AdS/CFT and condensed matter

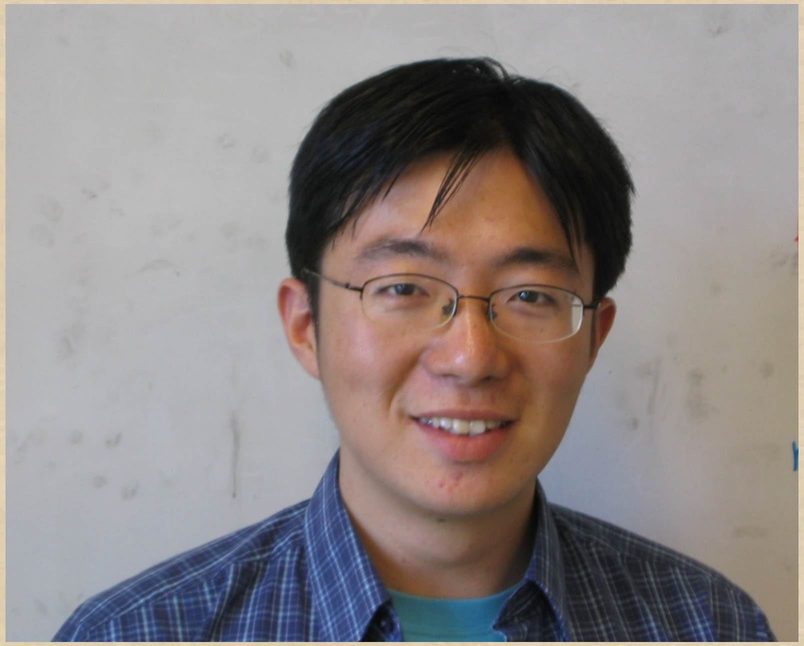
Talk online: sachdev.physics.harvard.edu

Particle theorists

Sean Hartnoll, KITP
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

Condensed matter theorists

Markus Mueller
Geneva



Cenke Xu
Harvard

Yang Qi
Harvard

Three foci of modern physics

Quantum phase transitions

Three foci of modern physics

Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

Three foci of modern physics

Quantum phase transitions

Black holes

Three foci of modern physics

Quantum phase transitions

Black holes

Bekenstein and Hawking originated the quantum theory, which has found fruition in string theory

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Universal description of fluids based upon conservation laws and positivity of entropy production

Black holes

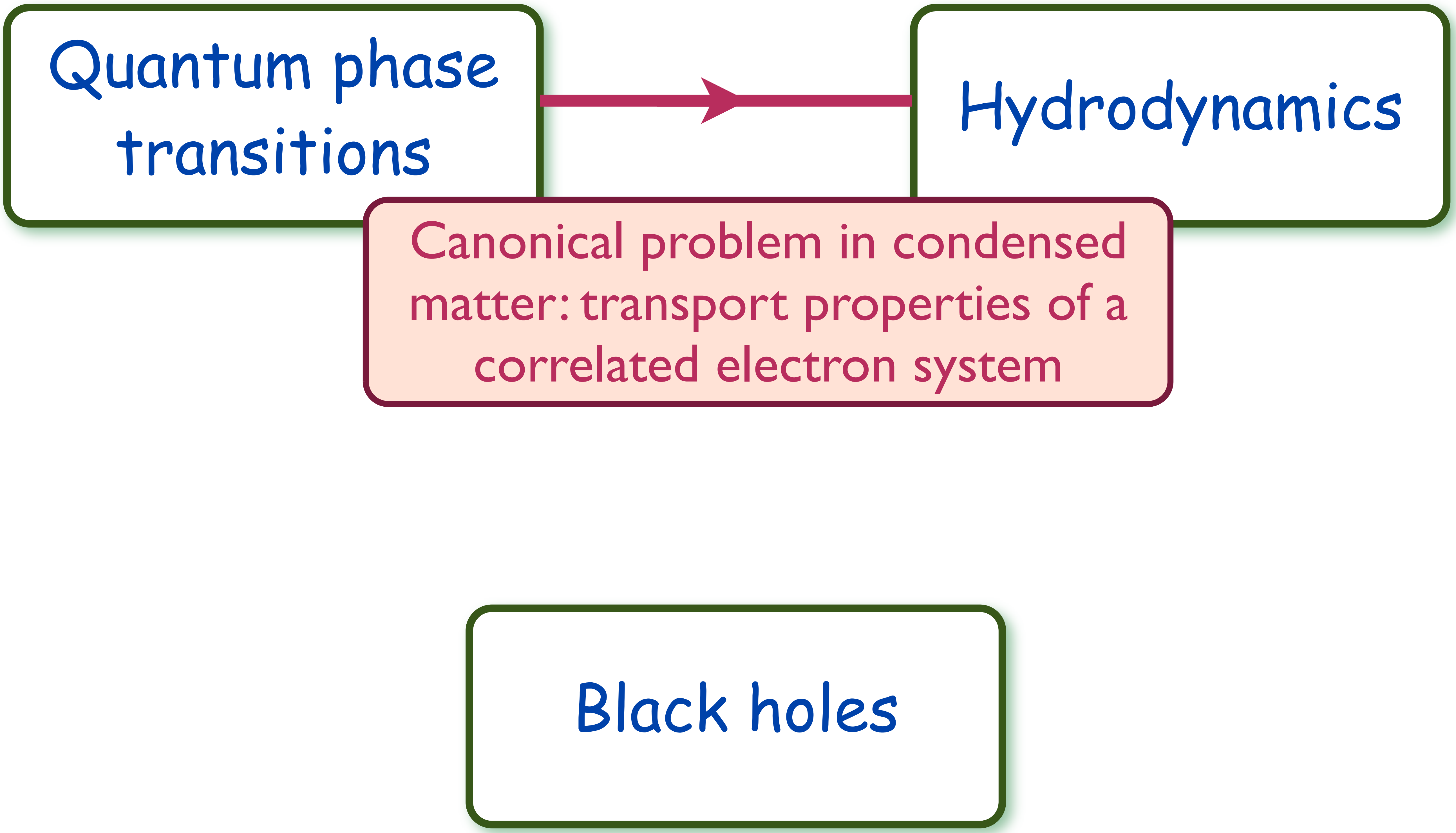
Three foci of modern physics

Quantum phase transitions

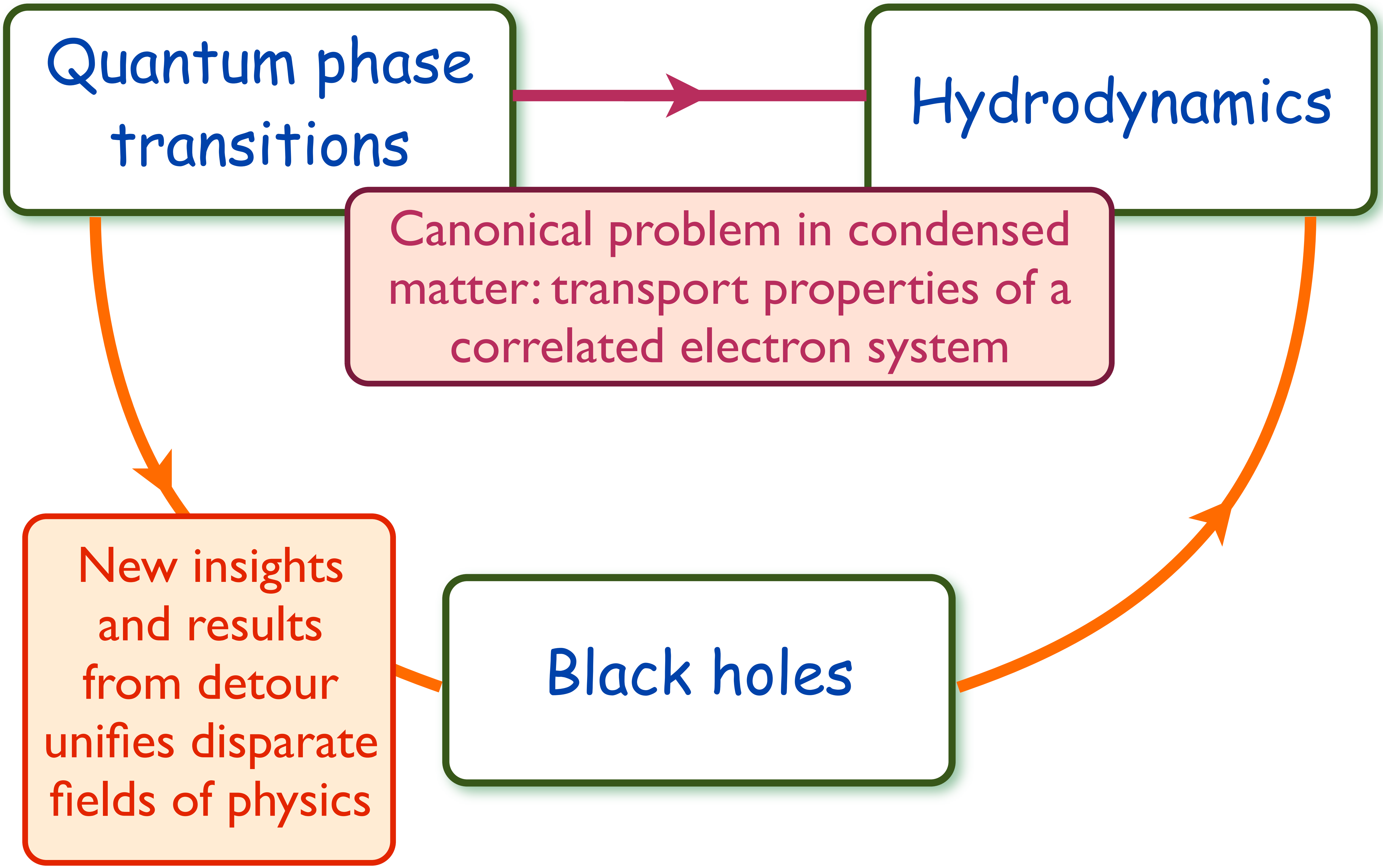
Hydrodynamics

Black holes

Three foci of modern physics



Three foci of modern physics



Three foci of modern physics

Quantum phase transitions

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Quantum phase transitions

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Three foci of modern physics

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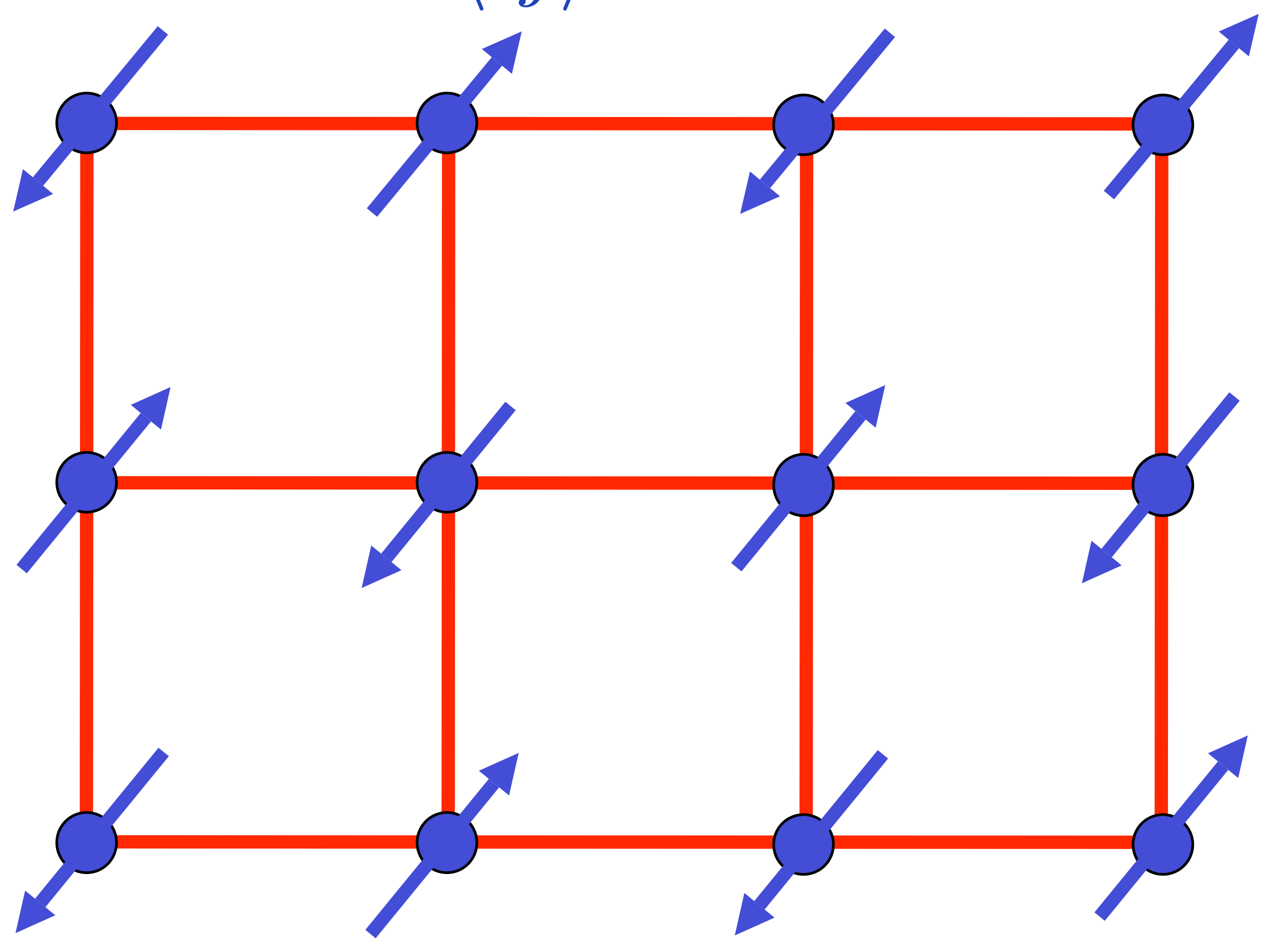
Hydrodynamics

Black holes

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

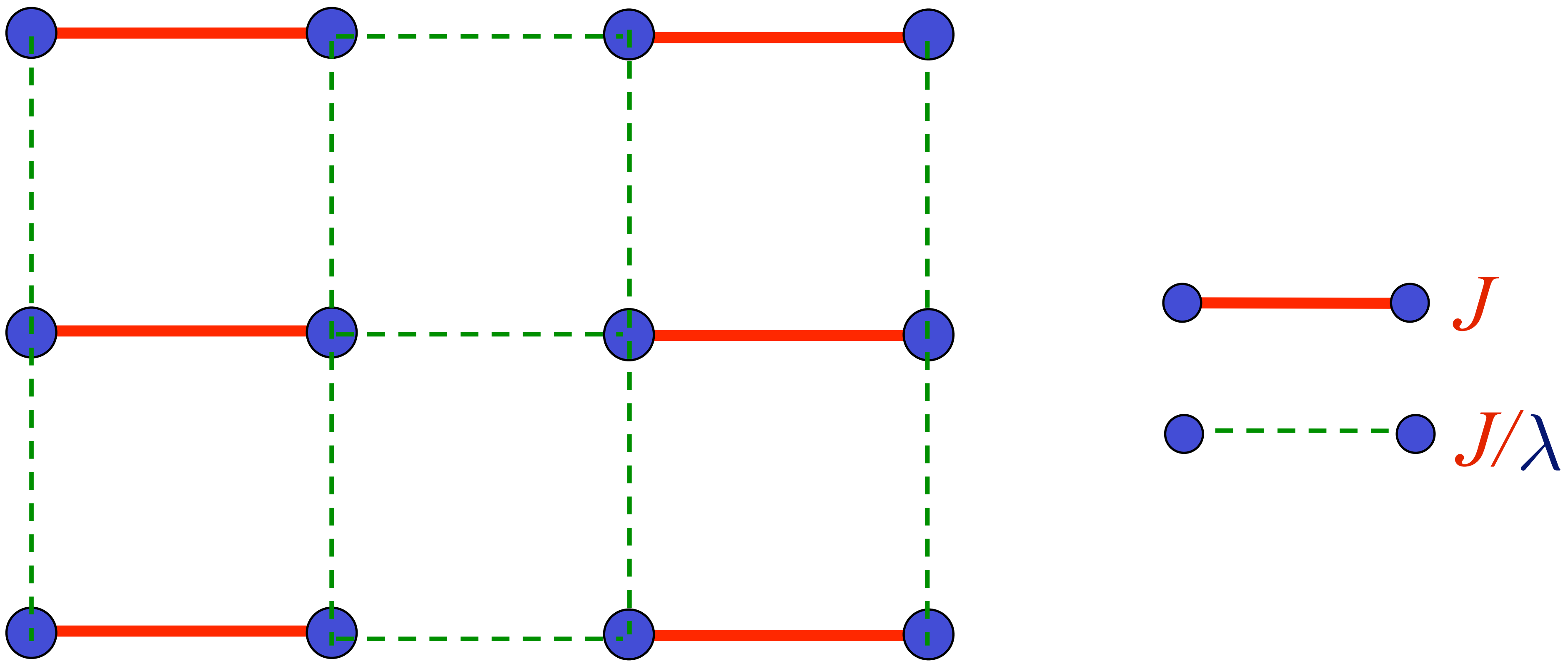
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

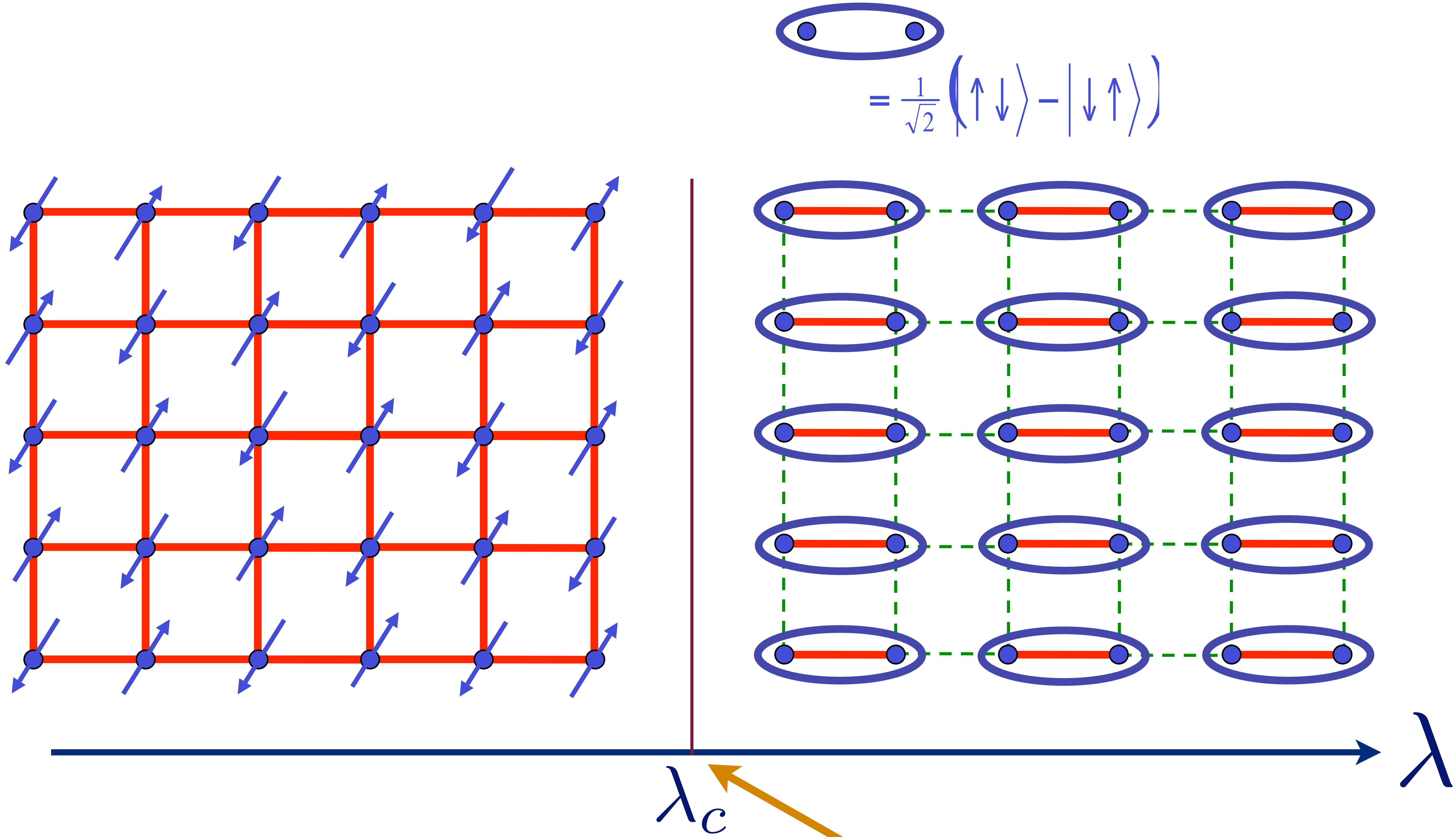
$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

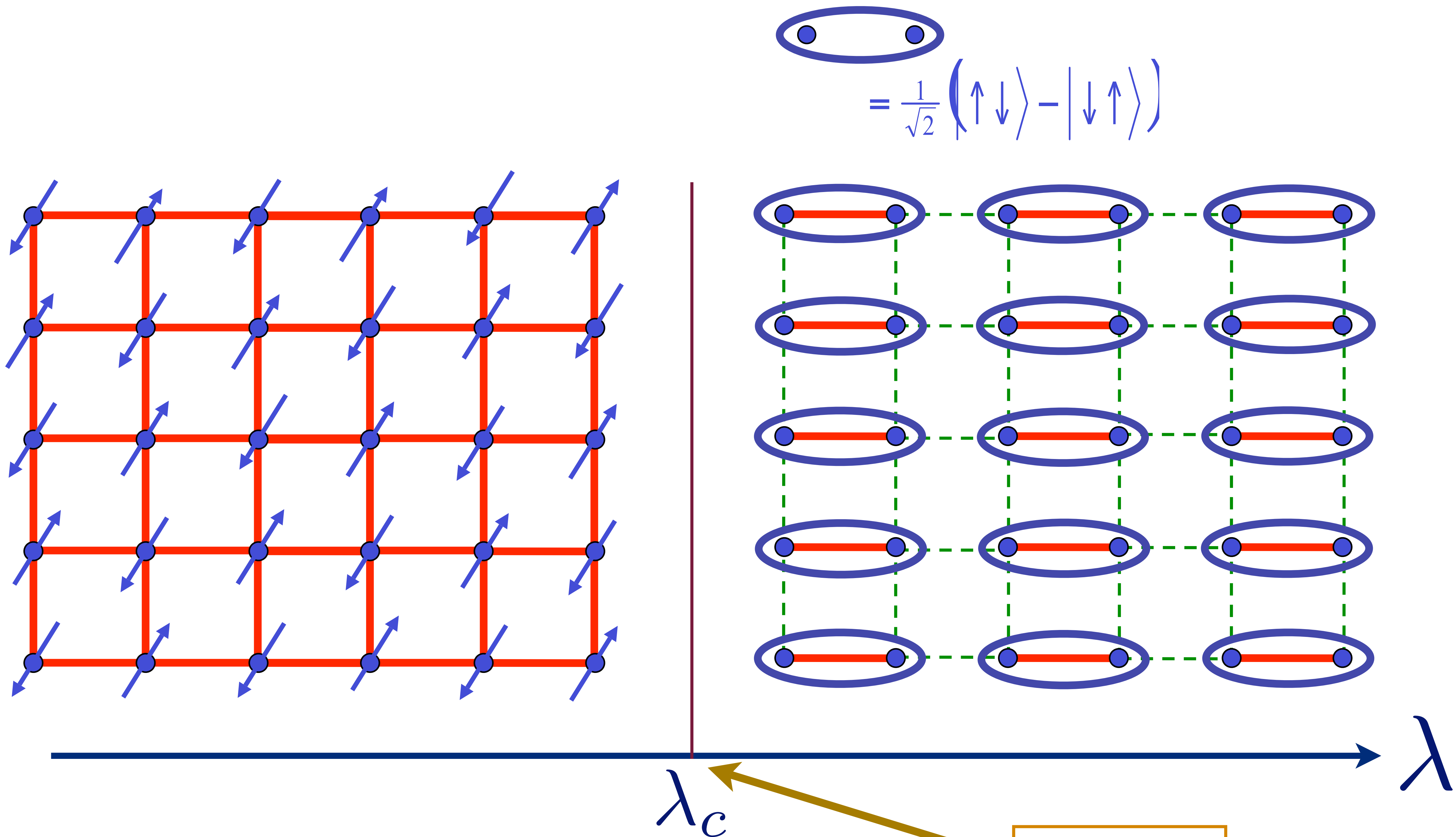


Weaken some bonds to induce spin entanglement in a new quantum phase



Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u \left(\vec{\varphi}^2\right)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [...] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096-\sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096+\sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107[d]	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230-\sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230+\sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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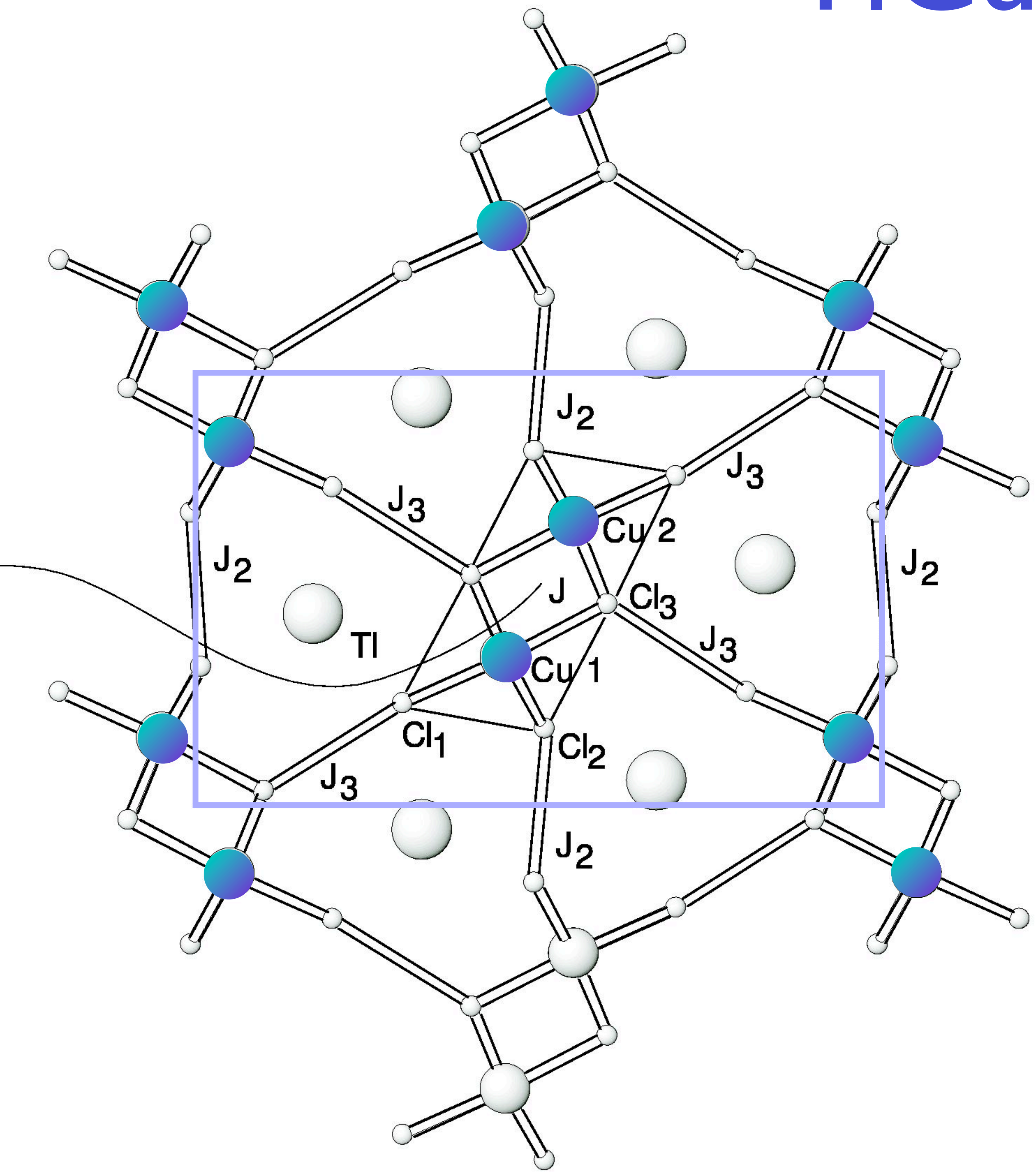
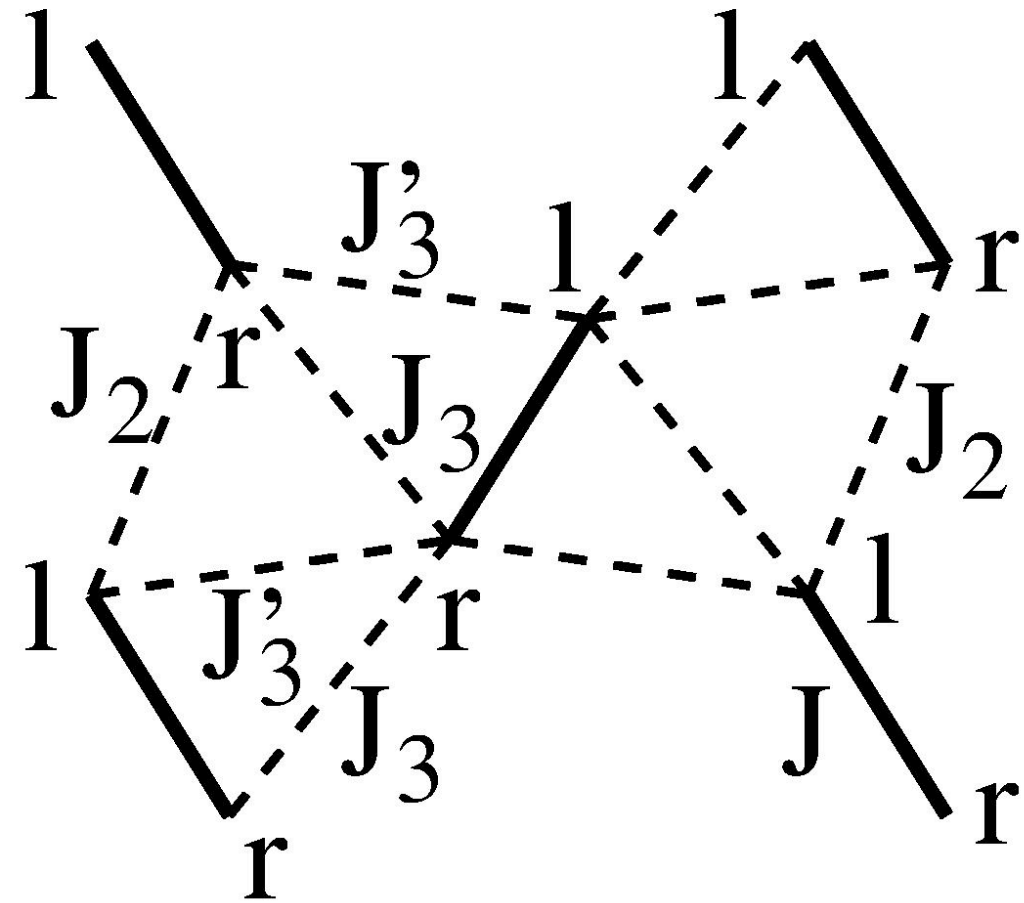
Field-theoretic RG of CFT3 E. Vicari *et al.*

S. Wenzel and W. Janke, arXiv:0808.1418
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

$TlCuCl_3$

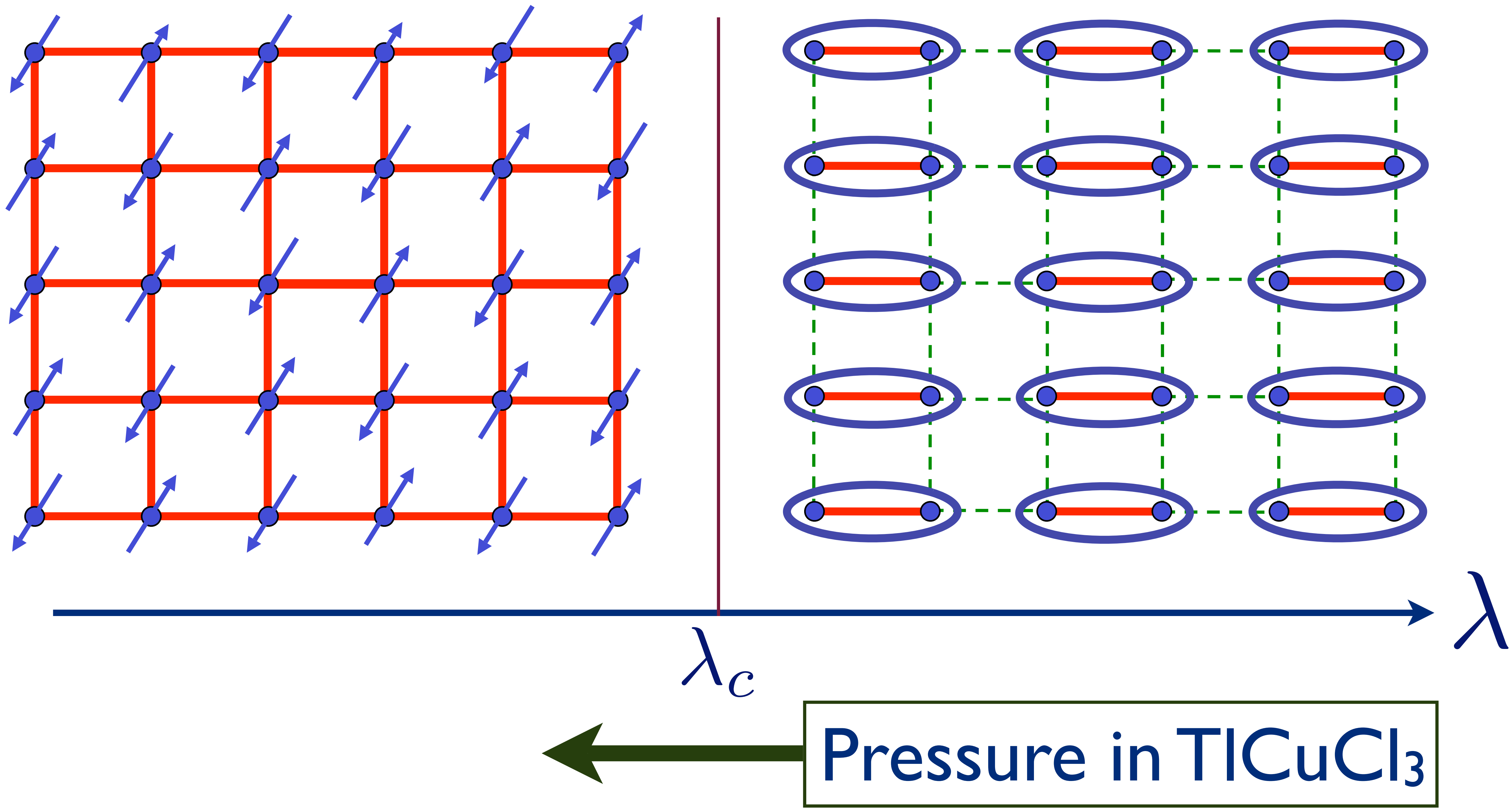


J2
J3
Cu 2
J2
J2
J
Cl3
Tl
J3
Cu 1
Cl1
Cl2
J3
J2



l
l
J'3
l
r
r
J2
J3
J2
l
r
l
J'3
J3
J
r
r

Phase diagram as a function of the ratio of exchange interactions, λ



$TlCuCl_3$ at ambient pressure

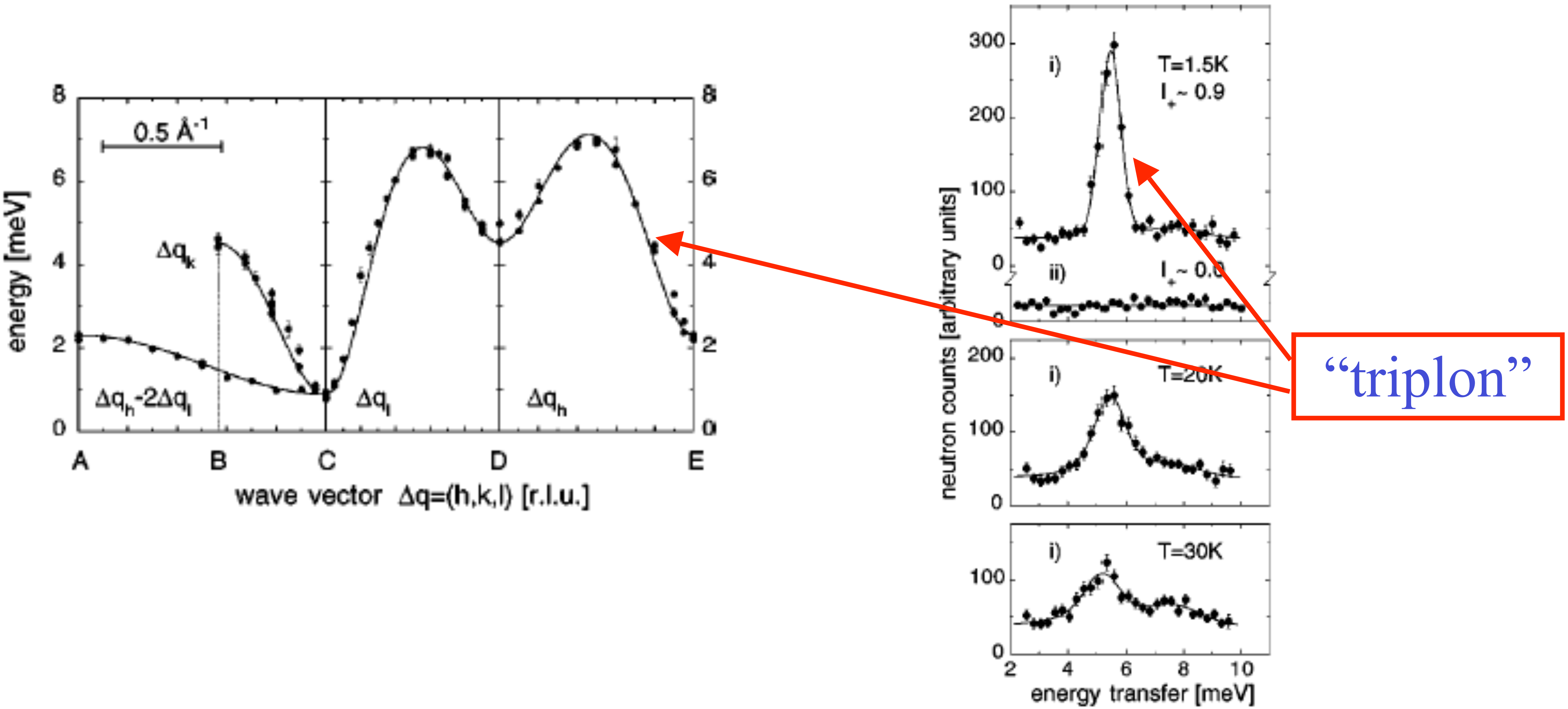
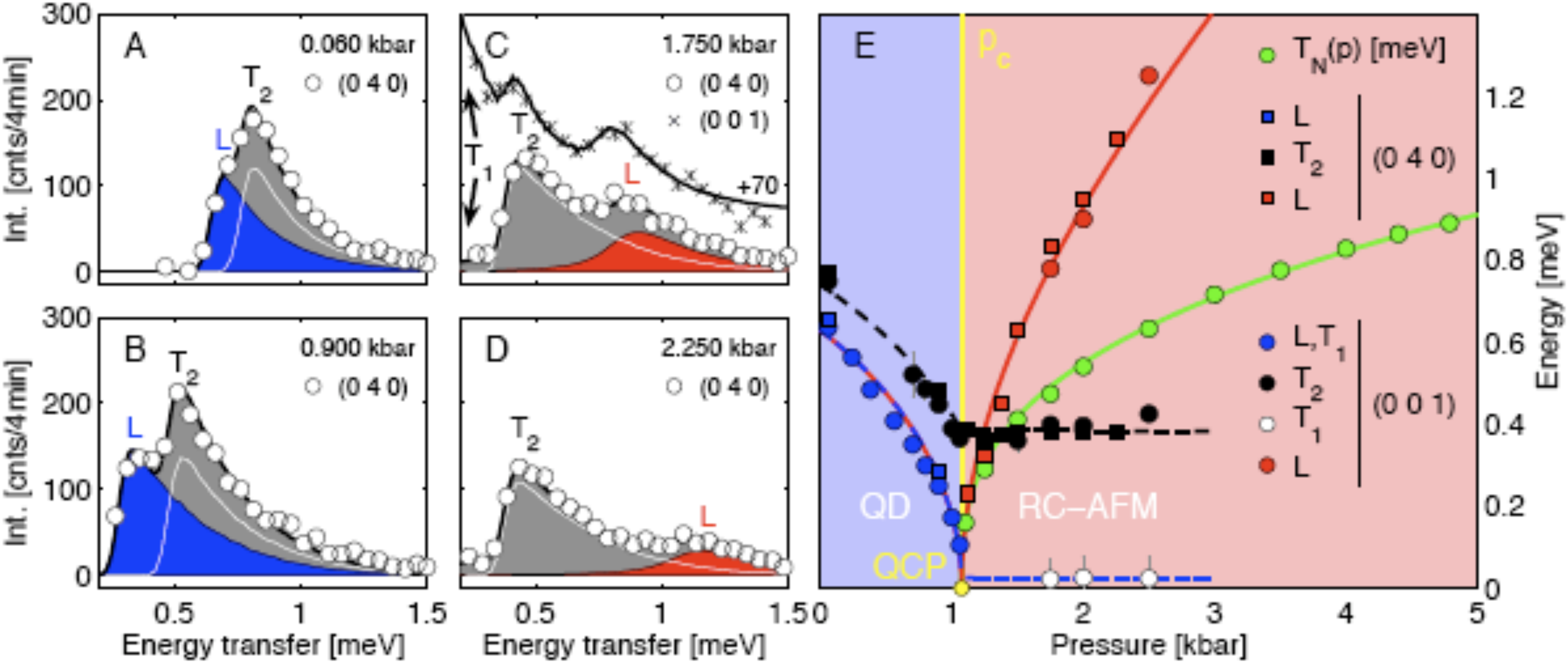


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev*. B 63 172414 (2001).

