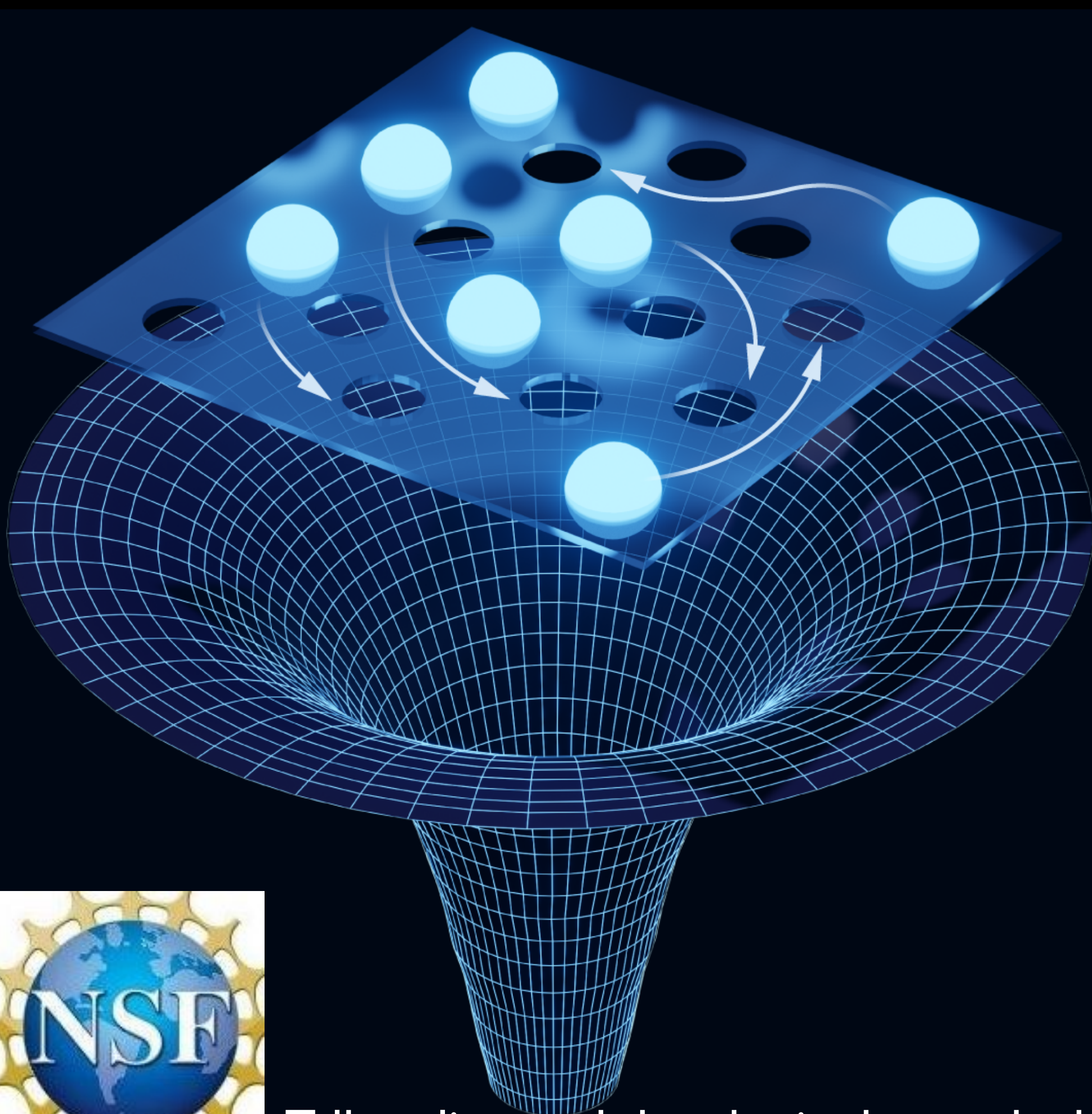


Quantum entanglement in nature:

superconductors and black holes



University of Michigan
April 3, 2024

Subir Sachdev



Talk online: sachdev.physics.harvard.edu



Great discoveries in physics

Quantum entanglement (1865, 1935)

Statistical mechanics (1870)

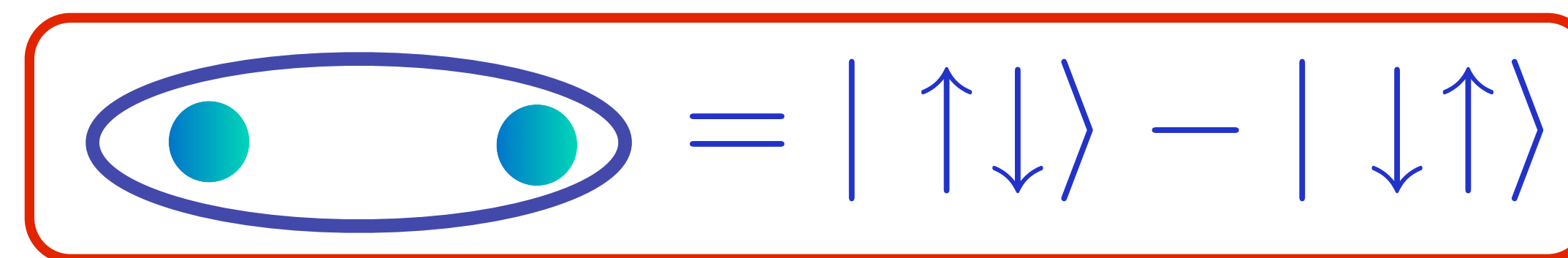
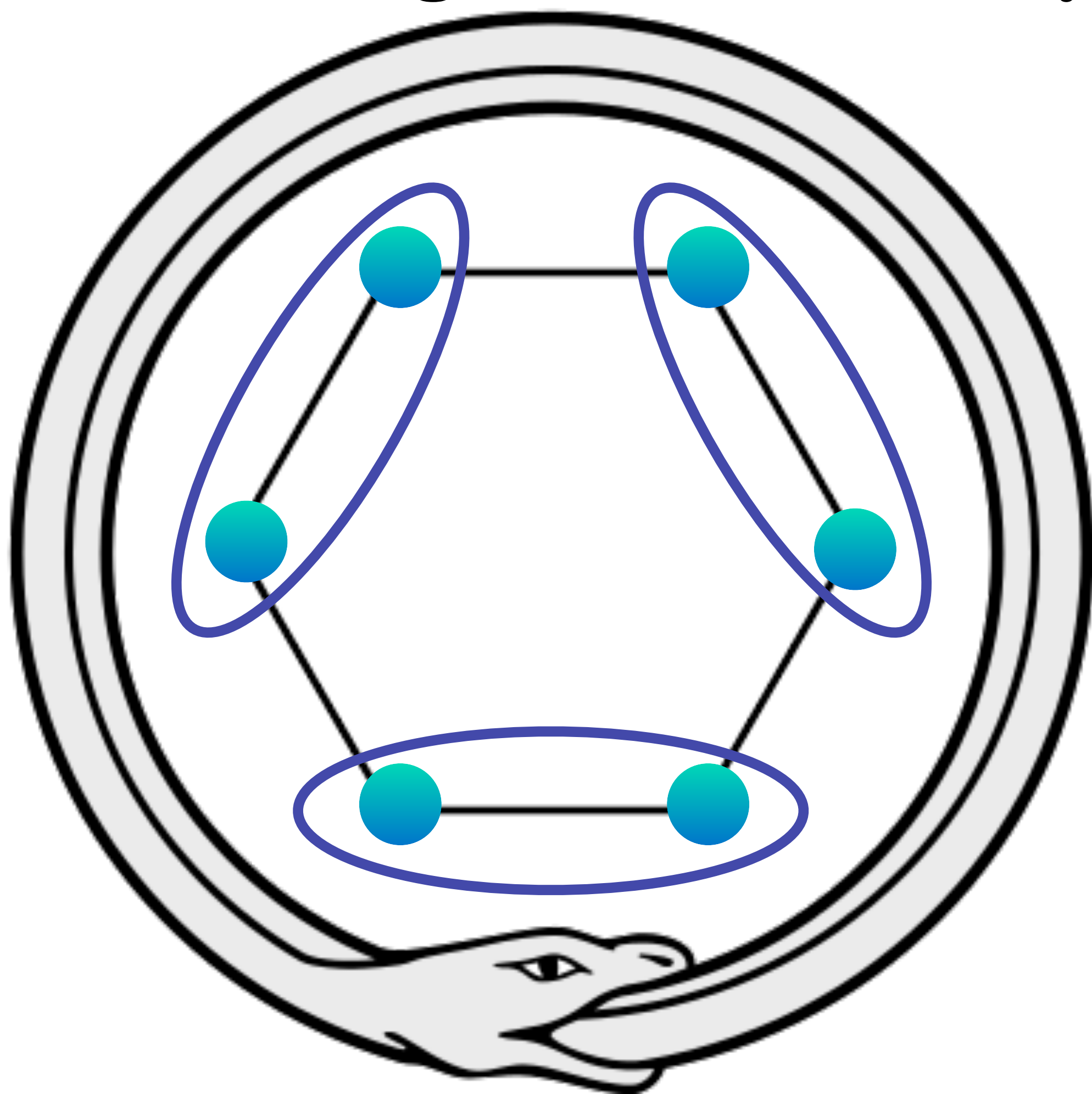
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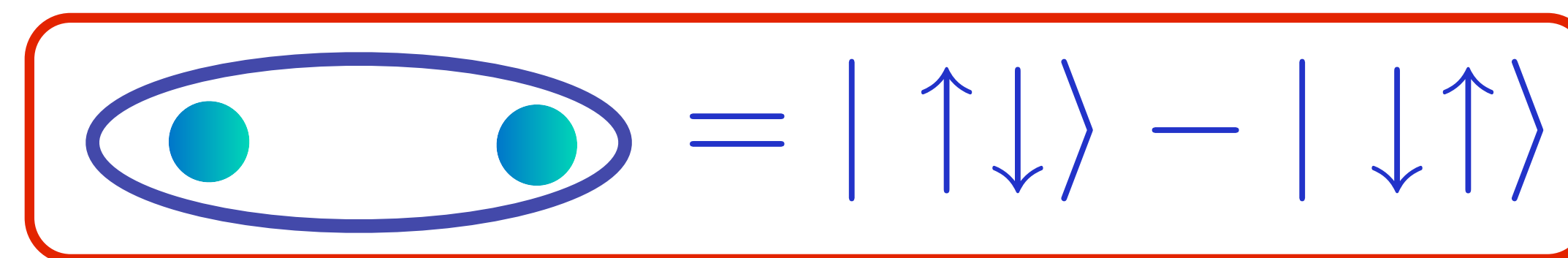
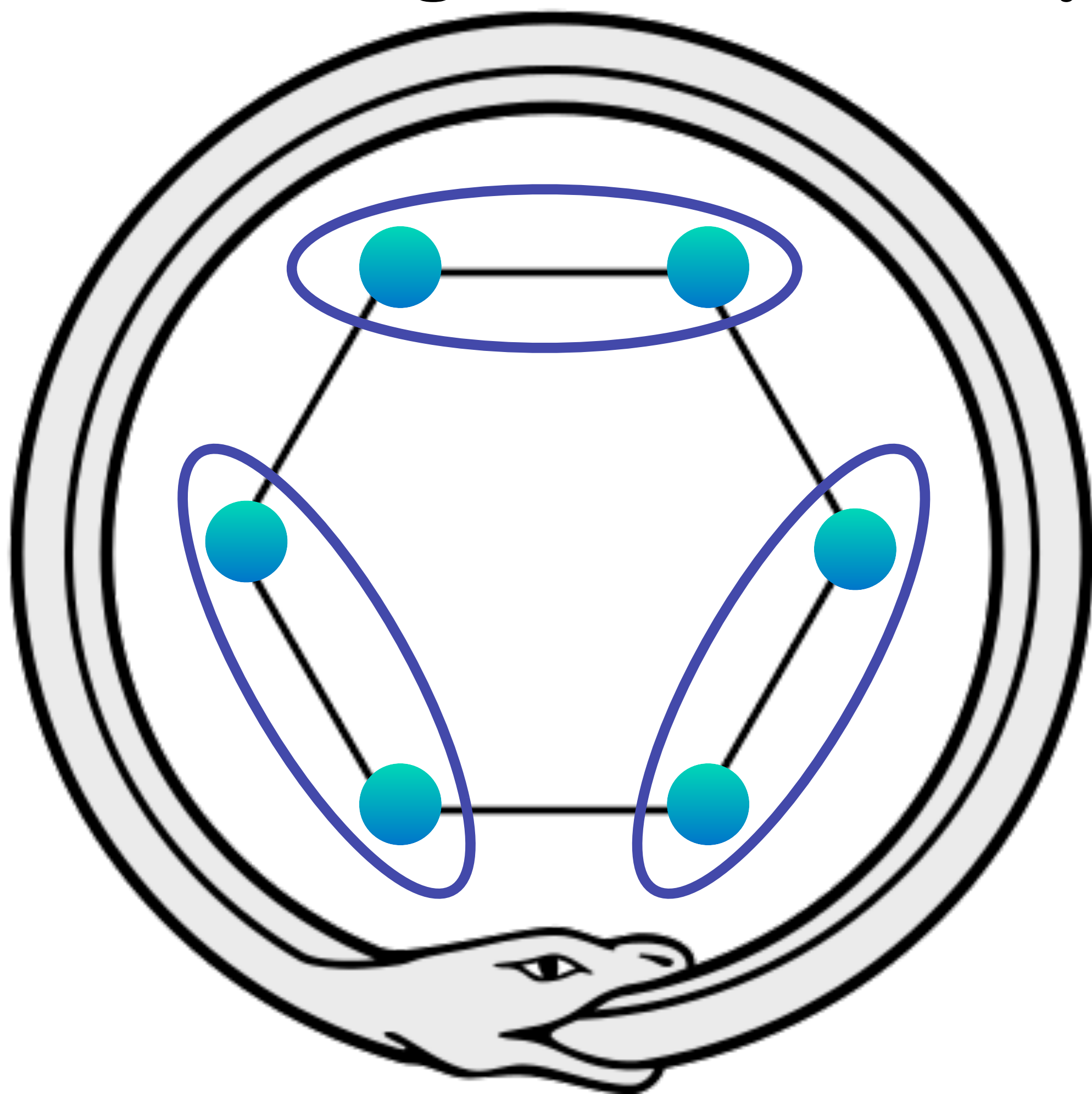
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Benzene

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Benzene

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

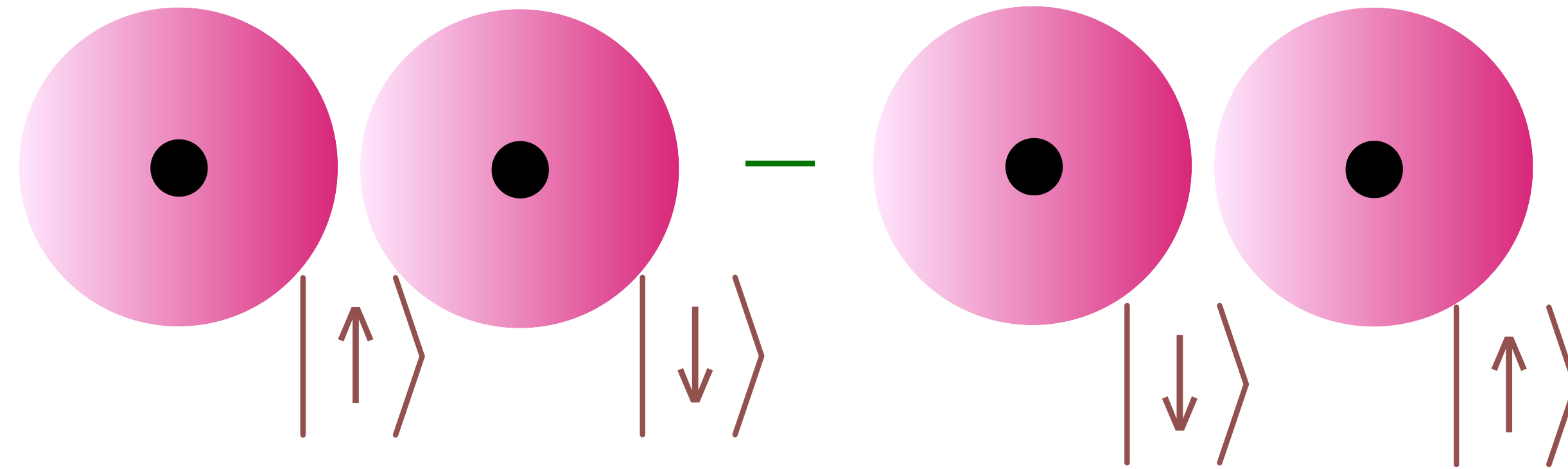
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

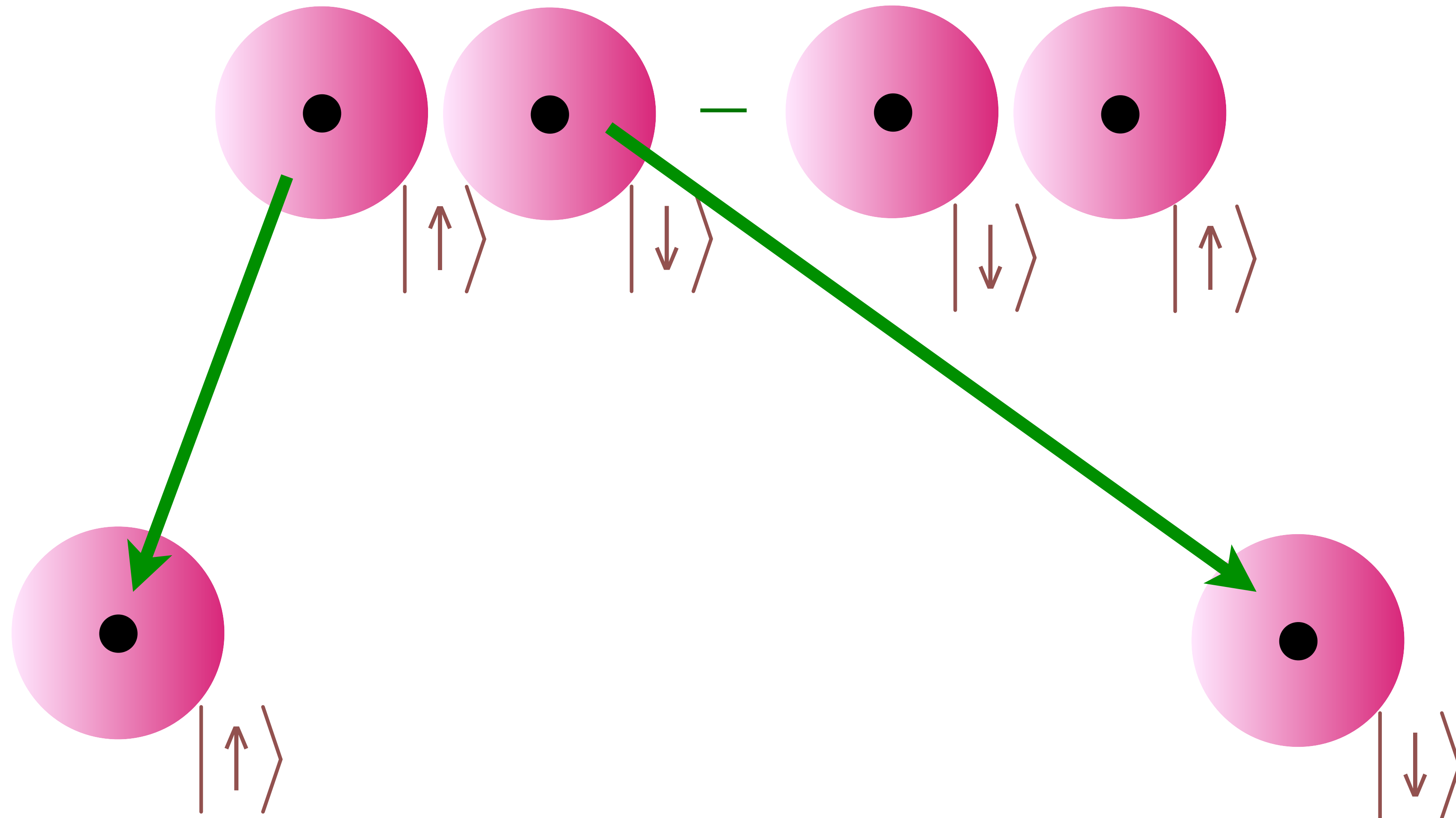
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



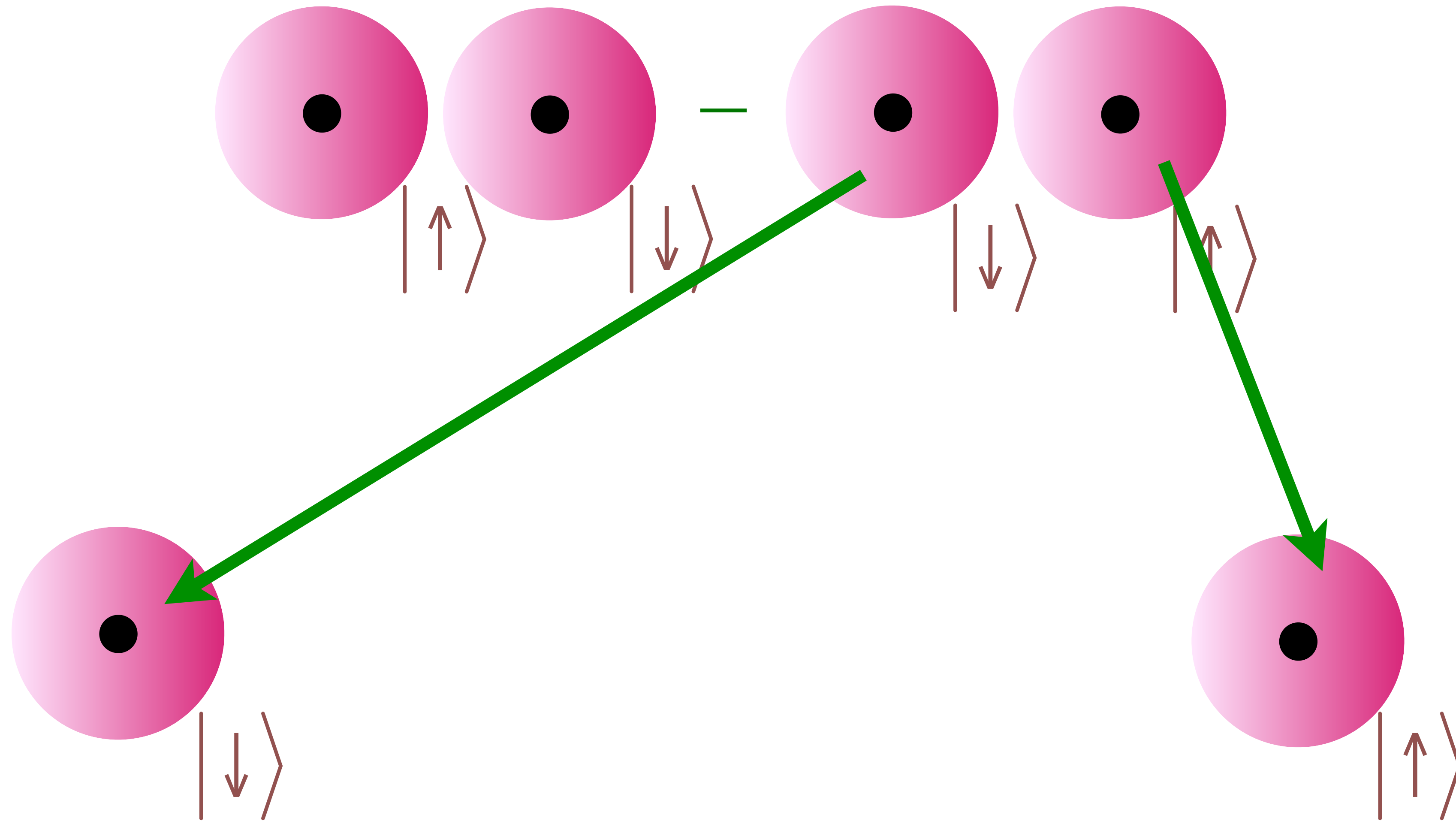
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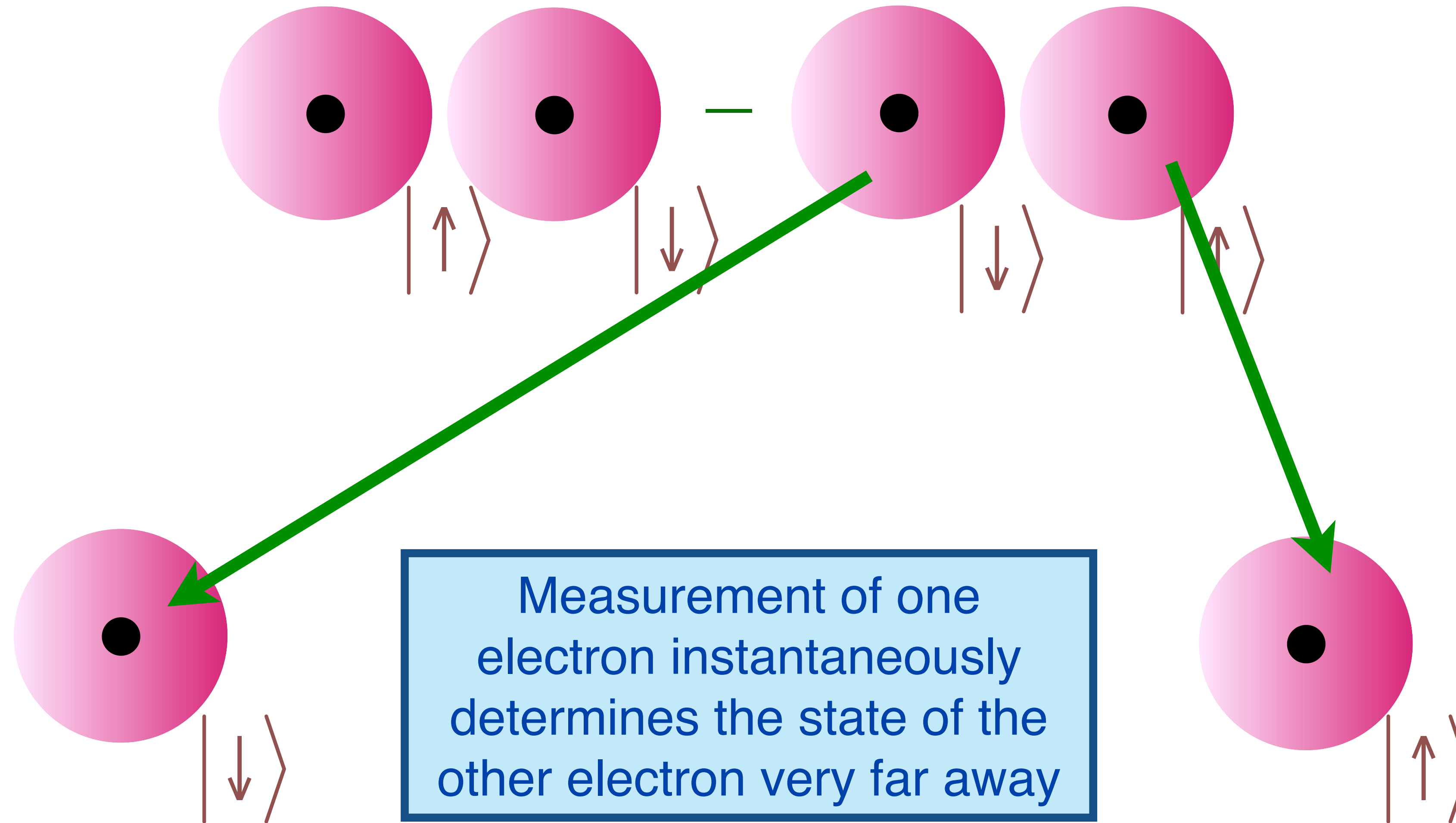
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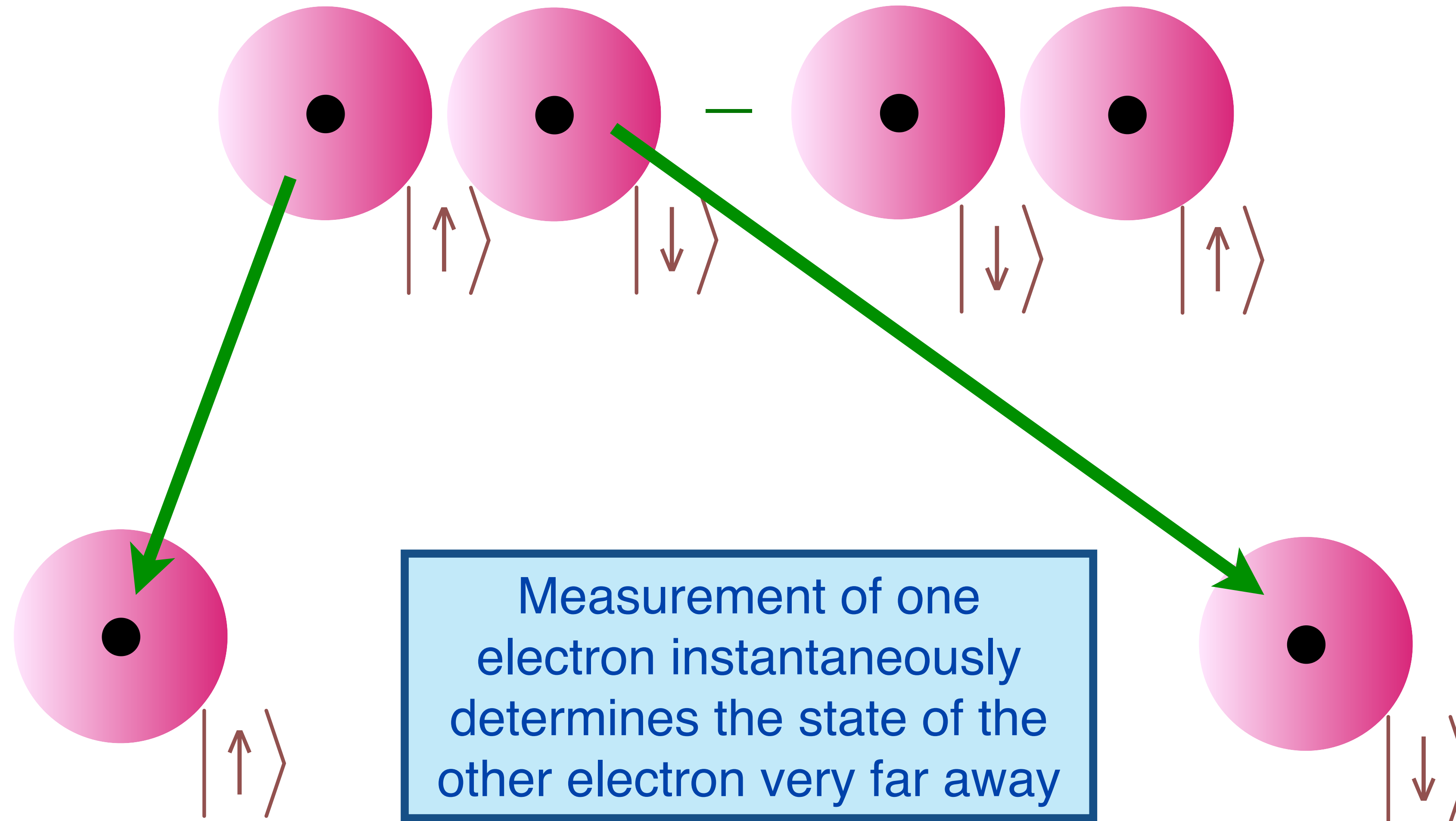
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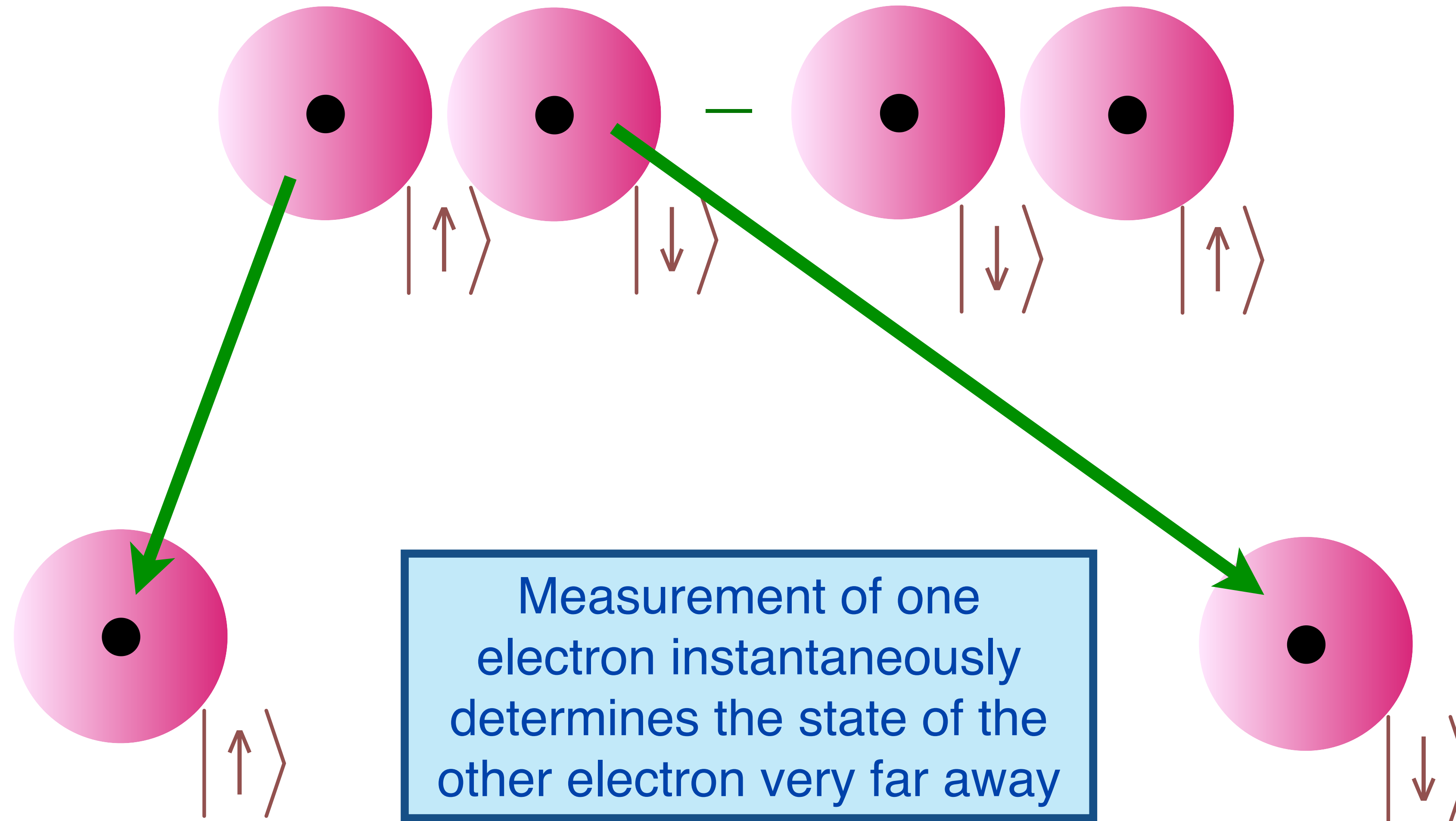
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Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Spooky action at a distance !

natürlicher
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt daß es wirklich mit der Theorie

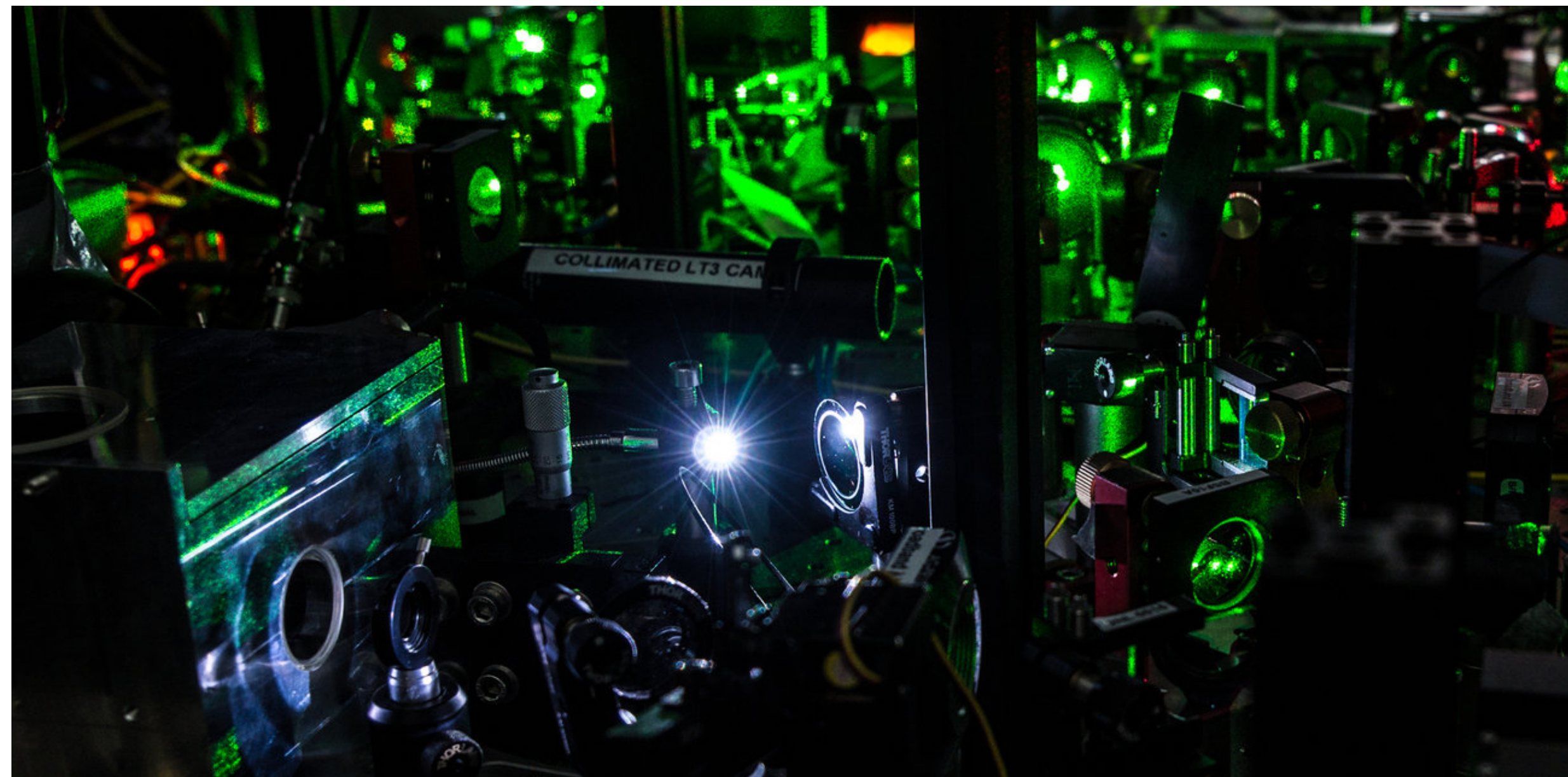
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

The New York Times

Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By JOHN MARKOFF OCT. 21, 2015



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Quantum entanglement plays a
central role in physics today:

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1. Qubit devices (quantum computers)

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3. Hawking's black hole quantum information problem

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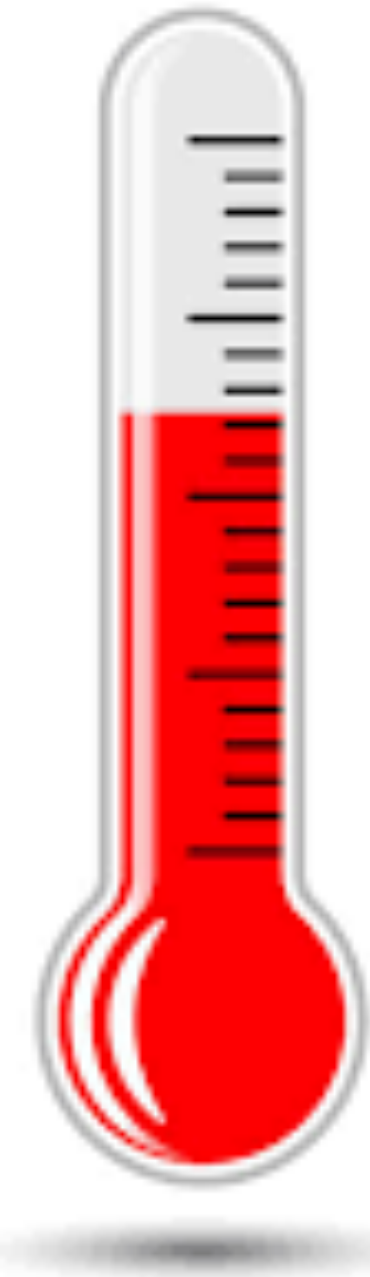
Superconductivity (1911)

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Statistical mechanics (1870)

Clausius (1865):
Second Law of Thermodynamics:
Every macroscopic system has an
“entropy” which cannot decrease.

➔ No perpetual motion machines!



Temperature

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$



Ludwig Boltzmann

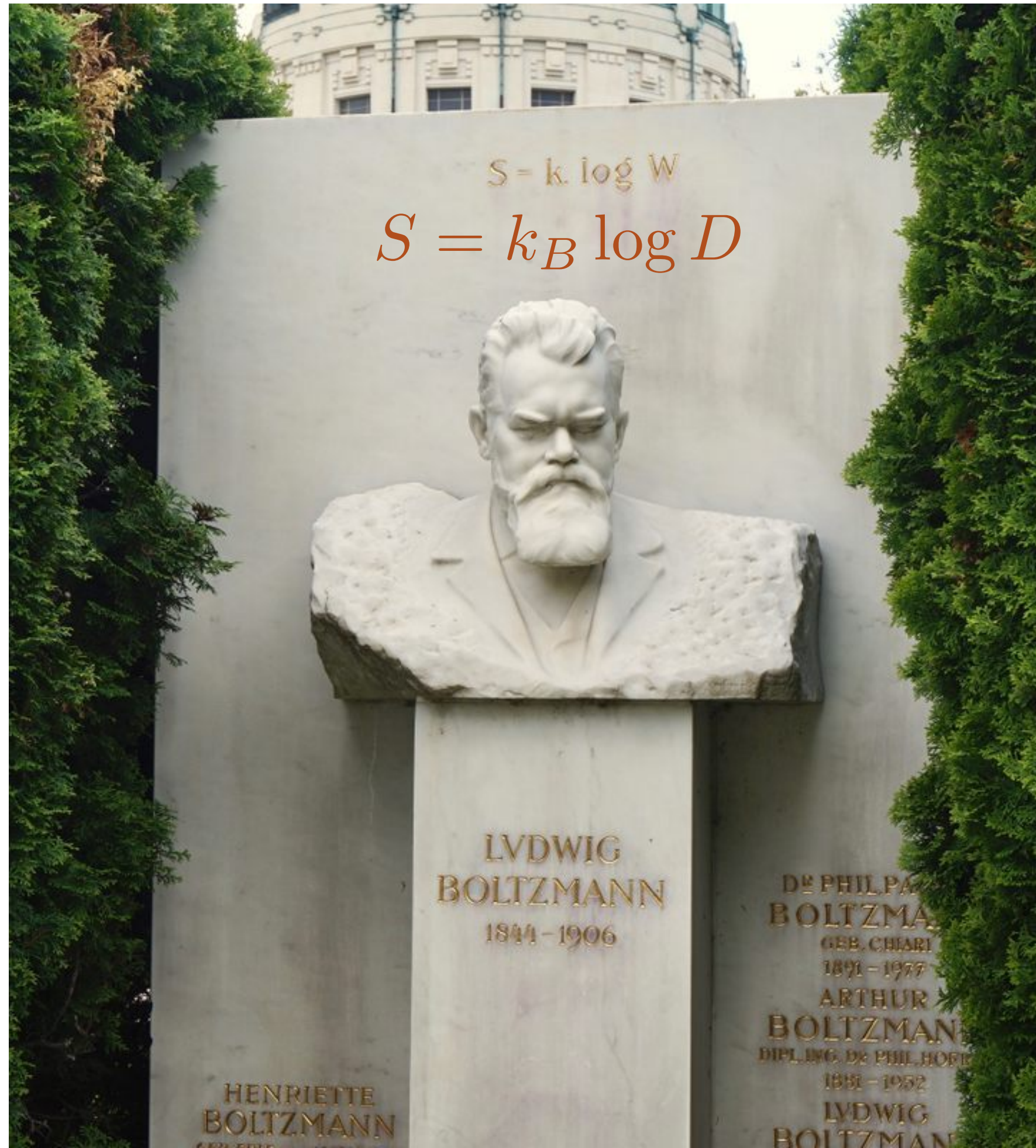
20 February 1844 - September 5, 1906

Vienna, Austria

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Density of quantum states $D(E) = \exp(S(E)/k_B)$



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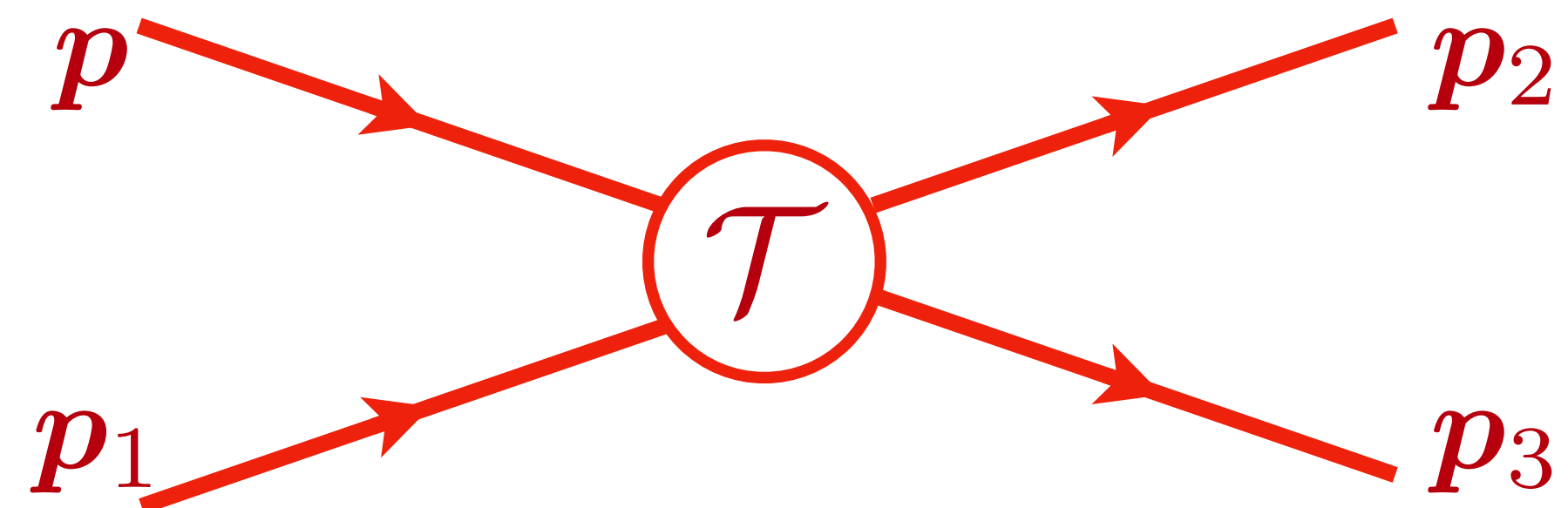
Vienna, Austria

Boltzmann equation (1872)

Dilute classical gas

Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
$$- 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$
$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} - f_{\mathbf{p}_2} f_{\mathbf{p}_3}]$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

