

$$|\Psi\rangle \Rightarrow \text{Ground state of entire system,}$$

$$\rho = |\Psi\rangle\langle\Psi|$$

$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B)$$

$$\begin{aligned} \text{Then } \rho_A = \text{Tr}_B \rho &= \text{density matrix of region } A \\ &= \frac{1}{2} (|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A) \end{aligned}$$

$$\begin{aligned} \text{Entanglement entropy } S_E &= -\text{Tr} (\rho_A \ln \rho_A) \\ &= \ln 2 \end{aligned}$$