

The pseudogap phase of the cuprate superconductors

University of Maryland
December 9, 2014

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PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS



JOHN TEMPLETON
FOUNDATION

PHYSICS



HARVARD

Talk online: sachdev.physics.harvard.edu

Theorists at Harvard



Max Metlitski
(KITP, UCSB)



Andrea Allais



Matthias Punk
(Innsbruck)



Debanjan
Chowdhury



Alexandra
Thomson



Jay Sau
(Maryland)

Cornell



Kazuhiro Fujita
Cornell/ BNL



Mohammad Hamidian
Cornell / BNL



Stephen Edkins
Cornell / St Andrews



Michael Lawler

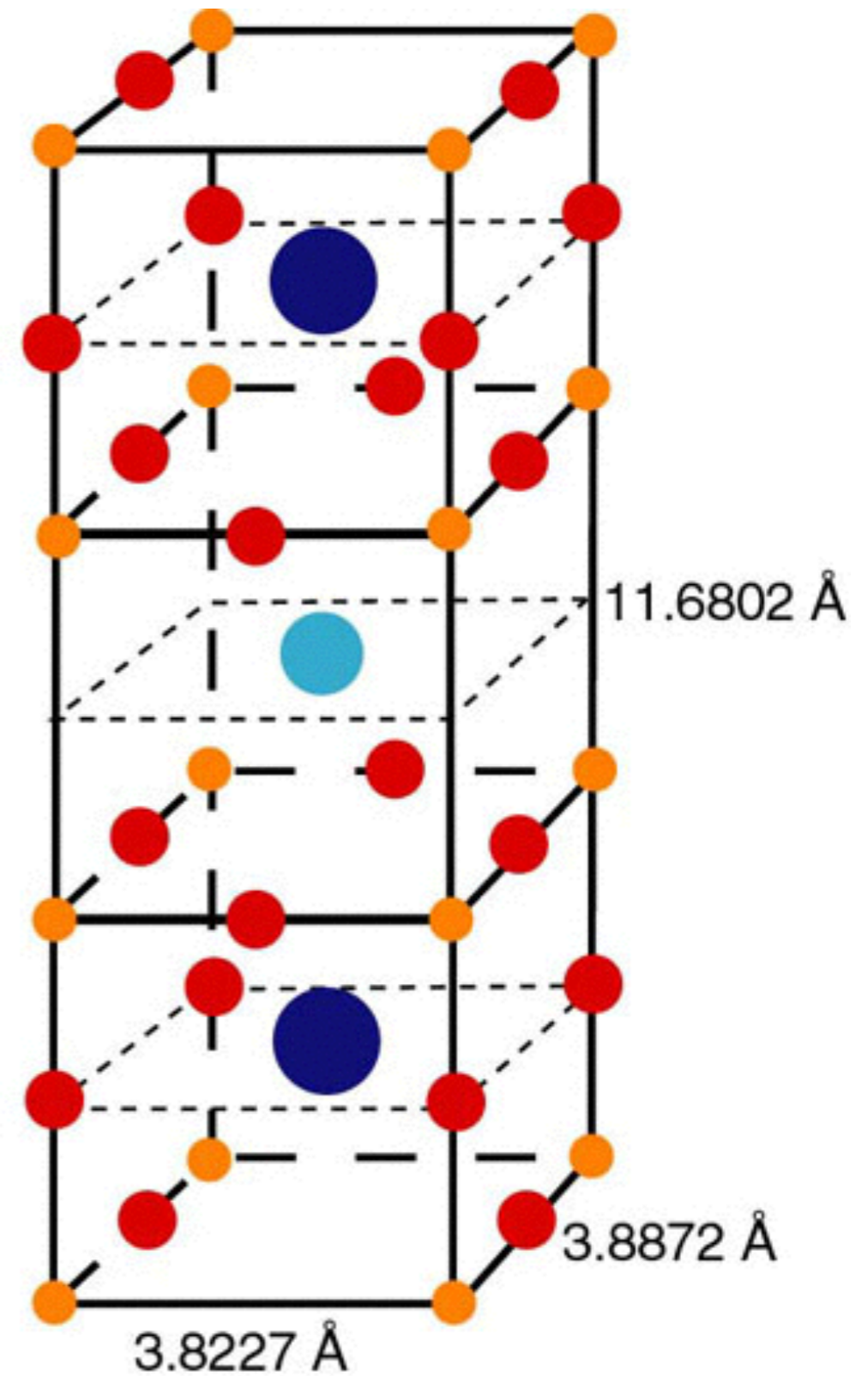
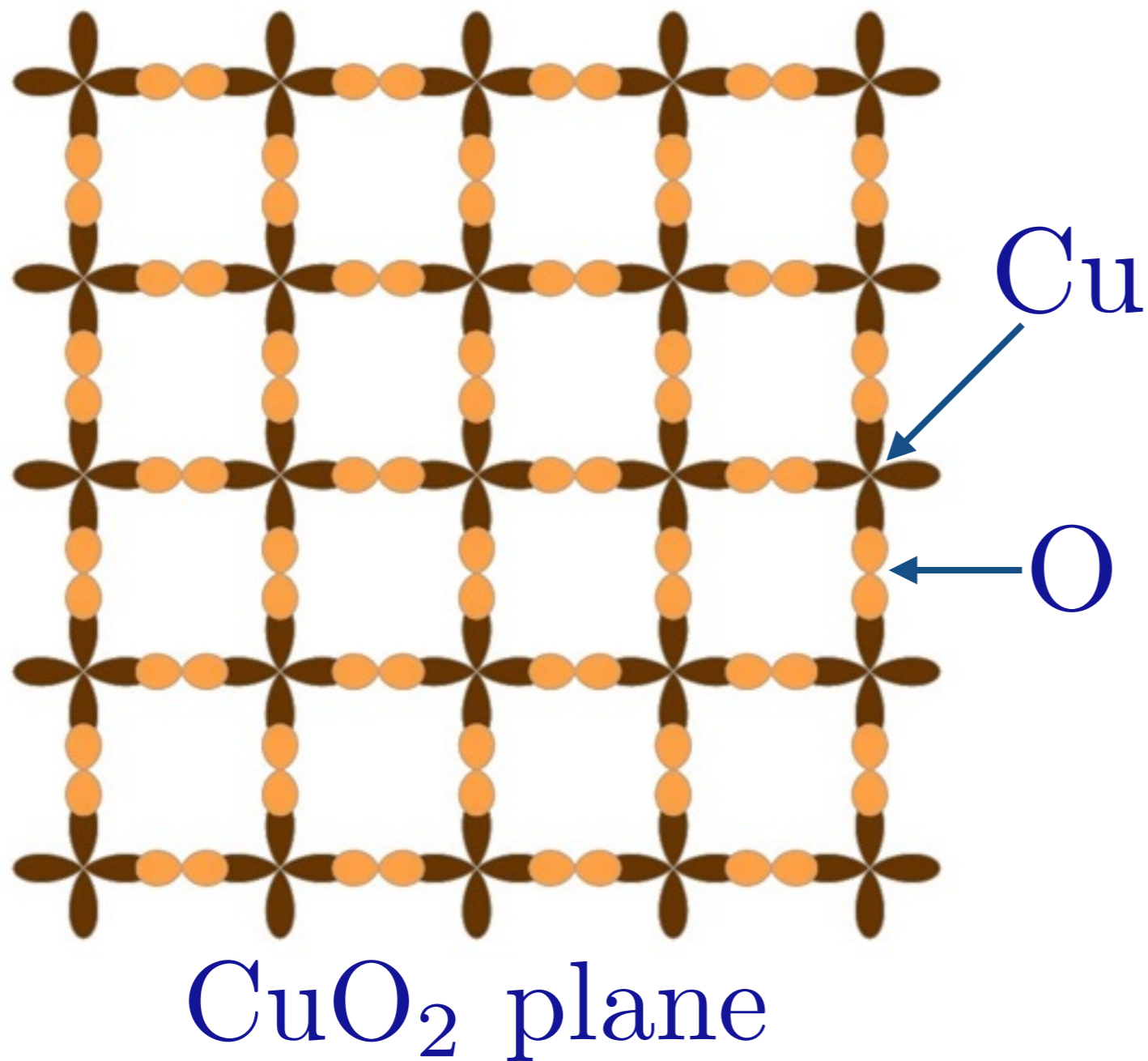


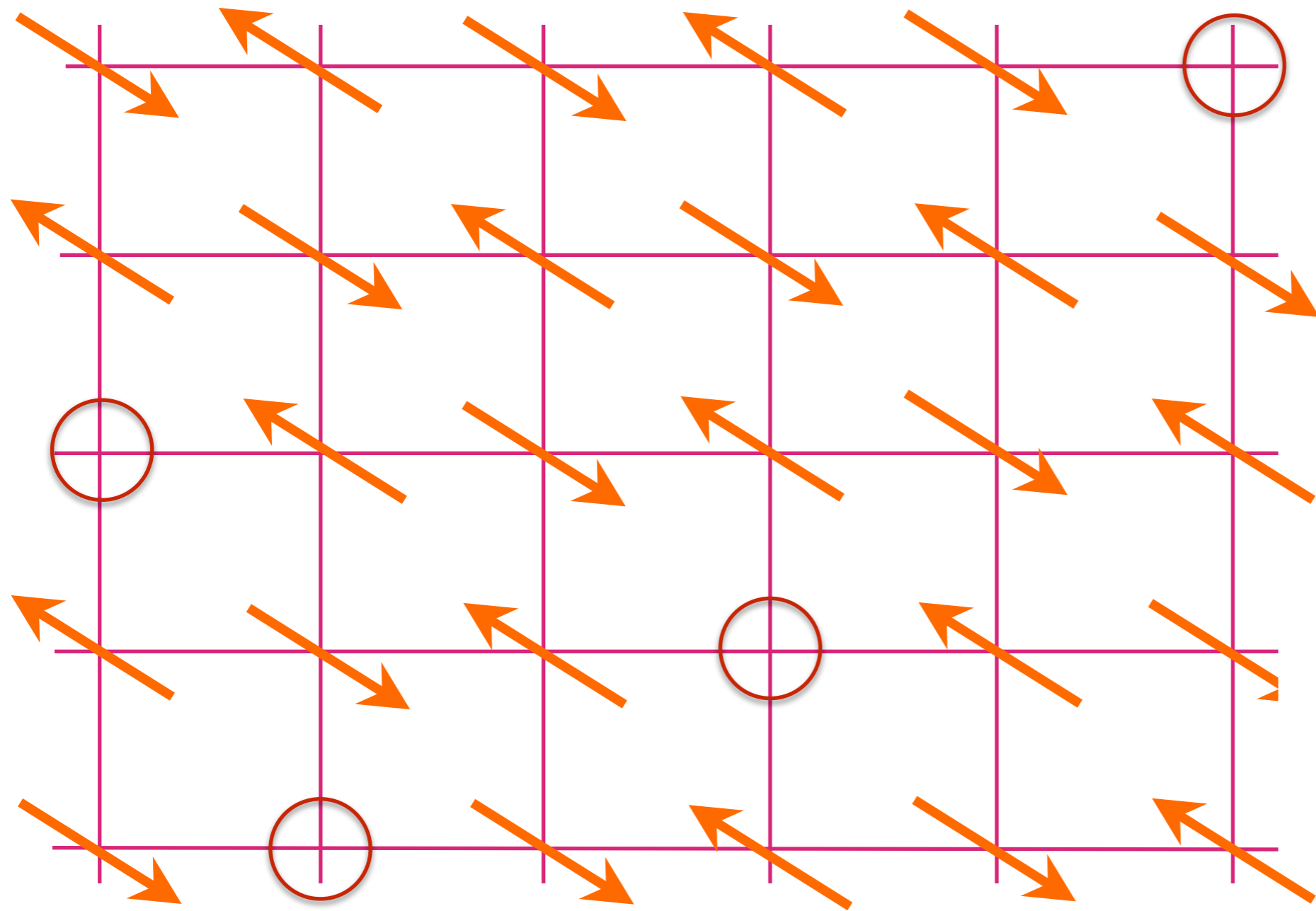
J. C. Seamus Davis



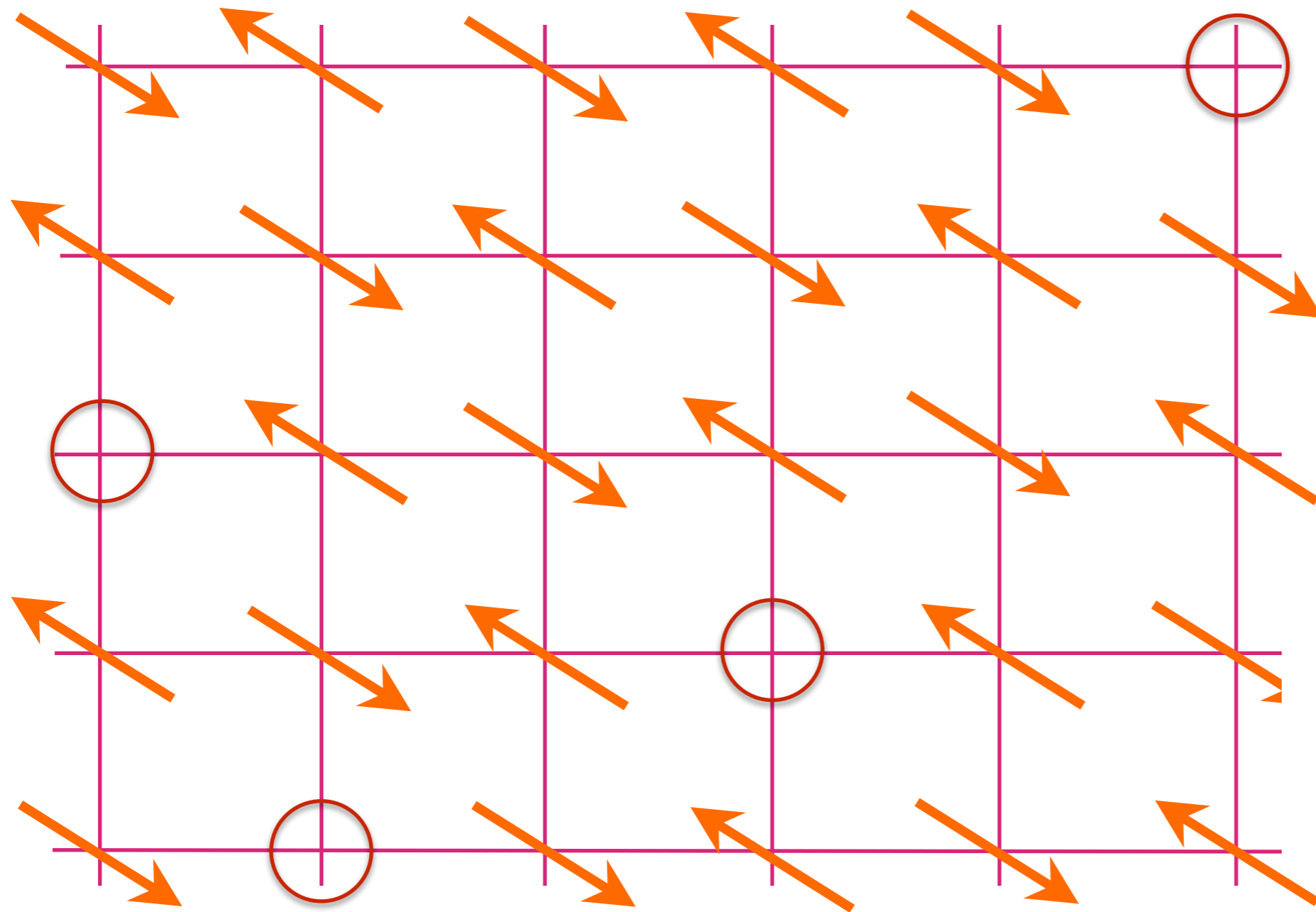
Eun-Ah Kim

High temperature superconductors



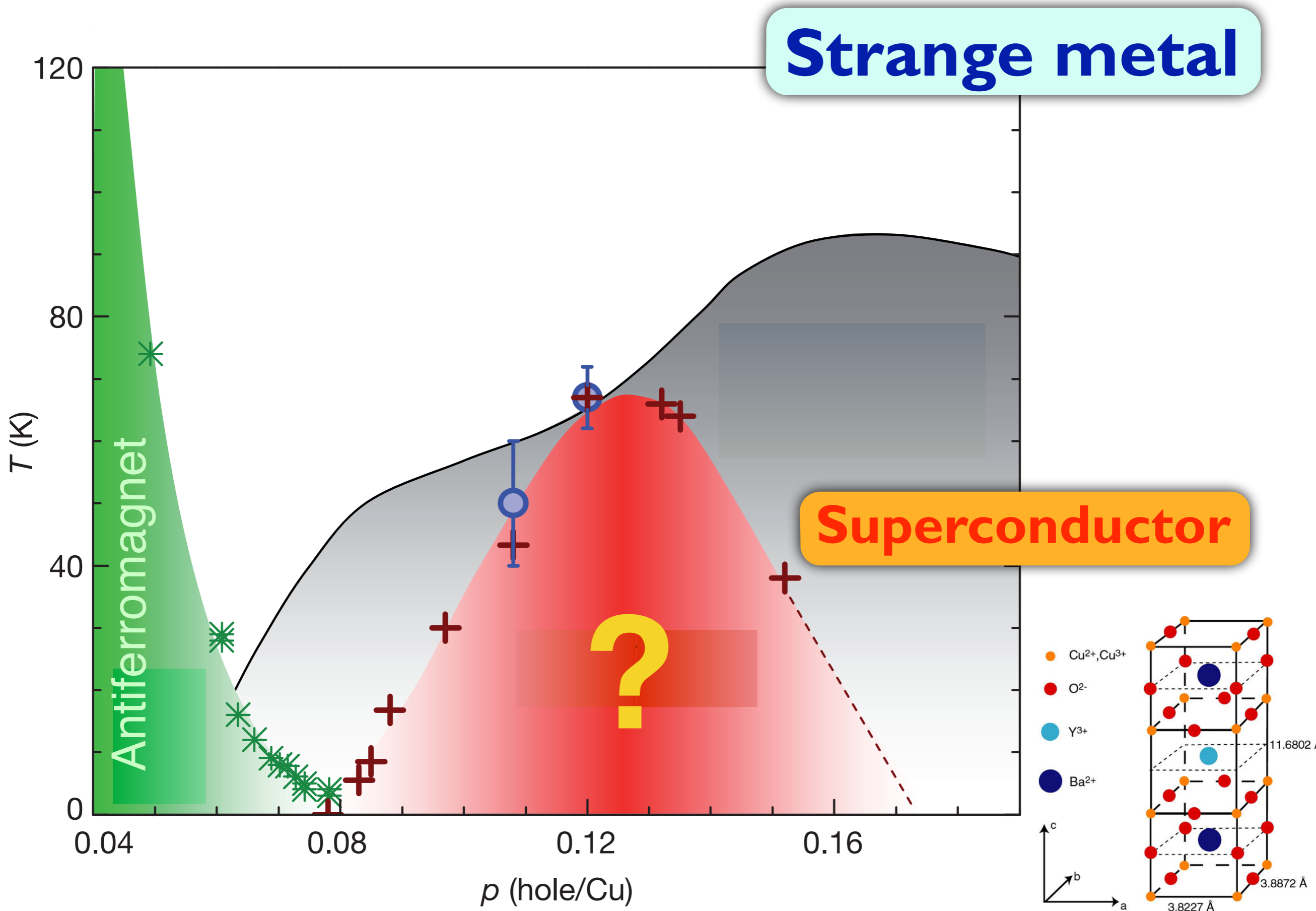


Anti-ferromagnet
with p holes
per square

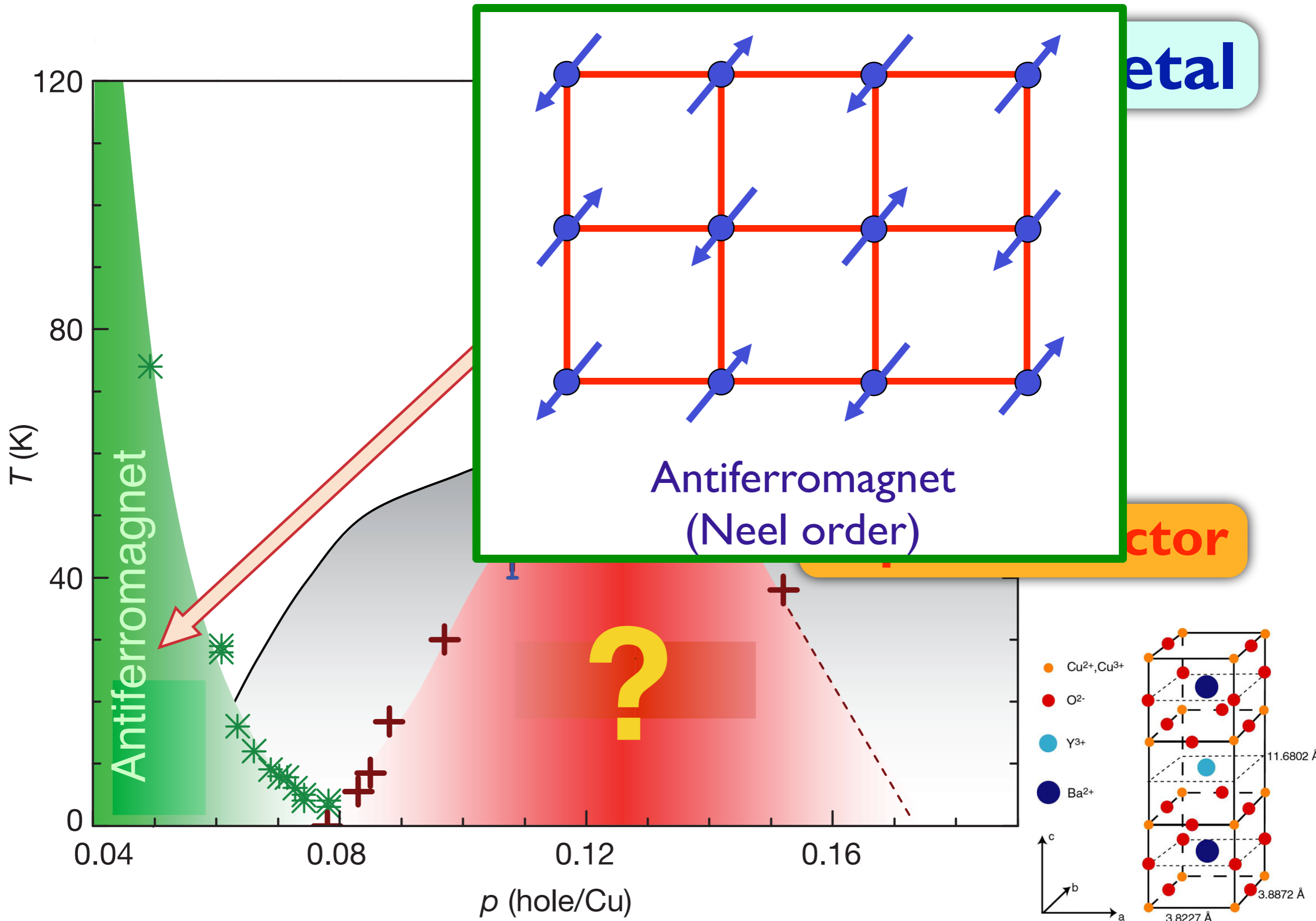


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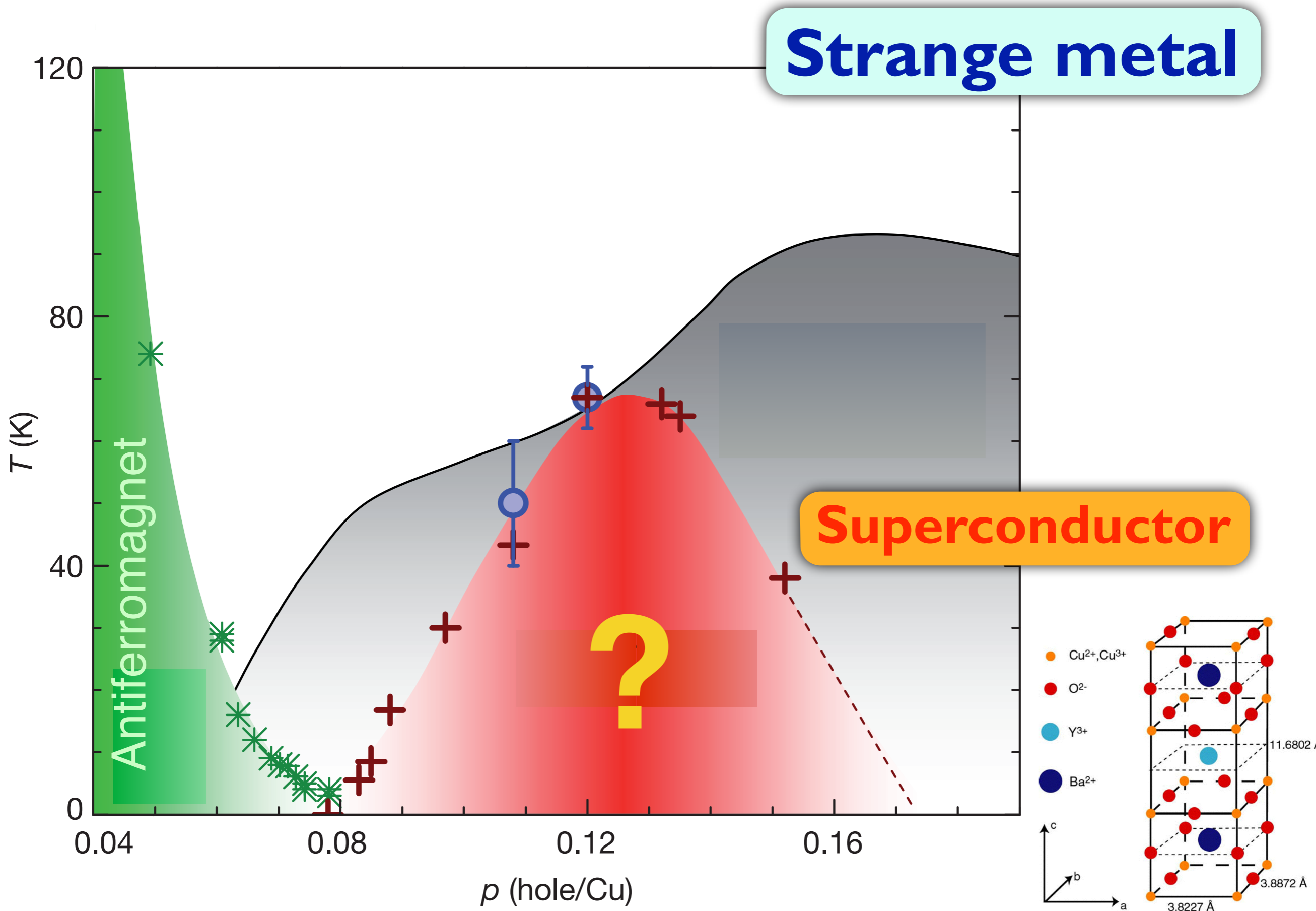
Note: relative to the fully-filled band insulator, there are $1+p$ holes per square



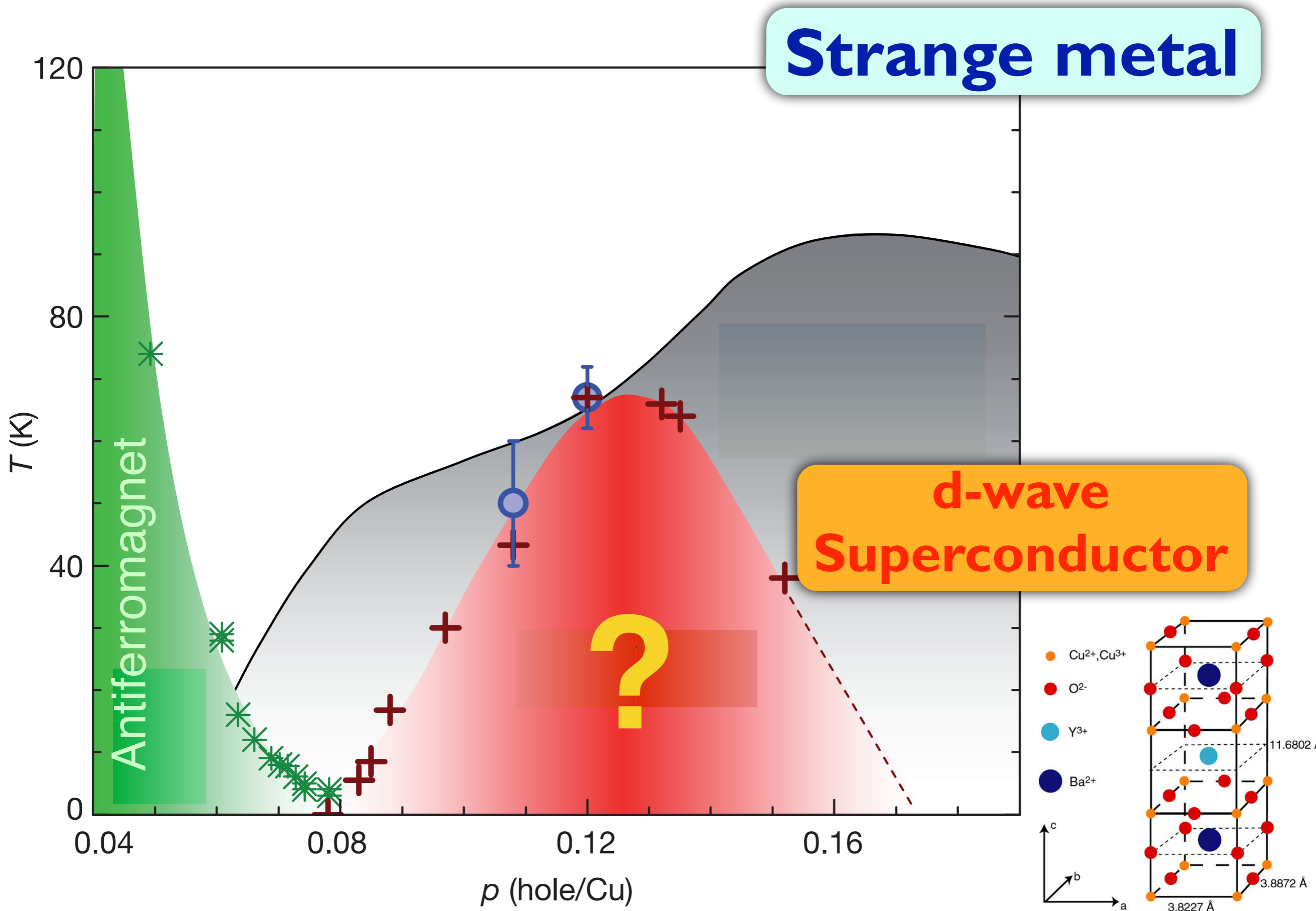
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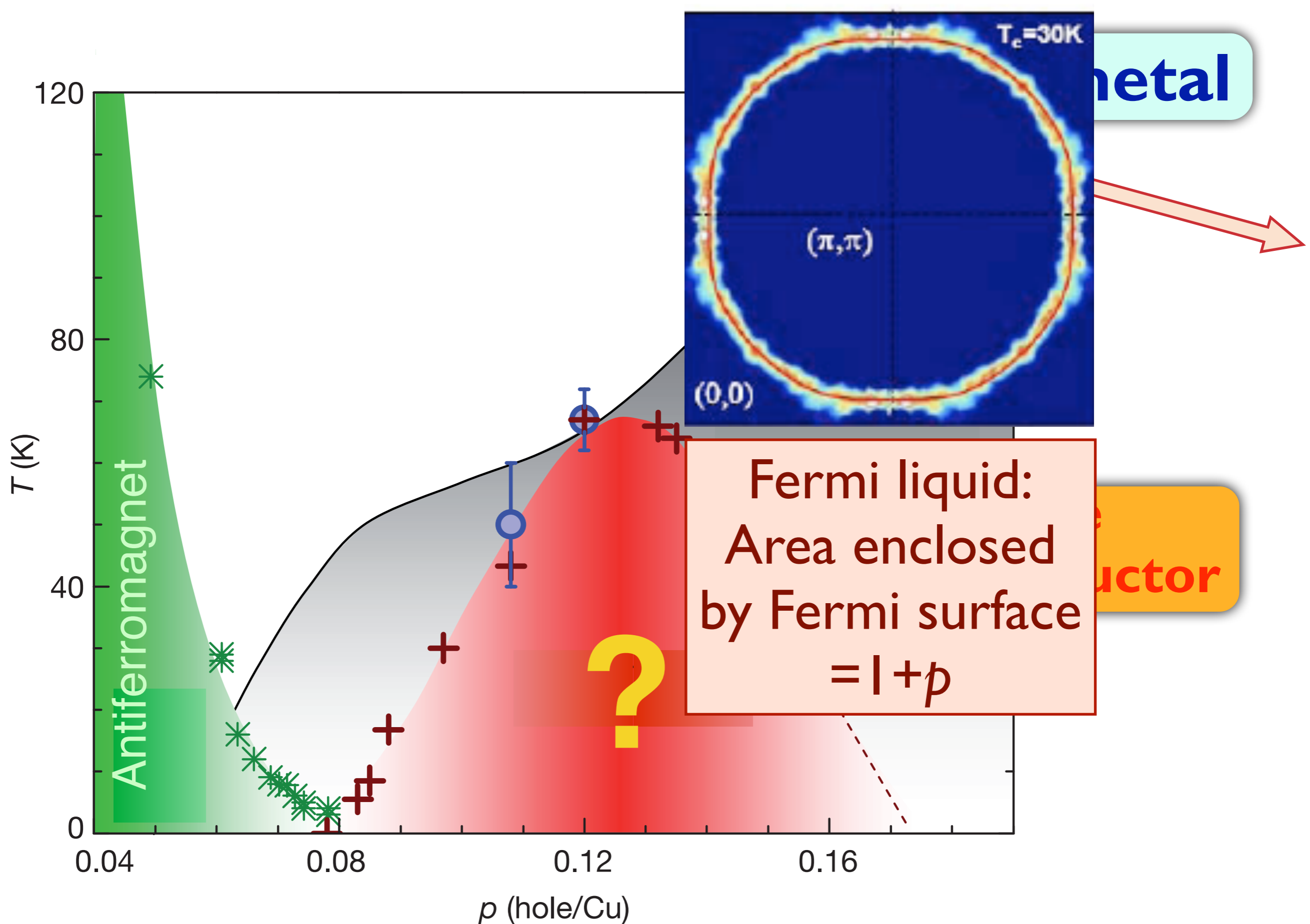
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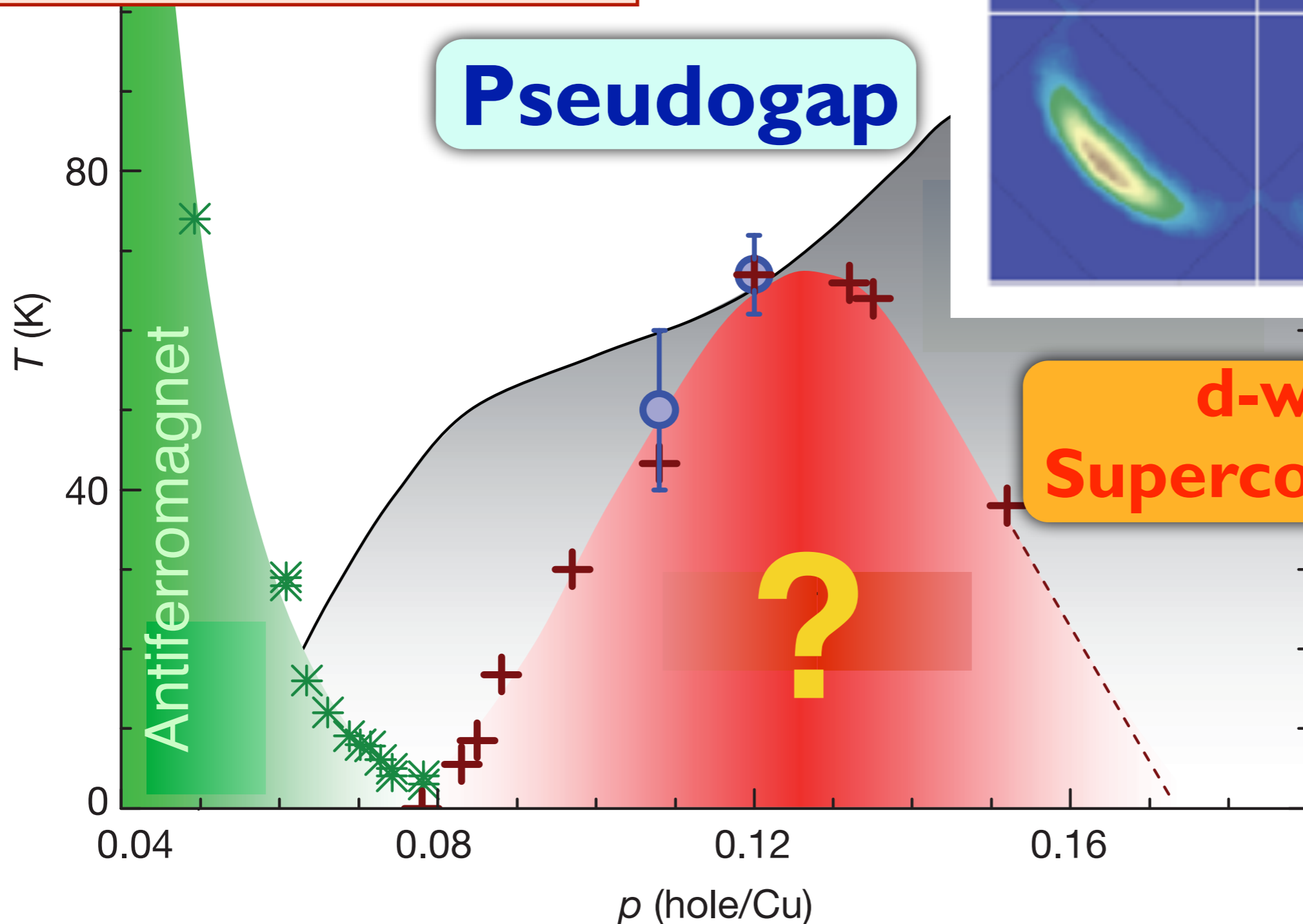
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M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

“Fermi arcs” at
low p

Pseudogap



d-wave
Superconductor

Key question

At low temperatures in the pseudogap regime, there is a broken symmetry at low temperatures associated with a charge density wave (to be described in more detail shortly).

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Key question

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Is the higher temperature pseudogap (with ``Fermi arc" spectra) described by

(A) Thermal fluctuations of the low temperature orders (superconductivity, charge density wave, antiferromagnetism...)

OR

(B) A new type of metal with “topological order”, which can be stable (in principle) as a quantum ground state

Outline

1. The low T pseudogap:
*STM observation of predicted
 d -form factor density wave*
2. The high T pseudogap:
*A metal with topological order:
the Fractionalized Fermi liquid: FL**
3. Connecting high and low T :
Density wave instabilities
4. Quantum critical point near optimal p :
A Higgs critical point

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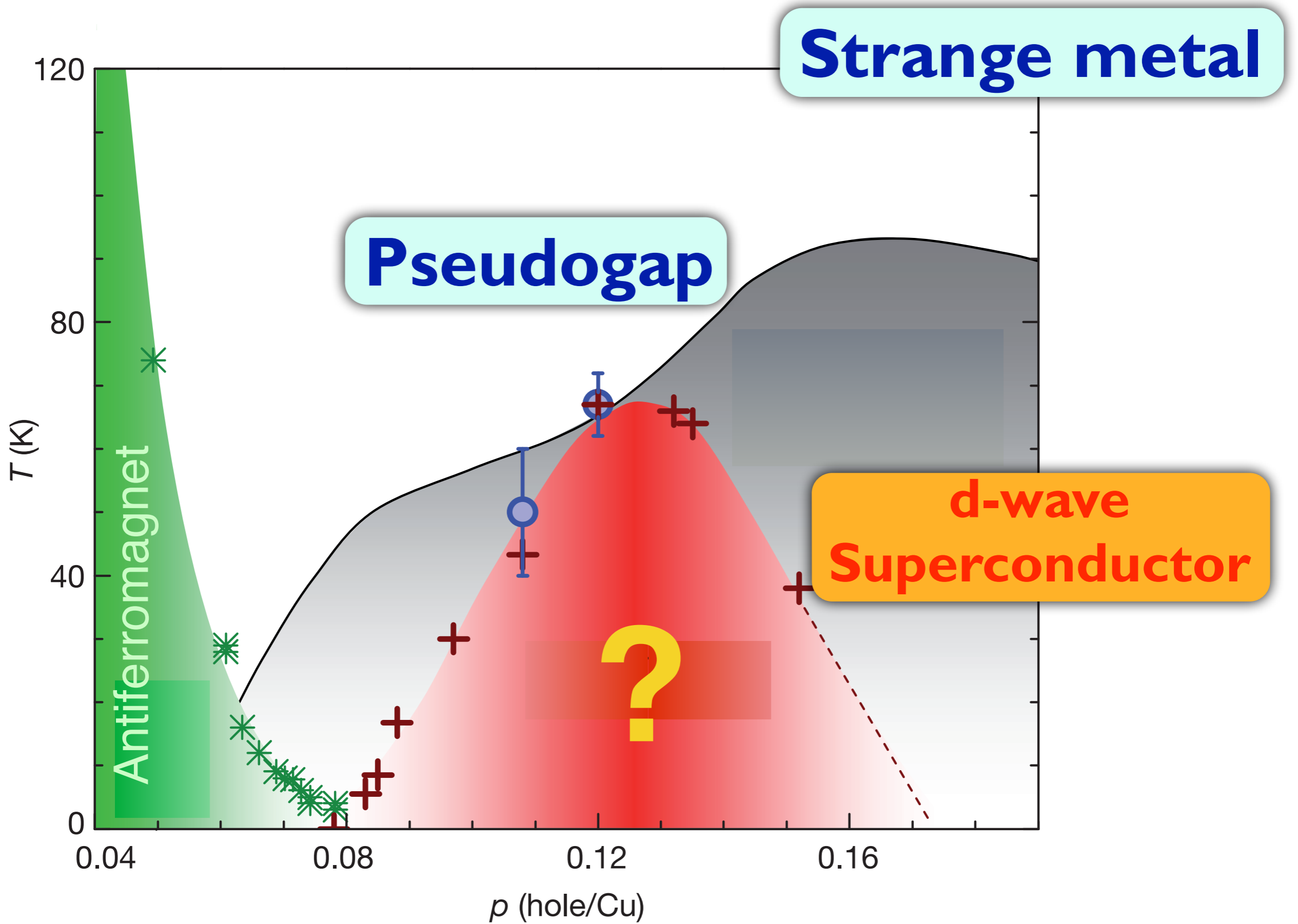
*A metal with topological order:
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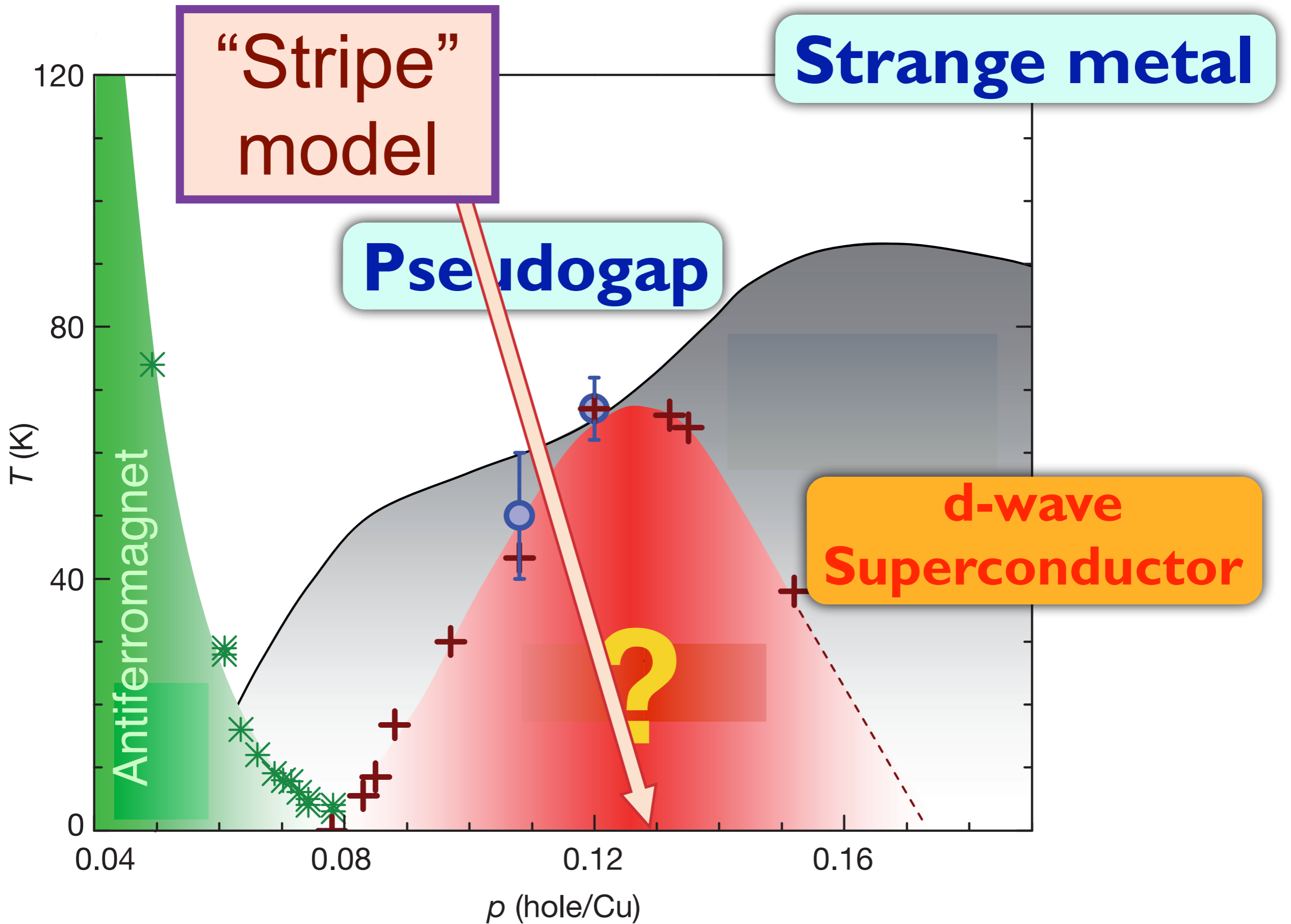
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Density wave instabilities

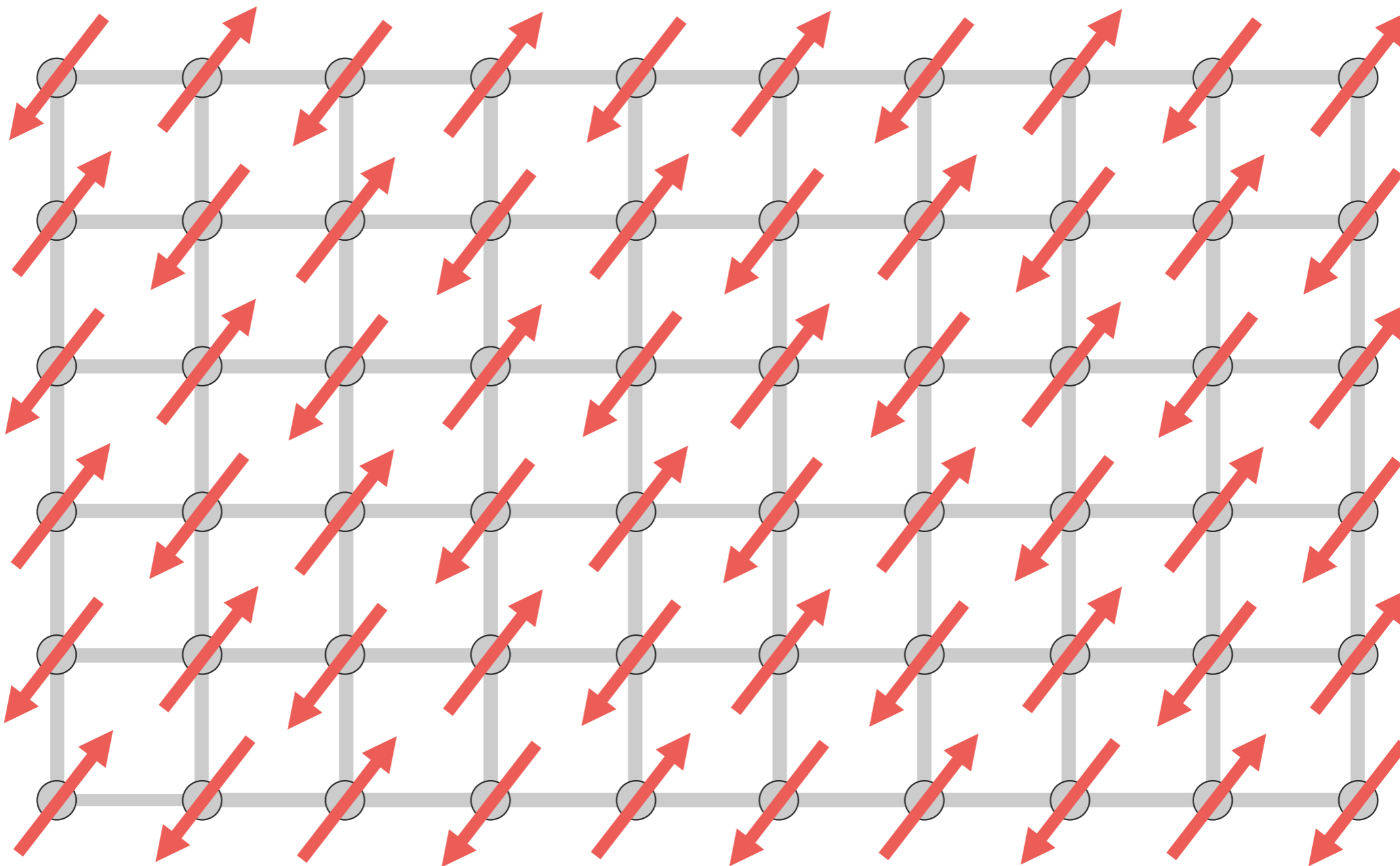
4. Quantum critical point near optimal p :

