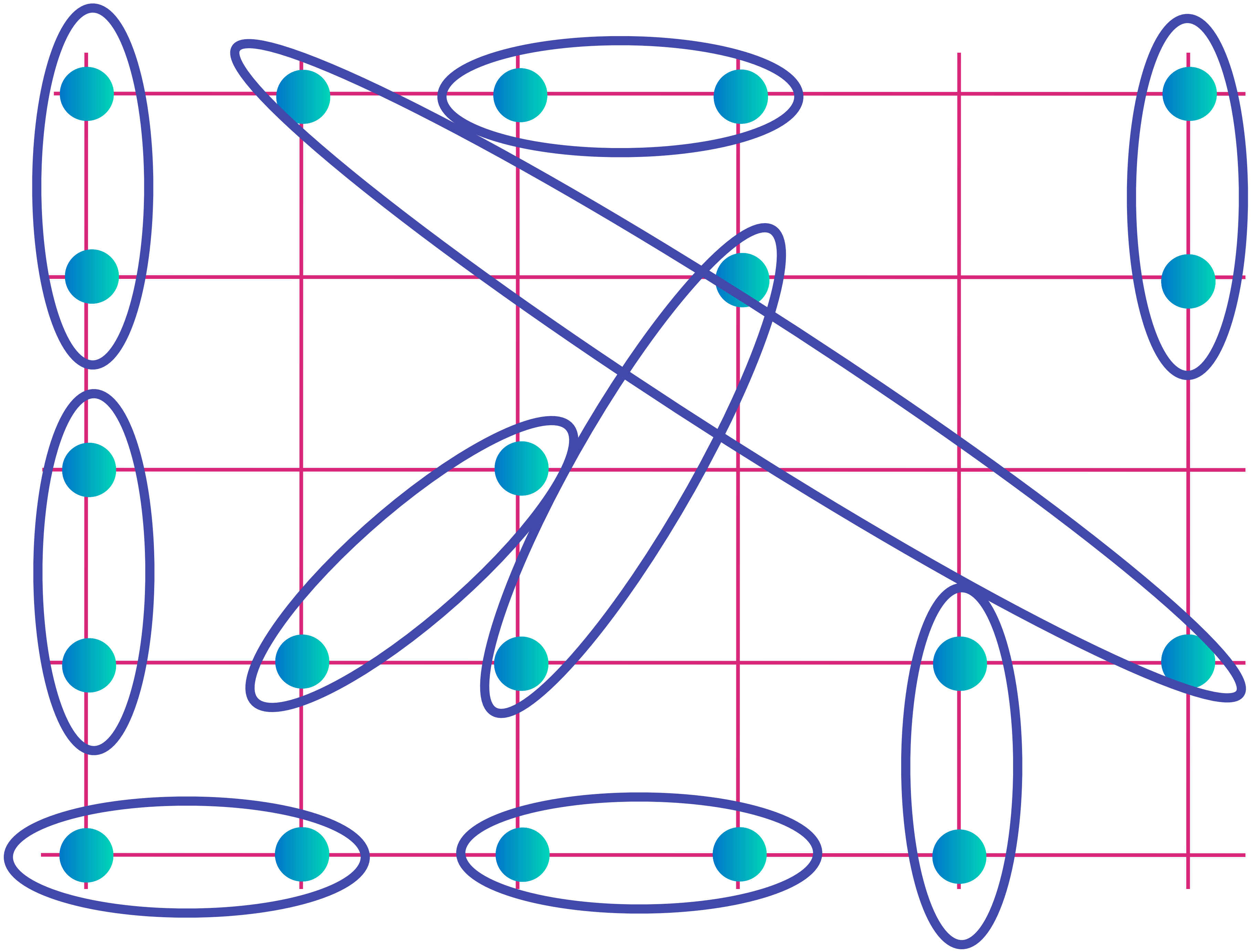

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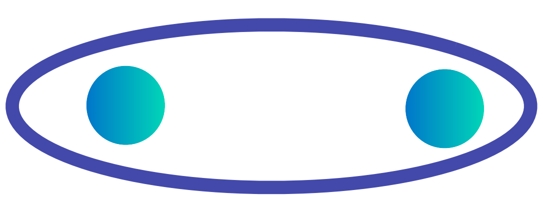
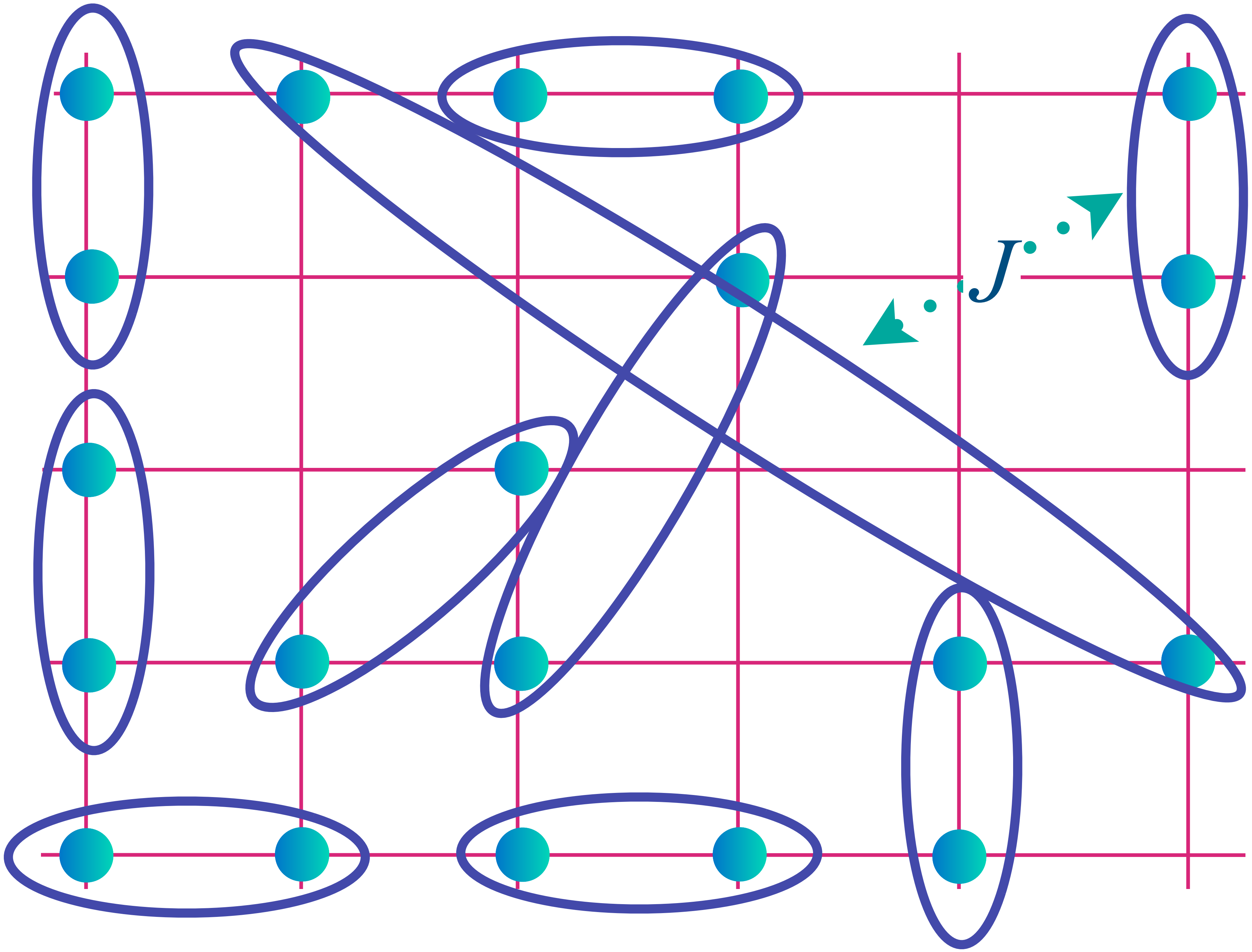
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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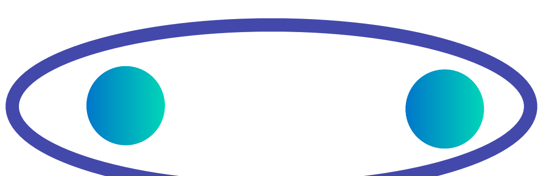
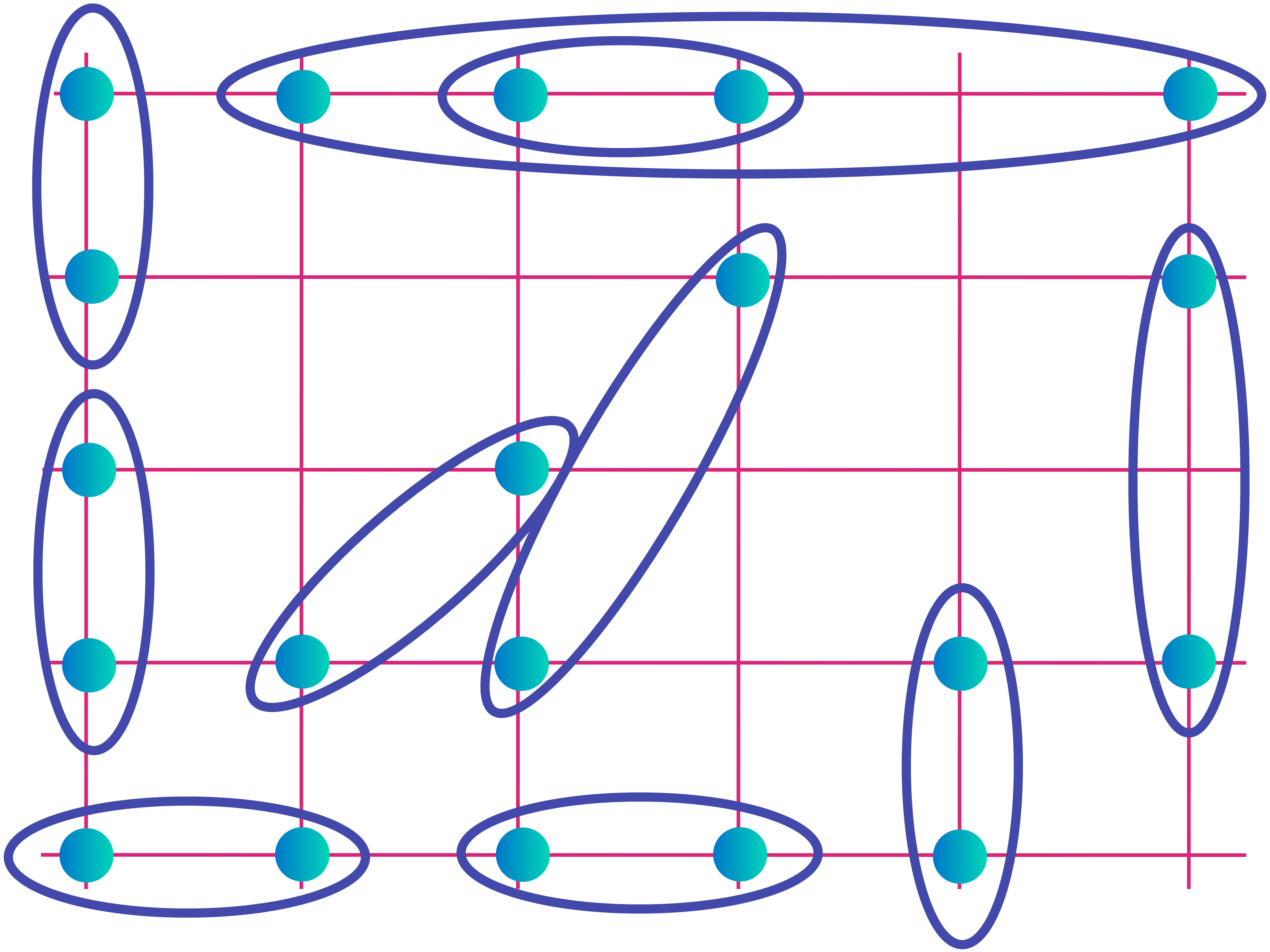
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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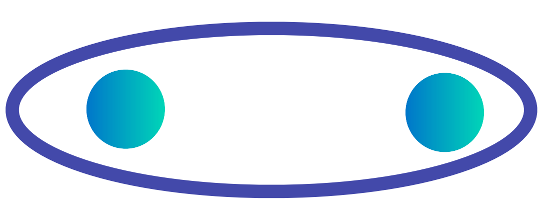

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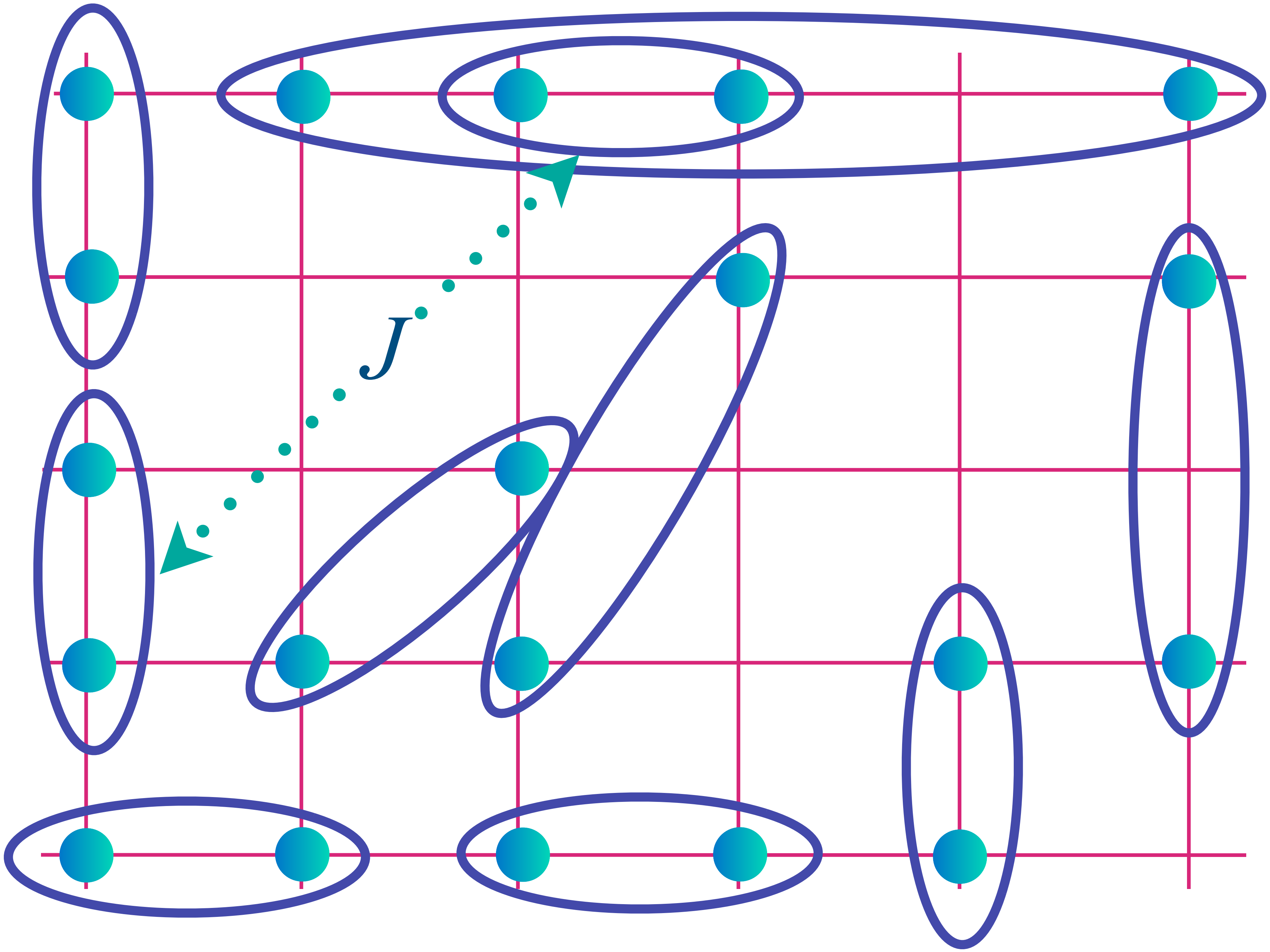

