The spin density wave quantum phase transition in two-dimensional metals

APS meeting, March 24, 2011

Talk online: sachdev.physics.harvard.edu

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Max Metlitski, Harvard

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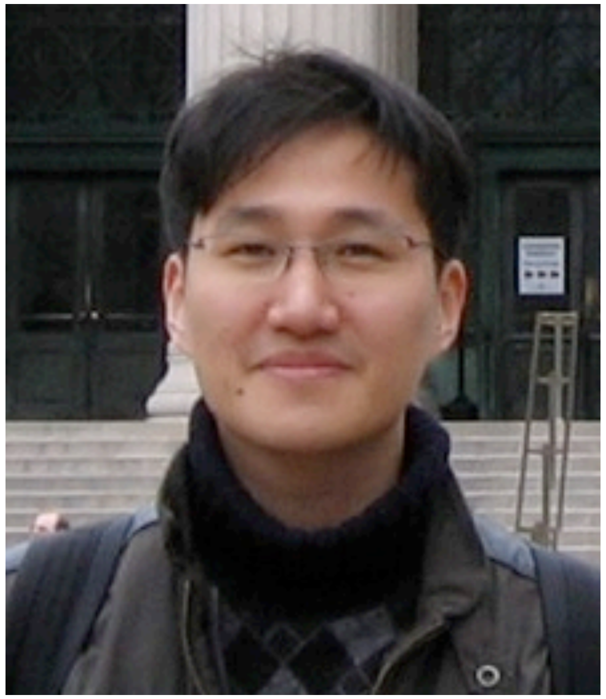

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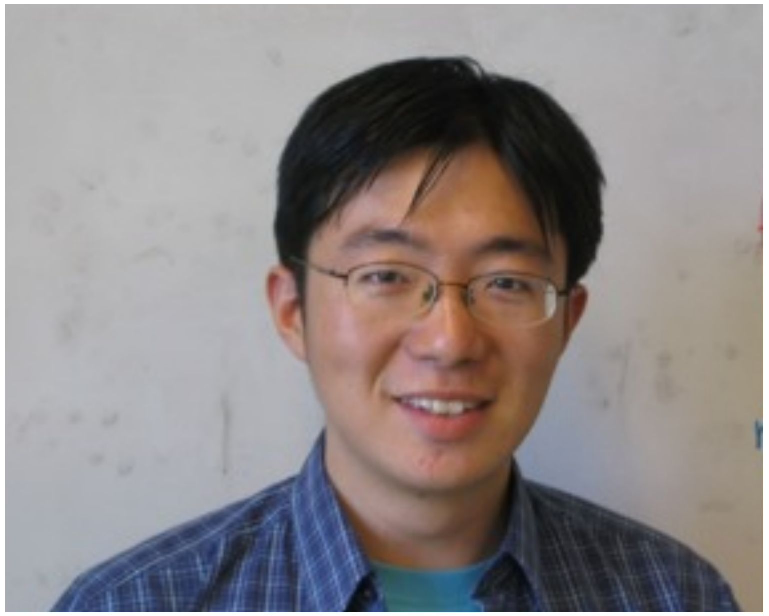

Cenke Xu
UCSB

Yang Qi
Tsinghua

Outline

1. Low energy theory of spin density wave quantum critical point

2. Instabilities near the quantum critical point:
   A. Unconventional superconductivity
   B. $2k_F$ bond-nematic ordering

3. Phase diagram in a magnetic field
   Electron versus hole doping

4. Possible exotic intermediate phases
   Fermi pockets without spin density wave order

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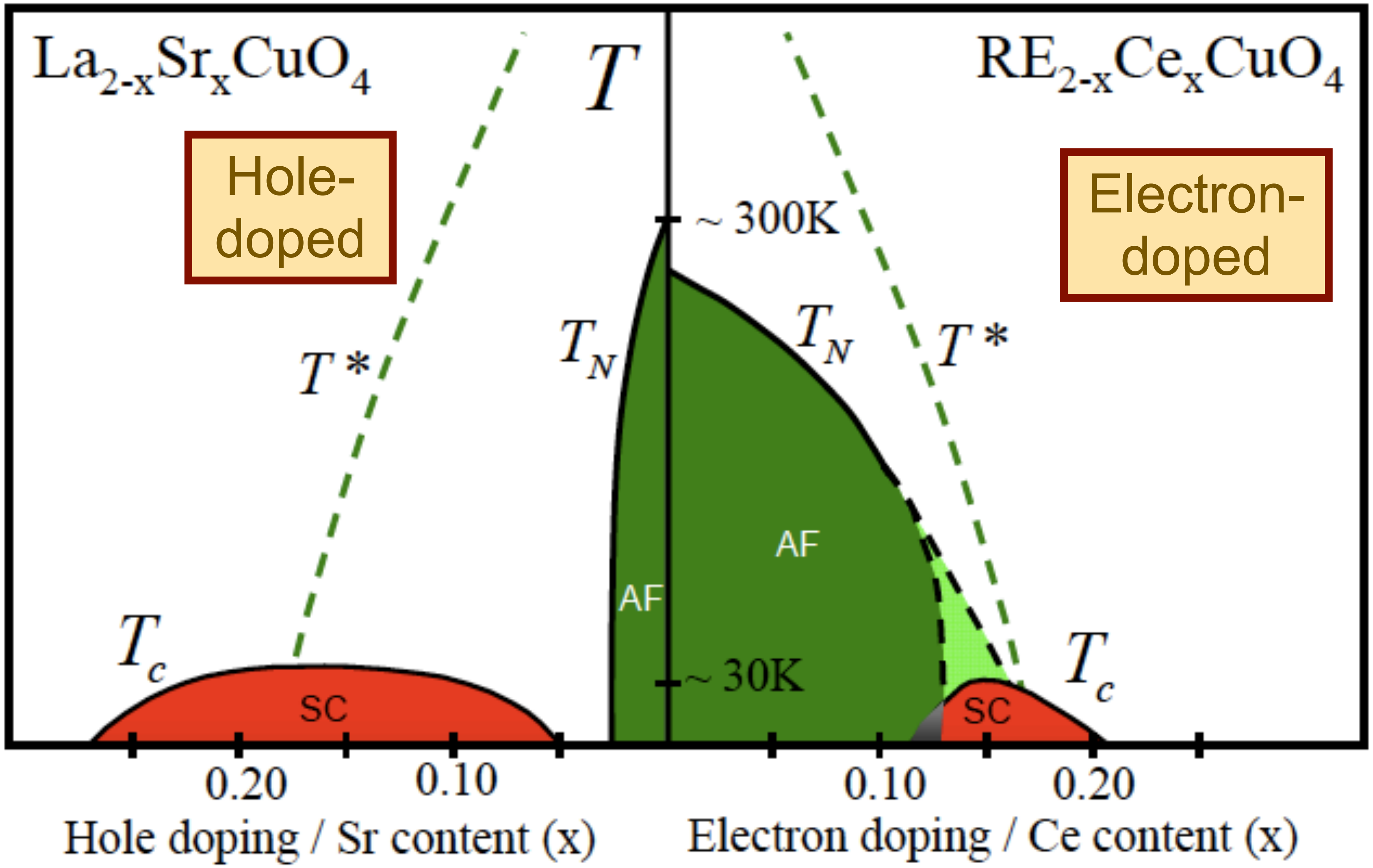
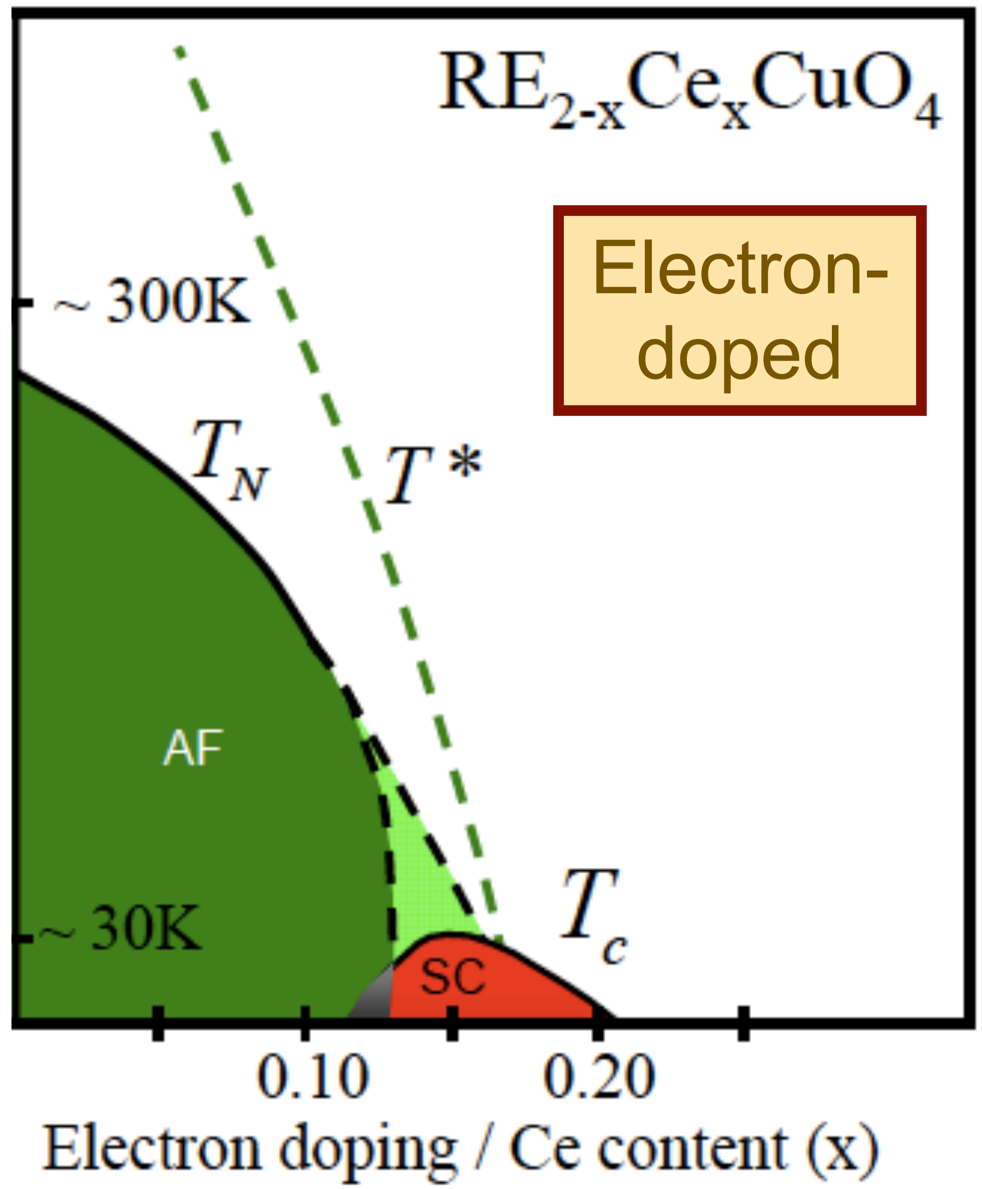


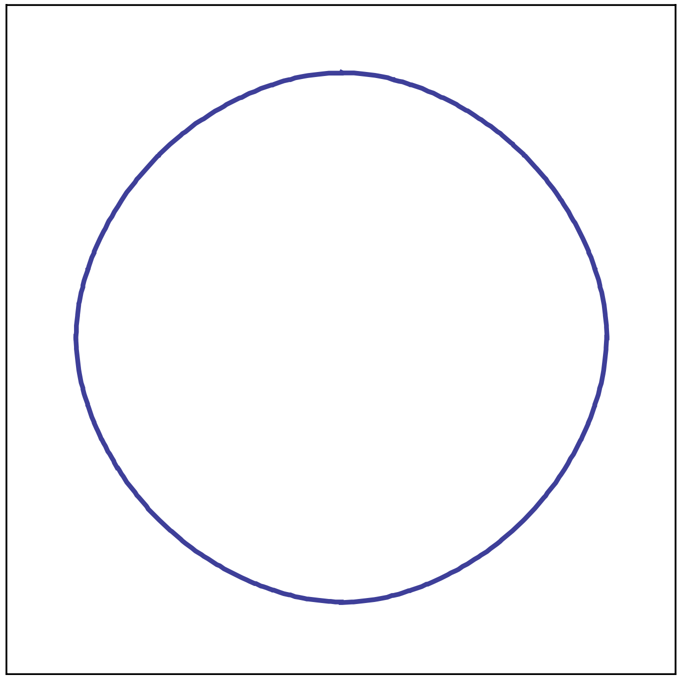
$La_{2-x}Sr_xCuO_4$
$RE_{2-x}Ce_xCuO_4$
Hole-doped
Electron-doped
T
~ 300K
~ 30K
T_N
T_N
T^*
T^*
T_c
T_c
AF
AF
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (x)
Electron doping / Ce content (x)

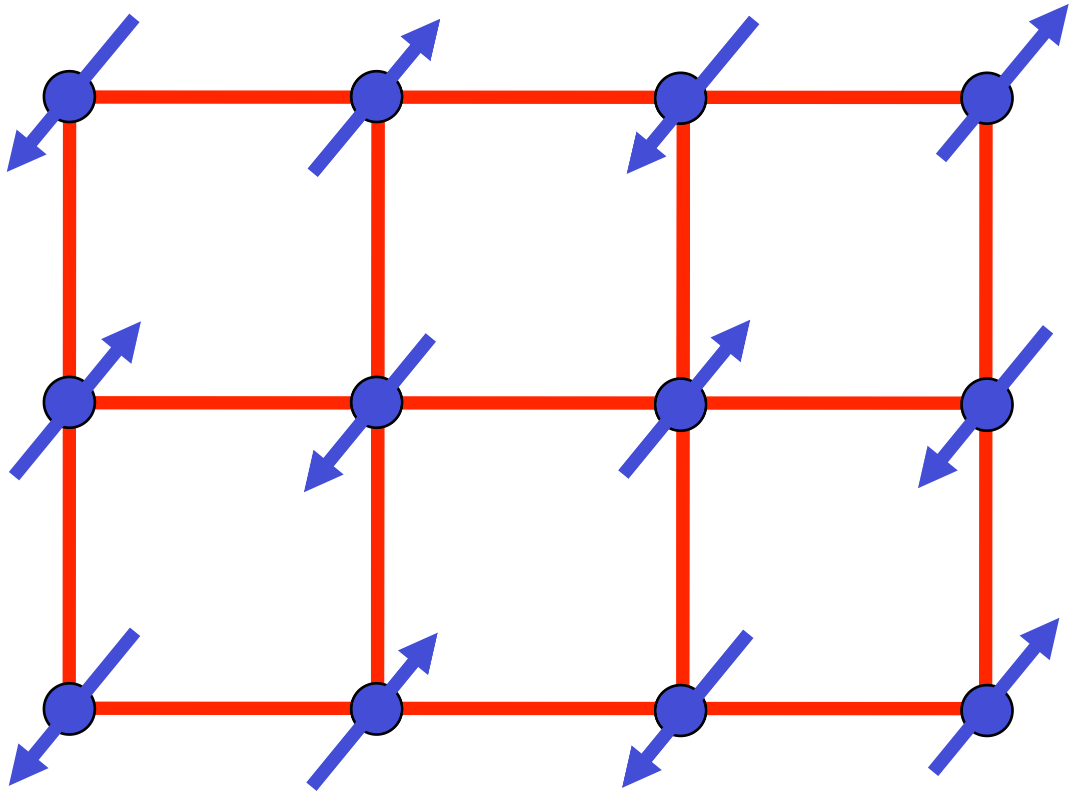
Electron-doped cuprate superconductors



Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



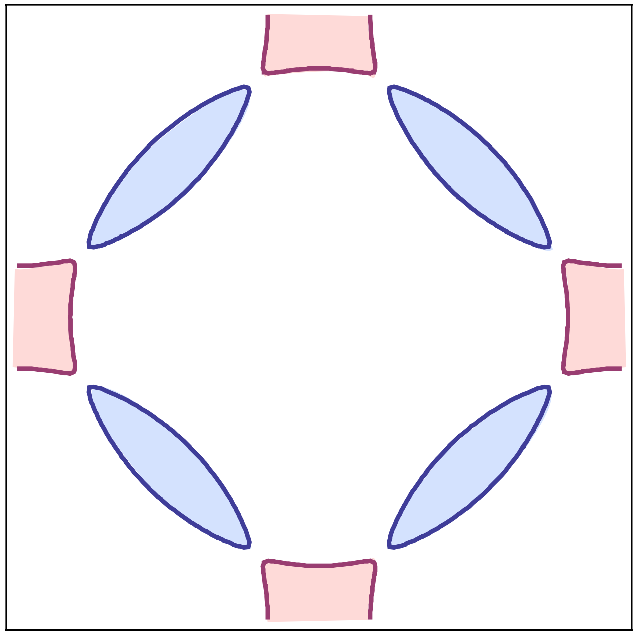


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

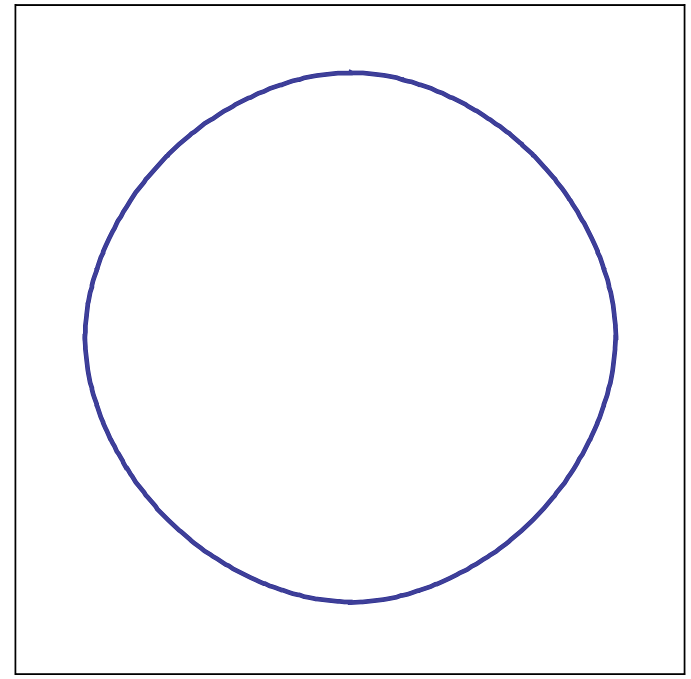
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



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Metal with "large" Fermi surface

Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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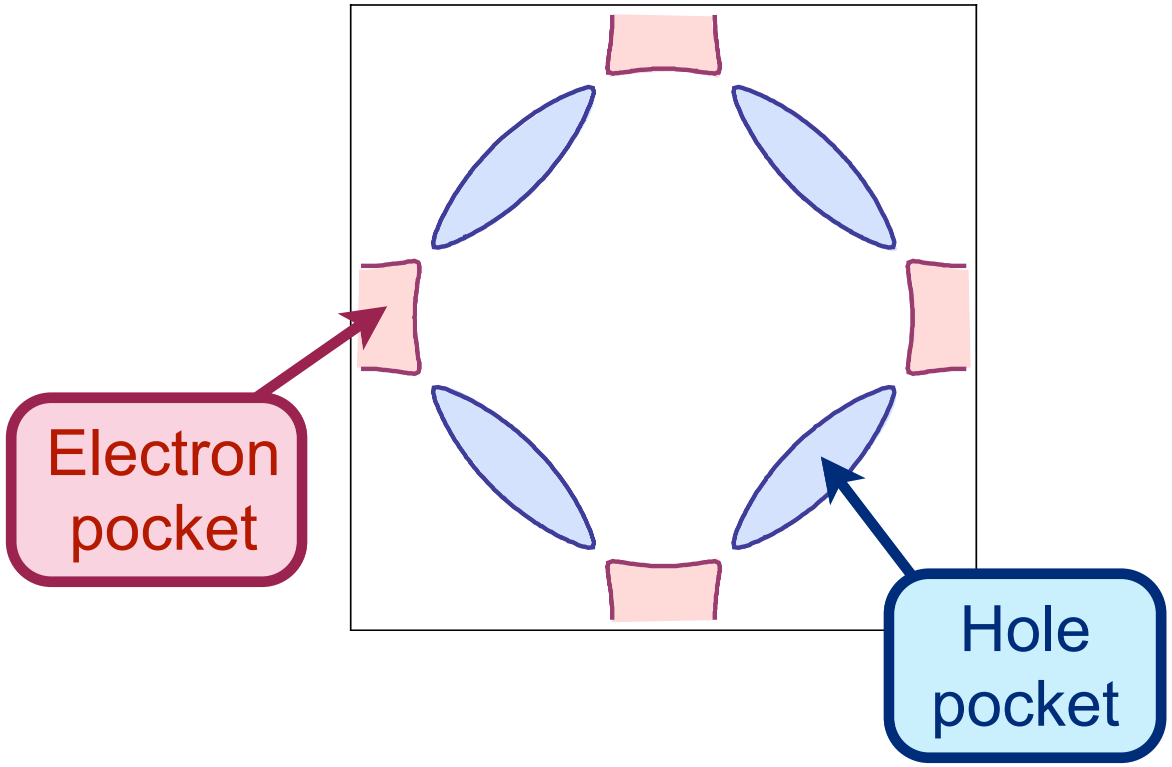

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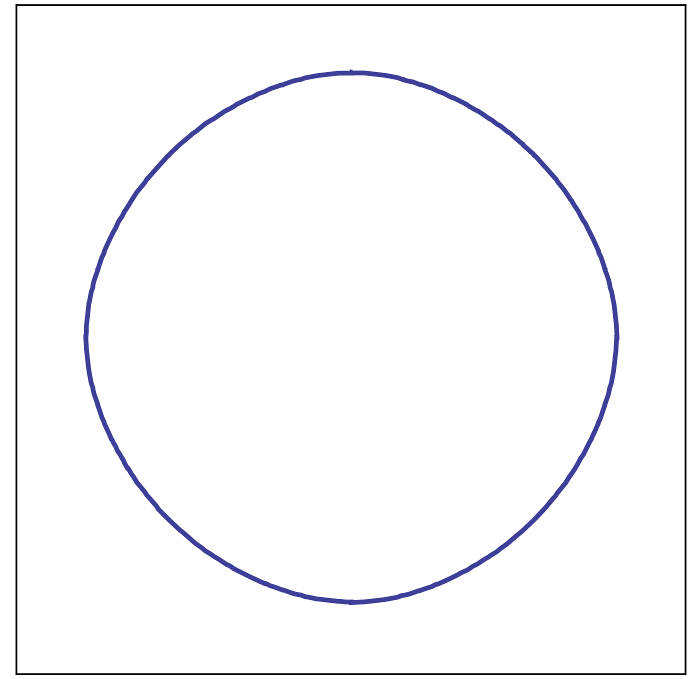

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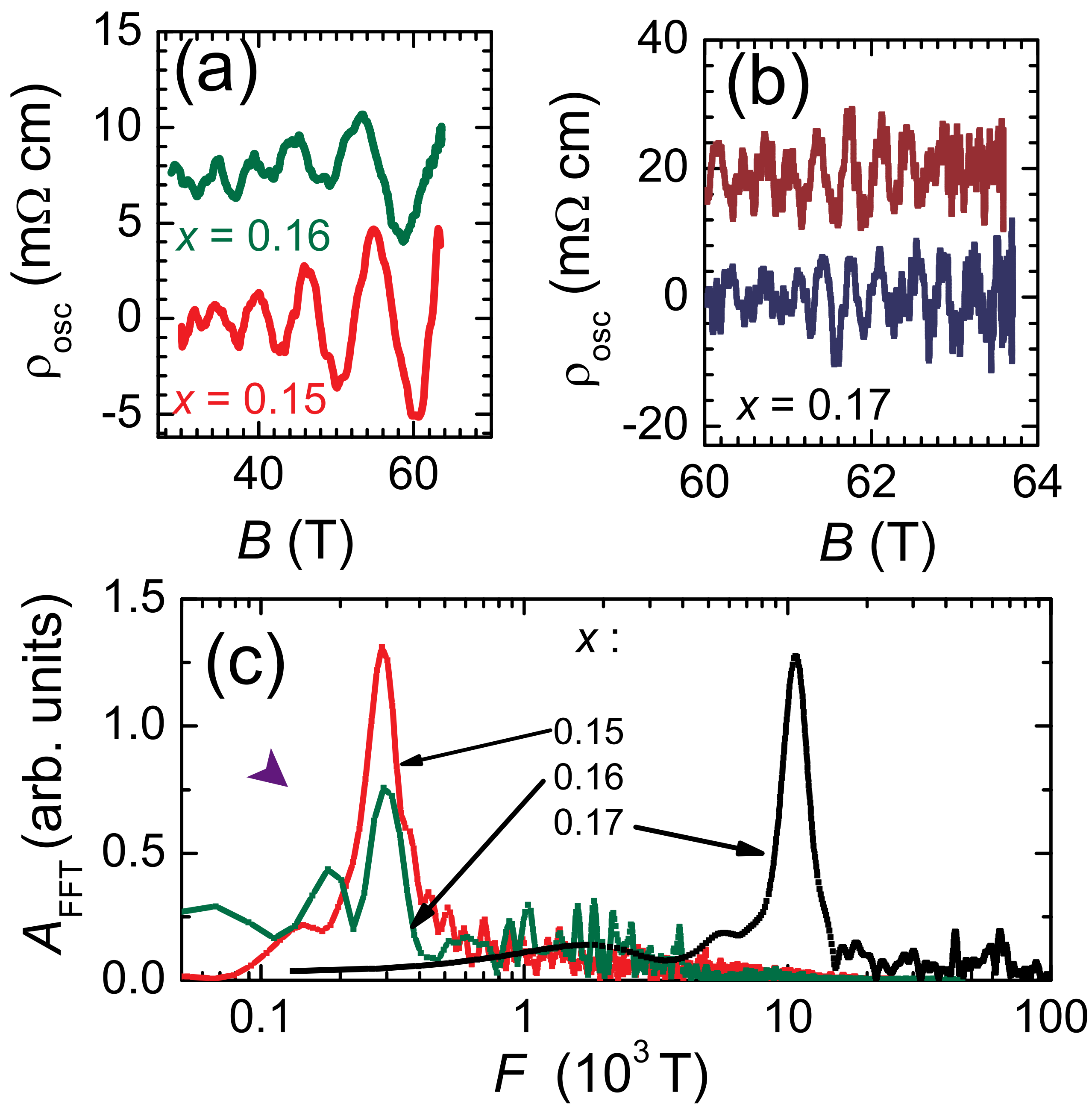
Saturday, March 26, 2011

Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$



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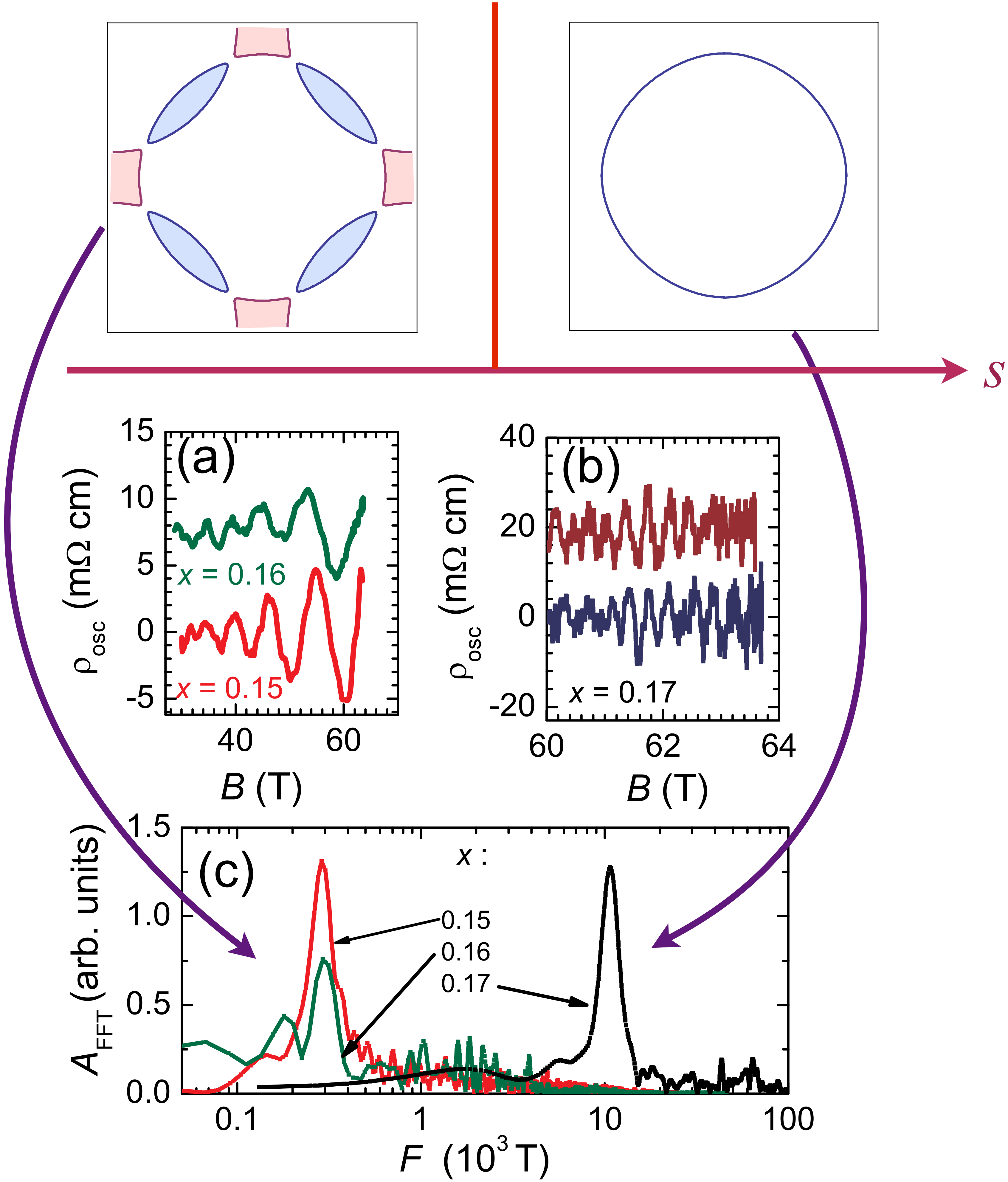

