

Quantum criticality and the phase diagram of the cuprates

Talk online: sachdev.physics.harvard.edu



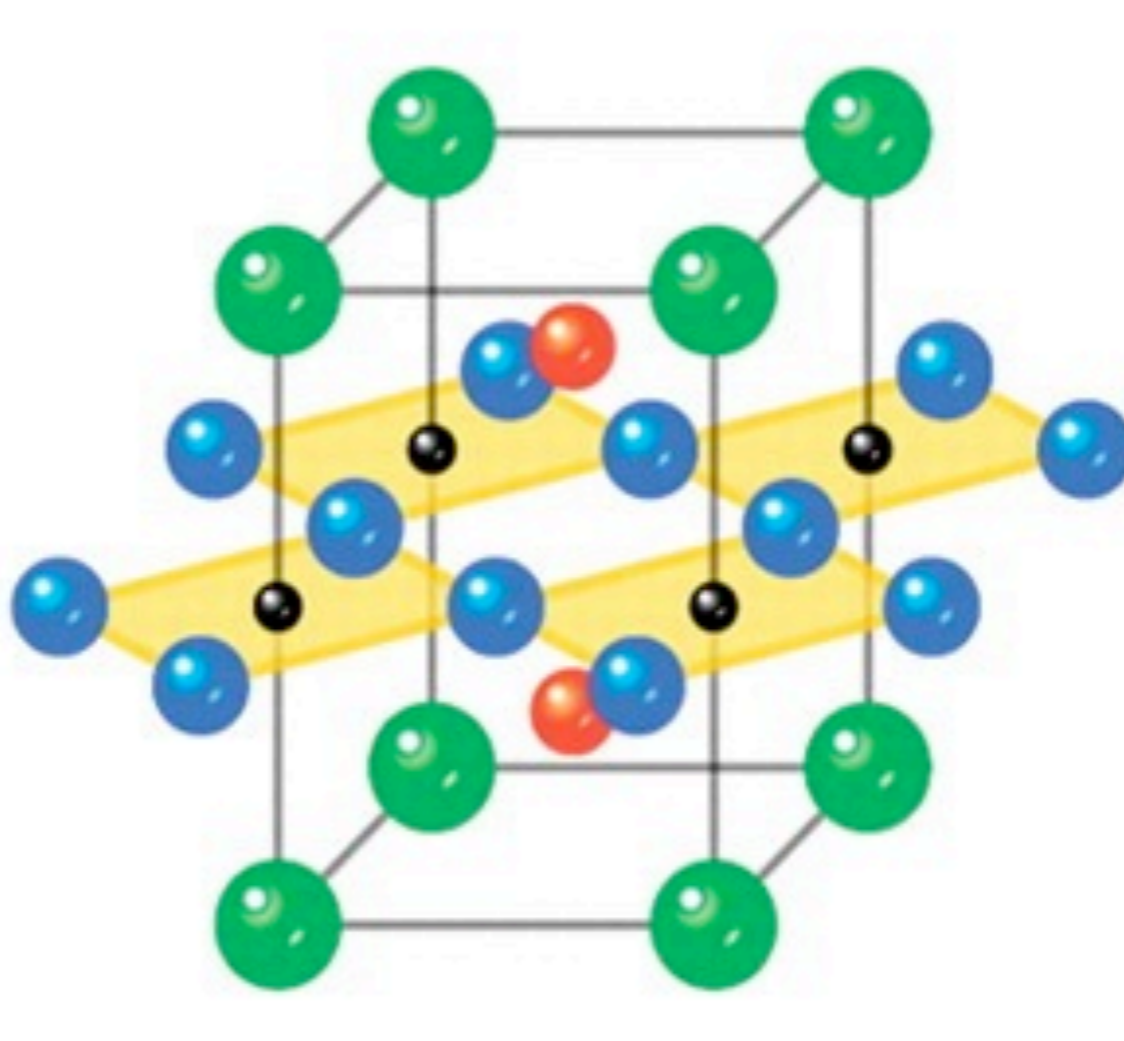
Victor Galitski, Maryland
Ribhu Kaul, Harvard → Kentucky
Max Metlitski, Harvard
Eun Gook Moon, Harvard
Yang Qi, Harvard
Cenke Xu, Harvard → Santa Barbara



The cuprate superconductors

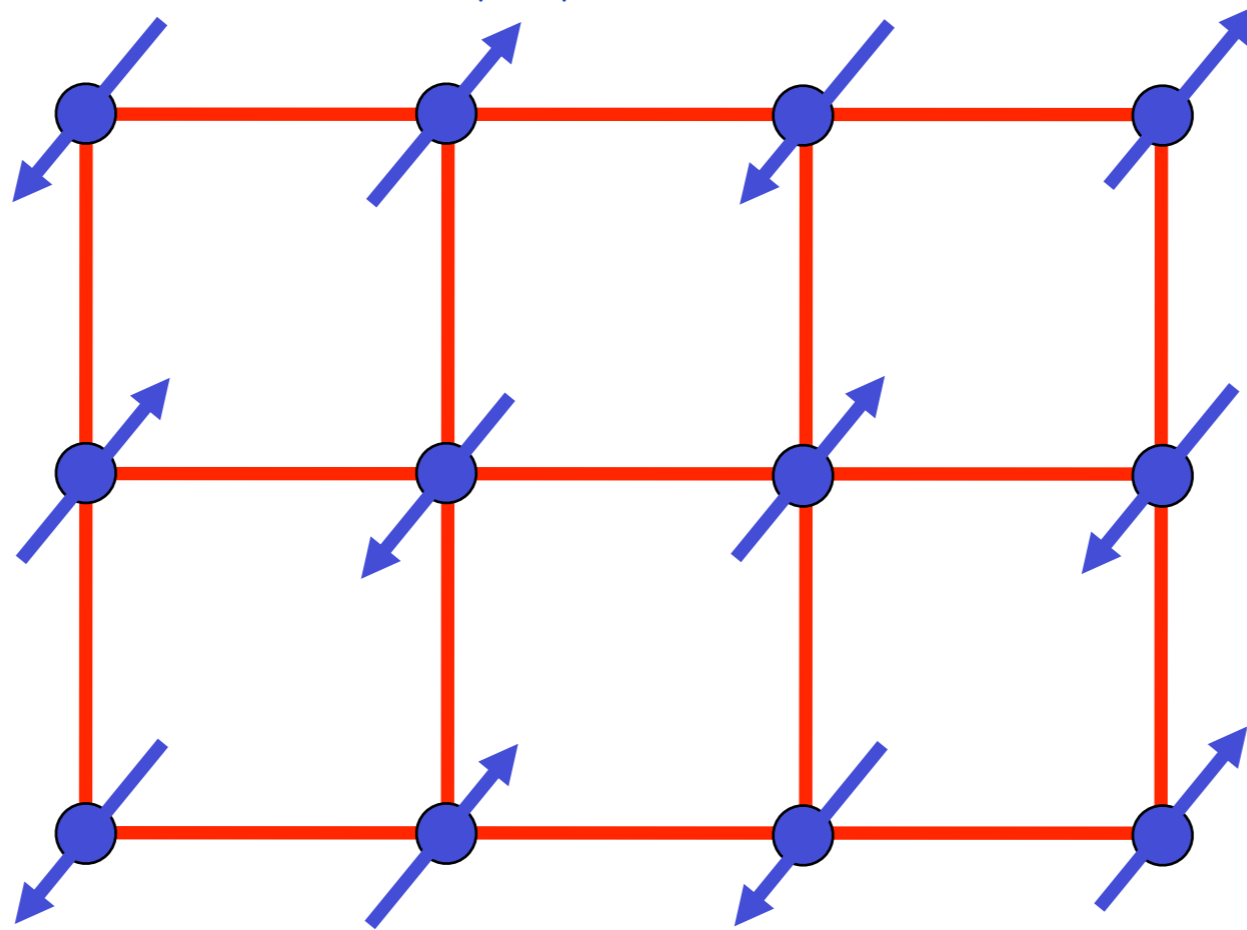
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



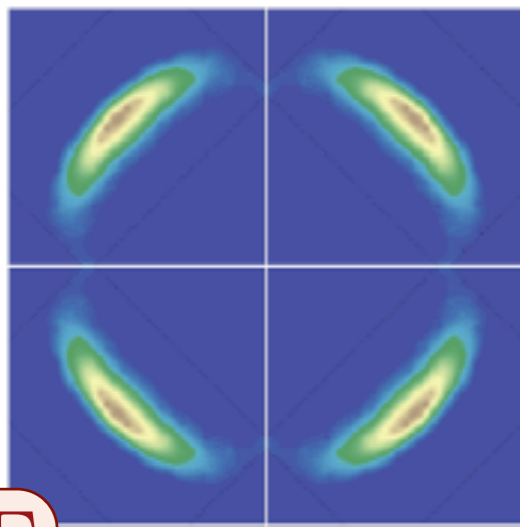
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

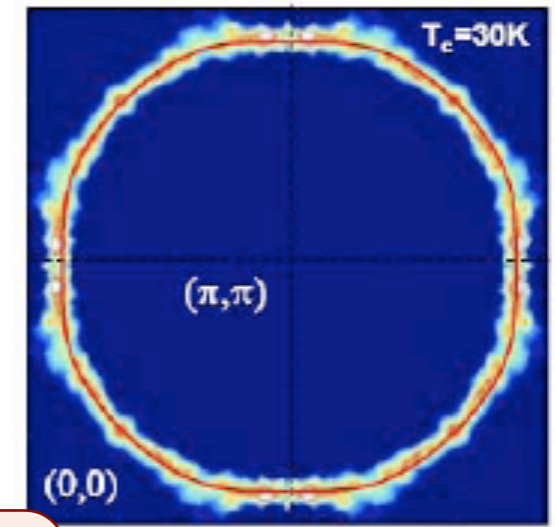
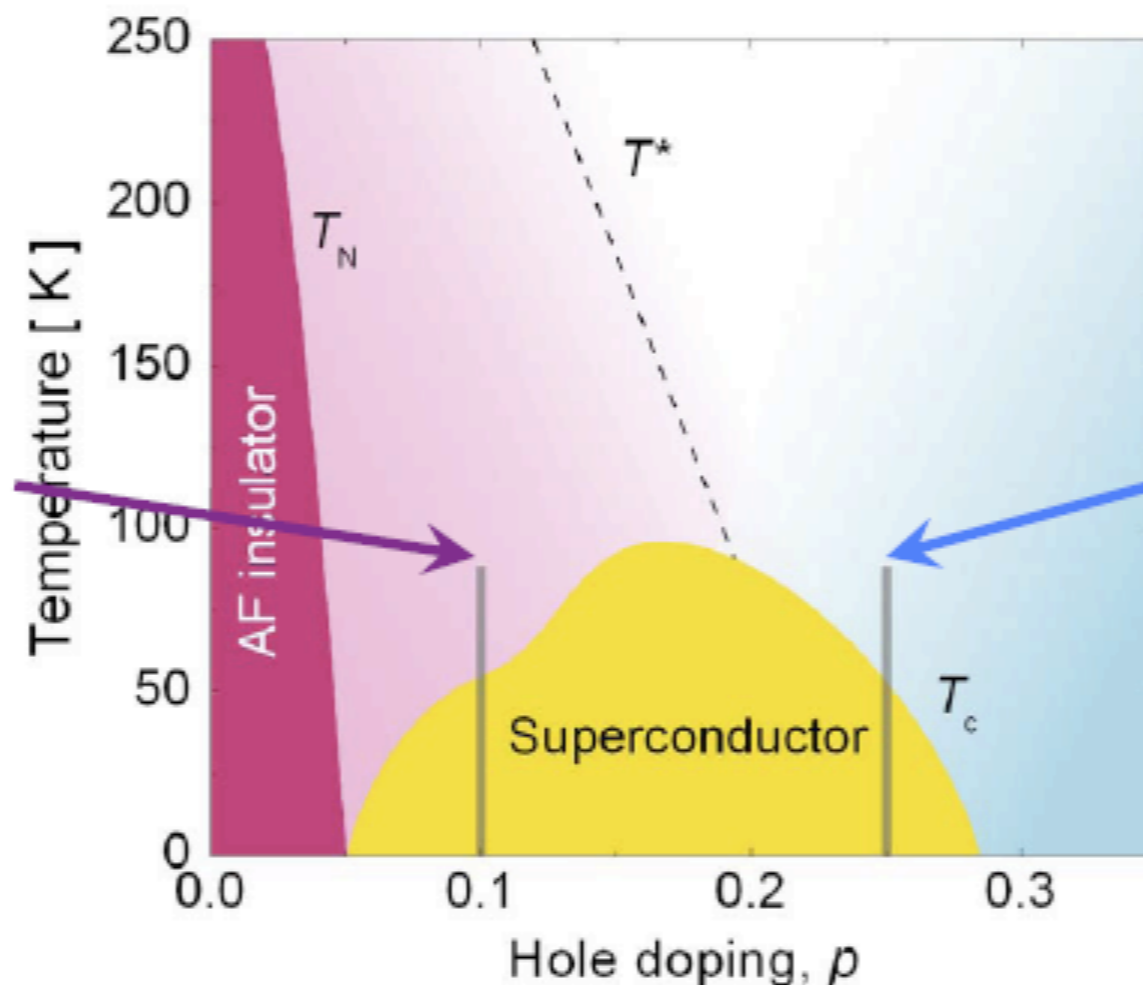
$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Γ

K.M. Shen et al., Science 2005



Γ

M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

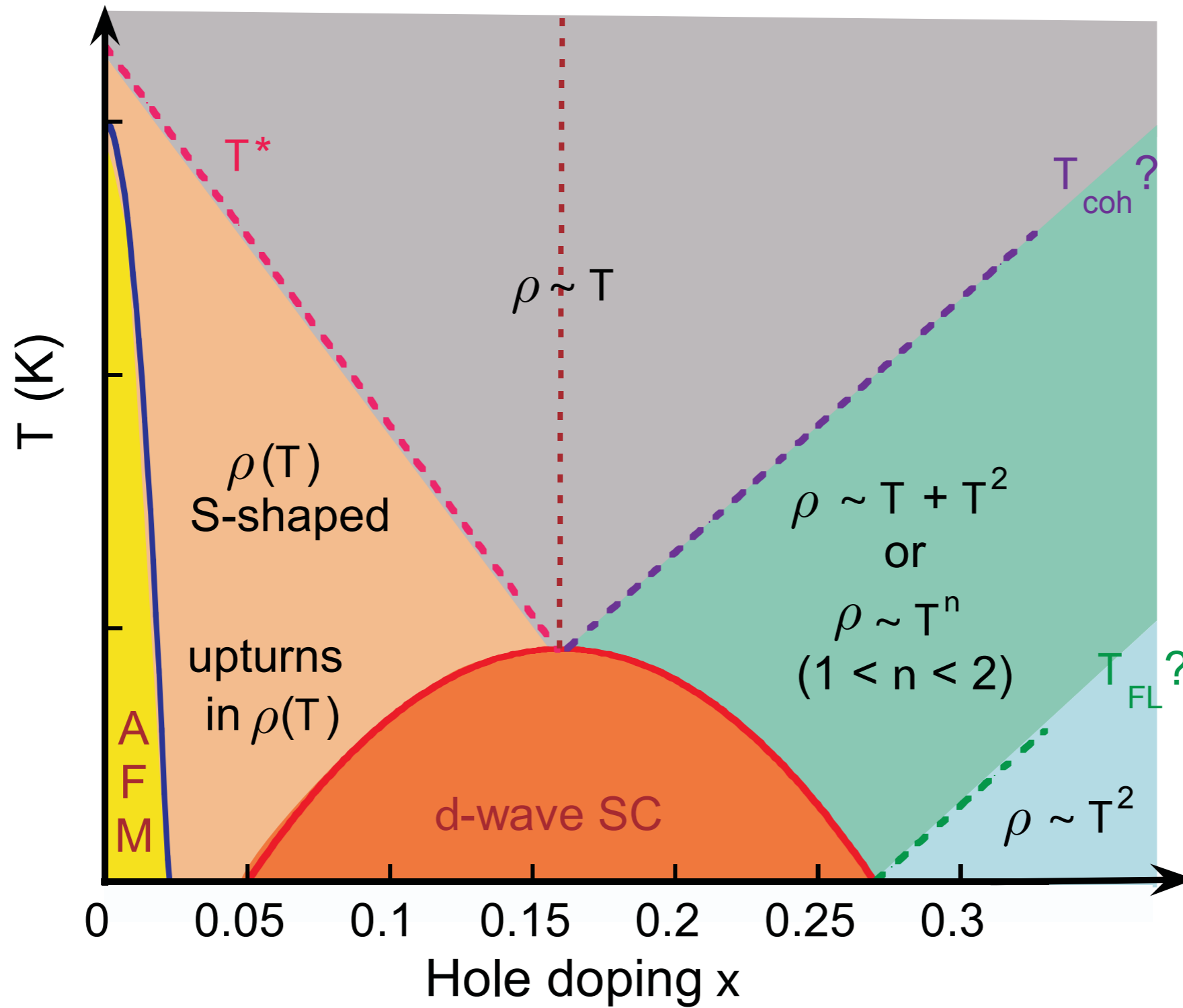
Large hole
Fermi surface

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

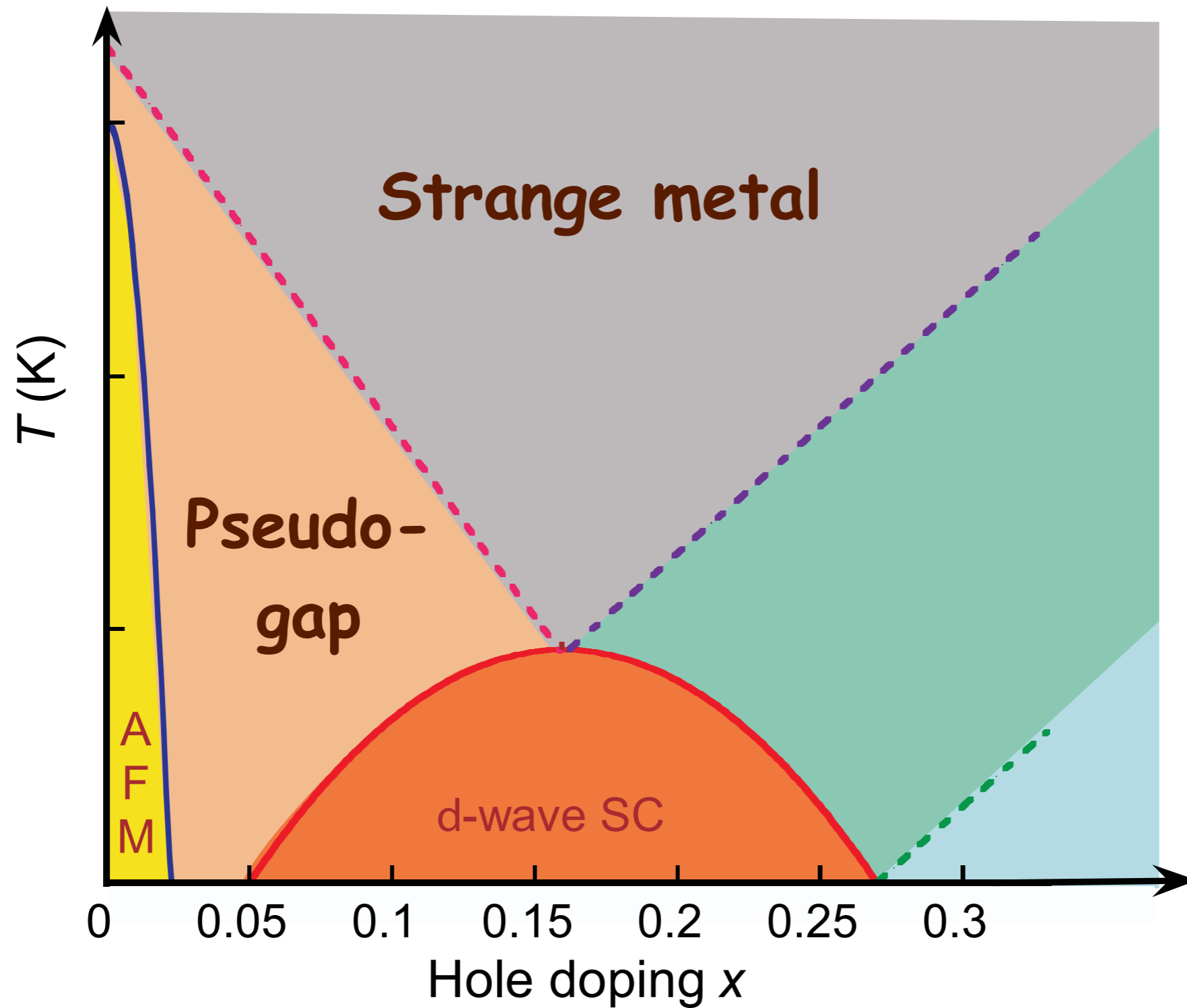
**Fermi
surface**

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

**Fermi
surface**

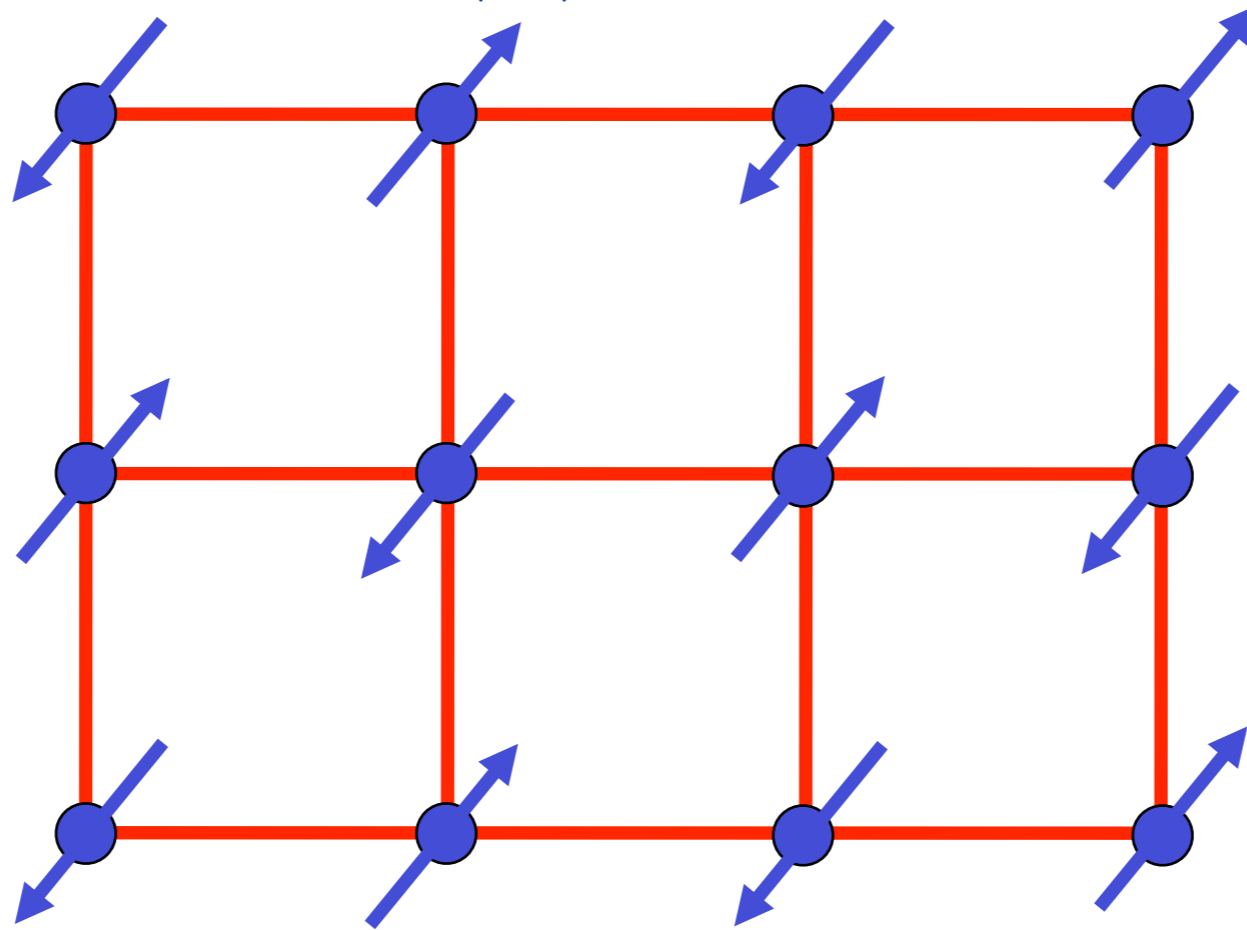
**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

**Fermi
surface**

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

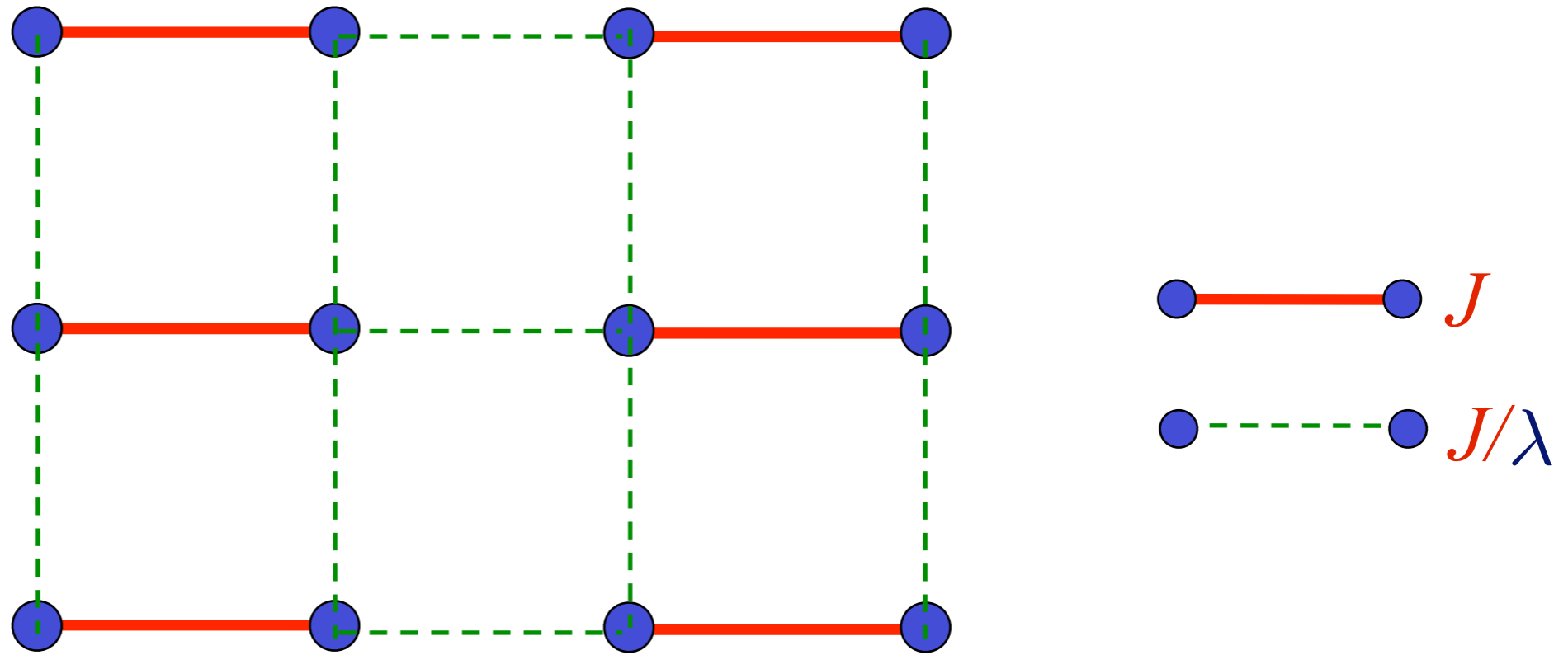
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

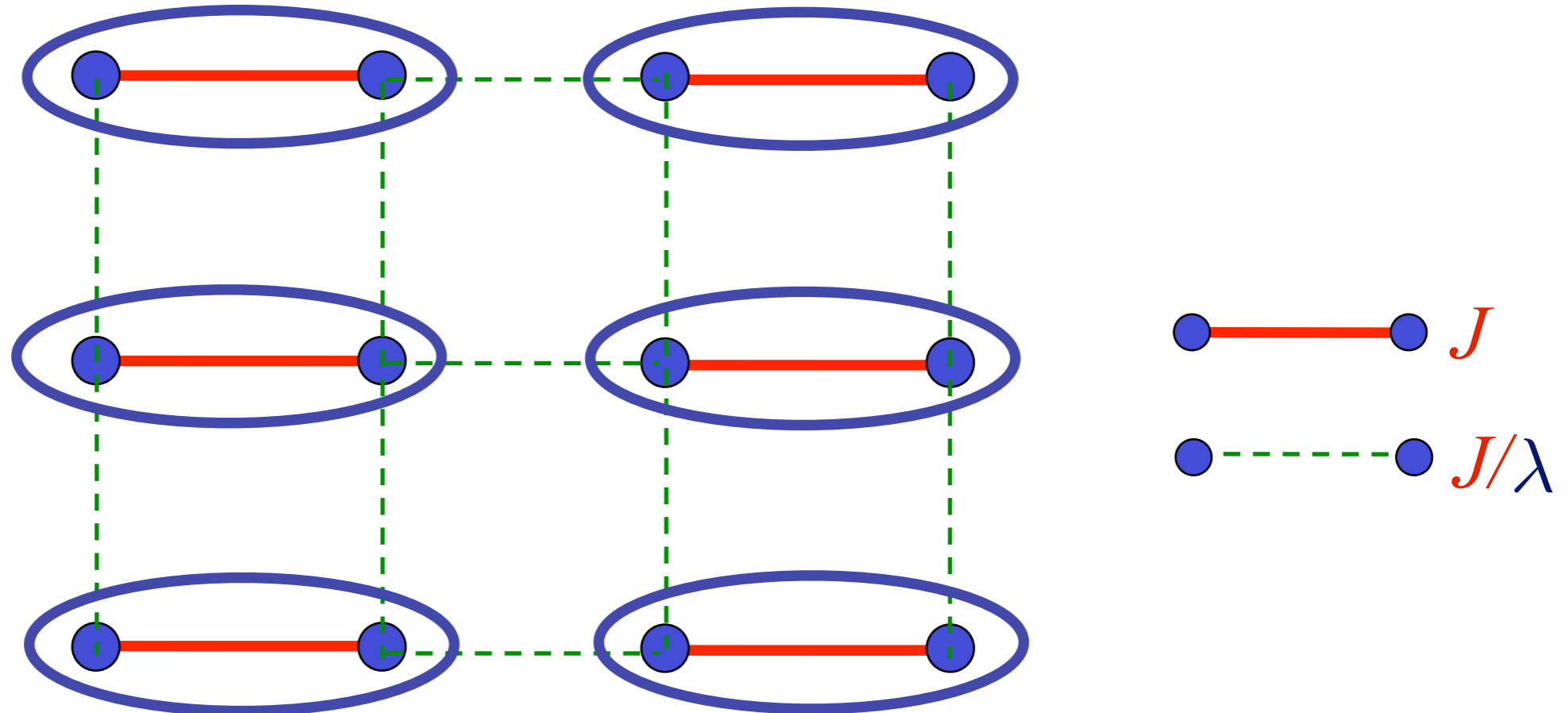
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

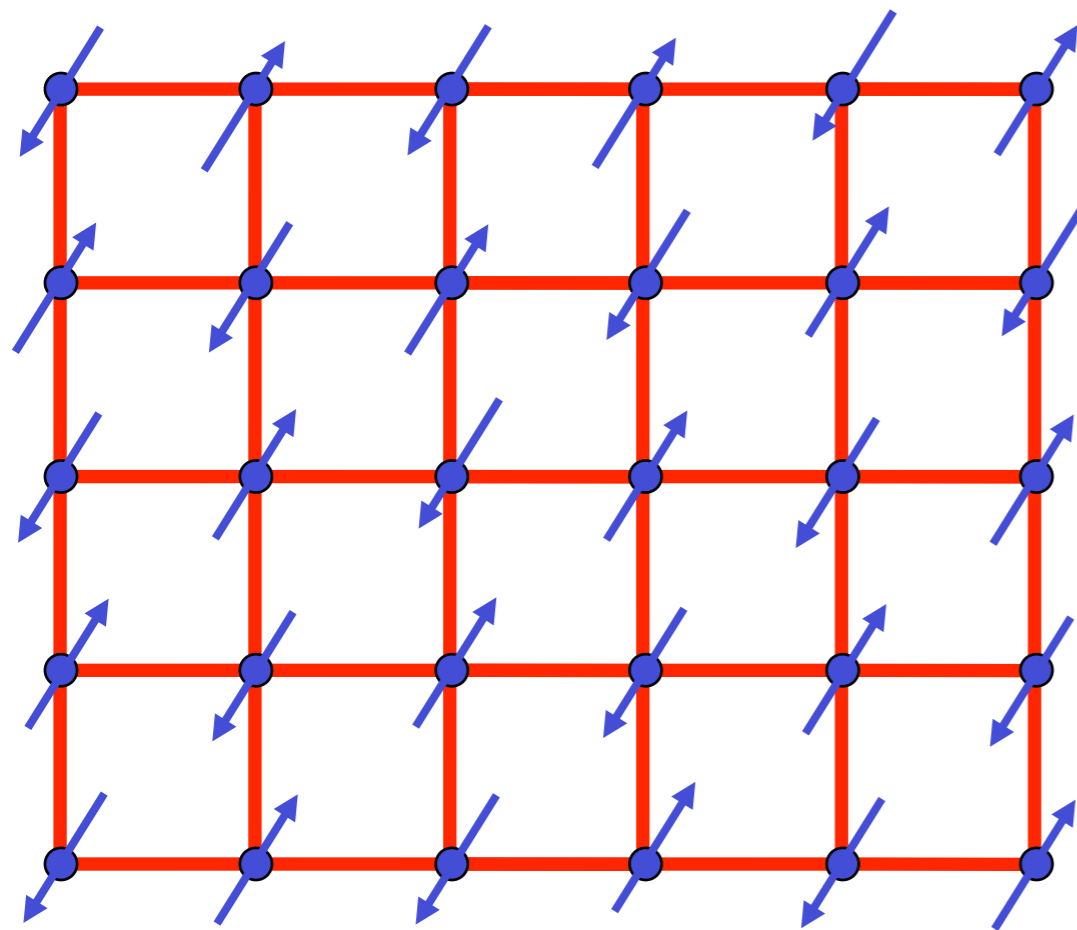
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

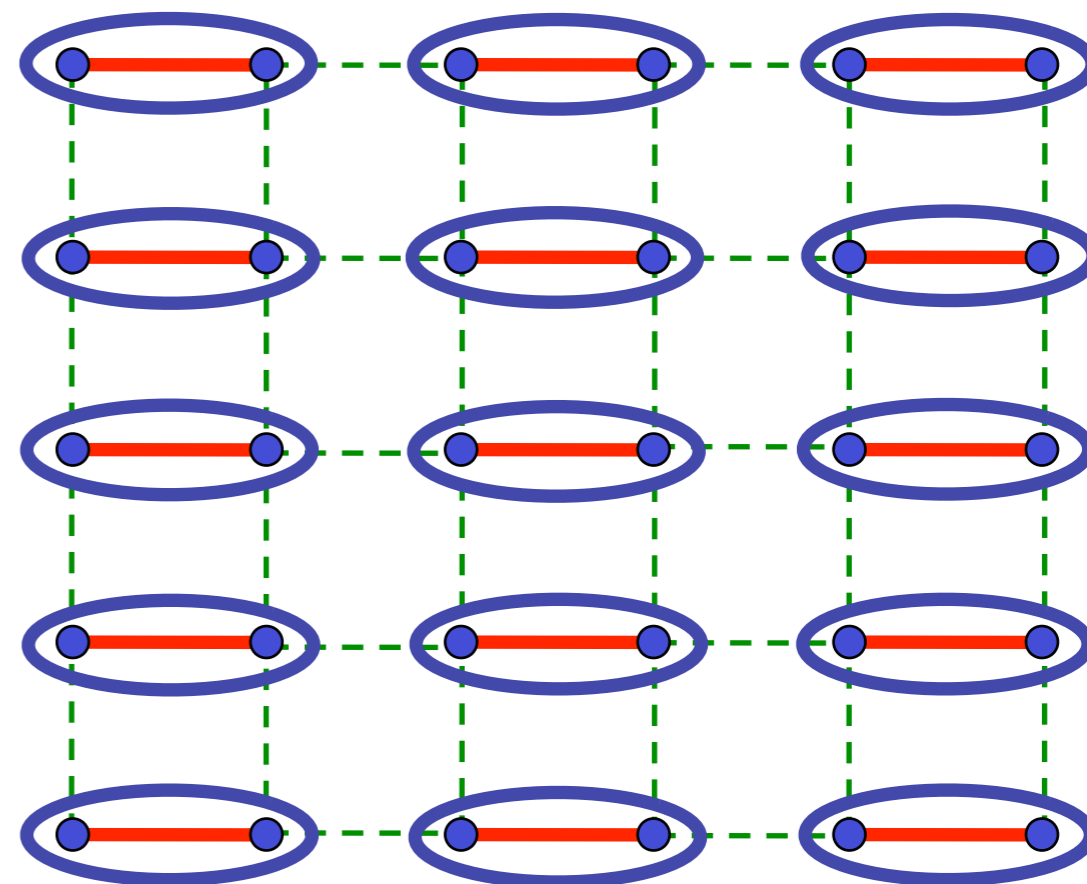


Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



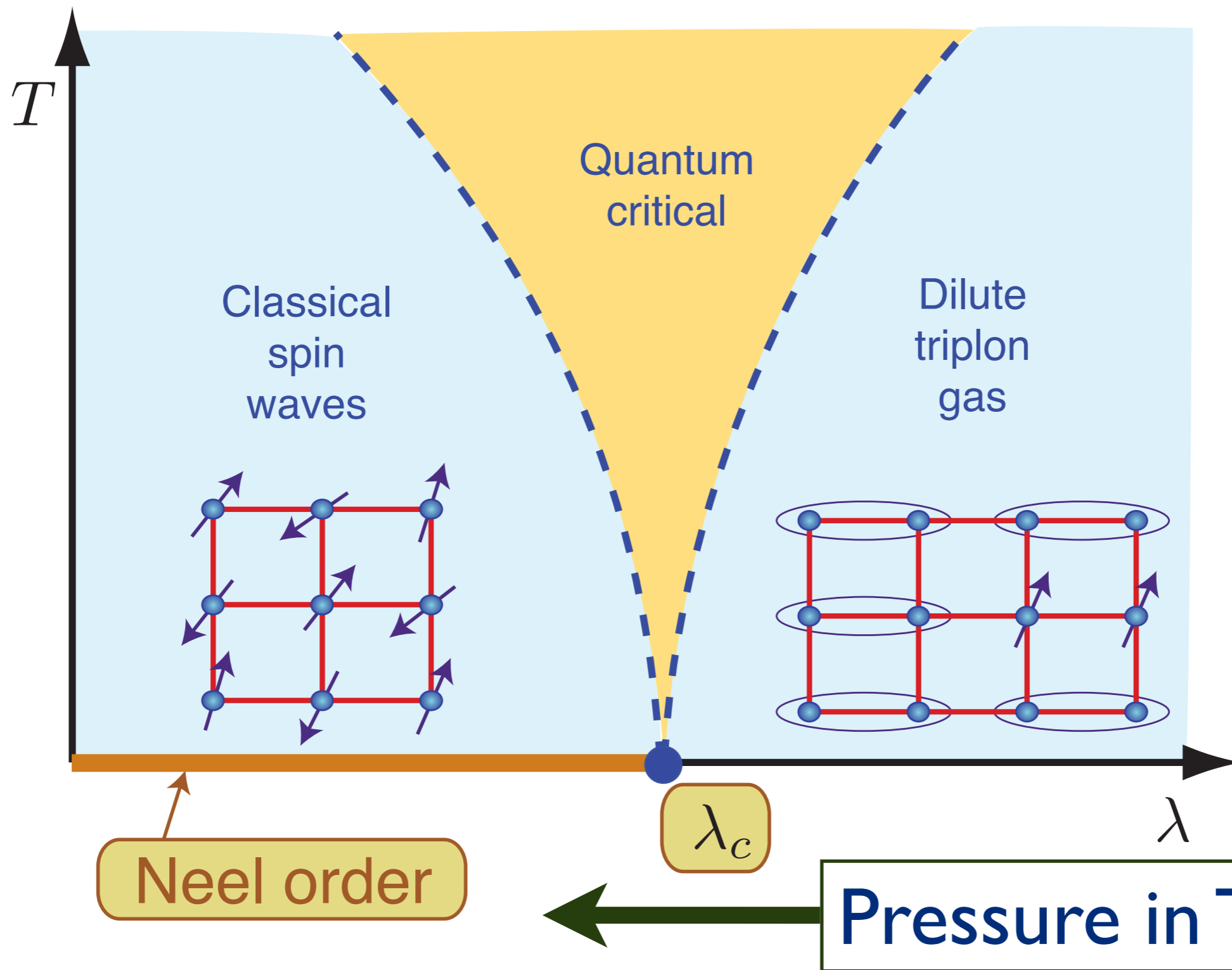
$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Pressure in TlCuCl_3

Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

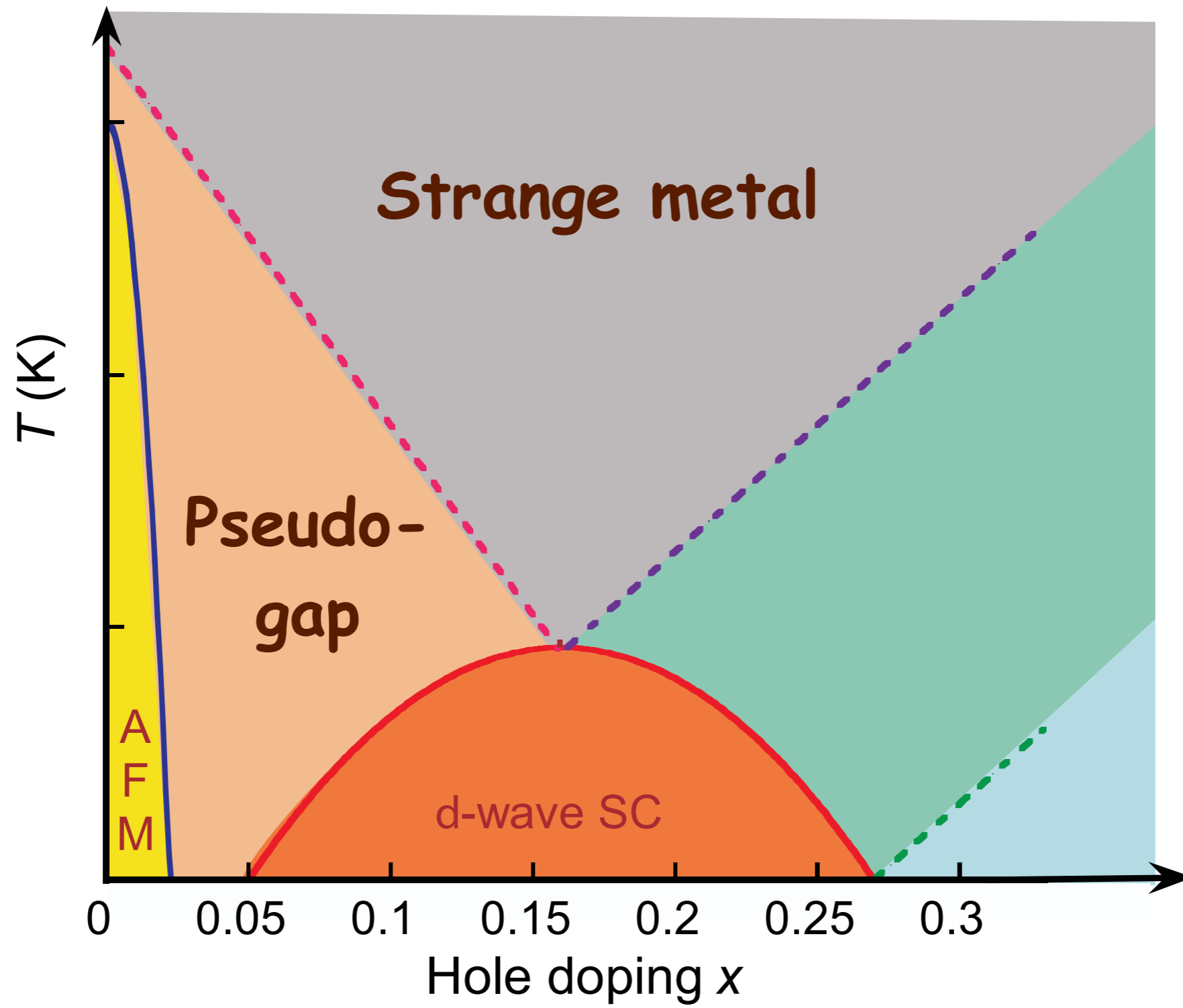
Canonical quantum critical phase diagram of coupled-dimer antiferromagnet



