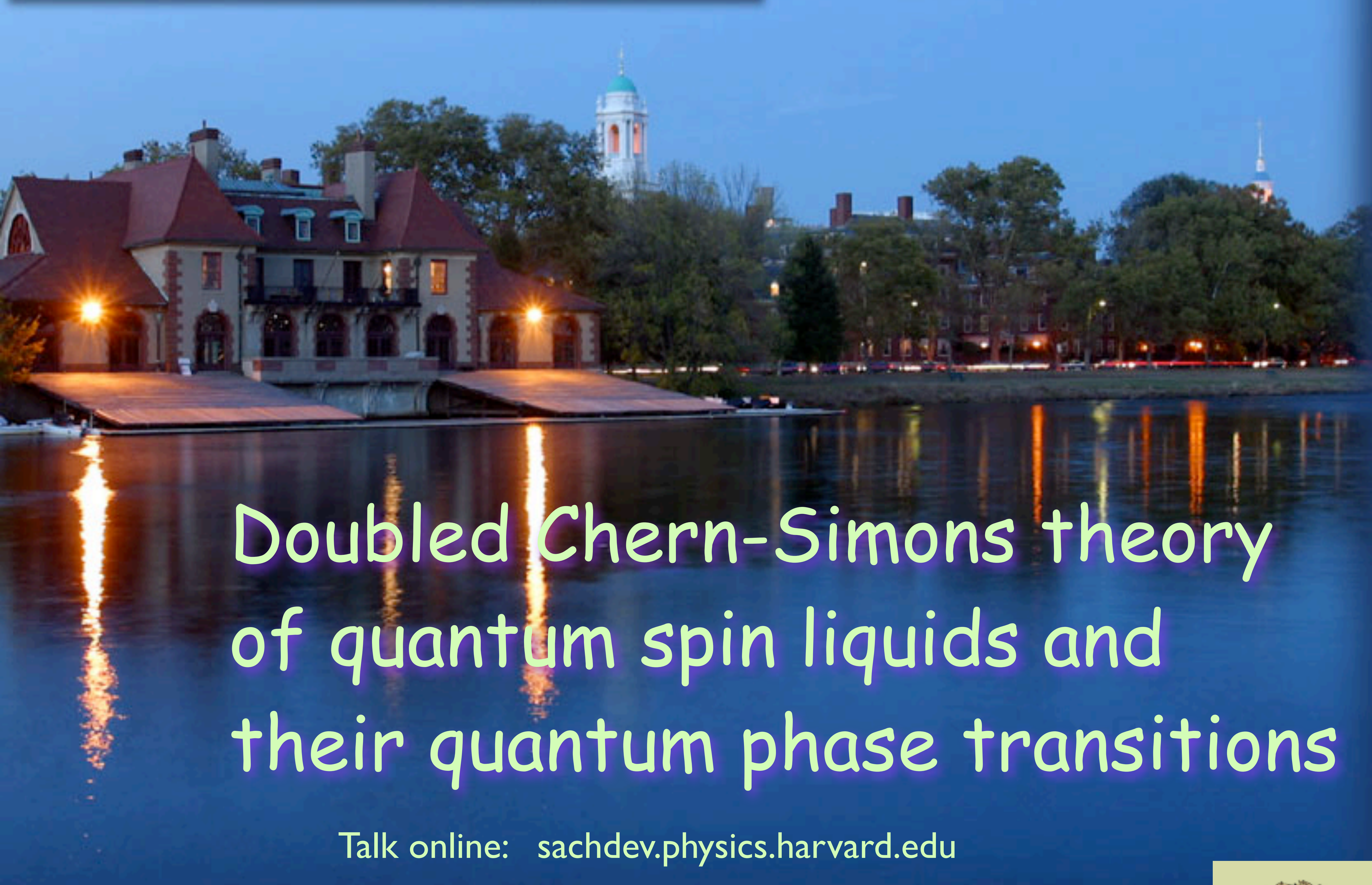
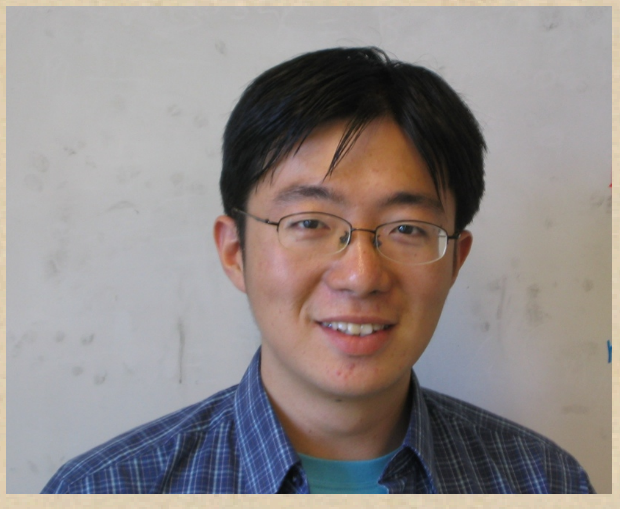


Doubled Chern-Simons theory of quantum spin liquids and their quantum phase transitions

Talk online: sachdev.physics.harvard.edu

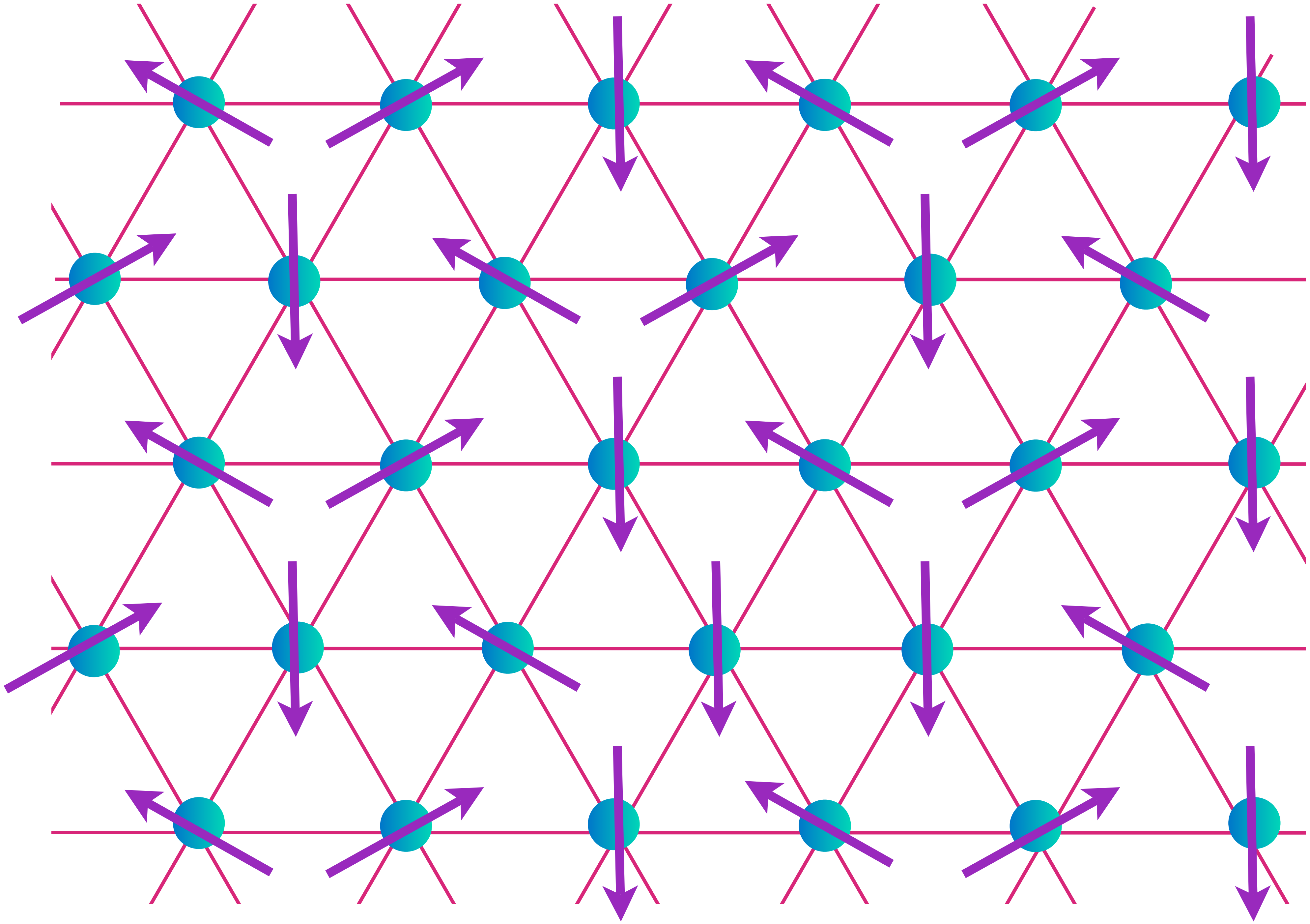


Yang Qi

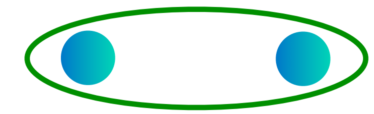
Cenke Xu

arXiv:0809.0694

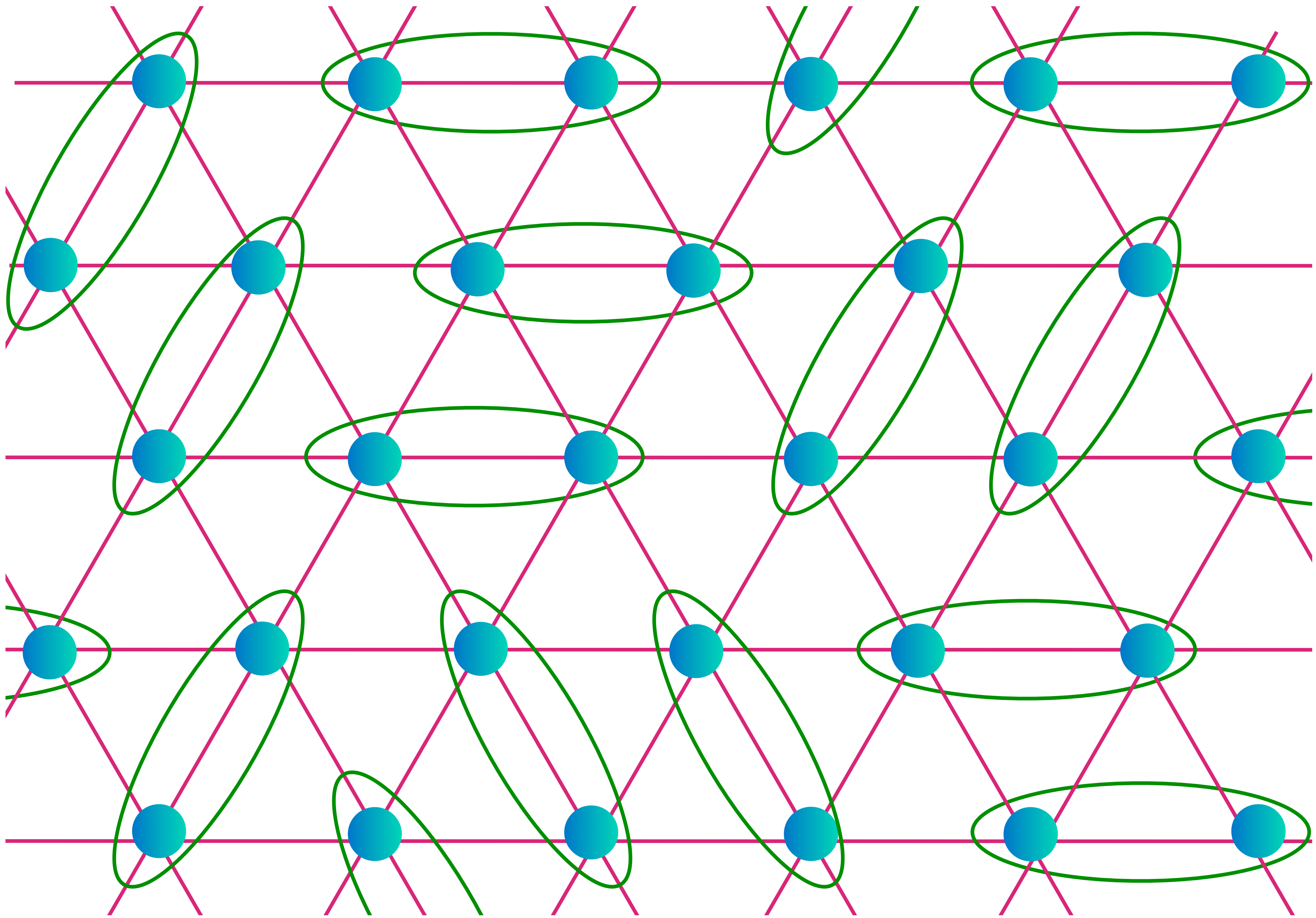
Antiferromagnet



Spin liquid



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



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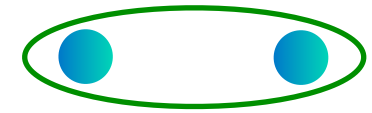

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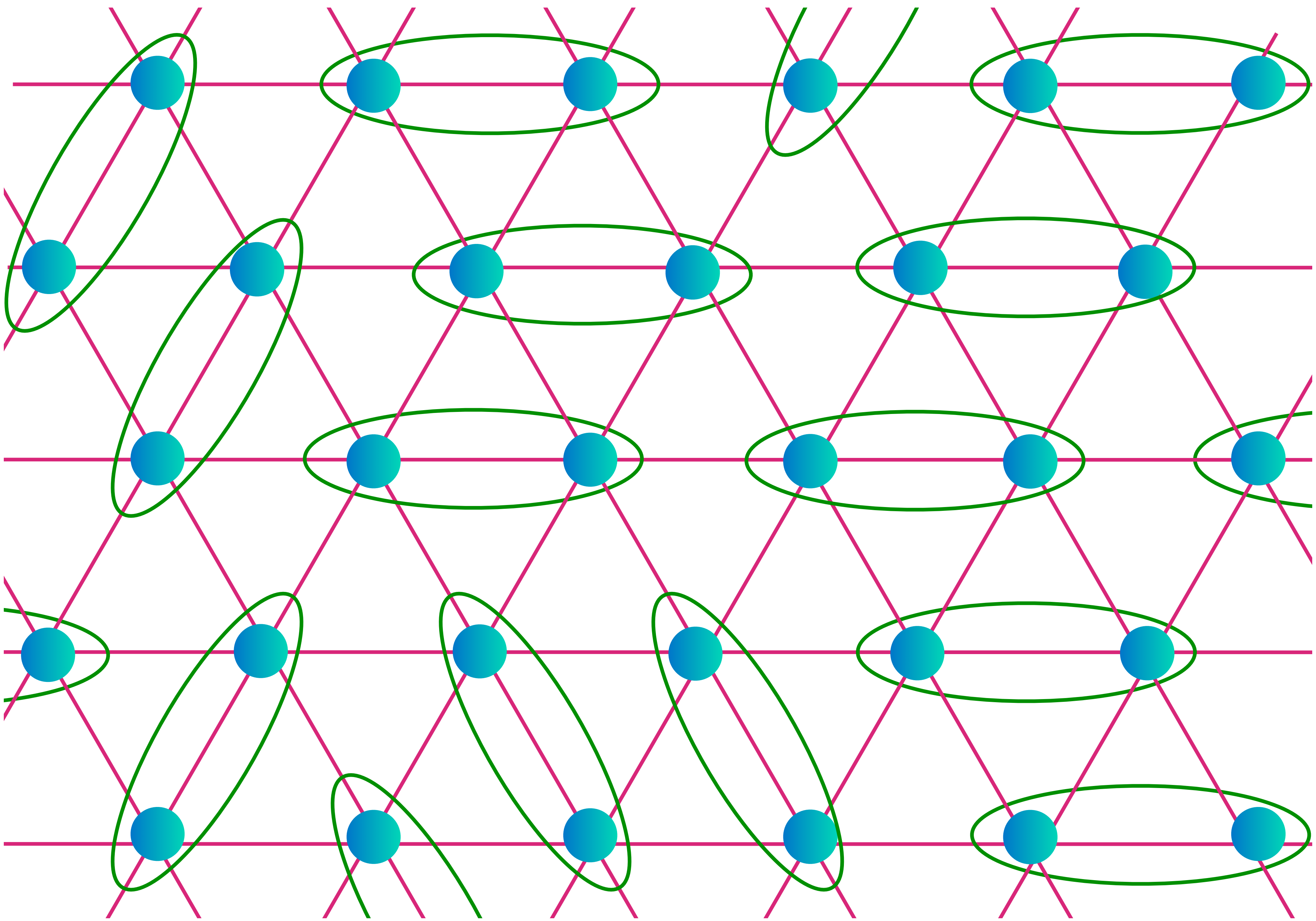