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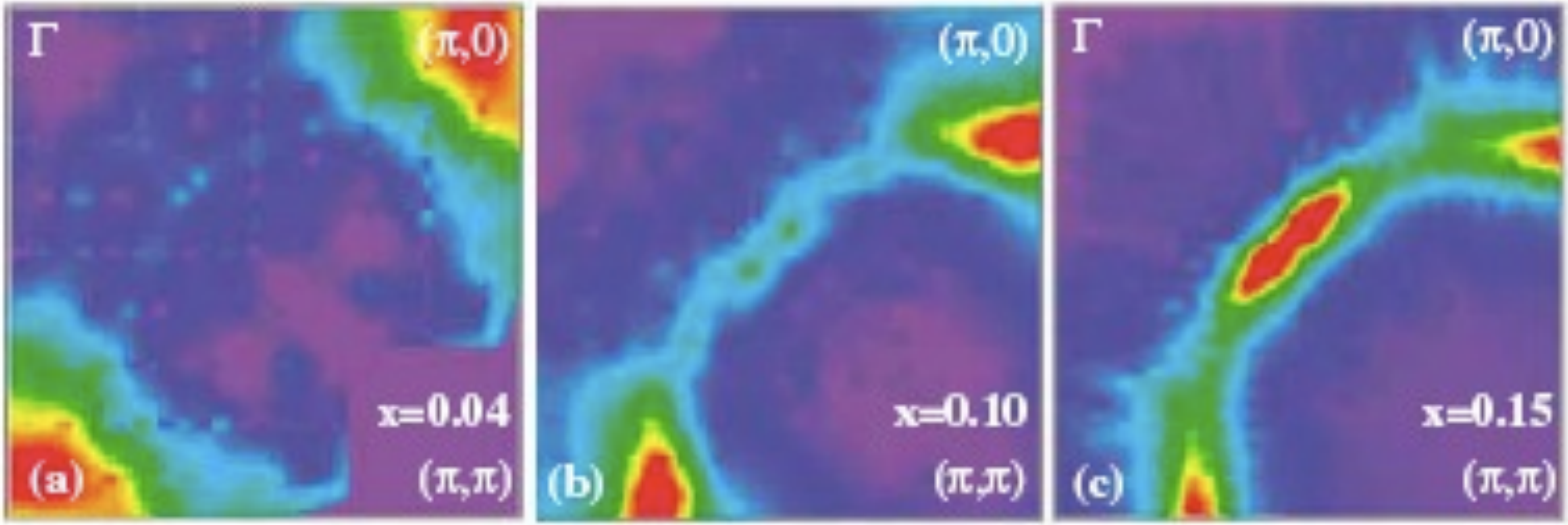
Photoemission in $Nd_{2-x}Ce_xCuO_4$



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^{\dagger}_{i\alpha} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^{\dagger}_{i\alpha} c_{i\alpha}$$

$$c^{\dagger}_{i\alpha} c_{j\beta} + c_{j\beta} c^{\dagger}_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Decouple Hubbard interaction with the antiferromagnetic order parameter $\vec{\varphi}$

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{r}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

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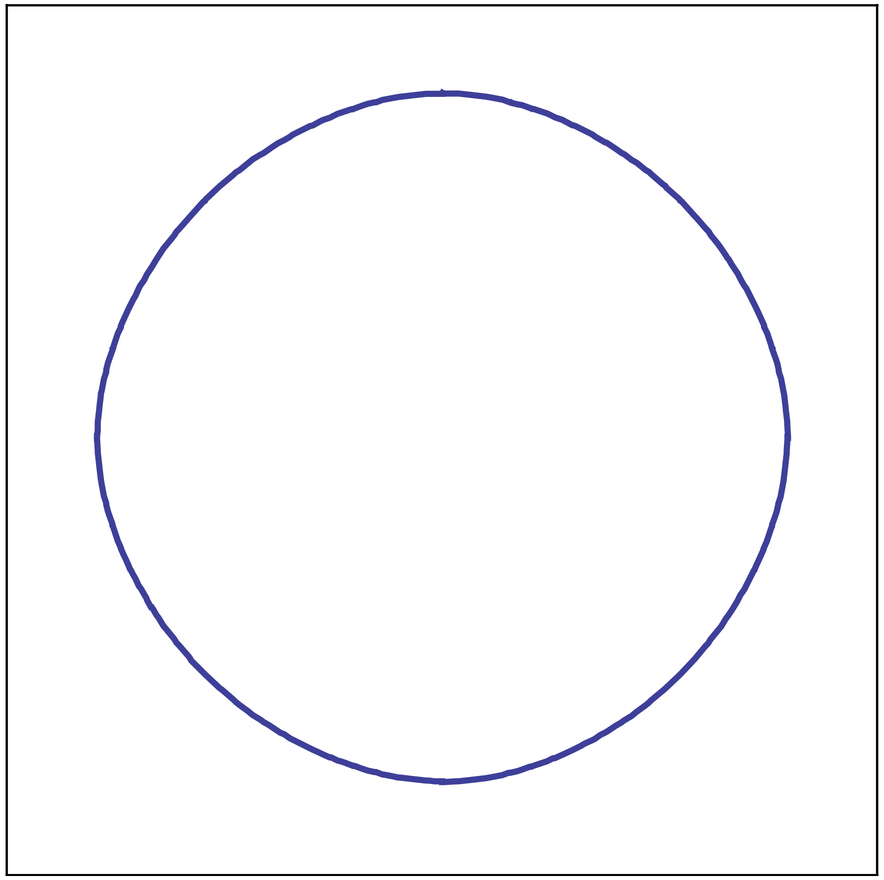
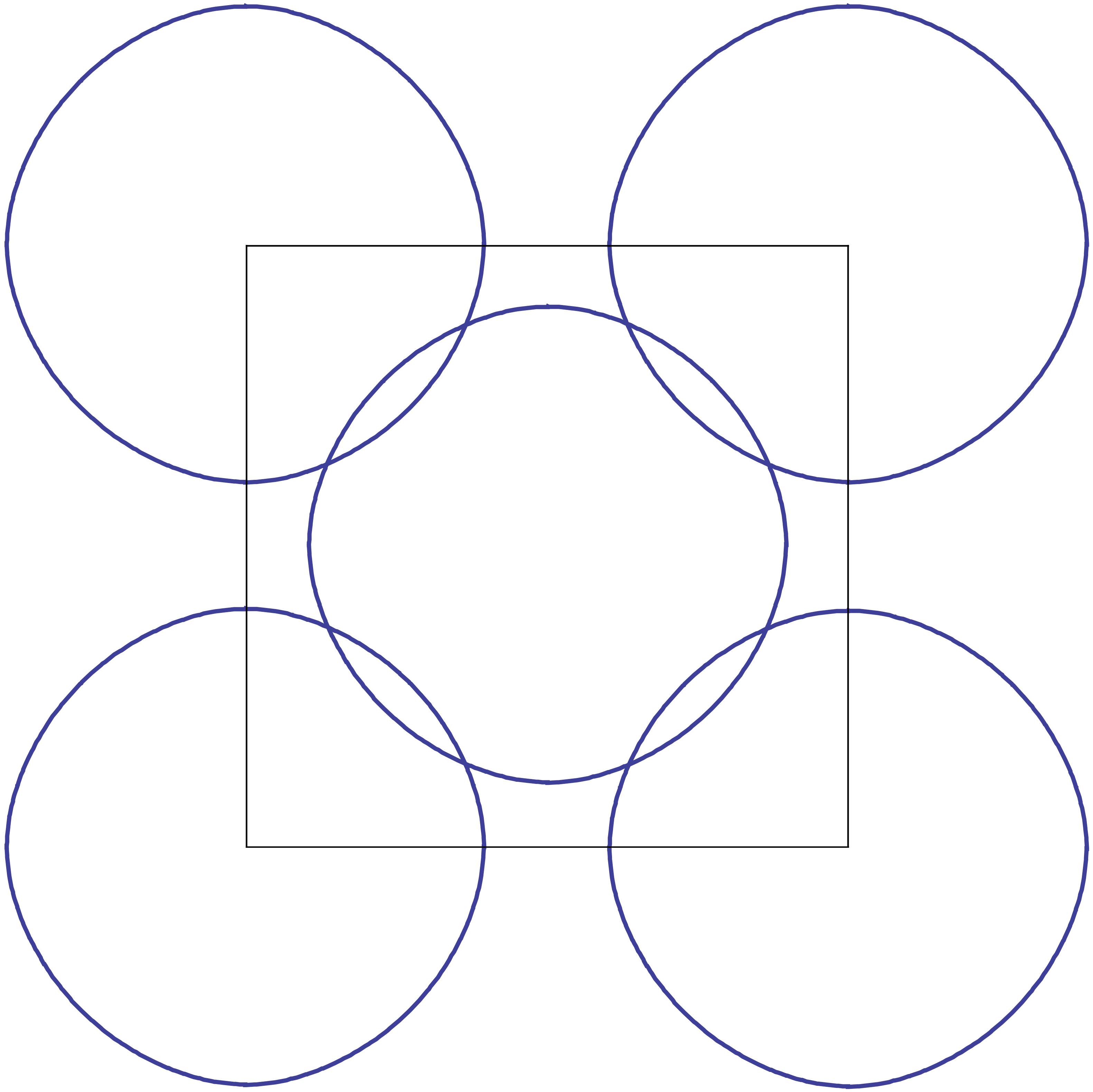
Coupling between fermions and antiferromagnetic order:

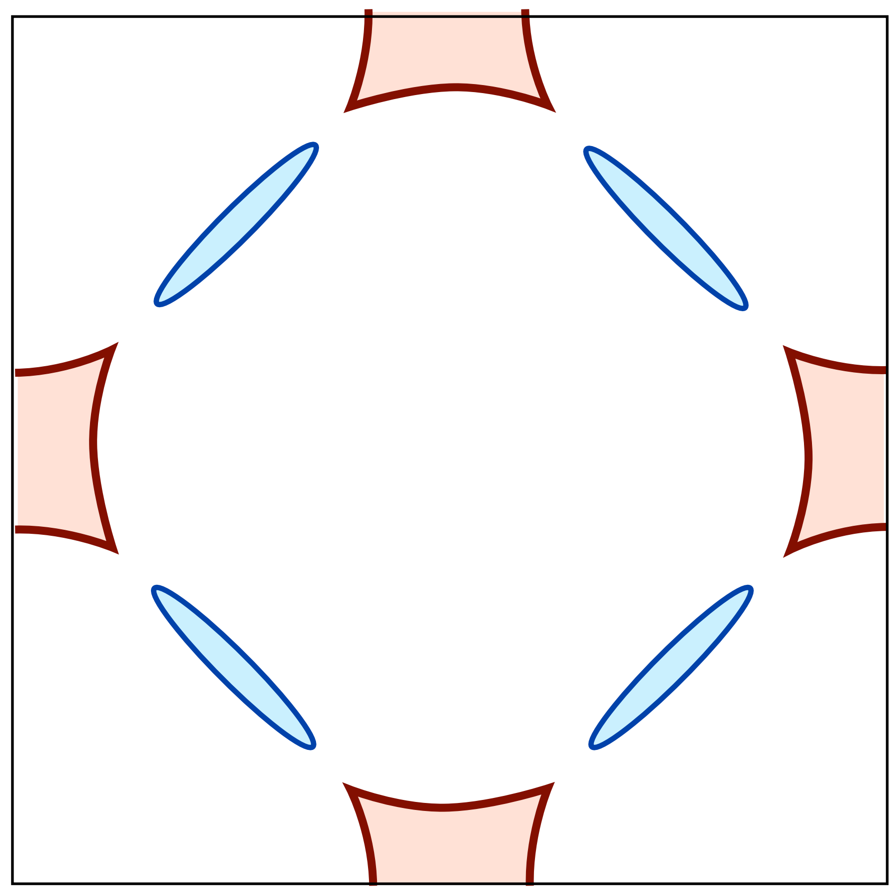
$\lambda^2 \sim U$



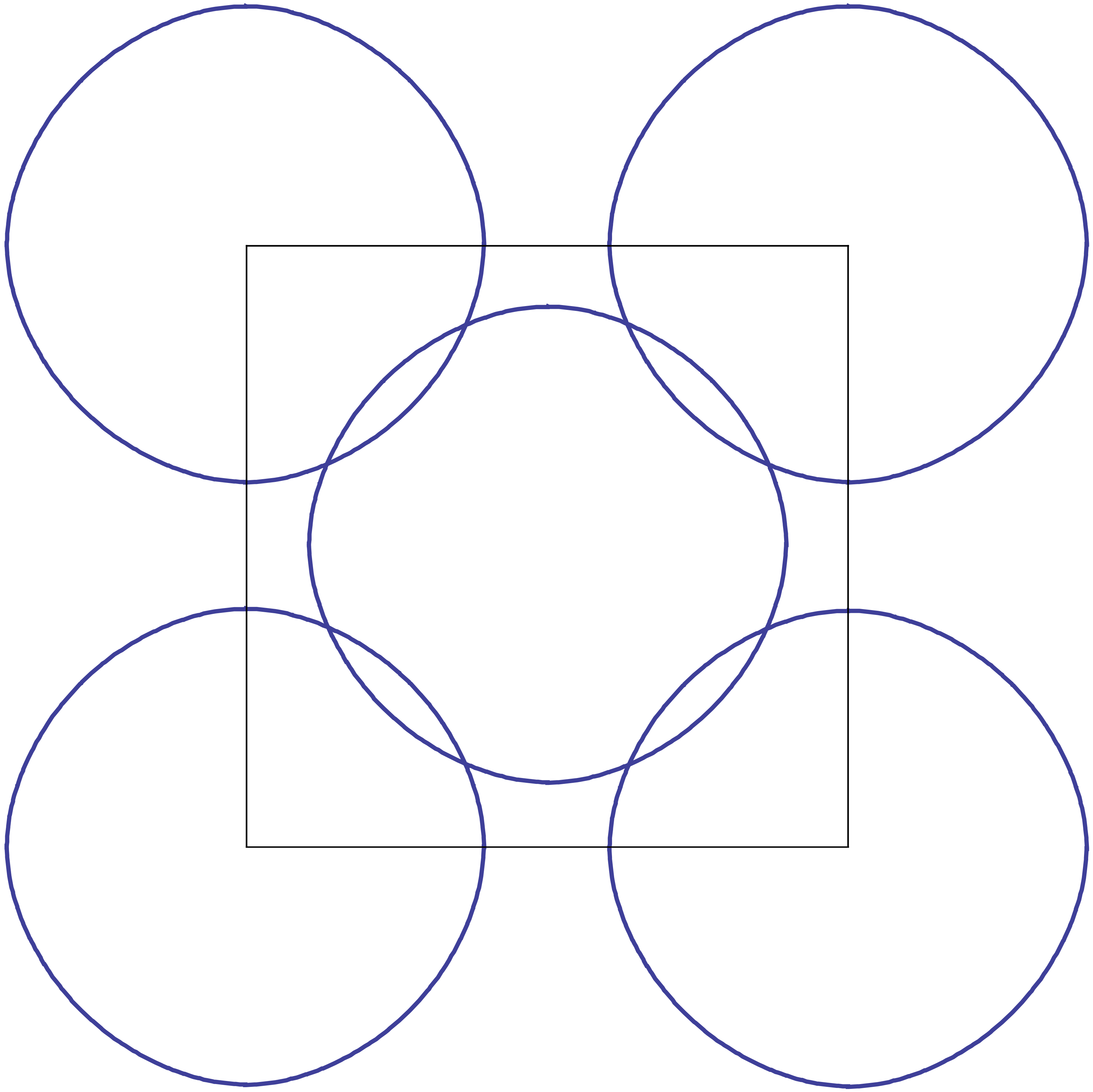
Metal with “large” Fermi surface



Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$



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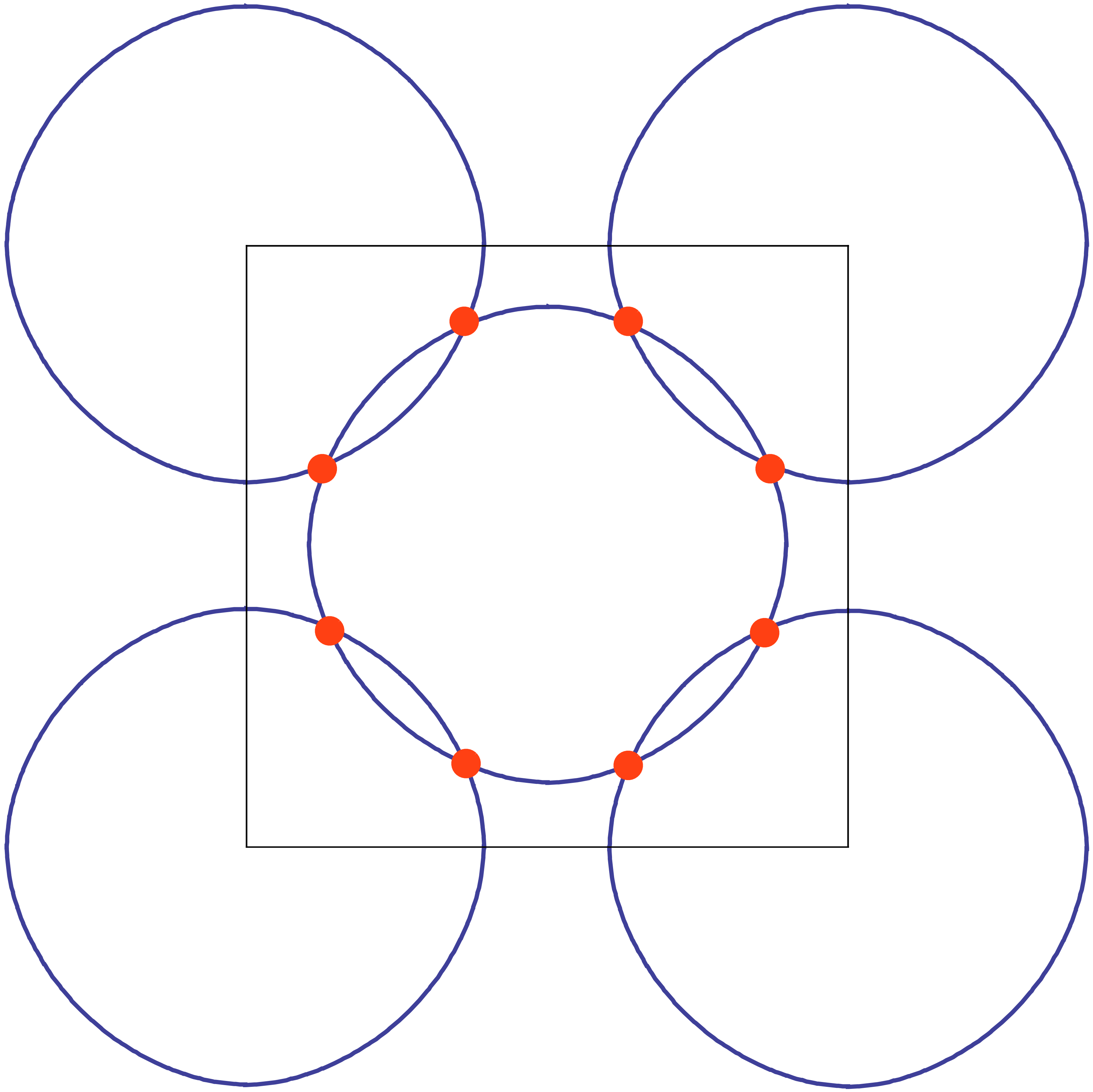
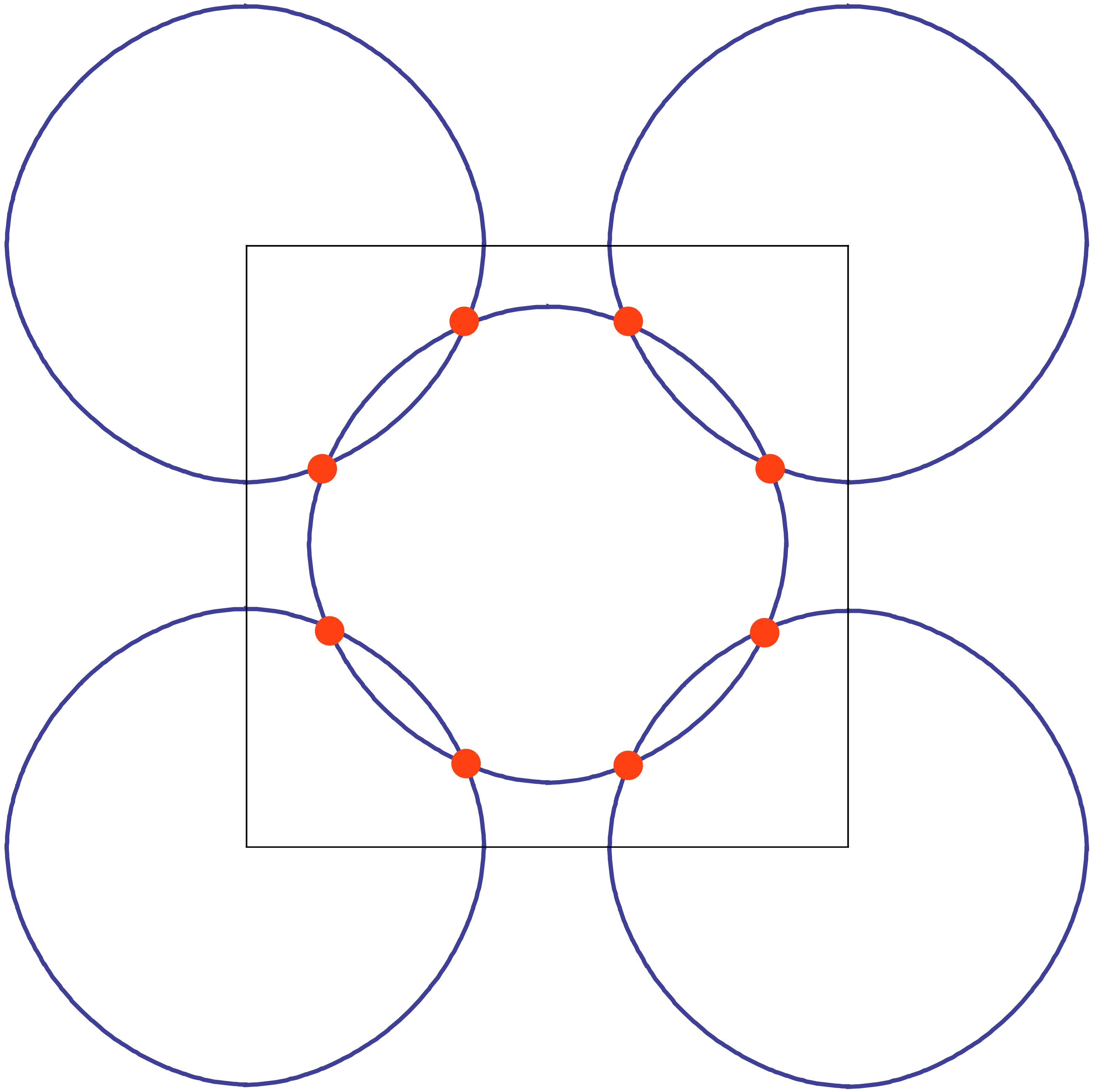
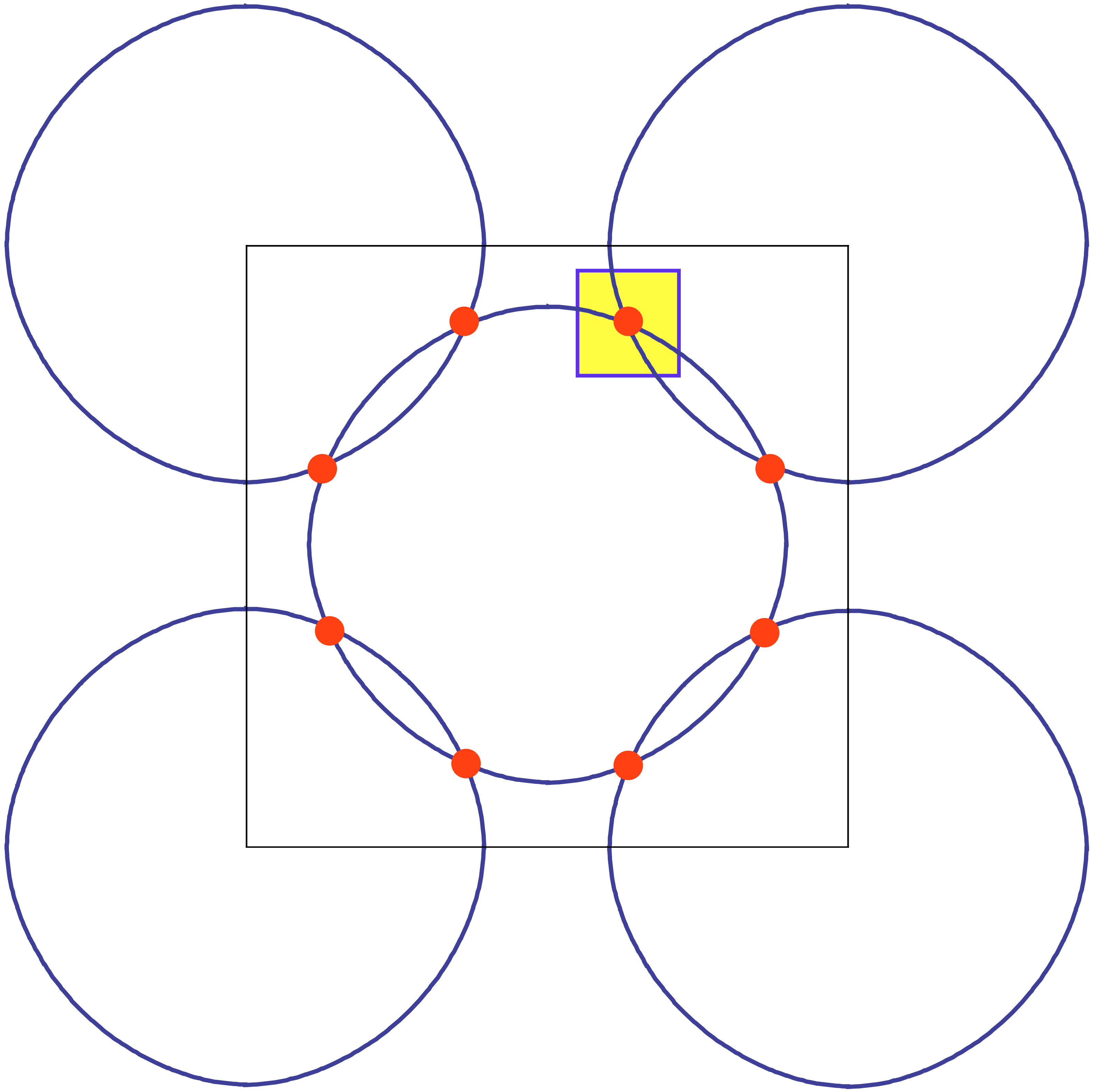


