

# Quantum criticality and high temperature superconductivity

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Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)





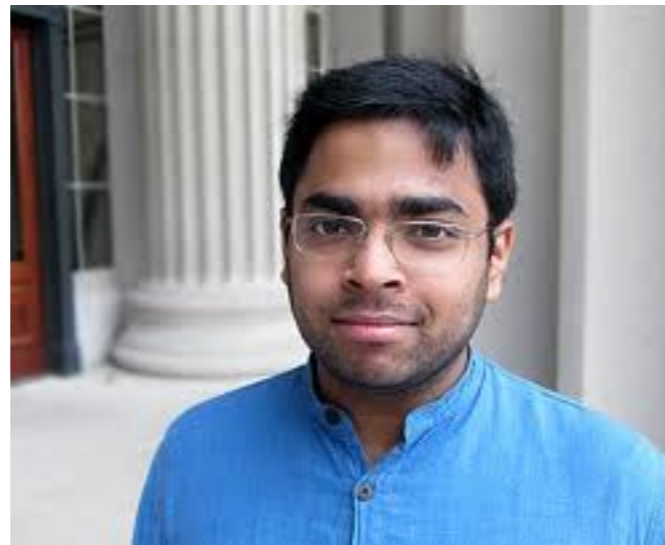
**William Witczak-Krempa**  
**Perimeter**



**Erik Sorensen**  
**McMaster**



**Sean Hartnoll**  
**Stanford**



**Raghu Mahajan**  
**Stanford**

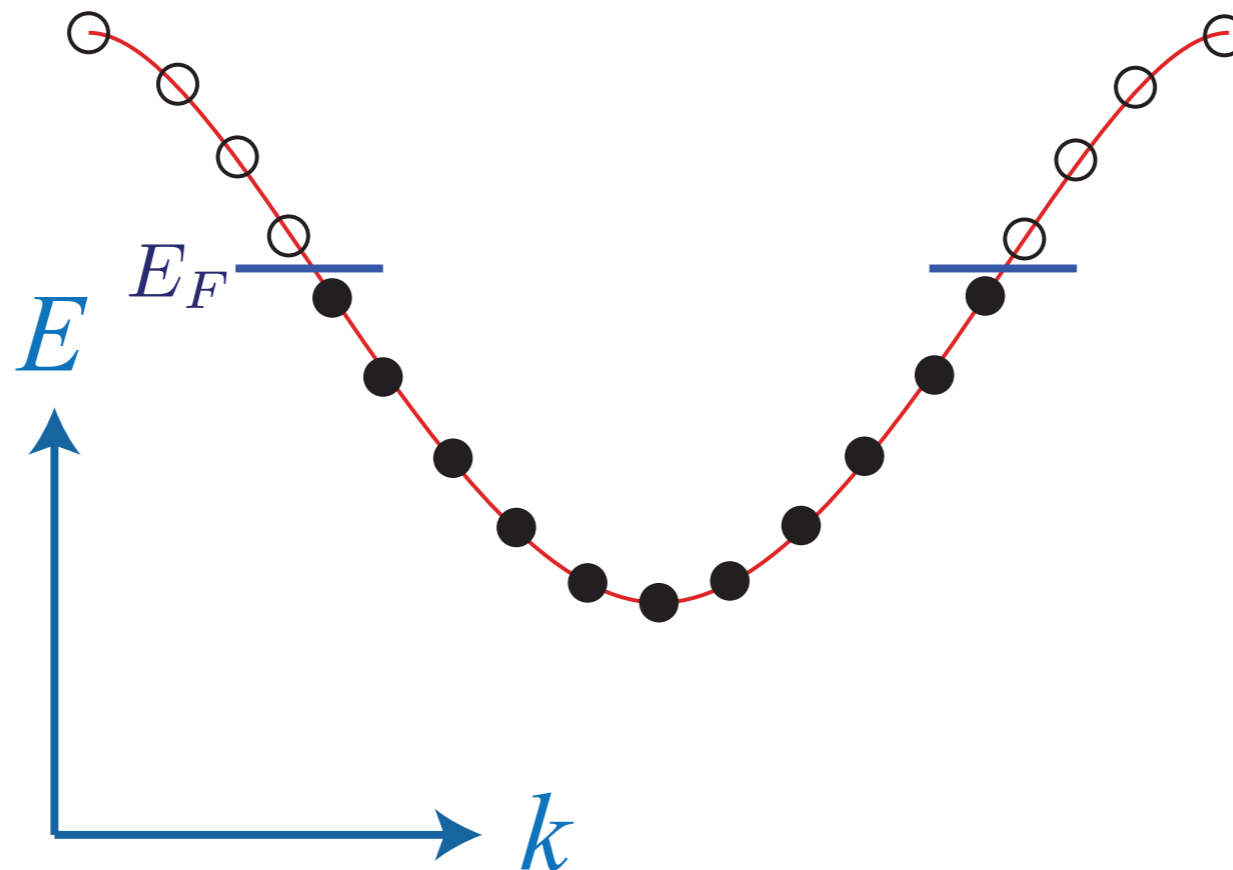


**Matthias Punk**  
**Innsbruck**

# *Foundations of quantum many body theory:*

## *I. Ground states connected adiabatically to independent electron states*

### Metals

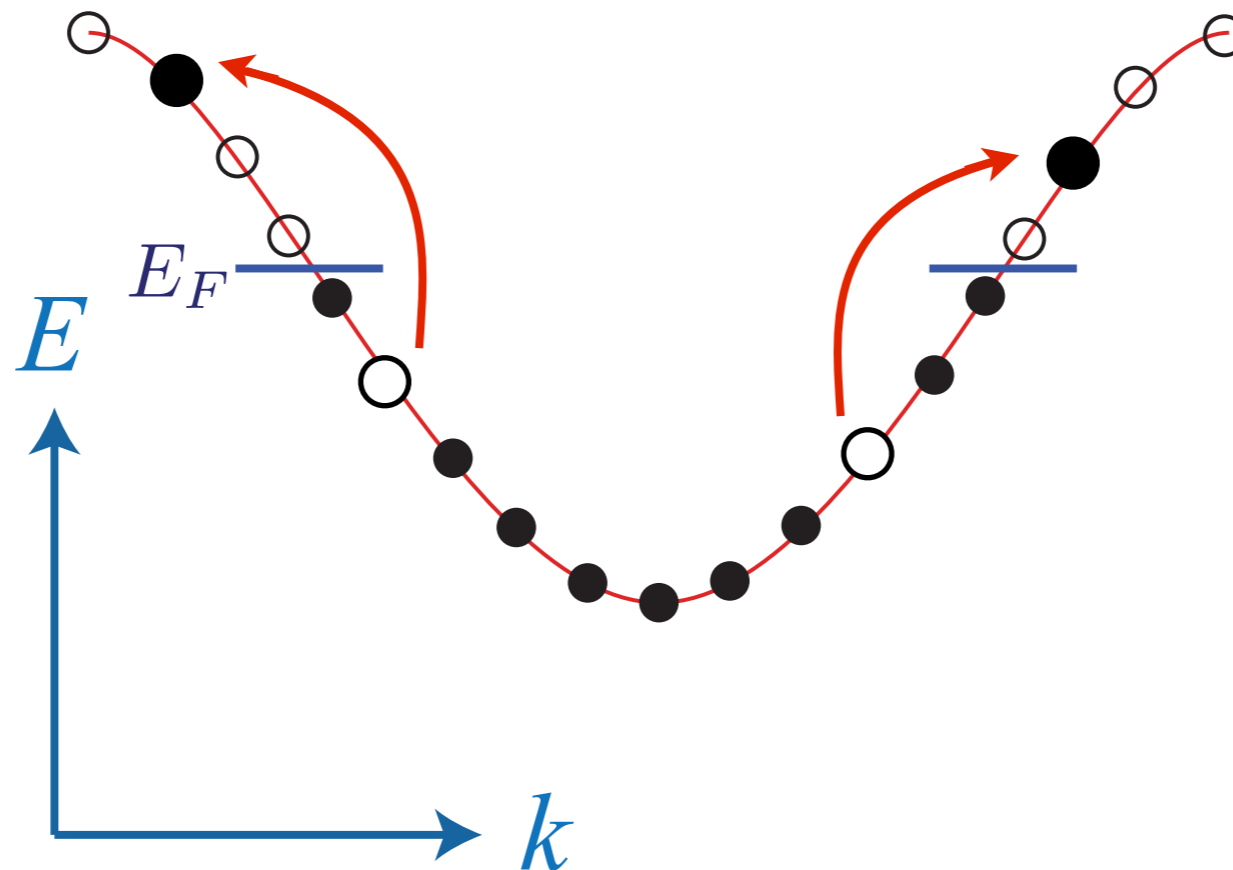


# *Foundations of quantum many body theory:*

*1. Ground states connected adiabatically to independent electron states*

*2. Quasiparticle structure of excited states*

## Metals



## Modern phases of quantum matter:

- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. Quasiparticle structure of excited states*

## Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

## Modern phases of quantum matter:

1. *Ground states disconnected from independent electron states: many-particle entanglement*
2. *Quasiparticle structure of excited states*

## Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations (“phonons”) are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

*Modern phases of quantum matter:*

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- 2. No quasiparticles**

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## Only 2 examples:

1. Conformal field theories in spatial dimension  $d > 1$
2. Quantum critical metals in dimension  $d=2$

# Outline

## 1. The simplest model without quasiparticles

*A. Superfluid-insulator transition*

*of ultracold bosonic atoms in an optical lattice*

*B. Conformal field theories in  $2+1$  dimensions and  
the AdS/CFT correspondence*

## 2. Metals without quasiparticles

*A “non-Fermi” liquid in the*

*high temperature superconductors:*

*the Ising-nematic quantum critical point*

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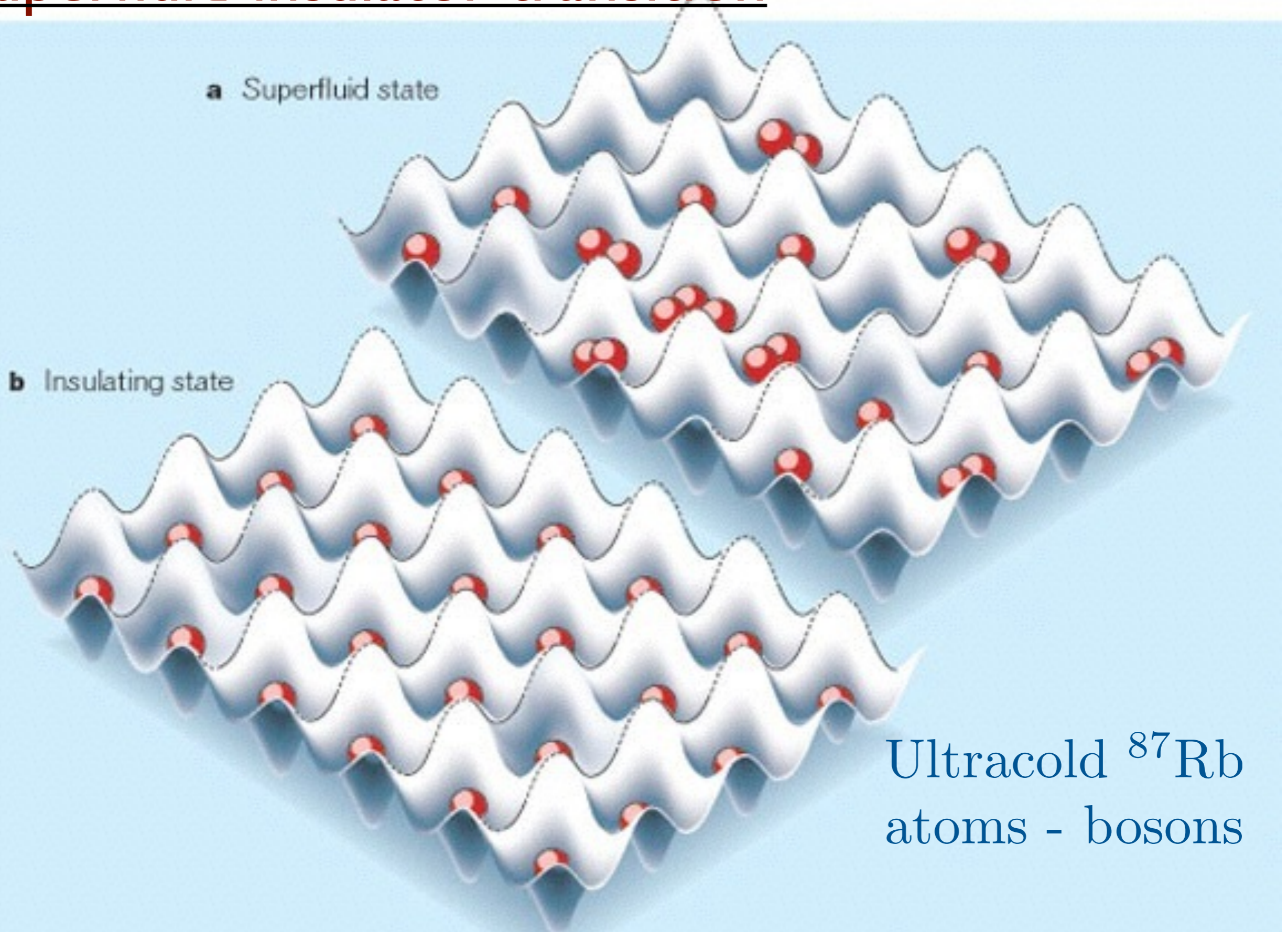
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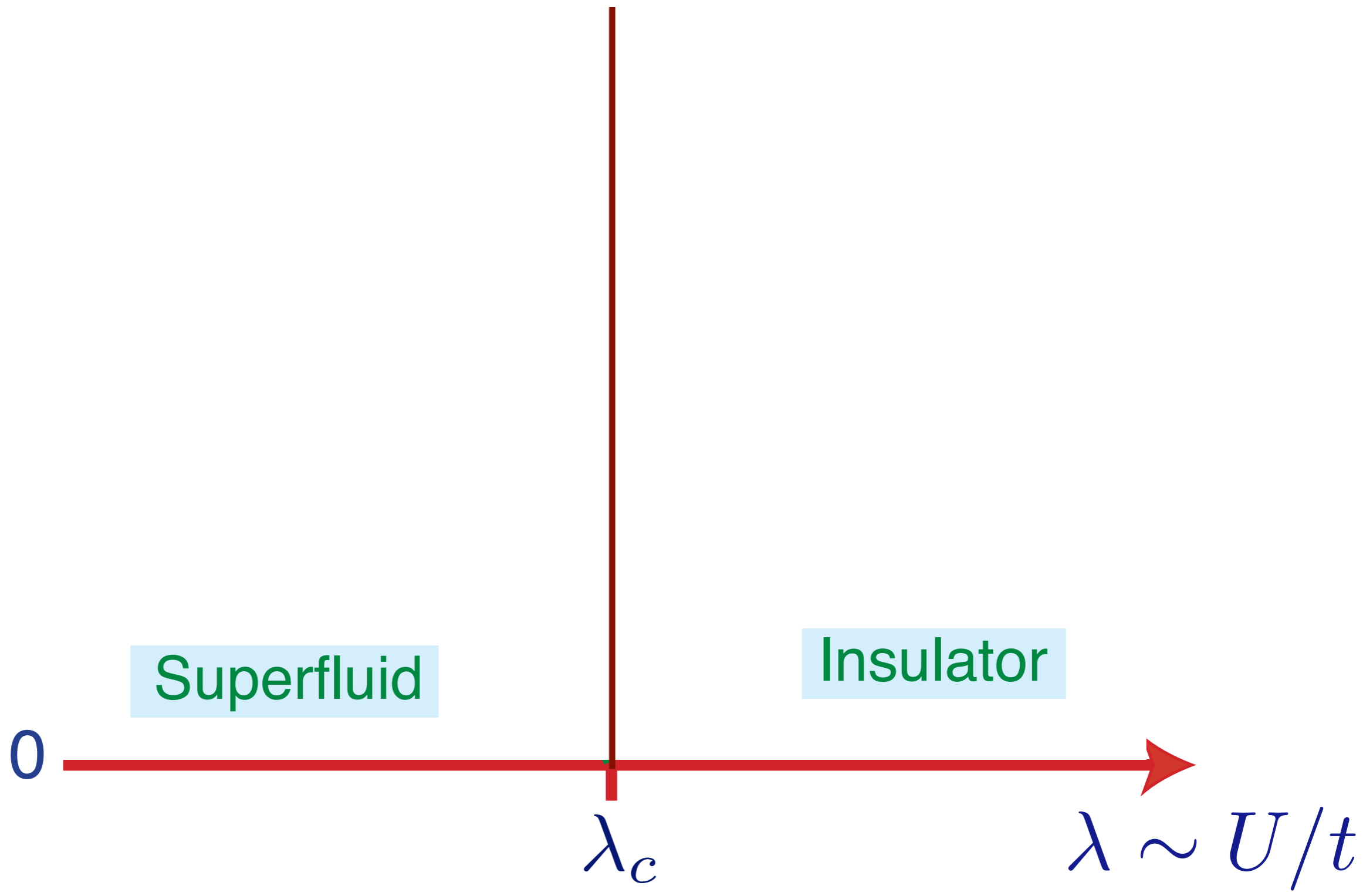
*high temperature superconductors:*

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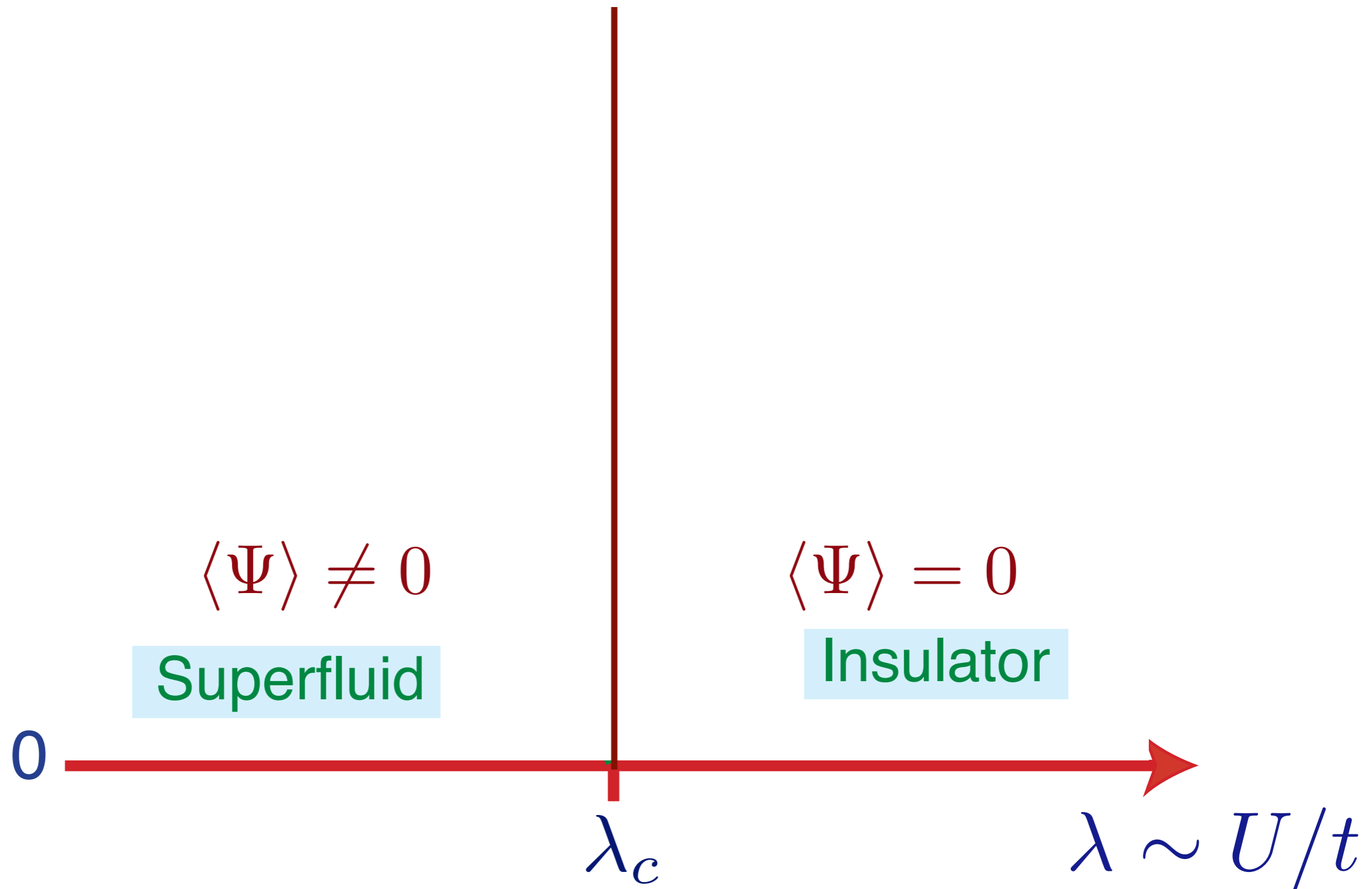
# Superfluid-insulator transition



Ultracold  $^{87}\text{Rb}$   
atoms - bosons

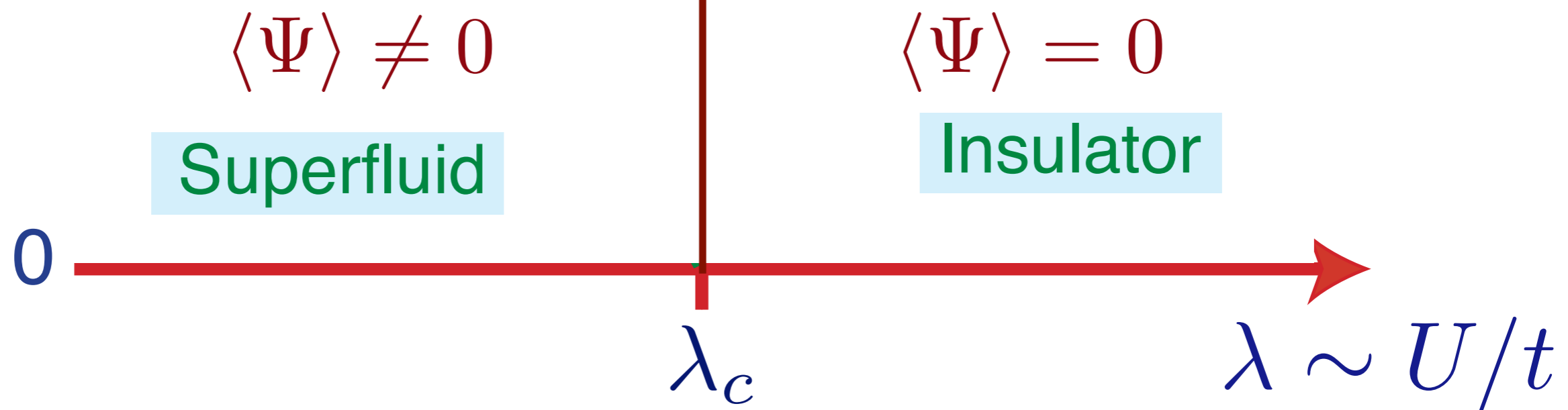


$\Psi \rightarrow$  a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

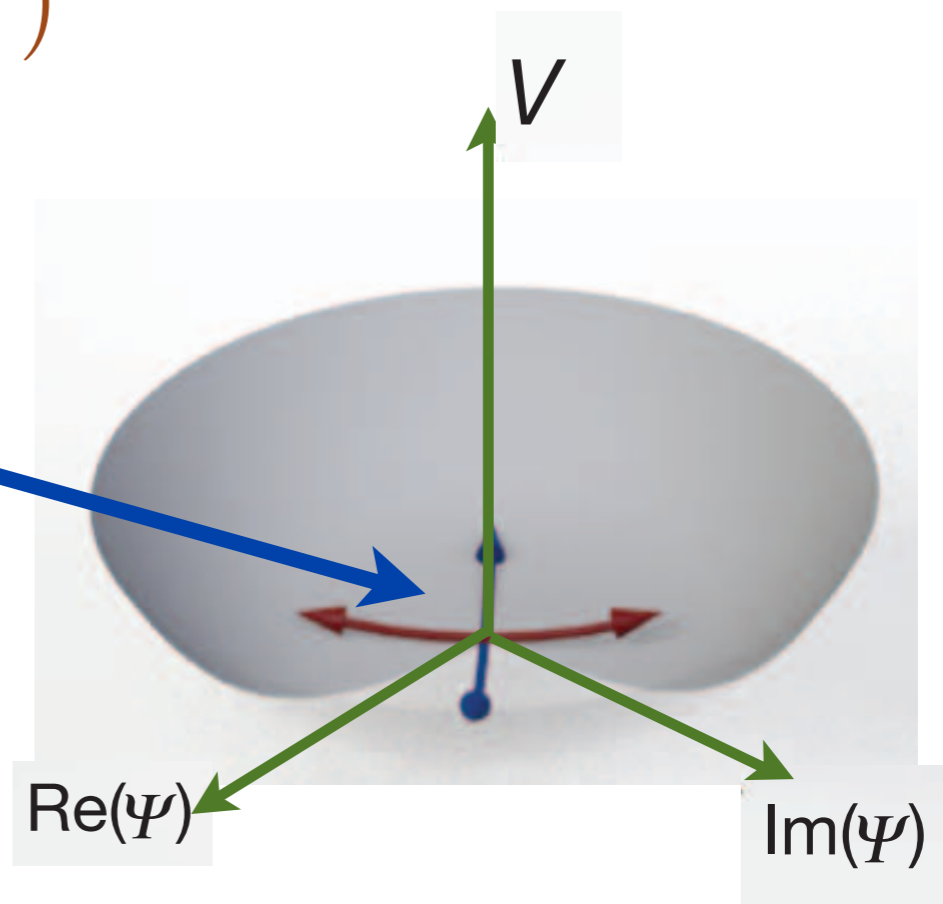
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



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Particles and holes correspond to the 2 normal modes in the oscillation of  $\Psi$  about  $\Psi = 0$ .



