Quantum vortices and competing orders

cond-mat/0408329 and cond-mat/0409470

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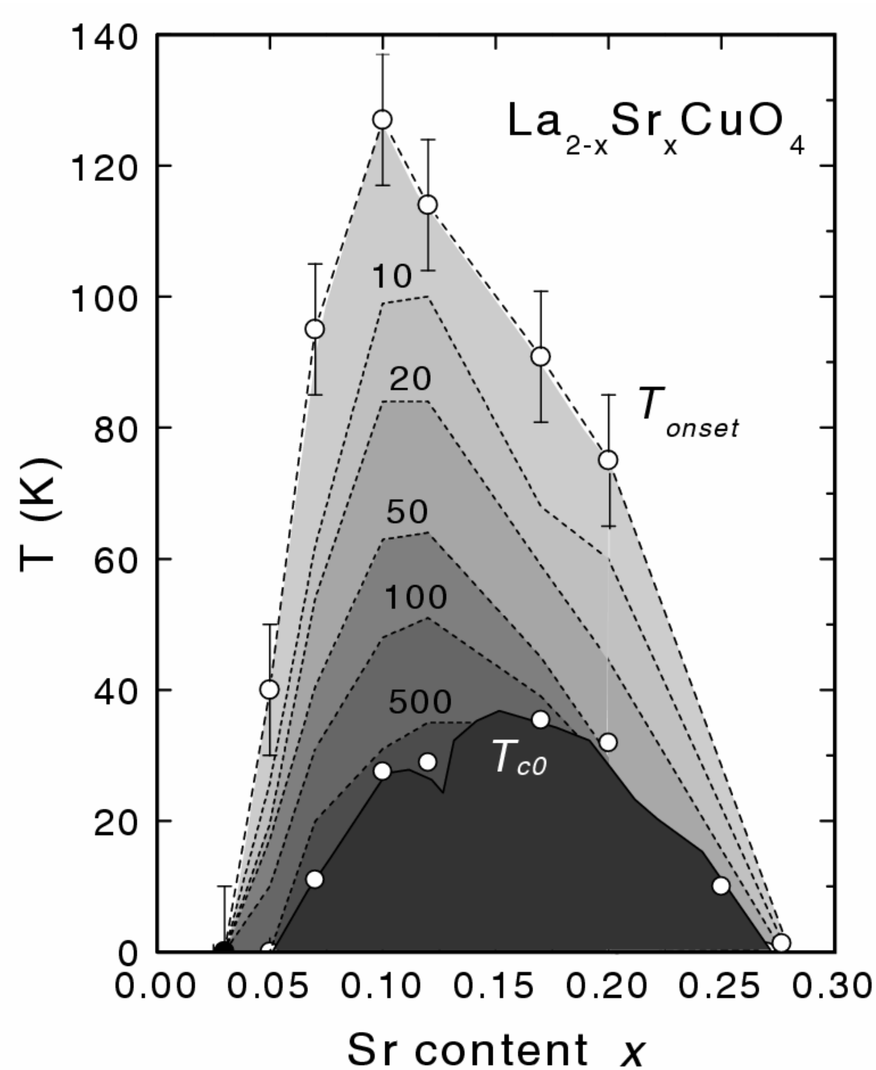
Talk online: Google Sachdev

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

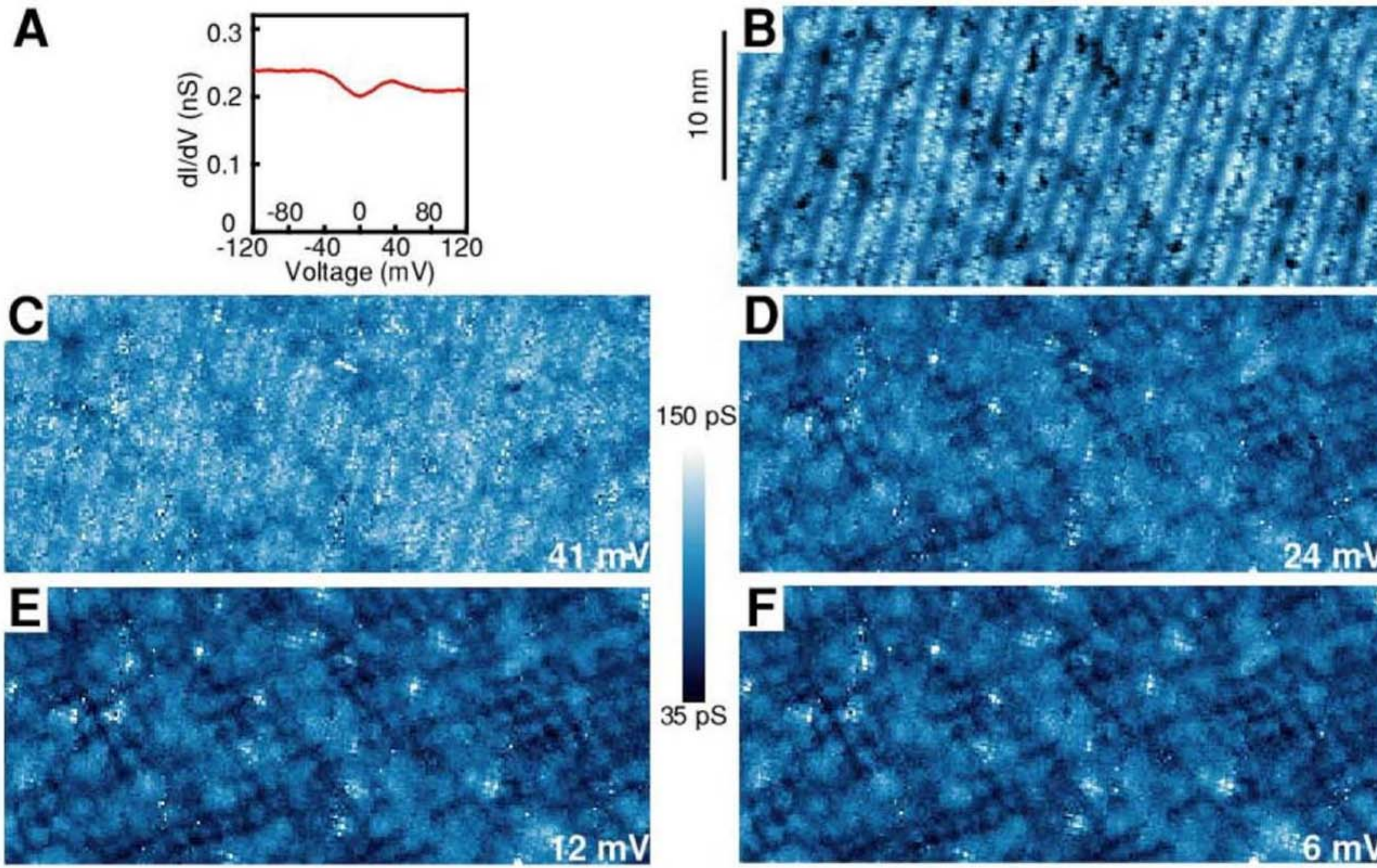


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

STM measurements observe "density" modulations with a period of $\approx$ 4 lattice spacings



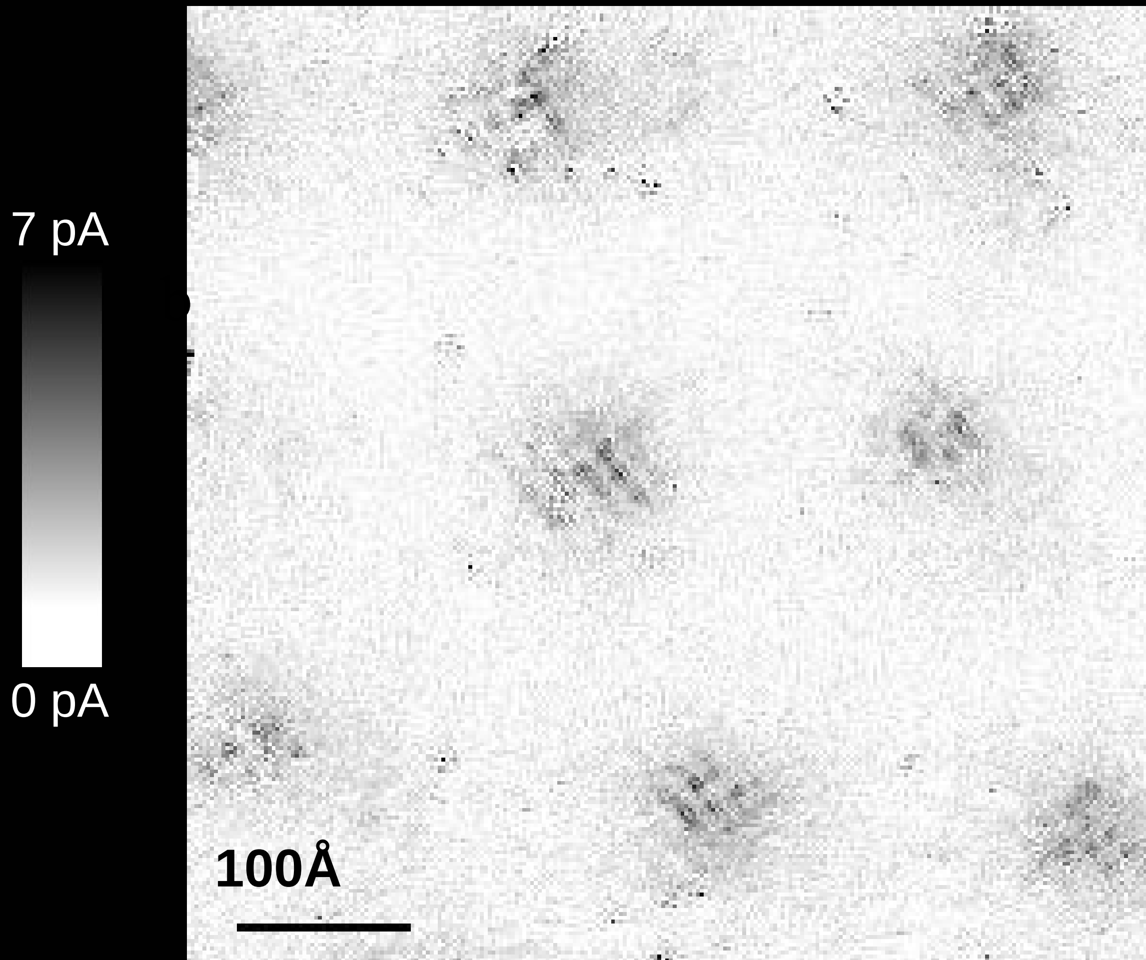
LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Is there a connection between vorticity and "density" wave modulations?

"Density" wave order---modulations in pairing amplitude, exchange energy, or hole density. Equivalent to valence-bond-solid (VBS) order (except at the special period of 2 lattice spacings)

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

Landau-Ginzburg-Wilson theory of multiple order parameters:

- "Vortex/phase fluctuations" ("preformed pairs")

  Complex superconducting order parameter: Ψ_{sc}

  $\Psi_{sc} \to \Psi_{sc} e^{i\theta}$ symmetry encodes number conservation

- "Charge/valence-bond/pair-density/stripe" order

  Order parameters:

$$\rho\left(\boldsymbol{r}\right) = \sum_{\boldsymbol{Q}} \rho_{\boldsymbol{Q}} e^{i\boldsymbol{Q}.\boldsymbol{r}}$$

$\rho_{\boldsymbol{Q}} \to \rho_{\boldsymbol{Q}} e^{i\vartheta}$ encodes space group symmetry

Landau-Ginzburg-Wilson theory of multiple order parameters:

LGW free energy:

$$F = F_{sc}\left[\Psi_{sc}\right] + F_{\text{charge}}\left[\rho_{\mathbf{Q}}\right] + F_{\text{int}}$$

$$F_{sc}\left[\Psi_{sc}\right] = r_1\left|\Psi_{sc}\right|^2 + u_1\left|\Psi_{sc}\right|^4 + \cdots$$

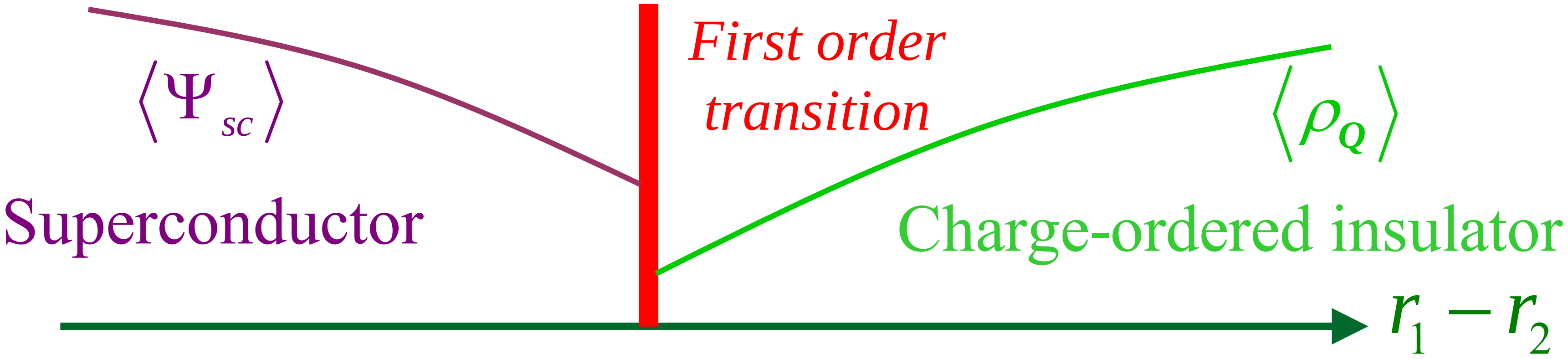
$$F_{\text{charge}}\left[\rho_{\mathbf{Q}}\right] = r_2\left|\rho_{\mathbf{Q}}\right|^2 + u_2\left|\rho_{\mathbf{Q}}\right|^4 + \cdots$$

$$F_{\text{int}} = v\left|\Psi_{sc}\right|^2\left|\rho_{\mathbf{Q}}\right|^2 + \cdots$$