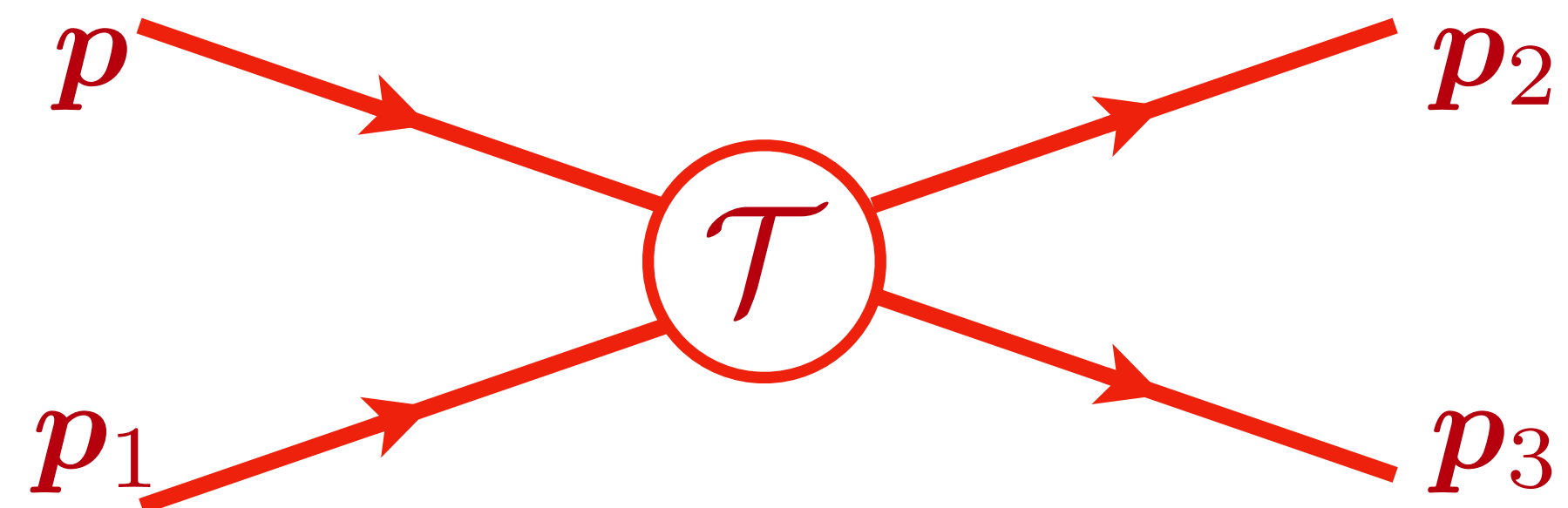
Vienna, Austria

Quantum Boltzmann equation (Landau)

Dense gas of electrons

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
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$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} (1 - f_{\mathbf{p}_2}) (1 - f_{\mathbf{p}_3}) - f_{\mathbf{p}_2} f_{\mathbf{p}_3} (1 - f_{\mathbf{p}}) (1 - f_{\mathbf{p}_1})]$$

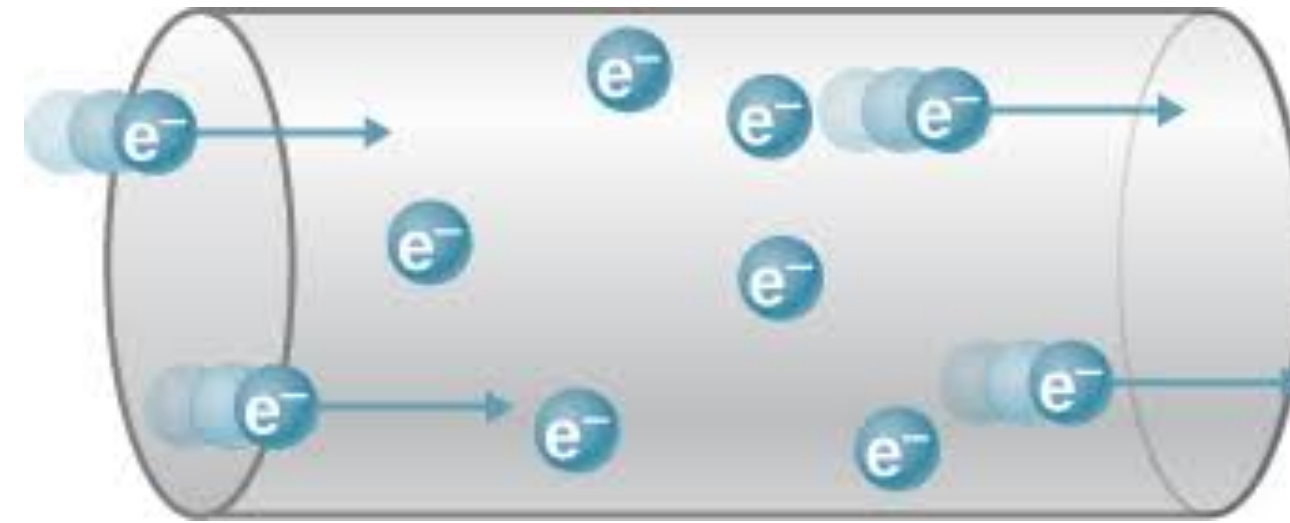


Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation \Rightarrow
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting ‘Planckian time’ $\frac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

Great discoveries in physics

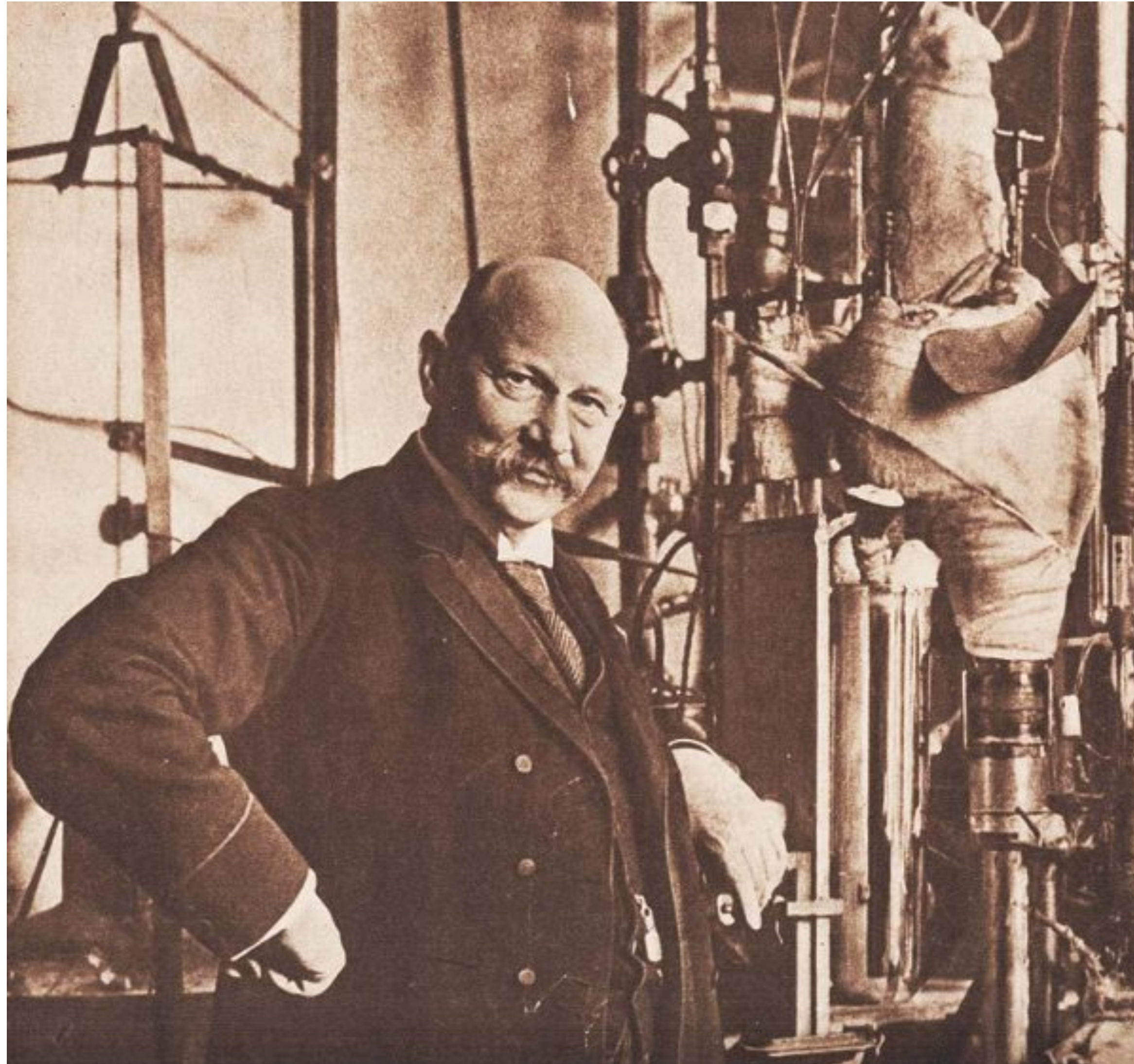
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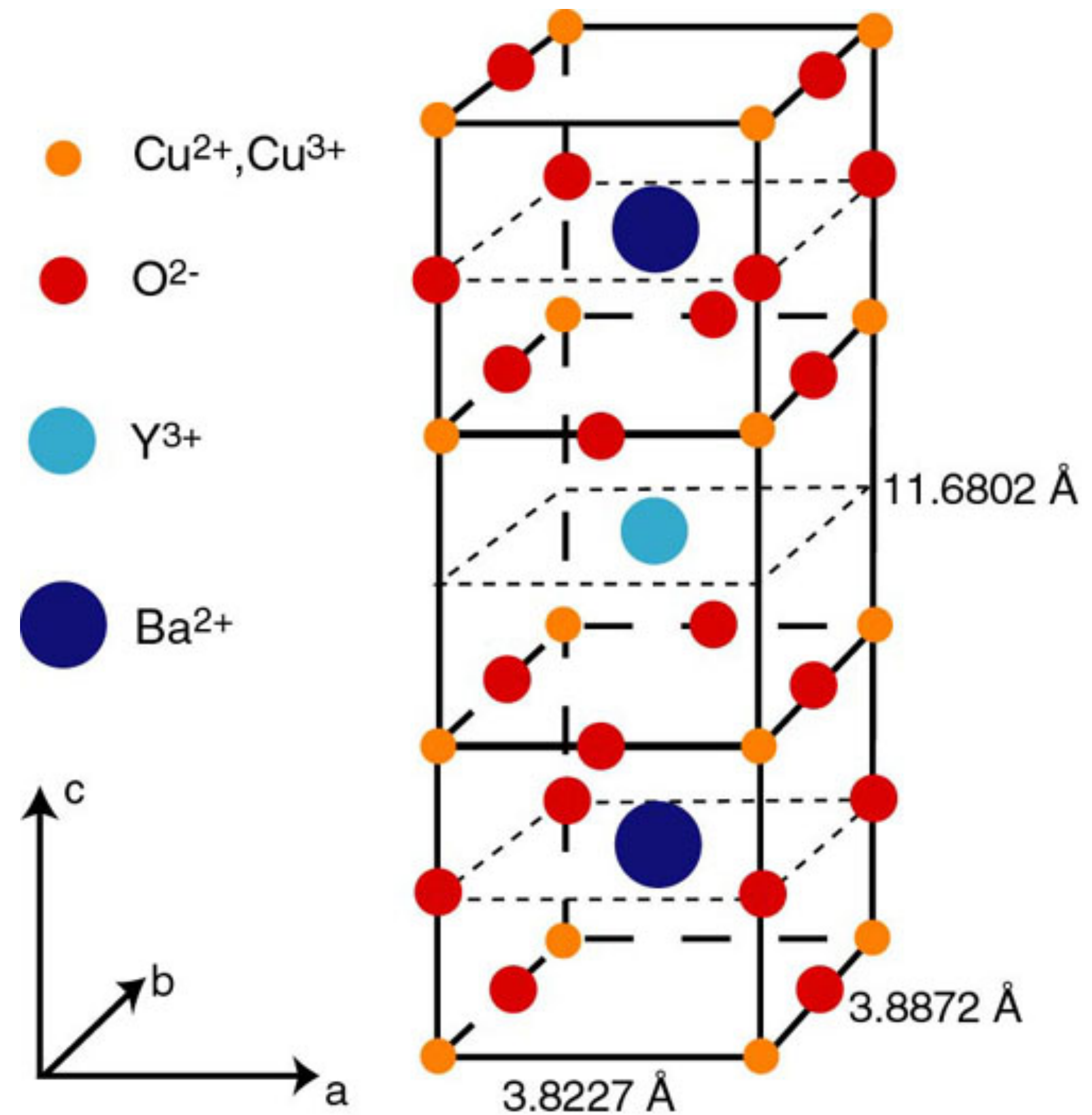
Black holes (1916)

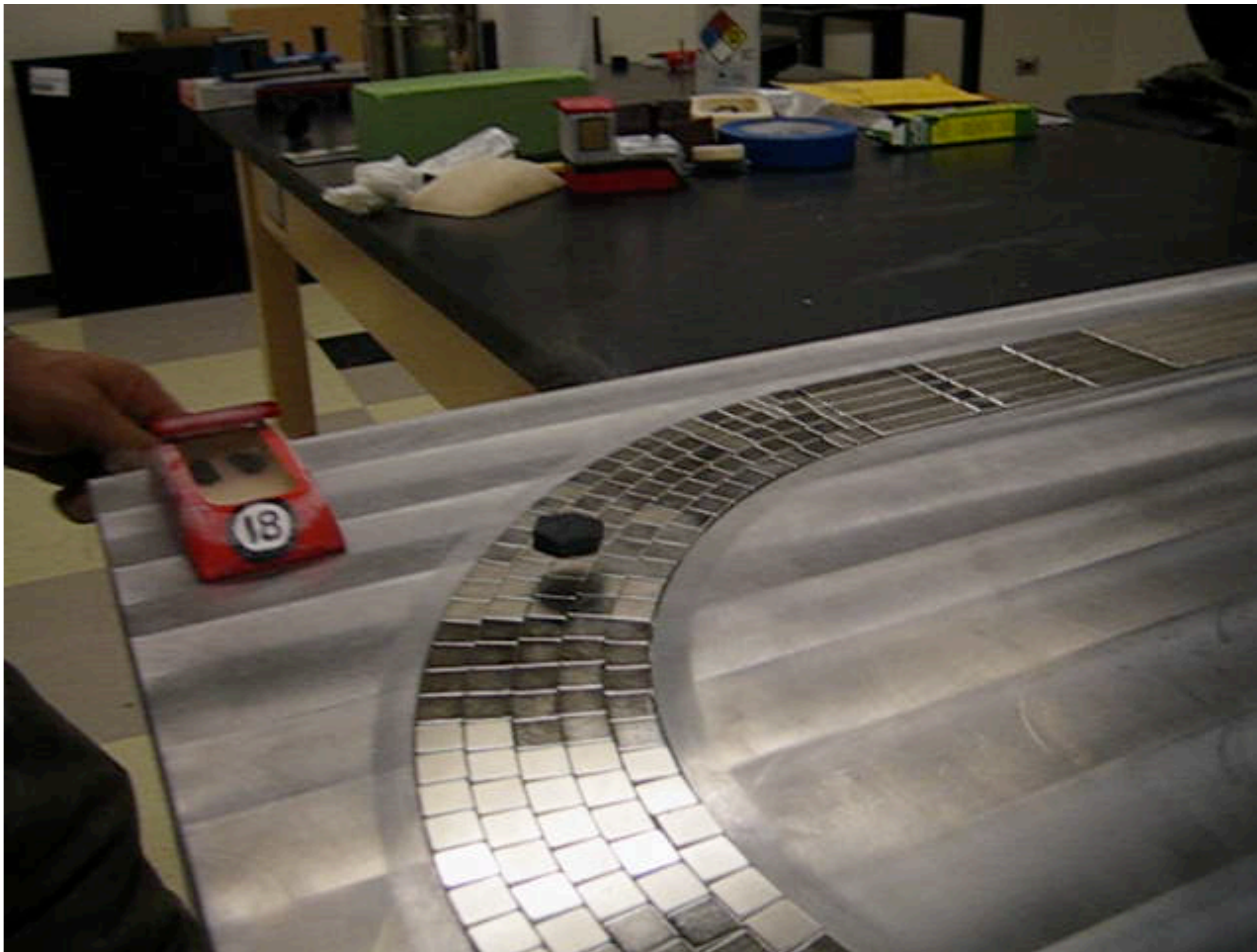
Superconductors (1911 - today)



Kamerlingh Onnes 1911:
Mercury is a superconductor below $-269\text{ }^{\circ}\text{C}$

Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

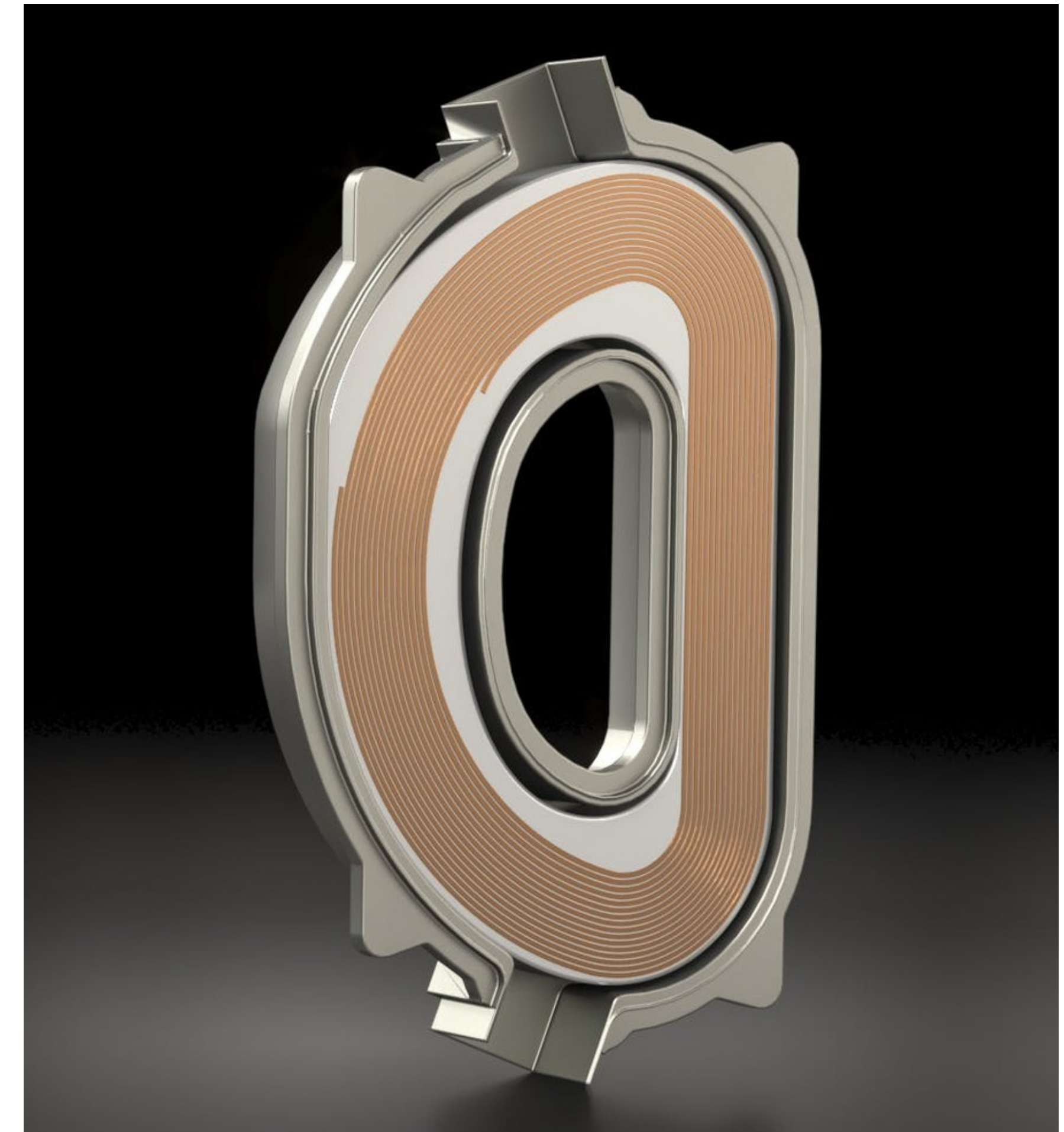
HTS Magnets: Enabling Technology

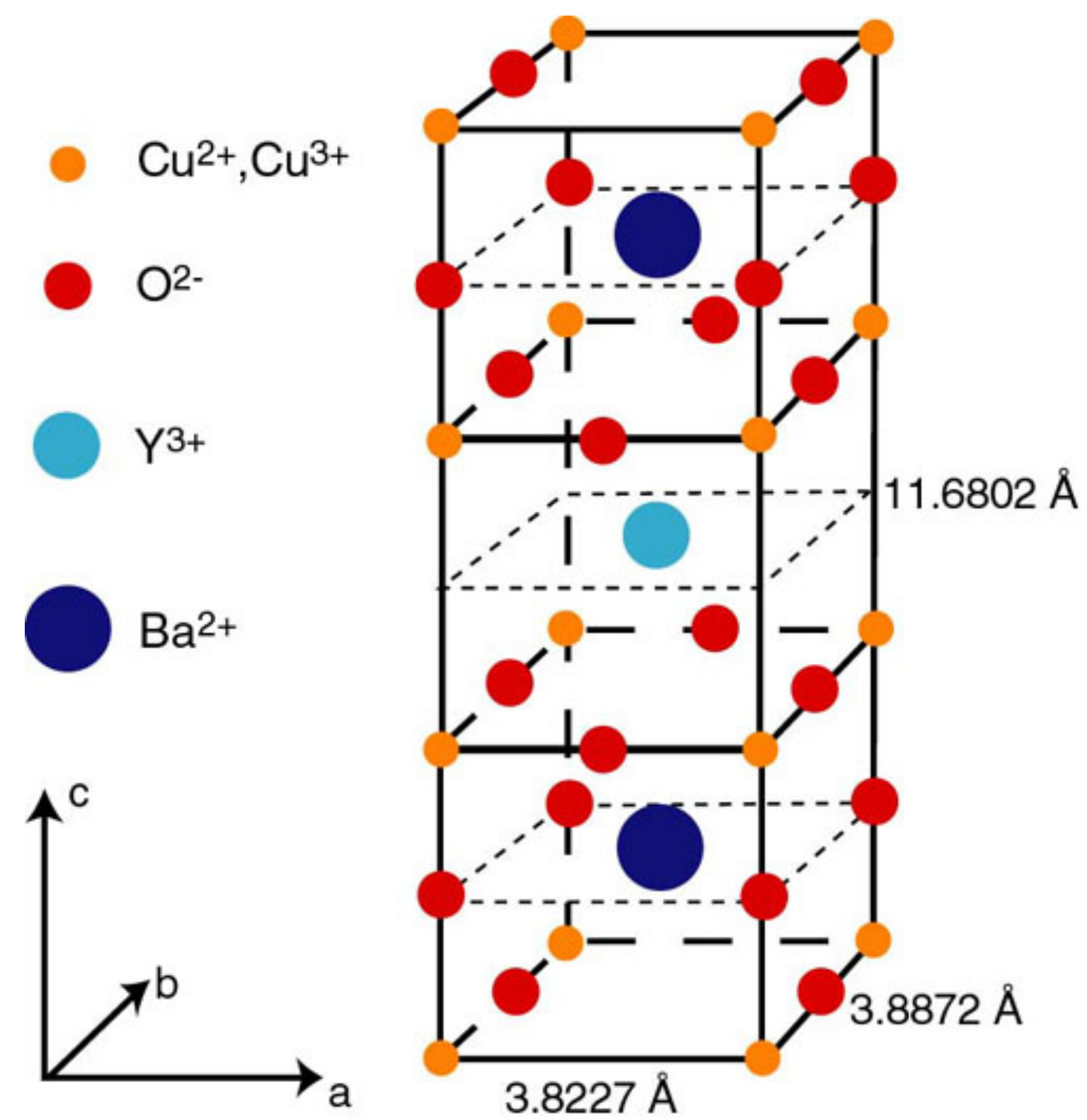
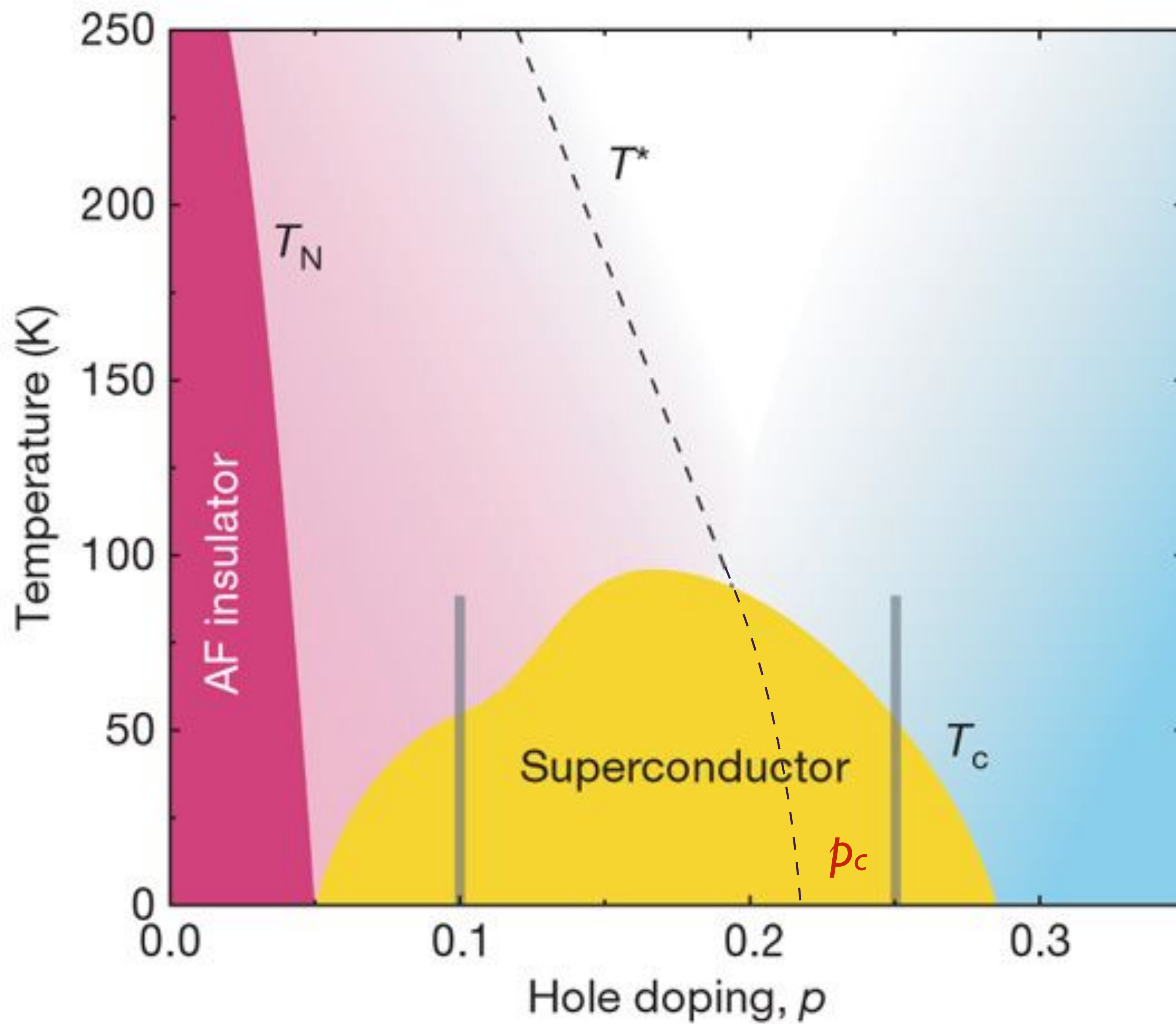
The surest path to limitless,
clean, fusion energy

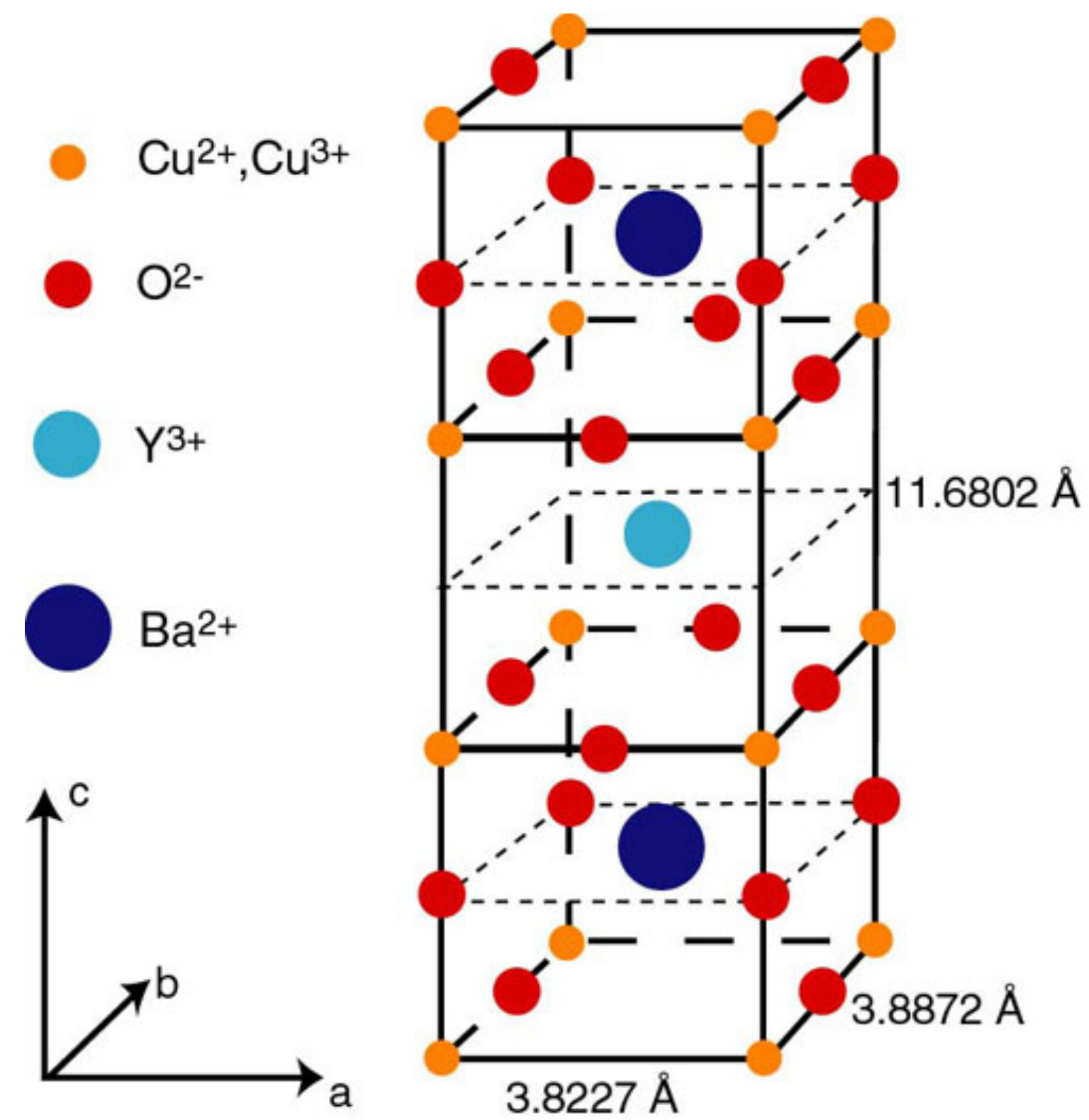
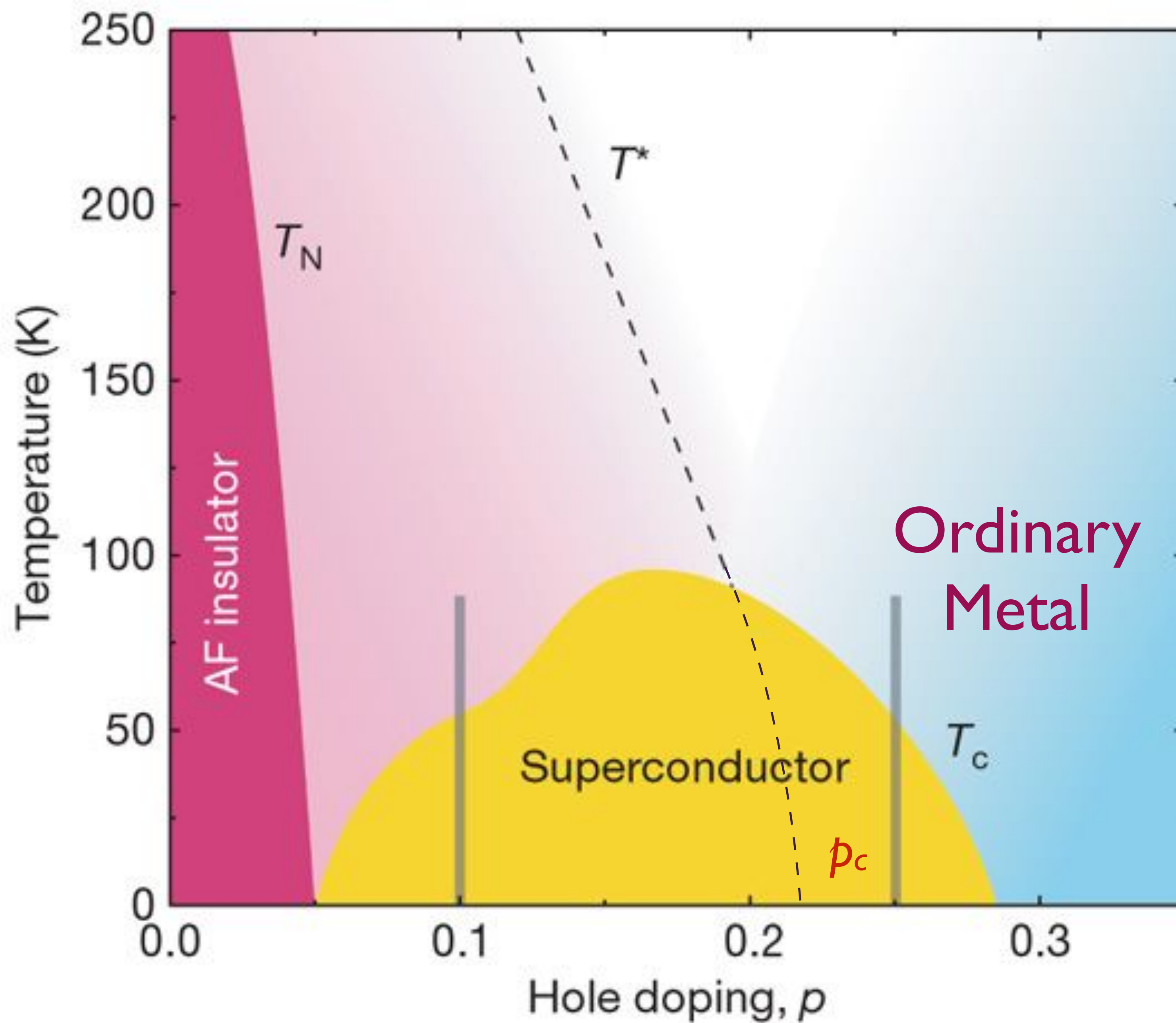
YBCO magnets allow for smaller,
faster, and less expensive
tokamaks for plasma fusion

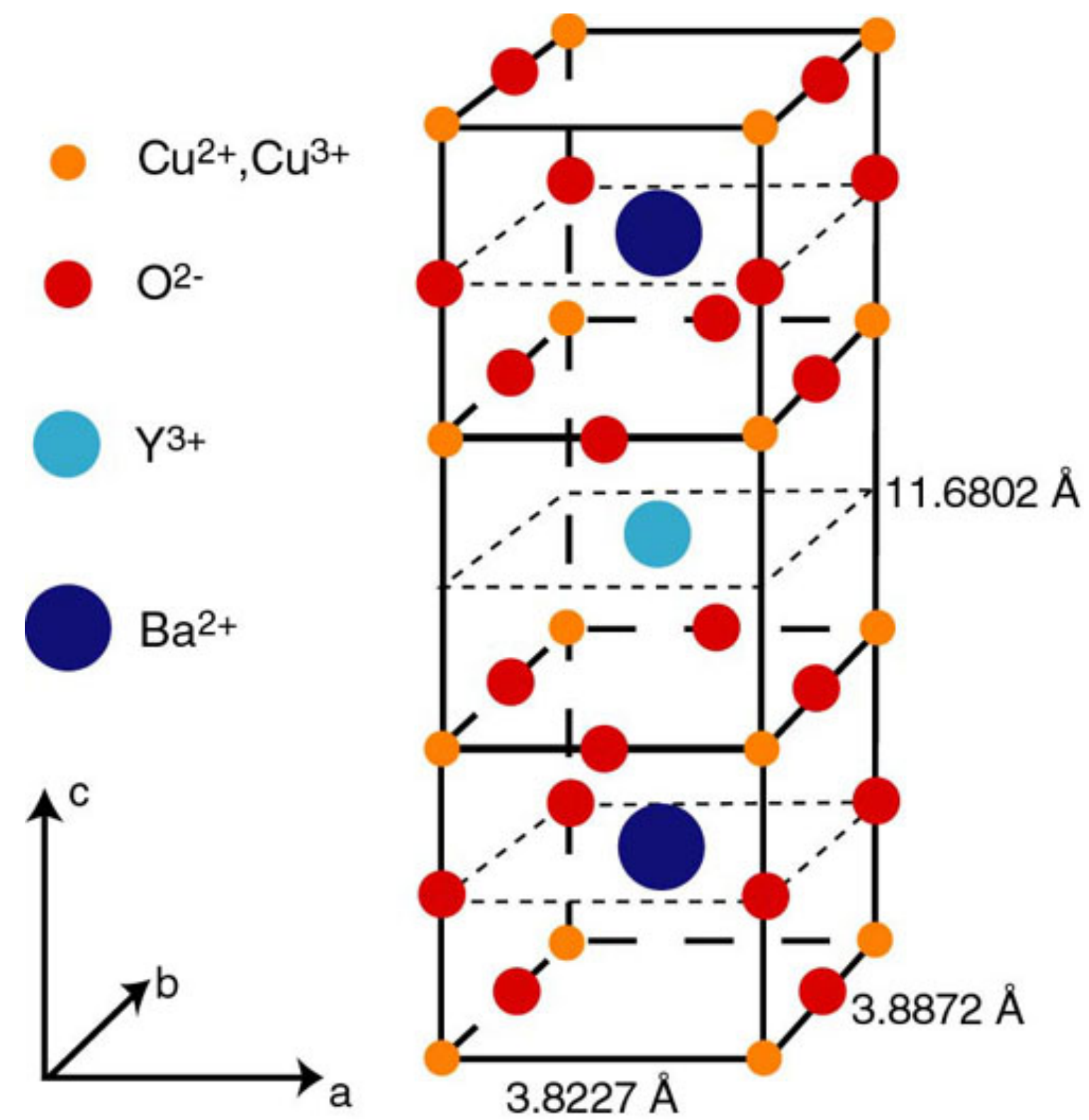
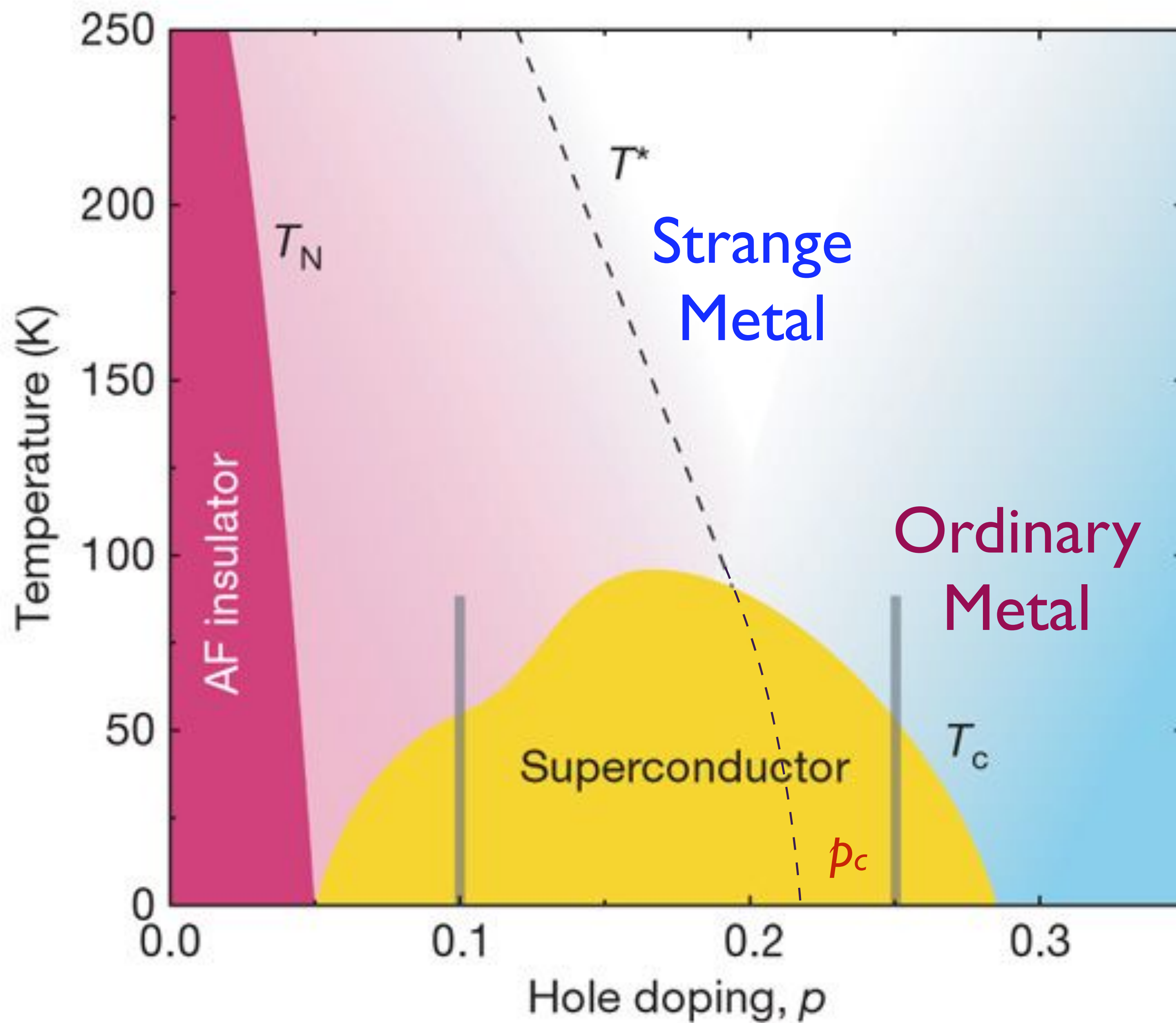


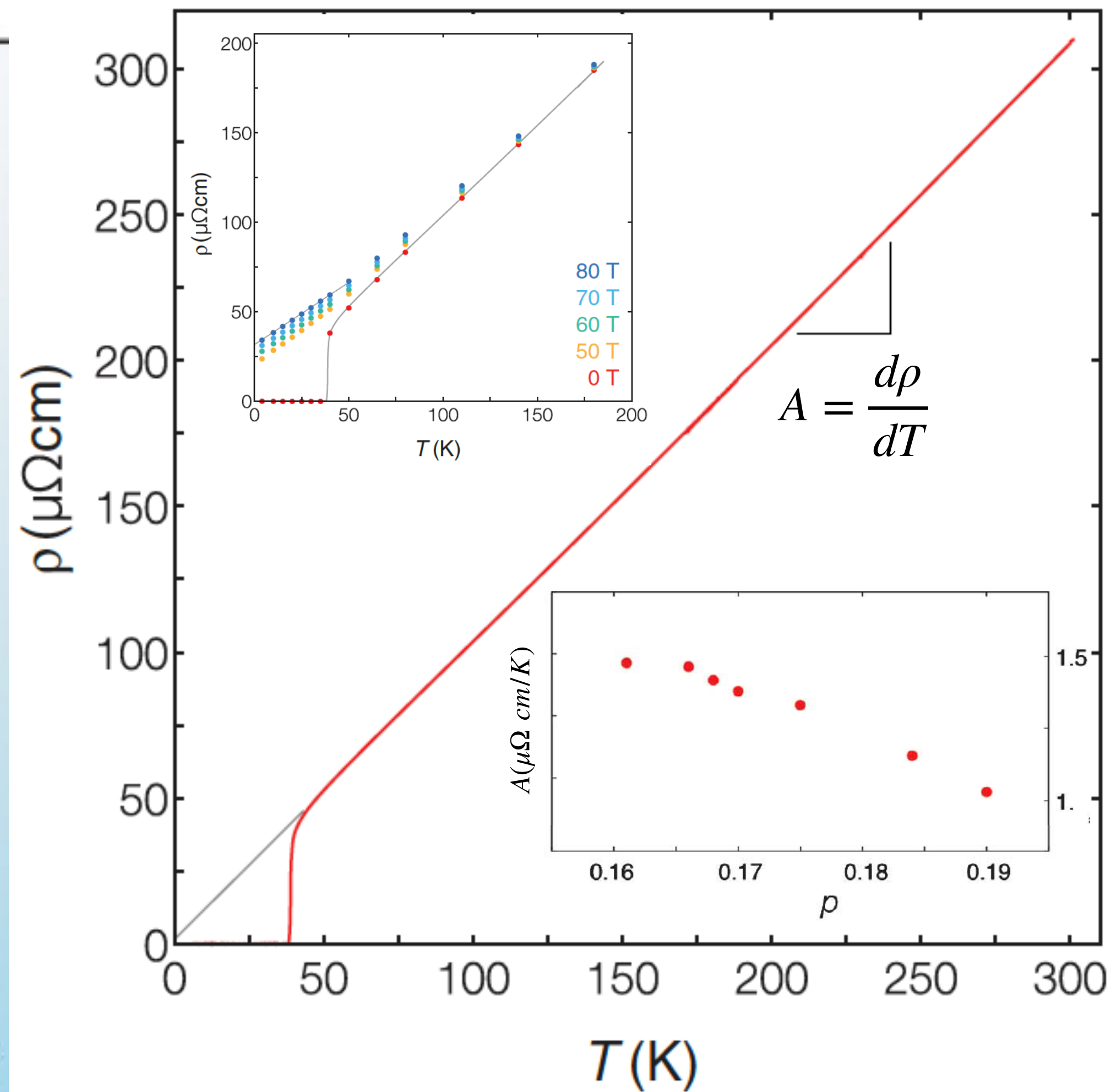
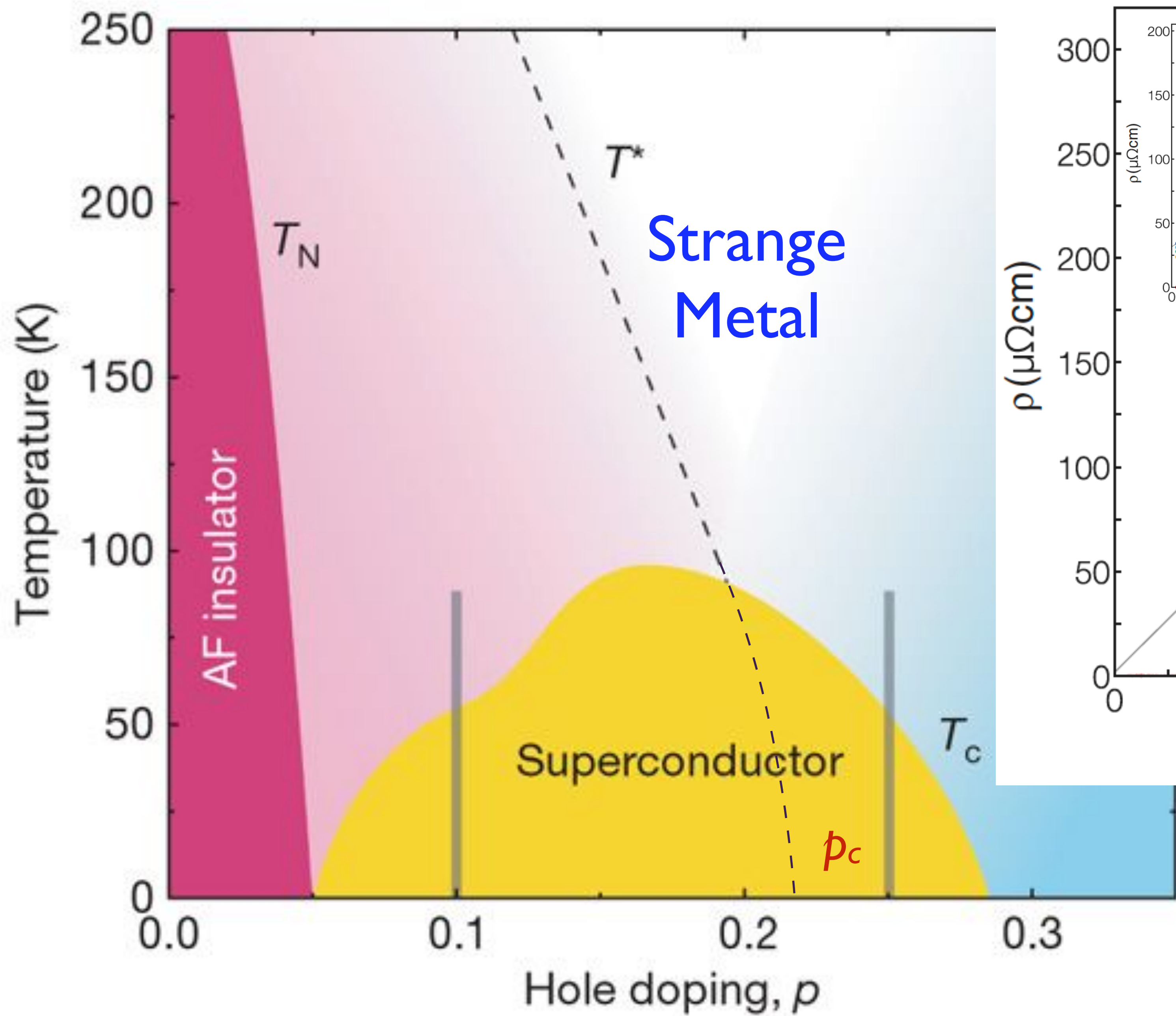
Commonwealth
Fusion Systems