$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

Insulator

0

$\lambda_c$

$\lambda \sim U/t$

$$\underline{U \gg t}$$

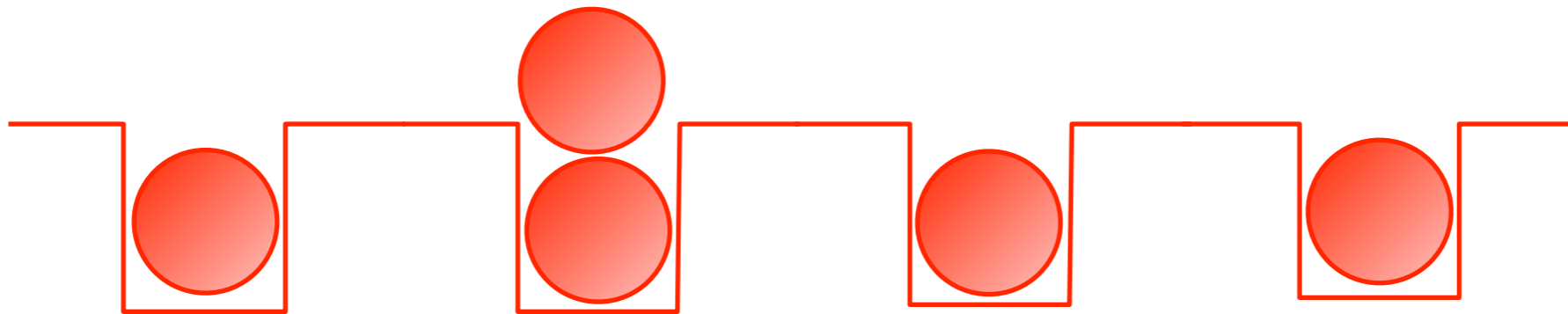


Insulator (the vacuum)  
at large repulsion between bosons

$$|\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle$$

$$\underline{U \gg t}$$

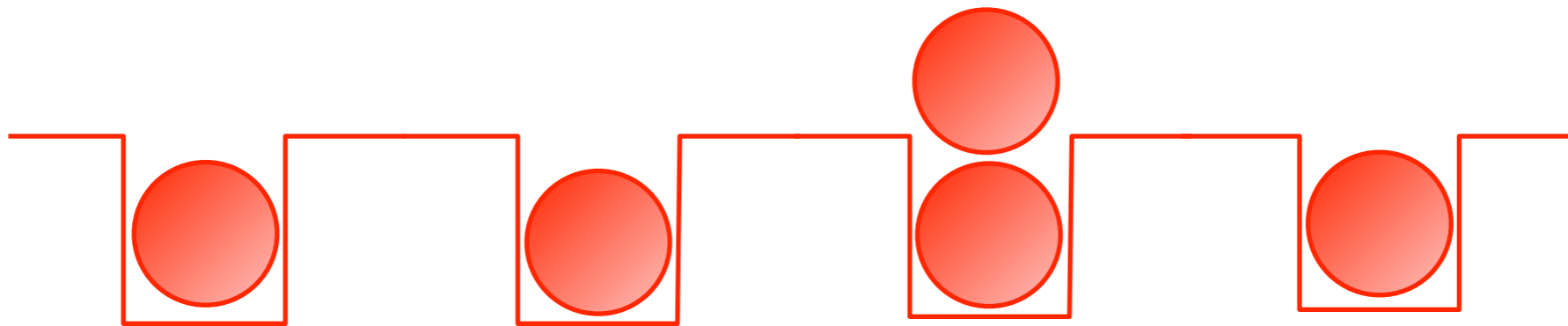
Excitations of the insulator:



Particles  $\sim \psi^\dagger$

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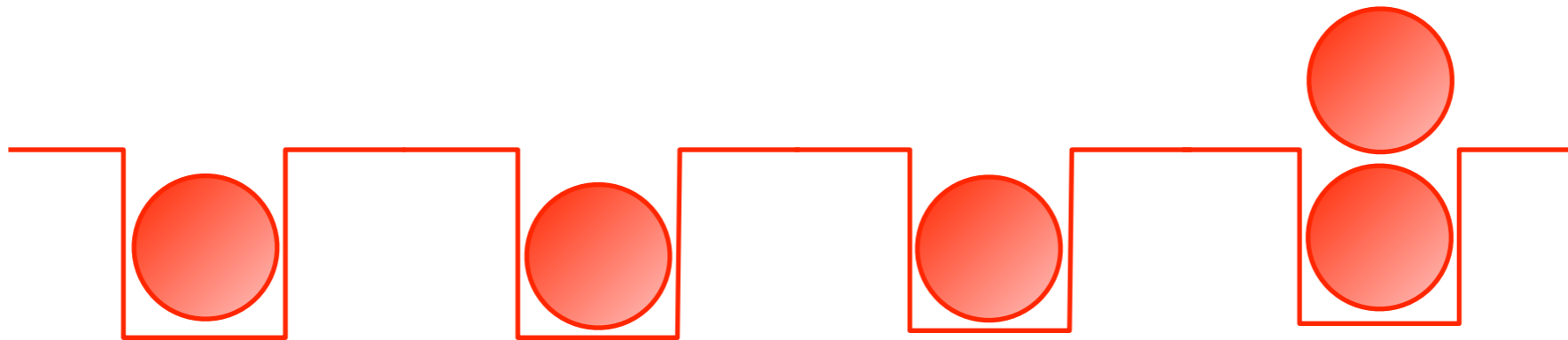
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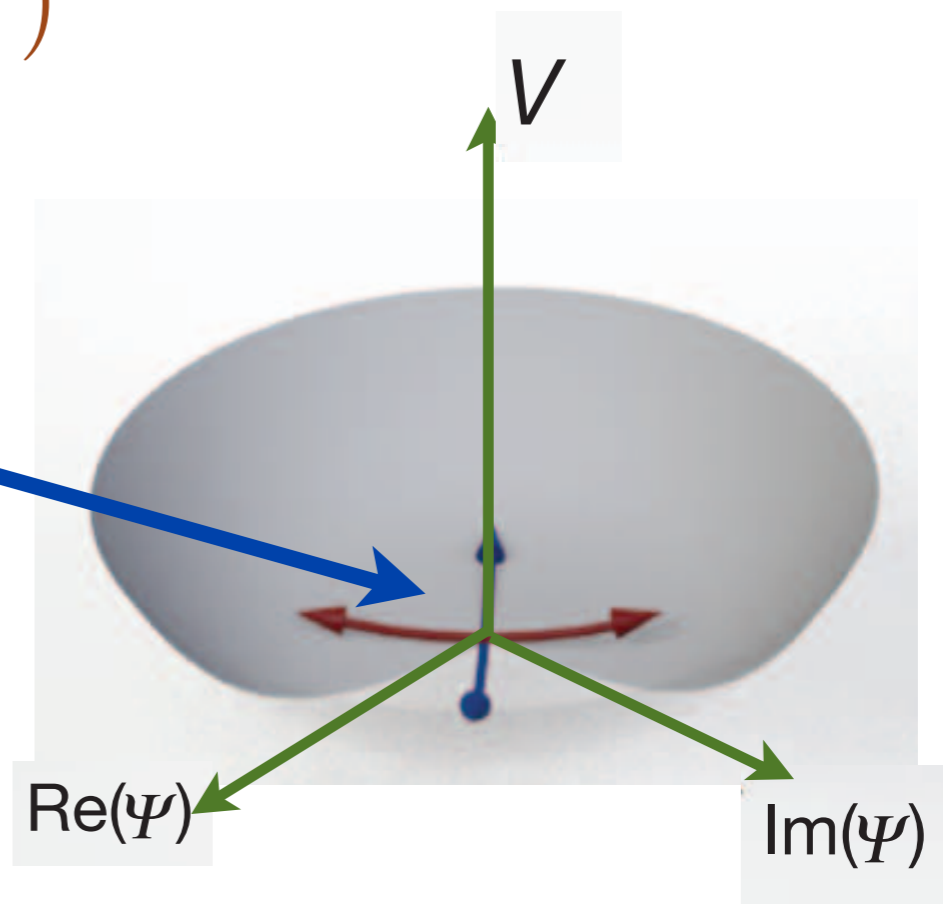


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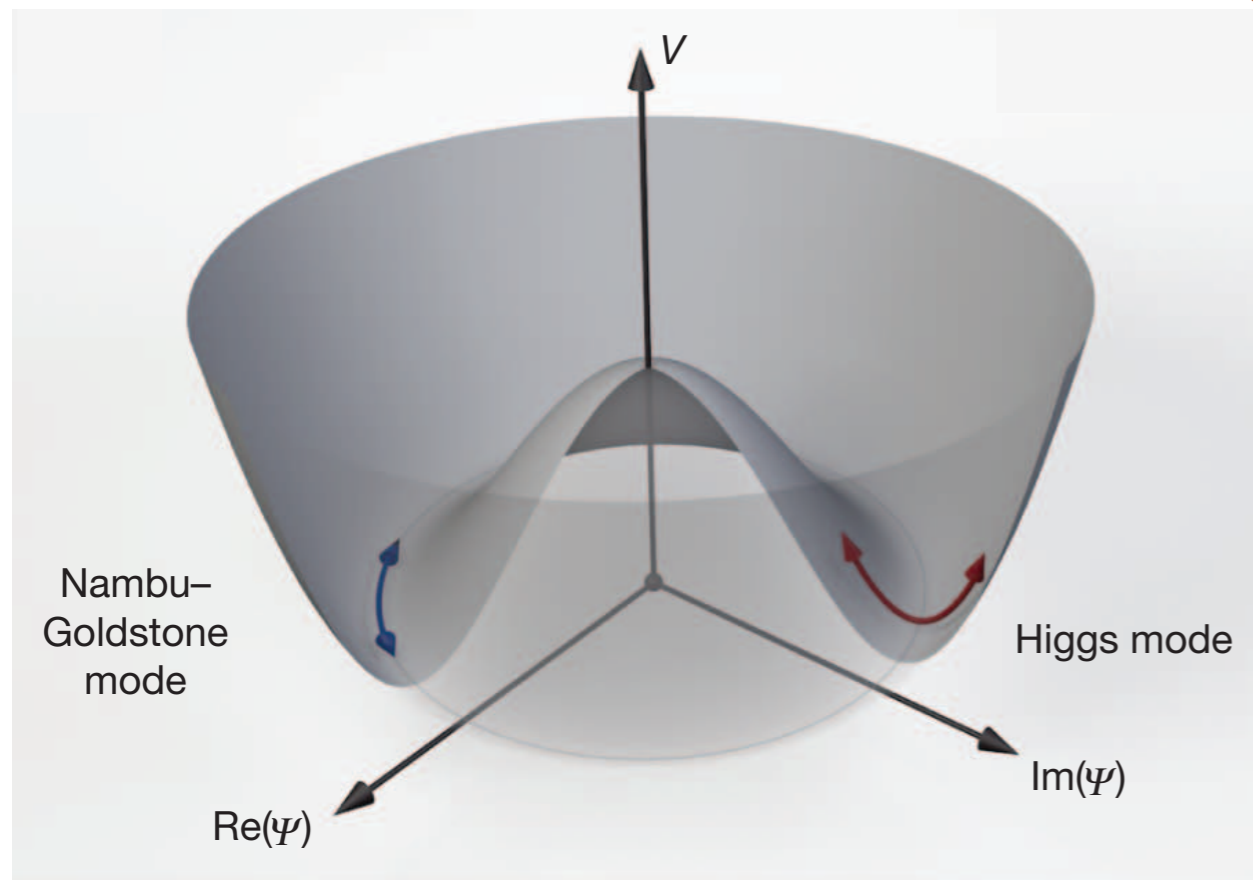
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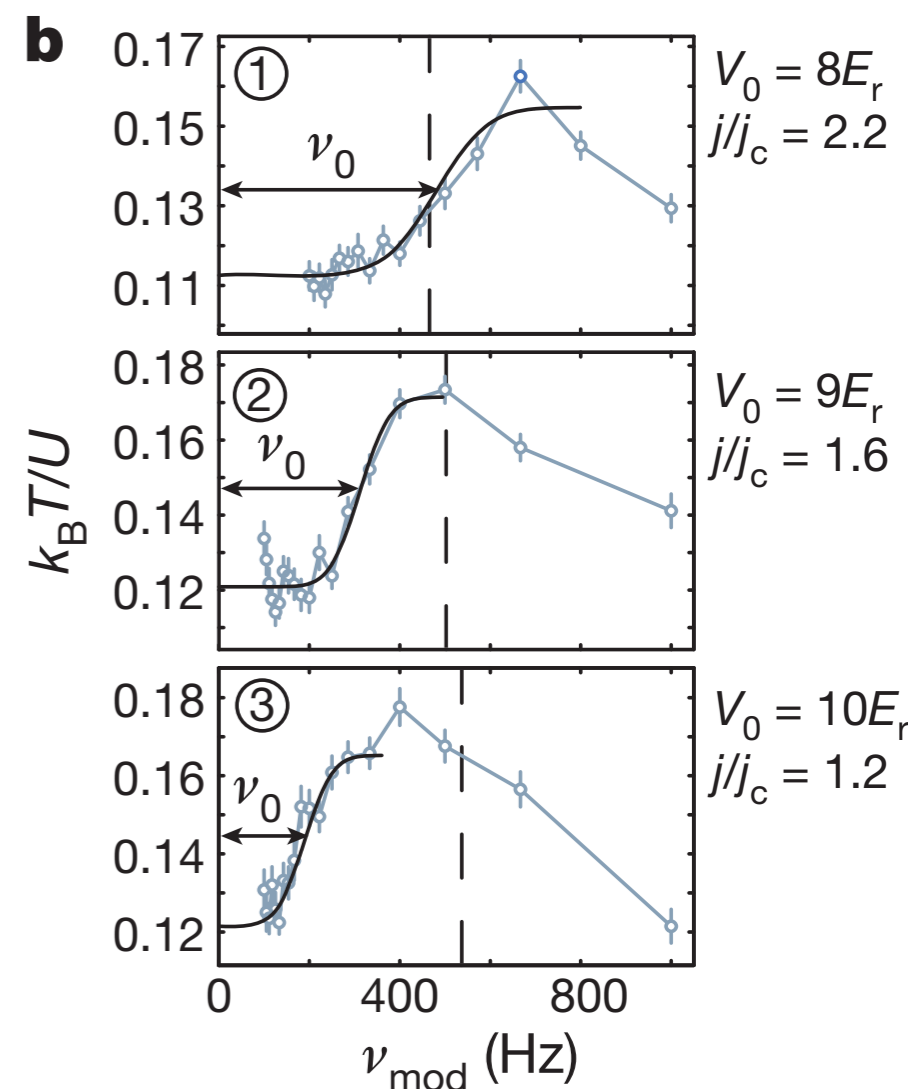
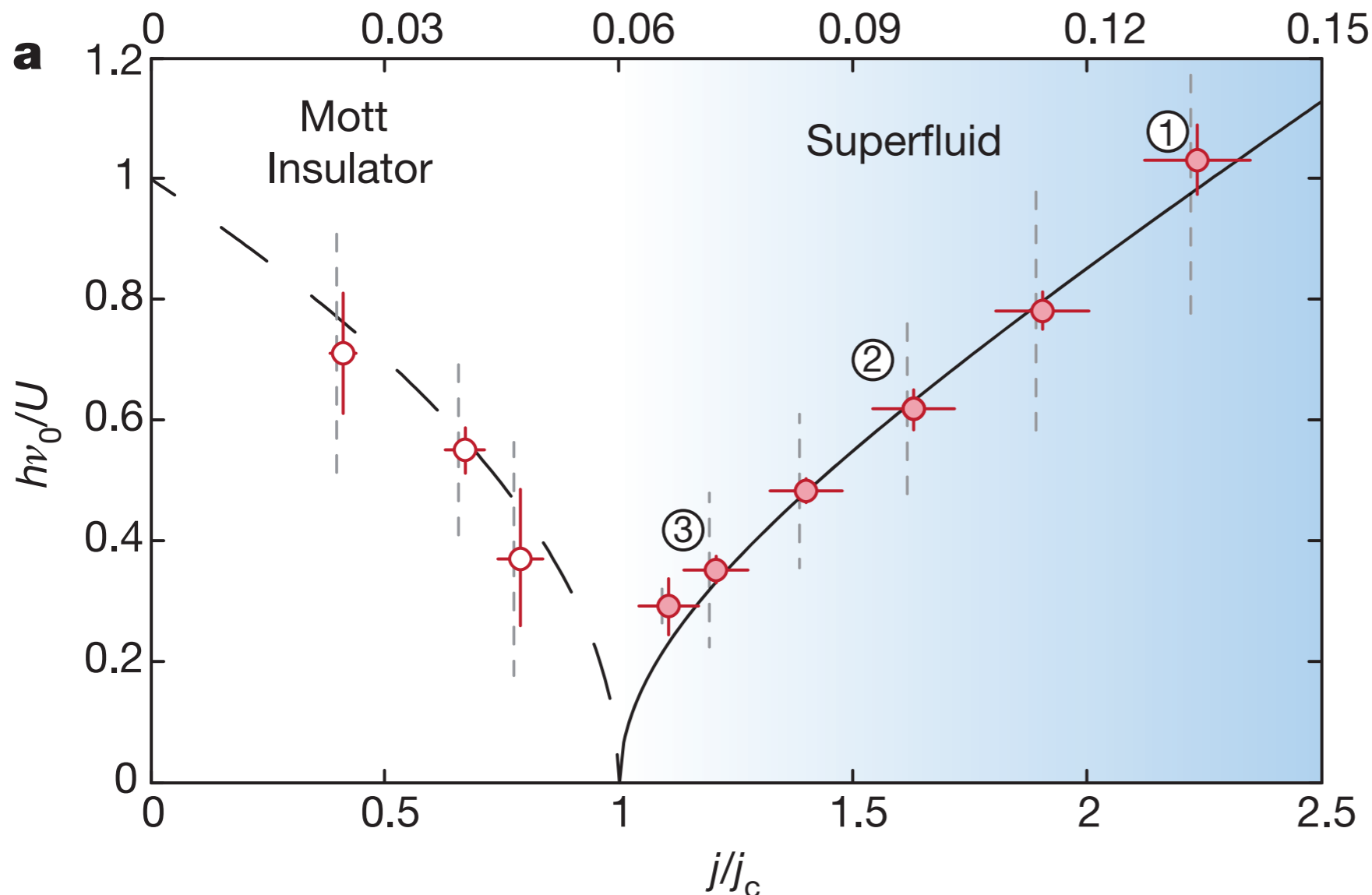
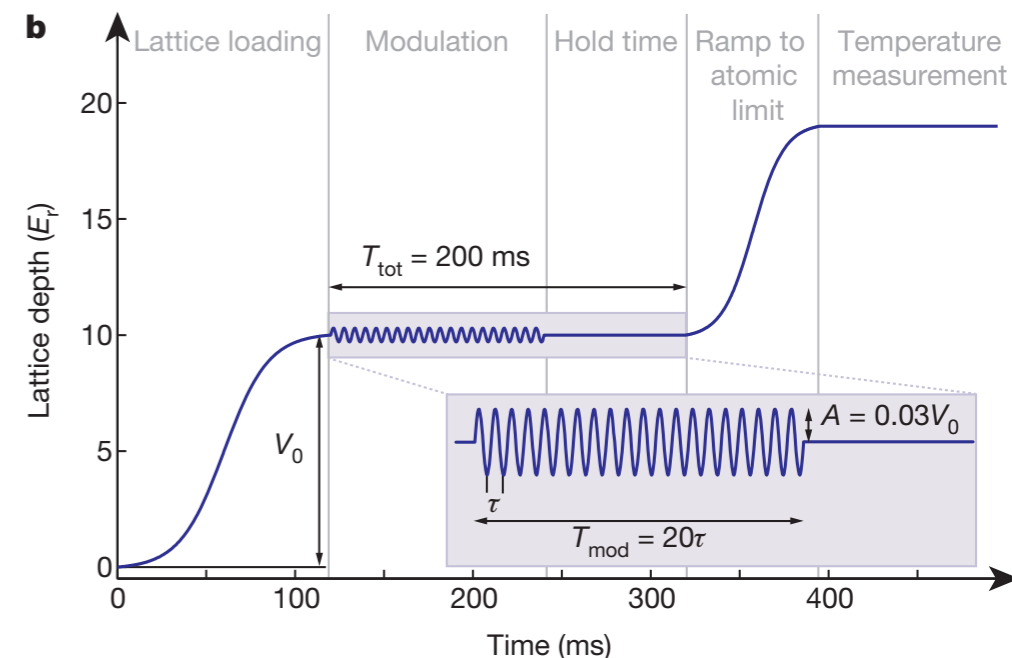
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Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

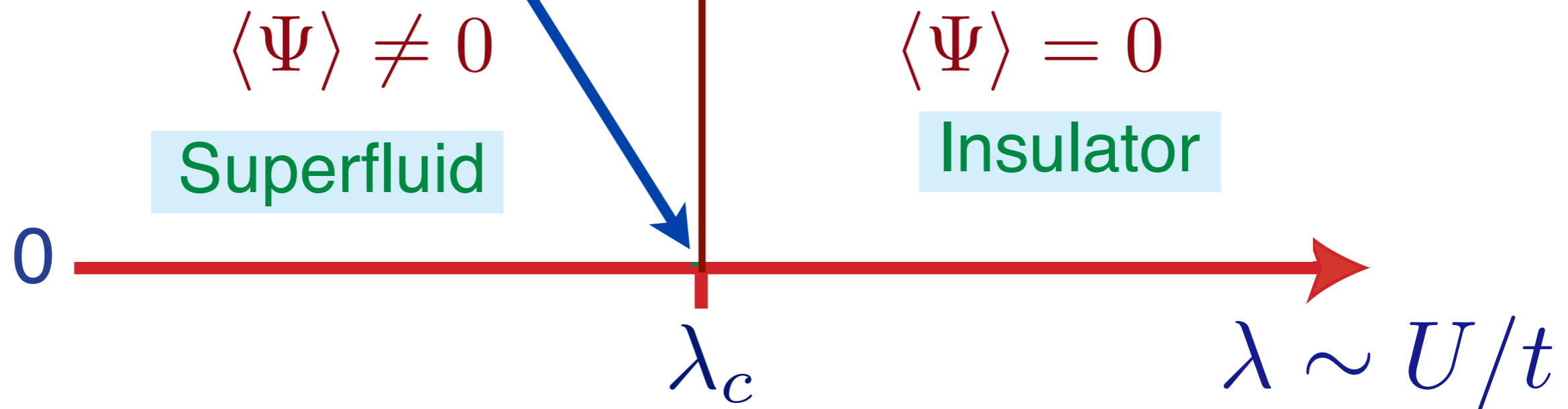


Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

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Quantum state with  
complex, many-body,  
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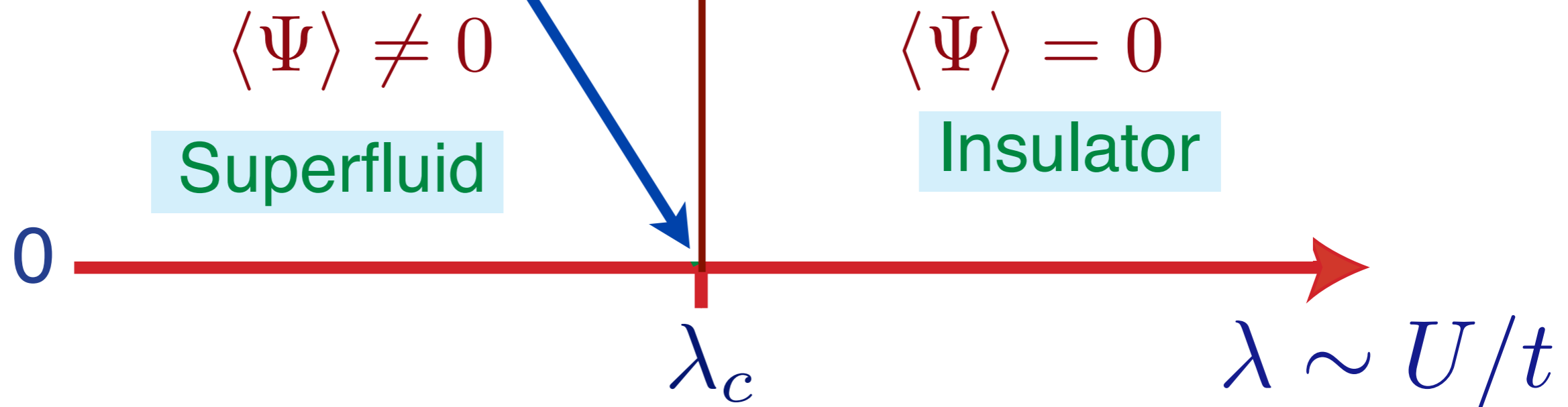
## Characteristics of quantum critical point

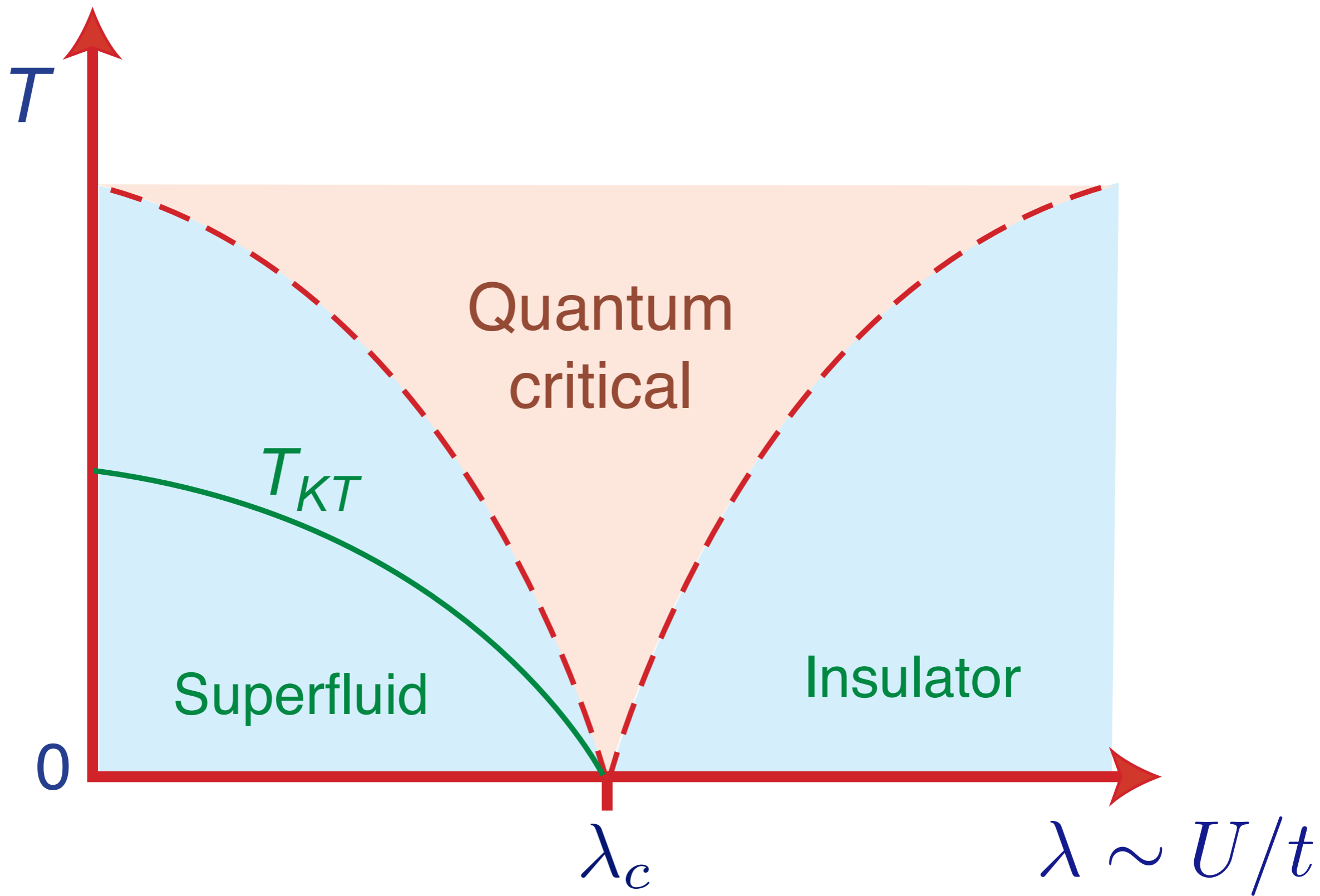
- Long-range entanglement
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- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
- The theory of the critical point is strongly-coupled because the quartic-coupling  $u$  flows to a renormalization group fixed point (the Wilson-Fisher fixed point). This fixed point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a **CFT<sub>3</sub>**

