Strange metals and black holes

Bell Lecture

McGill University, Montreal
February 22, 2019

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Ordinary metals:
quasiparticles

Strange metals:
no quasiparticles

Black
holes

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quasiparticles

Strange metals:

no quasiparticles

Black

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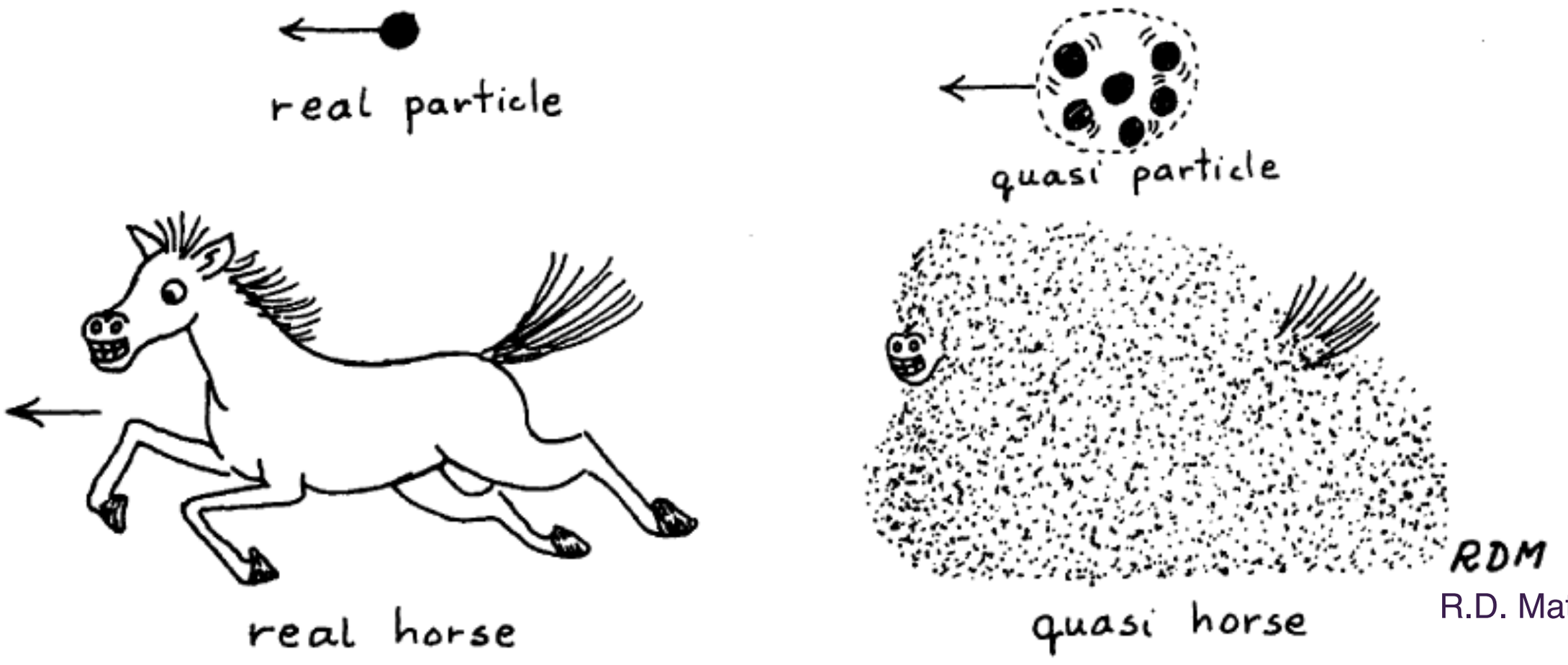
Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

What are quasiparticles ?

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \dots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

What are quasiparticles ?

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

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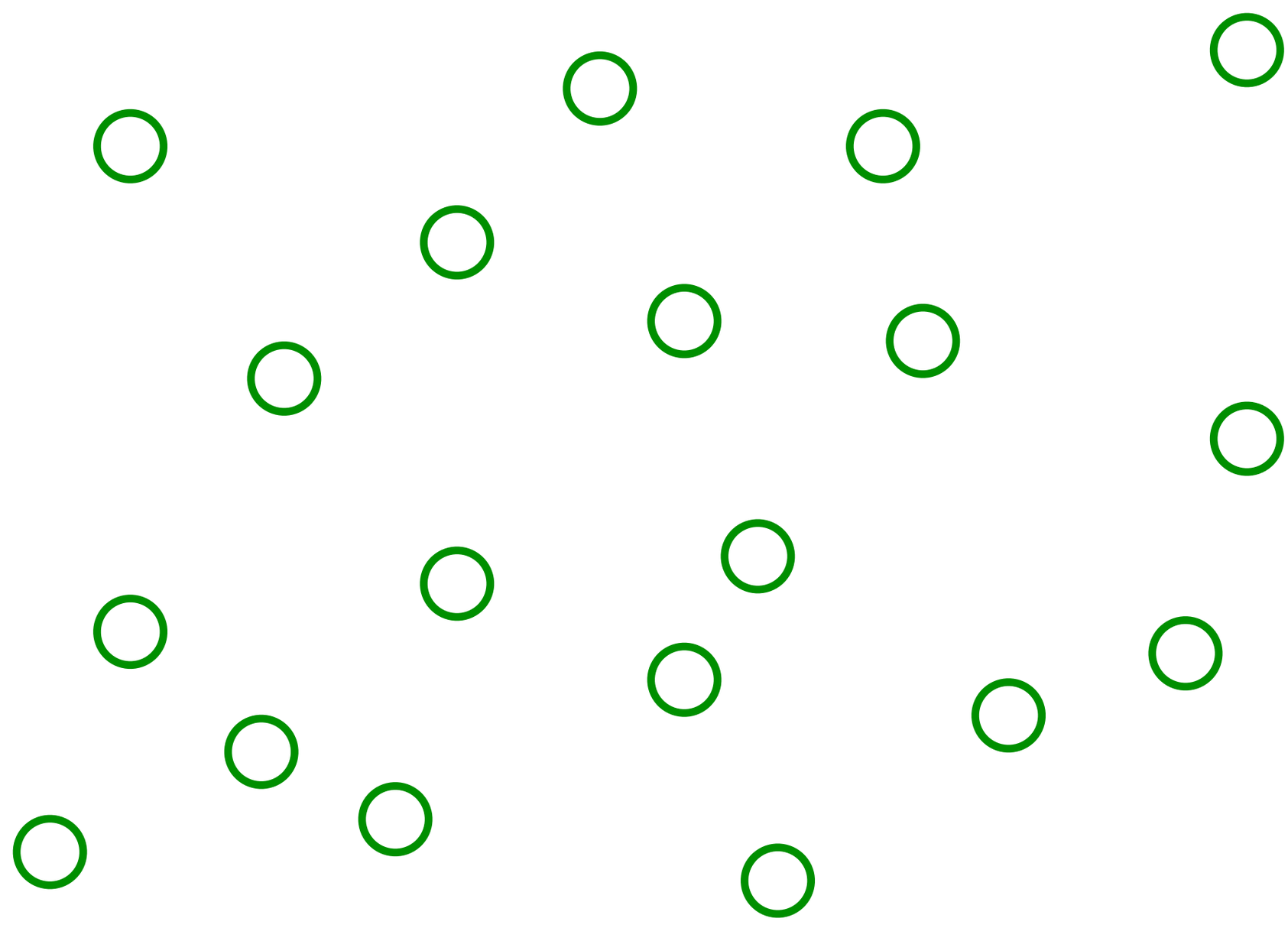
- This time is much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

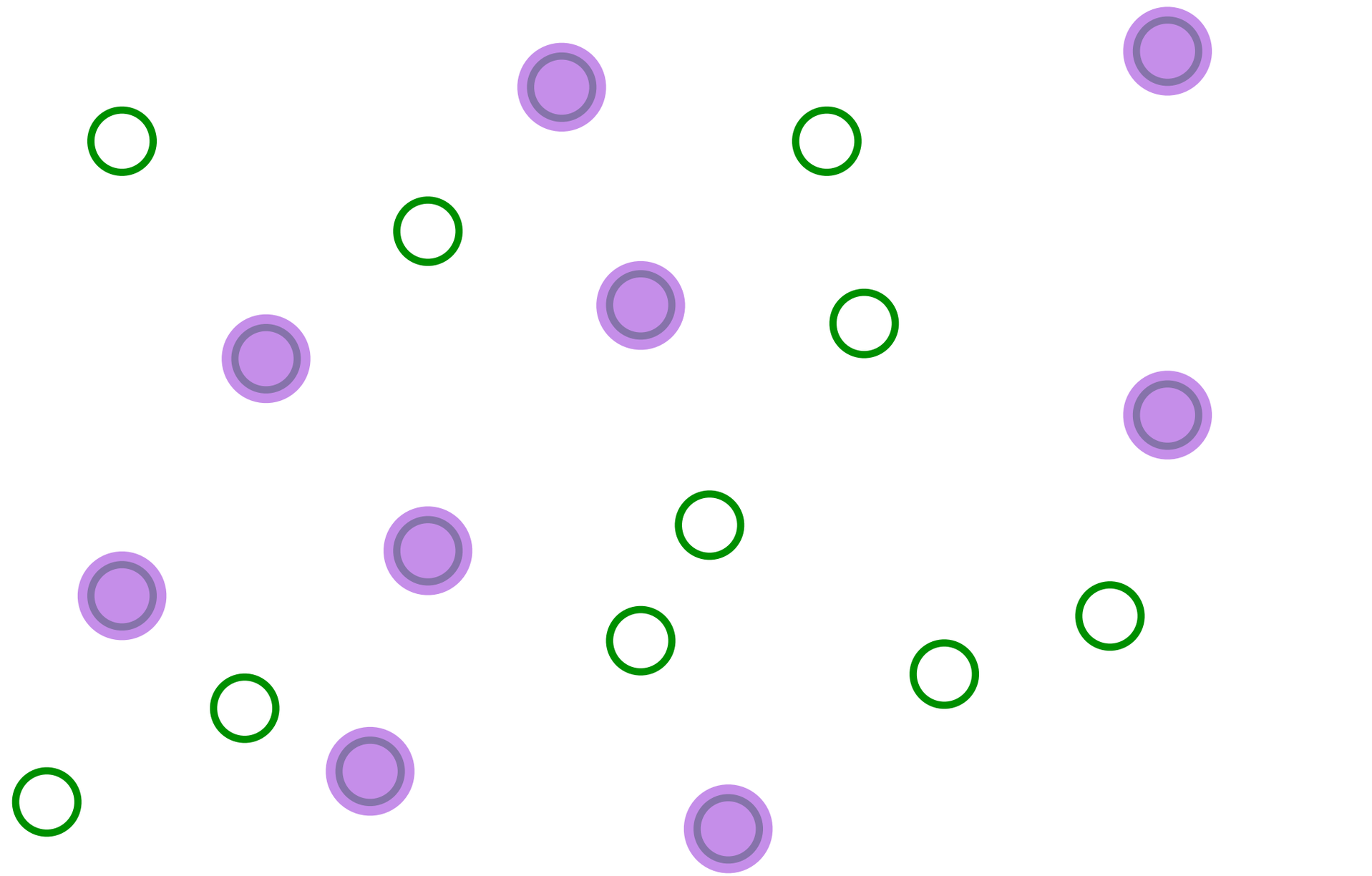
$$\tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A simple model of a metal with quasiparticles



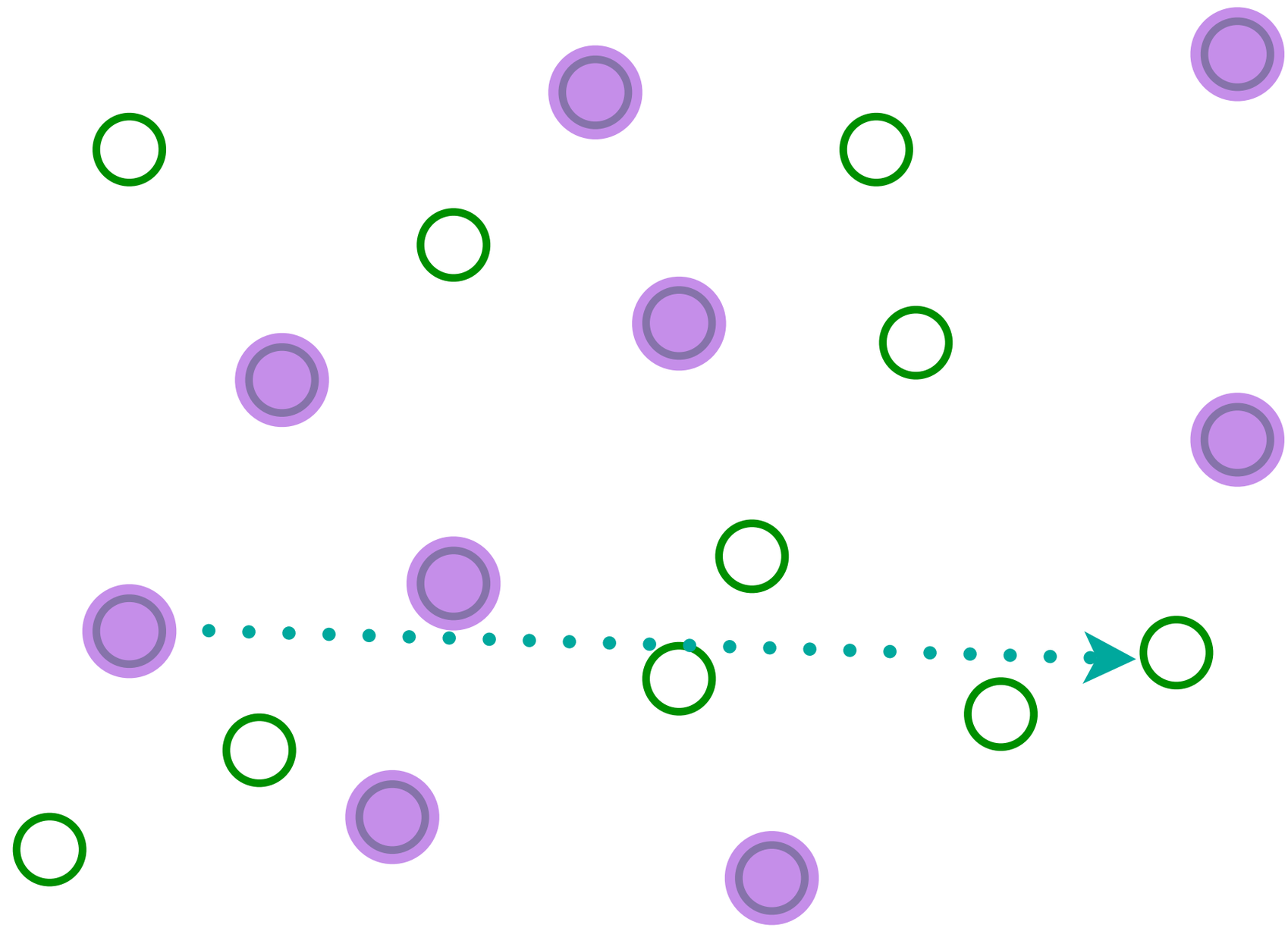
Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

A simple model of a metal with quasiparticles



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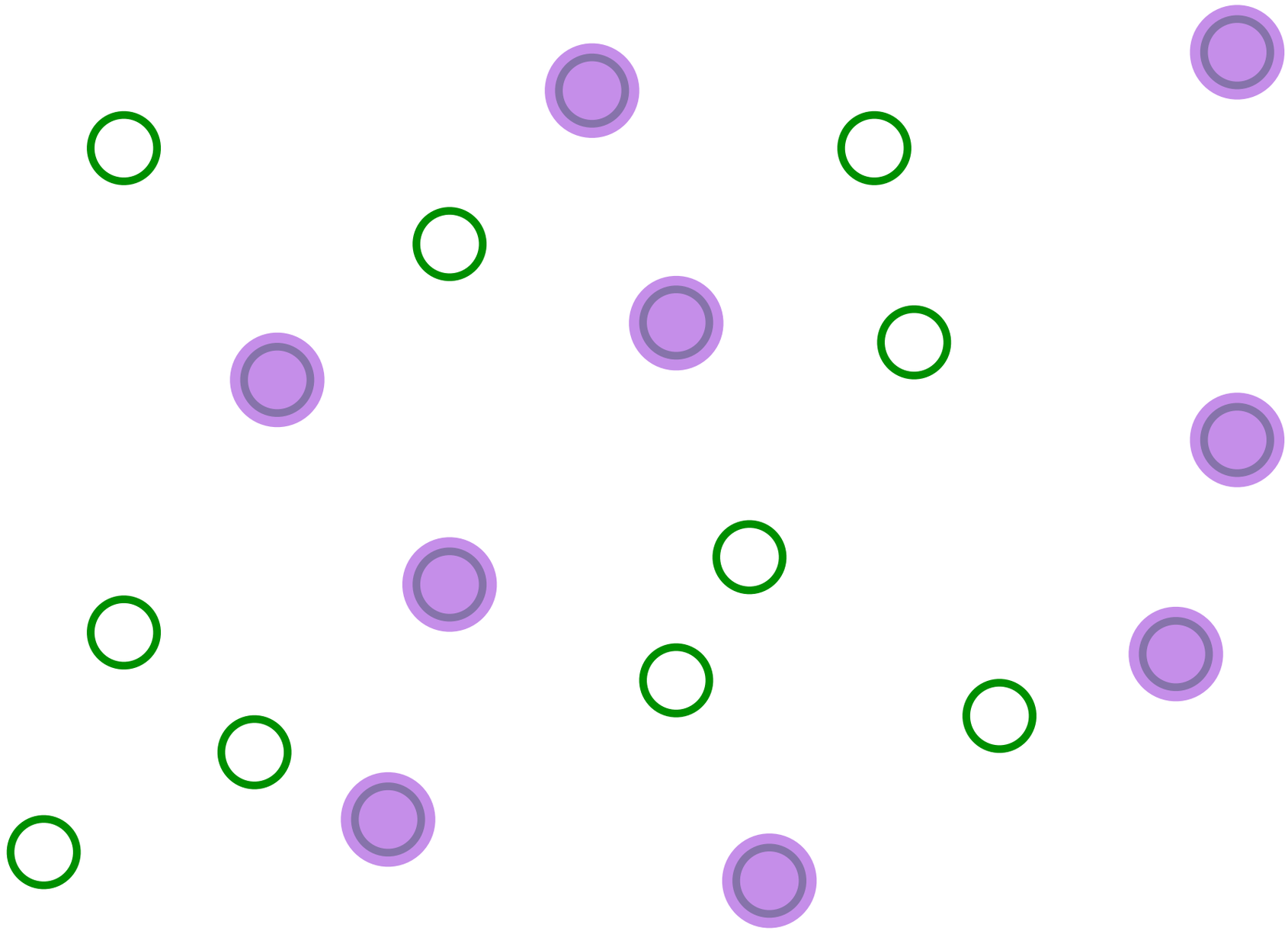
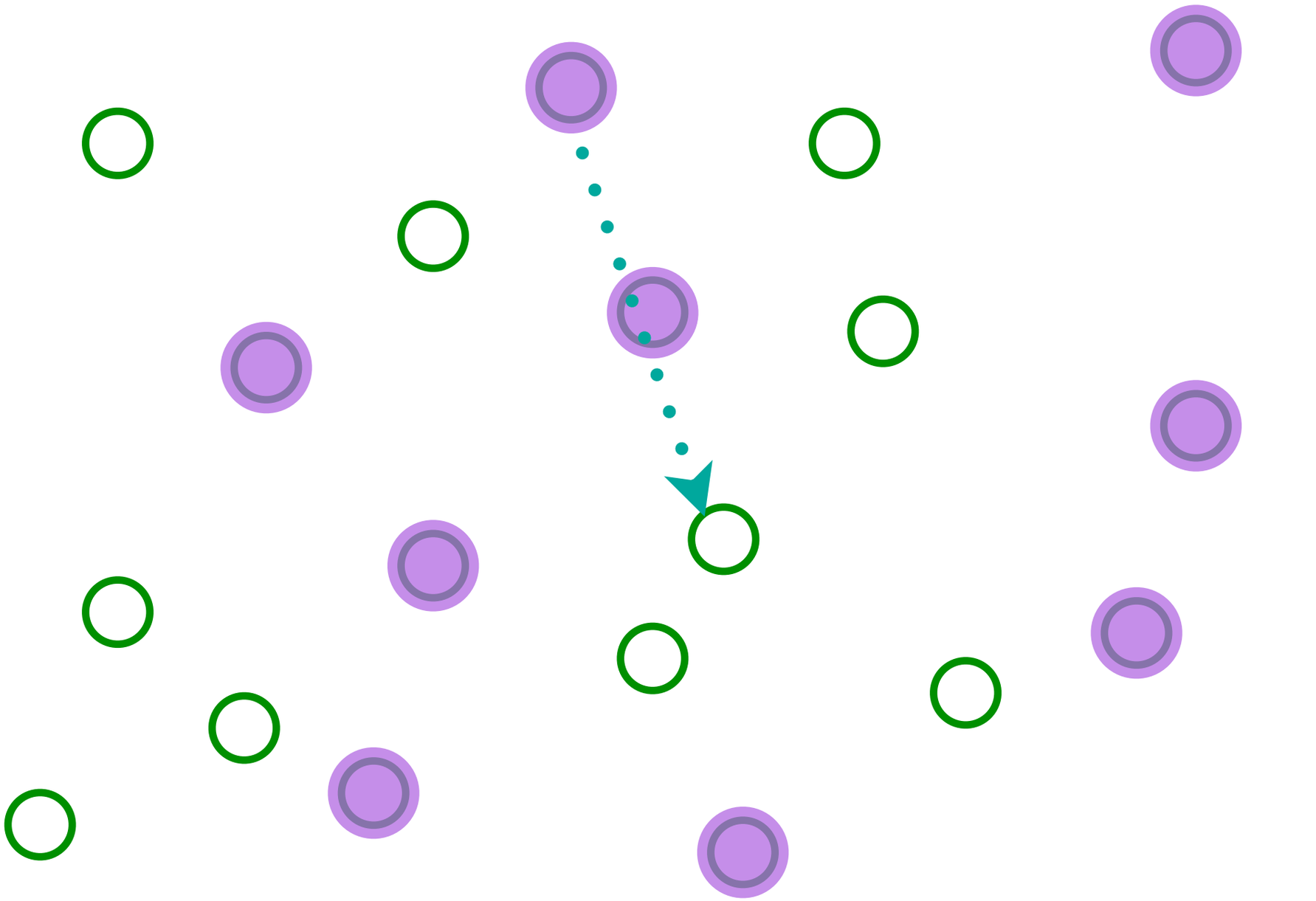


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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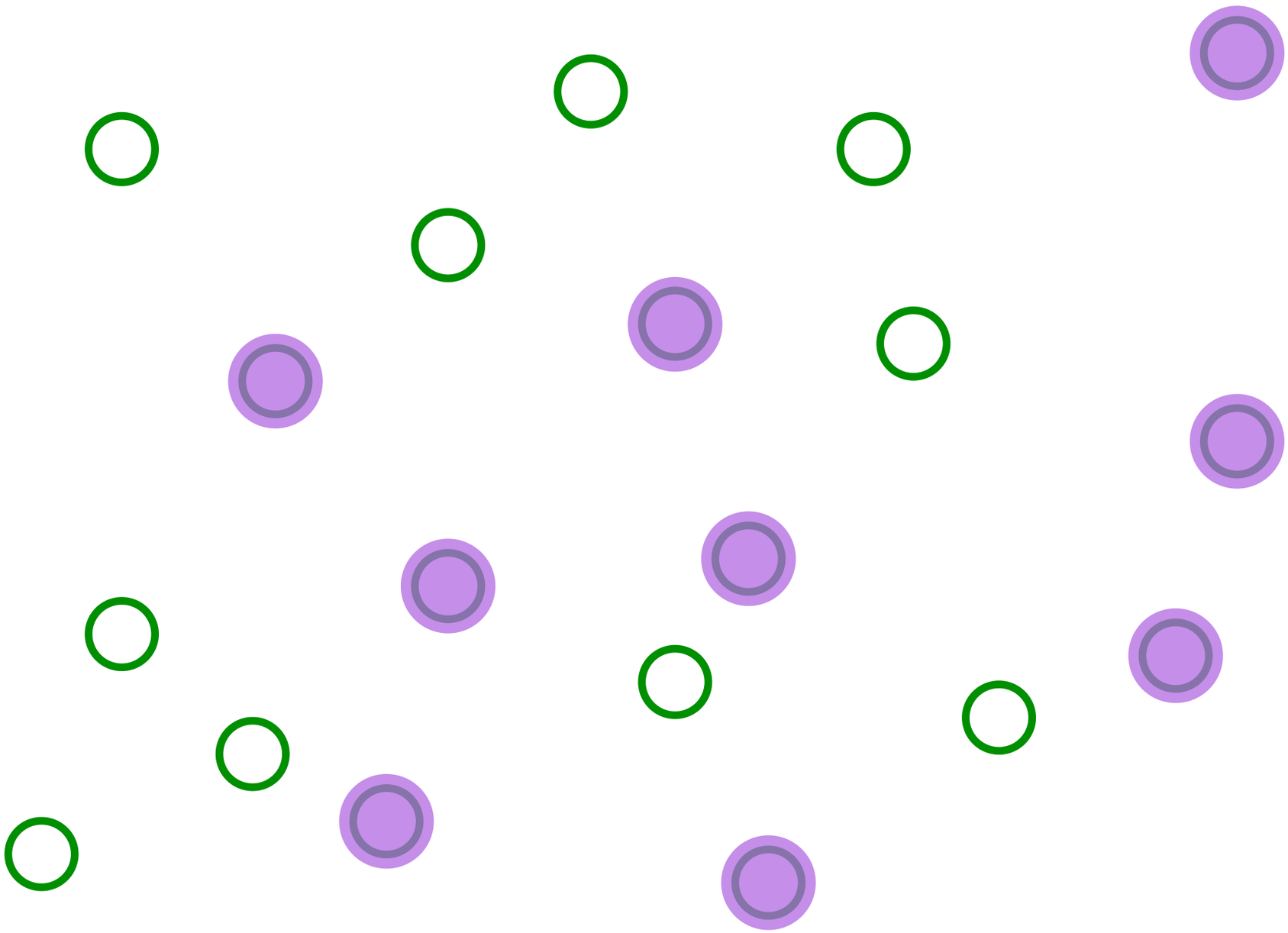

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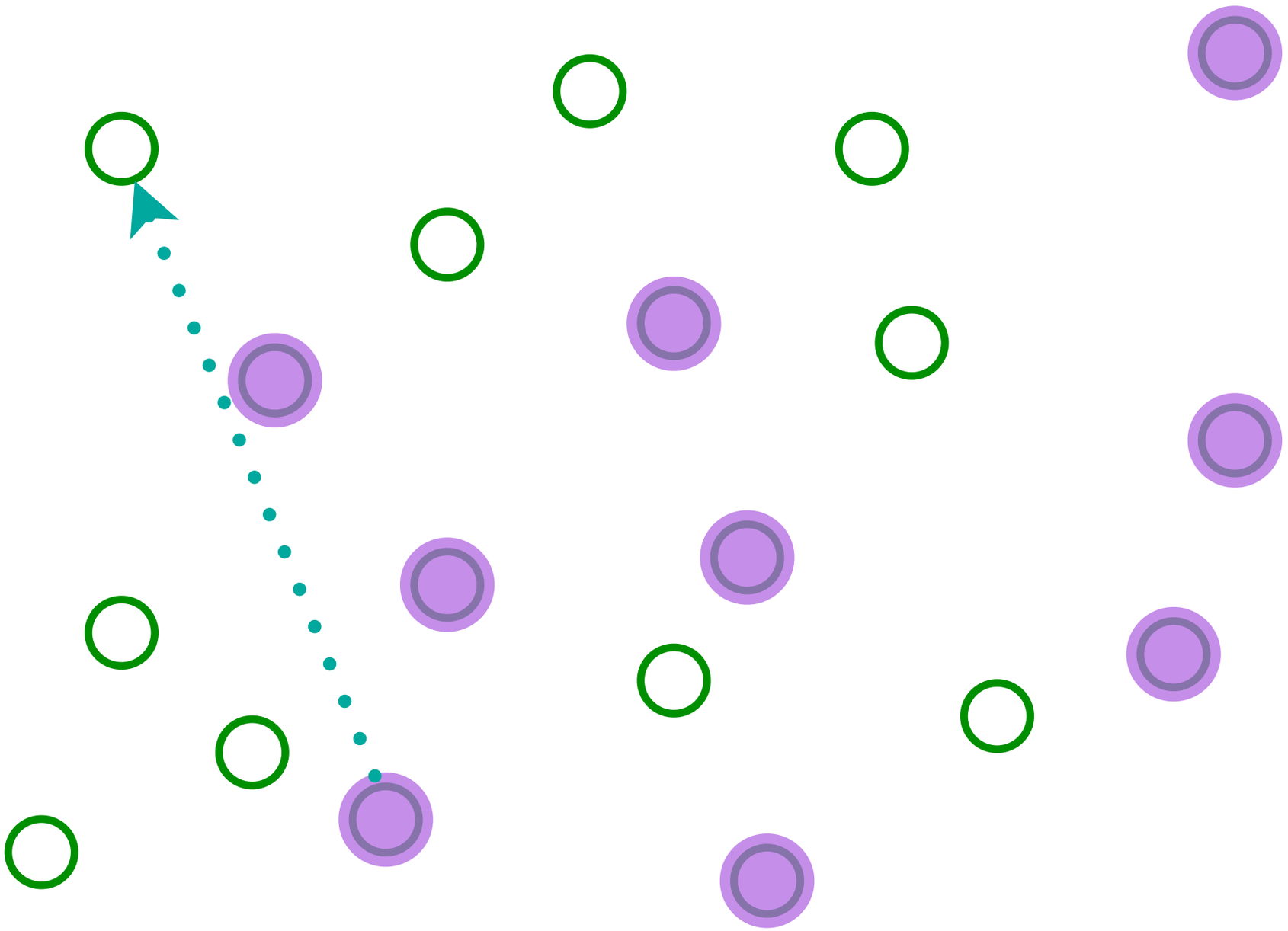