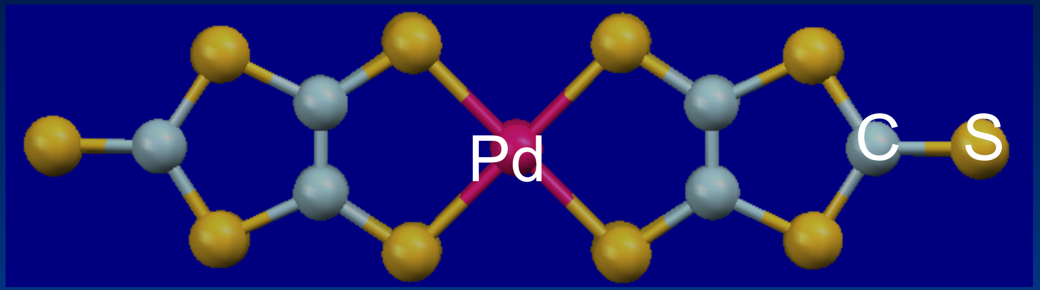
$TlCuCl_3$ with varying pressure

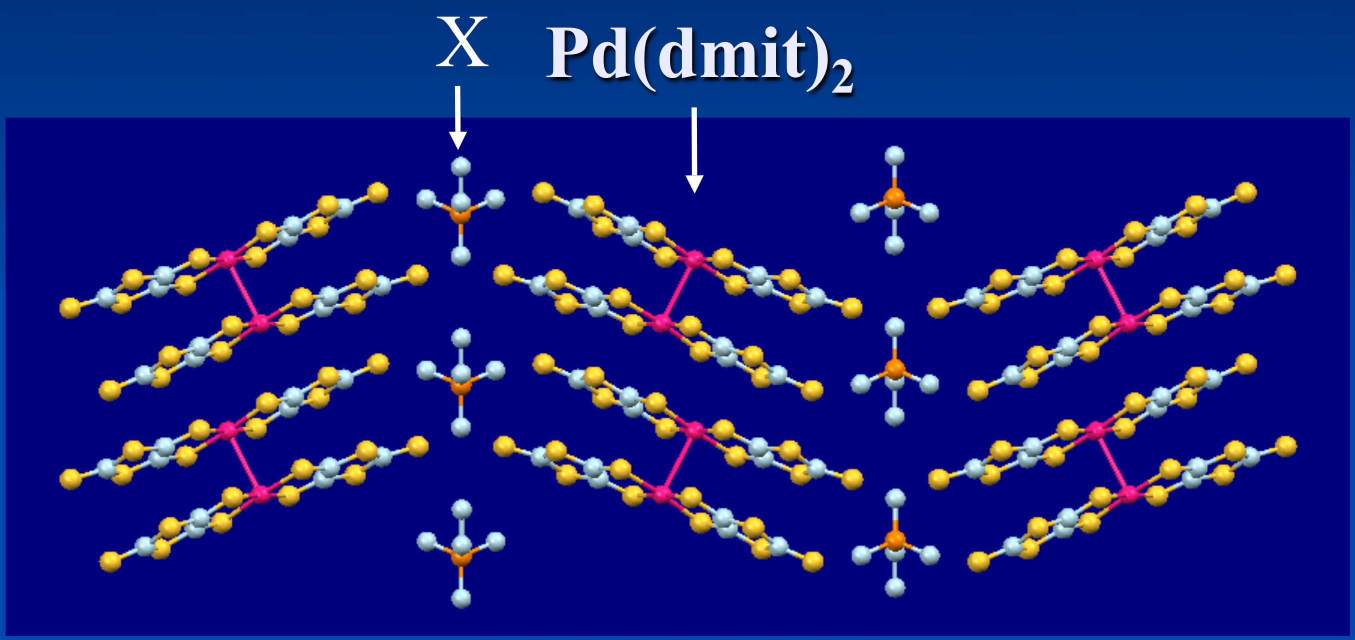


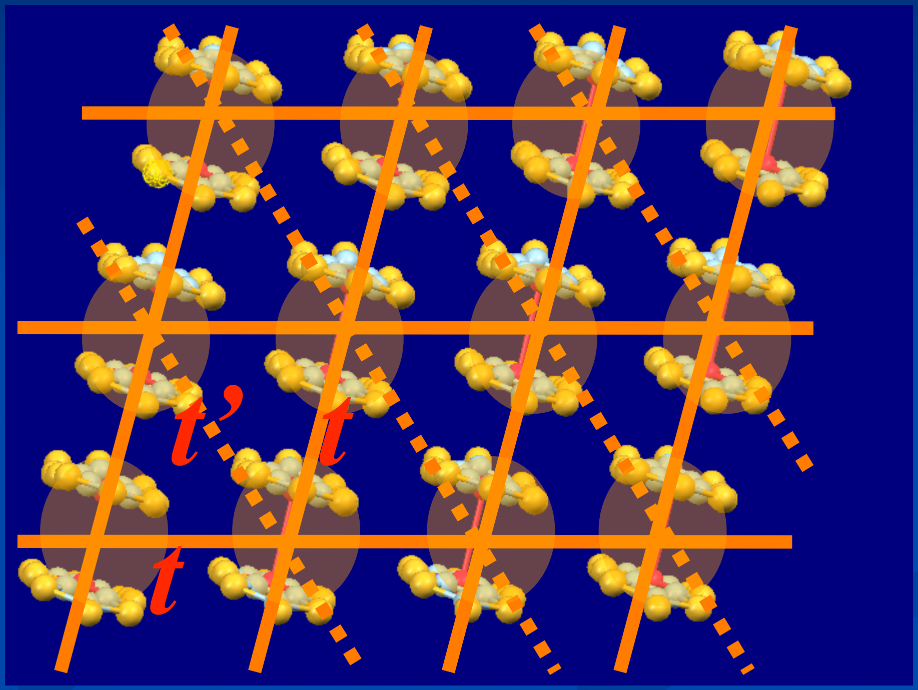
Observation of 3 → 2 low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

$X[Pd(dmit)_2]_2$







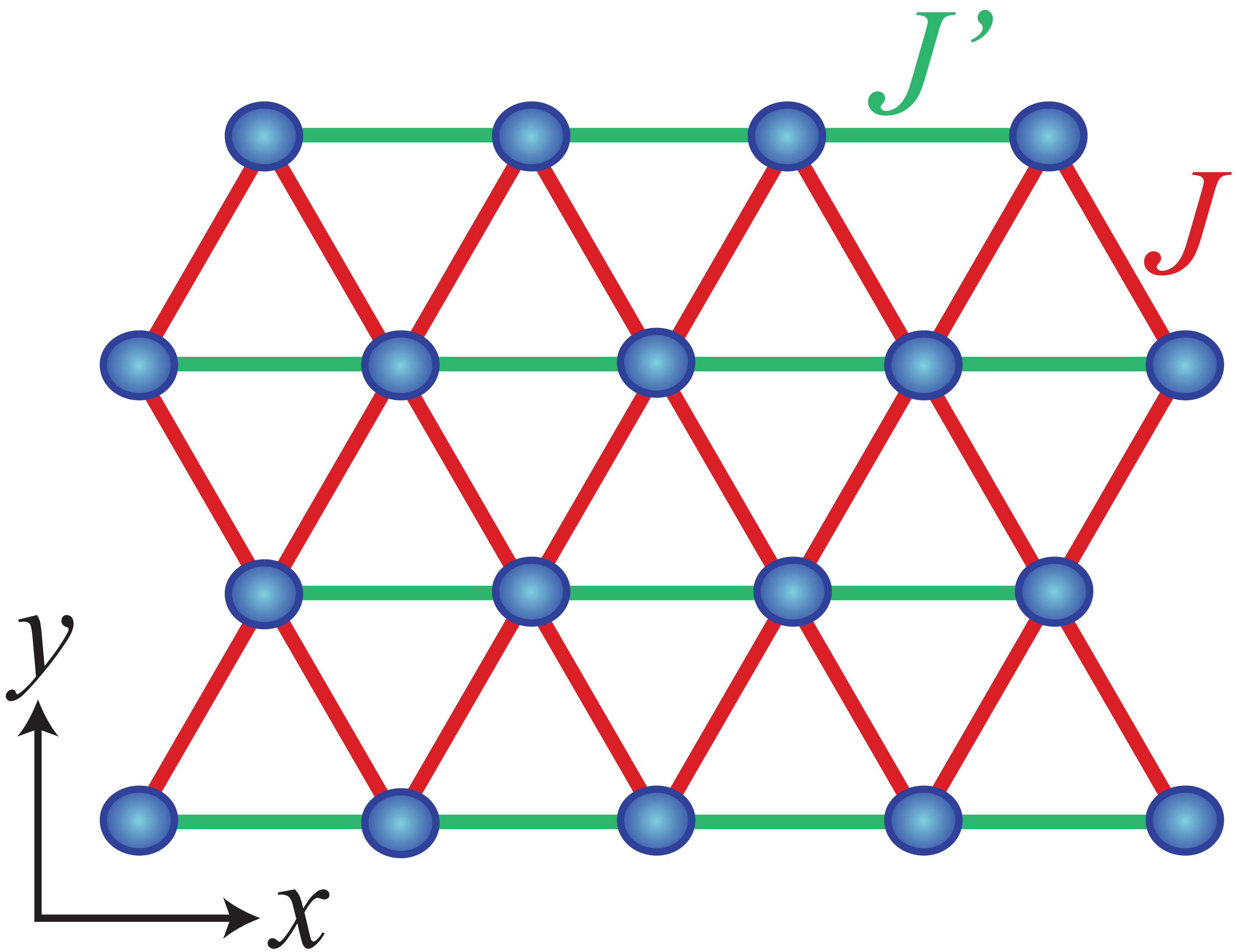
Half-filled band → Mott insulator with spin S = 1/2

Triangular lattice of $[Pd(dmit)_2]_2$
→ frustrated quantum spin system

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



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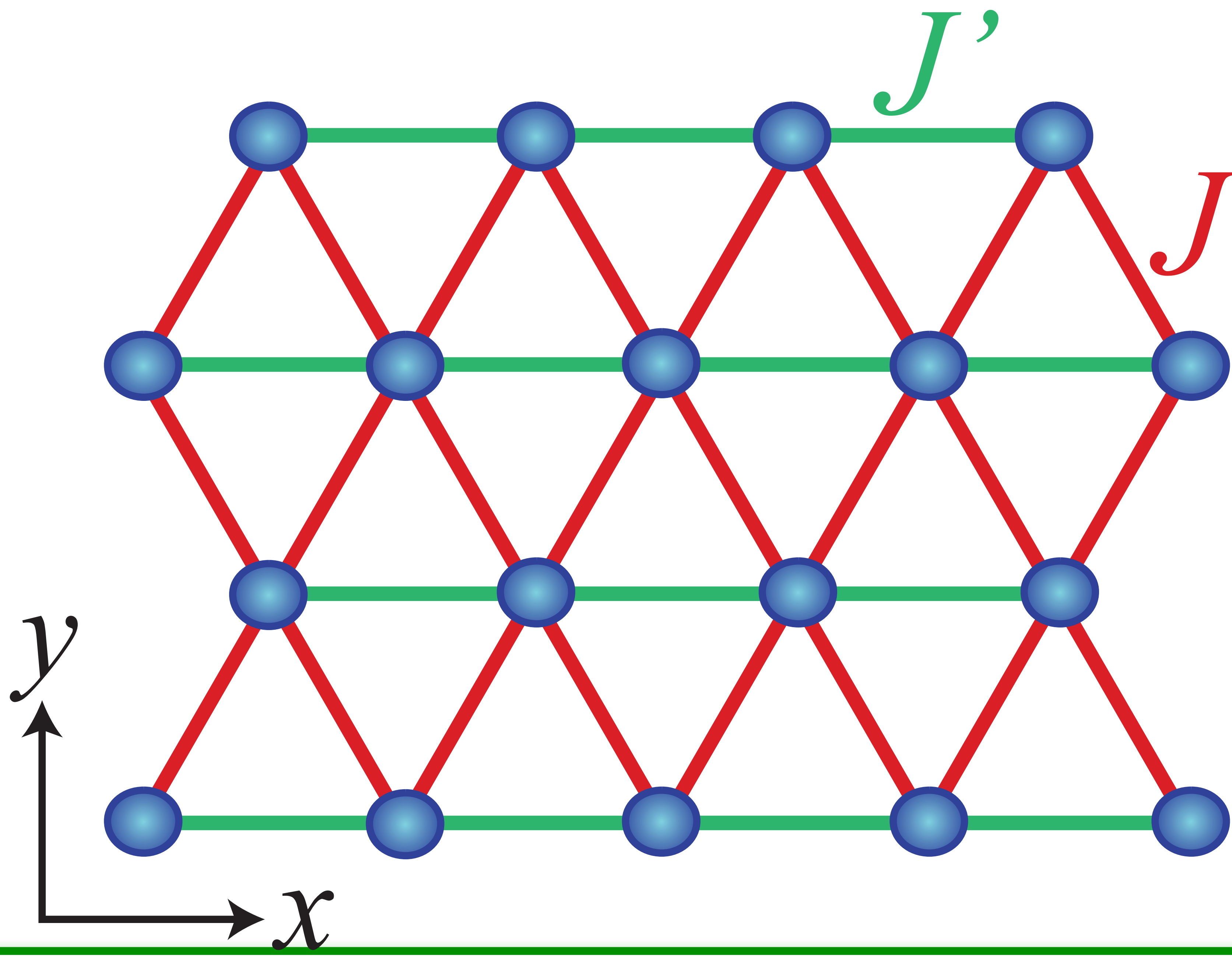
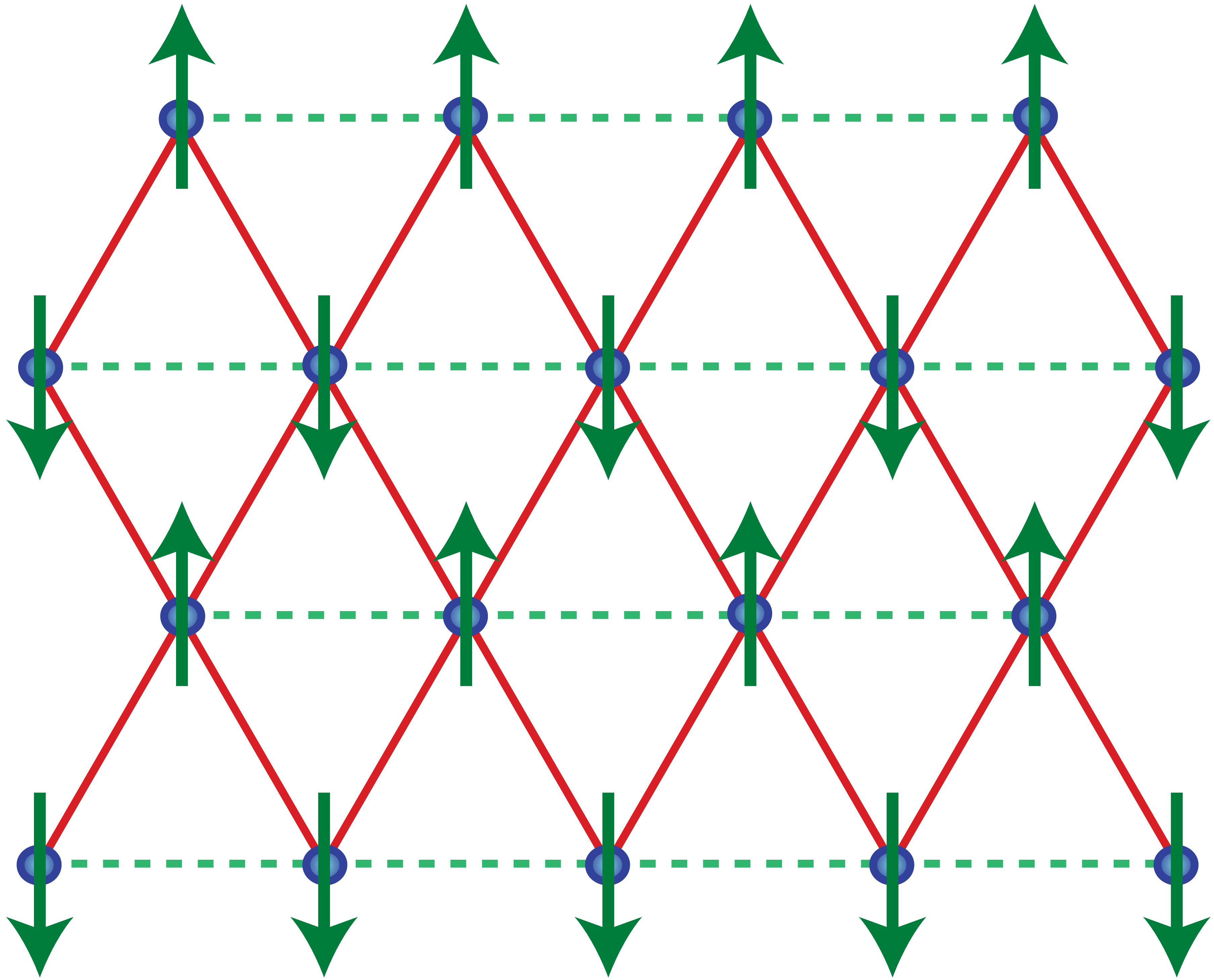


What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



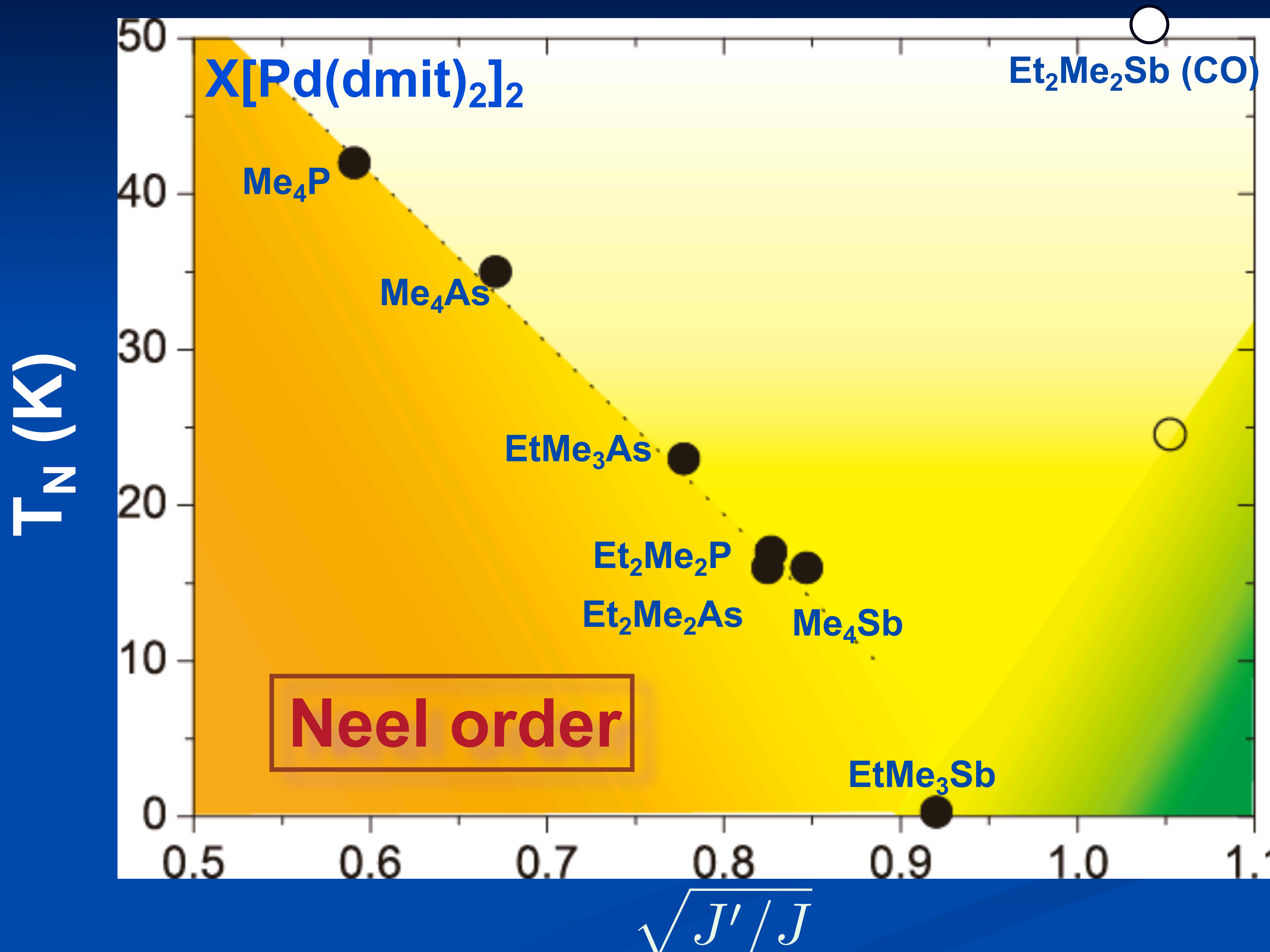
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

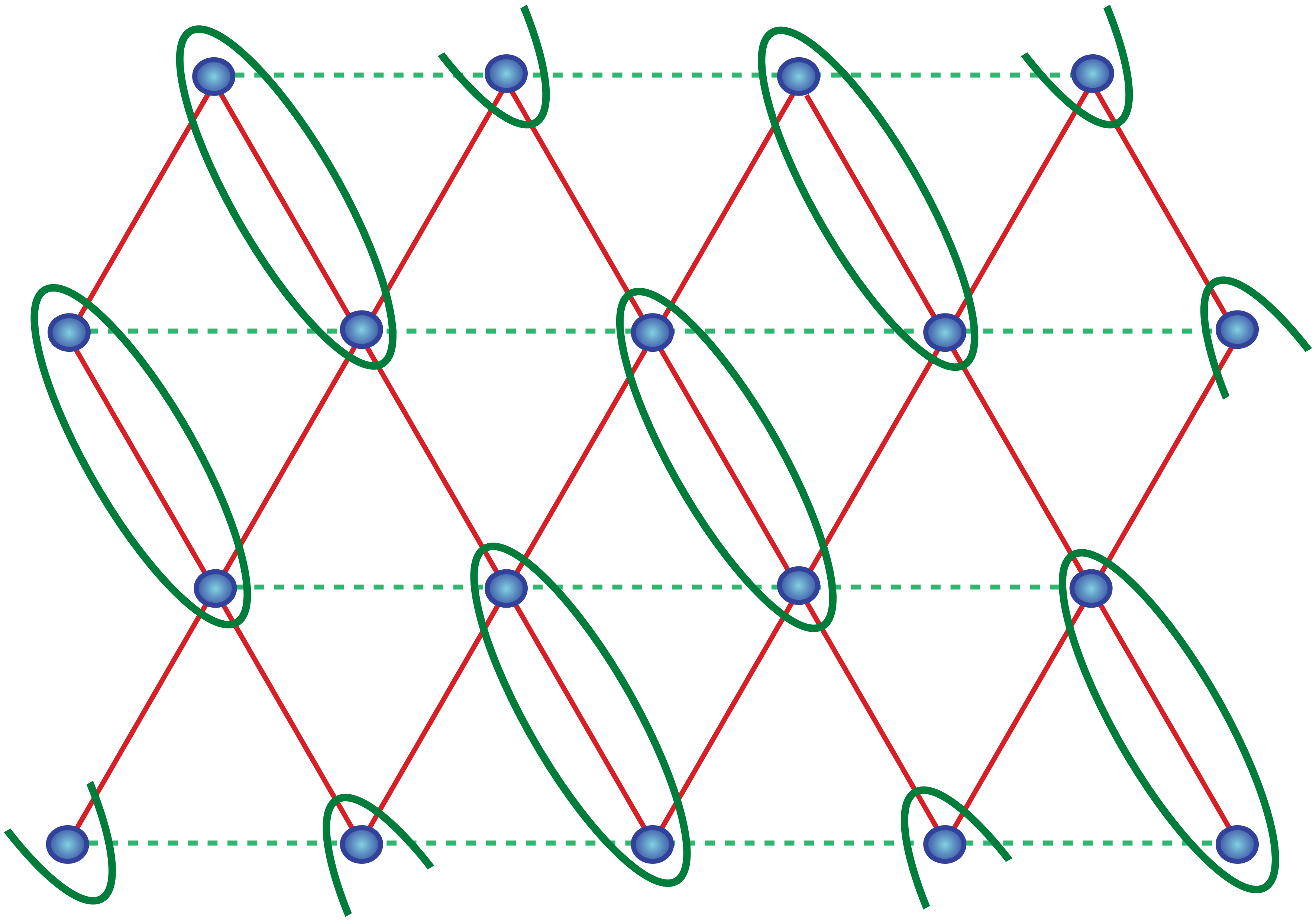
- Néel antiferromagnetic LRO

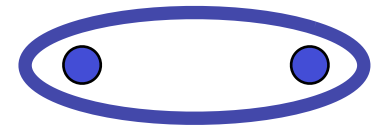
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet



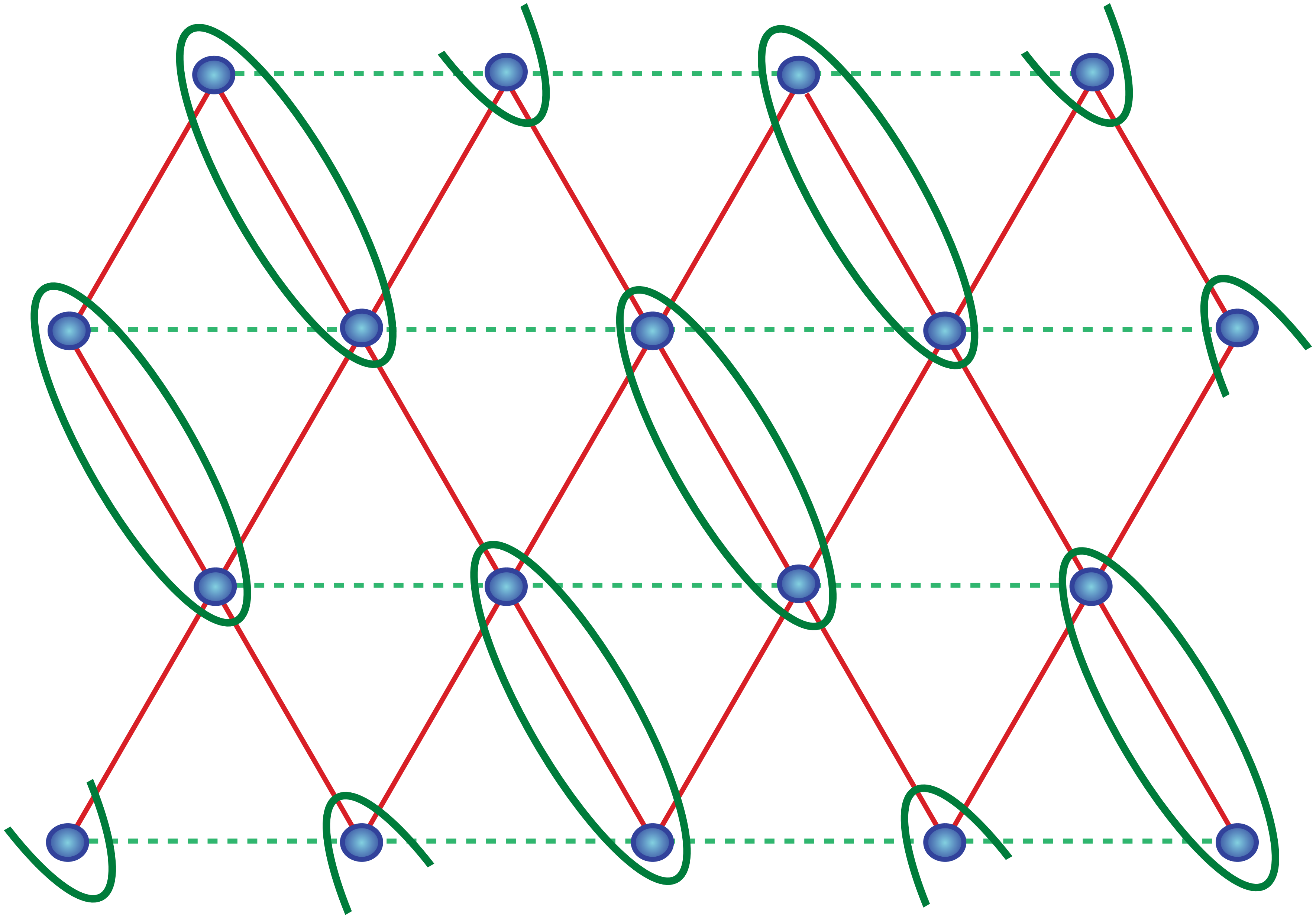


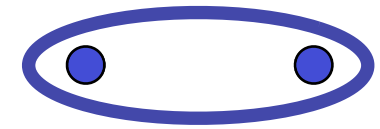
$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

Possible ground state for intermediate *J'/J*

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry





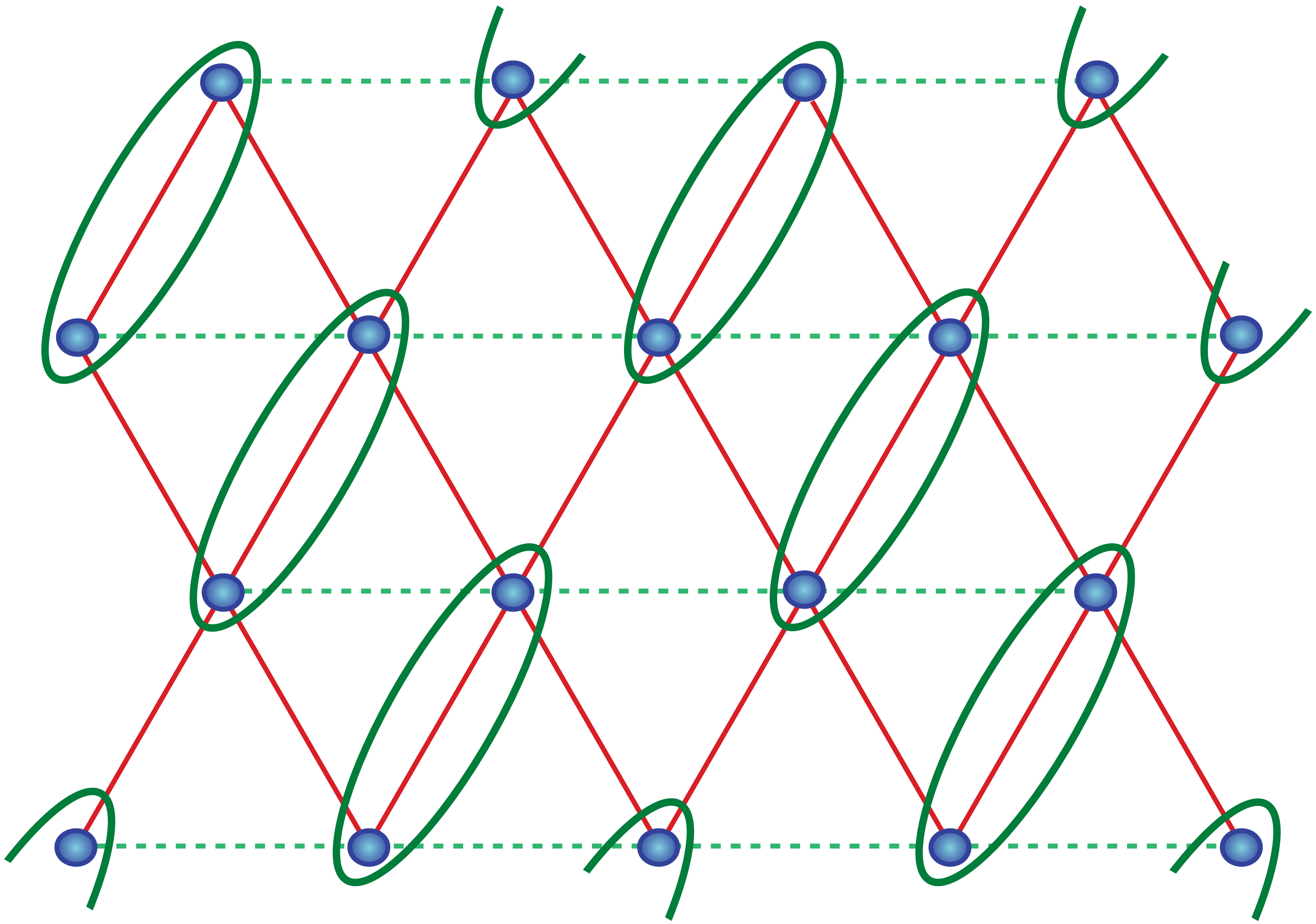
$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

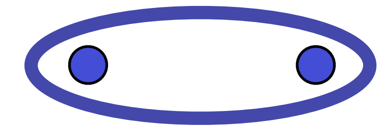
Valence bond solid (VBS)

Possible ground state for intermediate J'/J

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry





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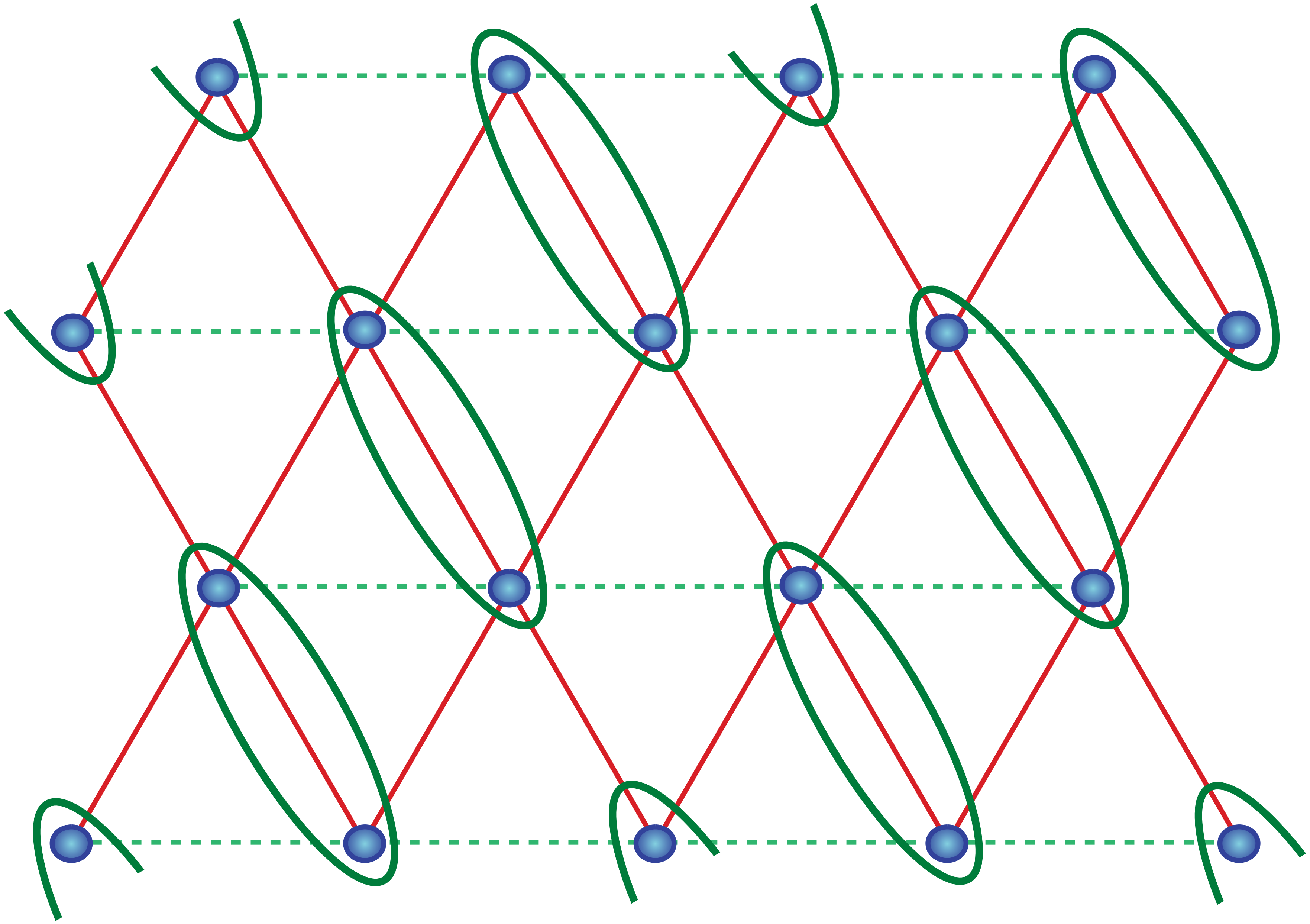
Valence bond solid (VBS)

Possible ground state for intermediate *J'/J*

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry



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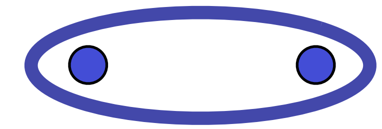
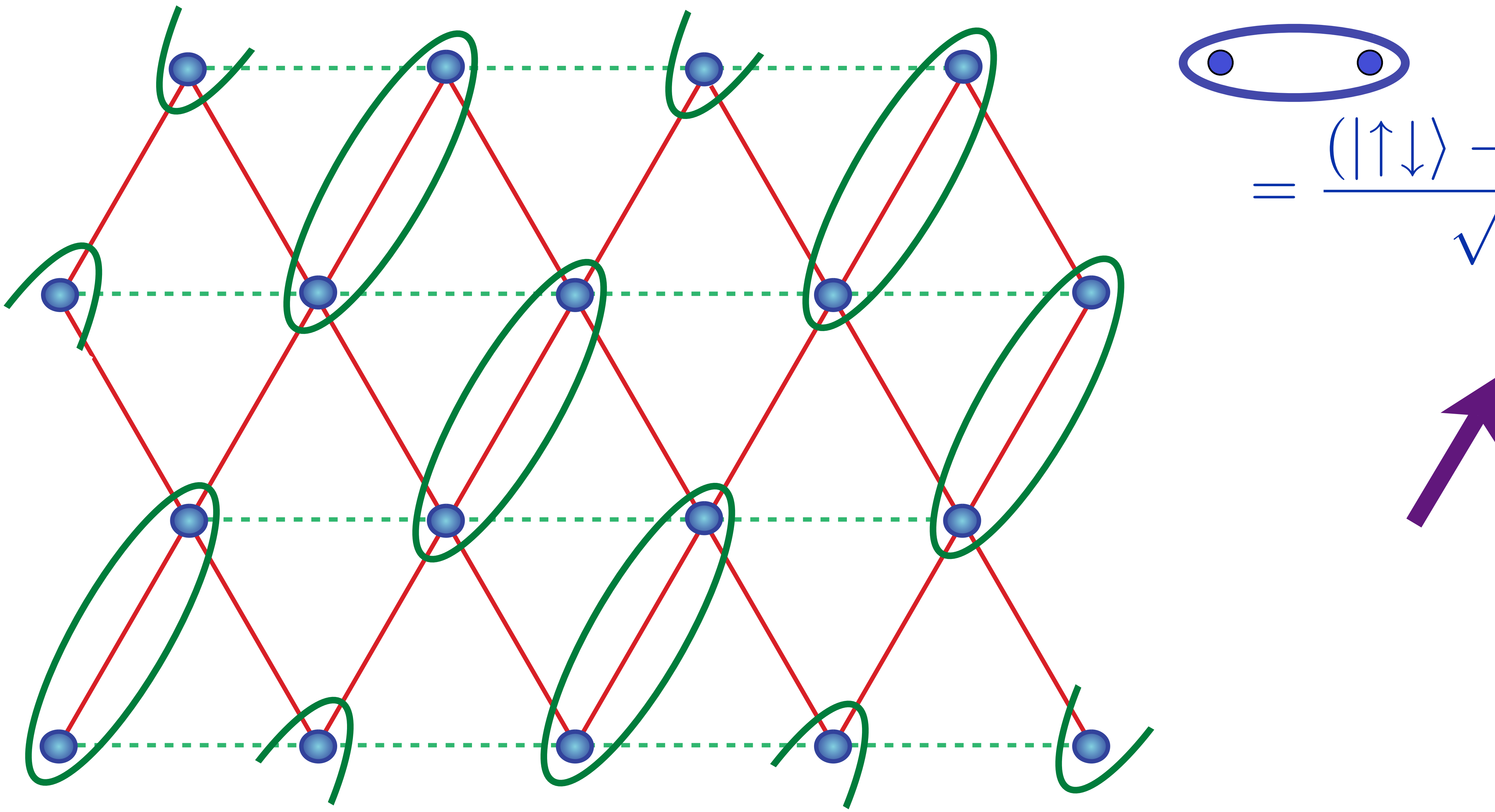


