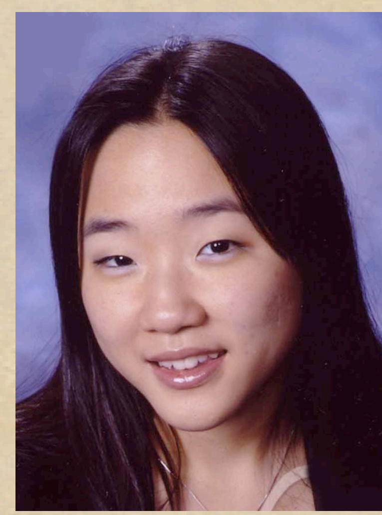
Electronic quasiparticles and competing orders in the cuprate superconductors



Ettore Vicari
Pisa

Yejin Huh
Harvard

Andrea Pelissetto
Rome

Subir Sachdev
Harvard

Gapless nodal quasiparticles in *d*-wave superconductors

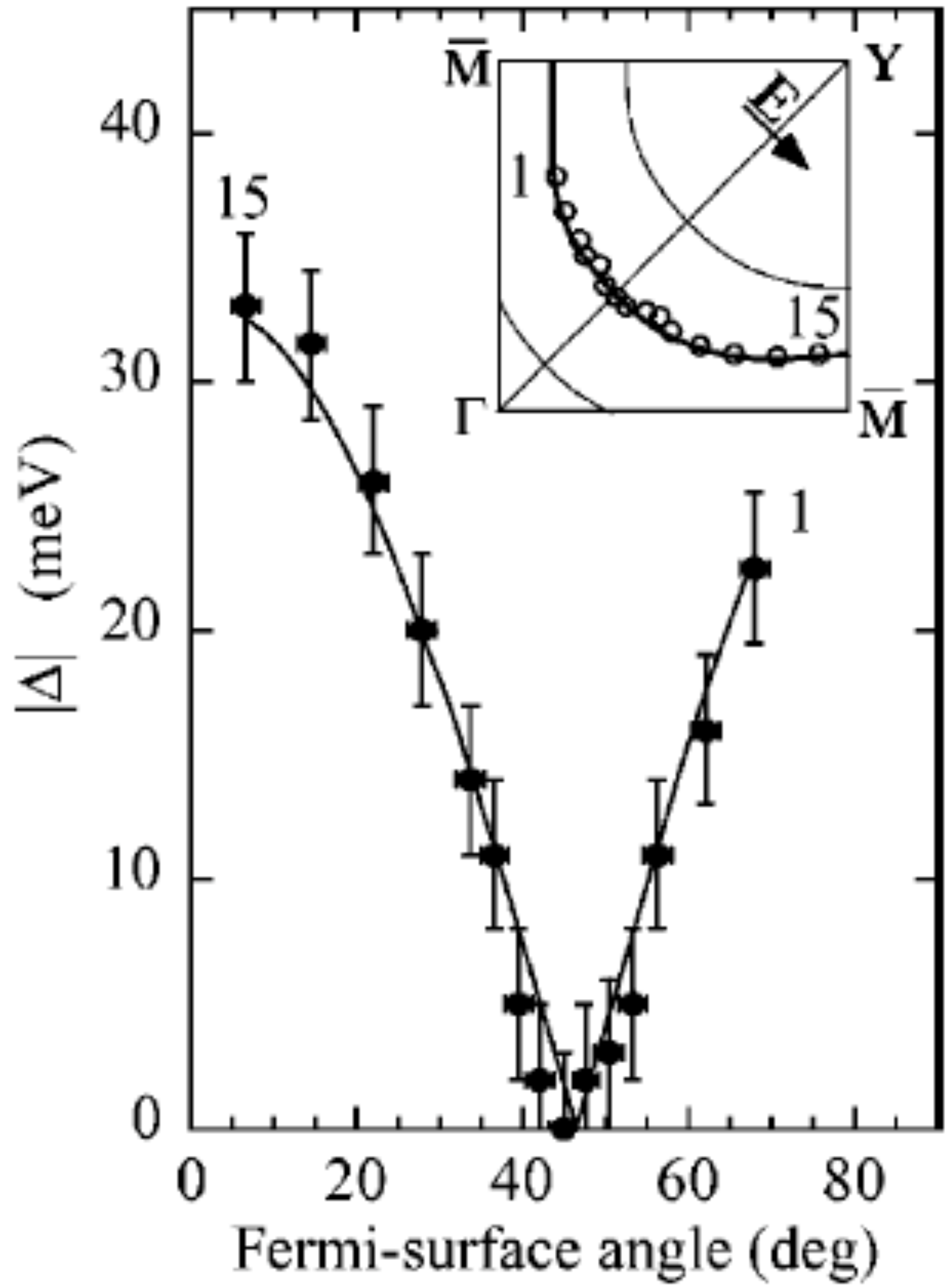
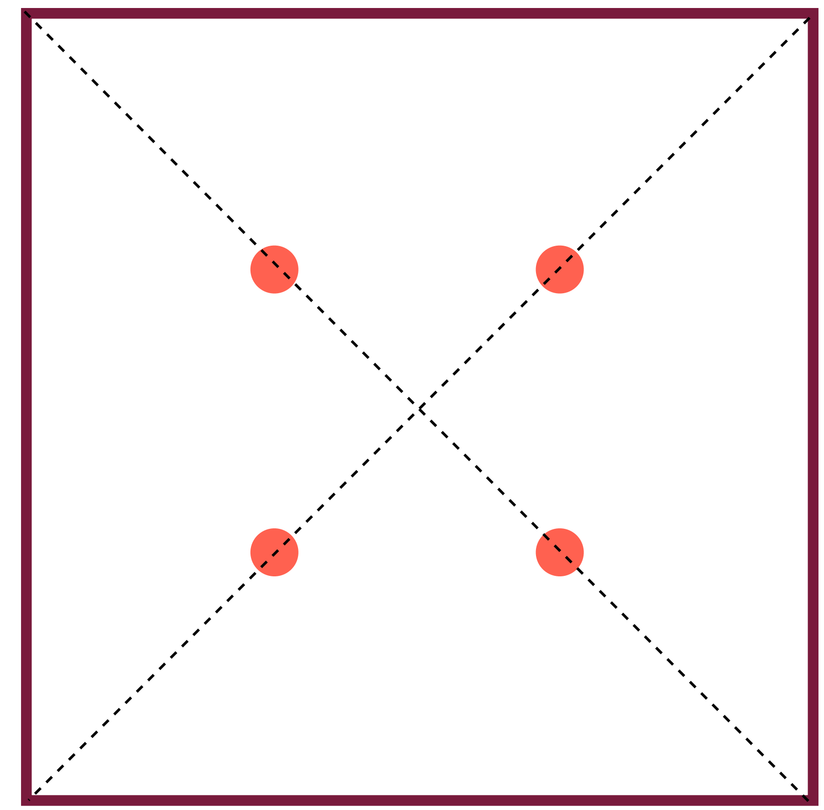
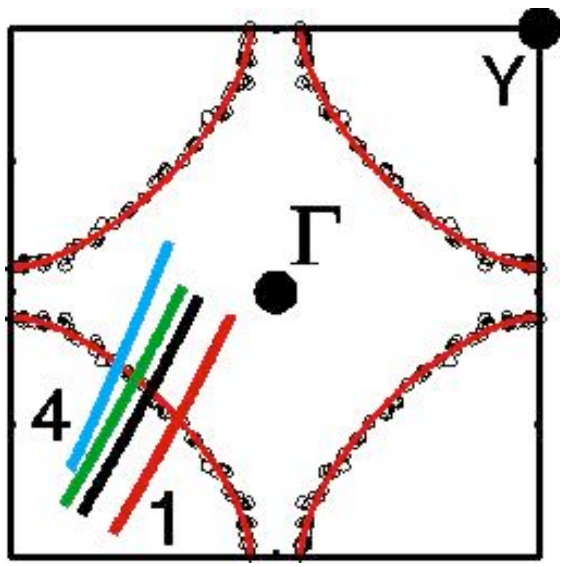


FIG. 46. Superconducting gap measured at 13 K on Bi2212 ($T_c = 87$ K) plotted vs the angle along the normal-state Fermi surface (see sketch of the Brillouin zone), together with a *d*-wave fit. From Ding, Norman, *et al.*, 1996.

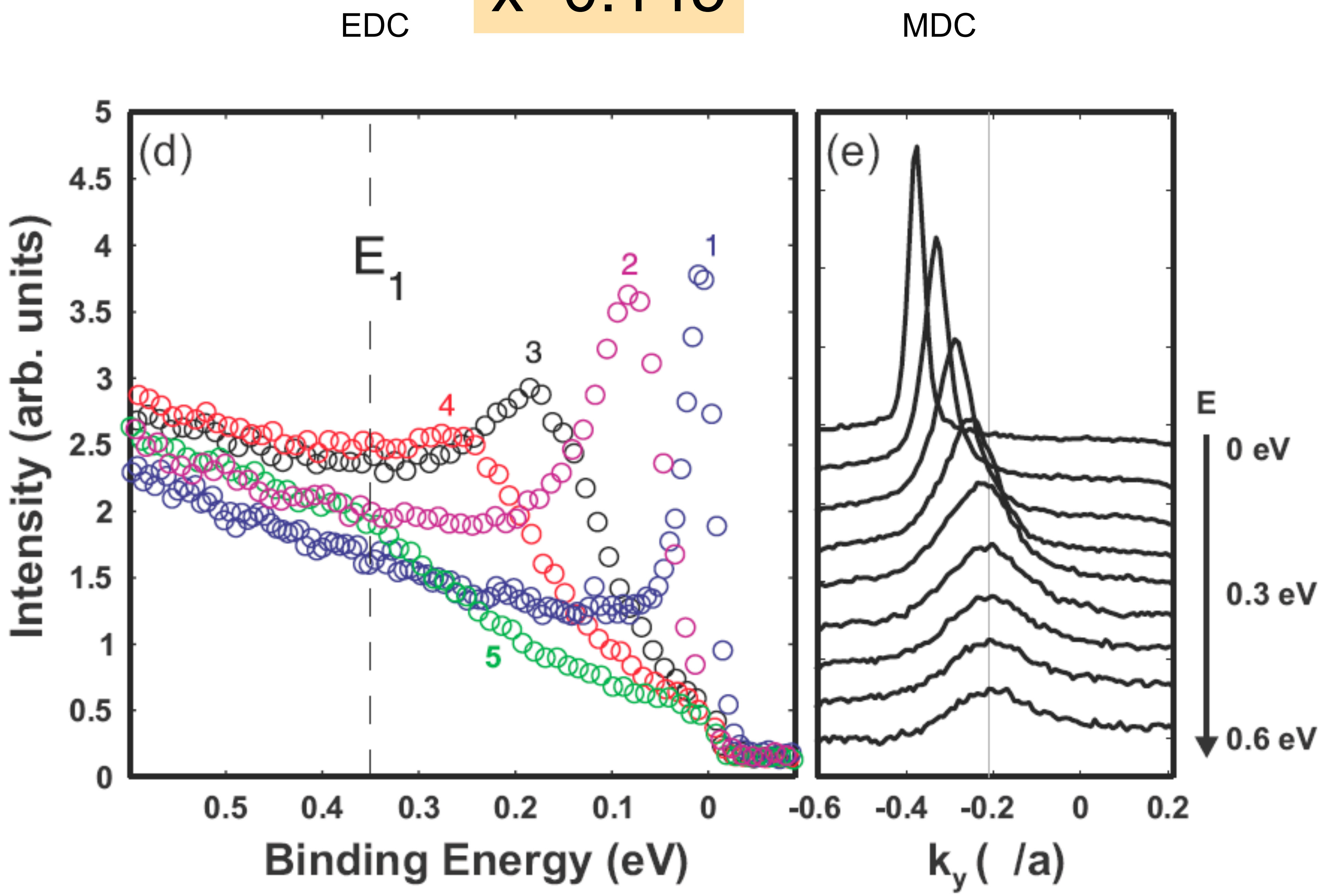


Brillouin zone

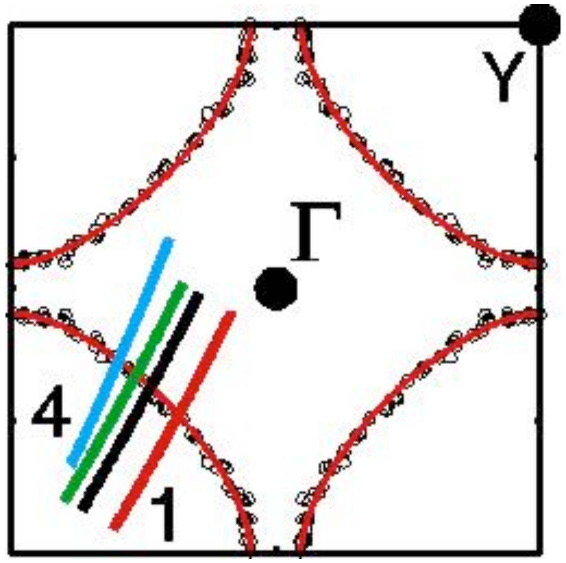
Photoemission spectra of $La_{2-x}Sr_xCuO_4$



