

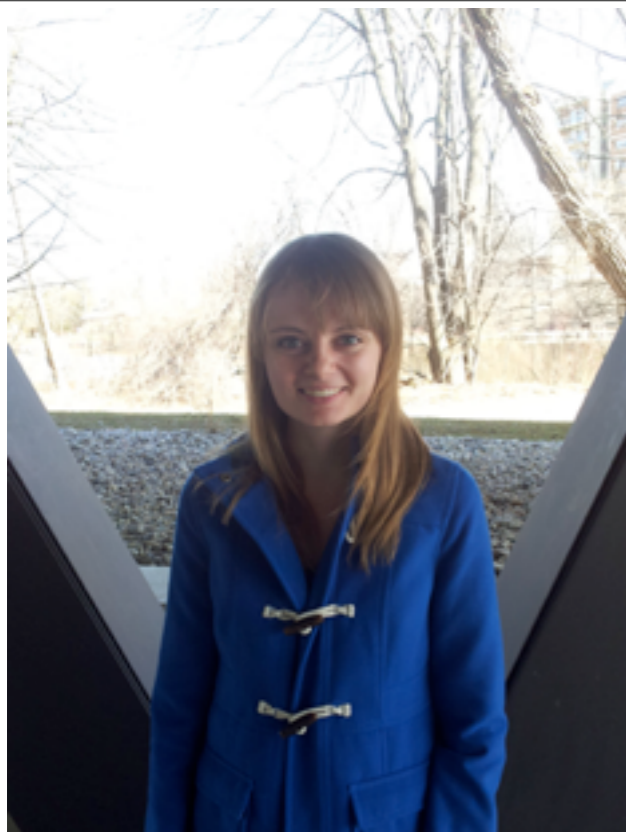
Quantum criticality in the high temperature superconductors

University of Minnesota
October 2, 2013

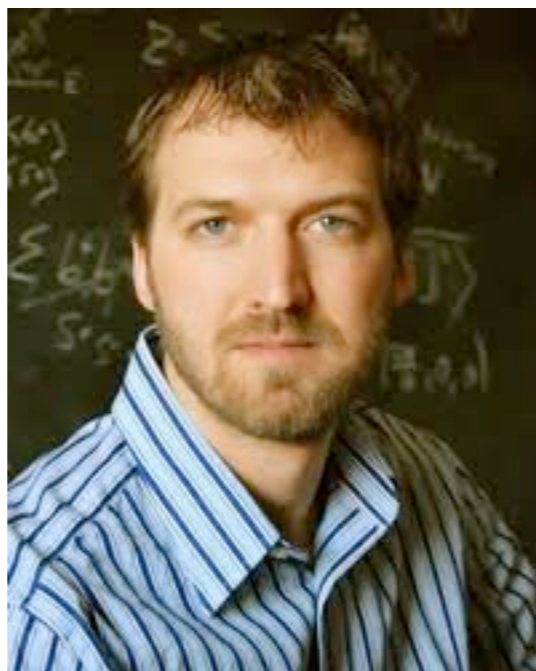
Subir Sachdev

Talk online: sachdev.physics.harvard.edu





Lauren
Hayward



Roger Melko



David
Hawthorn



Jay Deep Sau



Erez Berg



Max Metlitski



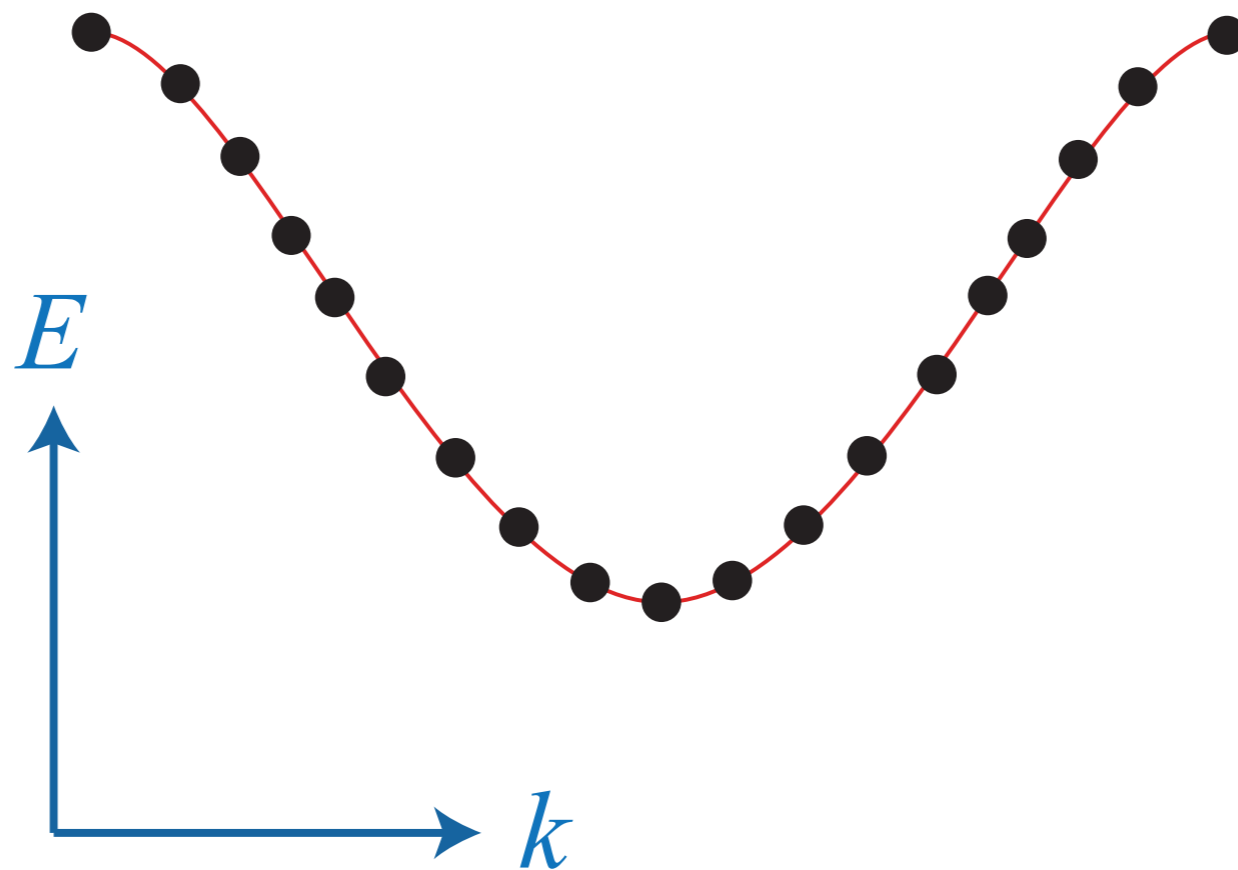
Rolando
La Placa



Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

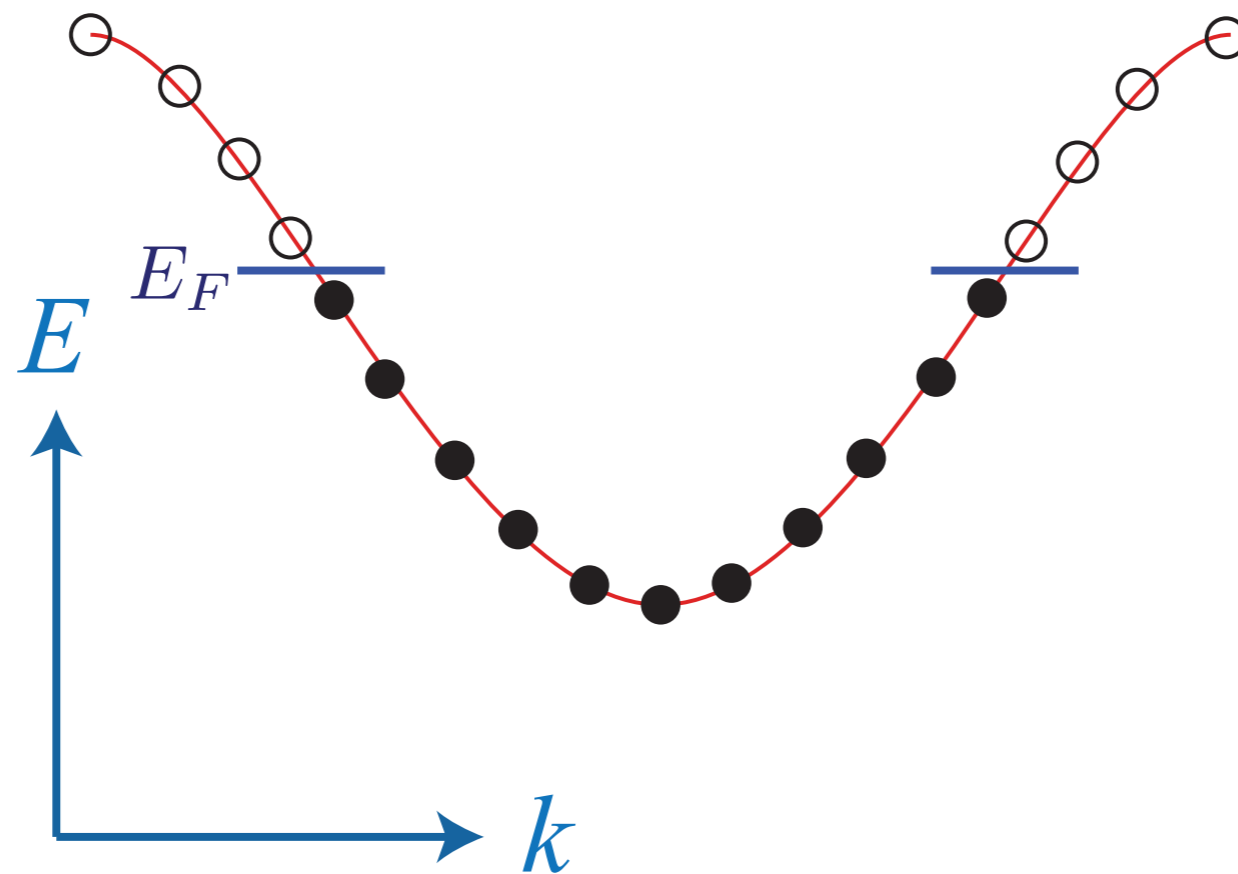
Band insulators



An even number of electrons per unit cell

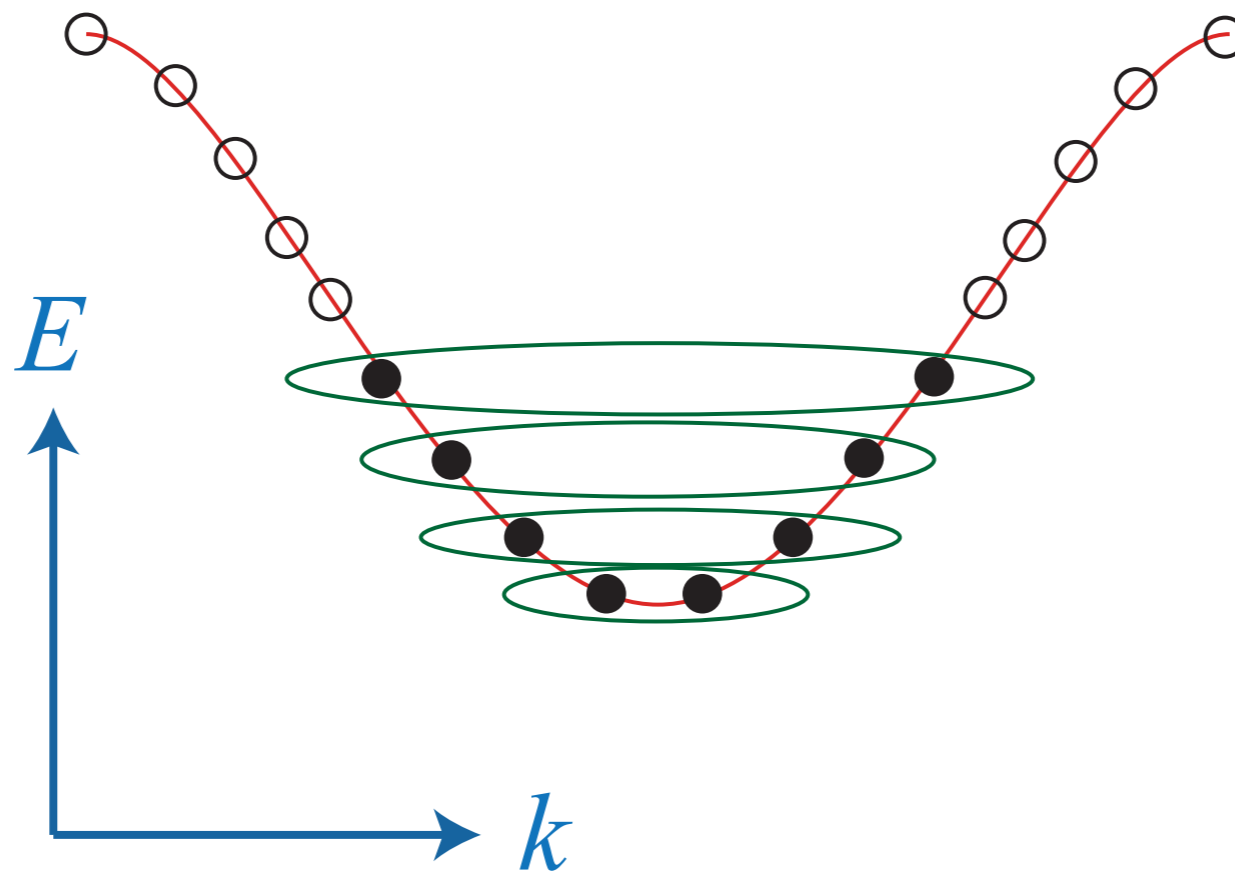
Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Metals



Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

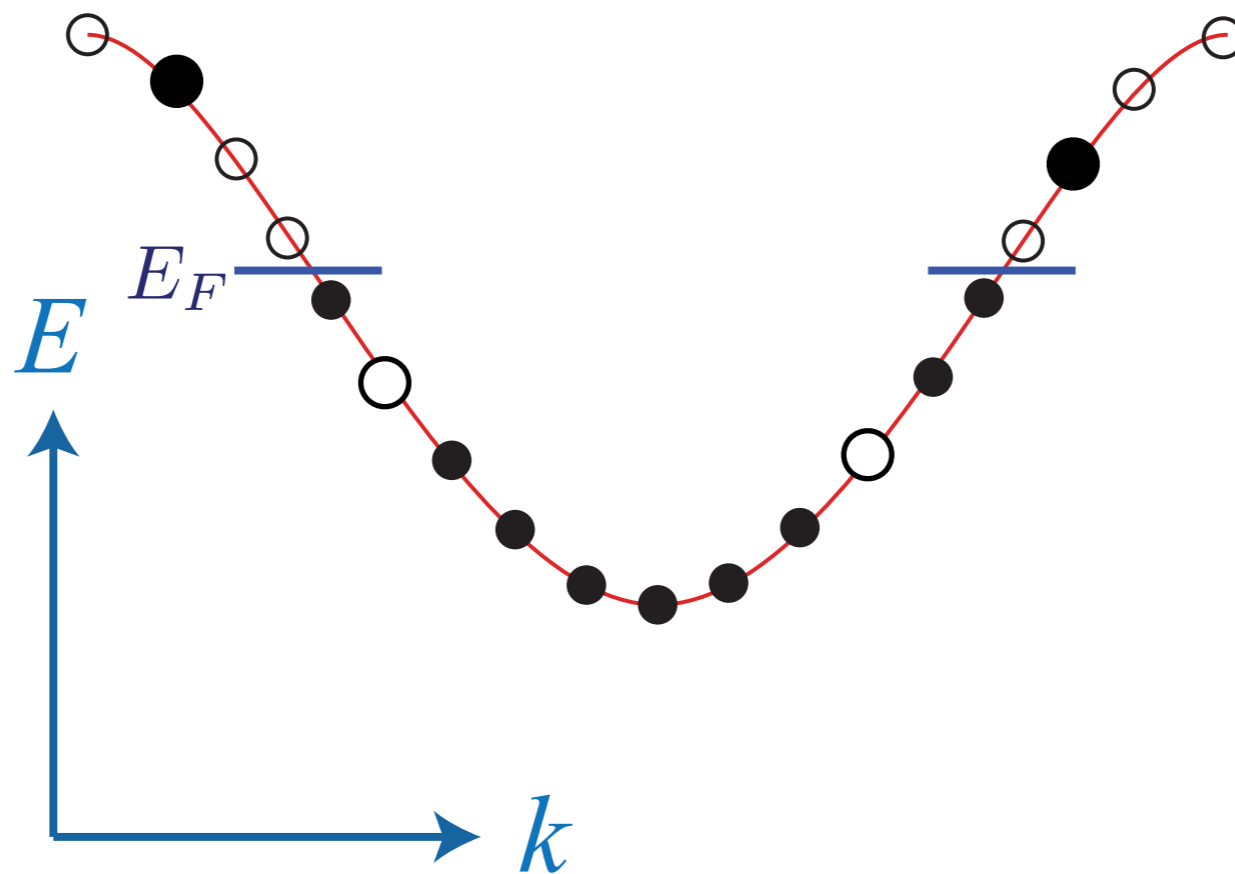
Superconductors



Boltzmann-Landau theory of dynamics of metals:

Long-lived **quasiparticles** (and **quasiholes**) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

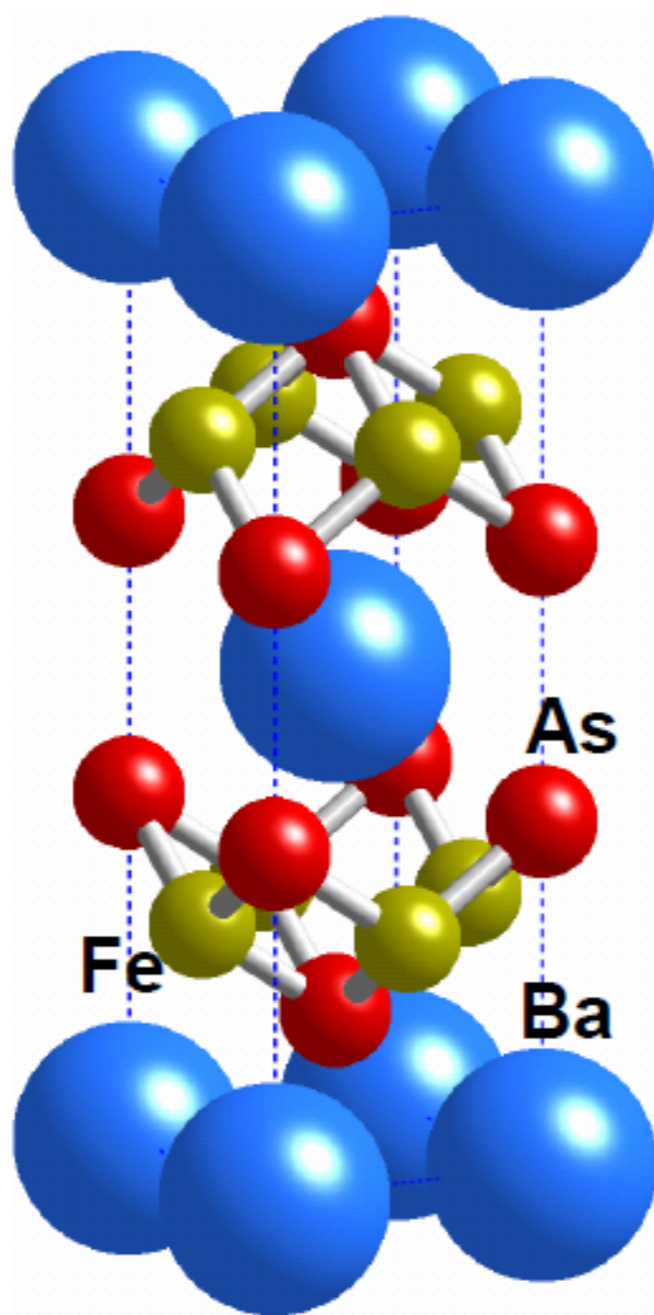
many-particle

quantum entanglement,

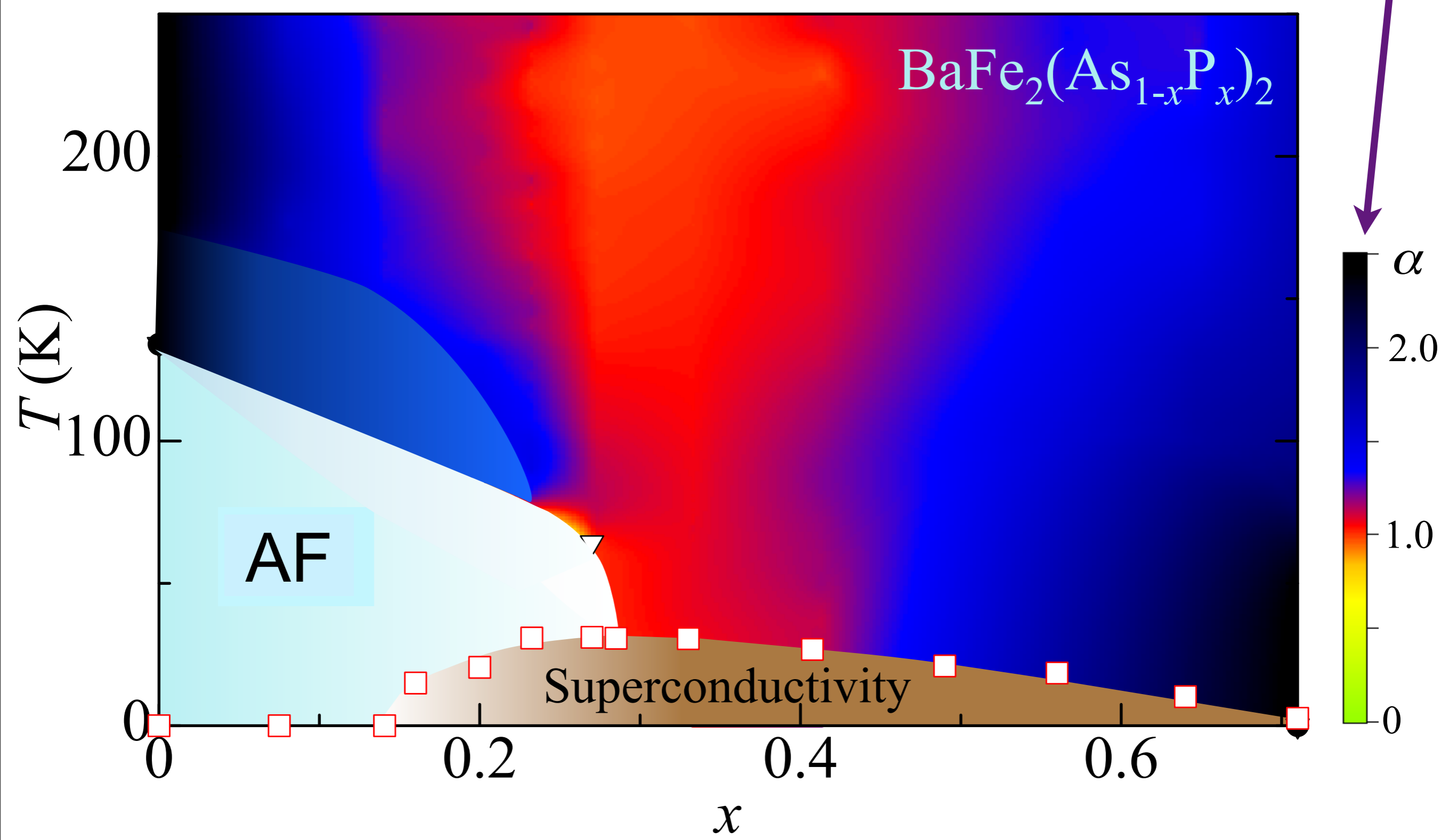
and no quasiparticles

Iron pnictides:

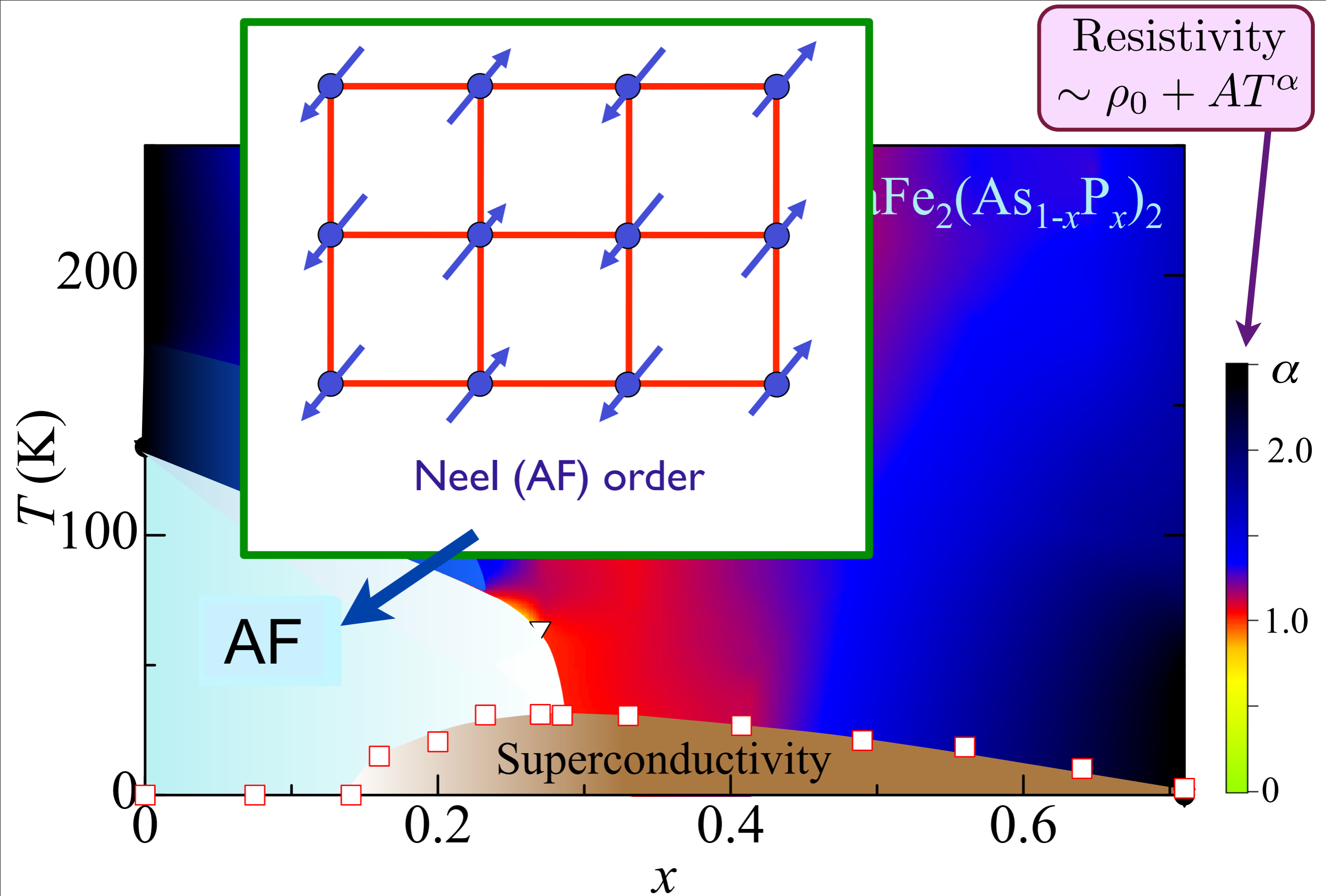
a new class of high temperature superconductors



Resistivity
 $\sim \rho_0 + AT^\alpha$

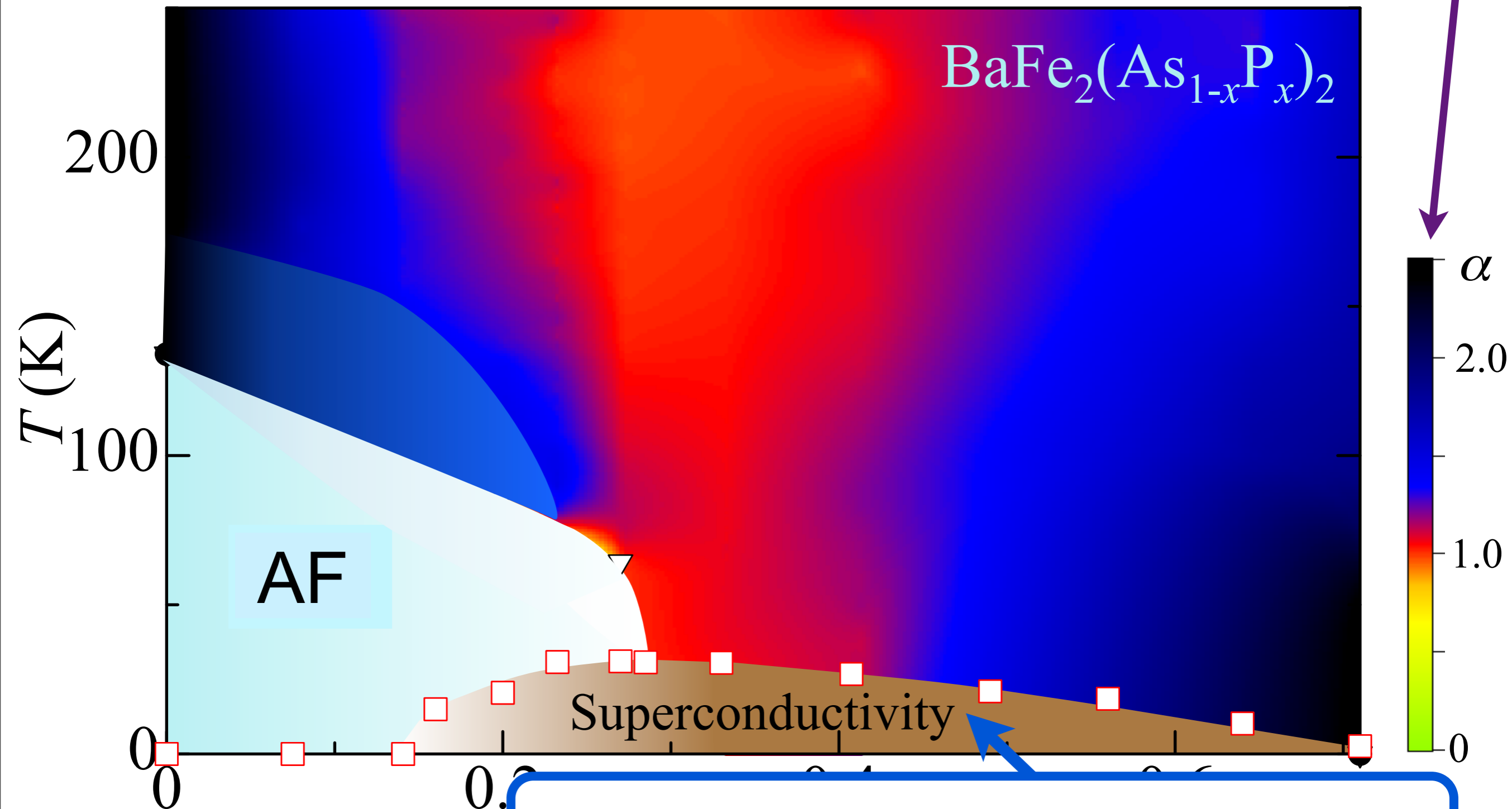


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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 H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
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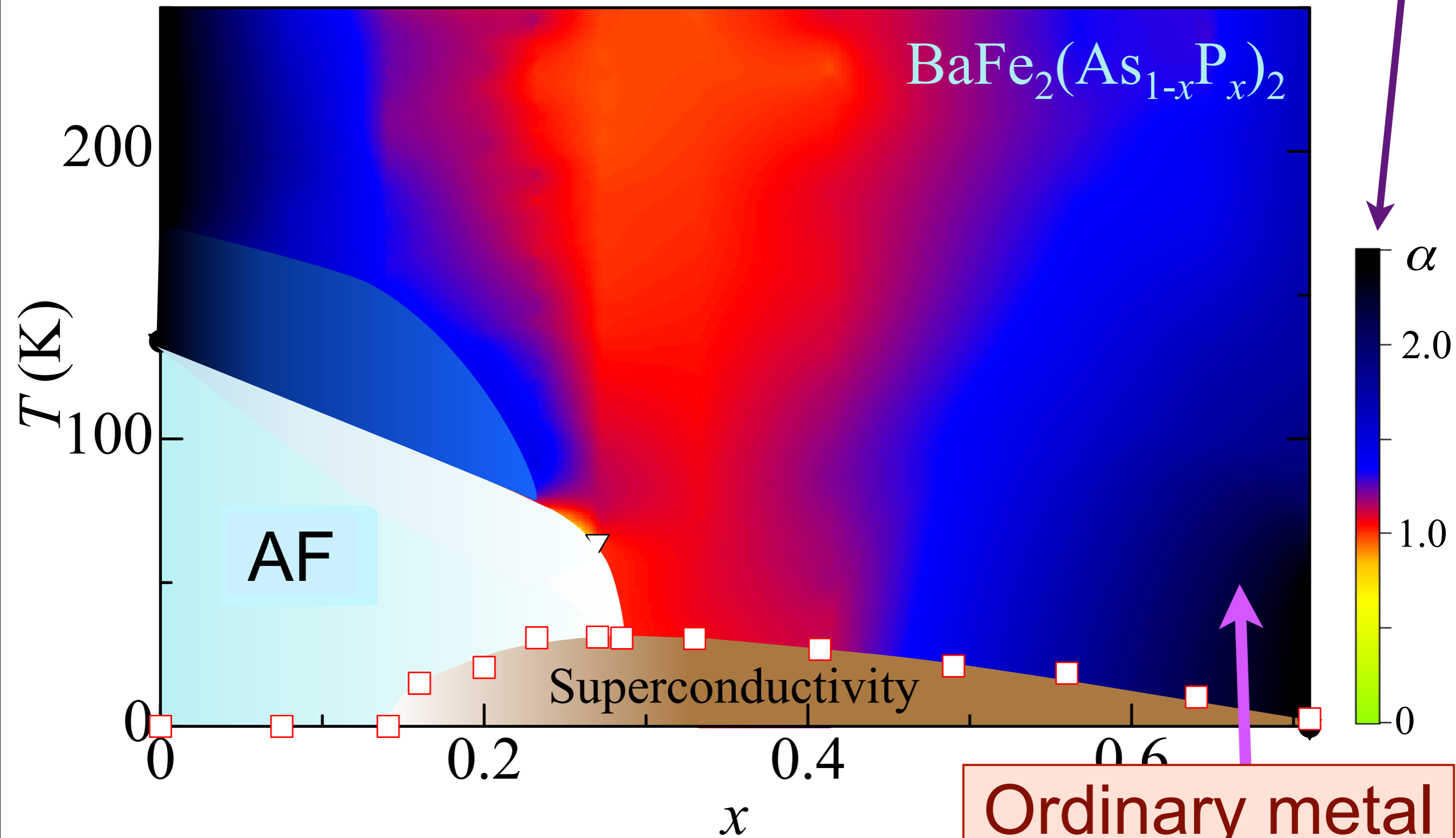
Resistivity
 $\sim \rho_0 + AT^\alpha$



Superconductor
Bose condensate of pairs of electrons

S. Kasahara, T. Shiba
H. Ike

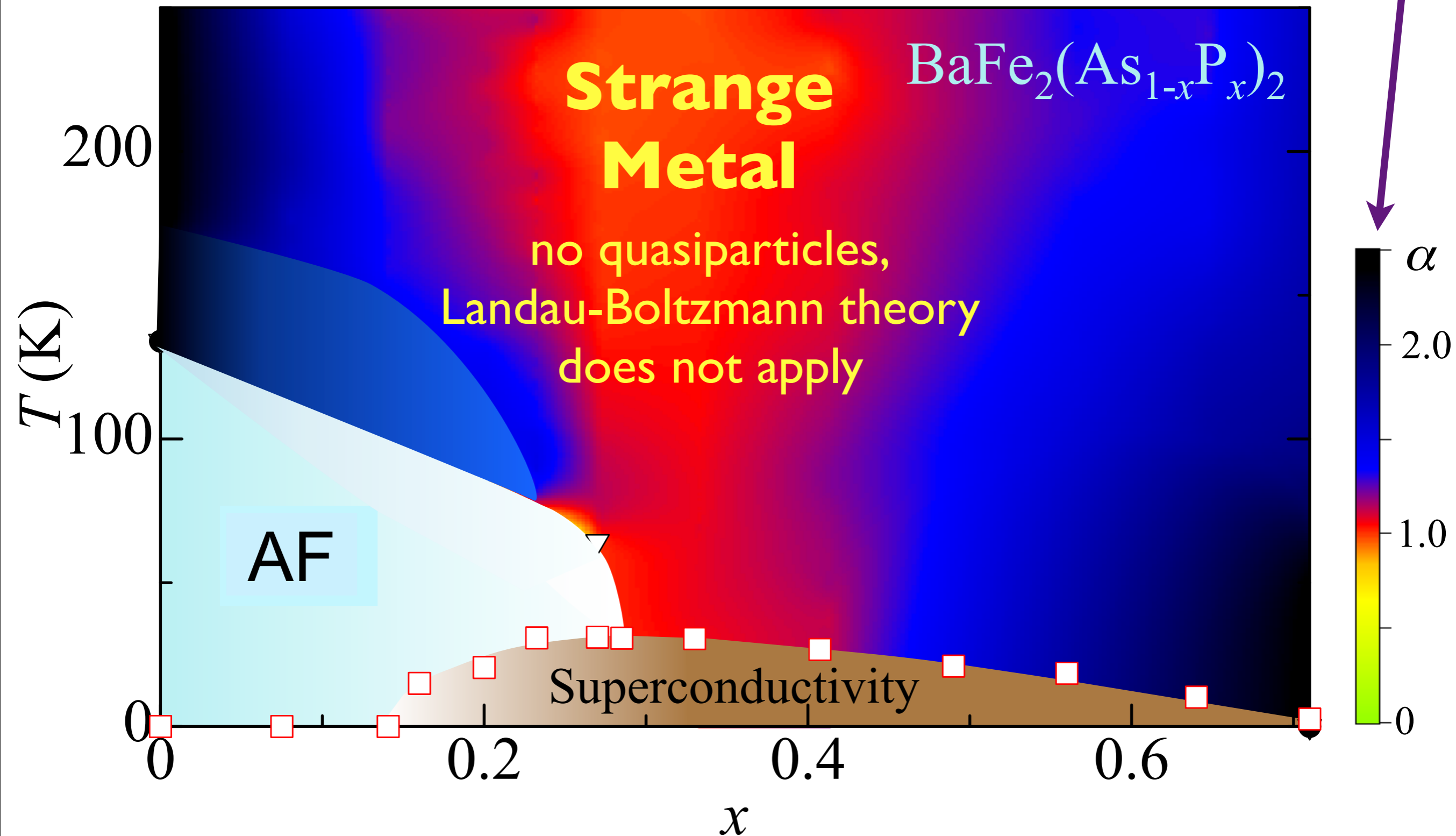
Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O.
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
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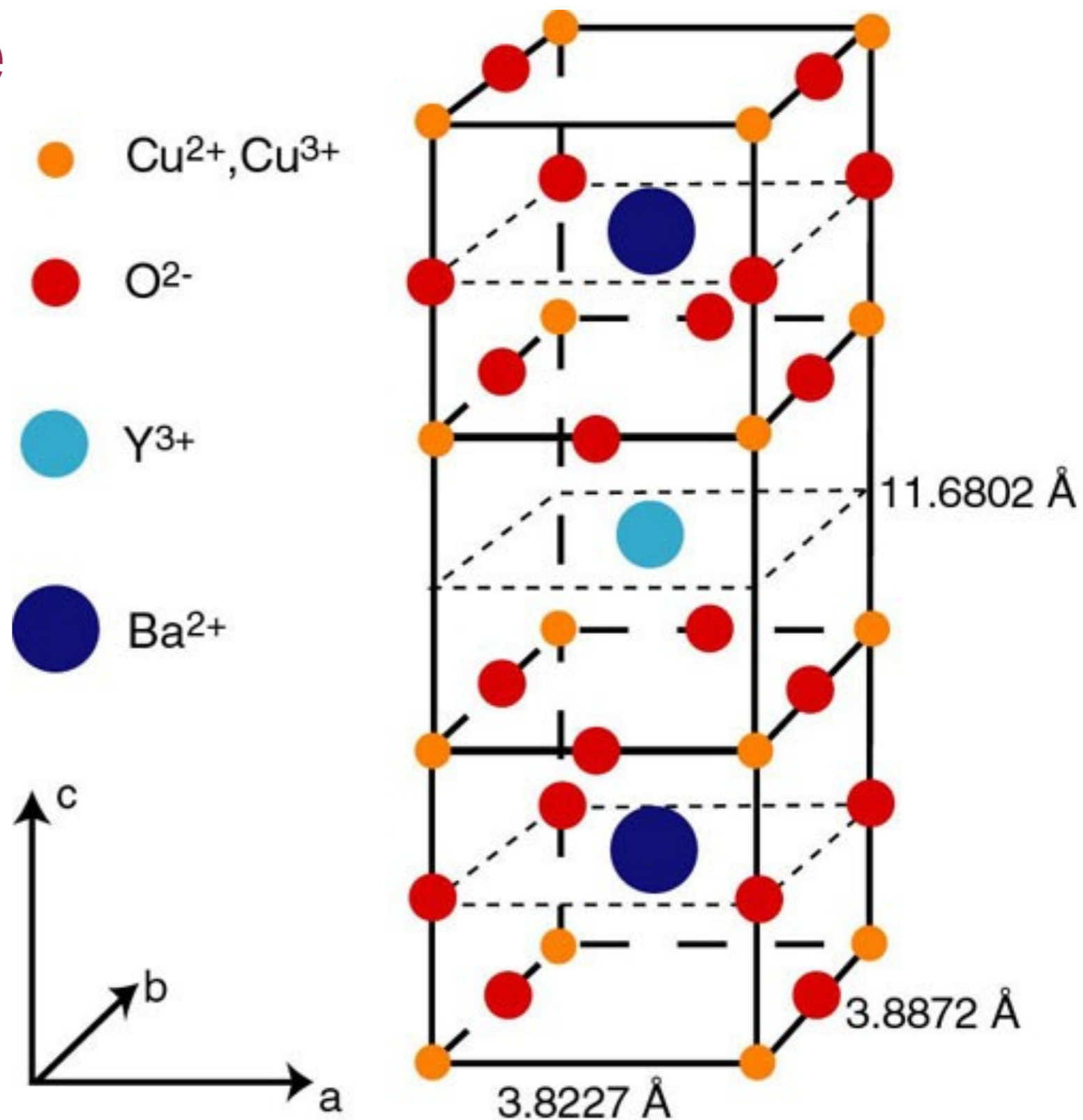
Ordinary metal
(Fermi liquid)