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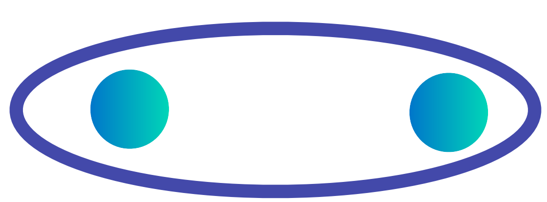
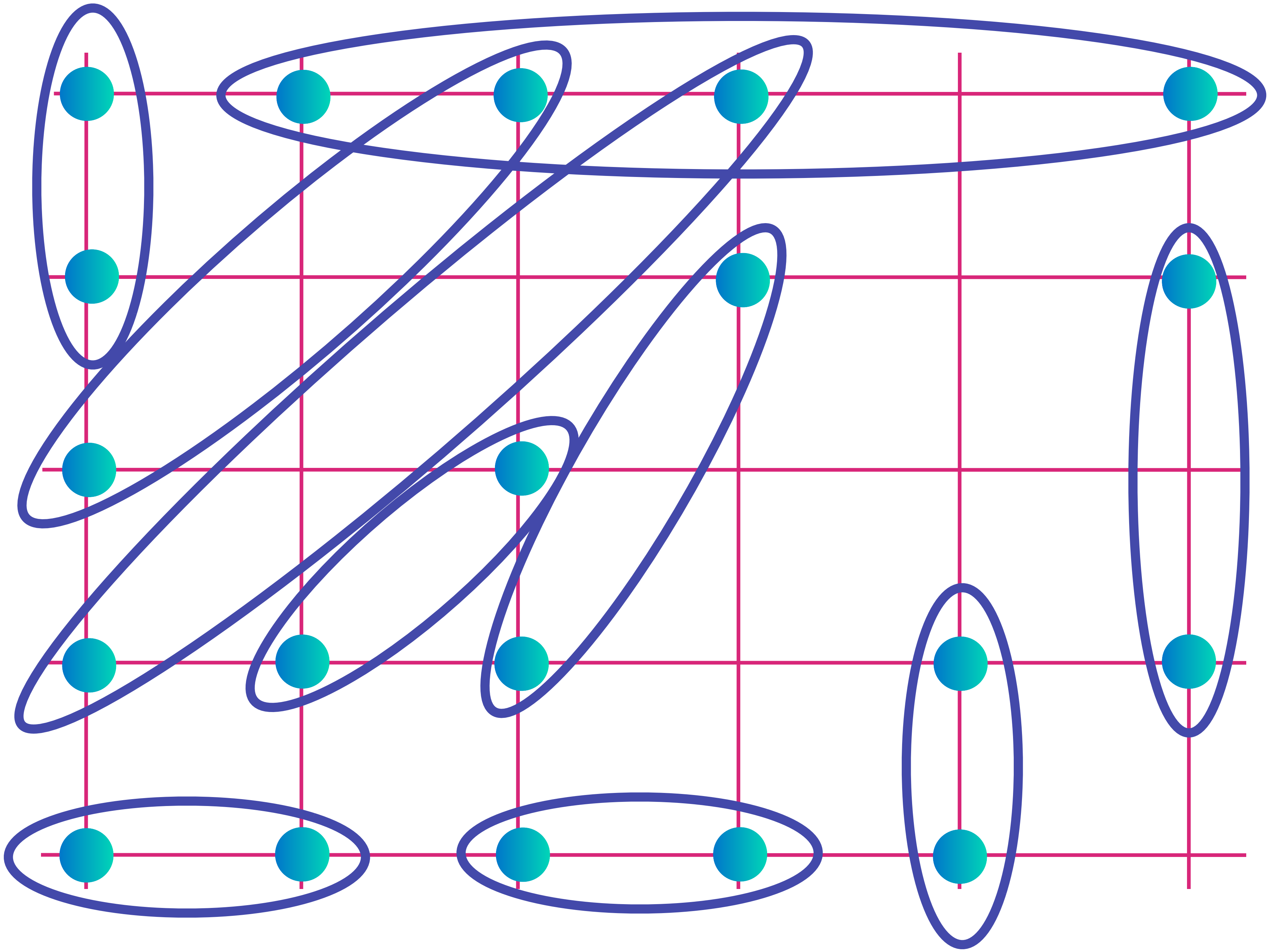
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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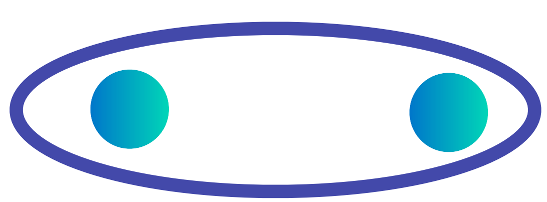
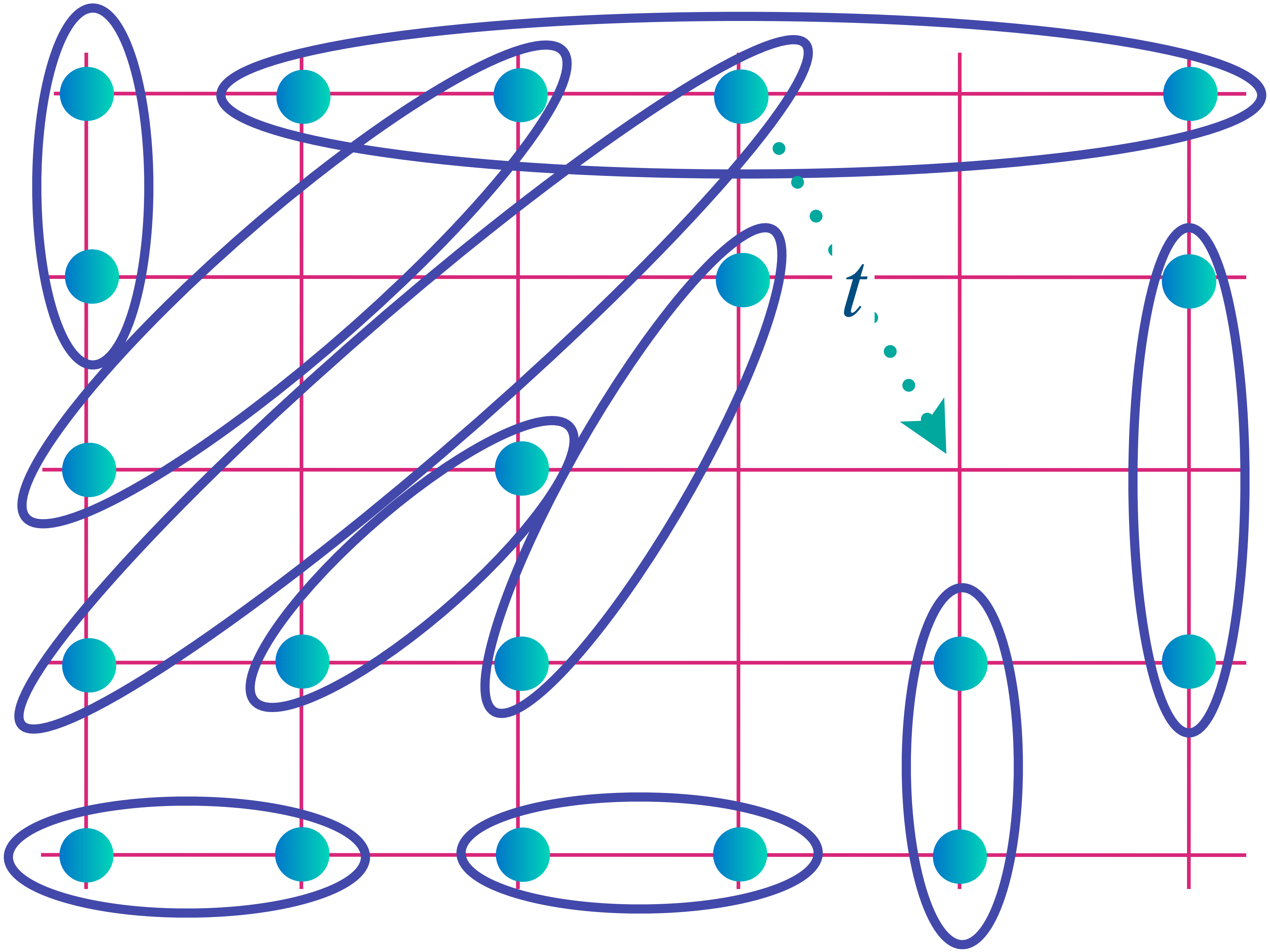
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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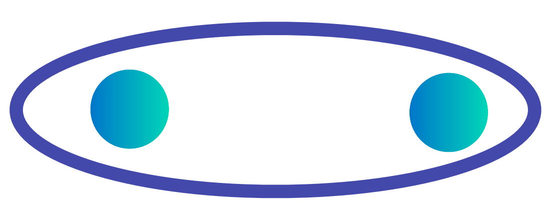

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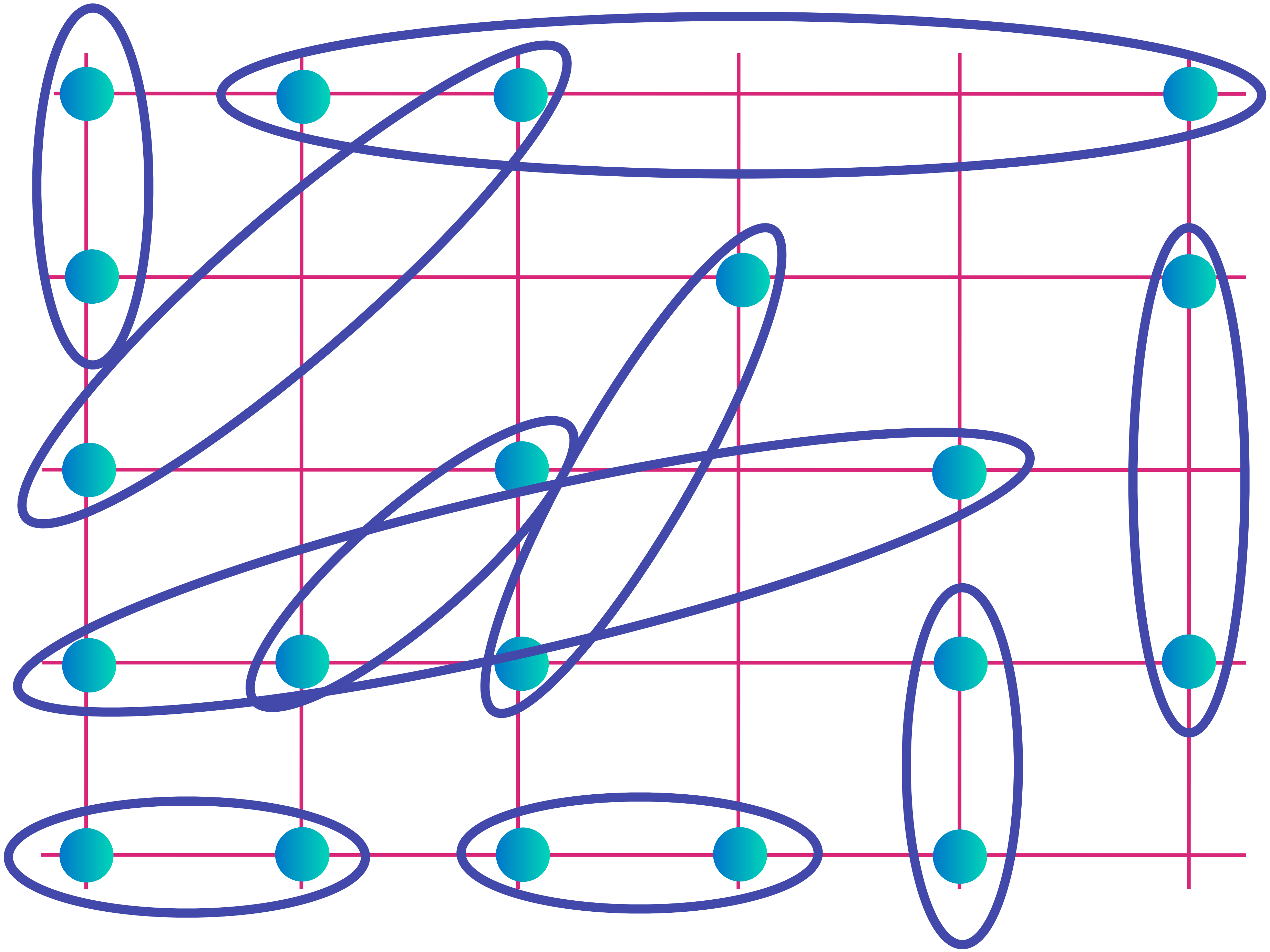