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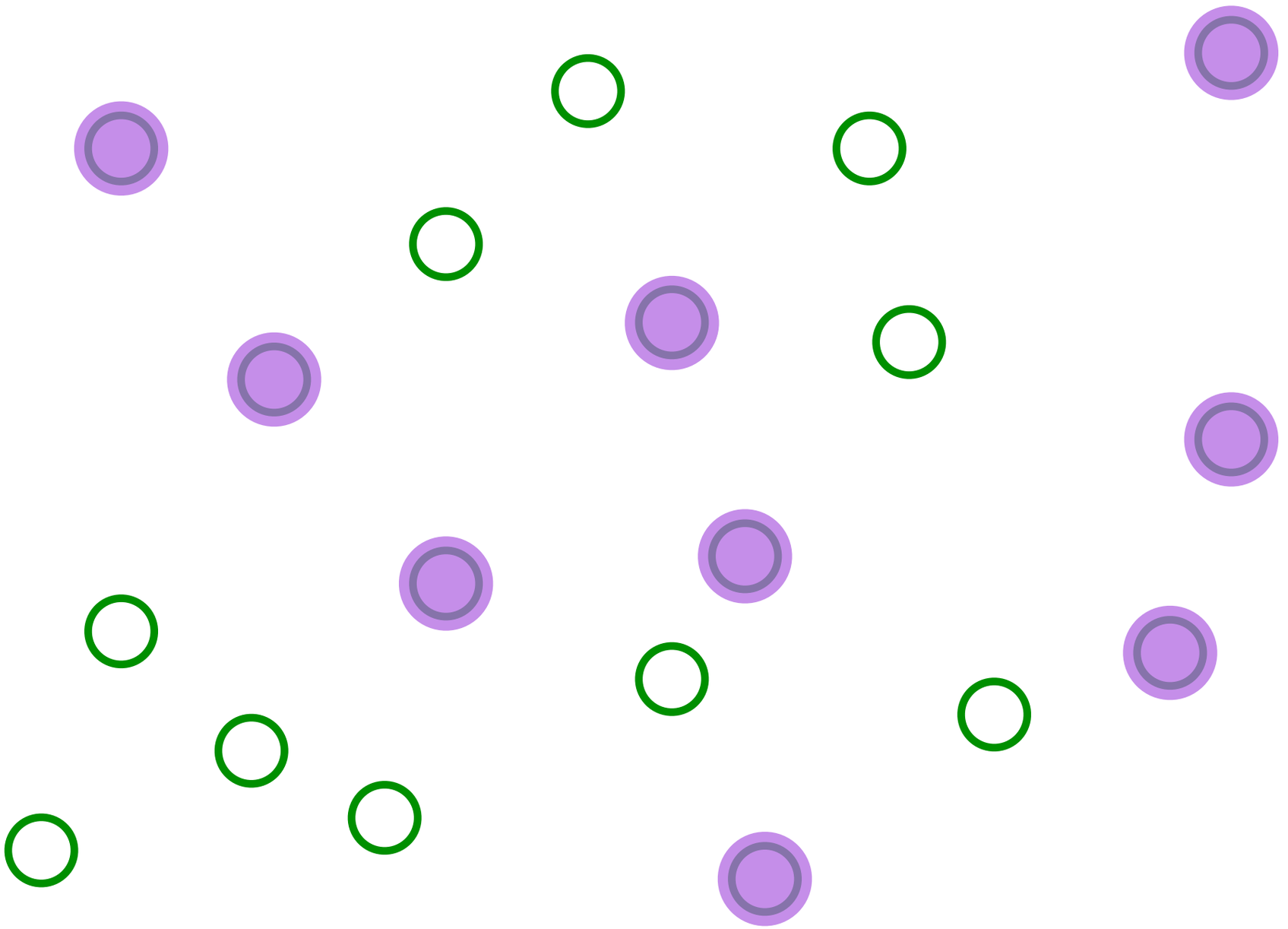

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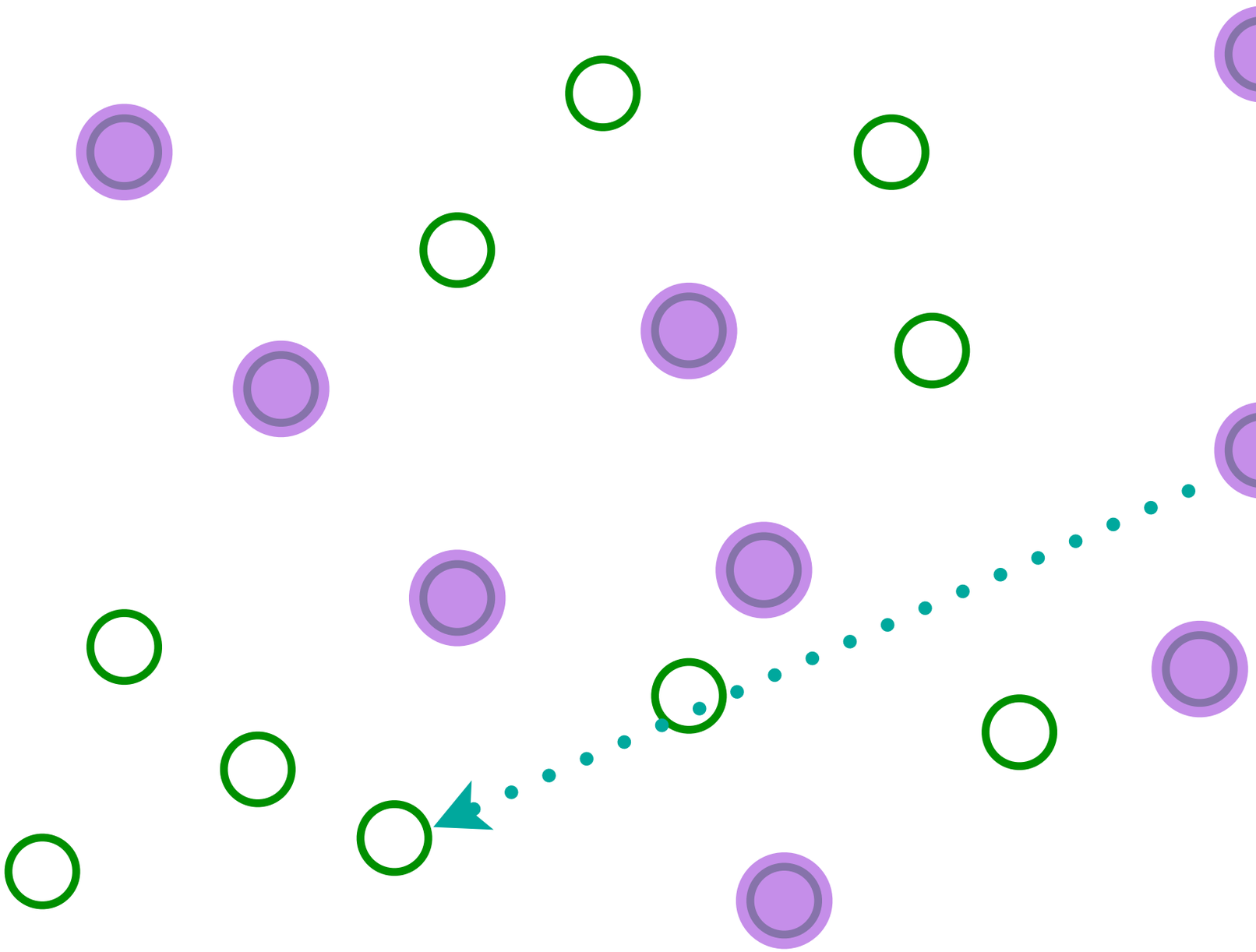

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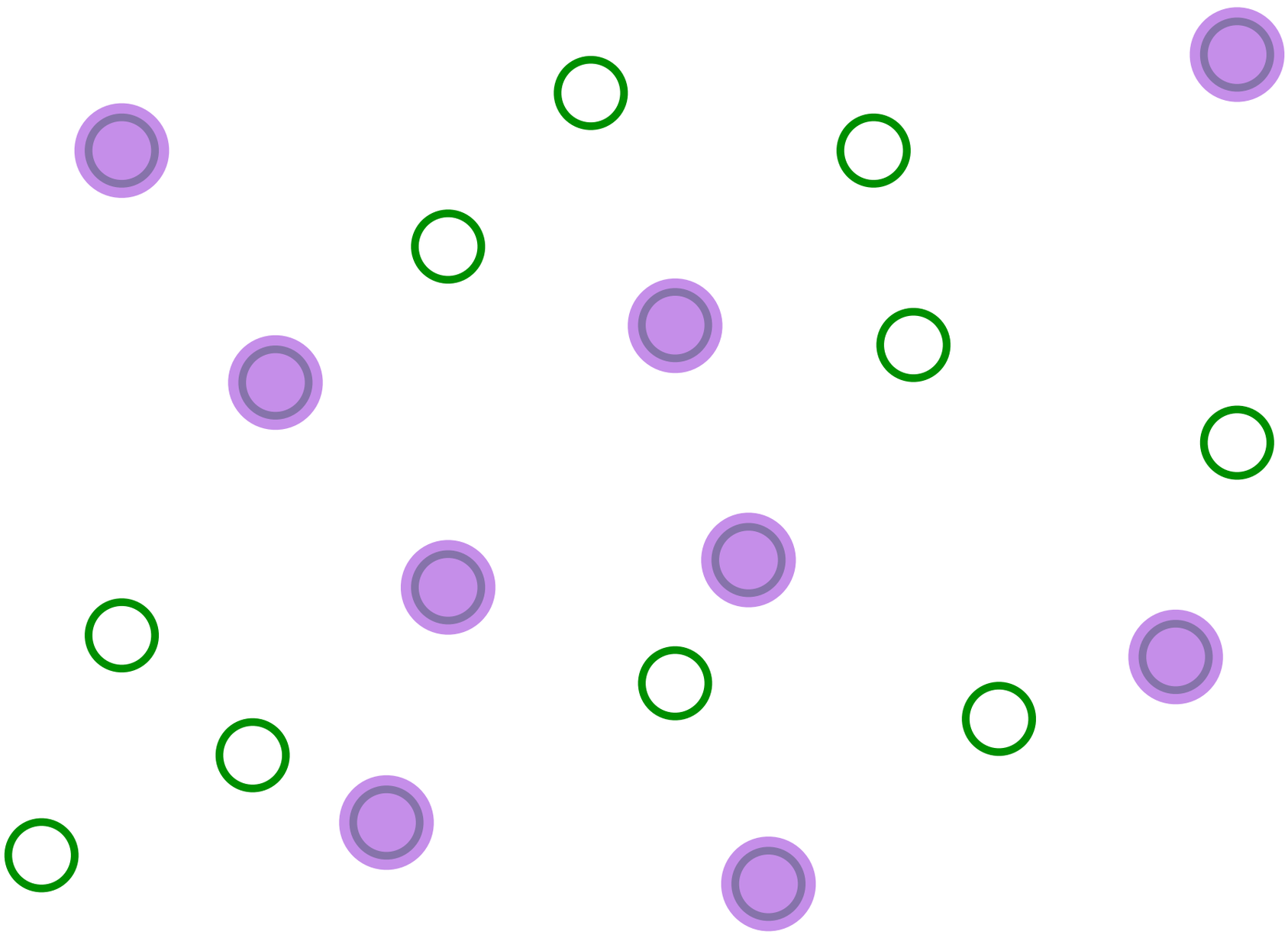


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

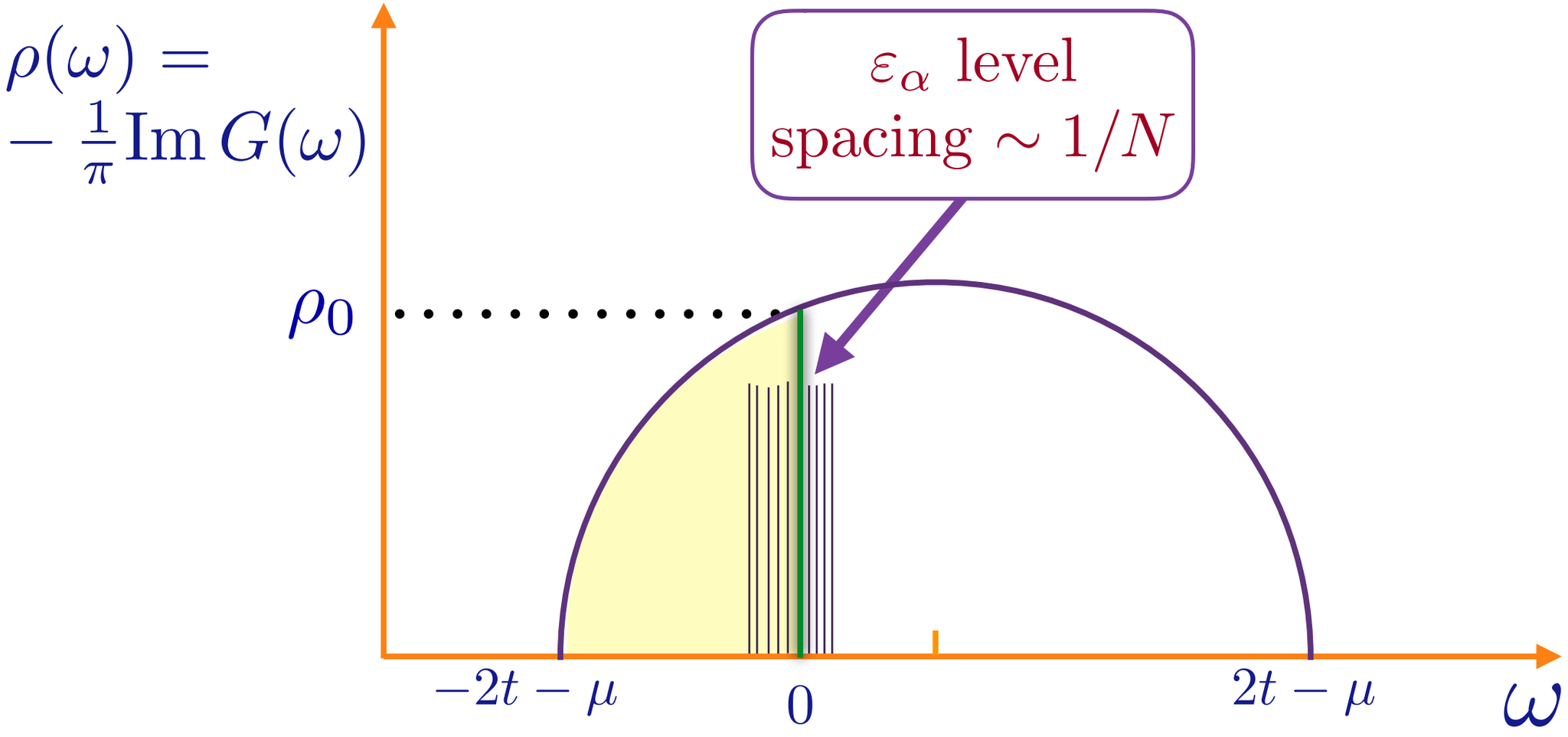
$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a
N x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The single-particle density of states is
$\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$, and $\rho_0 \equiv \rho(\omega = 0)$.



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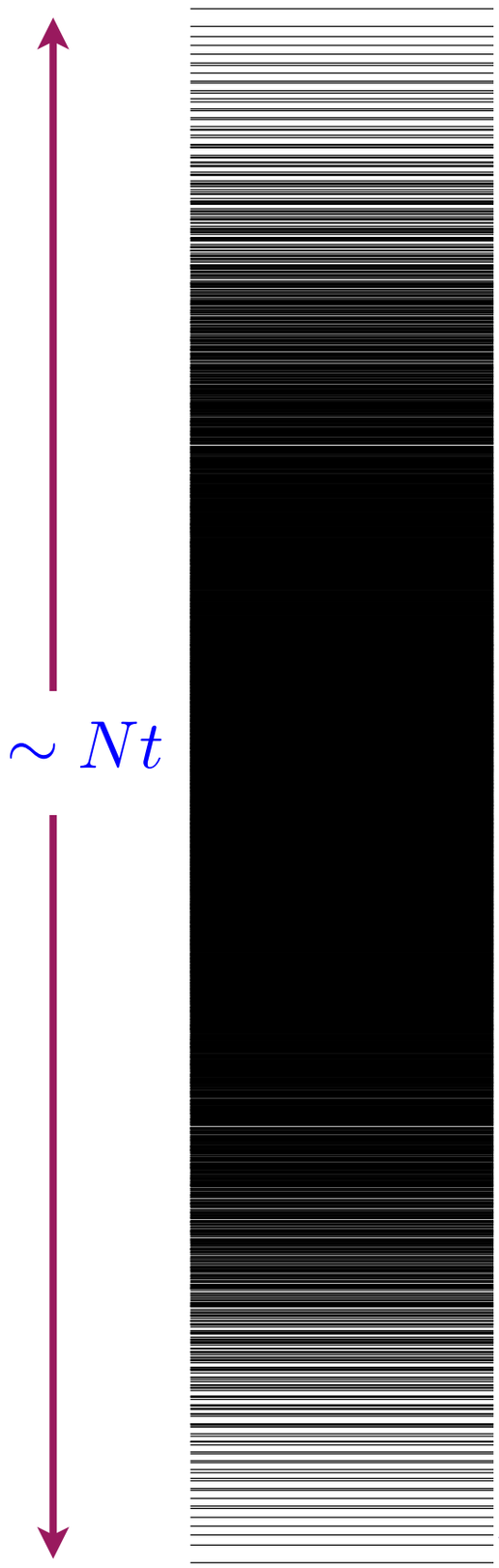

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

$$\begin{aligned}\frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta))\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy, and $f(\epsilon) = 1/(e^{\epsilon/T} + 1)$ is the Fermi function.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

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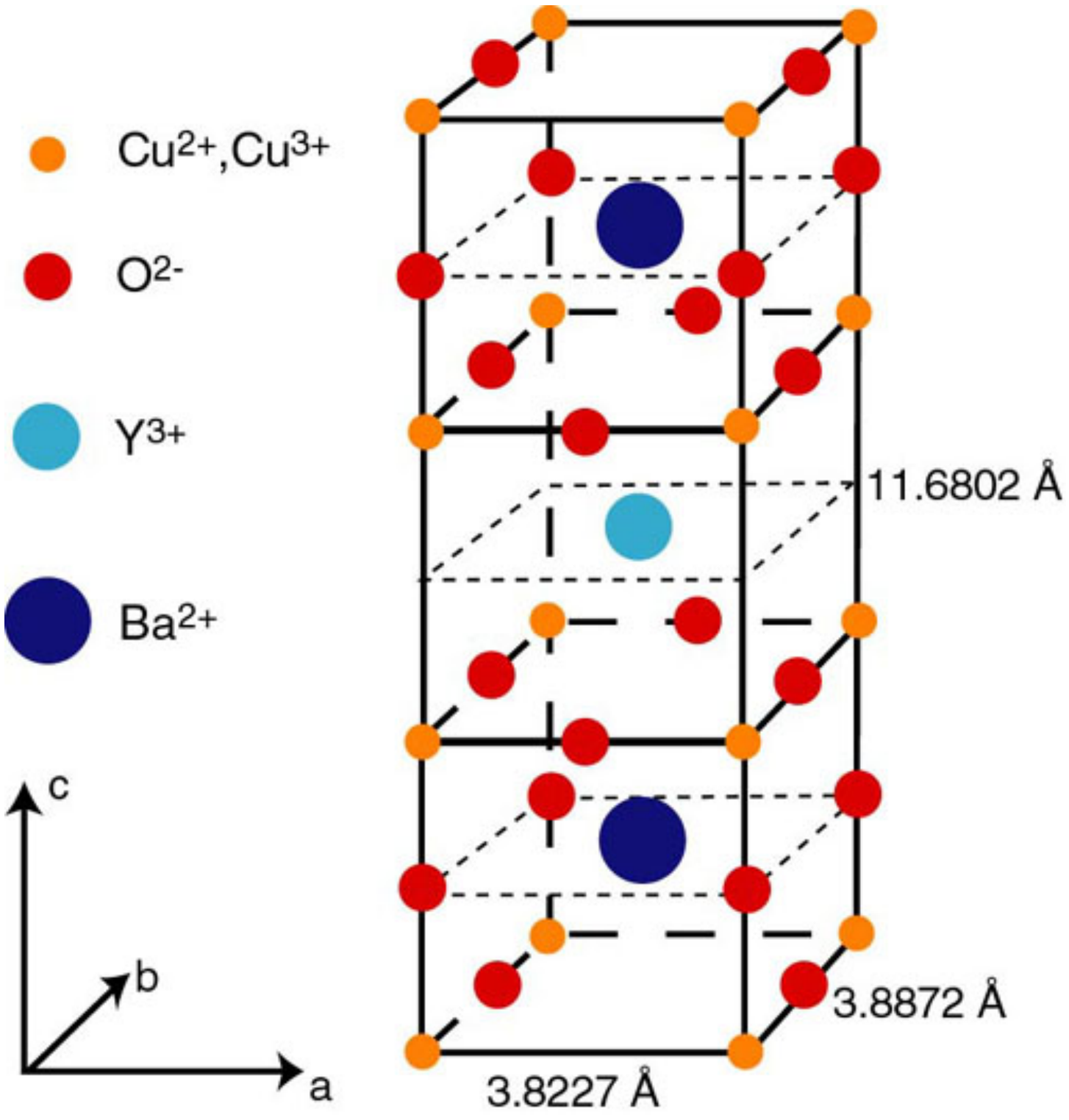
quasiparticles

Strange metals:
no quasiparticles

Black
holes

High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

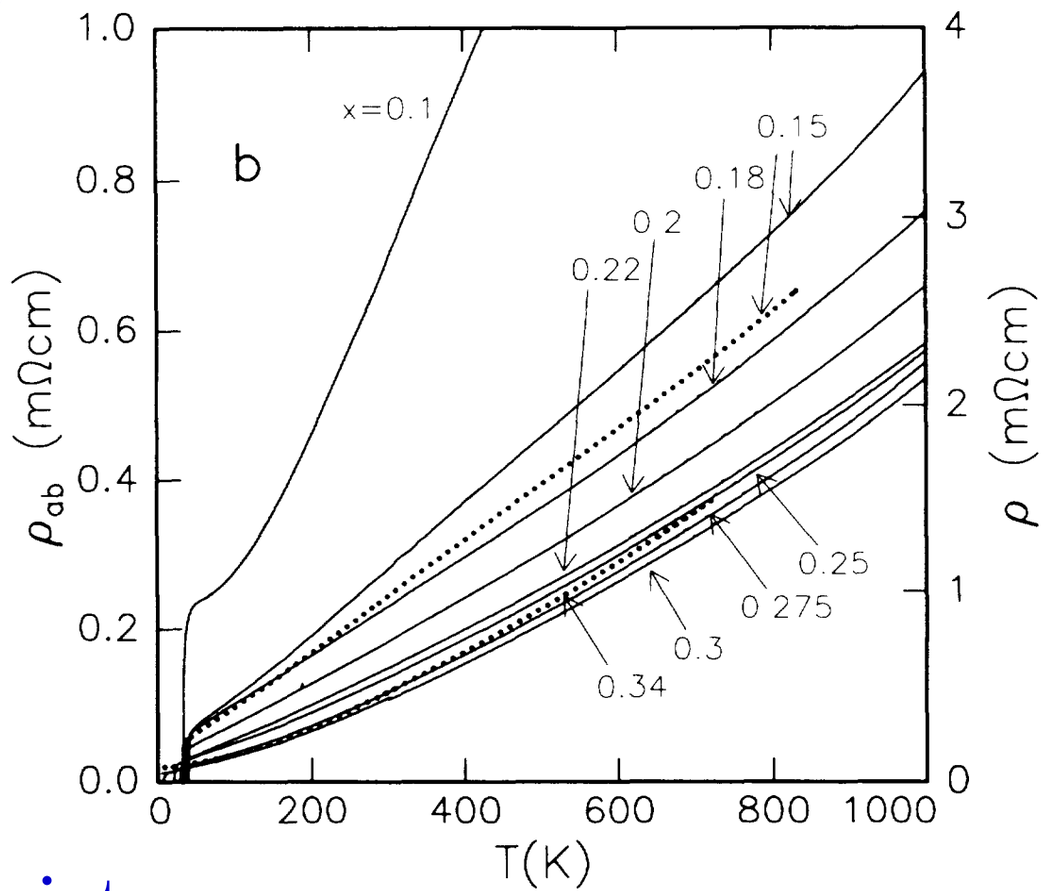
"Strange",

"Bad",

or "Incoherent",

metal found ubiquitously at temperatures
$T > T_c$ (the superconducting critical temperature)
has a resistivity, ρ, which obeys

H. Takagi, B. Batlogg,
H. L. Kao, J. Kwo,
R. J. Cava,
J. J. Krajewski, and
W. F. Peck, Jr.,
Phys. Rev. Lett. **69**,
2975 (1992)



$$\rho \sim T,$$

and

in some cases $\rho \gg h/e^2$

(in two dimensions),
where h/e^2 is the quantum unit of resistance.

Remarkable recent observation of
'Planckian' strange metal transport in cuprates,
pnictides, magic-angle graphene, and
ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}$$

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Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142–147

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6]* and C. Proust[3,6]*

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arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,[1,2] Tristin Metz,[2] Christopher Eckberg,[2] Kevin Kirshenbaum,[2] Alex Hughes,[2] Renxiong Wang,[2] Limin Wang,[2] Shanta R. Saha,[2] I-Lin Liu,[2,3,4] Nicholas P. Butch,[2,4] Zhonghao Liu,[5,6] Sergey V. Borisenko,[5] Peter Y. Zavalij,[7] and Johnpierre Paglione[2,8]

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Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigordà,[1] Kenji Watanabe,[2] Takashi Taniguchi,[2] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,†]

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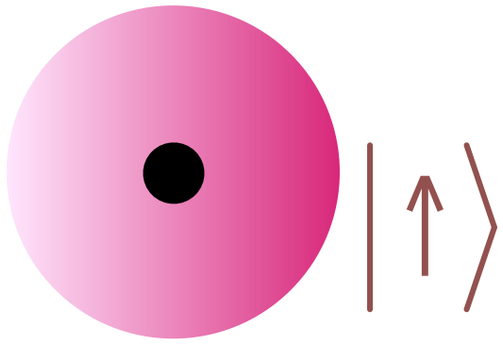
Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

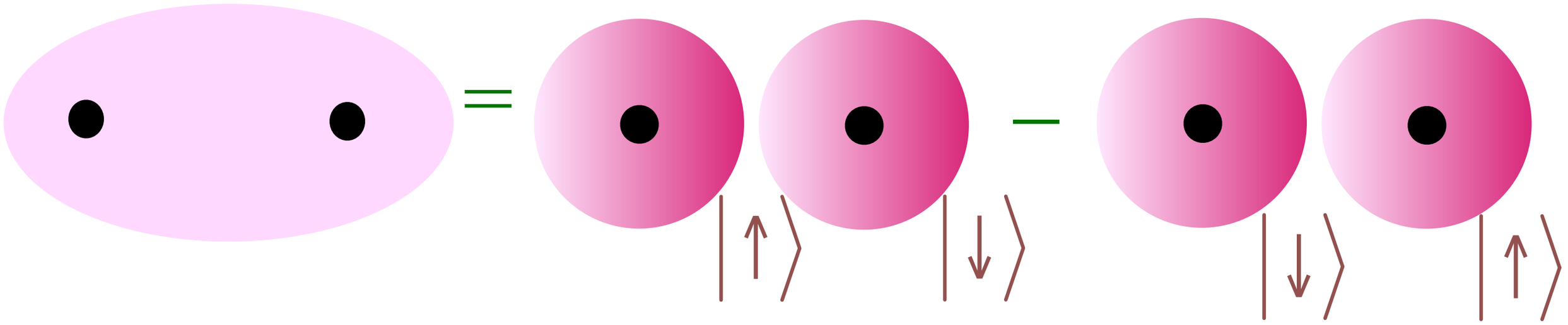
Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1†]

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

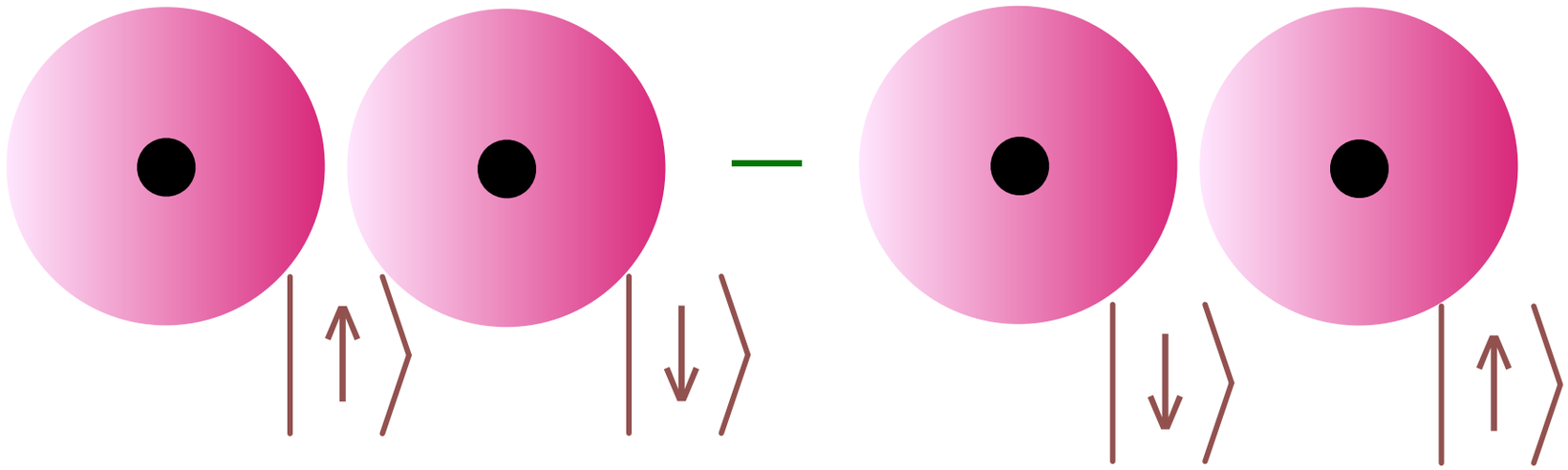


Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



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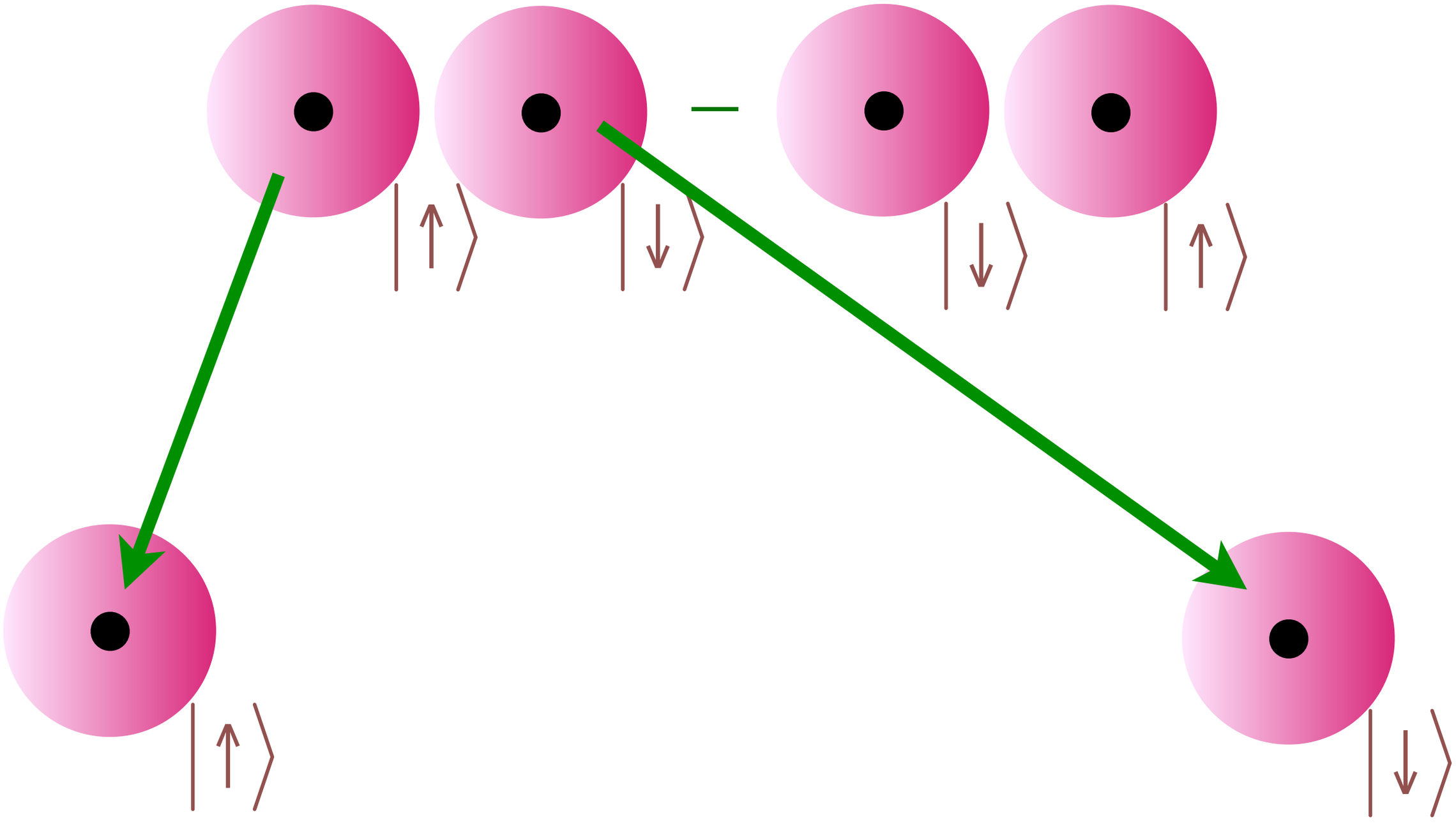

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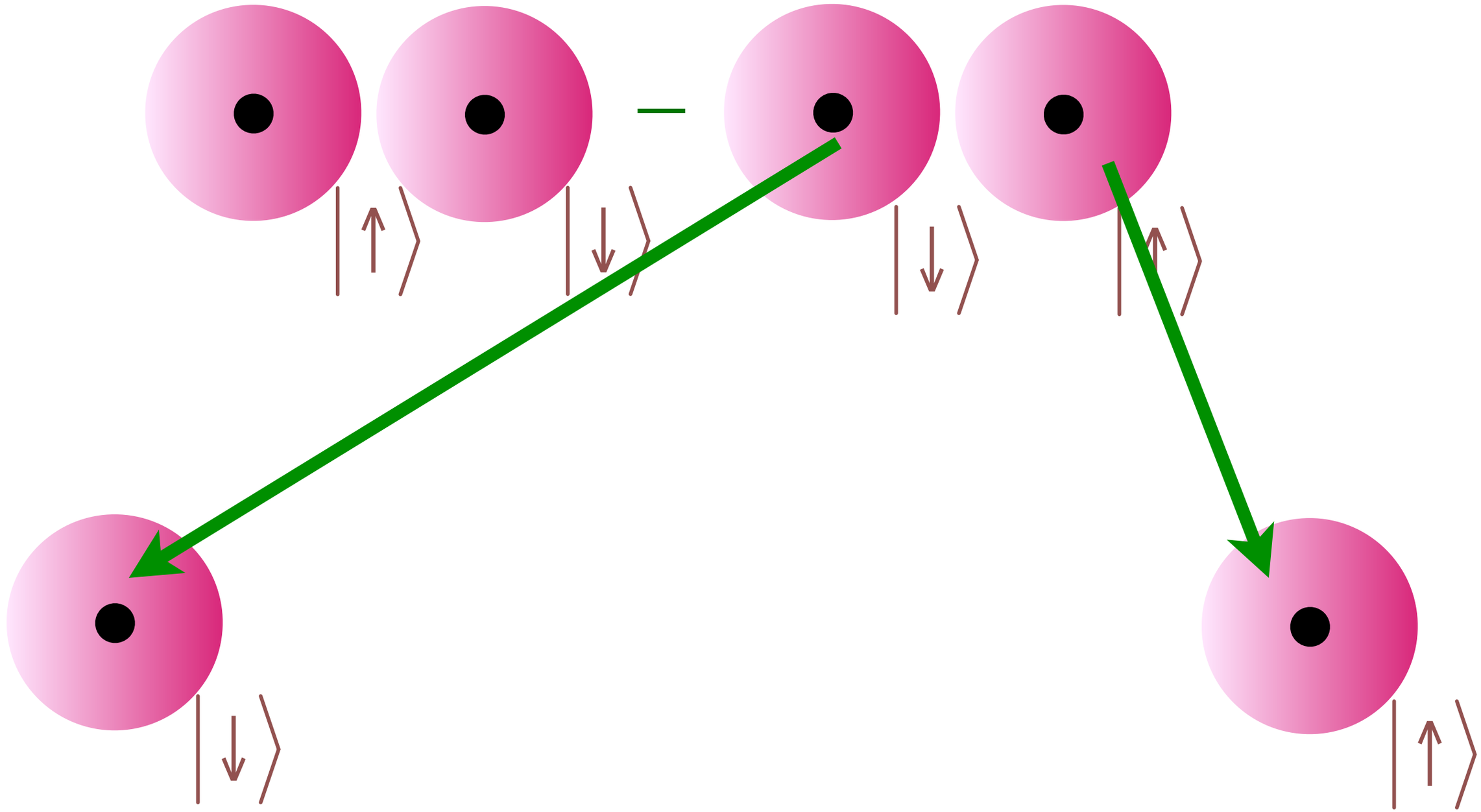

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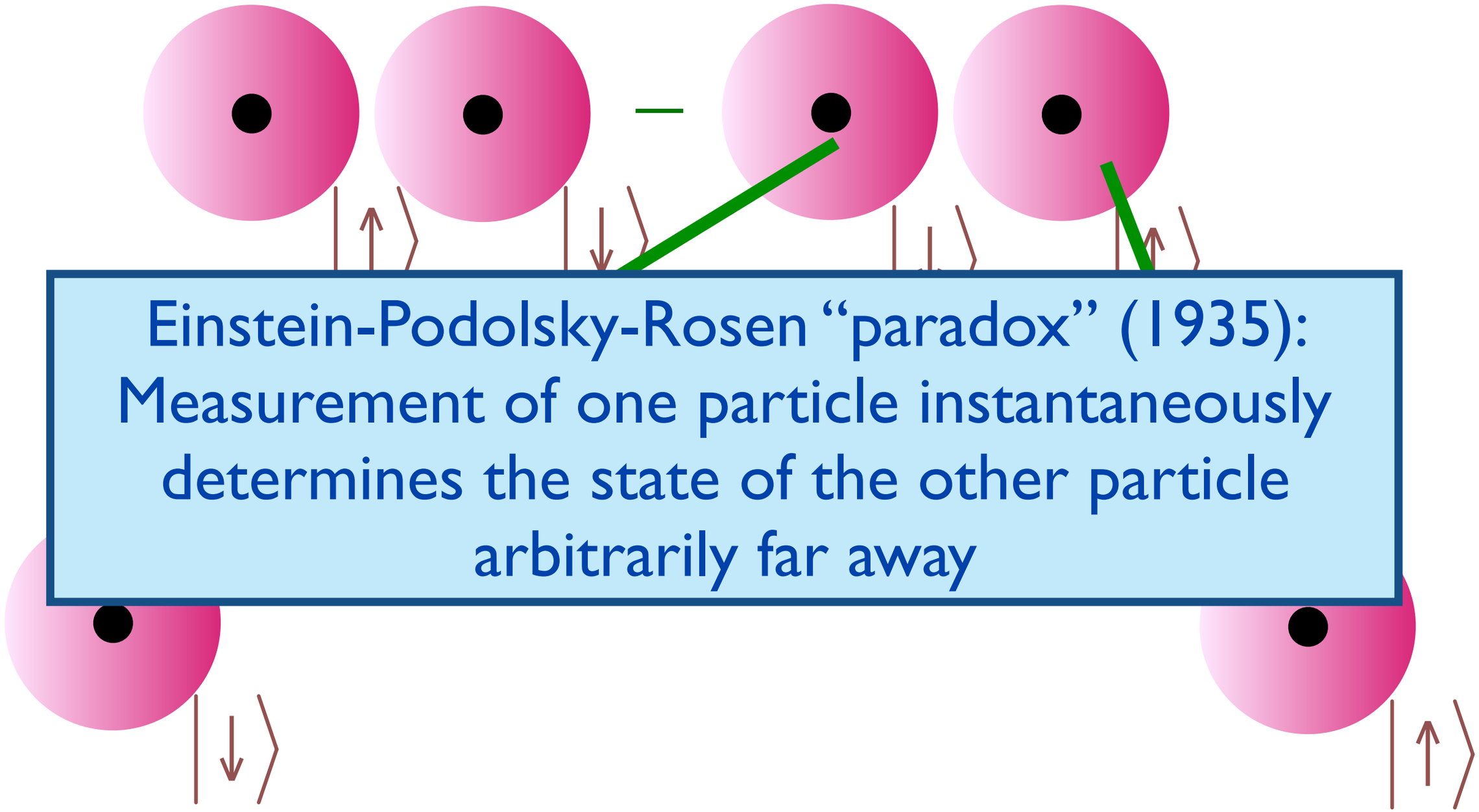
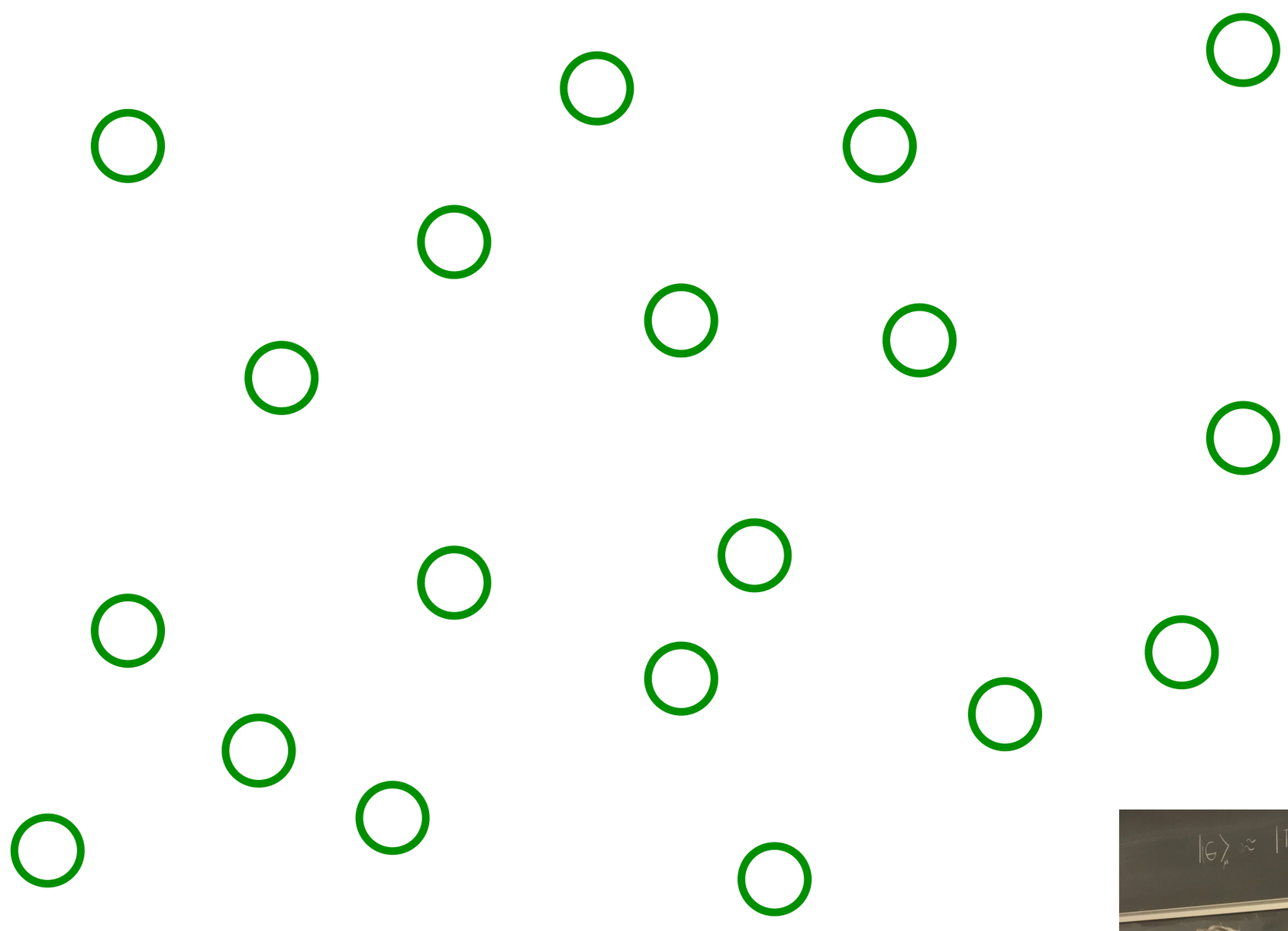
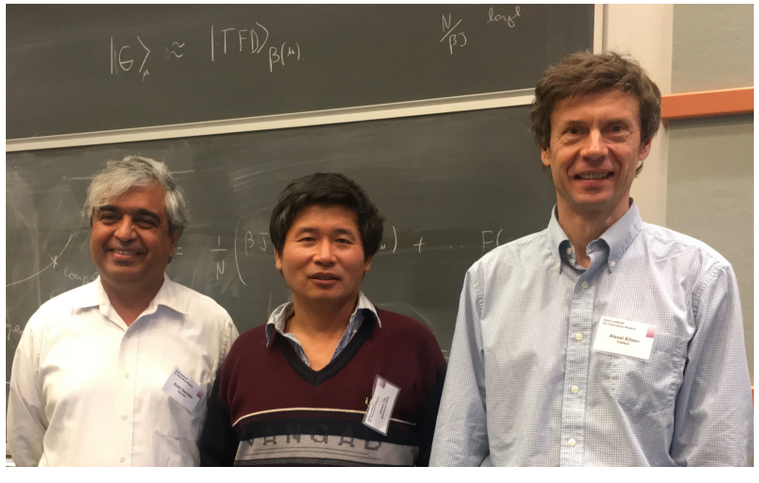


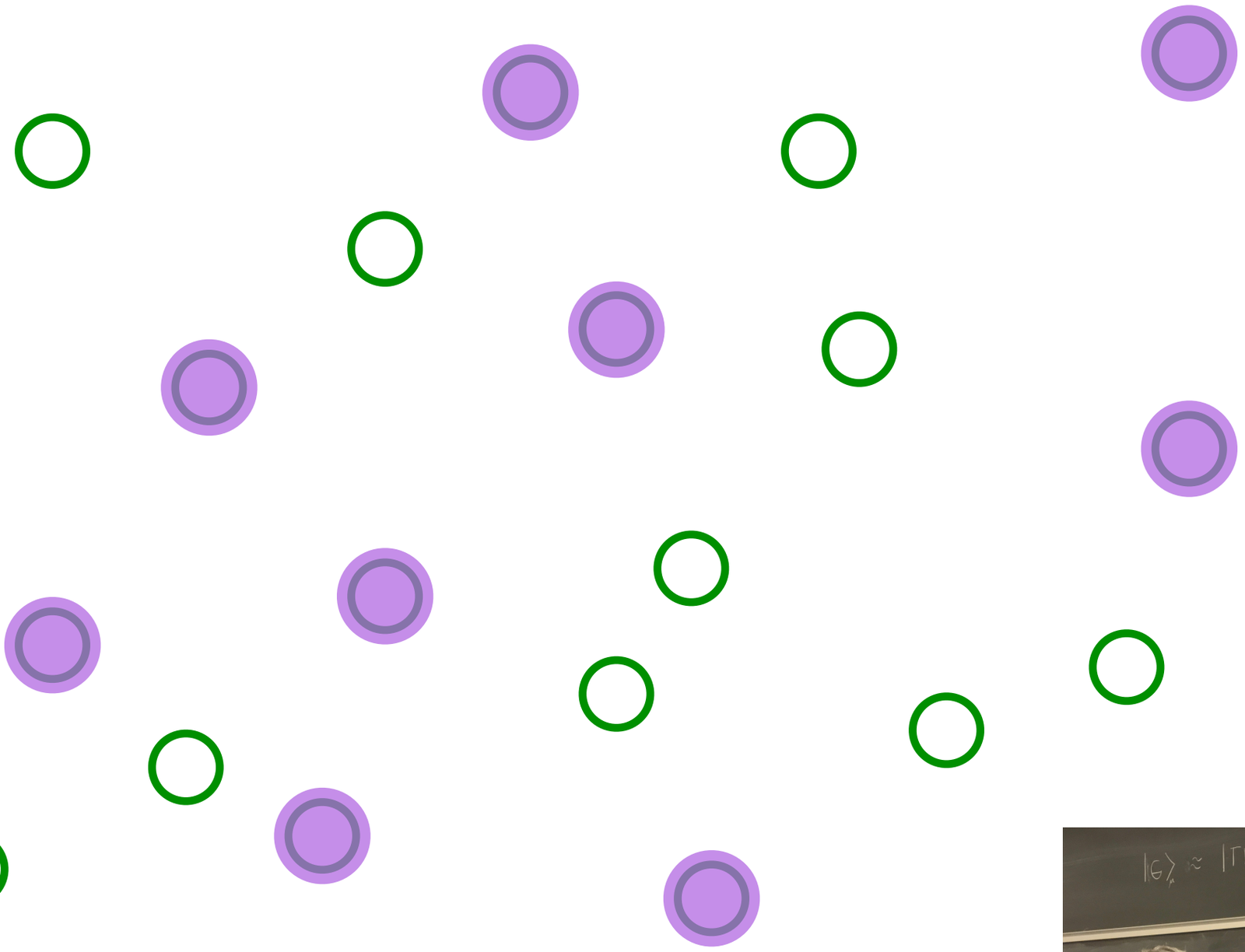
The Sachdev-Ye-Kitaev (SYK) model



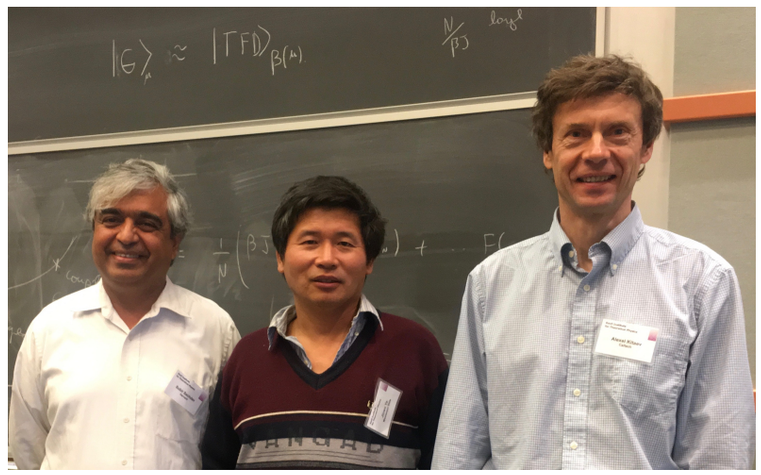
Pick a set of random positions



The SYK model



Place electrons randomly on some sites



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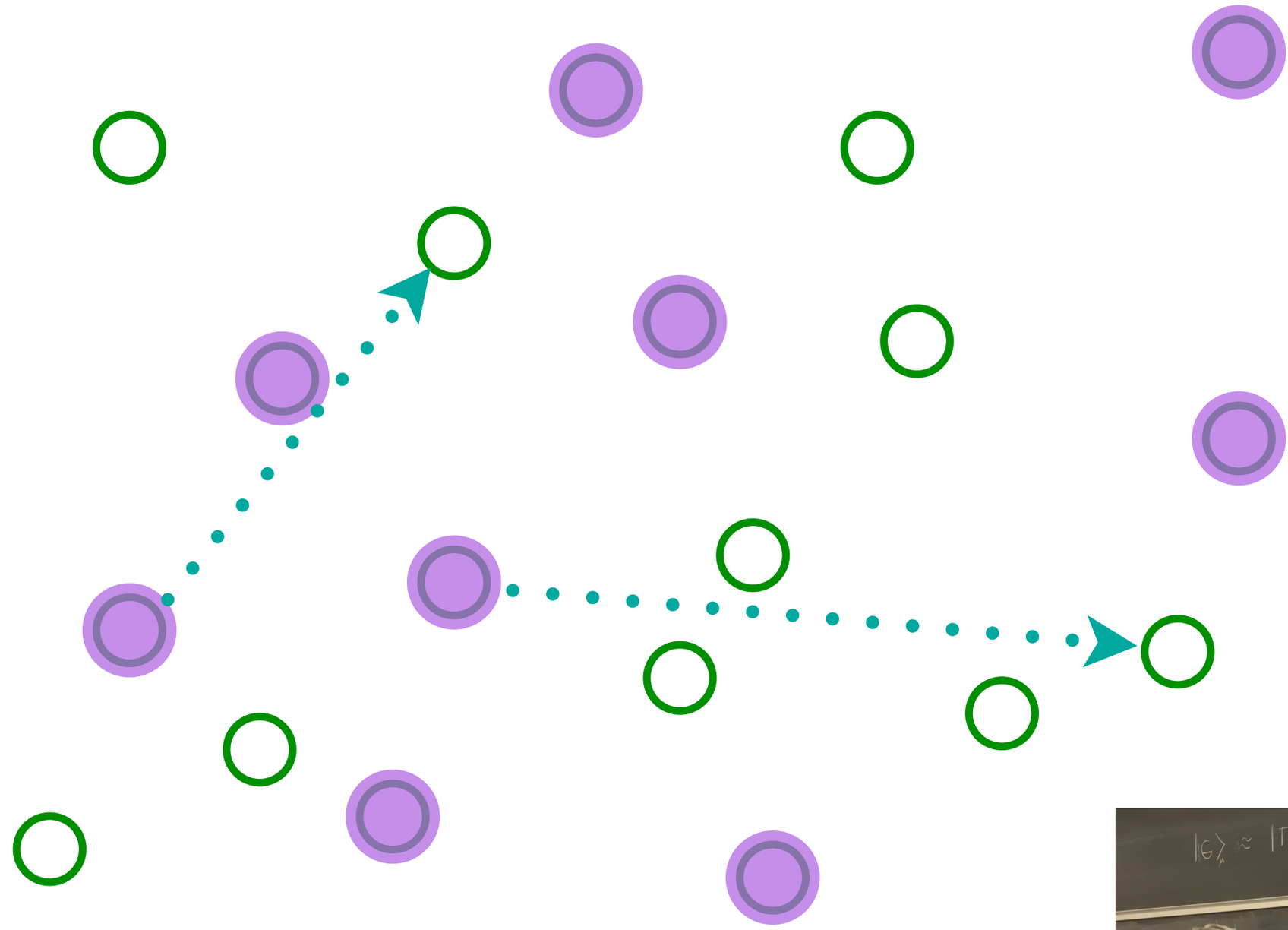

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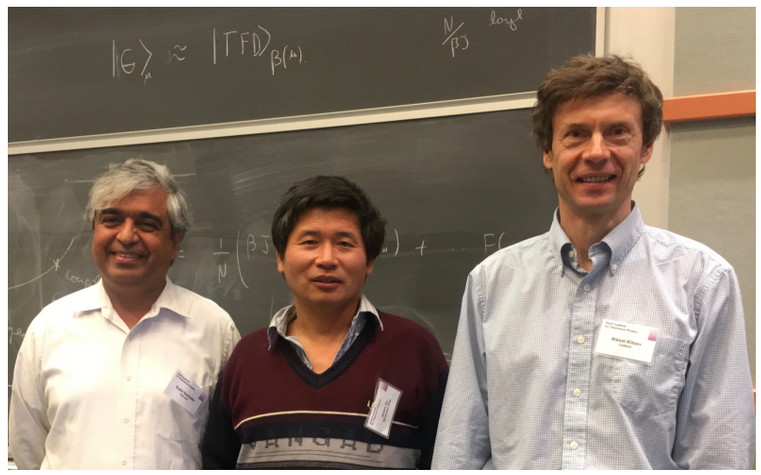

