

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

The Fermi liquid has  $\theta = d - 1$  and  $z = 1$ : so the Fermi liquid does *not* have such a gravity dual.

The non-Fermi liquid in  $d = 2$  has  $\theta = d - 1$ , and this implies  $z \geq 3/2$ . So the lower bound is precisely the value obtained for the non-Fermi liquid!