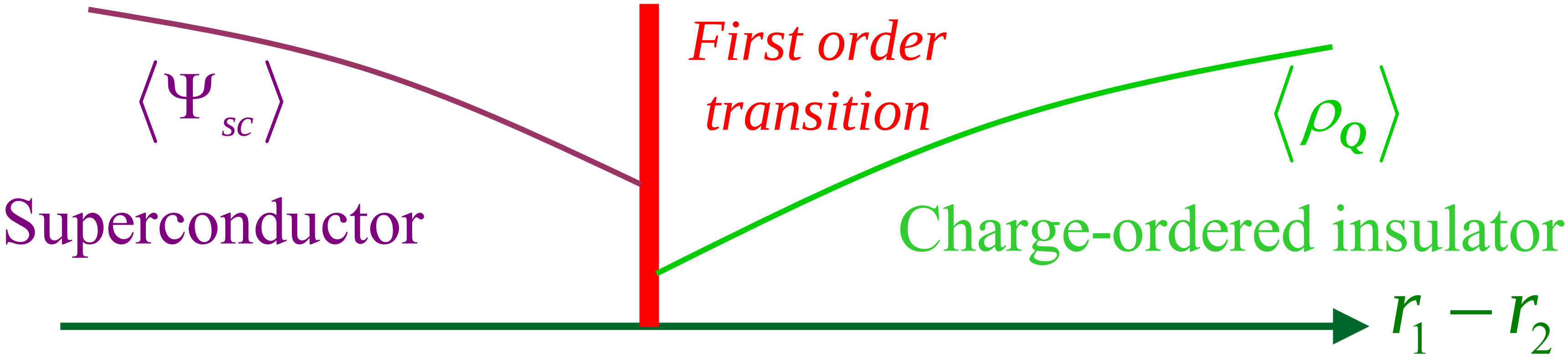
Distinct symmetries of order parameters permit couplings only between their energy densities (there are no symmetries which "rotate" two order parameters into each other)

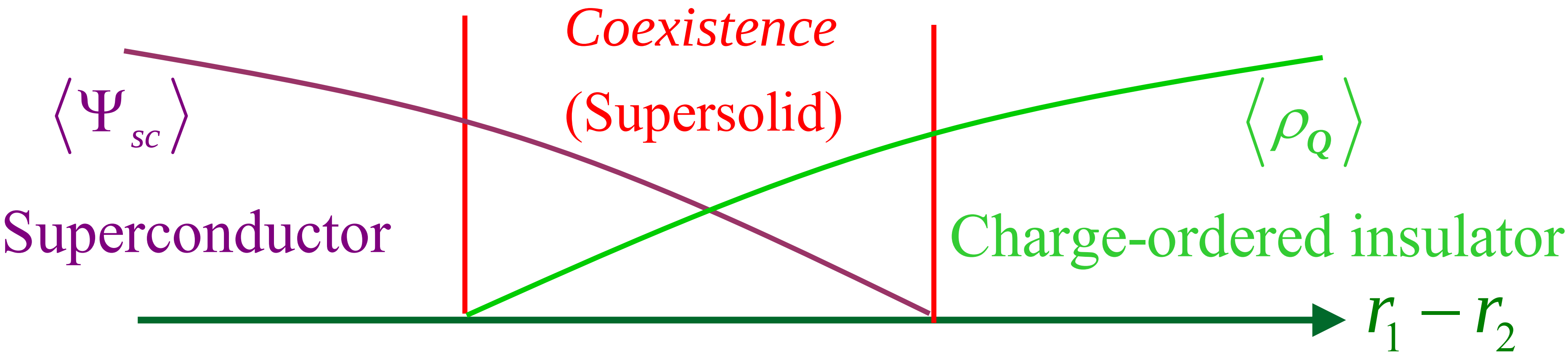
For large positive *v*, there is a correlation between vortices and density wave order

Predictions of LGW theory



Predictions of LGW theory





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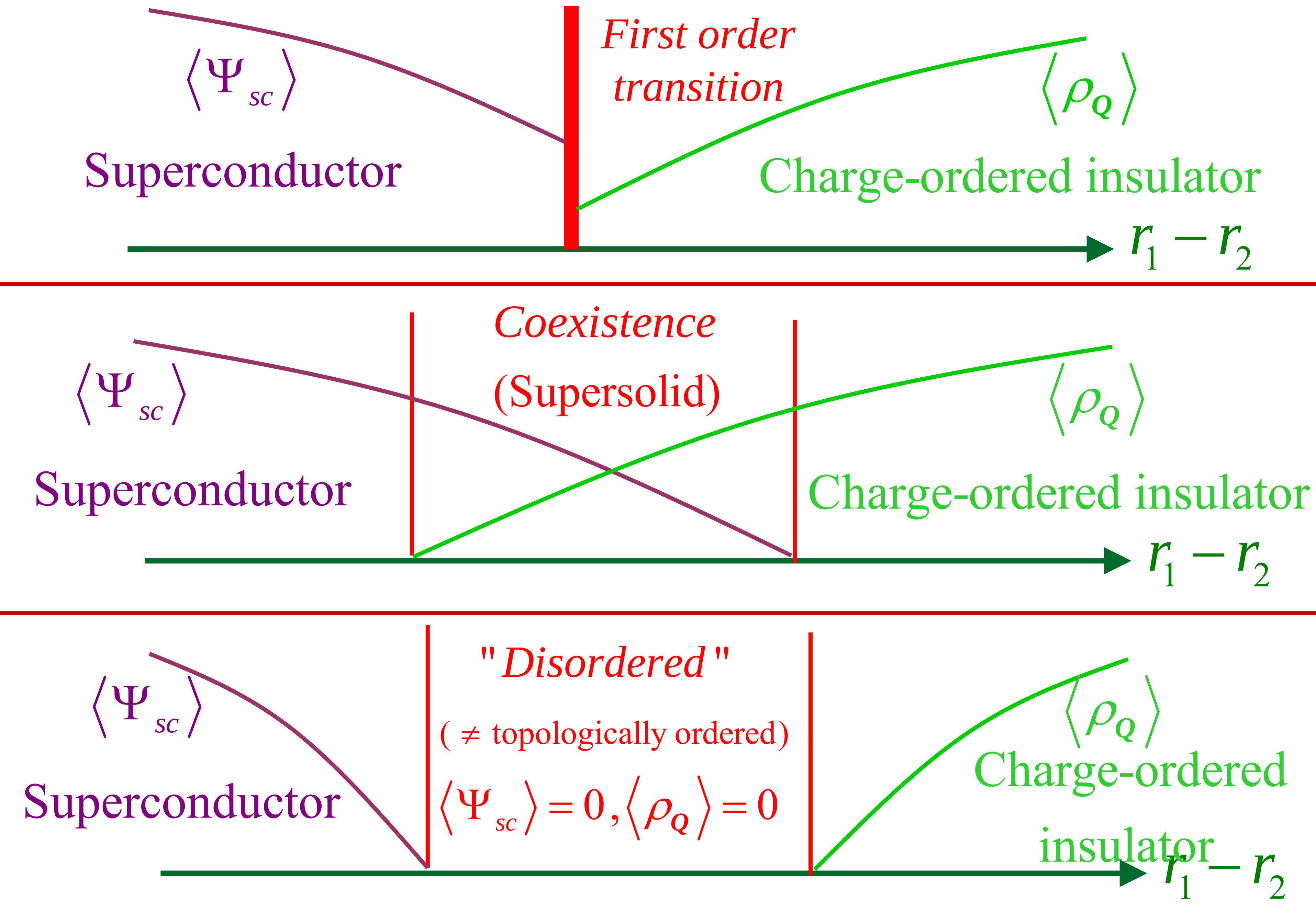
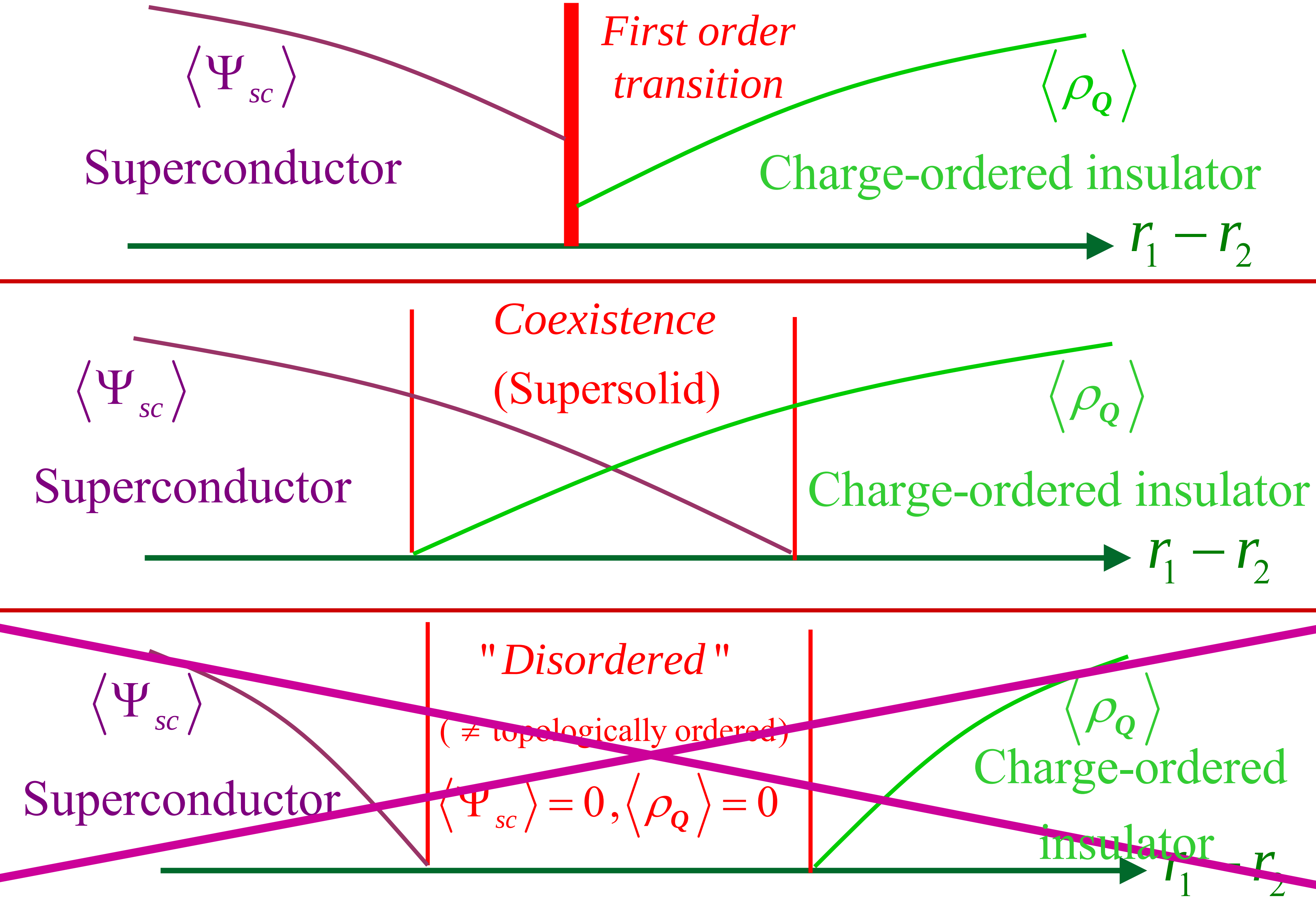


Predictions of LGW theory



Non-superconducting quantum phase must have some other "order":

- Charge order in an insulator
- Fermi surface in a metal
- "Topological order" in a spin liquid
- ……………

This requirement is *not* captured by LGW theory.

Needed: a theory of precursor fluctuations of the density wave order of the insulator within the superconductor.

i.e. a connection between vortices and density wave order

Outline

A. Superfluid-insulator transitions of bosons
on the square lattice at fractional filling
Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator

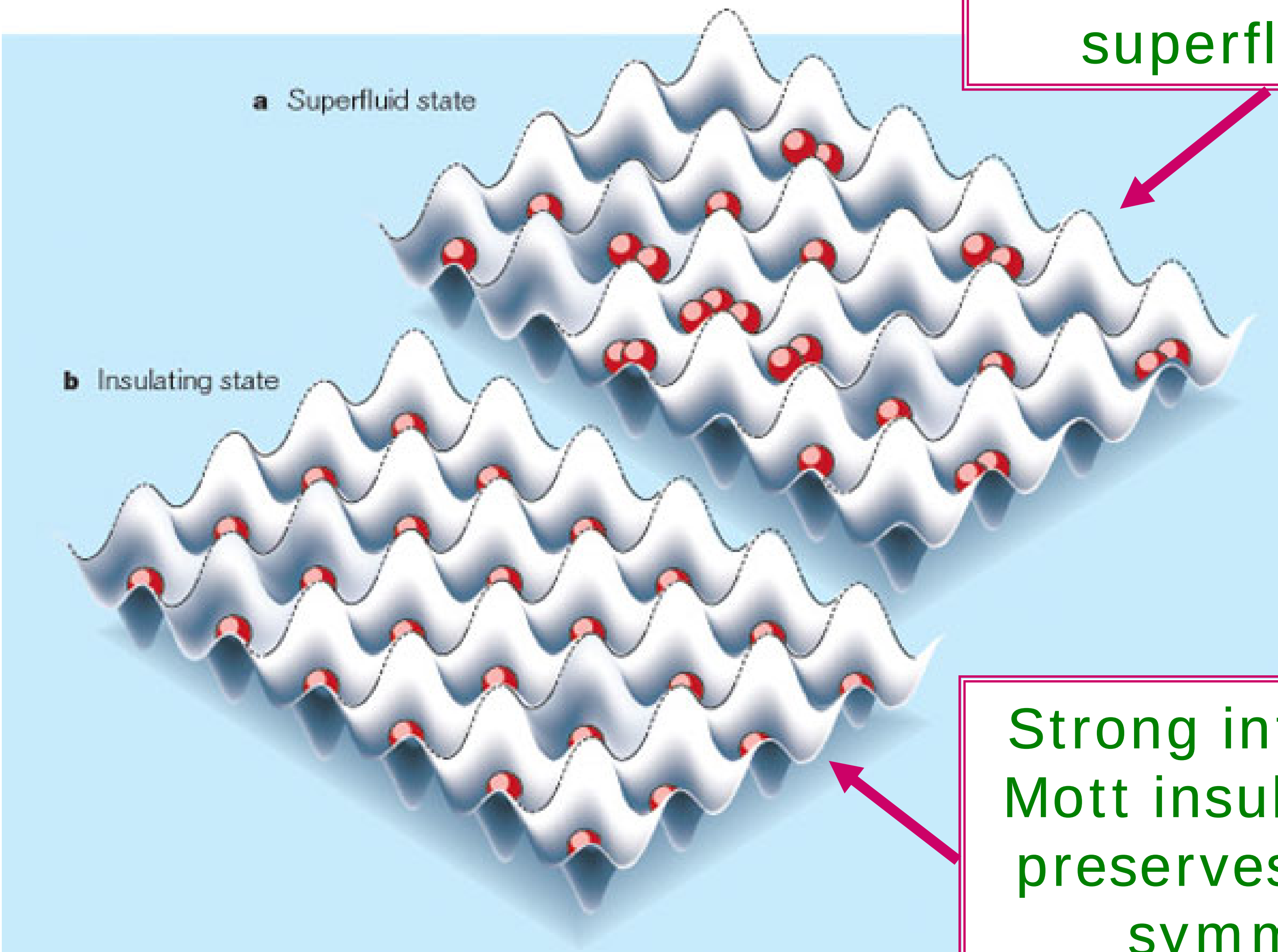
B. Application to a short-range pairing model for the cuprate superconductors
Competition between VBS order and d-wave superconductivity

A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling

Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator

Bosons at density $f = 1$

Weak interactions: superfluidity



Strong interactions: Mott insulator which preserves all lattice symmetries

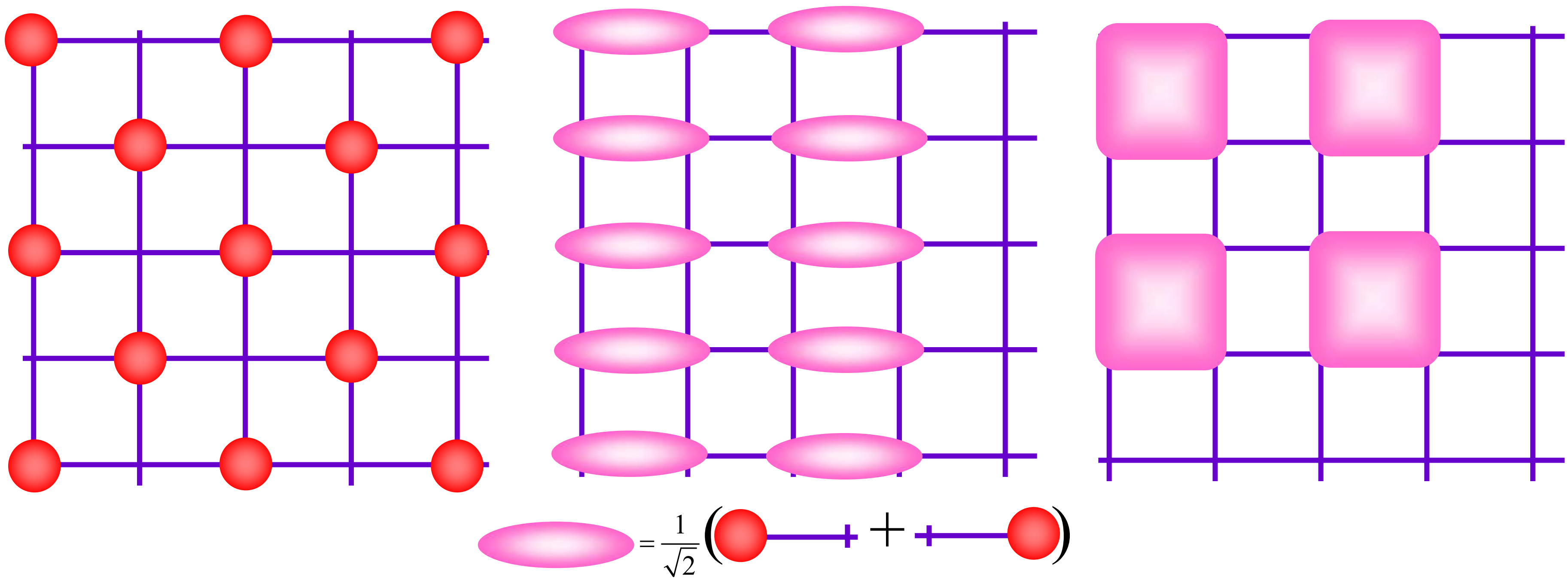
LGW theory: continuous quantum transitions between these states

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Bosons at density $f = 1/2$ (equivalent to S=1/2 AFMs)

Weak interactions: superfluidity $\langle \Psi_{sc} \rangle \neq 0$

Strong interactions: Candidate insulating states

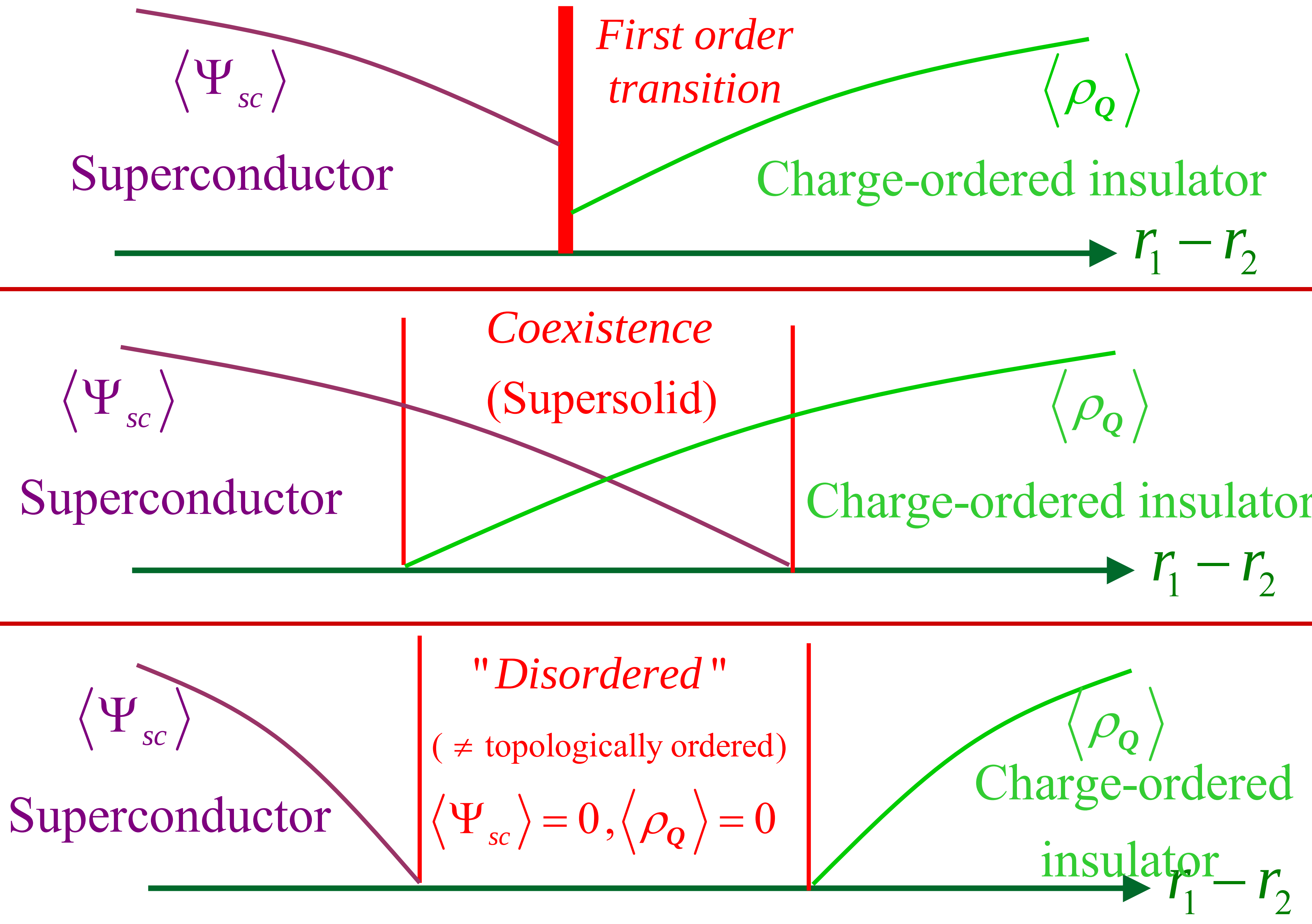


All insulating phases have density-wave order $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$ with $\langle \rho_{\mathbf{Q}} \rangle \neq 0$

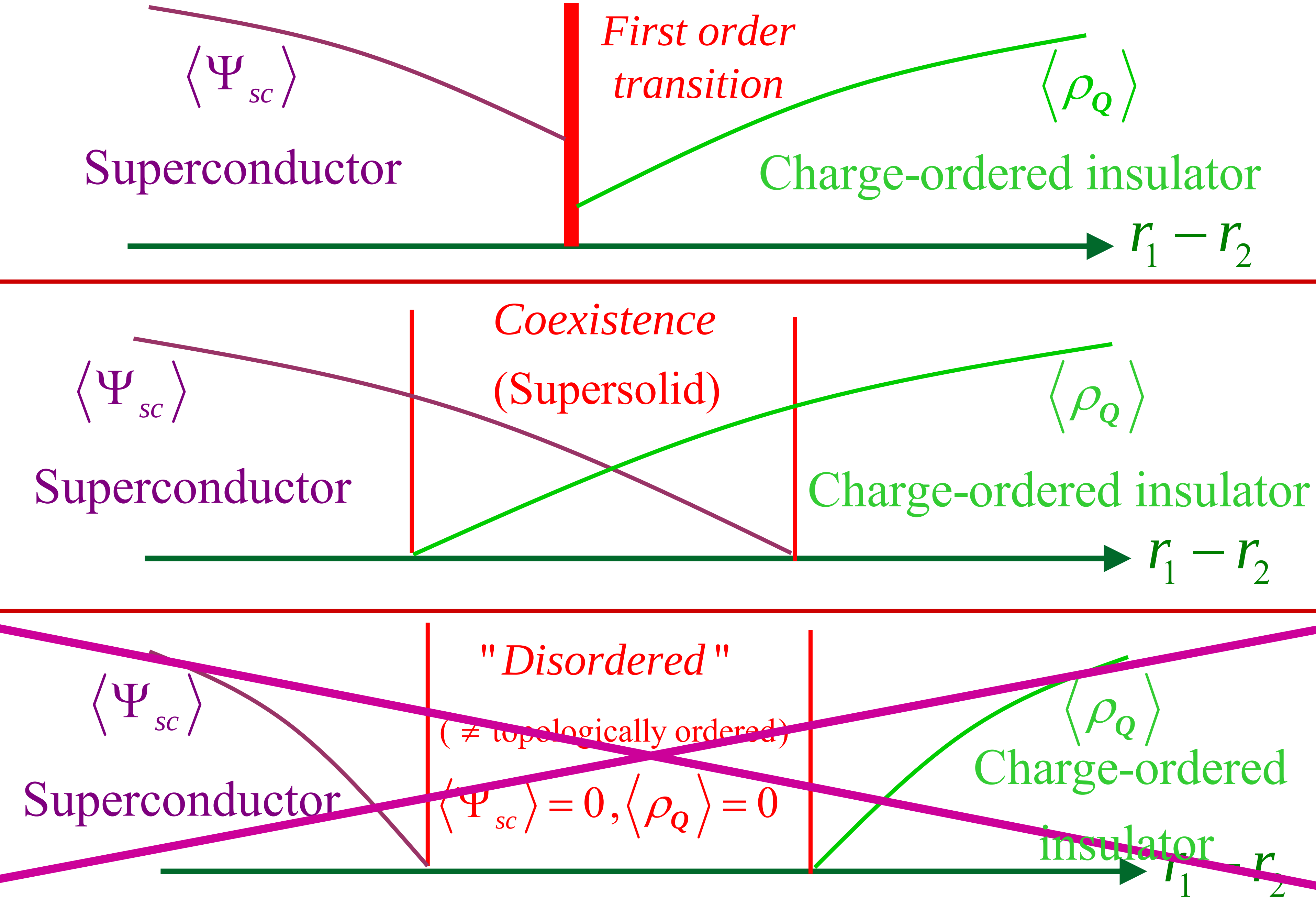
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

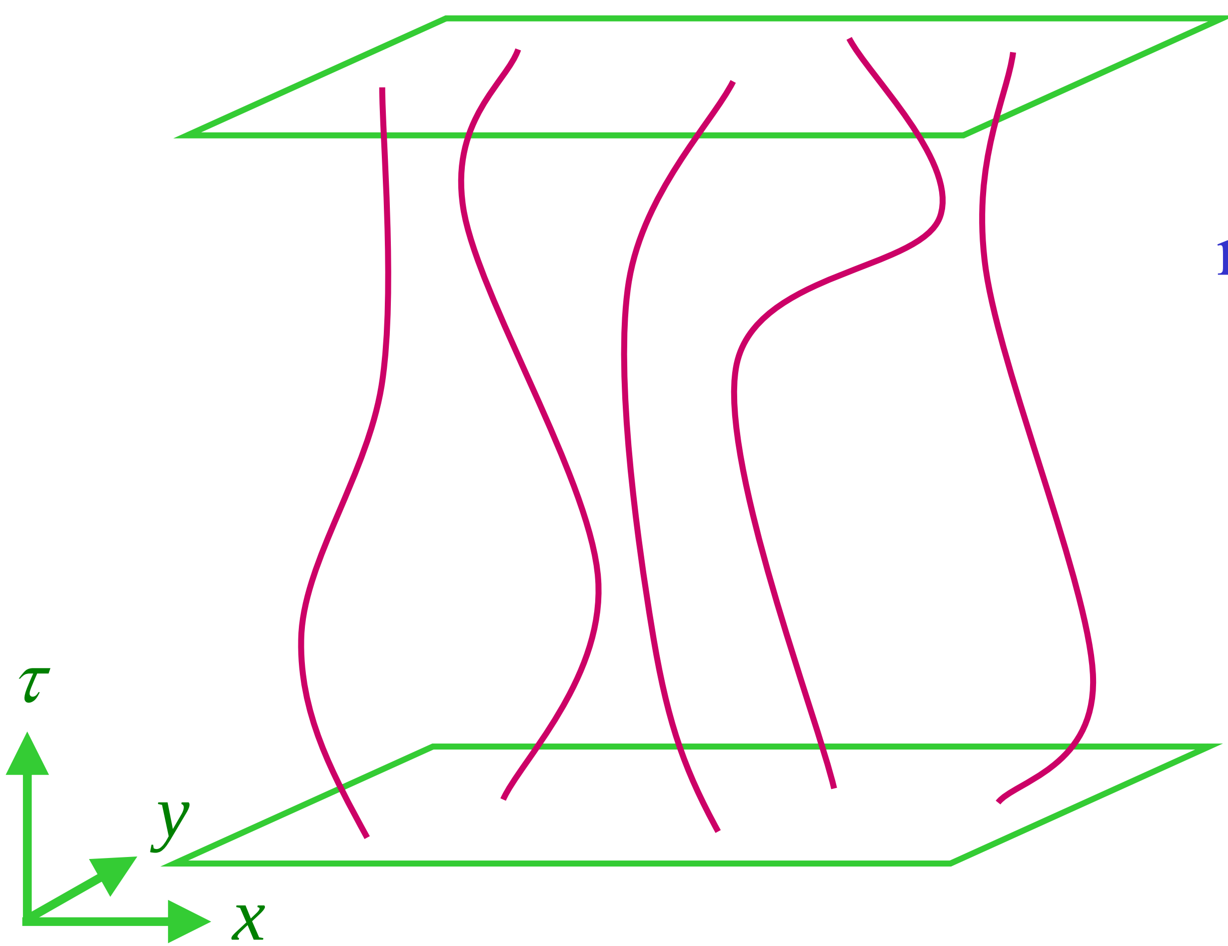
Predictions of LGW theory



Predictions of LGW theory



Boson-vortex duality



Quantum mechanics of two-dimensional bosons: world lines of bosons in spacetime

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

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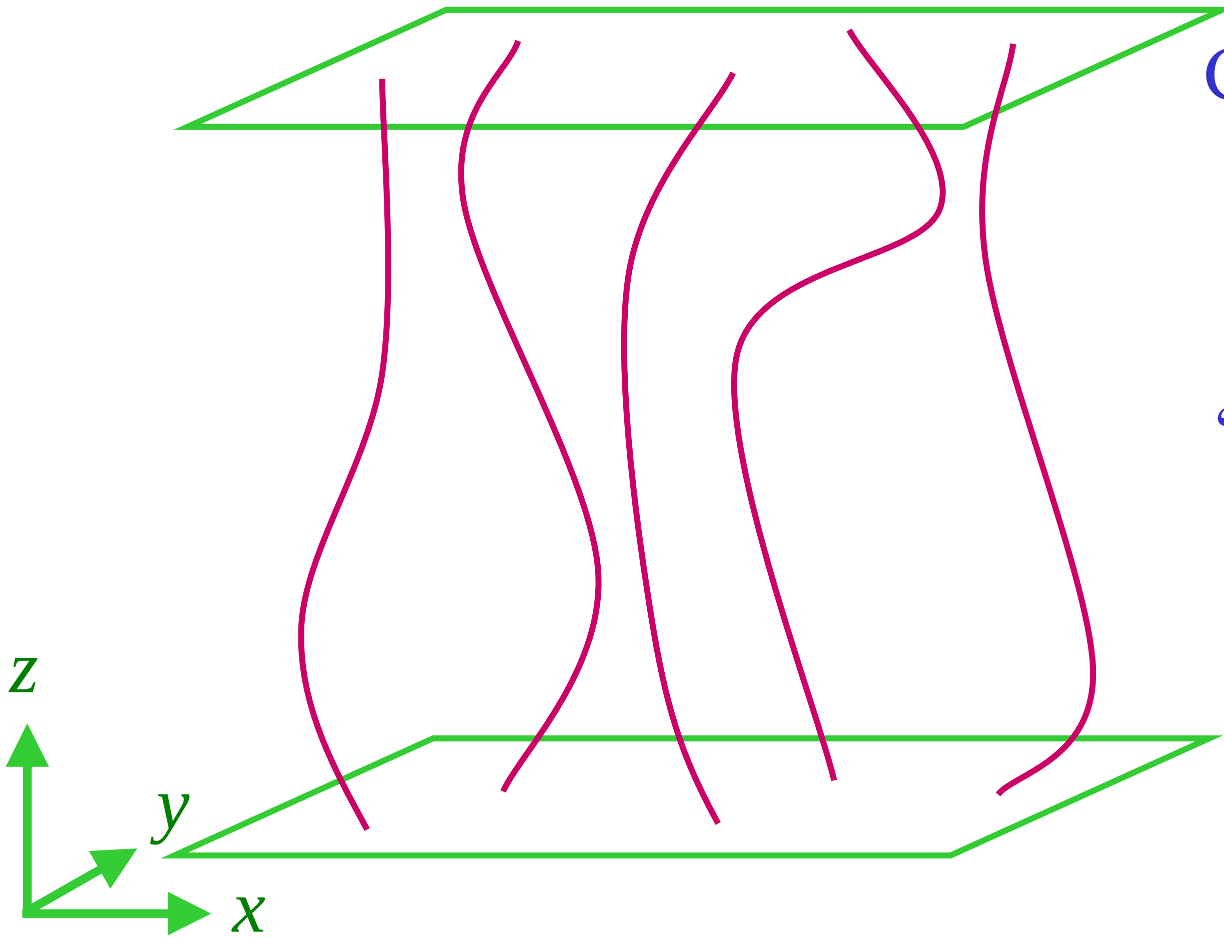

Classical statistical mechanics of a "dual" three-dimensional "superconductor", with order parameter φ : trajectories of vortices in a "magnetic" field

Strength of "magnetic" field on dual superconductor φ = density of bosons = f flux quanta per plaquette

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

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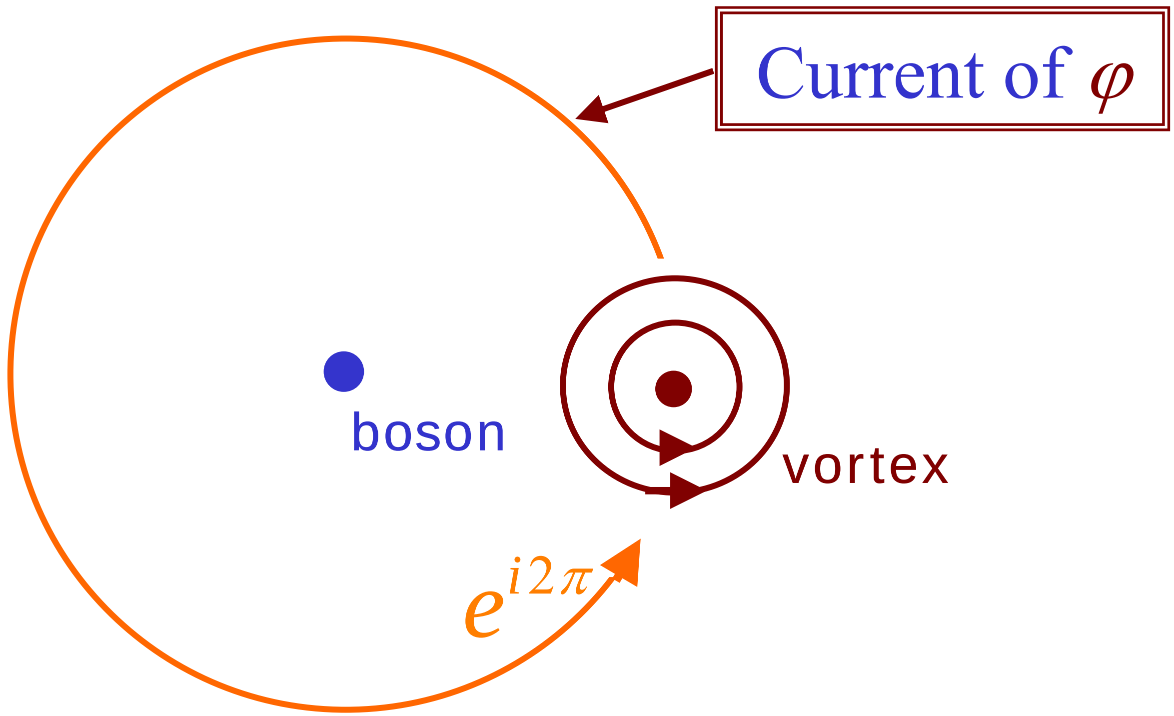


The wavefunction of a vortex acquires a phase of 2π each time the vortex encircles a boson

Strength of "magnetic" field on dual superconductor φ = density of bosons = f flux quanta per plaquette

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

Boson-vortex duality

Statistical mechanics of dual "superconductor" φ, is invariant under the square lattice space group:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

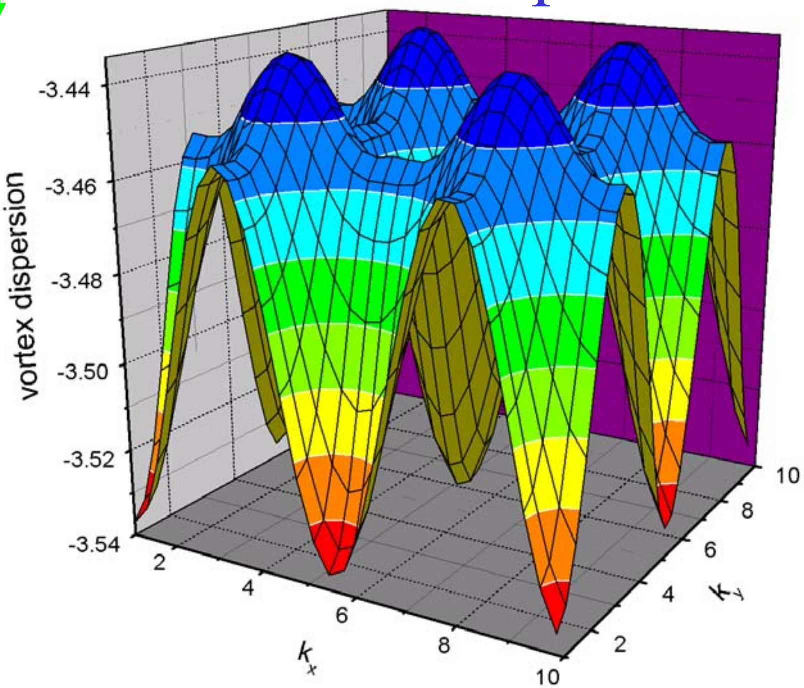
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Strength of "magnetic" field on dual superconductor φ = density of bosons = f flux quanta per plaquette

Boson-vortex duality

Hofstadter spectrum of dual "superconducting" order φ



At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \dots q$), associated with q gauge-equivalent regions of the Brillouin zone

Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

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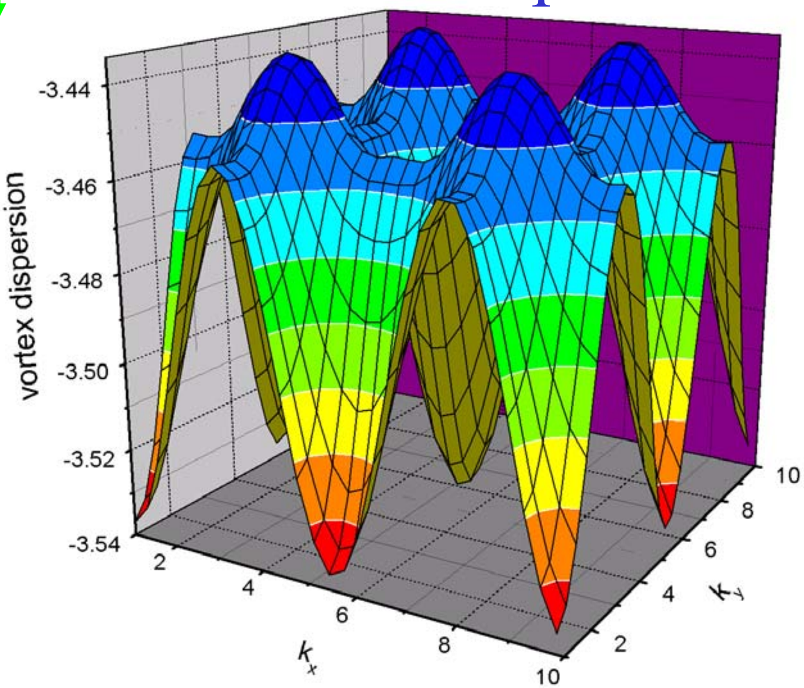

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The q vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

See also X.-G. Wen, *Phys. Rev.* B **65**, 165113 (2002)

Boson-vortex duality

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

$$\underline{\text{Superconductor/insulator}} : \left\langle \varphi_\ell \right\rangle = 0 \big/ \left\langle \varphi_\ell \right\rangle \neq 0$$

Boson-vortex duality

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

Density wave order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

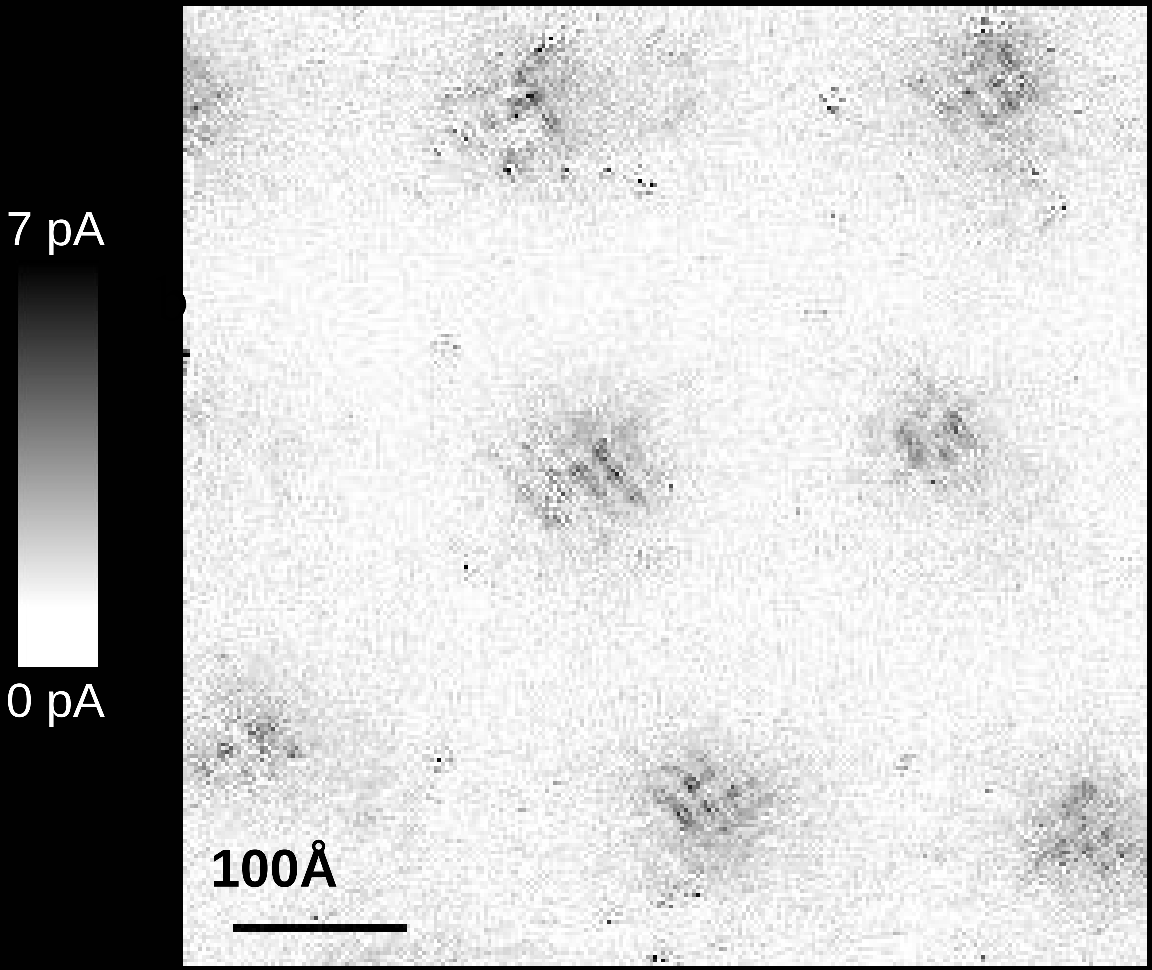
$$\boxed{\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}}$$

$$T_x : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

$$R : \rho(\mathbf{Q}) \to \rho(R\mathbf{Q})$$

Each pinned vortex in the superfluid has a halo of density wave order over a length scale ≈ the zero-point quantum motion of the vortex. This scale diverges upon approaching the Mott insulator

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



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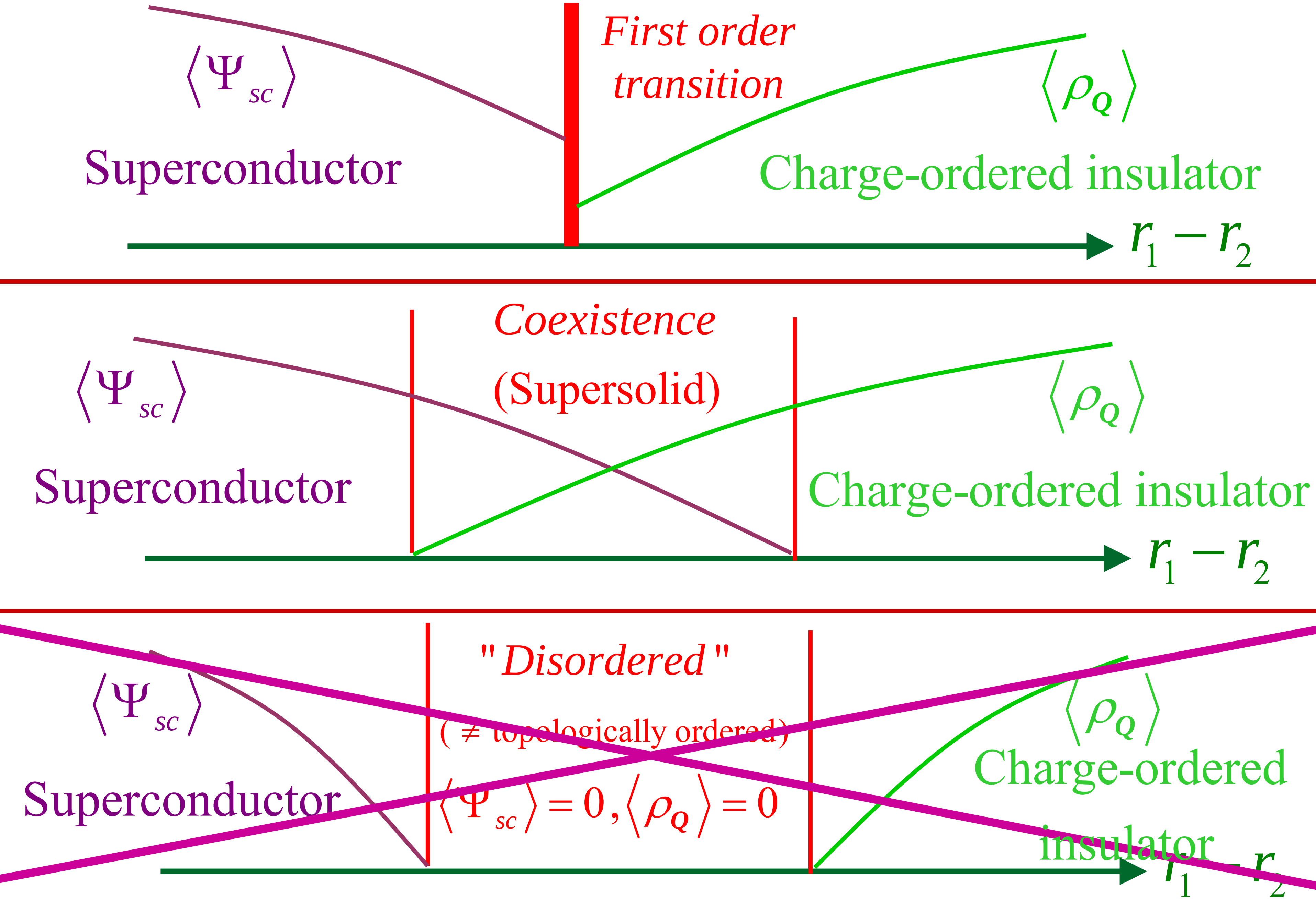
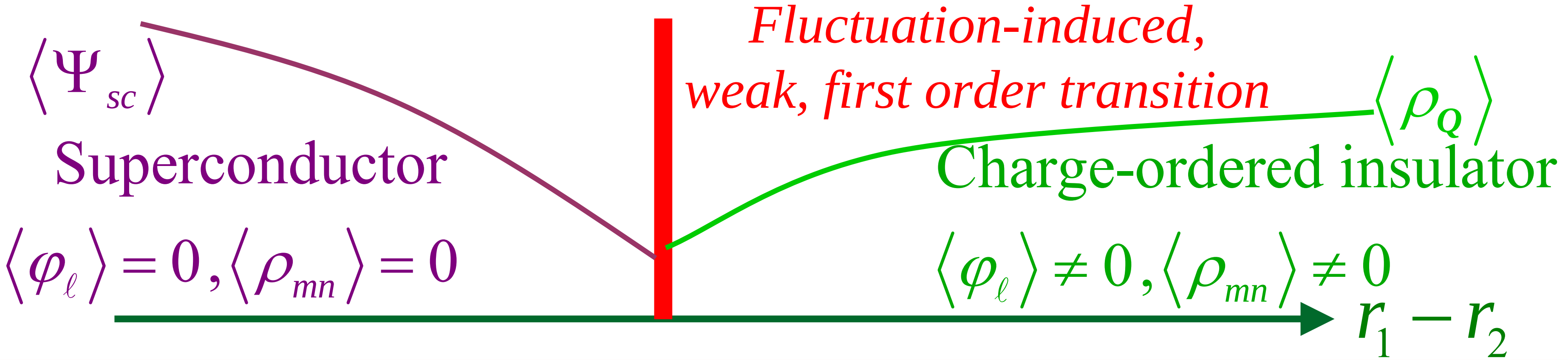


Analysis of "extended LGW" theory of projective representation



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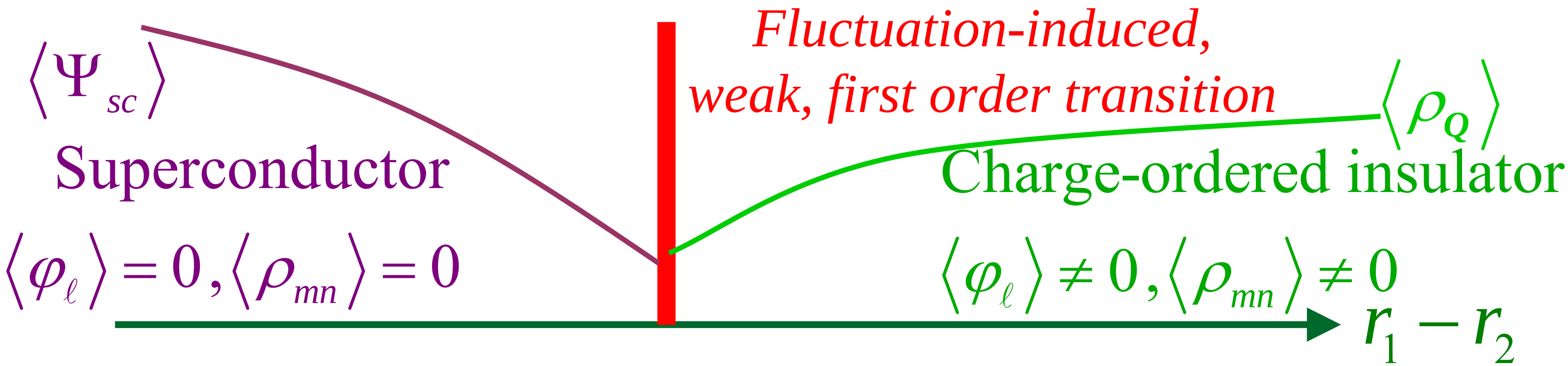

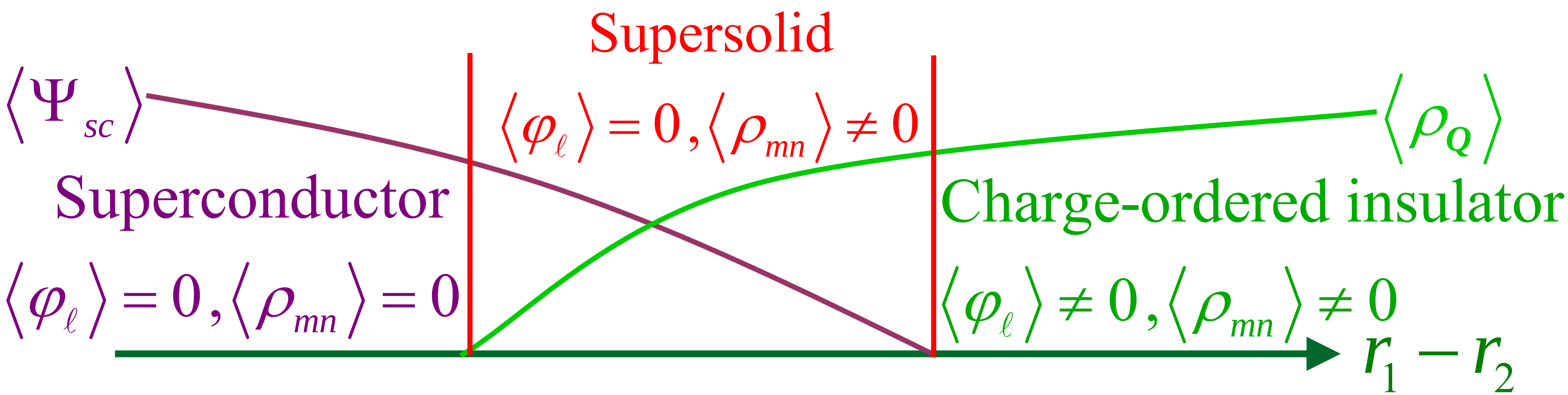

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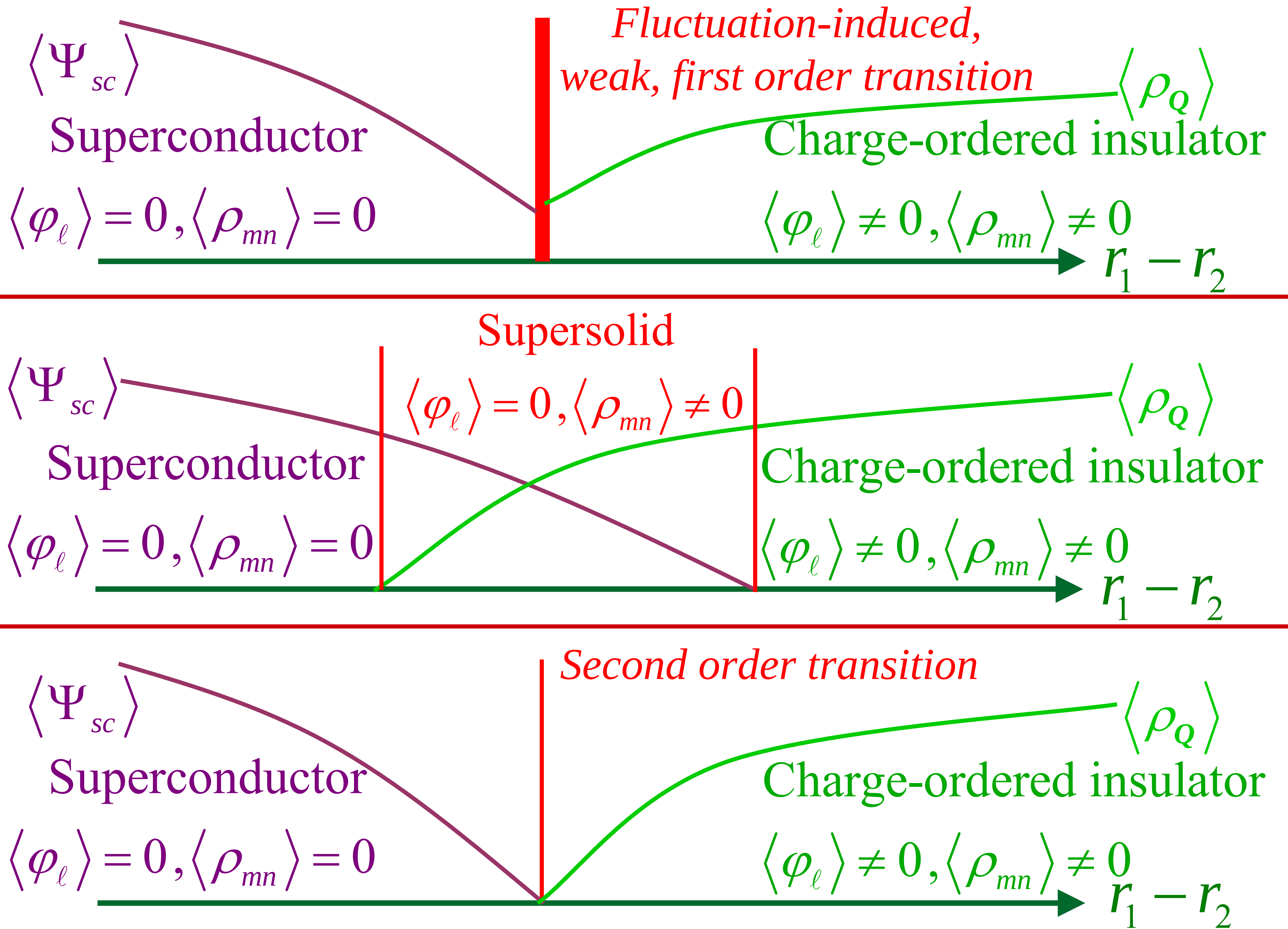
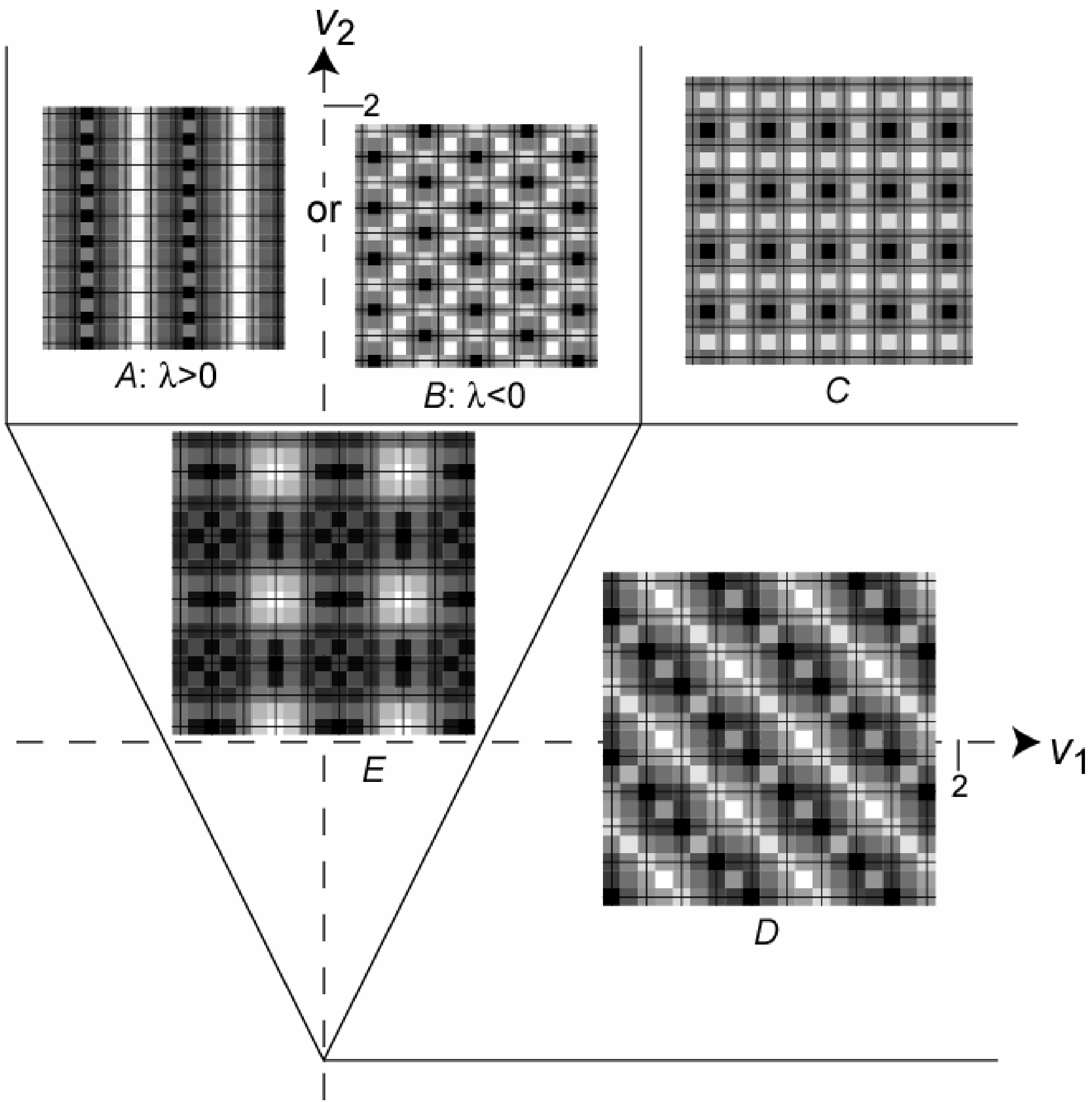


Analysis of “extended LGW” theory of projective representation

Spatial structure of insulators for q=4 (f=1/4 or 3/4)



$a \times b$ unit cells;

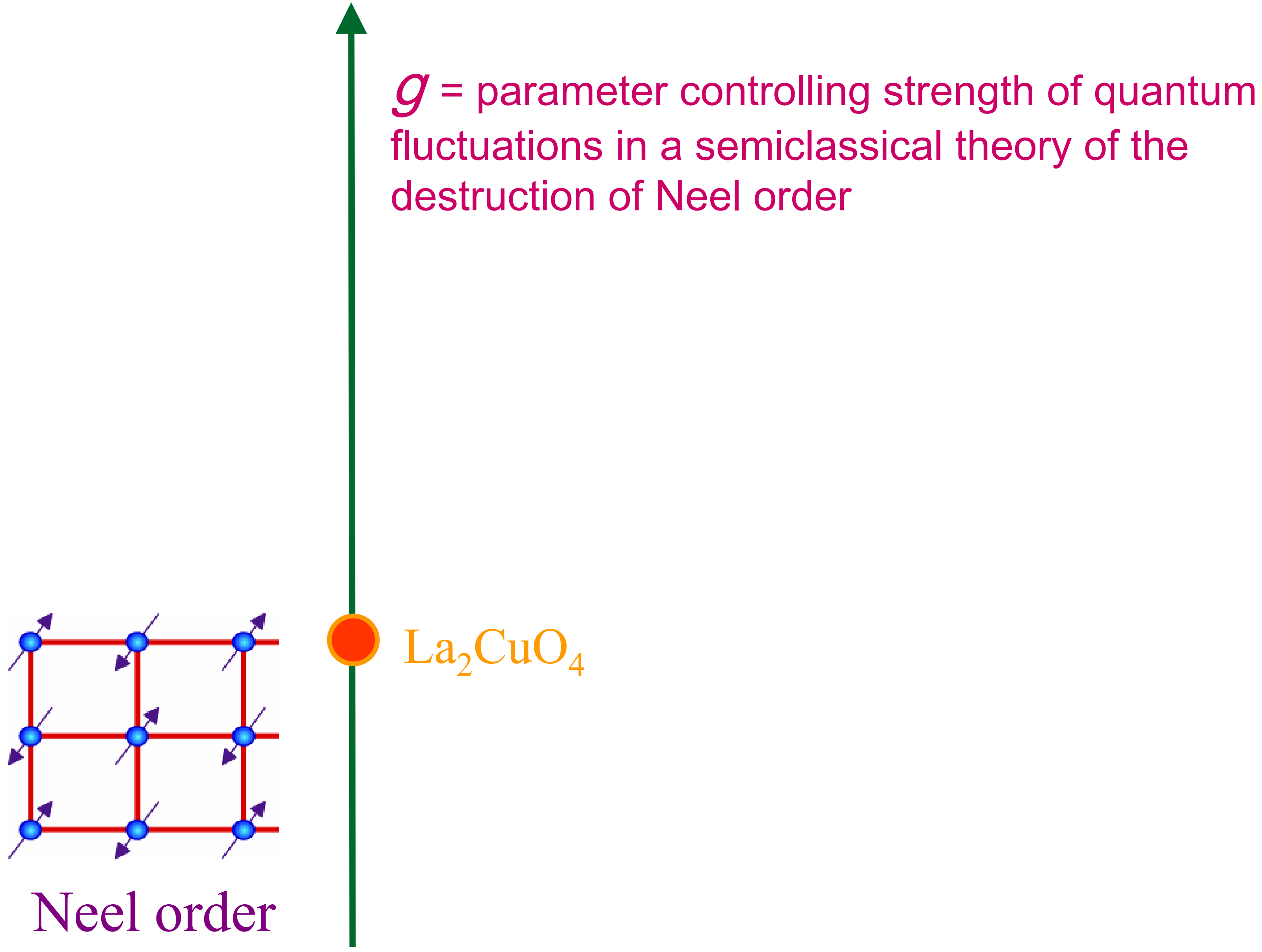
q/a, q/b, ab/q,

all integers

B. Application to a short-range pairing model for the cuprate superconductors

Competition between VBS order and d-wave superconductivity

Phase diagram of doped antiferromagnets



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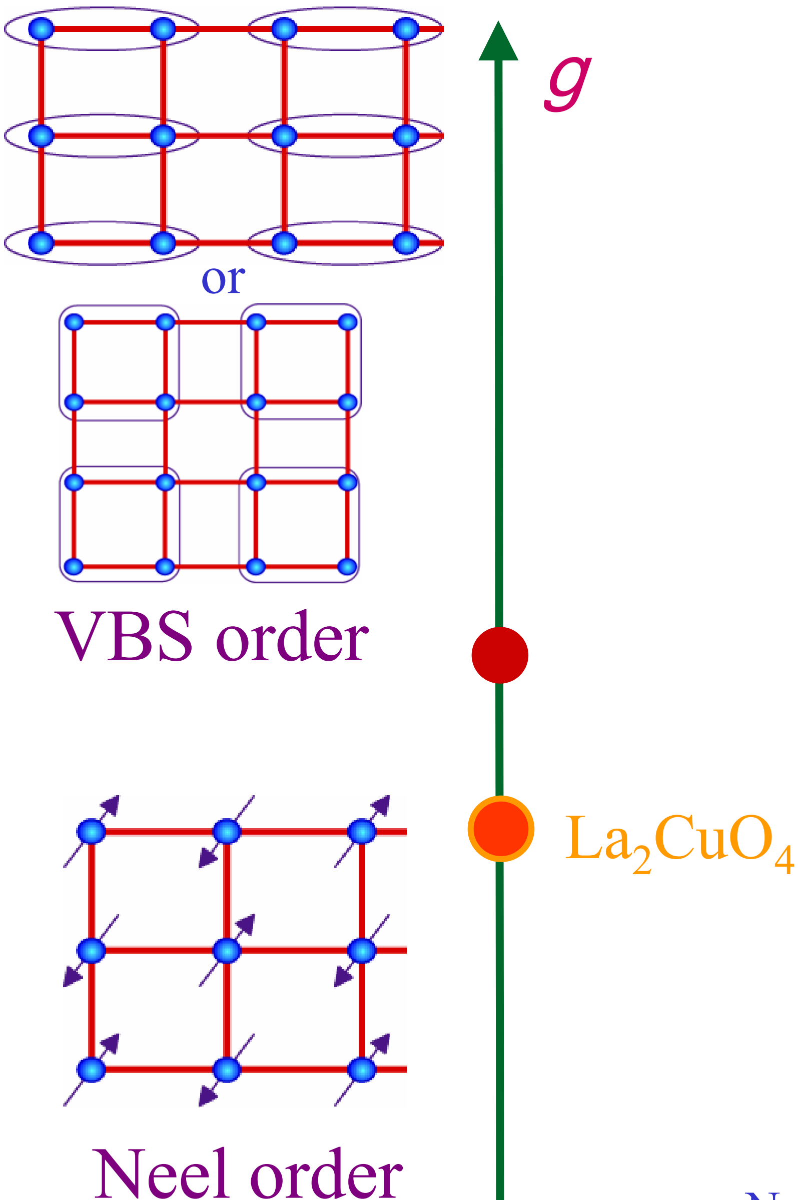

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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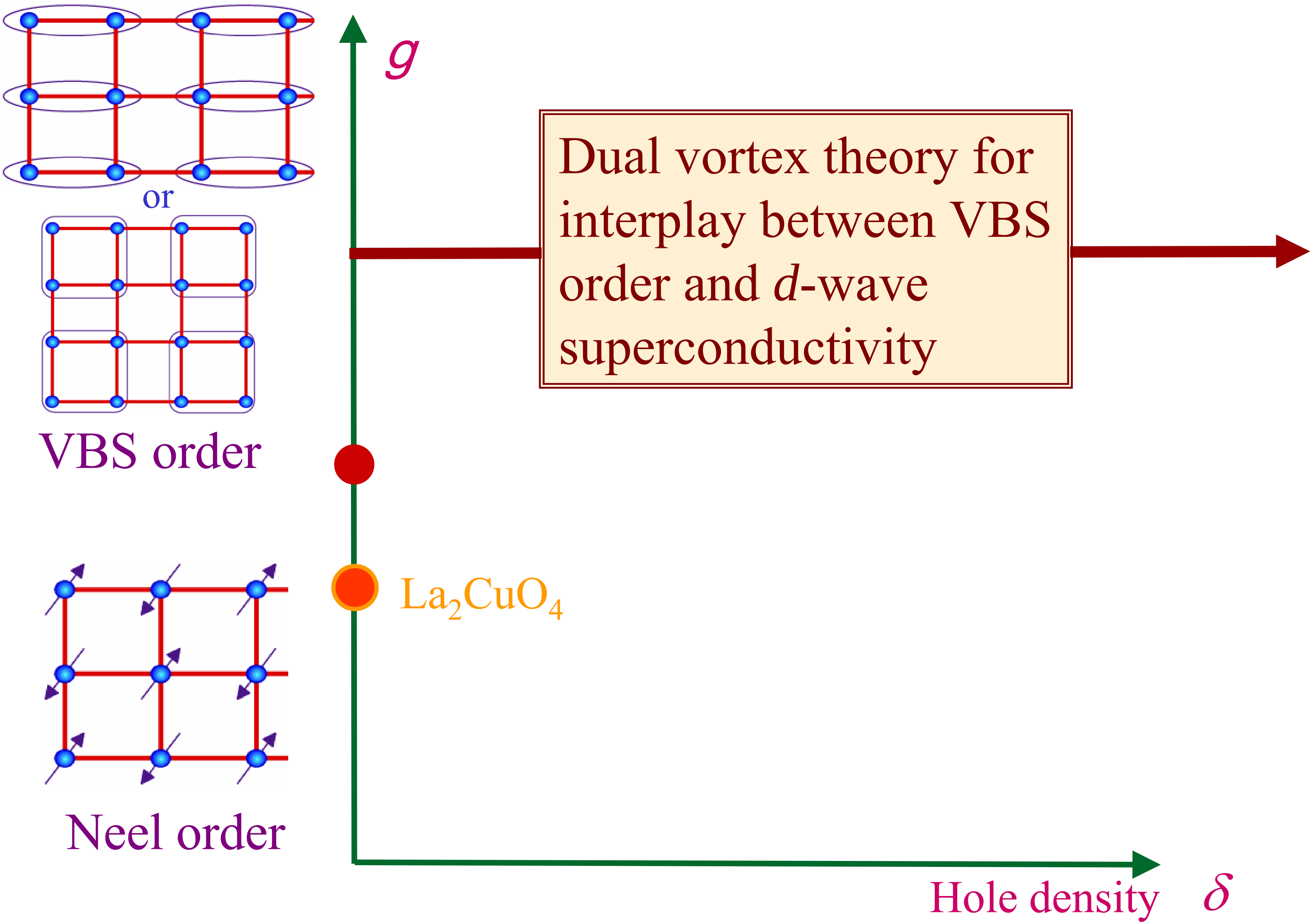
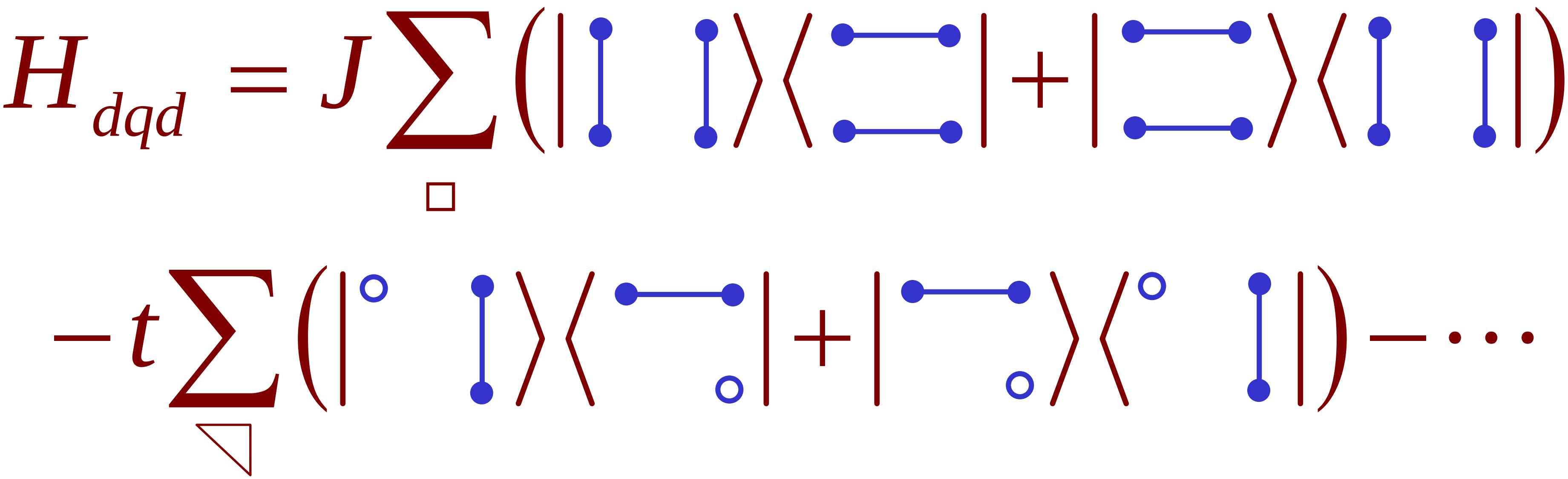


A convenient derivation of the dual theory for vortices is obtained from the doped quantum dimer model



Density of holes = δ

E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett.* B **4**, 225 (1990).

Duality mapping of doped dimer model shows:

Vortices in the superconducting state obey the magnetic translation algebra

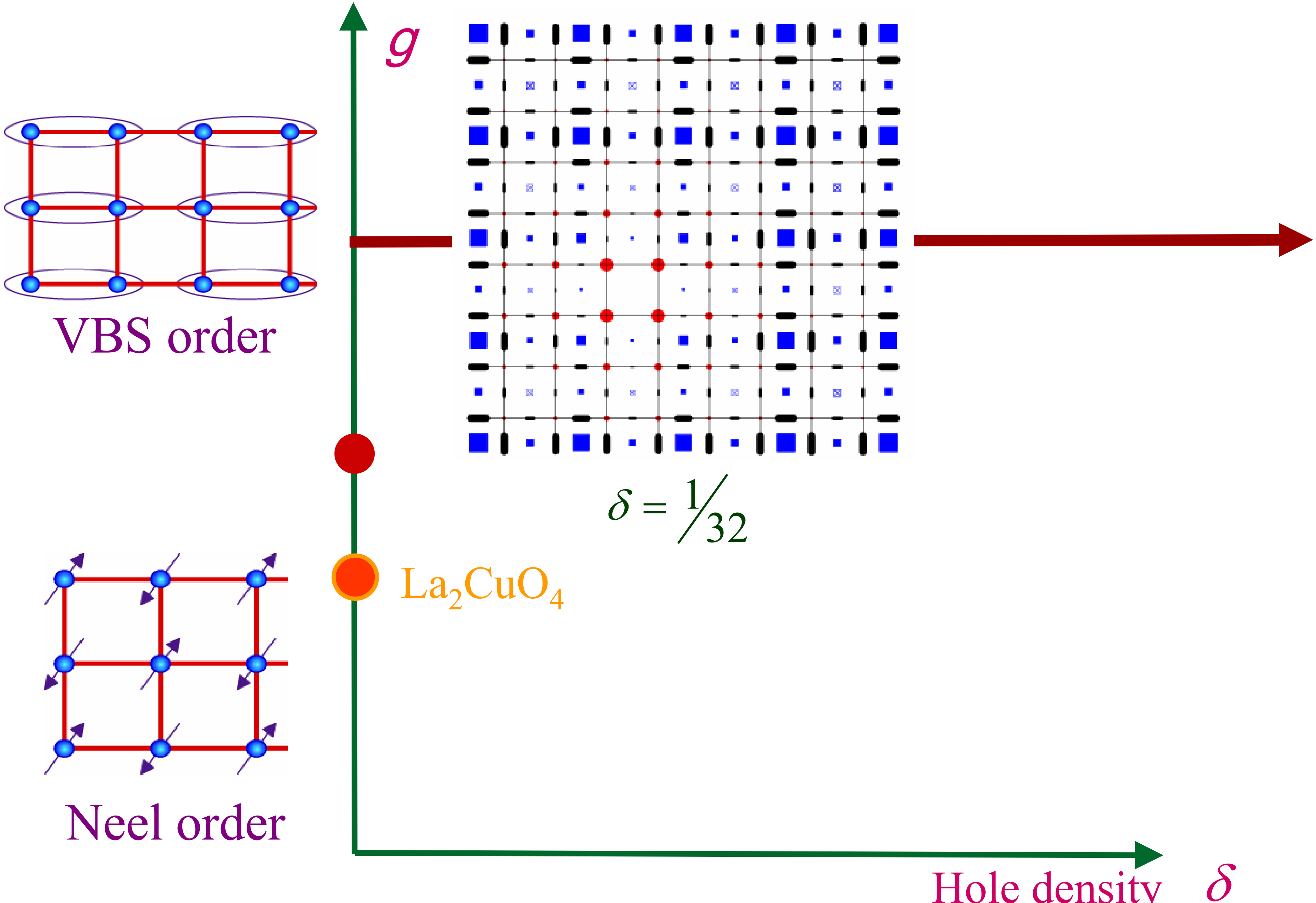
$$T_x T_y = e^{2\pi i f} T_y T_x$$

$$\text{with} \quad f = \frac{p}{q} = \frac{1-\delta_{MI}}{2}$$

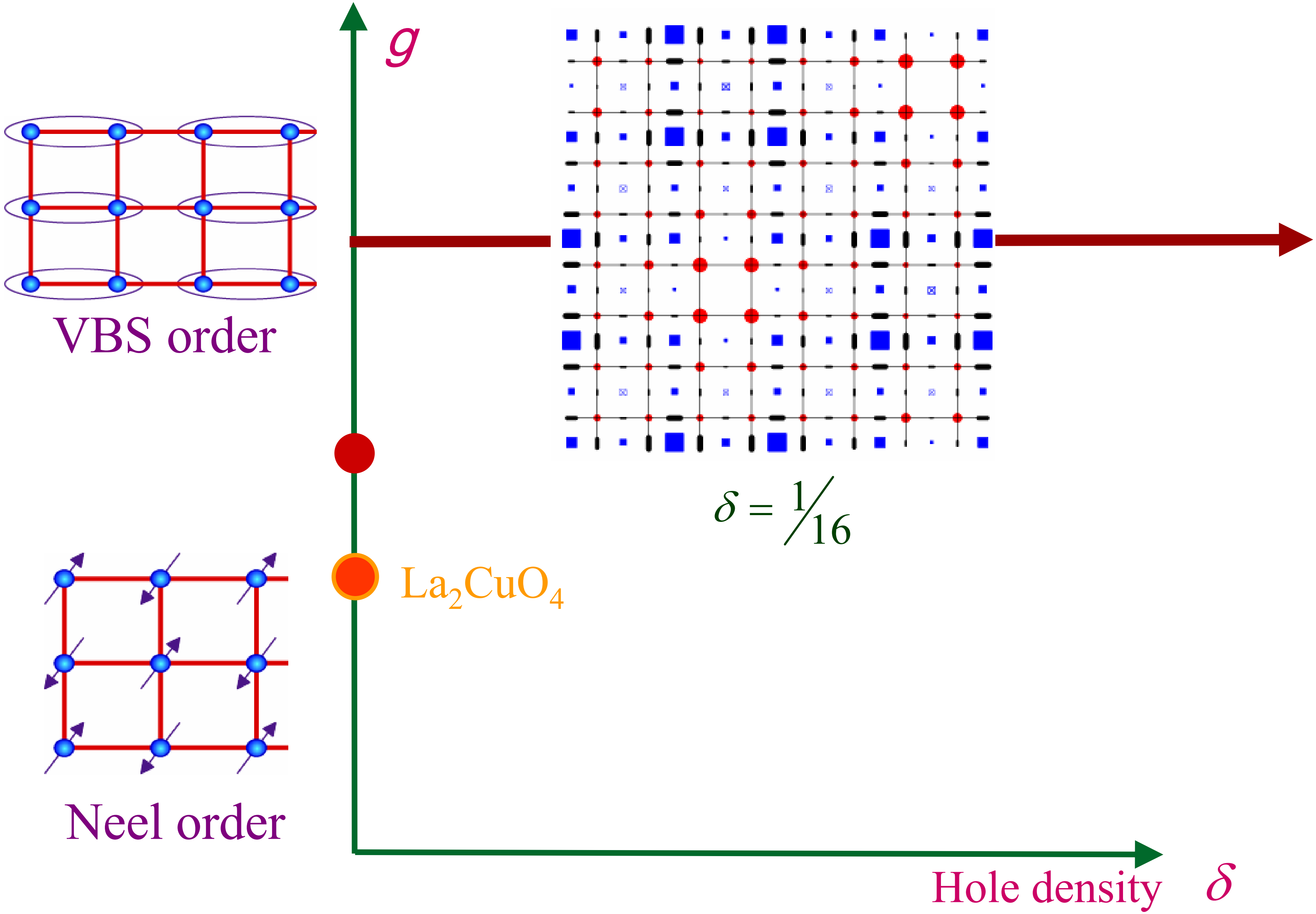
where δ_{MI} is the density of holes in the proximate Mott insulator (for $\delta_{MI} = 1/8,\ f = 7/16 \Rightarrow q = 16$)

Most results of Part A on bosons can be applied unchanged with q as determined above

Phase diagram of doped antiferromagnets



Phase diagram of doped antiferromagnets



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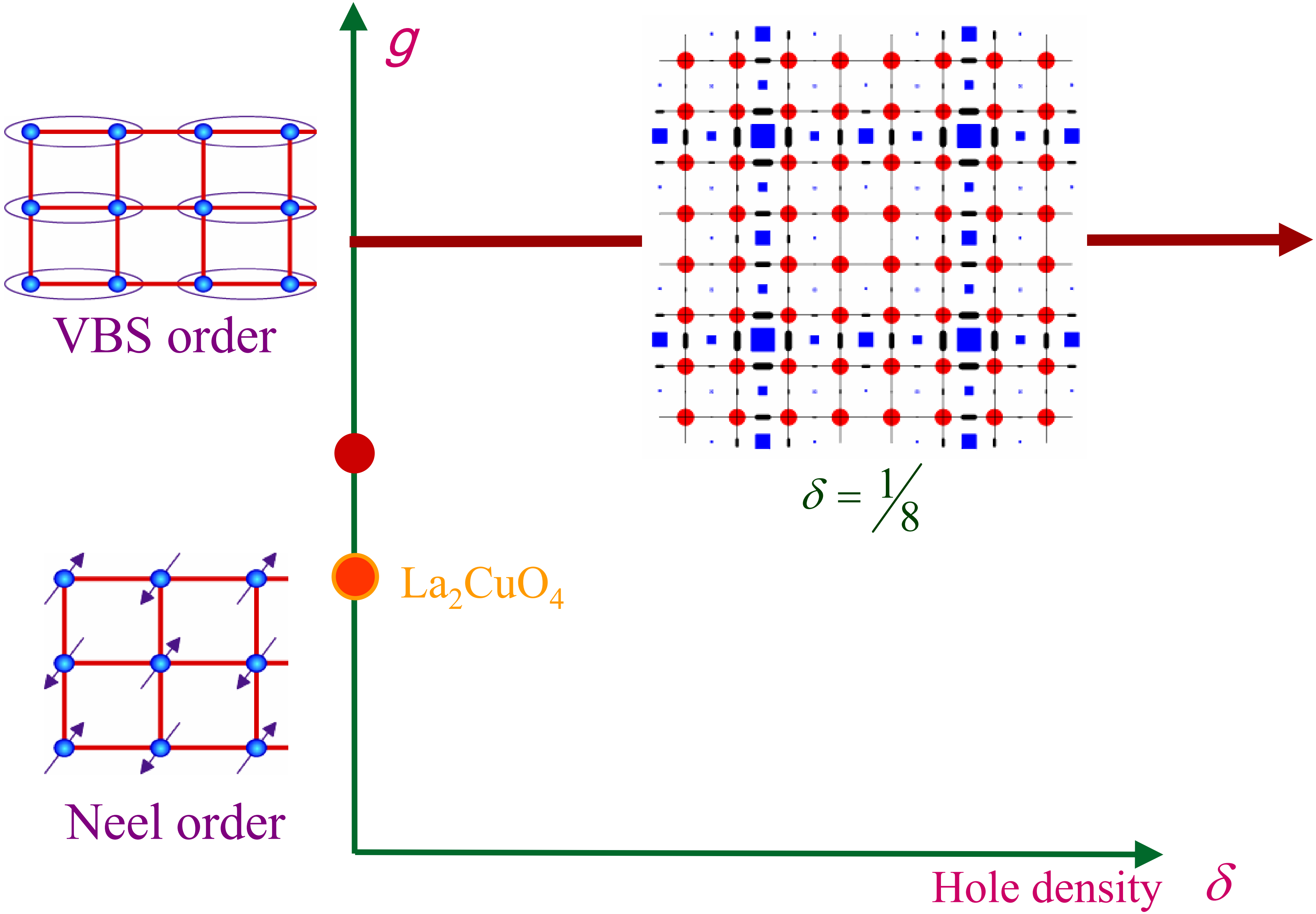

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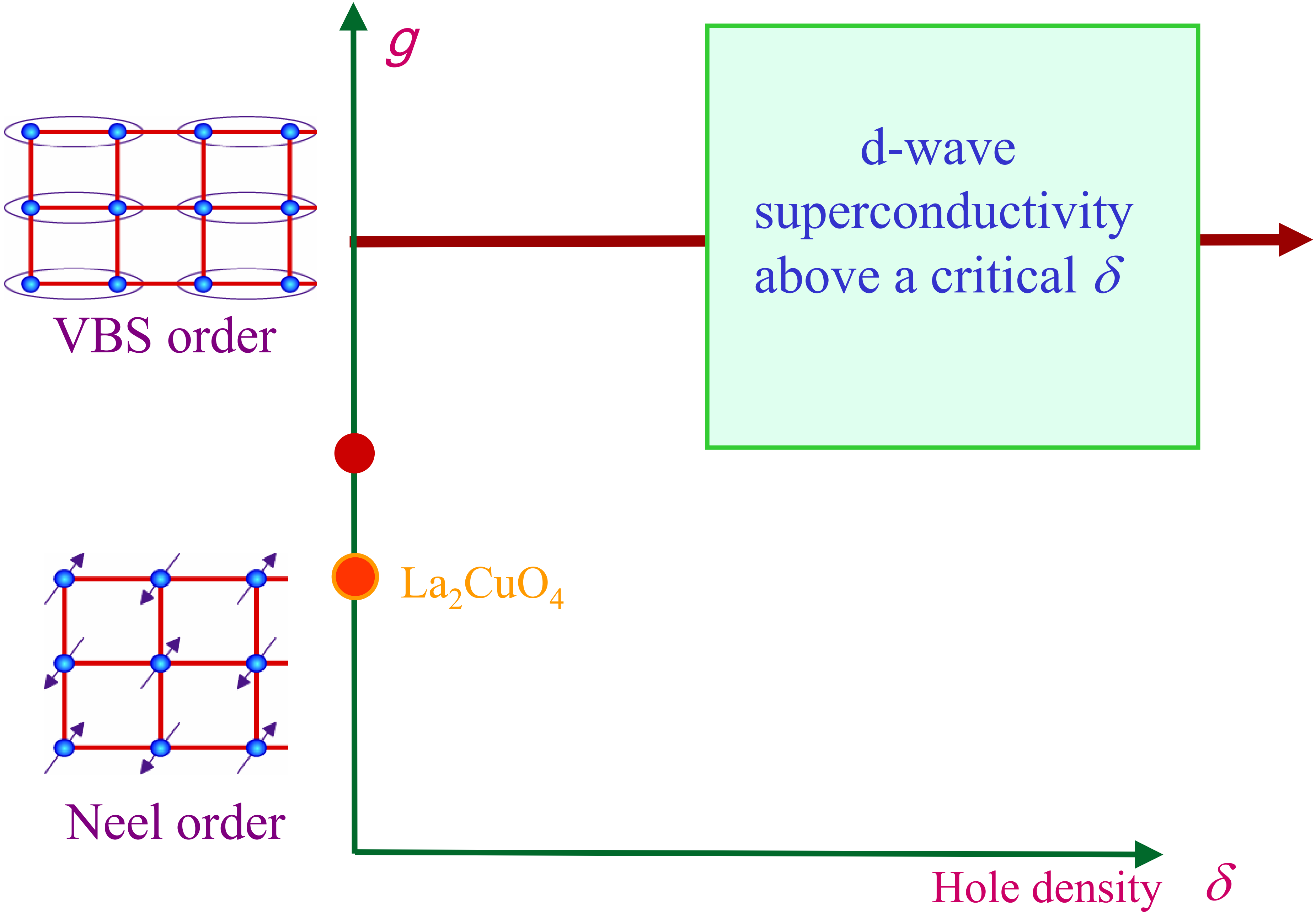

Conclusions

I. Description of the competition between superconductivity and density wave order in term of defects (vortices). Theory naturally excludes "disordered" phase with no order.

II. Vortices carry the quantum numbers of *both* superconductivity *and* the square lattice space group (in a projective representation).

III. Vortices carry halo of charge order, and pinning of vortices/anti-vortices leads to a unified theory of STM modulations in zero and finite magnetic fields.

IV. Conventional (LGW) picture: density wave order causes the transport energy gap, the appearance of the Mott insulator.
Present picture: Mott localization of charge carriers is more fundamental, and (weak) density wave order emerges naturally in theory of the Mott transition.