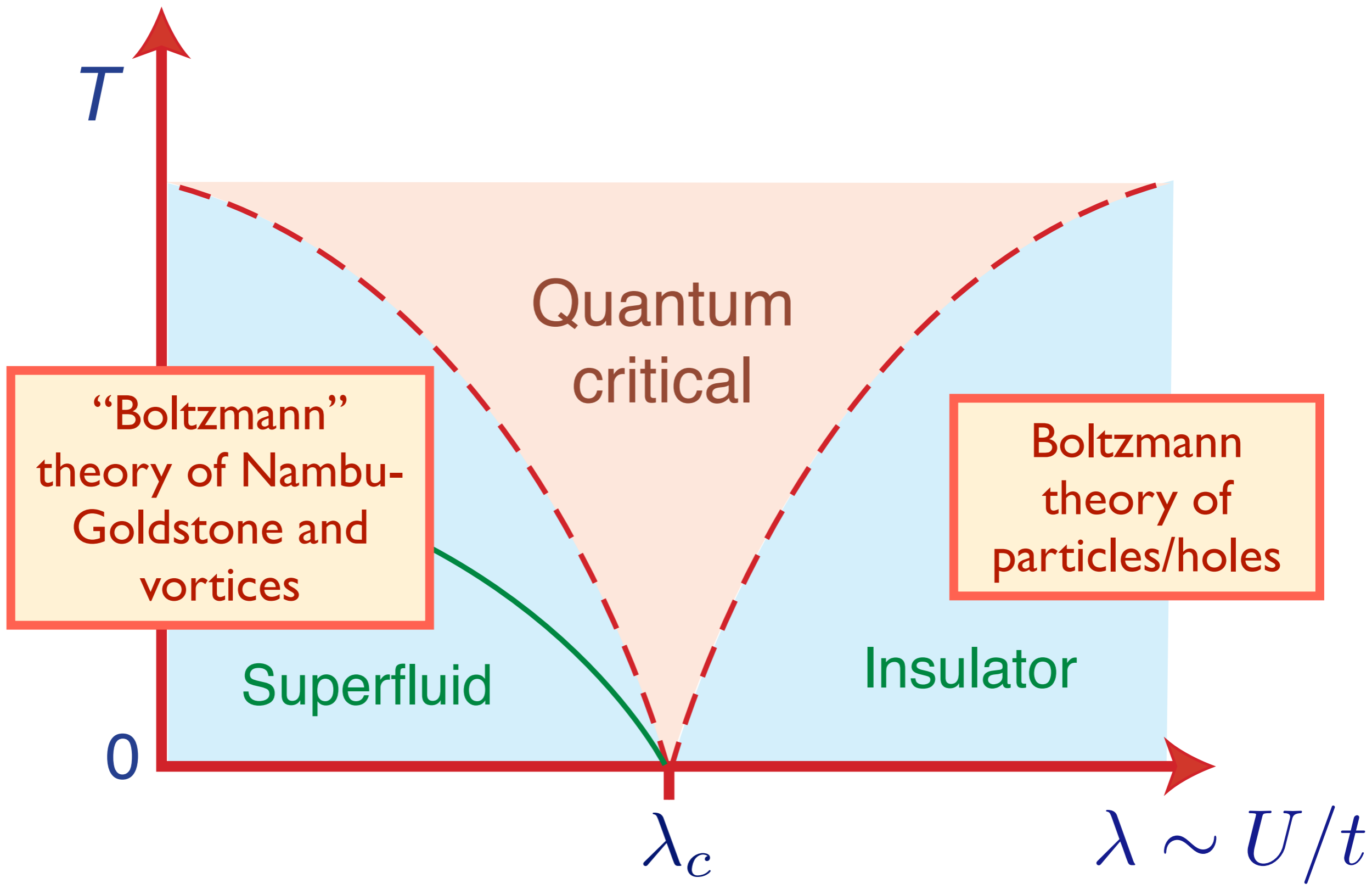
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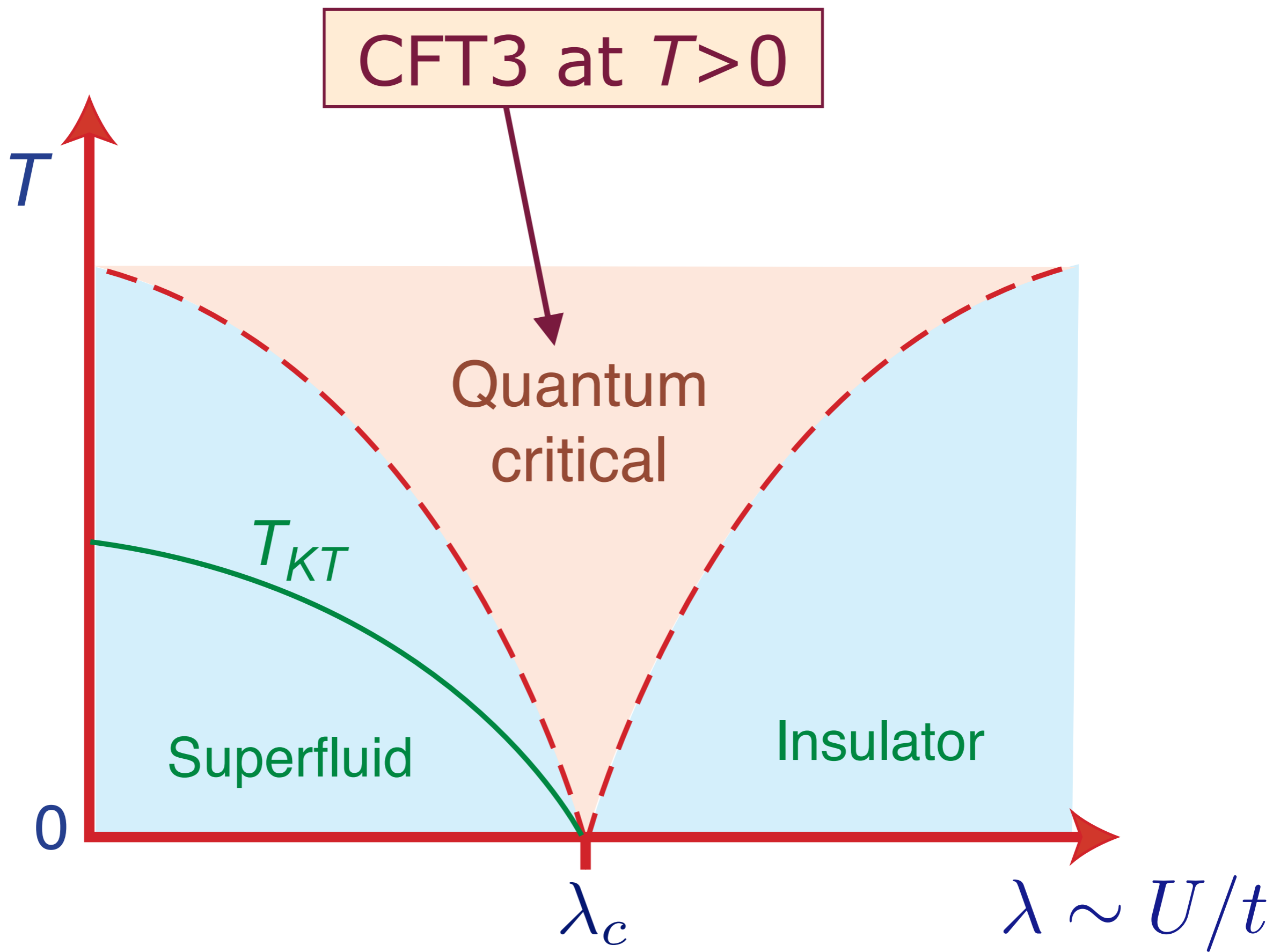
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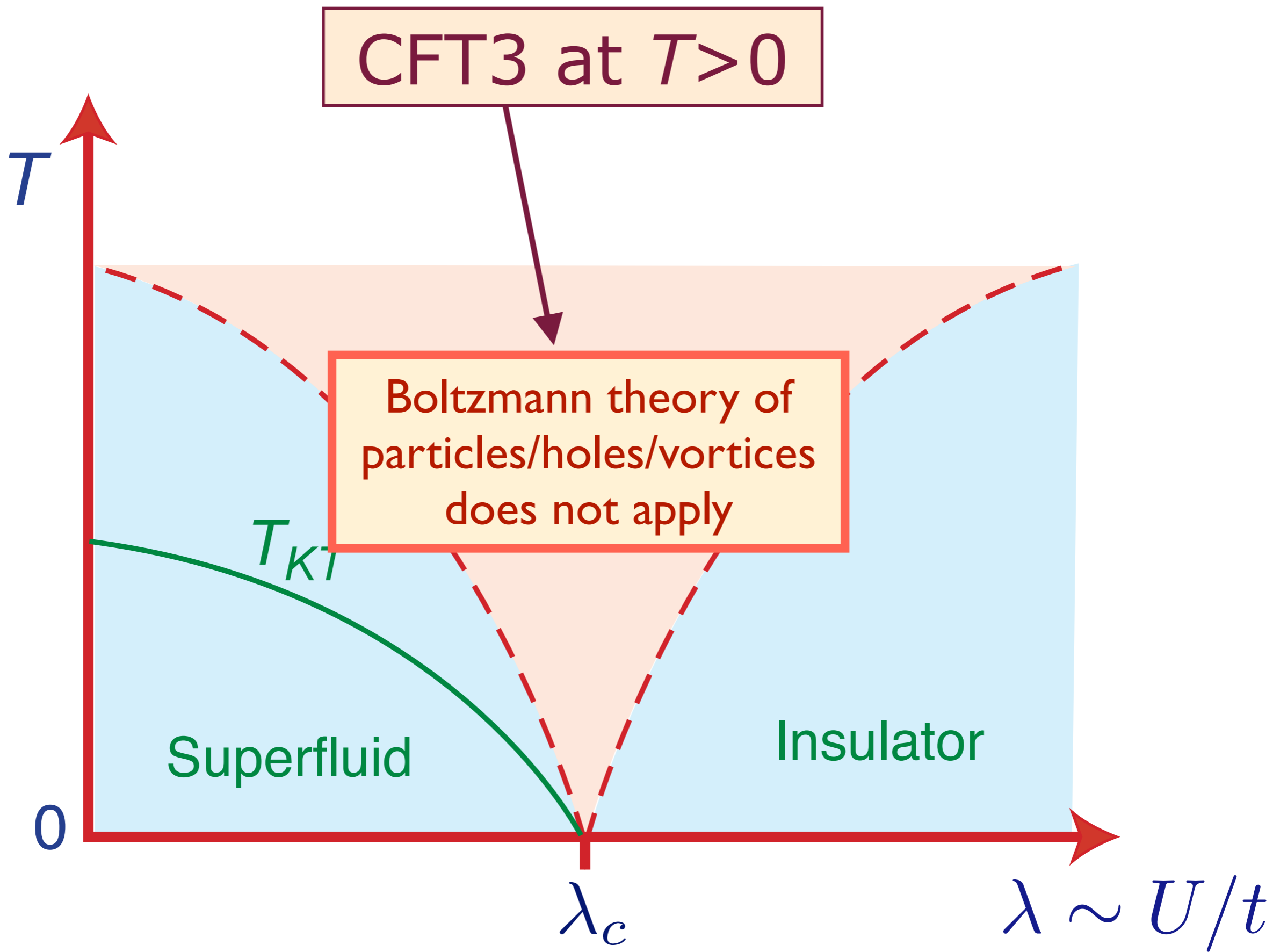
A conformal field theory  
in 2+1 spacetime dimensions:  
a CFT3



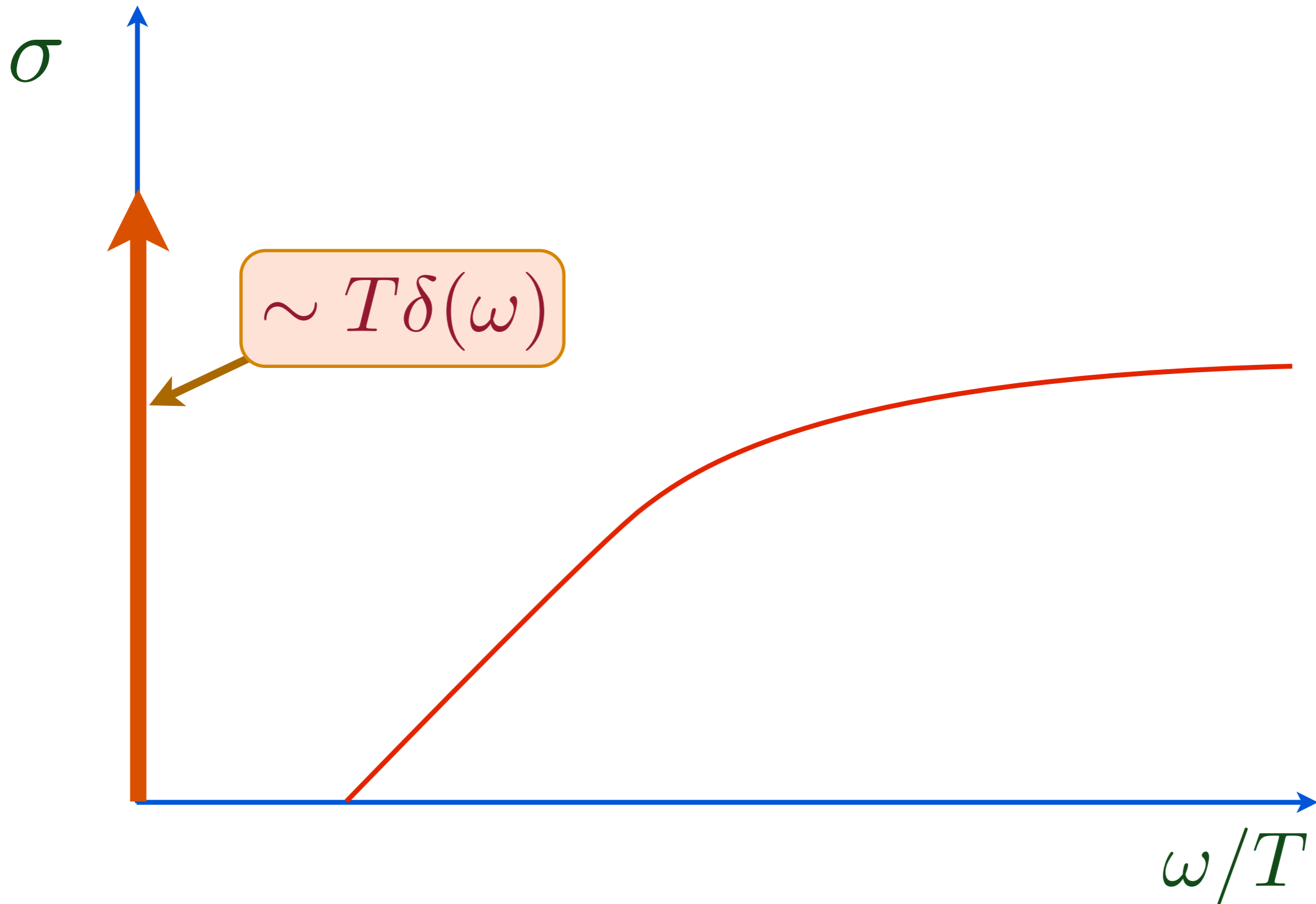




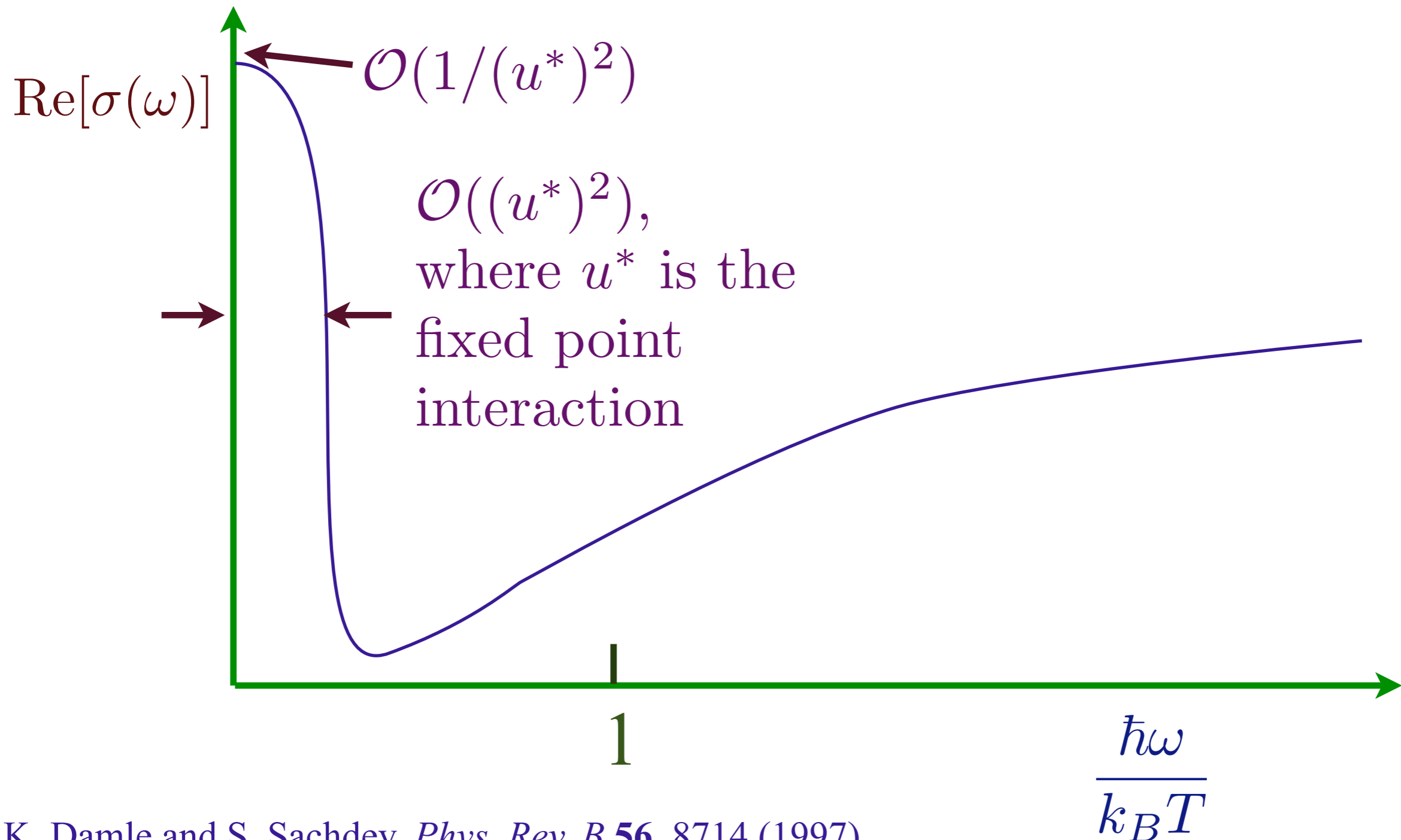




# Electrical transport in a free quasiparticle CFT3 for $T > 0$



# Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

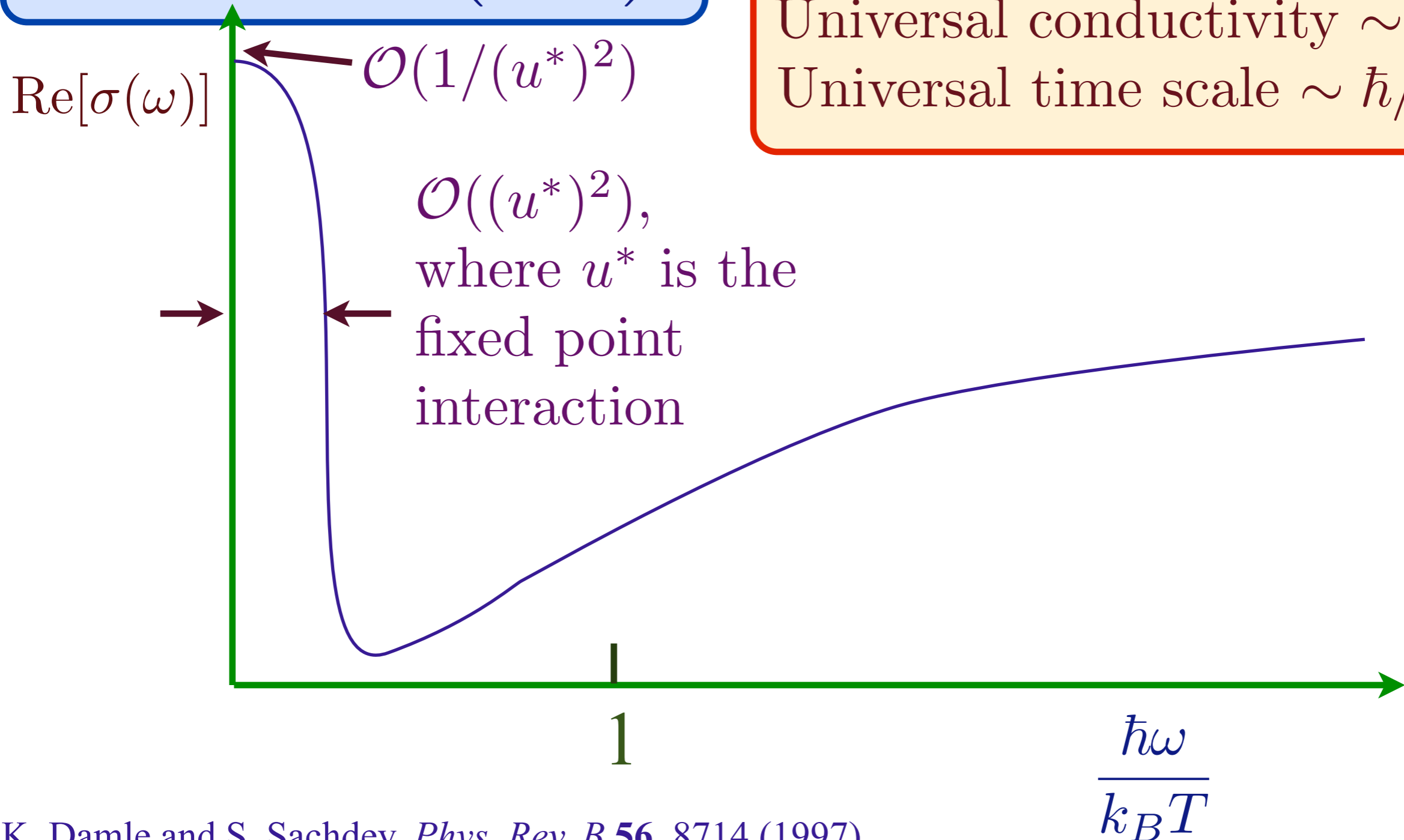


# Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left( \frac{\hbar\omega}{k_B T} \right)$$

$\Sigma \rightarrow$  a universal function

Universal conductivity  $\sim e^2/h$   
Universal time scale  $\sim \hbar/k_B T$



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$\text{Re}[\sigma(\omega)] \leftarrow \mathcal{O}(1/(u^*)^2)$

$\mathcal{O}((u^*)^2),$

v  
f  
i

Needed:

a method for computing  
 the universal conductivity  
 of strongly interacting CFT3s

1

$\frac{\hbar\omega}{k_B T}$

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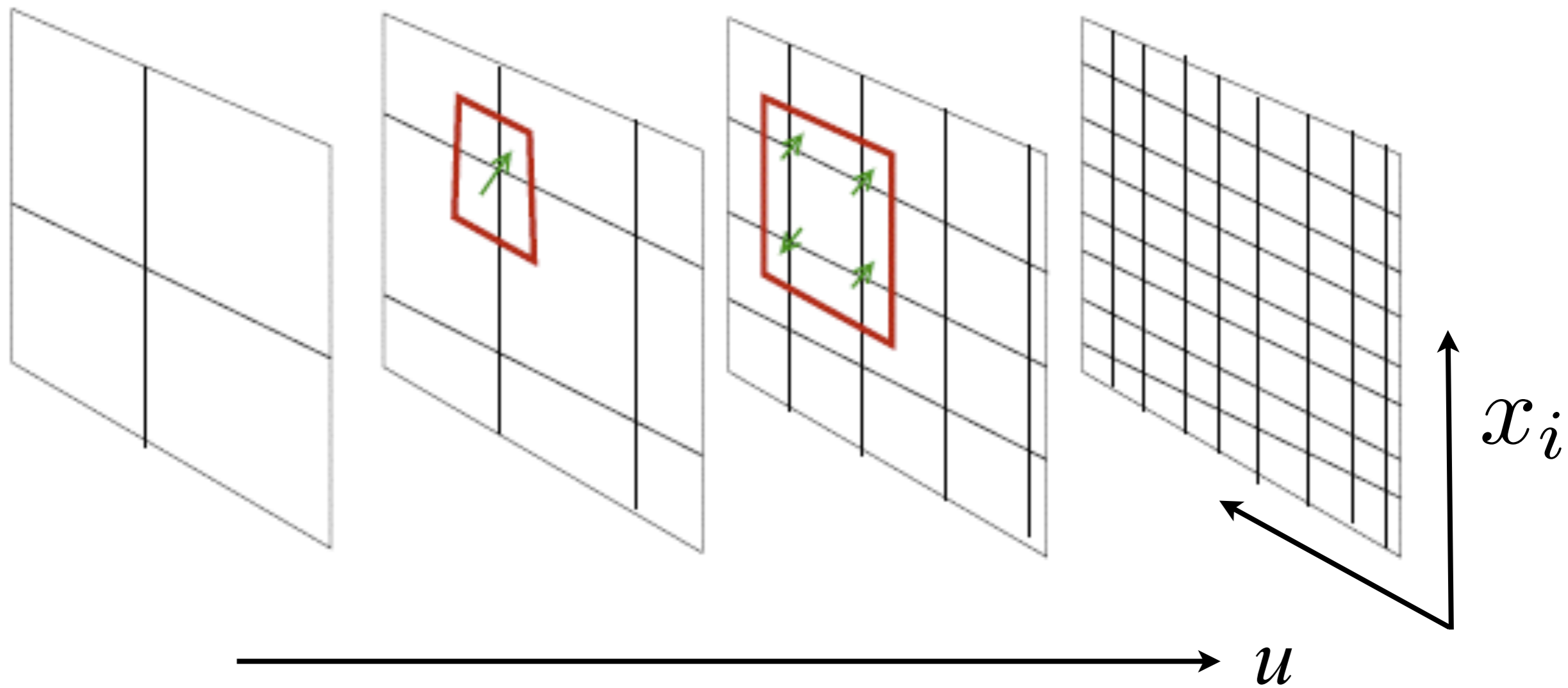
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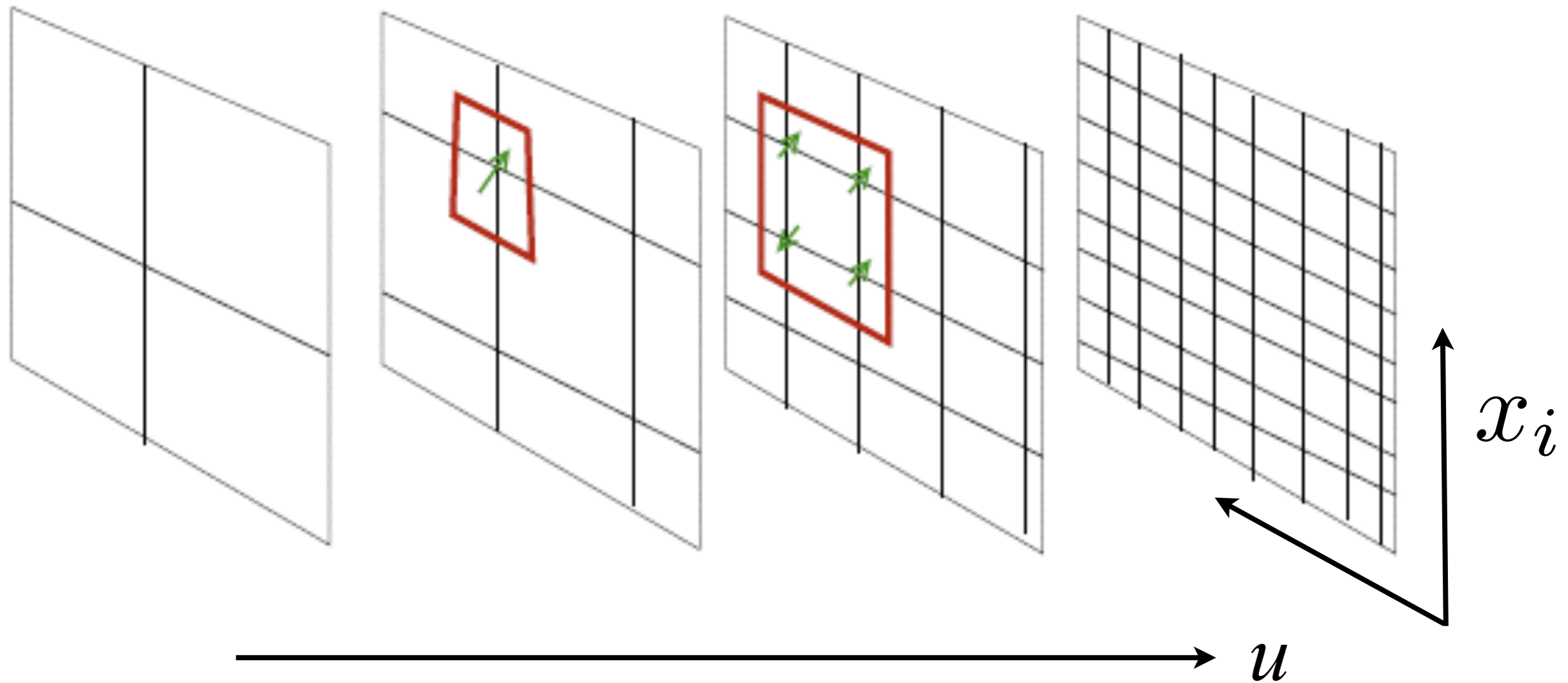
Field theories in  $d + 1$  spacetime dimensions are characterized by couplings  $g$  which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where  $u$  is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon  $u$ .