Valence bond solid (VBS)

Possible ground state for intermediate J'/J

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry



$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

Valence bond solid (VBS)

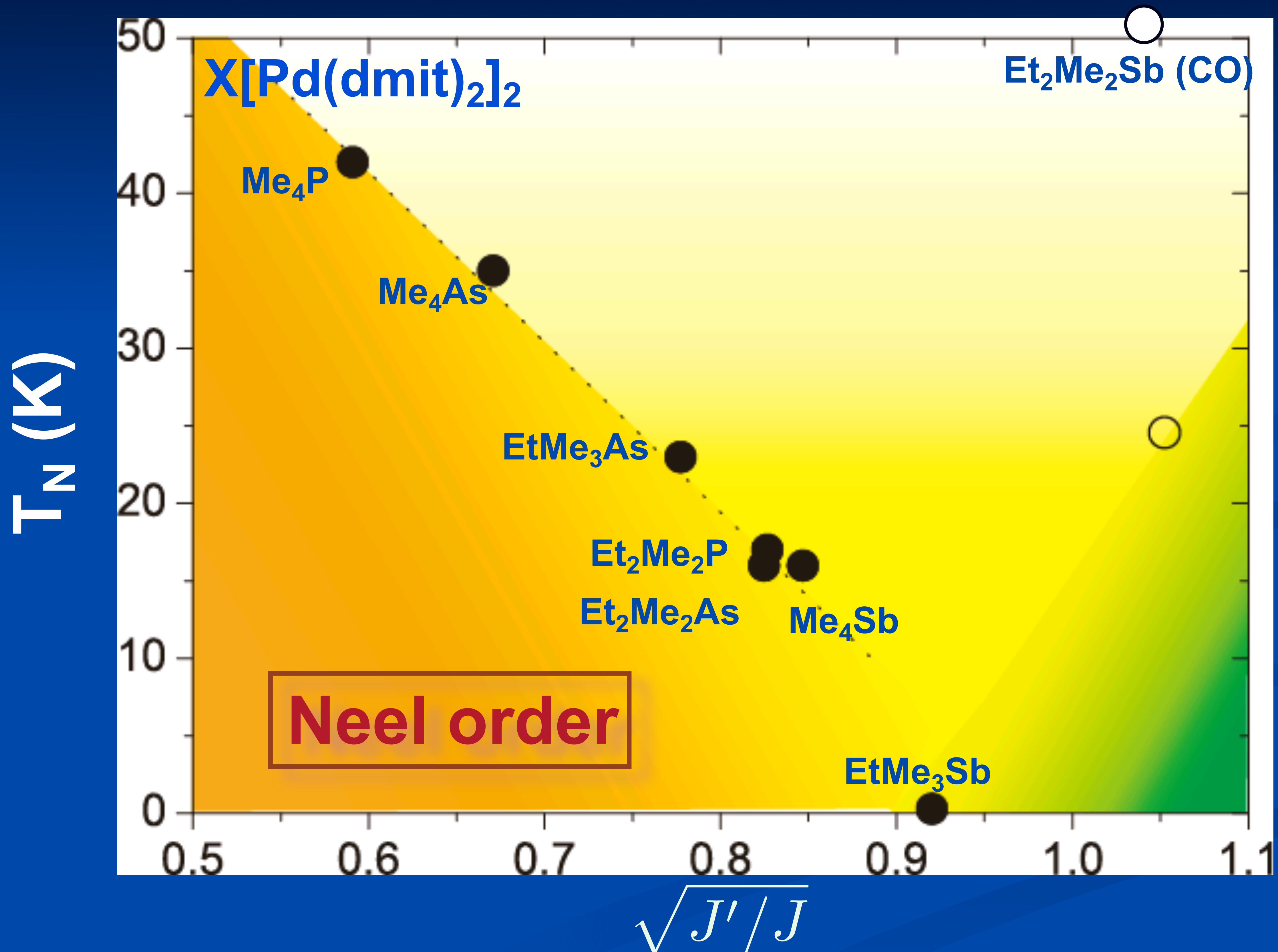
Possible ground state for intermediate J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

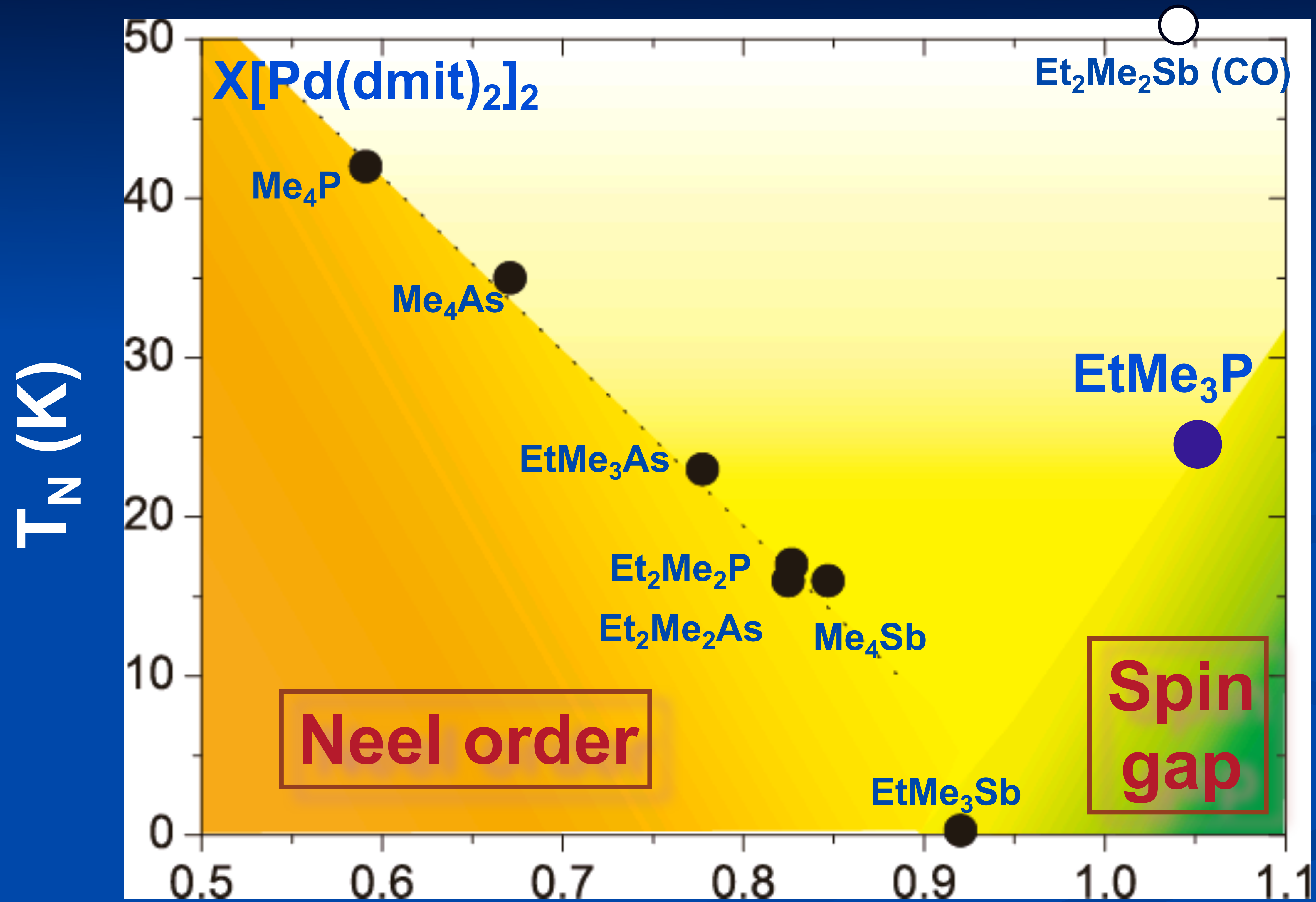
- Néel antiferromagnetic LRO
- Valence bond solid

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

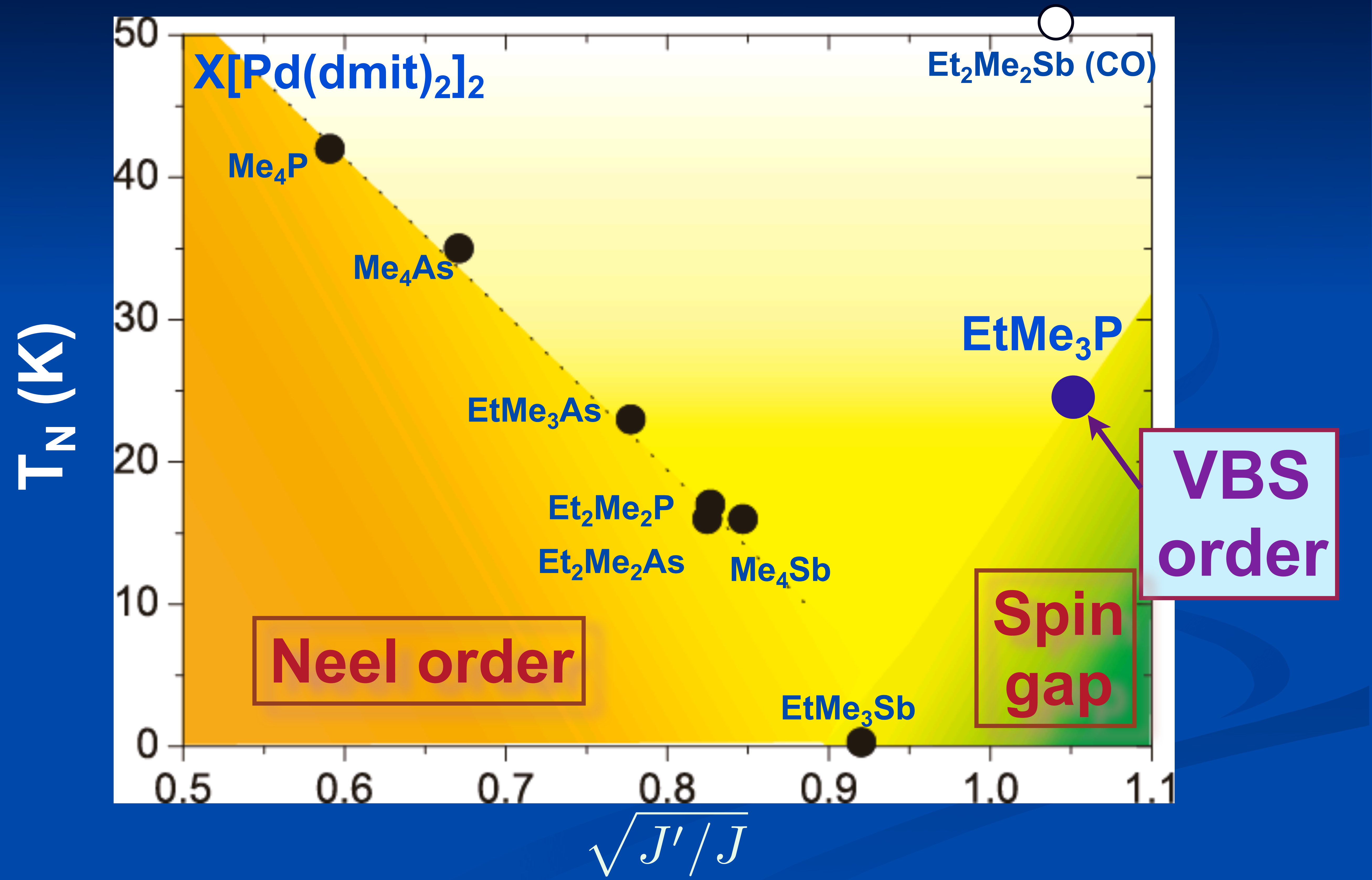
Magnetic Criticality



$\sqrt{J'/J}$

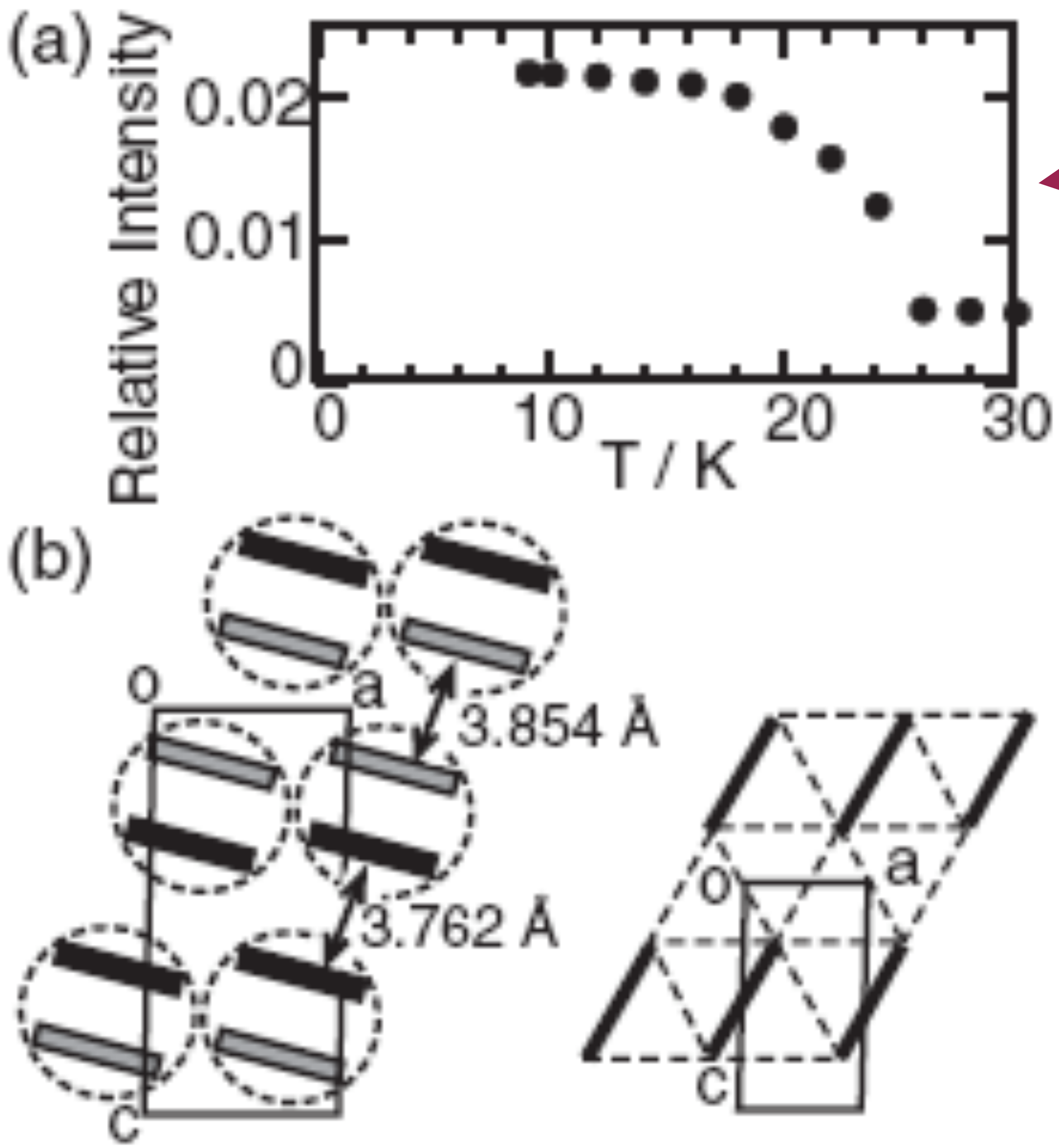
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$



X-ray scattering

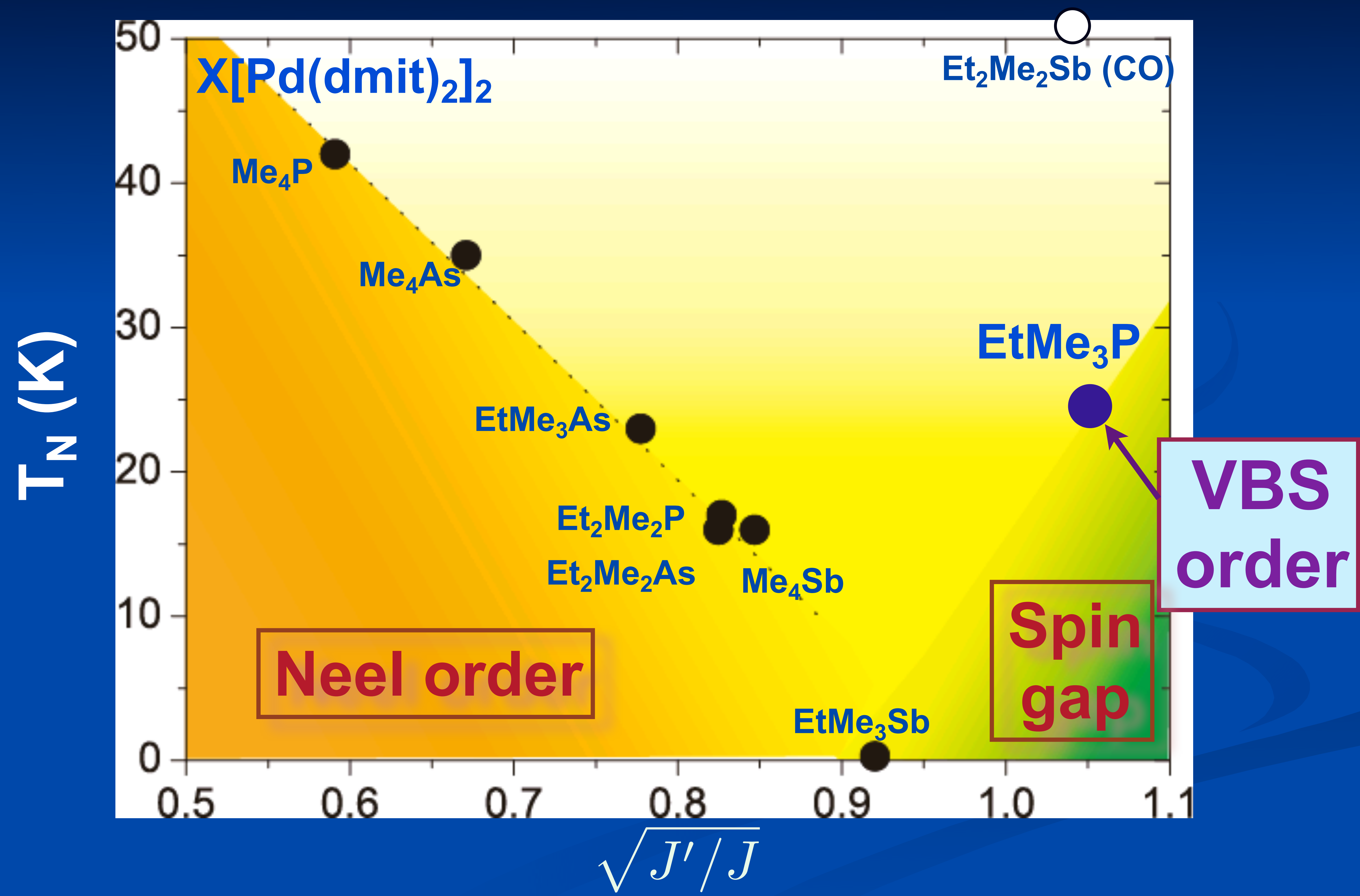
Spin gap ~ 40 K

J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality



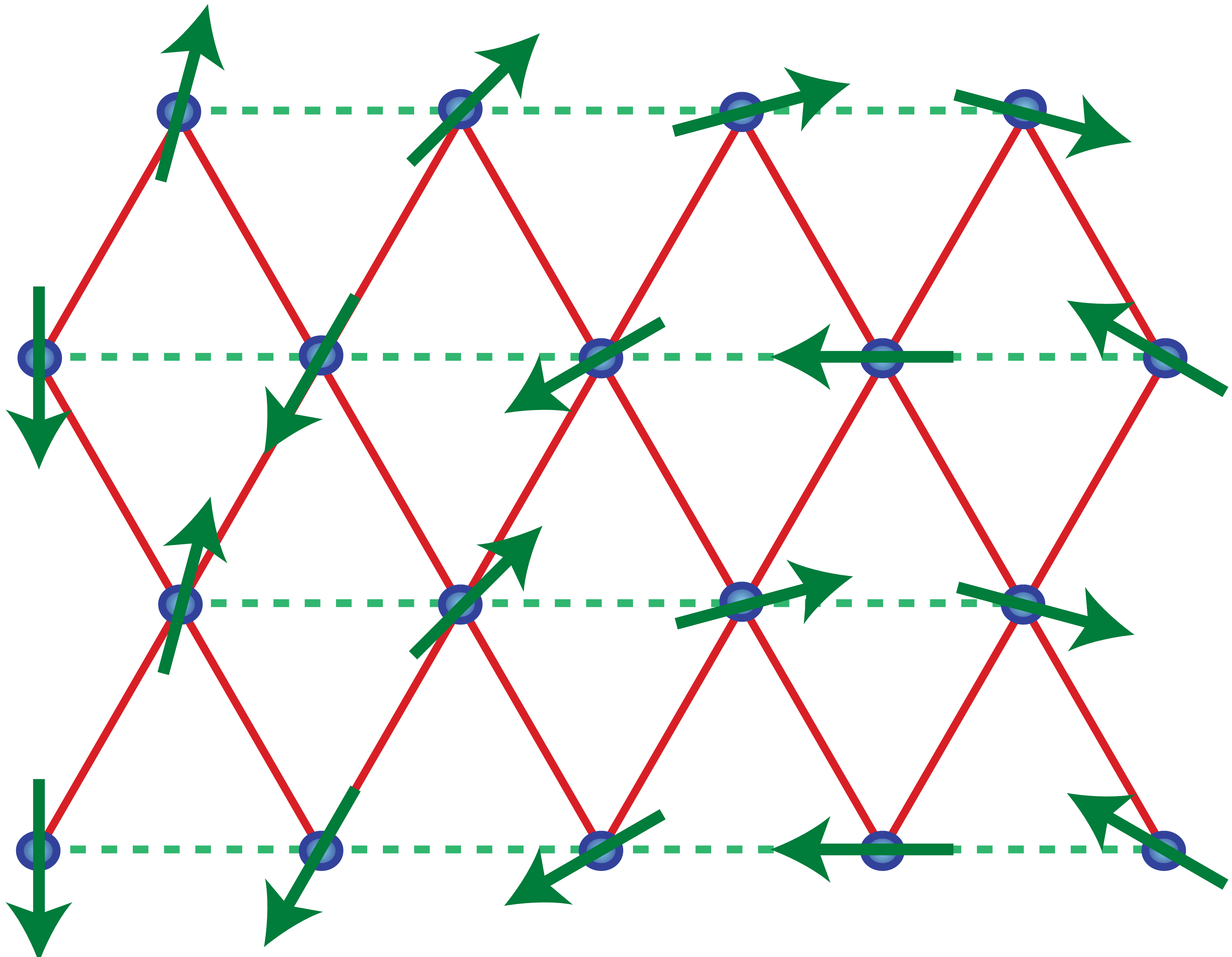
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in Cs_2CuCl_4

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

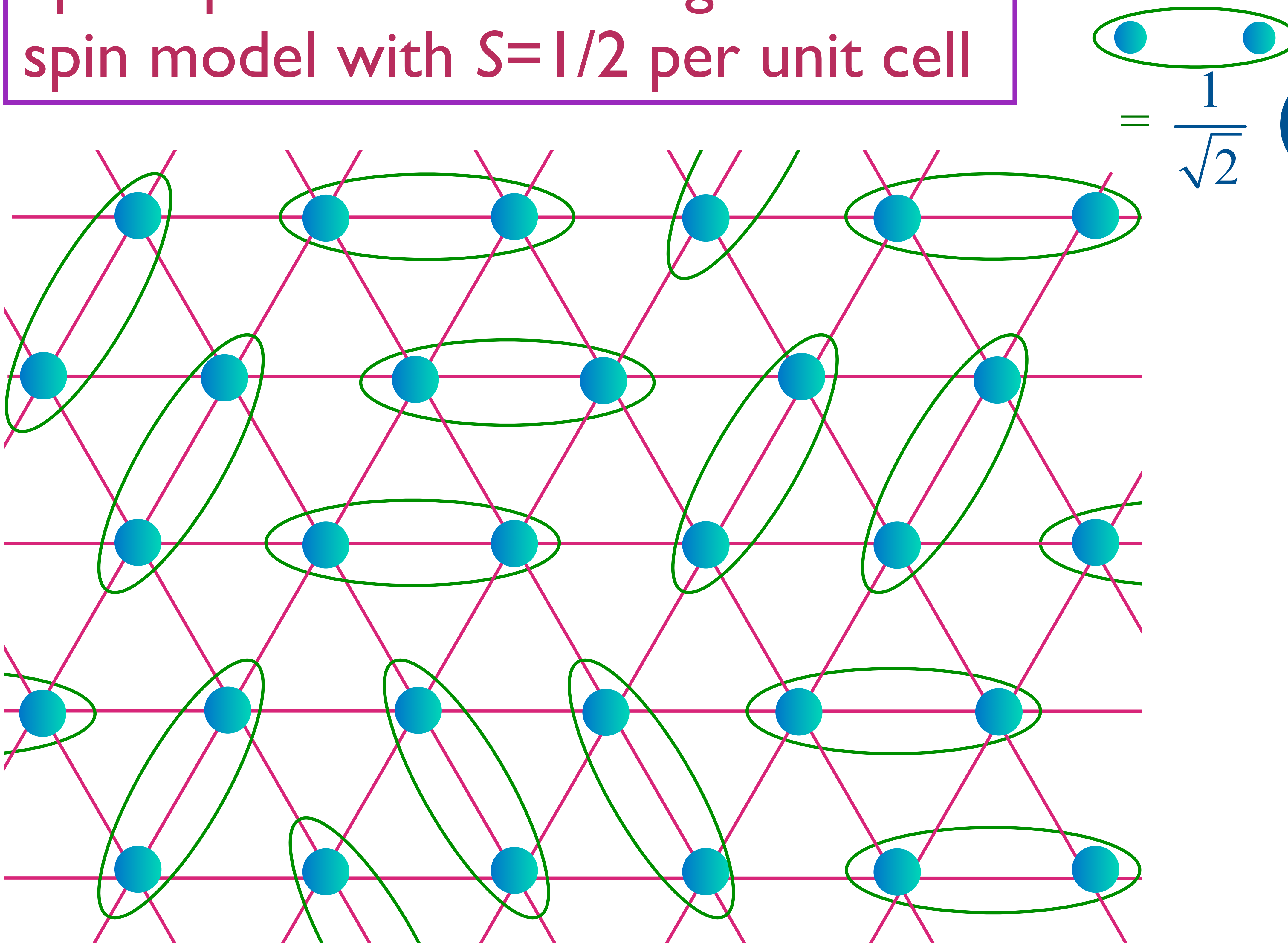
- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO
- Z_2 spin liquid: preserves all symmetries of Hamiltonian

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Triangular lattice antiferromagnet

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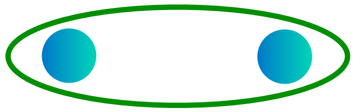

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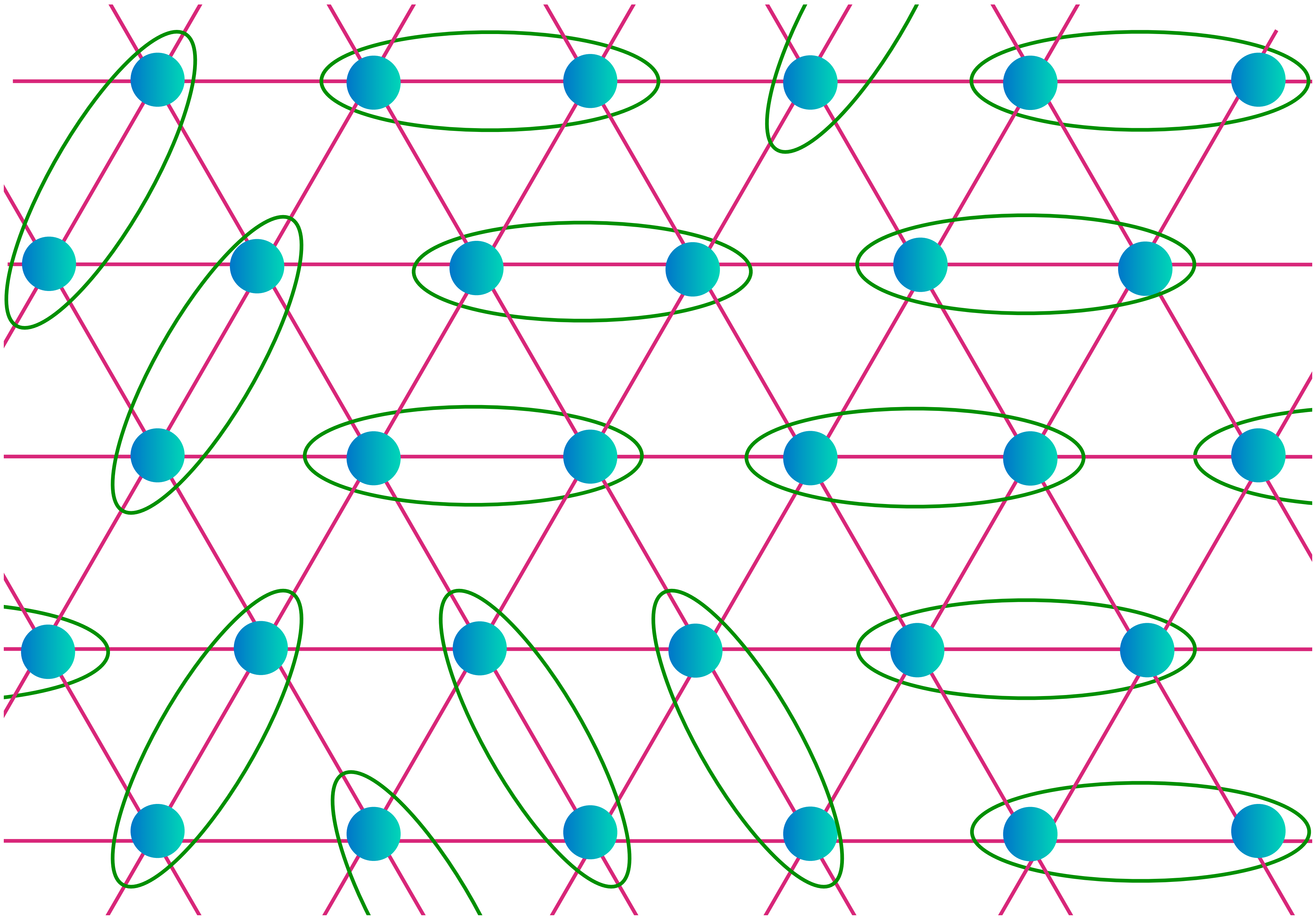

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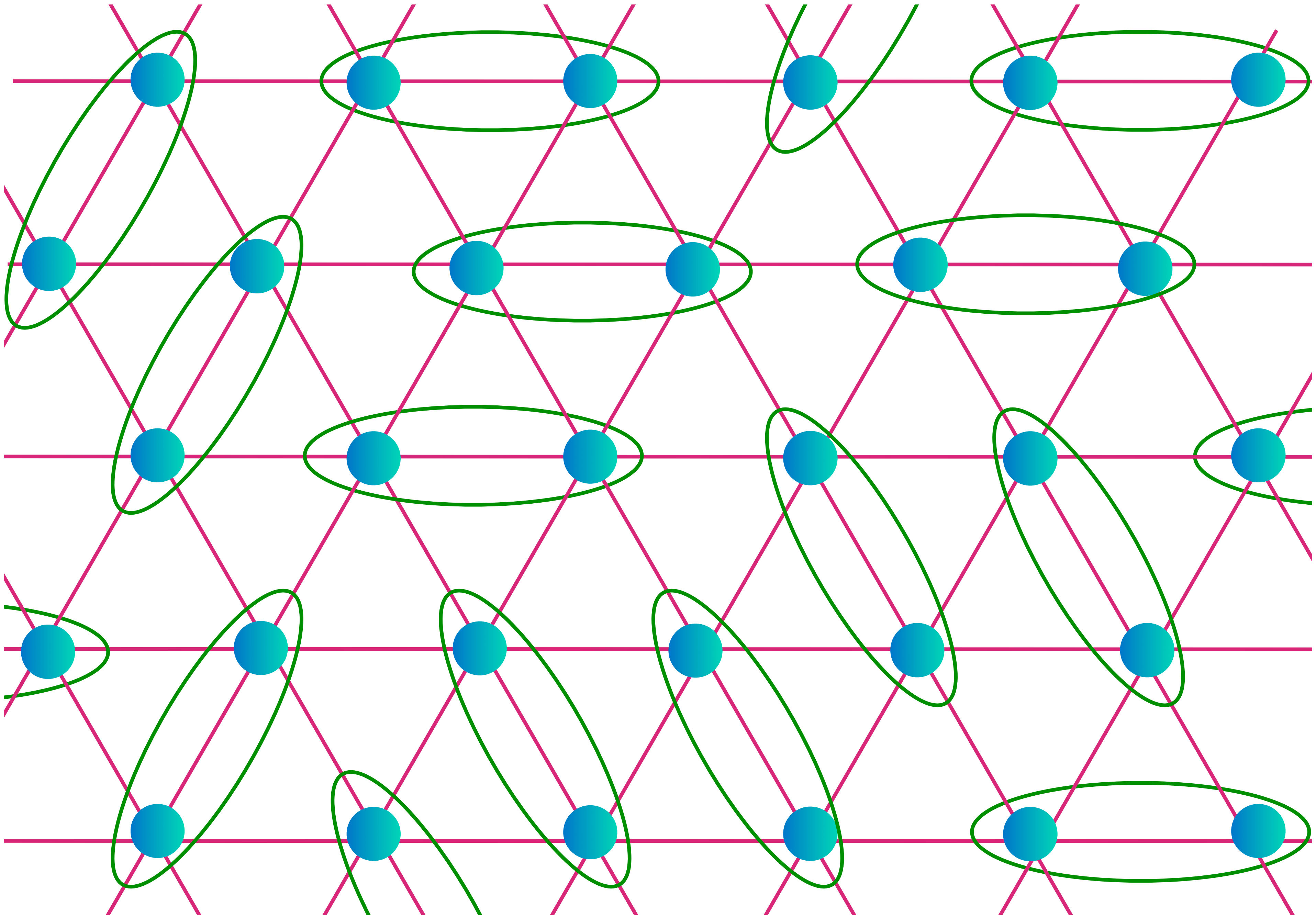

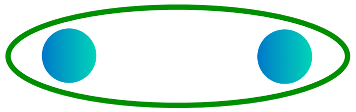

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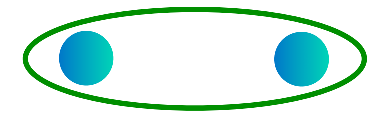

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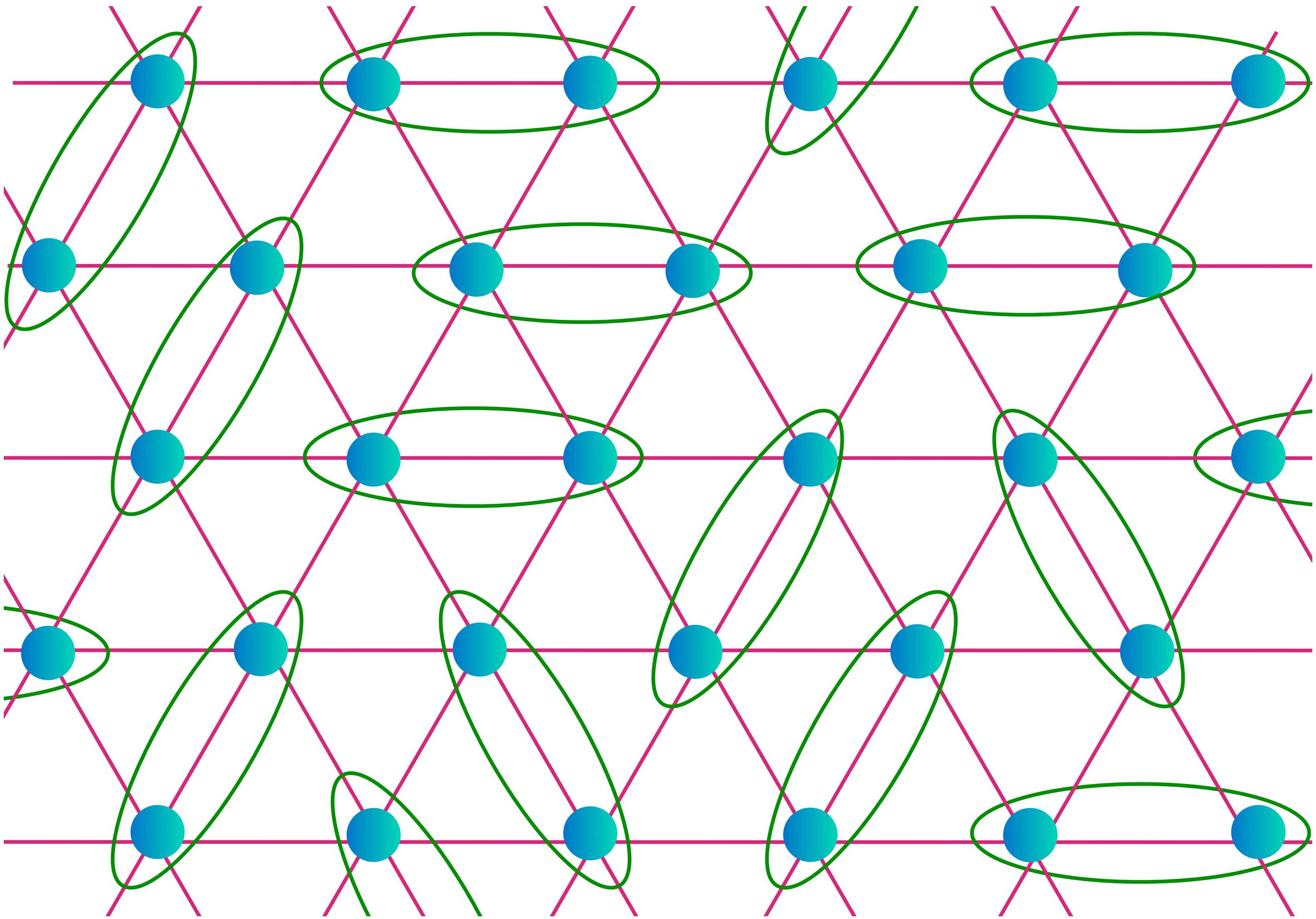

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Triangular lattice antiferromagnet

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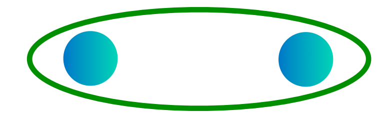

