Yukawa-SYK models and a universal theory of strange metals

Conformal field theory and quantum many-body physics
Centre de Recherches Mathematiques, University de Montreal
Aug 25, 2022

Subir Sachdev

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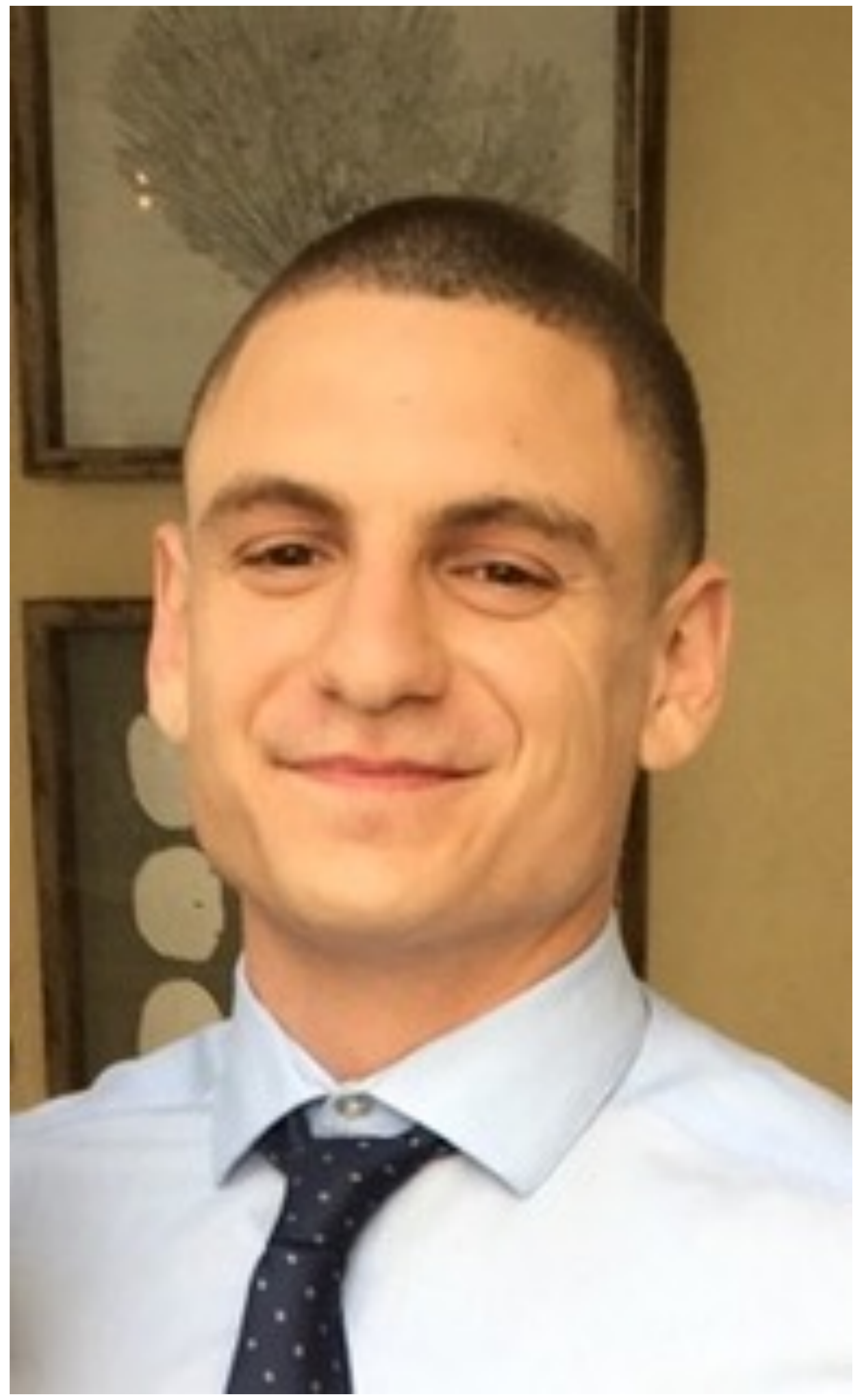
Talk online: sachdev.physics.harvard.edu





Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Harvard

Ilya Esterlis
Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

A solvable quantum many body system without quasiparticle excitations

Yukawa-SYK models

Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_{\ell} \frac{1}{2} \left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2 \right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

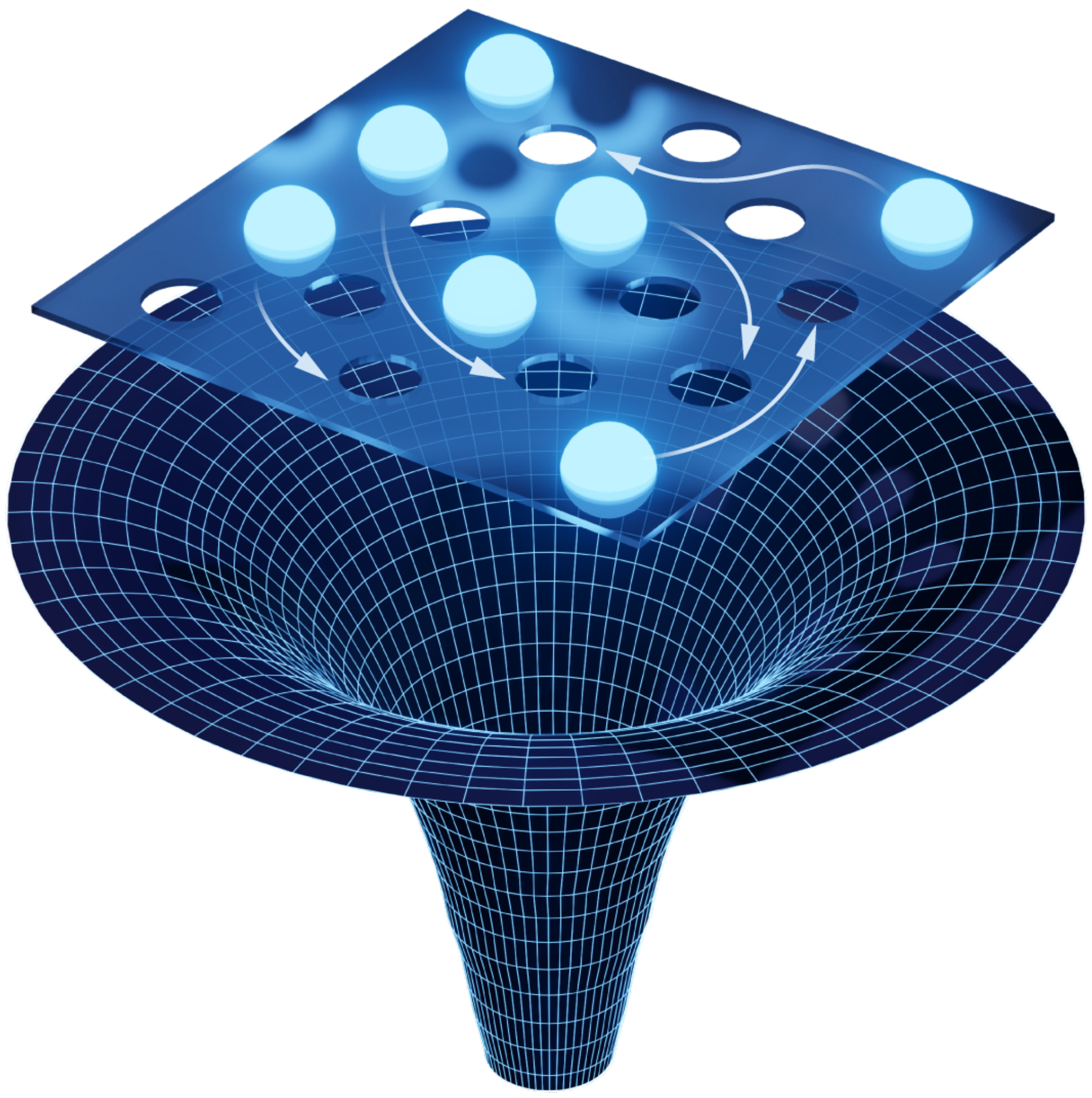
- Emergent conformal invariance in 0+1 dimensions at low energies

- There is a time reparameterization soft mode (associated with local conformal symmetry) which becomes strongly coupled at $\omega \sim 1/N$ and breaks conformal symmetry.

- The SL(2,R) global conformal symmetry is identical to the isometry group of AdS_2, and so there is a mapping to black holes with $\mathrm{AdS}_2 \times S^d$ near-horizon geometry.

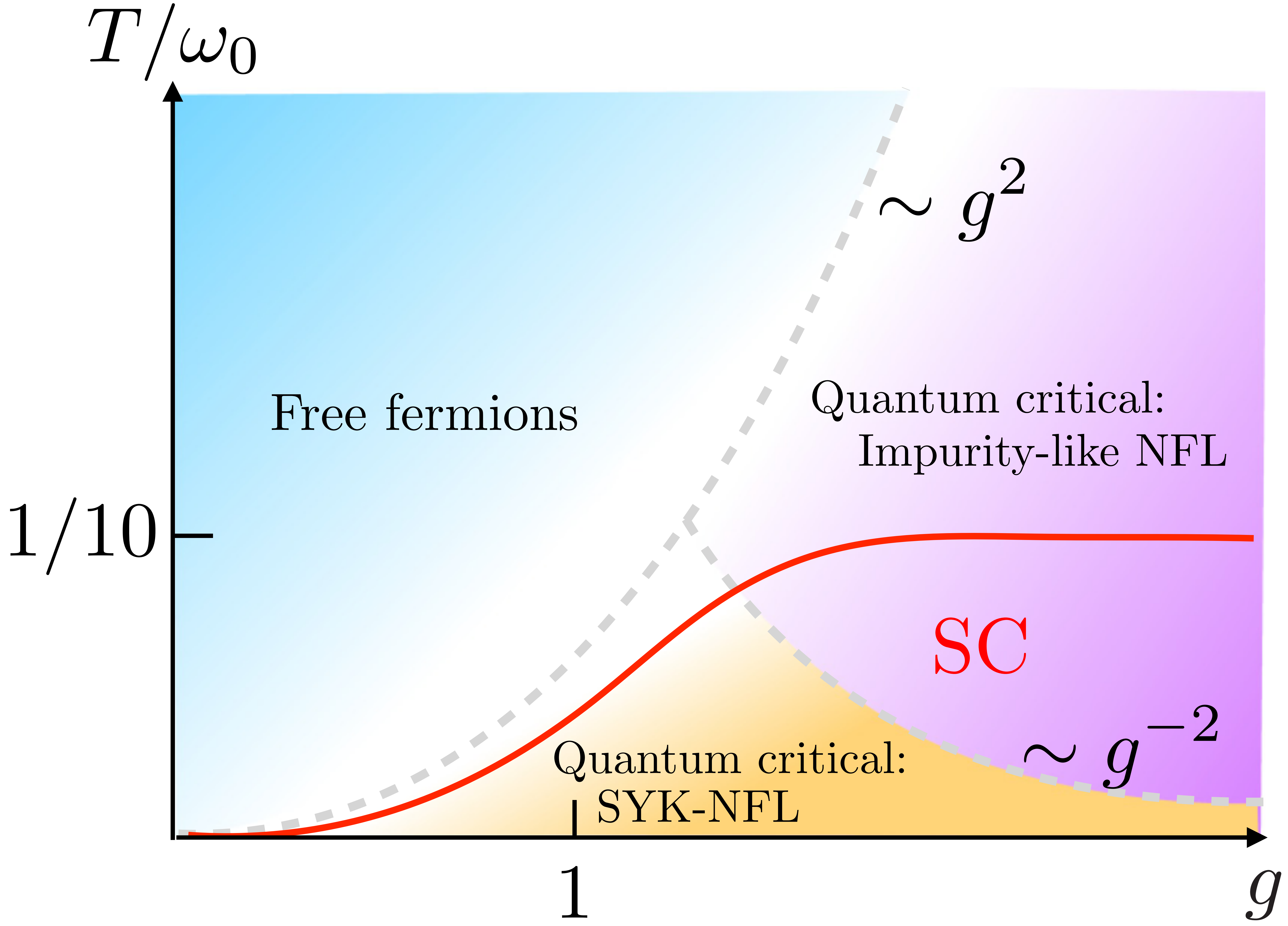
- The time reparameterization soft mode is the *boundary graviton* where the $\mathrm{AdS}_2 \times S^d$ crosses over to some other $d+2$ dimensional geometry.

- The conformal theory and its breaking at $\omega \sim 1/N$ is dual to JT-gravity in $D = 2$.