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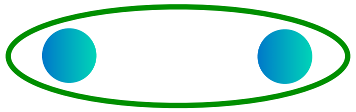

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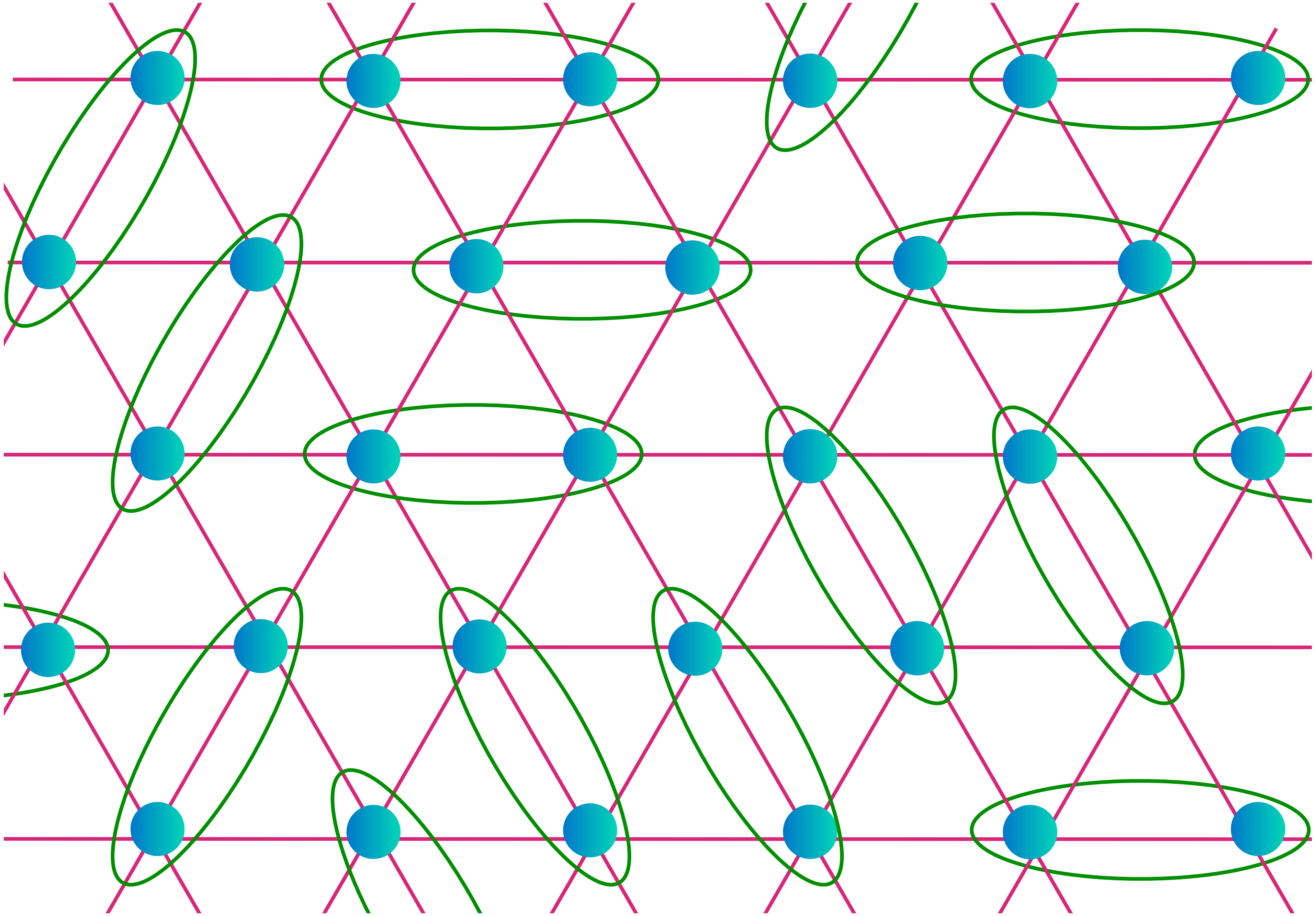

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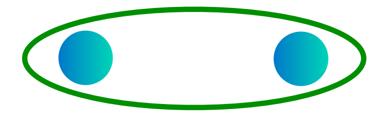

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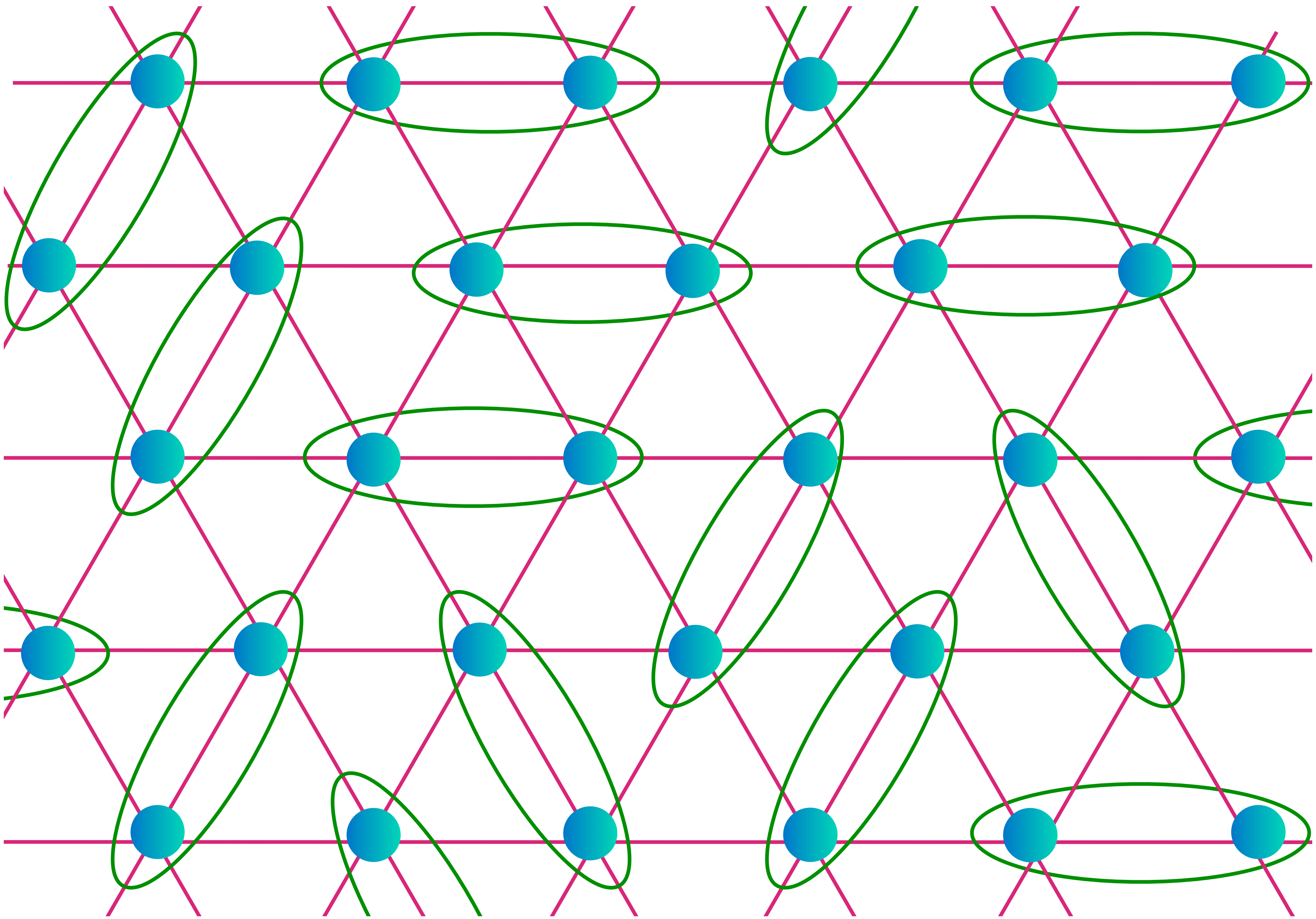

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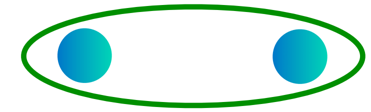

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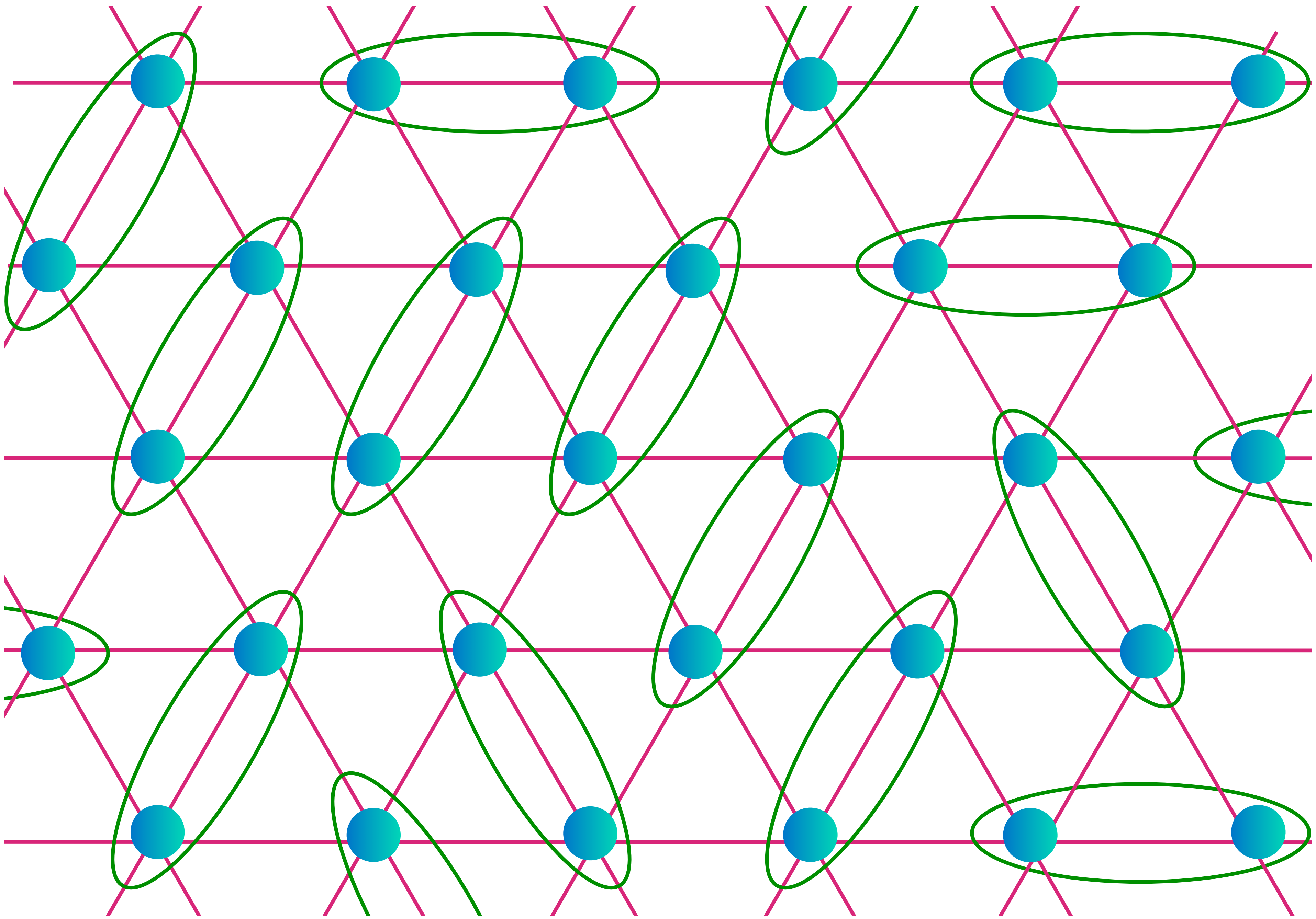

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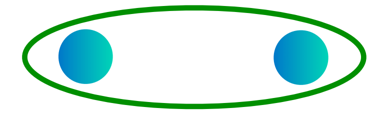

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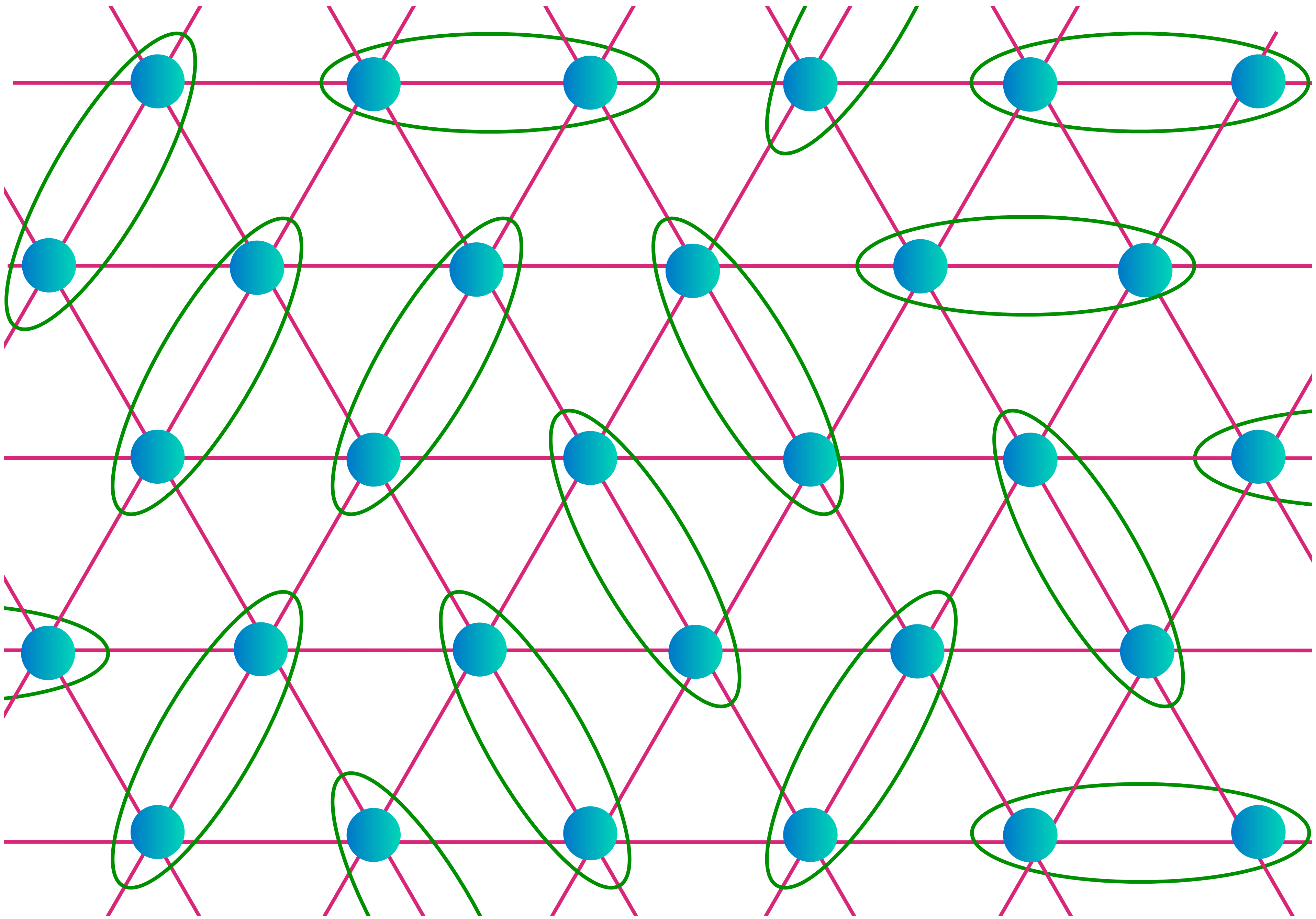
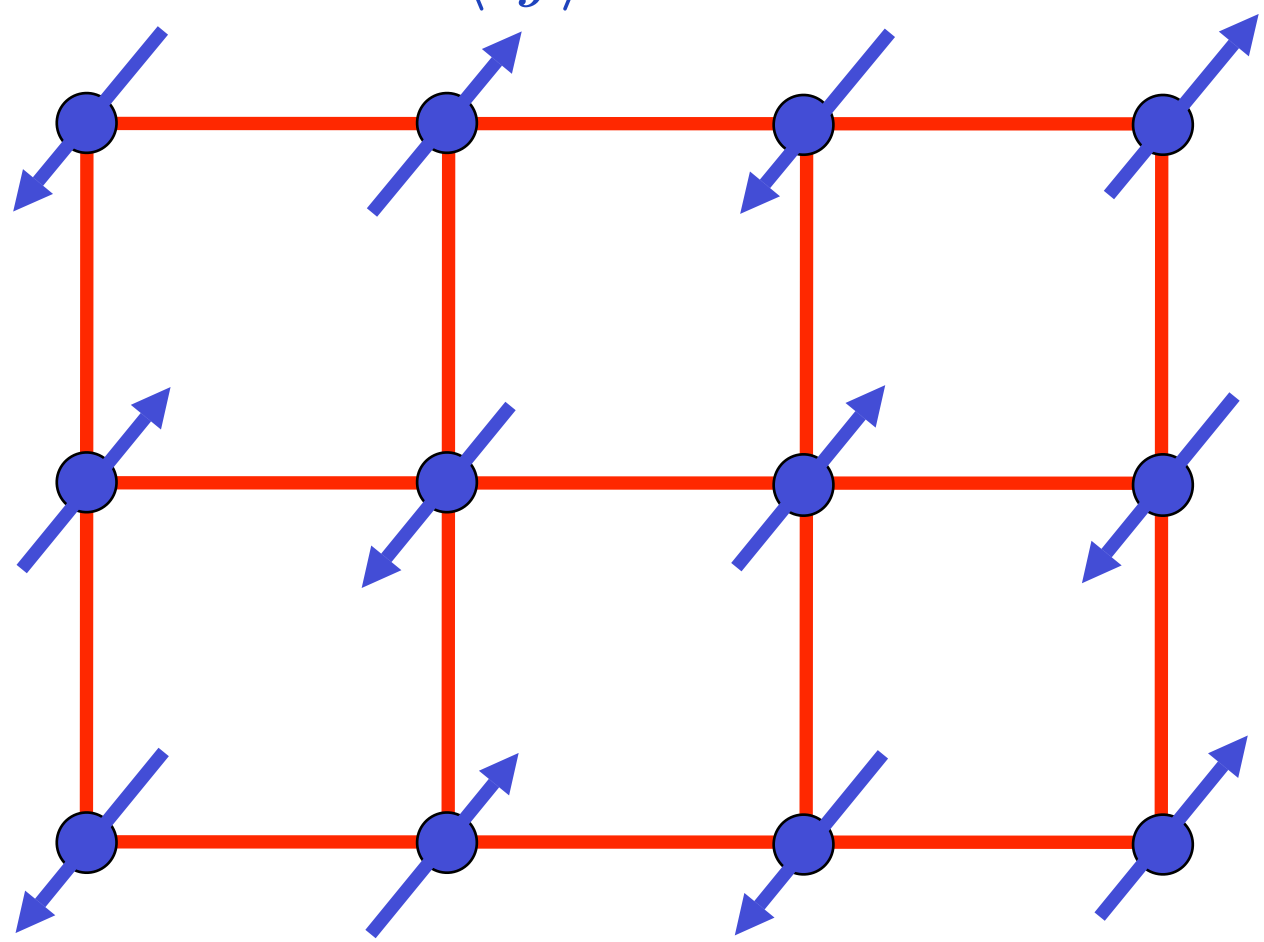


General approach

Look for spin liquids across continuous (or weakly first-order) quantum transitions from antiferromagnetically ordered states

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



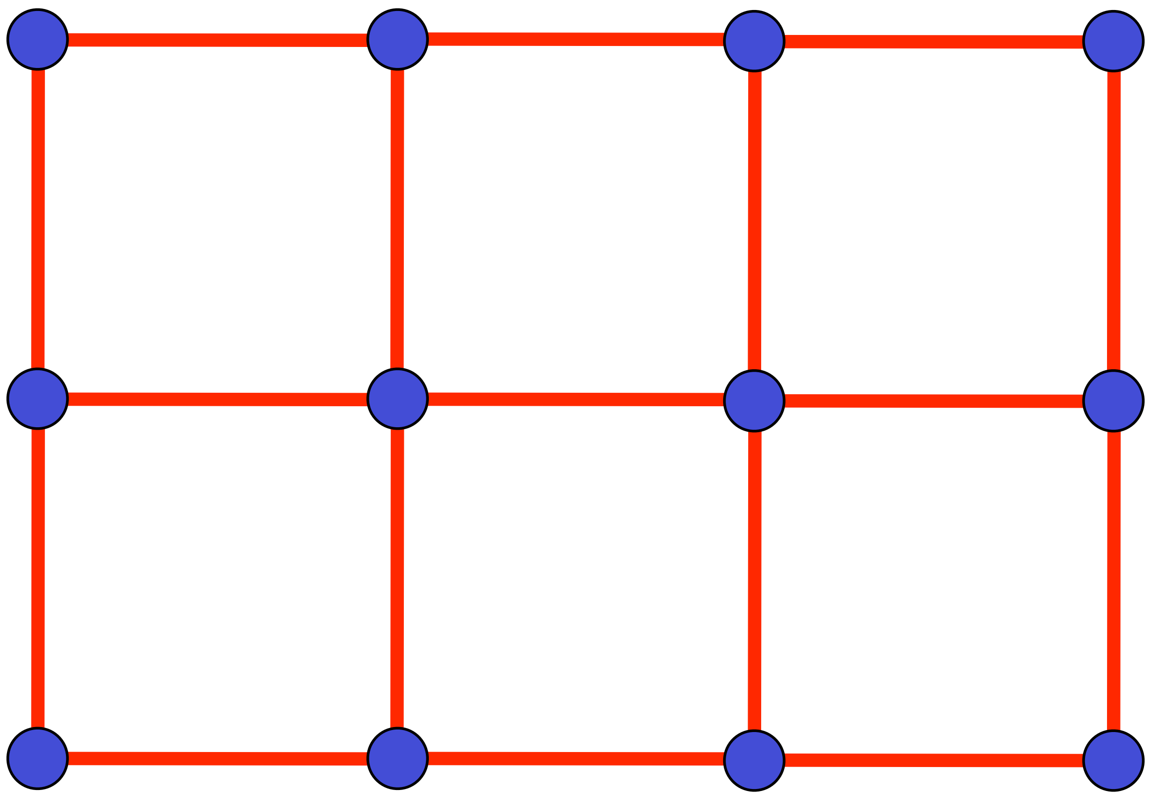
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet



Destroy Neel order by perturbations which preserve full square lattice symmetry

Theory for loss of Neel order

Write the spin operator in terms of
Schwinger bosons (spinons) $z_{i\alpha}$, $\alpha = \uparrow, \downarrow$:

$$\vec{S}_i = z_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} z_{i\beta}$$

where $\vec{\sigma}$ are Pauli matrices, and the bosons obey the local constraint

$$\sum_\alpha z_{i\alpha}^\dagger z_{i\alpha} = 2S$$

Effective theory for spinons must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \rightarrow e^{i\theta} z_{i\alpha}$$

Perturbation theory

Low energy spinon theory for "quantum disordering" the Néel state is the CP^1 model

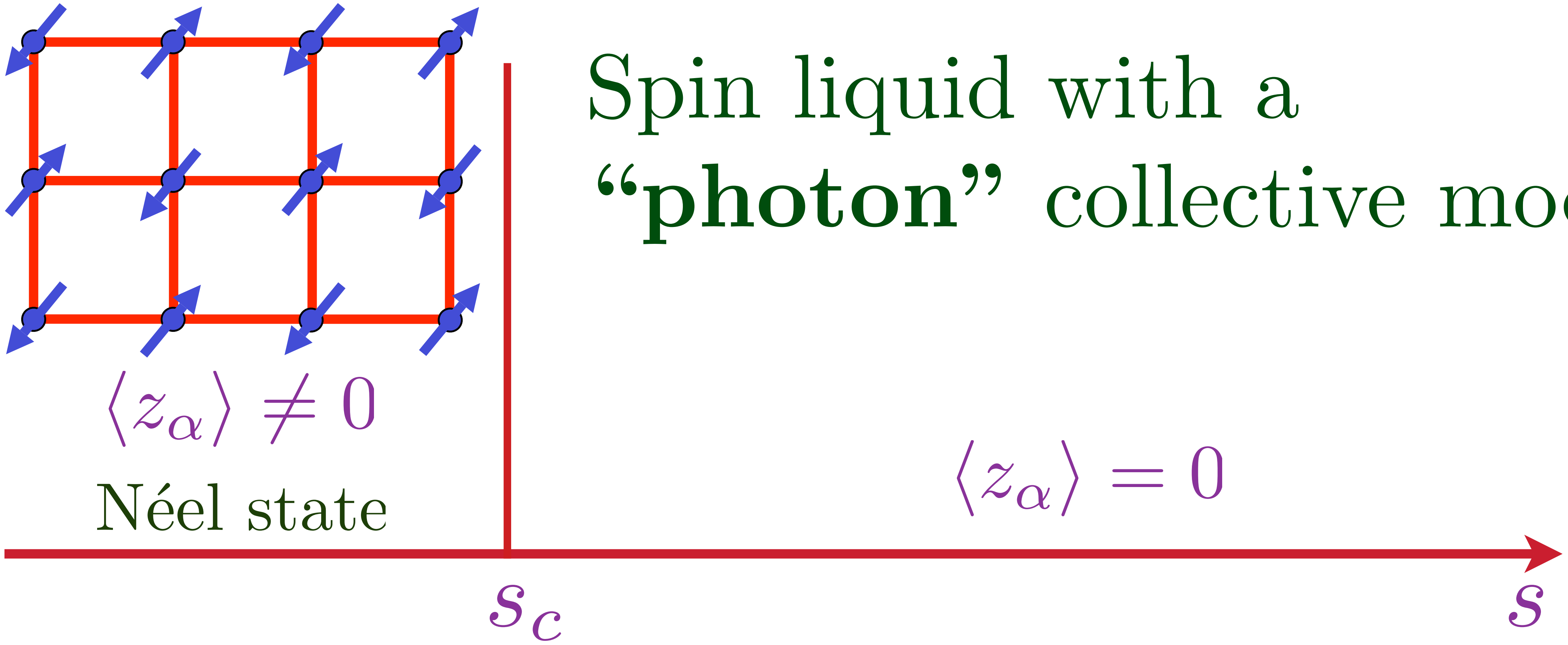
$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s \left|z_\alpha\right|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \Big]$$

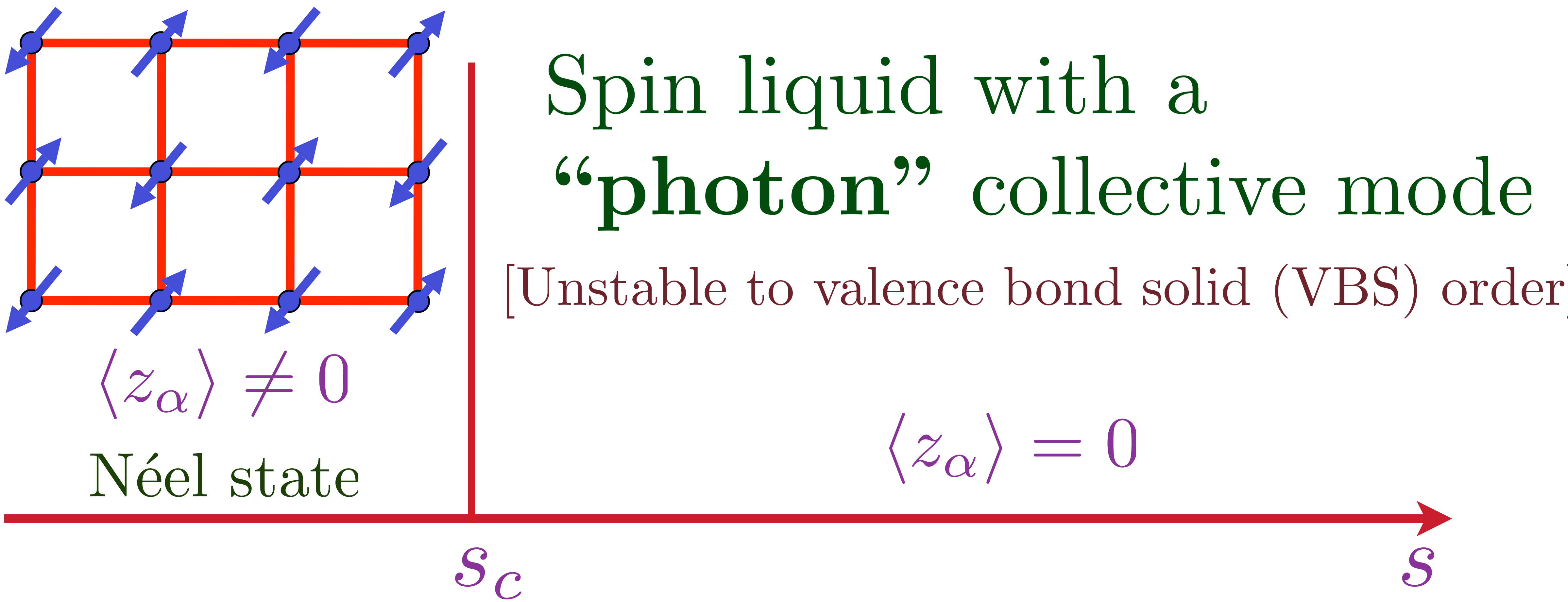
where A_μ is an emergent U(1) gauge field (the **"photon"**) which describes low-lying spin-singlet excitations.

Phases:

$\langle z_\alpha \rangle \neq 0$	$\Rightarrow$	Néel (Higgs) state
$\langle z_\alpha \rangle = 0$	$\Rightarrow$	Spin liquid (Coulomb) state

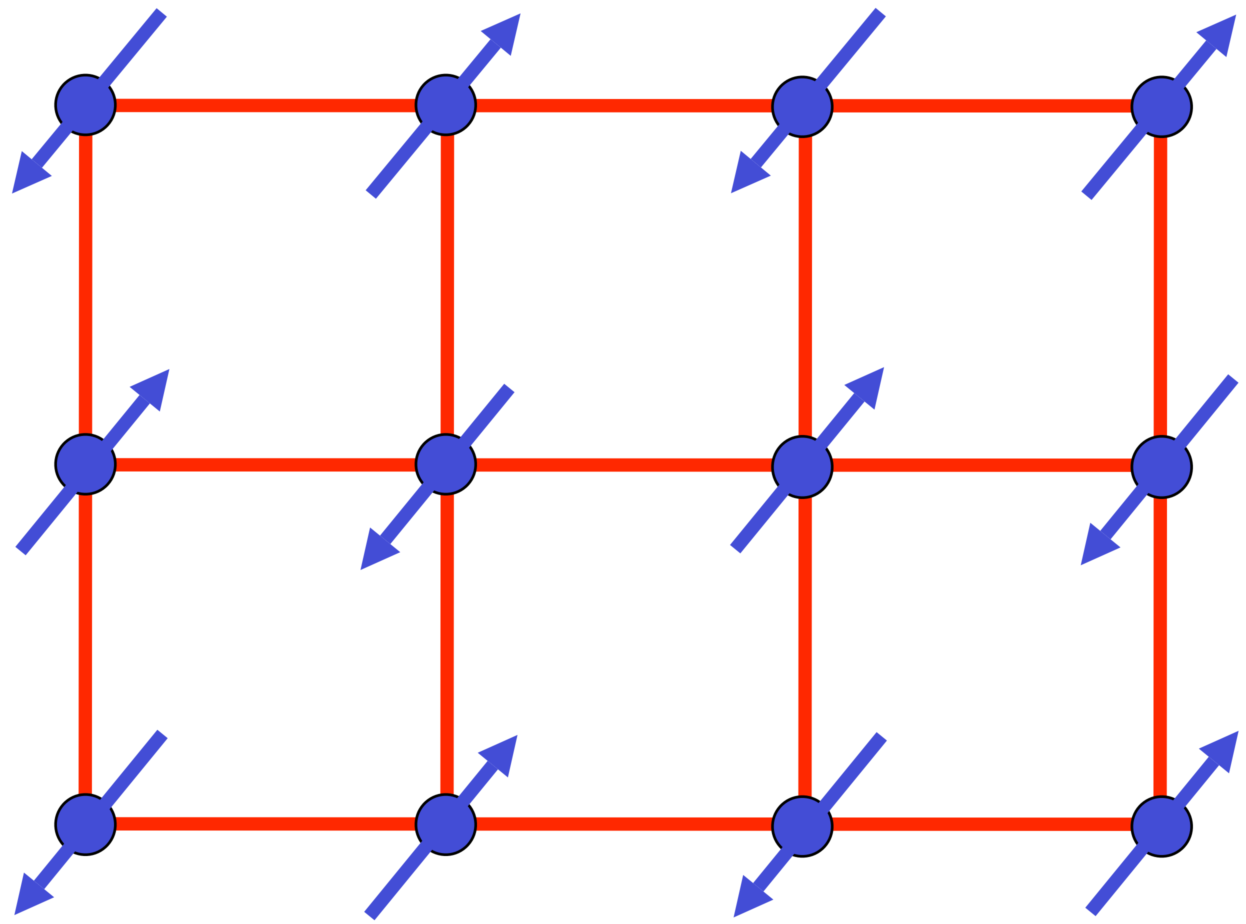
Spin liquid with a **"photon"** collective mode





N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

From the square to the triangular lattice

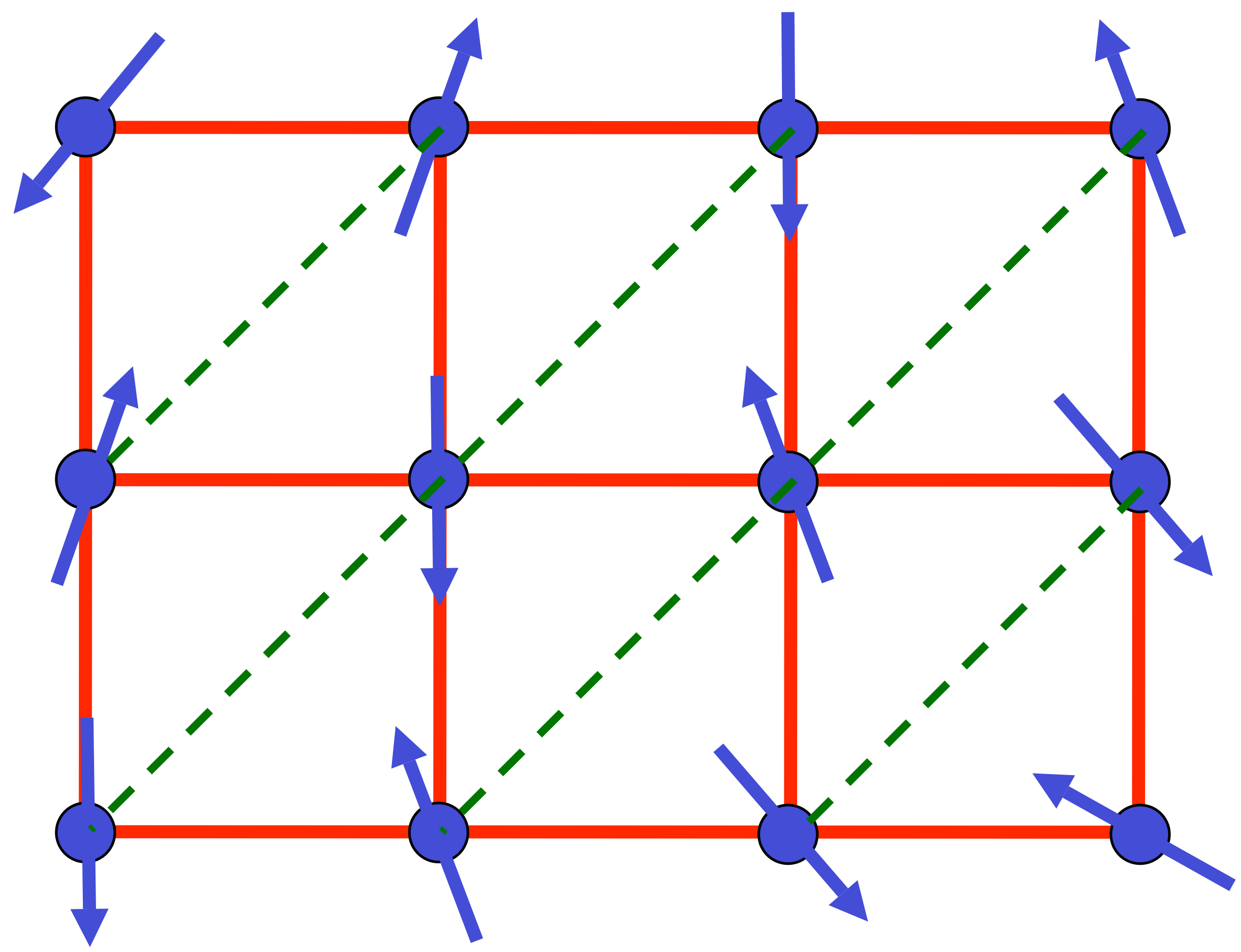


A spin density wave with

$$\langle \vec{S}_i \rangle \propto (\cos(\mathbf{K} \cdot \mathbf{r}_i, \sin(\mathbf{K} \cdot \mathbf{r}_i))$$

and $\mathbf{K} = (\pi, \pi)$.

From the square to the triangular lattice



A spin density wave with

$$\langle \vec{S}_i \rangle \propto (\cos(\mathbf{K} \cdot \mathbf{r}_i, \sin(\mathbf{K} \cdot \mathbf{r}_i))$$

and $\mathbf{K} = (\pi + \Phi, \pi + \Phi)$.

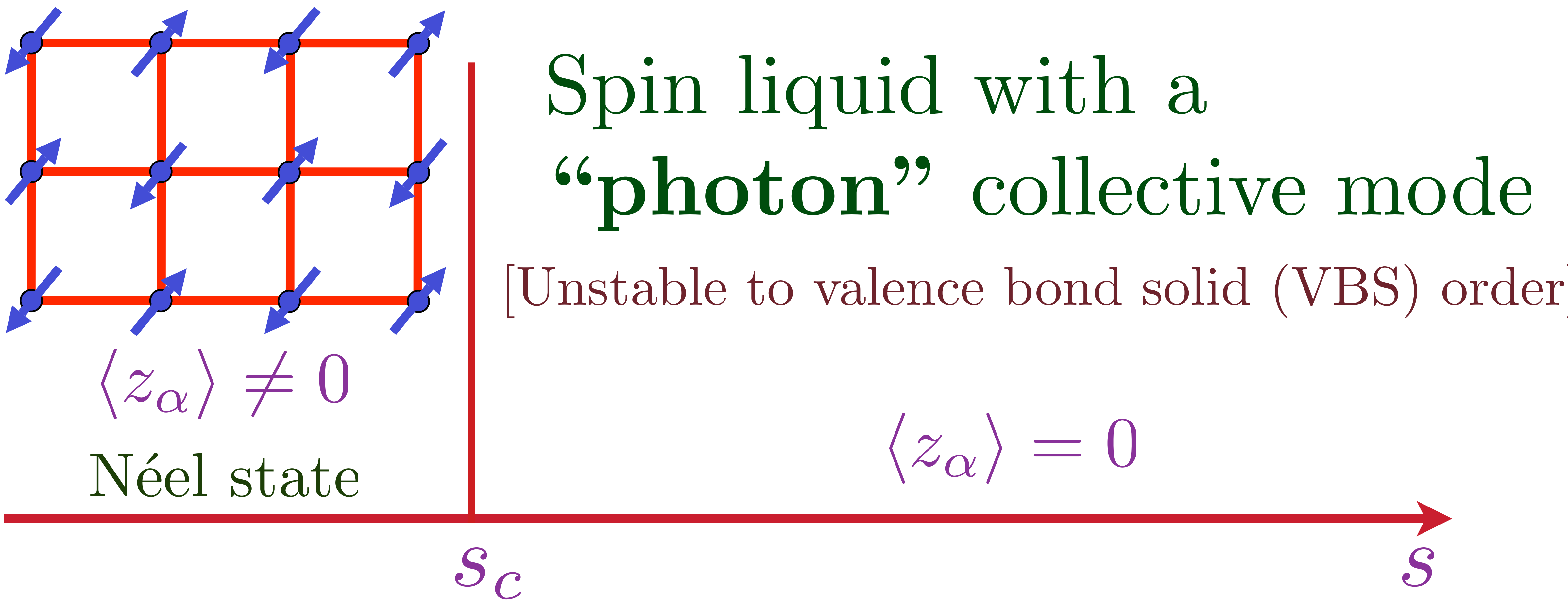
Interpretation of non-collinearity Φ

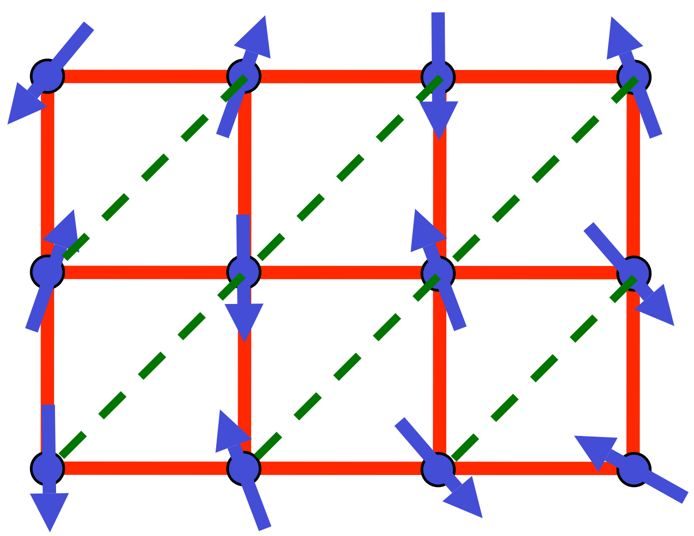
Its physical interpretation becomes clear from the allowed coupling to the spinons:

$$\mathcal{S}_{z,\Phi} = \int d^2 r d\tau \left[\lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \right]$$

Φ is a spinon pair field

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)


Spin liquid with a
"photon" collective mode
[Unstable to valence bond solid (VBS) order]
$\langle z_\alpha \rangle \neq 0$
Néel state
$\langle z_\alpha \rangle = 0$
s_c
s



$\langle z_\alpha \rangle \neq 0$, $\langle \Phi \rangle \neq 0$

non-collinear Néel state

Z_2 spin liquid with a **vison** excitation

$\langle z_\alpha \rangle = 0$, $\langle \Phi \rangle \neq 0$

s_c

s

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

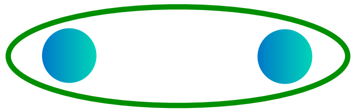
What is a vison ?

A vison is an Abrikosov vortex in the spinon pair field Φ.

In the Z_2 spin liquid, the vison is $S = 0$ quasiparticle with a finite energy gap

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

What is a vison ?



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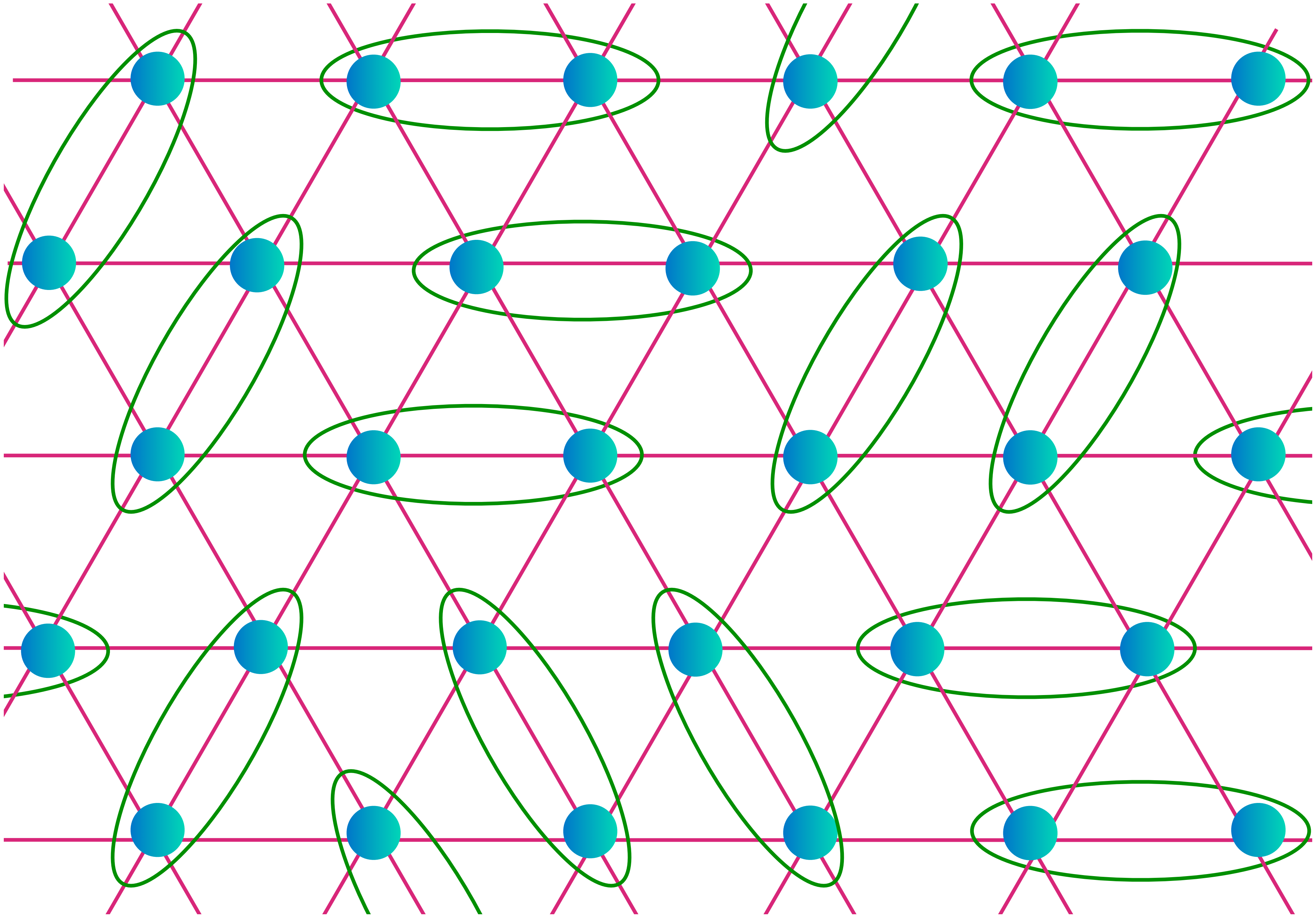
Z_2 Spin liquid



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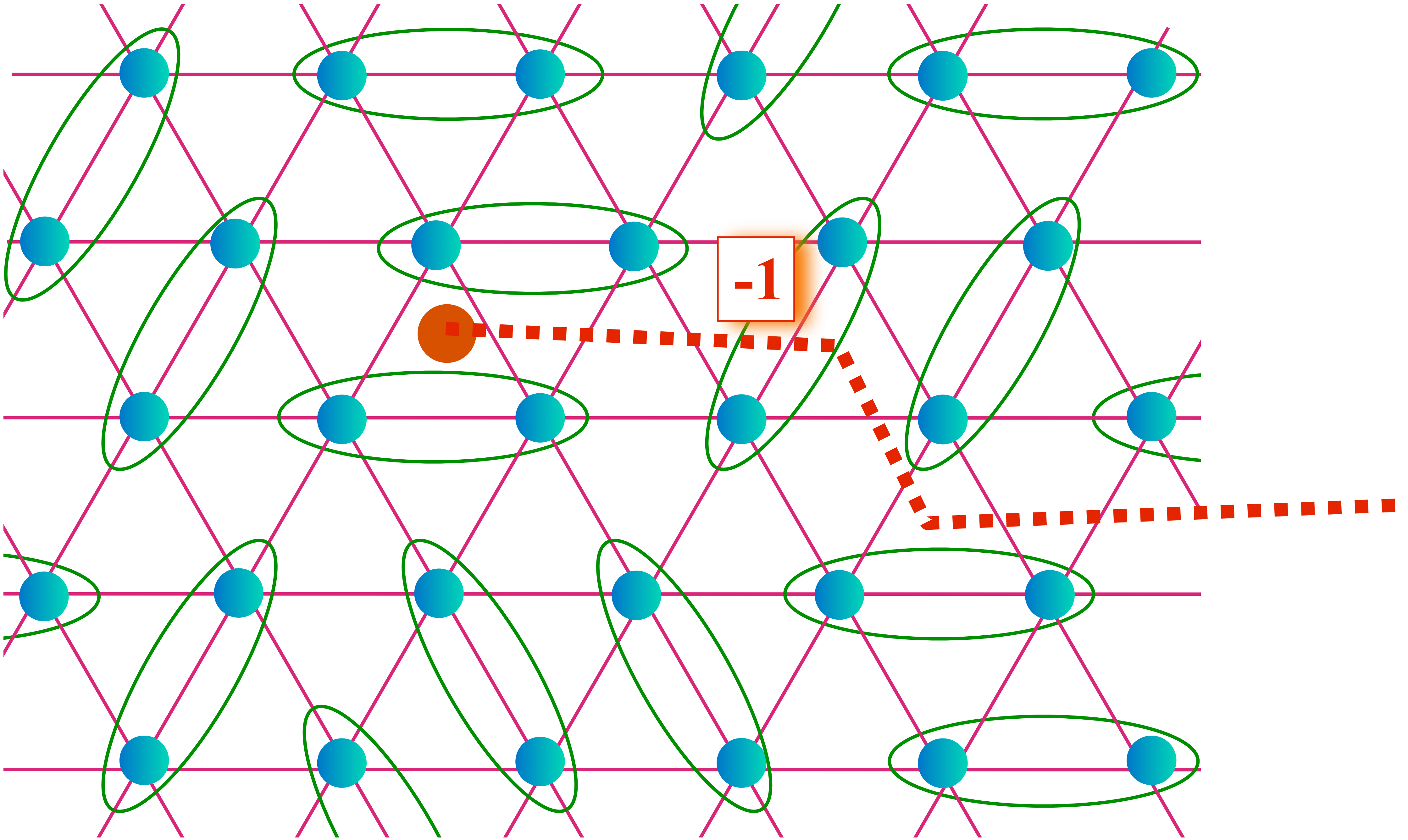

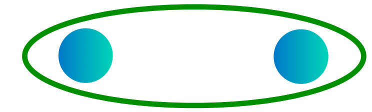

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N. Read and B. Chakraborty, *Phys. Rev. B* **40,** 7133 (1989)
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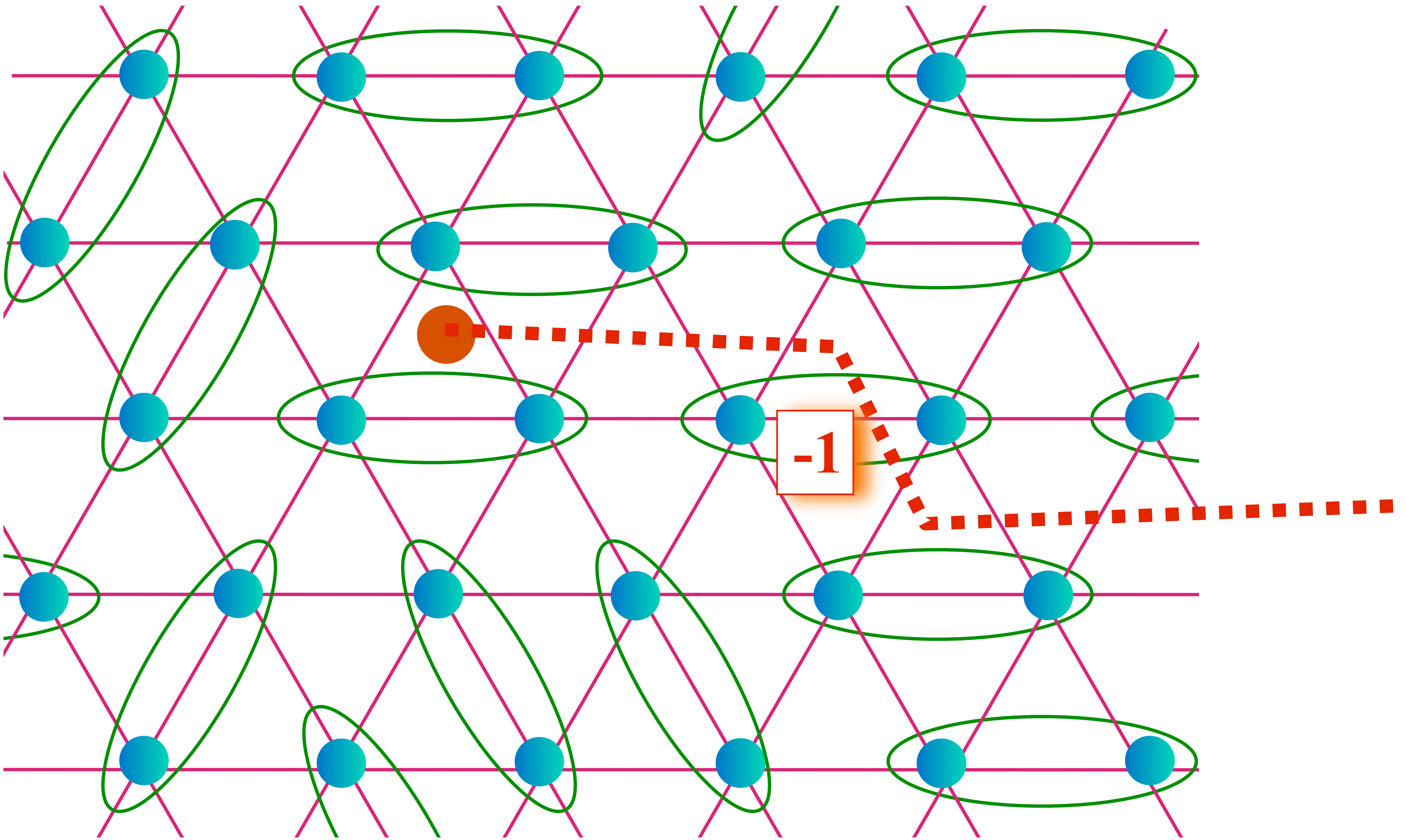

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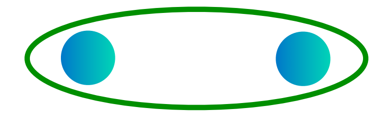

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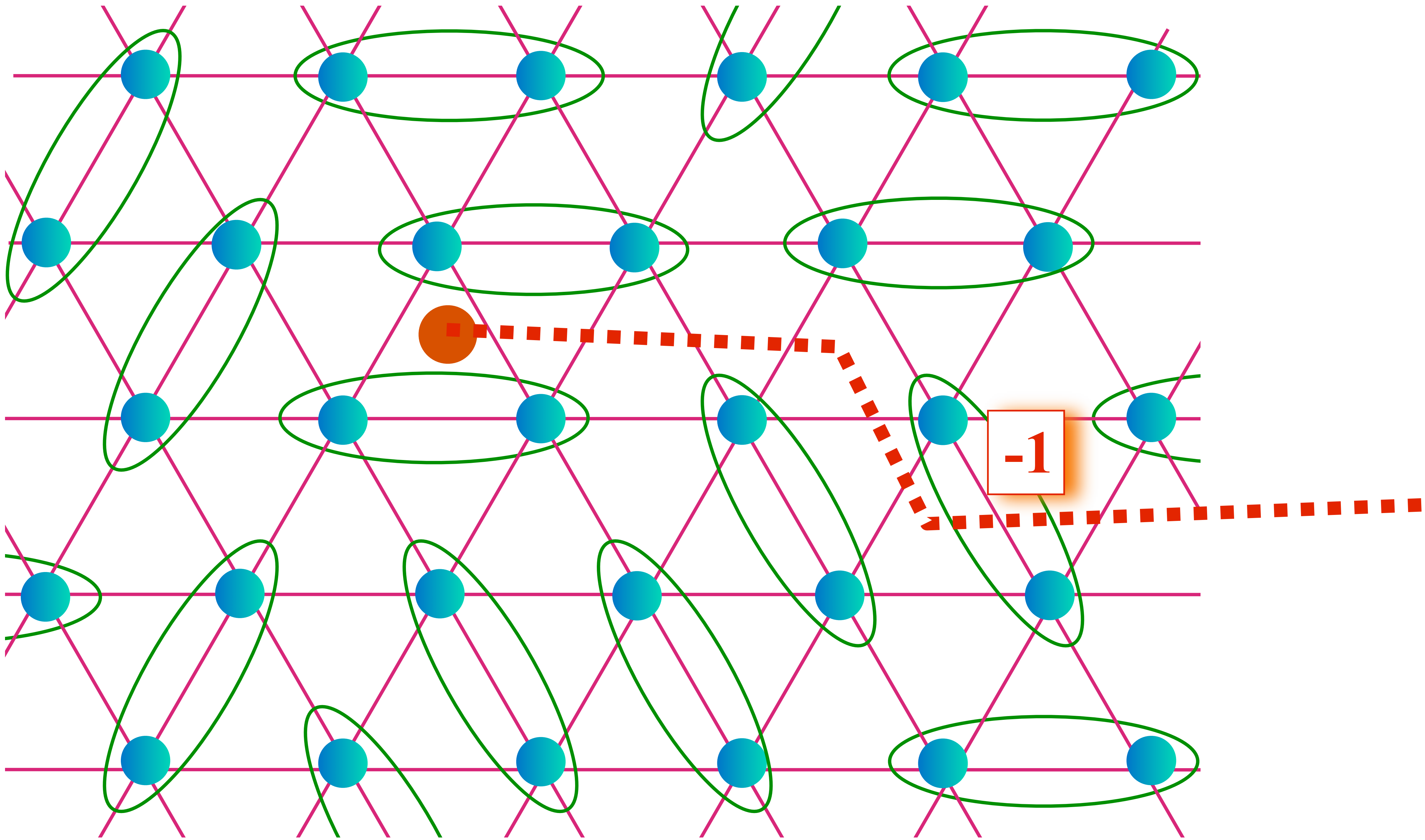

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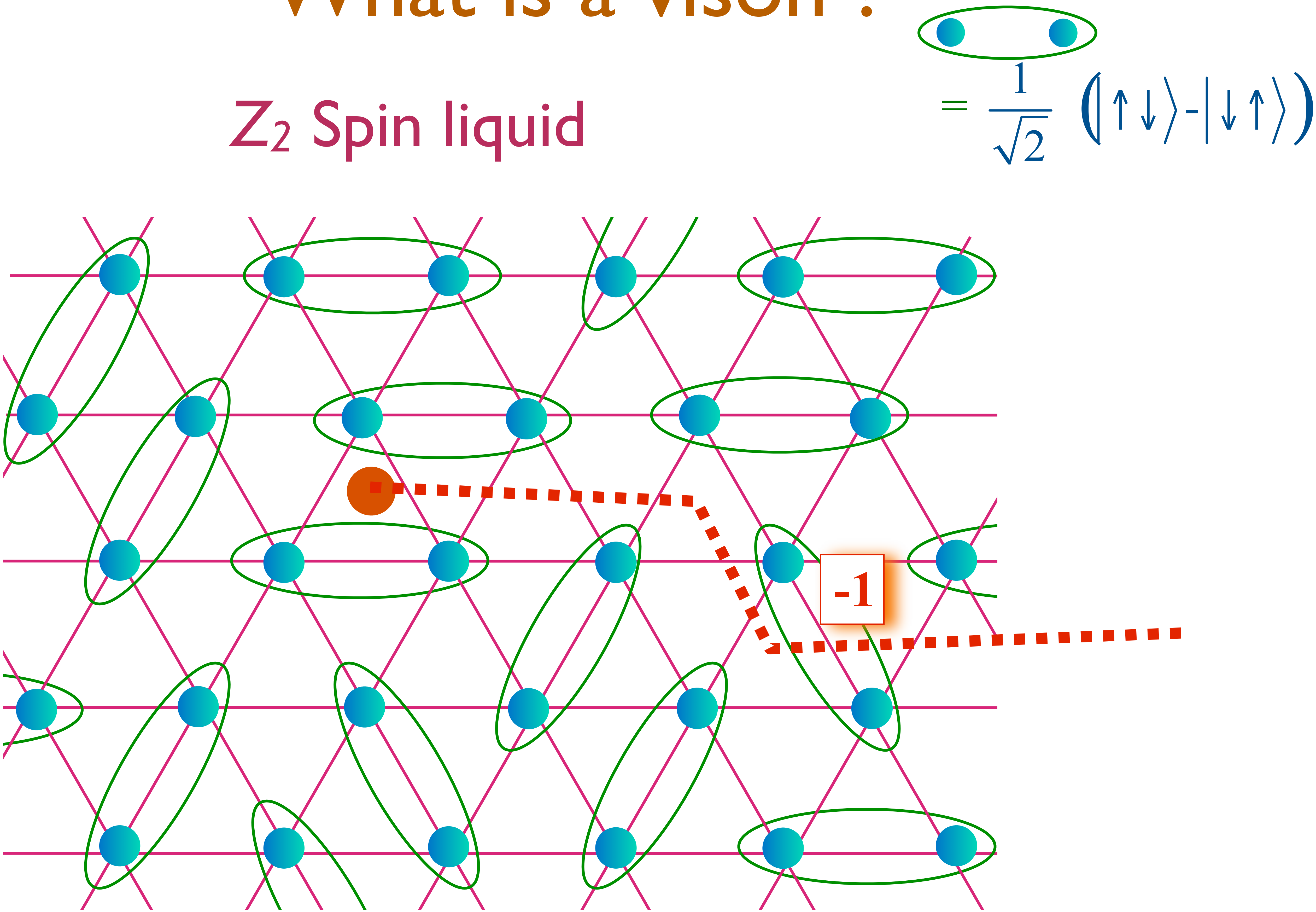

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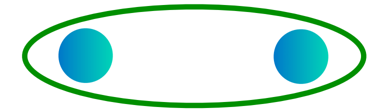

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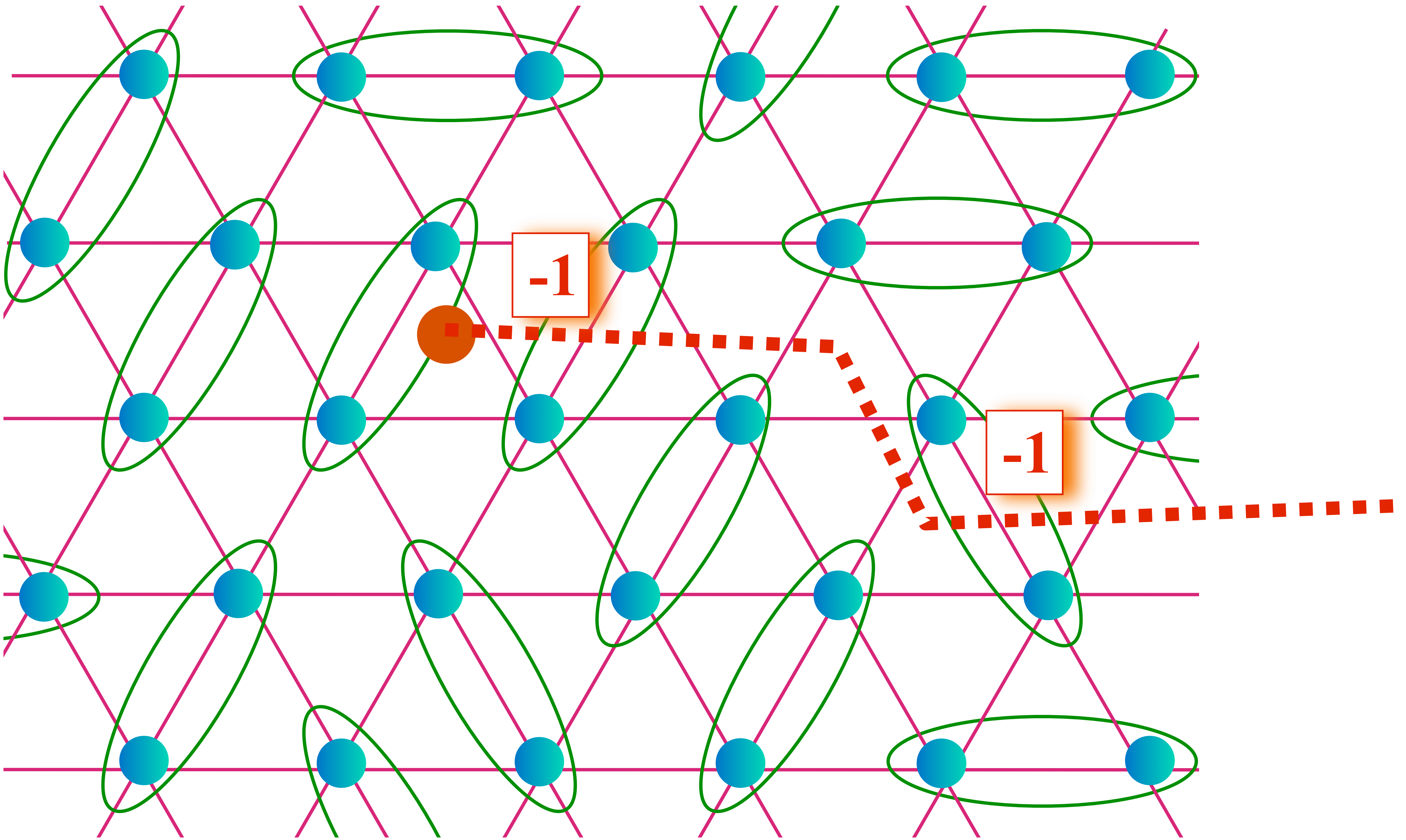

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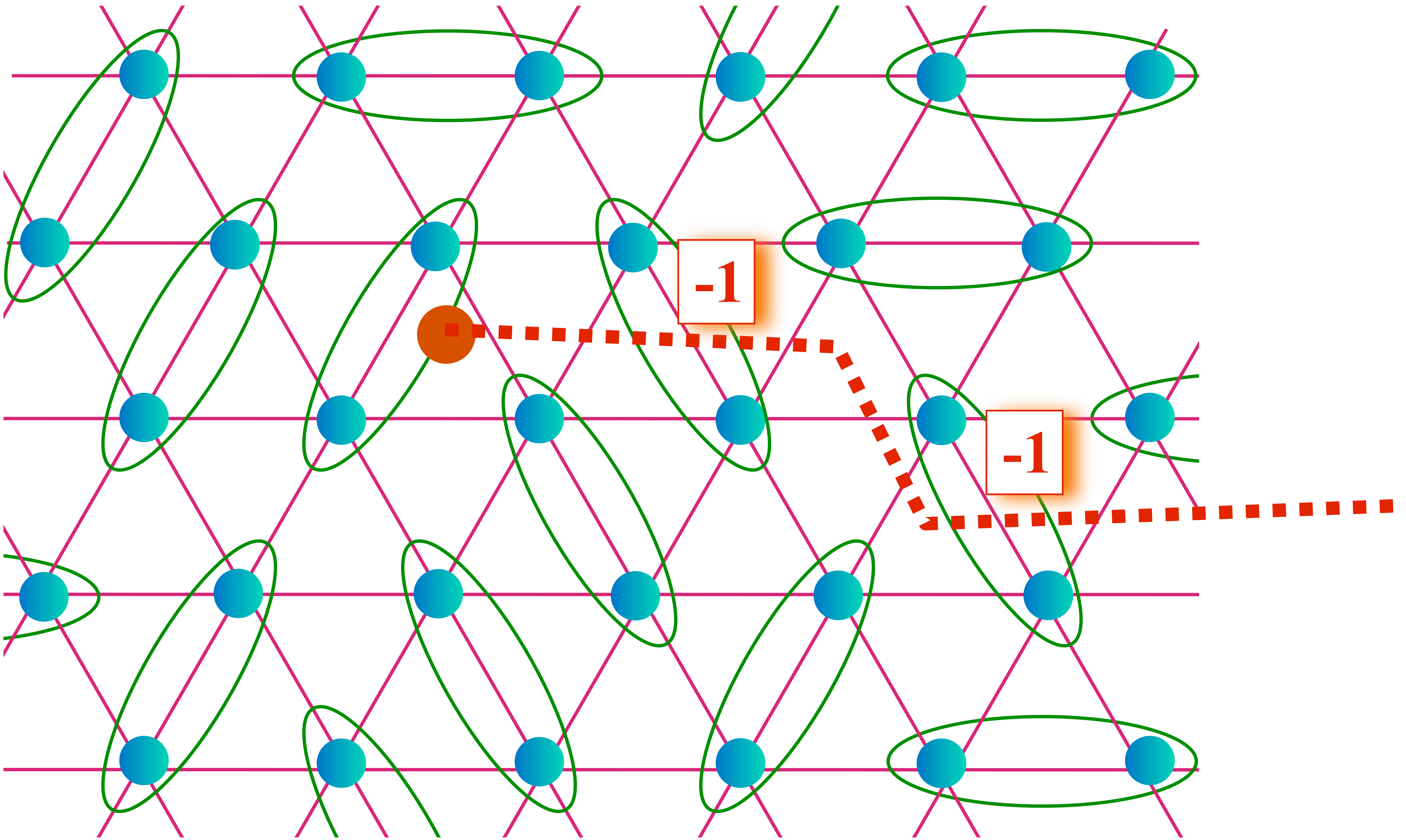

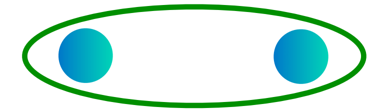

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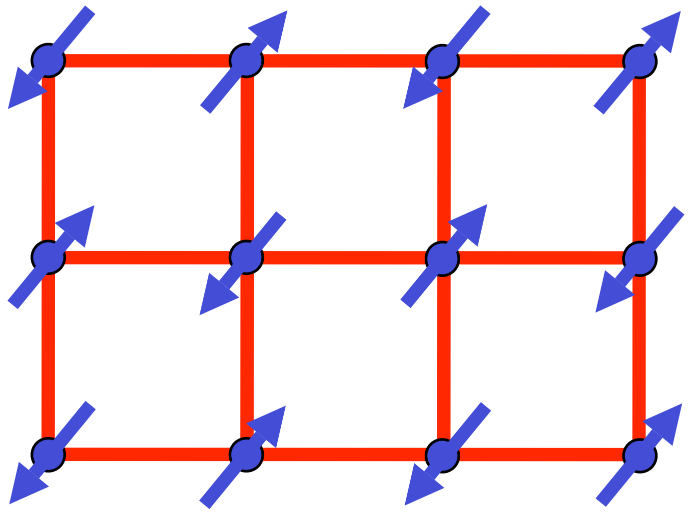
Global phase diagram



Spin liquid with a **"photon"** collective mode

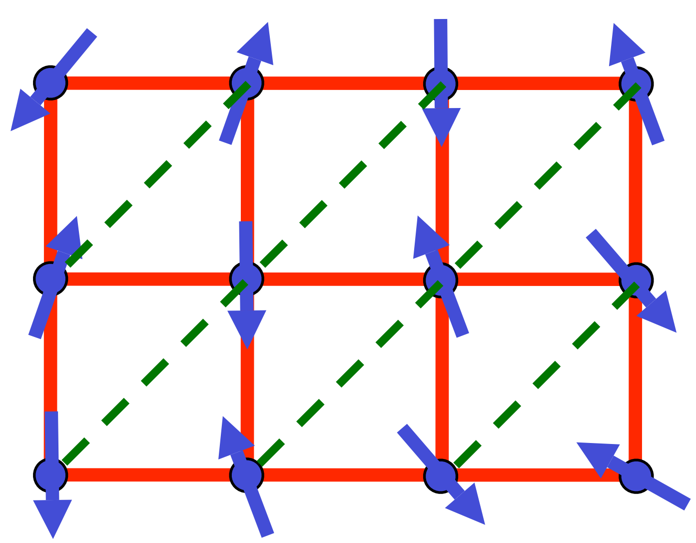
[Unstable to valence bond solid (VBS) order]

$\langle z_\alpha \rangle \neq 0$, $\langle \Phi \rangle = 0$

Néel state

$\langle z_\alpha \rangle = 0$, $\langle \Phi \rangle = 0$

s



Z_2 spin liquid with a **vison** excitation

$\langle z_\alpha \rangle \neq 0$, $\langle \Phi \rangle \neq 0$

non-collinear Néel state

$\langle z_\alpha \rangle = 0$, $\langle \Phi \rangle \neq 0$

$\widetilde{s}$

S. Sachdev and N. Read, *Int. J. Mod. Phys B* **5**, 219 (1991), cond-mat/0402109

A Simple Toy Model (A. Kitaev, 1997)

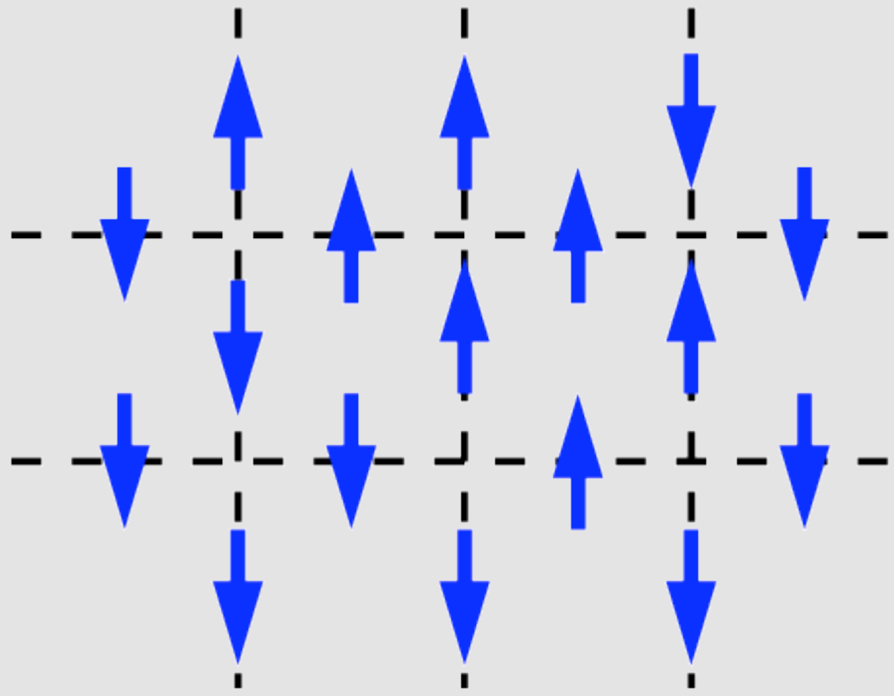
- Spins S_α living on the links of a square lattice:

$$H = -J_1 \sum_i A_i - J_2 \sum_p F_p$$

$$A_i \equiv \prod_{\alpha \in \mathcal{N}(i)} 2S^z_\alpha$$

$$F_p \equiv \prod_{\alpha \in p} 2S^x_\alpha$$

$$[F_p, F_{p'}] = [A_i, A_j] = [F_p, A_j] = 0$$



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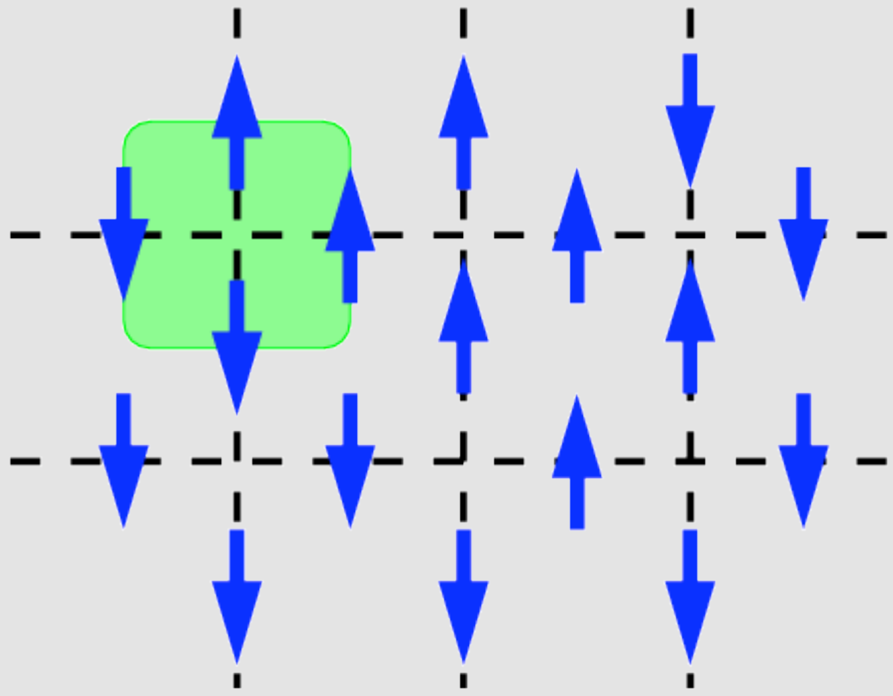

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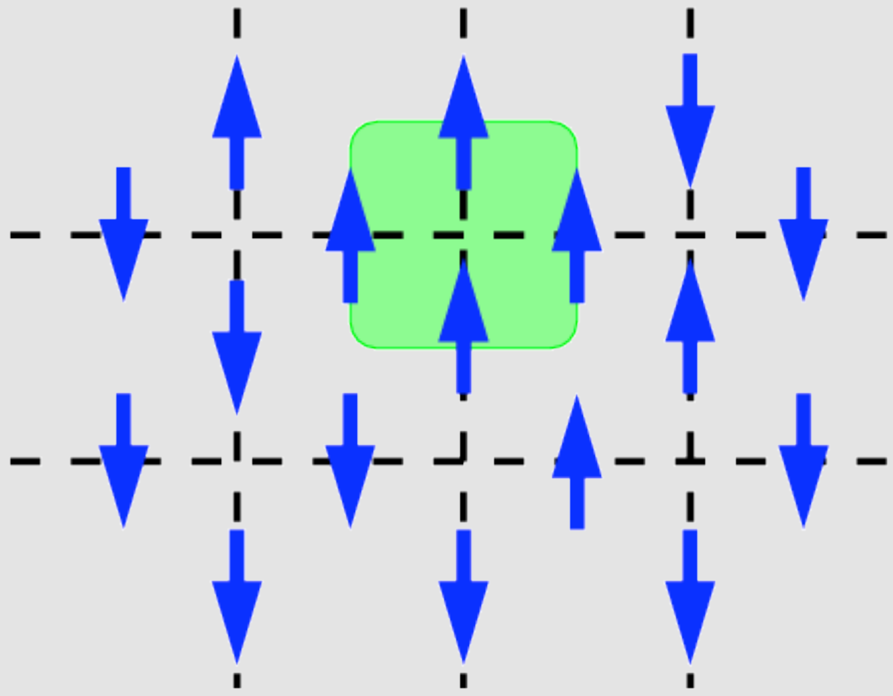

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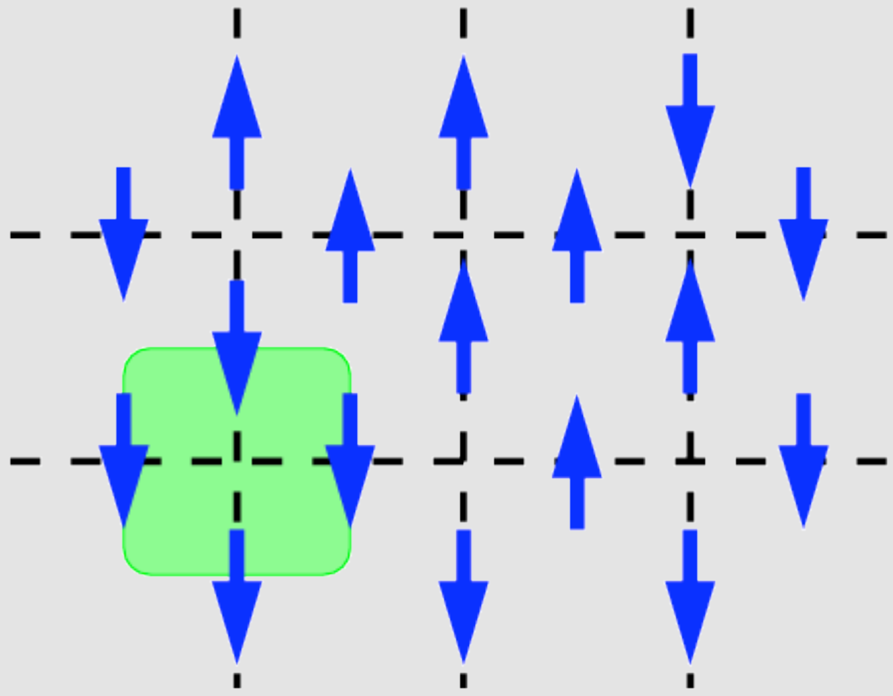

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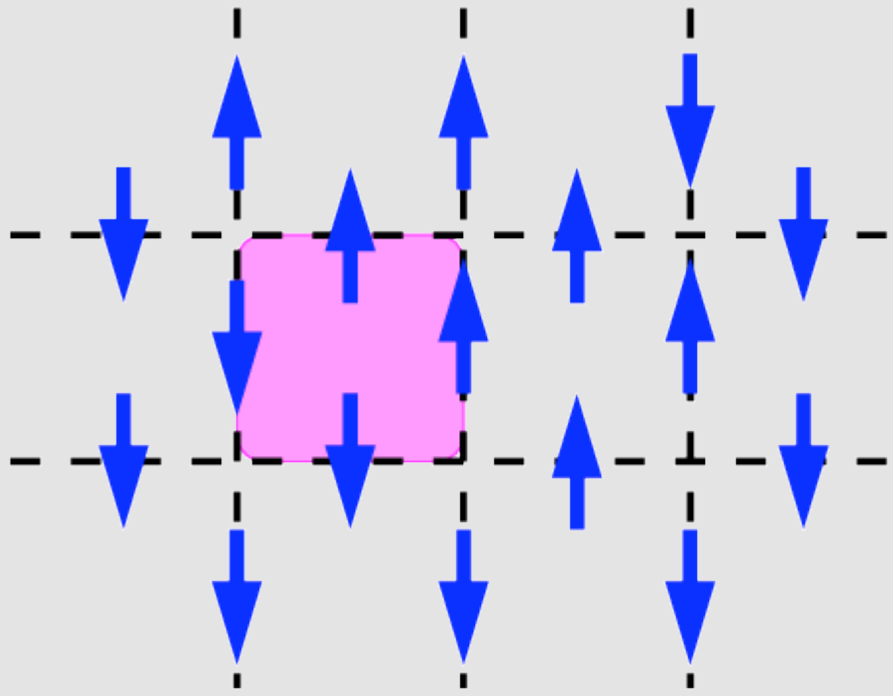

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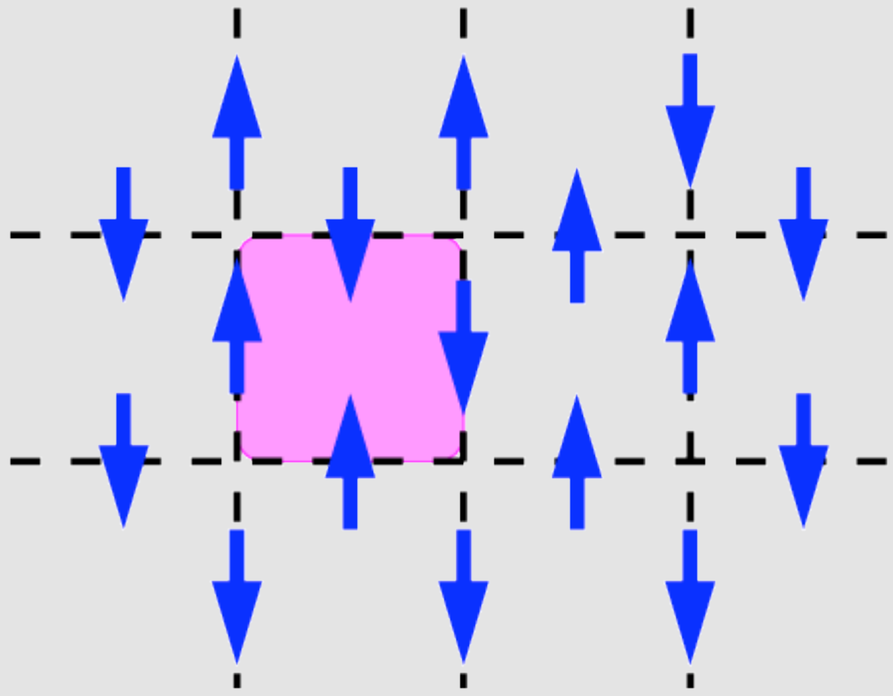

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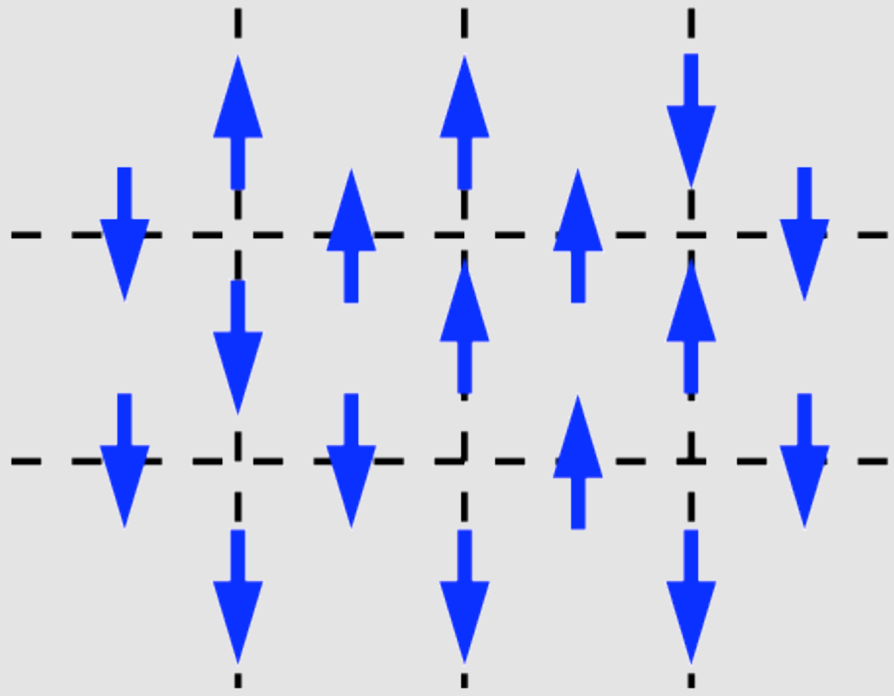

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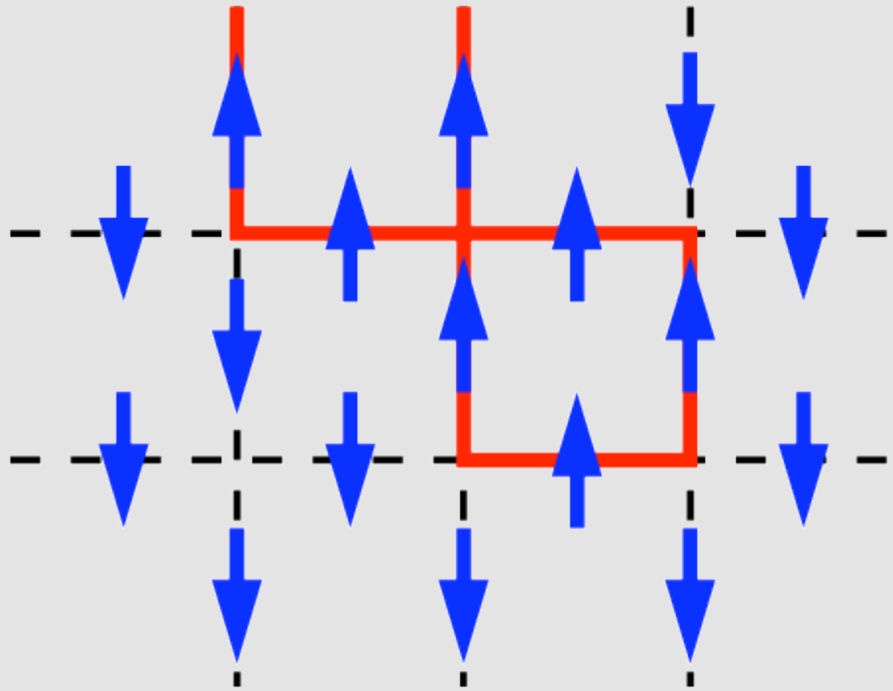

  - Hence, F_p's and A_i's form a set of conserved quantities.

A Simple Toy Model (A. Kitaev, 1997)

- Spins S_α living on the links of a square lattice:

$$H = -J_1 \sum_i A_i - J_2 \sum_p F_p$$

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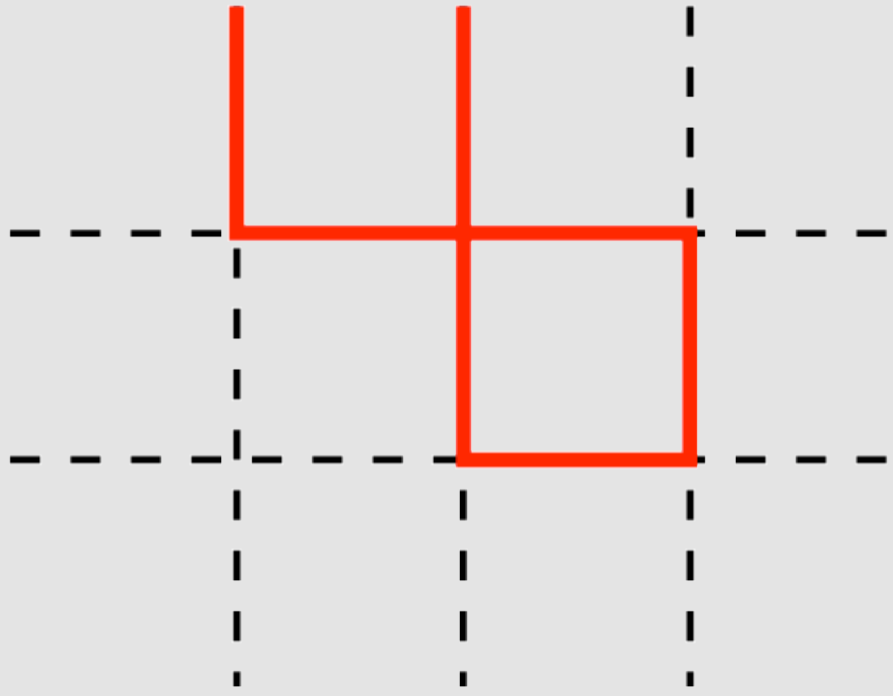

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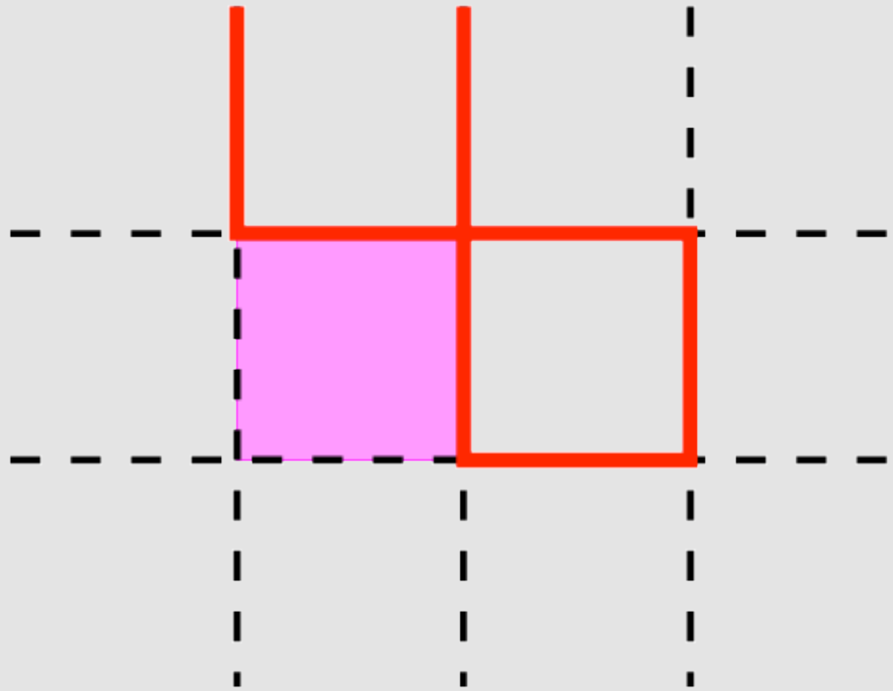

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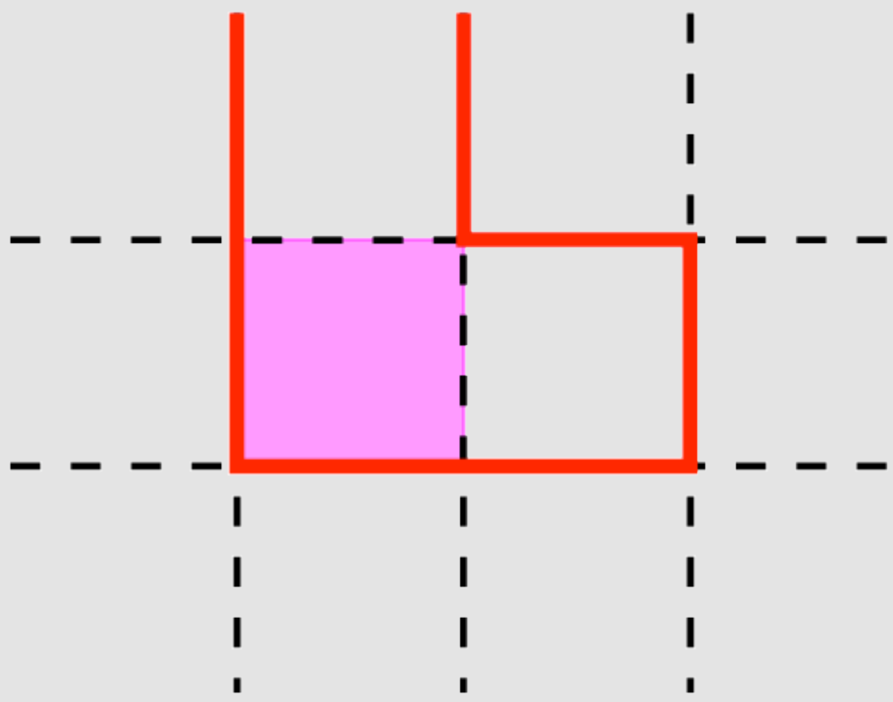

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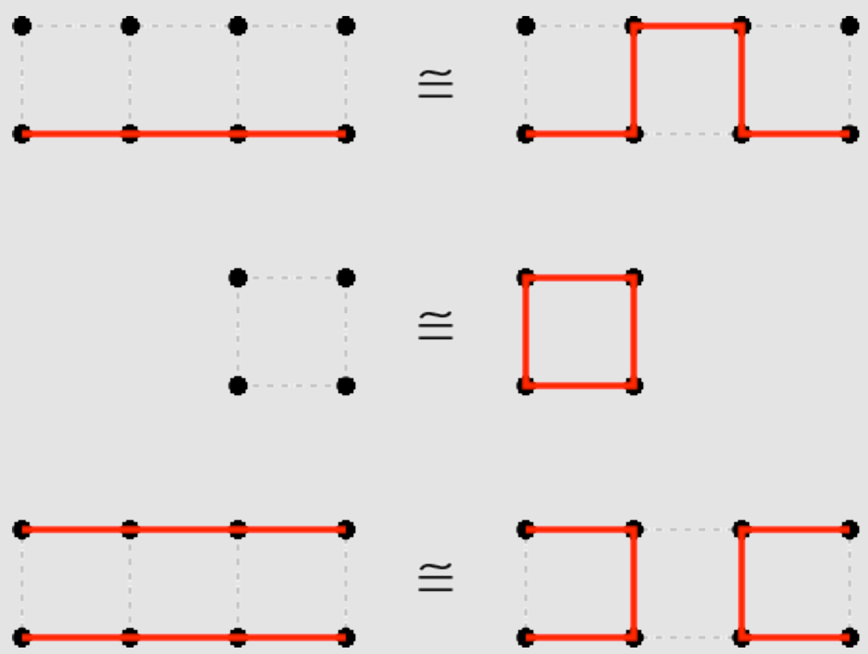
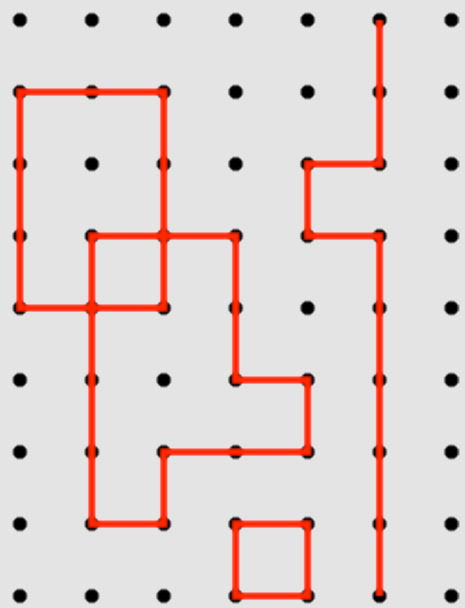
Properties of the Ground State

- Ground state: all A_i=1, F_p=1
- Pictorial representation: color each link with an up-spin.
- A_i=1 : closed loops.
- F_p=1 : every plaquette is an equal-amplitude superposition of inverse images.

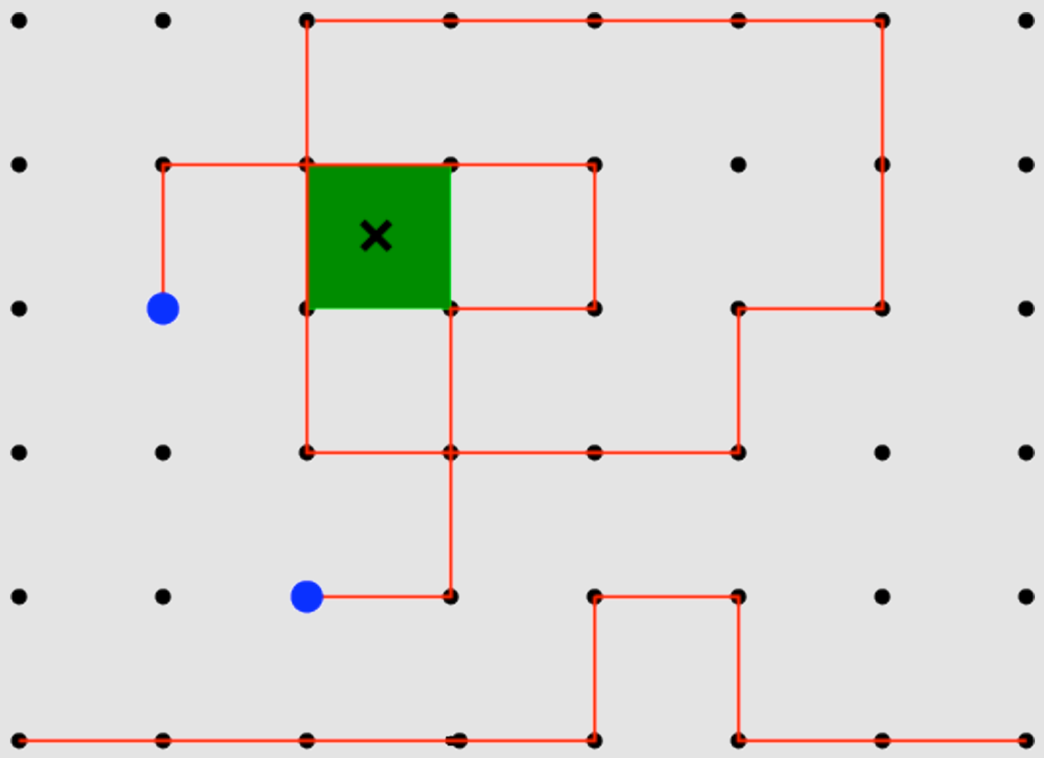




The GS wavefunction takes the same value on configurations connected by these operations. It does not depend on the geometry of the configurations, only on their topology.