LSCO: Giraldo-Gallo et al. 2018

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

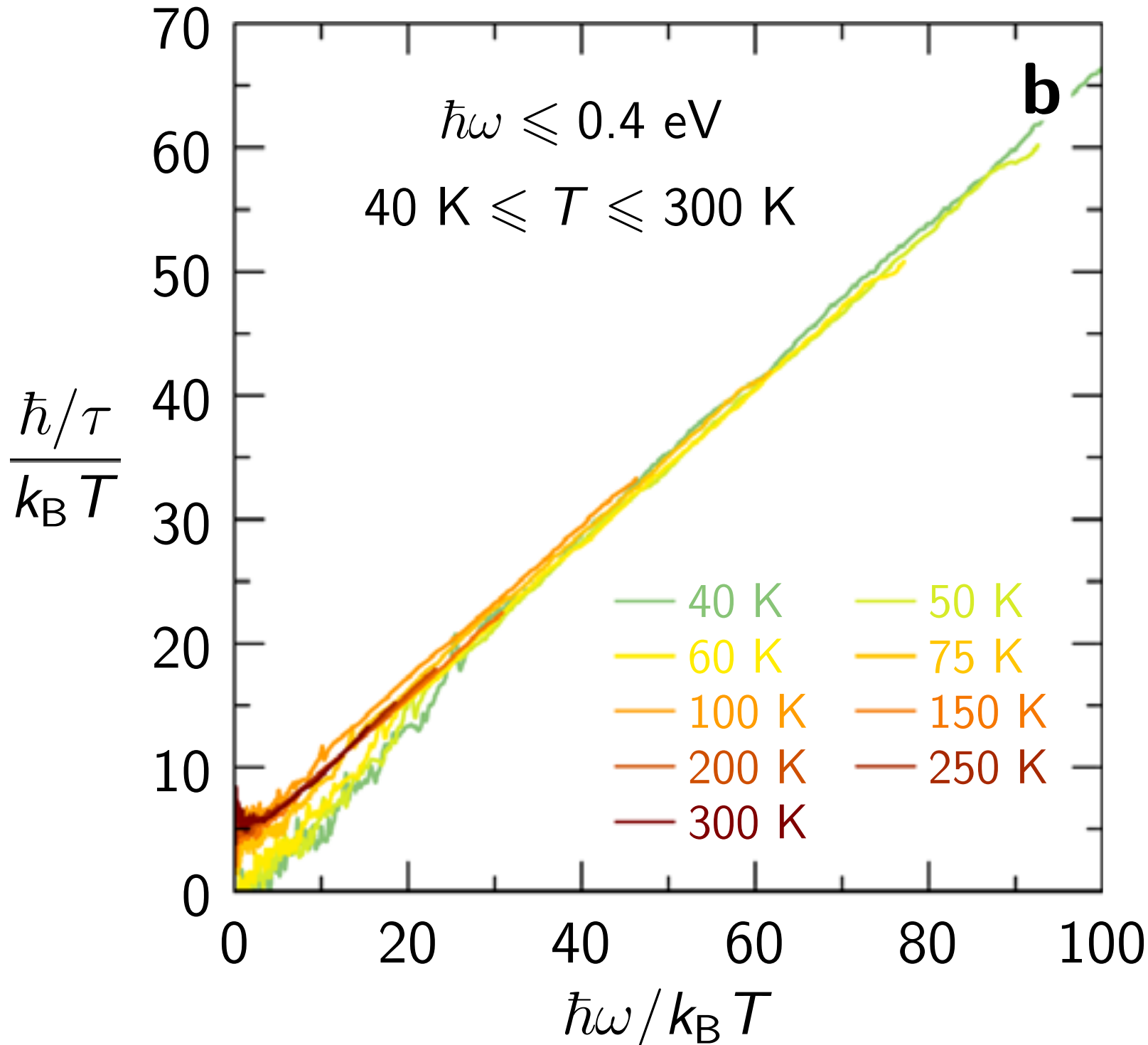
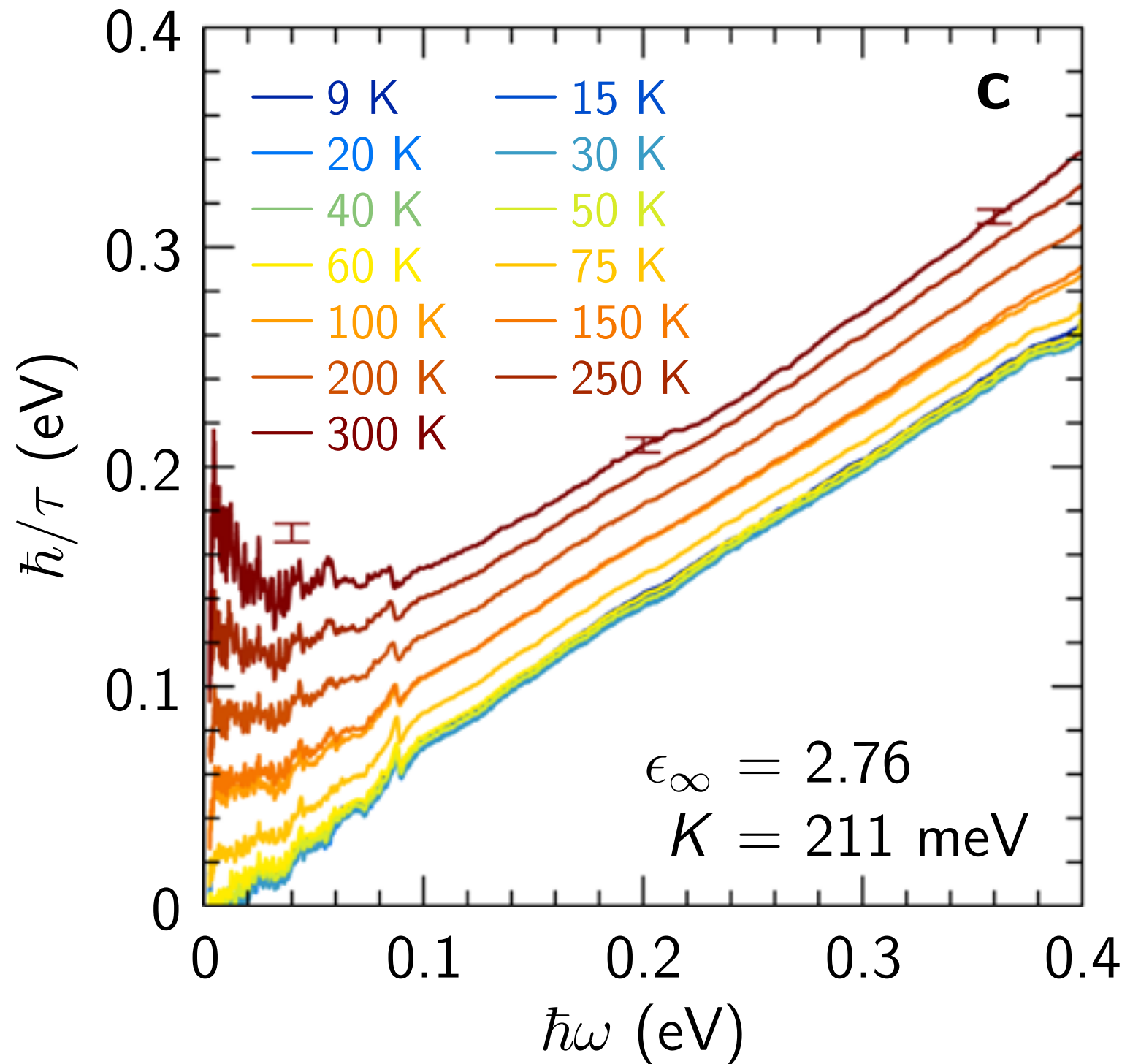
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar \omega}{k_B T}\right)$$



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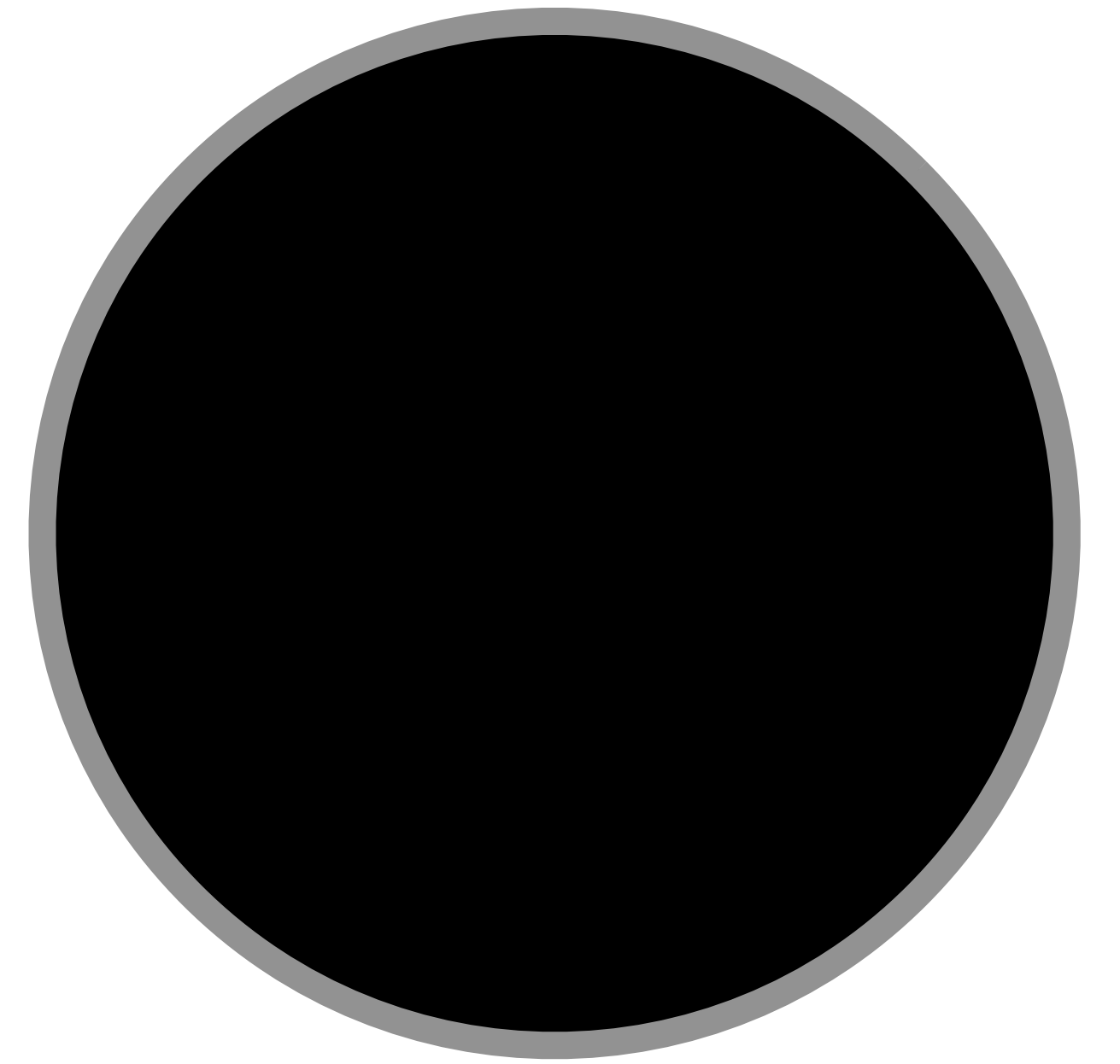
**Black holes
(1916-today)**

Black Holes

Objects so dense that light is gravitationally bound to them.



Horizon radius $R = \frac{2GM}{c^2}$



Karl Schwarzschild (1916)

G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$



The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses

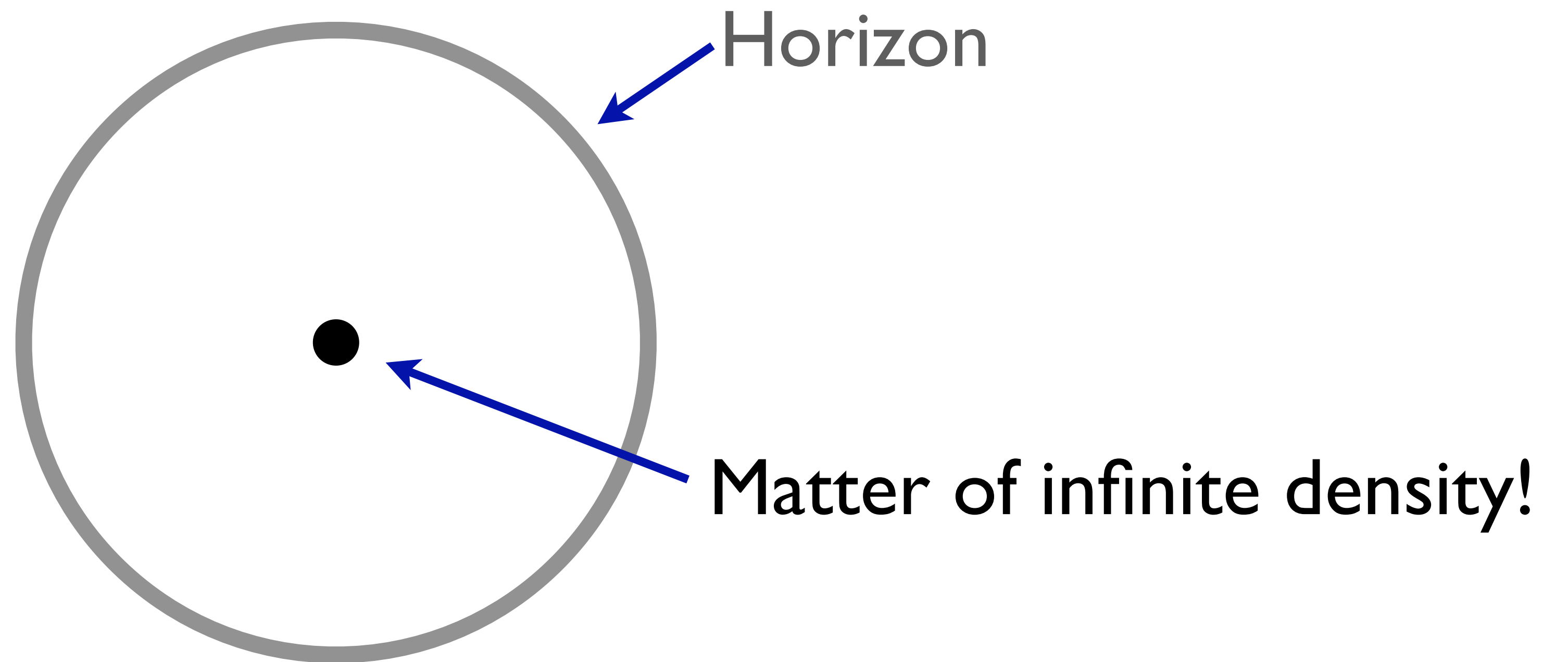
$$R = 1.3 \times 10^{11} \text{ m}$$

\approx earth's orbit

Event Horizon Telescope
May 12, 2022

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



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This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

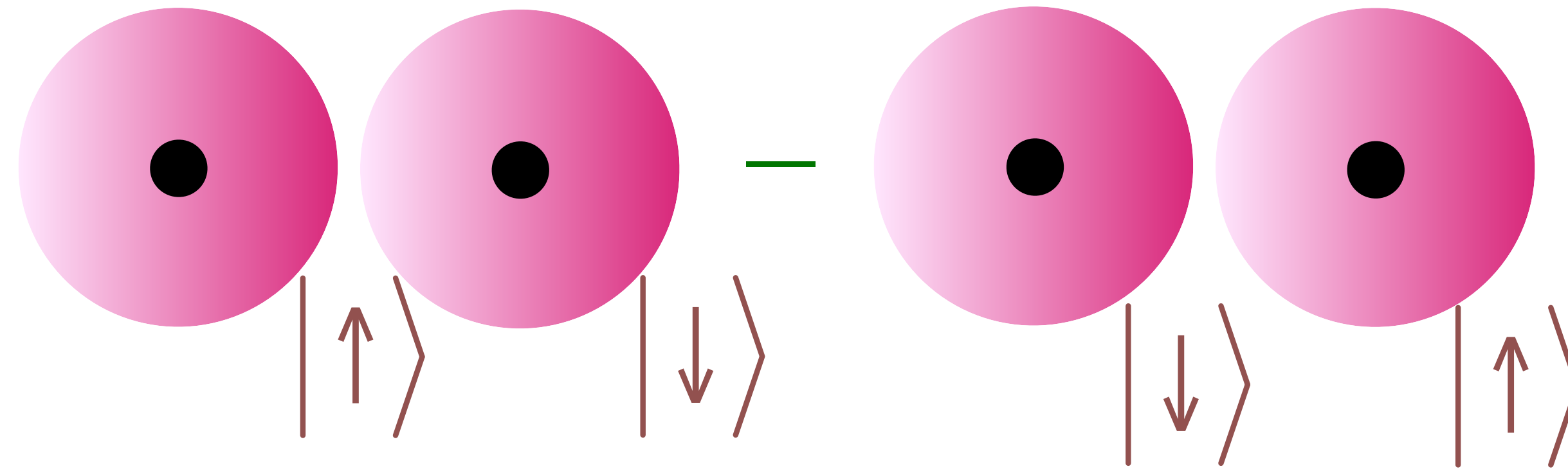
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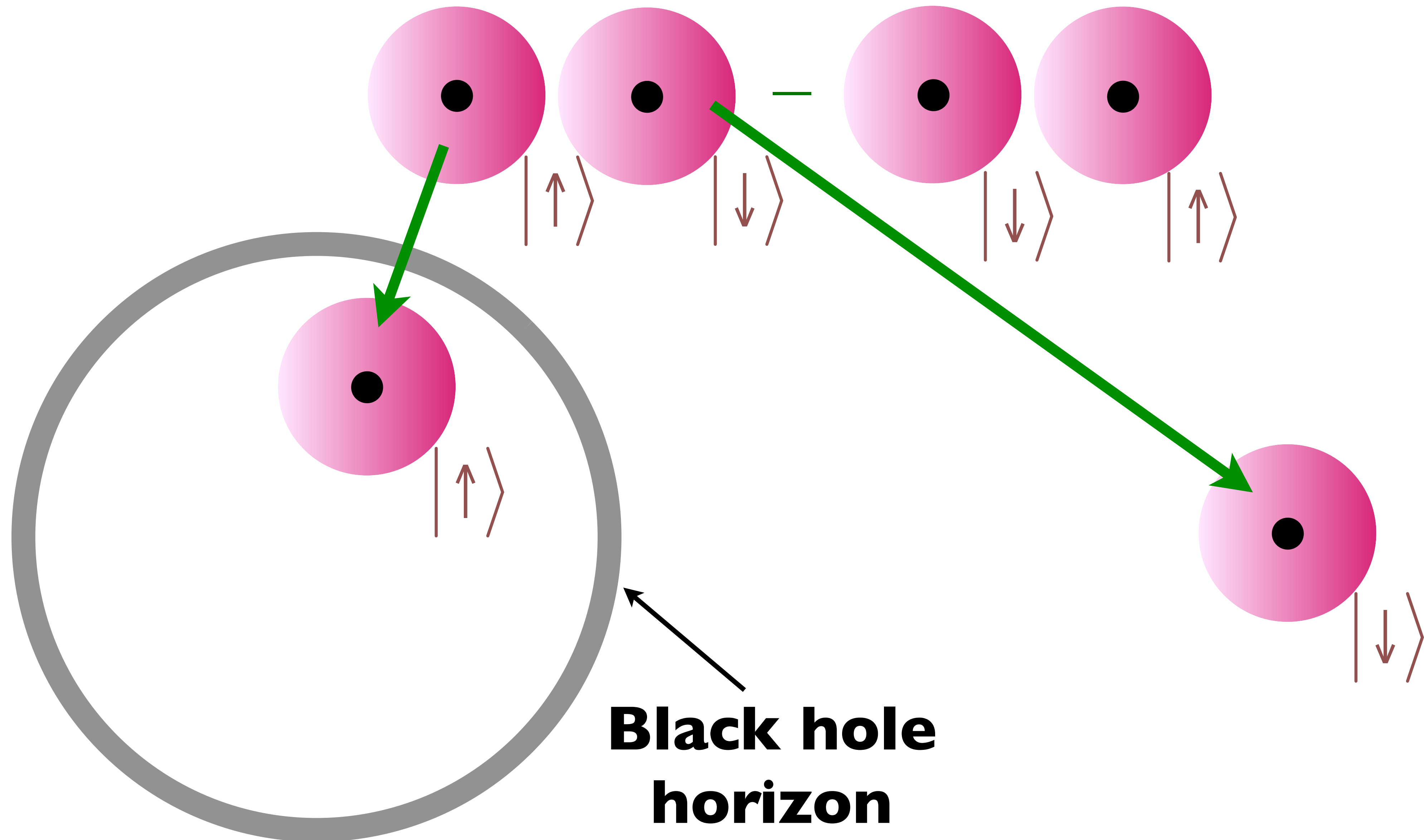
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In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

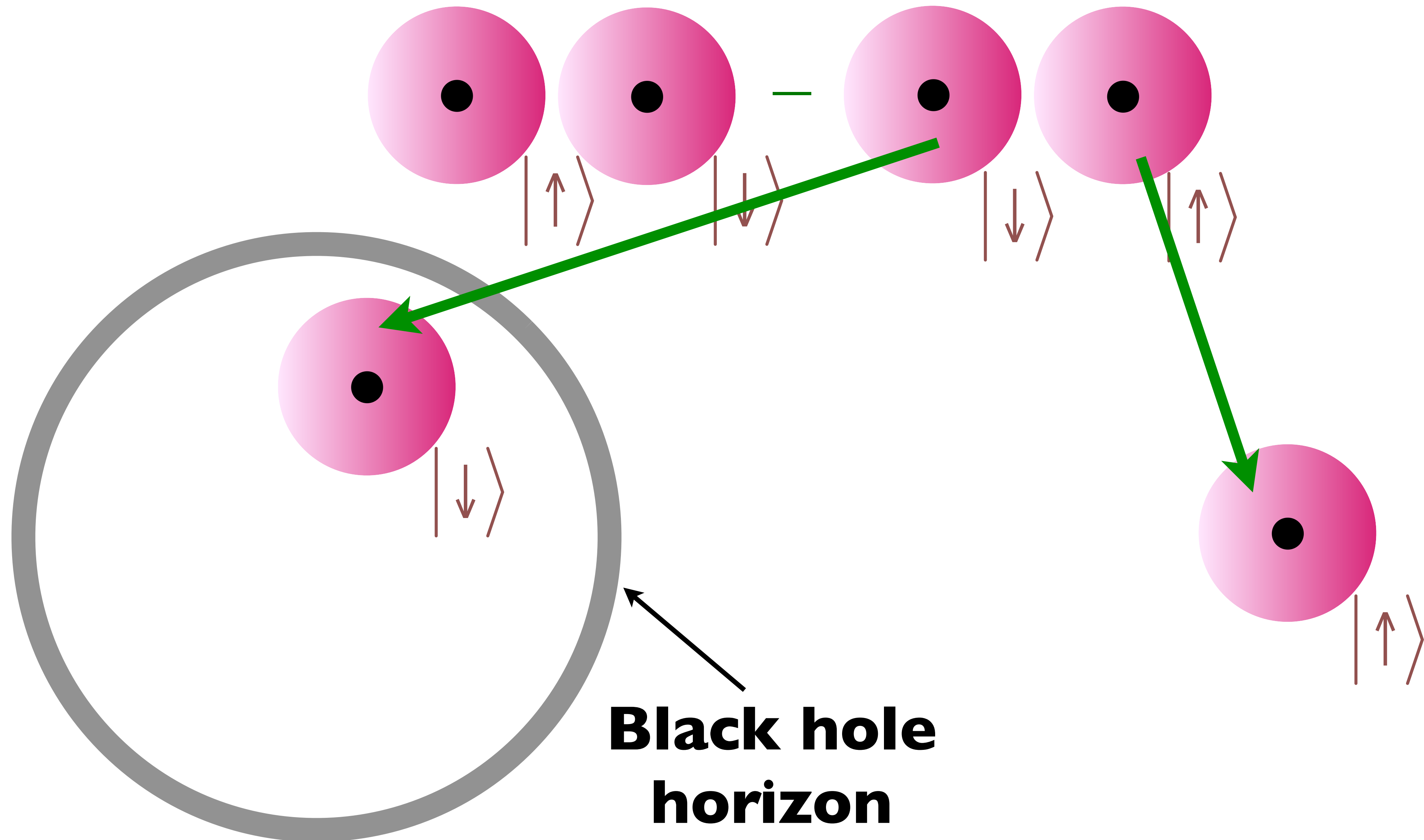
Quantum Entanglement across a black hole horizon



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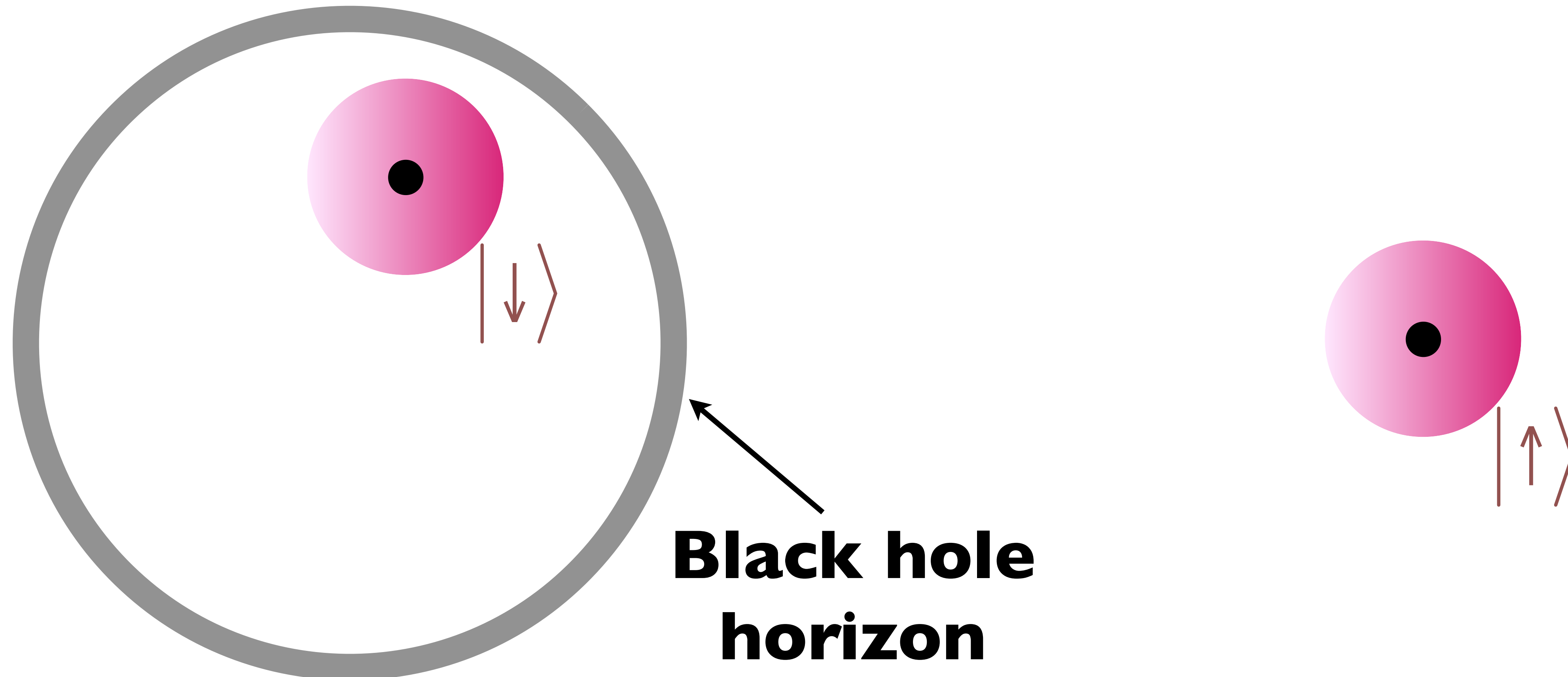


Quantum Entanglement across a black hole horizon



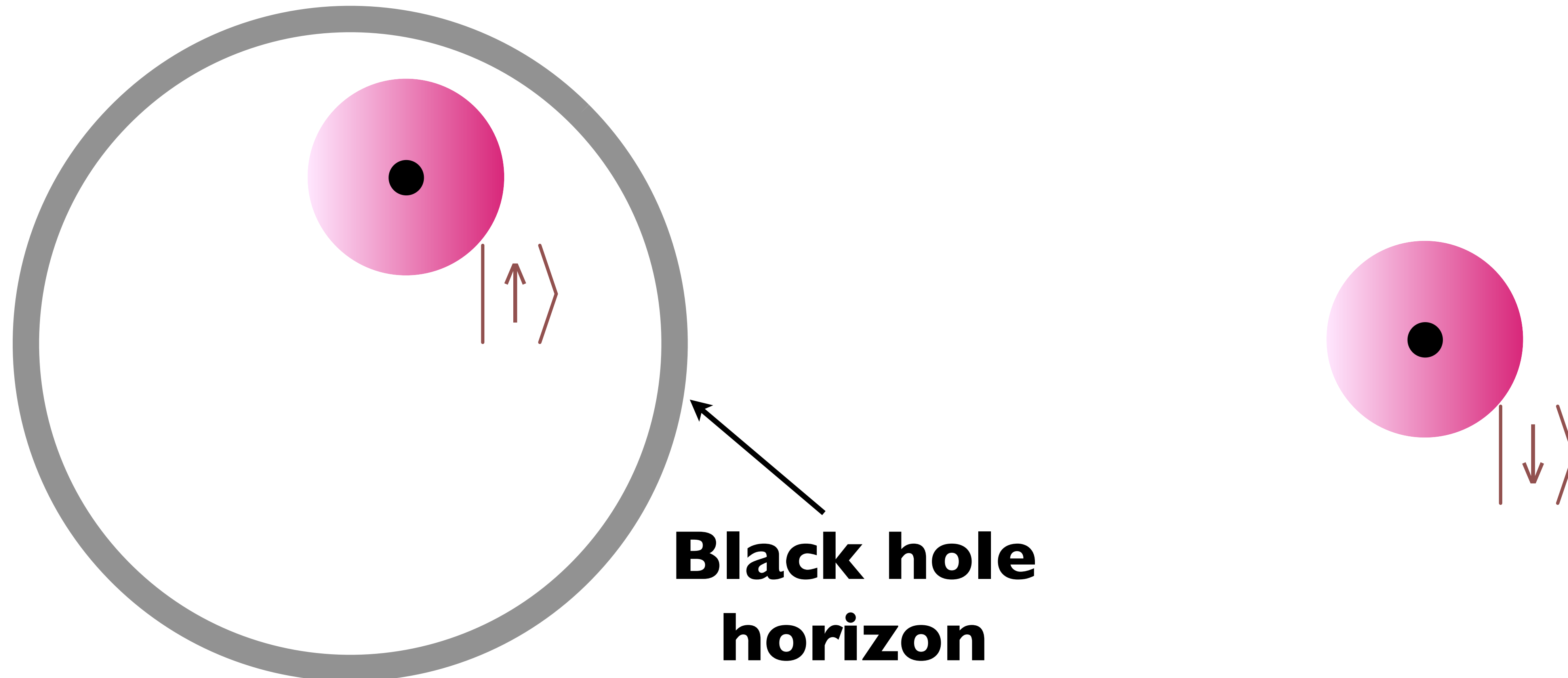
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There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

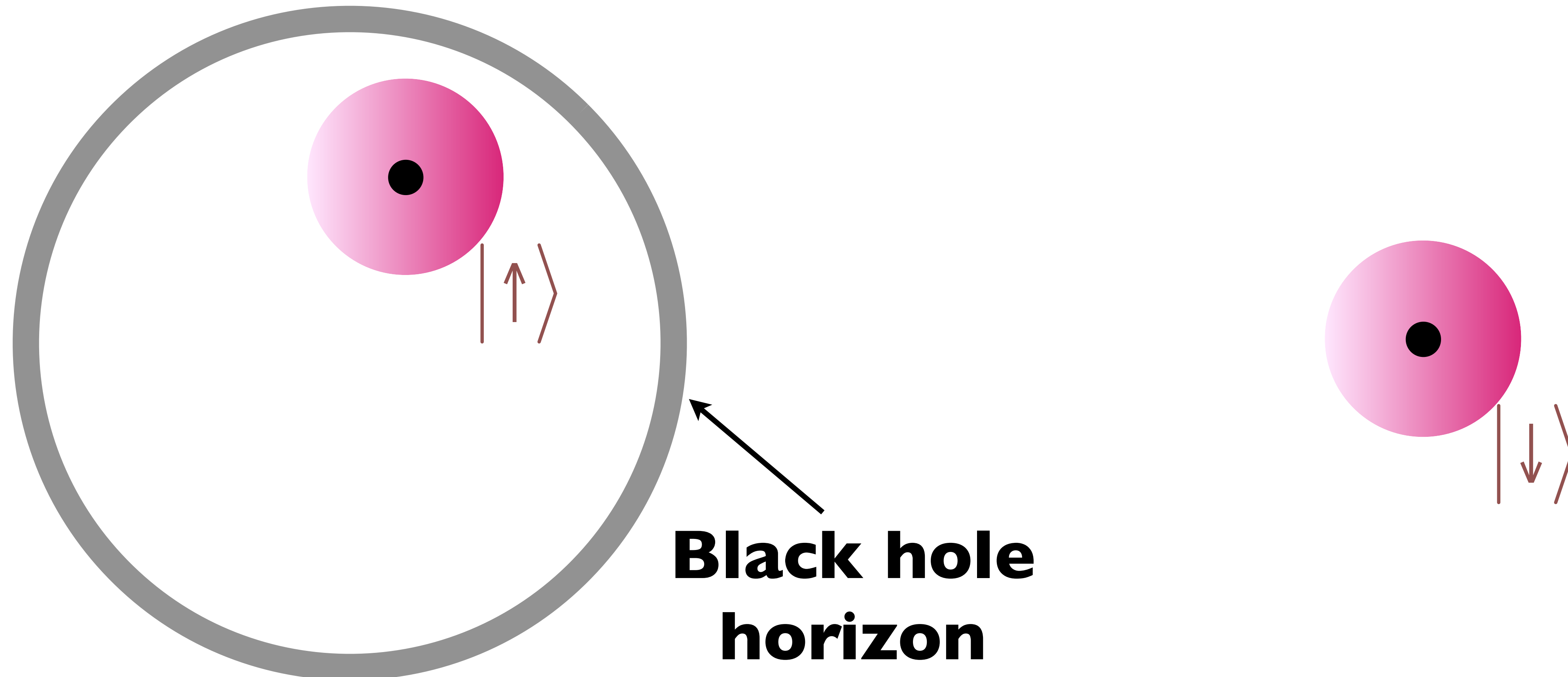
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Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

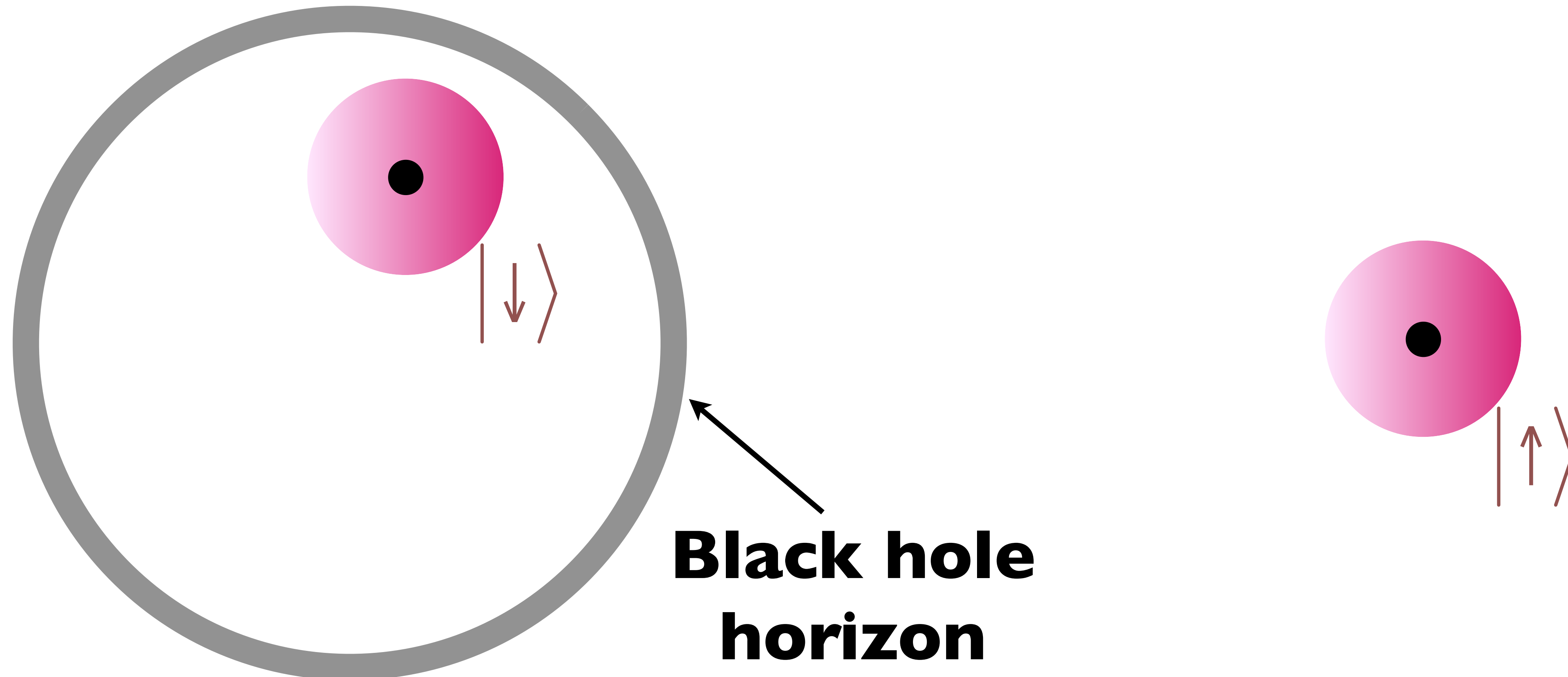
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

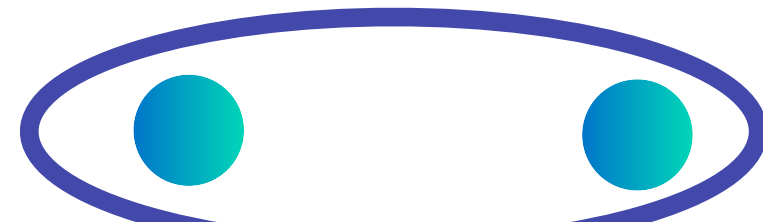
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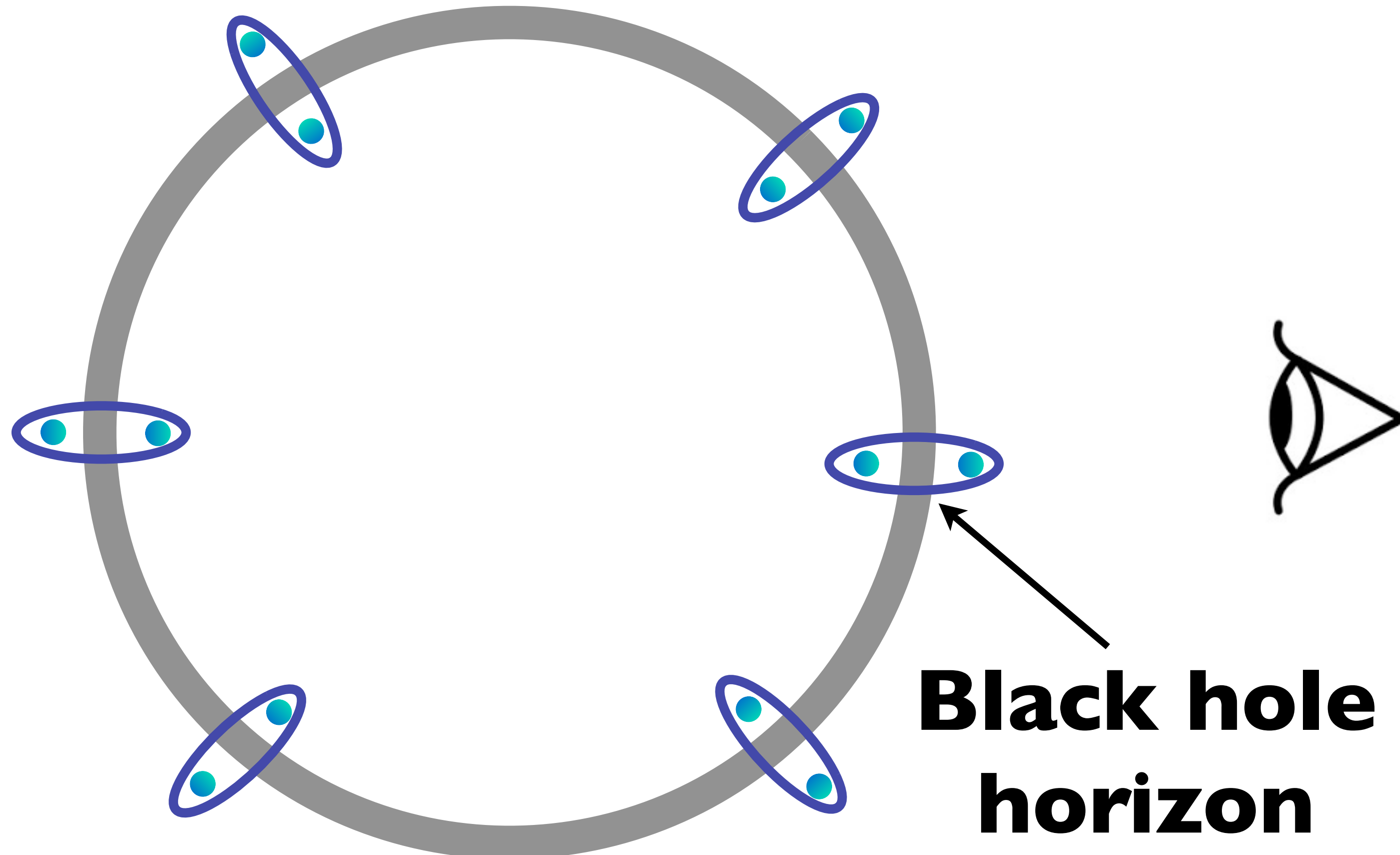


Quantum Entanglement across a black hole horizon

Quantum entanglement
on the surface



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside*
the black hole,
Hawking obtained

$$S = \frac{k_B A c^3}{4G\hbar}$$

where A is area of the
black hole horizon.

All other systems have
entropy proportional to
their volume.

What is inside a black hole ???

Hawking (1975): when viewed from the outside, black holes have an entropy and a temperature, and slowly evaporate like any thermal object



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

$$S = \frac{k_B A c^3}{4G\hbar}$$

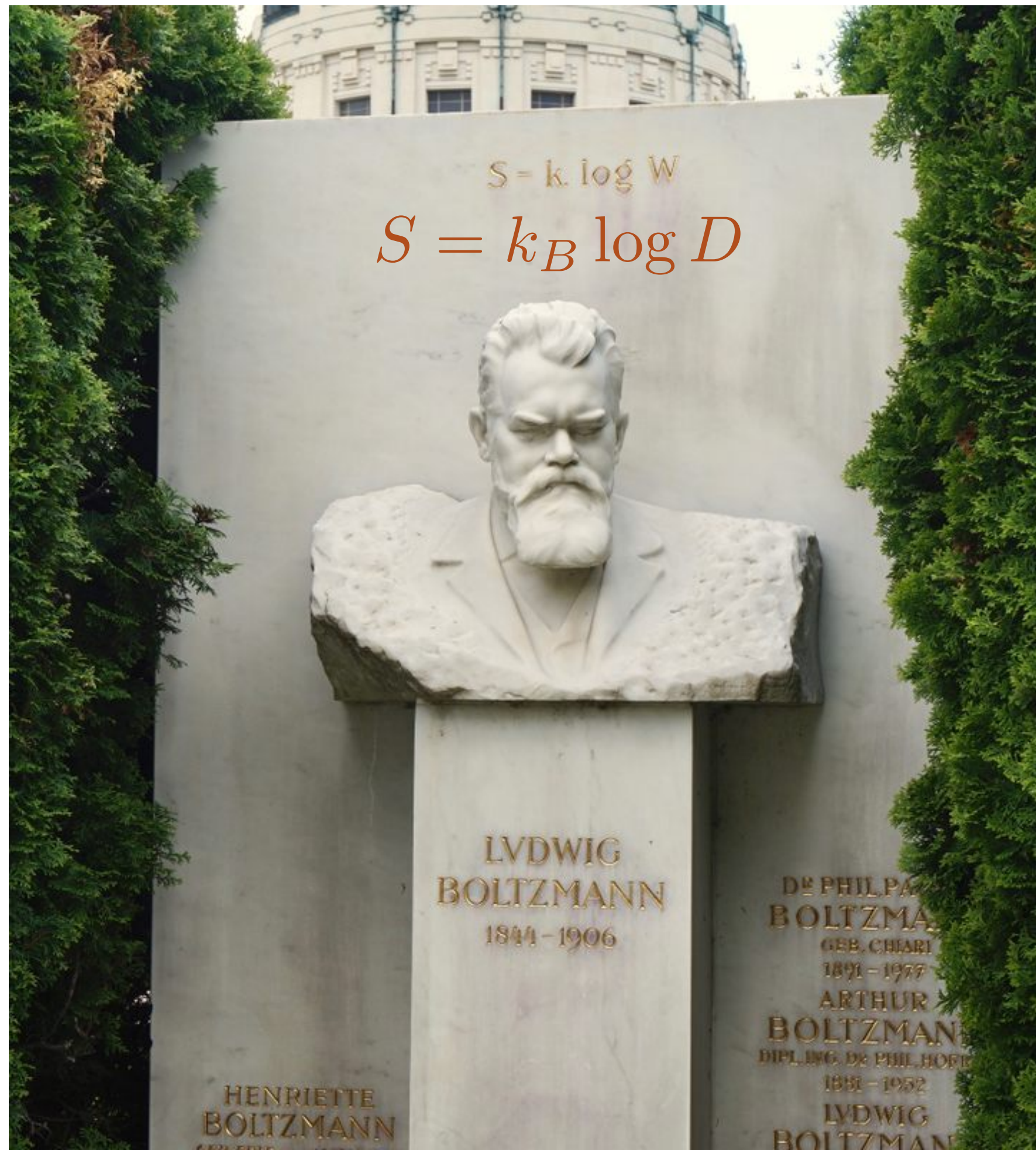
$$\tau_{\text{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!