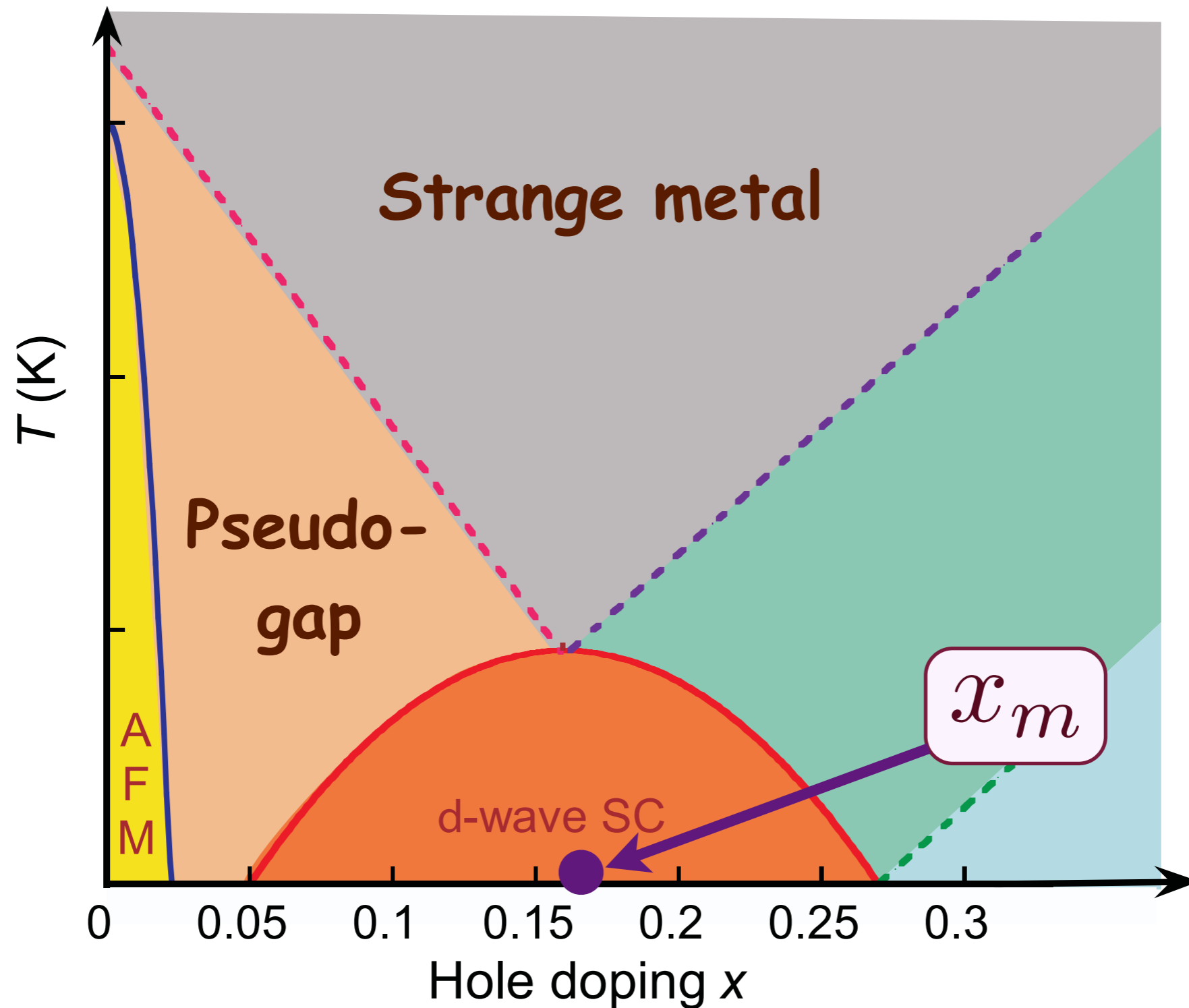
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Christian Rugg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

Crossovers in transport properties of hole-doped cuprates



Crossovers in transport properties of hole-doped cuprates



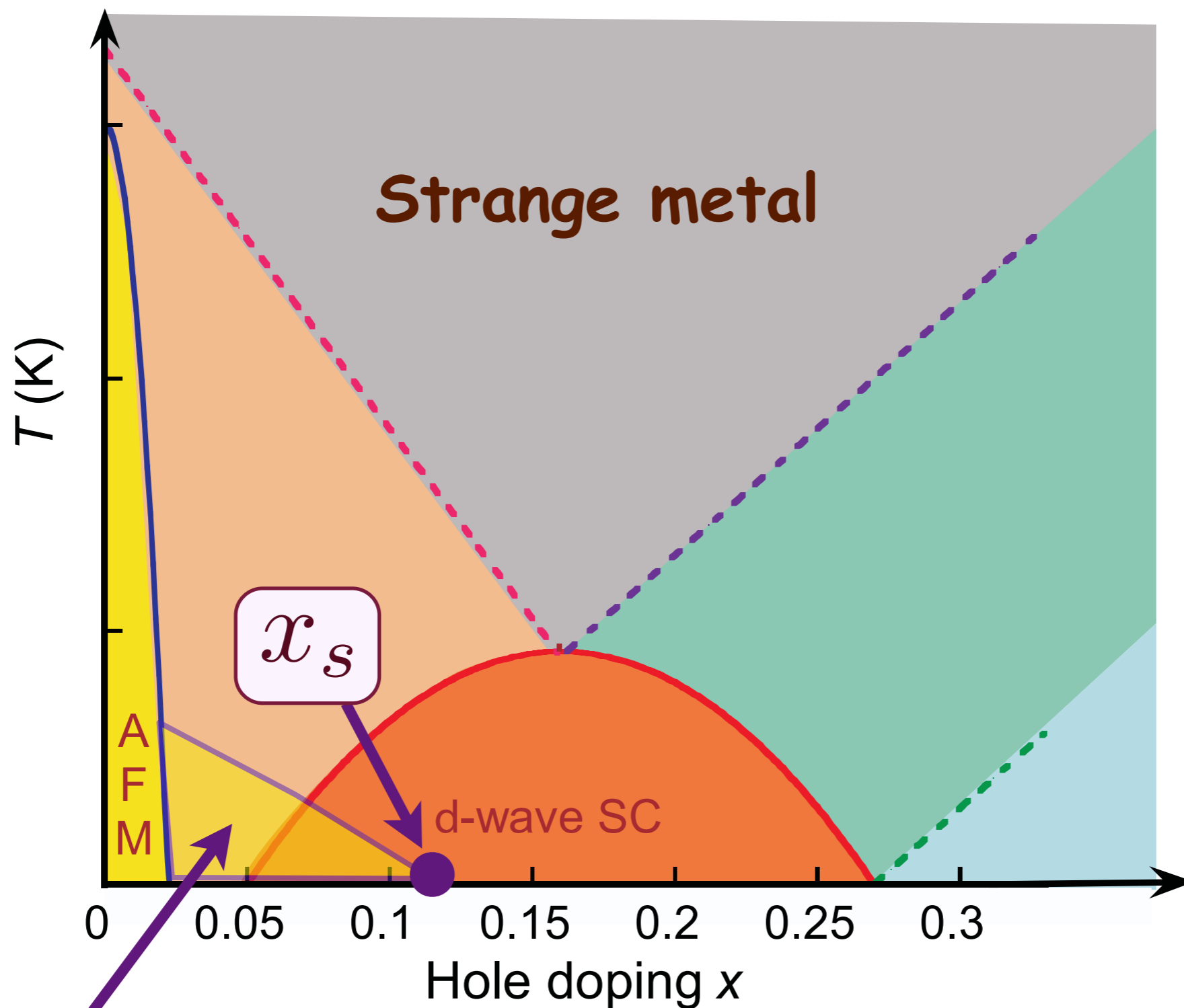
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

A. J. Millis, *Phys. Rev. B* **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

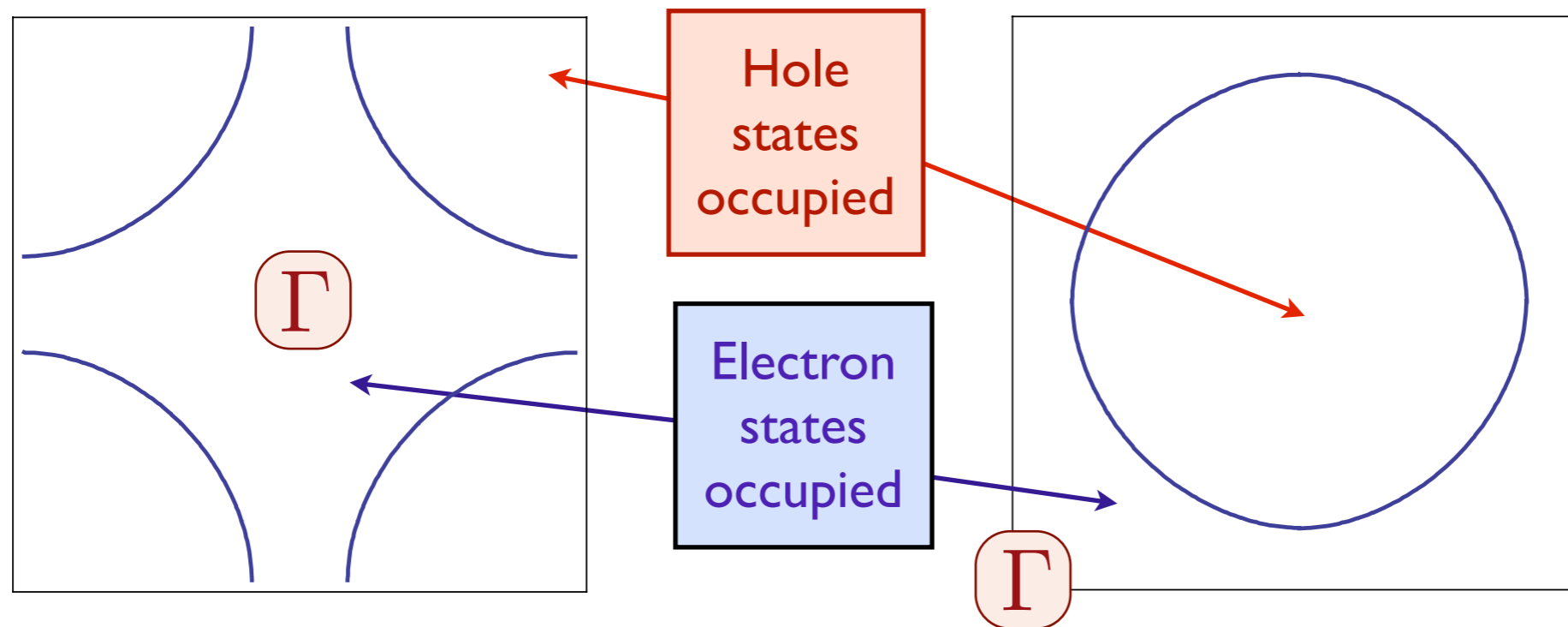
**Fermi
surface**

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

**Fermi
surface**

“Large” Fermi surfaces in cuprates



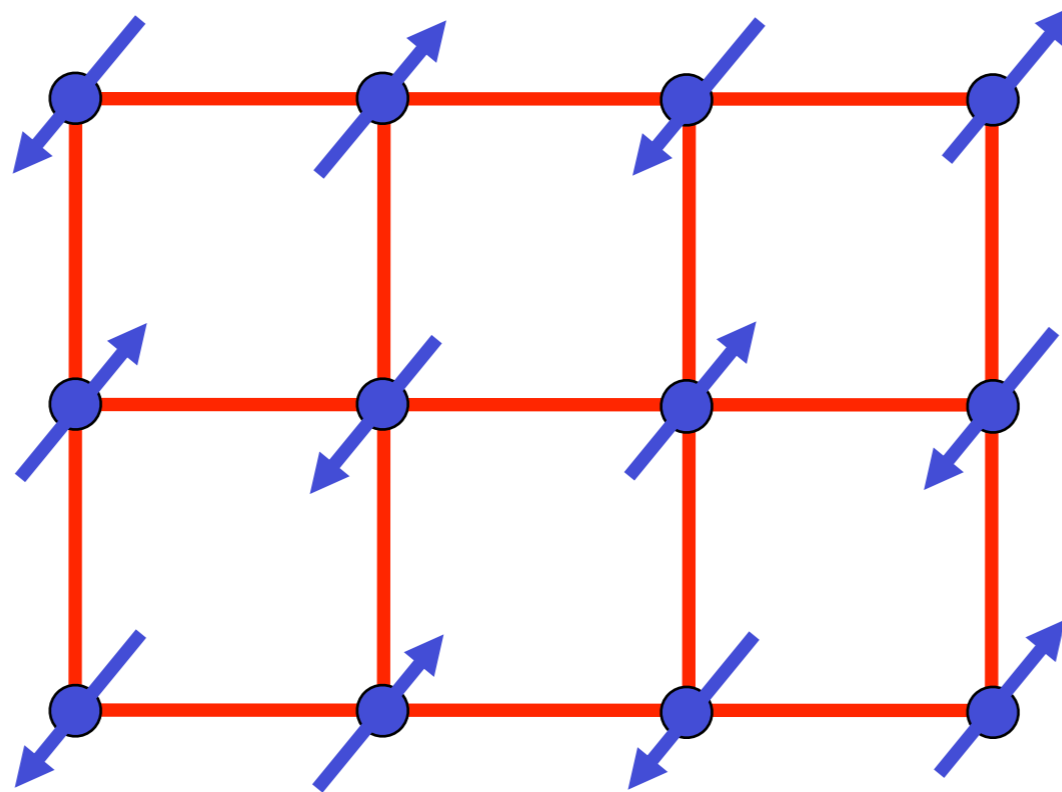
$$H_0 = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$A_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$A_h = 4\pi^2 - A_e$$

Spin density wave theory

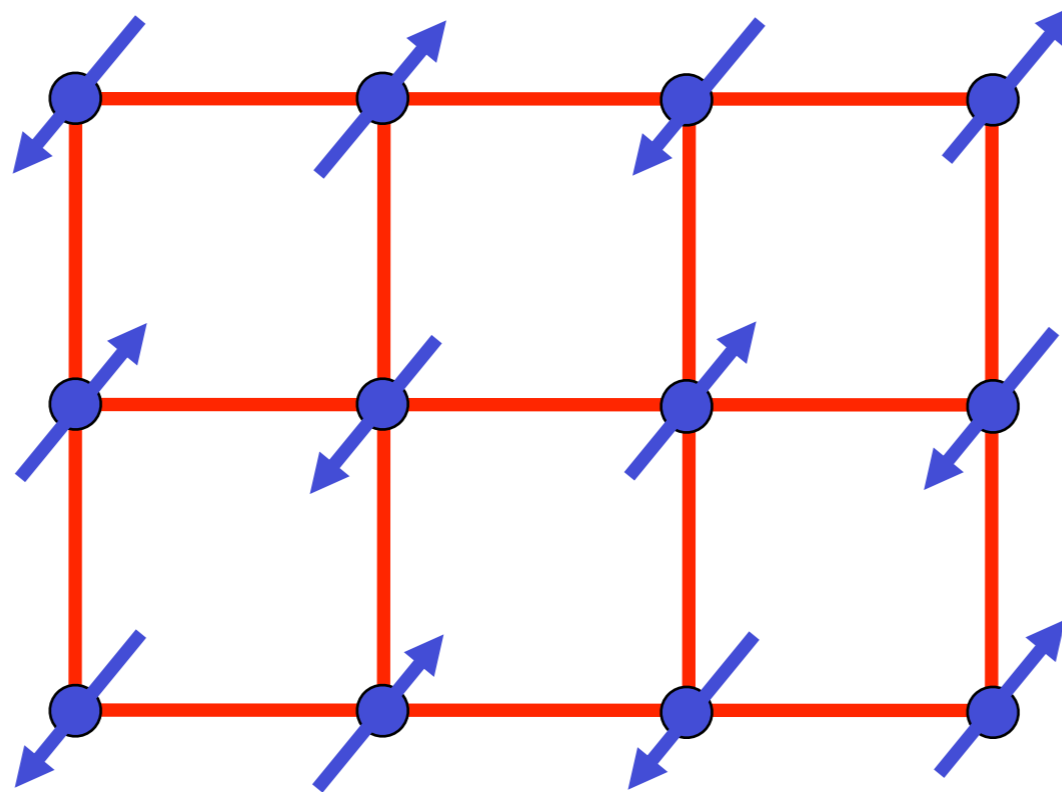


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and \mathbf{K} is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



Spin density wave Hamiltonian

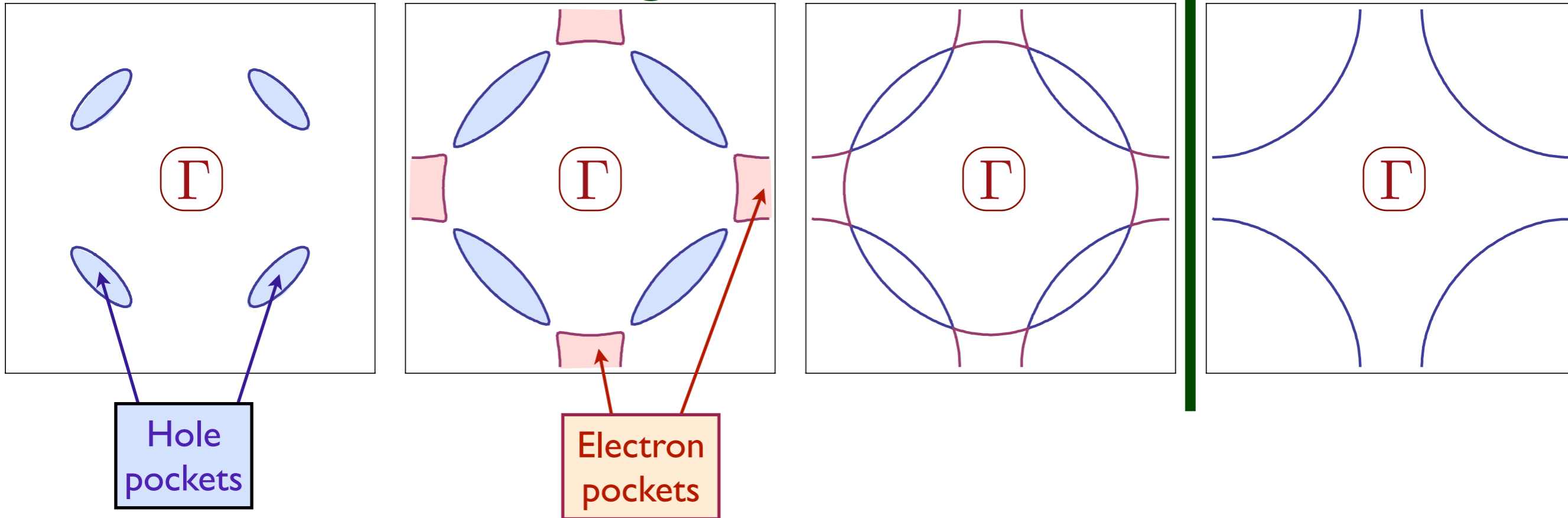
$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}, \beta}$$

Diagonalize $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \varphi^2}$$

Hole-doped cuprates

← Increasing SDW order →

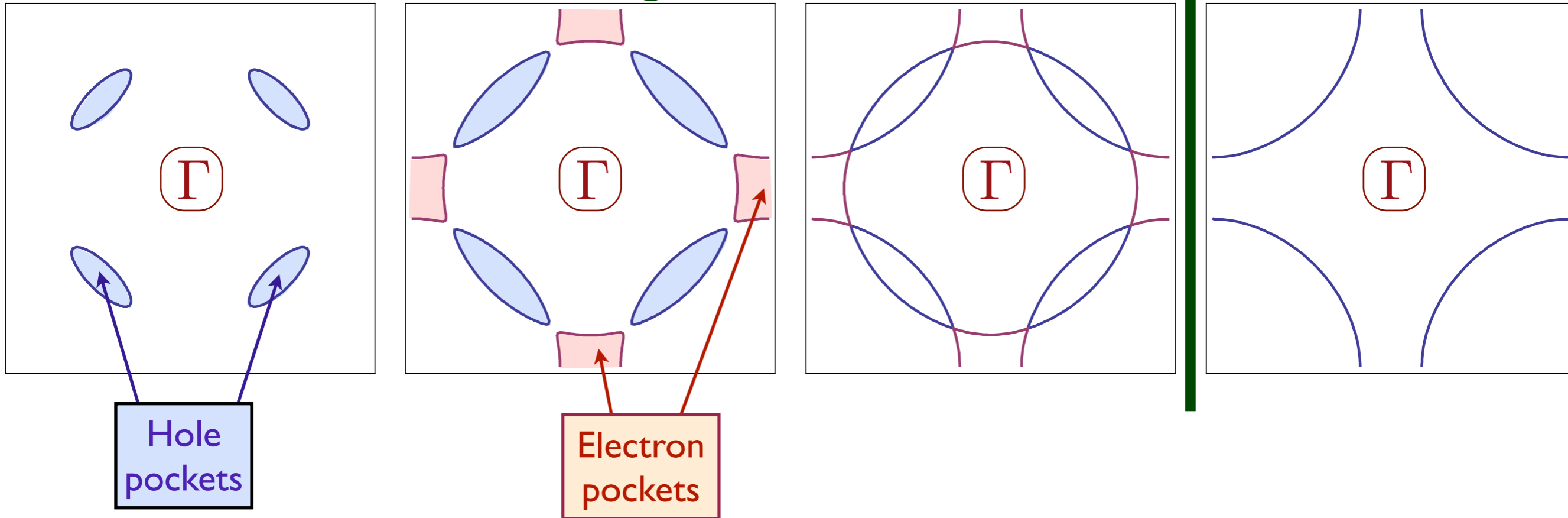


Large Fermi surface breaks up into
electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates

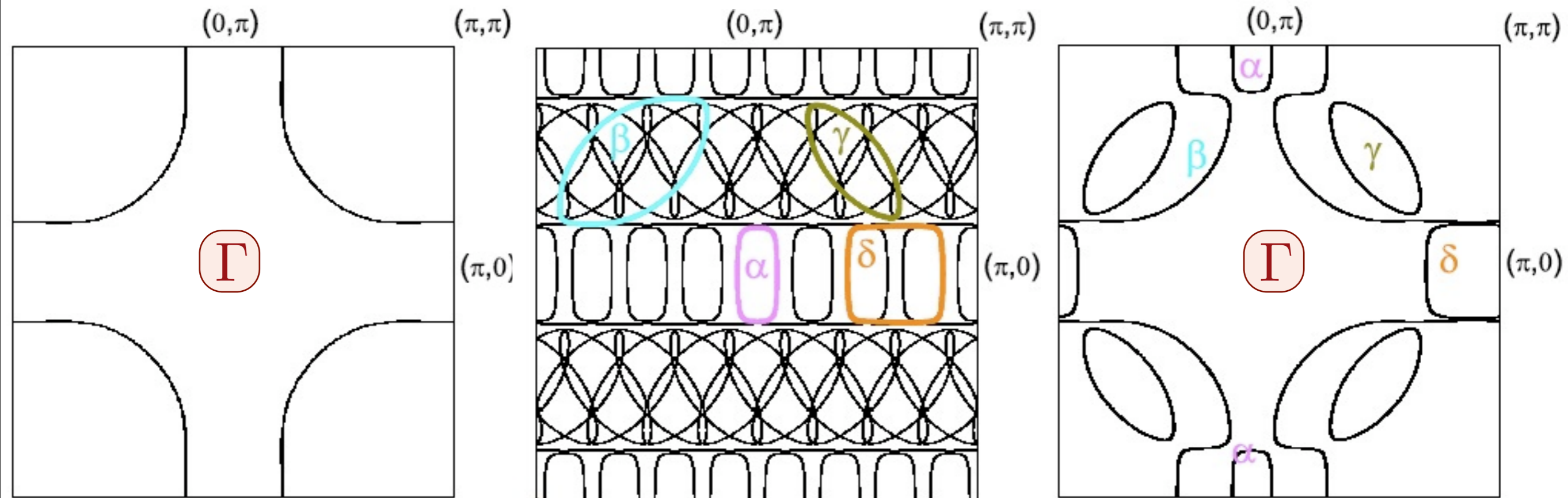
← Increasing SDW order →



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



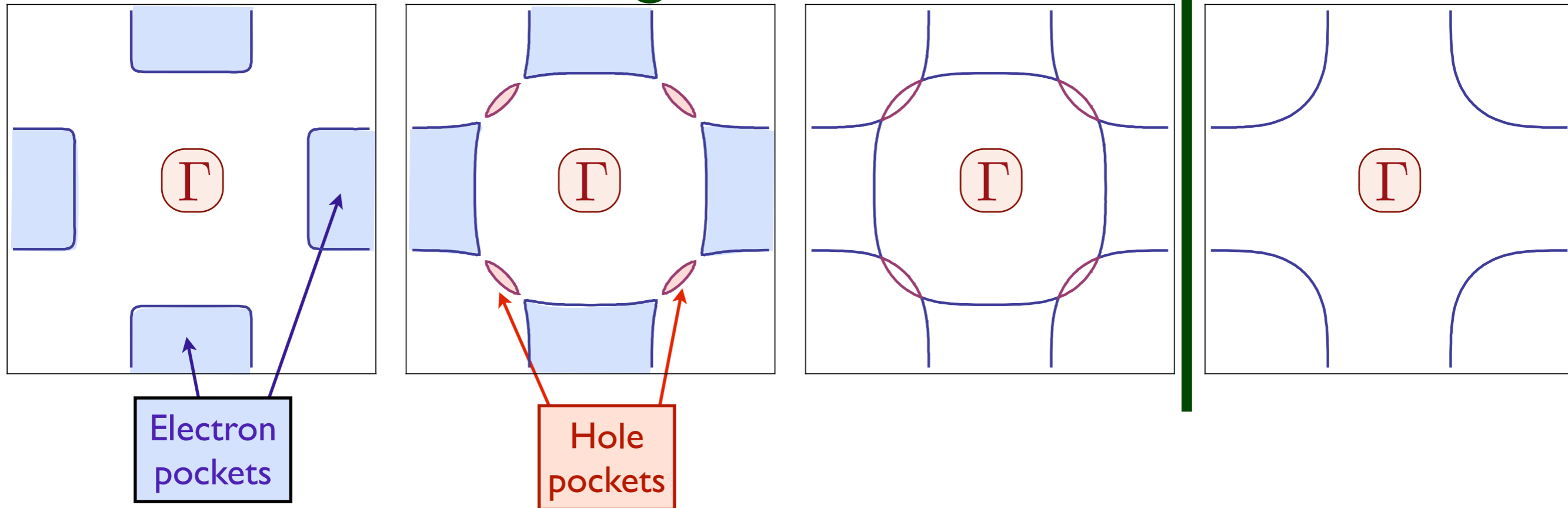
Incommensurate order in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).

N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Electron-doped cuprates

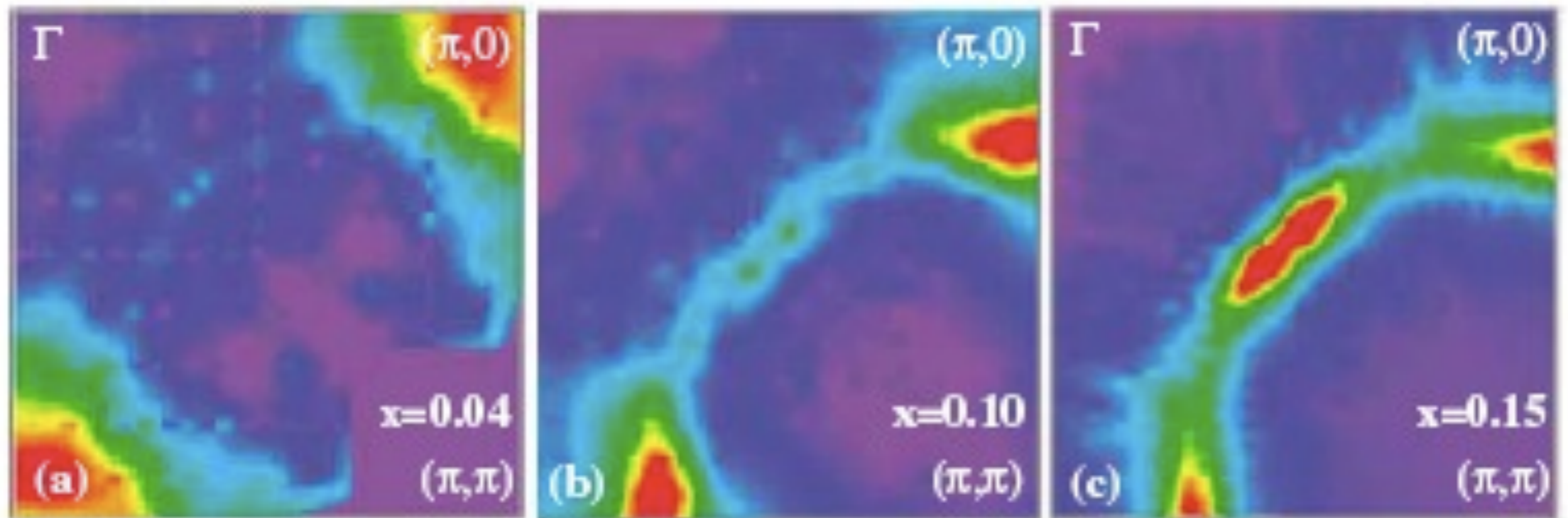
← Increasing SDW order →



Large Fermi surface breaks up into
electron and hole pockets

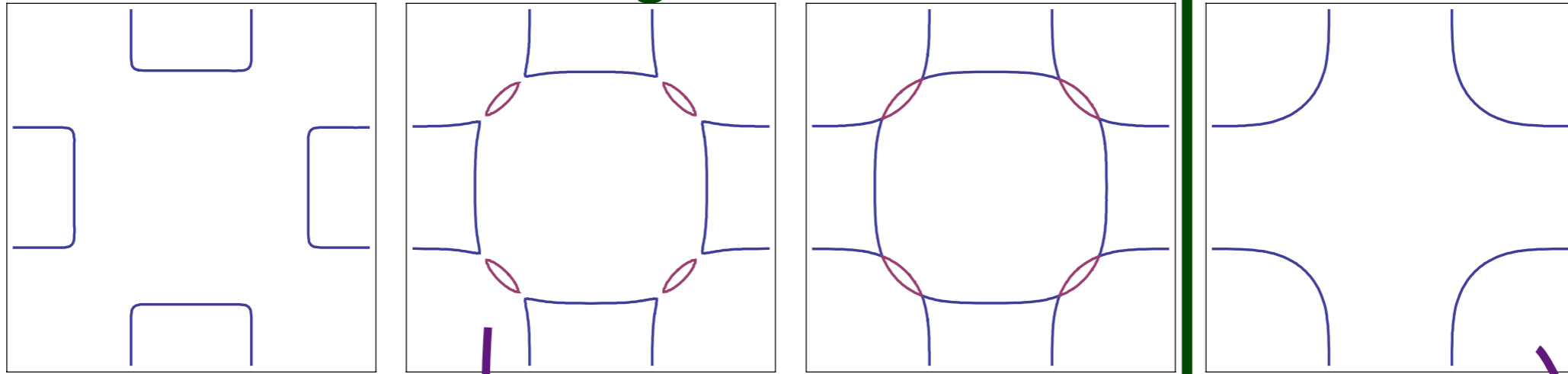
D. Senechal and A.-M. S. Tremblay, *Physical Review Letters* **92**, 126401 (2004)
J. Lin, and A. J. Millis, *Physical Review B* **72**, 214506 (2005).

Photoemission in NCCO



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

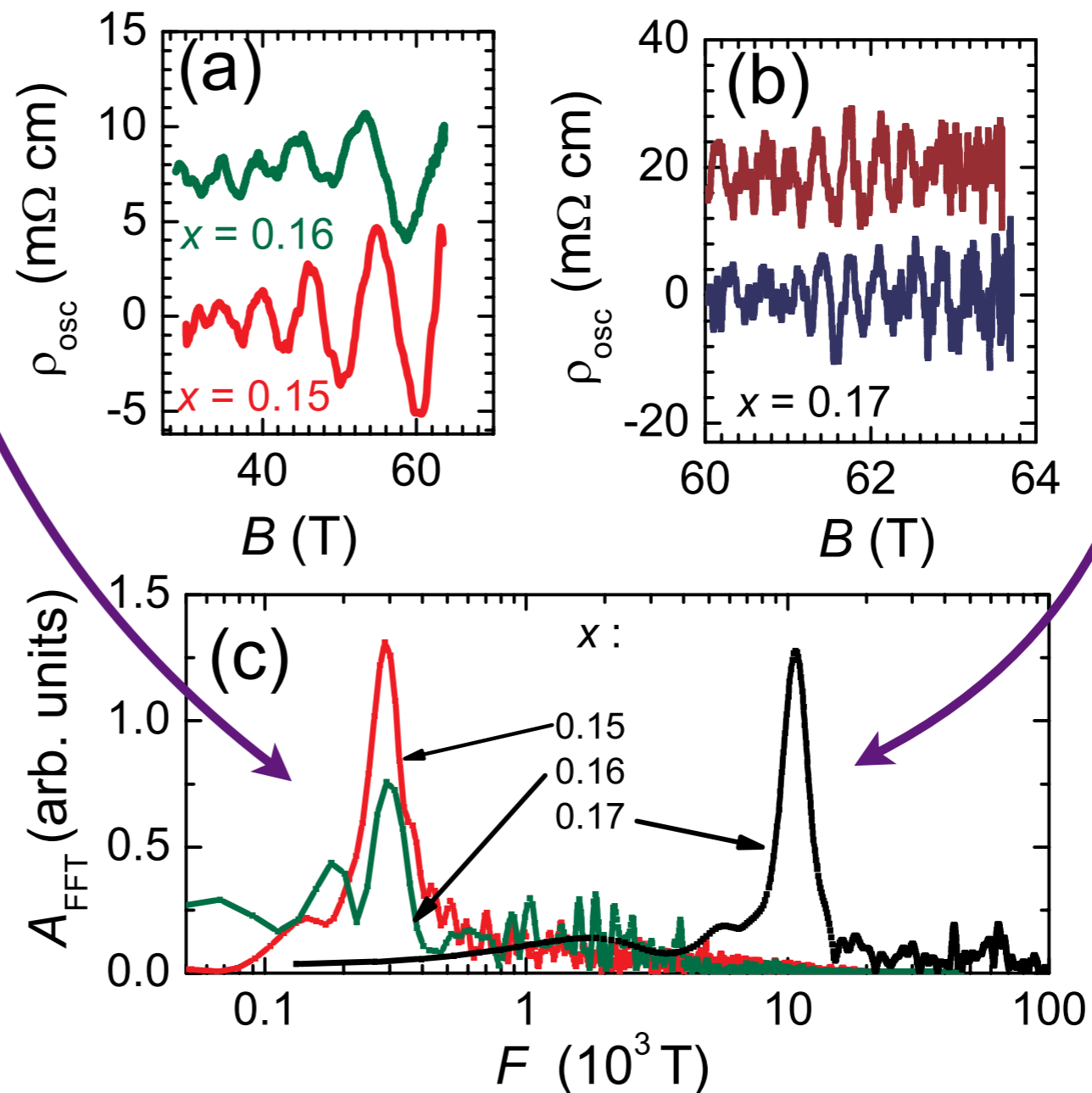
← Increasing SDW order →



Quantum oscillations



T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

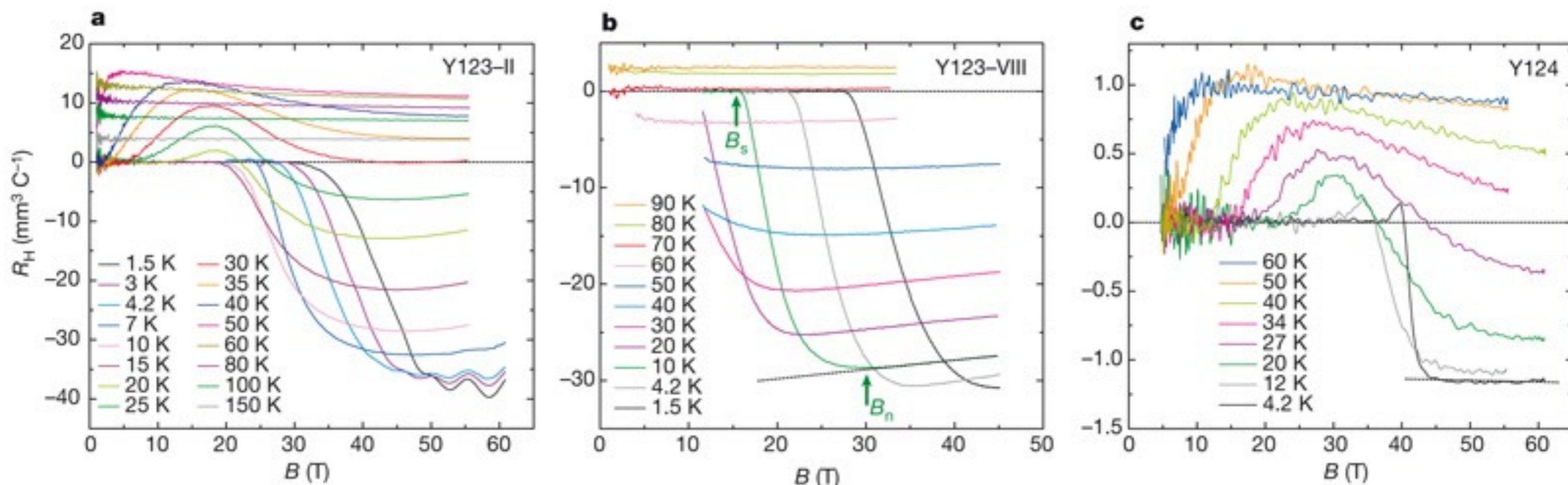


Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high- T_c superconductors

David LeBoeuf¹, Nicolas Doiron-Leyraud¹, Julien Levallois², R. Daou¹, J.-B. Bonnemaïson¹, N. E. Hussey³, L. Balicas⁴, B. J. Ramshaw⁵, Ruixing Liang^{5,6}, D. A. Bonn^{5,6}, W. N. Hardy^{5,6}, S. Adachi⁷, Cyril Proust² & Louis Taillefer^{1,6}

Nature **450**, 533 (2007)

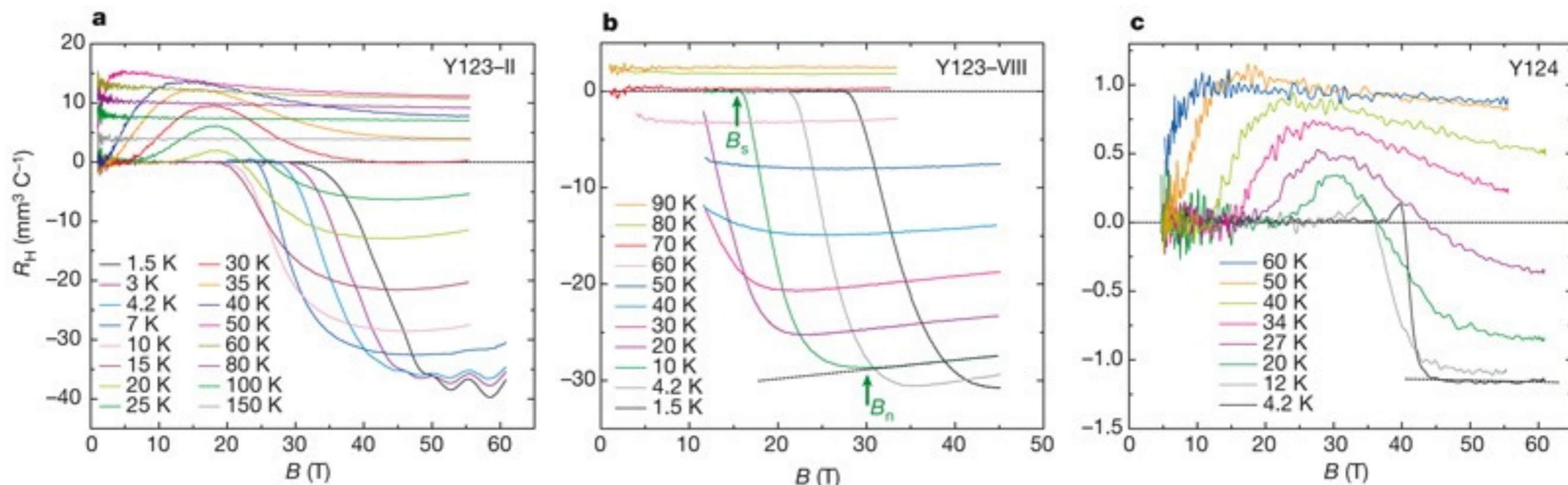


Quantum oscillations

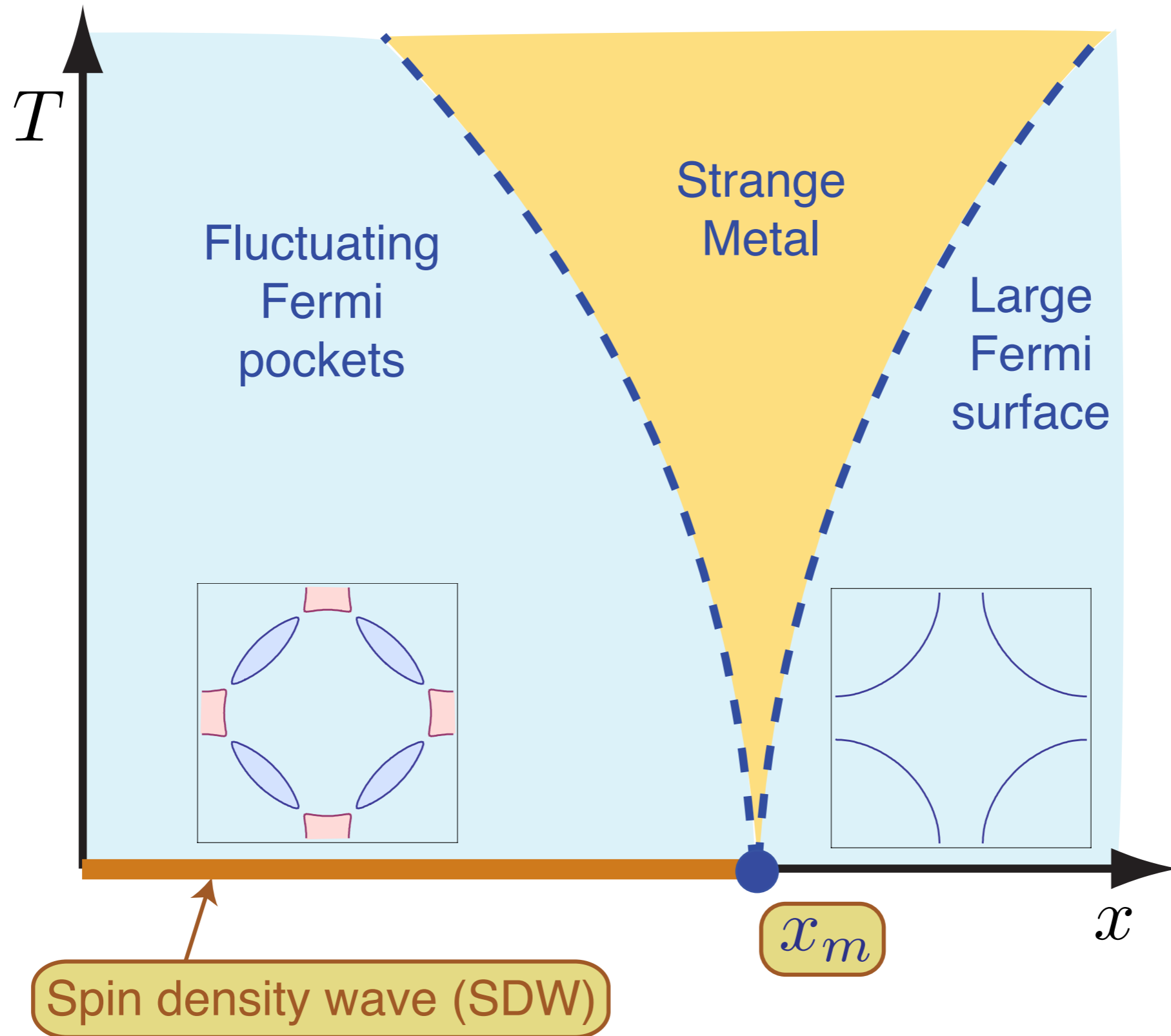
Electron pockets in the Fermi surface of hole-doped high- T_c superconductors