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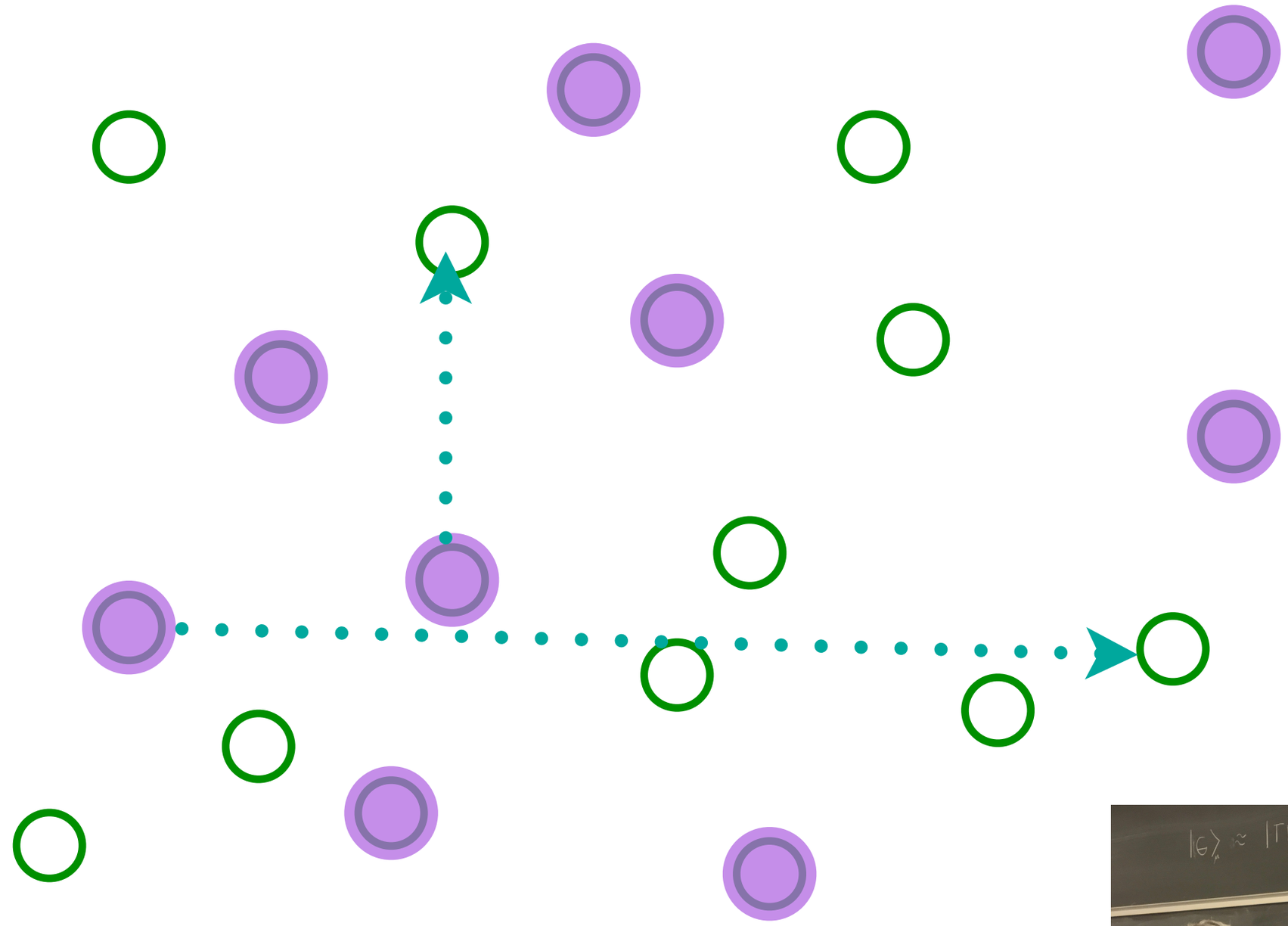

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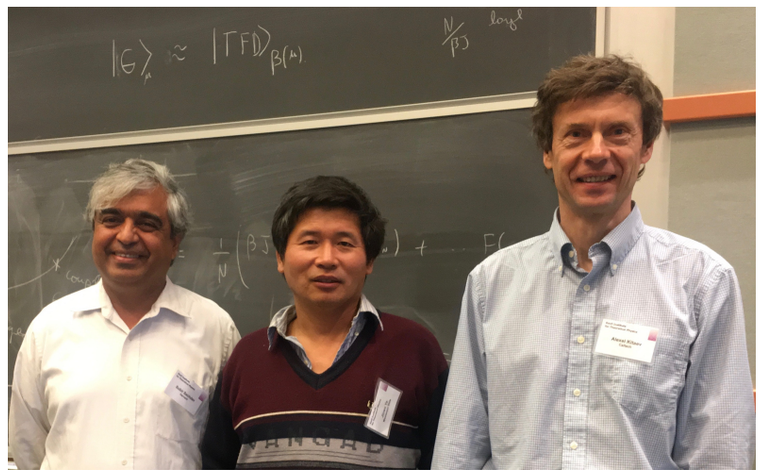

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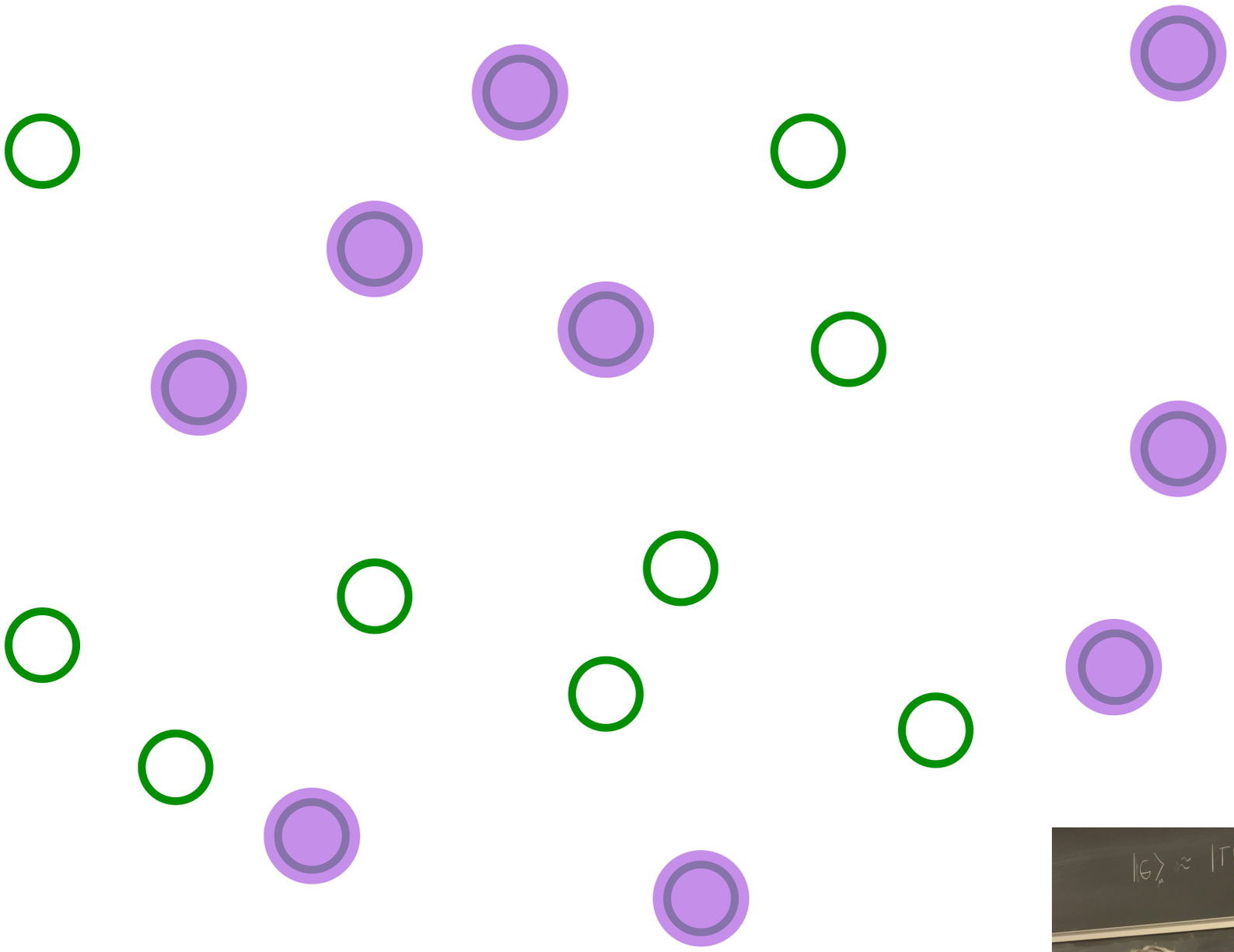
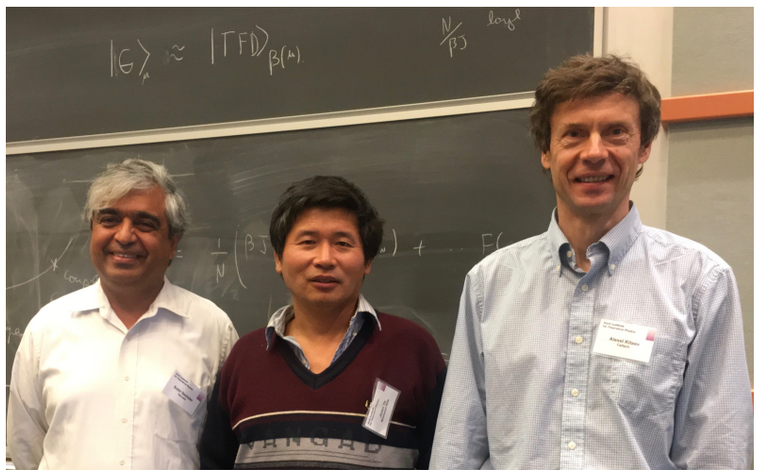
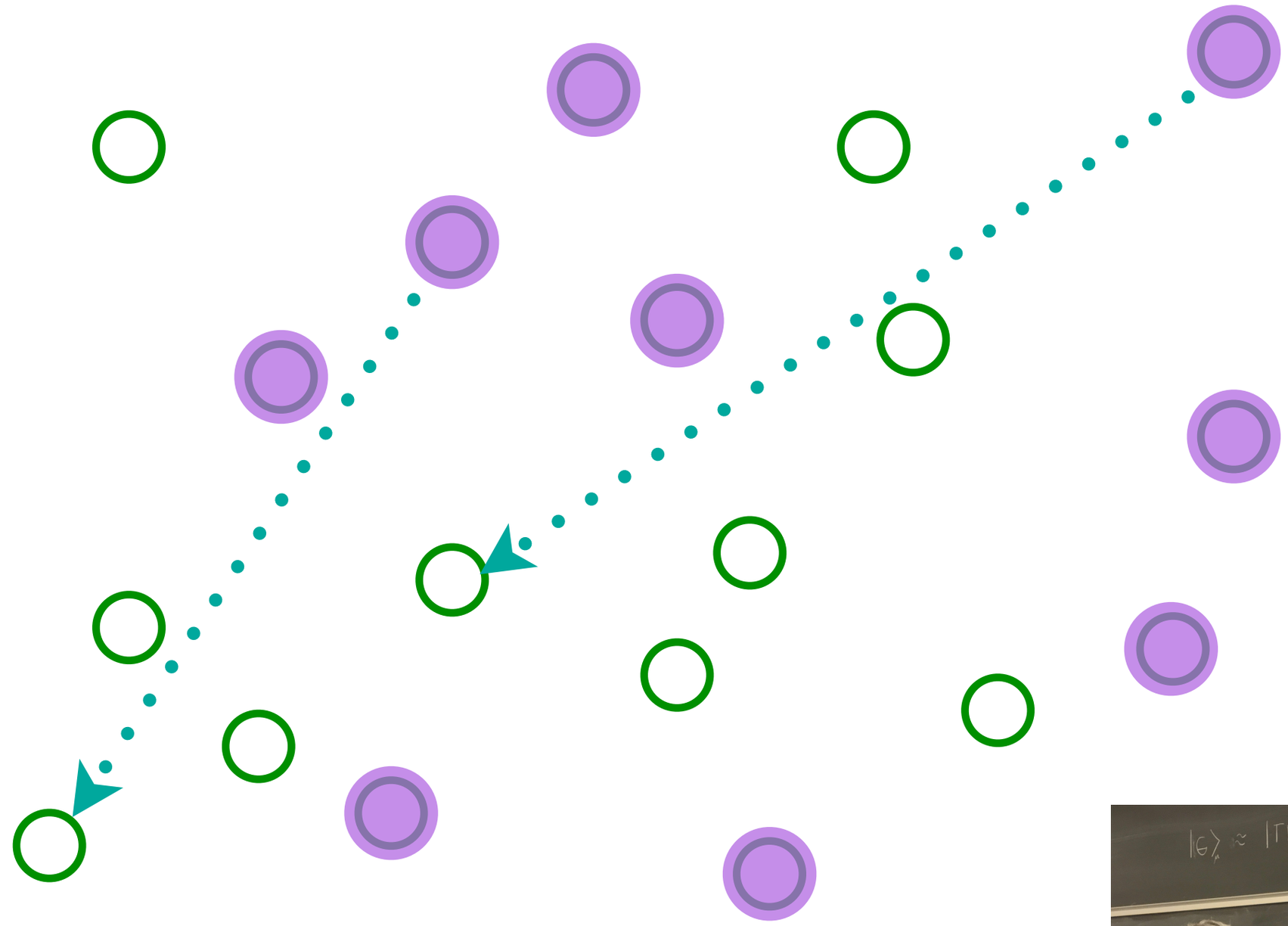


Entangle electrons pairwise randomly



The SYK model



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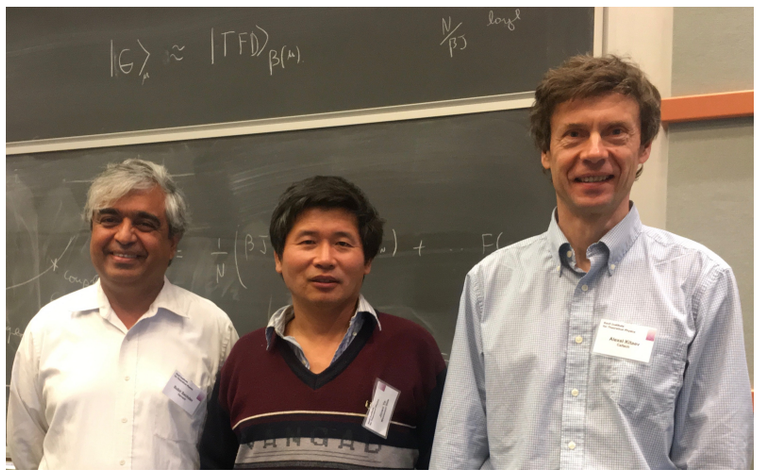

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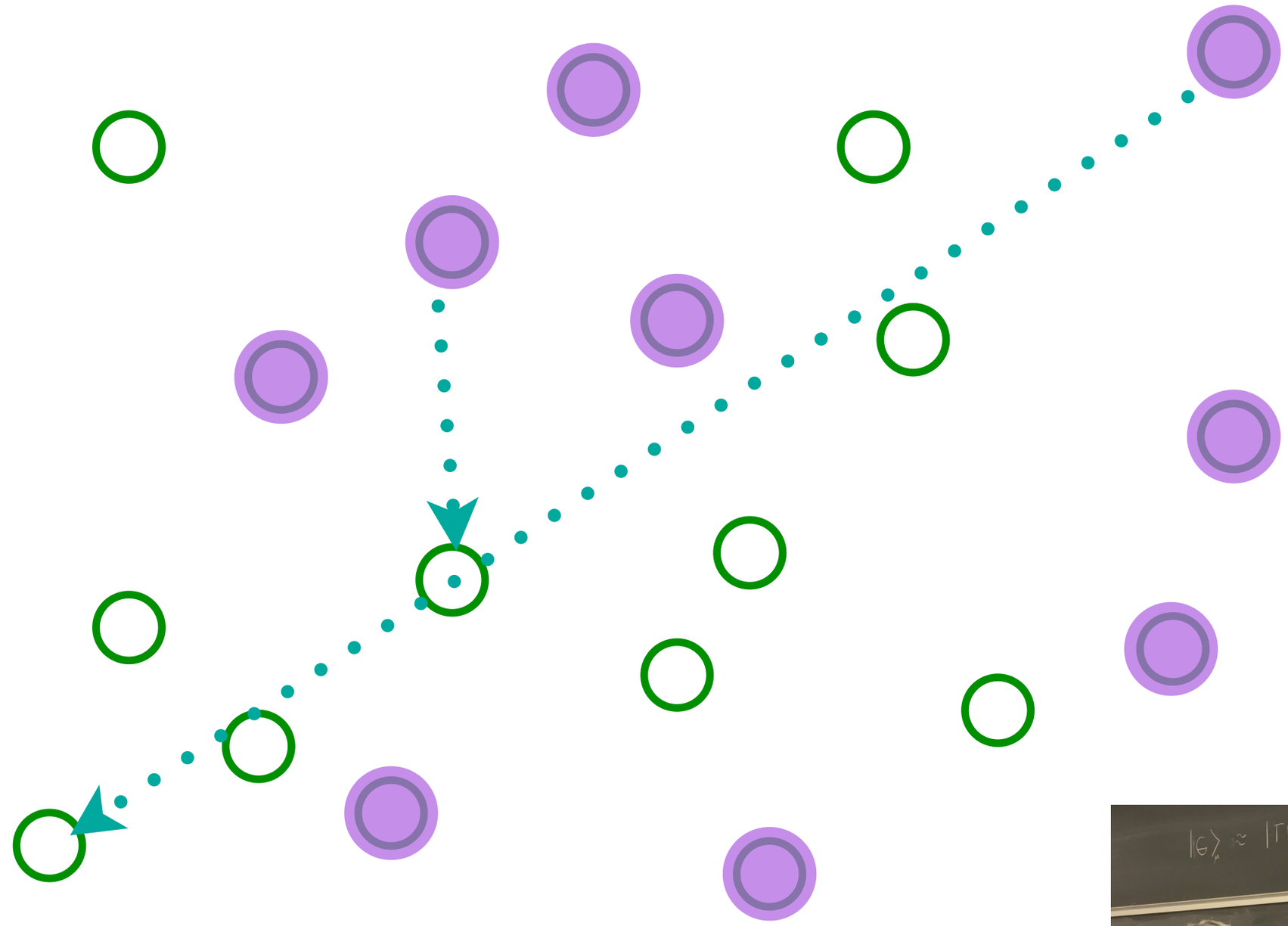

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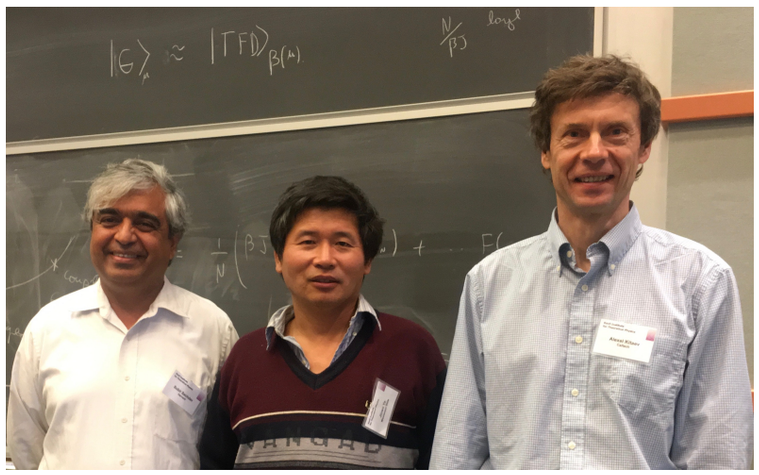

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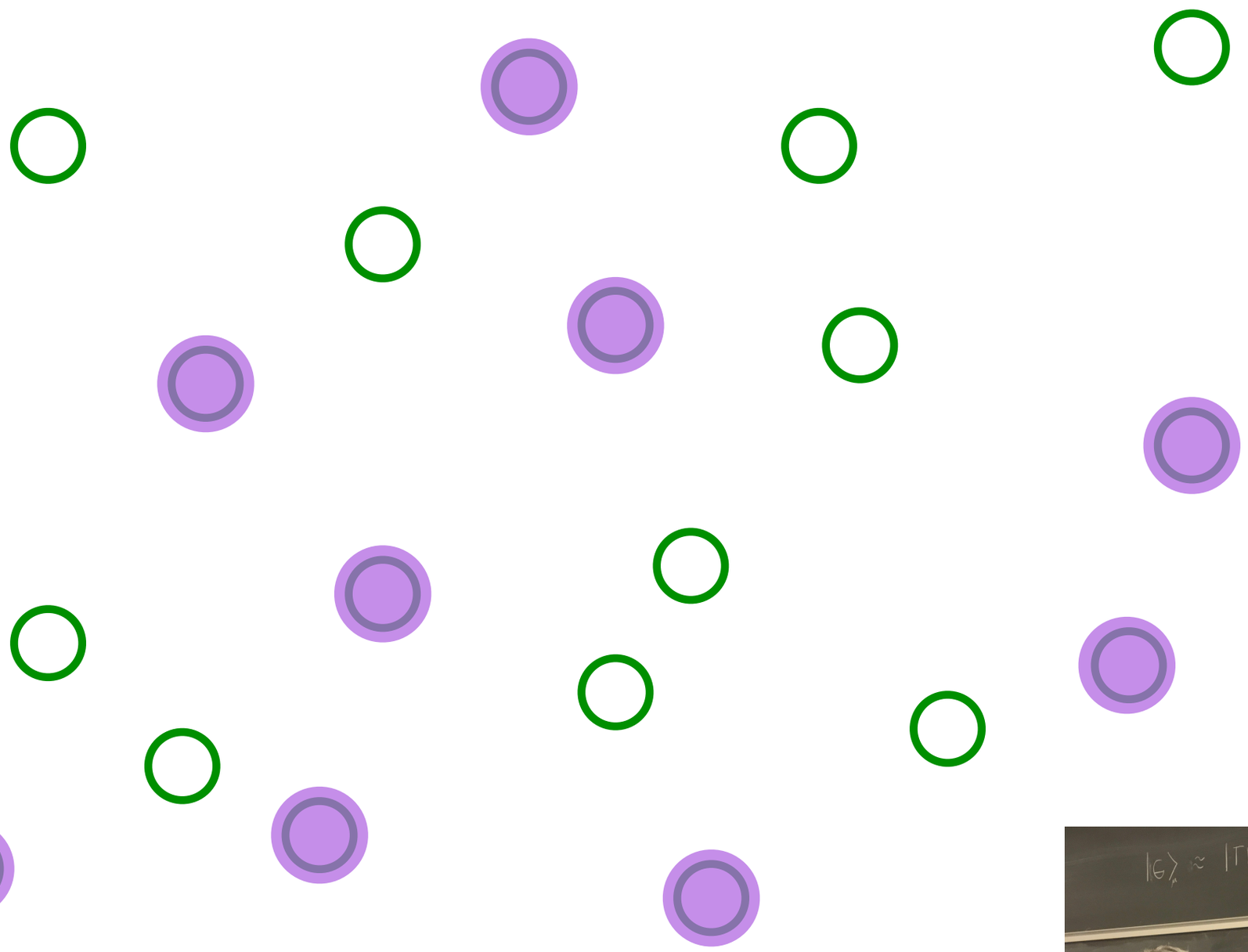

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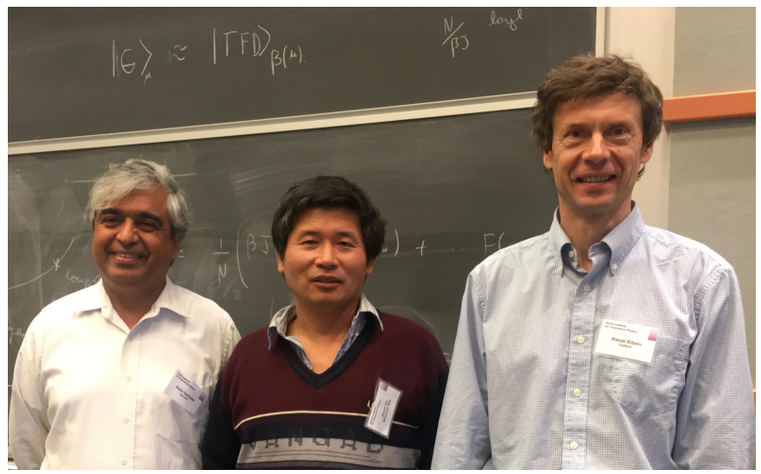

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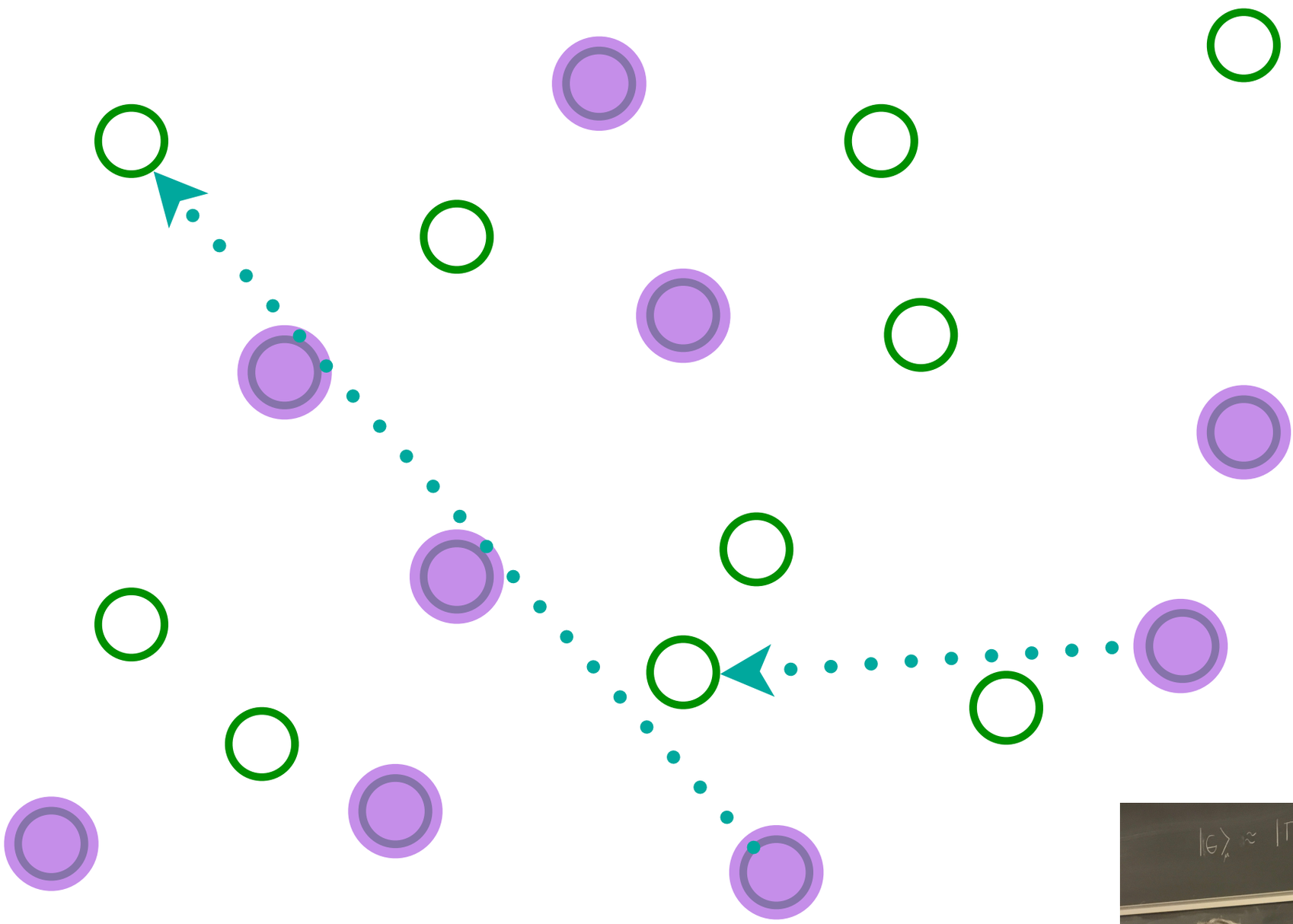