Statistical interpretation of entropy (1870)

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Density of quantum states $D(E) = \exp(S(E)/k_B)$



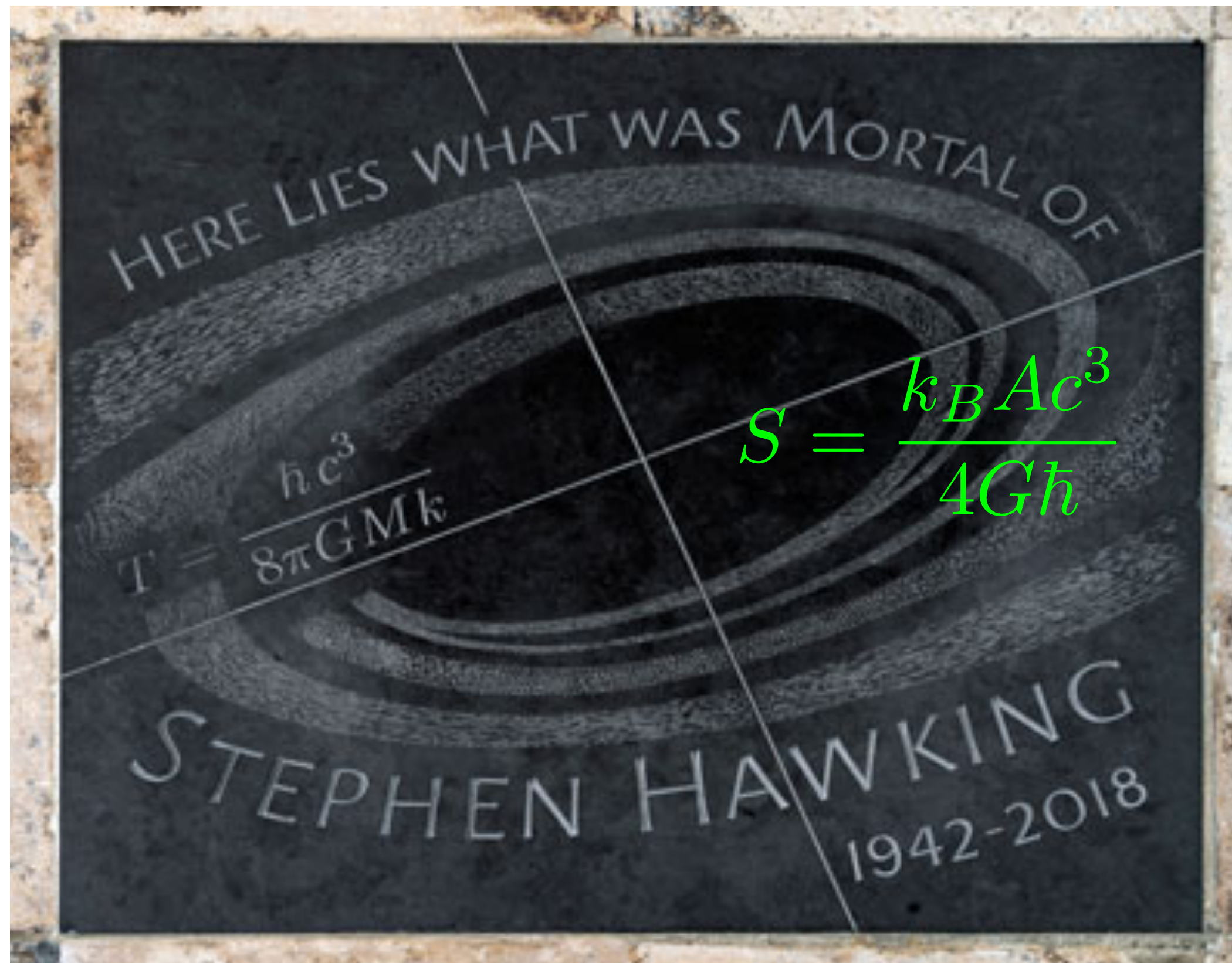
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Needed,
to solve open problems in the theory of
superconductivity and black holes:

A solvable model of quantum entanglement
of 3, 4, 5, ... ∞ particles

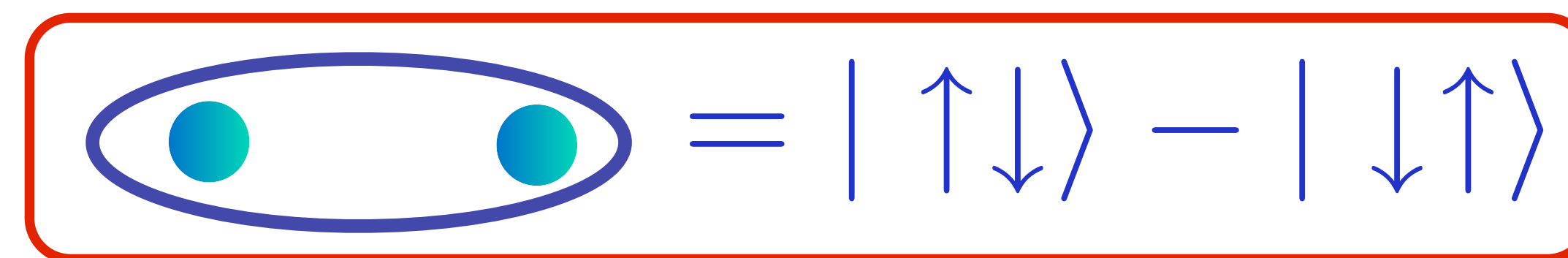
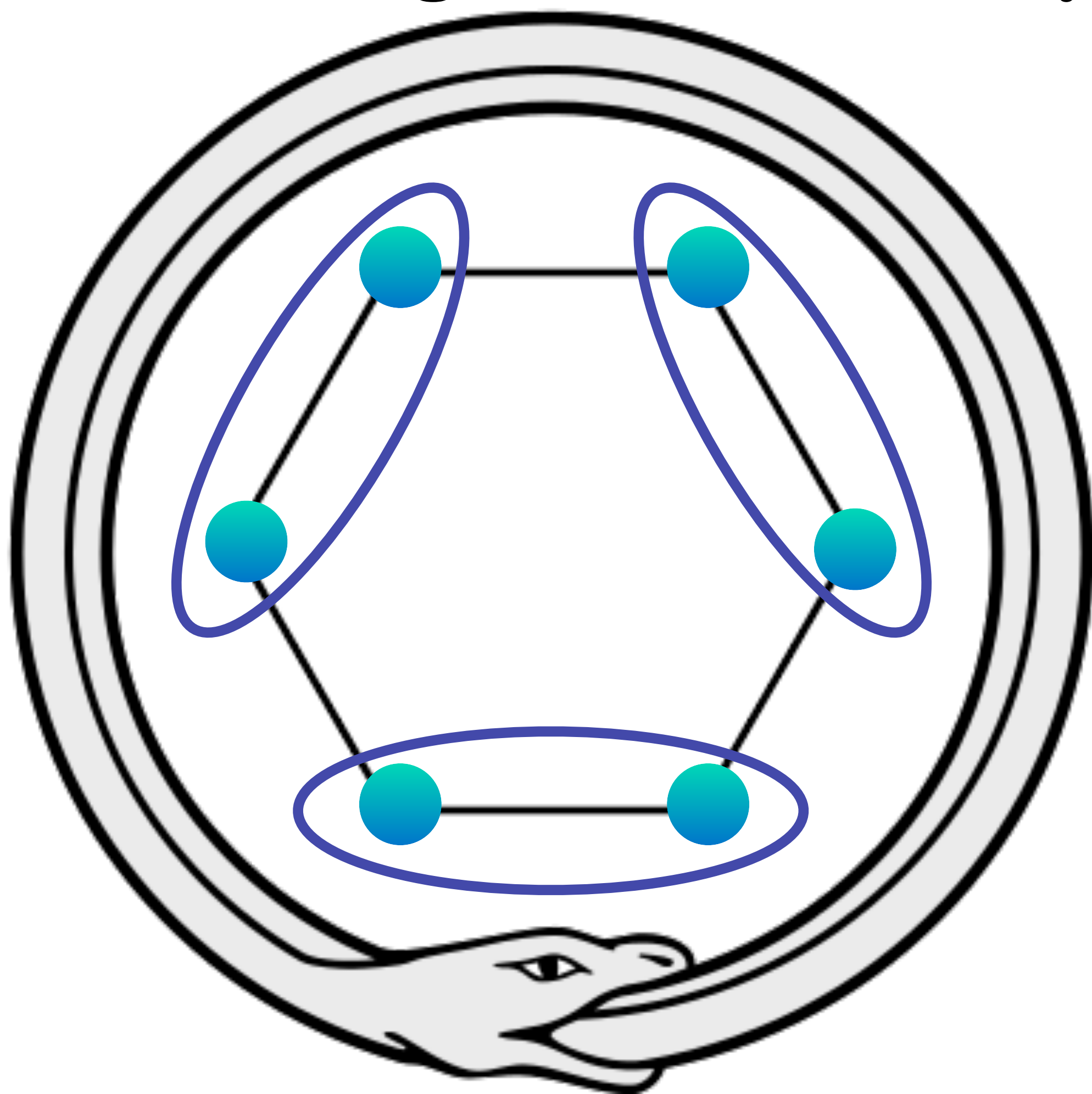
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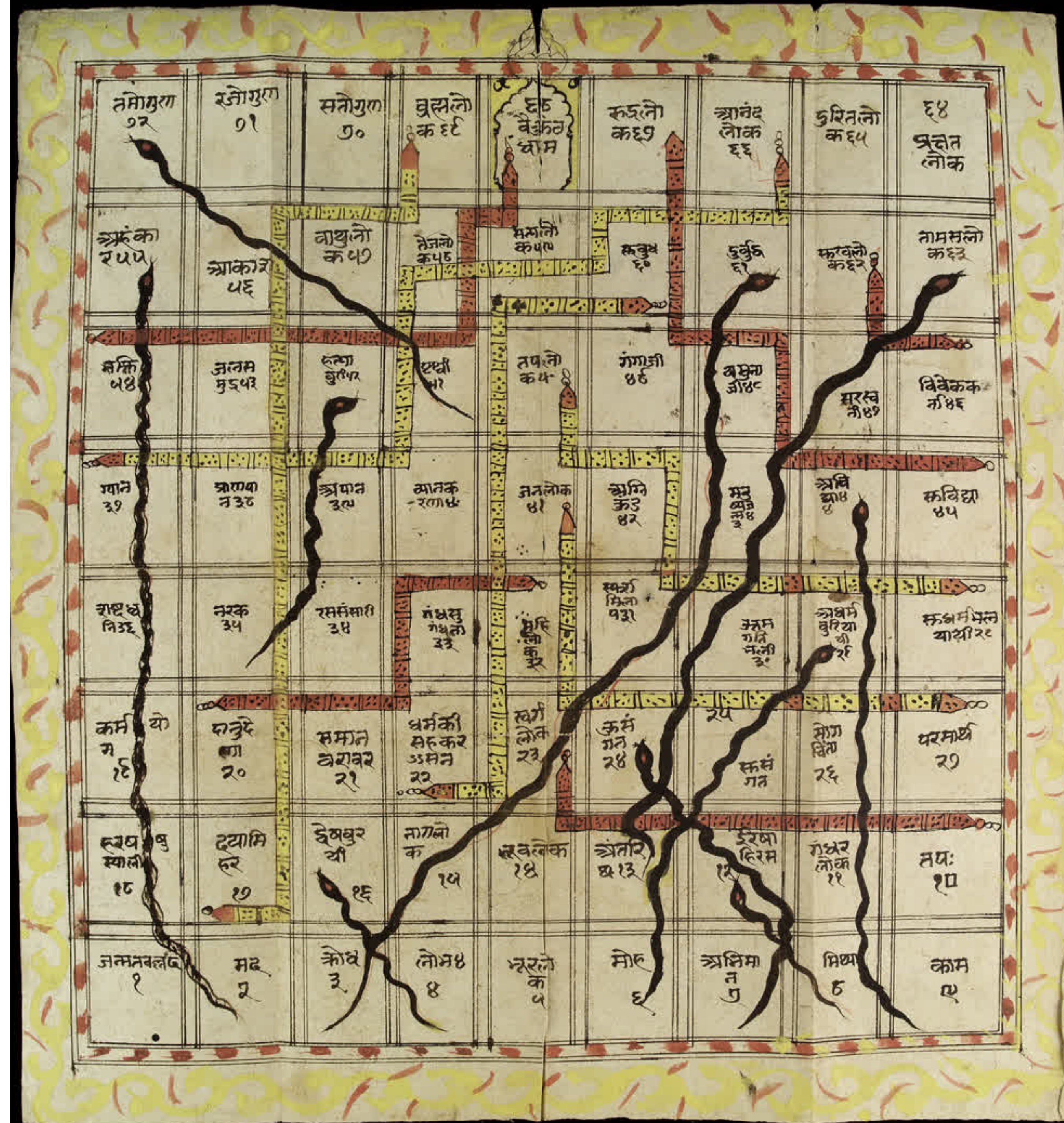
**The Sachdev-Ye-Kitaev model
of many-particle entanglement**

Kekulé's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



Benzene



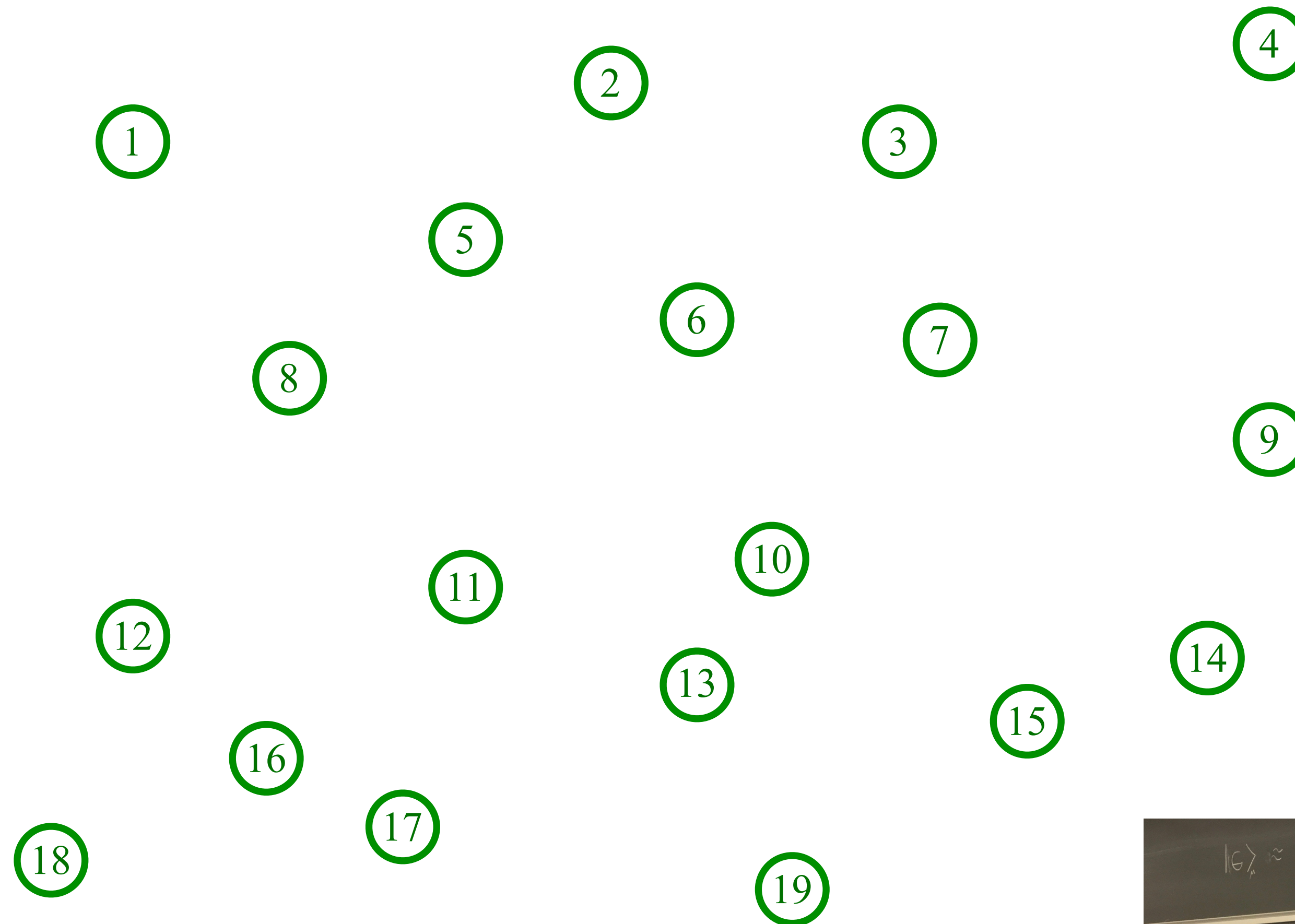
My
spooky
dream*

Ancient
Indian
game of
Snakes
and
Ladders

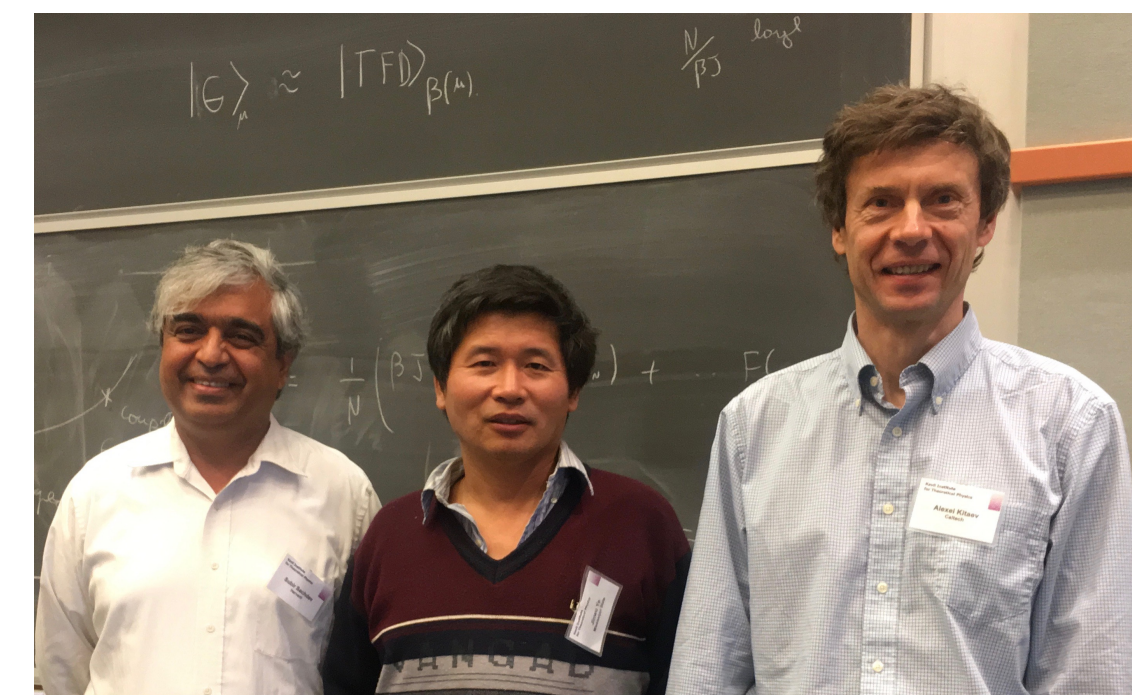
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

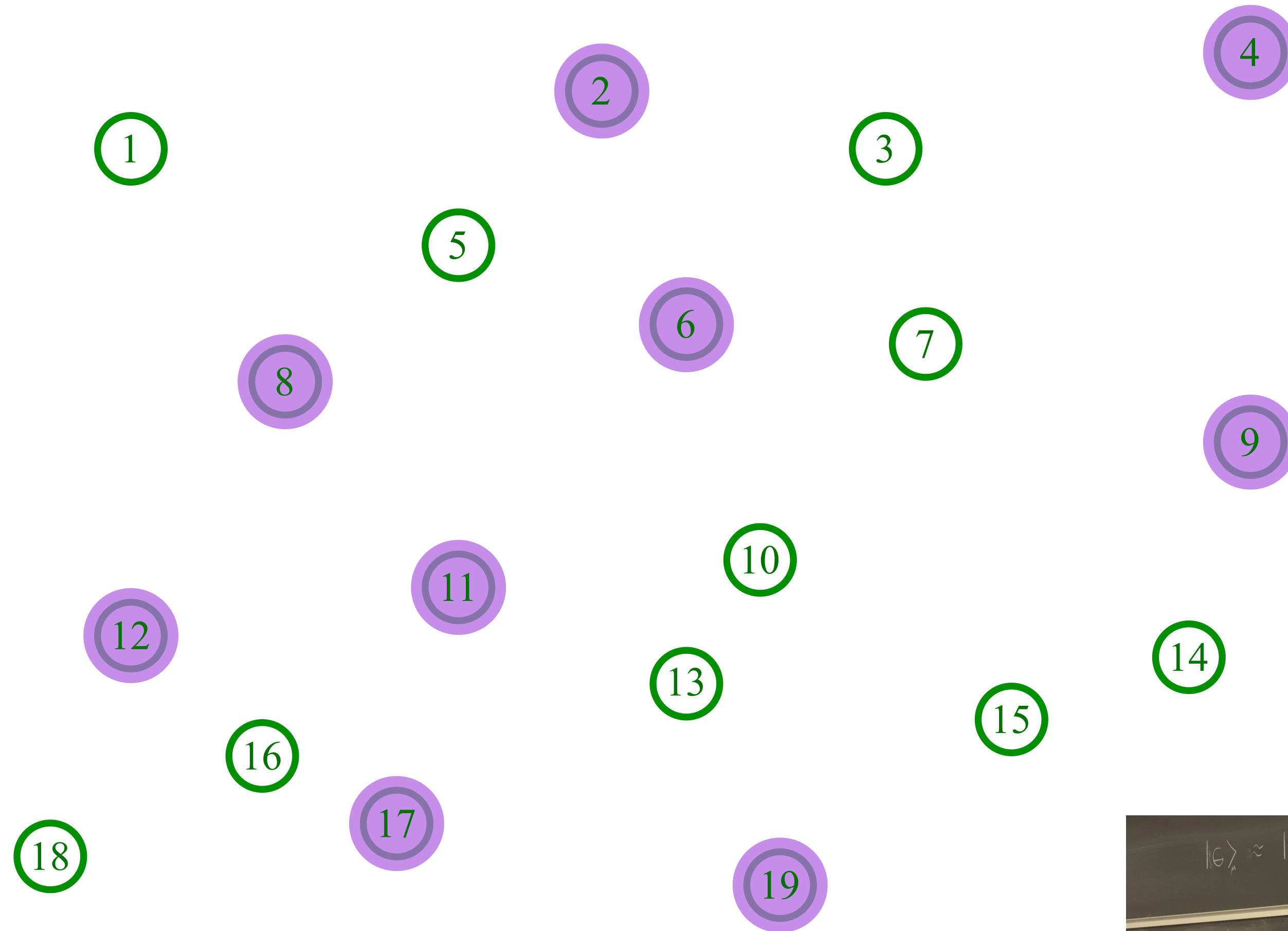


Pick a set of random positions

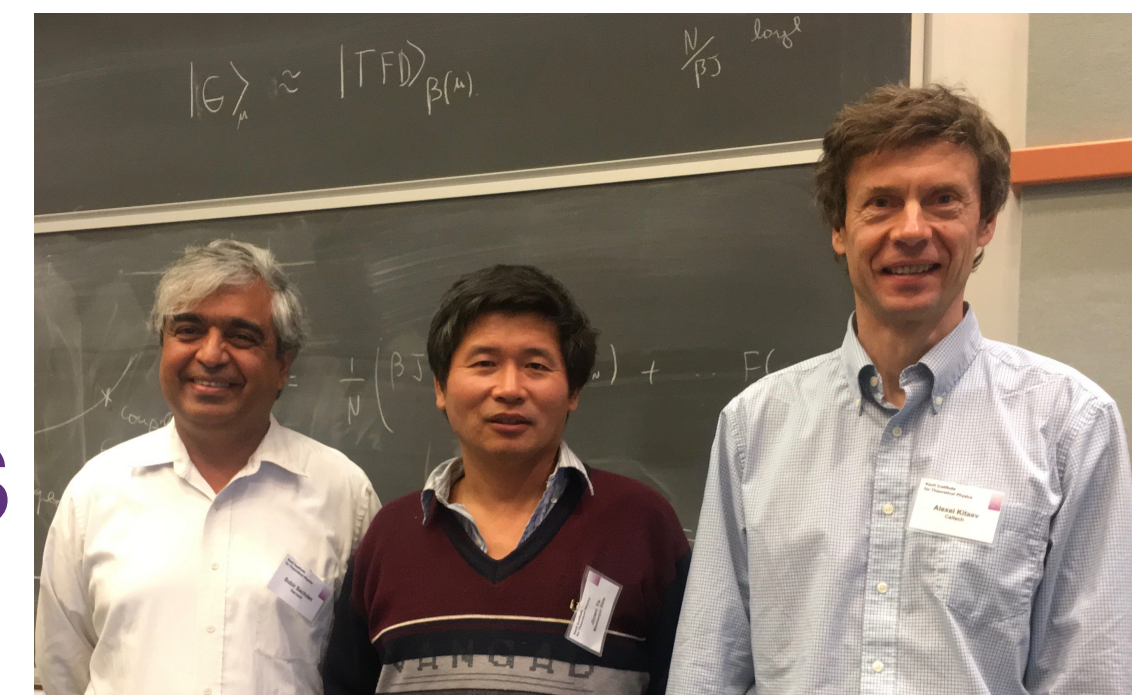


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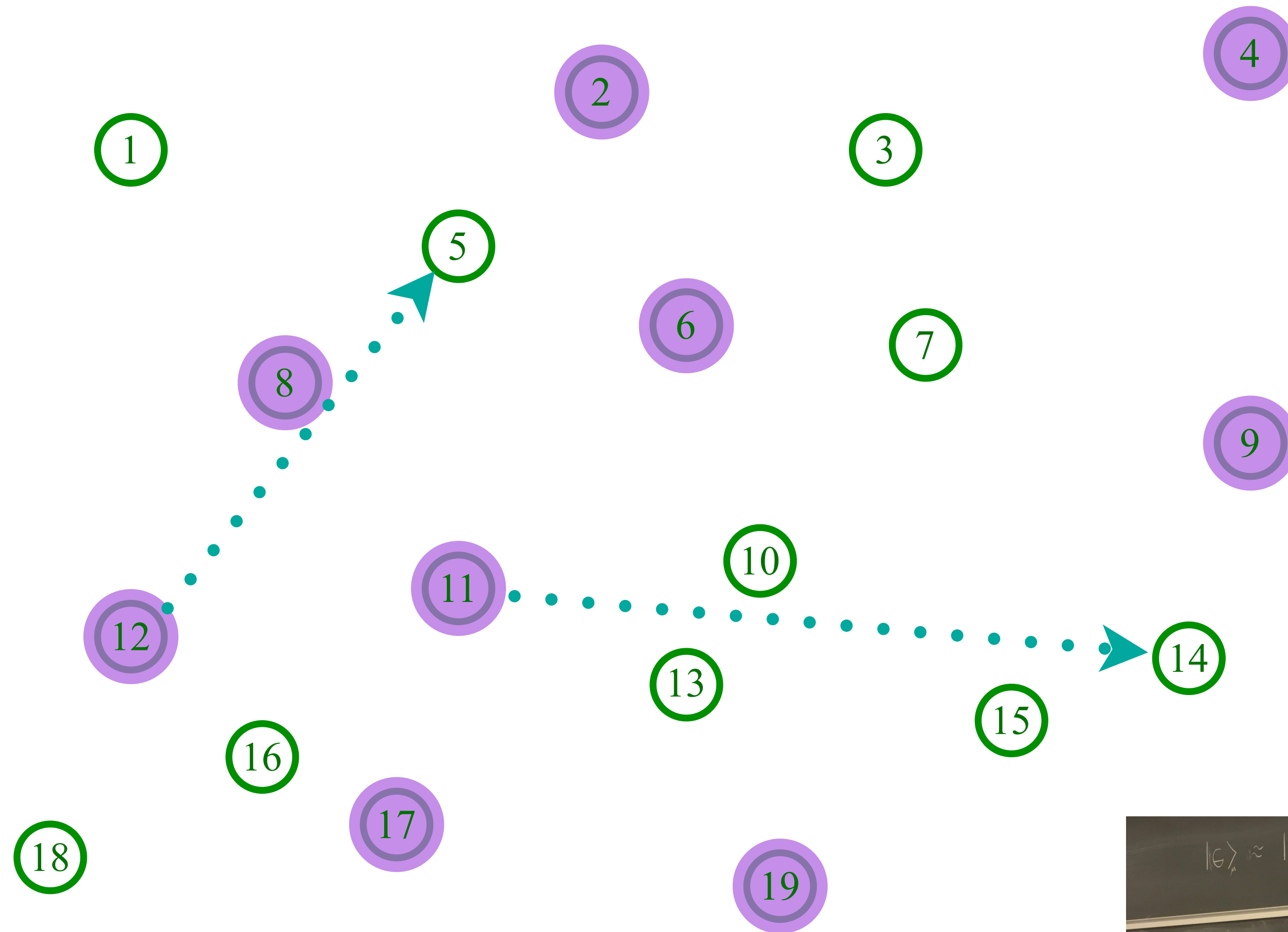
Place electrons randomly on some sites



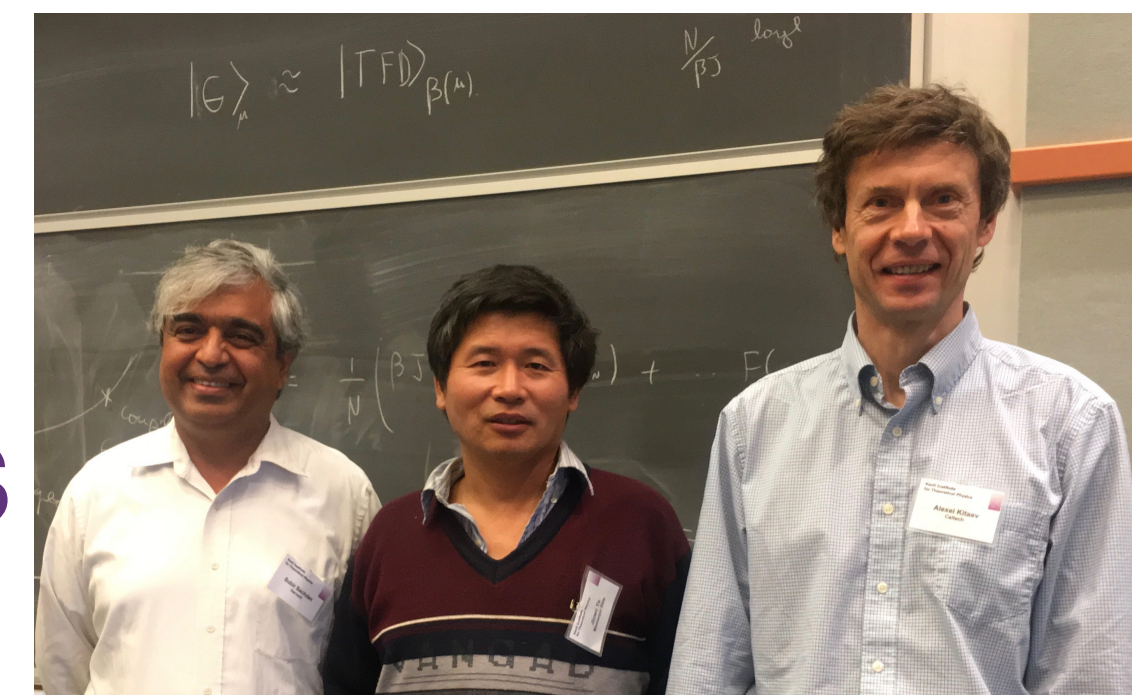
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$$U_{11,12;5,14}$$



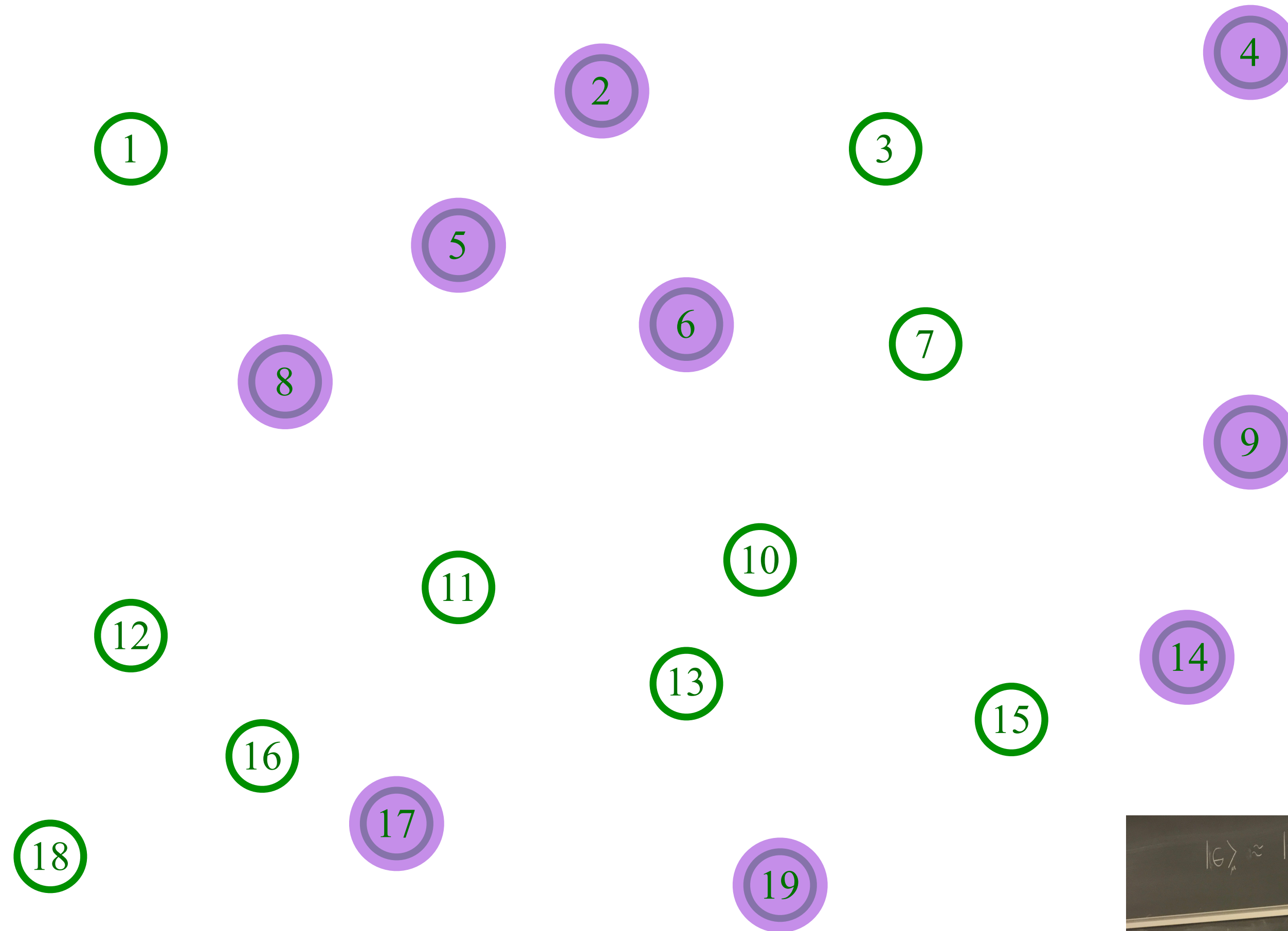
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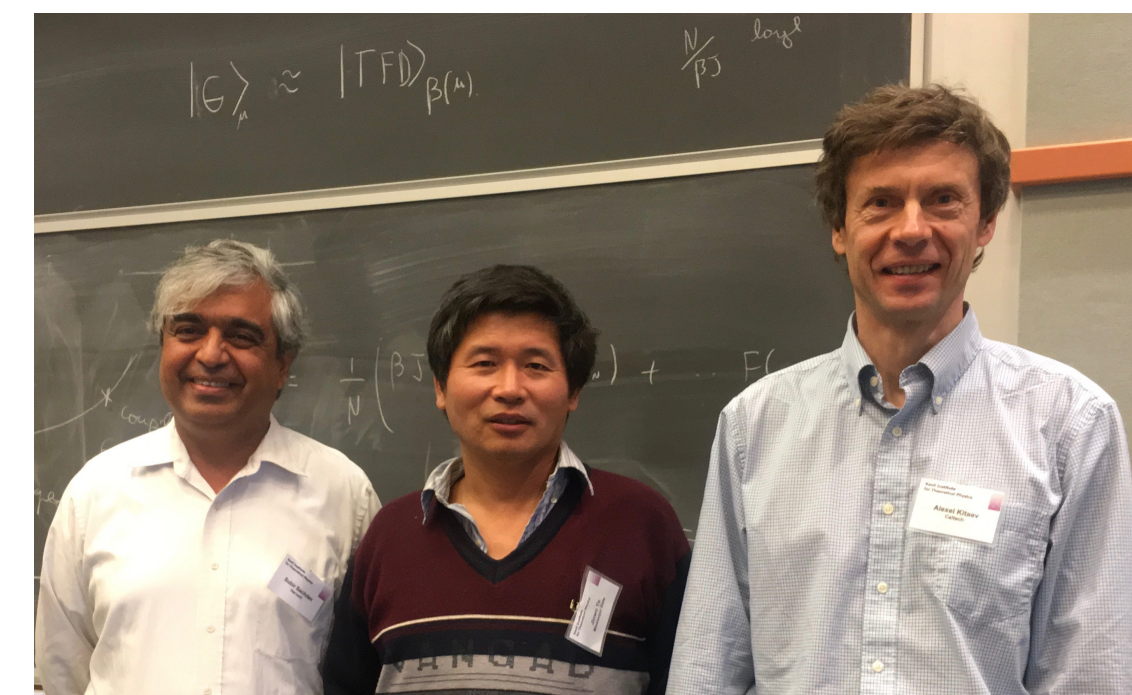
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$$U_{11,12;5,14}$$



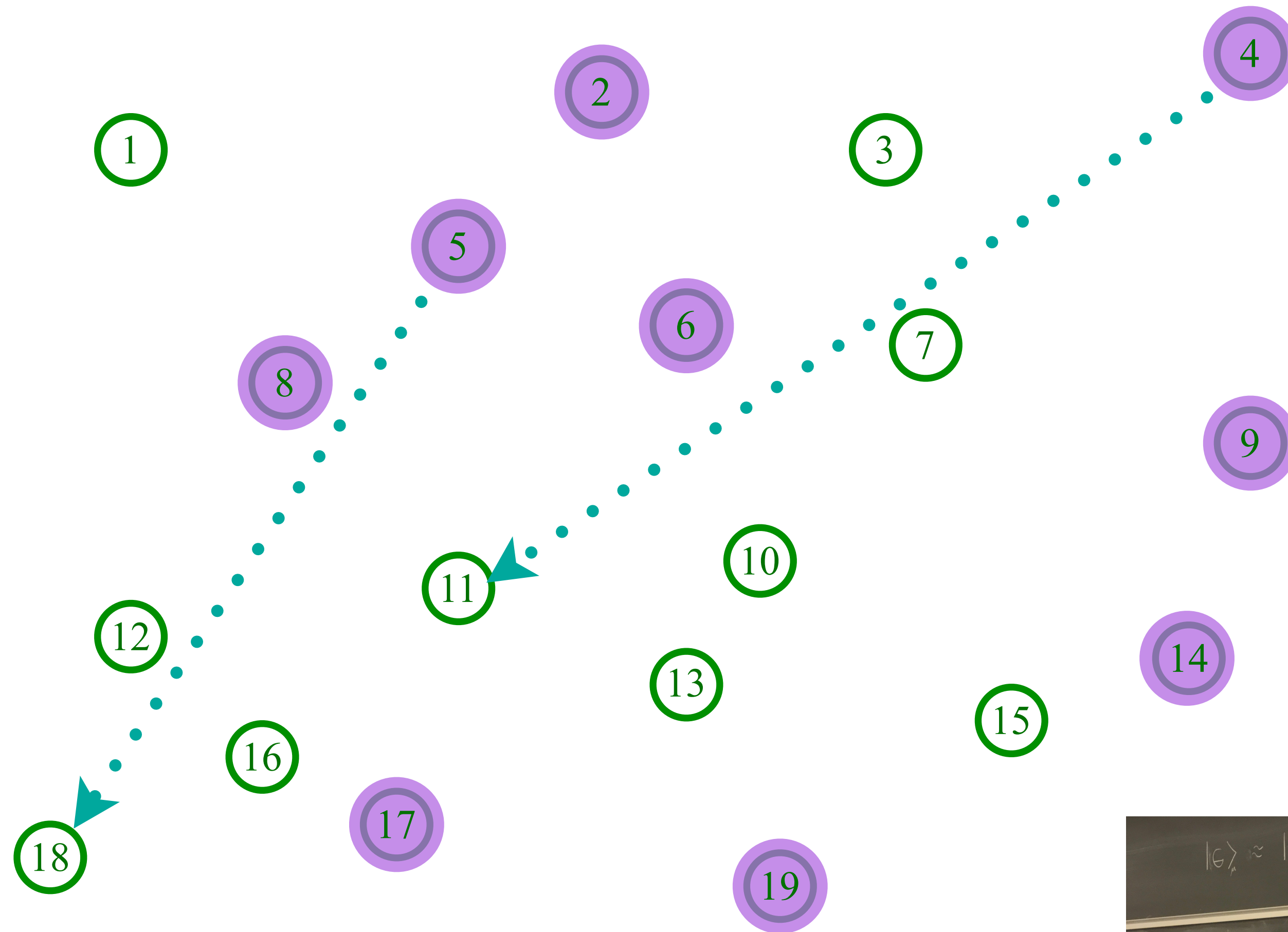
Entangle electrons pairwise randomly



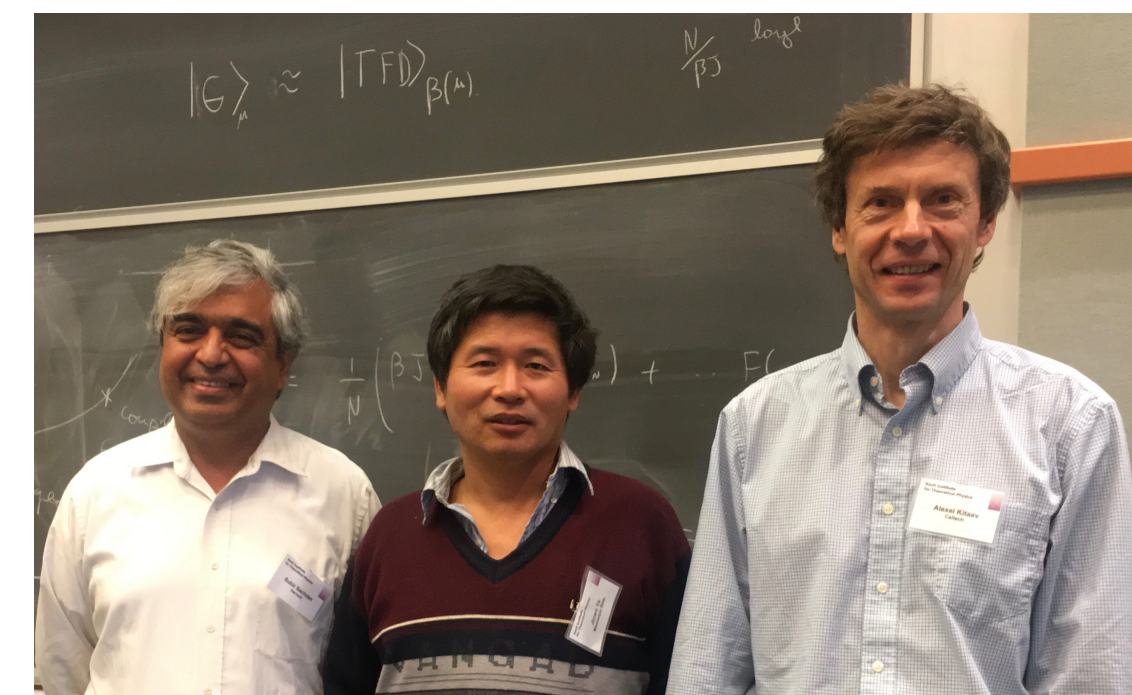
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



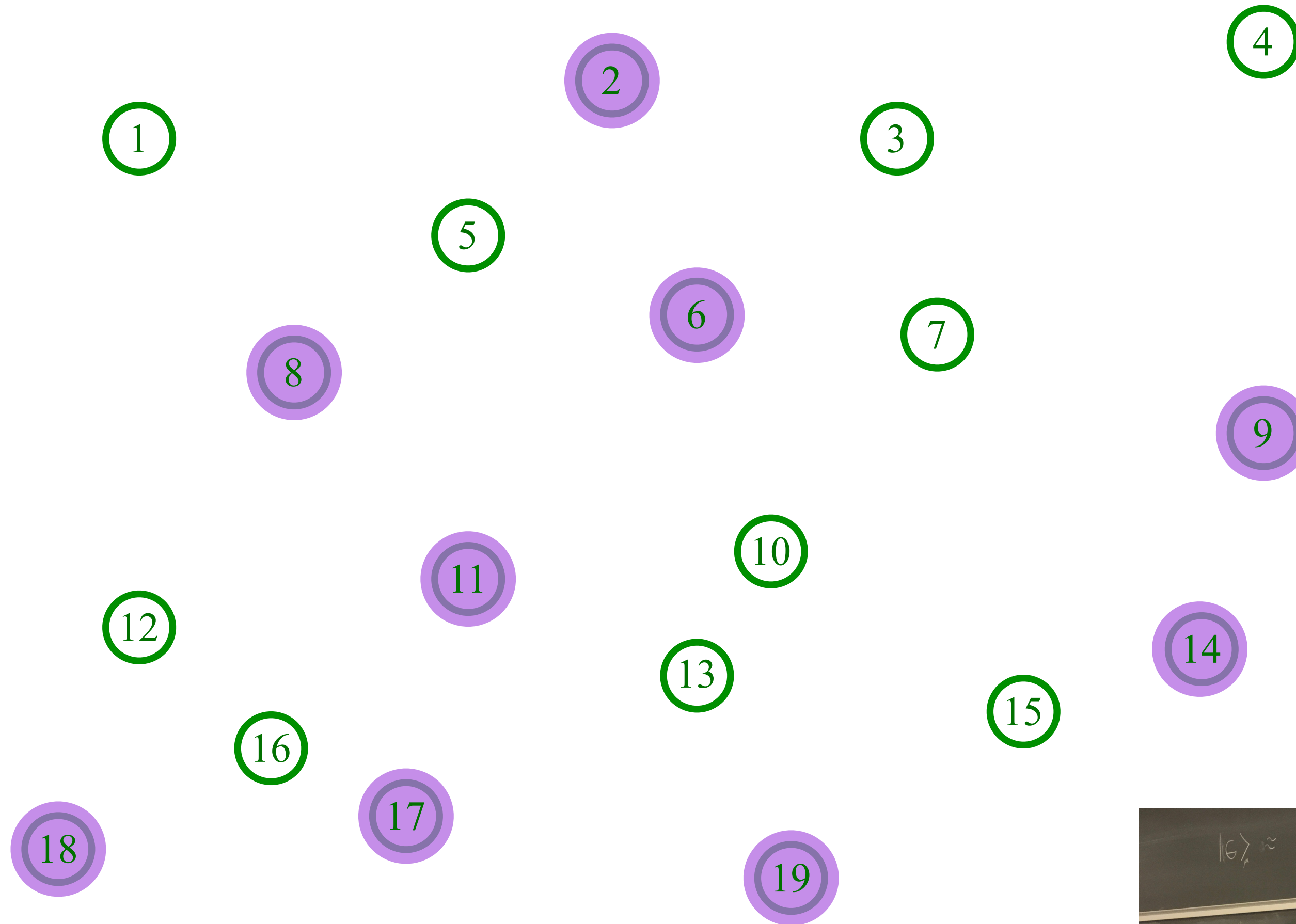
Entangle electrons pairwise randomly



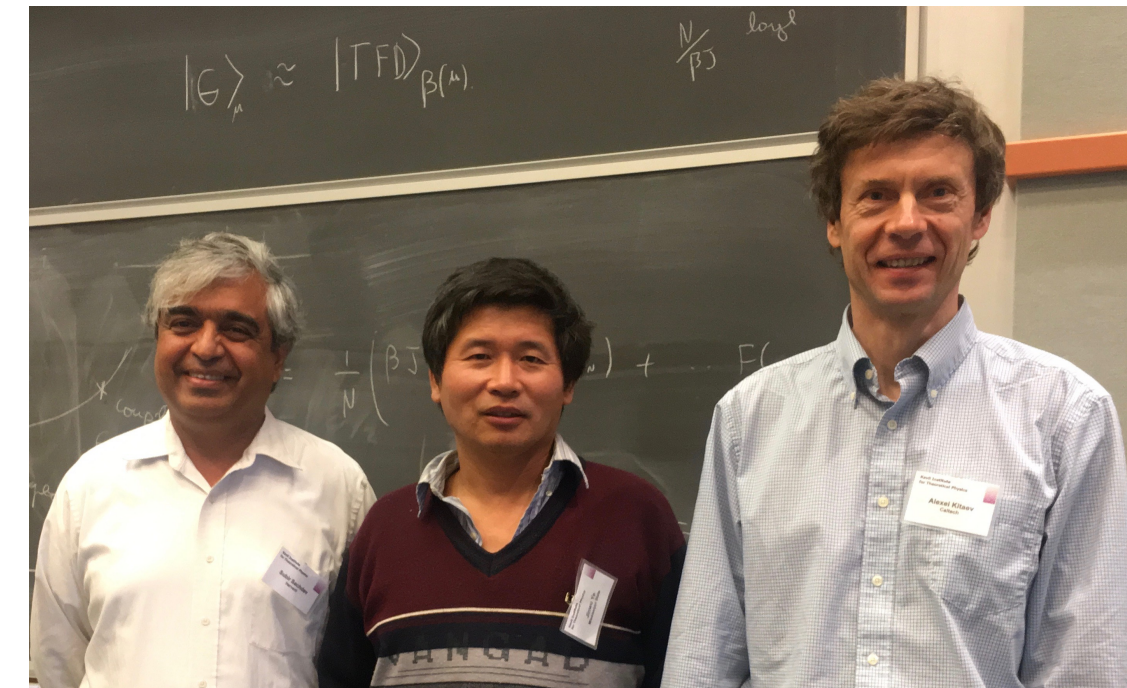
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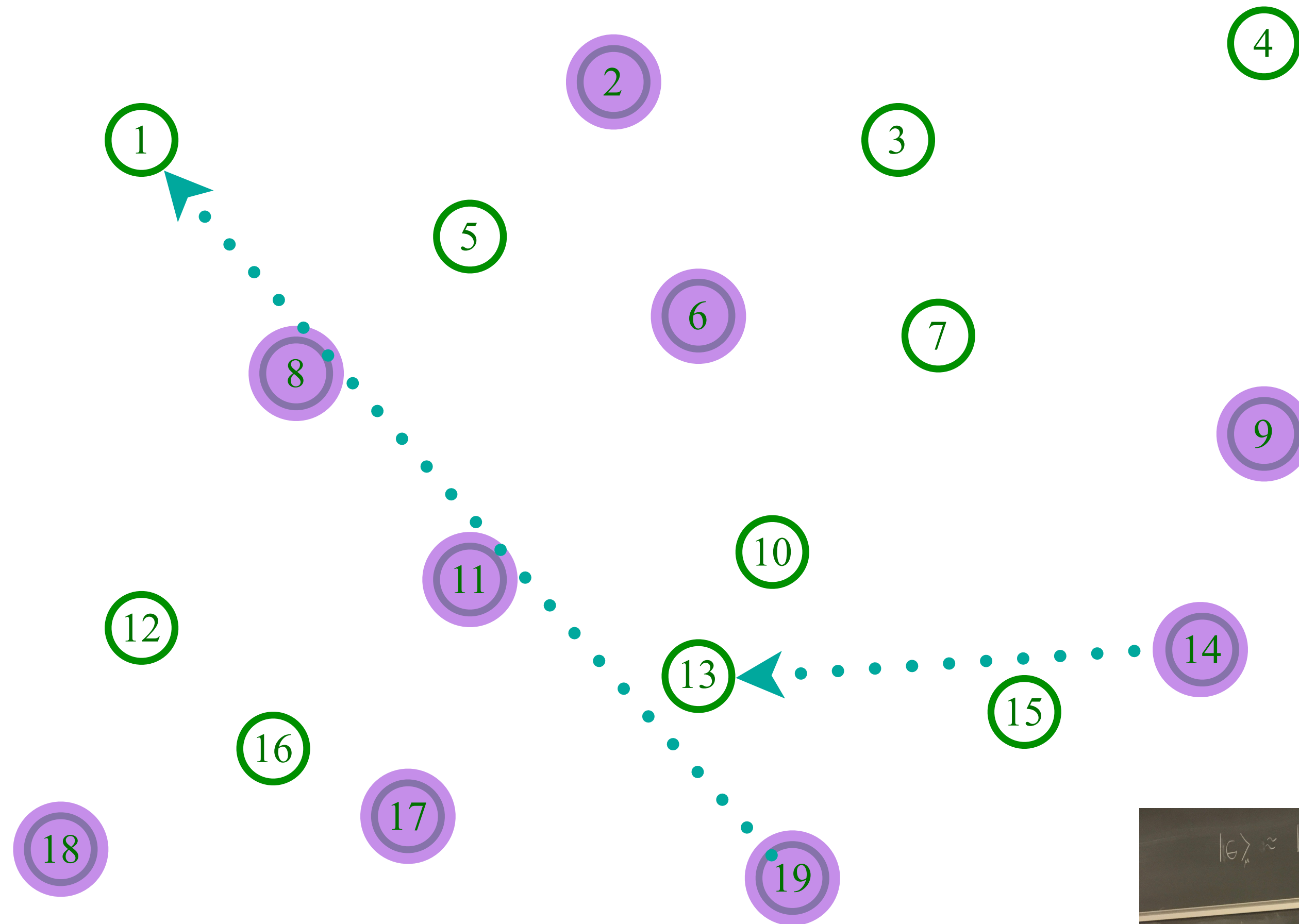
Entangle electrons pairwise randomly



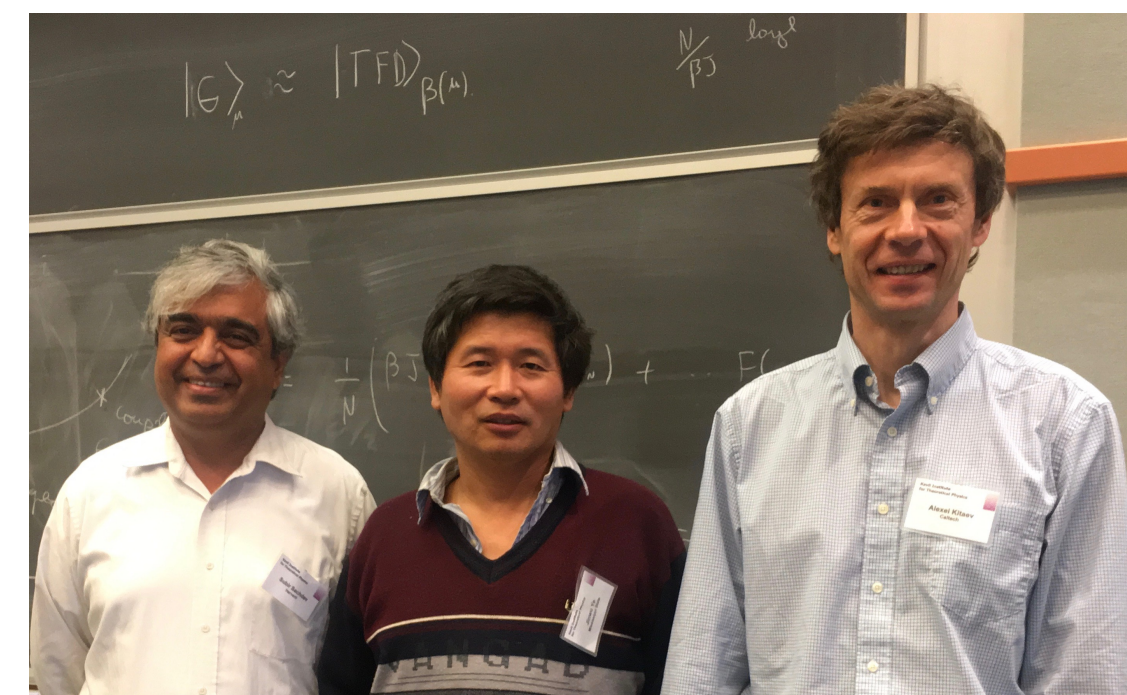
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



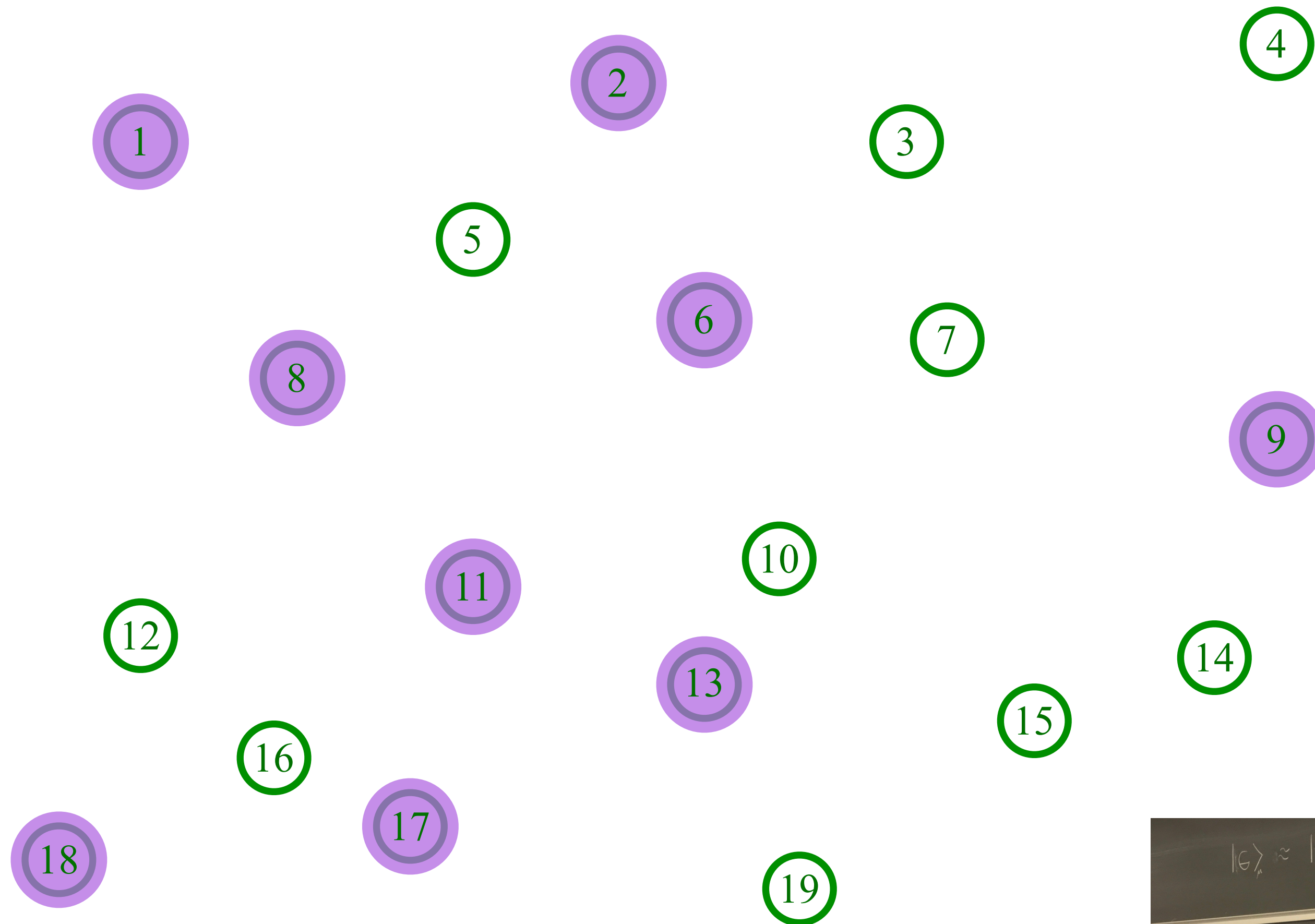
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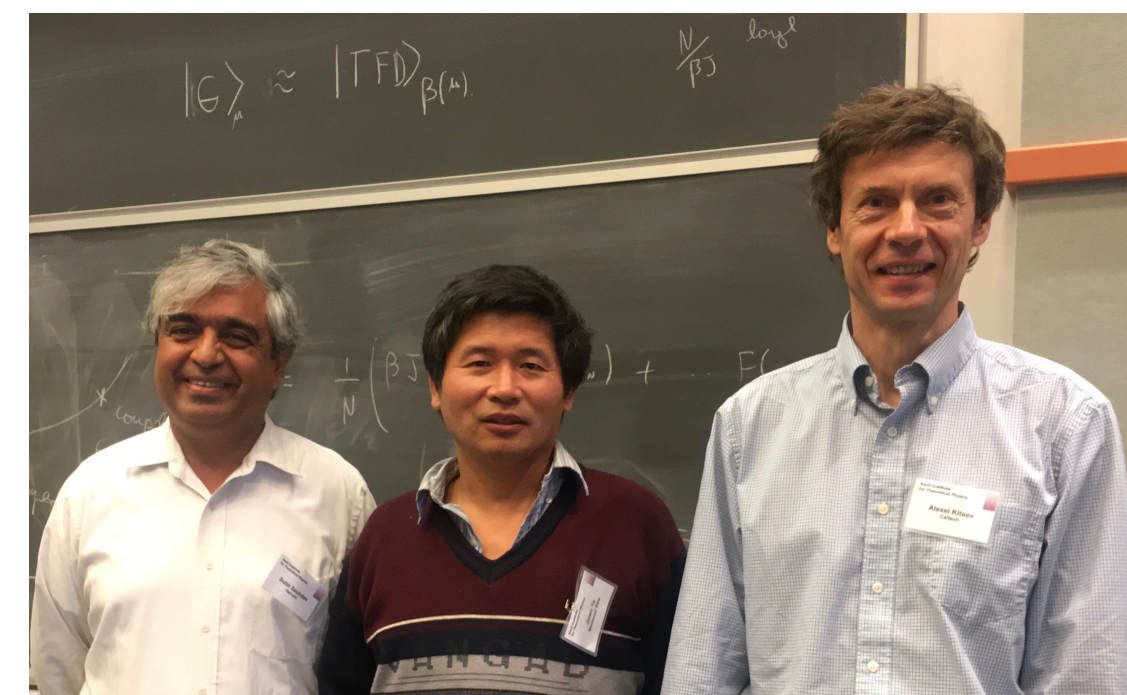
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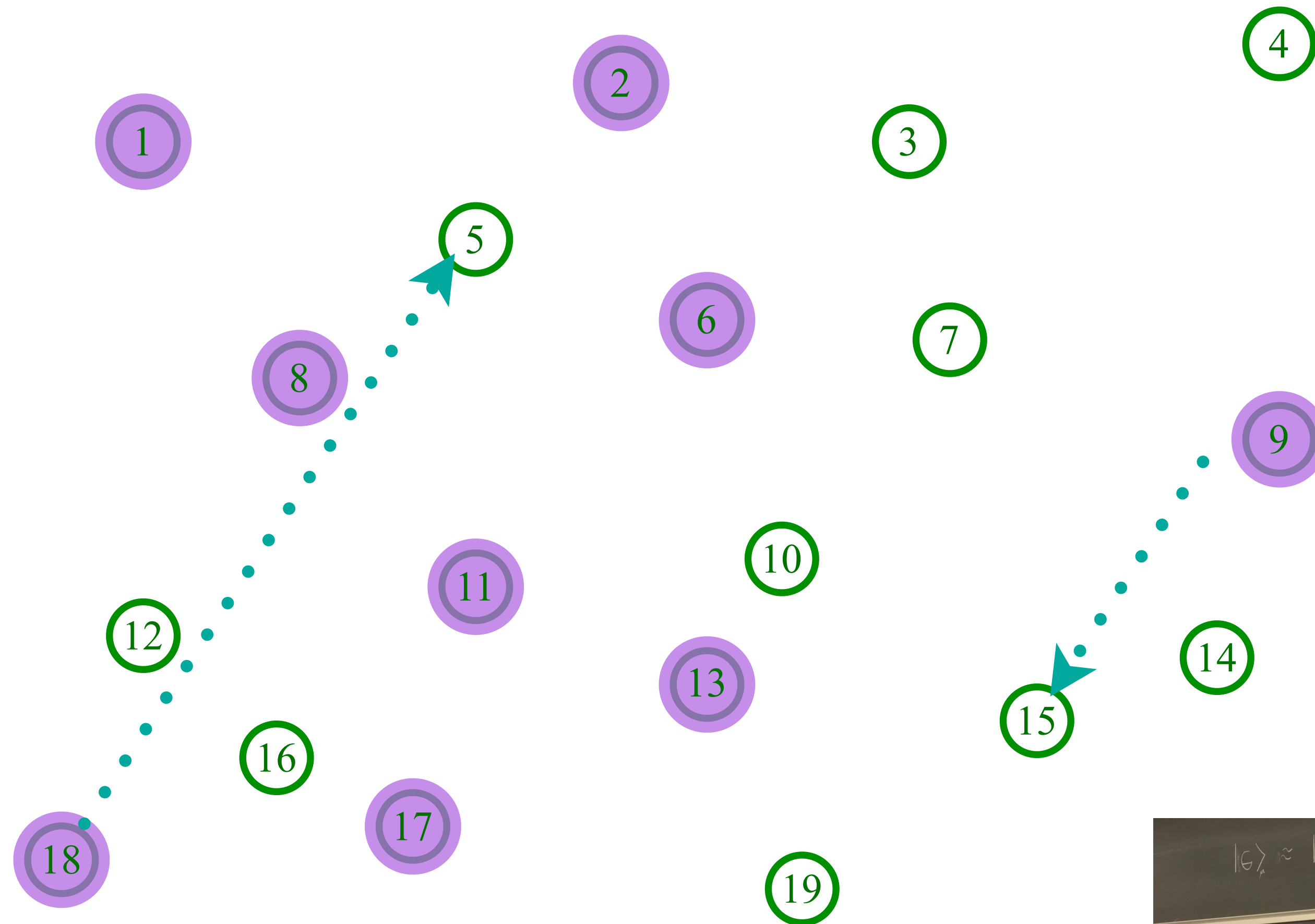
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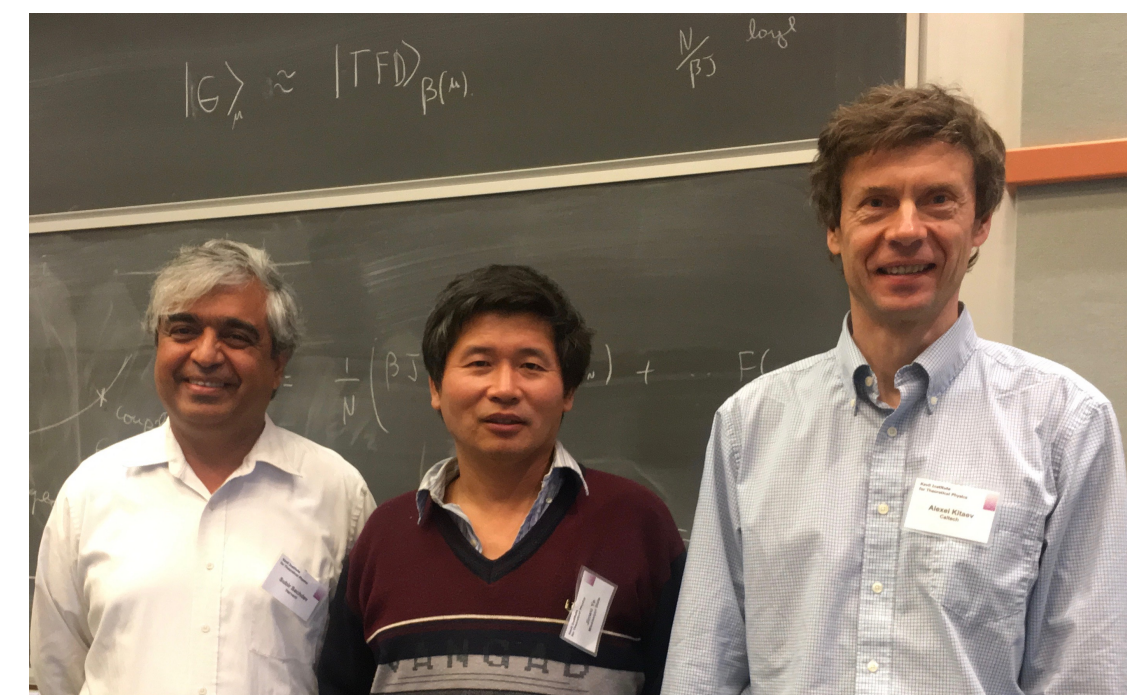
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



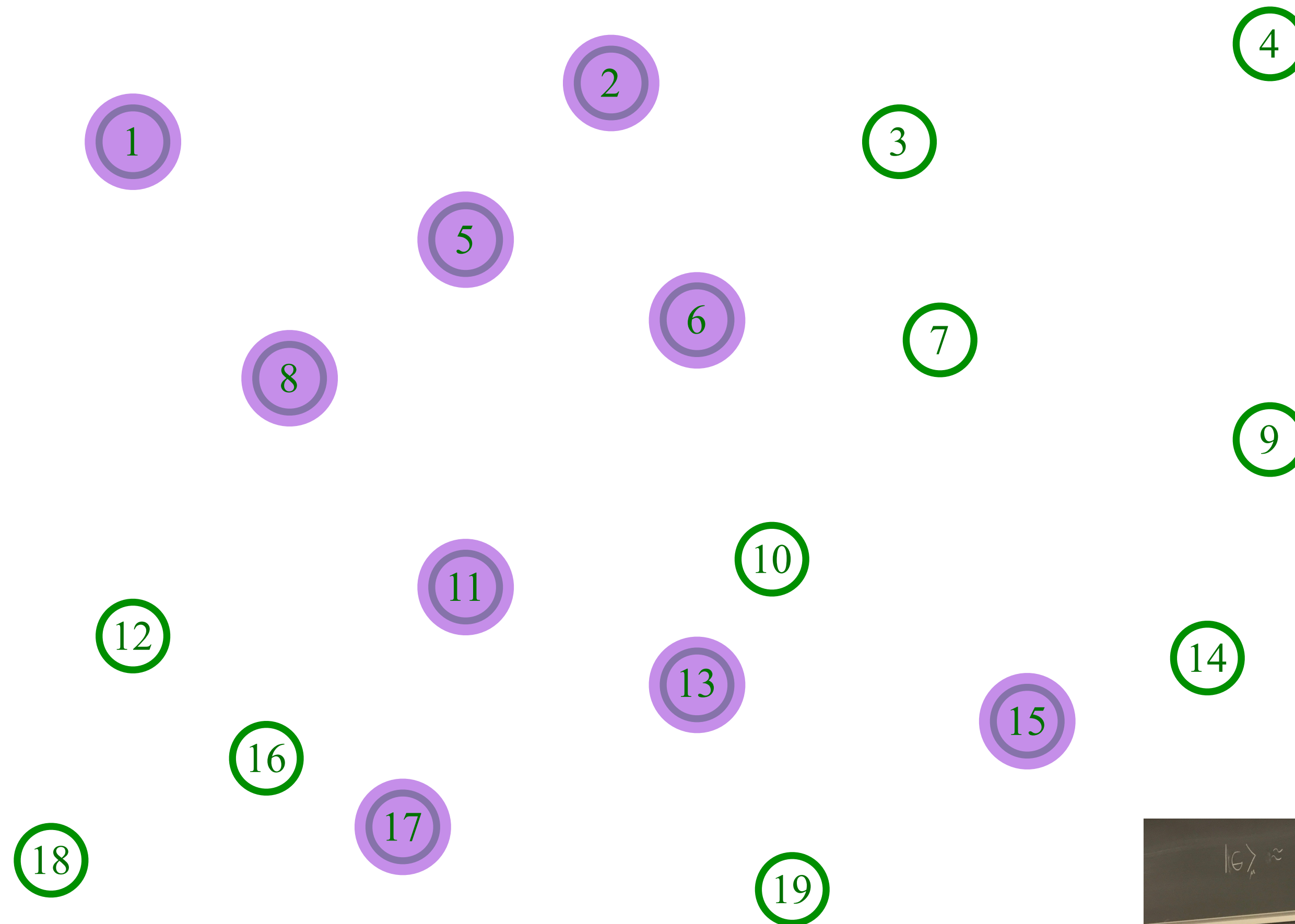
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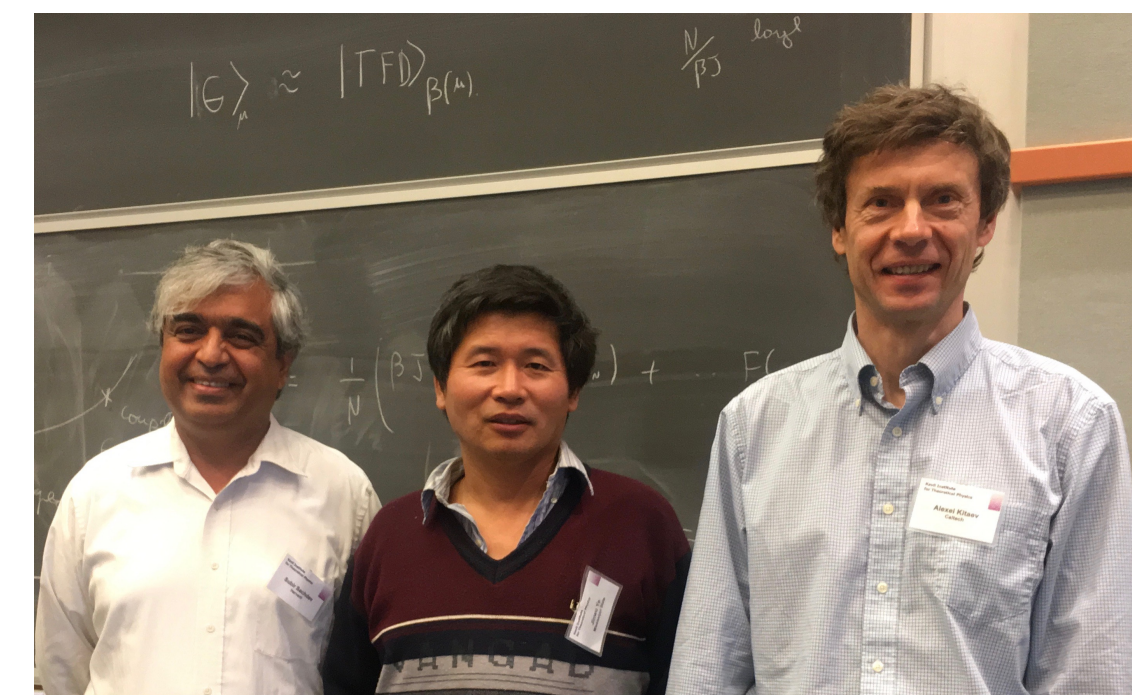
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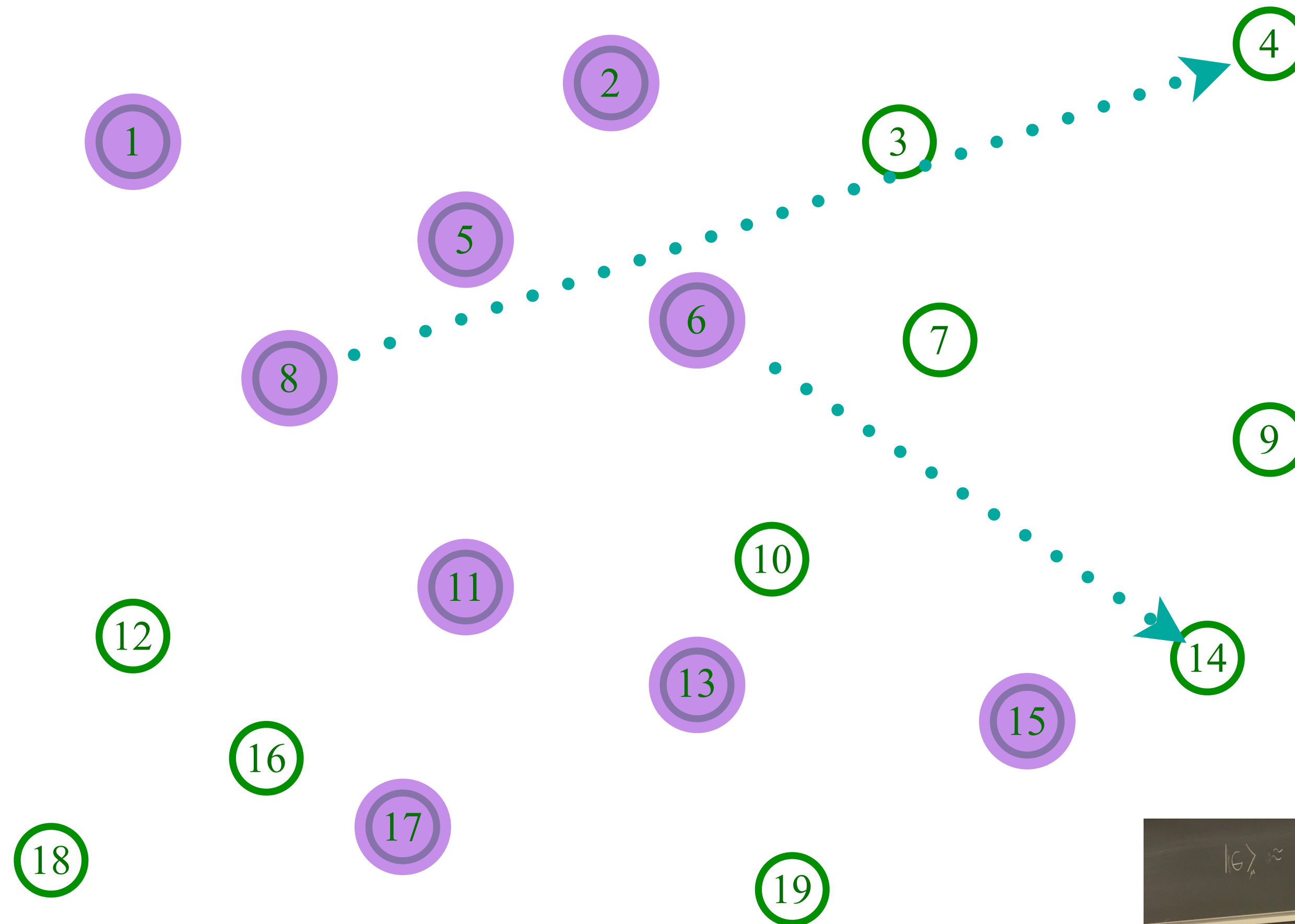
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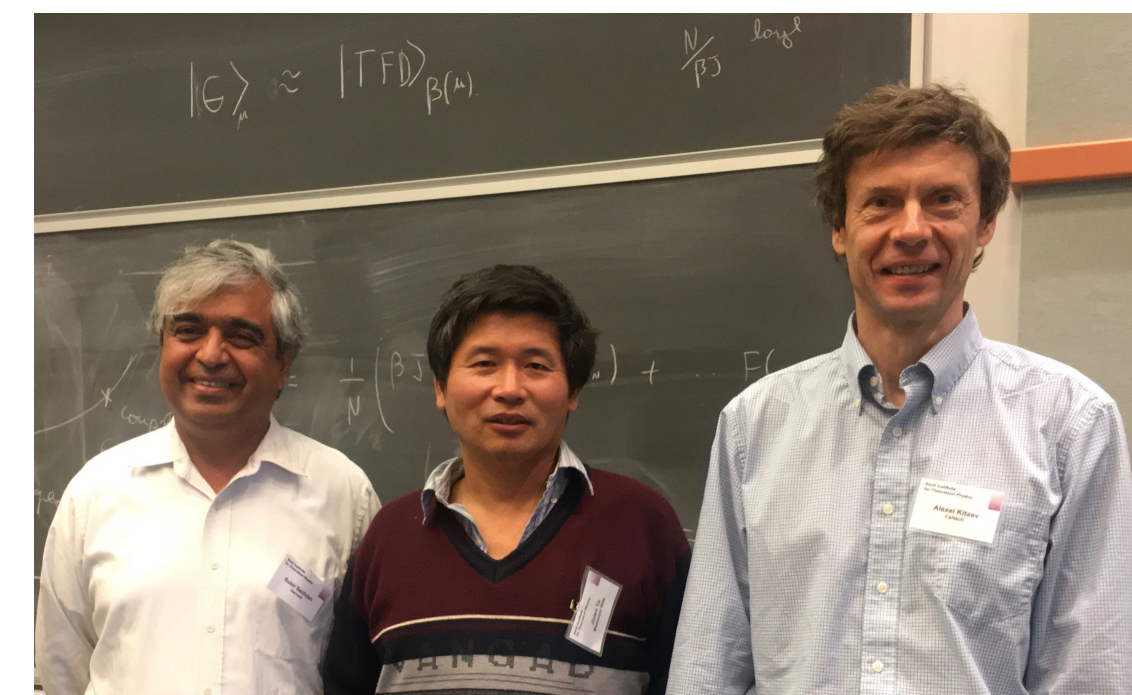
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



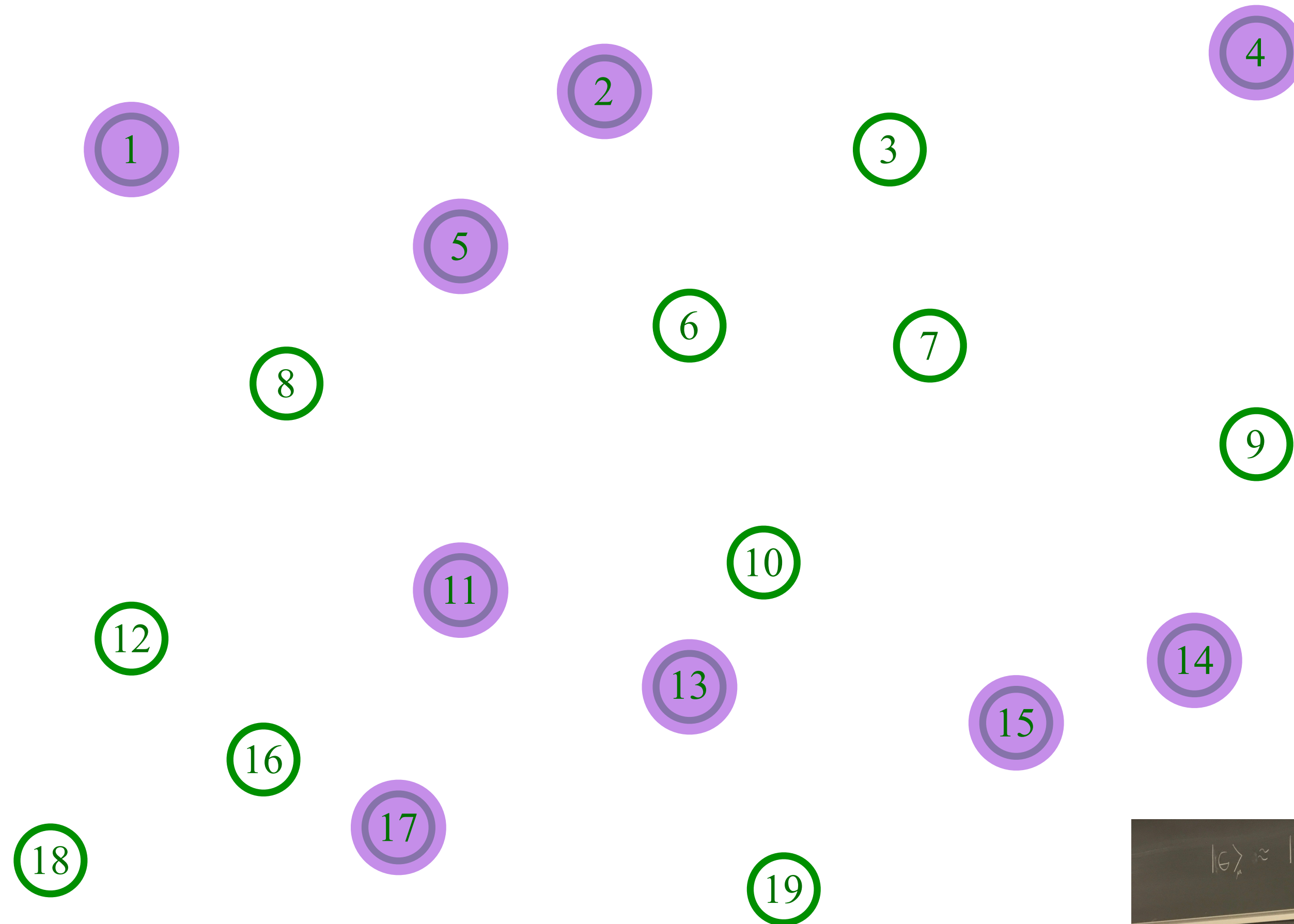
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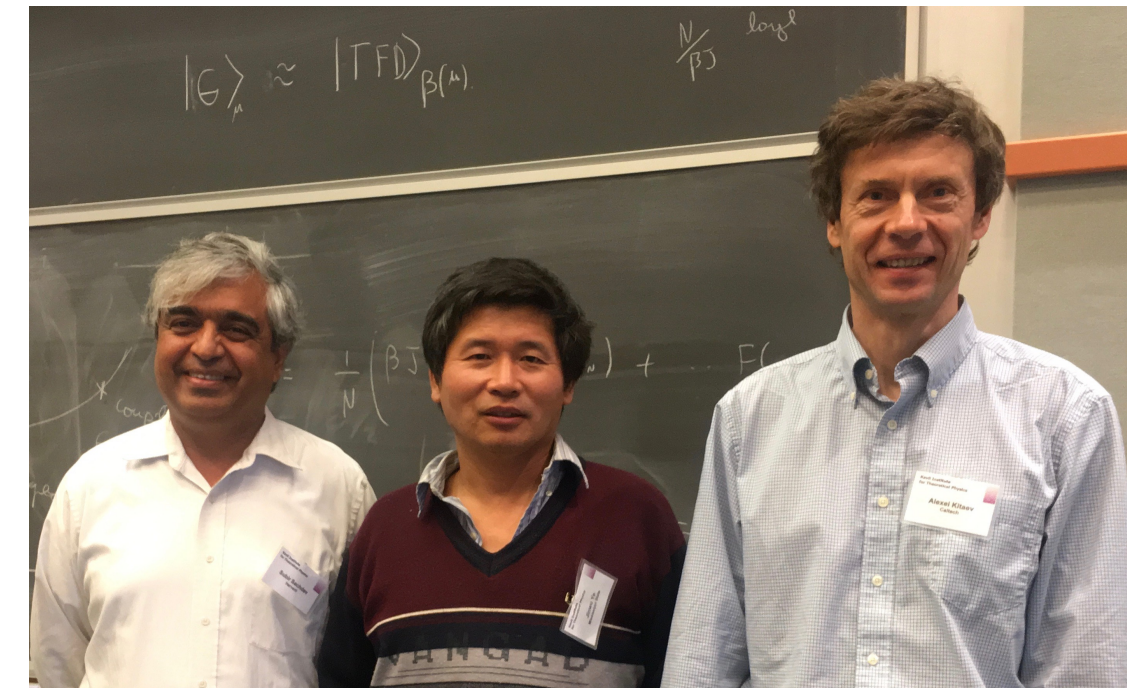
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Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle
quantum entanglement.

No quasiparticles: yields a metal in which
current is carried
not by individual electrons,
but by an entangled “quantum soup”

The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

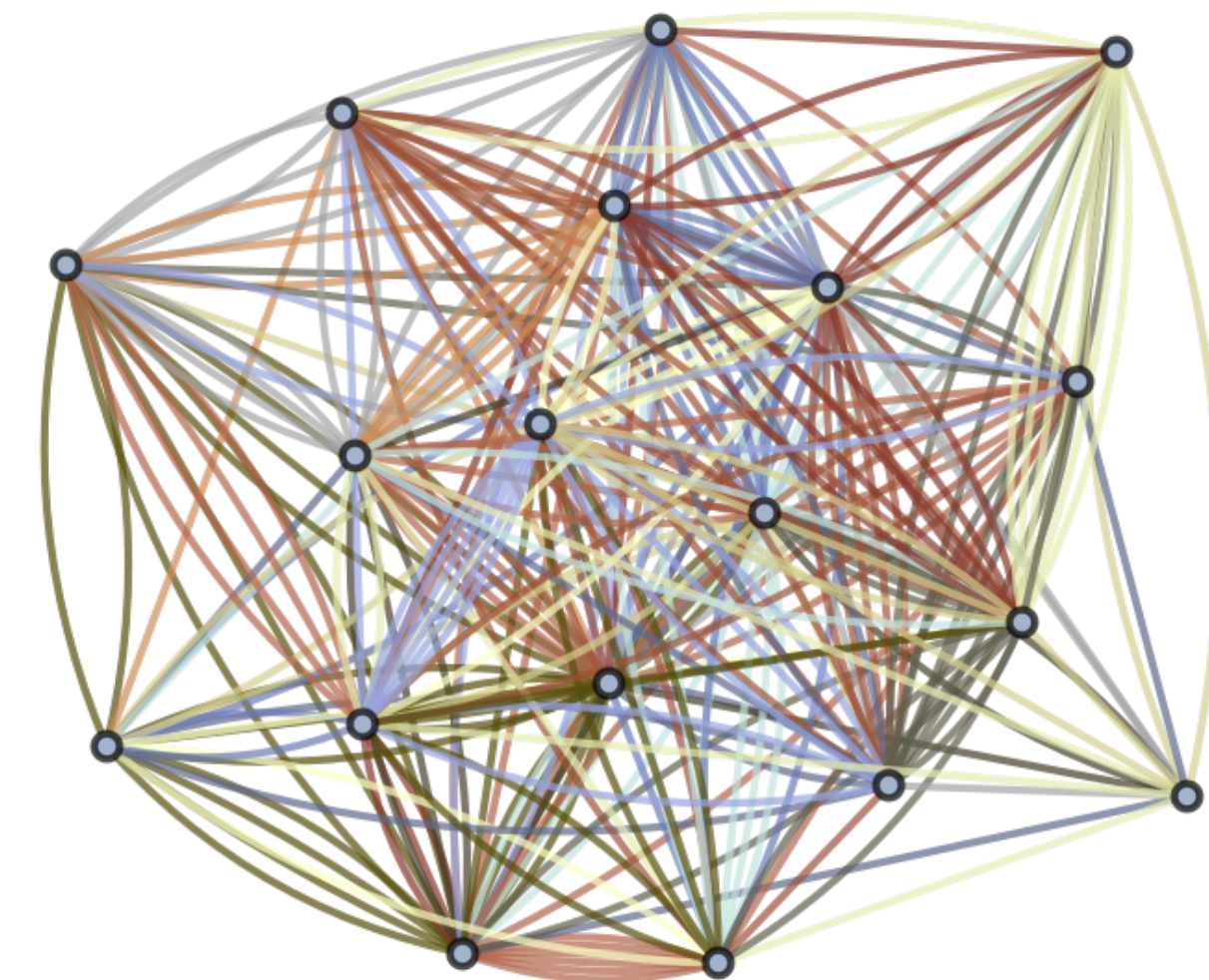
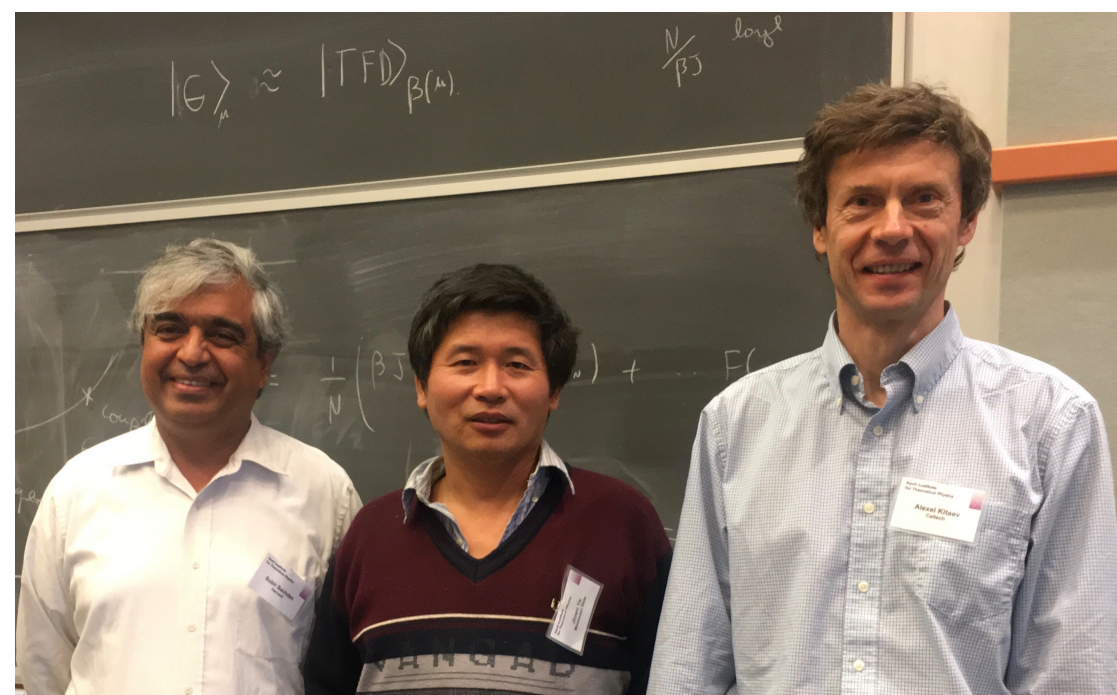
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

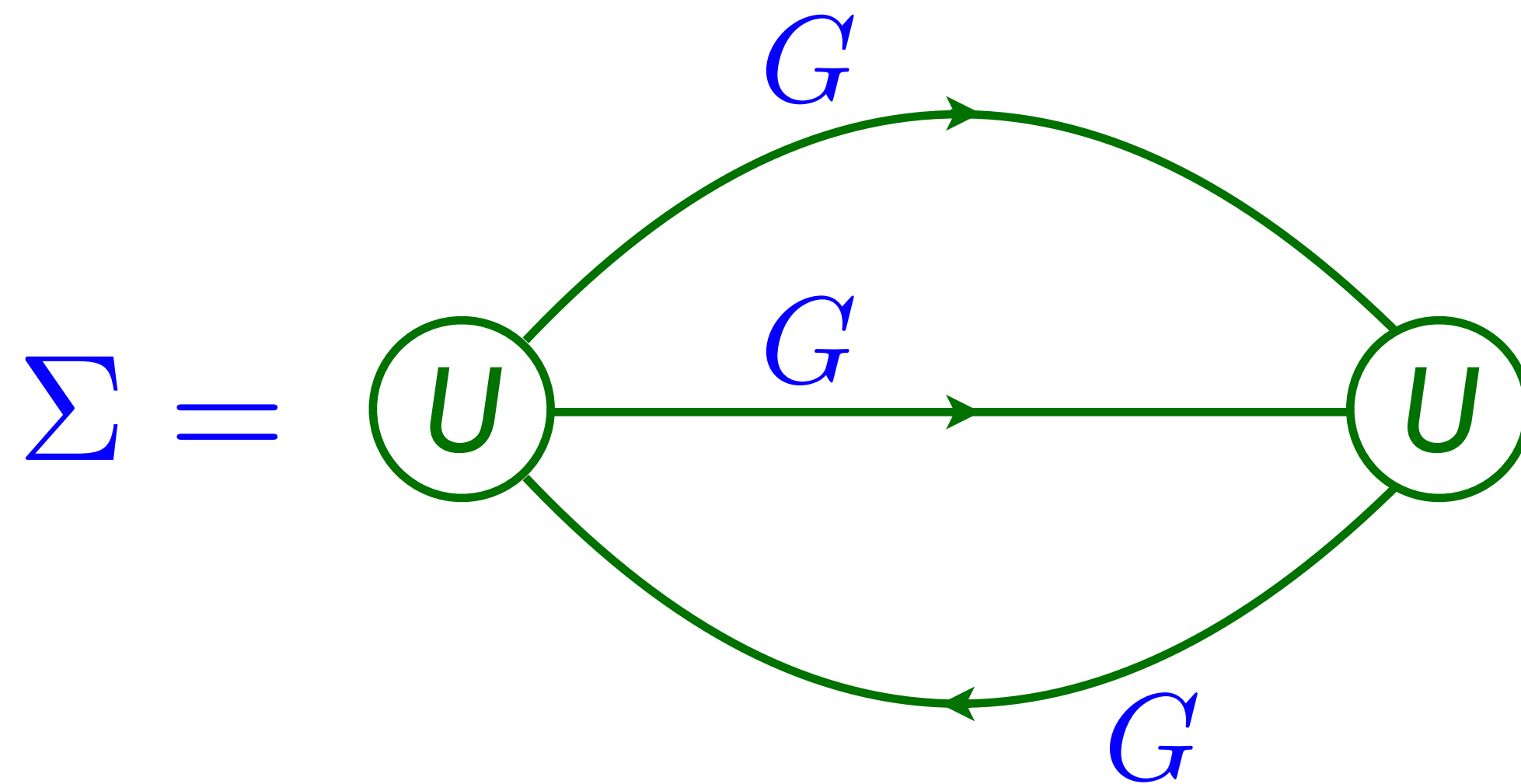


The SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function

$G_s(\tau) = -\sum_{\alpha} \langle c_{\alpha}(\tau) c_{\alpha}^{\dagger}(0) \rangle / N$ in the large N limit:

$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma_s(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$
$$G_s(\tau = 0^-) = Q.$$



S. Sachdev and J. Ye,
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1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

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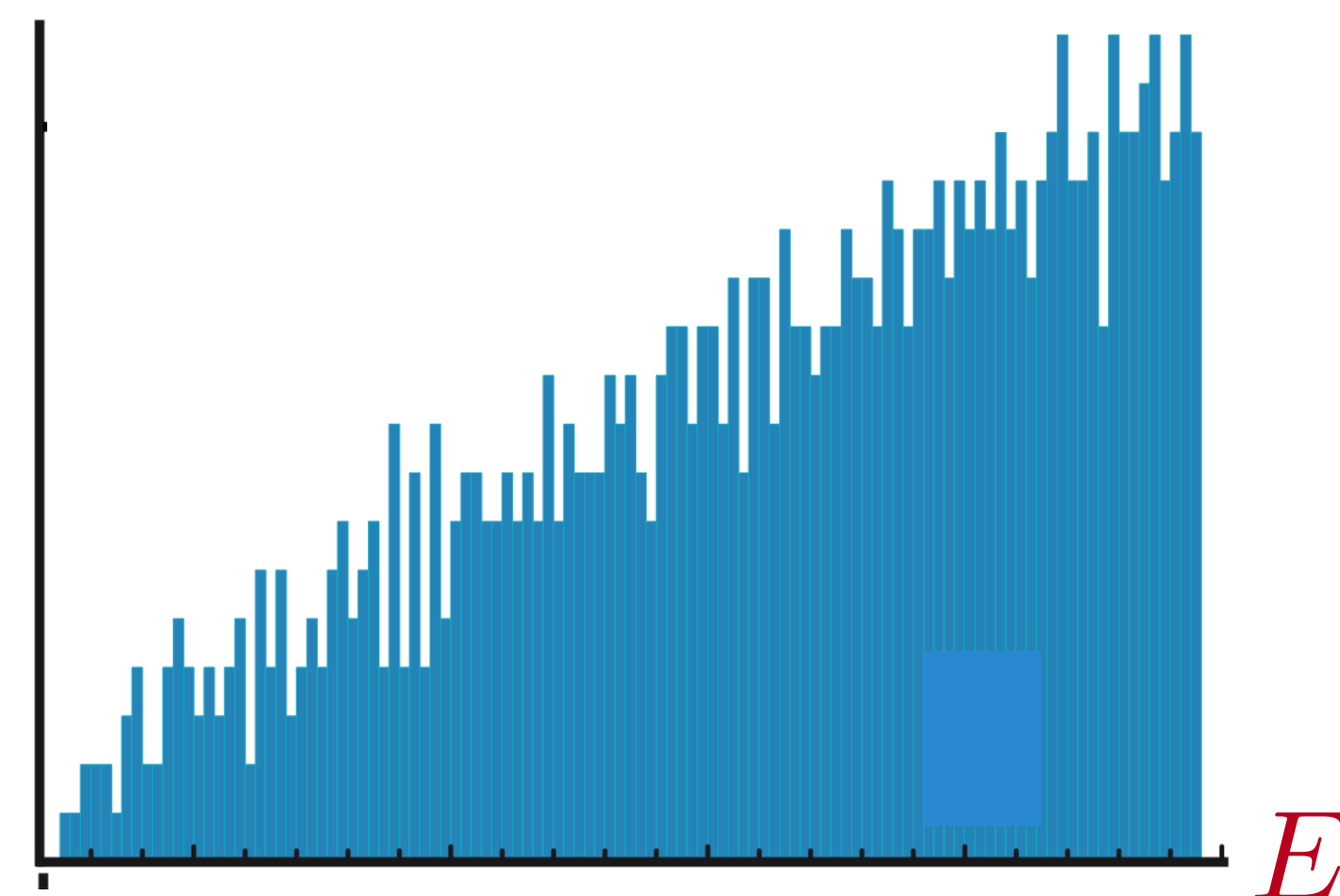
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$$G_s(\tau = 0^-) = Q.$$

2. Zero temperature entropy

$$\lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \rightarrow 0) \sim e^{N s_0} \quad D(E)$$

$$s_0 = 0.46484769917080510749\dots \text{ for } Q = 1/2.$$



The SYK model

The (averaged) partition function can be written as path integral over the bilocal fermion Green's function $G(\tau_1, \tau_2) \sim \frac{1}{N} \sum_{\alpha} c_{\alpha}(\tau_1) c_{\alpha}^{\dagger}(\tau_2)$

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(\tau_1, \tau_2) \exp(-N S_{\text{eff}}[G])$$

The large N saddle point equation $\delta S_{\text{eff}}/\delta G = 0$ yields $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$.

3. Time reparameterization symmetry:

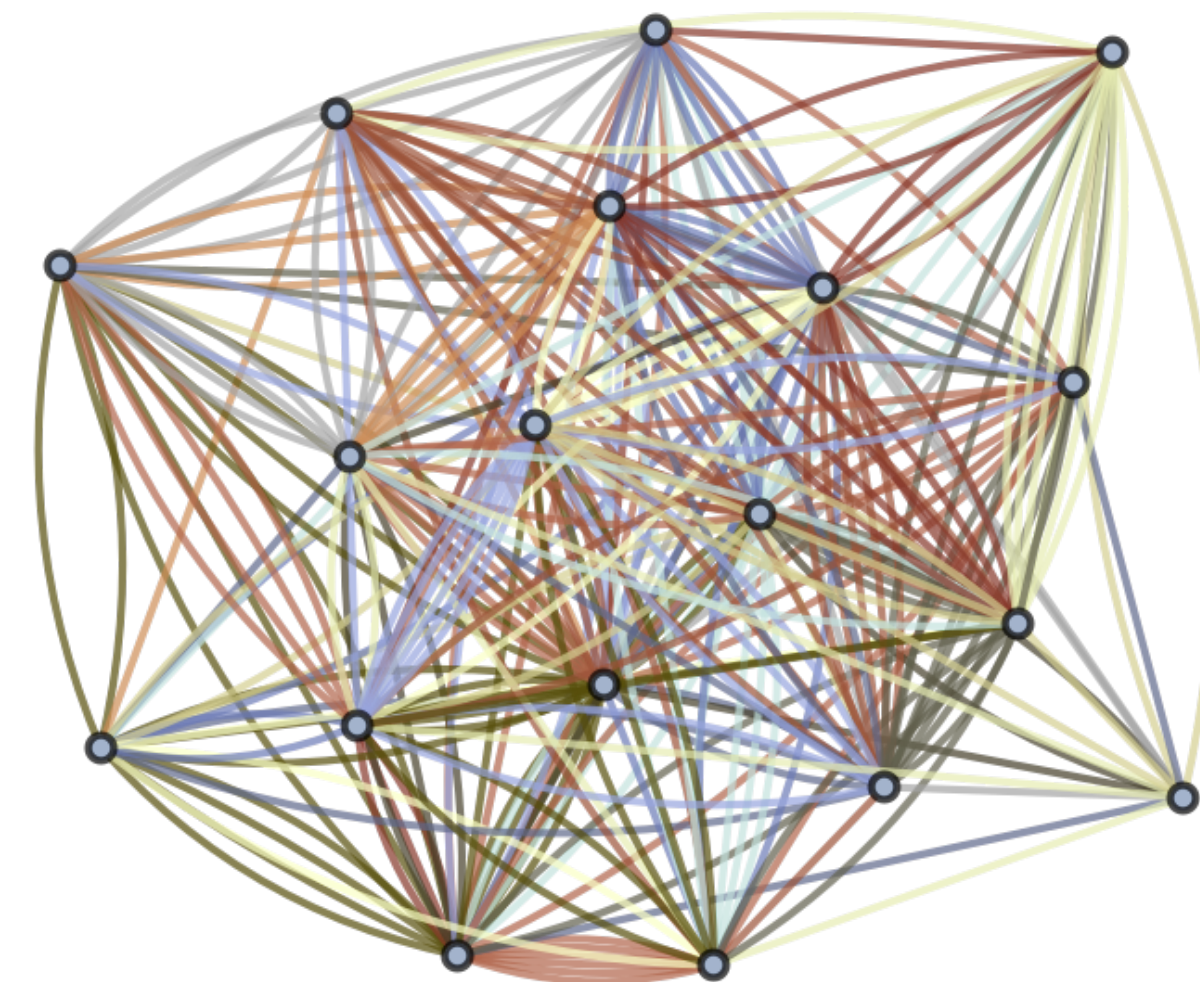
At frequencies $\ll U$, the path integral for is invariant under time reparameterization $f(\sigma)$

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \tilde{G}(\sigma_1, \sigma_2)$$

There is also an emergent $U(1)$ gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!

A. Georges, O. Parcollet, S. Sachdev 2001
A. Kitaev, 2015
J. Maldacena, D. Stanford 2016



The SYK model

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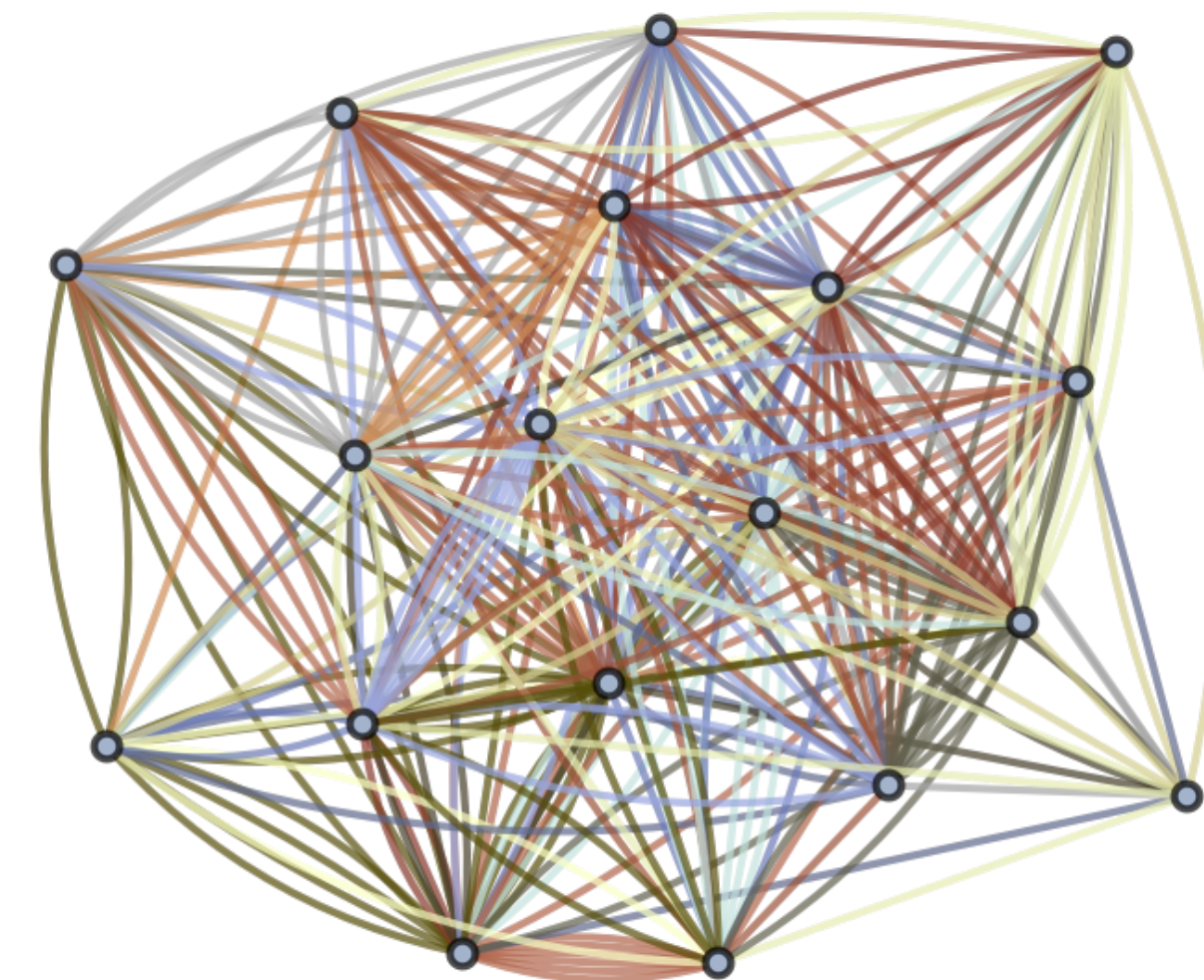
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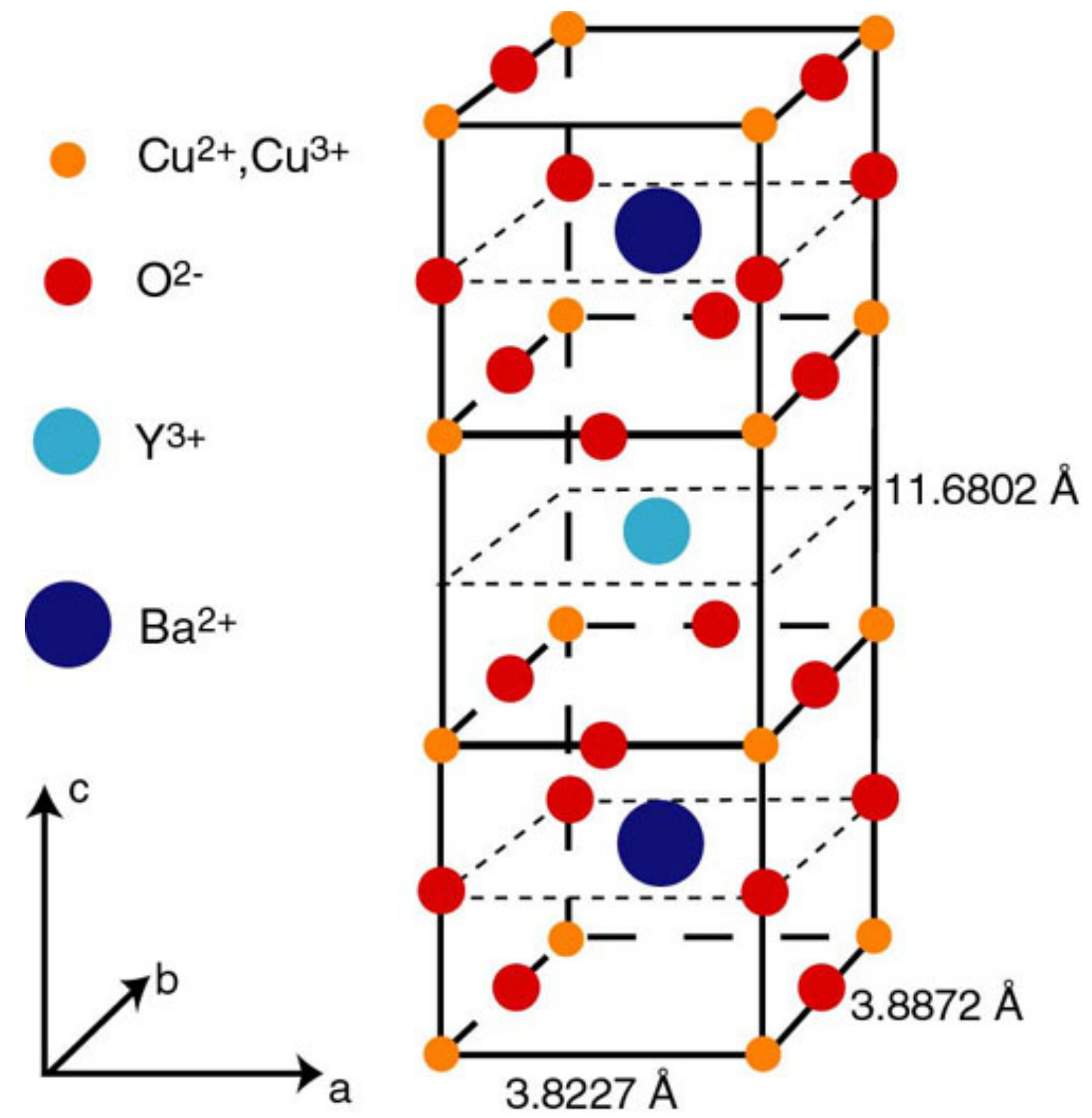
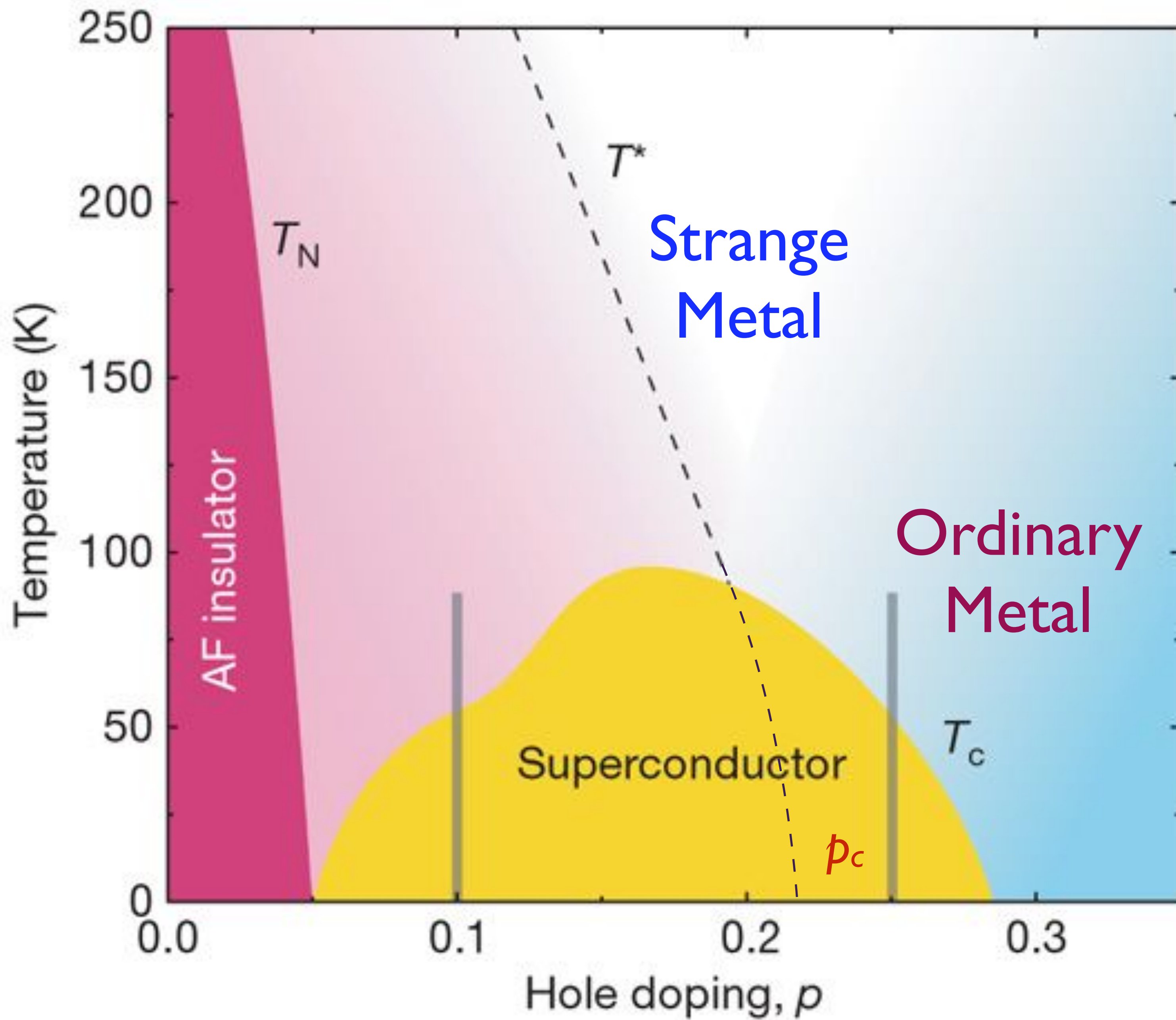
4. Conformal symmetry of saddle point:

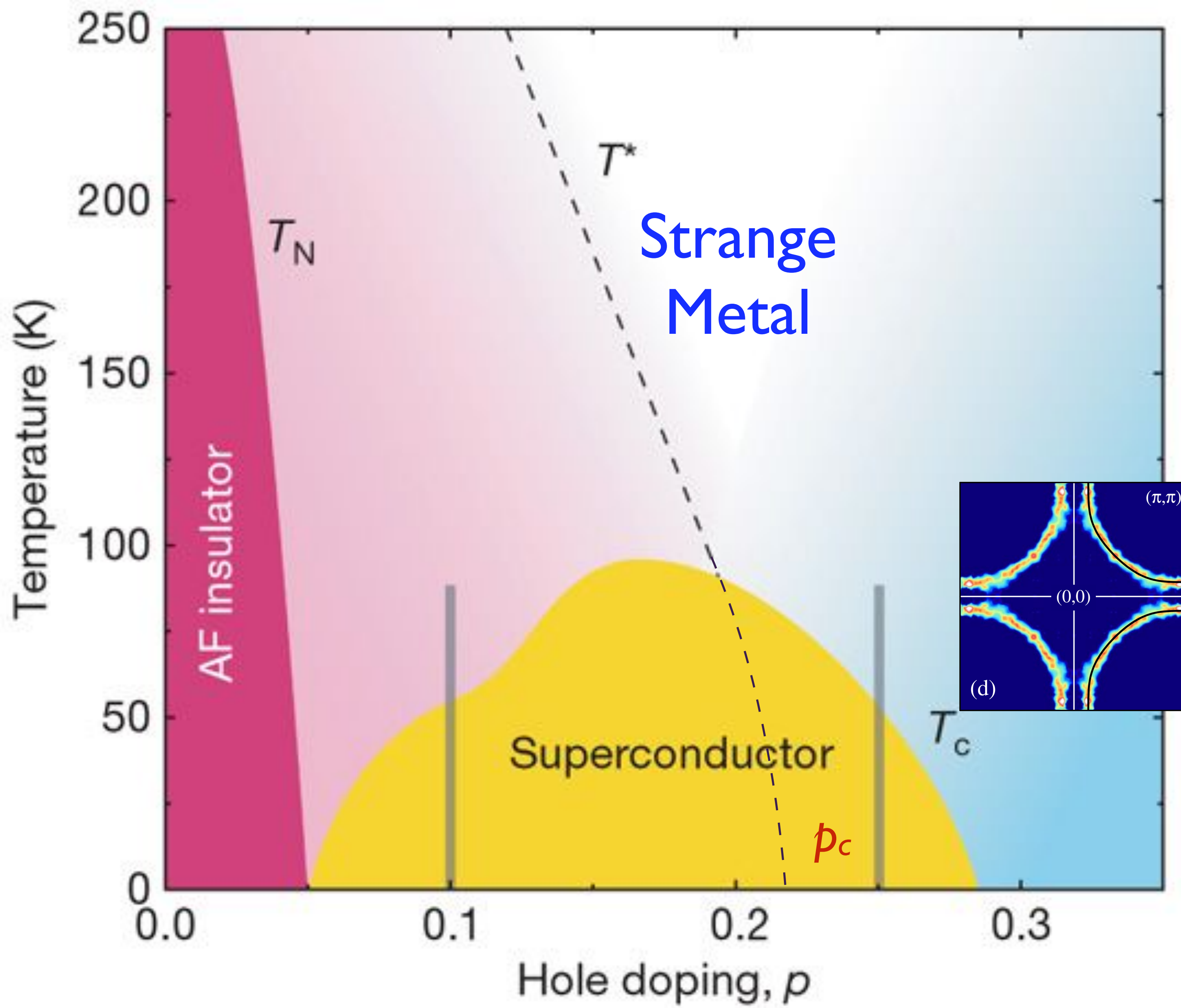
$$G_s(\tau_1 - \tau_2) = \tilde{G}_s(\sigma_1 - \sigma_2) \text{ for } \tau = \frac{a\sigma + b}{c\sigma + d}, \quad ad - bc = 1.$$

A. Georges, O. Parcollet, S. Sachdev 2001
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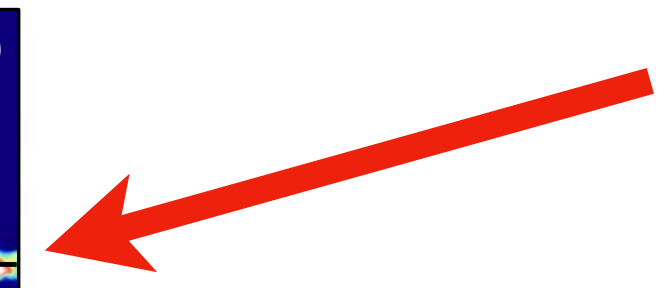


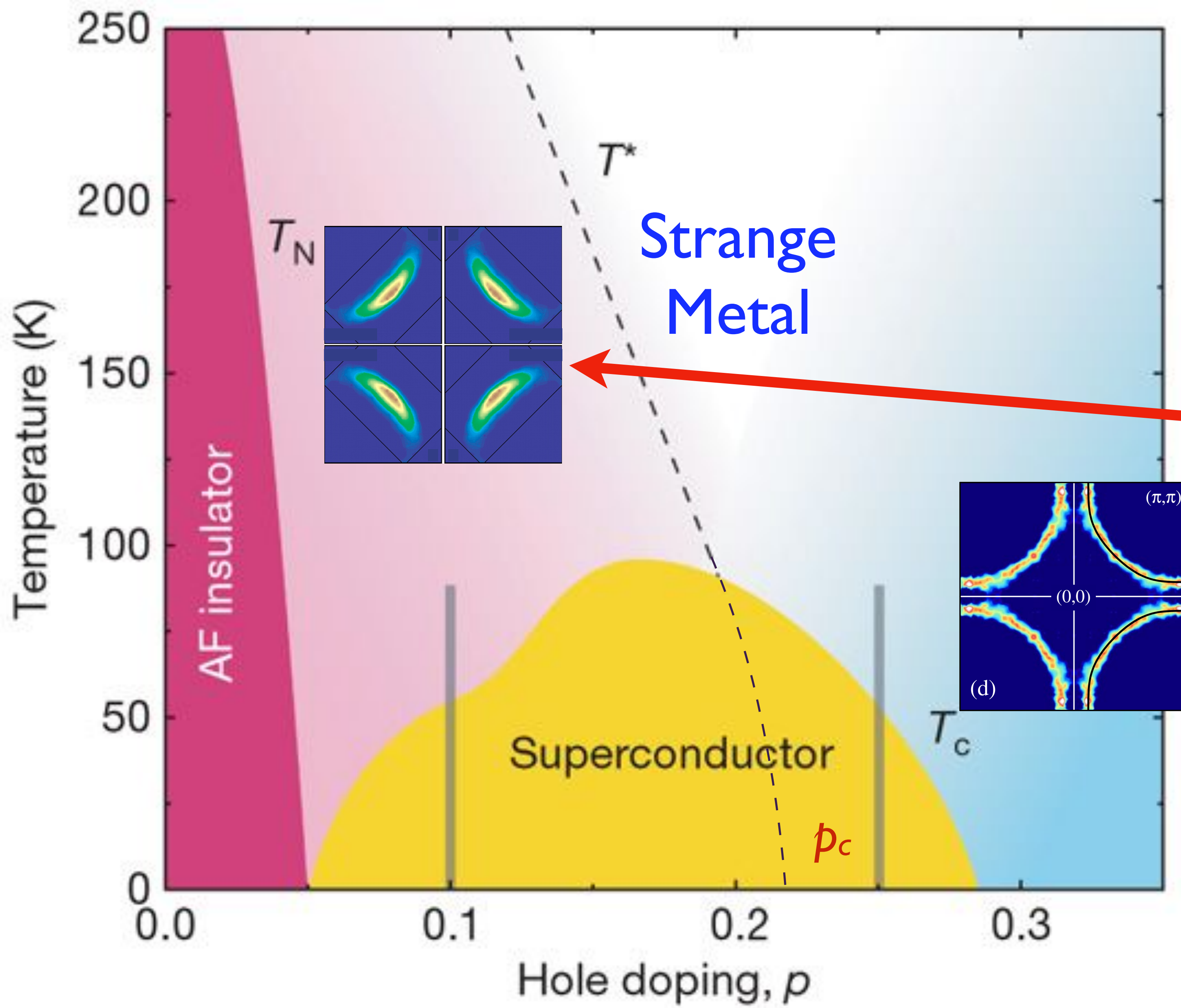
Quantum entanglement and superconductivity



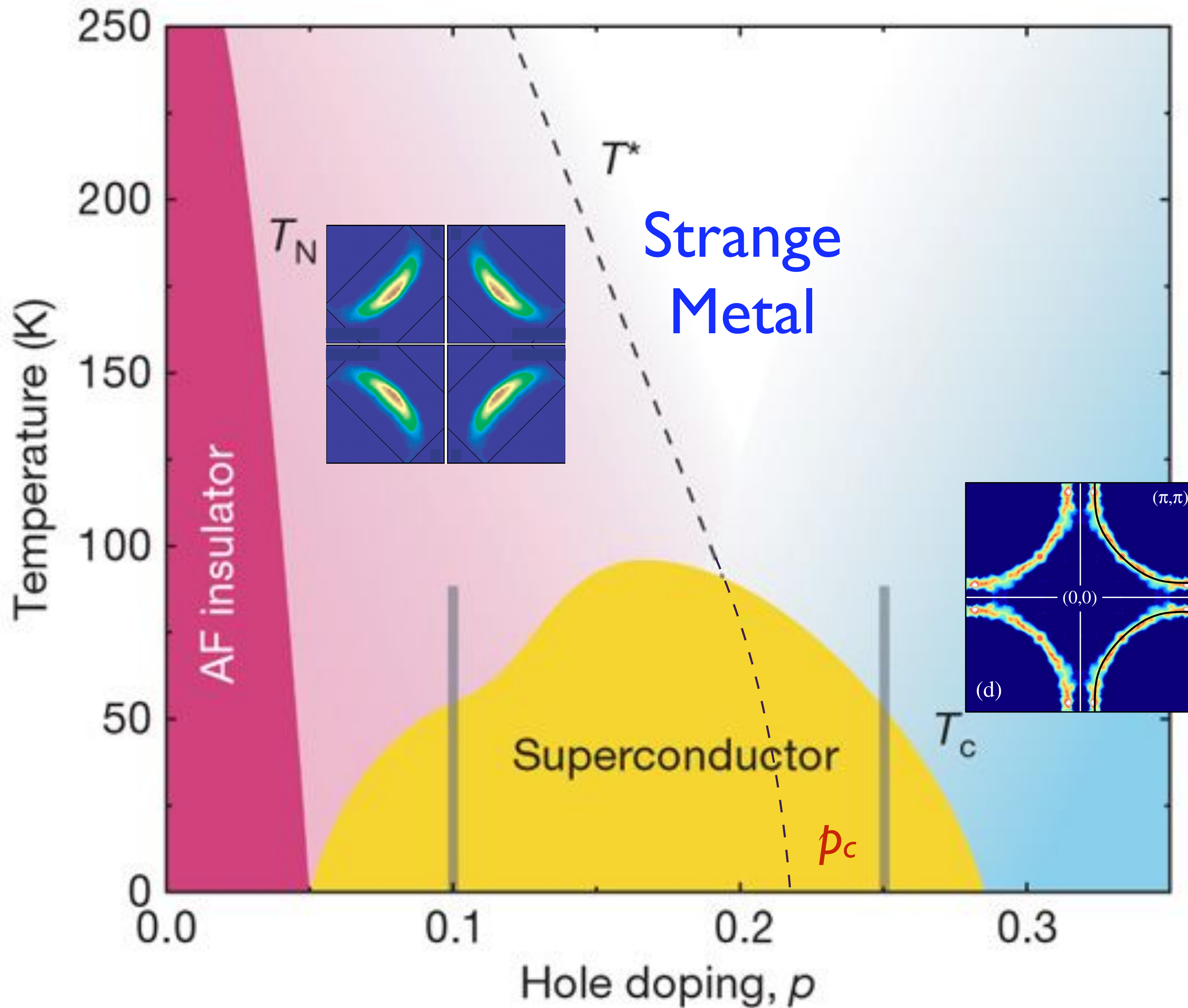


Fermi surface
as expected
in a model
of free electrons





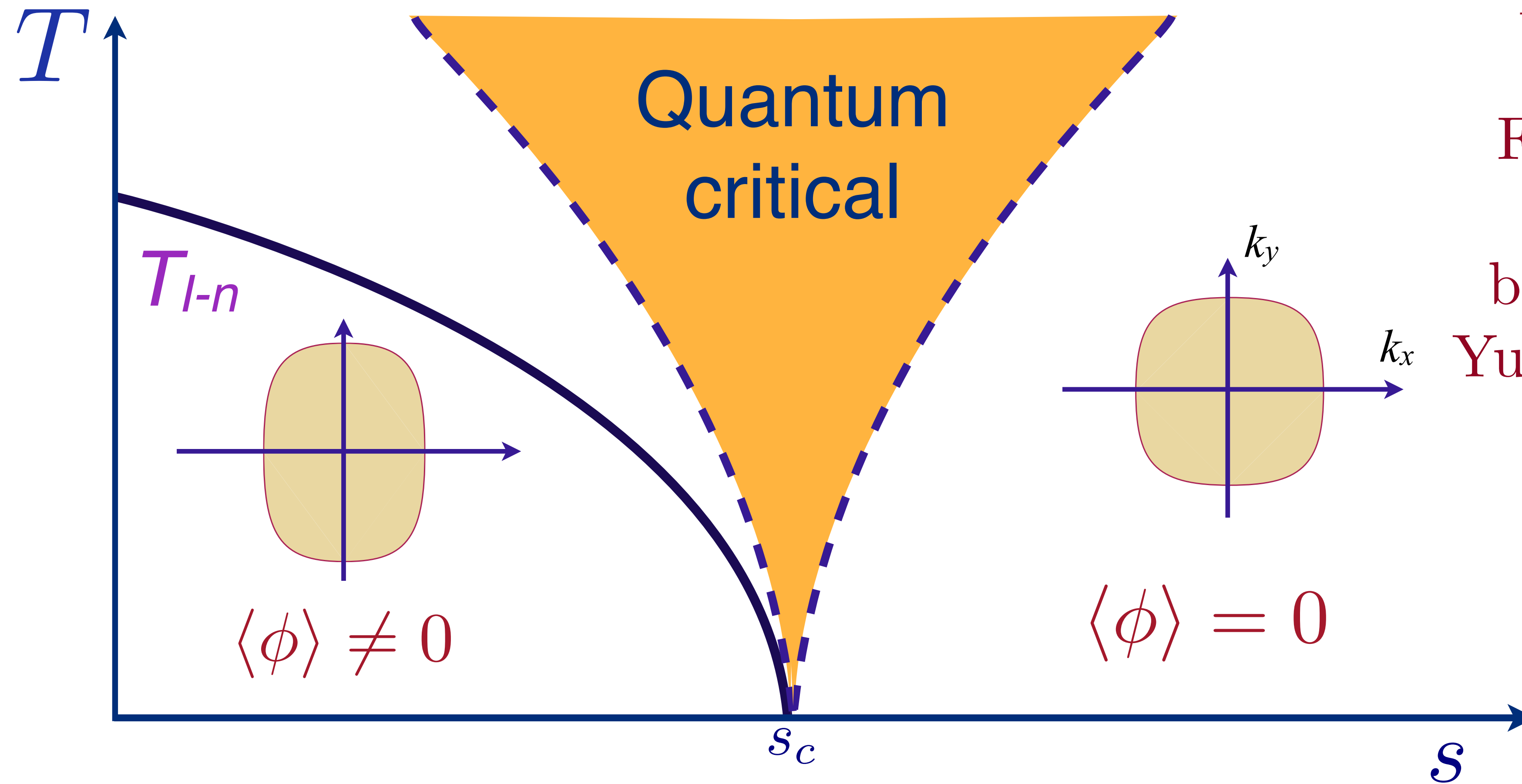
“Pseudogap metal”
 Fermi surface
 modified by
 electron-electron
 interactions



View the strange metal as a property of a $T = 0$ quantum phase transition involving change in the Fermi surface.

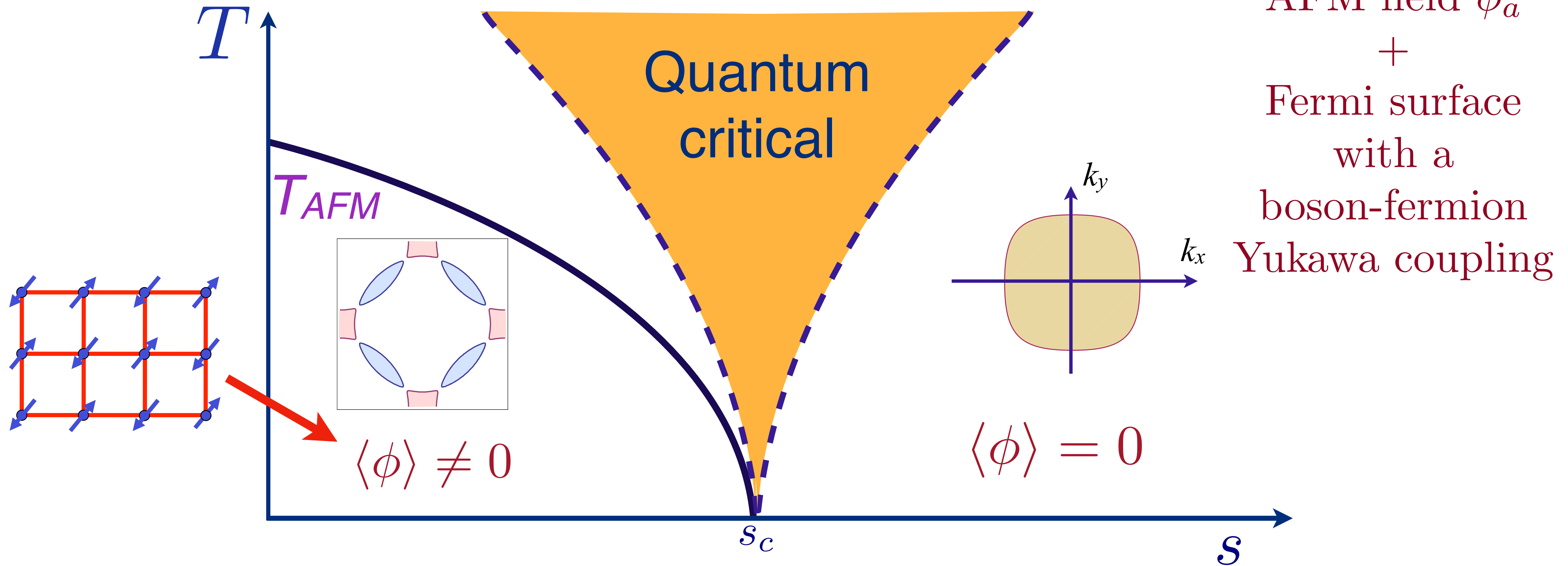
The onset of superconductivity may “hide” this quantum transition.