David LeBoeuf¹, Nicolas Doiron-Leyraud¹, Julien Levallois², R. Daou¹, J.-B. Bonnemaïson¹, N. E. Hussey³, L. Balicas⁴, B. J. Ramshaw⁵, Ruixing Liang^{5,6}, D. A. Bonn^{5,6}, W. N. Hardy^{5,6}, S. Adachi⁷, Cyril Proust² & Louis Taillefer^{1,6}

Nature **450**, 533 (2007)

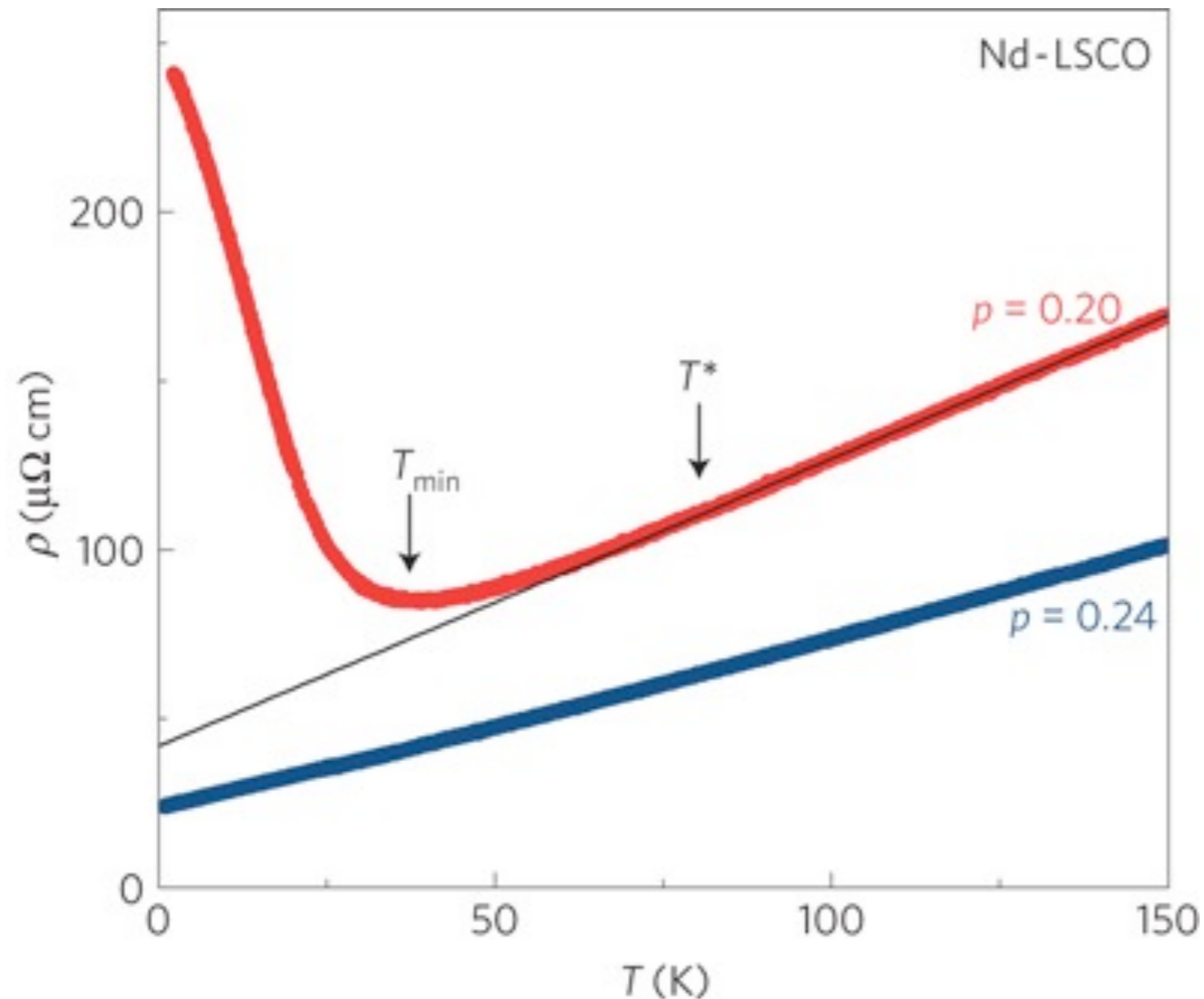


Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c

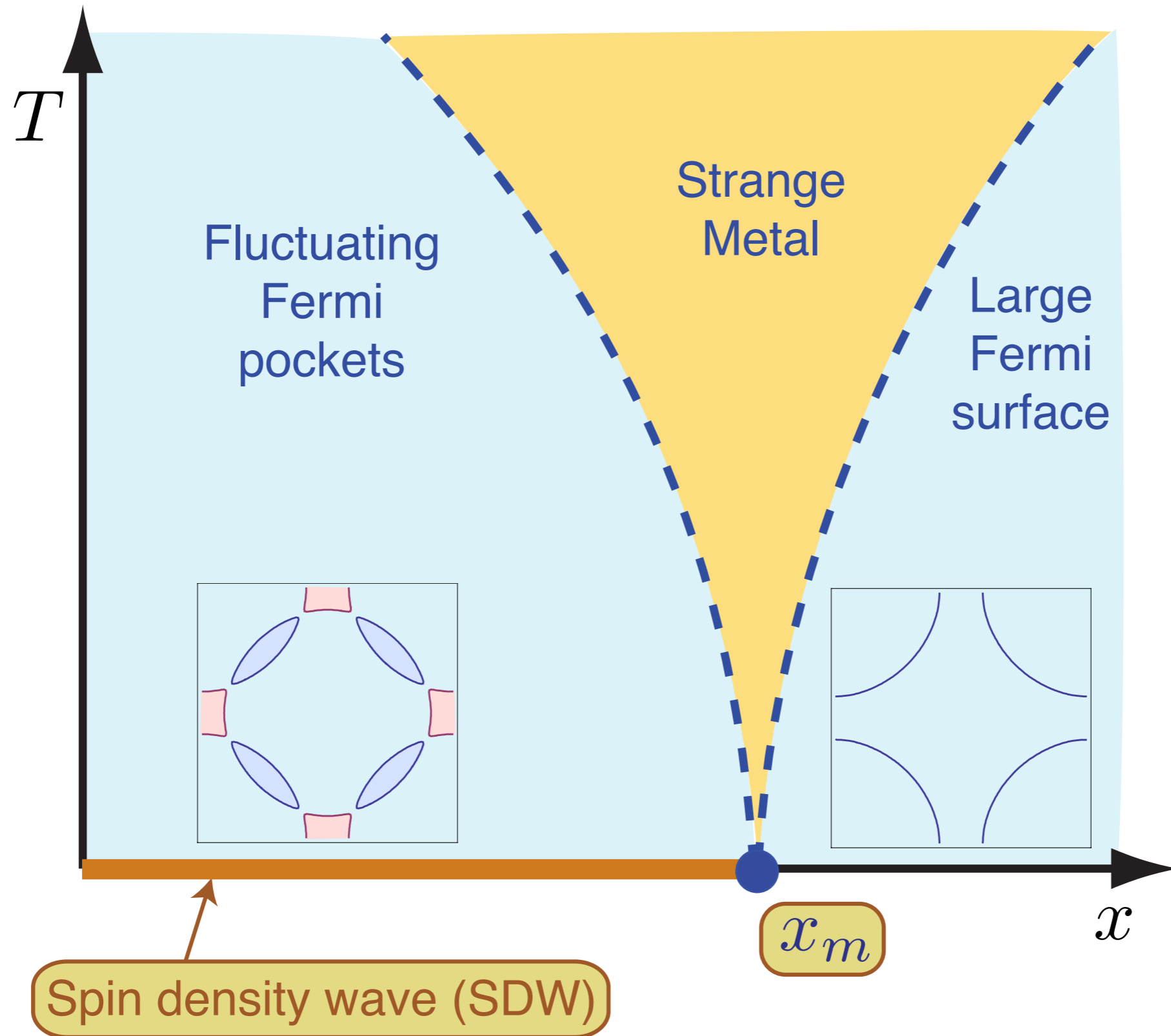


- Magnetic field of upto 35 T used to suppress superconductivity
- Identifies $x_m \approx 0.24$

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high- T_c superconductor

R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in metal at $x = x_m$



**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

**Fermi
surface**

**Spin
density
wave**

**d-wave
supercon-
ductivity**

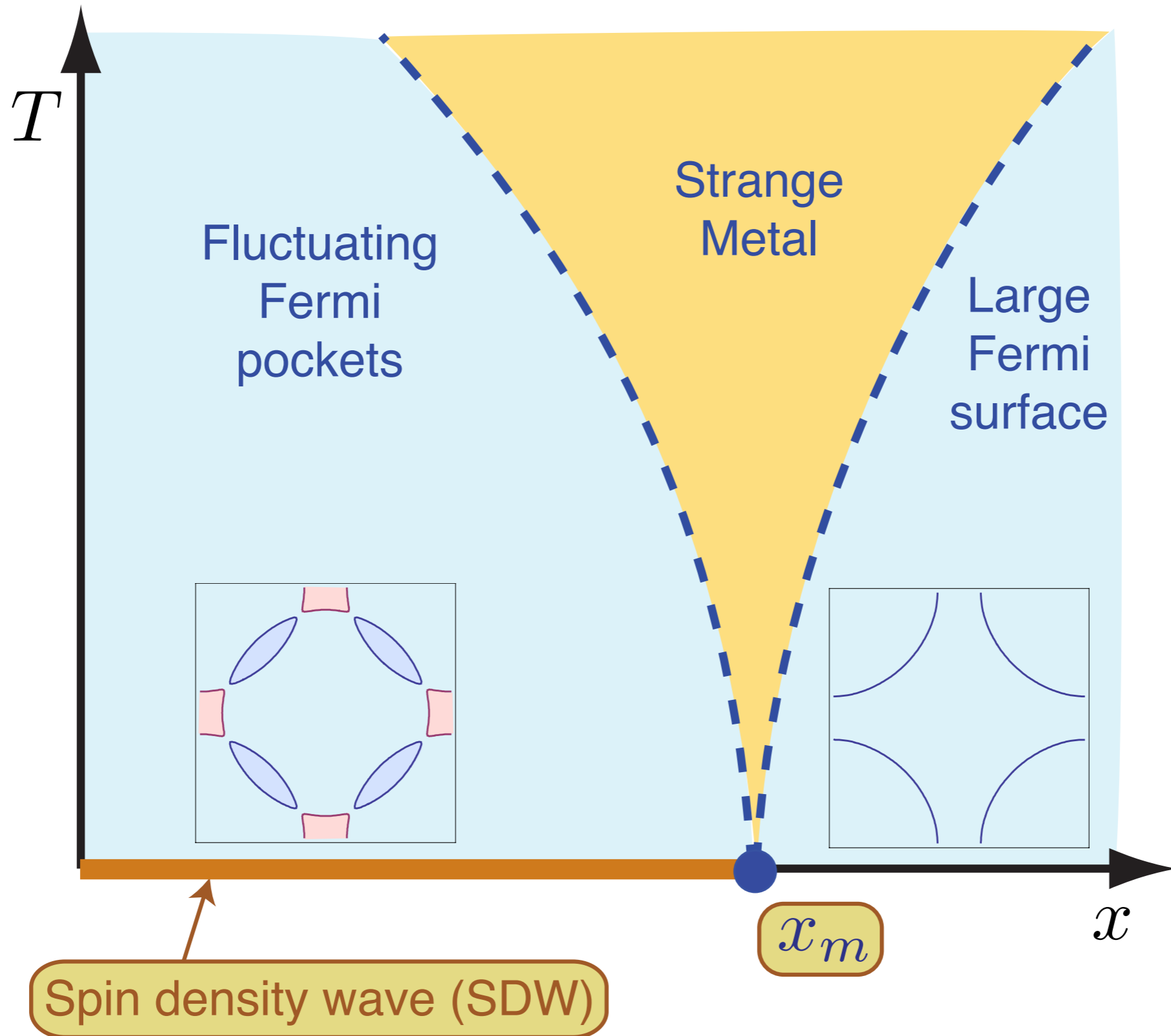
**Fermi
surface**

**Spin
density
wave**

**d-wave
supercon-
ductivity**

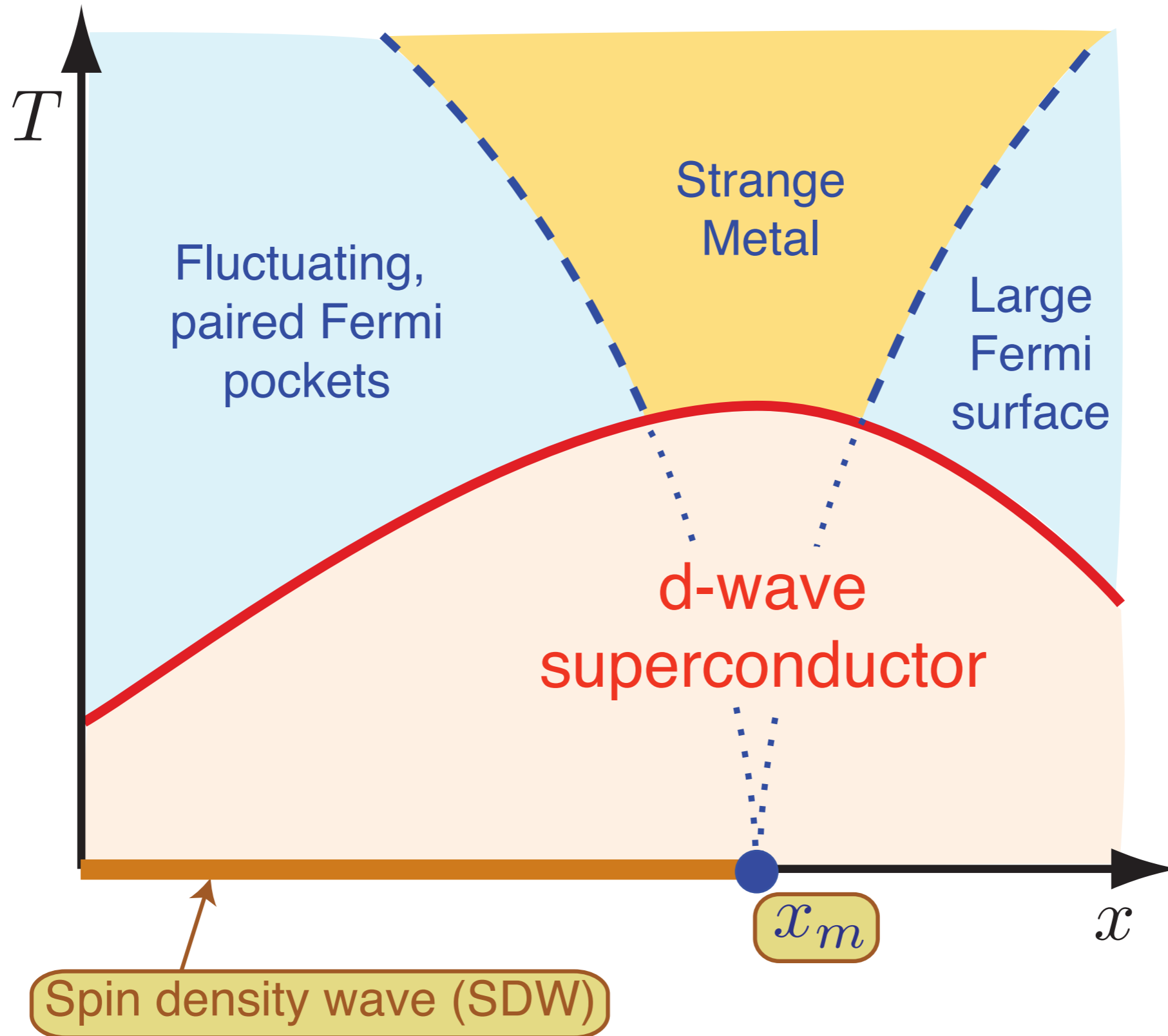
**Fermi
surface**

Theory of quantum criticality in the cuprates



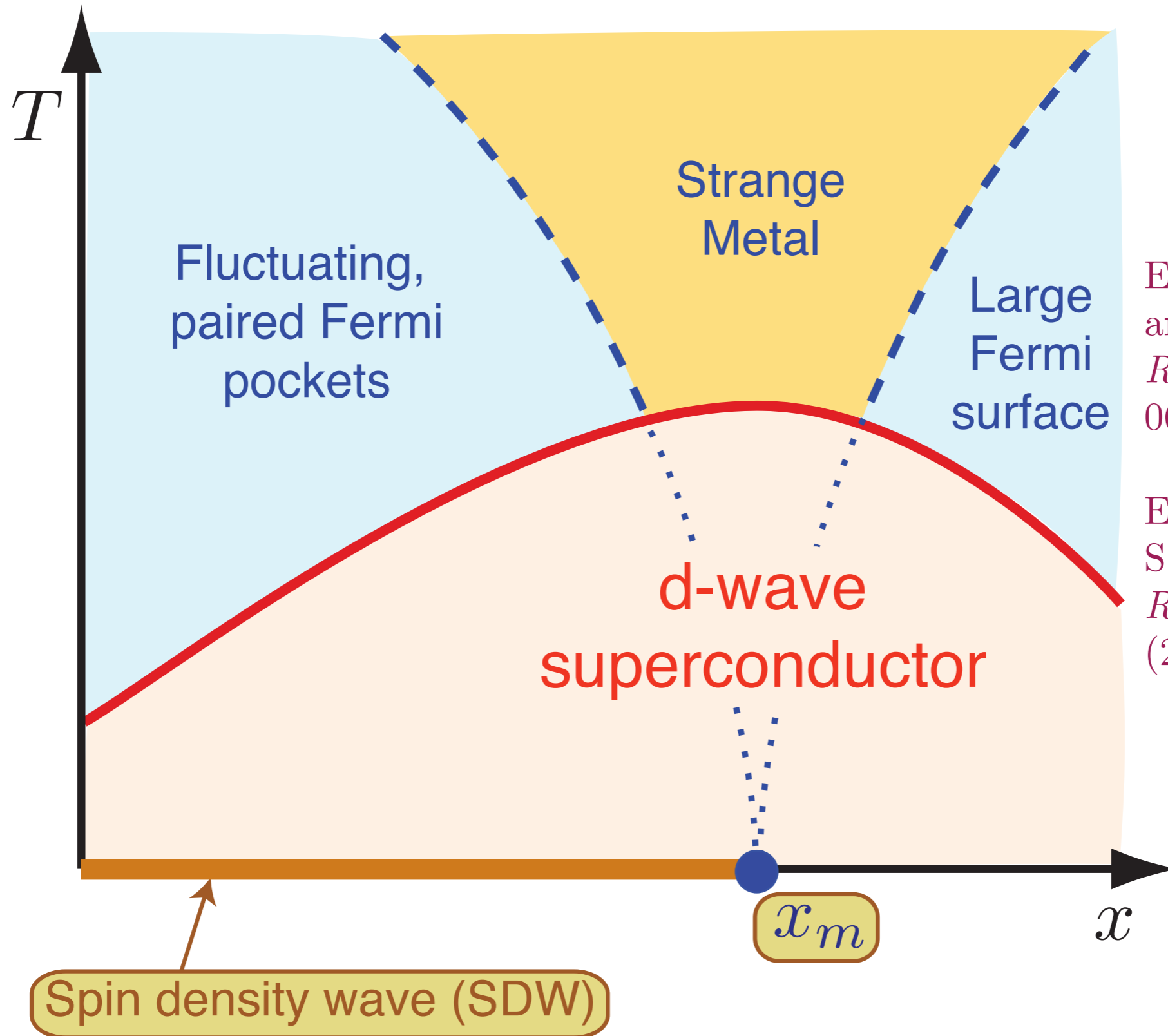
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



Onset of d -wave superconductivity
hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates

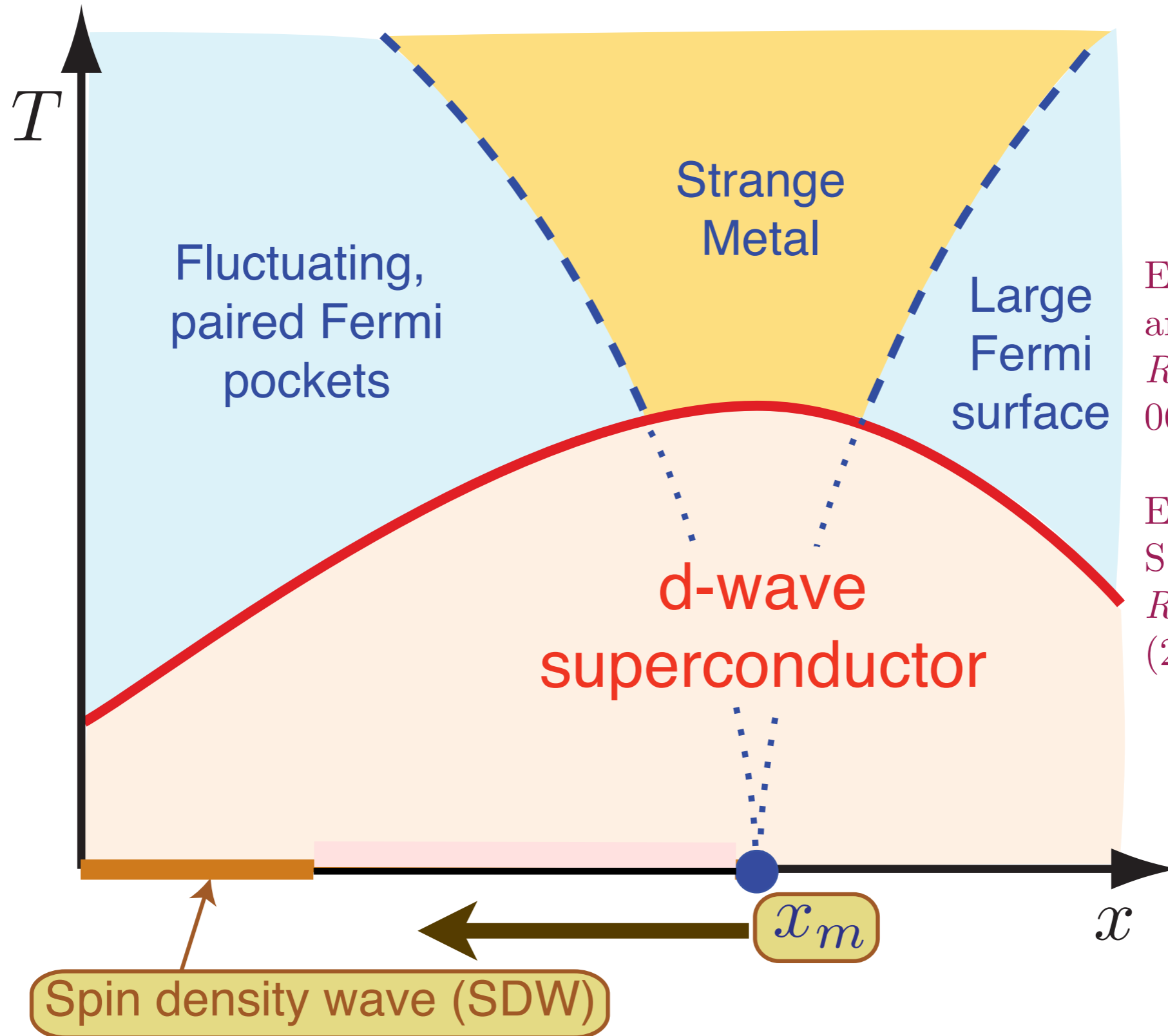


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

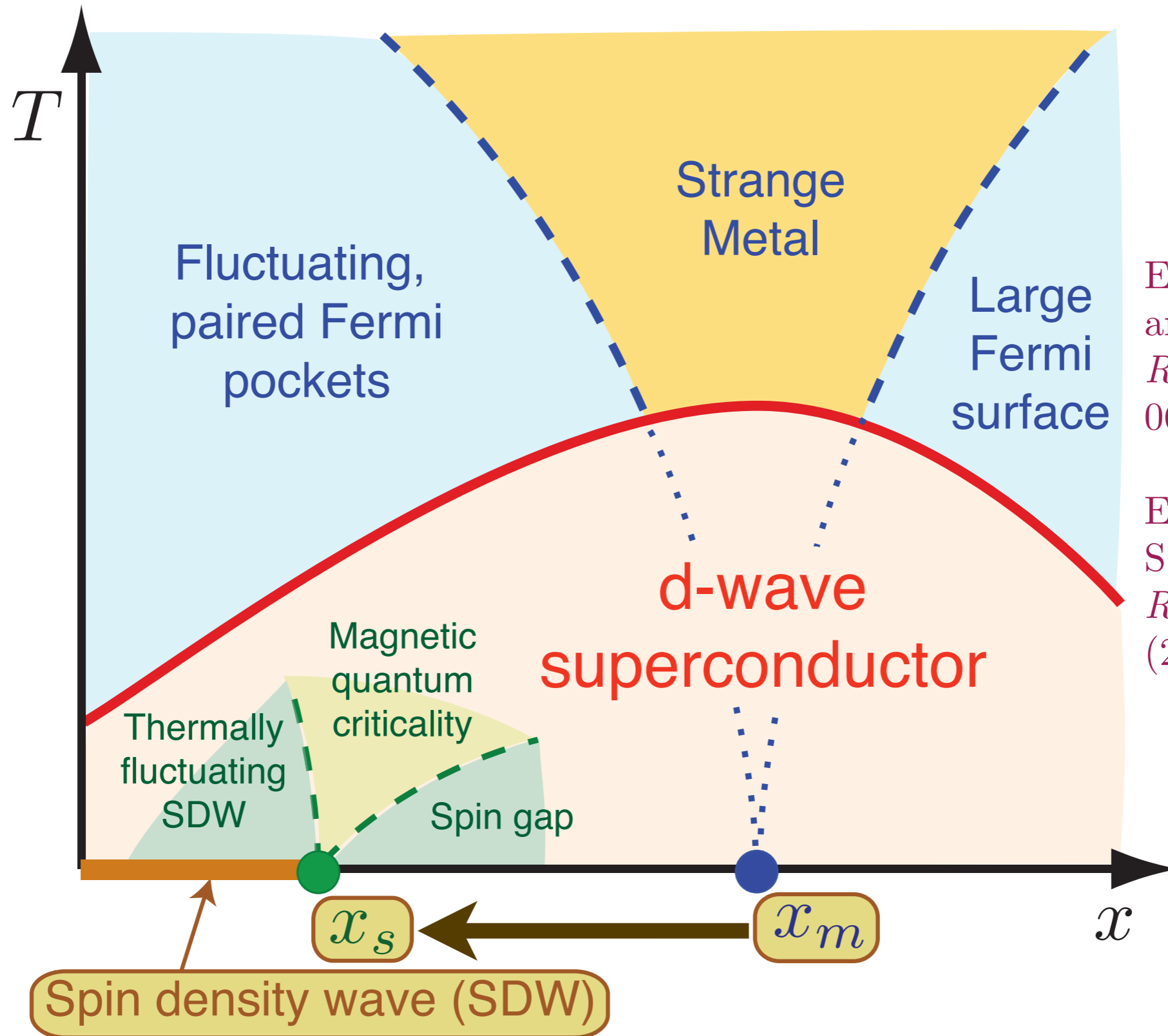


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

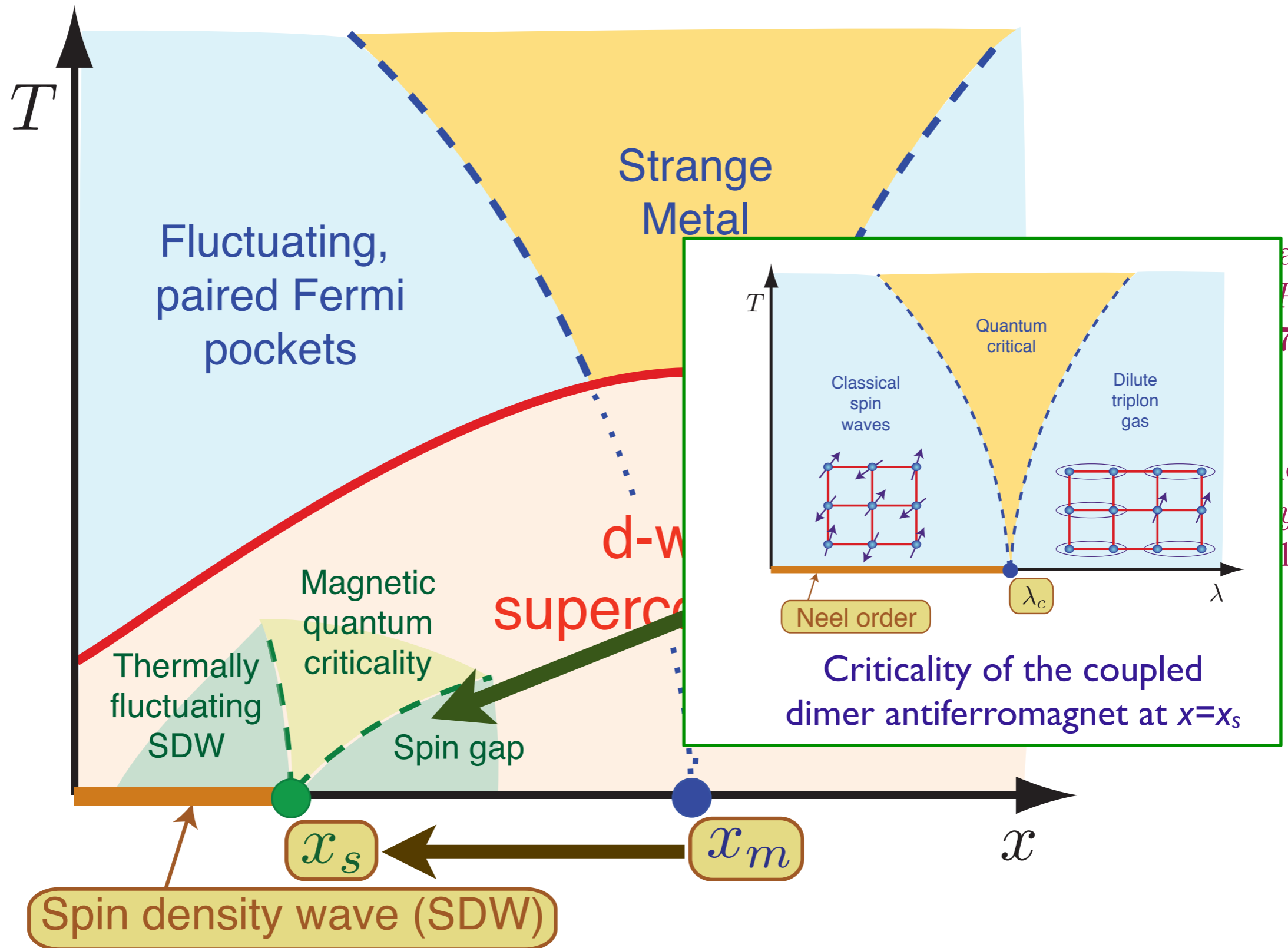


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

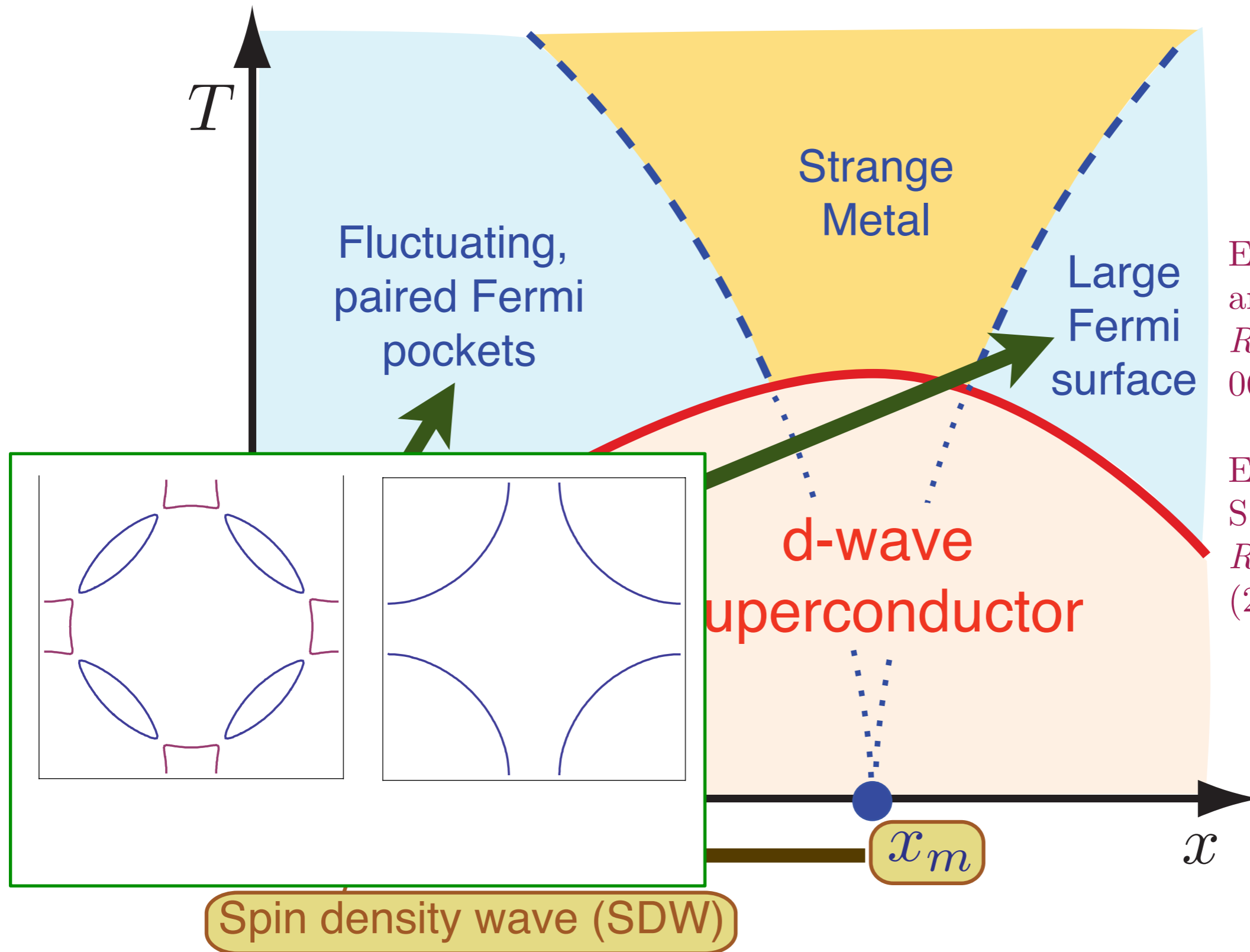
Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

achdev
Phys.
7,
d
y.
117

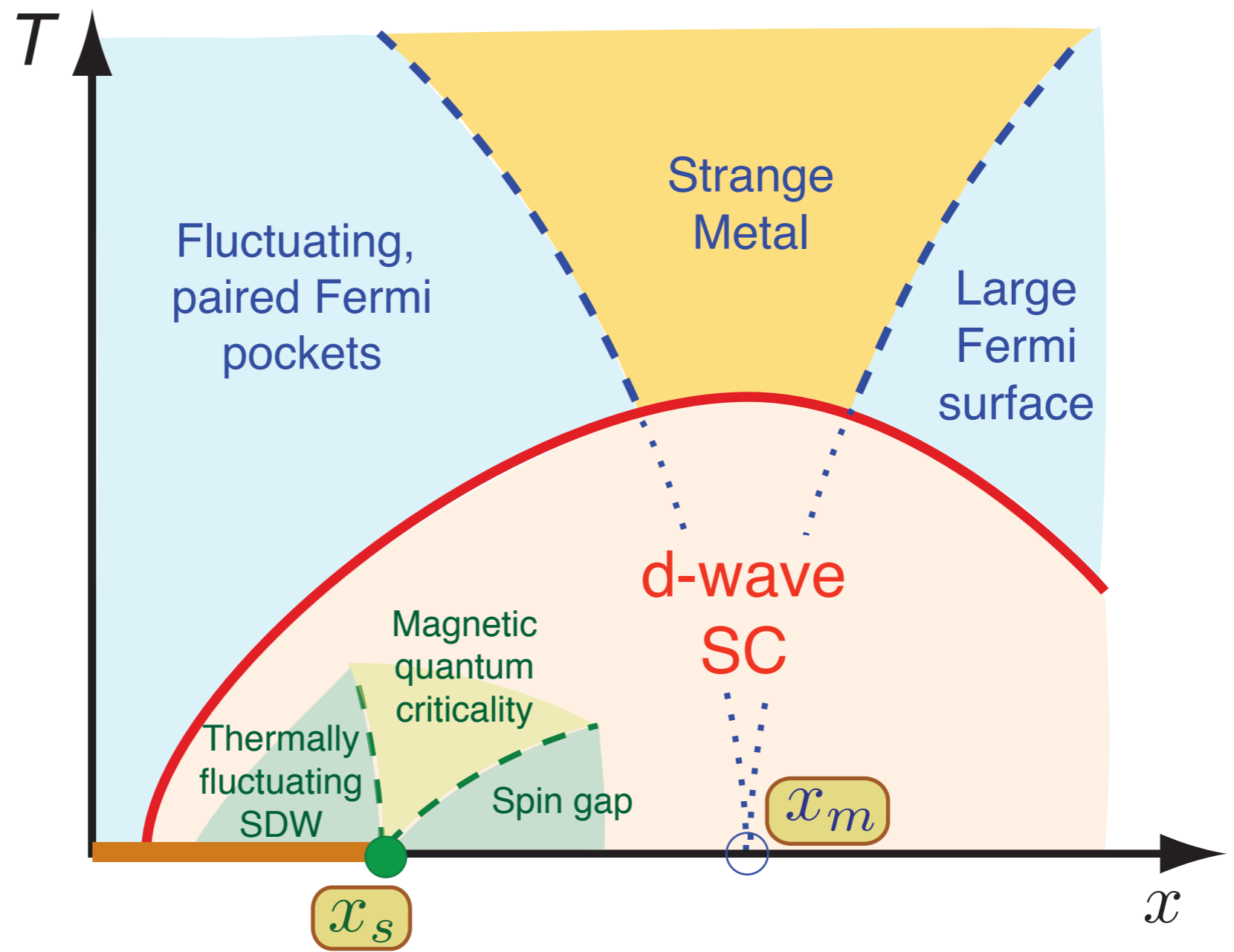
Theory of quantum criticality in the cuprates