A Higgs critical point



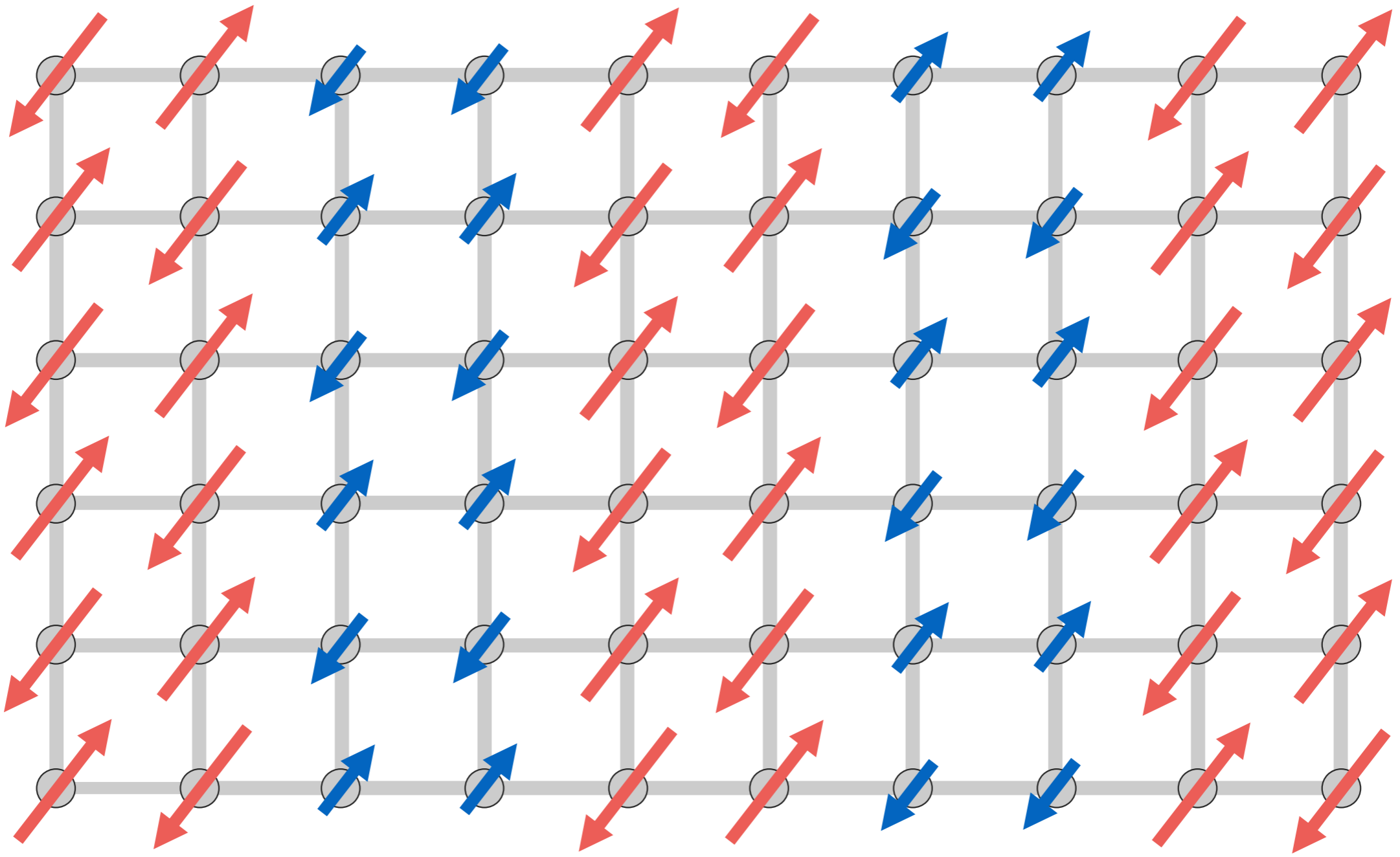


“Stripe” model



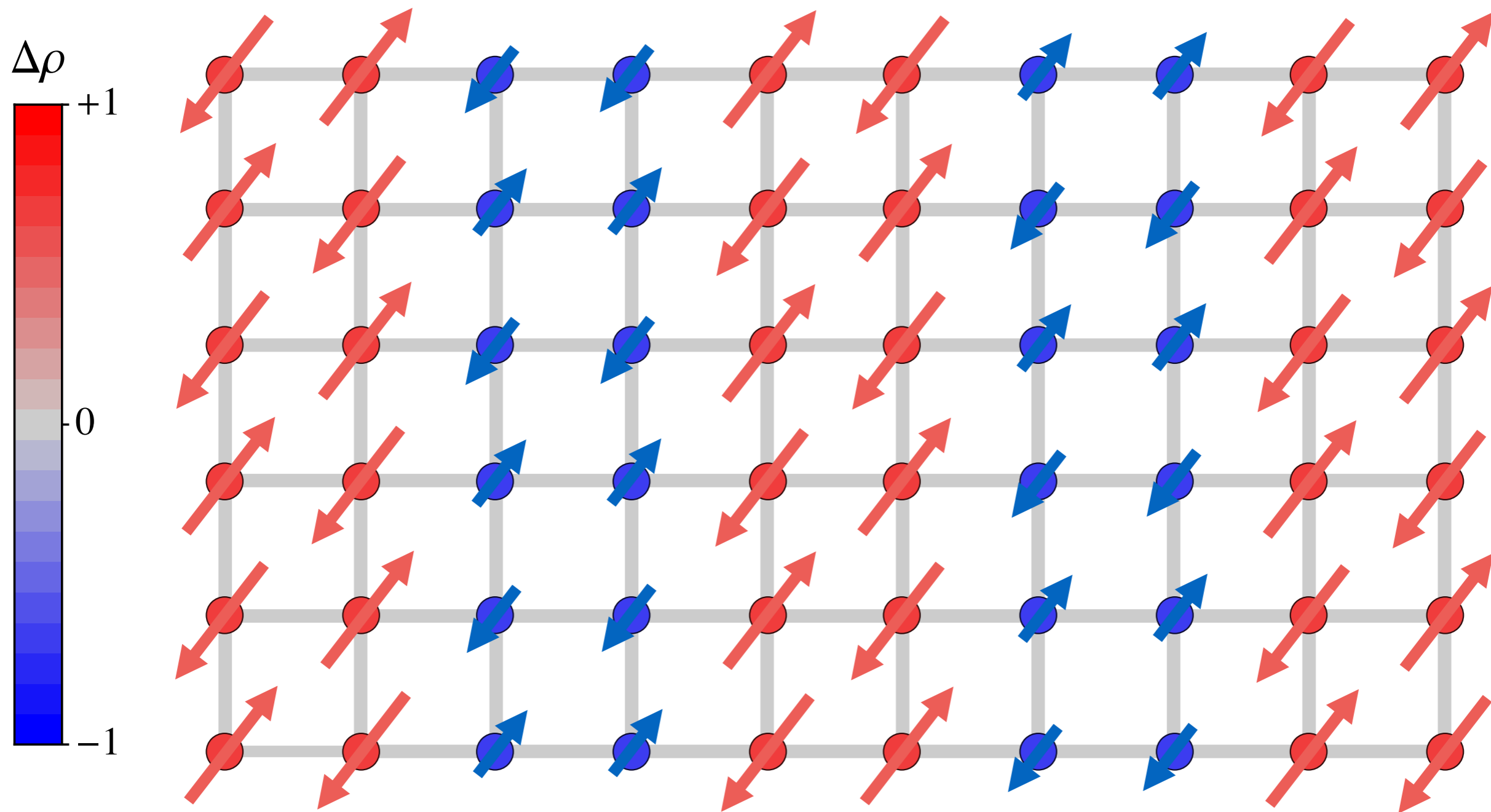
Start with an antiferromagnet

“Stripe”
model



Domain walls 4 lattice spacings apart

“Stripe” model

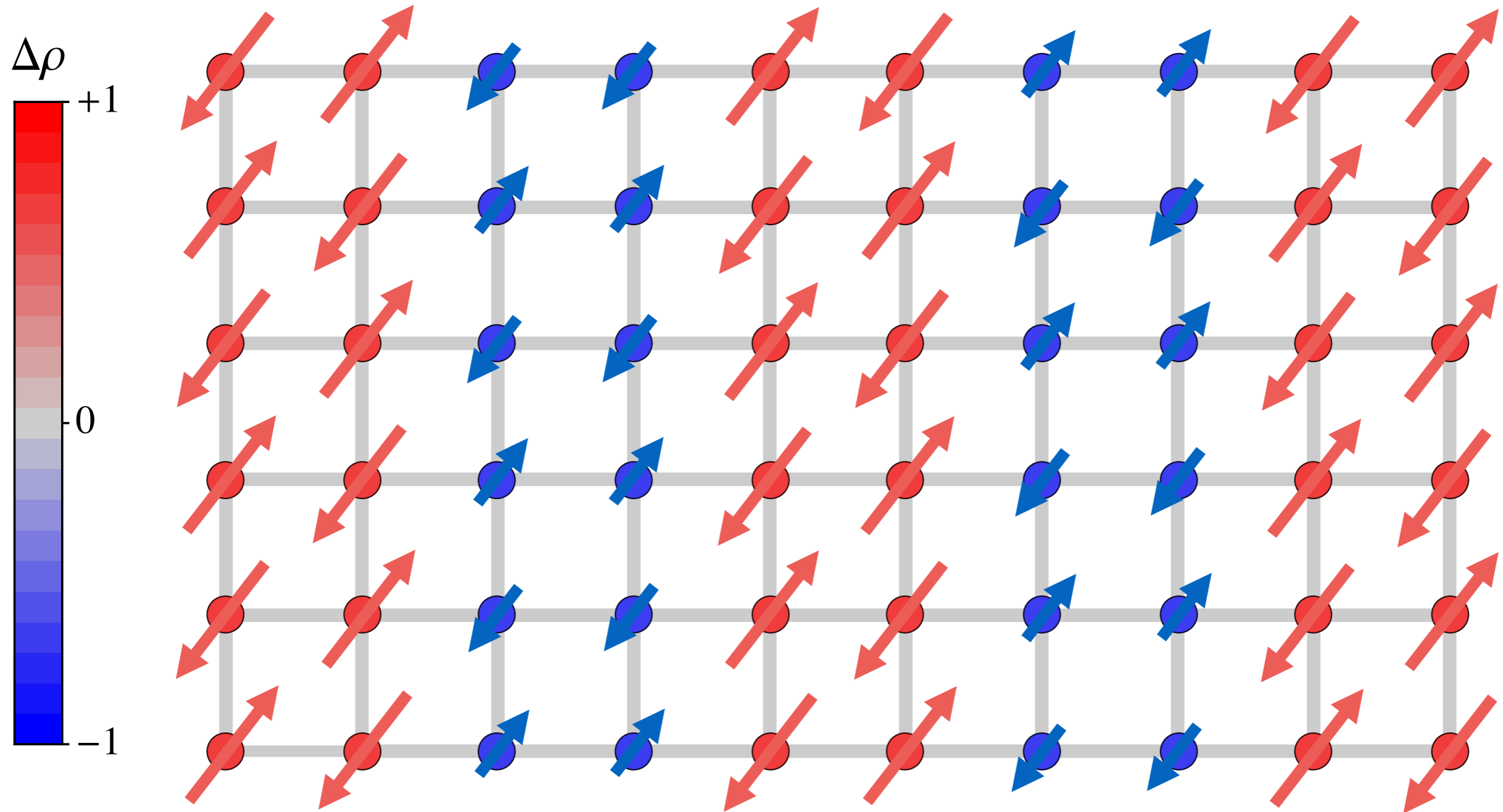


Put the holes in the domain walls

“Stripe” model

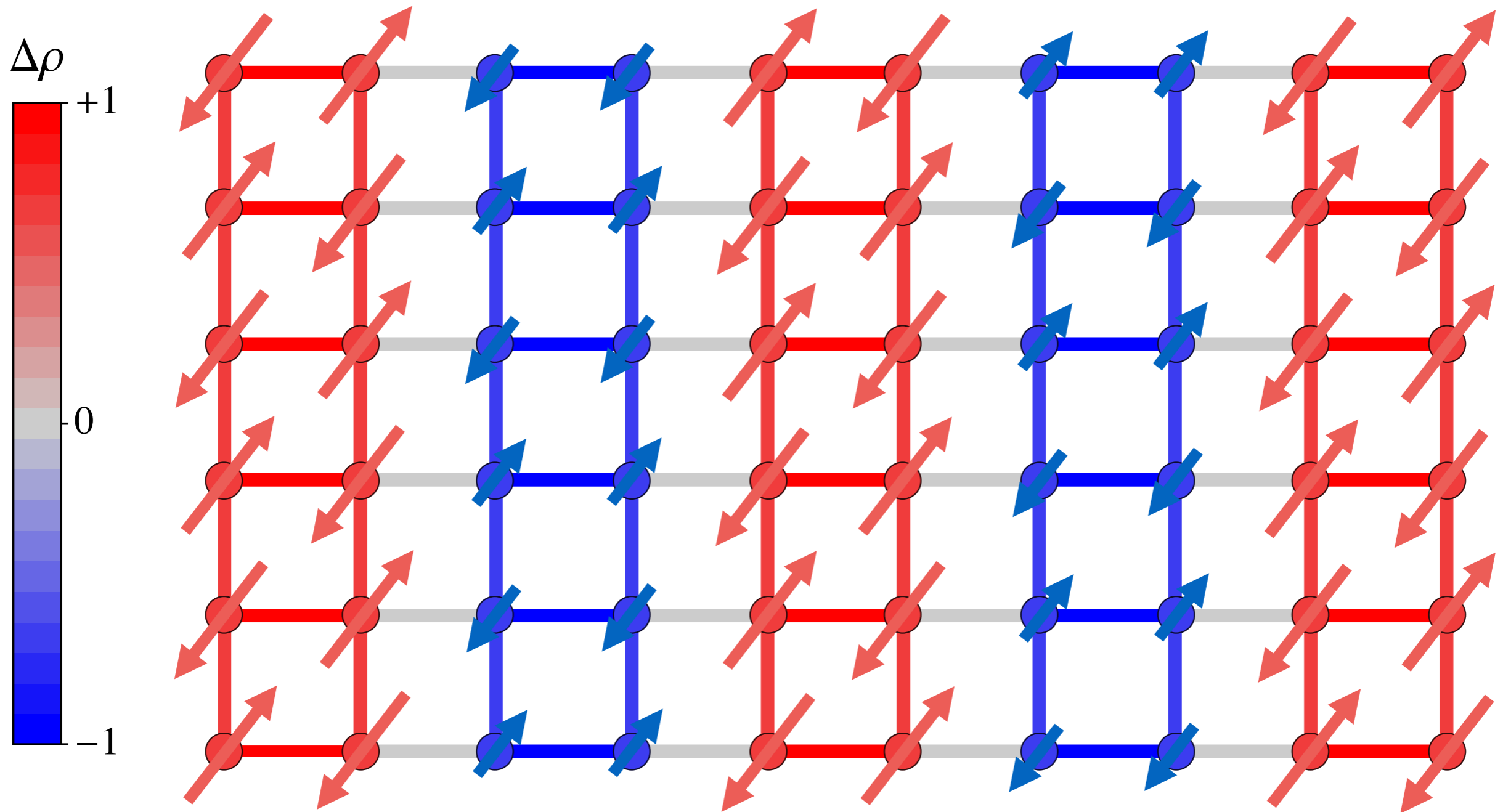
Observed in La-based
compounds (Tranquada..)

Theory: Zaanen, Kivelson, Fradkin....

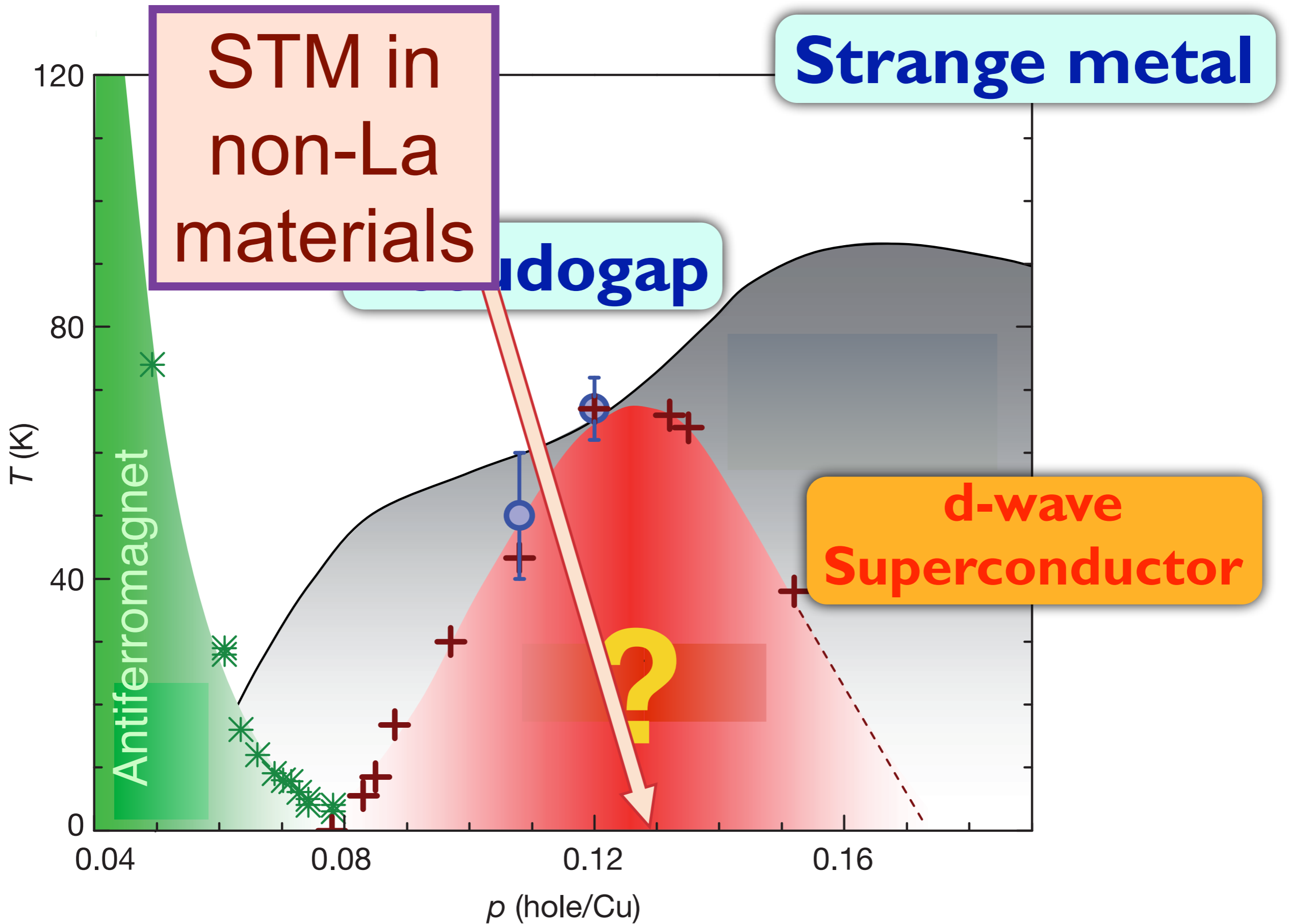


Put the holes in the domain walls

“Stripe” model



Colors on the bonds map the local exchange energy

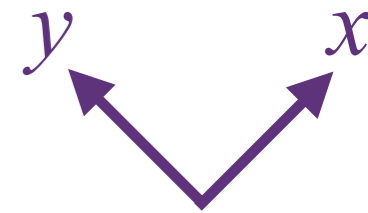
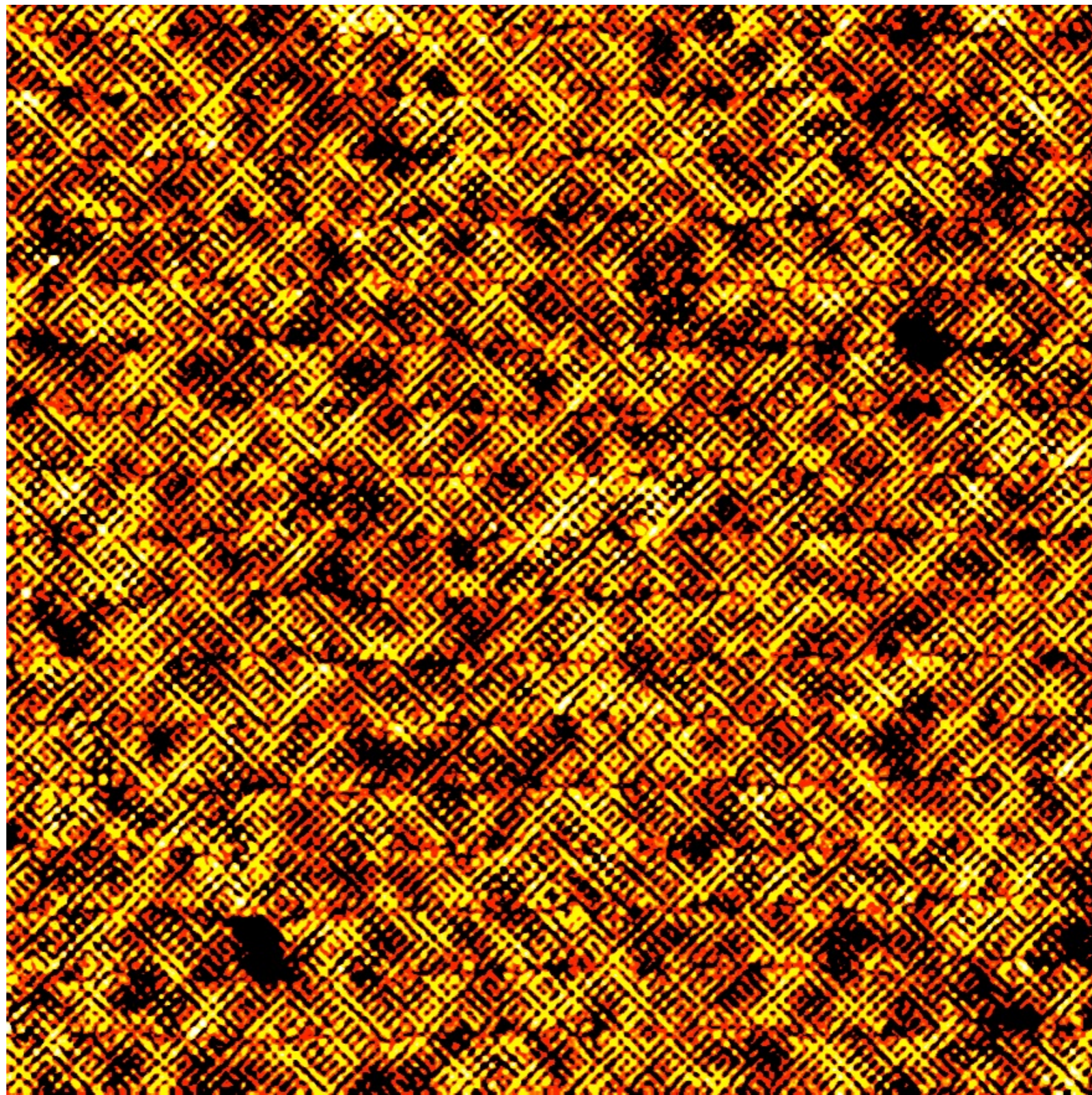


See also

C. Howald, H. Eisaki,
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“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.

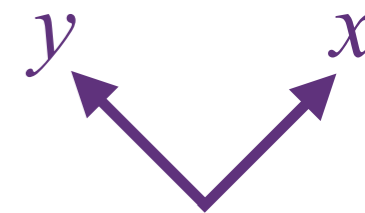
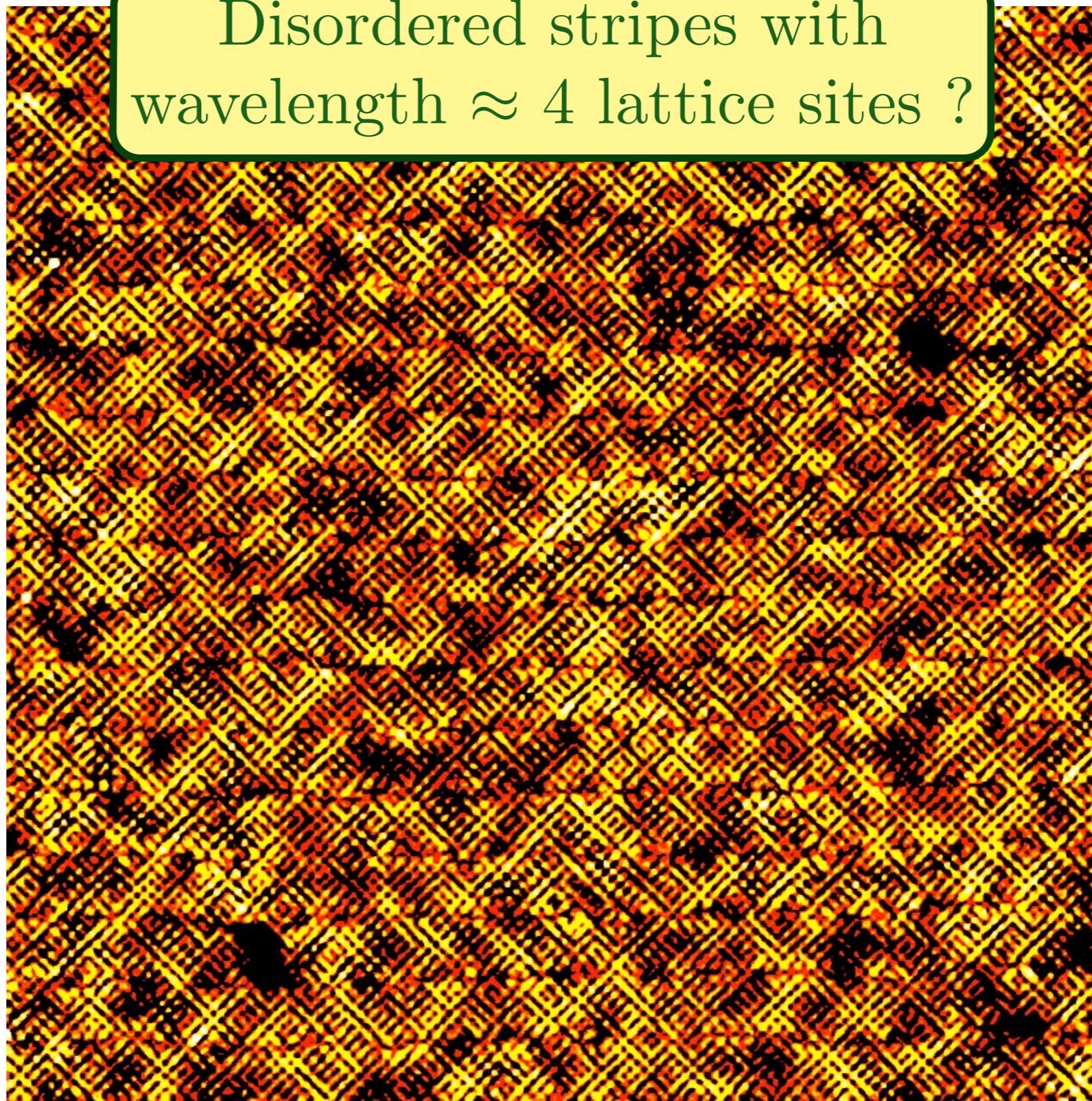
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Disordered stripes with
wavelength ≈ 4 lattice sites ?



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Unconventional density wave (DW) :
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2) / 2}$$

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Crucial “center-of-mass” co-ordinate.
(Not used in previous work)
Simplifies action of time-reversal

Unconventional density wave (DW) :
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Density wave form factor (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for \mathbf{Q} along axes or diagonals)

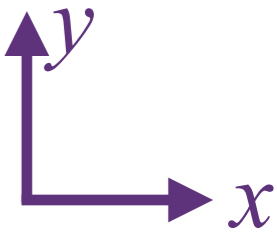
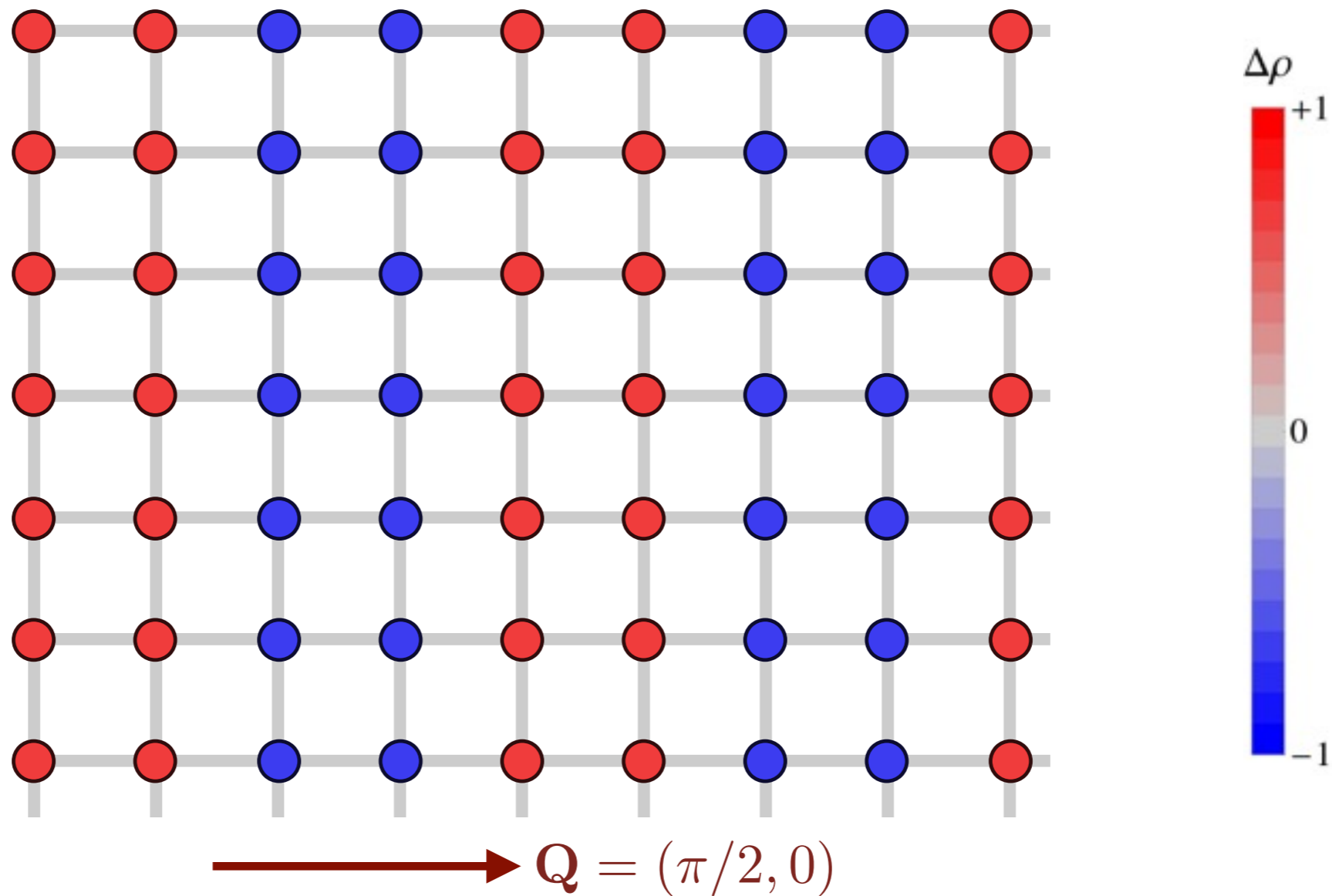
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

Conventional CDW order: s -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

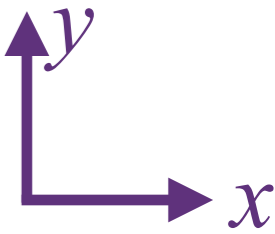
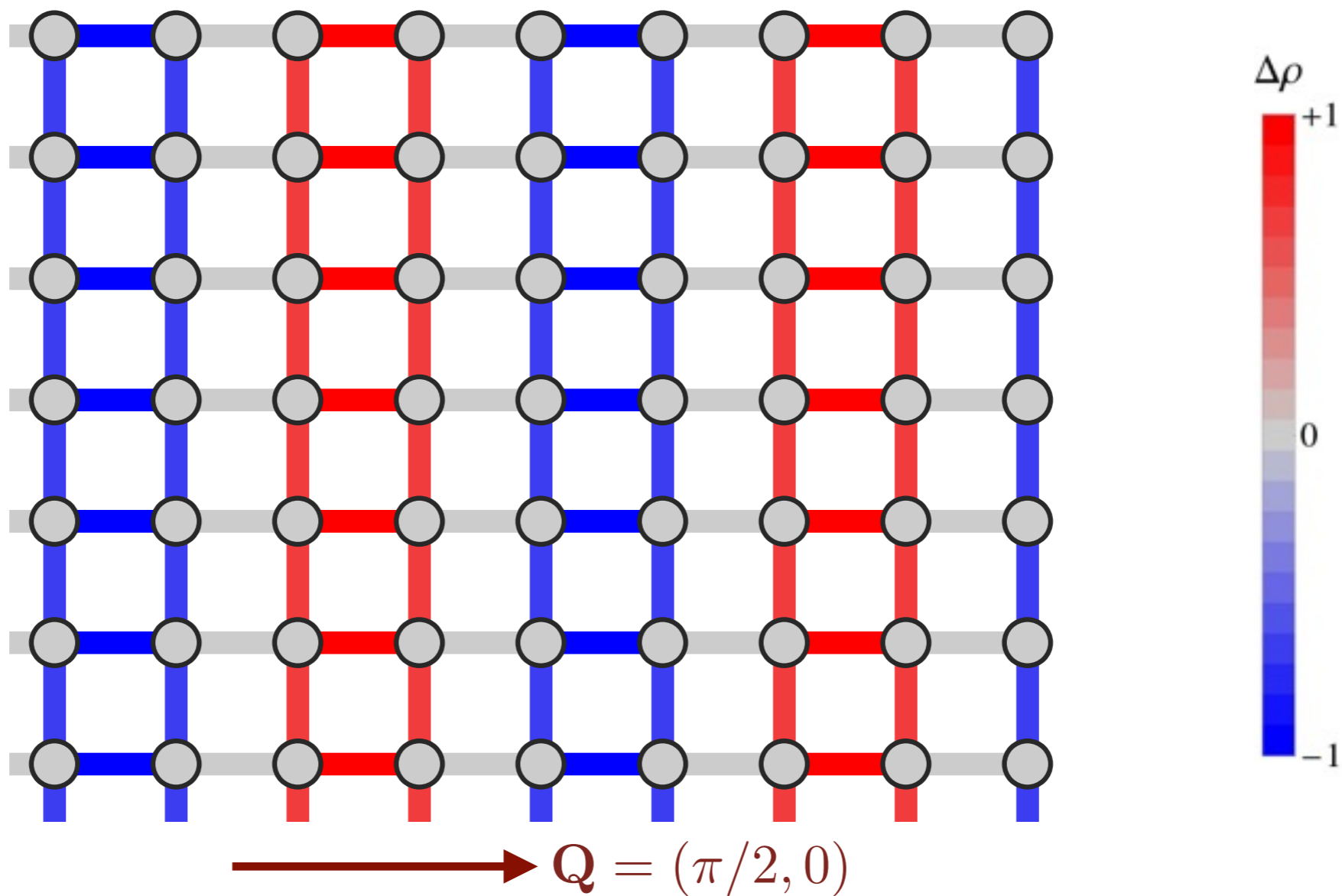


Unconventional DW order: s' -form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

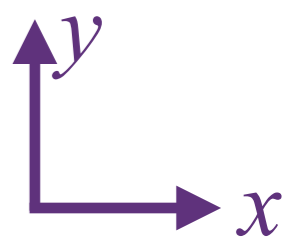


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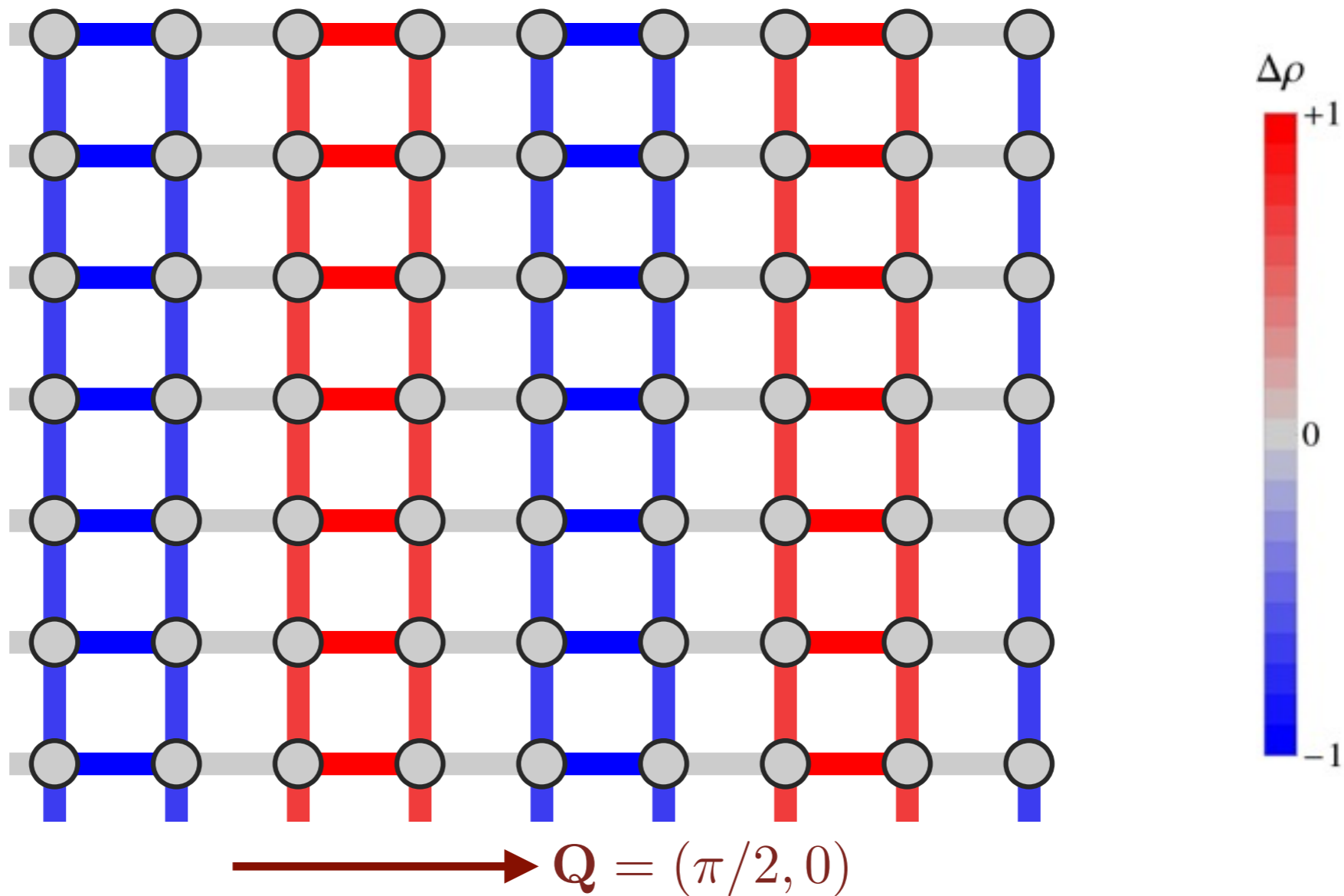
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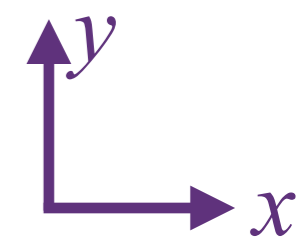
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“Stripe”
model



Unconventional DW order: s' -form factor



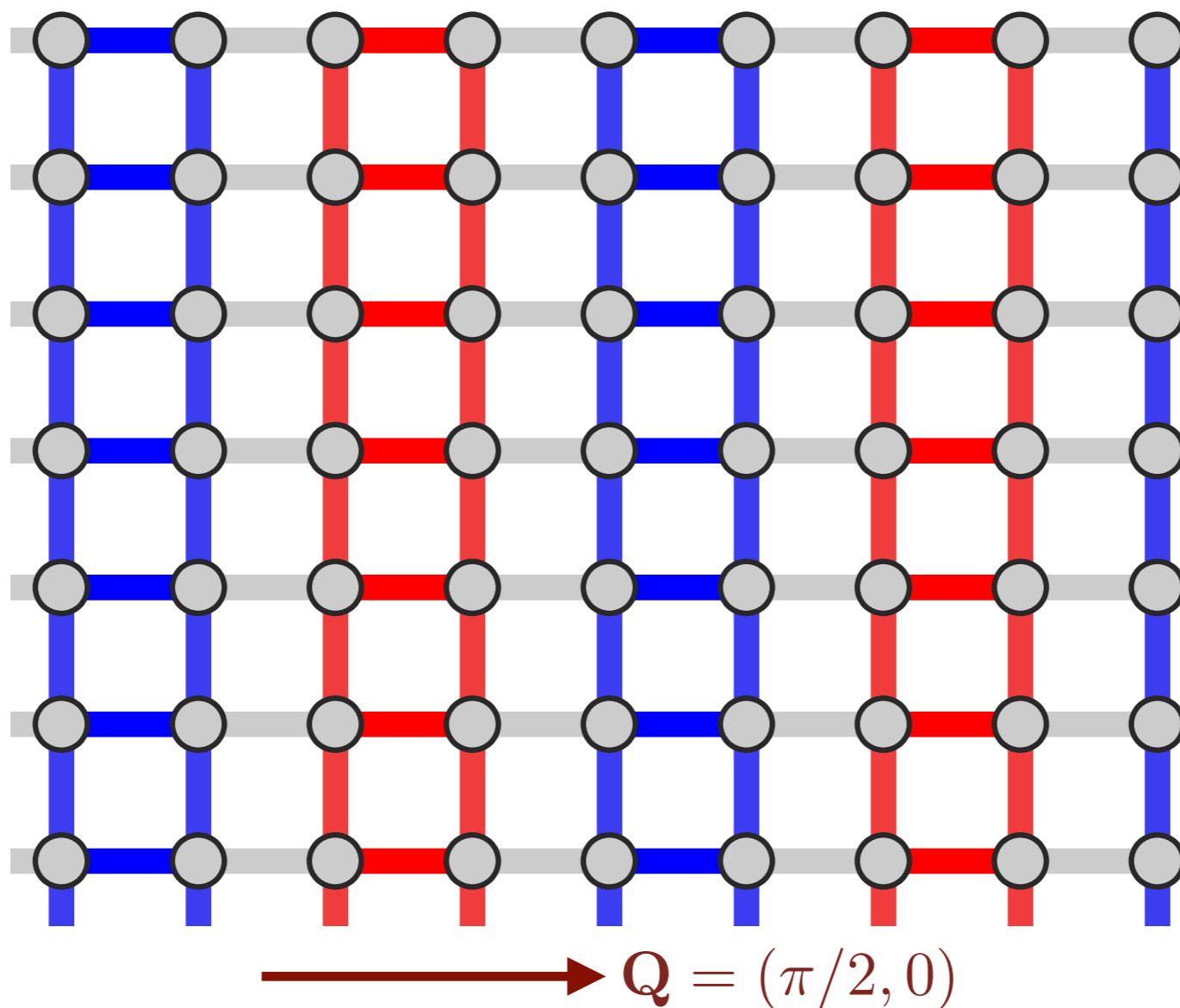
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“Stripe”
model

X-ray
observations
indicate
strong s'
component in
LBCO



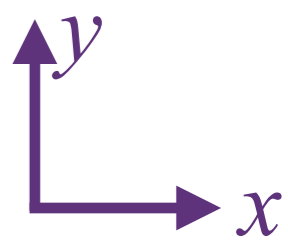
David
Hawthorn,
Waterloo

Unconventional DW order: d -form factor

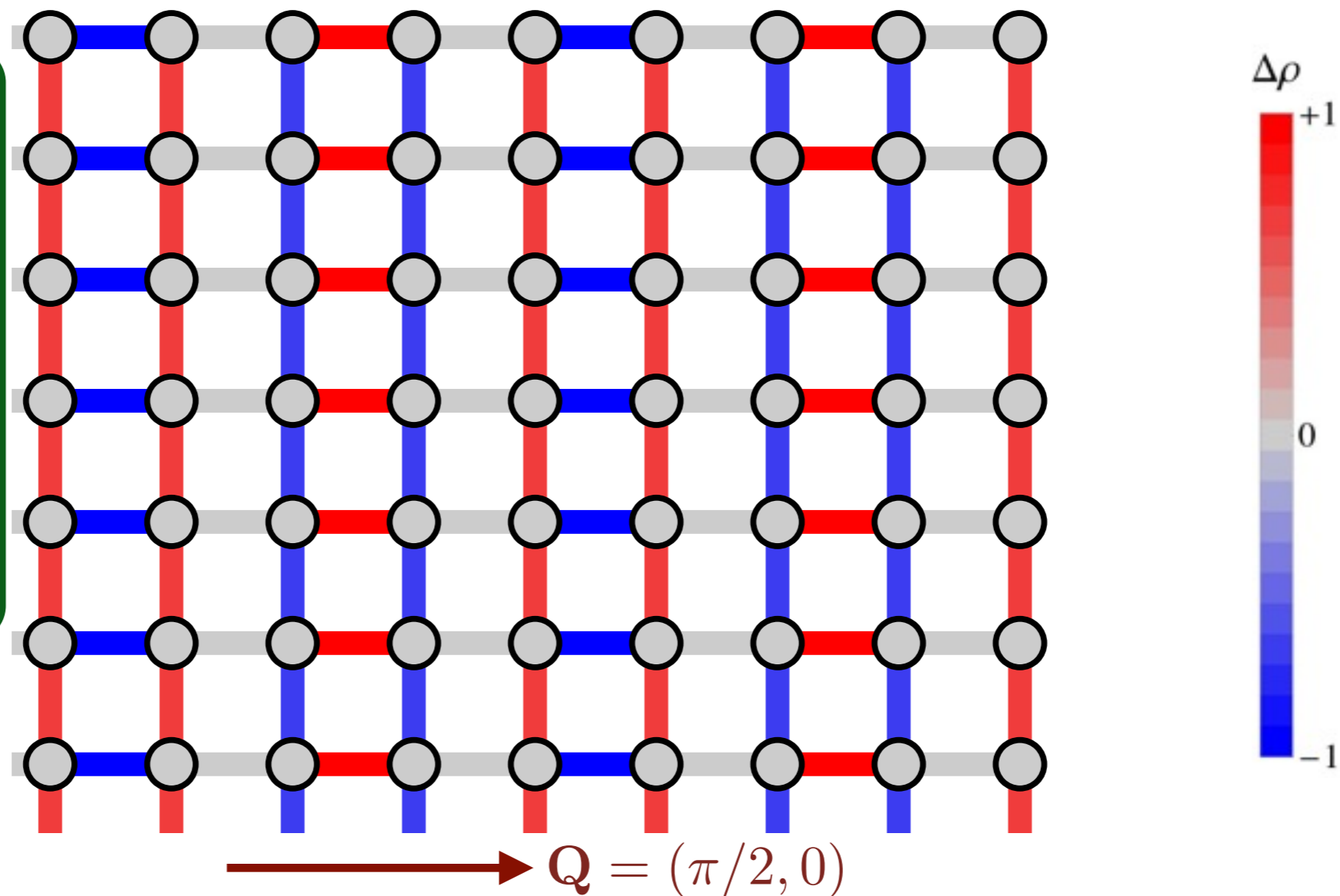
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Our prediction:
Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



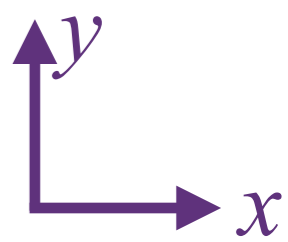
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
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Unconventional DW order: d -form factor

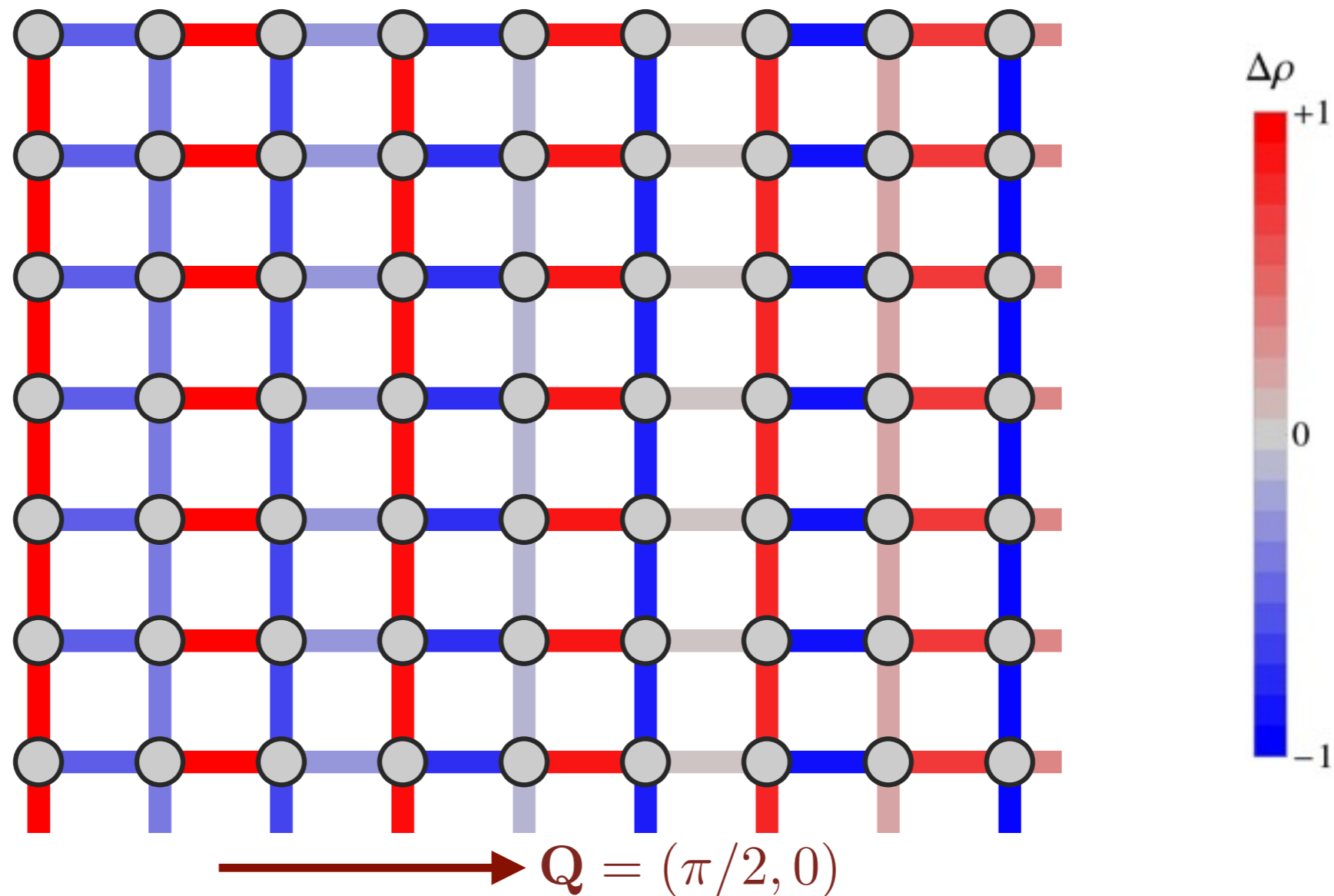
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$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$



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Direct phase-sensitive identification of a d -form factor density wave in underdoped cuprates

Kazuhiro Fujita^{a,b,c,1}, Mohammad H. Hamidian^{a,b,1}, Stephen D. Edkins^{b,d}, Chung Koo Kim^a, Yuhki Kohsaka^e, Masaki Azuma^f, Mikio Takano^g, Hidenori Takagi^{c,h,i}, Hiroshi Eisaki^j, Shin-ichi Uchida^c, Andrea Allais^k, Michael J. Lawler^{b,l}, Eun-Ah Kim^b, Subir Sachdev^{k,m}, and J. C. Séamus Davis^{a,b,d,2}

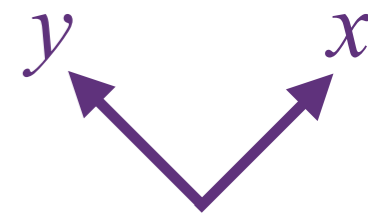
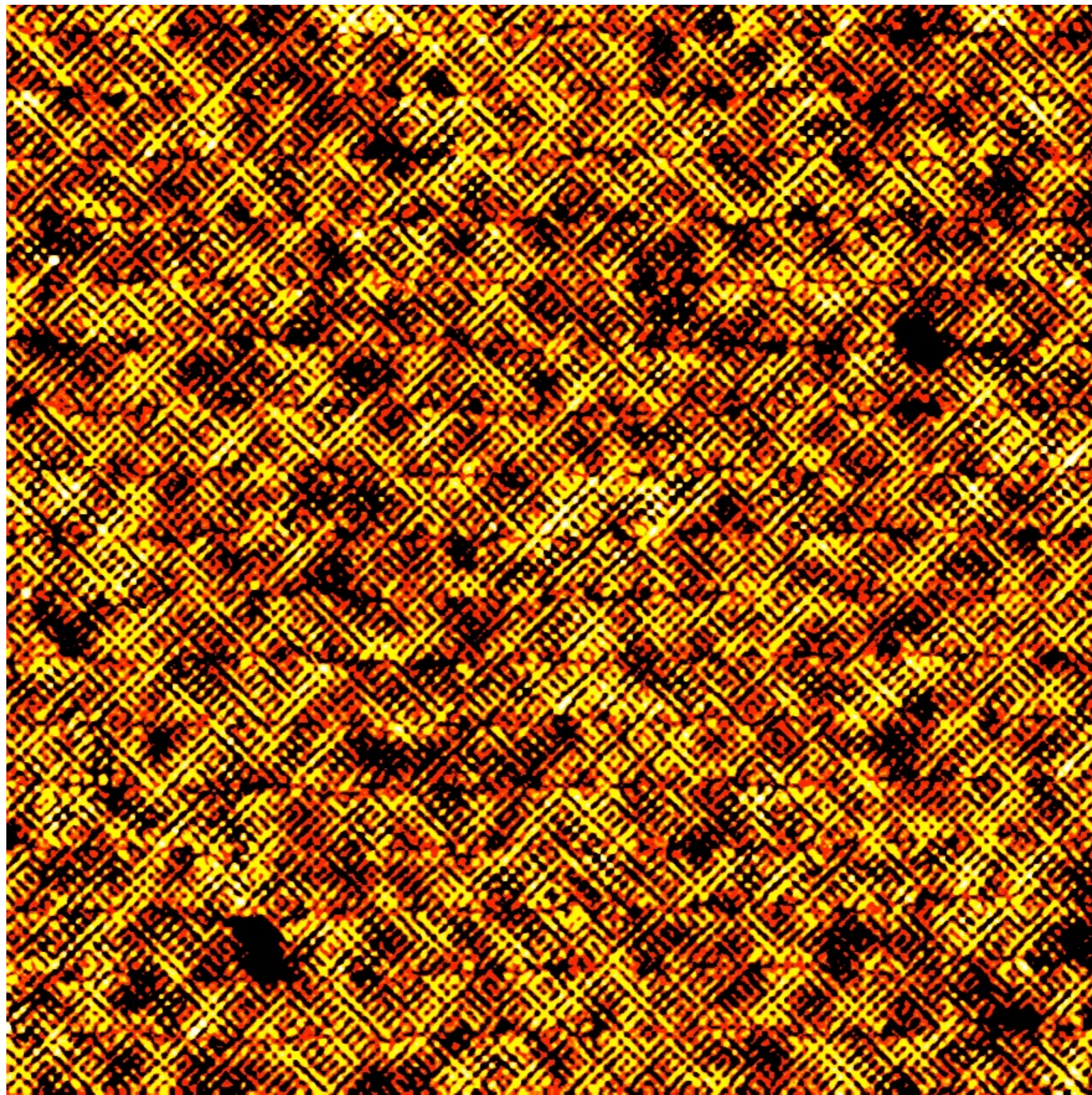
The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each CuO_2 unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [$\text{Cu}(r)$] and only the x/y axis O sites [$\text{O}_x(r)$ and $\text{O}_y(r)$]. Phase-resolved Fourier analysis reveals directly that the modulations in the $\text{O}_x(r)$ and $\text{O}_y(r)$ sublattice images consistently exhibit a relative phase of π . We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly d -symmetry form factor.