Type I: Fermi surface distortion

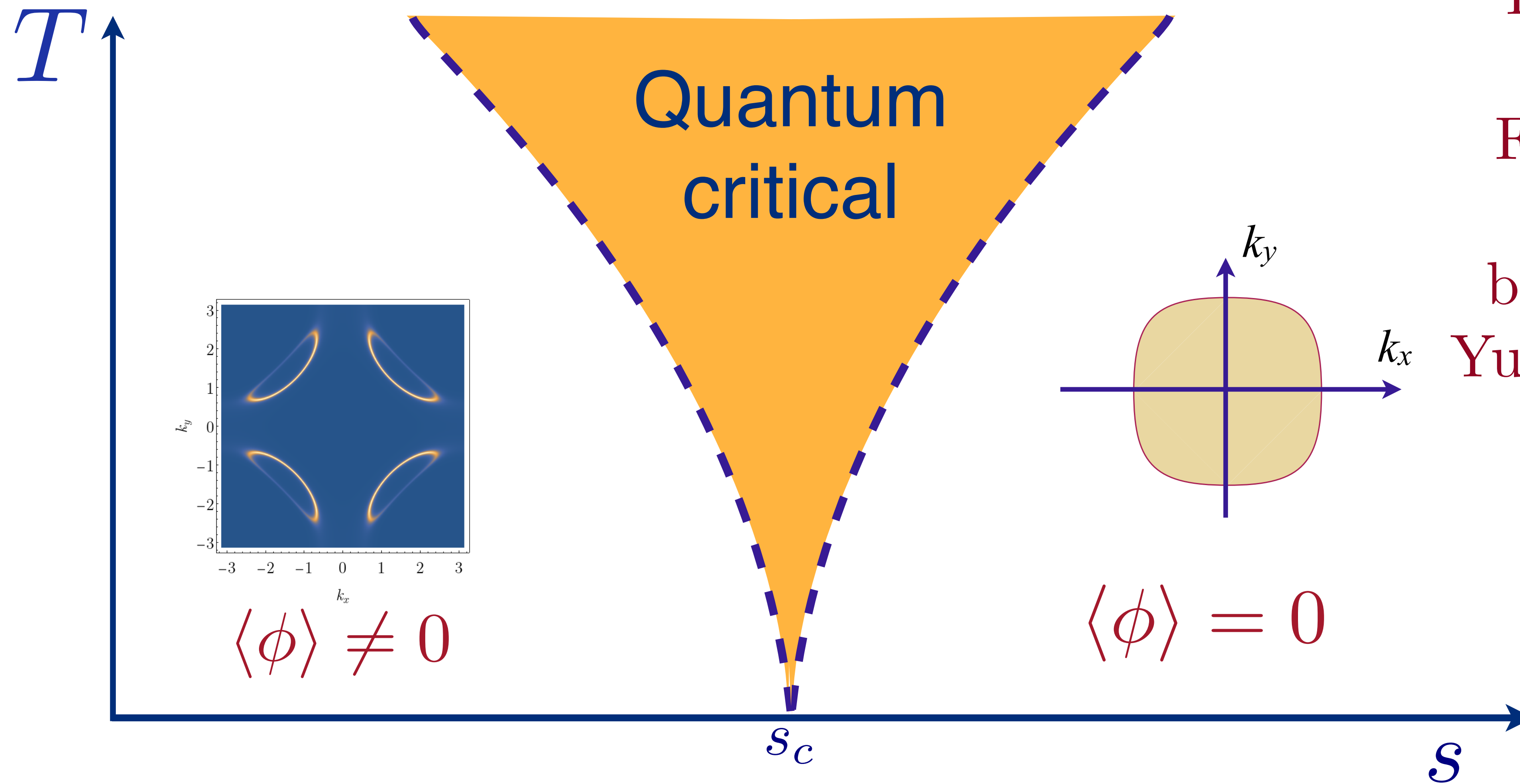


Ising field ϕ
+
Fermi surface
with a
boson-fermion
Yukawa coupling

Type II: Fermi surface reconstruction

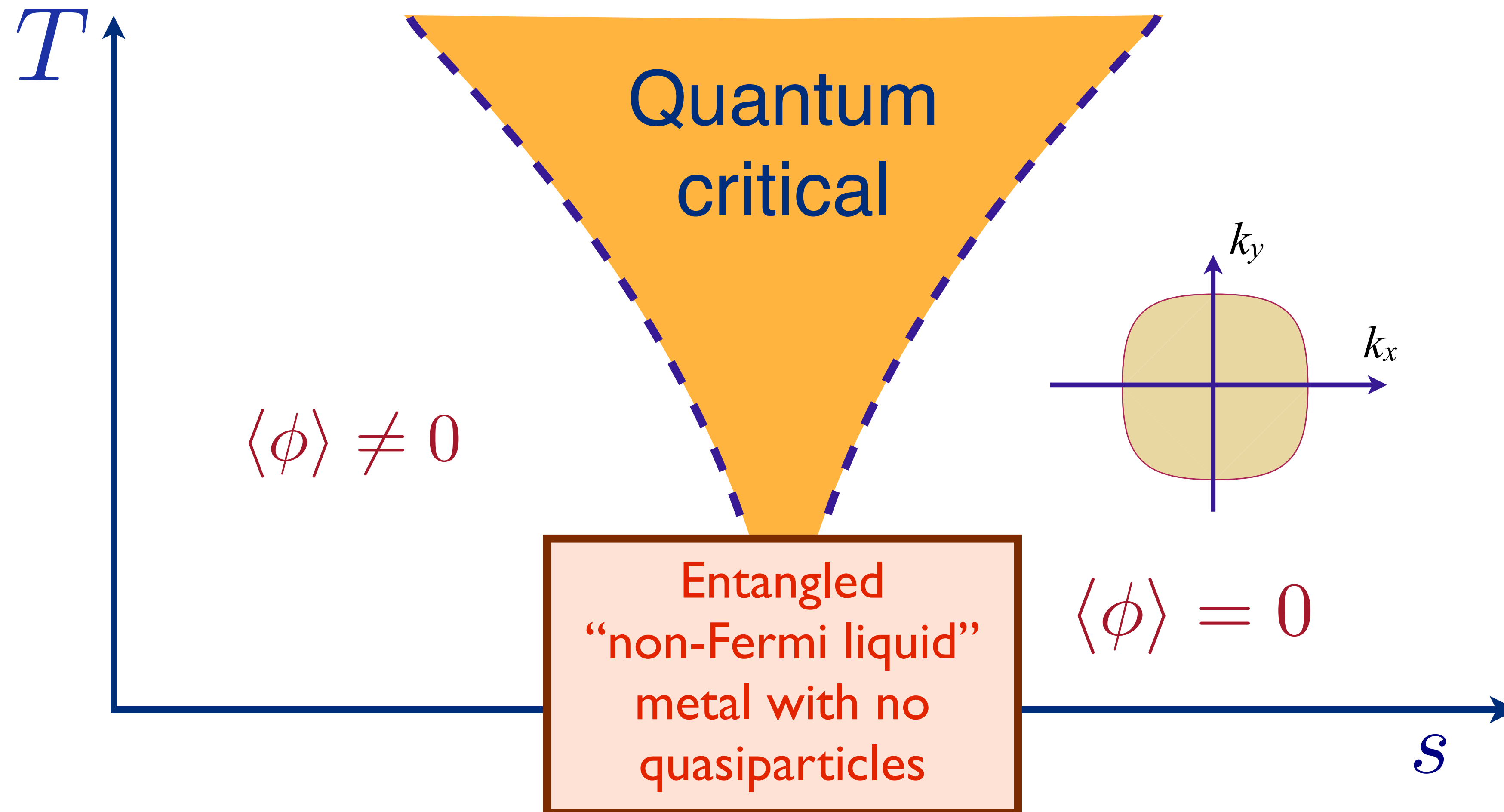


Type III: Fermi surface jump

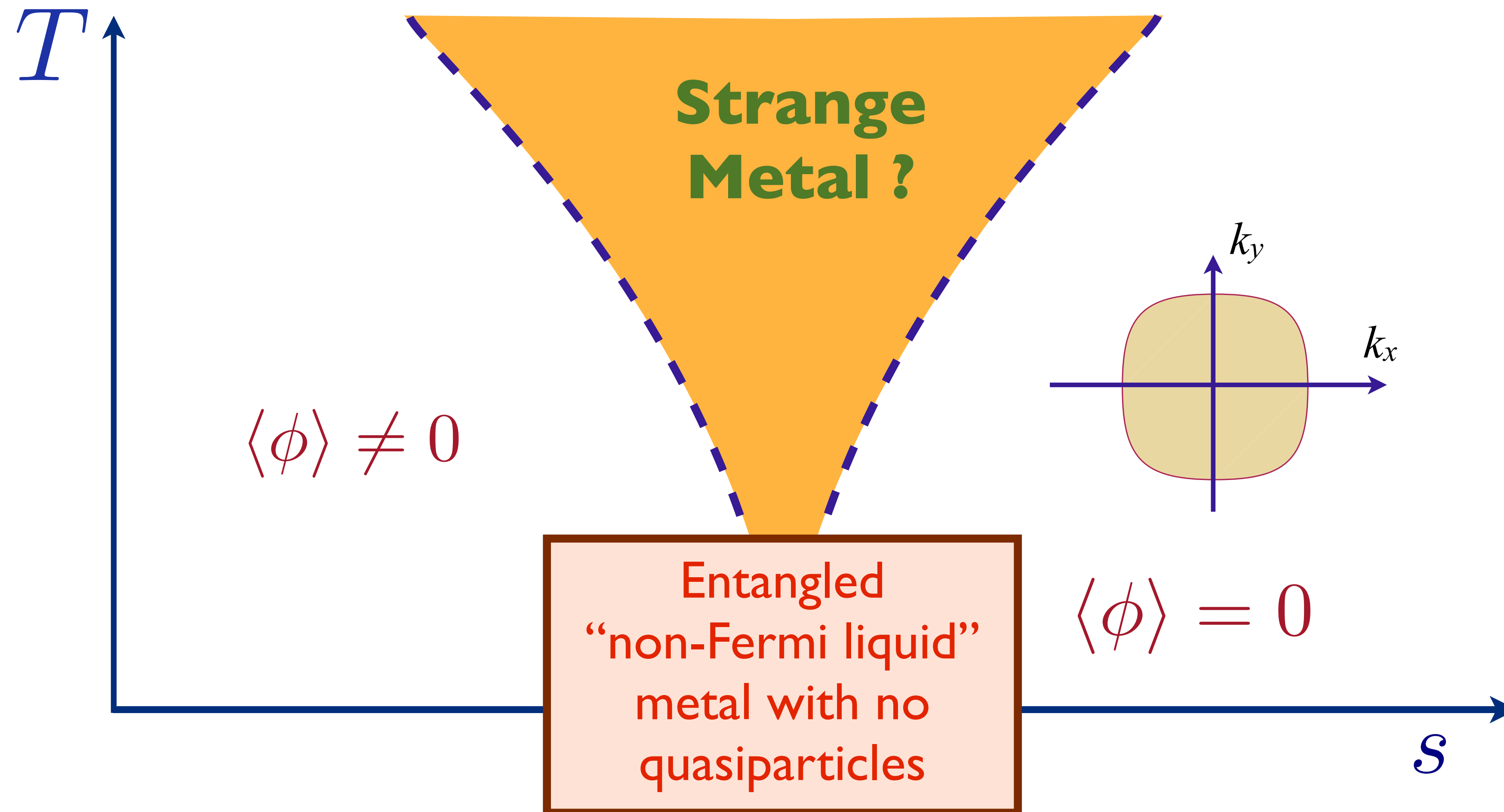


Spin liquid
+
Higgs field ϕ
+
Fermi surface
with a
boson-fermion
Yukawa coupling

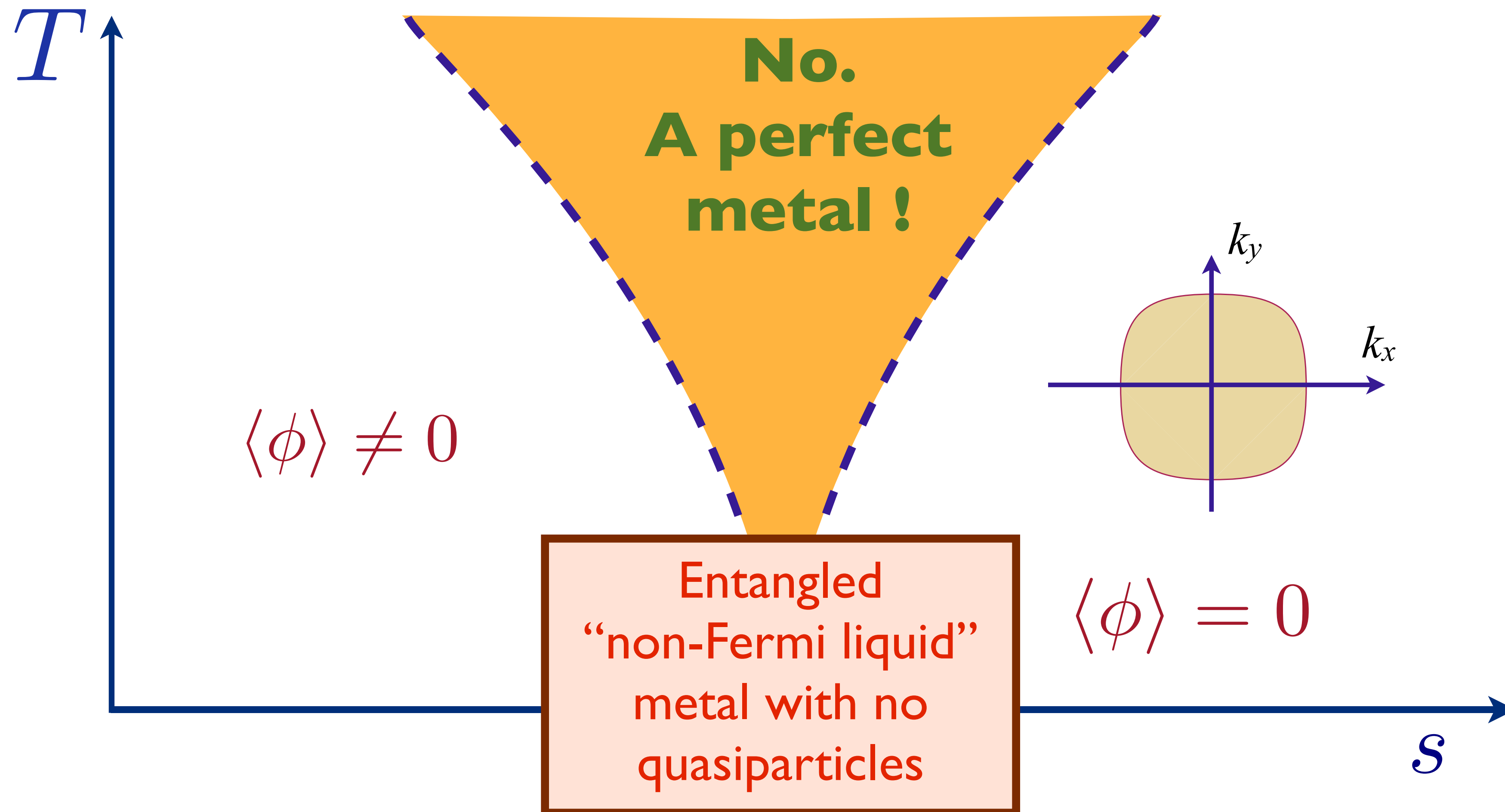
Type I, II, or III



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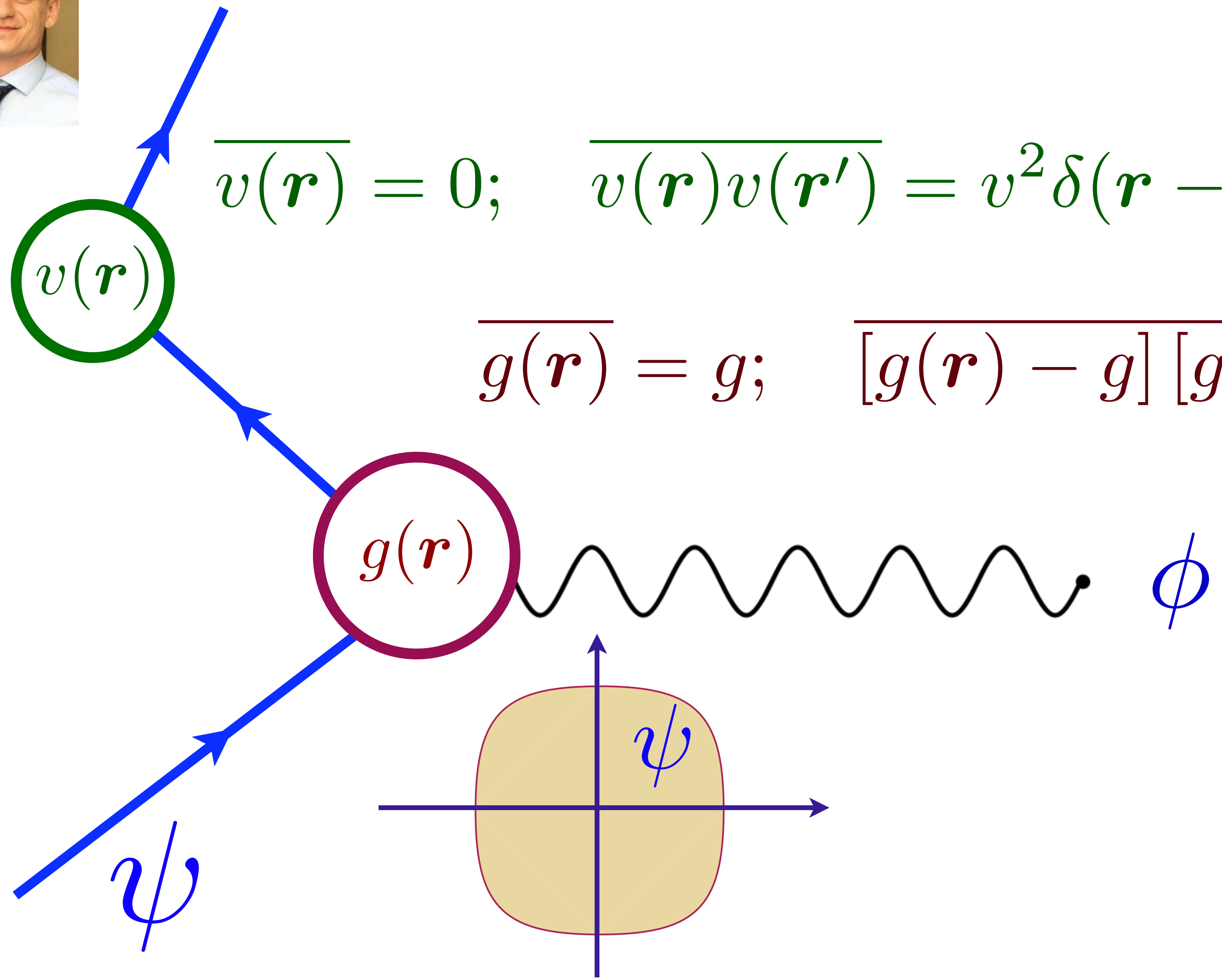
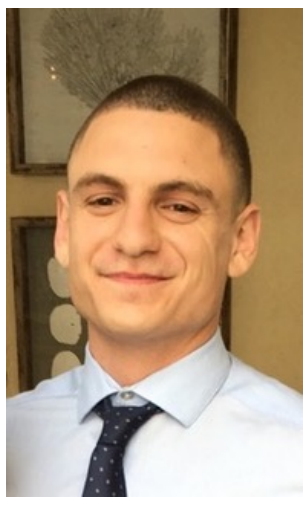
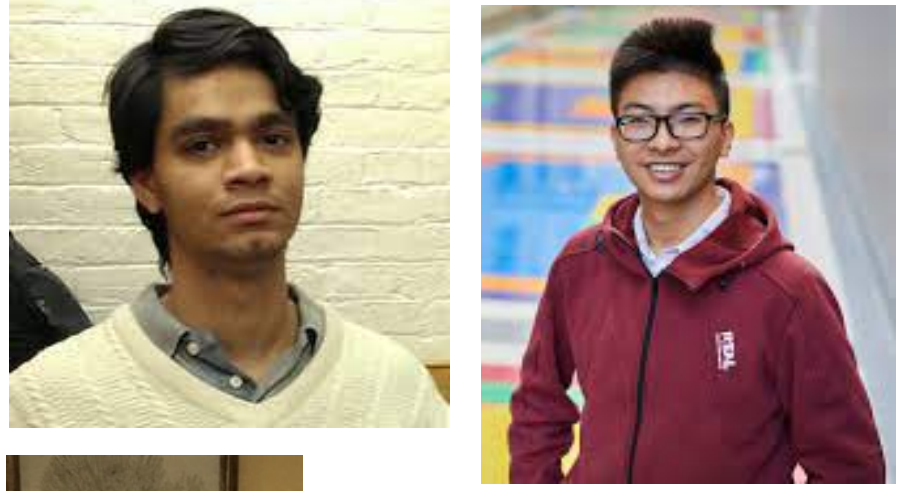


Type I, II, or III



Type I, II, or III: Universal theory of strange metals

Spatial randomness from impurities in the Yukawa coupling $g(\mathbf{r})$.



$$\overline{v(\mathbf{r})} = 0; \quad \overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$$

$$\overline{g(\mathbf{r})} = g; \quad \overline{[g(\mathbf{r}) - g][g(\mathbf{r}') - g]} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$$

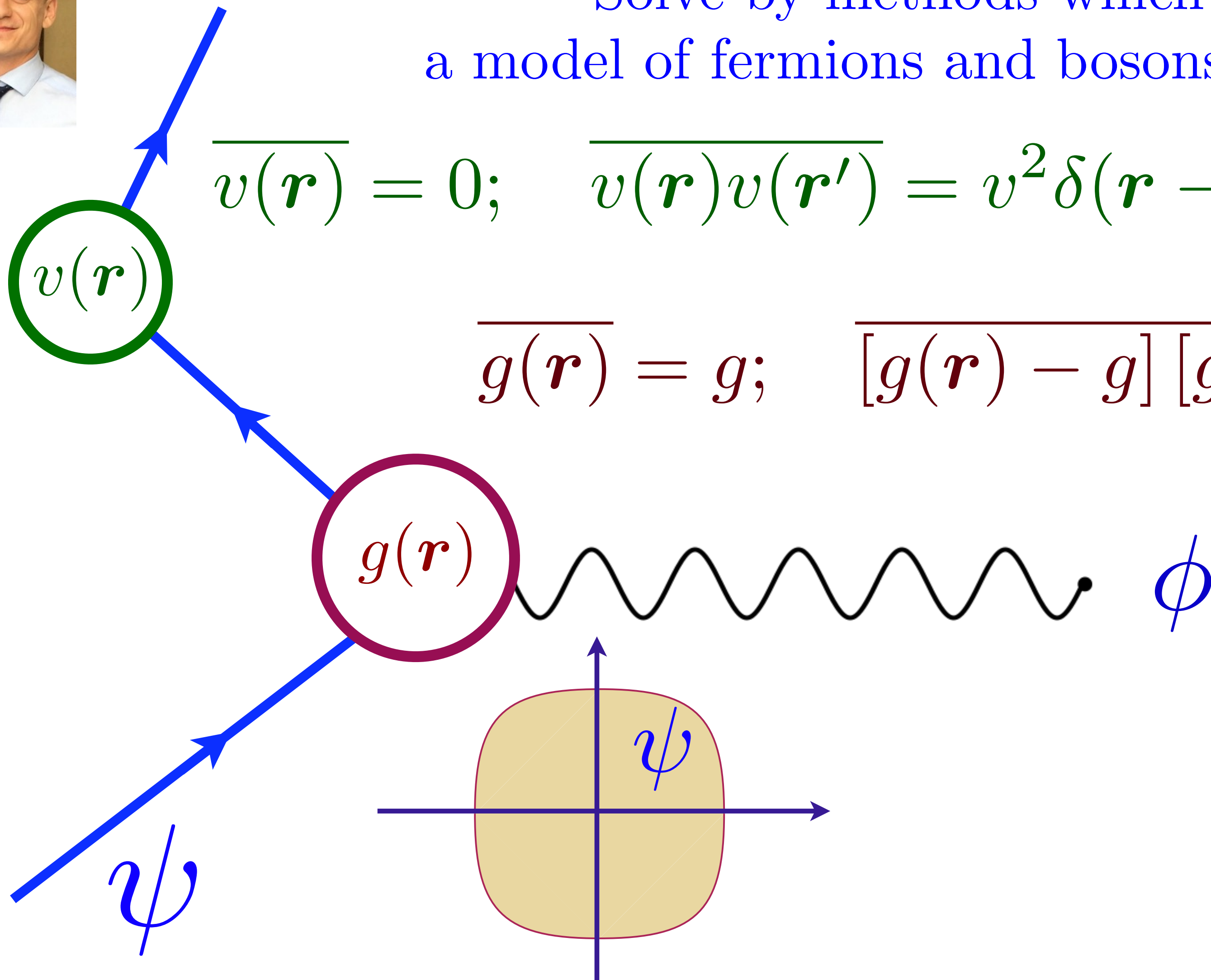
Type I, II, or III: Universal theory of strange metals

Spatial randomness from impurities in the Yukawa coupling $g(\mathbf{r})$.

Solve by methods which are exact for 'Yukawa-SYK',
a model of fermions and bosons with random Yukawa interactions.

$$\overline{v(\mathbf{r})} = 0; \quad \overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$$

$$\overline{g(\mathbf{r})} = g; \quad \overline{[g(\mathbf{r}) - g][g(\mathbf{r}') - g]} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$$



$$\text{Resistivity } \rho(T) \sim v^2 + g'^2 T$$

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

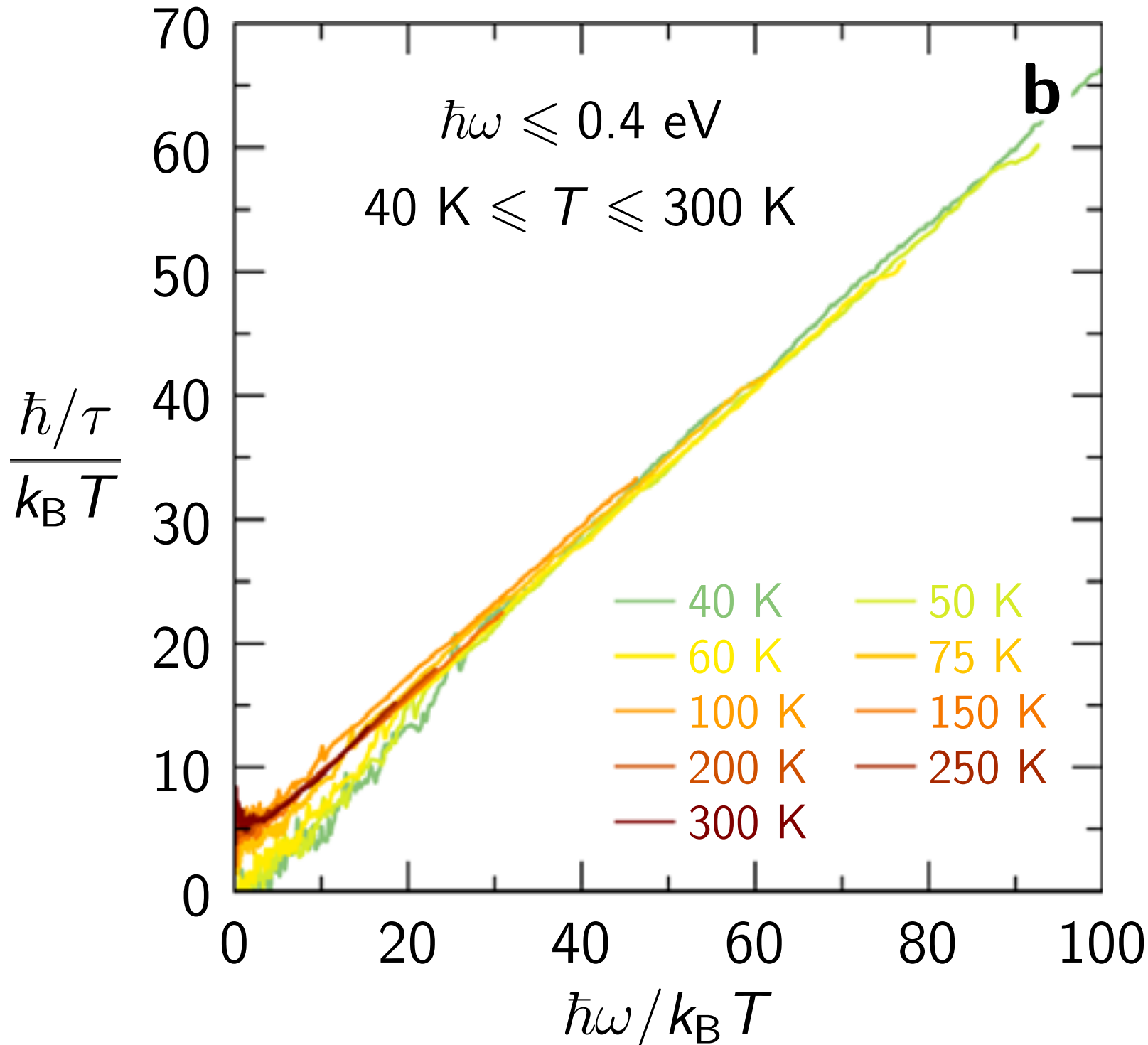
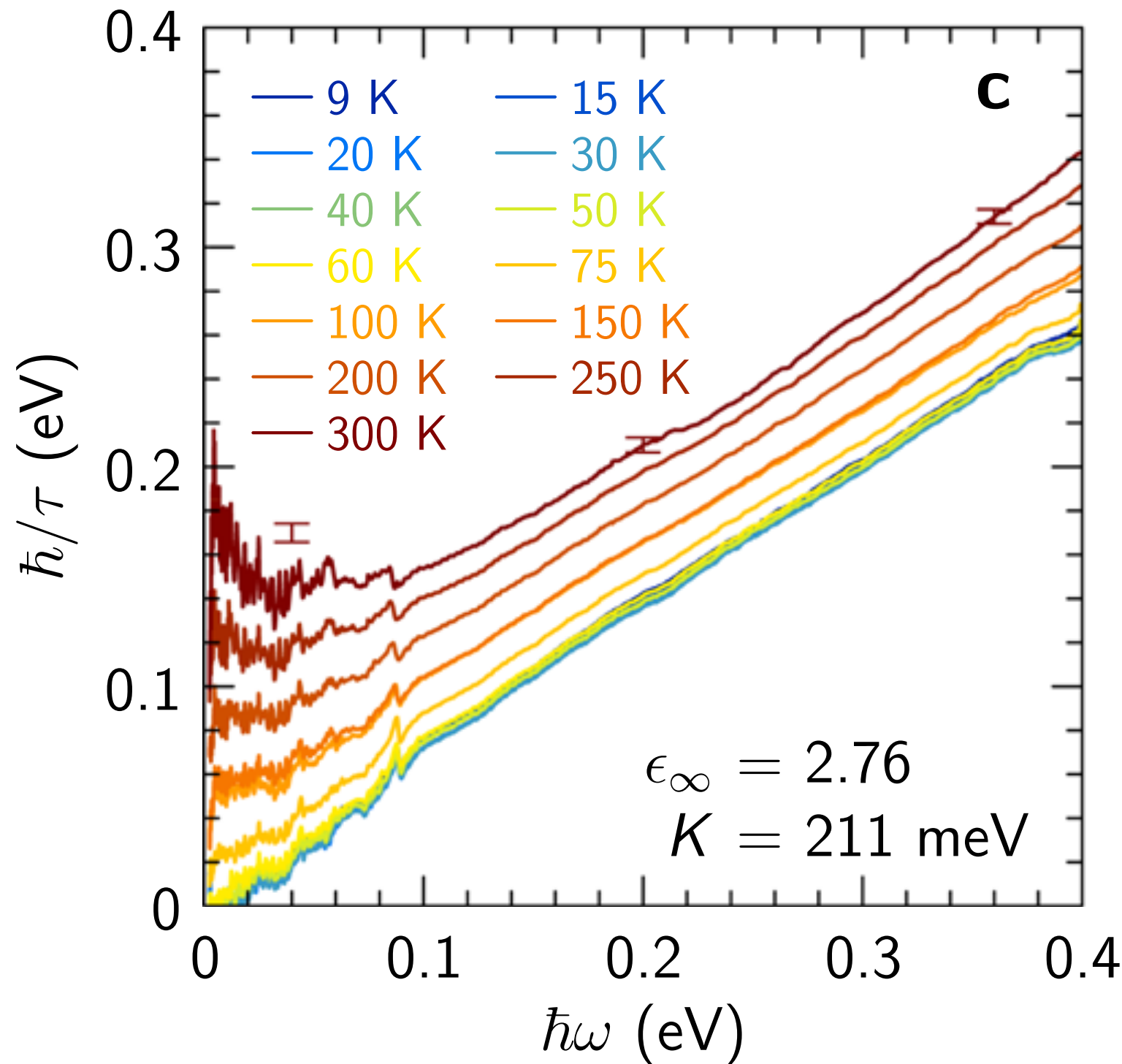
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F \left(\frac{\hbar \omega}{k_B T} \right)$$



Strange metal and superconductor

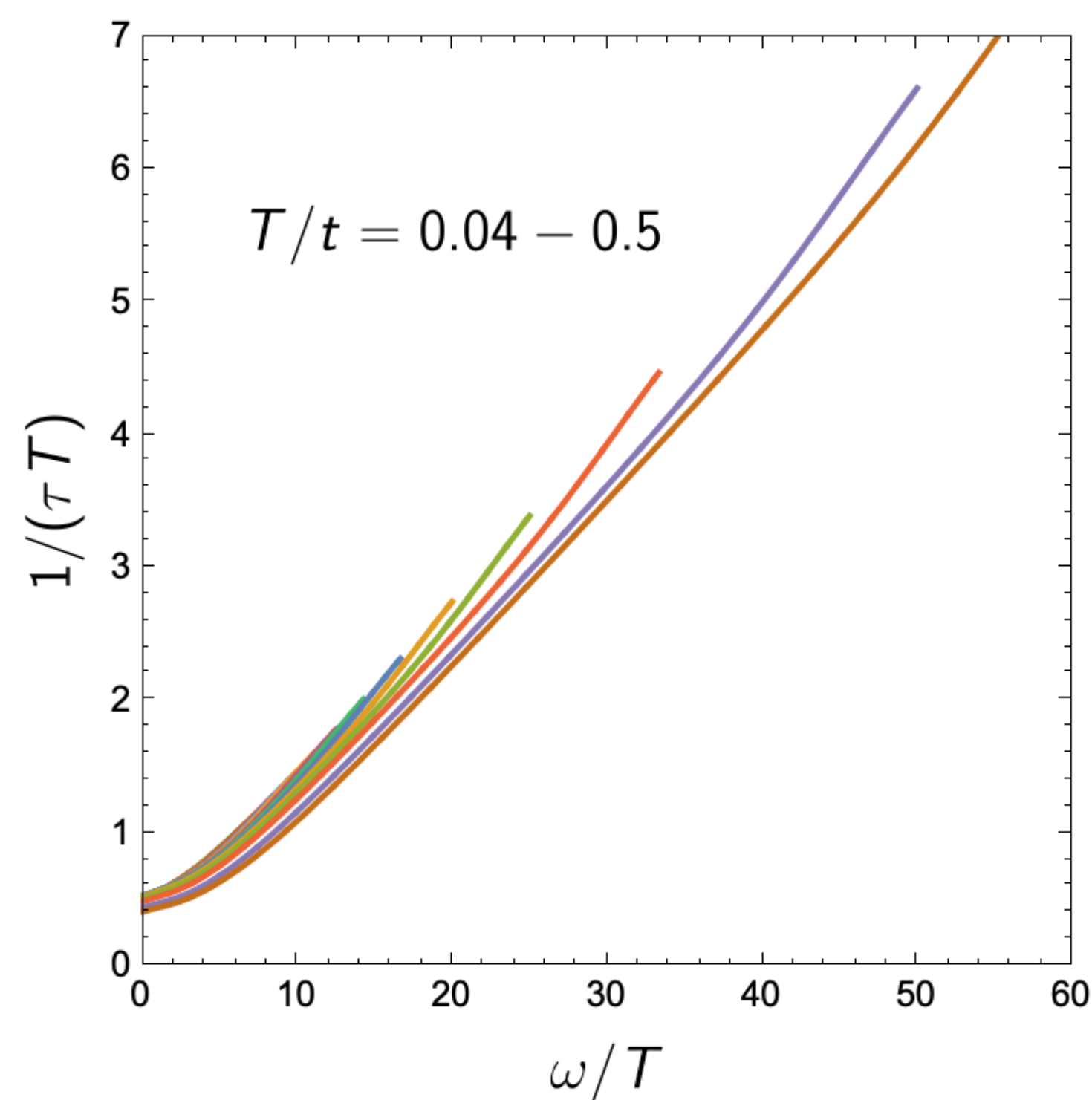
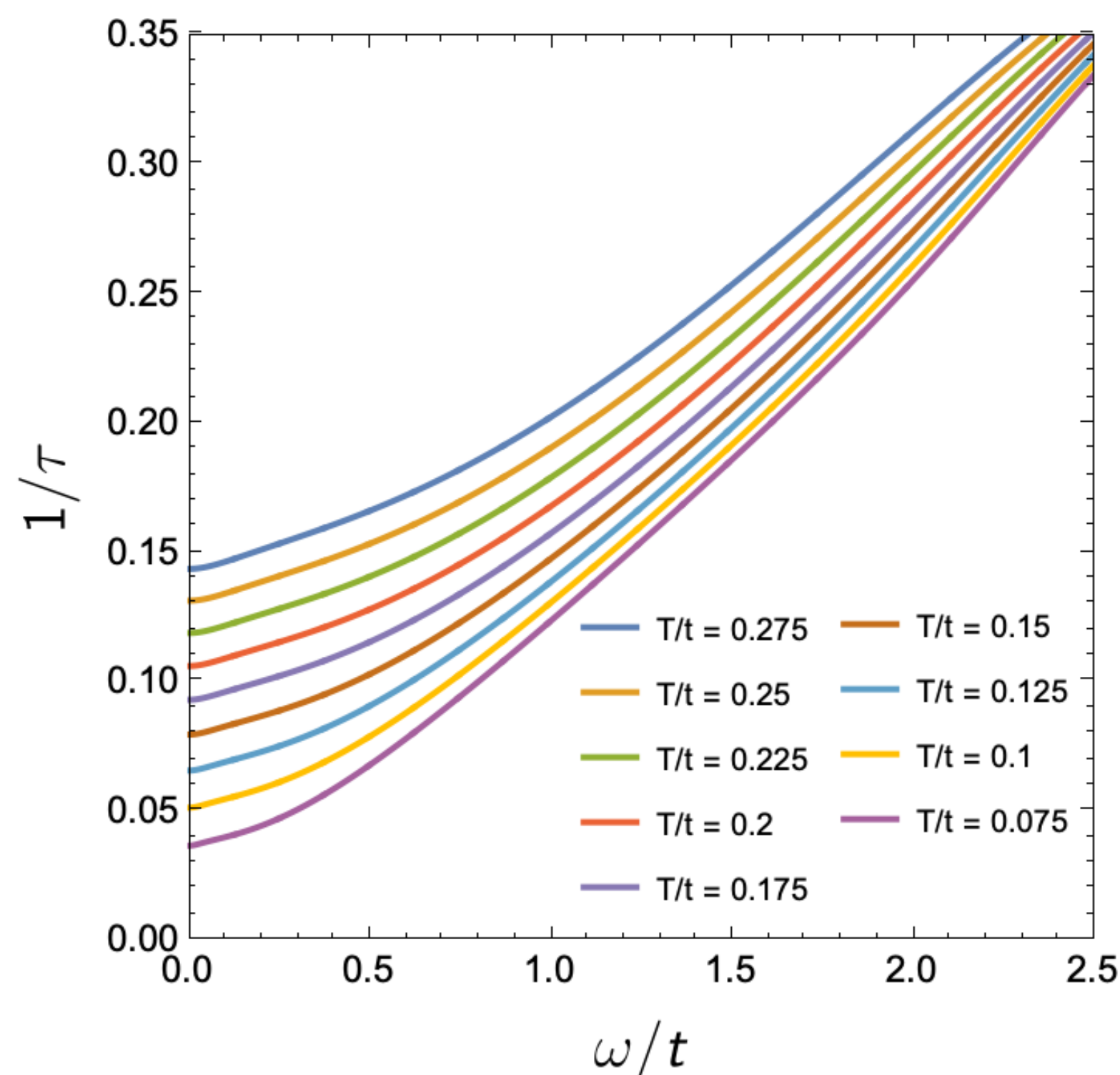
$$g = 0$$

in the two-dimensional Yukawa-SYK model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, to appear

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

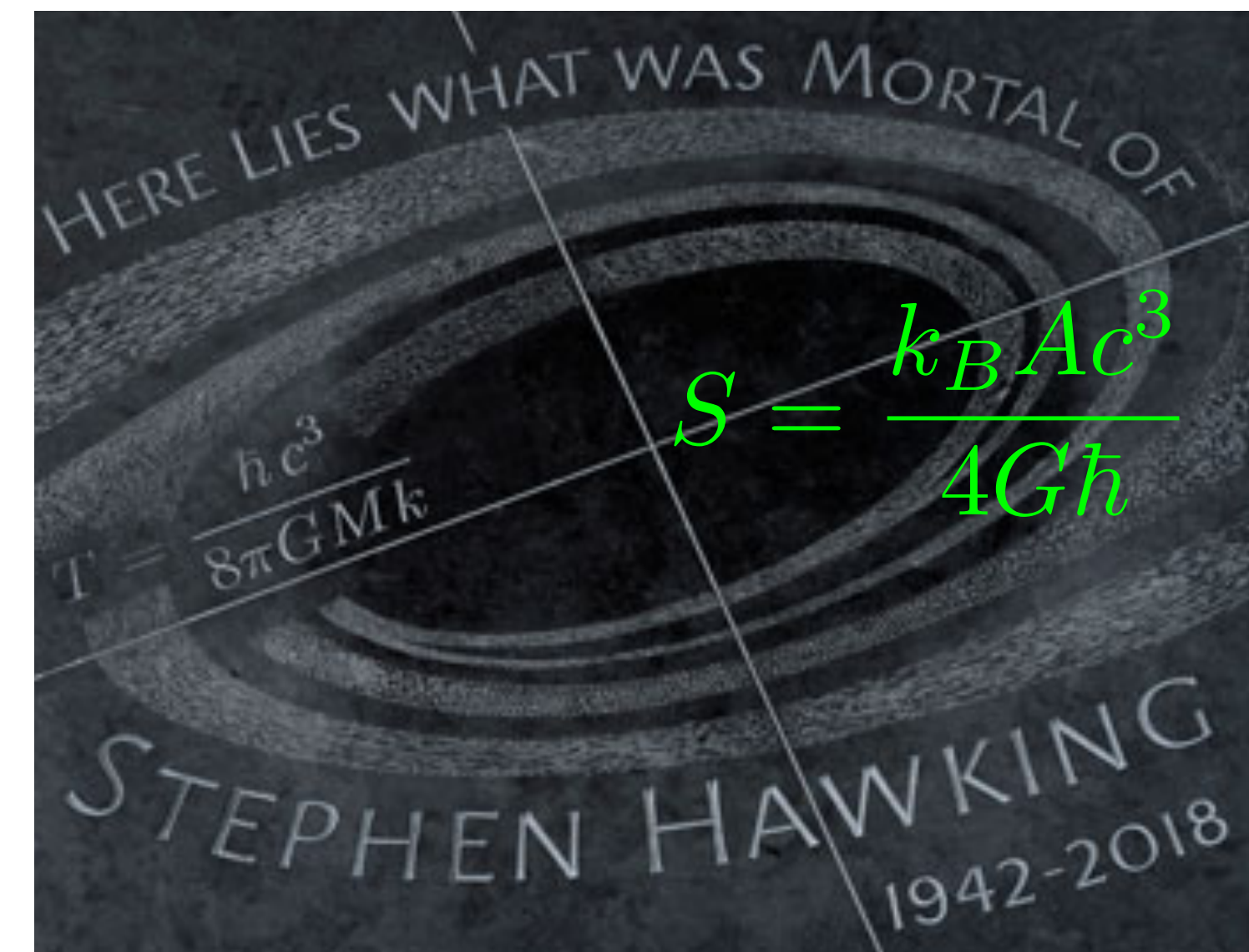
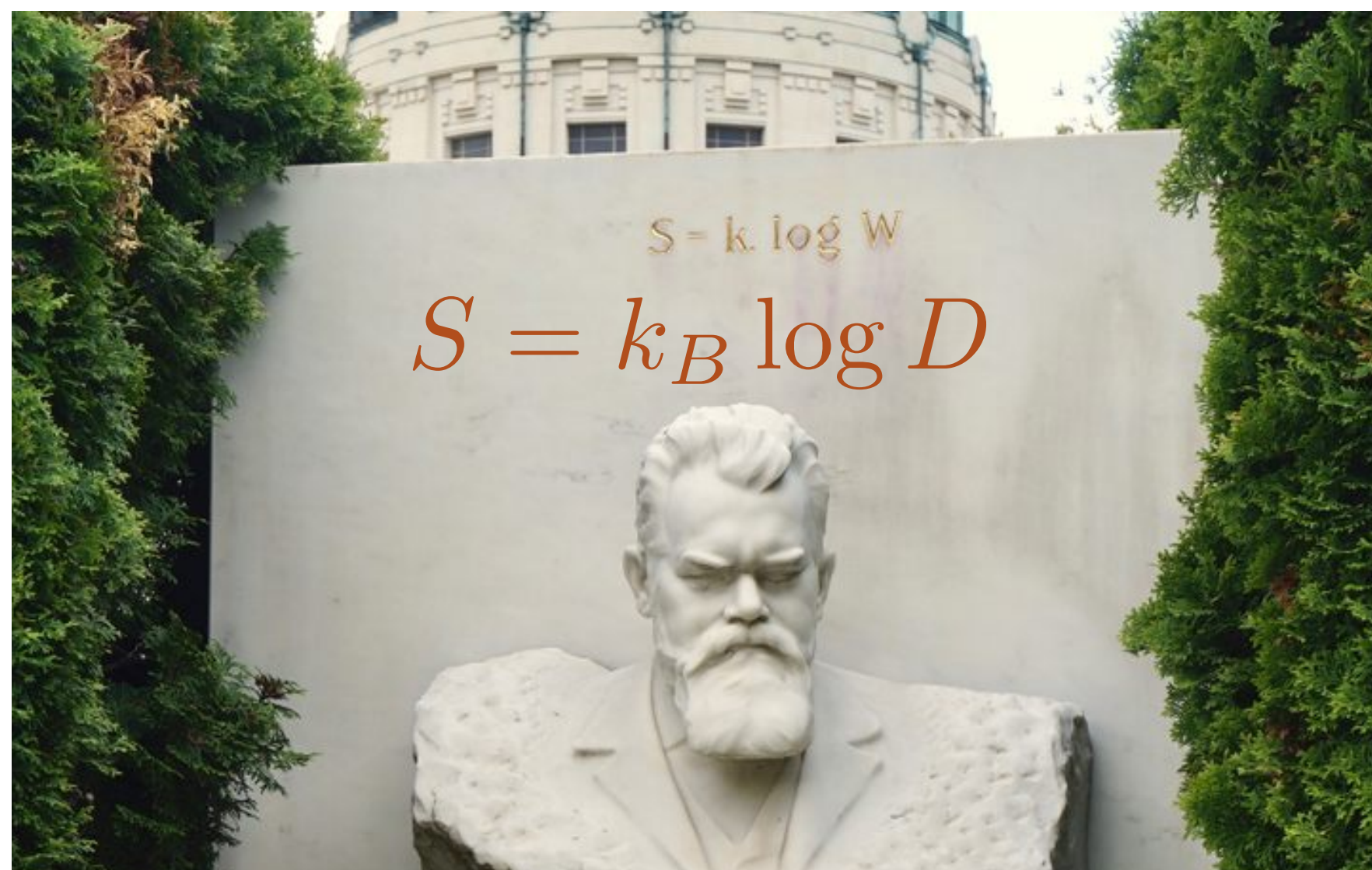


$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar \omega}{k_B T}\right)$$

The SYK model,
and black holes

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Hawking's entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?



Quantum Black Holes

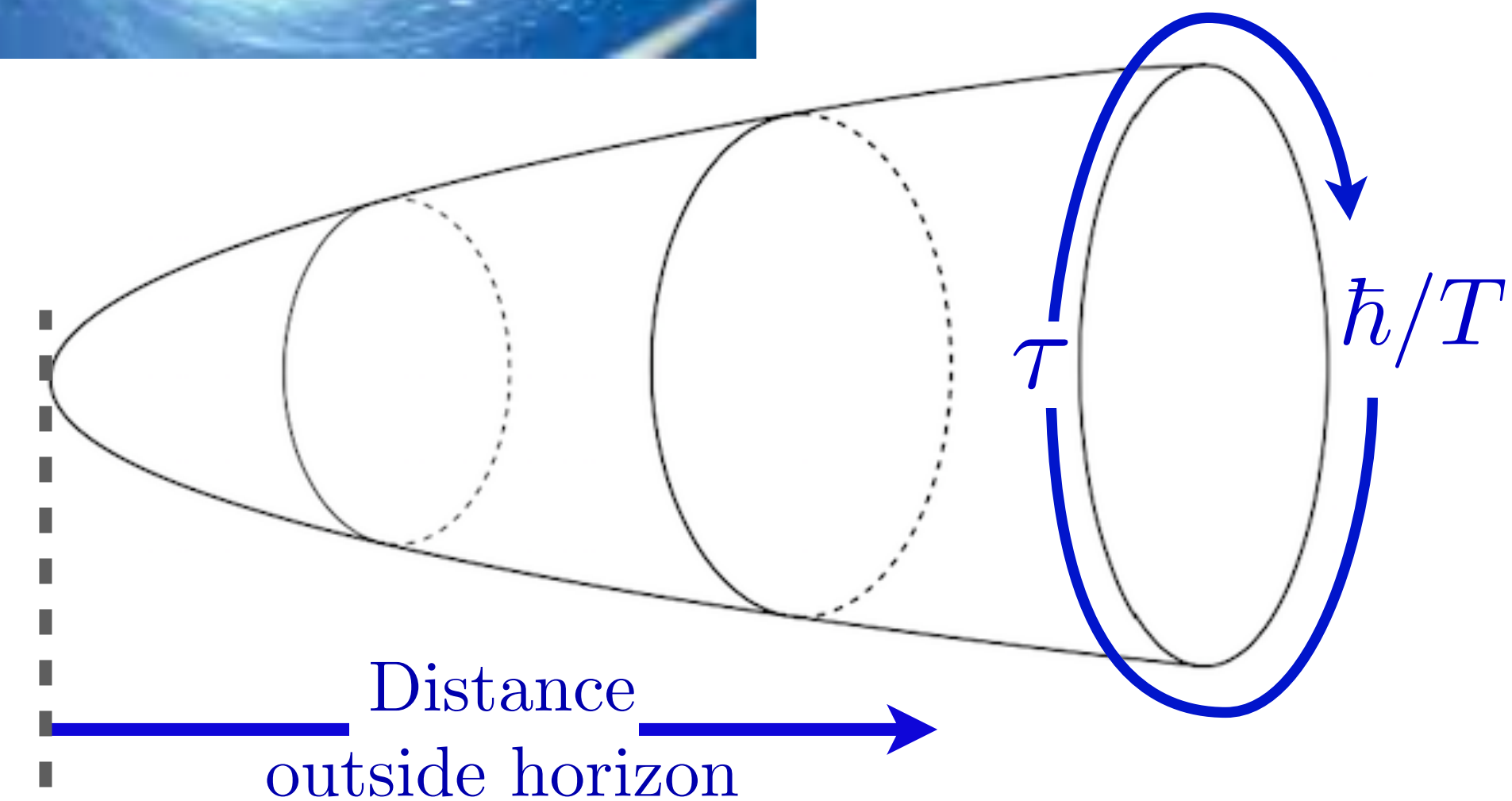
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- Answer from string theory for 'supersymmetric' charged black holes: $D(E) = e^S \delta(E)$ *i.e.* all the states required by Hawking's entropy have exactly the same energy.

Strominger, Vafa (1996)

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$



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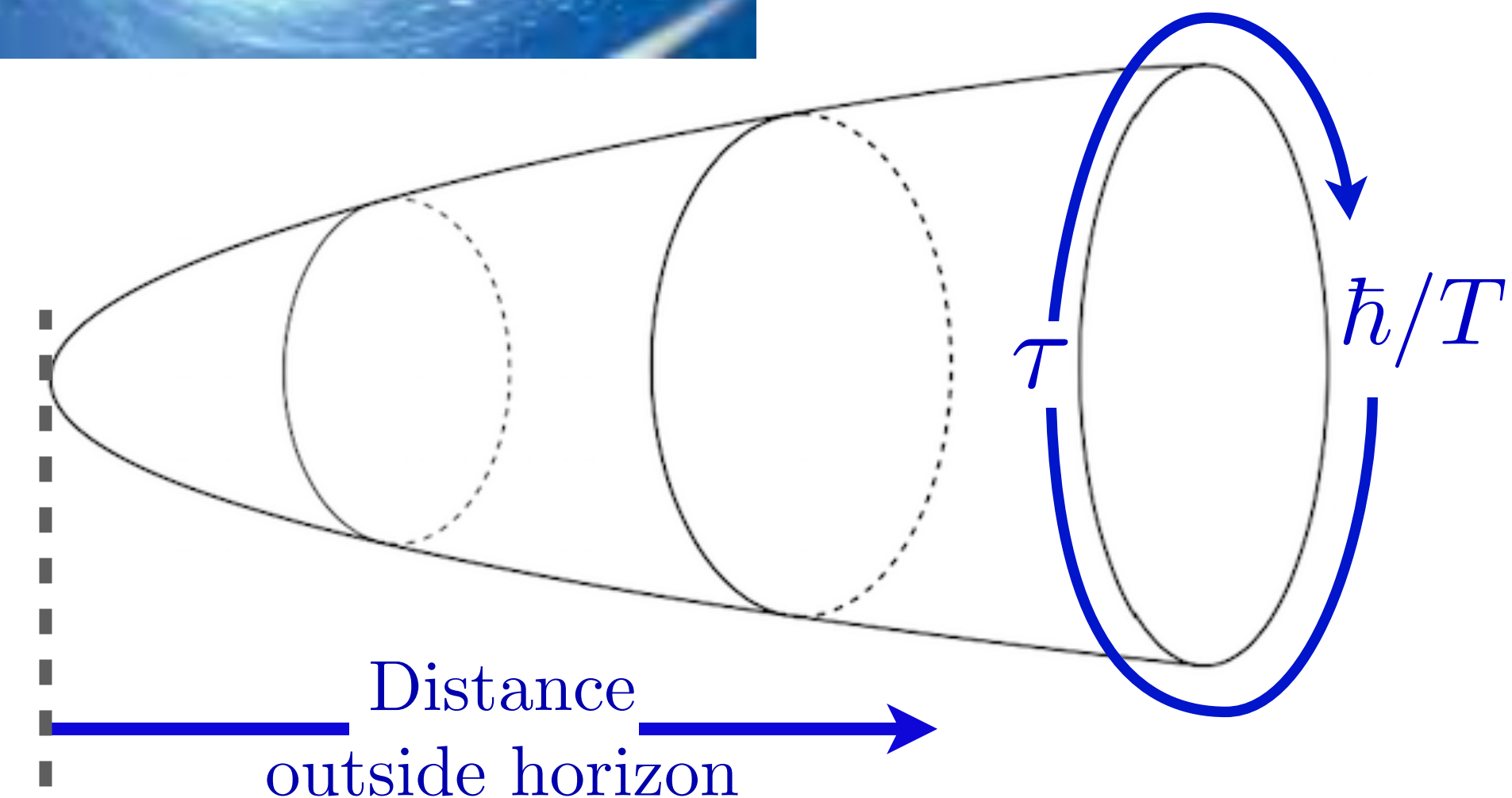
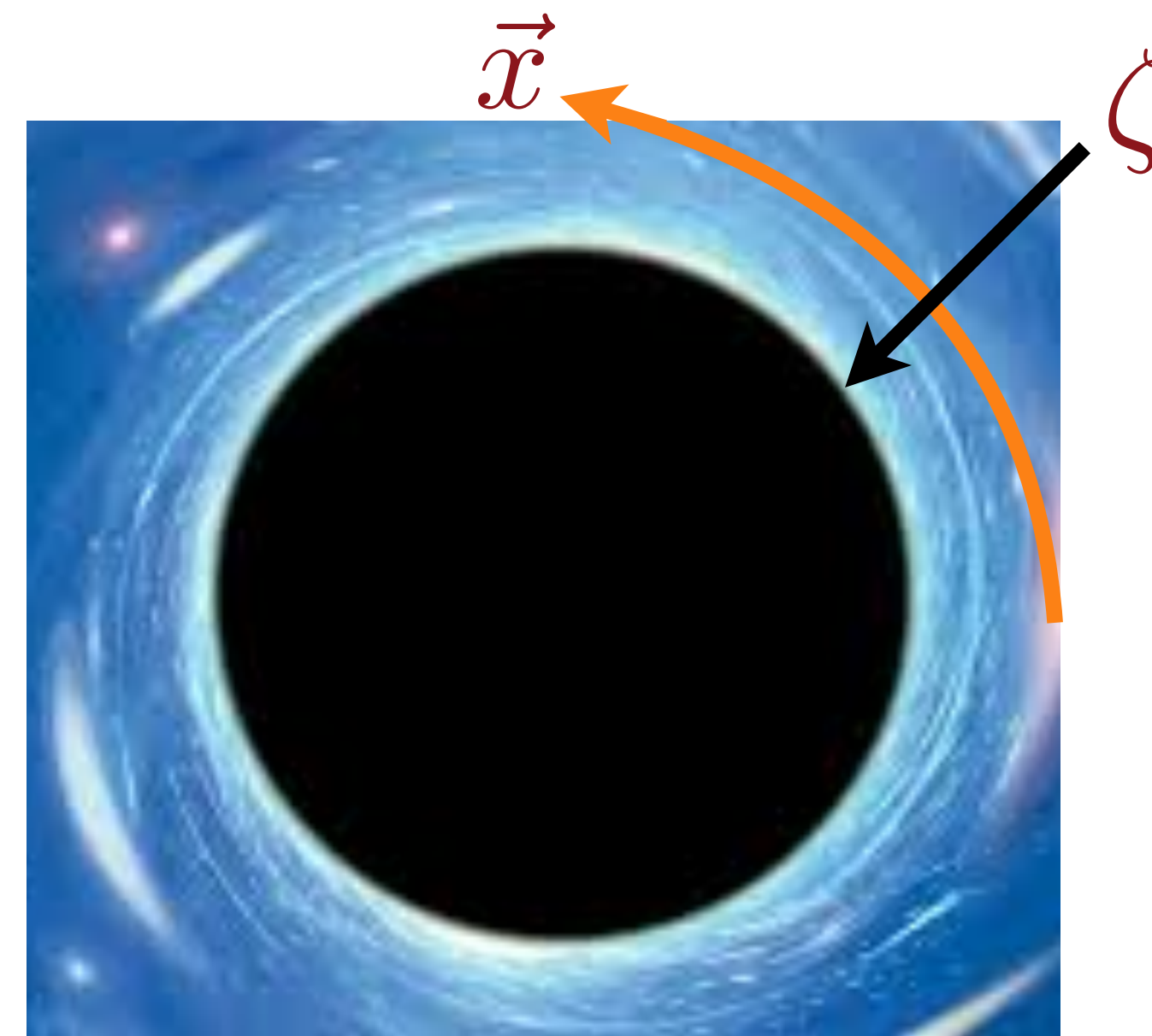
The near-horizon metric is $\text{AdS}_2 \times S^2$,
and the AdS_2 quantum fluctuations
dominate at low energies.