D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, Reviews of Modern Physics

Yukawa-SYK models



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
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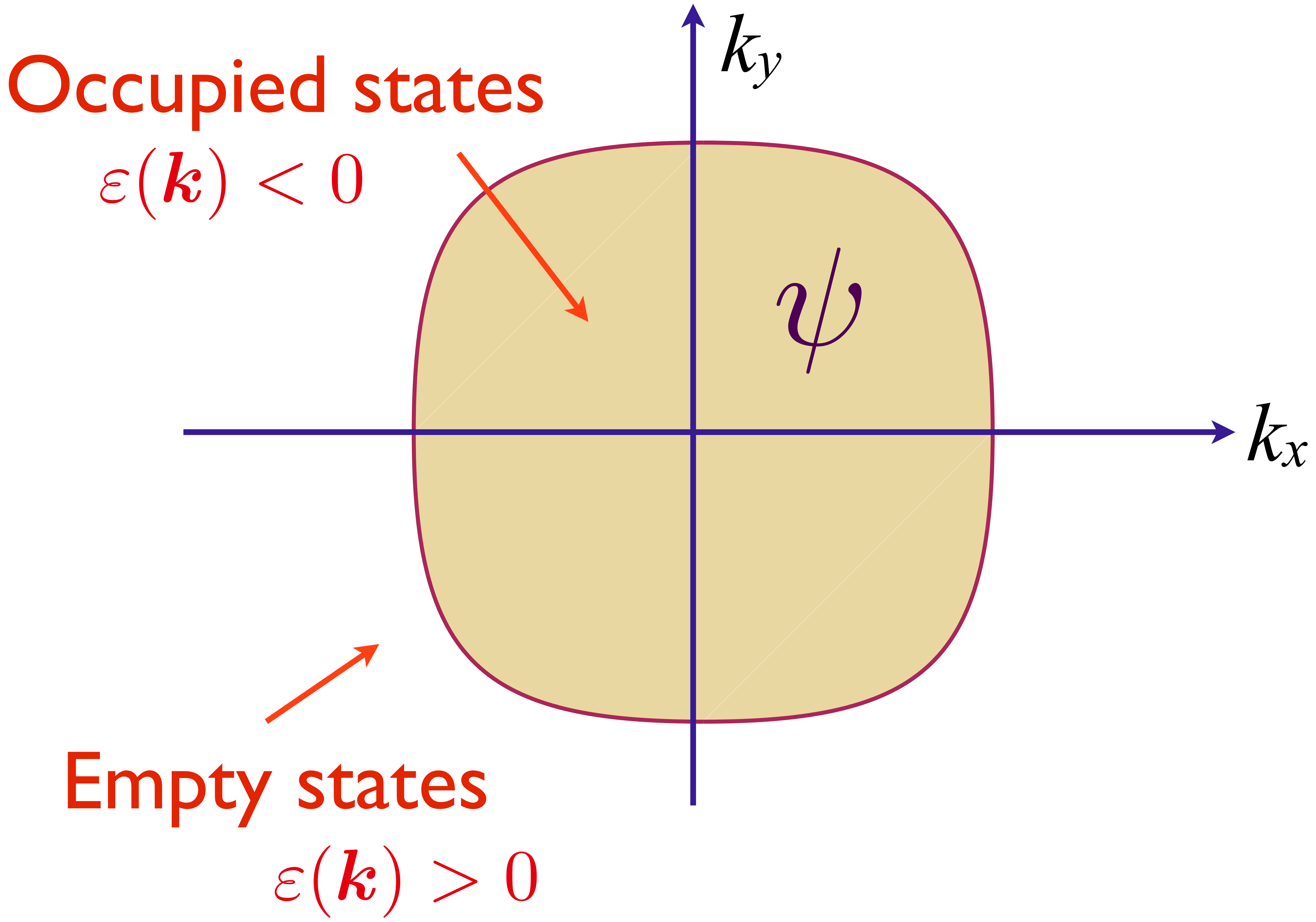
Non-Fermi liquids, Marginal Fermi liquids, and Strange metals

Fermi surface coupled to a critical boson



$+$

a critical boson

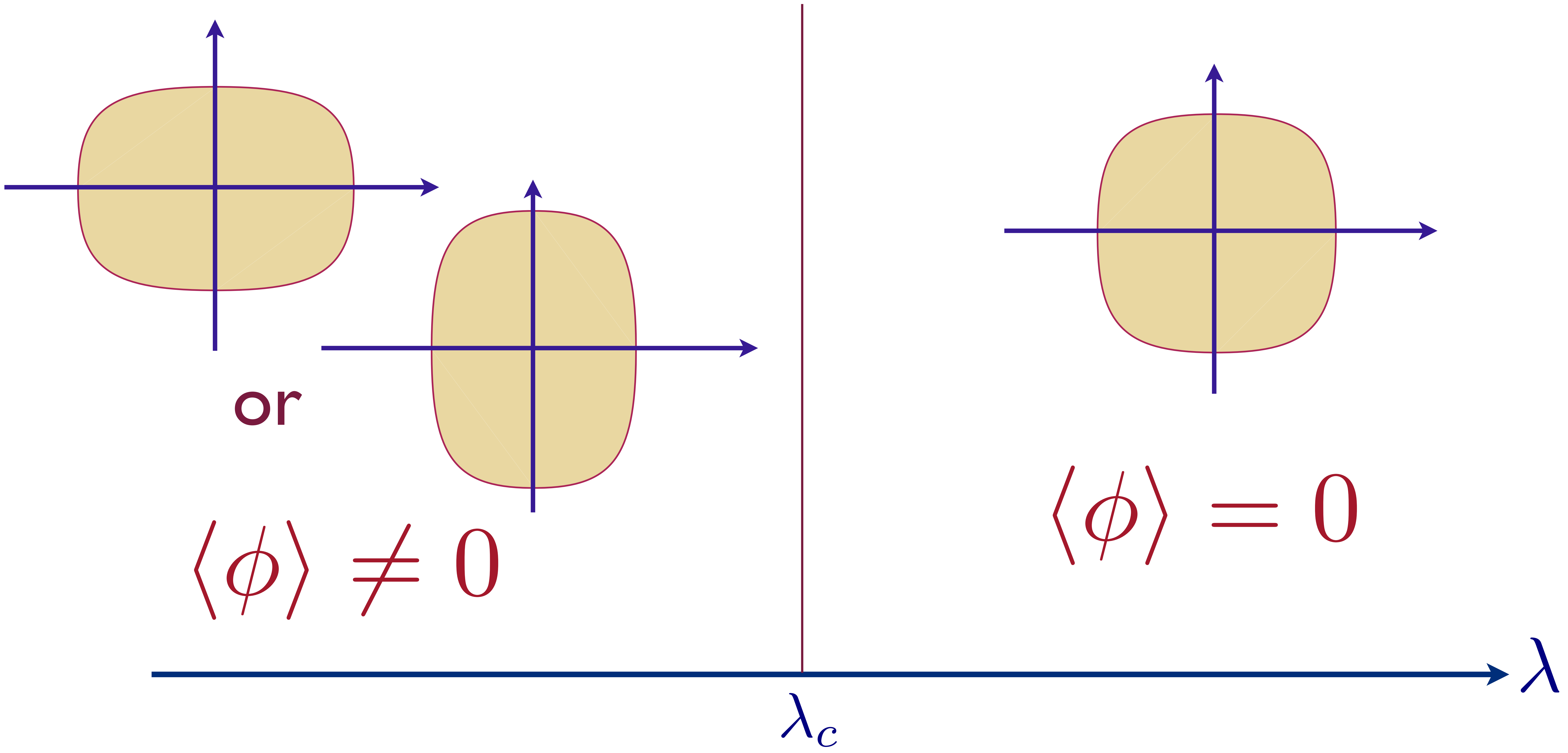
ϕ

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
- Antiferromagnetic order...

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

$$\mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + (\lambda - \lambda_c) \phi^2 \right]$$

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

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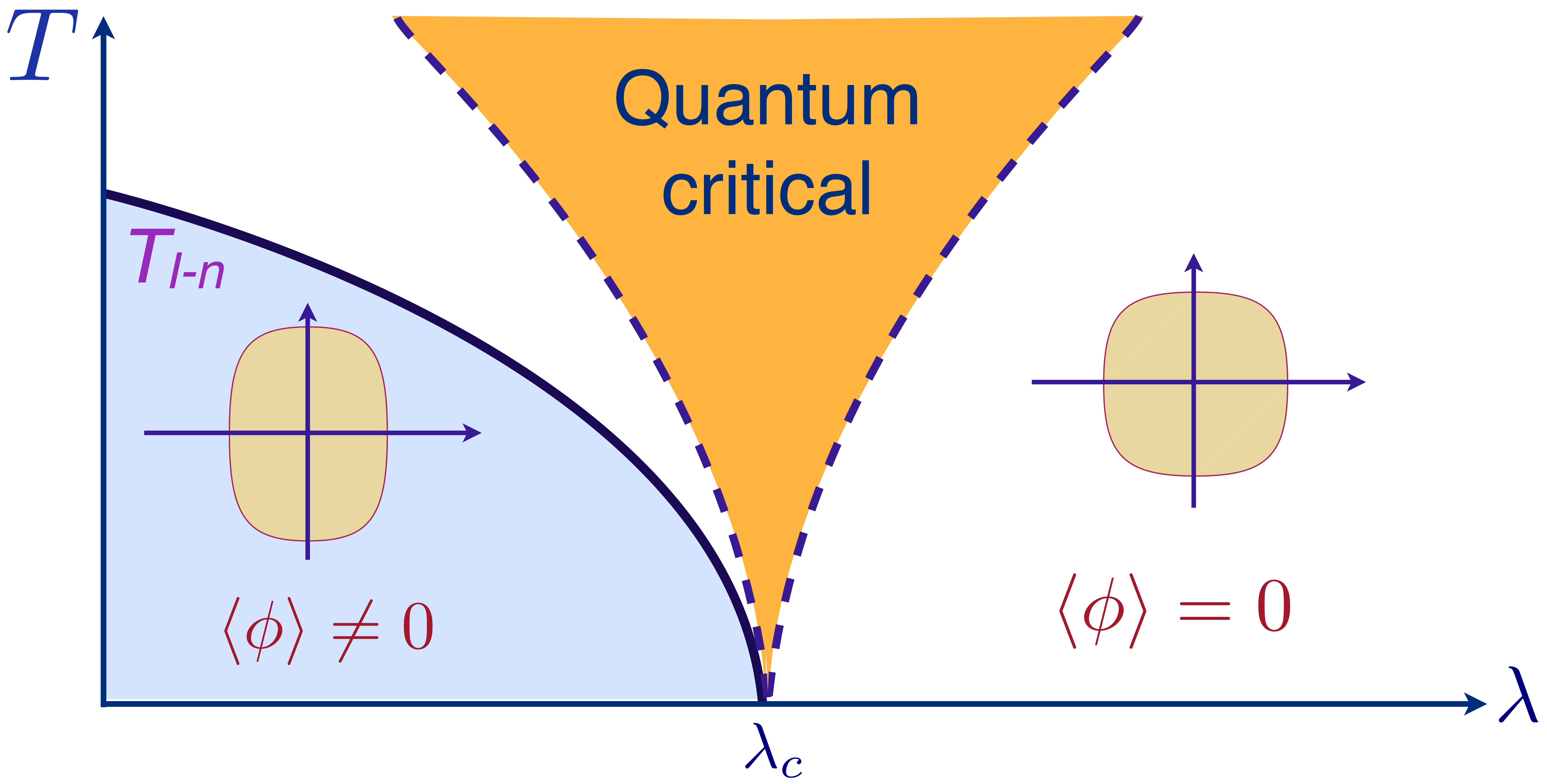

Phase diagram as a function of T and λ

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \qquad \mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + (\lambda - \lambda_c)\phi^2 \right]$$

"Yukawa" coupling: $g \int d^2 r d\tau\, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

Fermi liquids and their cousins: (defined by single-particle properties)

- Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \ll |\varepsilon|$ and a local density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