x=0.145

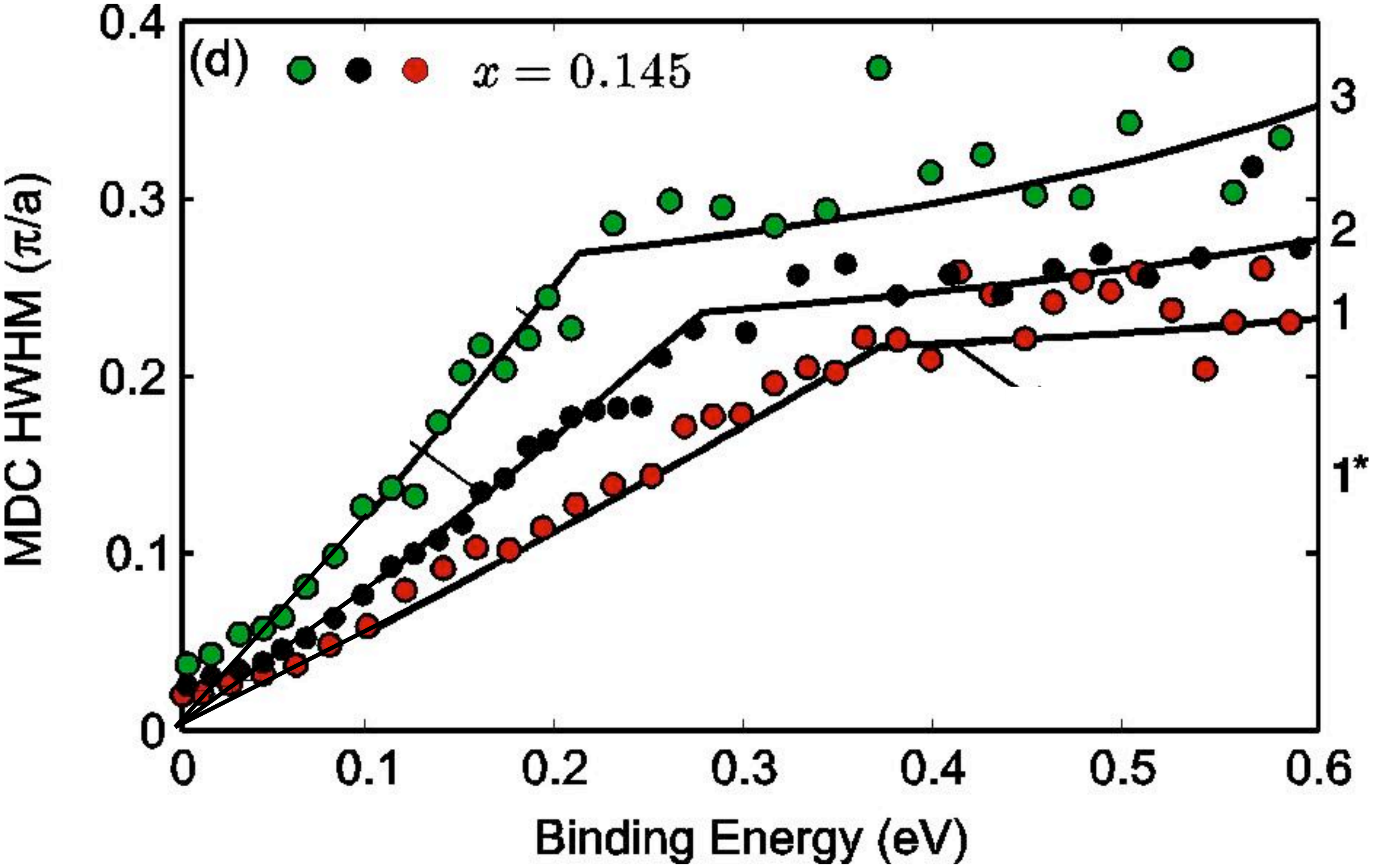


J. Chang, M. Shi, S. Pailhes, M. Maansson, T. Claesson, O. Tjernberg, A. Bendounan, L. Patthey, N. Momono, M. Oda, M. Ido, C. Mudry, and J. Mesot, arXiv:0708.2782



Photoemission spectra of $La_{2-x}Sr_xCuO_4$

x=0.145



J. Chang, M. Shi, S. Pailhes, M. Maansson, T. Claesson, O. Tjernberg, A. Bendounan, L. Patthey, N. Momono, M. Oda, M. Ido, C. Mudry, and J. Mesot, arXiv:0708.2782

Scanning tunneling microscopy of BSCCO

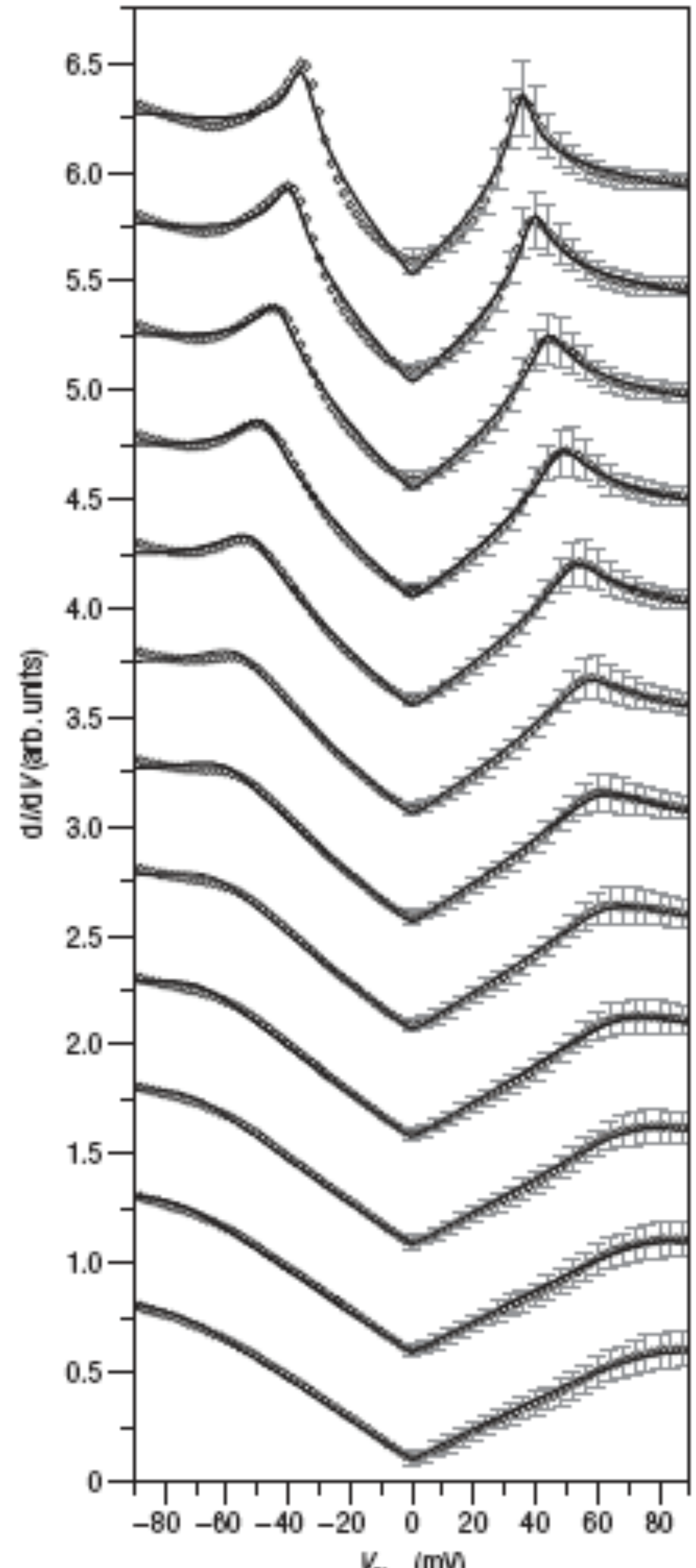
$$N(E,\Gamma_2) = A \times \mathrm{Re}\left(\left\langle \frac{E+i\Gamma_2(E)}{\sqrt{(E+i\Gamma_2(E))^2-\Delta(k)^2}}\right\rangle_{fs}\right) + I$$

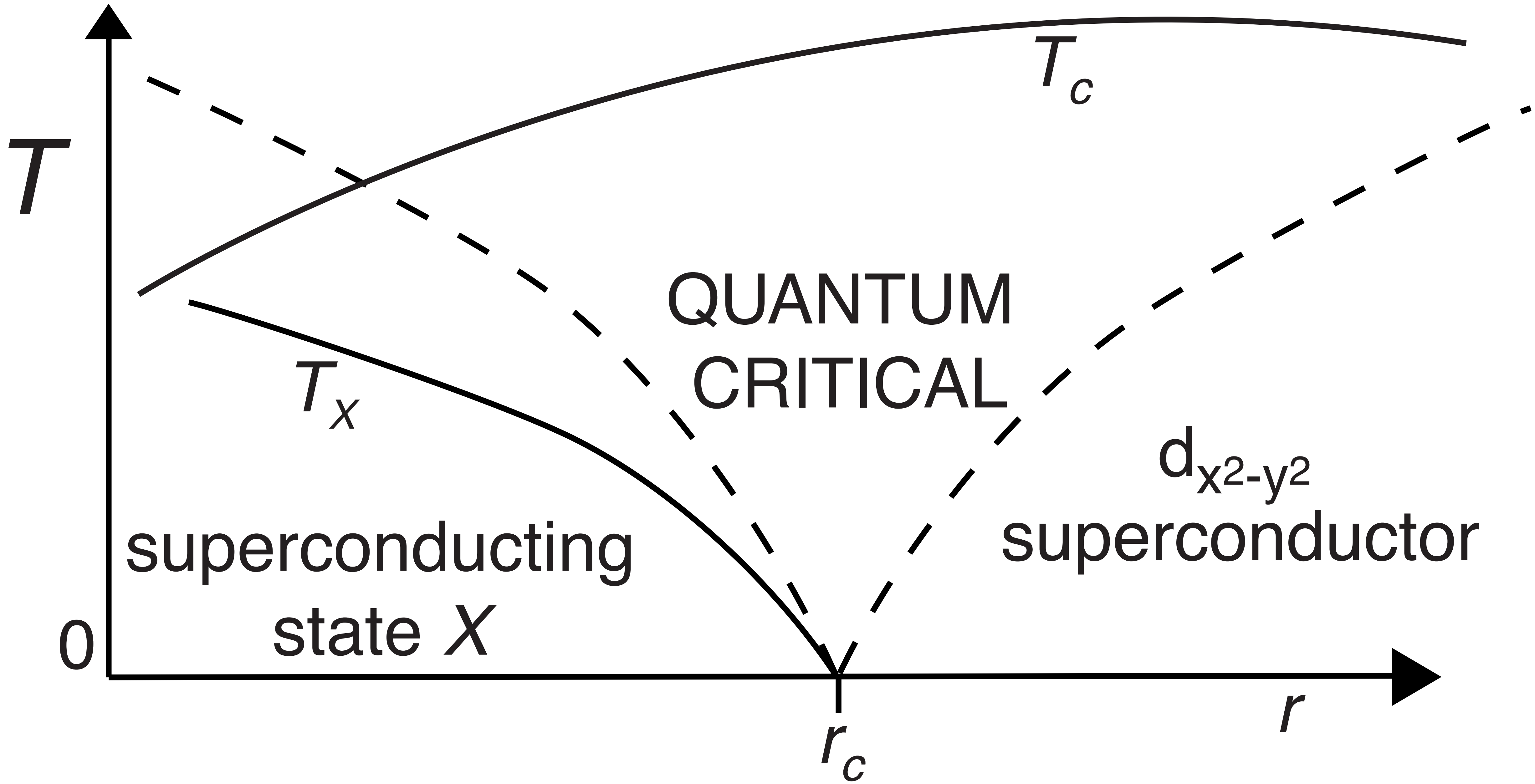
Good fit with

$$\Gamma_2(E) = \alpha\, E$$

J. W. Alldredge, Jinho Lee, K. McElroy, M. Wang, K. Fujita, Y. Kohsaka, C. Taylor, H. Eisaki, S. Uchida, P. J. Hirschfeld, and J. C. Davis

Nature Physics **4**, 319 (2008)





Needed:
Quantum critical point with nodal quasiparticles part of the critical theory