“Hot” spots

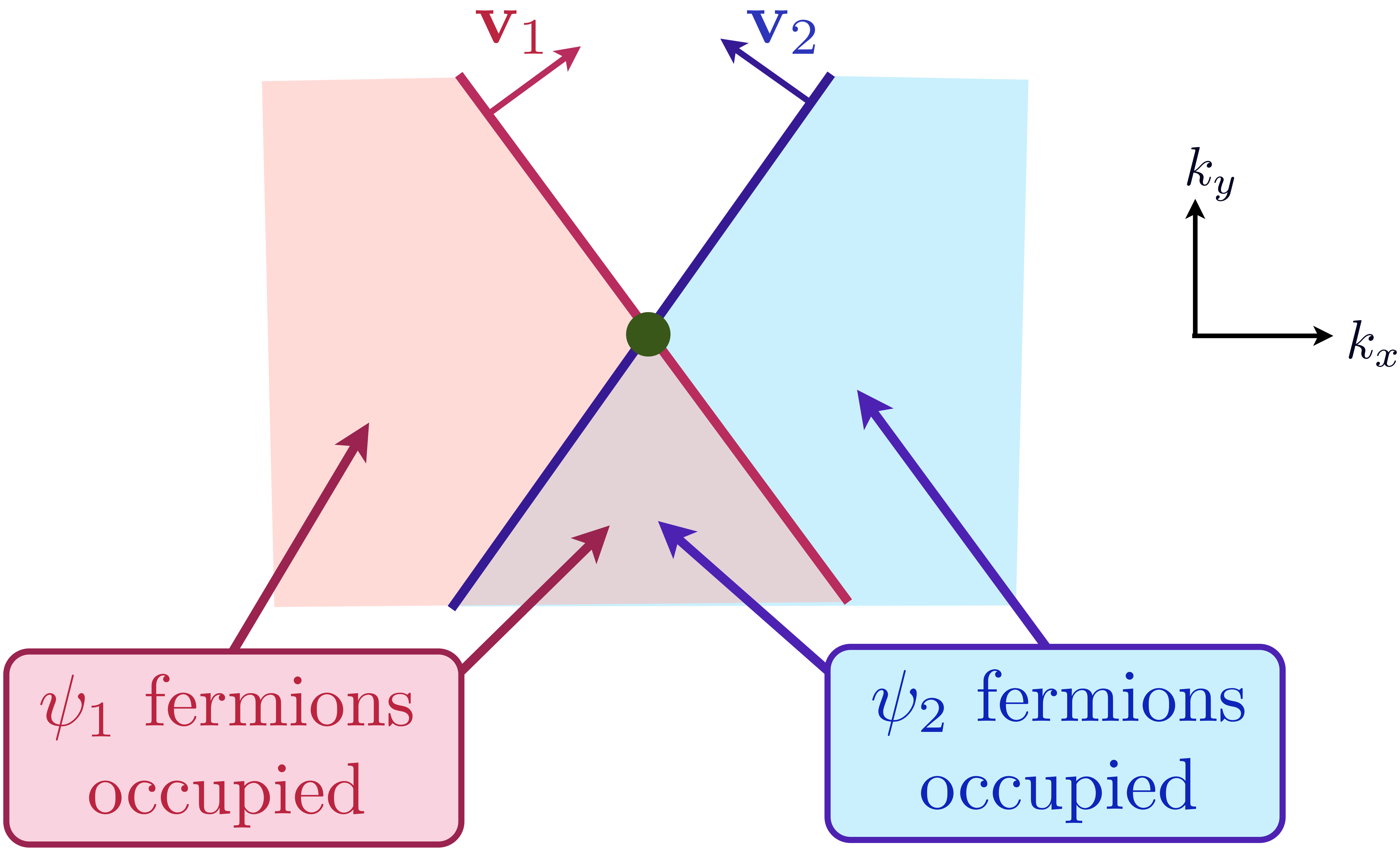


Low energy theory for critical point near hot spots

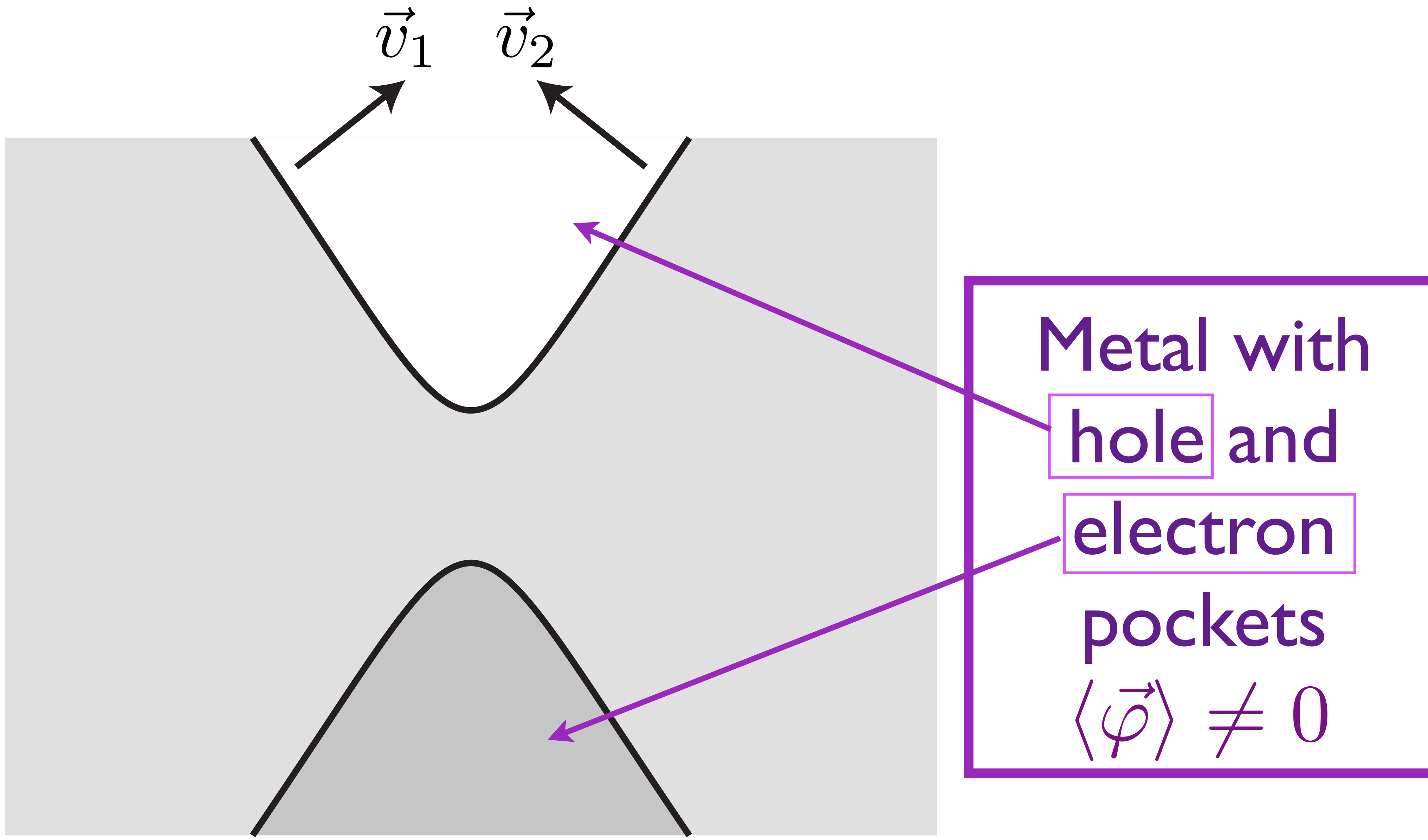


Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



Fermi lines reconnect in antiferromagnetic phase



Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

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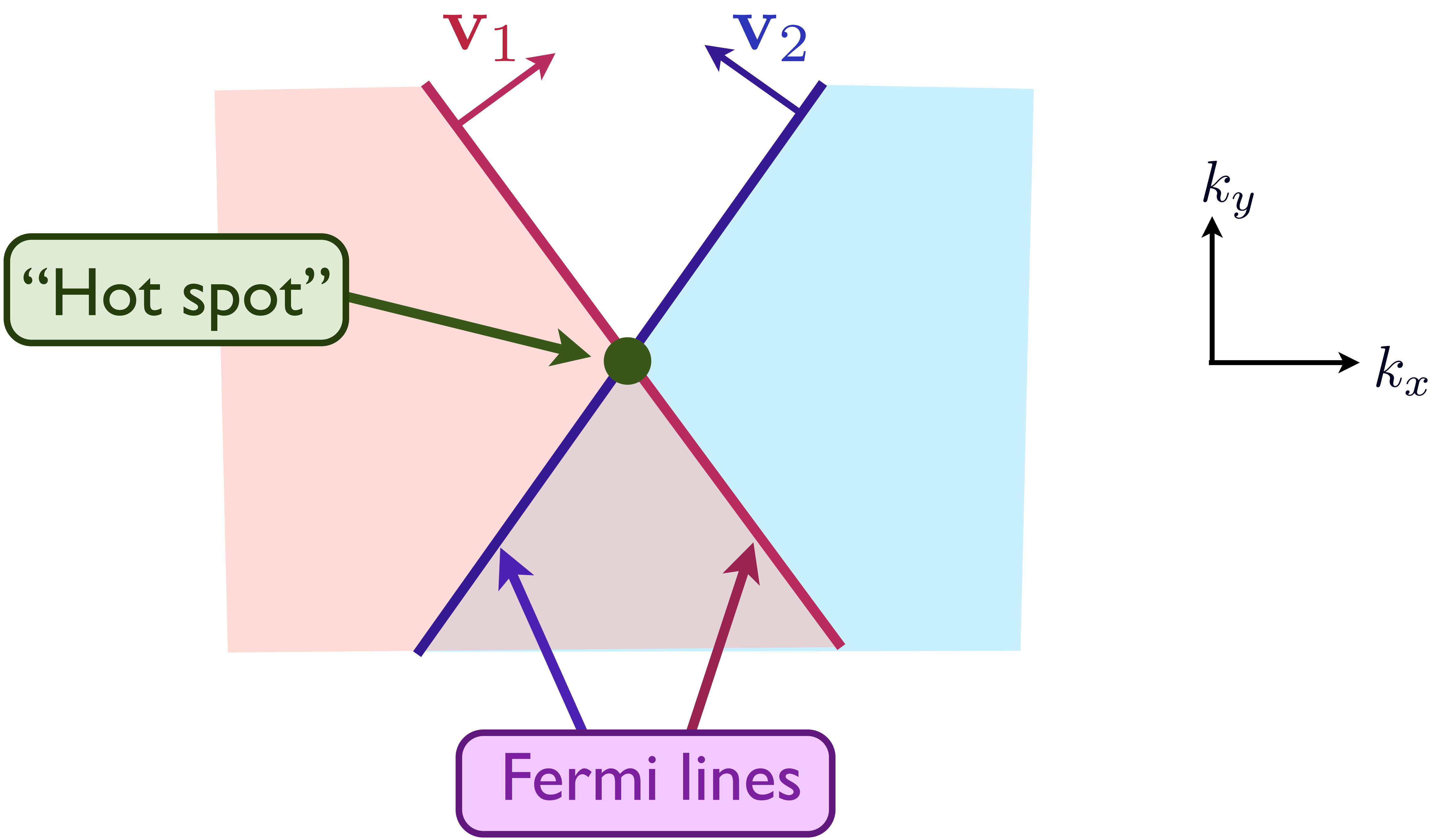

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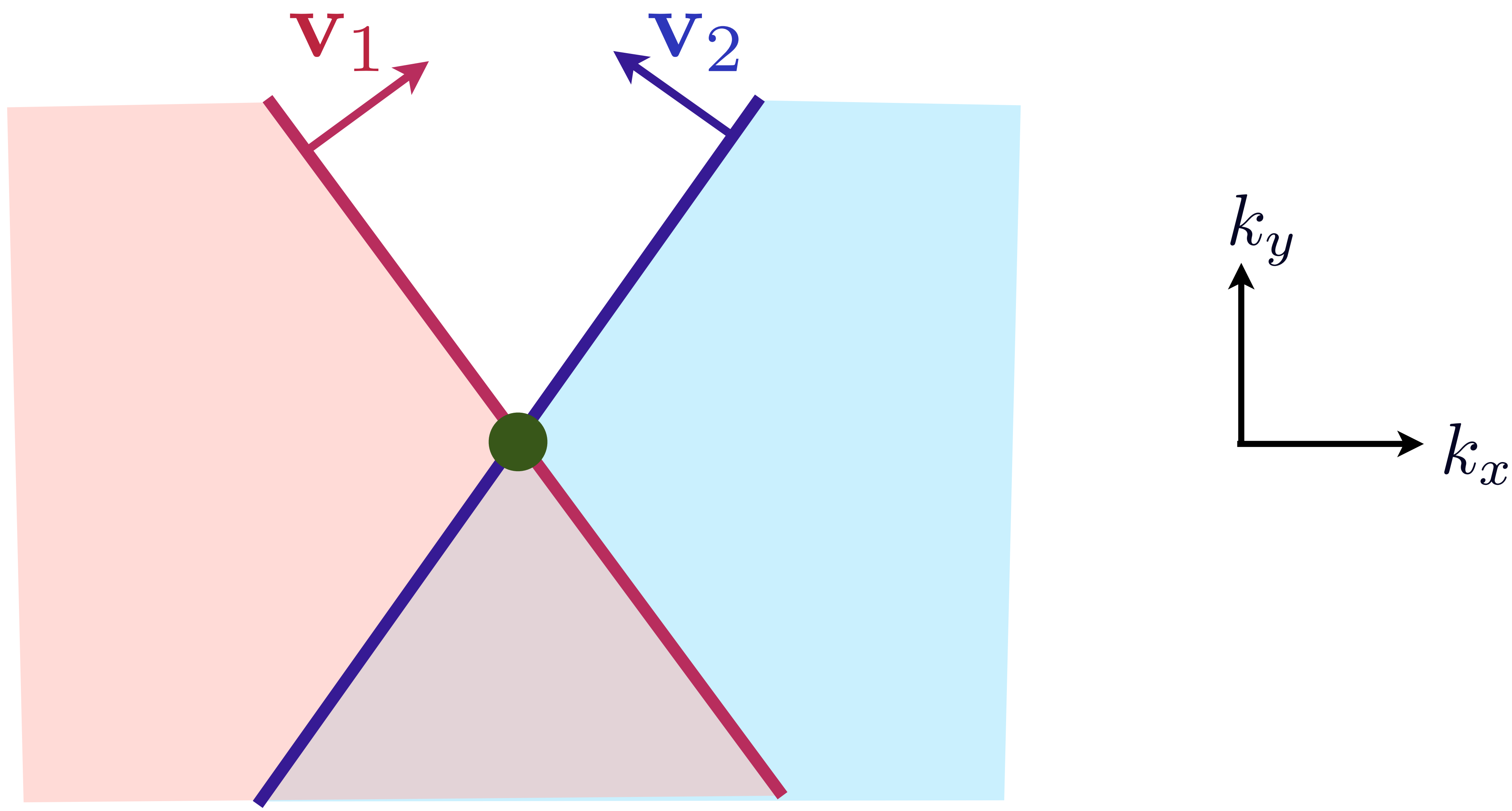
Critical point theory is strongly coupled in $d = 2$
Results are *independent* of coupling λ



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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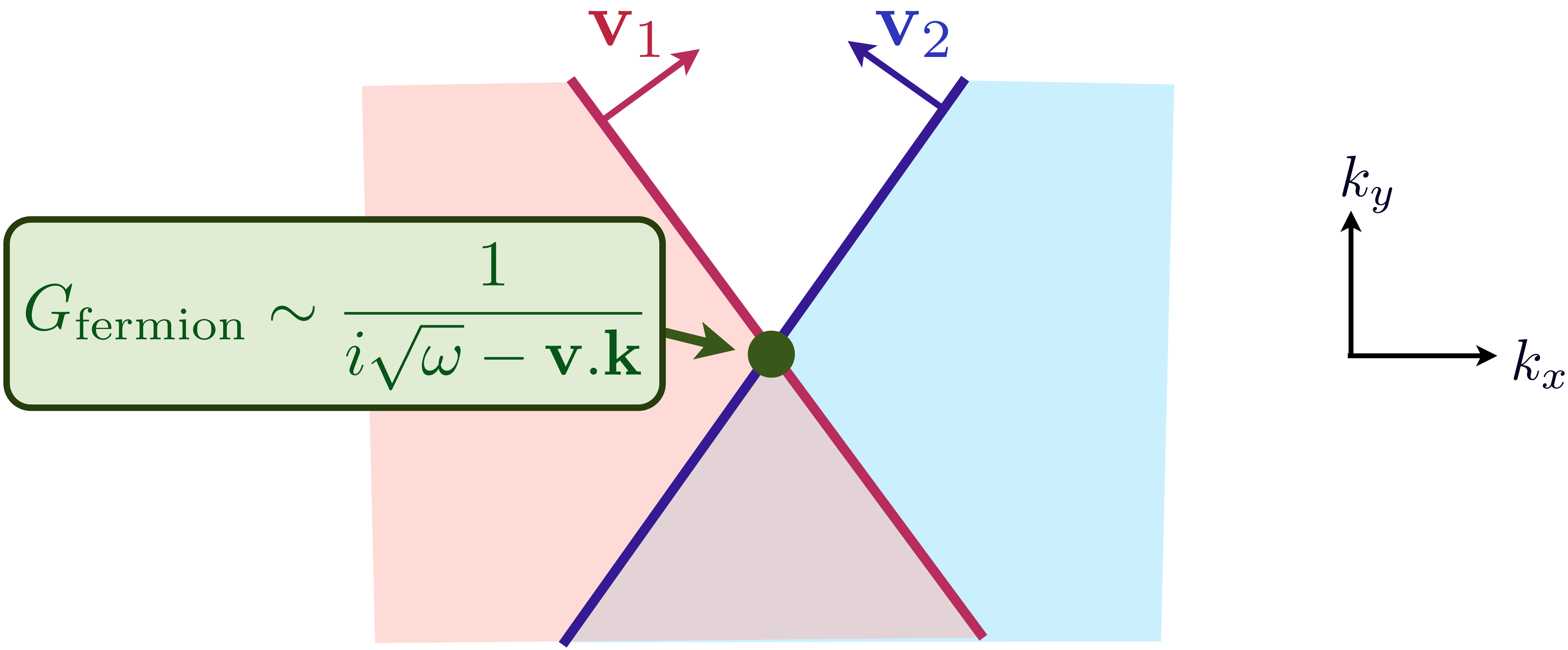

A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)
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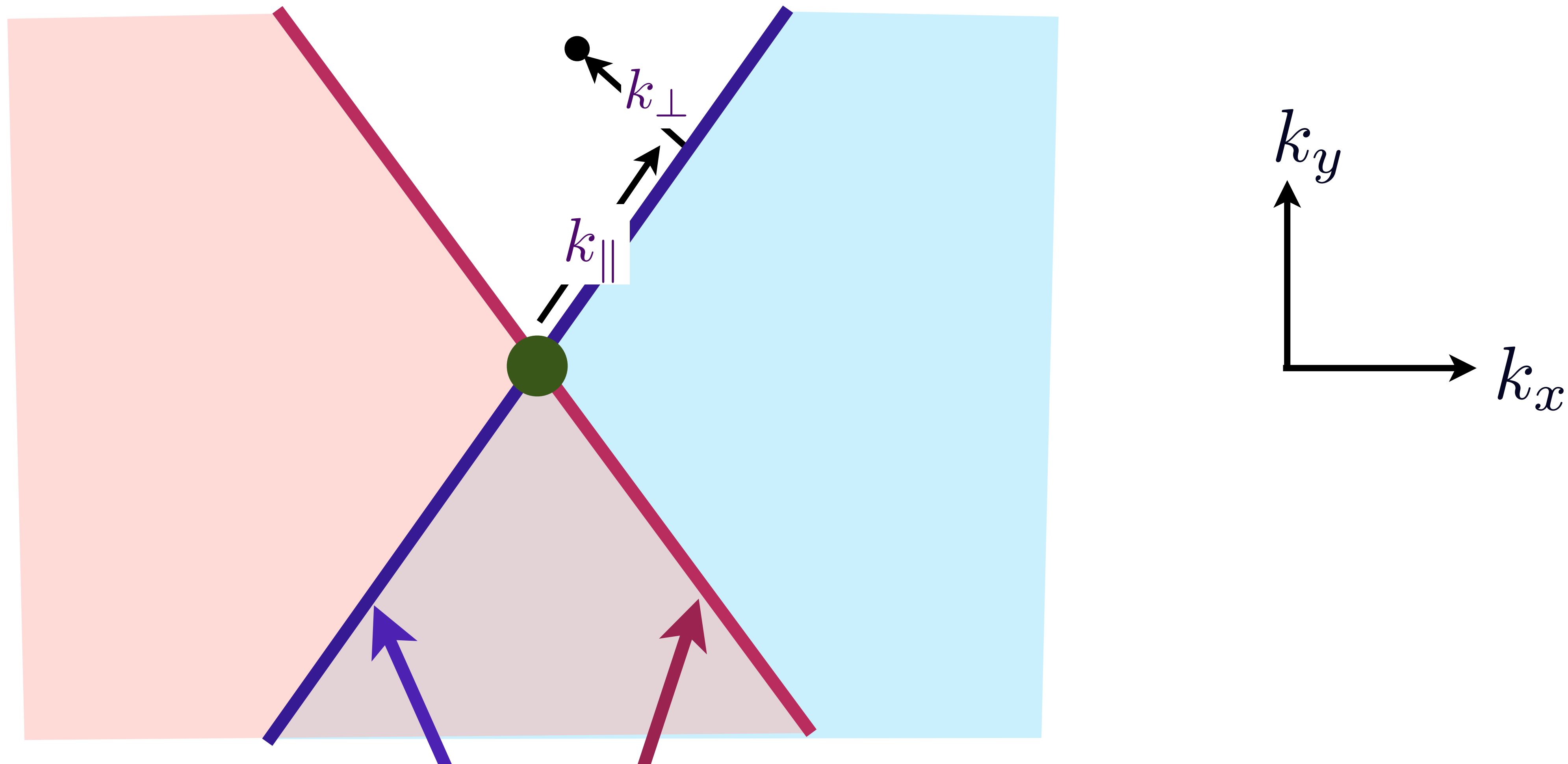

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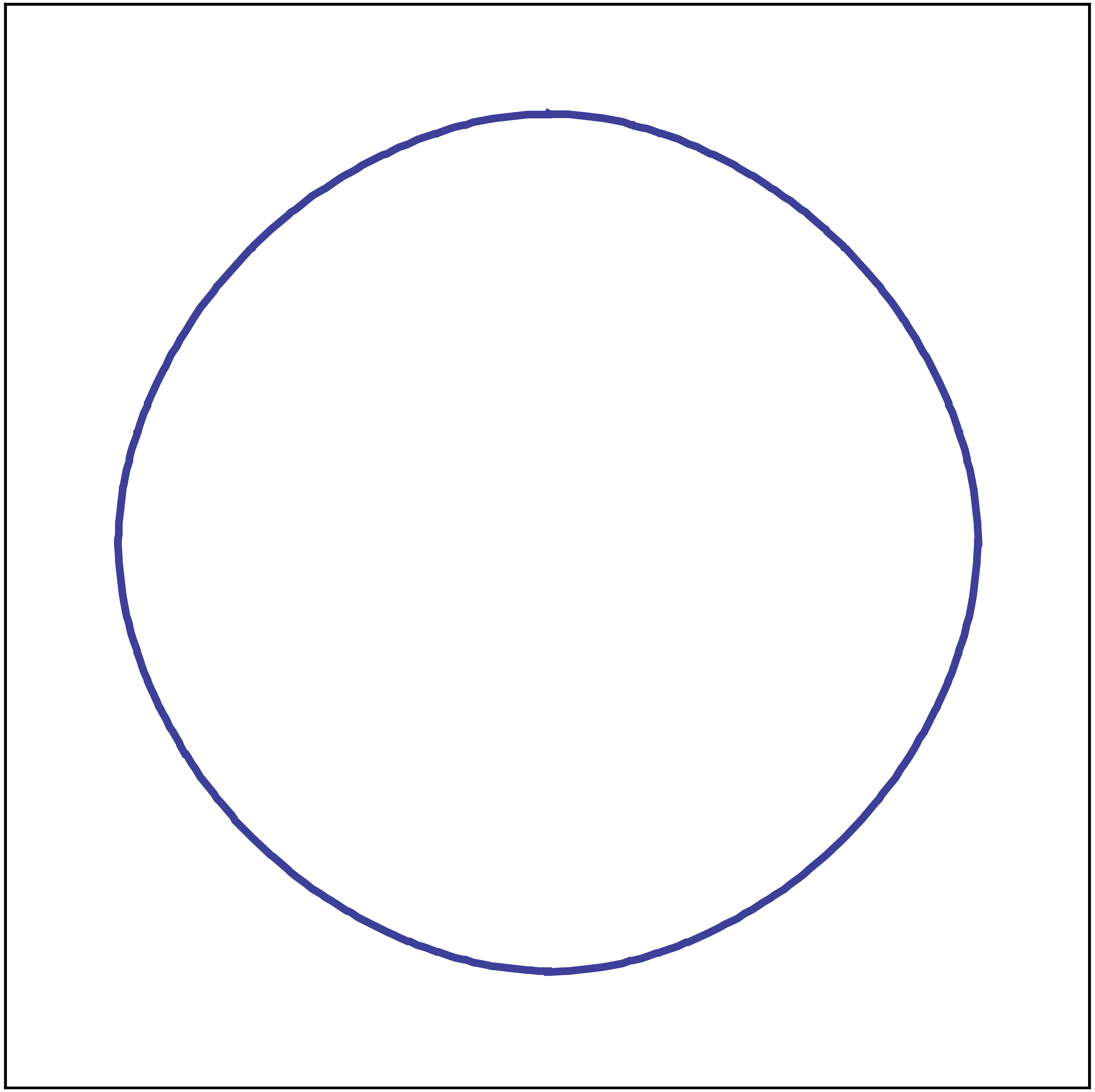

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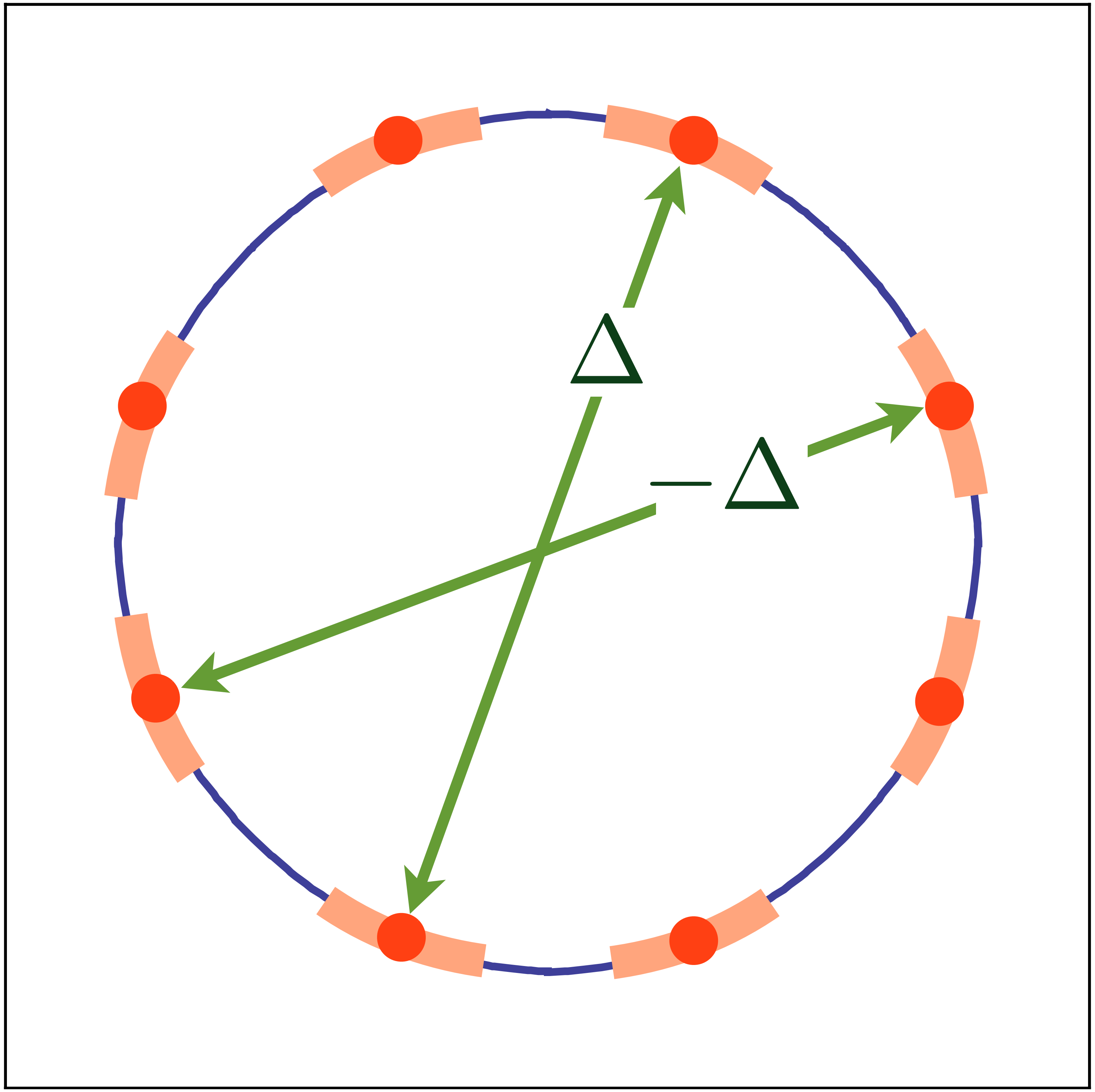
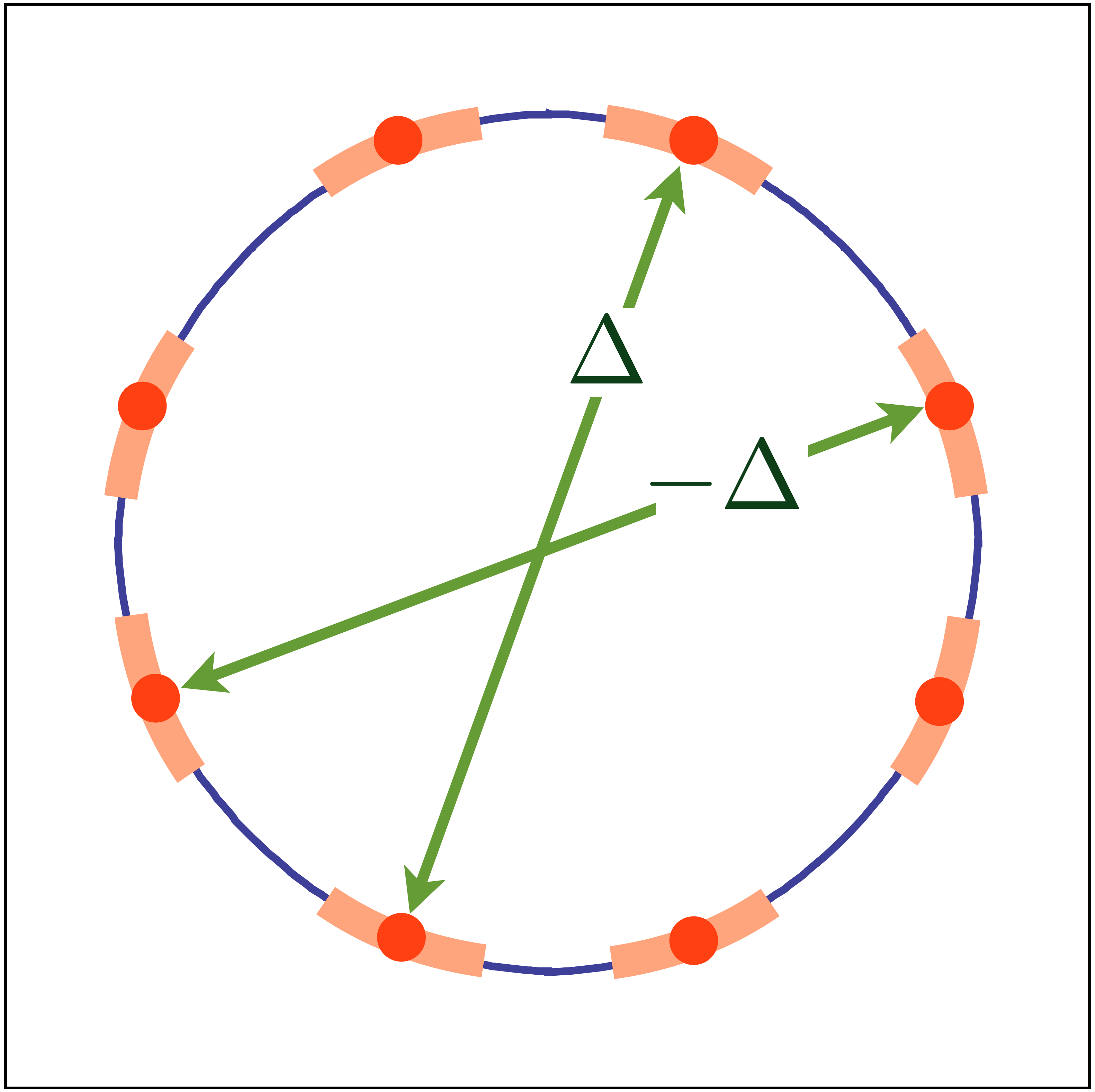


Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta(\cos k_x - \cos k_y)$$

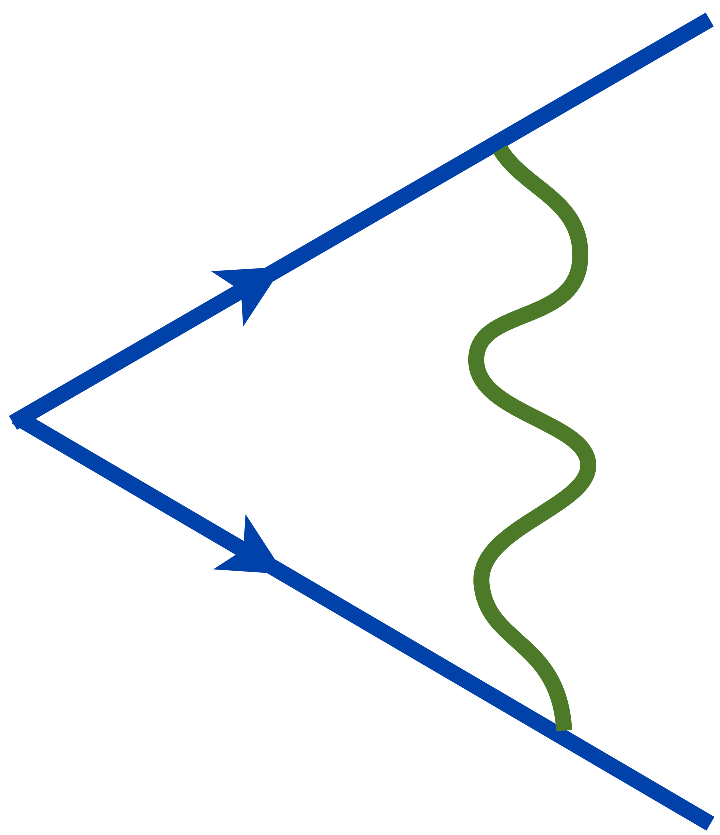


Unconventional pairing at *and near* hot spots

BCS theory

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

Cooper logarithm



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Electron-phonon coupling

BCS theory

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Electron-phonon coupling

Debye frequency

BCS theory

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Electron-phonon coupling

Debye frequency

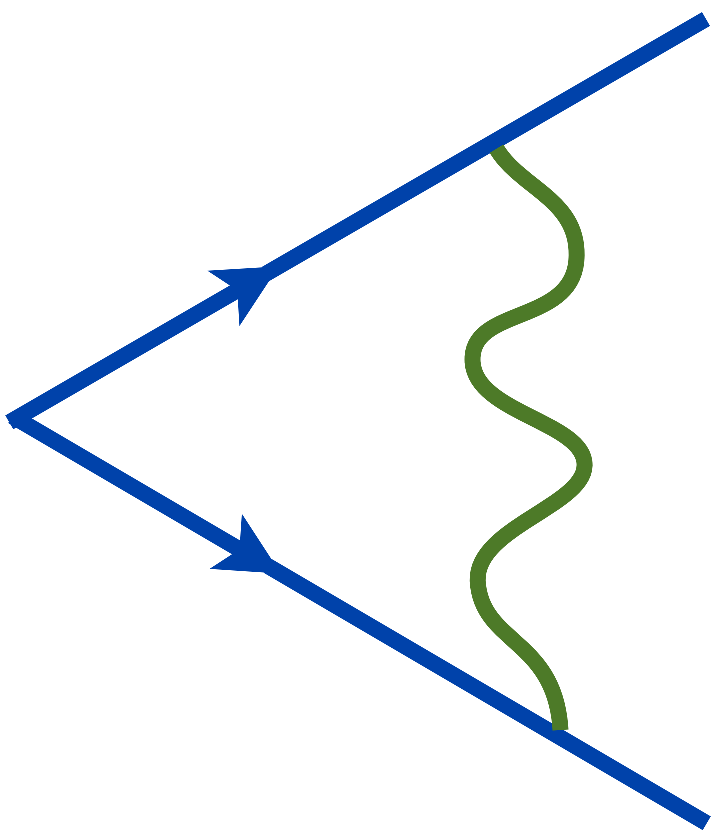
Implies

$$T_c \sim \omega_D \exp\left(-1/\lambda\right)$$

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \underbrace{\log\left(\frac{E_F}{\omega}\right)}_{\text{Cooper logarithm}}$$



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Fermi energy

Implies $T_c \sim E_F \exp\left(-\left(t/U\right)^2\right)$

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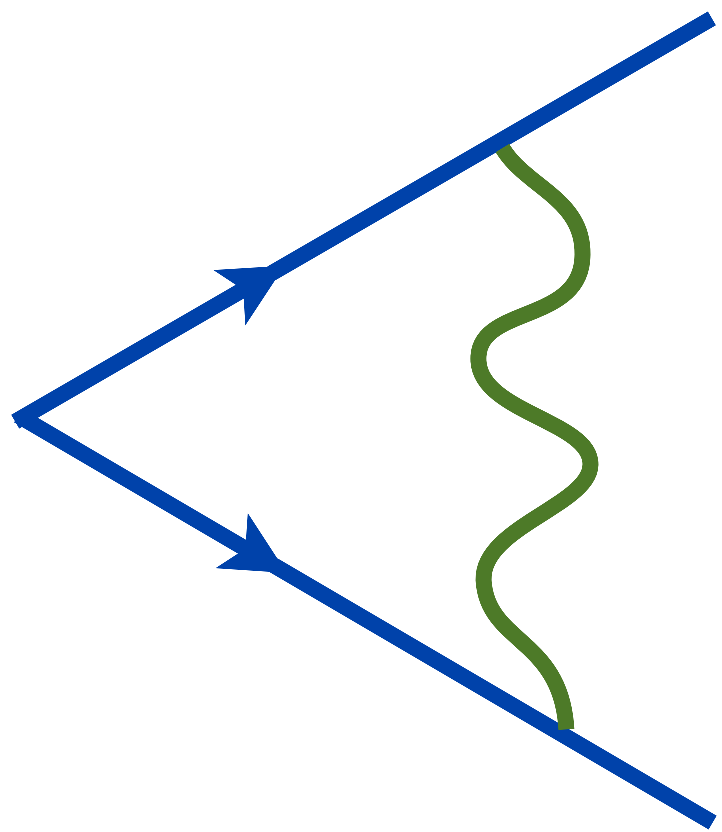
Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{E_F}{\omega}\right)$$



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Fermi energy

$\alpha = \tan\theta$, where 2θ is the angle between Fermi lines. *Independent* of interaction strength U in 2 dimensions.