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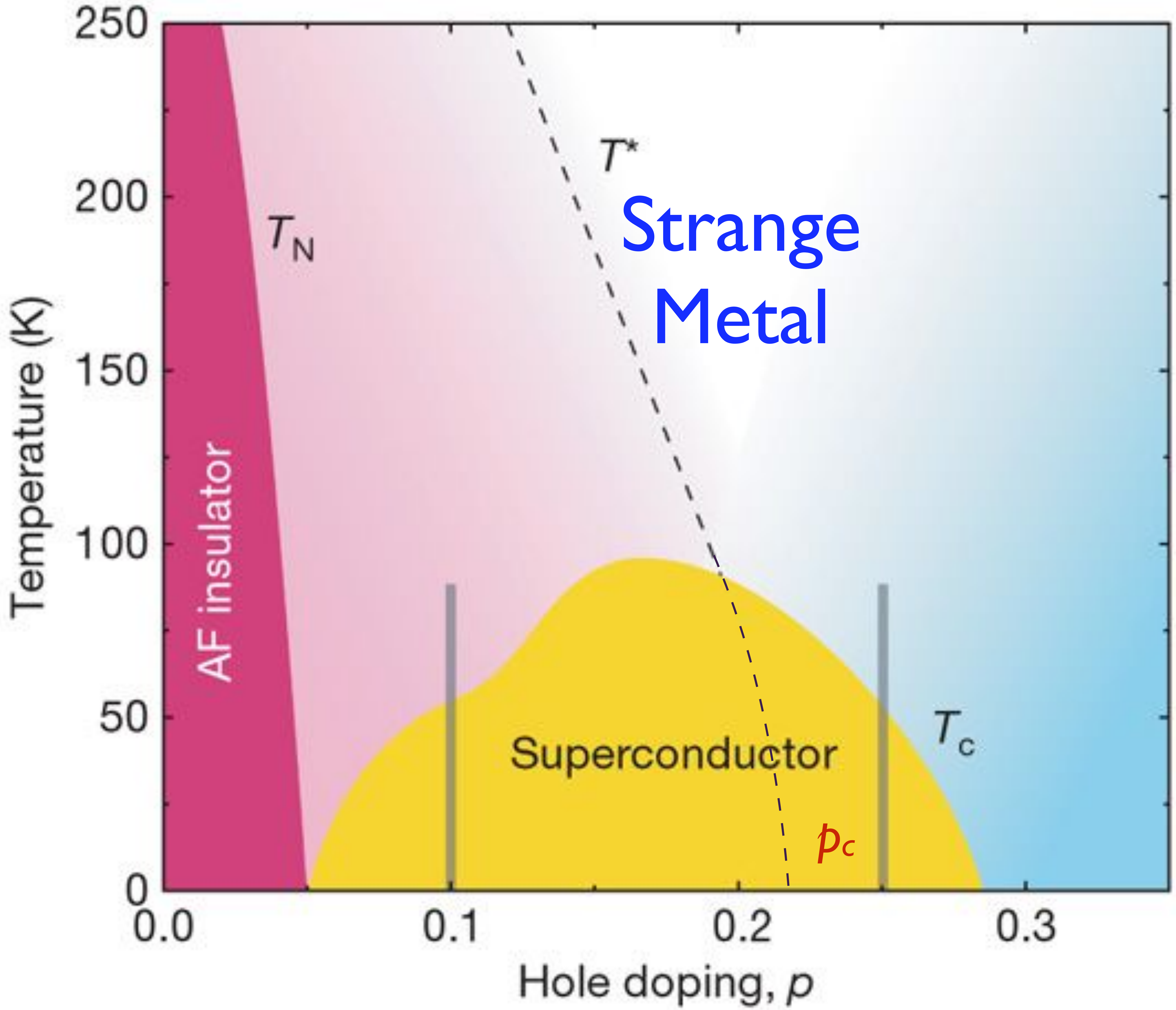
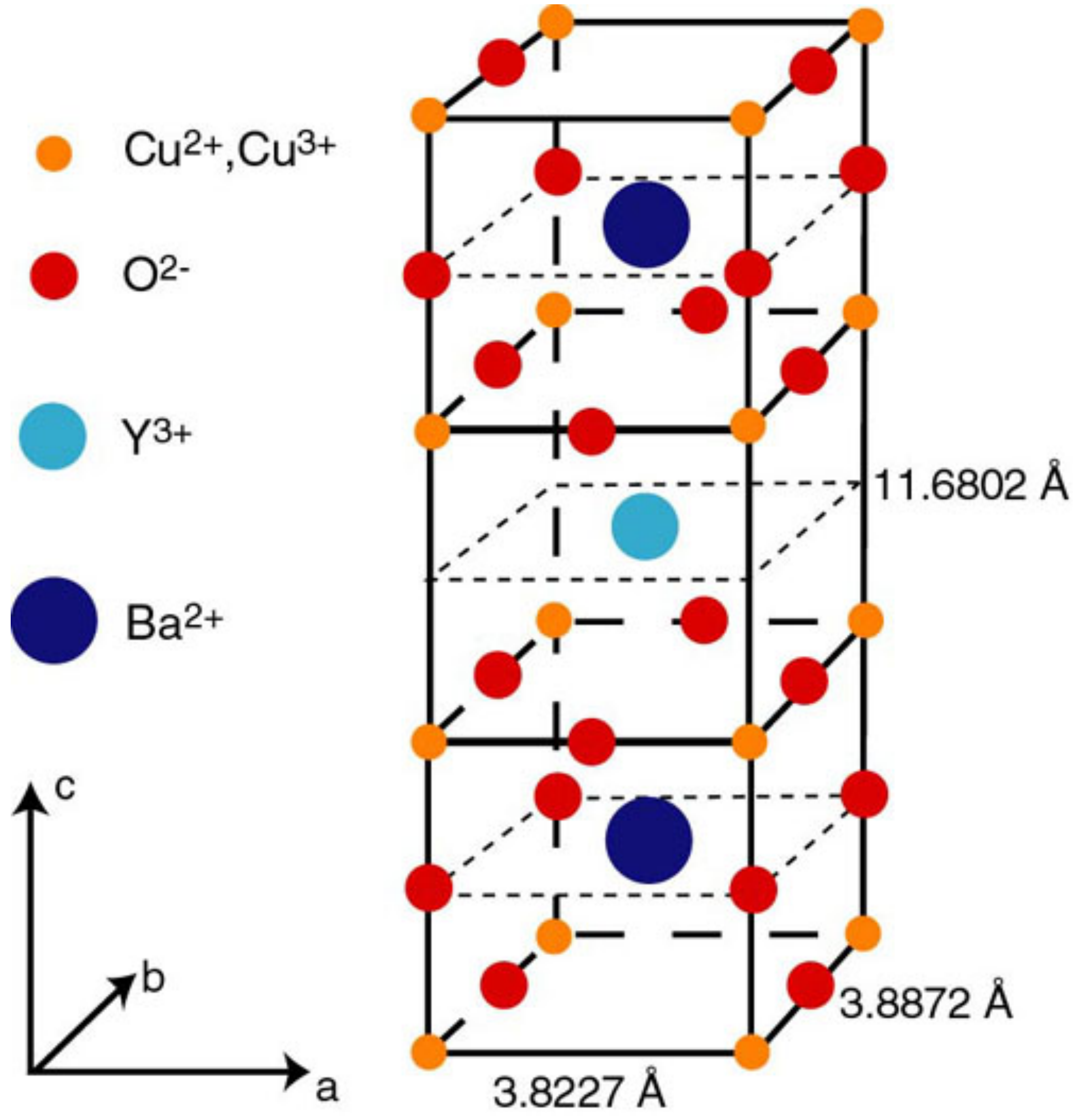
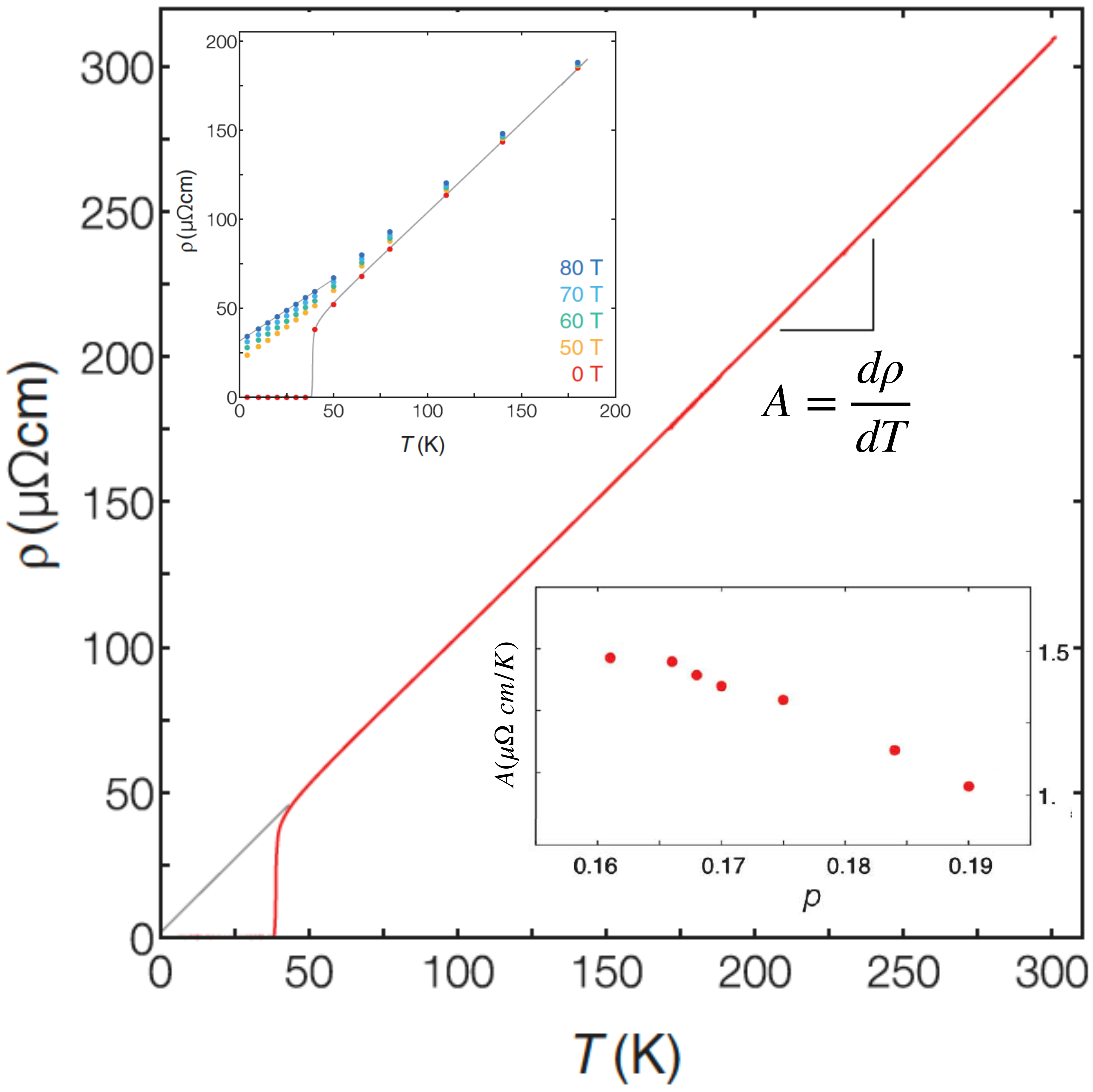
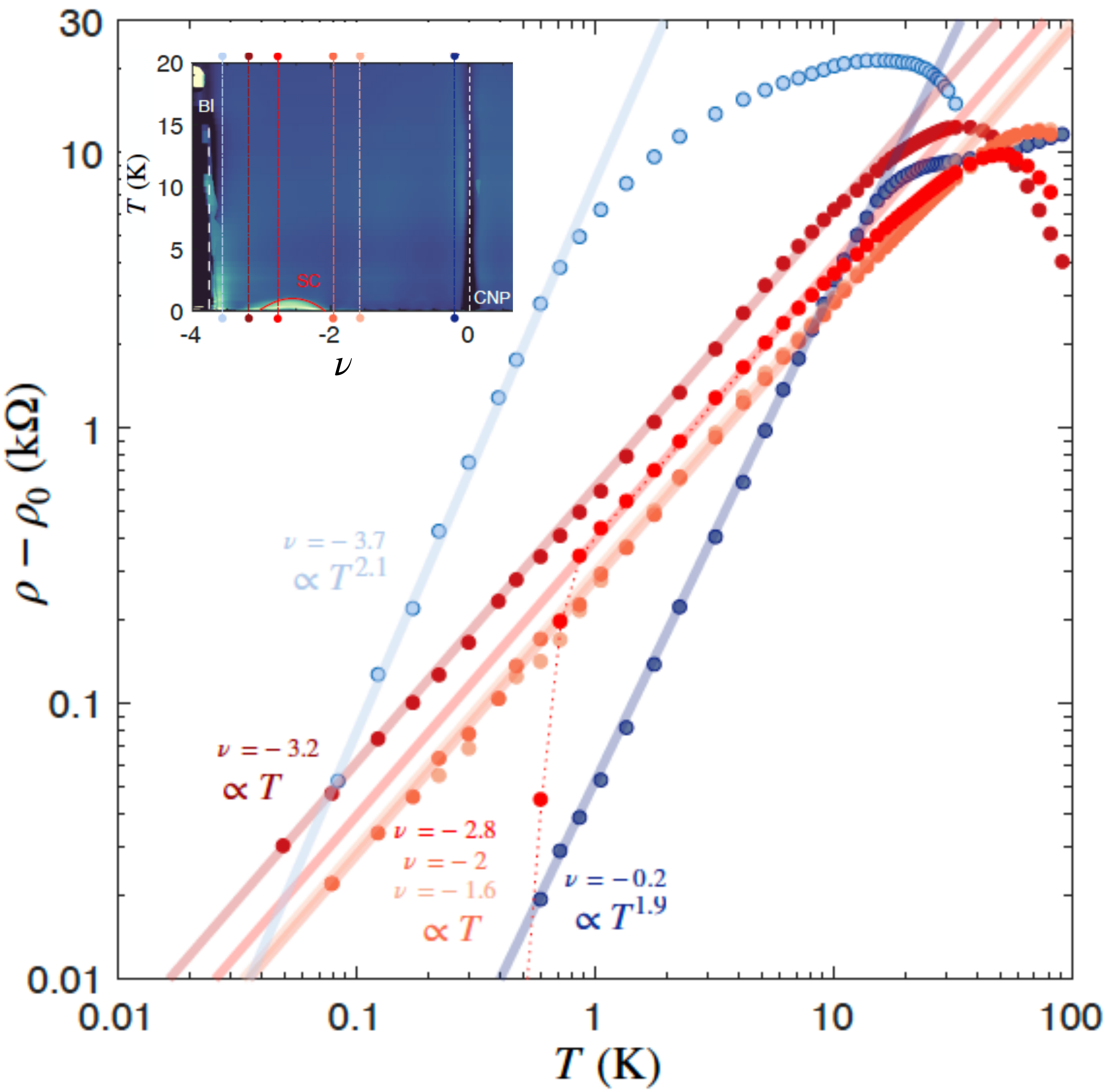
- Marginal Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \sim |\varepsilon|$ and a local density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Properties of a strange metal: (defined by transport and thermo)

- Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
  and $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are bad metals.

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- Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

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- Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} = \frac{k_B T}{\hbar} G\left(\frac{\hbar\omega}{k_B T}\right)$$

B. Michon……A. Georges, arXiv:2205.04030

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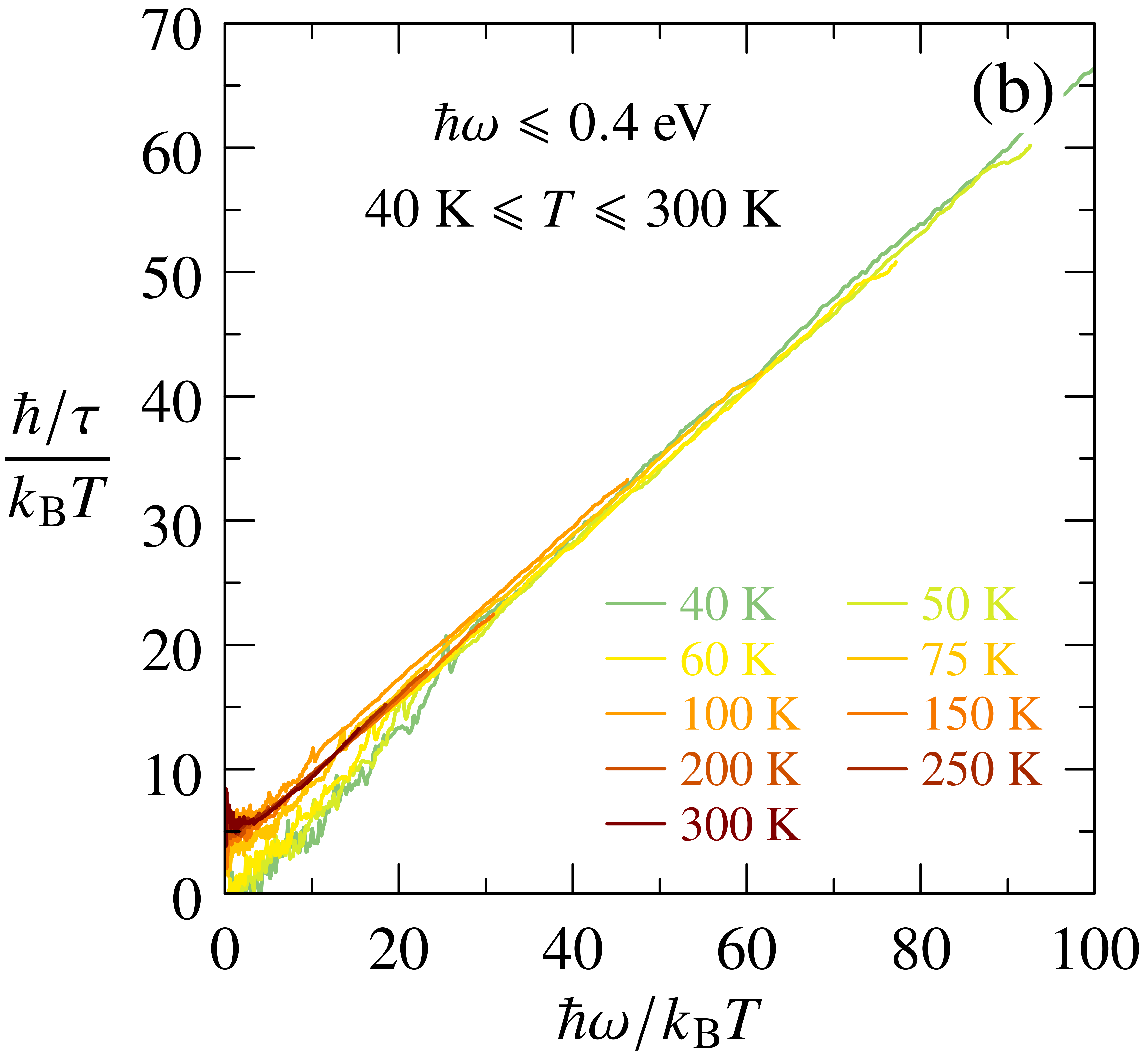