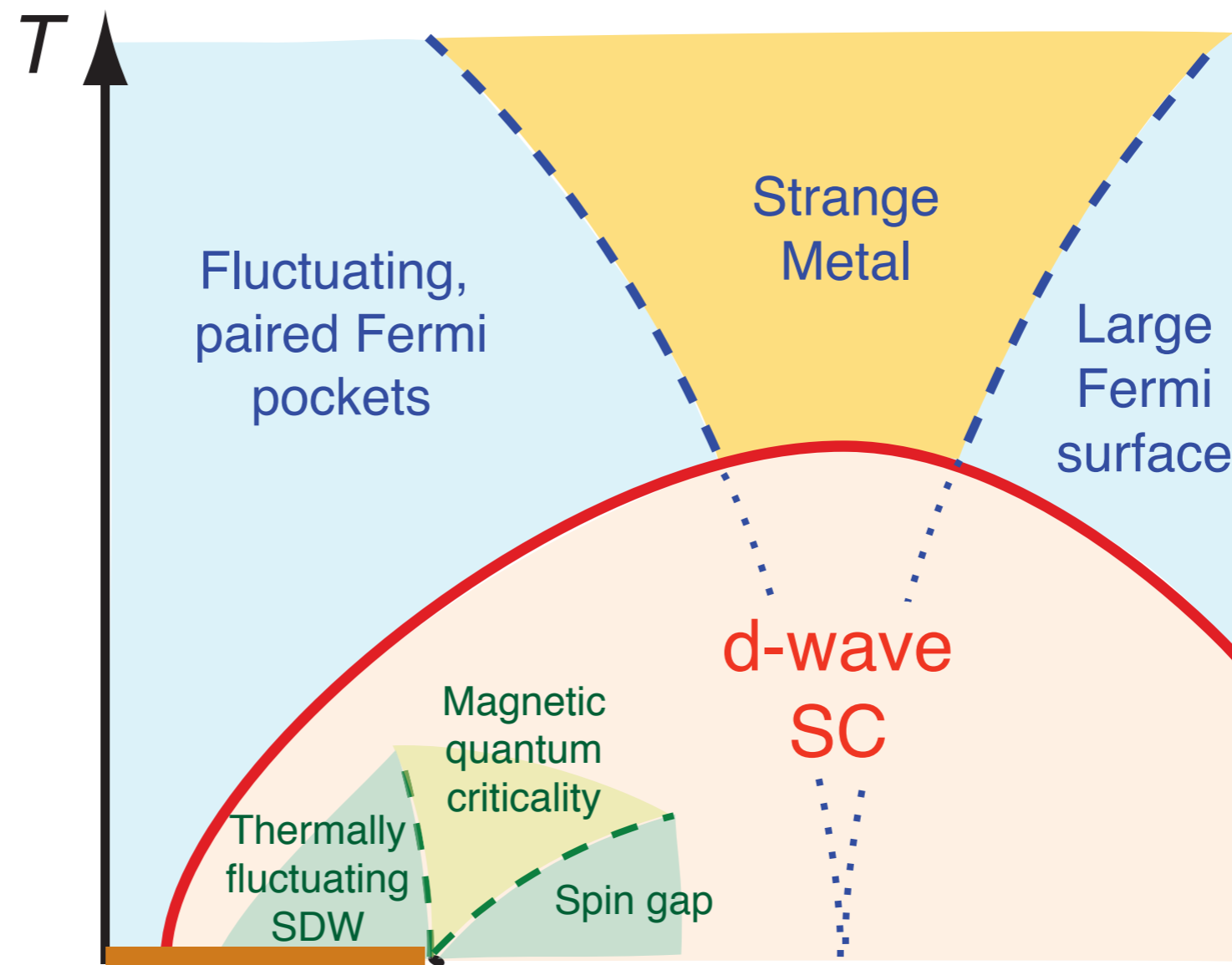


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

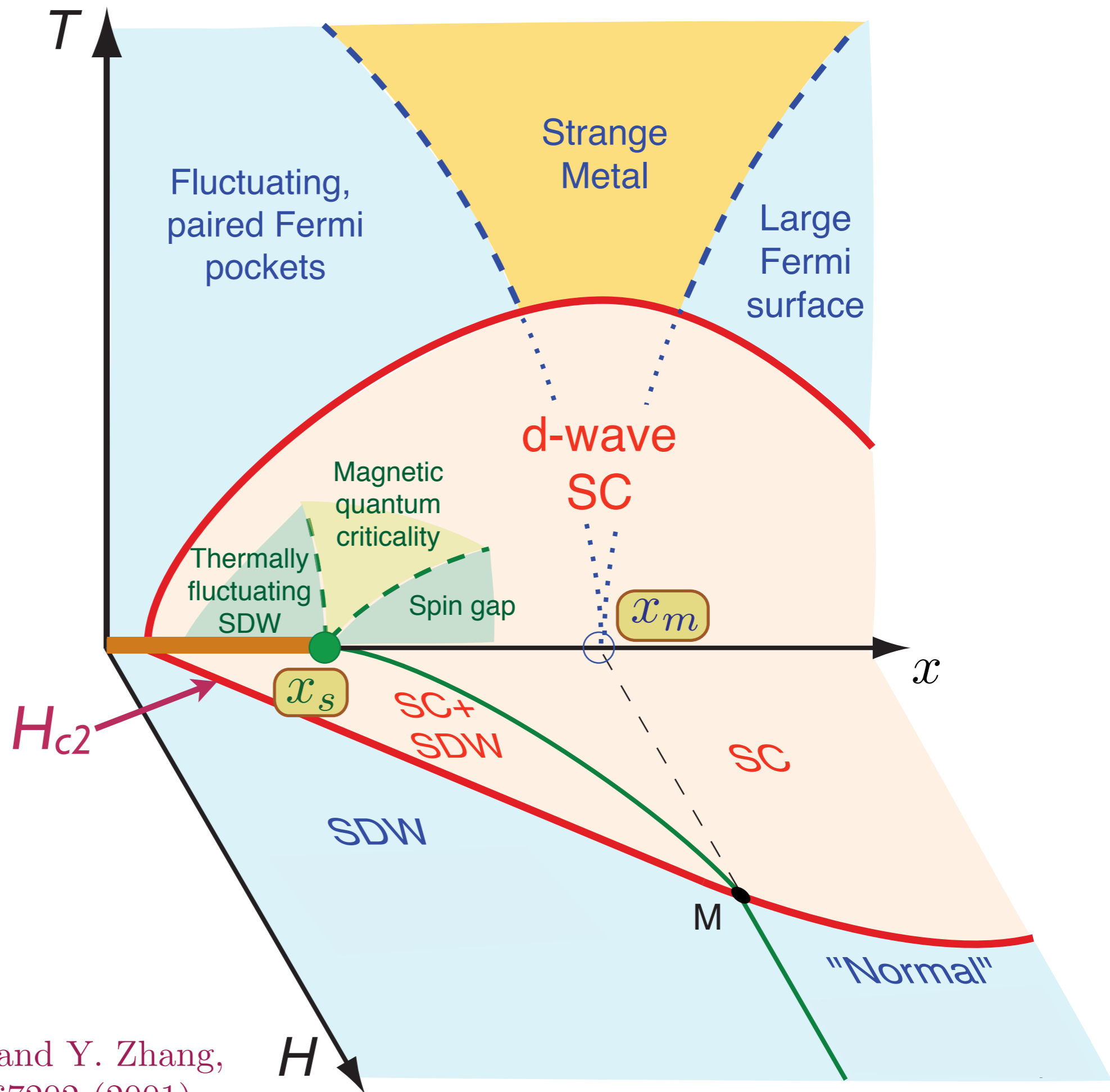
E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

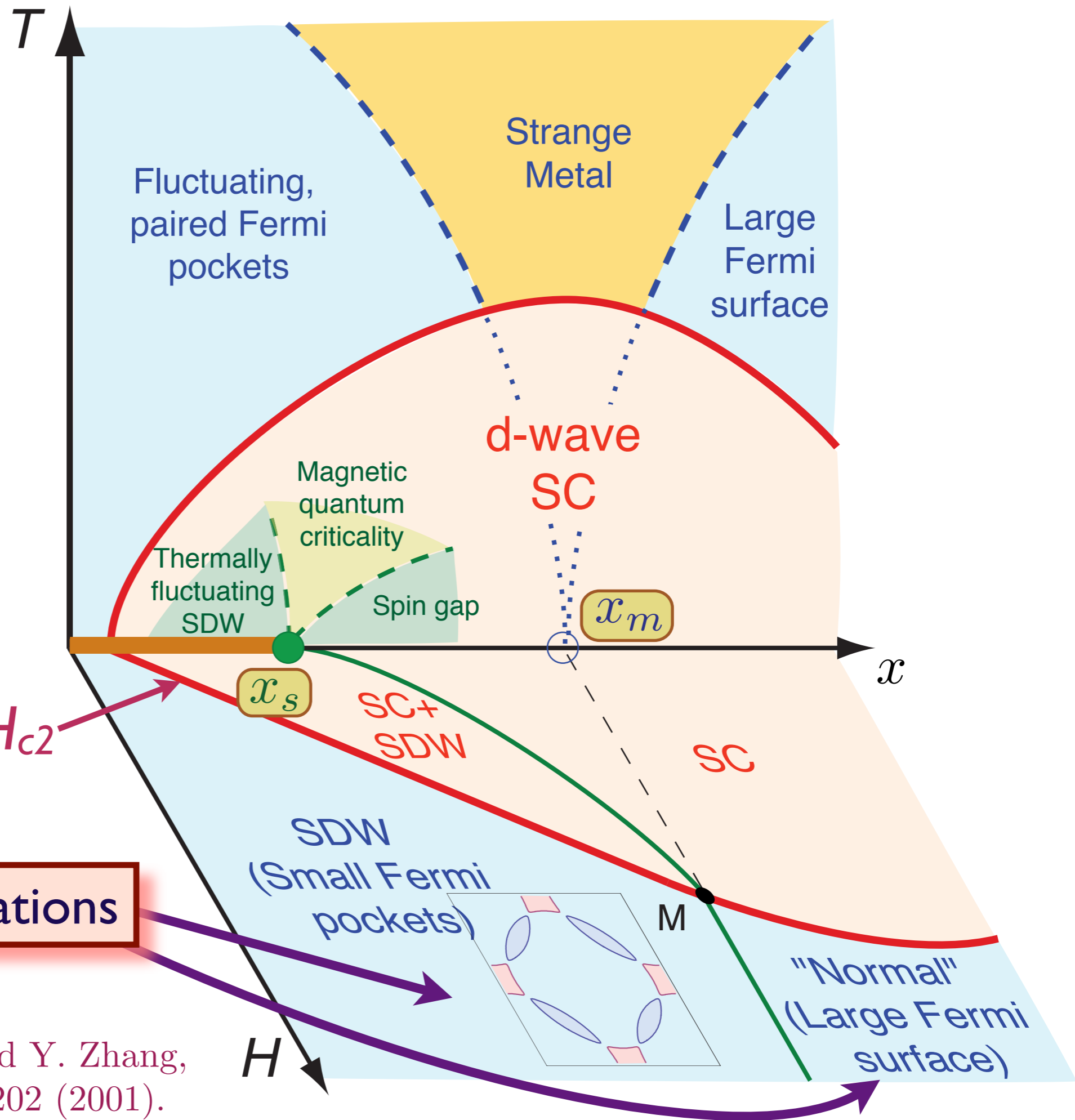




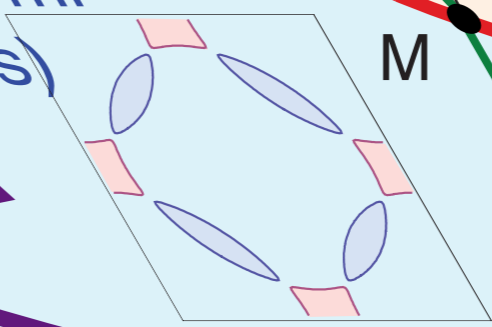
E. Demler, S. Sachdev and Y. Zhang,
Phys. Rev. Lett. **87**, 067202 (2001).



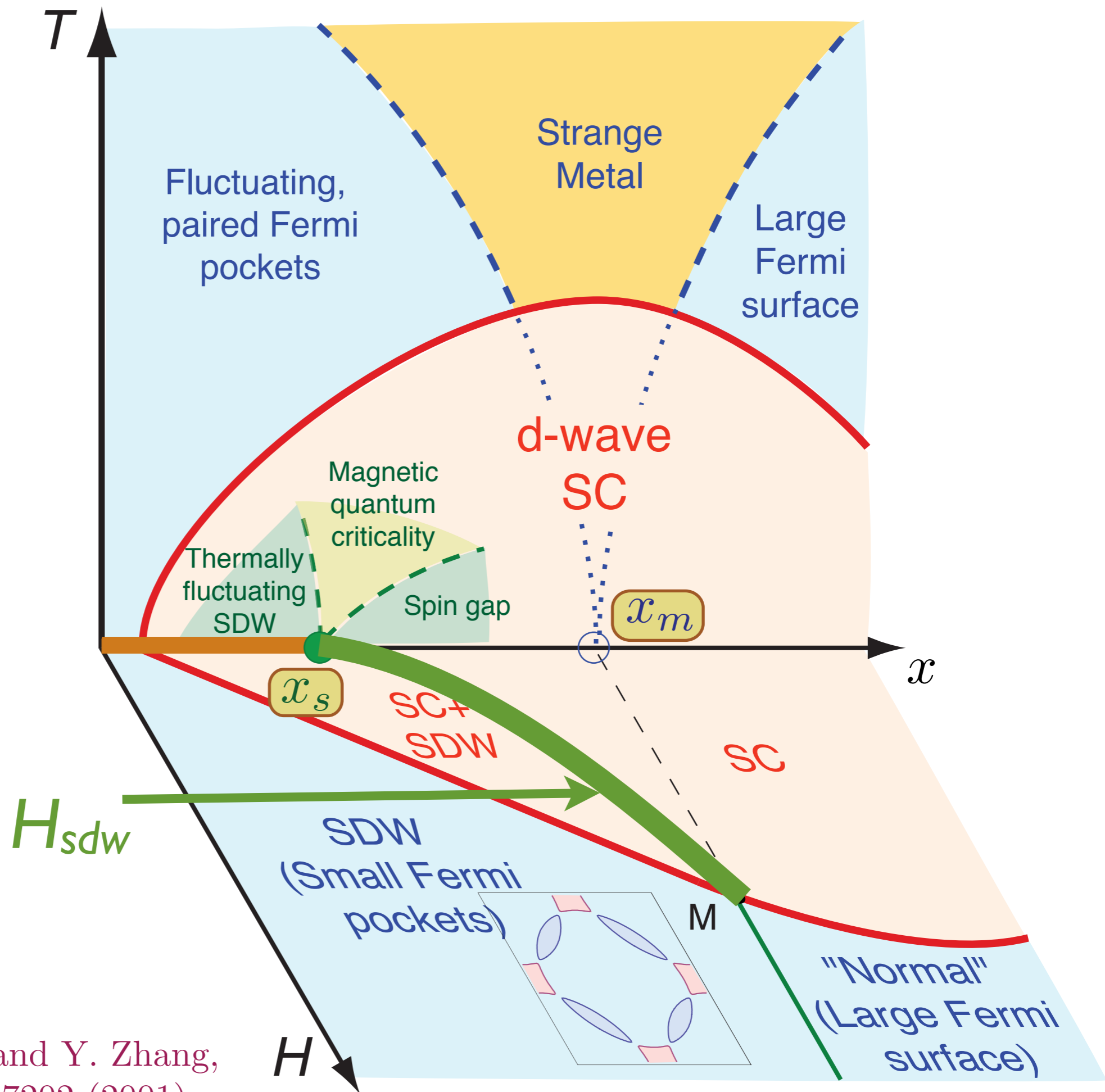
E. Demler, S. Sachdev and Y. Zhang,
Phys. Rev. Lett. **87**, 067202 (2001).



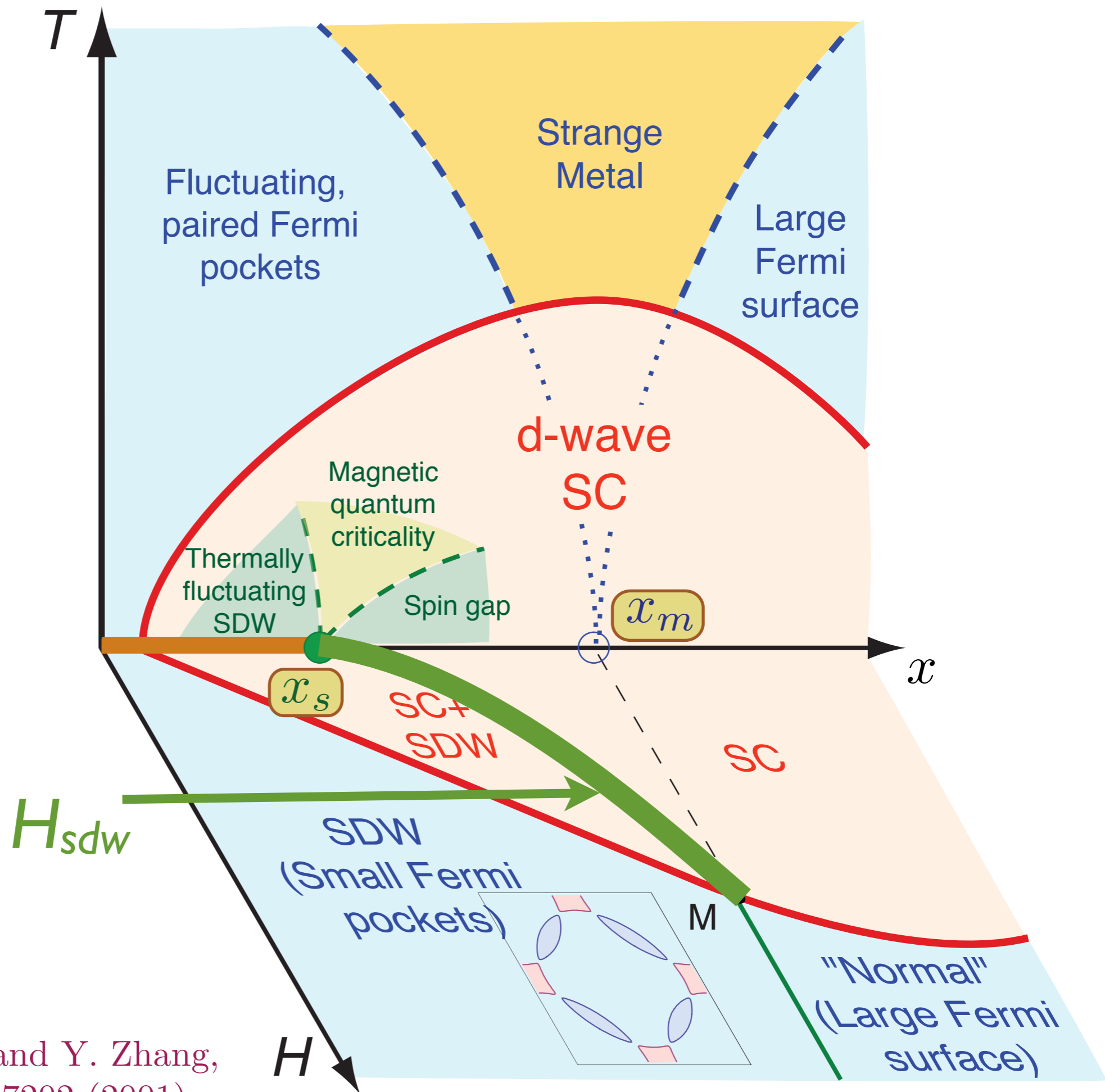
Quantum oscillations



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

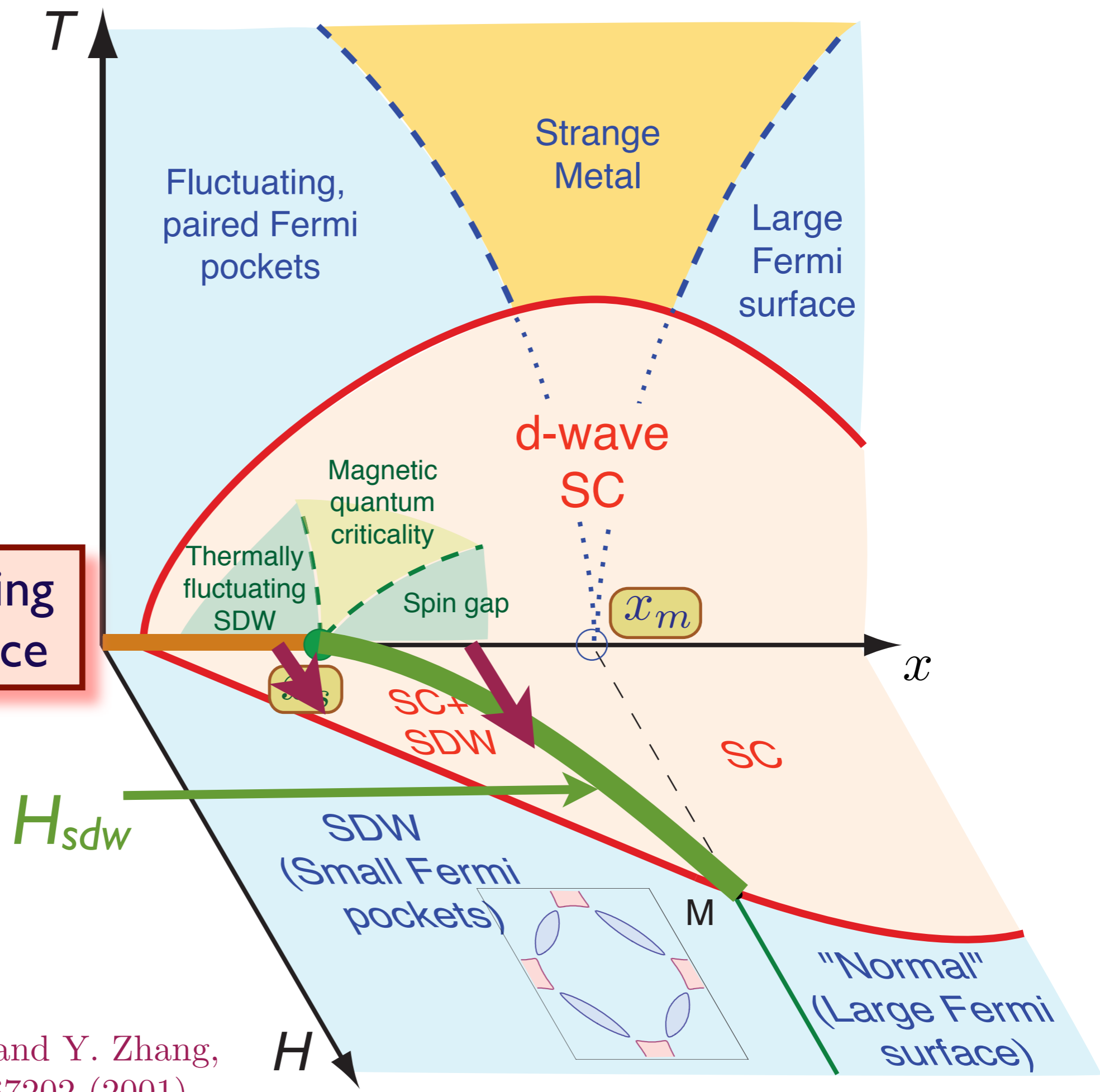


E. Demler, S. Sachdev and Y. Zhang,
Phys. Rev. Lett. **87**, 067202 (2001).

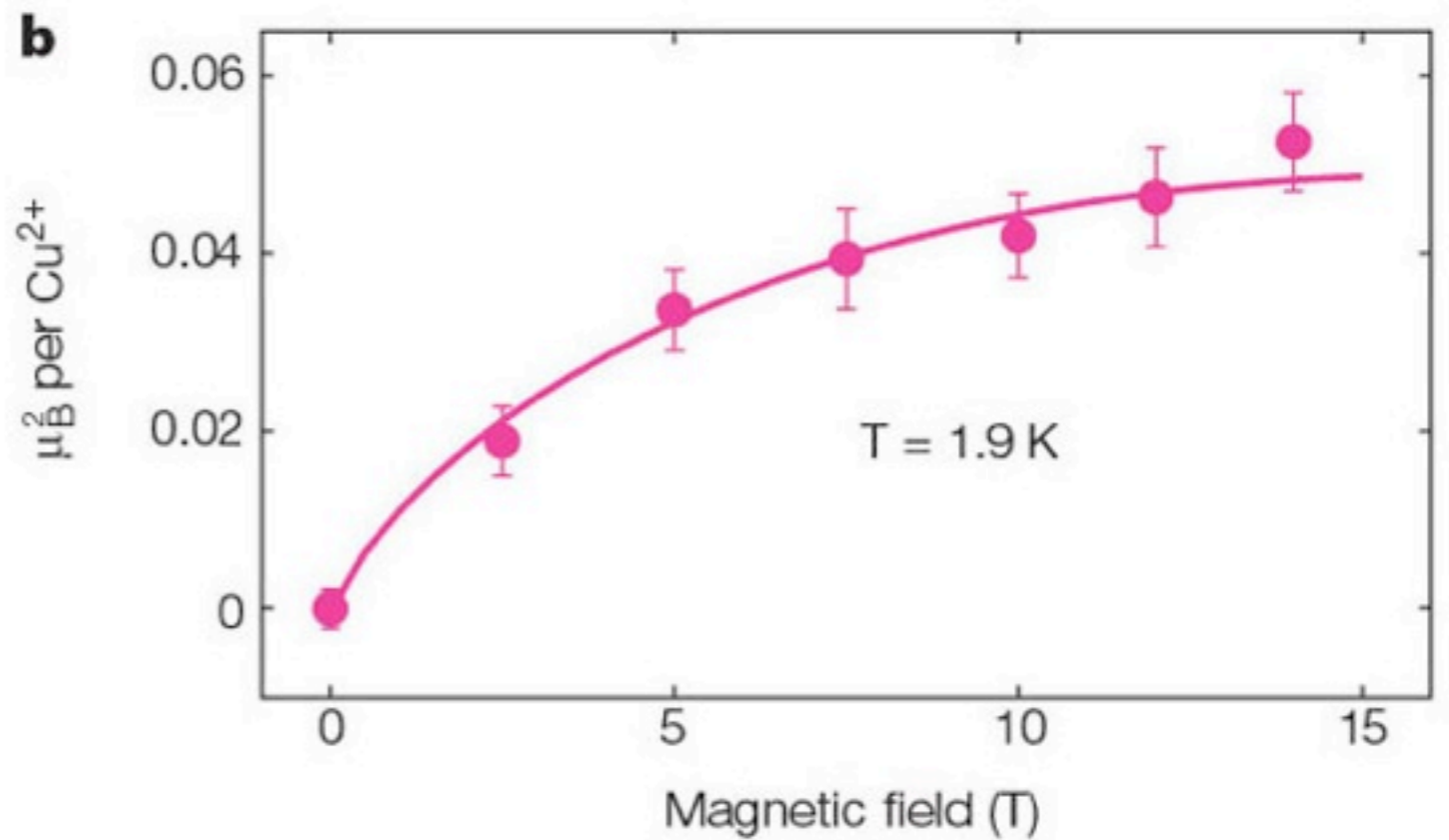
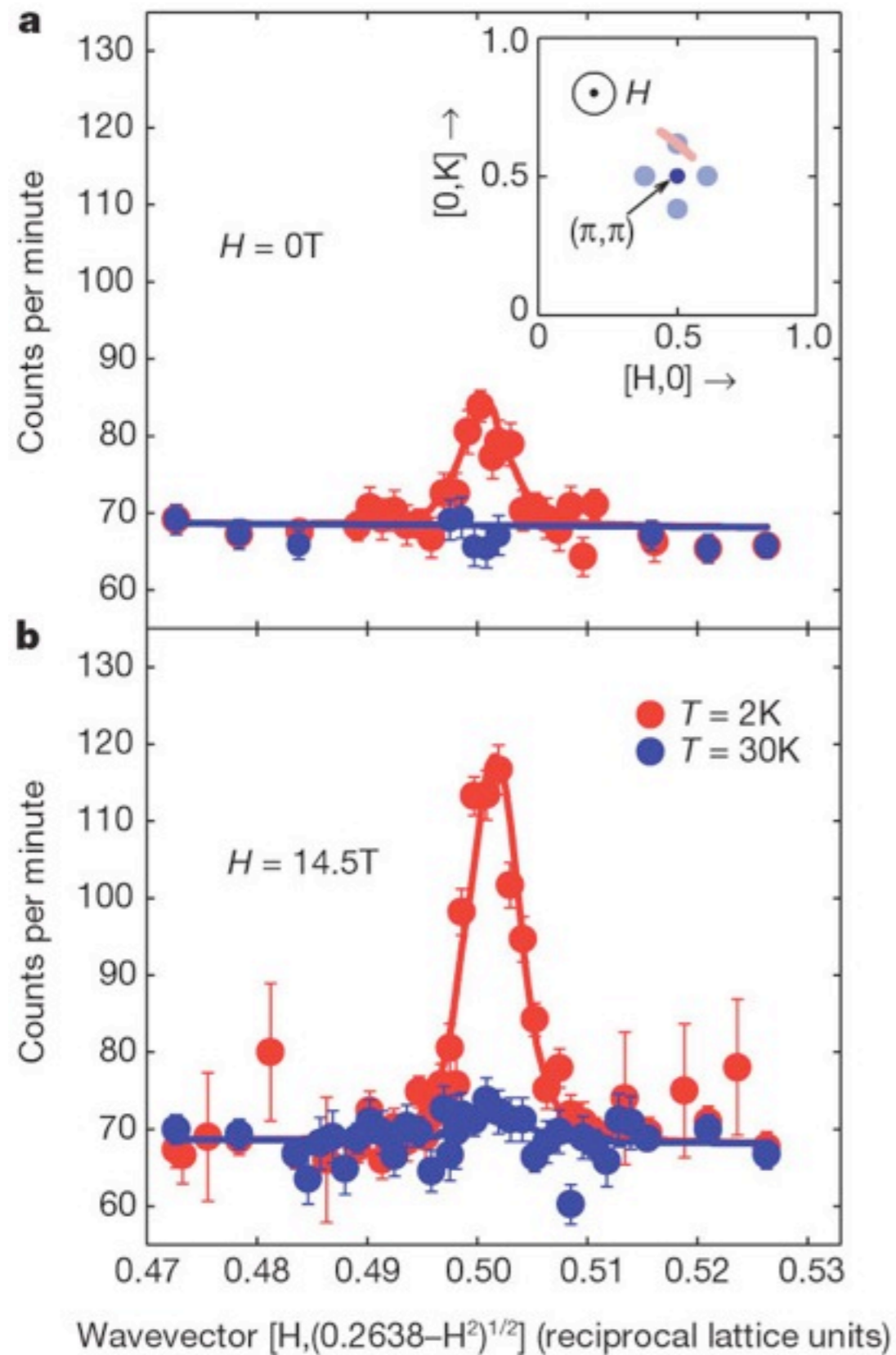


E. Demler, S. Sachdev and Y. Zhang,
Phys. Rev. Lett. **87**, 067202 (2001).

Neutron scattering & muon resonance



E. Demler, S. Sachdev and Y. Zhang,
Phys. Rev. Lett. **87**, 067202 (2001).



*B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, and T. E. Mason, Nature **415**, 299 (2002)*

*B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder Science **291**, 1759 (2001).*

Field-induced transition between magnetically disordered and ordered phases in underdoped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

B. Khaykovich,¹ S. Wakimoto,² R. J. Birgeneau,³ M. A. Kastner,¹ Y. S. Lee,¹ P. Smeibidl,⁴ P. Vorderwisch,⁴ and K. Yamada⁵

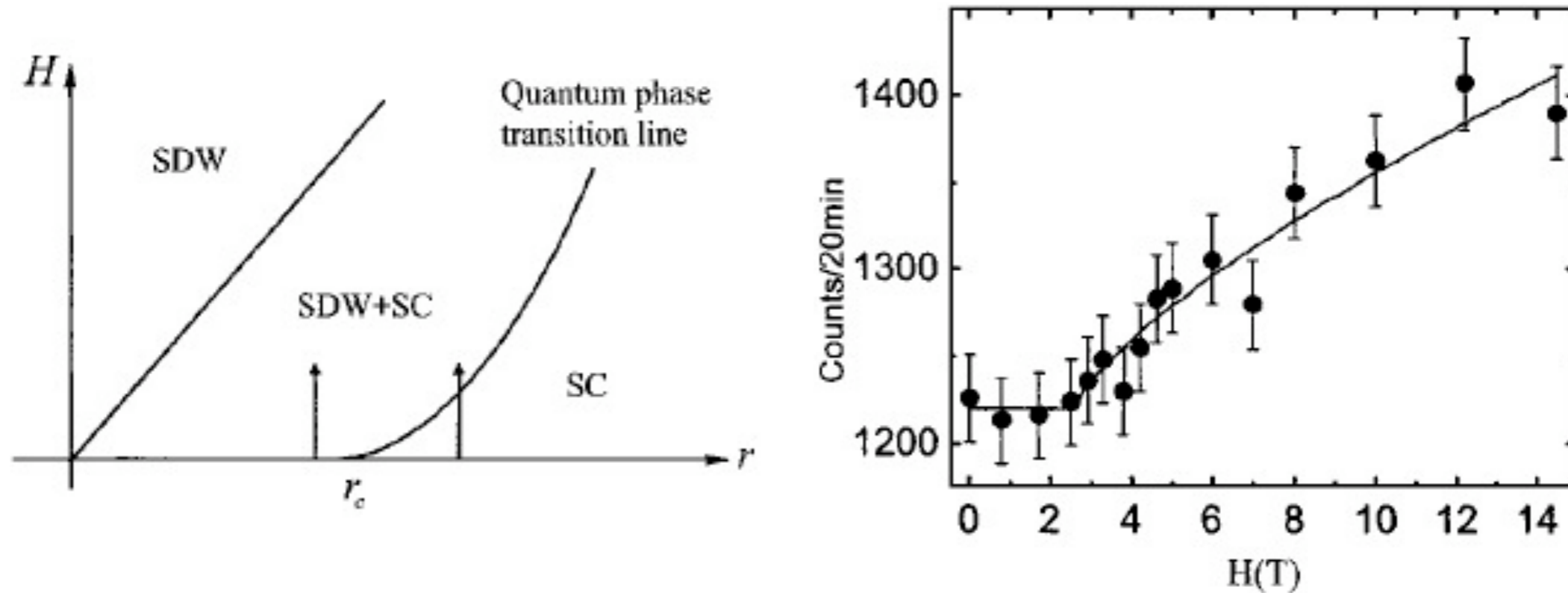
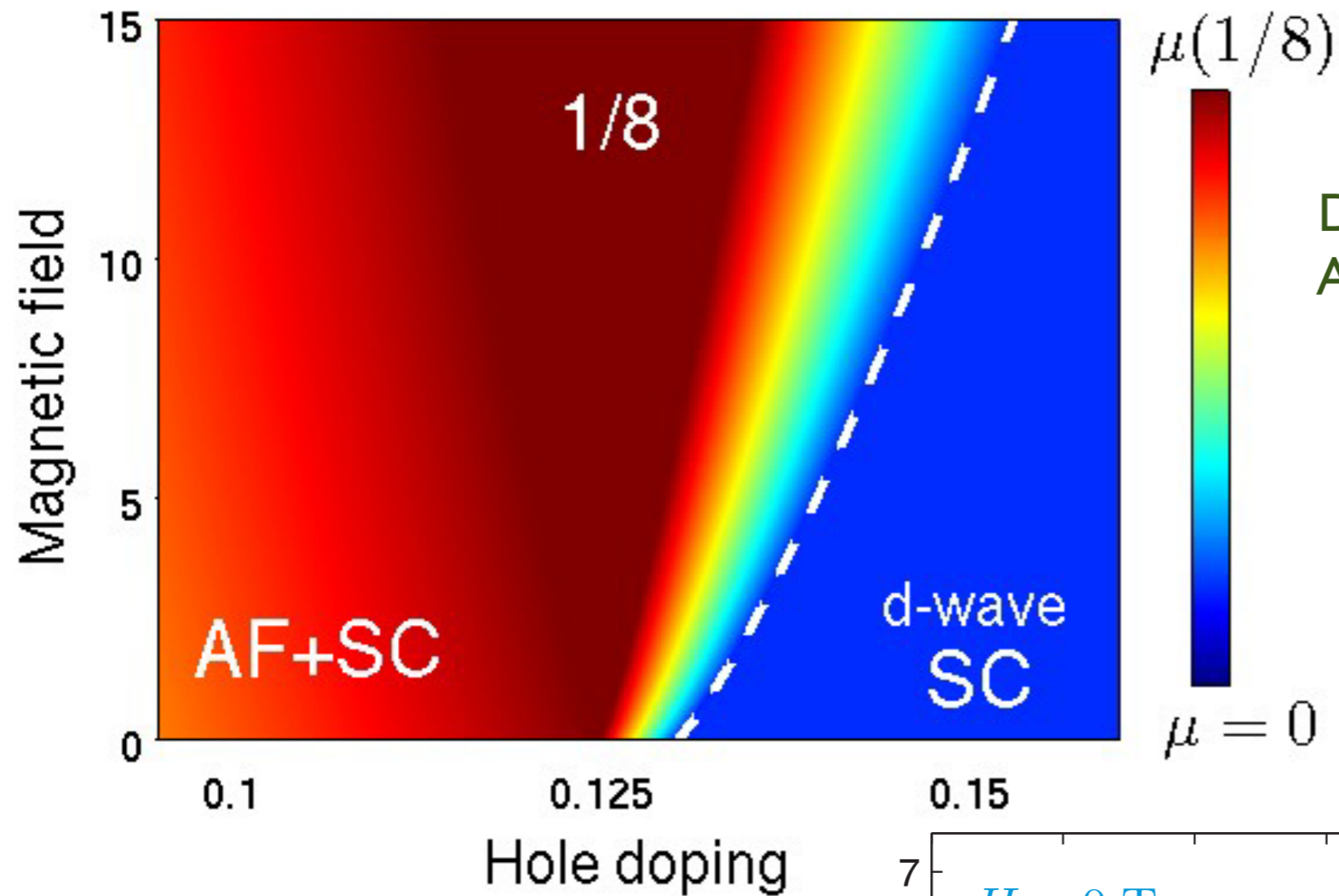
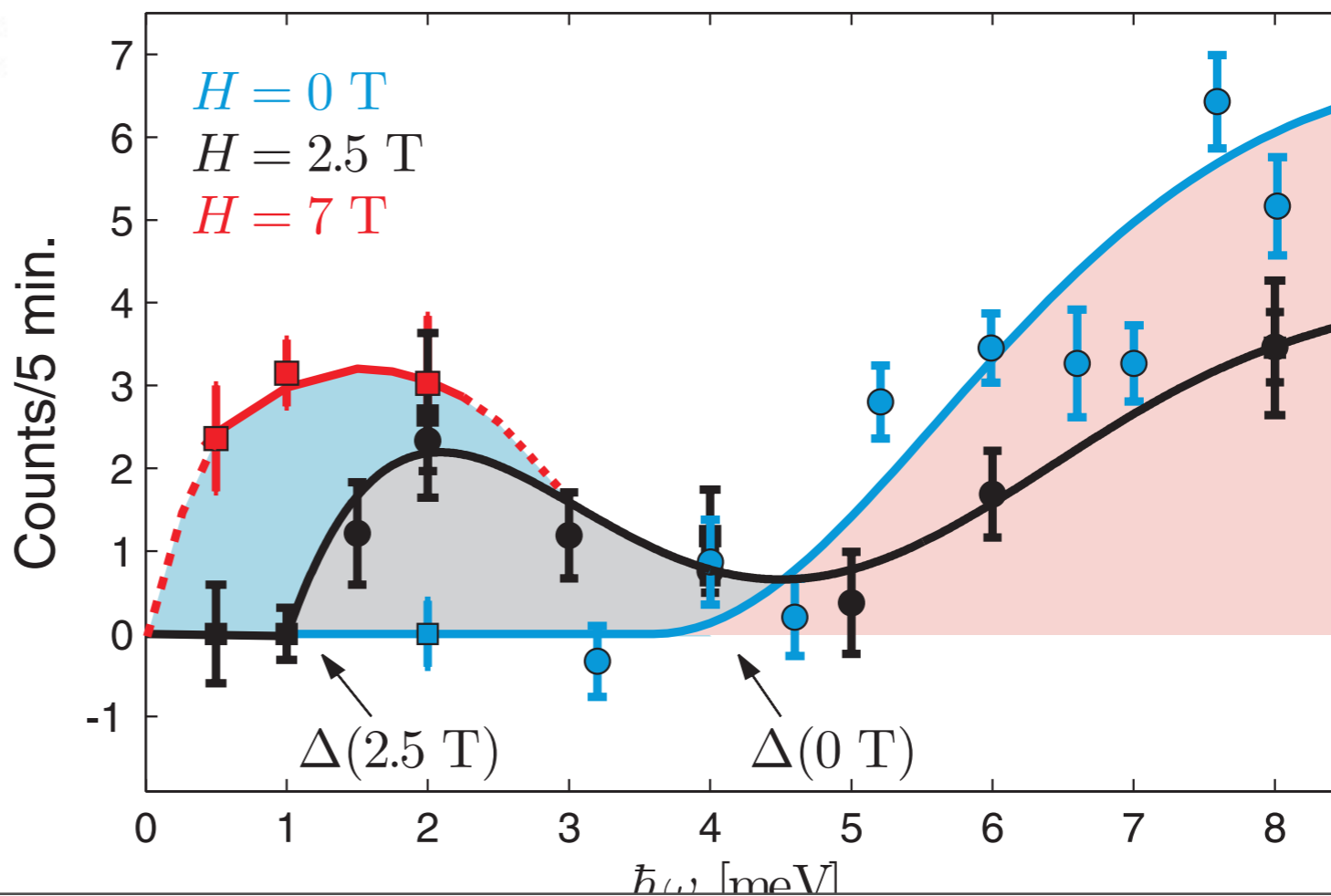