Outline

1. SDW order in LSCO

   Emergent O(4) symmetry

2. Nodal quasiparticles at the O(4) critical point

   Unique selection of quasiparticle coupling to (composite) nematic order

3. Nematic order in YBCO and BSCCO

   Broken lattice symmetry but no spin order

4. Theory of the onset of nematic order in a d-wave superconductor

   Infinite anisotropy fixed point

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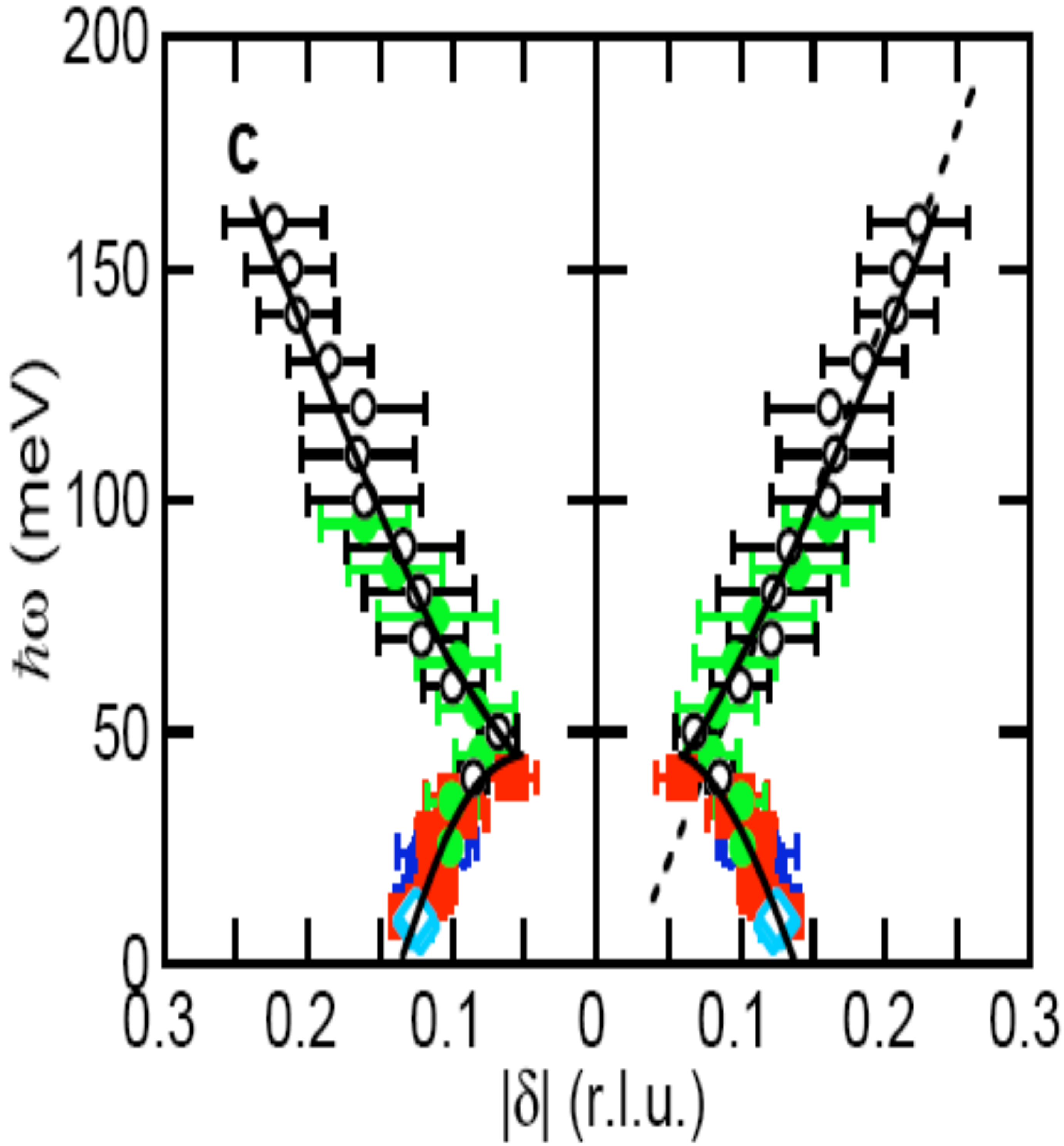
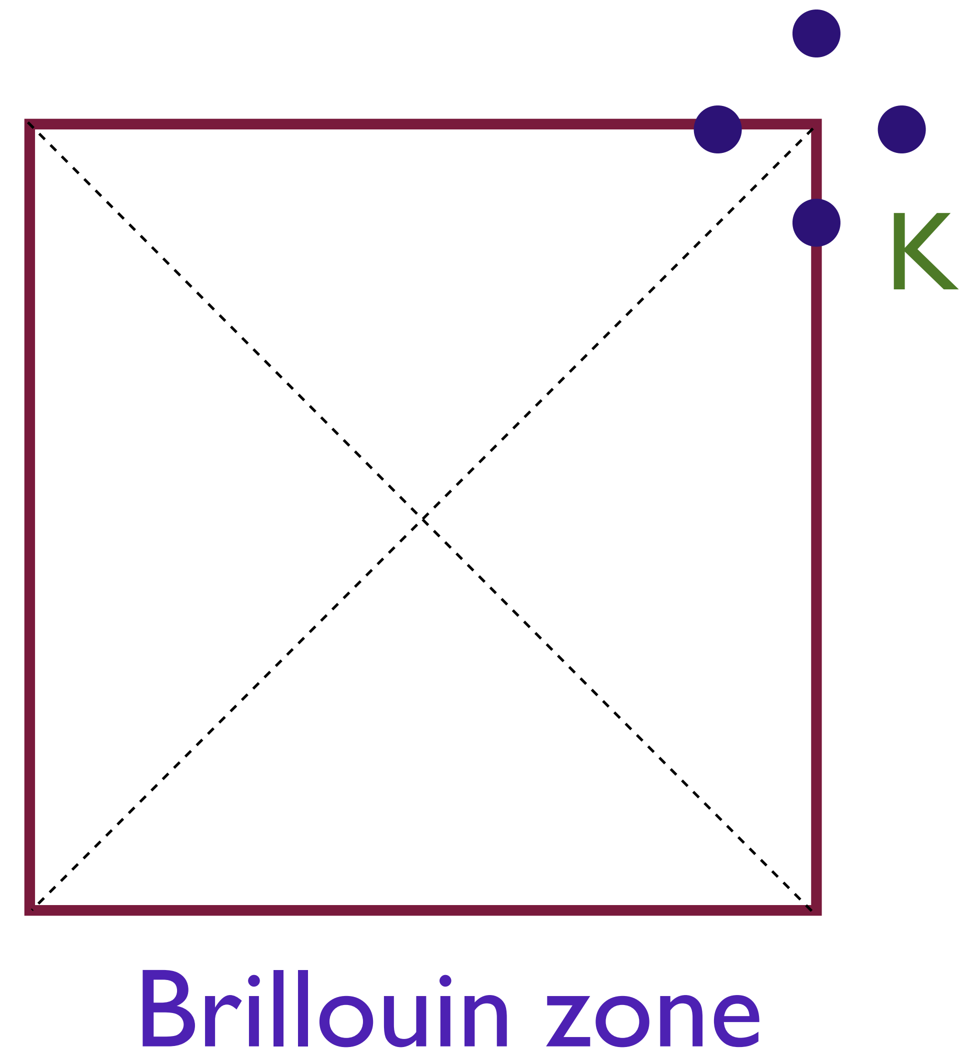
Neutron Scattering-LSCO



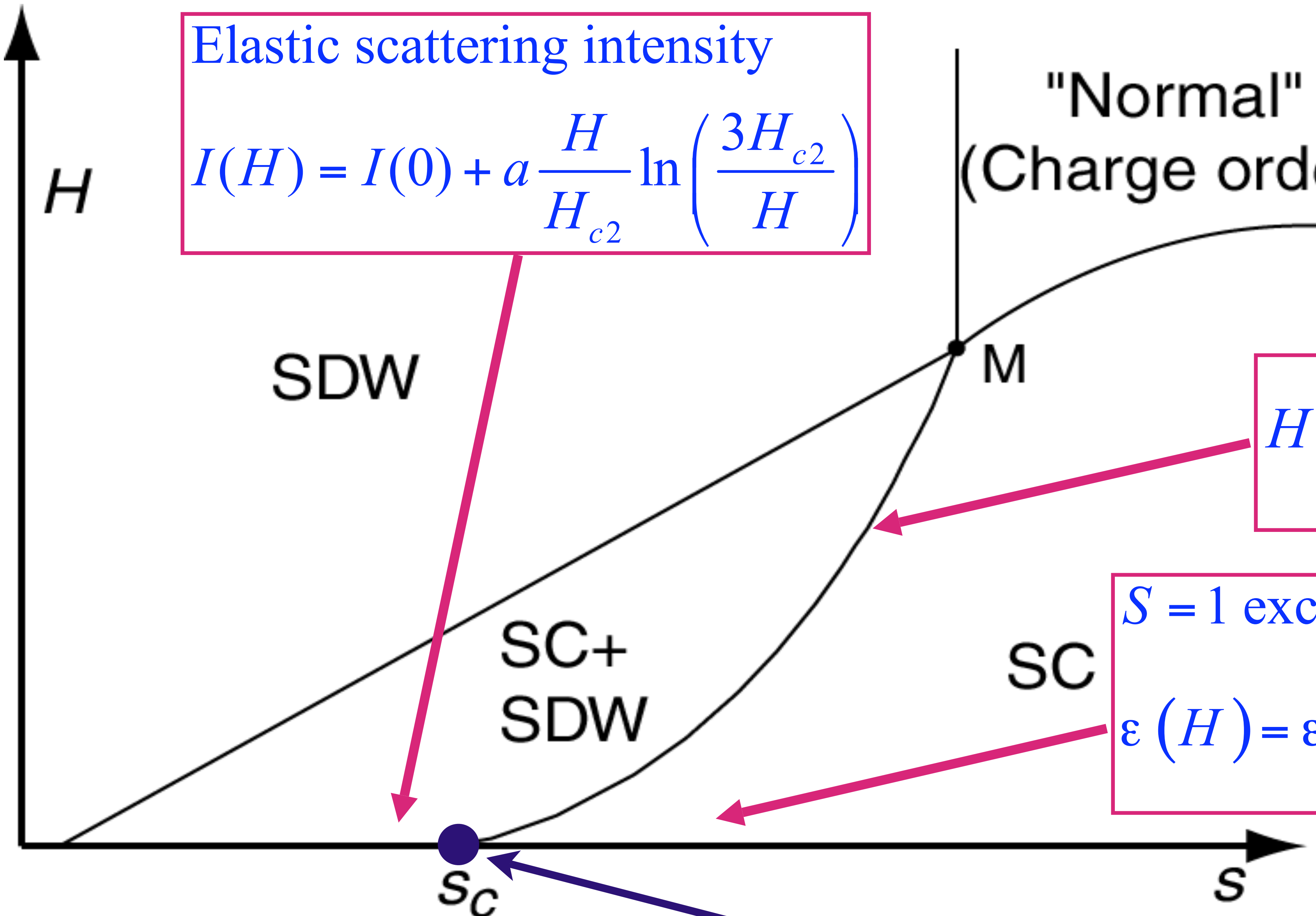
Vignolle *et al*., Nature Phys. 07
Christensen *et al*., PRL 04
Hayden *et al*., Nature 04
Tranquada *et al*., Nature 04



T=0

Elastic scattering intensity

$$I(H) = I(0) + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$



$$H \sim \frac{(s-s_c)}{\ln\left(1/(s-s_c)\right)}$$

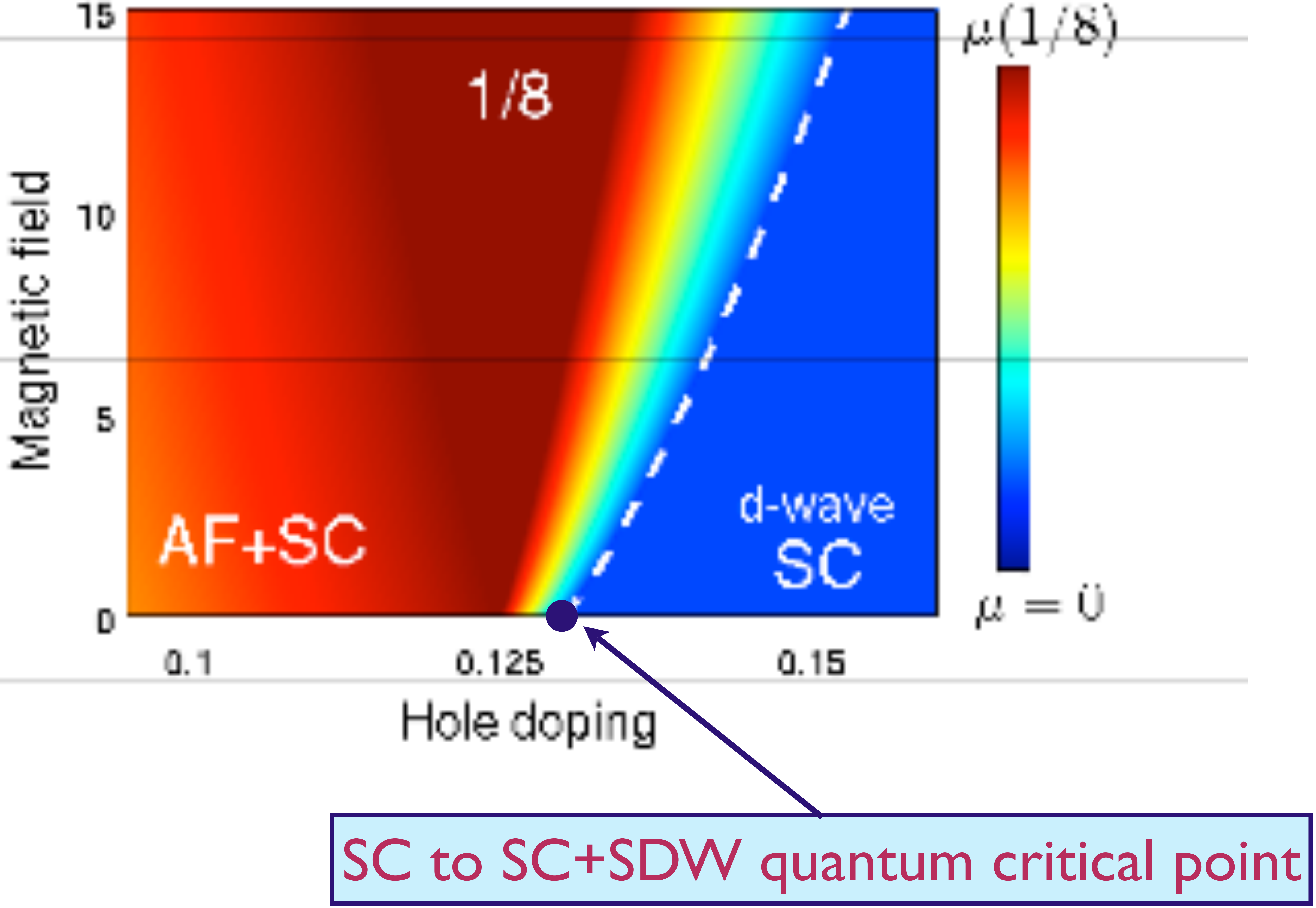
$S=1$ exciton energy

$$\varepsilon(H) = \varepsilon(0) - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

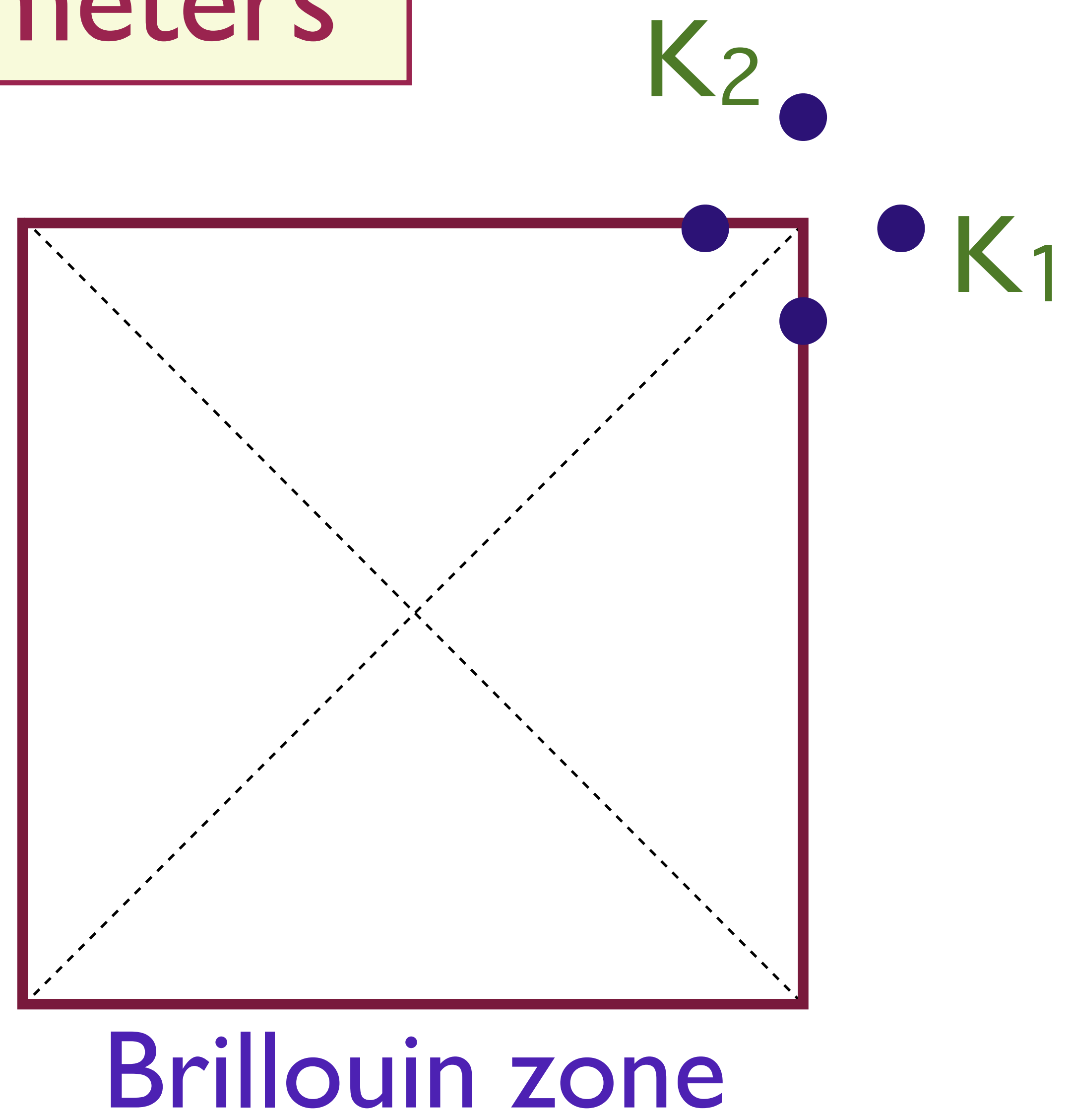
SC to SC+SDW quantum critical point

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, arXiv:0712.2181



SDW order parameters



$$S_i(\mathbf{r},\tau) = \mathrm{Re}\left[e^{i\mathbf{K}_1\cdot\mathbf{r}}\,\Phi_{1i}(\mathbf{r},\tau) + e^{i\mathbf{K}_2\cdot\mathbf{r}}\,\Phi_{2i}(\mathbf{r},\tau)\right].$$

$$\mathbf{K}_1 = \left(\frac{2\pi}{a}\right)\left(\frac{1}{2}-\vartheta, \frac{1}{2}\right) \quad , \quad \mathbf{K}_2 = \left(\frac{2\pi}{a}\right)\left(\frac{1}{2}, \frac{1}{2}-\vartheta\right),$$

