

Ancilla theory of the pseudogap and of the cuprate phase diagram

Max Planck Institute of Quantum Optics, Garching
November 17, 2023
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang,
Mathias Scheurer, and S. Sachdev, *Proc. Nat. Acad. Sci.* **120**, e2302701120 (2023)

Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi,
and S. Sachdev, *Phys. Rev. B* **108**, 045123 (2023)

Maine Christos and S. Sachdev, [arXiv:2308.03835](https://arxiv.org/abs/2308.03835)



PHYSICS



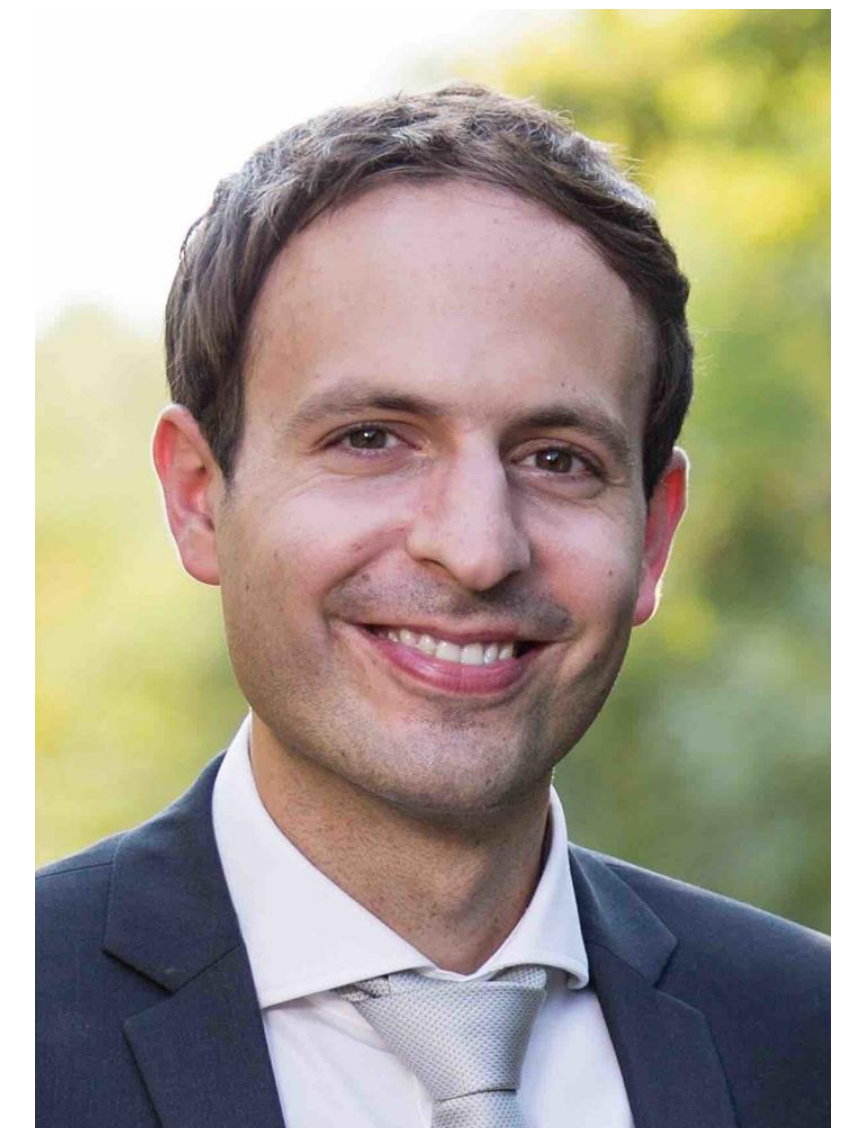
HARVARD



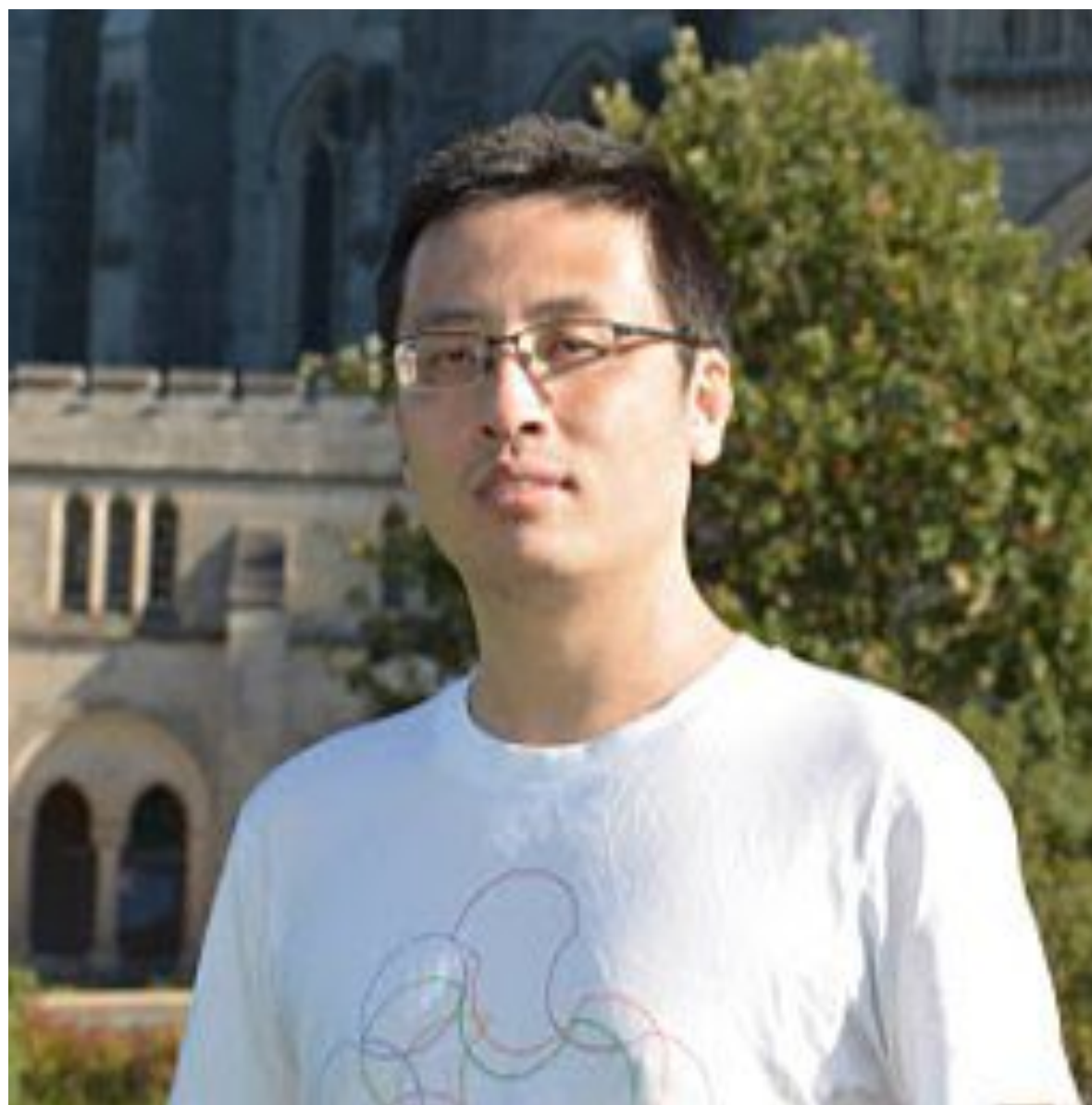
Maine Christos



Zhu-Xi Luo



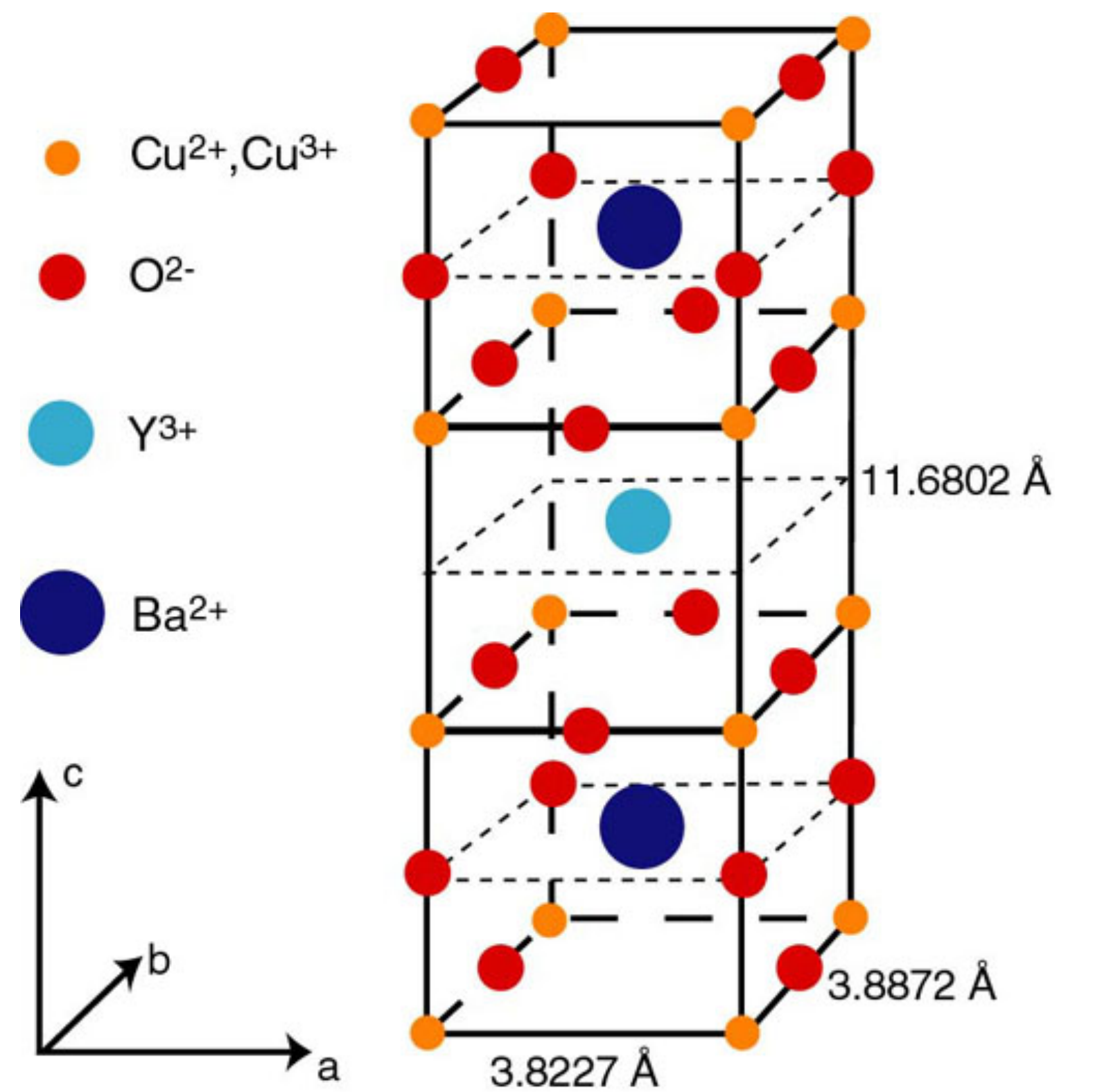
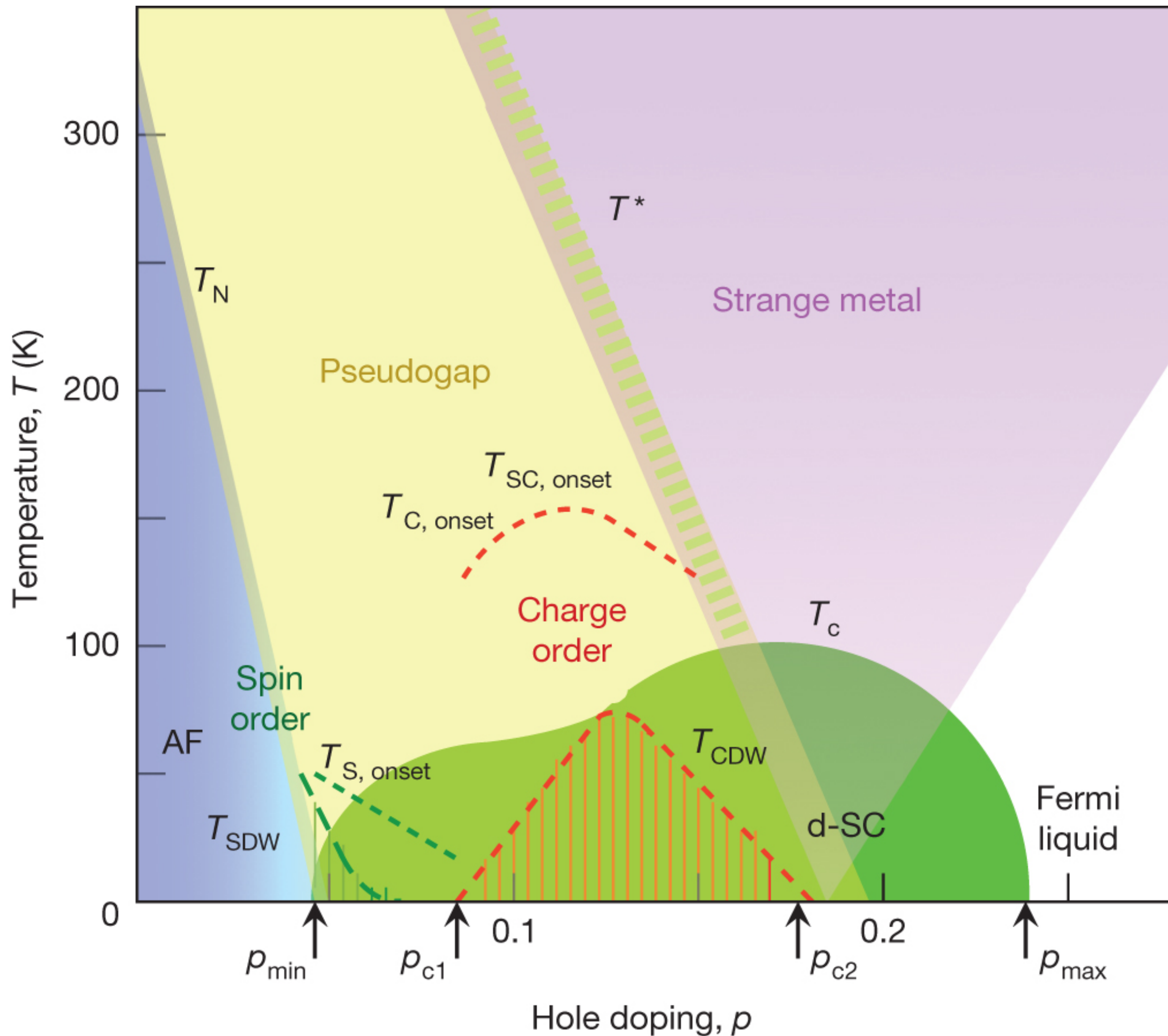
Mathias Scheurer



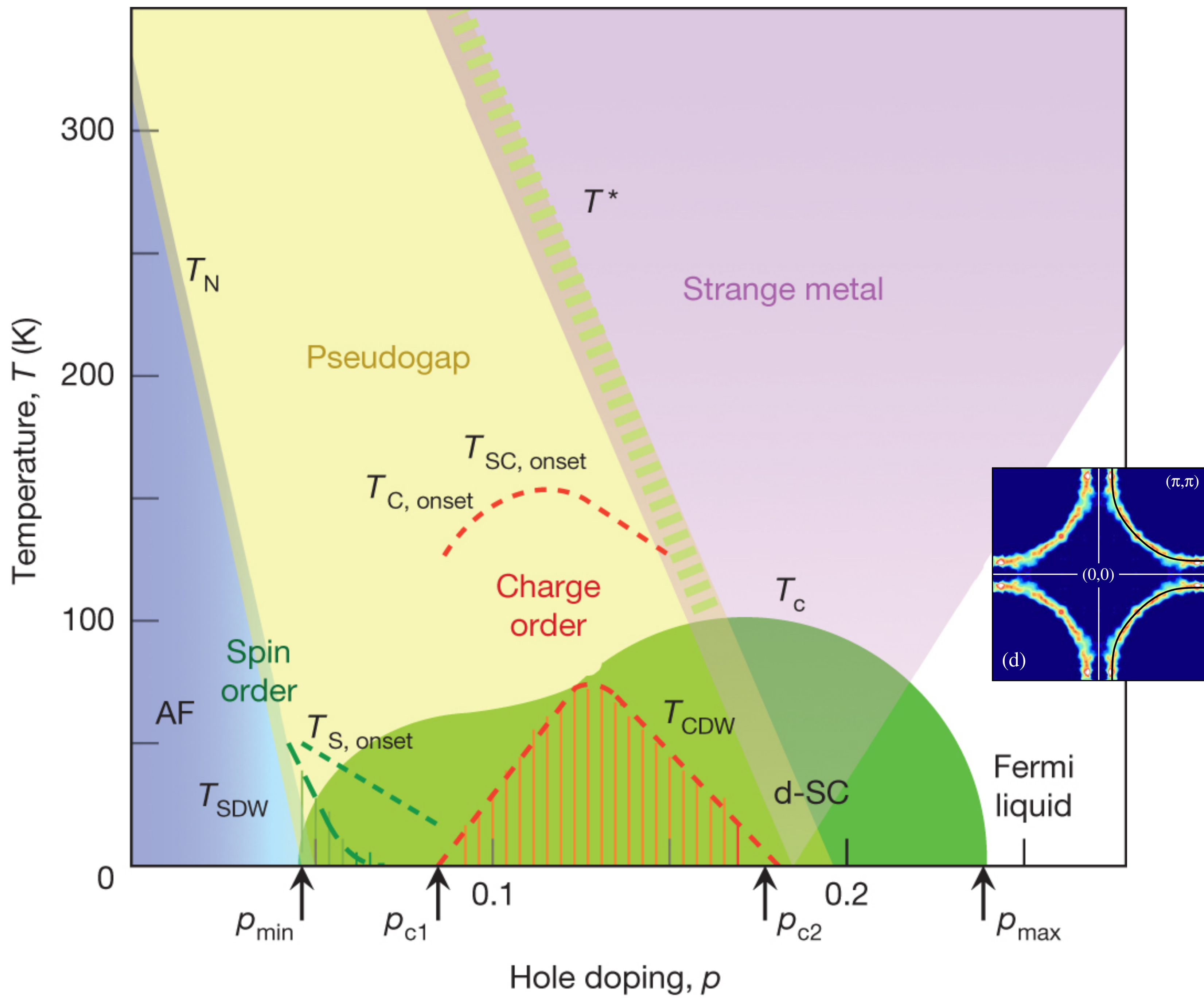
Ya-Hui Zhang



Henry Shackleton

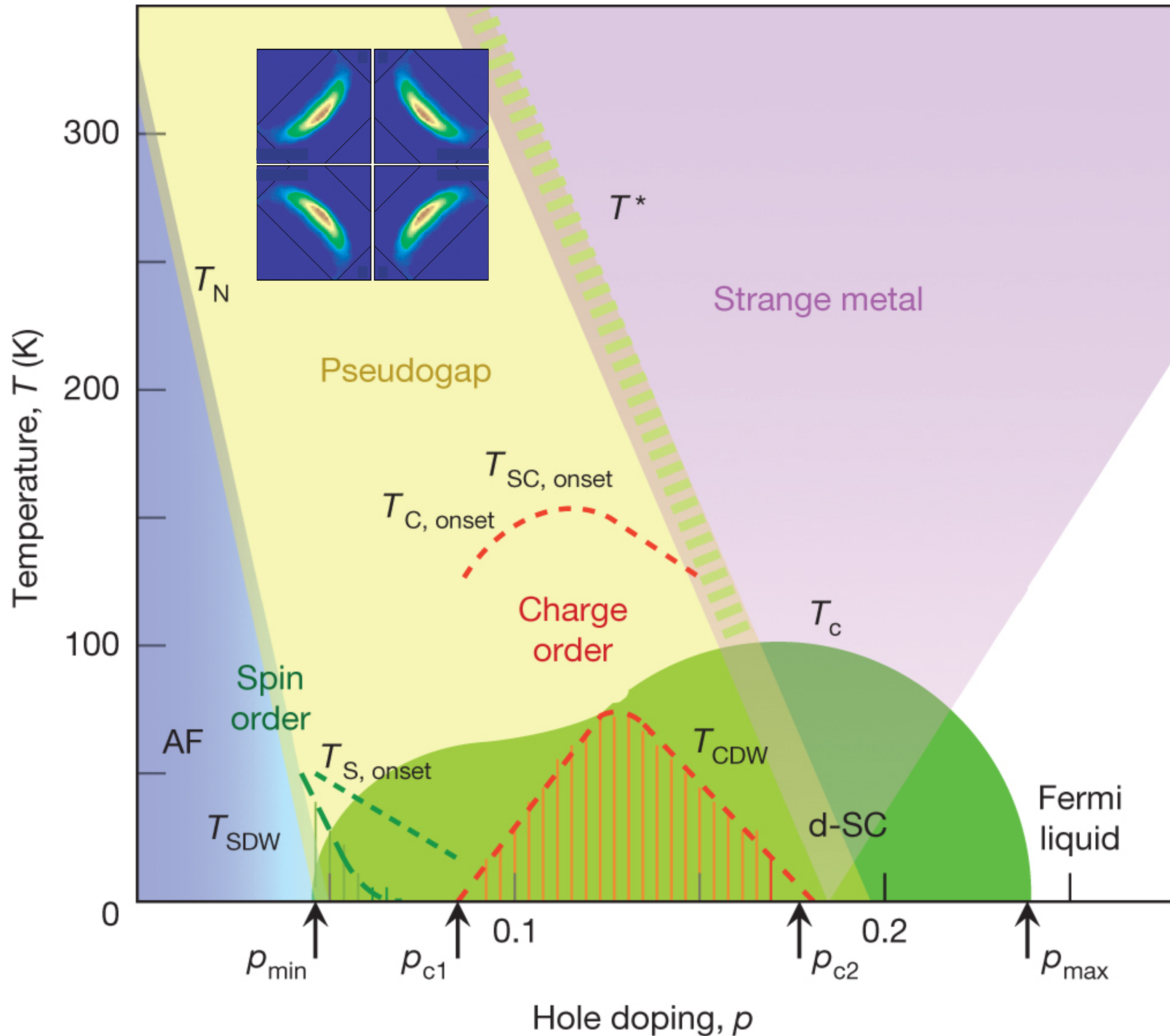


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Fermi liquid
in the
overdoped metal

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Pseudogap metal
with “Fermi arcs”

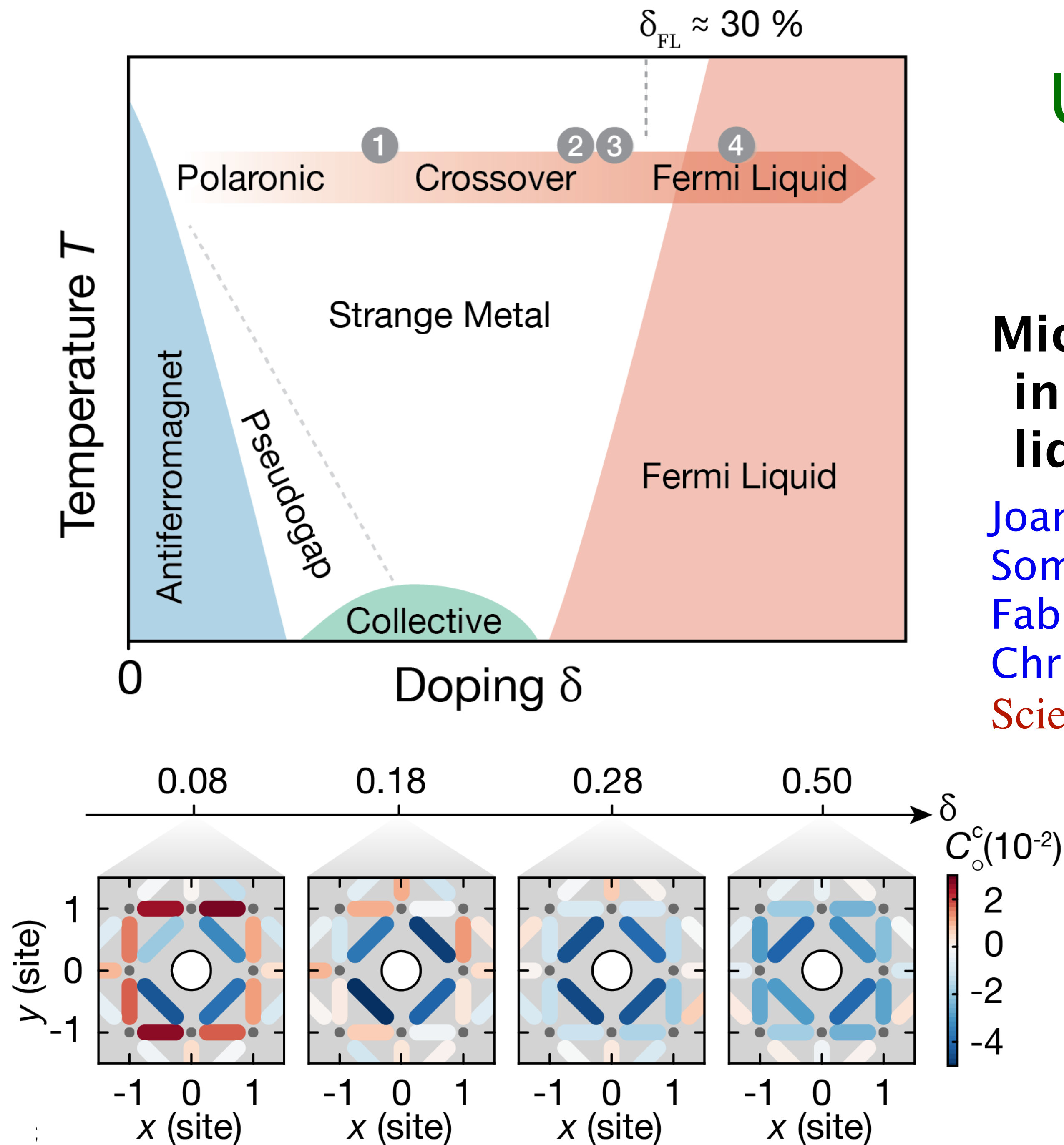
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Ultracold fermionic atoms in optical lattices

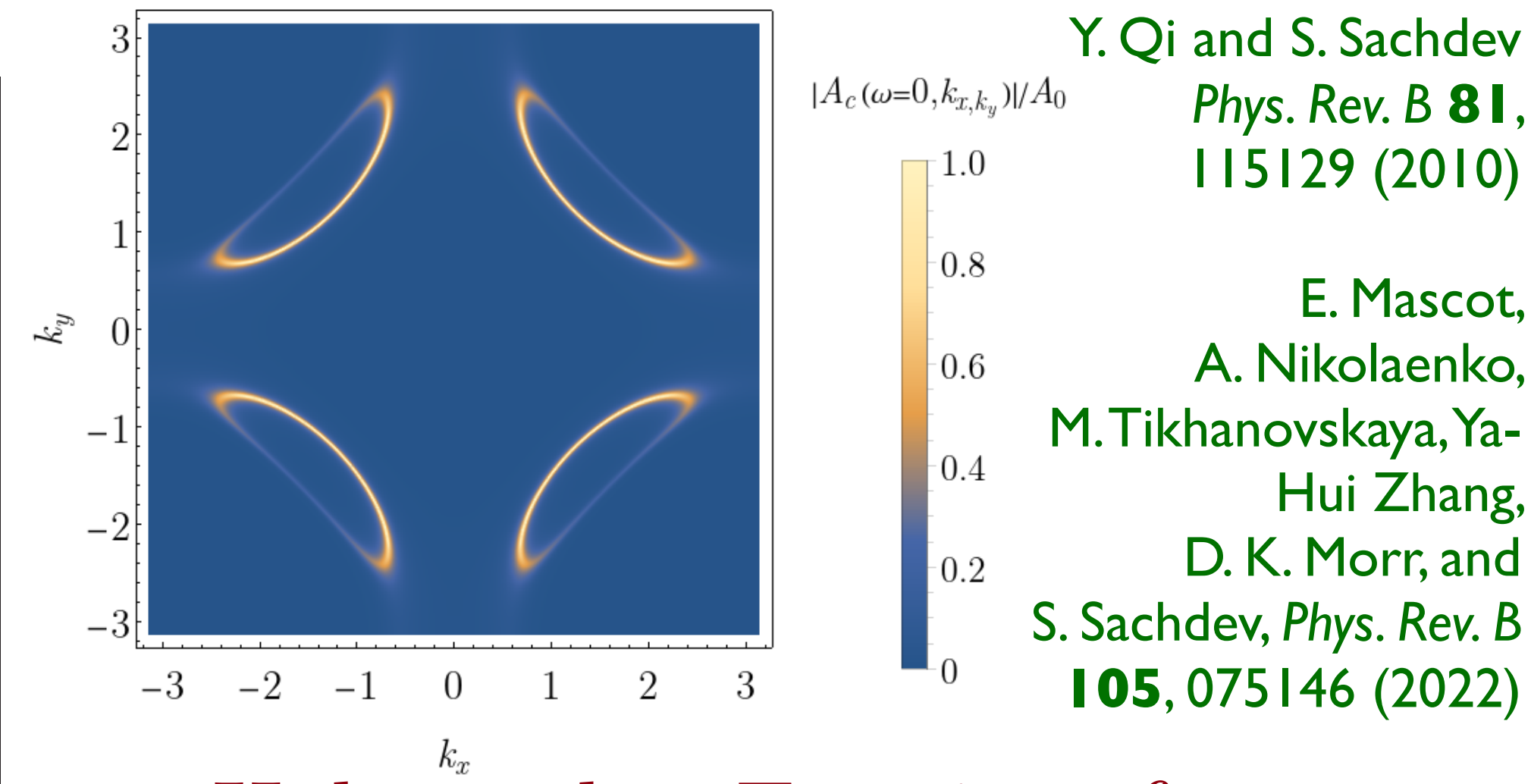
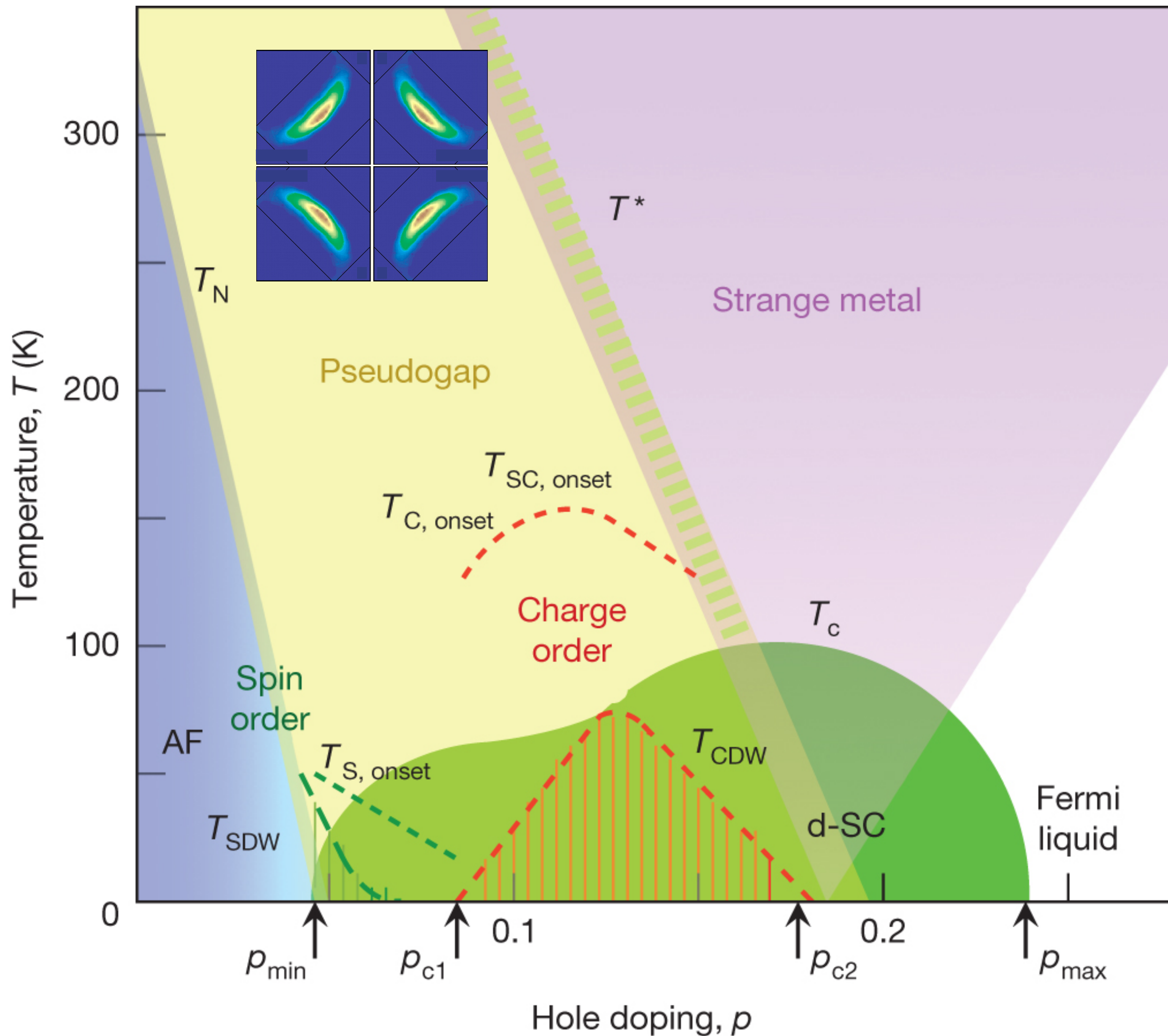
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82



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Y. Qi and S. Sachdev
Phys. Rev. B **81**,
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E. Mascot,
A. Nikolaenko,
M. Tikhanovskaya, Ya-
Hui Zhang,
D. K. Morr, and
S. Sachdev, *Phys. Rev. B*
105, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e , spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,
Phys. Rev. B **73**, 174501 (2006).

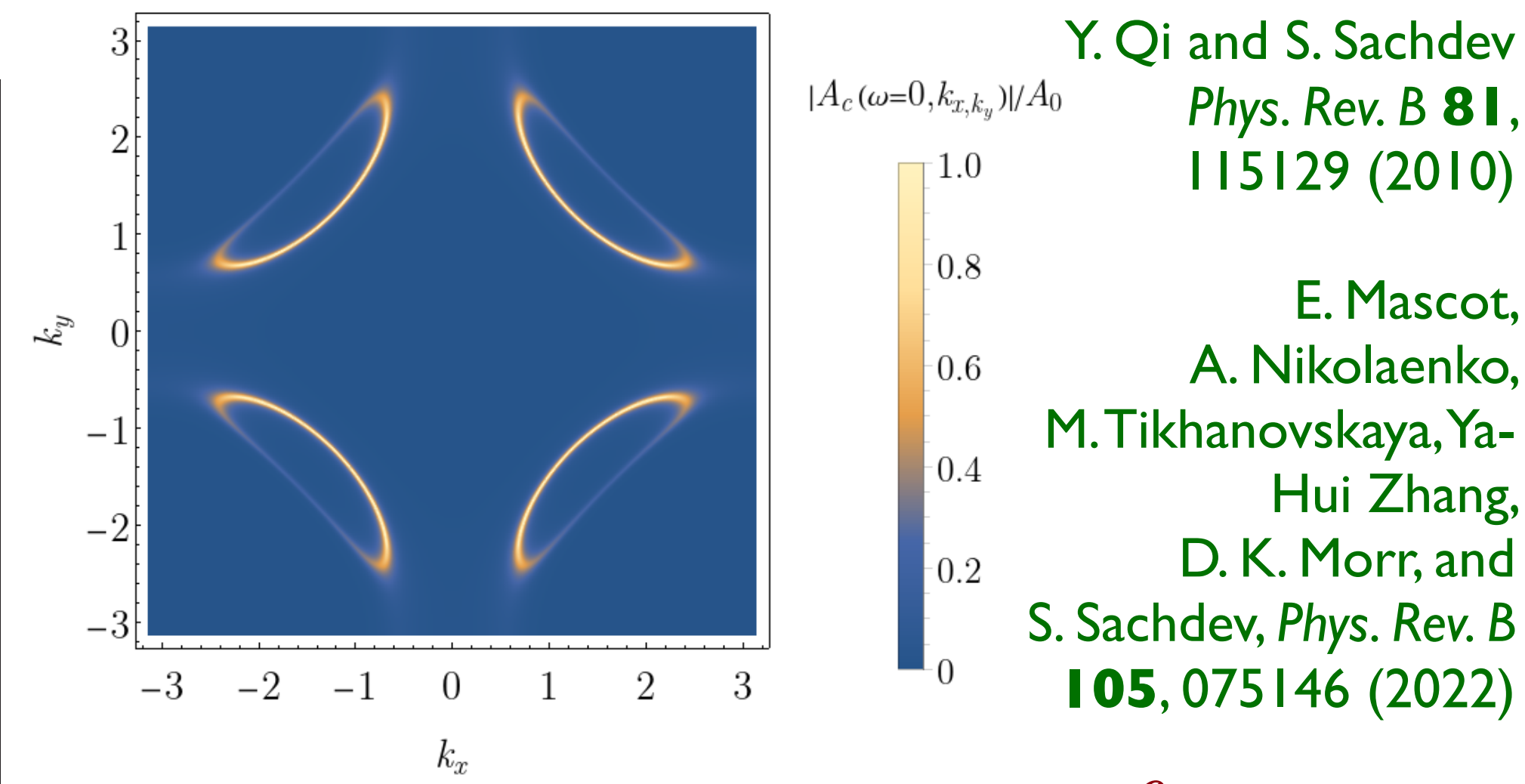
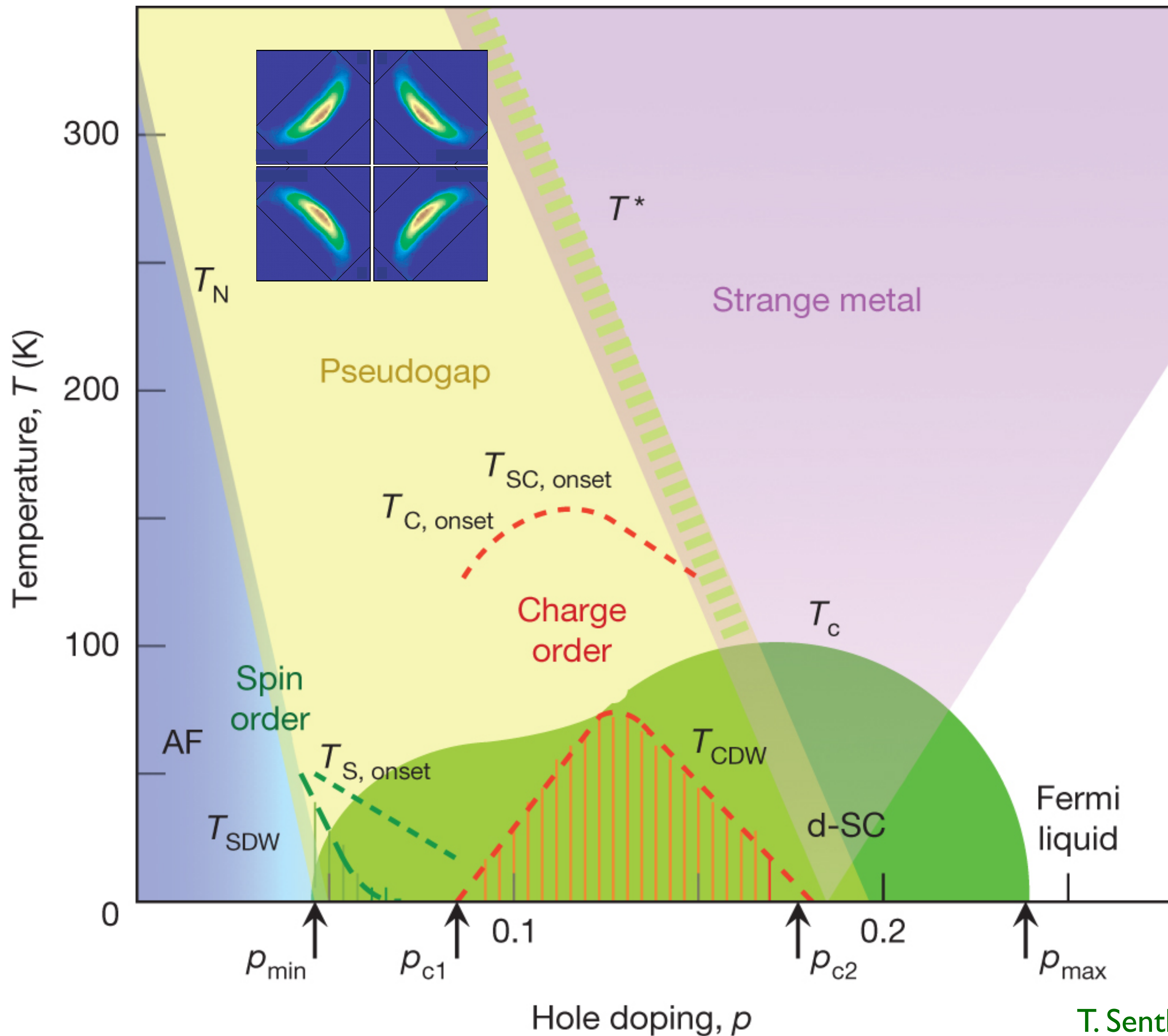
T. D. Stanescu and G. Kotliar,
Phys. Rev. B **74**, 125110 (2006).