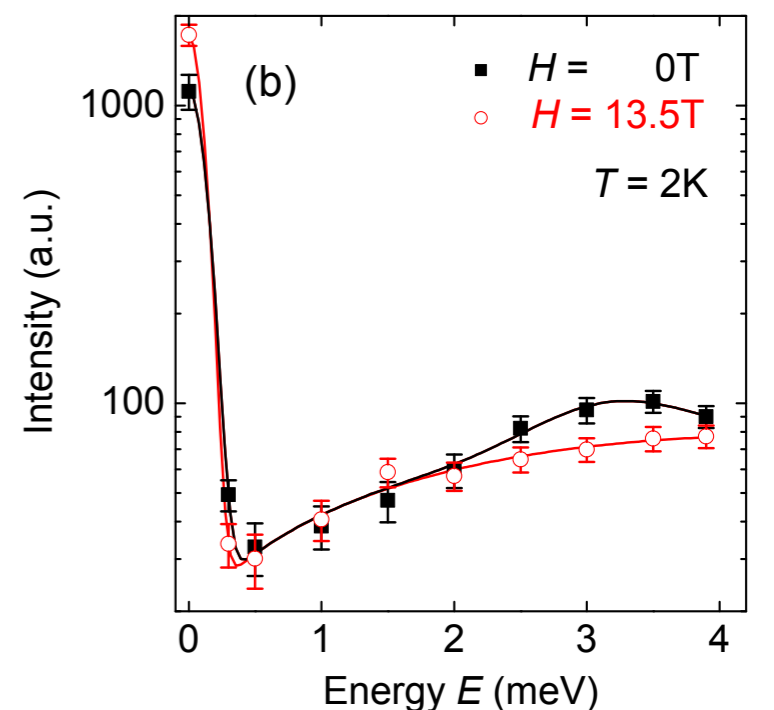
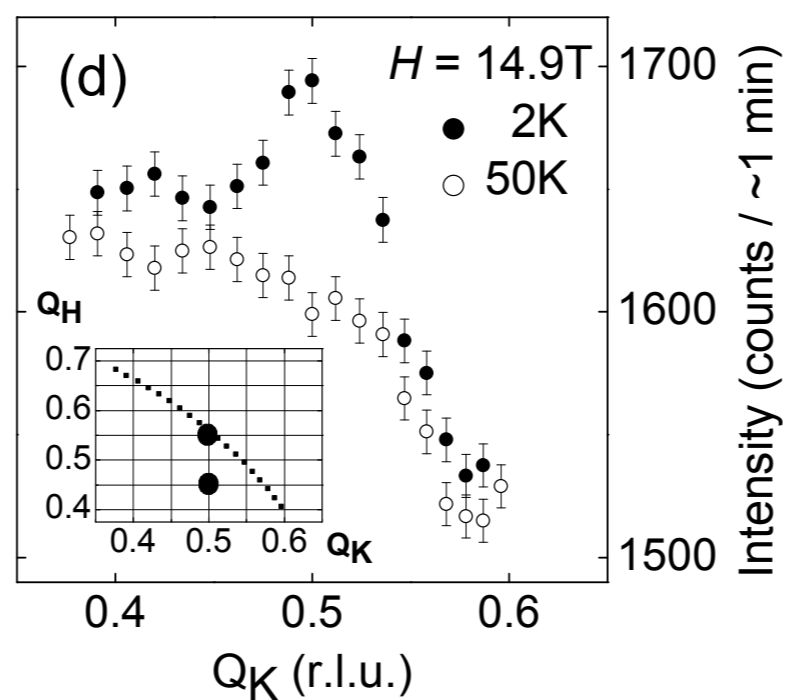
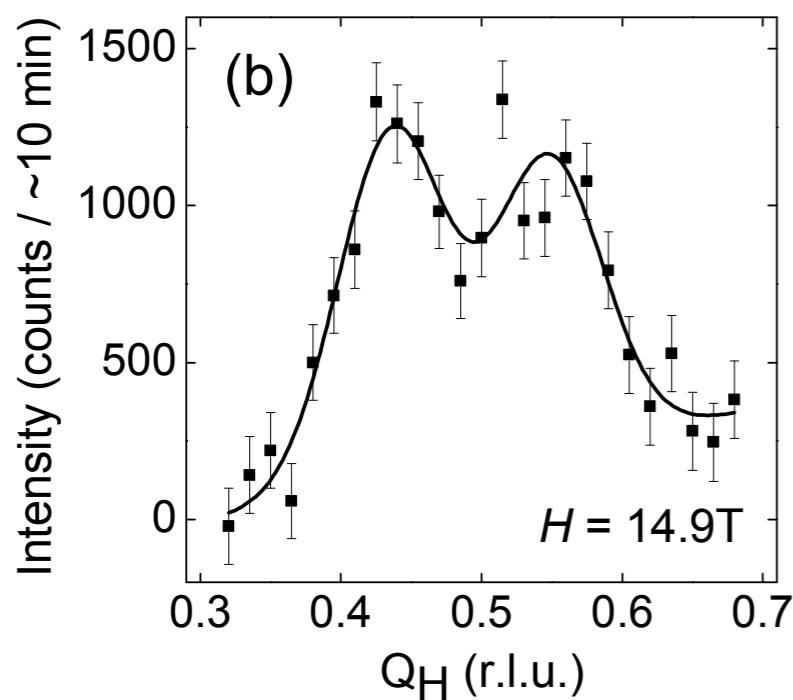
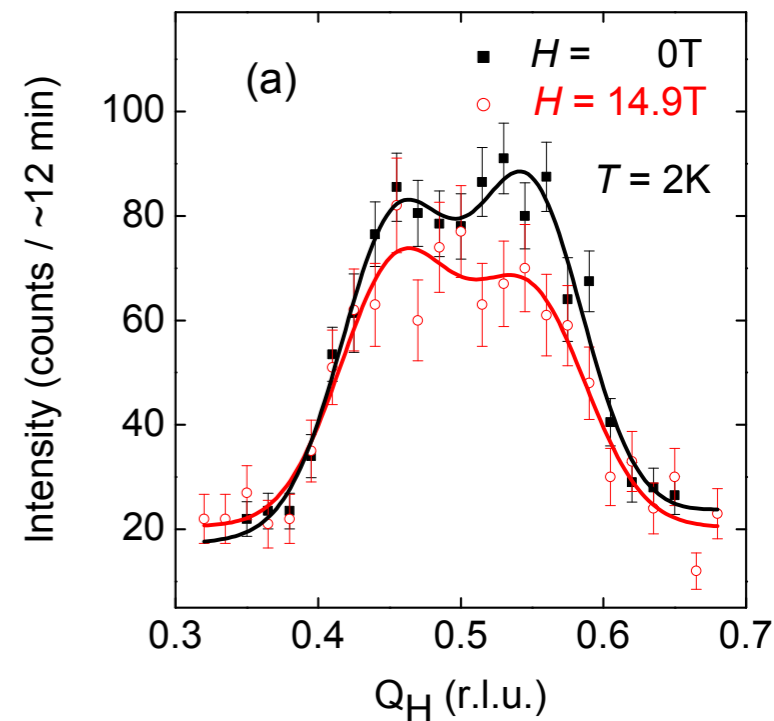
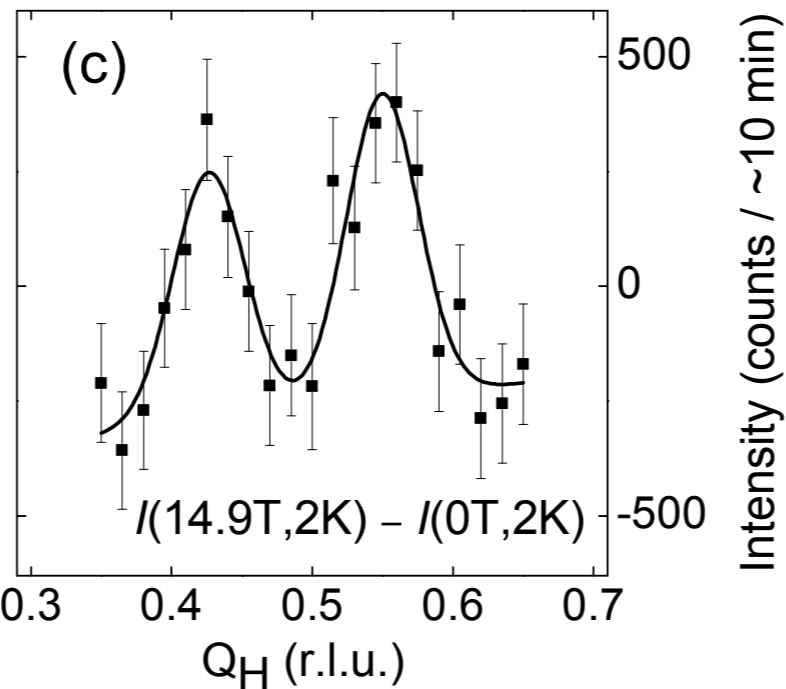
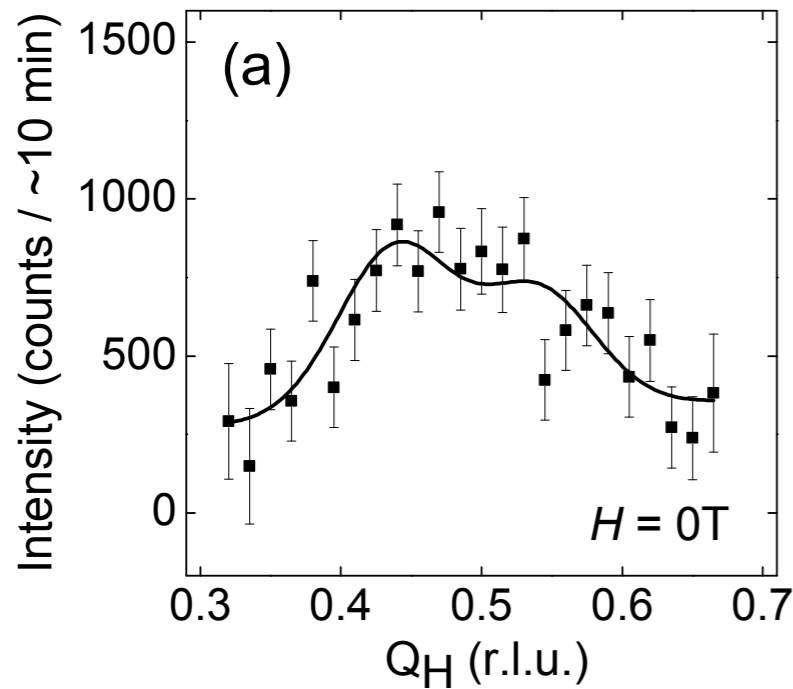
FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125, 0.125, 0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.



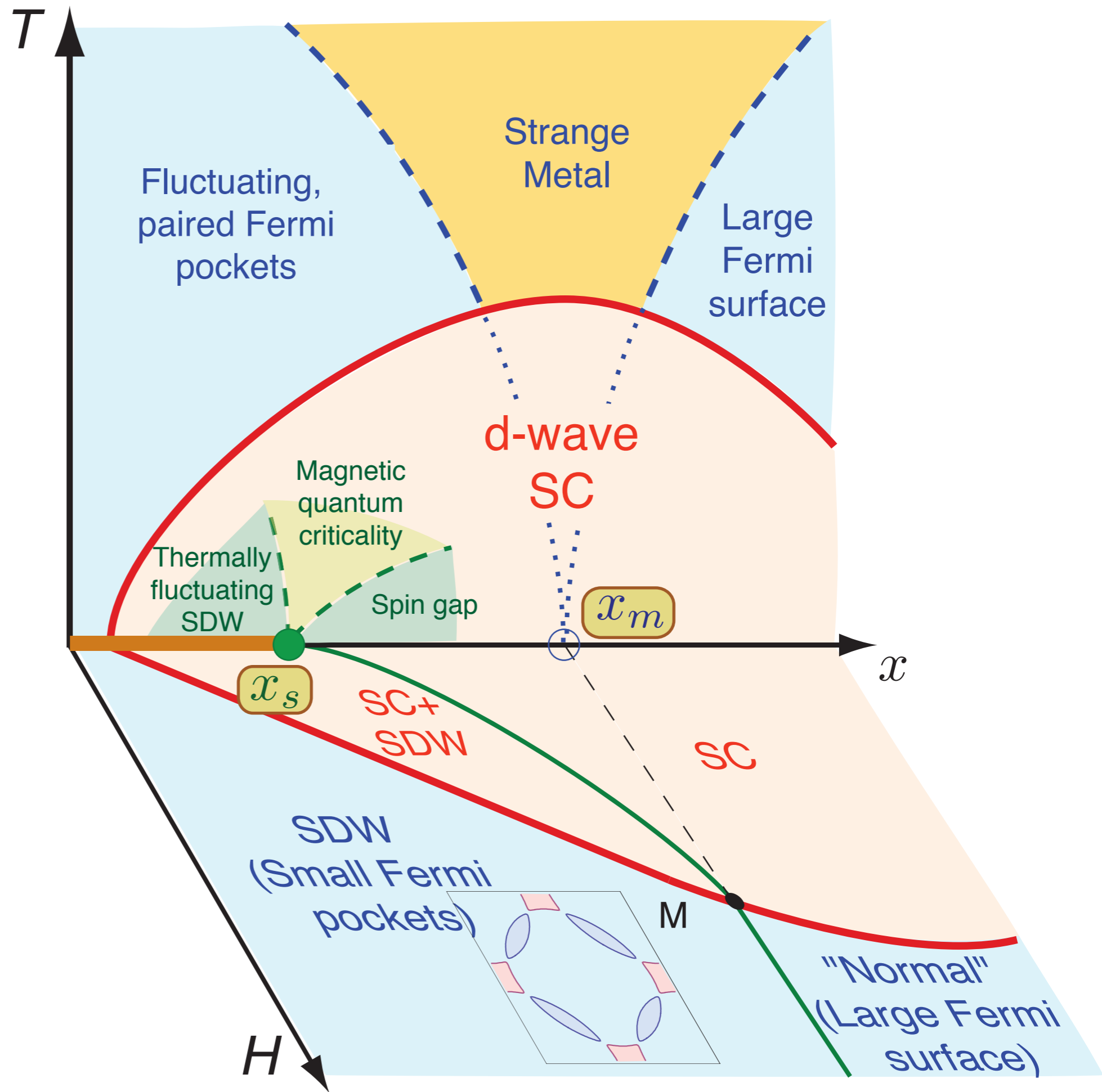
J. Chang, Ch. Niedermayer, R. Gilardi,
 N.B. Christensen, H.M. Ronnow,
 D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev,
 A. Hiess, S. Pailhes, C. Baines, N. Momono,
 M. Oda, M. Ido, and J. Mesot,
Physical Review B **78**, 104525 (2008).

J. Chang, N. B. Christensen,
 Ch. Niedermayer, K. Lefmann,
 H. M. Roennow, D. F. McMorrow,
 A. Schneidewind, P. Link, A. Hiess,
 M. Boehm, R. Mottl, S. Pailhes,
 N. Momono, M. Oda, M. Ido, and
 J. Mesot,
Phys. Rev. Lett. **102**, 177006
 (2009).

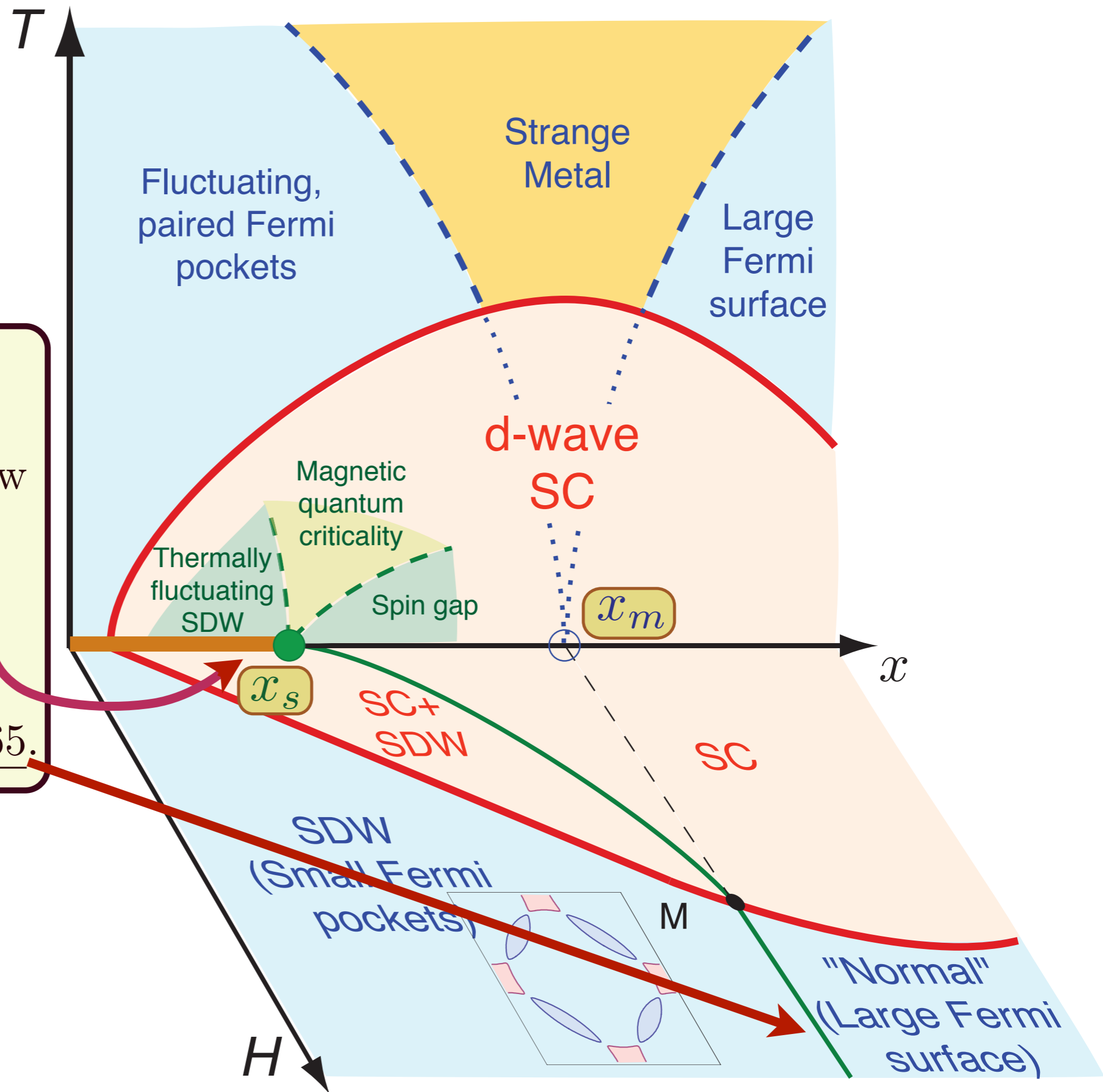


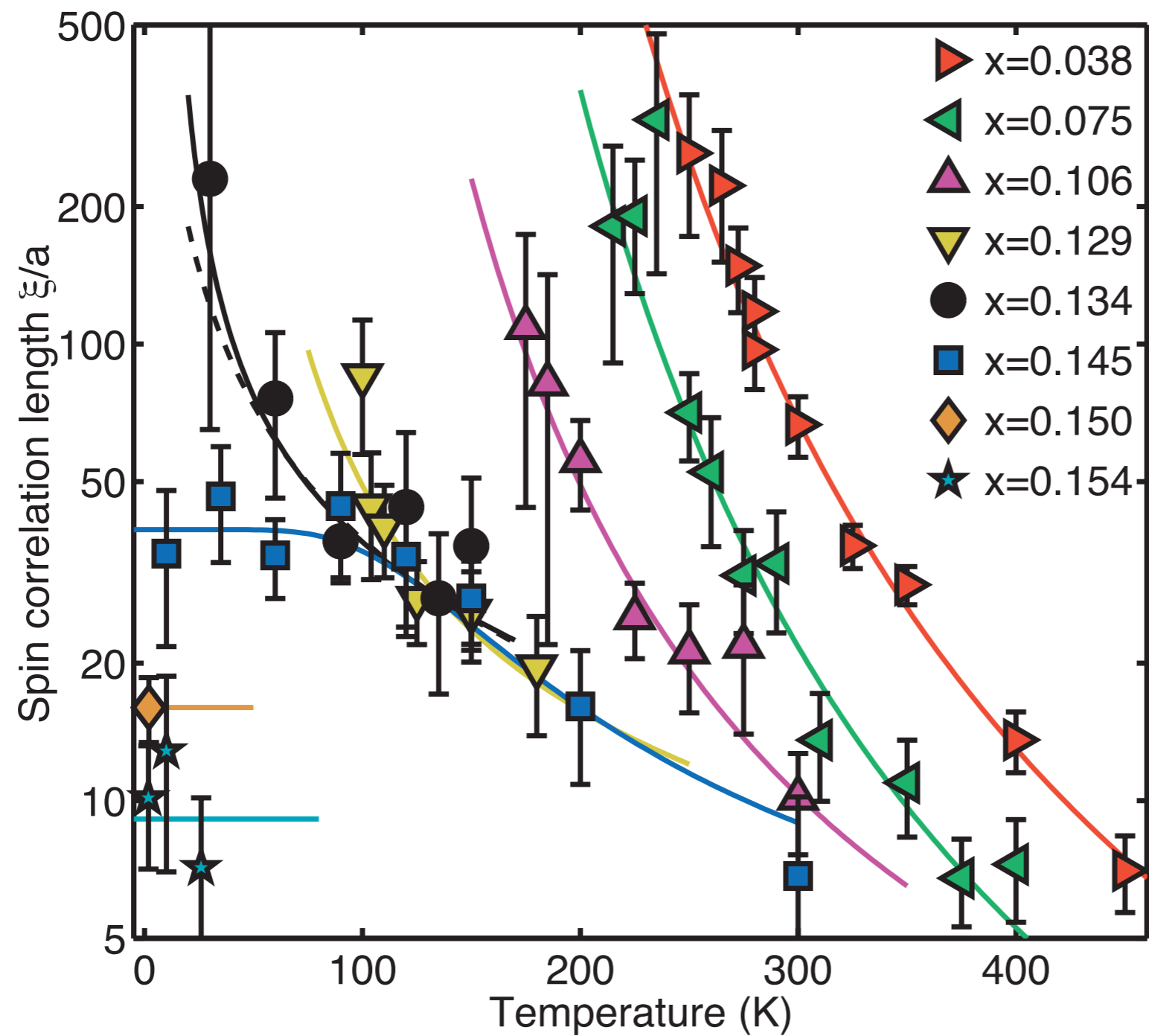


D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

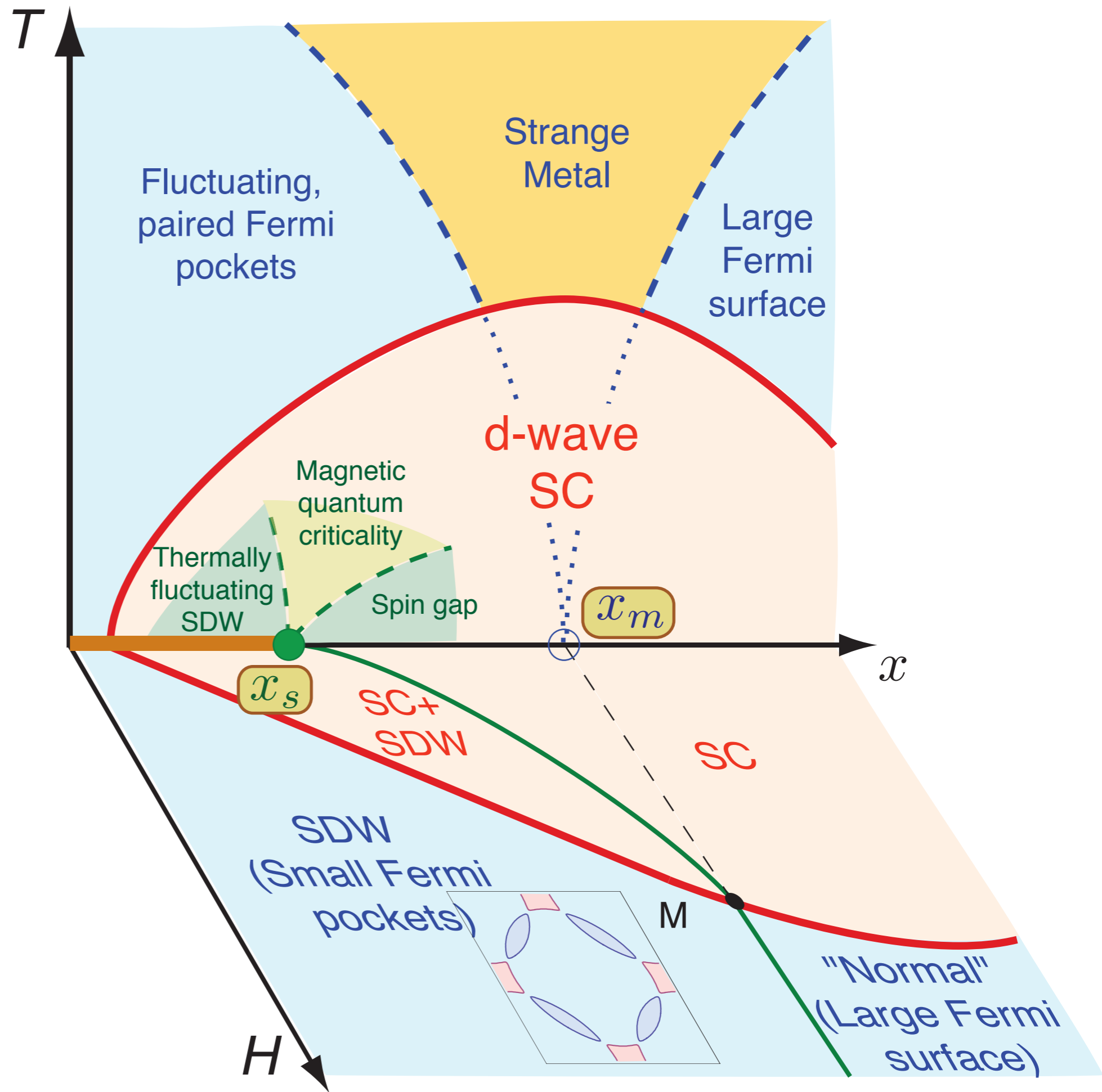


Neutron scattering experiments on $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.

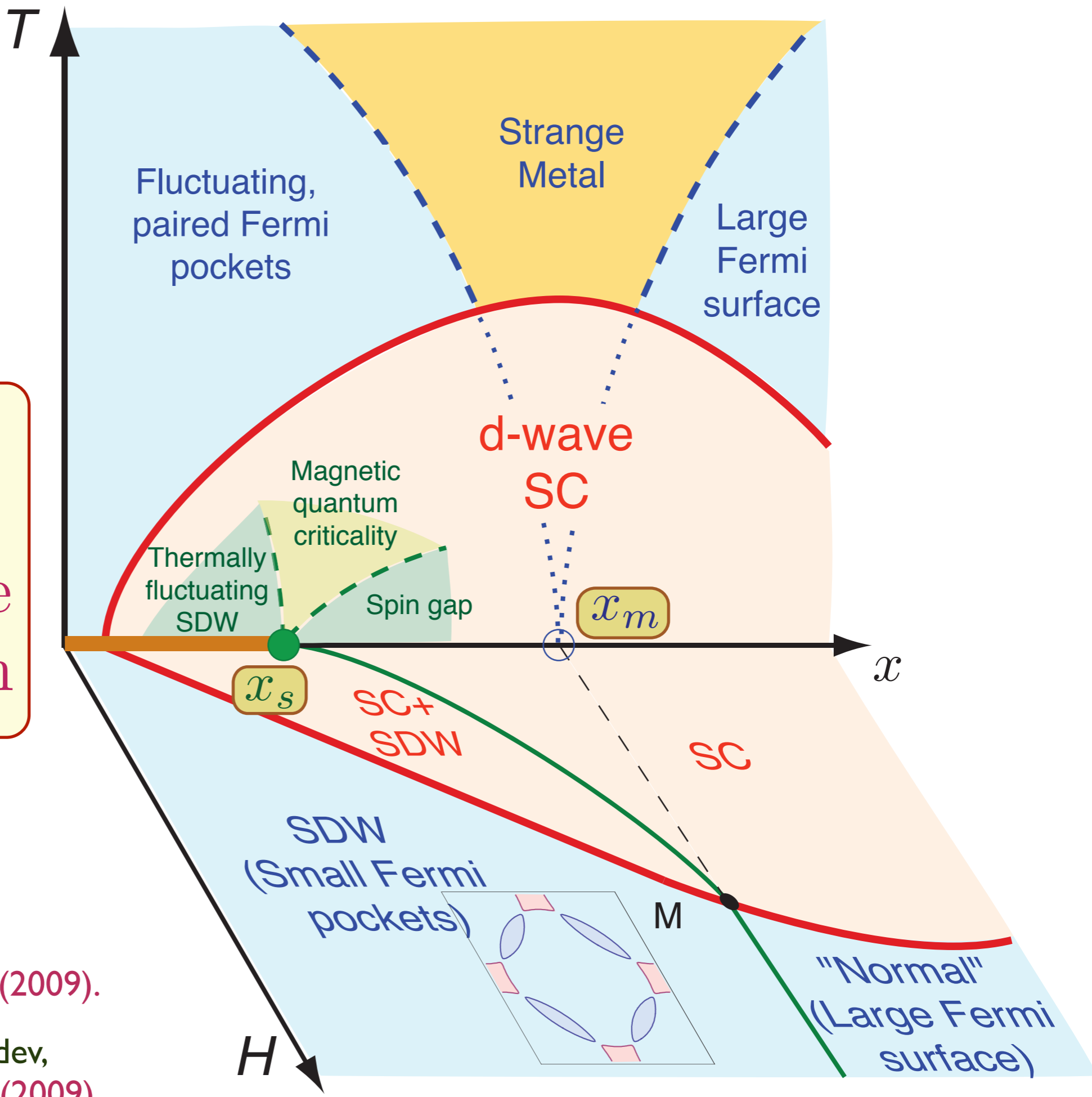




E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).



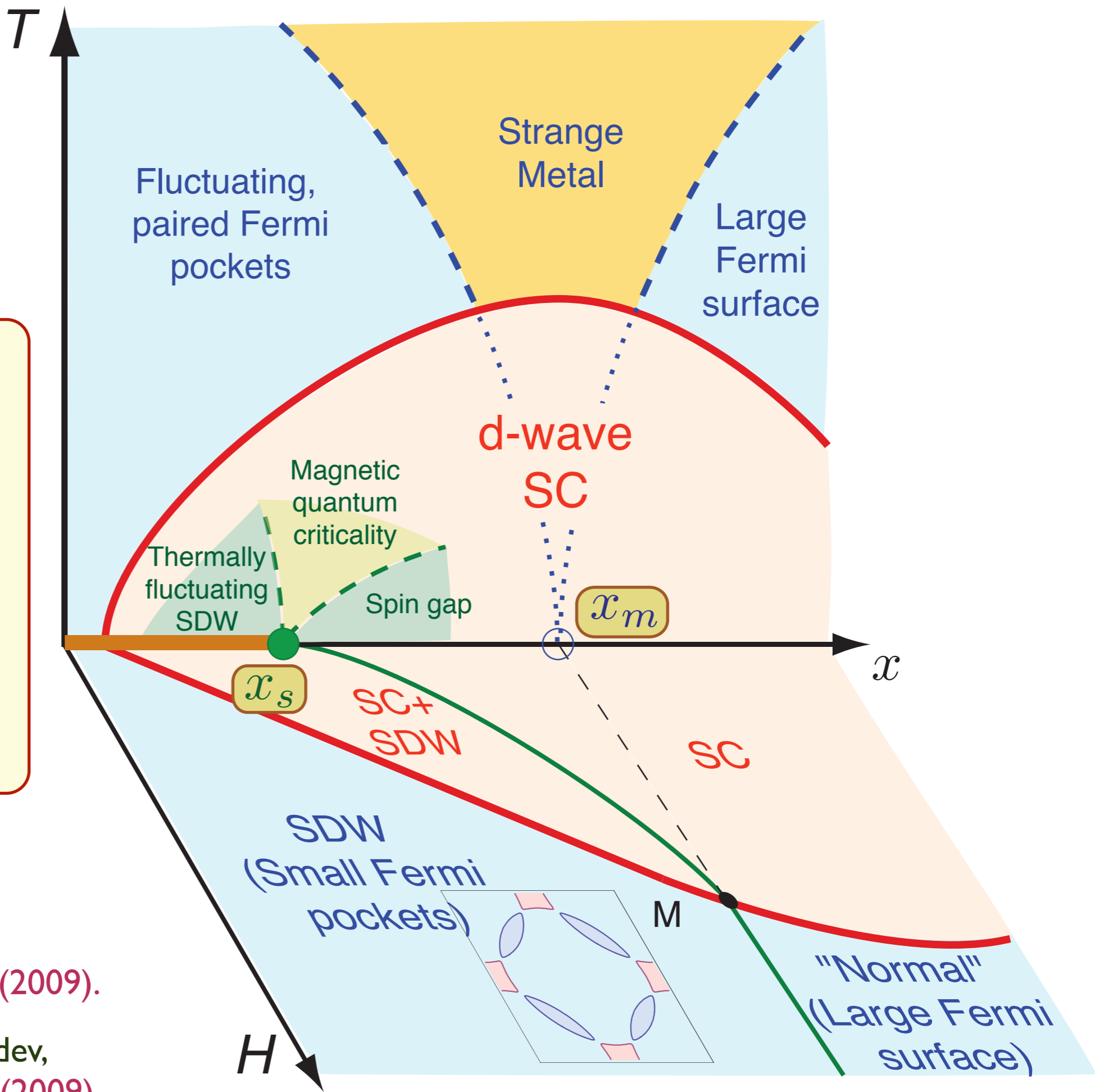
Theory of qualitative features of the phase diagram



V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

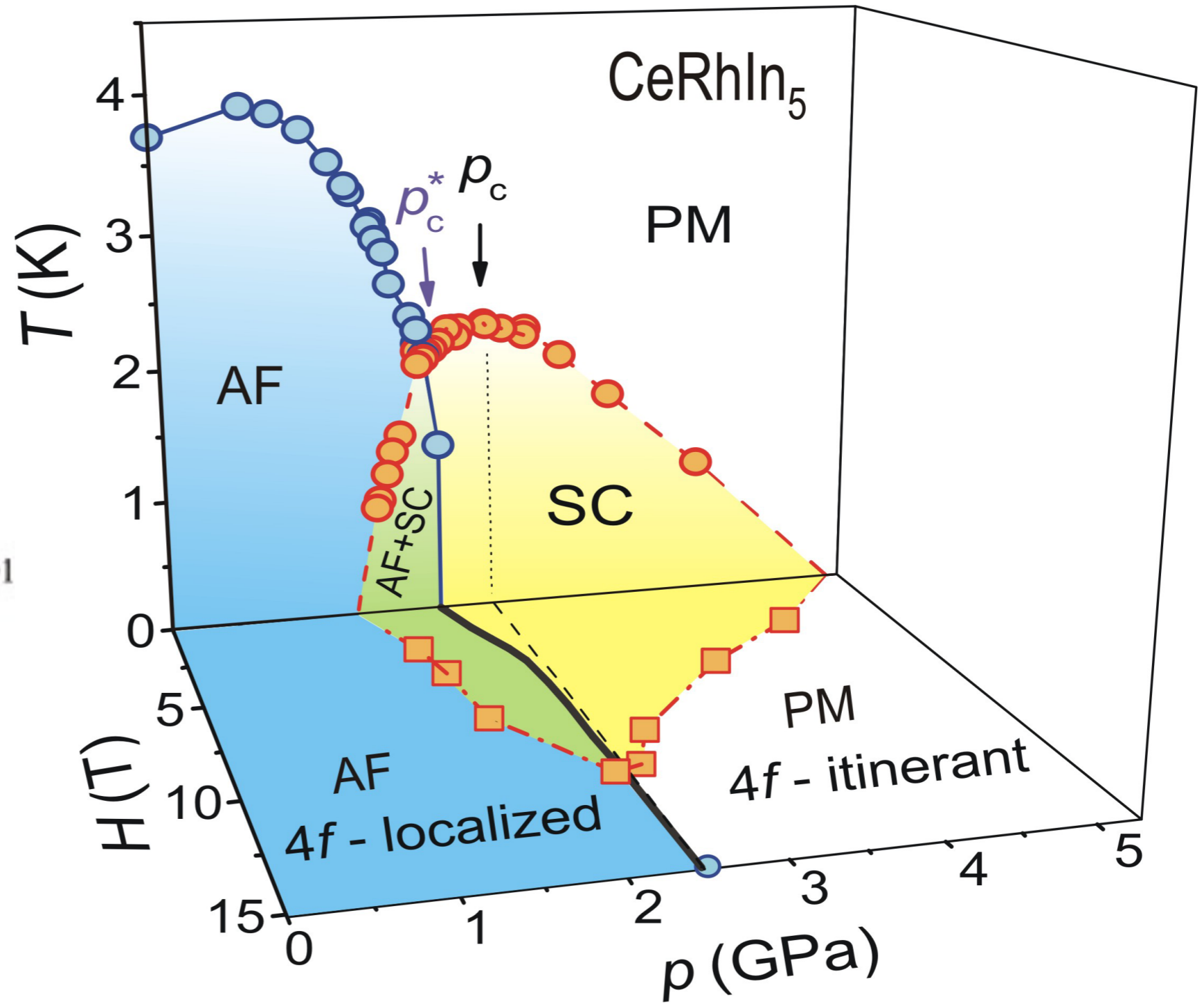
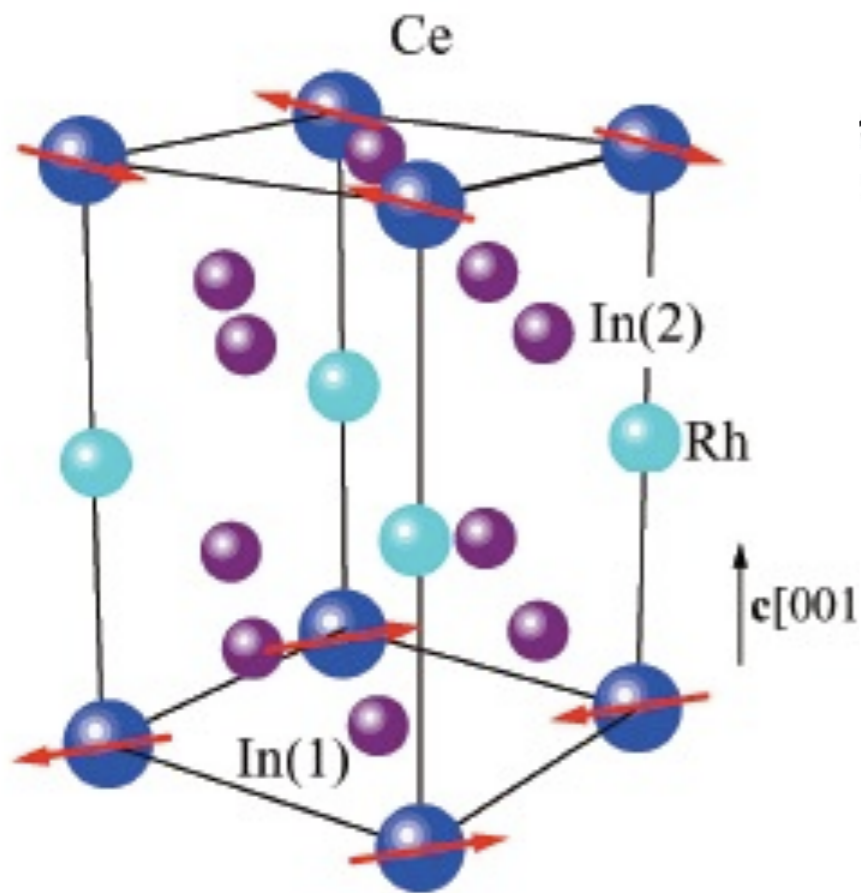
Physics of competition:
d-wave SC and SDW “eat up” same pieces of the large Fermi surface.



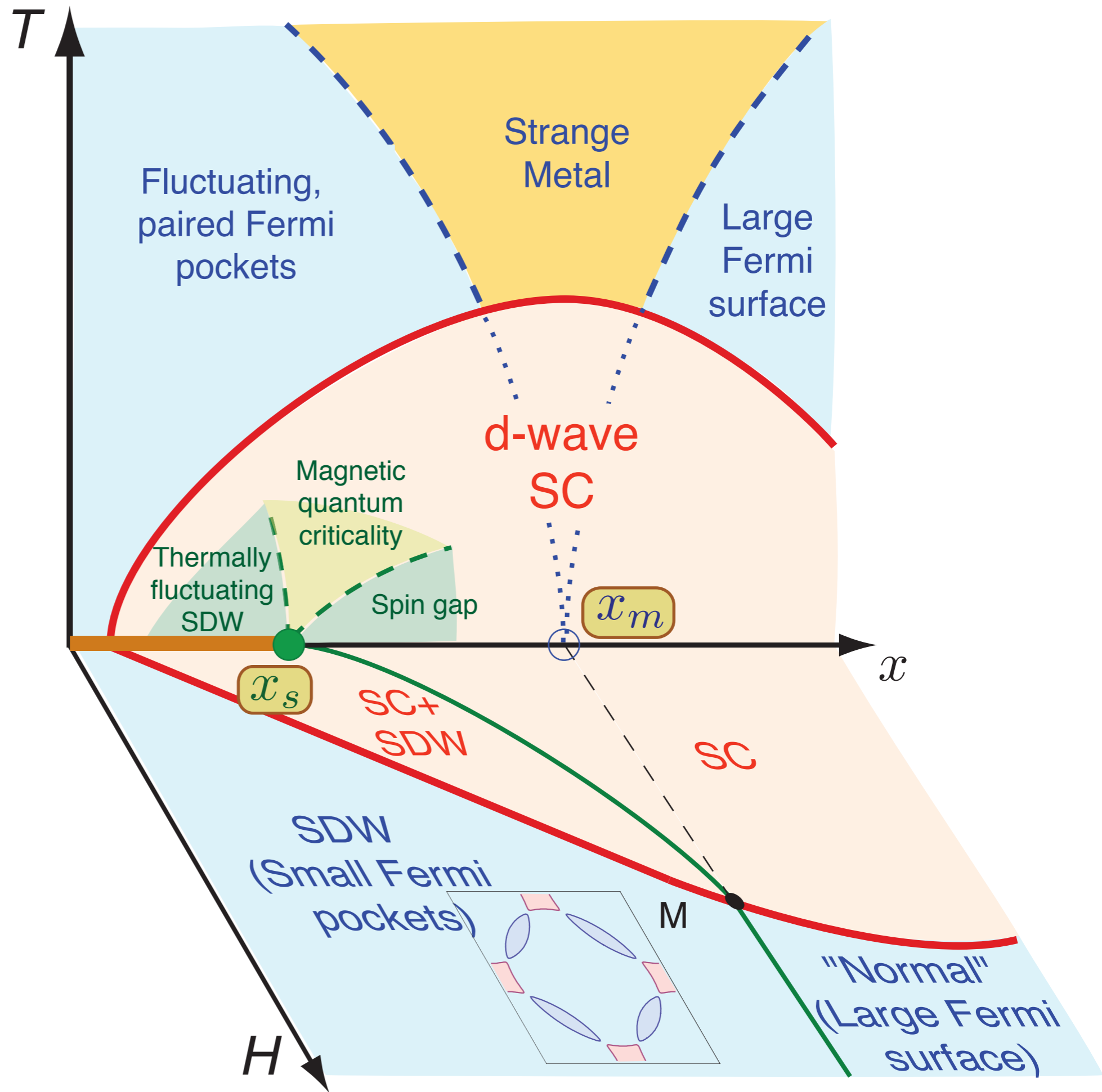
V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