SDW field theory

$$\mathcal{L}_\Phi = |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2$$
$$+|\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2)$$
$$+\frac{u_1}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2}(|\Phi_1^2|^2 + |\Phi_2^2|^2)$$
$$+w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2$$

Most general theory invariant under spin rotation, square lattice space group, and time-reversal symmetries

	T_x	T_y	R	I	$\mathcal{T}$
Φ_{1i}	$-e^{-i\vartheta}\Phi_{1i}$	$-\Phi_{1i}$	Φ_{2i}	Φ_{1i}^*	$-\Phi_{1i}$
Φ_{2i}	$-\Phi_{2i}$	$-e^{-i\vartheta}\Phi_{2i}$	Φ_{1i}^*	Φ_{2i}^*	$-\Phi_{2i}$

SDW field theory

$$
\begin{aligned}
\mathcal{L}_\Phi = & \; |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2 \\
& + |\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2) \\
& + \frac{u_1}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2}(|\Phi_1^2|^2 + |\Phi_2^2|^2) \\
& + w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2
\end{aligned}
$$

Symmetries:

$$\mathrm{U}(1) \otimes \mathrm{U}(1) \otimes Z_4 \otimes \mathrm{O}(3)$$

- U(1) → *x*-translations
- U(1) → *y*-translations
- Z_4 → lattice rotations
- O(3) → spin rotations

SDW field theory

Stable fixed point in a 6-loop RG analysis:
$w_1^* = u_1^* - u_2^*,\ w_2^* = w_3^* = u_2^*,\ v_1^* = v_2^*$

O(4)⊗O(3) invariant theory for φ_{ai}, with $a = 1 \ldots 4$ an O(4) index, and $i = 1 \ldots 3$ an O(3) index, and

$$\Phi_{1i} = \varphi_{1i} + i\varphi_{2i}, \quad \Phi_{2i} = \varphi_{3i} + i\varphi_{4i}.$$

M. De Prato, A. Pelissetto, and E. Vicari
Phys. Rev. B **74**, 144507 (2006).

SDW field theory

Symmetries:

$$\mathrm{U}(1)\otimes\mathrm{U}(1)\otimes Z_4\otimes\mathrm{O}(3)$$

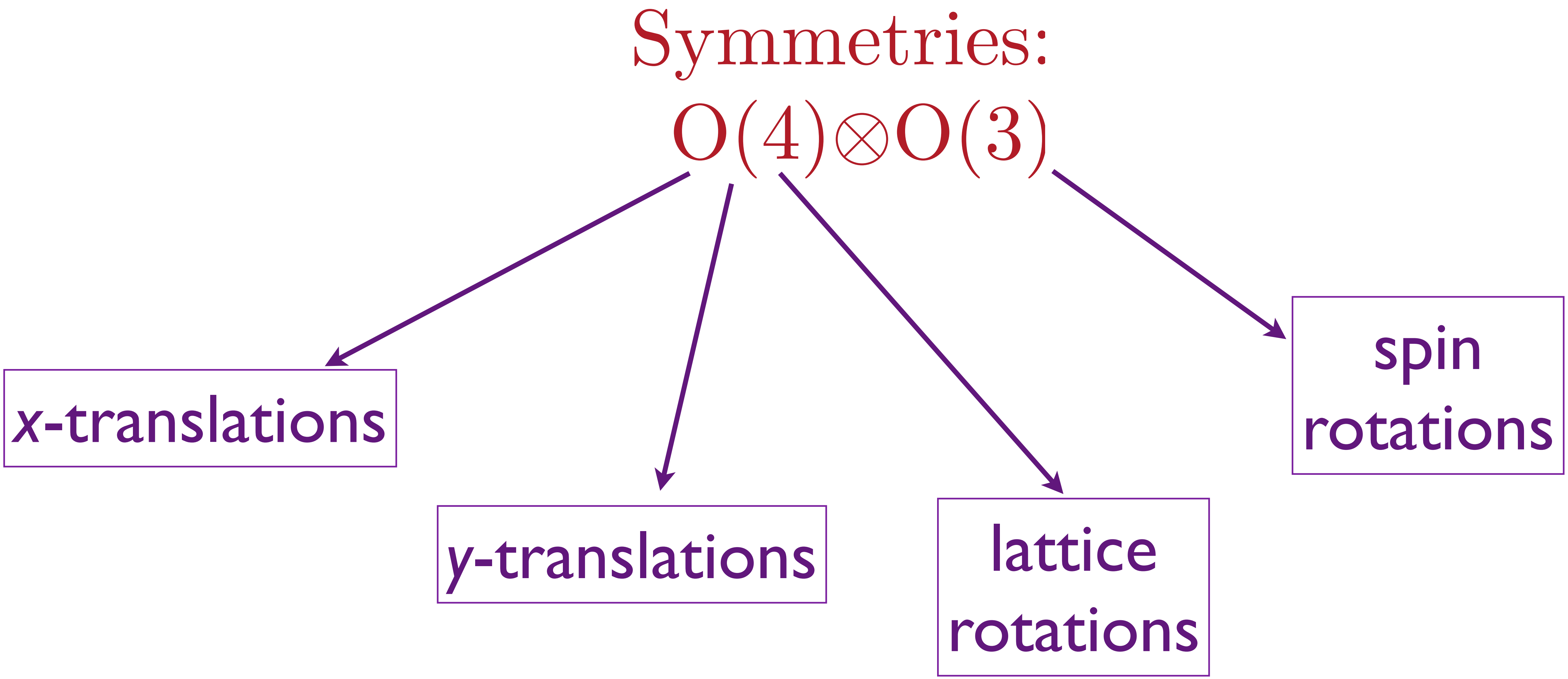
x-translations

y-translations

lattice rotations

spin rotations

SDW field theory



Properties of O(4)⊗O(3) fixed point:

The following 9 order parameters have divergent fluctuations at the spin-density wave ordering transition with the same exponent $\overline{\gamma}$:

- The real 'nematic' order parameter, $\phi \equiv \sum_i \left(|\Phi_{1i}|^2 - |\Phi_{2i}|^2 \right)$ which measures breaking of Z_4 symmetry
- Charge density waves at $2\mathbf{K}_1$ and $2\mathbf{K}_2$: $\sum_i \Phi_{1i}^2$ and $\sum_i \Phi_{2i}^2$
- Charge density waves at $\mathbf{K}_1 \pm \mathbf{K}_2$: $\sum_i \Phi_{1i}\Phi_{2i}$ and $\sum_i \Phi_{1i}^*\Phi_{2i}$

At the quantum critical point, the susceptibilities of these orders *all* diverge as $\chi \sim T^{-\overline{\gamma}}$, with

$$\overline{\gamma} = \begin{cases} 0.90(36) & \text{MZM}, 6 \text{ loops} \\ 0.80(54) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases}$$

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

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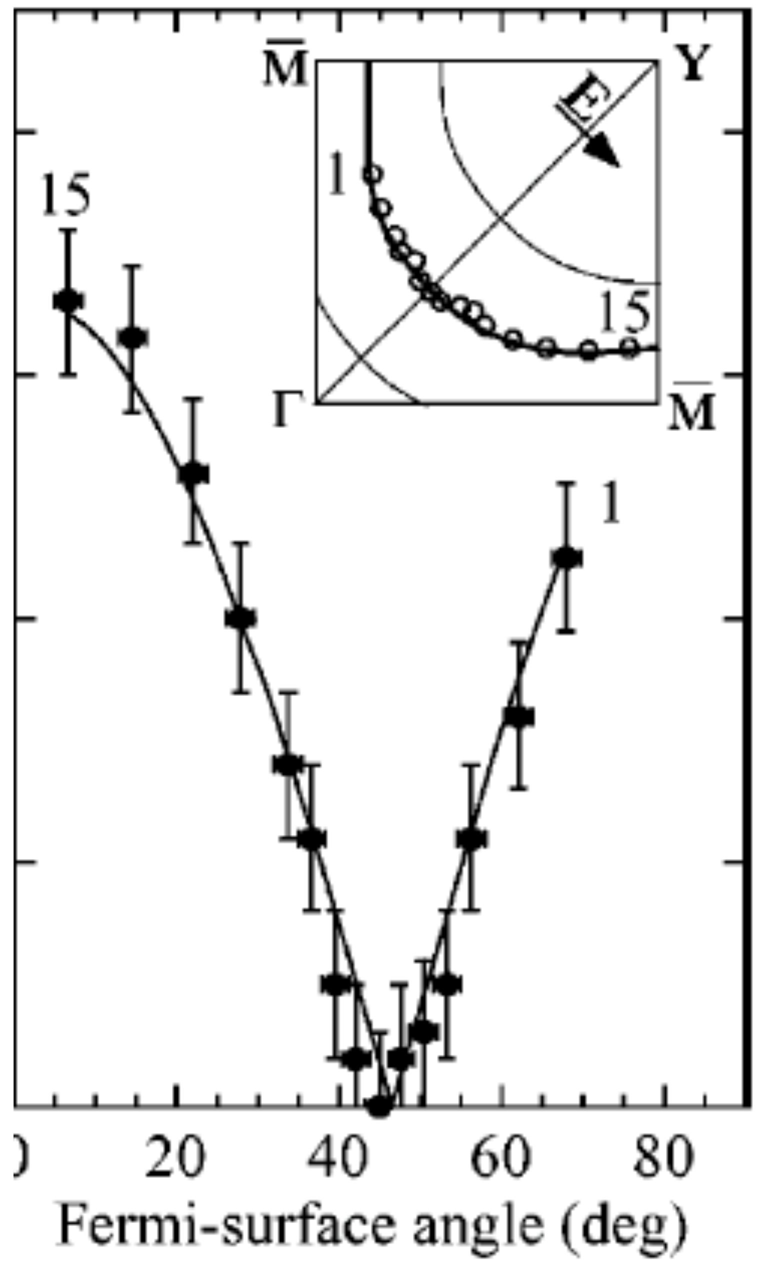
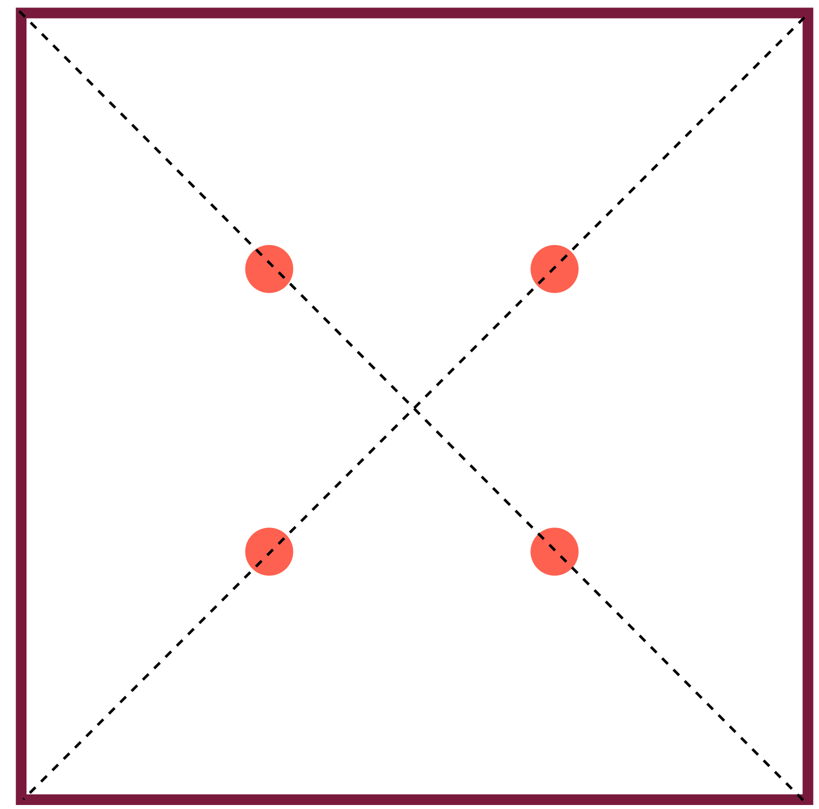