Properties of Excitations

- “Electric” particle, or $A_i = -1$ – endpoint of a line
- “Magnetic particle”, or *vortex*: $F_p = -1$ – a “flip” of this plaquette changes the sign of a given term in the superposition.
- Charges and vortices interact via topological Aharonov-Bohm interactions.



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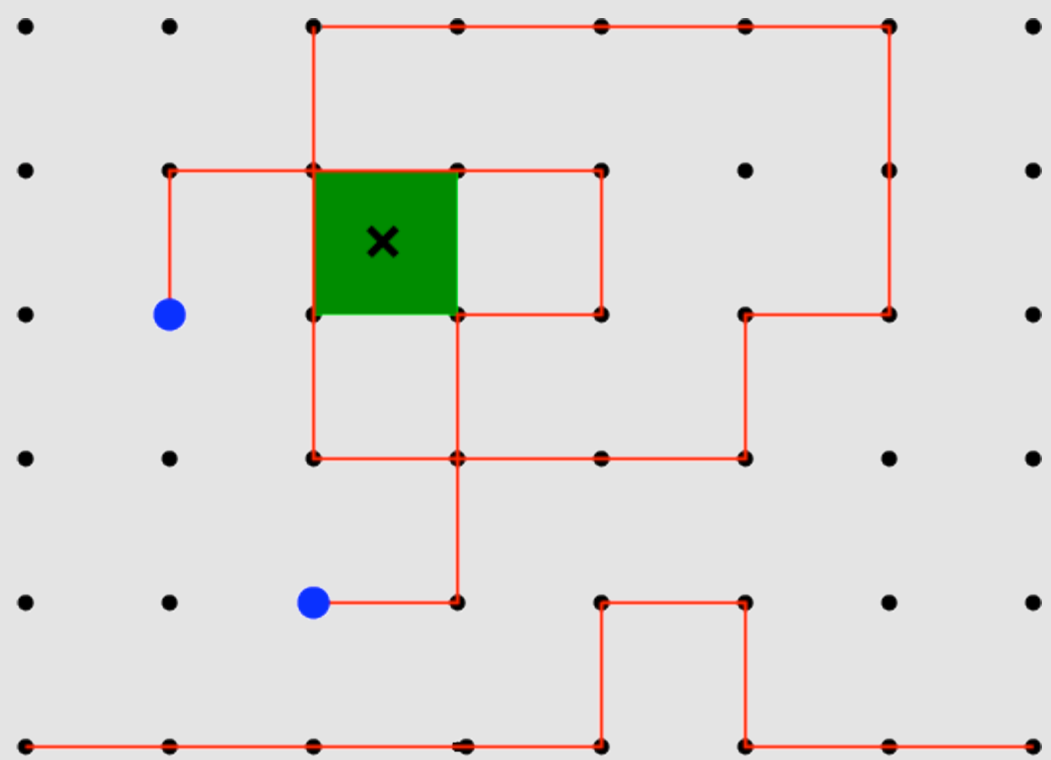

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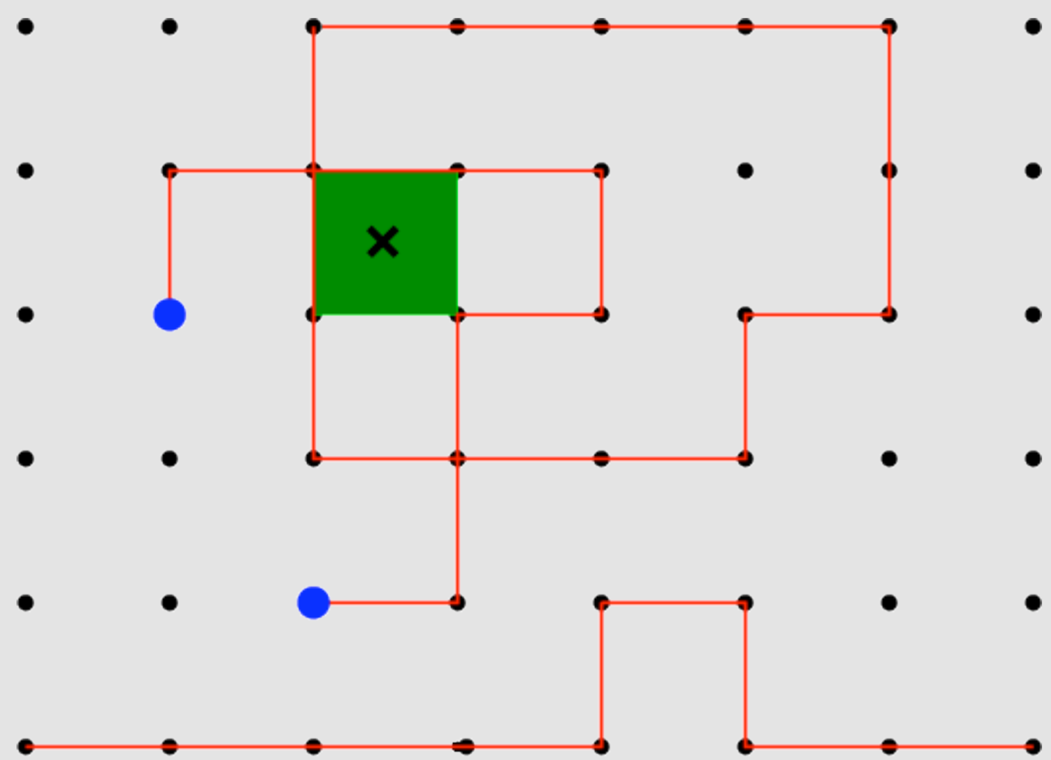

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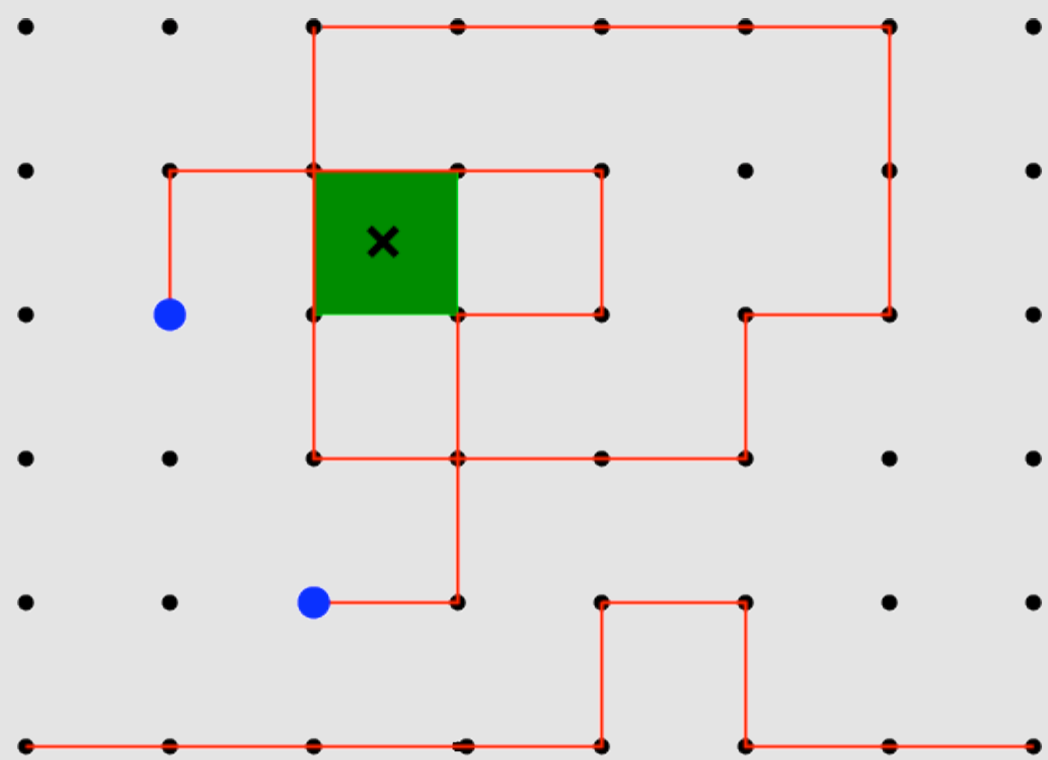
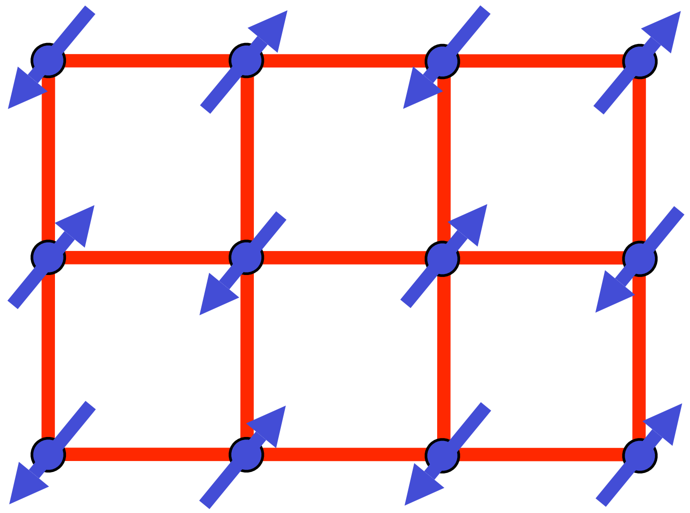


- *Spinons and visons are mutual semions*

Global phase diagram



$\langle z_\alpha \rangle \neq 0 \, , \; \langle \Phi \rangle = 0$

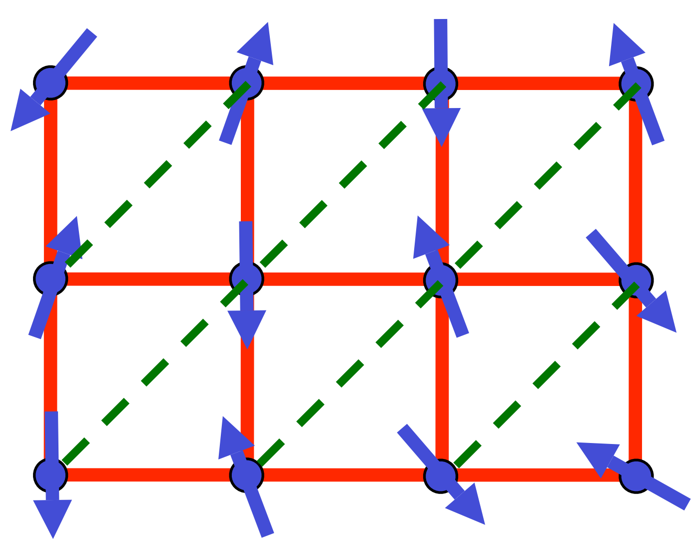
Néel state

Spin liquid with a **"photon"** collective mode

[Unstable to valence bond solid (VBS) order]

$\langle z_\alpha \rangle = 0 \, , \; \langle \Phi \rangle = 0$

s



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non-collinear Néel state

Z_2 spin liquid with a **vison** excitation

$\langle z_\alpha \rangle = 0 \, , \; \langle \Phi \rangle \neq 0$

$\widetilde{s}$

S. Sachdev and N. Read, *Int. J. Mod. Phys B* **5**, 219 (1991), cond-mat/0402109

Mutual Chern-Simons Theory

Express theory in terms of the physical excitations: the spinons, z_α, and the visons. After accounting for Berry phase effects, the visons can be described by a complex field v, which transforms non-trivially under the square lattice space group operations.

The spinons and visons have mutual semionic statistics, and this leads to the continuum theory:

$$\begin{aligned}\mathcal{S} &= \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + \ldots \\ &\quad + \widetilde{c}^2 \left|(\nabla_x - iB_x) v\right|^2 + \left|(\partial_\tau - iB_\tau) v\right|^2 + \widetilde{s}\left|v\right|^2 + \ldots \\ &\quad + \frac{i}{\pi}\epsilon_{\mu\nu\lambda} B_\mu \partial_\nu A_\lambda \Big]\end{aligned}$$

Cenke Xu and S. Sachdev, to appear

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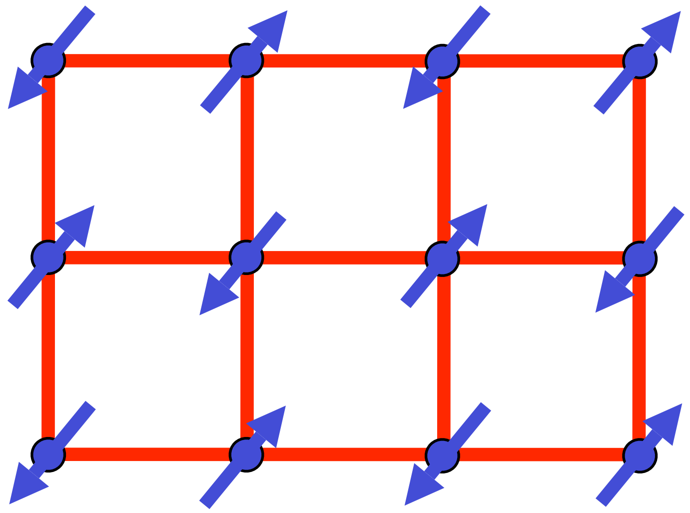
This theory fully accounts for all the phases, including their global topological properties and their broken symmetries. It also completely describe the quantum phase transitions between them.

Cenke Xu and S. Sachdev, to appear

Global phase diagram



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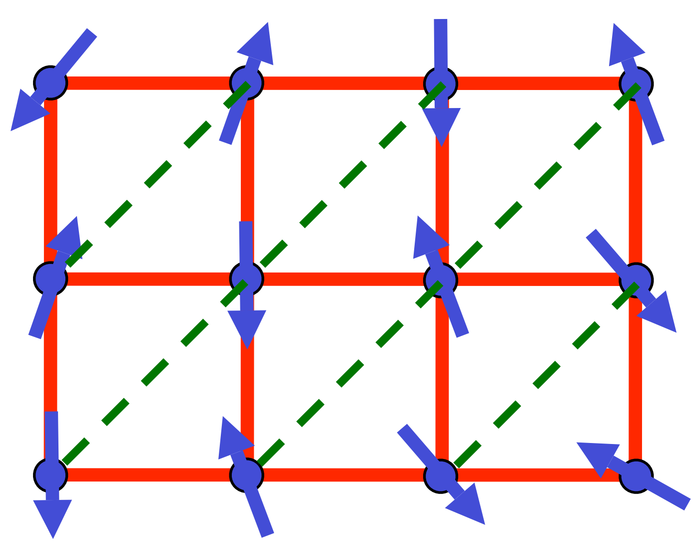

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$$+ \widetilde{c}^2 \left|(\nabla_x - iB_x) v\right|^2 + \left|(\partial_\tau - iB_\tau) v\right|^2 + \widetilde{s}\left|v\right|^2 + \ldots + \frac{i}{\pi}\epsilon_{\mu\nu\lambda} B_\mu \partial_\nu A_\lambda \Big]$$

Low energy states on a torus:

- Z_2 spin liquid has a 4-fold degeneracy.
- Non-collinear Néel state has low-lying tower of states described by a broken symmetry with order parameter S_3/Z_2.
- "Photon" spin liquid has a low-lying tower of states described by a broken symmetry with order parameter S_1/Z_2. This is the VBS order $\sim v^2$.
- Néel state has a low-lying tower of states described by a broken symmetry with order parameter $S_3 \times S_1/(U(1) \times U(1)) \equiv S_2$. This is the usual vector Néel order.

Cenke Xu and S. Sachdev, to appear