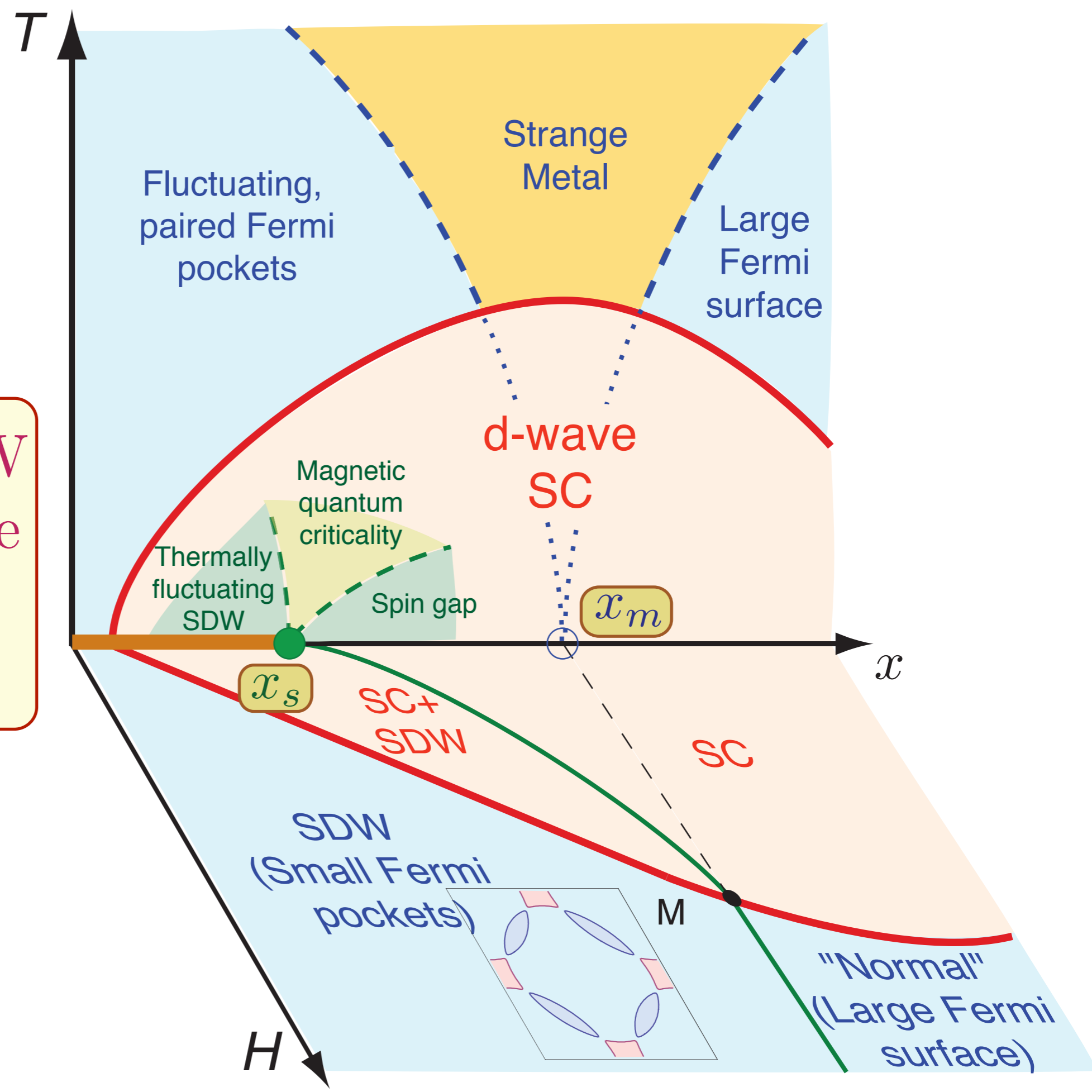
Similar phase diagram for CeRhIn₅



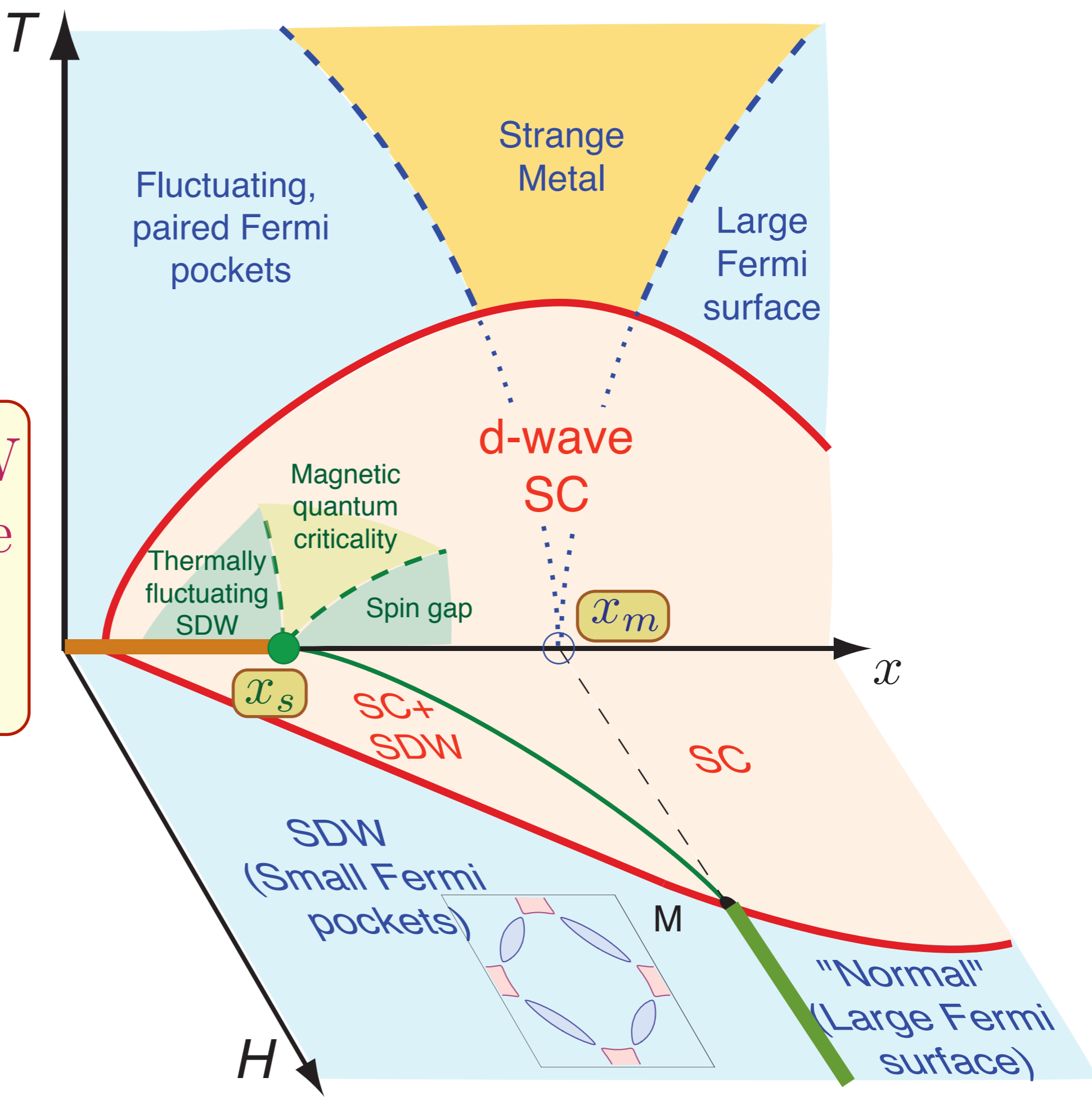
G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223



Theory of SDW quantum phase transition in metal

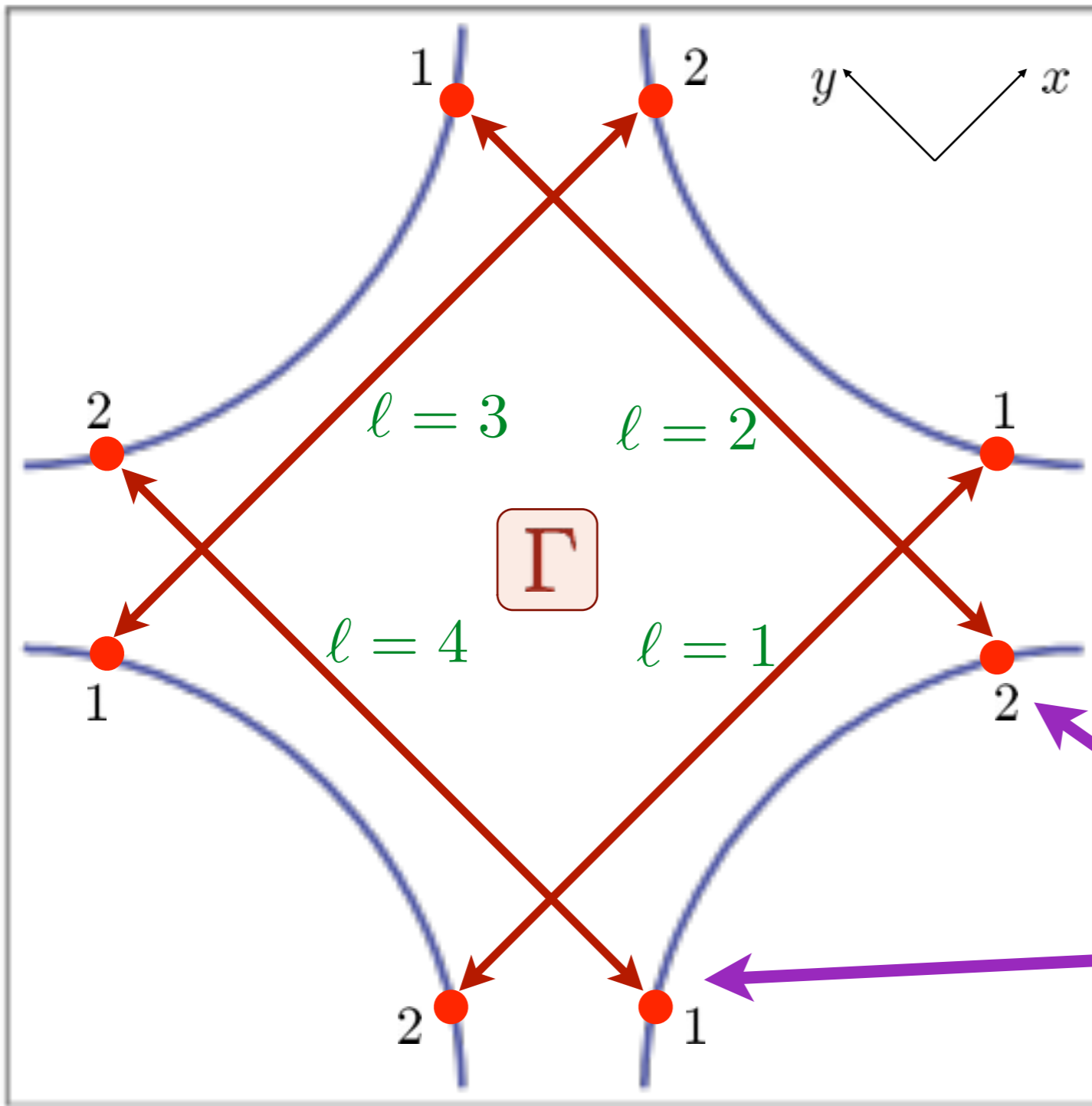


Theory of SDW quantum phase transition in metal



Start from the “spin-fermion” model

$$\begin{aligned}
 \mathcal{Z} &= \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp(-\mathcal{S}) \\
 \mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\
 &\quad - \lambda \int d\tau \sum_i c_{i\alpha}^\dagger \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\
 &\quad + \int d\tau d^2r \left[\frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]
 \end{aligned}$$



Low energy fermions

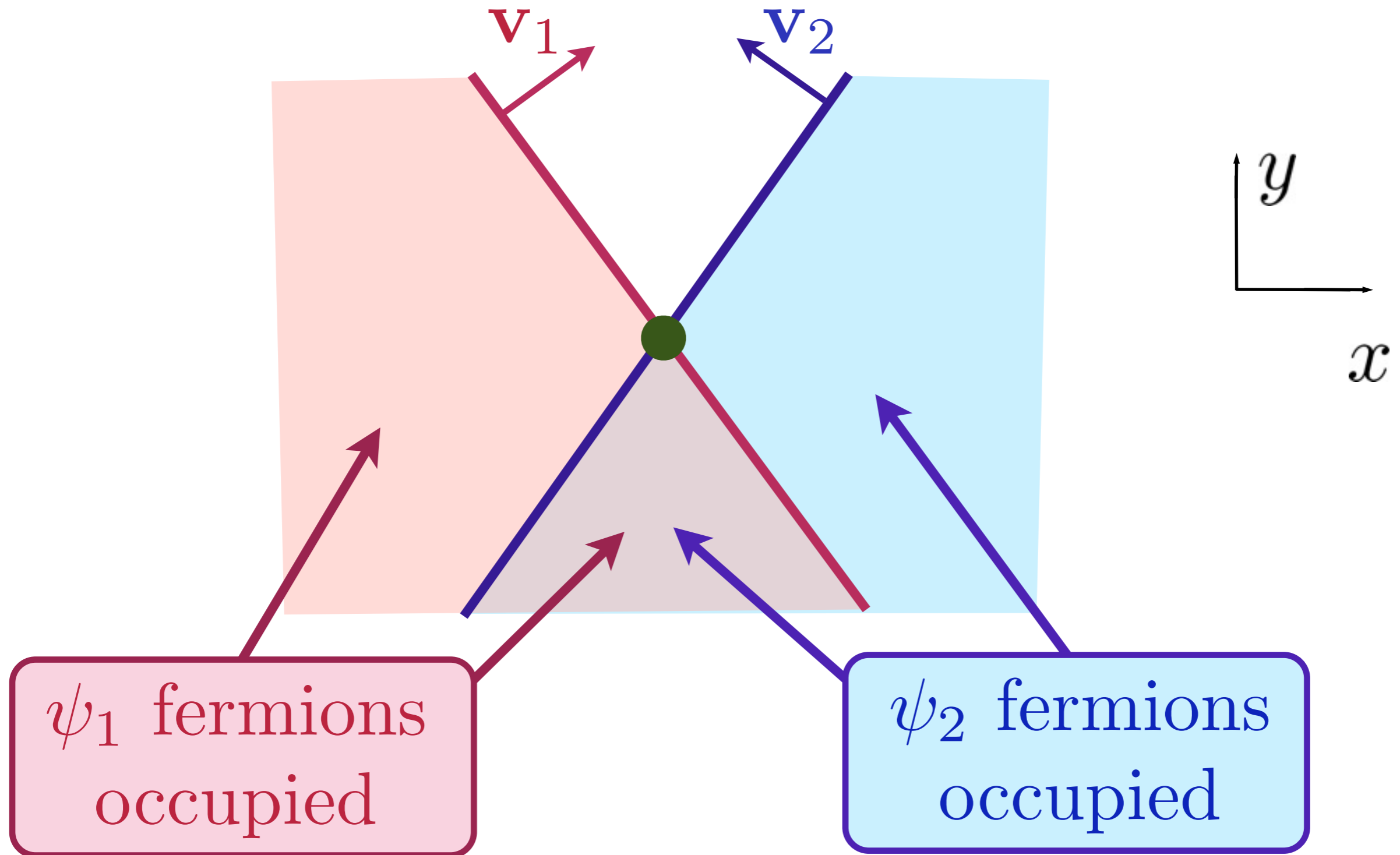
$$\psi_{1\alpha}^l, \psi_{2\alpha}^l$$

$$l = 1, \dots, 4$$

$$\mathcal{L}_f = \psi_{1\alpha}^{l\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^l \cdot \nabla_r) \psi_{1\alpha}^l + \psi_{2\alpha}^{l\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^l \cdot \nabla_r) \psi_{2\alpha}^l$$

$$\mathbf{v}_1^{l=1} = (v_x, v_y), \quad \mathbf{v}_2^{l=1} = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$



$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i\mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i\mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Hertz-Moriya-Millis (HMM) theory

Integrate out fermions and obtain non-local corrections to \mathcal{L}_φ

$$\mathcal{L}_\varphi = \frac{1}{2} \vec{\varphi}^2 [\mathbf{q}^2 + \gamma |\omega|] / 2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Hertz-Moriya-Millis (HMM) theory

Integrate out fermions and obtain non-local corrections to \mathcal{L}_φ

$$\mathcal{L}_\varphi = \frac{1}{2} \vec{\varphi}^2 [\mathbf{q}^2 + \gamma |\omega|] / 2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings

Ar.Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Perform RG on both fermions and $\vec{\varphi}$,
using a *local* field theory.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Under the rescaling $x' = x e^{-\ell}$, $\tau' = \tau e^{-z\ell}$, the spatial gradients are fixed if the fields transform as

$$\vec{\varphi}' = e^{(d+z-2)\ell/2} \vec{\varphi} \quad ; \quad \psi' = e^{(d+z-1)\ell/2} \psi.$$

Then the Yukawa coupling transforms as

$$\lambda' = e^{(4-d-z)\ell/2} \lambda$$

For $d = 2$, with $z = 2$ the Yukawa coupling is invariant, and the bare time-derivative terms ζ , $\tilde{\zeta}$ are irrelevant.

Two approaches:

- Ⓐ Fix $\lambda = 1$ and perform RG in a $1/N$ expansion, where N is the number of fermion flavors
- Ⓑ Make λ part of the bare fermion dispersion by transforming electrons to a ‘rotating reference frame’ determined by the local orientation of the SDW order $\vec{\varphi}$.

Two approaches:

- Ⓐ Fix $\lambda = 1$ and perform RG in a $1/N$ expansion, where N is the number of fermion flavors
- Ⓑ Make λ part of the bare fermion dispersion by transforming electrons to a ‘rotating reference frame’ determined by the local orientation of the SDW order $\vec{\varphi}$.



Max Metlitski

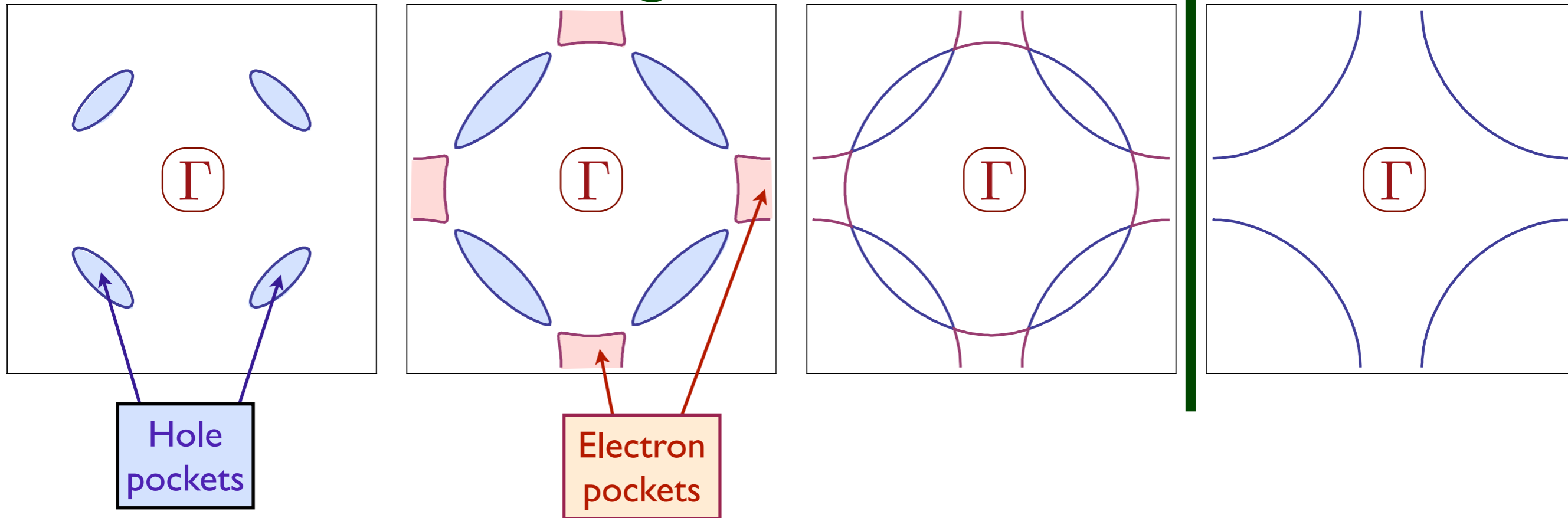
M. Metlitski and S. Sachdev, *to appear*

Ar. Abanov, A.V. Chubukov, and J. Schmalian,
Advances in Physics **52**, 119 (2003)

Sung-Sik Lee, arXiv:0905.4532.

Hole-doped cuprates

← Increasing SDW order →

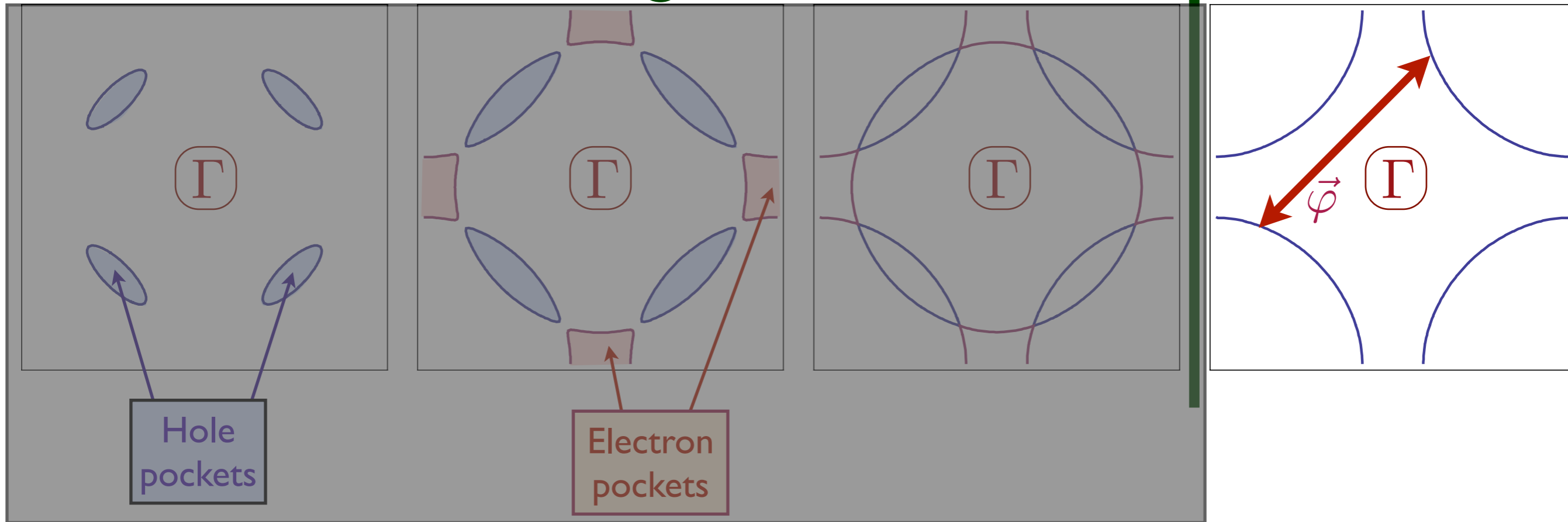


Large Fermi surface breaks up into
electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates

← Increasing SDW order →



$\vec{\varphi}$ fluctuations act on the
large Fermi surface

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$

“Yukawa” coupling: $\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$

With $z = 2$ scaling, ζ is irrelevant.

So we take $\zeta \rightarrow 0$

(watch for dangerous irrelevancy).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$

“Yukawa” coupling: $\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$

Set $\vec{\varphi}$ wavefunction renormalization by keeping co-efficient of $(\nabla_r \vec{\varphi})^2$ fixed (as usual).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Set fermion wavefunction renormalization by keeping Yukawa coupling fixed.

Y. Huh and S. Sachdev, *Phys. Rev. B* **78**, 064512 (2008).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

We find consistent two-loop RG factors, as $\zeta \rightarrow 0$, for the velocities v_x , v_y , and the wavefunction renormalizations.

Consistency check: the expression for the boson damping constant, $\gamma = \frac{2}{\pi v_x v_y}$, is preserved under RG.

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N} \frac{\alpha^2}{1 + \alpha^2}$$

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N} \frac{\alpha^2}{1 + \alpha^2}$$

The velocities flow as

$$\frac{1}{v_x} \frac{dv_x}{d\ell} = \frac{\mathcal{A}(\alpha) + \mathcal{B}(\alpha)}{2} ; \quad \frac{1}{v_y} \frac{dv_y}{d\ell} = \frac{-\mathcal{A}(\alpha) + \mathcal{B}(\alpha)}{2}$$

$$\mathcal{A}(\alpha) \equiv \frac{3}{\pi N} \frac{\alpha}{1 + \alpha^2}$$

$$\mathcal{B}(\alpha) \equiv \frac{1}{2\pi N} \left(\frac{1}{\alpha} - \alpha \right) \left(1 + \left(\frac{1}{\alpha} - \alpha \right) \tan^{-1} \frac{1}{\alpha} \right)$$

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N} \frac{\alpha^2}{1 + \alpha^2}$$

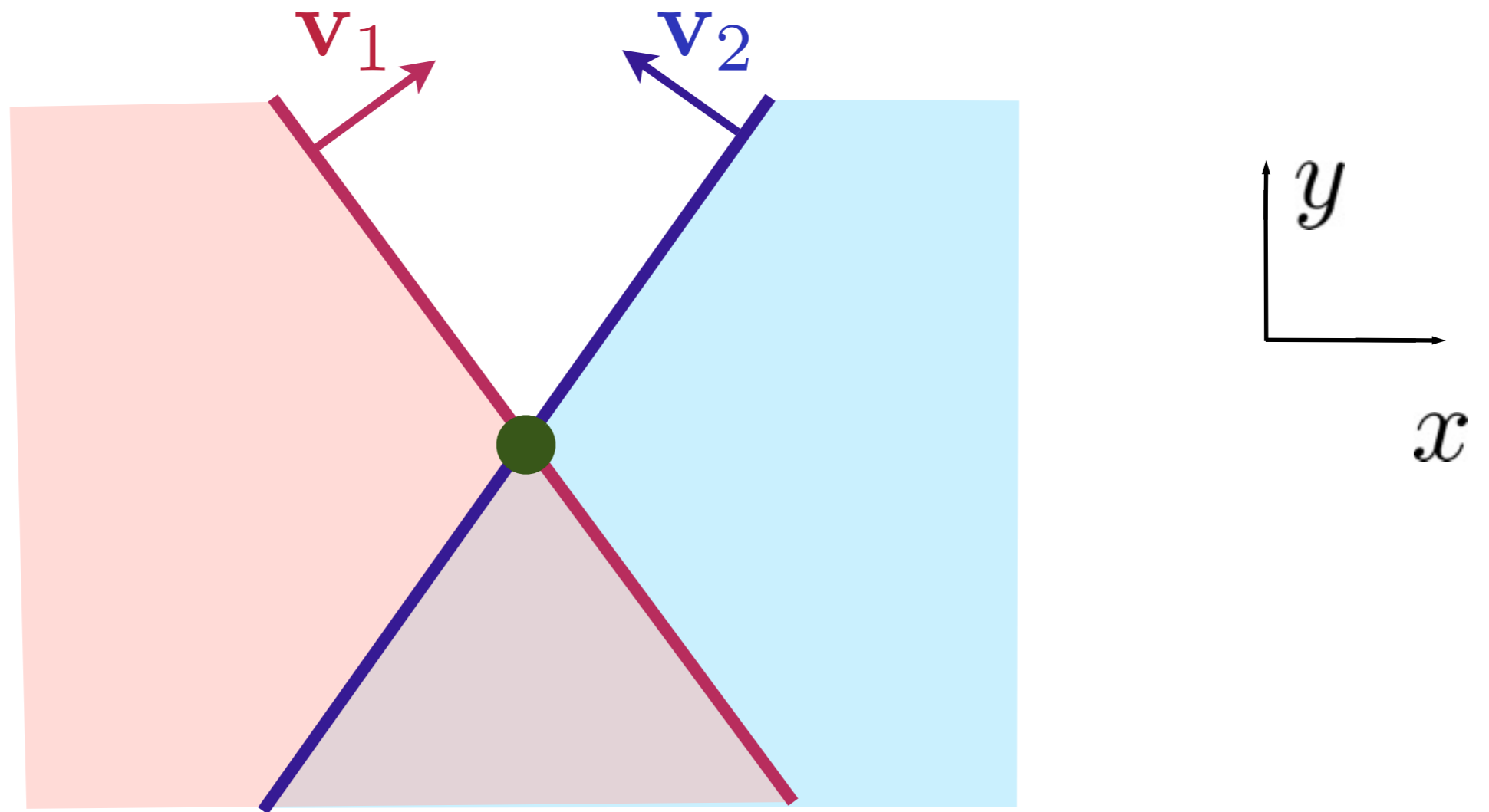
The anomalous dimensions of $\vec{\varphi}$ and ψ are

$$\eta_\varphi = \frac{1}{2\pi N} \left(\frac{1}{\alpha} - \alpha + \left(\frac{1}{\alpha^2} + \alpha^2 \right) \tan^{-1} \frac{1}{\alpha} \right)$$
$$\eta_\psi = -\frac{1}{4\pi N} \left(\frac{1}{\alpha} - \alpha \right) \left(1 + \left(\frac{1}{\alpha} - \alpha \right) \tan^{-1} \frac{1}{\alpha} \right)$$

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \rightarrow 0$ logarithmically in the infrared.

Dynamical Nesting

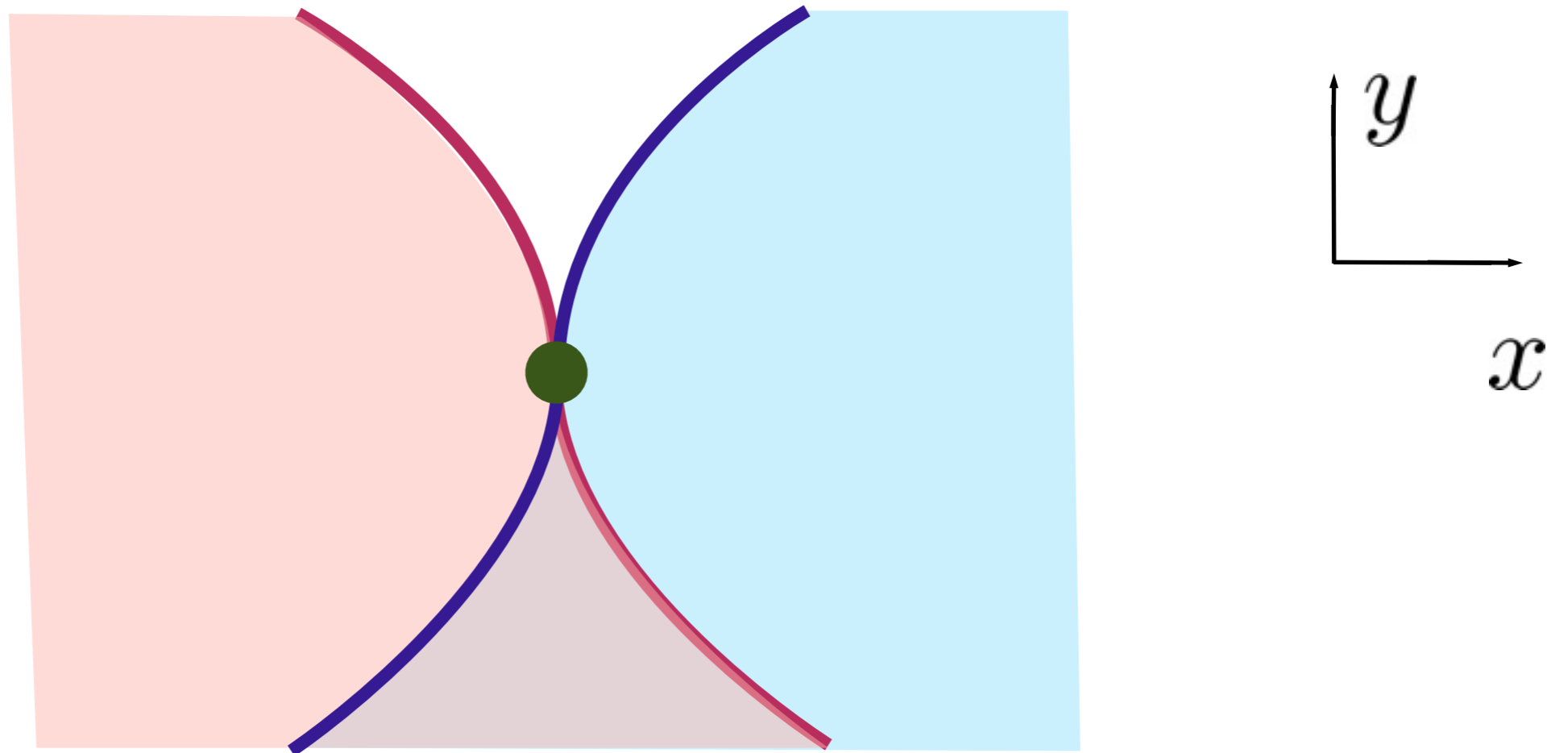


Bare Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \rightarrow 0$ logarithmically in the infrared.

Dynamical Nesting

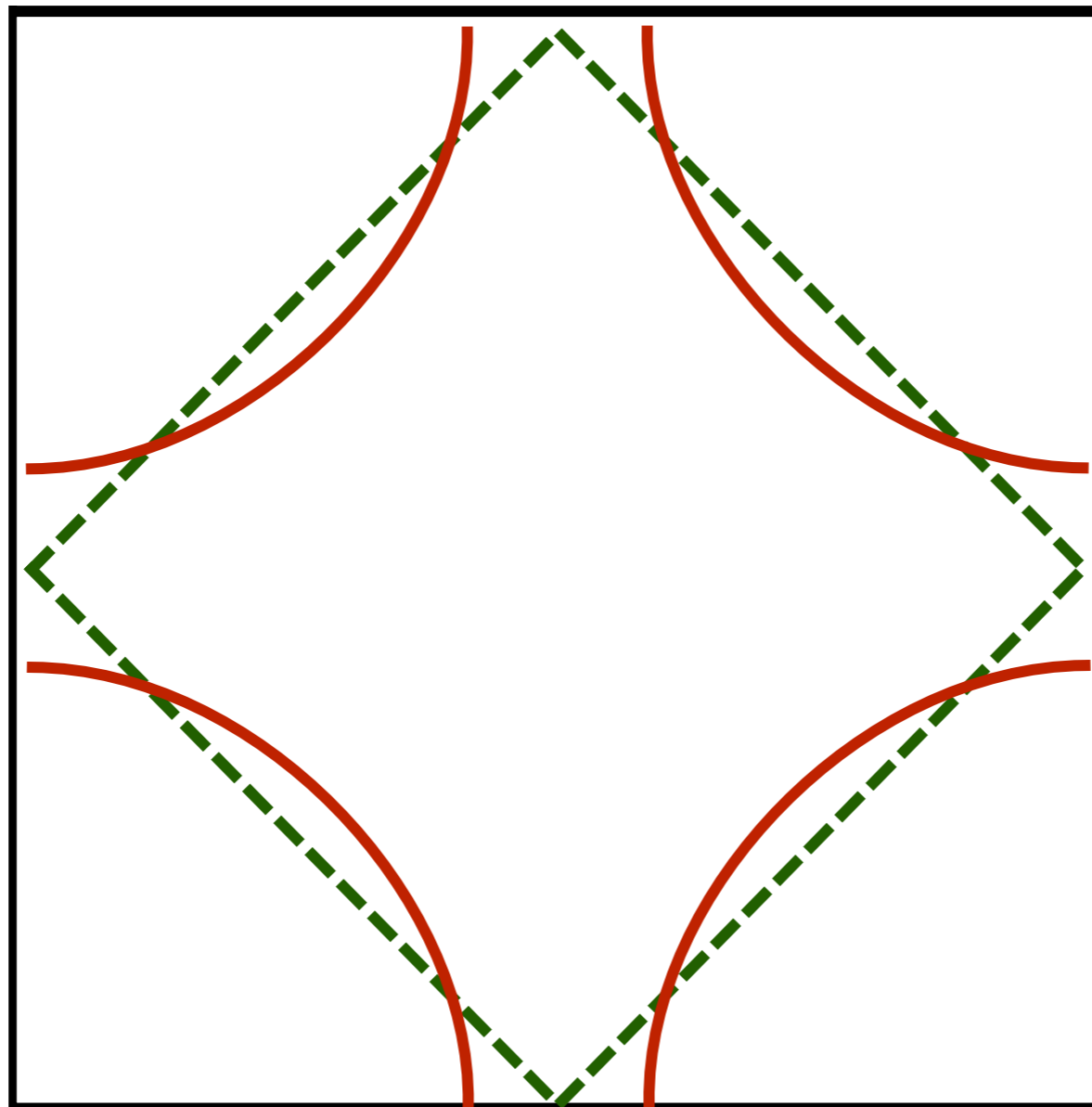


Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \rightarrow 0$ logarithmically in the infrared.

Dynamical Nesting

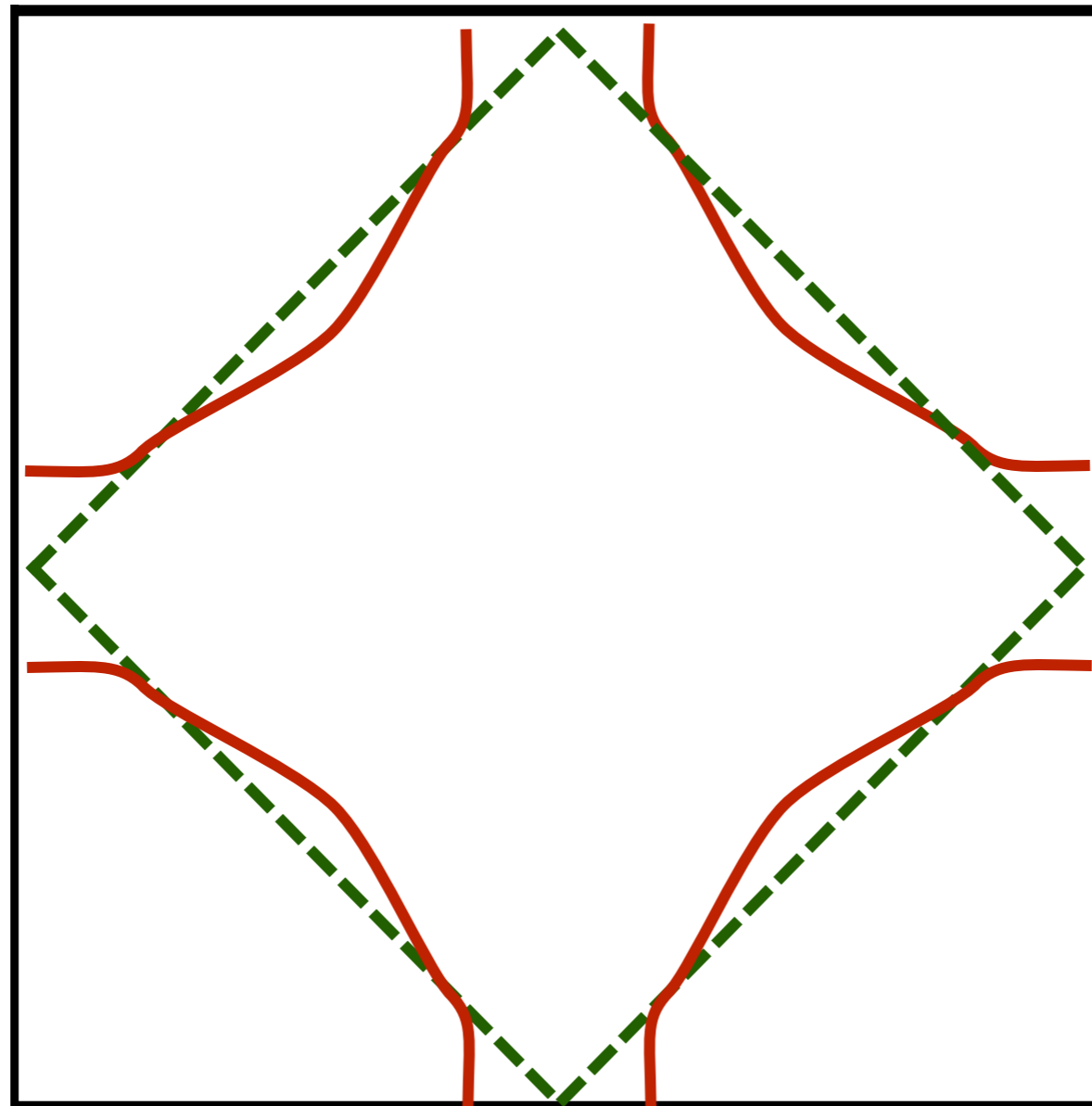


Bare Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \rightarrow 0$ logarithmically in the infrared.

Dynamical Nesting



Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \rightarrow 0$ logarithmically in the infrared.

In $\vec{\varphi}$ SDW fluctuations, characteristic q and ω scale as

$$q \sim \omega^{1/2} \exp\left(-\frac{3}{64\pi^2} \left(\frac{\ln(1/\omega)}{N}\right)^3\right).$$

However, $1/N$ expansion cannot be trusted in the asymptotic regime.