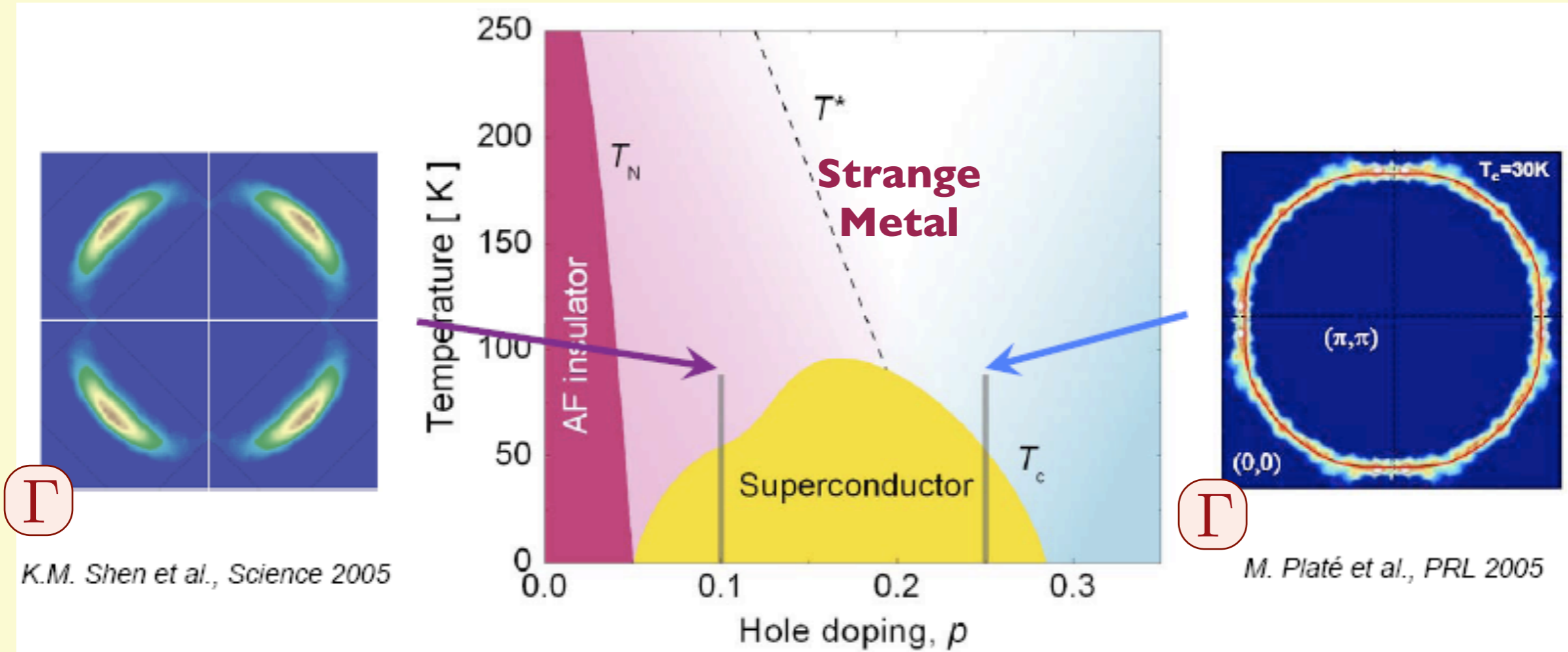
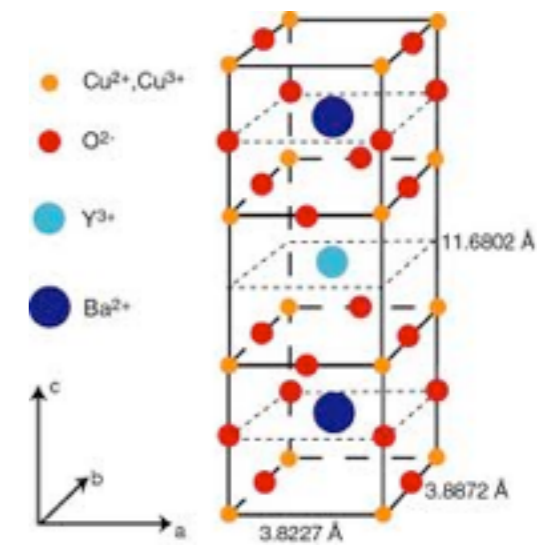
Resistivity
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H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
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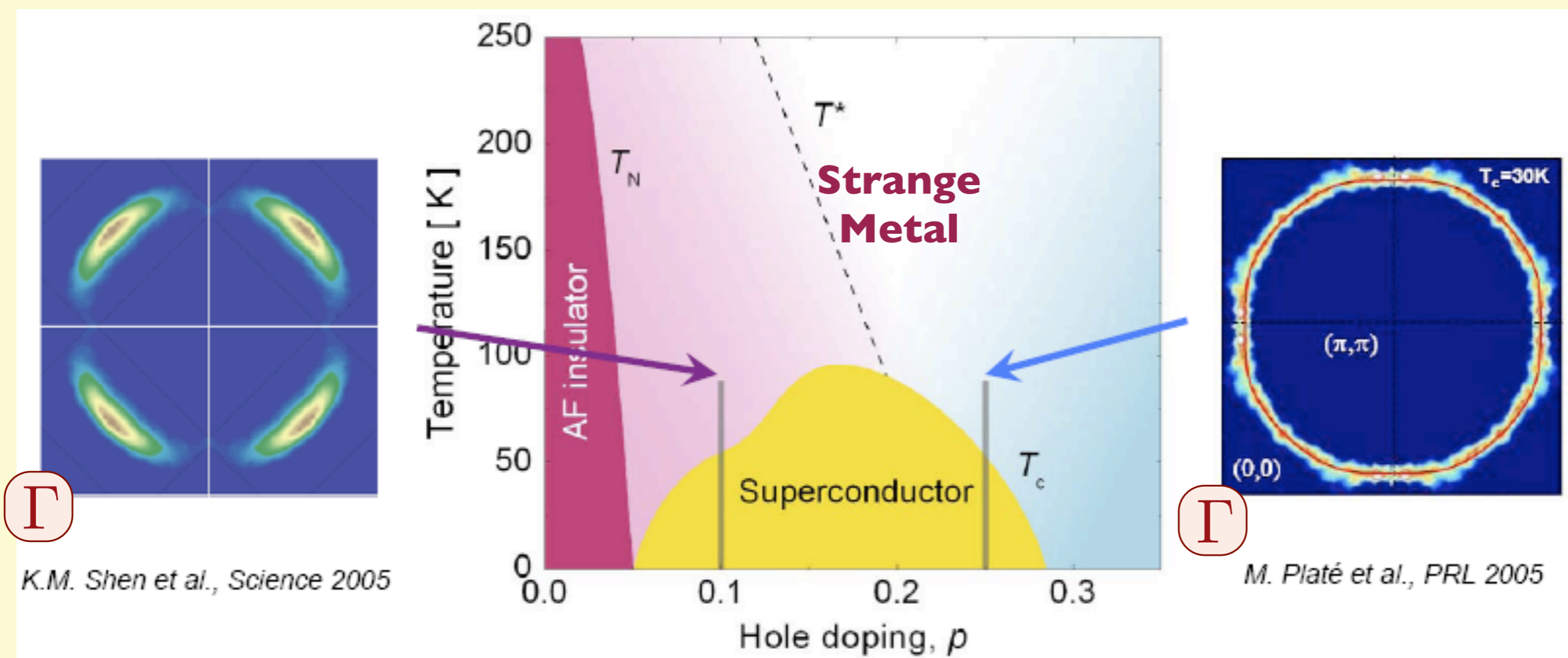
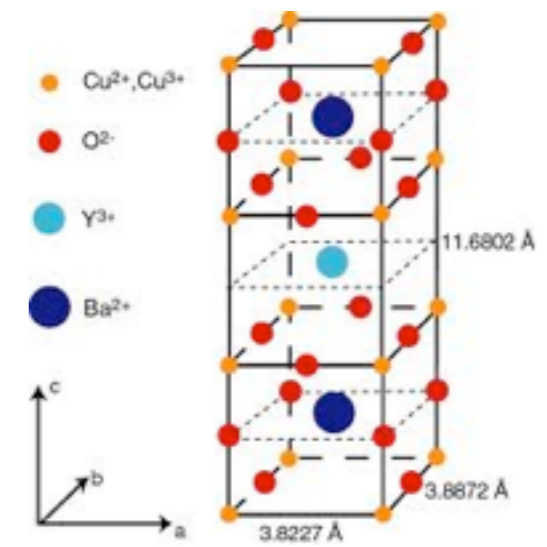
High temperature superconductors





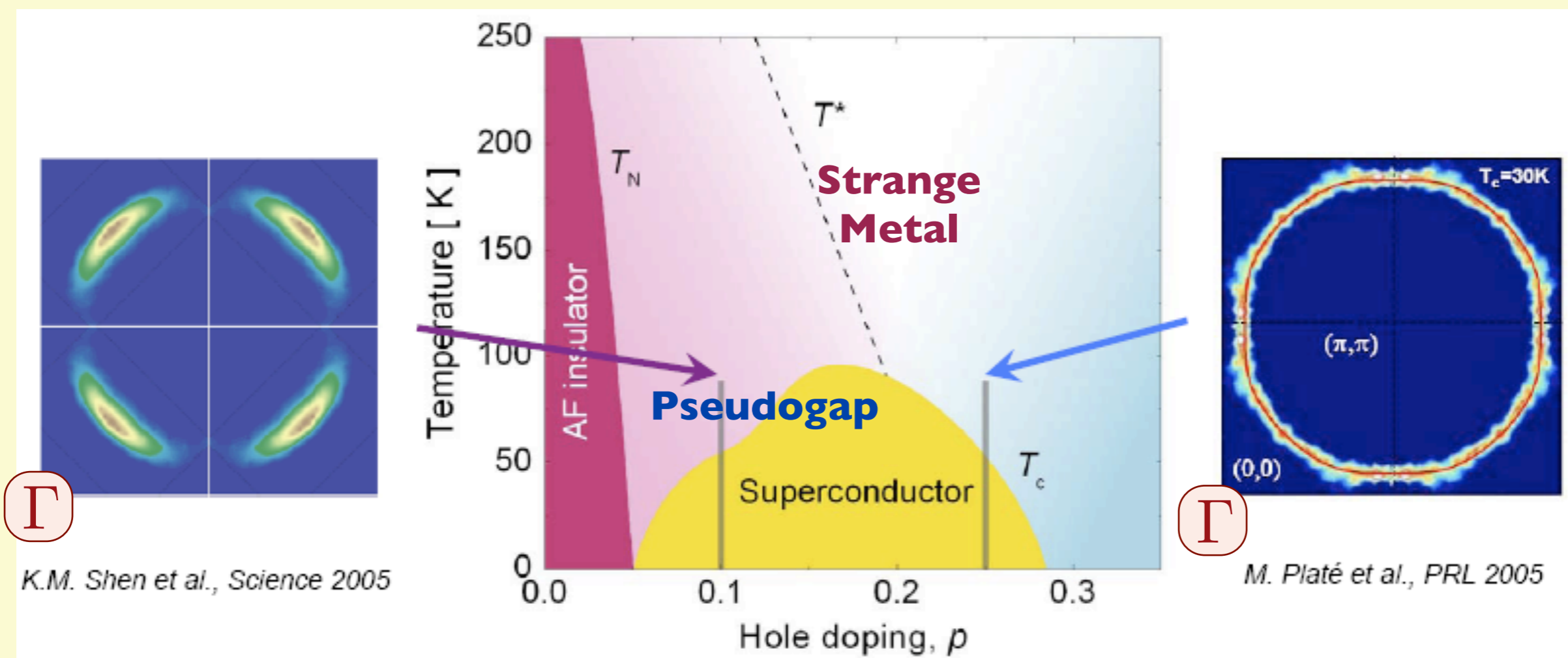
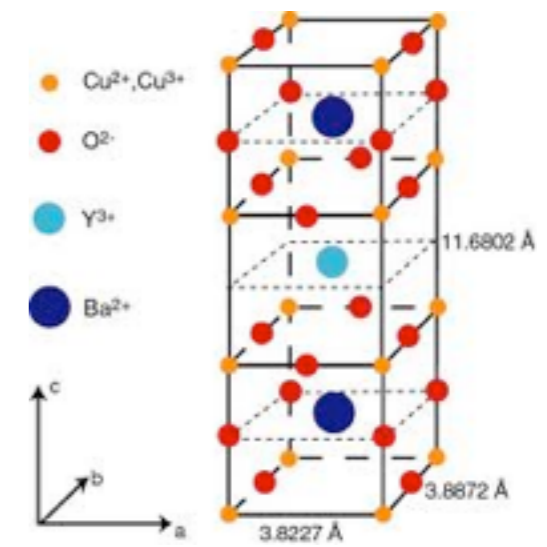
Smaller hole Fermi-pockets

Large hole Fermi surface



Smaller hole Fermi-pockets

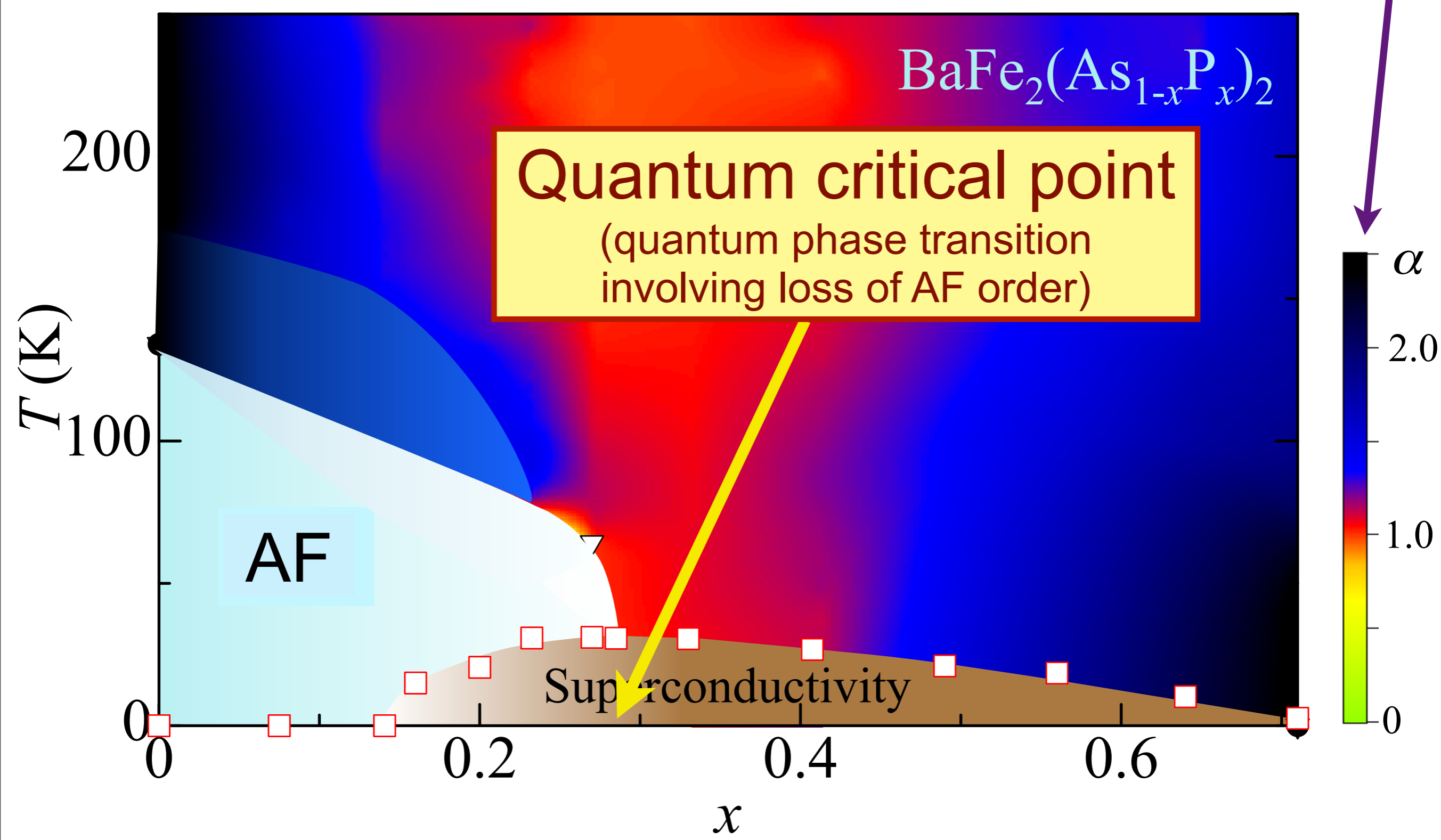
Large hole Fermi surface



Smaller hole Fermi-pockets

Large hole Fermi surface

Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Outline

1. Quantum critical point in an insulator

*Entanglement and
non-quasiparticle dynamics*

2. Quantum critical point in a metal

The iron-based superconductors

3. The pseudogap regime of the hole-doped cuprate superconductors

*Angular fluctuations of a
multicomponent order*

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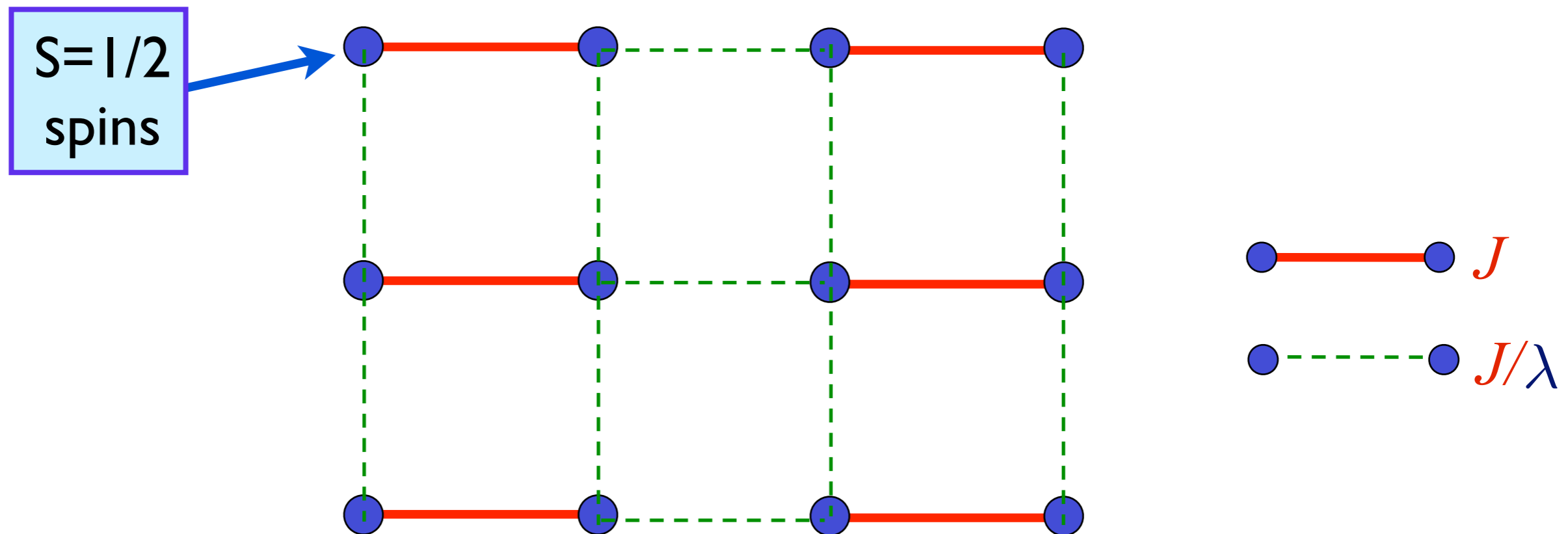
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3. The pseudogap regime of the hole-doped cuprate superconductors

*Angular fluctuations of a
multicomponent order*

Square lattice antiferromagnet

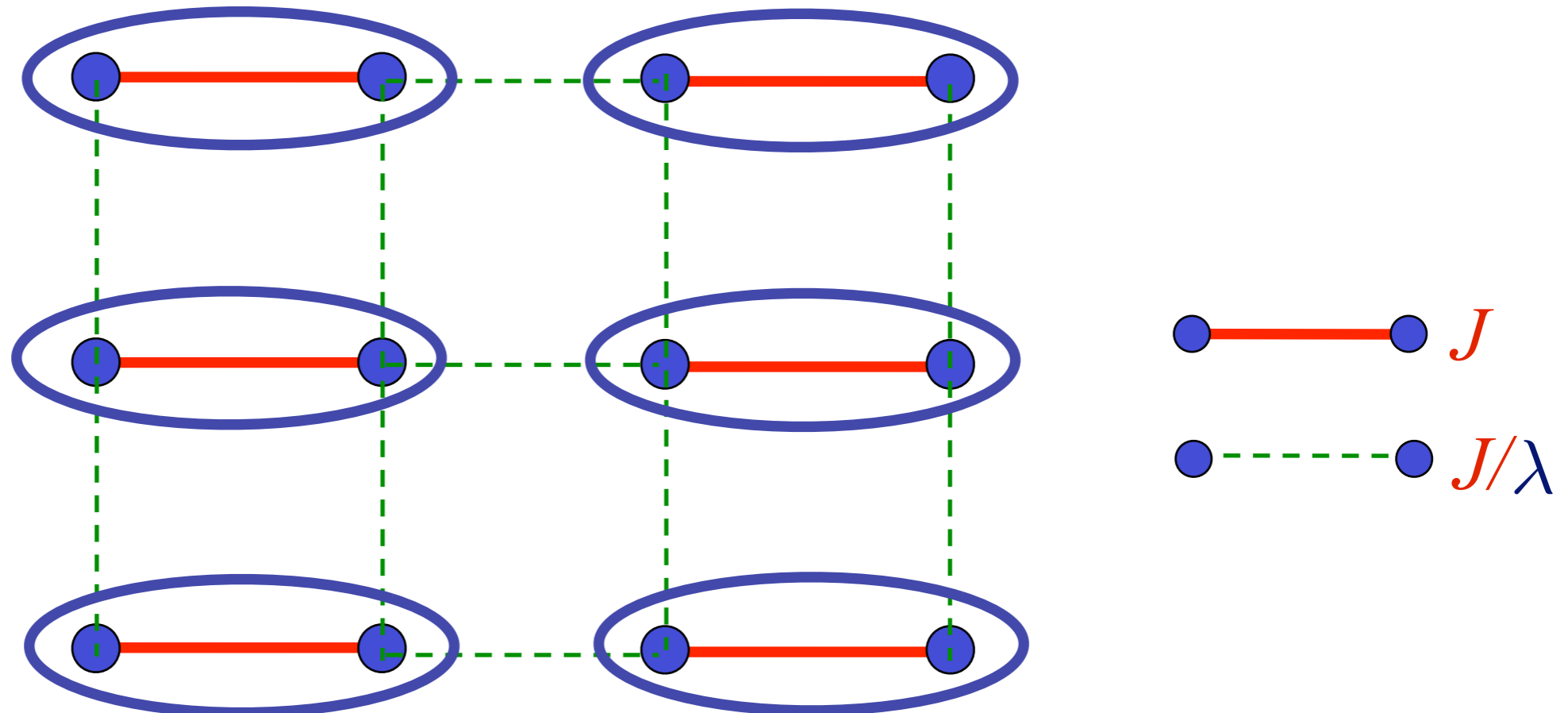
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

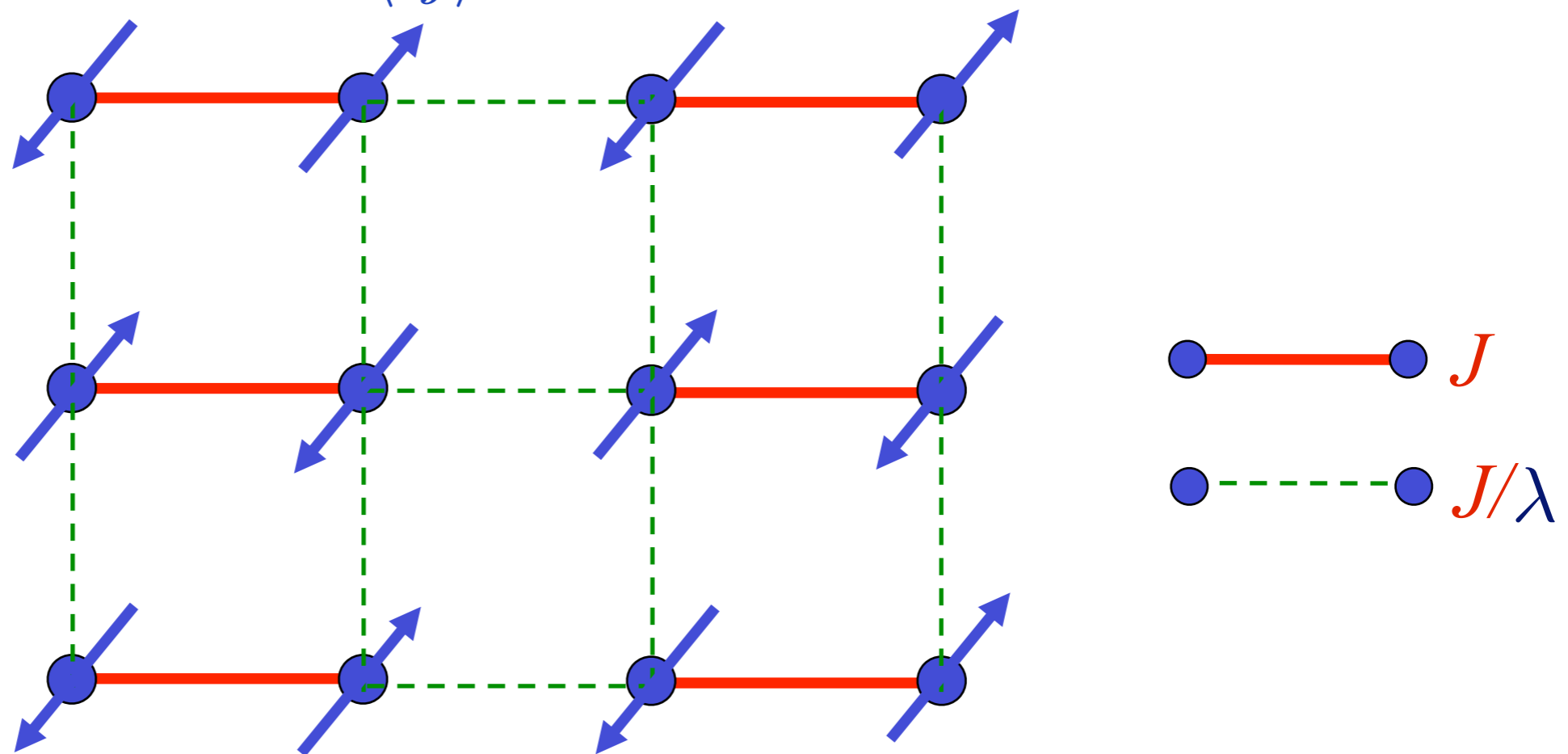


$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

At large λ ground state is a “quantum paramagnet” with spins locked in valence bond singlets

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



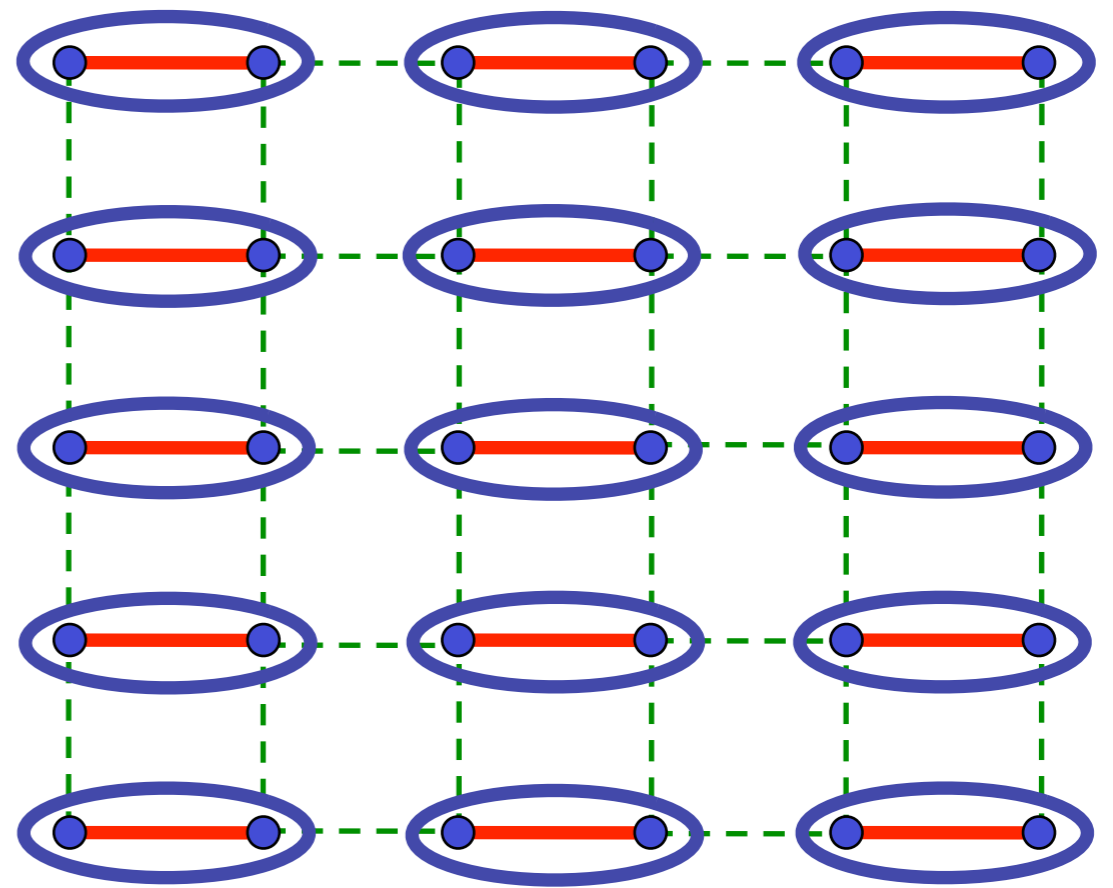
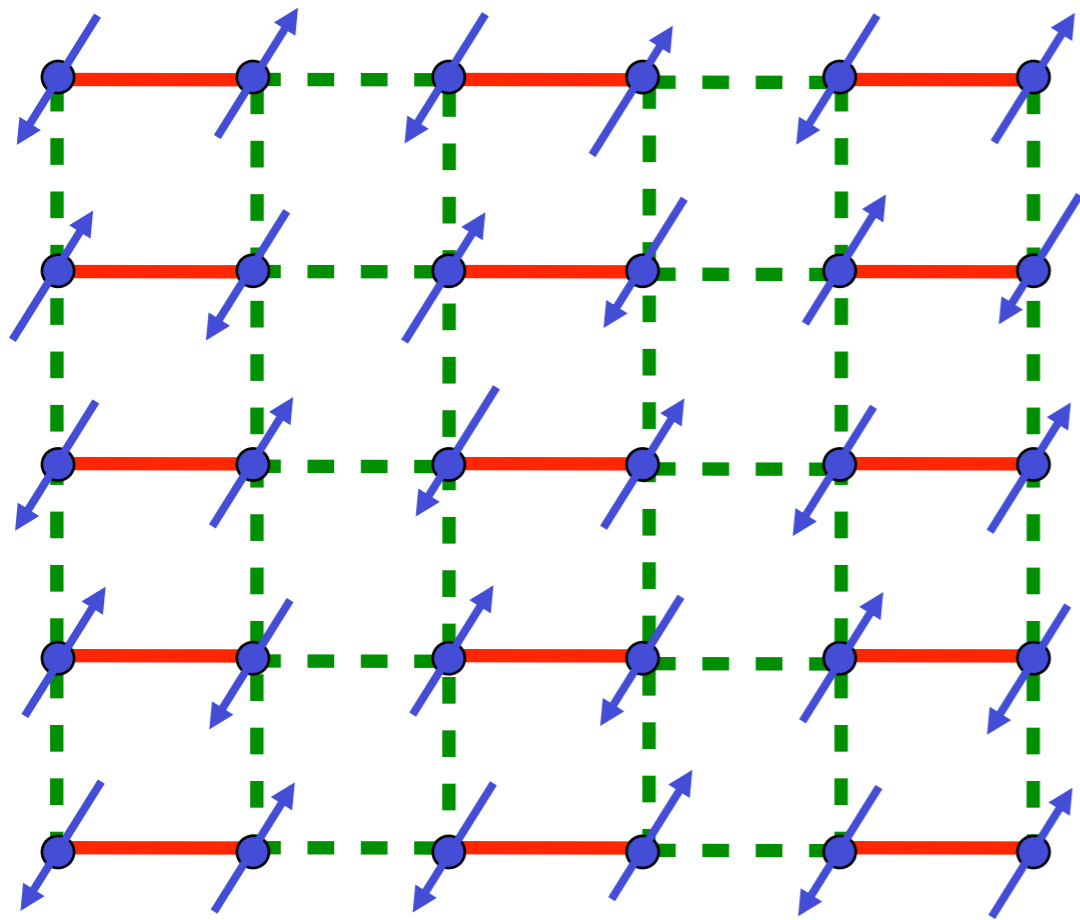
For $\lambda \approx 1$, the ground state has antiferromagnetic (“Néel”) order, and the spins align in a checkerboard pattern

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

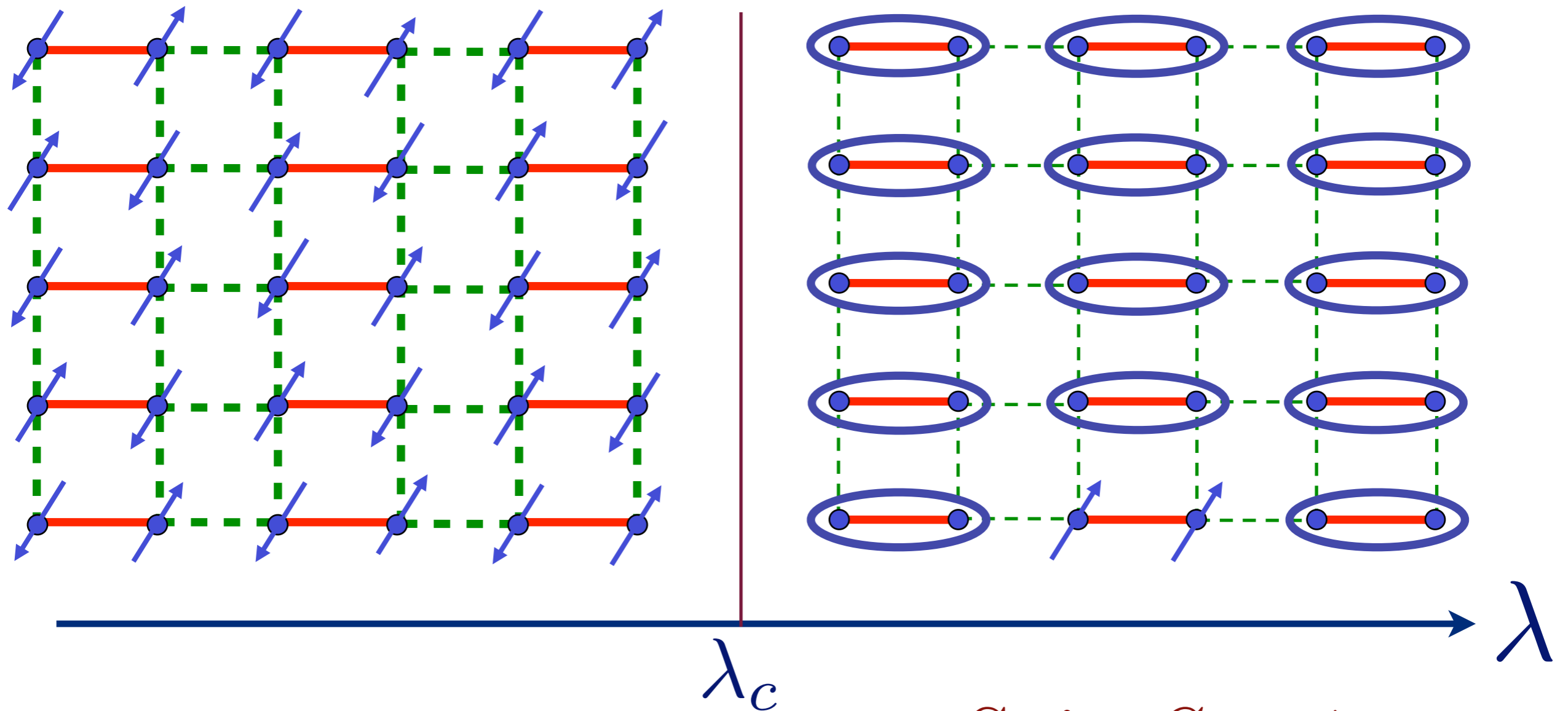
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

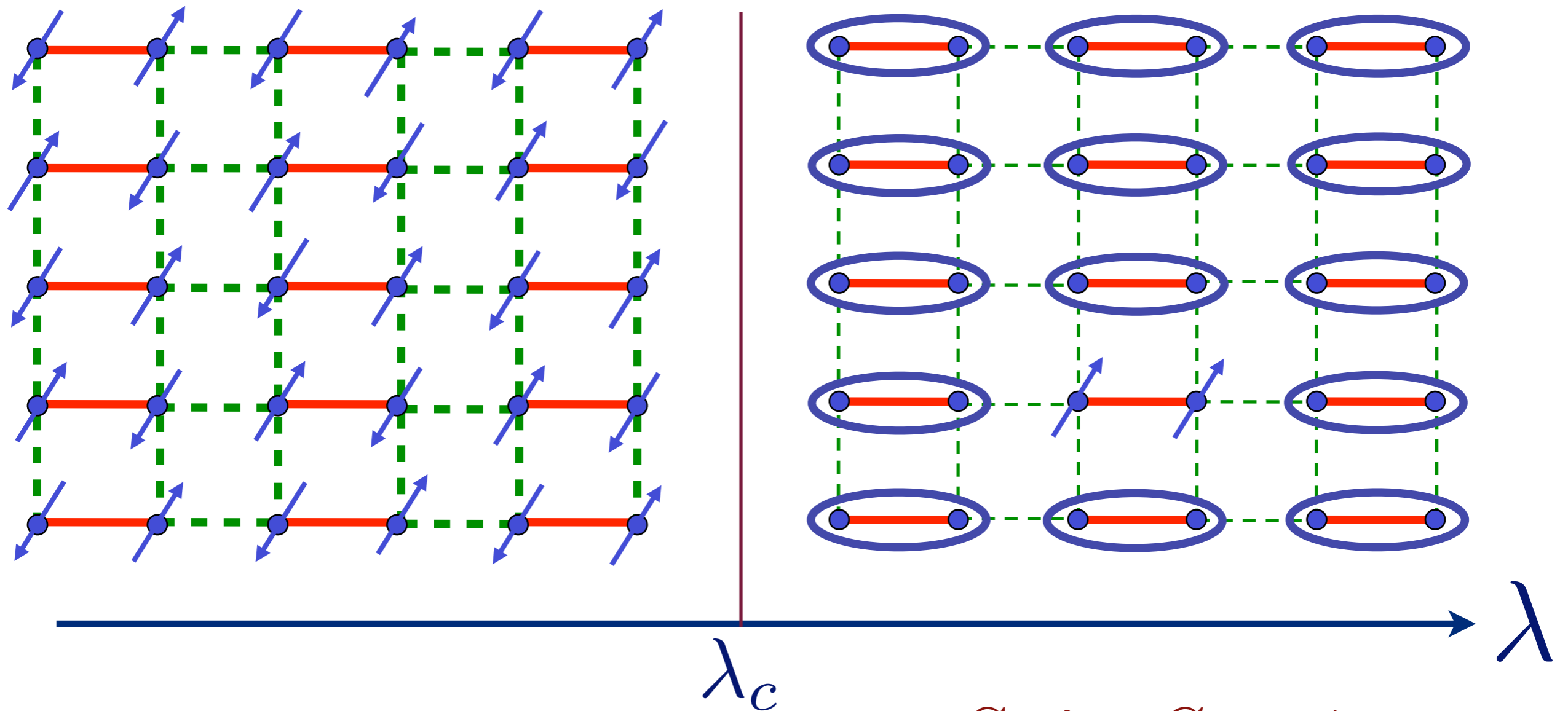


Excitation spectrum in the paramagnetic phase



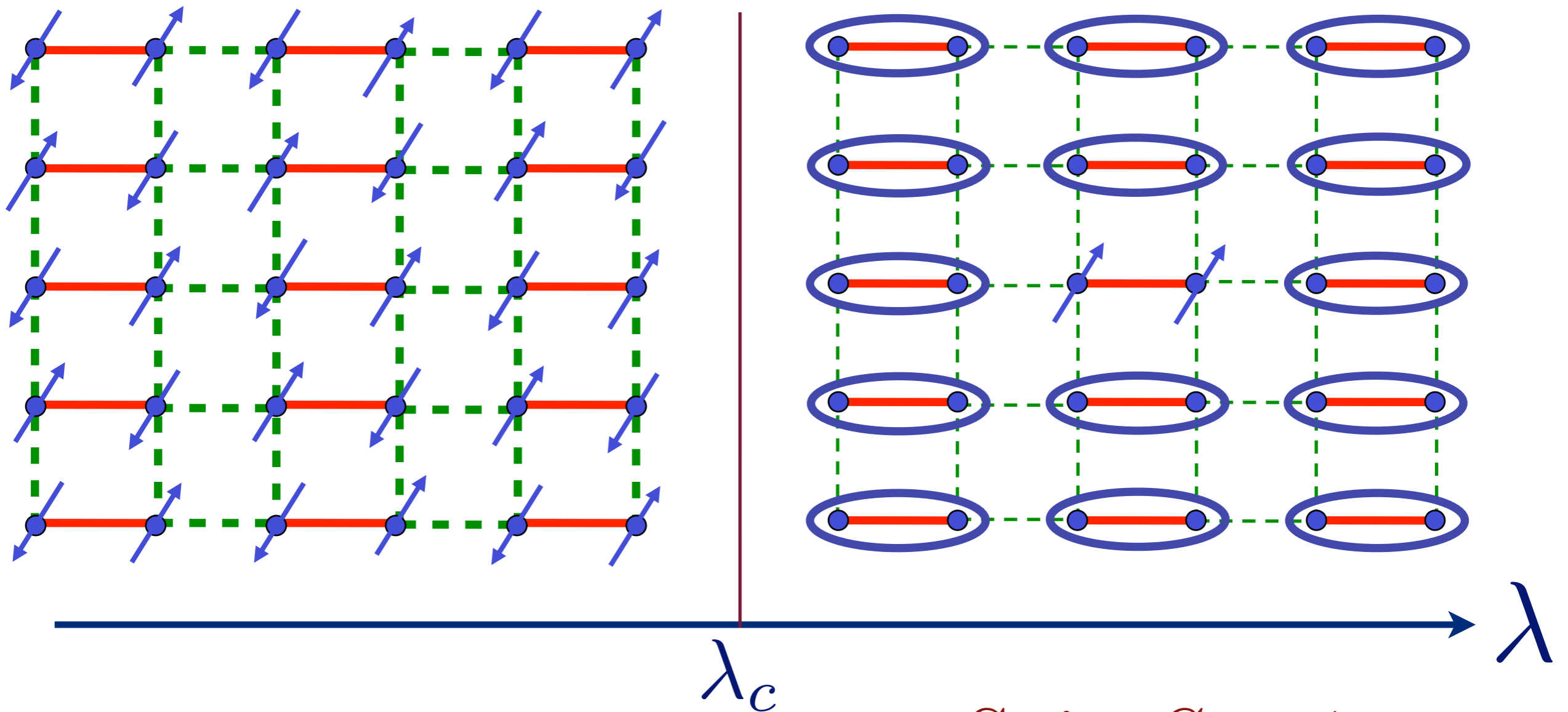
Spin $S = 1$
“triplon”

