Fermi surfaces and gauge-gravity duality

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PHYSICS
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Compressible quantum matter

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There are only a few established examples of such phases in condensed matter physics.
However, they appear naturally as duals of gravitational theories, and we want to interpret them in the gauge theory.

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All known examples of such phases have a *Fermi Surface*

(even in systems with only bosons in the Hamiltonian)

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

- Couple fermions to a dynamical gauge field A_a.

$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - i\sigma A_\tau - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right) f_\sigma \quad ; \quad \sigma = \pm 1$$

- Couple fermions to a dynamical gauge field A_a.
- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions.*

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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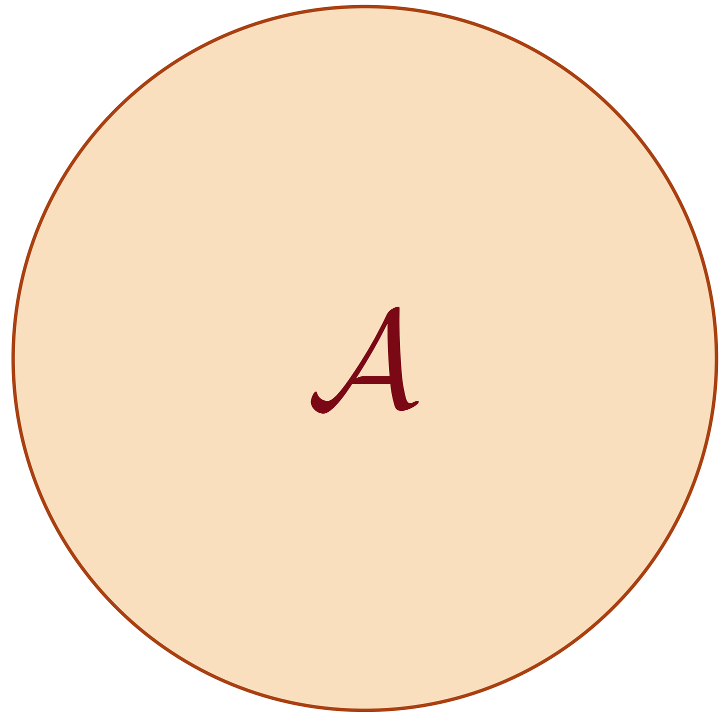
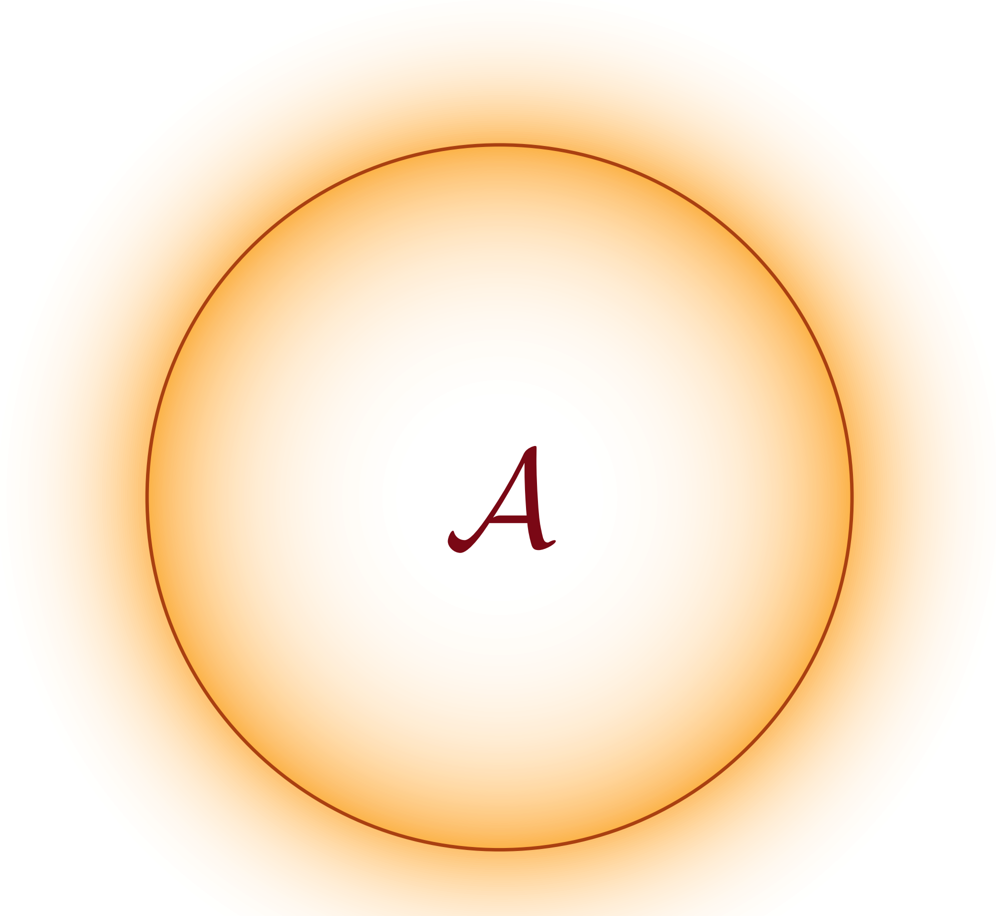


$$\mathcal{A} = \langle f^\dagger f \rangle = \langle \mathcal{Q} \rangle$$

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$



$$2\mathcal{A} = \left\langle f_\sigma^\dagger f_\sigma \right\rangle = \langle \mathcal{Q} \rangle$$

The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.