See also

C. Howald, H. Eisaki,
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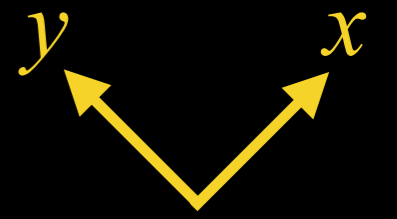
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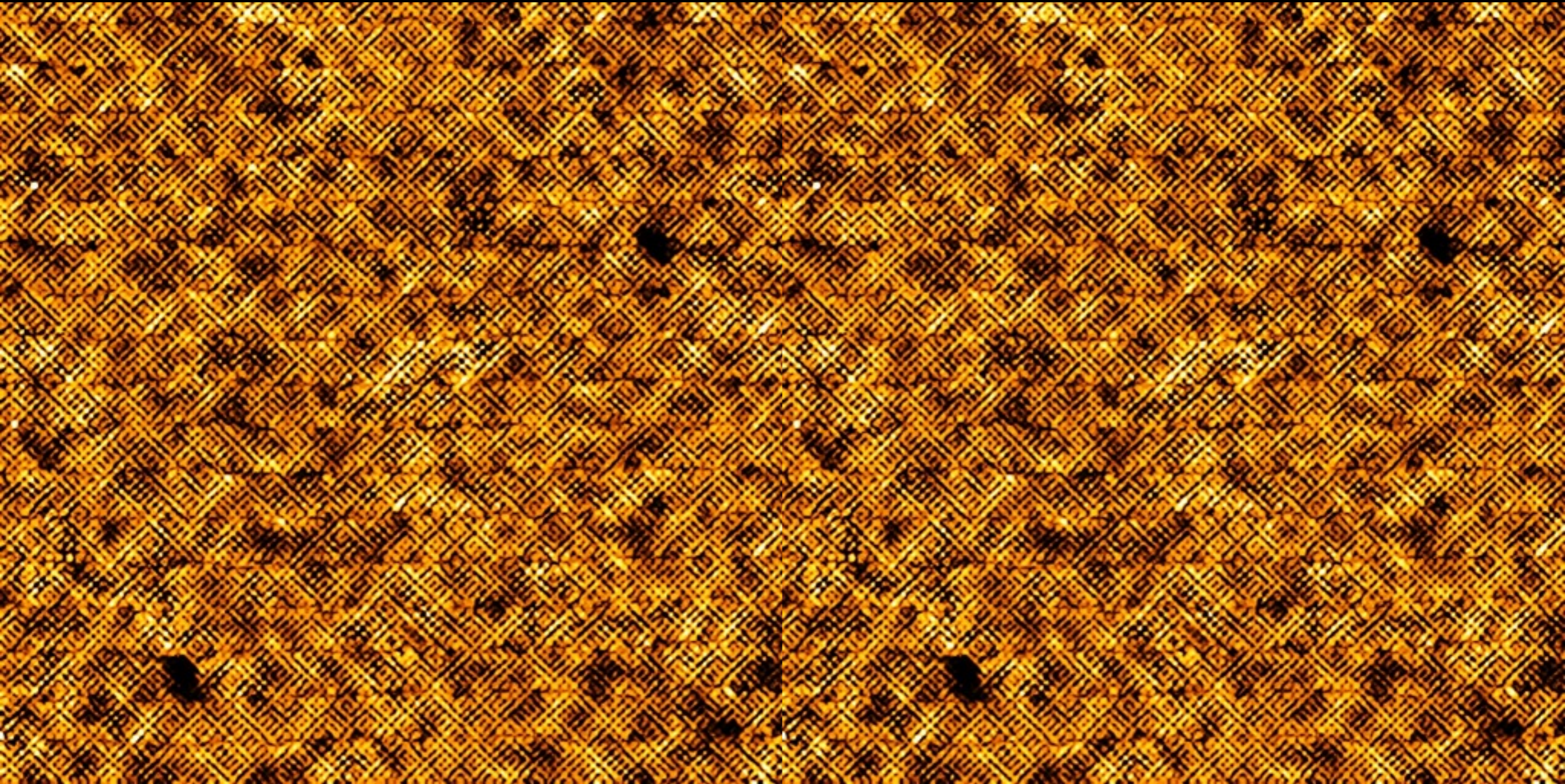
UD45K
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



Note that these are identical images.

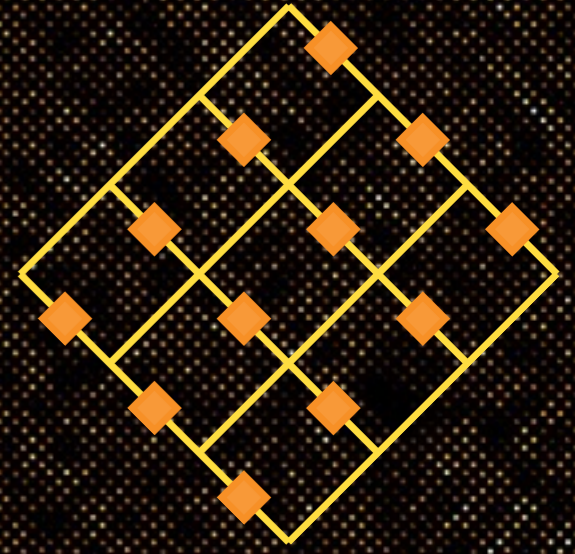
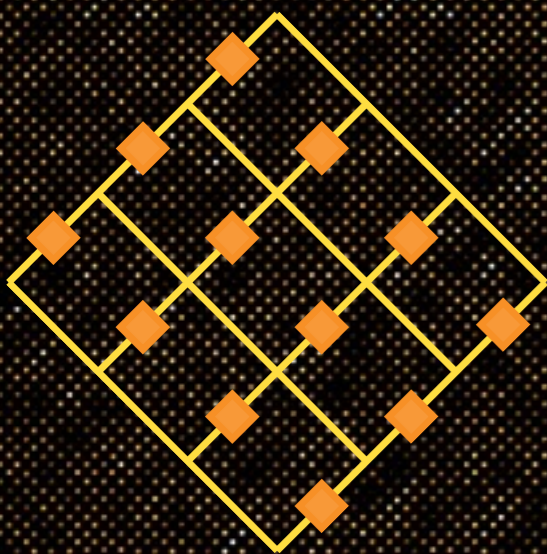
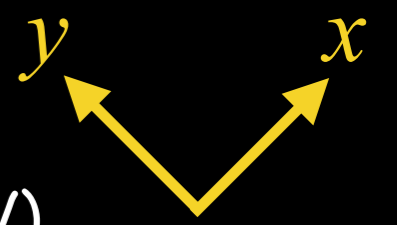
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

UD45K

$R(r=0, 150\text{mV})$

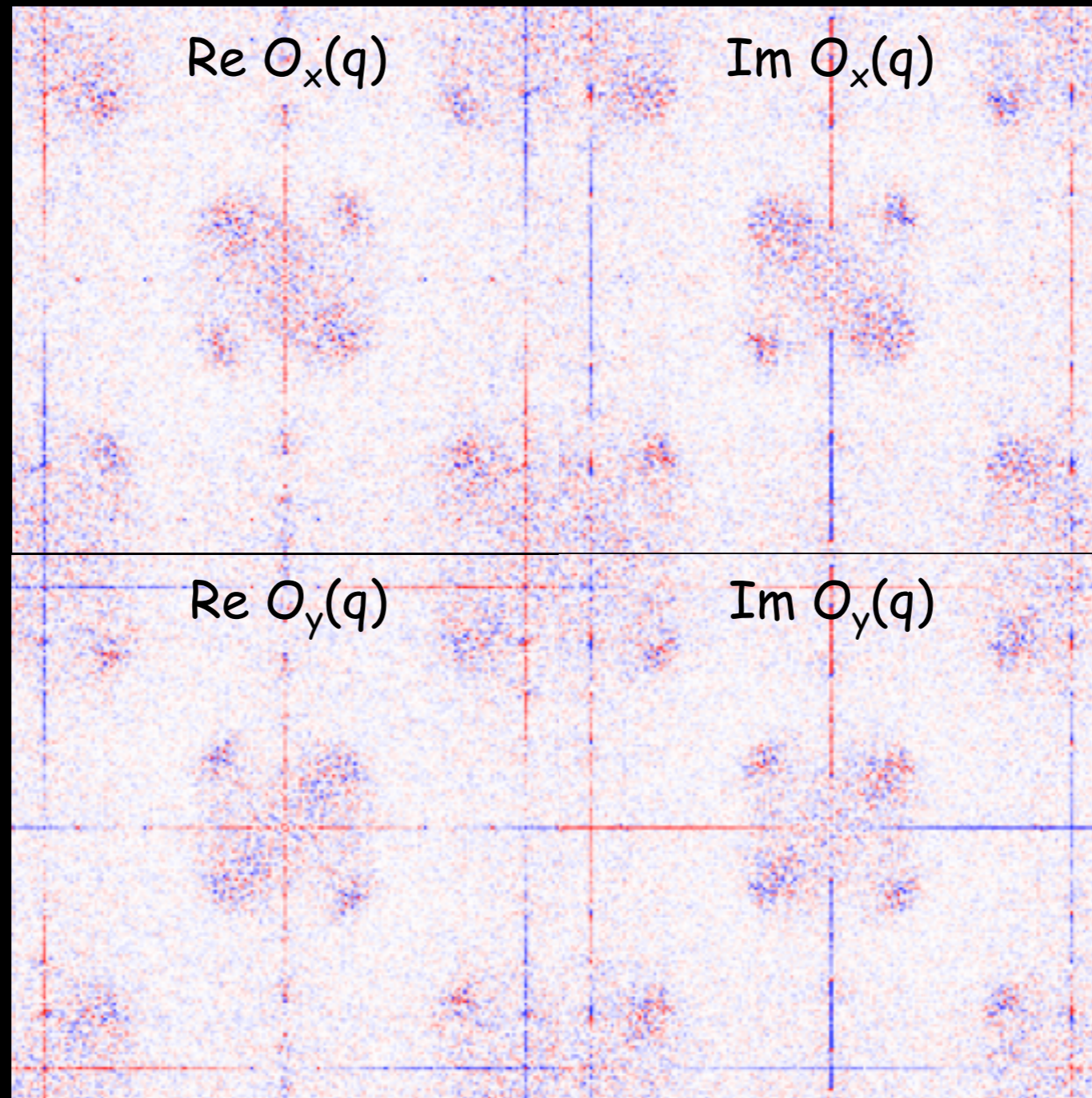
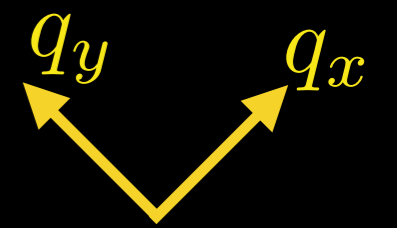
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

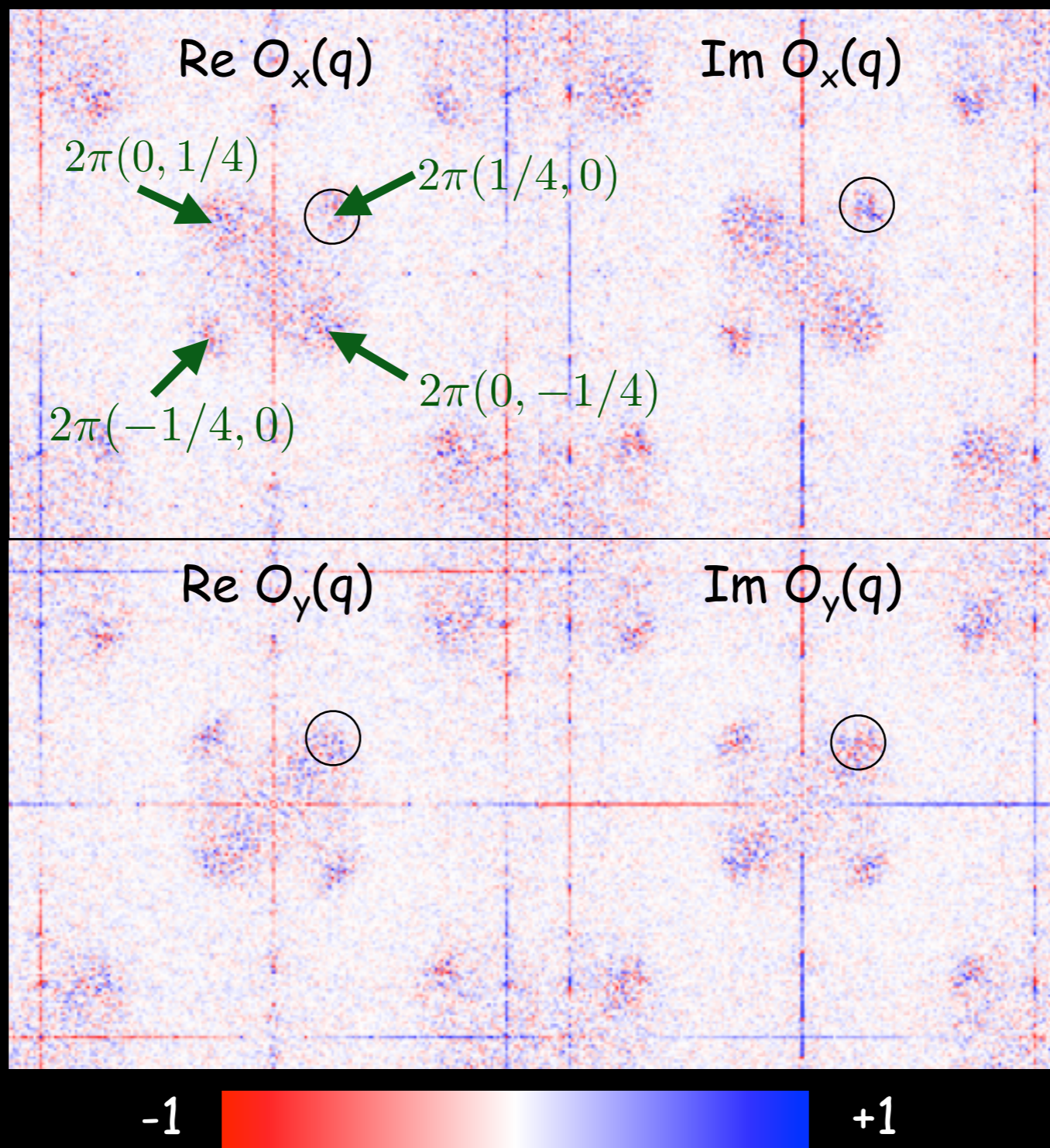
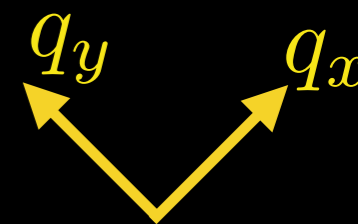


UD45K

Broad (0,Q) and (Q,0) DW Features

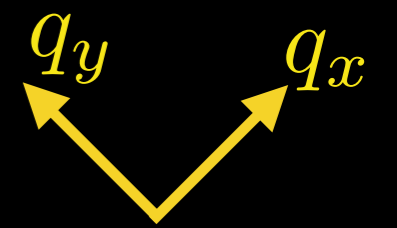


Broad (0,Q) and (Q,0) DW Features

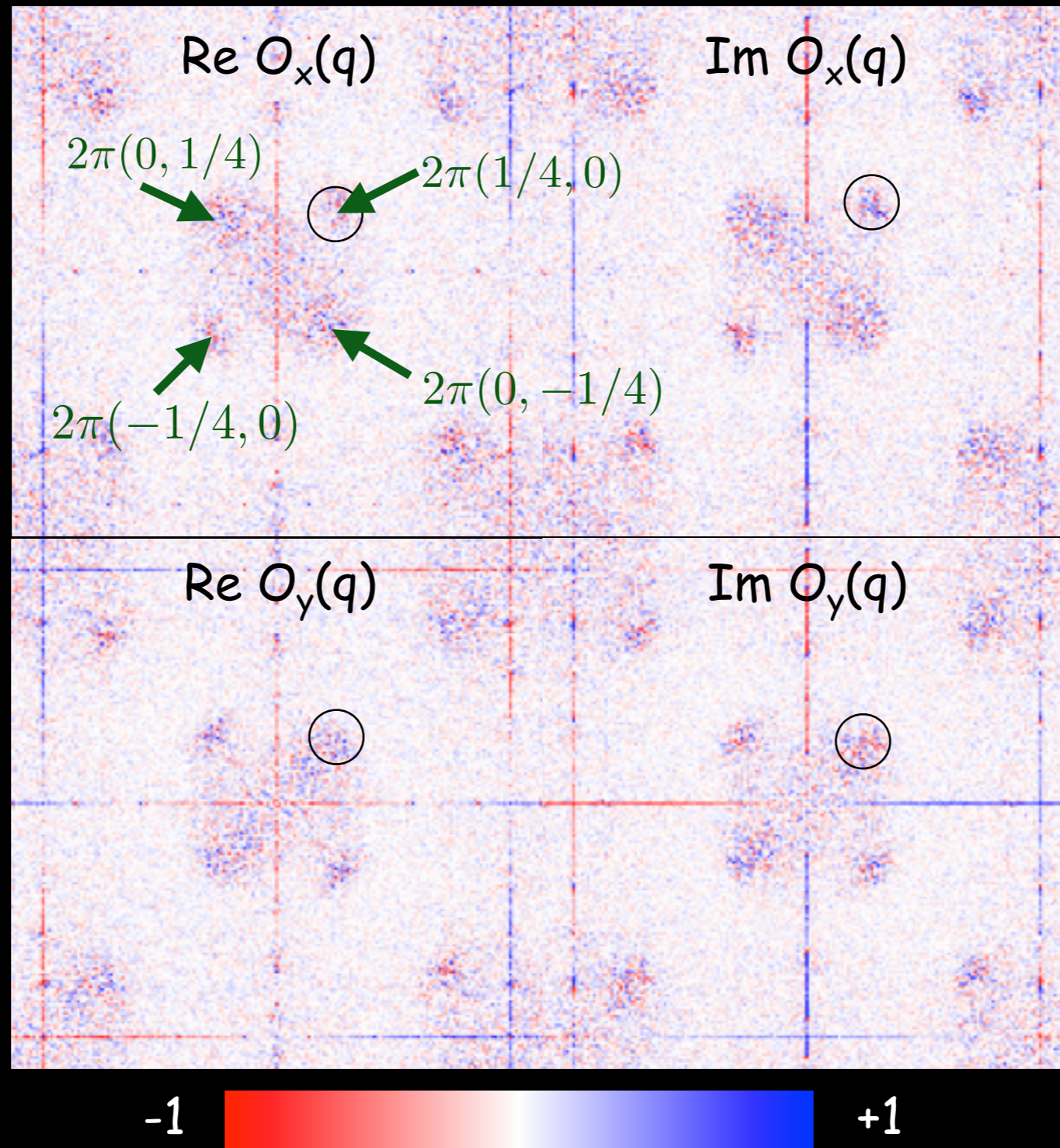


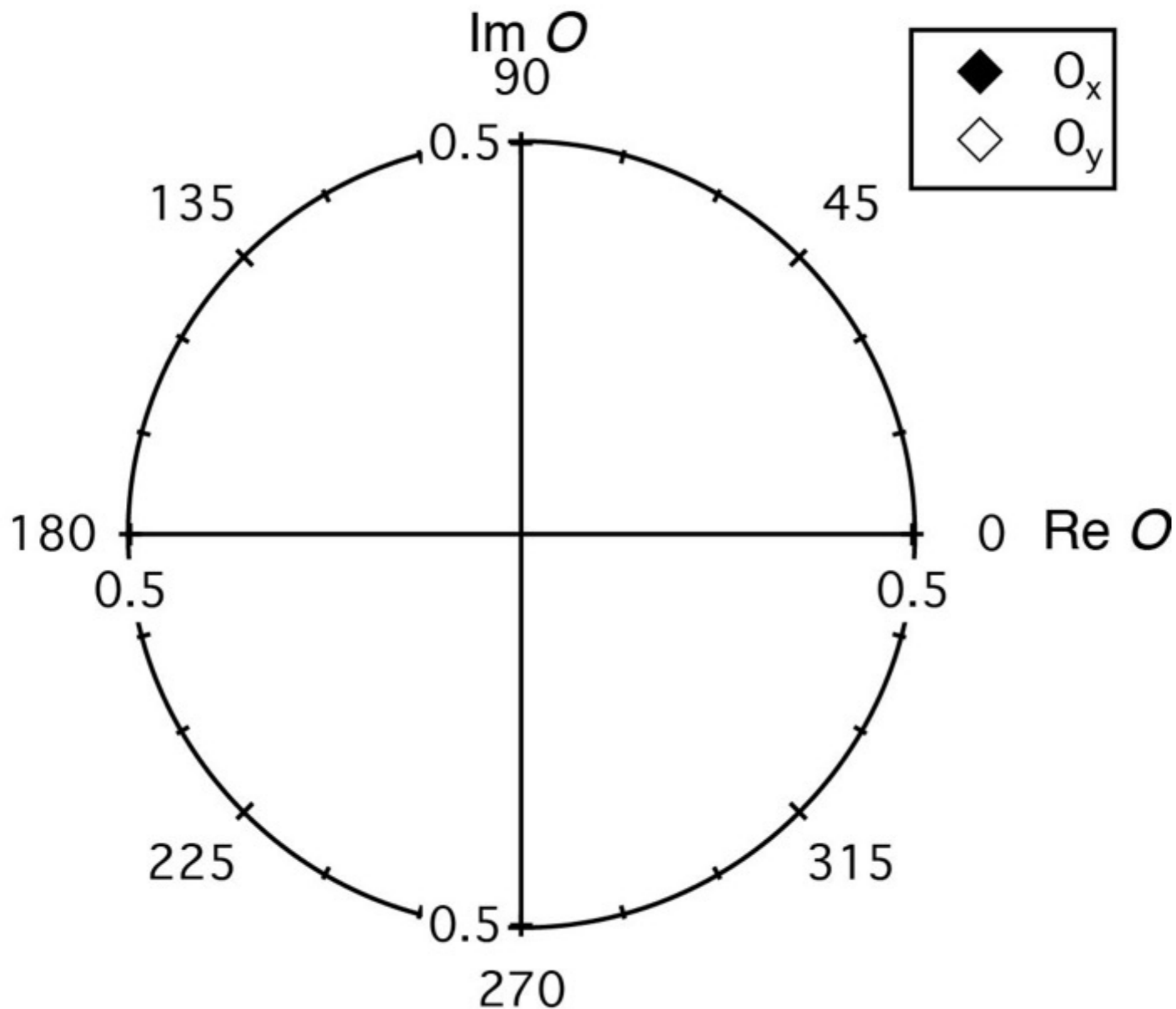
UD45K

Broad (0,Q) and (Q,0) DW Features

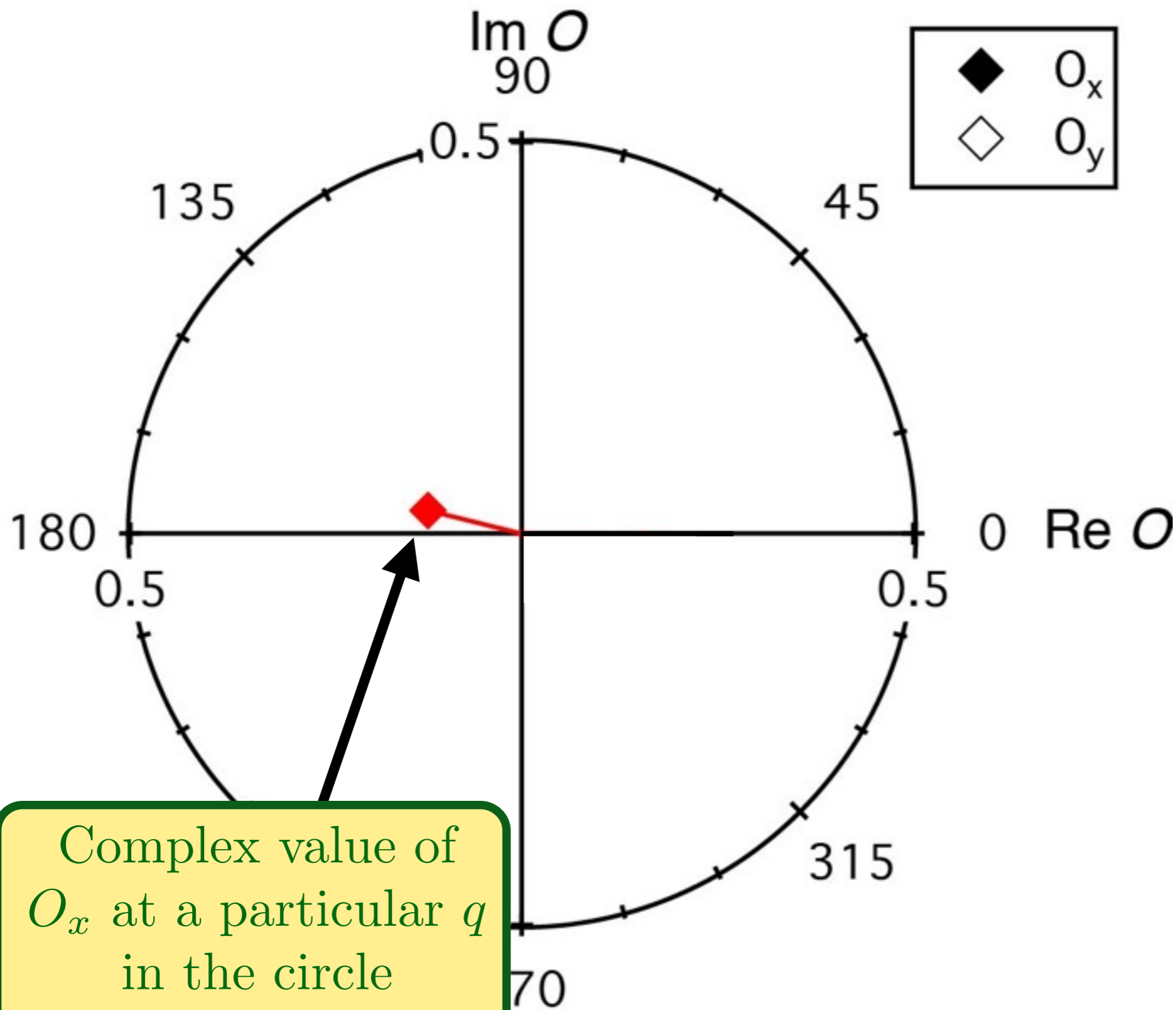


For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.



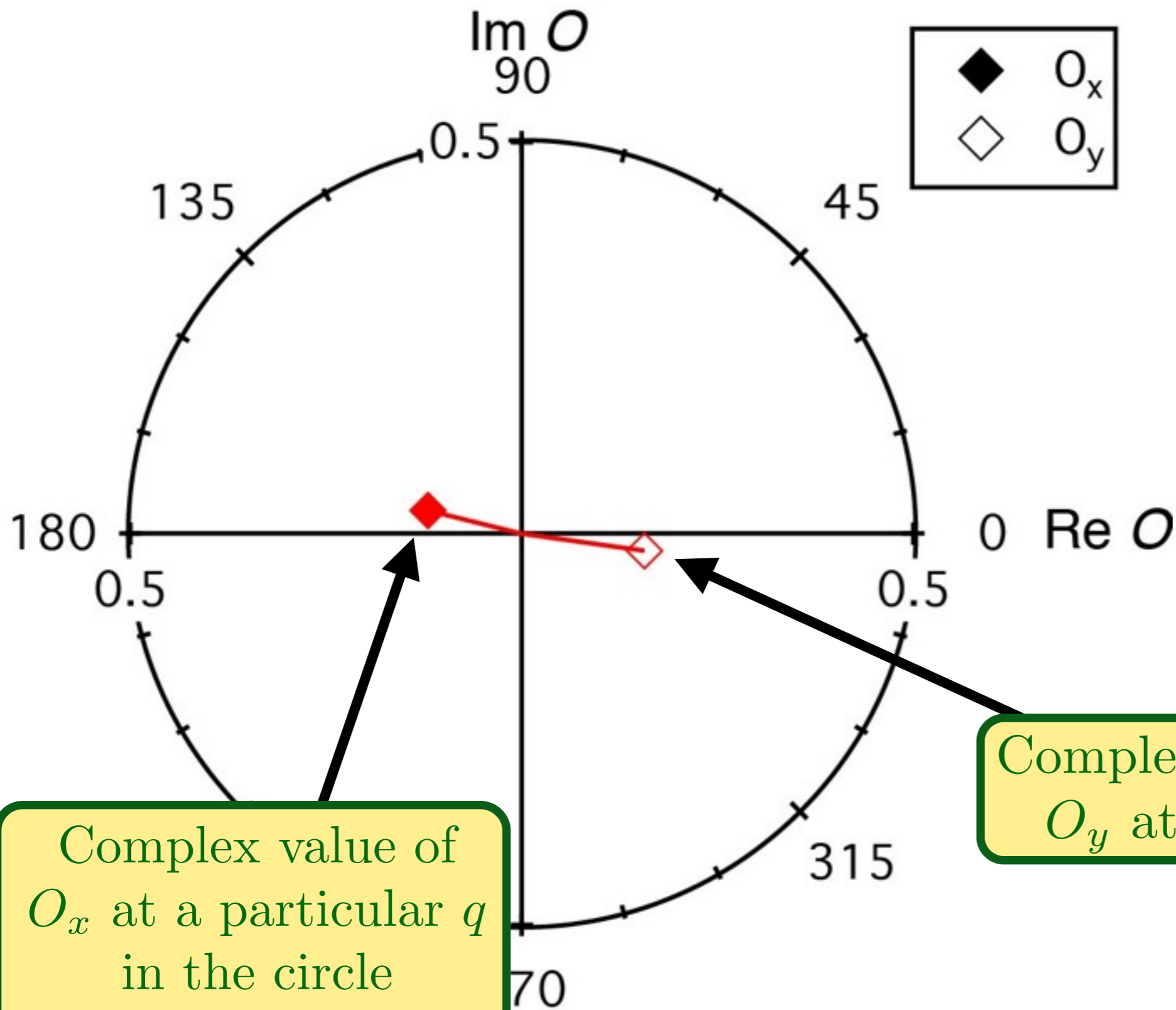


Phase-sensitive measurement of the d -form factor of density wave order



Phase-sensitive measurement of the d -form factor of density wave order

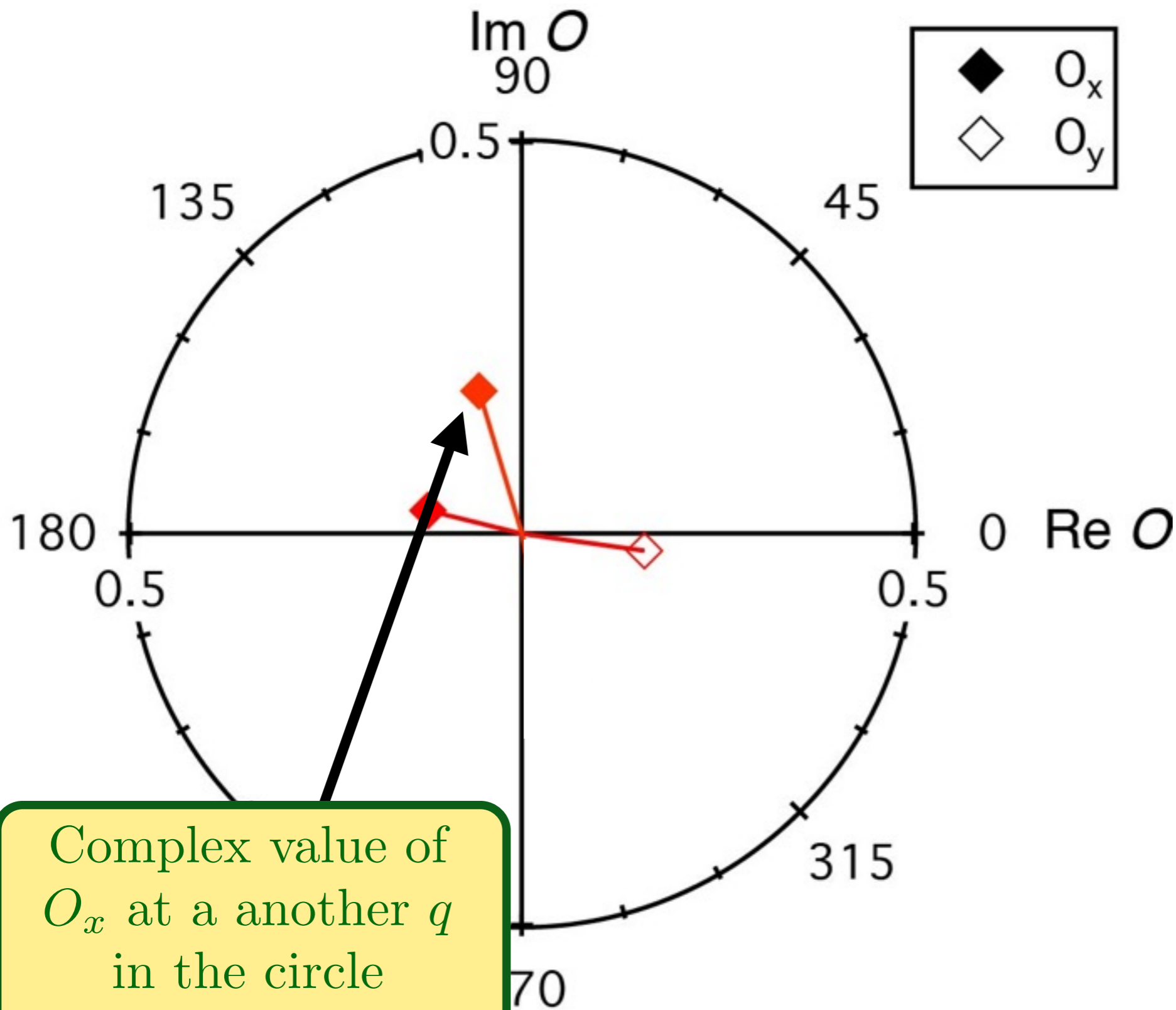
Complex value of O_x at a particular q in the circle around $2\pi(1/4, 0)$.



Phase-sensitive measurement of the d -form factor of density wave order

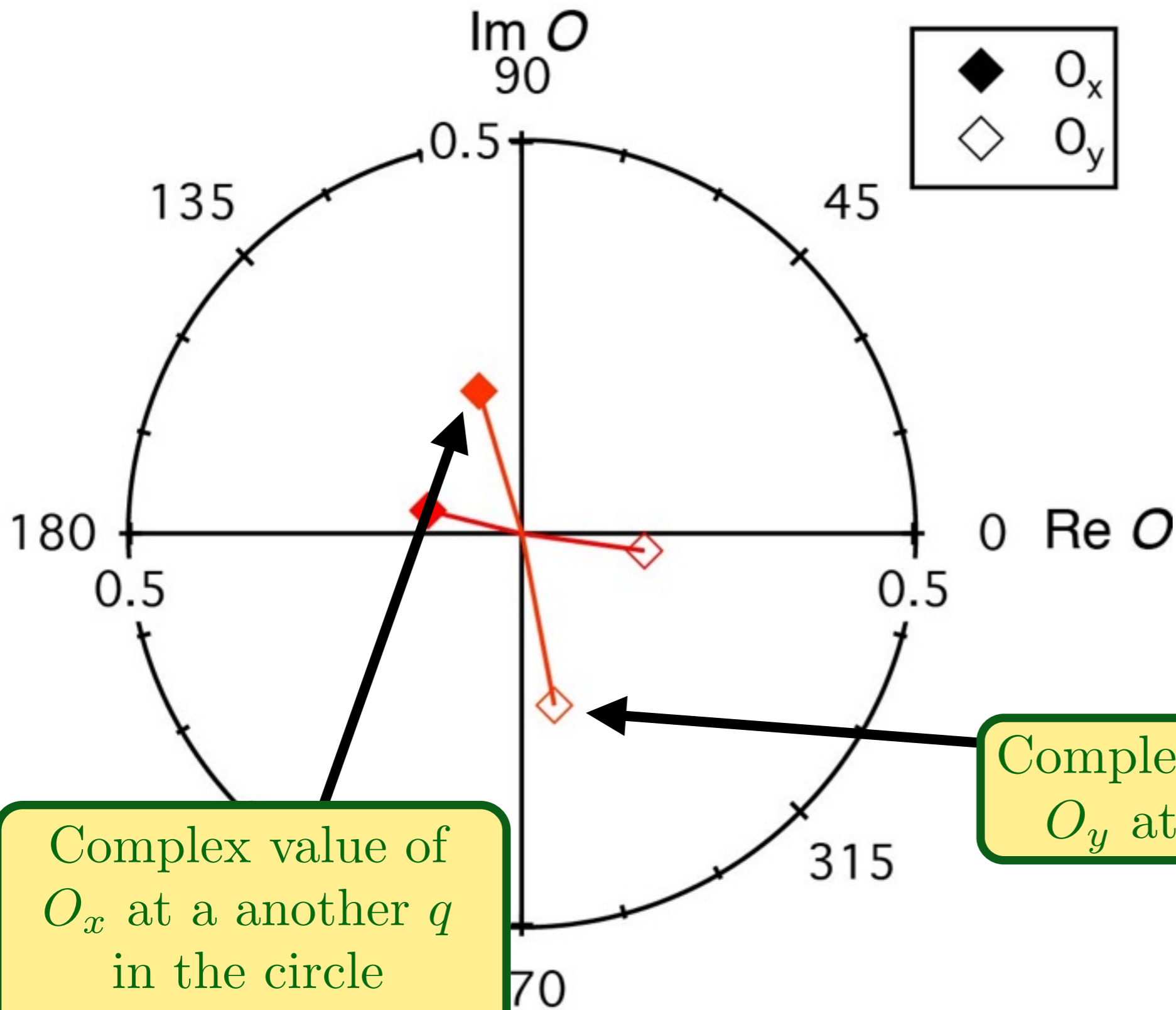
Complex value of O_x at a particular q in the circle around $2\pi(1/4, 0)$.

Complex value of O_y at same q



Phase-sensitive measurement of the d -form factor of density wave order

Complex value of O_x at a another q in the circle around $2\pi(1/4, 0)$.

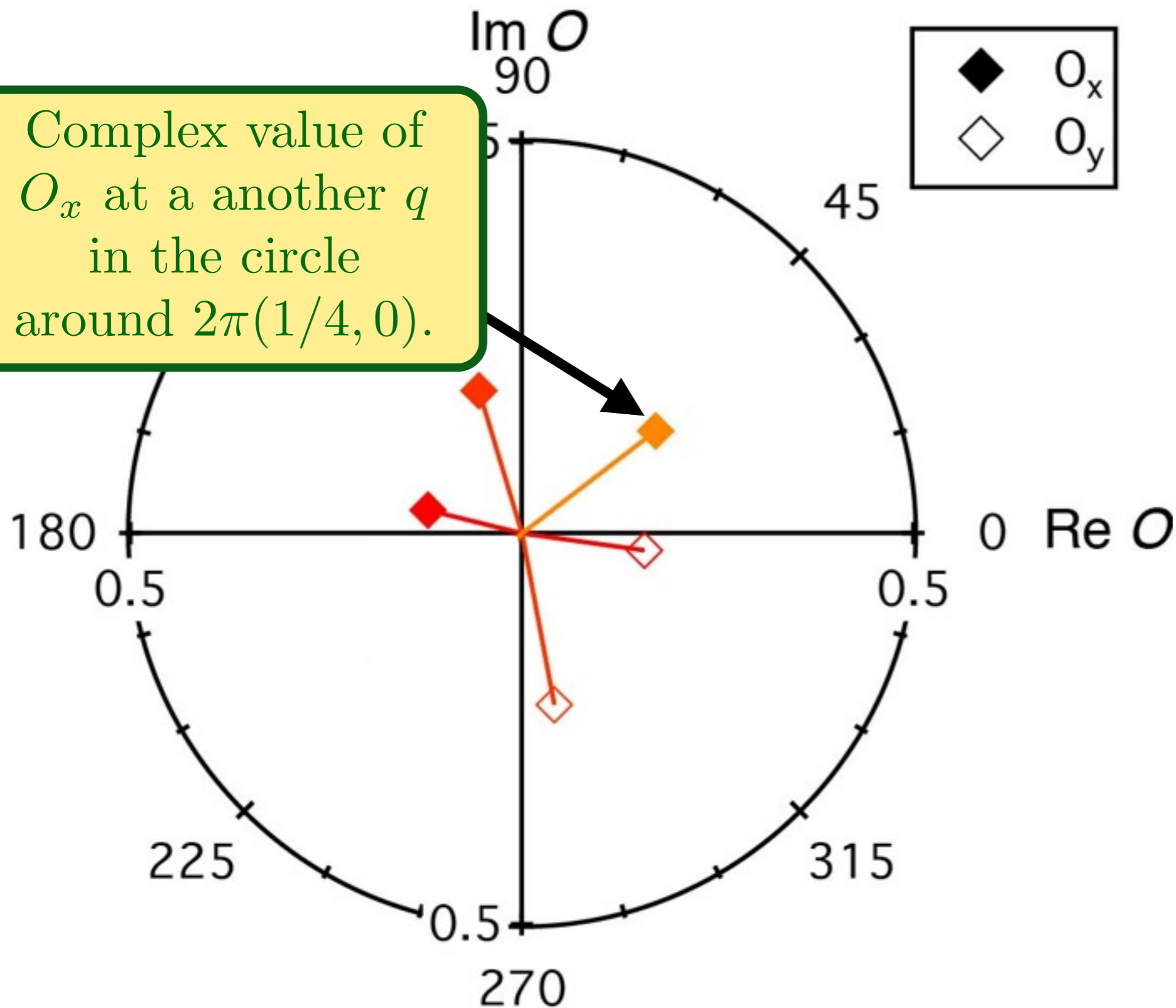


Phase-sensitive measurement of the d -form factor of density wave order

Complex value of O_x at a another q in the circle around $2\pi(1/4, 0)$.