Excitation spectrum in the paramagnetic phase



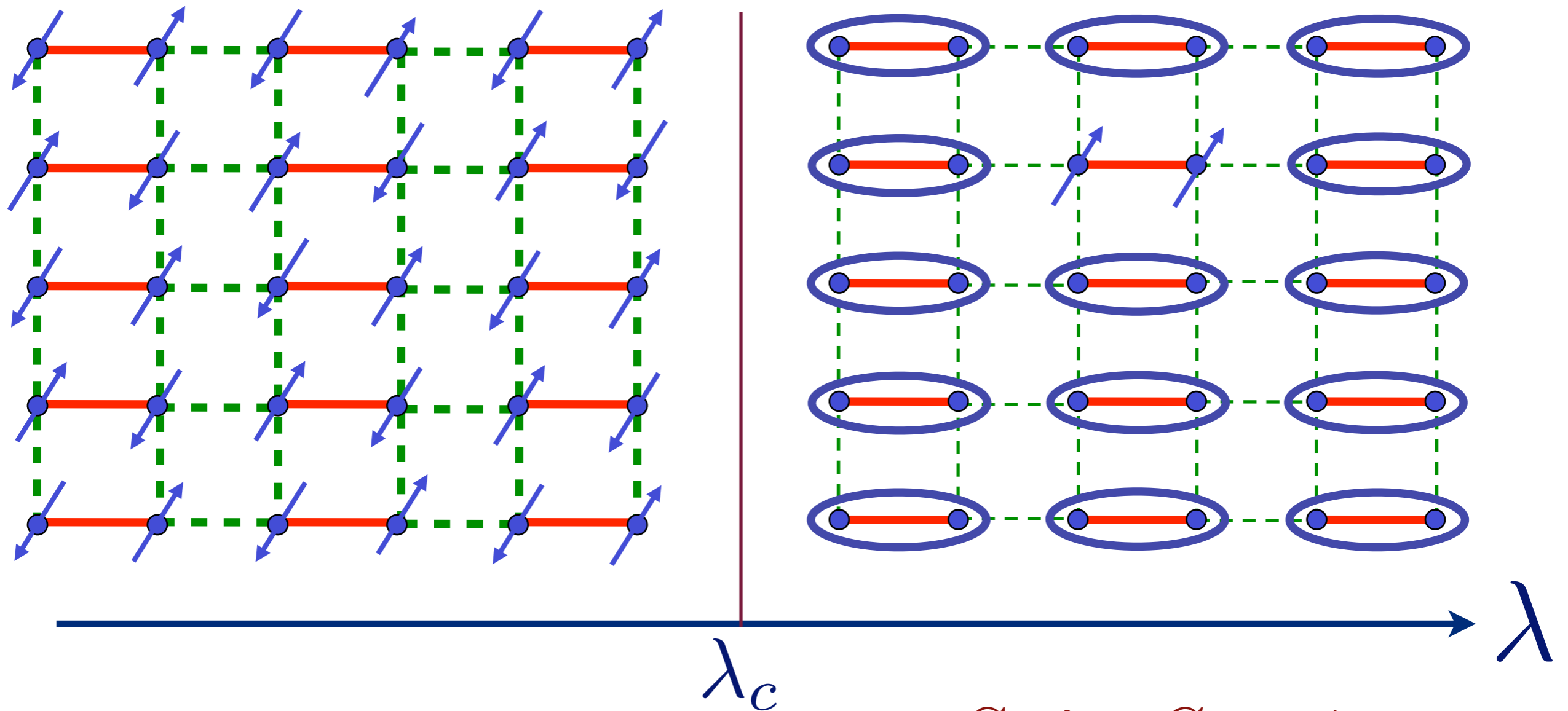
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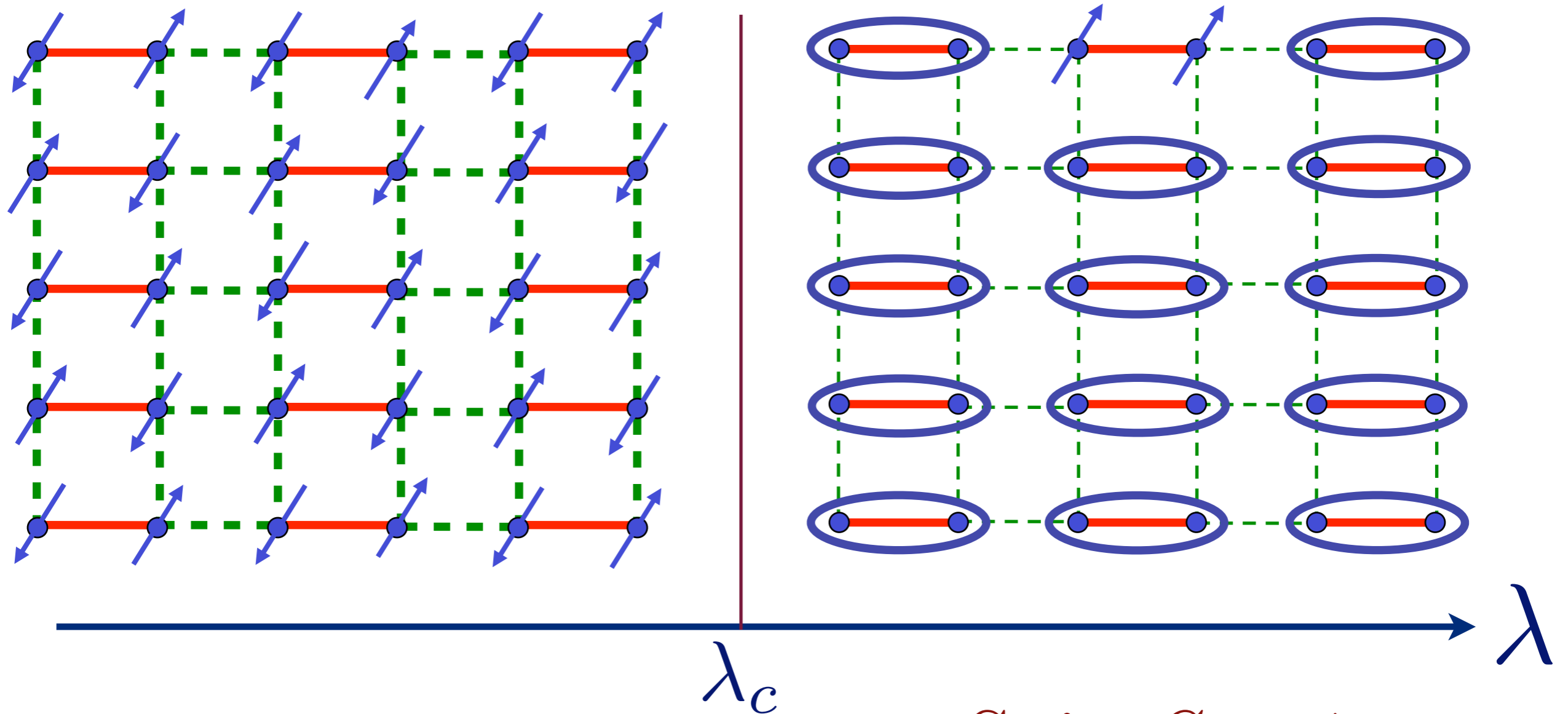
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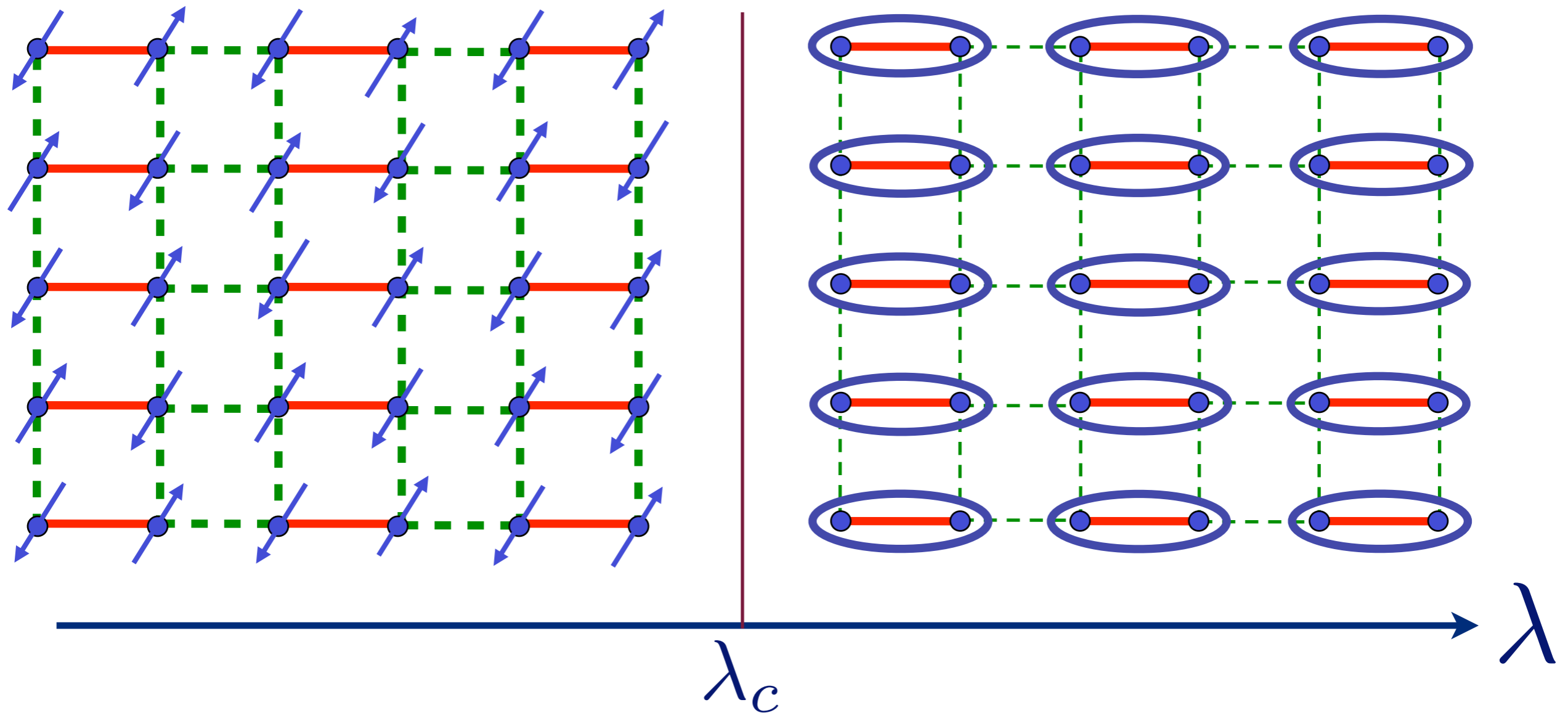
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Excitation spectrum in the paramagnetic phase



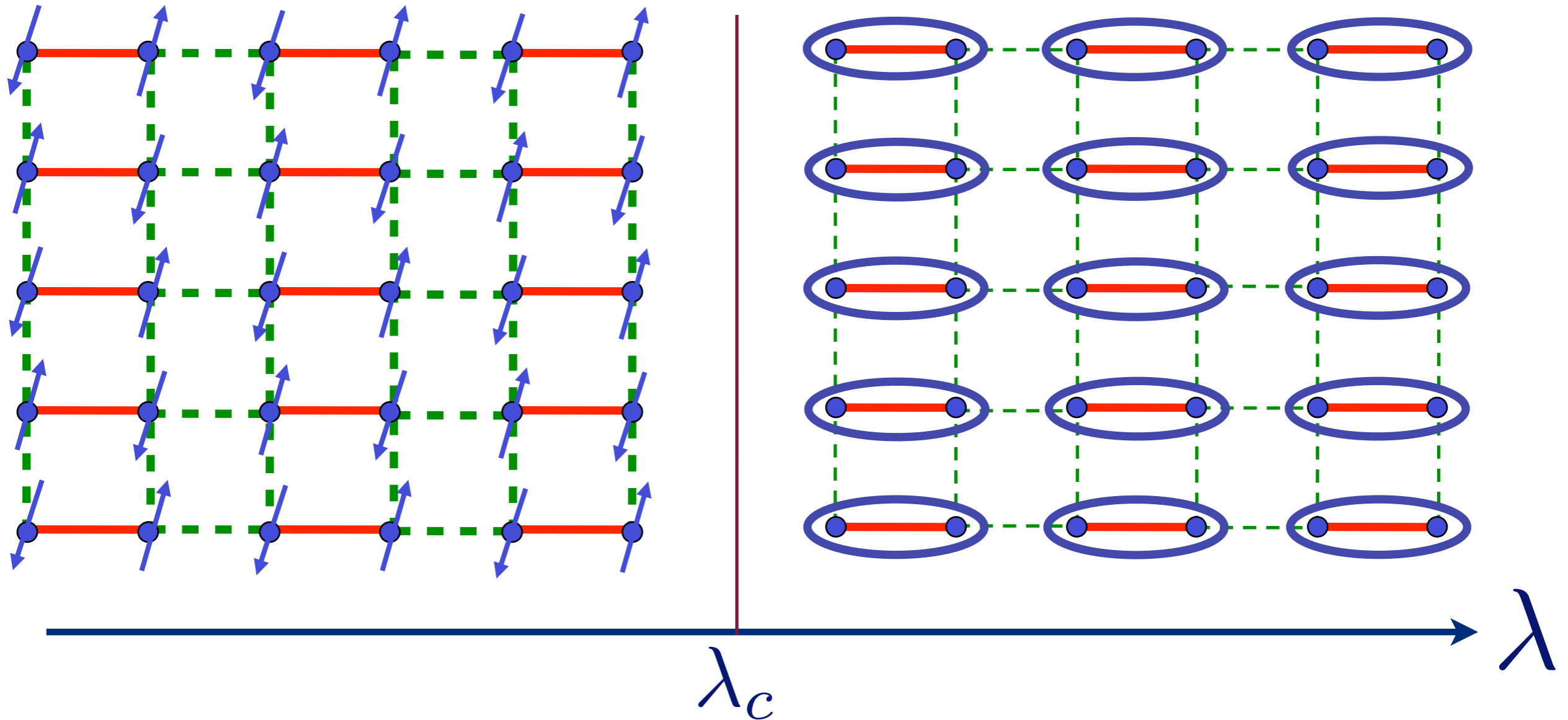
Spin $S = 1$
"triplon"

Excitation spectrum in the Néel phase



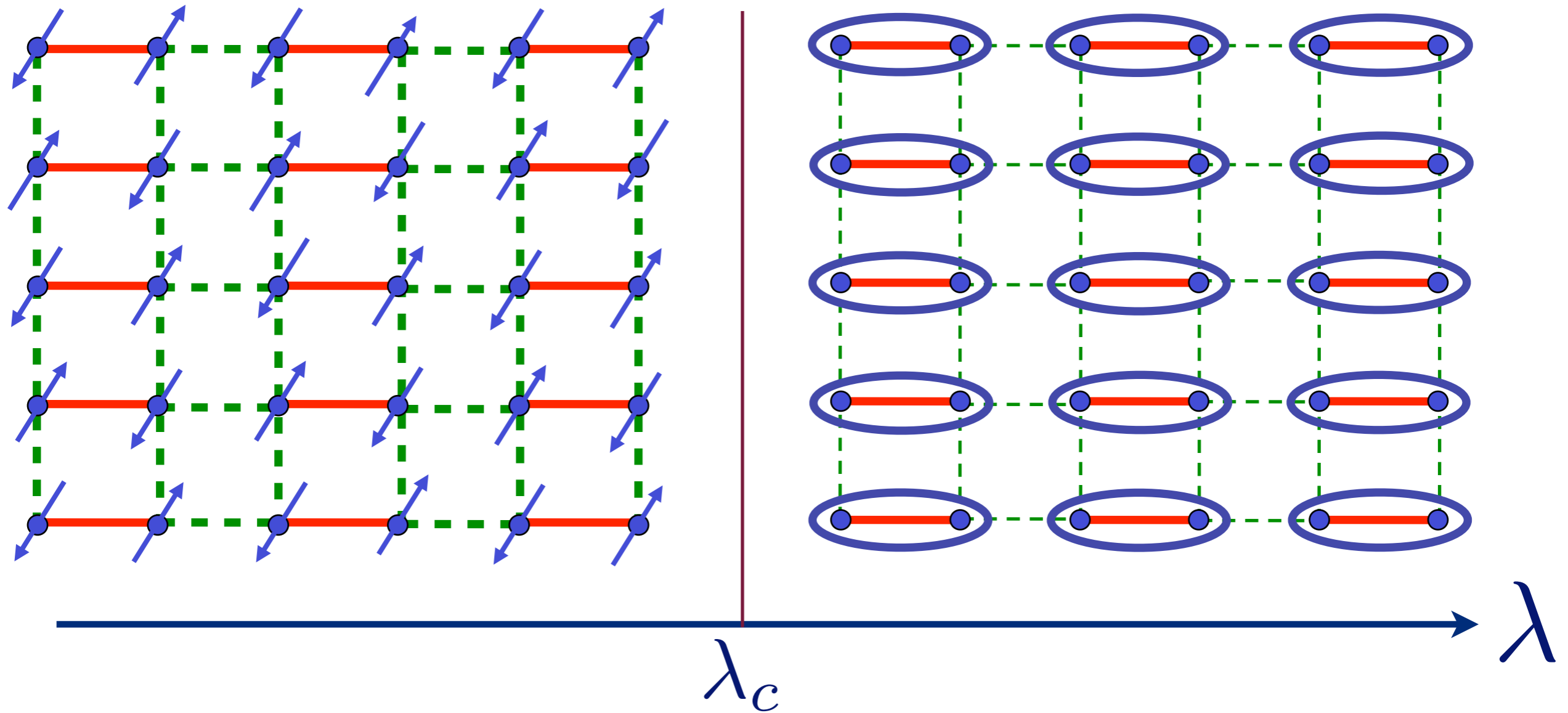
Spin waves

Excitation spectrum in the Néel phase



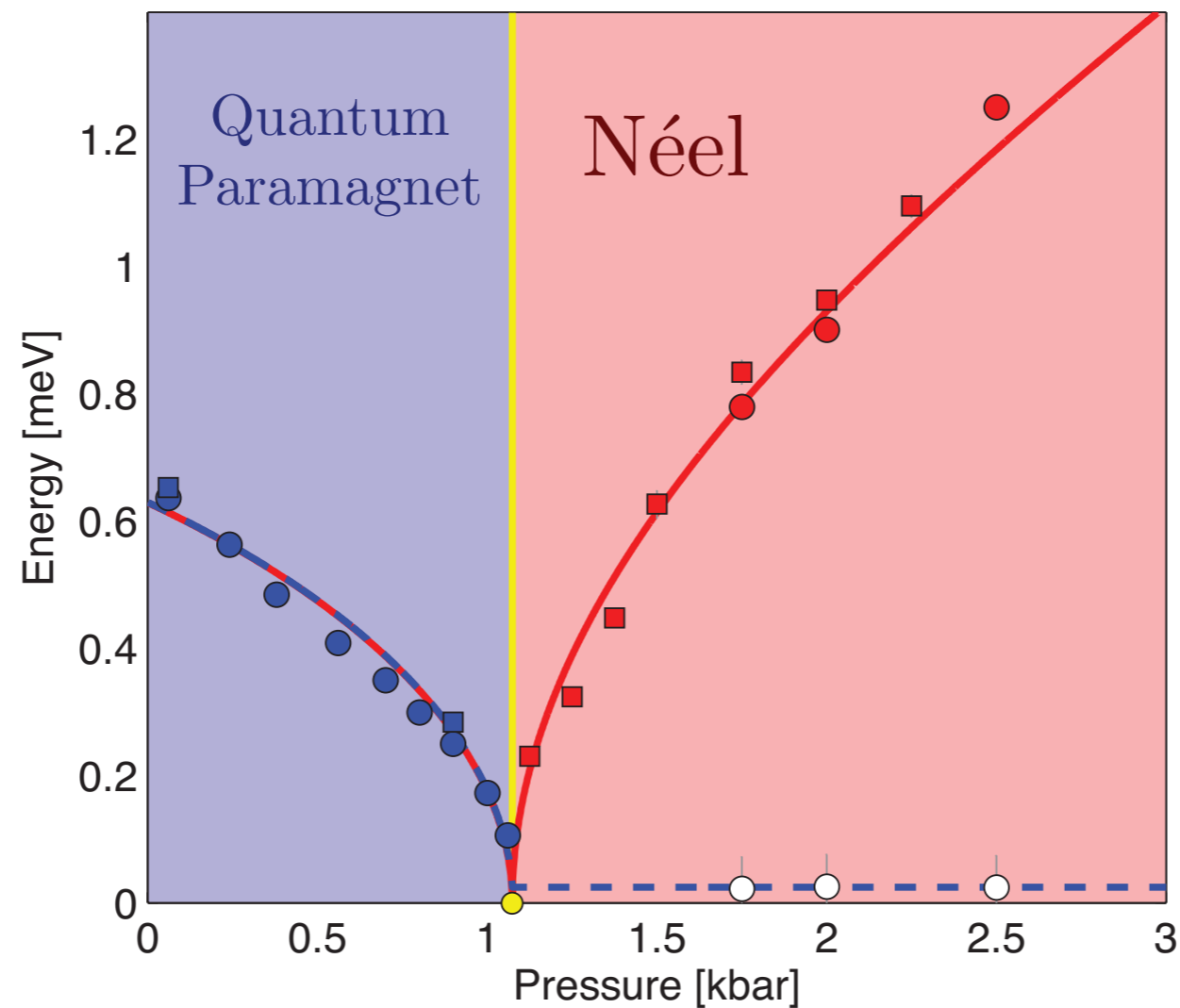
Spin waves

Excitation spectrum in the Néel phase



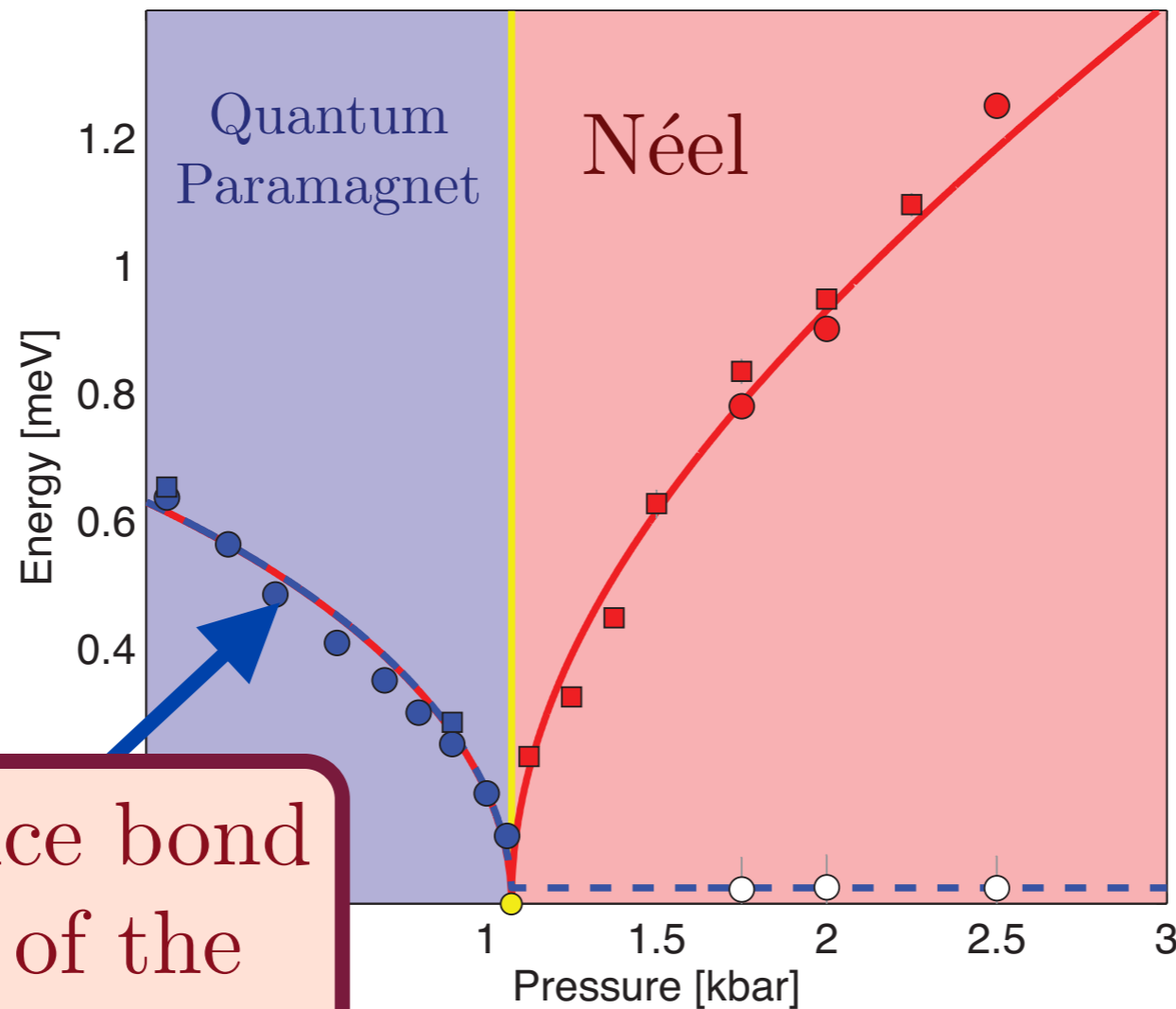
Spin waves

Excitations of TlCuCl_3 with varying pressure



Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

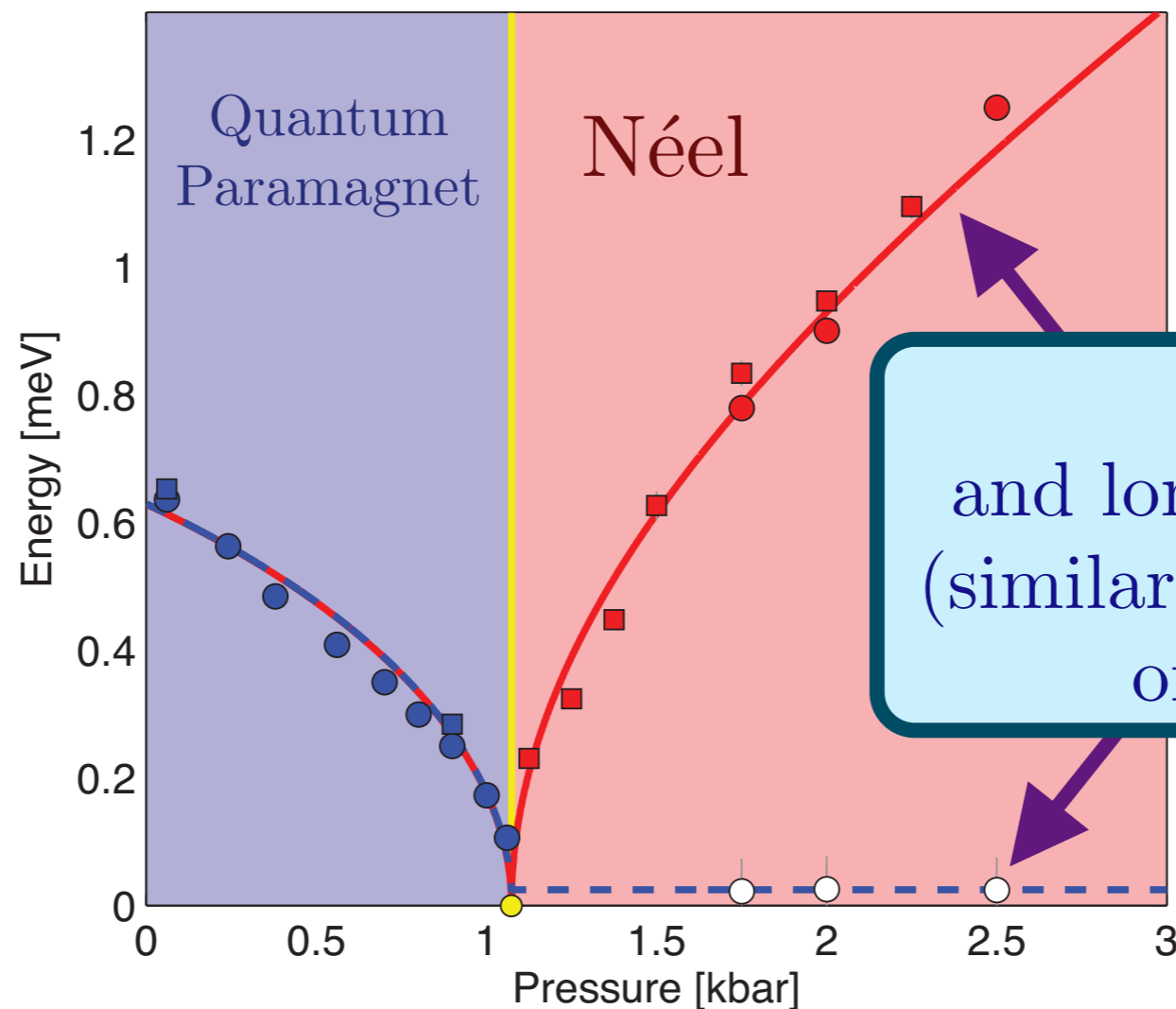
Excitations of TlCuCl_3 with varying pressure



Broken valence bond excitations of the quantum paramagnet

Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Excitations of TlCuCl_3 with varying pressure



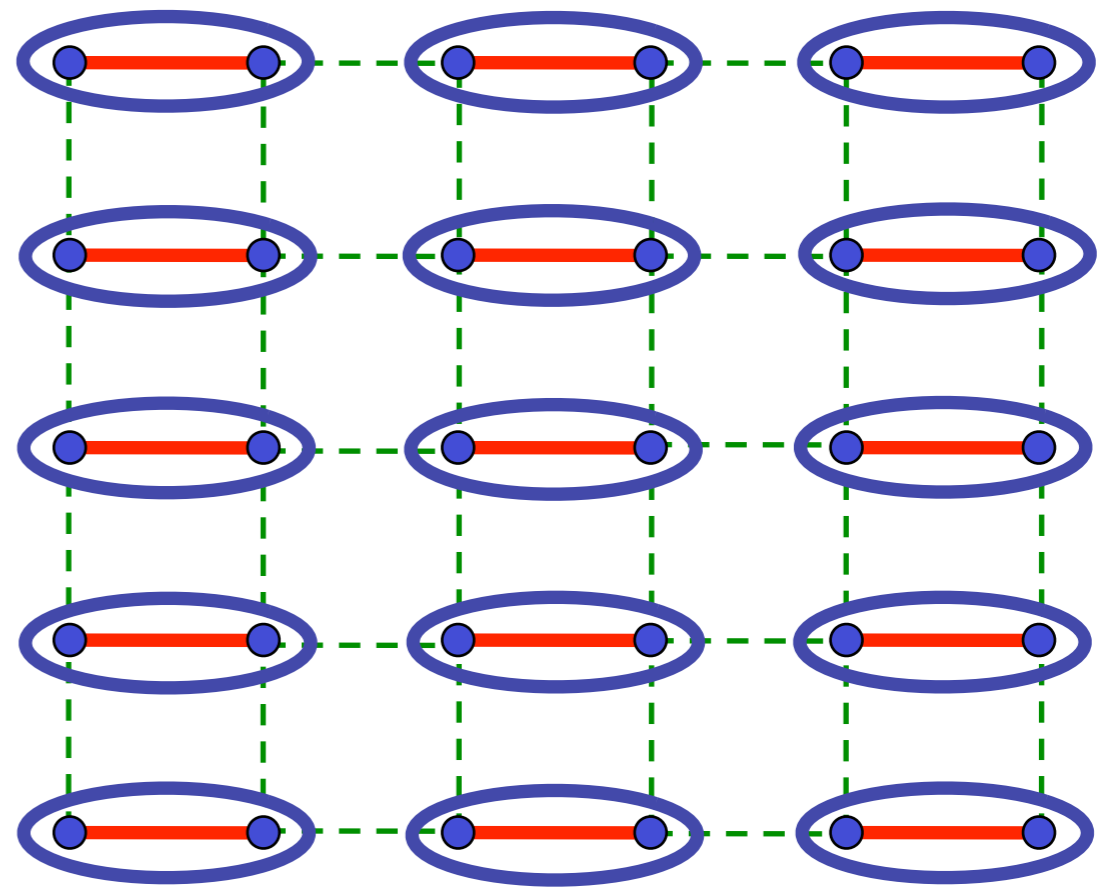
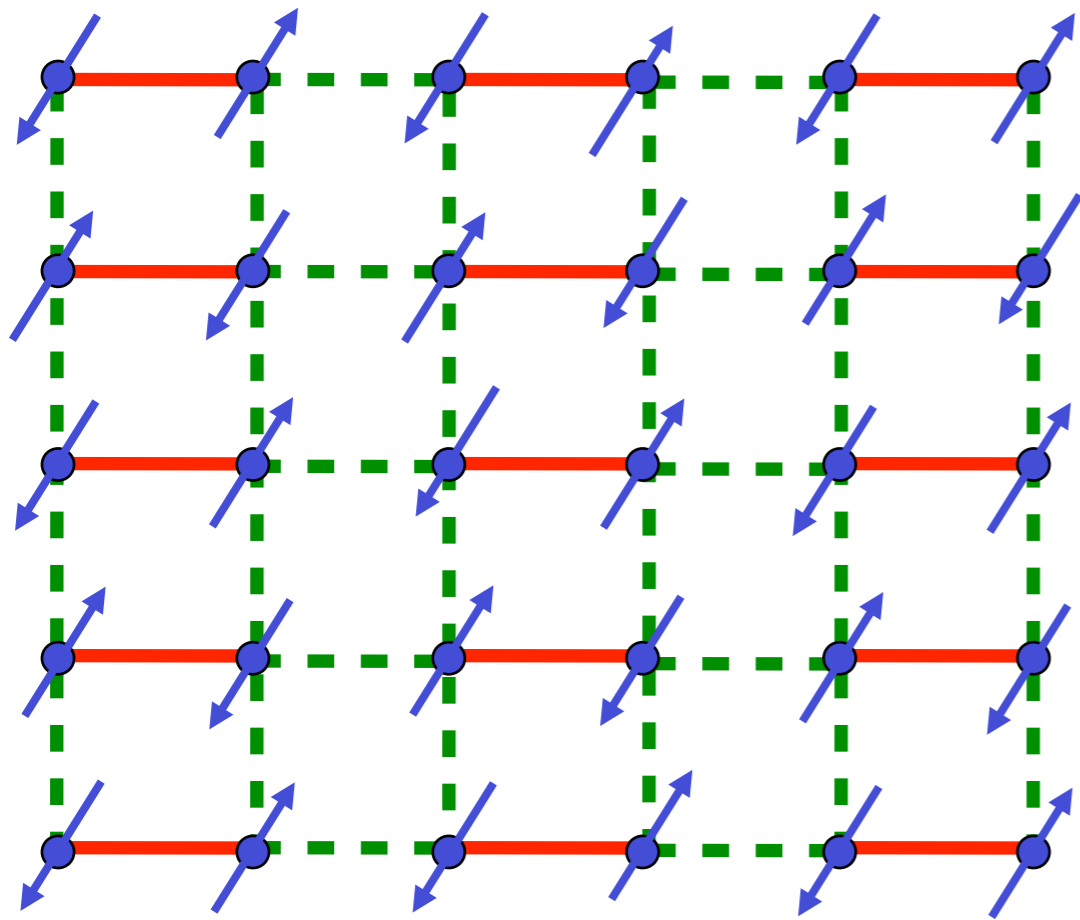
Spin wave
and longitudinal excitations
(similar to the Higgs particle)
of the Néel state.