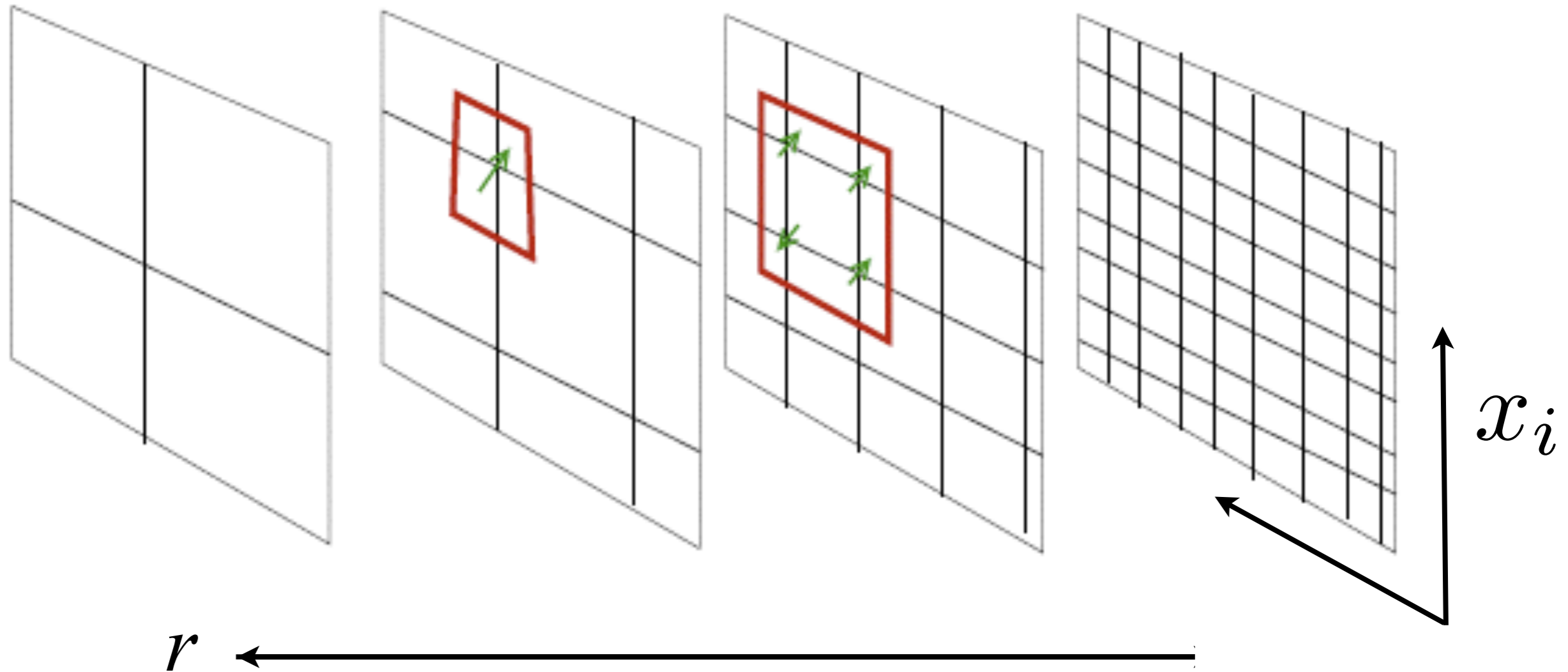


# Holography



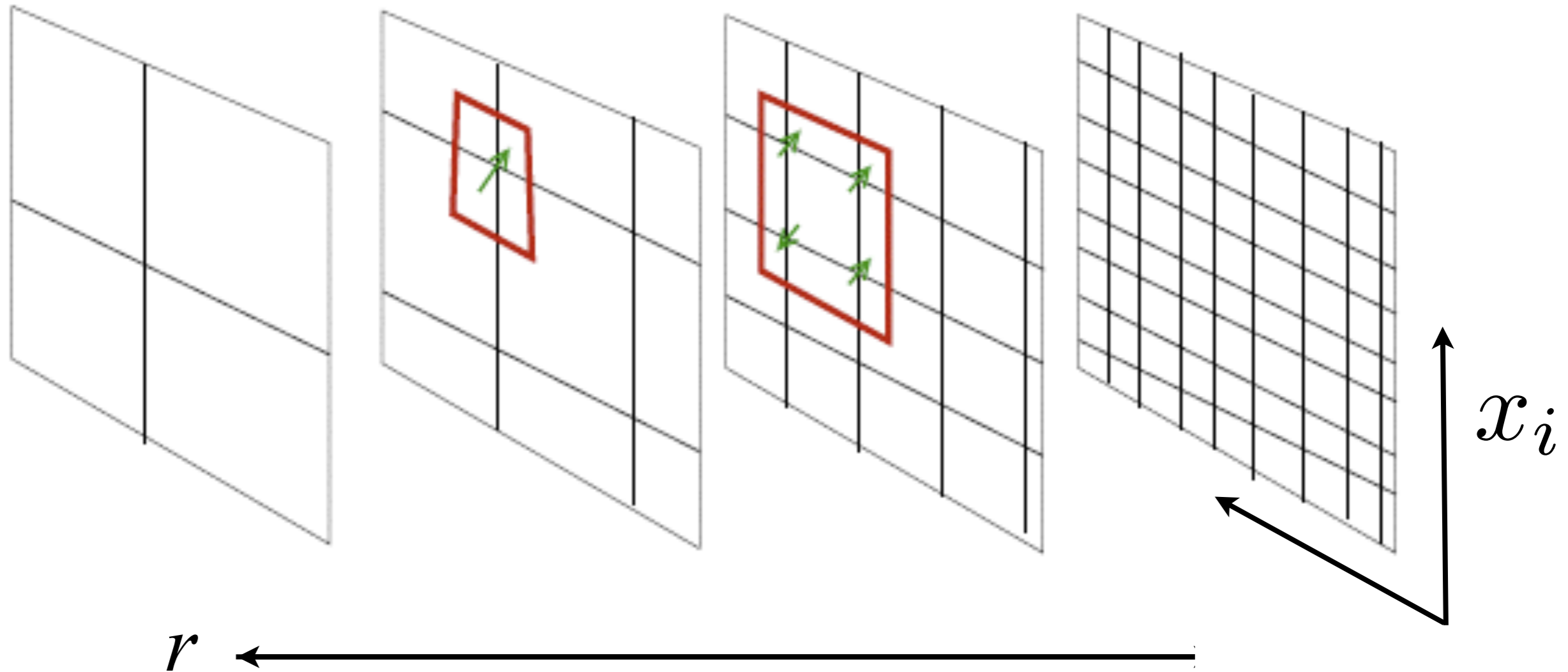
**Key idea:**  $\Rightarrow$  Implement  $u$  as an extra dimension, and map to a local theory in  $d+2$  spacetime dimensions.

# Holography



**Key idea:**  $\Rightarrow$  Implement  $u$  as an extra dimension, and map to a local theory in  $d+2$  spacetime dimensions. We identify the extra-dimensional co-ordinate  $r = 1/u$ .

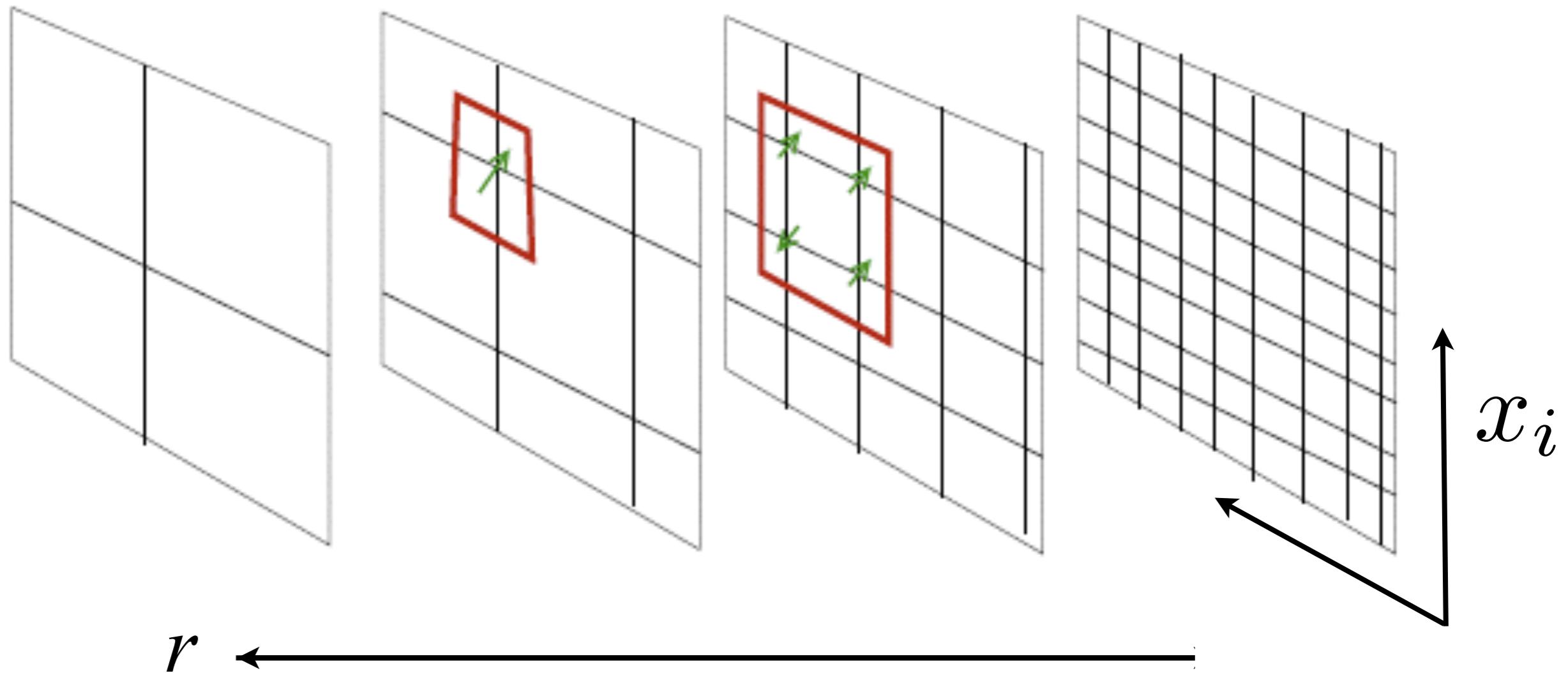
# Holography



For a relativistic CFT in  $d$  spatial dimensions, the proper length,  $ds$ , in the holographic space is fixed by demanding the scale transformation ( $i = 1 \dots d$ )

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

# Holography

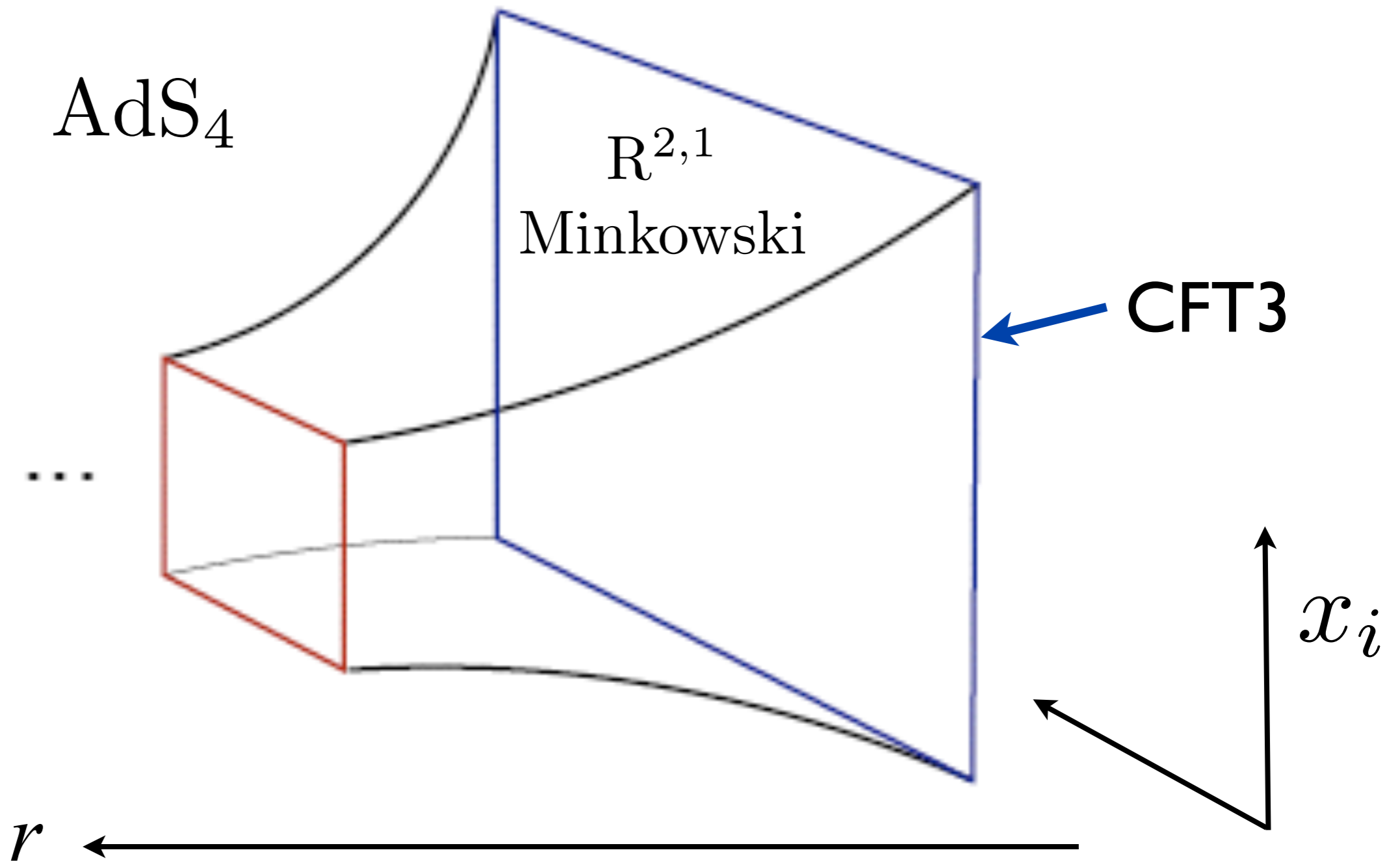


This gives the unique metric

$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

This is the metric of anti-de Sitter space  $\text{AdS}_{d+2}$ .

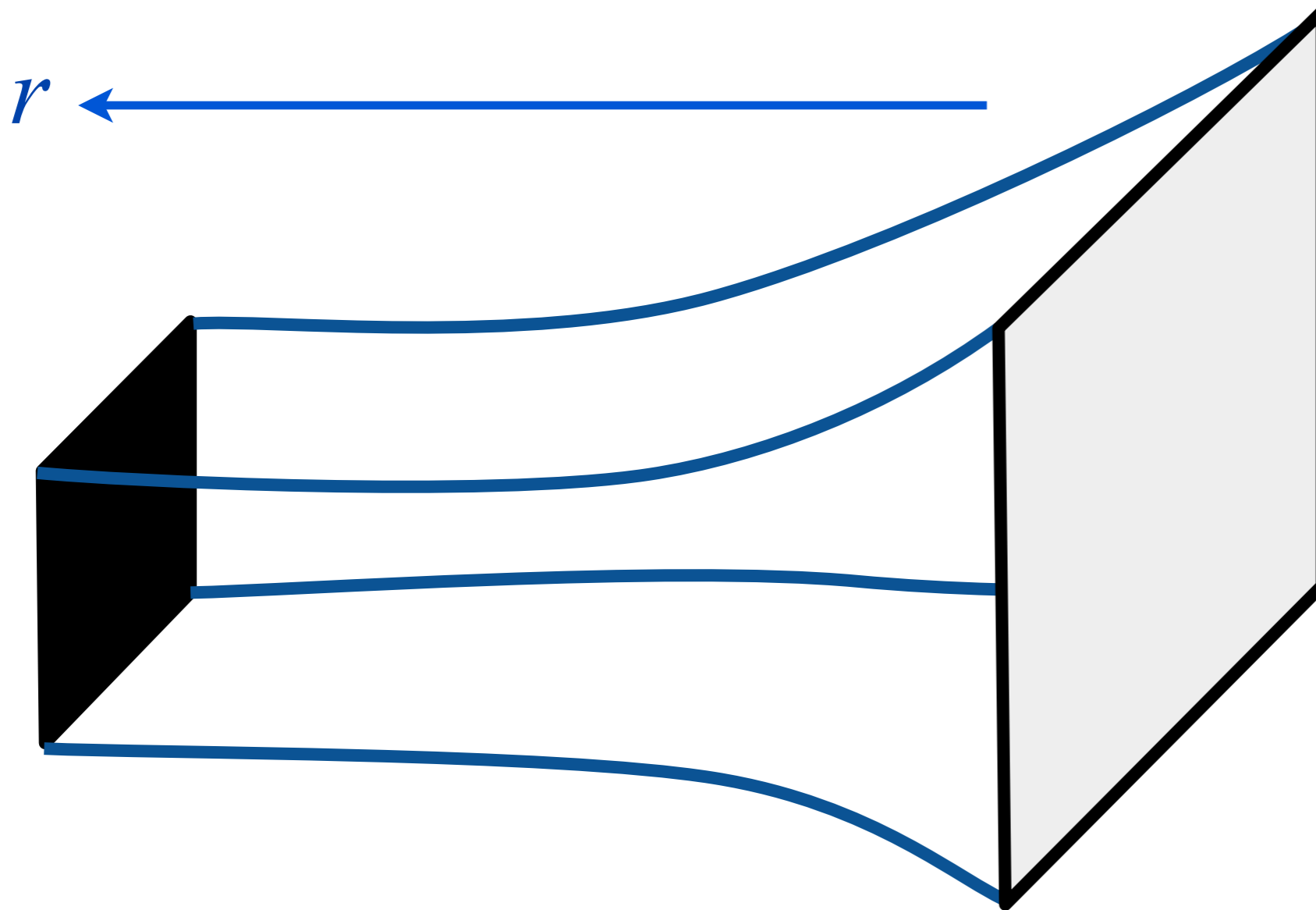
# AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$

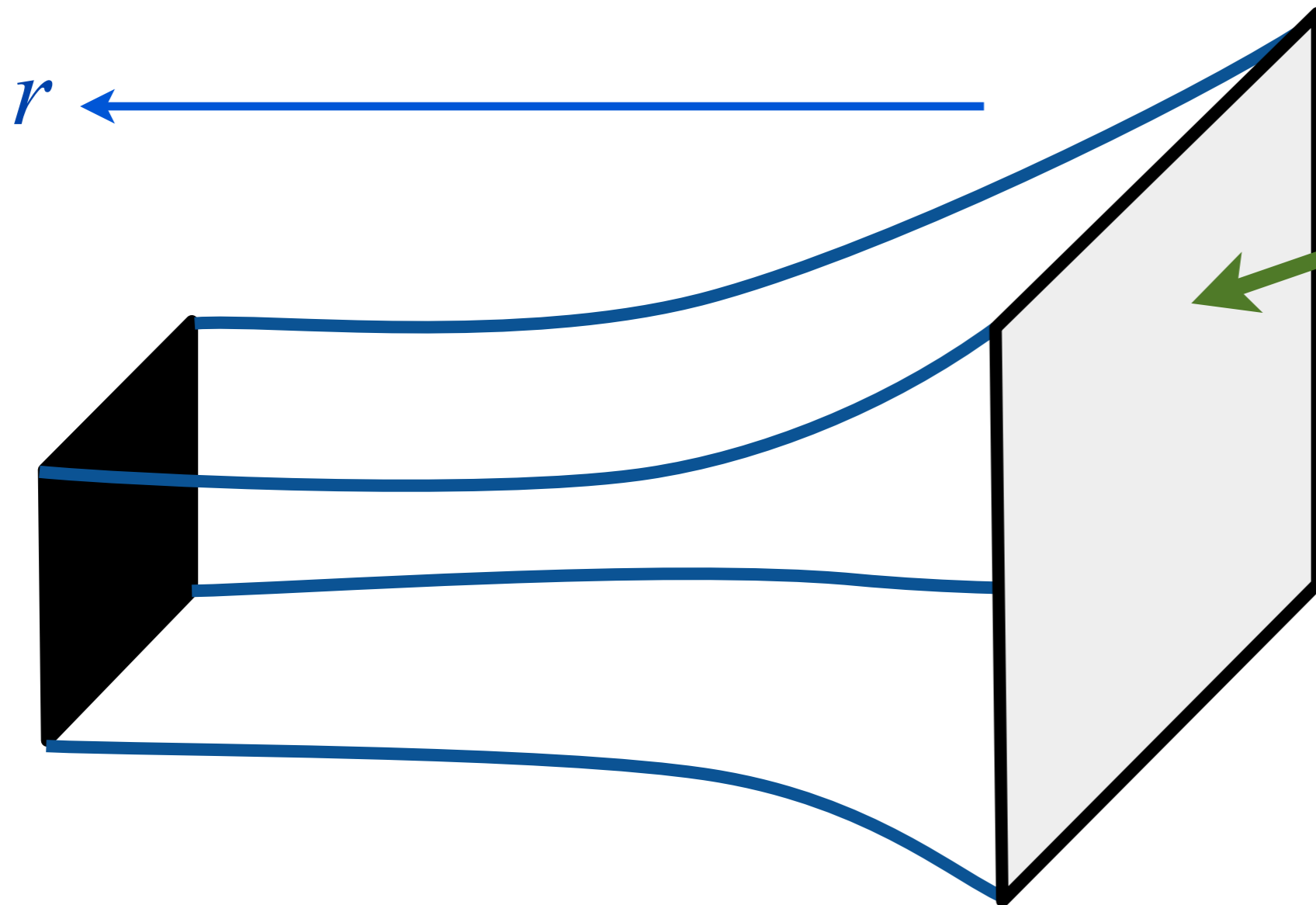
# Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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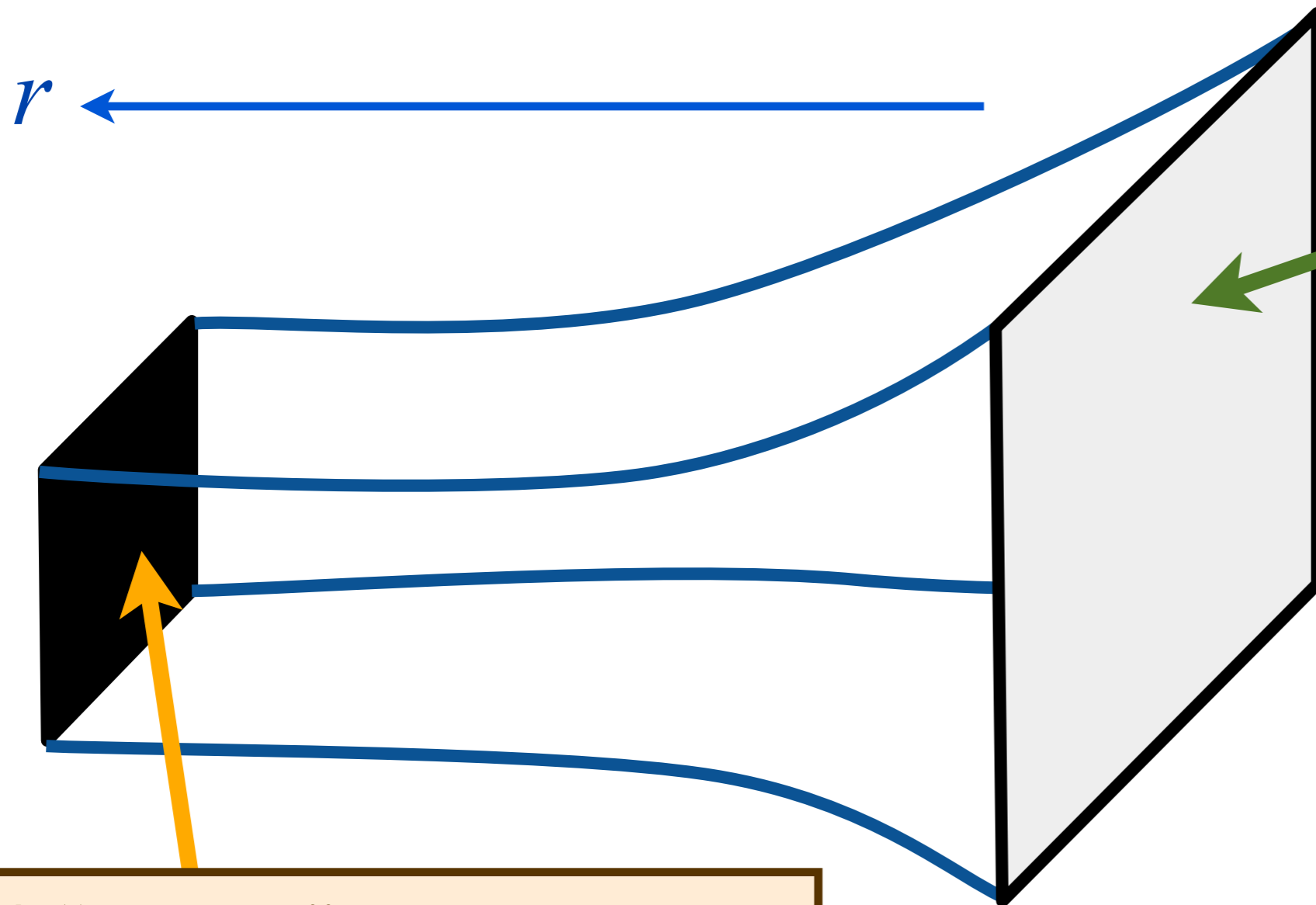
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A 2+1  
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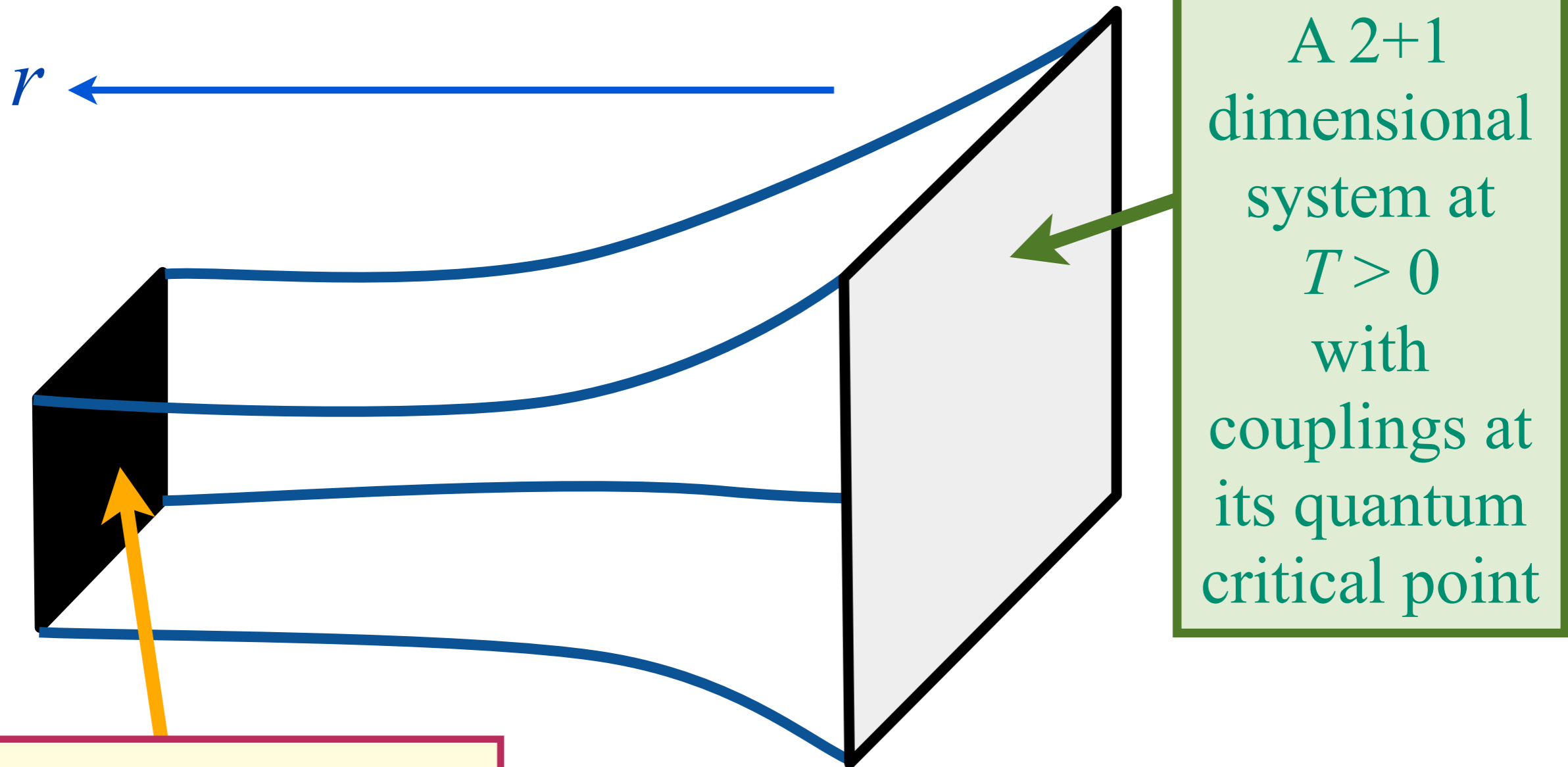


A 2+1  
dimensional  
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A "horizon", similar to the  
surface of a black hole !