The AdS_2 metric in Euclidean time is

$$ds^2 = \frac{d\tau^2 + d\zeta^2}{\zeta^2},$$

which has the same conformal symmetry
as the SYK saddle point!

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}, \quad ad - bc = 1.$$

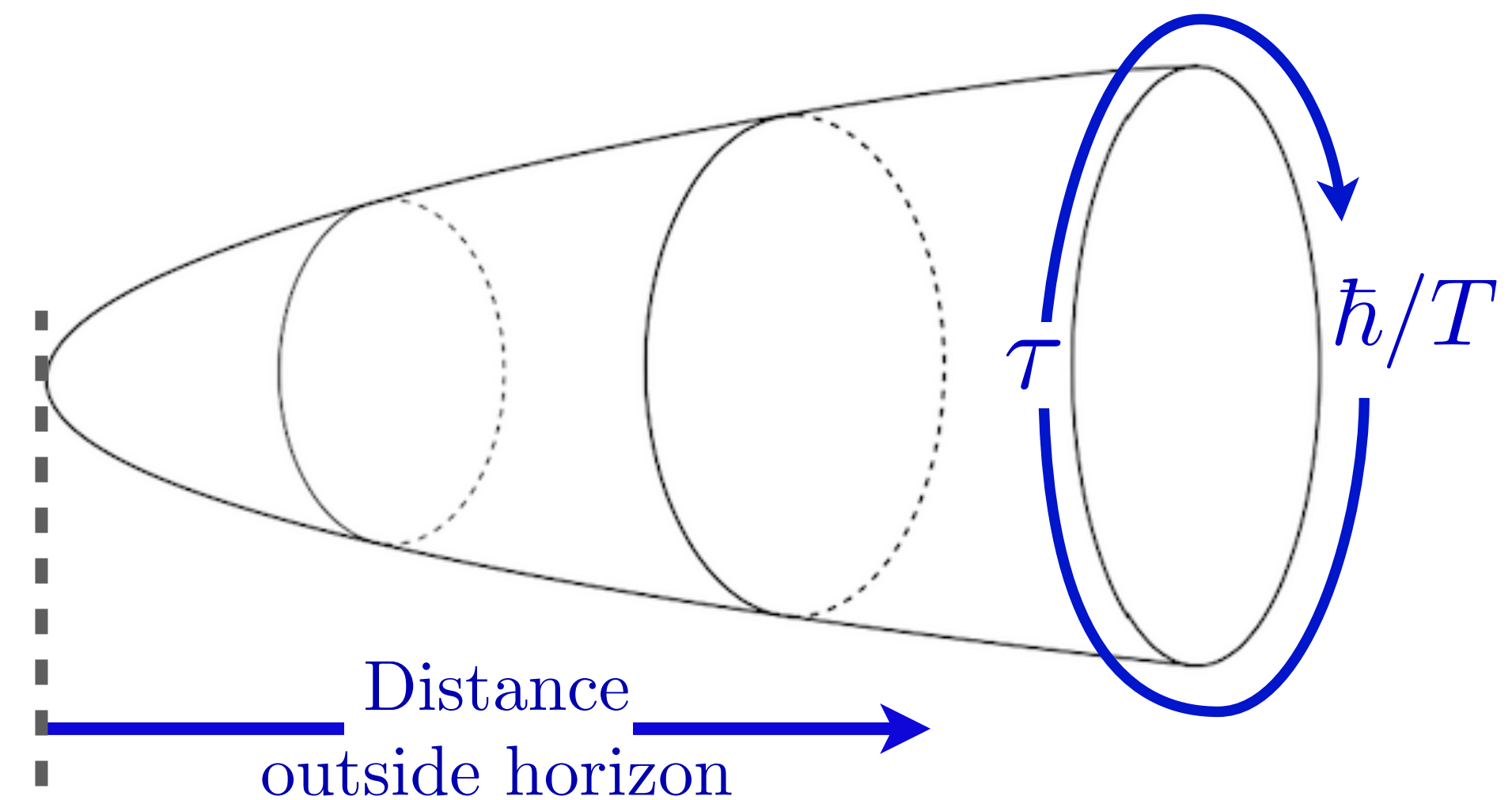


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$$\approx \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{JT gravity of AdS}_2+\text{boundary graviton}}^{(1+1)}[g_{\mu\nu}, A_\mu] \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.



Thermodynamics of quantum black holes with charge Q :

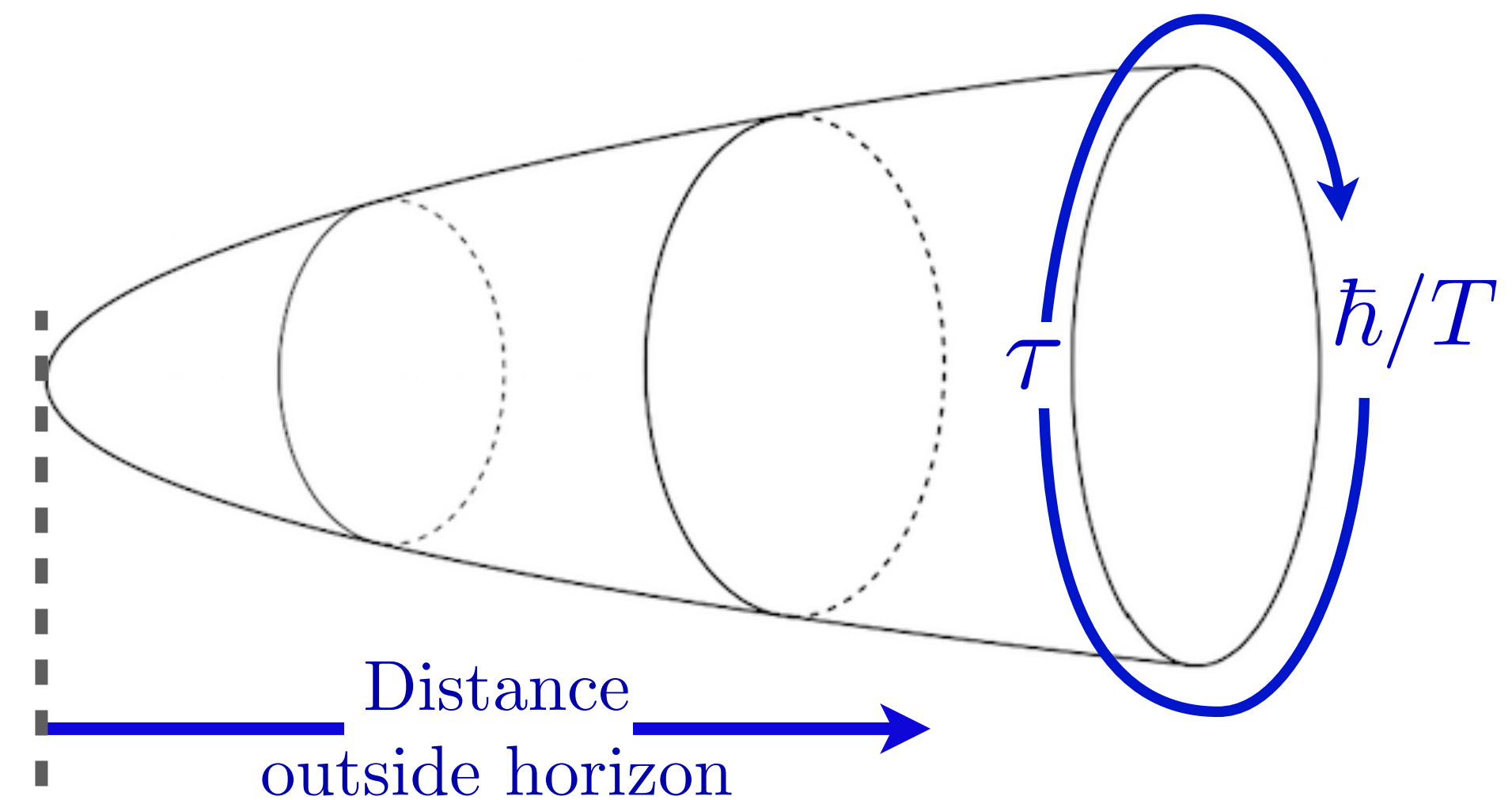
$$\begin{aligned} \mathcal{Z}(Q, T) &= \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_\mu] \right) \\ &\approx \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{JT gravity of AdS}_2+\text{boundary graviton}}^{(1+1)}[g_{\mu\nu}, A_\mu] \right) \\ &= \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) \exp \left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)] \right) \end{aligned}$$

The path integral over the action I_{SYK} can be evaluated exactly,

and leads to a computation of $D(E)$

$$\mathcal{Z}(Q, T) = \int dE D(E) \exp \left(-\frac{E}{k_B T} \right)$$

Maldacena, Stanford, Yang (2016); Cotler *et al.* (2017)

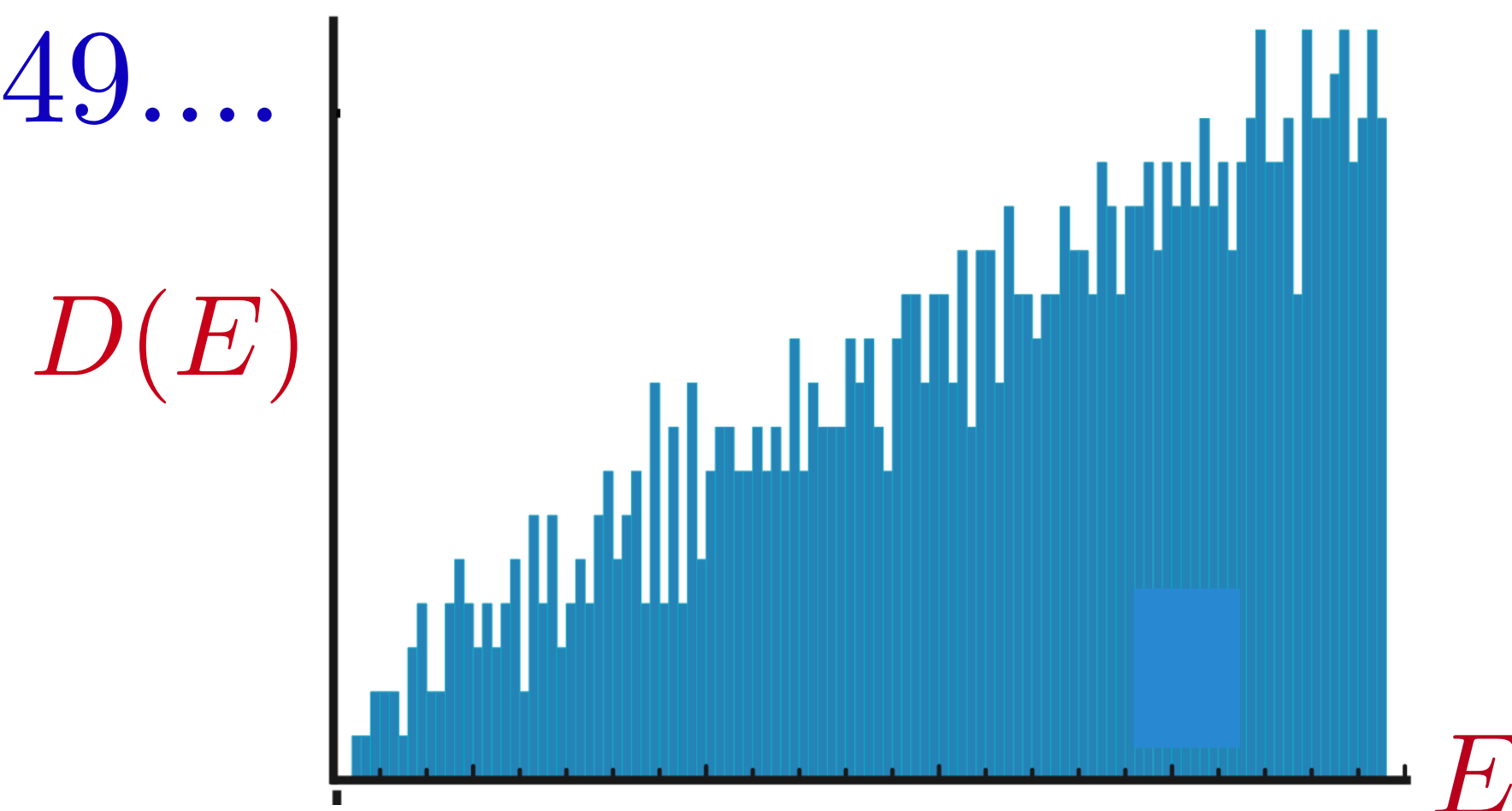
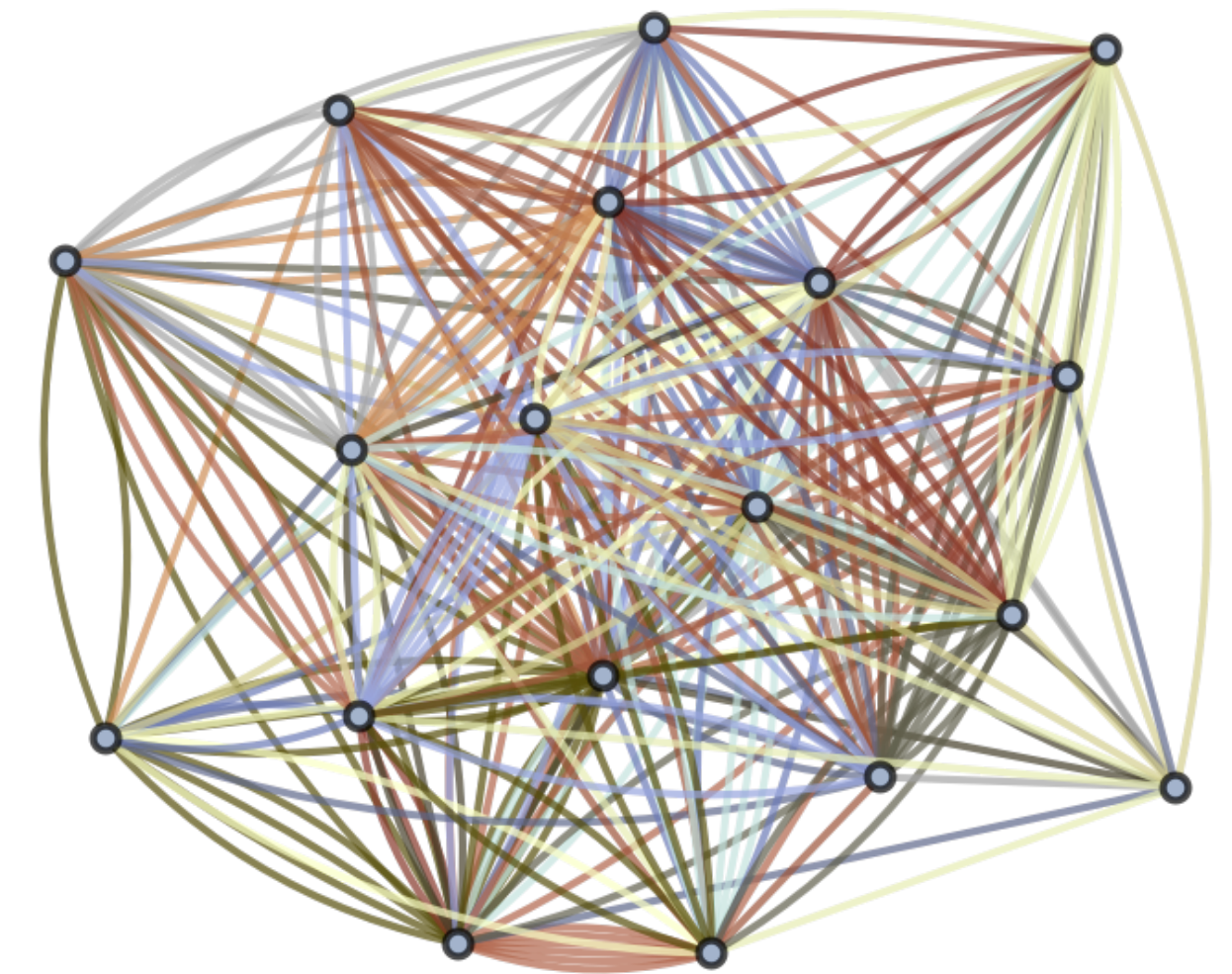


The Sachdev-Ye-Kitaev (SYK) model

- Density of quantum states of the SYK model with N sites

$$D(E) \sim \frac{1}{N} \exp(N s_0) \sinh\left([2N\gamma E]^{1/2}\right)$$

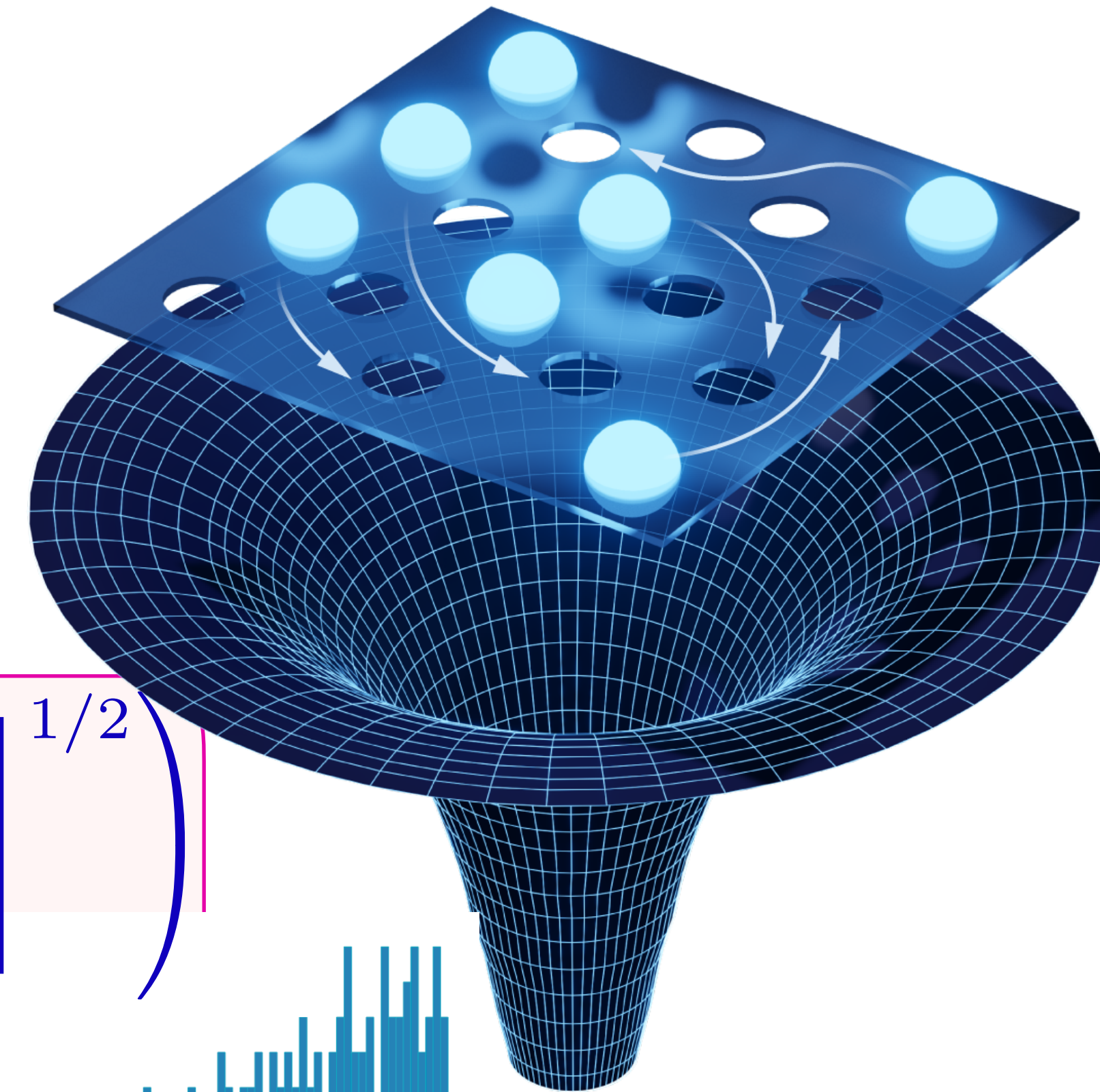
where $s_0 = 0.46484769917080510749\dots$



D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge Q ($A_0 = 2GQ^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

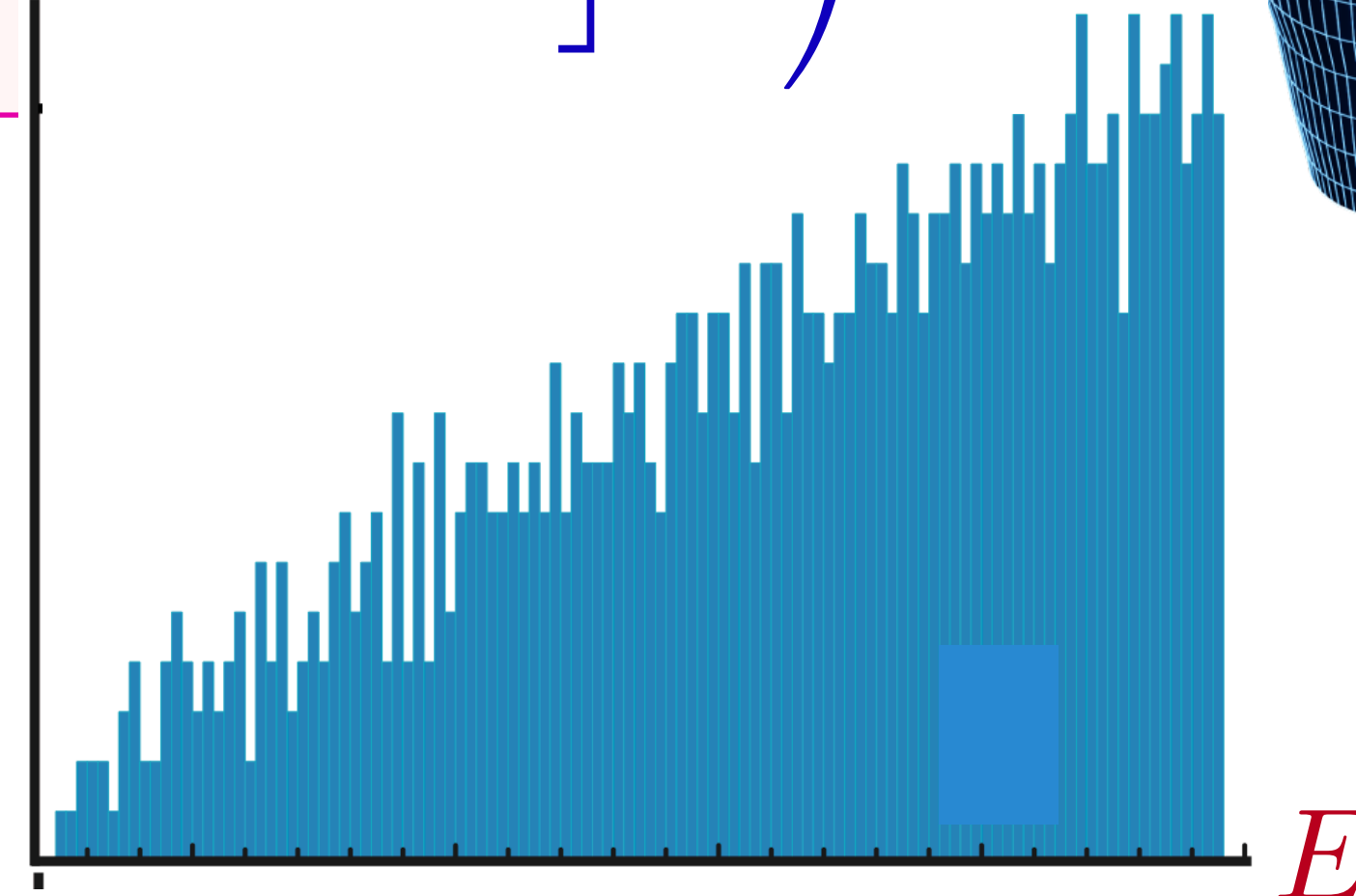


Bekenstein-Hawking

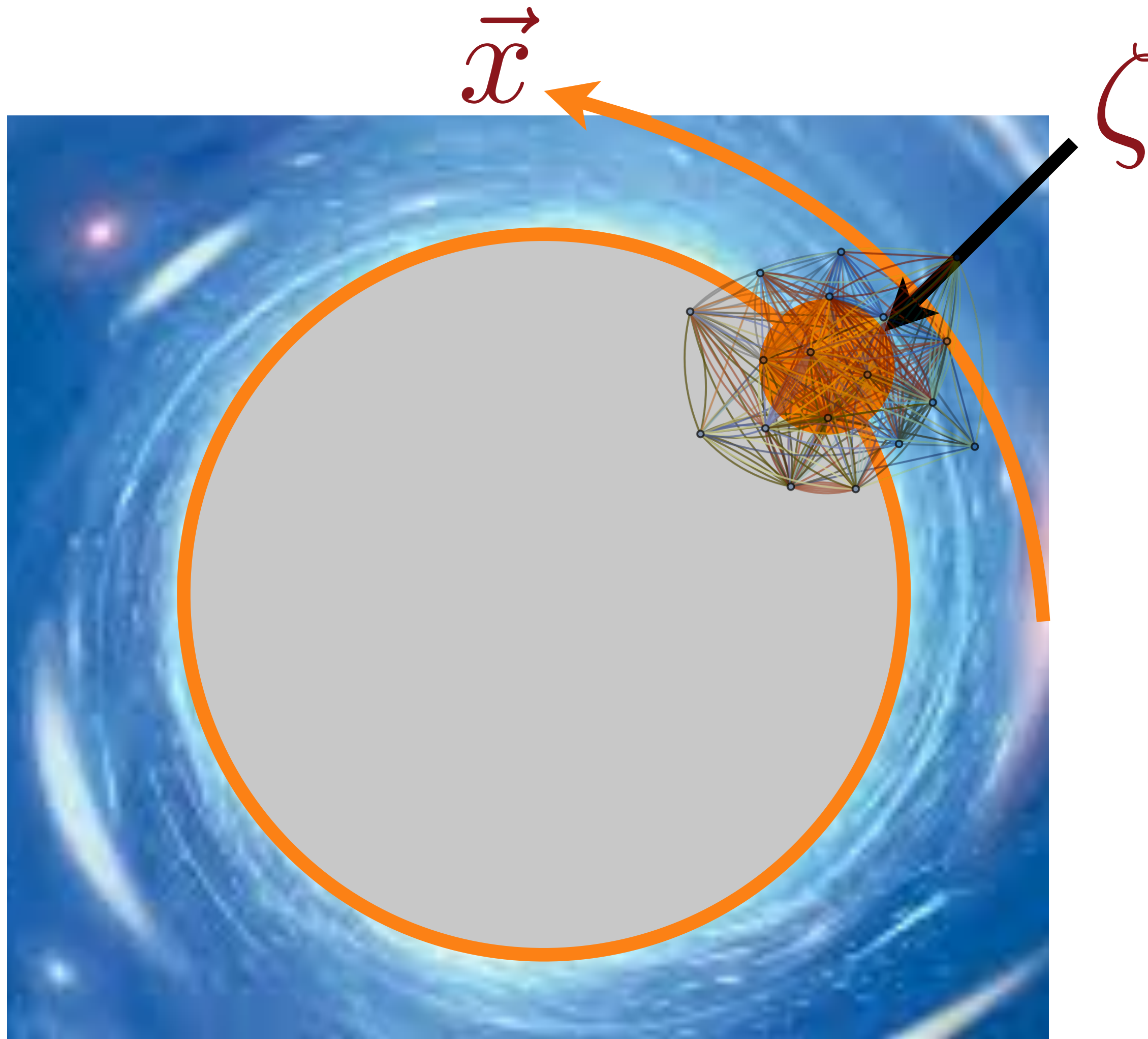
Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model

$D(E)$

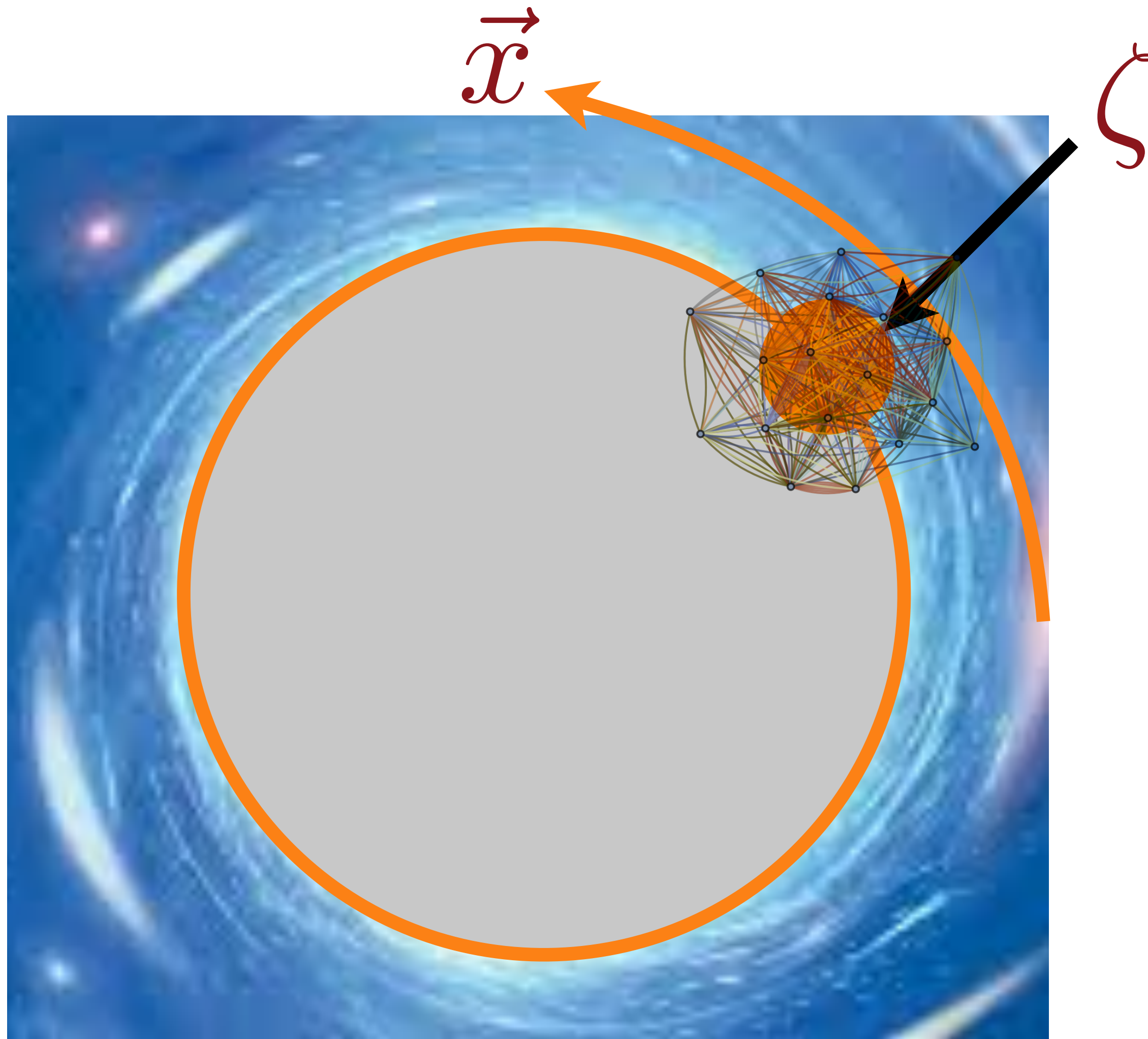


Quantum simulation of charged black holes by the SYK model



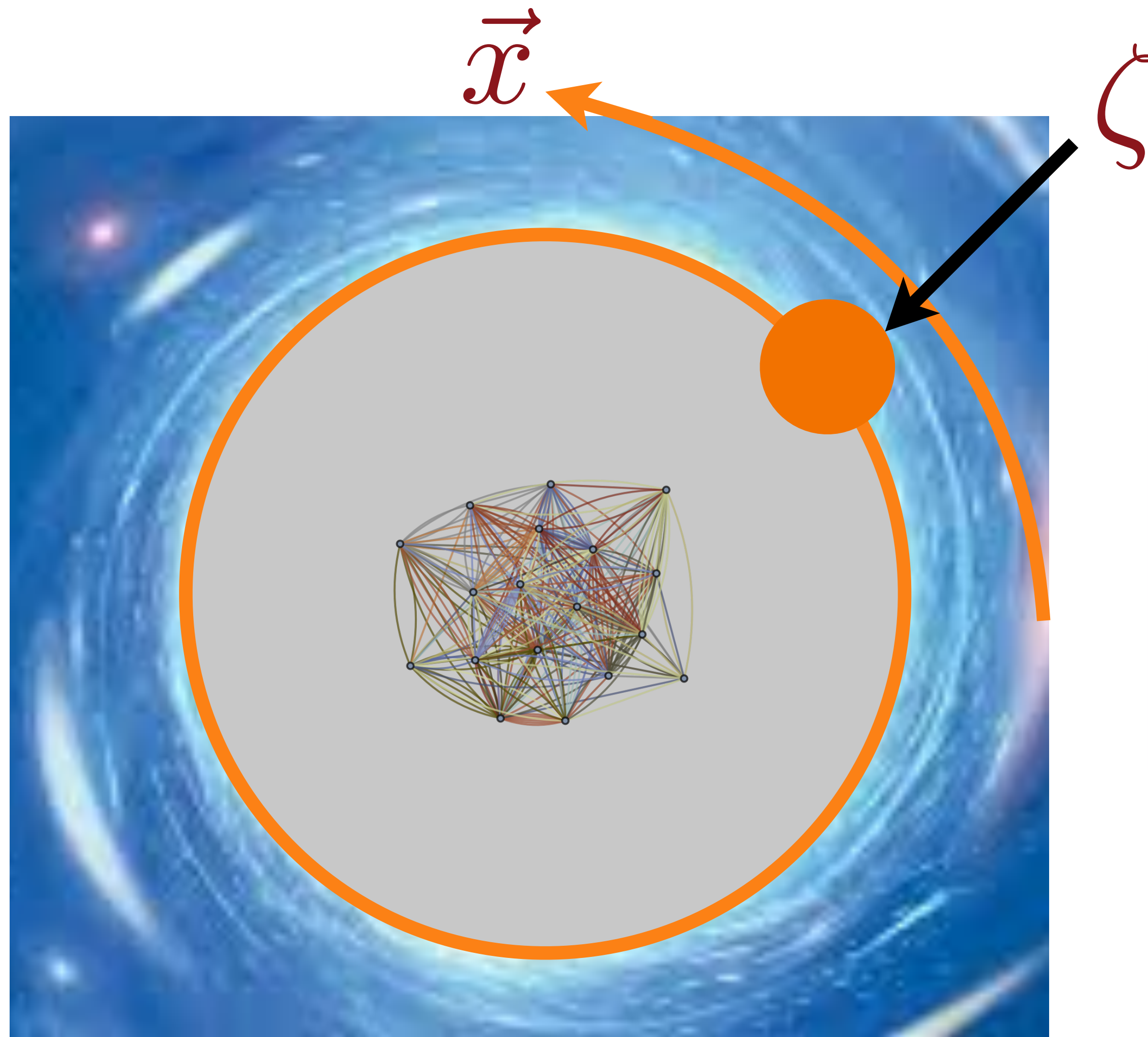
The SYK model
simulates the
interior of the black hole
to an outside observer

Quantum simulation of charged black holes by the SYK model



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Recap

Great discoveries in physics

Quantum entanglement (1865, 1935)

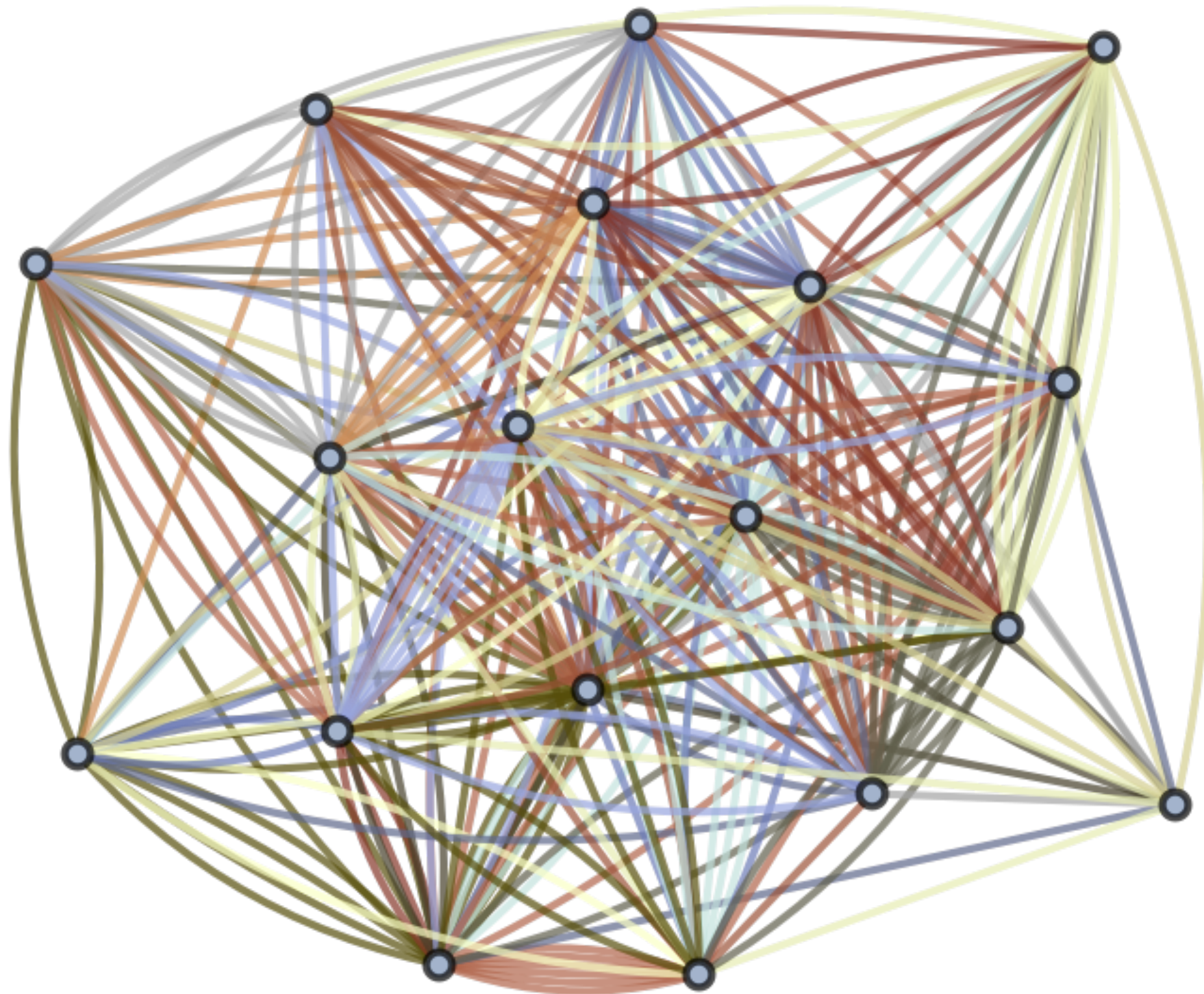
Statistical mechanics (1870)

Superconductivity (1911)

Black holes (1916)

The Sachdev-Ye-Kitaev (SYK) model

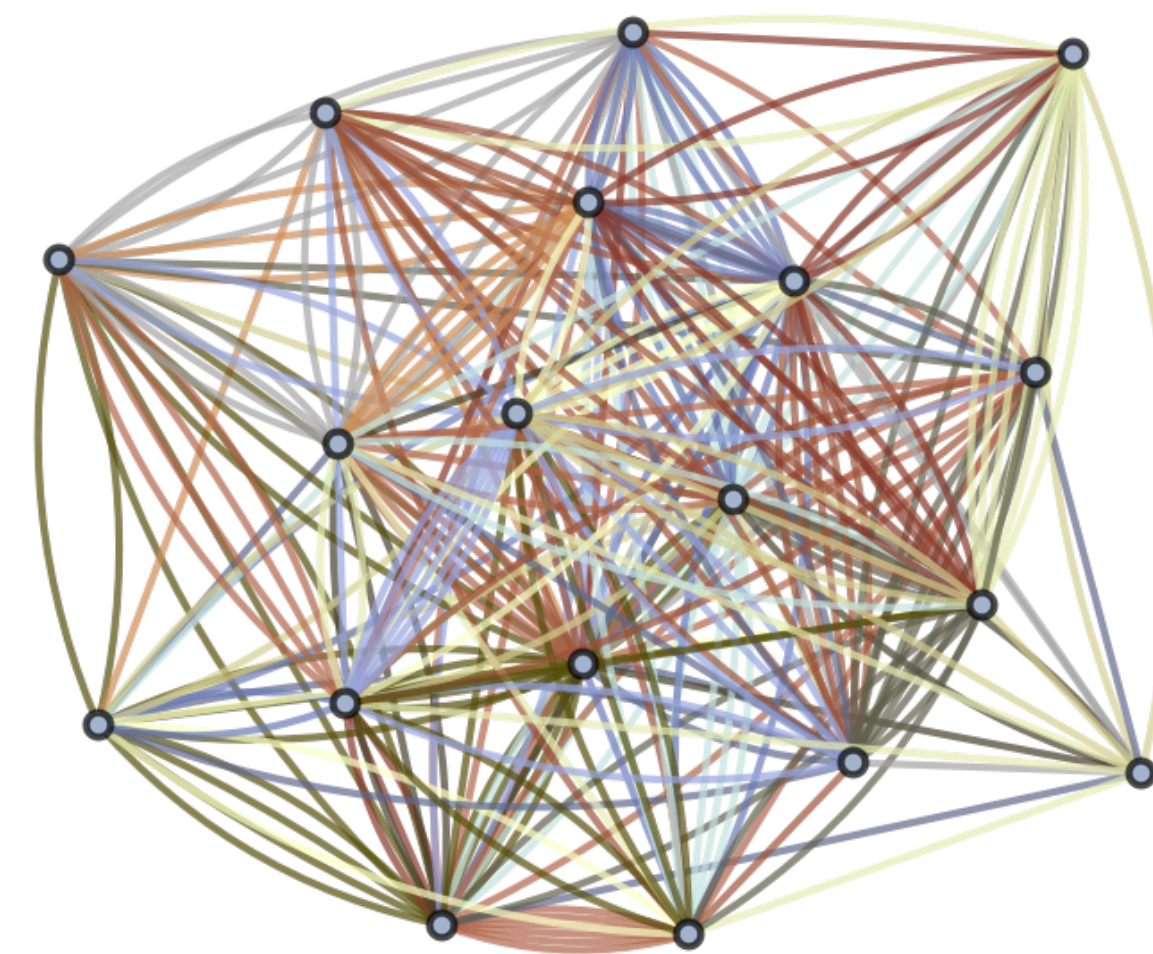
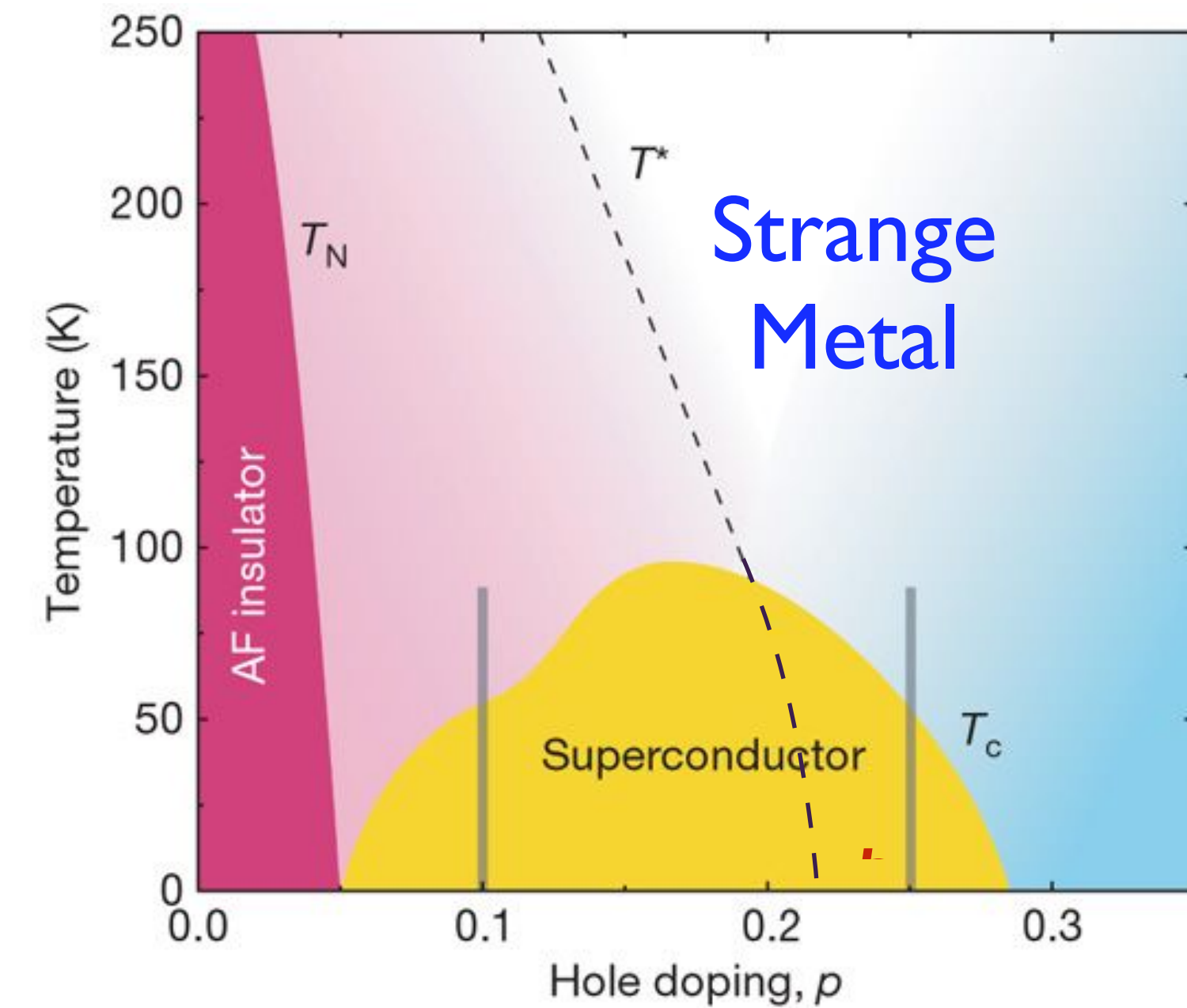
The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

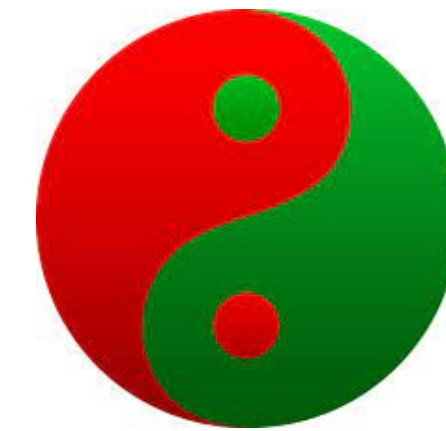
In one set of variables, it helps describe the *strange* electrical properties of YBCO



The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

In one set of variables, it helps describe the *strange* electrical properties of YBCO



In a *dual* set of variables it simulates the quantum degrees of freedom of the interior of *charged black holes*