“Higgs” particle appears at theoretically predicted energy

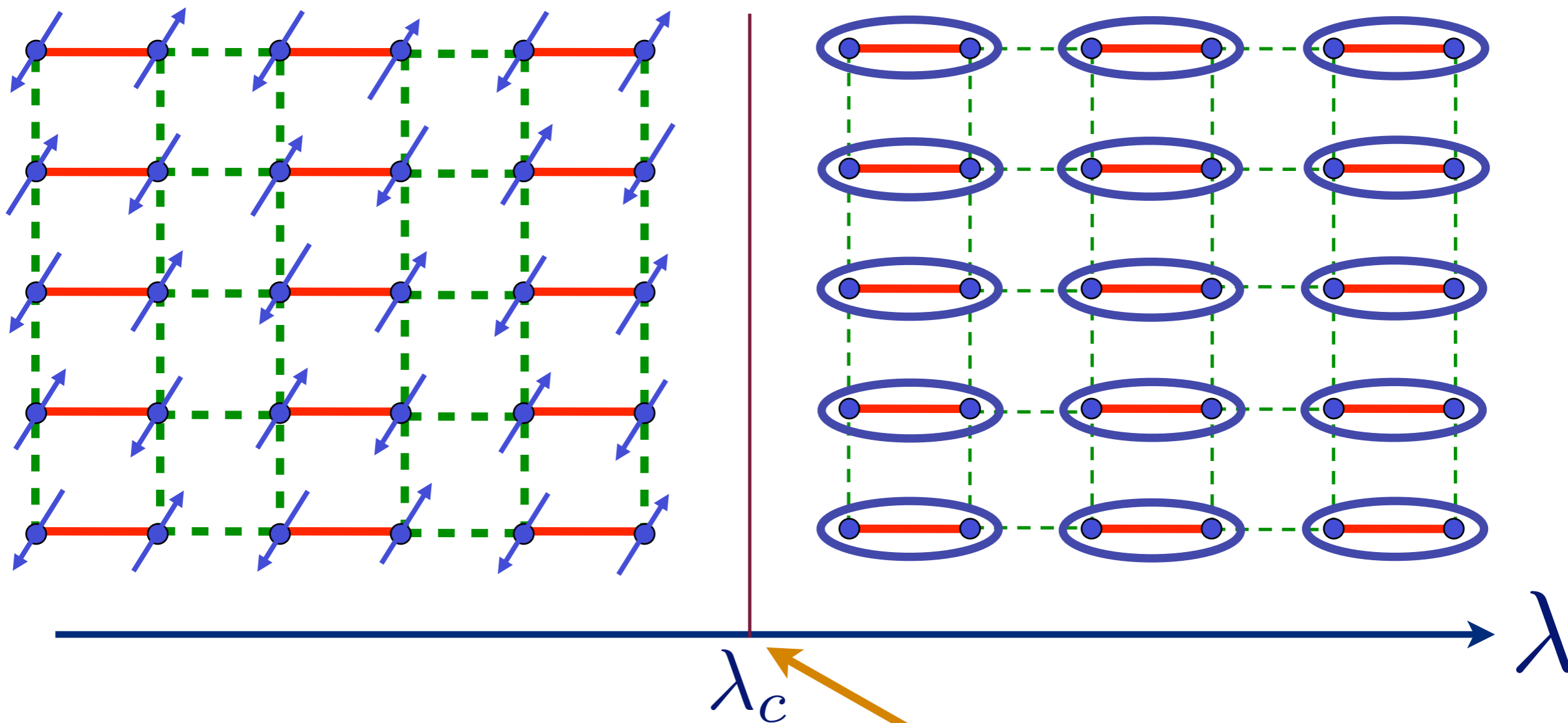
S. Sachdev, arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

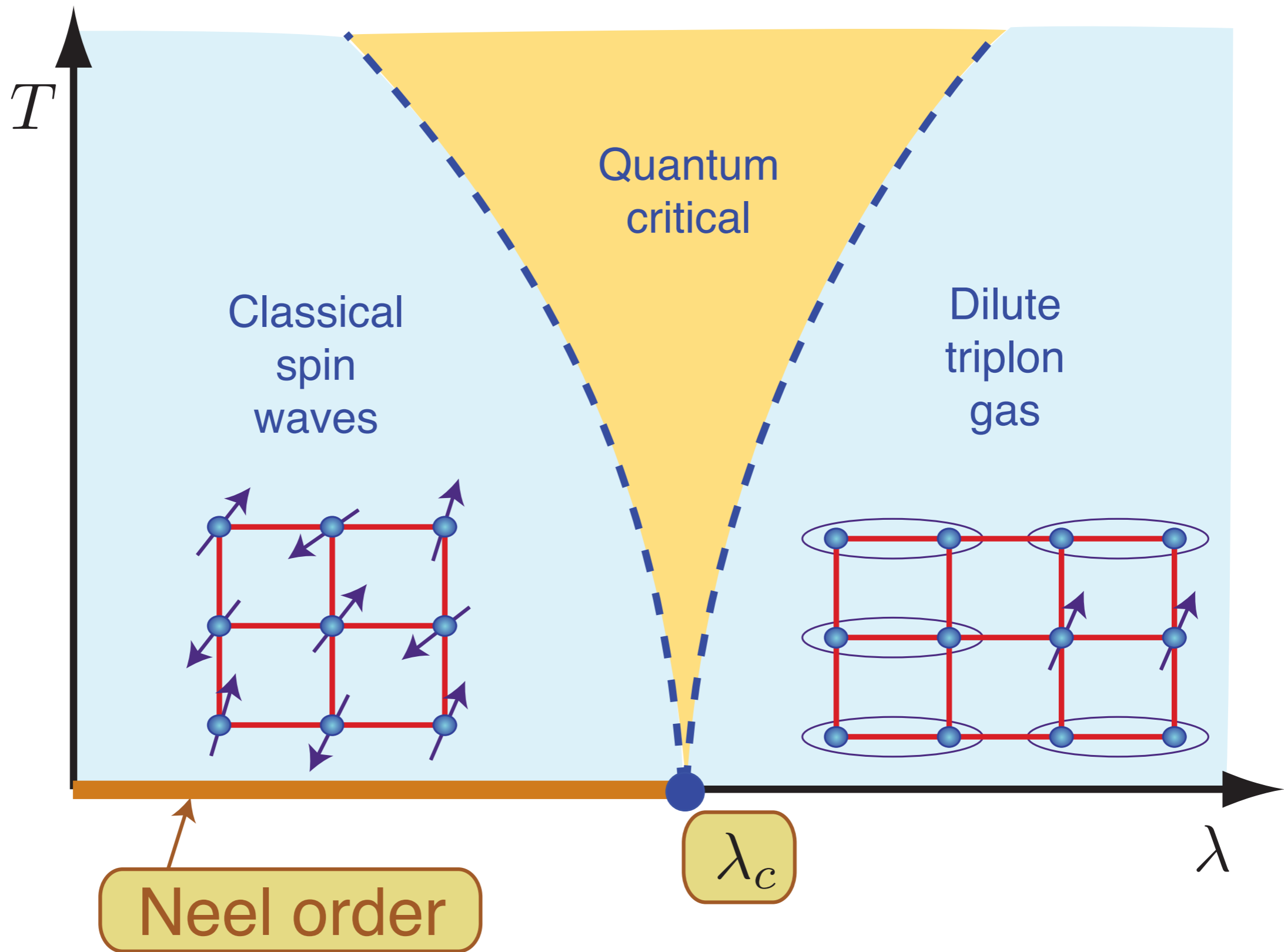


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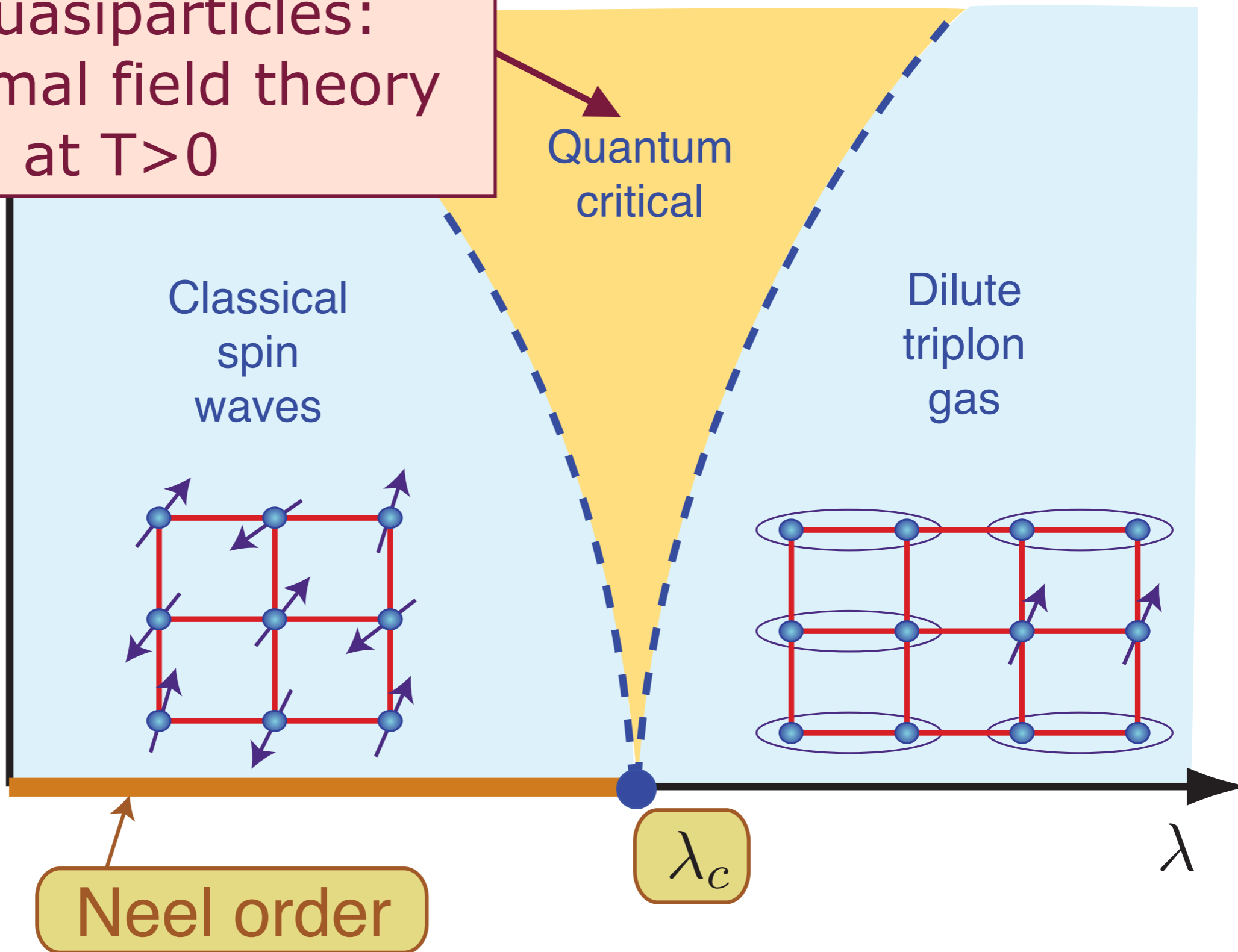
**Quantum critical point:
A new state of matter
with long-range quantum entanglement
and no quasiparticles**

S. Sachdev and
J. Ye, *Phys. Rev. Lett.*
69, 2411 (1992).



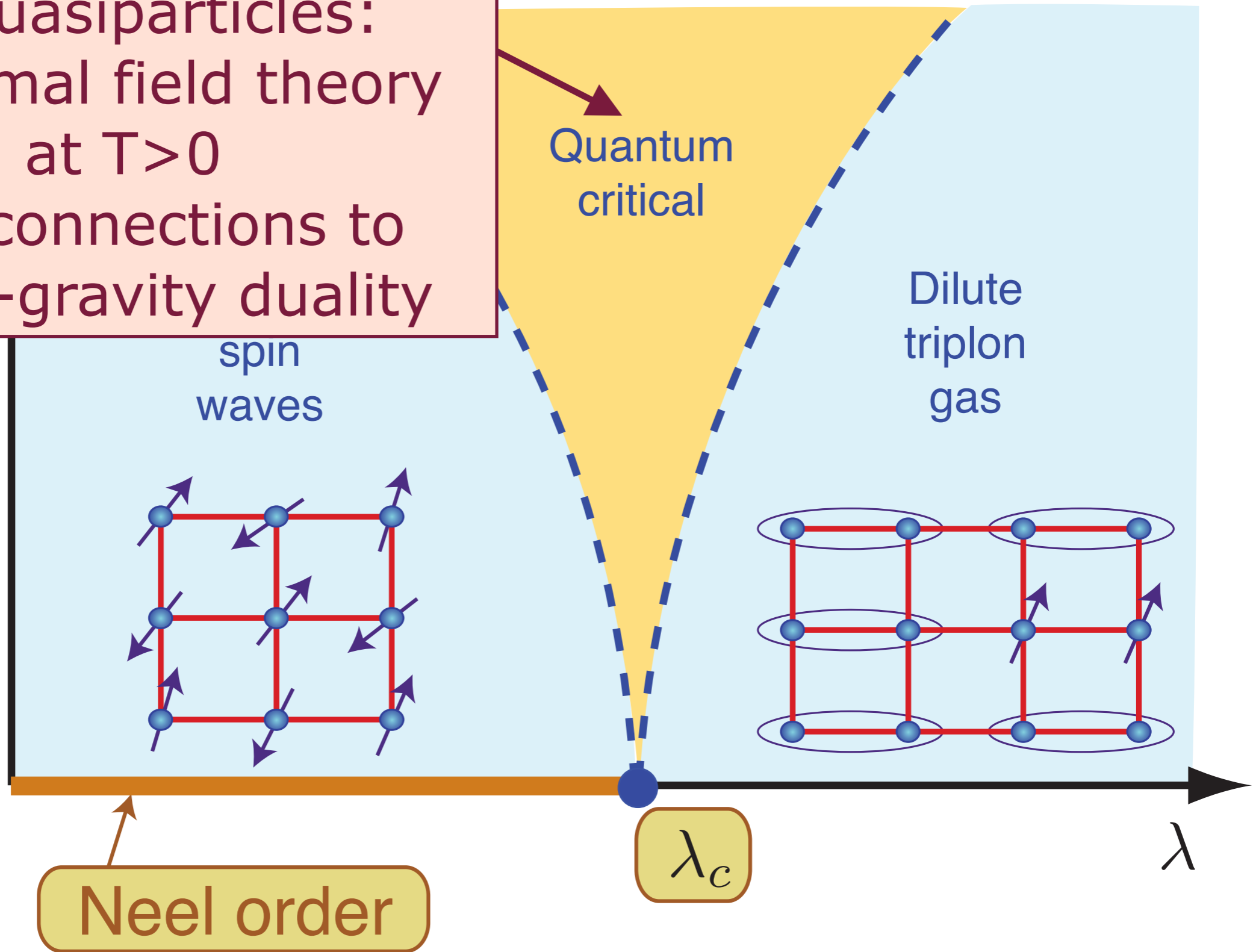
S. Sachdev and
 J. Ye, *Phys. Rev. Lett.*
69, 2411 (1992).

Dynamics not described
by quasiparticles:
conformal field theory
at $T > 0$



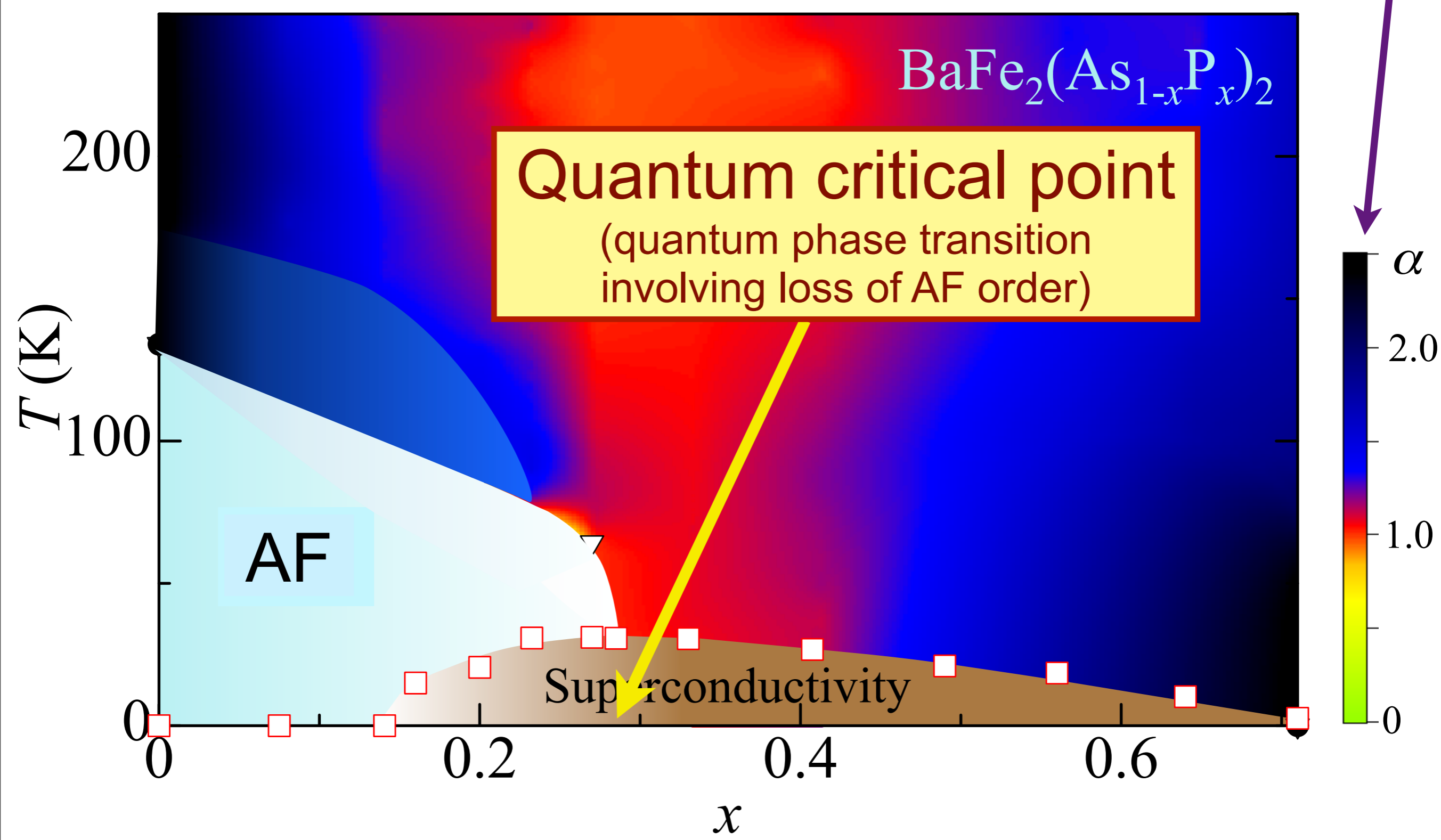
S. Sachdev and
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Dynamics not described
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conformal field theory
at $T > 0$
with connections to
gauge-gravity duality



S. Sachdev and
J. Ye, *Phys. Rev. Lett.*
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Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Outline

1. Quantum critical point in an insulator
*Entanglement and
non-quasiparticle dynamics*
2. Quantum critical point in a metal
The iron-based superconductors
3. The pseudogap regime of the hole-doped cuprate superconductors
*Angular fluctuations of a
multicomponent order*

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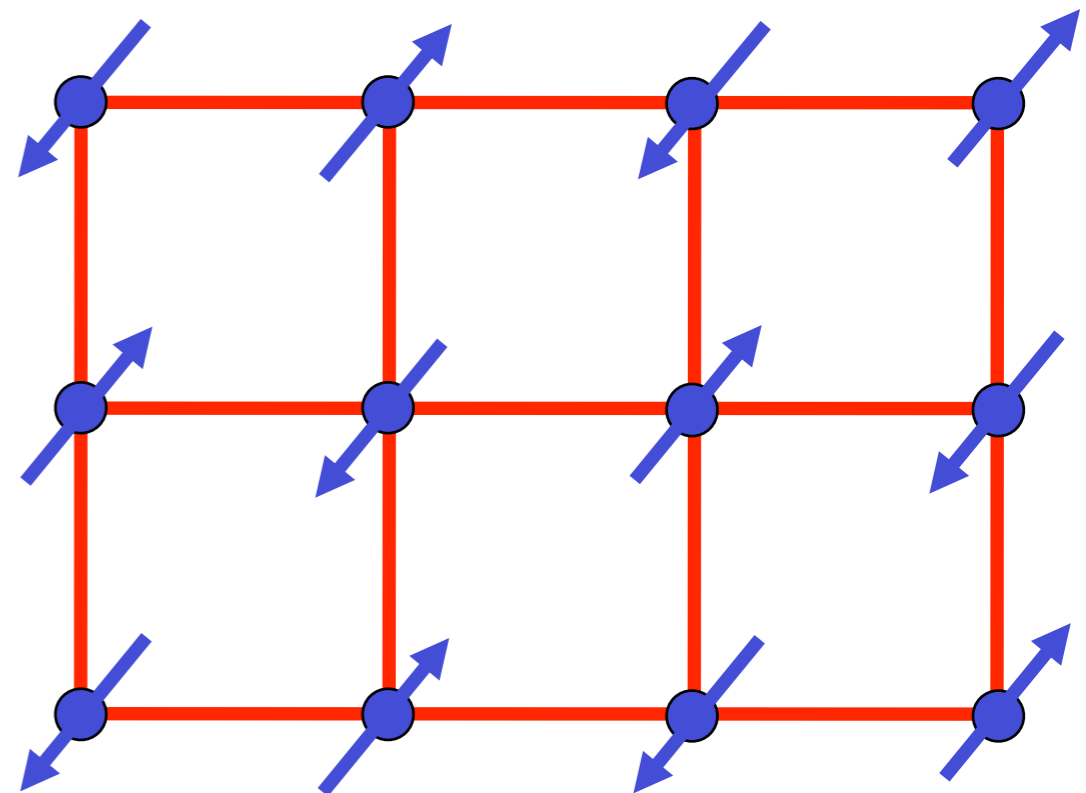
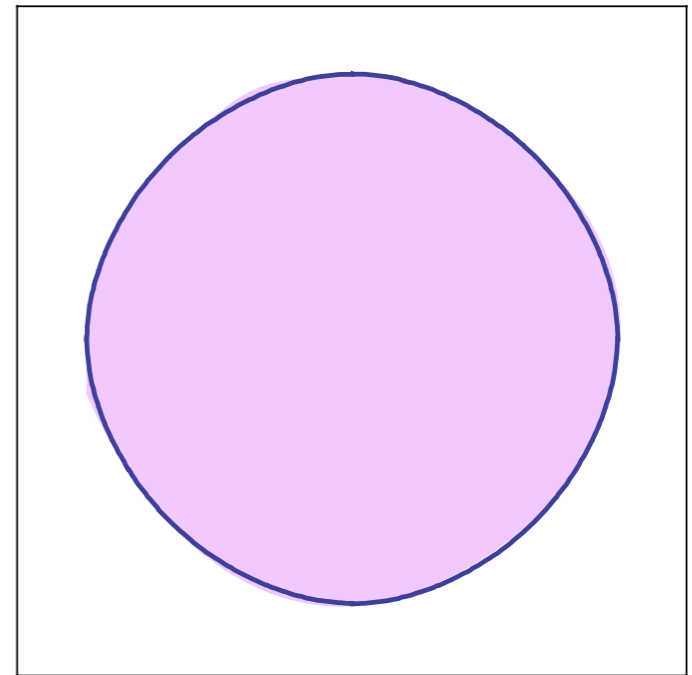
The iron-based superconductors

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*Angular fluctuations of a
multicomponent order*

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

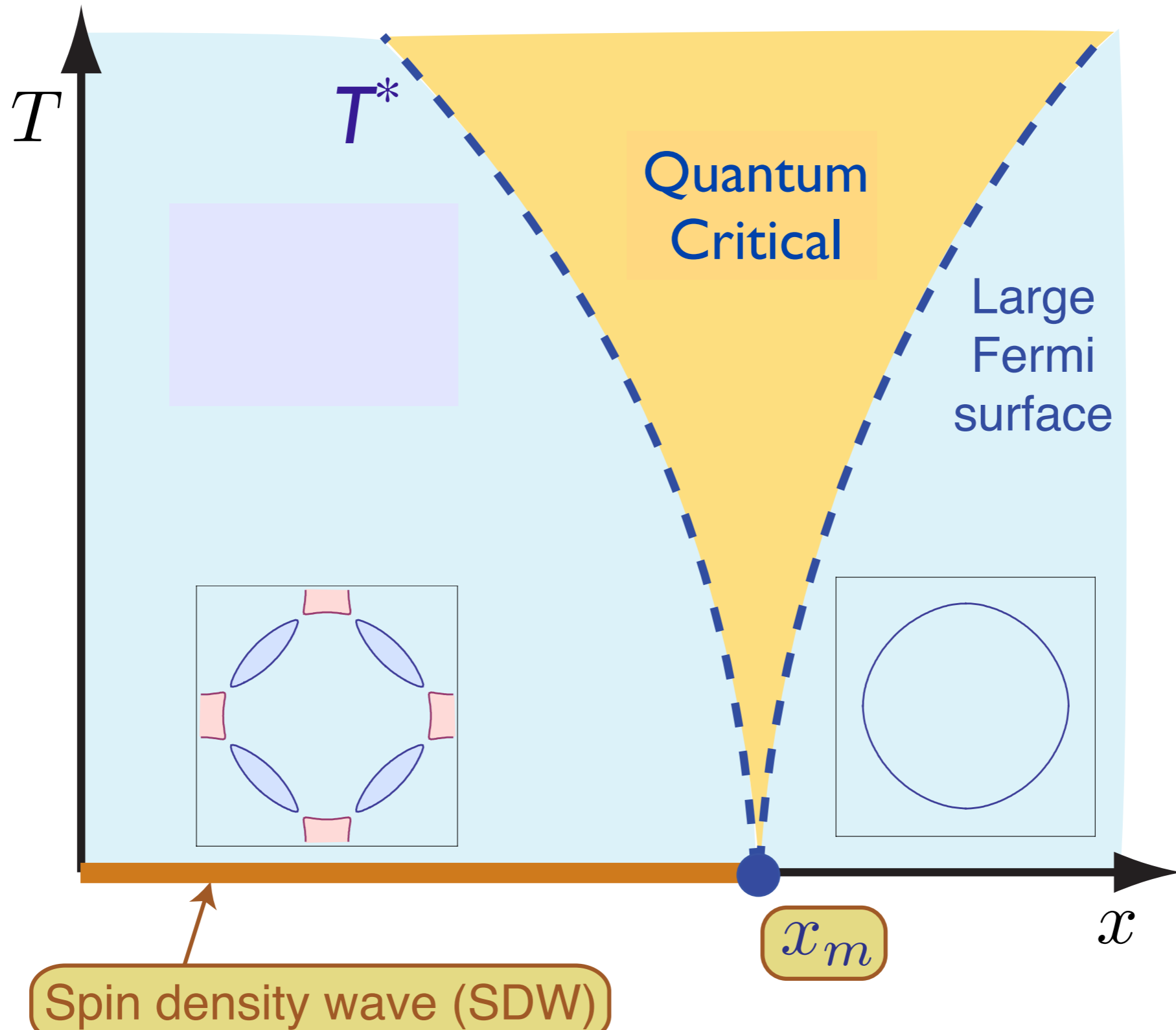


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

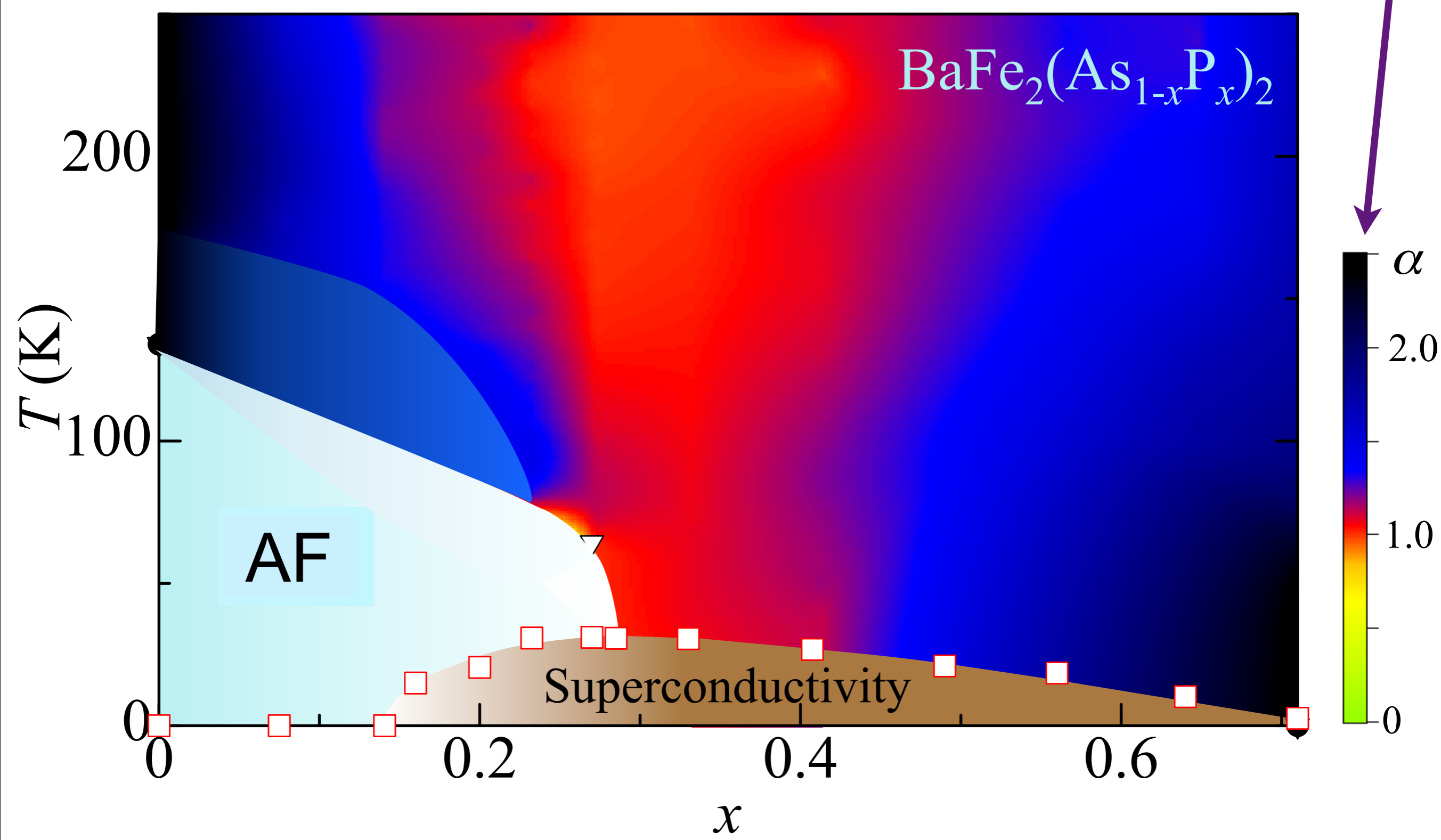
where \mathbf{K} is the ordering wavevector.

Fermi surface+antiferromagnetism



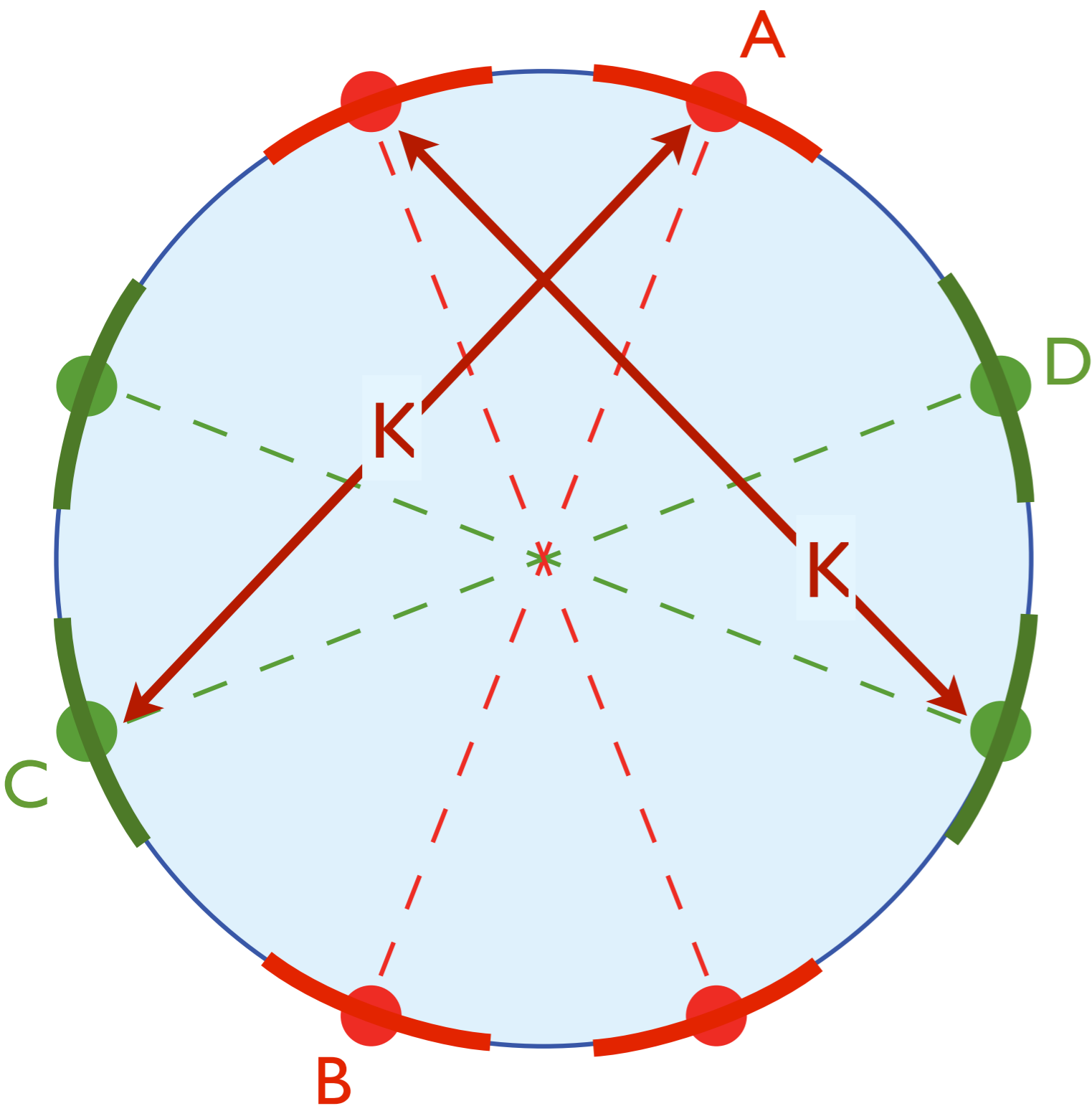
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Resistivity
 $\sim \rho_0 + AT^\alpha$



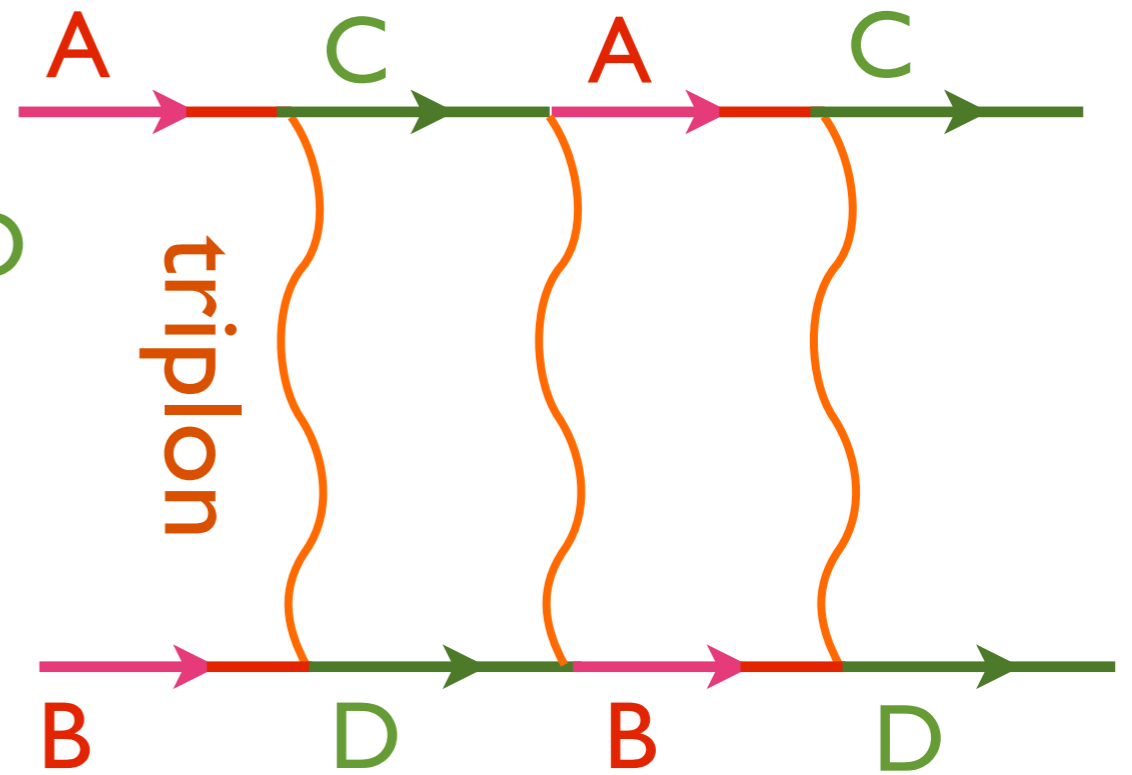
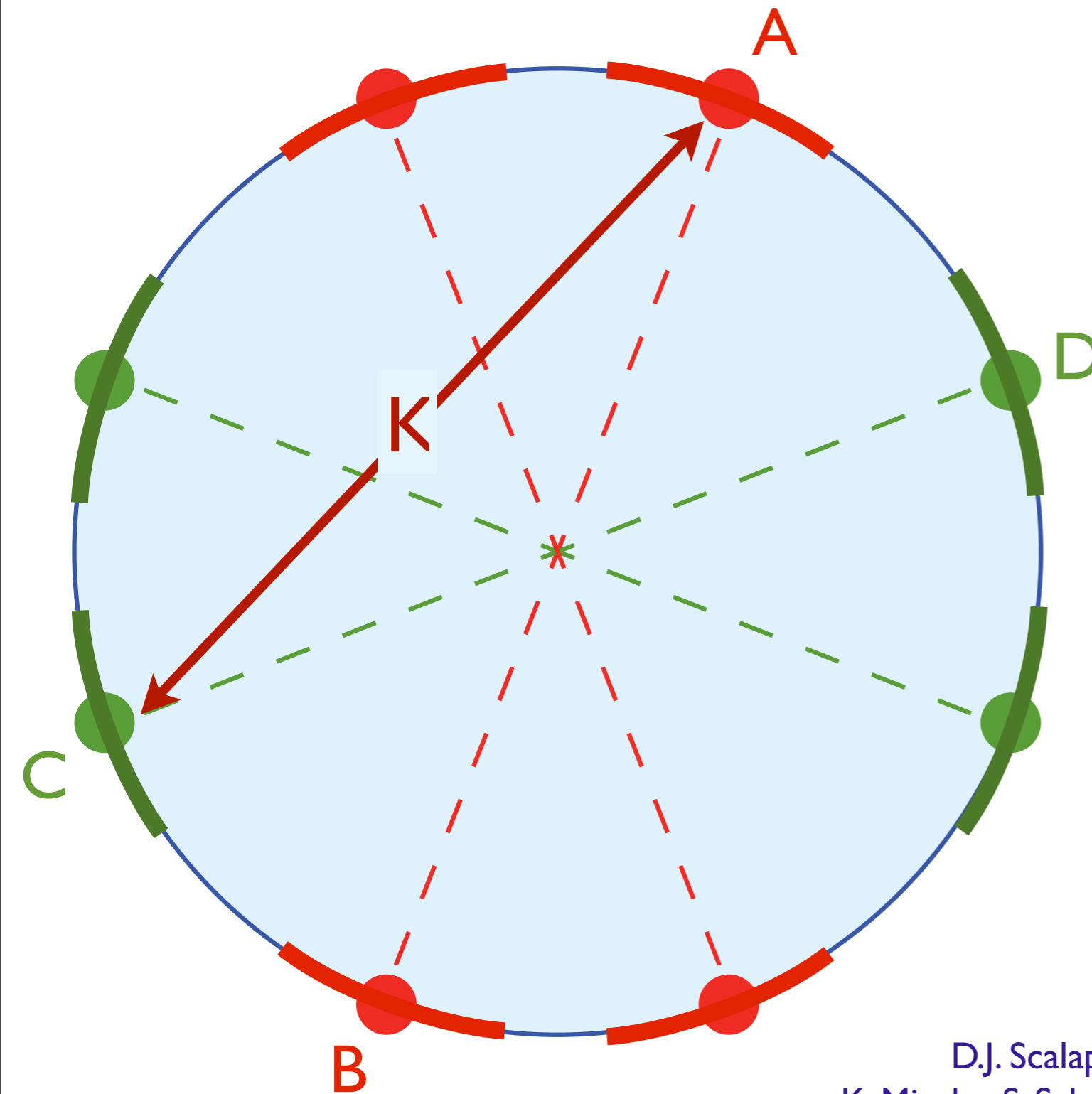
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Focus on points on the Fermi surface separated by \mathbf{K}



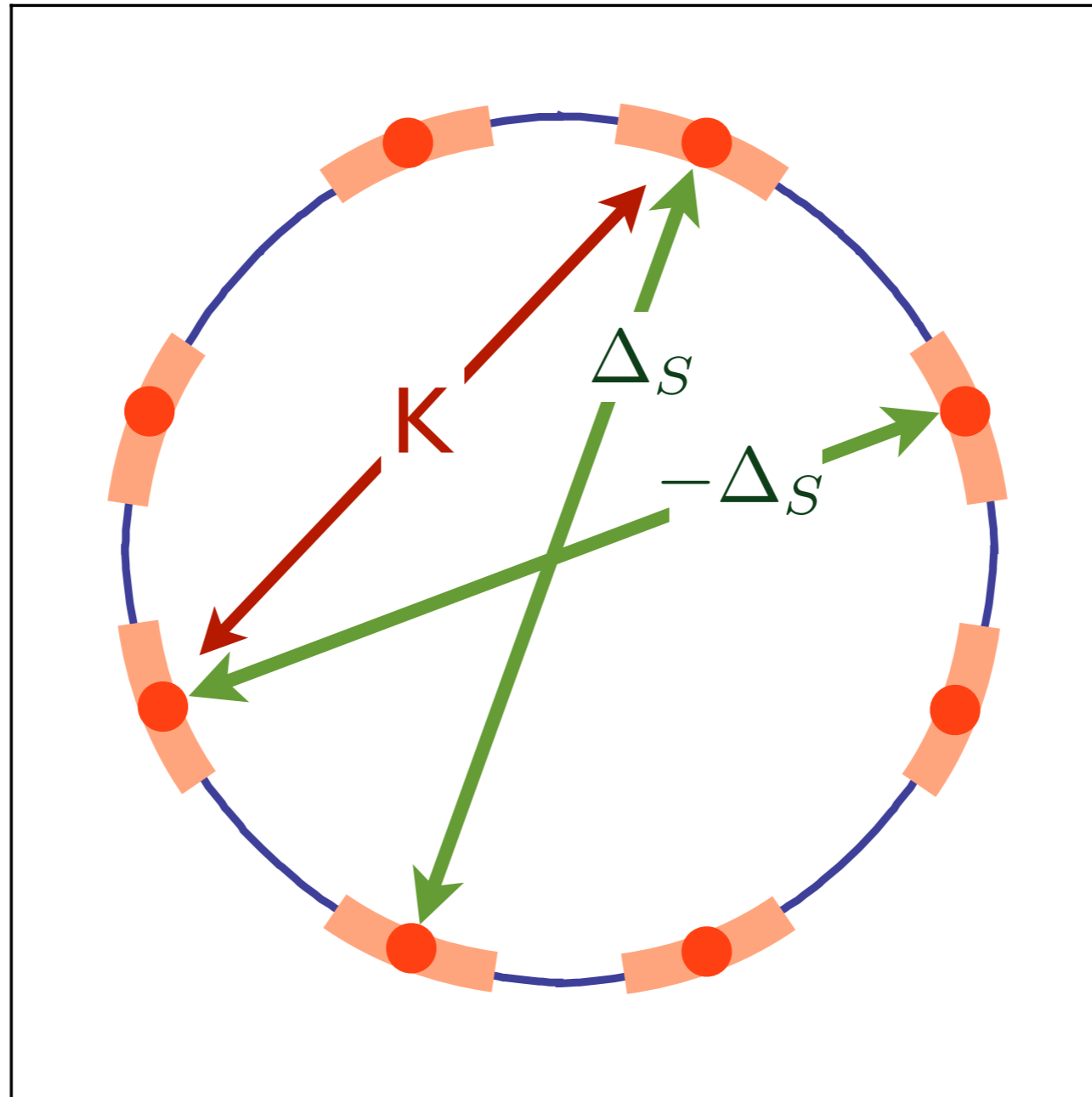
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

Pairing “glue” from triplon (paramagnon) exchange



- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
Ar. Abanov, A.V. Chubukov, and A.M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$



**d-wave superconductor: particle-particle pairing
with sign-changing pairing amplitude**

Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

- Pairing glue becomes stronger.



Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

- Pairing glue becomes stronger.
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

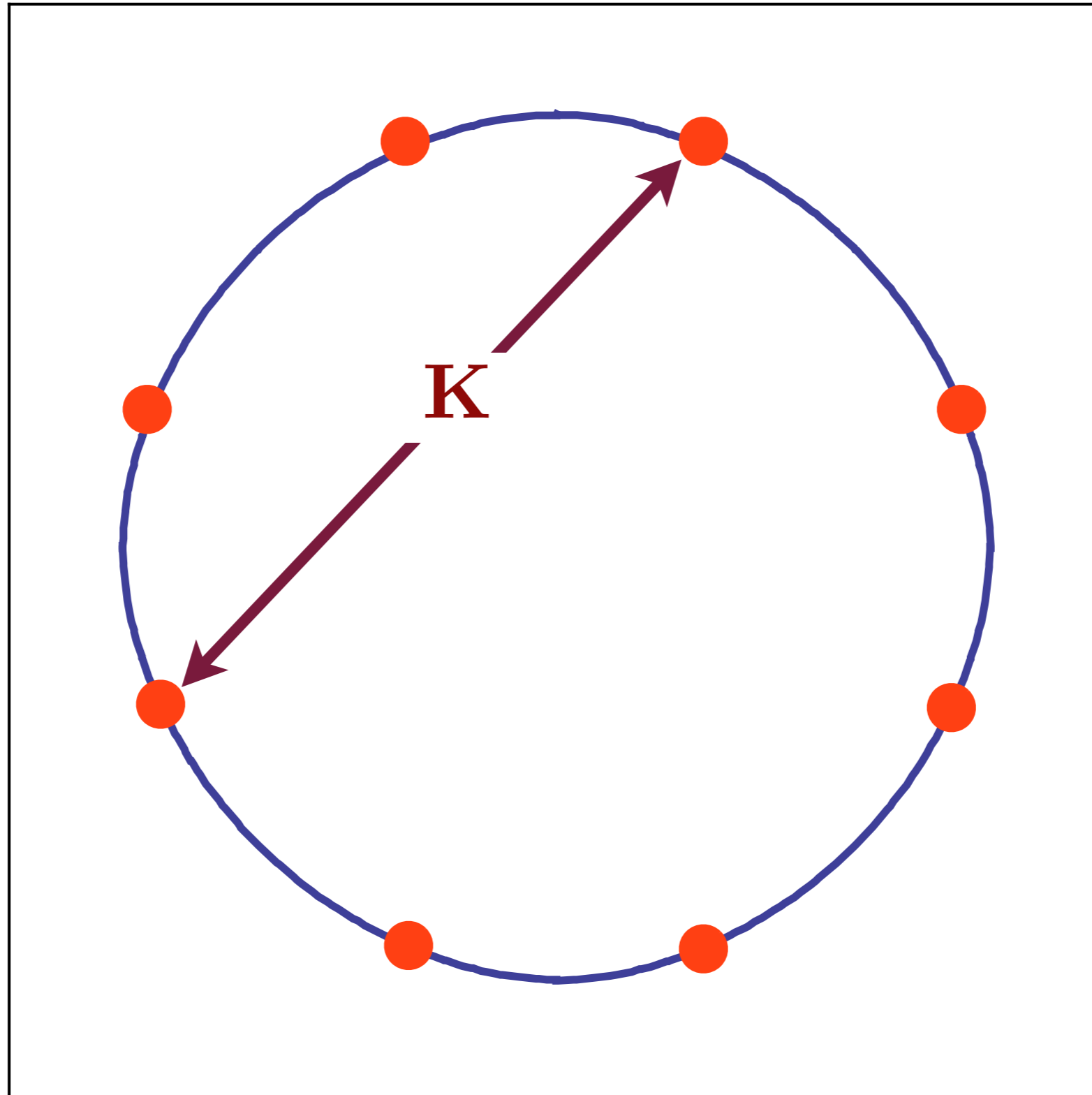


Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

- Pairing glue becomes stronger.
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.
- Other instabilities can appear *e.g.* to charge density waves/stripe order.

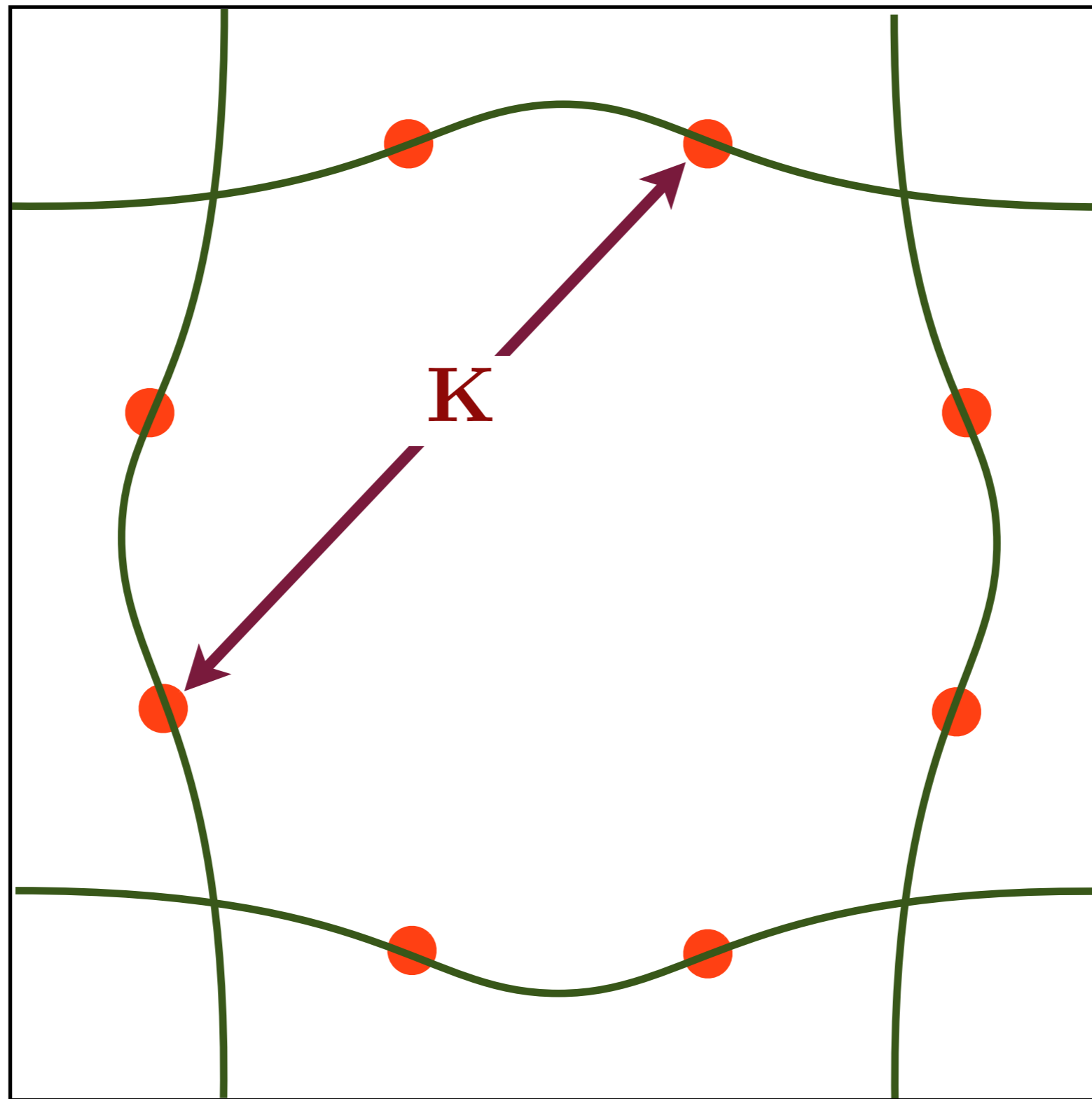


QMC for the onset of antiferromagnetism



Hot spots in a single band model

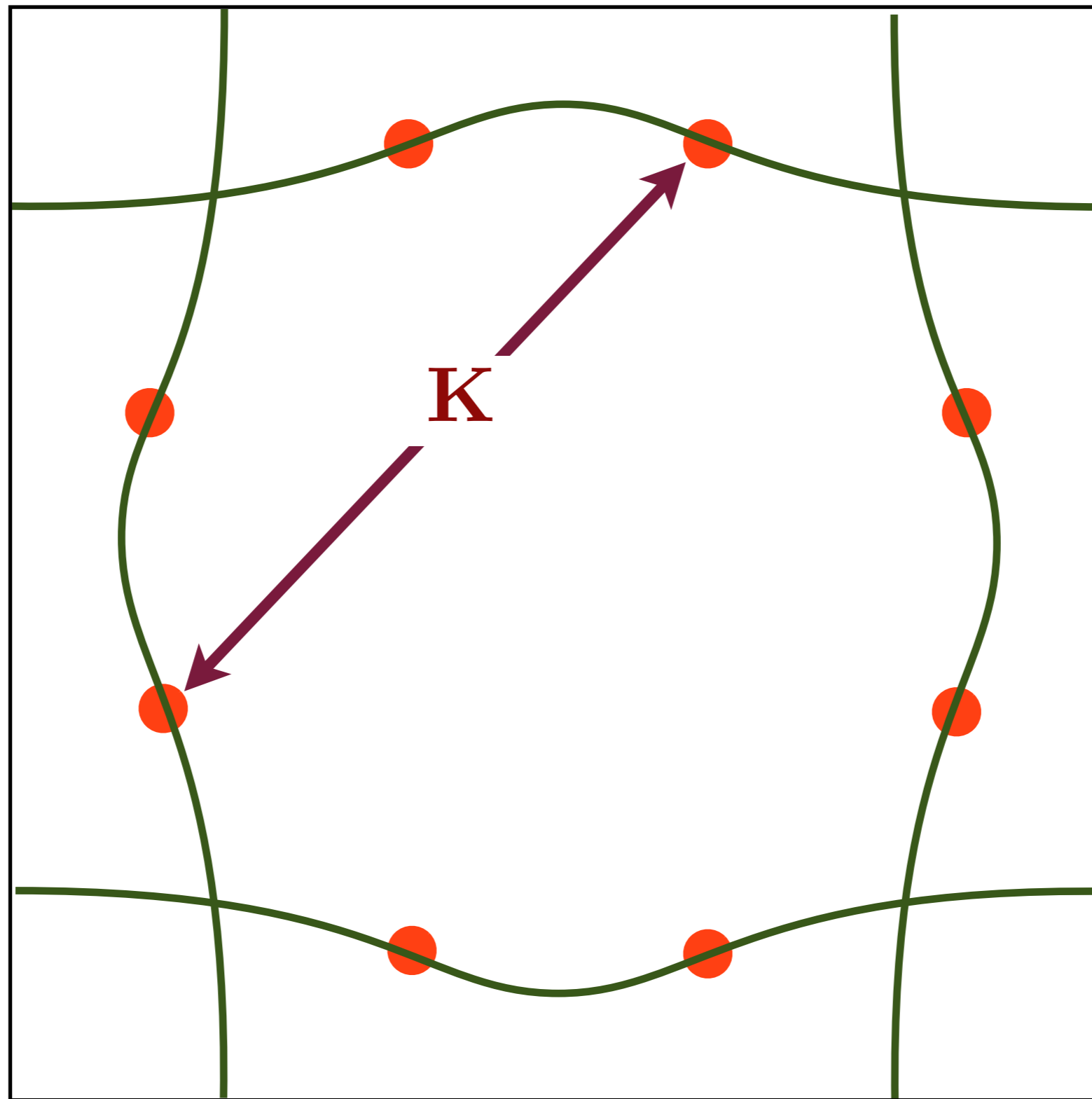
QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Hot spots in a two band model

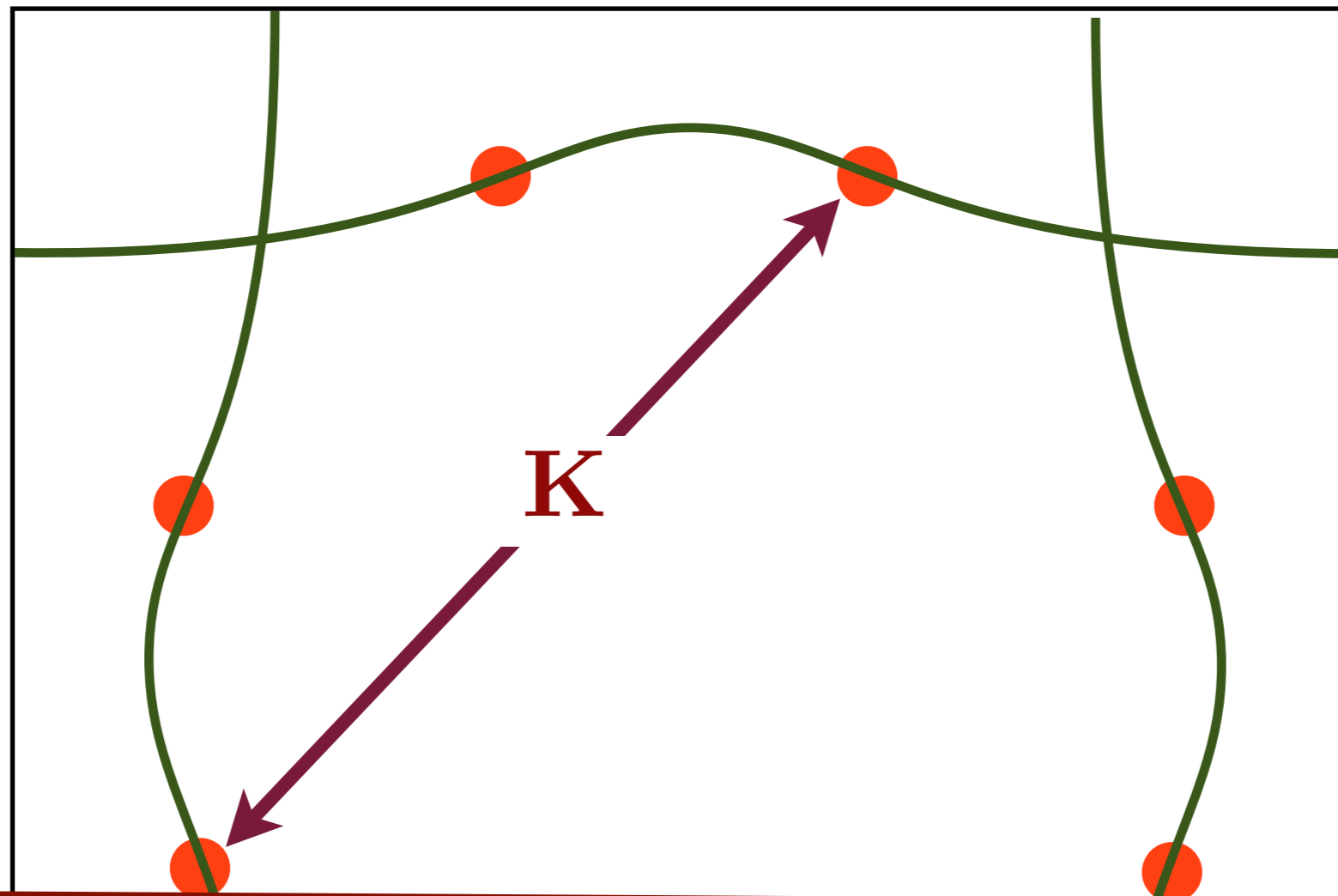
QMC for the onset of antiferromagnetism



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arXiv:1206.0742

Hot spots in a two band model

QMC for the onset of antiferromagnetism

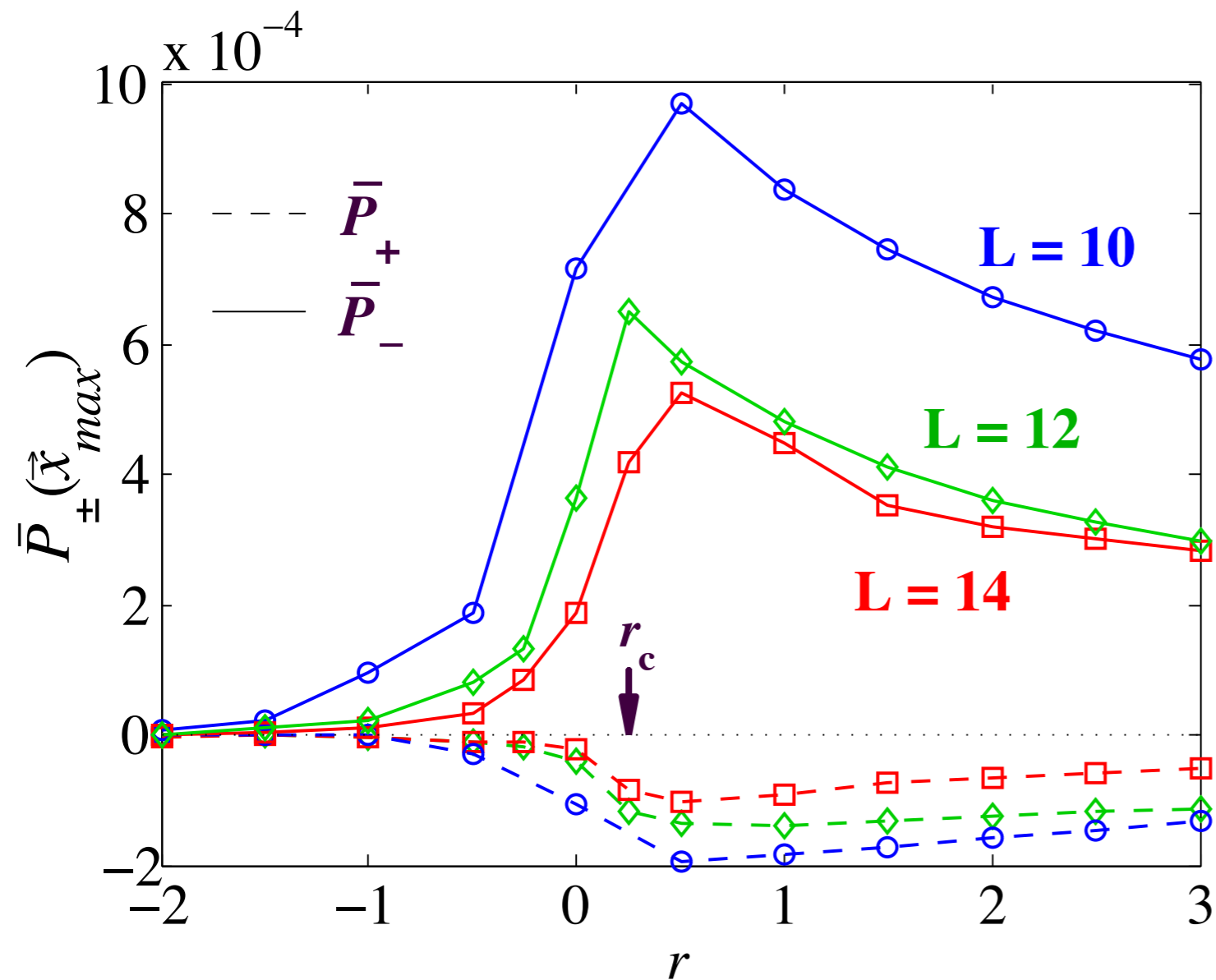


E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

No sign problem in
fermion determinant Monte Carlo !
Determinant is positive because of Kramer's
degeneracy, and no additional symmetries are needed; holds for
arbitrary band structure and band filling, provided K only
connects hot spots in distinct bands

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals

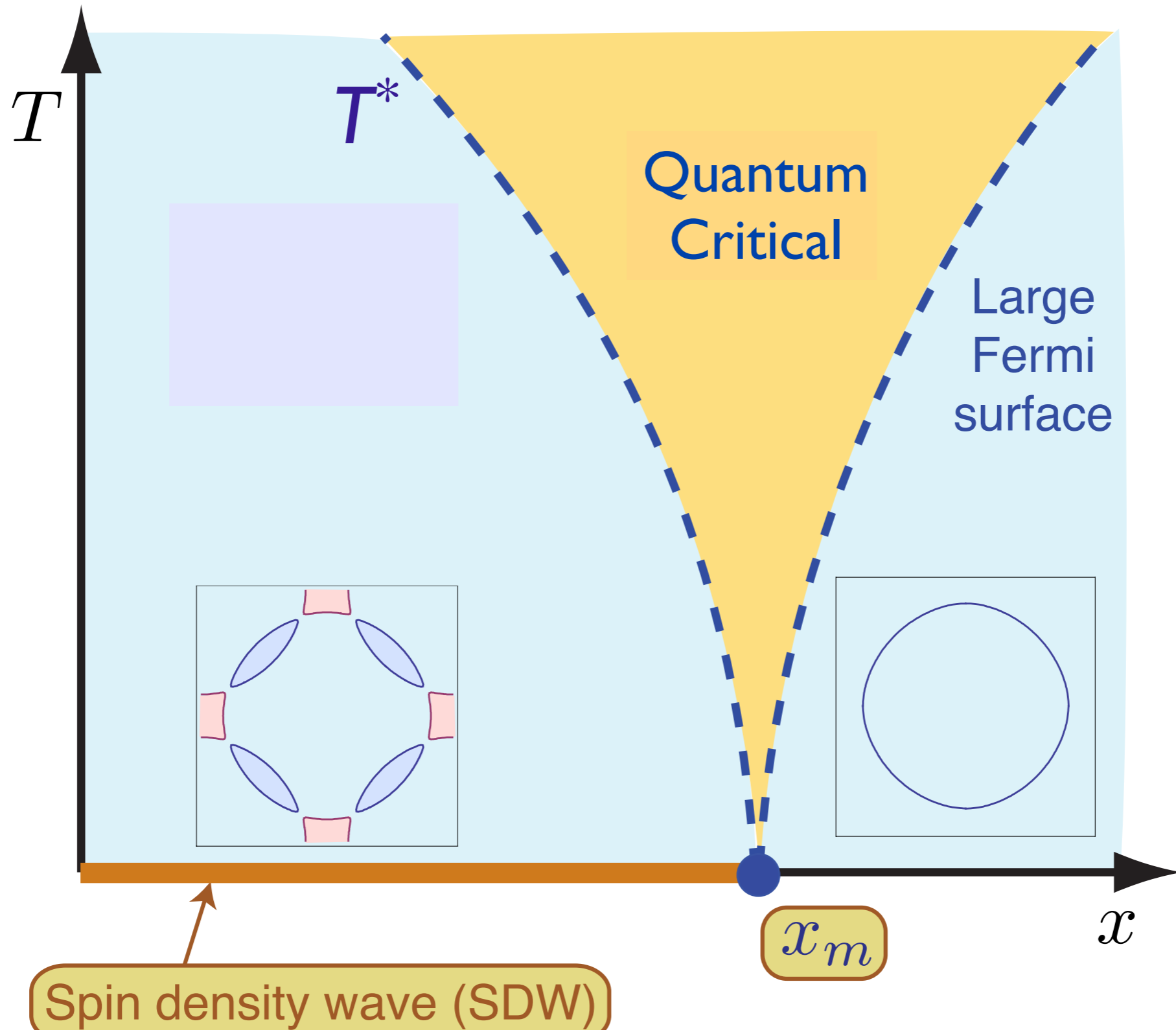
d-wave superconducting survives in the strong-coupling region across the quantum critical point



s/d pairing amplitudes P_{+}/P_{-} as a function of the tuning parameter r

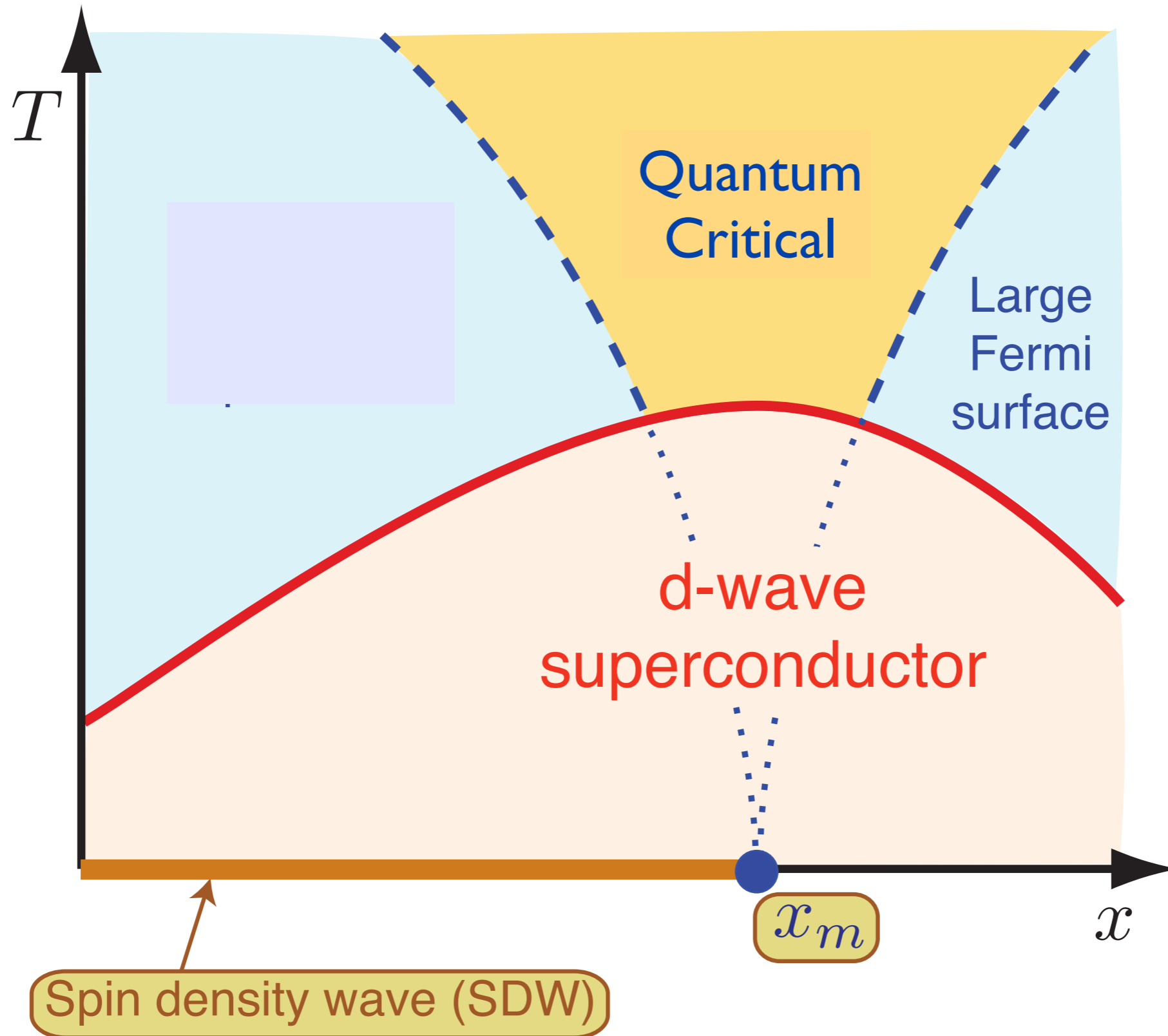
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism



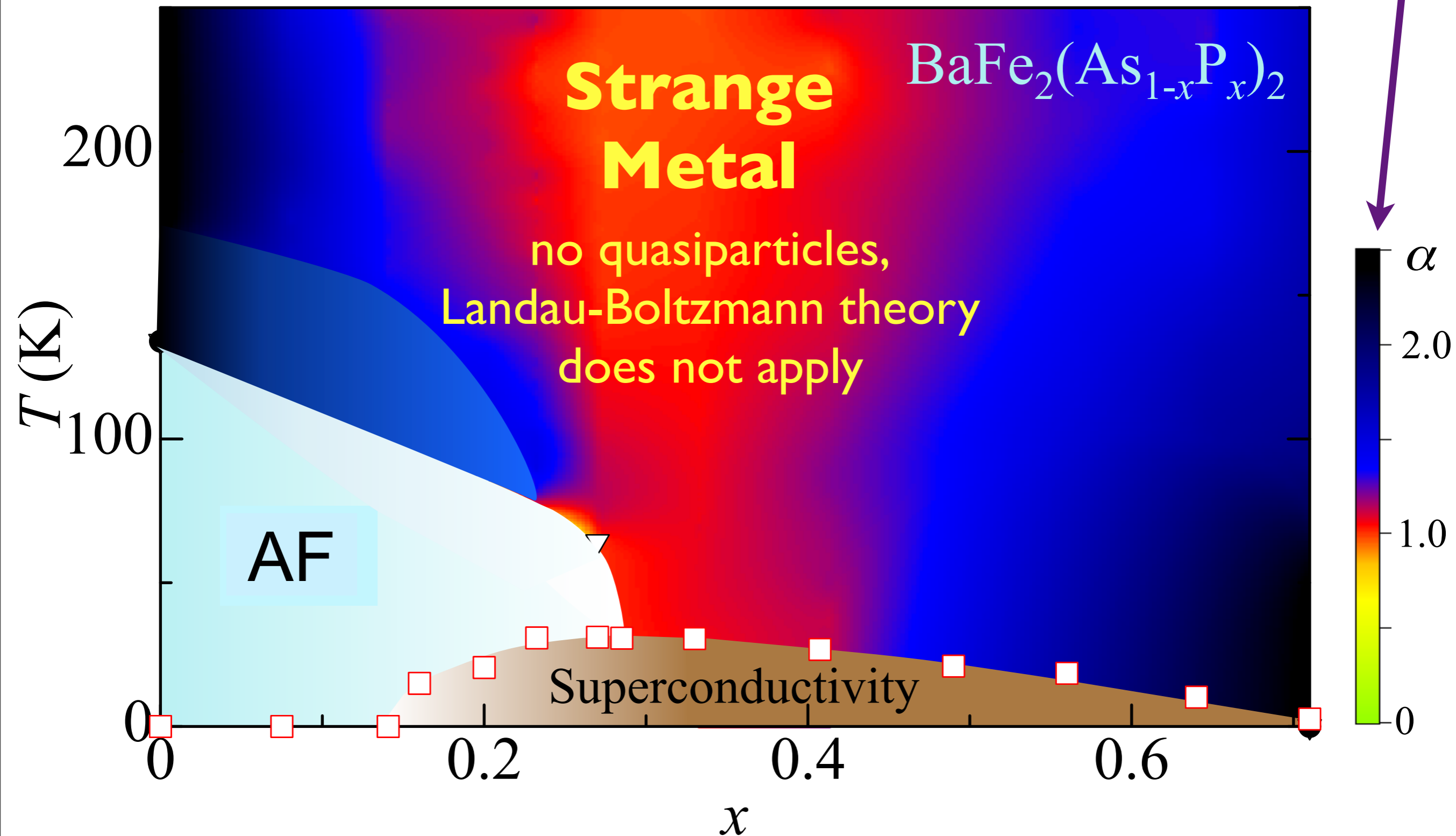
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor

Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

Outline

1. Quantum critical point in an insulator
*Entanglement and
non-quasiparticle dynamics*
2. Quantum critical point in a metal
The iron-based superconductors
3. The pseudogap regime of the hole-doped cuprate superconductors
*Angular fluctuations of a
multicomponent order*

Outline

1. Quantum critical point in an insulator

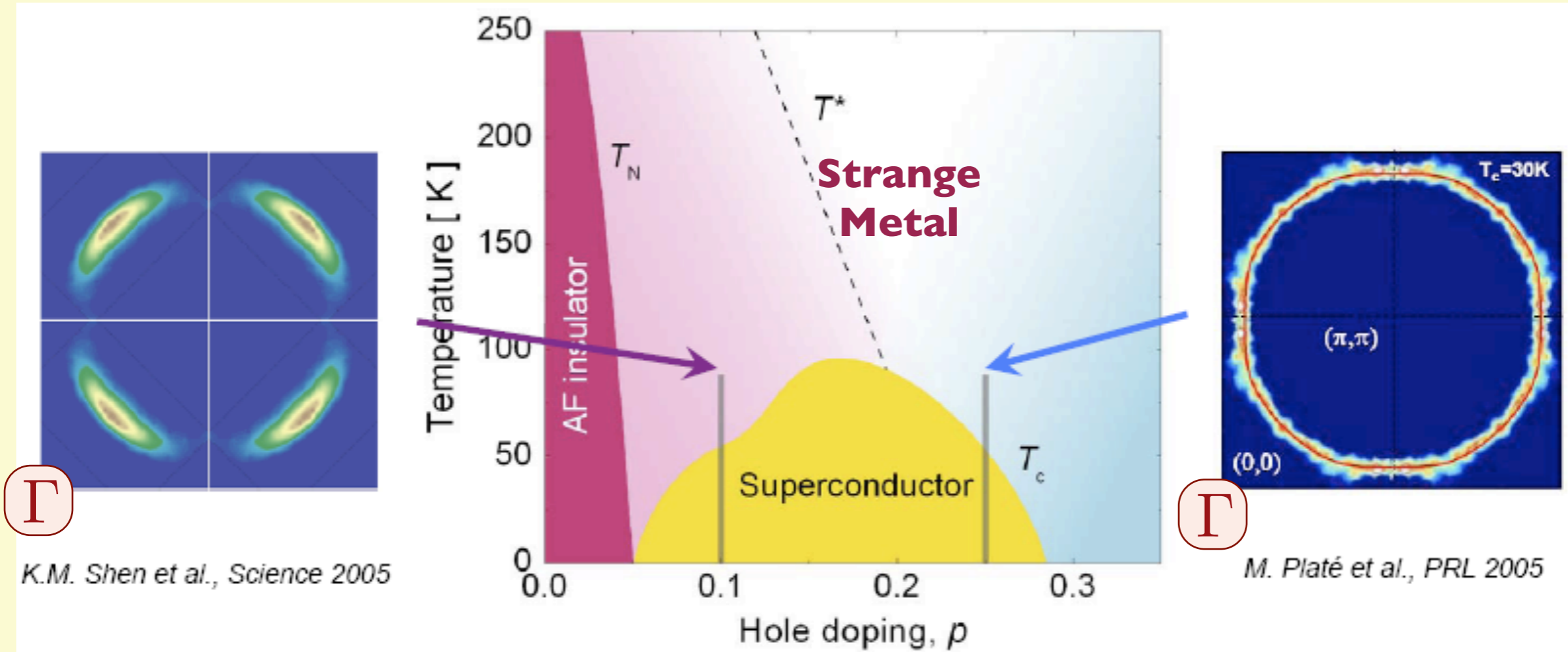
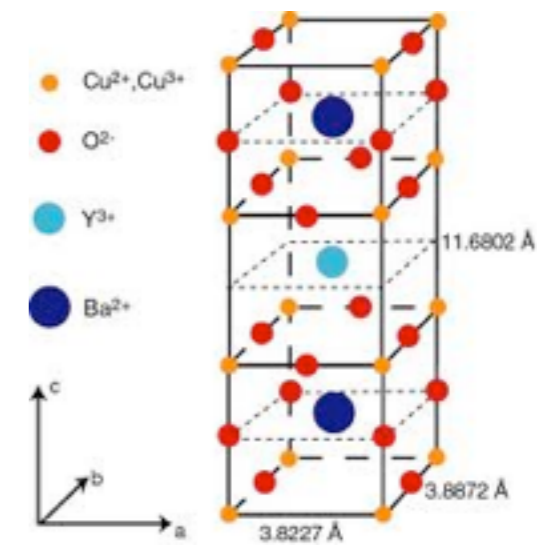
*Entanglement and
non-quasiparticle dynamics*

2. Quantum critical point in a metal

The iron-based superconductors

3. The pseudogap regime of the hole-doped cuprate superconductors

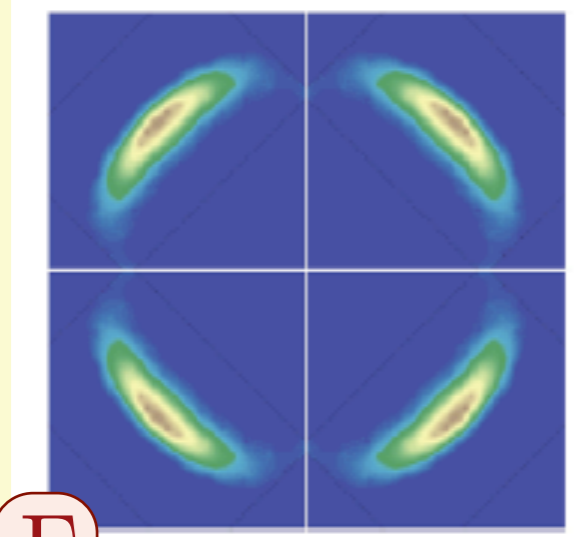
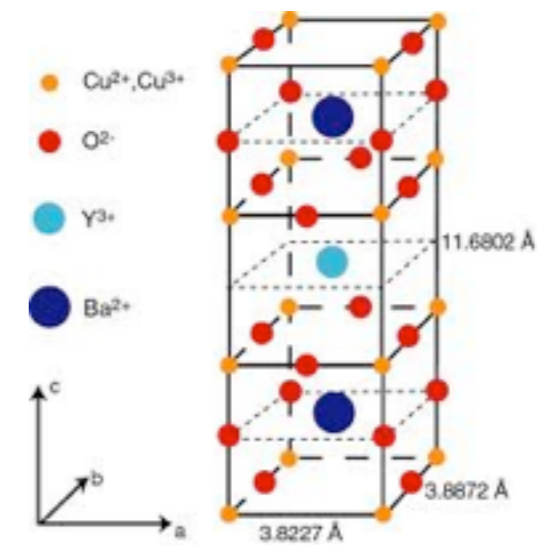
*Angular fluctuations of a
multicomponent order*



Smaller hole Fermi-pockets

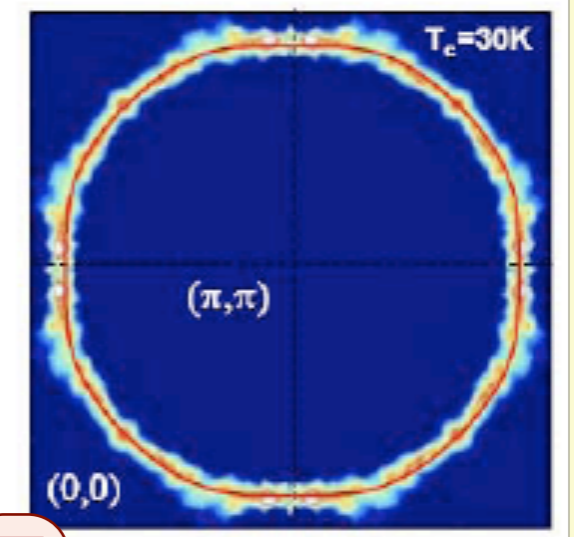
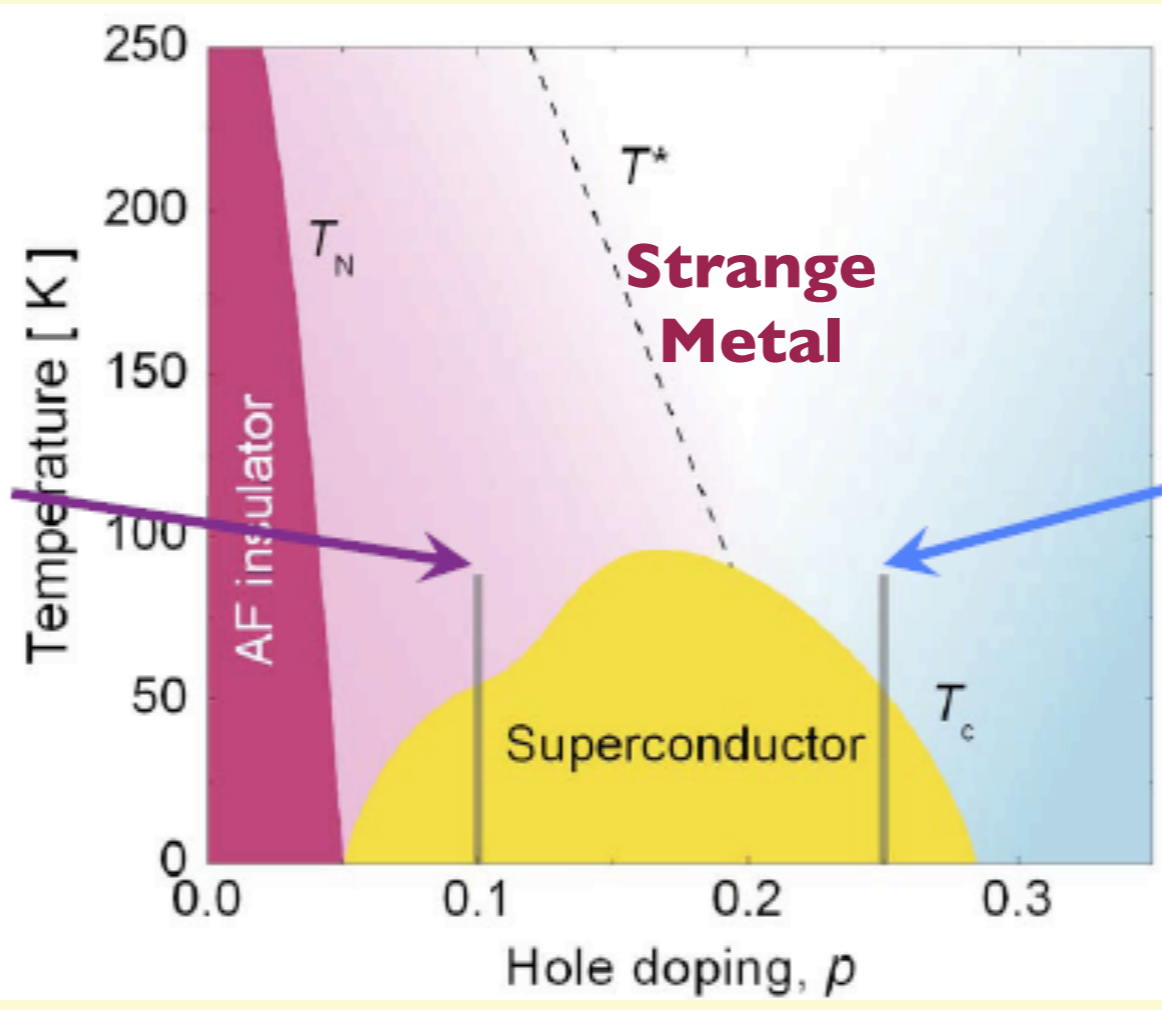
Large hole Fermi surface

What about the pseudogap ?