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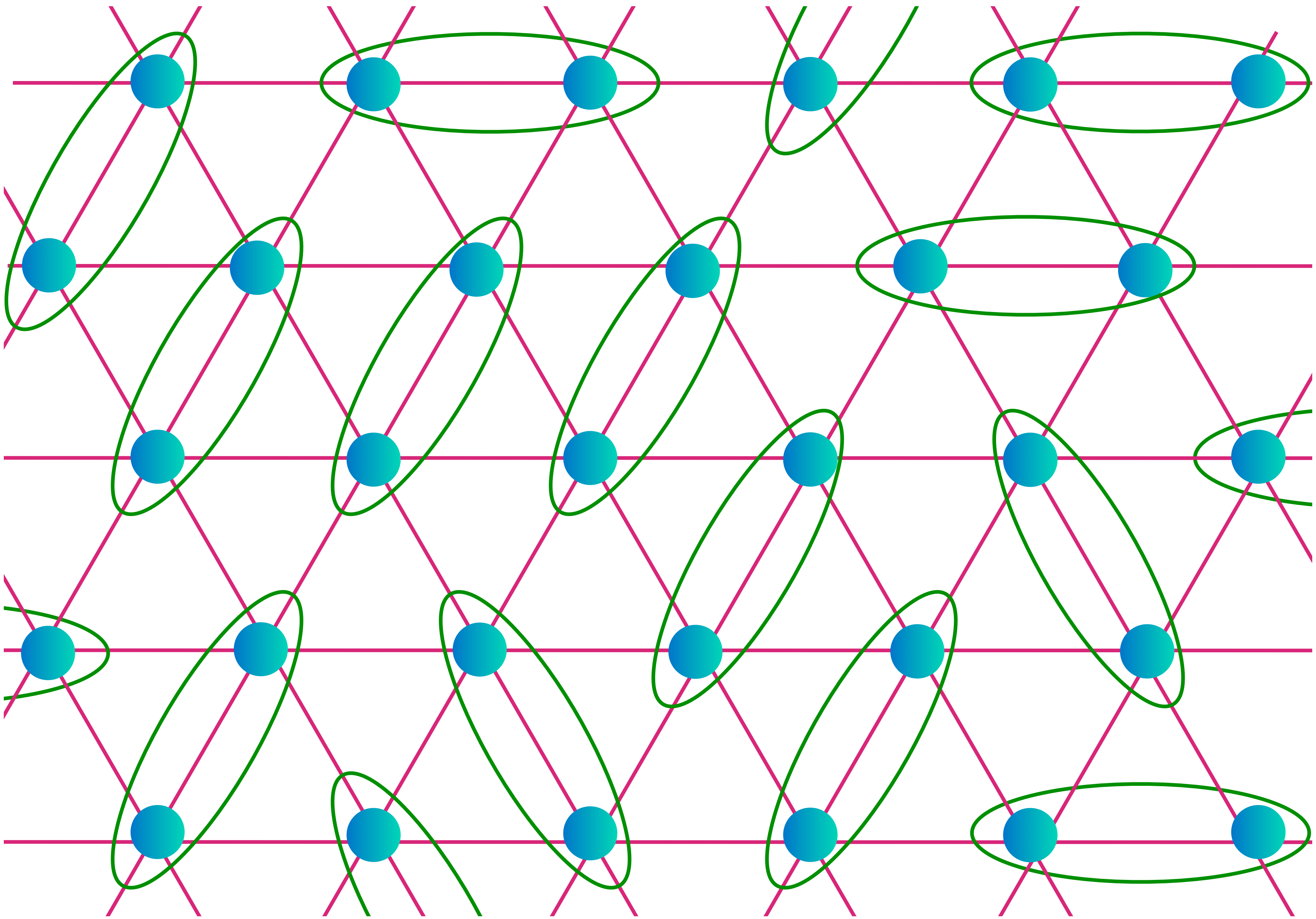

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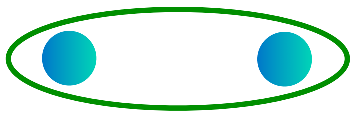

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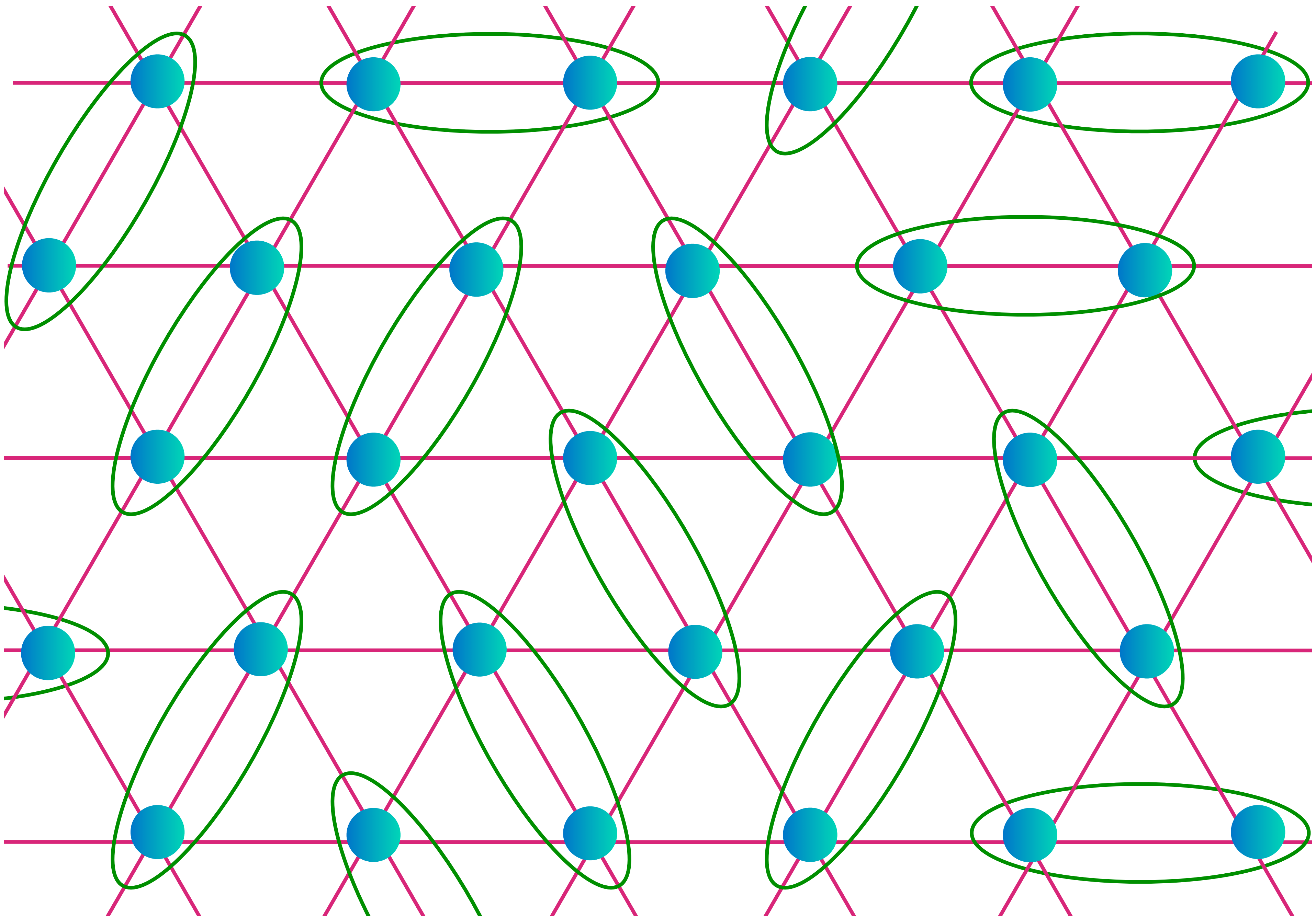

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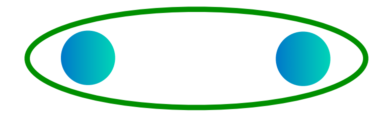
Excitations of the Z_2 Spin liquid

A spinon



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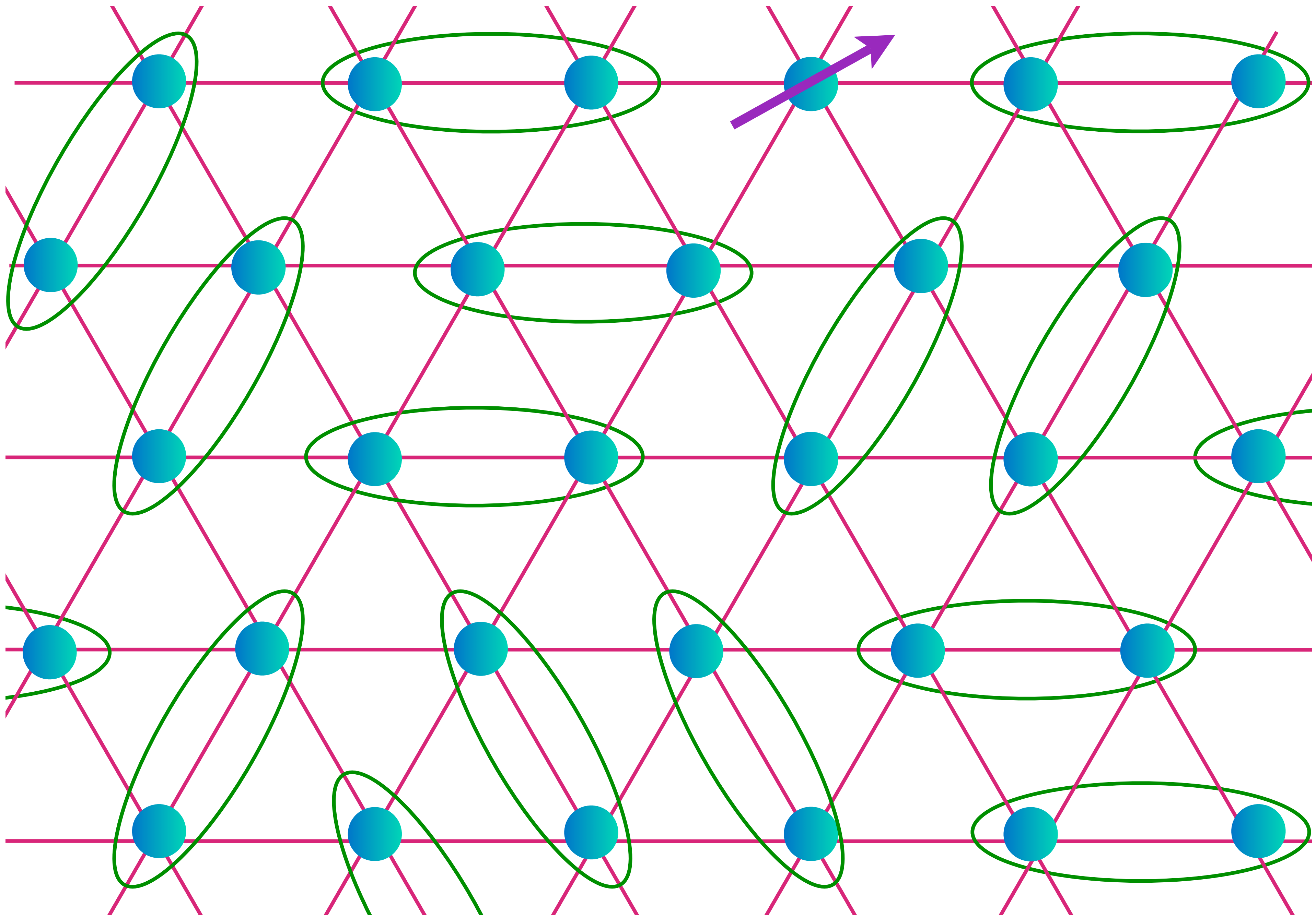

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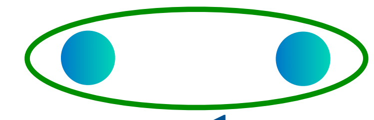

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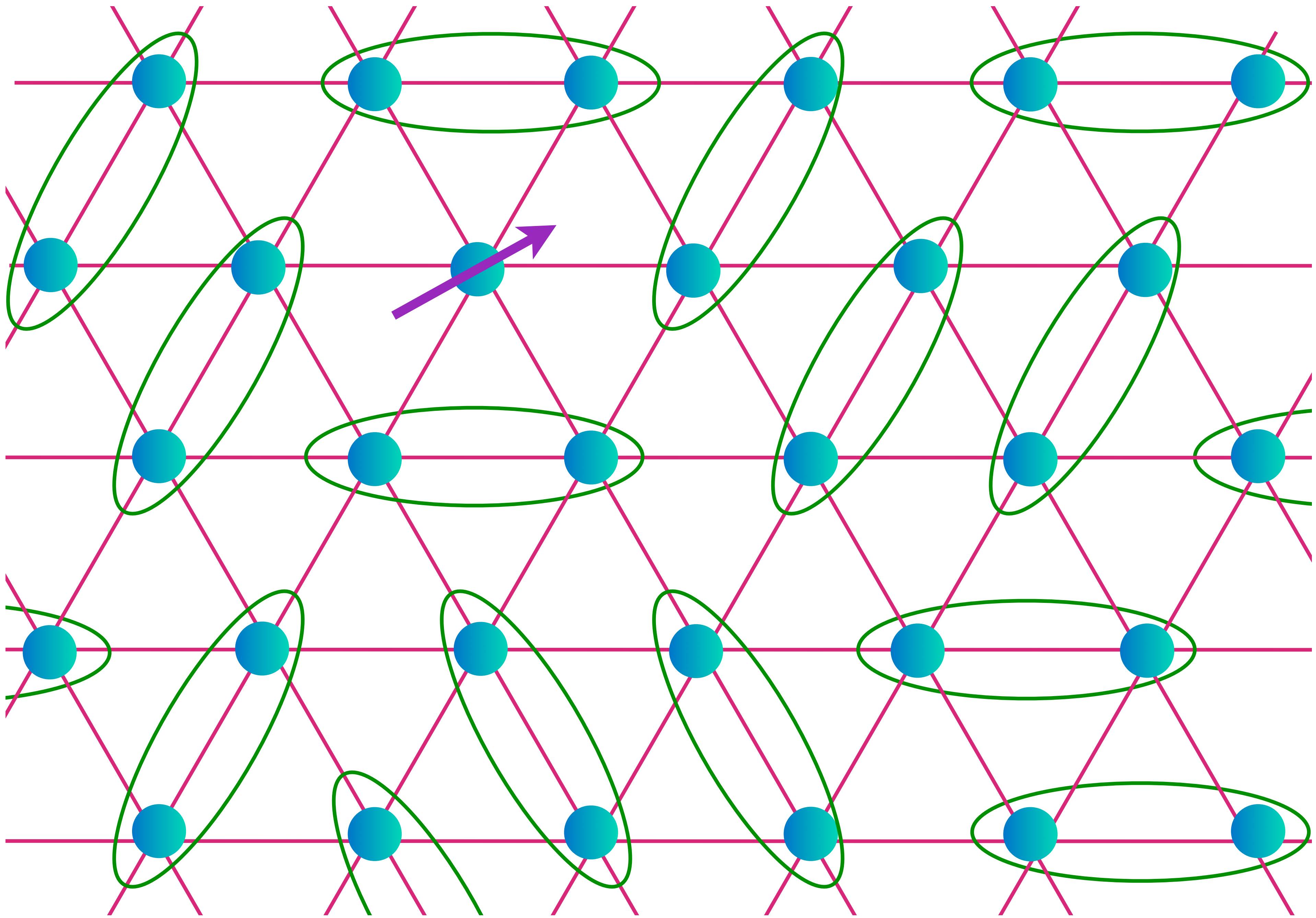

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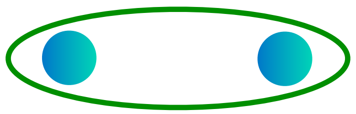

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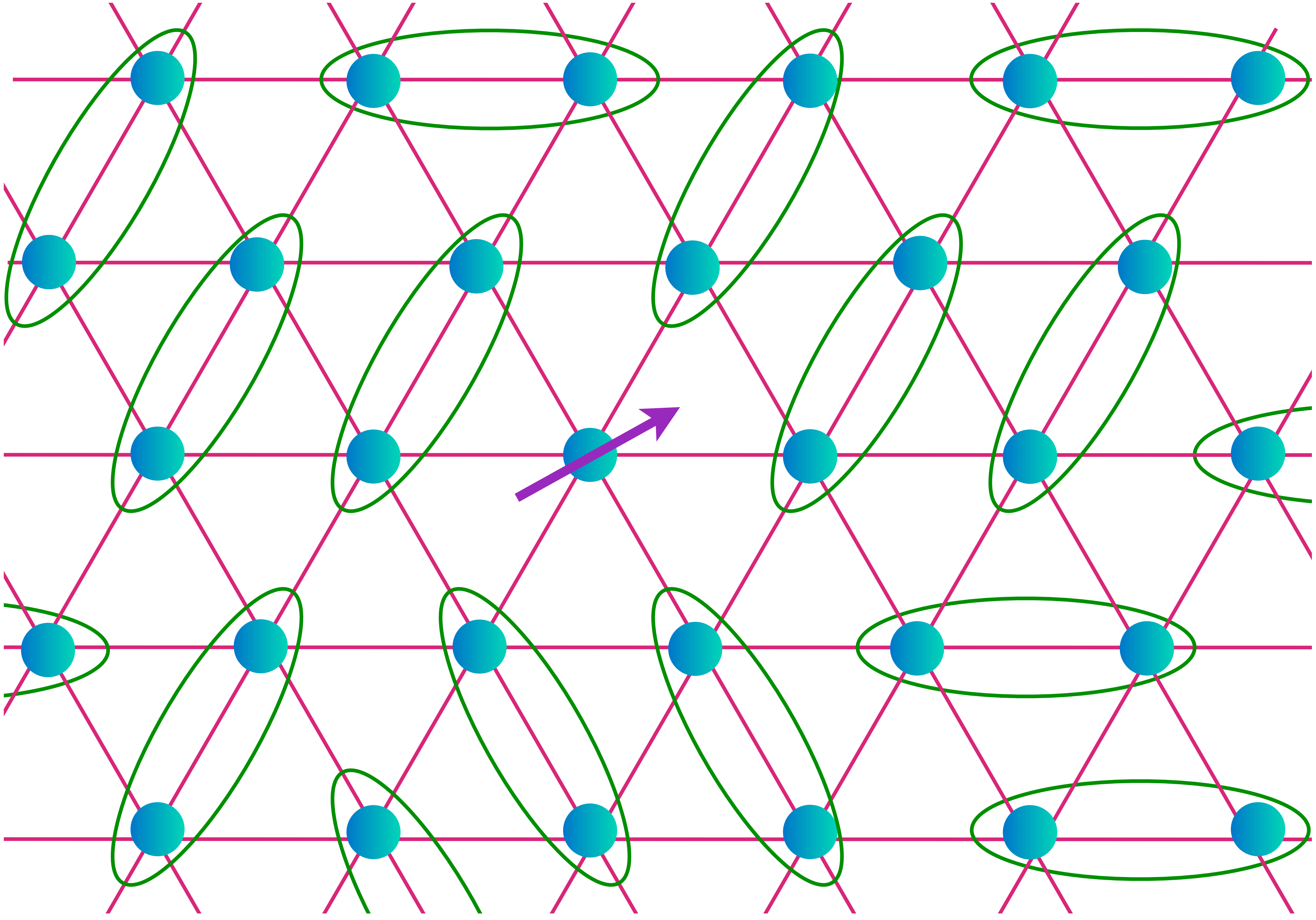

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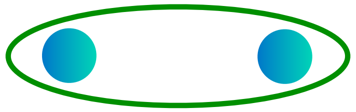

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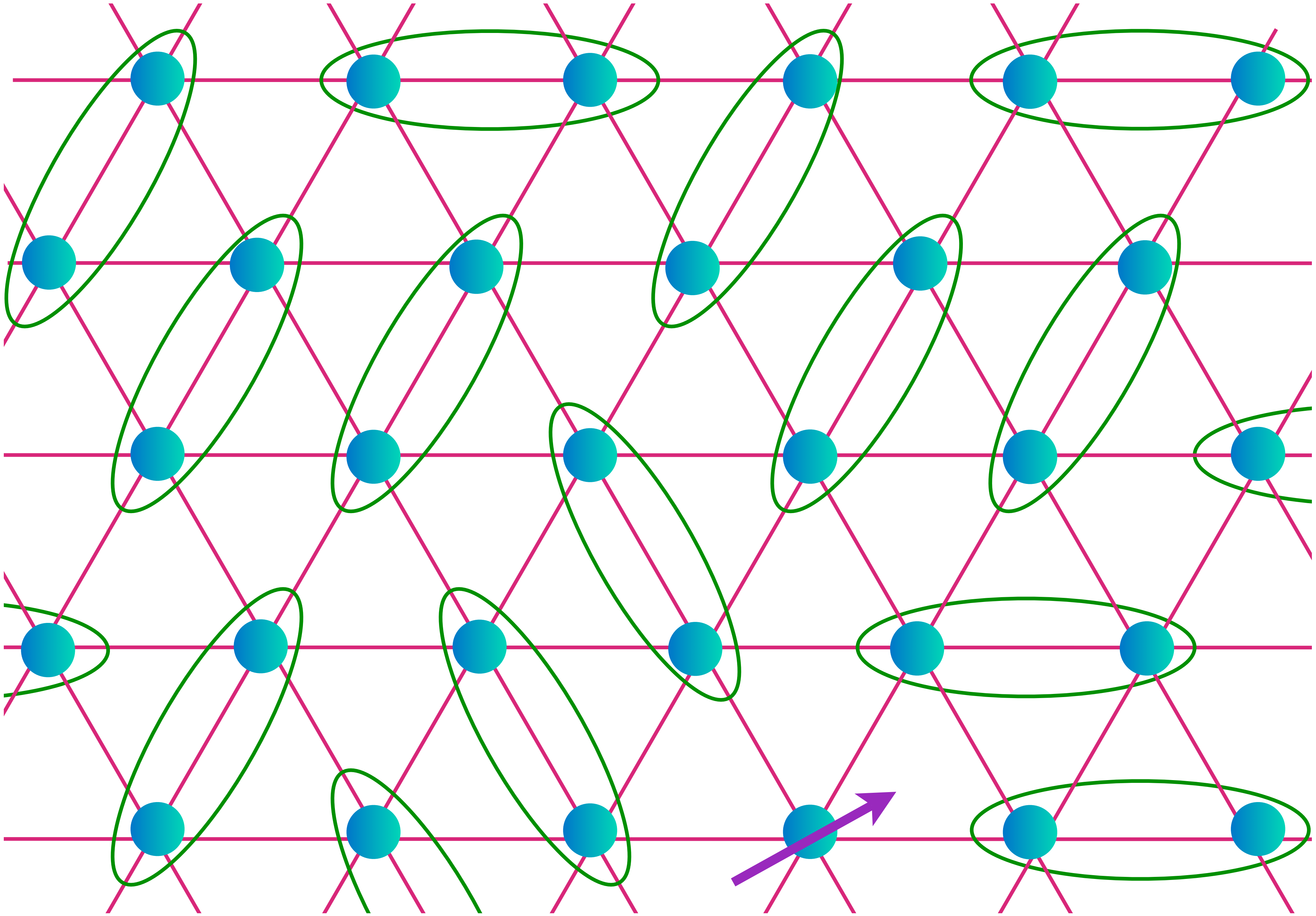

Excitations of the Z_2 Spin liquid

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The Néel order parameter, $\vec{\varphi}$ is a composite of the spinons:

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The theory for quantum phase transitions is expressed in terms of fluctuations of z_α, and *not* the order parameter $\vec{\varphi}$.

Effective theory for z_α must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \rightarrow e^{i\theta} z_{i\alpha}$$

Excitations of the Z_2 Spin liquid

A vison

- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate
- A vison is an Abrikosov vortex in the pair condensate of spinons
- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

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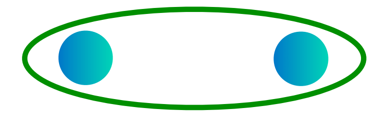

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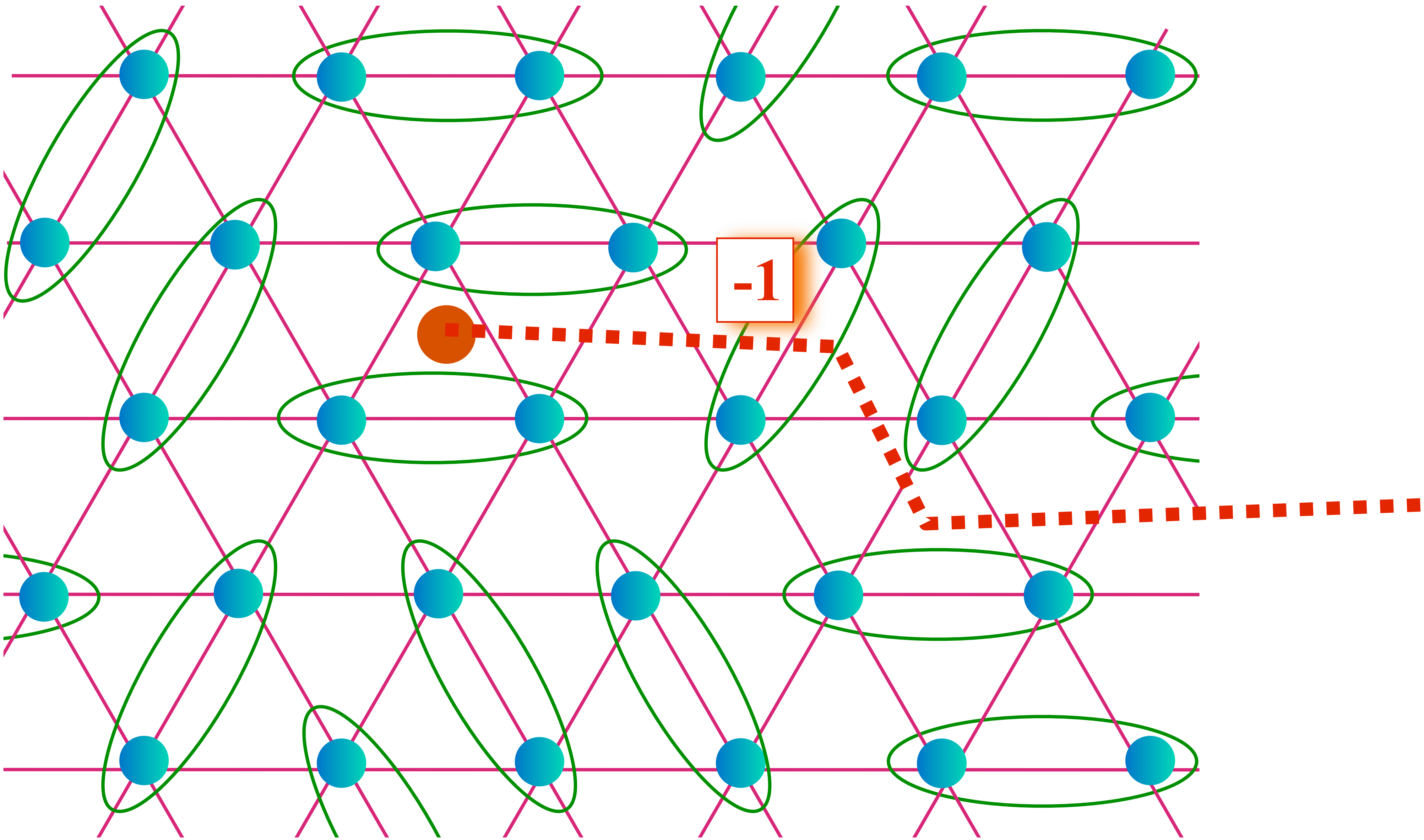

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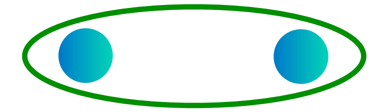

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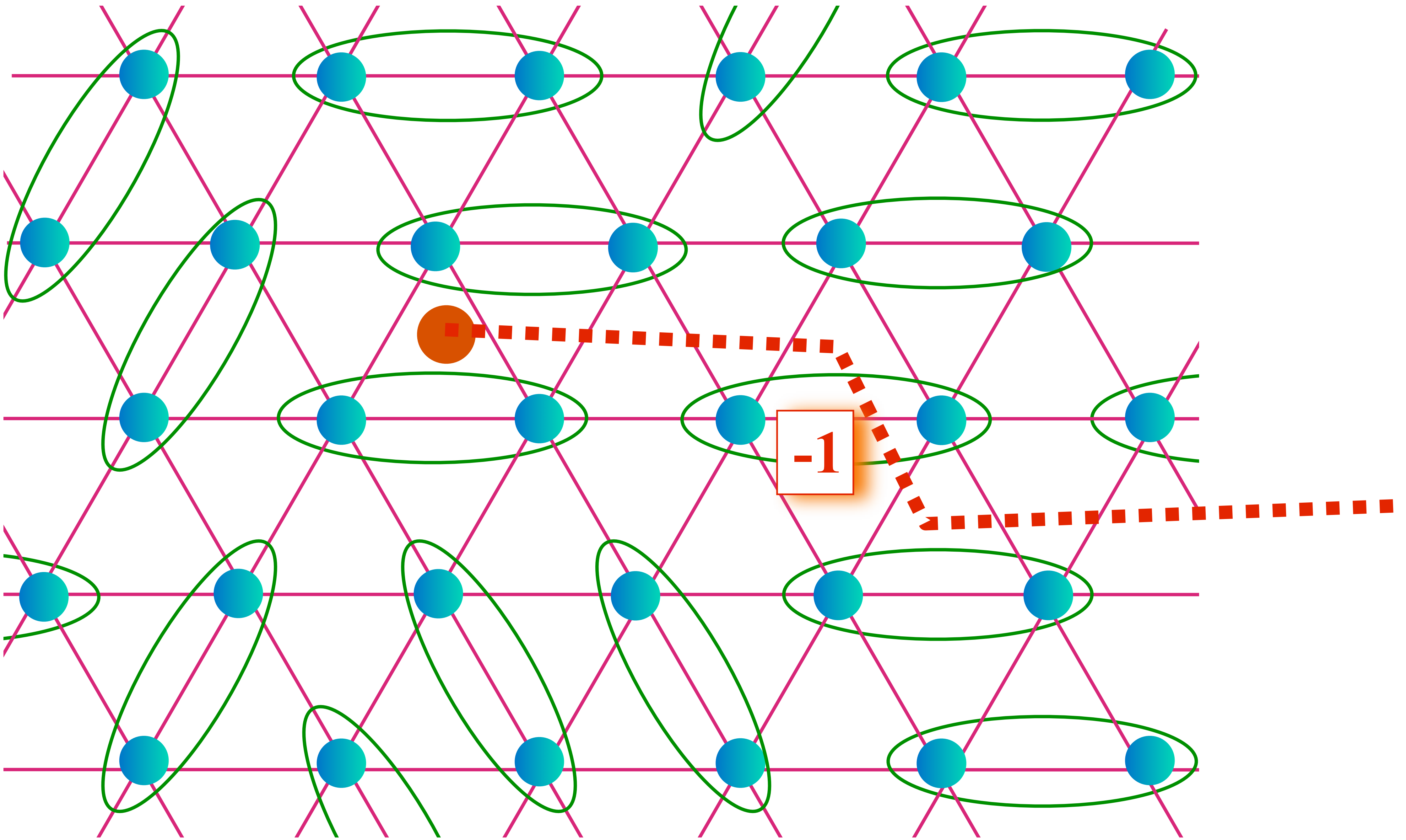

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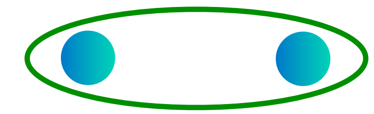

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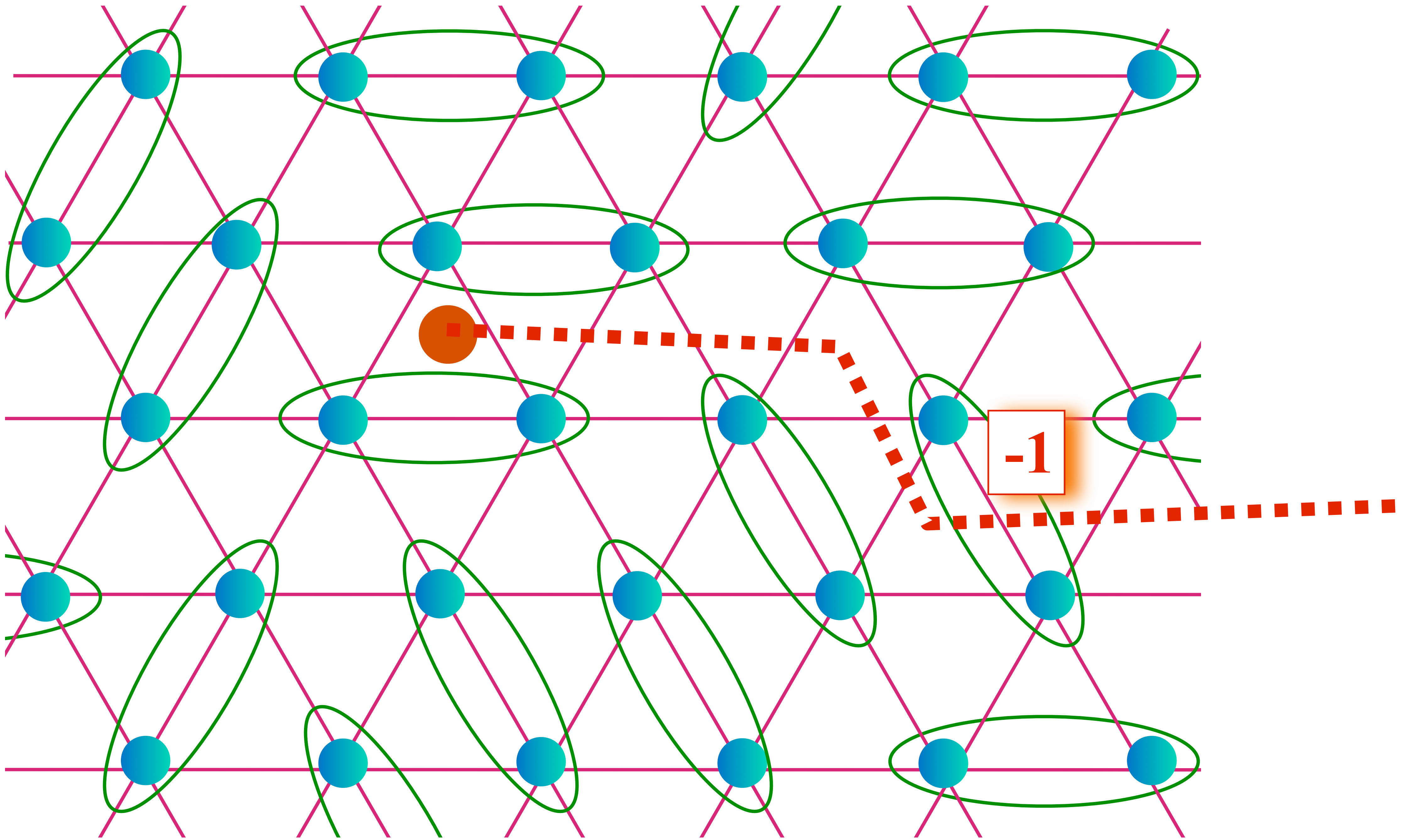

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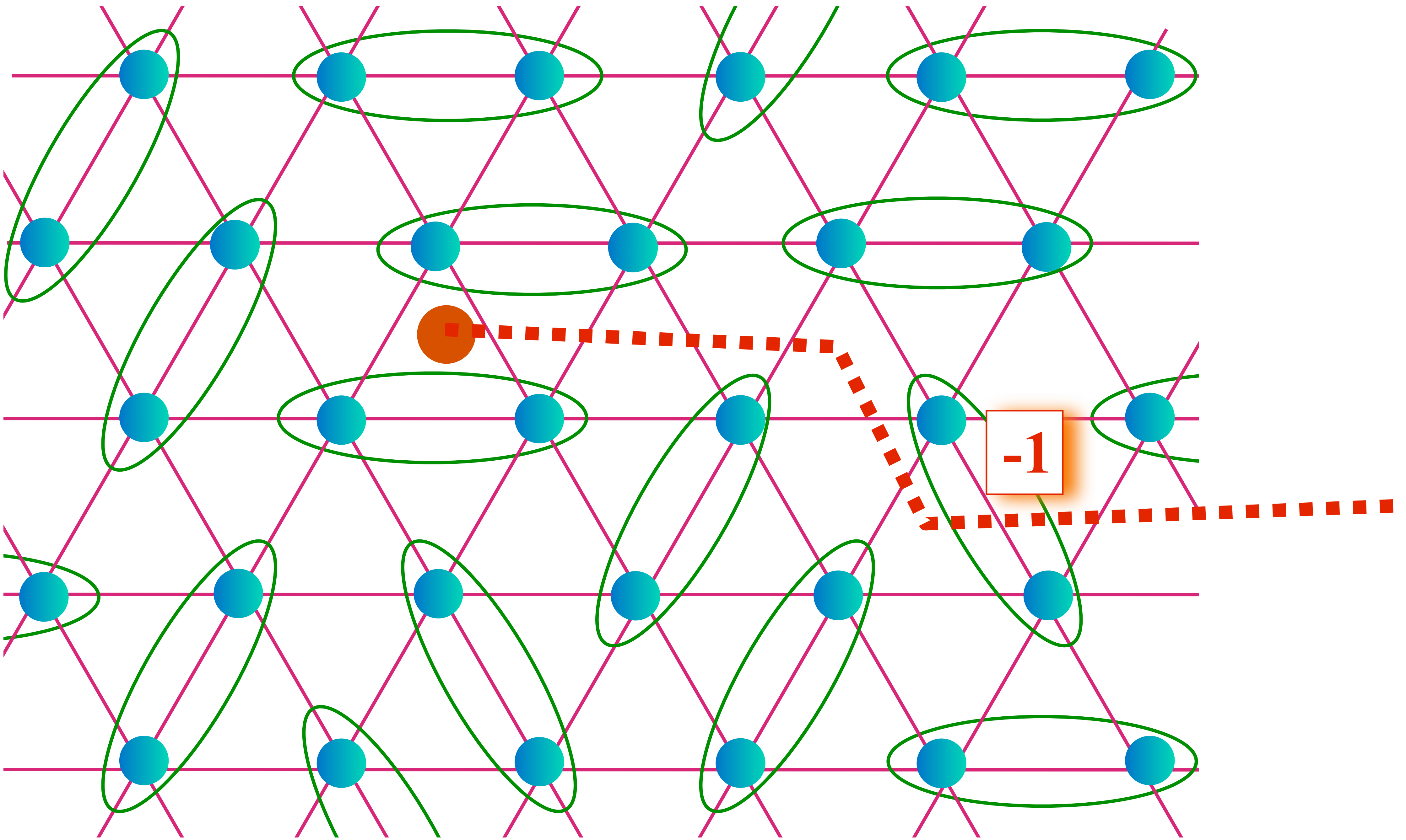

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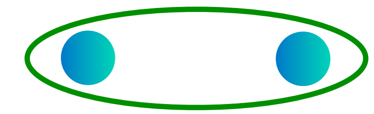

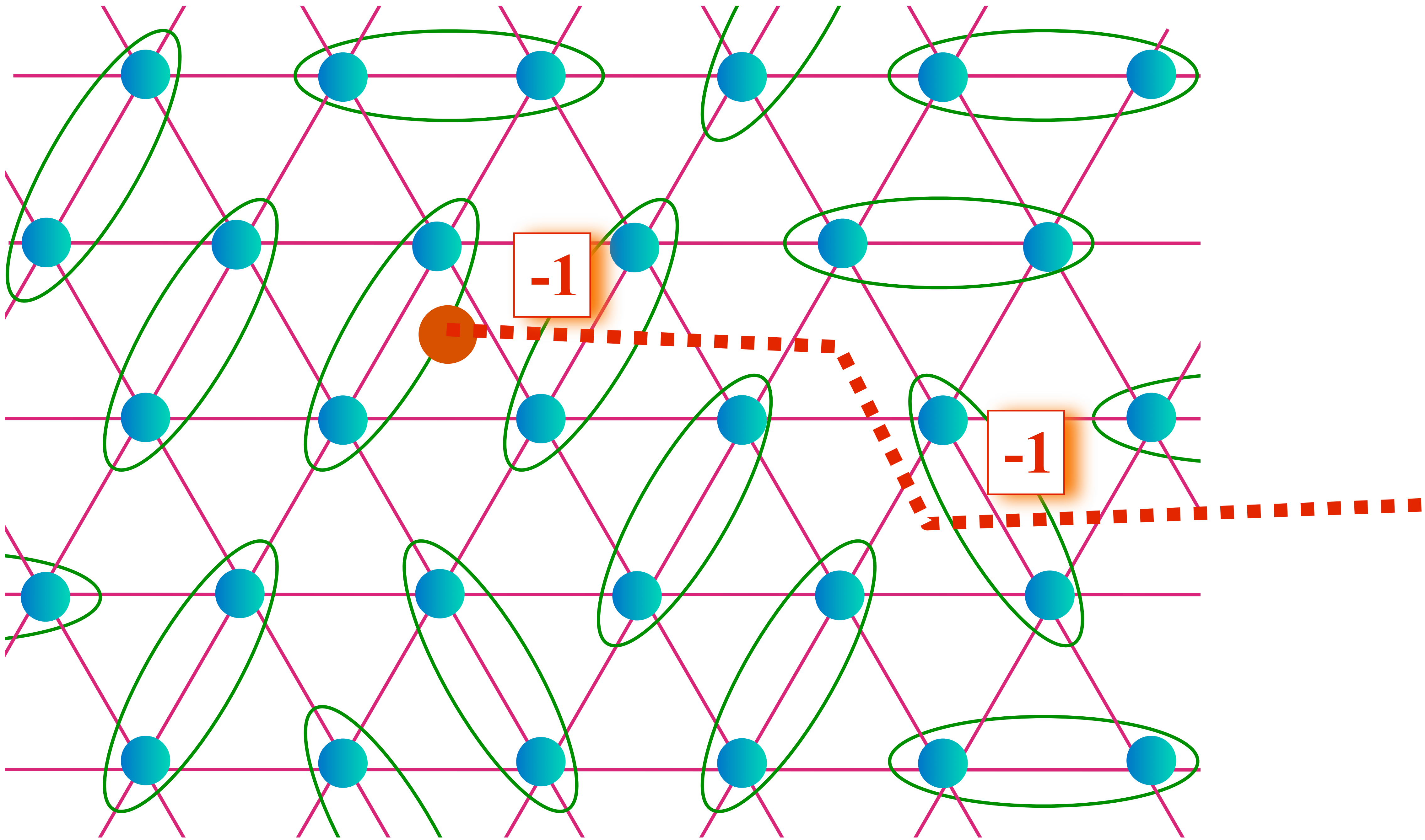

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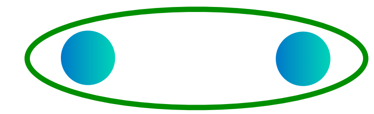