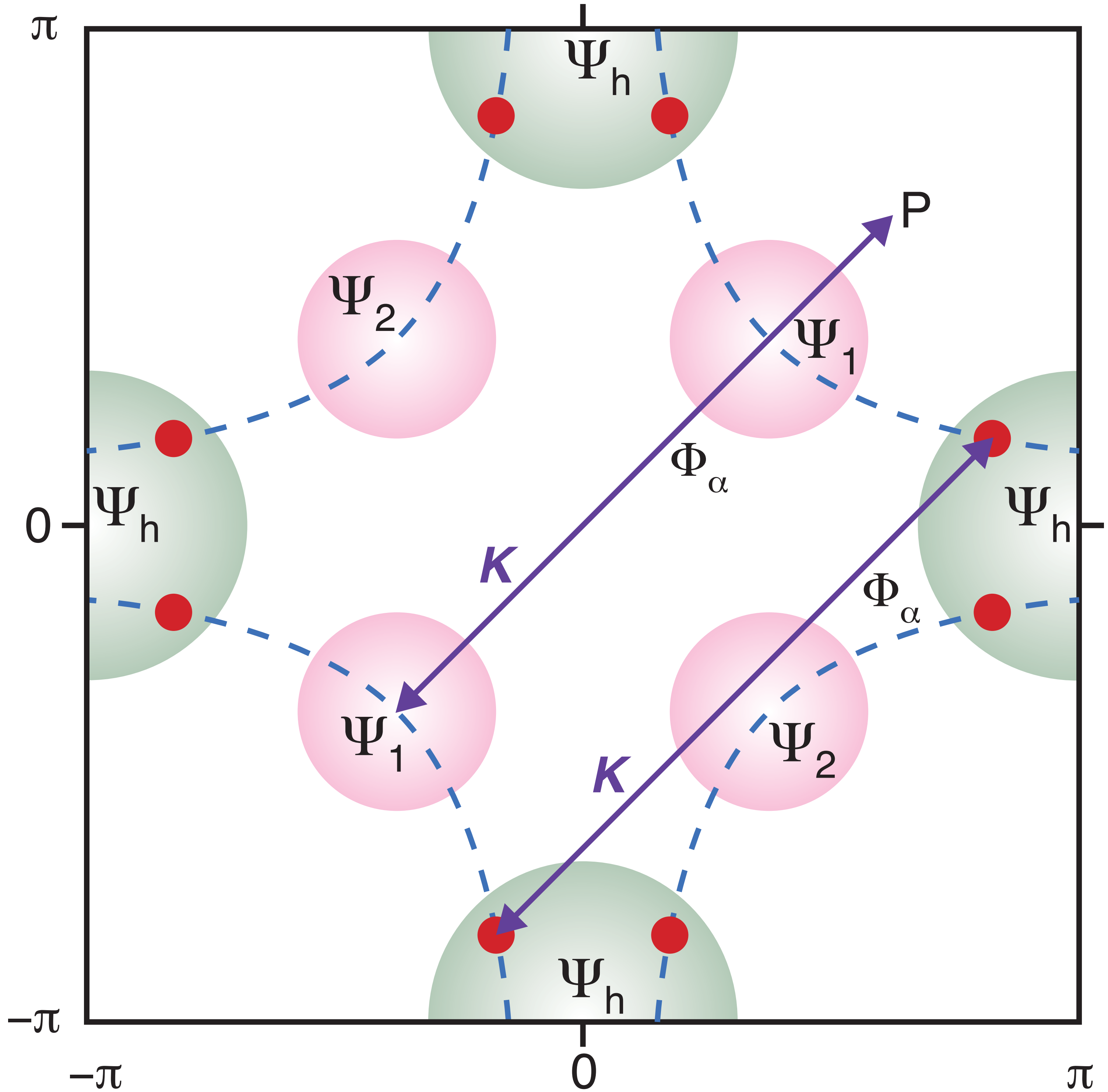


Brillouin zone

$$\mathcal{L}_\Psi = \Psi_1^\dagger \left(\partial_\tau - i\frac{v_F}{\sqrt{2}}(\partial_x + \partial_y)\tau^z - i\frac{v_\Delta}{\sqrt{2}}(-\partial_x + \partial_y)\tau^x \right) \Psi_1$$

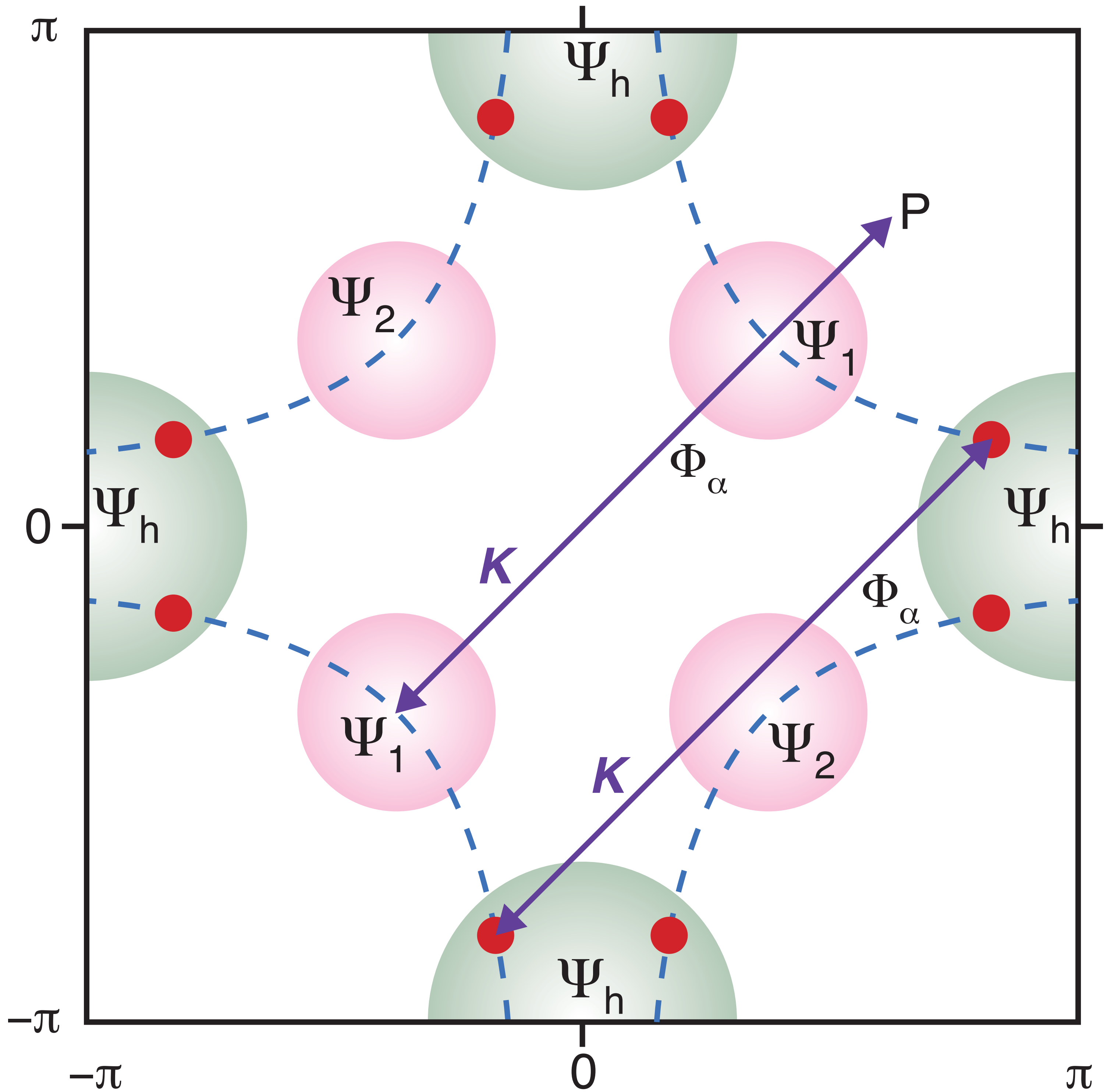
$$+\Psi_2^\dagger \left(\partial_\tau - i\frac{v_F}{\sqrt{2}}(-\partial_x + \partial_y)\tau^z - i\frac{v_\Delta}{\sqrt{2}}(\partial_x + \partial_y)\tau^x \right) \Psi_2.$$

Coupling of quasiparticles to SDW order



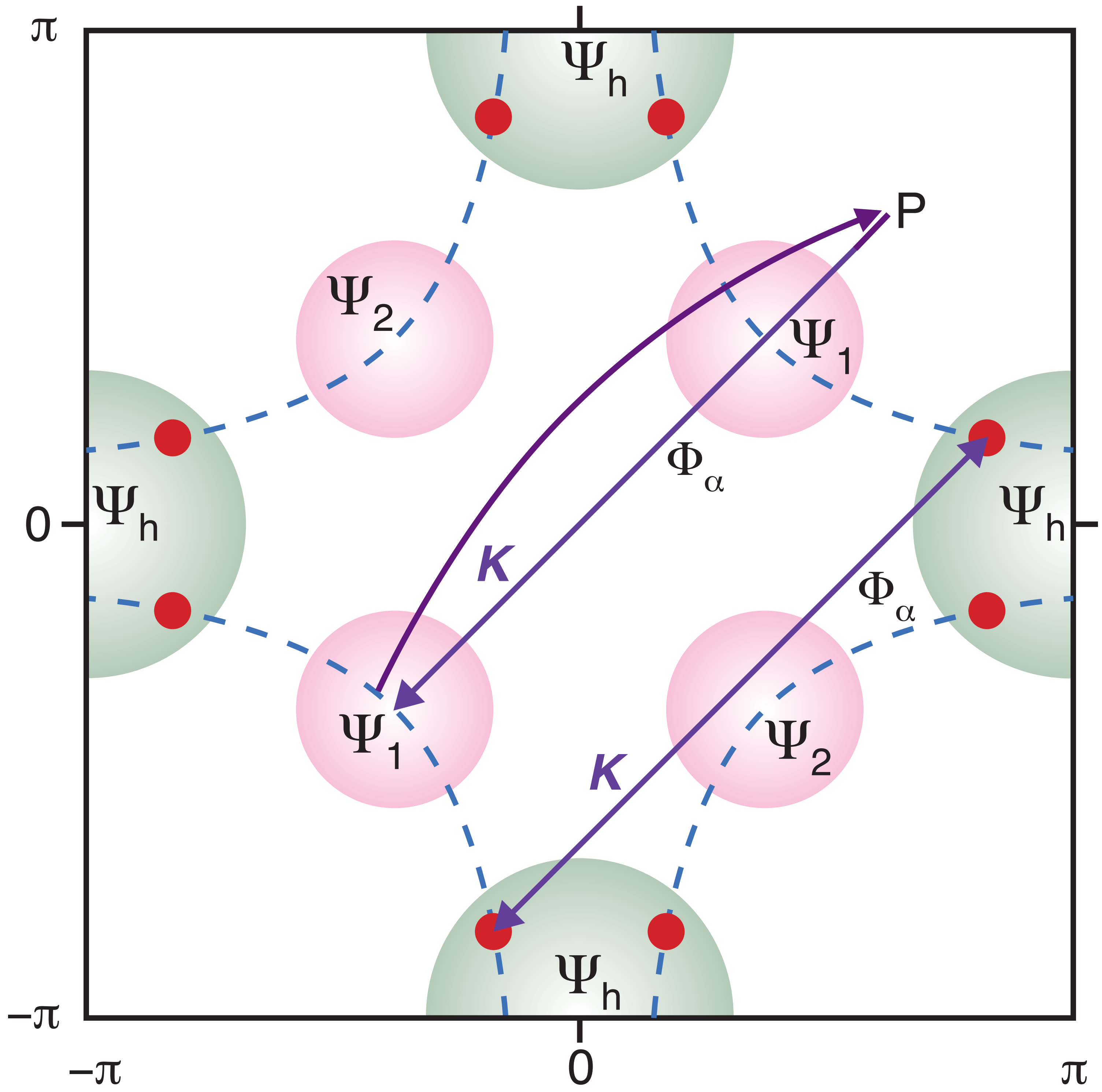
Wavevector mismatch suggests SDW order and nodal quasiparticles are decoupled

Coupling of quasiparticles to SDW order



No “Yukawa” coupling $\Phi\Psi^\dagger\Psi$

Coupling of quasiparticles to SDW order



Possible higher order coupling ~ $|\Phi|^2 \Psi^\dagger \Psi$

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_1 = \lambda_1 \left(|\Phi_1|^2 + |\Phi_2|^2\right) \left(\Psi_1^\dagger \tau^z \Psi_1 + \Psi_2^\dagger \tau^z \Psi_2\right)$$

Energy-energy coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_2 = \lambda_2 \left(|\Phi_1|^2 - |\Phi_2|^2\right) \left(\Psi_1^\dagger \tau^x \Psi_1 + \Psi_2^\dagger \tau^x \Psi_2\right)$$

Nematic coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_3 = \epsilon_{ijk} \Big[\left(\Phi_{1j}^* \Phi_{1k} + \Phi_{2j}^* \Phi_{2k}\right) \left(-\lambda_3 \Psi_2^\dagger \tau^x \sigma^i \Psi_2 + \lambda_3' \Psi_1^\dagger \tau^z \sigma^i \Psi_1\right) + \left(\Phi_{1j}^* \Phi_{1k} - \Phi_{2j}^* \Phi_{2k}\right) \left(\lambda_3 \Psi_1^\dagger \tau^x \sigma^i \Psi_1 - \lambda_3' \Psi_2^\dagger \tau^z \sigma^i \Psi_2\right) \Big].$$

Spiral spin order coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Scaling dimensions of these couplings:

$$\begin{aligned}
\dim[\lambda_1] &= \frac{1}{\nu} - 2 = -0.9(2) \\
\dim[\lambda_2] &= \frac{(\overline{\gamma} - 1)}{2} = \begin{cases} -0.05(18) & \text{MZM}, 6 \text{ loops} \\ -0.10(27) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases} \\
\dim[\lambda_3, \lambda_3'] &= \begin{cases} -0.84(8) & \text{MZM}, 6 \text{ loops} \\ -0.76(8) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases}
\end{aligned}$$

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Coupling of nematic order is nearly marginal:

Quantum-critical features appear in fermion spectrum via coupling to nematic fluctuations of spin density wave order.