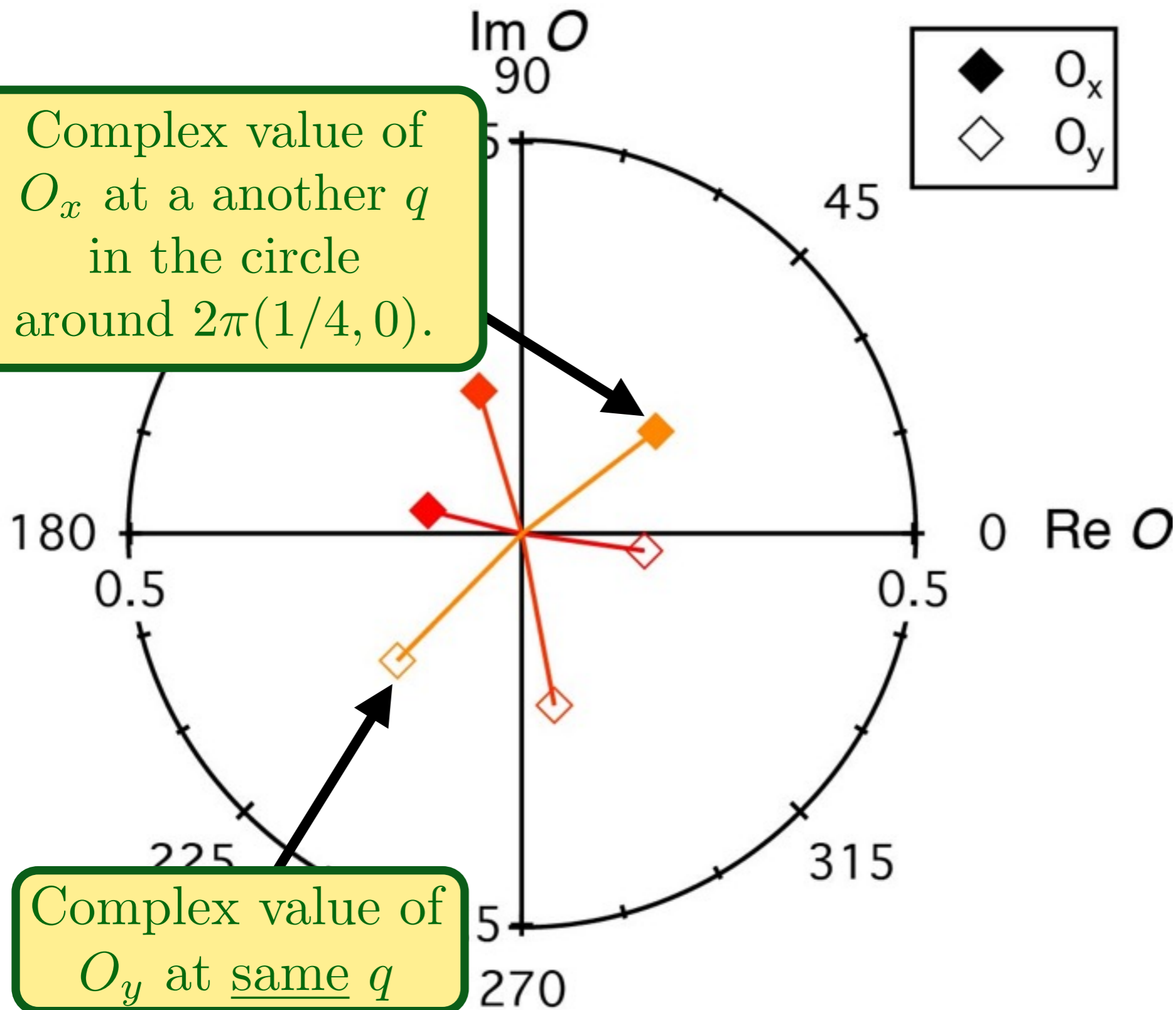
Complex value of O_y at same q

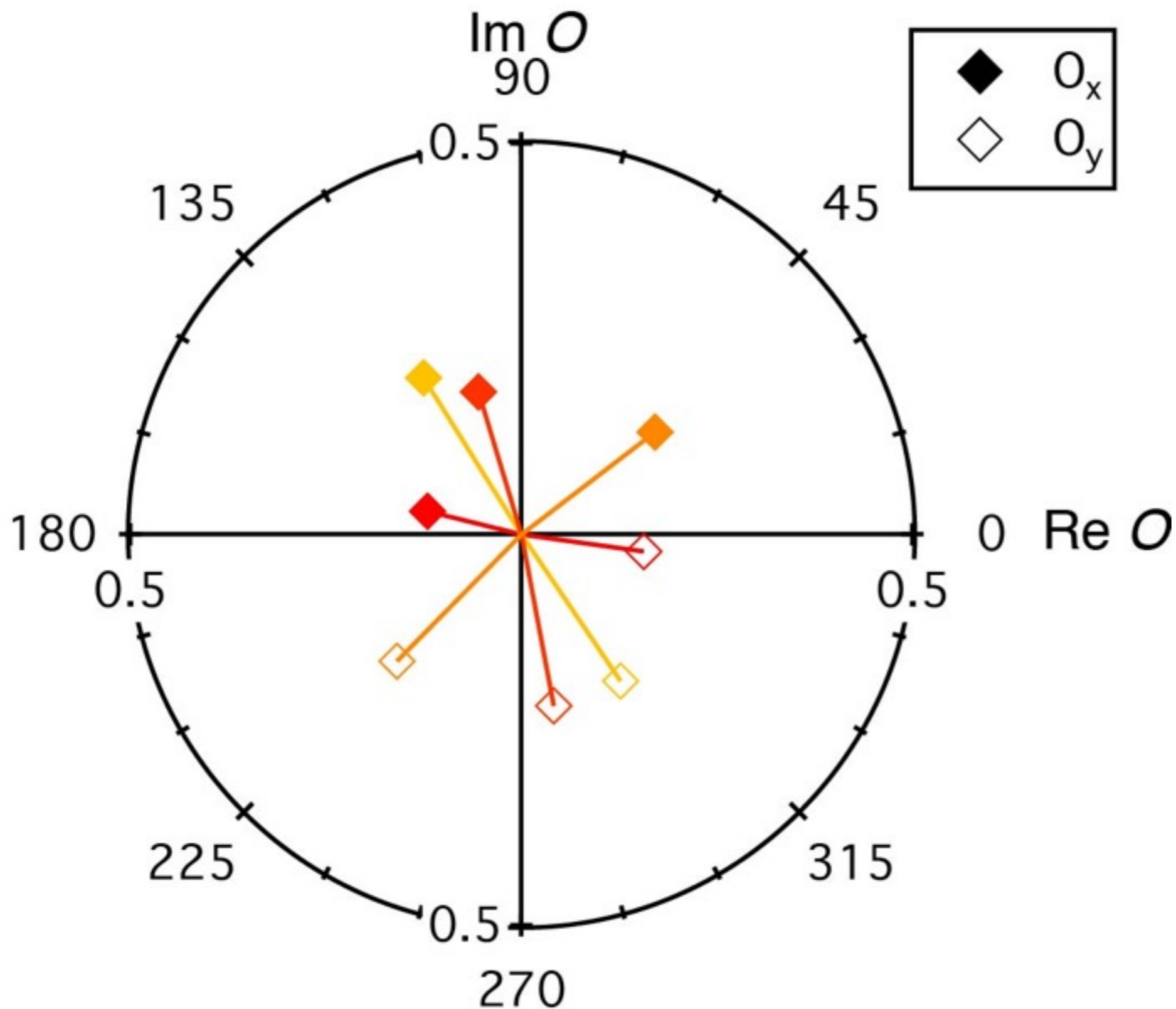
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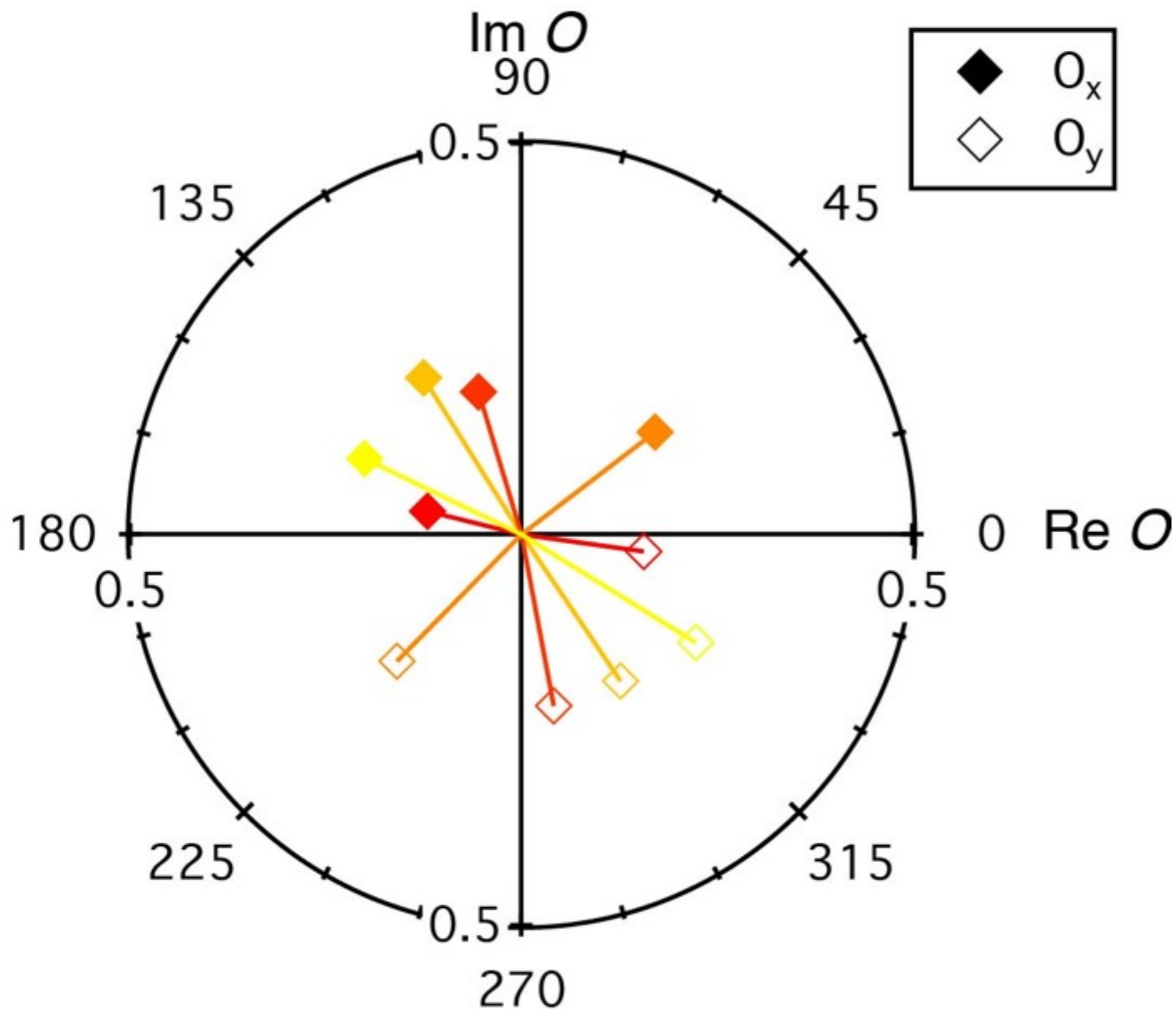
Phase-sensitive measurement of the d -form factor of density wave order

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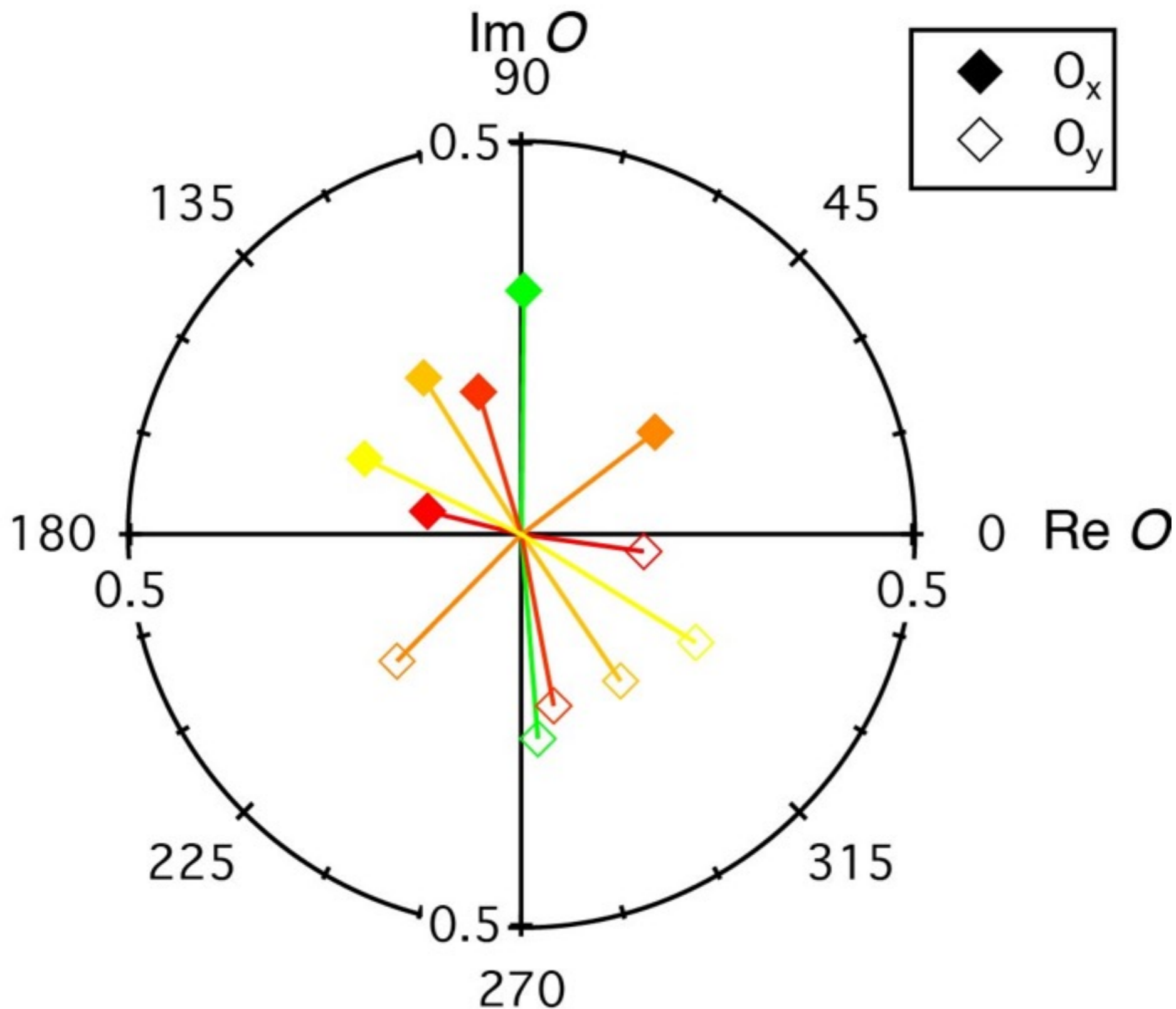




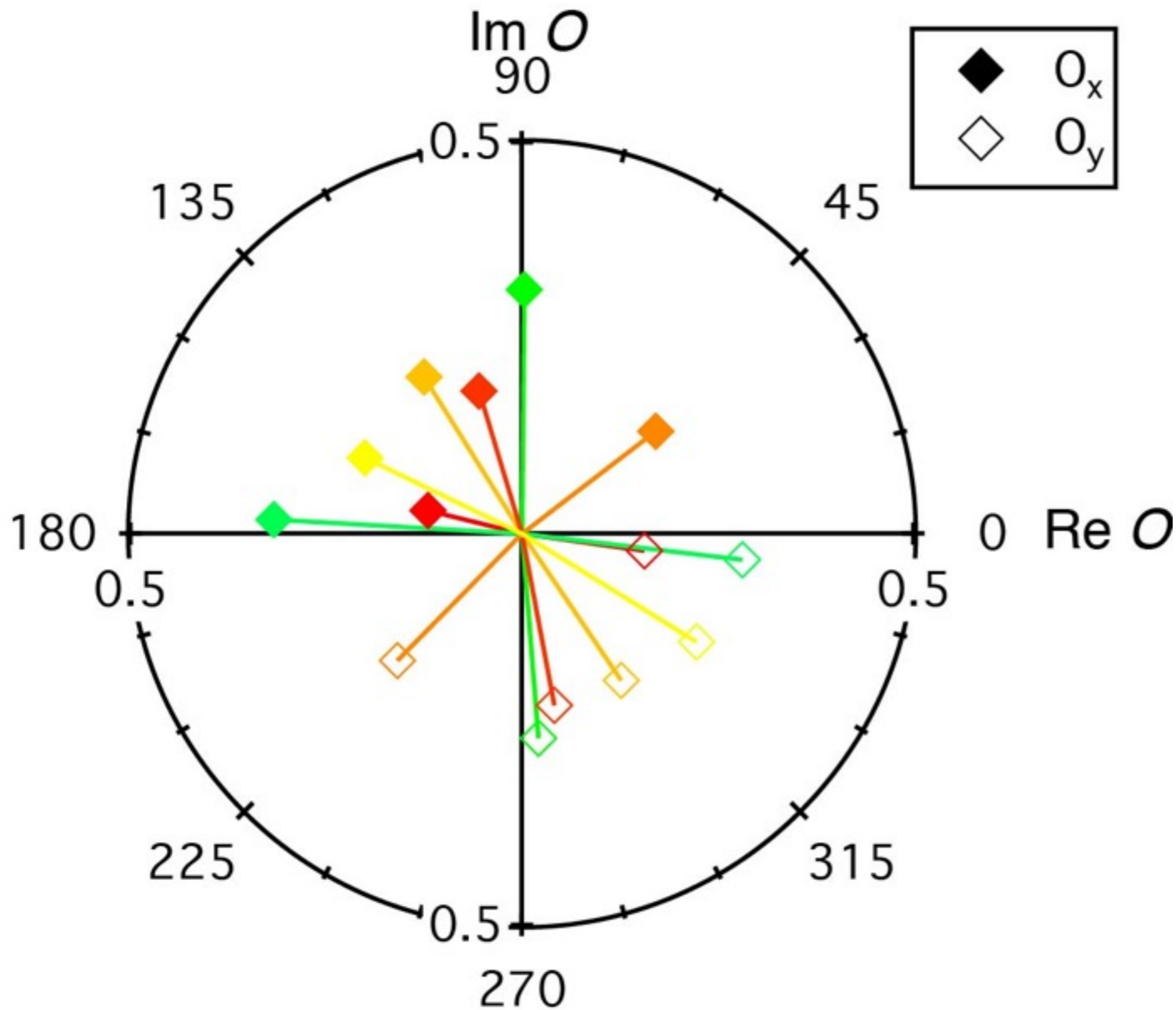
Phase-sensitive measurement of the d -form factor of density wave order



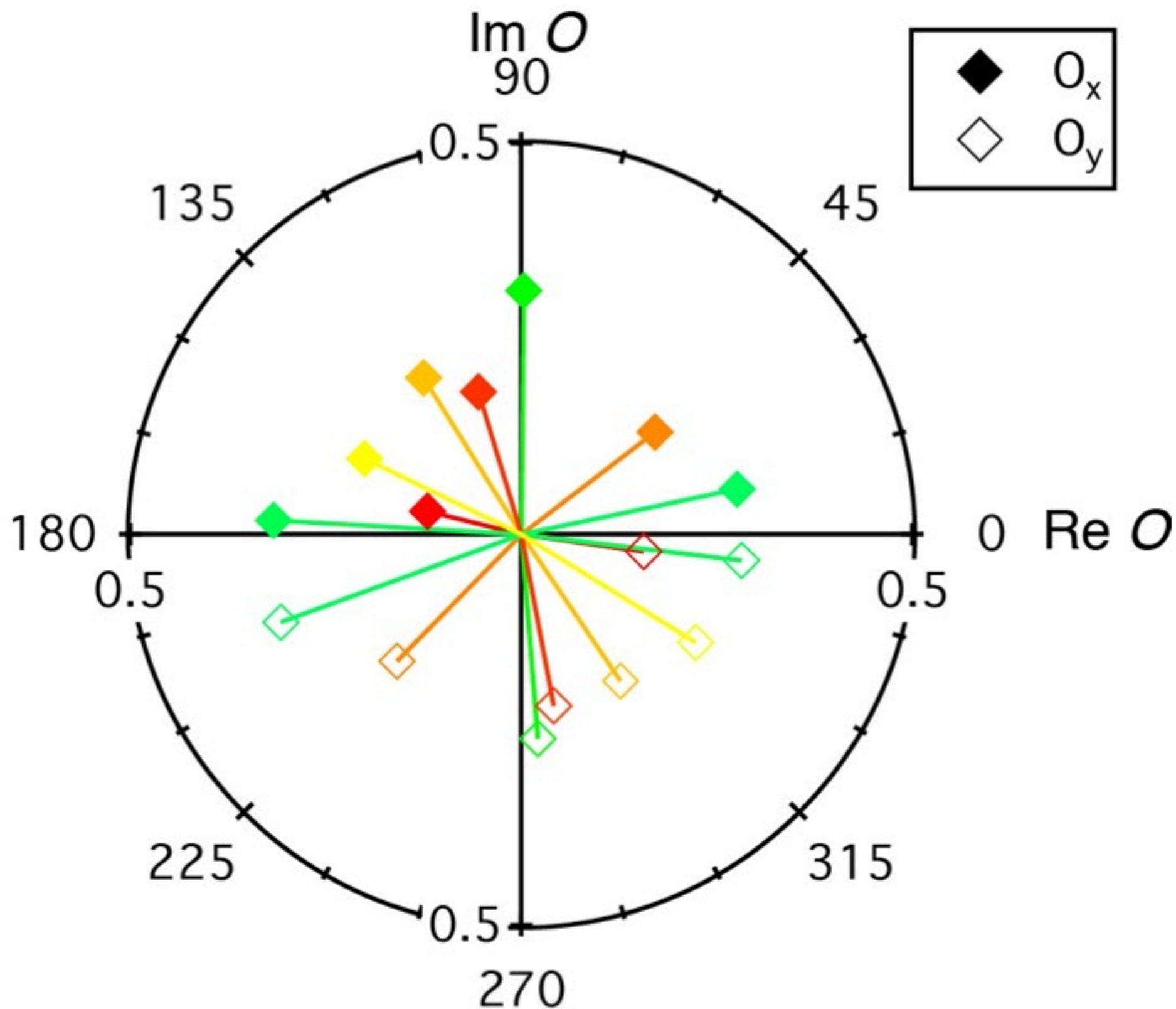
Phase-sensitive measurement of the d -form factor of density wave order



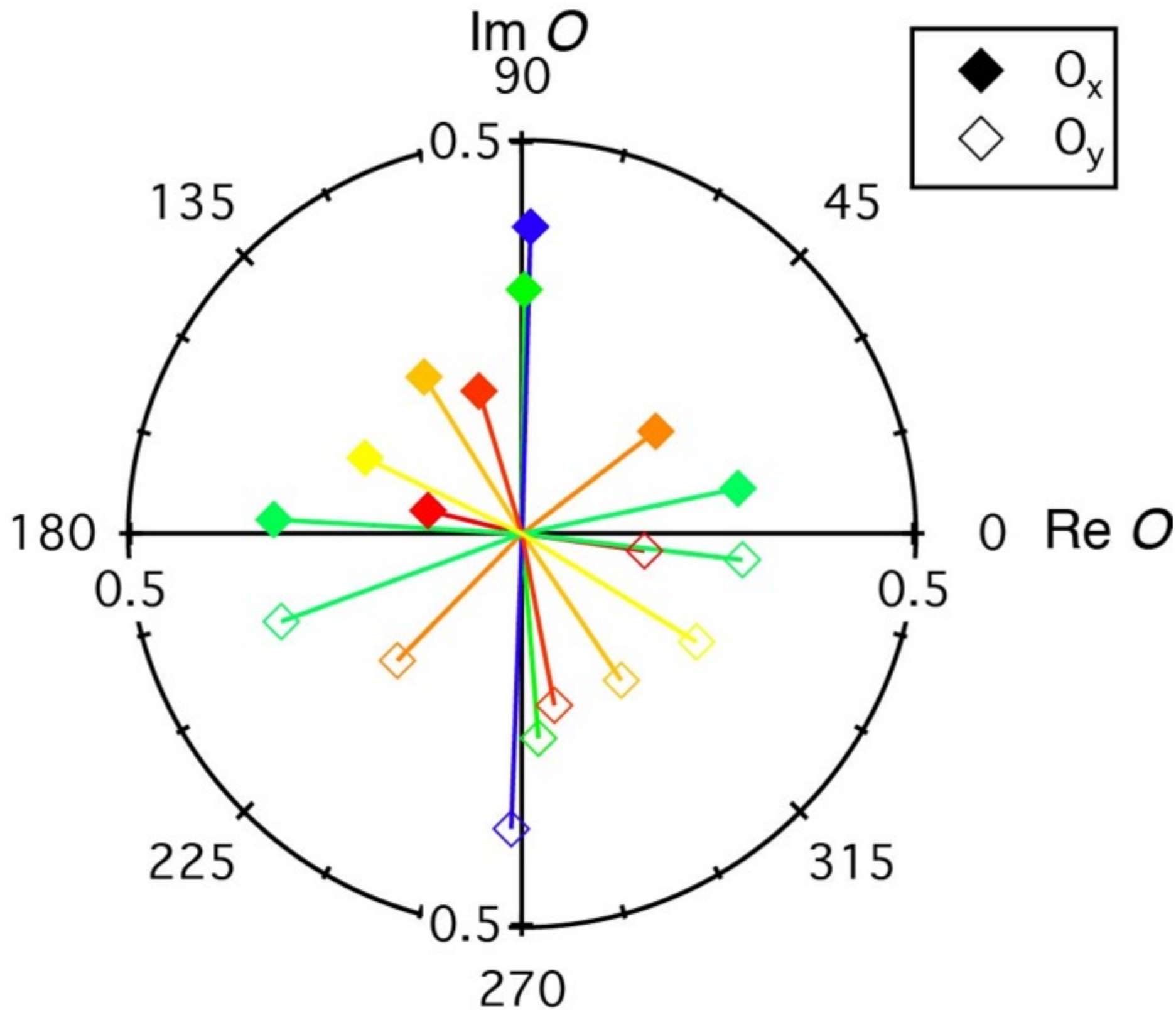
Phase-sensitive measurement of the d -form factor of density wave order



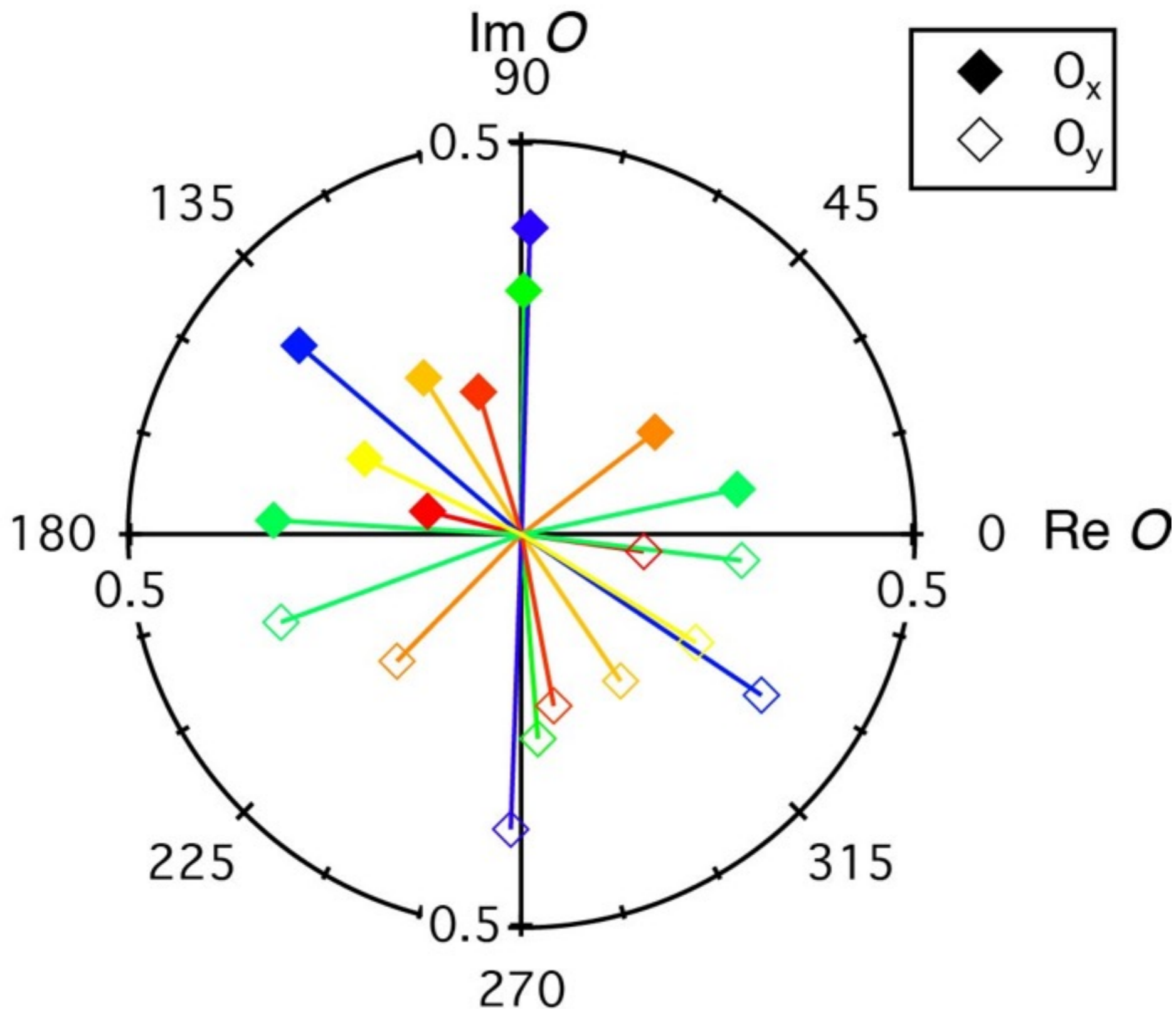
Phase-sensitive measurement of the *d*-form factor of density wave order



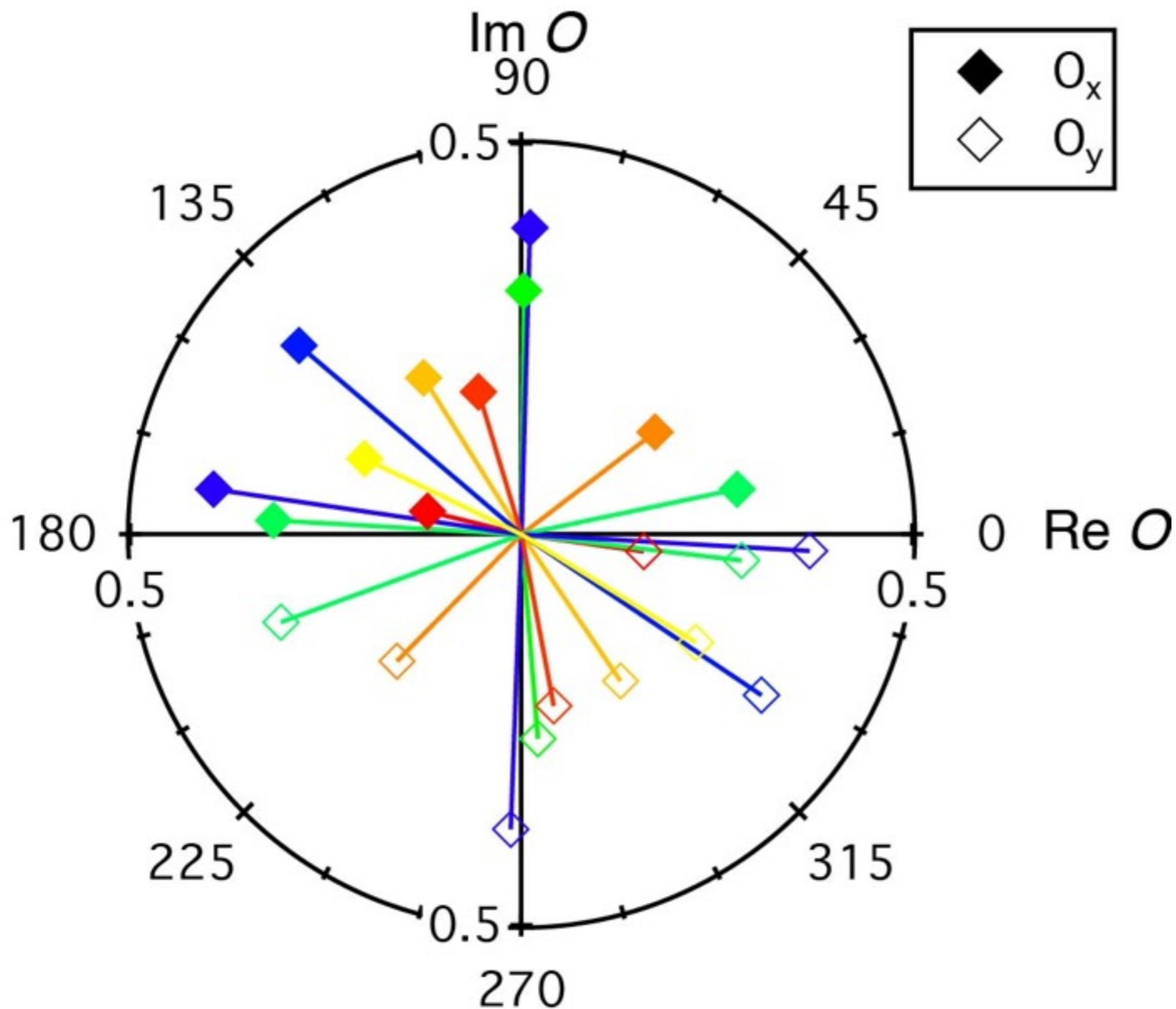
**Phase-sensitive
measurement of
the d -form factor
of density wave
order**



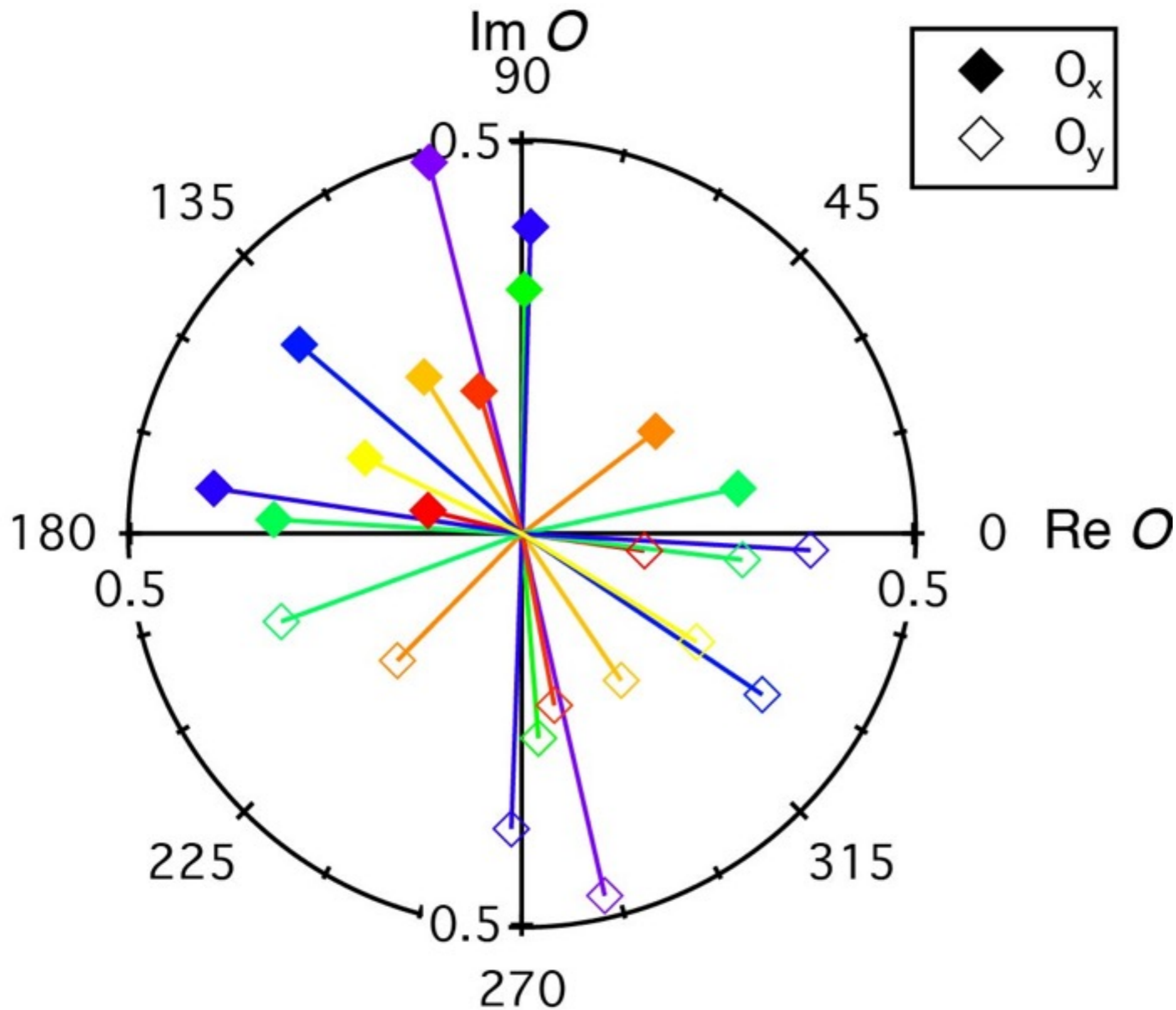
**Phase-sensitive
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**Phase-sensitive
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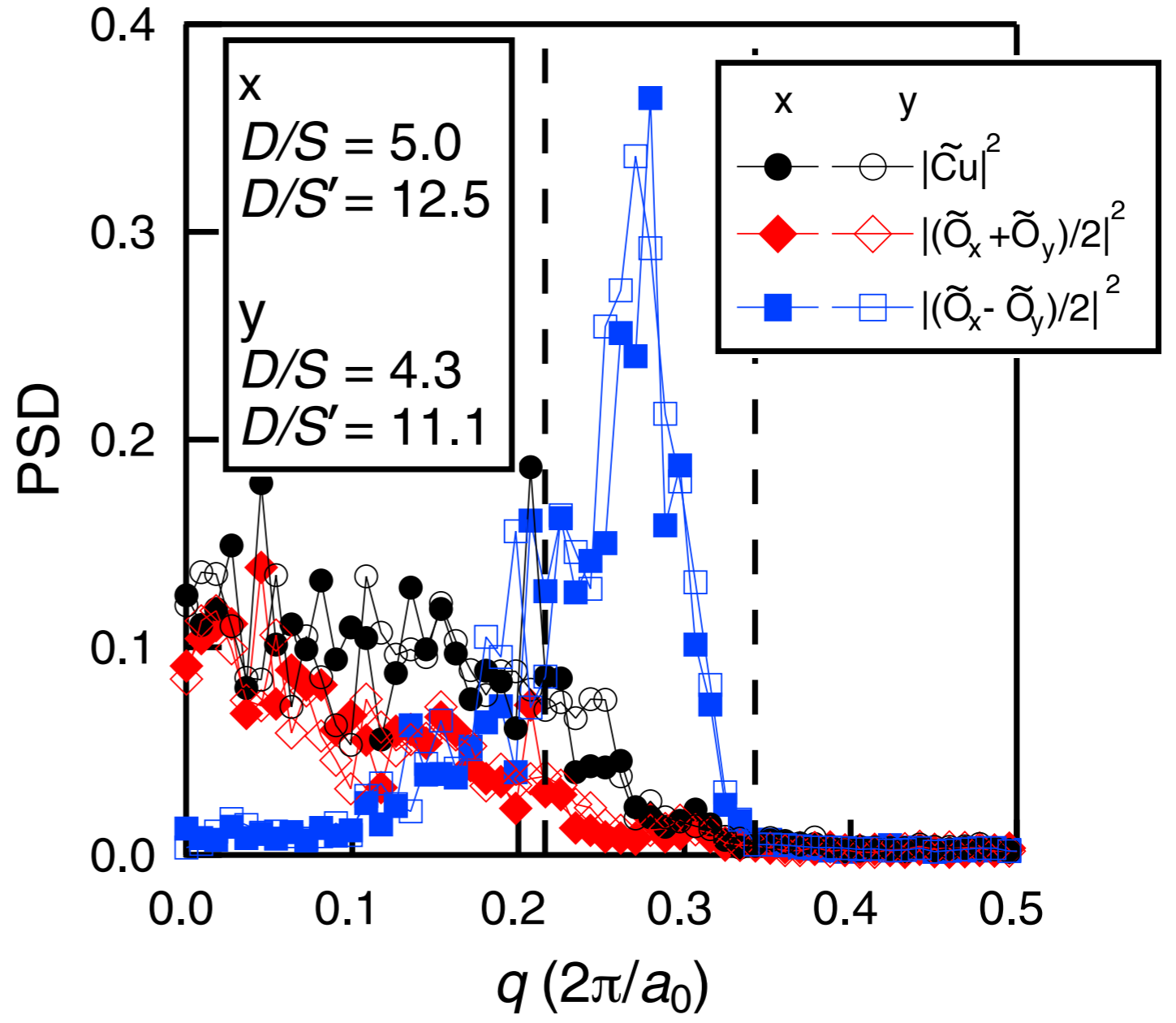
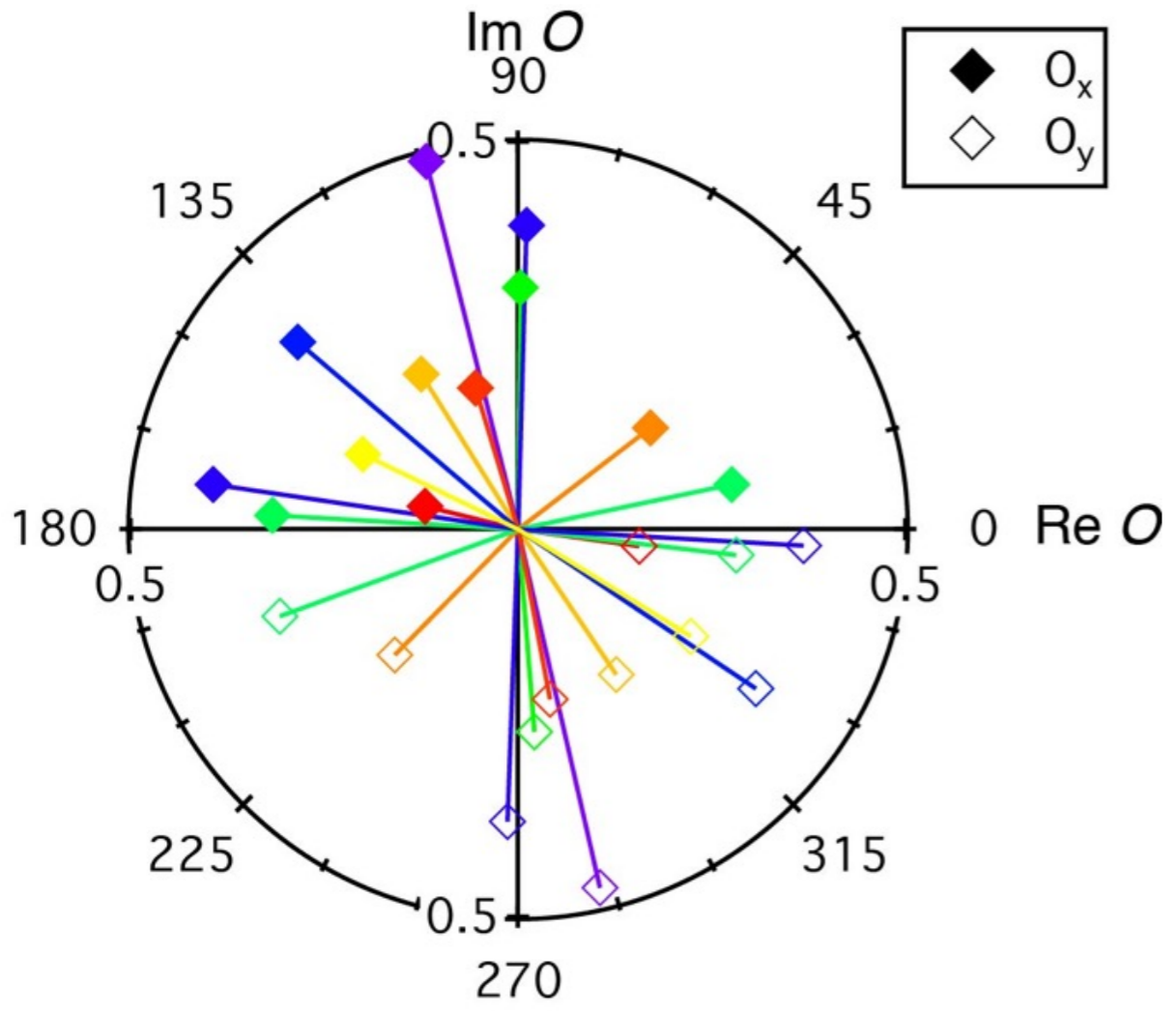


**Phase-sensitive
measurement of
the d -form factor
of density wave
order**



Phase-sensitive measurement of the *d*-form factor of density wave order

Phase-sensitive measurement of the d -form factor of density wave order



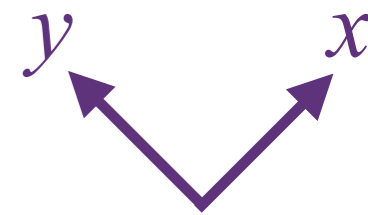
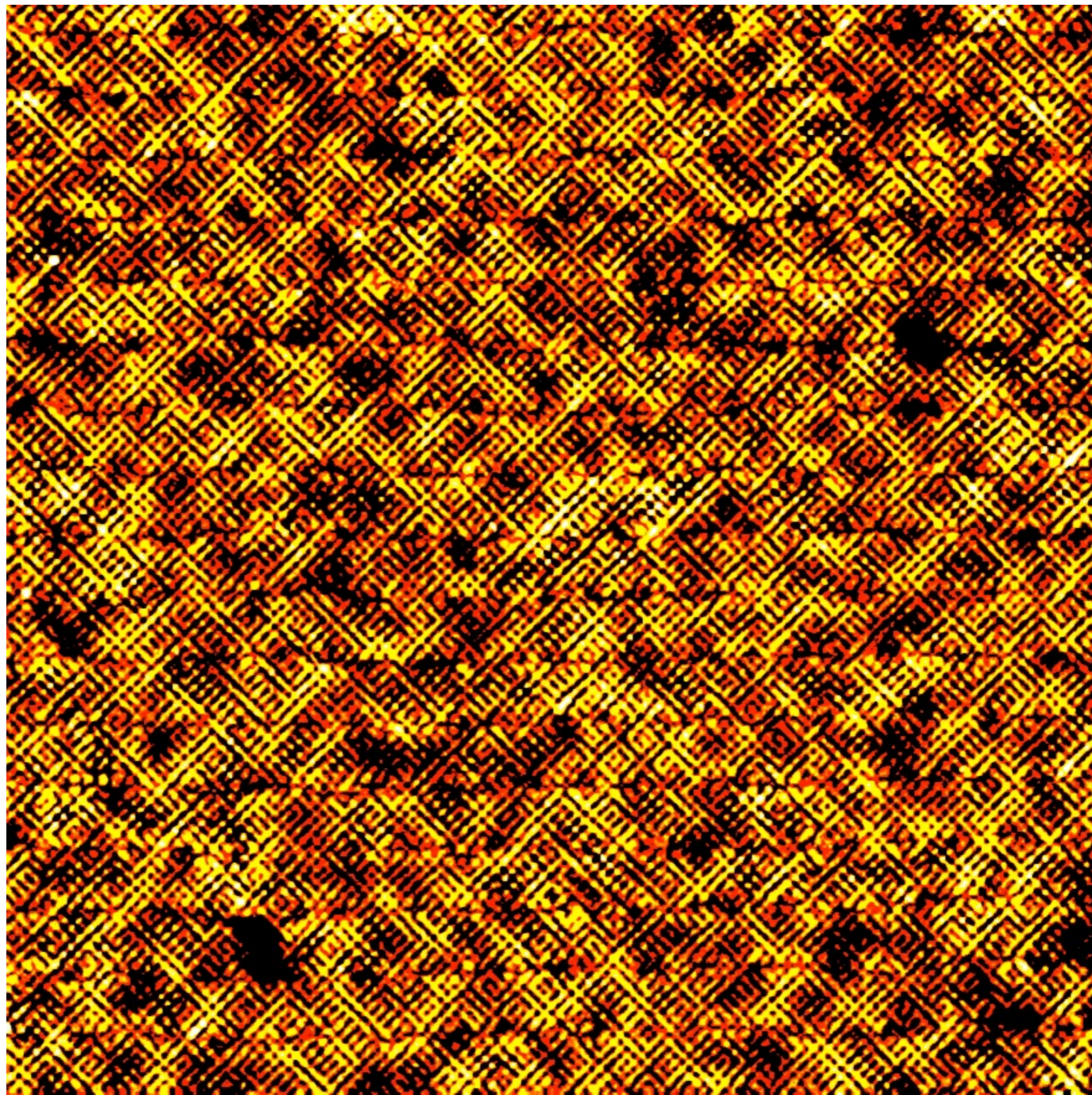
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See also