K.M. Shen et al., Science 2005

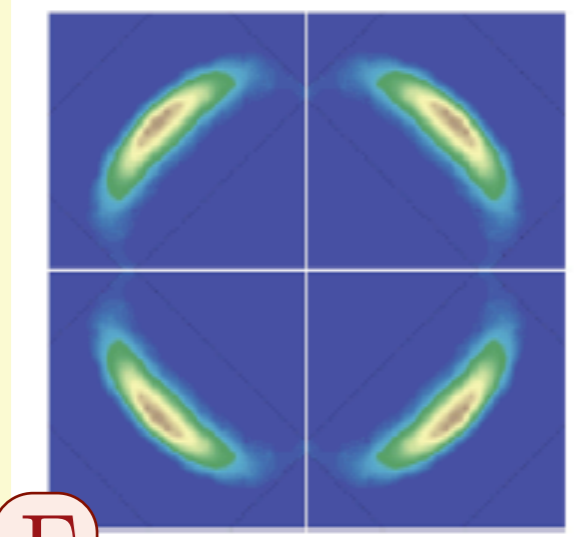
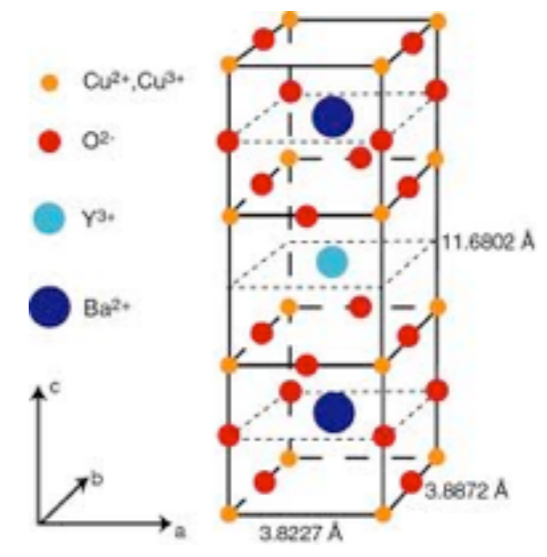
Smaller hole Fermi-pockets



M. Platé et al., PRL 2005

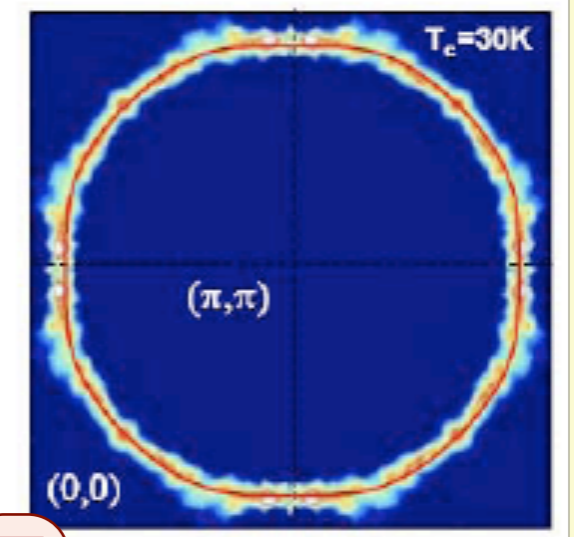
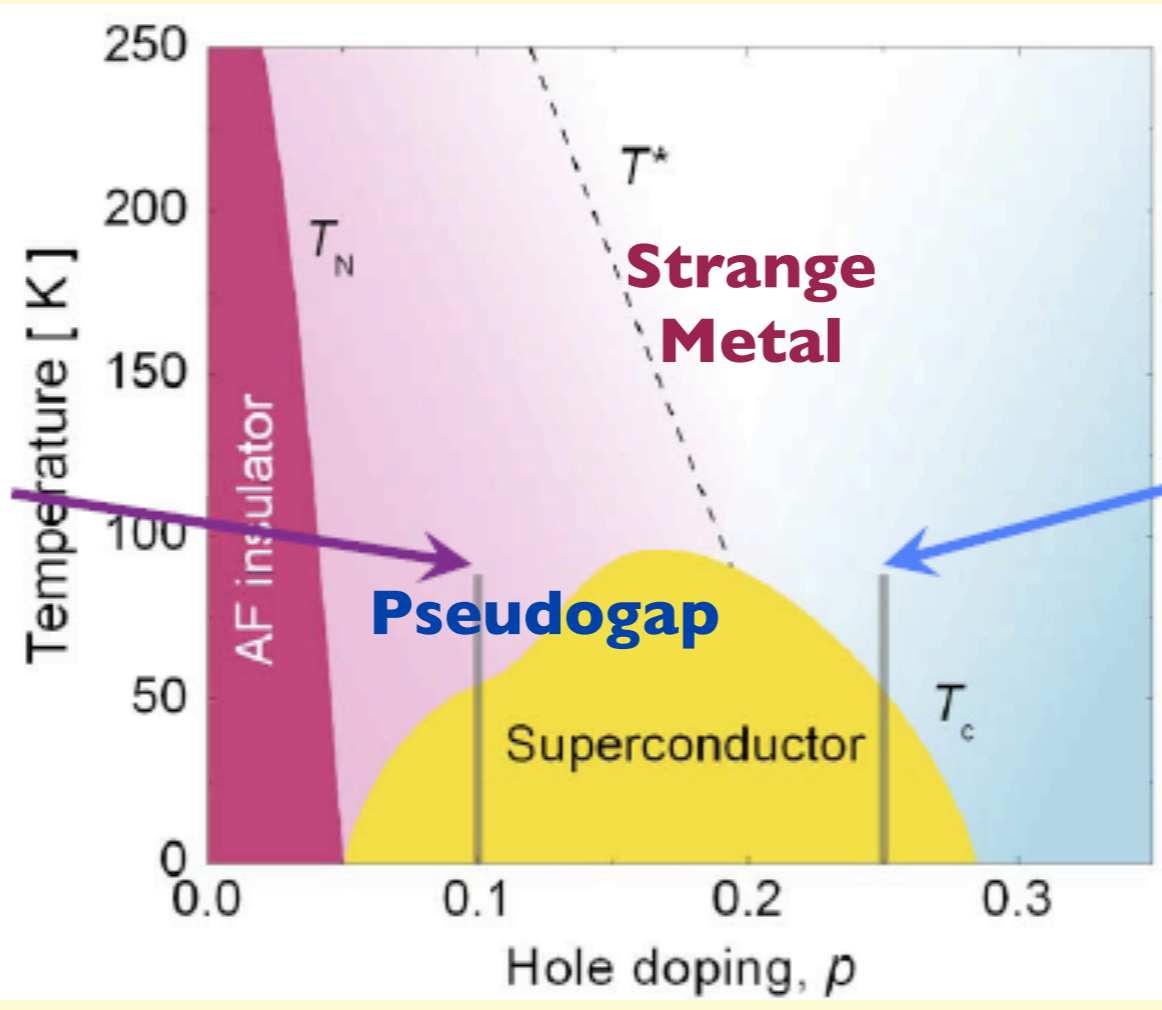
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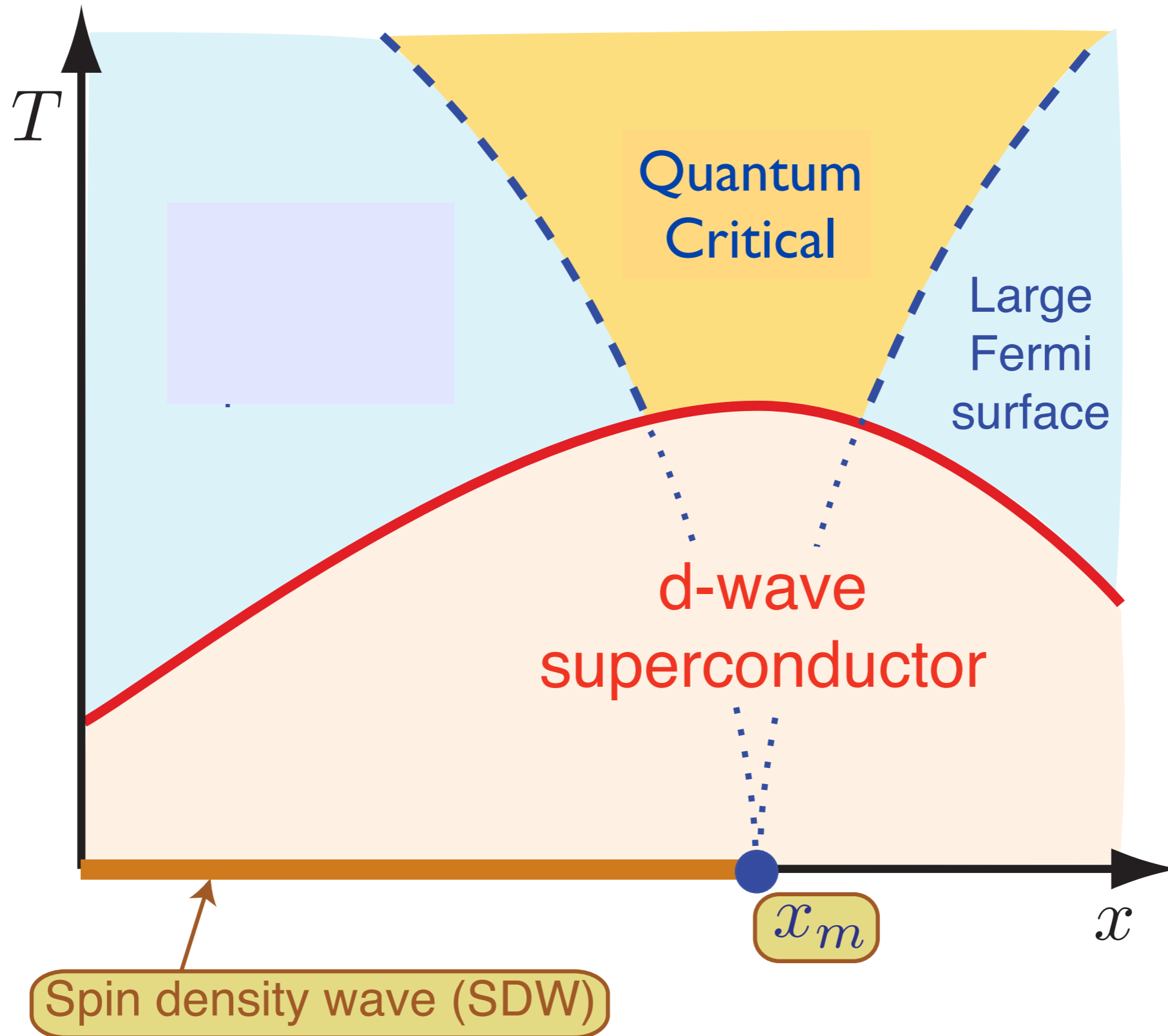
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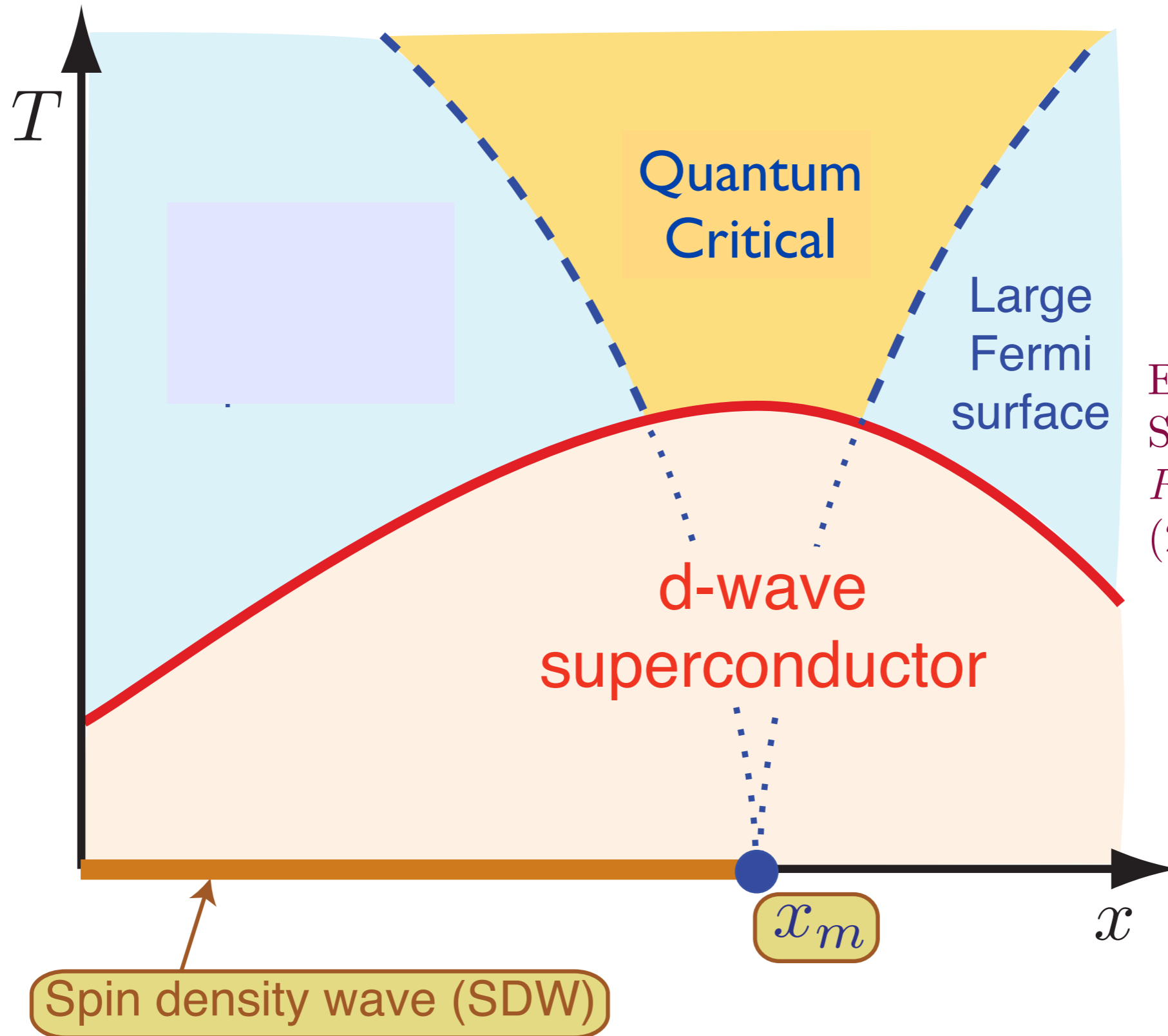
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QCP for the onset of SDW order is actually within a superconductor

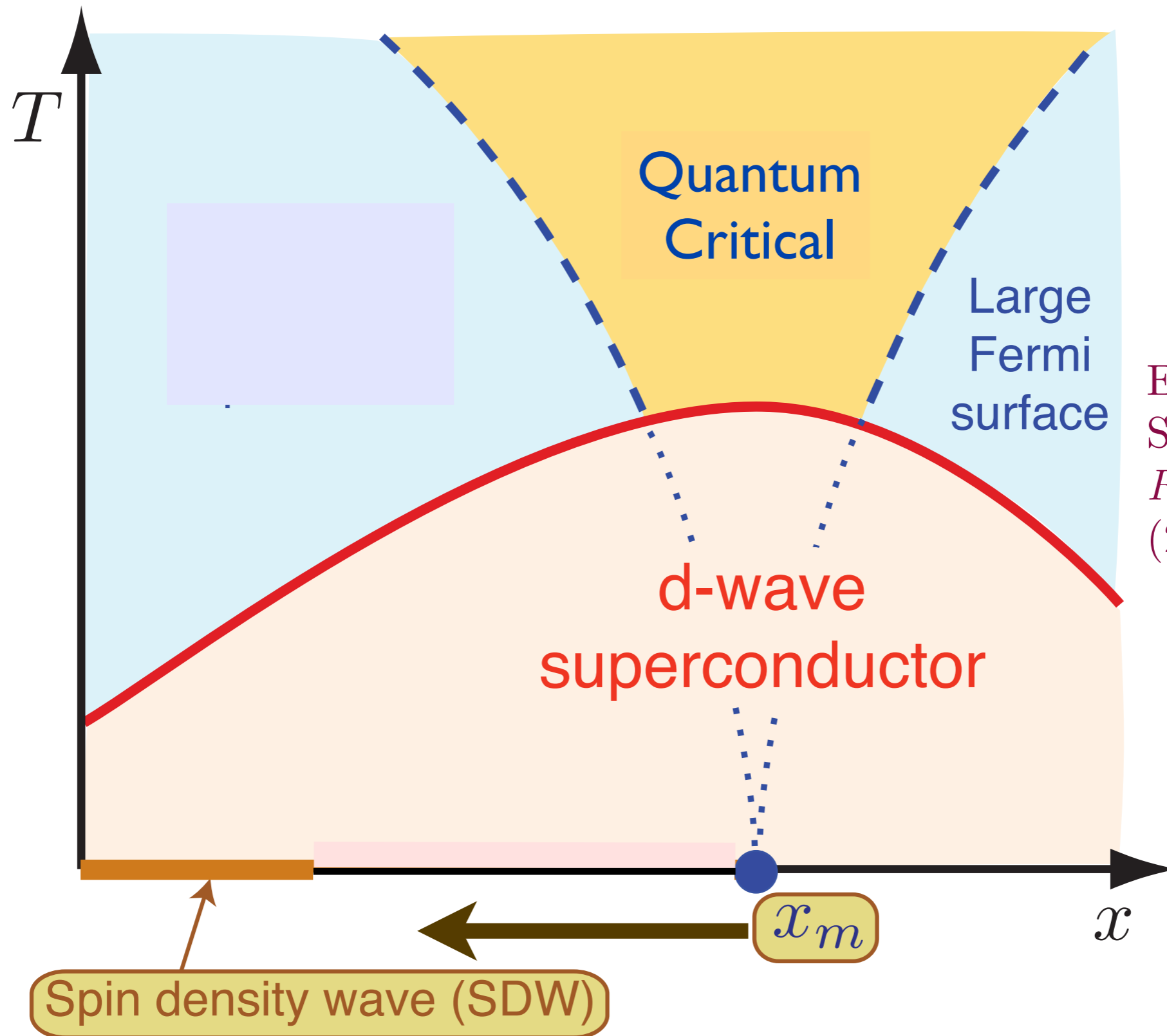
Theory of quantum criticality in the cuprates



E. G. Moon and
S. Sachdev, *Phy.
Rev. B* **80**, 035117
(2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

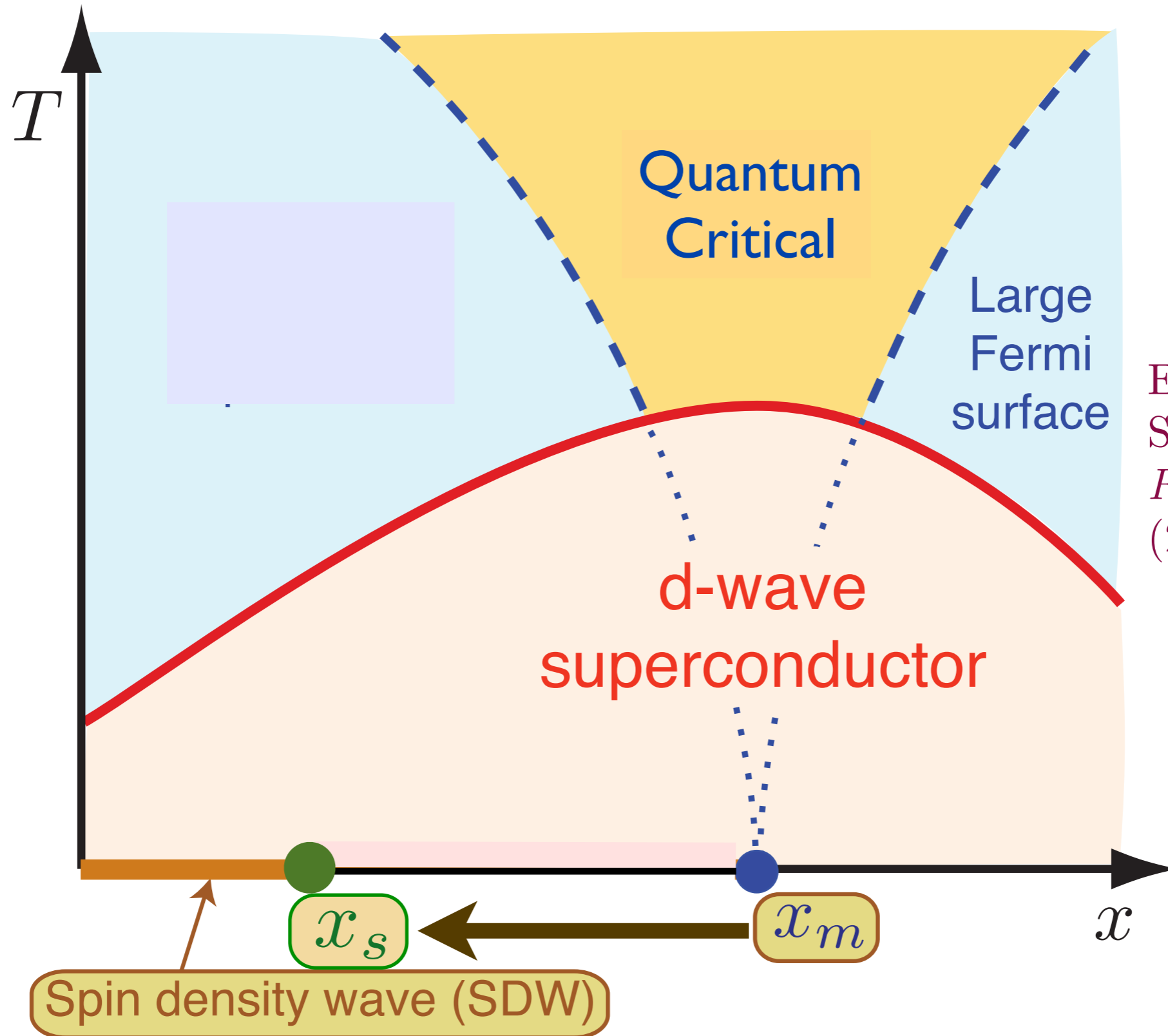
Theory of quantum criticality in the cuprates



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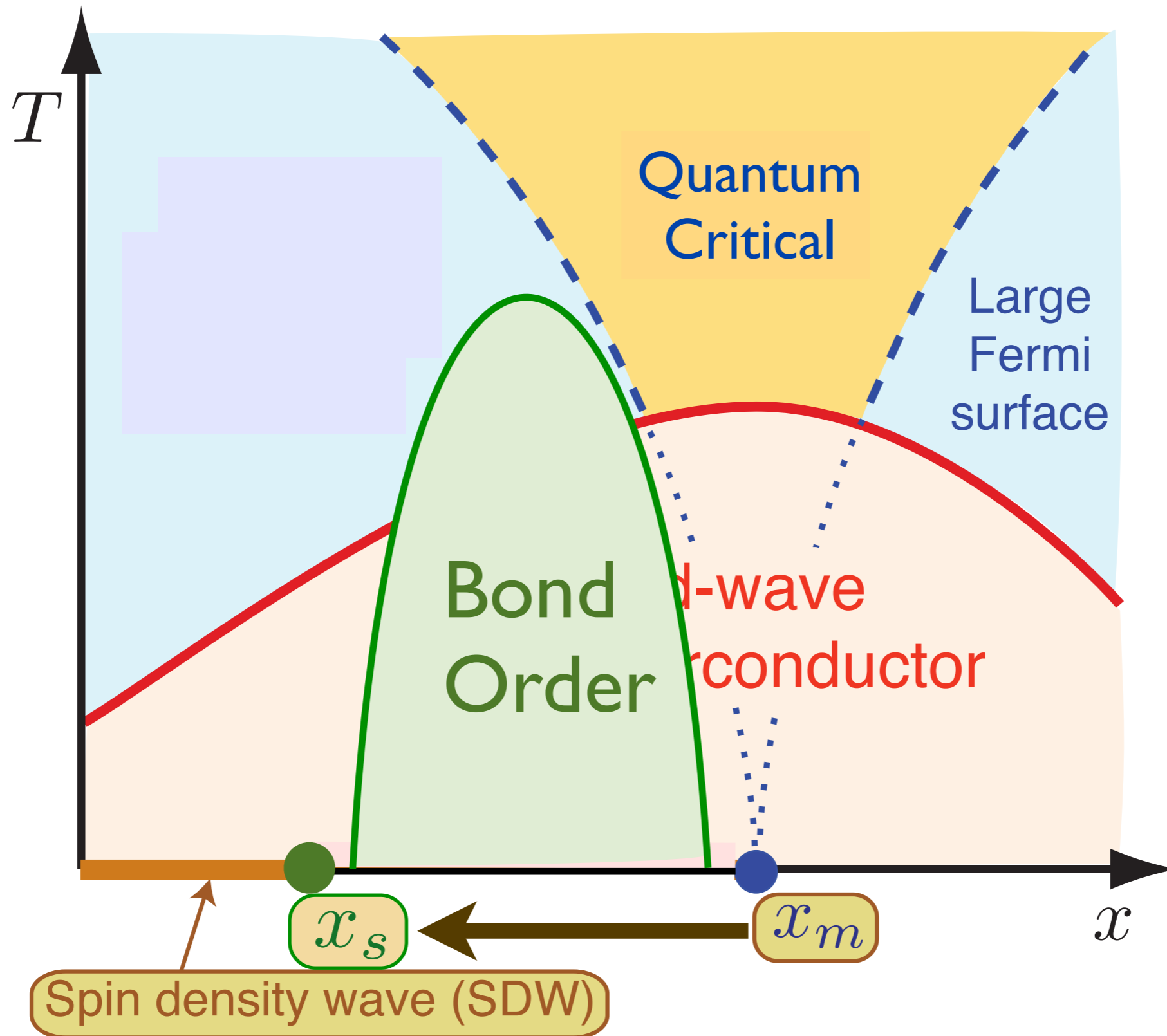
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Theory of quantum criticality in the cuprates



M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

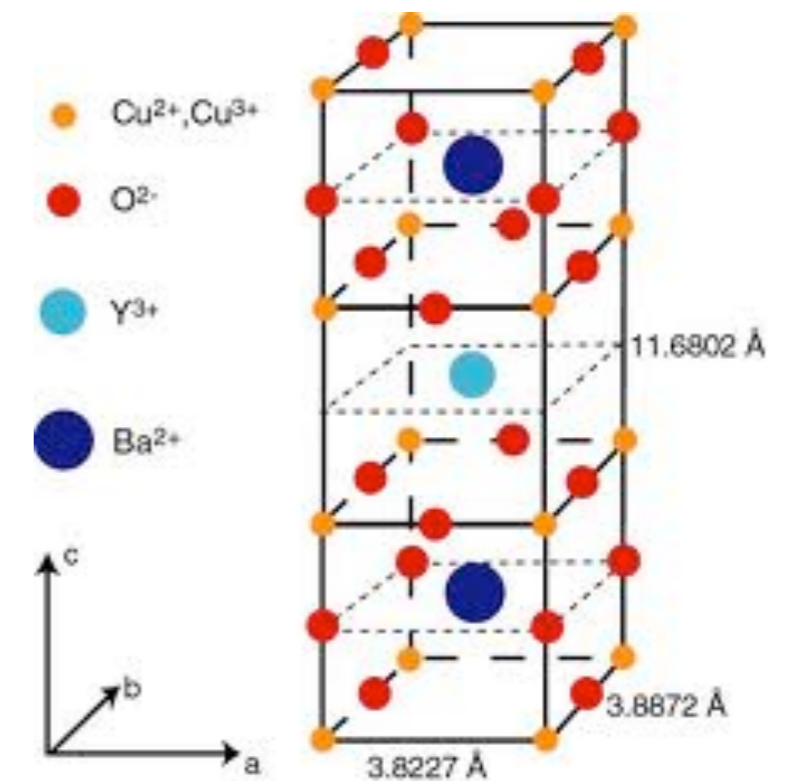
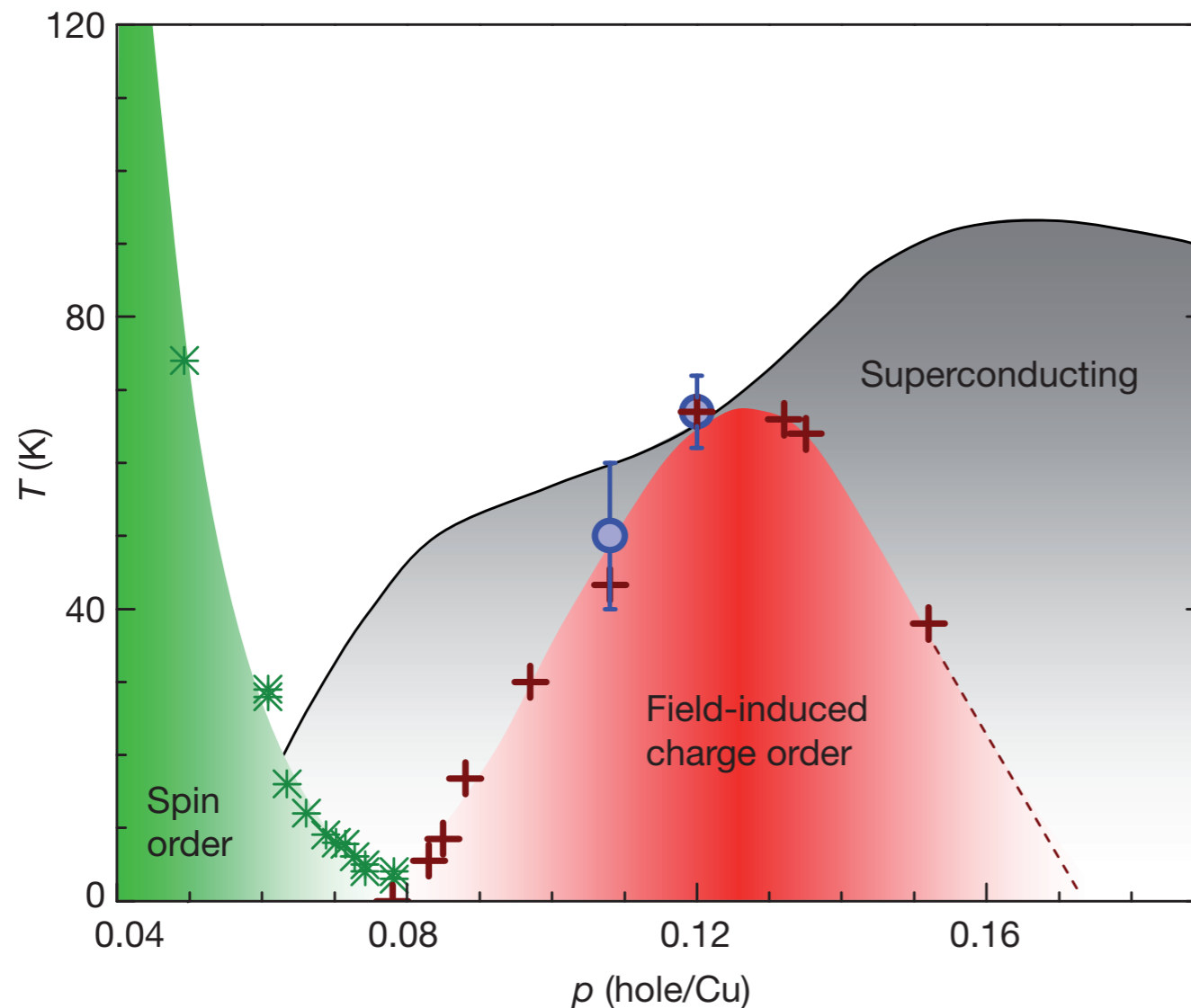
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

The metal has an instability to *both* d -wave superconductivity and a d -wave charge density wave (bond order).

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$

Tao Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Mladen Horvatić¹, Claude Berthier¹, W. N. Hardy^{2,3}, Ruixing Liang^{2,3}, D. A. Bonn^{2,3} & Marc-Henri Julien¹

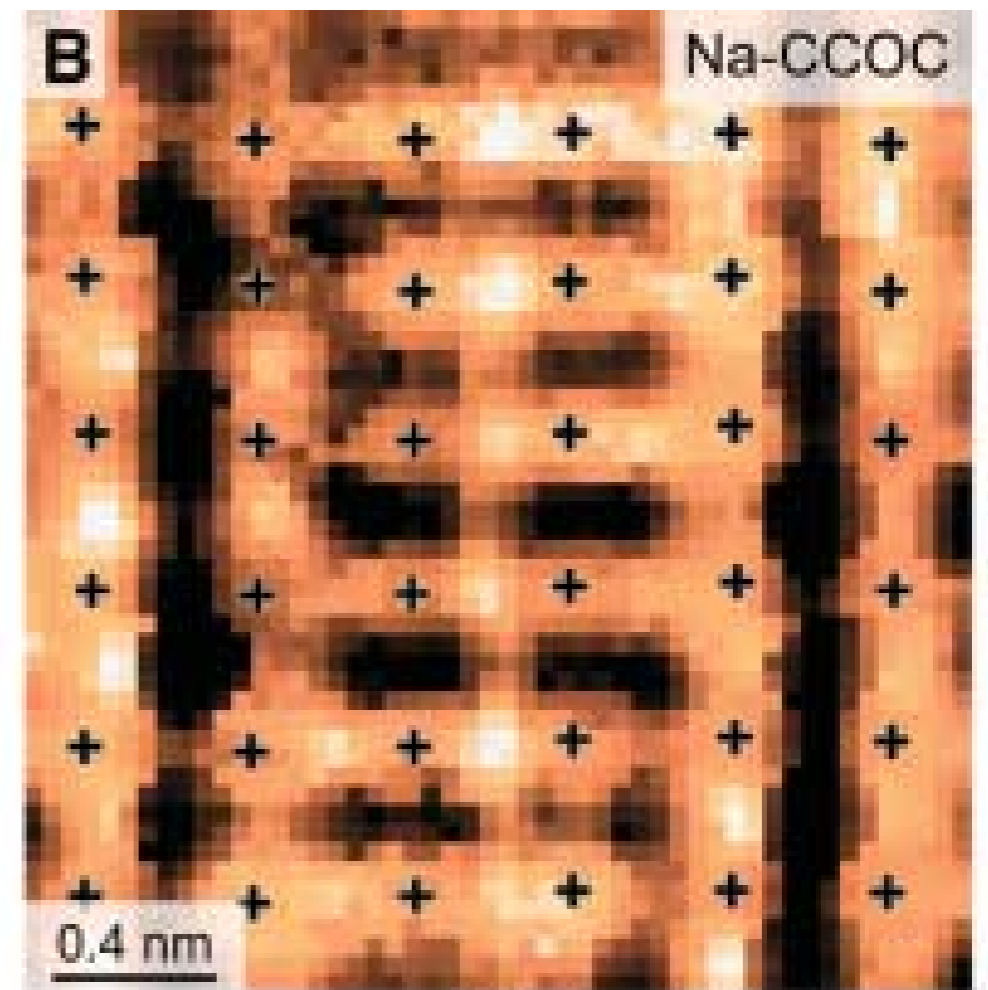
8 SEPTEMBER 2011 | VOL 477 | NATURE | 191



An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,¹ C. Taylor,¹ K. Fujita,^{1,2} A. Schmidt,¹ C. Lupien,³ T. Hanaguri,⁴ M. Azuma,⁵ M. Takano,⁵ H. Eisaki,⁶ H. Takagi,^{2,4} S. Uchida,^{2,7} J. C. Davis^{1,8*}

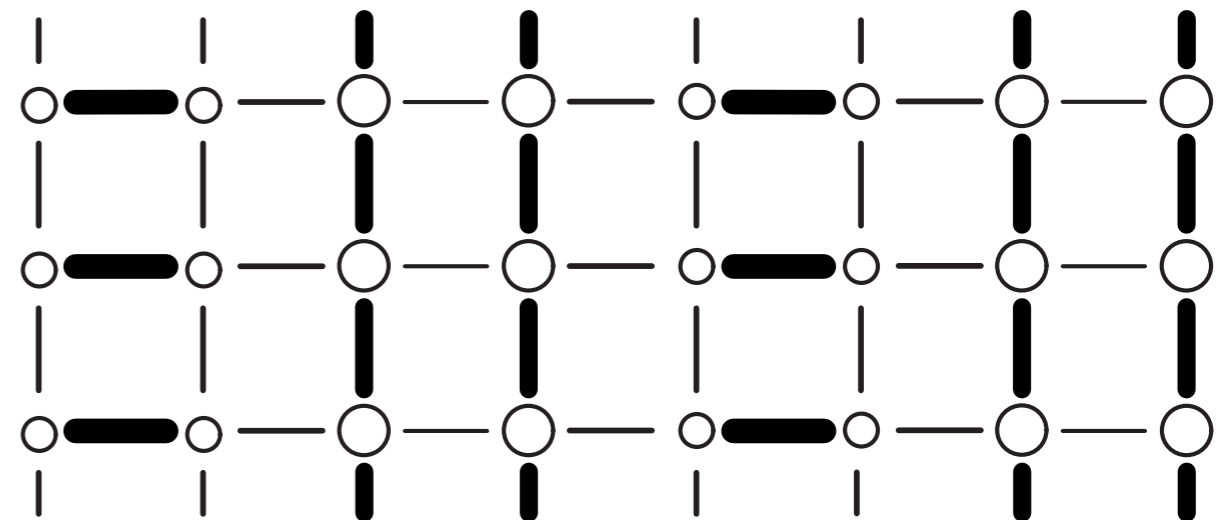
9 MARCH 2007 VOL 315 SCIENCE



PHYSICAL REVIEW B 77, 094504 (2008)