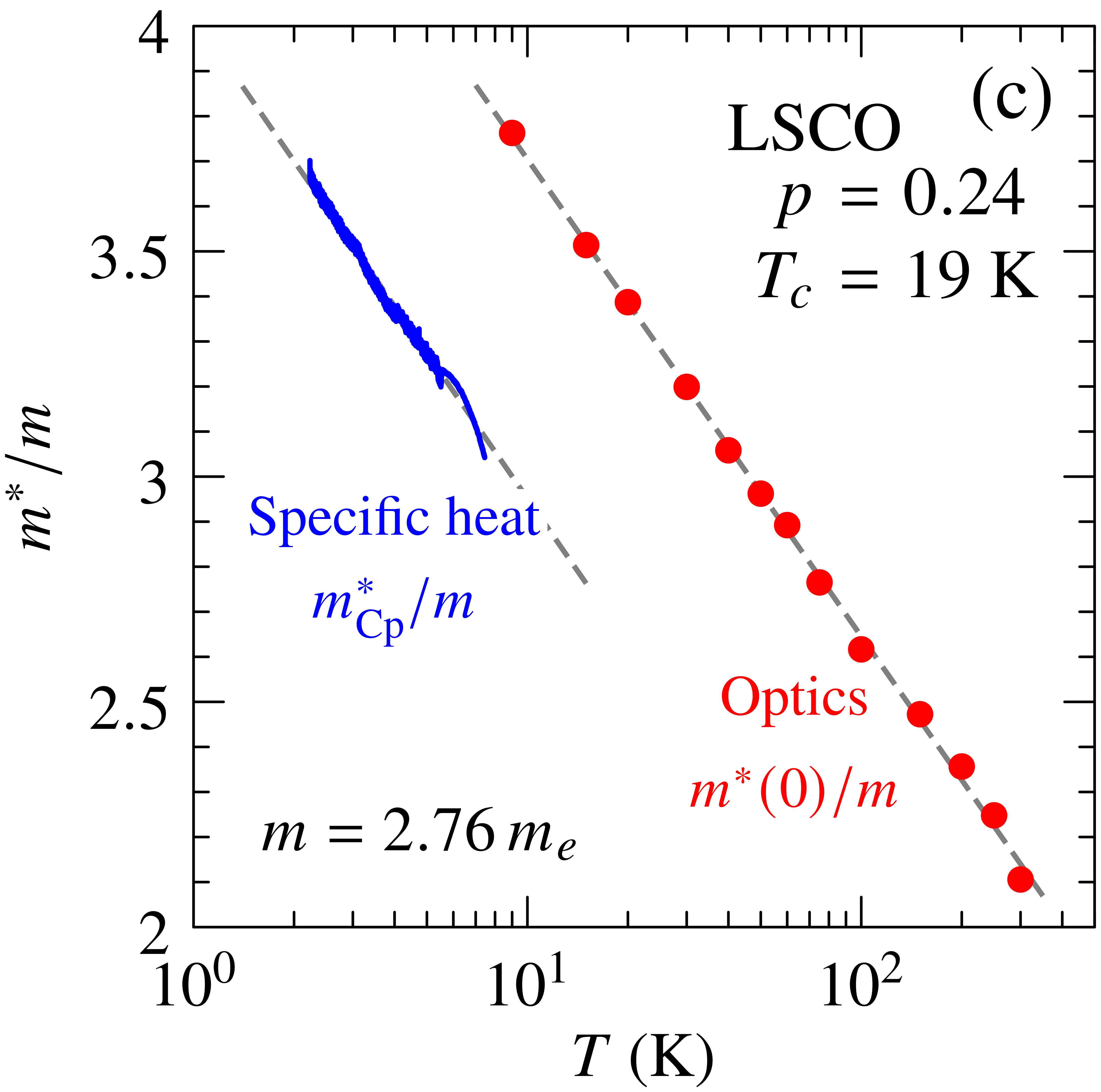

Beyond 0-d "toy" models....

Large-N theories of Non-Fermi liquids, Marginal Fermi liquids, and Strange metals

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson

$$\mathcal{L}_{\psi} = \psi_{\boldsymbol{k}}^{\dagger} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

$$\mathcal{L}_{\phi} = \frac{1}{2} \left[(\partial_{\tau} \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + (\lambda - \lambda_c) \phi^2 \right]$$

"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^{\dagger}(r, \tau) \psi(r, \tau) \phi(r, \tau)$

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$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

<u>Application of Yukawa-SYK approach:</u>
Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory

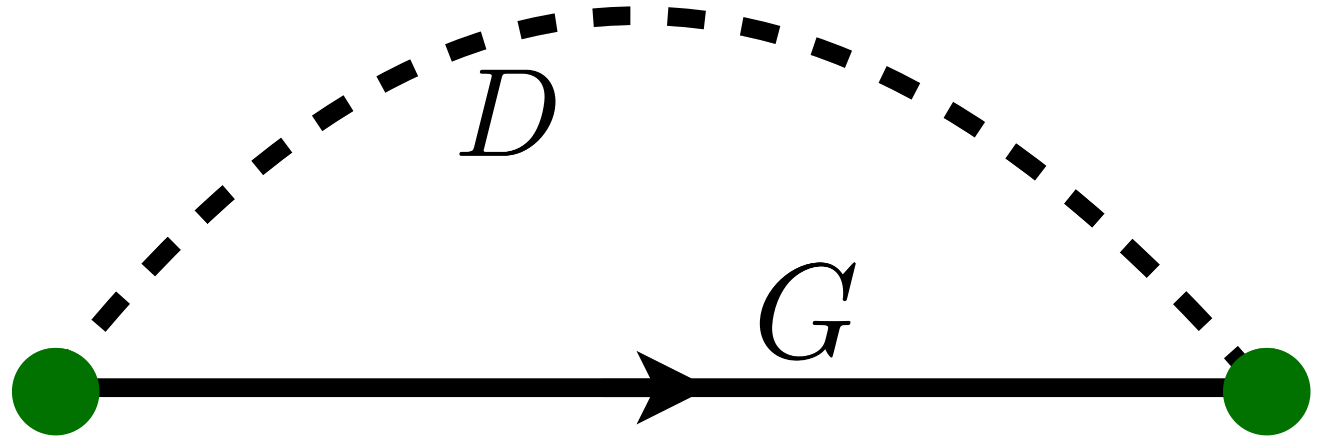
The saddle point equations are

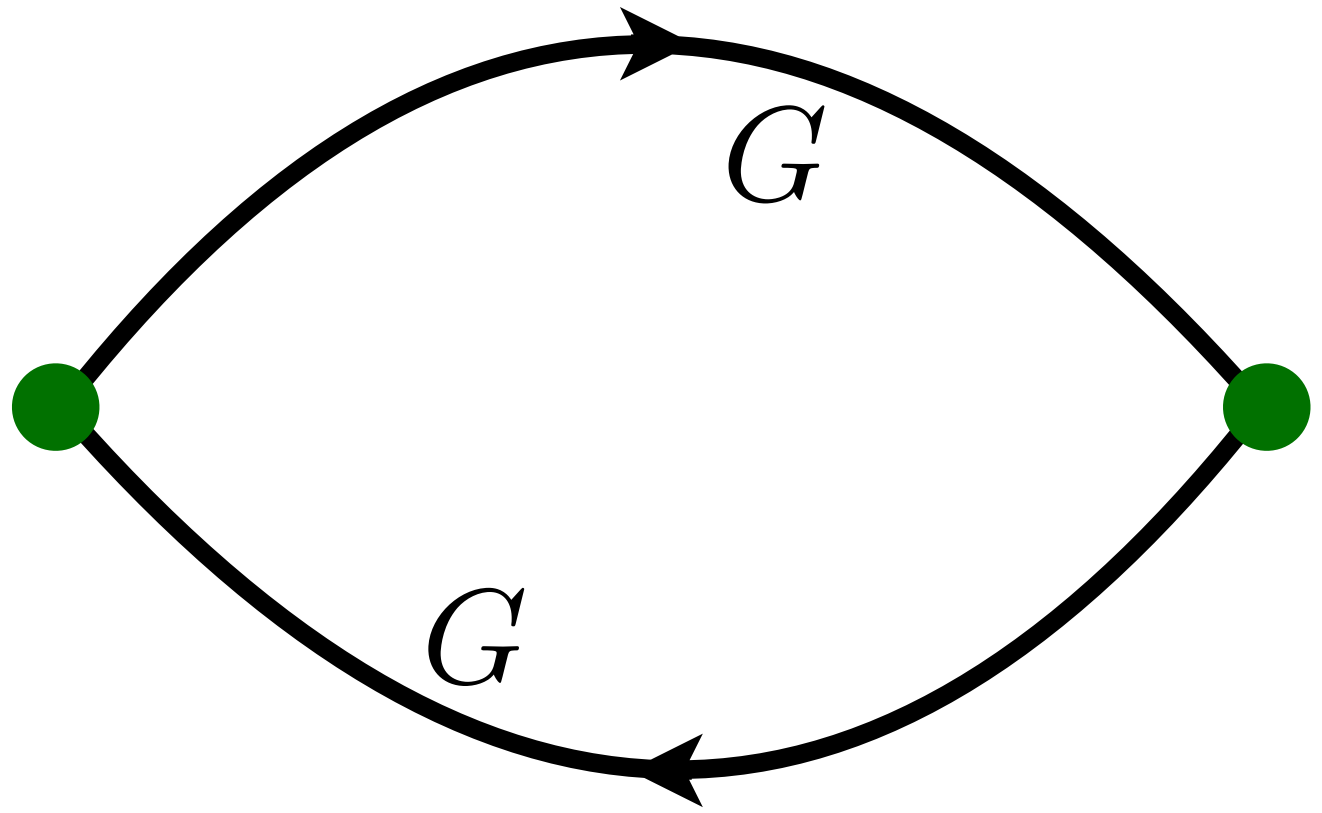
$$\Sigma(\boldsymbol{r},\tau) = g^2 \lambda D(\boldsymbol{r},\tau) G(\boldsymbol{r},\tau),$$

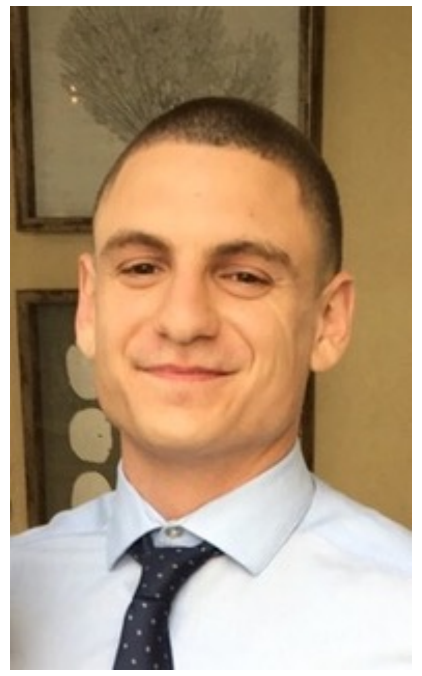
$$\Pi(\boldsymbol{r},\tau) = -g^2 G(-\boldsymbol{r},-\tau) G(\boldsymbol{r},\tau),$$

$$G(\boldsymbol{k}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k}, i\omega_n)},$$

$$D(\boldsymbol{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\boldsymbol{q}, i\Omega_m)}.$$

$\Sigma =$ 

$\Pi =$ 



Exact Solution at small ω:

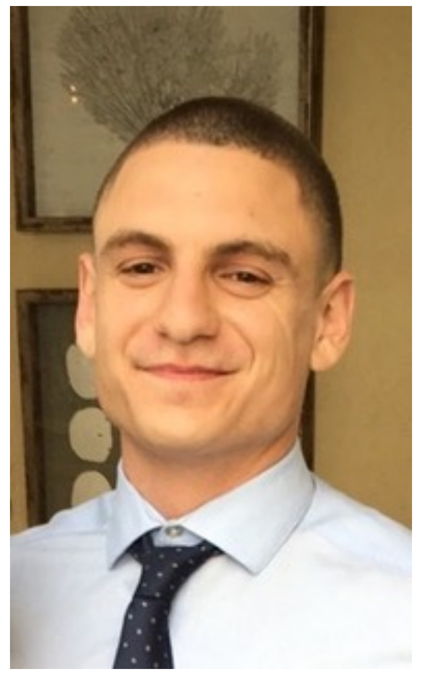
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\text{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{-1}{\varepsilon(\boldsymbol{k}) + \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\boldsymbol{k}}$.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

Fermi surface coupled to a critical boson



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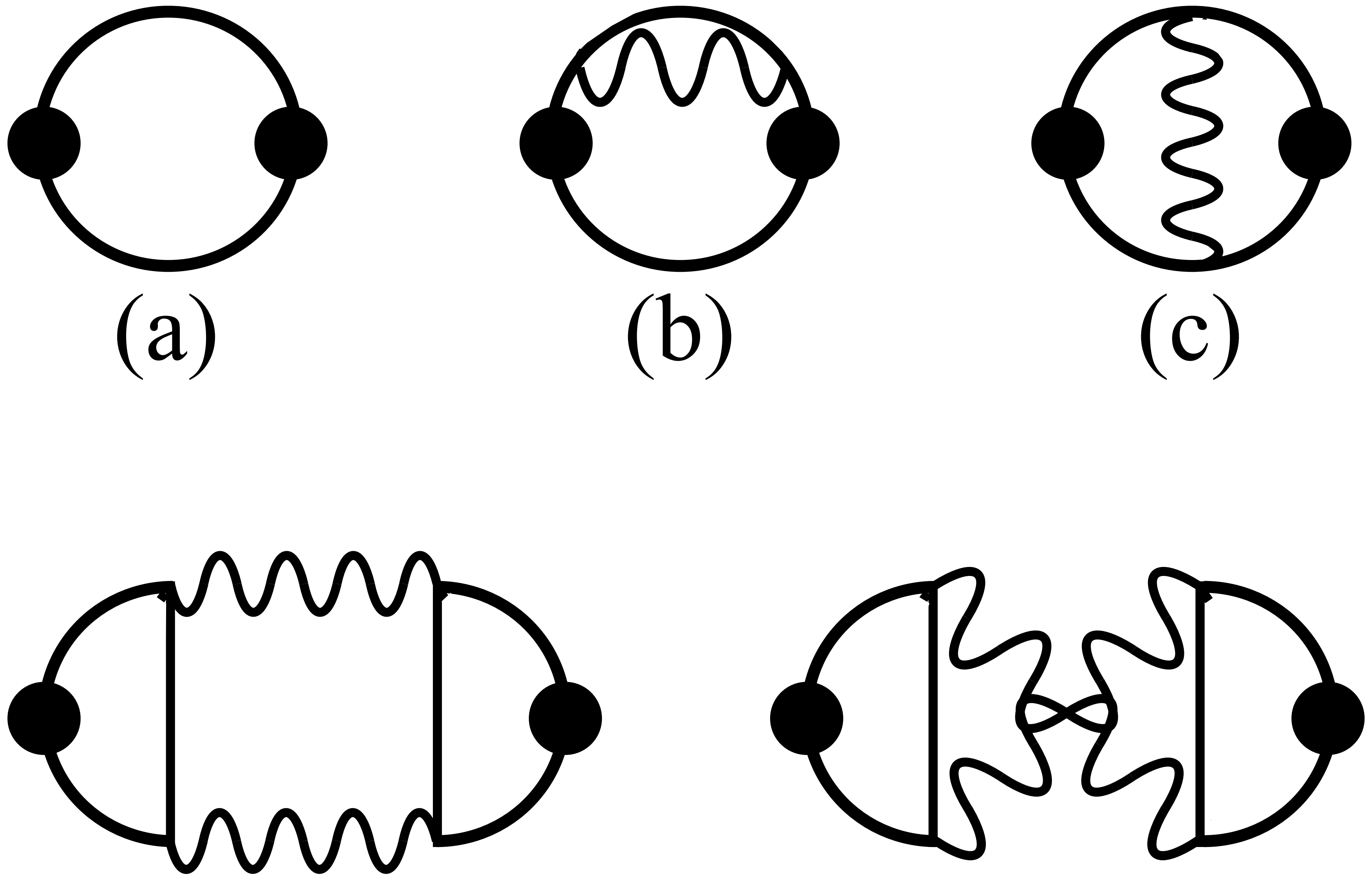
$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Transport:



(a) (b) (c) (d) (e)

+ all ladders and bubbles…..