C. Howald, H. Eisaki,
N. Kaneko, M. Greven,
and A. Kapitulnik,
Phys. Rev. B **67**,
014533 (2003);

M. Vershinin, S. Misra,
S. Ono, Y. Abe, Yoichi
Ando, and
A. Yazdani, *Science*
303, 1995 (2004).

W. D. Wise, M. C. Boyer,
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Nature Phys. **4**, 696
(2008).



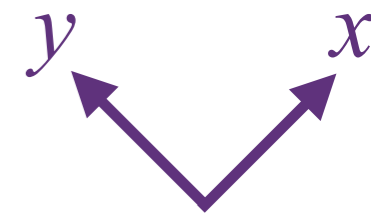
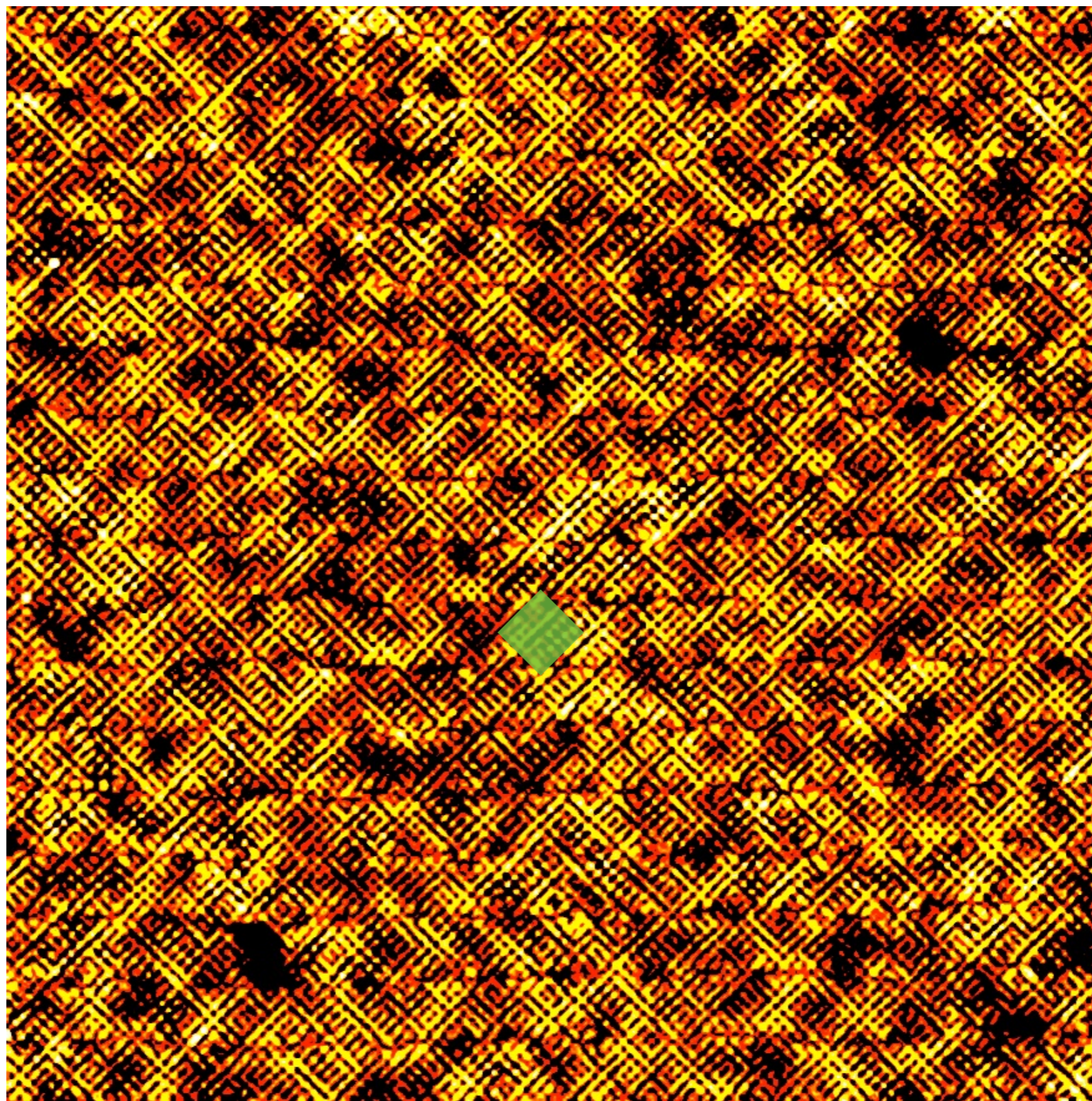
“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

See also

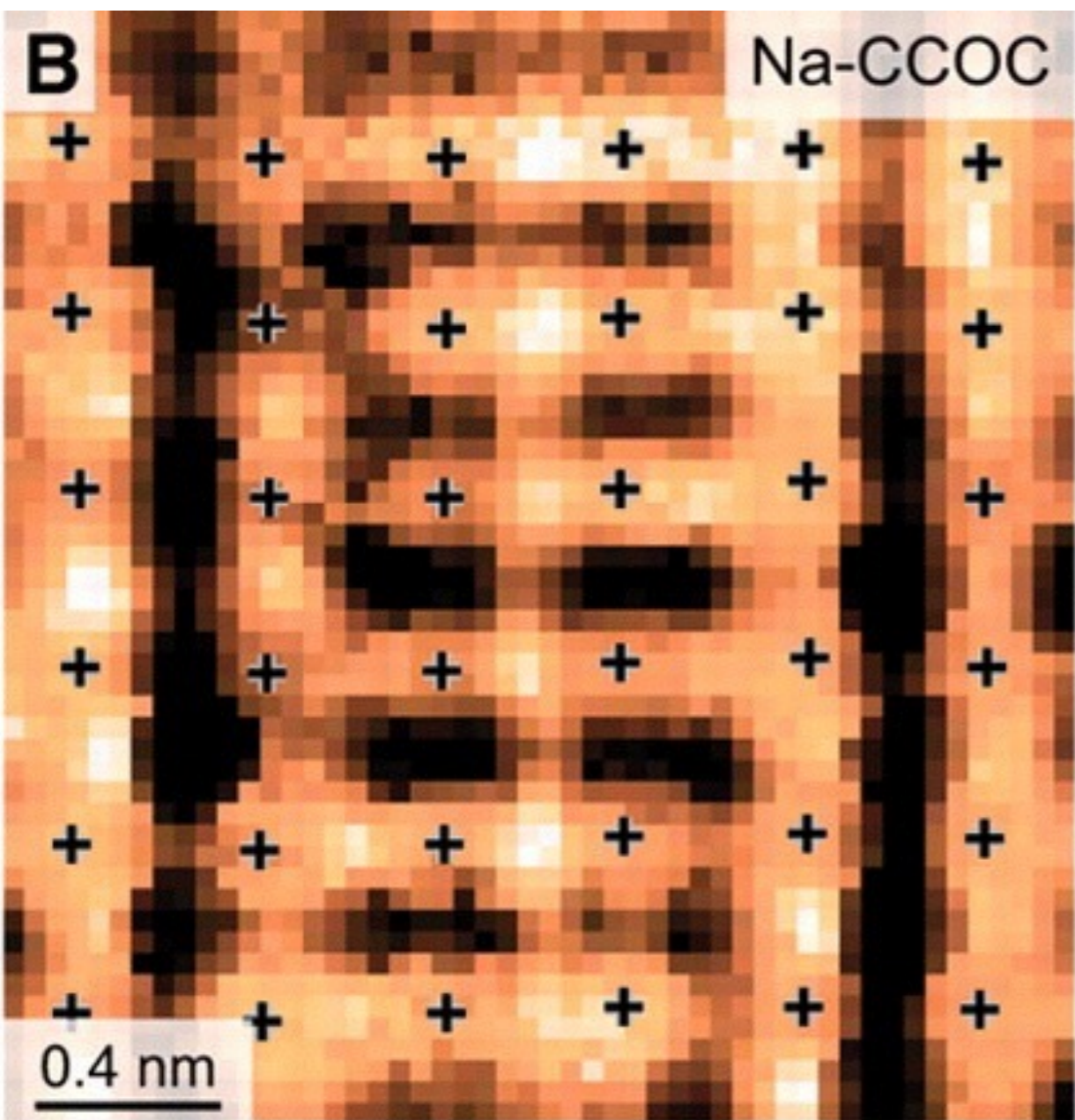
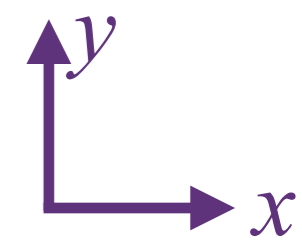
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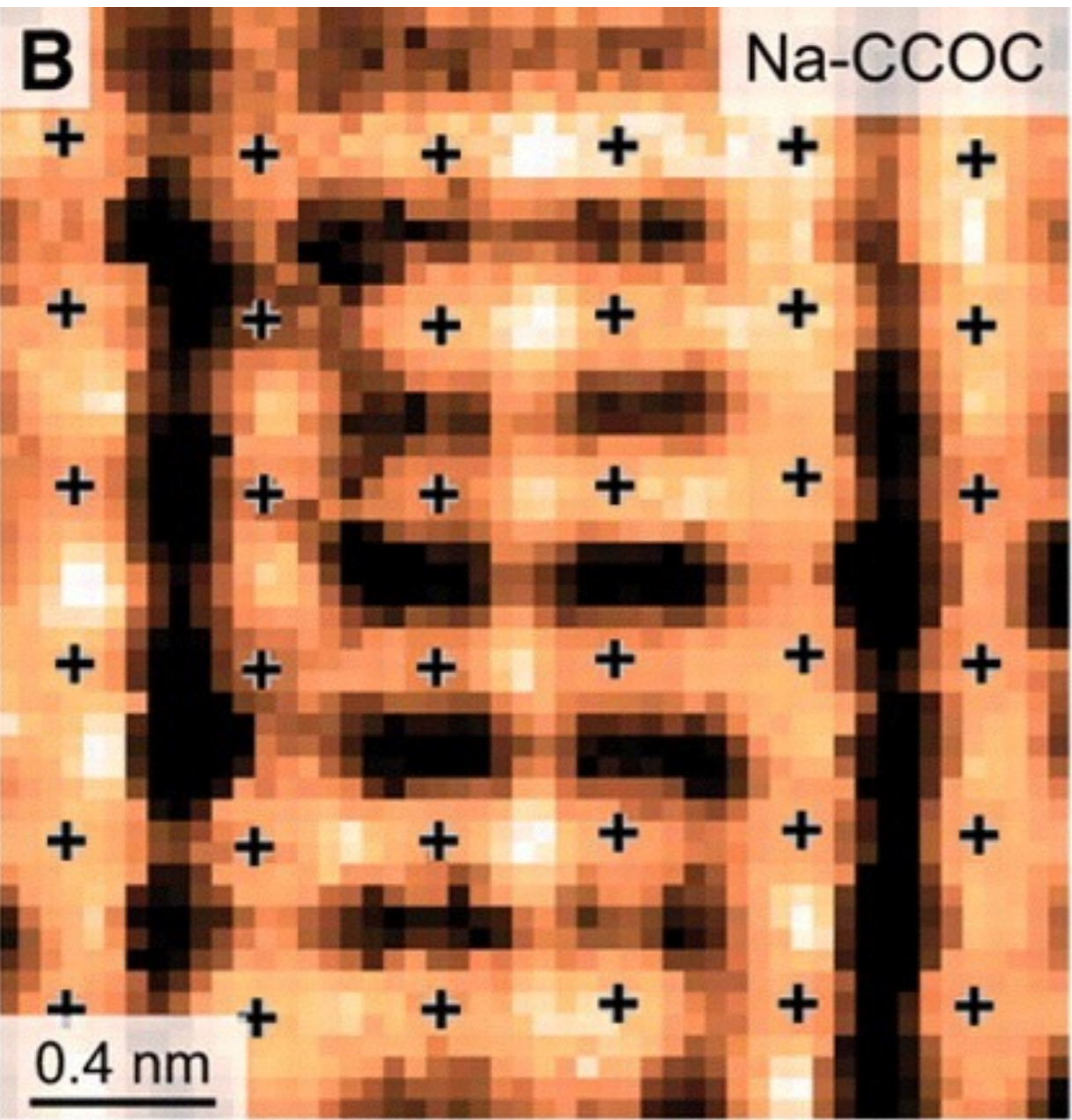
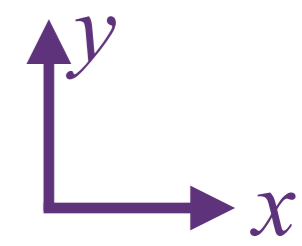
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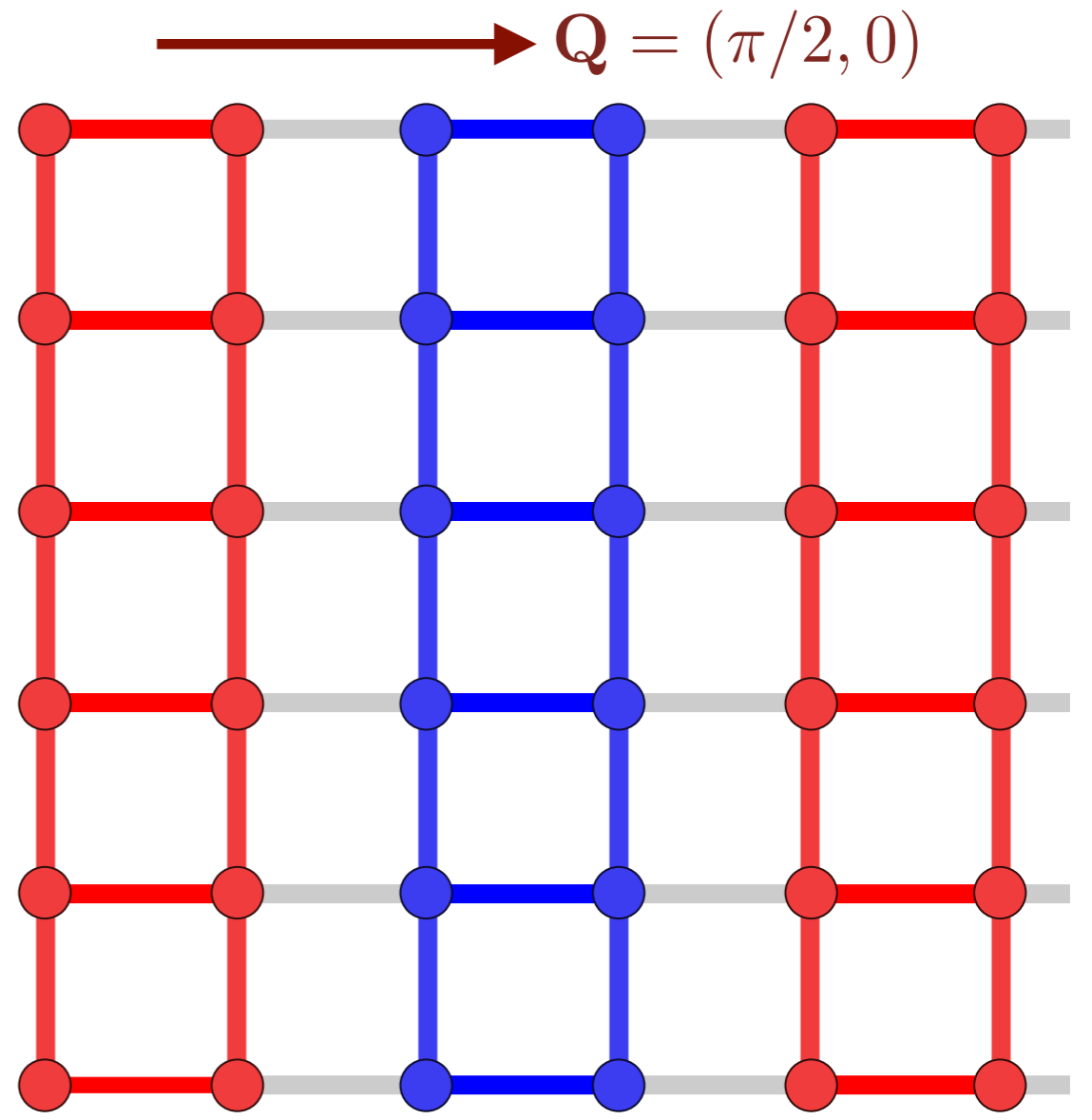
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Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

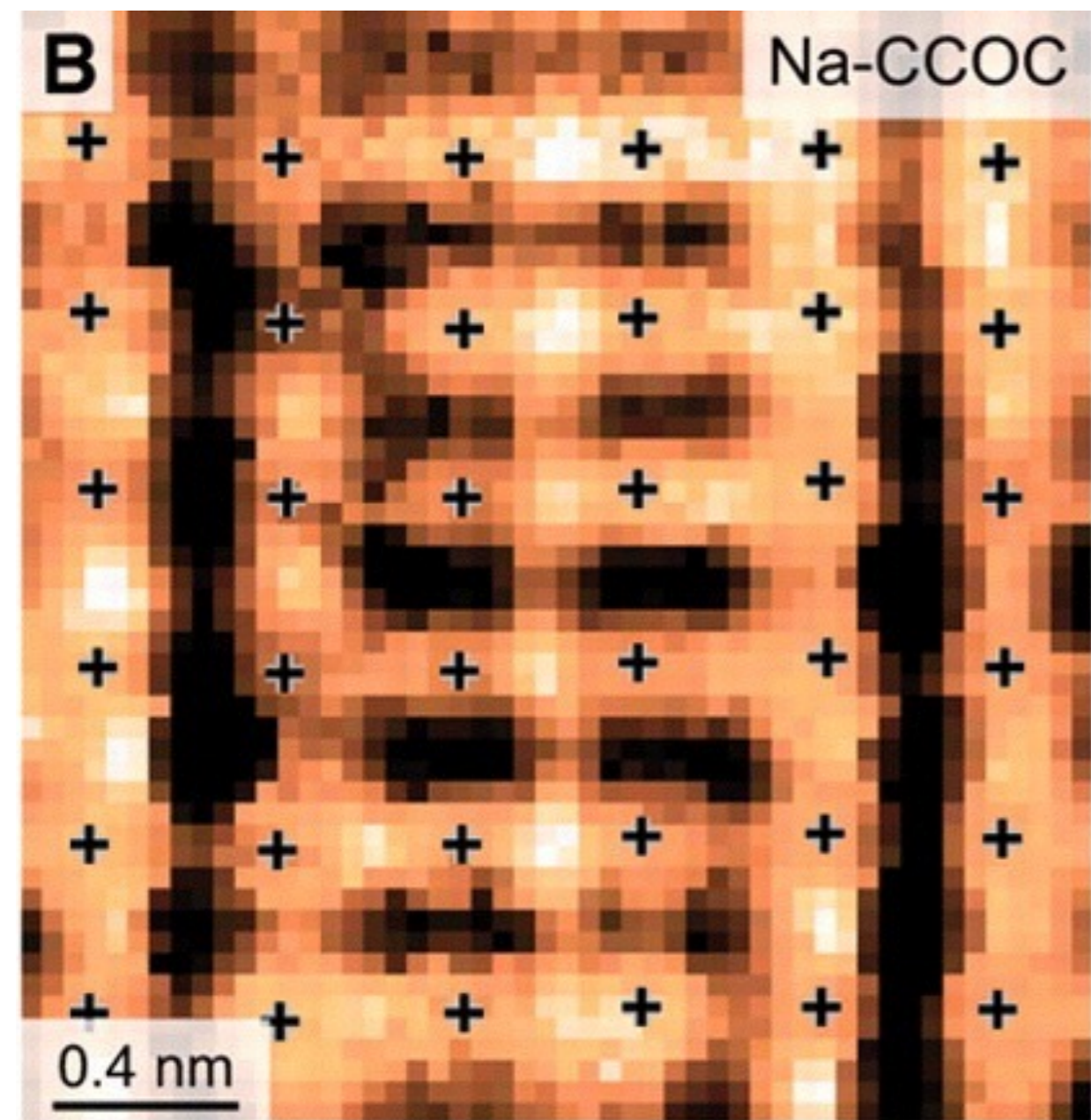


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

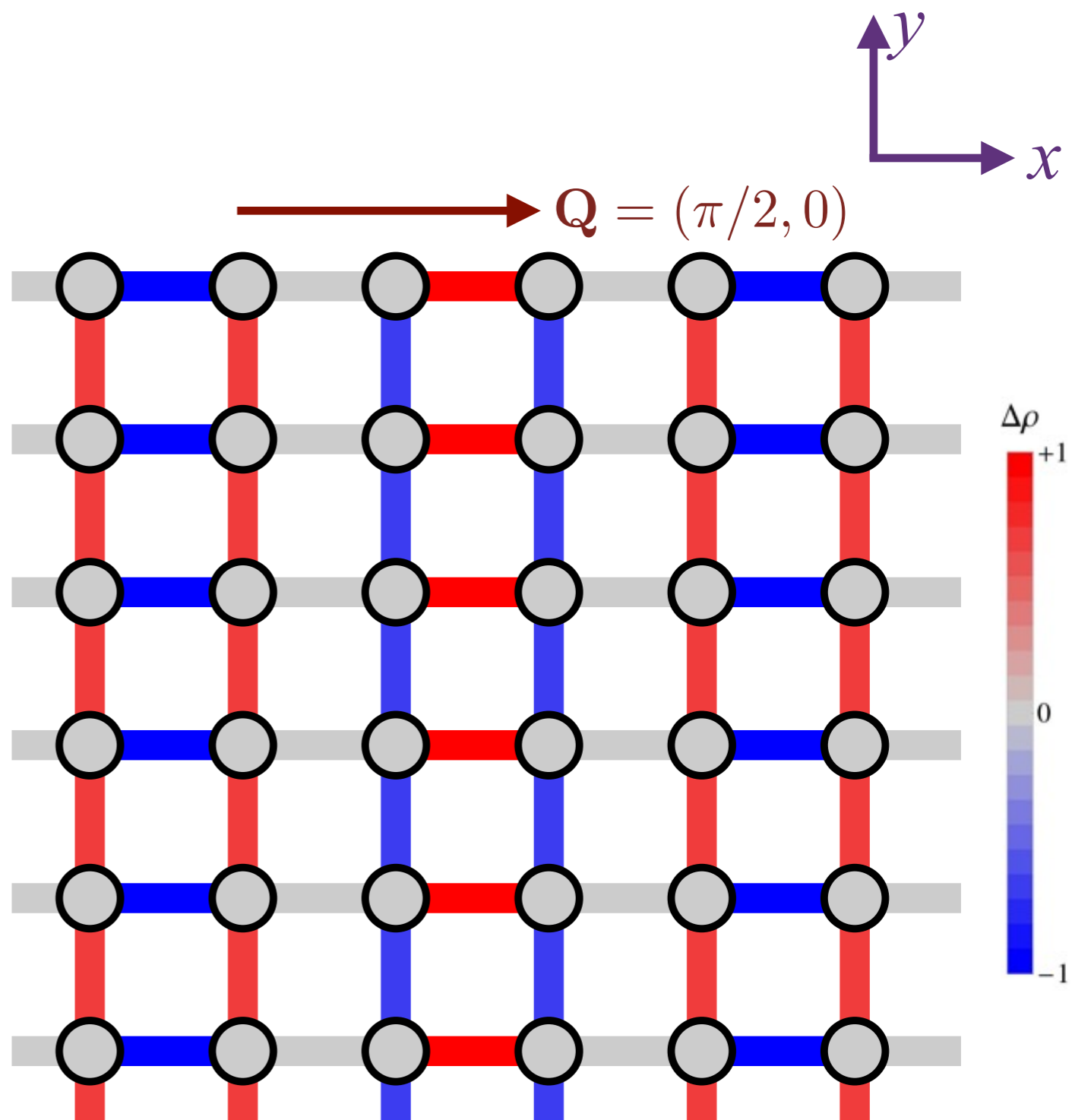


$s + s'$ -form factor density wave

$s + s'$ form factor (stripe model) does not match STM measurements on BSCCO, Na-CCOC.

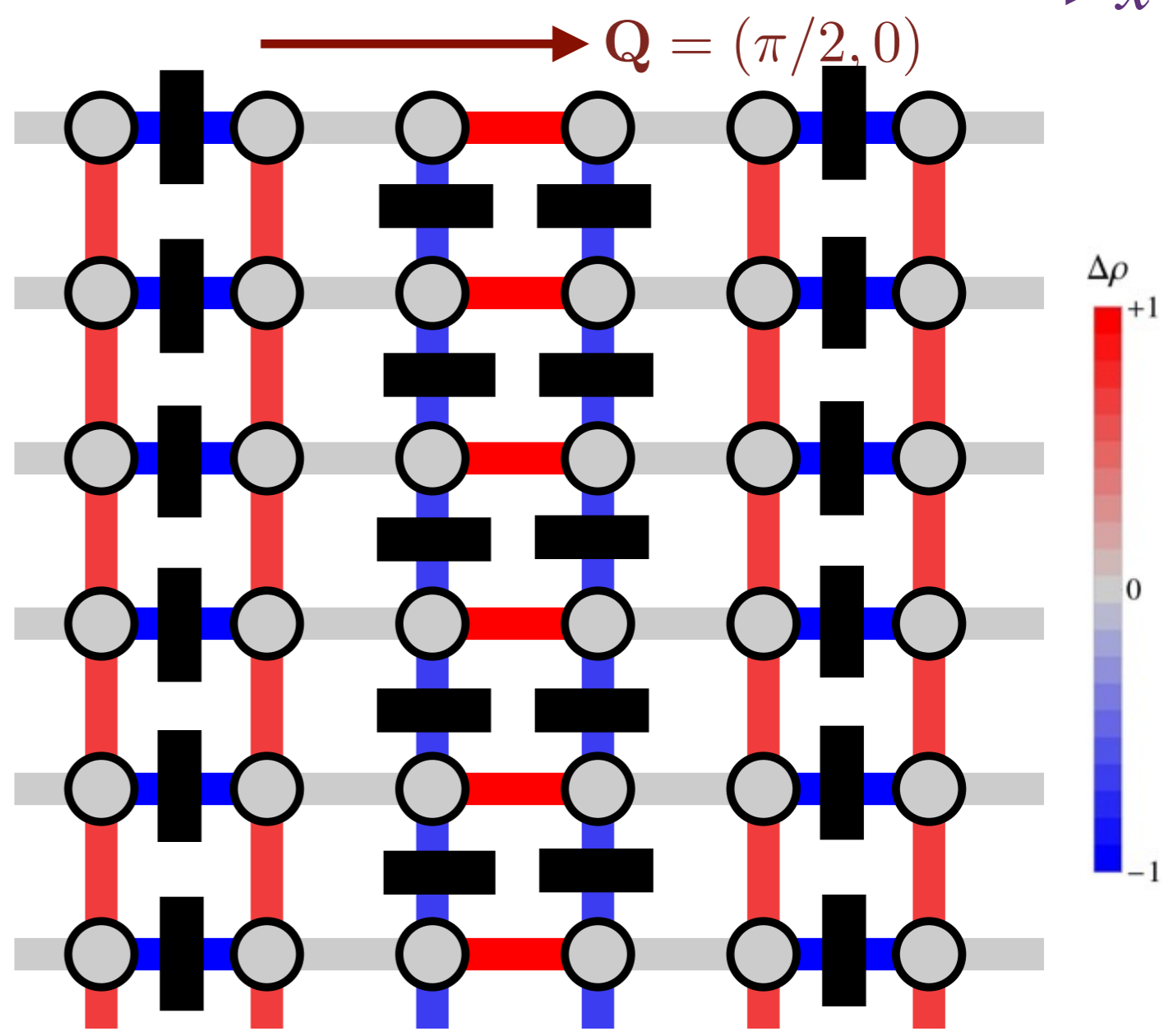
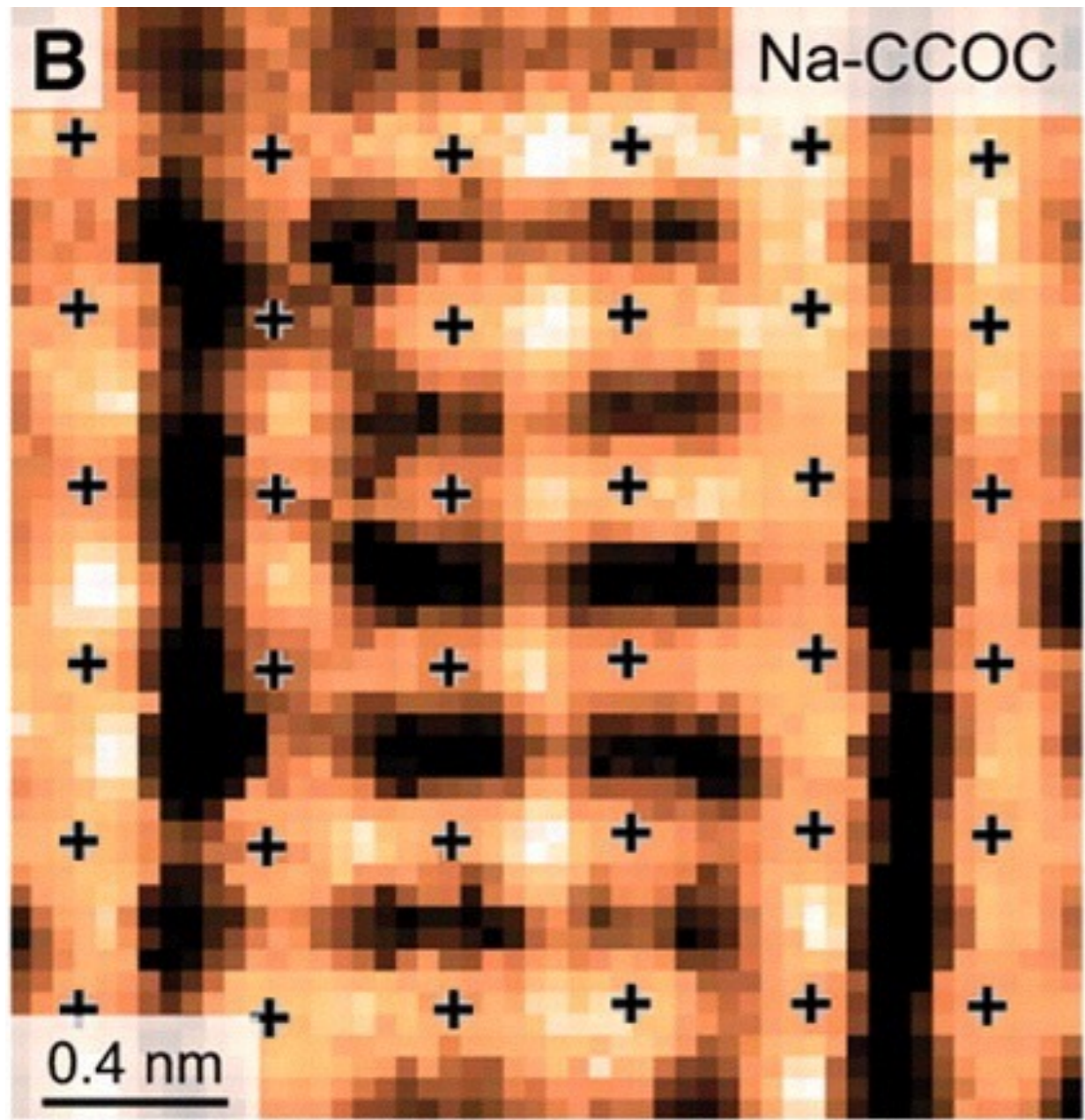
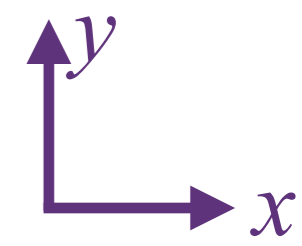


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



d-form factor density wave order

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
 S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

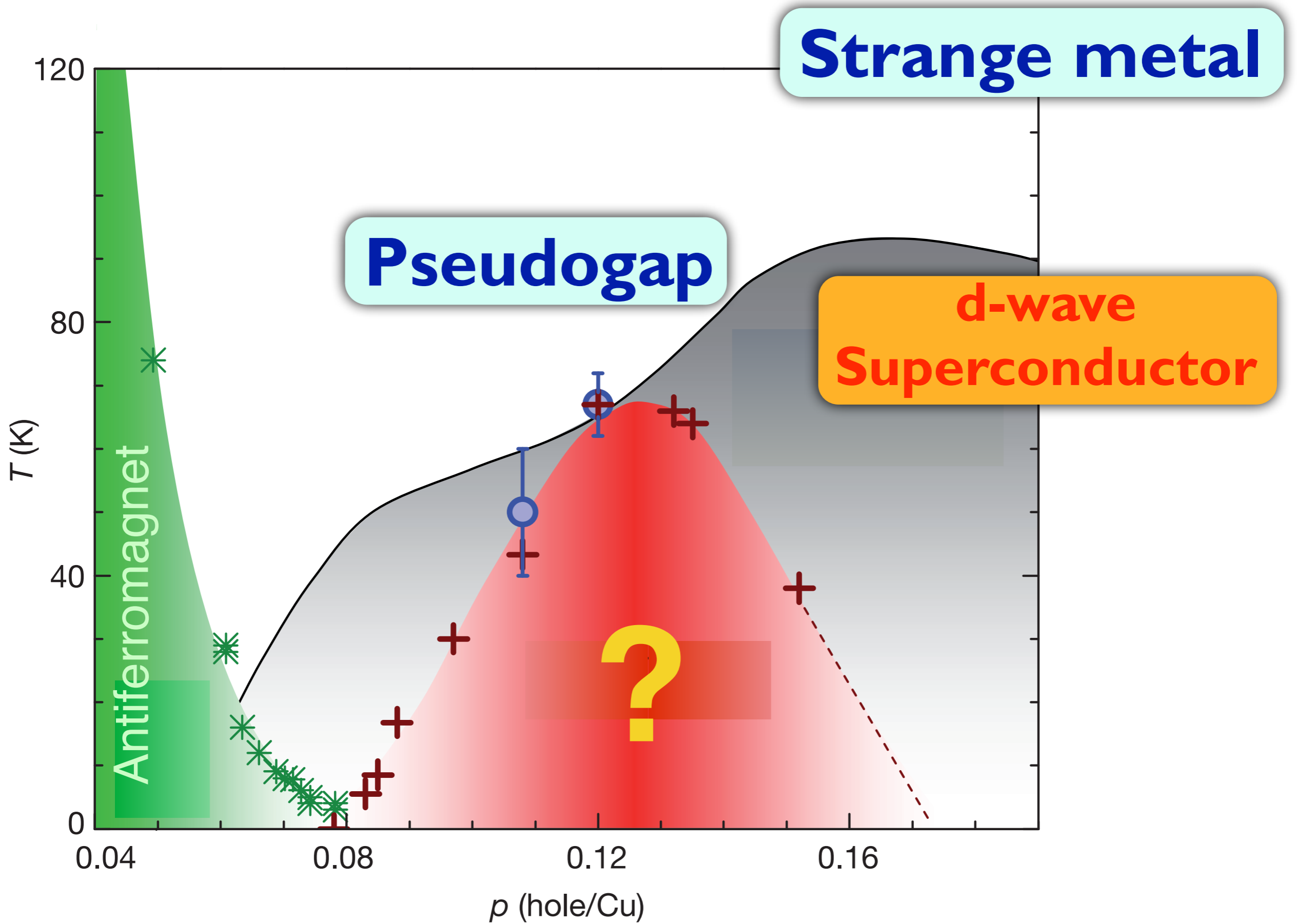


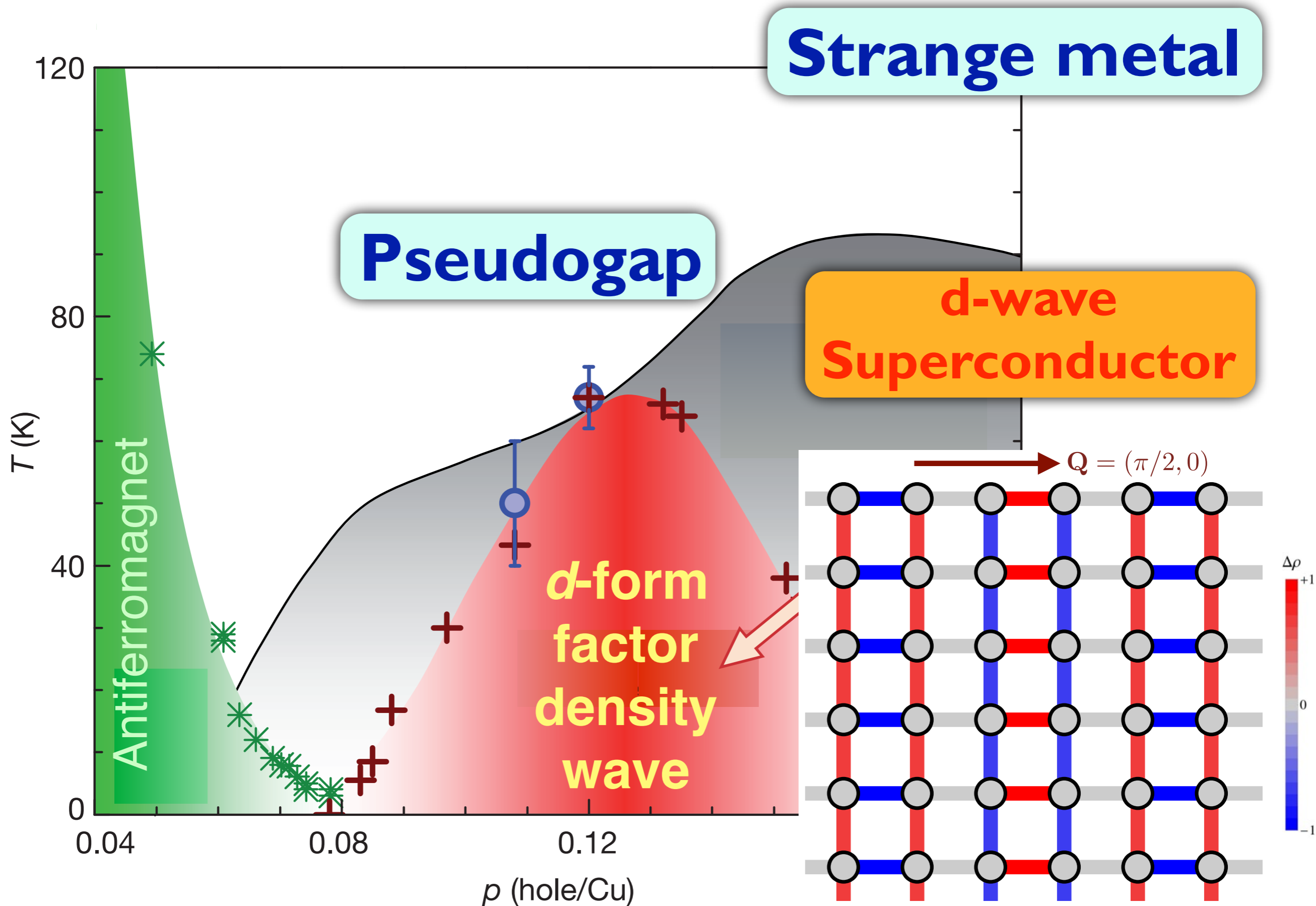
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

d-form factor density wave order

Predicted *d* form factor observed
in STM measurements on BSCCO, Na-CCOC !

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).





K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS 111, E3026 (2014)

Outline

1. The low T pseudogap:

*STM observation of predicted
d-form factor density wave*

2. The high T pseudogap:

*A metal with topological order:
the Fractionalized Fermi liquid: FL**

3. Connecting high and low T :

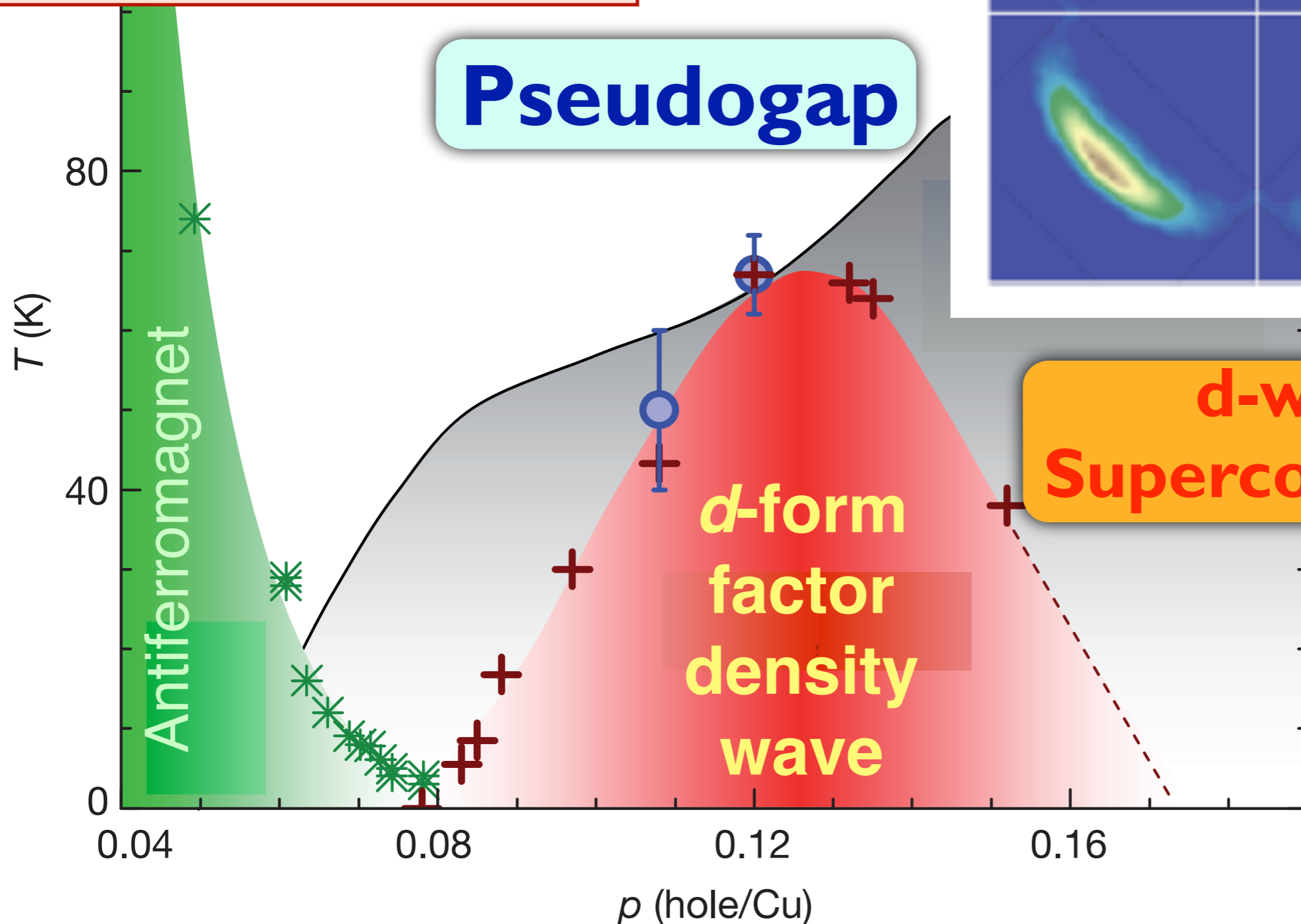
Density wave instabilities

4. Quantum critical point near optimal p :

A Higgs critical point

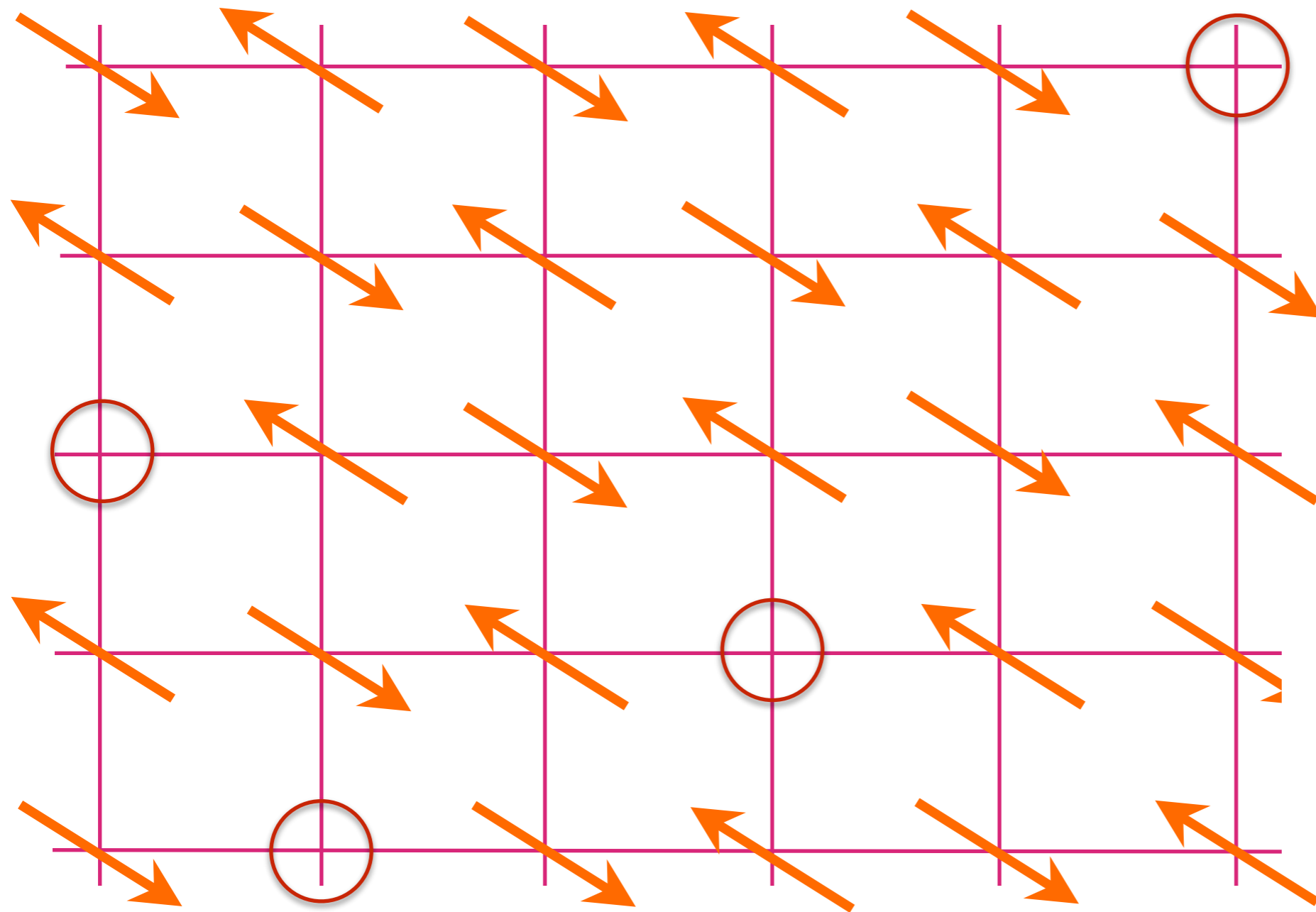
“Fermi arcs” at
low p

Pseudogap



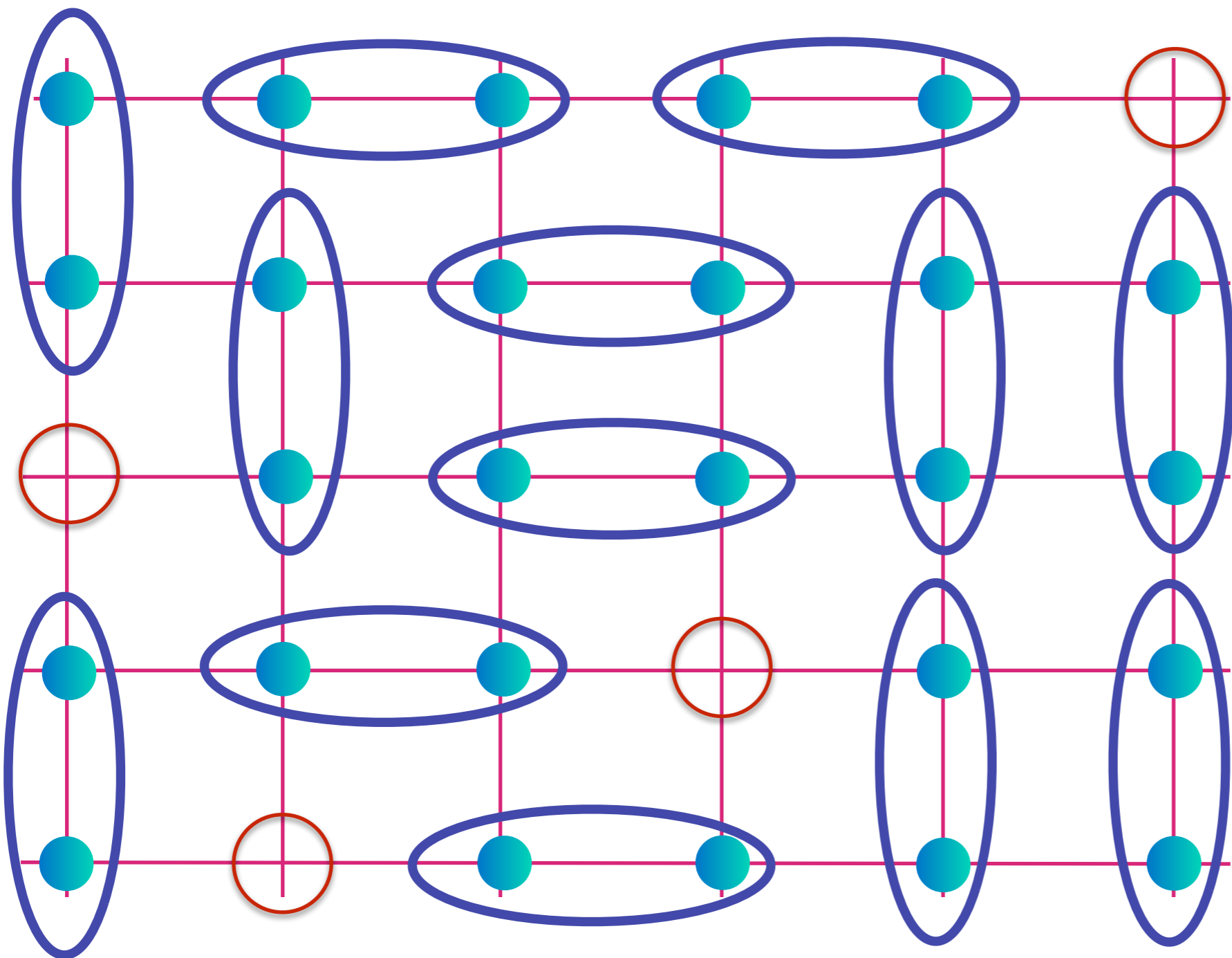
d-wave
Superconductor

d-form
factor
density
wave



Anti-ferromagnet with p holes per square

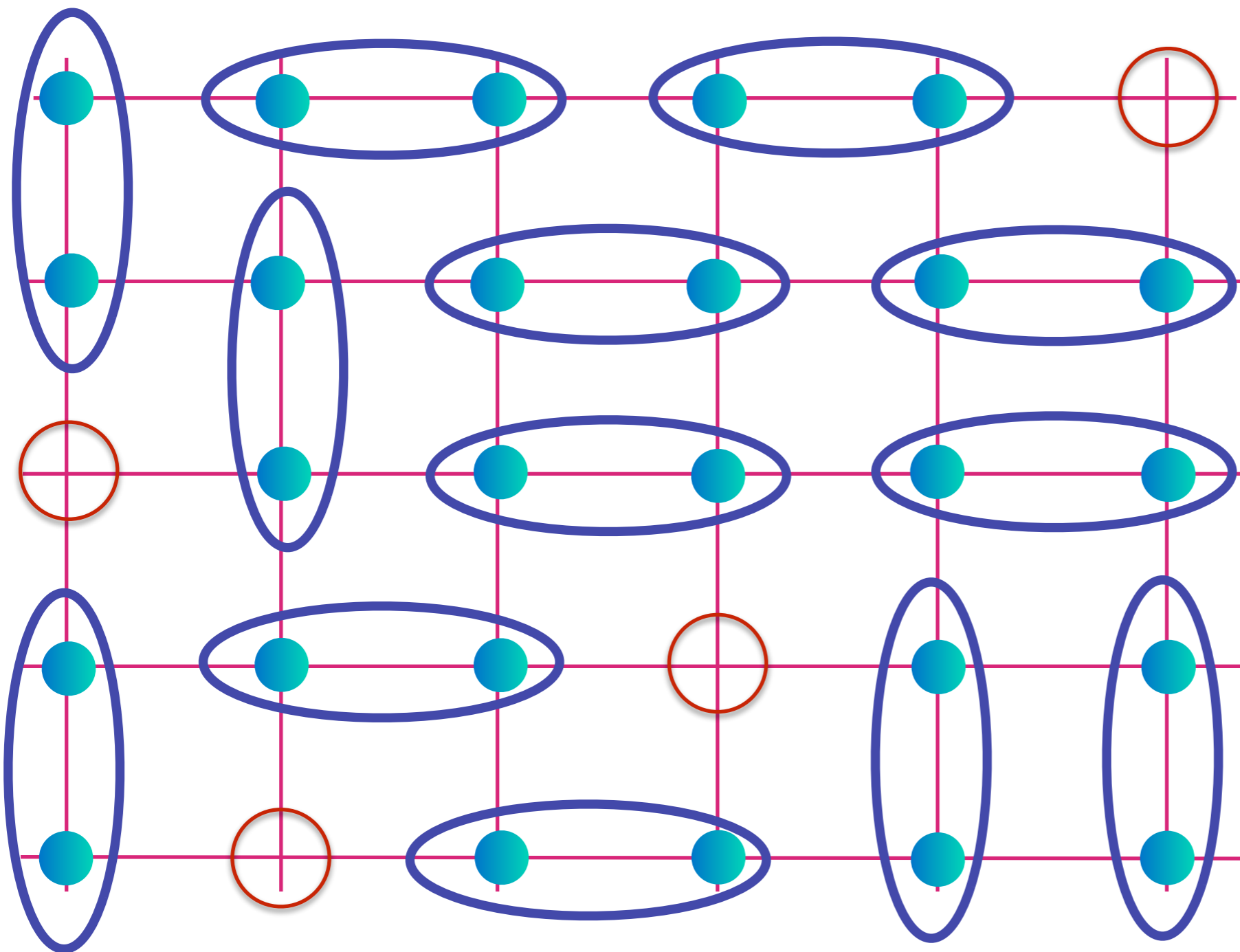
Note: relative to the fully-filled band insulator, there are $1+p$ holes per square



Spin liquid
with
 p "holons"
(spinless,
charge $+e$
quasiparticles)
per square

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

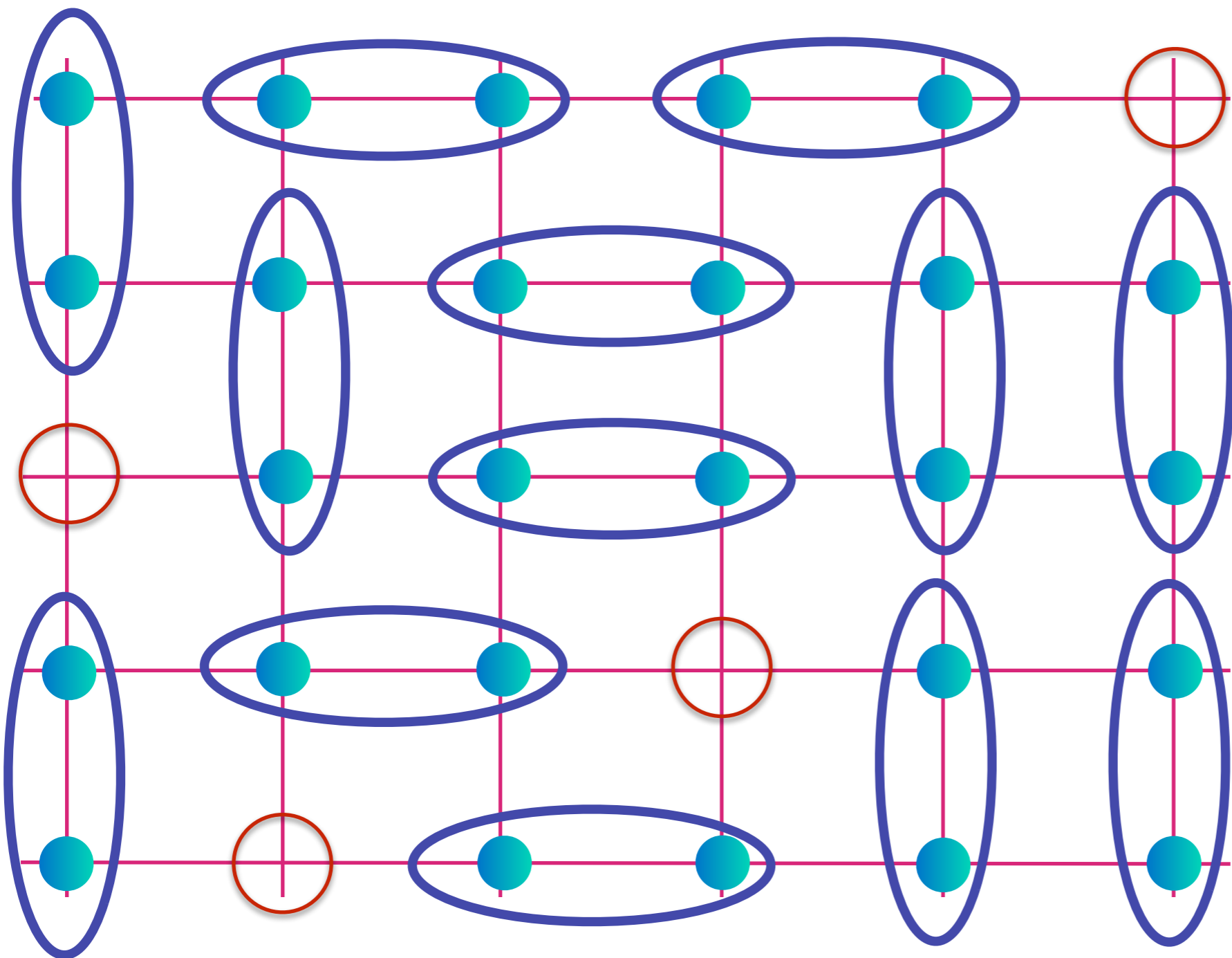
Baskaran, Zou, Anderson, Fradkin, Kivelson...



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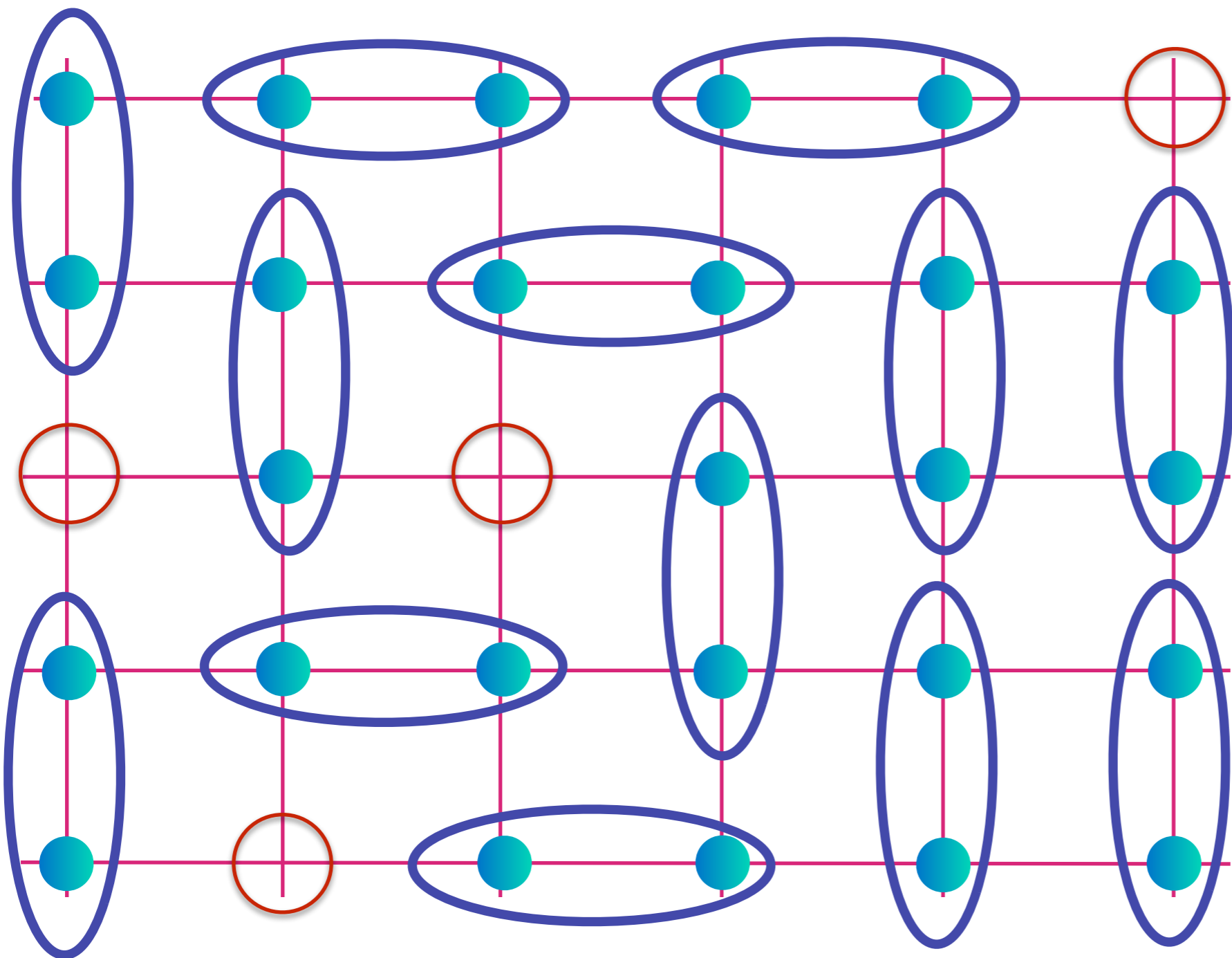
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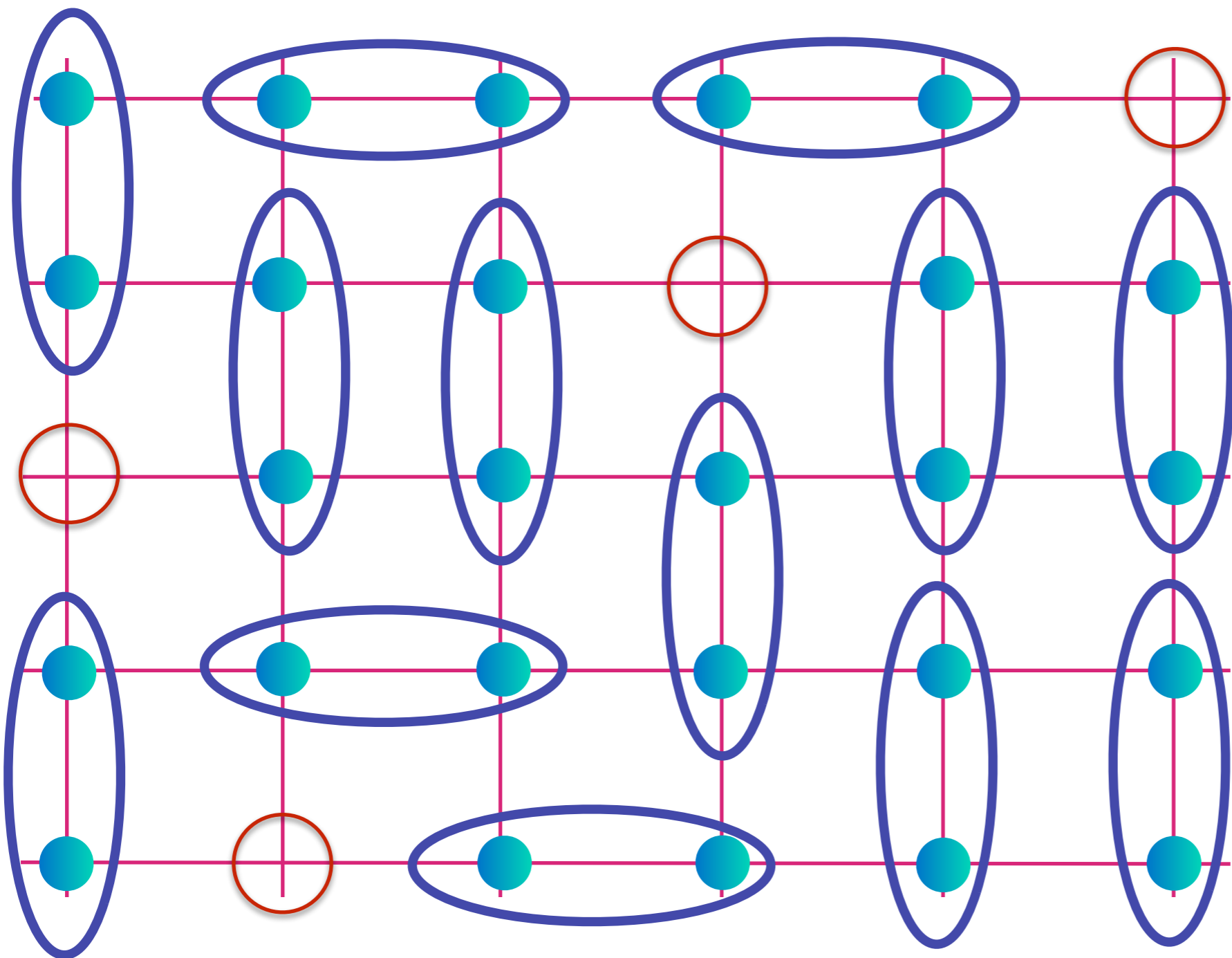
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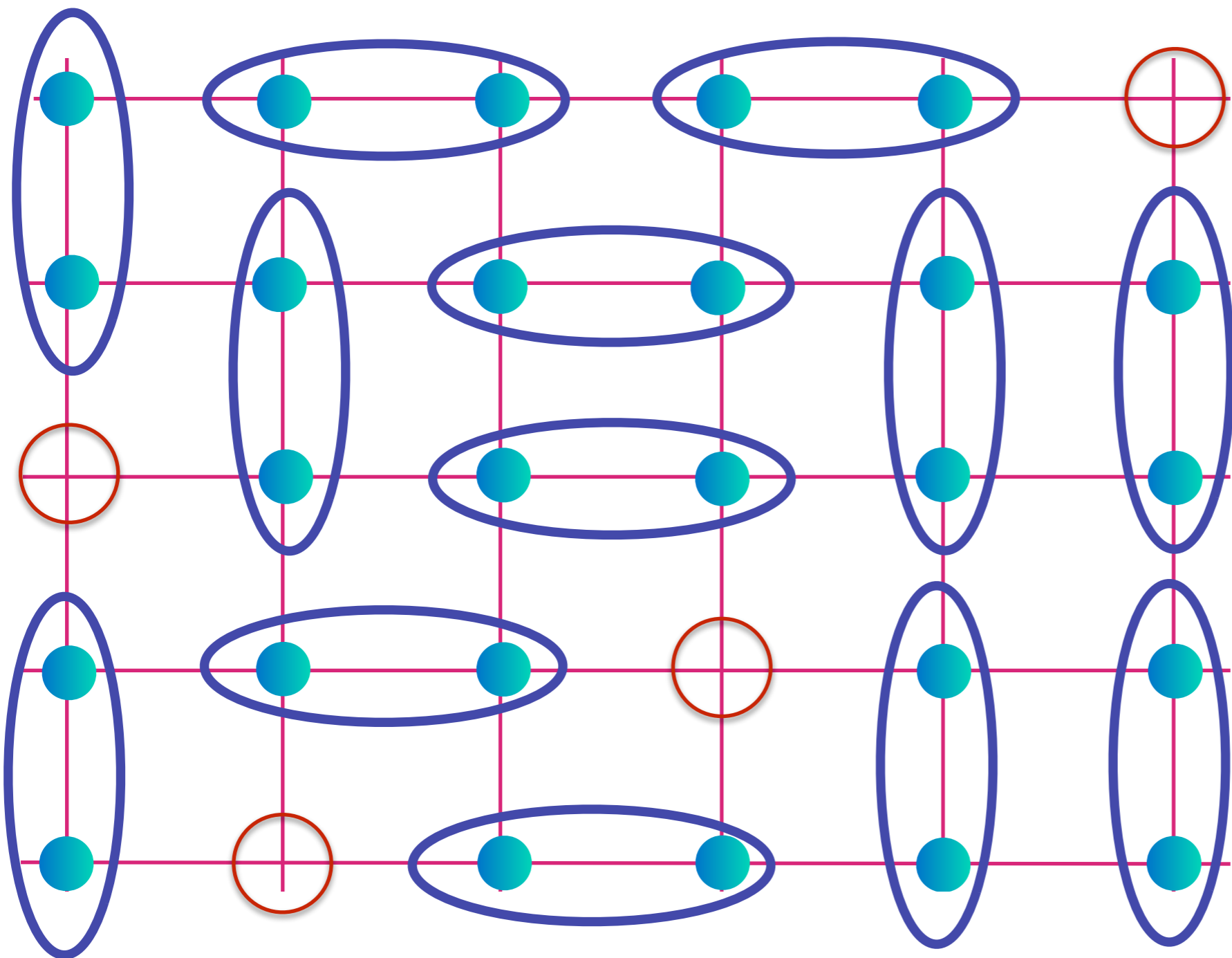
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Baskaran, Zou, Anderson, Fradkin, Kivelson...

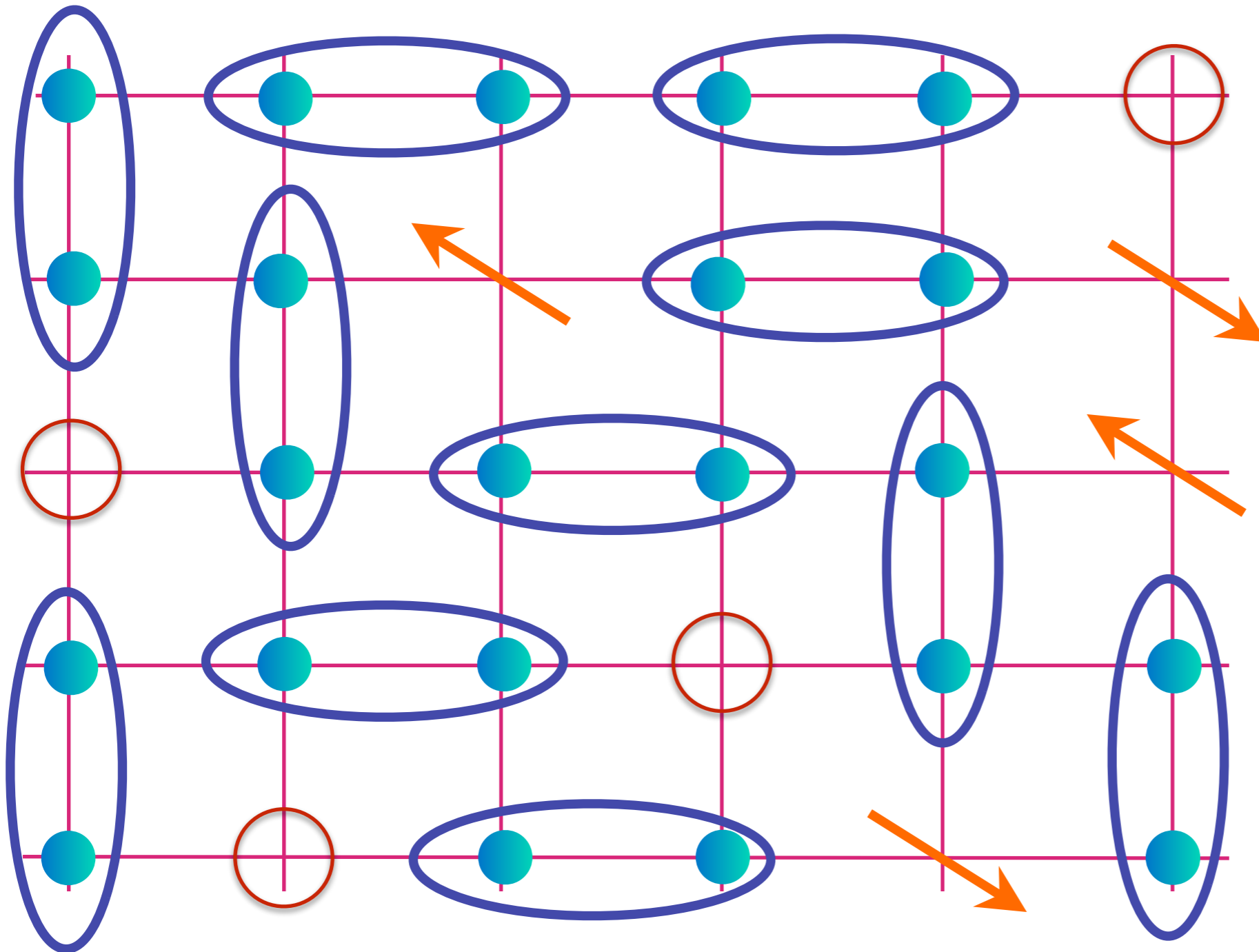


Spin liquid
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per square

$$\text{[blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

Neutral spin $S=1/2$ “spinon” excitations

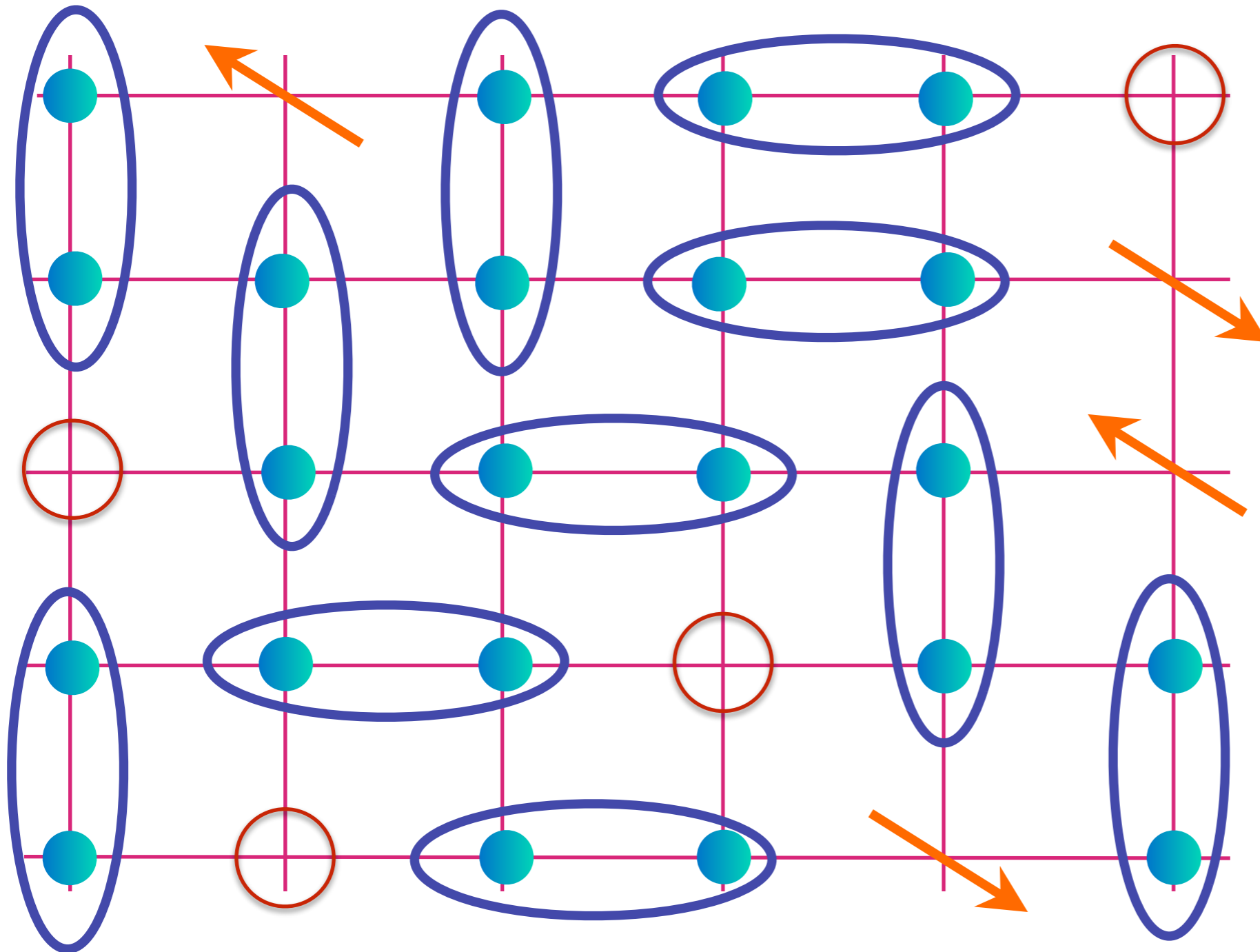


Spin liquid
with
 p “holons”
(spinless,
charge $+e$
quasiparticles)
per square

$$\text{[Blue oval with two cyan dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

Neutral spin $S=1/2$ “spinon” excitations

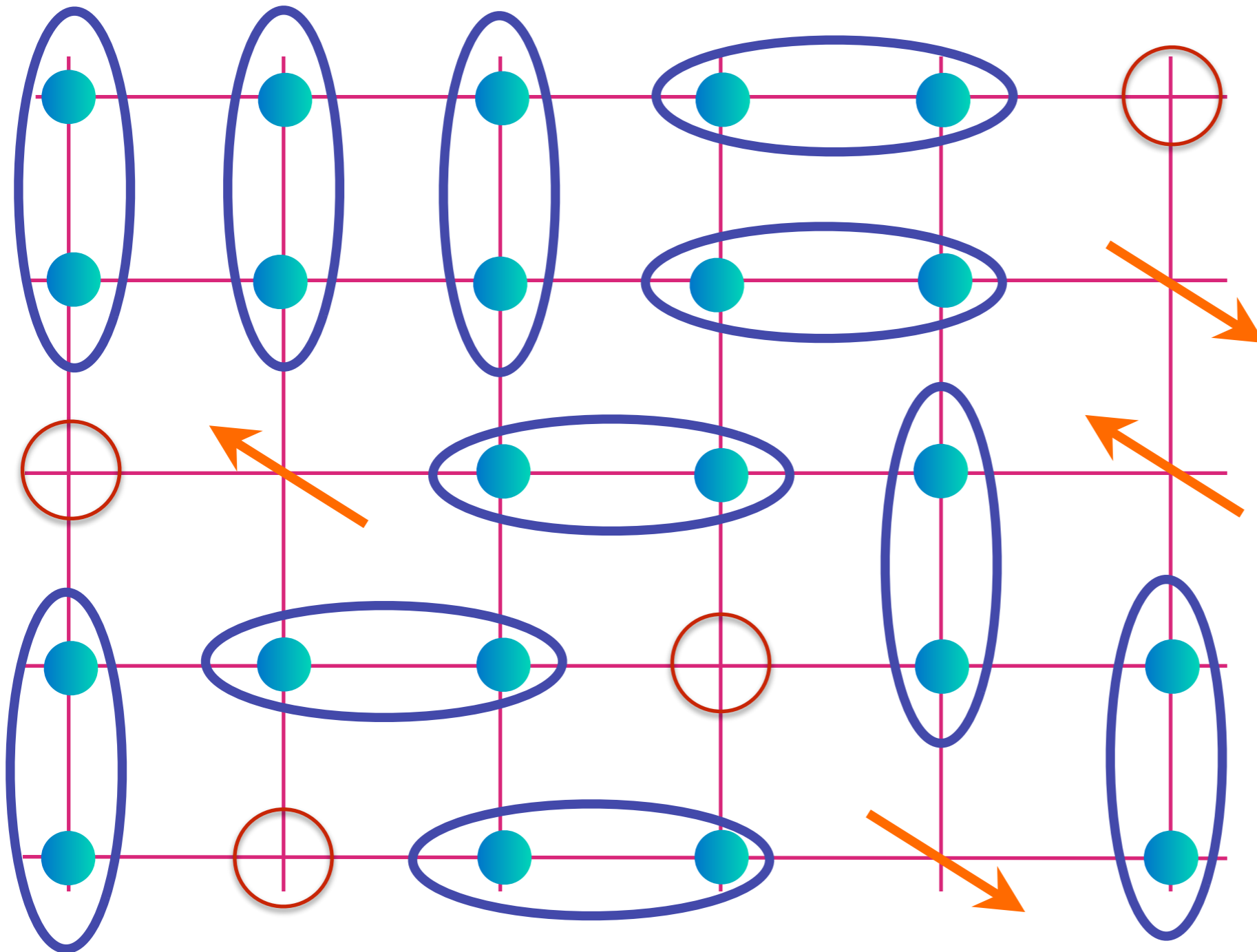


Spin liquid
with
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charge $+e$
quasiparticles)
per square

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

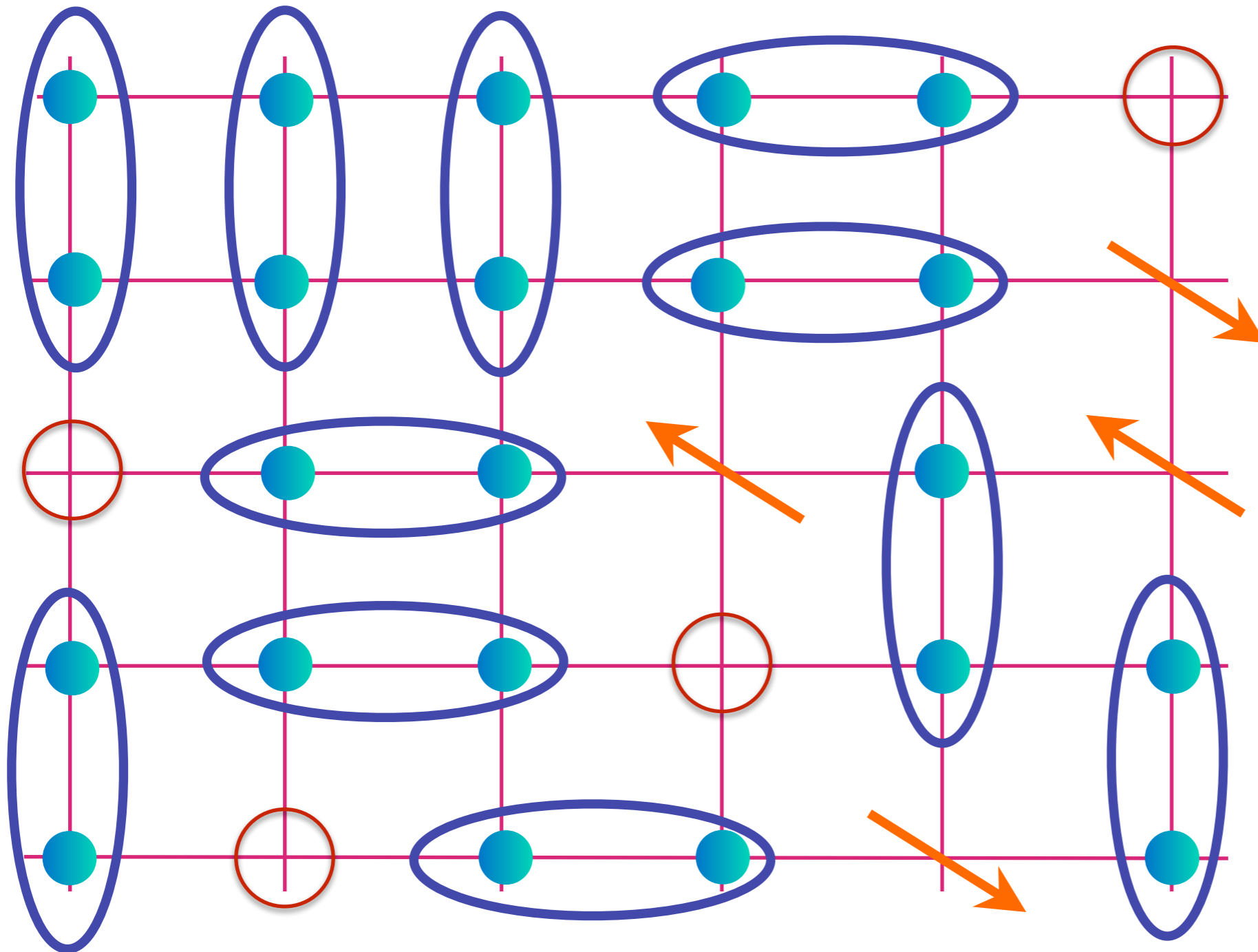
Neutral spin $S=1/2$ “spinon” excitations



Spin liquid
with
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charge $+e$
quasiparticles)
per square

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Neutral spin $S=1/2$ “spinon” excitations

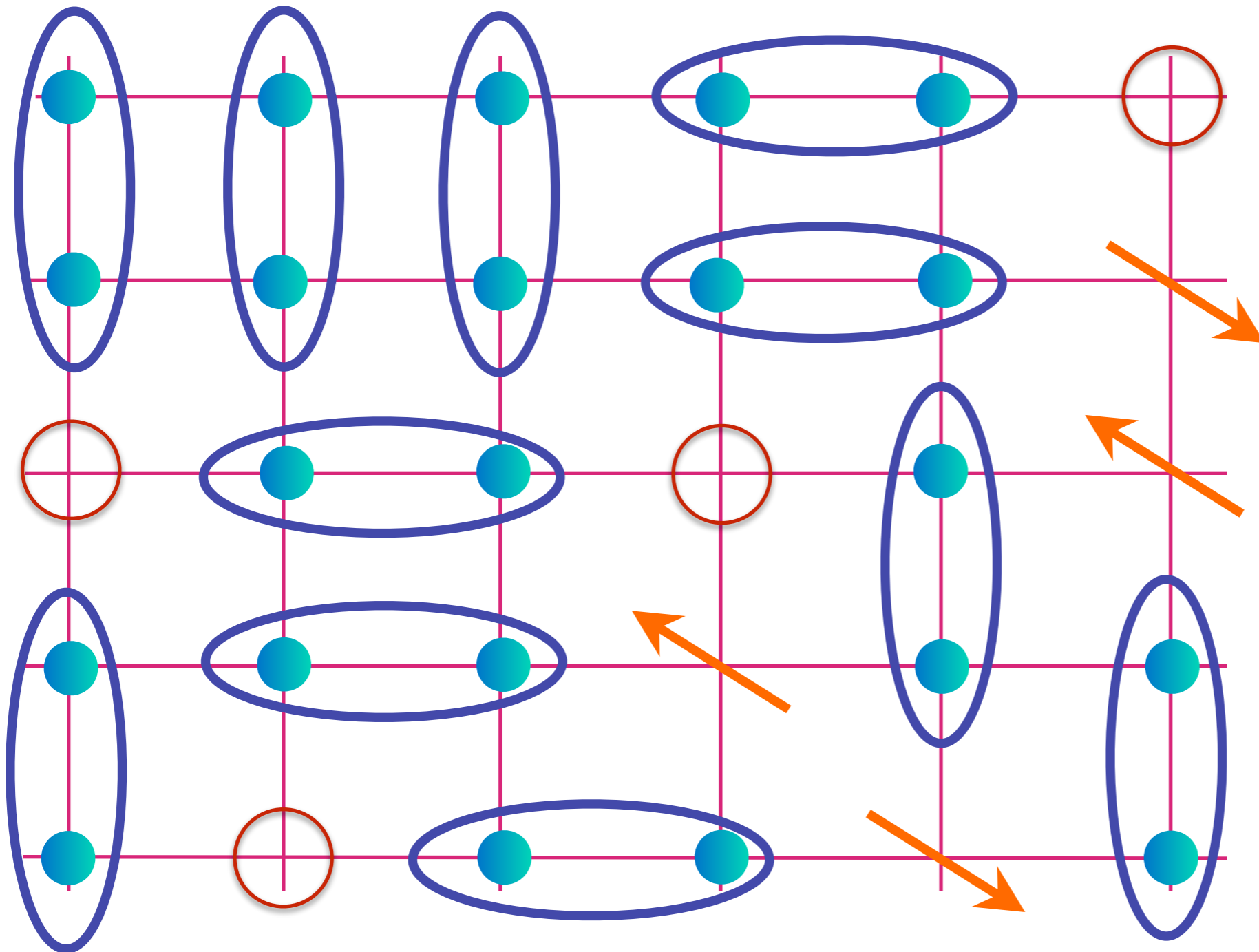


Spin liquid
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charge $+e$
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$$\text{[Blue oval with two cyan circles]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

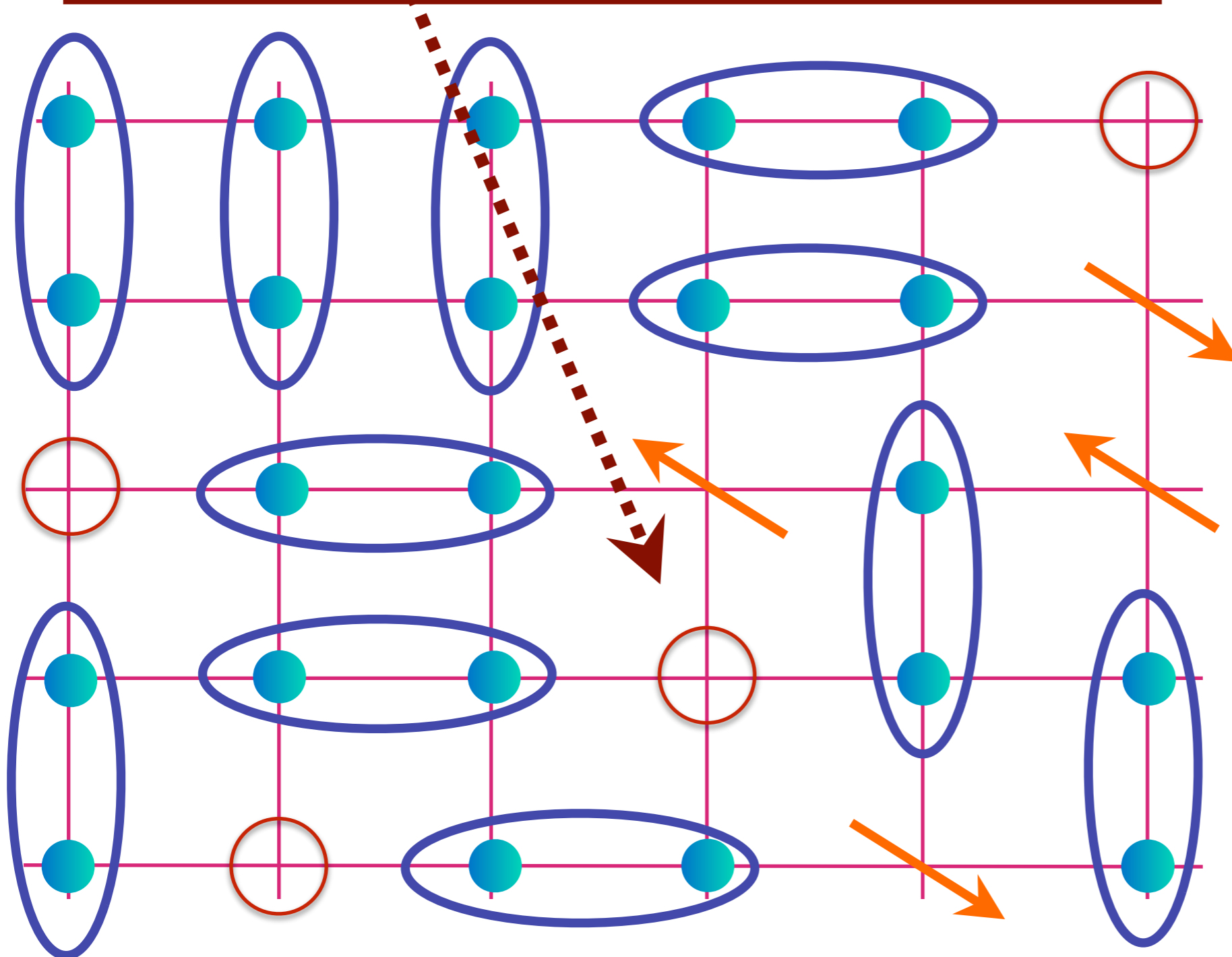
Neutral spin $S=1/2$ “spinon” excitations



Spin liquid
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$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

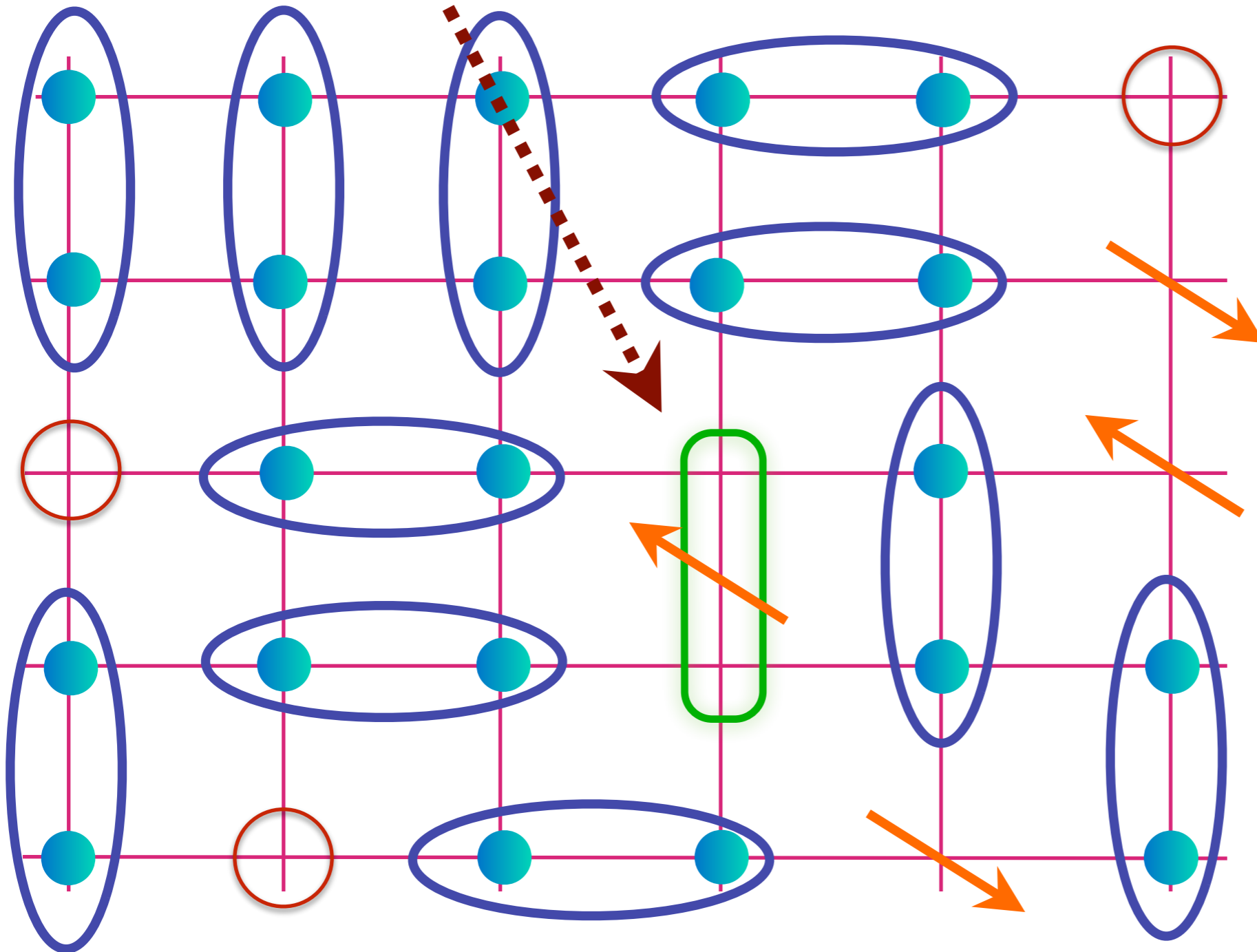
Nearest-neighbor hopping leads to attraction between holon and spinon, which can pay for the energy needed to create the spinon



Spin liquid with p "holons" (spinless, charge $+e$ quasiparticles) per square

$$\text{[Holon]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

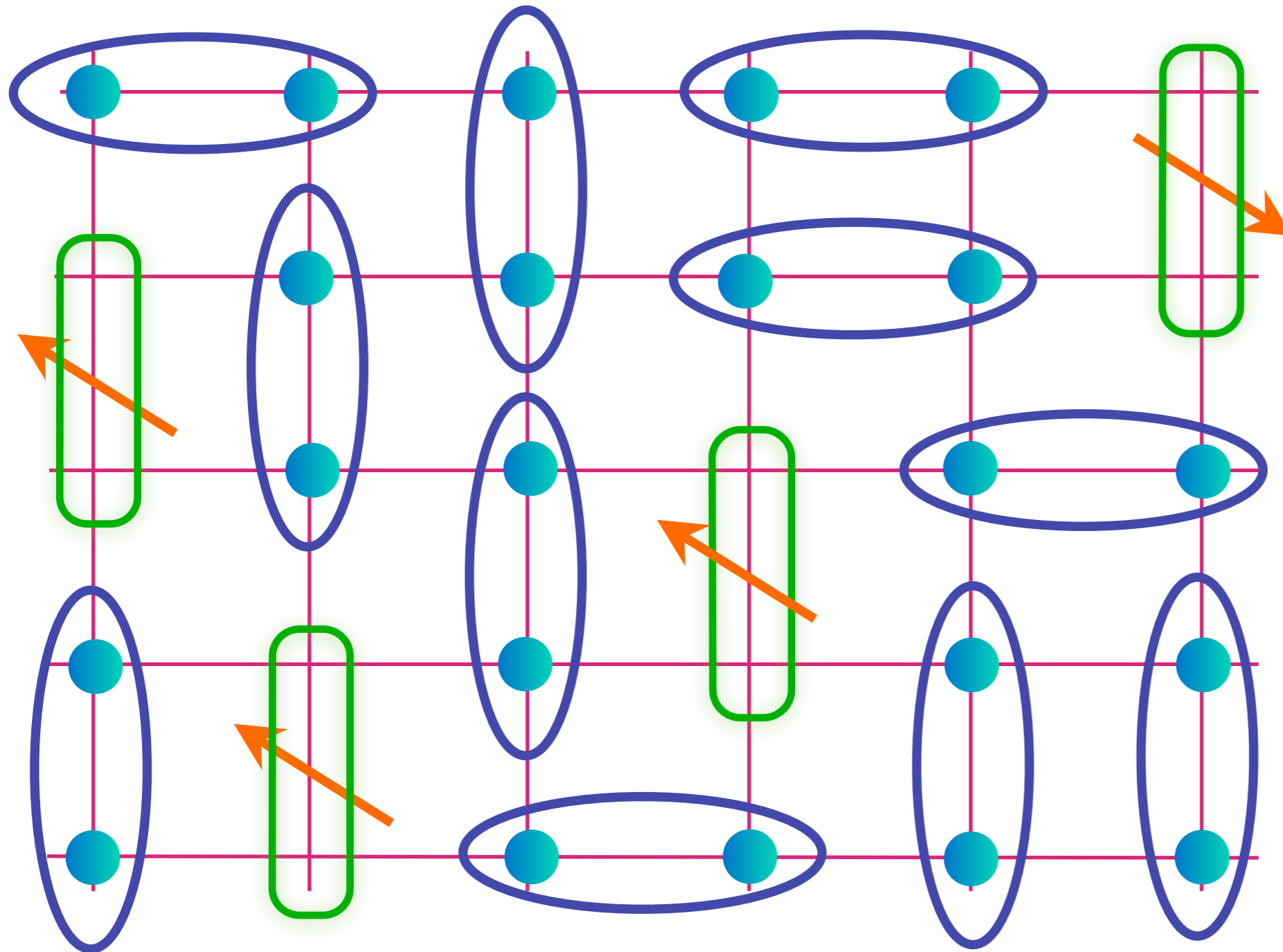
Electron (or hole) resides on a “bonding” orbital between two sites