$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - i\sigma A_\tau - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right) f_\sigma \quad ; \quad \sigma = \pm 1$$

ABJM theory in D=2+1 dimensions

- $4N^2$ Weyl fermions carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
- $4N^2$ complex bosons carrying fundamental charges of $\mathrm{U}(N)\times\mathrm{U}(N)\times\mathrm{SU}(4)_R$.
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Adding a chemical potential coupling to a SU(4) charge breaks supersymmetry and SU(4) invariance

Theory similar to ABJM

- U(1) gauge invariance and U(1) global symmetry
- Fermions, f_+ and f_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
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- No supersymmetry
- Fermions, c, gauge-invariant bound states of fermions and bosons carrying global U(1) charge 2.

Theory similar to ABJM

$$\mathcal{L} = f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma$$
$$+ \; b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma$$
$$+ \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right)$$

The index $\sigma = \pm 1$

Theory similar to ABJM

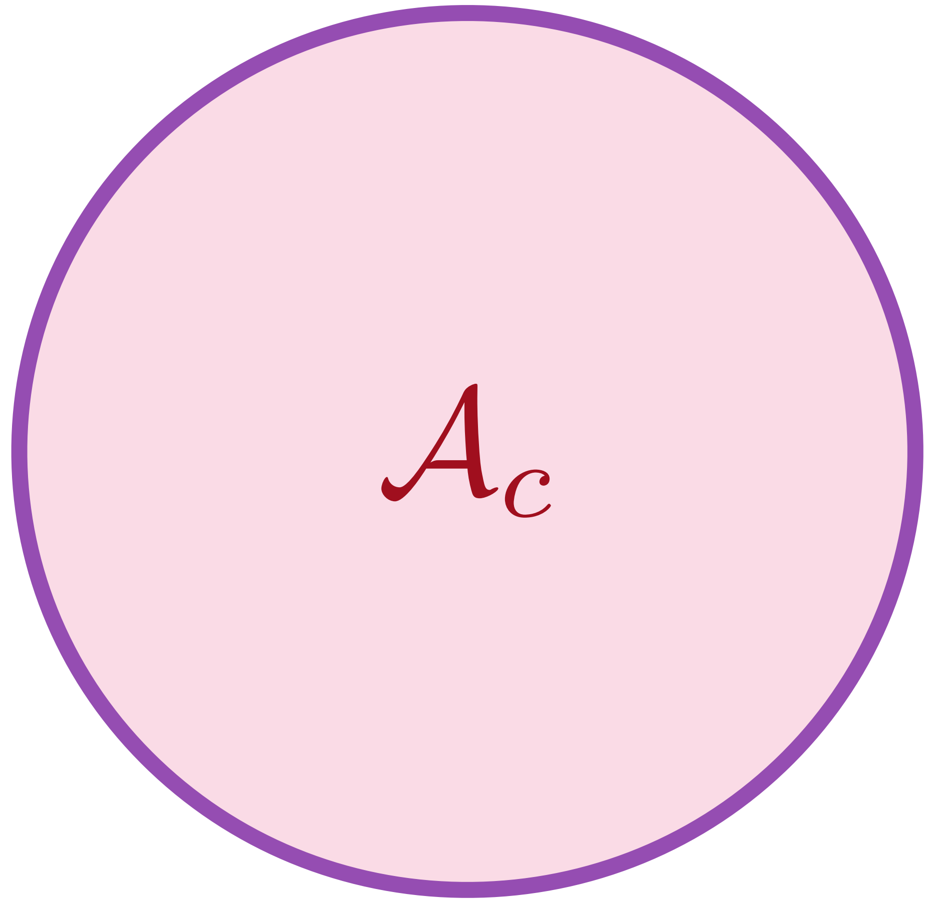
$$
\begin{aligned}
\mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\
&+ b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\
&\quad + \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right) \\
&+ c^\dagger \left[\partial_\tau - \frac{\nabla^2}{2m_c} + \epsilon_2 - 2\mu \right] c \\
&\quad - g_2 \left[c^\dagger \left(f_+ b_- + f_- b_+ \right) + \text{H.c.} \right]
\end{aligned}
$$

The index $\sigma = \pm 1$, and $\epsilon_{1,2}$ are tuning parameters of phase diagram

$$
\mathcal{Q} = f_\sigma^\dagger f_\sigma + b_\sigma^\dagger b_\sigma + 2c^\dagger c
$$