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

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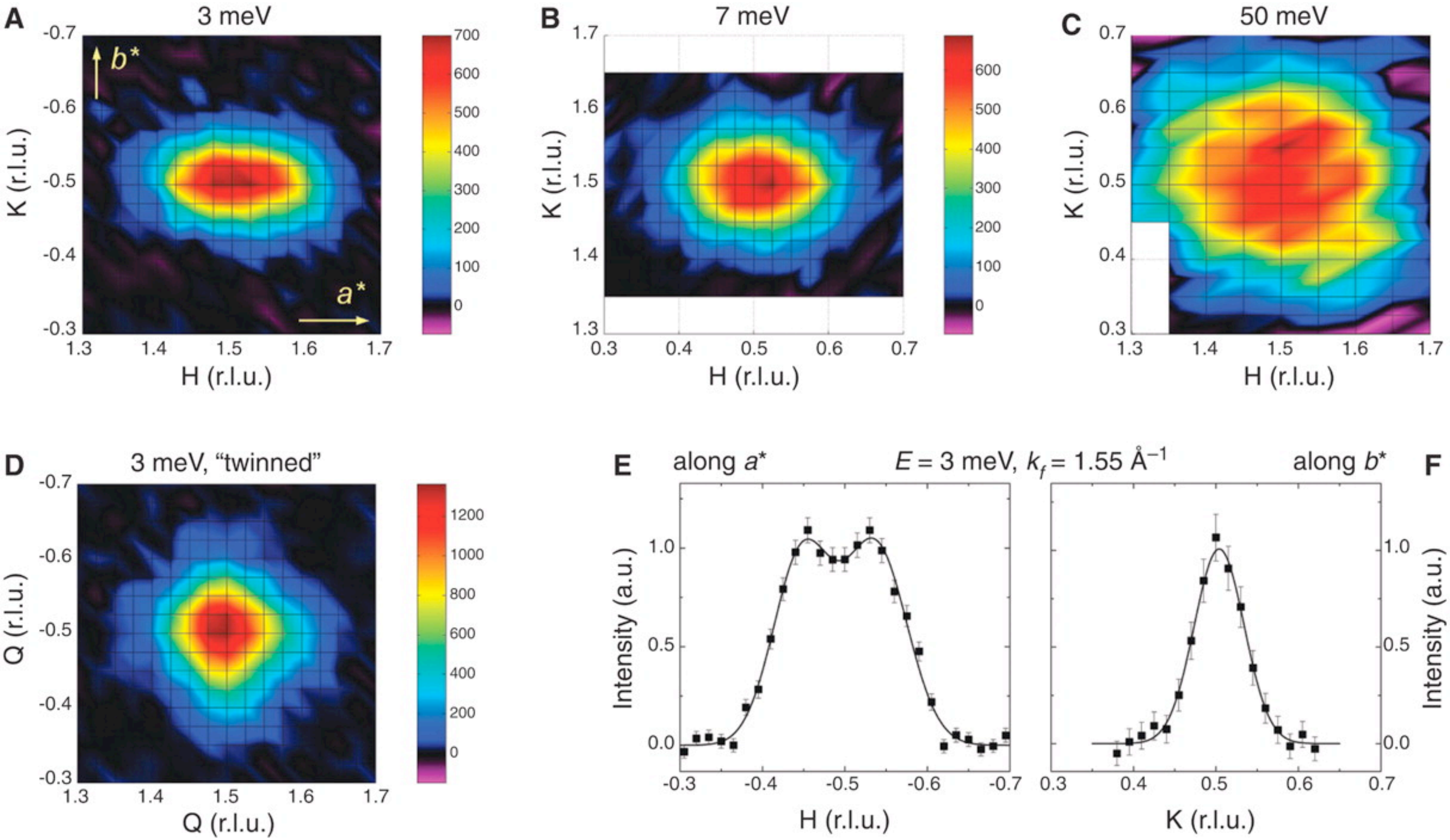
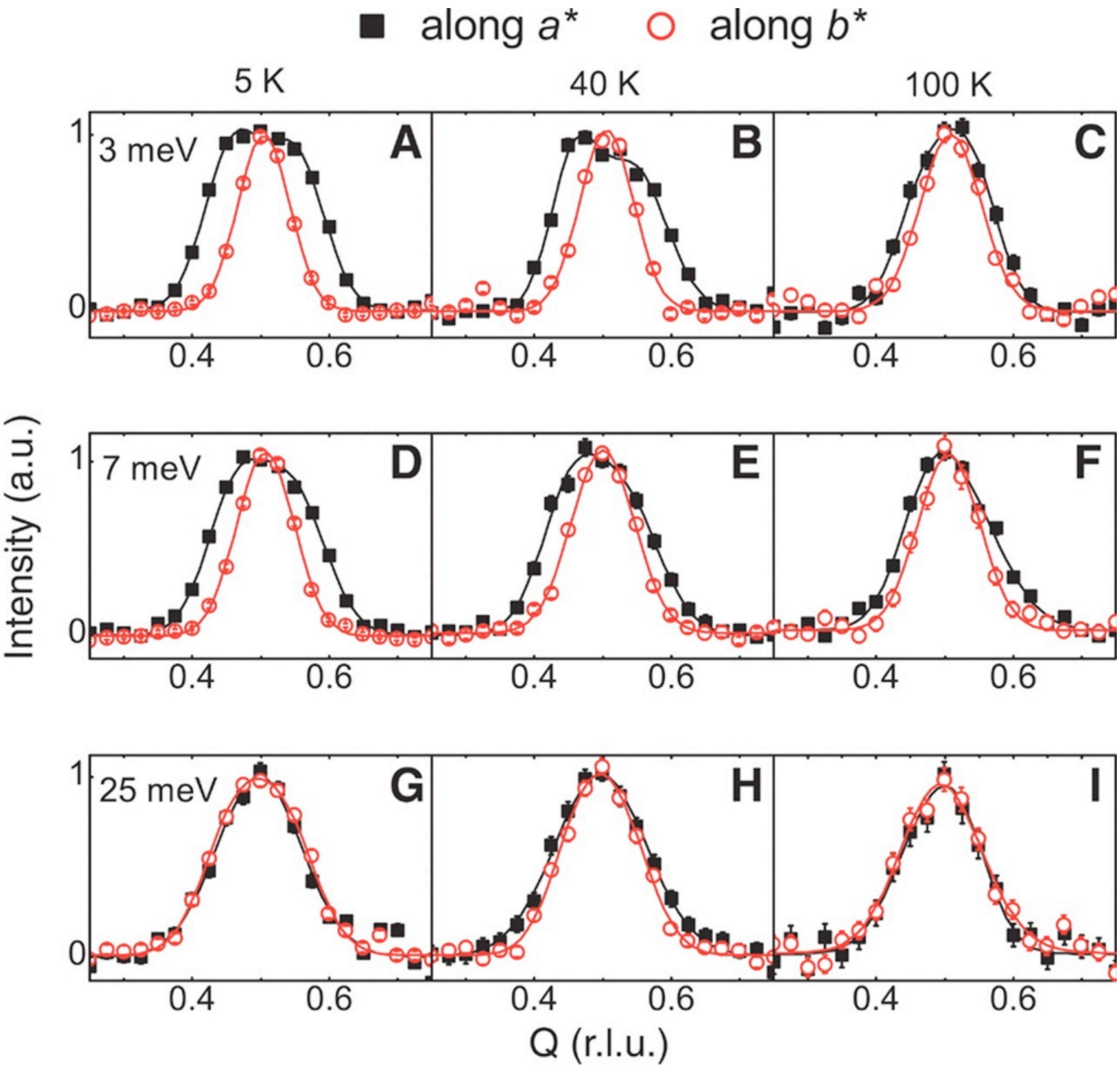


Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

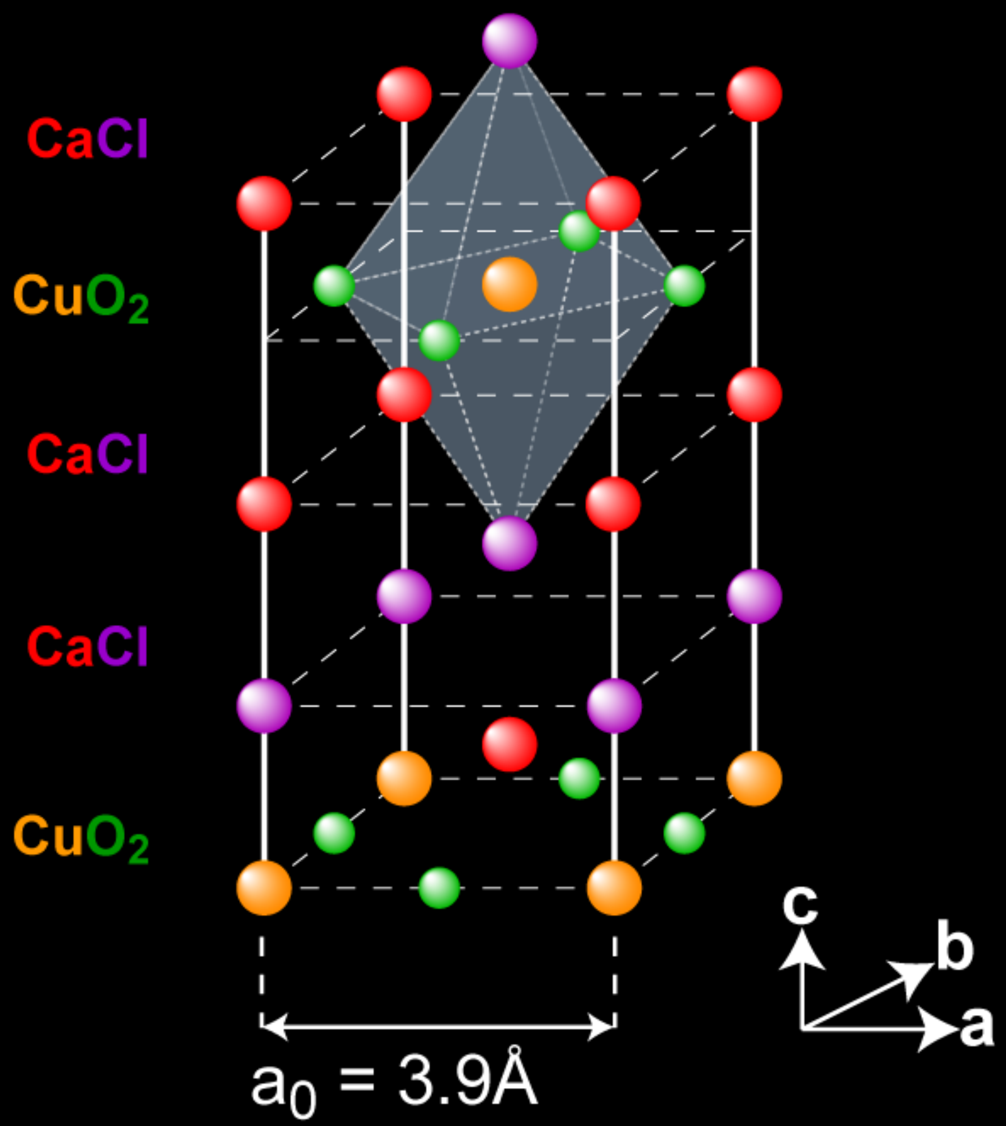


V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

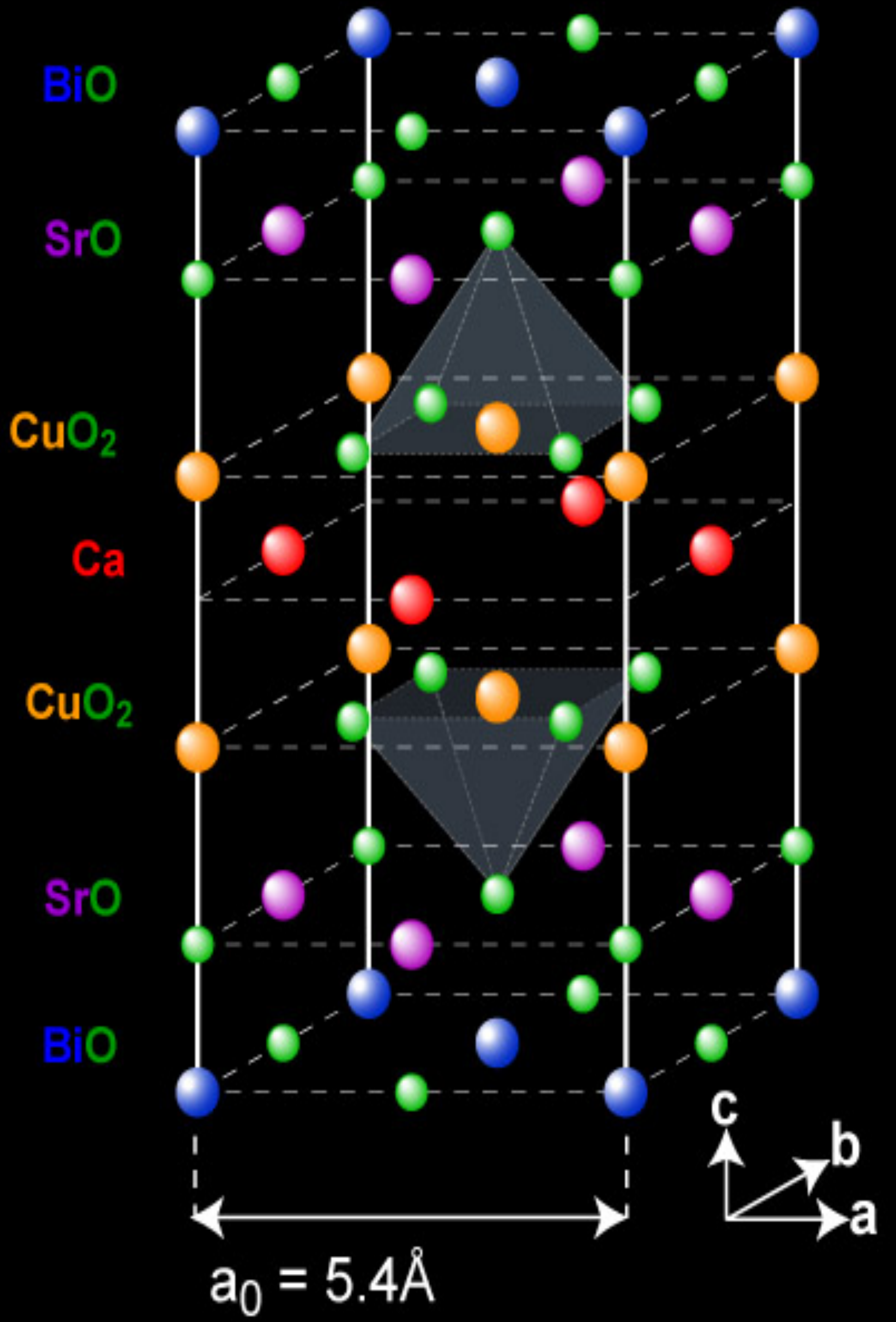
Nematic order in YBCO

STM studies of the underdoped superconductor

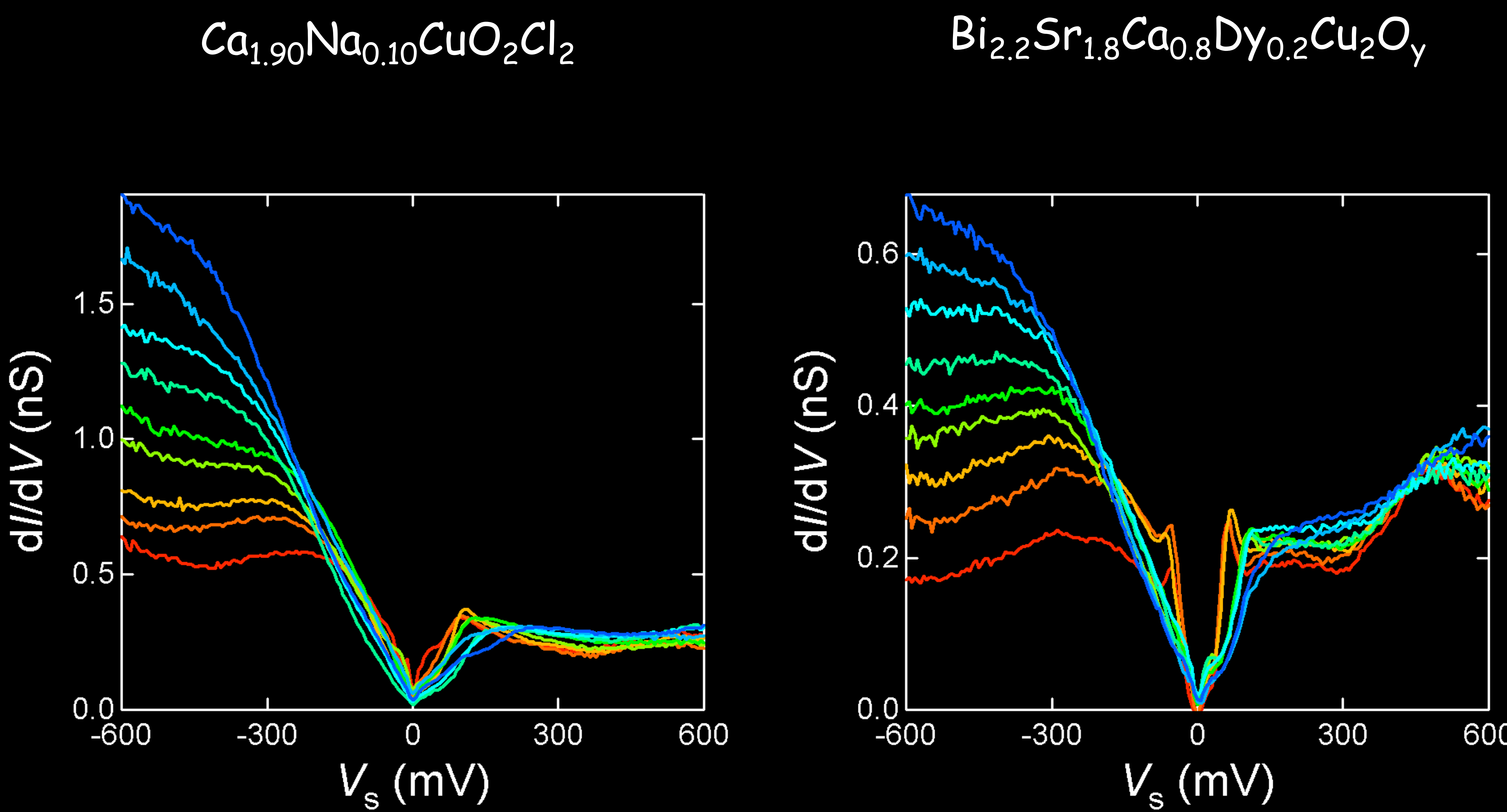
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



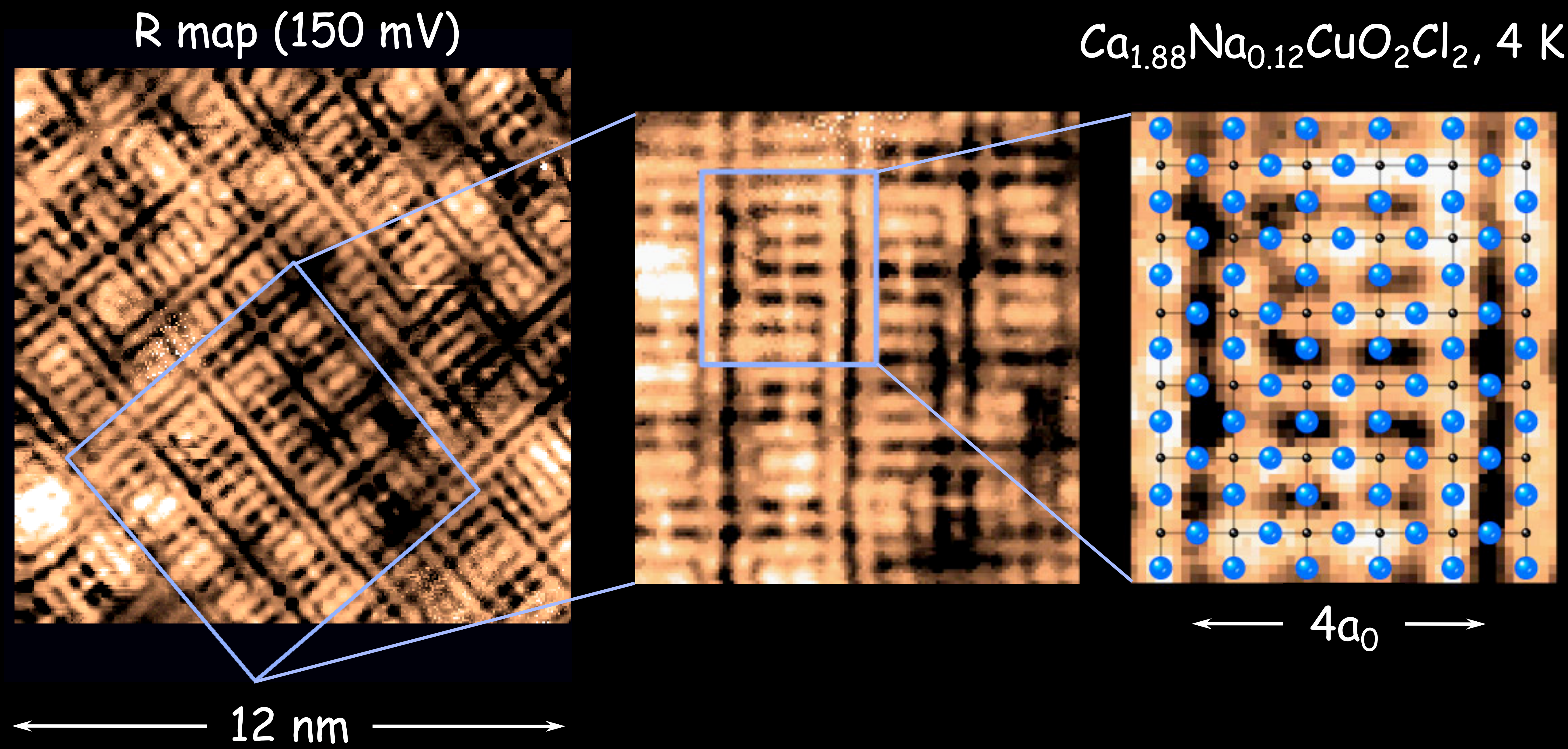
dI/dV Spectra



Intense Tunneling-Asymmetry (TA)
variation are highly similar

Y. Kohsaka et al. **Science 315**, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



Y. Kohsaka et al. Science 315, 1380 (2007)

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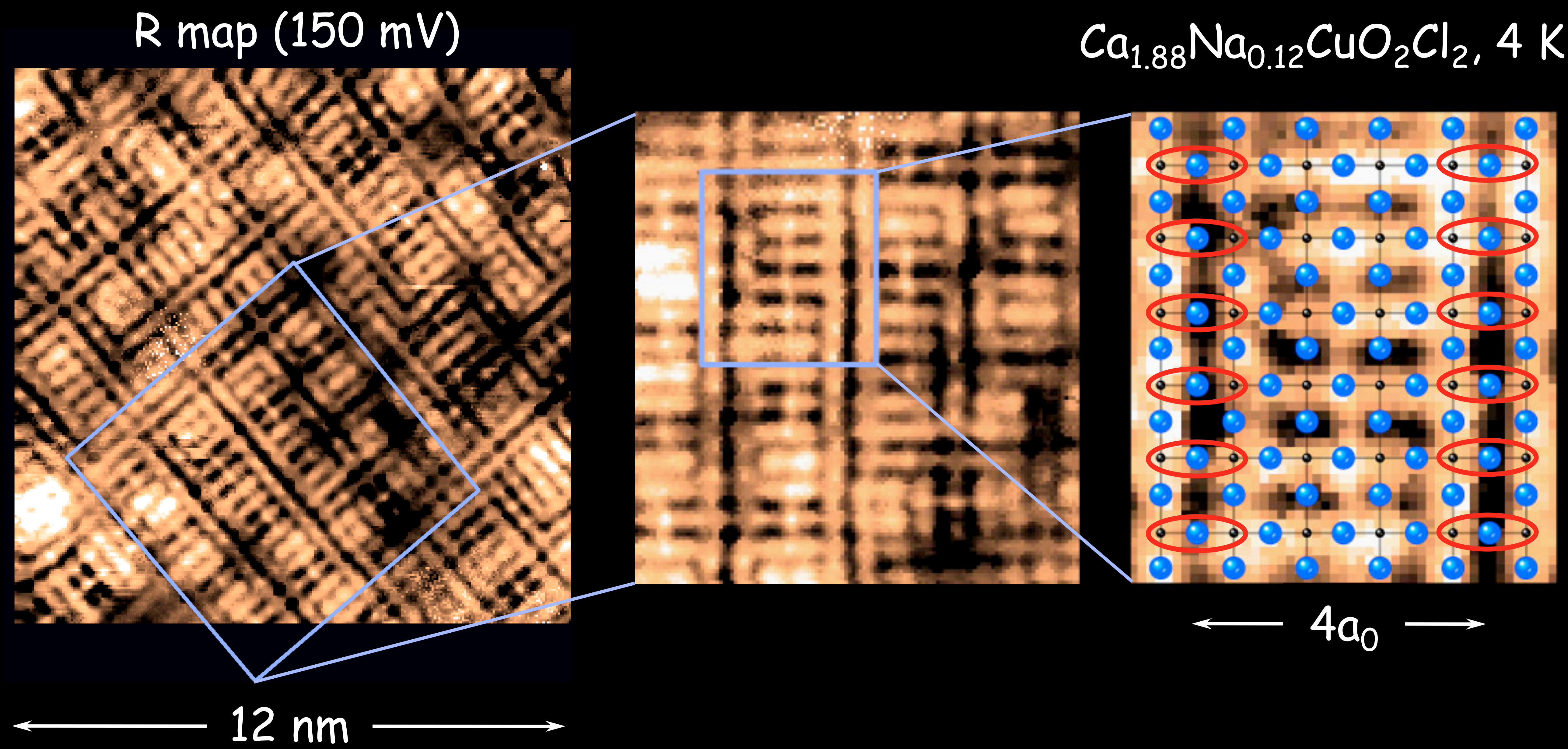

Broken lattice translation and rotation symmetries: valence bond supersolid or supernematic

S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991).
S. A. Kivelson, E. Fradkin, V. J. Emery, Nature 393, 550 (1998).
M. Vojta and S. Sachdev, Phys. Rev. Lett. 83, 3916 (1999).