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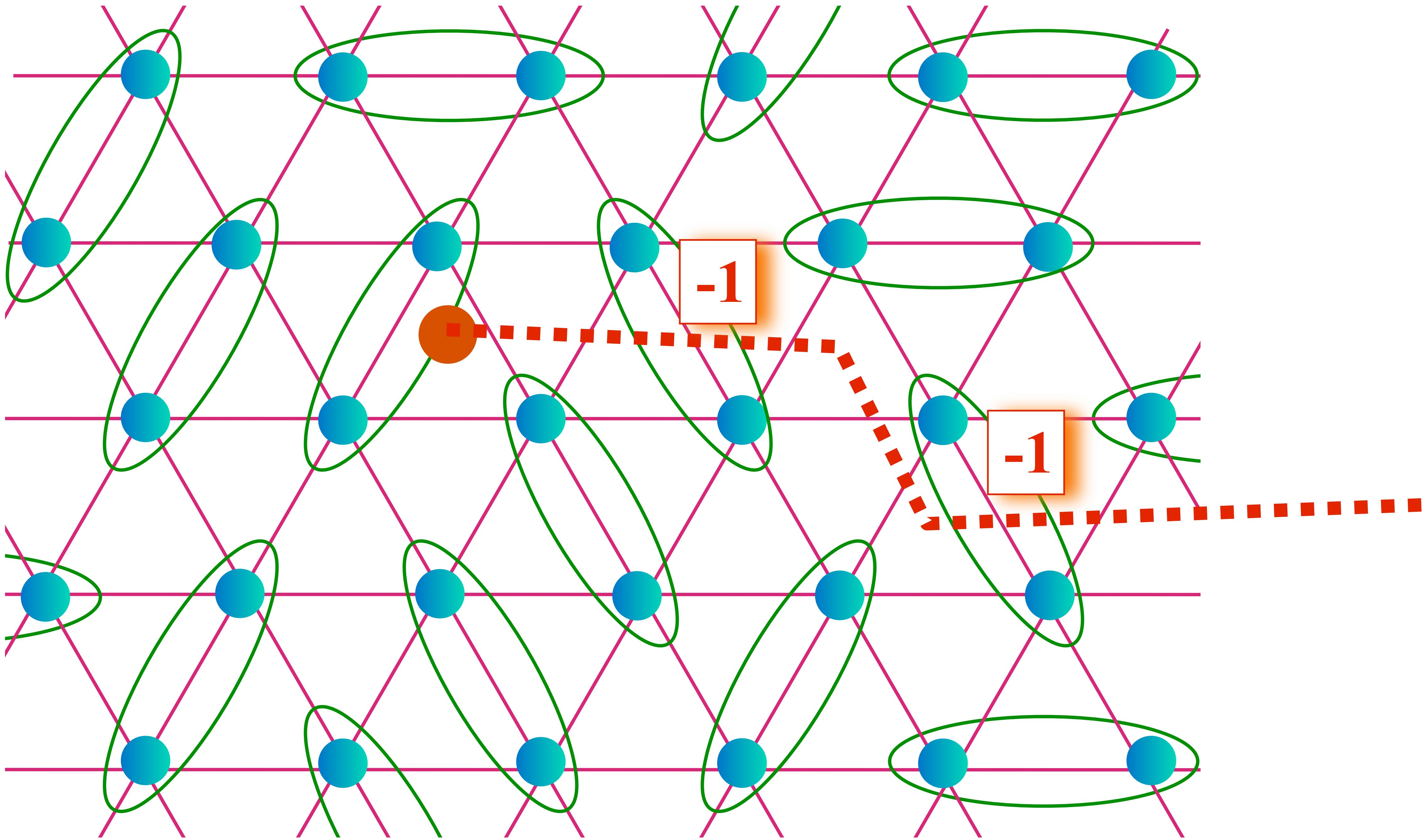

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A Simple Toy Model (A. Kitaev, 1997)

- Spins S_α living on the links of a square lattice:

$$H = -J_1 \sum_i A_i - J_2 \sum_p F_p$$

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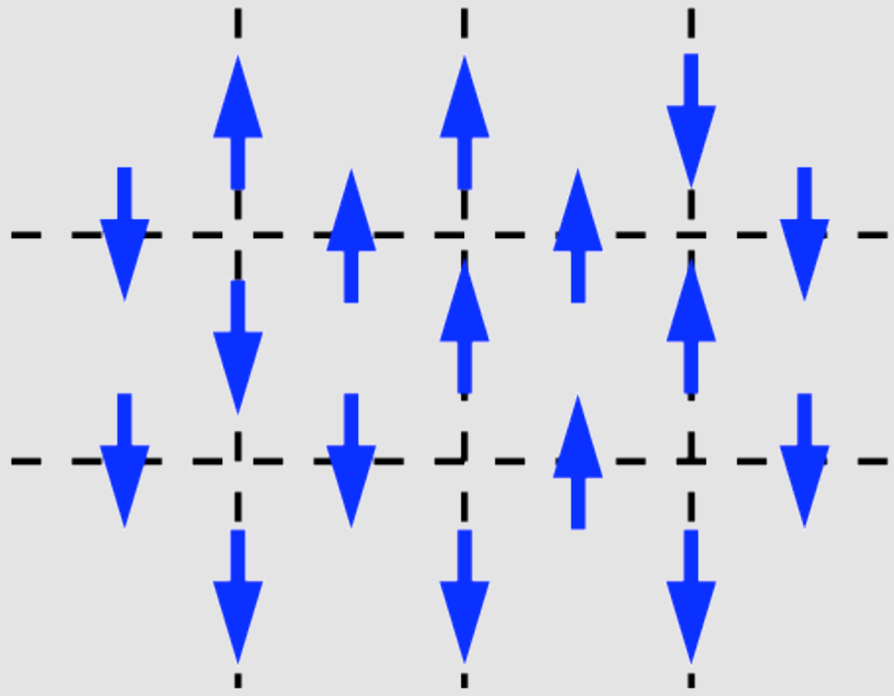

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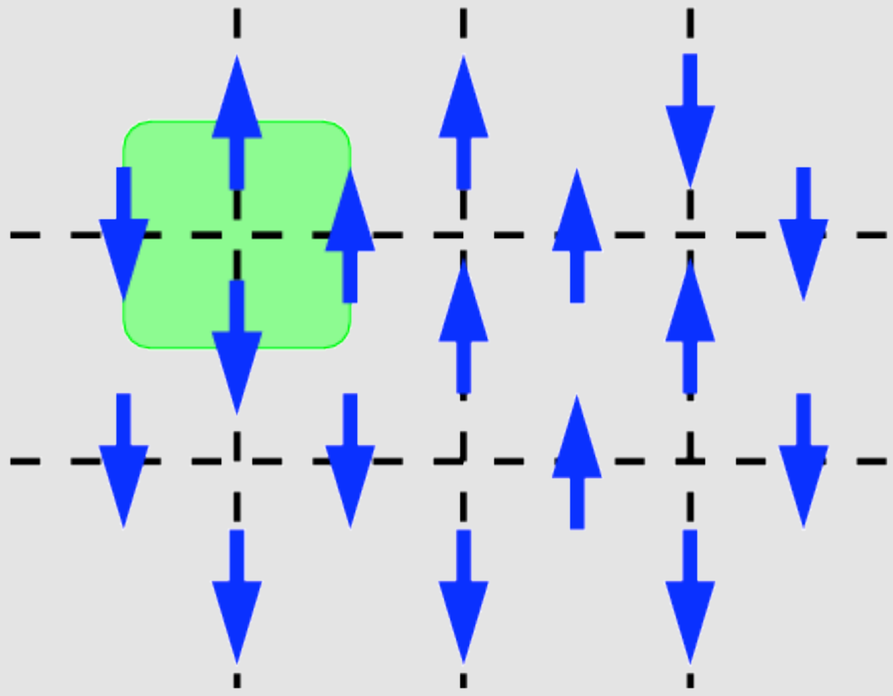

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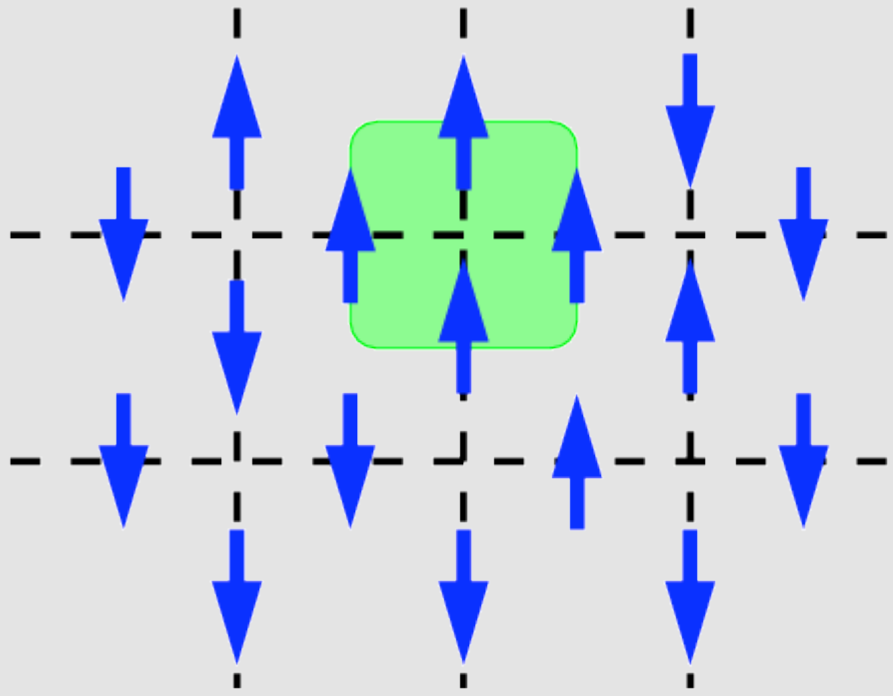

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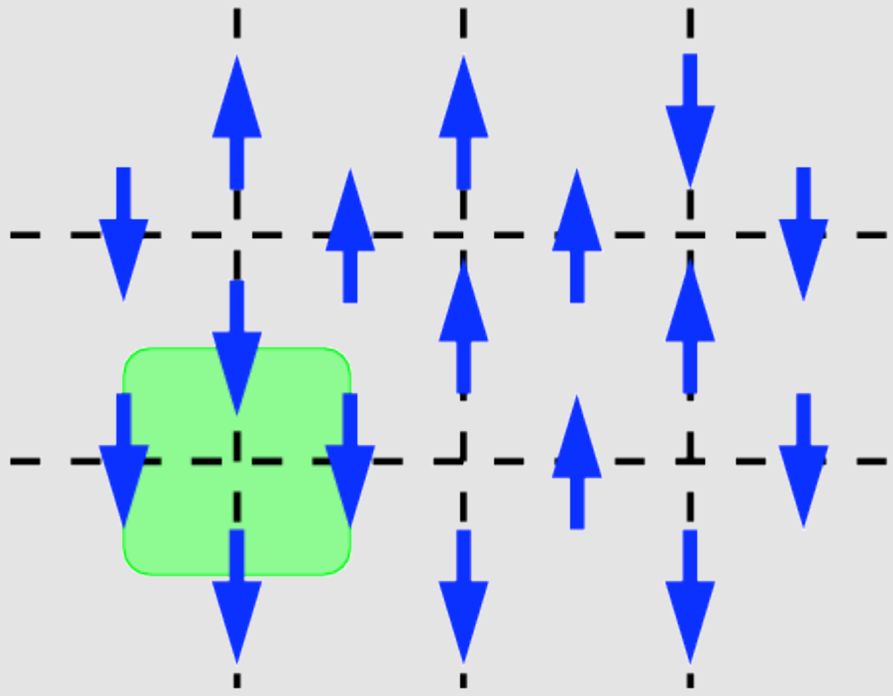

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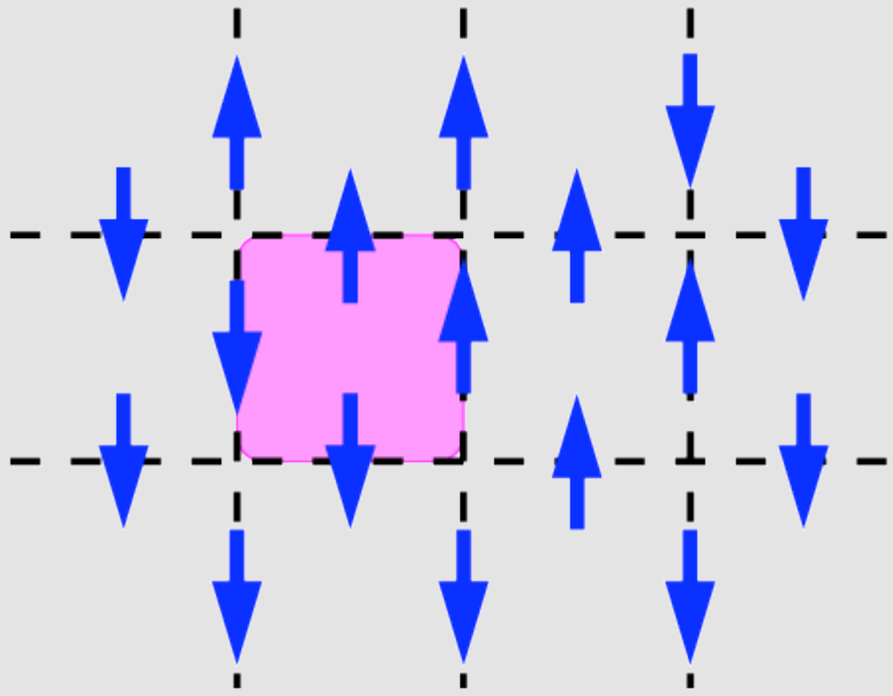

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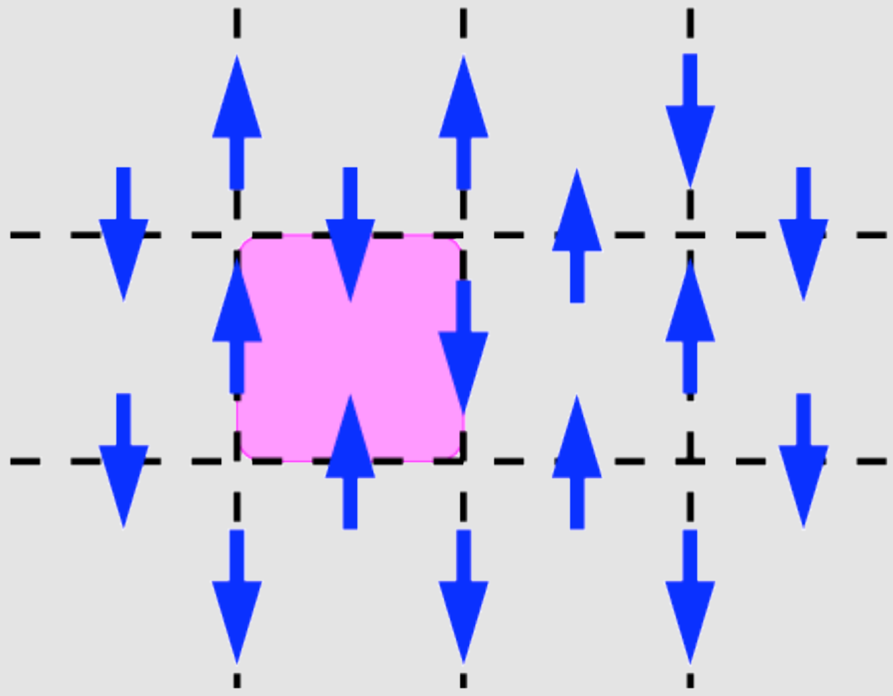

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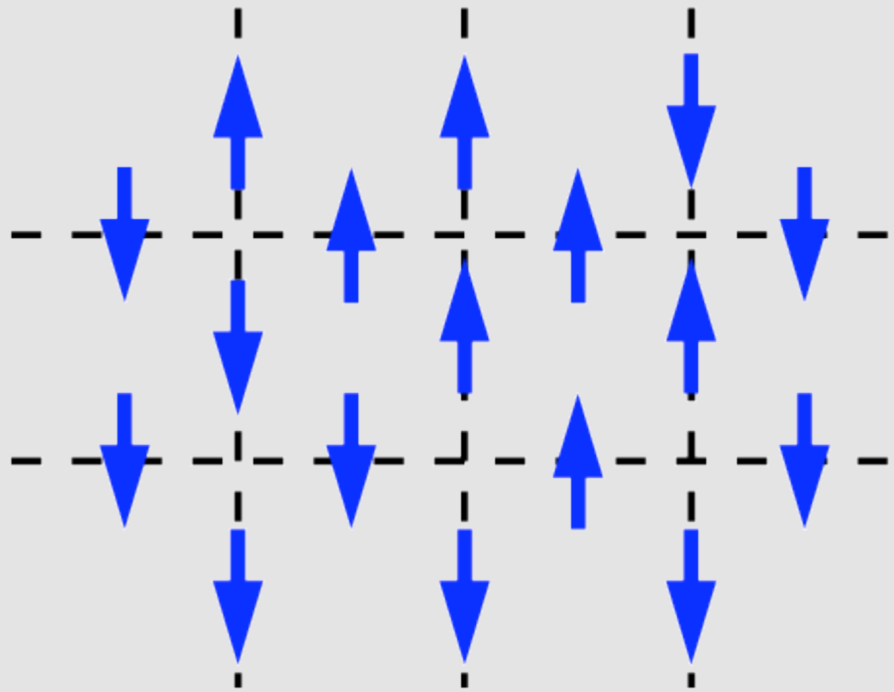

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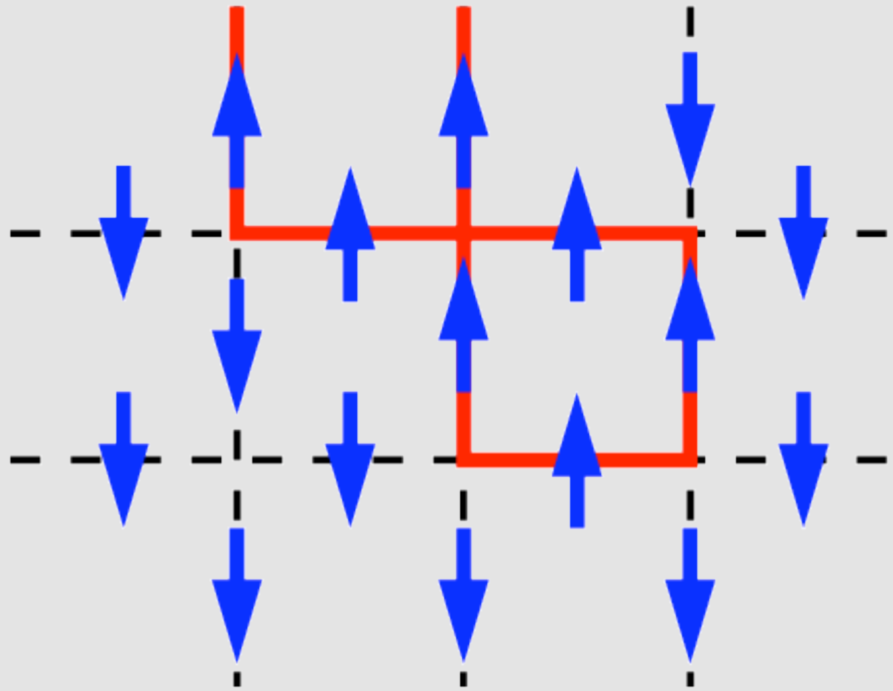

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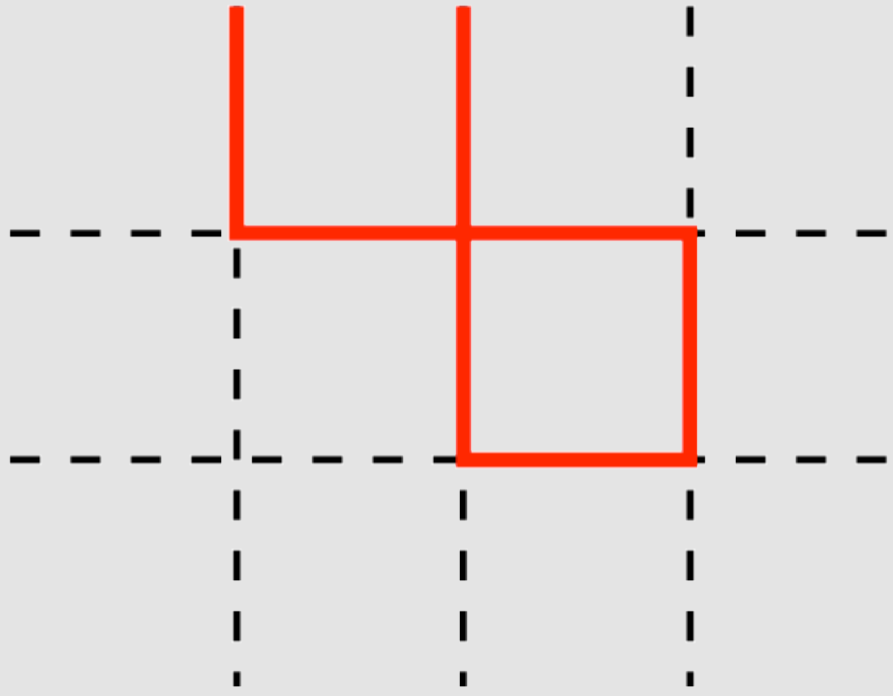

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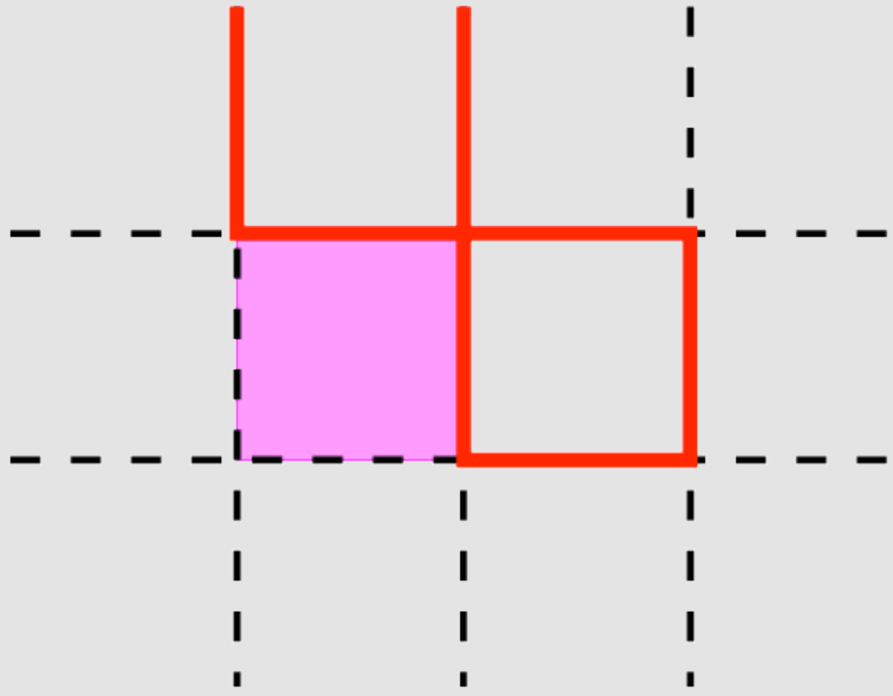

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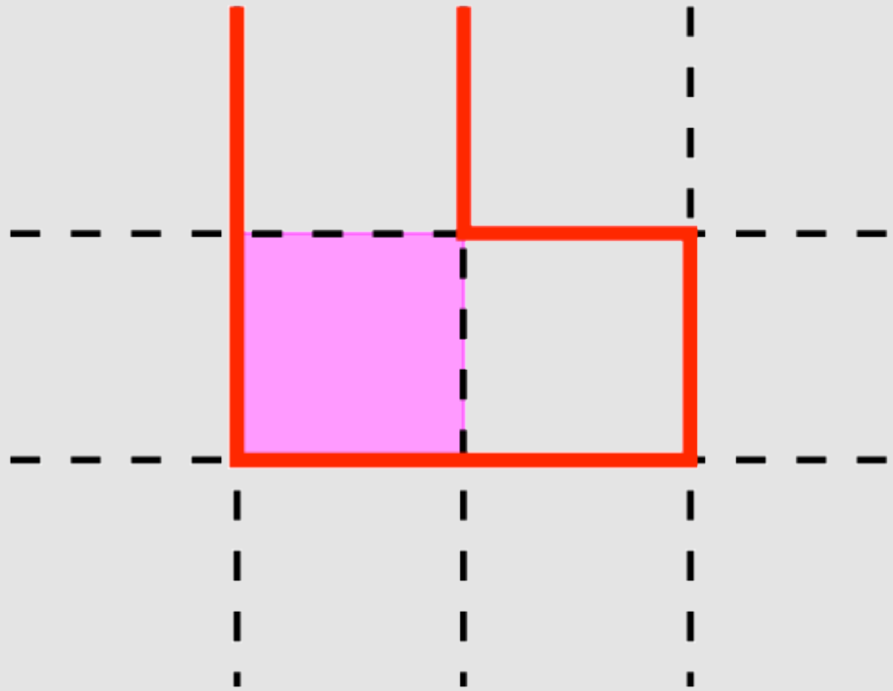

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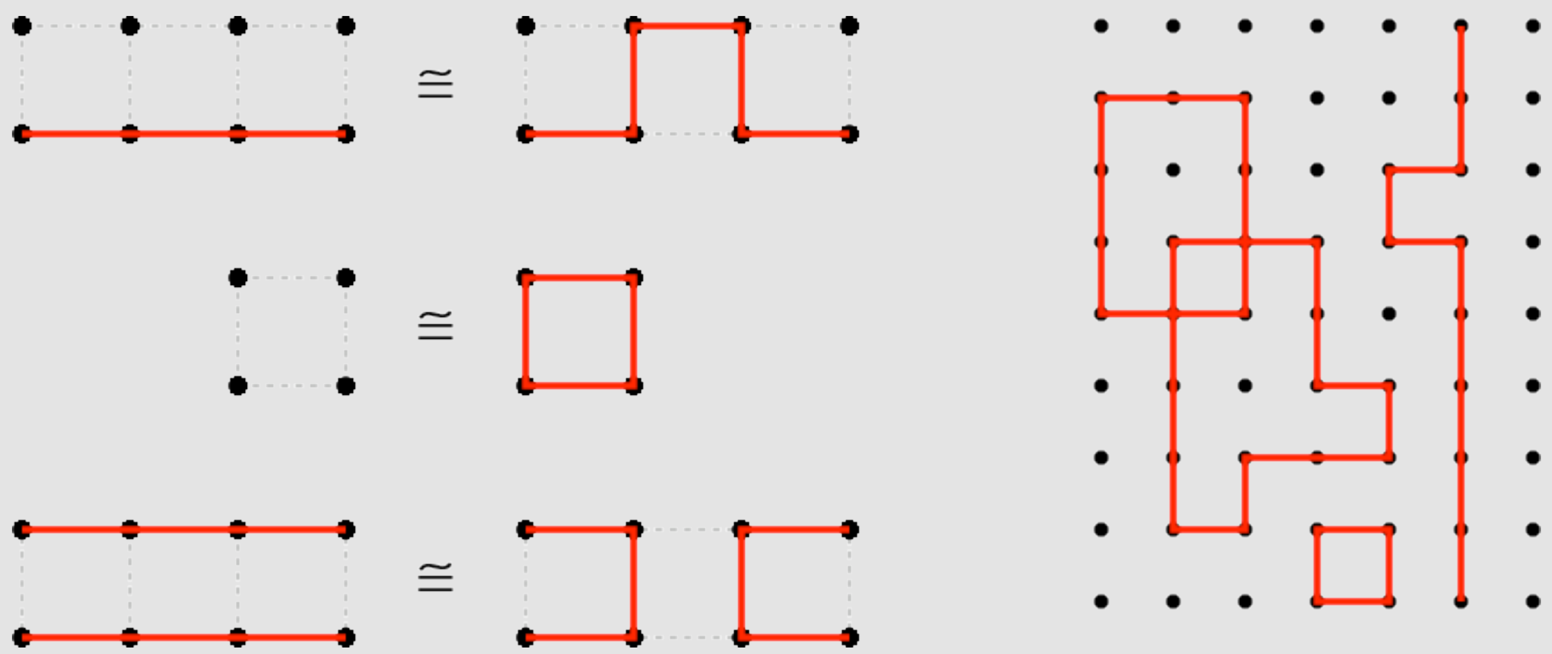
Properties of the Ground State

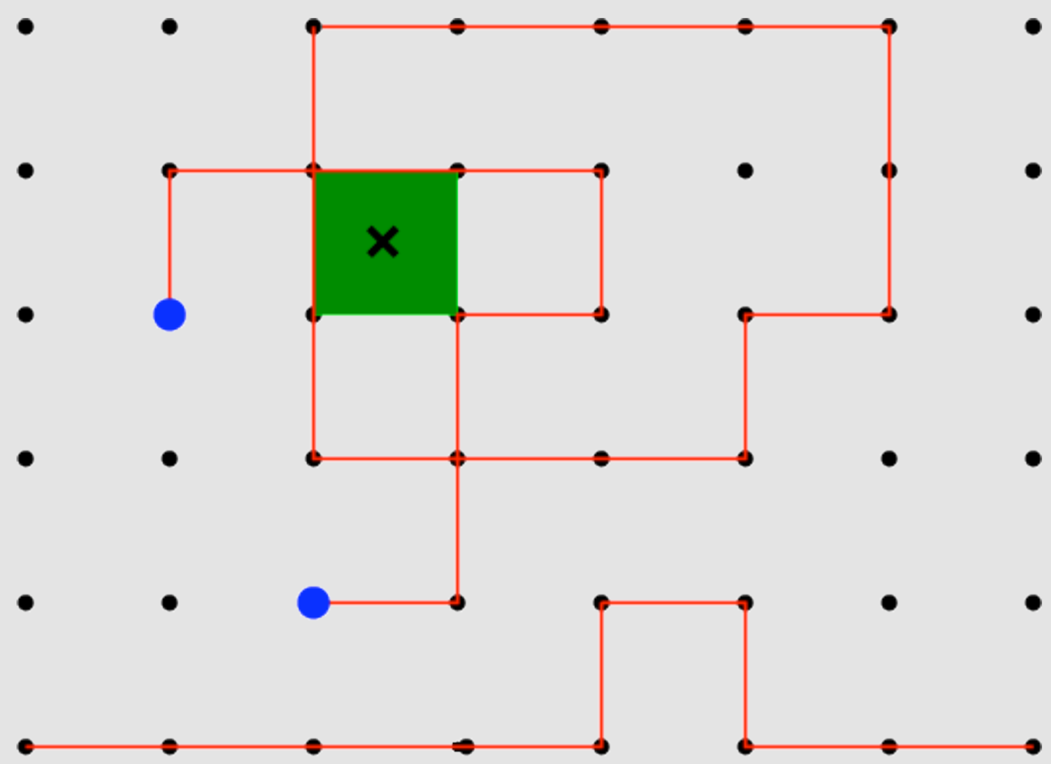
- Ground state: all $A_i=1$, $F_p=1$
- Pictorial representation: color each link with an up-spin.
- $A_i=1$: closed loops.
- $F_p=1$: every plaquette is an equal-amplitude superposition of inverse images.



The GS wavefunction takes the same value on configurations connected by these operations. It does not depend on the geometry of the configurations, only on their topology.

Properties of Excitations

- "Electric" particle, or $A_i = -1$ – endpoint of a line
- "Magnetic particle", or *vortex*: $F_p = -1$ – a "flip" of this plaquette changes the sign of a given term in the superposition.
- Charges and vortices interact via topological Aharonov-Bohm interactions.



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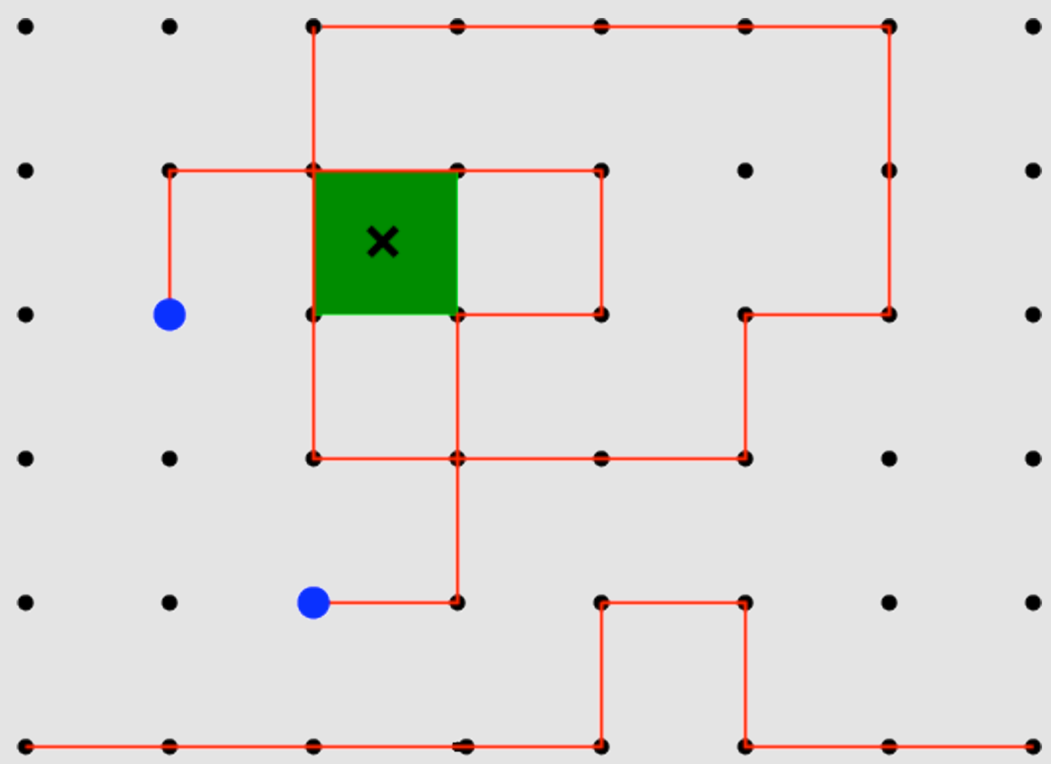

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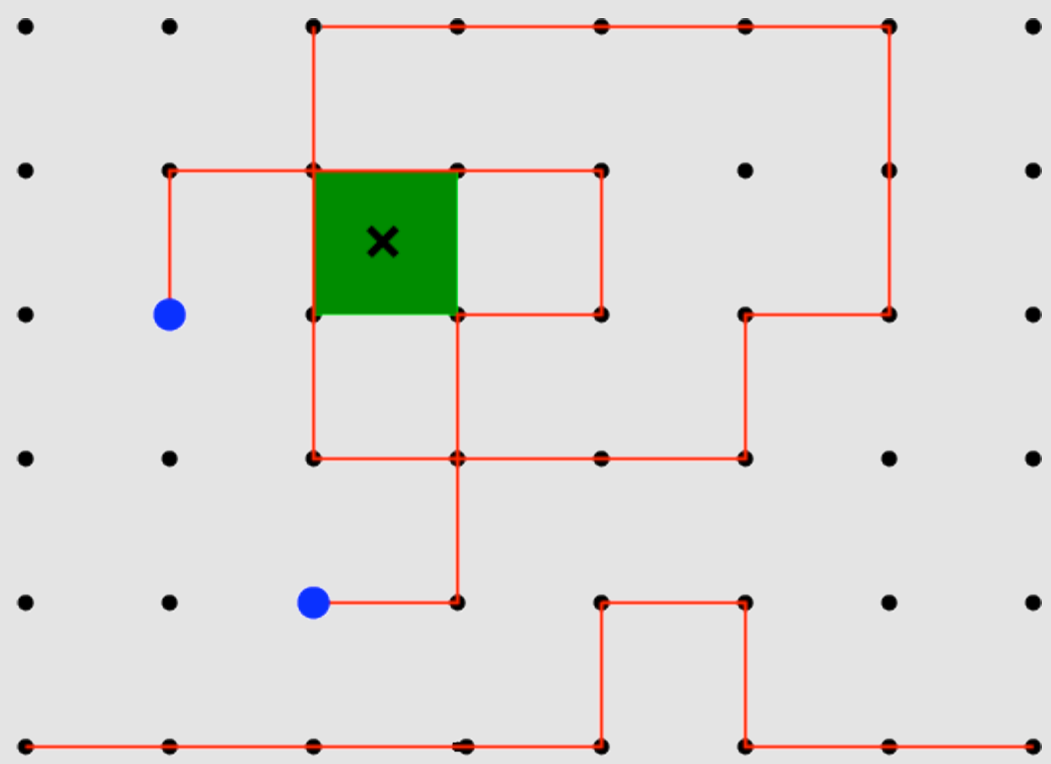

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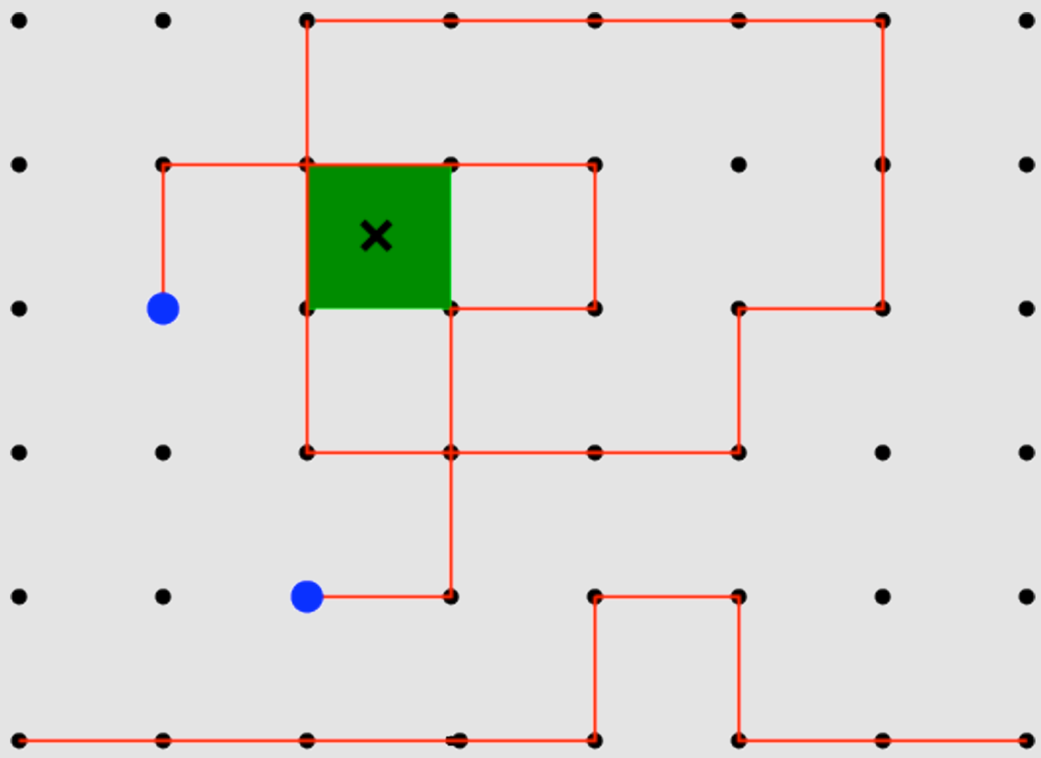


- *Spinons and visons are mutual semions*

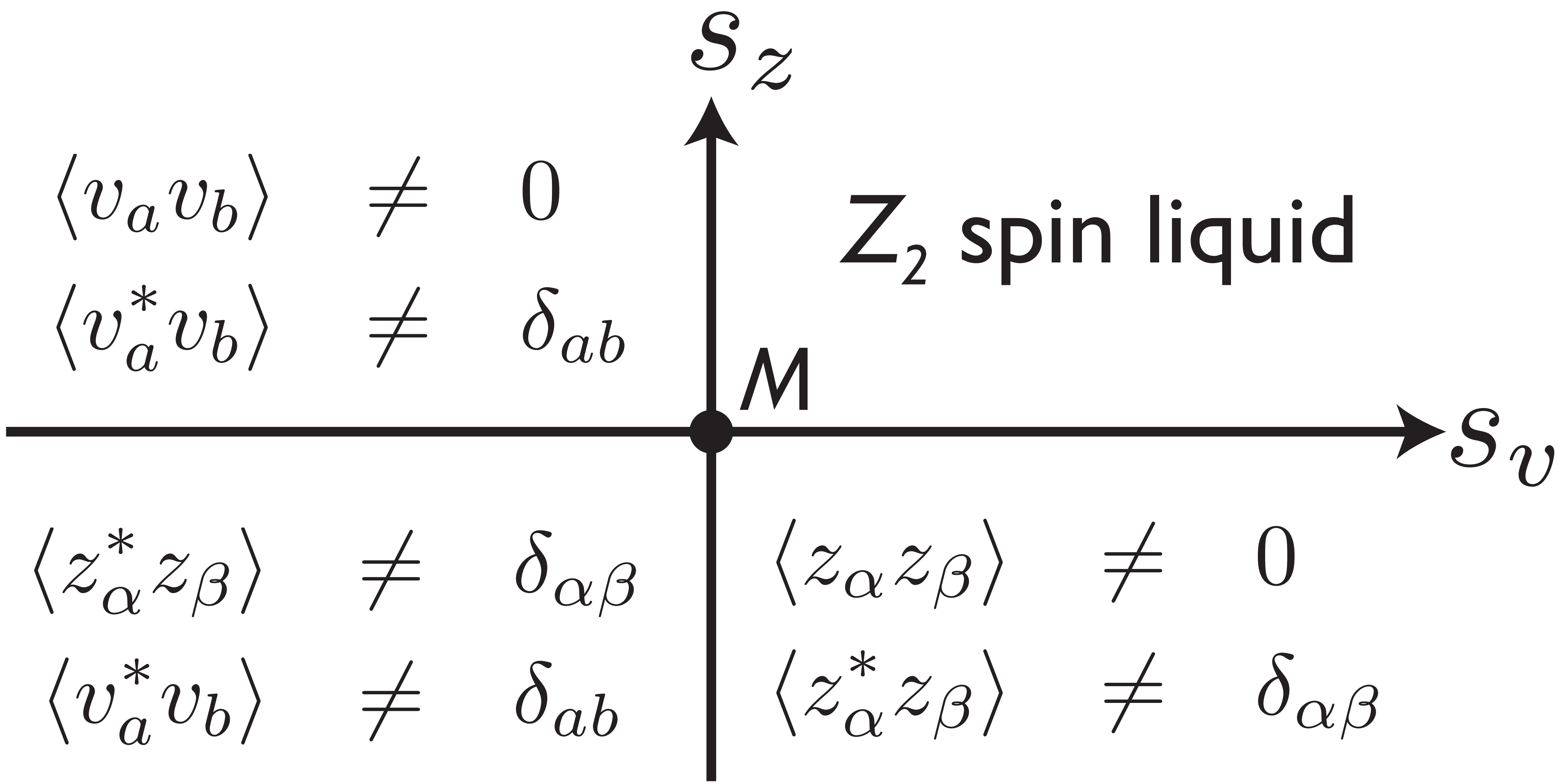
Mutual Chern-Simons Theory

Express theory in terms of the physical excitations of the Z_2 spin liquid: the spinons, z_α, and the visons. After accounting for Berry phase effects, the visons can be described by complex fields v_a, which transforms non-trivially under the square lattice space group operations.