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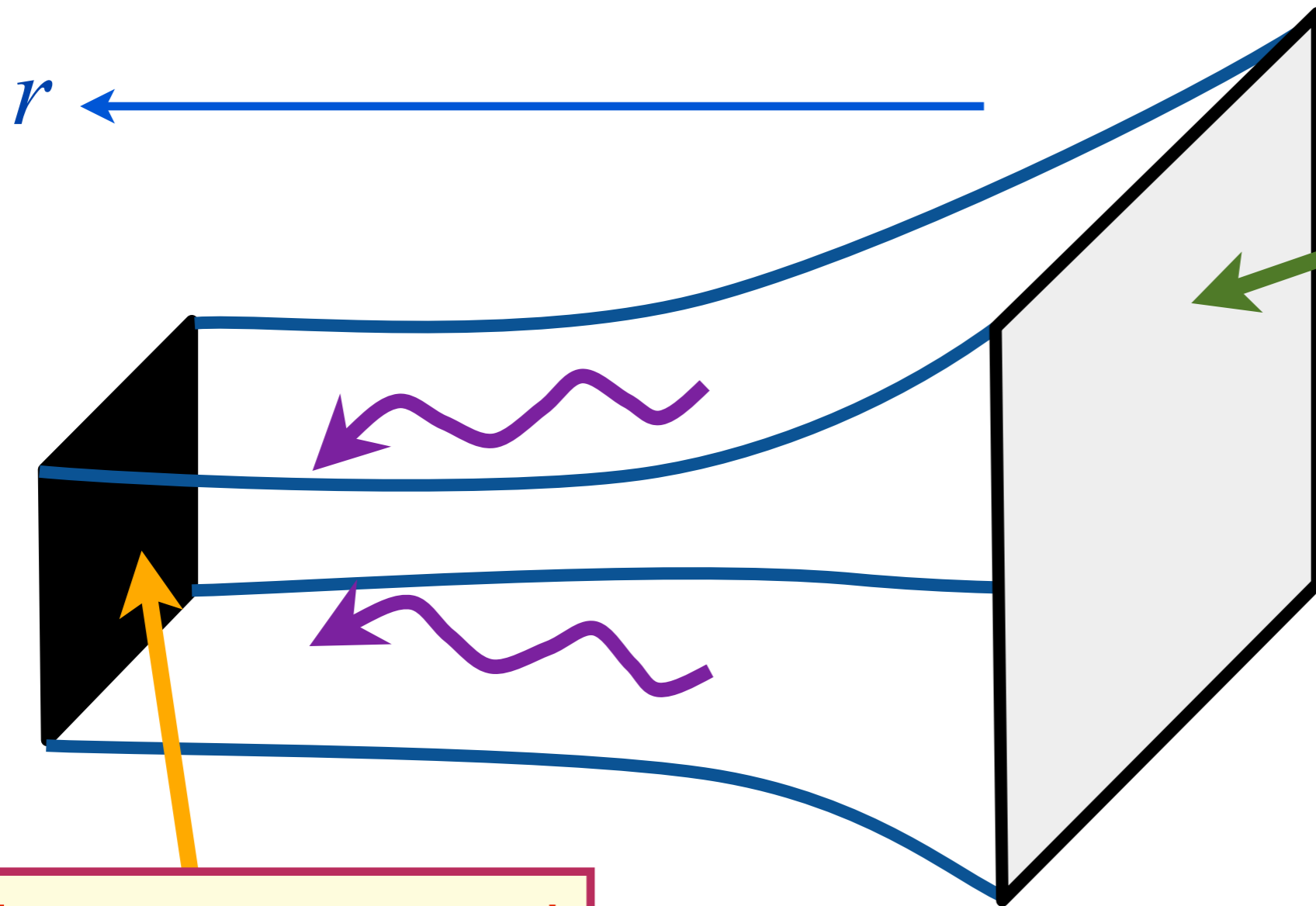
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The temperature and entropy of the horizon equal those of the quantum critical point

A 2+1 dimensional system at  $T > 0$  with couplings at its quantum critical point

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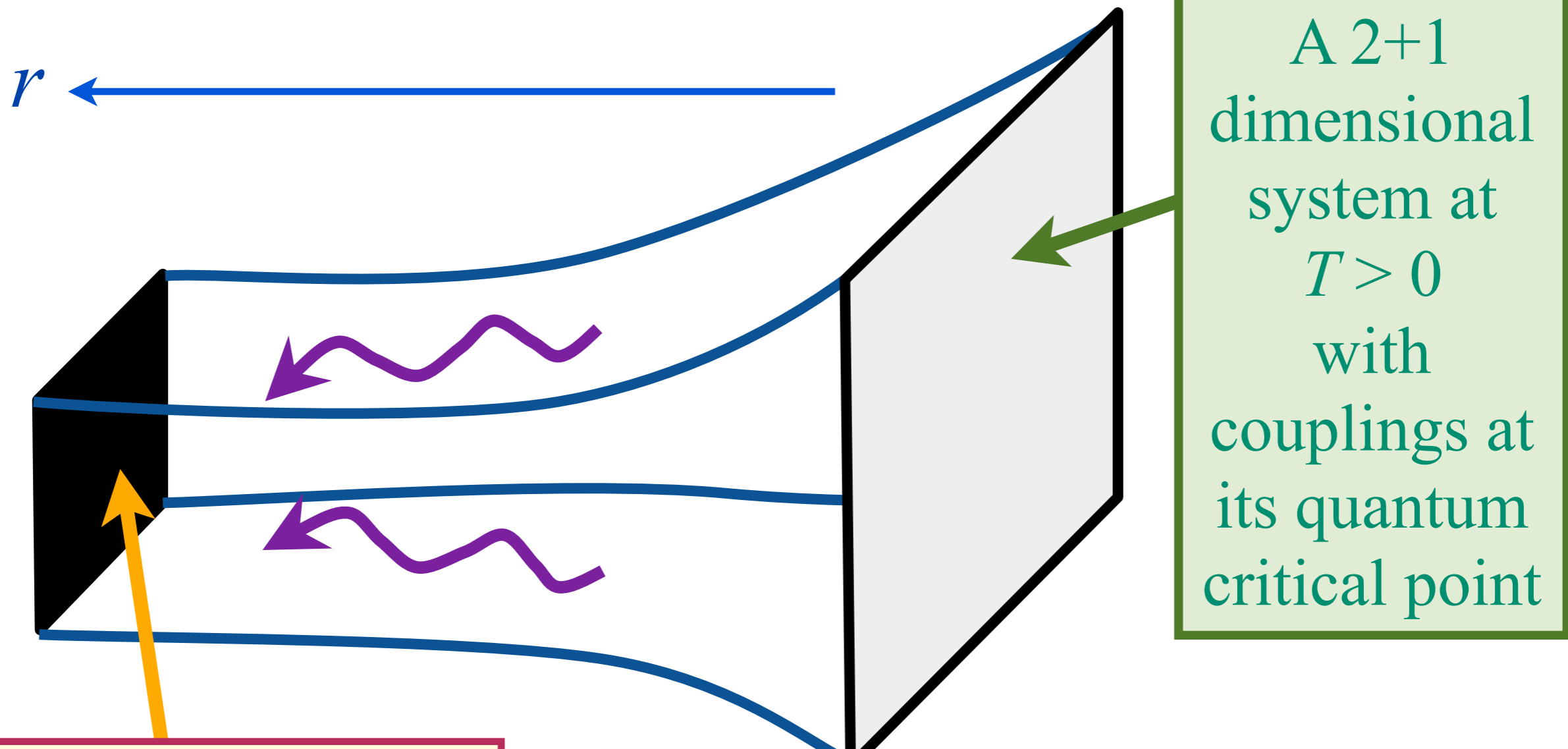


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Quasi-normal modes of quantum criticality = waves falling into black hole

# Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at  $T > 0$  with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Characteristic damping time of quasi-normal modes:  
 $(k_B / \hbar) \times$  Hawking temperature

# Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

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- Solve Einstein-Maxwell equations. Dynamics of quasi-normal modes of black branes.

# AdS<sub>4</sub> theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

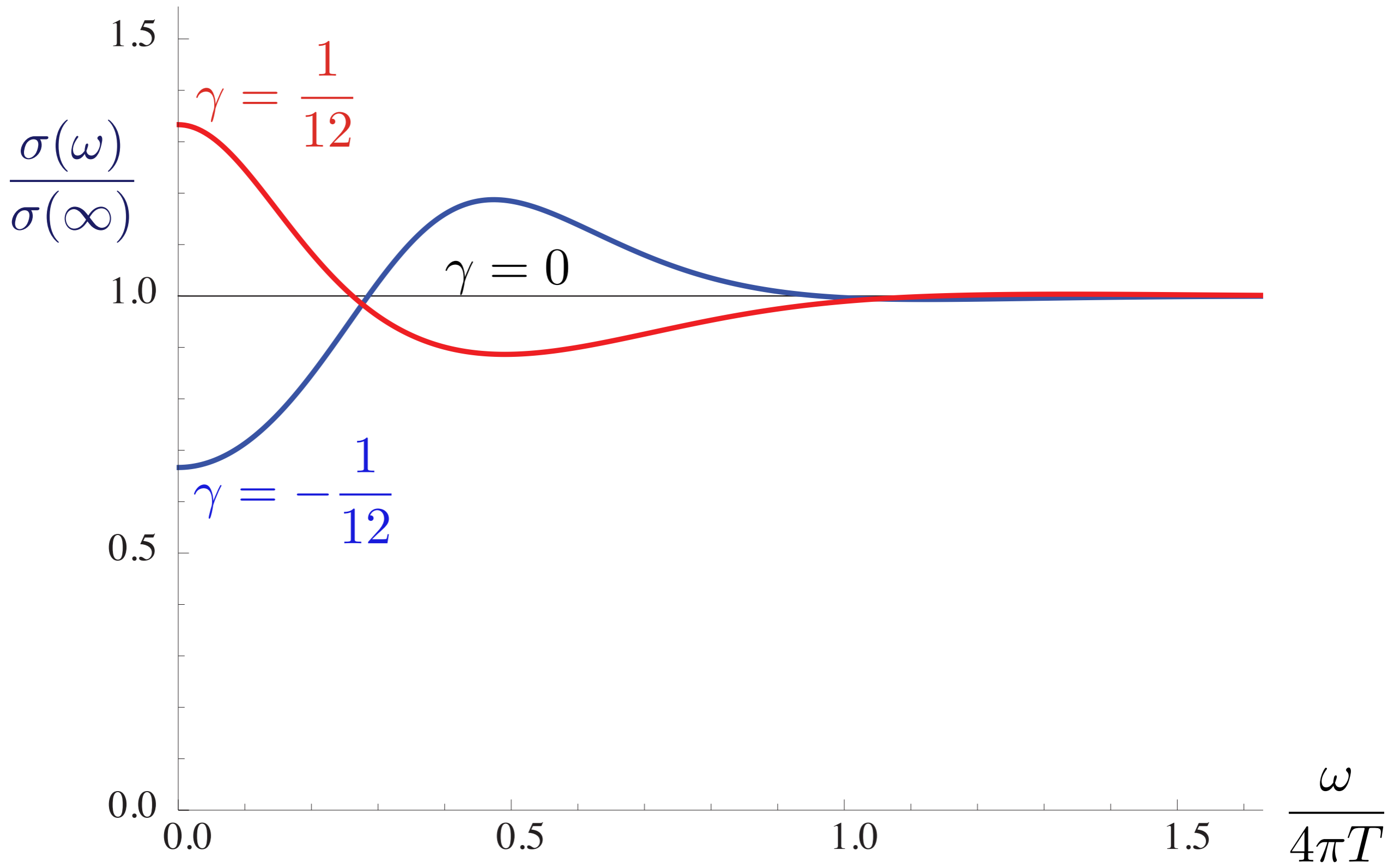
$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current  $J_\mu$  and the stress energy tensor  $T_{\mu\nu}$ , and a 3-point  $T, J, J$  correlator. Constraints from both the CFT and the gravitational theory bound  $|\gamma| \leq 1/12 = 0.0833..$

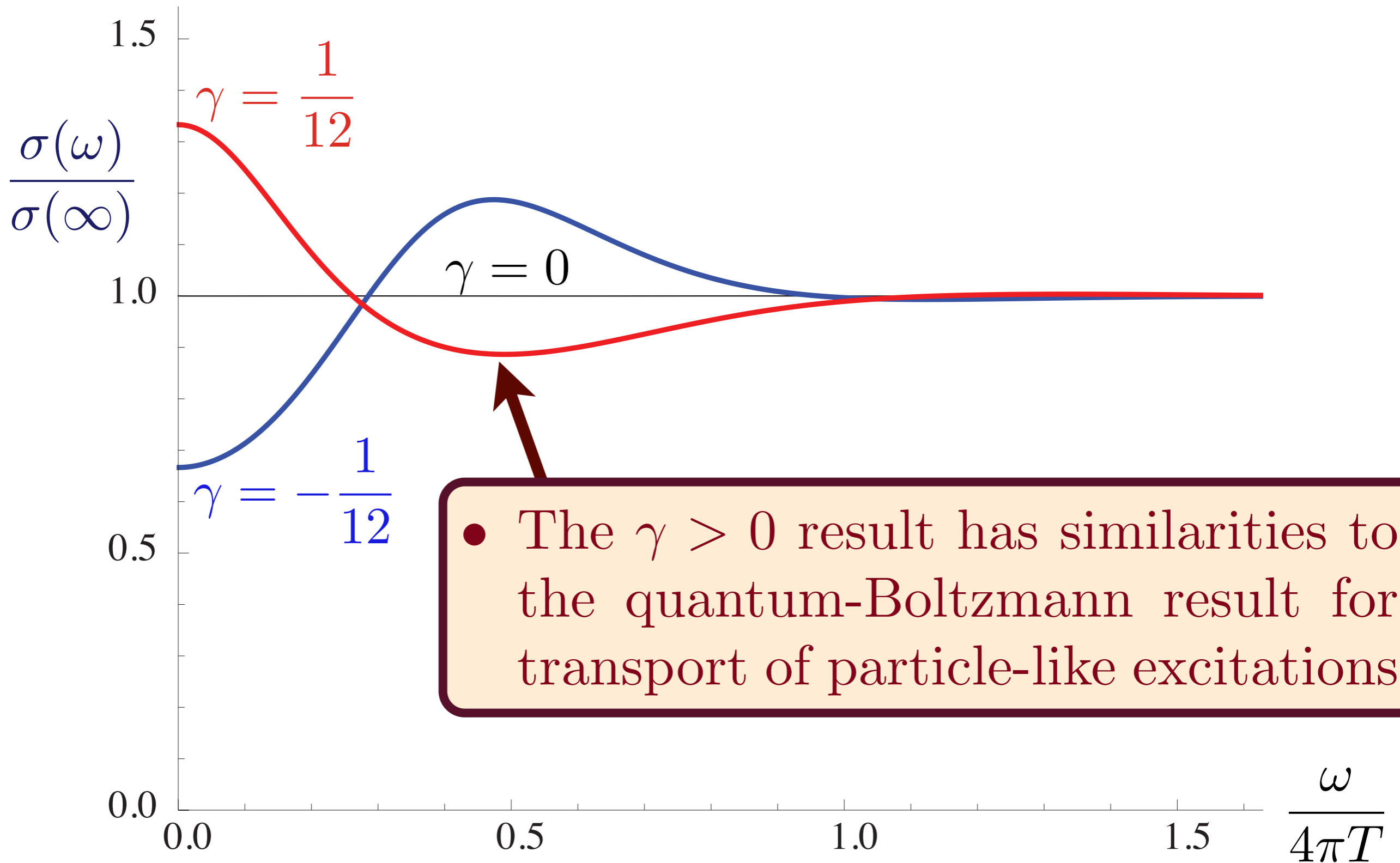
R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

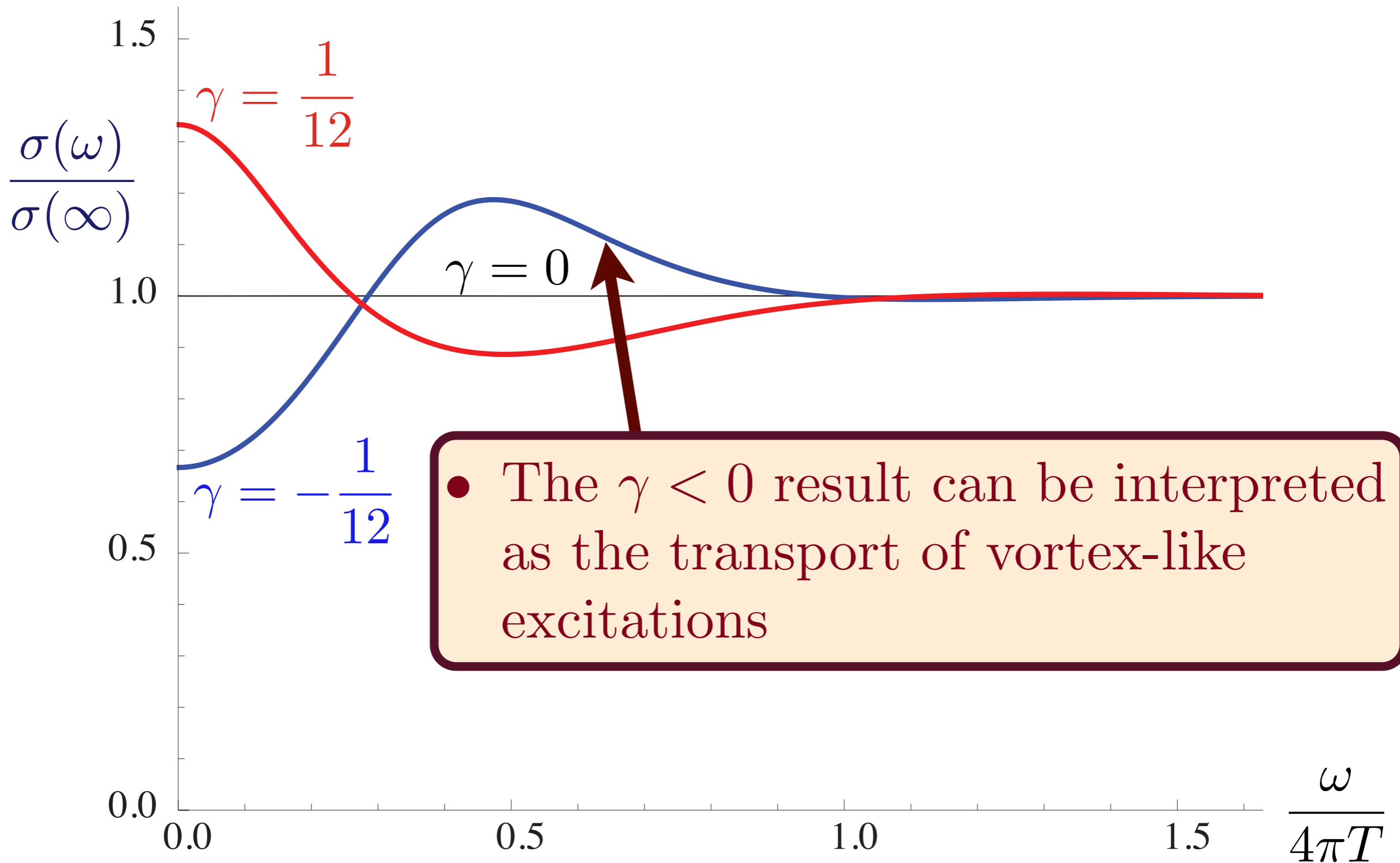
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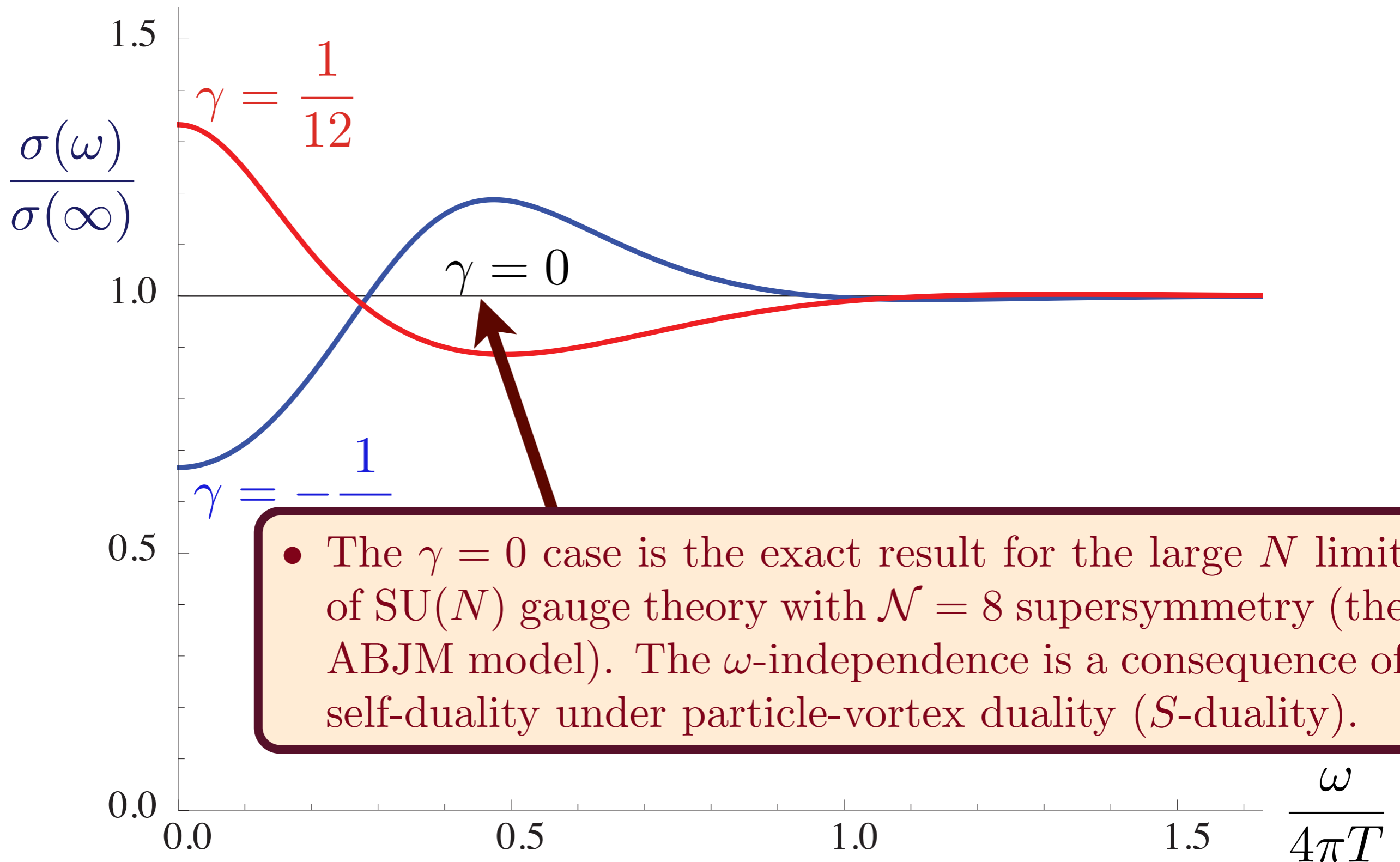
# AdS<sub>4</sub> theory of quantum criticality



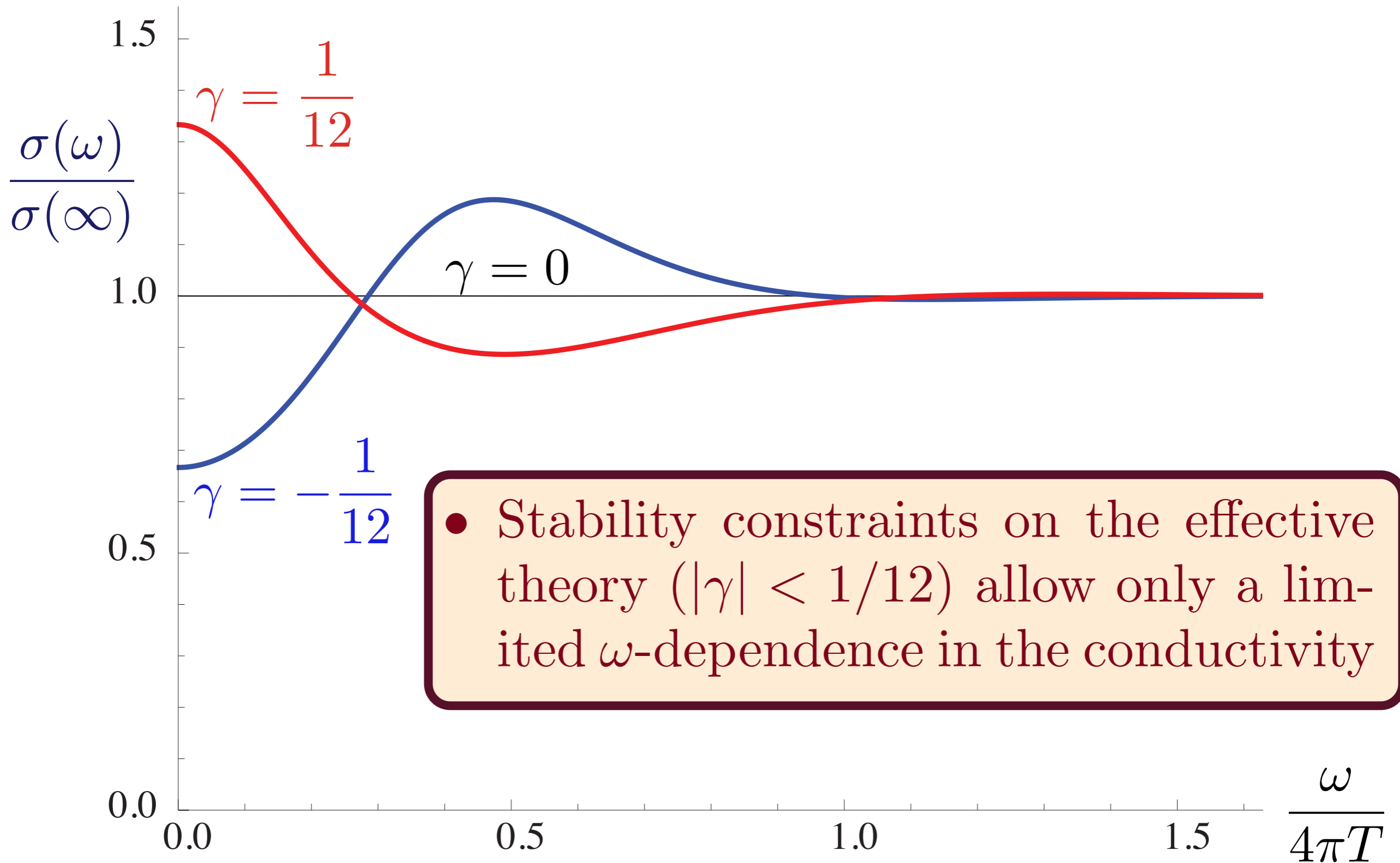
# AdS<sub>4</sub> theory of quantum criticality



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# The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