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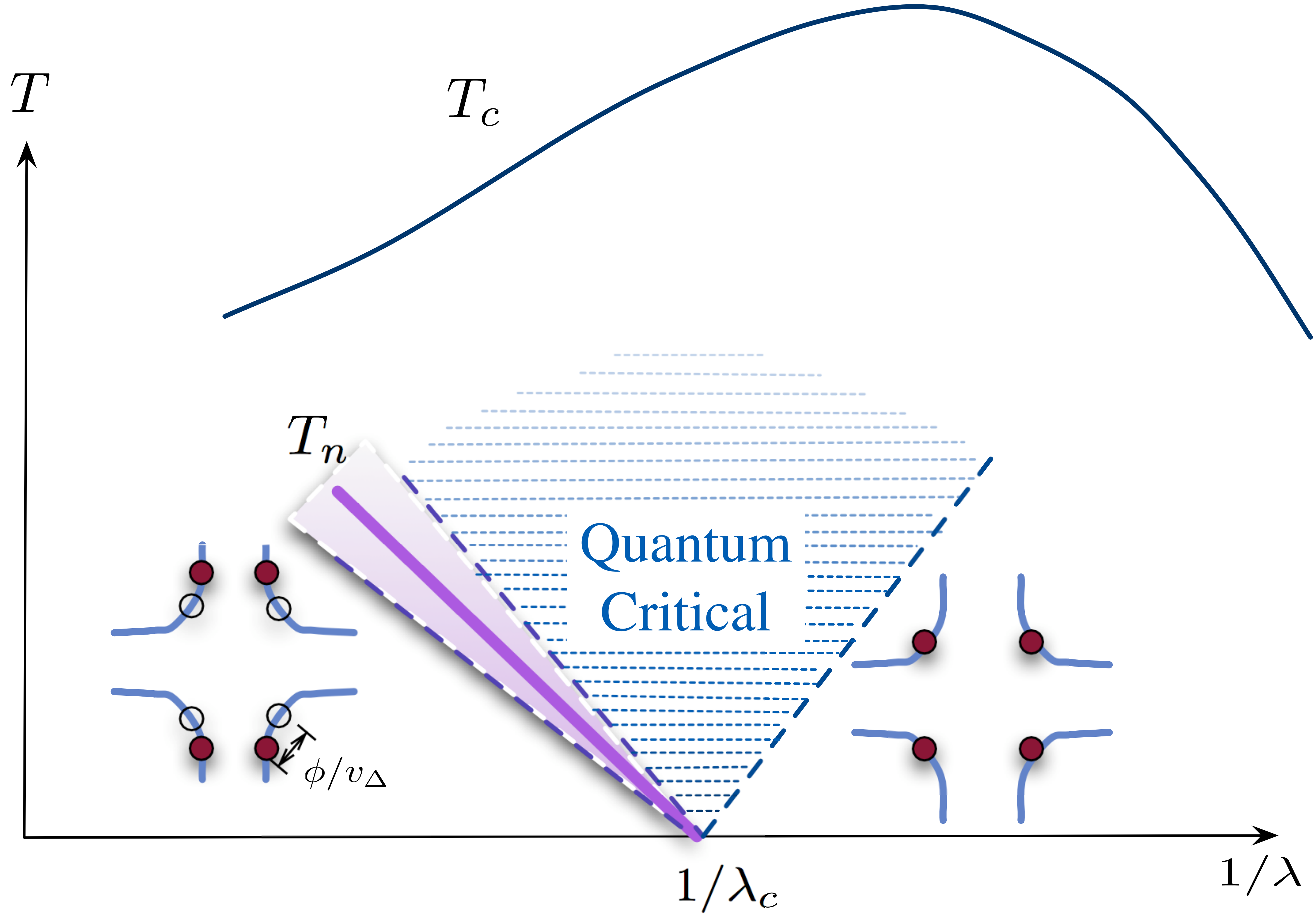
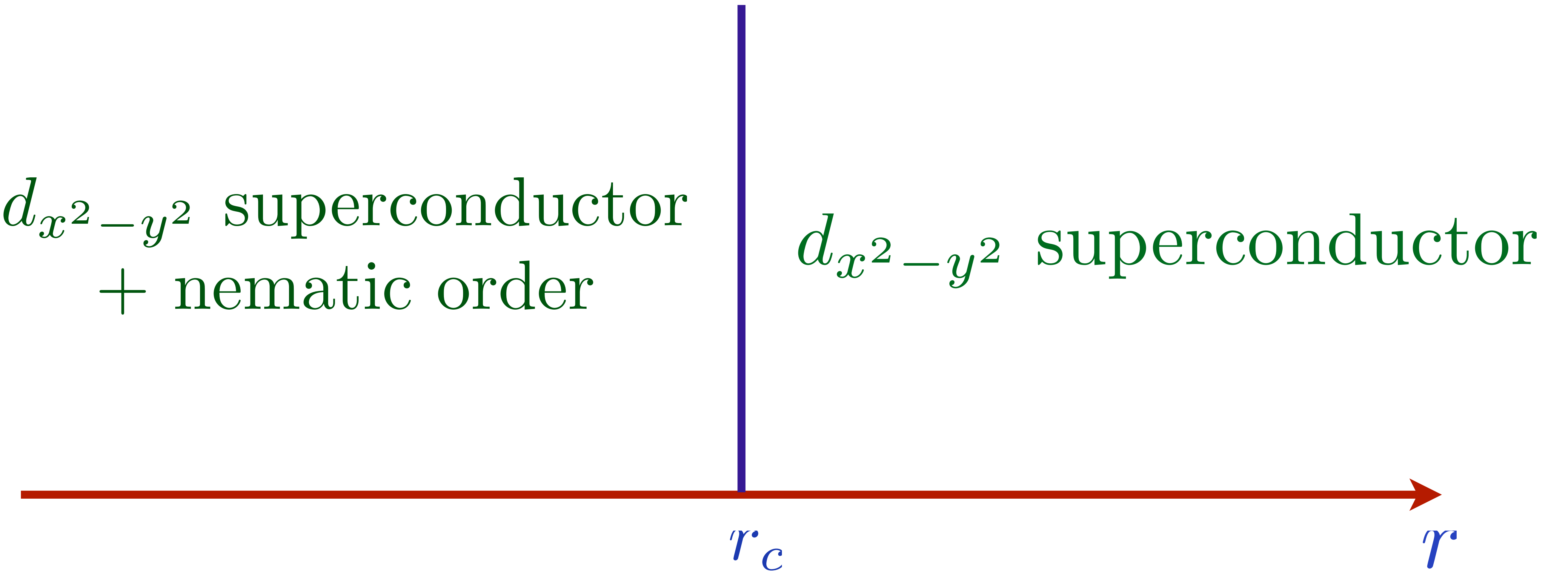
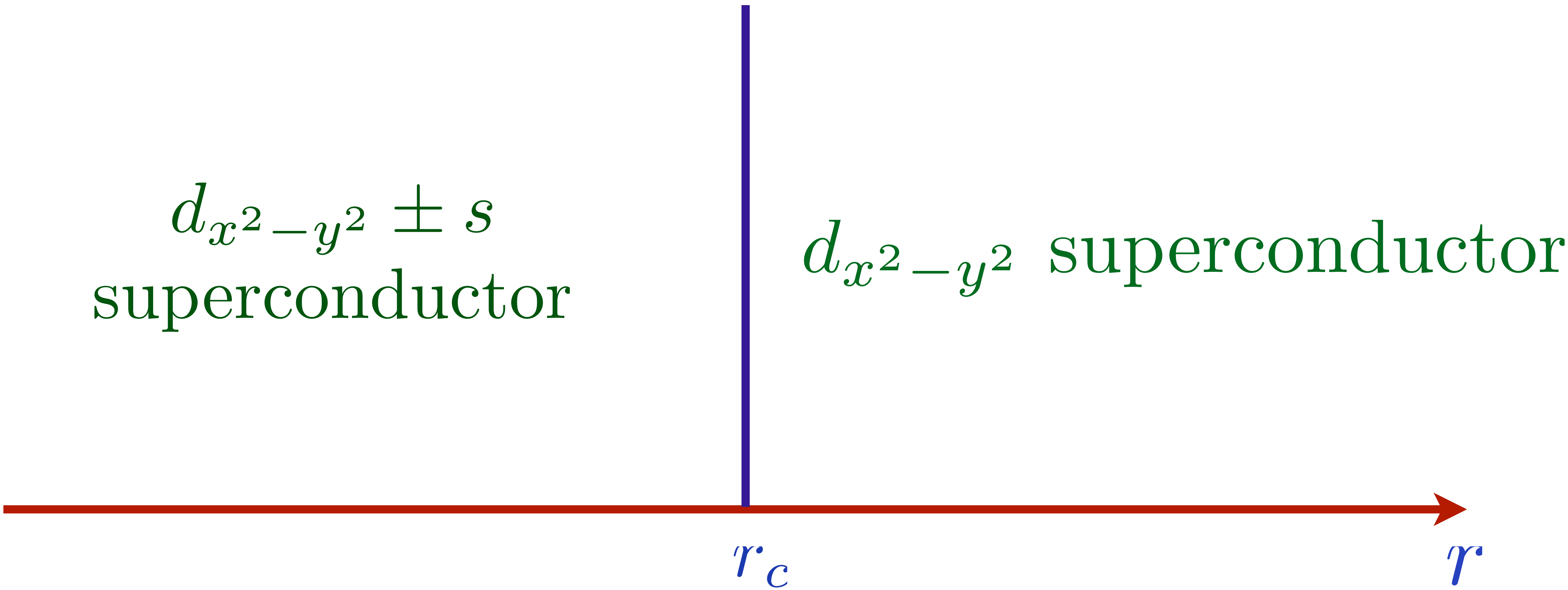


M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000)
E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099



$d_{x^2-y^2}$ superconductor
+ nematic order
$d_{x^2-y^2}$ superconductor
r_c
r



Order parameter - s pairing amplitude $\sim \phi$

Also $\phi \sim |\Phi_1|^2 - |\Phi_2|^2$

Field theory for dSC to dSC+nematic transition

$$S_\phi^0 = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

Ising theory for nematic ordering

$$\begin{aligned} S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{1a}^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x \right) \Psi_{1a} \\ &+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{2a}^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x \right) \Psi_{2a}. \end{aligned}$$

Free nodal quasiparticles

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Field theory for dSC to dSC+nematic transition

$$S_{\Psi\phi} = \int d^2x d\tau \Big[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^x \Psi_{1a} + \Psi_{2a}^\dagger \tau^x \Psi_{2a} \right) \Big],$$

Yukawa coupling is now permitted
and is strongly relevant

RG analysis close to 3 dimensions
yields runaway flow to strong coupling

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F}\Gamma\left[\lambda_0\phi(x,\tau);\frac{v_\Delta}{v_F}\right] + \frac{N_f}{2}\int d^2x d\tau \Big(r\phi^2(x,\tau)\Big) + \text{irrelevant terms}$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ/v_F.

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{2}\int_{k,\omega} |\phi(k,\omega)|^2 \Bigg[r + \frac{\lambda_0^2}{8 v_F v_\Delta}\left(\frac{\omega^2 + v_F^2 k_x^2}{\sqrt{\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2}} + (x \leftrightarrow y)\right)\Bigg]$$

$$+\text{higher order terms which cannot be neglected}$$

E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

Renormalization group analysis

Couplings are local in the fermion action, so perform RG on fermion self energy

The $1/N_f$ expansion has only one coupling constant at criticality: v_Δ/v_F.

The RG has the structure:

$$
\begin{aligned}
\text{dynamic critical exponent}: \quad z &= 1 + \frac{1}{N_f} F_1(v_\Delta/v_F) \\
\text{fermion anomalous dimension}: \quad \eta_f &= \frac{1}{N_f} F_2(v_\Delta/v_F) \\
\text{RG flow equation}: \quad \frac{d(v_\Delta/v_F)}{d\ell} &= \frac{1}{N_f} F_3(v_\Delta/v_F)
\end{aligned}
$$

where we have computed the functions $F_{1,2,3}(v_\Delta/v_F)$.

Renormalization group analysis

The RG flow is to $v_\Delta/v_F \to 0$ with

$$\frac{d(v_\Delta/v_F)}{d\ell} = -\frac{4}{\pi^2 N_f}\left(v_\Delta/v_F\right)^2 \ln\left(\frac{0.46987}{(v_\Delta/v_F)}\right)$$

This implies that at the critical point, as $T \to 0$ we have the asymptotic result

$$\frac{v_\Delta}{v_F} = \frac{\pi^2 N_f}{4}\frac{1}{\ln\left(\frac{\Lambda}{T}\right)\ln\left[\frac{0.1904}{N_f}\ln\left(\frac{\Lambda}{T}\right)\right]},$$

and a more precise result is obtained by numerically integrating the RG equation.

Renormalization group analysis

In the limit $v_\Delta / v_F \to 0$, the effective action becomes

$$
\begin{aligned}
S_\phi &= \frac{N_f}{v_\Delta v_F} \Gamma \left[\lambda_0 \phi(x,\tau); \frac{v_\Delta}{v_F} \right] \\
&\quad + \frac{N_f}{2} \int d^2x d\tau \Big(r \phi^2(x,\tau) \Big) \\
&\quad + \text{irrelevant terms}
\end{aligned}
$$

Renormalization group analysis

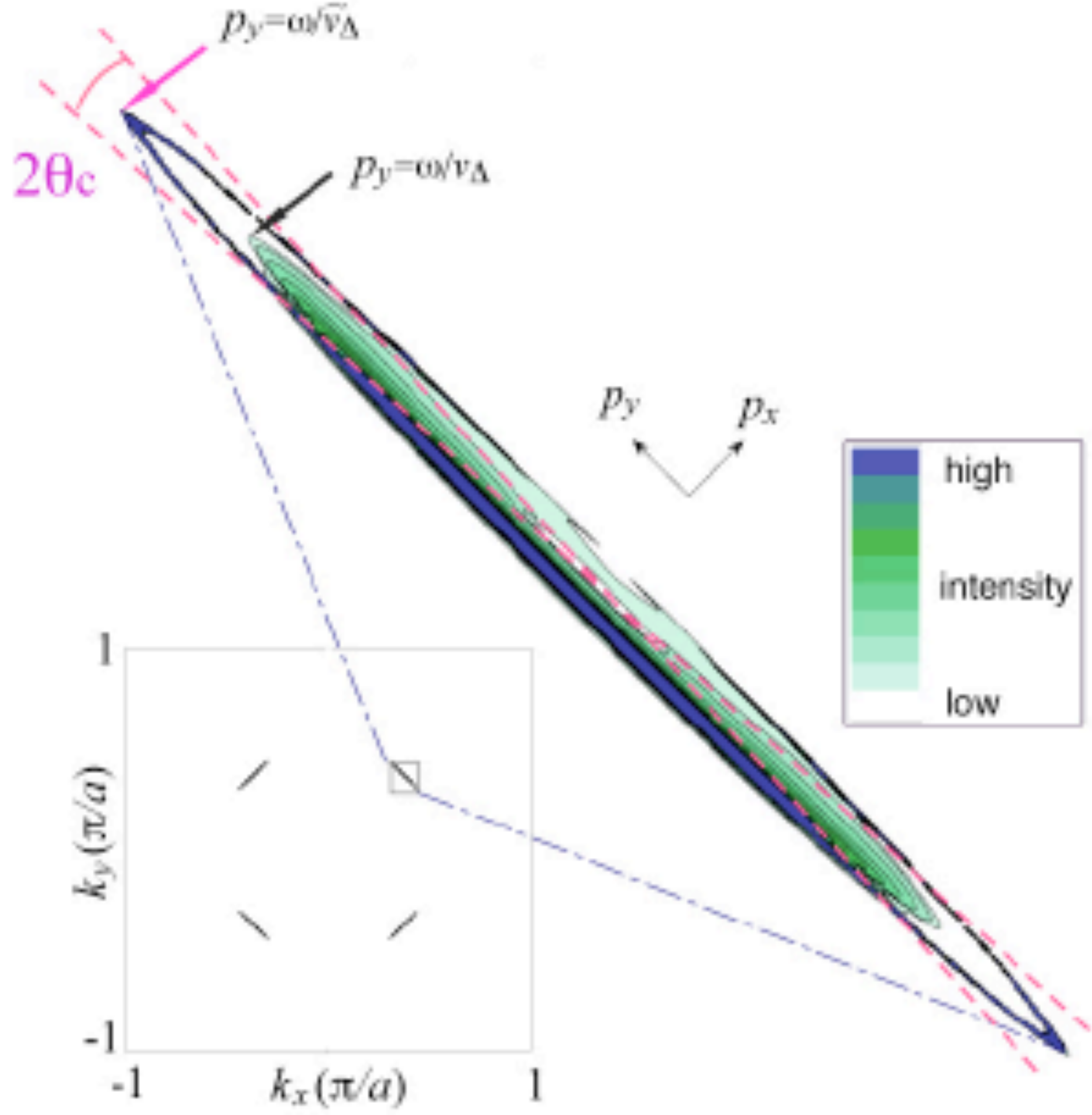
In the limit $v_\Delta/v_F \to 0$, the effective action becomes

$$S_\phi \sim \frac{N}{v_\Delta}\Big[\text{terms that have at most a divergence which is a power of} \ln(v_F/v_\Delta)\Big]$$

$\Rightarrow$ The theory is controlled by the expansion parameter v_Δ/N_f, and so results are asymptotically exact even for $N_f = 2$.

Fermion spectral functions

ϕ fluctuations broaden the fermion spectral functions except in a wedge near the nodal points



(a)$A(\vec{p}, -9meV)$

E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

Conclusions

1. Theories for damping of nodal quasiparticles in cuprates

2. SDW theory for LSCO yields (nearly) quantum critical spectral functions with arbitrary values of v_Δ / v_F

3. Nematic theory for YBCO/BSCCO has a fixed point with $v_\Delta / v_F = 0$ which is approached logarithmically. The theory is expressed as an expansion in v_Δ / v_F

4. Theories yield "Fermi arc" spectra.