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i \frac{1}{N\sqrt{\gamma}v} \sqrt{\omega} F \left(\frac{v^2 q^2}{\omega} \right)}$$

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

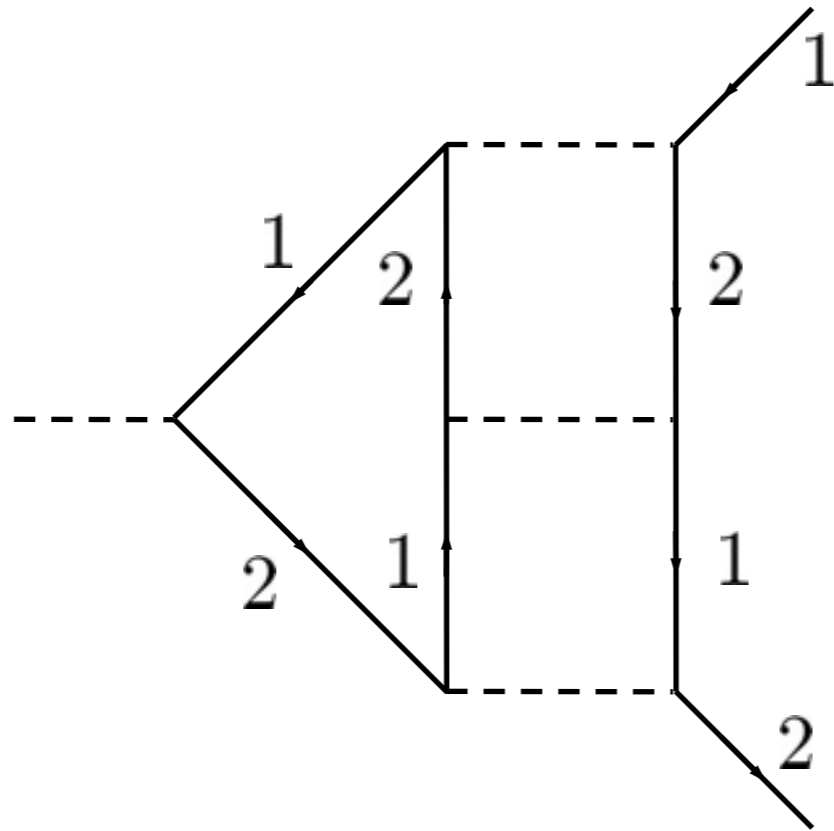
$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i \frac{1}{N\sqrt{\gamma}v} \sqrt{\omega} F\left(\frac{v^2 q^2}{\omega}\right)}$$

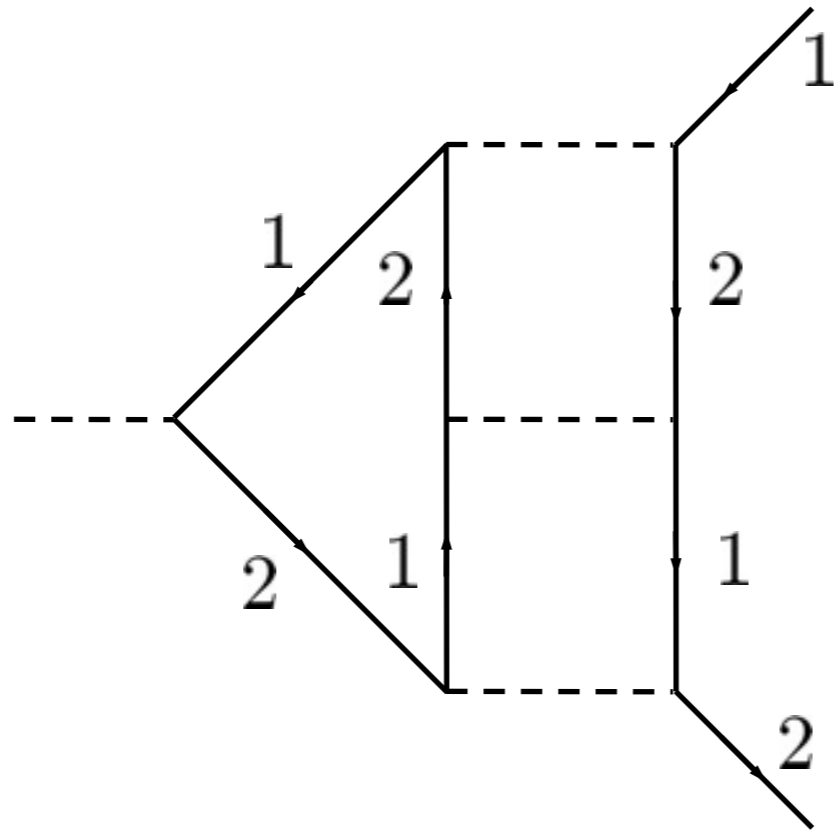
 **Dangerous**

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \frac{1}{N^2} \times \frac{1}{\zeta^2} \times \frac{1}{\omega}$

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \frac{1}{N^2} \times \frac{1}{\zeta^2} \times \frac{1}{\omega}$

Actual order $\sim \frac{1}{N^0}$

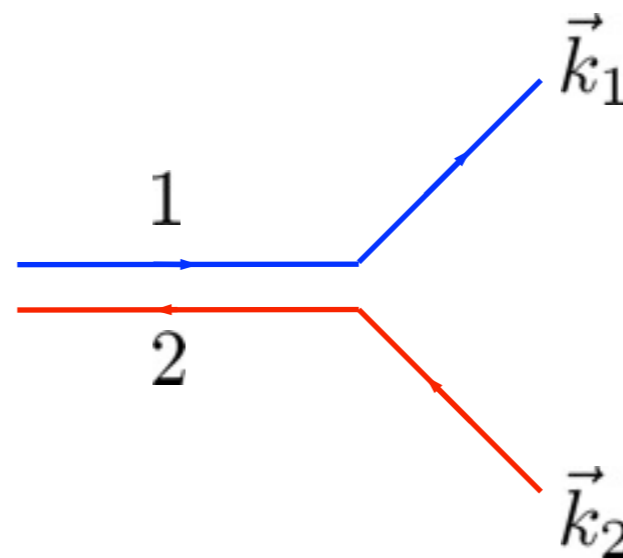
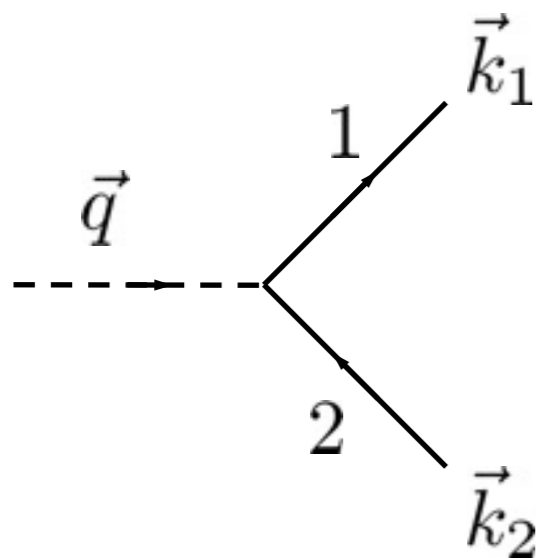
Double line representation

- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega, \vec{k}) = \frac{1}{-\Sigma_1(\omega, \vec{k}) - \vec{v} \cdot \vec{k}} \quad \Sigma_1 \sim \frac{1}{N}$$

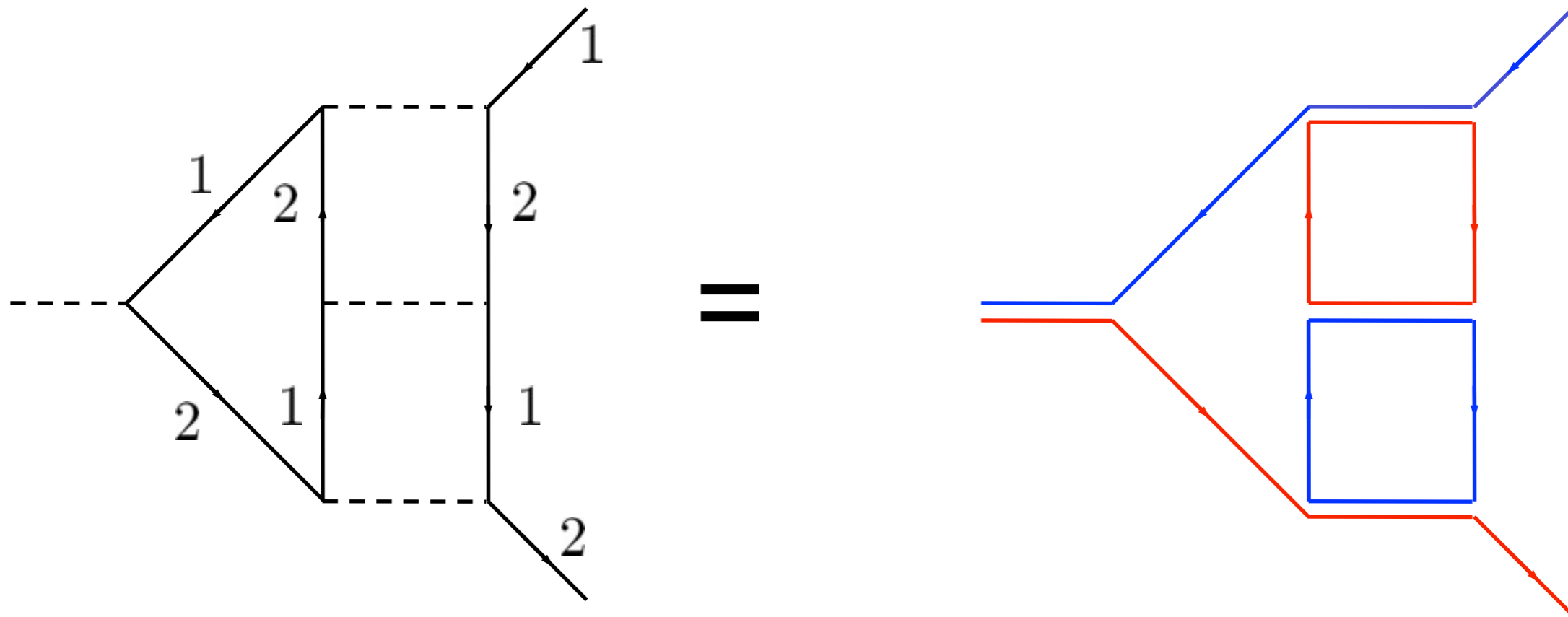
- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

$$\vec{q} = \vec{k}_1 - \vec{k}_2 \quad \vec{k}_1 \in \text{FS of } \psi_1 \quad \vec{k}_2 \in \text{FS of } \psi_2$$



R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
 S.W.Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

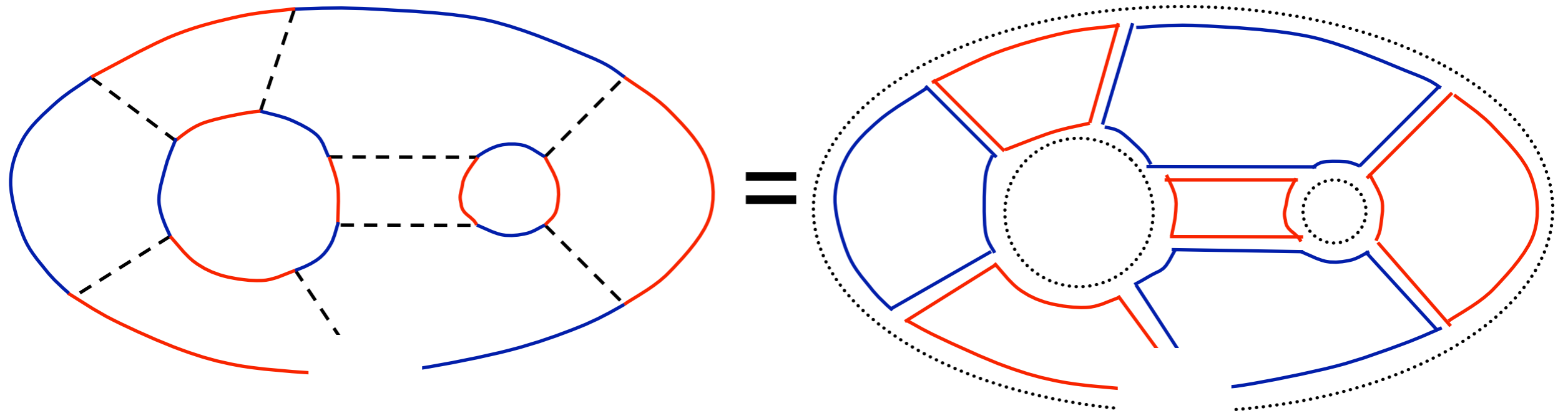
New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Singularities as $\zeta \rightarrow 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)

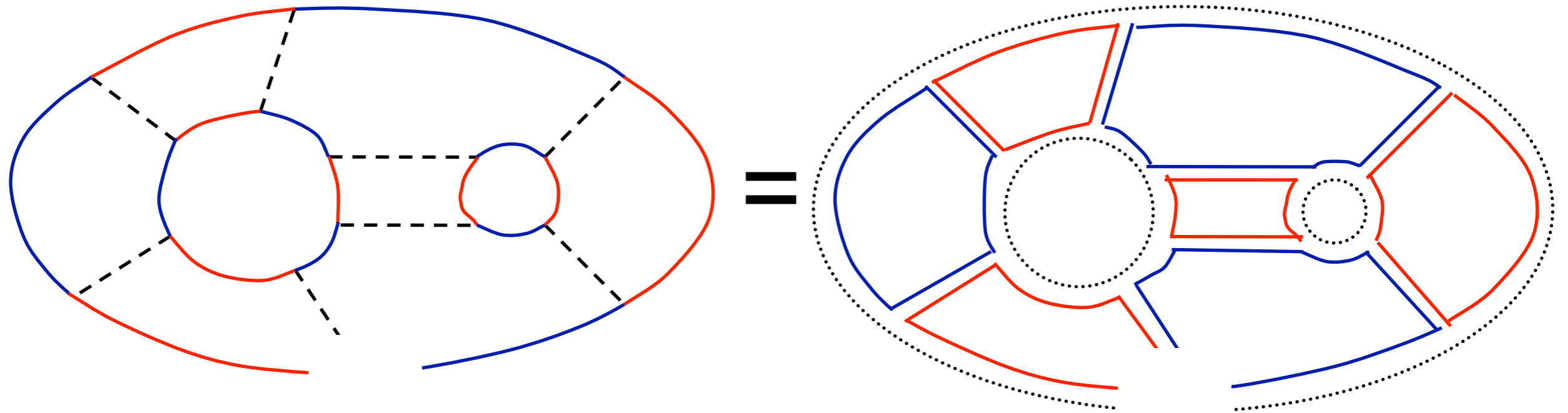


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines by drawing additional dotted single line loops for each fermion loop

Sung-Sik Lee, arXiv:0905.4532

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops
(Breakdown of Migdal-Eliashberg)



$$\text{Actual order} \sim \frac{1}{N^0}$$



A consistent analysis requires
resummation of all planar graphs



Theory for the onset
of spin density wave
order in metals is
strongly coupled in
two dimensions

Start from the “spin-fermion” model

$$\begin{aligned}
 \mathcal{Z} &= \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp(-\mathcal{S}) \\
 \mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\
 &\quad - \lambda \int d\tau \sum_i c_{i\alpha}^\dagger \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\
 &\quad + \int d\tau d^2r \left[\frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]
 \end{aligned}$$

Two approaches:

- Ⓐ Fix $\lambda = 1$ and perform RG in a $1/N$ expansion, where N is the number of fermion flavors
- Ⓑ Make λ part of the bare fermion dispersion by transforming electrons to a ‘rotating reference frame’ determined by the local orientation of the SDW order $\vec{\varphi}$.

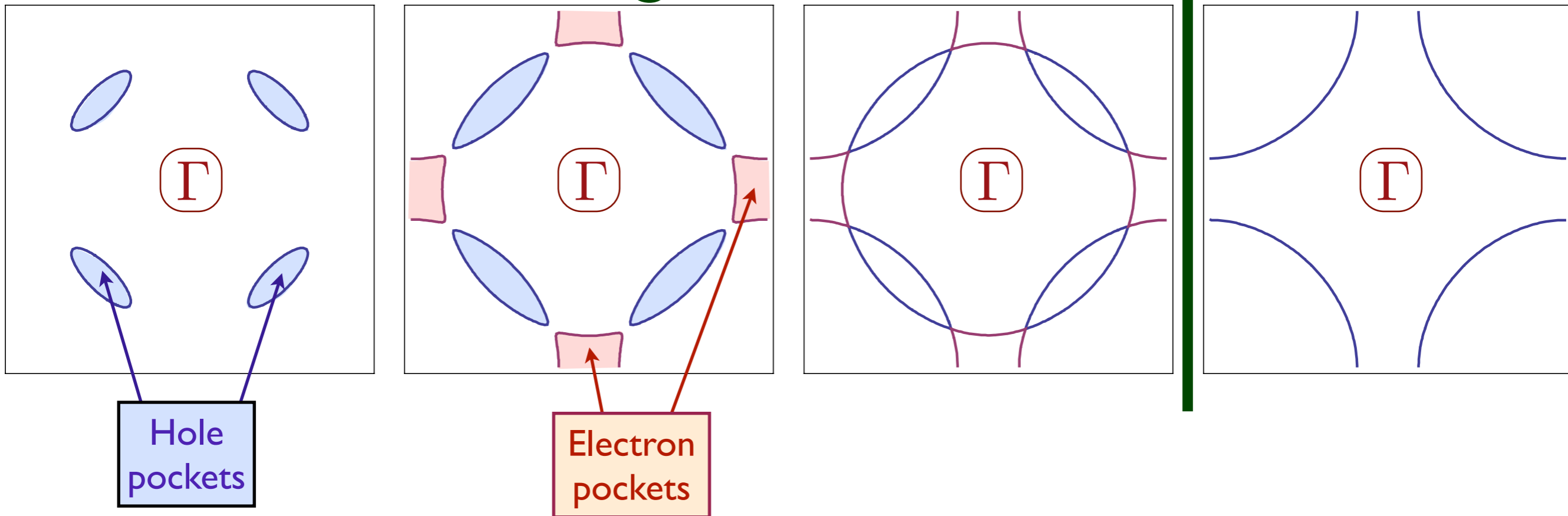
Two approaches:

Ⓐ Fix $\lambda = 1$ and perform RG in a $1/N$ expansion, where N is the number of fermion flavors

Ⓑ Make λ part of the bare fermion dispersion by transforming electrons to a ‘rotating reference frame’ determined by the local orientation of the SDW order $\vec{\varphi}$.

Theory of underdoped cuprates

← Increasing SDW order →

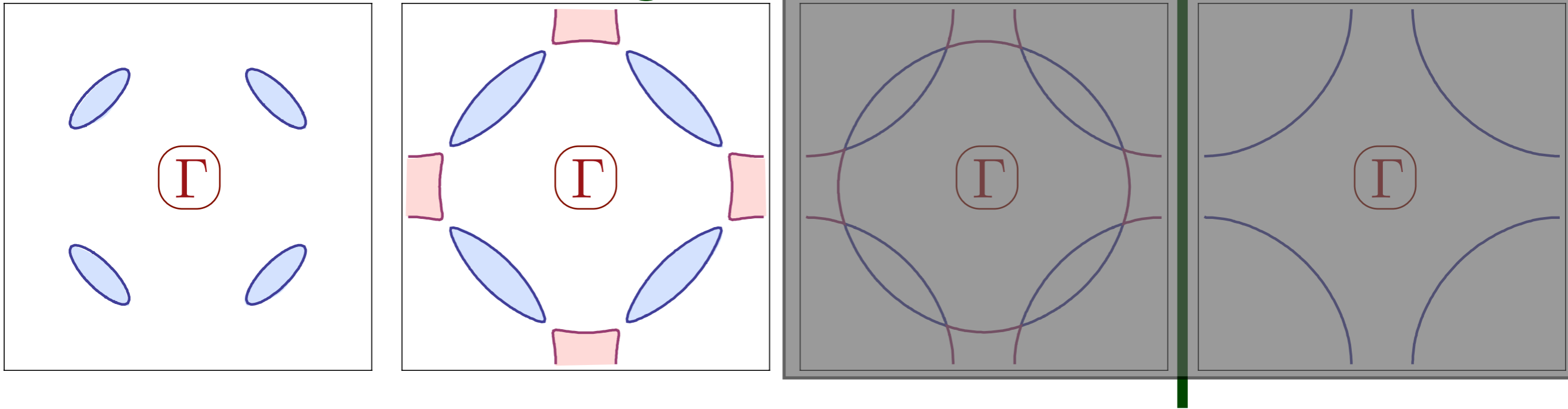


Large Fermi surface breaks up into
electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of underdoped cuprates

← Increasing SDW order →



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\varphi}$

$$\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = R \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix} ; \quad R^{\dagger} \hat{\varphi} \cdot \vec{\sigma} R = \sigma^z ; \quad R^{\dagger} R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)

B. I. Shraiman and E. D. Siggia, *Physical Review Letters* **61**, 467 (1988).

J. R. Schrieffer, *Journal of Superconductivity* **17**, 539 (2004)

Theory of underdoped cuprates

$$\text{With } R = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^* \\ z_{\downarrow} & z_{\uparrow}^* \end{pmatrix} \text{ or } \hat{\vec{\varphi}} = z_{\alpha}^* \vec{\sigma}_{\alpha\beta} z_{\beta}$$

the theory is invariant under

$$z_{\alpha} \rightarrow e^{i\theta} z_{\alpha} ; \psi_{+} \rightarrow e^{-i\theta} \psi_{+} ; \psi_{-} \rightarrow e^{i\theta} \psi_{-}$$

We obtain a U(1) gauge theory of

- bosonic neutral spinons z_{α} ;
- spinless, charged fermions ψ_{\pm} with small ‘pocket’ Fermi surfaces;
- an emergent U(1) gauge field A_{μ} .

X.-G. Wen, *Phys. Rev. B* **39**, 7223 (1989).

P. A. Lee, *Phys. Rev. Lett.* **63**, 680 (1989).

R. Shankar, *Phys. Rev. Lett.* **63**, 203 (1989).

L. B. Ioffe and P. B. Wiegmann, *Phys. Rev. Lett.* **65**, 653 (1990).

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007).

Theory of underdoped cuprates

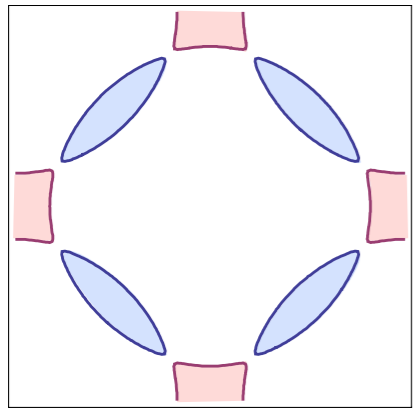
Phases of theory:

- $\langle z_\alpha \rangle \neq 0$:
Conventional SDW Fermi liquid with small pocket Fermi surfaces.
- $\langle z_\alpha \rangle = 0$:
A non-Fermi liquid ('algebraic charge liquid' ACL) with spinless 'pocket' fermions ψ_\pm , and spinful bosons z_α .

Quantum phase transitions in metal

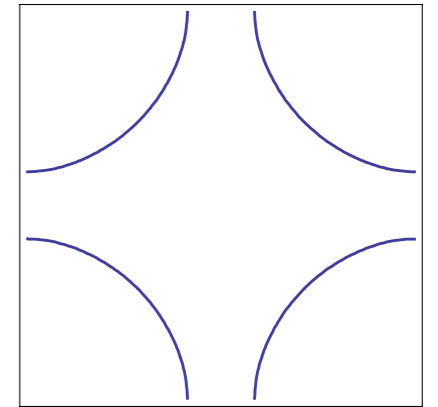
(A)

$\langle R \rangle \neq 0$; $\langle N \rangle \neq 0$
SDW order
small Fermi pockets



(C)

$\langle R \rangle \neq 0$; $\langle N \rangle = 0$
Fermi liquid
large Fermi surface



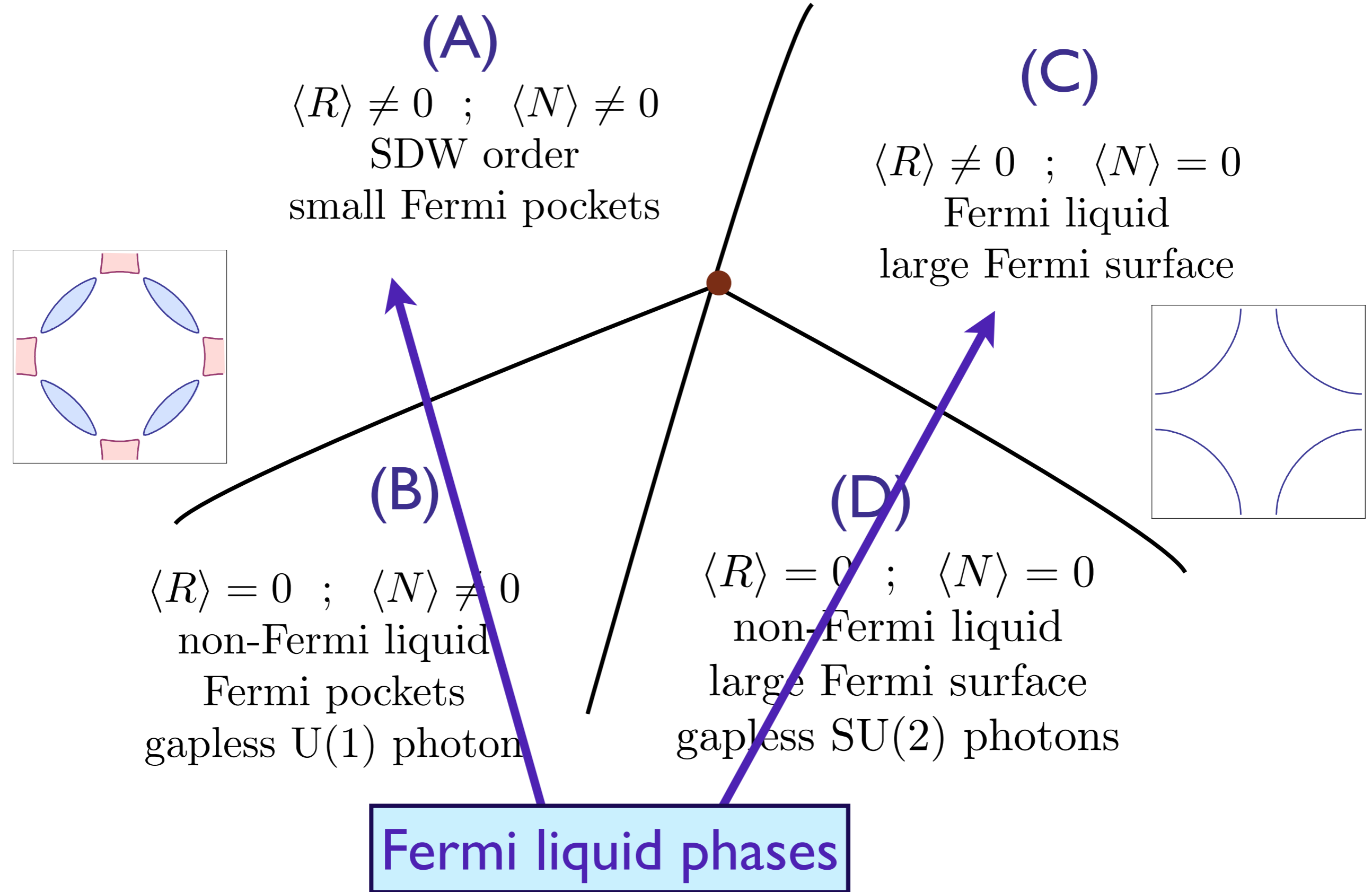
(B)

$\langle R \rangle = 0$; $\langle N \rangle \neq 0$
non-Fermi liquid
Fermi pockets
gapless U(1) photon

(D)

$\langle R \rangle = 0$; $\langle N \rangle = 0$
non-Fermi liquid
large Fermi surface
gapless SU(2) photons

Quantum phase transitions in metal

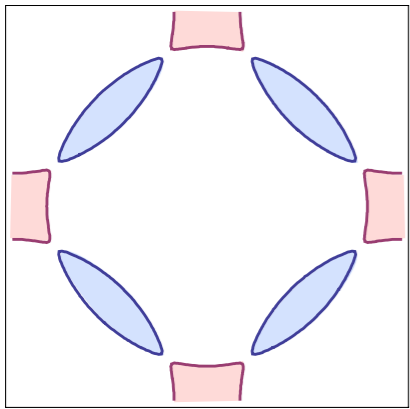


S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Quantum phase transitions in metal

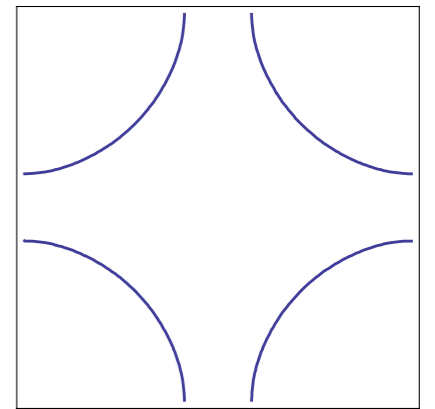
(A)

$\langle R \rangle \neq 0$; $\langle N \rangle \neq 0$
SDW order
small Fermi pockets



(C)

$\langle R \rangle \neq 0$; $\langle N \rangle = 0$
Fermi liquid
large Fermi surface



(B)

$\langle R \rangle = 0$; $\langle N \rangle \neq 0$
non-Fermi liquid
Fermi pockets
gapless U(1) photon

(D)

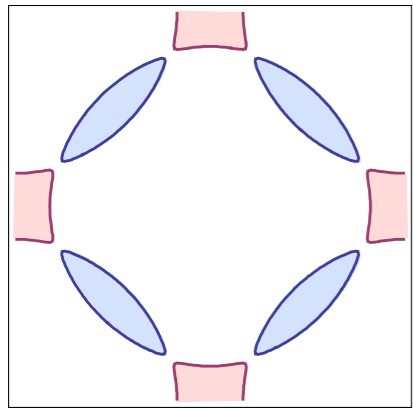
$\langle R \rangle = 0$; $\langle N \rangle = 0$
non-Fermi liquid
large Fermi surface
gapless SU(2) photons

Route A: $\vec{\varphi}$ fluctuations

Quantum phase transitions in metal

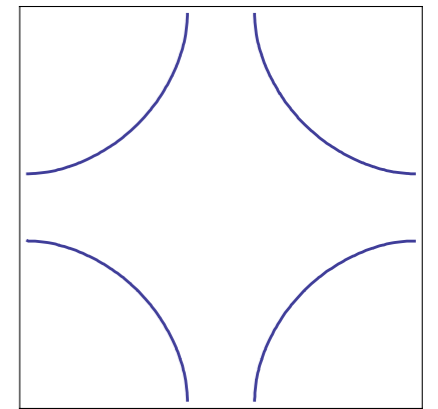
(A)

$\langle R \rangle \neq 0$; $\langle N \rangle \neq 0$
SDW order
small Fermi pockets



(C)

$\langle R \rangle \neq 0$; $\langle N \rangle = 0$
Fermi liquid
large Fermi surface



(B)

$\langle R \rangle = 0$; $\langle N \rangle \neq 0$
non-Fermi liquid
Fermi pockets
gapless U(1) photon

(D)

$\langle R \rangle = 0$; $\langle N \rangle = 0$
non-Fermi liquid
large Fermi surface
gapless SU(2) photons

Route B: non-Fermi liquids (ACLs)
with z_α fluctuations

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Theory of underdoped cuprates

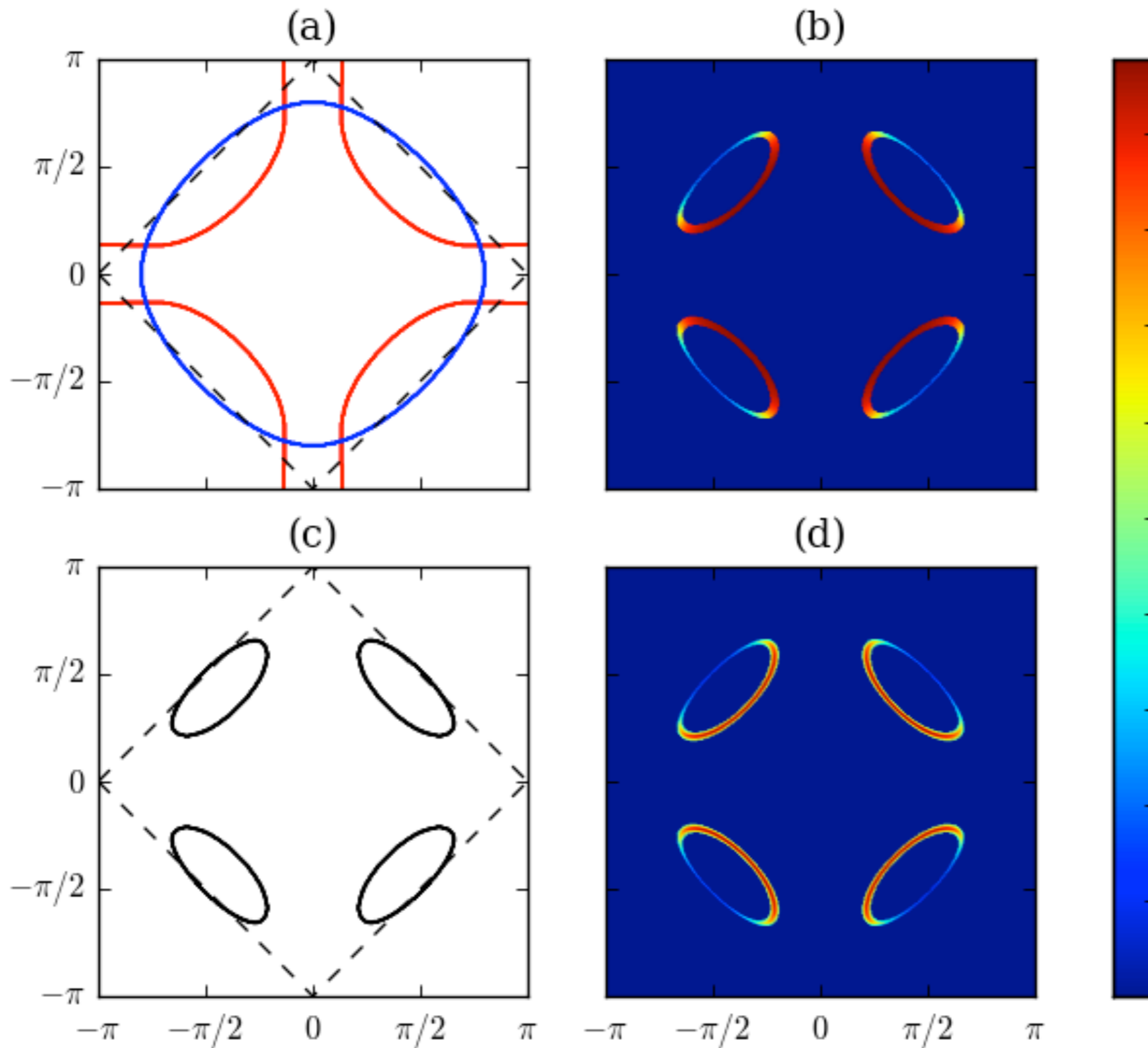
Phases of theory:

- $\langle z_\alpha \rangle \neq 0$:
Conventional SDW Fermi liquid with small pocket Fermi surfaces.
- $\langle z_\alpha \rangle = 0$:
A non-Fermi liquid ('algebraic charge liquid' ACL) with spinless 'pocket' fermions ψ_\pm , and spinful bosons z_α .

Theory of underdoped cuprates

Phases of theory:

- $\langle z_\alpha \rangle \neq 0$:
Conventional SDW Fermi liquid with small pocket Fermi surfaces.
- $\langle z_\alpha \rangle = 0$:
A non-Fermi liquid ('algebraic charge liquid' ACL) with spinless 'pocket' fermions ψ_\pm , and spinful bosons z_α .
 - Bound states of z_α and ψ_\pm lead to pocket Fermi surfaces not centered at Brillouin zone boundary.



R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)

Y. Qi and S. Sachdev, [arXiv:0912.xxxx](https://arxiv.org/abs/0912.xxxx)

Theory of underdoped cuprates

Phases of theory:

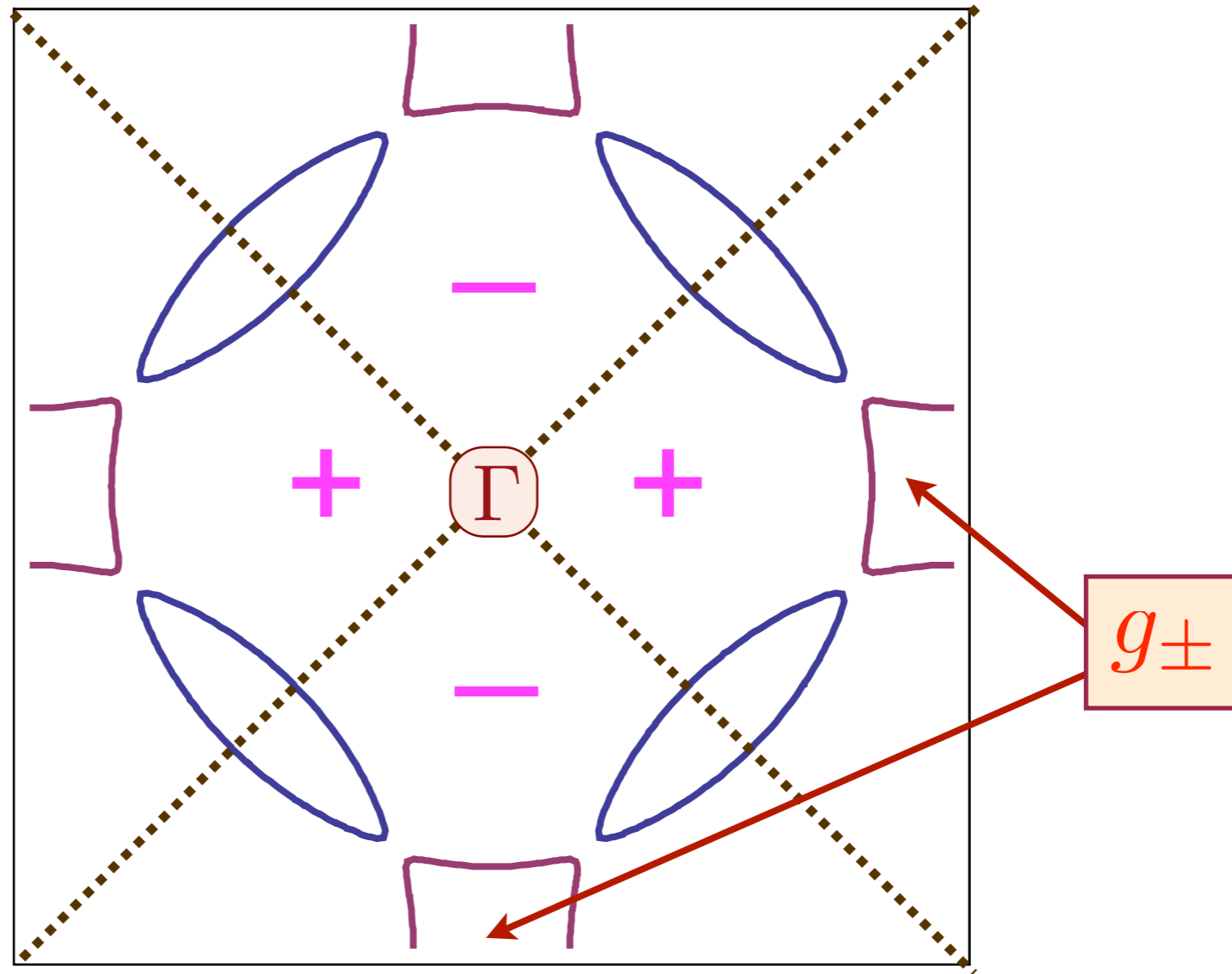
- $\langle z_\alpha \rangle \neq 0$:
Conventional SDW Fermi liquid with small pocket Fermi surfaces.
- $\langle z_\alpha \rangle = 0$:
A non-Fermi liquid ('algebraic charge liquid' ACL) with spinless 'pocket' fermions ψ_\pm , and spinful bosons z_α .
 - Bound states of z_α and ψ_\pm lead to pocket Fermi surfaces not centered at Brillouin zone boundary.

Theory of underdoped cuprates

Phases of theory:

- $\langle z_\alpha \rangle \neq 0$:
Conventional SDW Fermi liquid with small pocket Fermi surfaces.
- $\langle z_\alpha \rangle = 0$:
A non-Fermi liquid ('algebraic charge liquid' ACL) with spinless 'pocket' fermions ψ_\pm , and spinful bosons z_α .
 - Bound states of z_α and ψ_\pm lead to pocket Fermi surfaces not centered at Brillouin zone boundary.
 - Gauge fluctuations lead to d wave pairing of electrons, with strong (locally 's wave') pairing of electron pockets, and weak (locally 'p wave') pairing of hole pockets.

Strong pairing of the g_{\pm} electron pockets



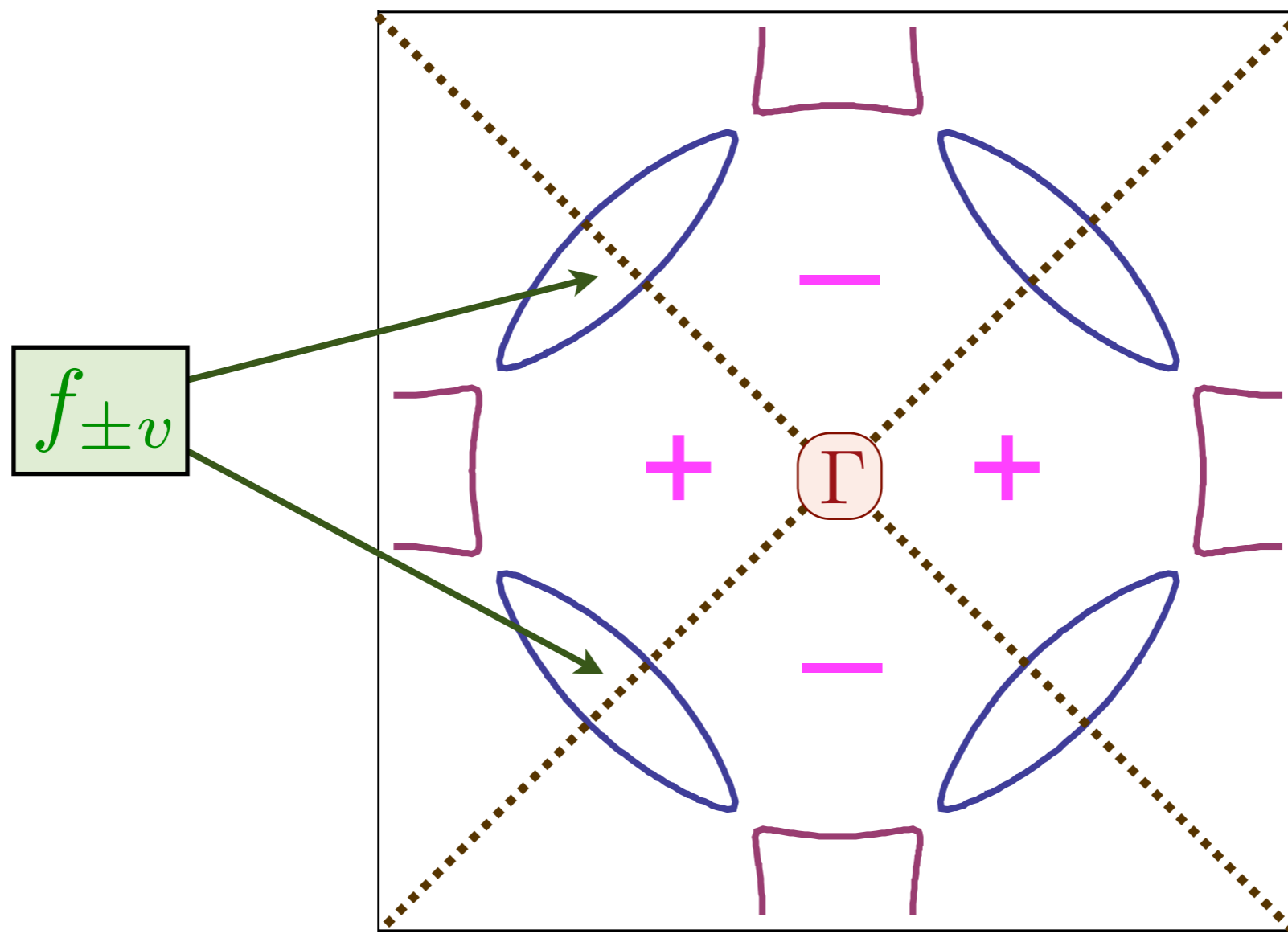
$$\langle g_{+}g_{-} \rangle = \Delta$$

R. K. Kaul, M. Metlitski, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Eun Gook Moon and S. Sachdev, *Physical Review B* **80**, 035117 (2009).

Weak pairing of the f_{\pm} hole pockets



$$\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle \sim (k_x - k_y) J \langle g_+ g_- \rangle;$$

$$\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle \sim (k_x + k_y) J \langle g_+ g_- \rangle;$$

$$\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle = 0,$$

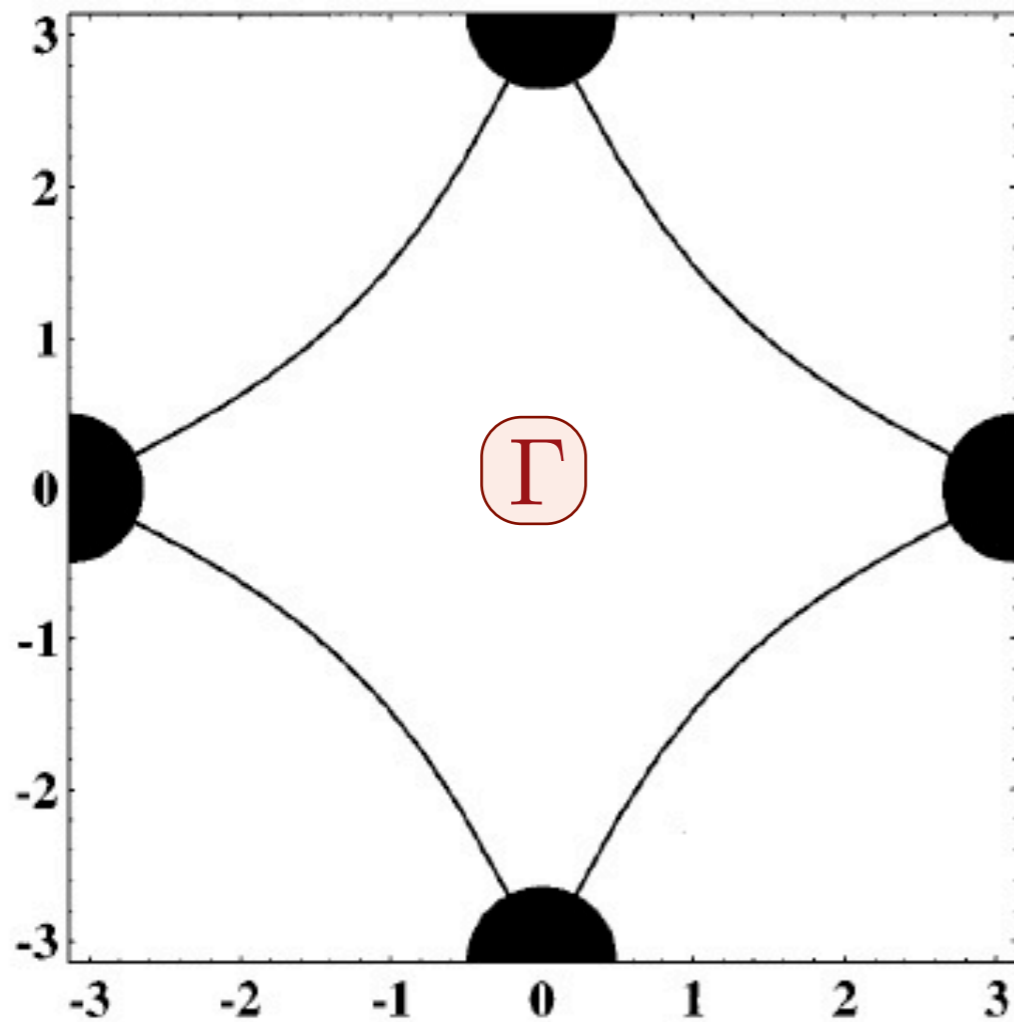


FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, *Phys. Rev. B* **55**, 3173 (1997).