A related Berry phase is the phase of -1 acquired by a spinon encircling a vortex. This is implemented in the following “mutual Chern-Simons” theory at $k = 2$:

$$\begin{aligned}\mathcal{L} &= \sum_{\alpha=1}^{2}\left\{|(\partial_\mu - ia_\mu)z_\alpha|^2 + s_z|z_\alpha|^2 + u_z(|z_\alpha|^2)^2\right\} \\ &+ \sum_{a=1}^{N_v}\left\{|(\partial_\mu - ib_\mu)v_a|^2 + s_v|v_a|^2 + u_v(|v_a|^2)^2\right\} \\ &+ \frac{ik}{2\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu b_\lambda + \cdots\end{aligned}$$

Cenke Xu and S. Sachdev, arXiv:0811.1220



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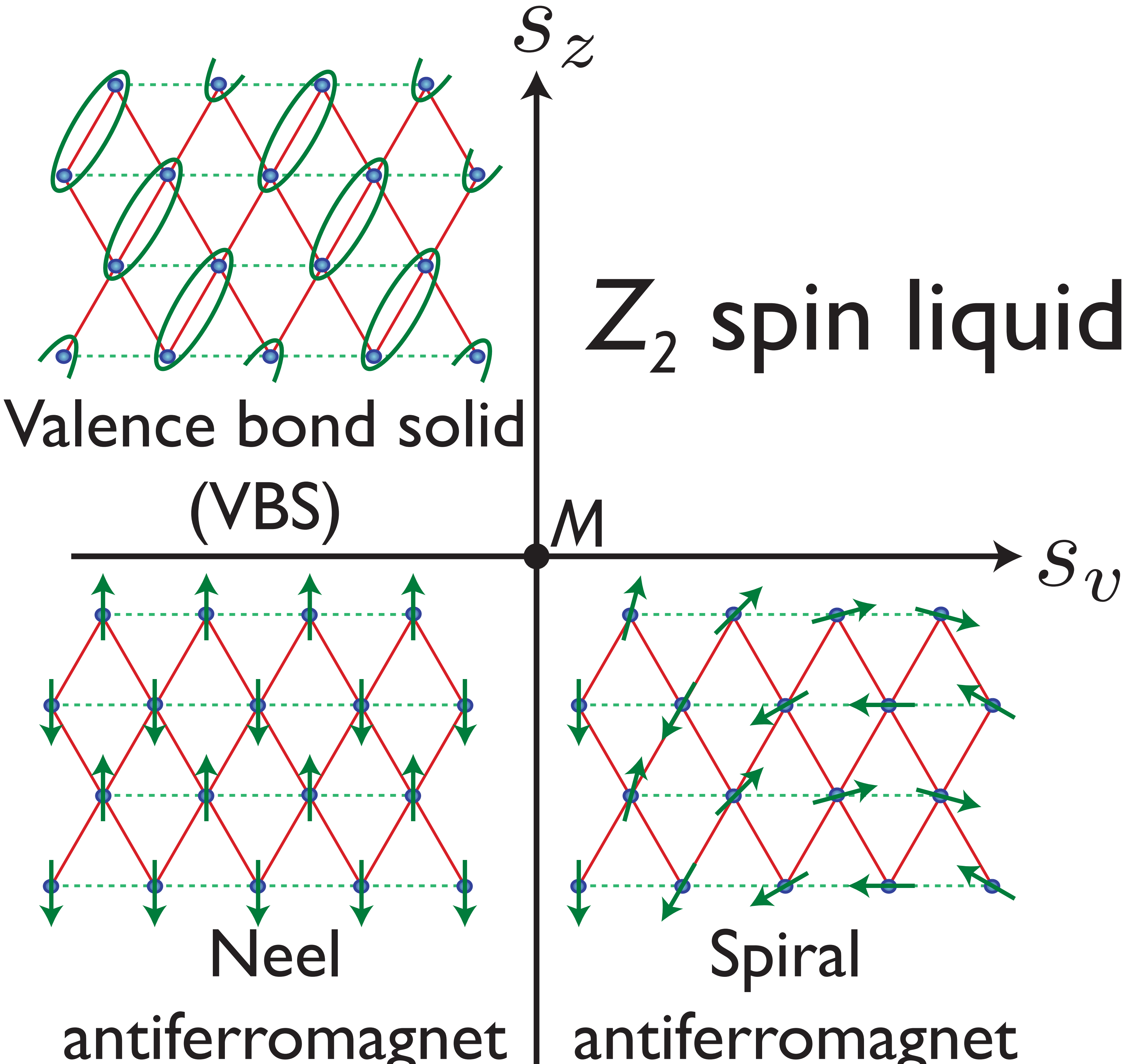
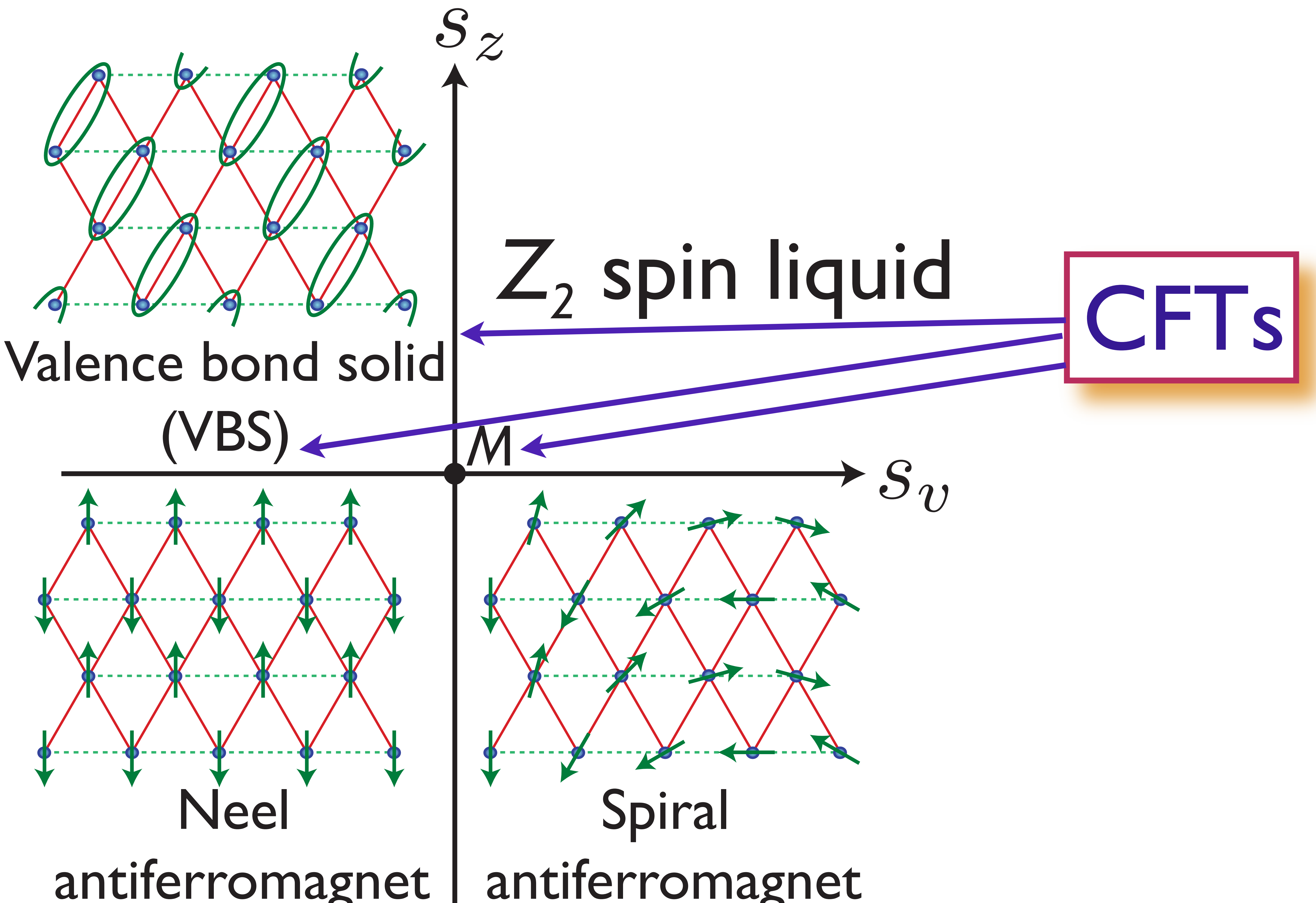
Theoretical global phase diagram



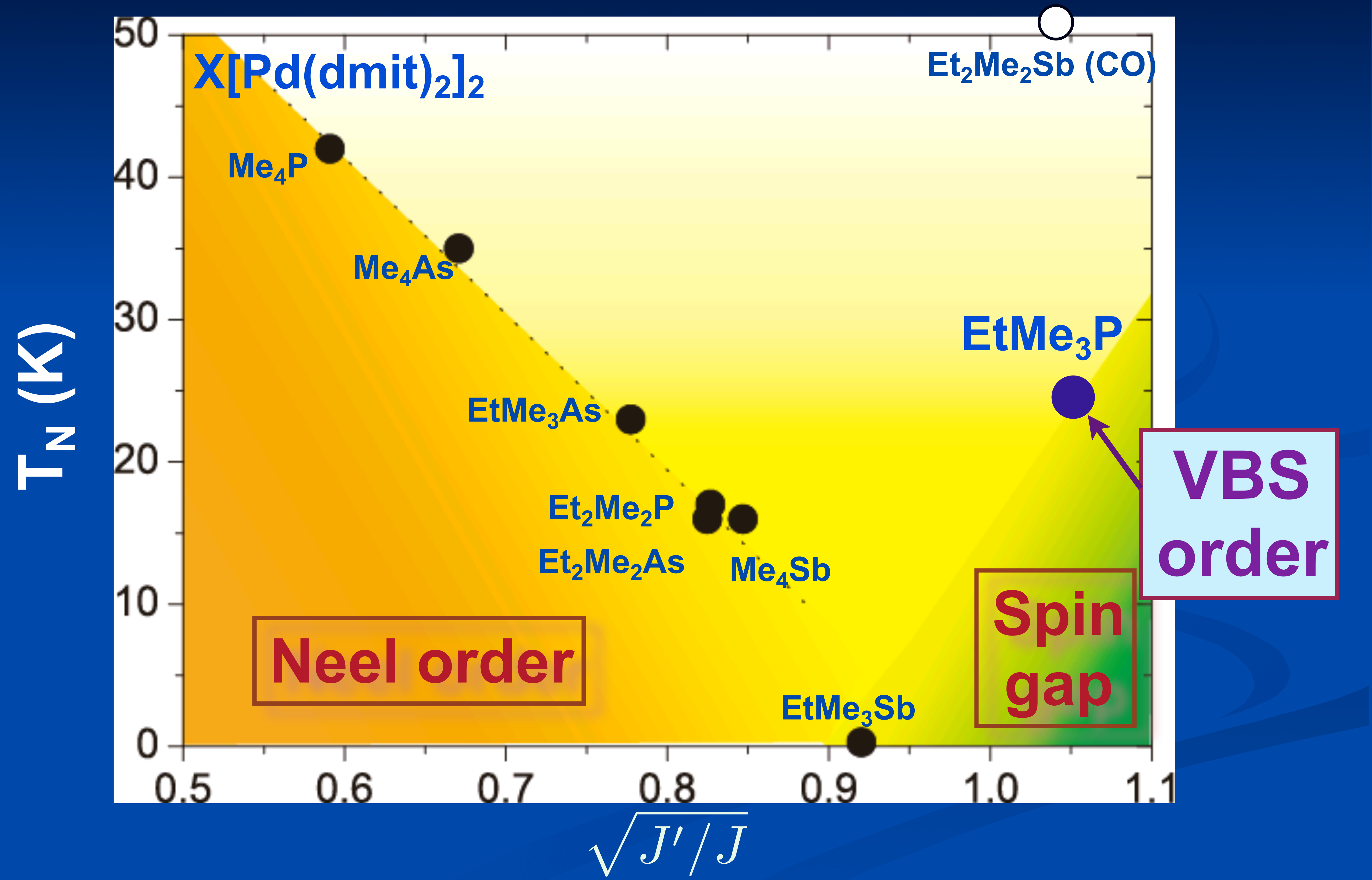
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).
Cenke Xu and S. Sachdev, arXiv:0811.1220

Theoretical global phase diagram



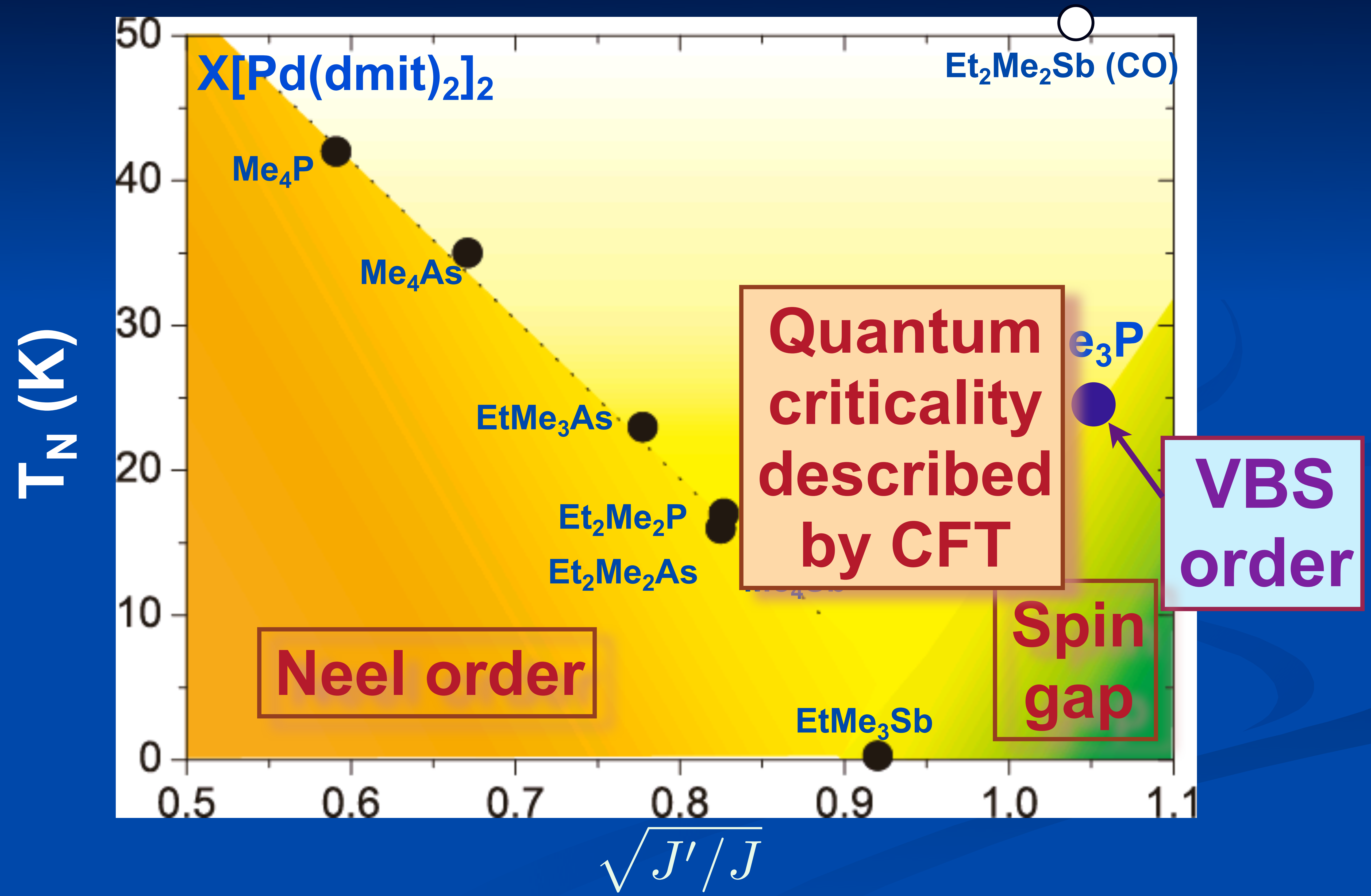
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).
Cenke Xu and S. Sachdev, arXiv:0811.1220

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

From quantum antiferromagnets to string theory

A direct generalization of the CFT of the multicritical point M ($s_z = s_v = 0$) to $\mathcal{N} = 4$ supersymmetry and the U(N) gauge group was shown by O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, JHEP **0810**, 091 (2008) to be dual to a theory of quantum gravity (M theory) on $\text{AdS}_4 \times \text{S}_7/Z_k$.

$$
\begin{aligned}
\mathcal{L} &= \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\
&+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - i b_\mu) v_a|^2 + s_v |v_a|^2 + u_v (|v_a|^2)^2 \right\} \\
&+ \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \cdots
\end{aligned}
$$

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

Black holes

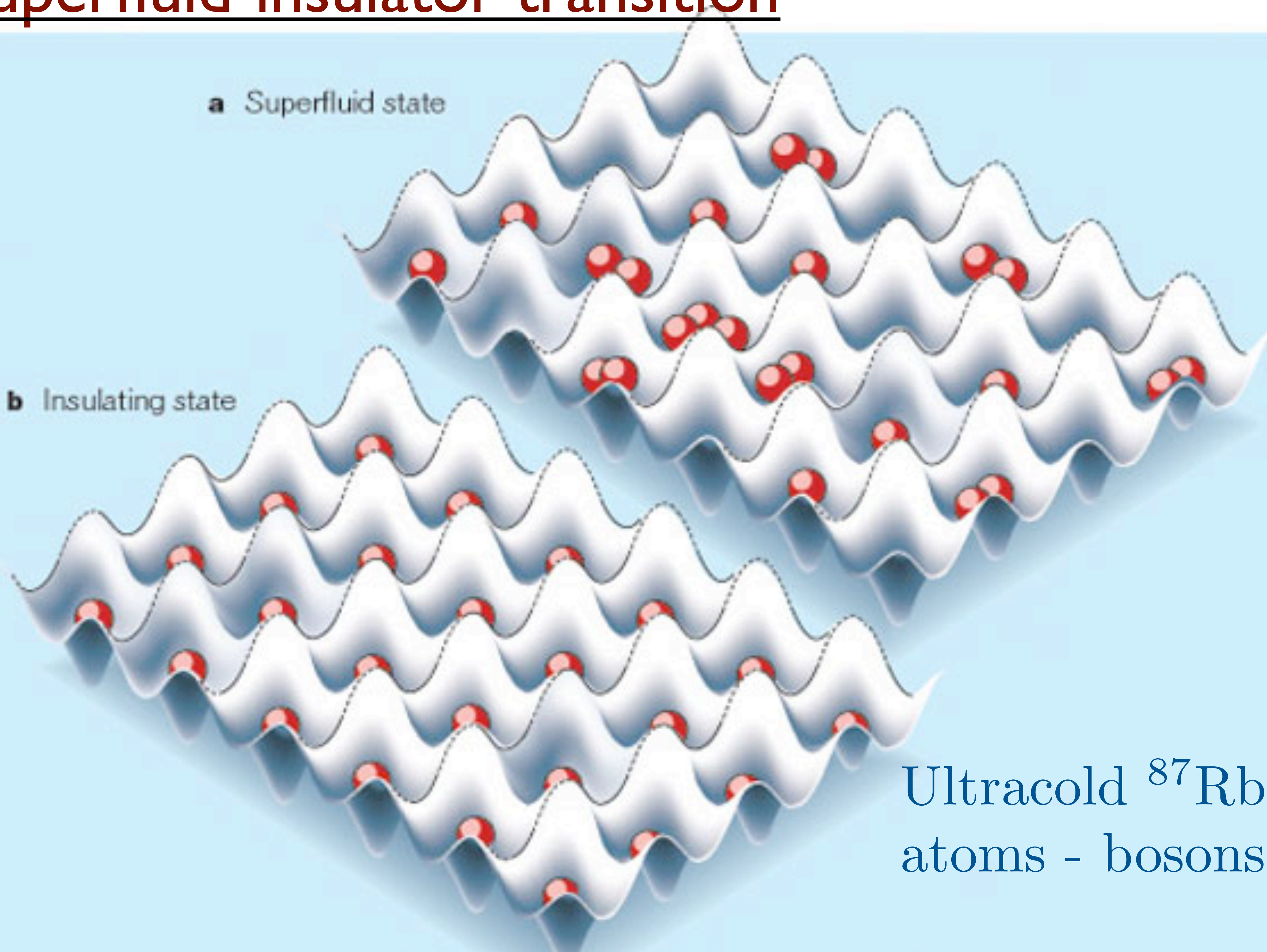
Three foci of modern physics

Quantum phase transitions → Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

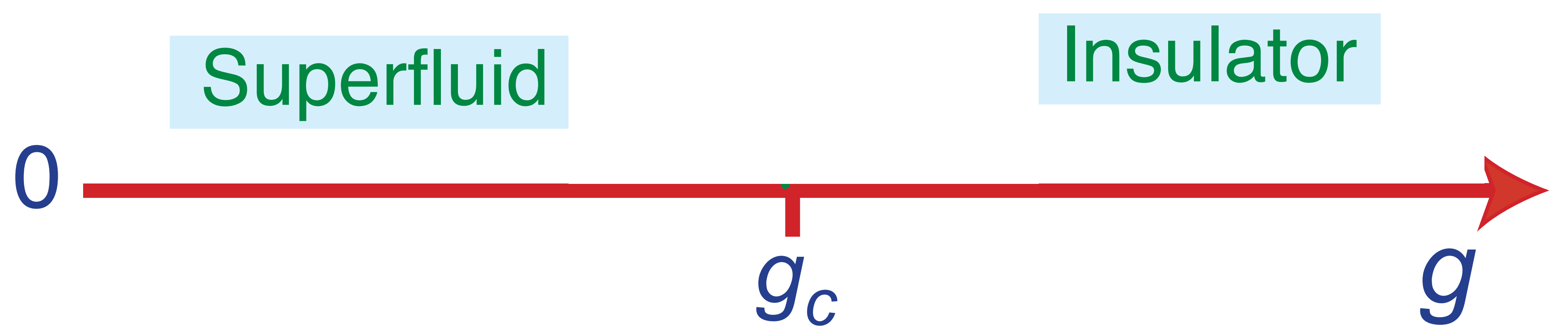
Black holes

Superfluid-insulator transition

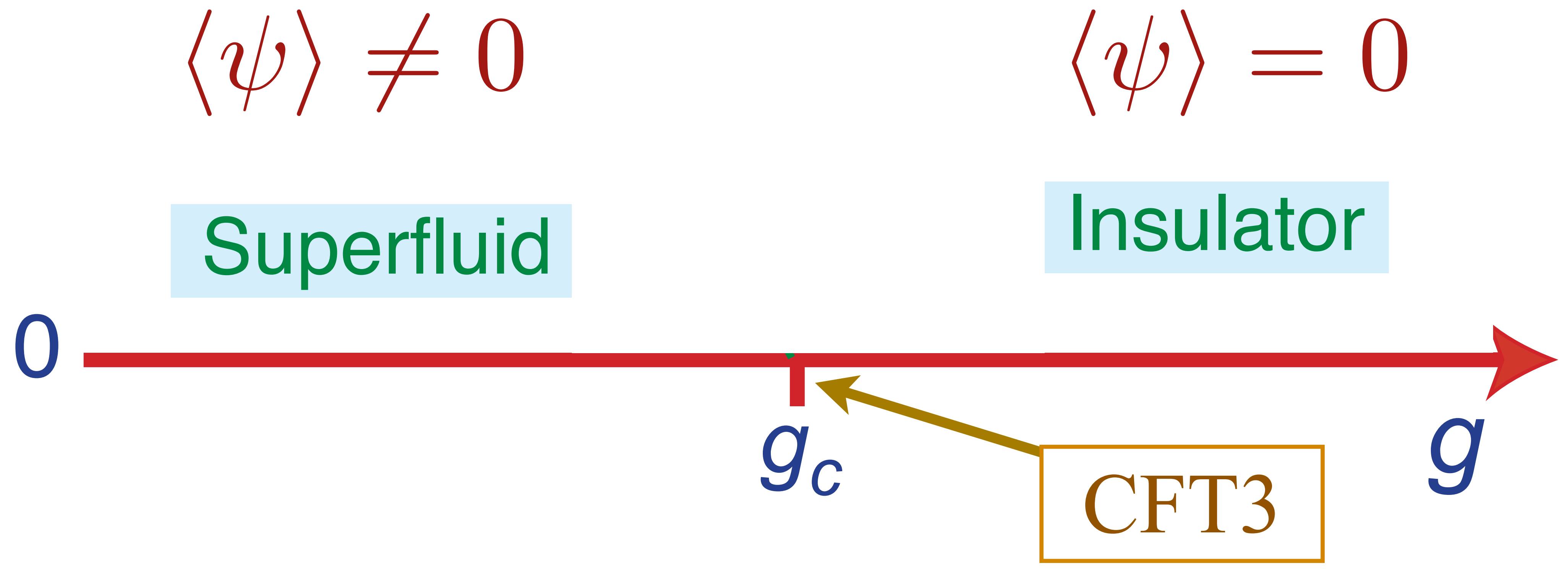


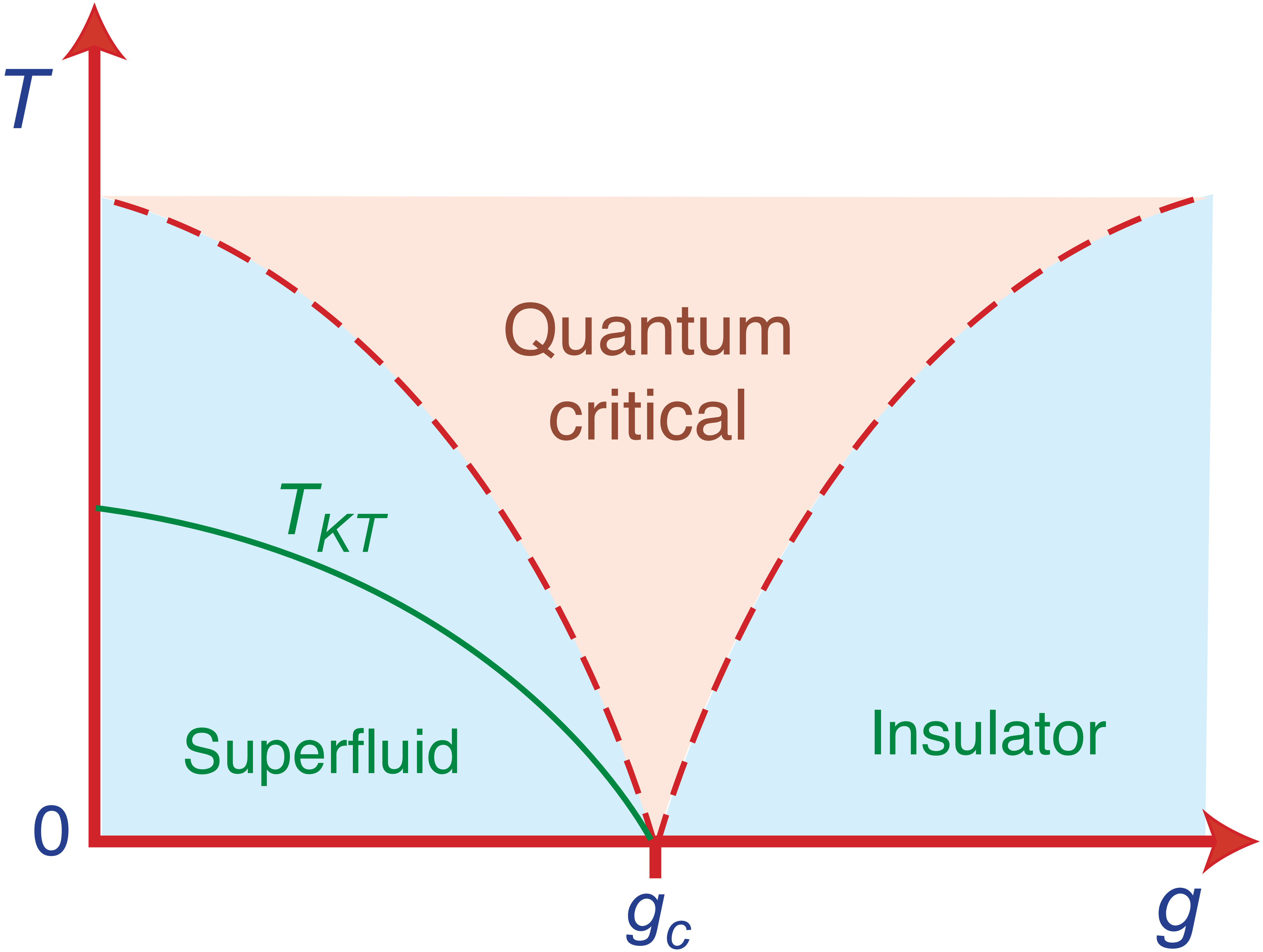
Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

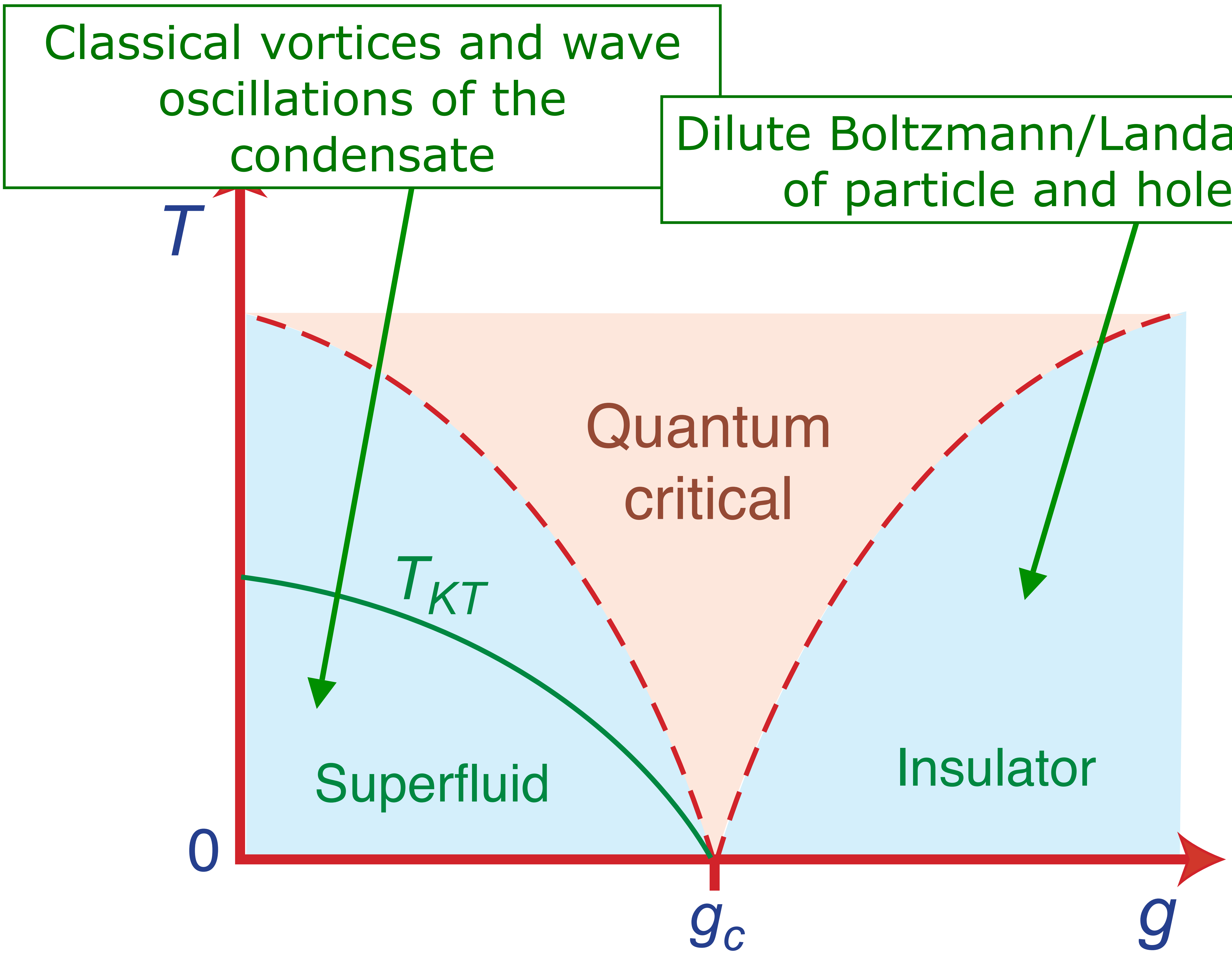

Superfluid
Insulator
0
g_c
g

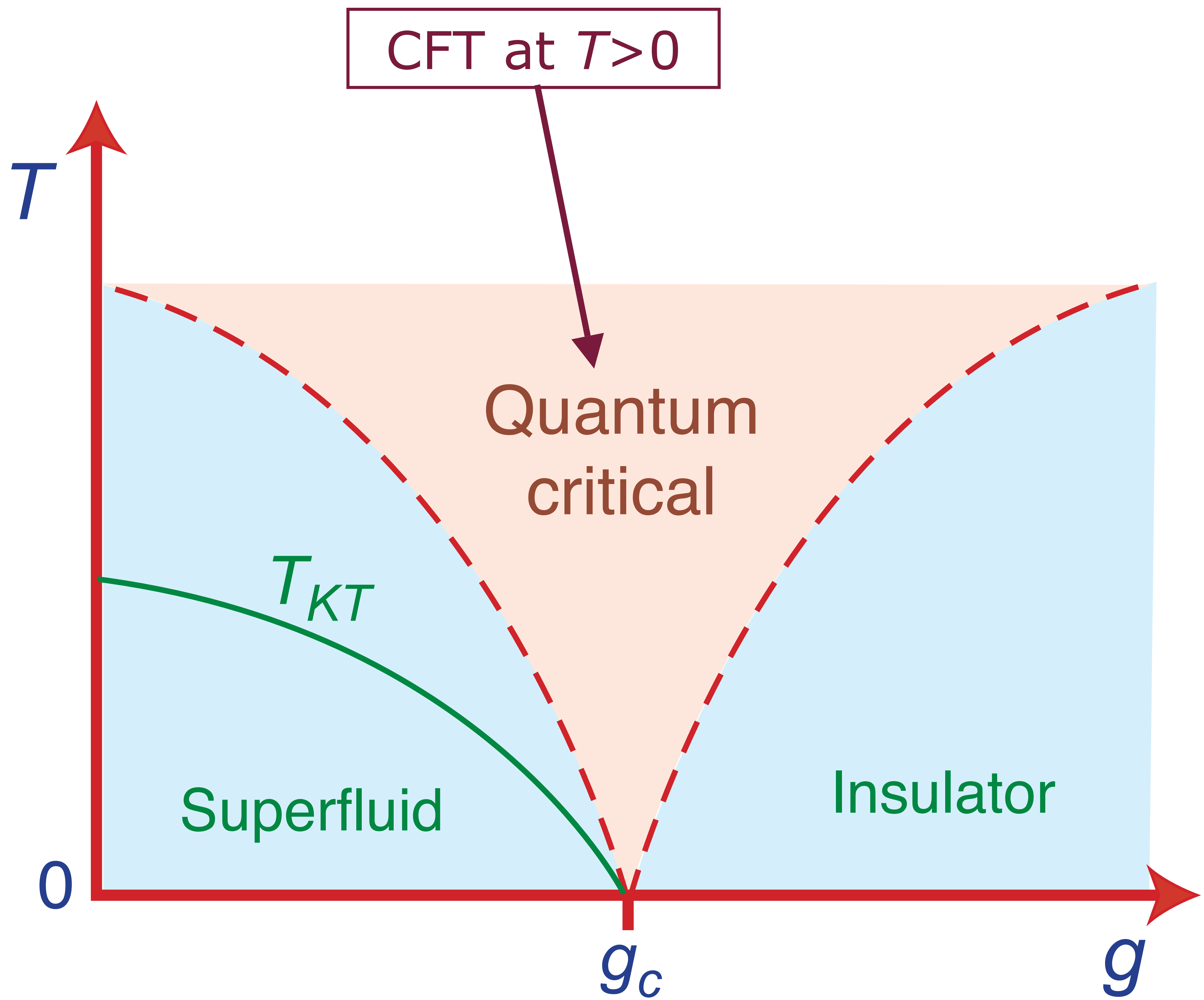
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$





T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


Classical vortices and wave oscillations of the condensate
Dilute Boltzmann/Landau gas of particle and holes
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