Spin liquid with p “holons” (spinless, charge $+e$ quasiparticles) per square

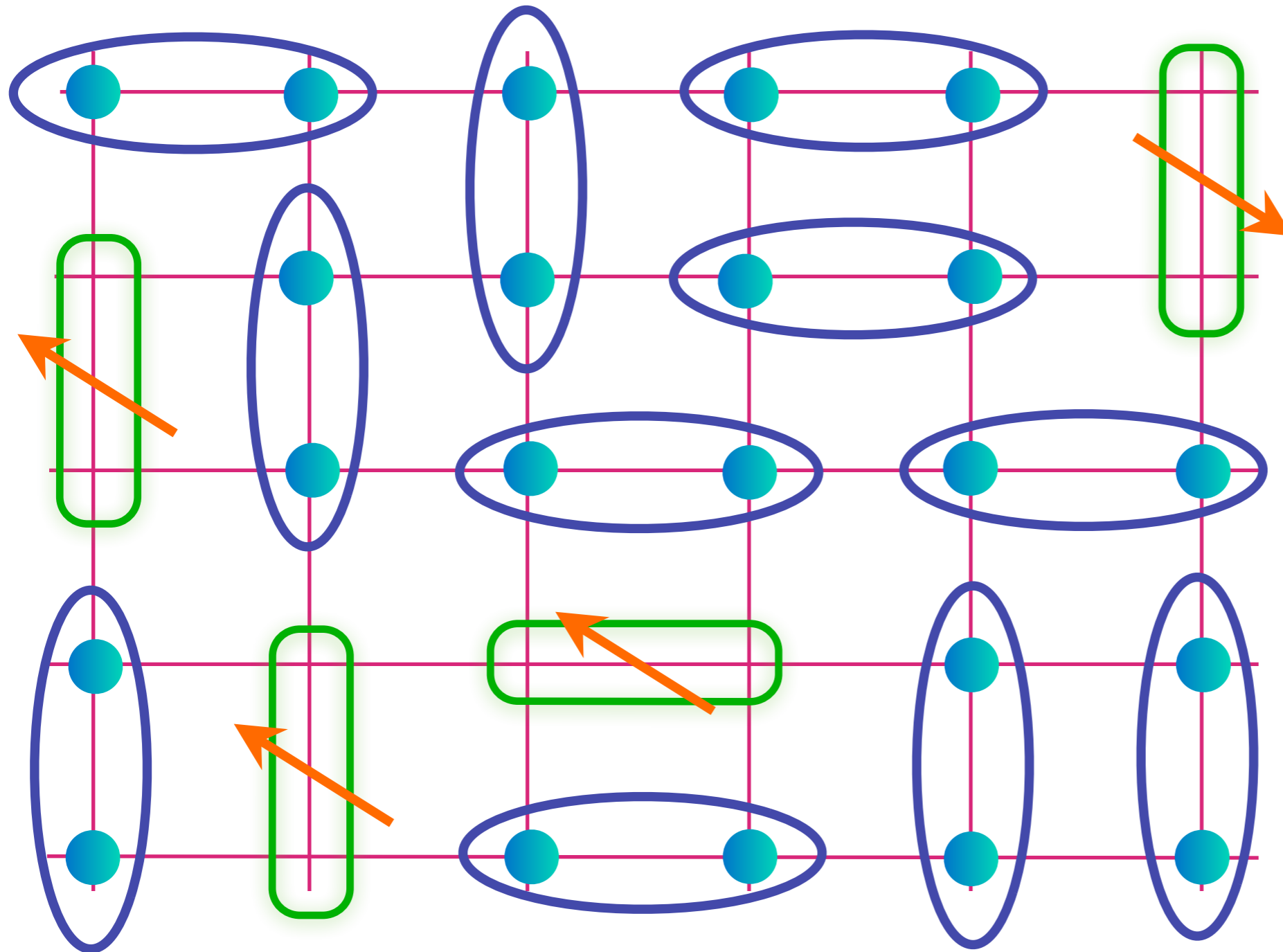
$$\text{Bonding orbital} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)



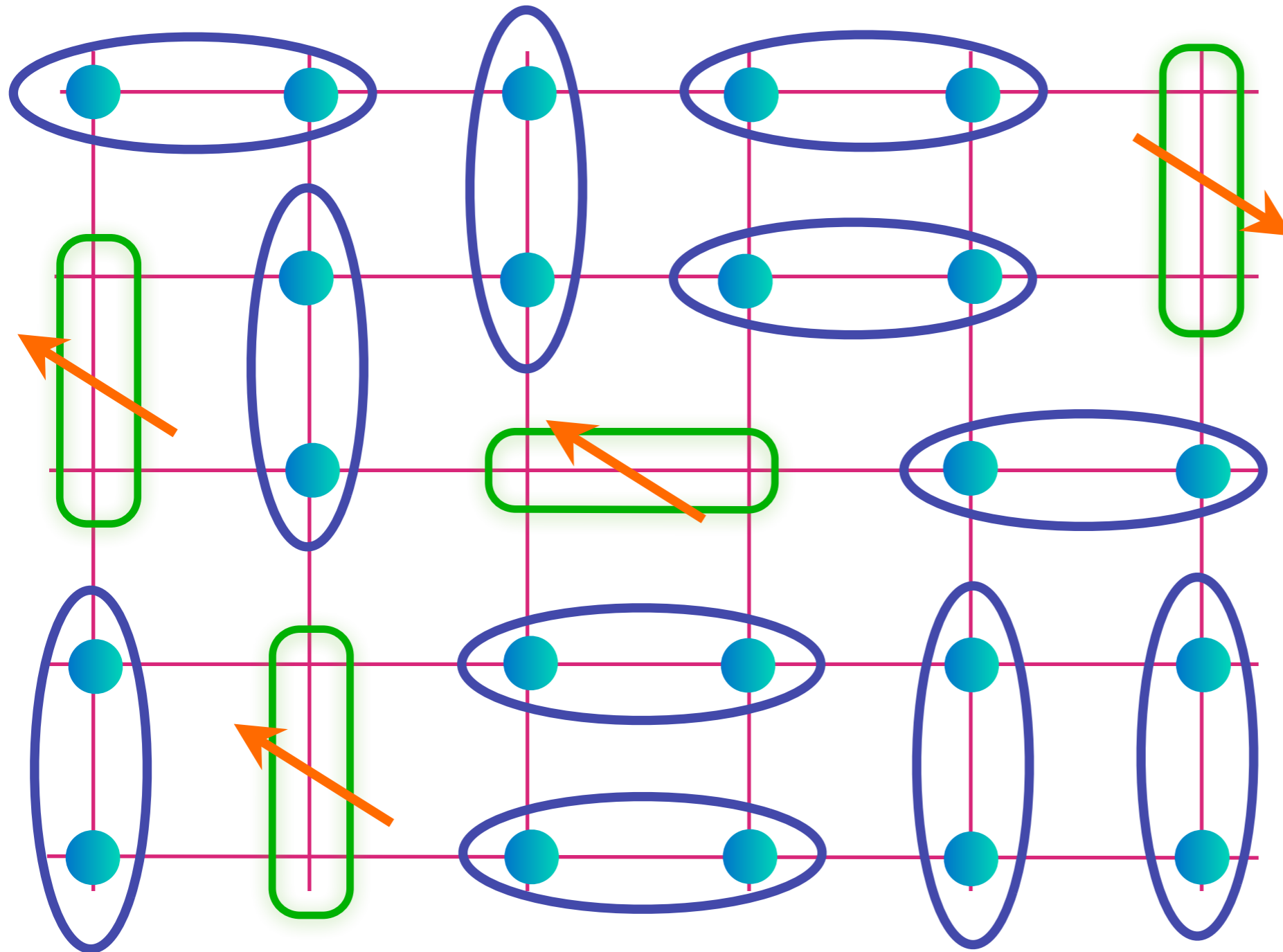
Spin
singlets
and
charge $+e$
 $S=1/2$
holes
of density p

Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)



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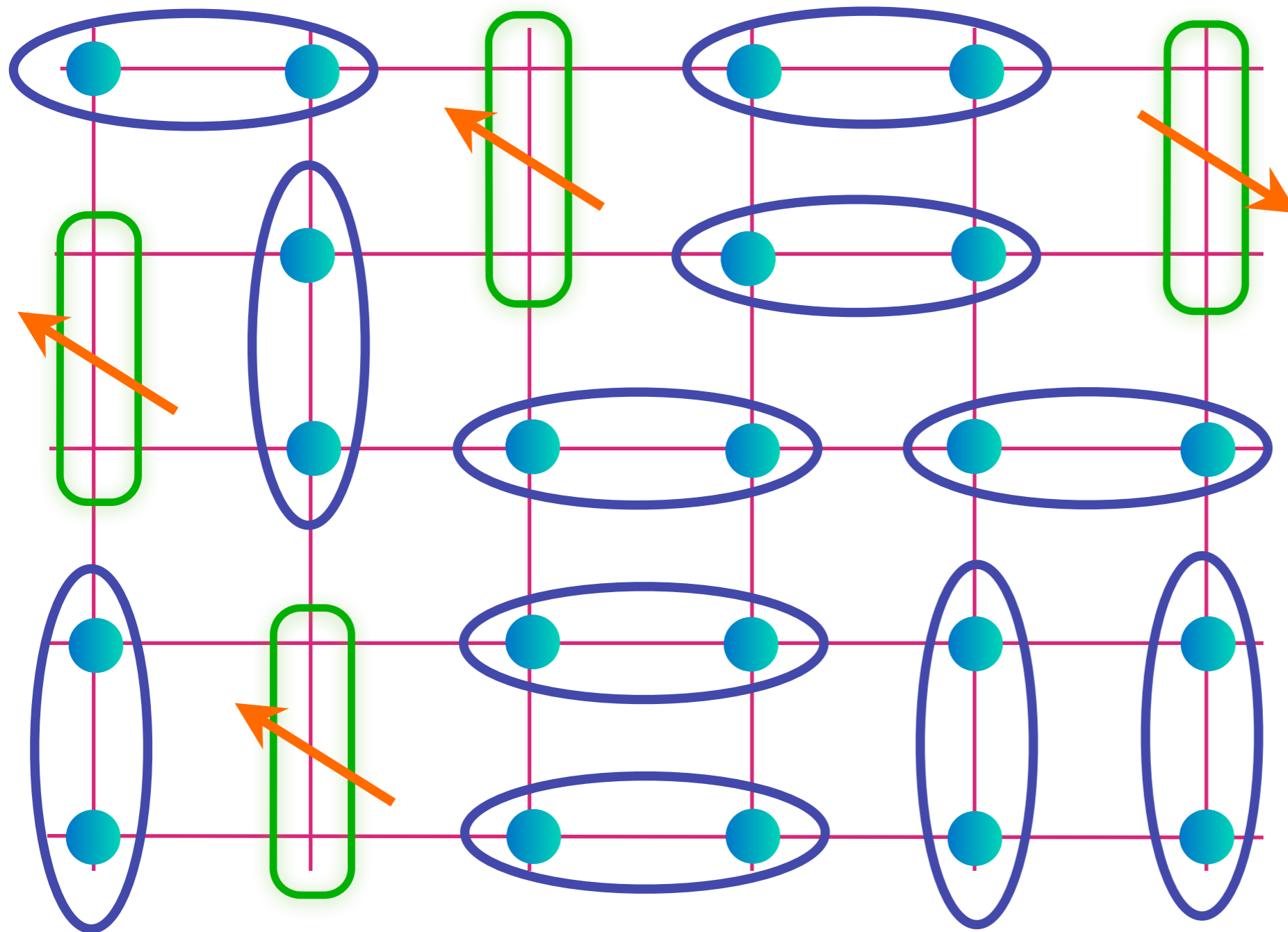
Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)



FL*!

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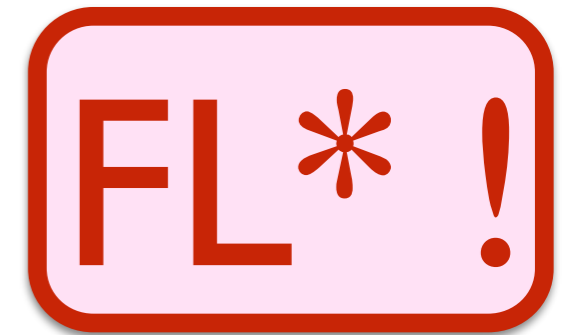
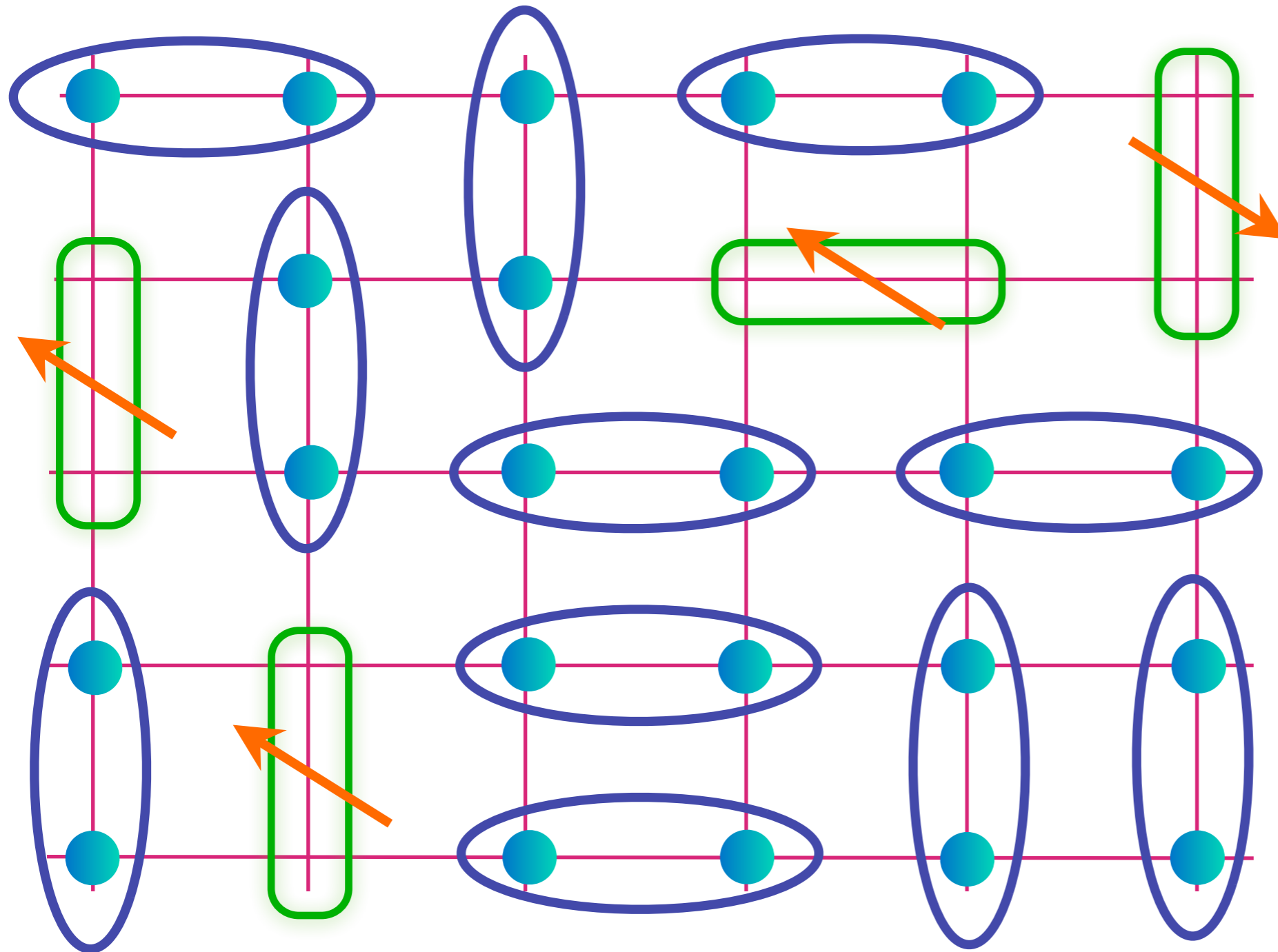
Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)



FL* !

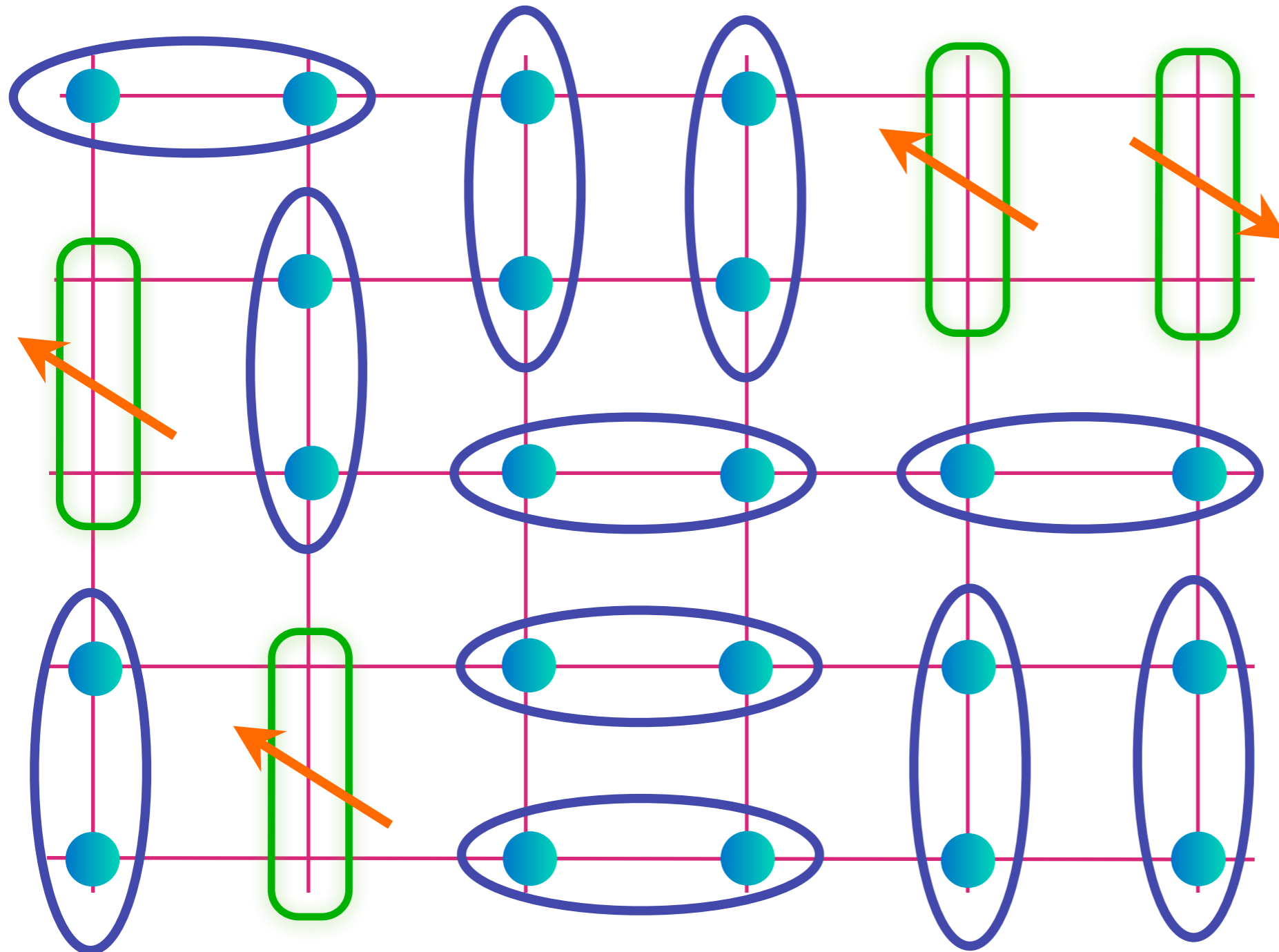
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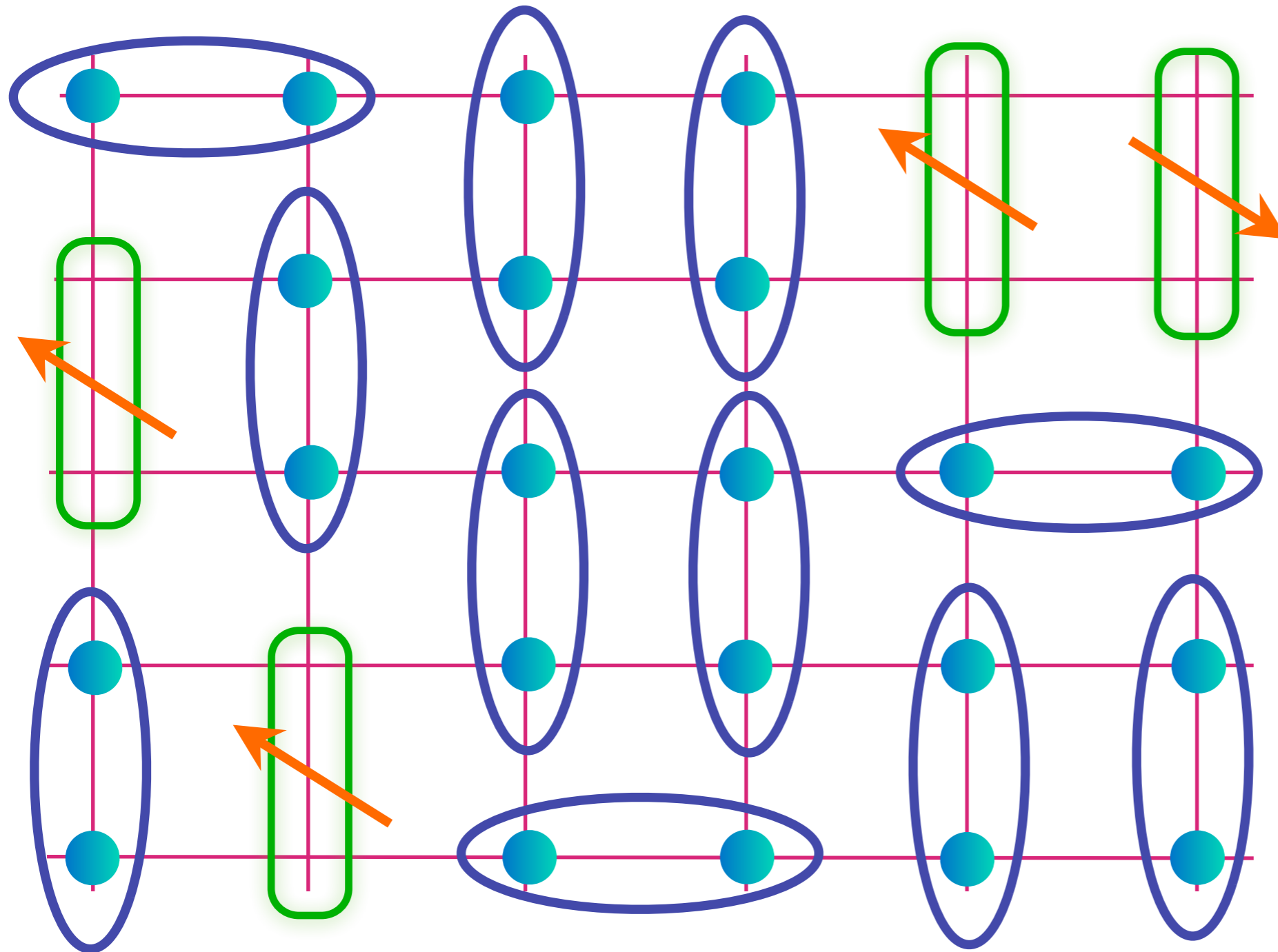
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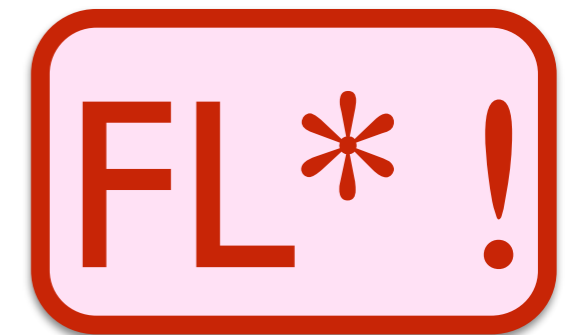
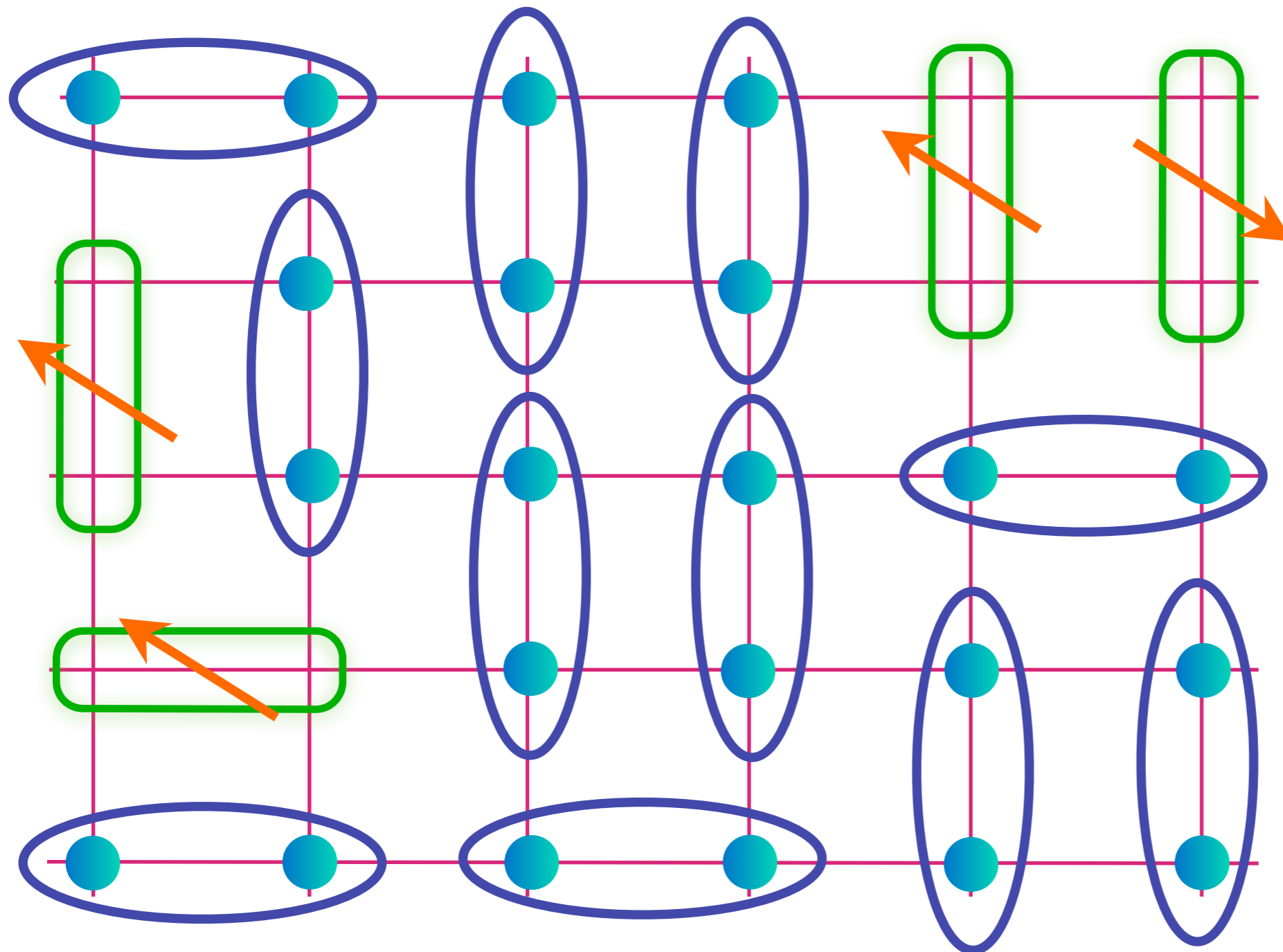
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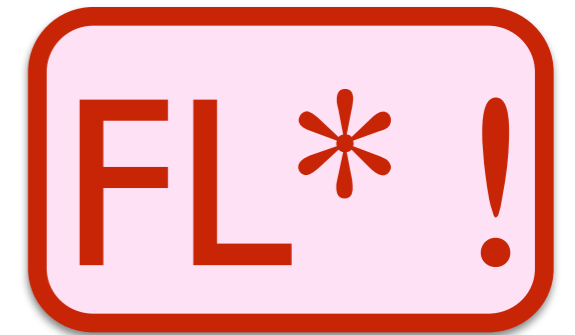
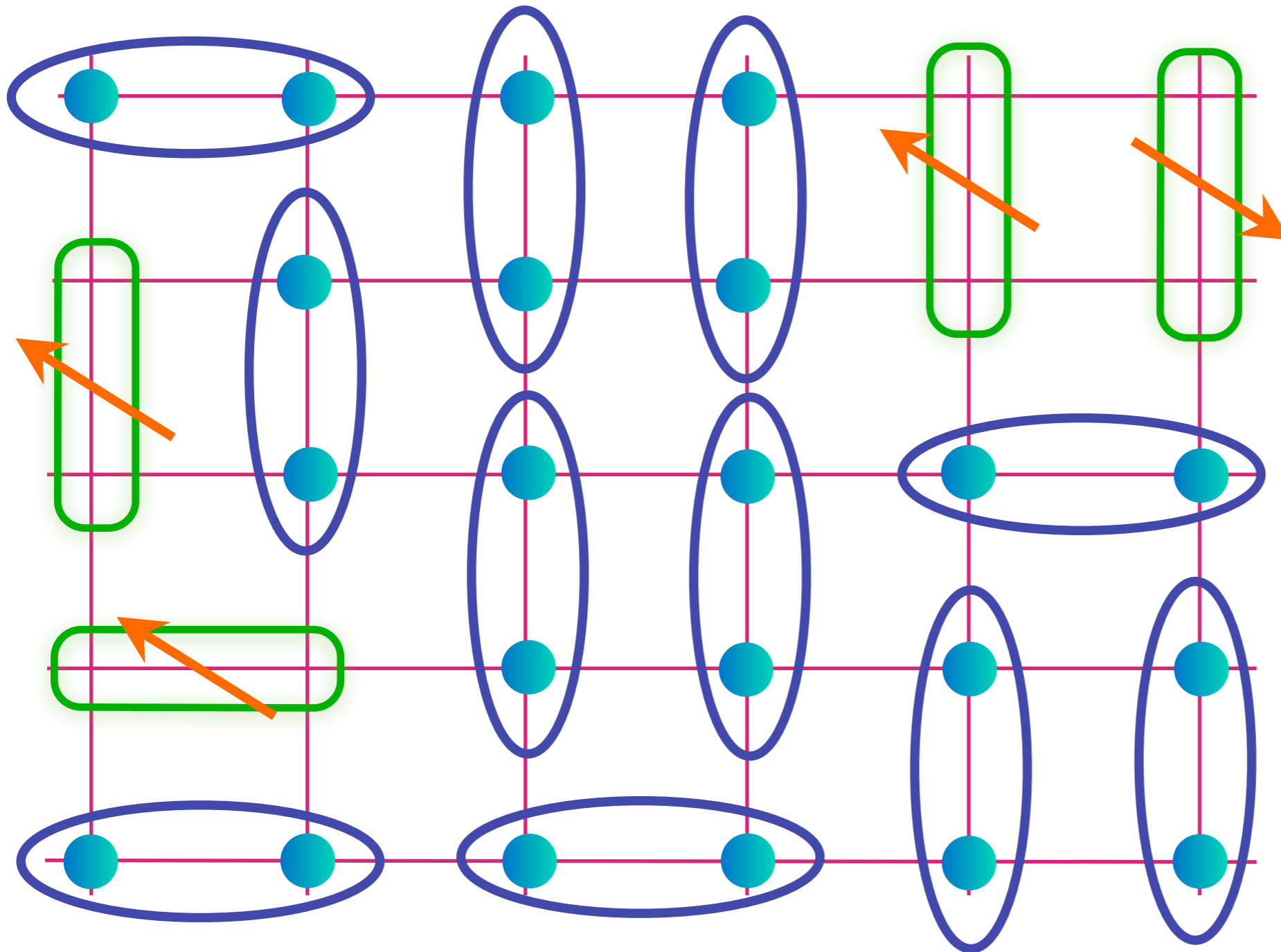
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Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)



Described by
a quantum
dimer model
with bosonic
and
fermionic
dimers

Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)

- Fermi surface of electron/hole-like quasiparticles (the green dimers) enclosing area p . Contrast this with the area $1 + p$ in a Fermi liquid.

Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)

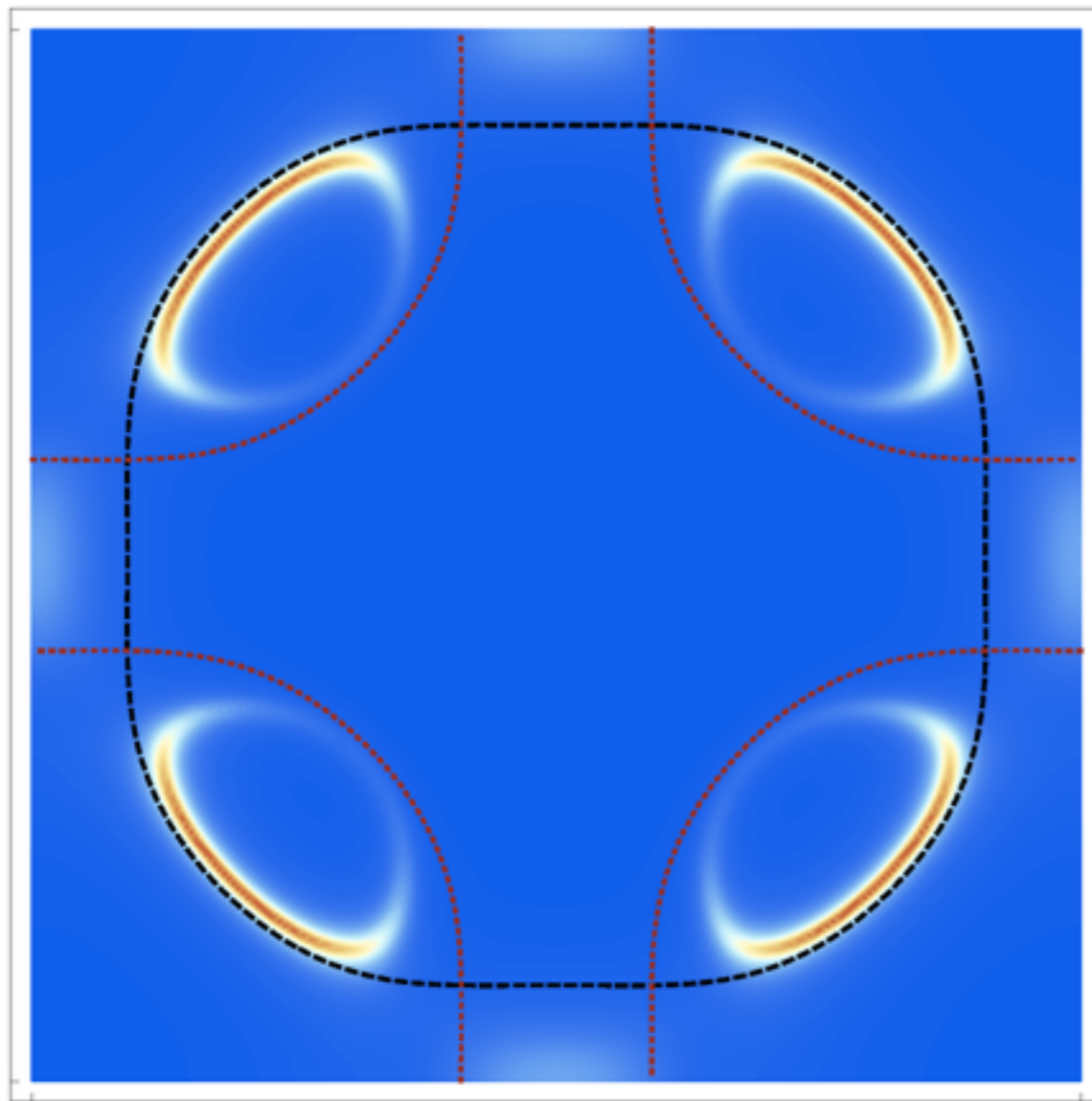
- Fermi surface of electron/hole-like quasiparticles (the green dimers) enclosing area p . Contrast this with the area $1 + p$ in a Fermi liquid.
- The green dimers can only move by resonating with a background of blue dimers. An *emergent gauge field* describes the dynamics of the blue dimers.

Theory of the Pseudogap: Fractionalized Fermi liquid (FL*)

- Fermi surface of electron/hole-like quasiparticles (the green dimers) enclosing area p . Contrast this with the area $1 + p$ in a Fermi liquid.
- The green dimers can only move by resonating with a background of blue dimers. An *emergent gauge field* describes the dynamics of the blue dimers.
- The emergent gauge excitations are needed to account for the breakdown of the Luttinger theorem.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

M. Punk and S. Sachdev, *Phys. Rev. B* **85**, 195123 (2012), and to appear.



Electron spectral
function of FL*

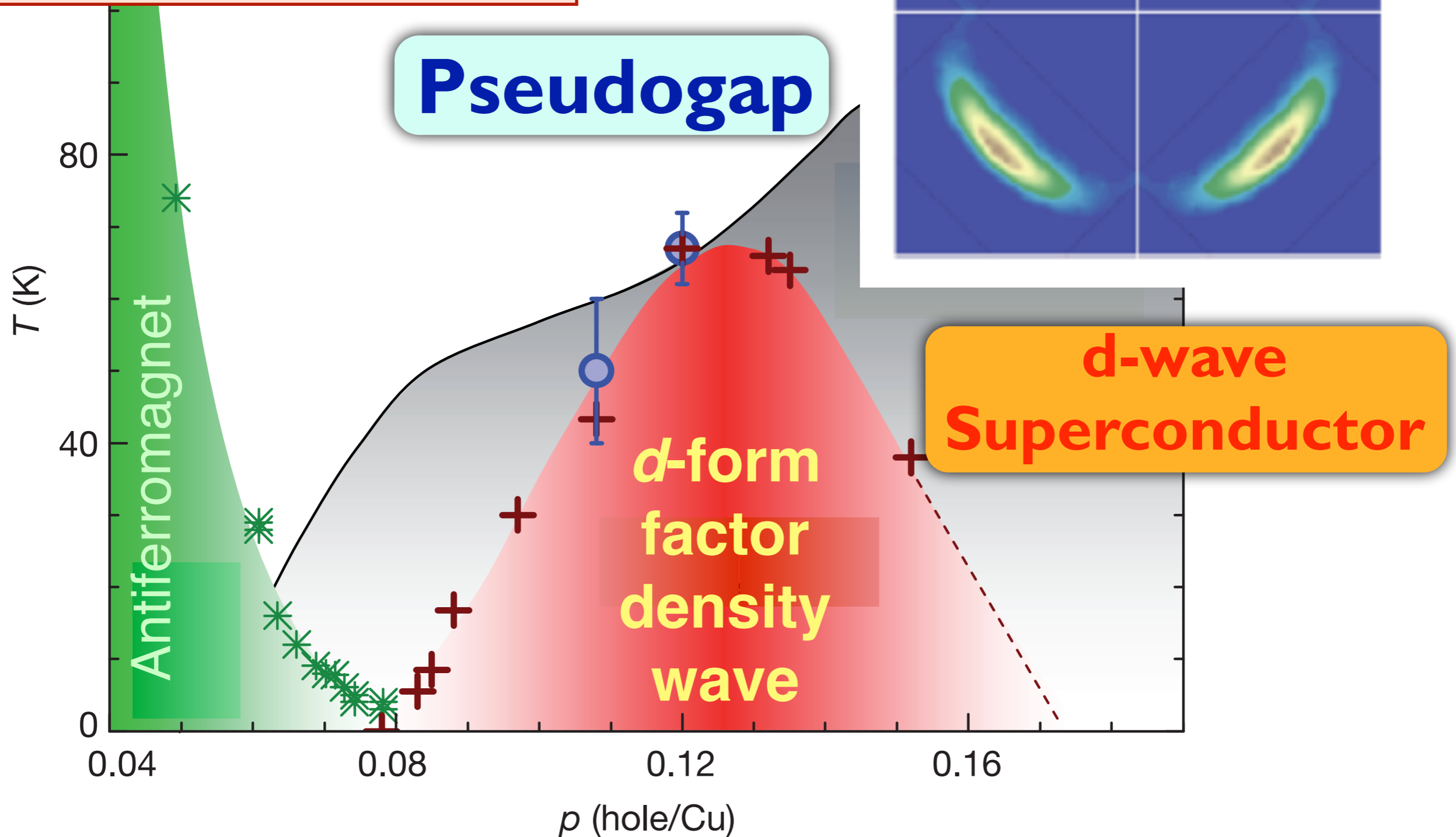
Semi-phenomenological theory of a FL* state with hole pockets of volume p , along with a background spin liquid with an emergent U(1) gauge field. Note that the quasiparticle excitations around the Fermi surface do not carry U(1) gauge charges.

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

M. Punk and S. Sachdev, Phys. Rev. B **85**, 195123 (2012), and to appear

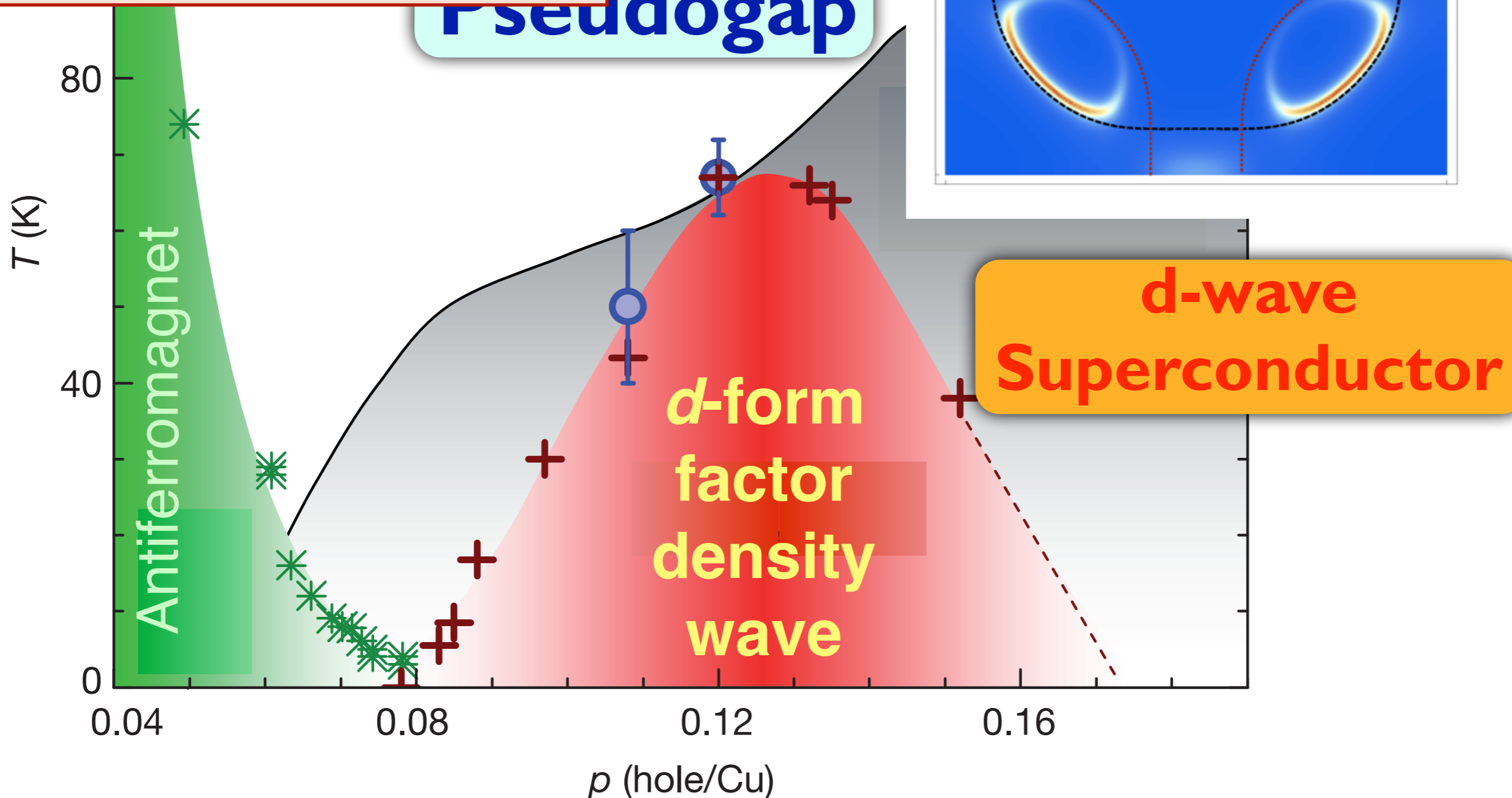
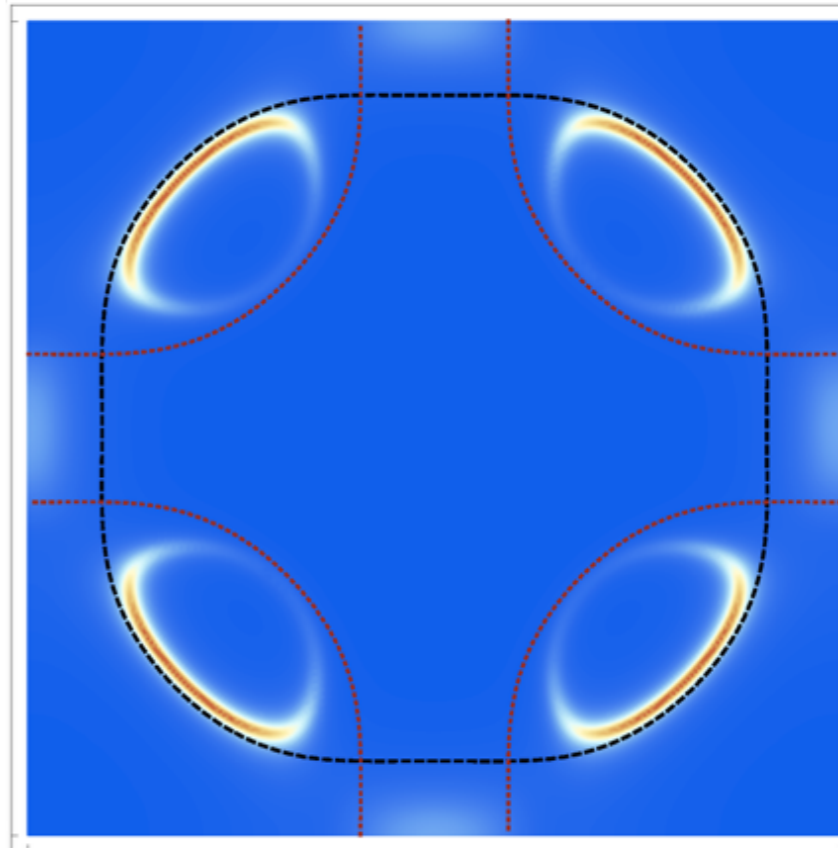
“Fermi arcs” at
low p

Pseudogap



Hole pockets with
“back-side”
suppressed by small
quasiparticle residue:

Pseudogap



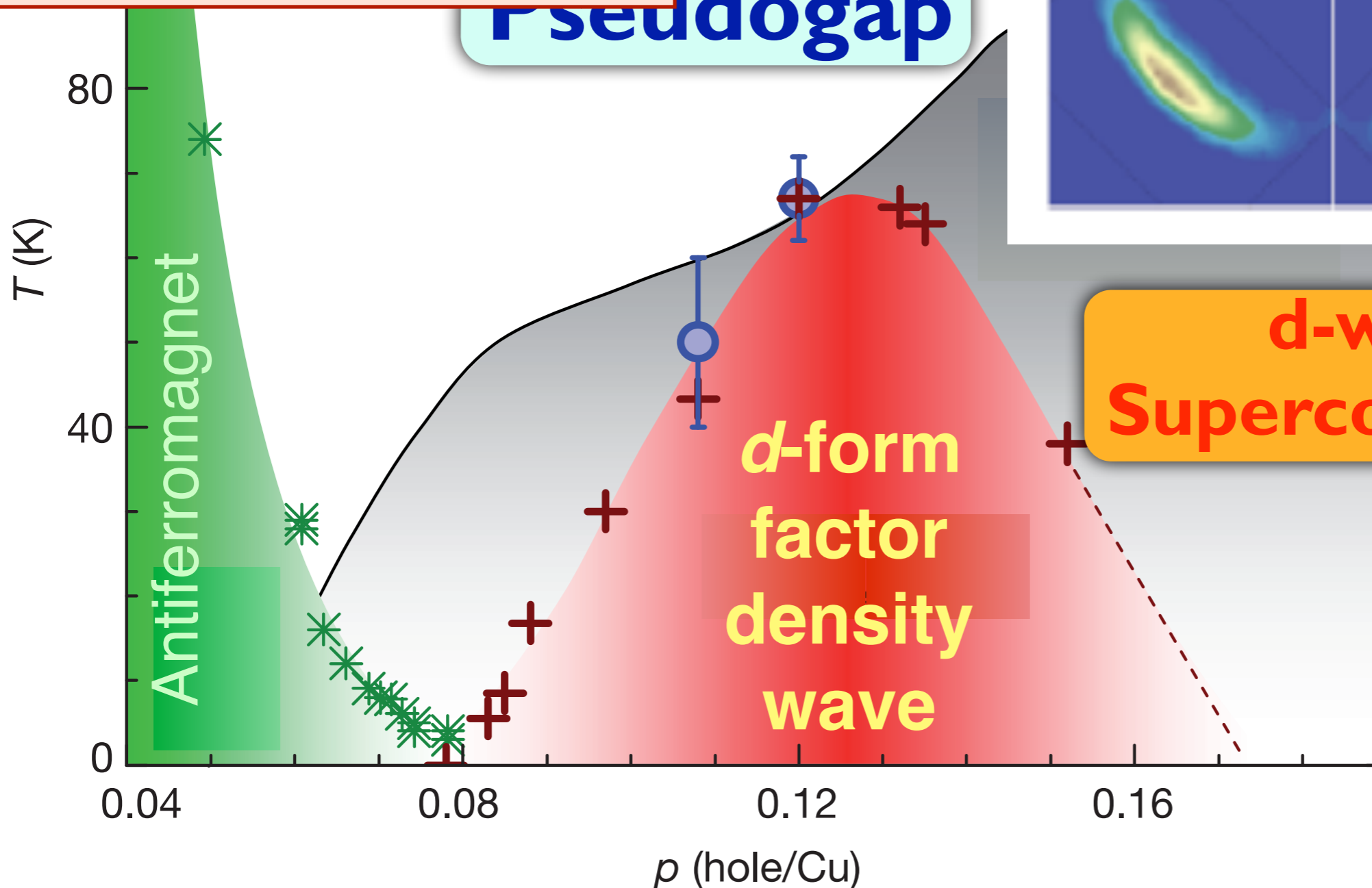
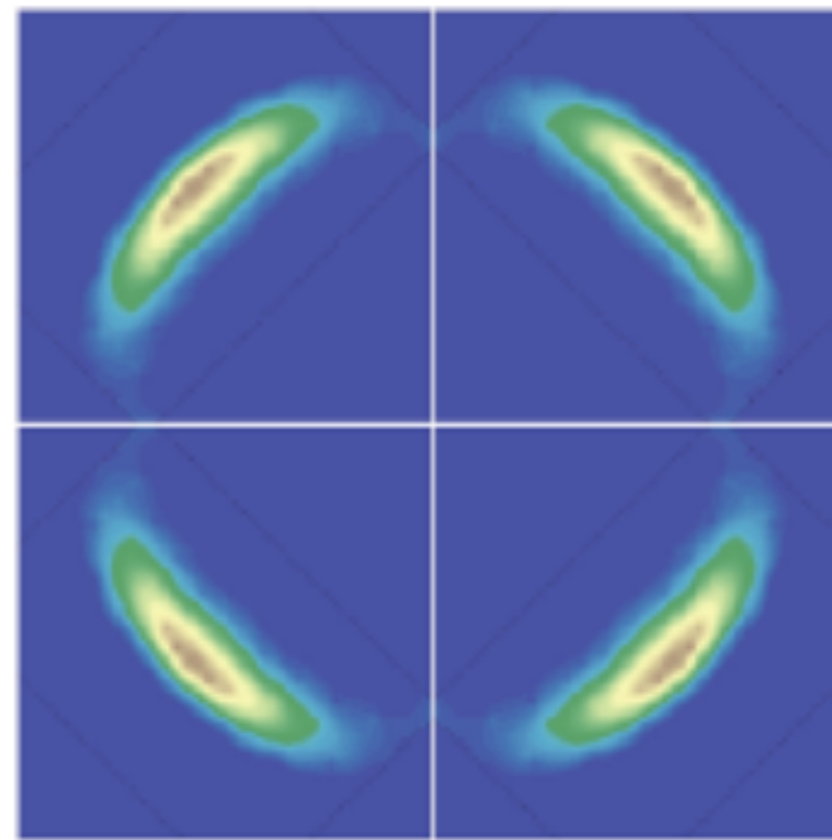
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