Similar features in our theory

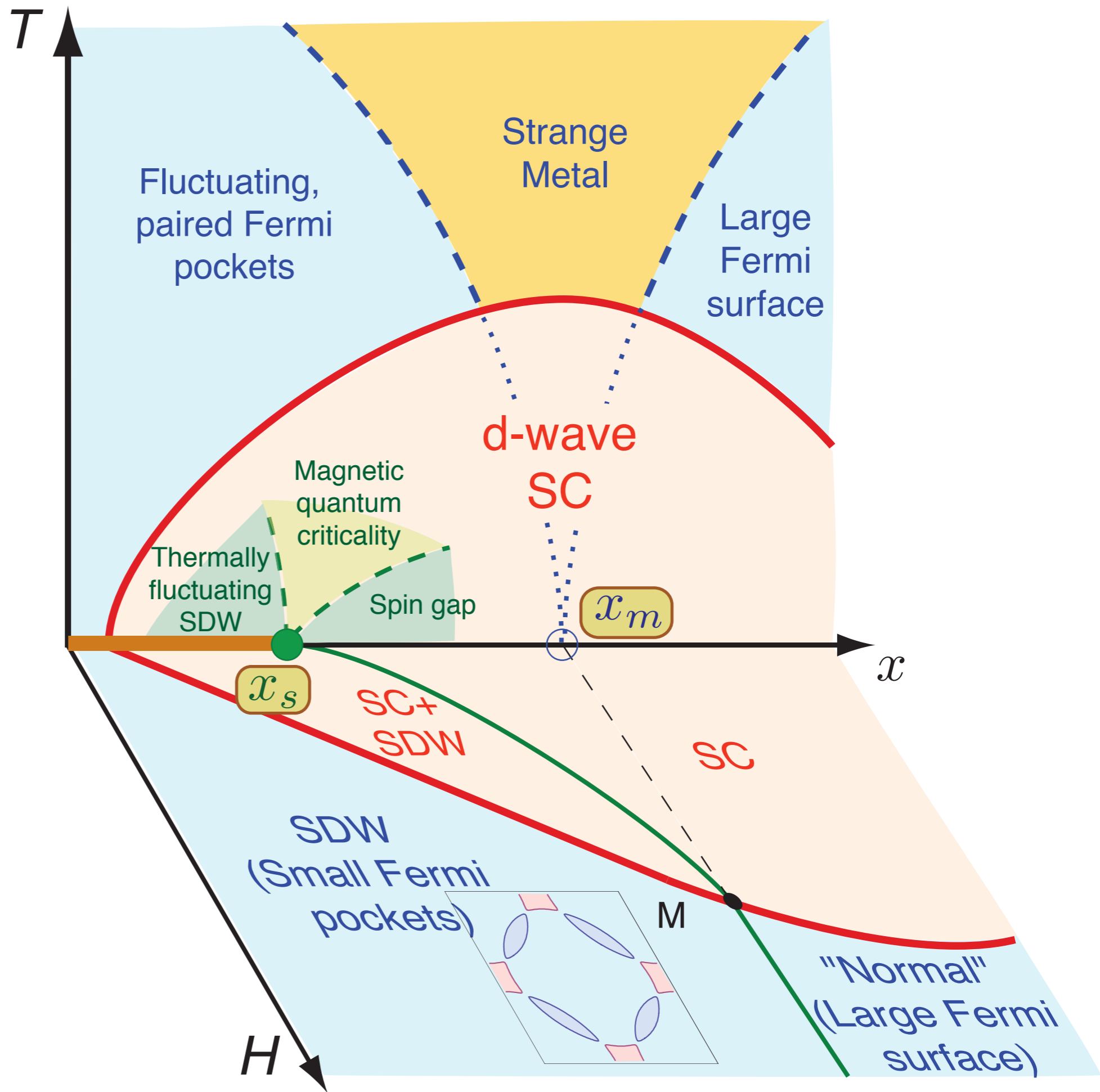
V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q}' V_{p,q} (b_q^\dagger c_{p\uparrow} c_{q-p\downarrow} + \text{H.c.})$$

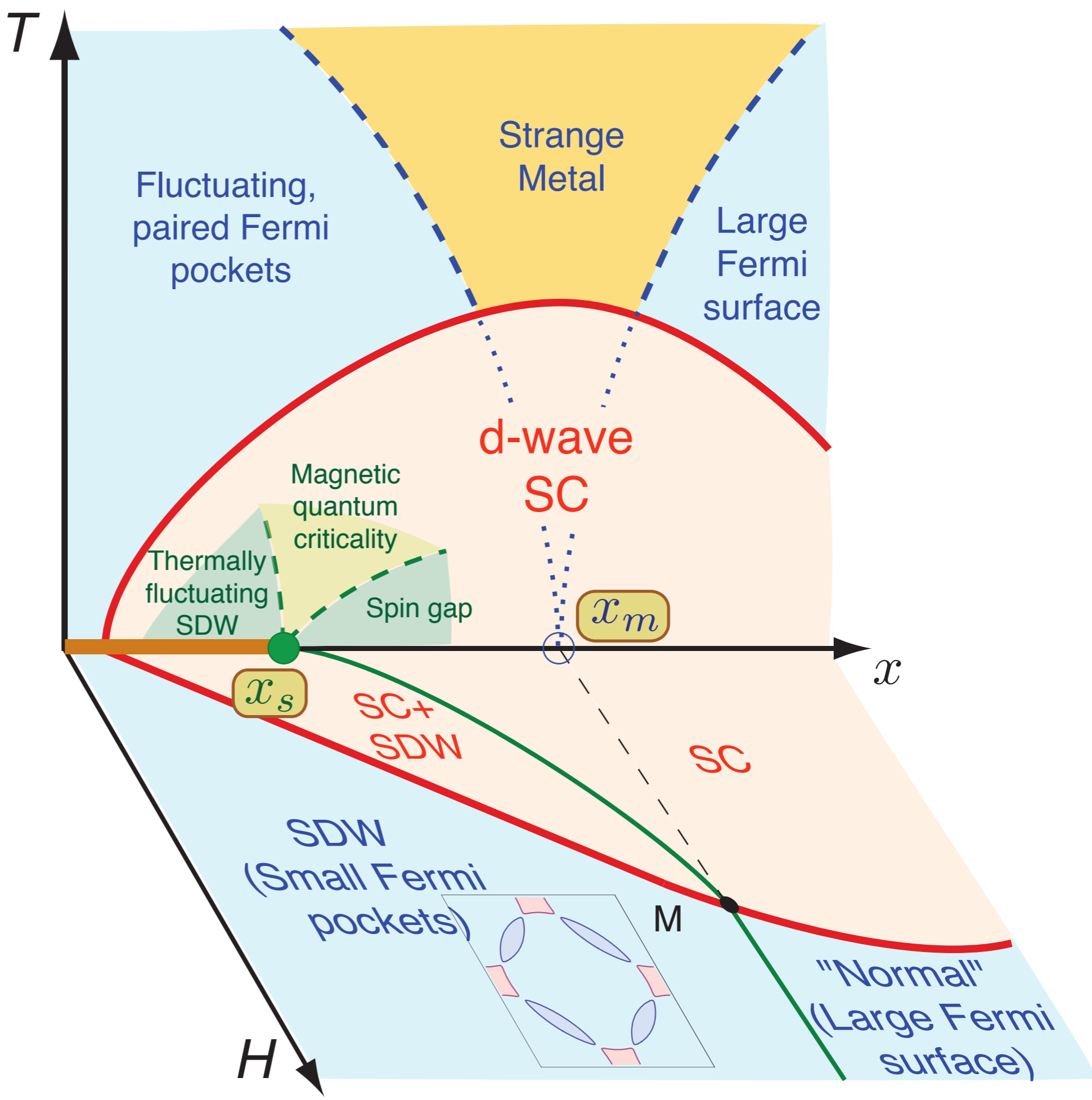
$$+ \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma};$$

$$V_{p,q} = Va^2(p_x^2 - p_y^2)$$

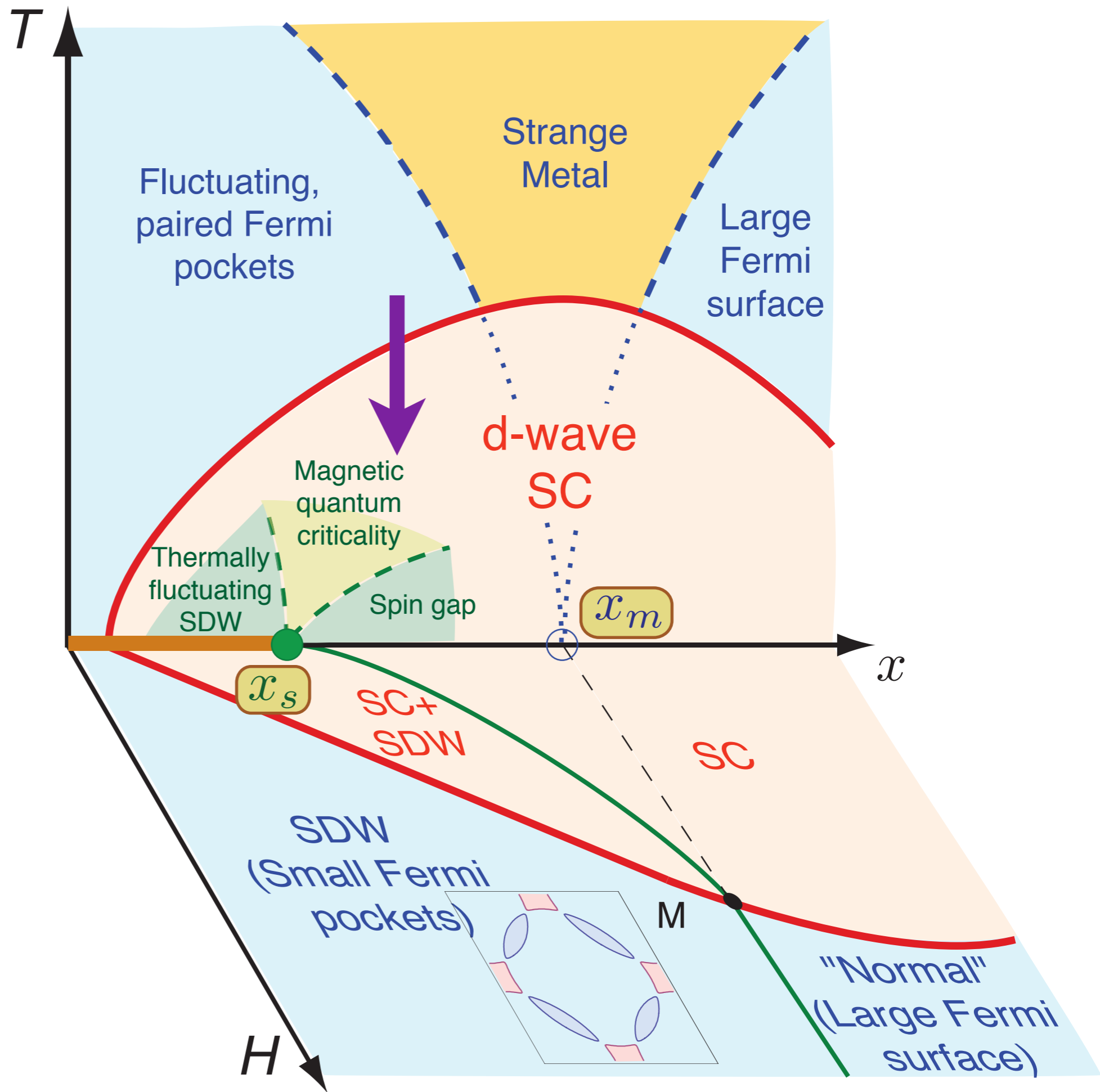
-2e bosons at antinodes,
+e fermion “arcs” at nodes,
and proximity “Josephson”
coupling



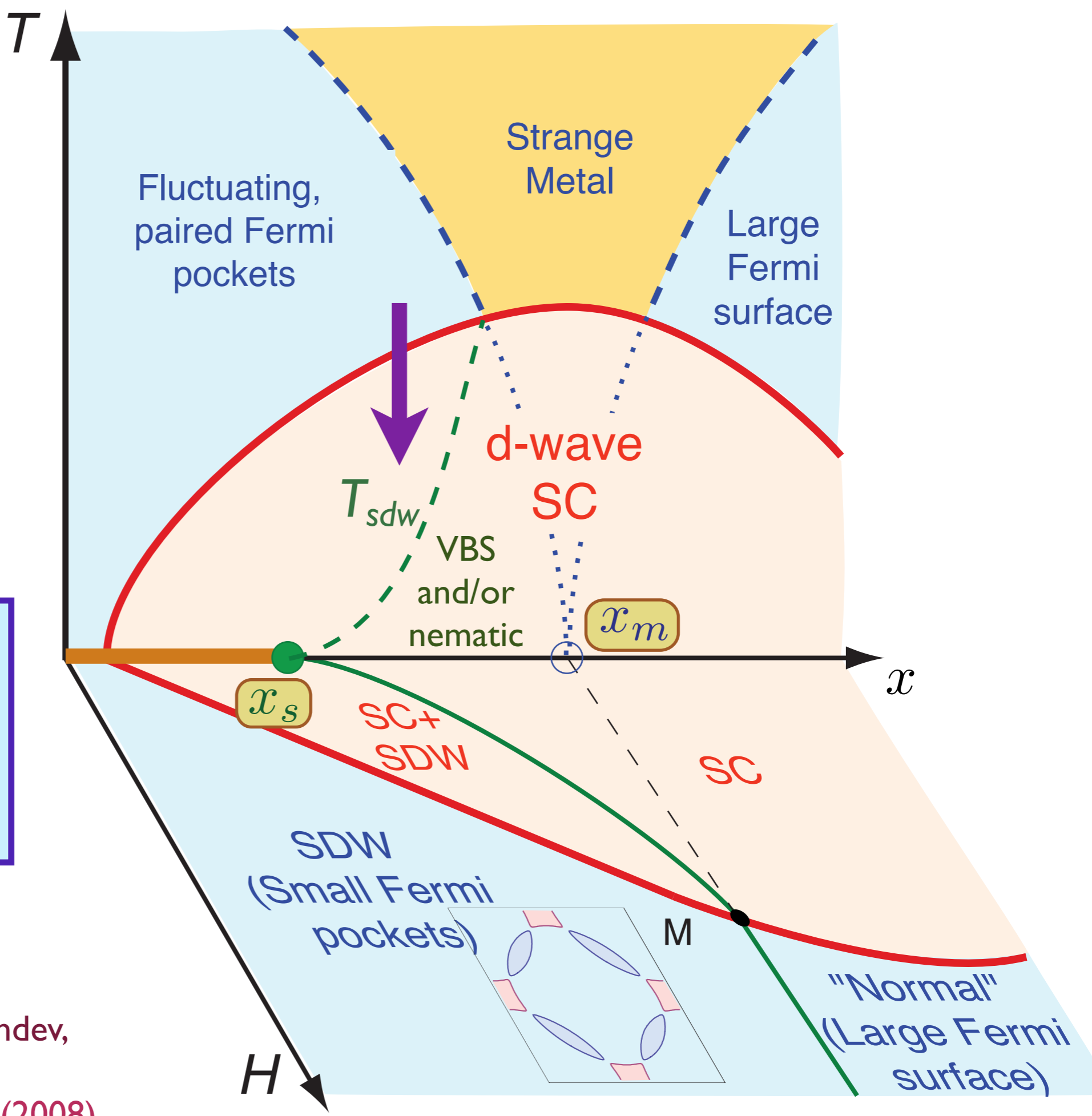
Naturally formulated in route B: theory of fluctuating Fermi pockets



Naturally formulated in route B: theory of fluctuating Fermi pockets



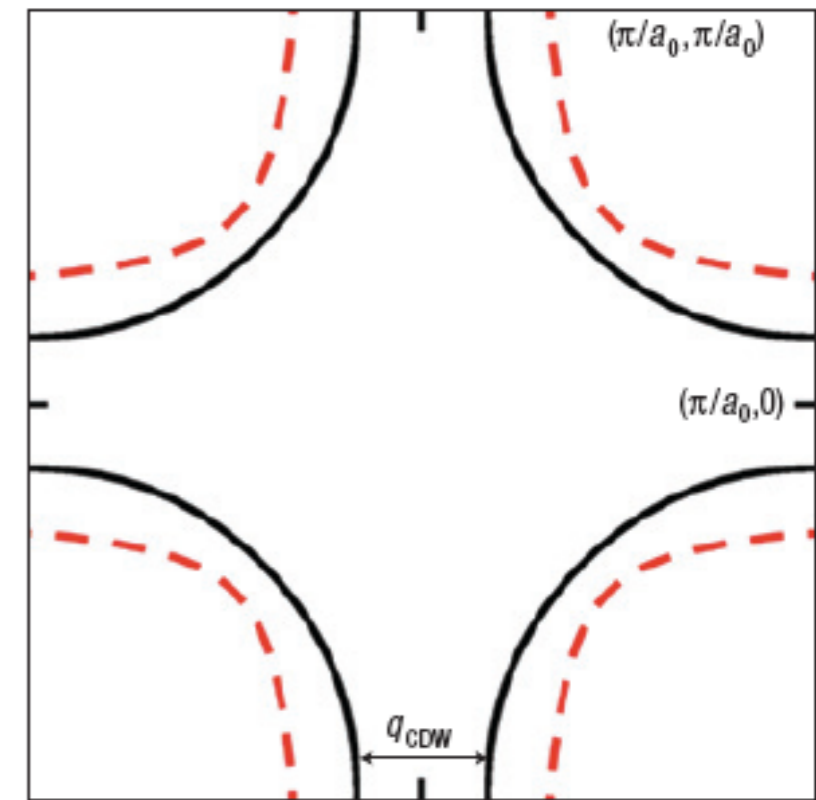
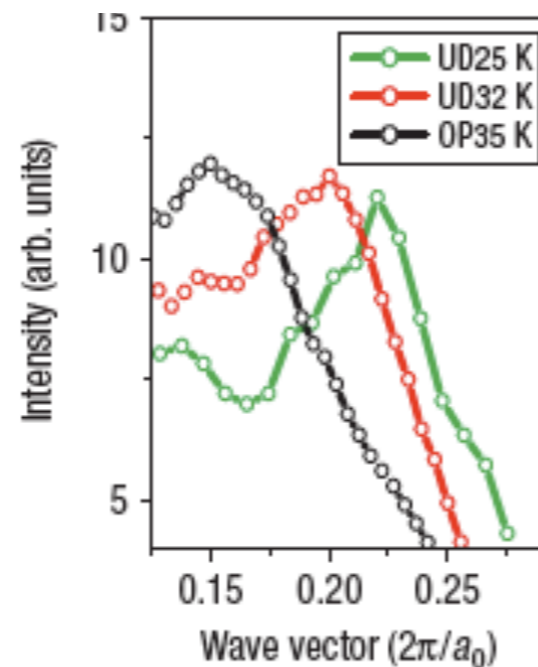
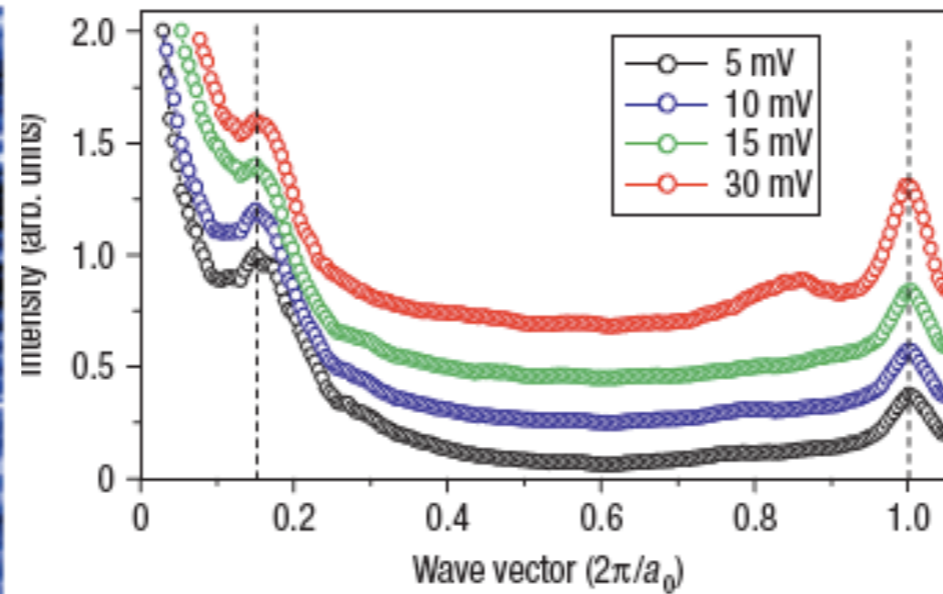
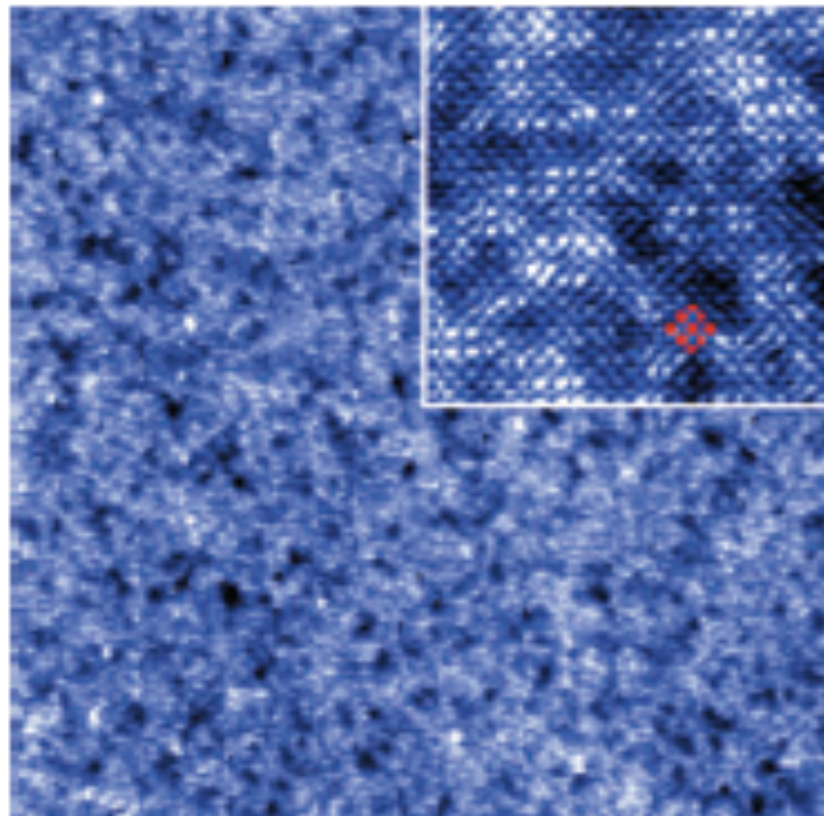
Onset of superconductivity induces confinement



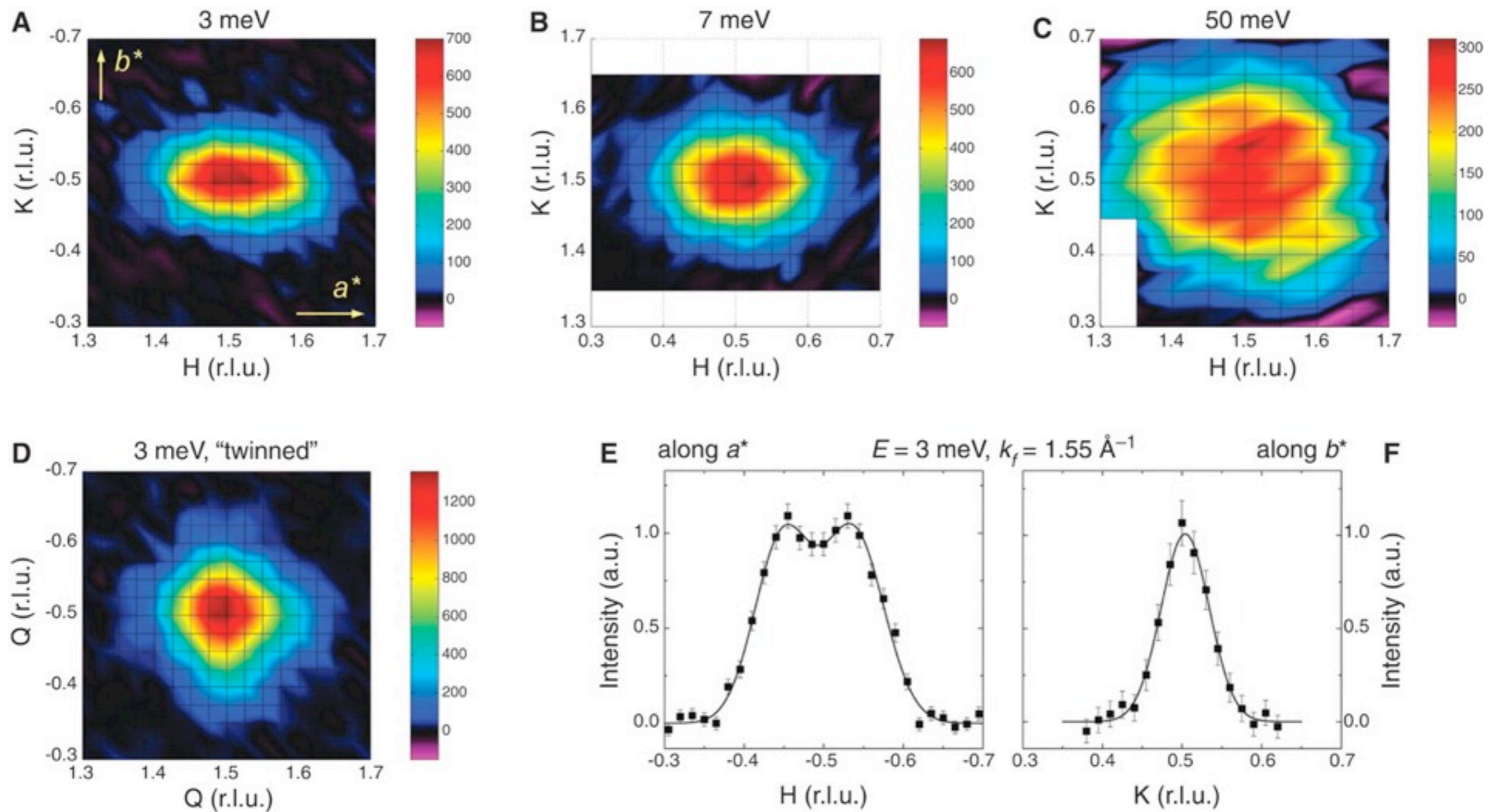
R. K. Kaul, M. Metlitski, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

Charge-density-wave origin of cuprate checkerboard visualized by scanning tunnelling microscopy

W. D. WISE¹, M. C. BOYER¹, KAMALESH CHATTERJEE¹, TAKESHI KONDO^{1,2*}, T. TAKEUCHI^{2,3}, H. IKUTA², YAYU WANG^{1*} AND E. W. HUDSON^{1†}



Nature Physics 4, 696 (2008)



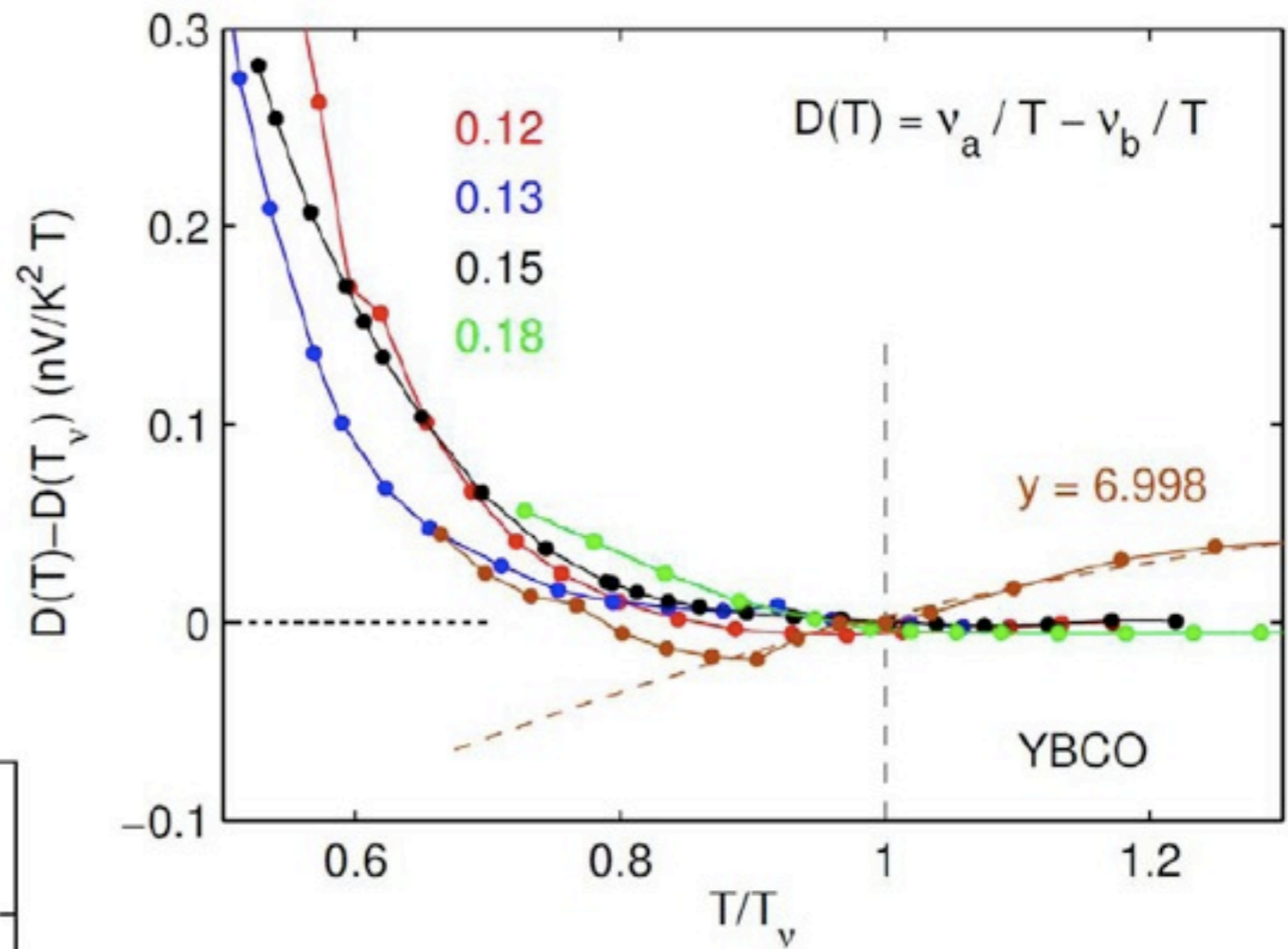
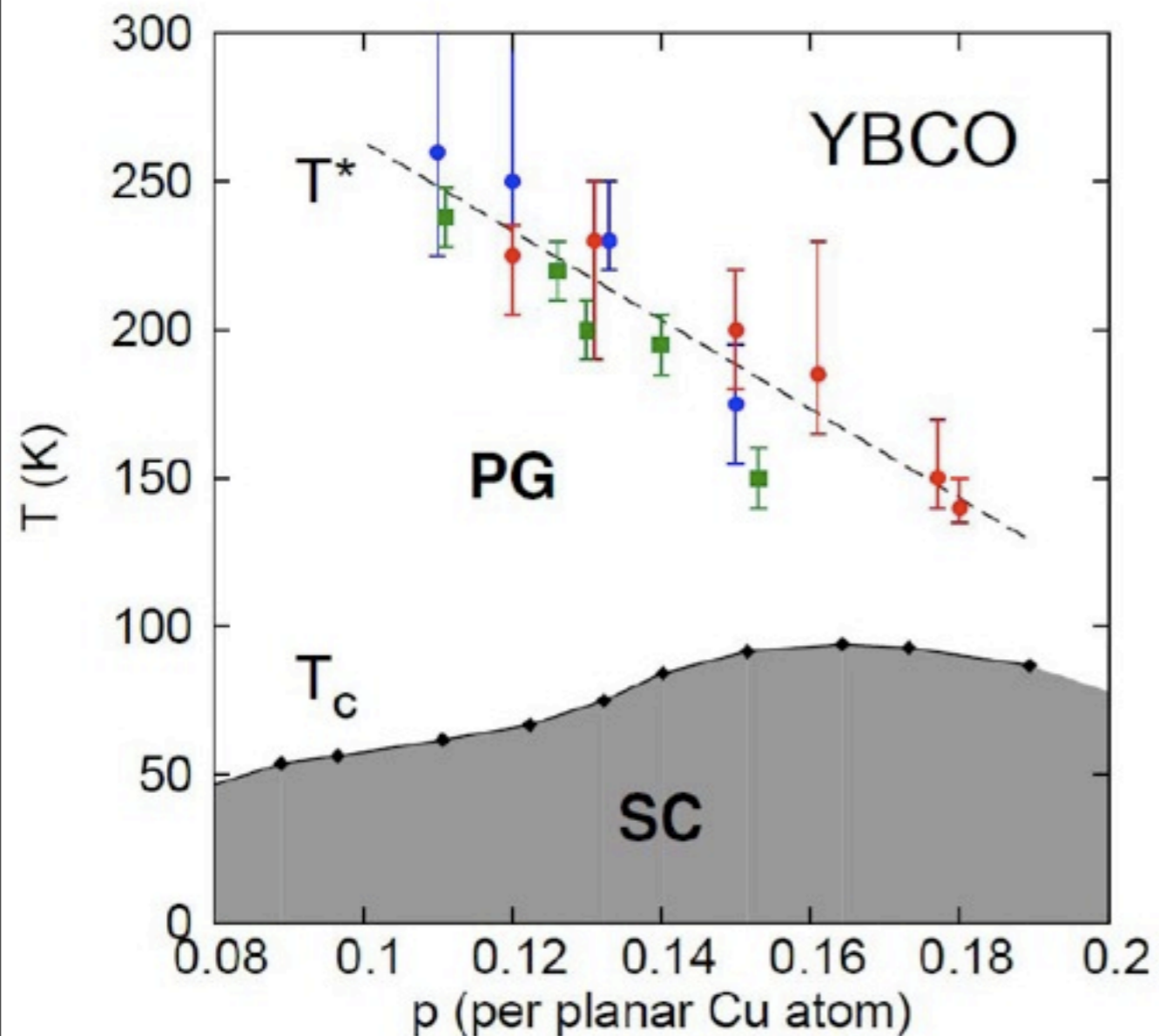
S.A. Kivelson,
 E. Fradkin, and
 V.J. Emery,
Nature **393**,
 550 (1998).

Nematic order in YBCO

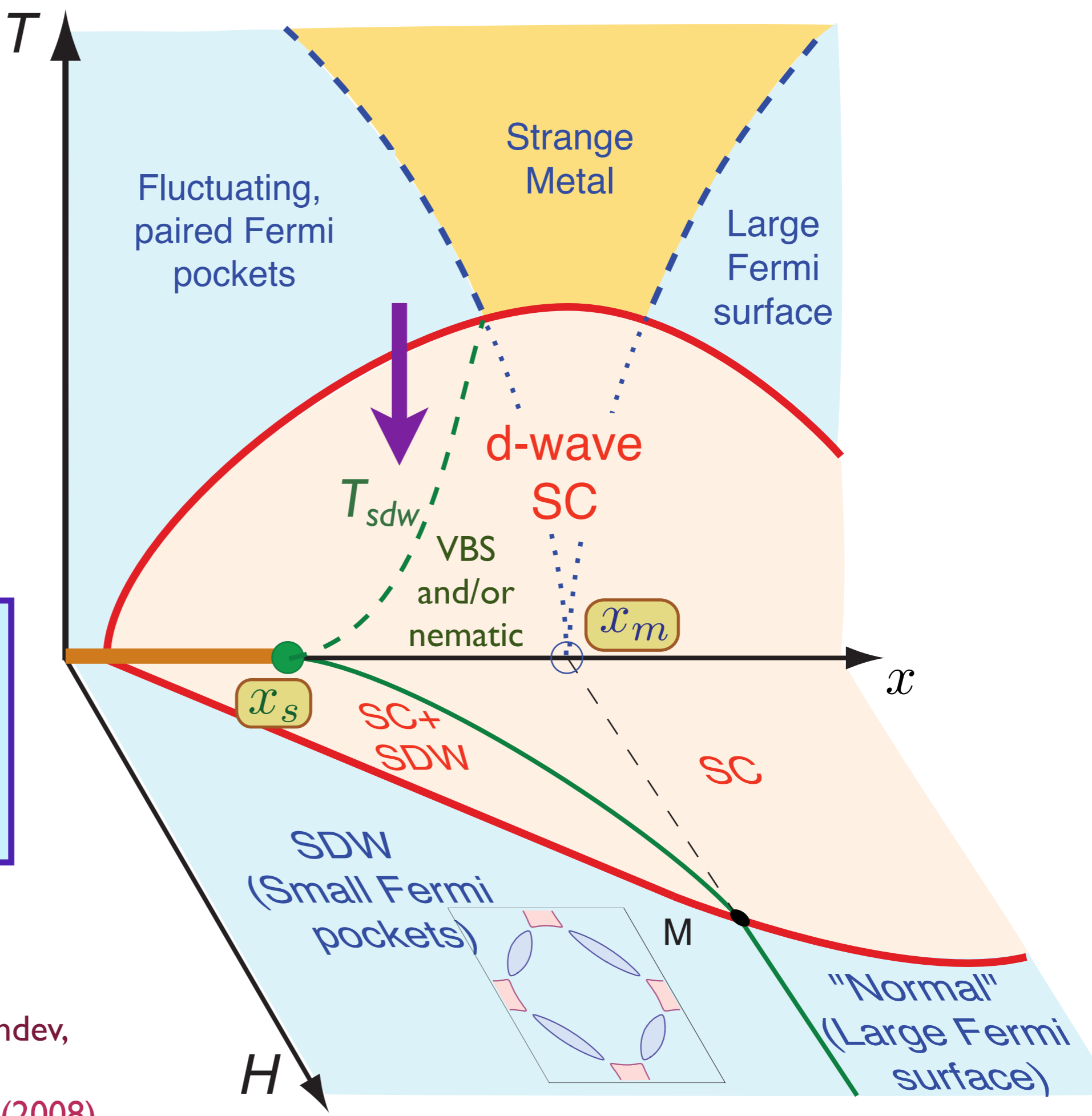
V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov,
 C. Bernhard, C. T. Lin, and B. Keimer, *Science* **319**, 597 (2008)

Broken rotational symmetry in the pseudogap phase of a high- T_c superconductor

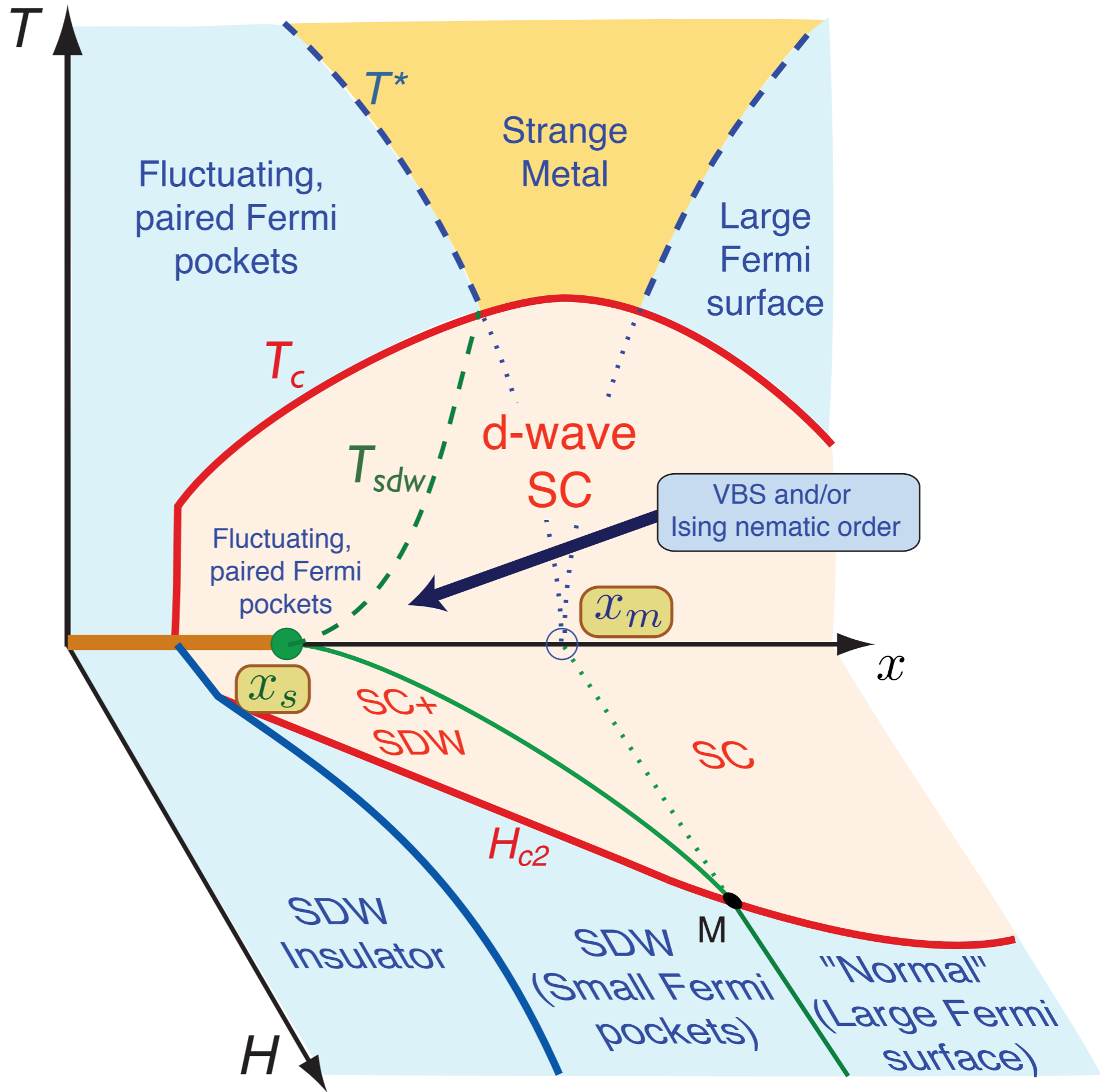
R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430, Nature, in press



Onset of superconductivity induces confinement



R. K. Kaul, M. Metlitski, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).



Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity

Conclusions

Theory for the onset
of spin density wave
order in metals is
strongly coupled in
two dimensions