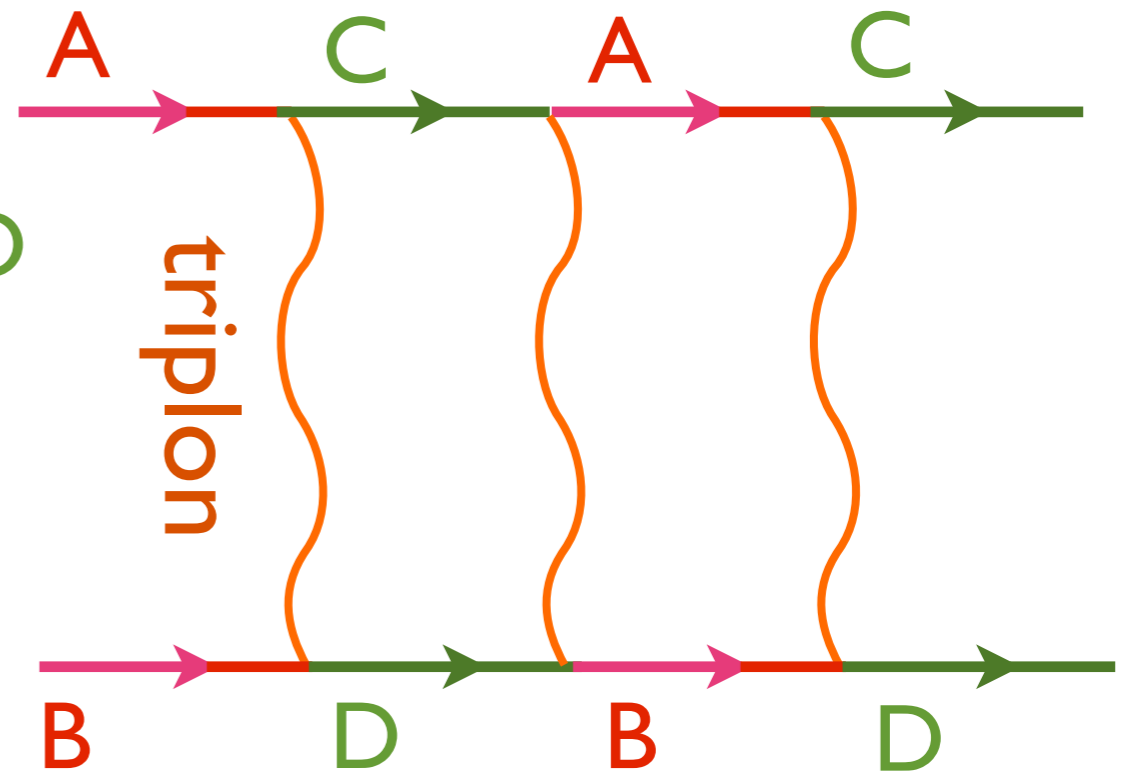
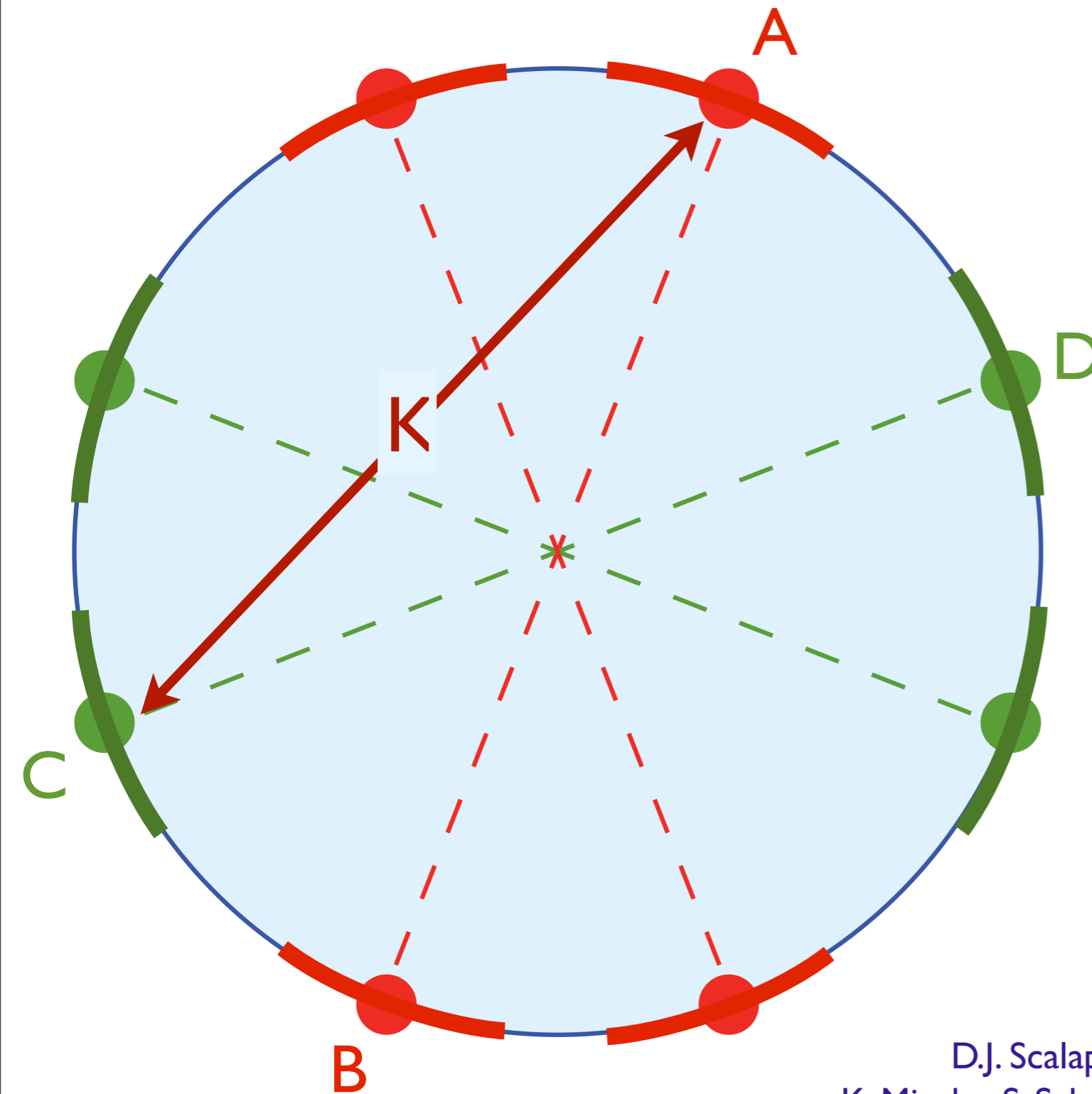
Superconducting *d*-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

Matthias Vojta and Oliver Rösch



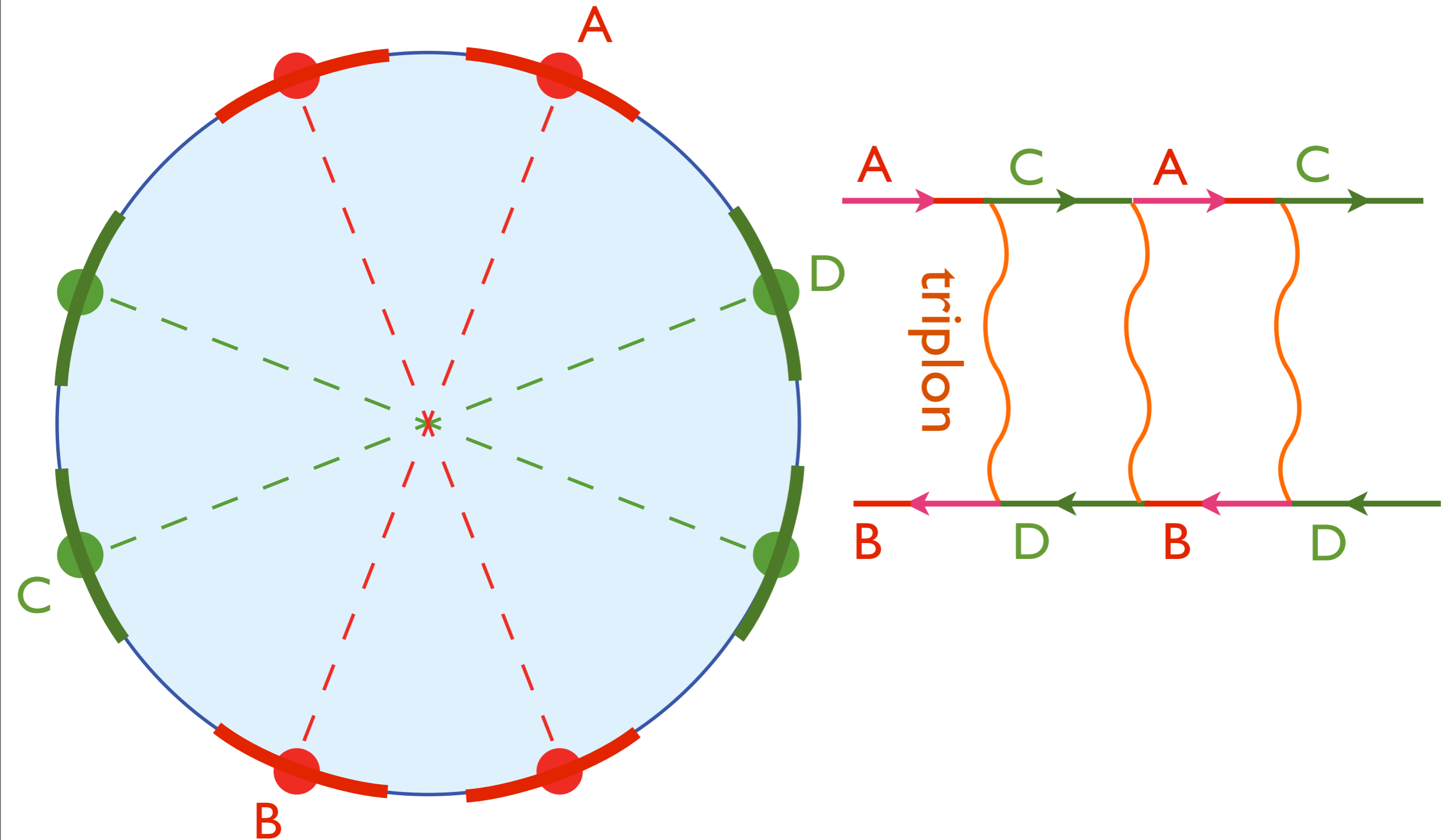
We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both *s*- and *d*-wave channels, with nonlocal Coulomb repulsion suppressing the *s*-wave component.

Pairing “glue” from triplon (paramagnon) exchange



- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
Ar. Abanov, A.V. Chubukov, and A.M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Same “glue” leads to bond/charge order !

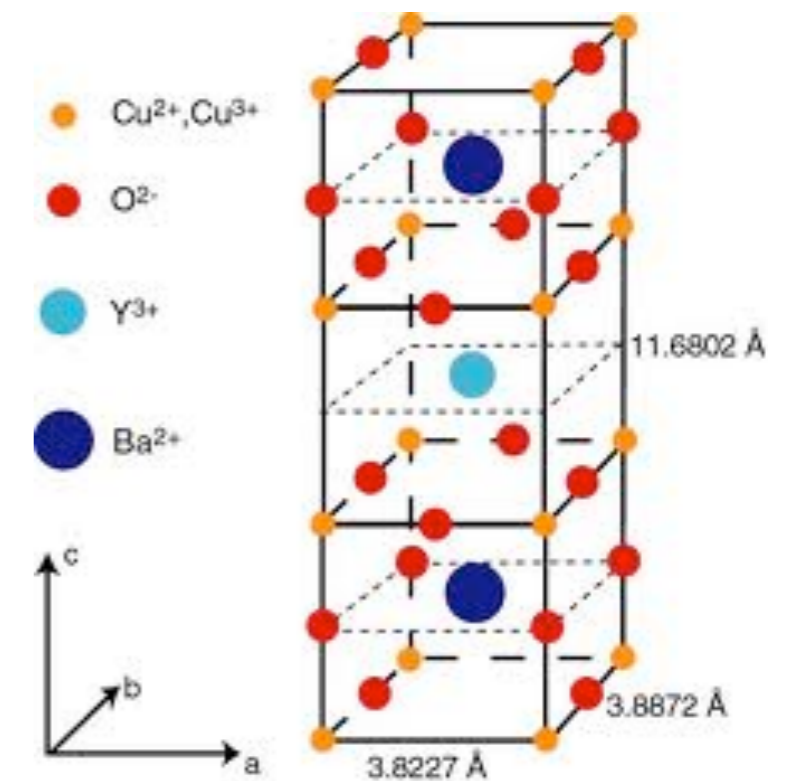
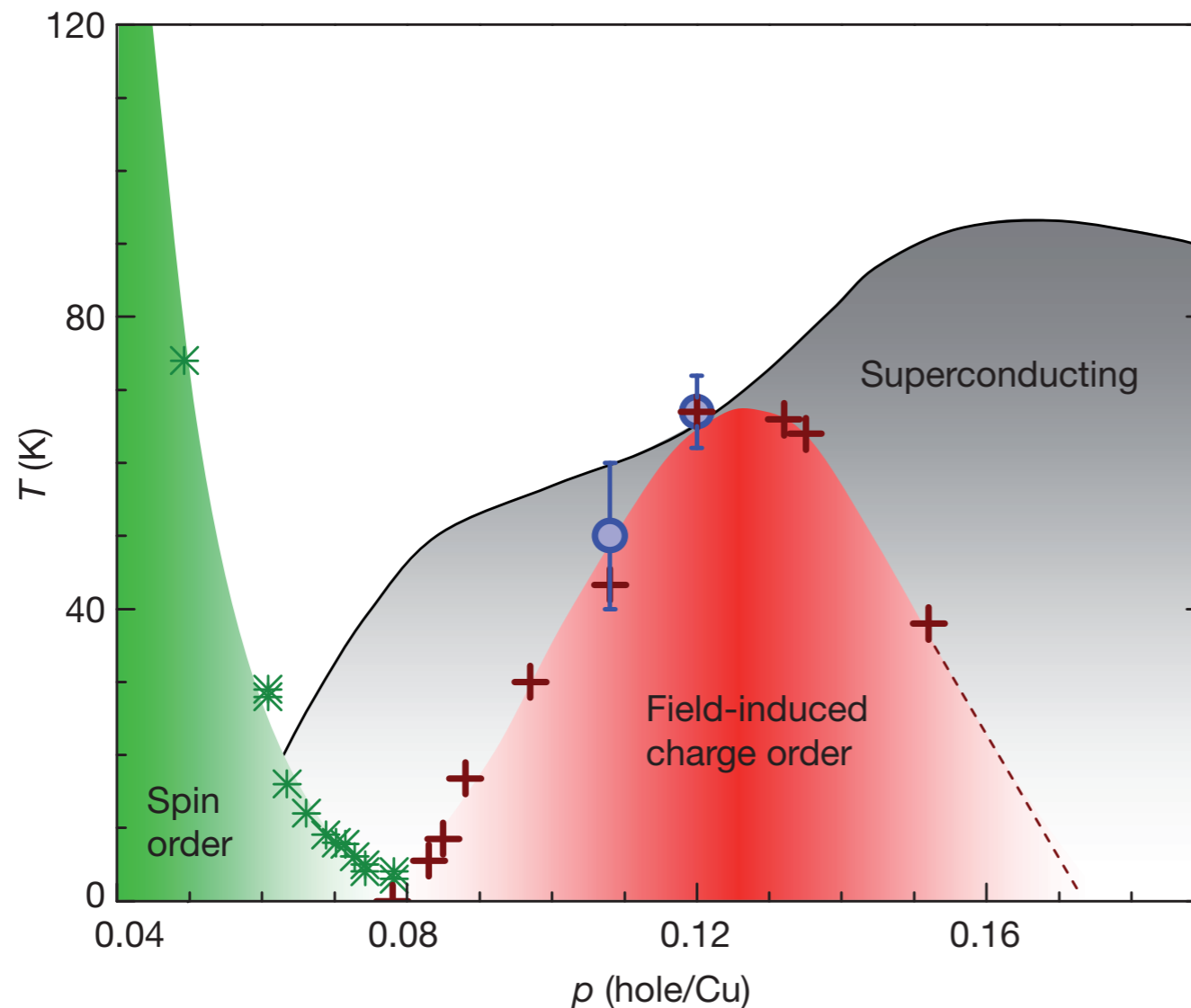


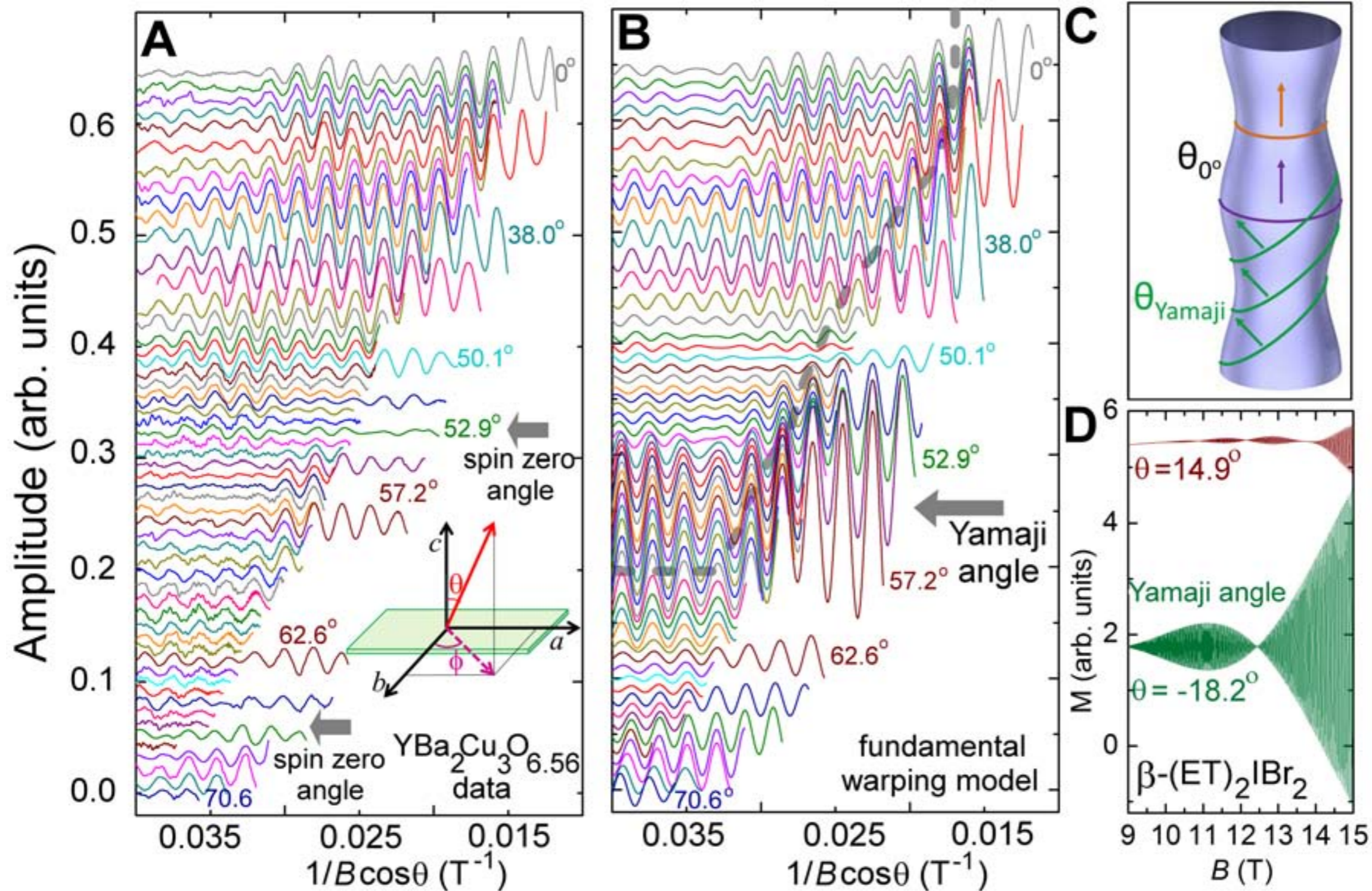
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$

Tao Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Mladen Horvatić¹, Claude Berthier¹, W. N. Hardy^{2,3}, Ruixing Liang^{2,3}, D. A. Bonn^{2,3} & Marc-Henri Julien¹

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





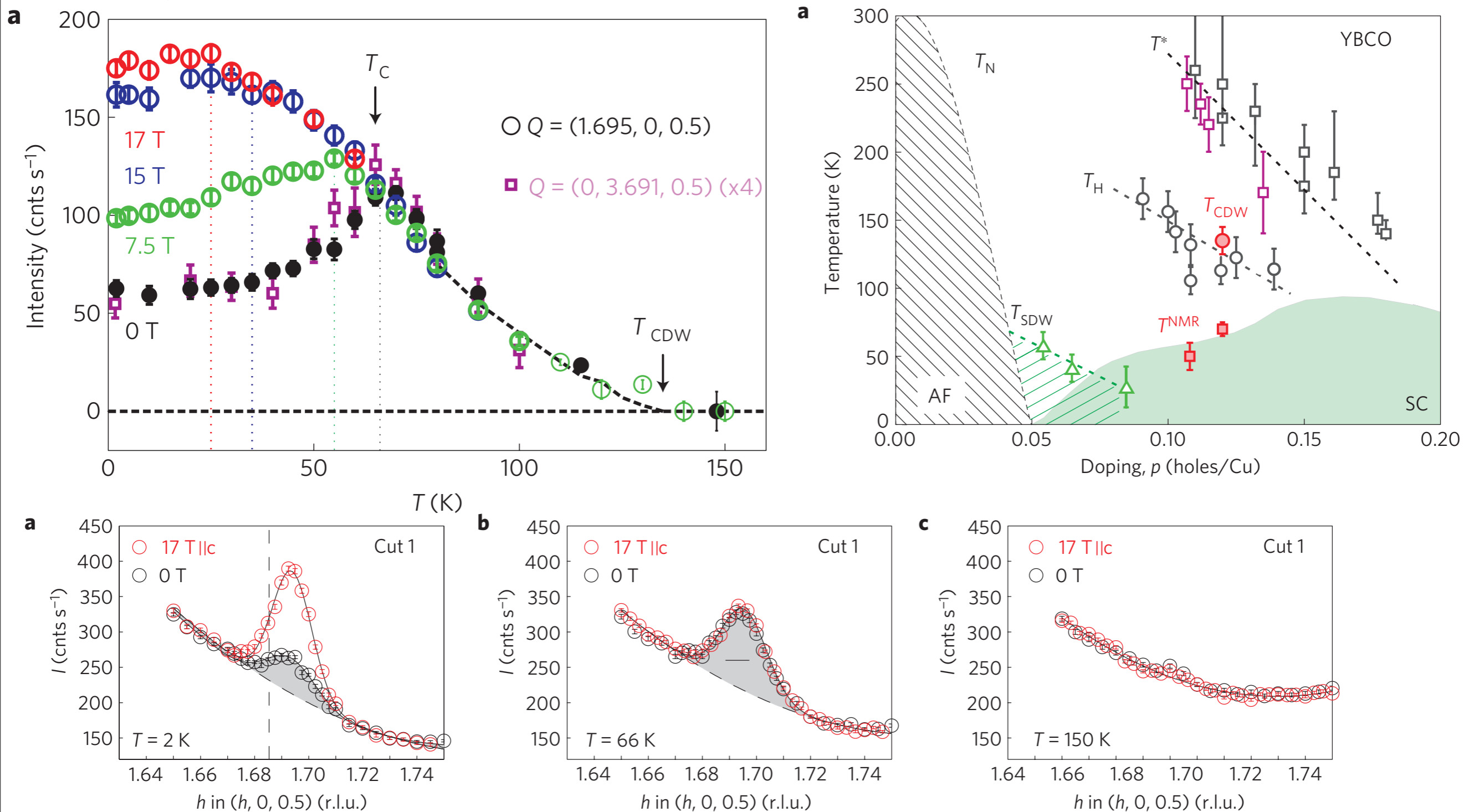
Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

APS March meeting 2013
B2.00004

Suchitra E. Sebastian,^{1*} N. Harrison,² F. F. Balakirev,² M. M. Altarawneh,^{2,3}
Ruixing Liang,^{4,5} D. A. Bonn,^{4,5} W. N. Hardy,^{4,5} G. G. Lonzarich,¹

Direct observation of competition between superconductivity and charge density wave order in $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$

J. Chang^{1,2*}, E. Blackburn³, A. T. Holmes³, N. B. Christensen⁴, J. Larsen^{4,5}, J. Mesot^{1,2}, Ruixing Liang^{6,7}, D. A. Bonn^{6,7}, W. N. Hardy^{6,7}, A. Watenphul⁸, M. v. Zimmermann⁸, E. M. Forgan³ and S. M. Hayden⁹



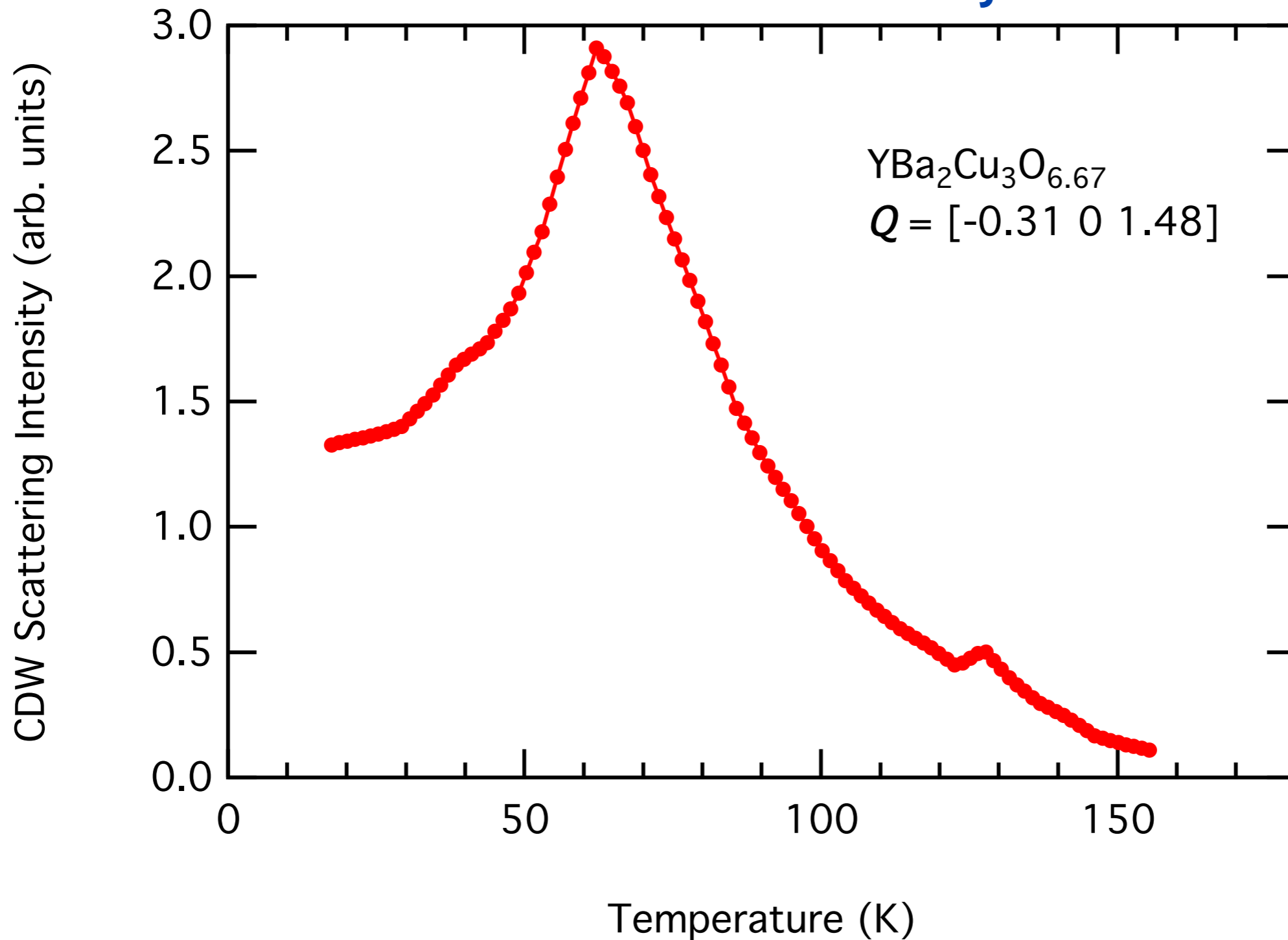


FIG. 1: The temperature dependence of the CDW scattering intensity at $\mathbf{Q} = [-0.31 \ 0 \ 1.48]$ in YBa₂Cu₃O_{6.67} measured by resonant x-ray scattering in Ref. [4]. This sample has $T_c \approx 65.5\text{K}$.

Competing orders in thermally fluctuating superconductors in two dimensions

Subir Sachdev

Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120, USA

Eugene Demler

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 6 August 2003; revised manuscript received 24 November 2003; published 6 April 2004)

We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the pseudogap regime where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.

Competing orders in thermally fluctuating superconductors in two dimensions

Subir Sachdev

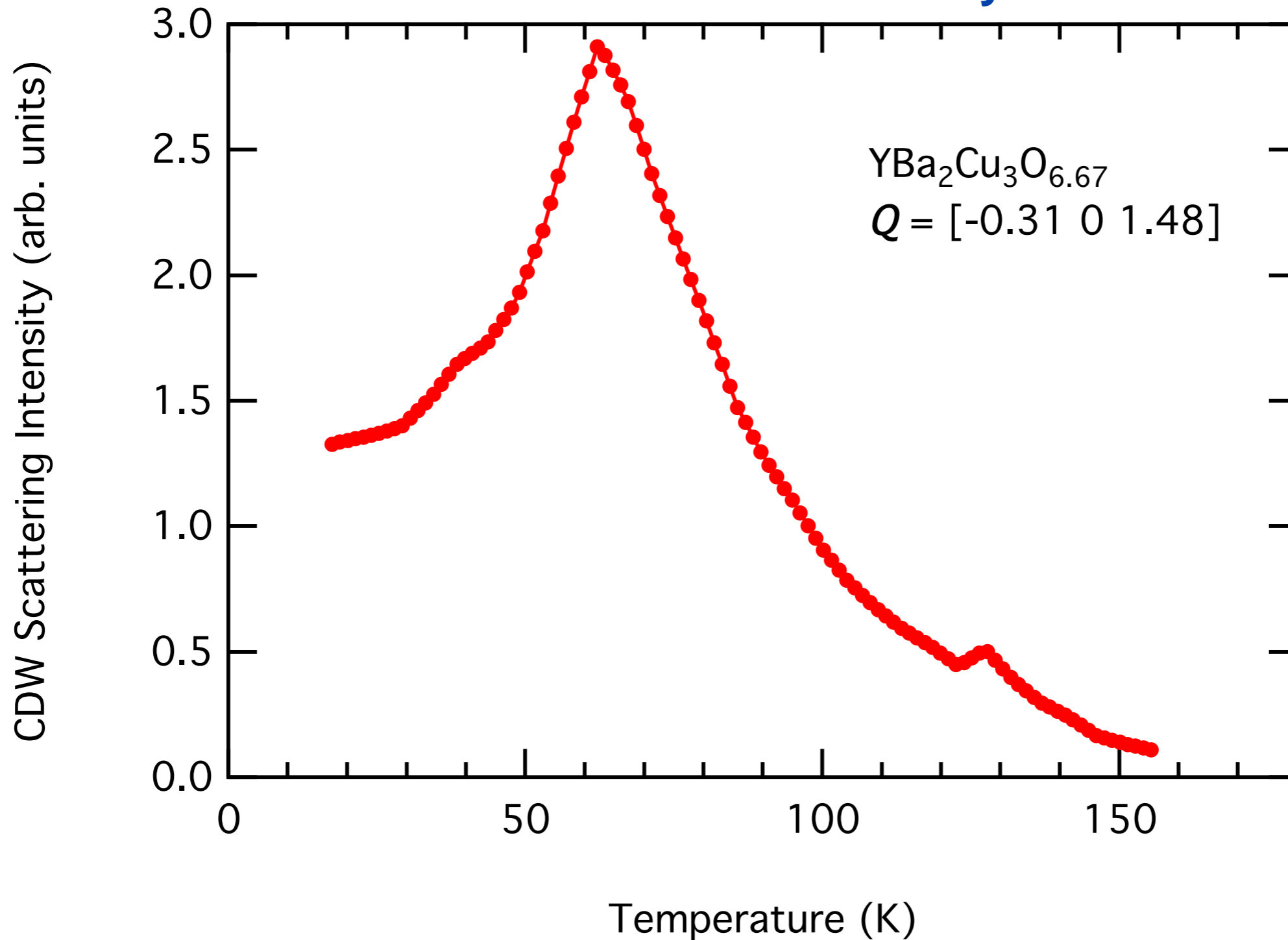
Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120, USA

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Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

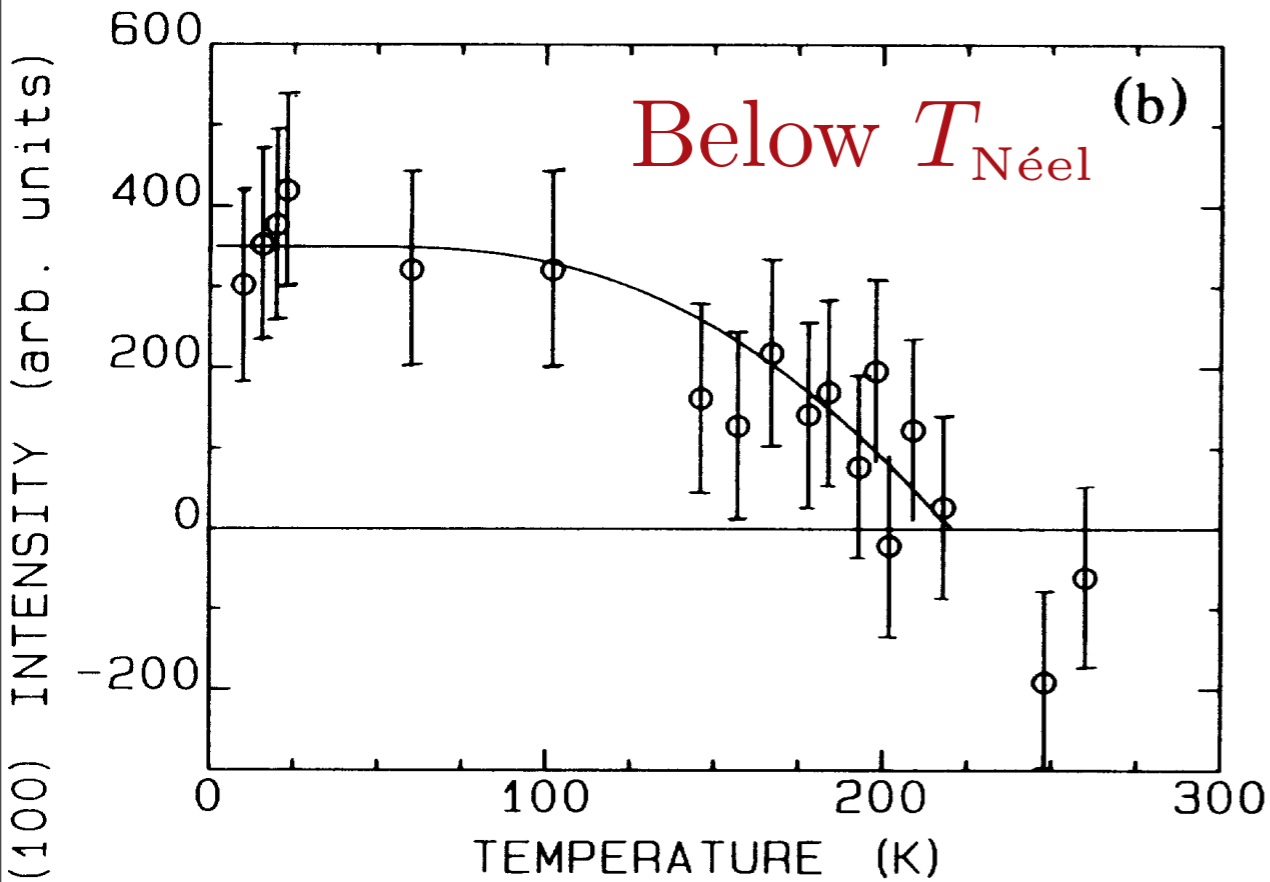
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Onset is unlike an arrested ordering transition,
or precursor critical fluctuations

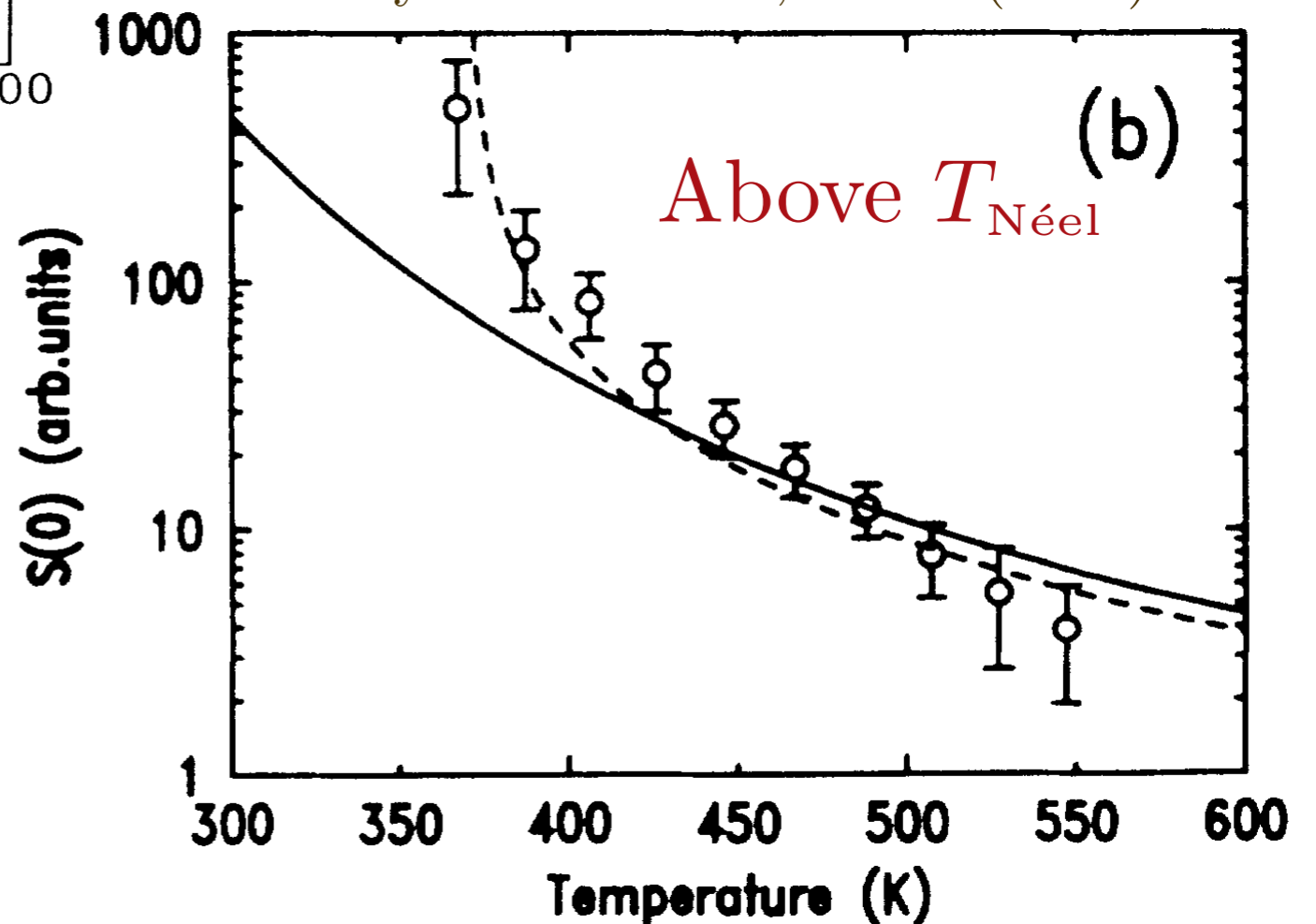
Key idea: analogy with the onset of antiferromagnetism in the insulator La_2CuO_4



D. Vaknin *et al.*,
Phys. Rev. Lett. **58**, 2802 (1987).

$$T_{Néel} = 325K$$

B. Keimer *et al.*,
Phys. Rev. B **46**, 14034 (1992).



Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4

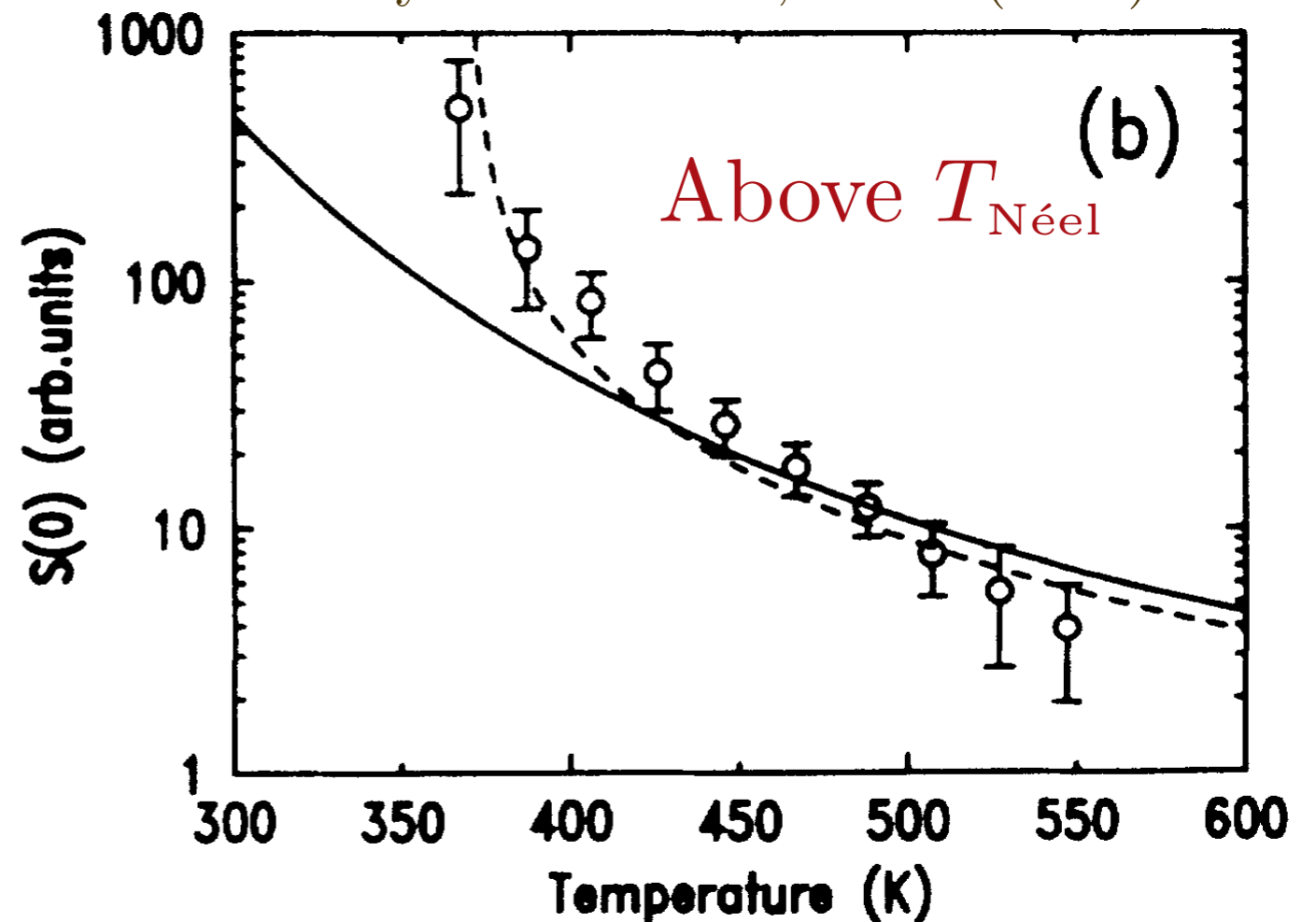
Gradual onset of intensity over a wide range of T is a consequence of angular thermal fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975