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994) examined these graphs and concluded that the d.c. resistivity $\rho(T) \sim T^{4/3}$ (analog of Bloch's law) and $\sigma(\omega \gg T) \sim \omega^{-2/3}$.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Key ingredient of our universal theory of strange metals:

Fermion-boson drag:

- For electron-phonon scattering in metals, we have "Bloch's law" (1931): a resistivity $\rho(T) \sim T^5$.

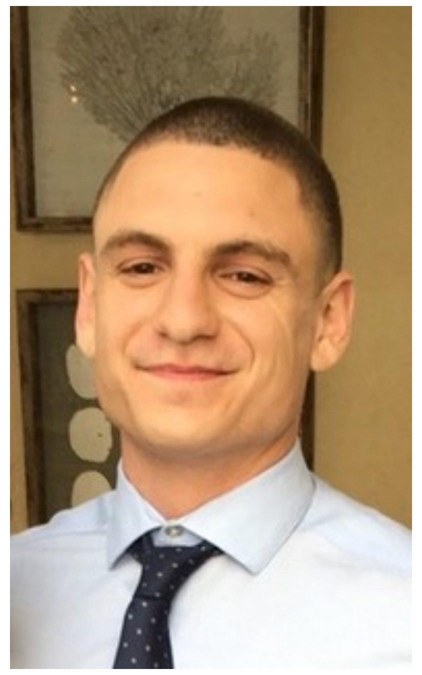
  However, Bloch's law ignores conservation of total momentum, or **phonon drag**. Peierls (1932) pointed out that the conservation of total momentum implies that an electrical current cannot decay, and so the resistance is practically zero in a pure sample. But because of the weak electron-phonon coupling, Bloch's law applies except in ultrapure crystals.

Key ingredient of our universal theory of strange metals:

Fermion-boson drag:

In a non-Fermi liquid, we cannot separate the momenta carried by the fermions and the bosons, because neither of them exists at low energies! We must treat the combined system together: *extreme drag*. The analog of Bloch's law does not apply.

Fermi surface coupled to a critical boson



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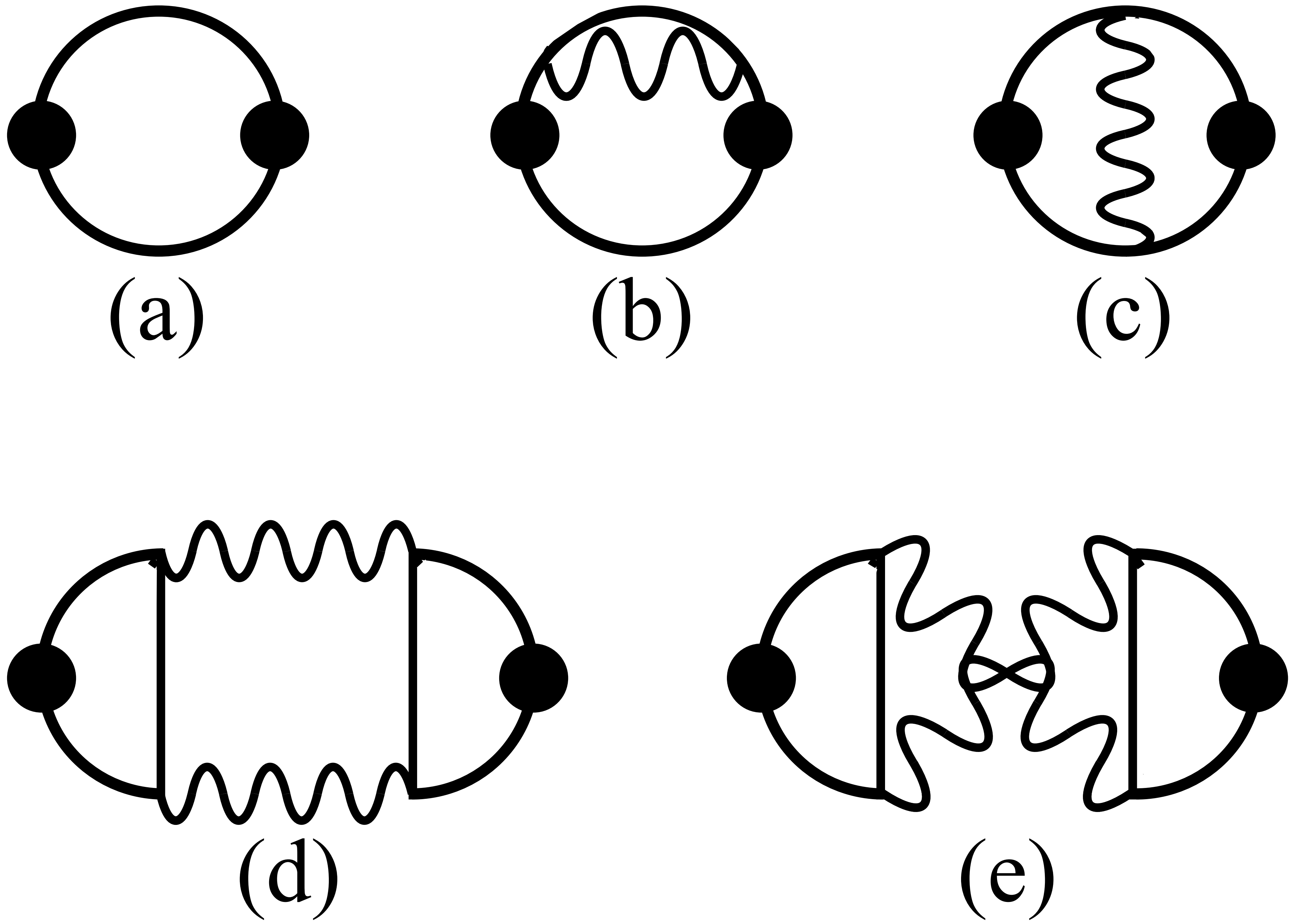

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These conclusions are not consistent with conservation of total momentum *i.e.* 'boson drag'.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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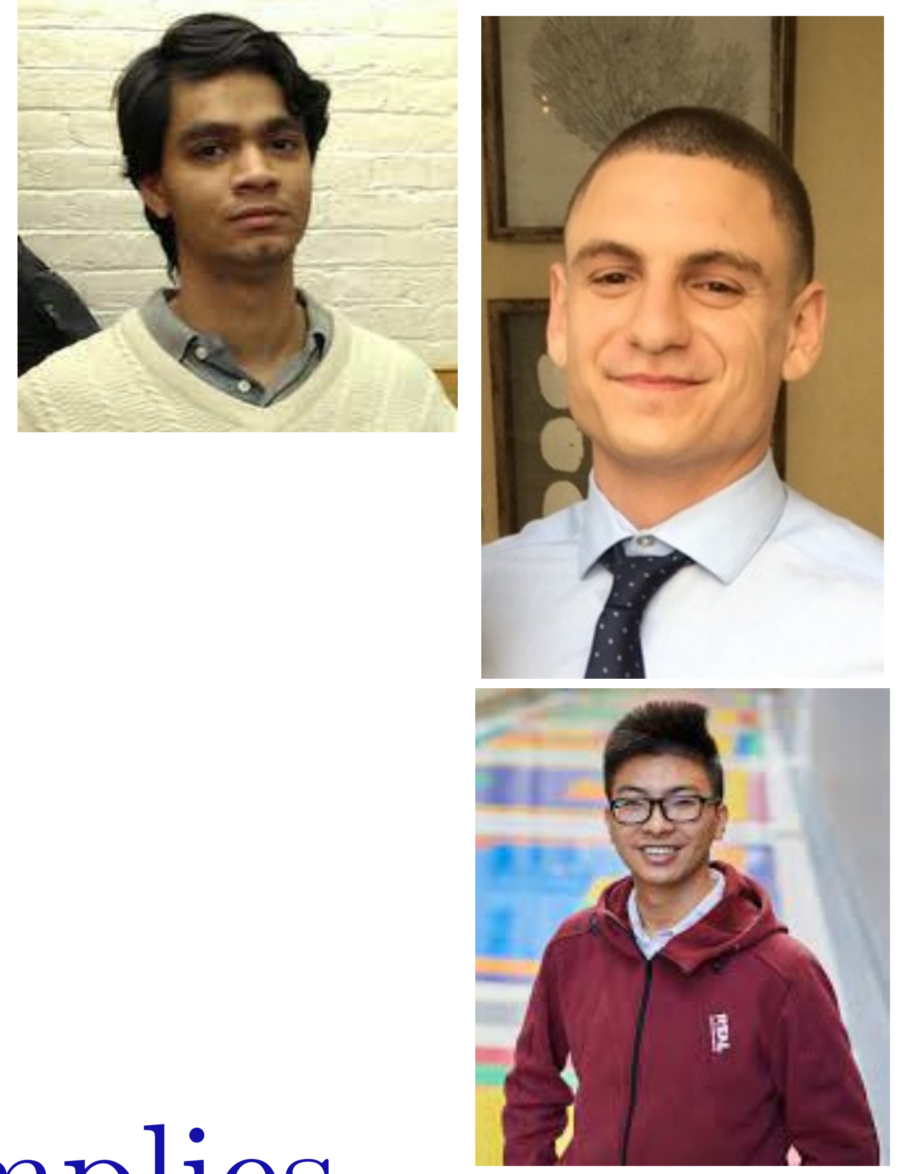

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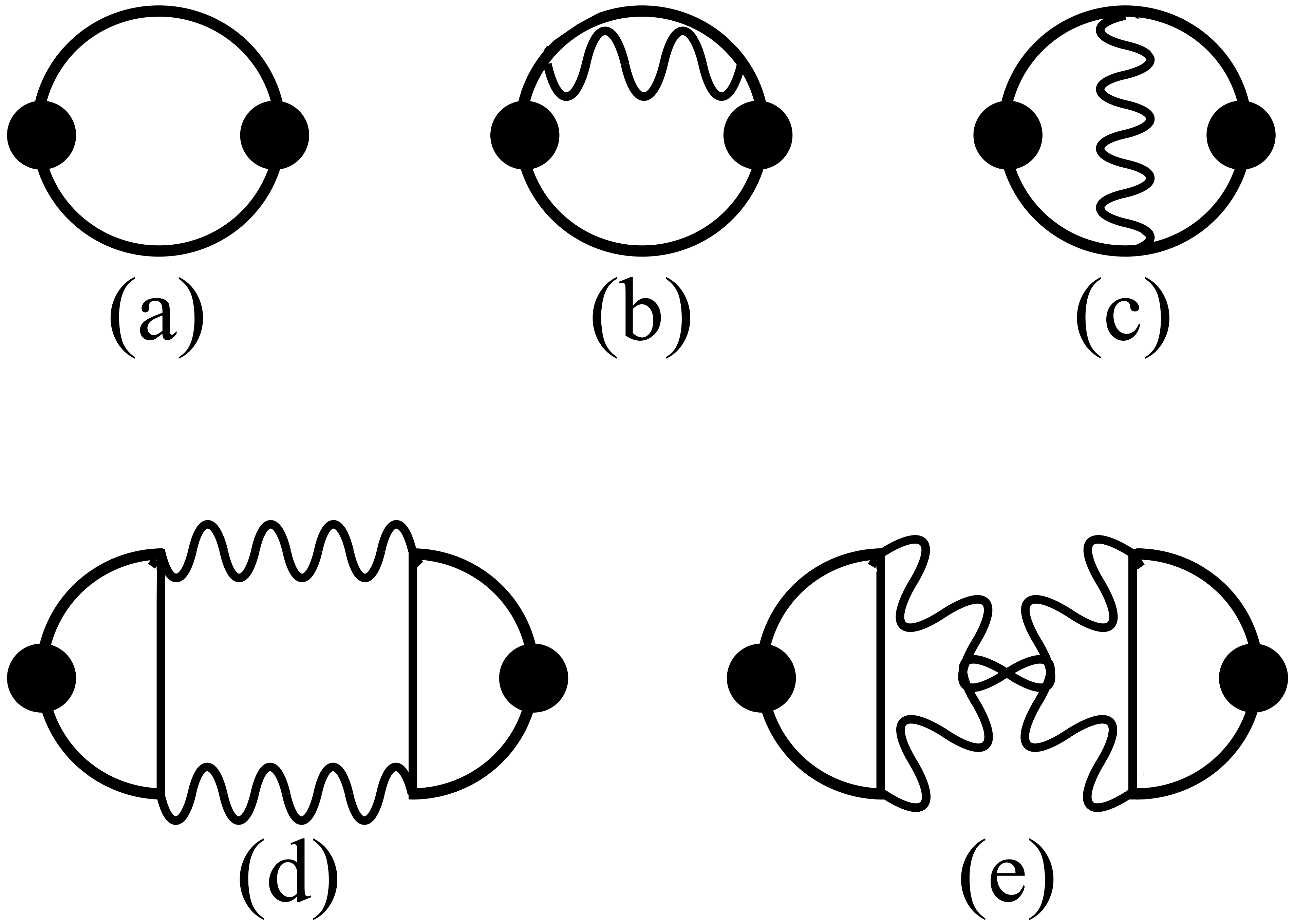

+ all ladders and bubbles…..

Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

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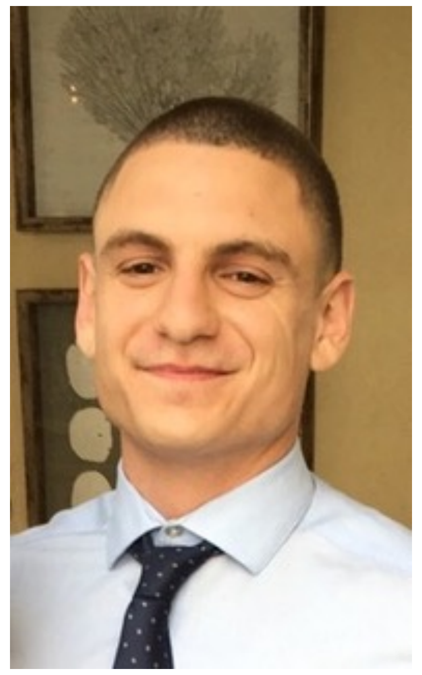

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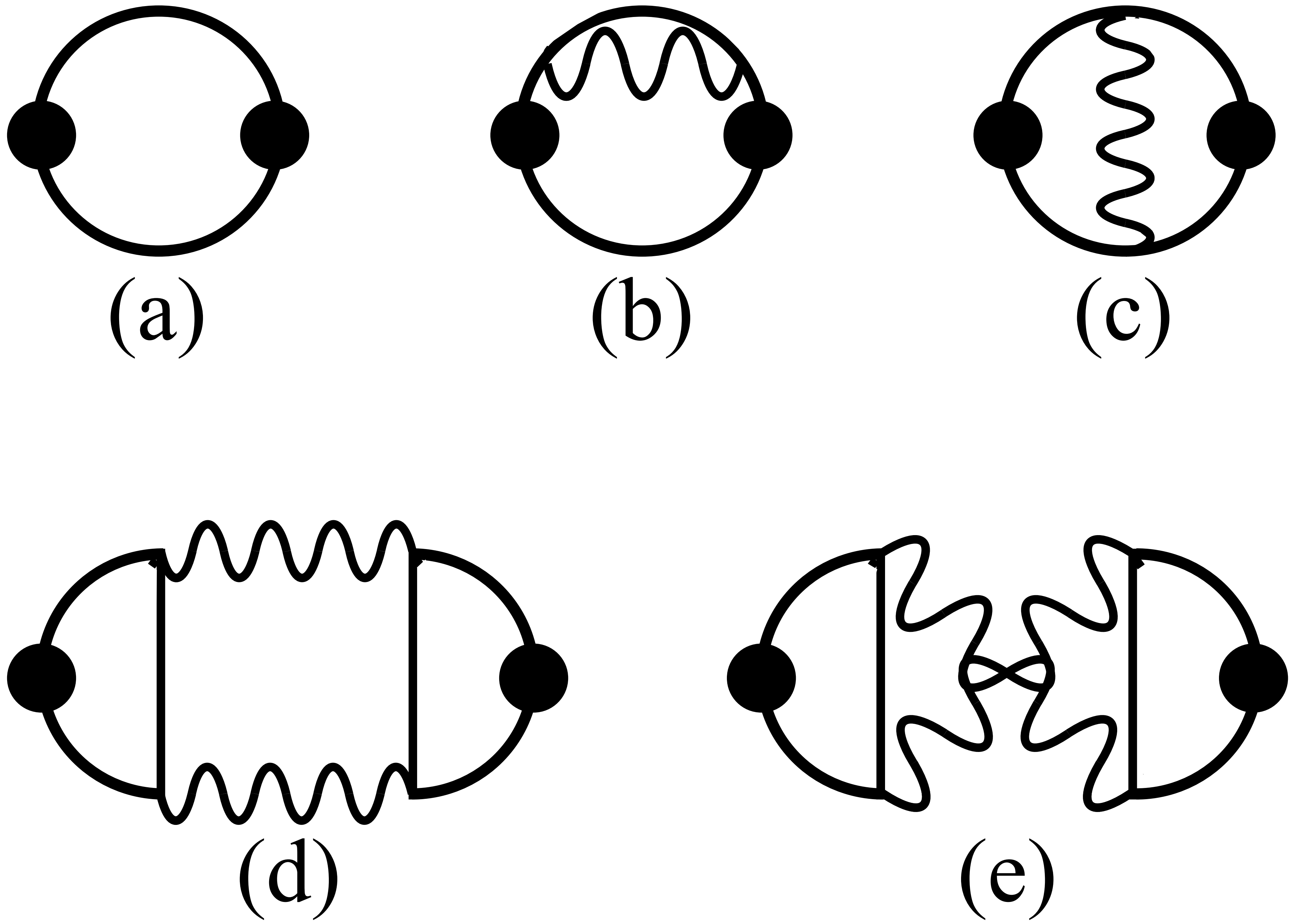

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A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \ldots$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990
Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson

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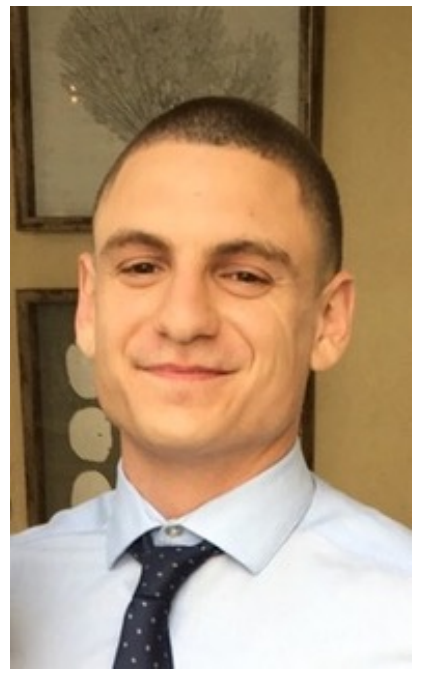
"Yukawa" coupling: $\quad g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Random potential $\quad \int d^2 r d\tau \, v(r) \, \psi^\dagger(r,\tau)\psi(r,\tau)$

Spatially random potential $v(r)$ with $\overline{v(r)} = 0$, $\overline{v(r)v(r')} = v^2 \delta(r - r')$

Fermi surface coupled to a critical boson with spatial disorder



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Random potential: $$+ \frac{1}{\sqrt{N}} \int d^2 r d\tau \, v_{ij}(r) \psi_i^\dagger(r,\tau) \psi_j(r,\tau)$$

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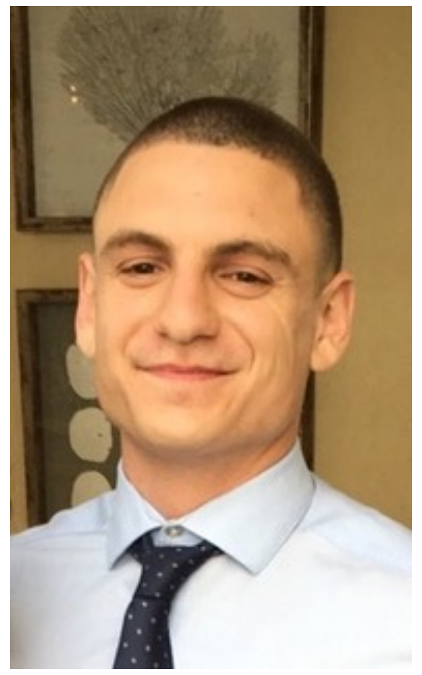
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder



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$$\text{Fermion self energy:} \; \Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|)$$

Marginal Fermi liquid self energy and $T \log T$ specific heat

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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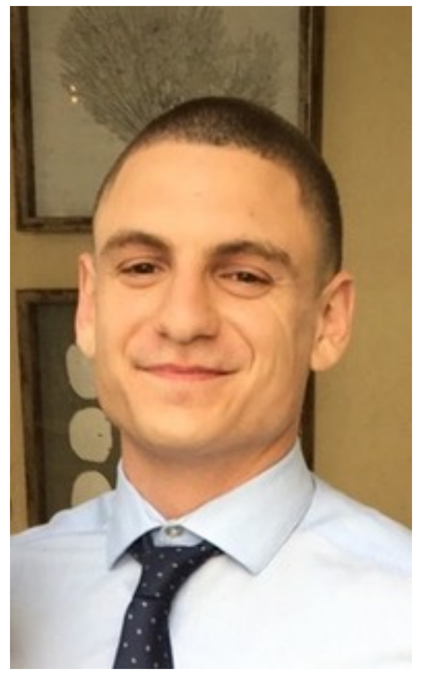

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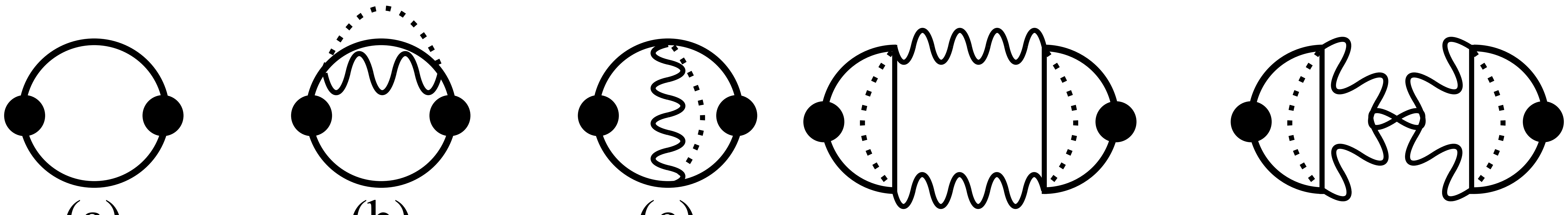
Conductivity:



(a) σ_v

(b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$

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+ all ladders and bubbles…..

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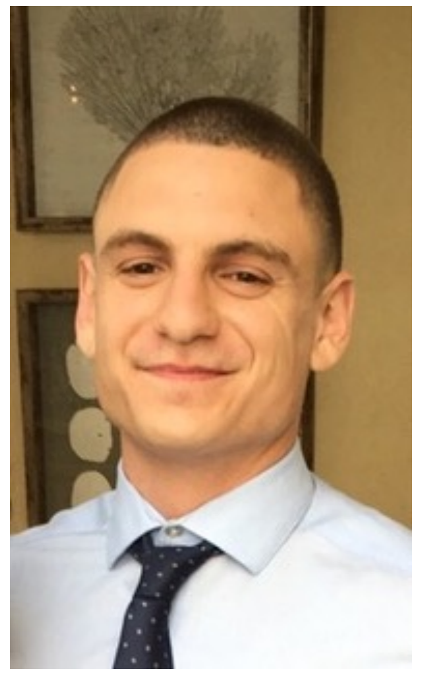

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The g^2log term does not contribute to transport:
With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \ldots.$$

with $1/\rho(0) \sim 1/\tau_{\text{trans}} \sim v^2$.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson:

Potential disorder

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Interaction disorder

A marginal Fermi liquid AND a strange metal

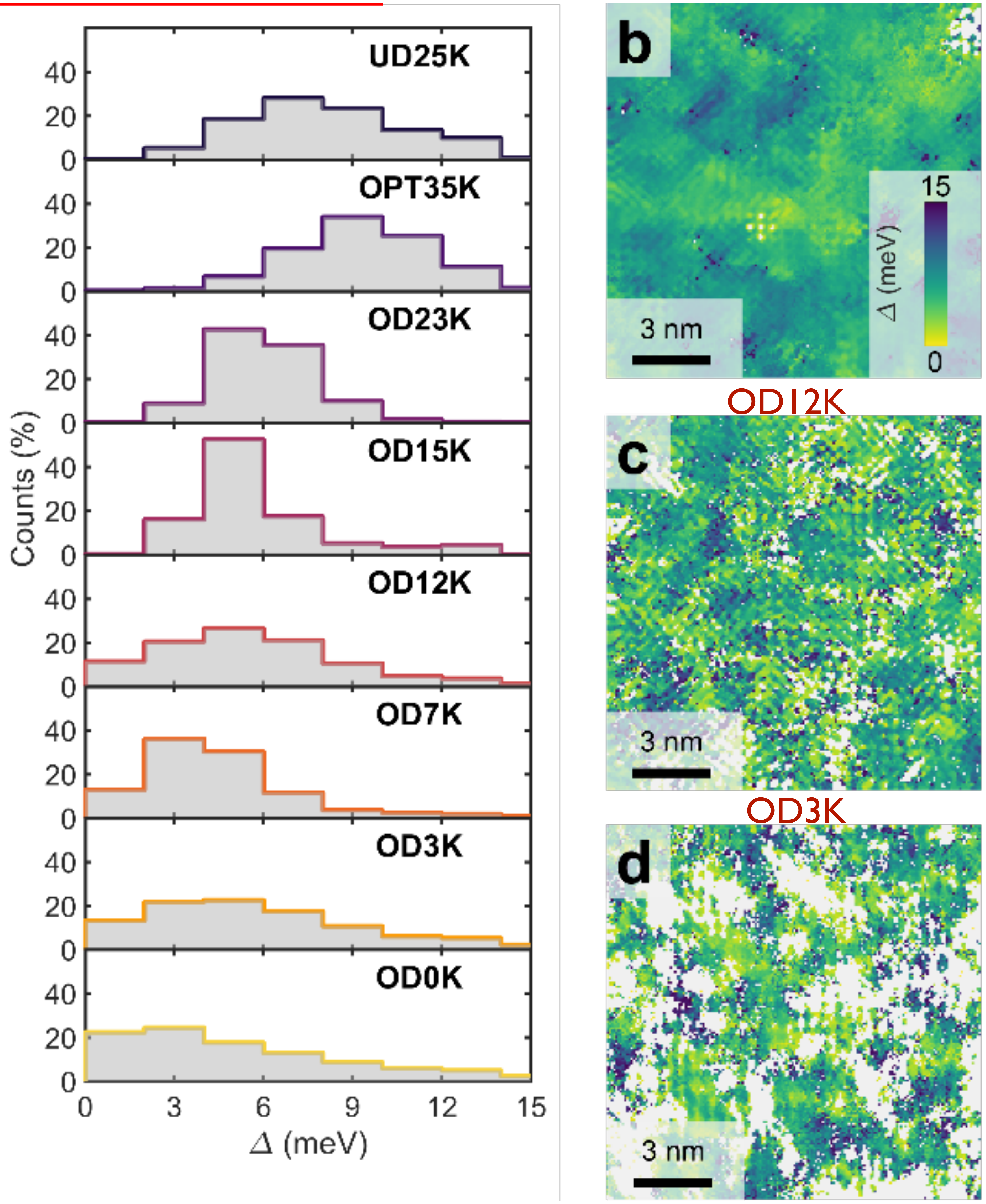
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740



Spatially random interactions!