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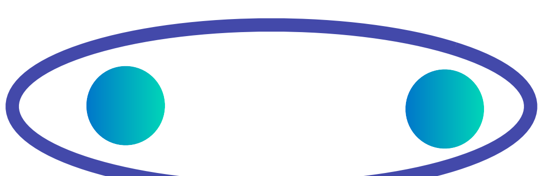
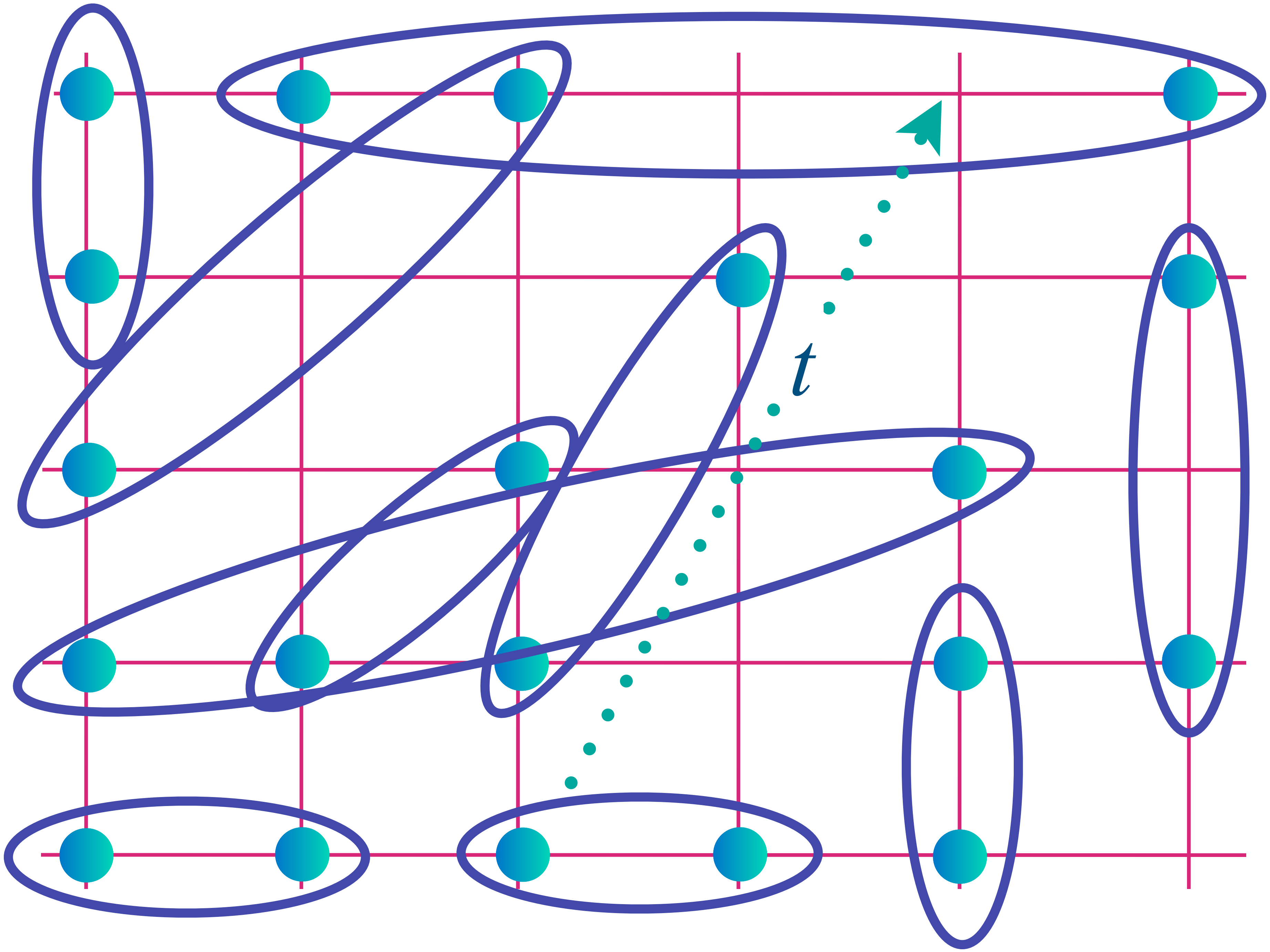
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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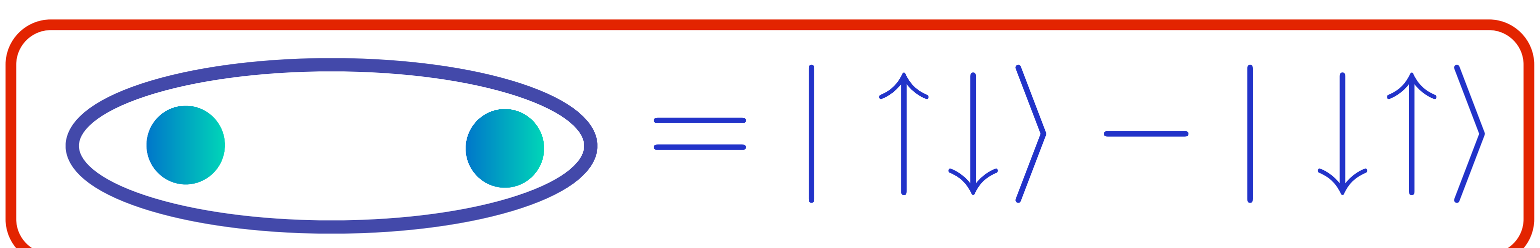

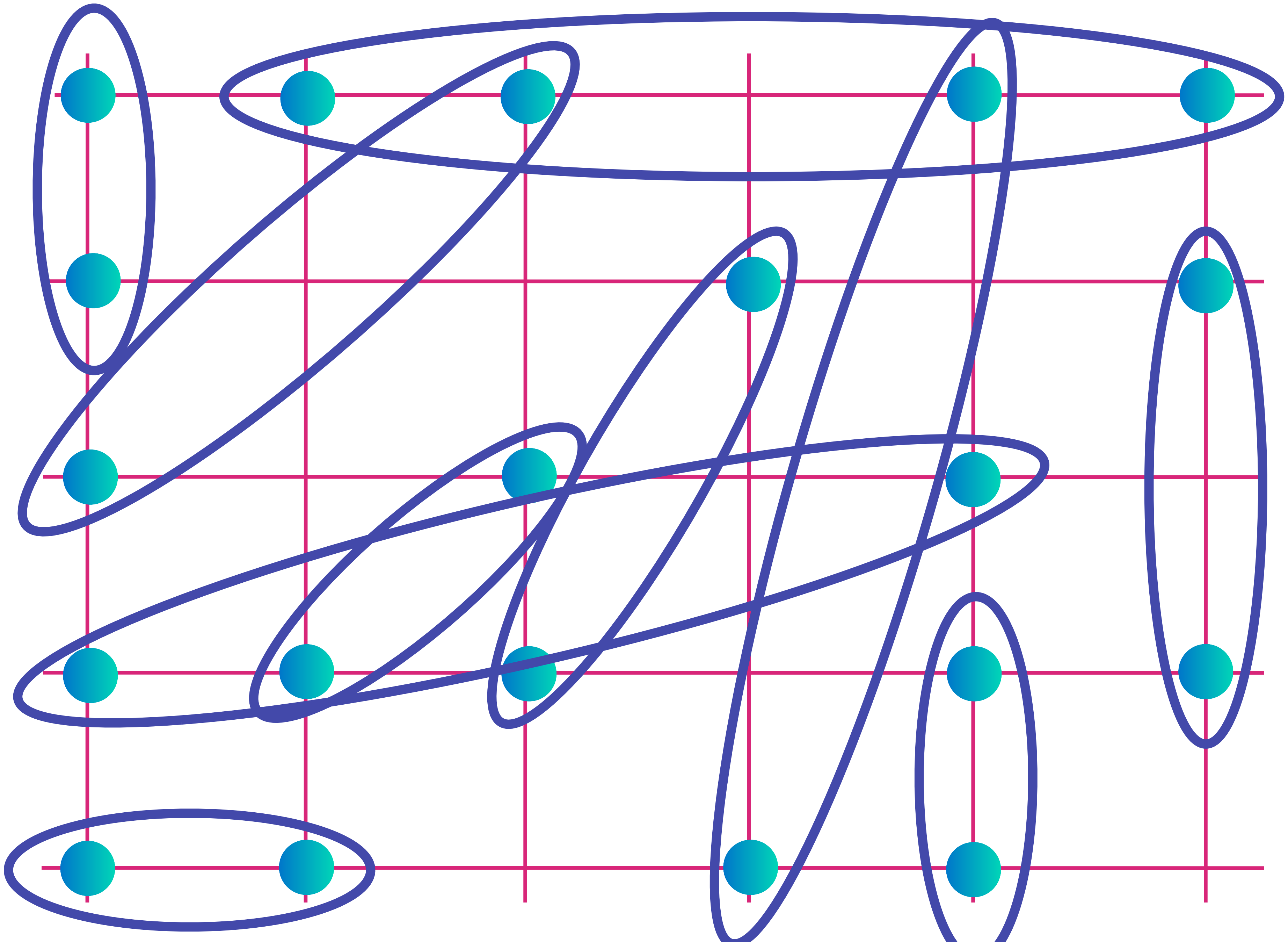

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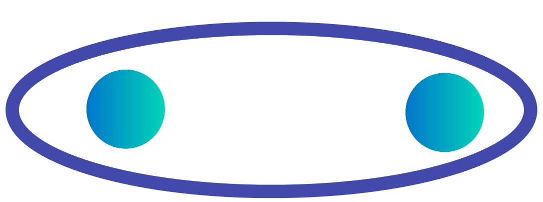

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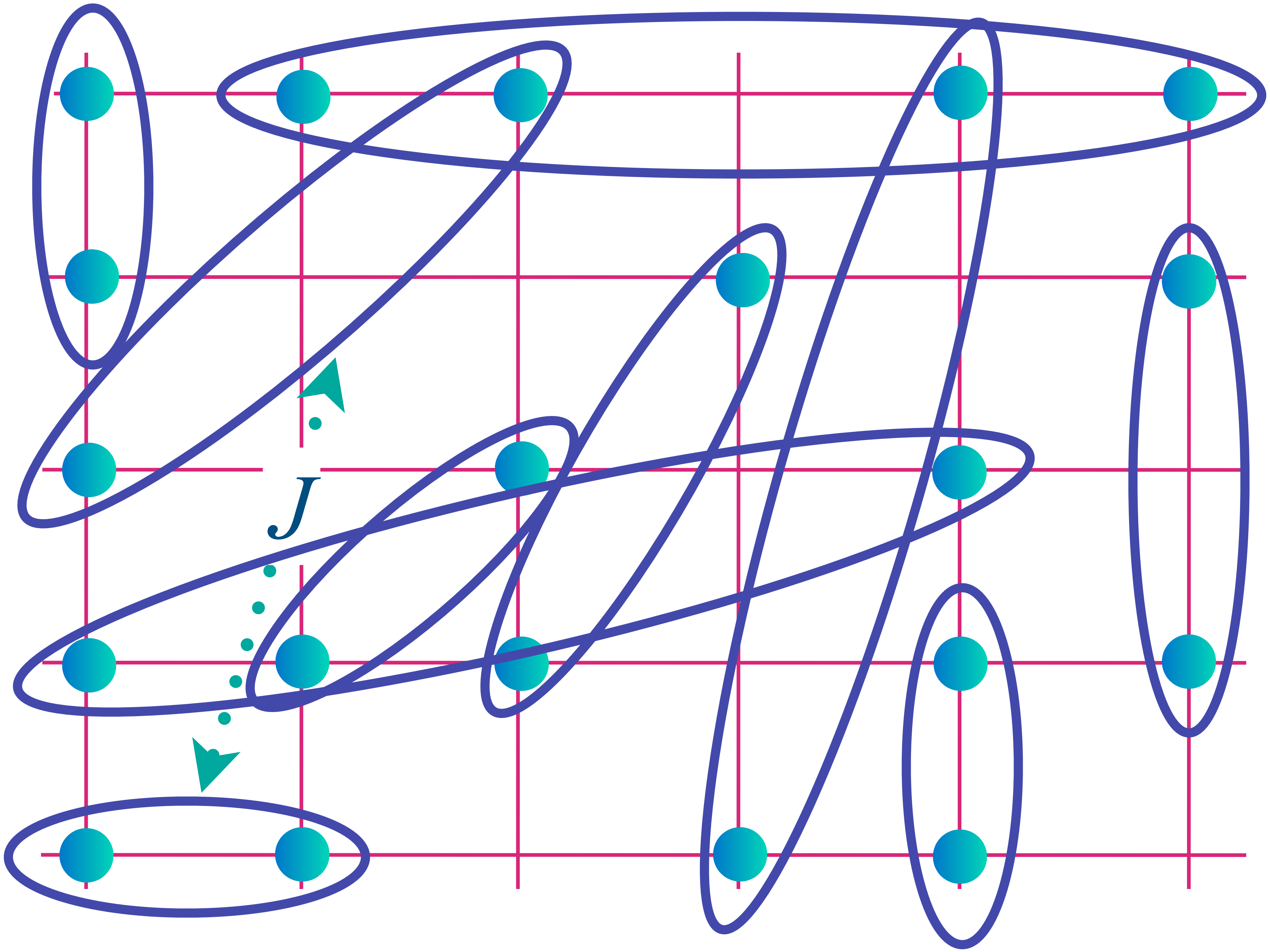