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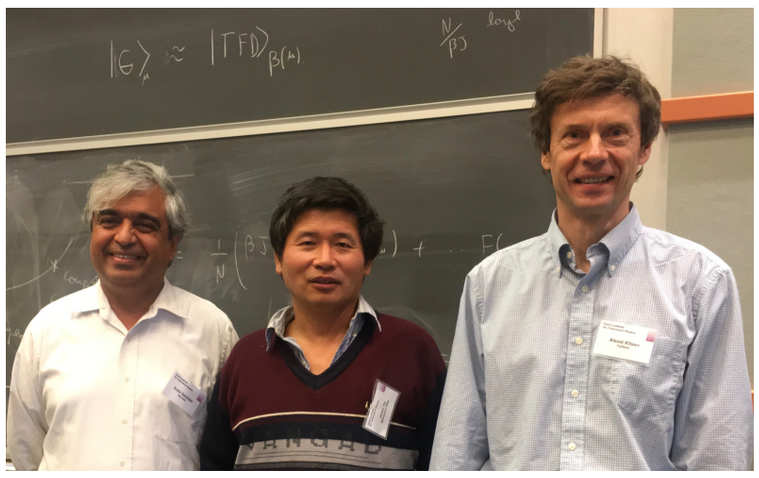

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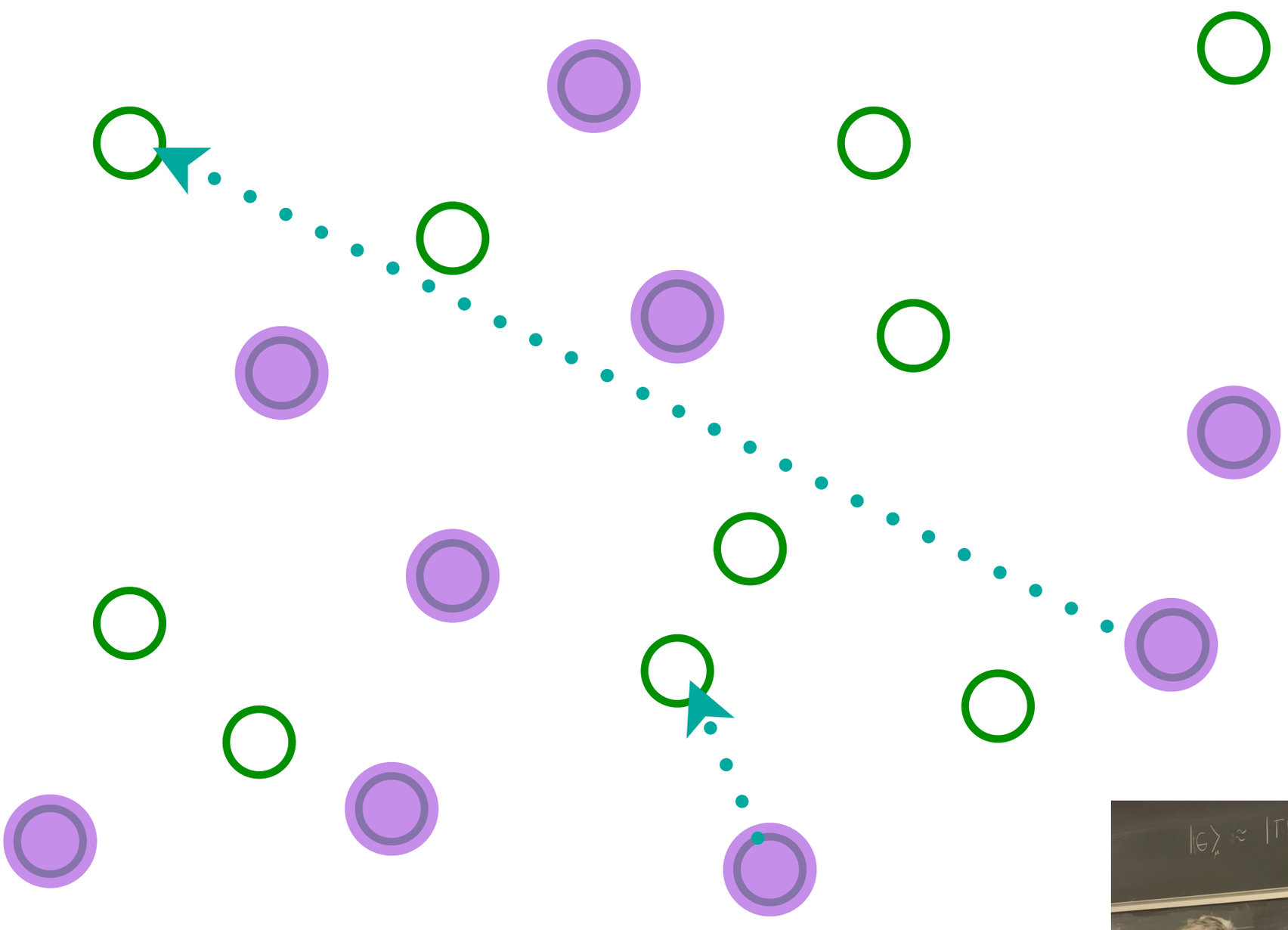

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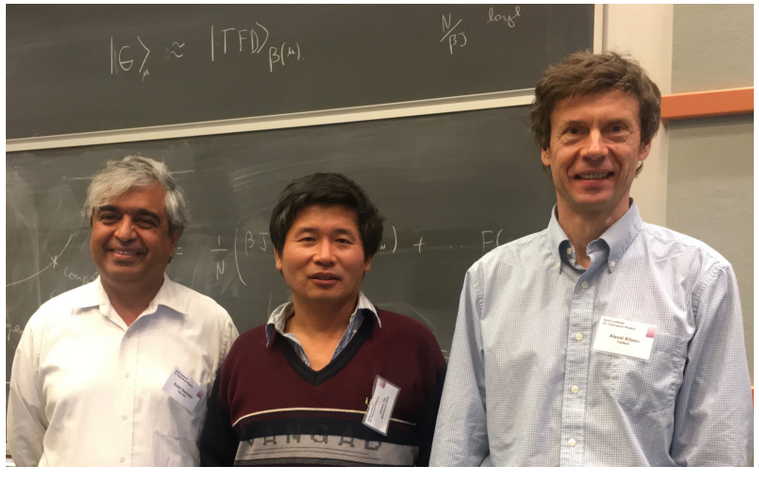

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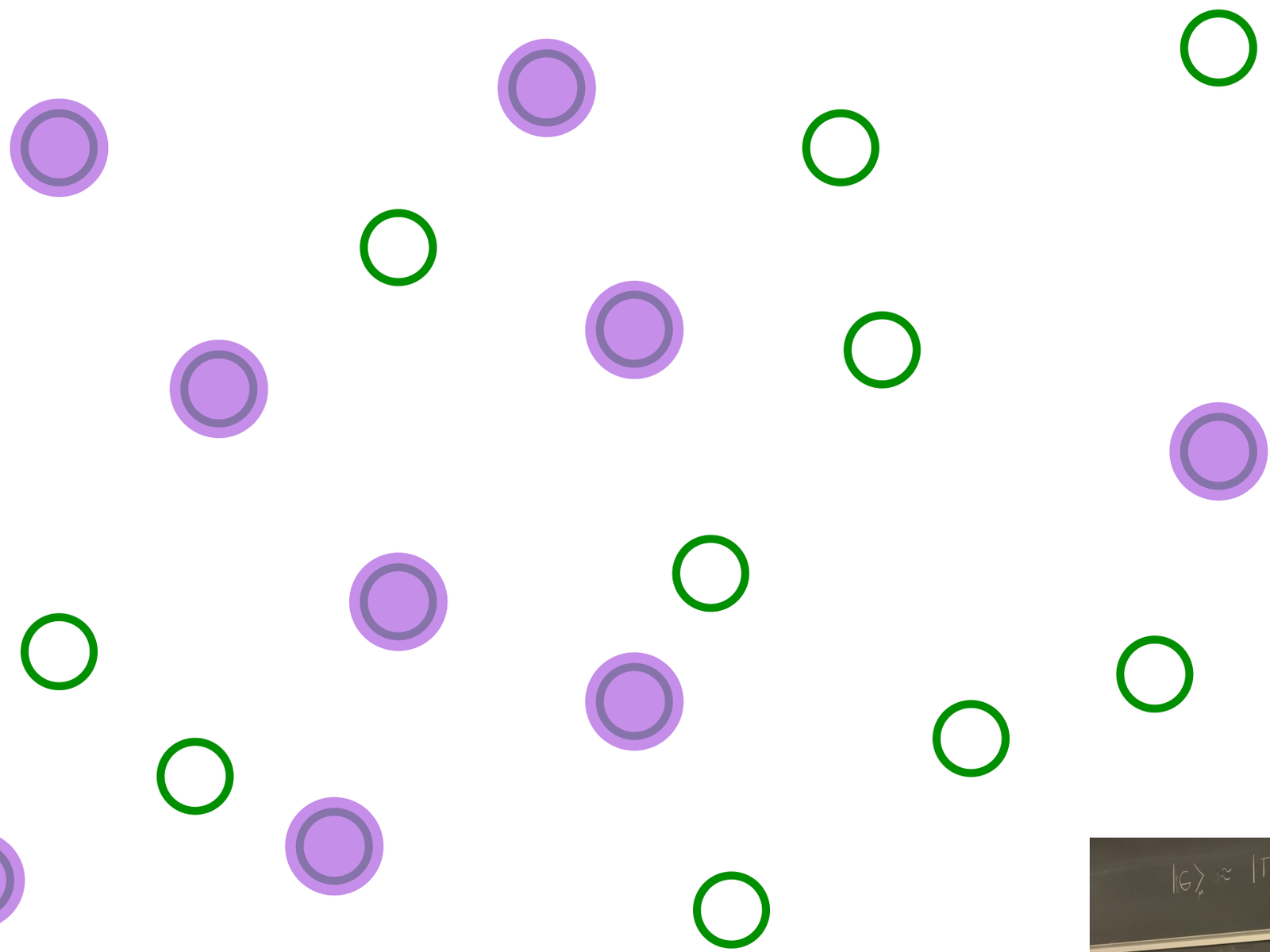

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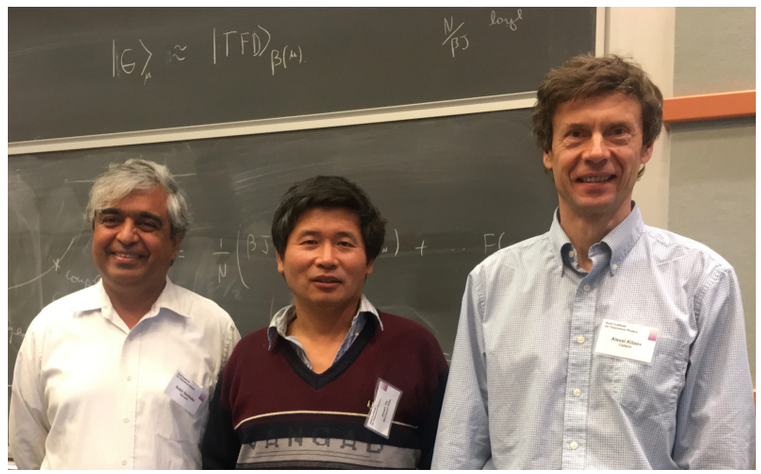

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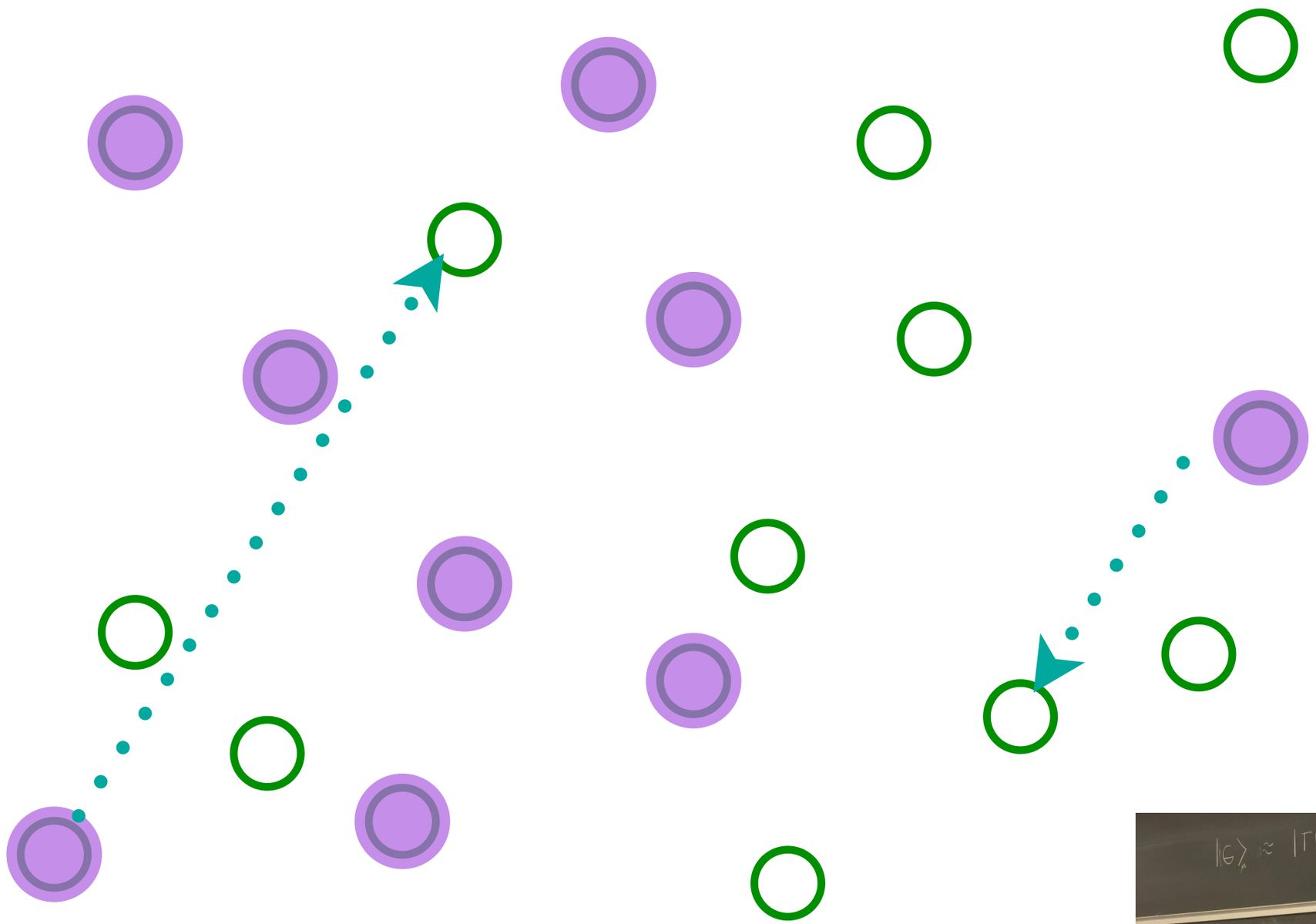

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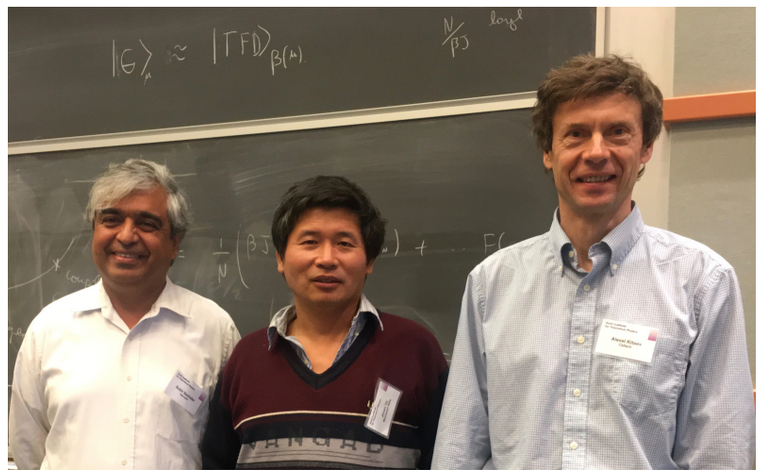

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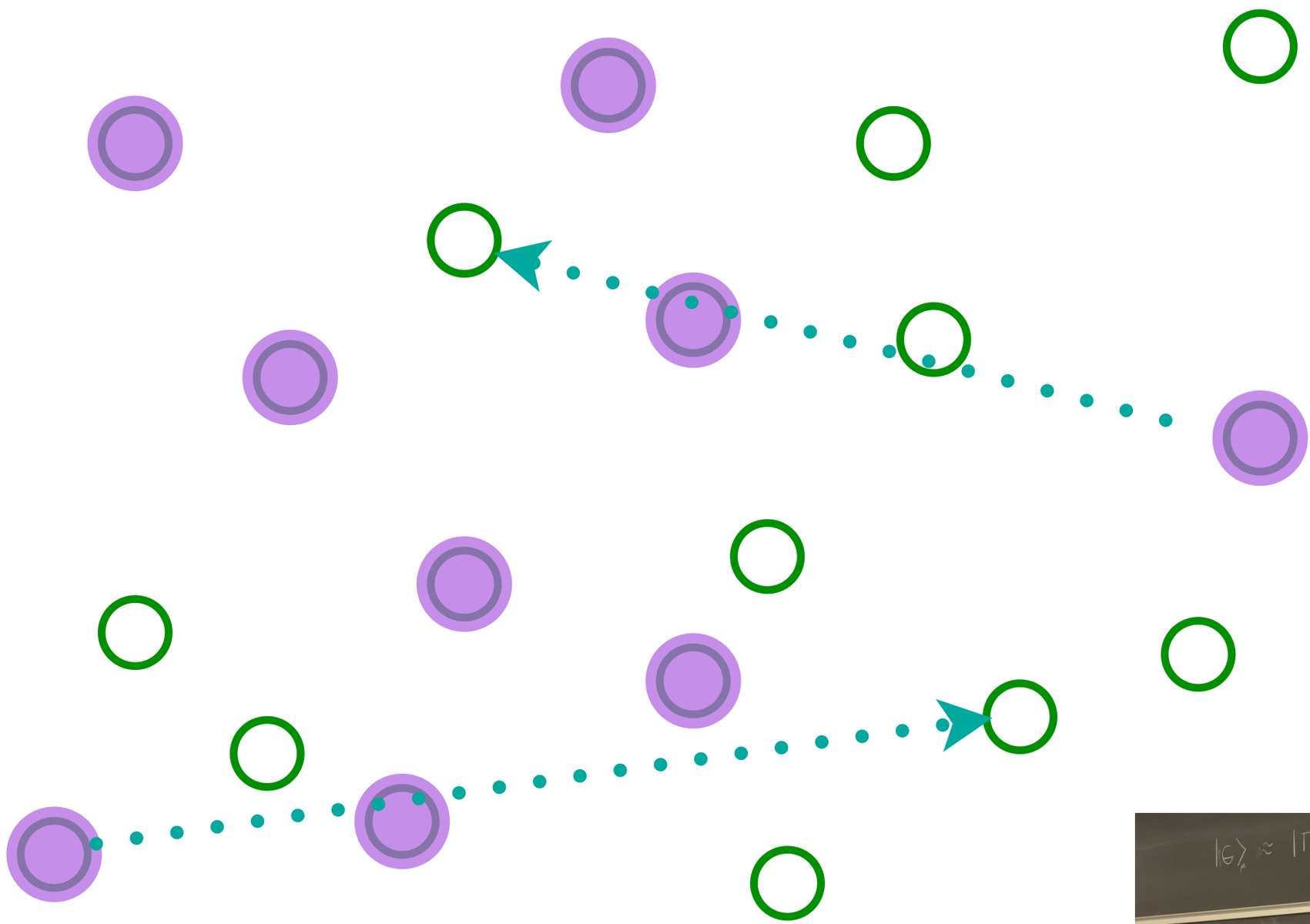

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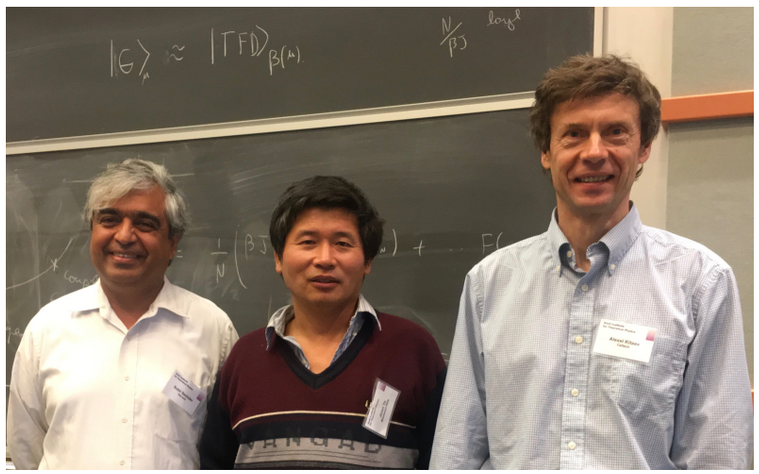

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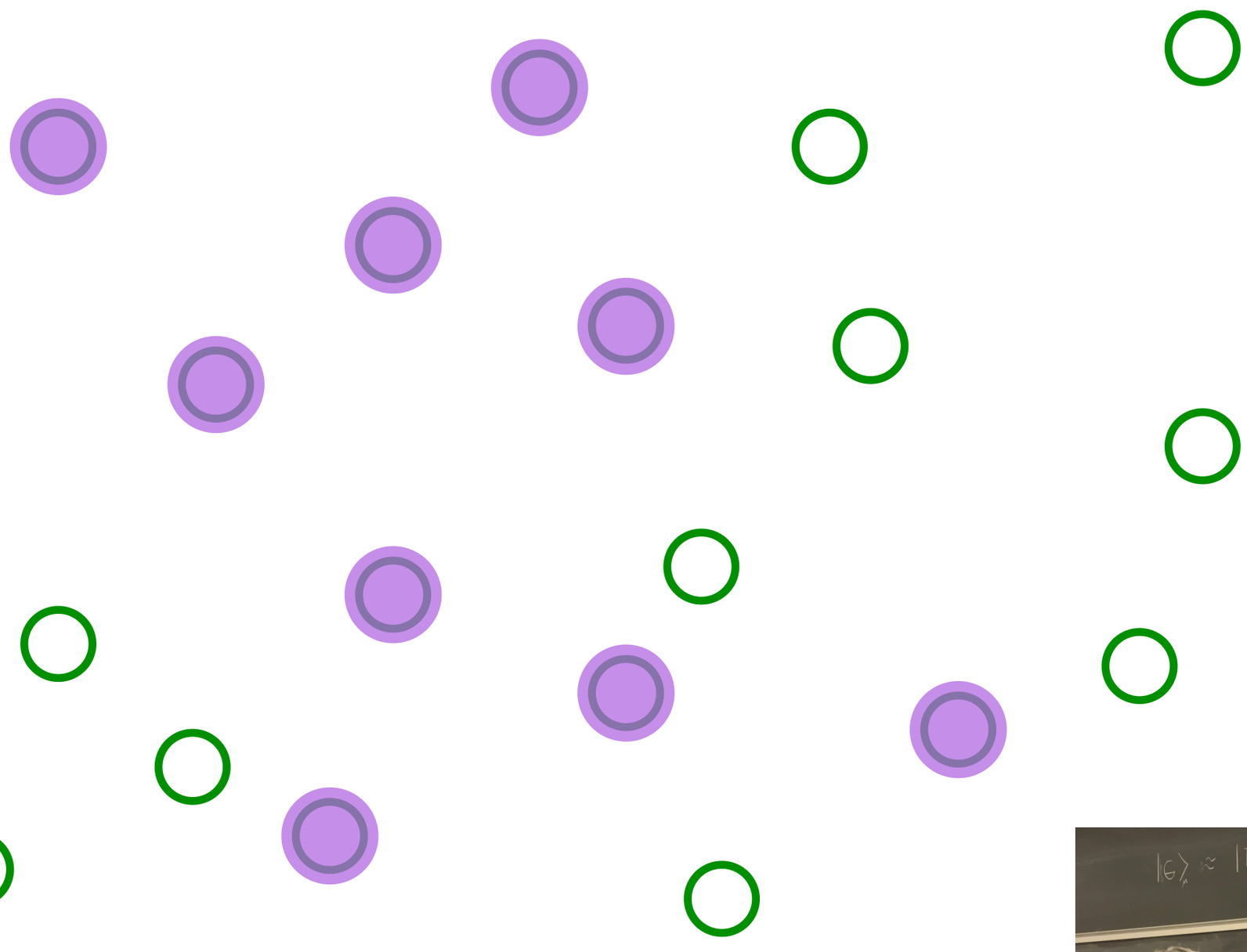

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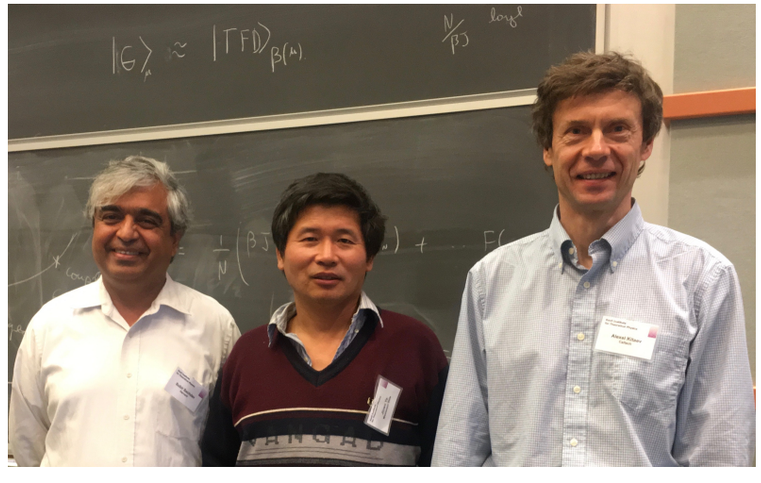

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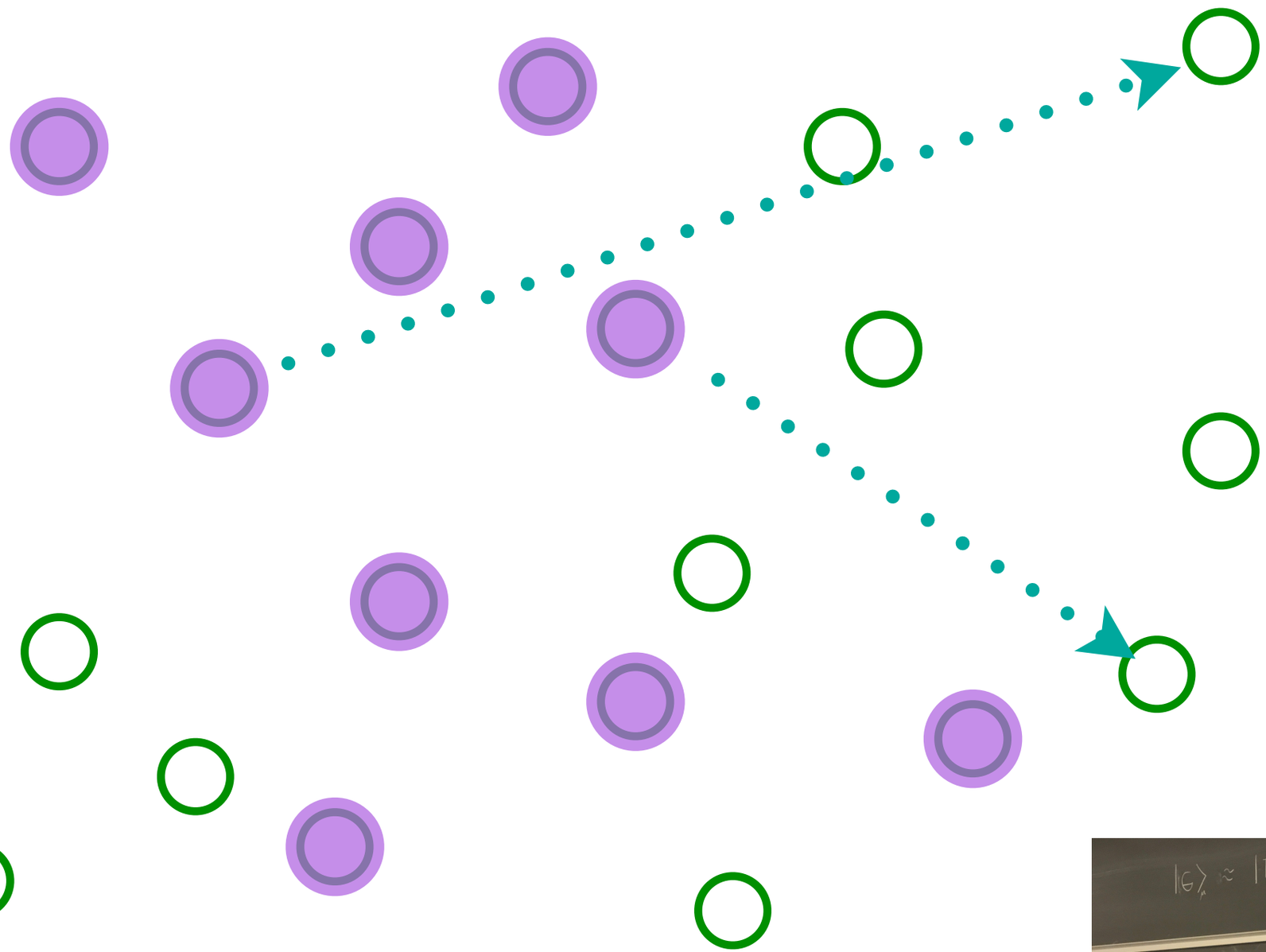