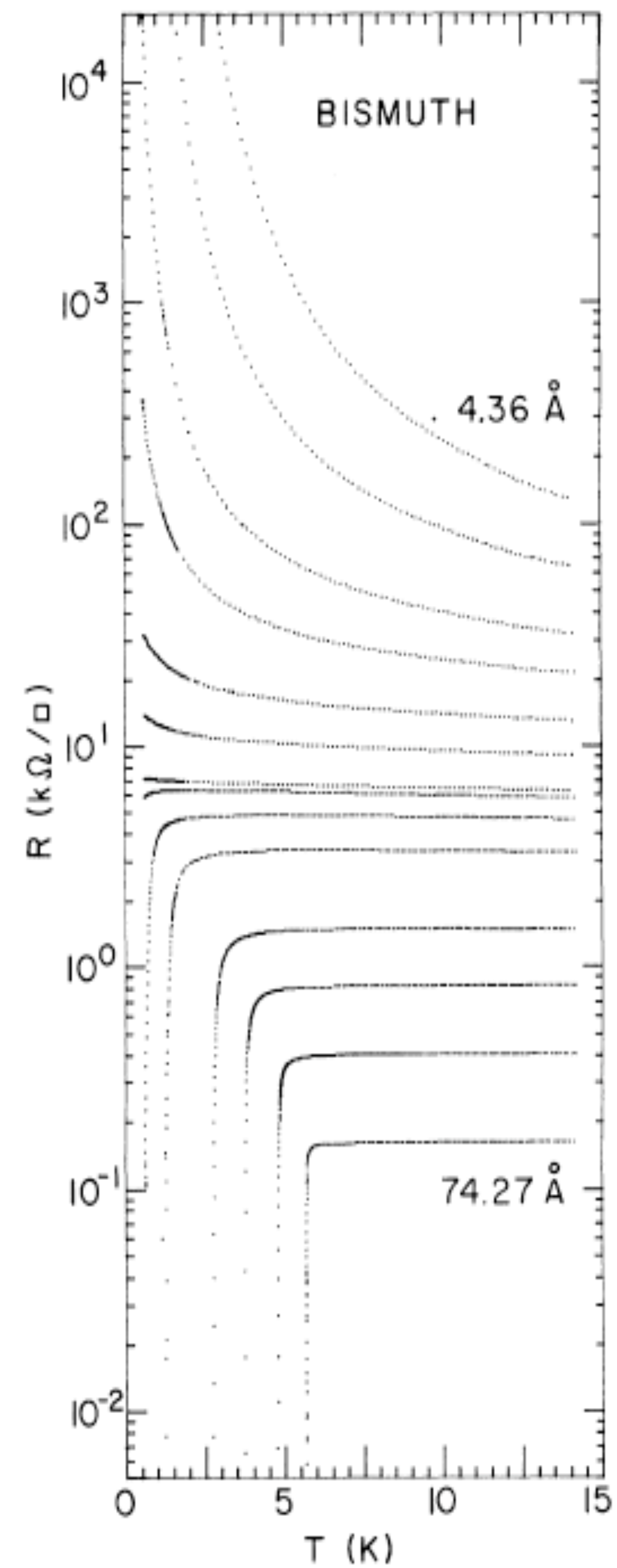


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k,\omega) = 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h} \Theta_1 \Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

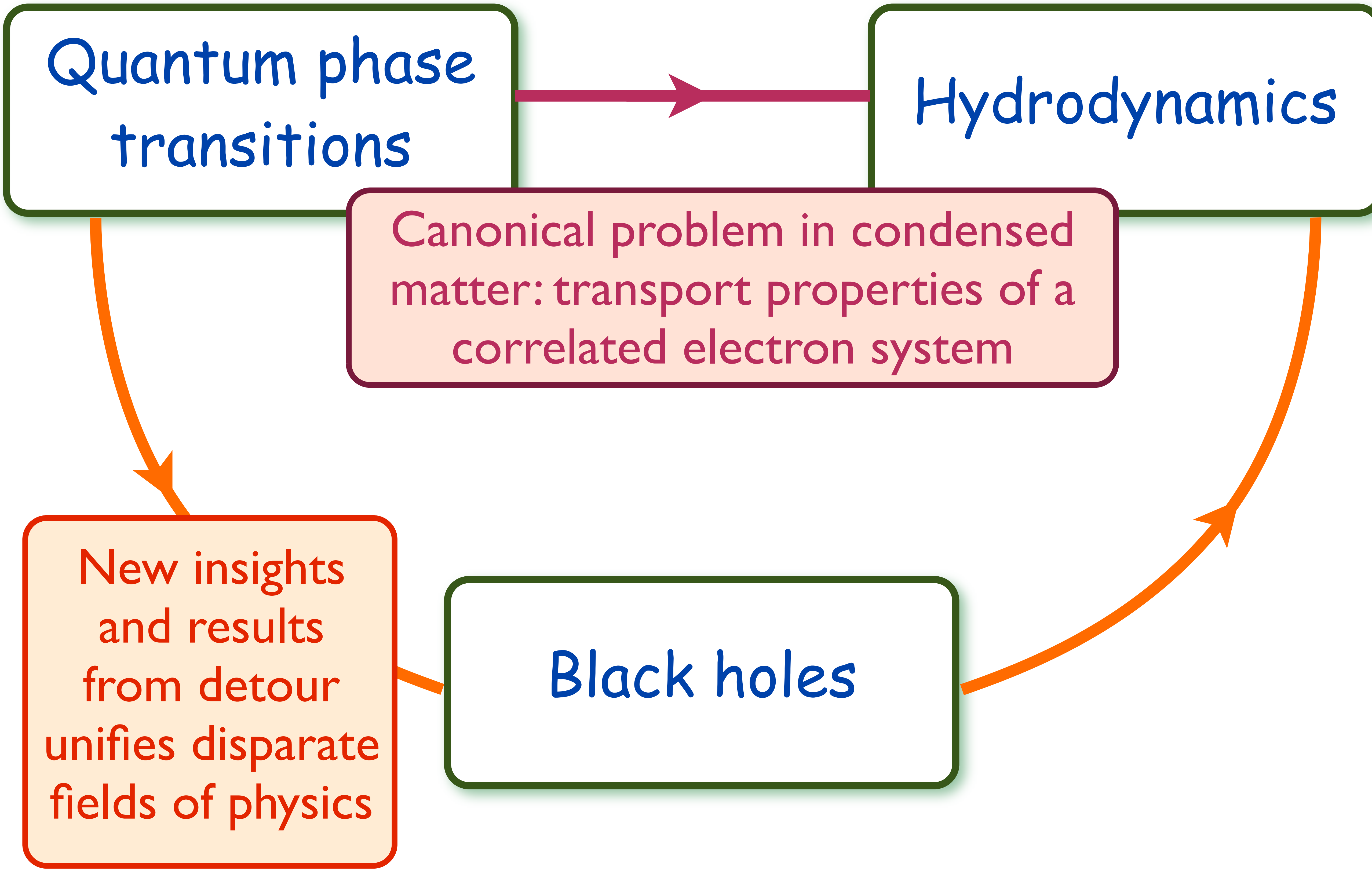
Three foci of modern physics

Quantum phase transitions

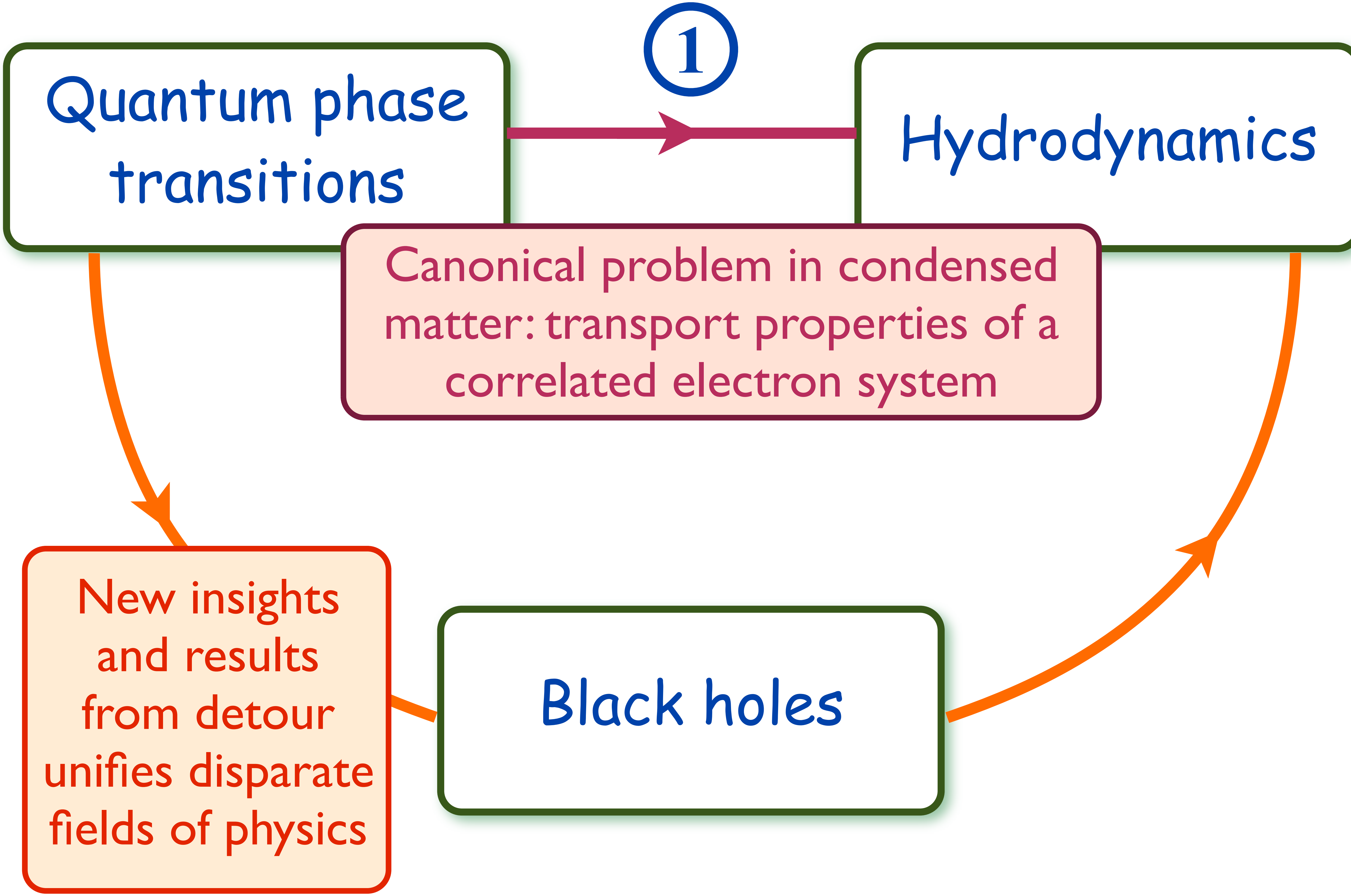
Hydrodynamics

Black holes

Three foci of modern physics



Three foci of modern physics

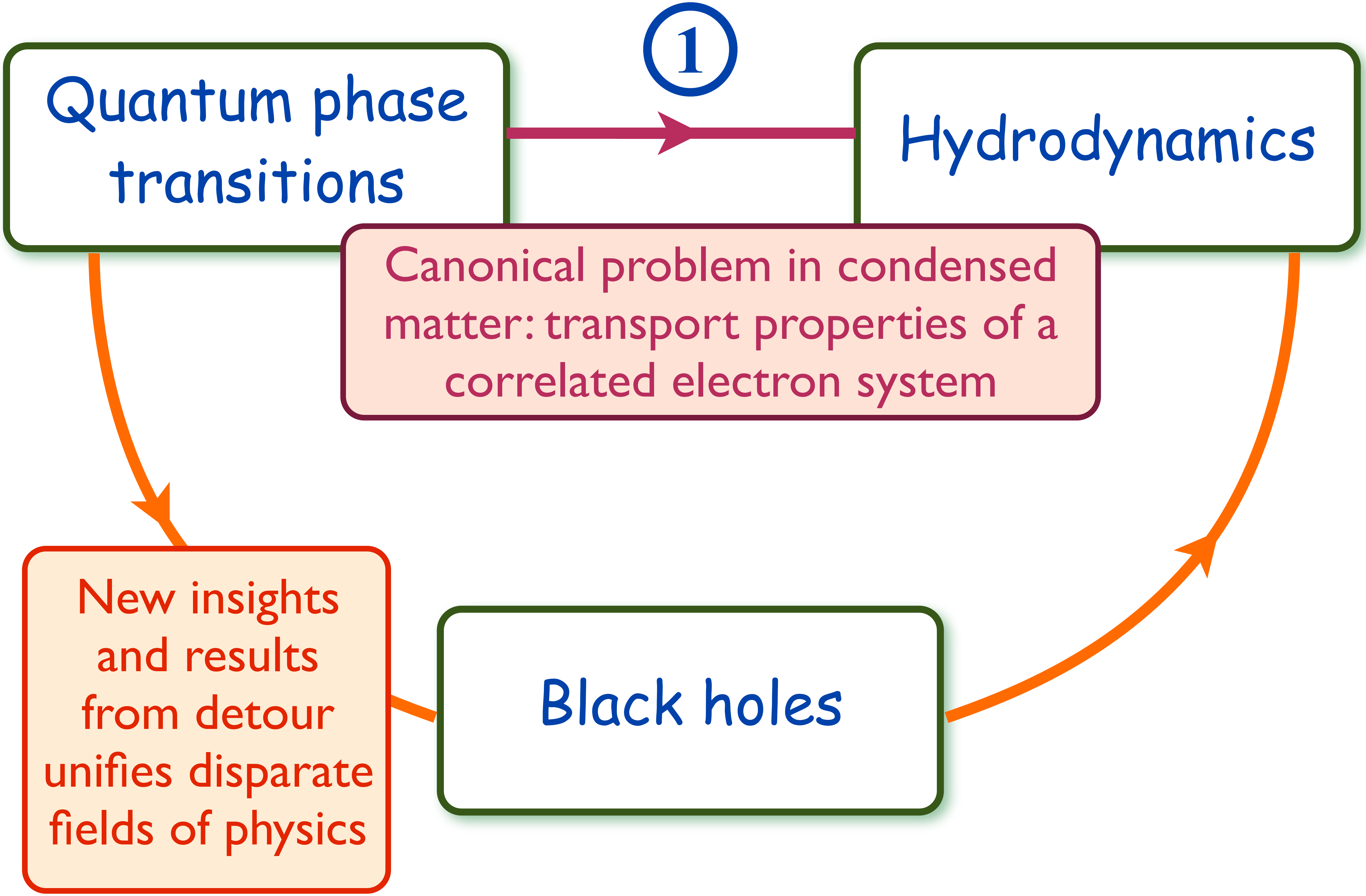


Hydrodynamics of quantum critical systems

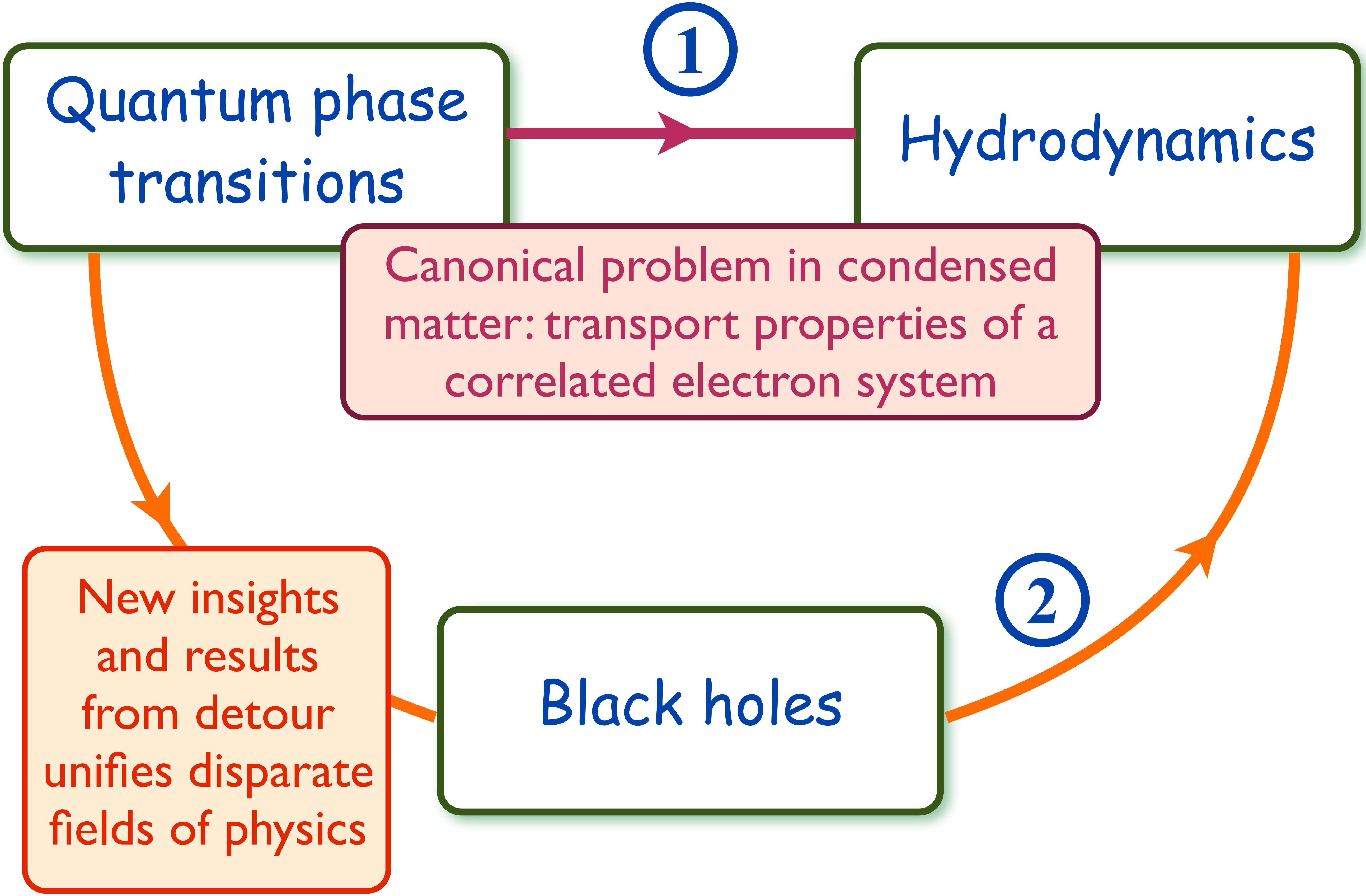
1. Use quantum field theory + quantum transport equations + classical hydrodynamics

   Uses physical model but strong-coupling makes explicit solution difficult

Three foci of modern physics



Three foci of modern physics



Hydrodynamics of quantum critical systems

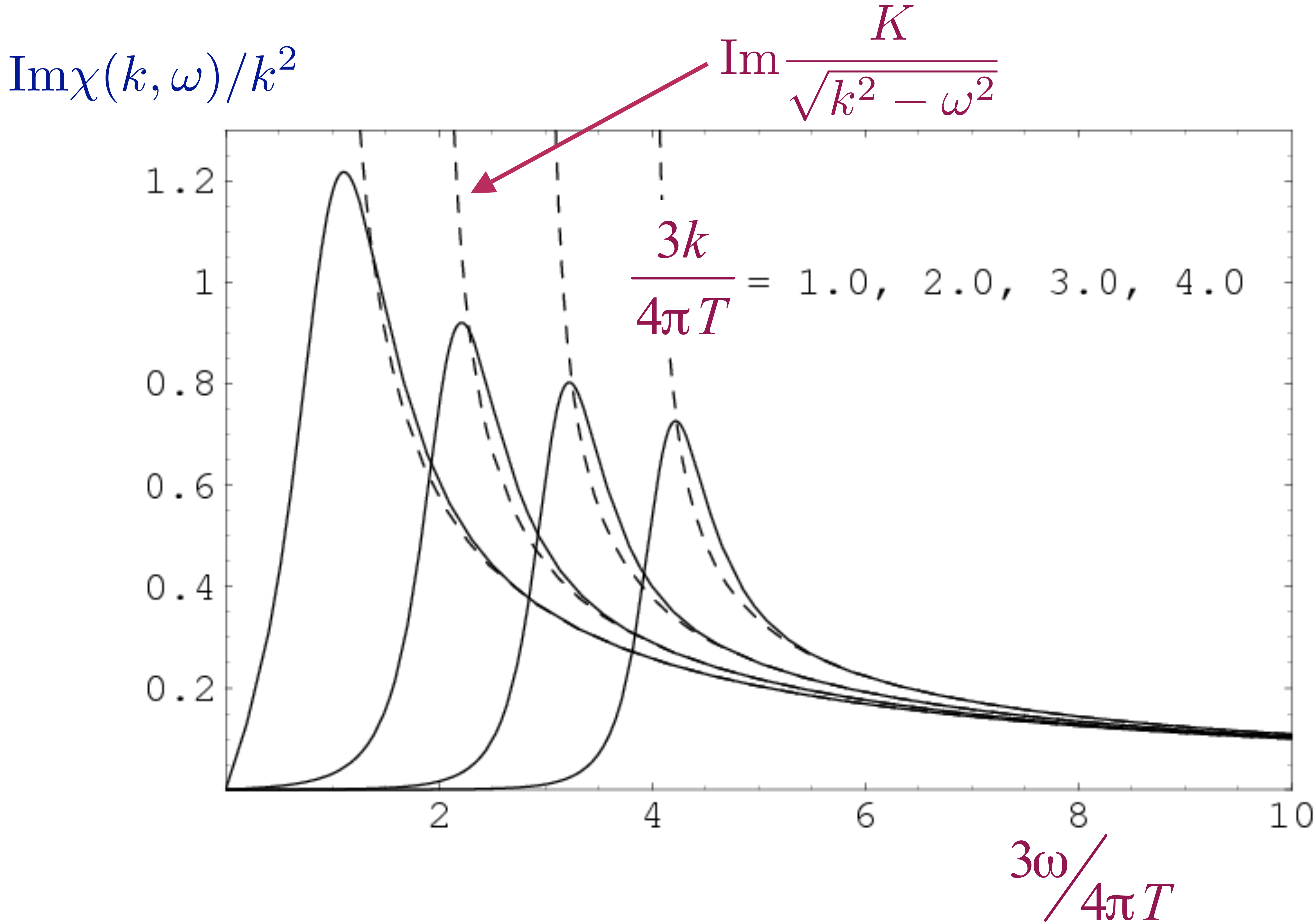
1. Use quantum field theory + quantum transport equations + classical hydrodynamics

   Uses physical model but strong-coupling makes explicit solution difficult

2. Solve Einstein-Maxwell equations in the background of a black hole in AdS space

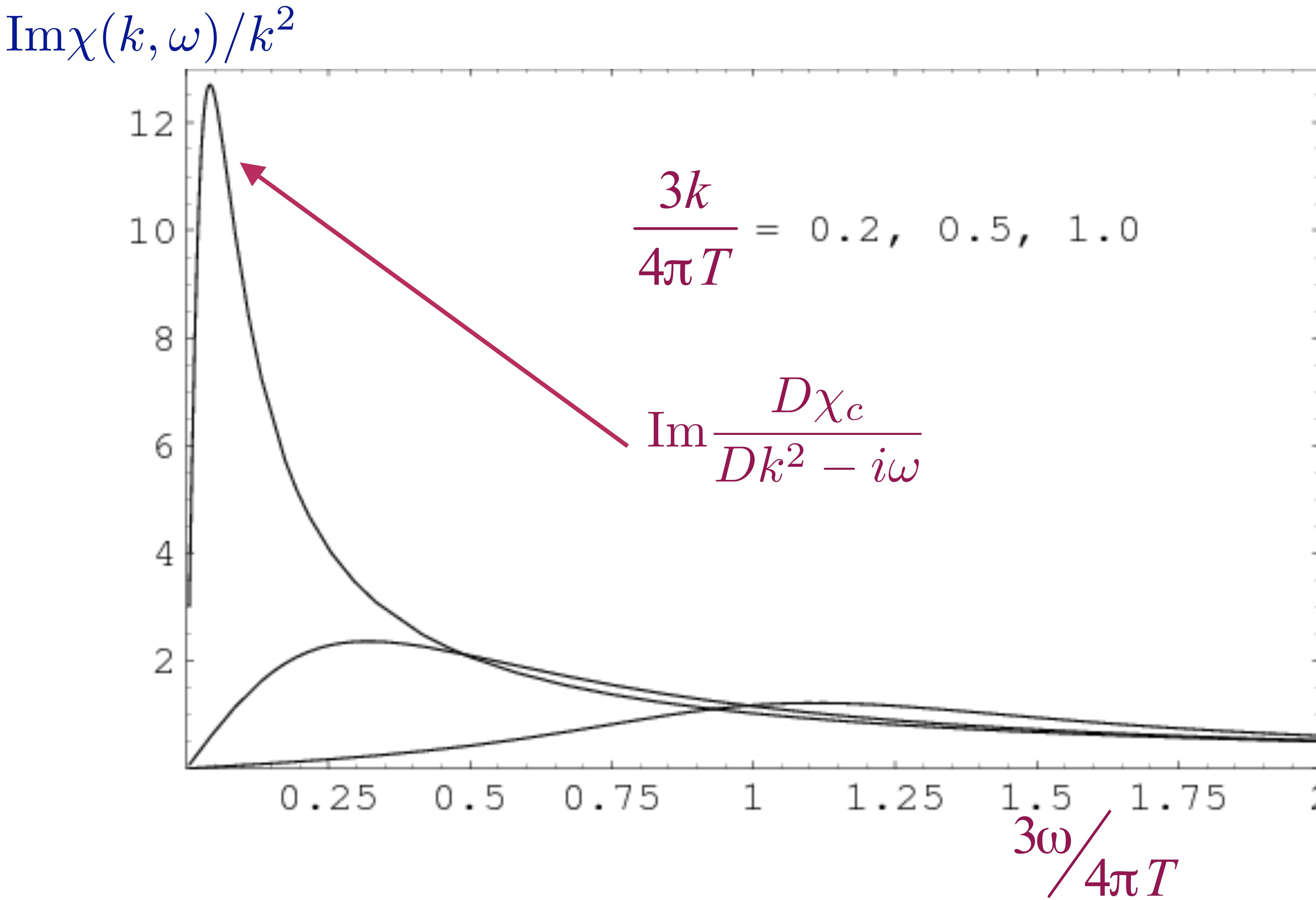
   Yields hydrodynamic relations which apply to general classes of quantum critical systems. First exact numerical results for transport co-efficients (for supersymmetric systems).

Collisionless to hydrodynamic crossover of SYM3



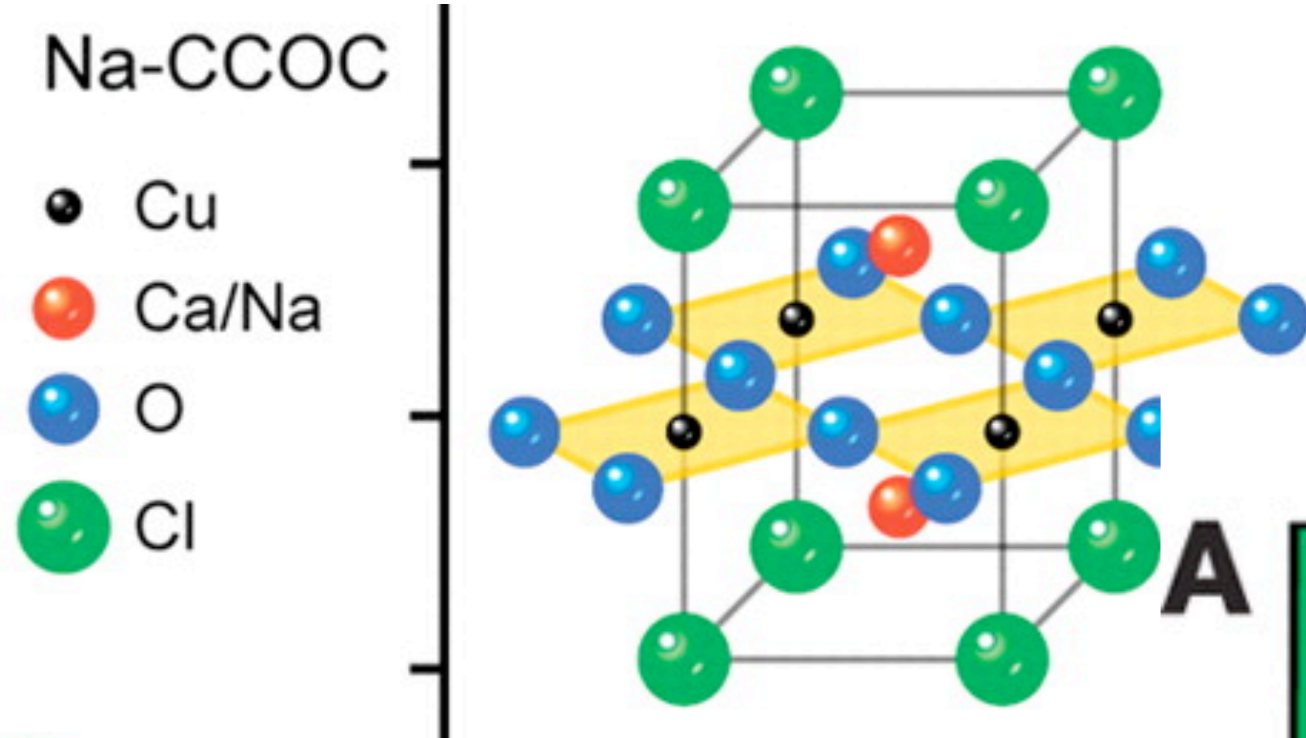
P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

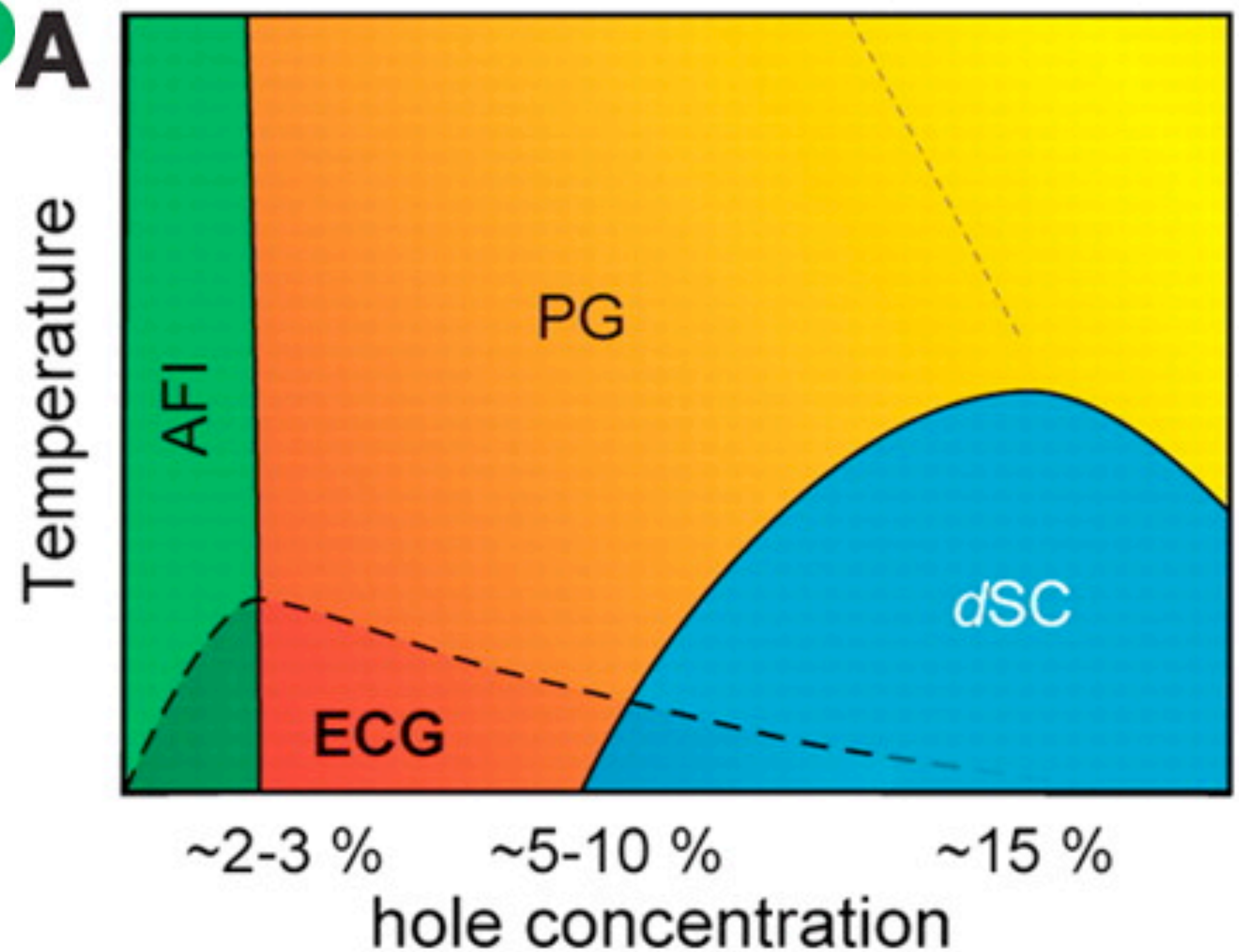
Collisionless to hydrodynamic crossover of SYM3



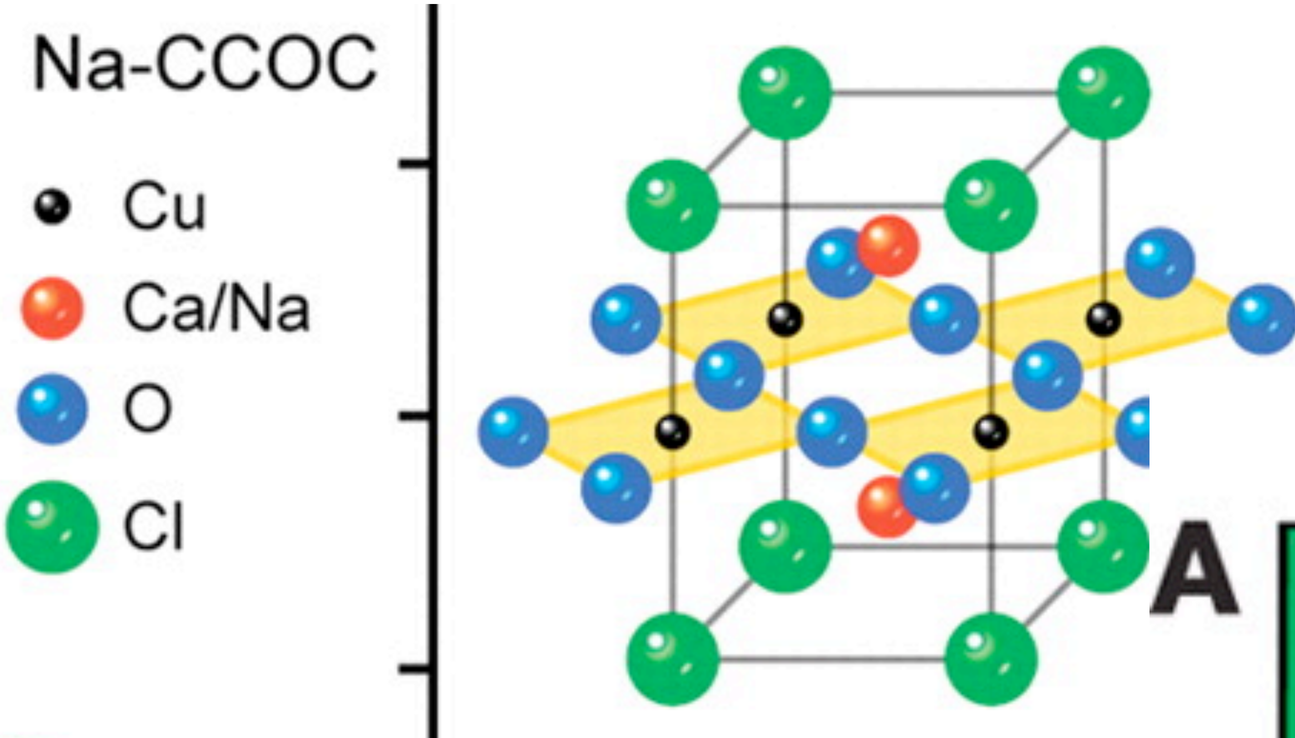
P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