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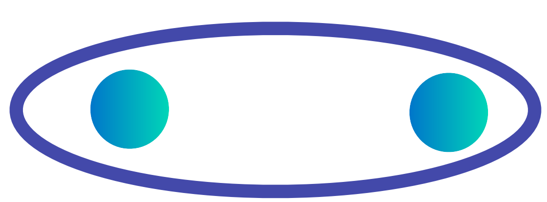
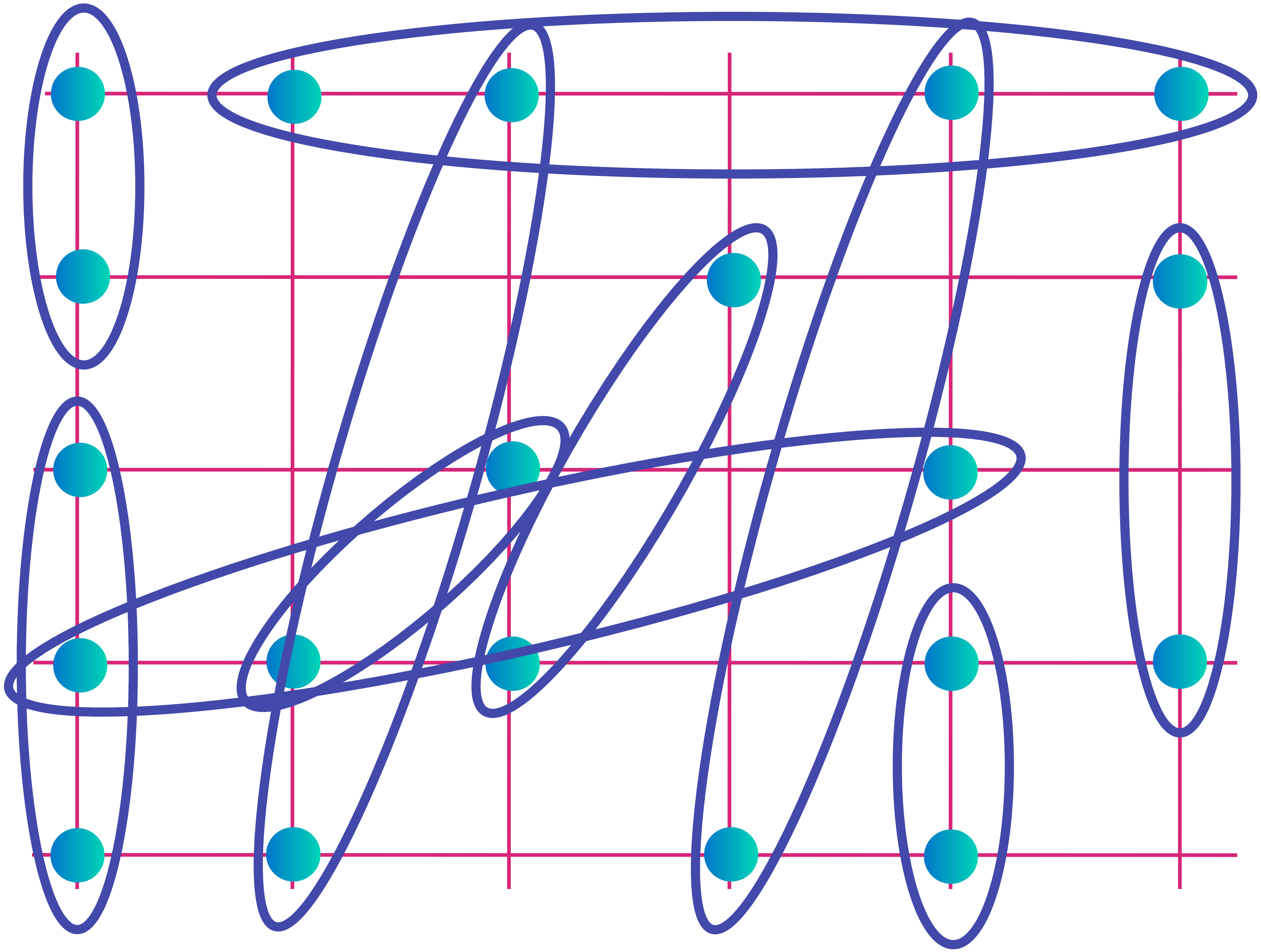
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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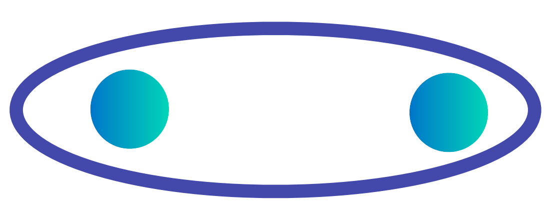 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$
\begin{aligned}
\left|0\right\rangle \Rightarrow b^{\dagger} \left|v\right\rangle \quad &, \quad c_{\alpha}^{\dagger} \left|0\right\rangle \Rightarrow f_{\alpha}^{\dagger} \left|v\right\rangle \\
c_\alpha &= f_\alpha b^\dagger \\
\vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\
f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \\
\text{U(1) gauge invariance,} \quad & b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}
\end{aligned}
$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

t-J model

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$$|0\rangle \Rightarrow \mathfrak{f}^\dagger\, |v\rangle \qquad , \qquad c_\alpha^\dagger\, |0\rangle \Rightarrow \mathfrak{b}_\alpha^\dagger\, |v\rangle$$

$$c_\alpha = \mathfrak{b}_\alpha \mathfrak{f}^\dagger$$

$$\vec{S} = \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta$$

$$\mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} = 1$$

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

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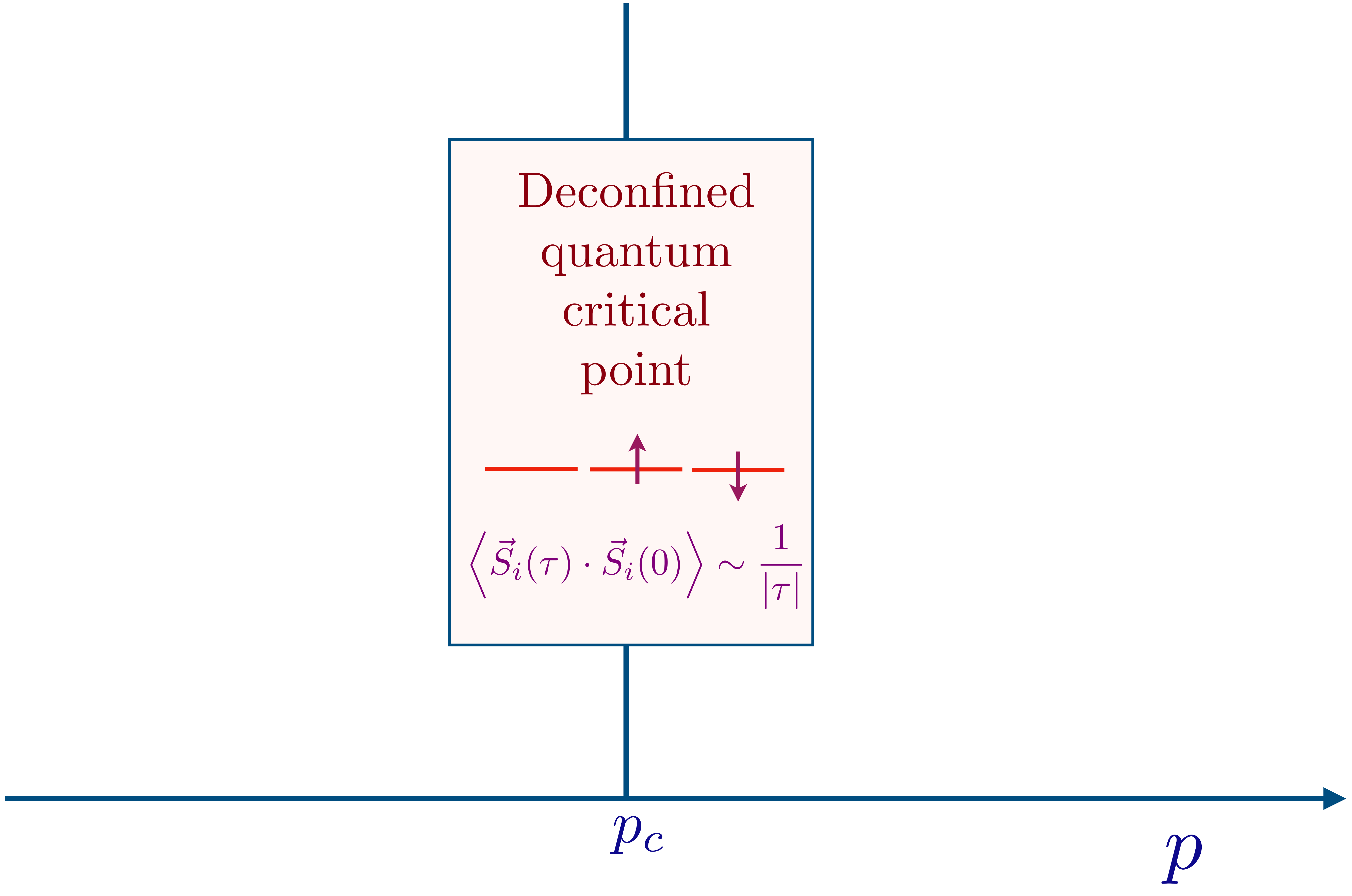
t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

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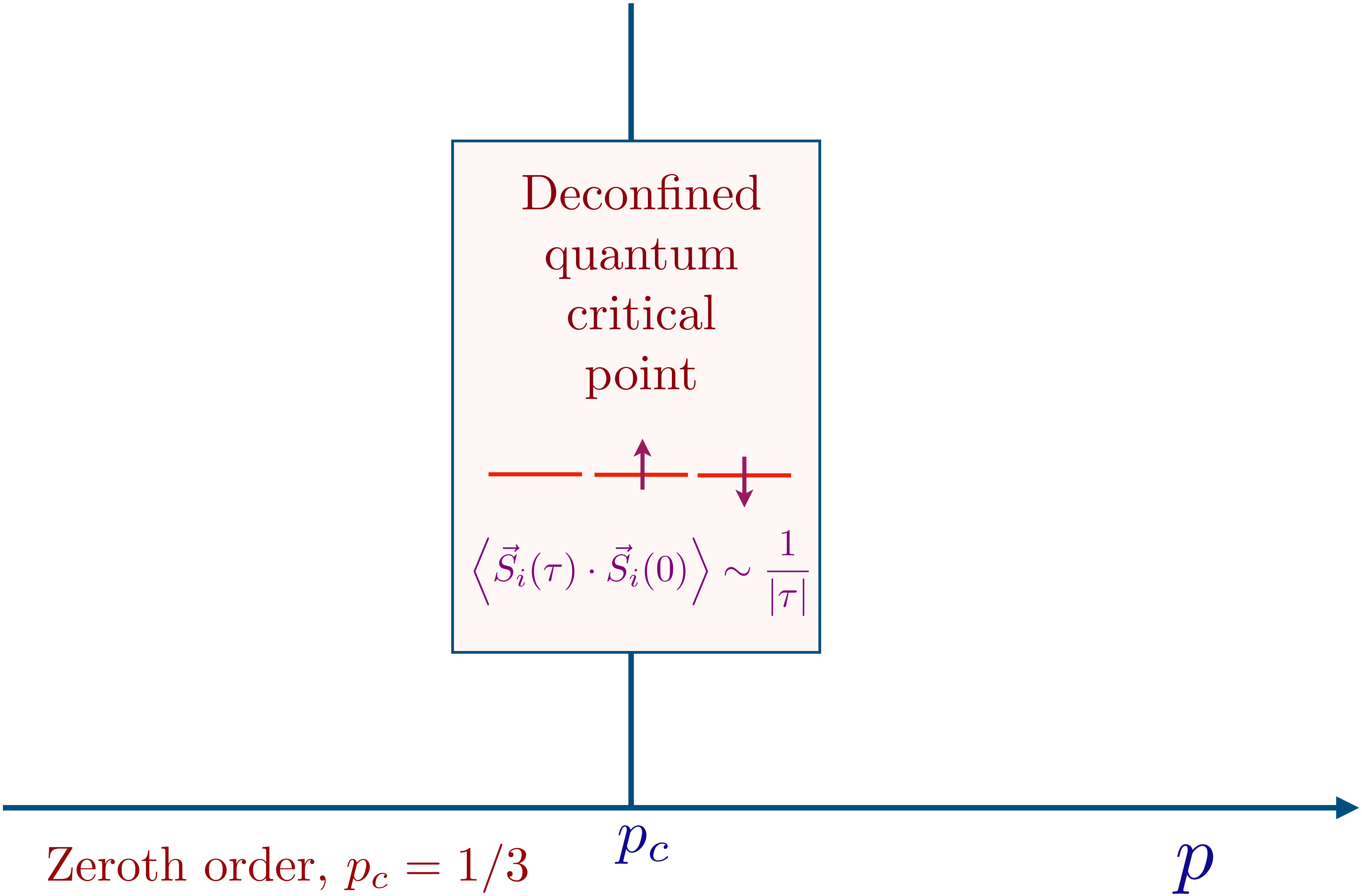

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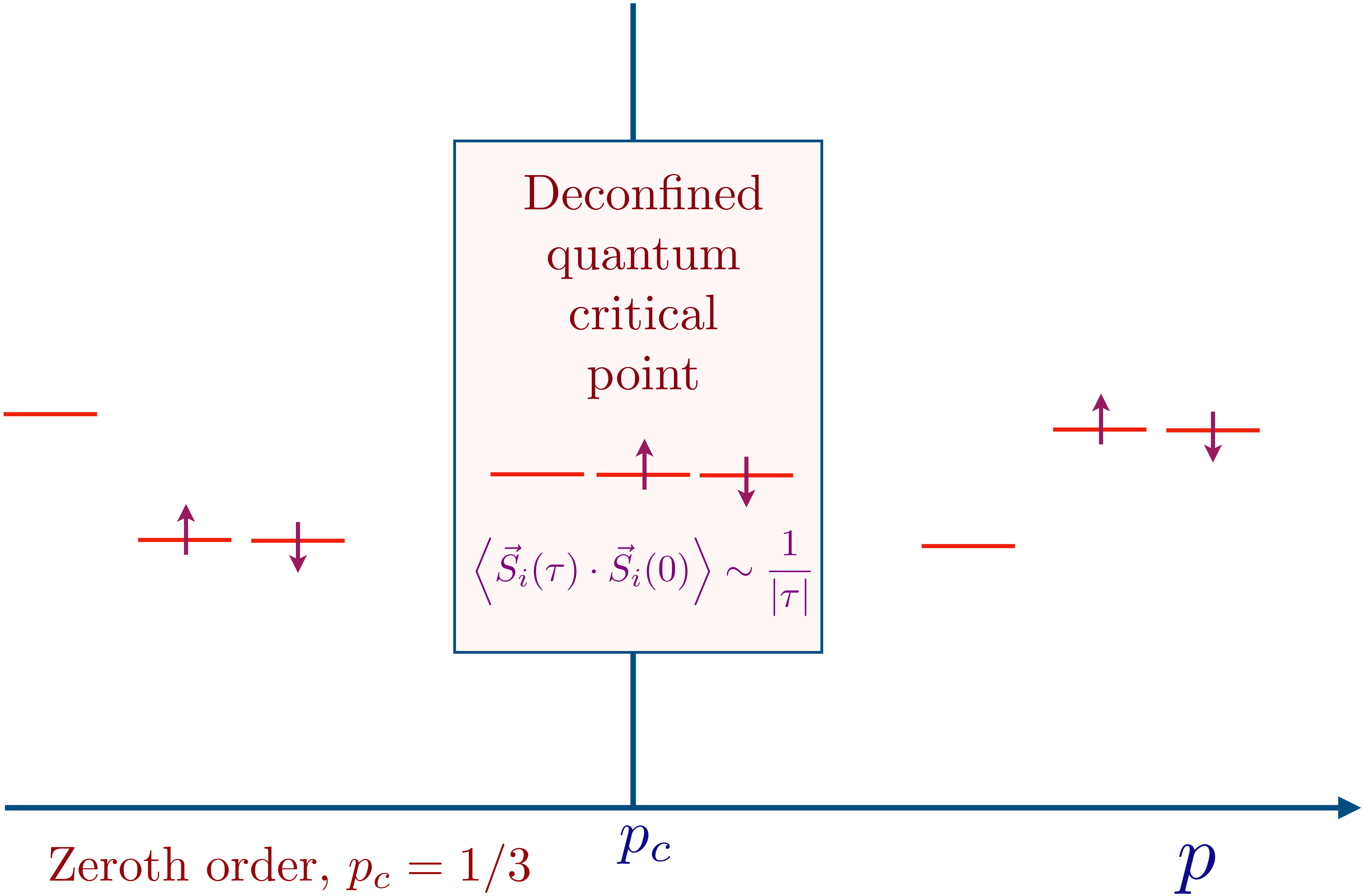
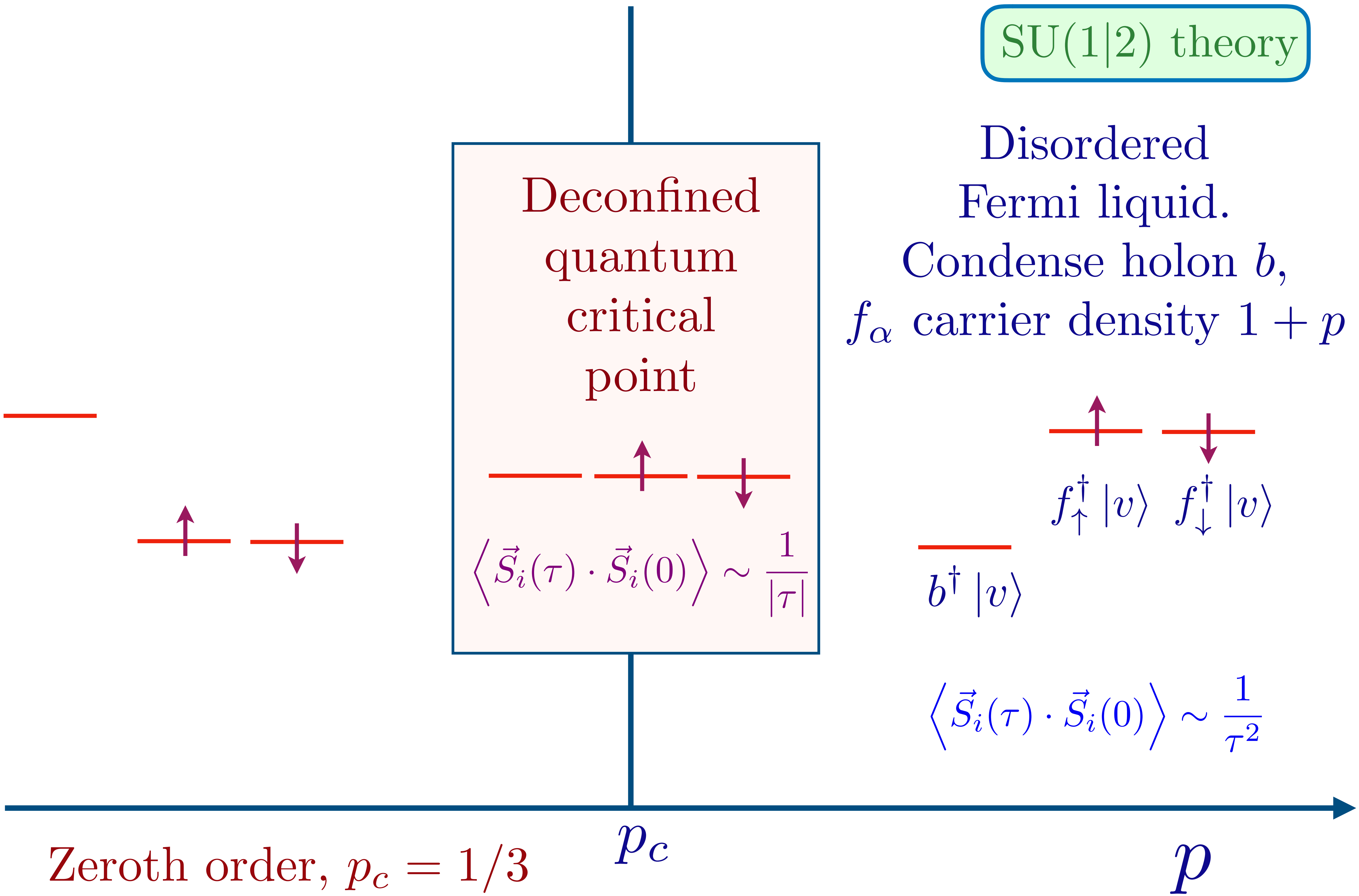


D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram

SU(2|1) theory

SU(1|2) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}^\dagger_\uparrow |v\rangle$ $\mathfrak{b}^\dagger_\downarrow |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

Deconfined quantum critical point

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$

$f^\dagger_\uparrow |v\rangle$ $f^\dagger_\downarrow |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

p_c

p

Zeroth order, $p_c = 1/3$

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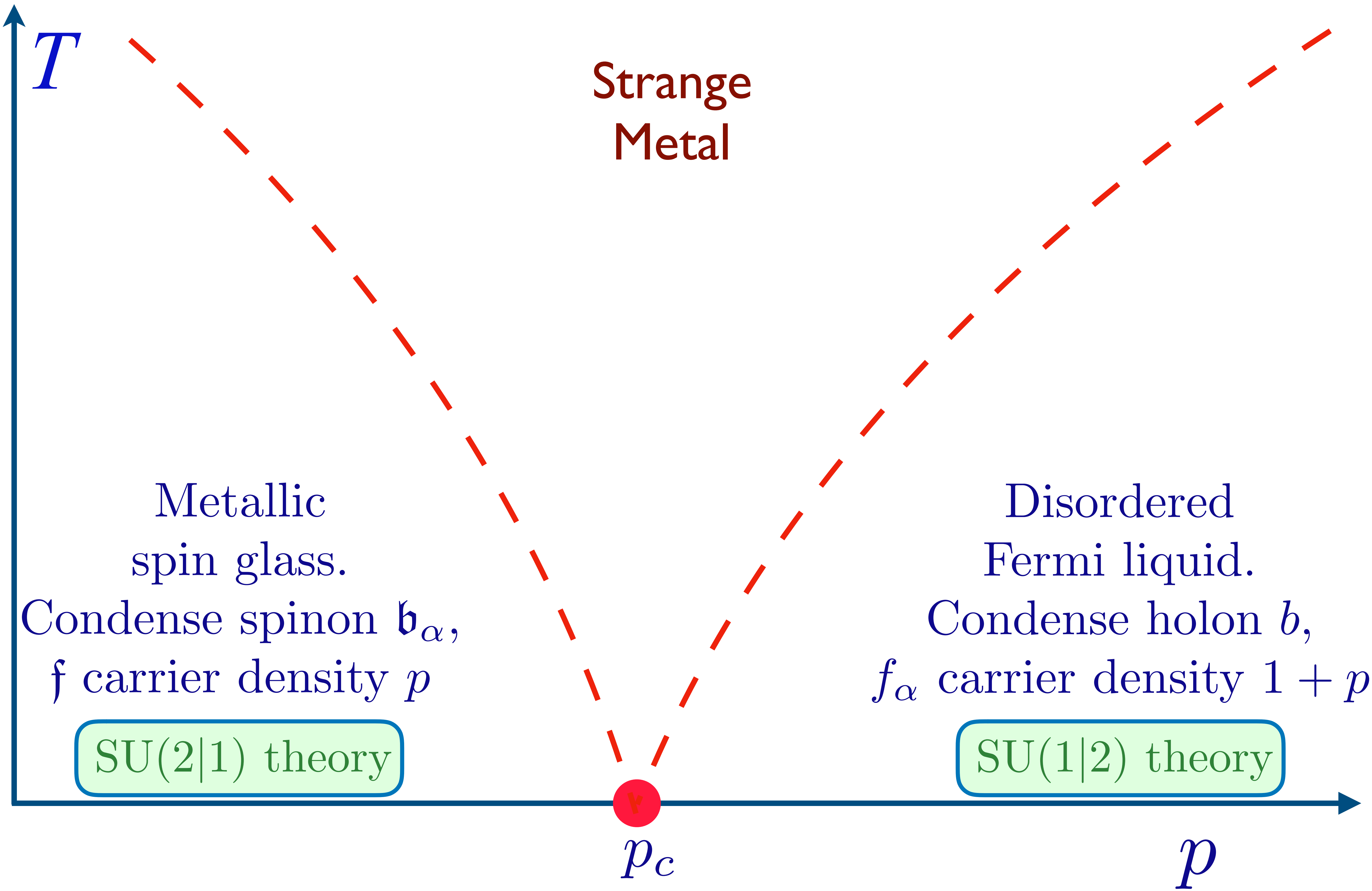
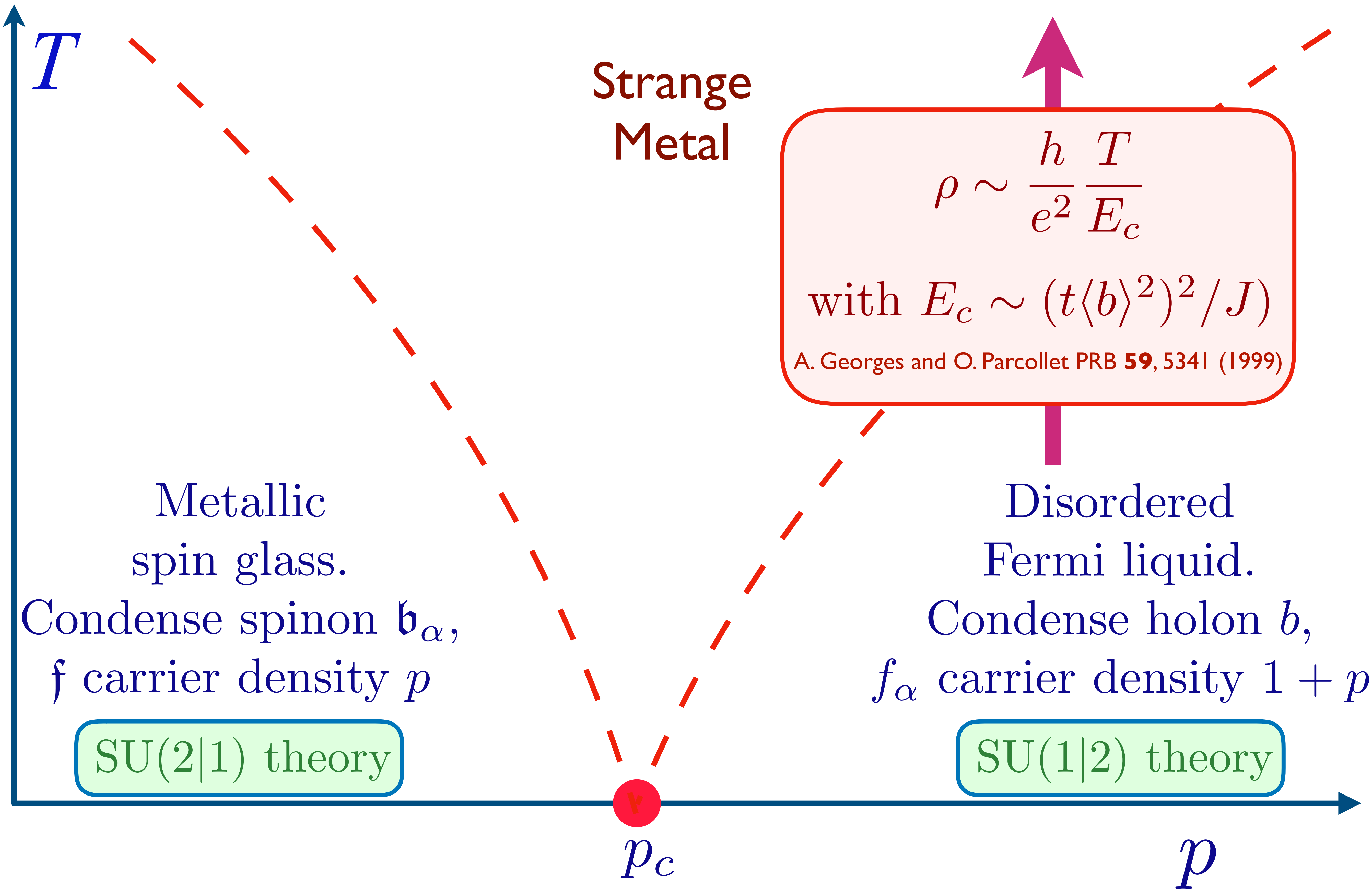
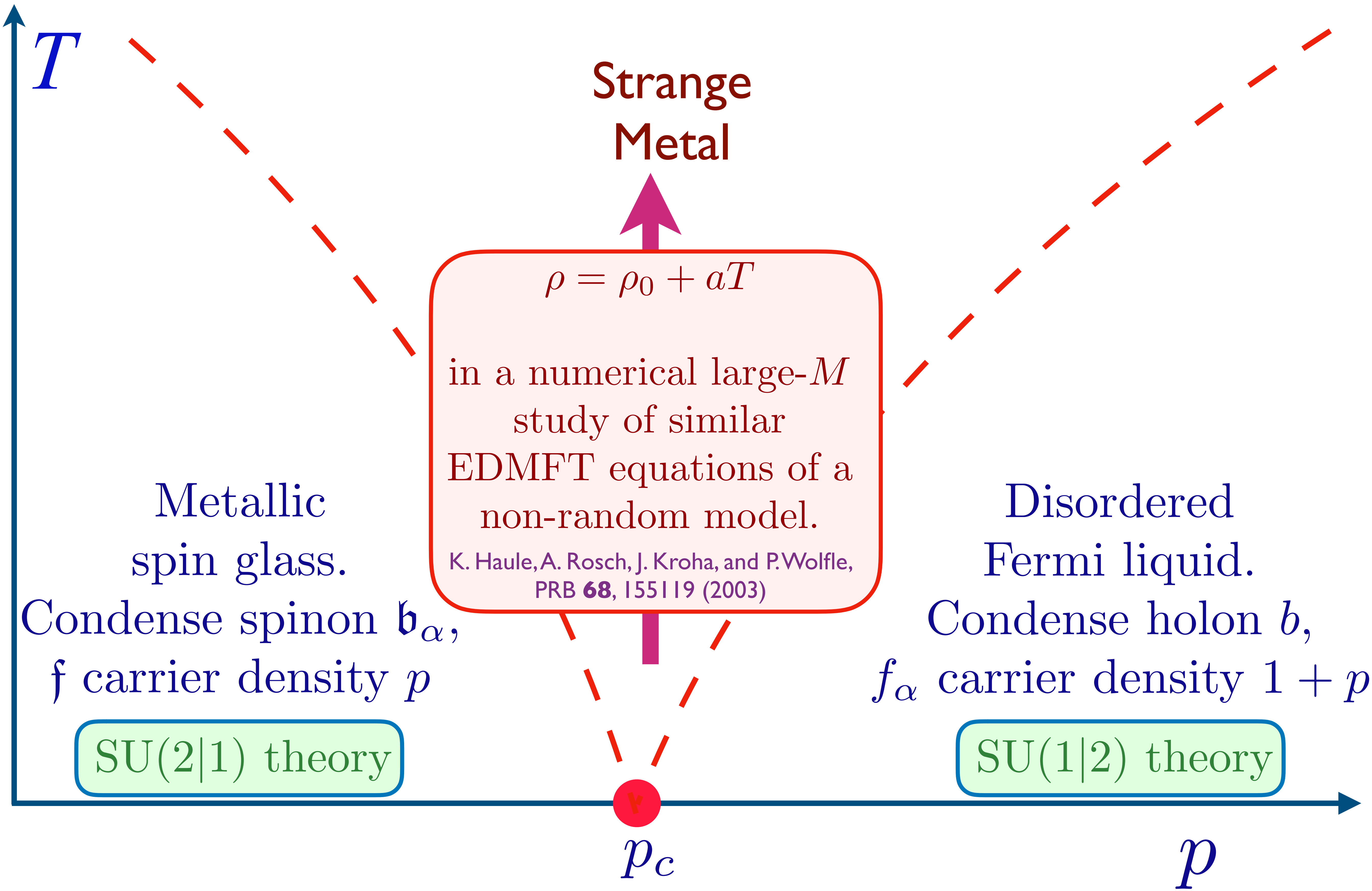
t-J model phase diagram



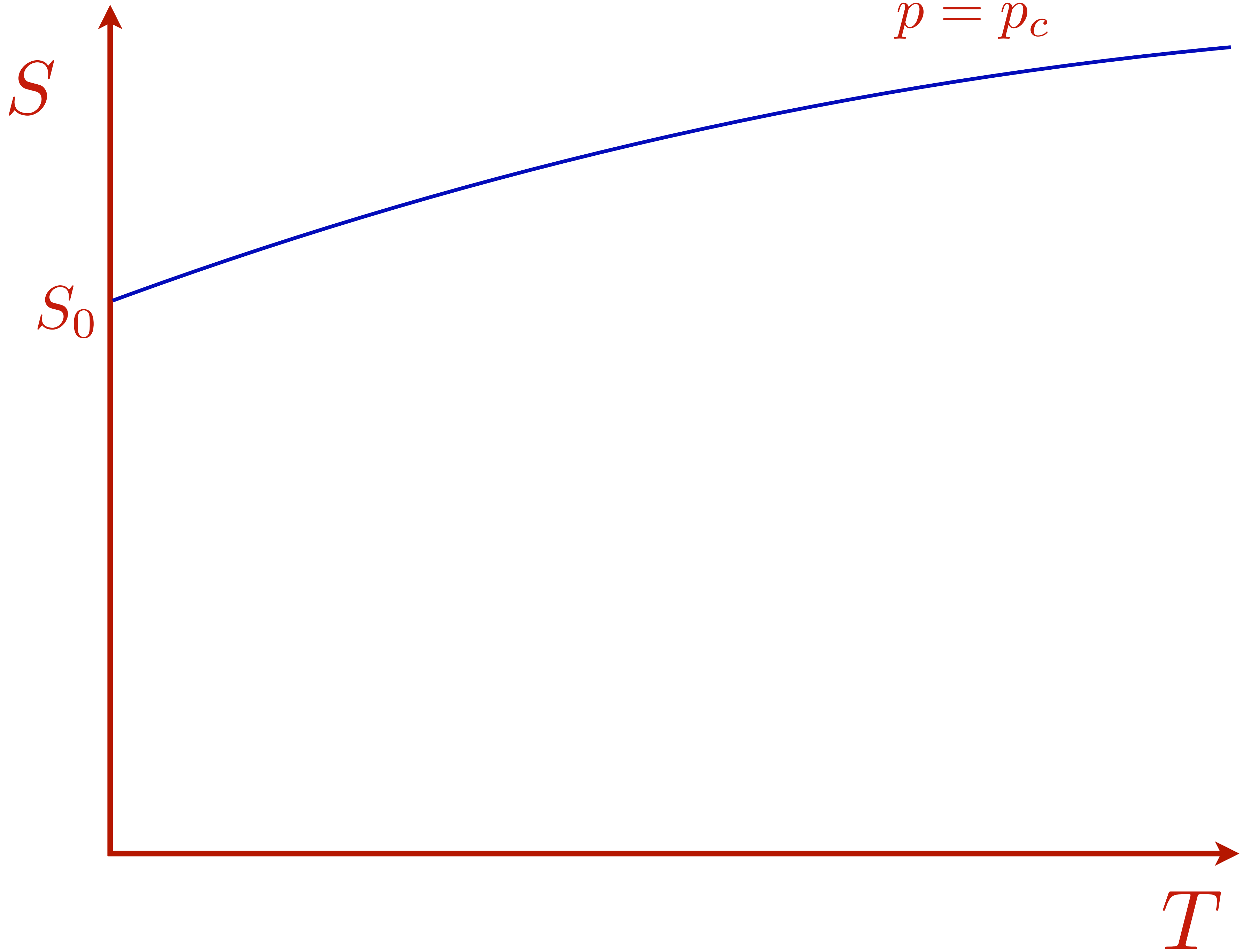
t-J model phase diagram



t-J model phase diagram

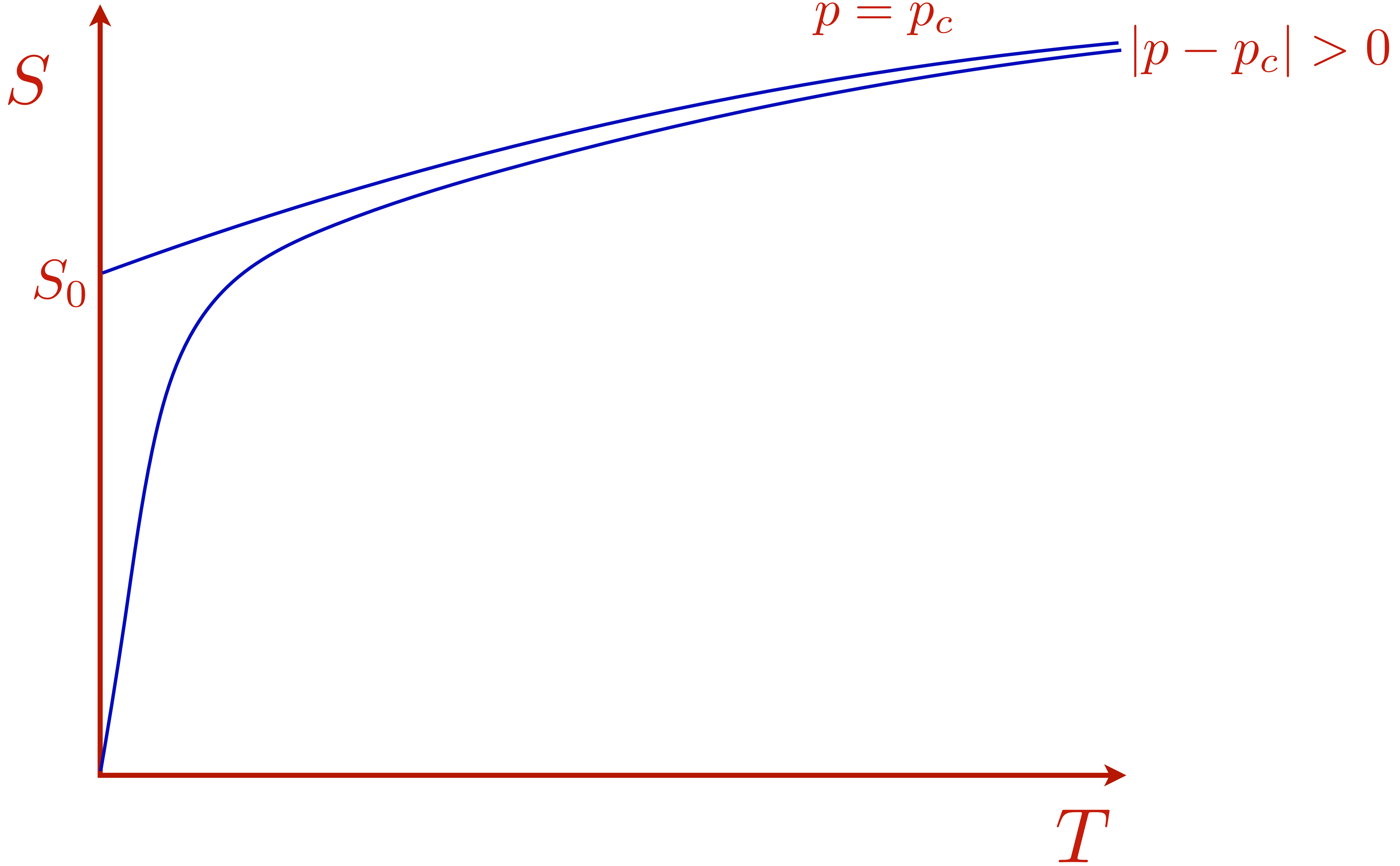


t-J model entropy



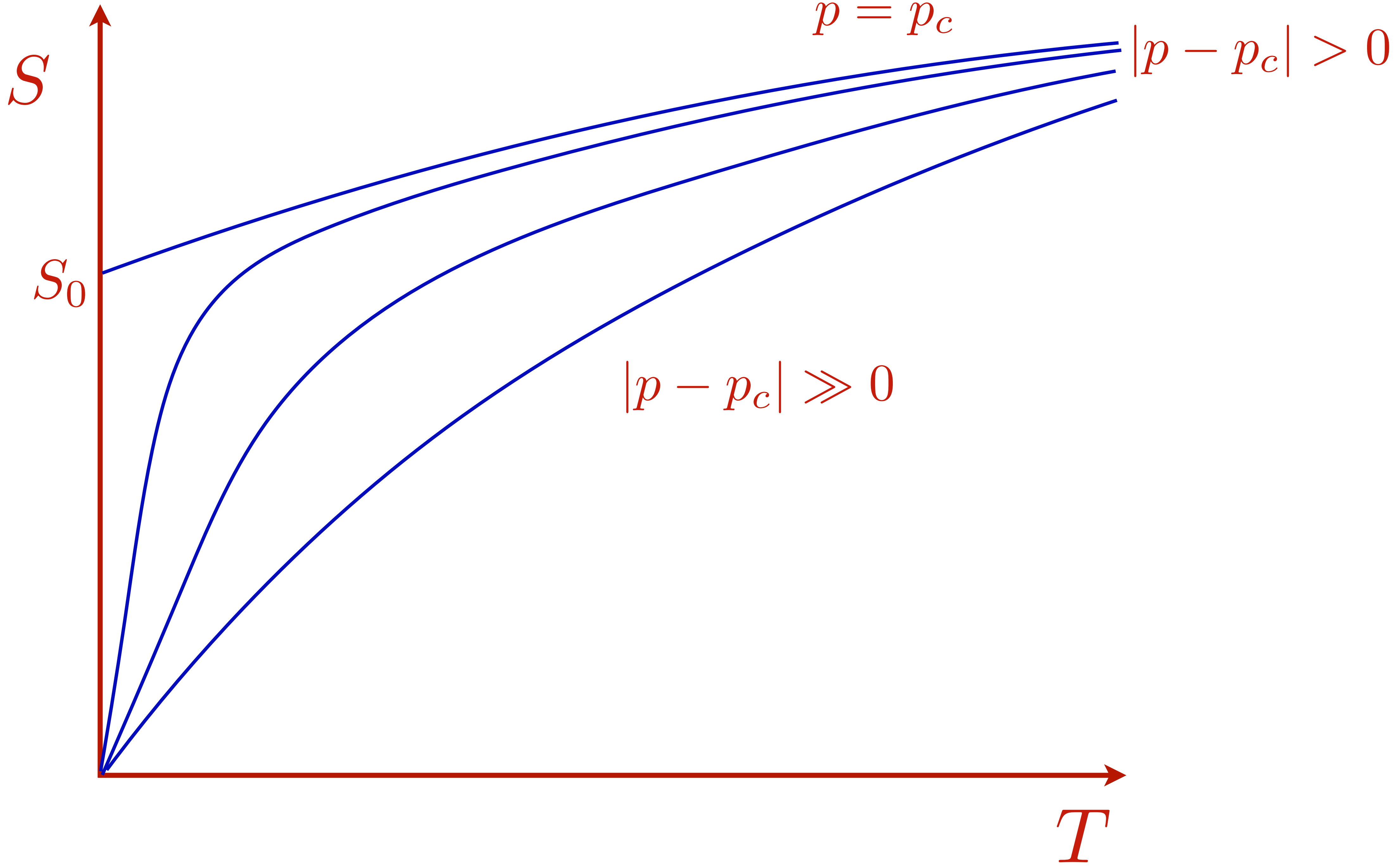
$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy



$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy

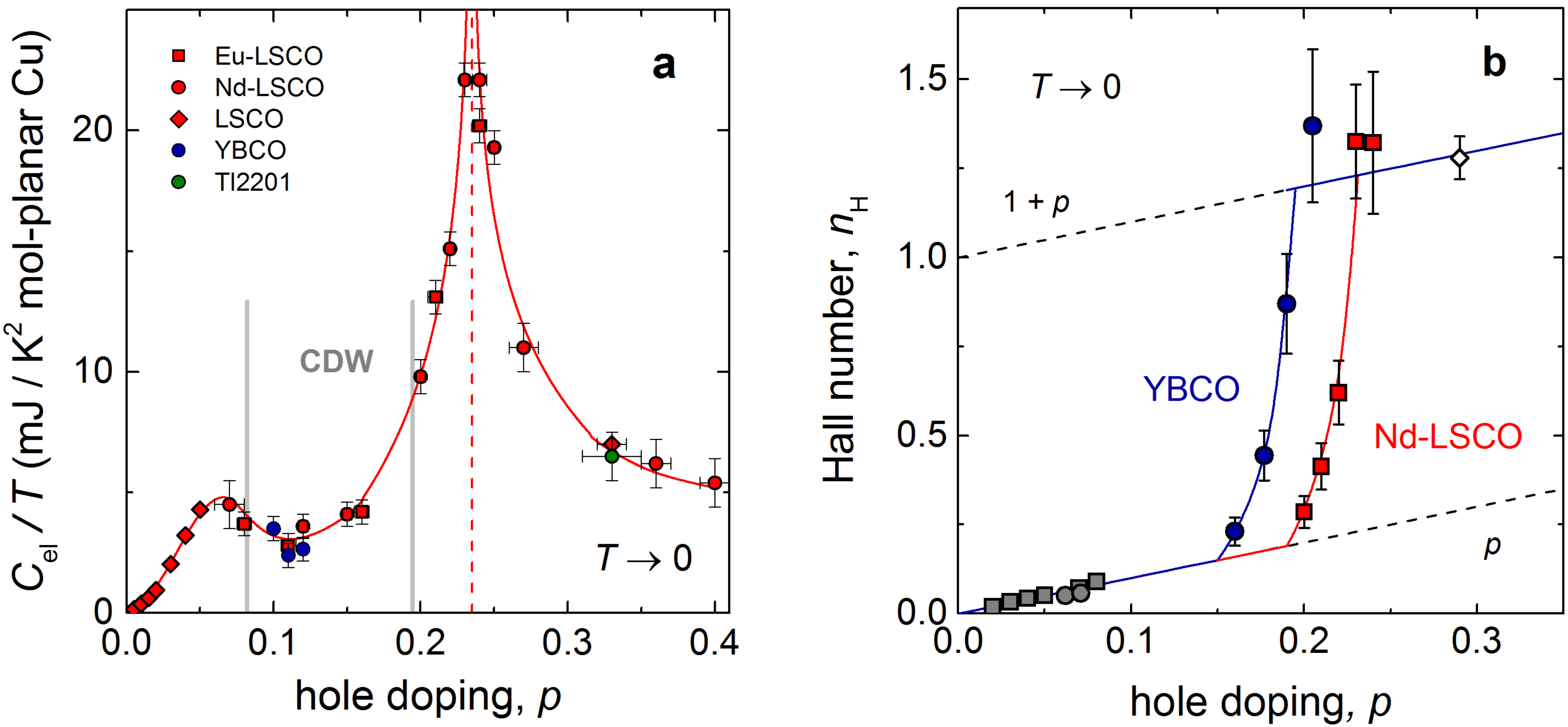


$$\frac{C}{T} = \frac{dS}{dT}$$

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates

The holographic connection between strange metals and black holes

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

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- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

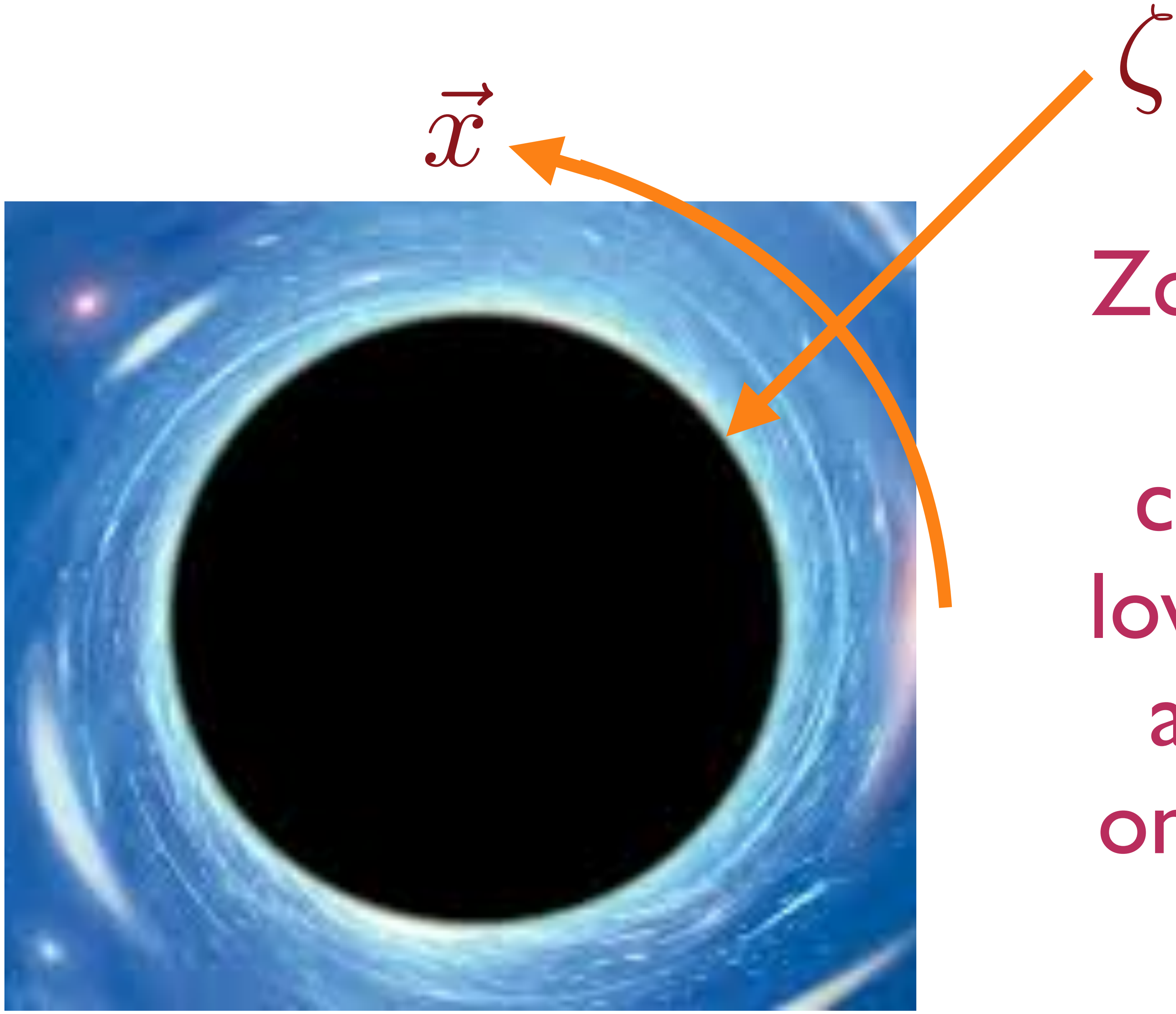
Holography:

Quantum black holes "look like" quantum-critical many-particle systems without quasiparticle excitations, residing "on" the surface of the black hole

J. Maldacena, IJTP **38**, 1113 (1999); S.S. Gubser, I.R. Klebanov, and A.M. Polyakov Phys. Lett. B **428**, 105 (1998);
E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)

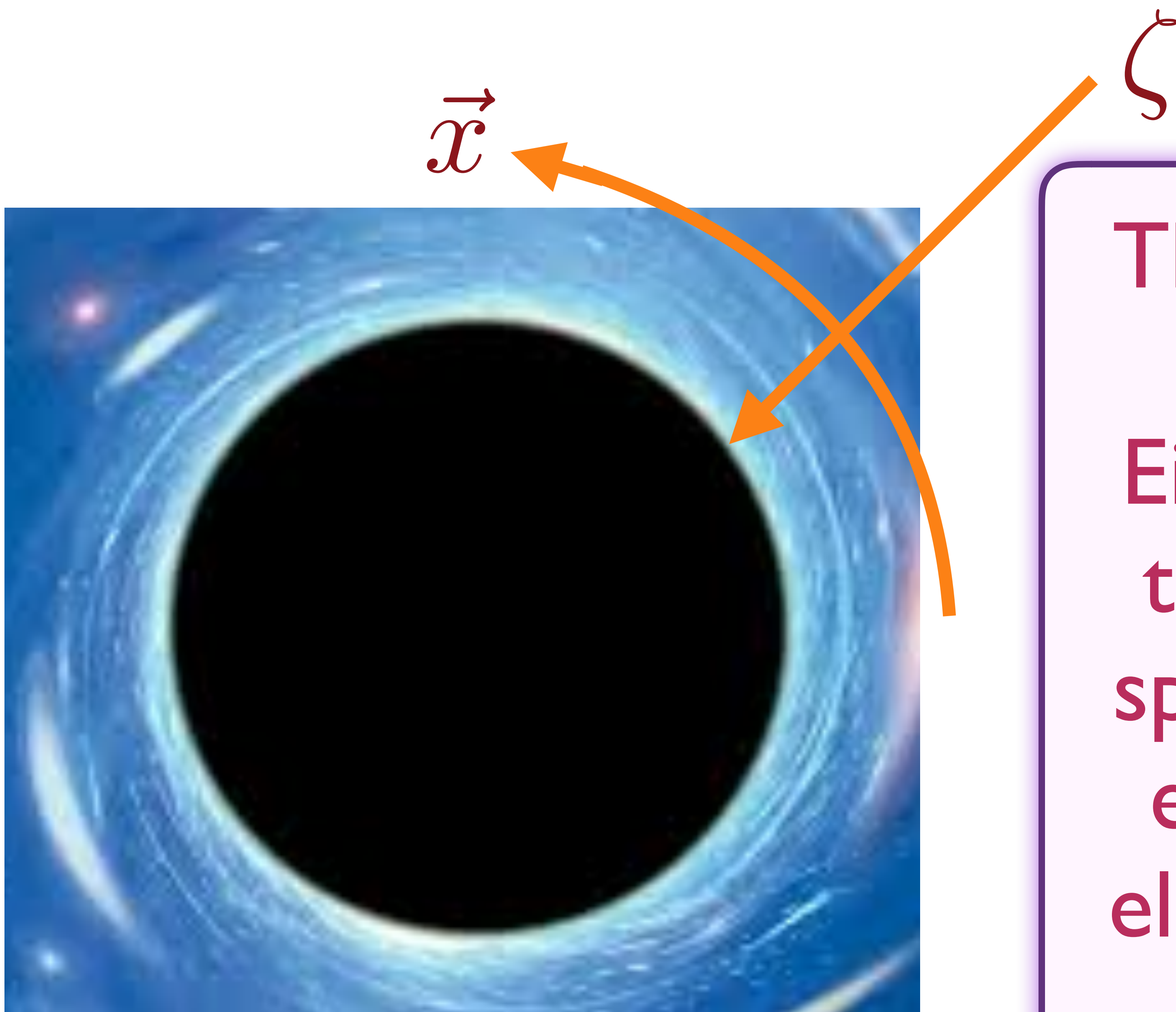
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



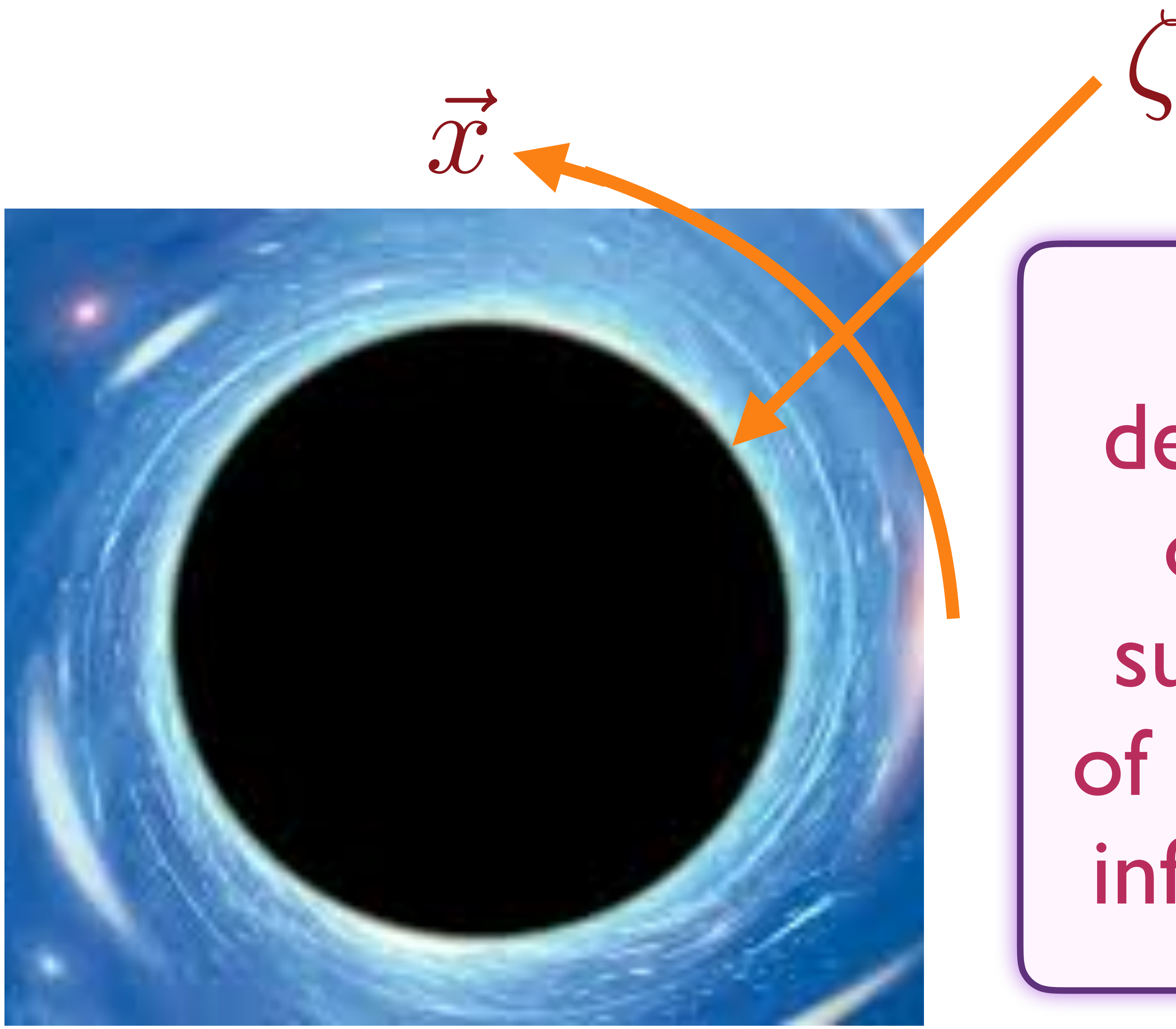
Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



This has led to a deeper understanding of entanglement in superconductors and of Hawking's black hole information "paradox"

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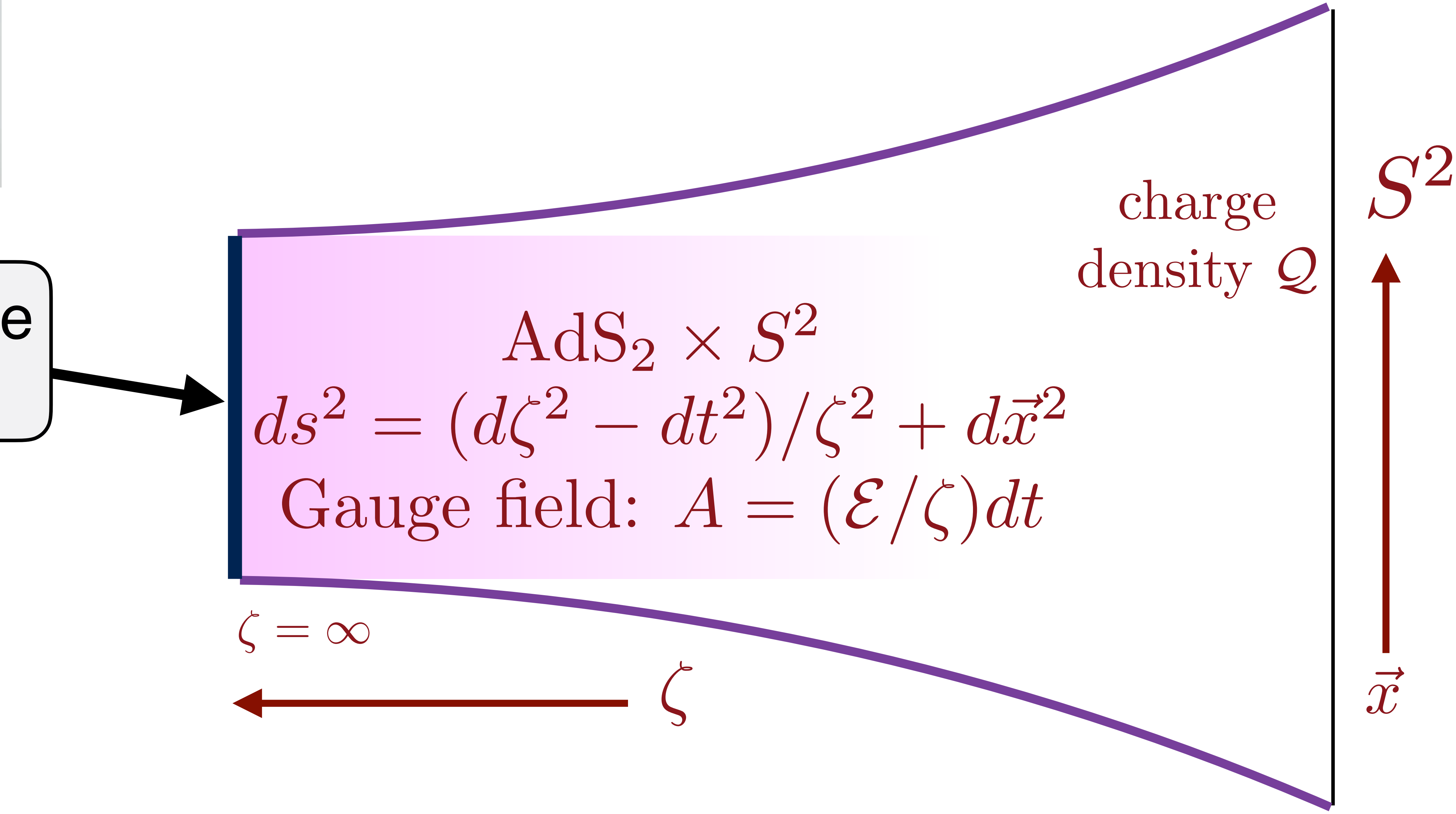
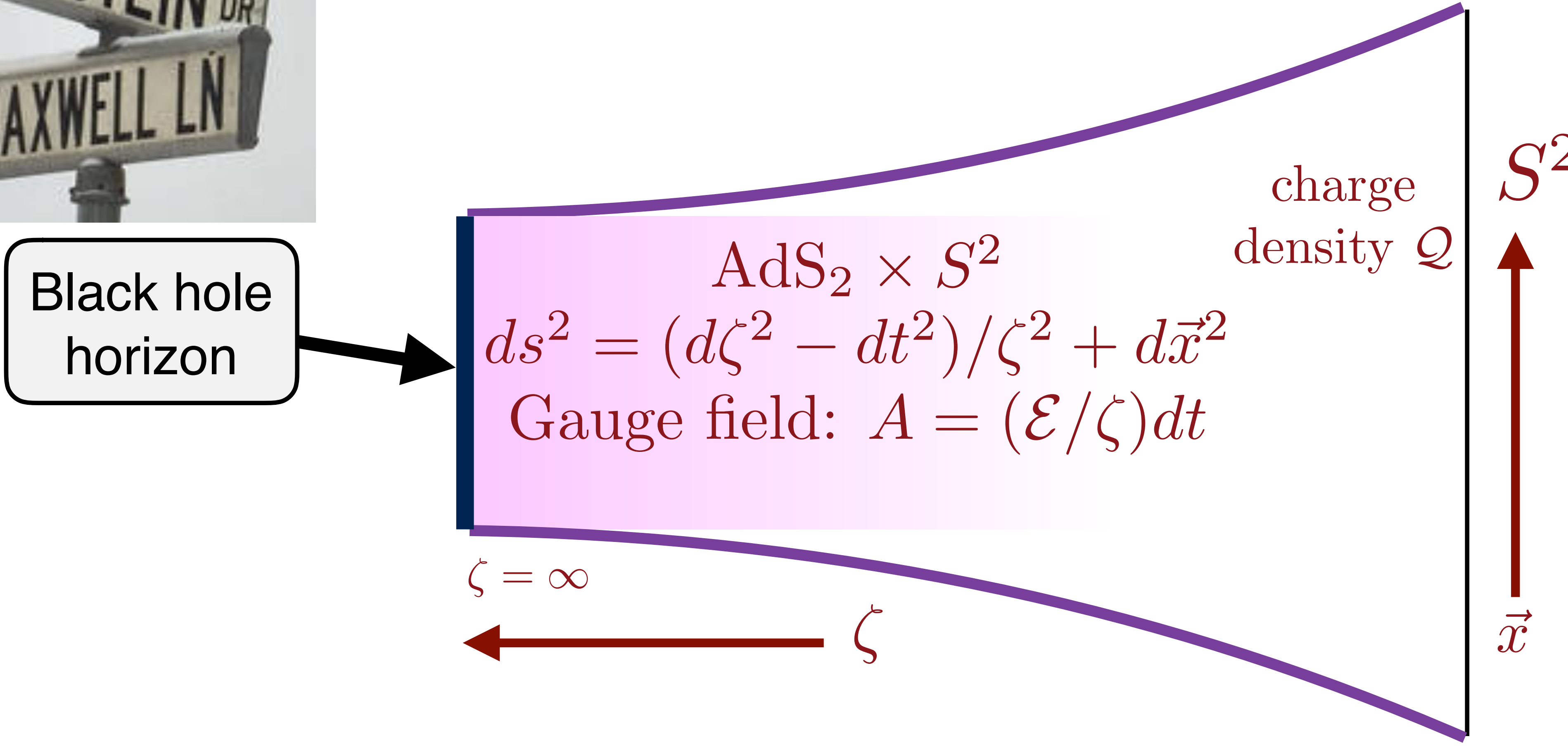
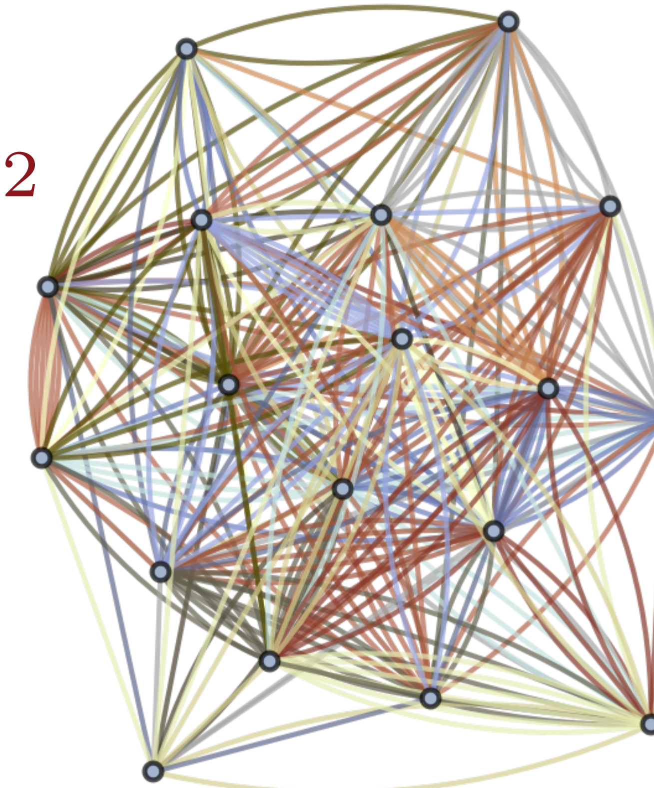
SYK model and charged black holes



The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model

SYK model and charged black holes





Bekenstein-Hawking entropy of AdS_2 horizon at $T = 0 \Leftrightarrow N s_0$ entropy of SYK model.

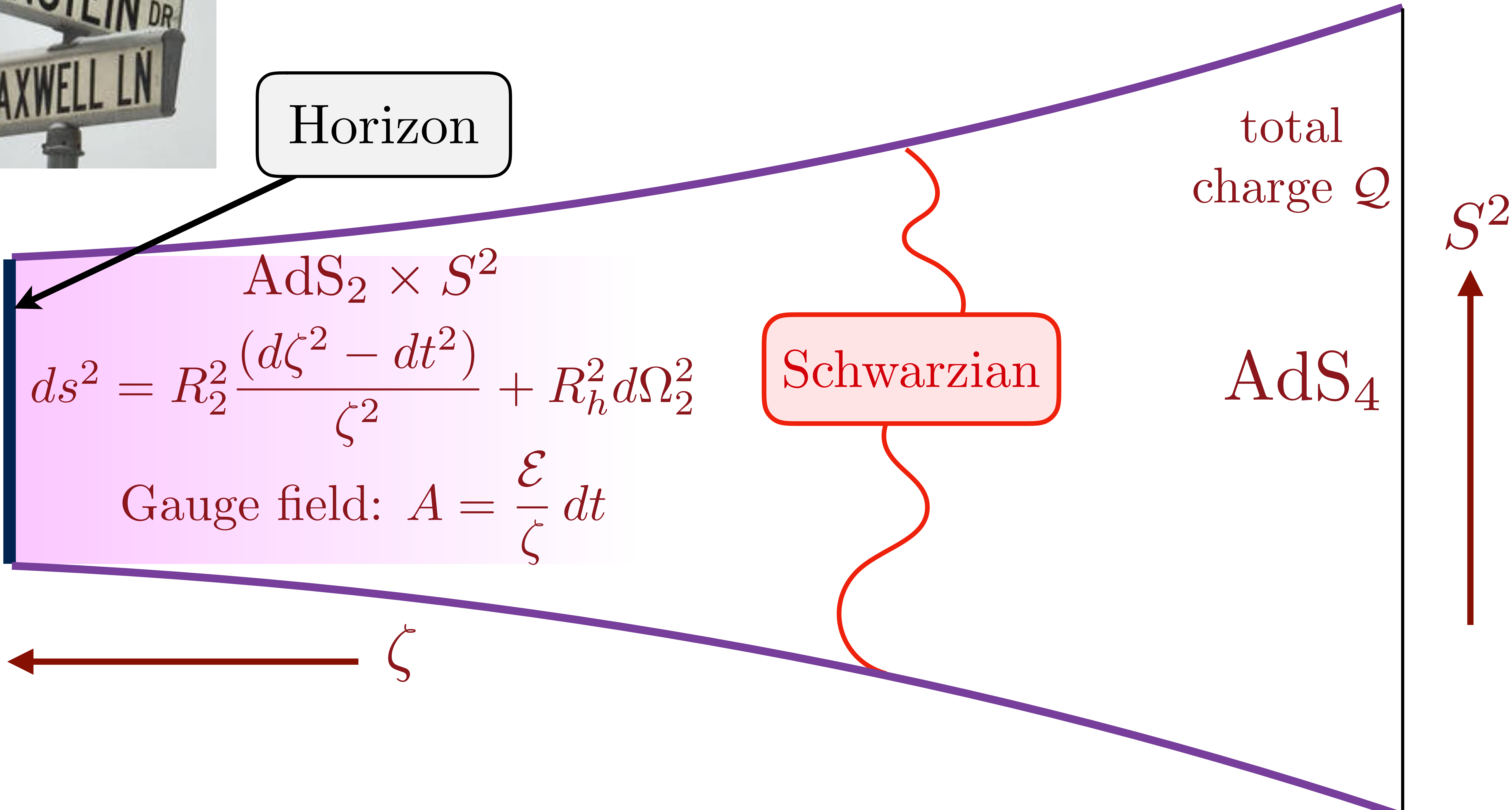
$\dfrac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}$ can be obtained from the Einstein equations for the black hole,

and the quantum theory of the SYK model,

and $\mathcal{E}$ determines identical fermion spectral functions.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

SYK model and charged black holes



Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a 'Schwarzian' theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS_2 and AdS_4.

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

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K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
Phys. Rev. B **95**, 155131 (2017)

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746 P. Nayak, A. Shukla,

R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius R_h, with total charge $\mathcal{Q}$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0 + 1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f \, \mathcal{D}\phi \, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \, \mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$f(\tau + 1/T) = f(\tau) + 1/T$,

ϕ is a phase conjugate to the charge density with

$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n$, n integer,

$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.

The couplings are related to the entropy $S(T, \mathcal{Q})$ and the chemical potential μ via

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{d\mathcal{Q}}$$

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