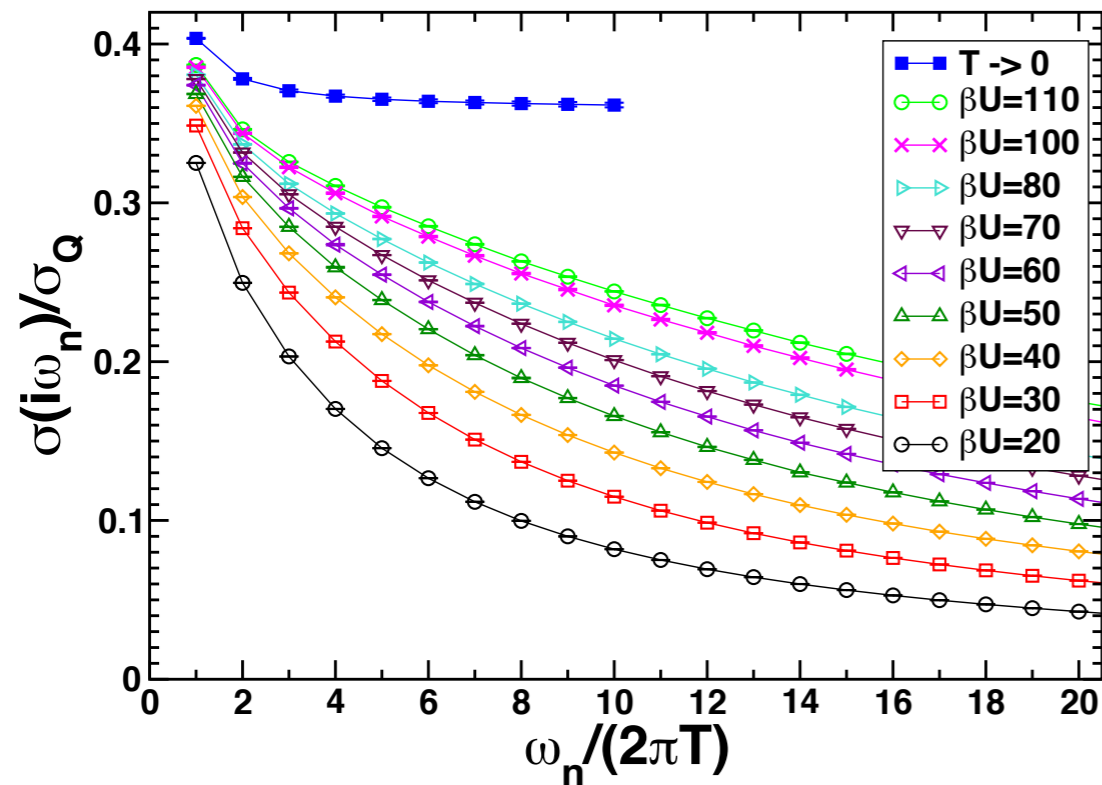
# Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

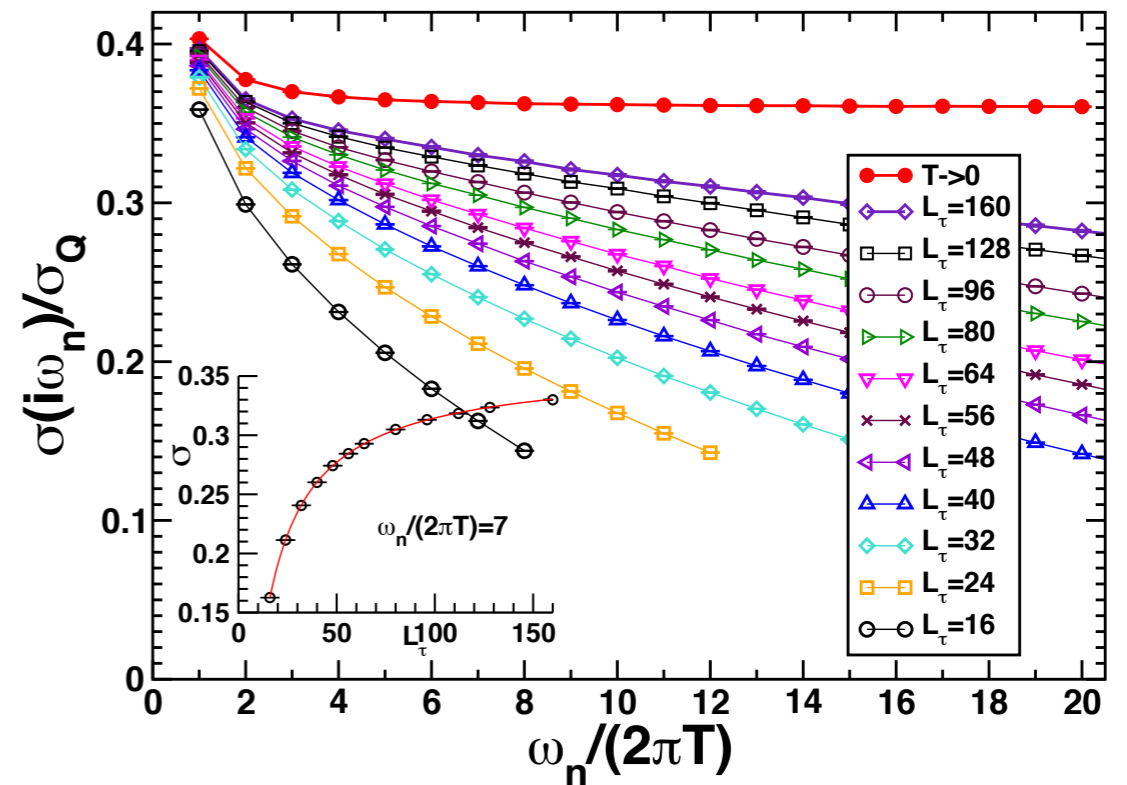
(Submitted on 22 Sep 2013)

We compute the universal conductivity of the  $(2+1)$ -dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical  $(2+1)$ -dimensional  $J$ -current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau,  $\sigma(\infty) = 0.359(4)\sigma_Q$  with  $\sigma_Q$  the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our  $\sigma(i\omega_n) - \sigma(\infty)$  function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.

# Quantum Monte Carlo for lattice bosons



(a)



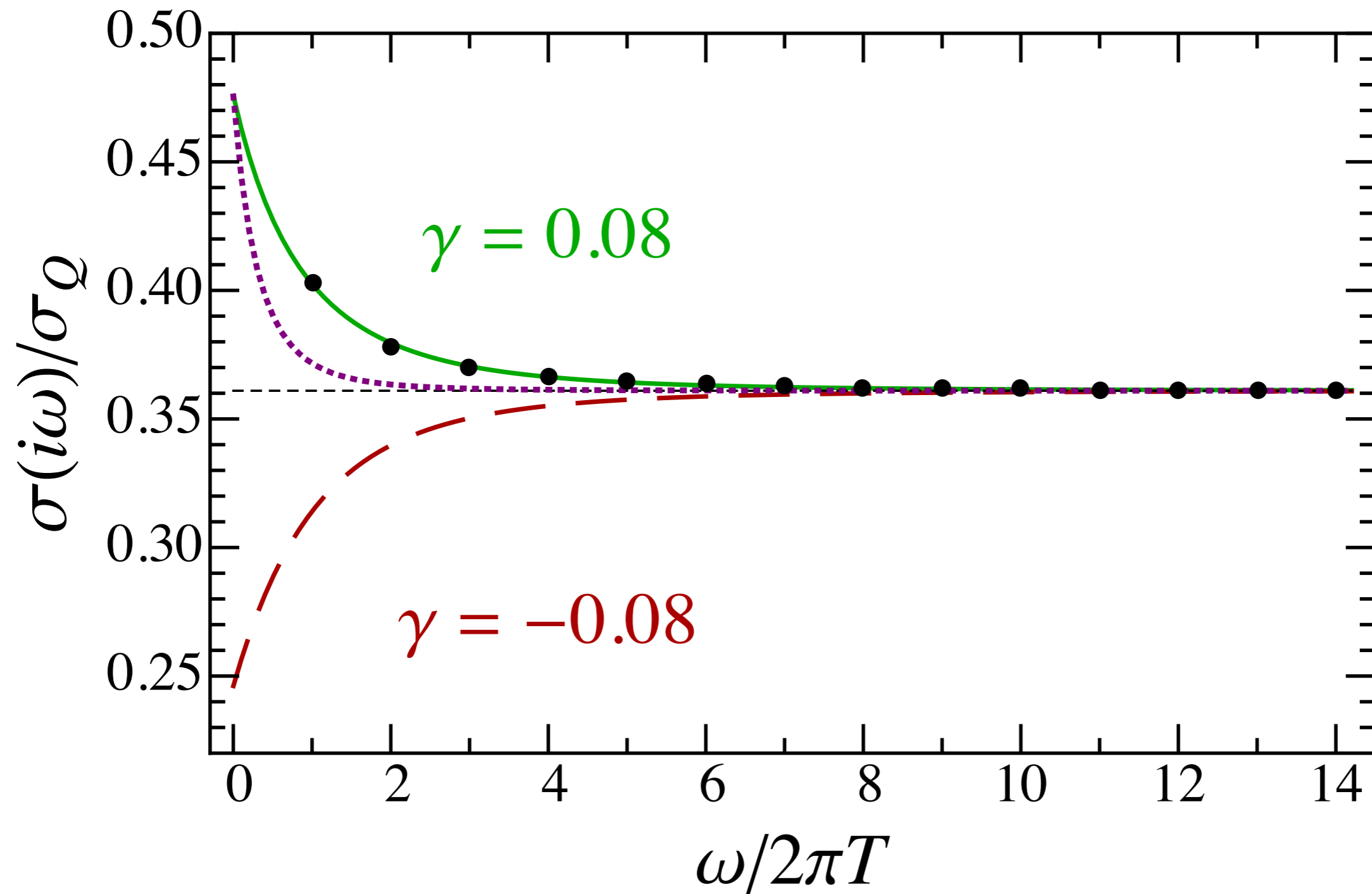
(b)

FIG. 2. **Quantum Monte Carlo data** (a) Finite-temperature conductivity for a range of  $\beta U$  in the  $L \rightarrow \infty$  limit for the quantum rotor model at  $(t/U)_c$ . The solid blue squares indicate the final  $T \rightarrow 0$  extrapolated data. (b) Finite-temperature conductivity in the  $L \rightarrow \infty$  limit for a range of  $L_\tau$  for the Villain model at the QCP. The solid red circles indicate the final  $T \rightarrow 0$  extrapolated data. The inset illustrates the extrapolation to  $T = 0$  for  $\omega_n/(2\pi T) = 7$ . The error bars are statistical for both a) and b).

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

# AdS<sub>4</sub> theory of quantum criticality

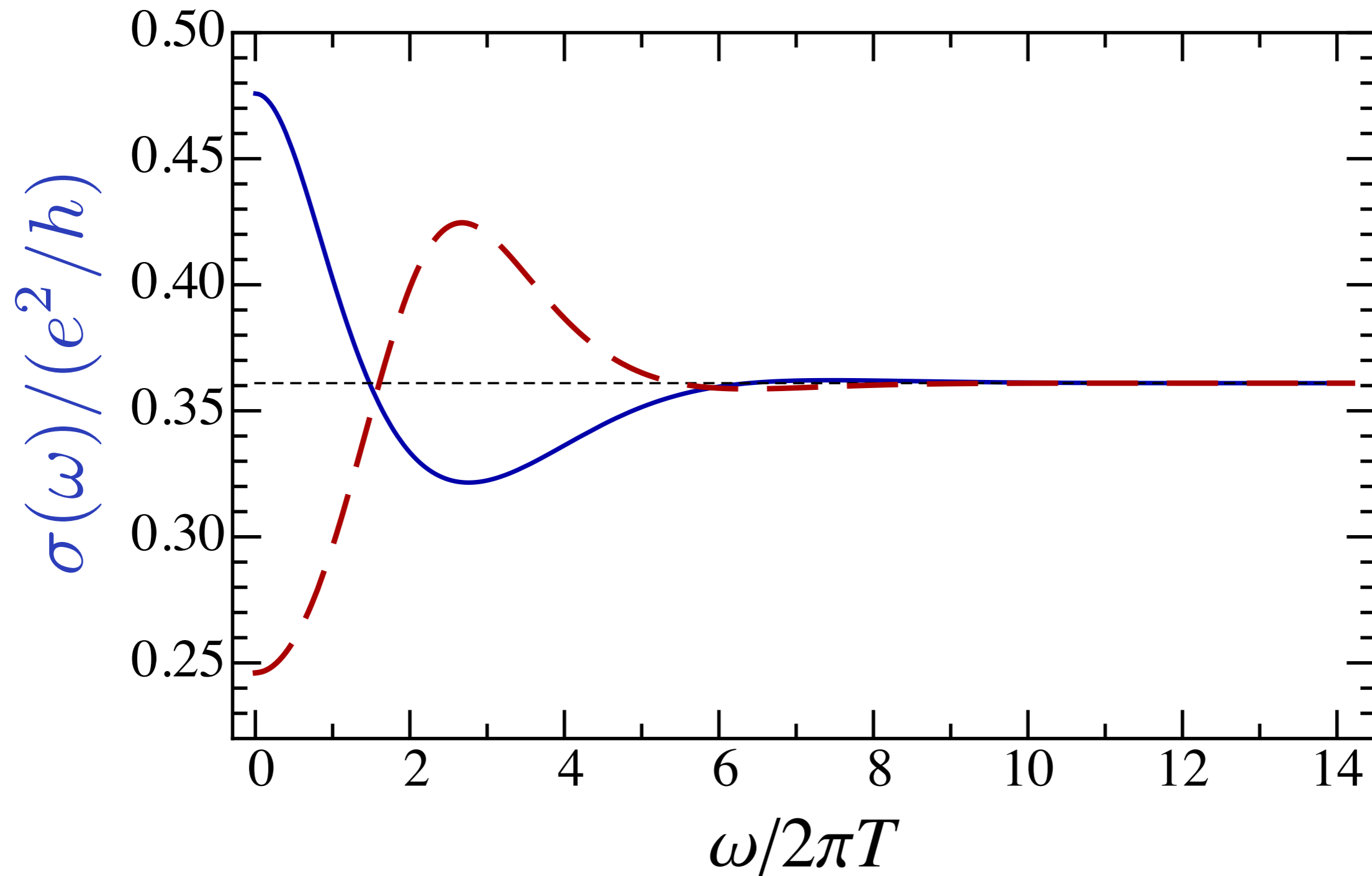


Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective  $T$  and taking  $\sigma_Q = 1/g_M^2$ .

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

# AdS<sub>4</sub> theory of quantum criticality



Predictions of holographic theory,  
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

# Outline

## 1. The simplest model without quasiparticles

### *A. Superfluid-insulator transition*

*of ultracold bosonic atoms in an optical lattice*

### *B. Conformal field theories in $2+1$ dimensions and the AdS/CFT correspondence*

## 2. Metals without quasiparticles

### *A “non-Fermi” liquid in the*

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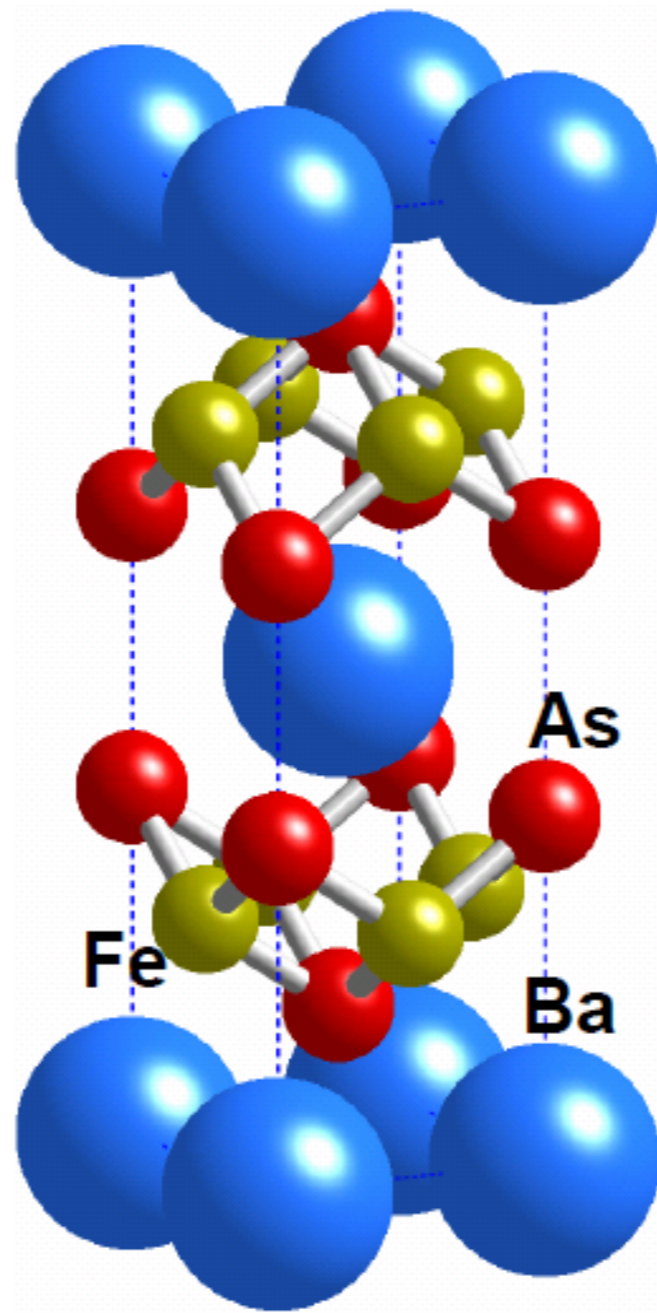
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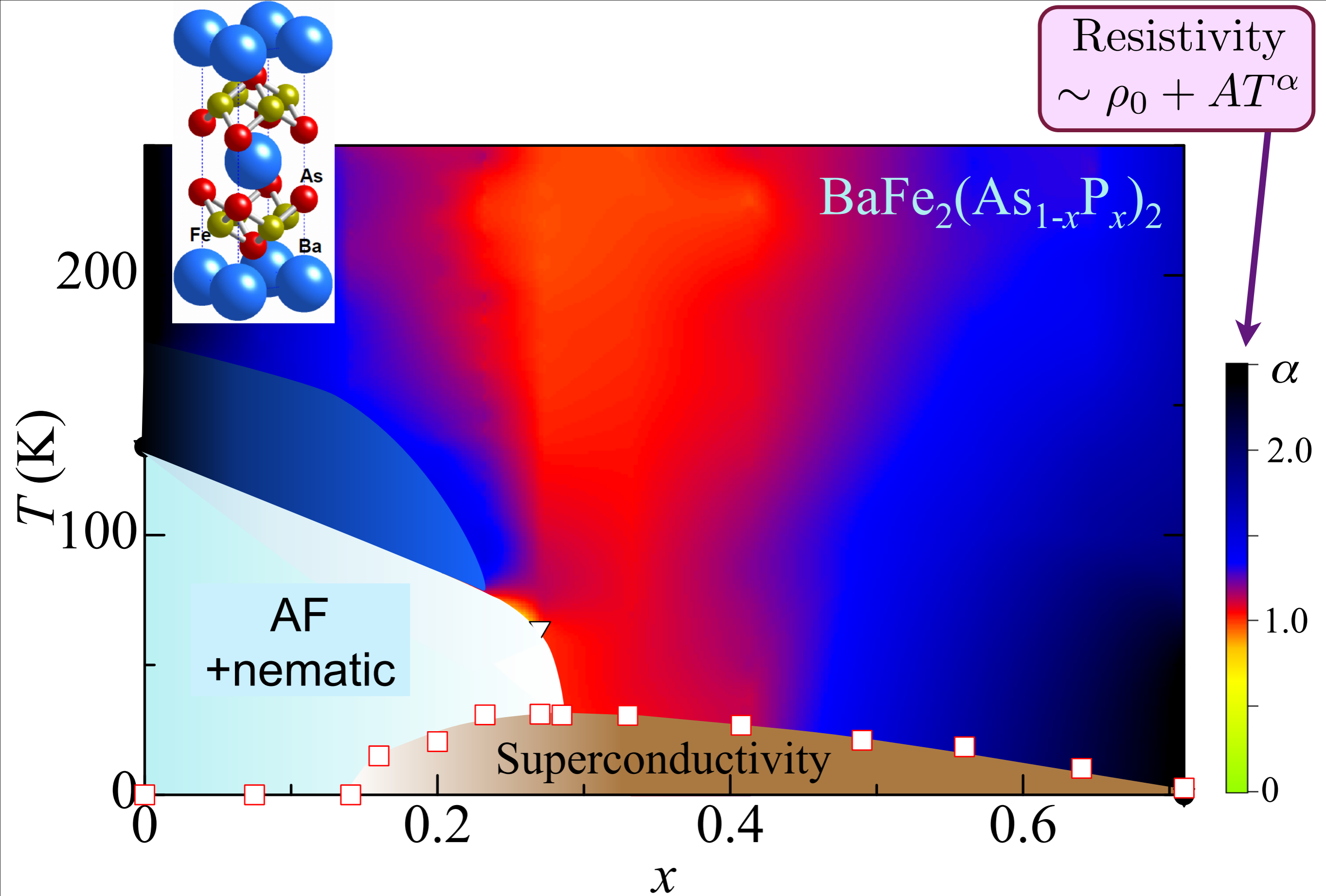
*high temperature superconductors:*

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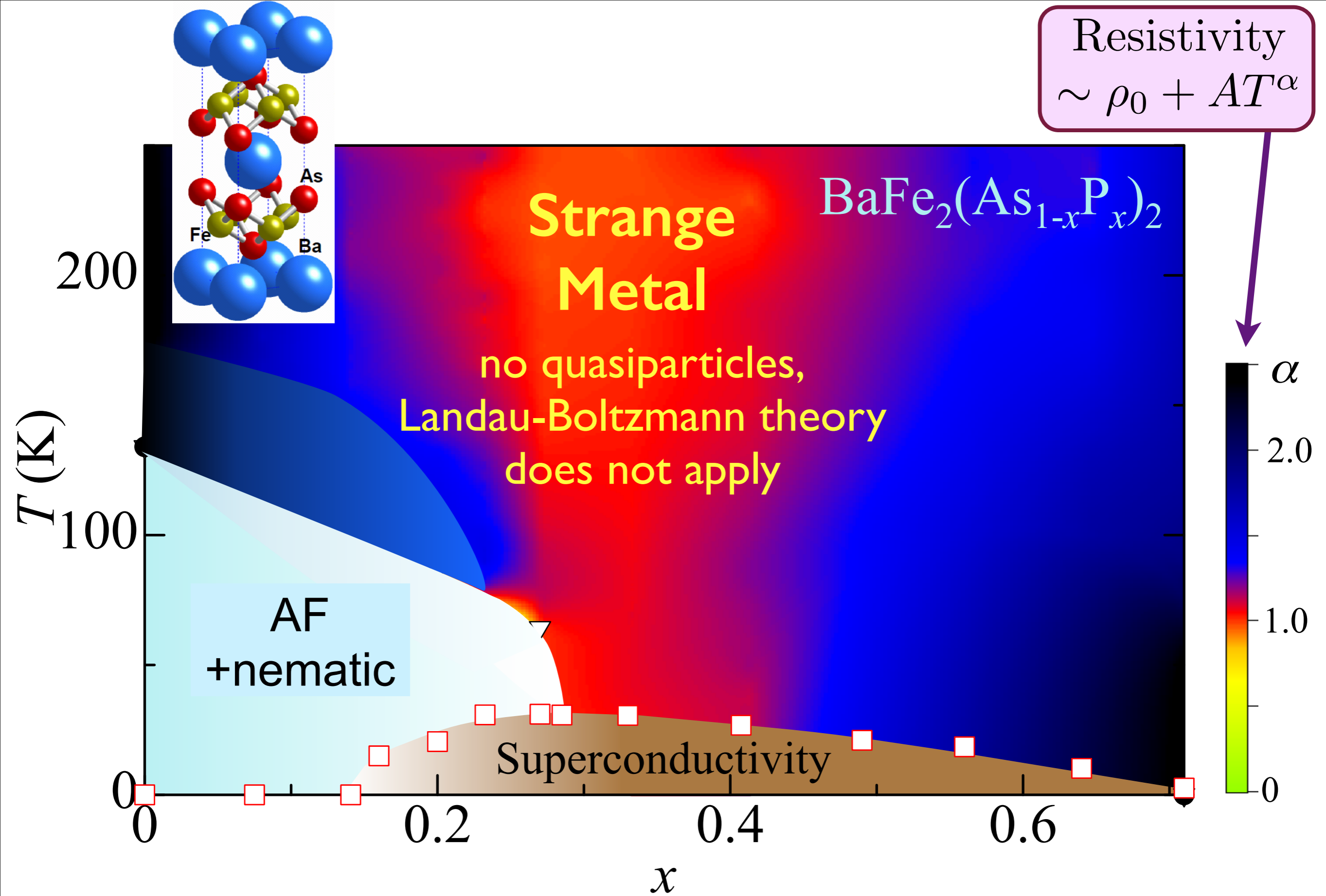
# Iron pnictides:

a new class of high temperature superconductors



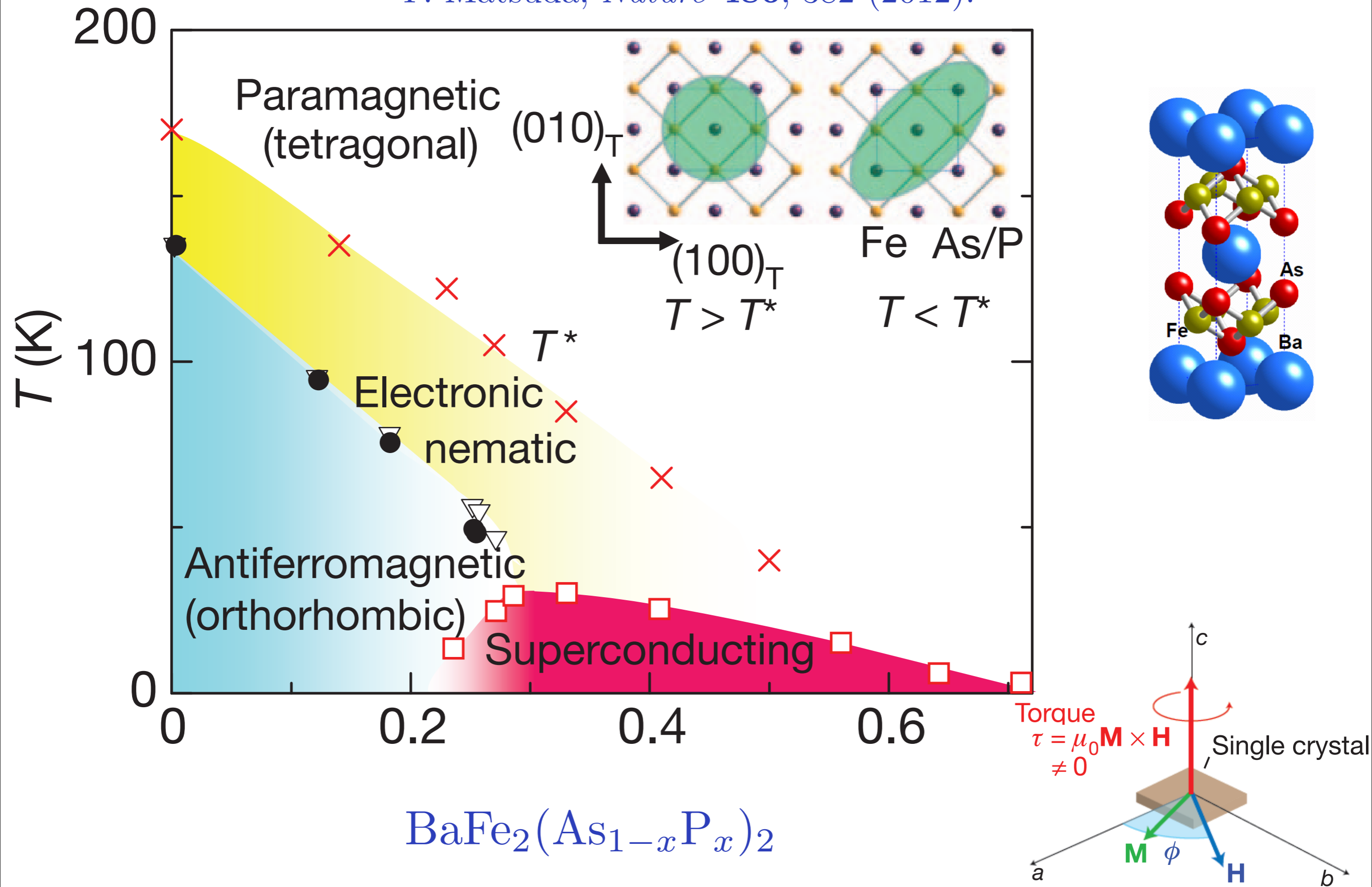


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

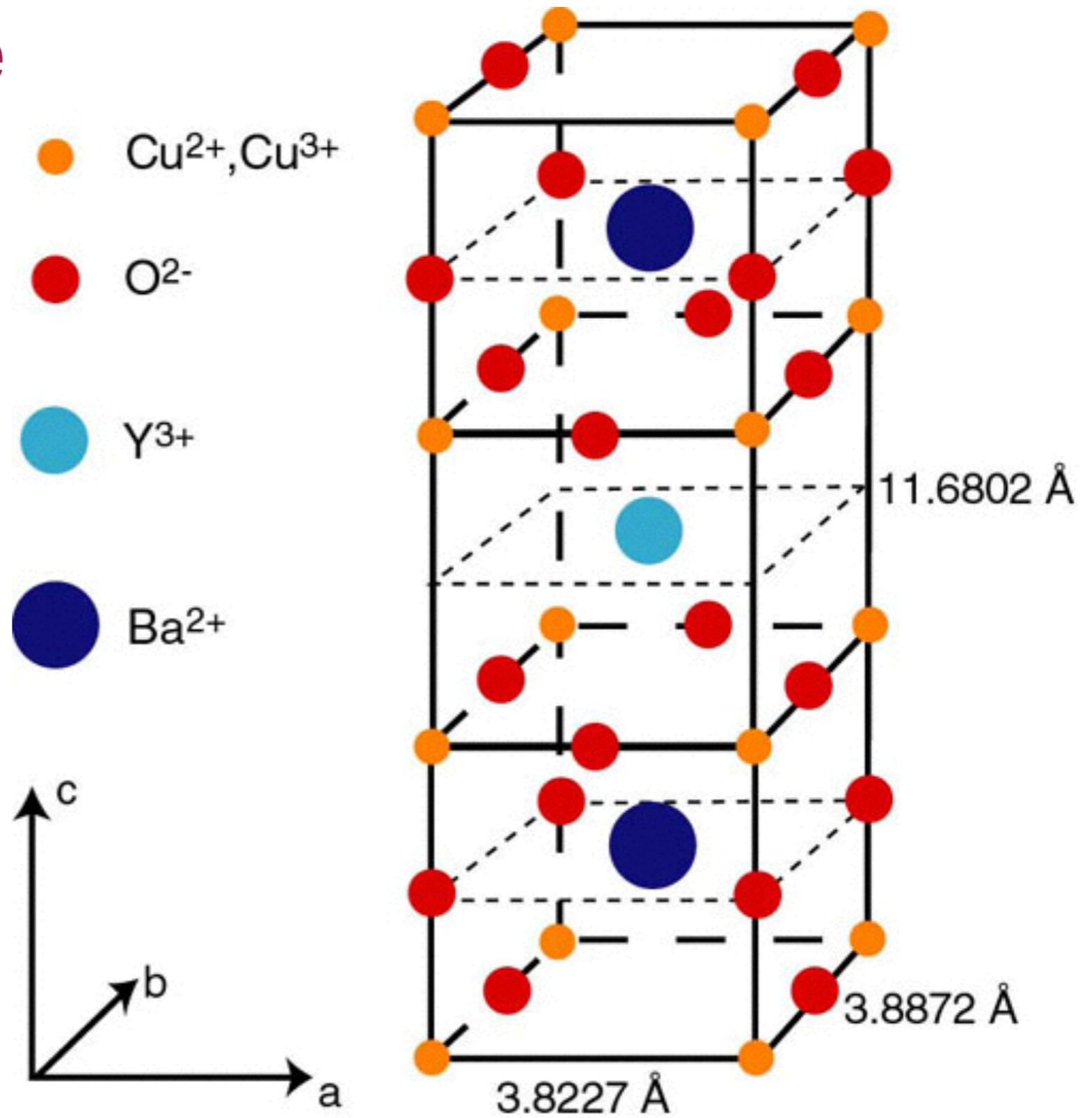


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S. Kasahara, H.J. Shi, K. Hashimoto, S. Tonegawa, Y. Mizukami, T. Shibauchi, K. Sugimoto, T. Fukuda, T. Terashima, A.H. Nevidomskyy, and Y. Matsuda, *Nature* **486**, 382 (2012).

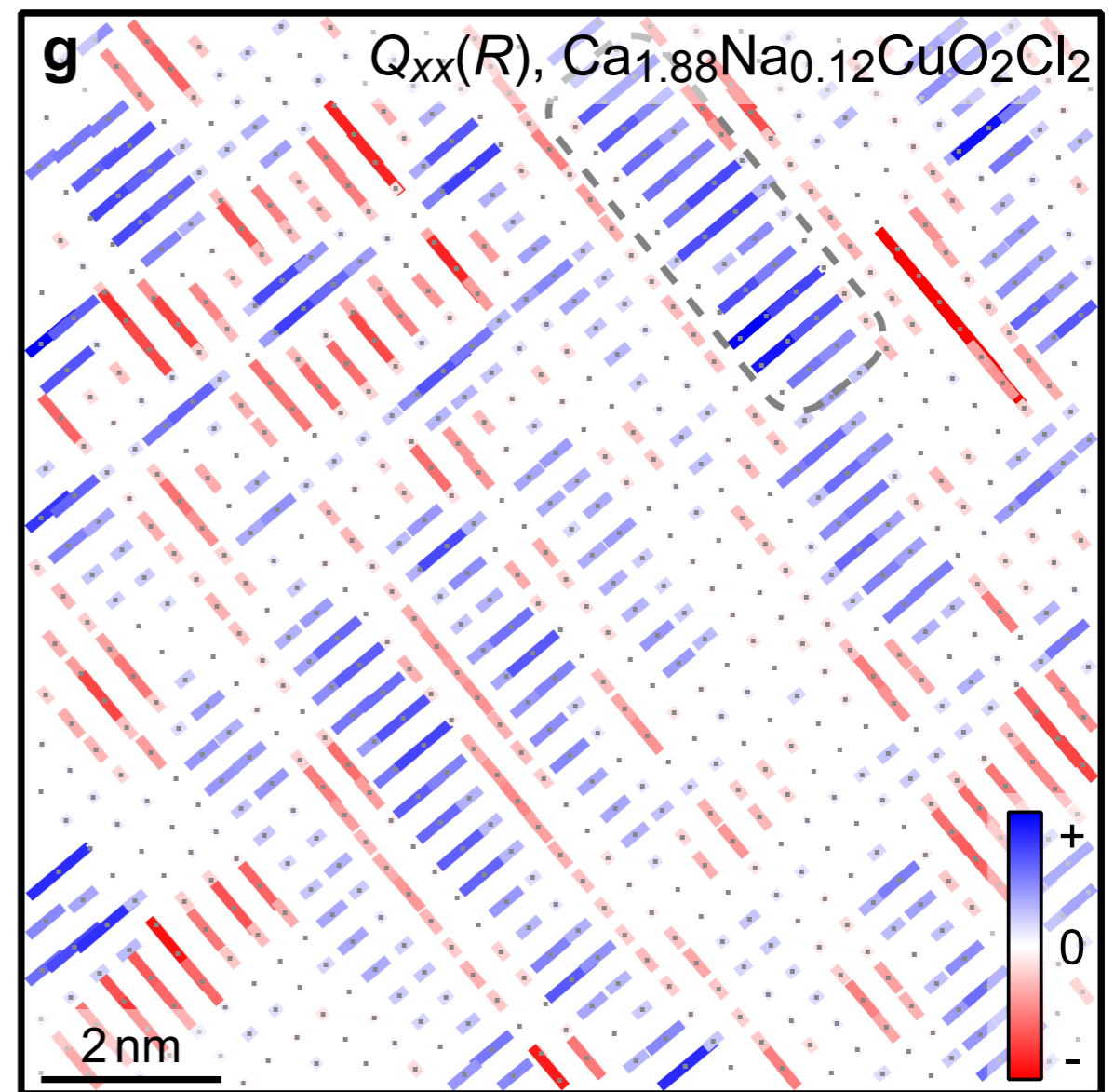
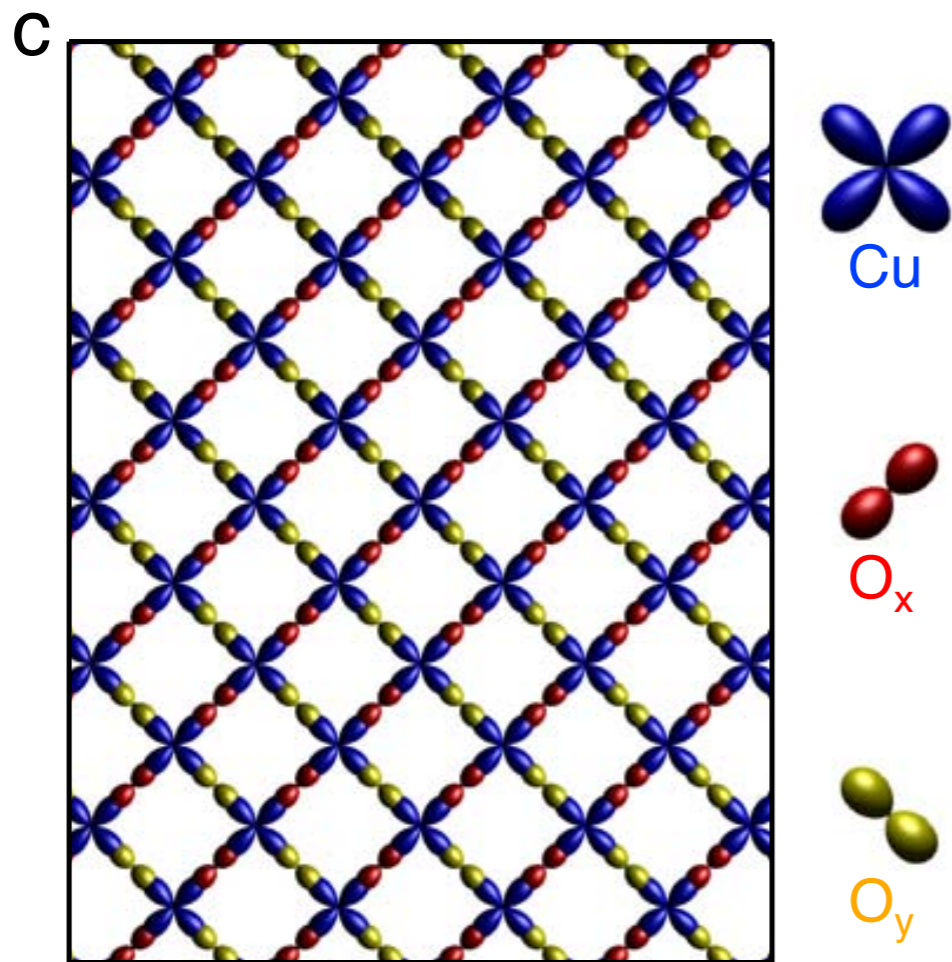


# High temperature superconductors



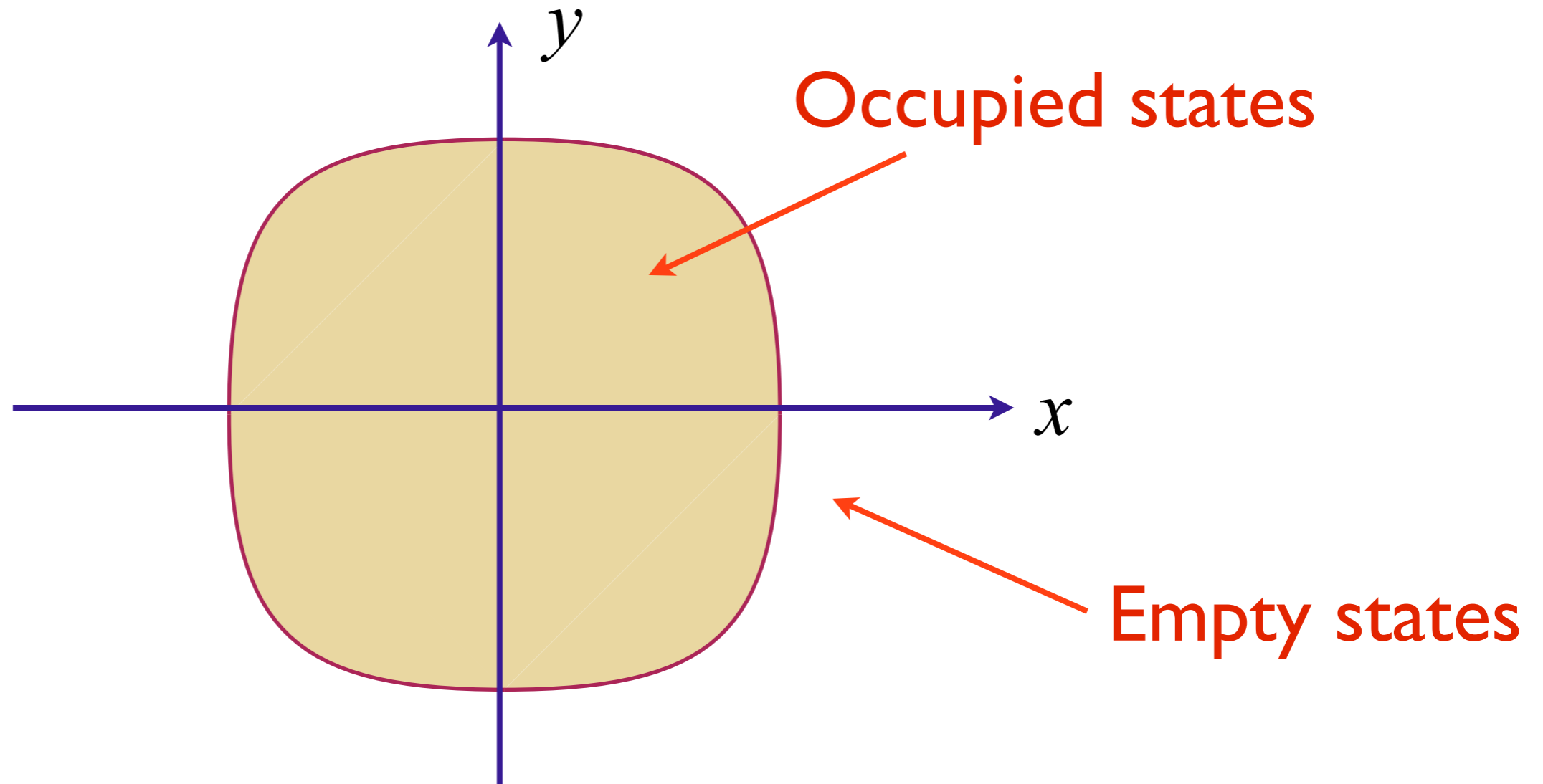
# Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi  
*Nature Physics*, 8, 534 (2012).



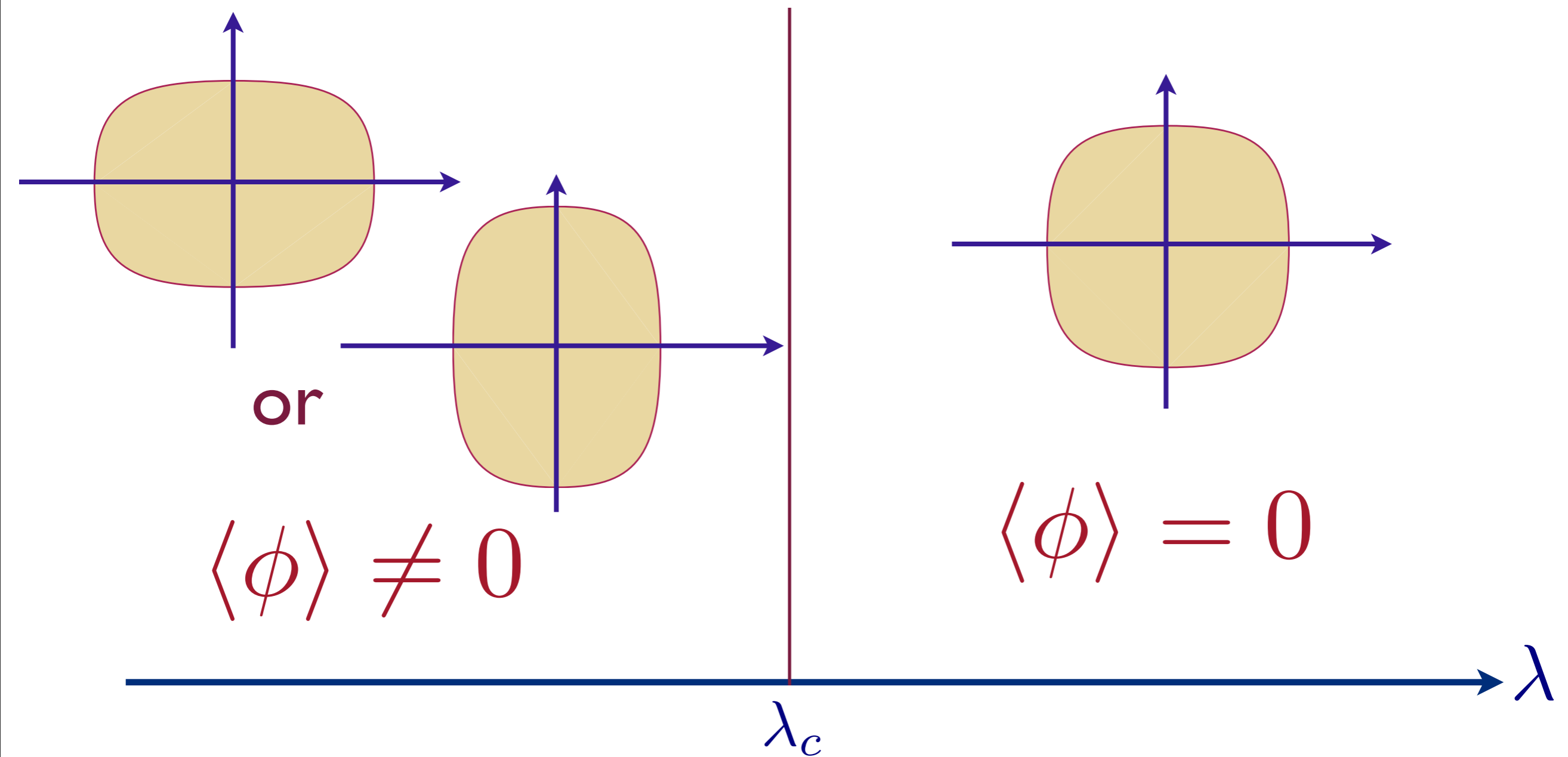
Evidence for “nematic” order (*i.e.* breaking of  $90^\circ$  rotation symmetry) in  $\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$ .

# Quantum criticality of Ising-nematic ordering in a metal



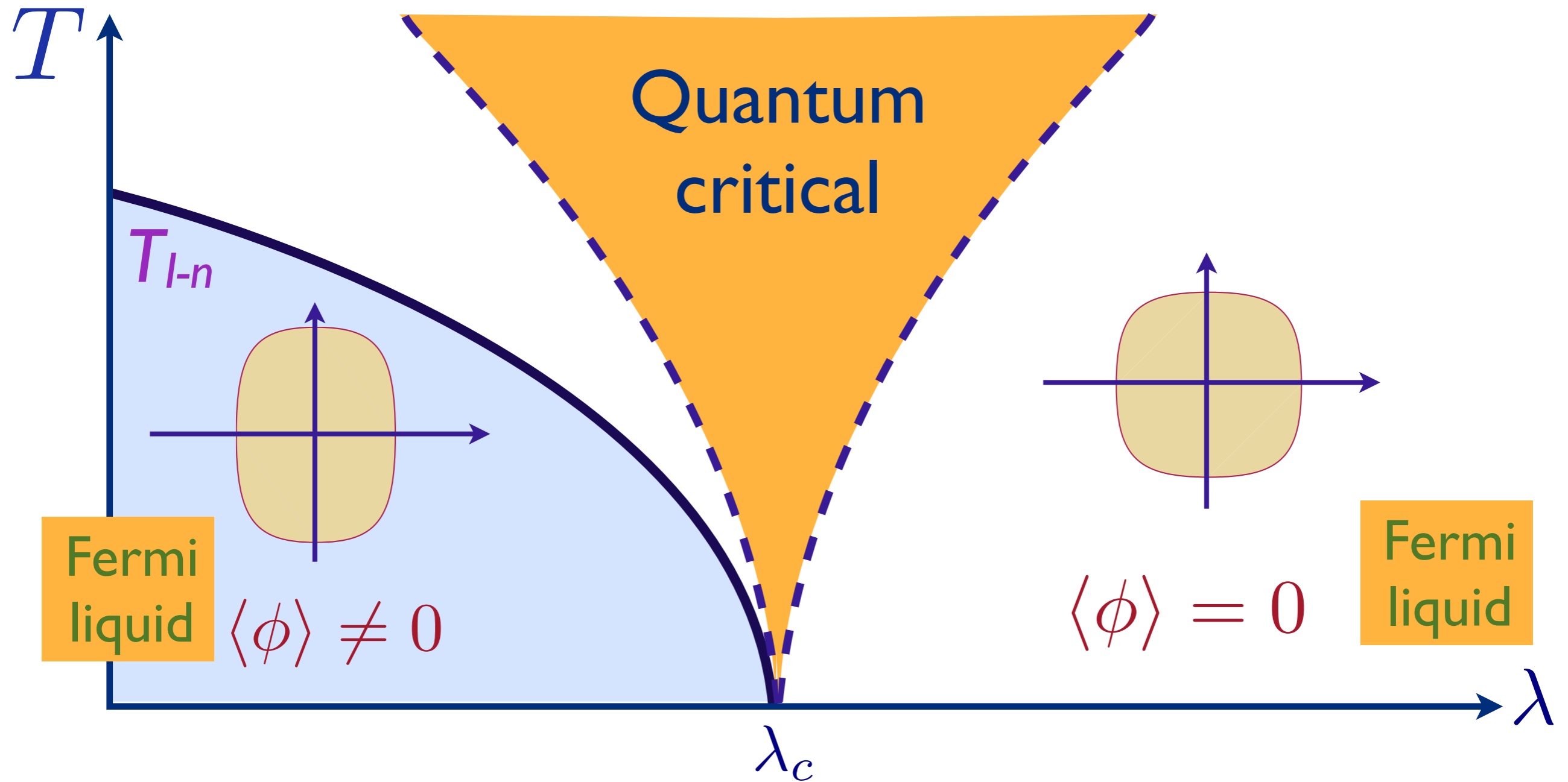
A metal with a Fermi surface  
with full square lattice symmetry

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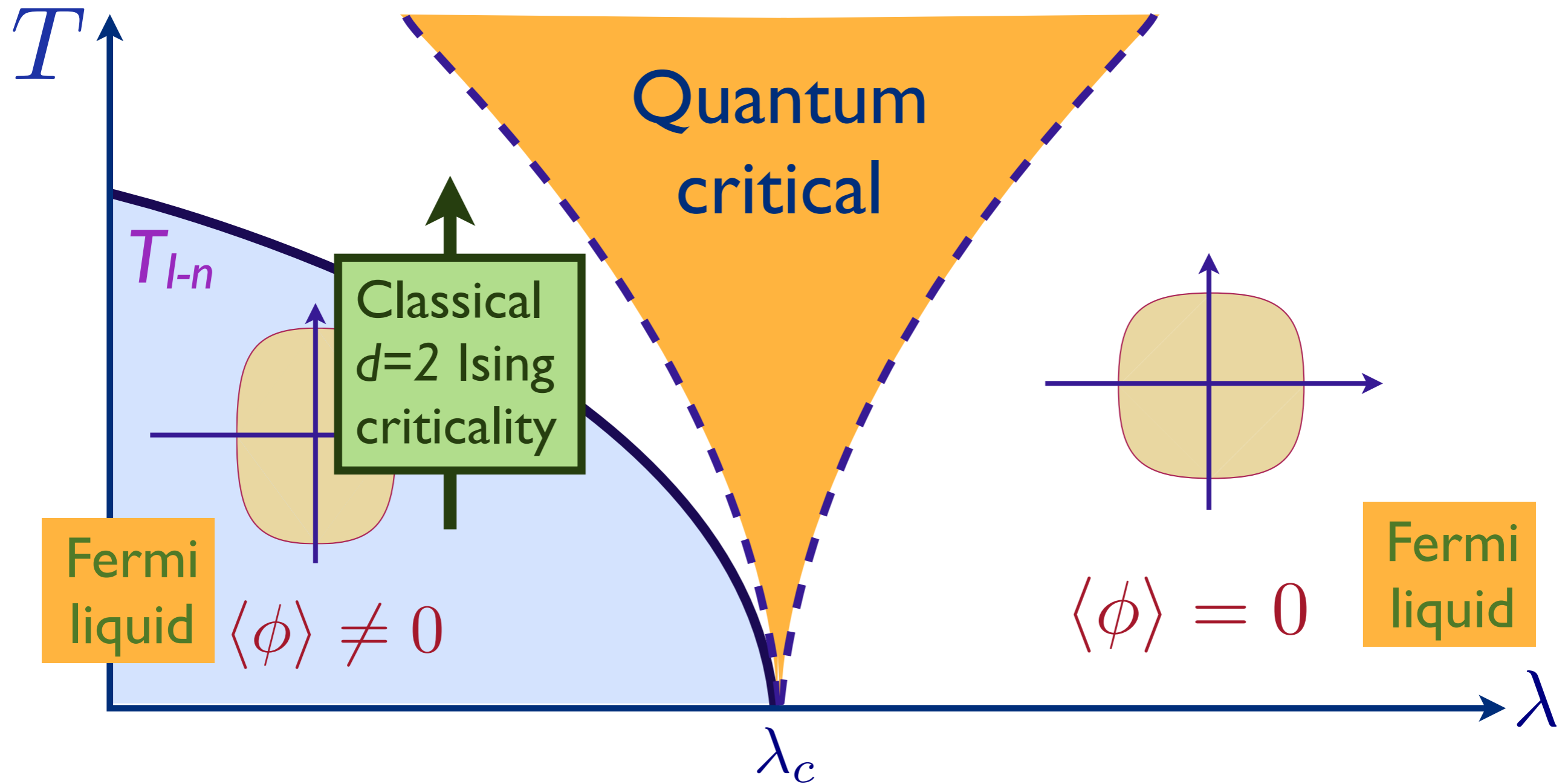
Pomeranchuk instability as a function of coupling  $\lambda$