C. Berthod, T. Giamarchi, S. Biermann, and A. Georges,
Phys. Rev. Lett. **97**, 136401 (2006).

S. Sakai, Y. Motome, M. Imada,
Phys. Rev. Lett. **102**, 056404 (2009).

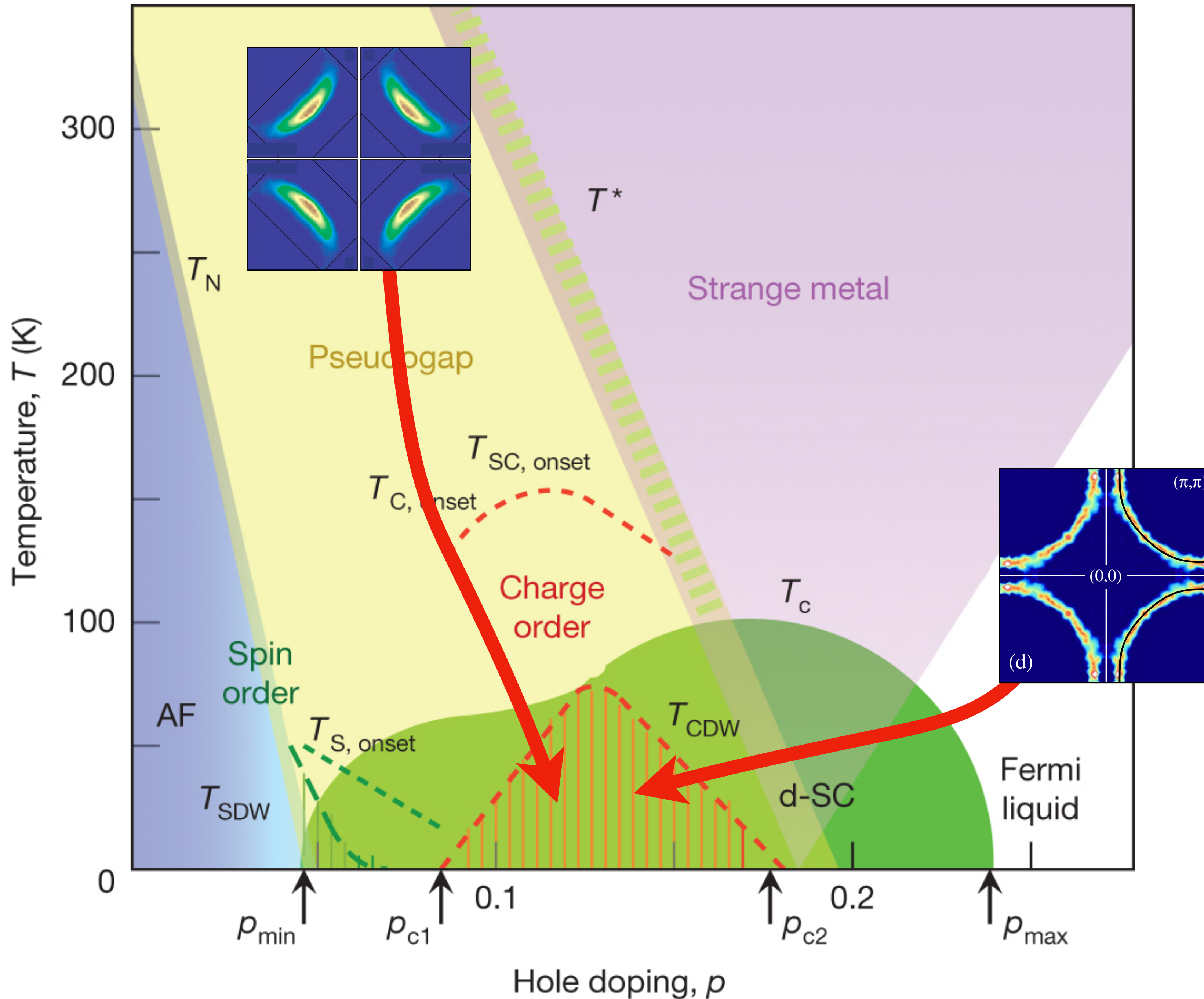
J. Skolimowski and M. Fabrizio,
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N. Wagner....A. Georges, G. Sangiovanni, arXiv:2301.05588
Jinchao Zhao, Gabriele La Nave, Philip Phillips,
arXiv:2304.04787.



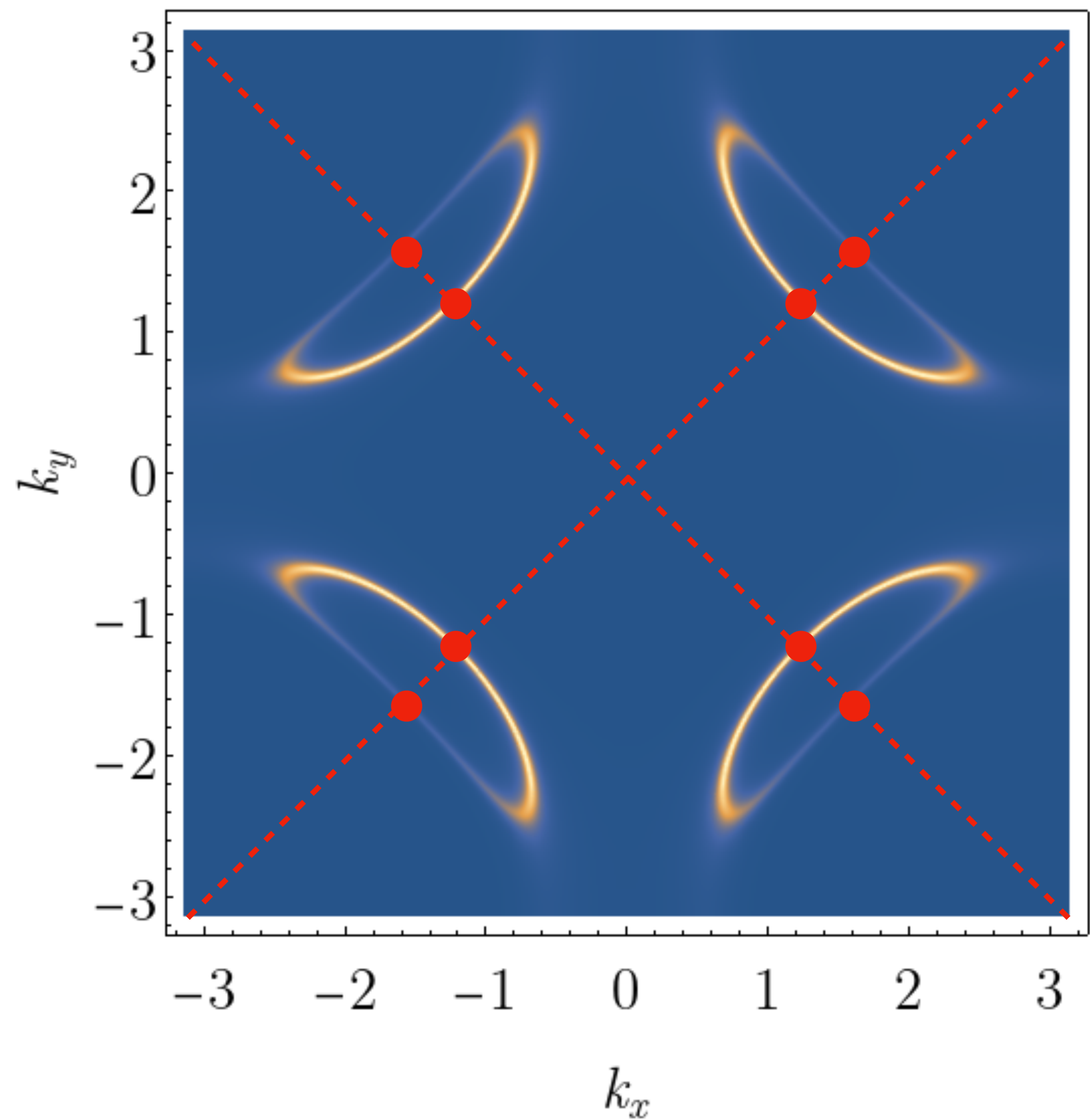
Hole pocket Fermi surfaces
of size p with
charge e , spin-1/2 quasiparticles
+
'spectator'
square lattice spin liquid
at half-filling.

FL*: Spin liquid is *required* because
the Fermi surface does not enclose
the Luttinger volume $(1 + p)$.

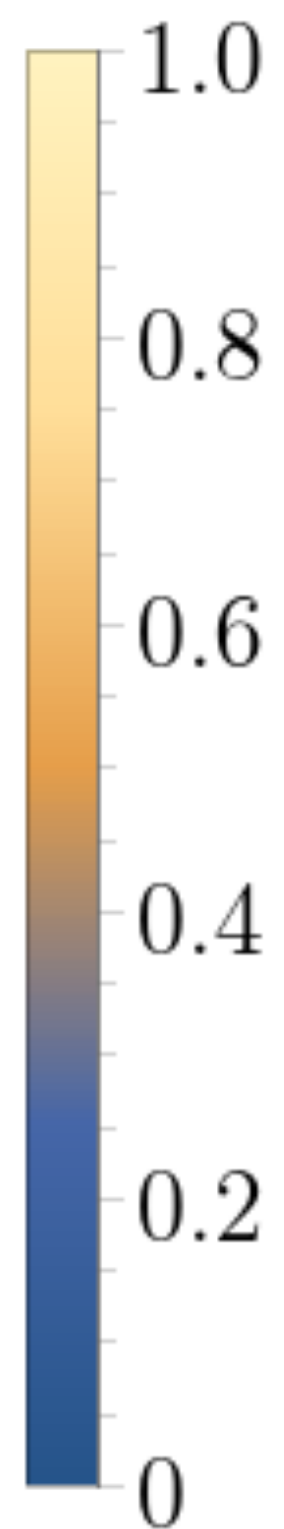


Build a theory for the phase diagram from a theory of the pseudogap metal as a ‘metastable’ $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.



$$|A_c(\omega=0, k_x, k_y)|/A_0$$



Adding *d*-wave pairing
to the hole pockets
leads to 8 nodal points???

From FL*

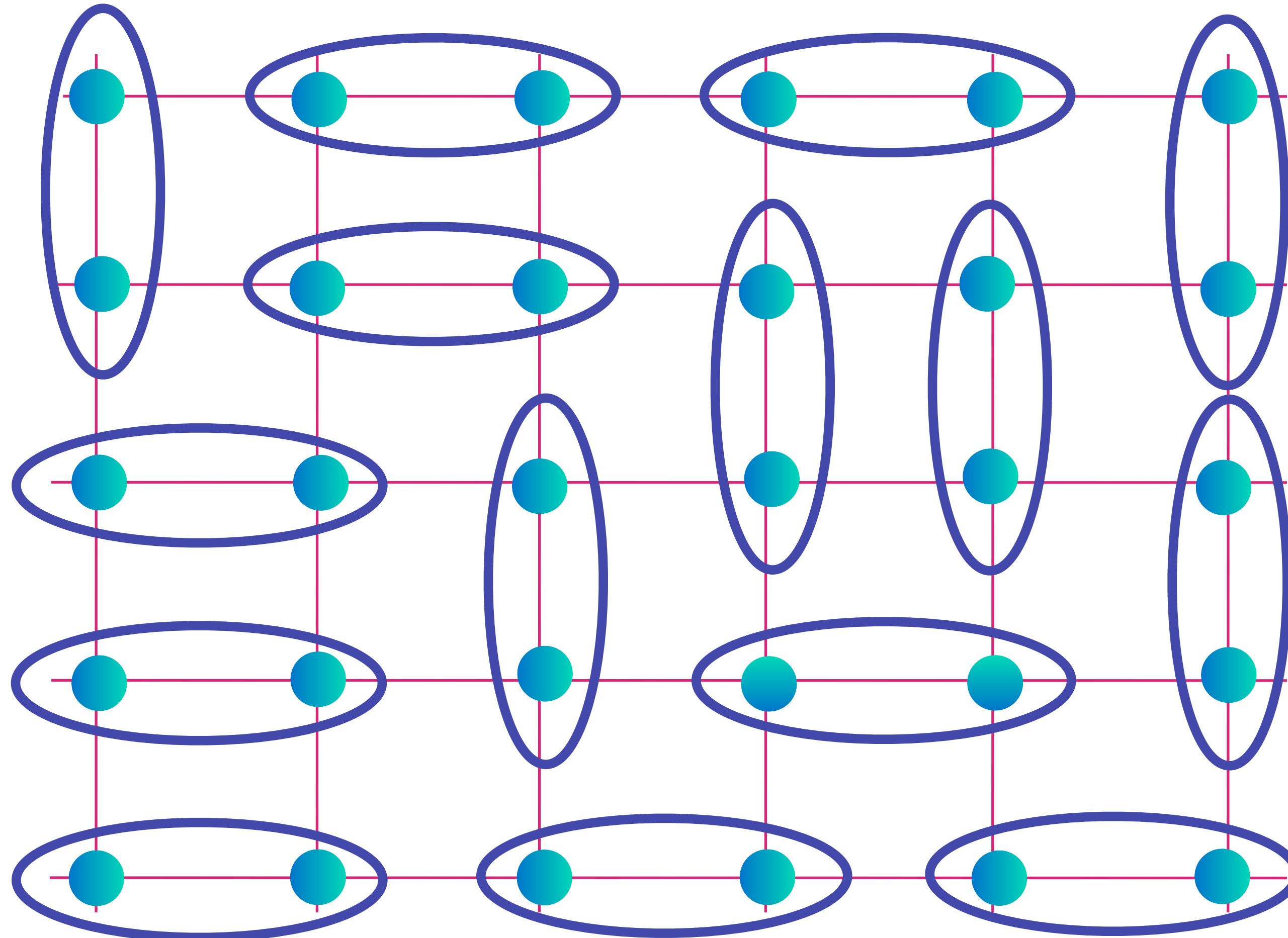
to

a cuprate phase diagram

The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample

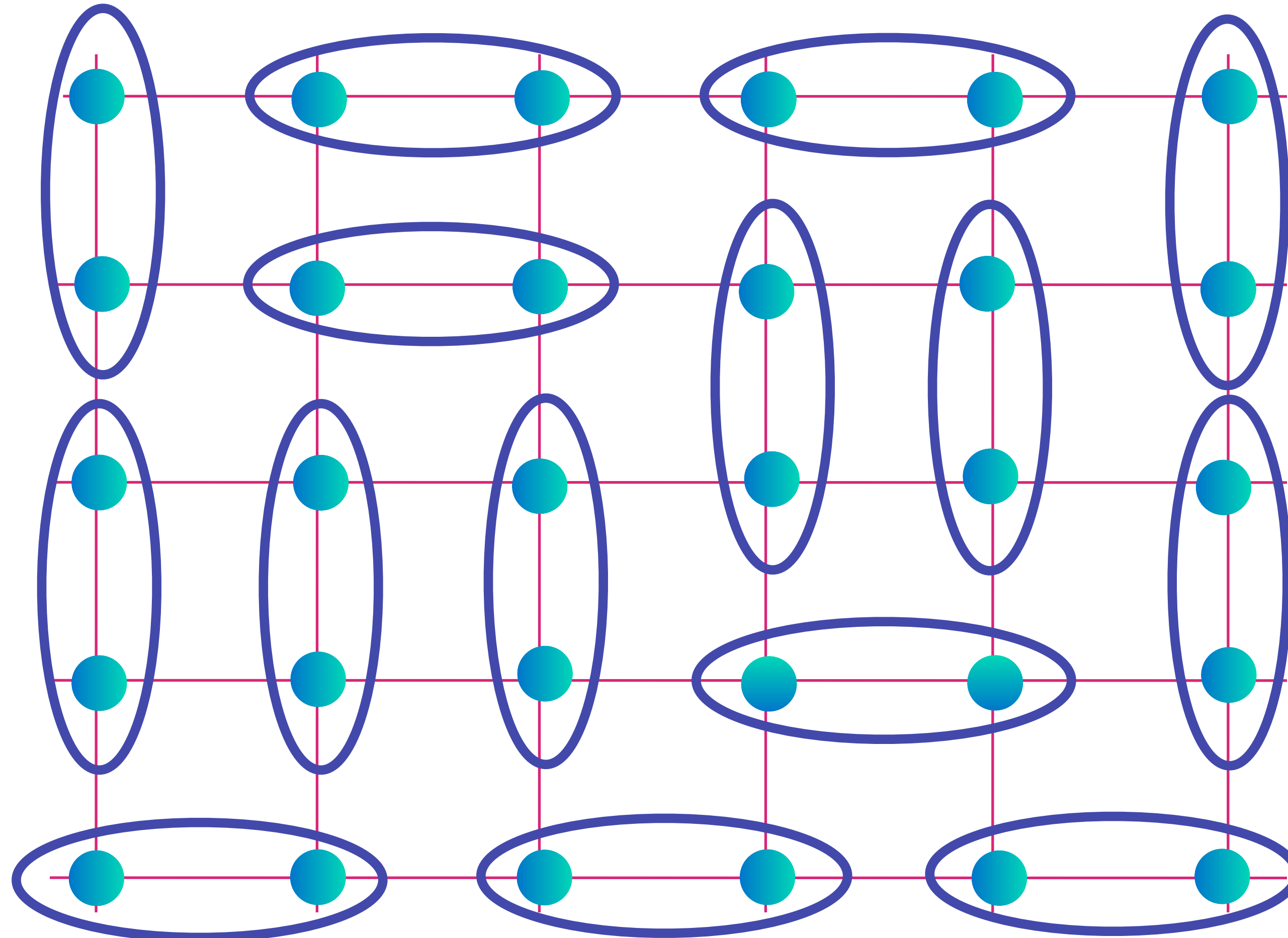
$$\text{[Diagram of two teal circles in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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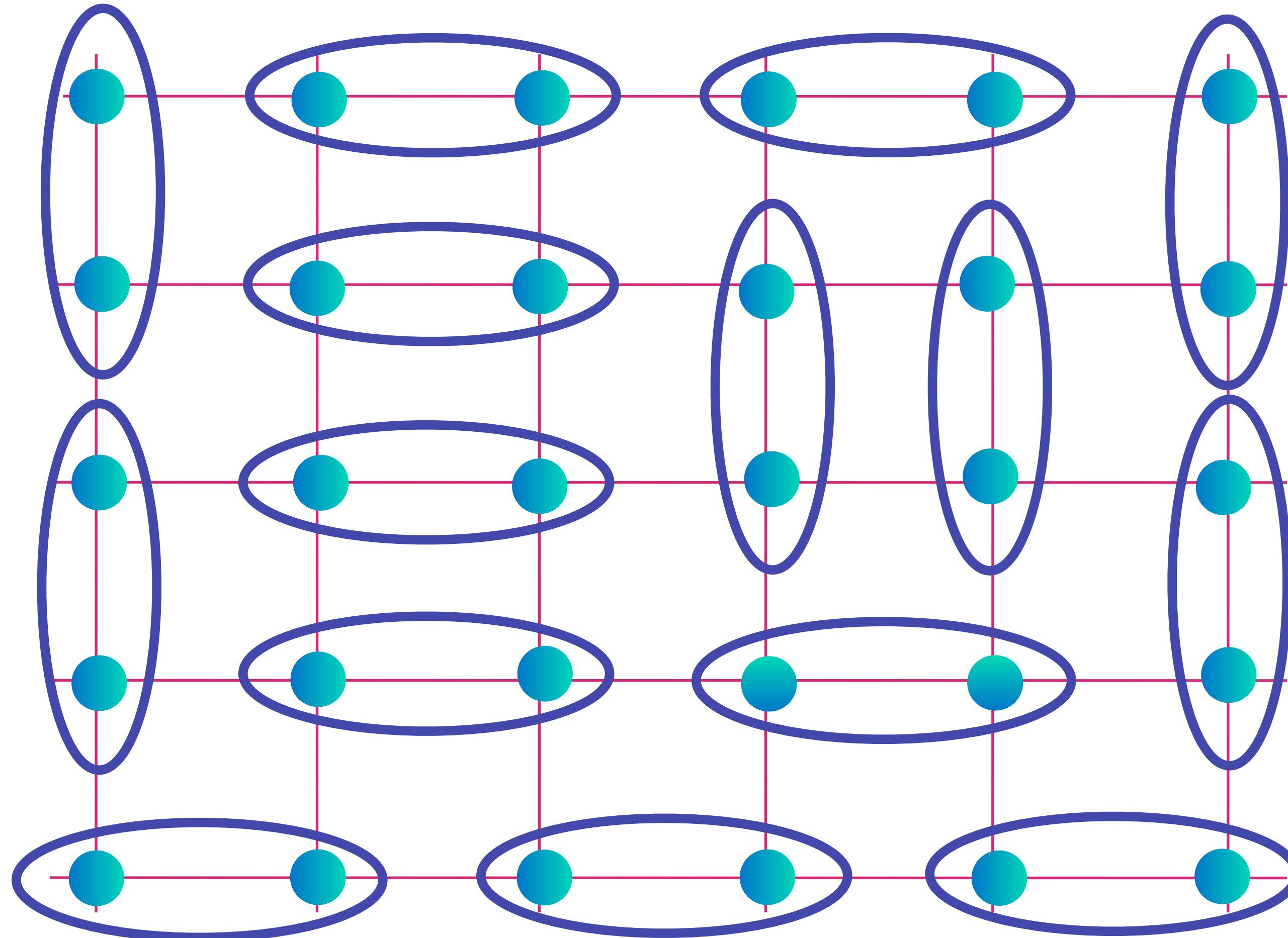
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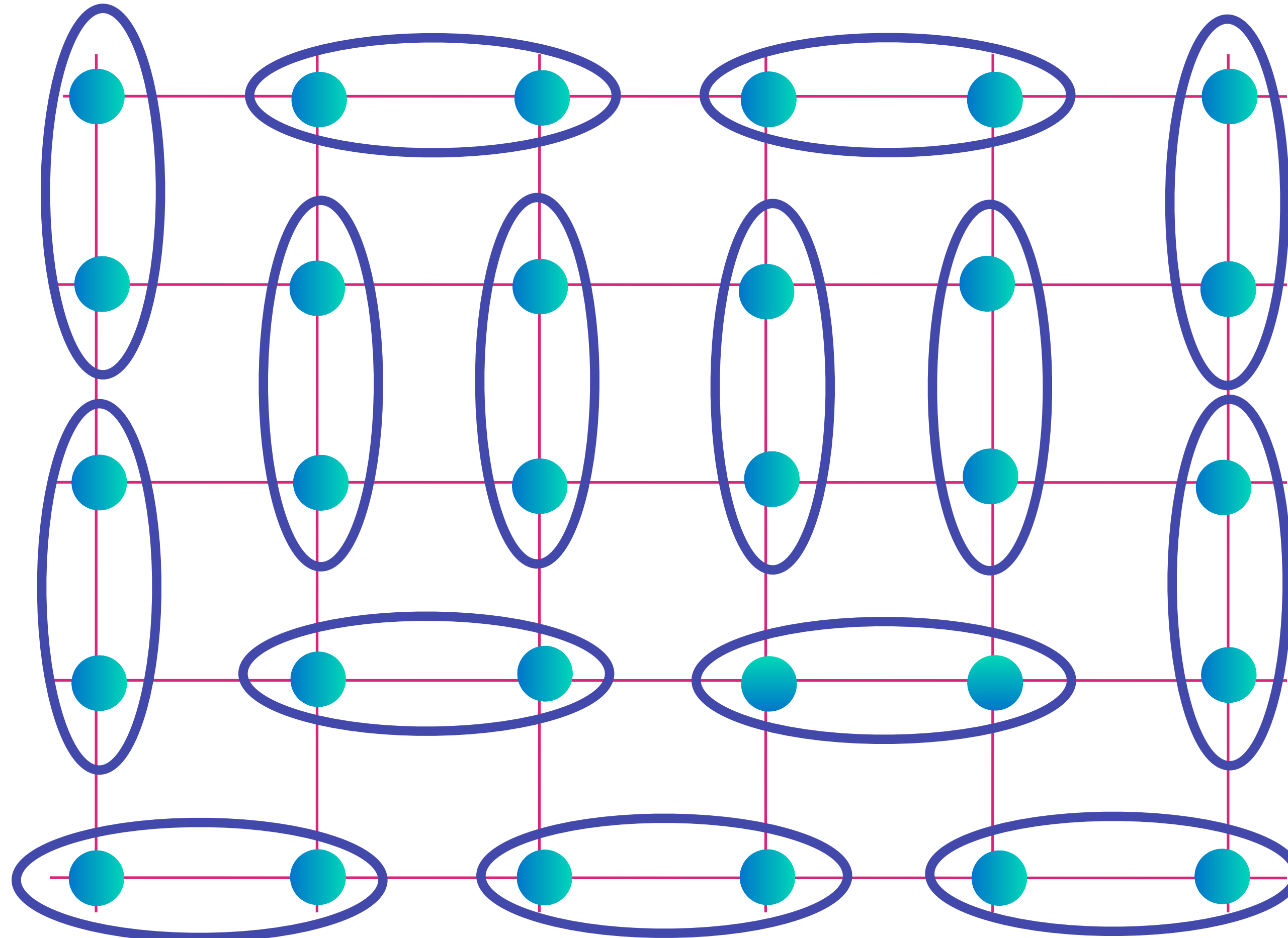
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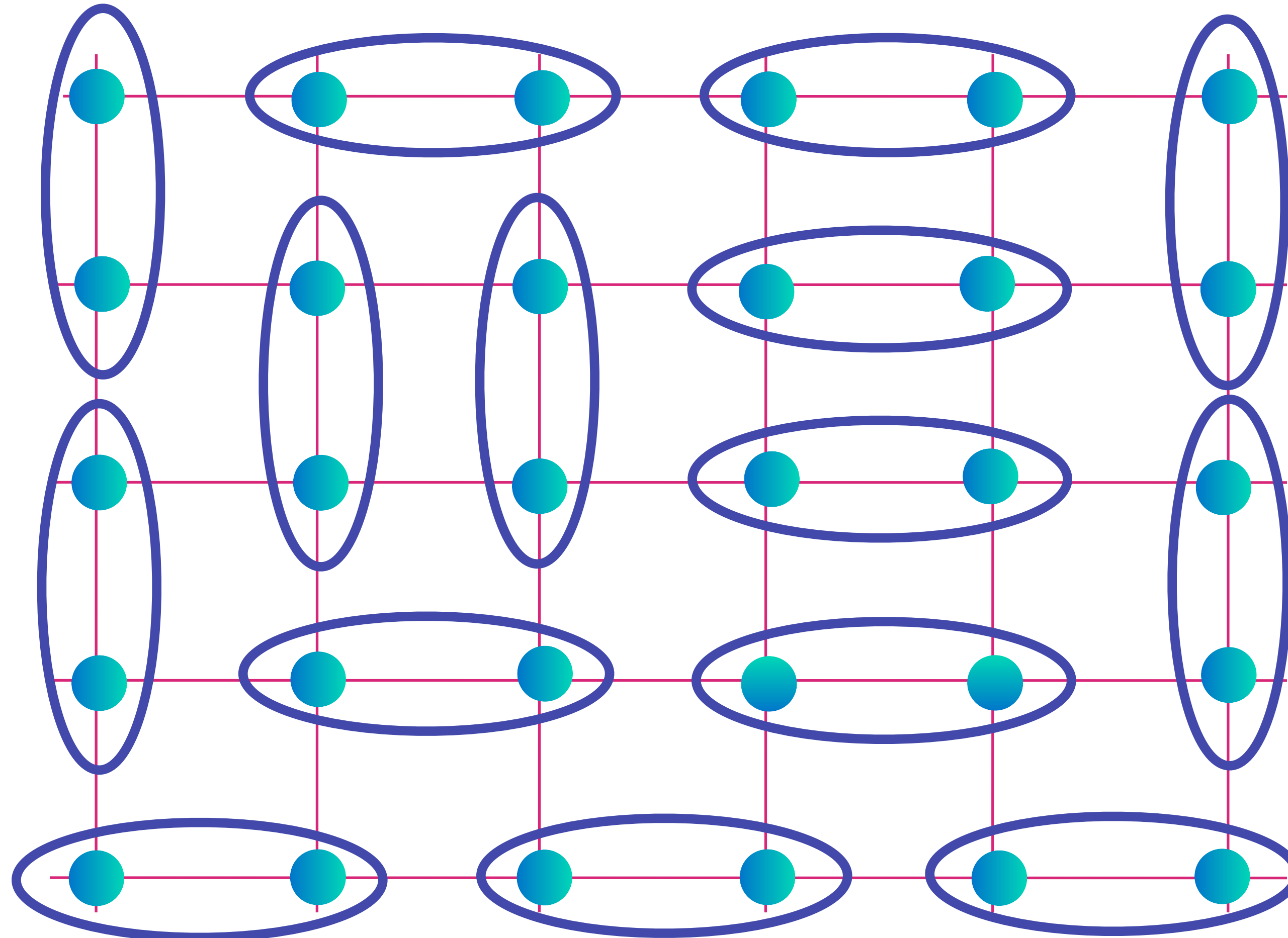
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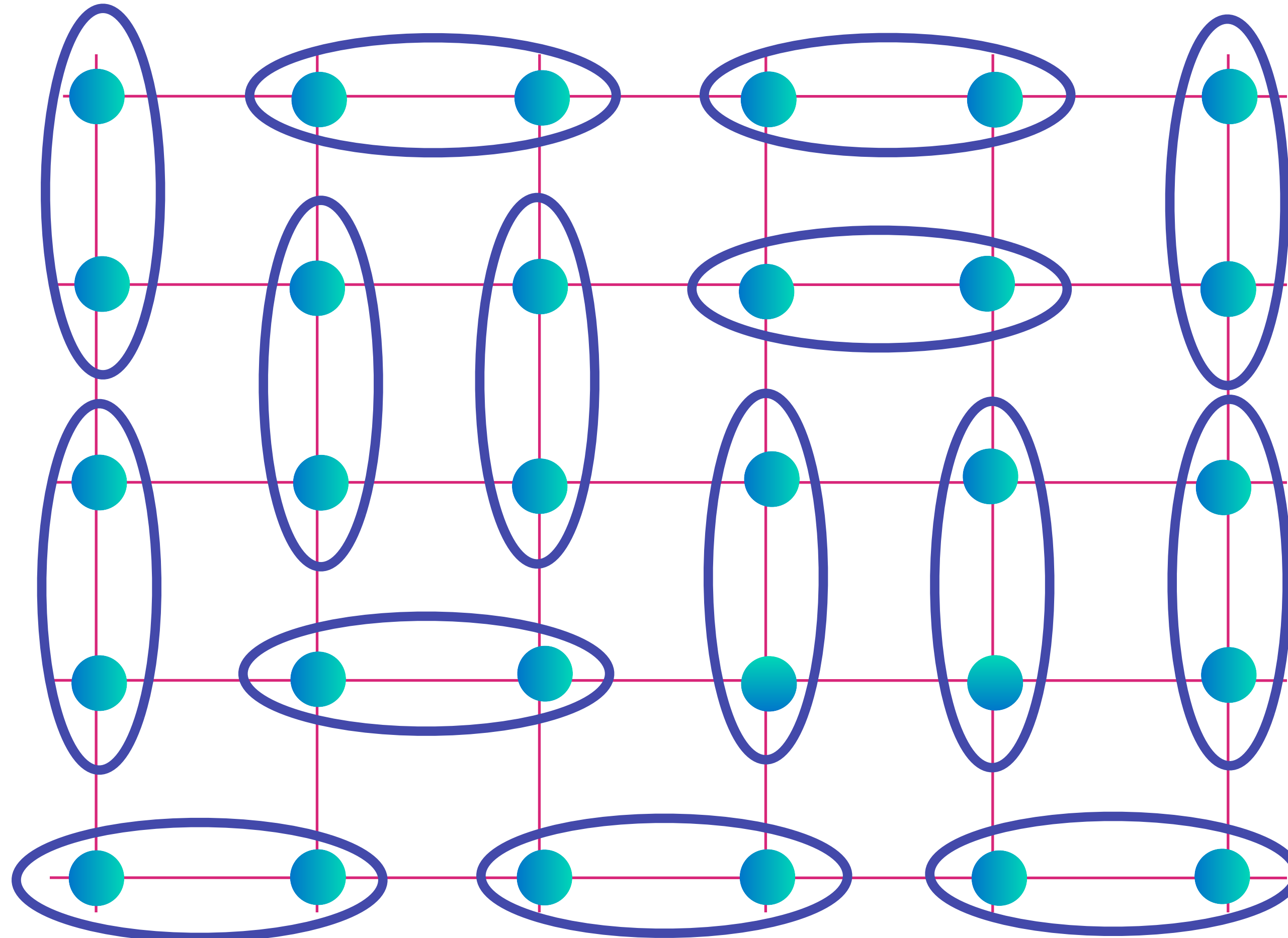
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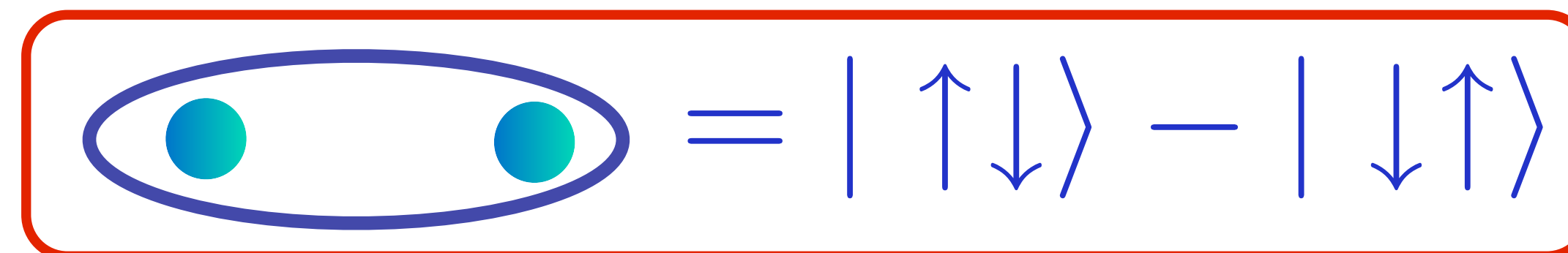
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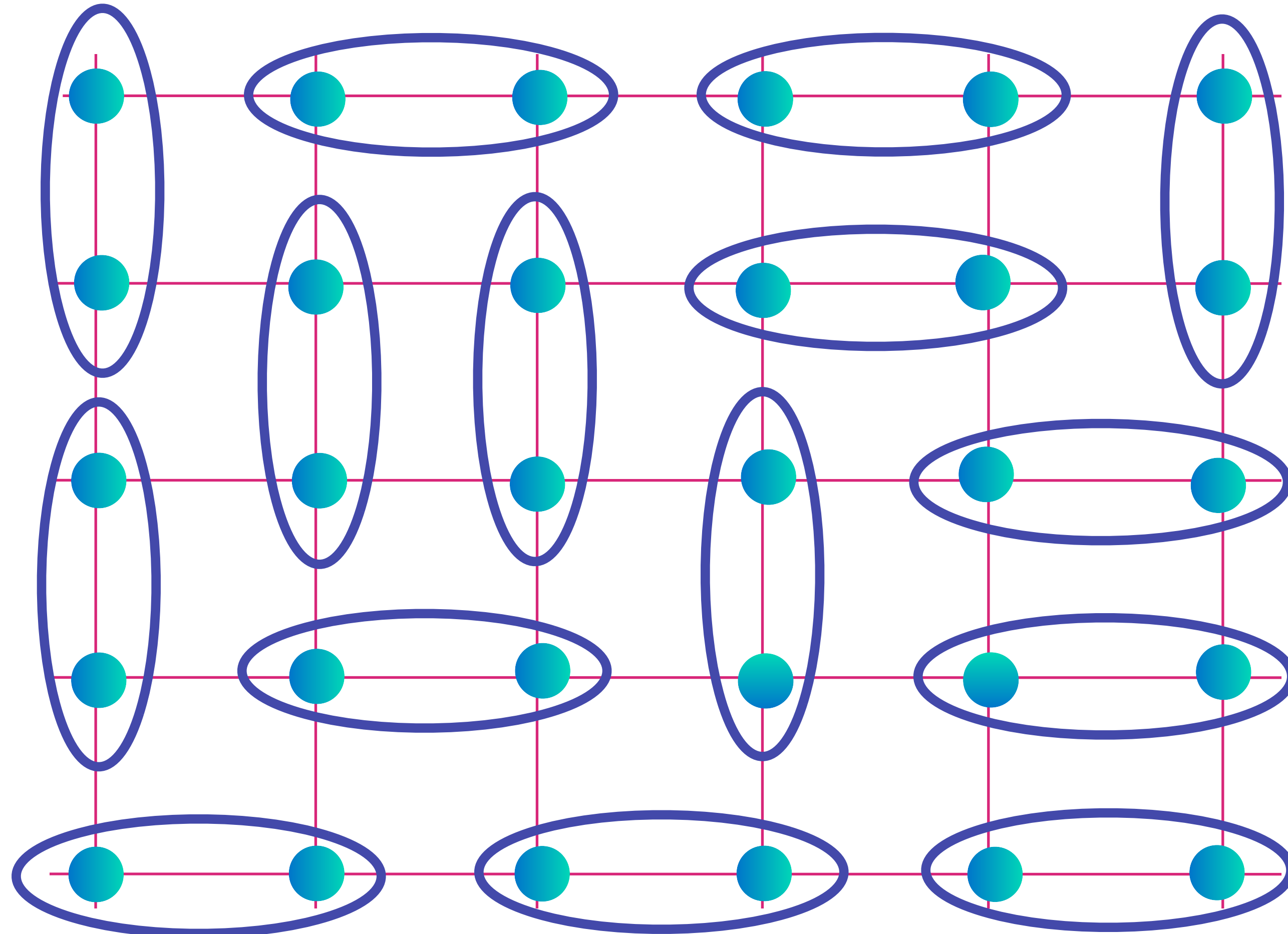


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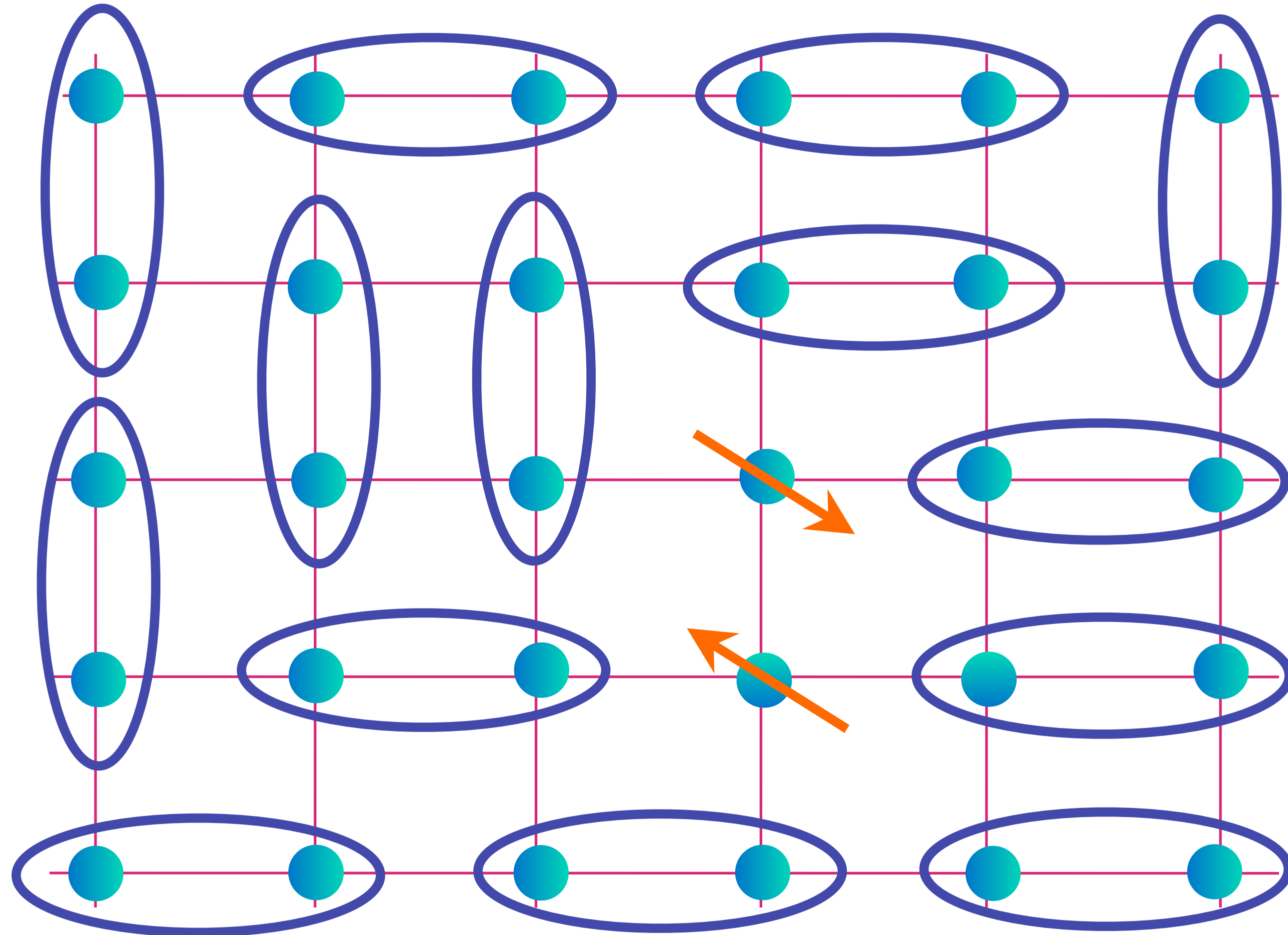


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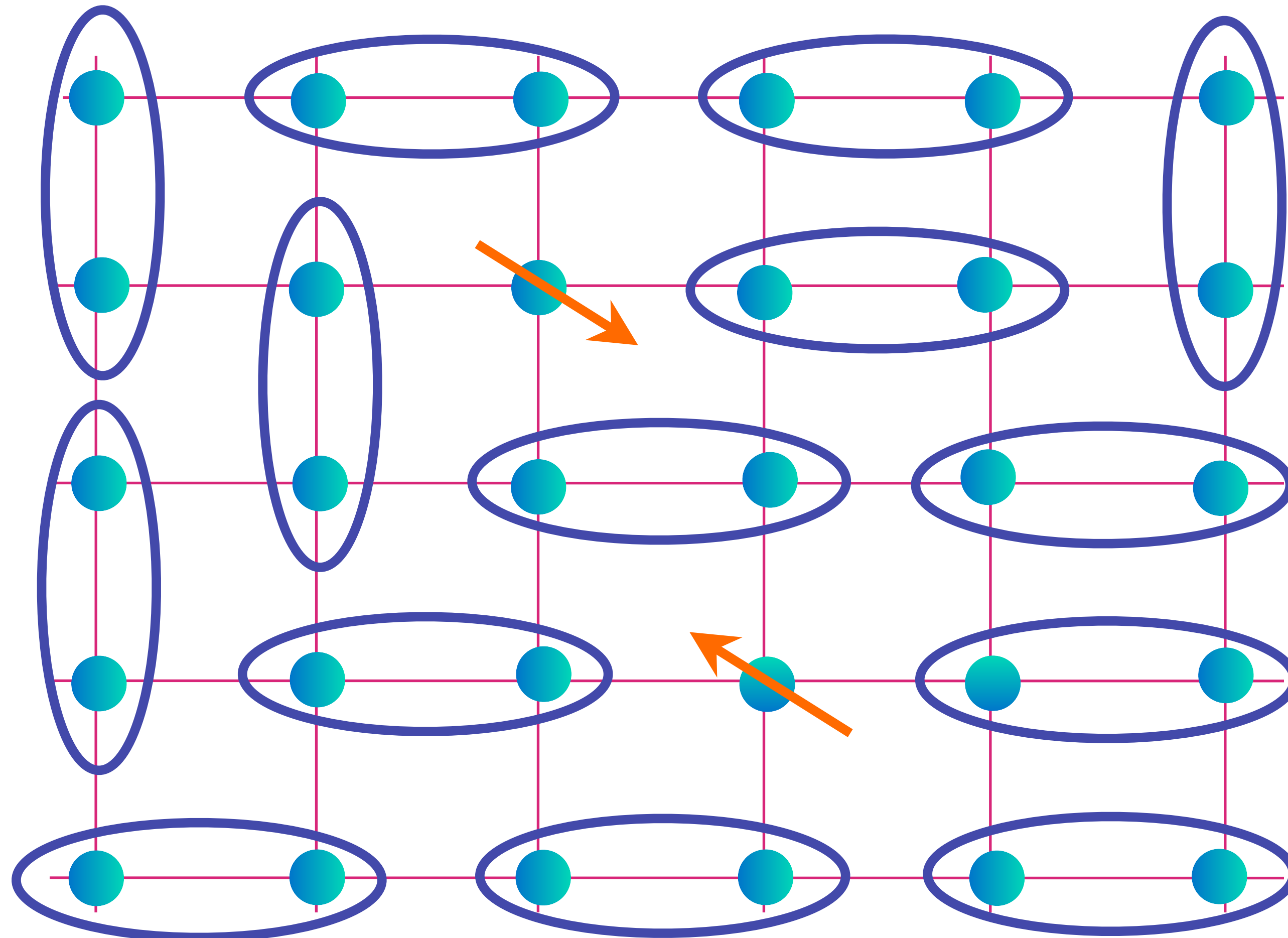
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Fractionalized
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excitations
with spin $S=1/2$
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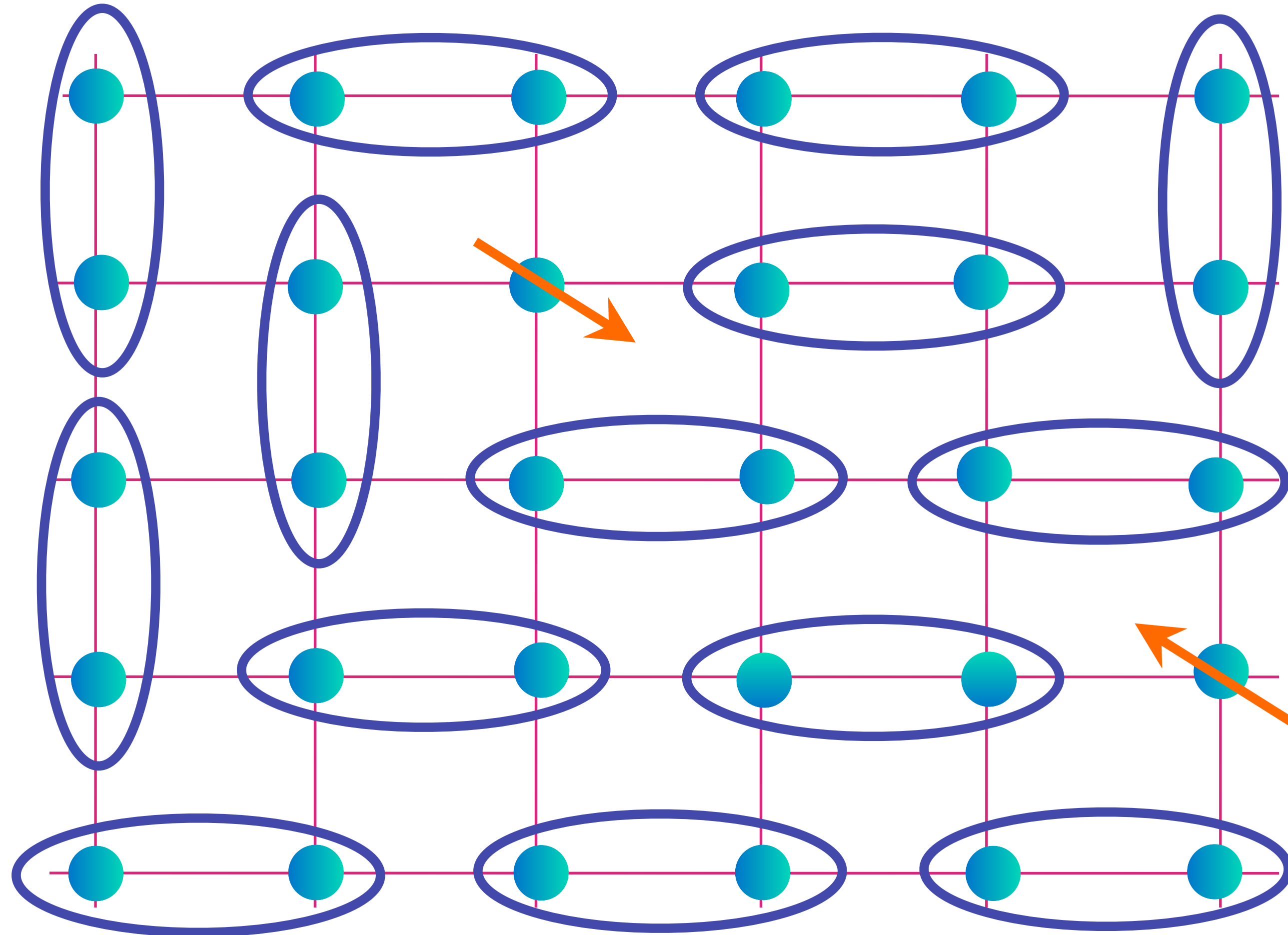
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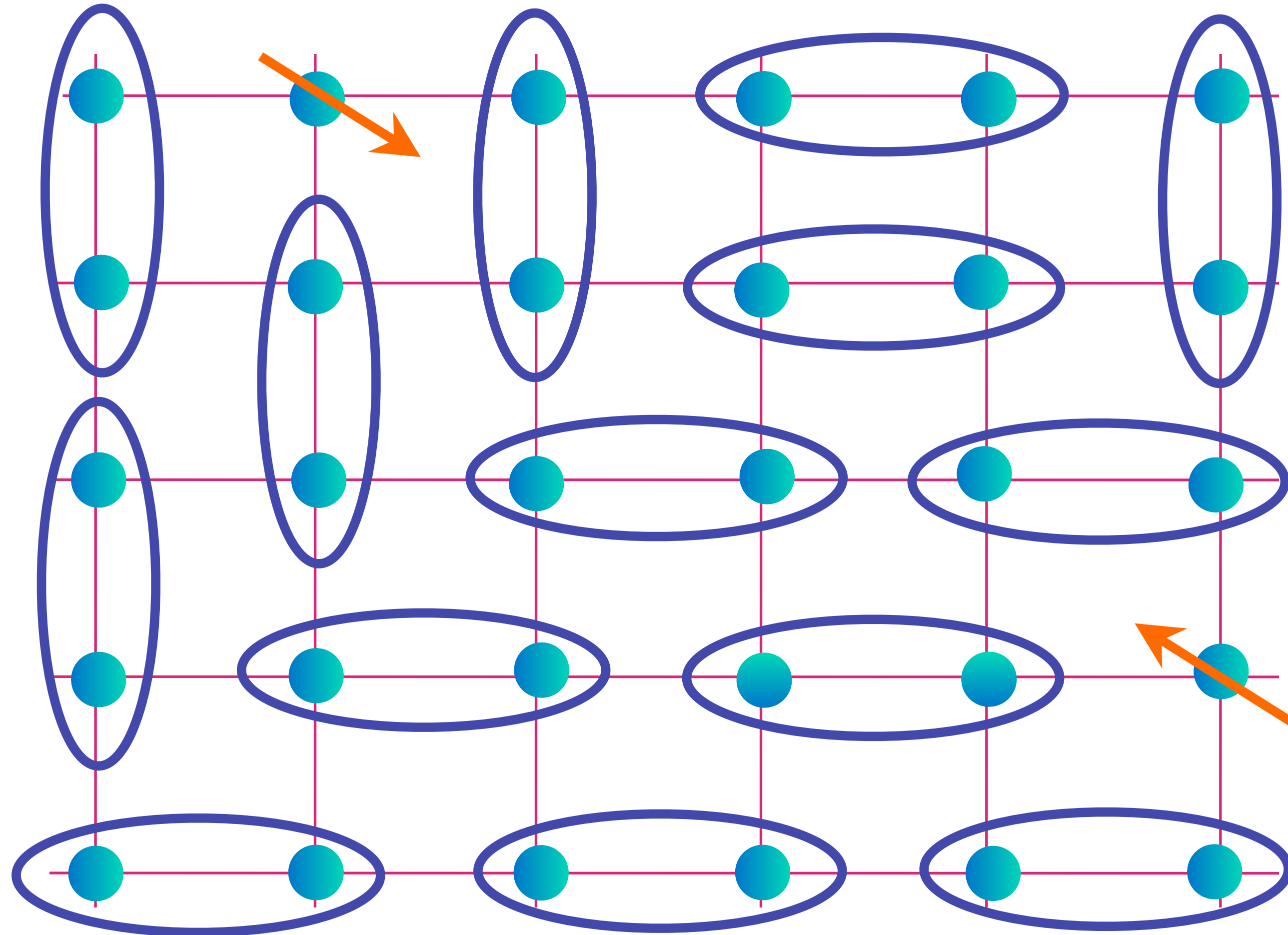
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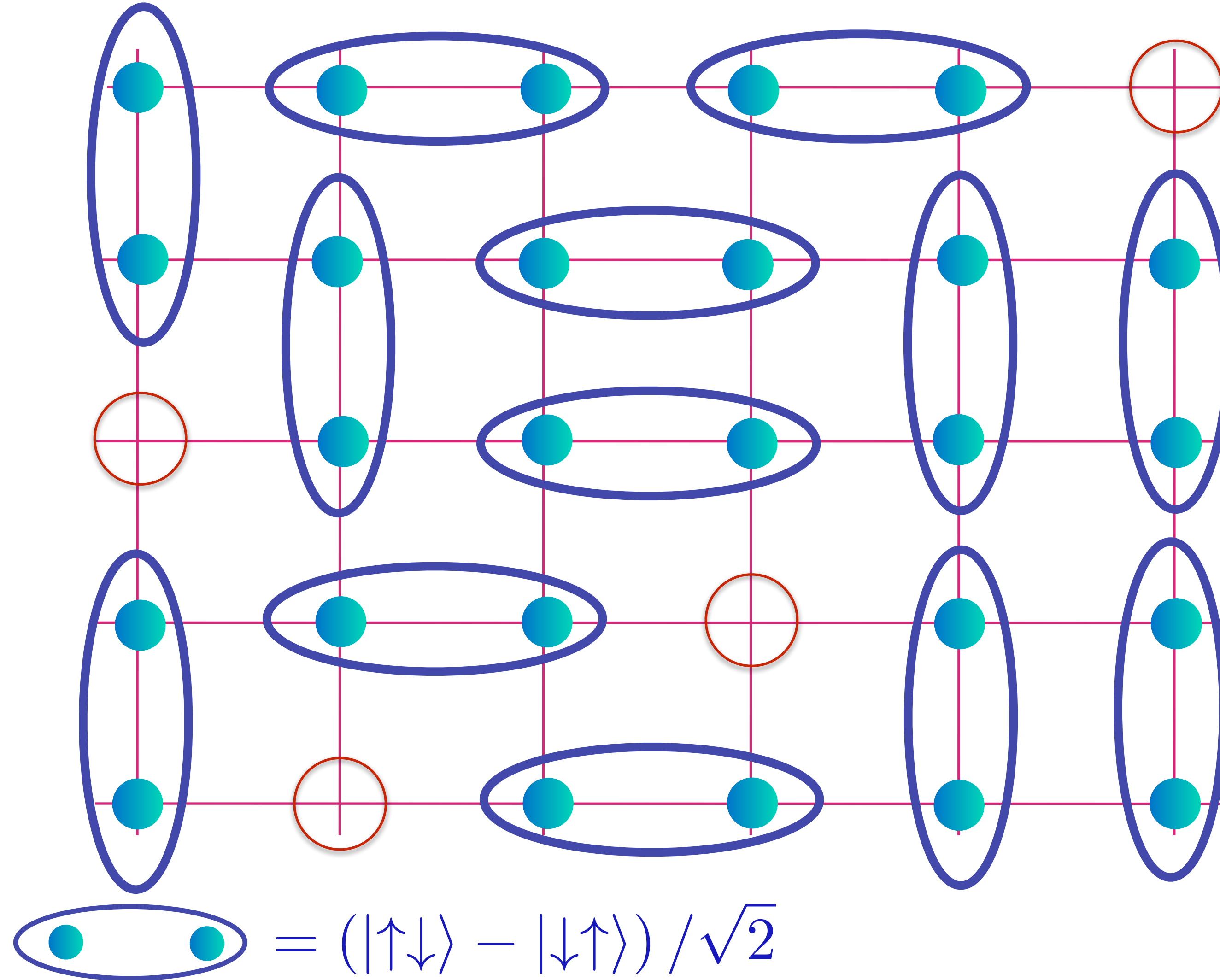
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Holons

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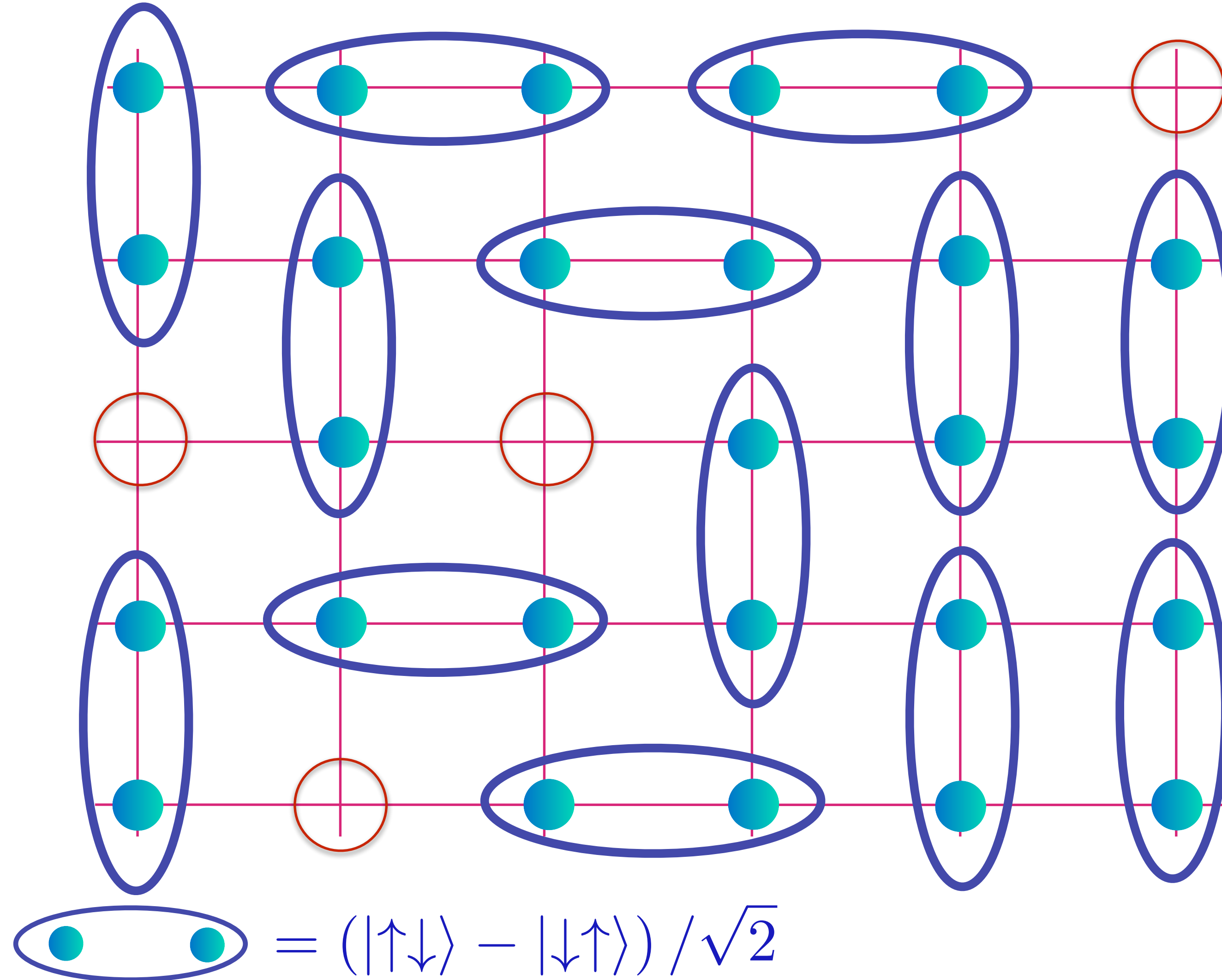
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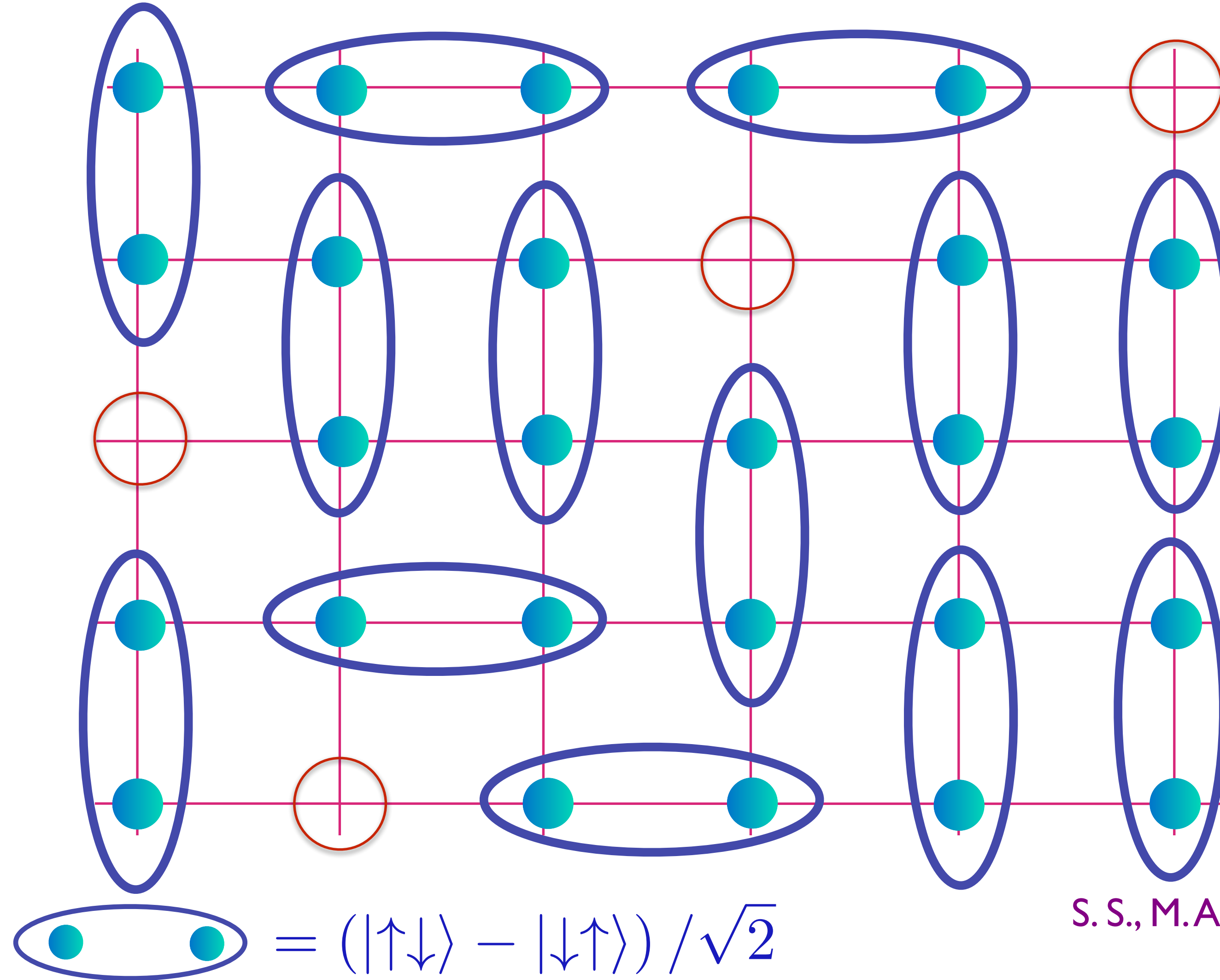
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If each holon is a fermion, we obtain a Fermi surface of holons of size p

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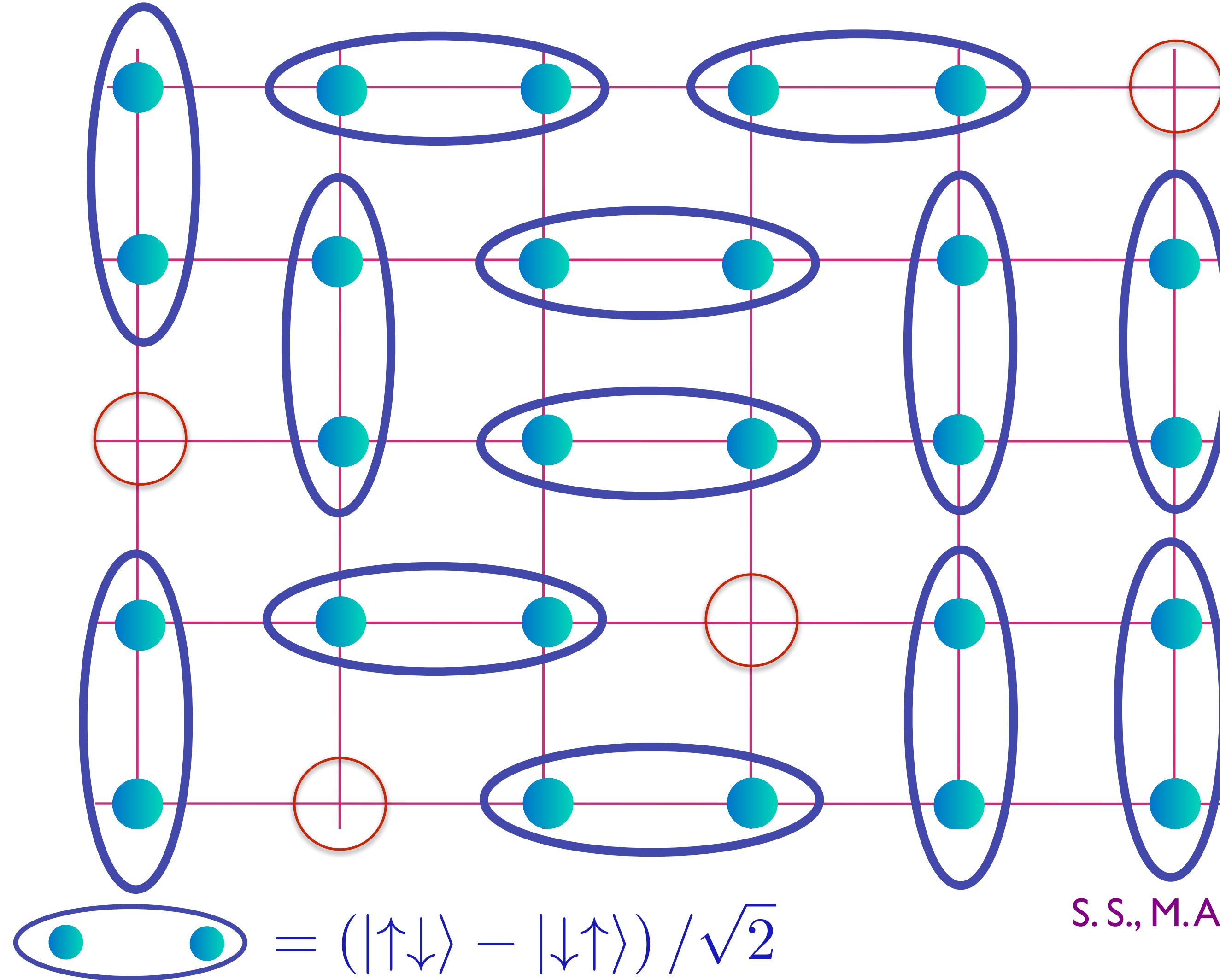
Pietro M. Bonetti and Walter Metzner, *PRB* **106**, 205152 (2022)

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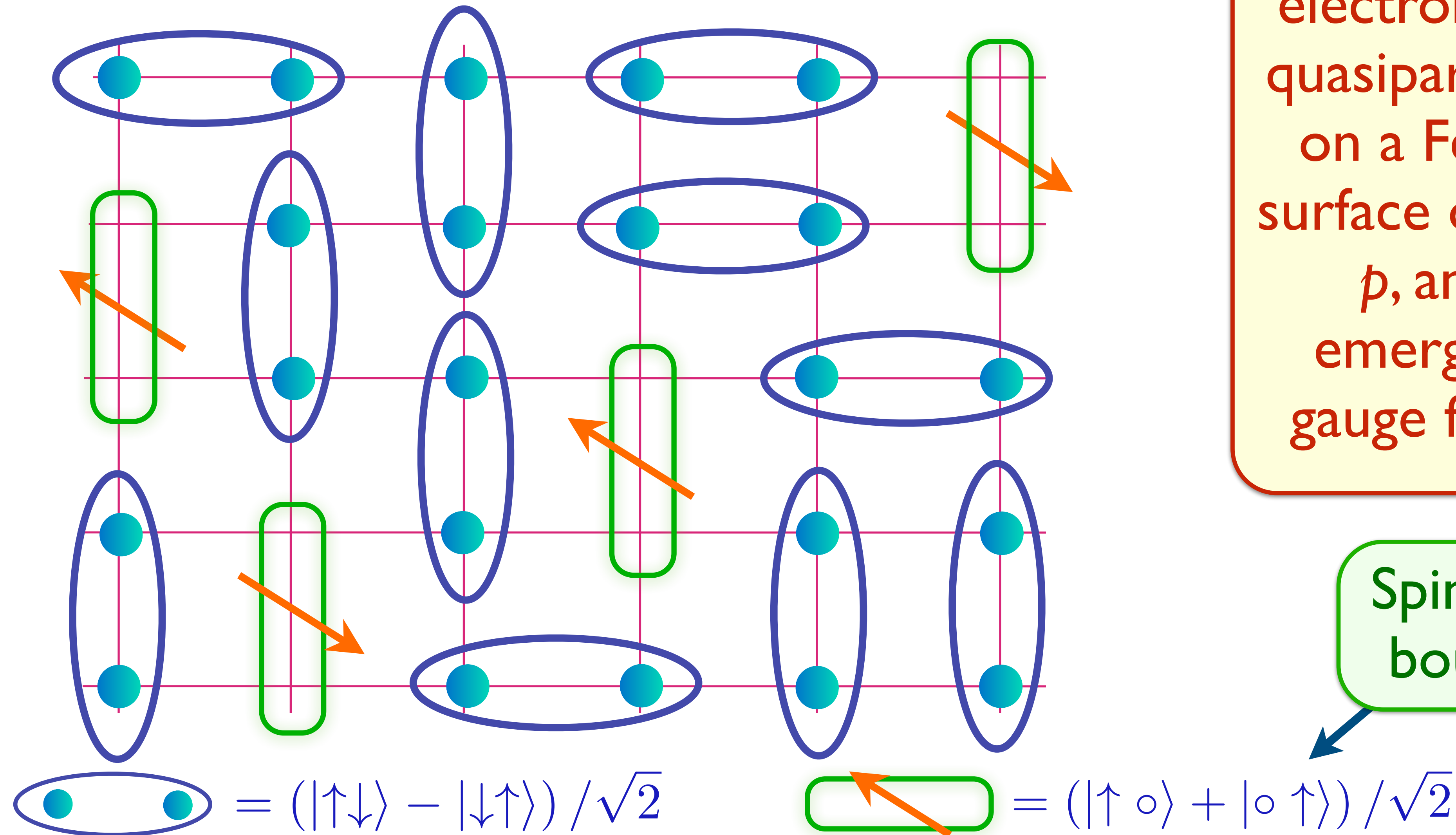
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FL* in a **one-band** model

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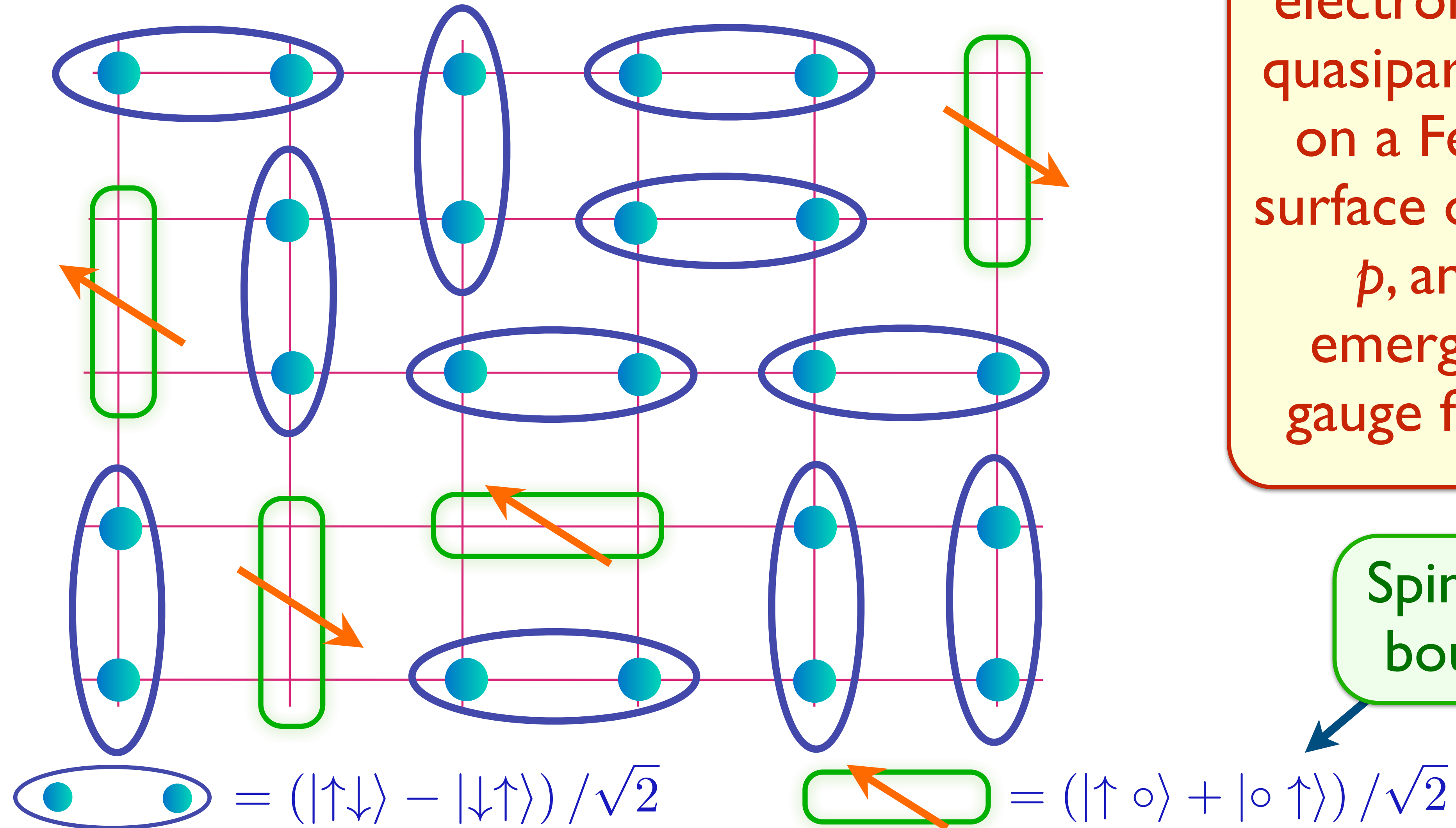
Metal with electron-like quasiparticles on a Fermi surface of size p , and emergent gauge fields

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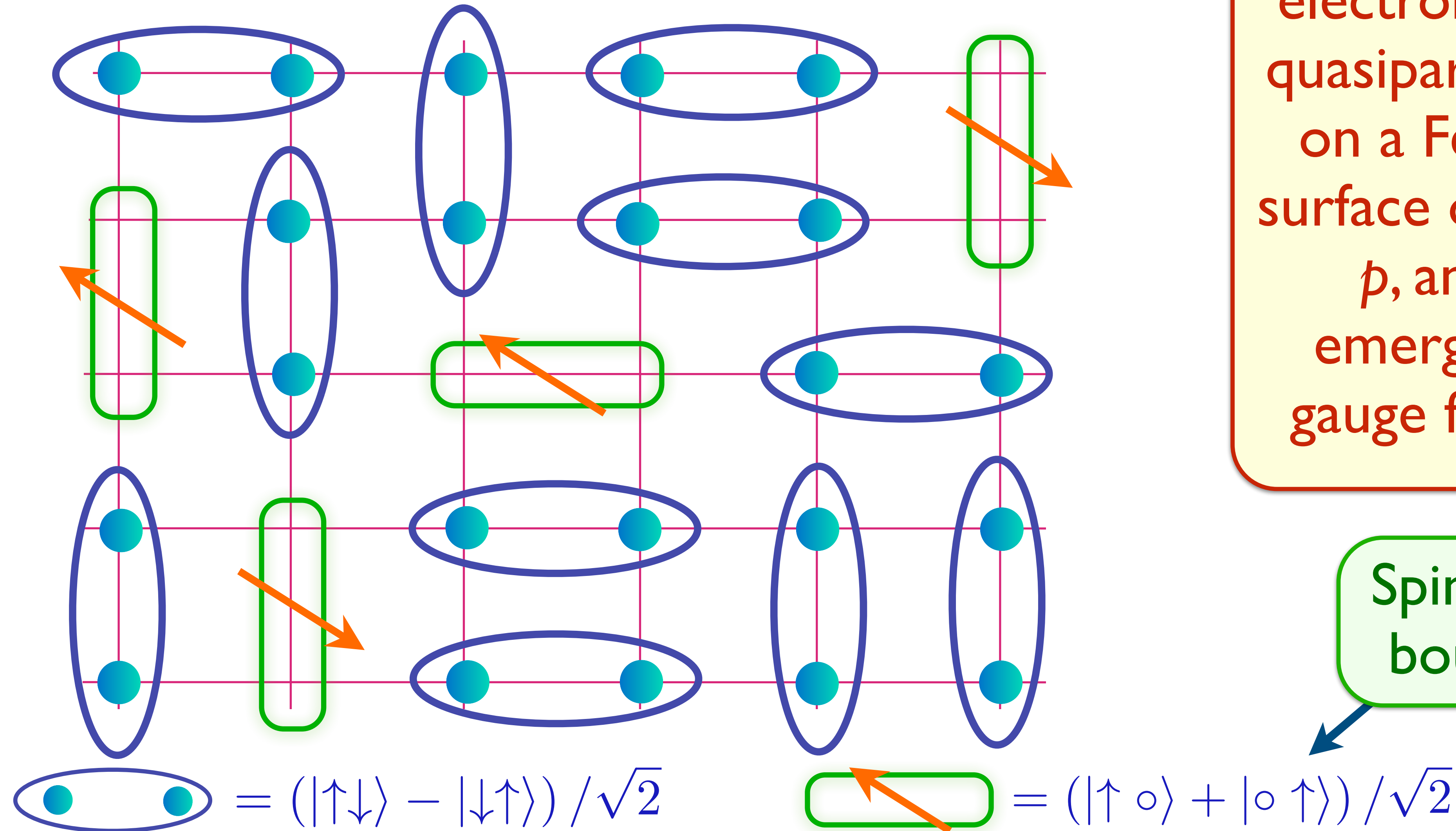
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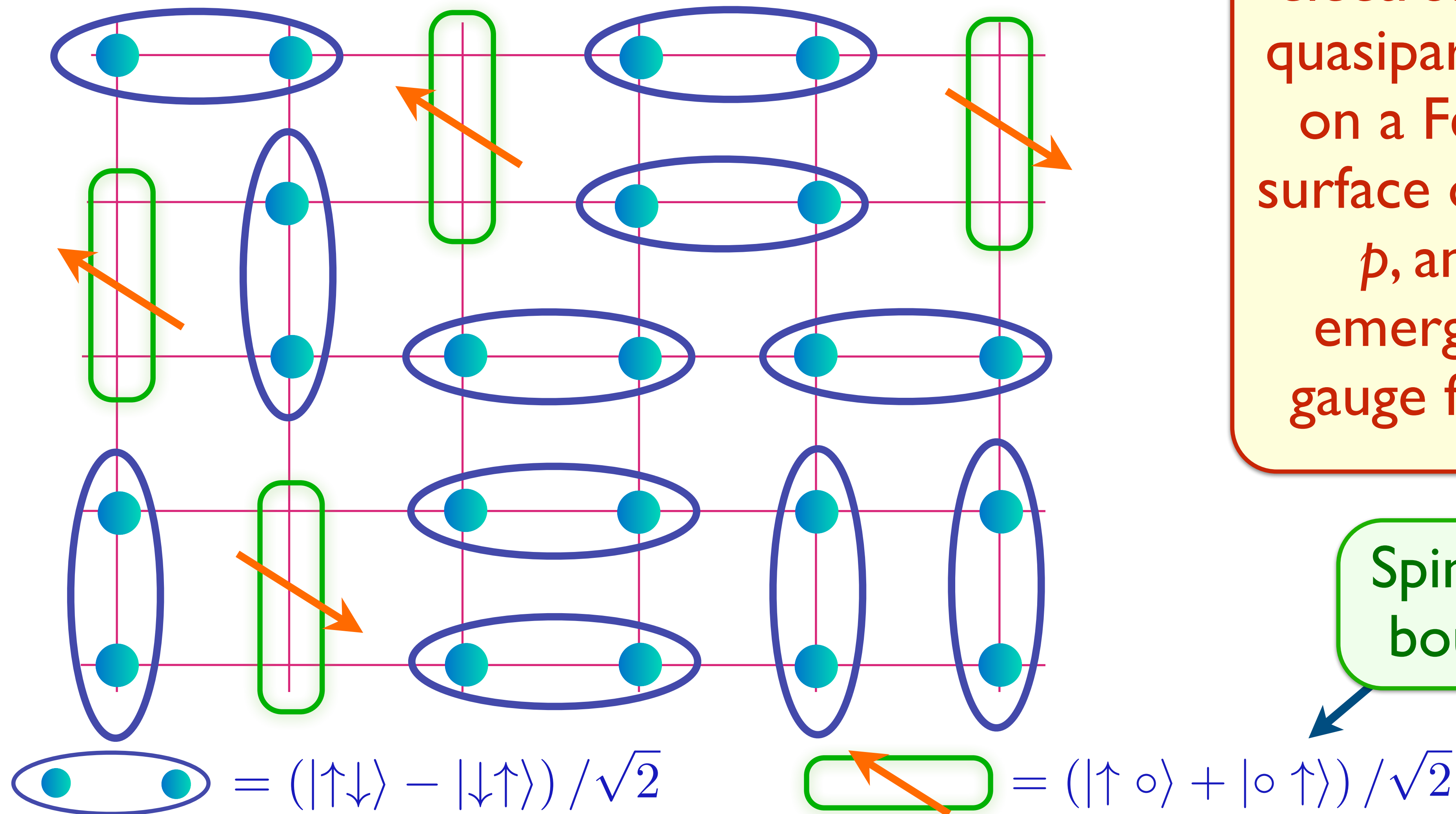
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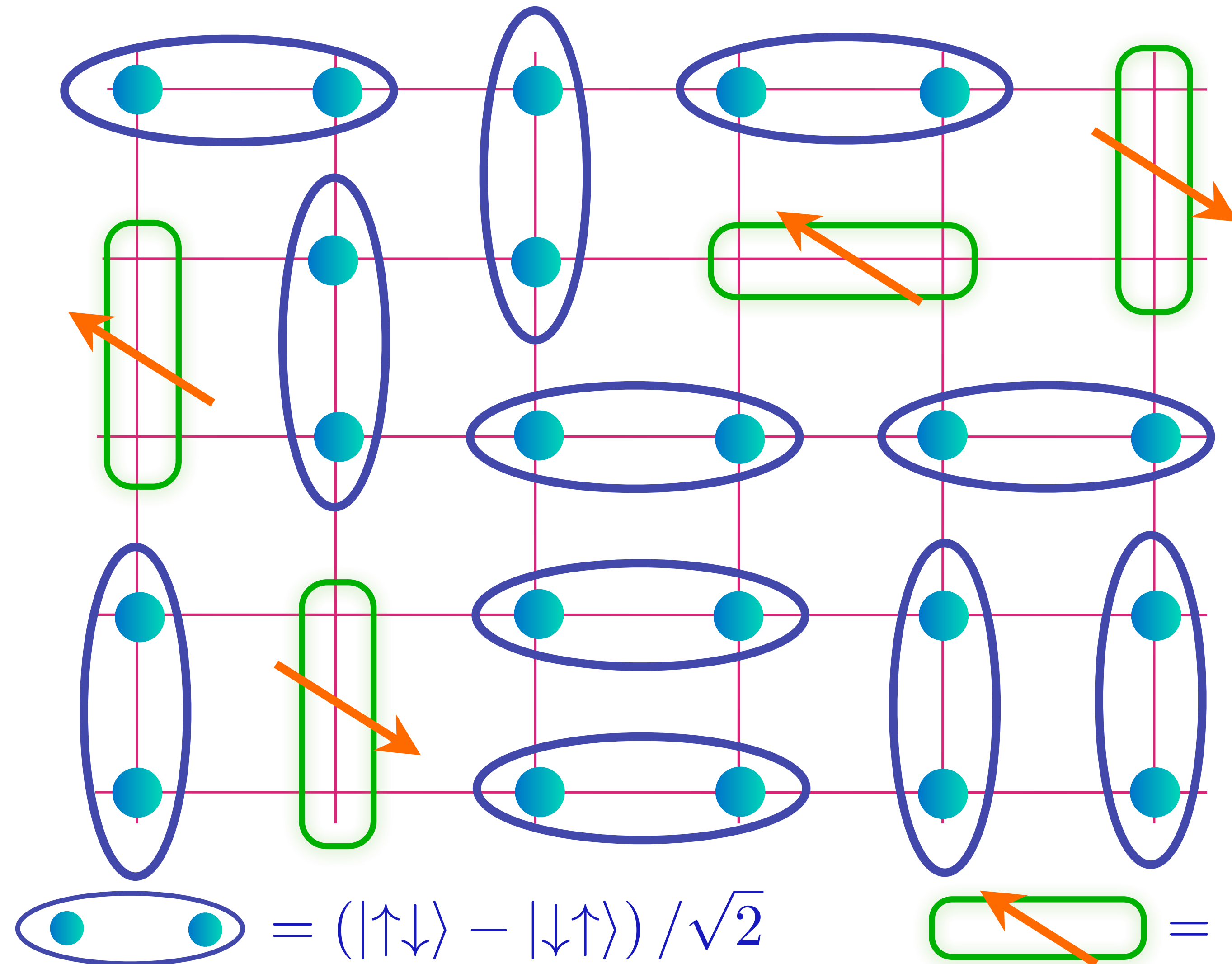
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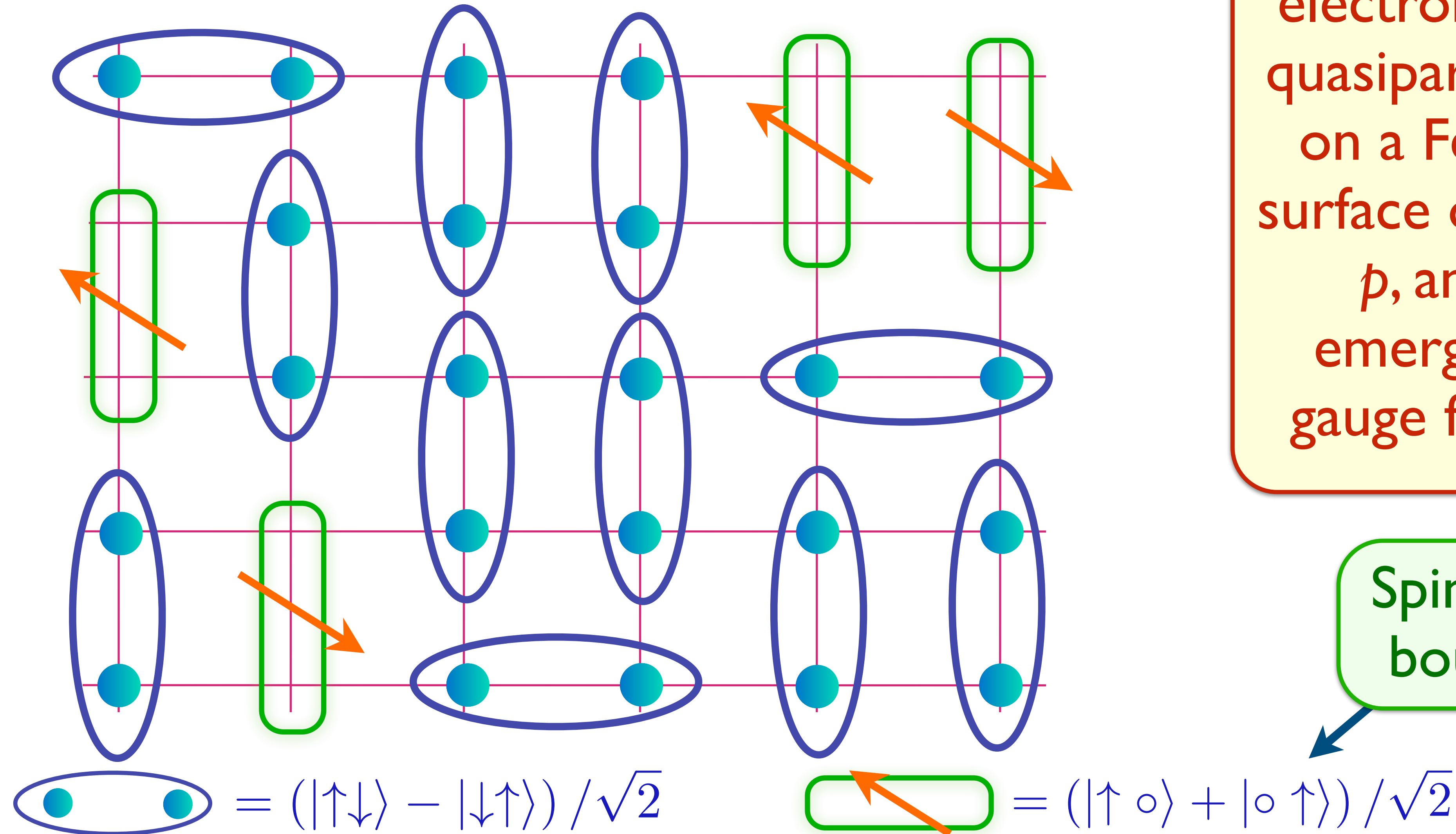
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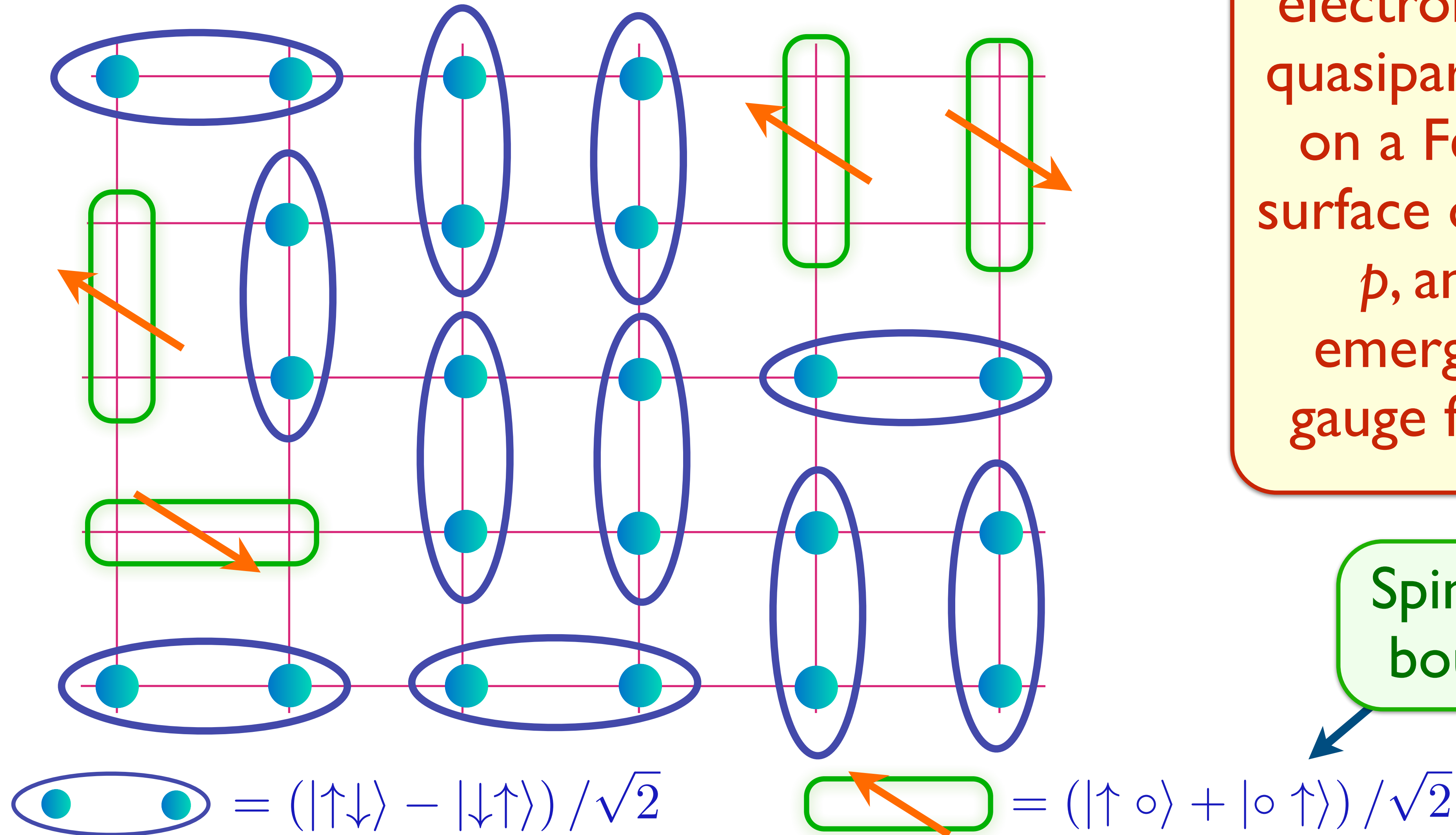
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FL* in a **one-band** model

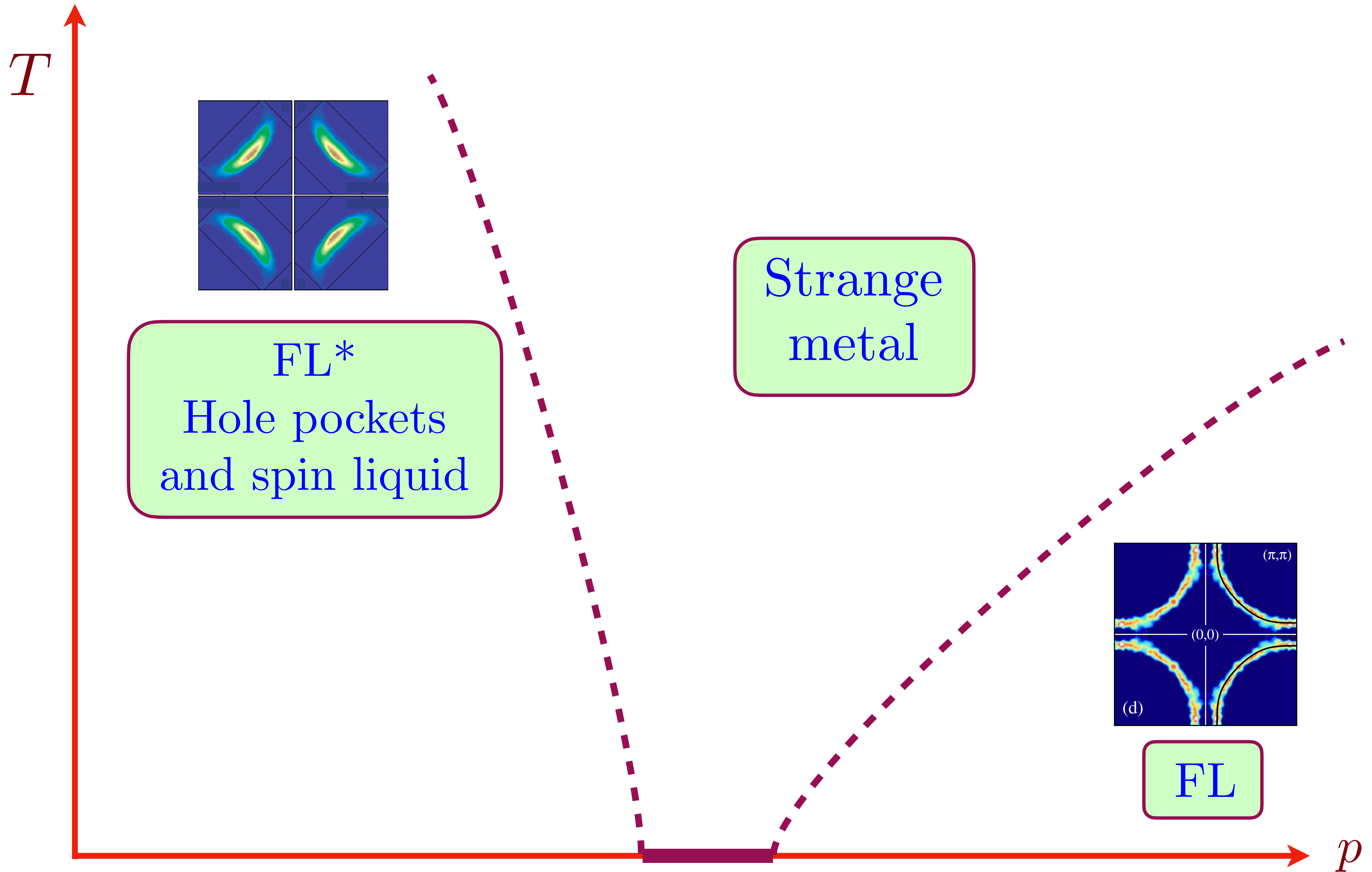
S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

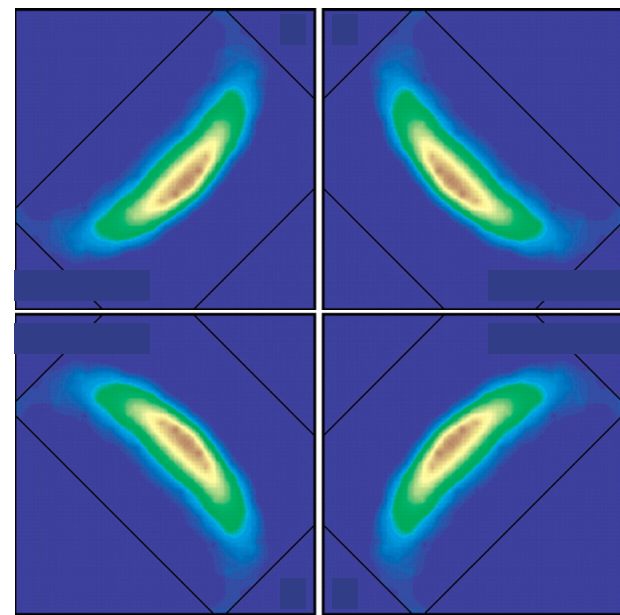


Metal with electron-like quasiparticles on a Fermi surface of size p , and emergent gauge fields

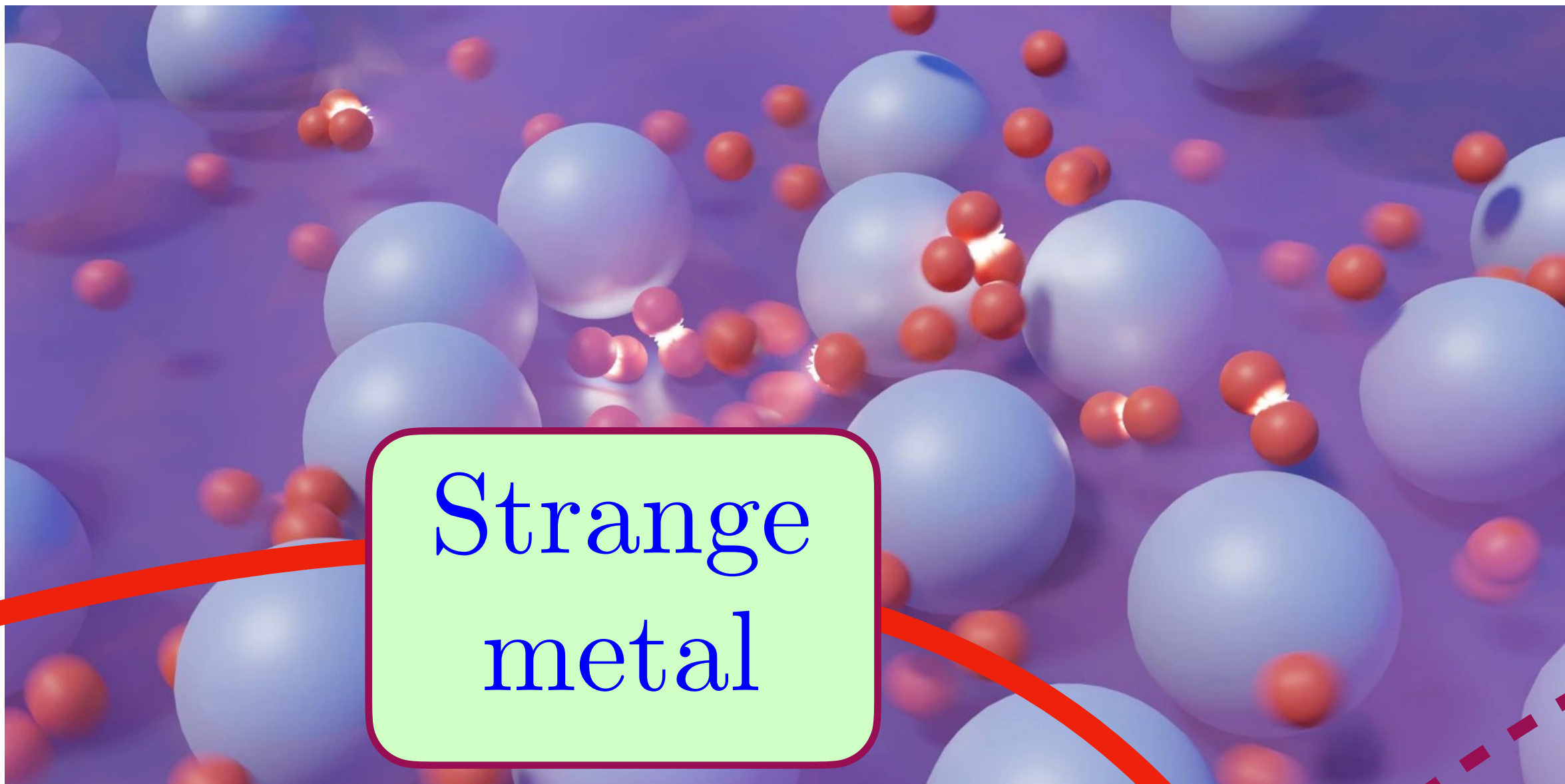
Spinon-holon bound state



T

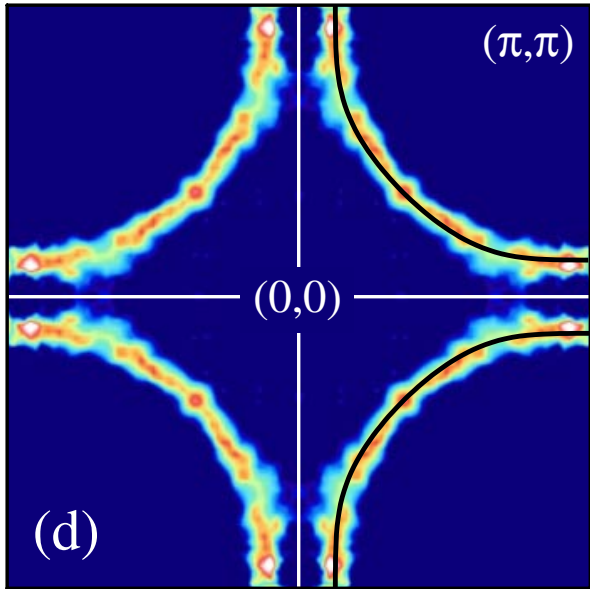


FL*
Hole pockets
and spin liquid



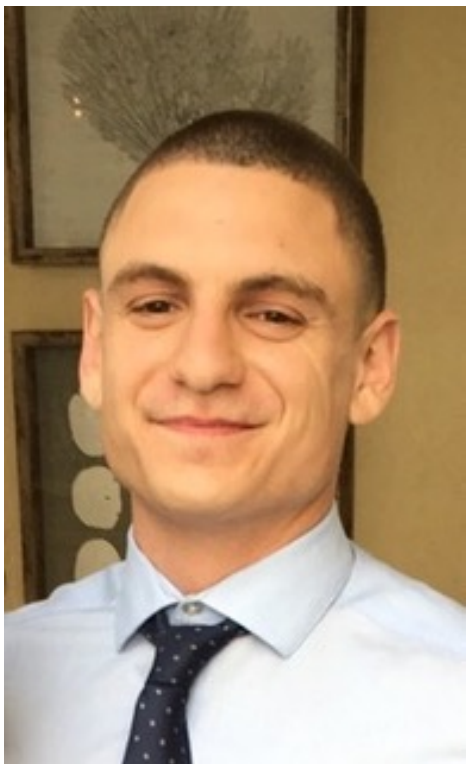
Strange
metal

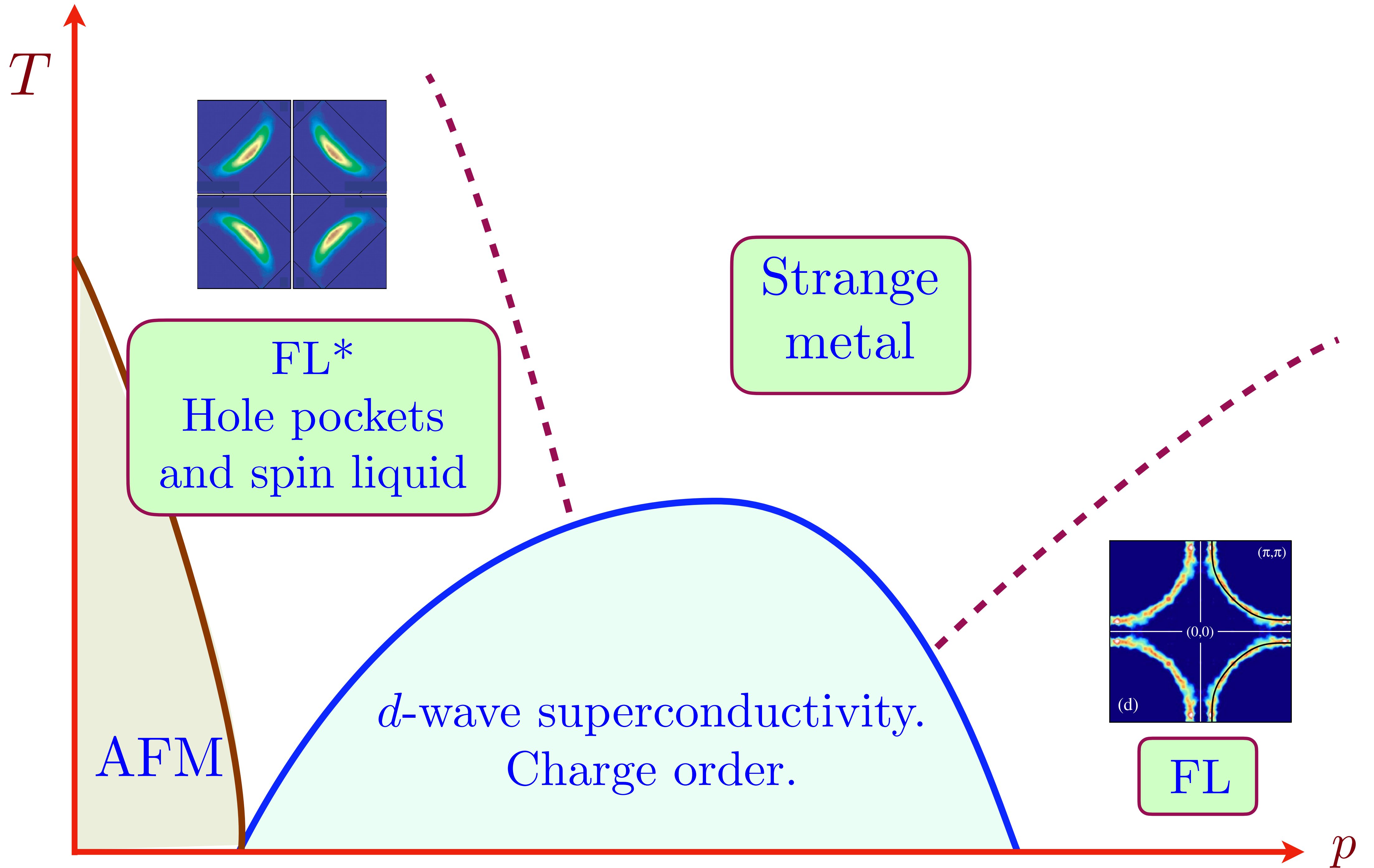
Patel,
Haoyu
Guo,
Esterlis,
Sachdev,
Science
381, 790
(2023)

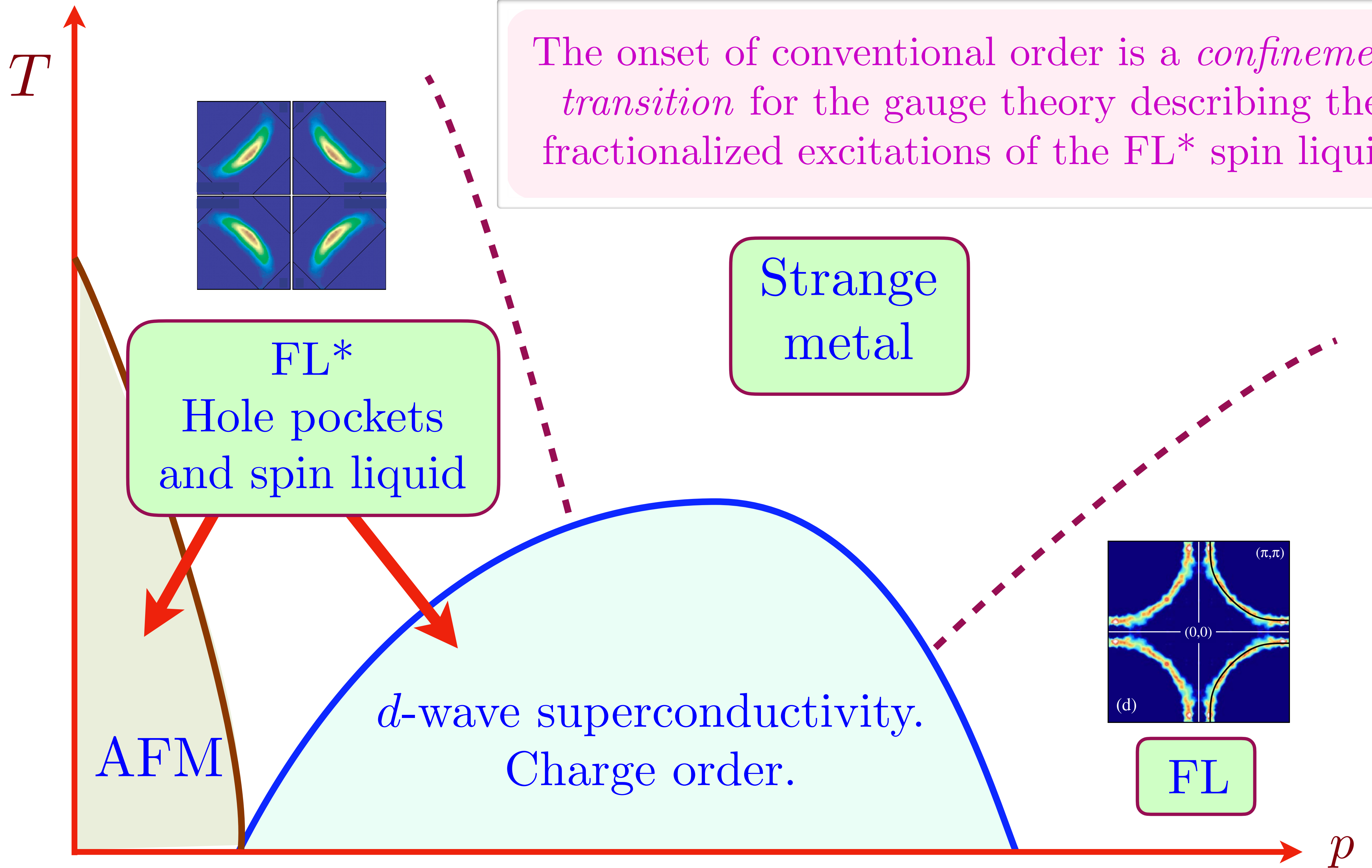


FL

p







The onset of conventional order is a *confinement transition* for the gauge theory describing the fractionalized excitations of the FL* spin liquid

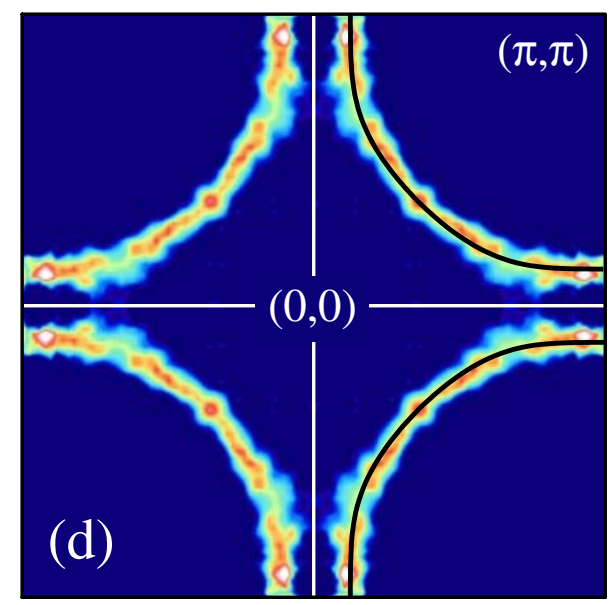
FL*
Hole pockets
and spin liquid

Strange
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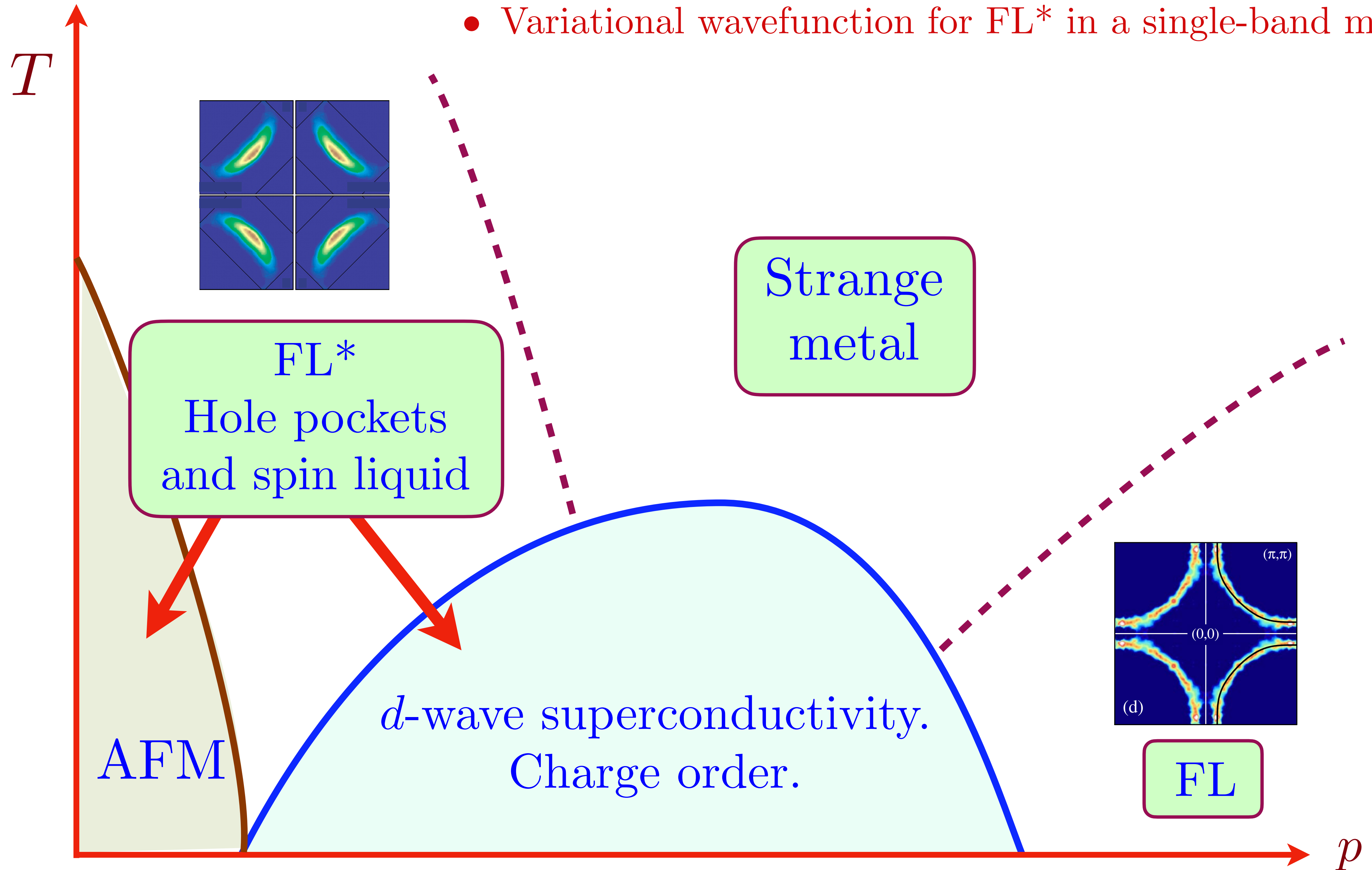
AFM

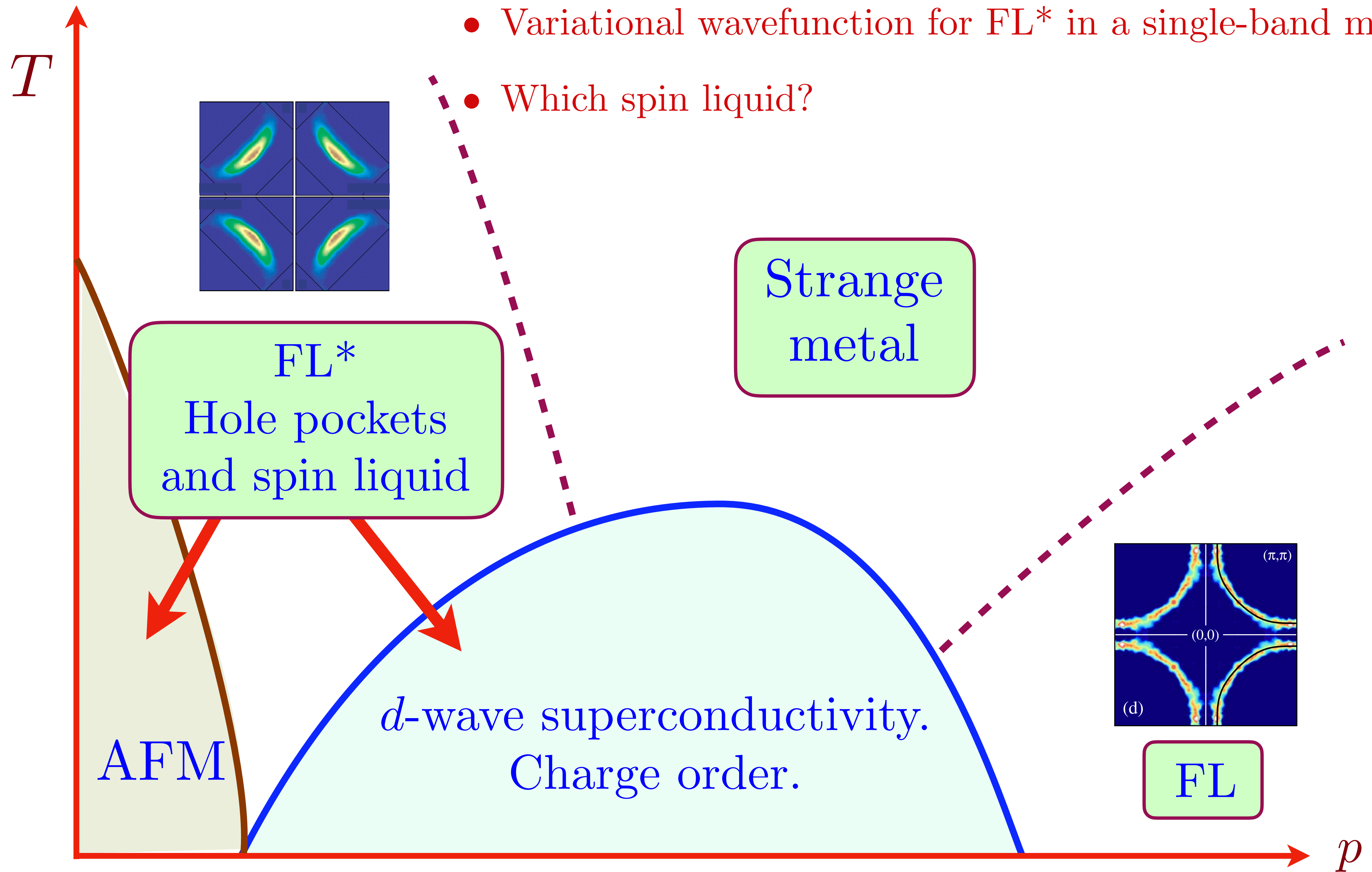
d -wave superconductivity.
Charge order.

FL



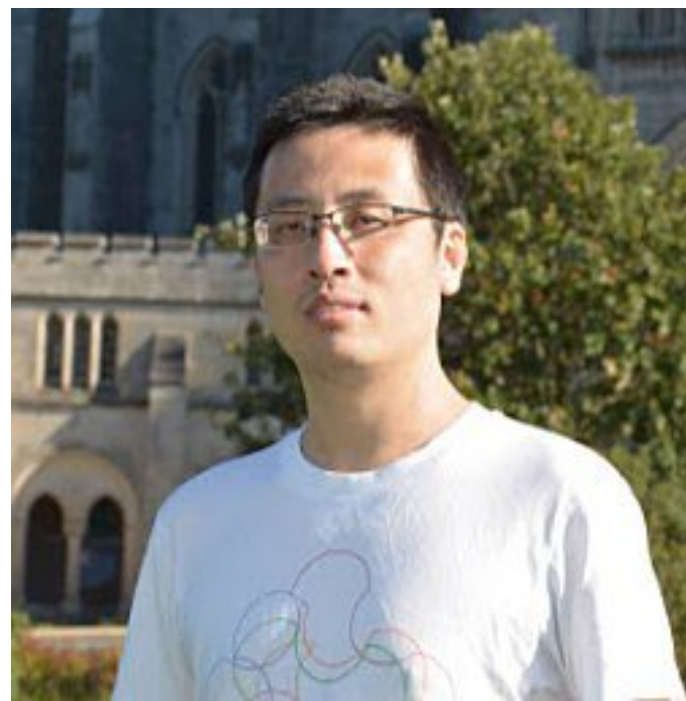
- Variational wavefunction for FL* in a single-band model?





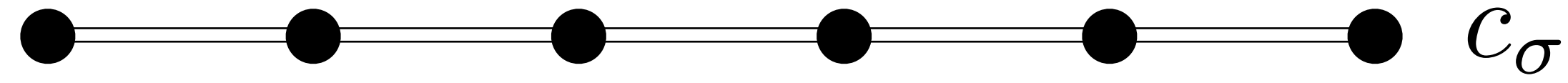
- Variational wavefunction for FL* in a single-band model?
- Which spin liquid?

Ancilla theory of FL^* in a single-band model



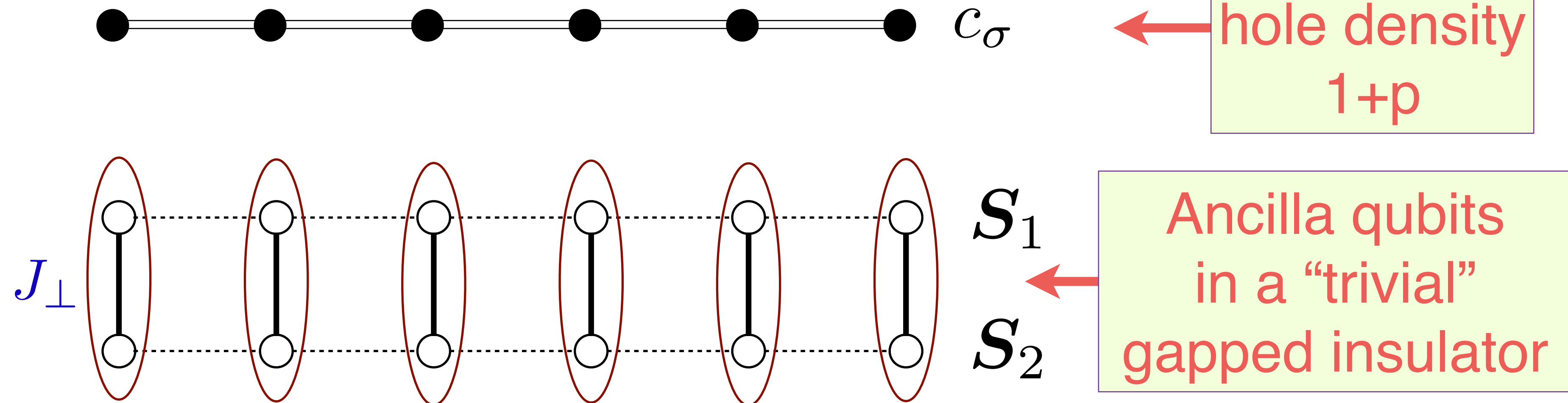
Ya-Hui Zhang

Ancilla theory of the Hubbard model



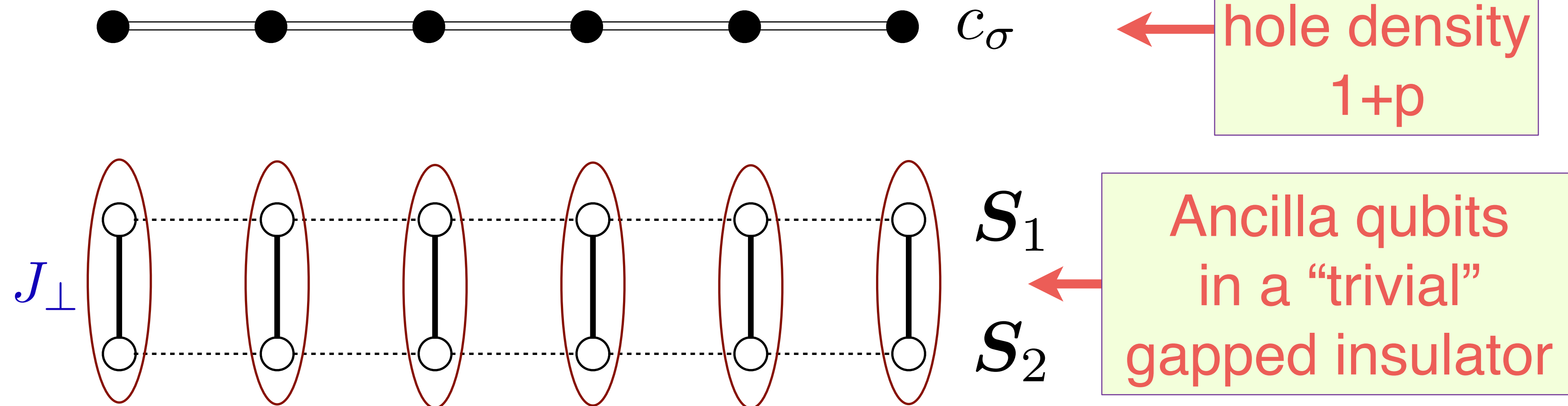
← Hubbard
model of
hole density
 $1+p$

Ancilla theory of the Hubbard model

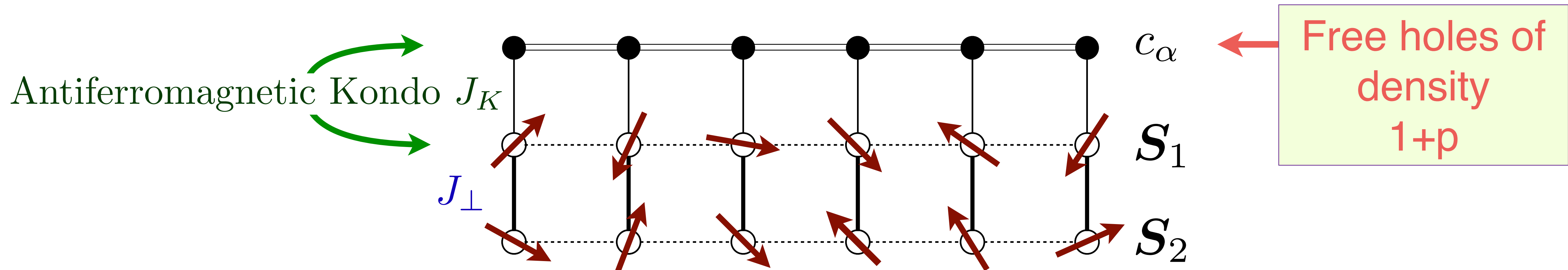


$$\mathcal{H}_{\text{Hubbard}} + \mathcal{H}_{\text{trivial insulator}}$$

Ancilla theory of the Hubbard model

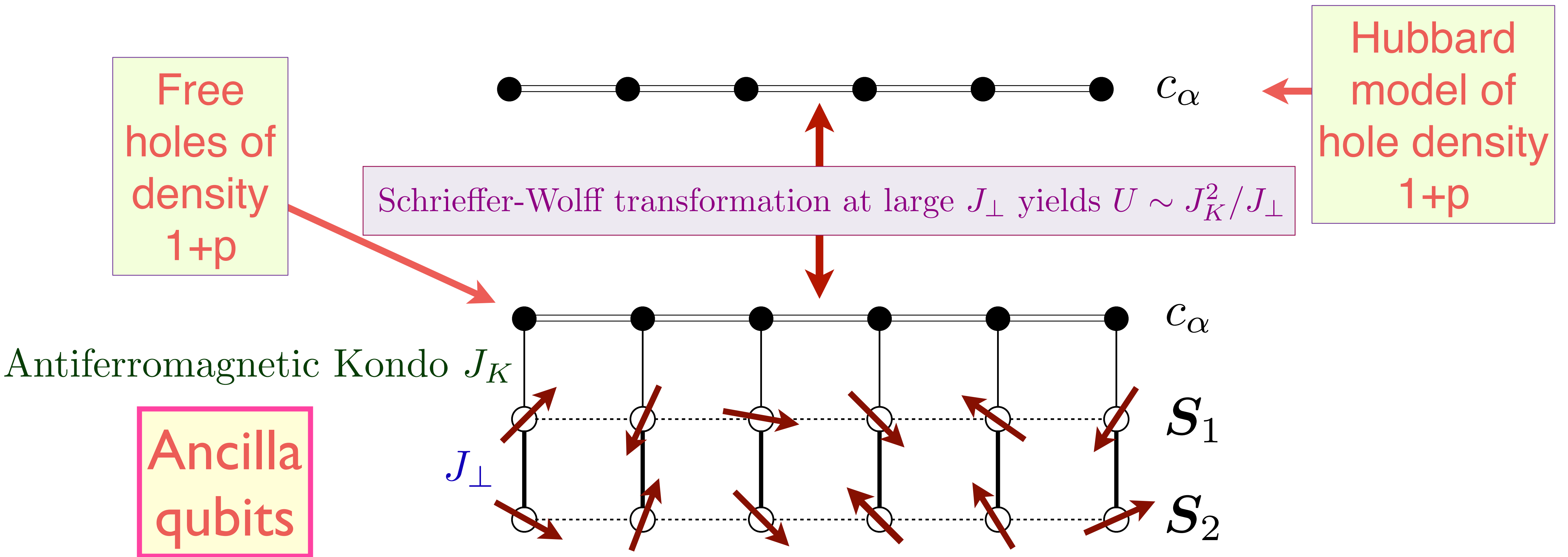


$$U (\mathcal{H}_{\text{Hubbard}} + \mathcal{H}_{\text{trivial insulator}}) U^{-1} = \mathcal{H}_{\text{ancilla}}$$



Ancilla theory of the Hubbard model

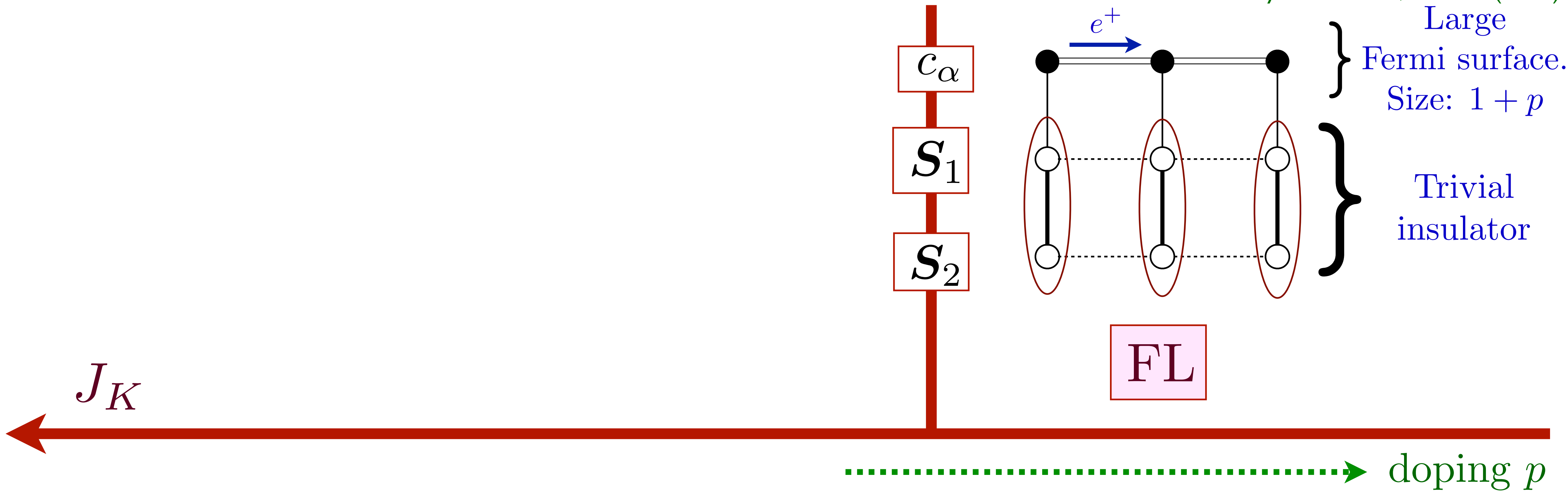
Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)



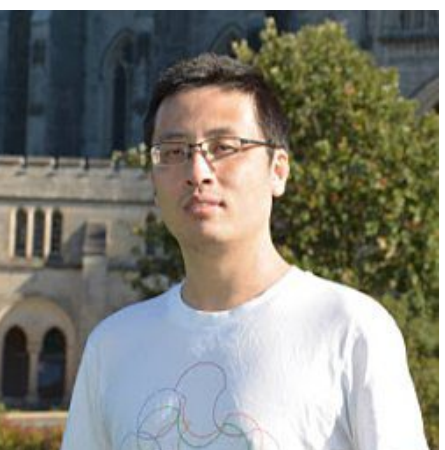
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

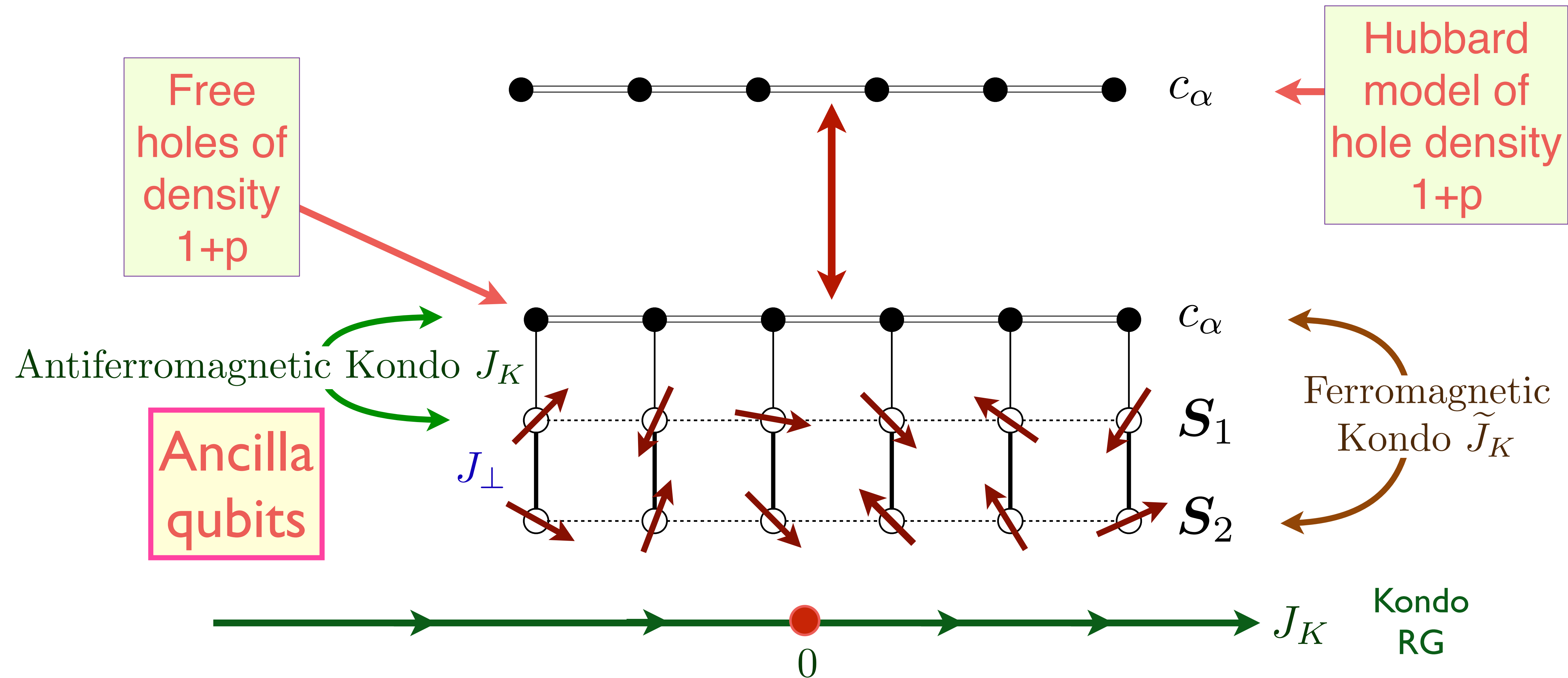


Ya-Hui
Zhang



Ancilla theory of the Hubbard model

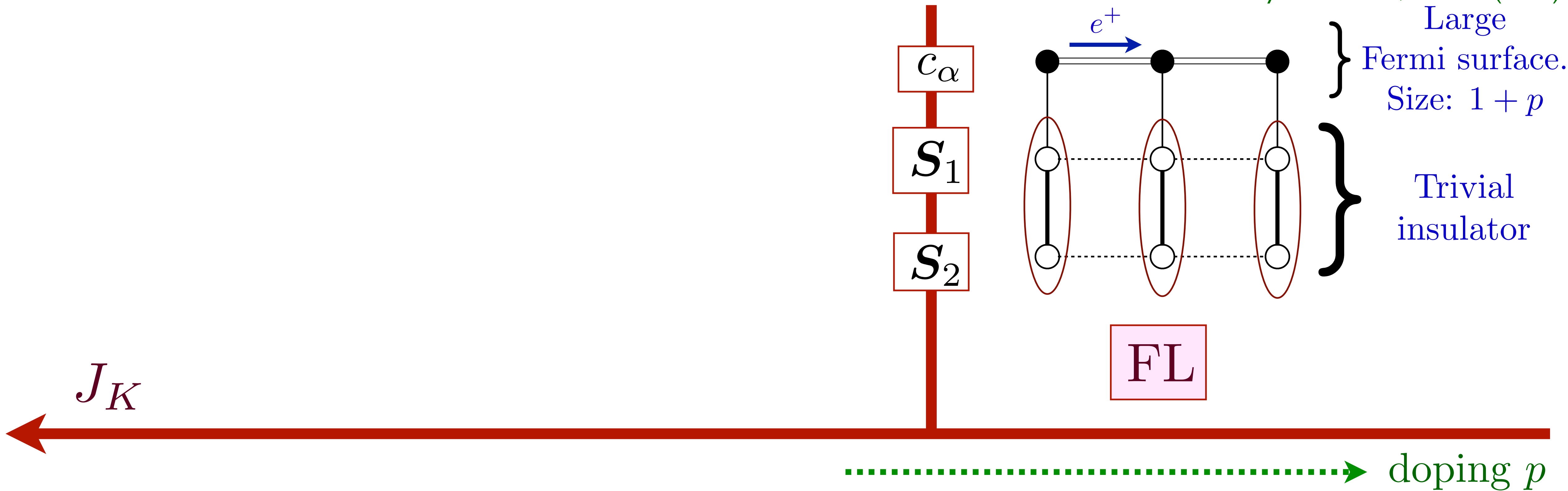
Ya-Hui Zhang and S. Sachdev,
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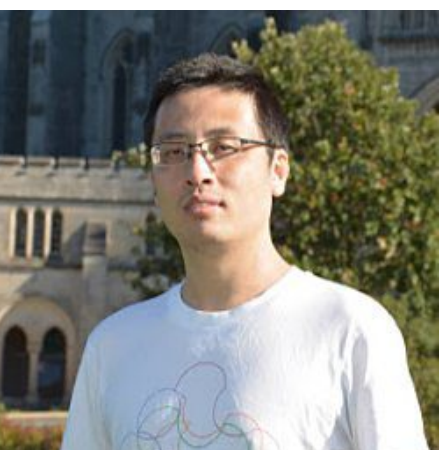
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Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)



Ya-Hui
Zhang

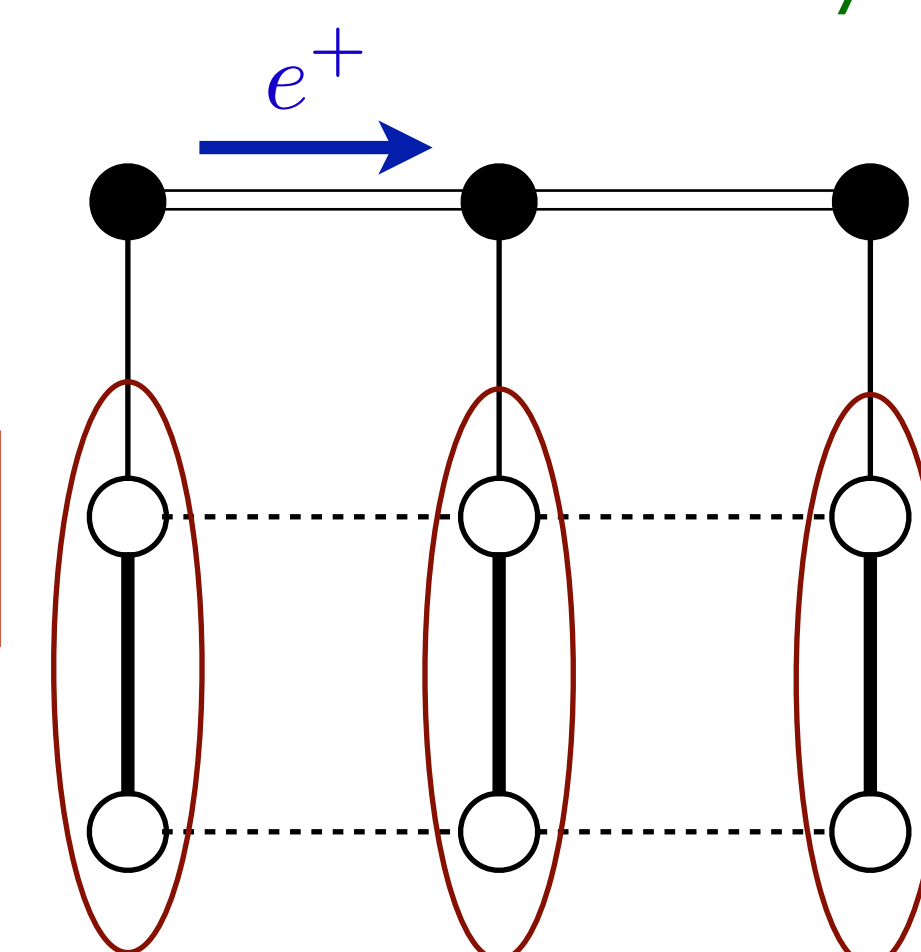
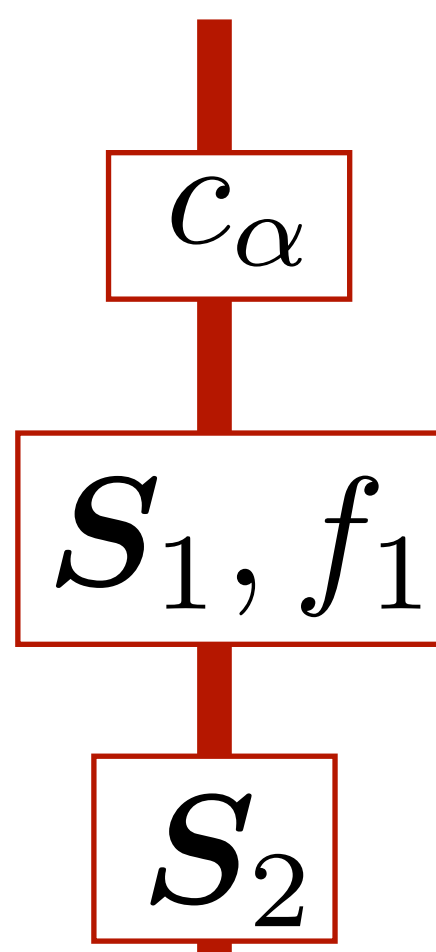
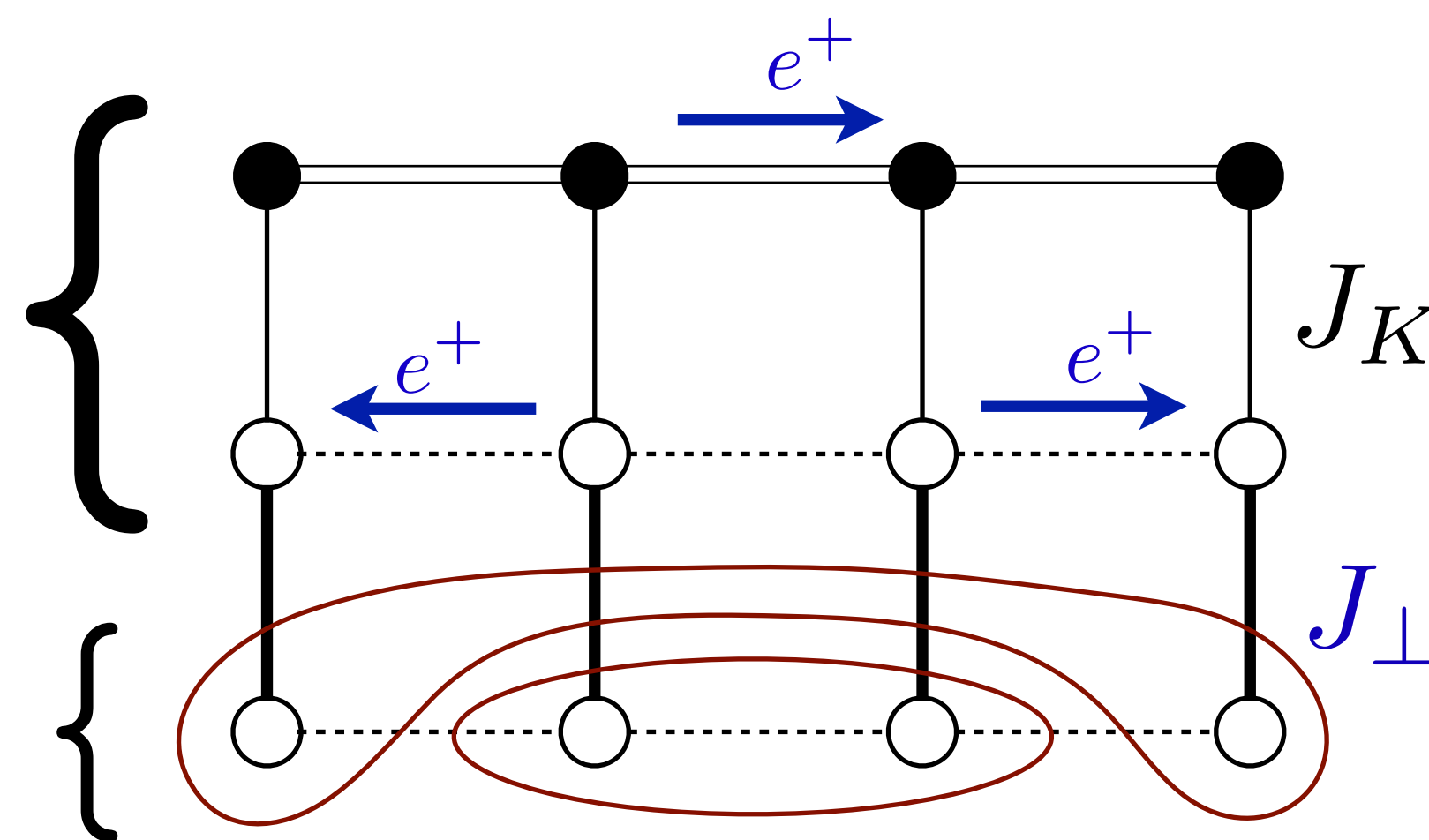


Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

Kondo lattice
 heavy Fermi liquid.
 Size $1 + p + 1$
 $= p \pmod{2}$.
Small Fermi surface!

Spin liquid



Large
 Fermi surface.
 Size: $1 + p$

Trivial
 insulator

FL*

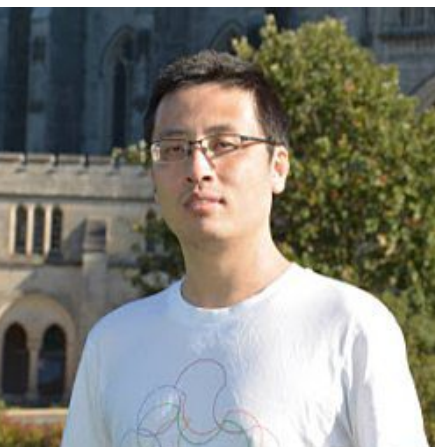
FL

J_K

doping p

Pseudogap metal =
 Kondo Lattice Heavy
 Fermi Liquid
 \oplus
 Spin Liquid

Ya-Hui
 Zhang

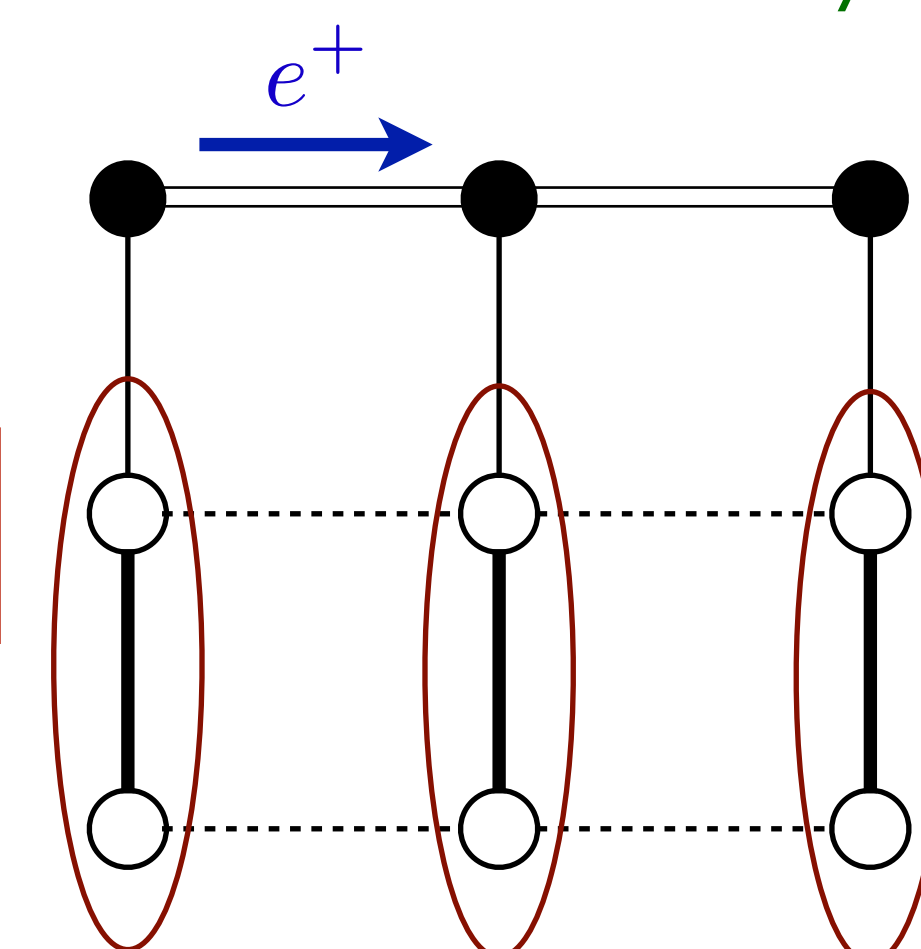
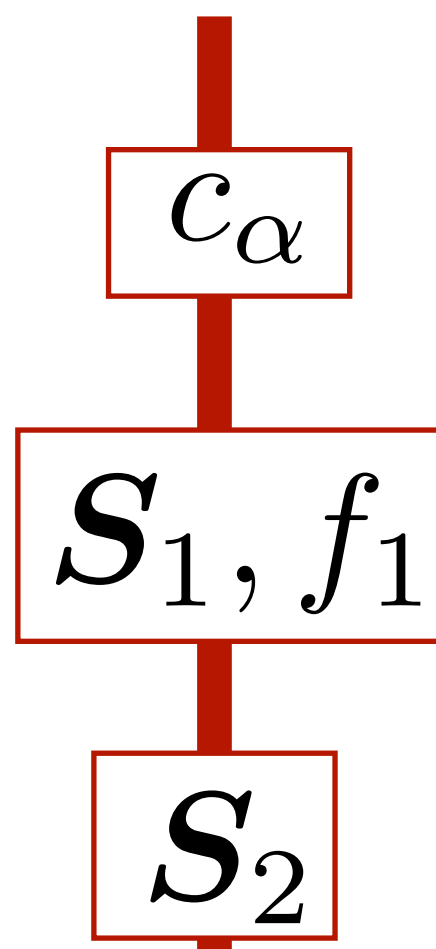
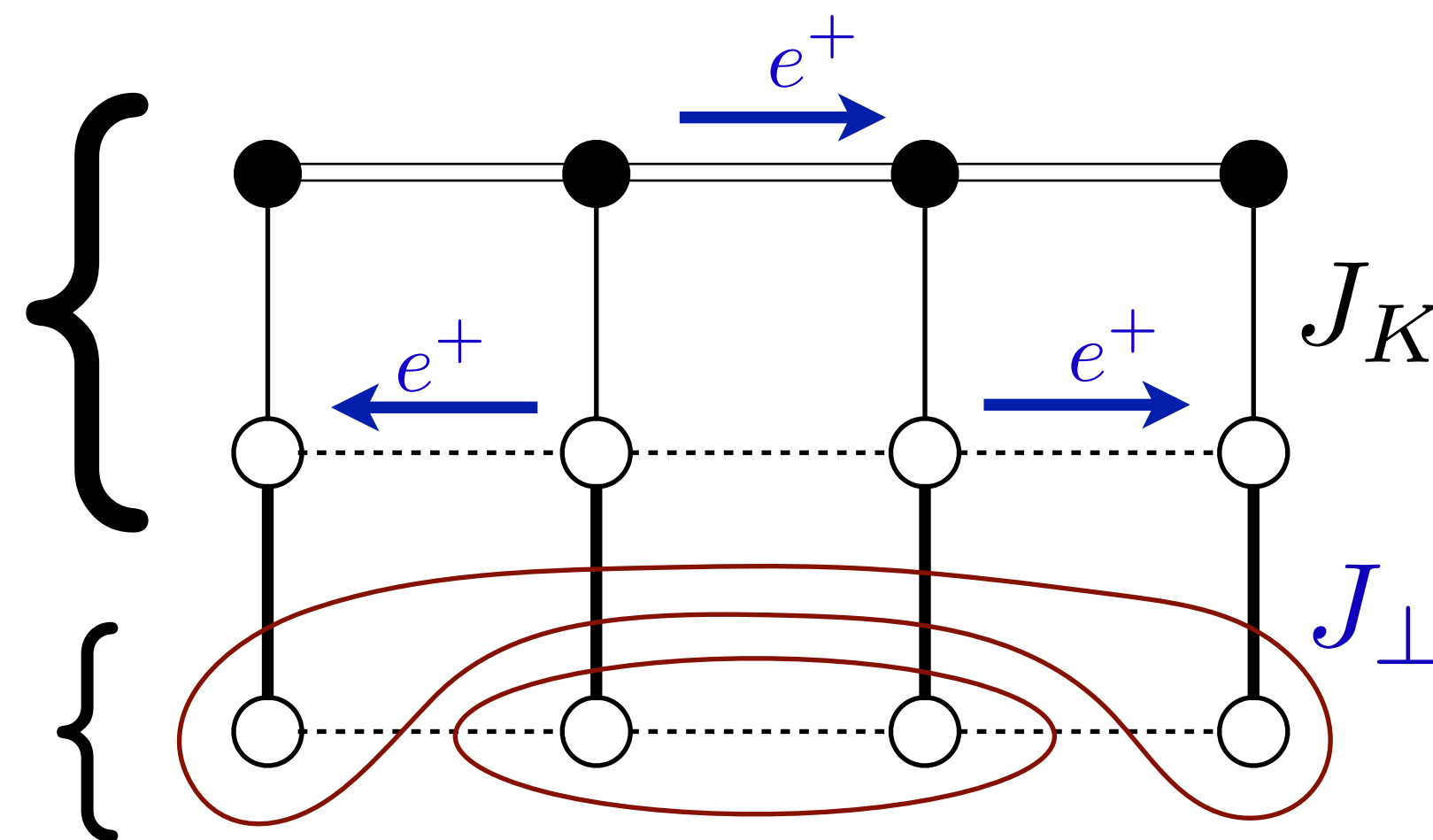


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Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)

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 Size $1 + p + 1$
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 Fermi surface.
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 insulator

FL*

FL

J_K

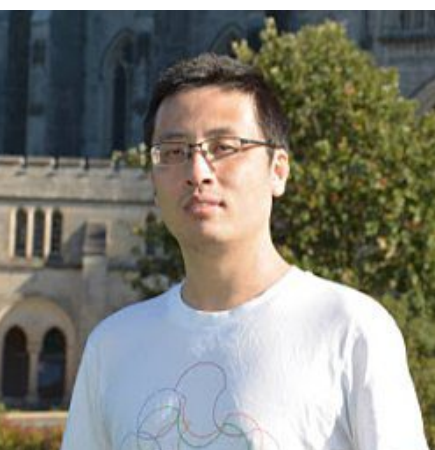
doping p

Pseudogap metal =
 Kondo Lattice Heavy
 Fermi Liquid
 \oplus
 Spin Liquid

$$|\text{FL}^*\rangle = [\text{Projection onto rung singlets of } S_1, S_2] \\
\otimes |\text{Slater determinant of } (c, f_1)\rangle \\
\otimes |\text{Spin liquid of } S_2\rangle$$

Replacement for “vanilla” Gutzwiller-projected Fermi liquid in the underdoped regime

Ya-Hui
 Zhang

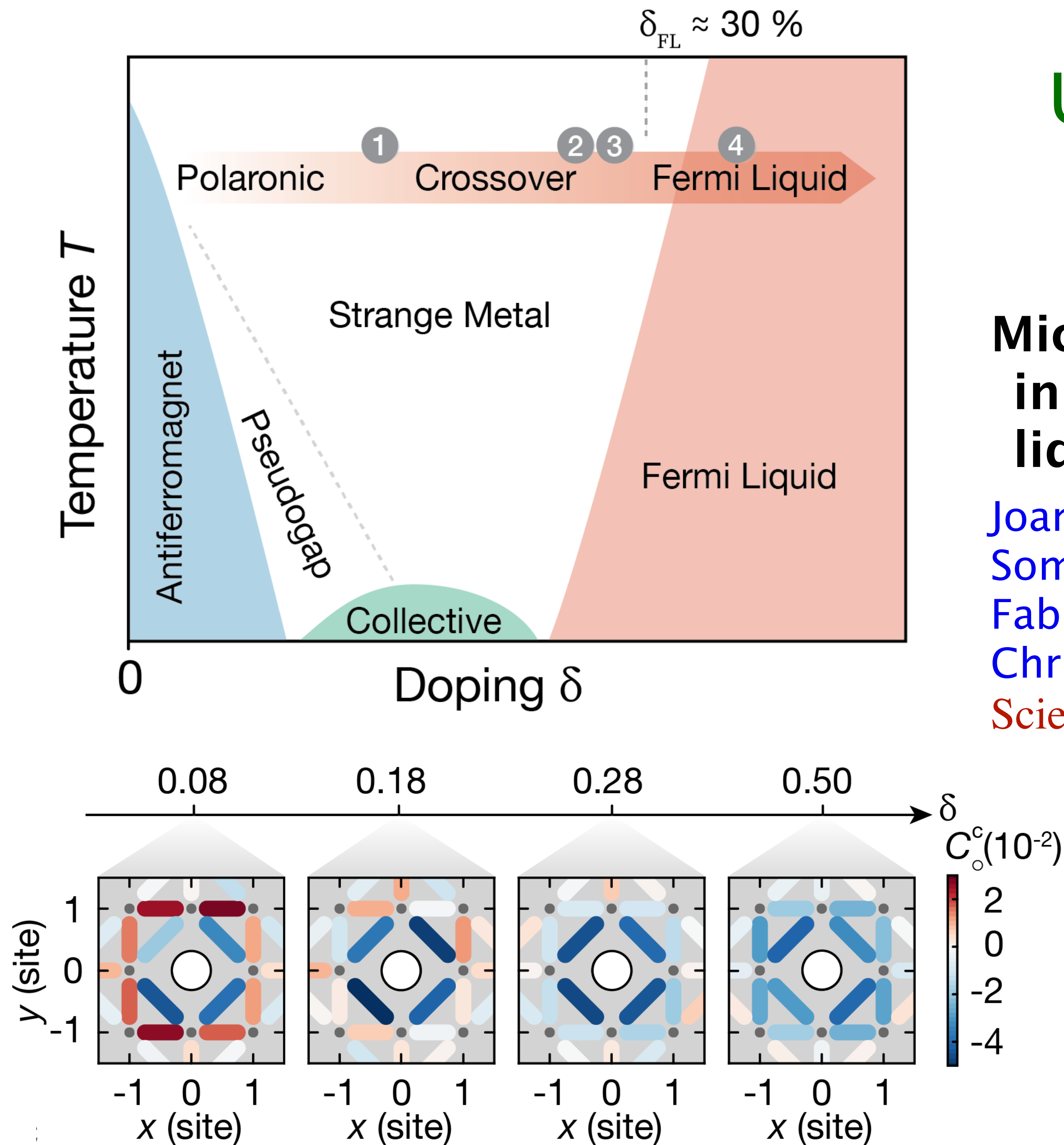


Ultracold fermionic atoms in optical lattices

Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

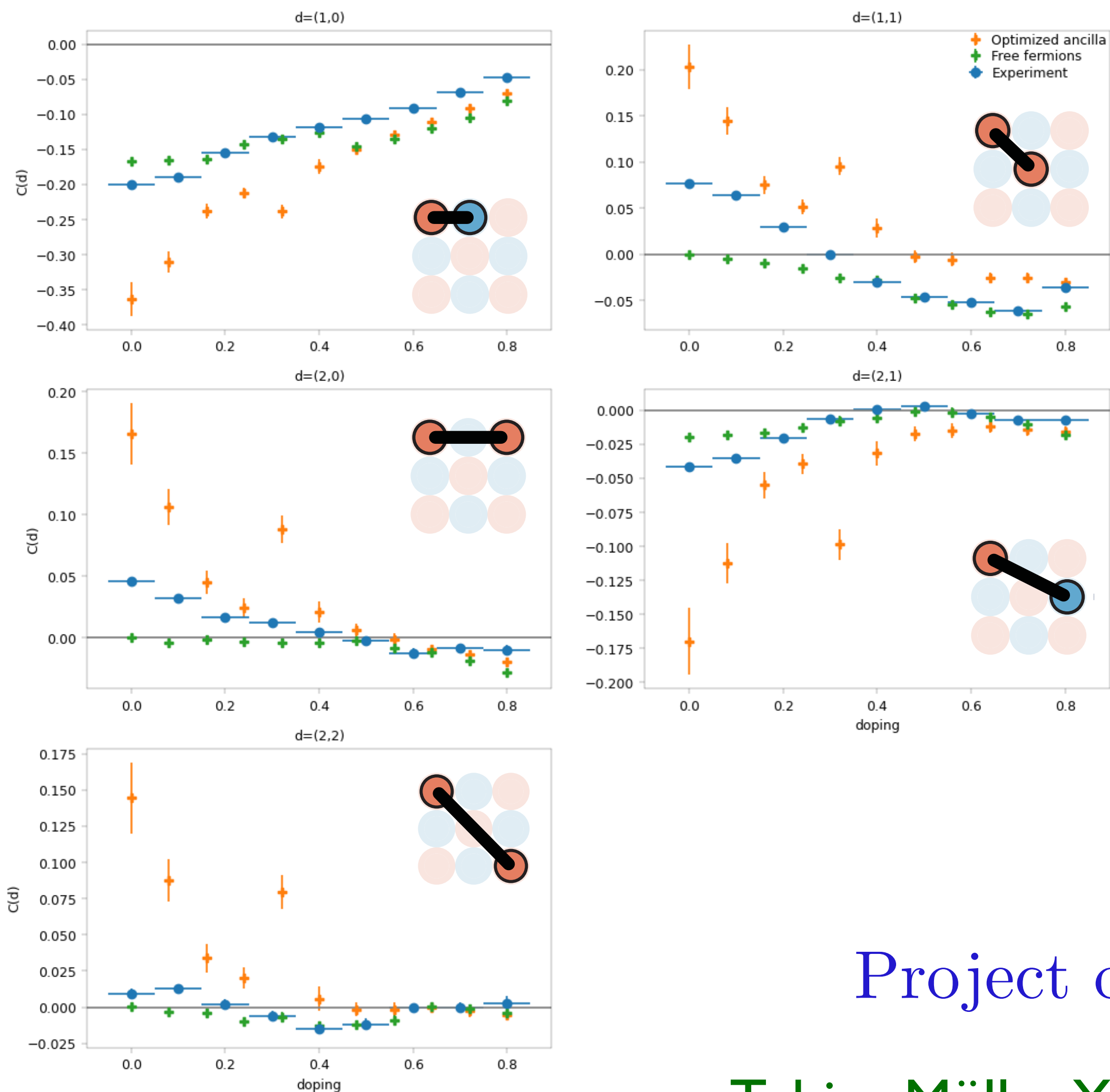
Science **374** (2021) 82



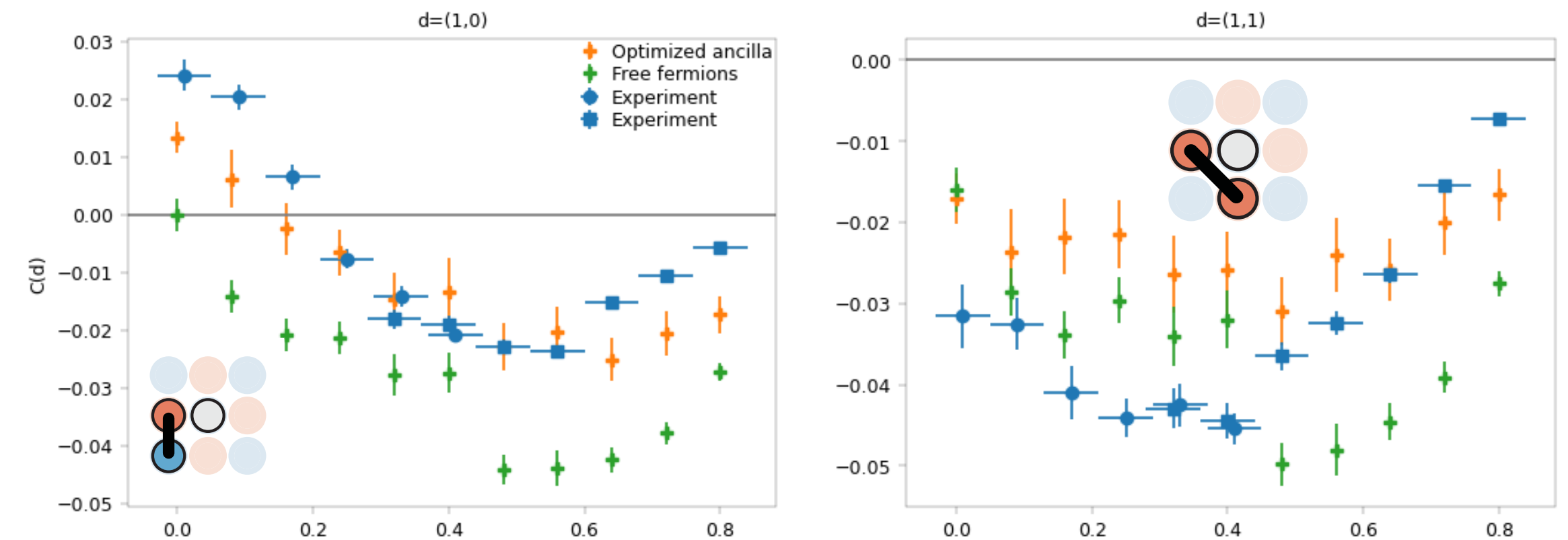
Max Planck Institute of
Quantum Optics,
Garching

Results for Connected Correlators: Comparison with experiment

Bare Spin-Spin Correlators



Hole-induced Spin-Spin Correlators



Preliminary data

- *Test different spin-liquid ansätze for second ancilla layer
- *Improve sampling of wave-functions
- *Better statistics to obtain more reliable error-bars

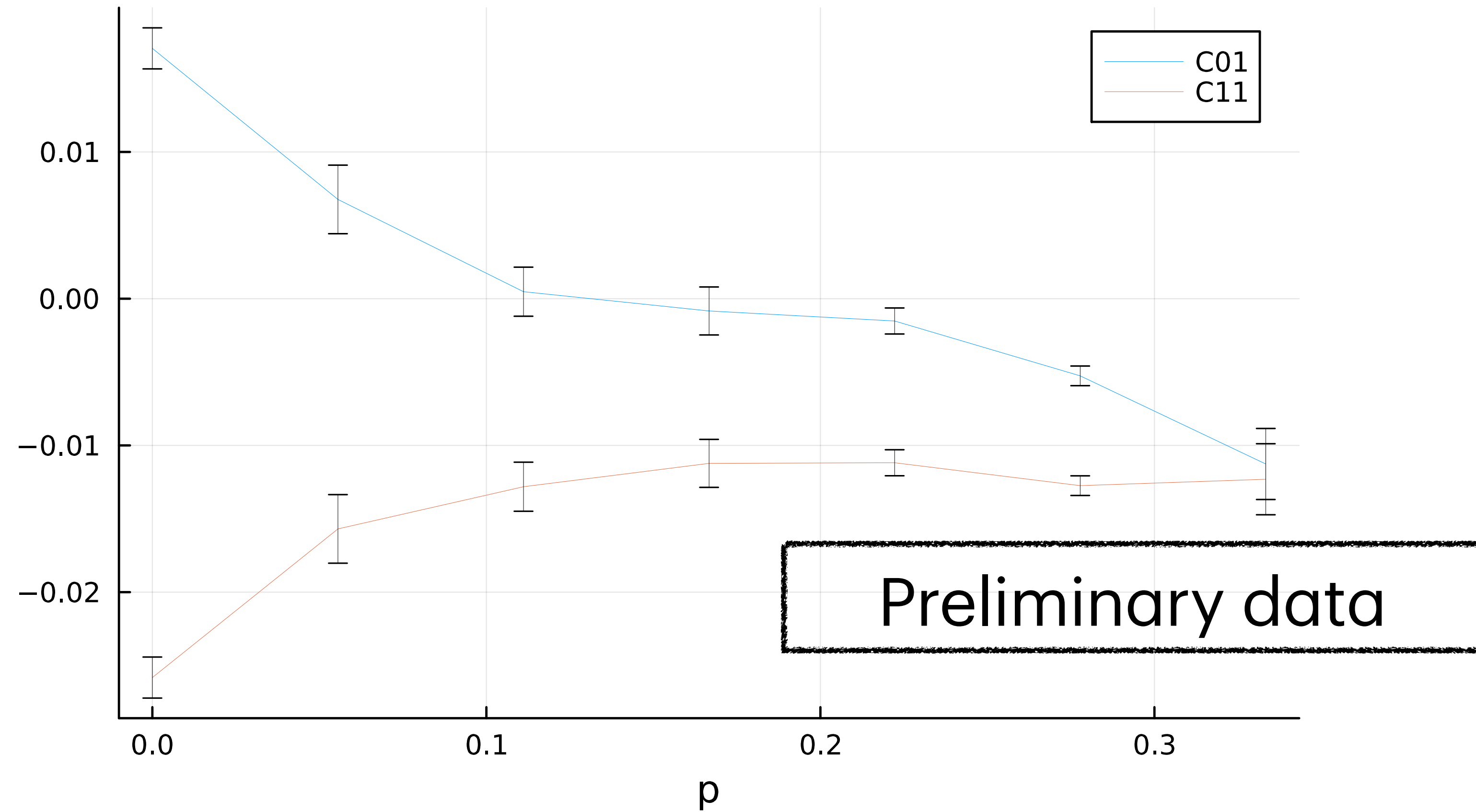
Project only to $S_{1z} + S_{2z} = 0$

Tobias Müller, Yasir Iqbal, Ronny Thomale to appear



Ancilla theory of the Hubbard model

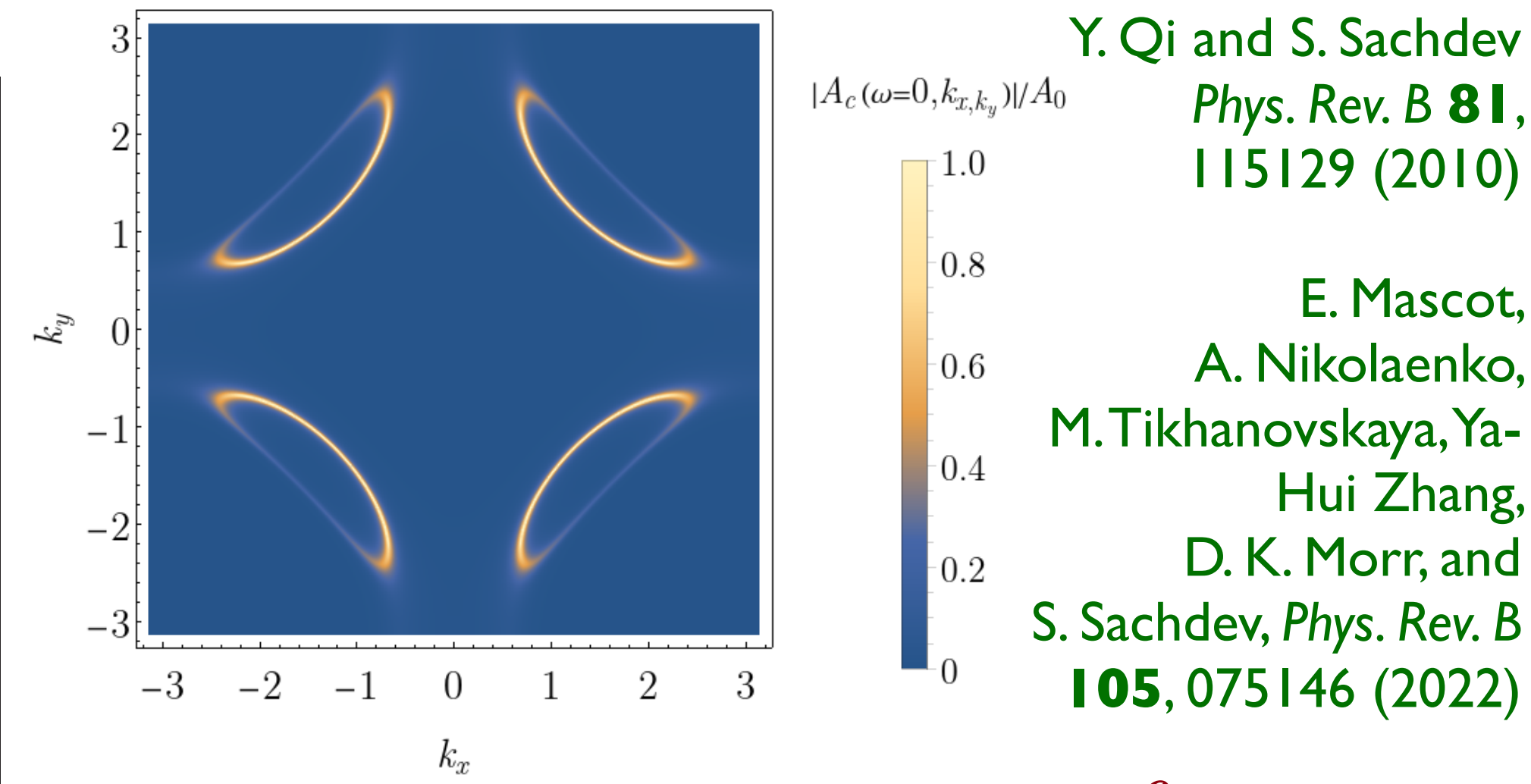
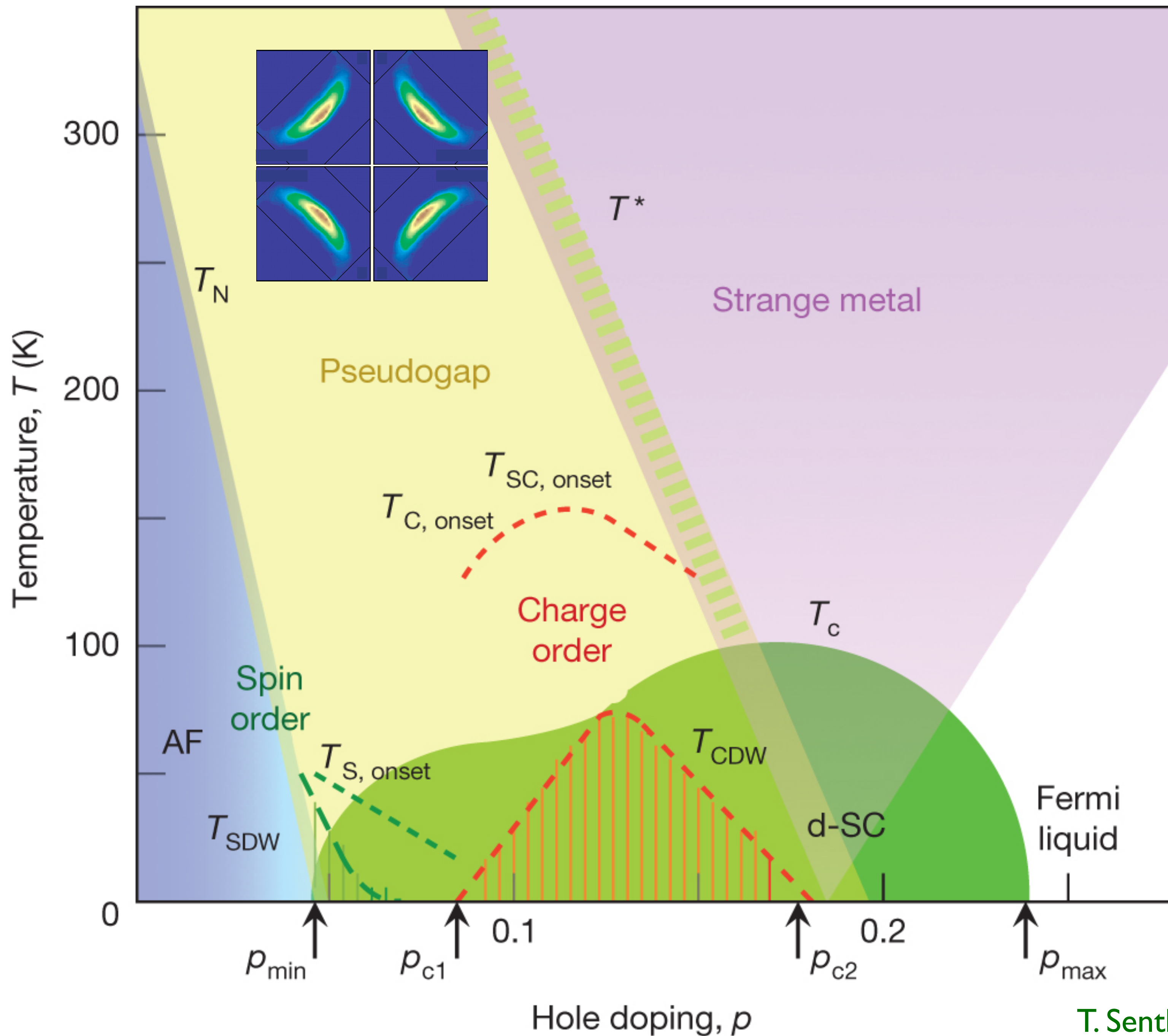
Hole-induced correlations



Project to $\mathcal{S}_1 + \mathcal{S}_2 = 0$

H. Shackleton, S.S., Shiwei Zhang, to appear



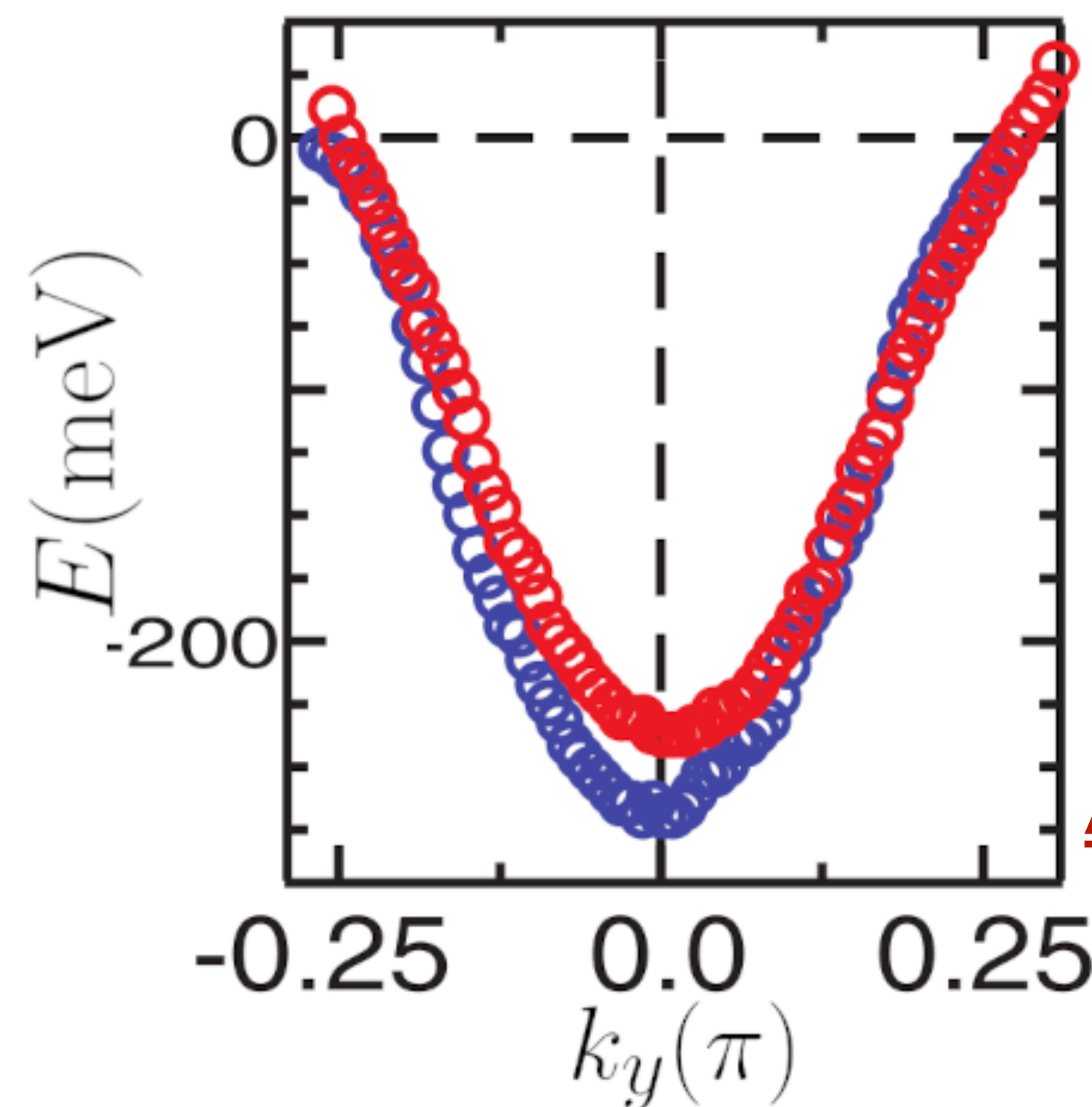
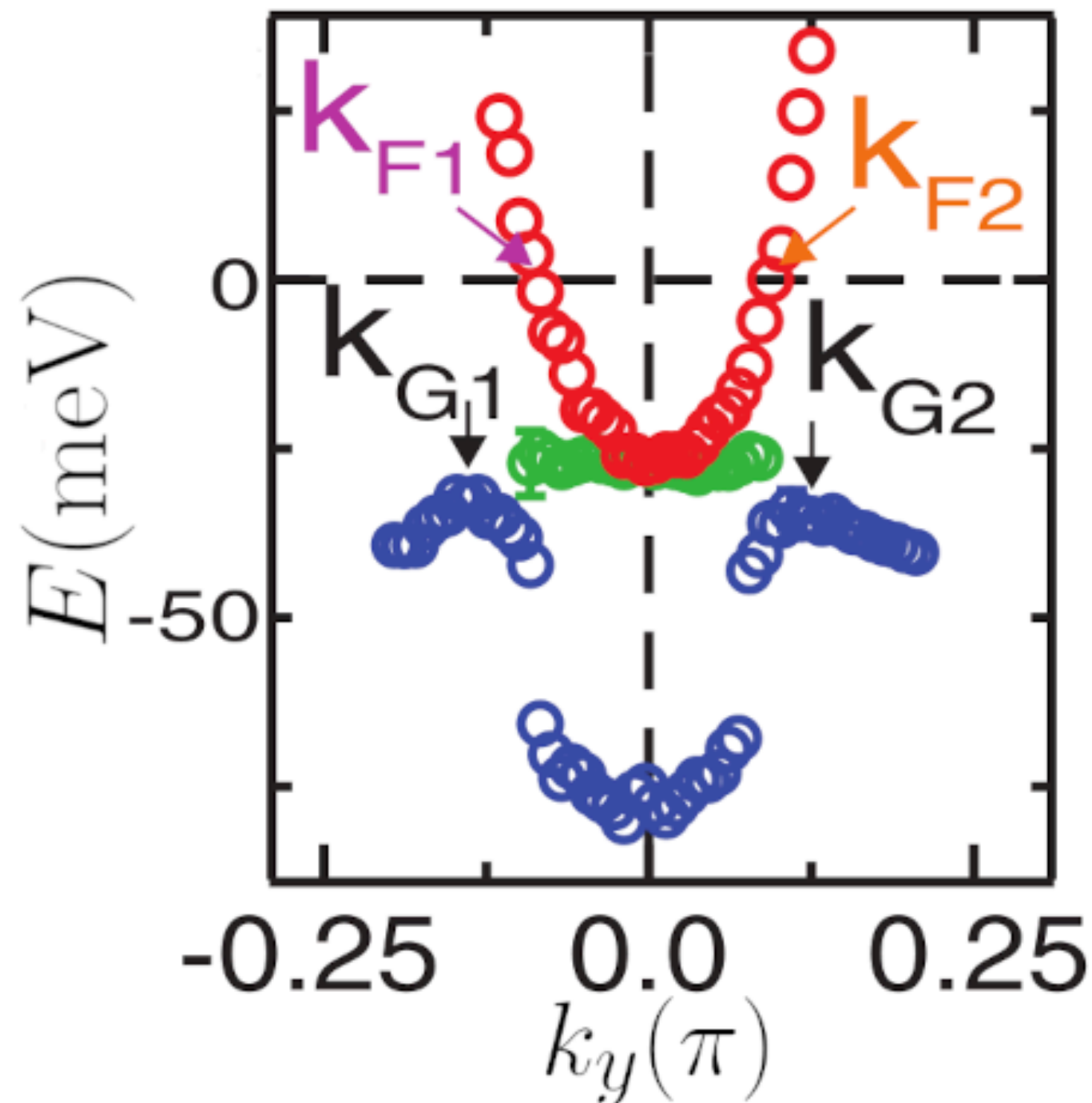
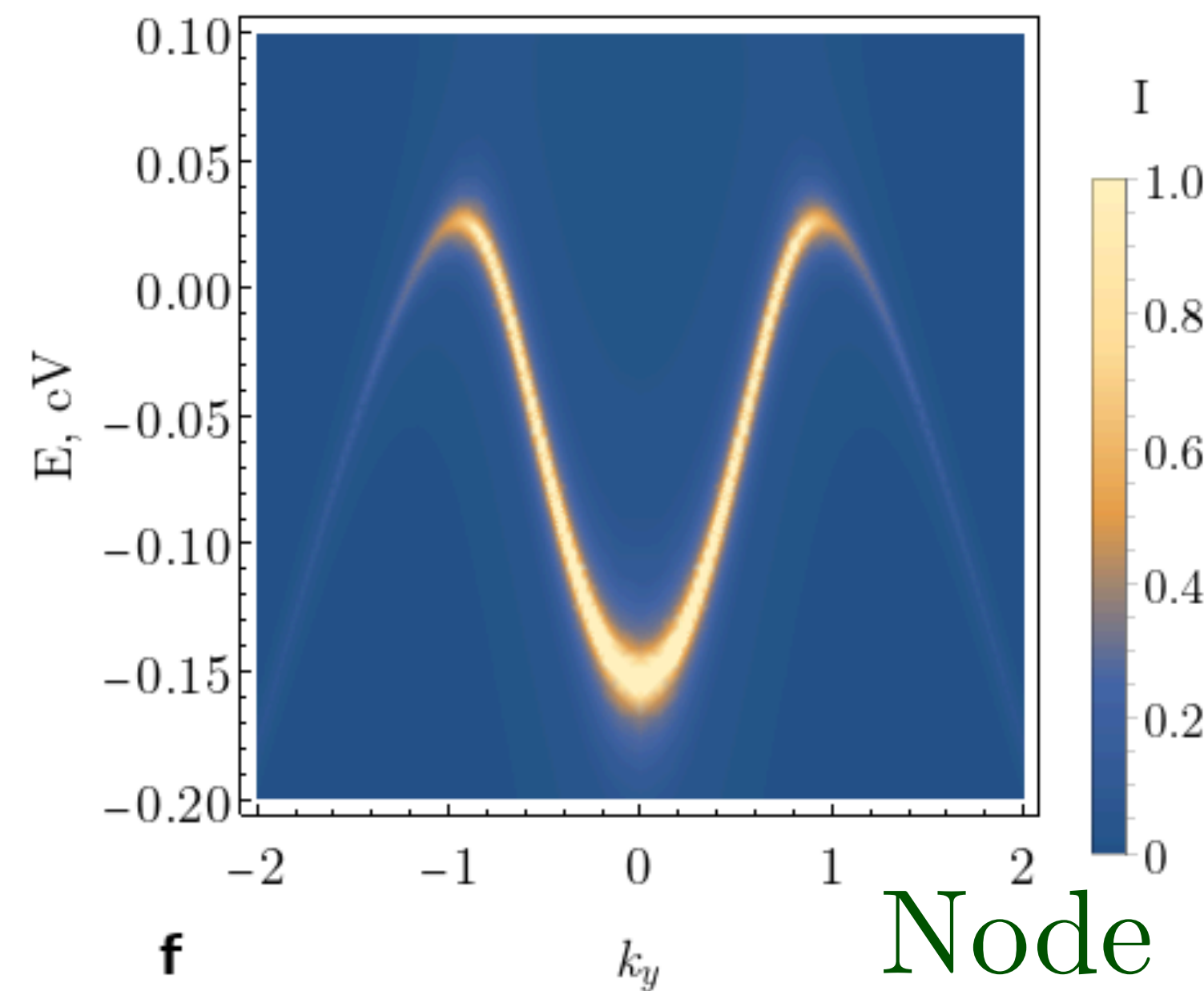
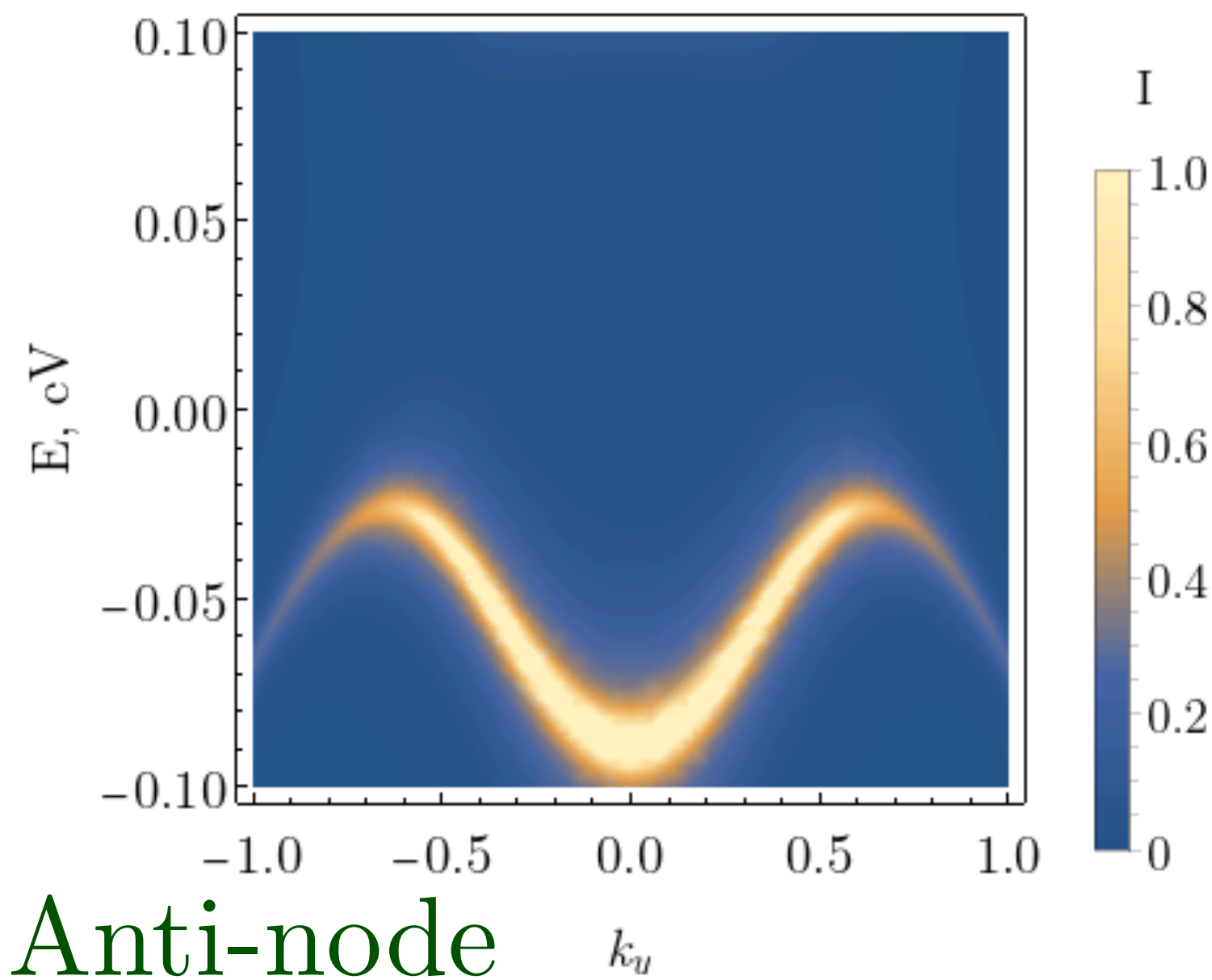


Hole pocket Fermi surfaces
of size p with
charge e , spin-1/2 quasiparticles
+
'spectator'
square lattice spin liquid
at half-filling.

FL*: Spin liquid is *required* because
the Fermi surface does not enclose
the Luttinger volume $(1 + p)$.

FL* in a one-band model

Broadening by second ancilla layer is needed to describe MDC and EDC

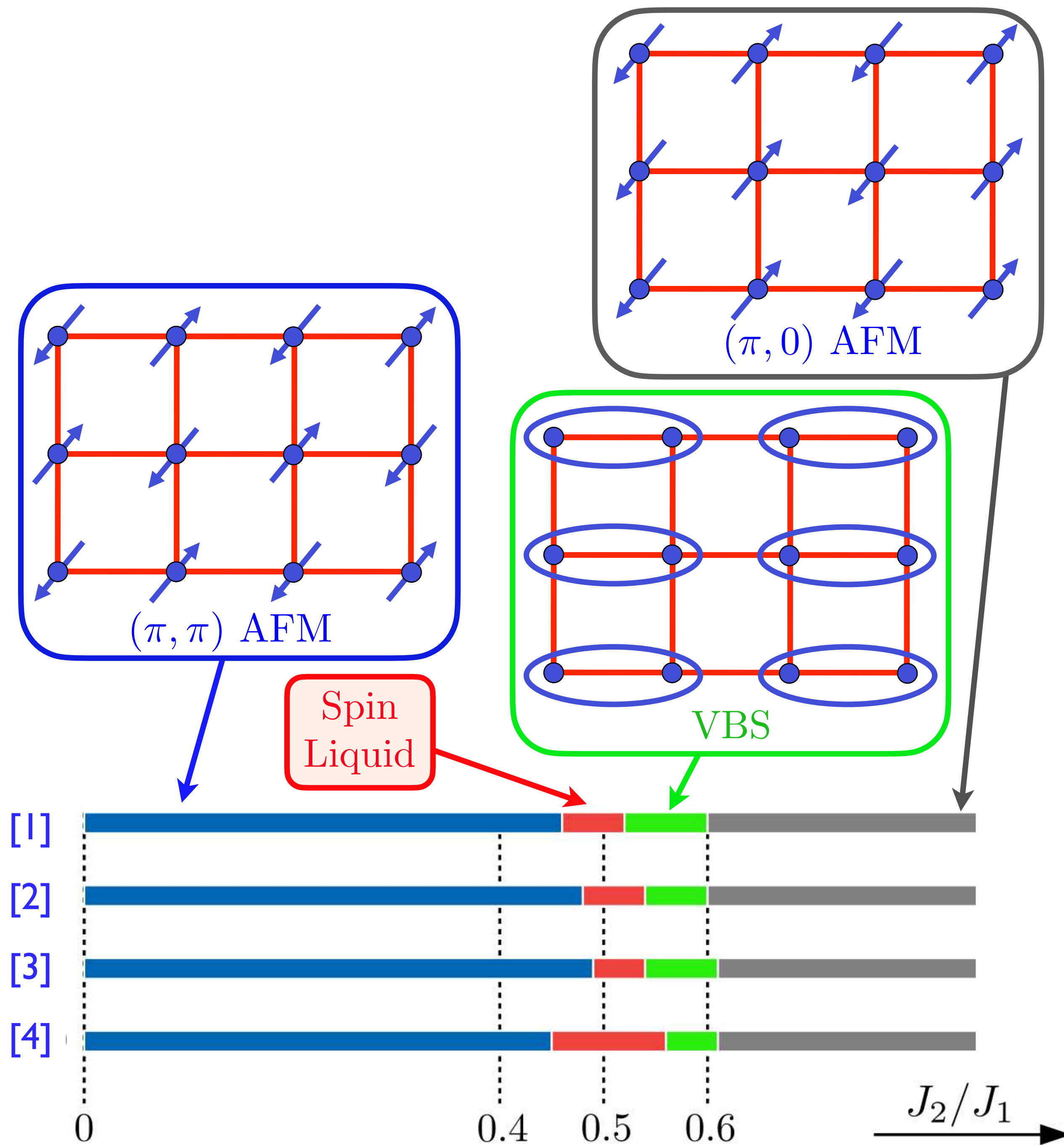


He, Hashimoto, Karapetyan, Koralek, Hinton, Testaud, Nathan, Yoshida, Yao, Tanaka, Meevasana, Moore, Lu, Mo, Ishikado, Eisaki, Hussain, Devereaux, Kivelson, Orenstein, Kapitulnik, and Shen, *Science* **331**, 1579 (2011)

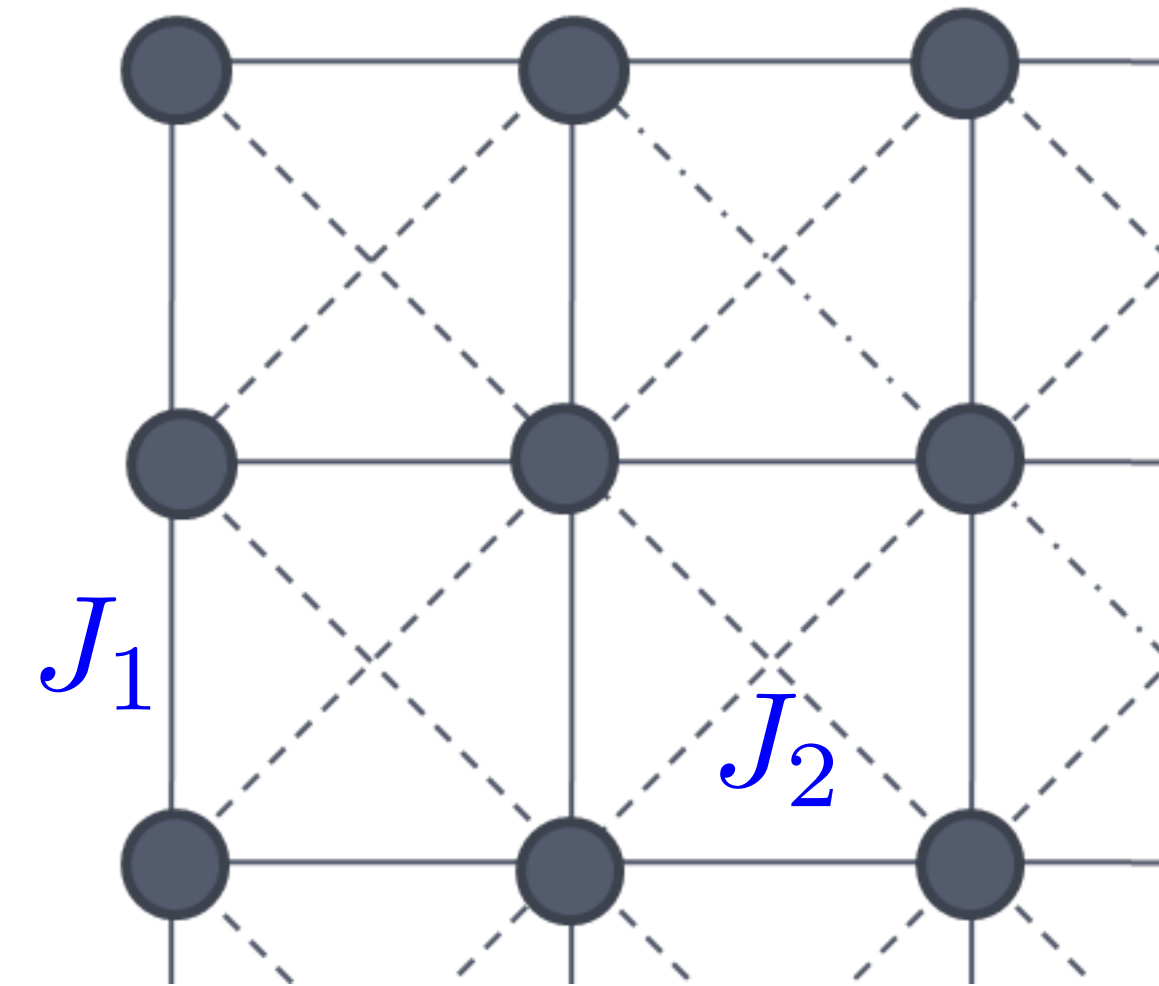
ARPES on Bi2201



The CP^1/π -flux spin liquid

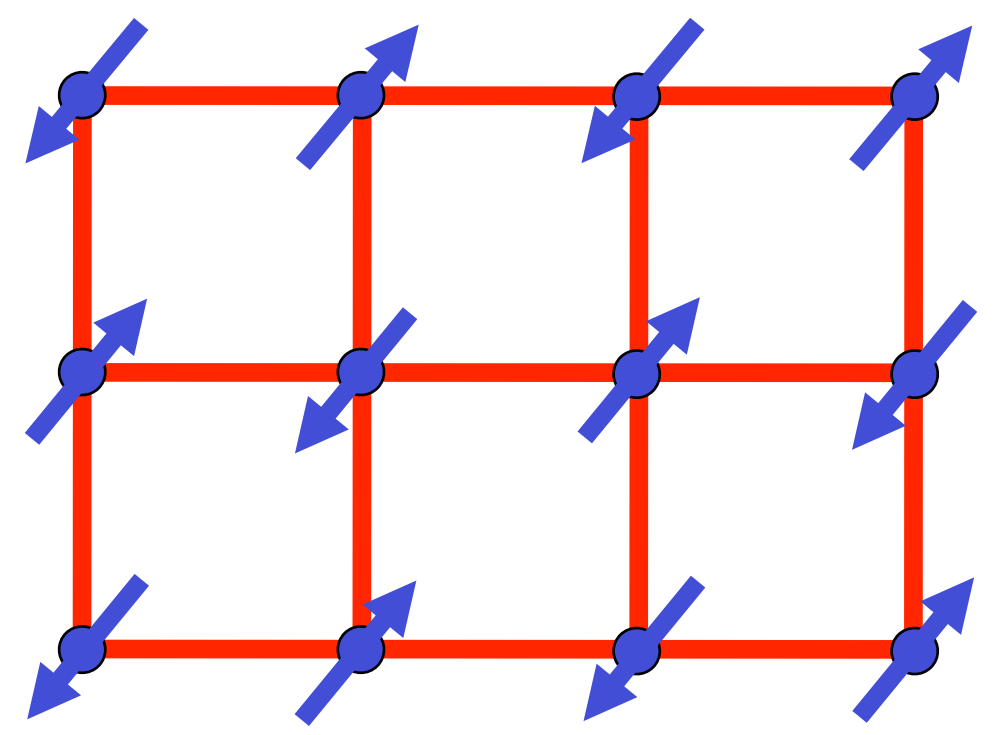


$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



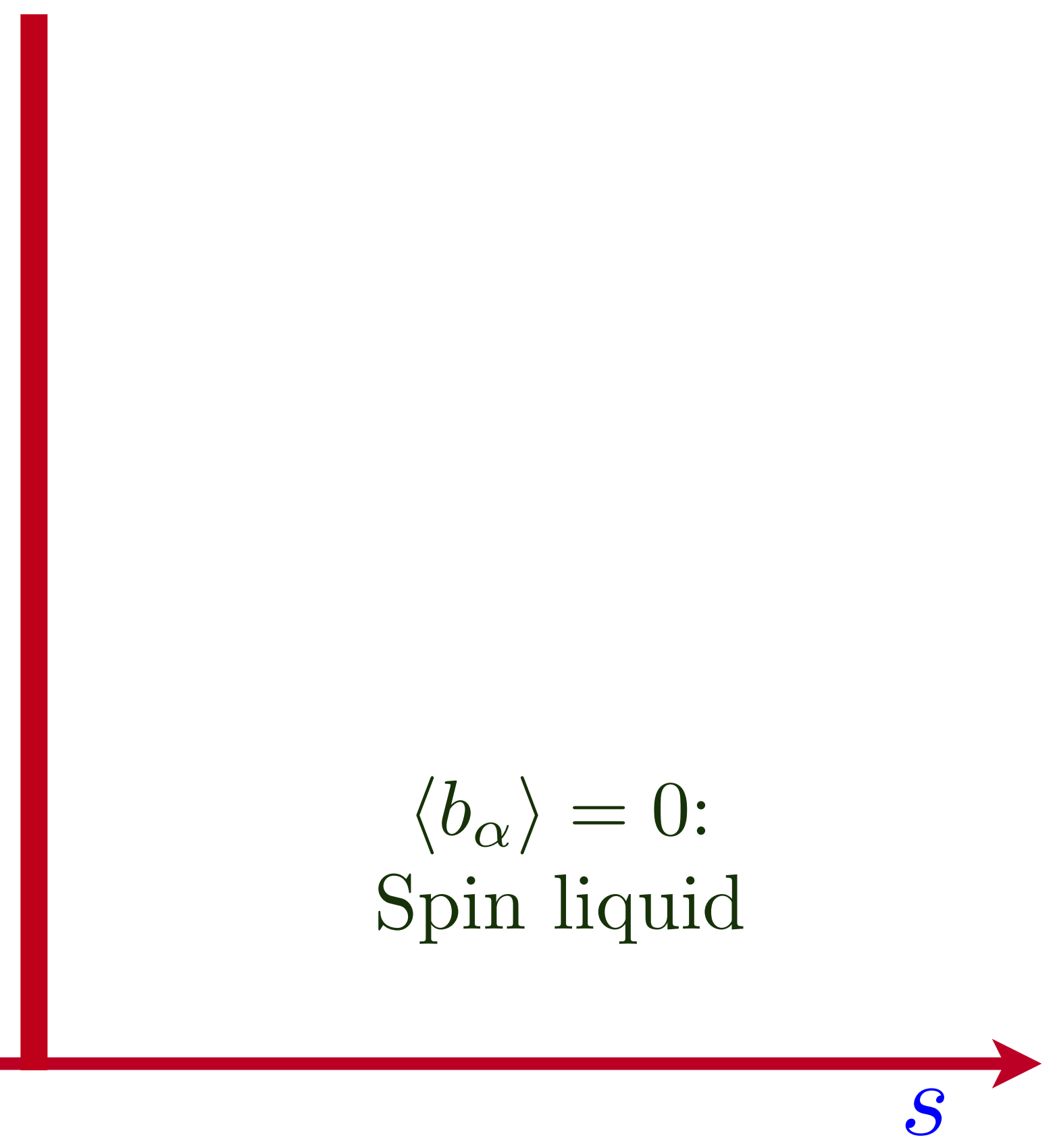
1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67**, 1034 (2022)

Insulating $S=1/2$ antiferromagnet



$\langle b_\alpha \rangle \neq 0$:
Néel order

$\langle b_\alpha \rangle = 0$:
Spin liquid



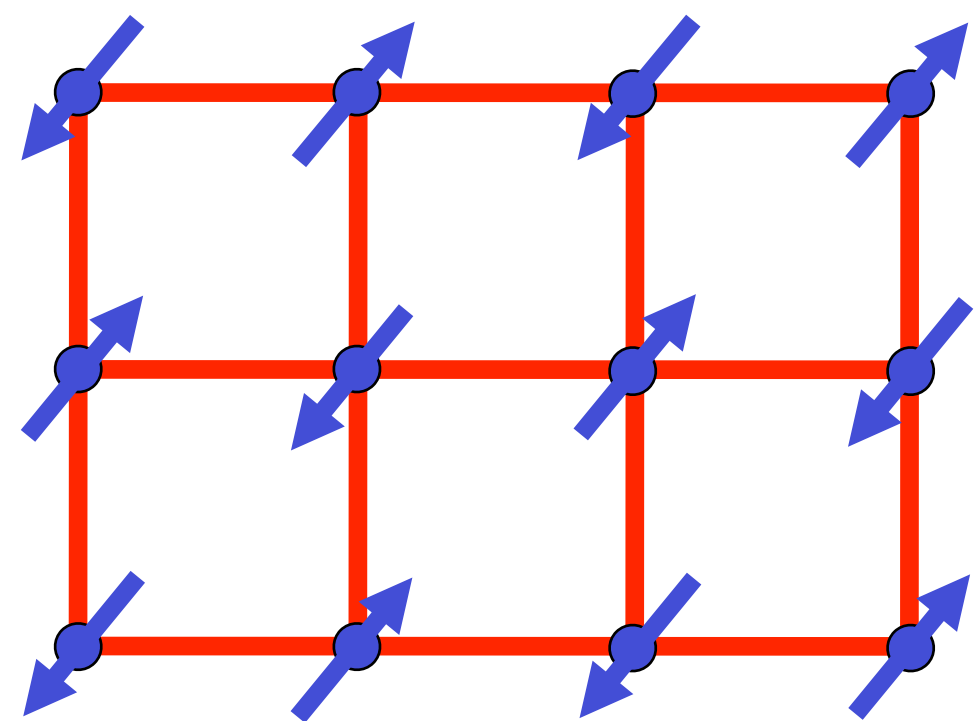
$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

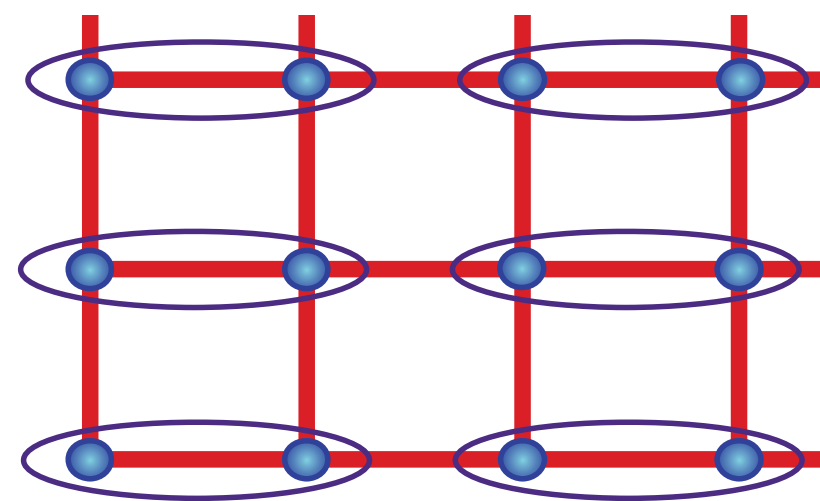
$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

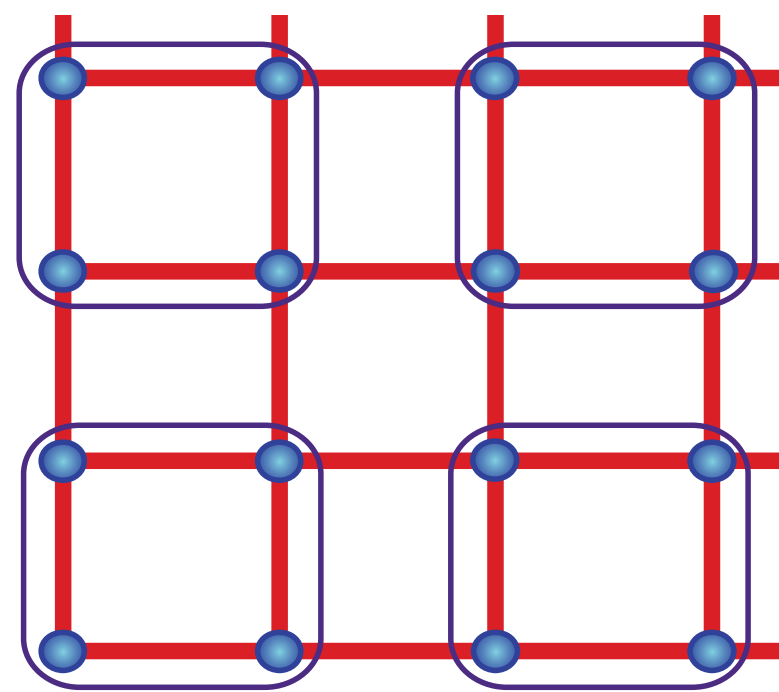
Insulating $S=1/2$ antiferromagnet



Spin liquid



or



Higgs phase, $\langle z_\alpha \rangle \neq 0$:
Néel order

Confining phase, $\langle z_\alpha \rangle = 0$:
VBS order



$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

Low energy $\mathbb{C}P^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - ia_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

$\mathbb{C}P^1$ U(1) gauge theory

$S=1/2$
square
lattice anti-
ferromagnet

SU(2) gauge theory of $N_f = 2$
fundamental, massless, Dirac fermions.

Obtained from a saddle-point of
fermionic spinons moving in π -flux.

I. Affleck and J.B. Marston, *Phys. Rev. B* **37**, 3774 (1988)

SO(5) non-linear σ -model
of Néel/VBS orders
with $k = 1$ WZW term

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Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Anders W. Sandvik *Phys. Rev. Lett.* **98**, 227202 (2007)

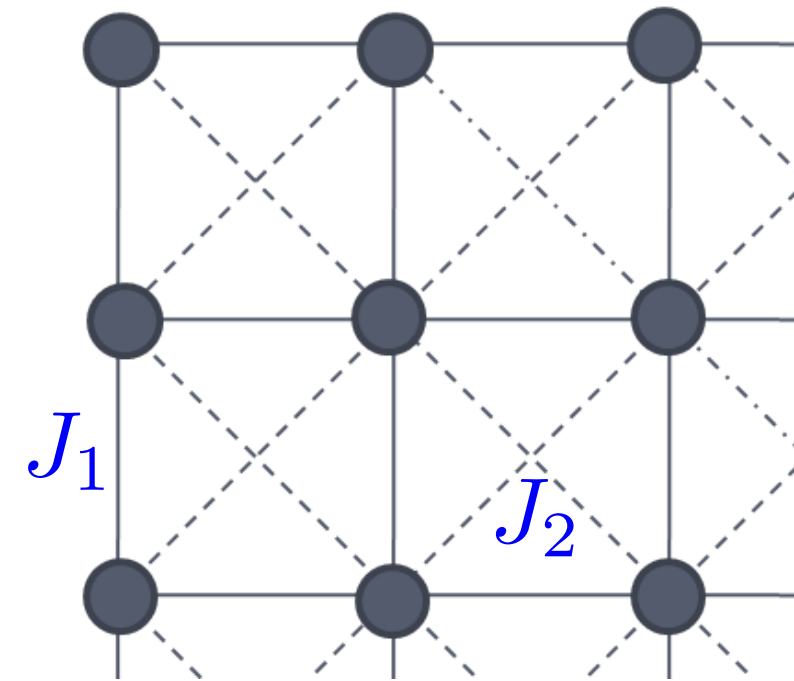
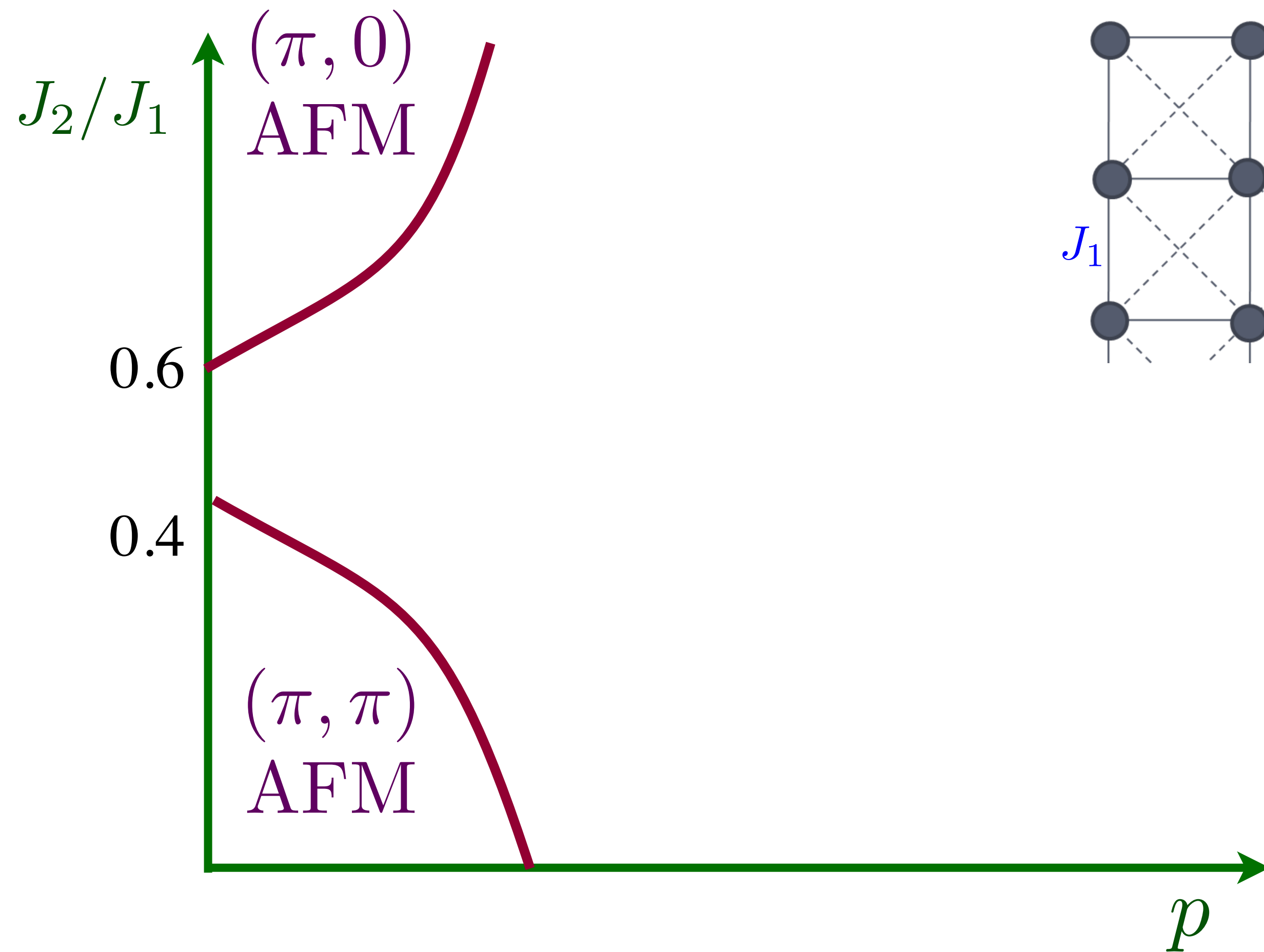
Adam Nahum, P. Serna, J. T. Chalker, M. Ortuño, and A. M. Somoza, *Phys. Rev. Lett.* **115**, 267203 (2015)

Z. Zhou, L. Hu, W. Zhu, and Yin-Chen He, arXiv:2306.16435

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang ^{1,*} and Steven A. Kivelson ²

Phys. Rev. Lett. **127**, 097002 (2021)



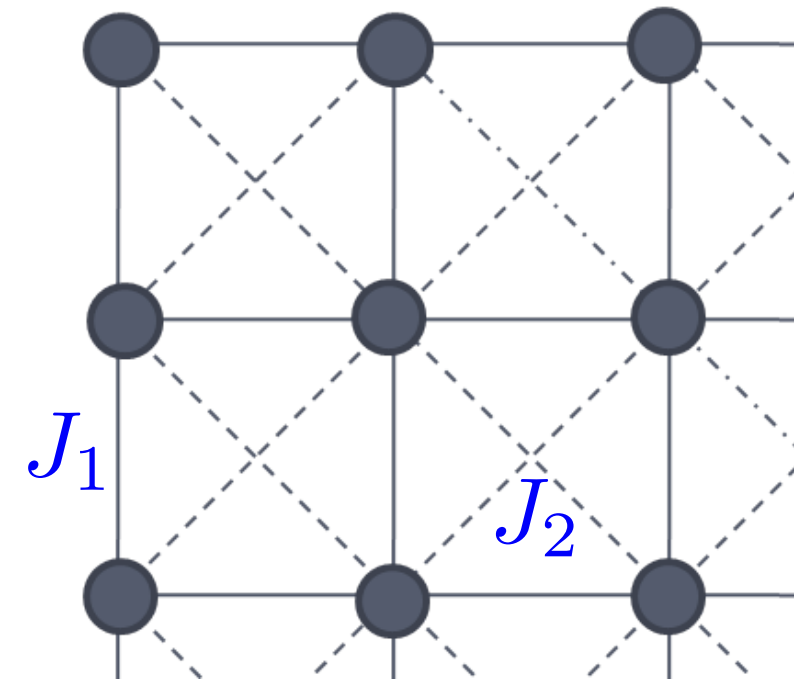
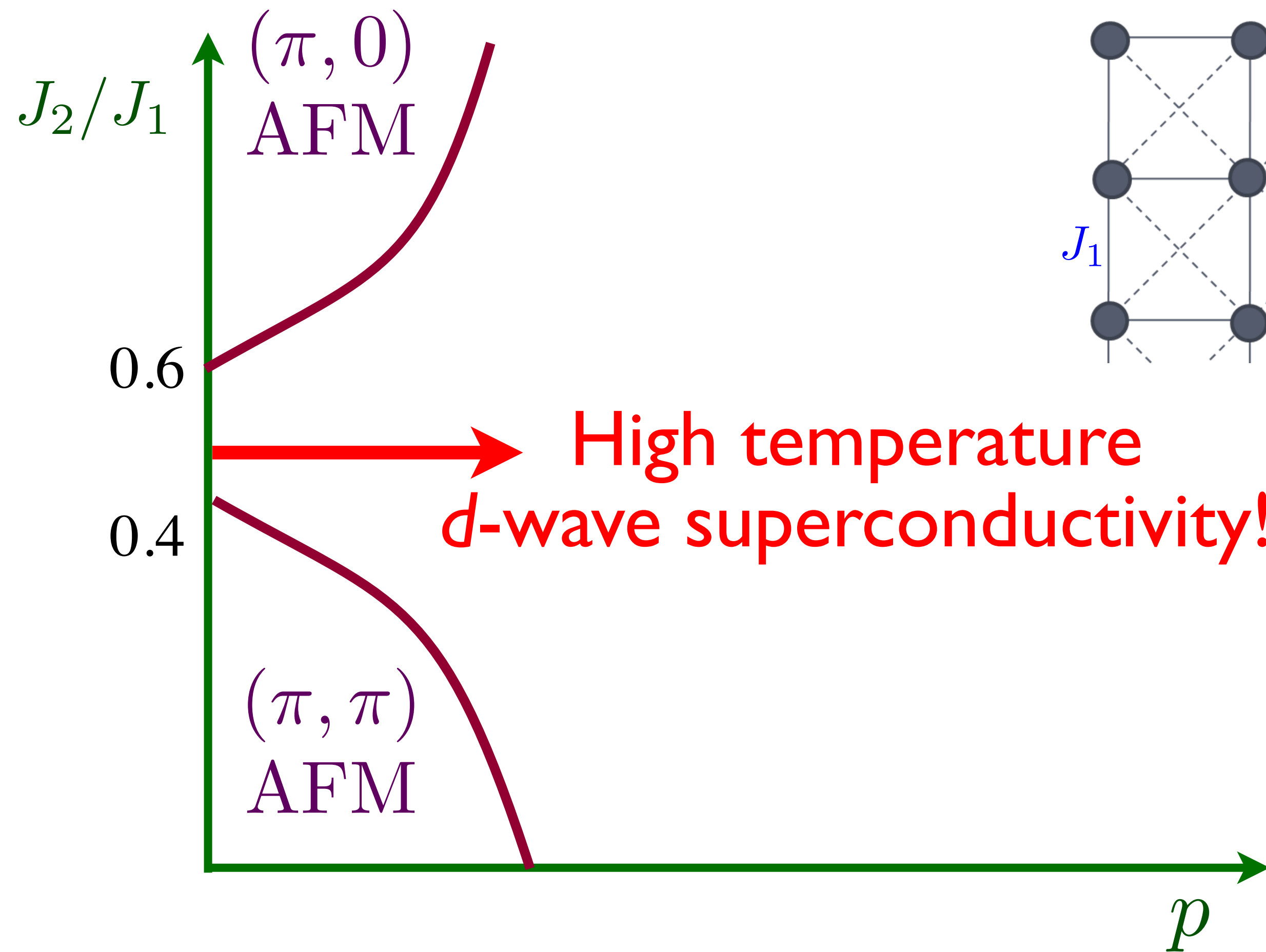
Superconducting valence bond fluid in
lightly doped 8-leg t - J cylinders
Hong-Chen Jiang, Steven A. Kivelson, and
Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any “competing” charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

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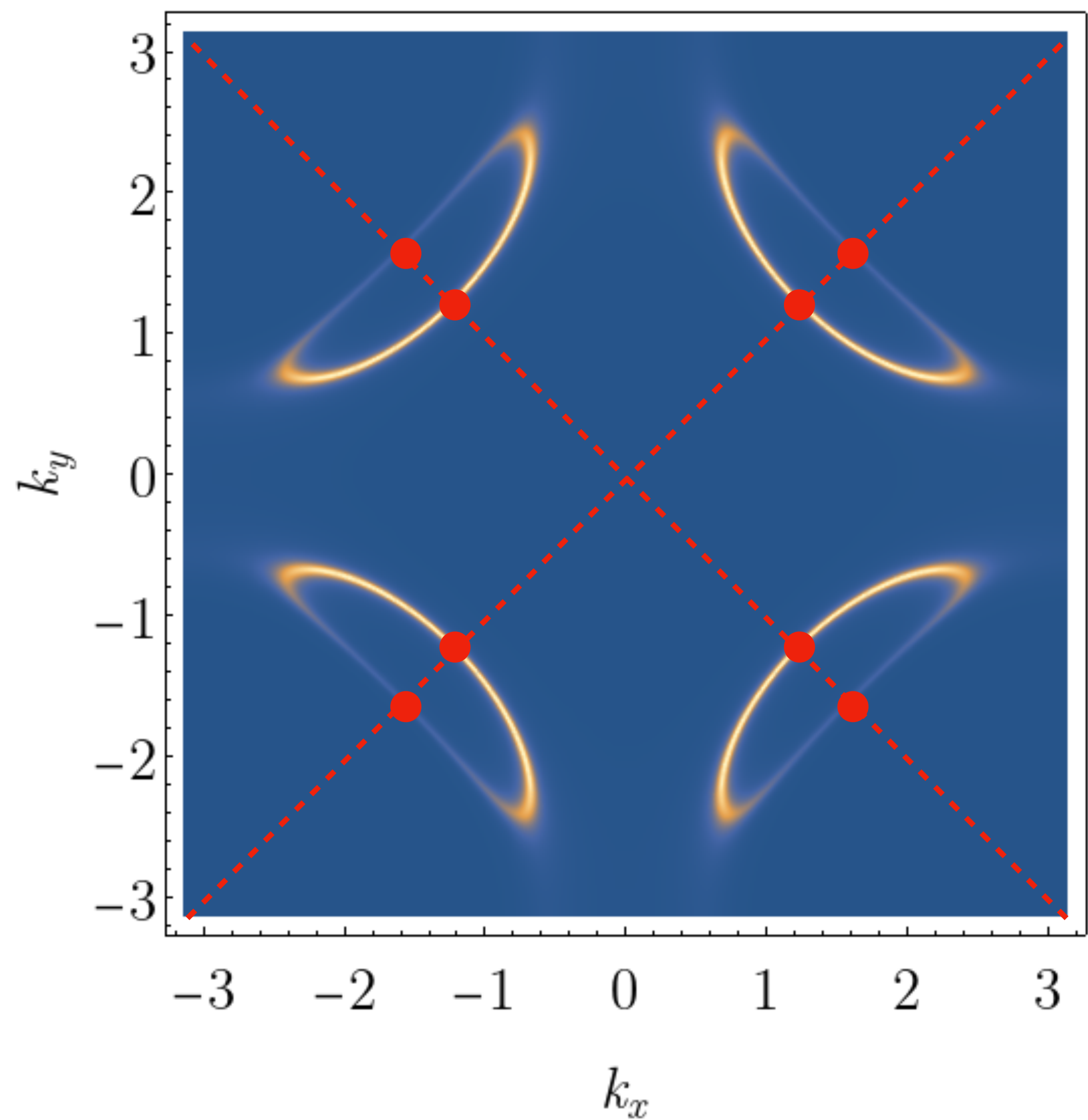
Phys. Rev. Lett. **127**, 097002 (2021)



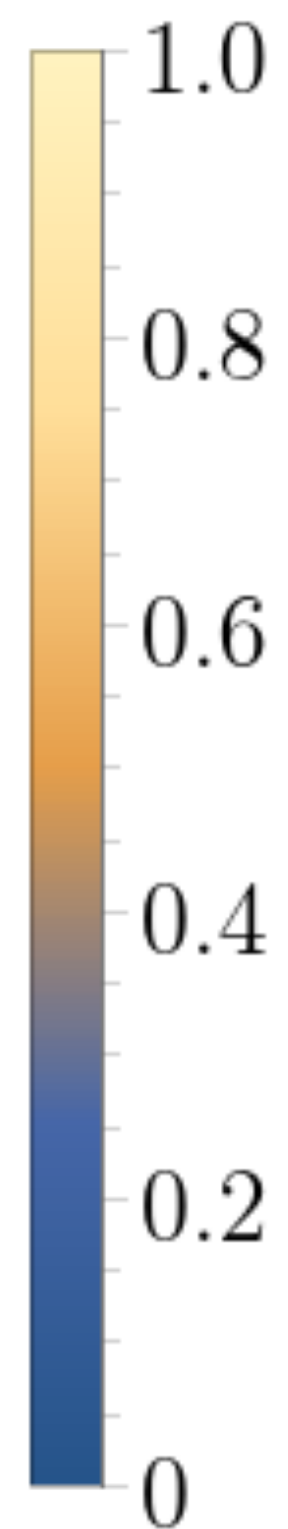
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From FL* to
d-wave superconductivity

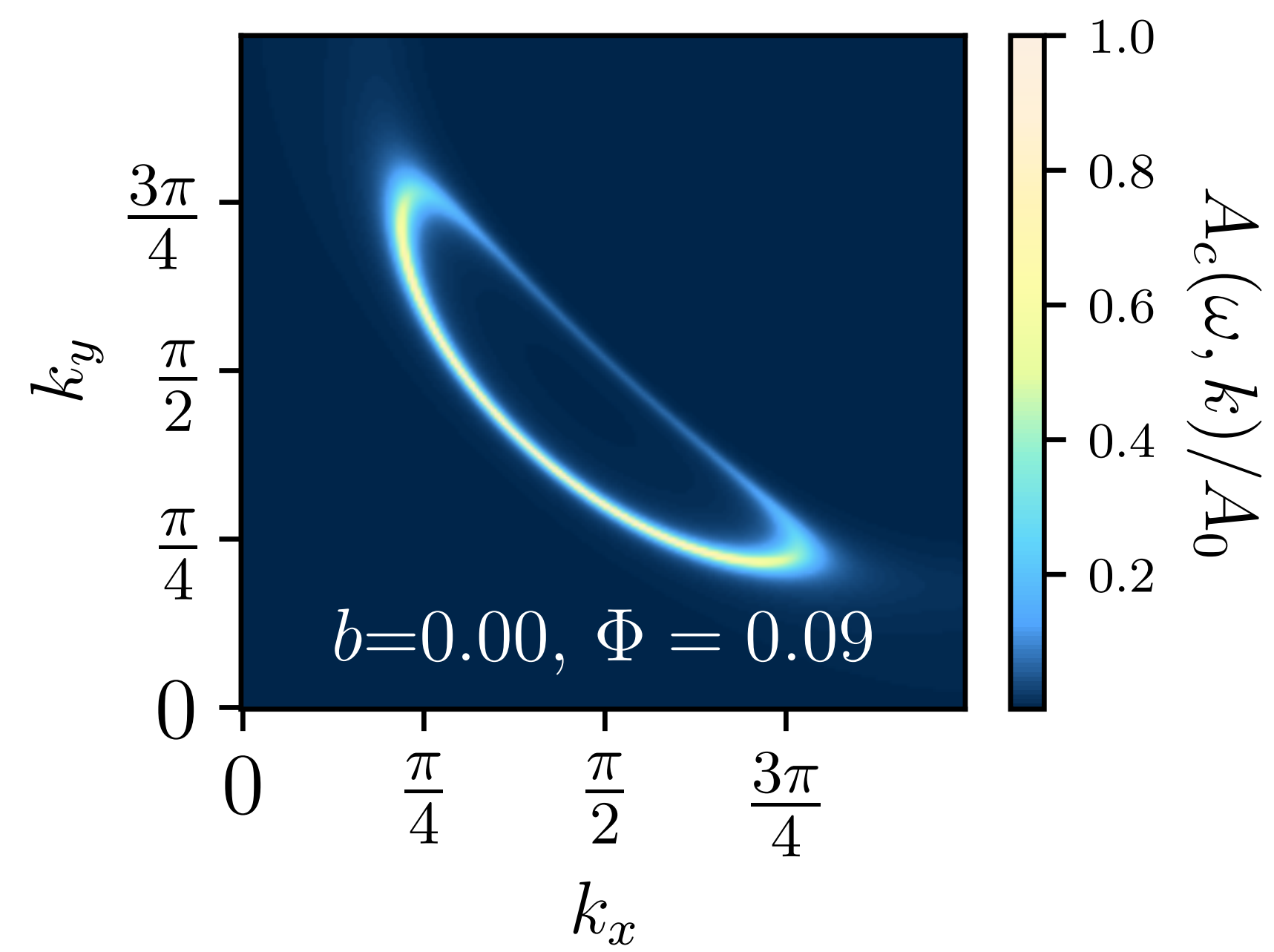


$$|A_c(\omega=0, k_x, k_y)|/A_0$$



Adding *d*-wave pairing
to the hole pockets
leads to 8 nodal points???

Electron spectral density in hole-doped cuprates



FL*

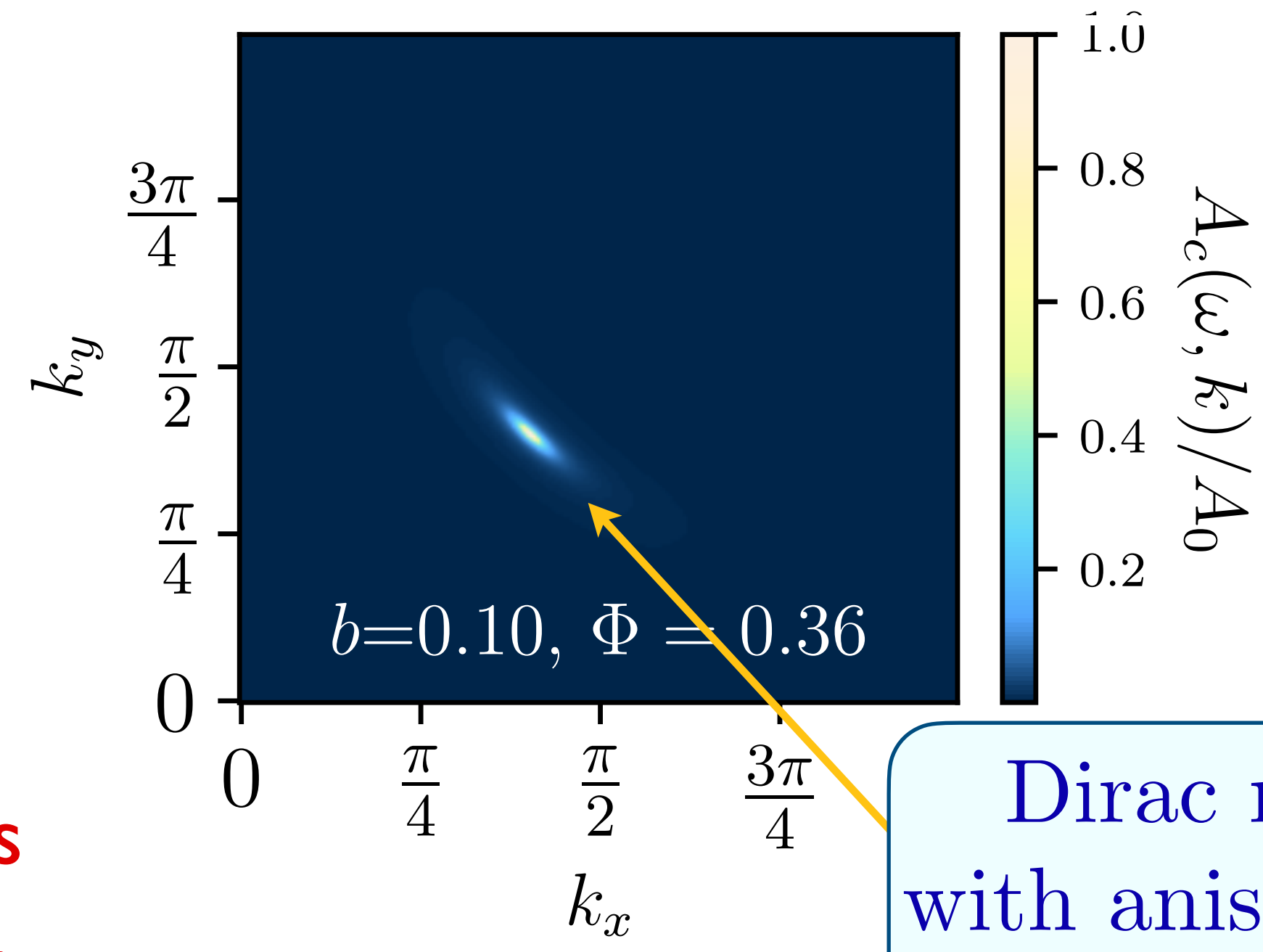
d-wave superconductor obtained by condensing charge-*e*, SU(2) fundamental boson.

Spinons of the π -flux state annihilate the extra nodes in the *d*-wave superconductor.

Shubhayu Chatterjee and S. S., PRB **94**, 205117 (2016)



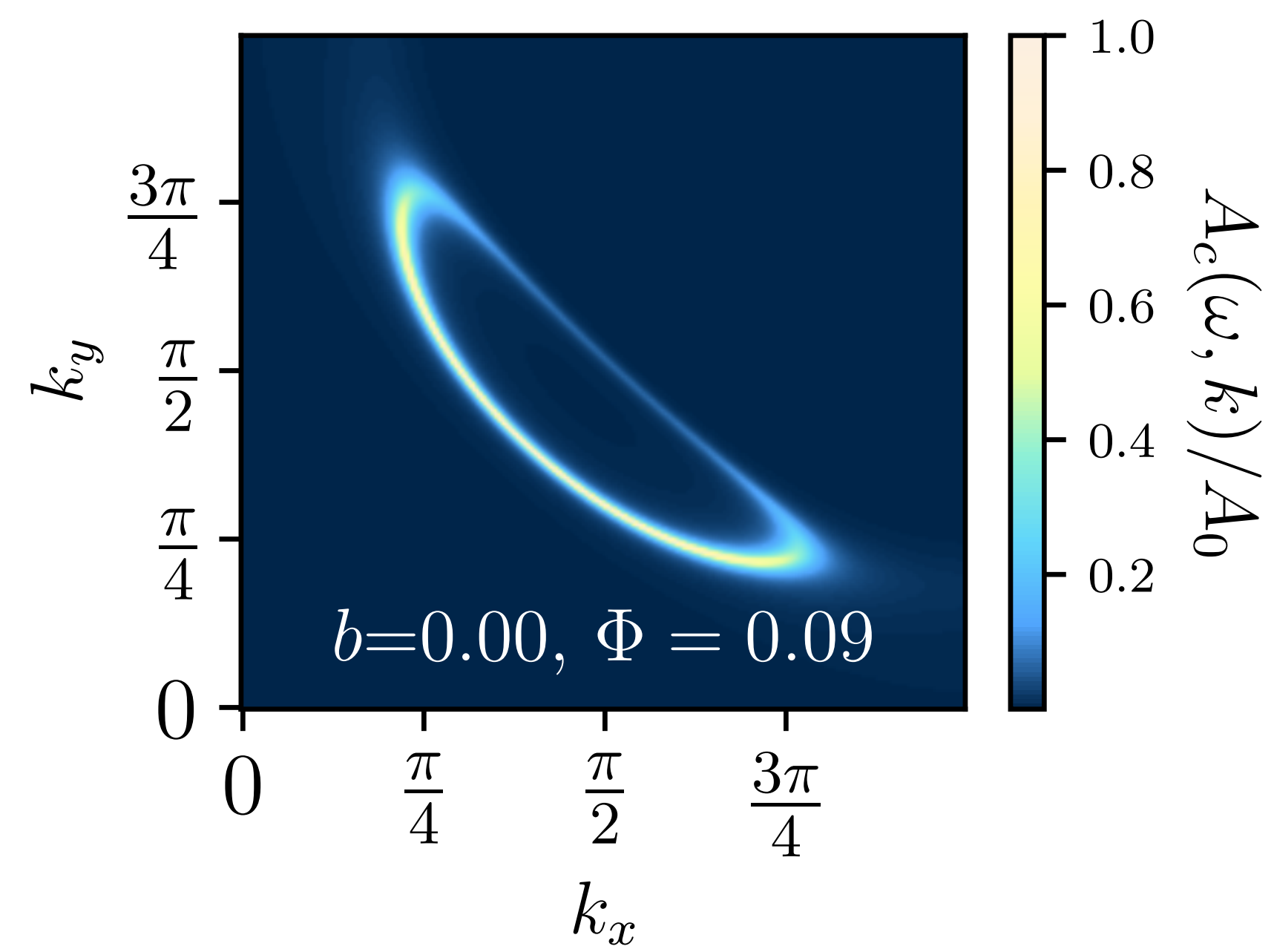
Maine Christos and S.Sachdev, arXiv:2308.03835



dSC

Dirac node with anisotropic velocities

Electron spectral density in hole-doped cuprates



FL*

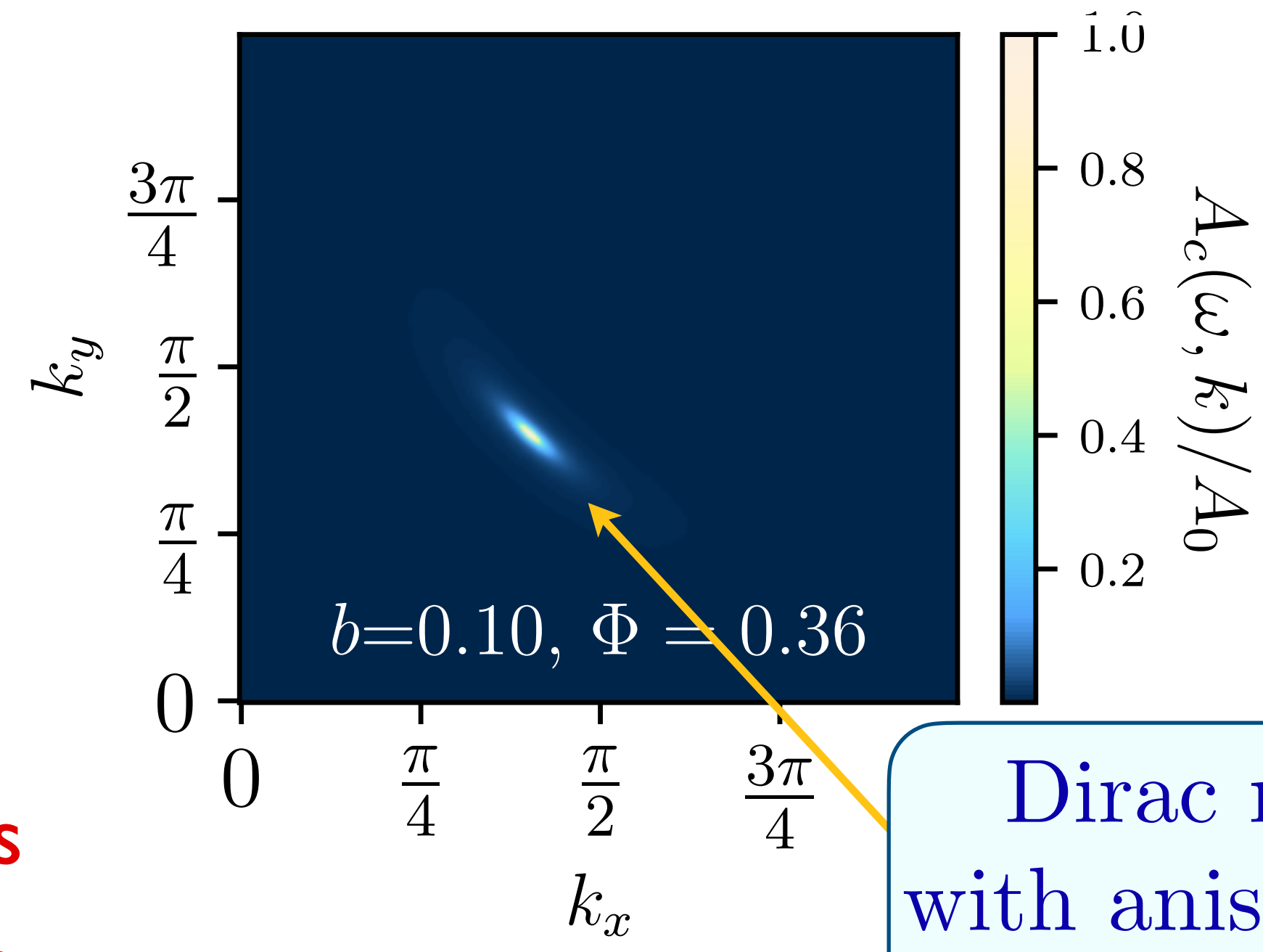
d-wave superconductor obtained by condensing charge-*e*, SU(2) fundamental boson.

Spinons of the π -flux state annihilate the extra nodes in the *d*-wave superconductor.

Shubhayu Chatterjee and S. S., PRB **94**, 205117 (2016)

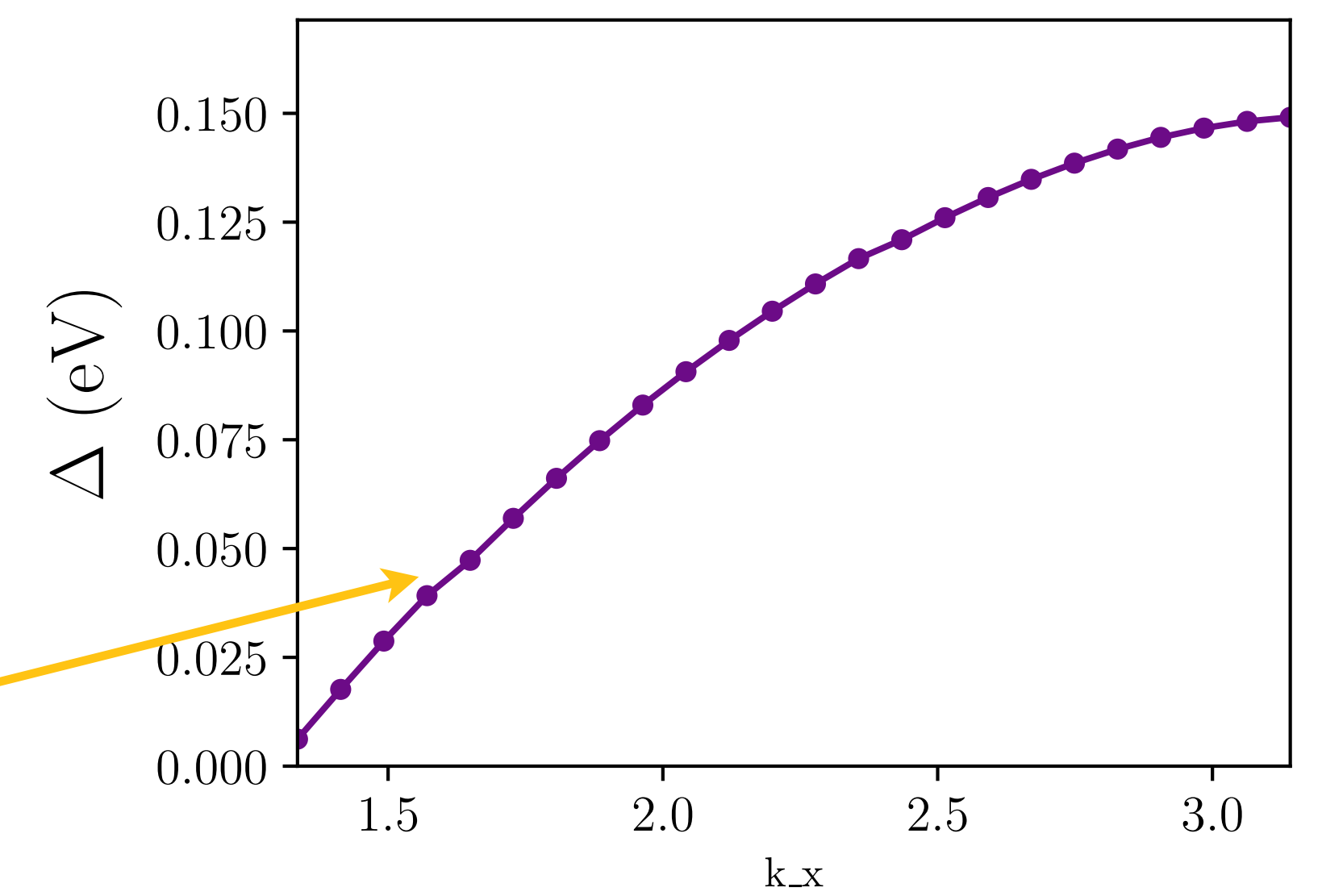


Maine Christos and S.Sachdev, arXiv:2308.03835



dSC

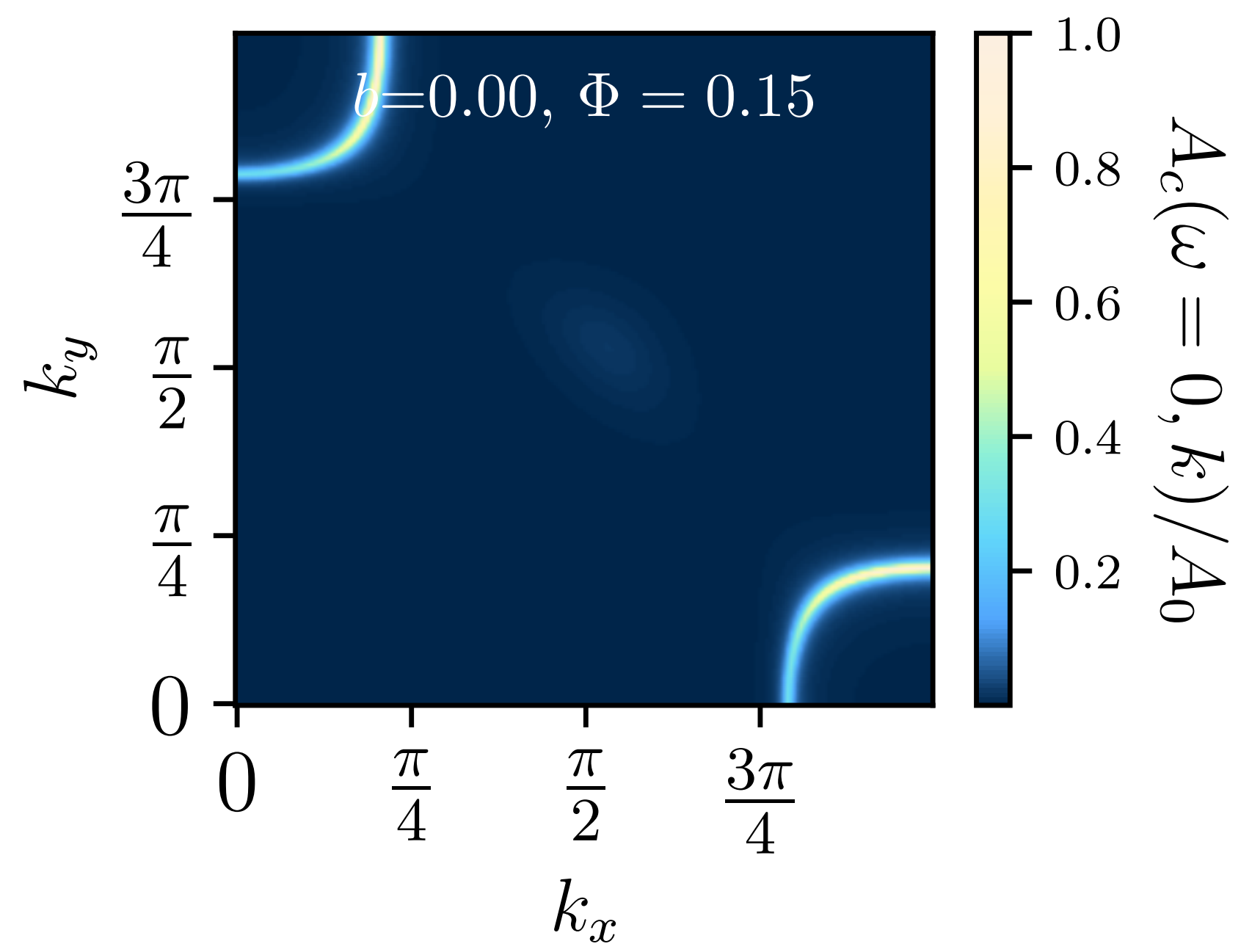
Dirac node with anisotropic velocities



Electron spectral density in electron-doped cuprates

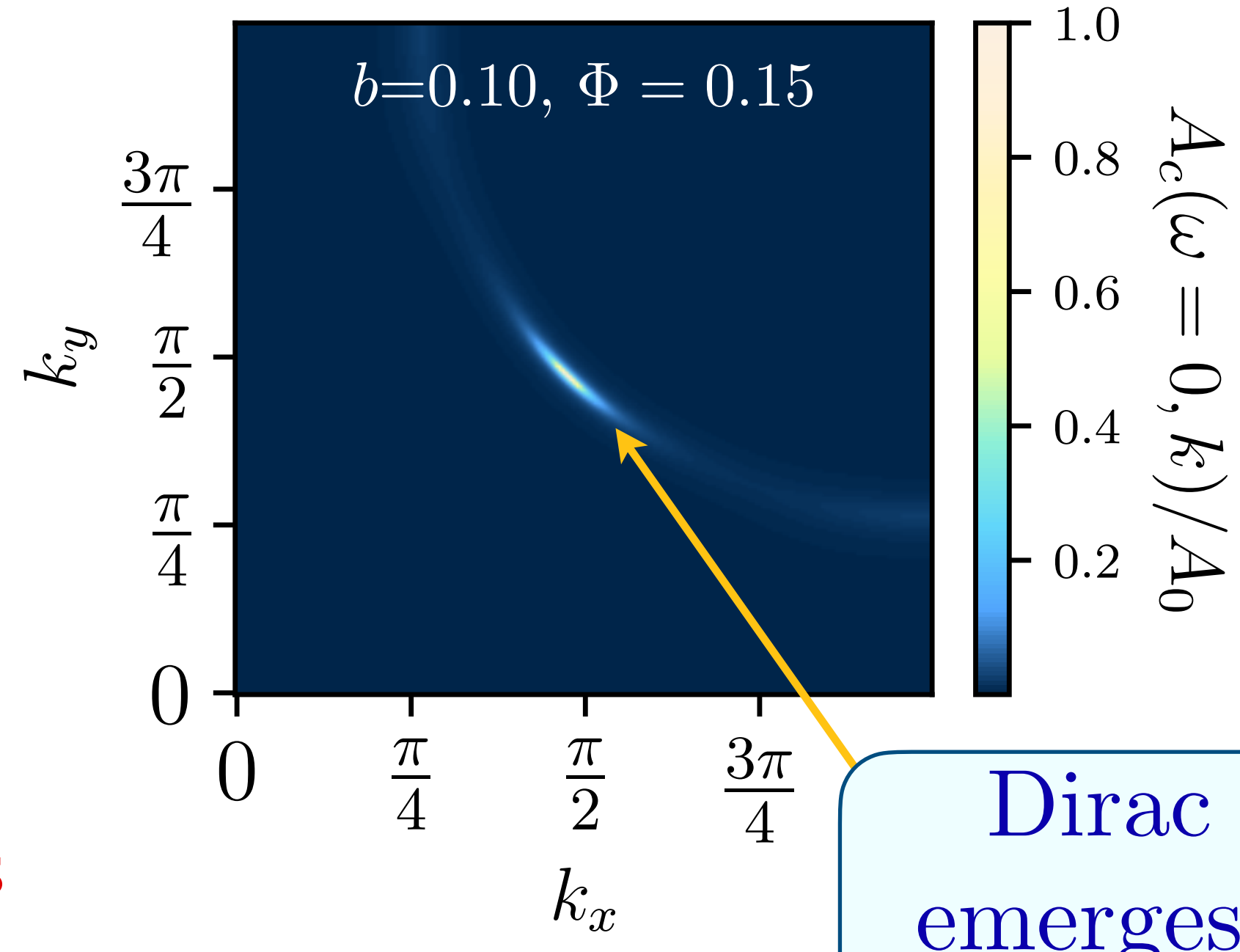


Maine Christos and S.Sachdev,
arXiv:2308.03835



FL*

d-wave superconductor obtained by condensing charge-*e*, SU(2) fundamental boson.

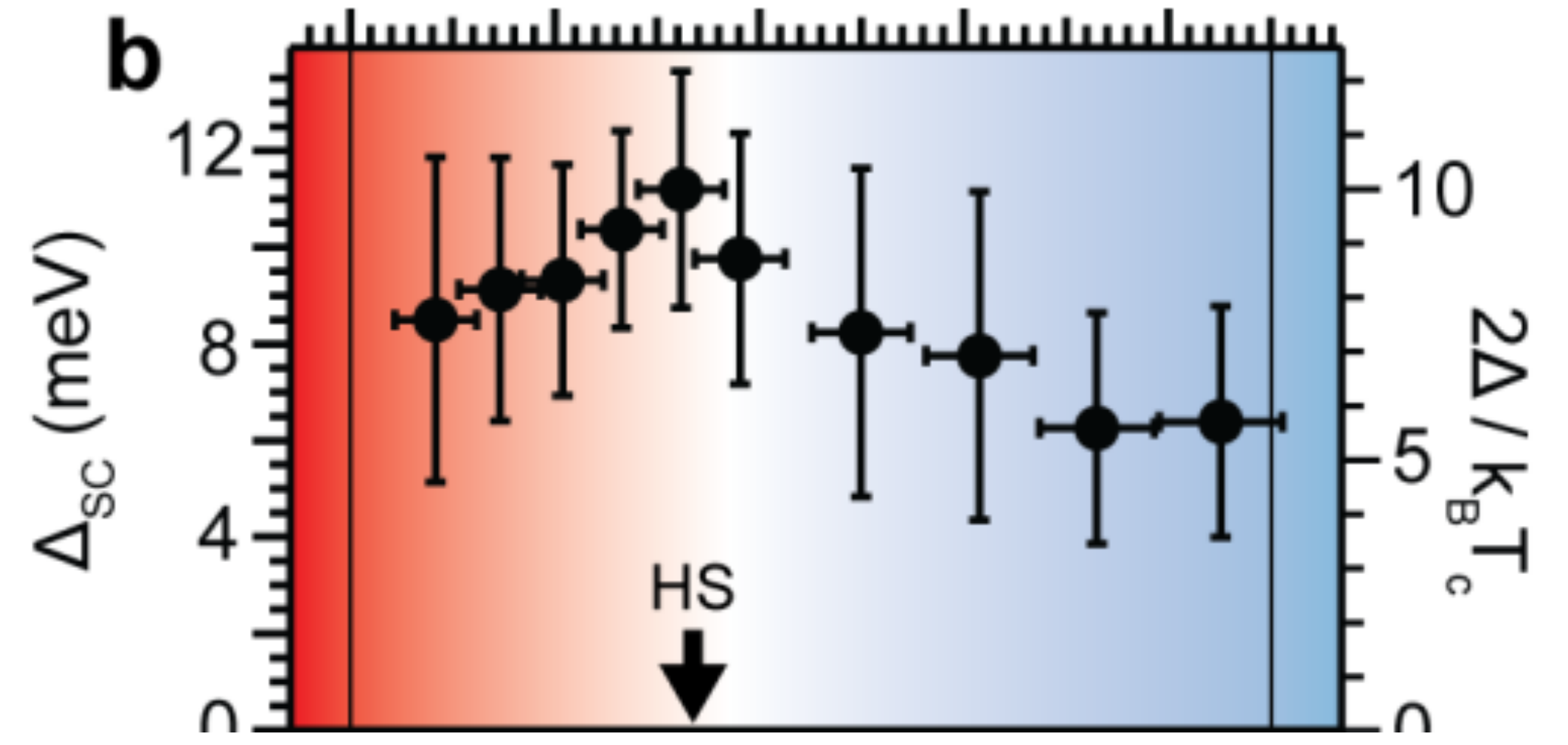
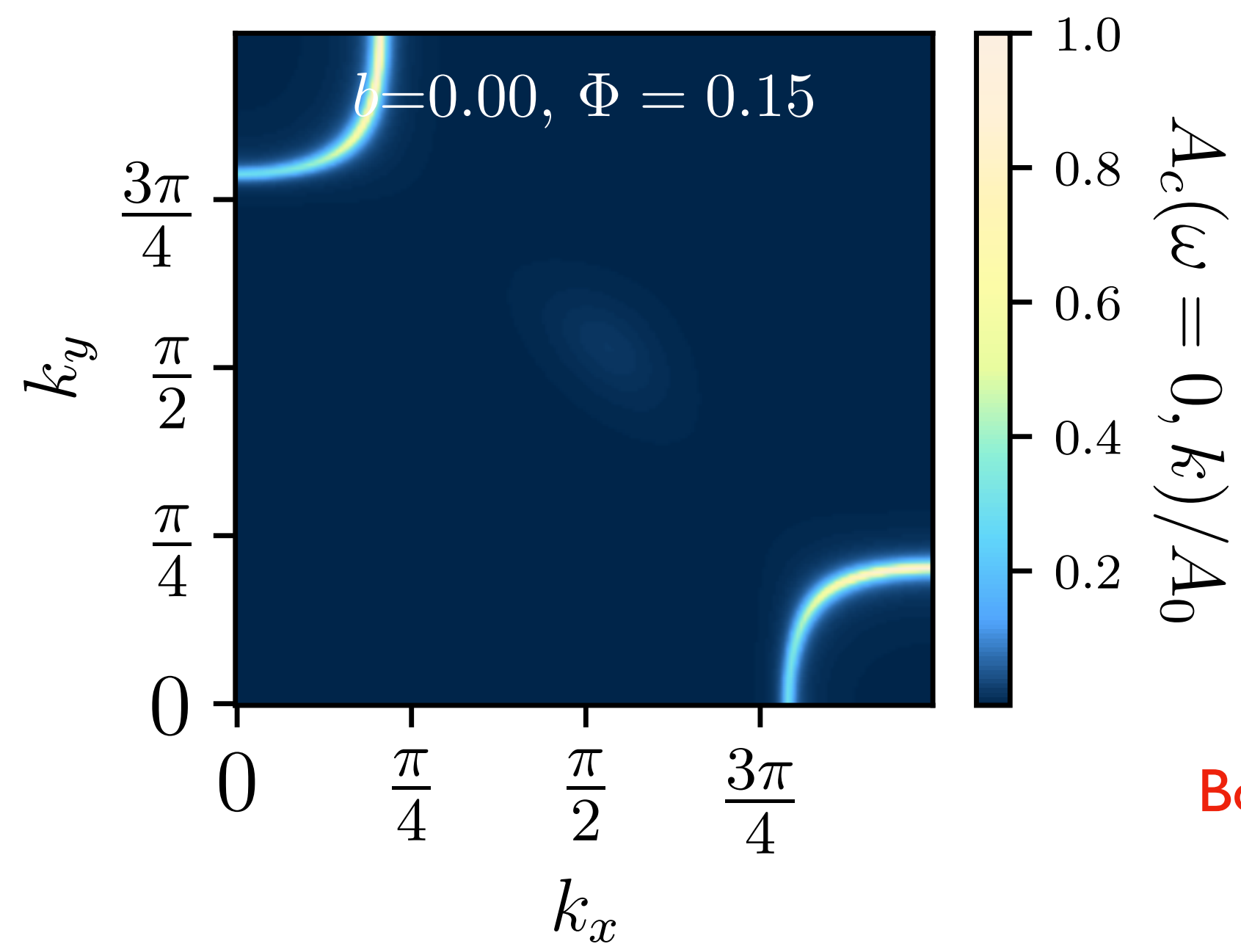


dSC

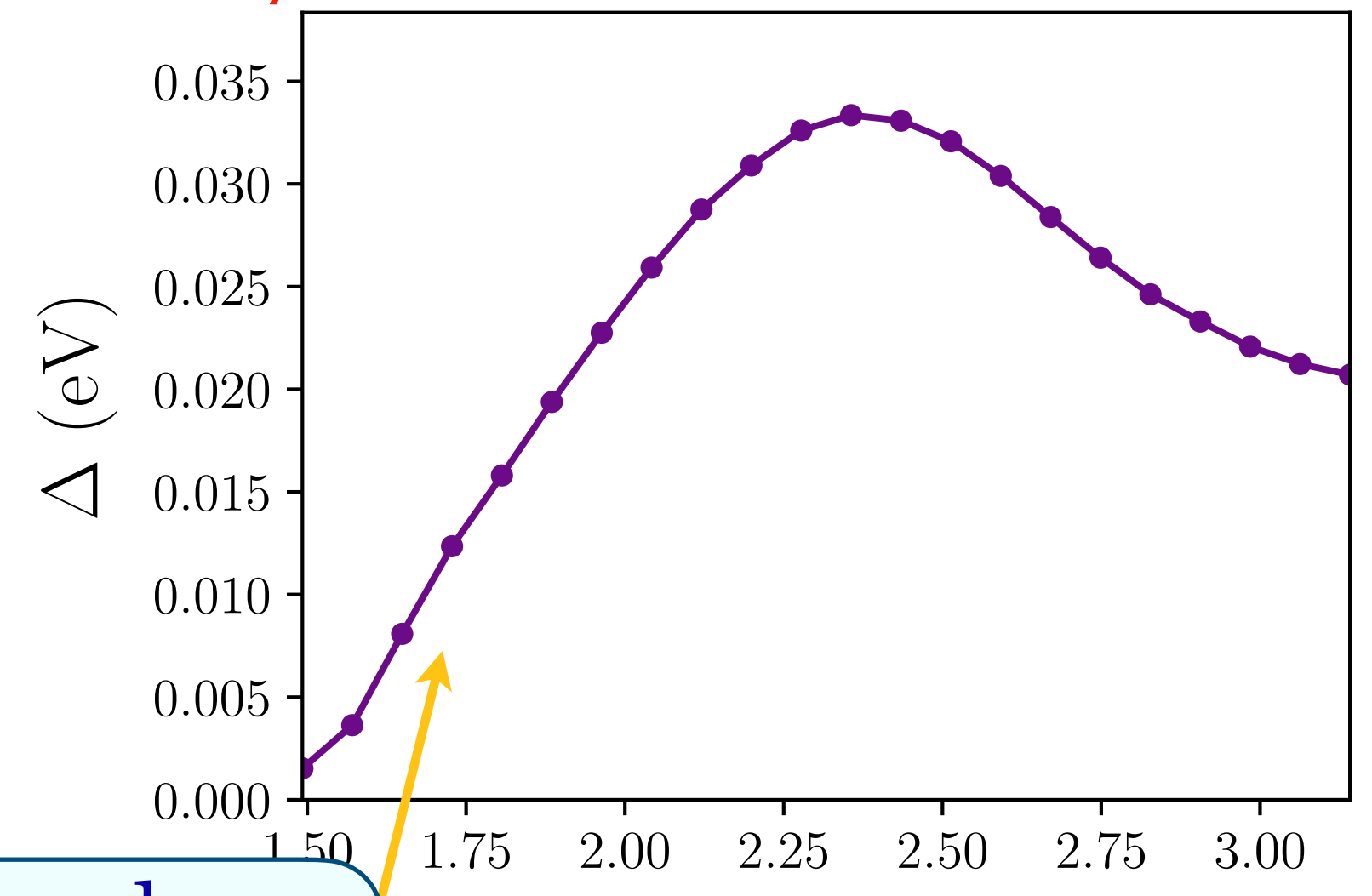
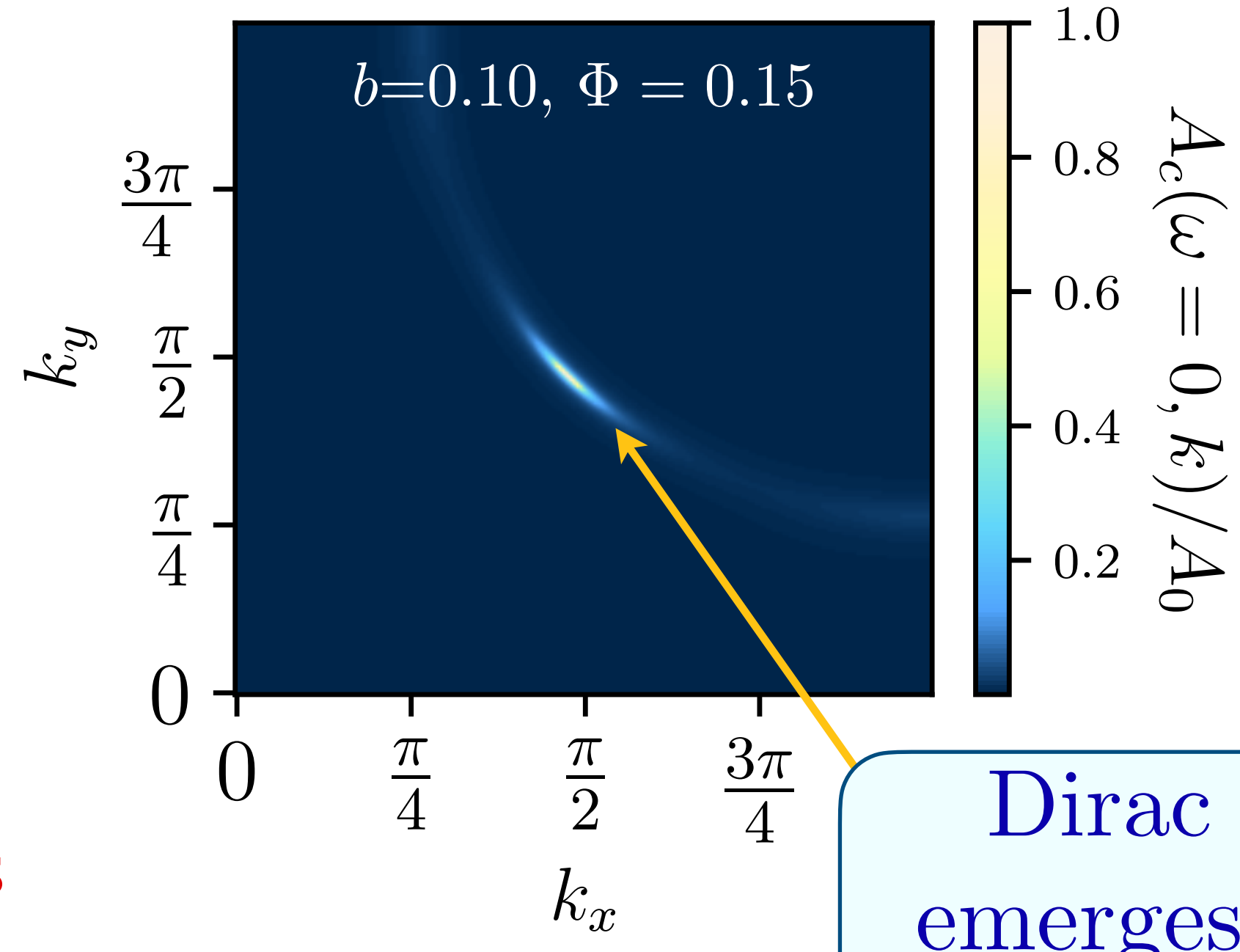
Nodes of the *d*-wave superconductor are remnants of the spinons of the π -flux state.

Dirac node emerges inside normal state gap

Electron spectral density in electron-doped cuprates



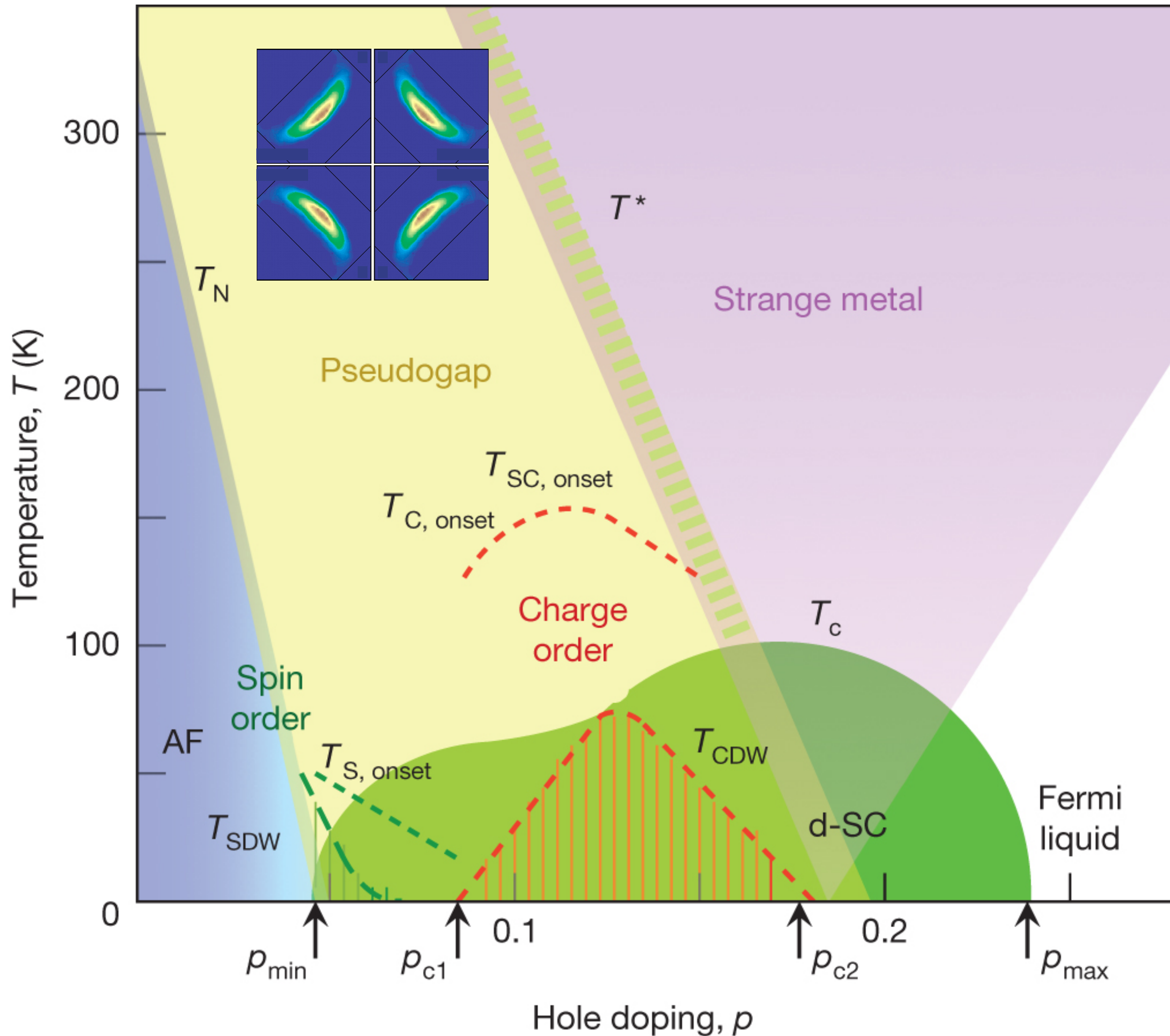
Bogoliubov Quasiparticle on the Gossamer Fermi Surface in Electron-Doped Cuprates, Ke-Jun Xu.....Z.-X. Shen, arXiv:2308.05313; Nature Physics



Dirac node emerges inside normal state gap

Maine Christos and S.Sachdev, arXiv:2308.03835

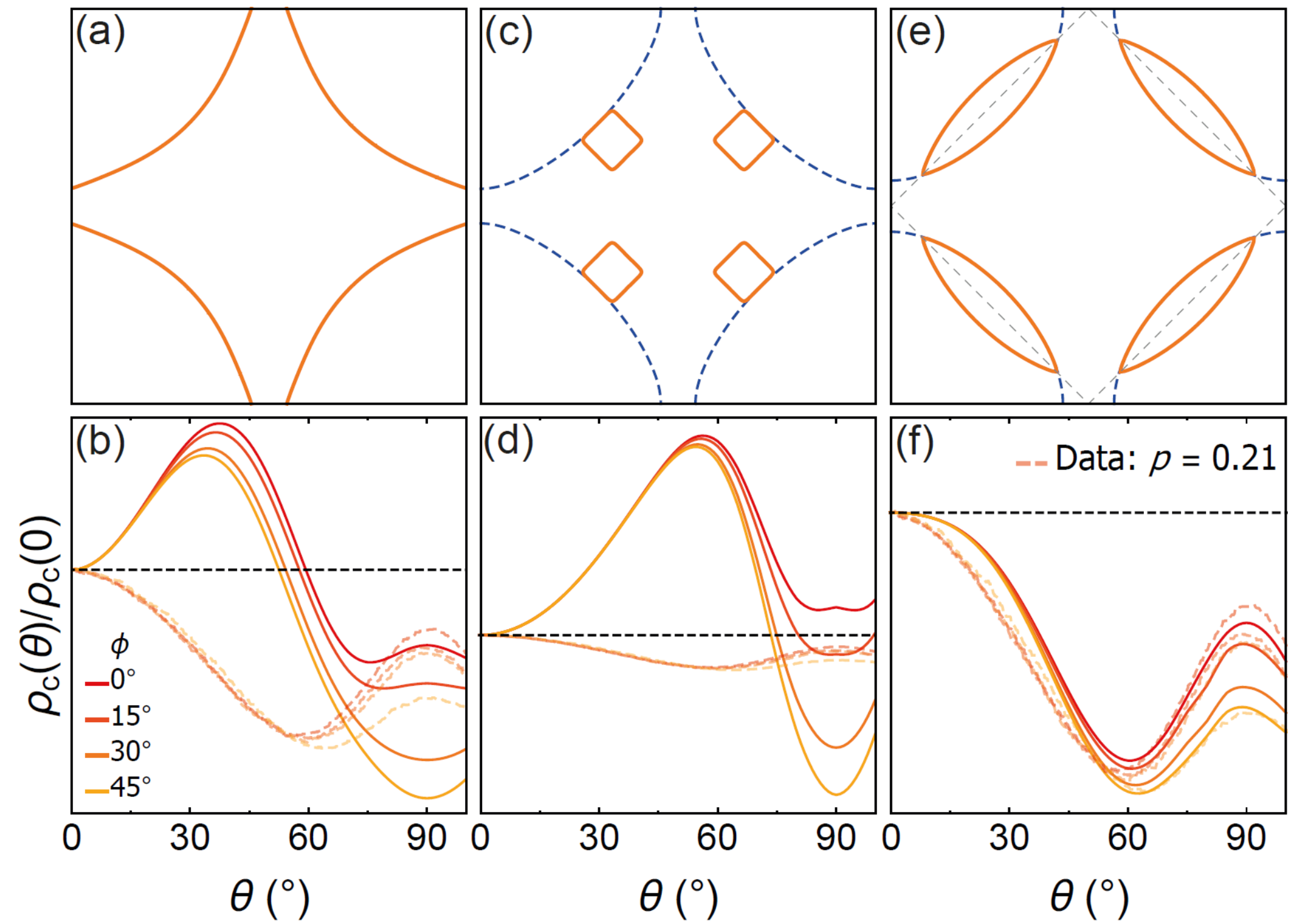
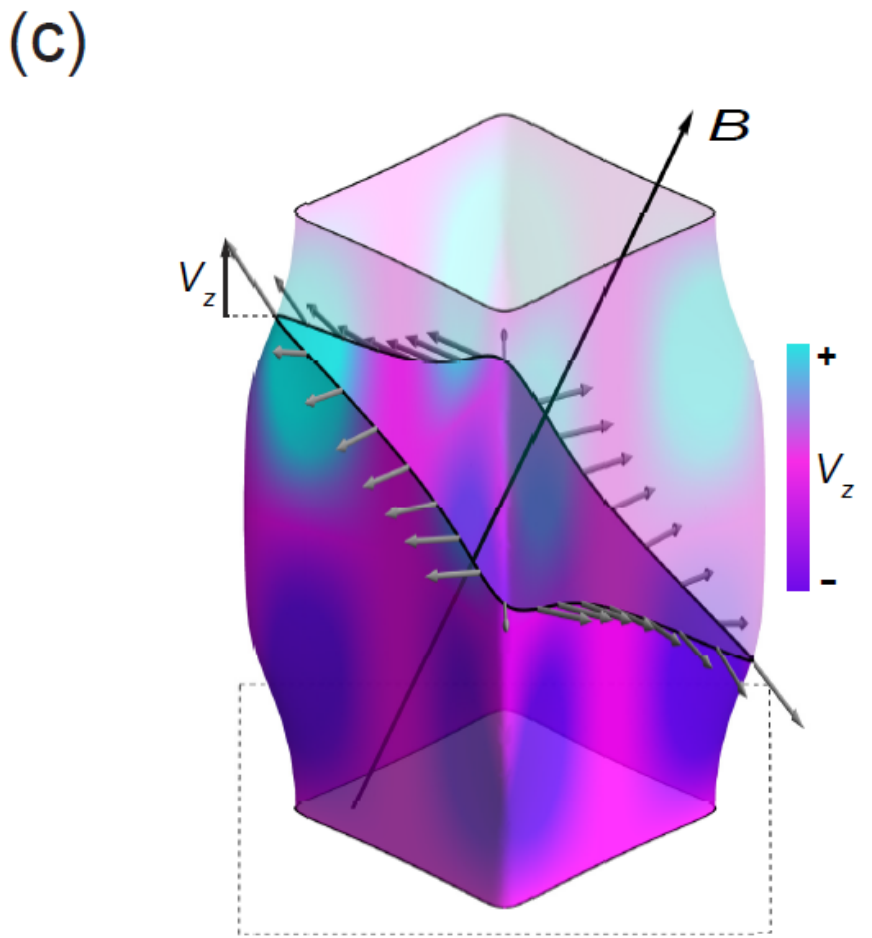
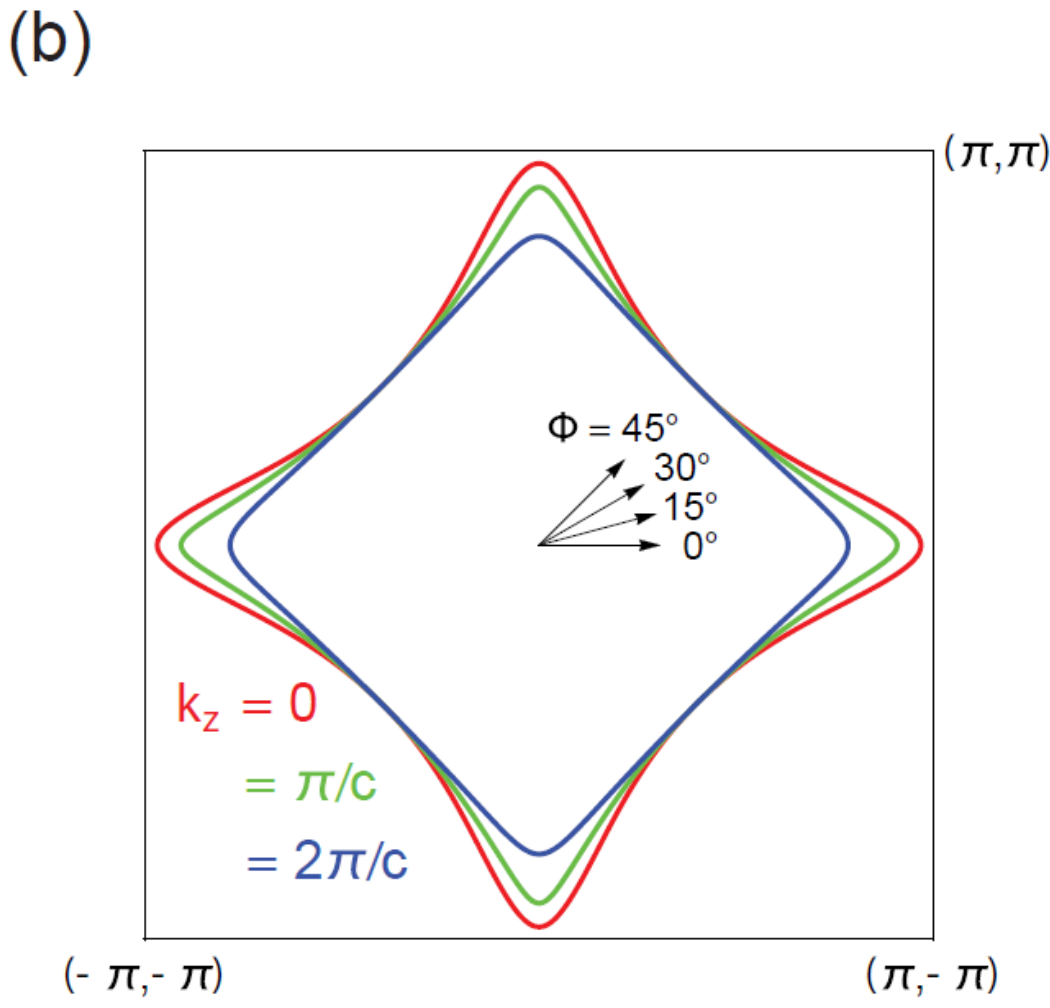
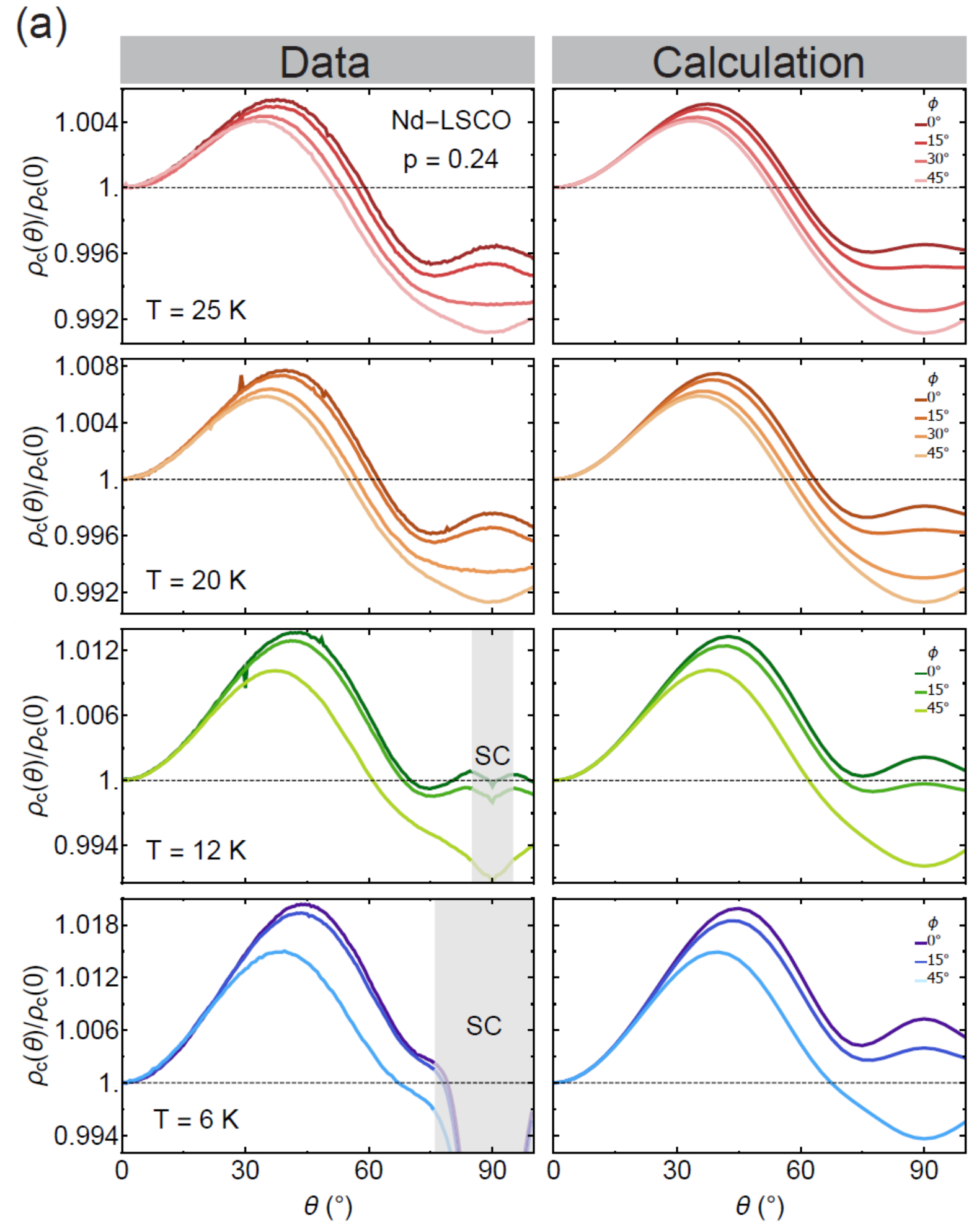
**More connections
to experiments**



Emergent symmetry of the charge- e , $SU(2)$ fundamental boson effective theory of the π -flux state provides a rationale for the near equality of the onset temperatures of d -wave superconductivity and charge order.

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)



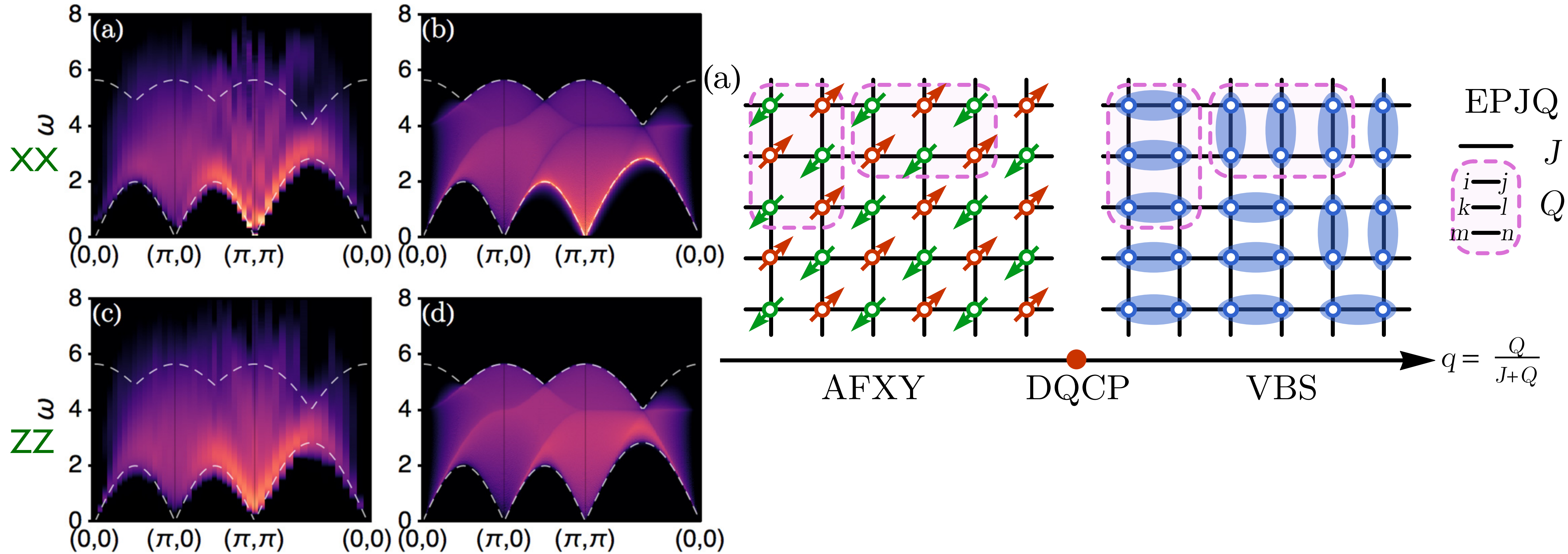
$p > p_c$ Large Fermi surface

$p < p_c$ Reconstructed Fermi surface

Observable by neutron scattering in pseudogap ?

QMC

Free fermion
spinons in π -flux



Summary

Ancilla theory of pseudogap metal with hole pockets and underlying π -flux spin liquid yields:

- Theory for Fermi arcs in hole-doped pseudogap metal.
- **ADMR in pseudogap.**
- Anti-nodal and nodal electronic dispersion.
- ***d*-wave superconductor with 4 nodal points in both electron- and hole-doped cuprates.**
- Near-equality of dSC and charge order onset temperatures
- **Multipoint correlators measured by cold atom experiments**
- Theory for strange metal in the crossover from FL* to FL