Phases of ABJM-like theories

$$\langle b_\pm \rangle = 0$$



$$2\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

Fermi liquid (FL) of gauge-neutral particles

U(1) gauge theory is in confining phase

Phases of ABJM-like theories

$$\langle b_{\pm} \rangle \neq 0$$

$$\langle b_+ b_- \rangle \neq 0$$

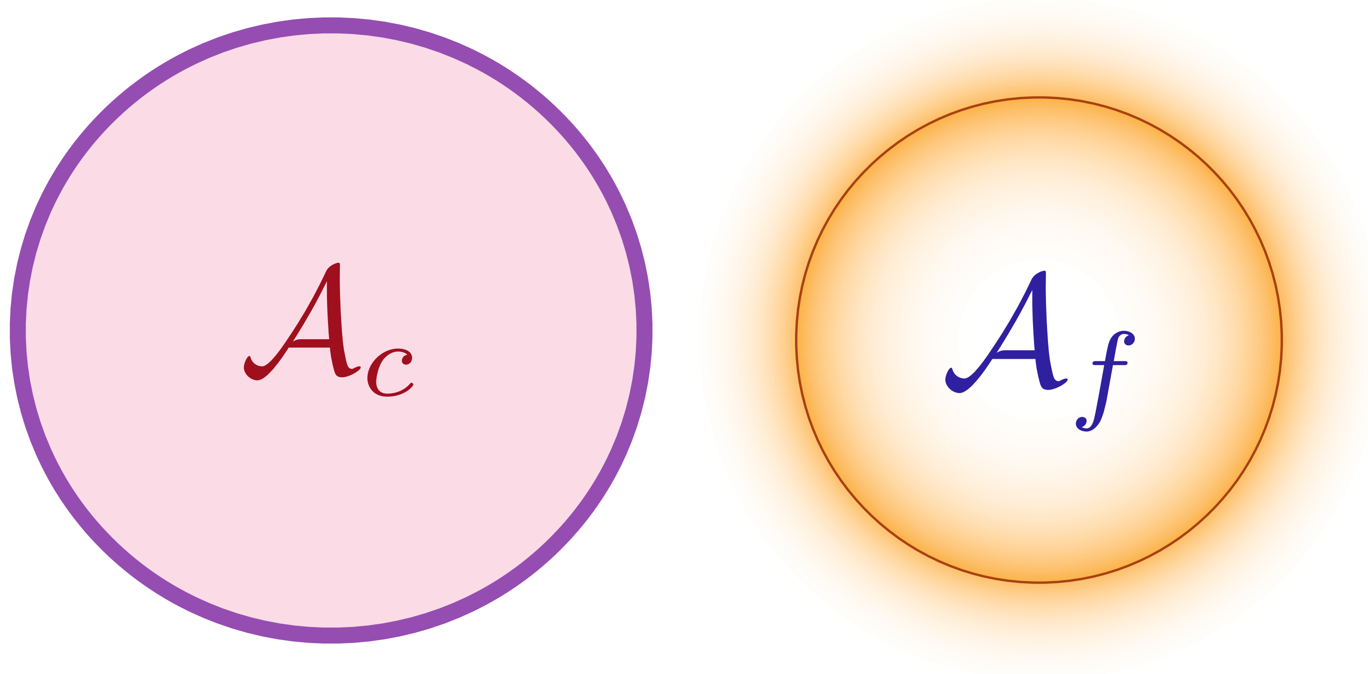
No constraint on Fermi surface area, which can be zero

Superconductor

U(*1*) gauge theory is in Higgs phase and global U(1) is broken

Phases of ABJM-like theories

$$\langle b_{\pm} \rangle = 0$$



$$2\mathcal{A}_c + 2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

Fractionalized Fermi liquid (FL*)

U(*1*) gauge theory is in deconfined phase

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Claim: the FL* phase underlies recent holographic theories of compressible metallic states.

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- The holographic background has a large entropy and compressibility: this is presumed to be that of the many f Fermi surfaces coupled to gauge fields.

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Claim: the FL* phase underlies recent holographic theories of compressible metallic states.

- The holographic background has a large entropy and compressibility: this is presumed to be that of the many f Fermi surfaces coupled to gauge fields.

- A probe gauge-invariant fermion has a Fermi surface: this is linked to the c fermions.

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Claim: the FL* phase underlies recent holographic theories of compressible metallic states.

- Dynamic mean-field solutions of FL* phases in models with infinite-range hopping have properties which match precisely with those of the $\mathrm{AdS}_2 \times R^2$ holographic solutions:
  - A non-zero ground state entropy.
  - Single fermion self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
  - A marginal Fermi liquid spectrum for the c fermions (for the holographic solution, this requires tuning a free parameter).
  - The low energy sector has conformally invariant correlations which are consistent with the AdS_2 geometry.

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Conclusions

- Compressible quantum matter is characterized by Fermi surfaces.

- Fermi surfaces can be removed from the Luttinger count if the fermions acquire gauge charges

- Phases of a strongly-coupled gauge theory: Fermi liquids (FL) and fractionalized Fermi liquids (FL*)

Conclusions

- Mean field Kondo lattice models capture the physics of holographic metals with a $AdS_2 \times R^d$ geometry

- Needed: Holographic theory for FL* or related compressible phases, without a factorized geometry. Challenge: detect Fermi surfaces of fermions with gauge charges