Holography of compressible quantum phases

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Compressible quantum matter

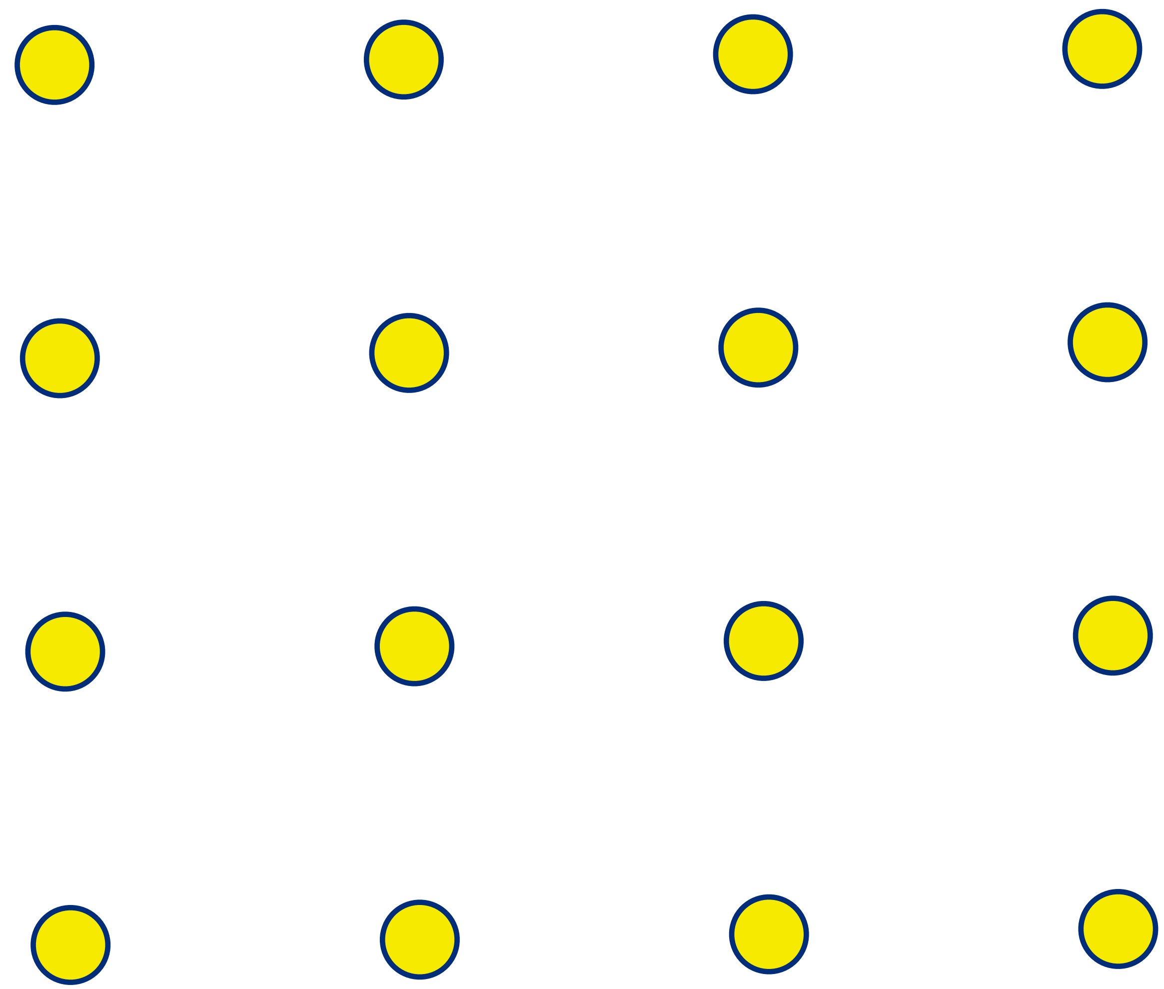
- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.

- Compressible systems must be gapless.

Compressible quantum matter

One compressible state is the **solid** (or “Wigner crystal” or “stripe”).
This state breaks translational symmetry.
Has integer number of particles per unit cell



Compressible quantum matter

Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

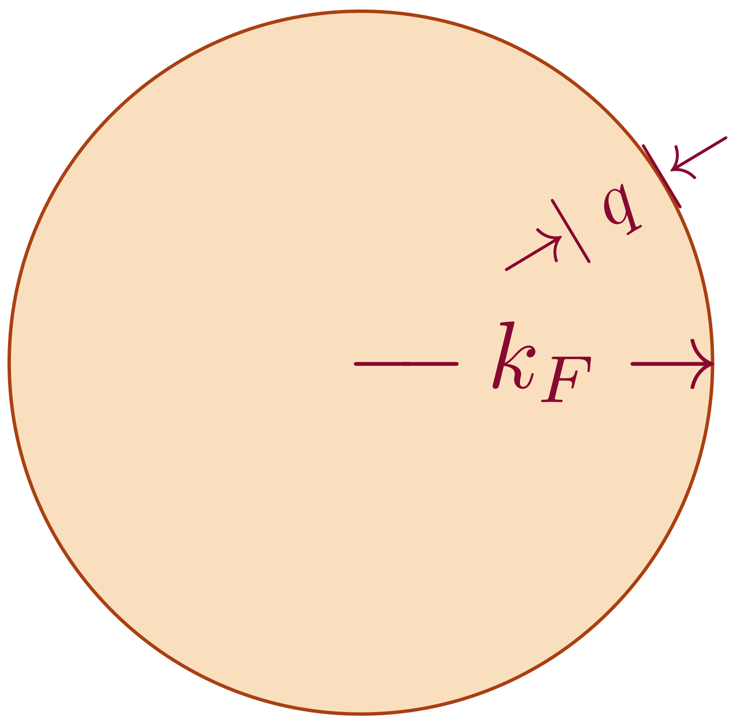
Compressible quantum matter

The only compressible phase of traditional condensed matter physics which does not break the translational or $\mathrm{U}(1)$ symmetries is the Landau Fermi liquid

The Fermi liquid

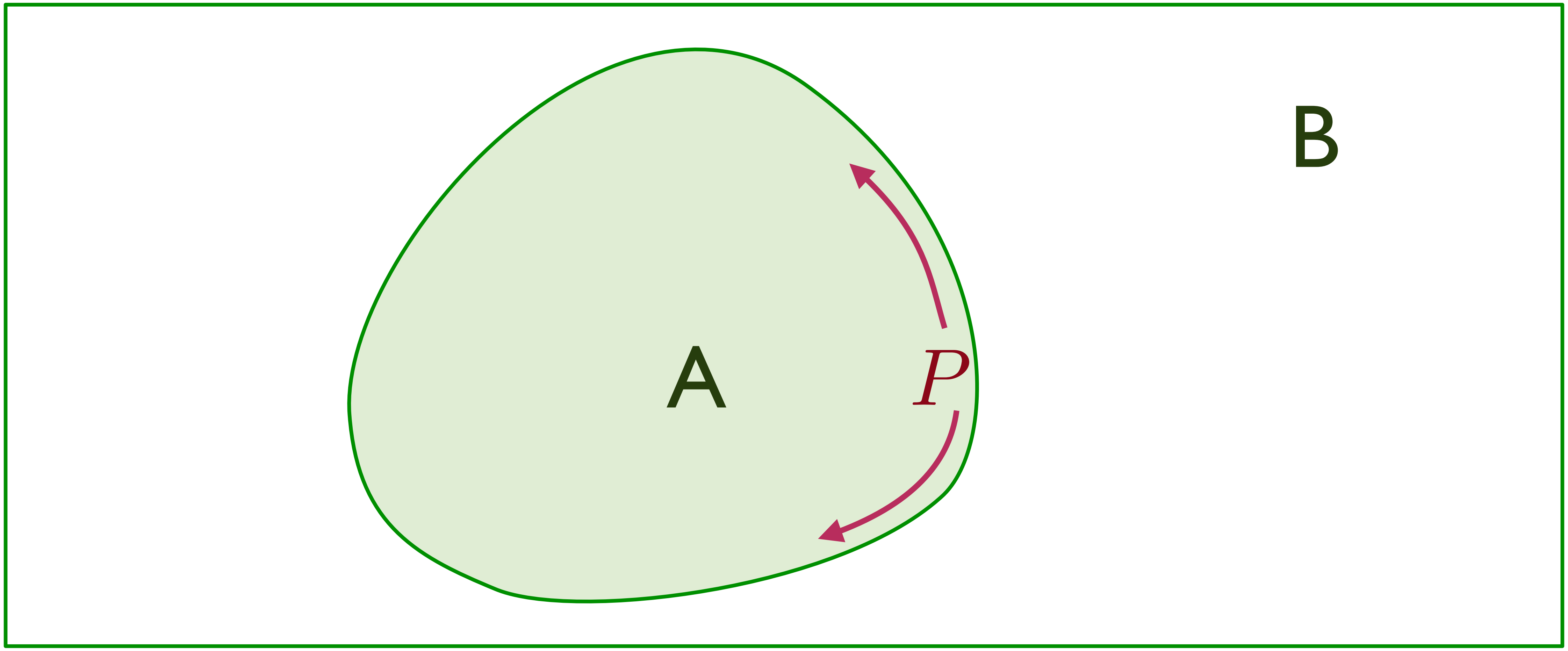
$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$



- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this is as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Entanglement entropy of the Fermi liquid



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor 1/12 is *universal*: it is independent of the shape of the entangling region, and of the strength of the interactions.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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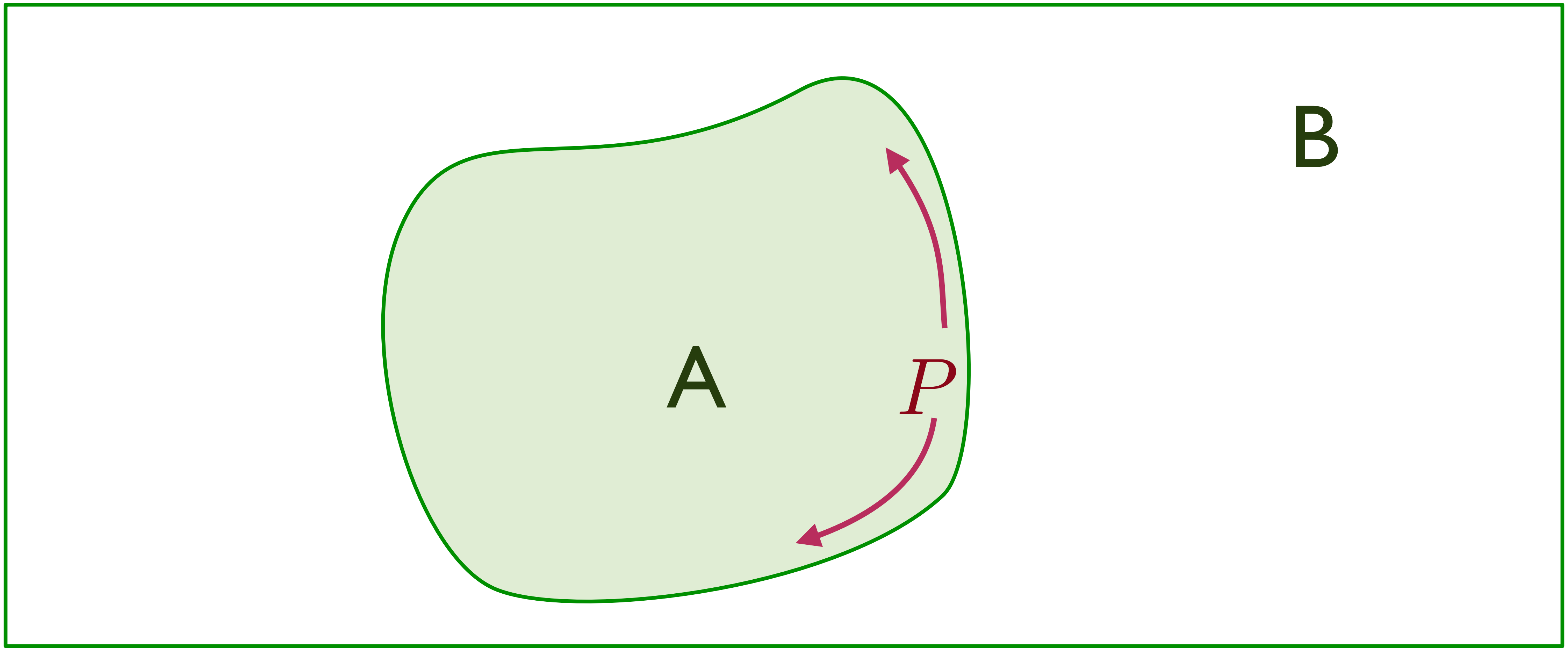

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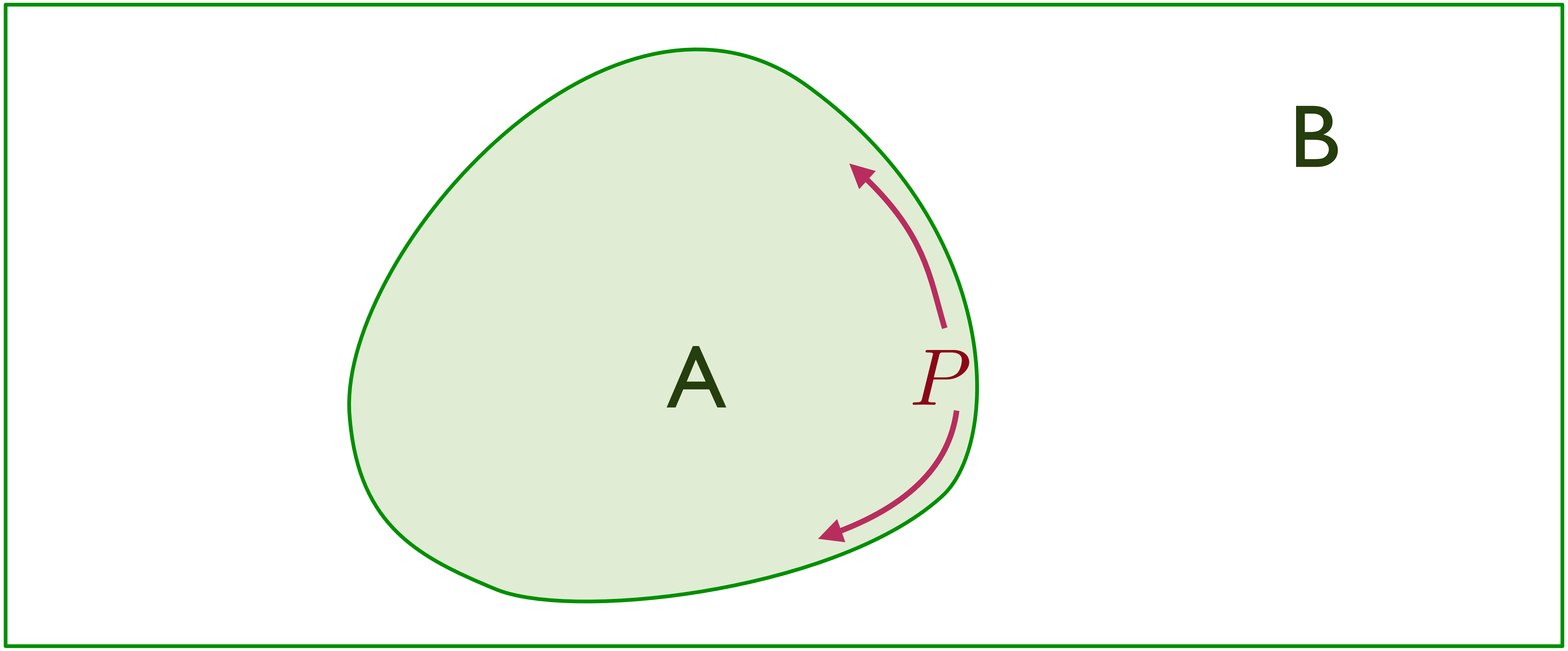

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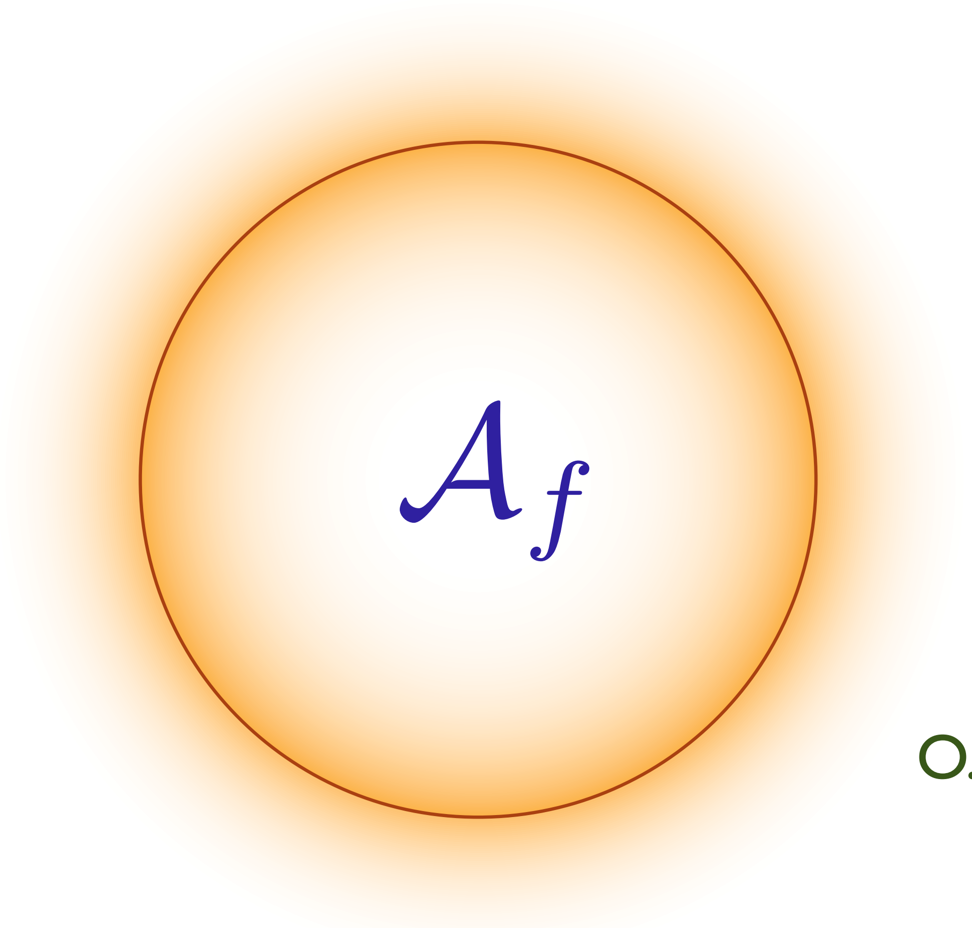
Bosons with correlated hopping

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$

- **NFL Bose metal:** We have the fractionalization $b \to f_1 f_2$, where the f_1, f_2 both form a Fermi surface. Both fermions are gauge-charged, and so the Fermi surfaces are partially "hidden".

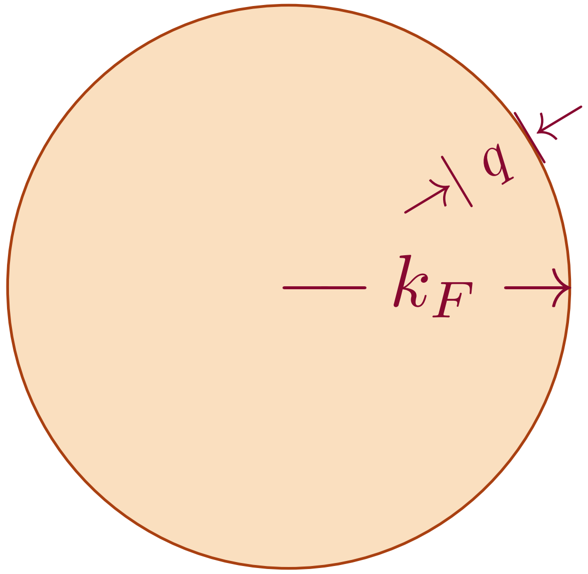


$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Phys. Rev.* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Phys. Rev.* D **84**, 026001 (2011)
S. Sachdev, arXiv:1209.1637

FL Fermi liquid

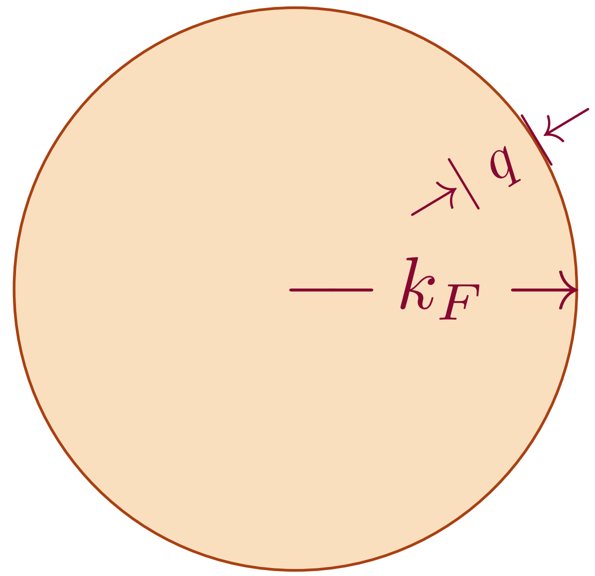
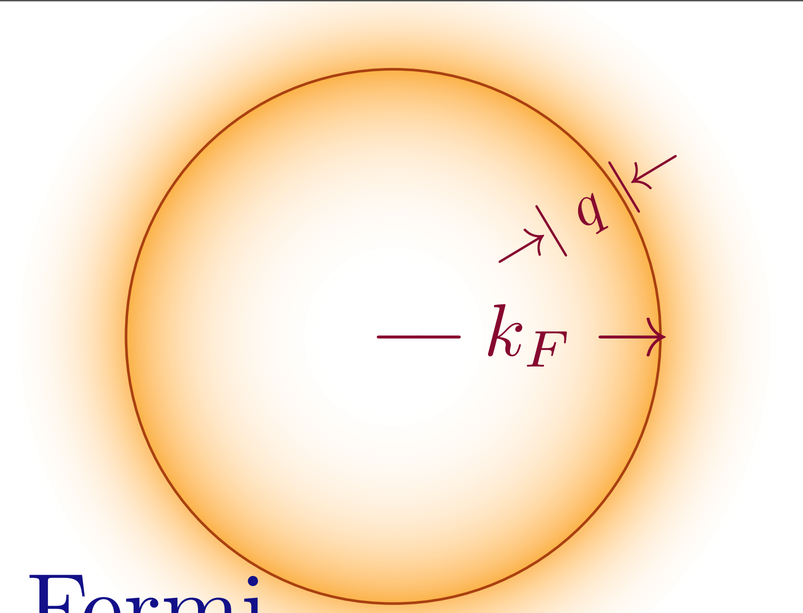


- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL Fermi liquid 	NFL Bose metal 
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \|q\|^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
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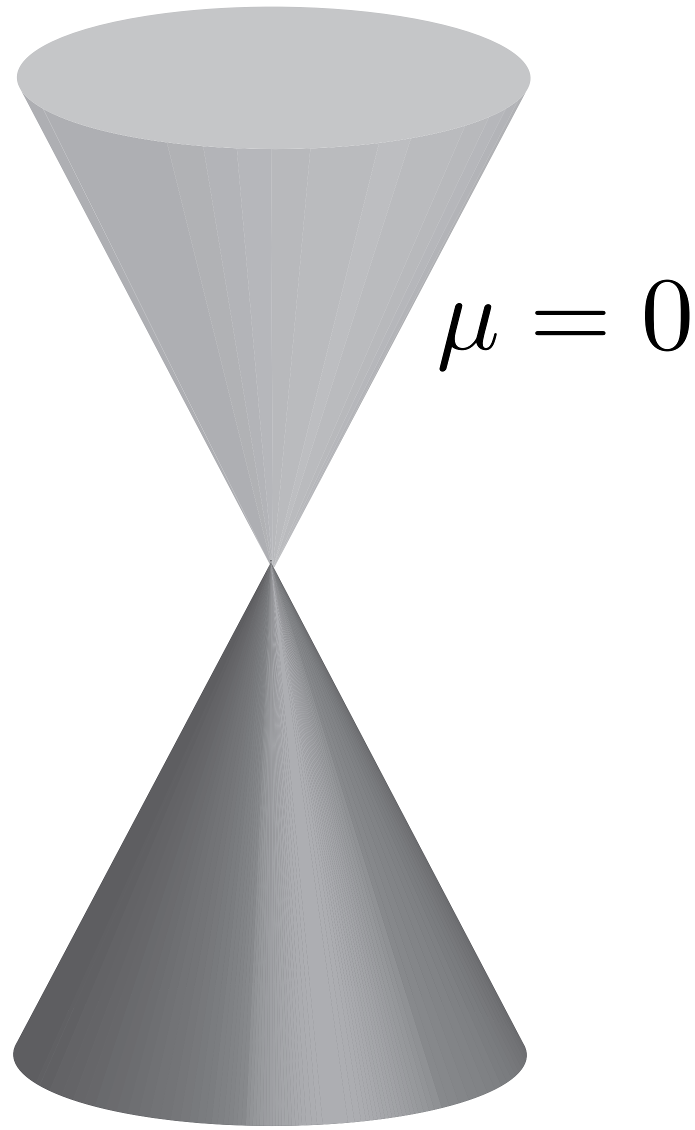
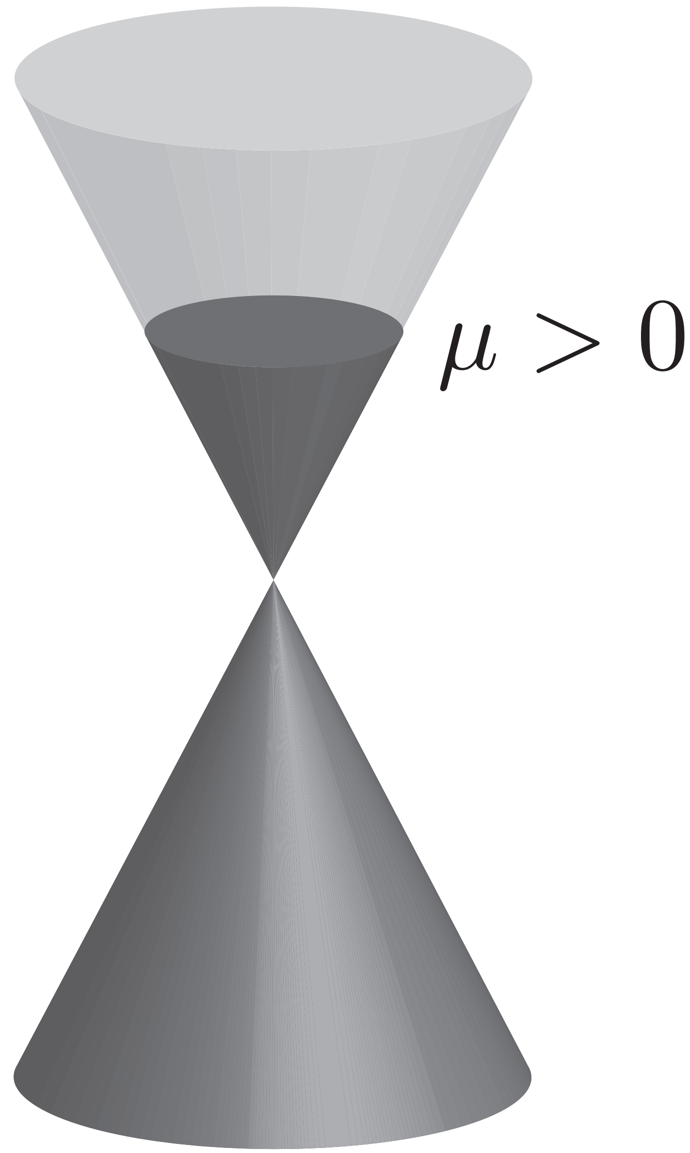
Start with a CFT



Turn on a chemical potential



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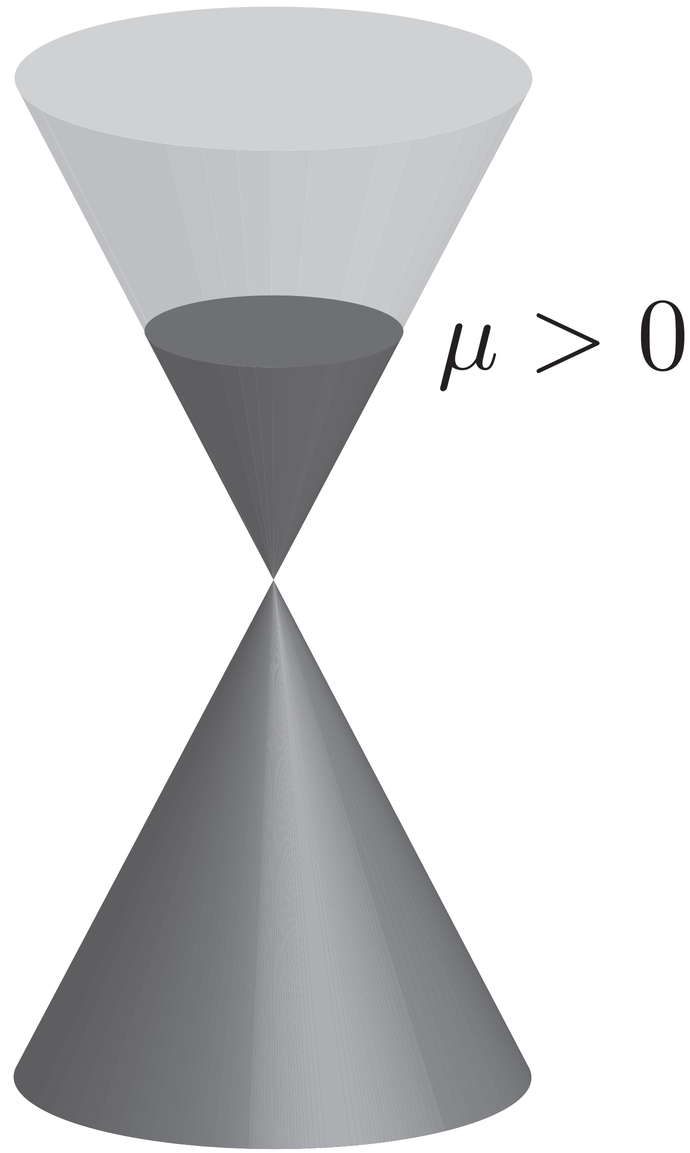
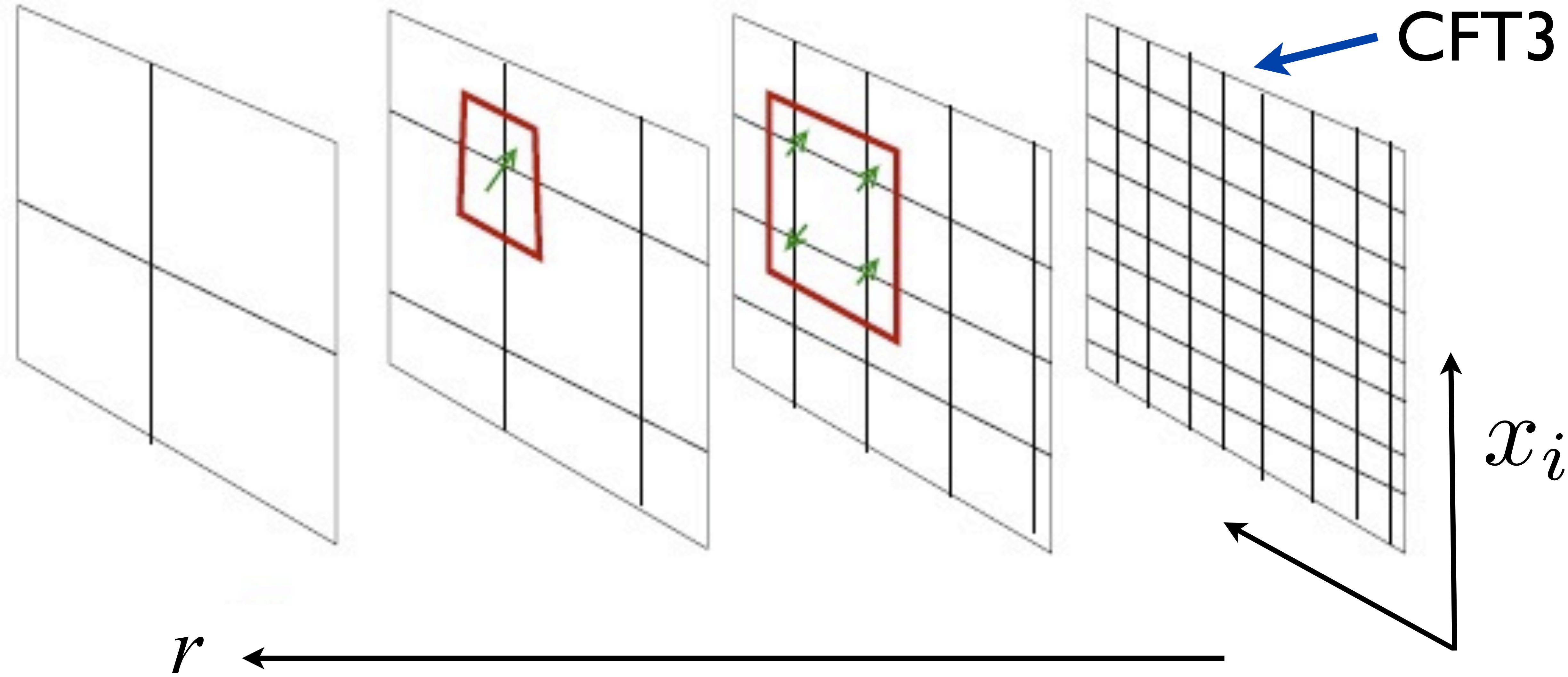


What happens when we start with strongly-coupled CFT ?

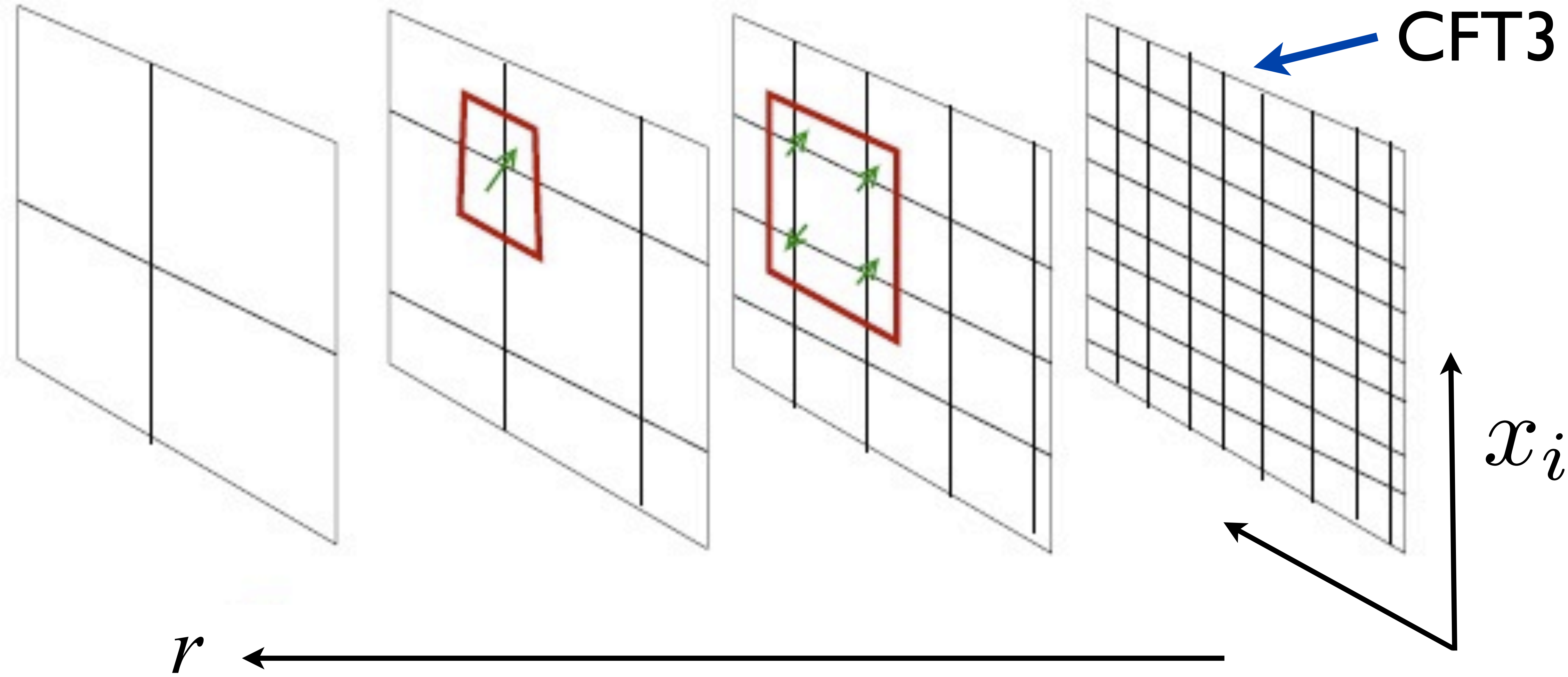
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

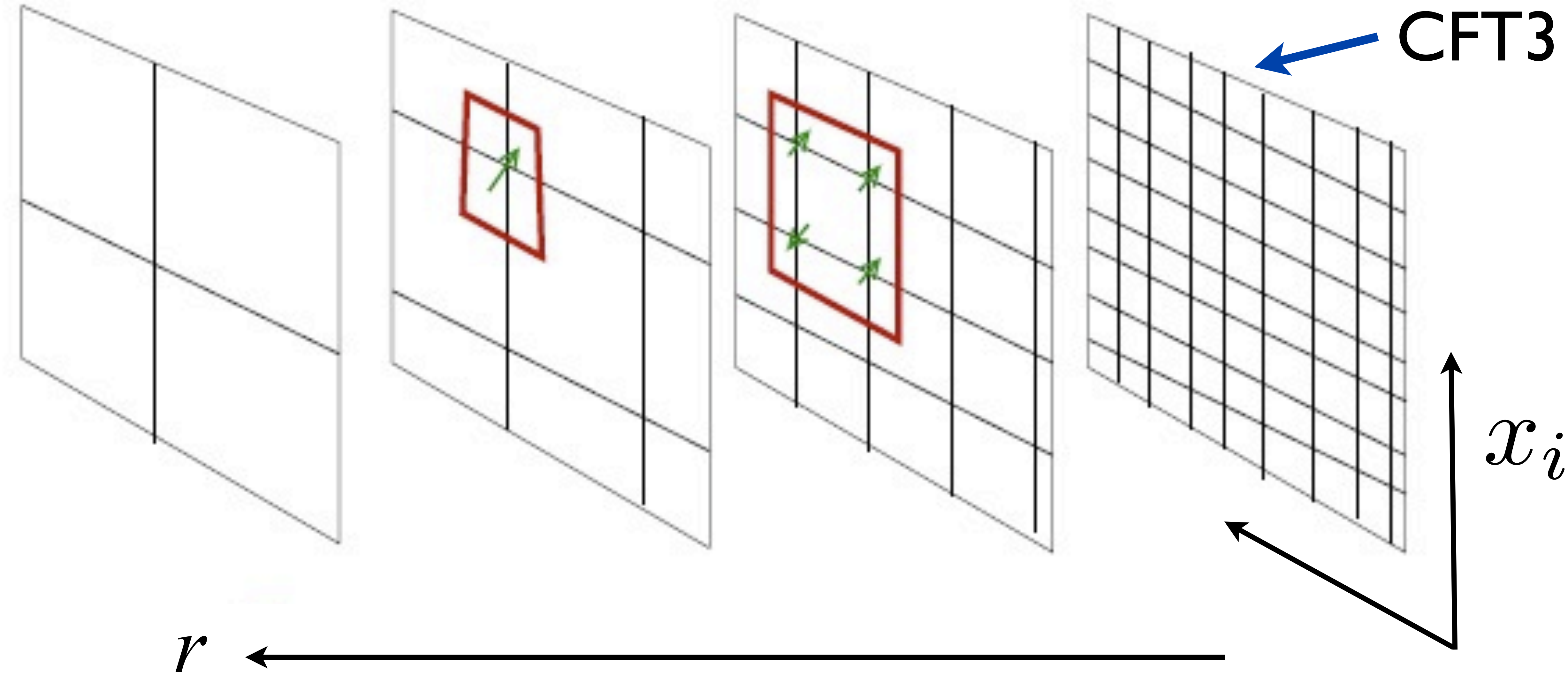


This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

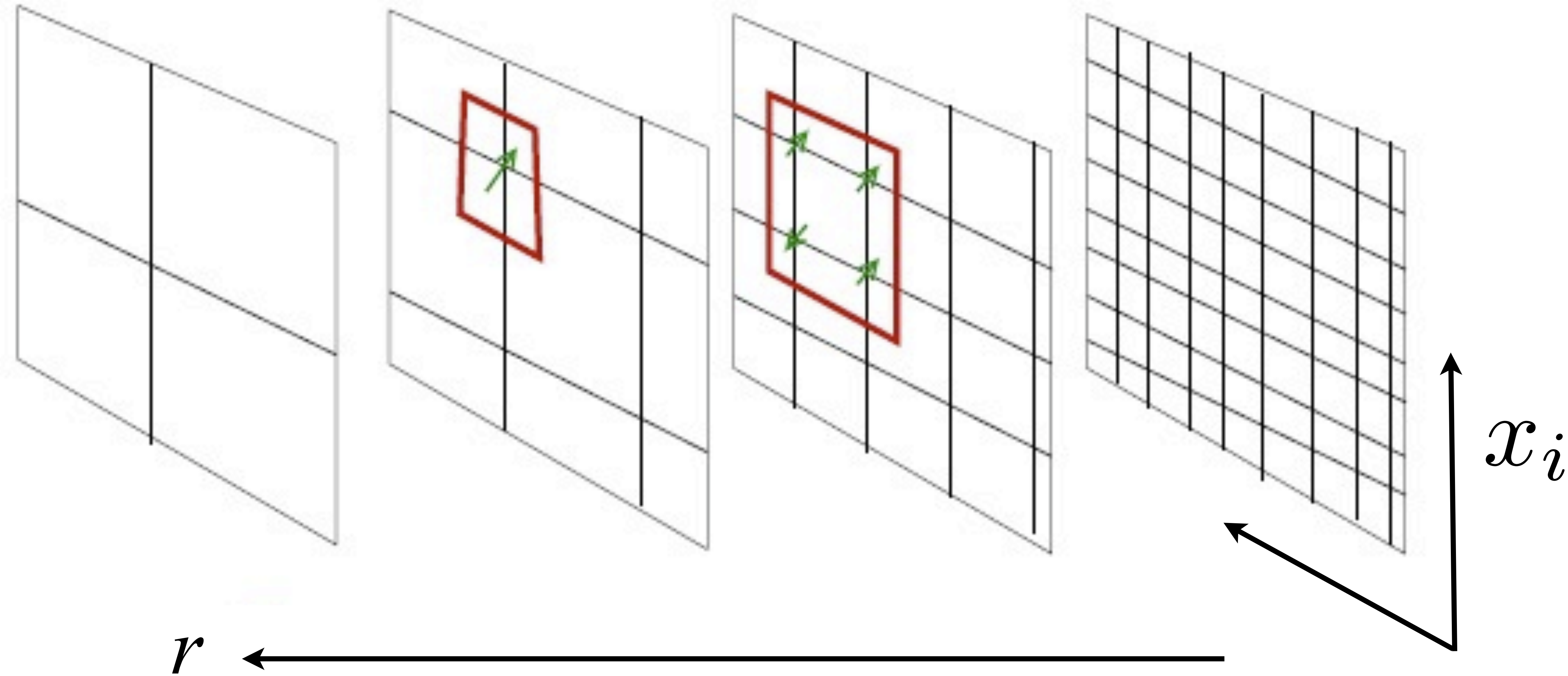
Holography



A conserved U(1) charge is represented by a bulk U(1) gauge flux $F = dA$.
A chemical potential corresponds to a boundary condition $A_t(r \to 0) = \mu$.

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) + \frac{1}{4g^2}F^2\right]$$

Holography

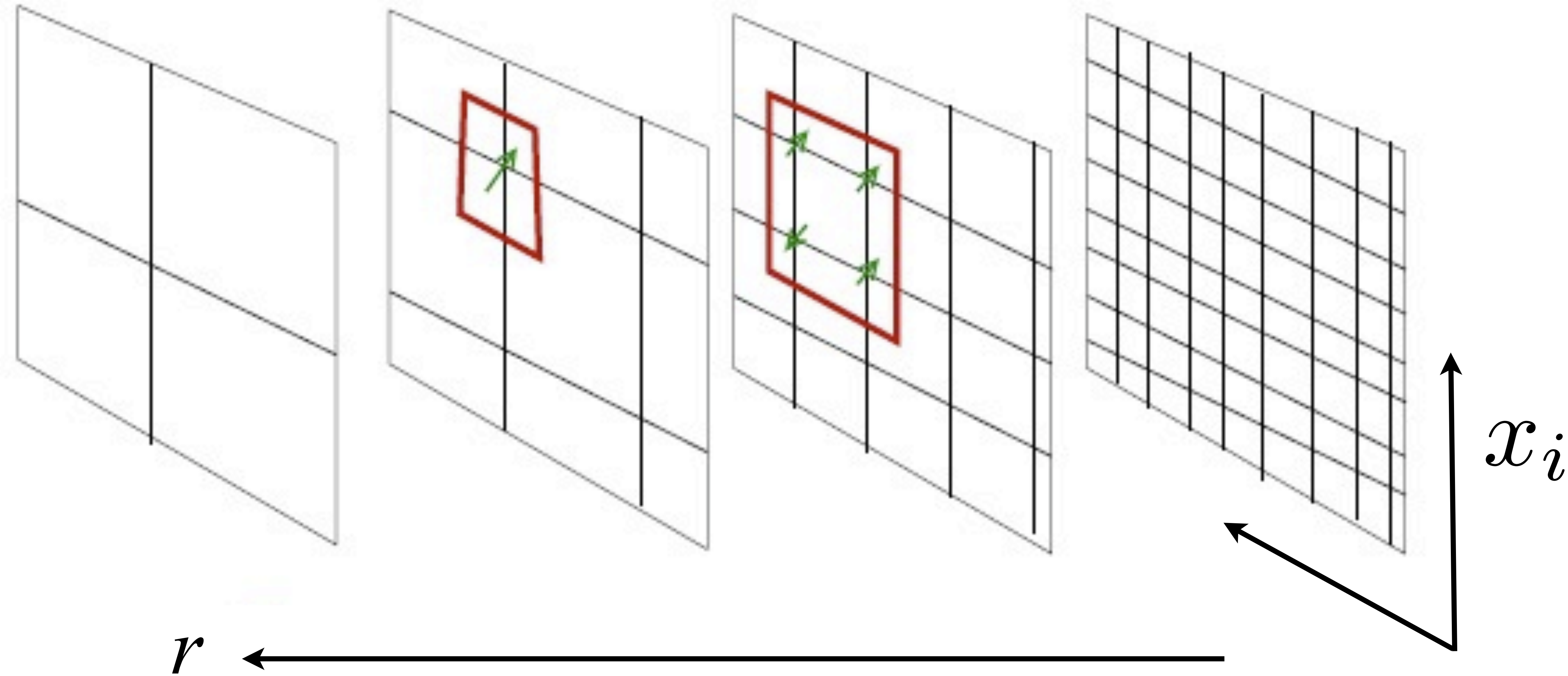


Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Holography



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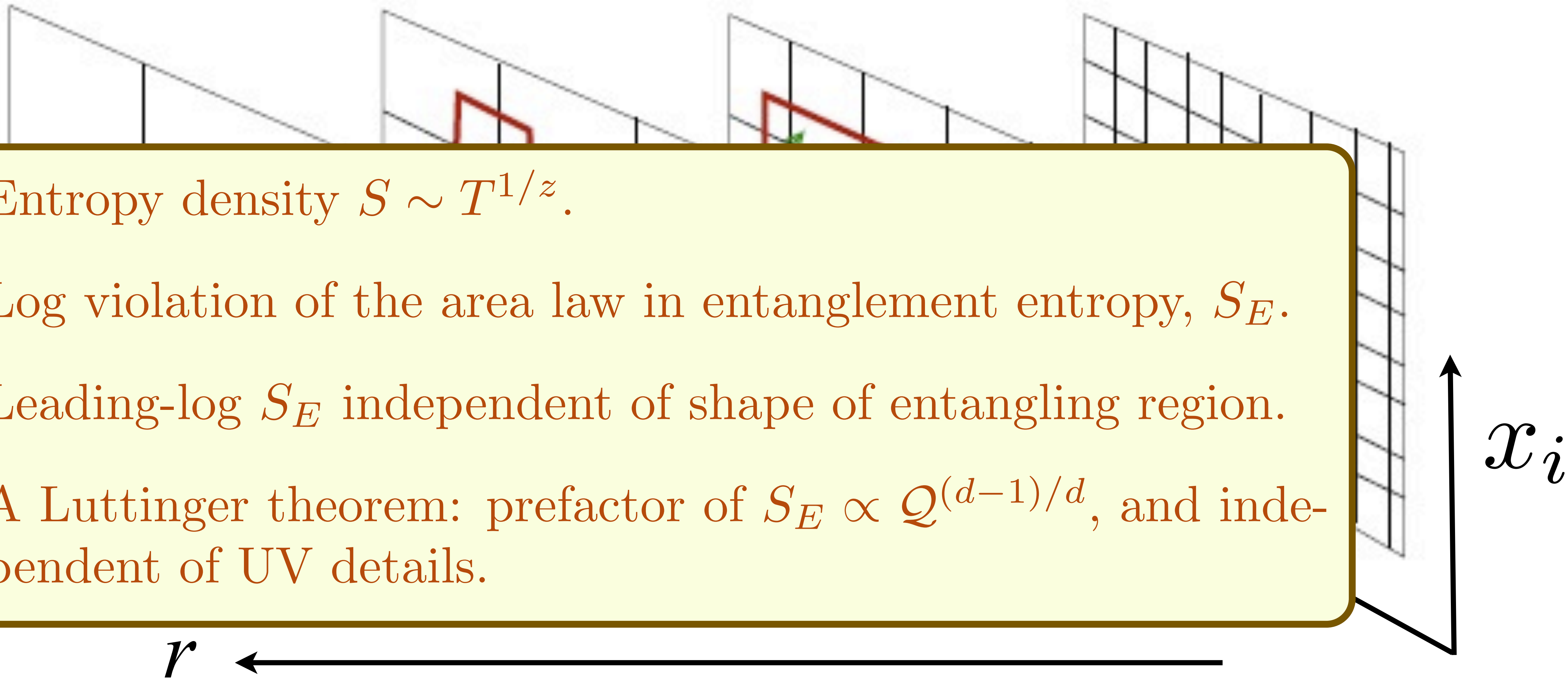
The value $\theta = d - 1$ reproduces *all* the essential characteristics of the entropy and entanglement entropy of a non-FL.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography



- Entropy density $S \sim T^{1/z}$.
- Log violation of the area law in entanglement entropy, S_E.
- Leading-log S_E independent of shape of entangling region.
- A Luttinger theorem: prefactor of $S_E \propto \mathcal{Q}^{(d-1)/d}$, and independent of UV details.

Consider a metric which transforms under rescaling as

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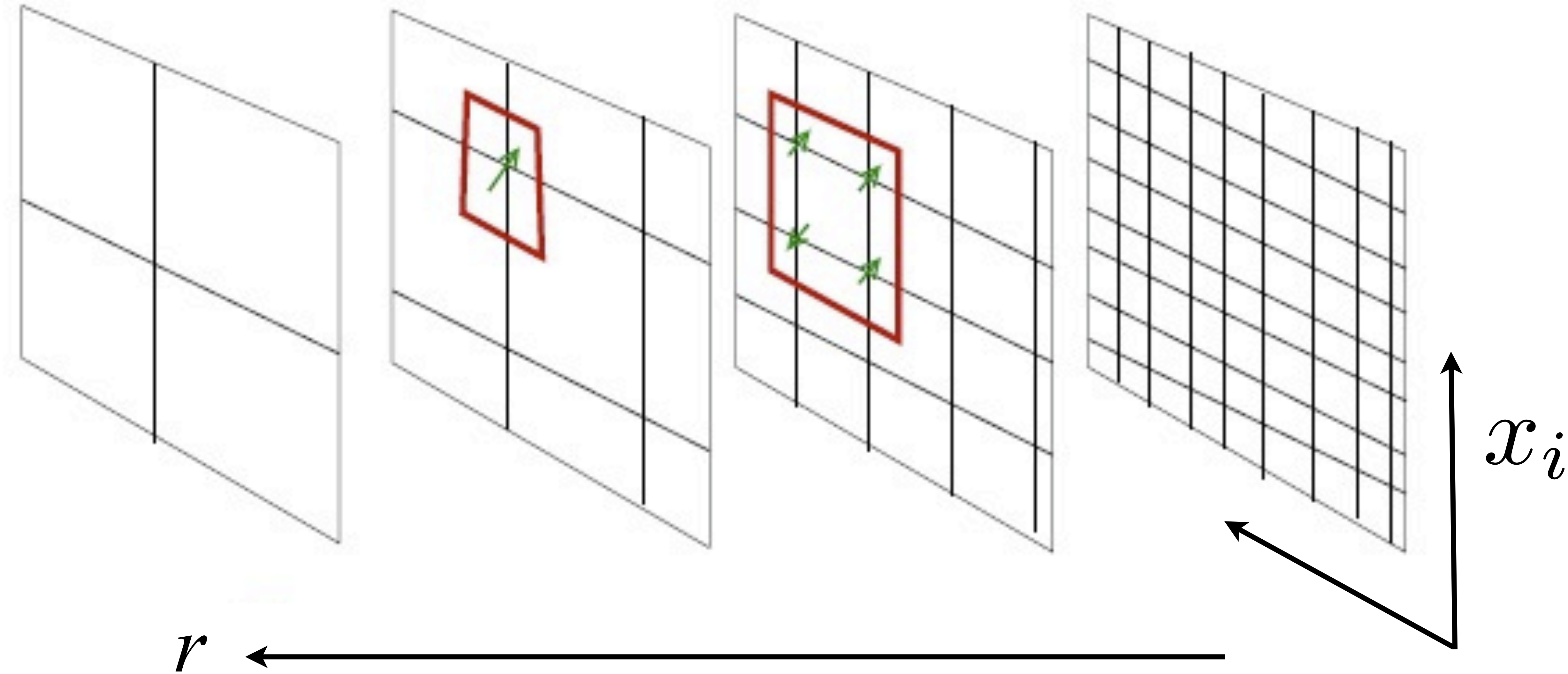

Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

The null-energy condition of gravity yields $z \geq 1 + \theta/d$. In $d = 2$, this leads to $z \geq 3/2$. Field theory on non-FL yields $z = 3/2$ to 3 loops!

P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
B. Blok and H. Monien, Phys. Rev. B **47**, 3454 (1993)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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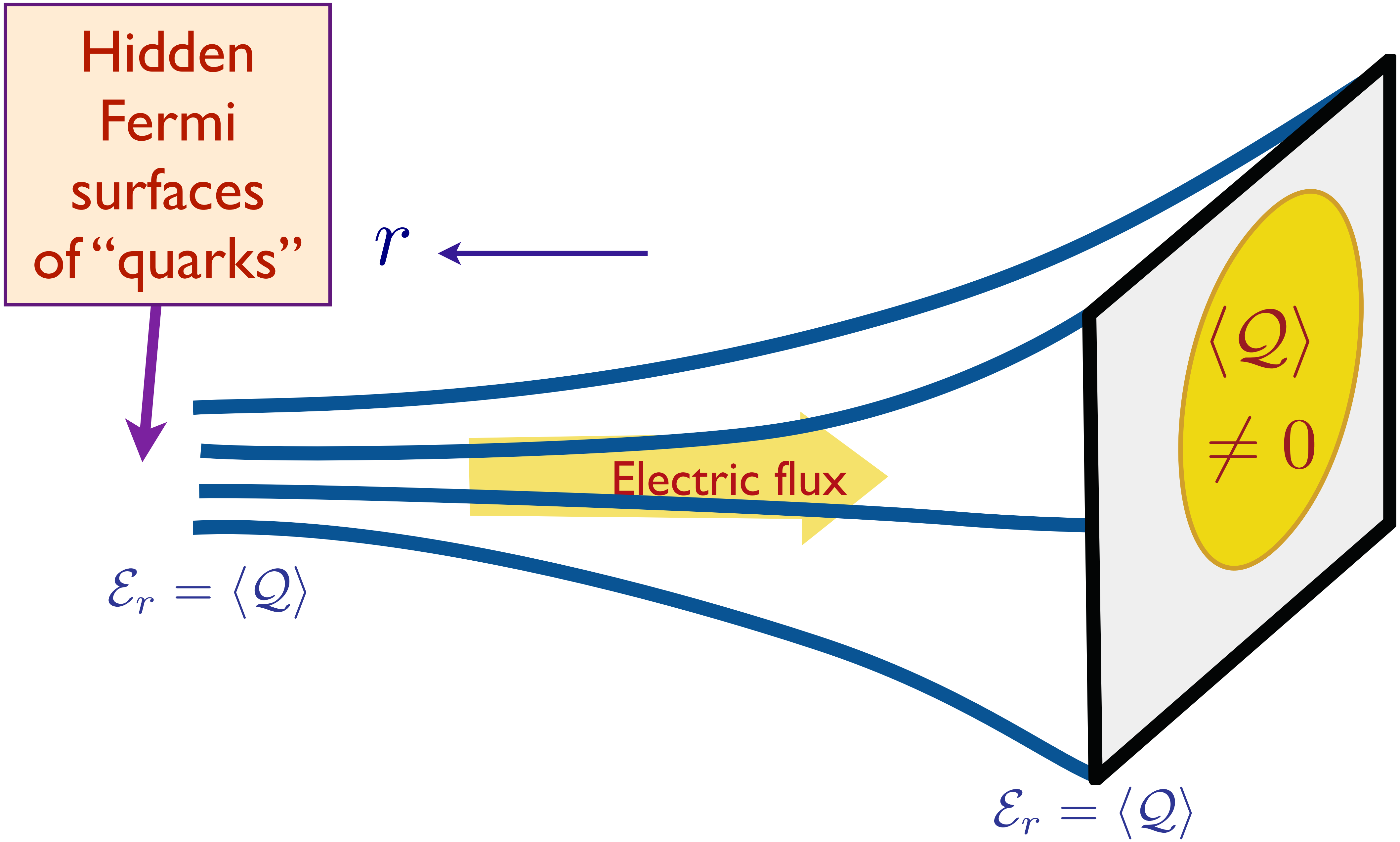
Holography of a non-Fermi liquid



This is a "bosonization" of the *hidden* Fermi surface

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension *d=1*

Monopoles in the 2+1 dimensional bulk U(1) gauge field acquire a Berry phase determined by the boundary U(1) charge density $\mathcal{Q}$, and a dilute gas theory of monopoles leads to Friedel oscillations with

$$\langle \rho(x)\rho(0)\rangle \sim \frac{\cos(2k_F x)}{|x|^{2\Delta_F}}$$

T. Faulkner and N. Iqbal, arXiv:1207.4208

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension d=2

- For every CFT in 2+1 dimensions with a globally conserved U(1), we can define a monopole operator which transforms as a scalar under conformal transformations.
  e.g. for the XY model, we insert a monopole at x_m by including a *fixed* background gauge flux α_μ so that

$$\mathcal{L} = |(\partial_\mu - i\alpha_\mu)\psi|^2 + s|\psi|^2 + u|\psi|^4$$

where the flux $\beta_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu\alpha_\lambda$ obeys

$$\partial_\mu\beta_\mu = 2\pi\delta(x - x_m) \quad , \quad \epsilon_{\mu\nu\lambda}\partial_\nu(\Omega\beta_\nu) = 0$$

where the CFT lives on the conformally flat space with is $ds^2 = \Omega^{-2}dx_\mu^2$.

S. Sachdev, arXiv:1209.1637

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension *d=2*

- In the holographic theory, we have a bulk scalar field Φ_m (conjugate to the monopole operator of the CFT) which carries the charge of the S-dual of the 4-dimensional bulk U(1) gauge field:

$$\mathcal{S}_m = \int d^4x\sqrt{-g}\left[|(\nabla - 2\pi i\widetilde{A})\Phi_m|^2 + \ldots\right]$$

where $\widetilde{F} = d\widetilde{A} = *F = *dA$.

S. Sachdev, arXiv:1209.1637

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension d=2

- When a chemical potential is applied to the boundary CFT, Φ_m experiences a magnetic flux. Consequently condensation of Φ_m leads to a vortex-lattice-like state, which corresponds to the formation of a *crystal* in the CFT. *The crystal has unit $\mathcal{Q}$ charge per unit cell.*

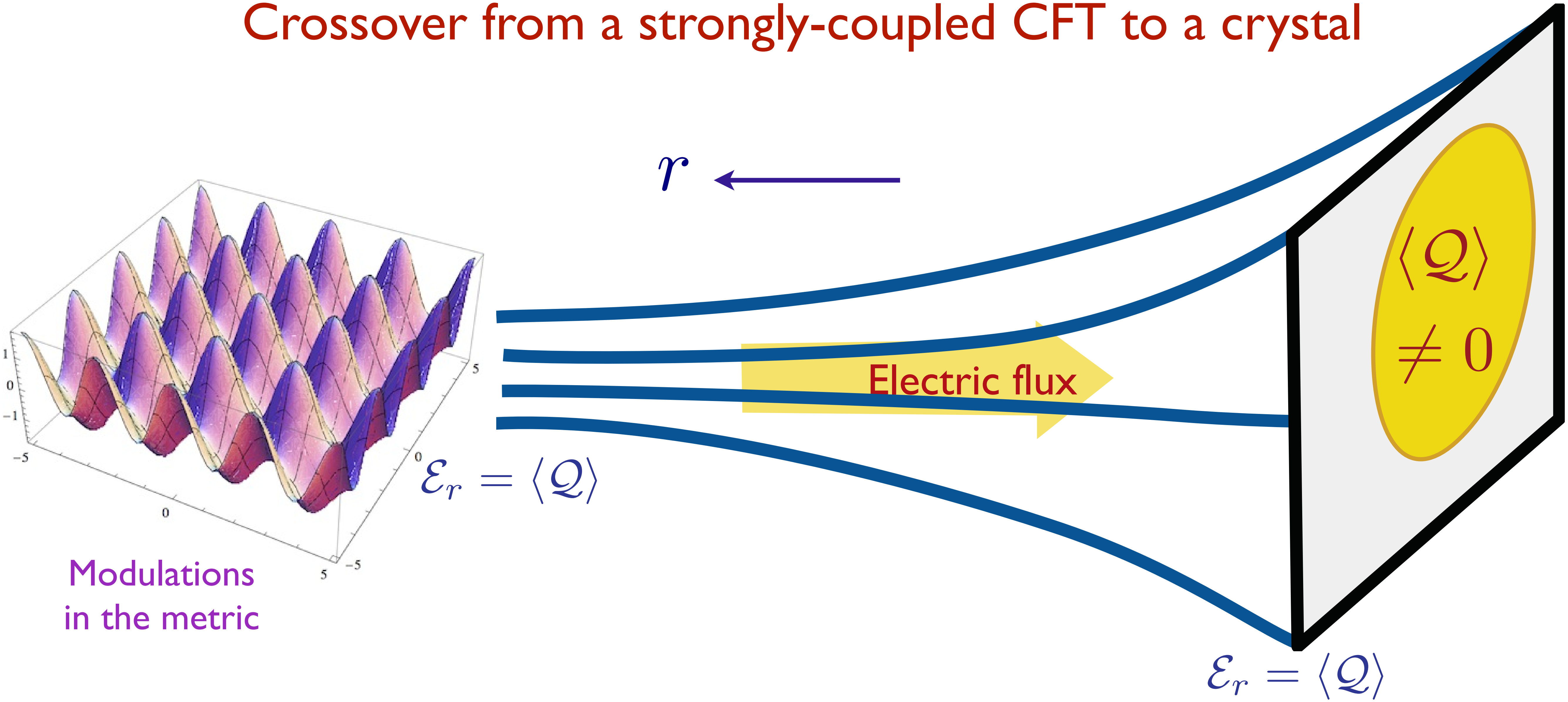
- We expect that a vortex-liquid-like state of the Φ_m will yield the Friedel oscillations of the Fermi surface, with the correct Fermi wavevector.

S. Sachdev, arXiv:1209.1637

N. Bao, S. Harrison, S. Kachru, and S. Sachdev, arXiv:1303.4390

Holography of a crystal

Crossover from a strongly-coupled CFT to a crystal



The modulations in the metric *grow* in the IR, as $r \to \infty$ (the electric flux is dual to a magnetic flux, and the latter cannot be screened by any bulk matter fields). This growth likely implies a "fragmentation" of space. Each lattice site is like a Kondo impurity with localized quantum degrees of freedom.

N. Bao, S. Harrison, S. Kachru, and S. Sachdev, arXiv:1303.4390