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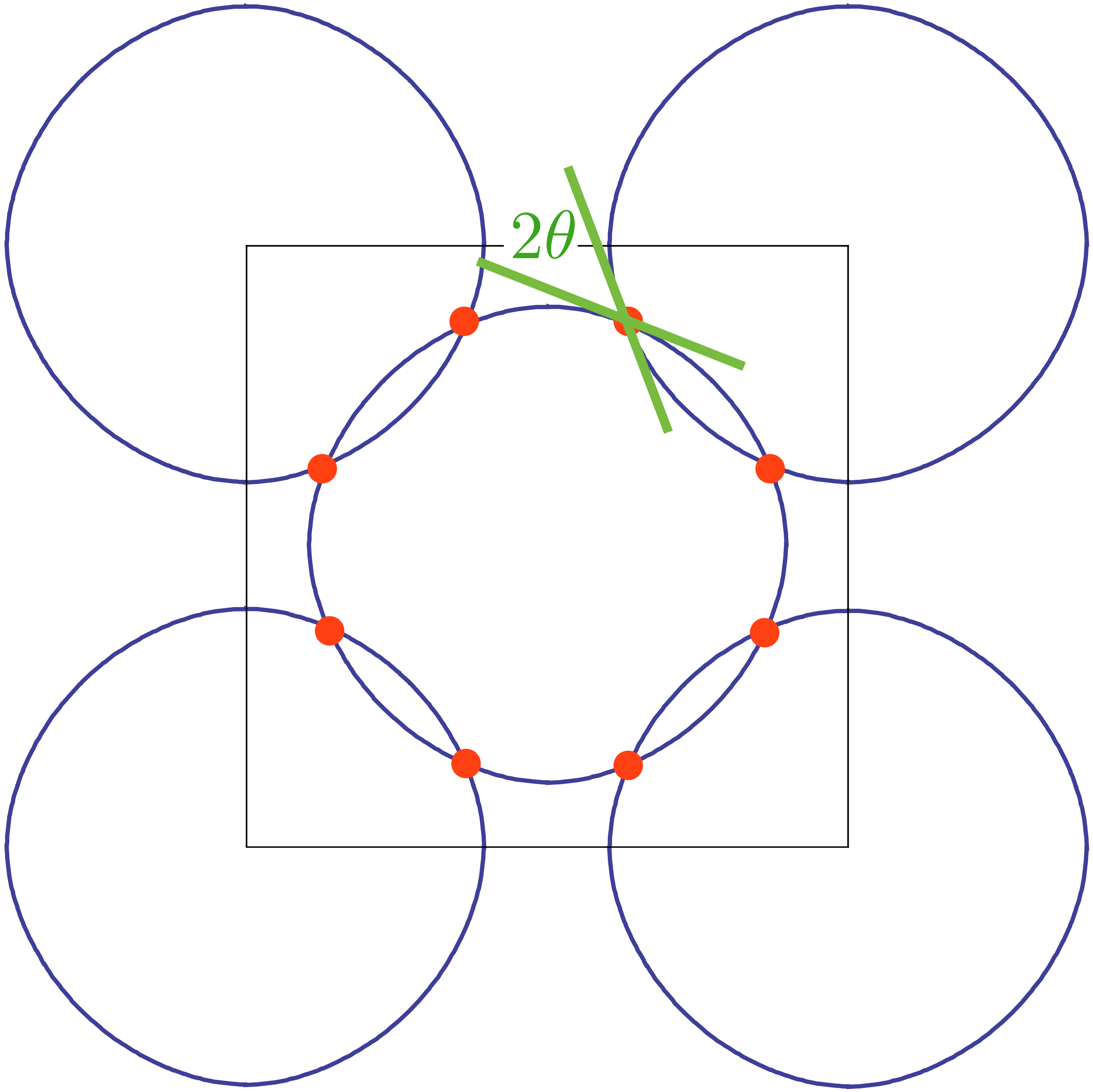
2θ

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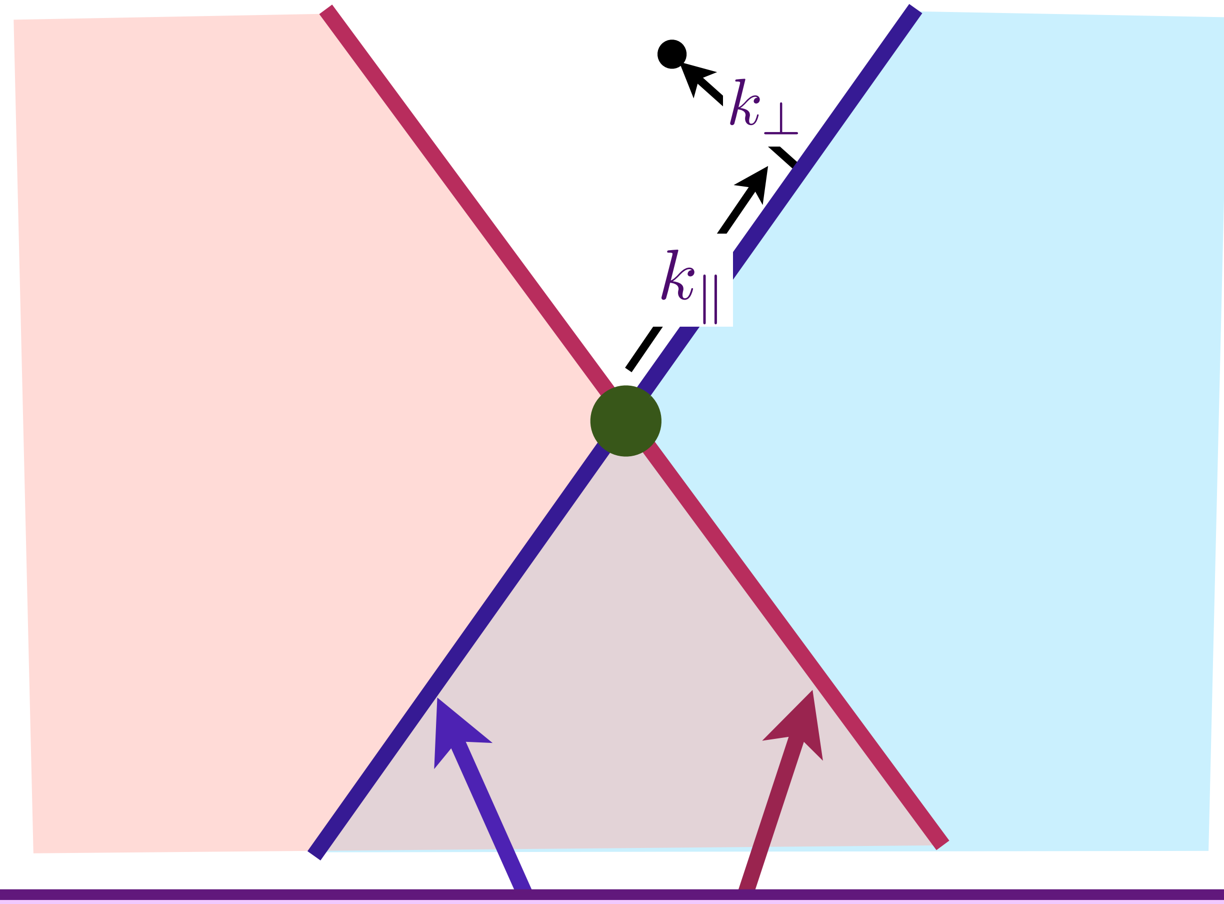

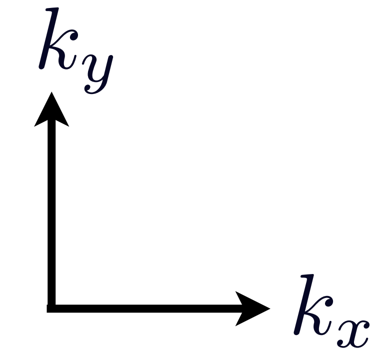

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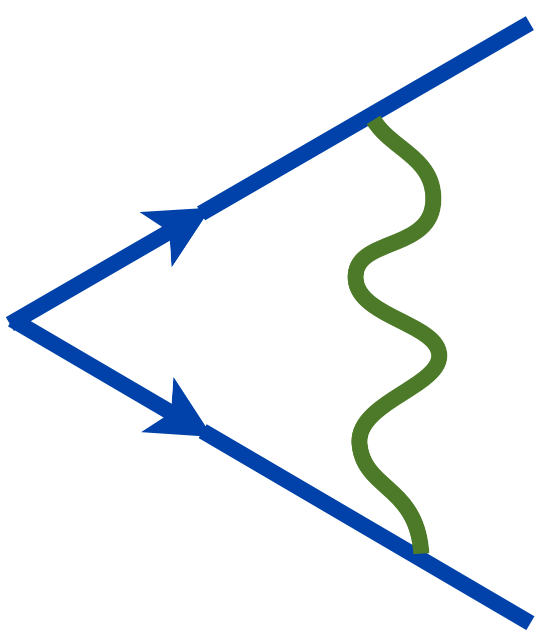

$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

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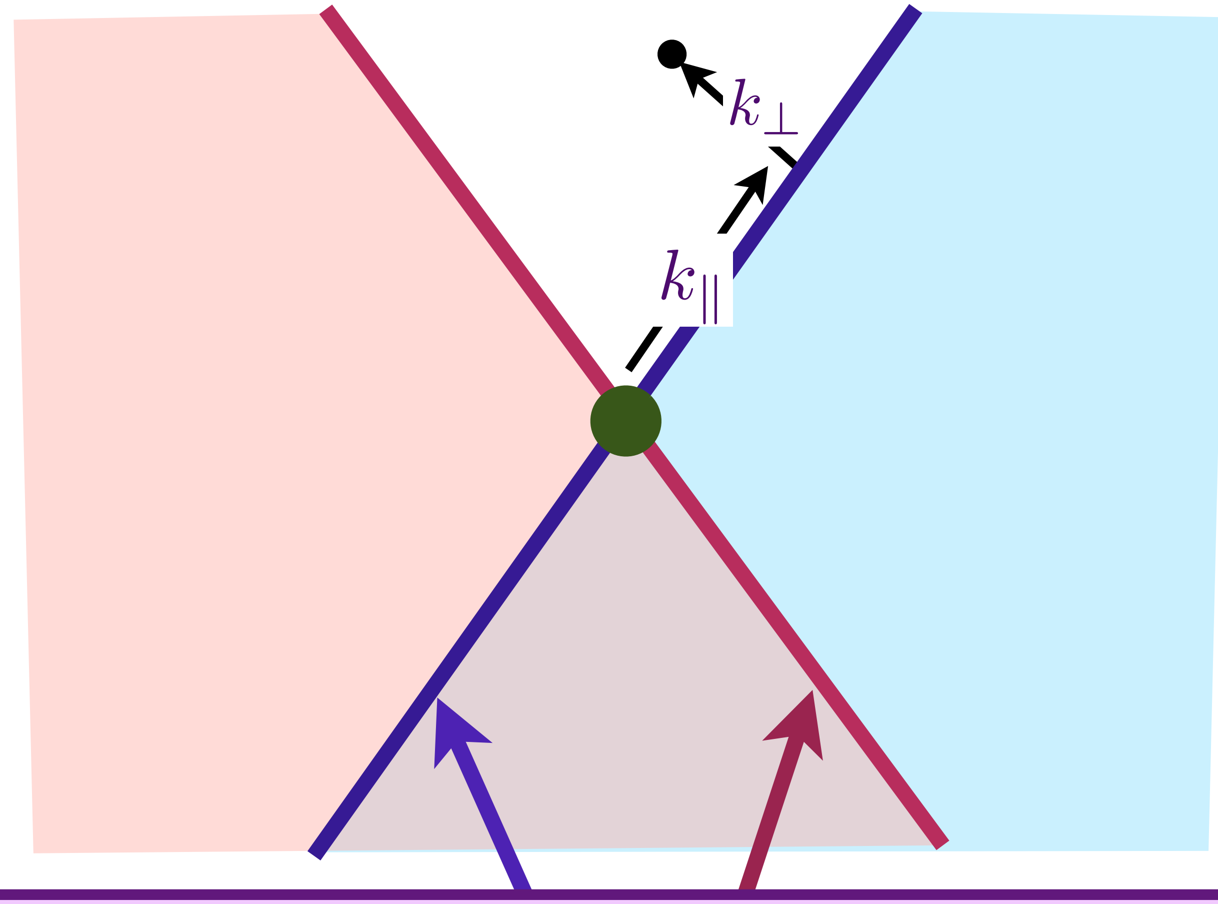

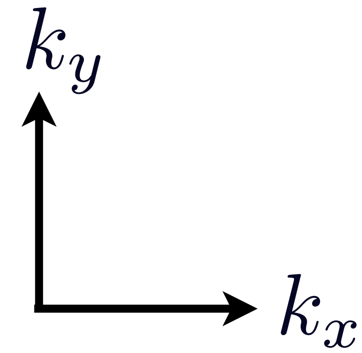

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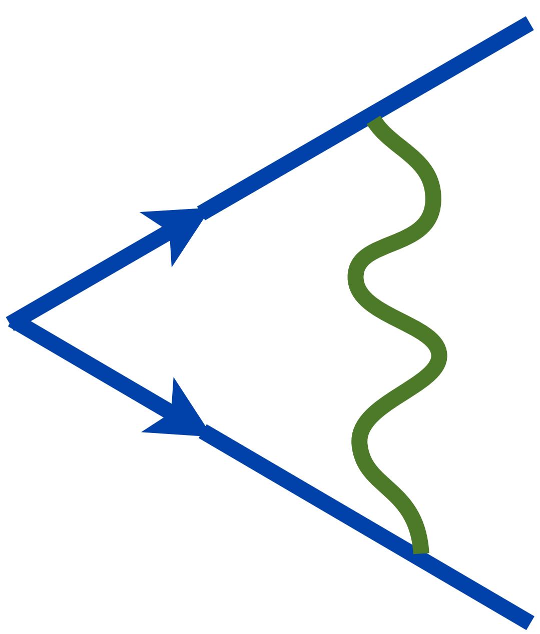
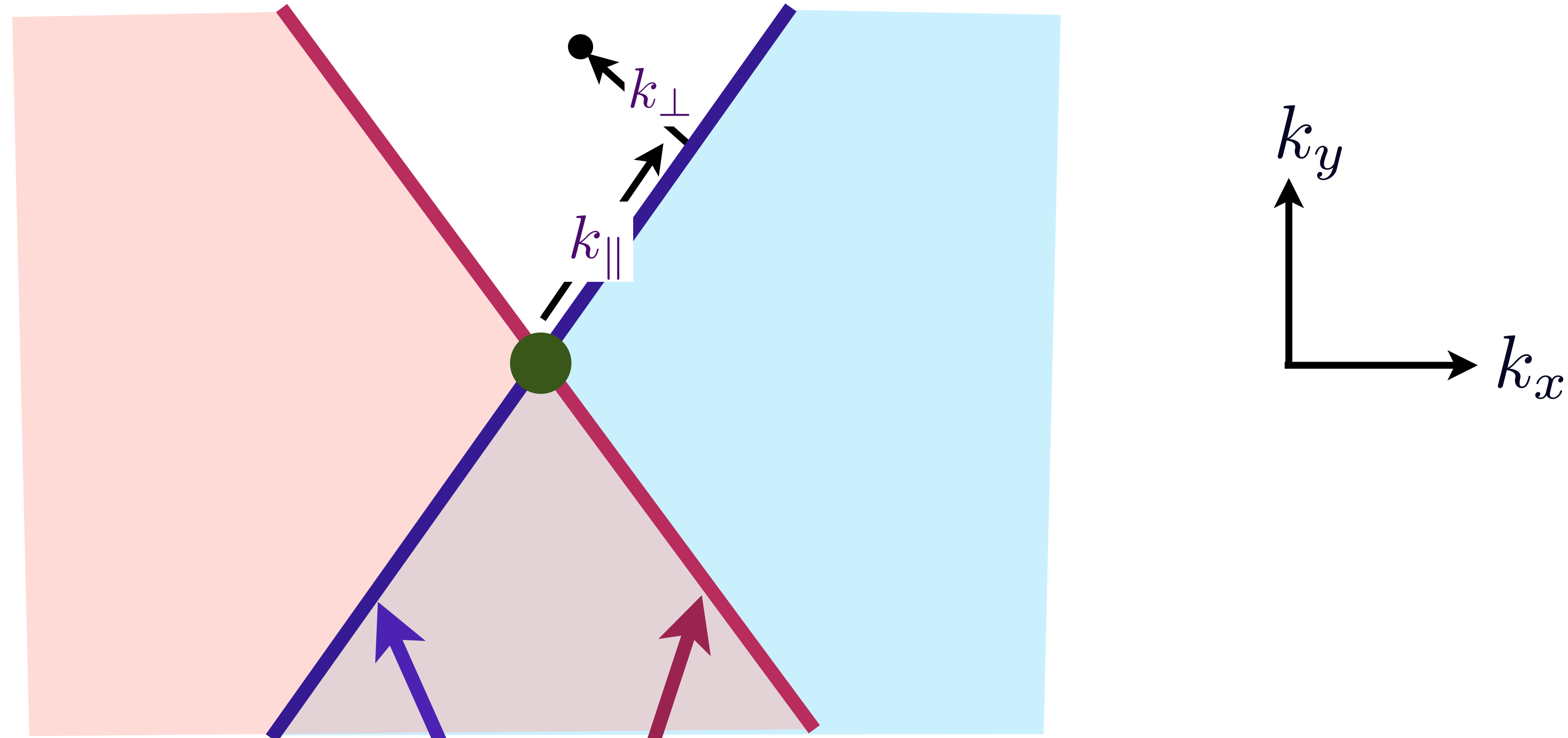


$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \underbrace{\left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}}$$

Cooper logarithm

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



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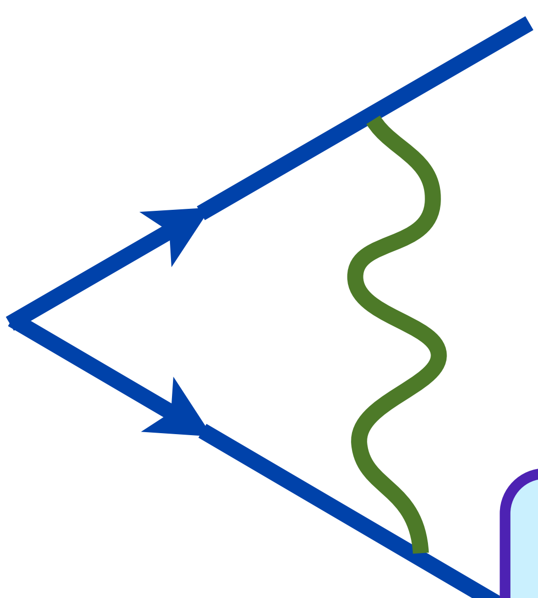

$$\int dk_\parallel \, \frac{1}{k_\parallel^2} \left(\frac{Z^2(k_\parallel)}{v_F(k_\parallel)} \right) \log \frac{k_\parallel^2}{\omega}$$

Spin fluctuation propagator

Cooper logarithm

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

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- $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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- Interference between BCS and quantum-critical logs.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Is there a log^2 for any other susceptibility ?

Is there a log^2 for any other susceptibility ?

Only one other

Outline

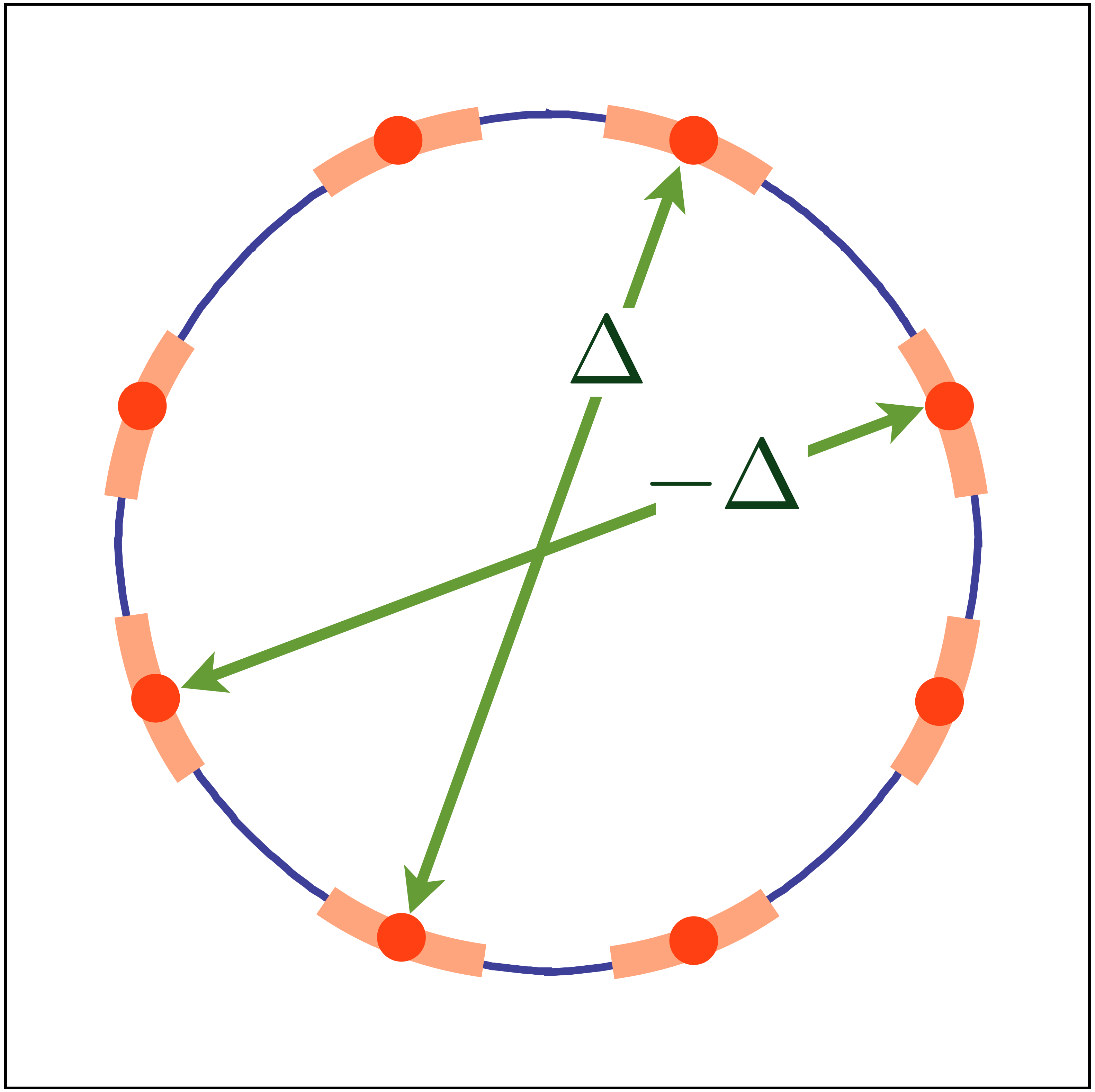
1. Low energy theory of spin density wave quantum critical point

2. Instabilities near the quantum critical point:
   A. Unconventional superconductivity
   B. $2k_F$ bond-nematic ordering

3. Phase diagram in a magnetic field
   Electron versus hole doping

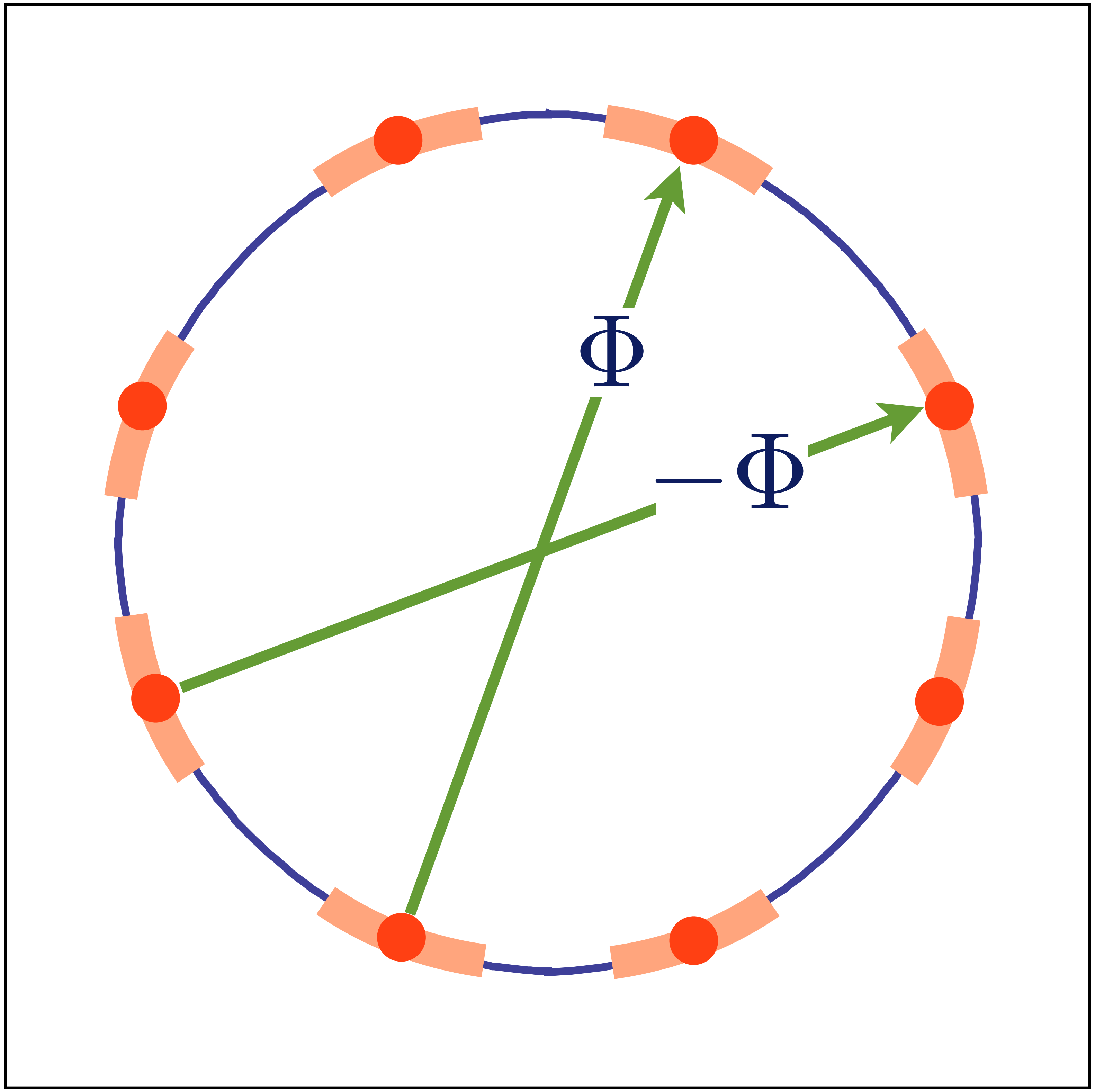
4. Possible exotic intermediate phases
   Fermi pockets without spin density wave order

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at ***and near*** hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



$\mathbf{Q}$ is '$2k_F$' wavevector

Unconventional particle-hole pairing at *and near* hot spots

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of Φ susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{3\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of Φ susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{3\pi(1+\alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

- Emergent pseudospin symmetry of low energy theory also induces $\log^2$ in a single "d-wave" particle-hole channel. Fermi-surface curvature reduces prefactor by 1/3.

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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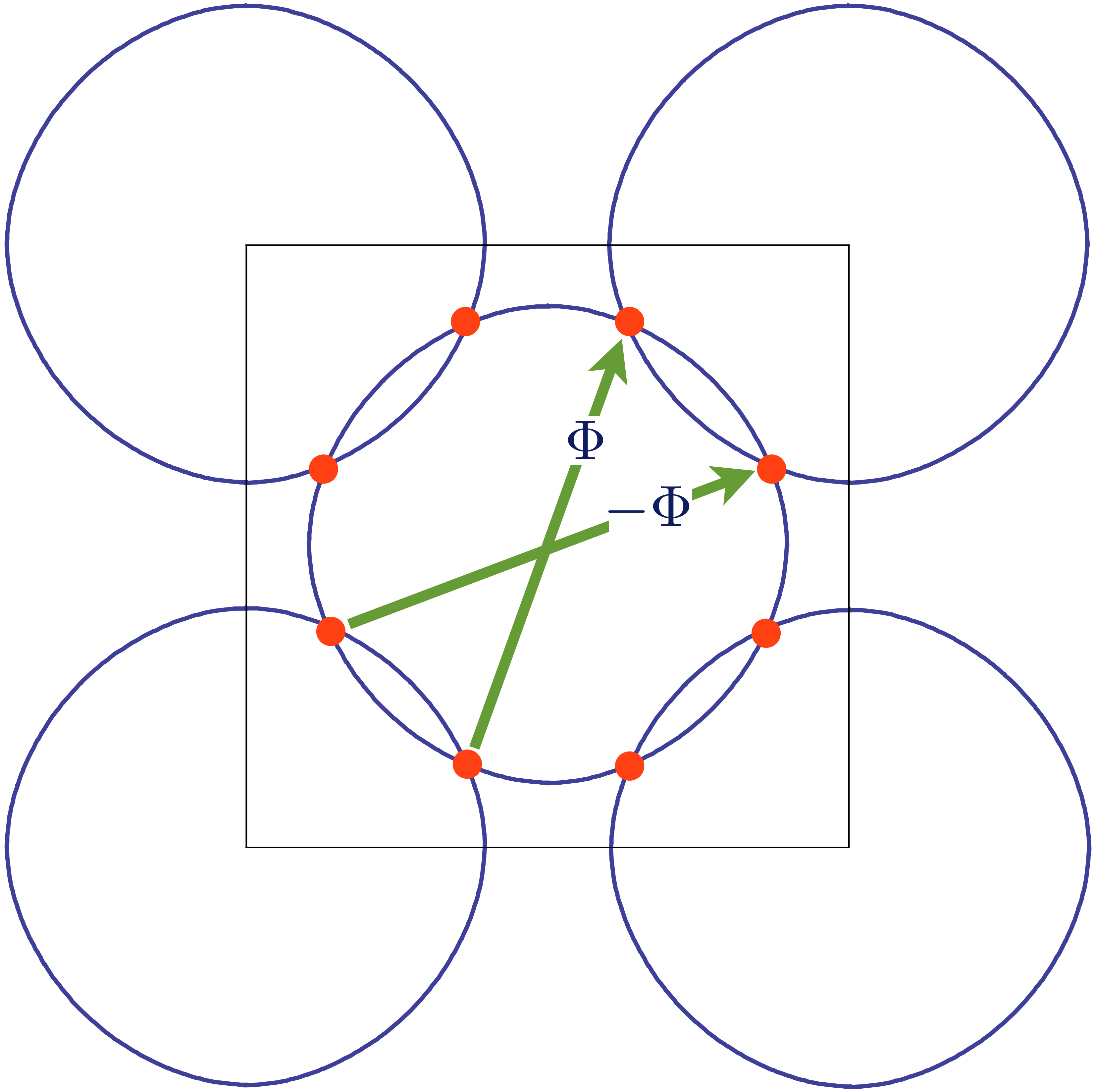
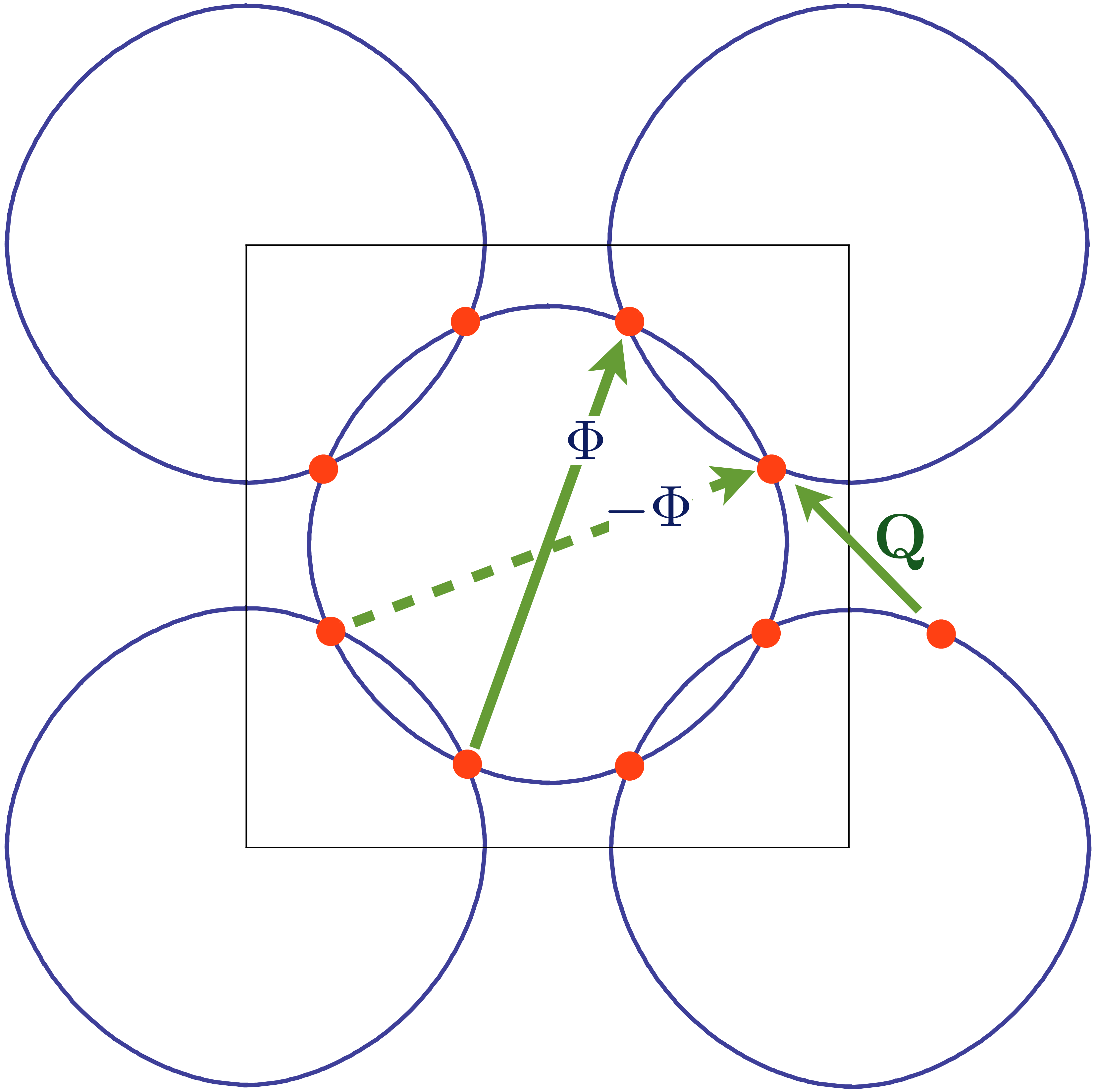
- Φ corresponds to a $2k_F$ *bond-nematic order*

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

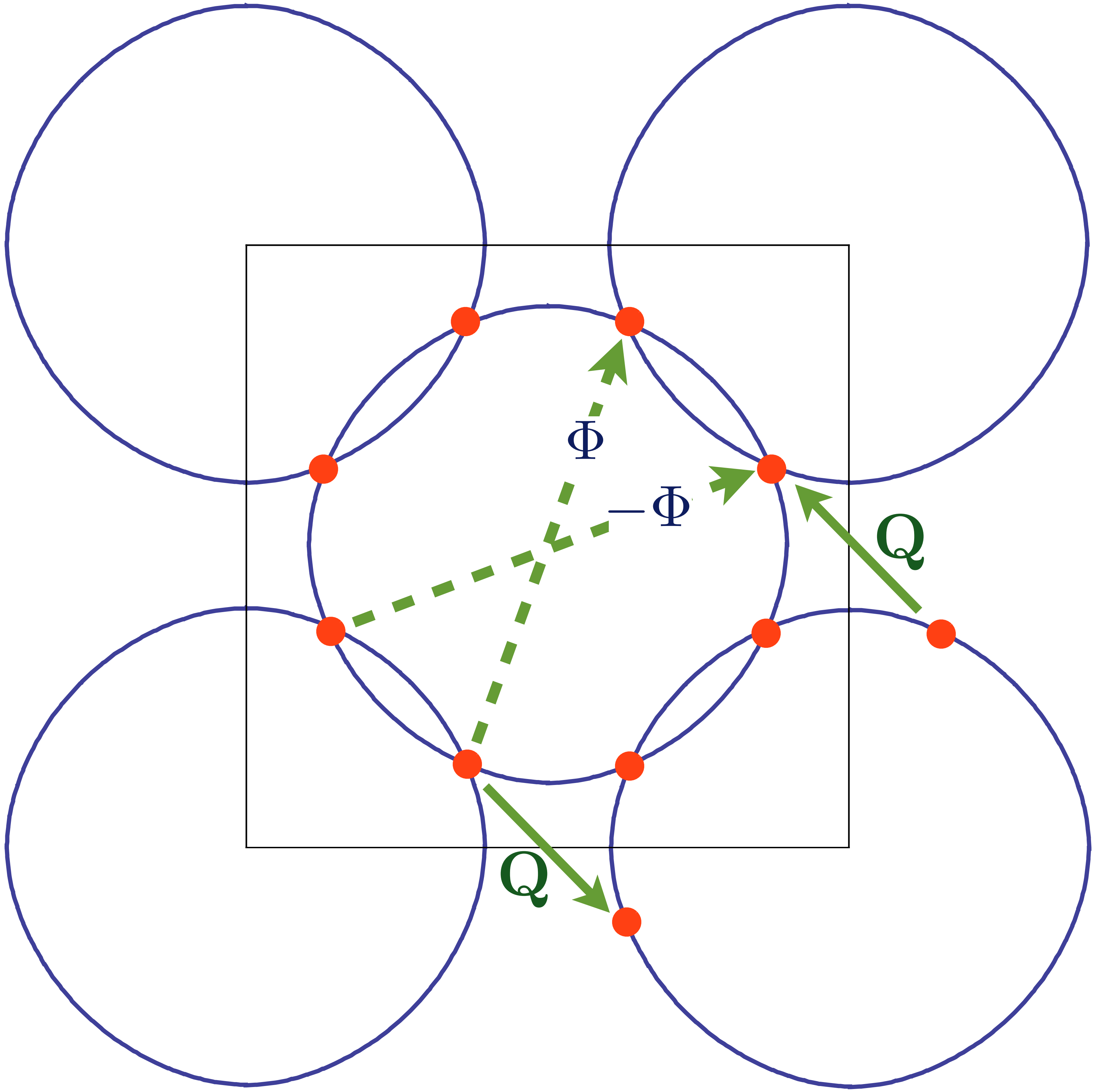
$2k_F$ bond-nematic order



$2k_F$ bond-nematic order



$2k_F$ bond-nematic order



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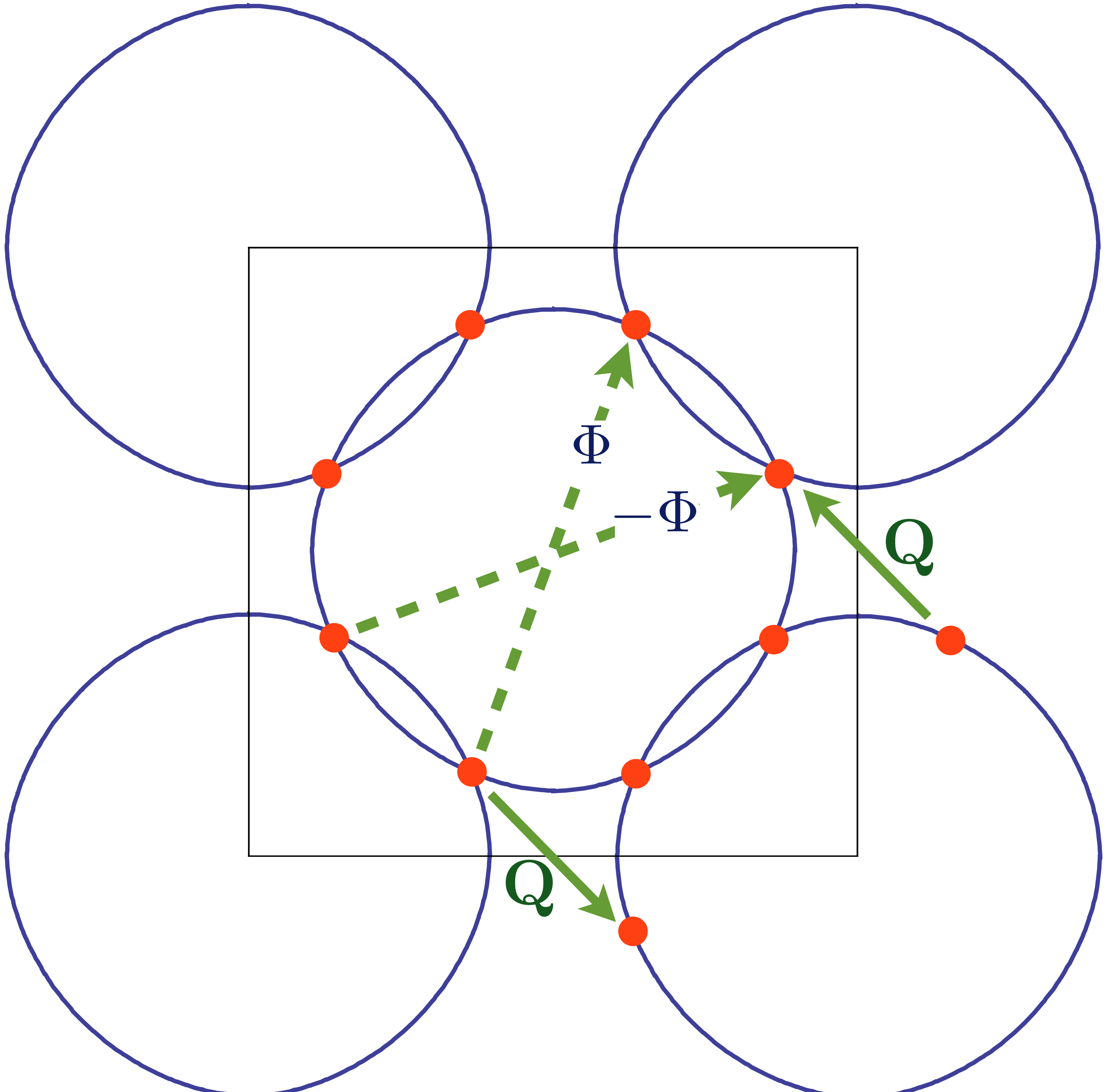
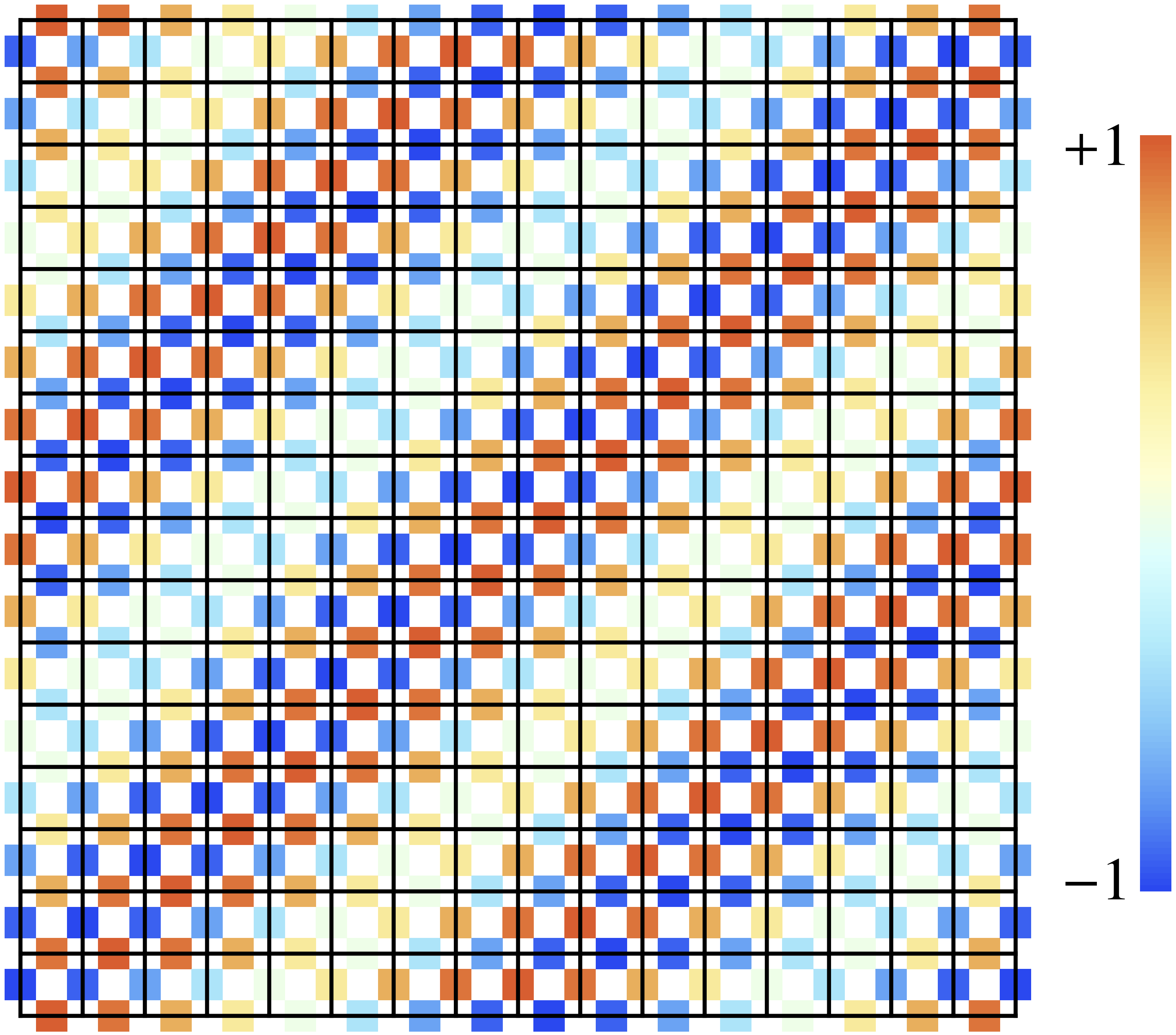


$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

$2k_F$ bond-nematic order



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

No modulations on sites, $\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle$ is modulated only for $\mathbf{r} \neq \mathbf{s}$.

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

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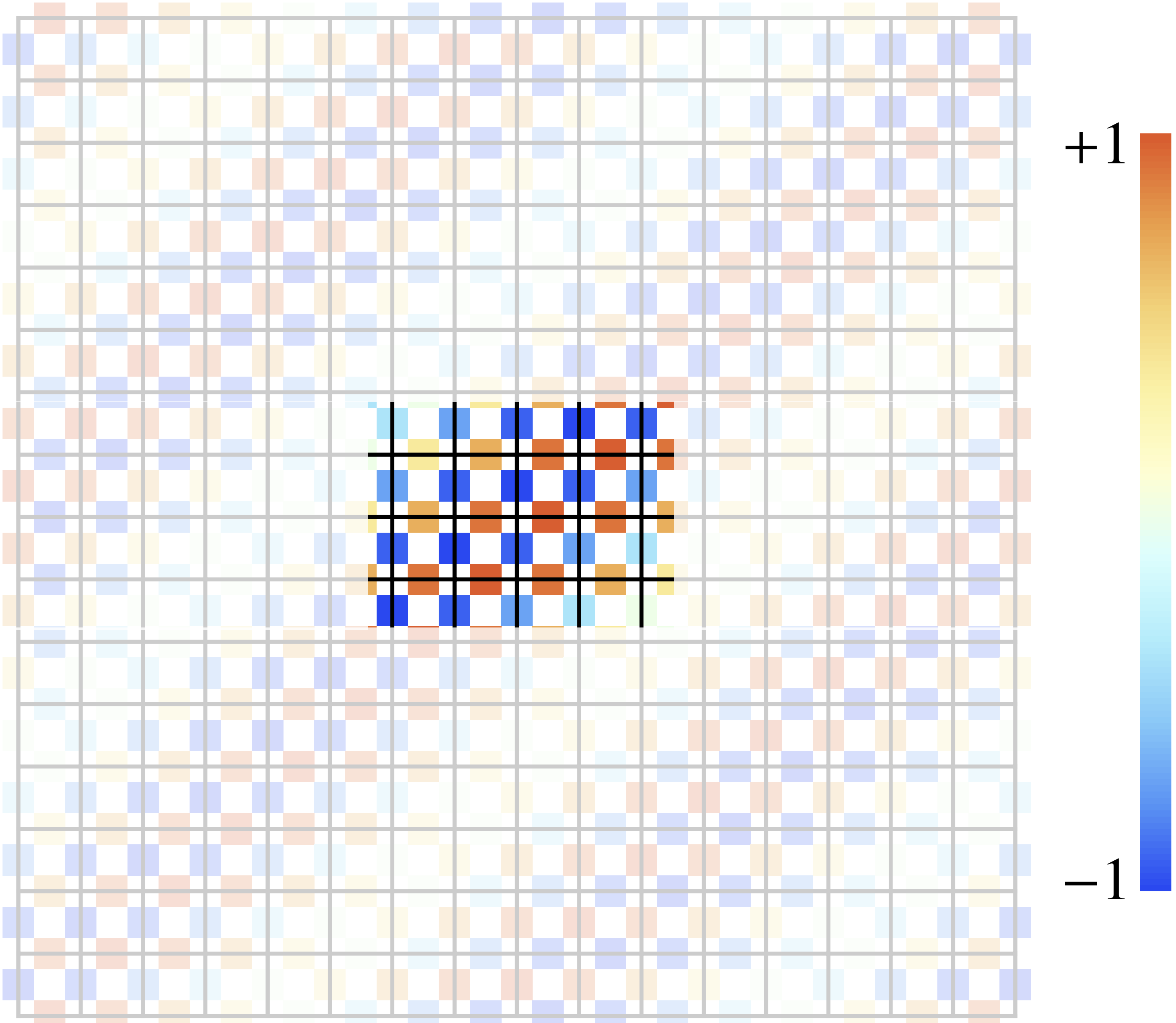

Local Ising nematic order with an envelope which oscillates

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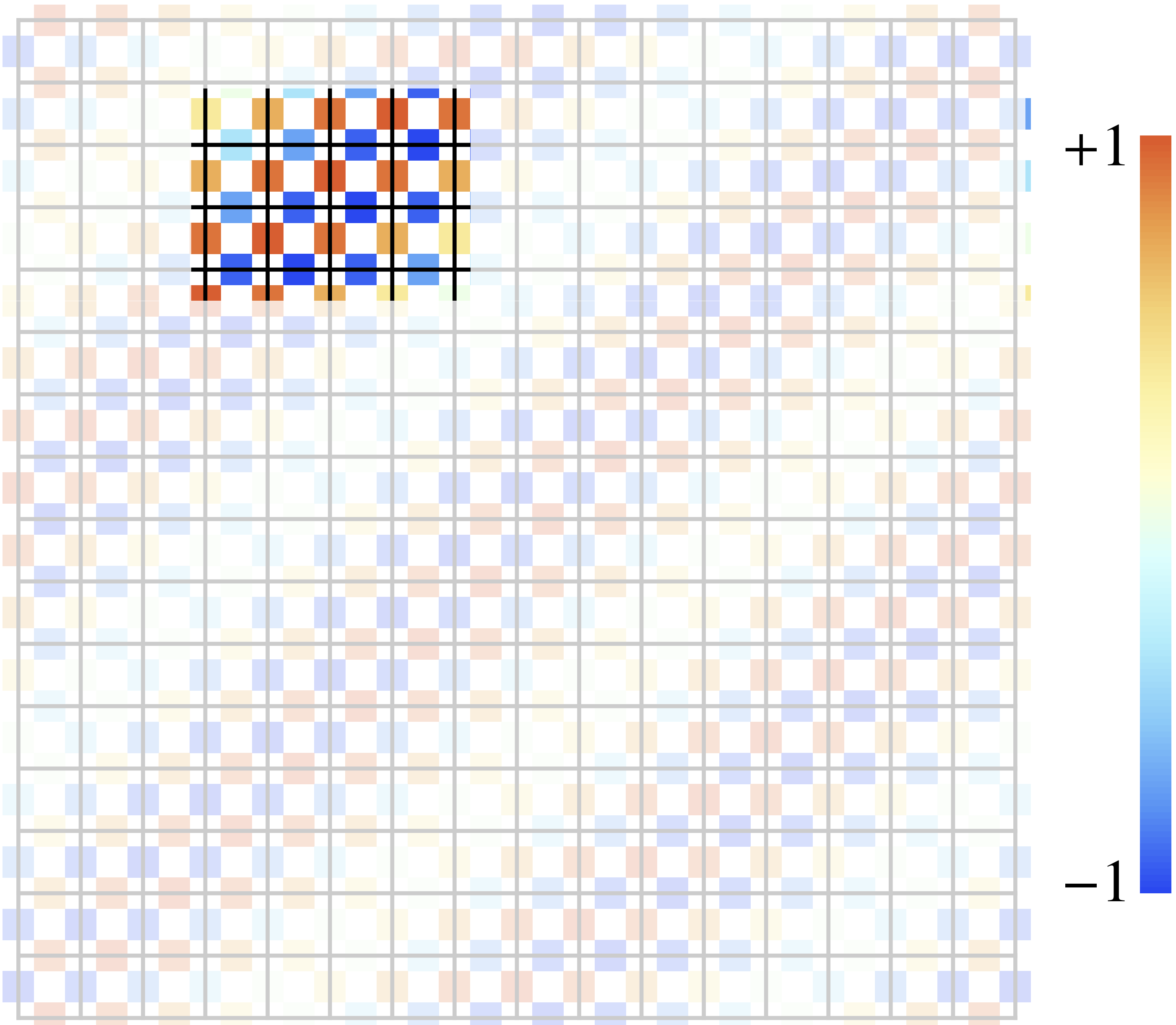

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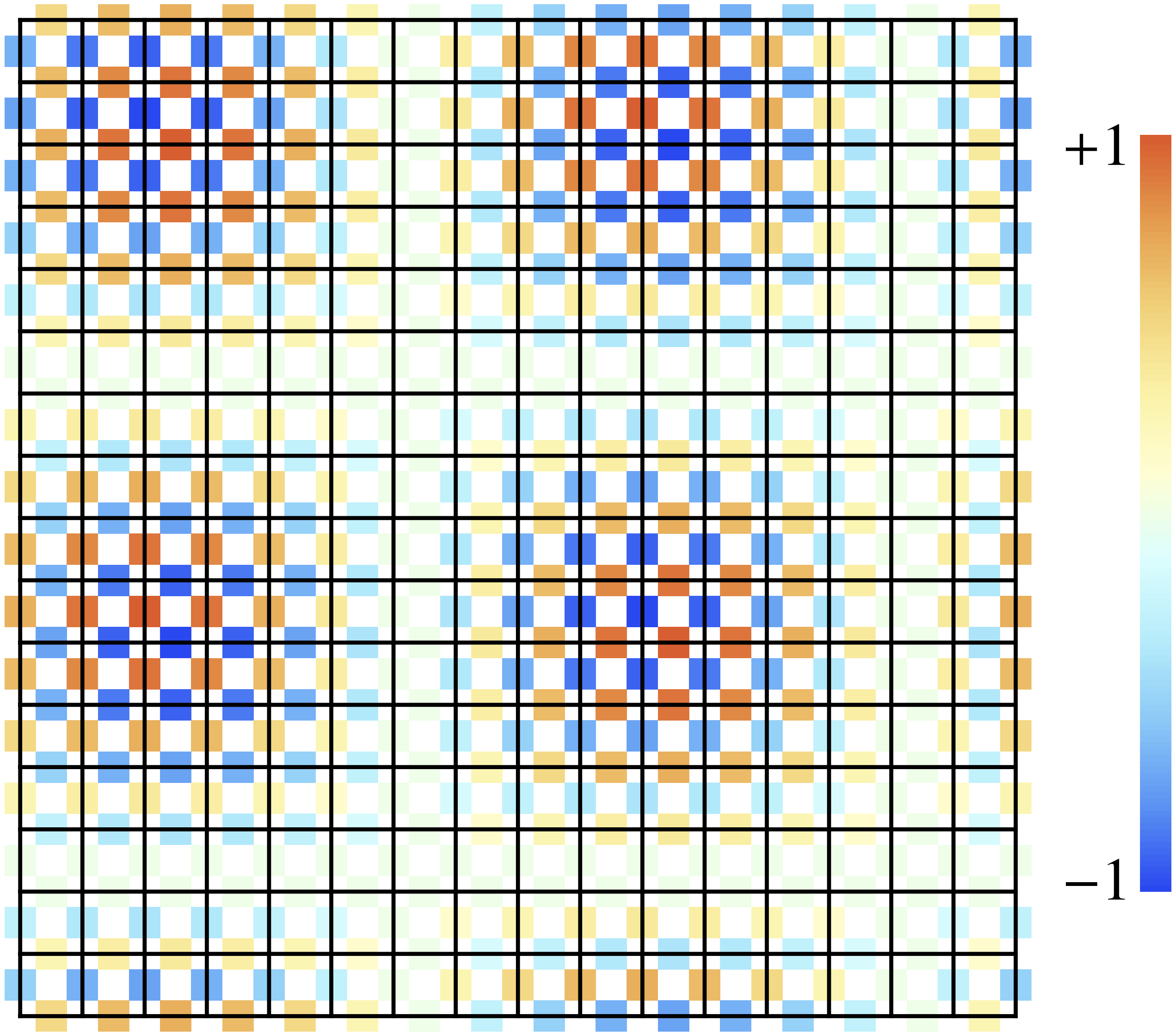

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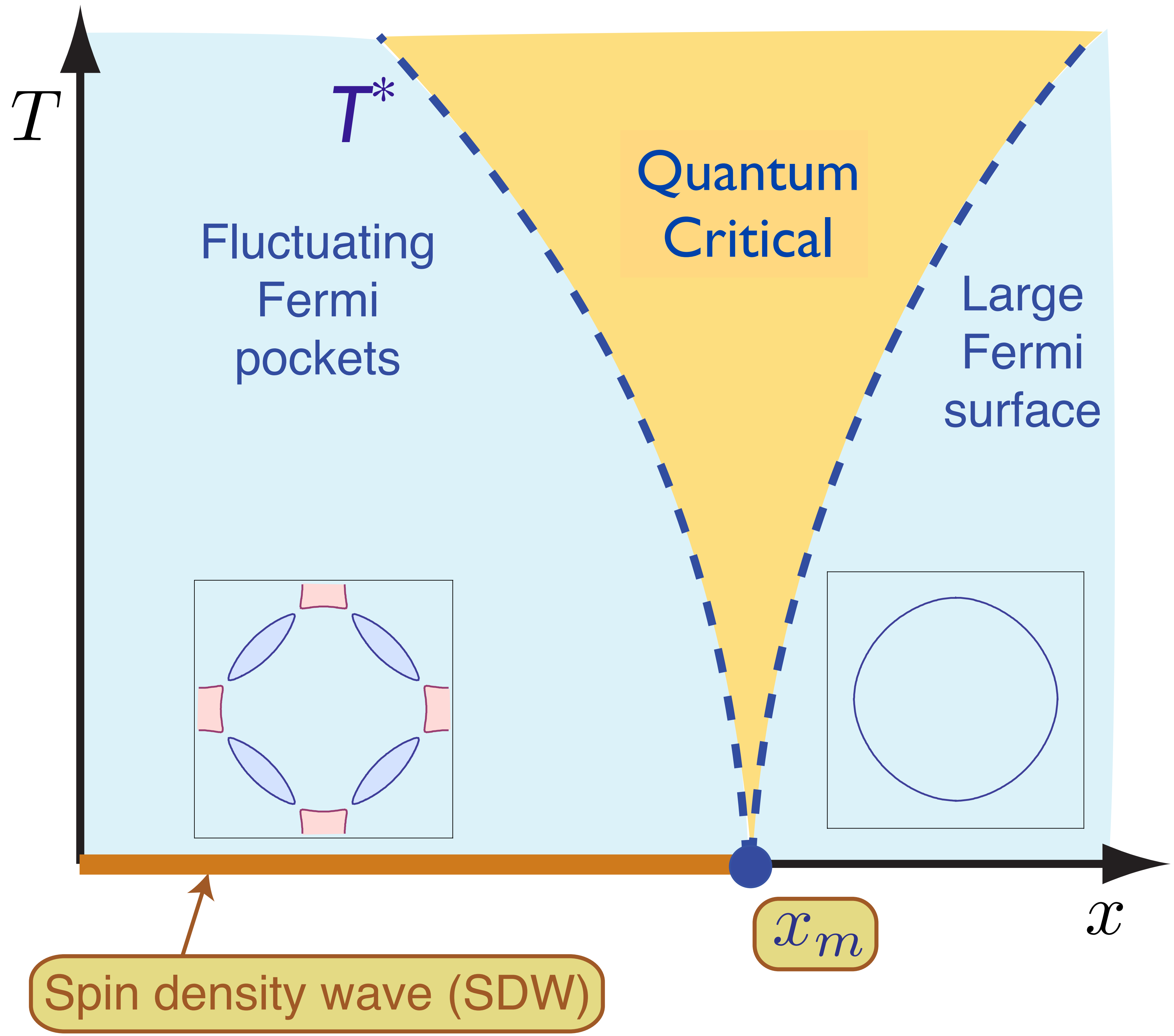
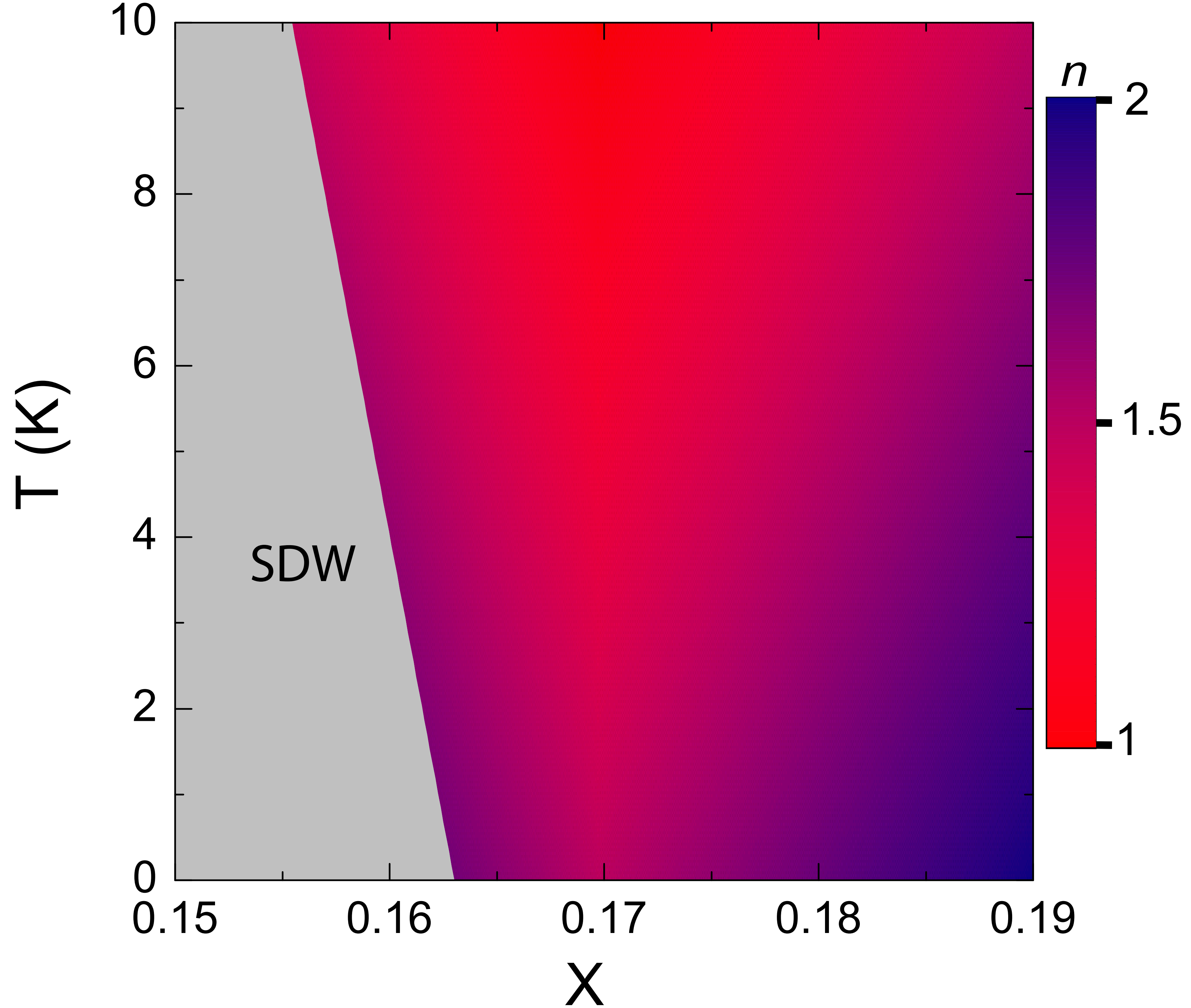
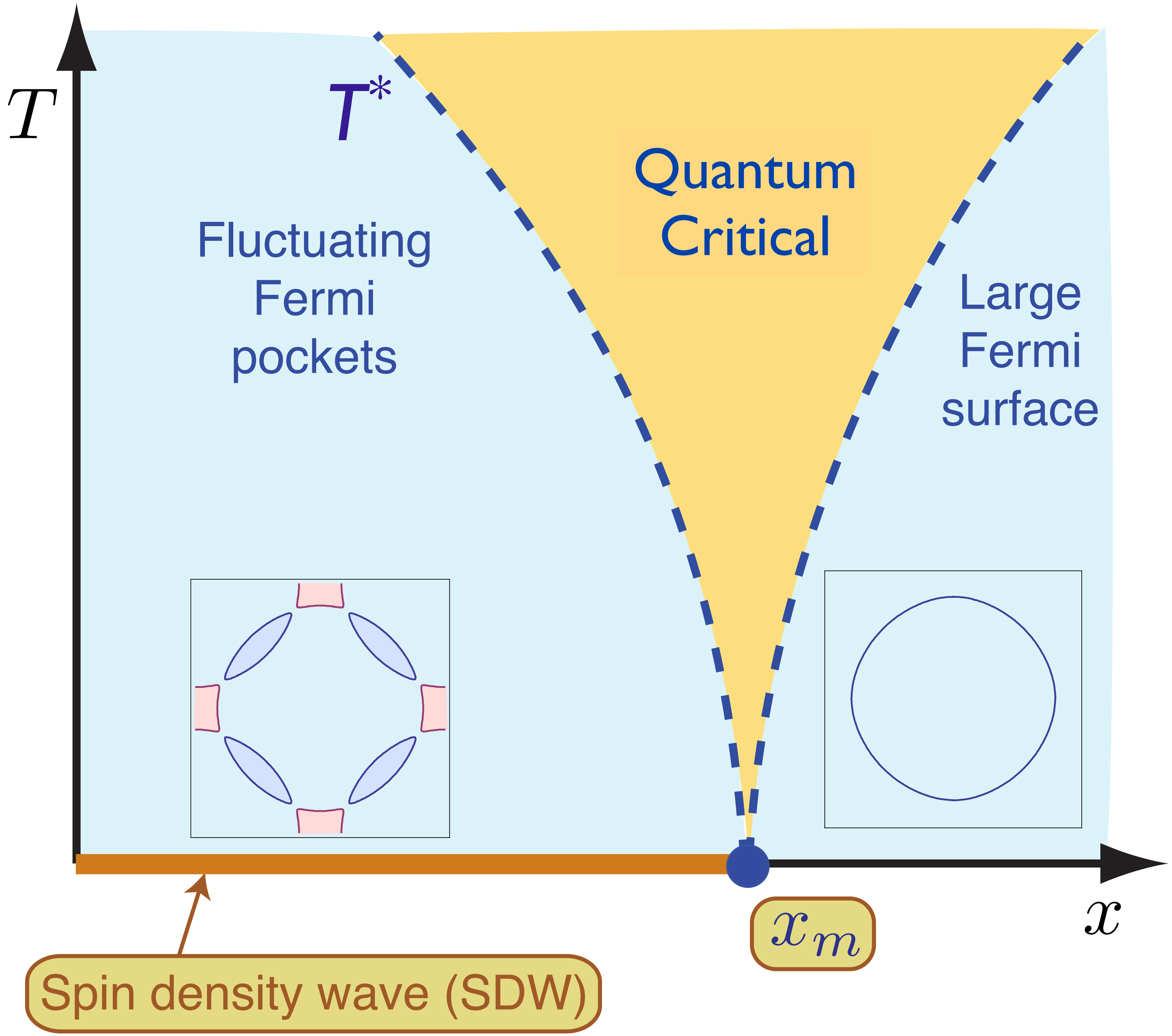


Underlying SDW ordering quantum critical point in **metal** at $x = x_m$



Resistivity
$\rho(T) \sim T^n$

The electron-doped copper oxide superconductor $Pr_{2-x}Ce_xCuO_4$, with superconductivity suppressed by an applied magnetic field.
Figure prepared by K. Jin and and R. L. Greene based on Fig. 56 in N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



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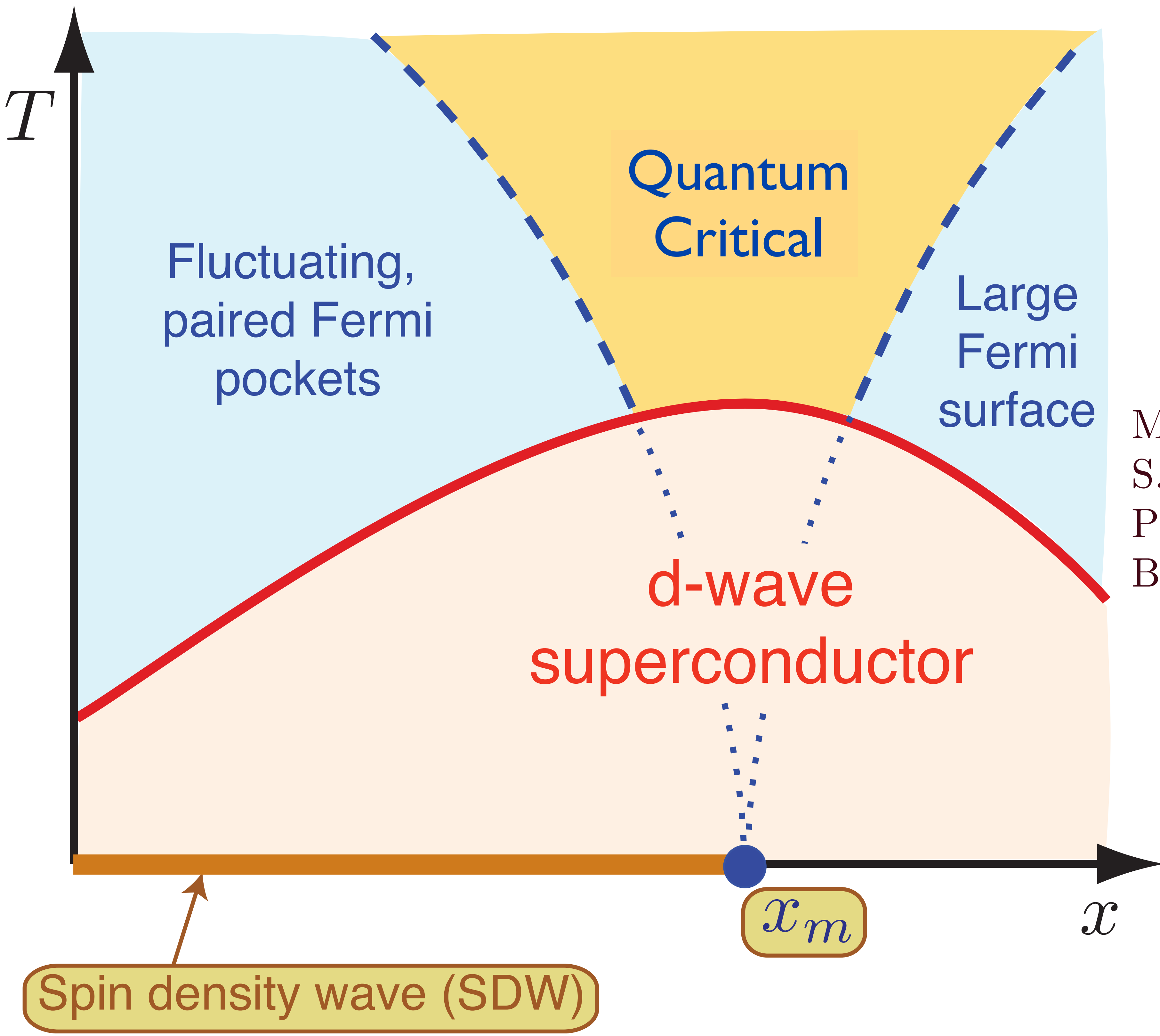
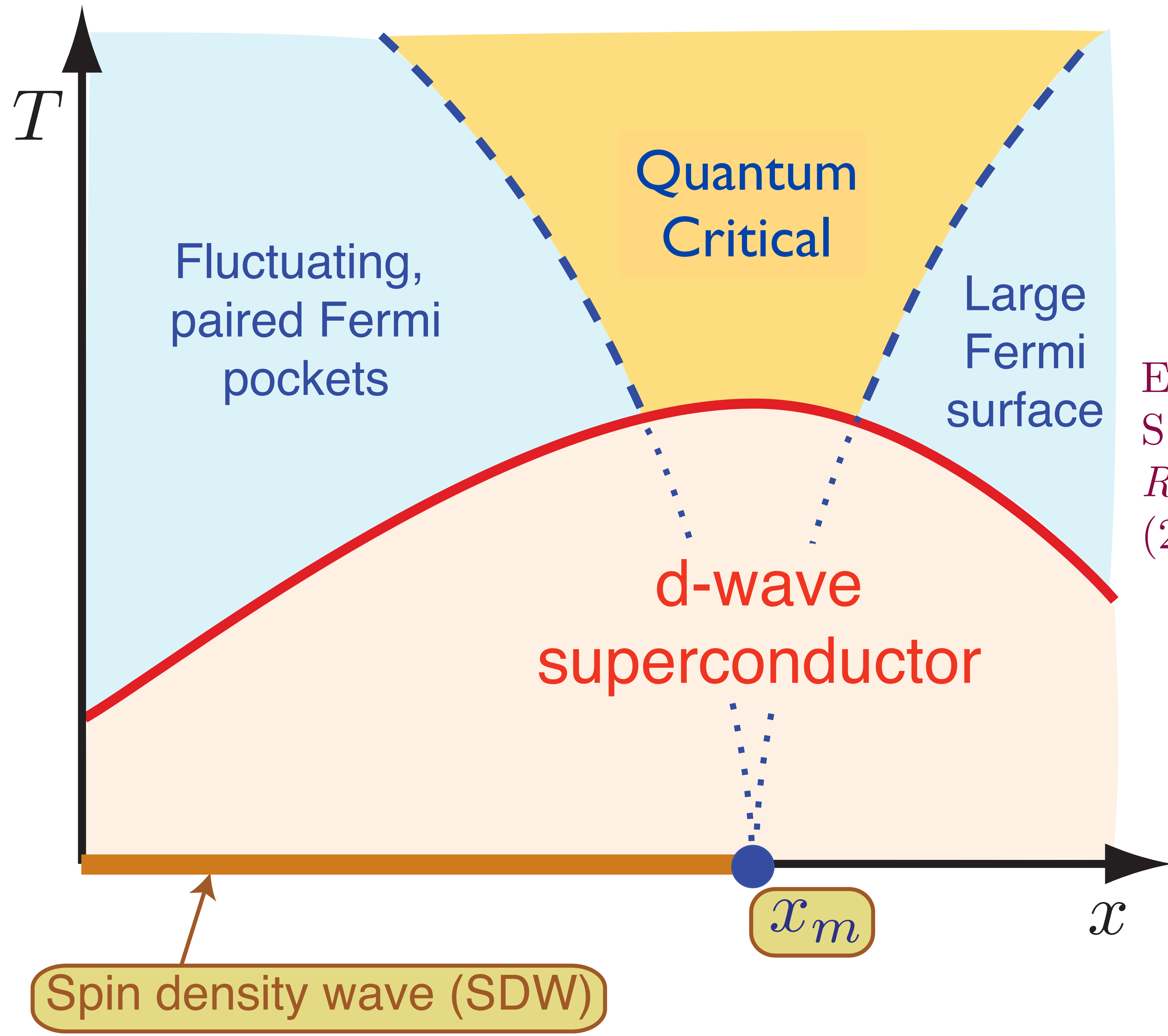


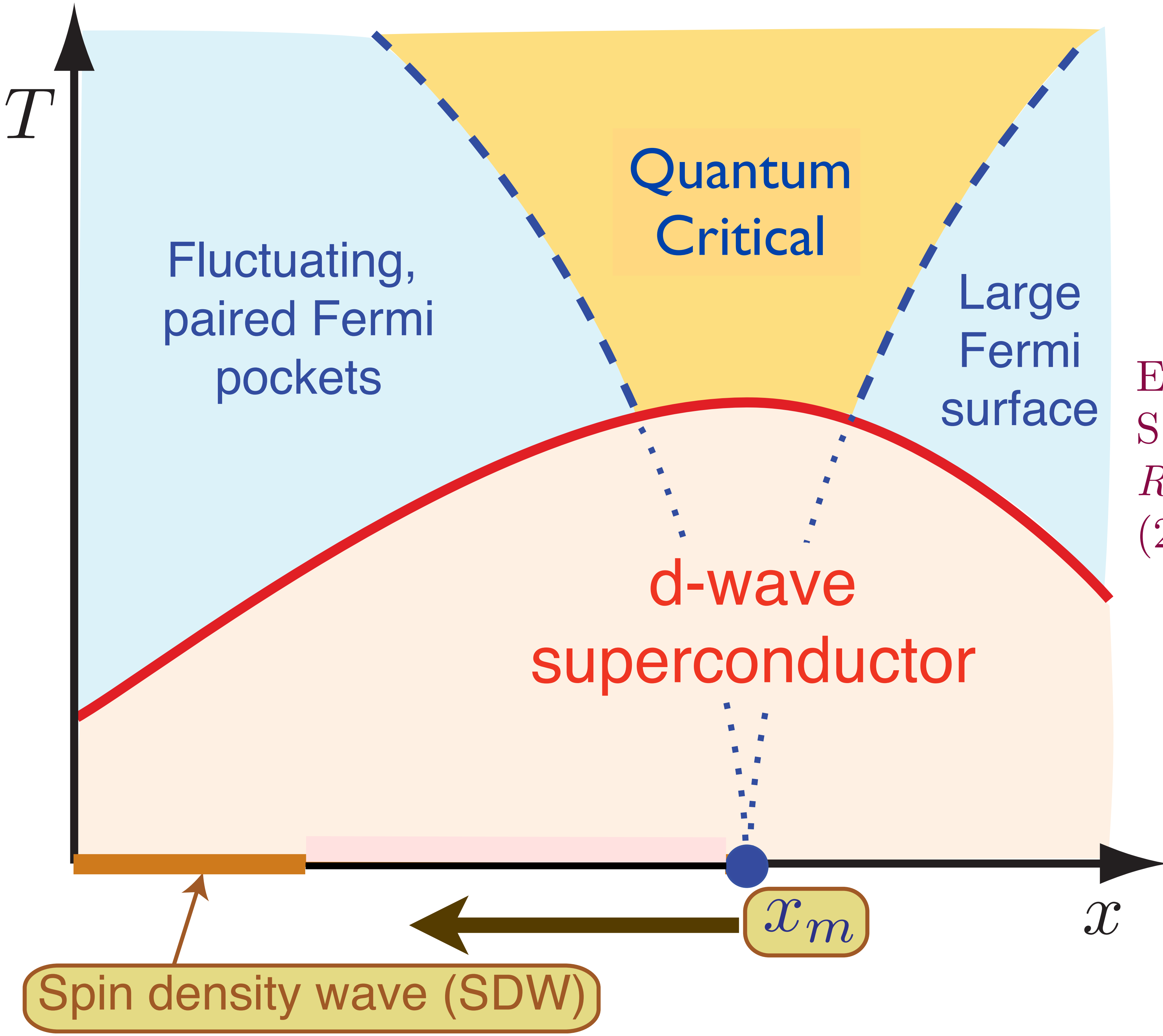
M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.



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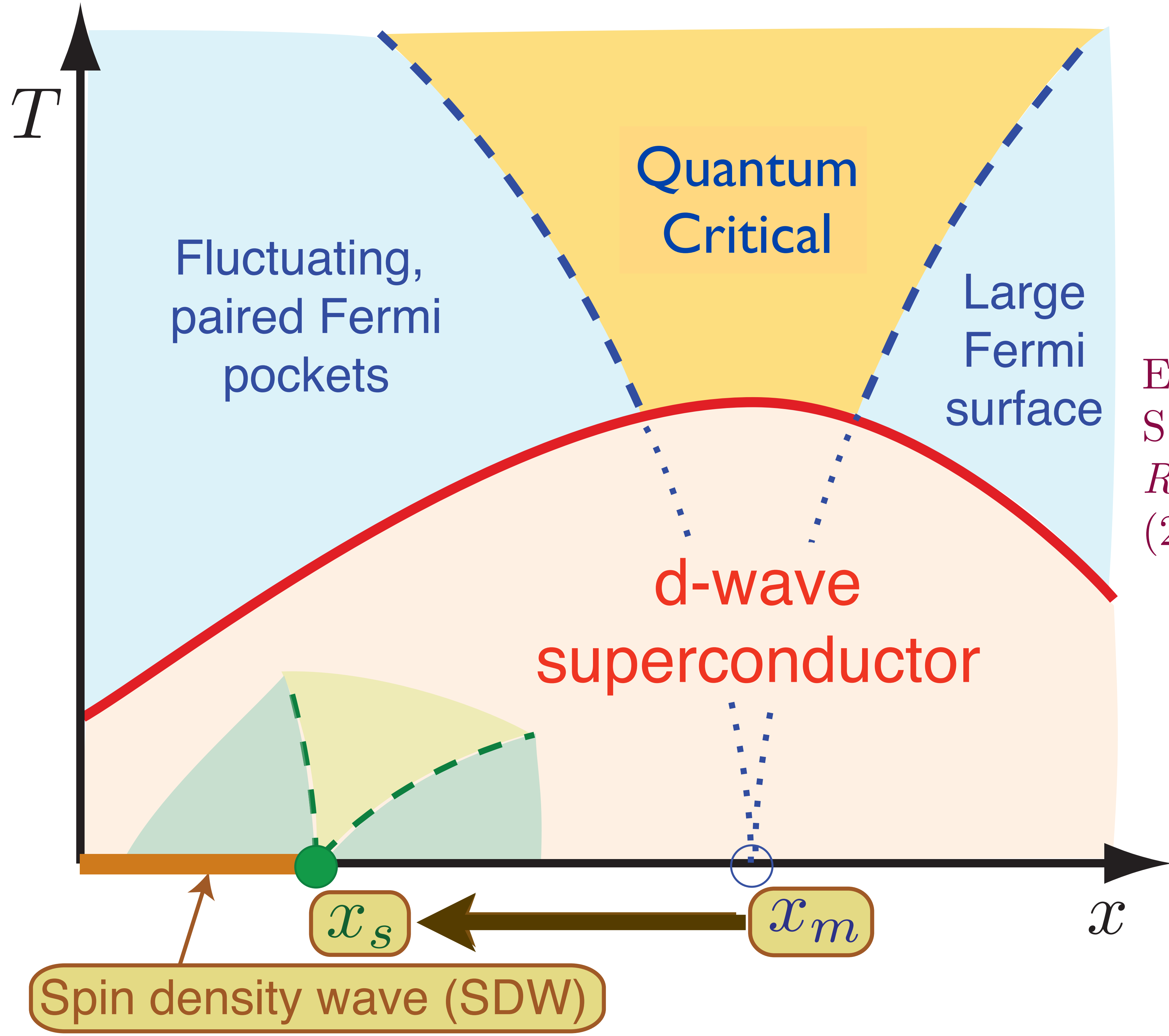

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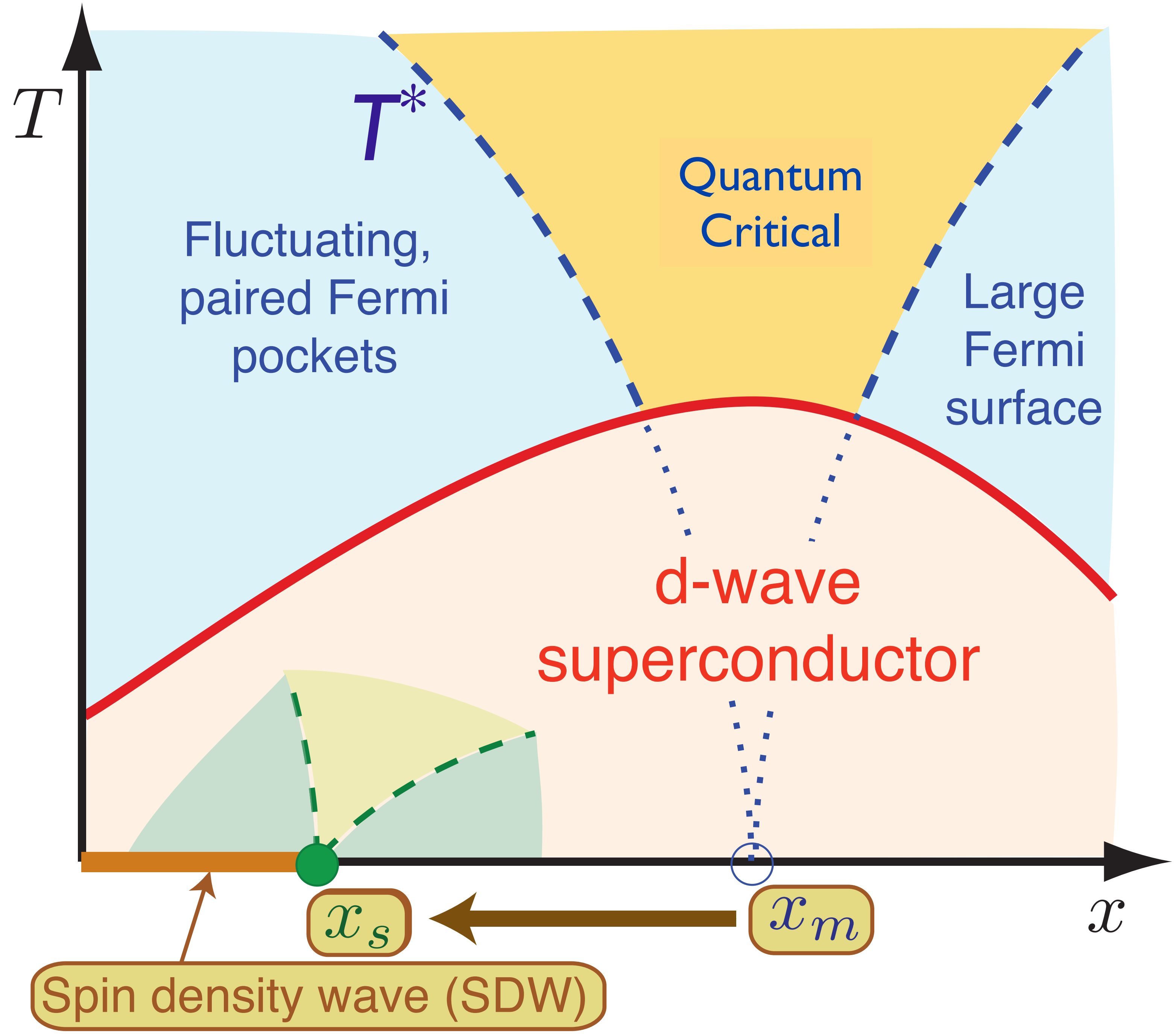
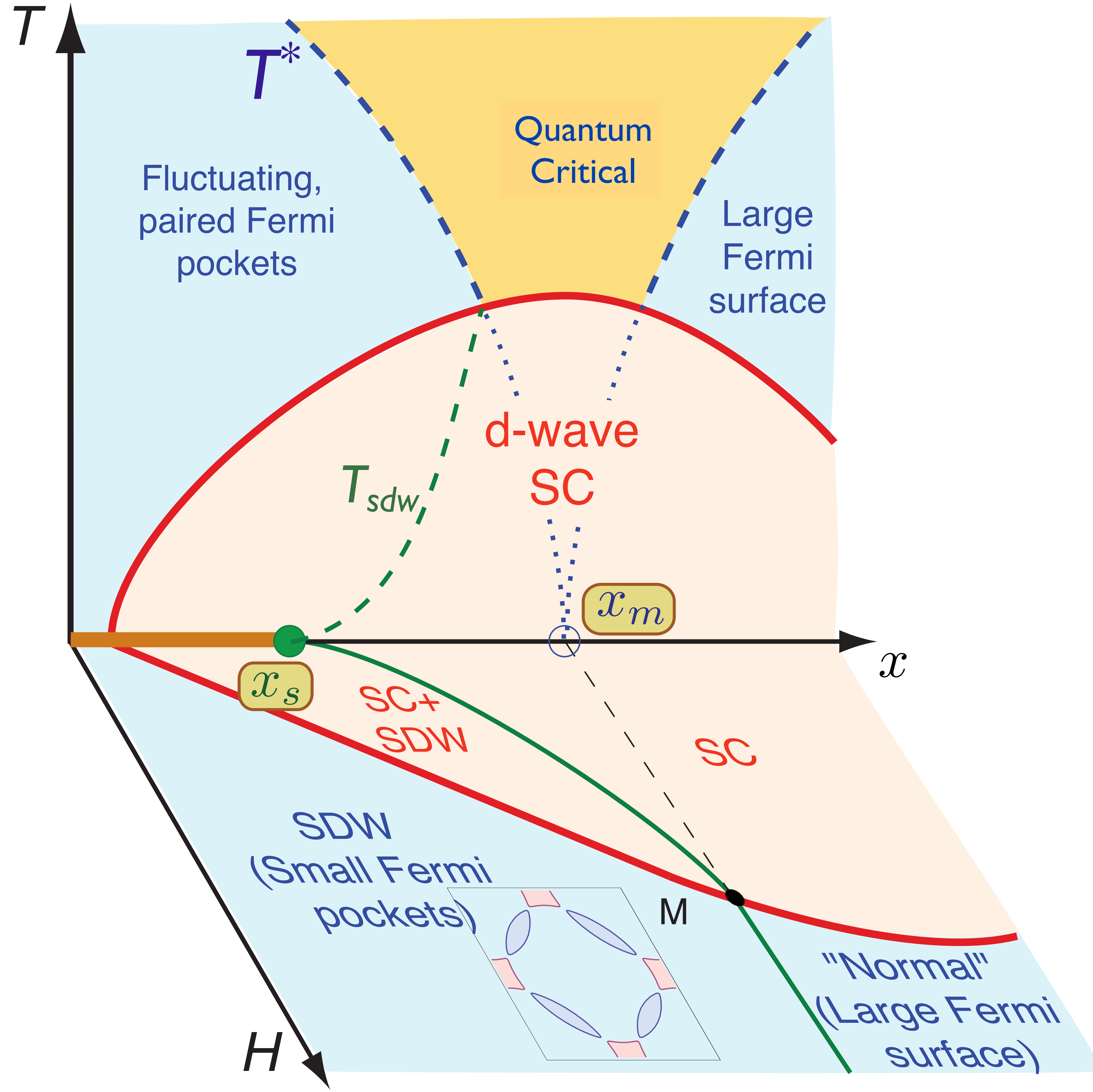


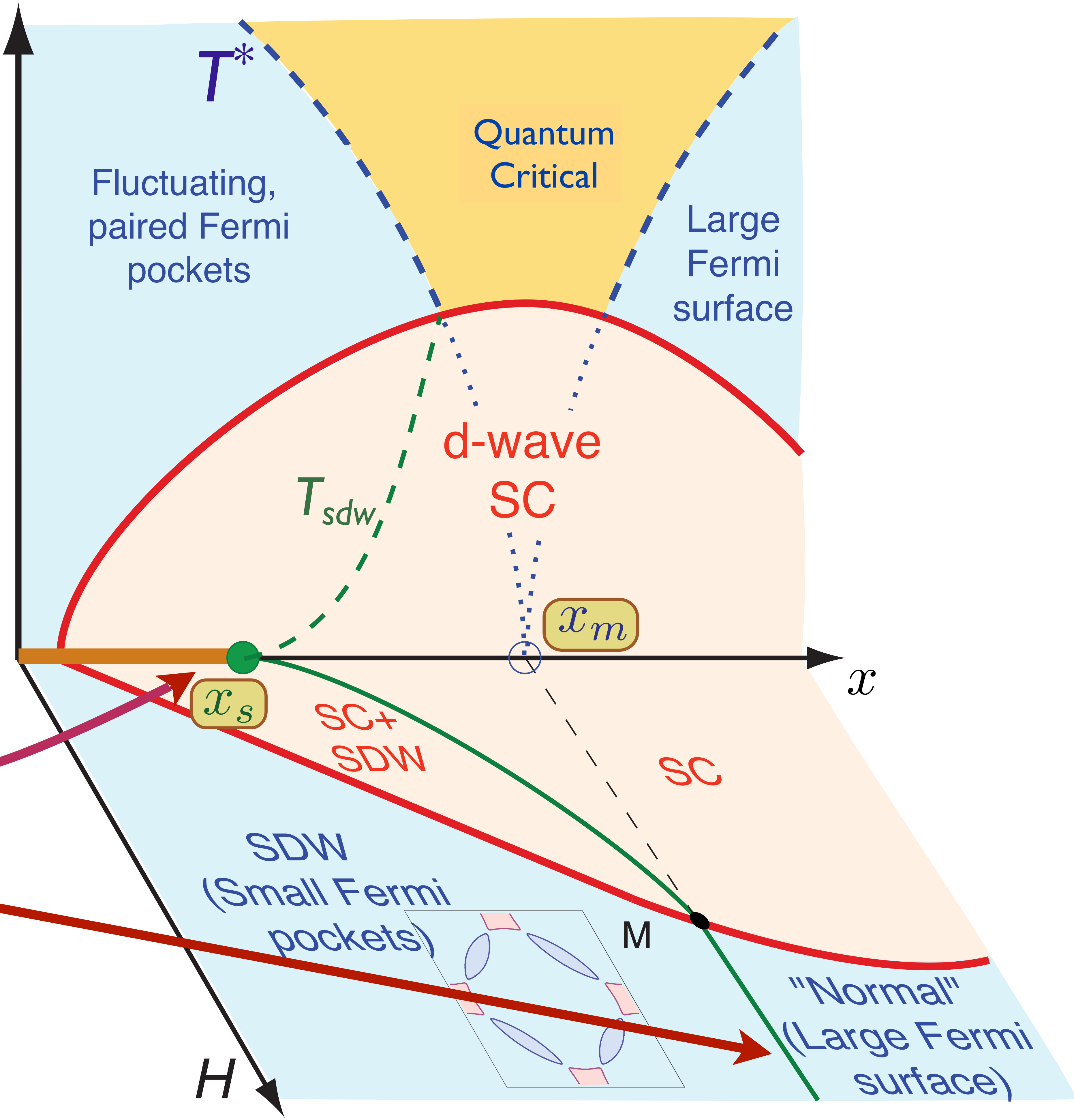
T
T^*
Fluctuating, paired Fermi pockets
Quantum Critical
Large Fermi surface
d-wave superconductor
x_s
x_m
x
Spin density wave (SDW)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

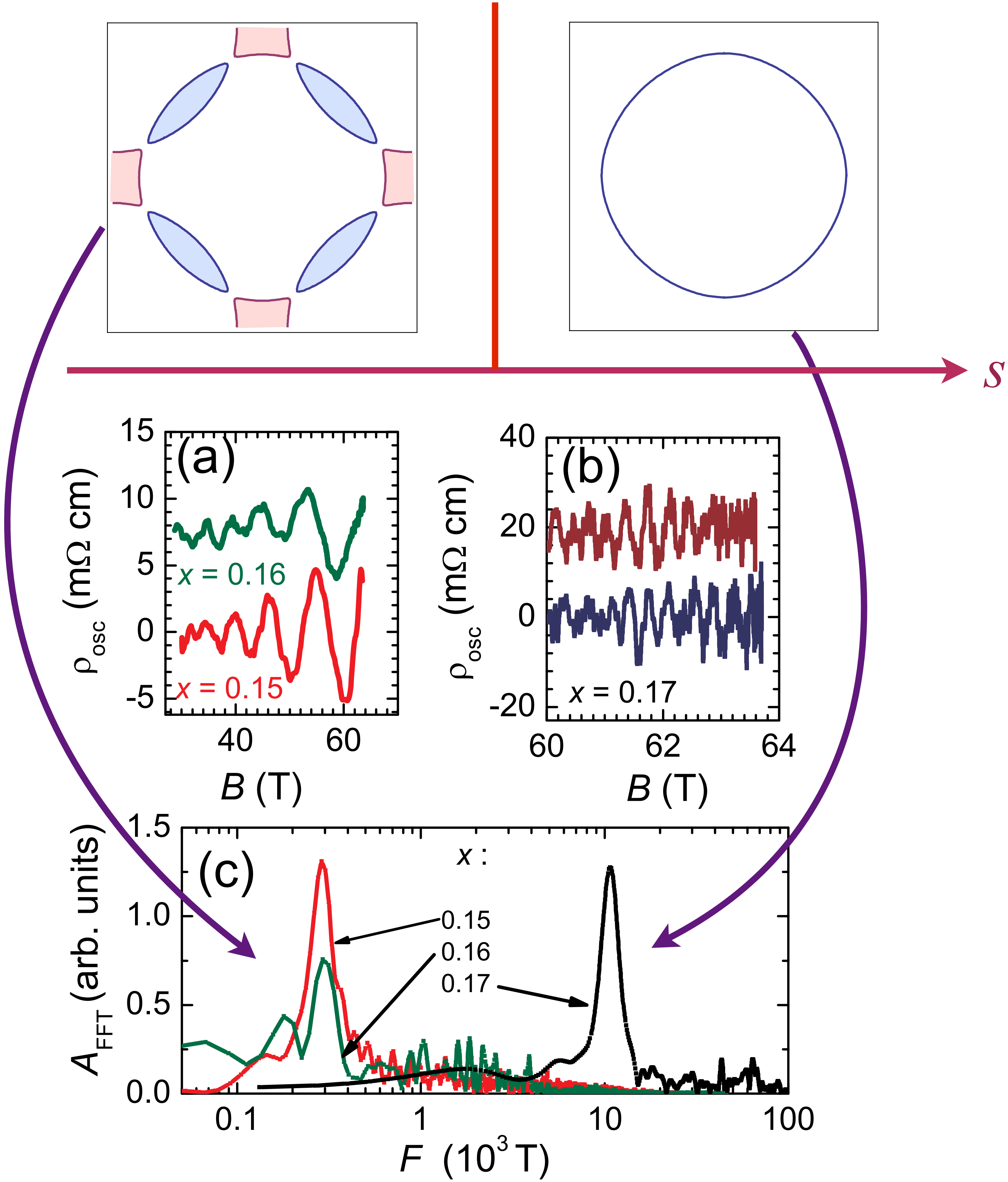


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.



Quantum oscillations



$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

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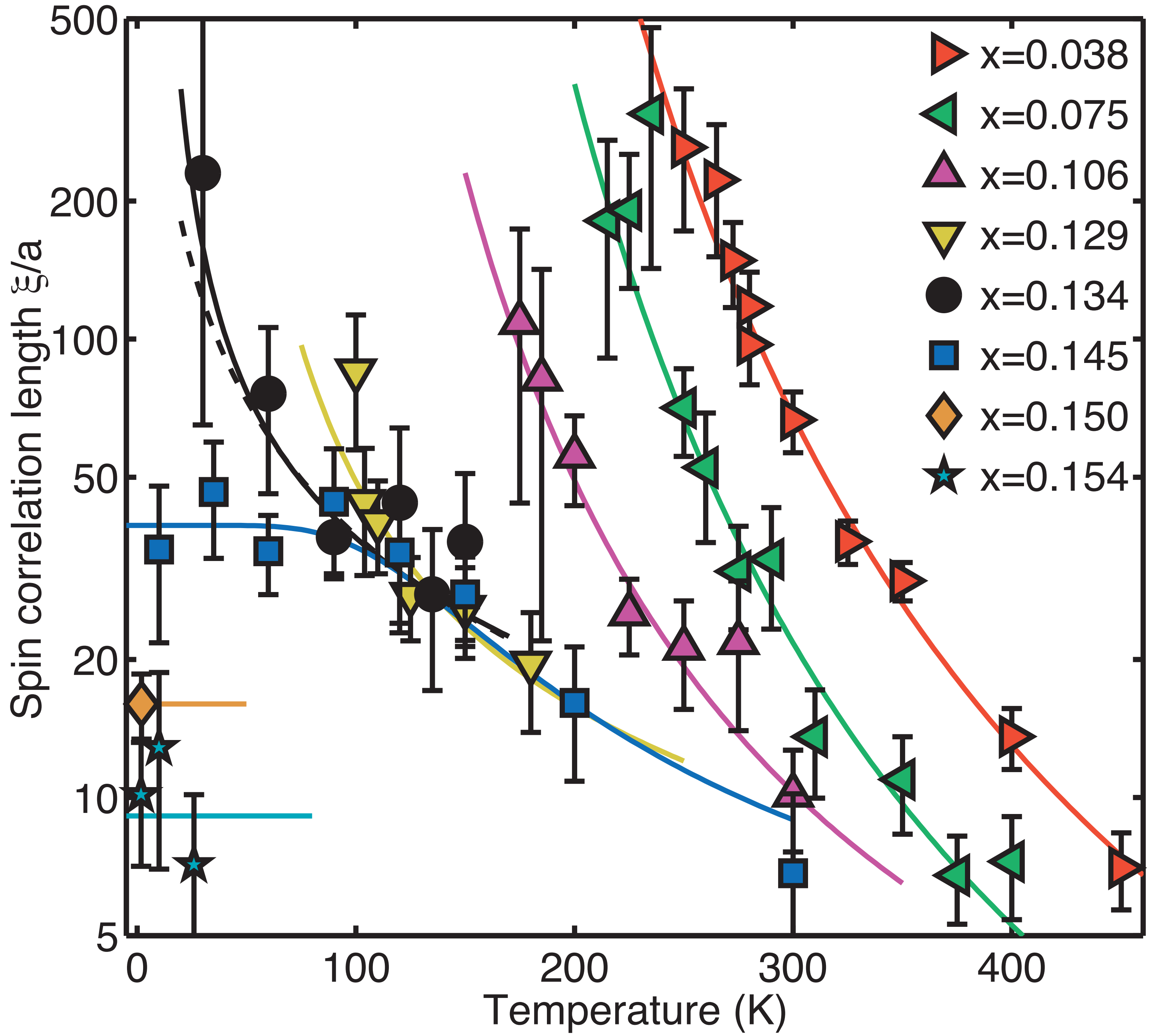

E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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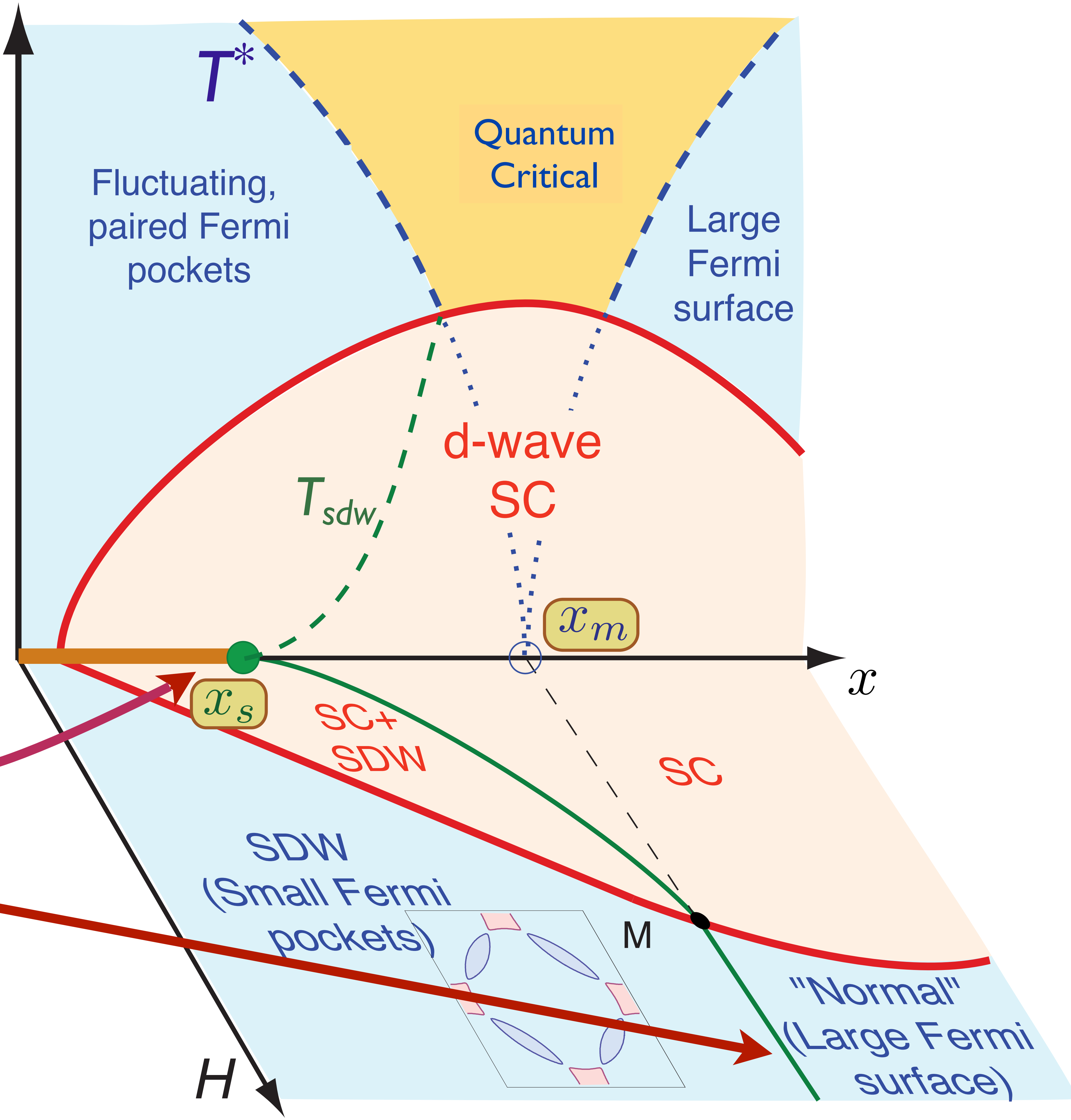
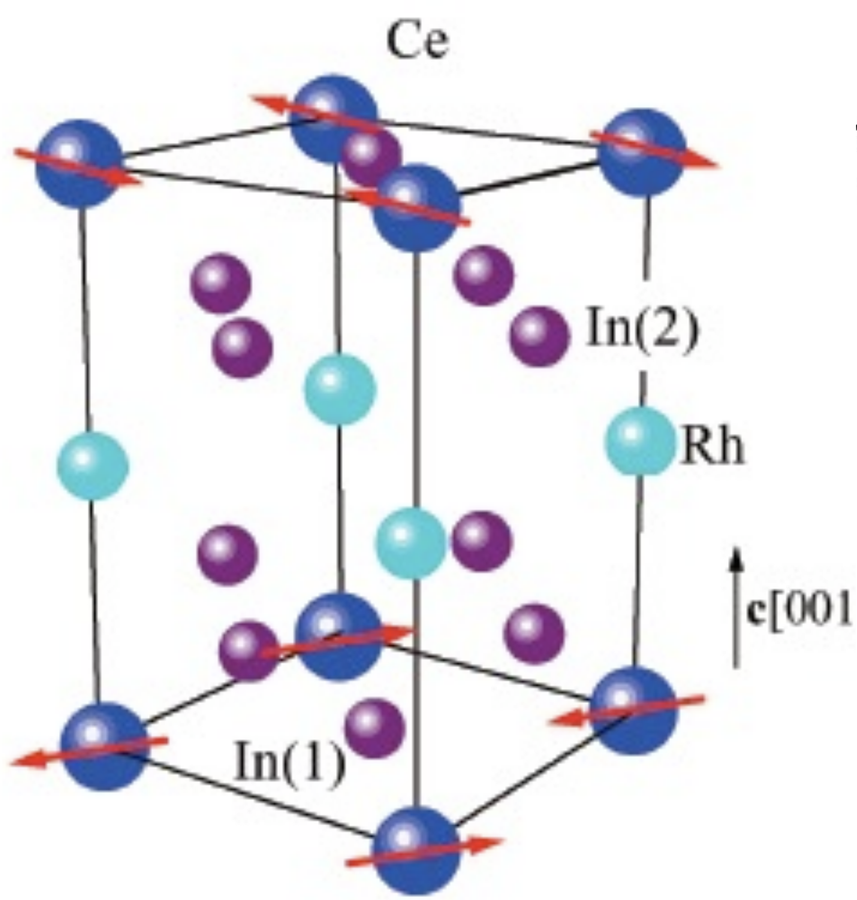
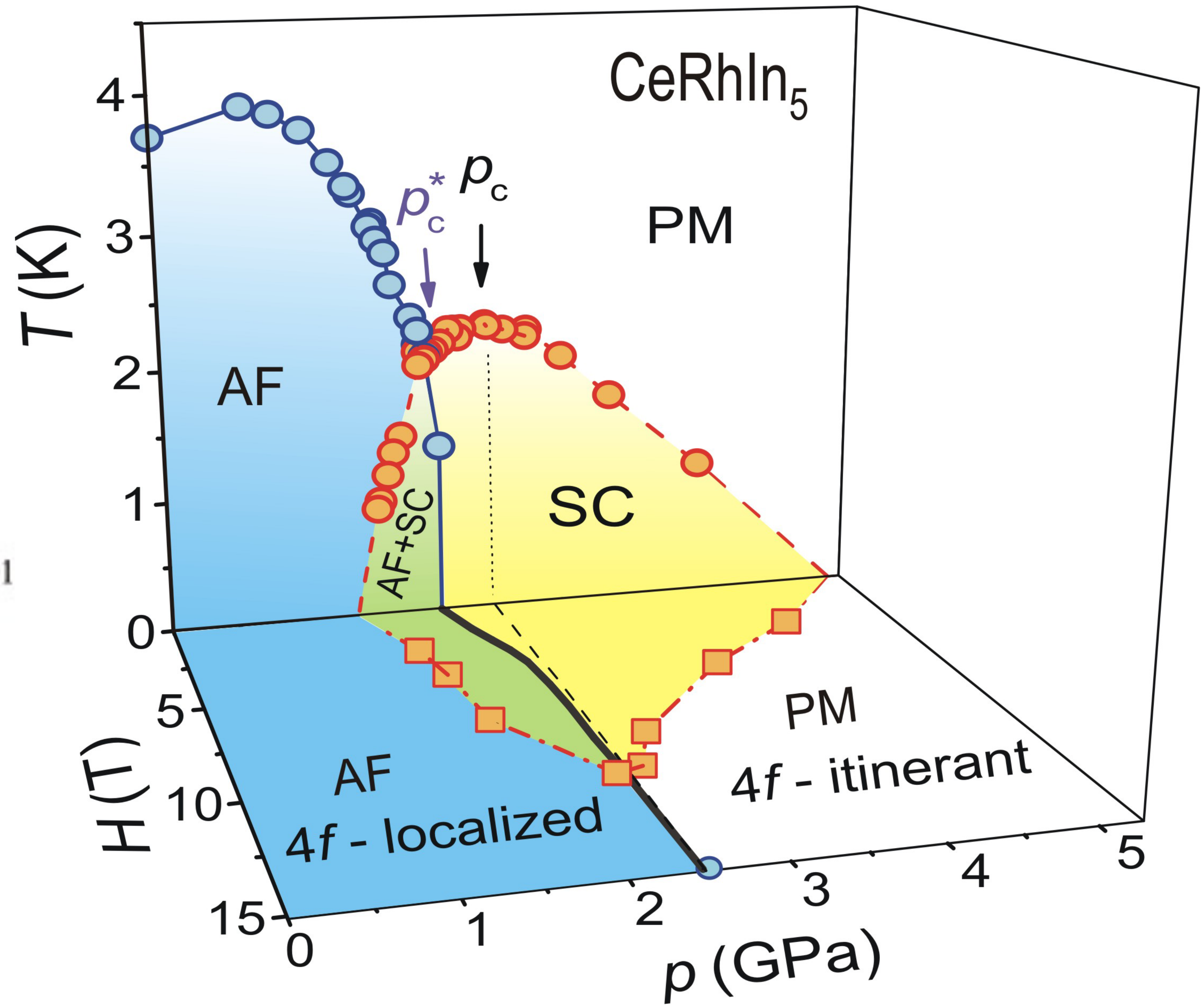


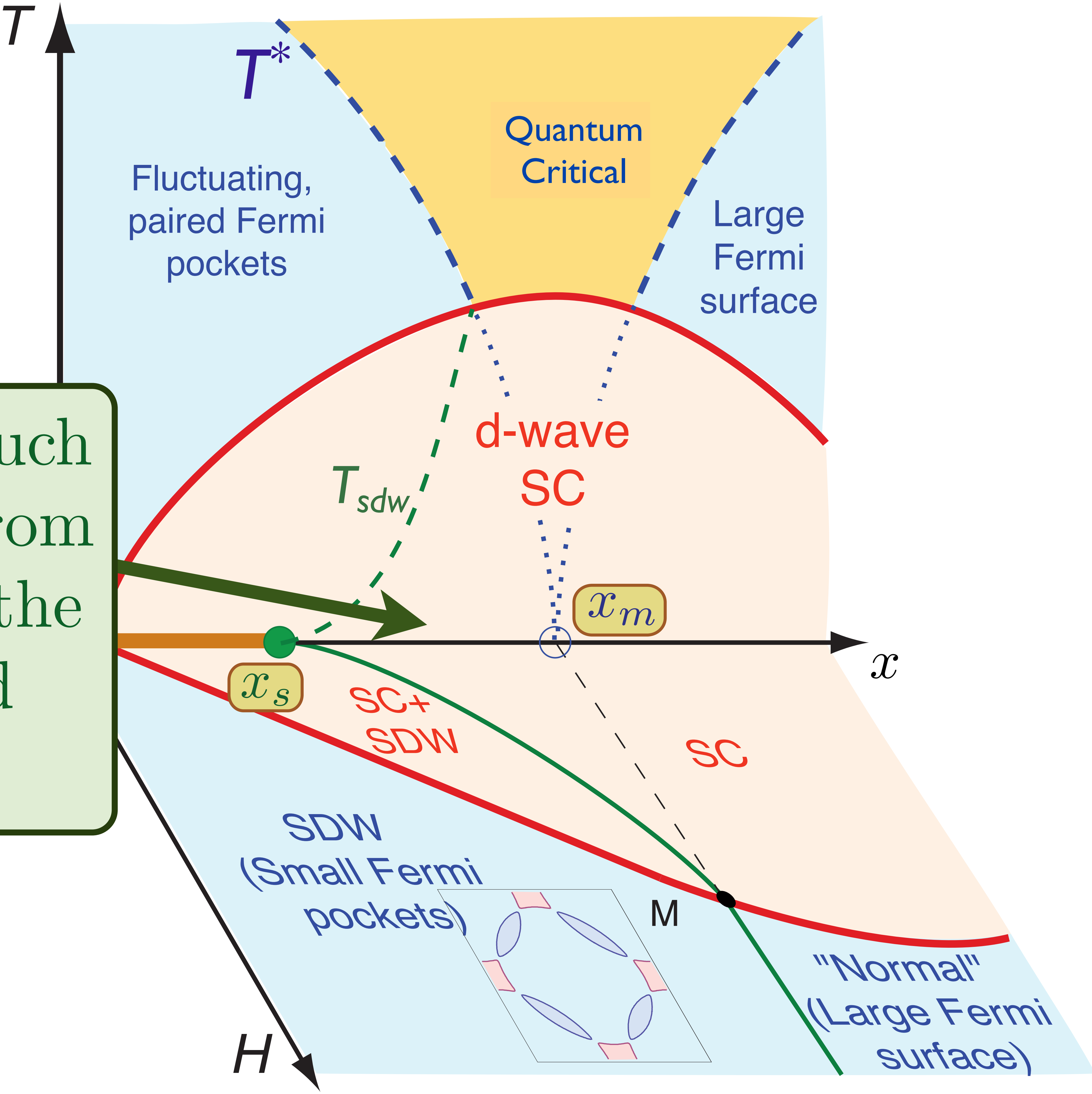
Similar phase diagram for $CeRhIn_5$



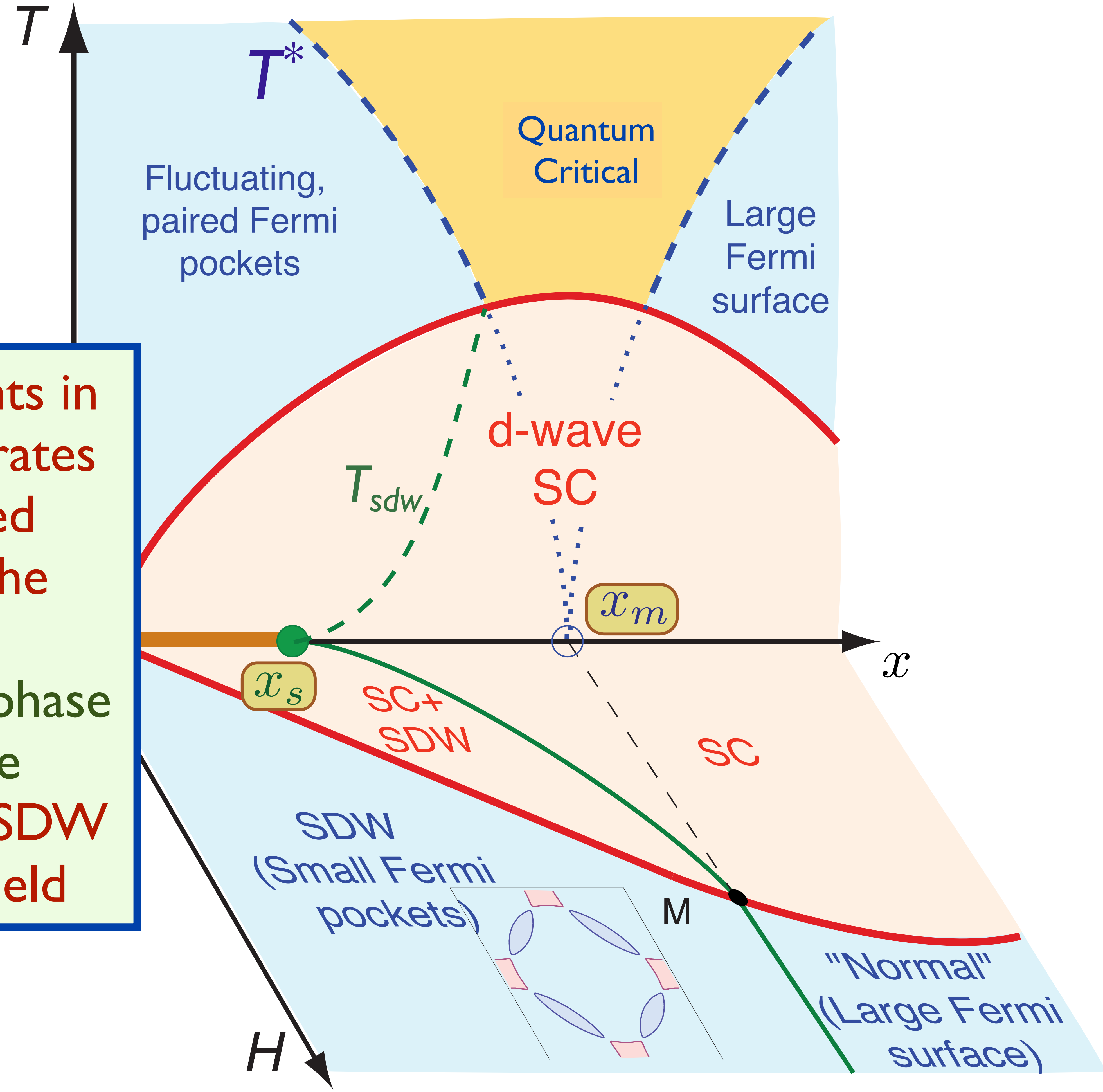


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

There is a much larger shift from x_m to x_s in the hole-doped cuprates.

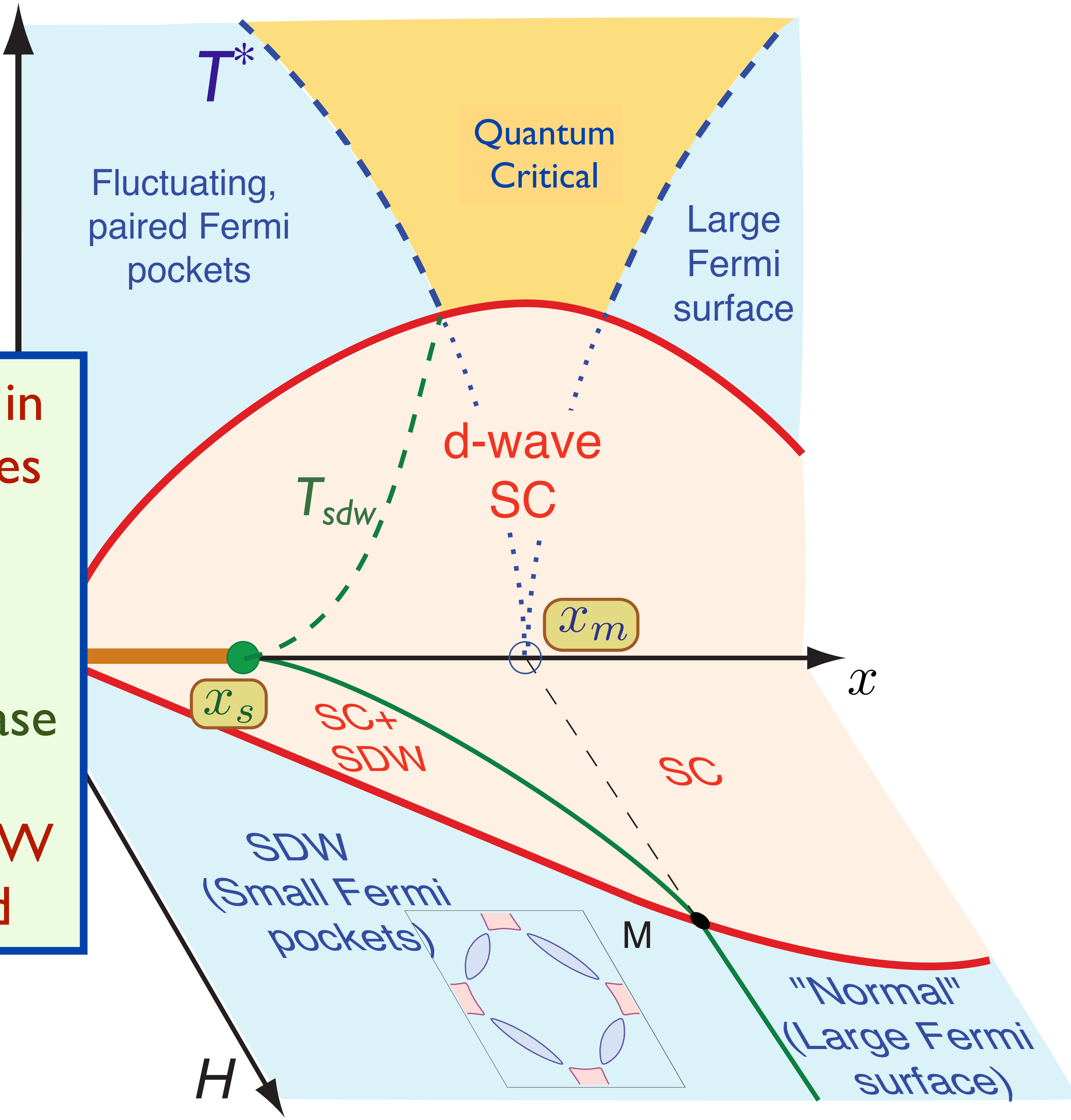


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Many experiments in hole-doped cuprates have presented evidence for the predicted green quantum phase transition line from SC to SC+SDW in a magnetic field

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



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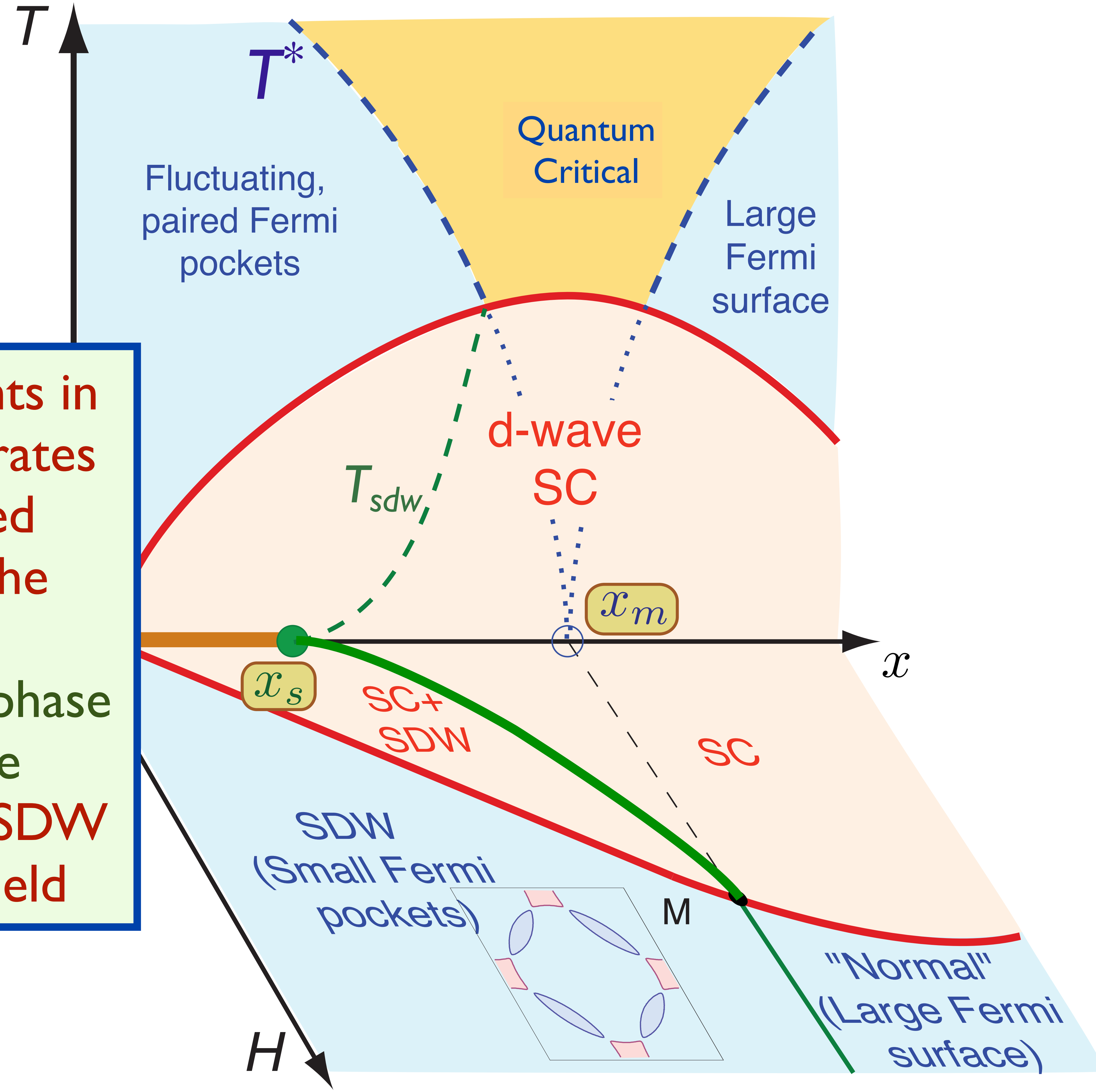

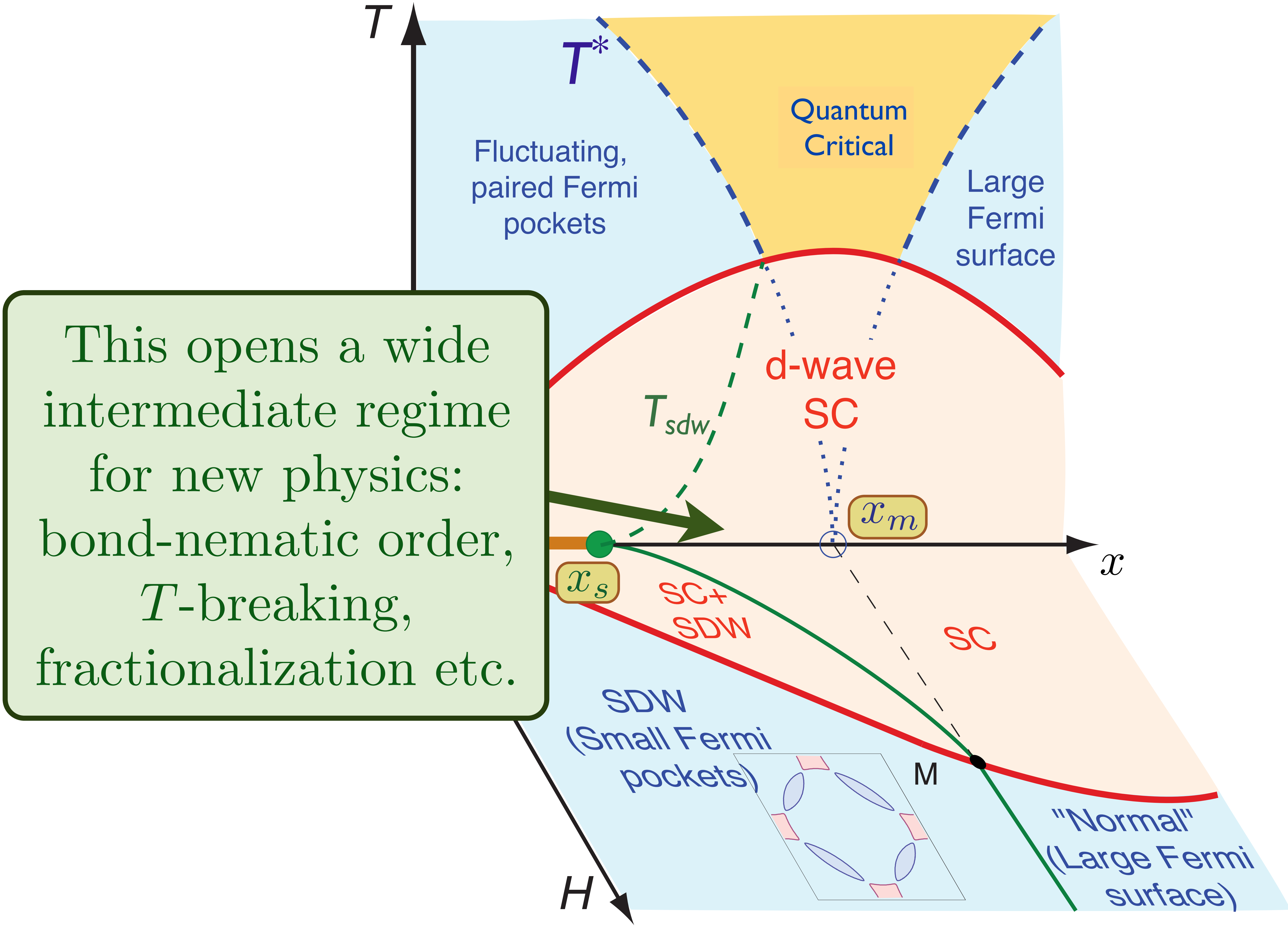

T
T^*
Fluctuating, paired Fermi pockets
Quantum Critical
Large Fermi surface
d-wave SC
T_{sdw}
This opens a wide intermediate regime for new physics: bond-nematic order, T-breaking, fractionalization etc.
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

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Transform electrons to a
"rotating reference frame",
quantizing spins in the direction of the
local antiferromagnetic order

Transform electrons to a "rotating reference frame", quantizing spins in the direction of the local antiferromagnetic order

This is facilitated by writing the vector antiferromagnetic order parameter $\vec{\varphi}$ in terms of a bosonic spinor z_α, with $\alpha = \uparrow, \downarrow$ and

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta.$$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \bullet \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \quad \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$\mathrm{SU(2)_{spin}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \bullet \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$\mathrm{U(1)_{charge}}$

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$U \times U^{-1}$

$\mathrm{SU(2)_{s;gauge}}$

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

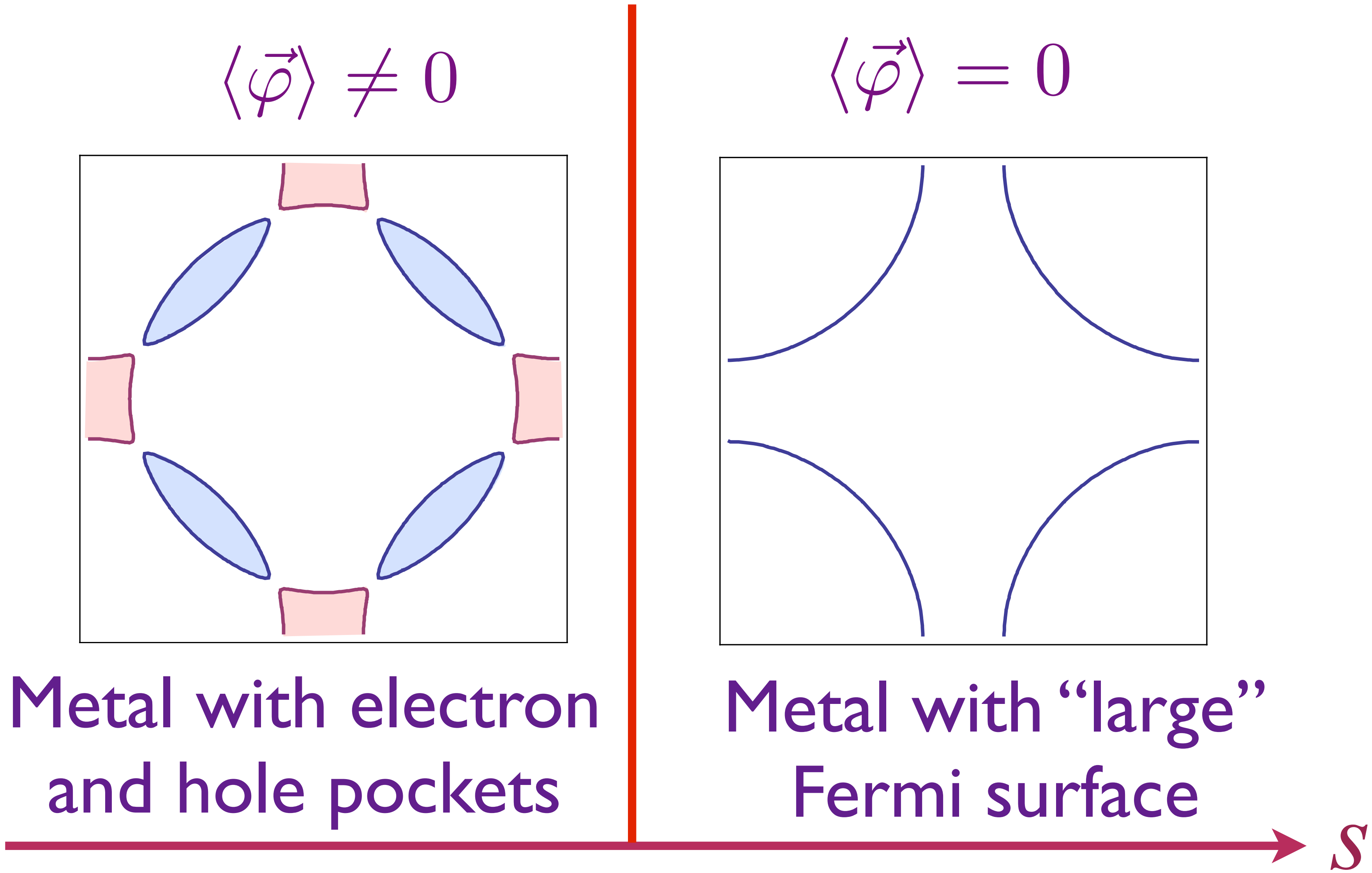
The Hubbard model can be written as a lattice gauge theory with a

$$\text{SU}(2)_{s;g} \times \text{SU}(2)_{\text{spin}} \times \text{U}(1)_{\text{charge}}$$

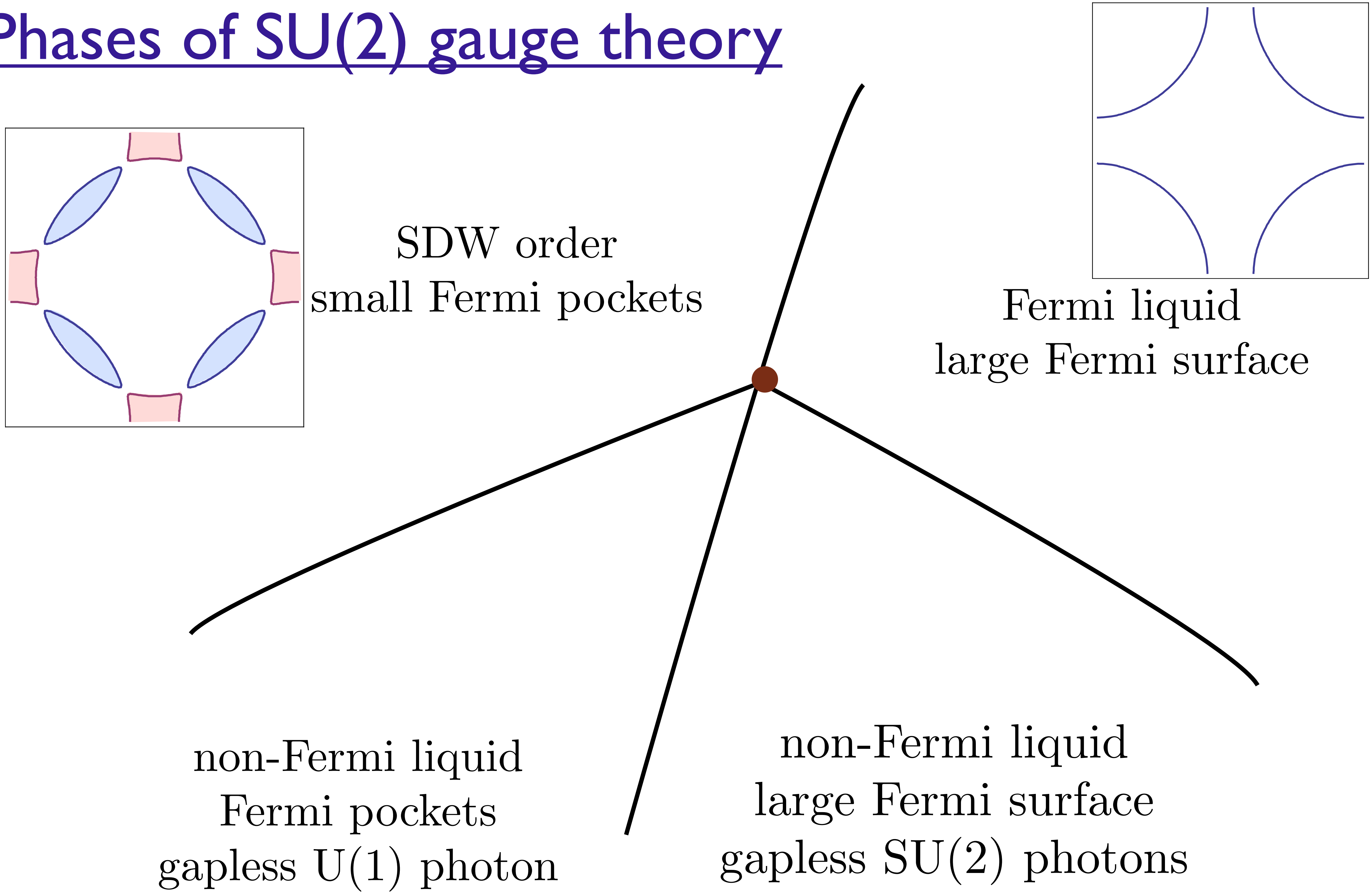
invariance.
The $\text{SU}(2)_{s;g}$ is a gauge invariance, while $\text{SU}(2)_{\text{spin}} \times \text{U}(1)_{\text{charge}}$ is a global symmetry

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Conventional phases considered so far



Phases of SU(2) gauge theory



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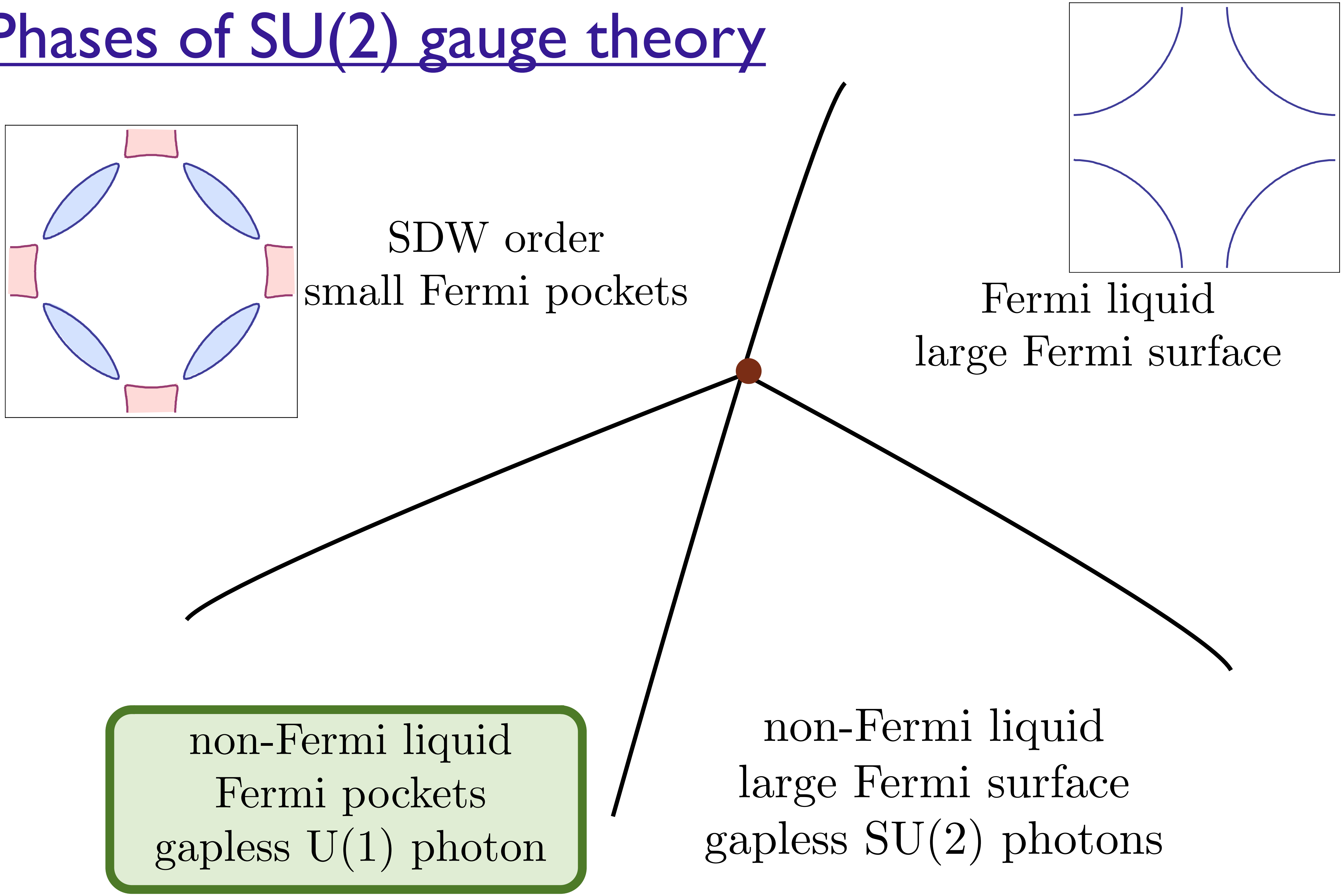
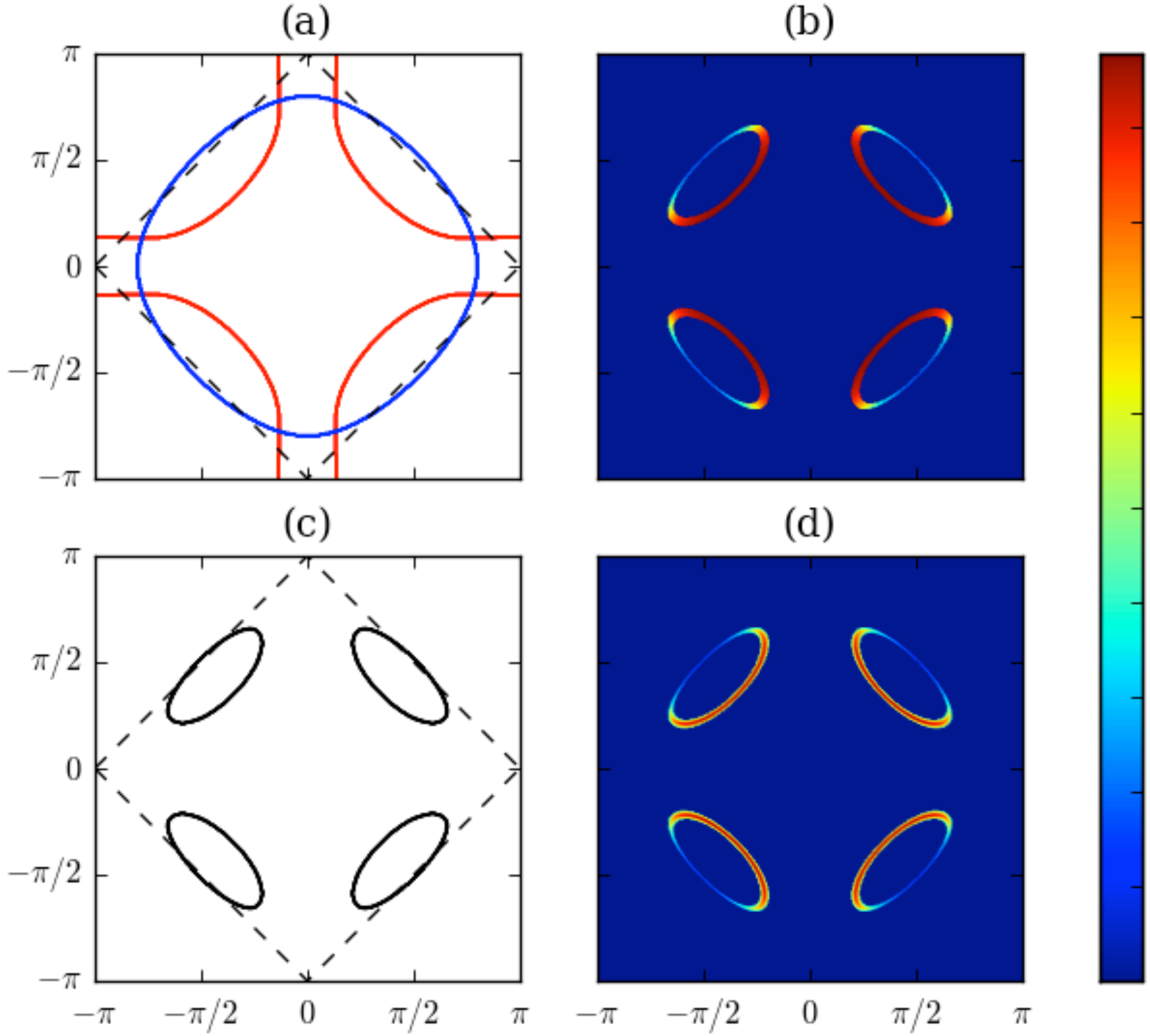
Phases of SU(2) gauge theory



S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Exotic non-Fermi liquid has Fermi pockets without long-range antiferromagnetism, along with emergent gauge excitations



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R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Physical Review B* **75** , 235122 (2007)

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Conclusions

The quantum critical point describing
the onset of spin-density-wave order
in metals is strongly coupled
in two spatial dimensions, and displays
universal non-Fermi liquid physics
which is independent of
electron interaction strength.

Conclusions

The quantum critical point has
an instability to
unconventional "*d*-wave" pairing, with a
universal *log-squared* enhancement
of the pairing susceptibility,
which is independent of
electron interaction strength.

Conclusions

The leading subdominant instability is to
a $2k_F$ bond-nematic ordering.
Its susceptibility also has a
universal *log-squared* enhancement ,
which is independent of
electron interaction strength.

Conclusions

The onset of superconductivity leads to a shift in the position of the spin density wave quantum critical point. This is a crucial ingredient in the phase diagram in an applied magnetic field.

Conclusions

There can be exotic intermediate phases, with pocket Fermi surfaces, no long-range spin-density-wave order, and emergent gauge excitations.