Chakravarty, Halperin, Nelson 1989

$$T_{\text{Néel}} = 325\text{K}$$

B. Keimer *et al.*,
Phys. Rev. B **46**, 14034 (1992).



Multi-component order parameter for the pseudogap

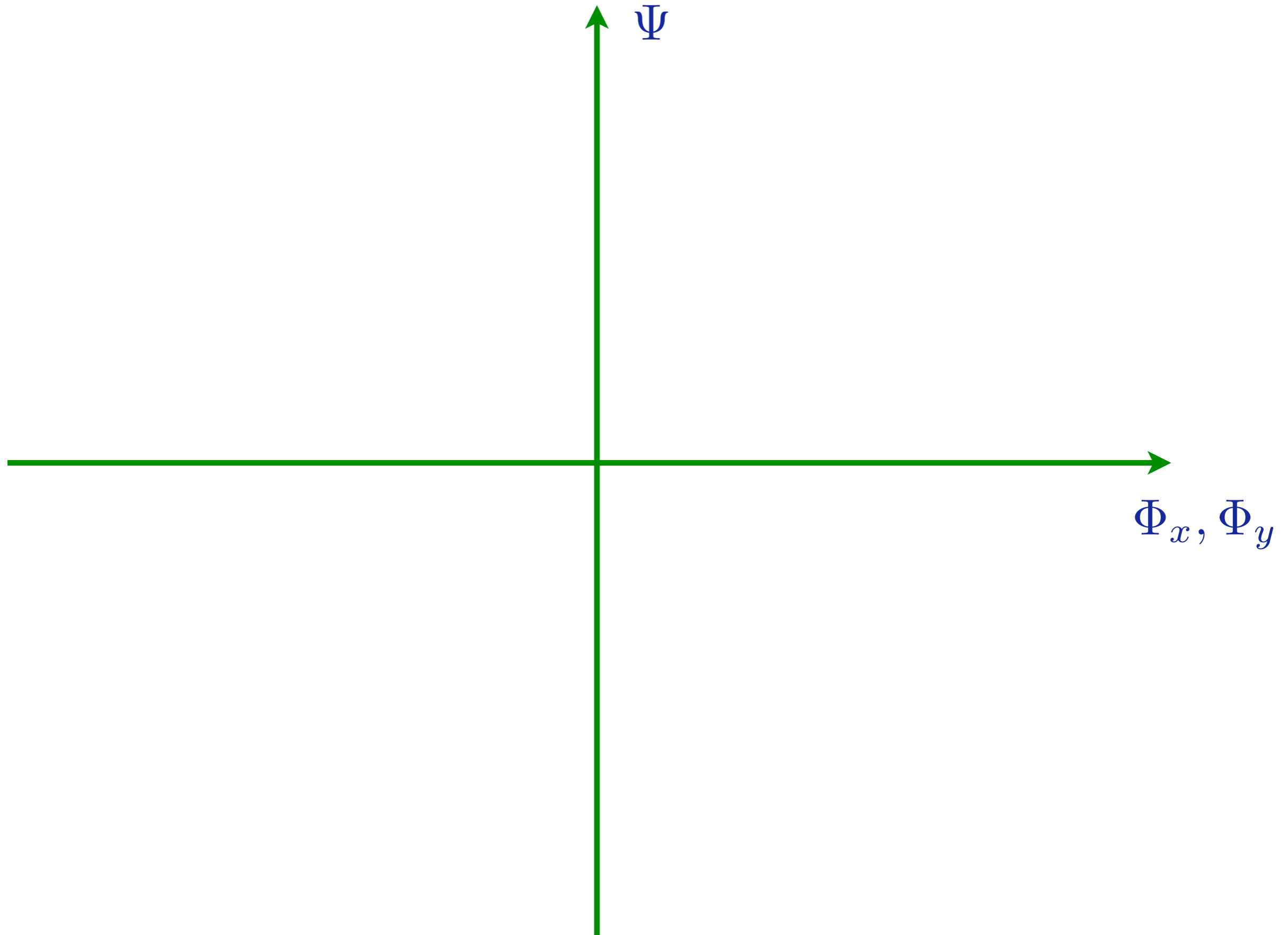
Superconducting order $\Psi(\mathbf{r})$:

$$\langle c_{i\alpha}^\dagger c_{j\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi((\mathbf{r}_i + \mathbf{r}_j)/2)$$

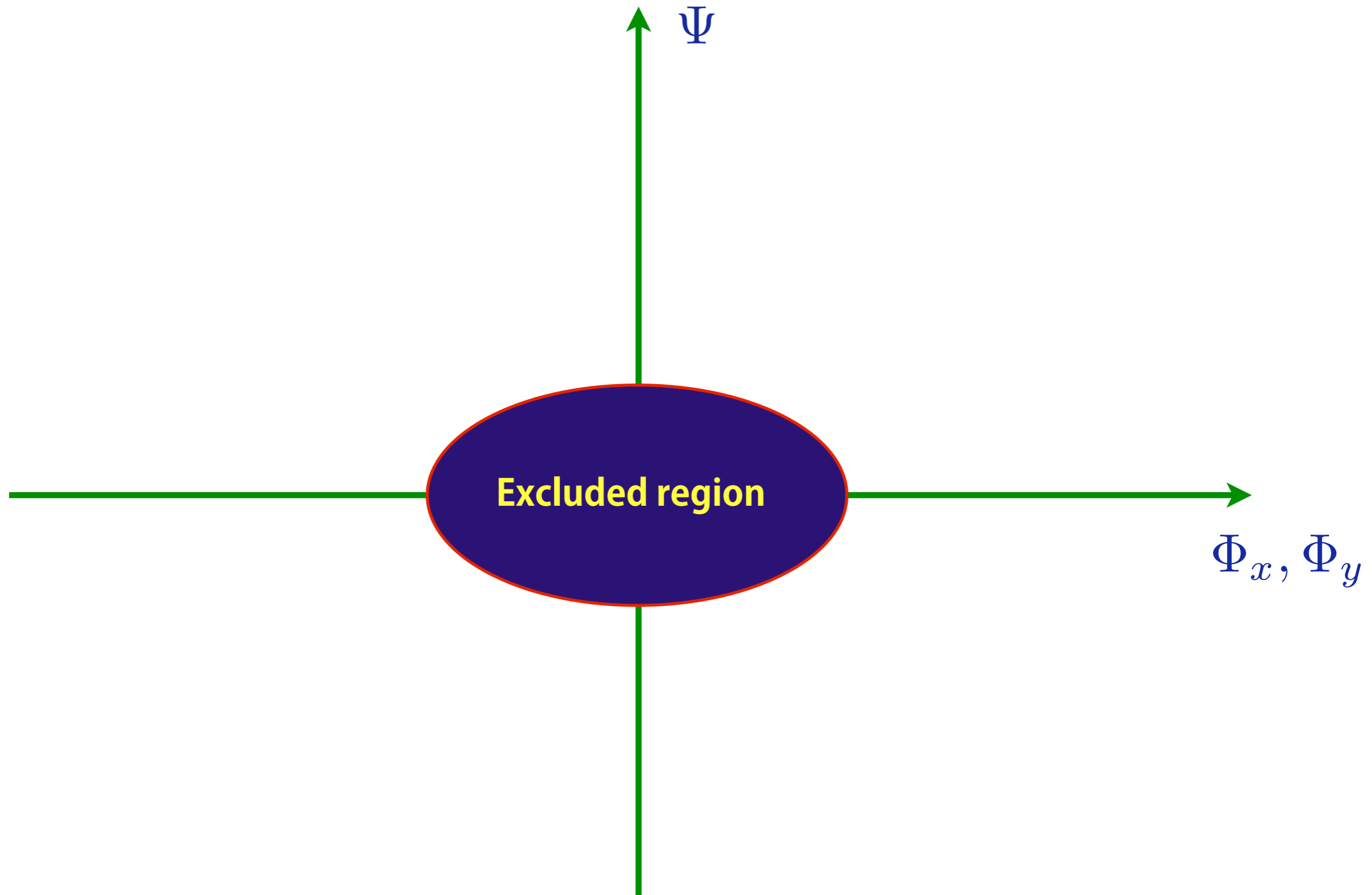
Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\begin{aligned} \langle c_{i\alpha}^\dagger c_{j\beta} \rangle &= \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x((\mathbf{r}_i + \mathbf{r}_j)/2) \\ &+ \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y((\mathbf{r}_i + \mathbf{r}_j)/2) \end{aligned}$$

Multi-component order parameter



Multi-component order parameter

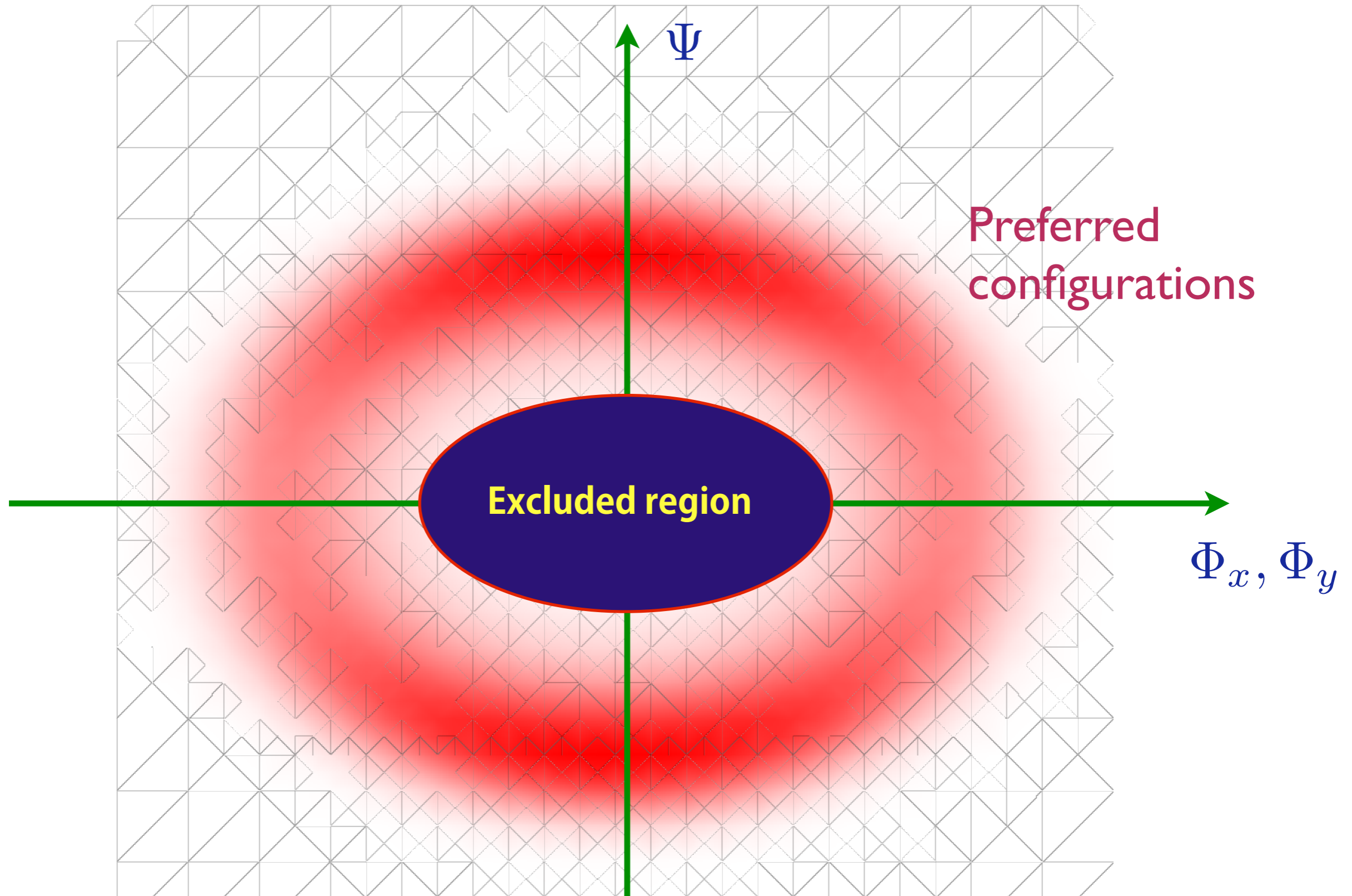


Support from theory of antiferromagnetic quantum criticality

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

Multi-component order parameter

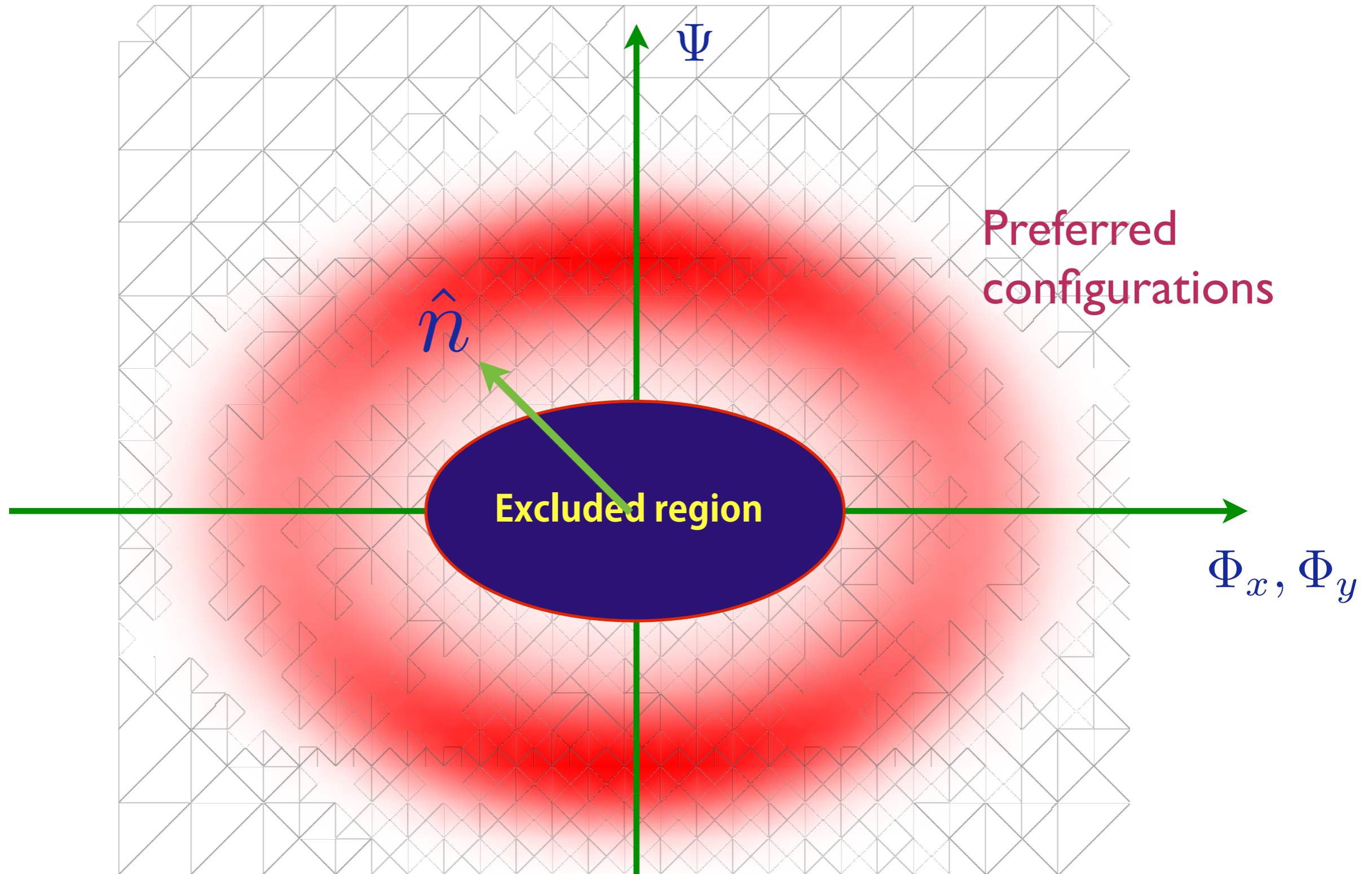


Support from theory of antiferromagnetic quantum criticality

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

Multi-component order parameter



Label order parameter by a 6-component unit vector n_α with $\sum_\alpha n_\alpha^2 = 1$

O(6) non-linear sigma model

$$\mathcal{Z} = \int \mathcal{D}n_\alpha(\mathbf{r}) \delta \left(\sum_{\alpha=1}^6 n_\alpha^2(\mathbf{r}) - 1 \right) \exp \left(- \frac{\rho_s}{2T} \int d^2r \left[\sum_{\alpha=1}^2 (\nabla n_\alpha)^2 + \lambda \sum_{\alpha=3}^6 (\nabla n_\alpha)^2 + g \sum_{\alpha=3}^6 n_\alpha^2 + w \left[(n_3^2 + n_4^2)^2 + (n_5^2 + n_6^2)^2 \right] \right] \right).$$

where $\Psi \propto n_1 + in_2$, $\Phi_x \propto n_3 + in_4$, $\Phi_y \propto n_5 + in_6$.

Describes $O(6) \Rightarrow O(2) \times O(2) \times O(2) \rtimes \mathbb{Z}_2$. The coupling g determines the anisotropy between superconductivity and charge order.

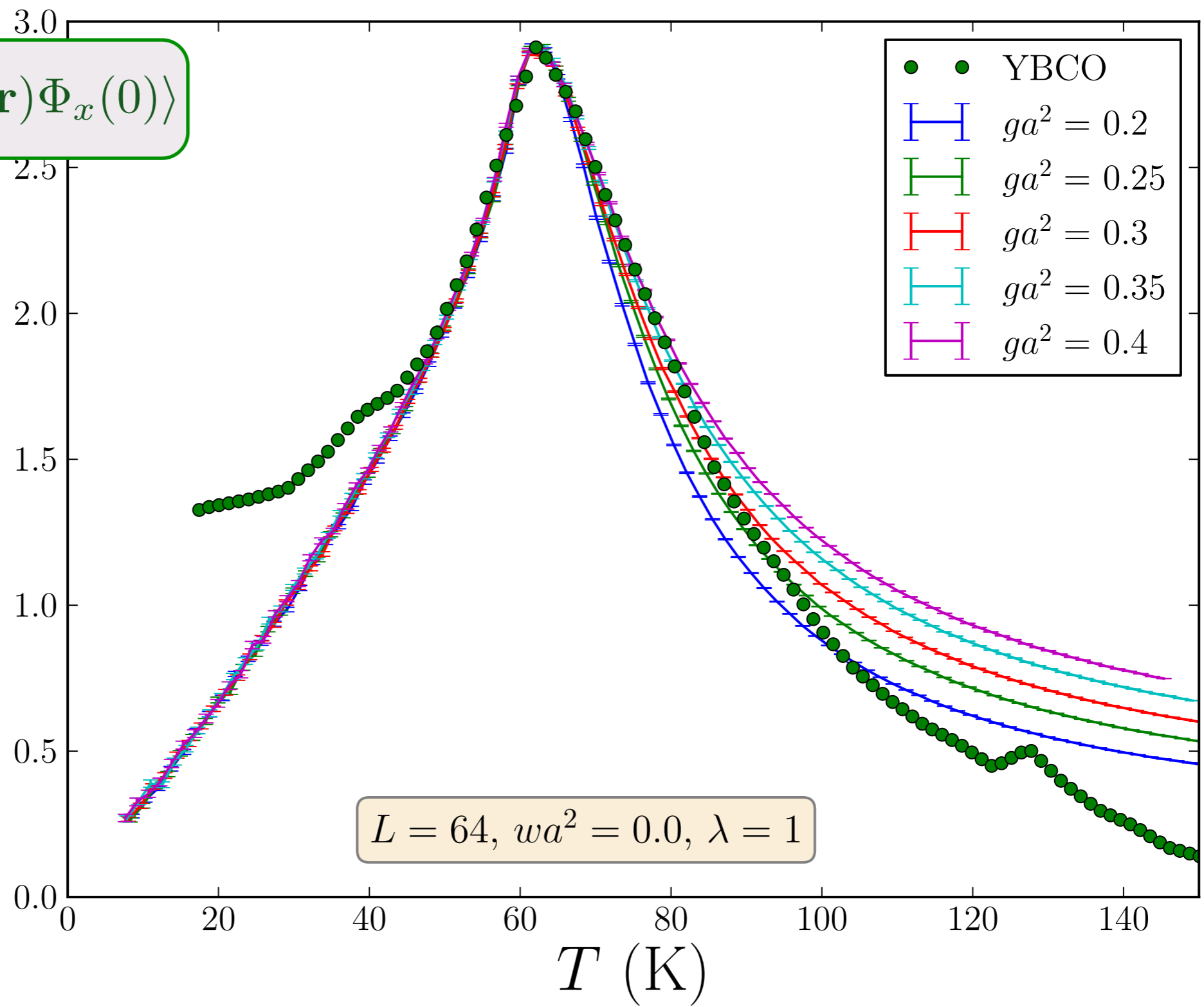
Solve by cluster Monte Carlo and $1/N$ expansion.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

Comparison of Monte Carlo with experiments

$$S_{\Phi_x} = \int d^2r \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order
structure
factor S_{Φ_x}



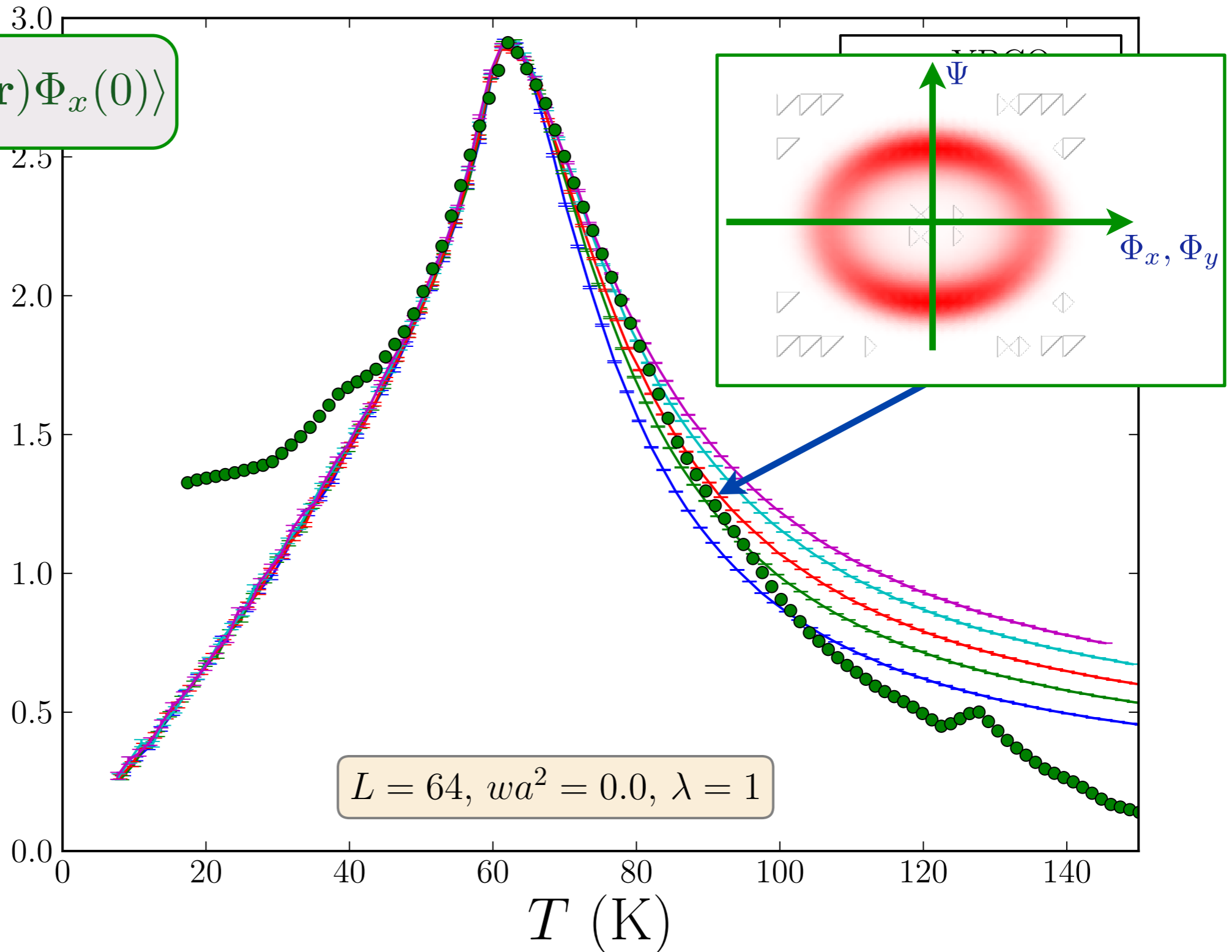
For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160\text{K}$.
The height was also rescaled to make the peak heights match.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

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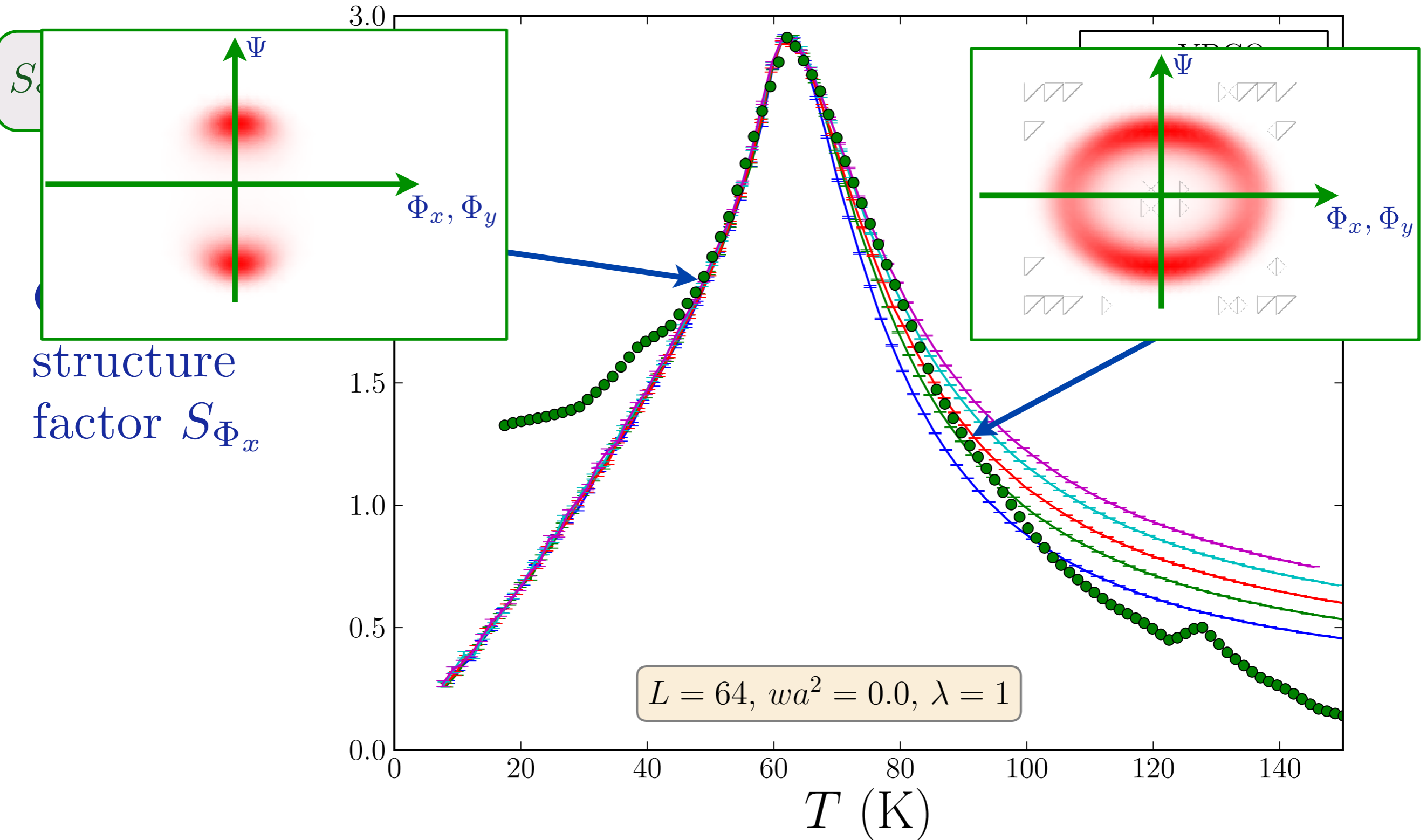
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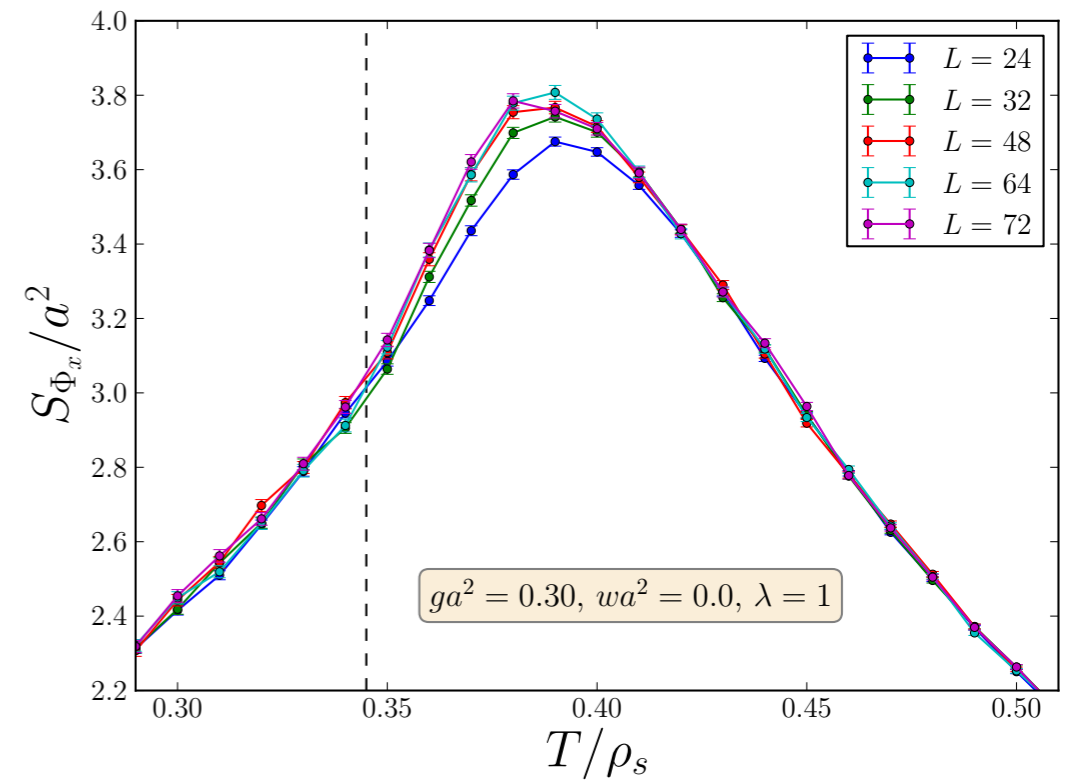
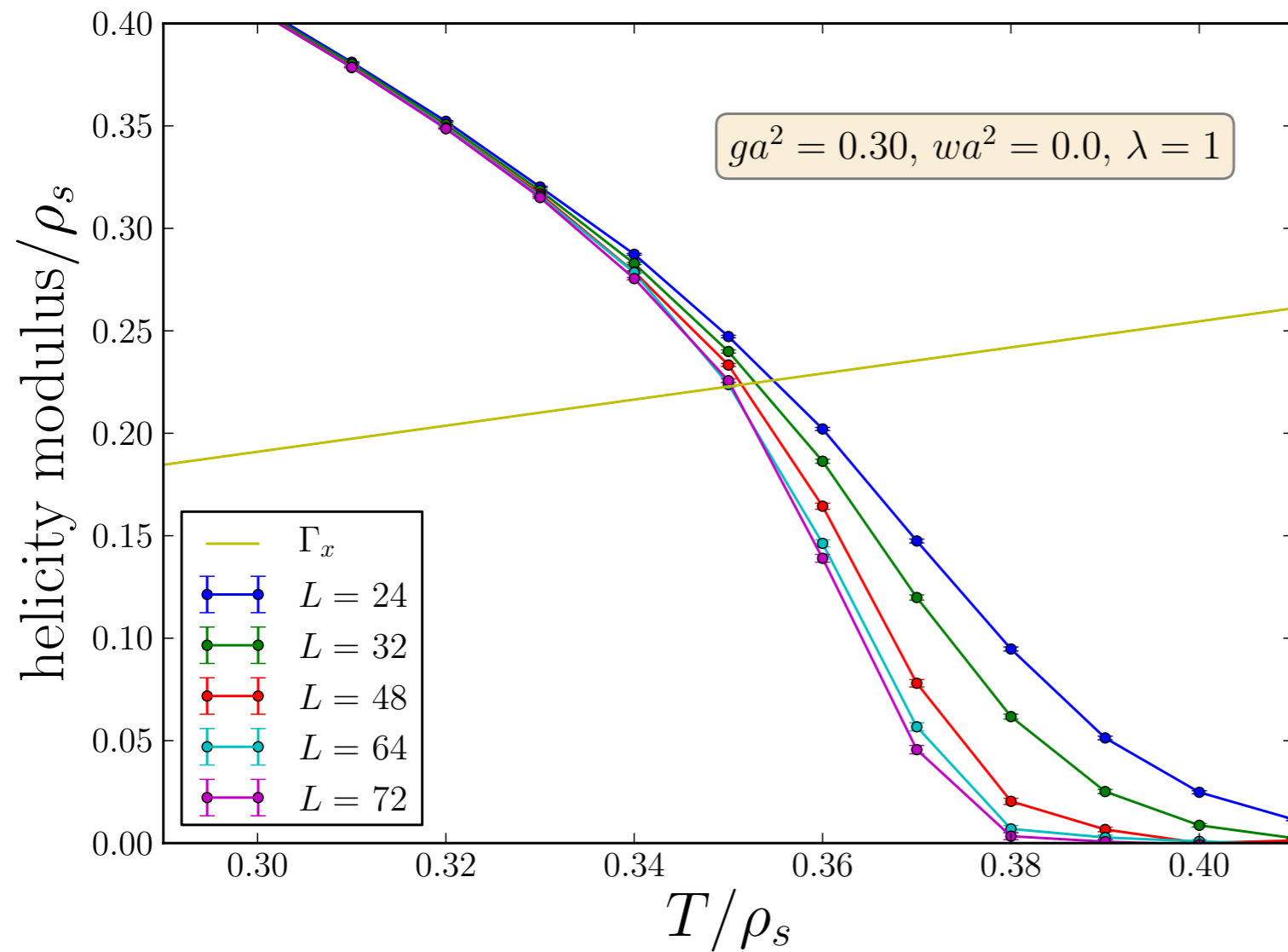


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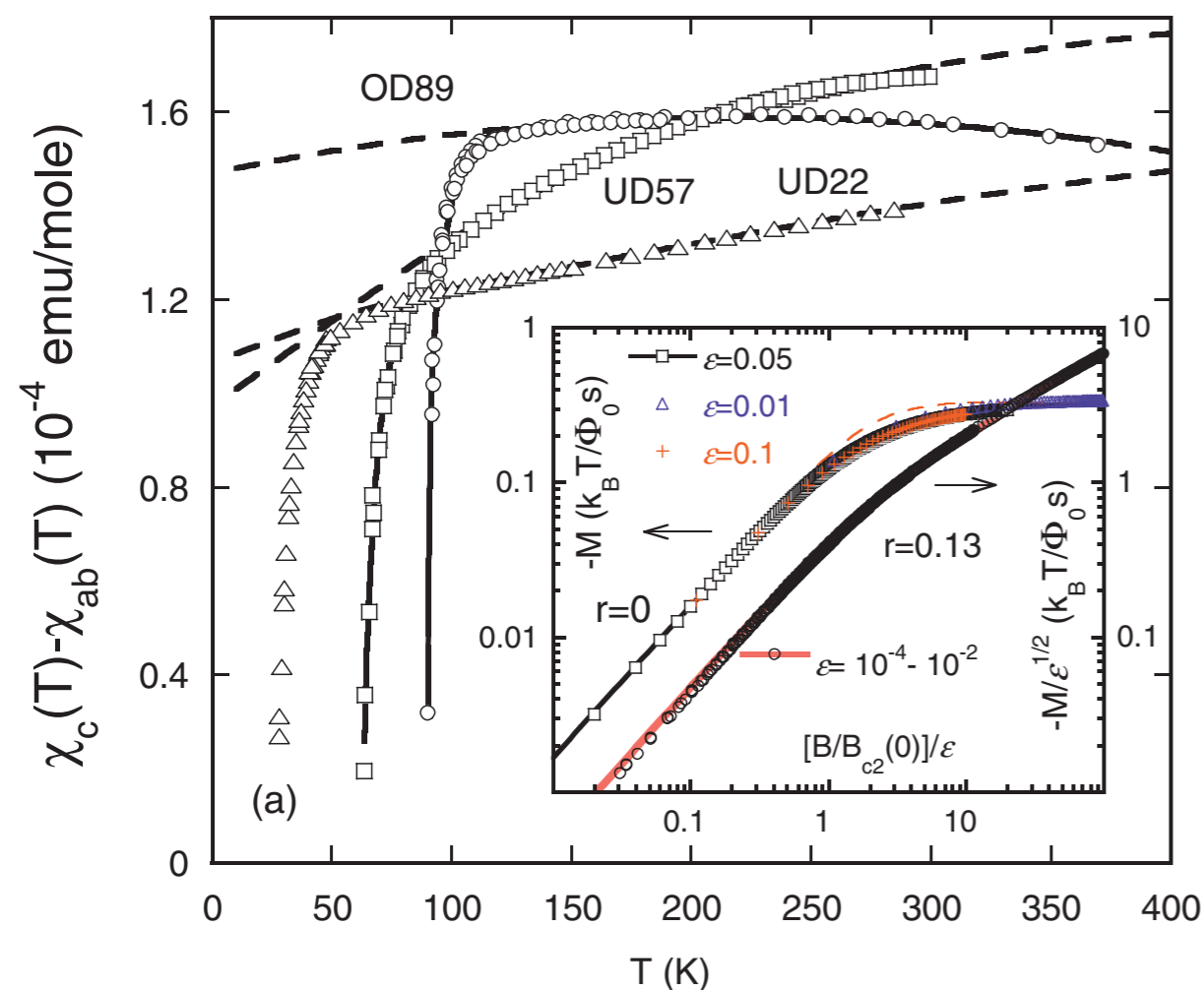
Onset of superconductivity in Monte Carlo



L. E. Hayward, D. G. Hawthorn,
R. G. Melko, and S. Sachdev, arXiv:1309.6639

Other experiments in the pseudogap

- The *same* set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above T_c . Indeed $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ shows significant fluctuation diamagnetism over the same range of temperatures. (S. Chatterjee et al, in progress).



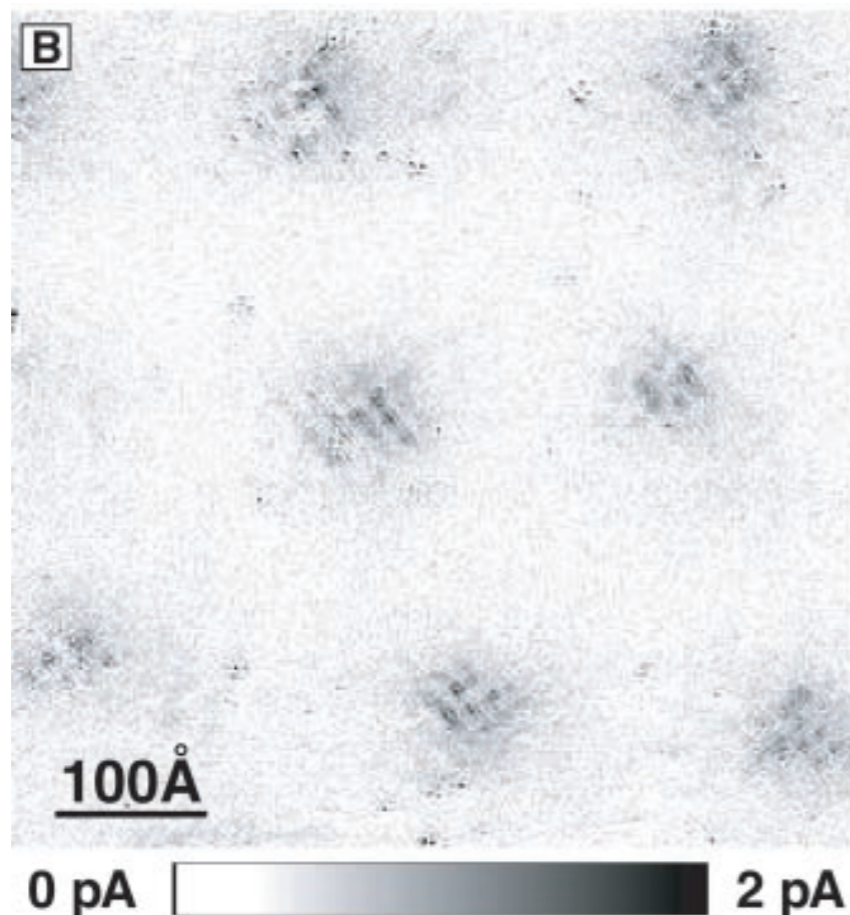
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Diamagnetism of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ crystals above T_c : Evidence for Gaussian fluctuations

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A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

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- Fluctuating 6-component order can explain “Fermi arc” photoemission spectra (Randeria; D. Chowdhury et al, in progress)

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- The “pseudogap” phase is described by the angular fluctuations of a 6-component vector representing the superconducting and bond orders