M. Punk and S. Sachdev, Phys. Rev. B **85**, 195123 (2012), and to appear

D. Chowdhury and S. Sachdev, arXiv:1409.5430

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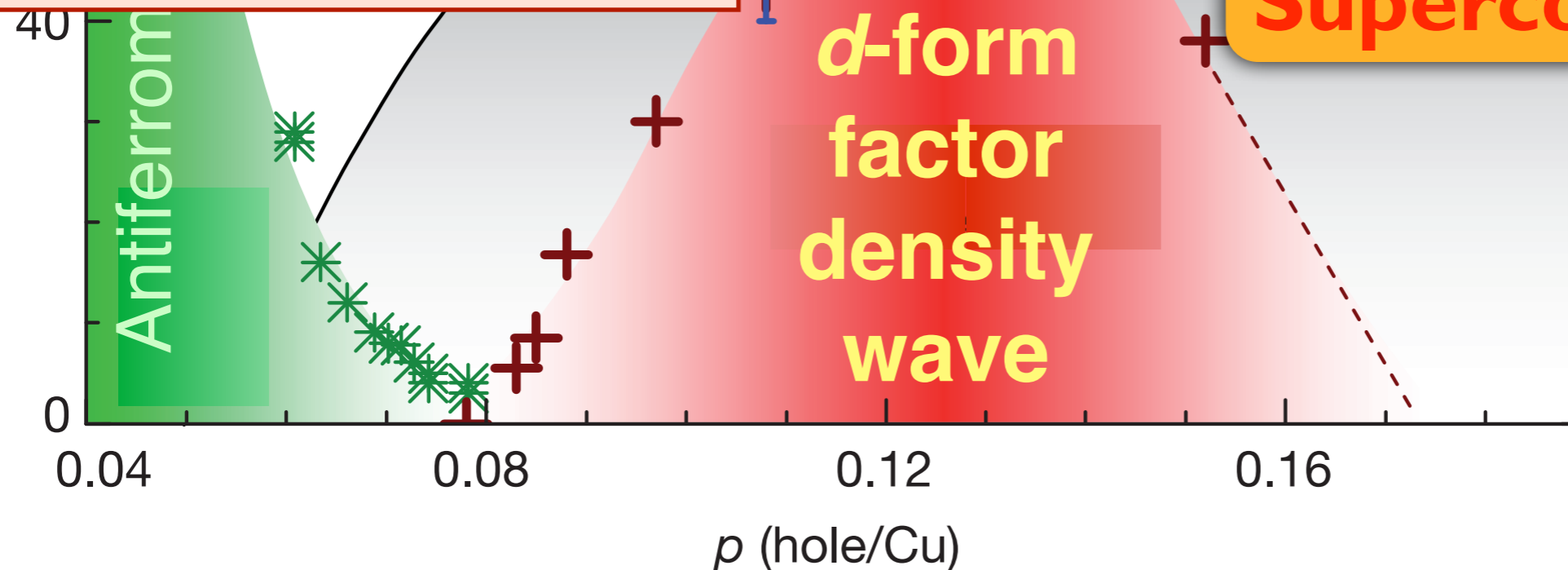
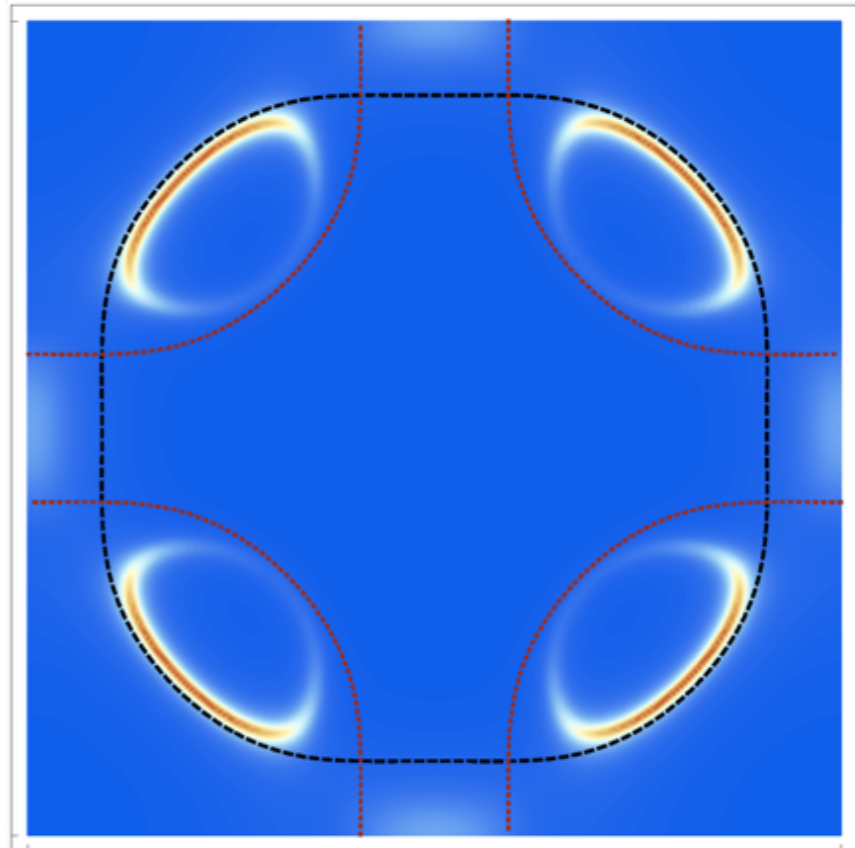
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Hole pockets with
“back-side”
suppressed by small
quasiparticle residue:

Detect by probes
which don't add or
remove
electrons.....

nodogap



d-wave
Superconductor

d-form
factor
density
wave

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk and S. Sachdev, Phys. Rev. B **85**, 195123 (2012), and to appear
D. Chowdhury and S. Sachdev, arXiv:1409.5430

Outline

1. The low T pseudogap:

*STM observation of predicted
d-form factor density wave*

2. The high T pseudogap:

*A metal with topological order:
the Fractionalized Fermi liquid: FL**

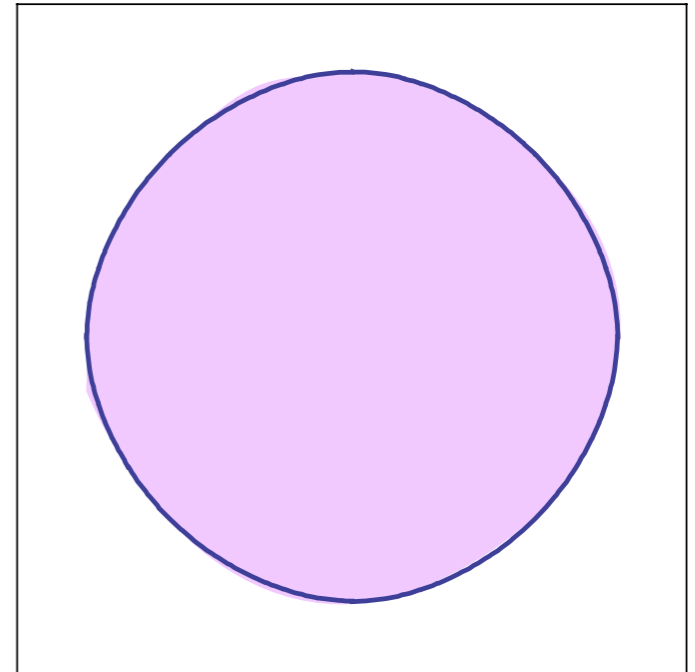
3. Connecting high and low T :
Density wave instabilities

4. Quantum critical point near optimal p :

A Higgs critical point

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

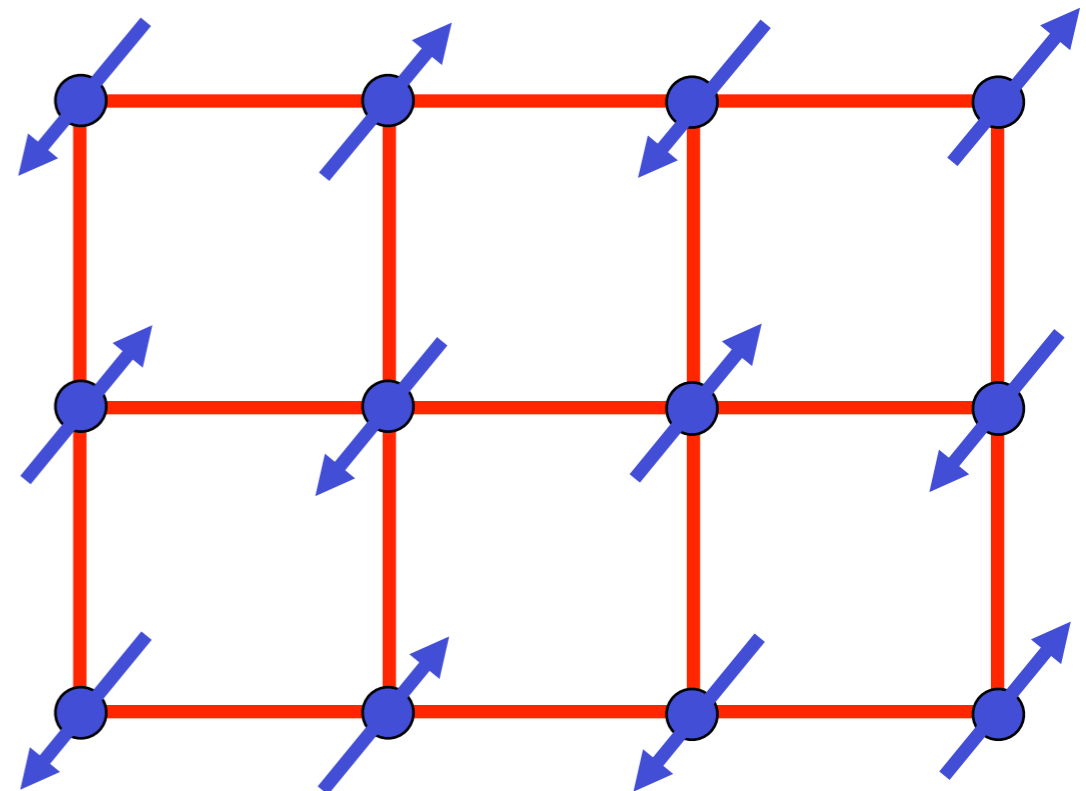


+

The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K} = (\pi, \pi)$ is the ordering
wavevector.



Pairing “glue” for d-wave superconductivity from antiferromagnetic fluctuations



- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
P. Monthoux, A. V. Balatsky, and D. Pines, *Phys. Rev. Lett.* **67**, 3448 (1991)

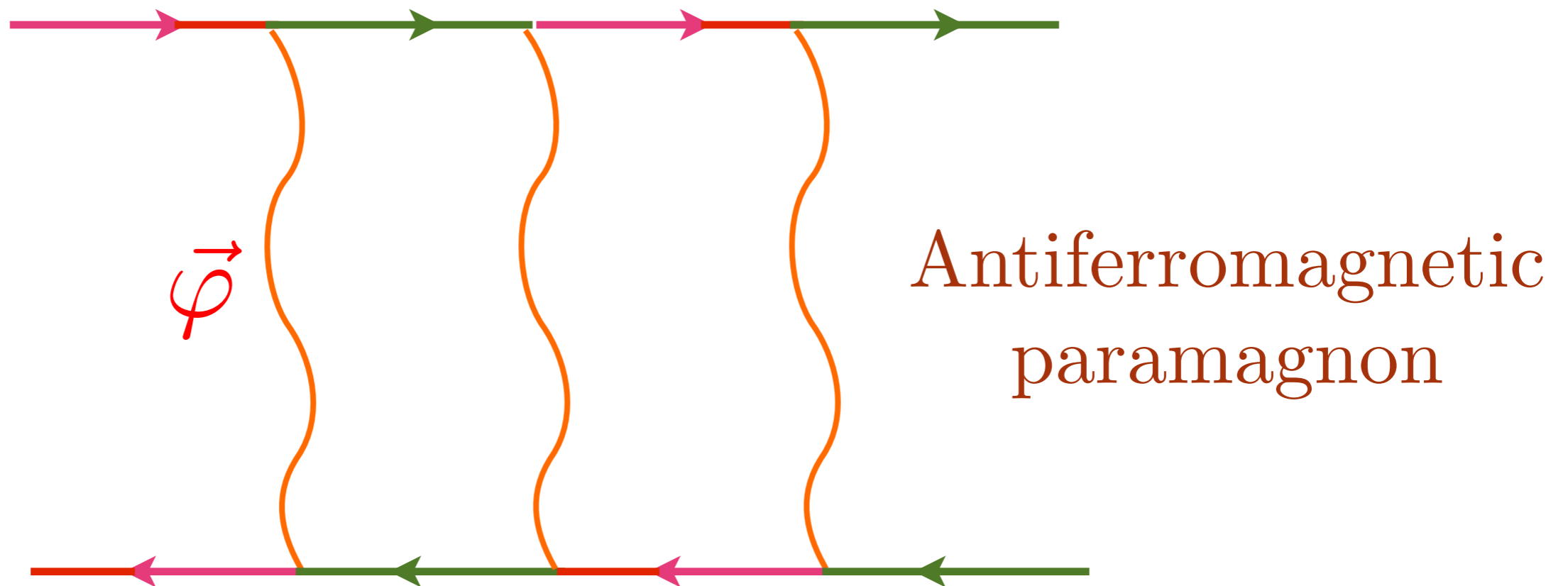
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Leads to $\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$

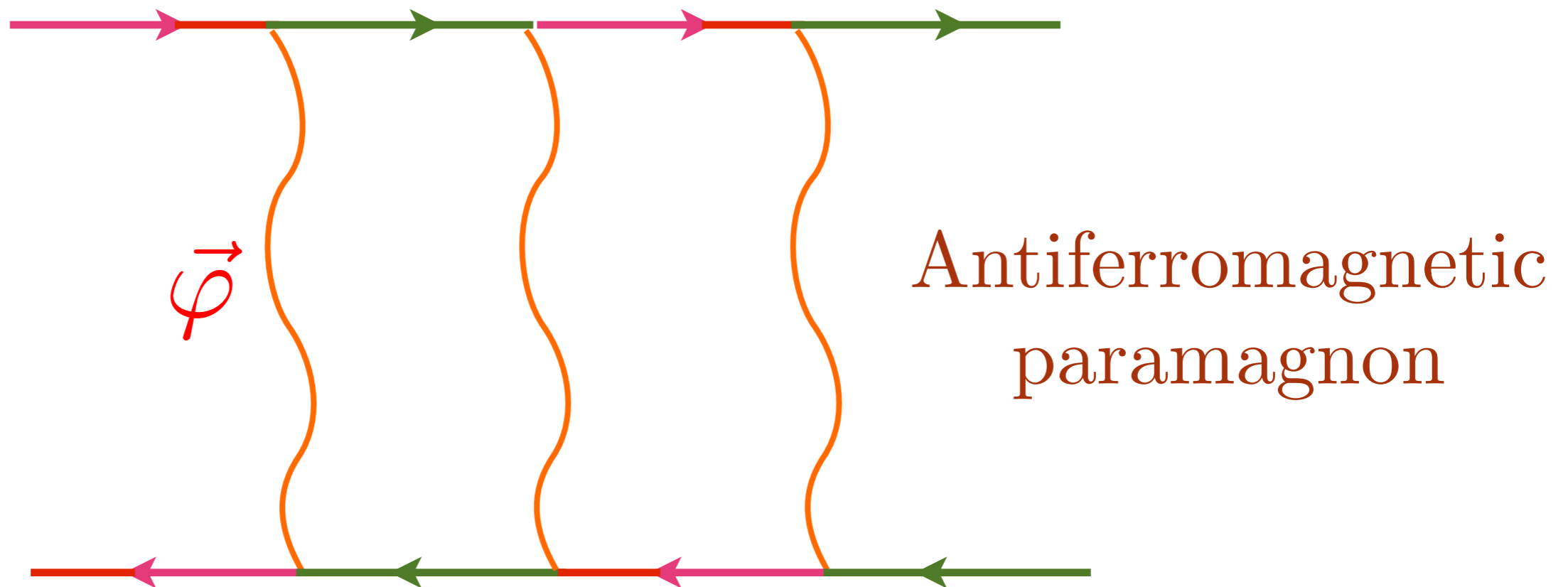
- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
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Same glue can lead to “d-wave” particle-hole pairing !



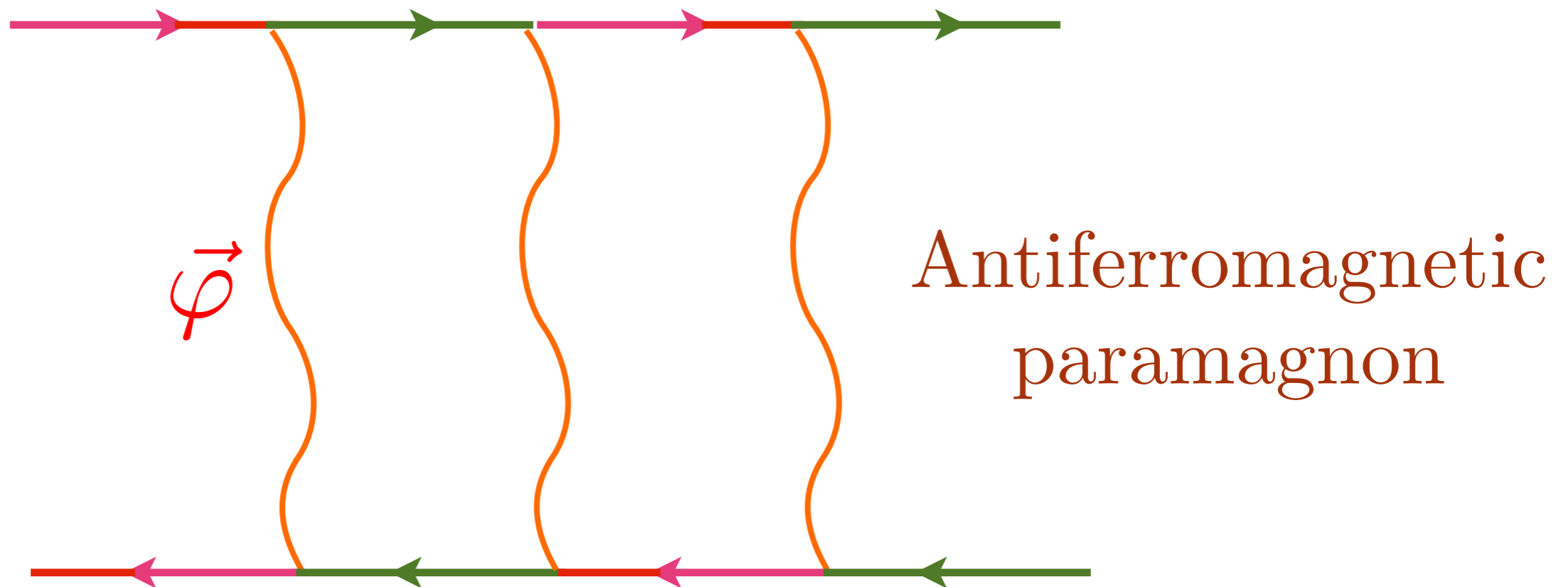
- M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)
K. B. Efetov, H. Meier, and C. Pépin, Nat. Phys. **9**, 442 (2013)
J. D. Sau and S. Sachdev, Phys. Rev. B **89**, 075129 (2014)
Y. Wang and A. V. Chubukov, Phys. Rev. B **90**, 035149 (2014)

Same glue can lead to “d-wave” particle-hole pairing !



Leads to $\left\langle c_{\mathbf{k}+\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{Q}/2,\alpha} \right\rangle =$
 $\mathcal{P}_s + \mathcal{P}_{s'} (\cos k_x + \cos k_y) + \mathcal{P}_d (\cos k_x - \cos k_y)$
 with \mathcal{P}_d dominant.

Same glue can lead to “d-wave” particle-hole pairing !

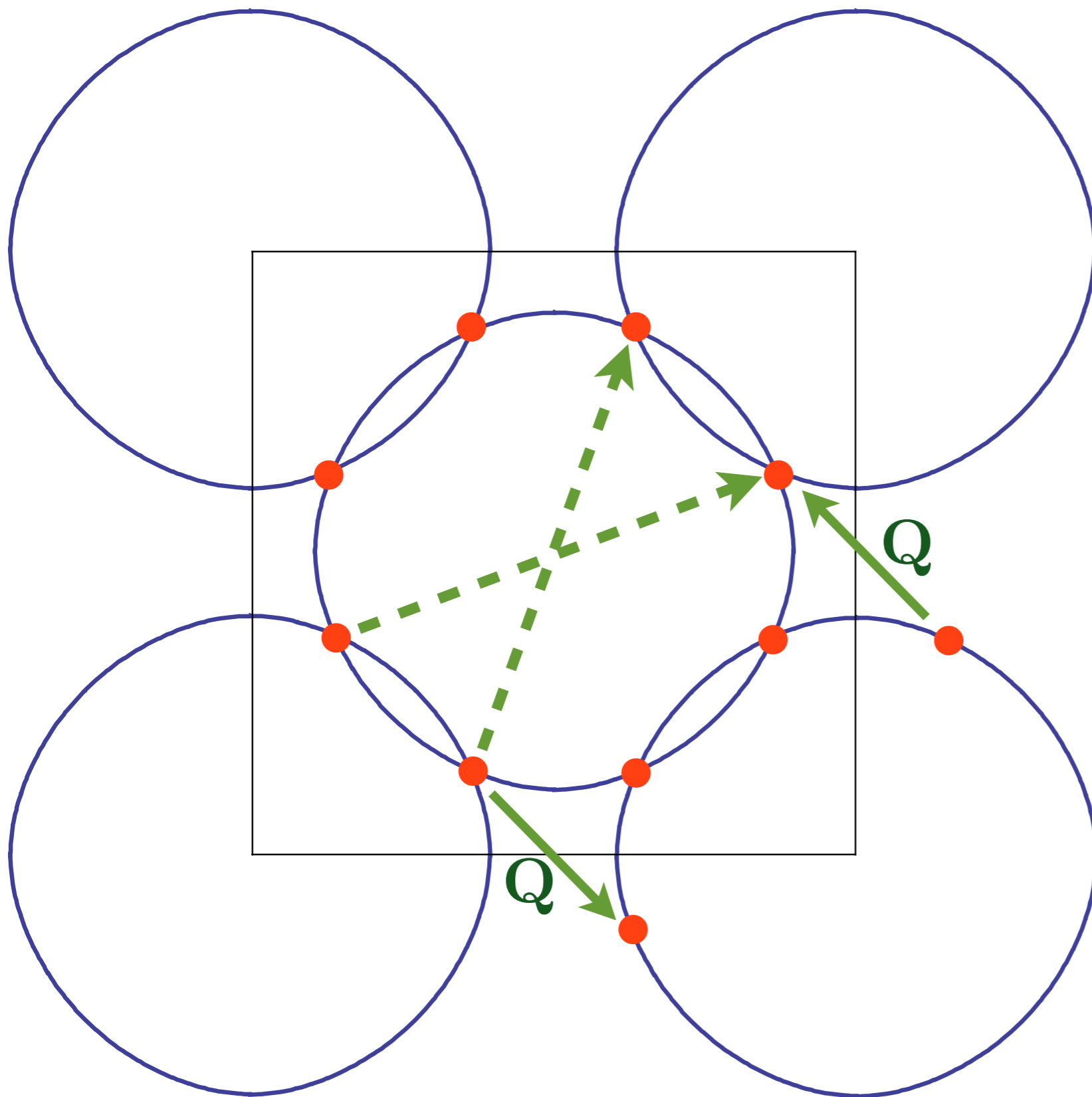


Note: previous work on density waves with non-zero angular momentum used $\langle c_{\mathbf{k}+\mathbf{Q},\alpha}^\dagger c_{\mathbf{k},\alpha} \rangle \sim (\cos k_x - \cos k_y)$; this is a *different* state which has components both even and odd under time-reversal.

Same glue can lead to “d-wave” particle-hole pairing !

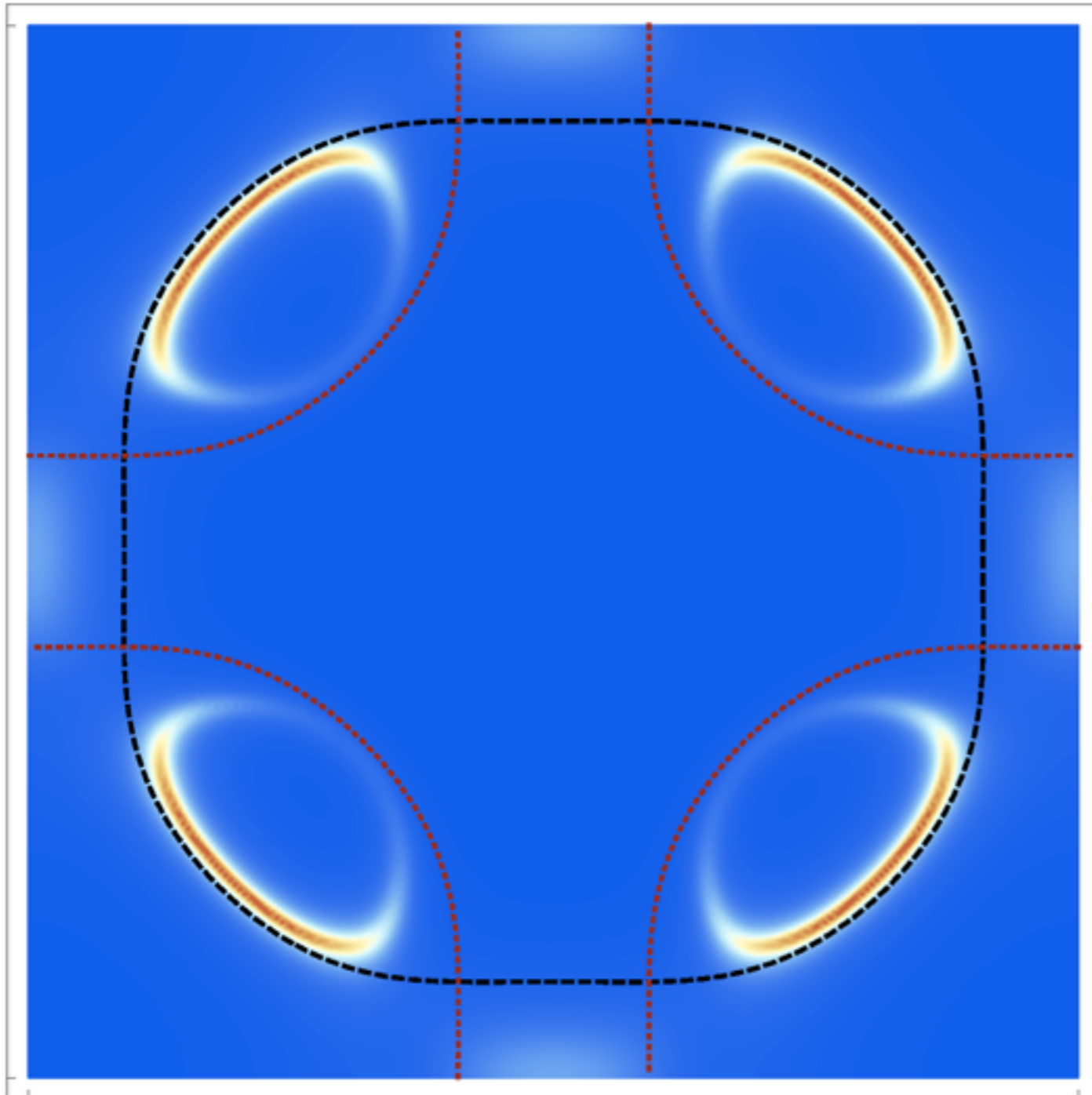


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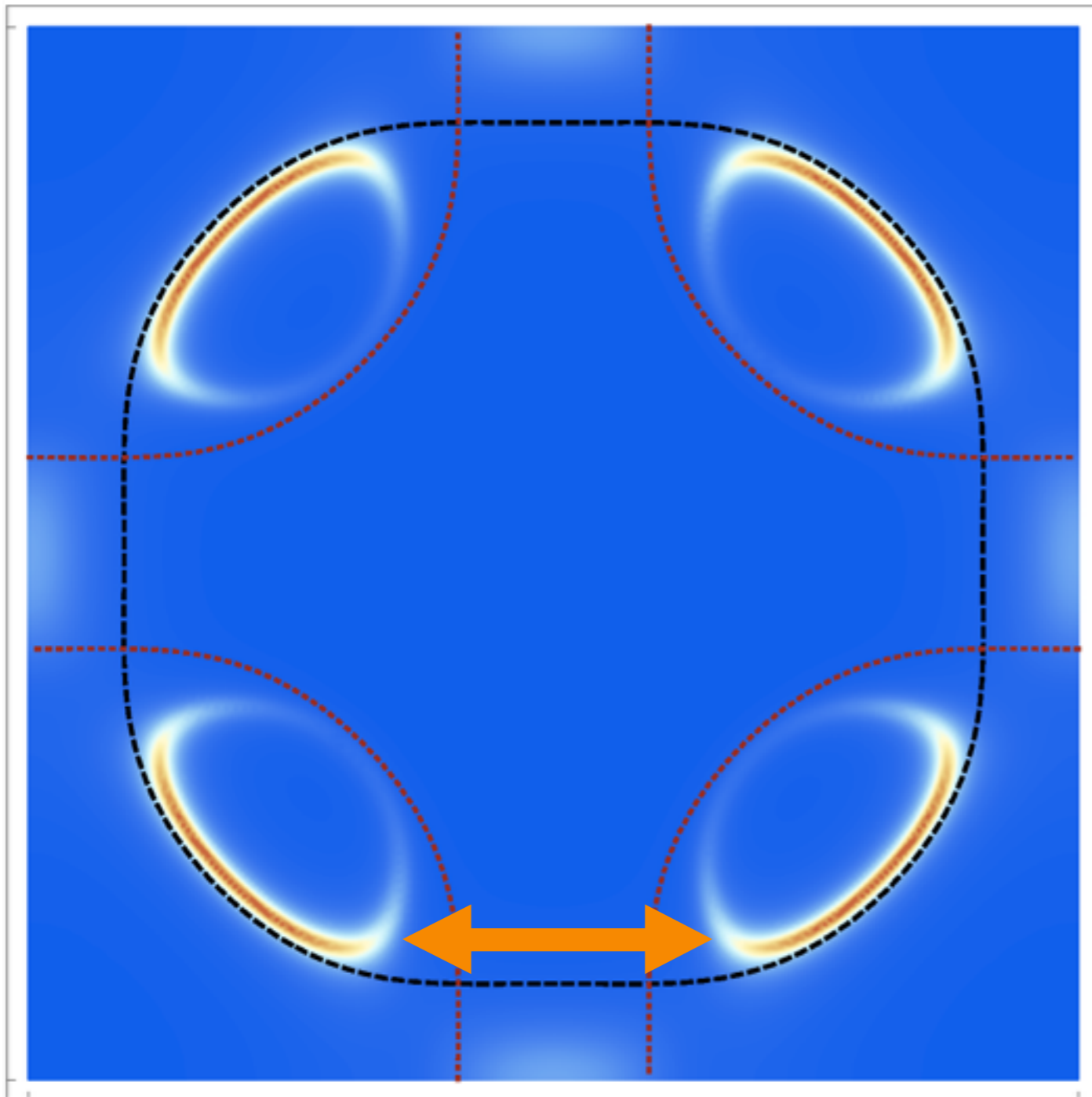


Density wave instability of large Fermi surface leads to an incorrect “diagonal” wavevector

$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \mathcal{P}_d(\cos k_x - \cos k_y)$$



Fermi
surface
of FL*



Density wave
instability of
 FL^* leads to the
observed
wavevector
and form-factor

Outline

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2. The high T pseudogap:

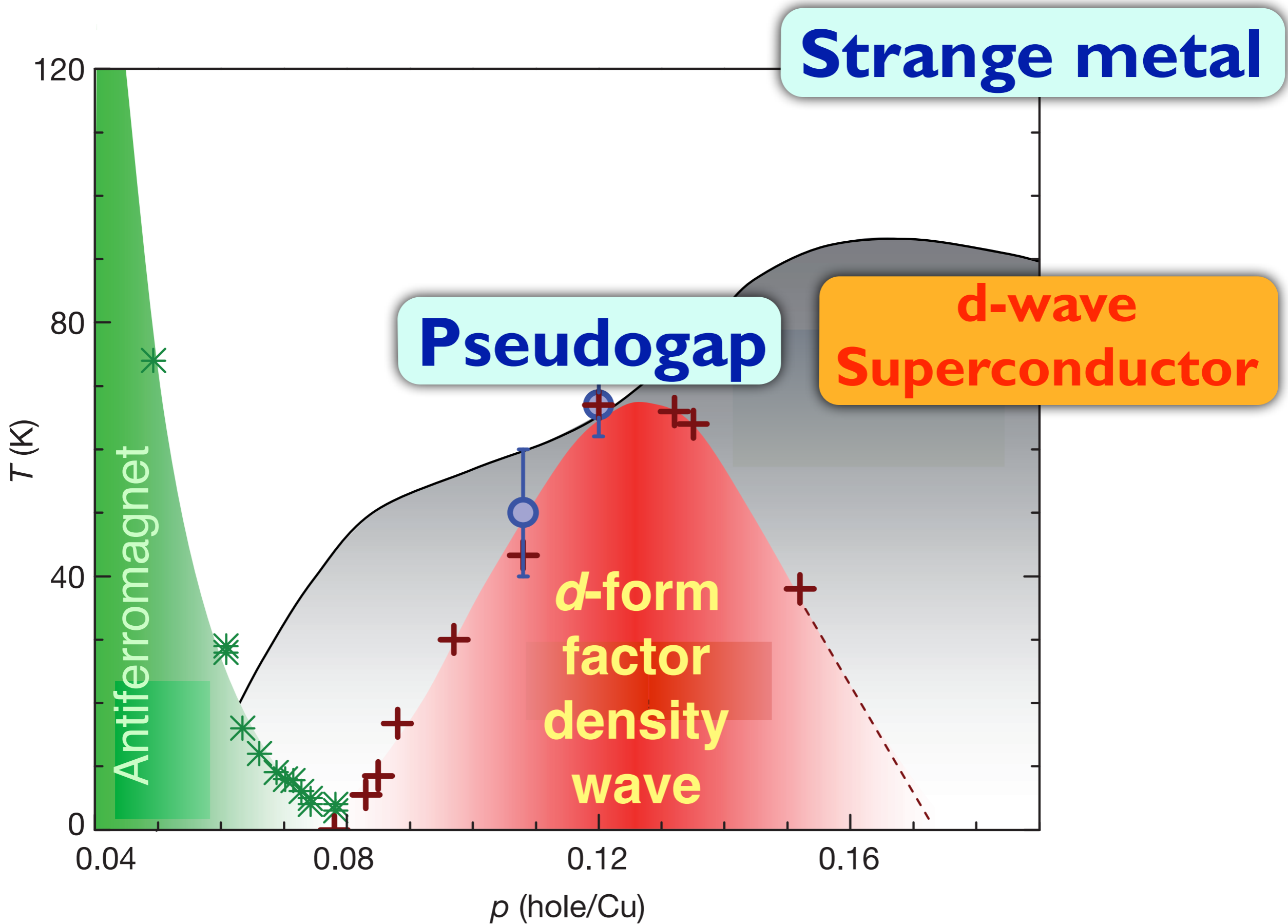
*A metal with topological order:
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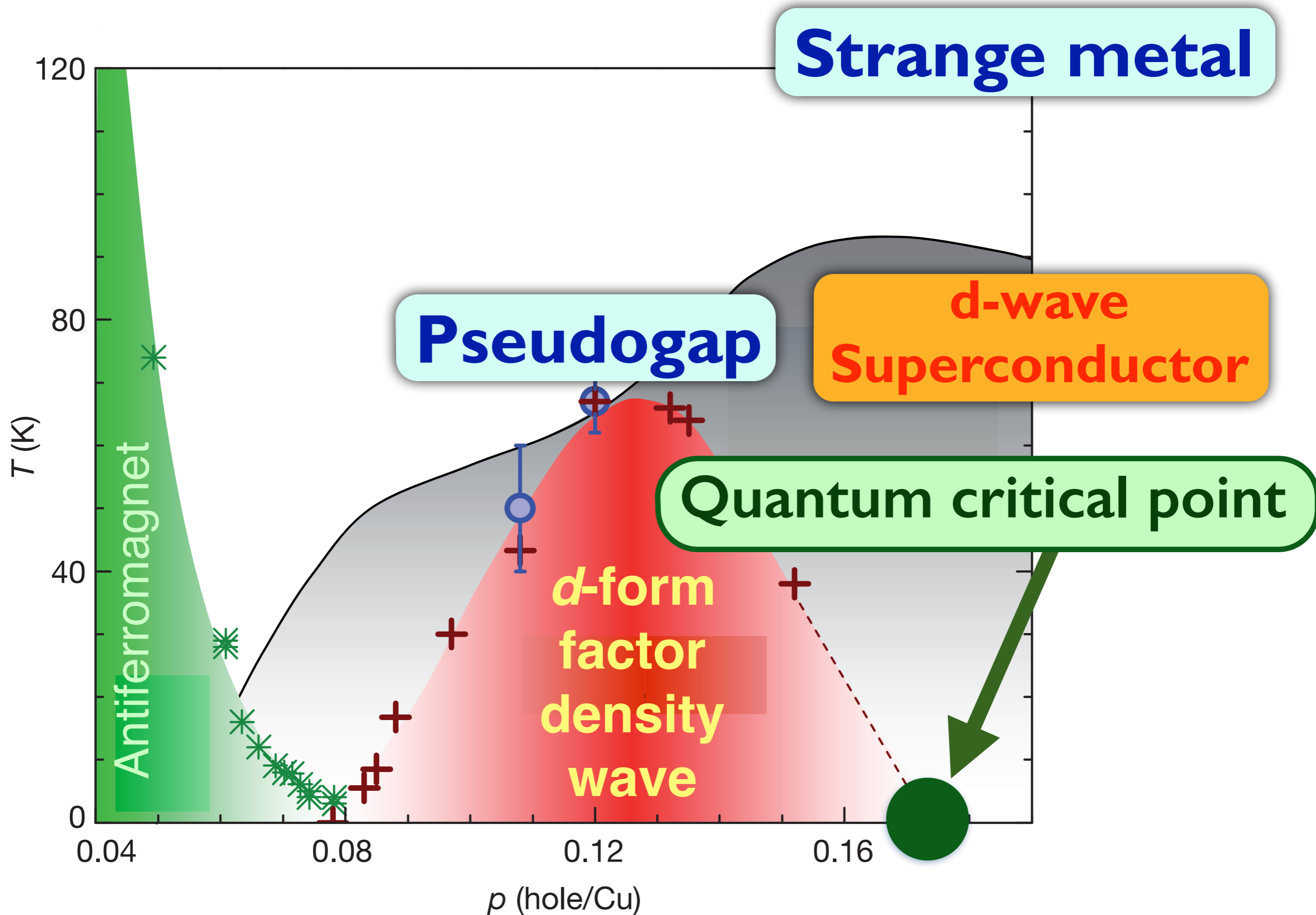
3. Connecting high and low T :

Density wave instabilities

4. Quantum critical point near optimal p :

A Higgs critical point





Y. He *et al.*, Science **344**, 608 (2014)
 K. Fujita *et al.*, Science **344**, 612 (2014)

SU(2) gauge theory for underlying quantum critical point

Write the electron operator c_α ($\alpha = \uparrow, \downarrow$ are spin indices) as

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

where R is a SU(2) matrix which determines the orientation of the local antiferromagnetic order, and ψ_\pm are spinless fermions which carry the global electron U(1) charge.

This parameterization is invariant under a SU(2) *gauge* transformation

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow U \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} ; \quad R \rightarrow RU^\dagger$$

SU(2) gauge theory for underlying quantum critical point

Assume field R is non-critical.

- Fermion ψ , transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a “large” Fermi surface.
- A SU(2) gauge boson.
- A real Higgs field, H , transforming as a gauge SU(2) adjoint, carrying lattice momentum (π, π) . Condensation of the Higgs breaks $SU(2) \rightarrow U(1)$, and transforms the large Fermi surface to a small Fermi surface.

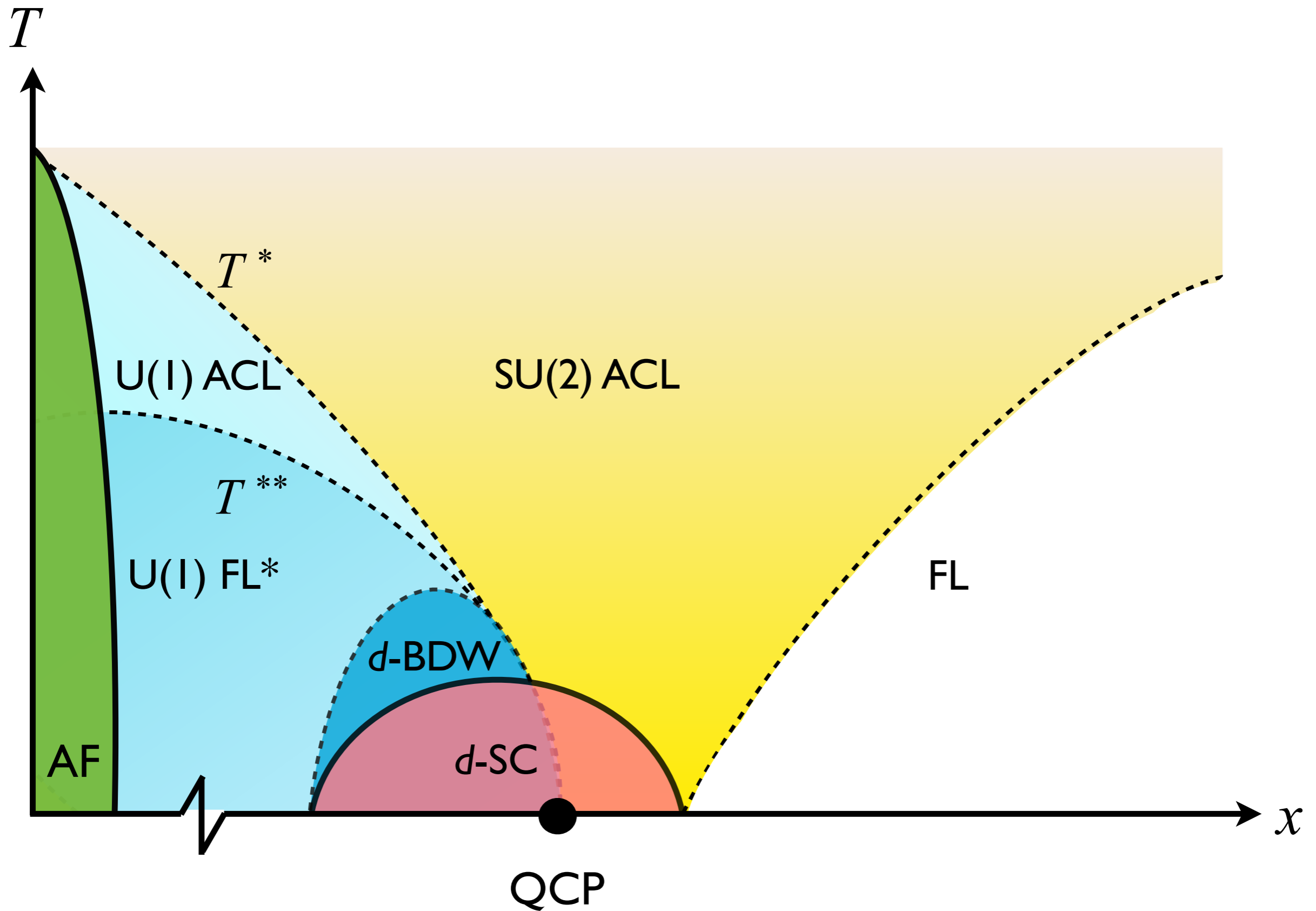
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- The quantum critical theory is the Higgs transition where the gauge “symmetry” breaks from SU(2) down to U(1), in the presence of a Fermi surface of fermions carrying fundamental SU(2) charges.

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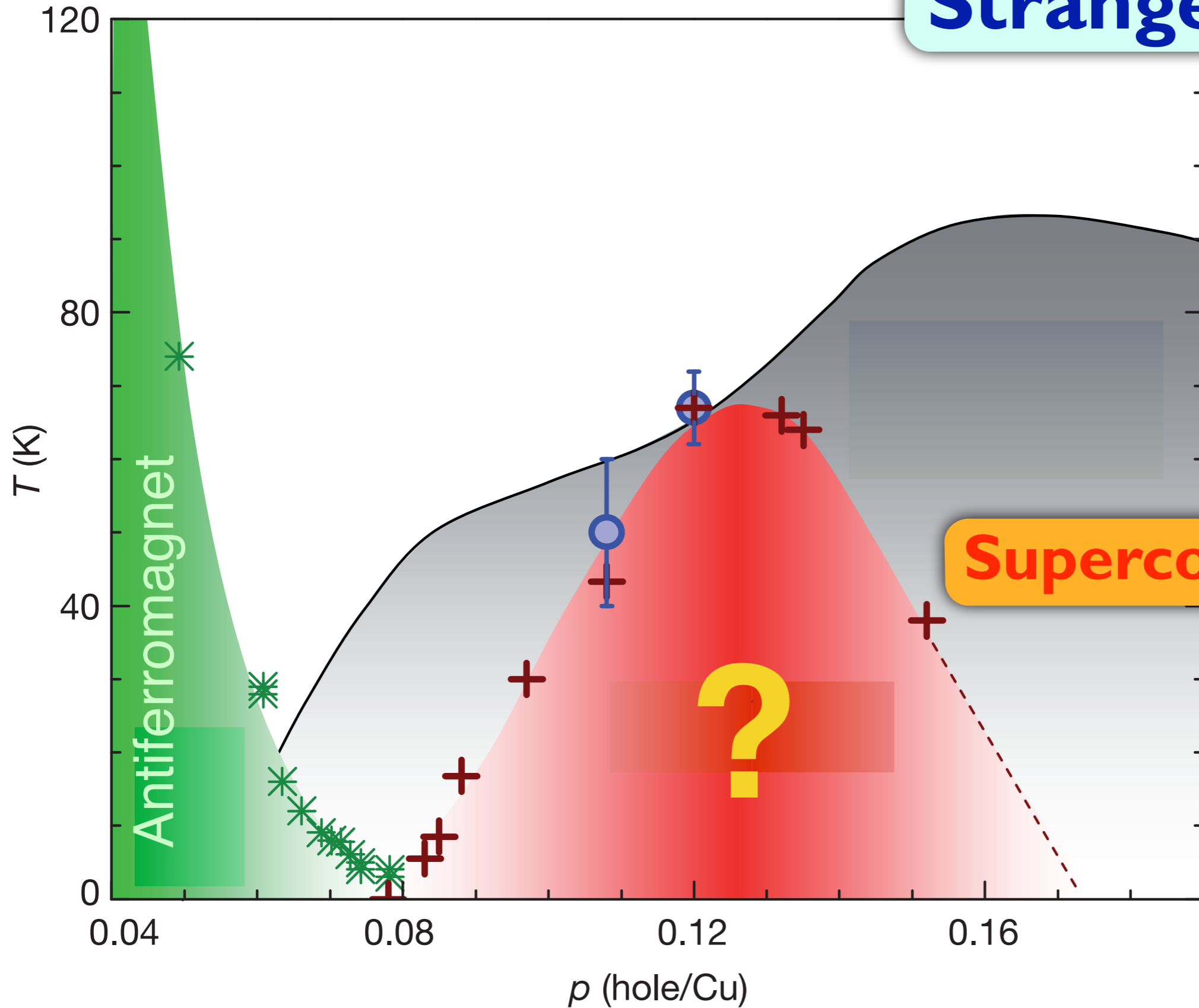
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- The Higgs condensation does not give the fermions a “mass”; instead it reconstructs the Fermi surface from *large* to *small*.
- The quantum phase transition has no gauge-invariant “order parameter”, and it does not break any global symmetries.

Conclusions

1. Predicted d -form factor density wave order observed in the non-La hole-doped cuprate superconductors.
2. The d -form factor is an unexpected window into the electronic structure of the pseudogap, with evidence for a fractionalized Fermi liquid (FL*) model.

Strange metal



Superconductor