The cuprate superconductors

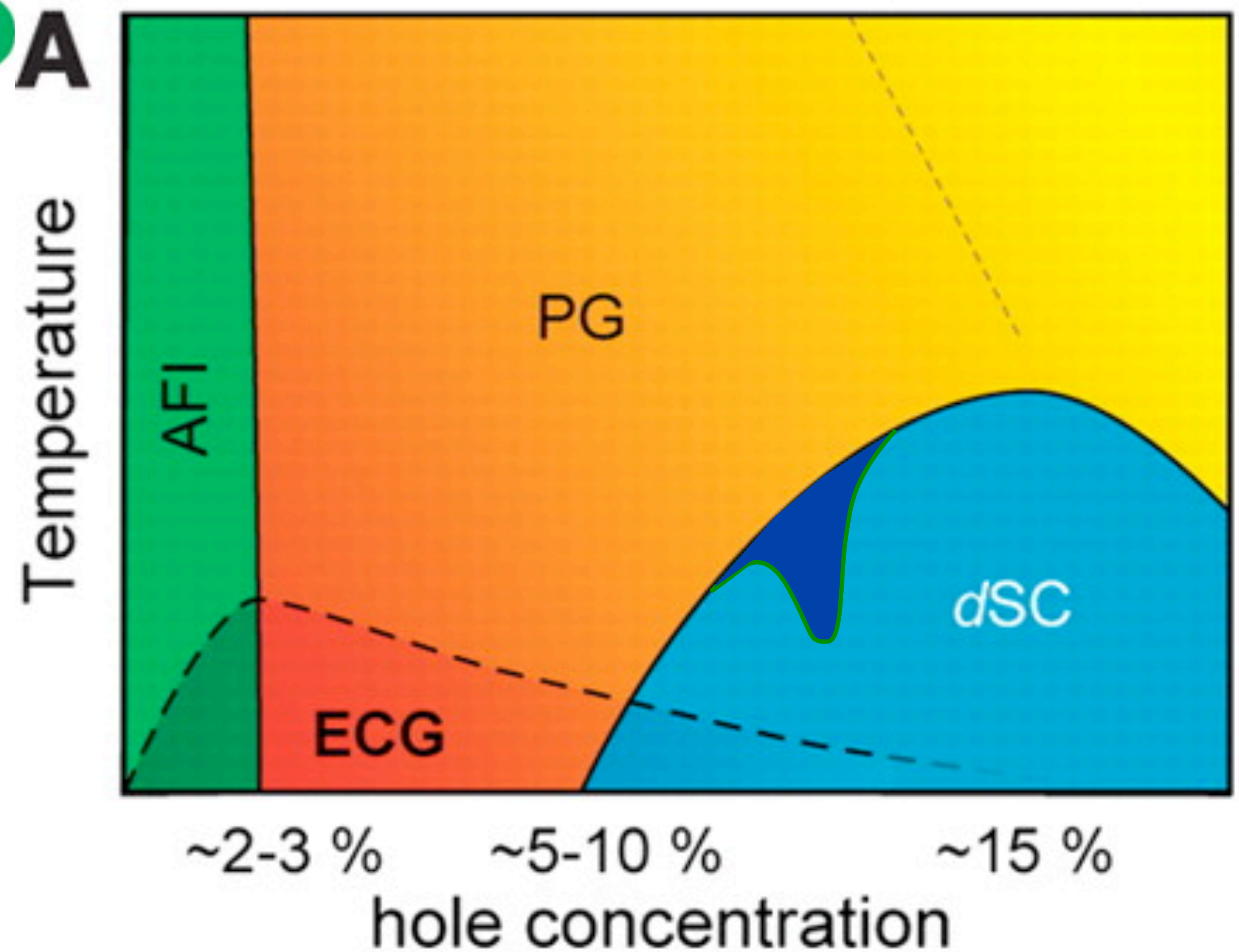




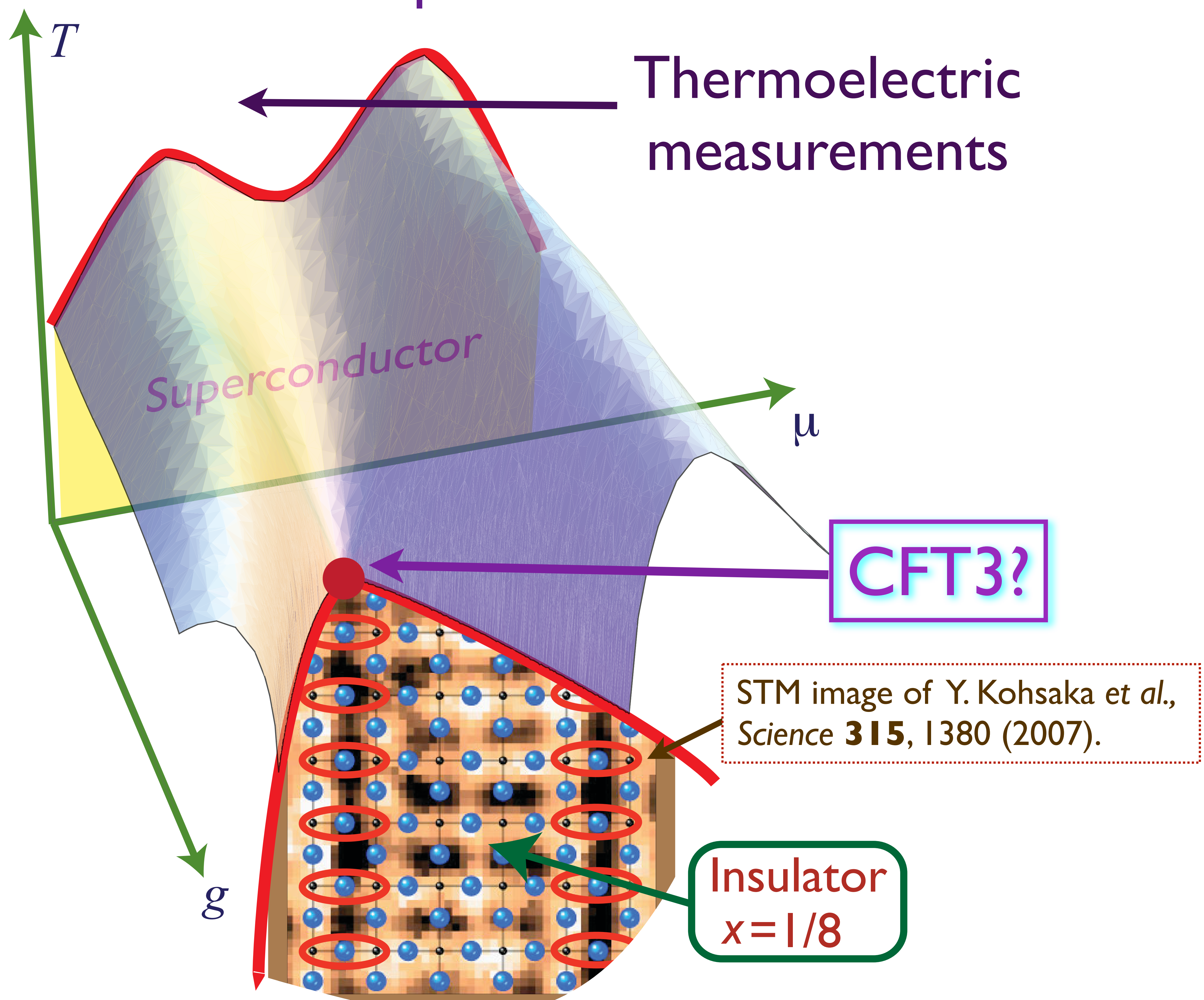
The cuprate superconductors



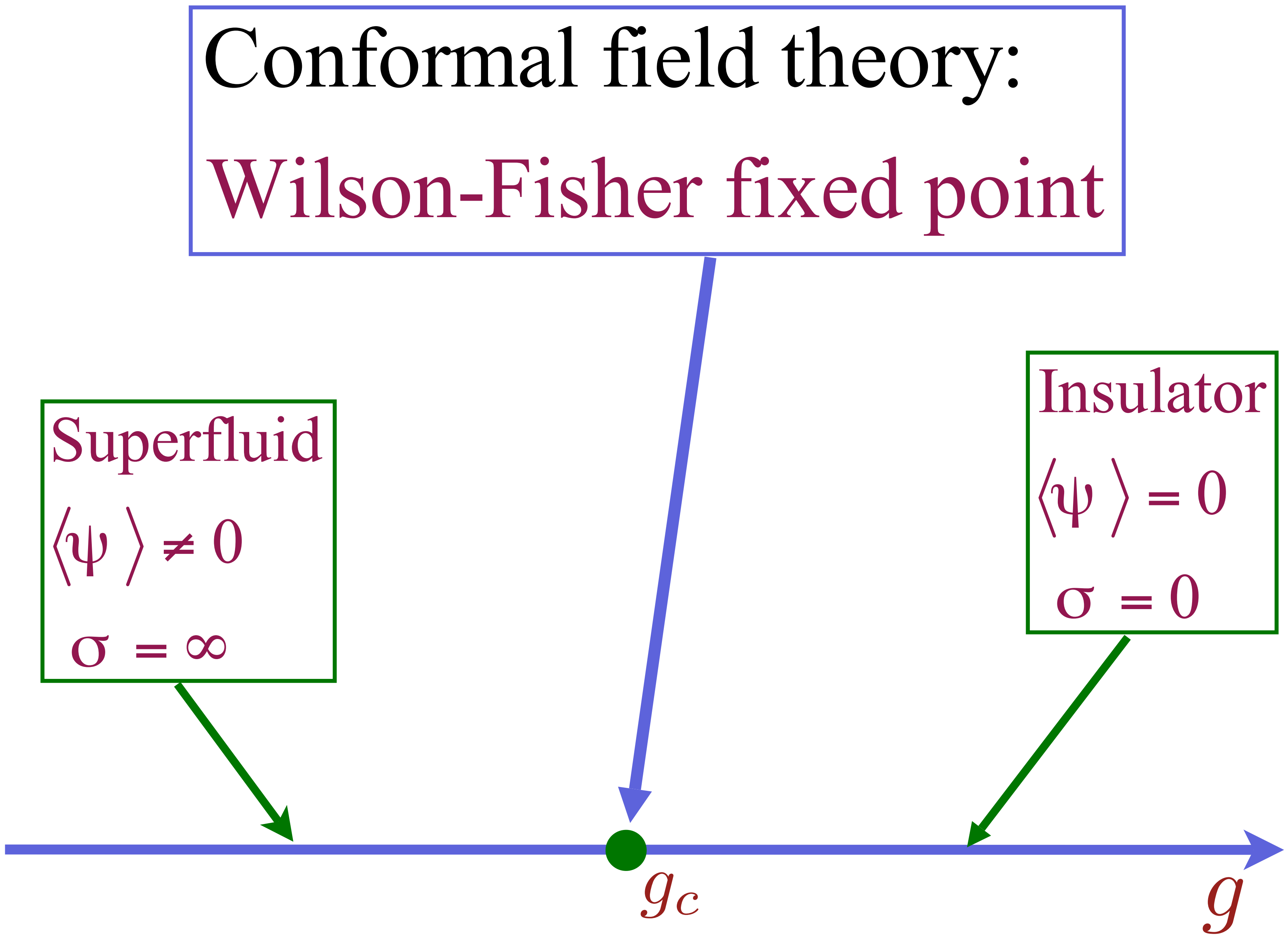
Proximity to an insulator at 12.5% hole concentration



Cuprates



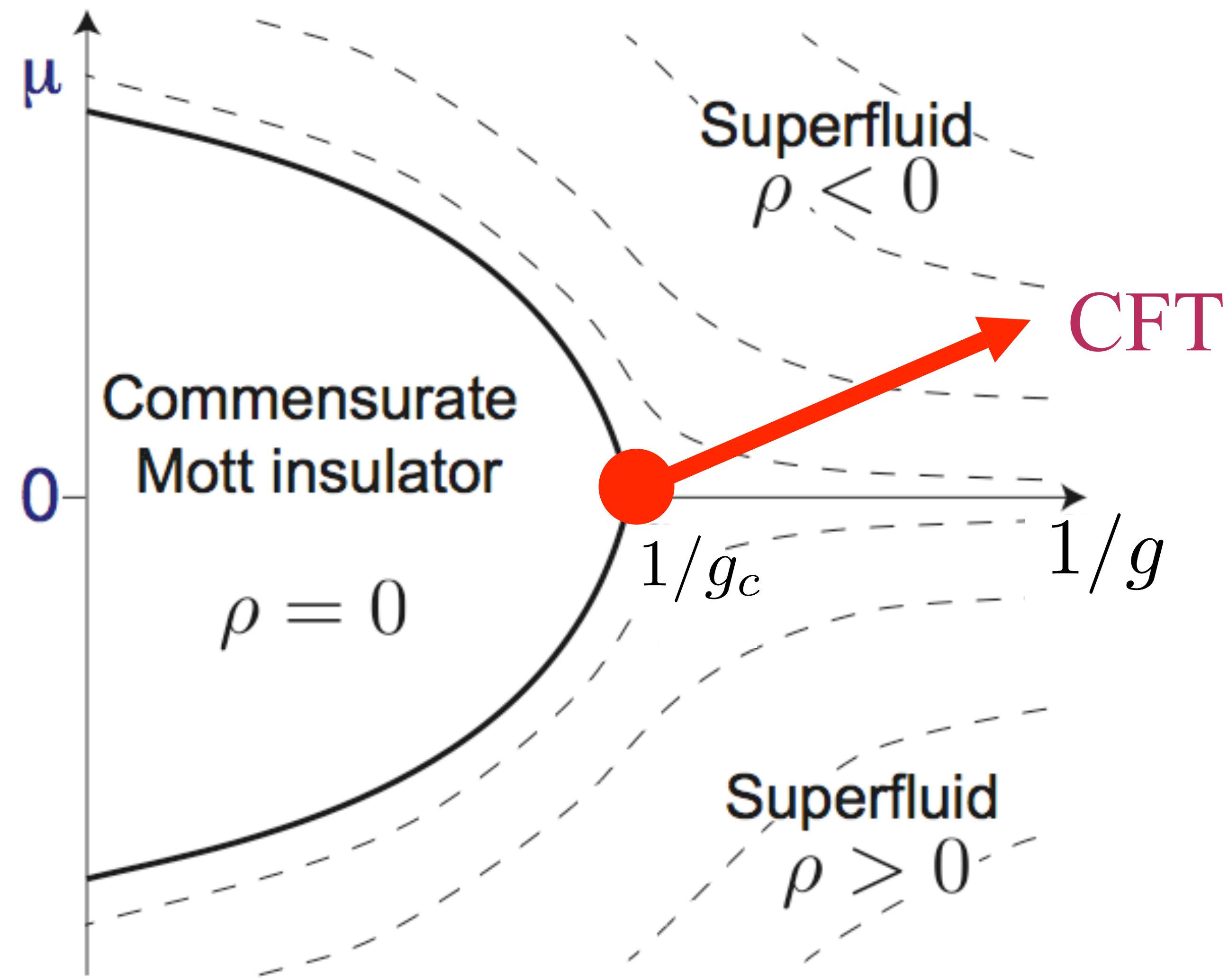
T
Thermoelectric measurements
Superconductor
μ
CFT3?
STM image of Y. Kohsaka et al., Science 315, 1380 (2007).
g
Insulator
x=1/8



$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ
- A magnetic field *B*



e.g.

$$\mathcal{S} = \int d^2 r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the regime $\hbar\omega \ll k_B T$, we can use the principles of hydrodynamics:

- Describe system in terms of local state variables which obey the equation of state
- Express conserved currents in terms of gradients of state variables using transport co-efficients. These are restricted by demanding that the system relaxes to *local equilibrium i.e.* entropy production is positive.
- The conservation laws are the equations of motion.

The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the **difference** in density from the Mott insulator.
- ε, the local energy
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \end{aligned}$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu
\end{aligned}
$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

For experimental applications, we must move away from the ideal CFT

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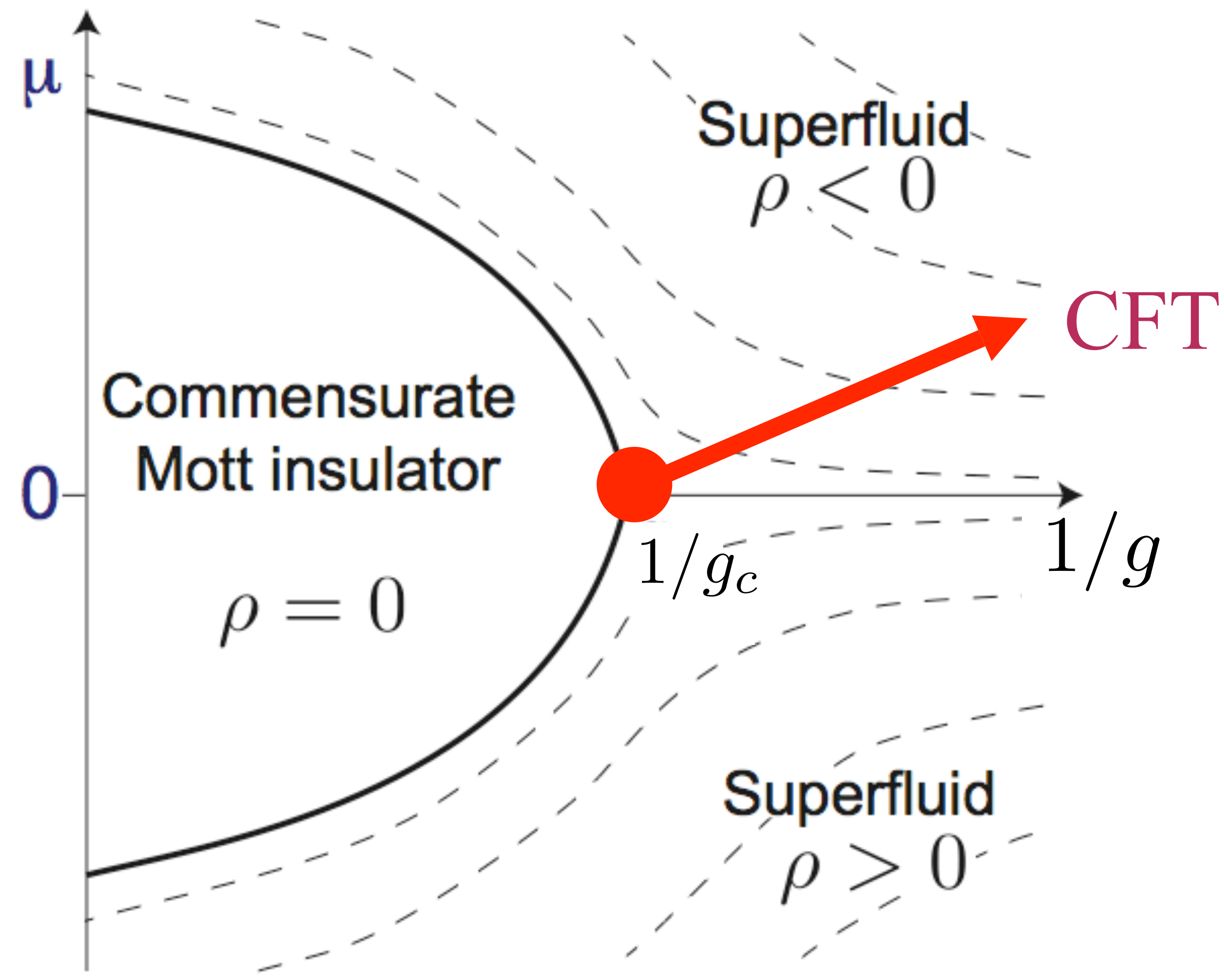
- A magnetic field B



e.g.

$$\mathcal{S} = \int d^2 r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

For experimental applications, we must move away from the ideal CFT

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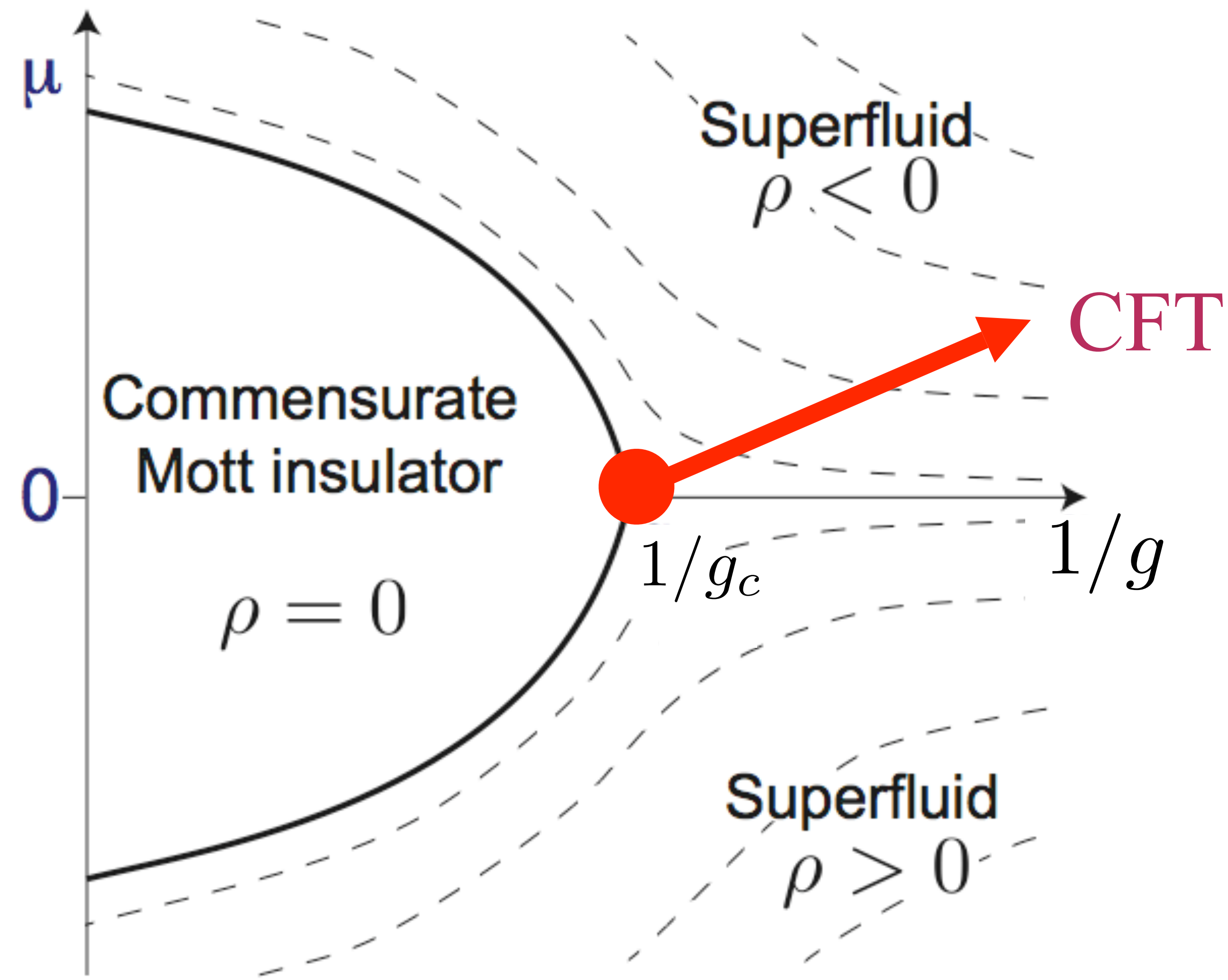
- An impurity scattering rate $1/\tau_{imp}$ (its T dependence follows from scaling arguments)



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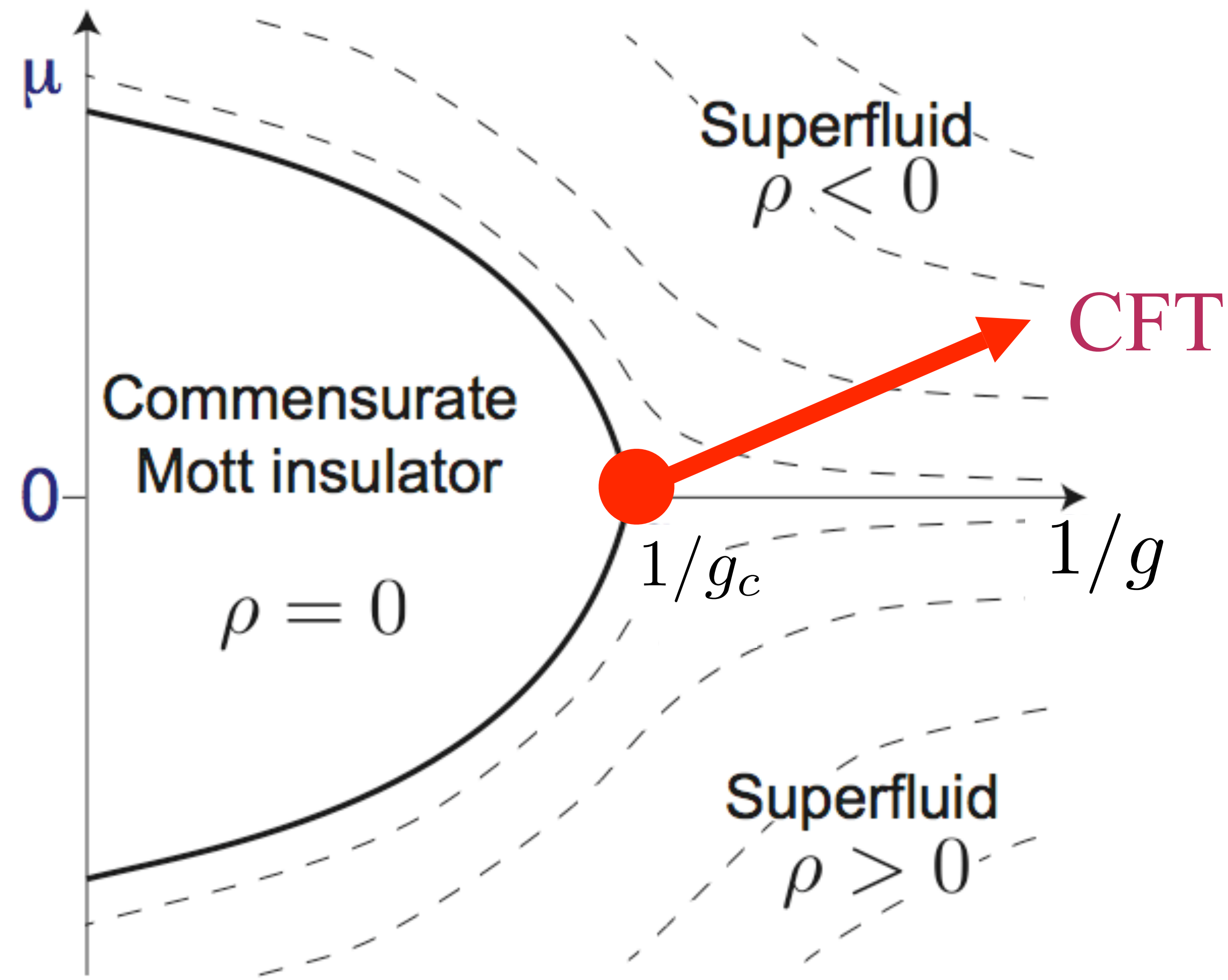

e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2|(\vec{\nabla} - i\vec{A})\psi|^2 + (g - g_c)|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\text{imp}}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right] \end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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\begin{aligned}
\partial_\mu J^\mu &= 0 \\
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T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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\end{aligned}
$$

Solve initial value problem and relate results to response functions (Kadanoff+Martin)

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\rm imp})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\rm imp})}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2} \right] .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$

$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Hydrodynamic cyclotron resonance at a frequency

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and with width

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

where B = magnetic field, ρ = charge density away from density of CFT, ε = energy density, P = pressure, v = velocity of "light" in CFT, and $\sigma_Q e^2/h$ is the universal conductivity of the CFT.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right) \left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right] \end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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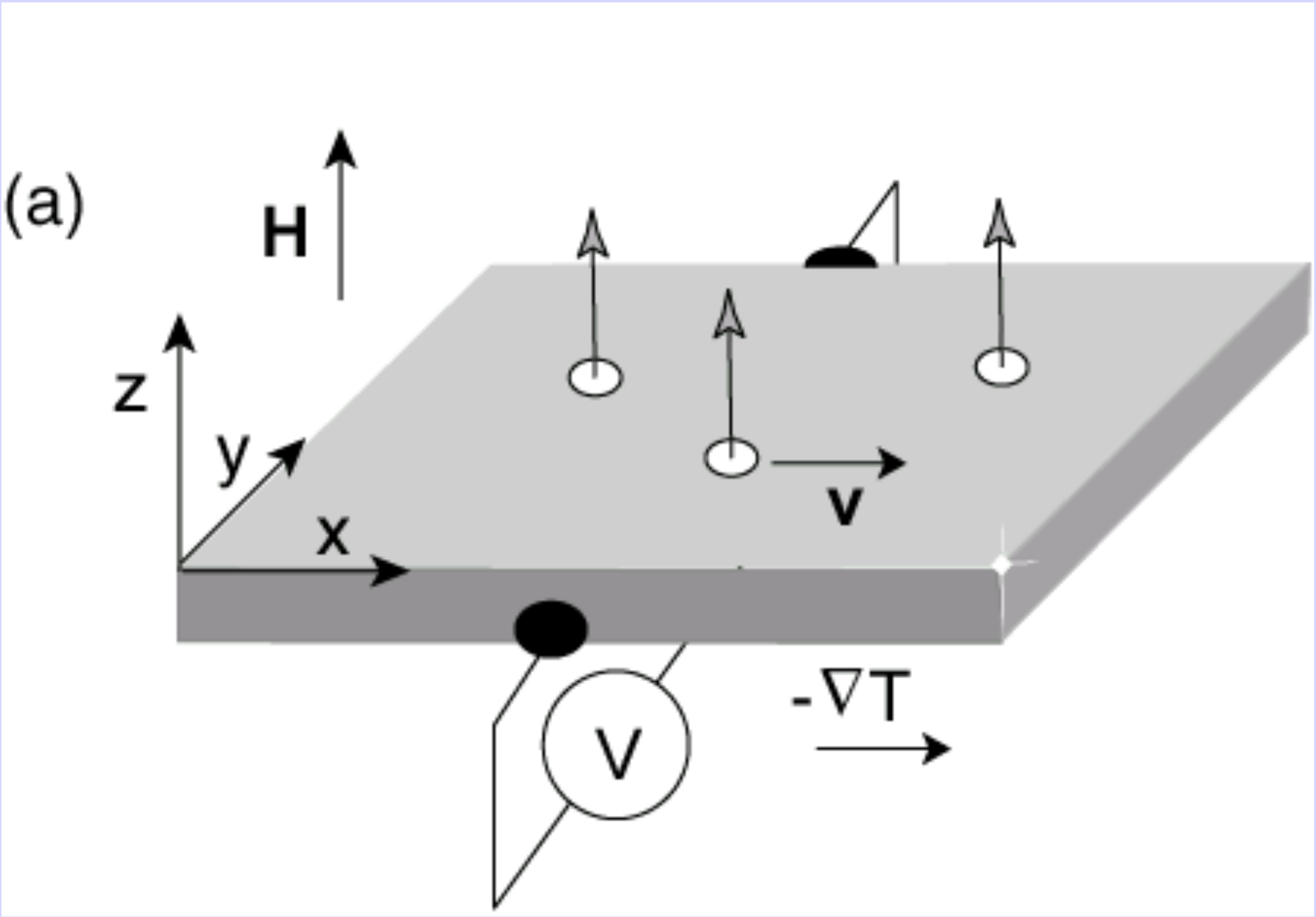
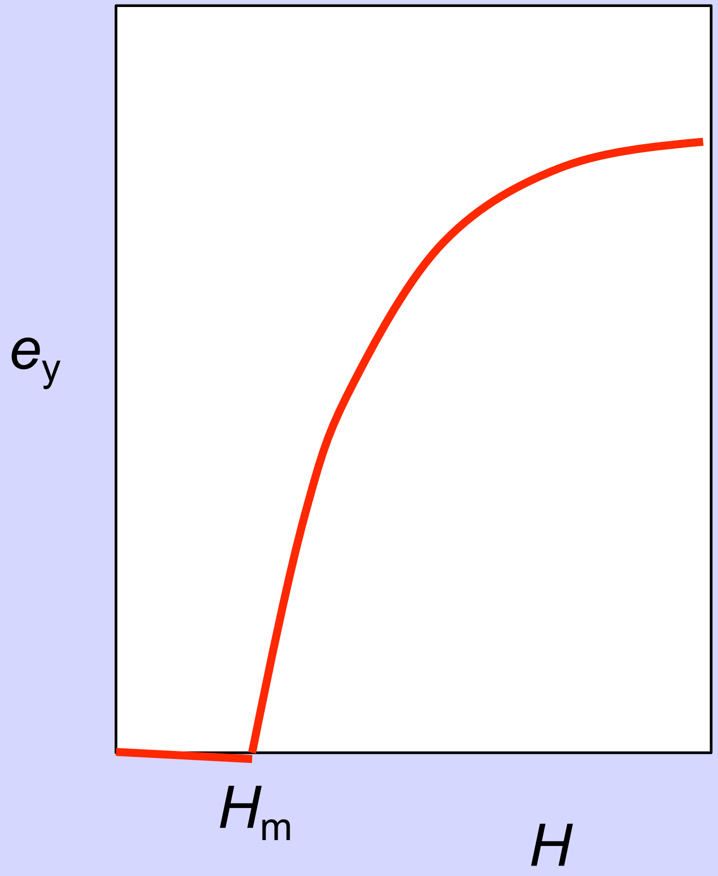
$\rightarrow 1$ as $\rho \rightarrow 0$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Nernst experiment





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$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086 \mu\text{V}/\text{K}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Exact Results

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

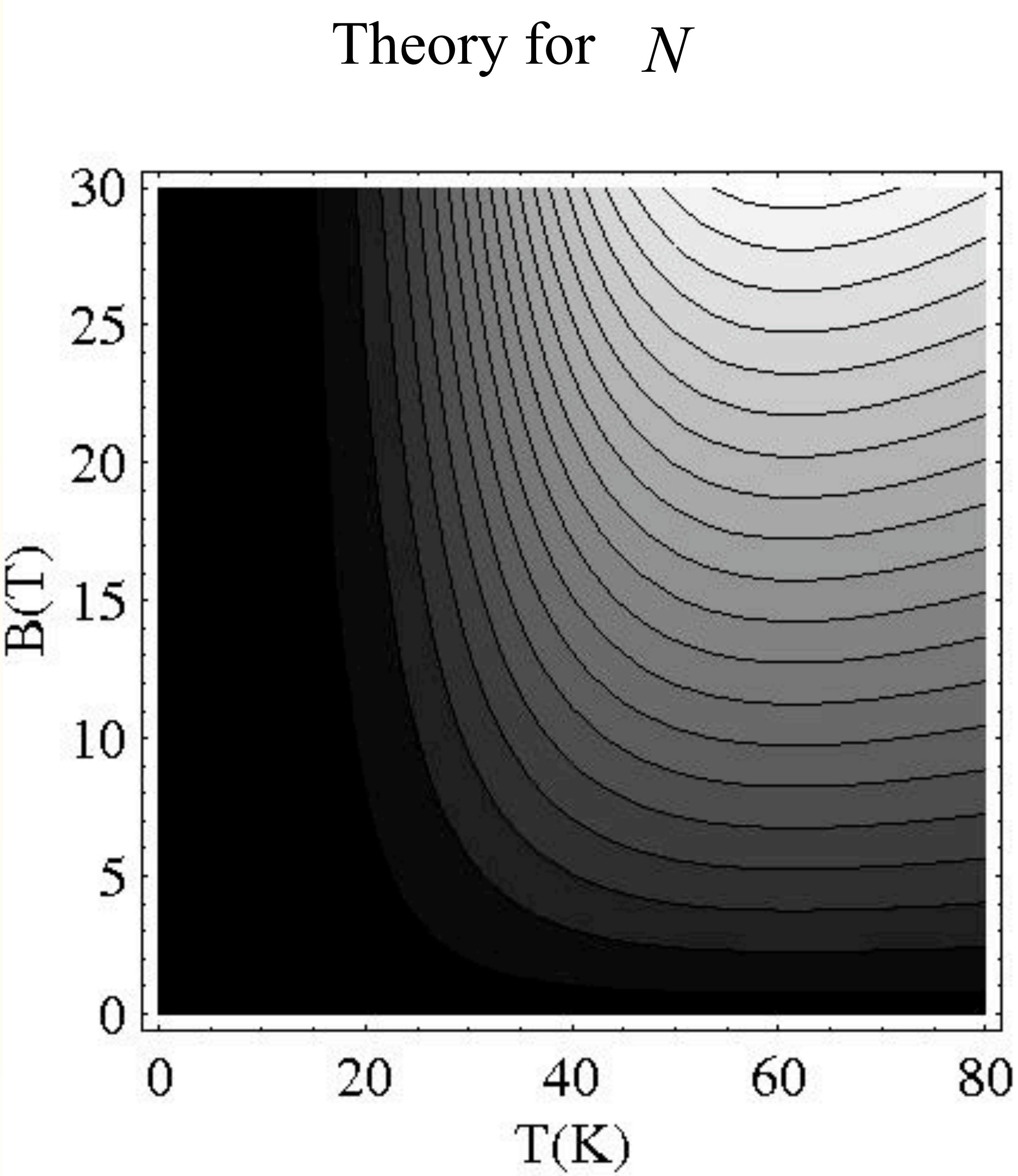
After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

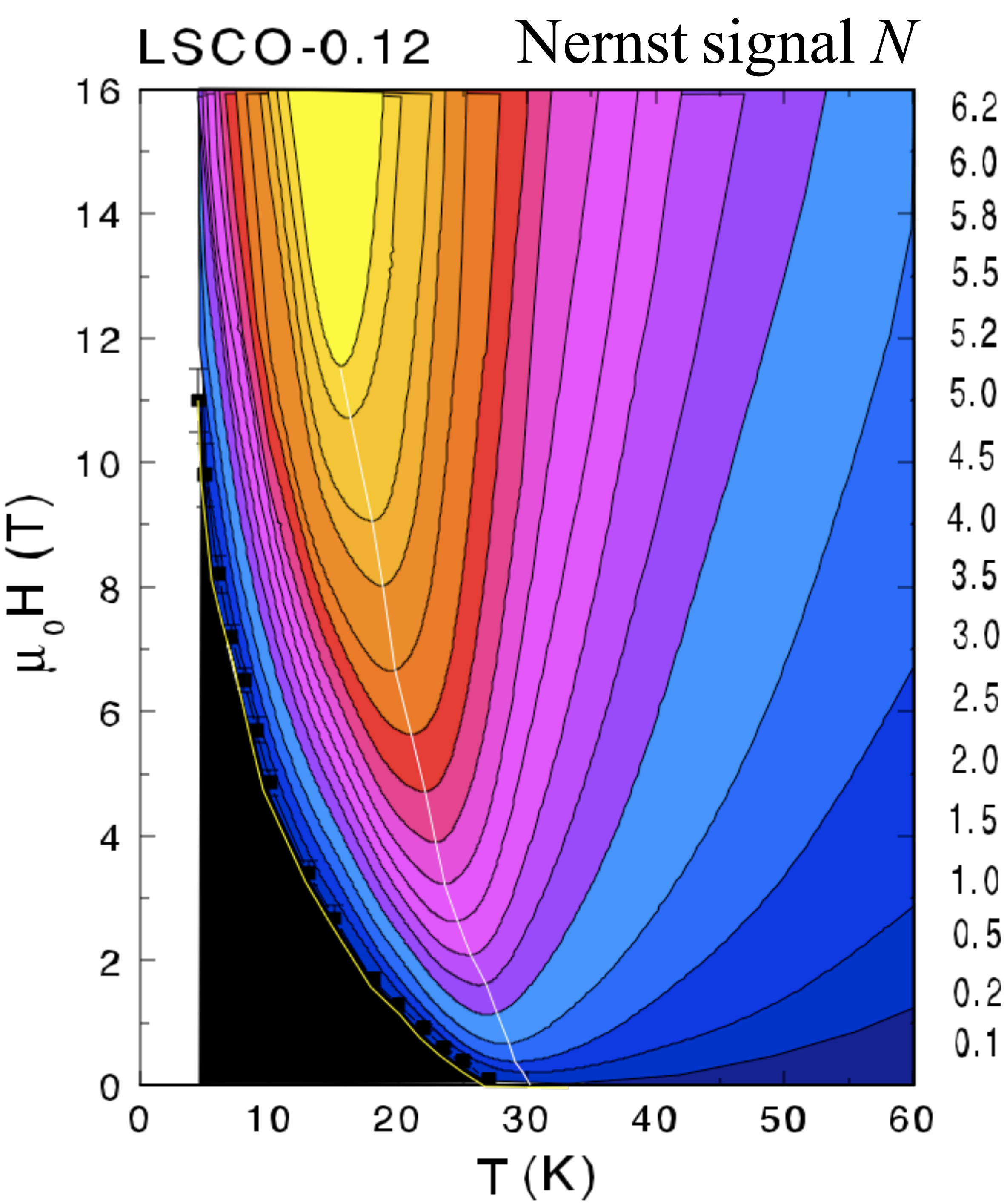
- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

LSCO Experiments





Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B 73, 024510 (2006).

B and T dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, τ_{imp} and v.

Specific quantitative predictions for THz experiments on graphene at room temperature.

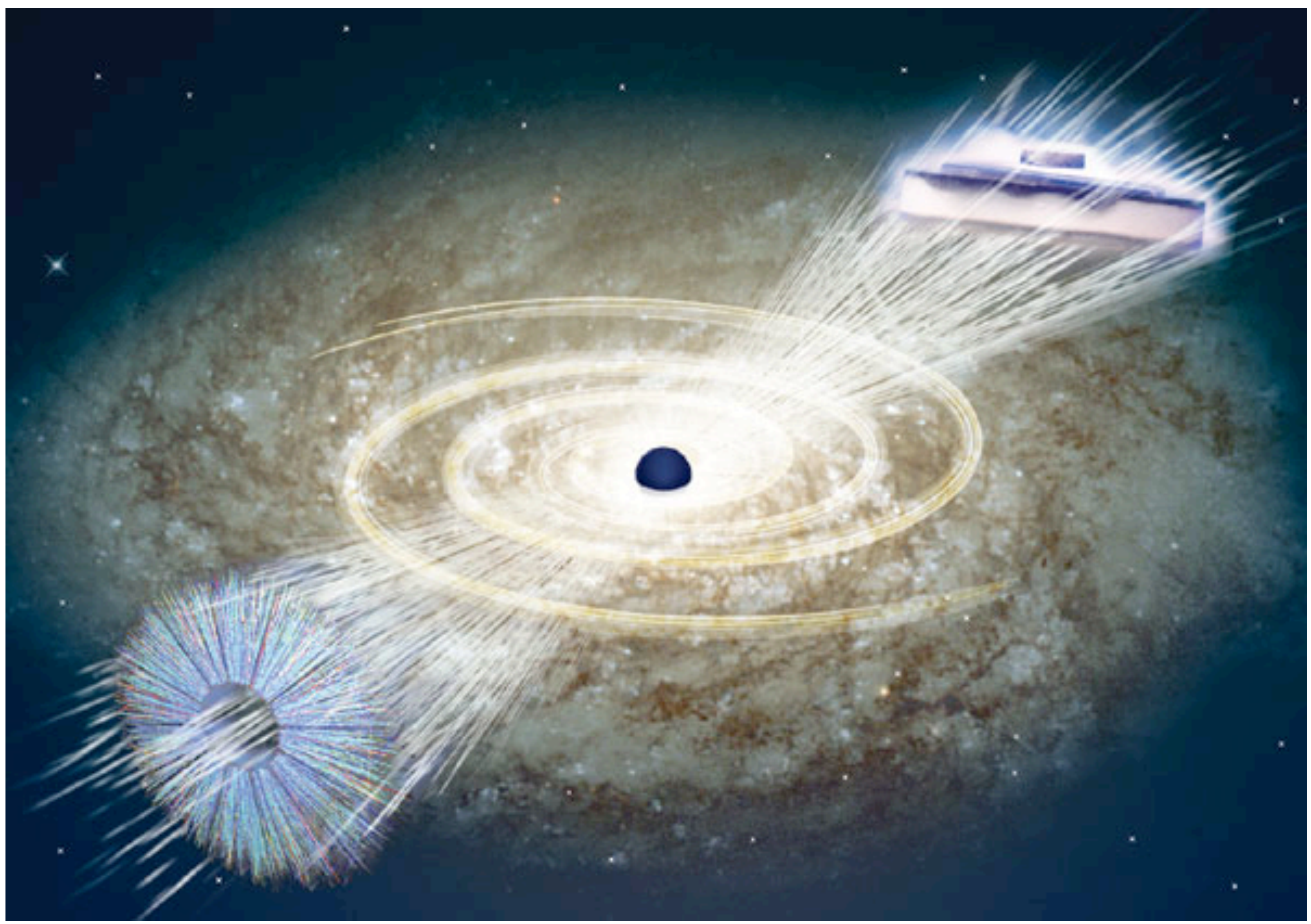
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Conclusions

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- Theory of Nernst effect near the superfluid-insulator transition, and connection to cuprates.