Randomness in hopping t_{ij} leads to randomness in exchange interactions t_{ij}^2/U. The interaction associated with the ϕ collective mode has the schematic form

$$- \int d^2 r d\tau \, J(r) \psi^\dagger \psi^\dagger \psi \psi$$

where we have omitted a local 'form factor' for the interaction, and the random strength of the overall interaction is determined by the coupling $J(r)$. Upon decoupling

$$\int d^2 r d\tau \left[\frac{\phi^2}{2J(r)} - \phi \psi^\dagger \psi \right]$$

This as a random 'mass' in the boson and is strongly relevant.

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This as a random 'mass' in the boson and is strongly relevant. A key idea is that we should account for the relevant disorder exactly by rescaling the field ϕ in a r-dependent manner so that

$$\int d^2r d\tau \left[\frac{\phi^2}{2} - \sqrt{J(r)}\phi\psi^\dagger\psi\right]$$

The disorder is in the boson-fermion coupling, and can be accounted for systematically.

Key ingredient of our universal theory of strange metals:

Spatially random Yukawa coupling $g'(r)$ with $\overline{g'(r)} = 0$, $\overline{g'(r)g'(r')} = g'^2\delta(r-r')$

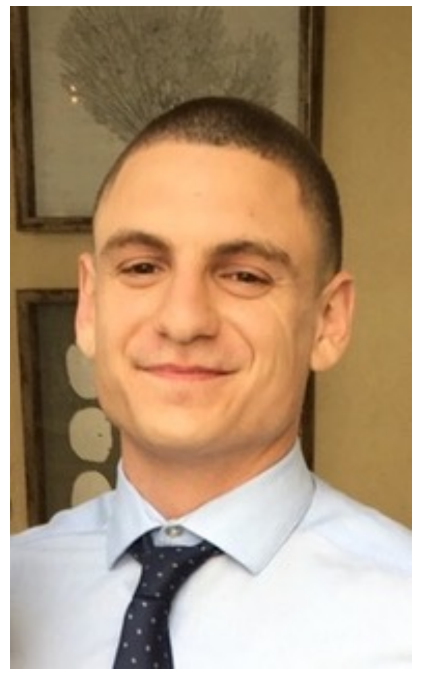
$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

$$\mathcal{L}_\phi = \frac{1}{2}\left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla}\phi)^2 + s\phi^2 \right]$$

"Yukawa" coupling: $\displaystyle\int d^2r d\tau \, \underline{[g + g'(r)]} \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Random potential $\displaystyle\int d^2r d\tau \, v(r) \, \psi^\dagger(r,\tau)\psi(r,\tau)$

Fermi surface coupled to a critical boson with spatial disorder



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Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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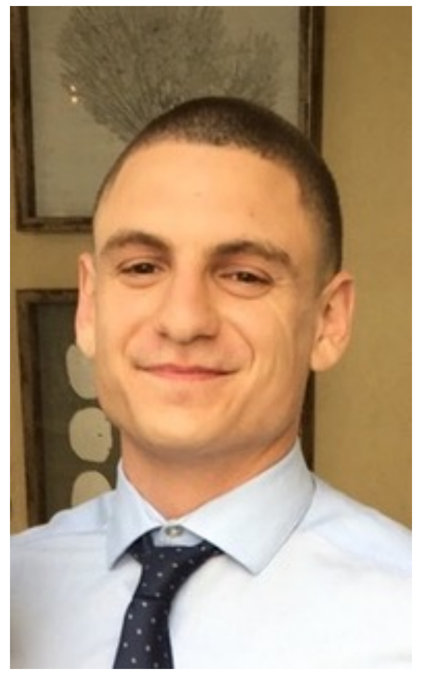

$$\text{Boson self energy: } \Pi = \Pi_g + \Pi_{g'}$$

$$\Pi_g(i\Omega) \sim -\frac{g^2}{v^2}|\Omega|, \qquad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|, \qquad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$$

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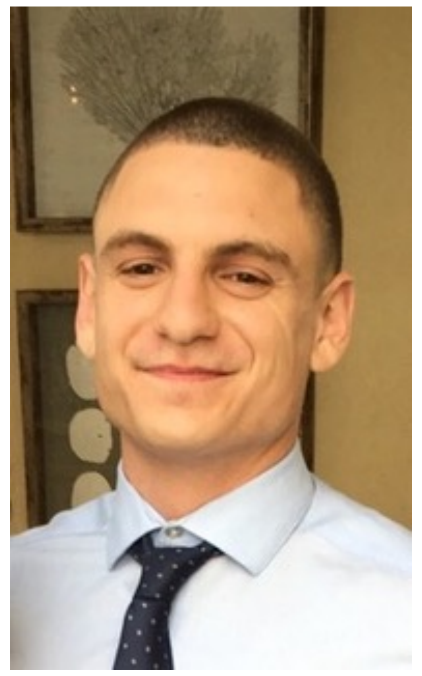

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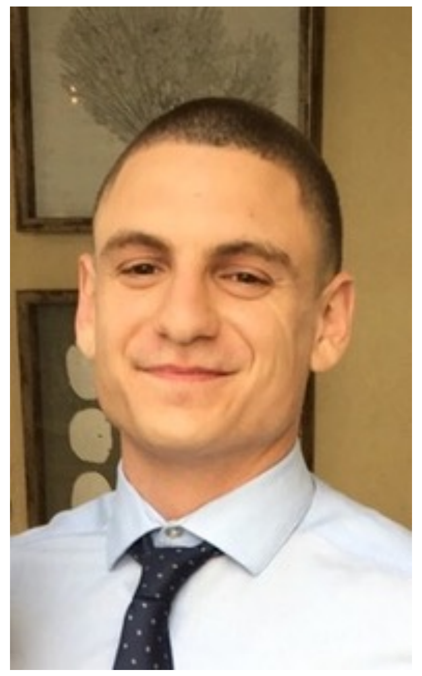

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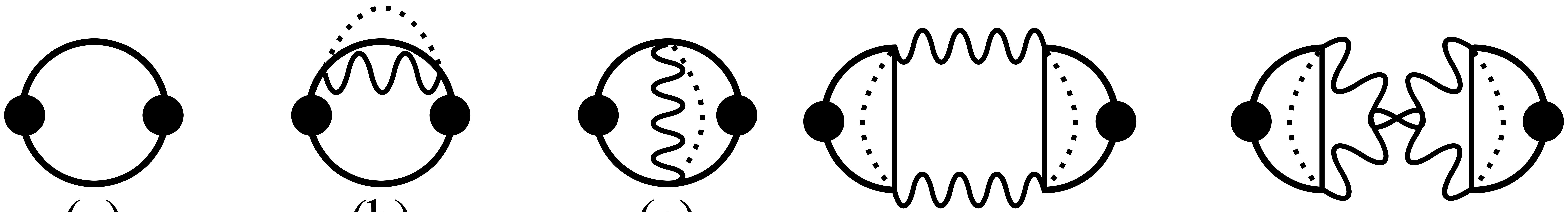

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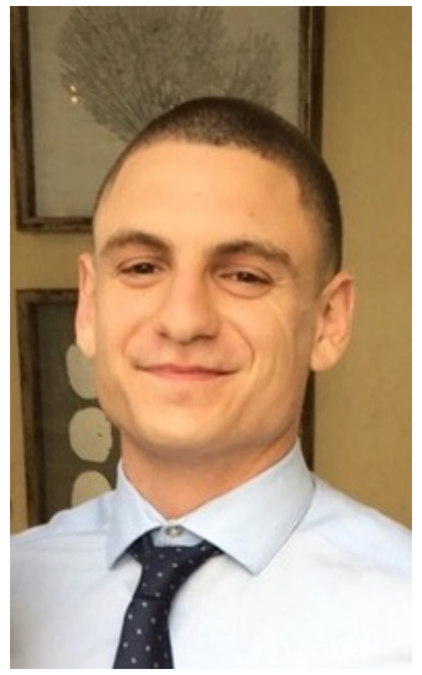

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The g^2 log term does not contribute to transport but the g'^2 log term does!

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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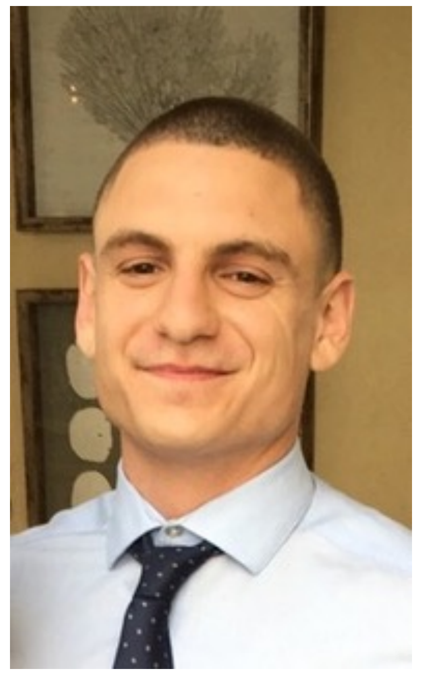


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$$\text{Conductivity:} \ \sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega\, m^*(\omega)/m]^{-1}$$

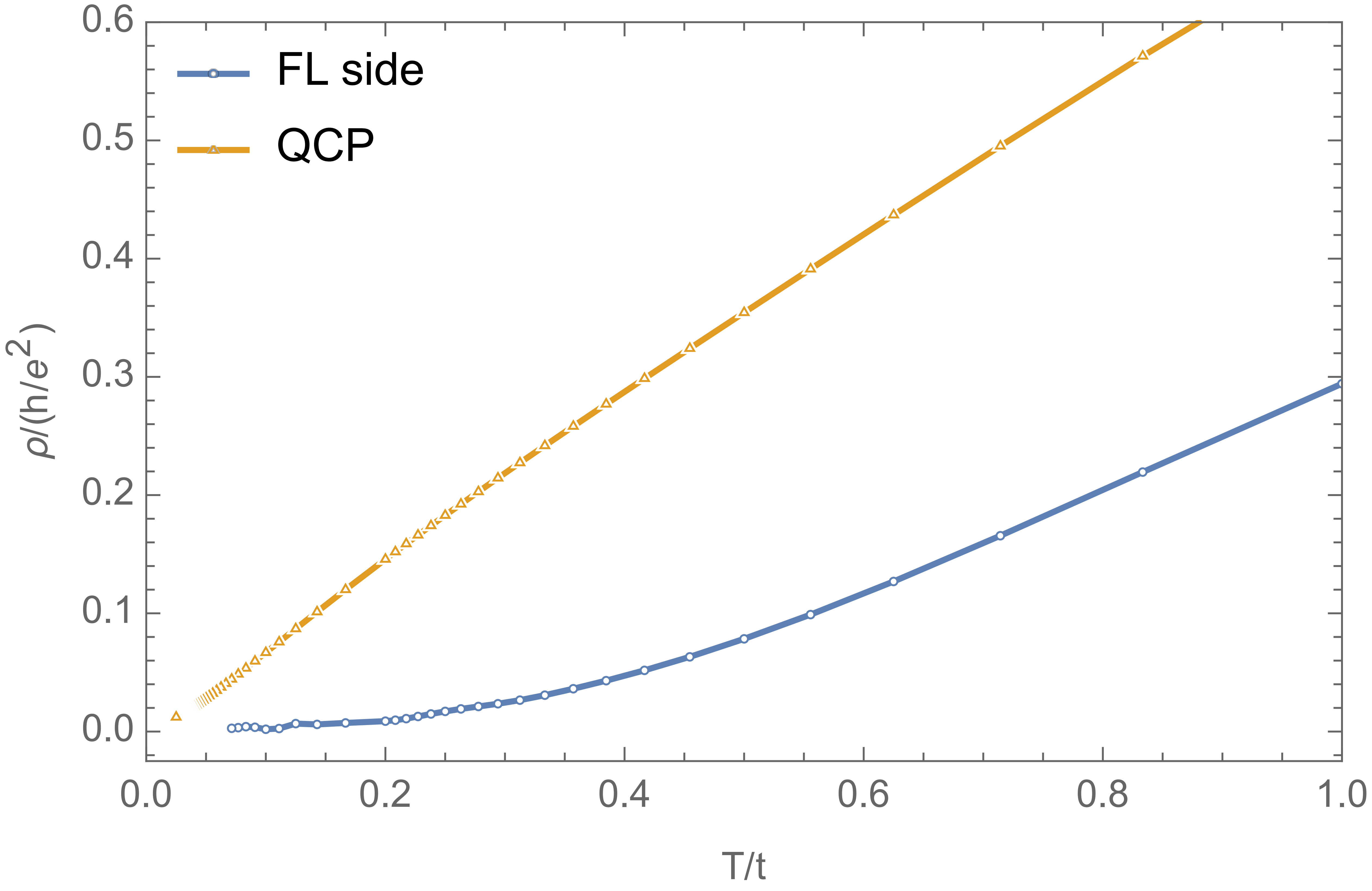
$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

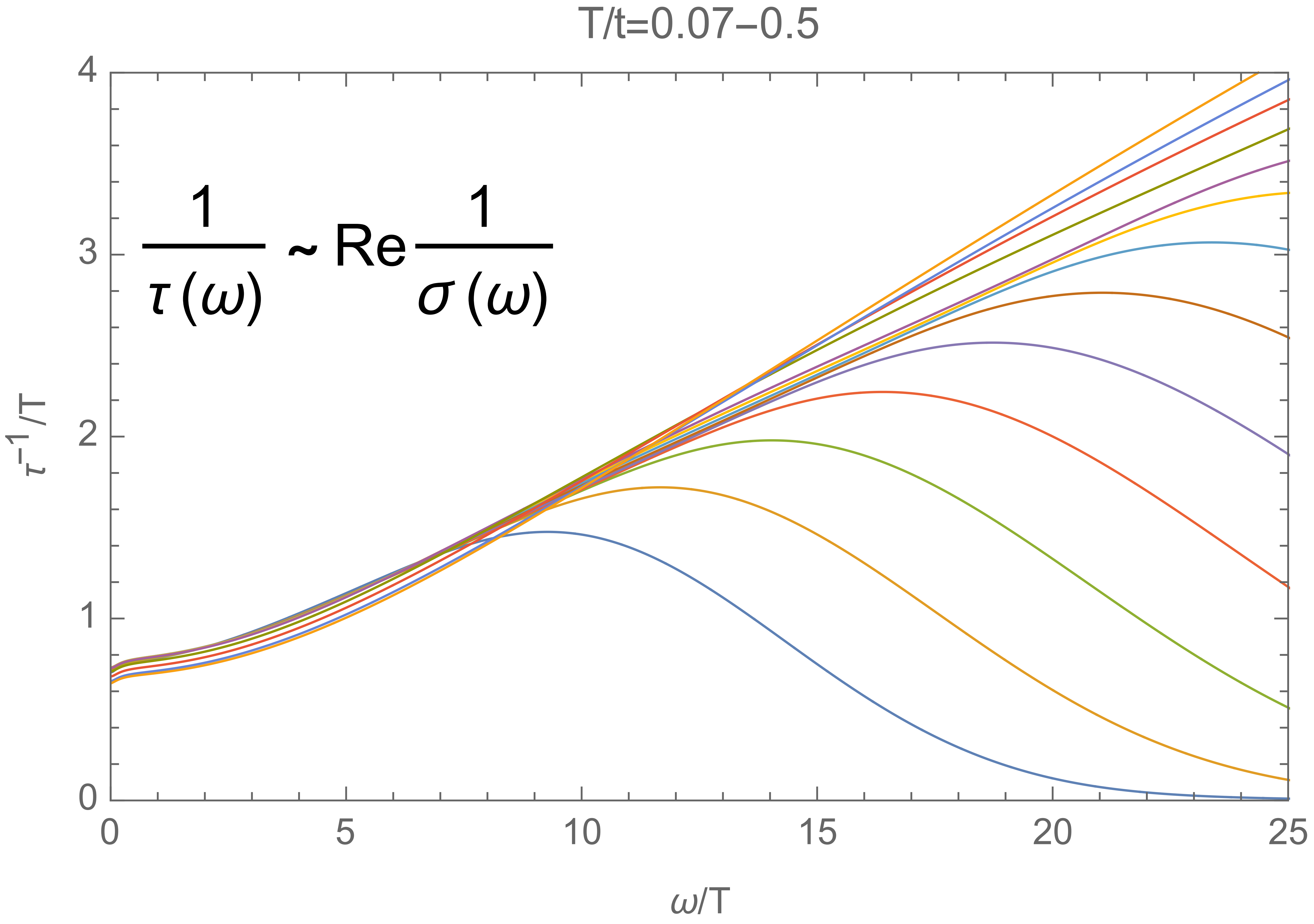
Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Strange metal from a Yukawa-SYK model

Full numerical solution of large N limit at $g' \neq 0$, $g = 0$
(the singular corrections from a non-zero g are expected to vanish in the conductivity).





Ilya Esterlis, Aavishkar Patel, Haoyu Guo,, S.S. to appear

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NOT a strange metal

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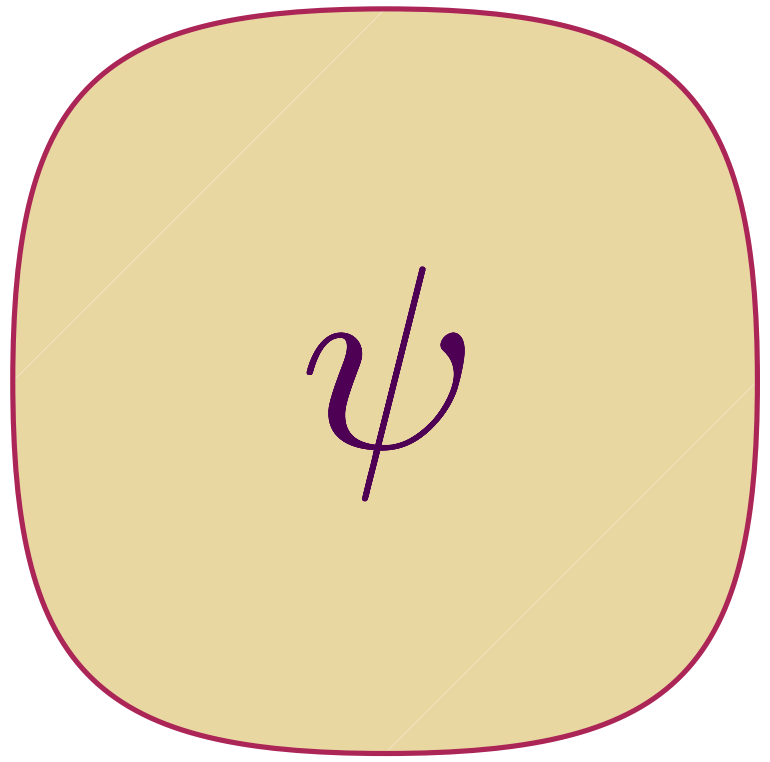

$+$ a critical boson ϕ

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Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Strange metal from a Yukawa-SYK model

ψ

+ a critical boson

ϕ

Non-Fermi liquid with $T^{2/3}$ specific heat,
but conductivity $\sigma(\omega) \sim \delta(\omega)$

"Yukawa" coupling: $$\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

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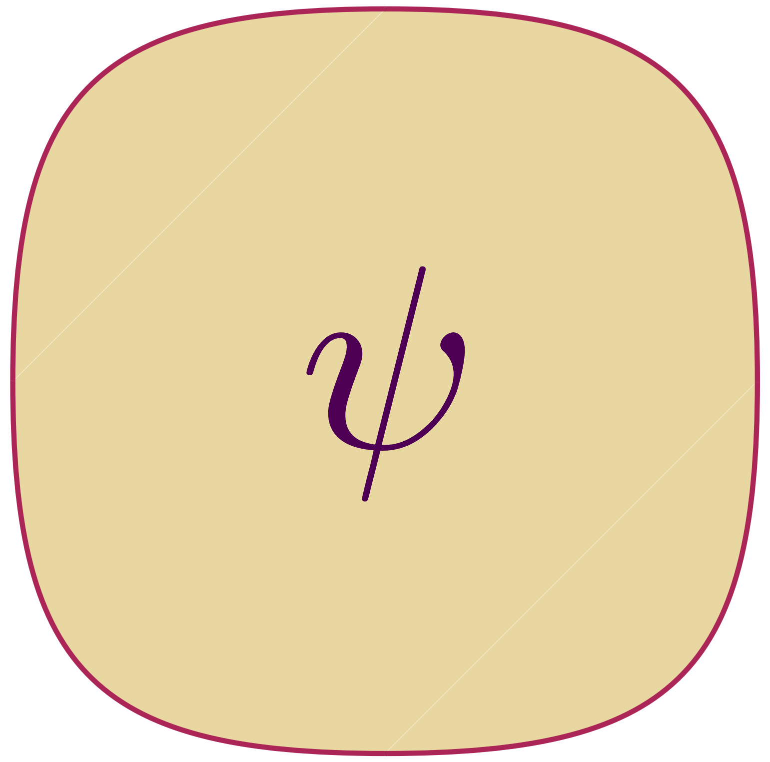

$+$ a critical boson ϕ

MFL self-energy, $T \ln(1/T)$ specific heat,
but T-independent 'residual' resistivity,
and negligible optical conductivity

"Yukawa" coupling: $\displaystyle \frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

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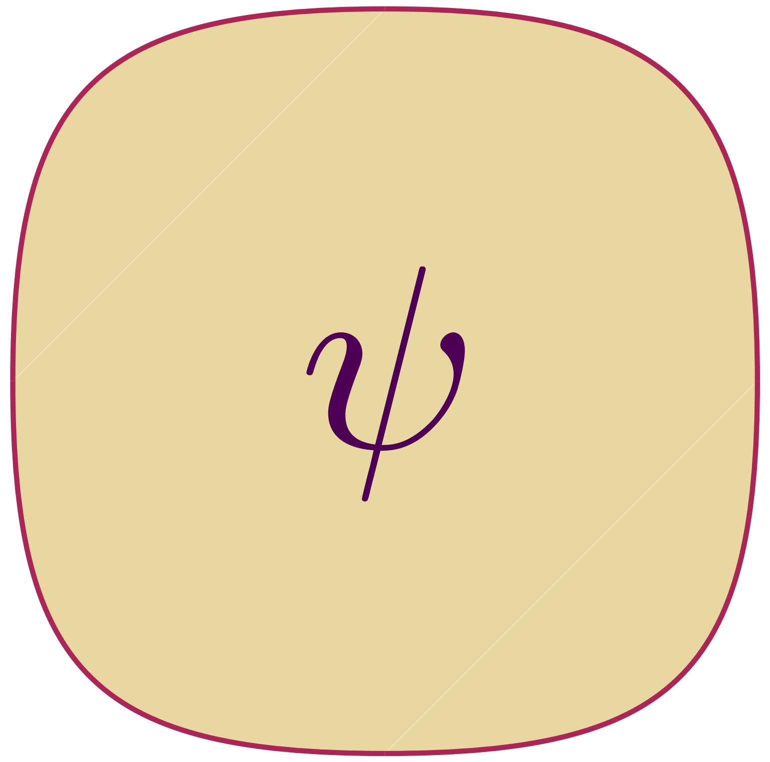

$+$ a critical boson ϕ

MFL self-energy, $T \ln(1/T)$ specific heat, linear-T resistivity and $1/[\omega - i(2\omega/\pi)\ln(\Lambda/\omega)]$ optical conductivity

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Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev,
arXiv: 2109.05037, Reviews of Modern Physics

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Universal theory of a marginal Fermi liquid and a strange metal (including linear-T resistivity): spatially random interactions in a two-dimensional quantum-critical metal, solvable in a Yukawa-SYK-like large N limit.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev,
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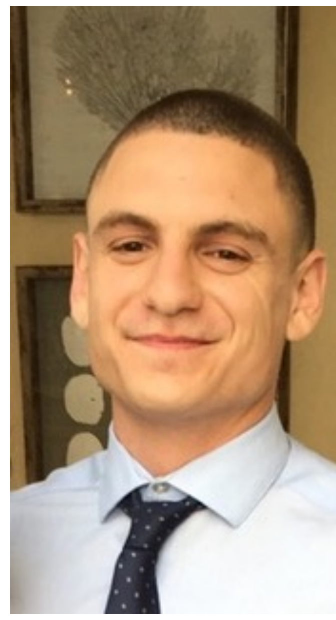
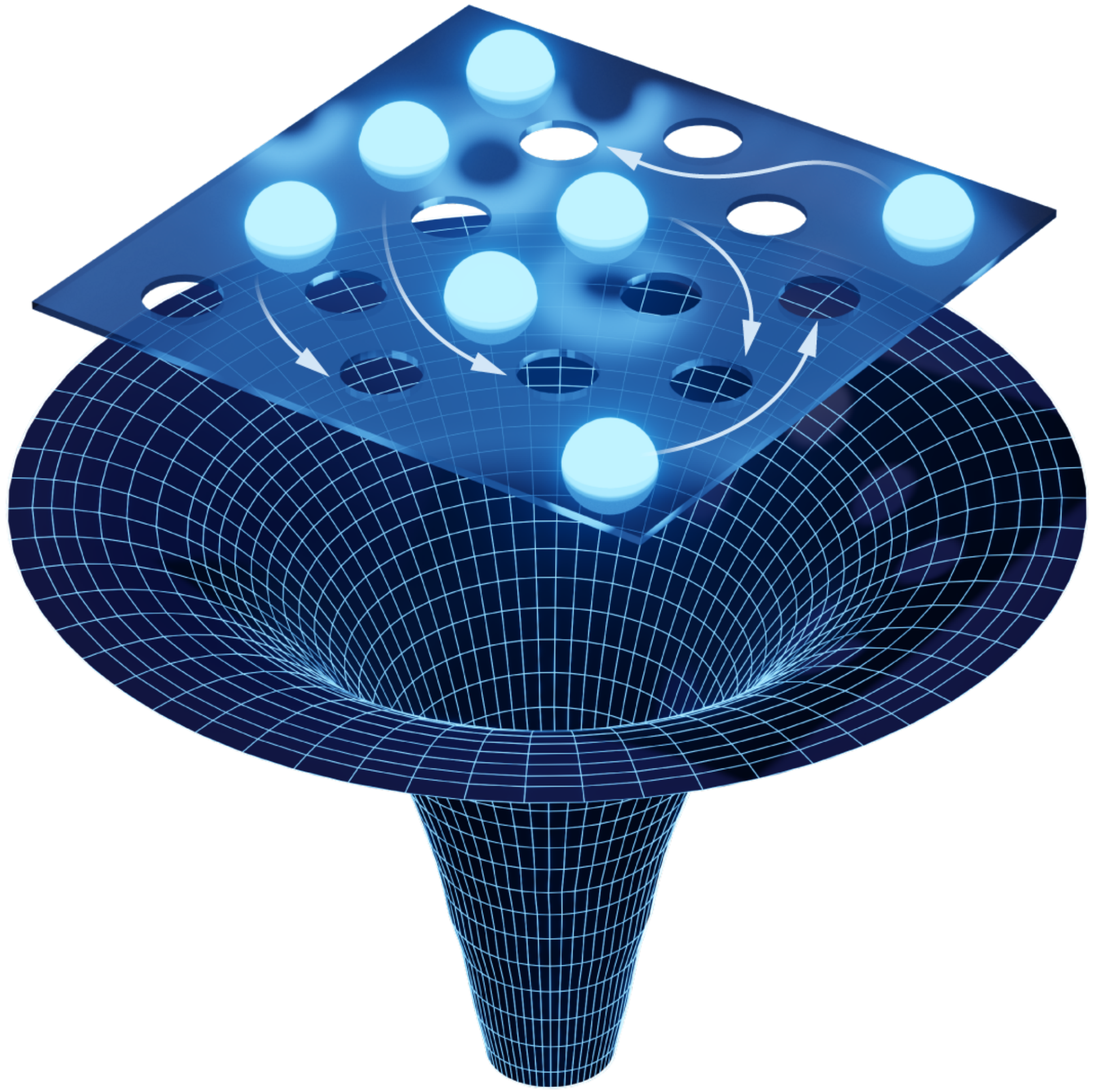


Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

Summary

- Black holes with a net charge in asymptotically Minkowski space have a near horizon $\text{AdS}_2 \times S^2$ geometry: this geometry has an emergemt time-reparameterization soft mode with an action identical to that of the SYK model. In other words, the SYK model is a quantum simulation of the low energy physics of charged black holes in Einstein-Maxwell theory.



D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, Reviews of Modern Physics