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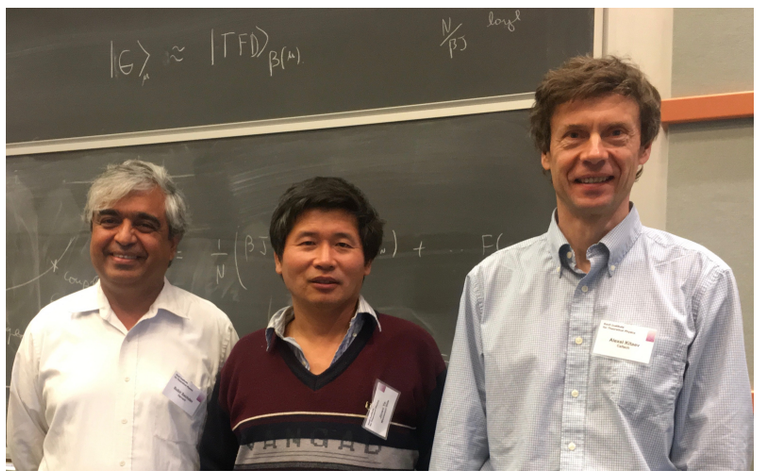

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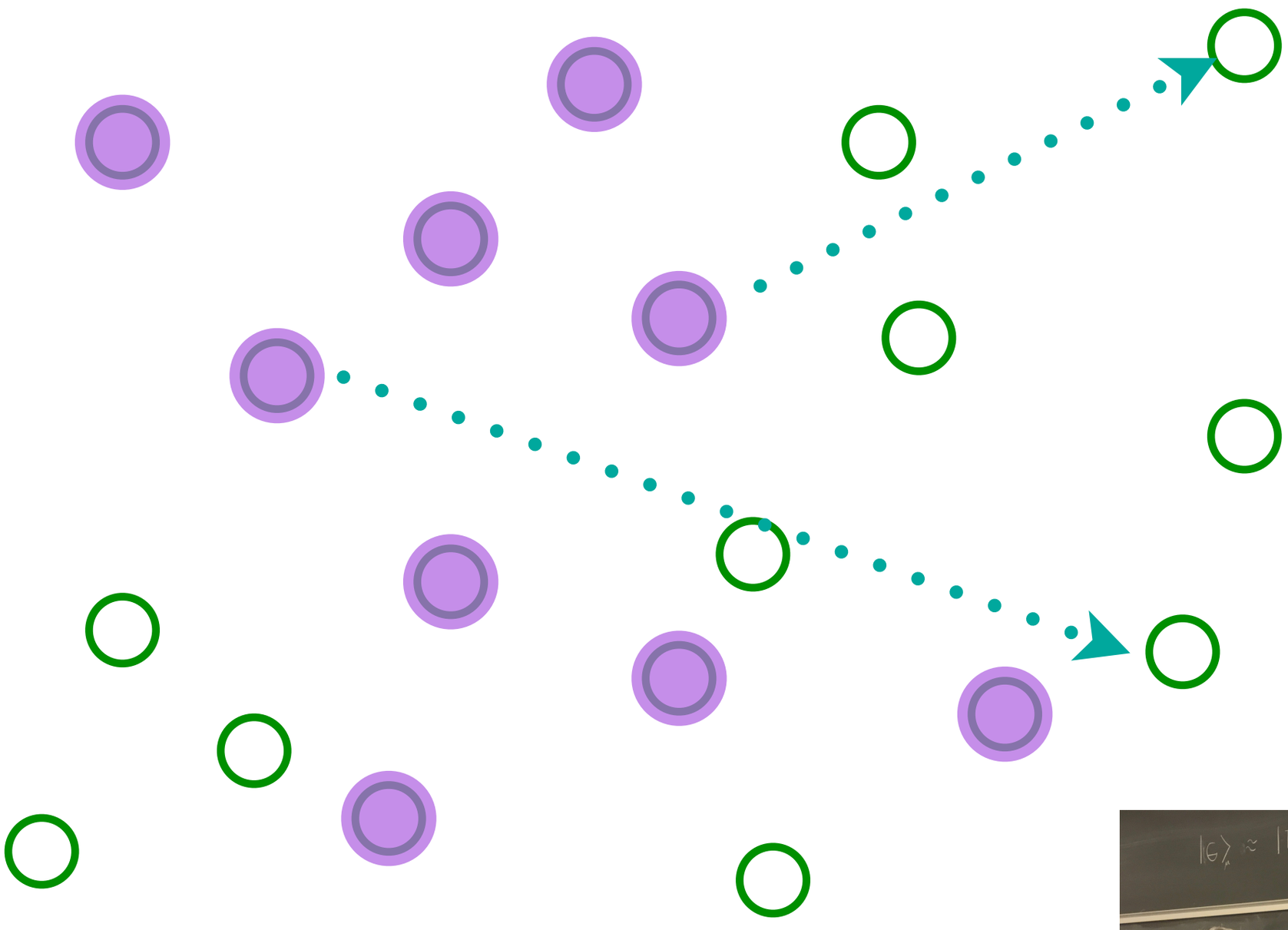

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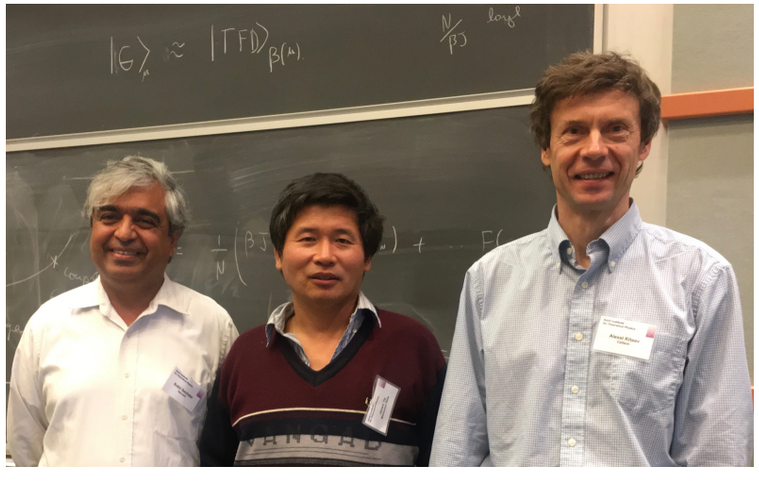

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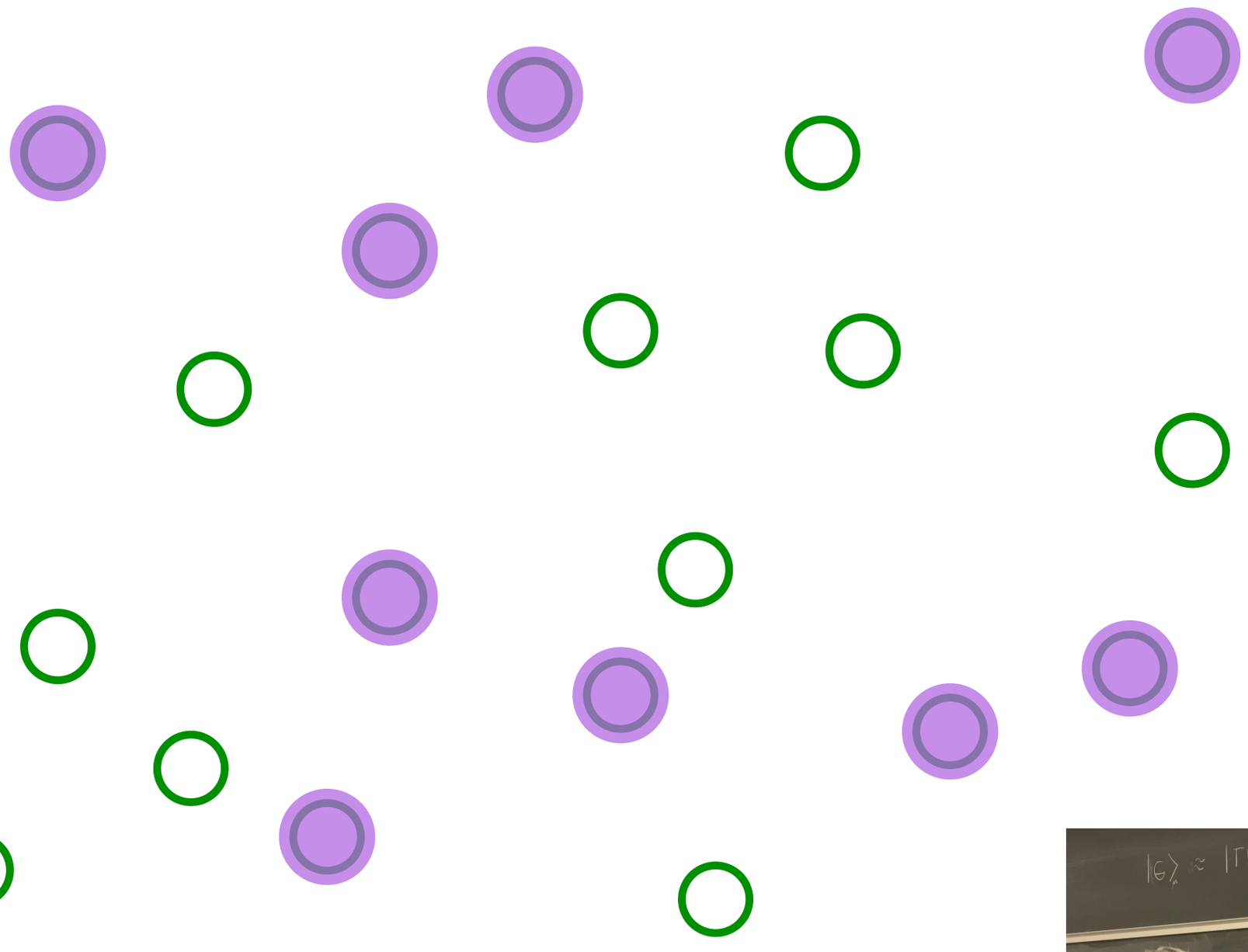

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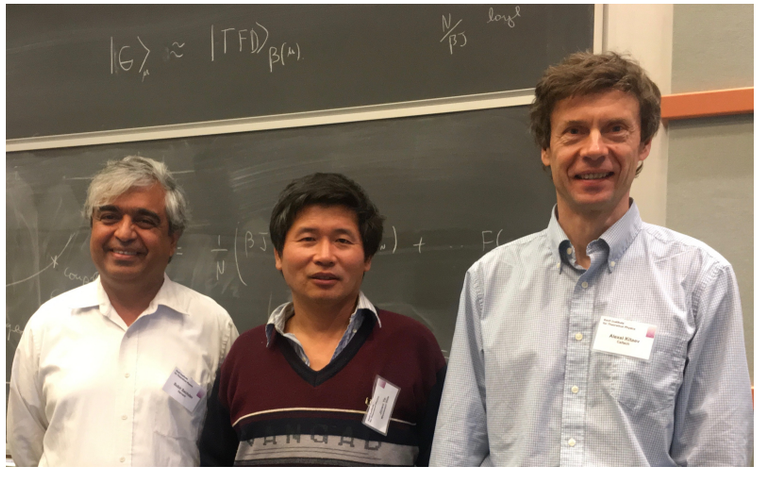

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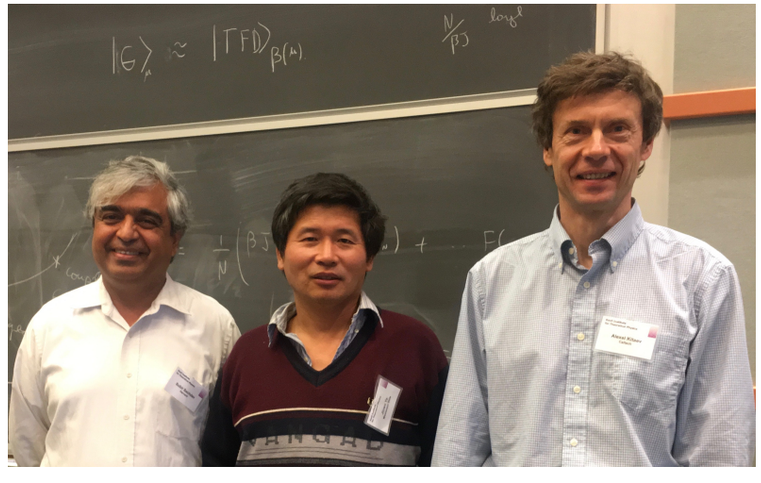

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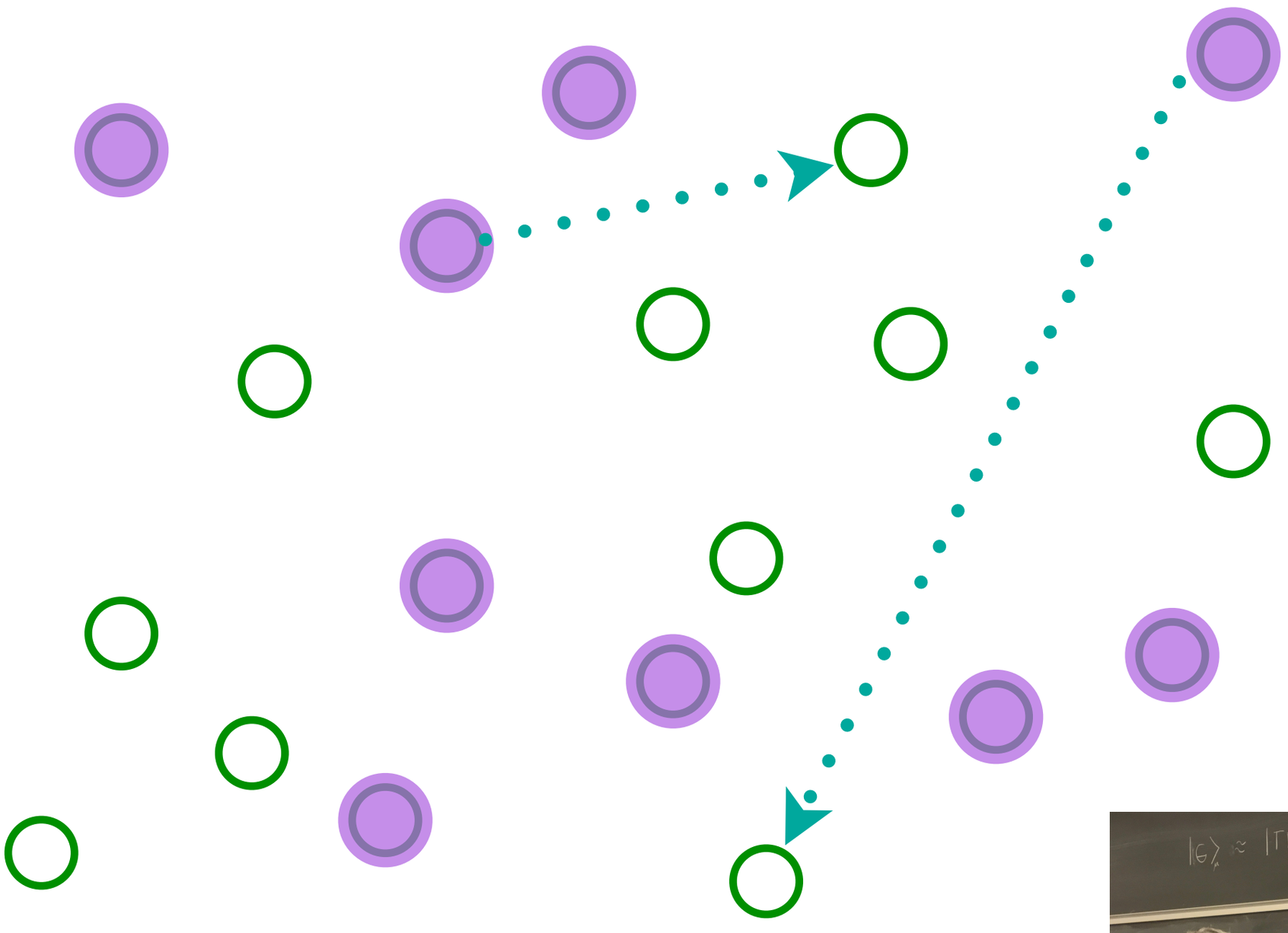

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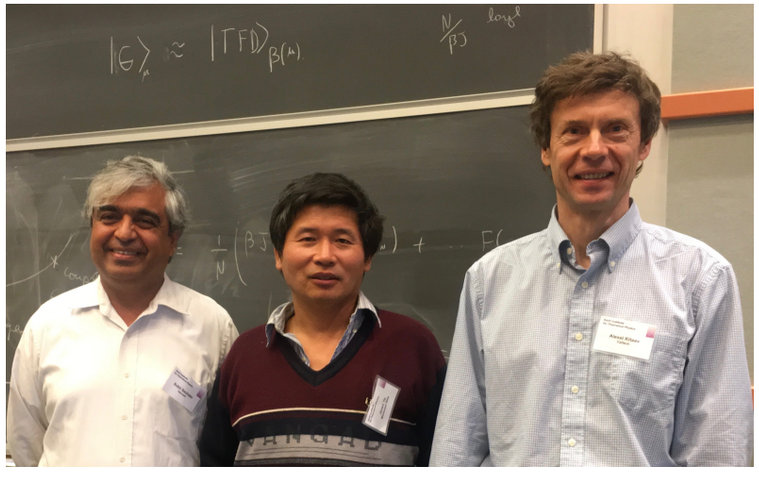

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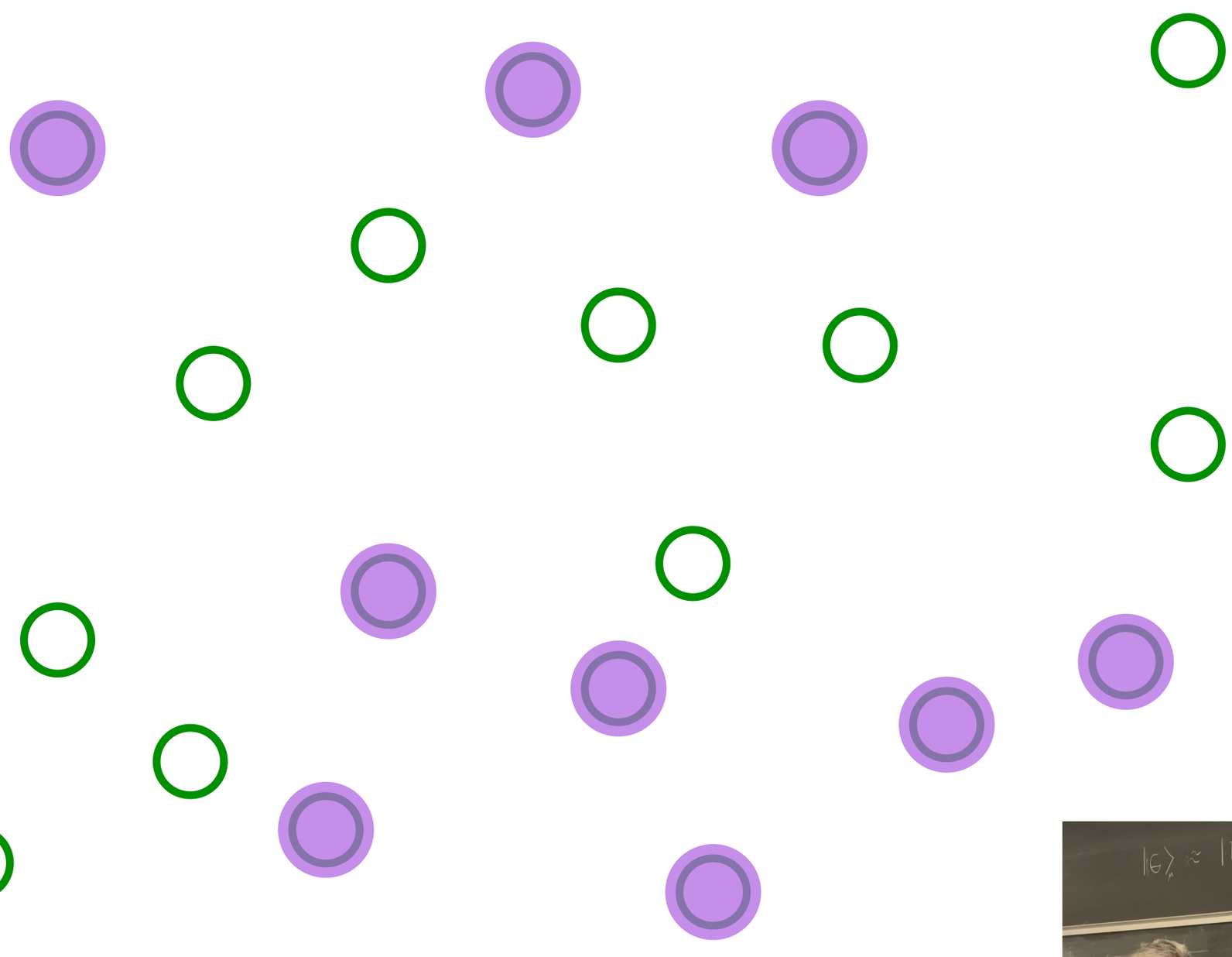

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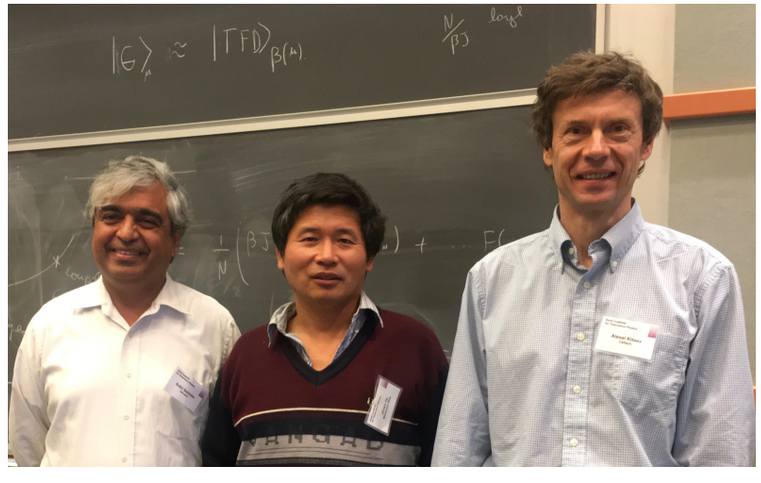

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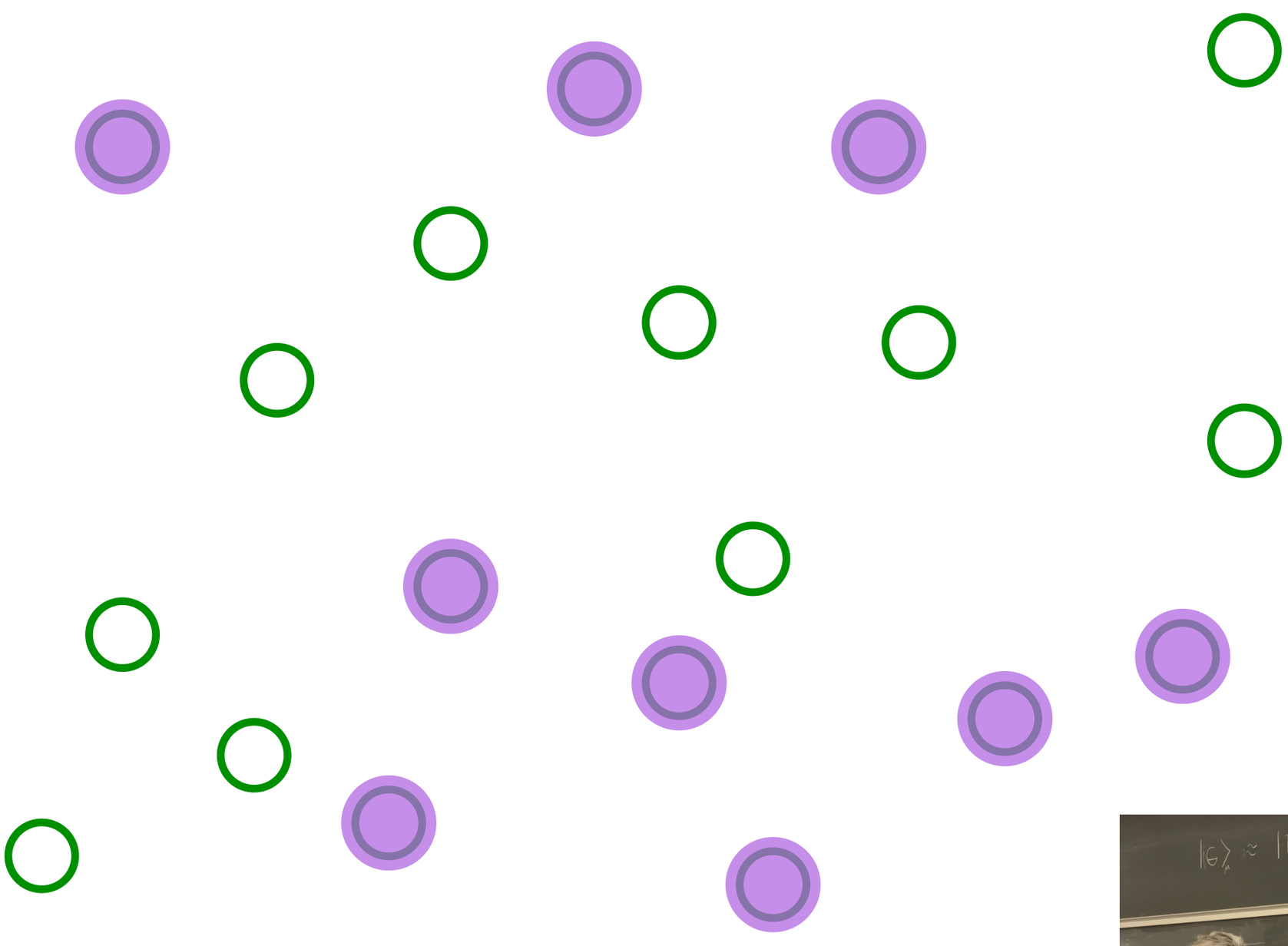
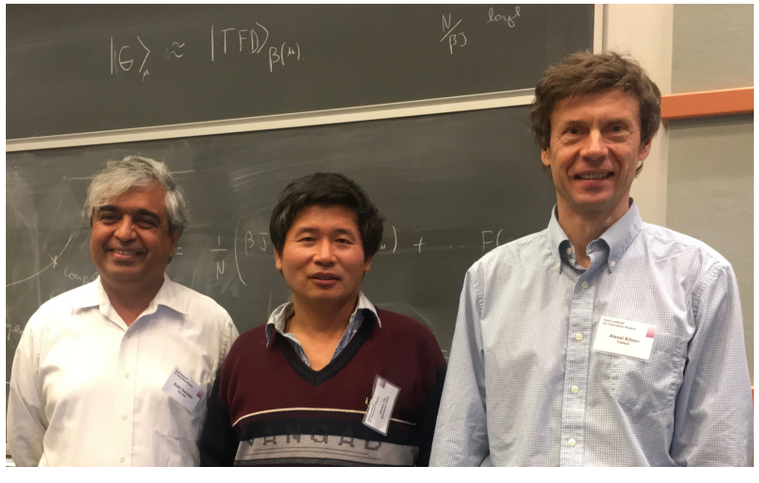


This describes both a strange metal and a black hole!



The SYK model

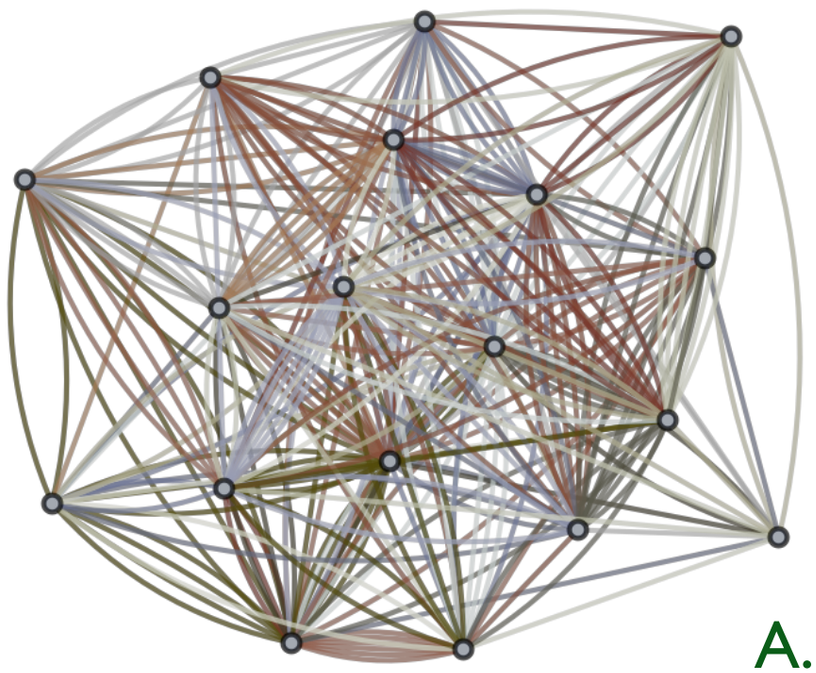
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

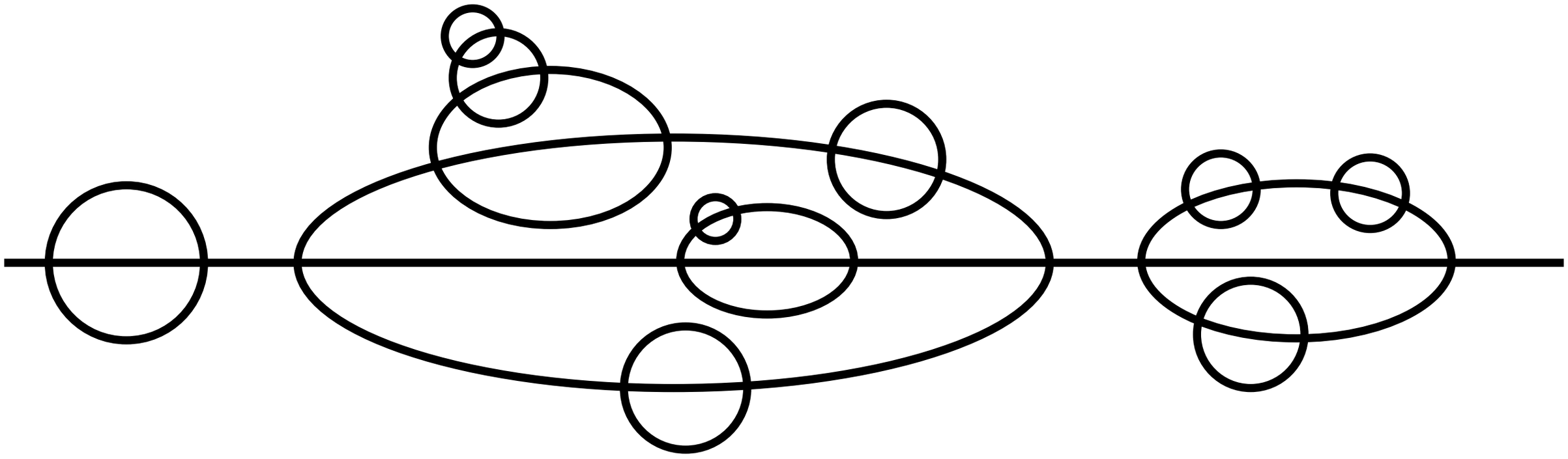


S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

The large N limit is given by the sum of "melon" Feynman graphs



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The SYK model

The large N limit is given by the sum of "melon" Feynman graphs

For long times $\tau > 0$

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle = \frac{A}{\sqrt{\tau}}$$

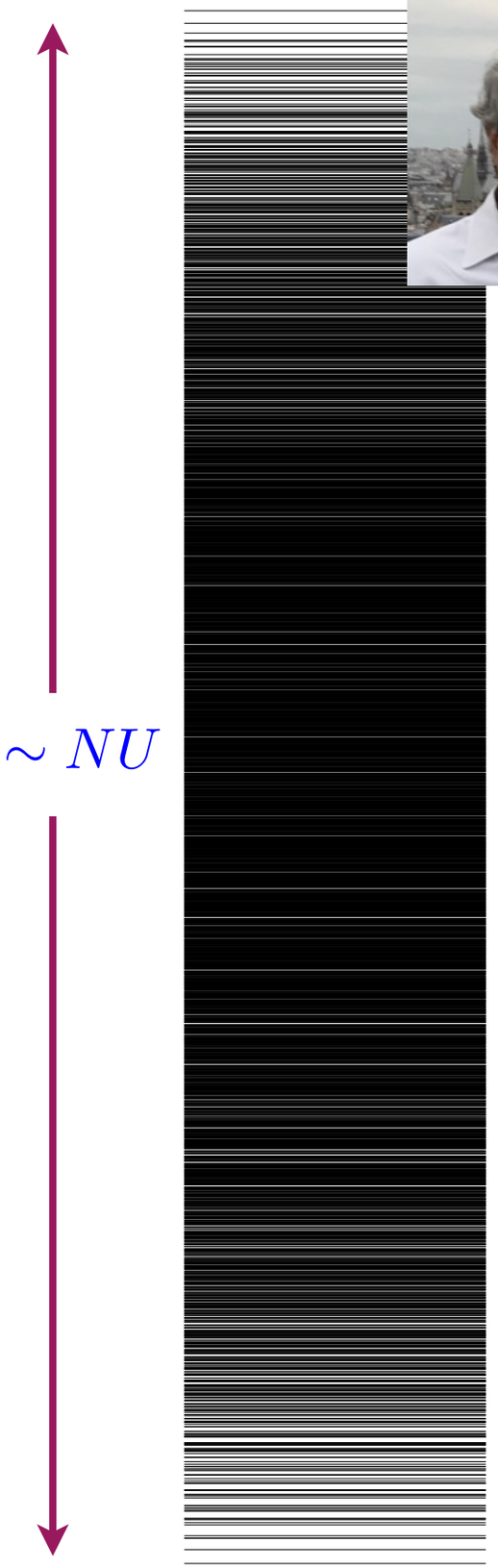
$$\left\langle c_i^\dagger(\tau) c_i(0) \right\rangle = e^{-2\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E}$ determines the particle-hole asymmetry.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The SYK model

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

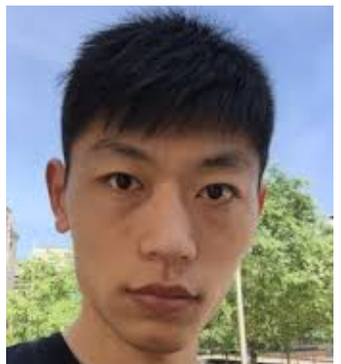
There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi \mathcal{E} \, .$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.



W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

No quasiparticles

Julia Steinberg

- Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and
J. Steinberg, PRB **96**, 205123 (2017)

Established by solution of Schwinger-Keldysh equations for a quench.

The SYK model

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Established by solution of Schwinger-Keldysh equations for a quench.

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

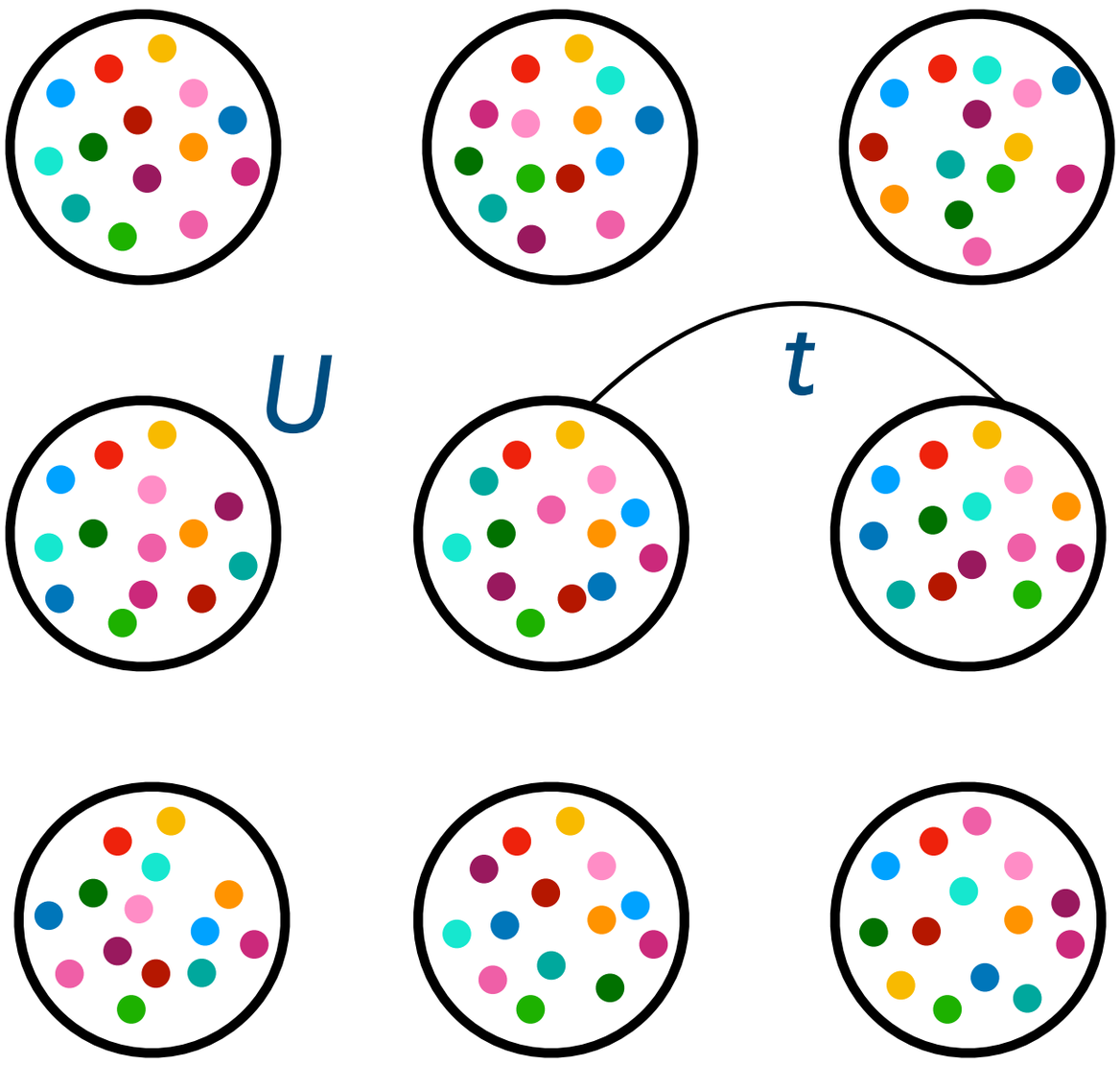
Absence of quasiparticles $\Leftrightarrow$ Fastest possible thermalization

Coupled SYK Islands

Xue-Yang Song

Aavishkar Patel

SYK quantum islands of electrons with random or regular hopping between them: lead to linear-in-T resistivity for $t_0^2/U < T < U$.



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Pengfei Zhang, PRB **96**, 205138 (2017)

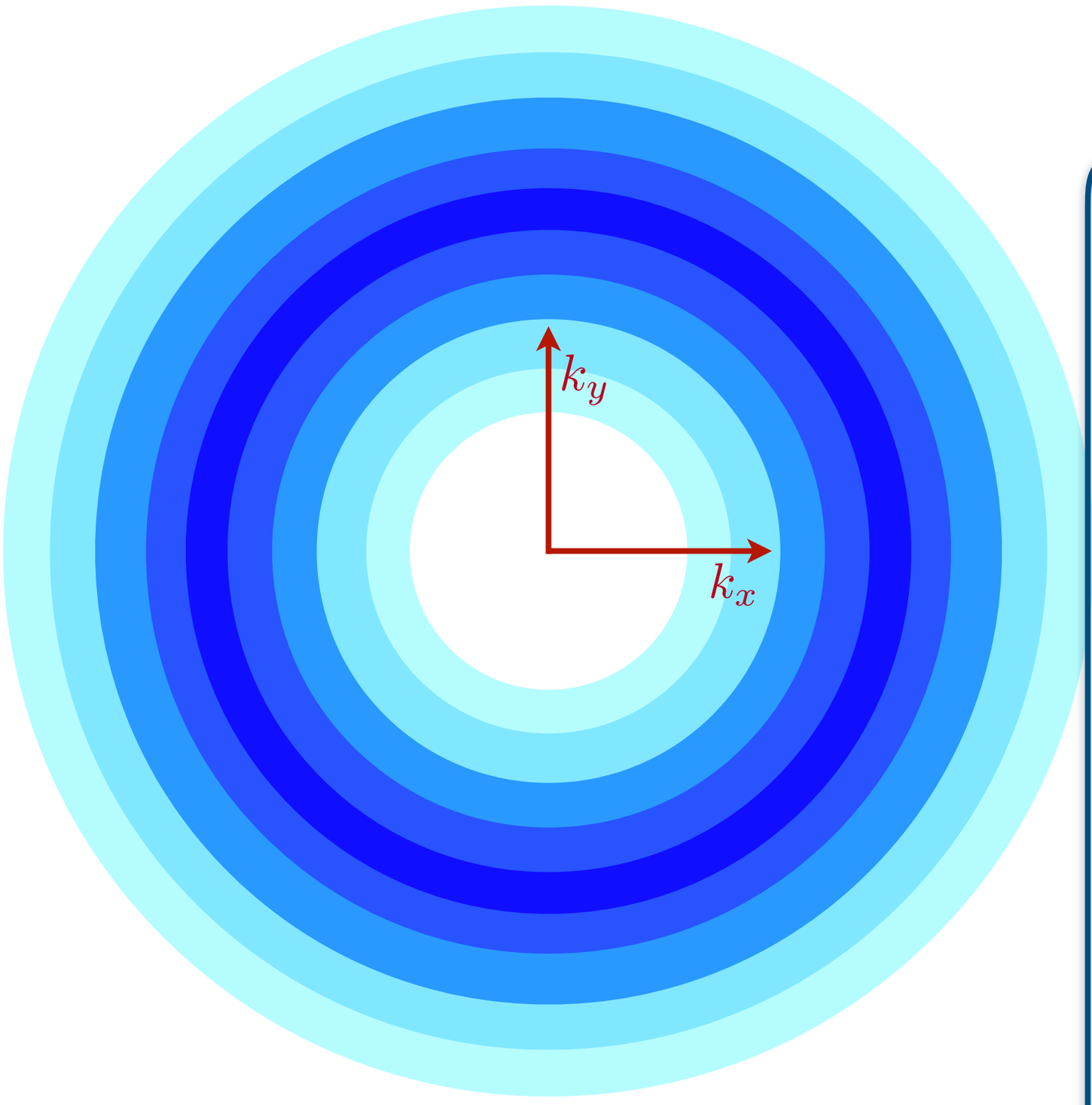
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 021049 (2018)

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

SYK model in momentum space

Aavishkar Patel



Random SYK interactions between each shell in momentum space lead to a resistivity

$$\rho = \frac{m^*}{ne^2}\left(1.11\frac{k_B T}{\hbar}\right)$$

independent of the interaction strength and spatial dimensionality, with very weak dependence on Fermi surface shape.

A. Patel *et al.*, to appear….

Ordinary metals:
quasiparticles

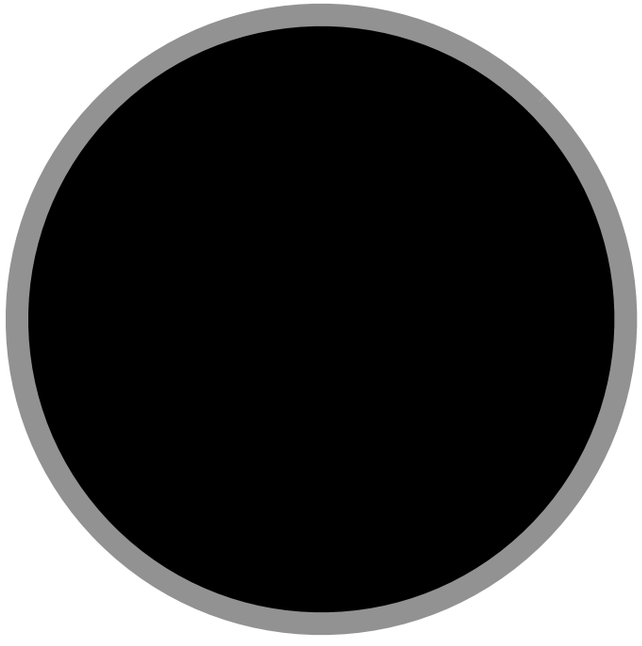
Strange metals:
no quasiparticles

Black
holes

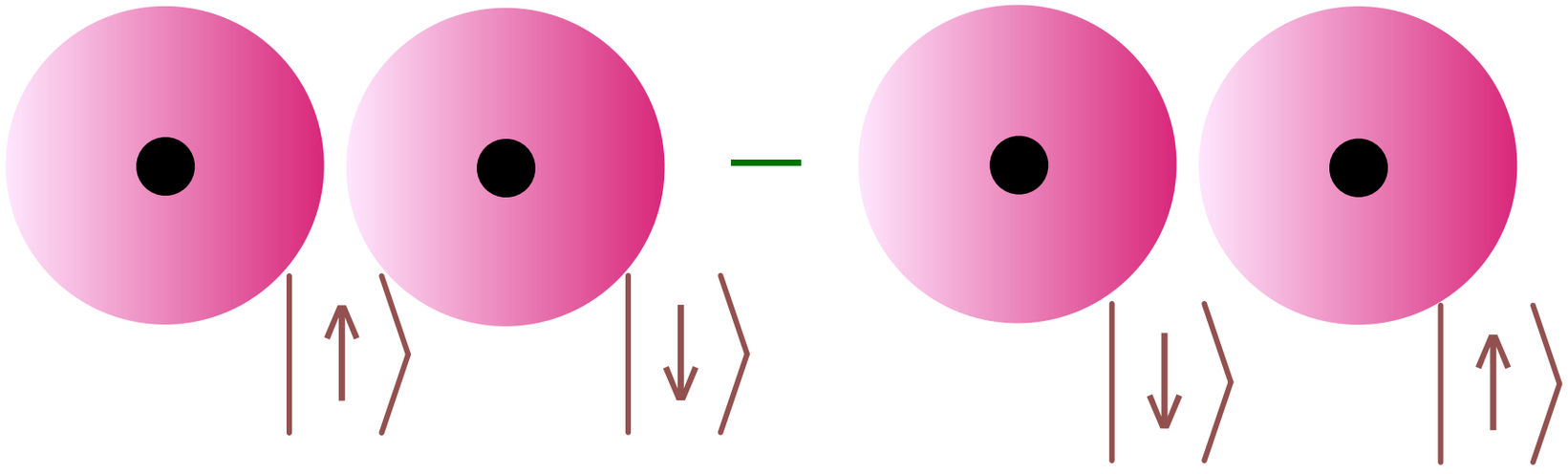
Black Holes

Objects so dense that light is gravitationally bound to them.

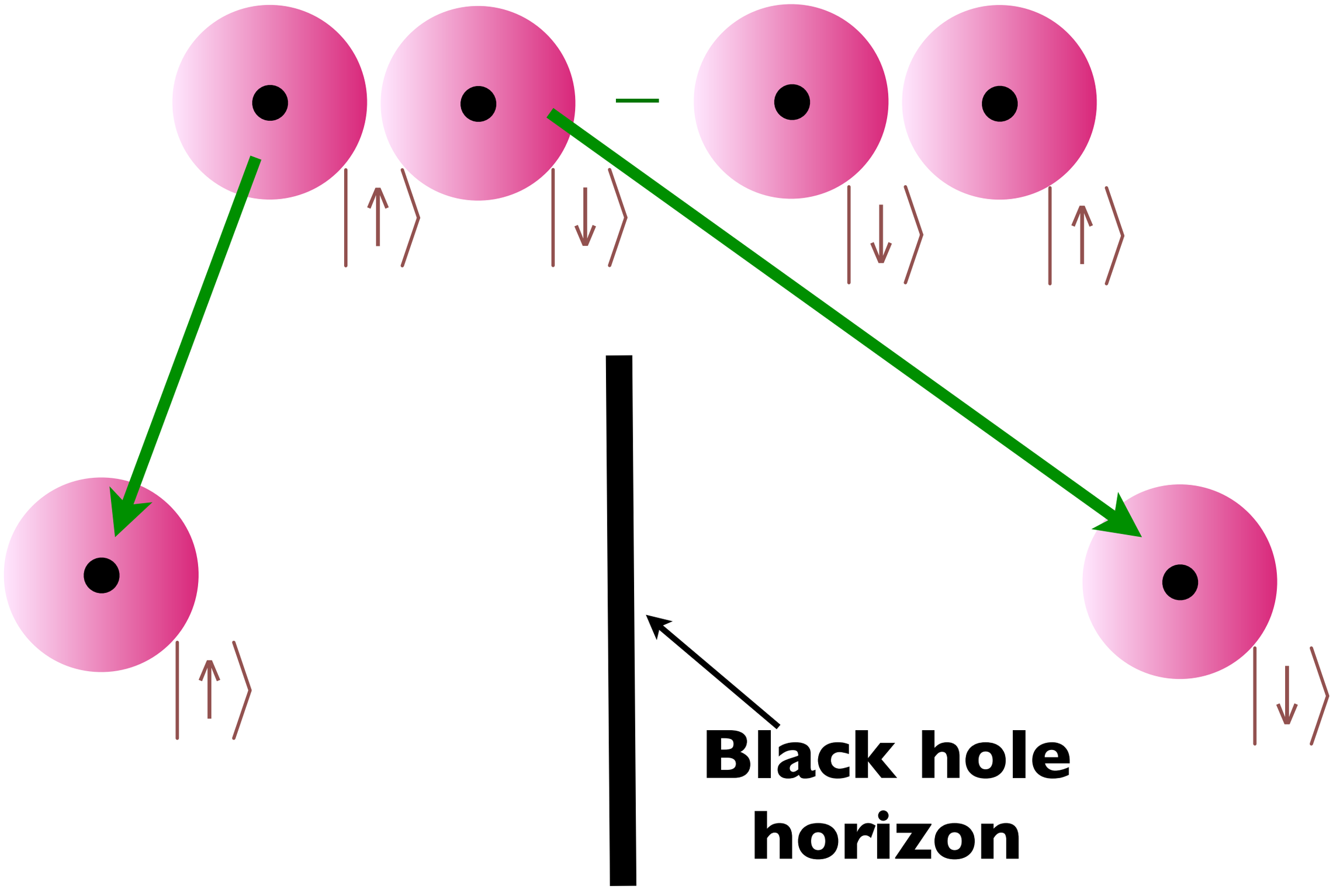
In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.



Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



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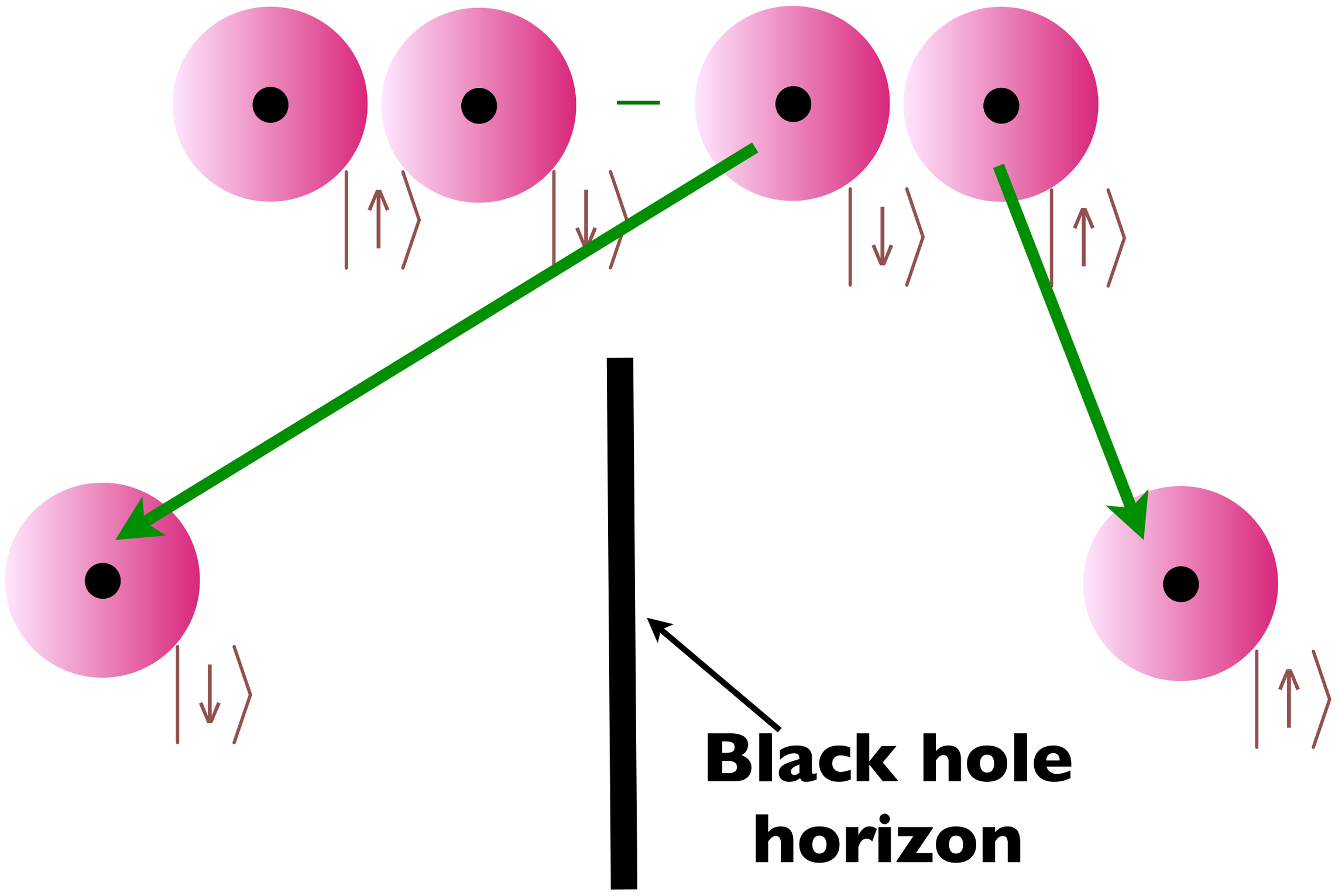
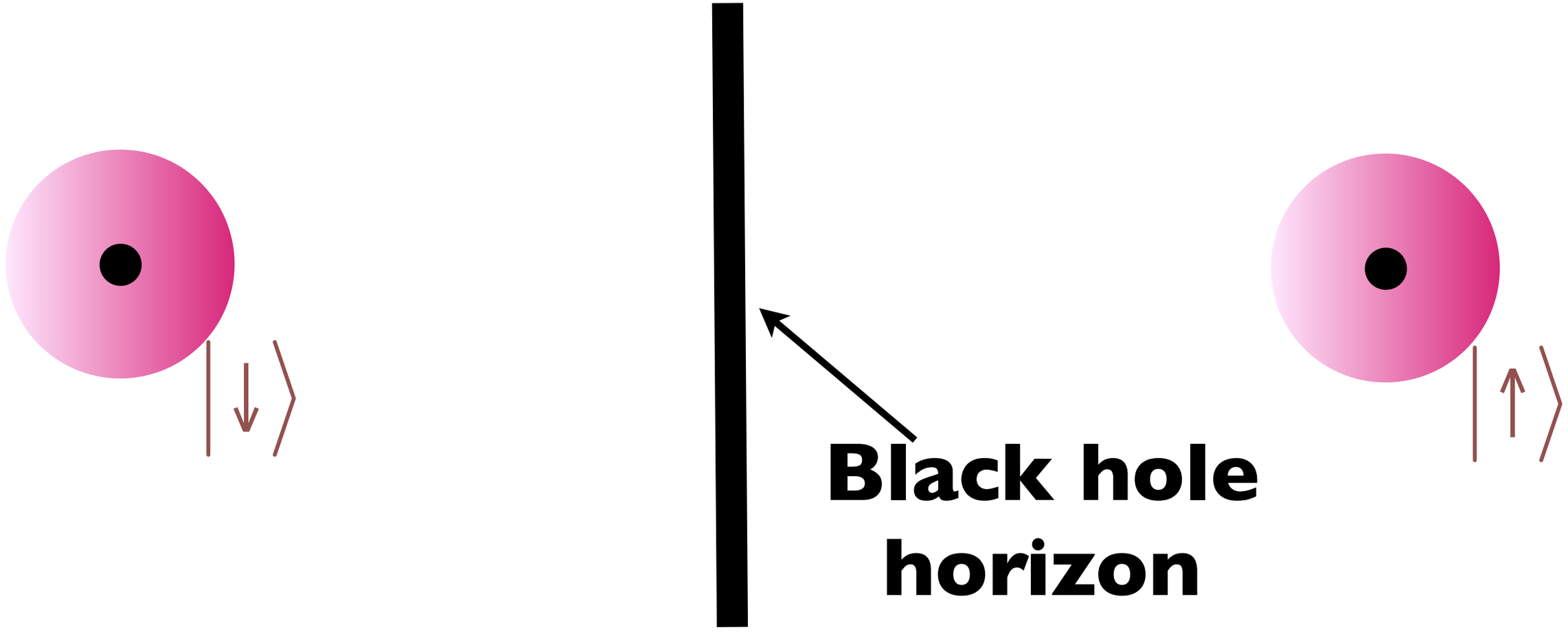


Quantum Entanglement across a black hole horizon

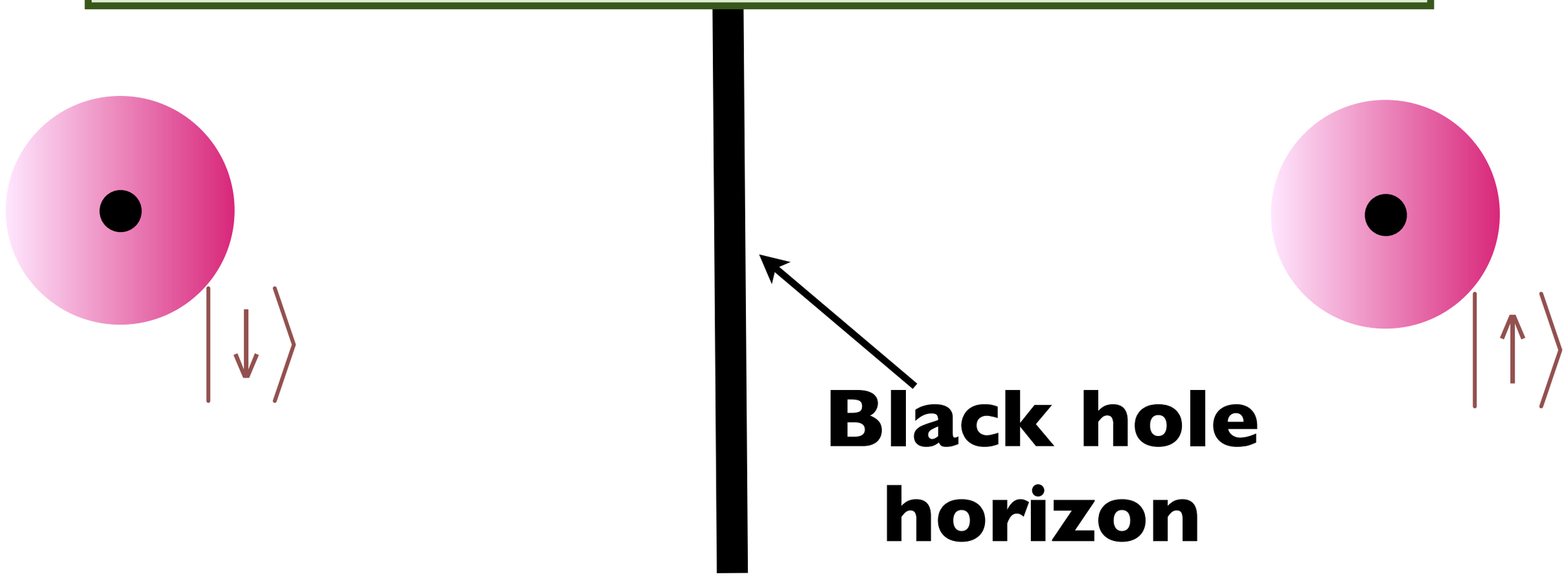
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

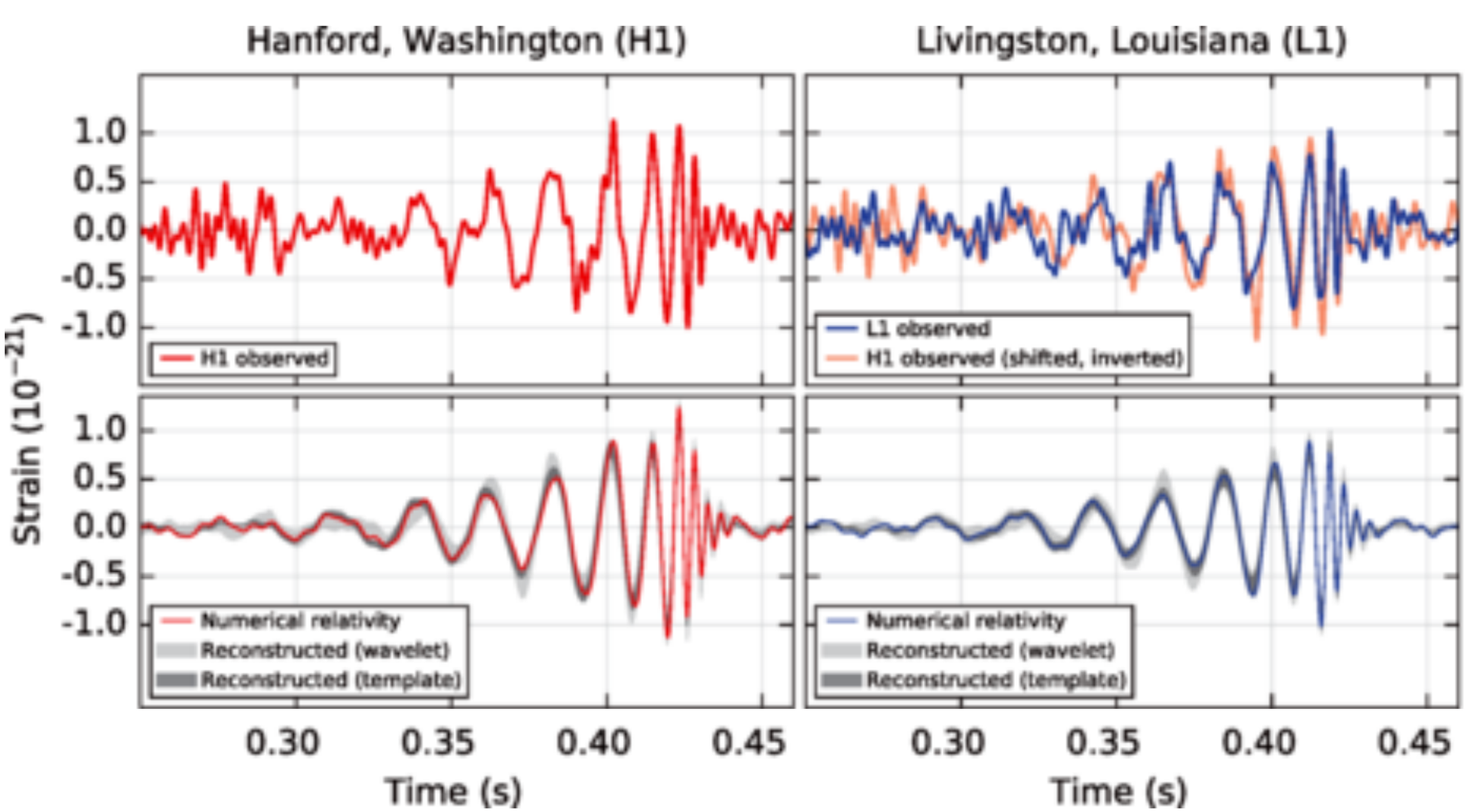
(because to an outside observer, the state of the electron inside the black hole is an unknown)

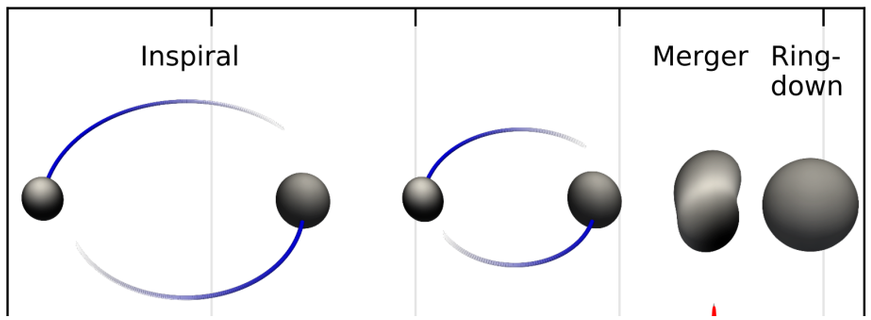


Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

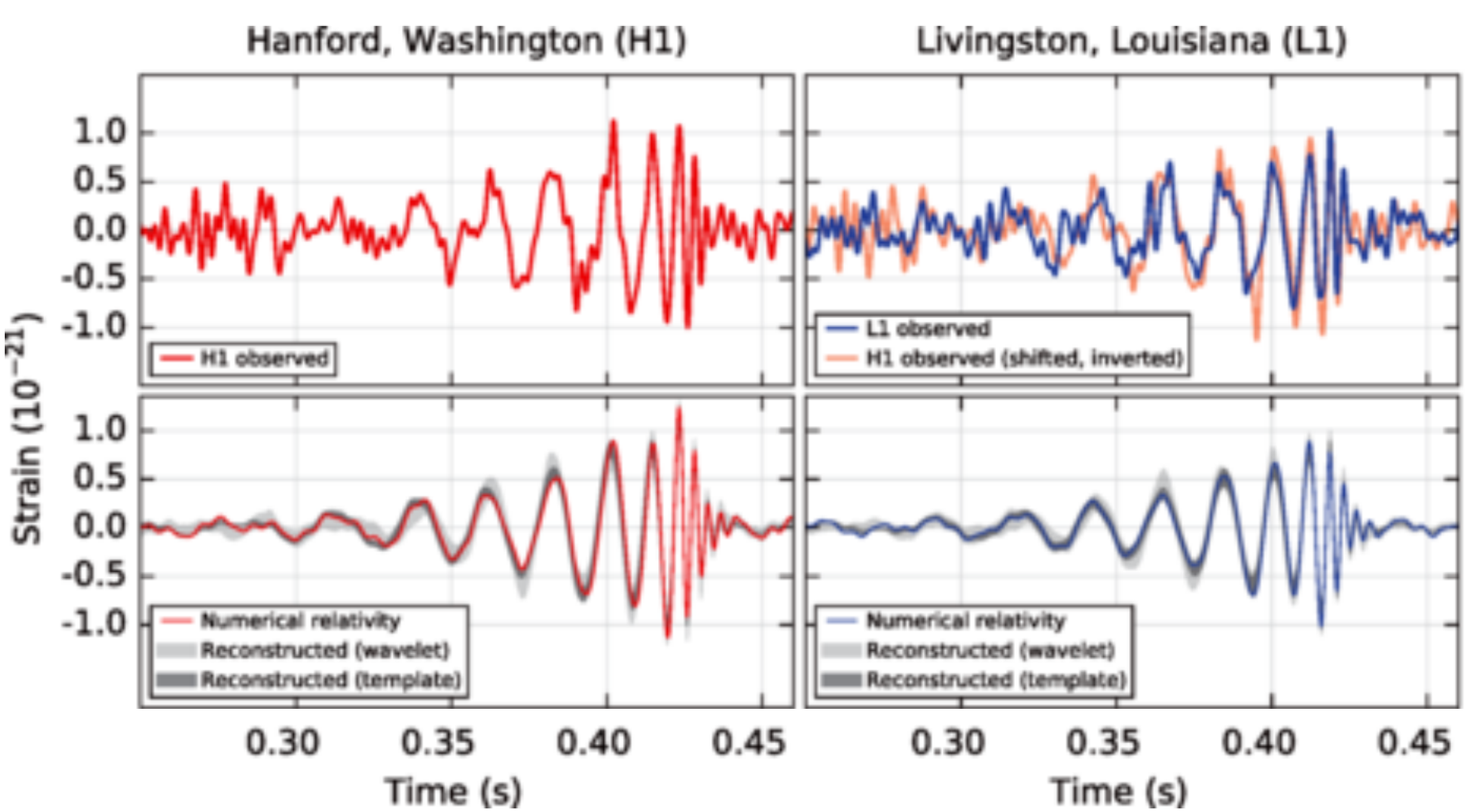


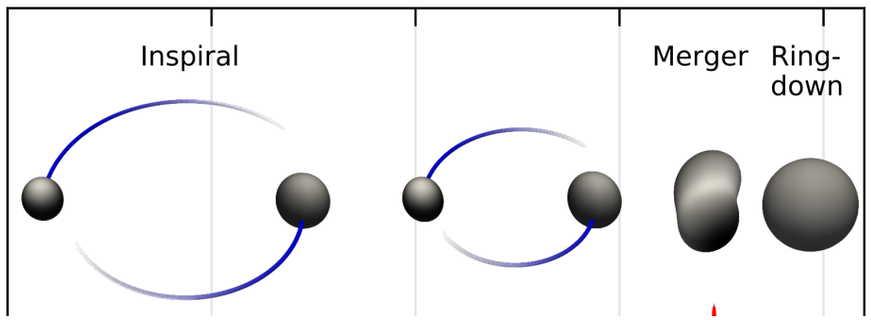


LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds.

C. V. Vishveshwara, Nature **227**, 936 (1970)





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- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\dfrac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

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Holography:
Quantum black holes "look like"
quantum many-particle systems
without quasiparticle excitations,
residing "on" the surface of the black hole

J. Maldacena, IJTP **38**, 1113 (1999); S.S. Gubser, I.R. Klebanov, and A.M. Polyakov Phys. Lett. B **428**, 105 (1998);
E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)

Ordinary metals:
quasiparticles

Strange metals:
no quasiparticles

Black
holes

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Black holes

The SYK model also describes charged black holes at low T !

We use a theory of Maxwell's electromagnetism and Einstein's general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge

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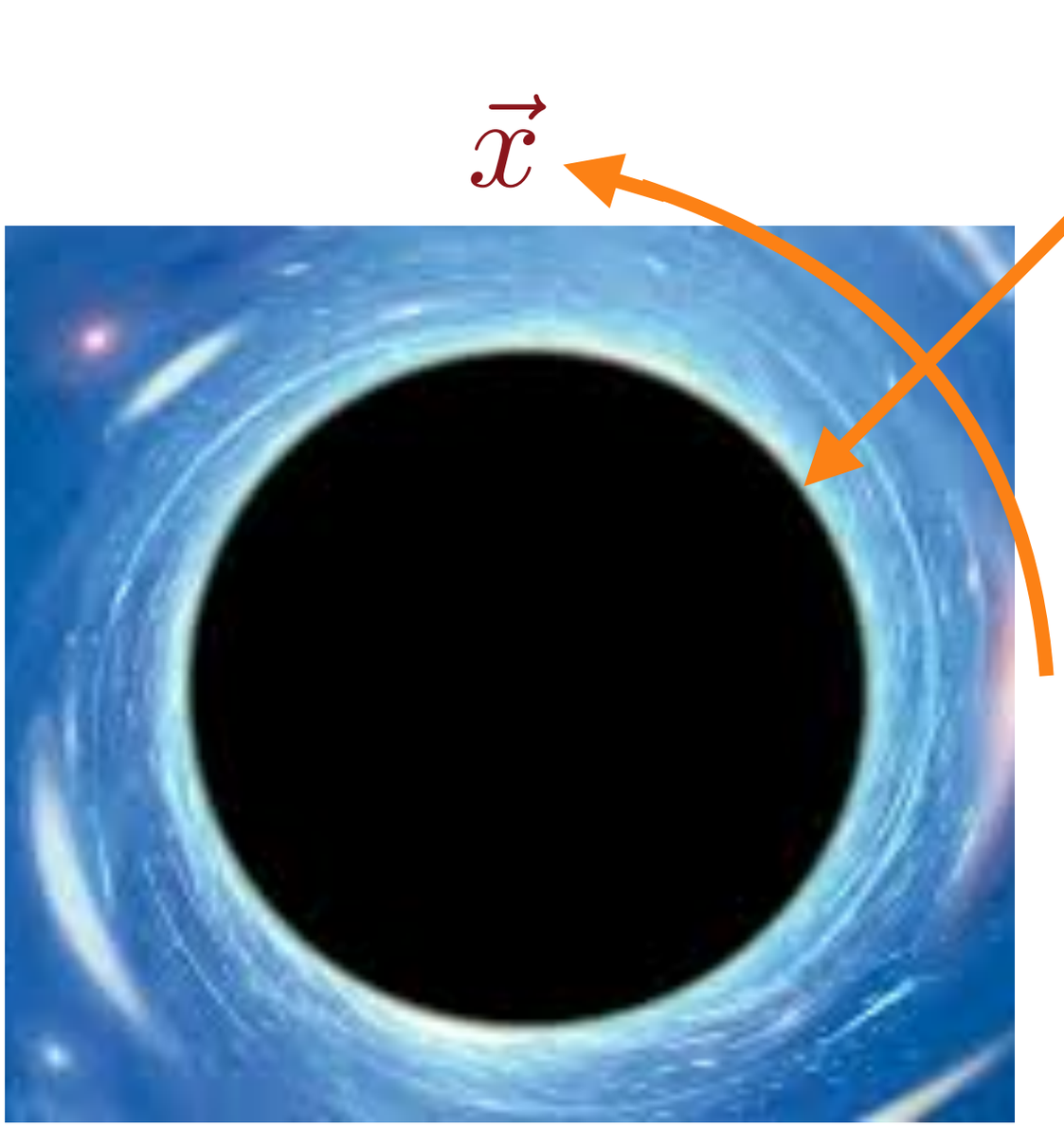
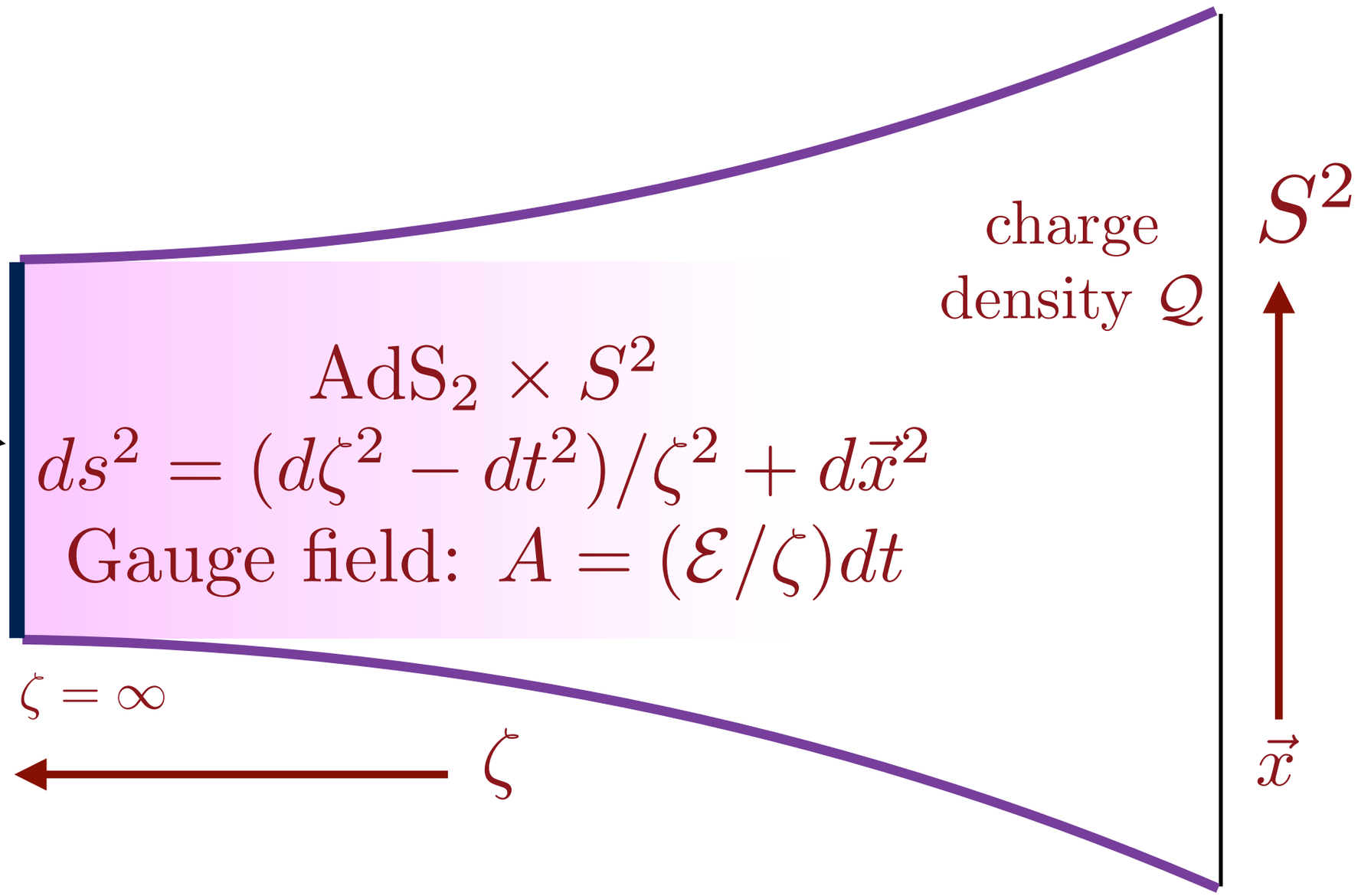


Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

SYK model and charged black holes

Black hole horizon



The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model

SYK model and charged black holes

Black hole horizon

$\mathrm{AdS}_2 \times S^2$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

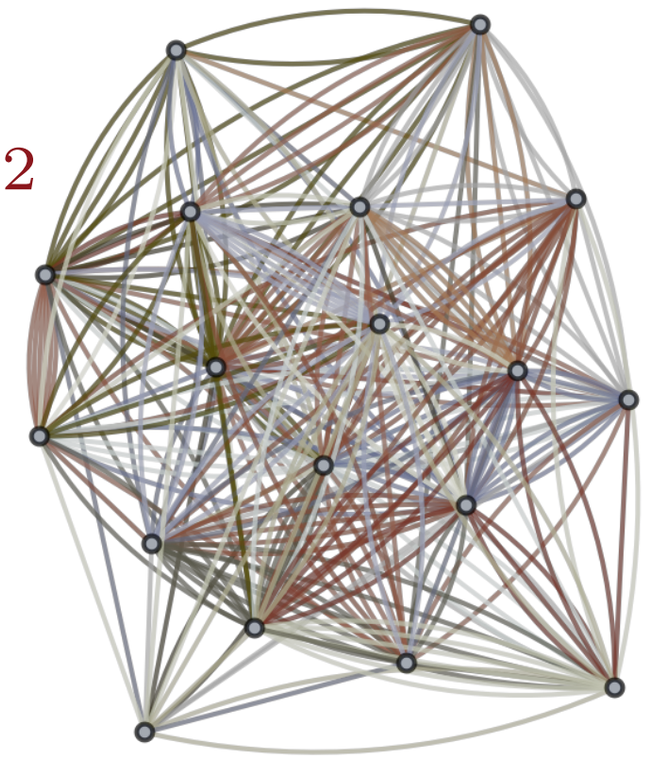
charge density $\mathcal{Q}$

$\zeta = \infty$

ζ

S^2

$\vec{x}$



Bekenstein-Hawking entropy of AdS_2 horizon at $T = 0 \Leftrightarrow N s_0$ entropy of SYK model

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

SYK model and charged black holes

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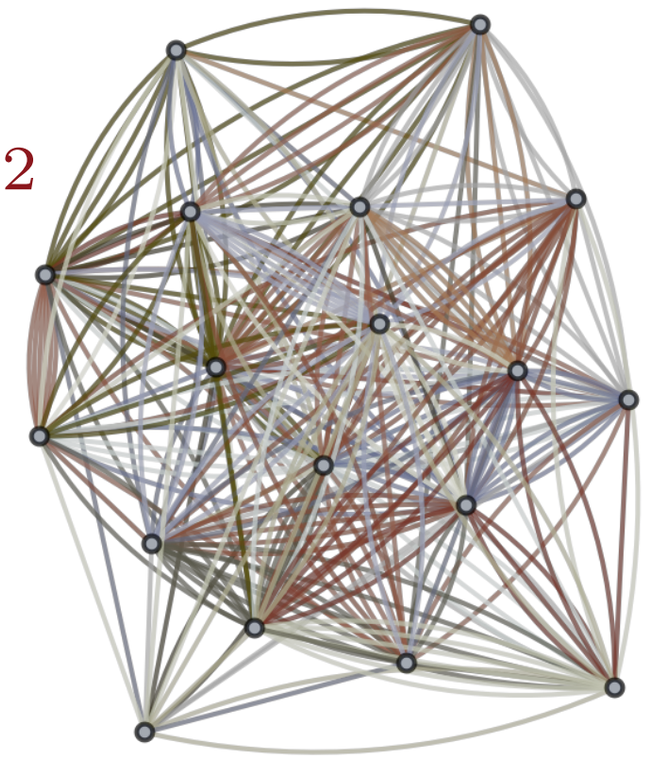

Einstein's equations imply that the growth of the Bekenstein-Hawking entropy with the black hole charge obeys the same relation as the SYK model:

$$\frac{\partial S_{BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

where $\mathcal{E}$, the near-horizon electric field, determines the particle-hole asymmetry, also as in the SYK model.

A Sen, JHEP **0509**, 038 (2005); S. Sachdev, PRX **5,** 041025 (2015)

SYK model and charged black holes

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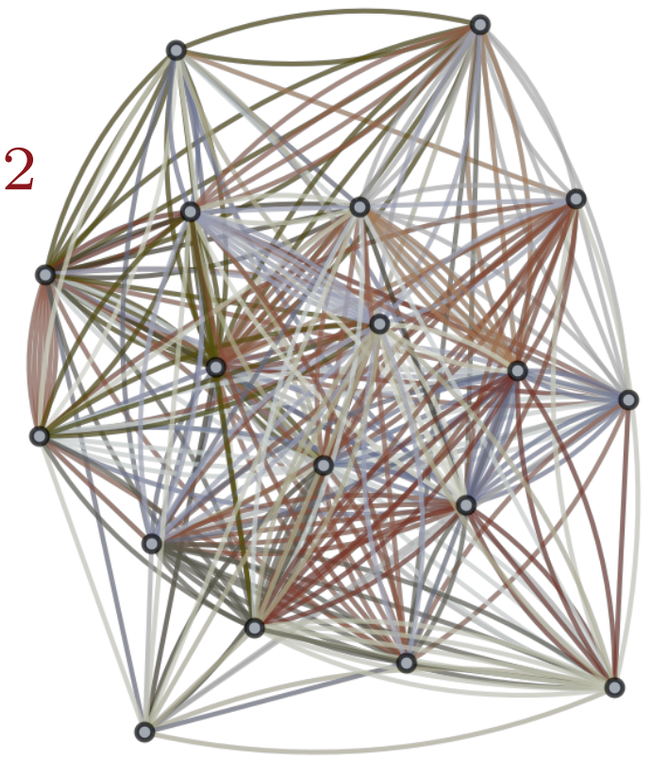

The correlator of a probe fermion near the black hole horizon is the same as that in the SYK model

For long times $\tau > 0$,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle = \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_i^\dagger(\tau) c_i(0) \right\rangle = e^{-2\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The particle-hole asymmetry parameter $\mathcal{E}$ also determines the electric field on the surface of the black hole.

T. Faulkner, Hong Liu, J. McGreevy, D.Vegh, PRD **83**, 125002 (2011)

SYK model and charged black holes

Horizon

total charge $\mathcal{Q}$

S^2

$\mathrm{AdS}_2 \times S^2$

$$ds^2 = R_2^2 \frac{(d\zeta^2 - dt^2)}{\zeta^2} + R_h^2 d\Omega_2^2$$

Gauge field: $A = \dfrac{\mathcal{E}}{\zeta}\, dt$

Schwarzian

AdS_4

ζ

Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a 'Schwarzian' theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS_2 and AdS_4.

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

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and

Charged black holes in 3+1 dimensions of radius R_h, with total charge $\mathcal{Q}$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0+1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f\, \mathcal{D}\phi\, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$$f(\tau + 1/T) = f(\tau) + 1/T,$$

ϕ is a phase conjugate to the charge density with

$$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n,\ n \text{ integer},$$

$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.
The couplings are related to the entropy $S(T, \mathcal{Q})$ and the chemical potential μ via

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{d\mathcal{Q}}$$

Main result

- Not the usual AdS/CFT correspondence, which involves only neutral black holes at $T > 0$.
- Unlike the AdS/CFT correspondence, *both* sides of the duality are solvable. This has enabled numerous recent studies of black holes quantum information.

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

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R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

Quantum matter without quasiparticles

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- Black holes thermalize in a Planckian time $\sim \hbar/(k_B T_H)$, where T_H is the Hawking temperature.

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, (along with a phase fluctuating mode) describes the quantum dynamics of

  - the SYK models
  - black holes with near-extremal AdS_2 horizons