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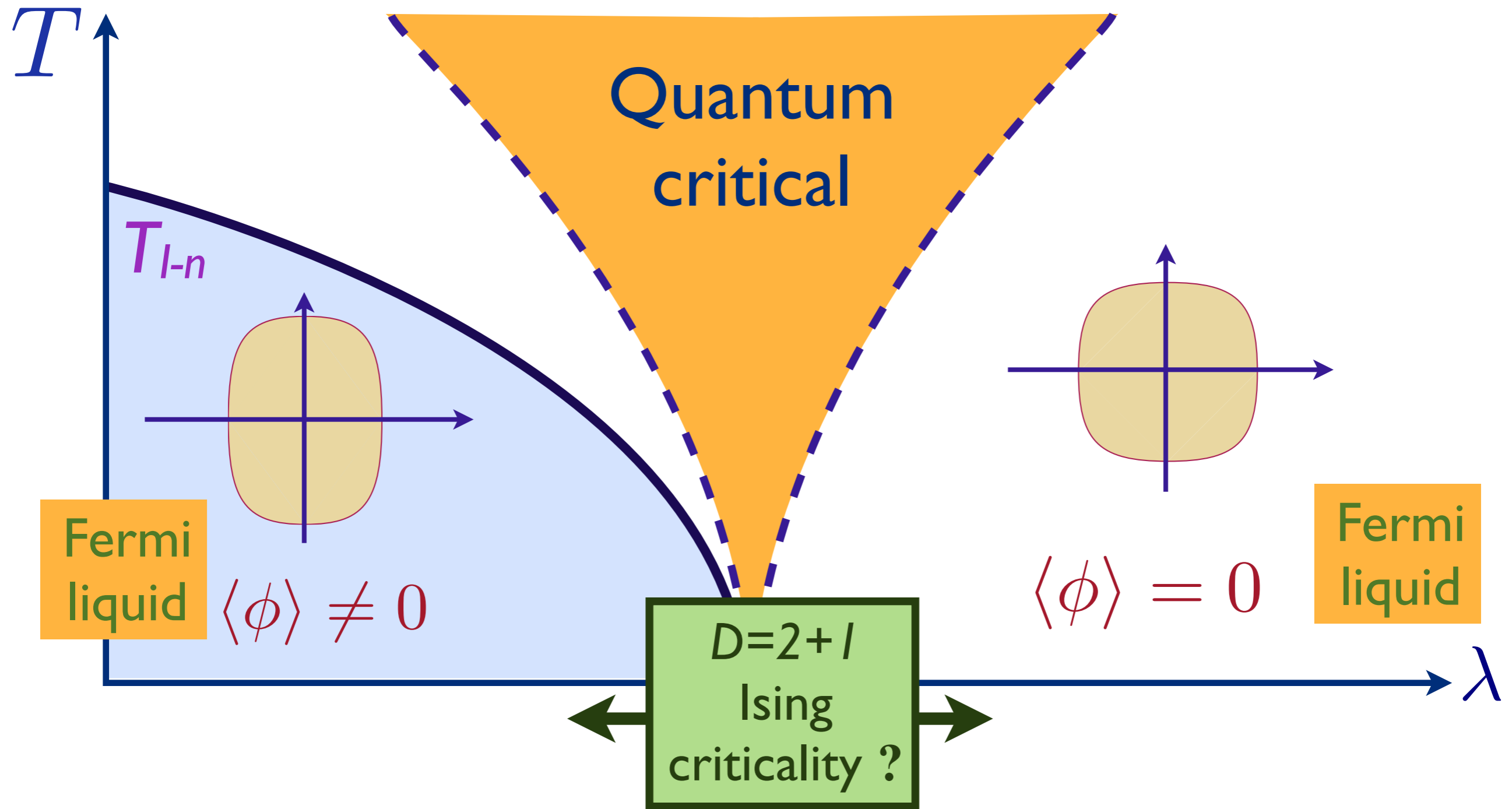
Phase diagram as a function of  $T$  and  $\lambda$

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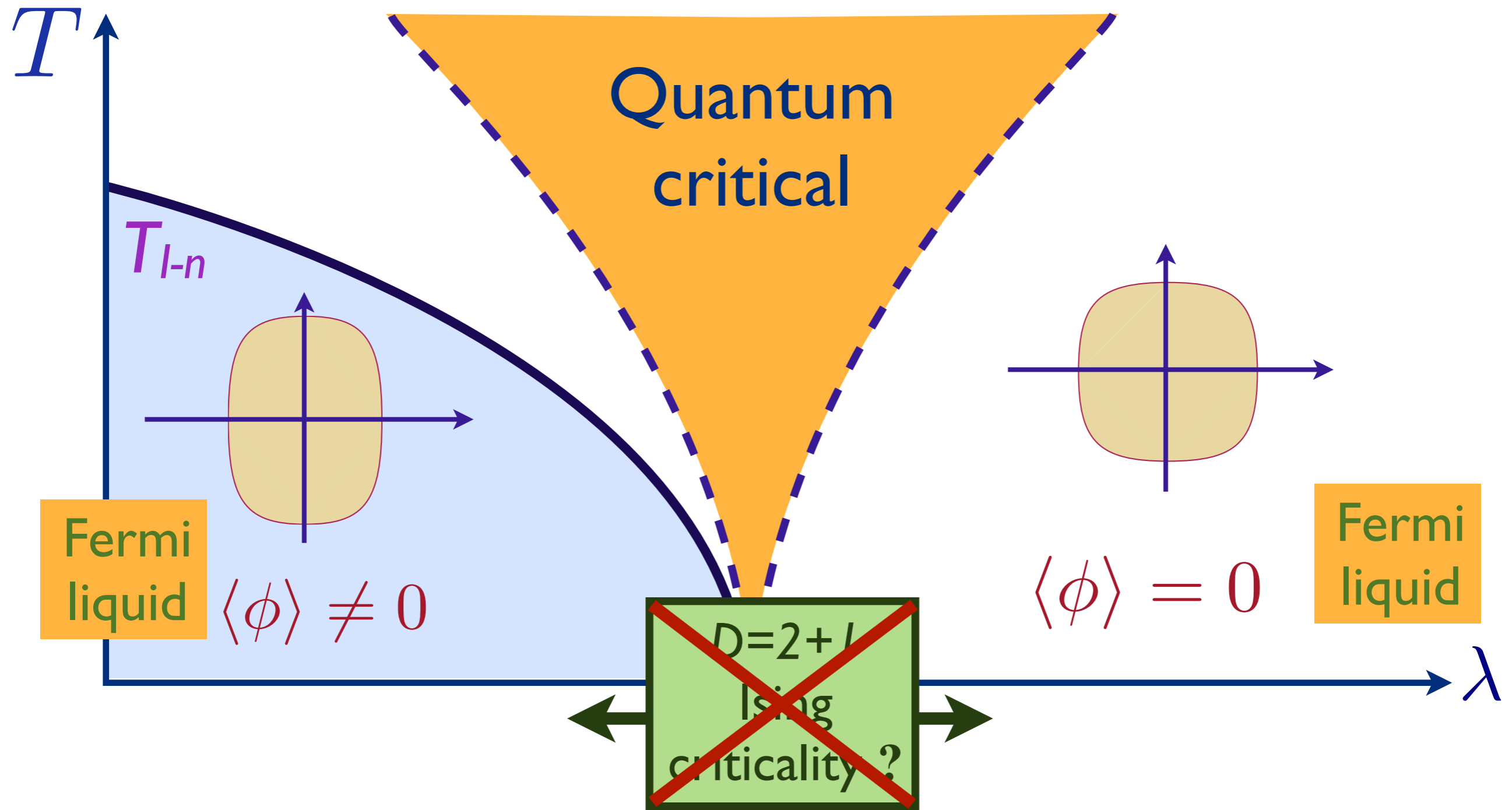
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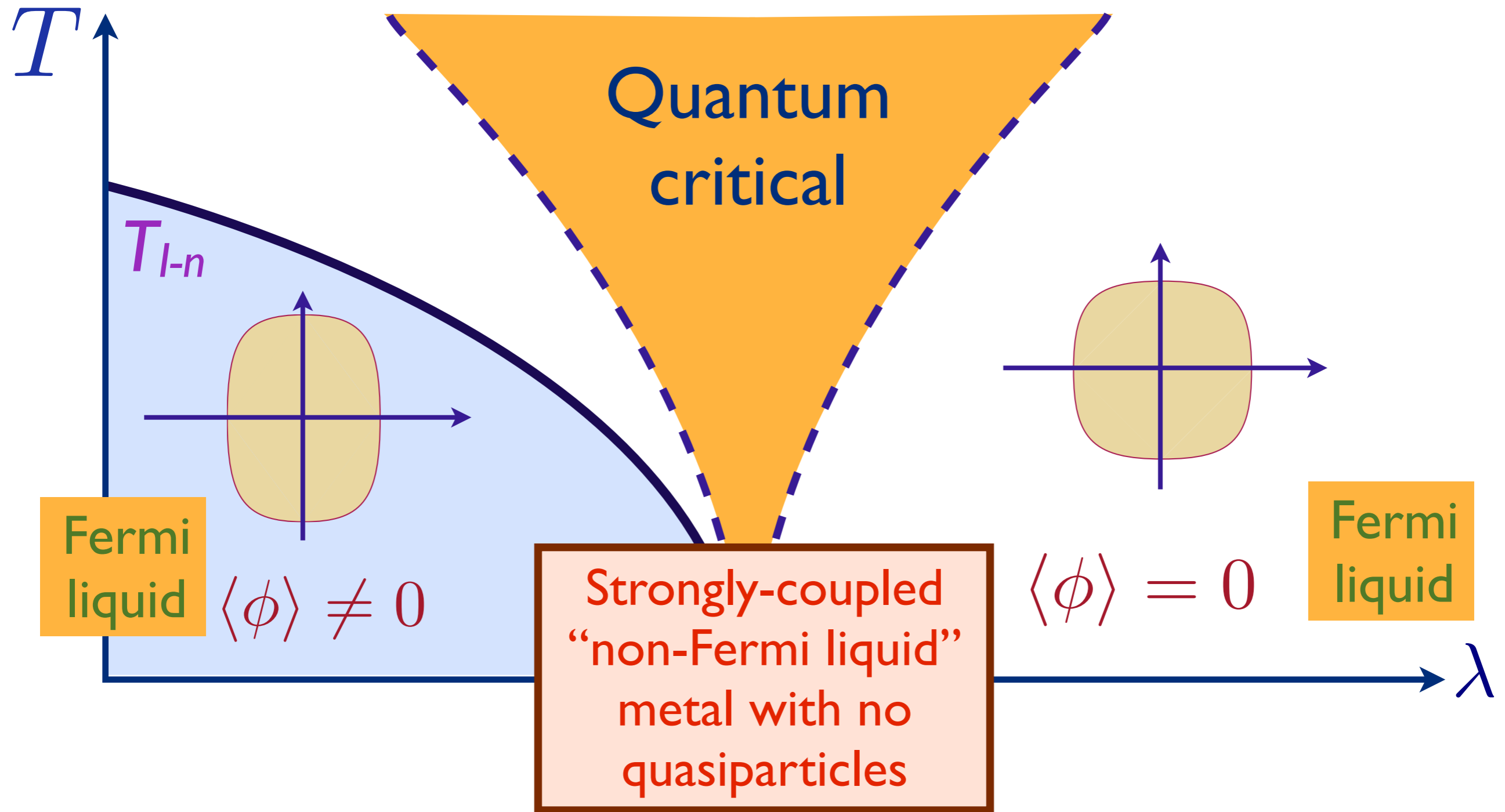
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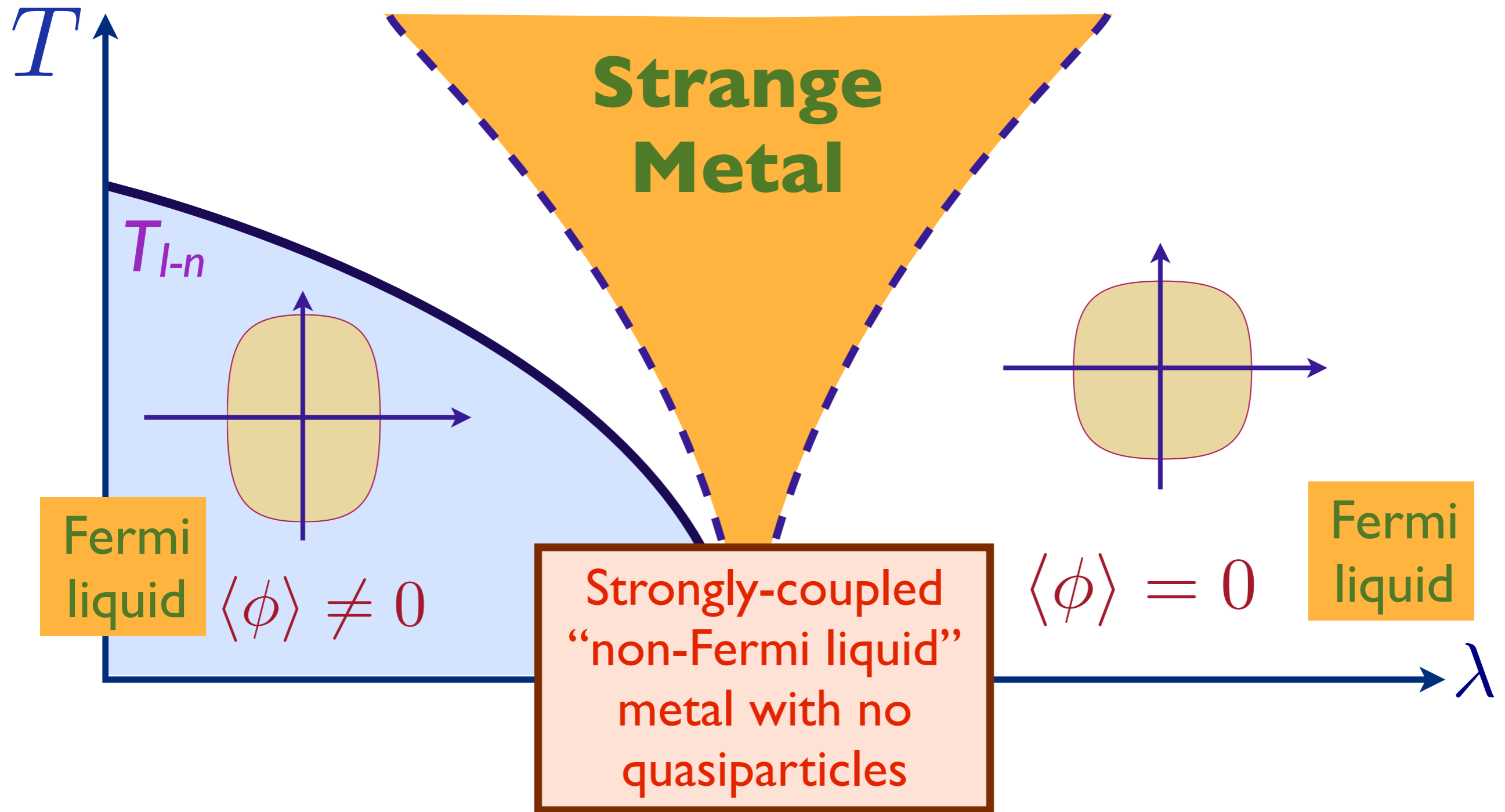
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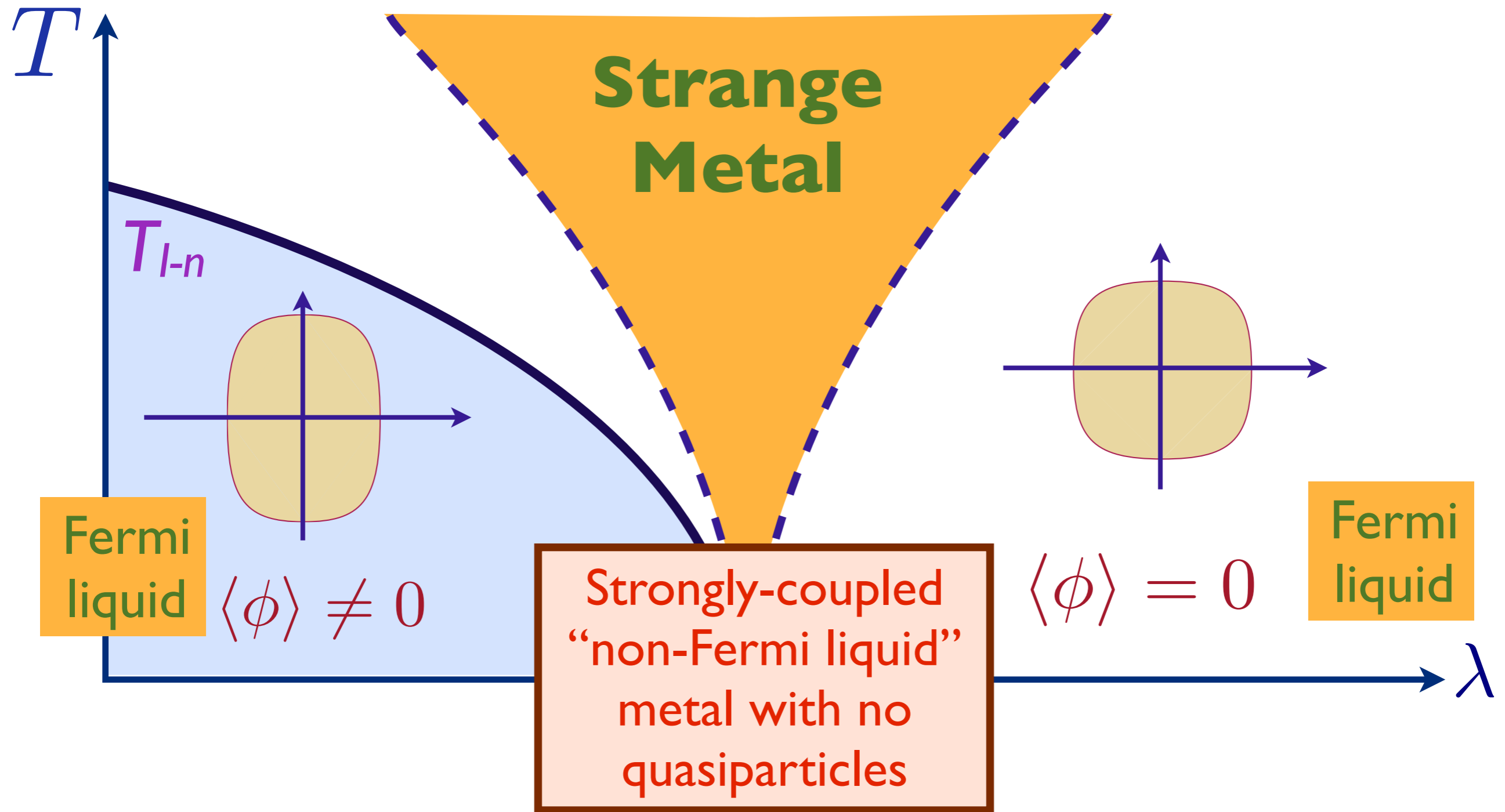
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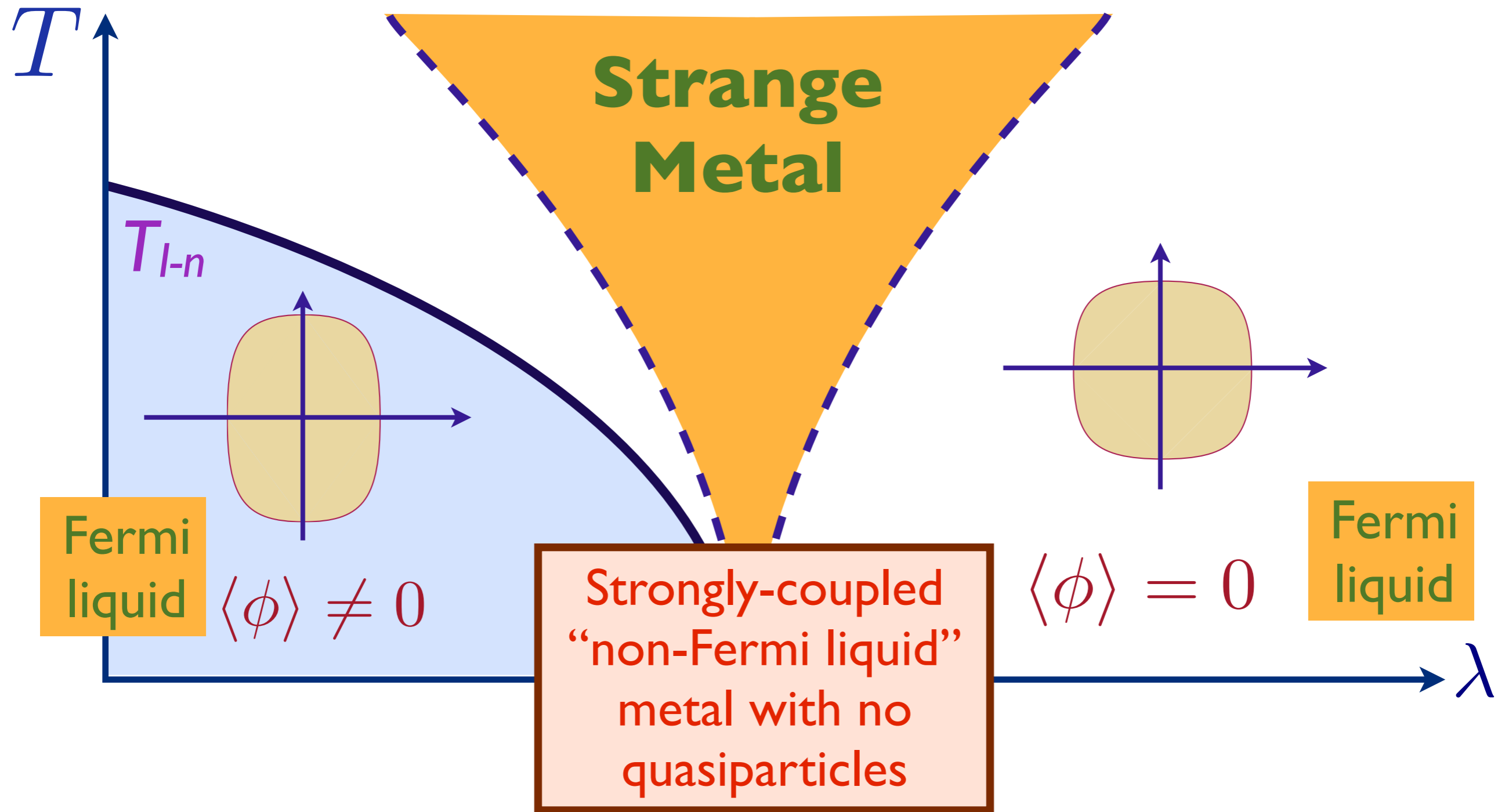
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# Quantum criticality of Ising-nematic ordering in a metal



Common theoretical belief from a quasiparticle-like analysis:  
resistivity of strange metal  $\rho(T) \sim T^{4/3}$ .

# Quantum criticality of Ising-nematic ordering in a metal



Common theoretical belief from a quasiparticle-like analysis:  
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Ignores conservation of total momentum (analog of "phonon drag")  
D. L. Maslov, V. I. Yudson, and A. V. Chubukov, Phys. Rev. Lett. **106**, 106403 (2011).

# Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

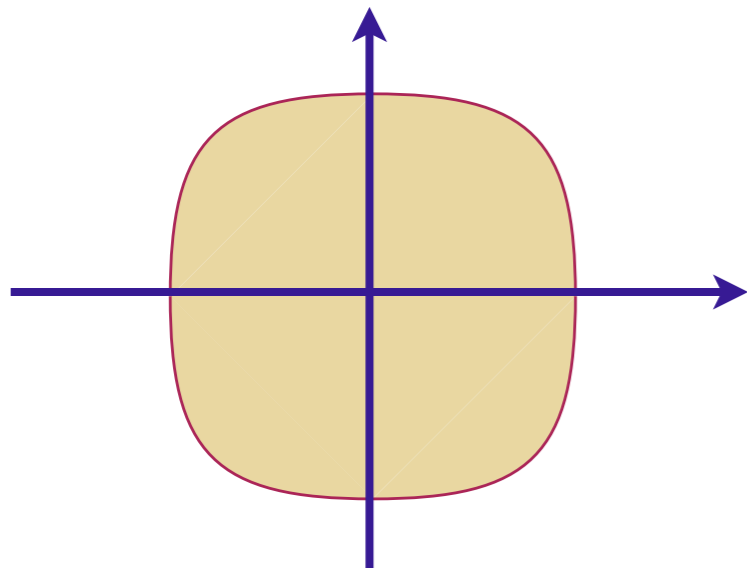
$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

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$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

Effective action for electrons, with Fermi surface:



$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

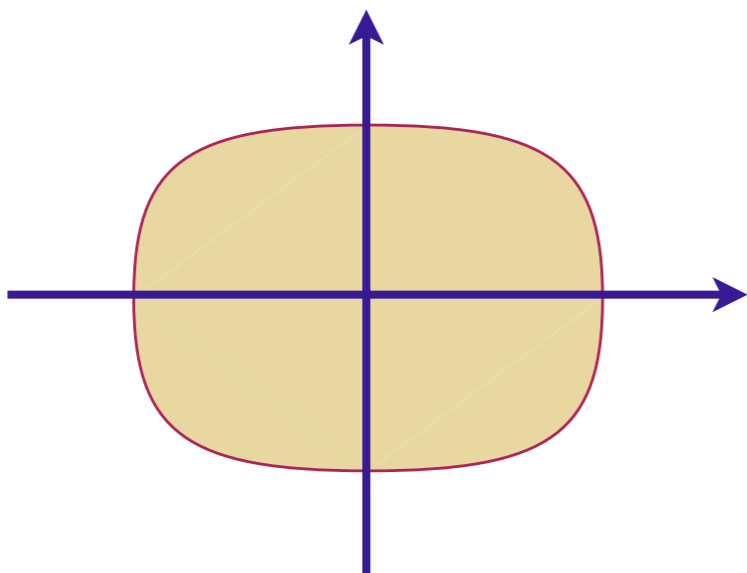
where  $\varepsilon_{\mathbf{k}} = -\mu - 2t(\cos k_x + \cos k_y) + \dots$

# Quantum criticality of Ising-nematic ordering in a metal

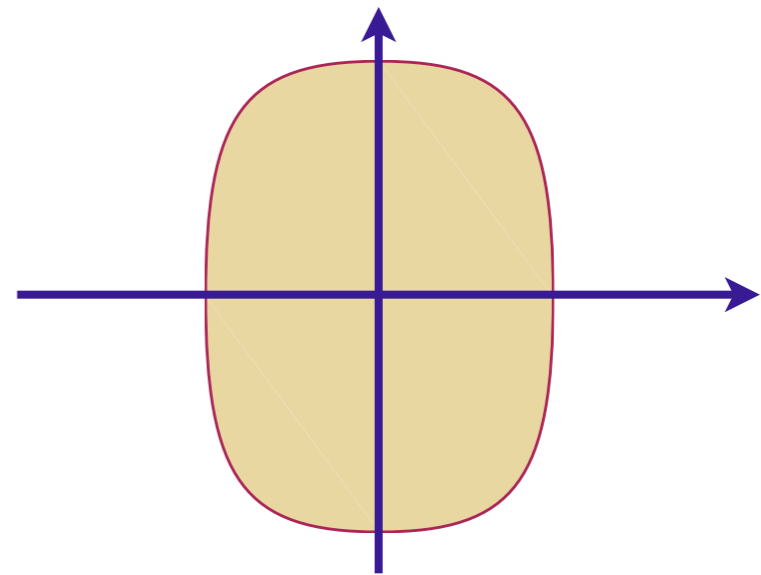
“Yukawa” coupling between Ising order and Fermi surface

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

for spatially dependent  $\phi$



$$\langle \phi \rangle > 0$$



$$\langle \phi \rangle < 0$$

# Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

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$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \int d^2r d\tau c_\alpha^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) c_\alpha$$

$$\mathcal{S}_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi [c_\alpha^\dagger \{(\partial_x^2 - \partial_y^2) c_\alpha\} + \{(\partial_x^2 - \partial_y^2) c_\alpha^\dagger\} c_\alpha]$$

This continuum theory has a conserved momentum  $\mathbf{P}$ , and  $\chi_{\mathbf{J}, \mathbf{P}} \neq 0$ , and so the resistivity  $\rho(T) = 0$

# Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\text{dis}} = \int d^2r d\tau [V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi] ,$$

$$\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2 \delta(\mathbf{r} - \mathbf{r}') ,$$

$$\overline{h(\mathbf{r})} = 0 \quad ; \quad \overline{h(\mathbf{r})h(\mathbf{r}')} = h_0^2 \delta(\mathbf{r} - \mathbf{r}') ,$$

we obtain the resistivity for current along angle  $\vartheta$

$$\rho(T) = \frac{1}{\chi_{\mathbf{J},\mathbf{P}}^2} \lim_{\omega \rightarrow 0} \int \frac{d^2k}{(2\pi)^2} k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi_{c^\dagger c}^R(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} D_\phi^R(\omega, \mathbf{k})}{\omega} \right)$$

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Fermi surface term: Obtain  $T$ -dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)

I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

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Bosonic term: Dominant contribution:

$$\rho(T) \sim T^{(d-z+\eta)/z}$$

Crosses over from the “relativistic” form ( $z = 1, \eta \approx 0$ ) with  $\rho(T) \sim T$  at higher  $T$ ,

to the “Landau-damped” form ( $z = 3, \eta = 0$ ) with  $\rho(T) \sim (T \ln(1/T))^{-1/2}$  at lower  $T$  (subtle corrections to scaling specific to this field theory).

# Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\text{dis}} = \int d^2r d\tau [V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi] ,$$

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Also obtained in holographic theory of a generalized compressible quantum state (A. Lucas, S. Sachdev, and K. Schalm, [arXiv:1401.7933](https://arxiv.org/abs/1401.7933)).

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- Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography