Sign-problem-free Quantum Monte Carlo of the onset of antiferromagnetism in metals

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Diego Hofman

Matthias Punk

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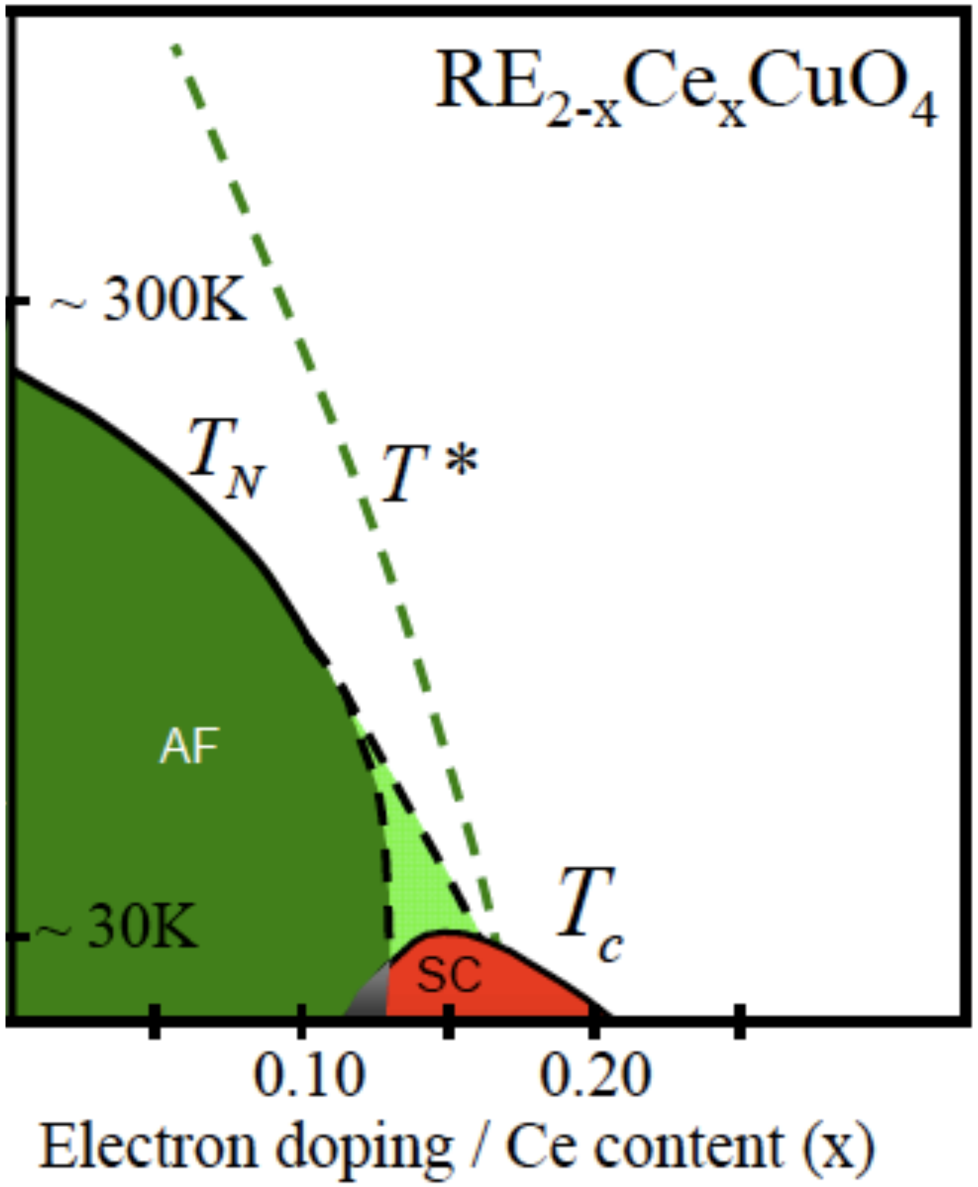
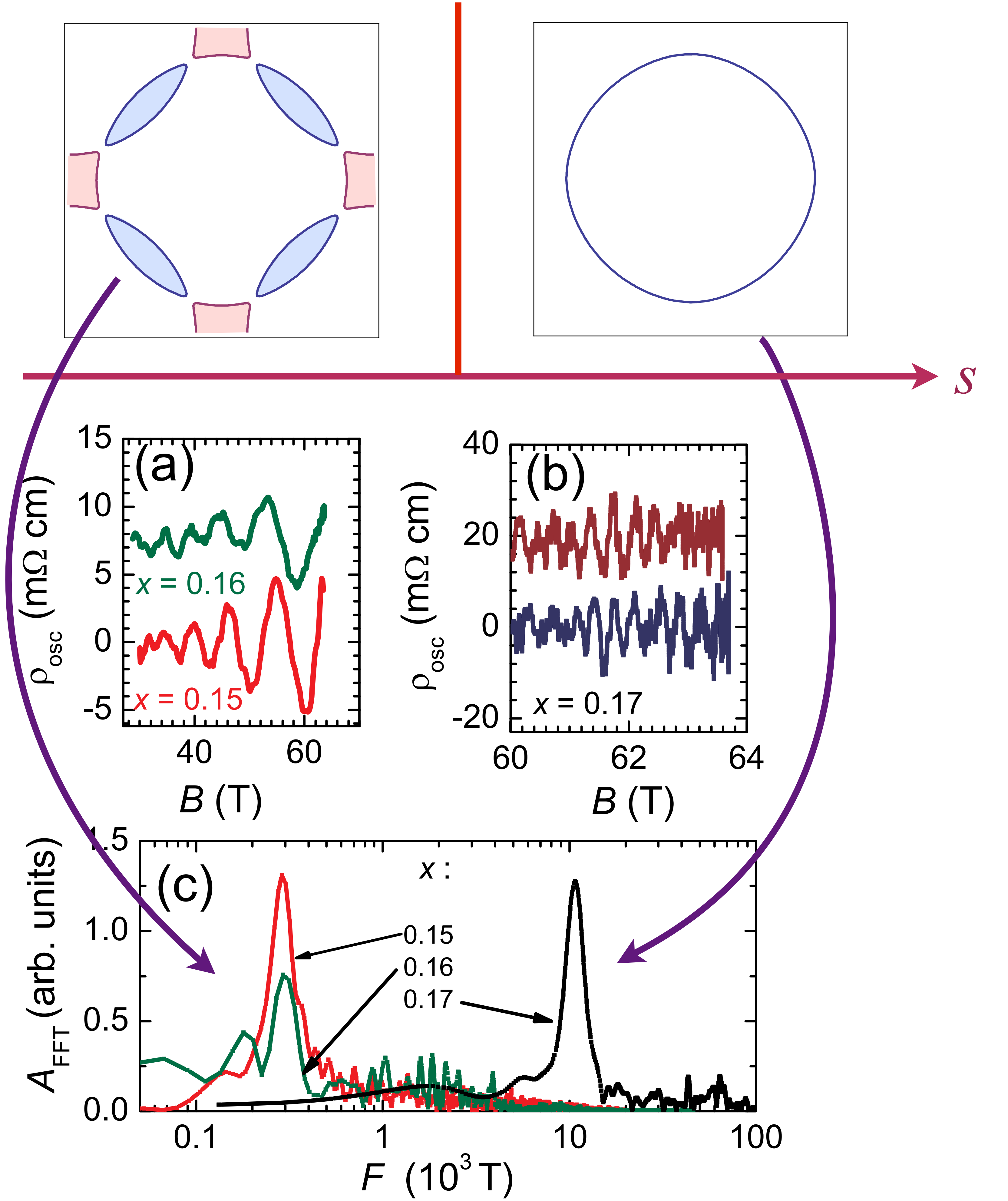
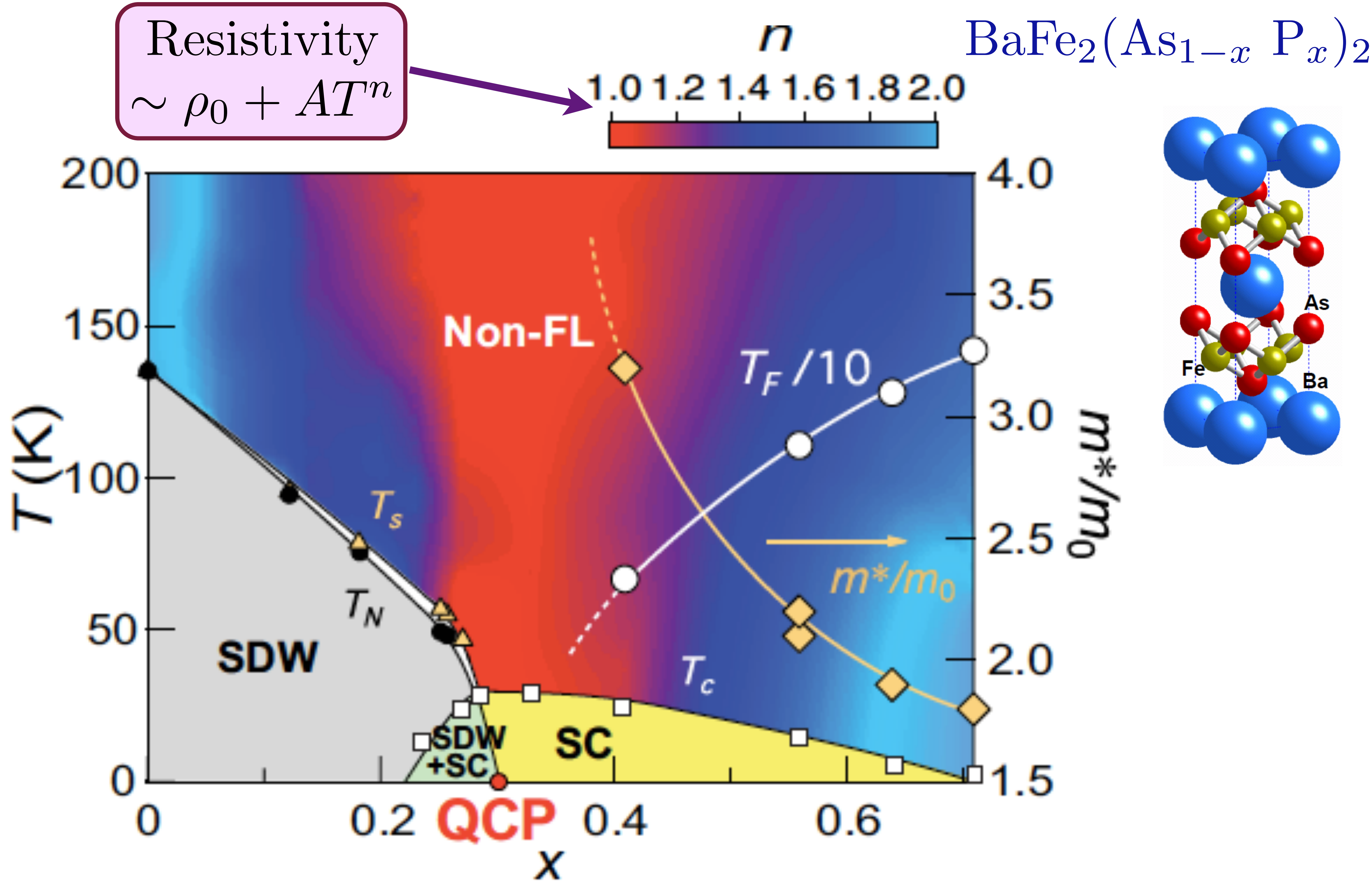




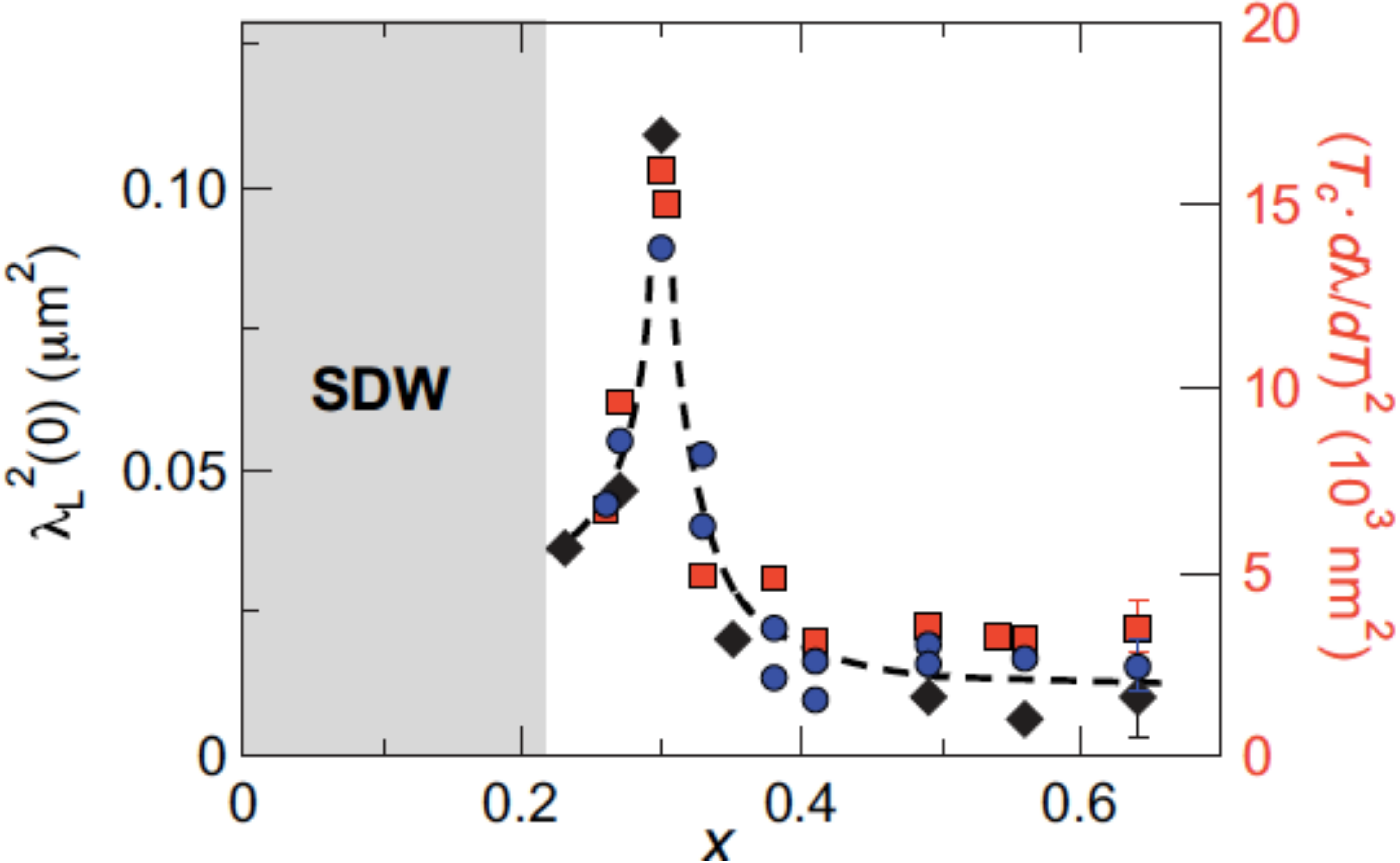
$Nd_{2-x}Ce_xCuO_4$

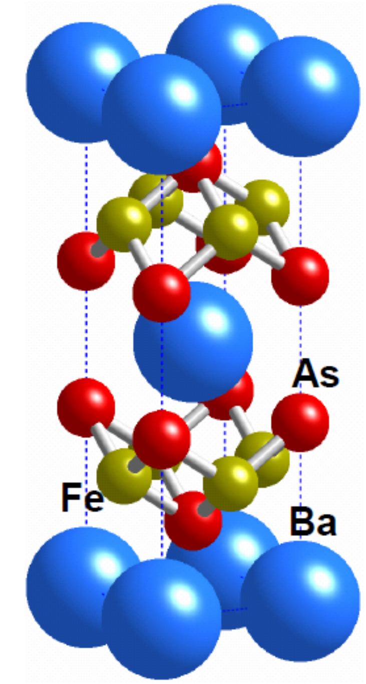
T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

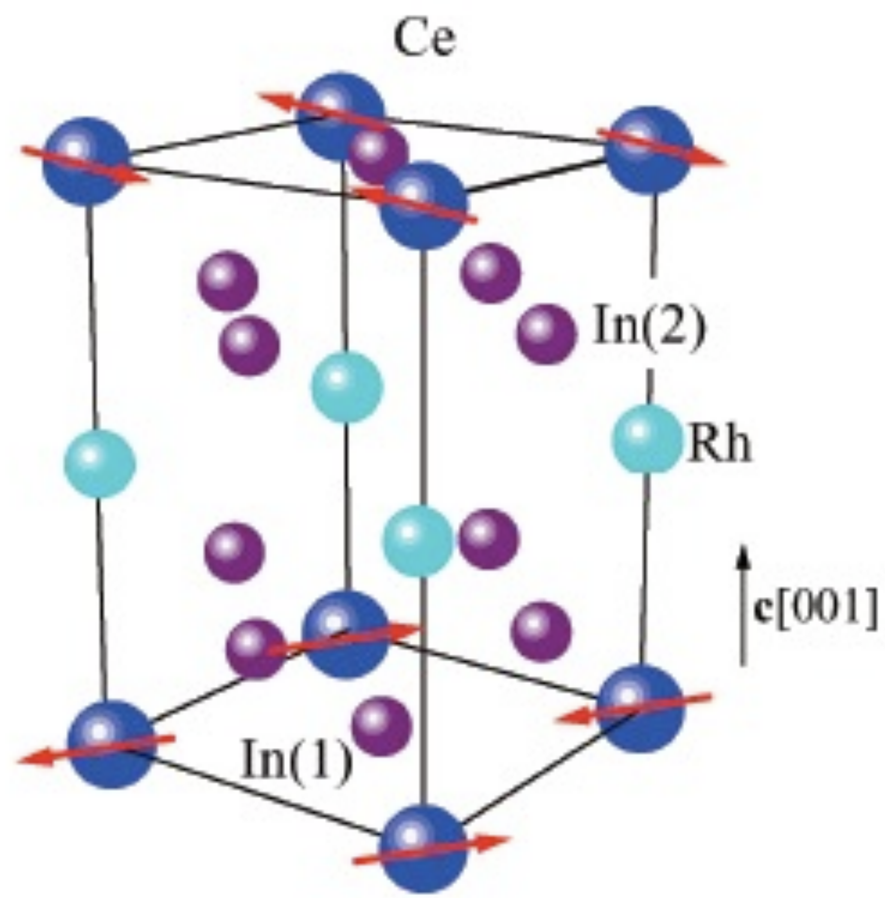
$BaFe_2(As_{1-x}P_x)_2$

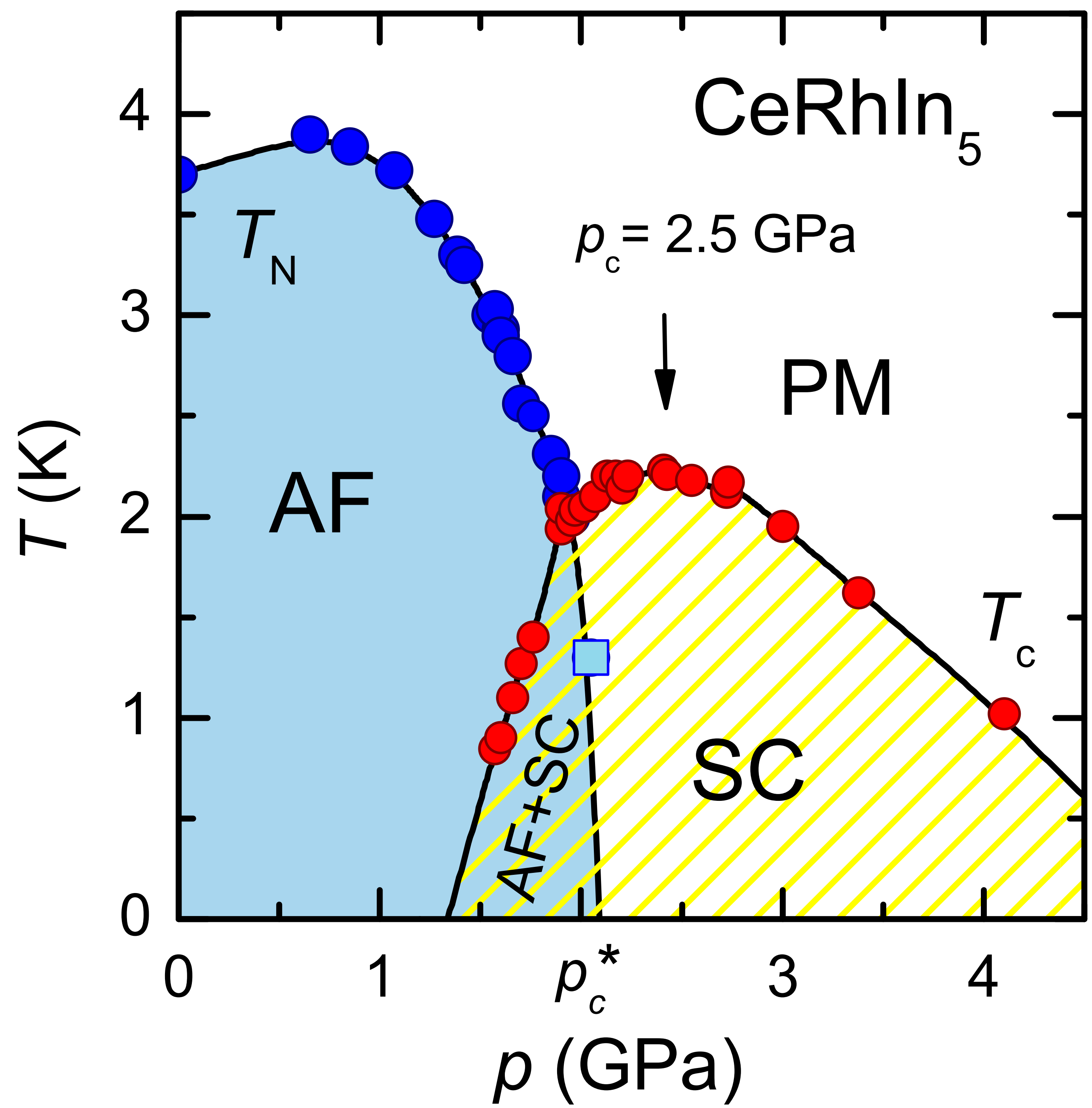




K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

Lower T_c superconductivity in the heavy fermion compounds





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

Outline

1. Weak coupling theory

2. Universal critical theory

3. Quantum Monte Carlo
   without the sign problem

4. Features of strong coupling

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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_{i} c^{\dagger}_{i\alpha} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

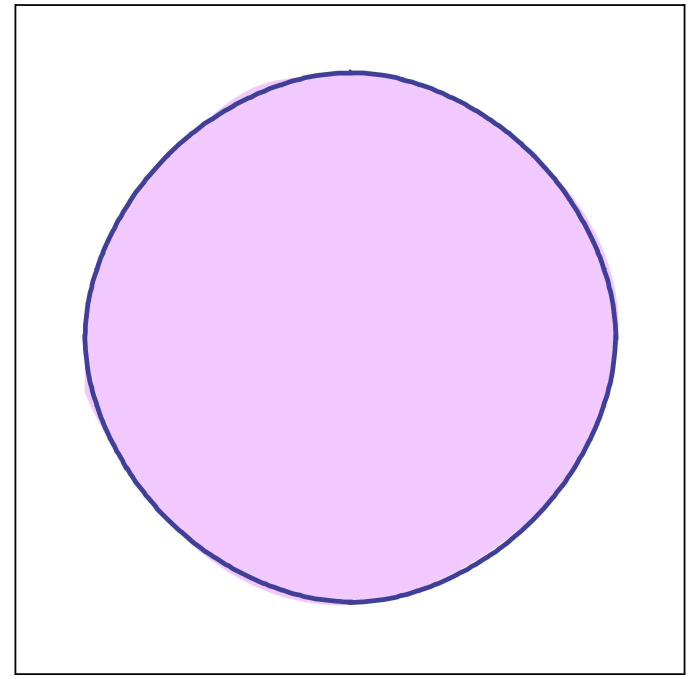
Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^{\dagger}_{i\alpha} c_{i\alpha}$$

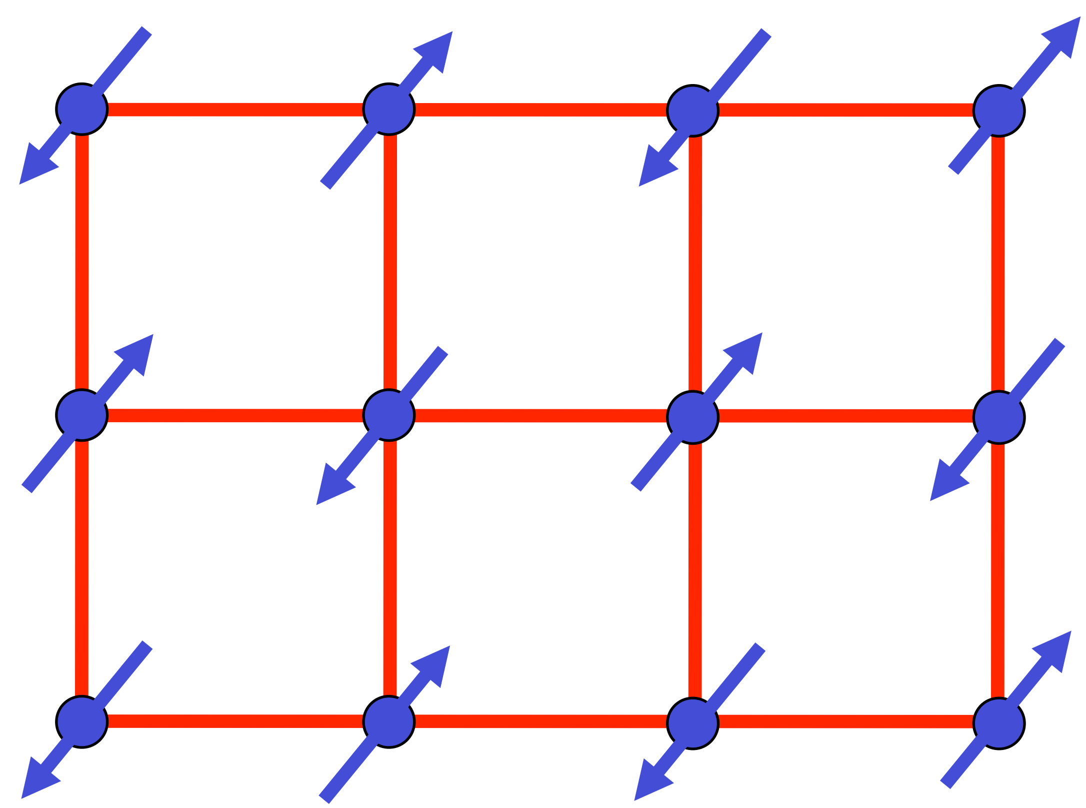
$$c^{\dagger}_{i\alpha} c_{j\beta} + c_{j\beta} c^{\dagger}_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



+



The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

The Hubbard Model

Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2 r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi} \right]$$

$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda\, \varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b.$$

"Yukawa" coupling between fermions and antiferromagnetic order:
$\lambda^2 \sim U$, the Hubbard repulsion

The Hubbard Model

Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2 r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi} \right]$$

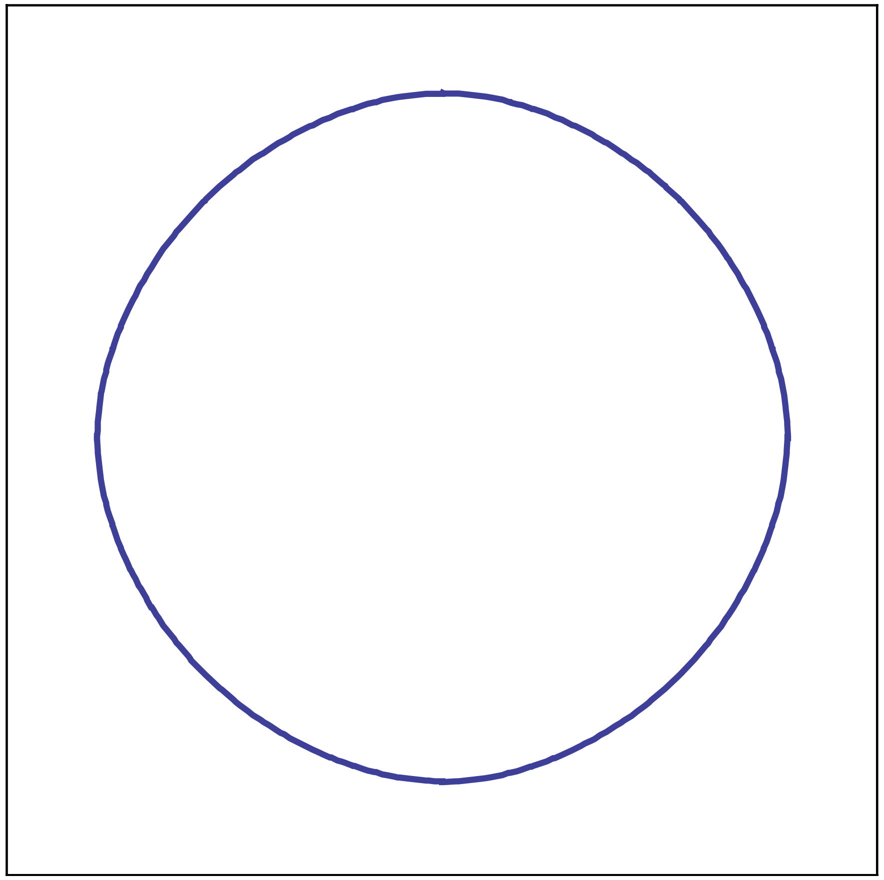
$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2} (\boldsymbol{\nabla} \varphi_\alpha)^2 + \frac{r}{2} \varphi_\alpha^2 + \frac{u}{4} \left(\varphi_\alpha^2 \right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda \, \varphi_\alpha \, e^{i\mathbf{K}\cdot\mathbf{r}} \, c_a^\dagger \, \sigma_{ab}^\alpha \, c_b .$$

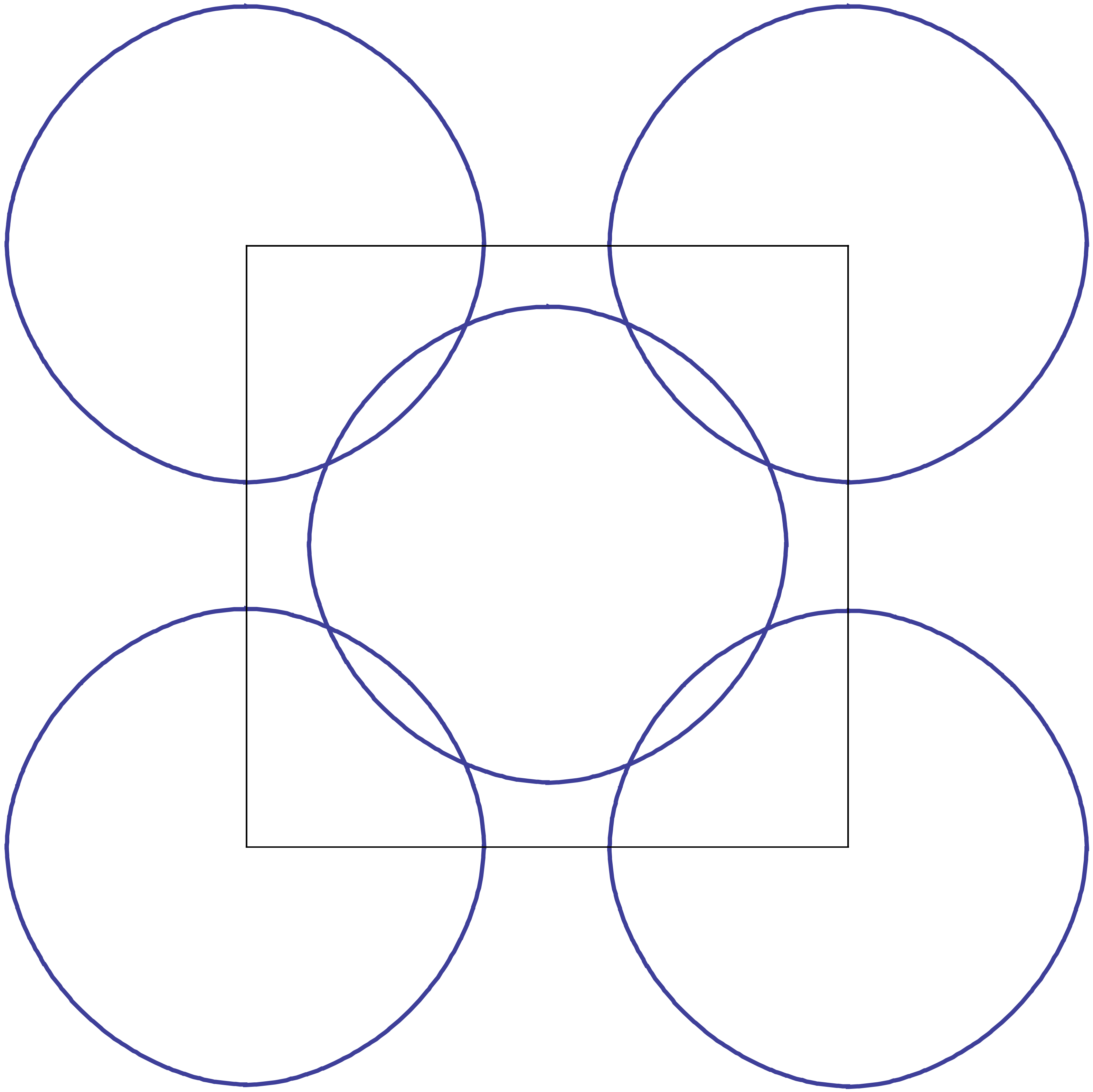
Hertz, Moriya, Millis: integrate out fermions, and focus on effective theory of damped excitations of the order parameter φ_α. Method *fails* in $d = 2$.

Fermi surface+antiferromagnetism



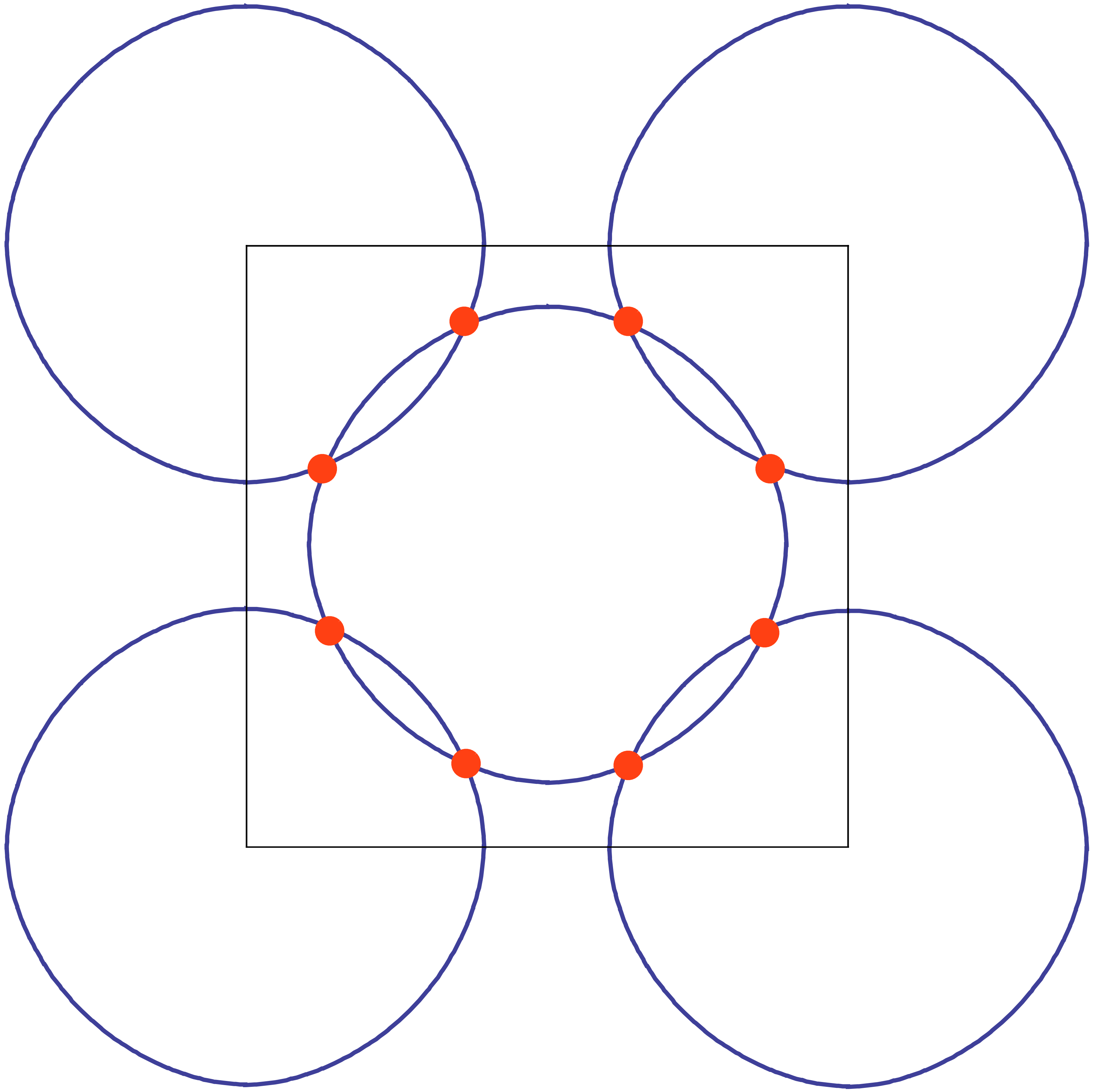
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



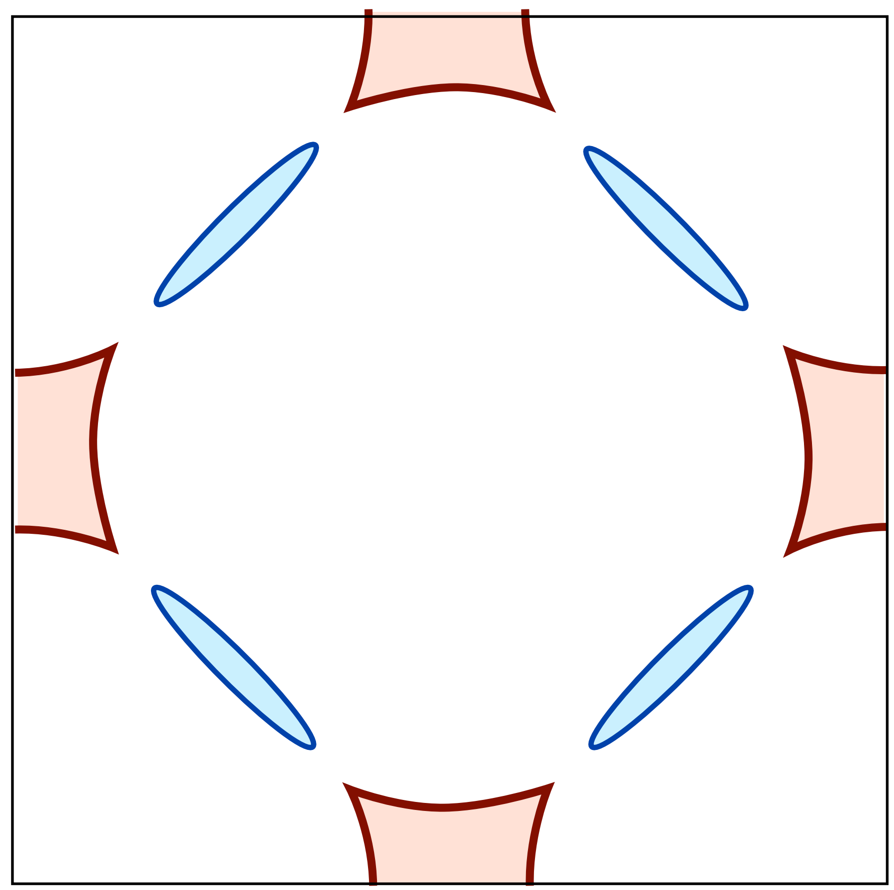
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



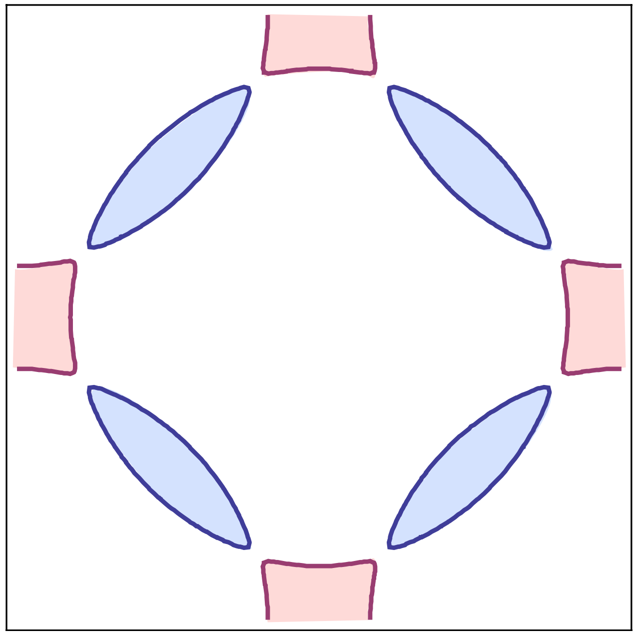
"Hot" spots

Fermi surface+antiferromagnetism



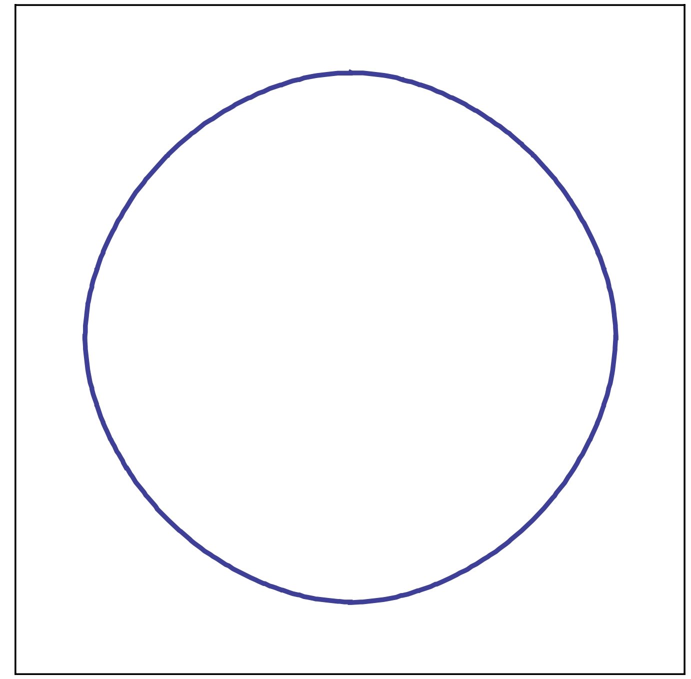
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



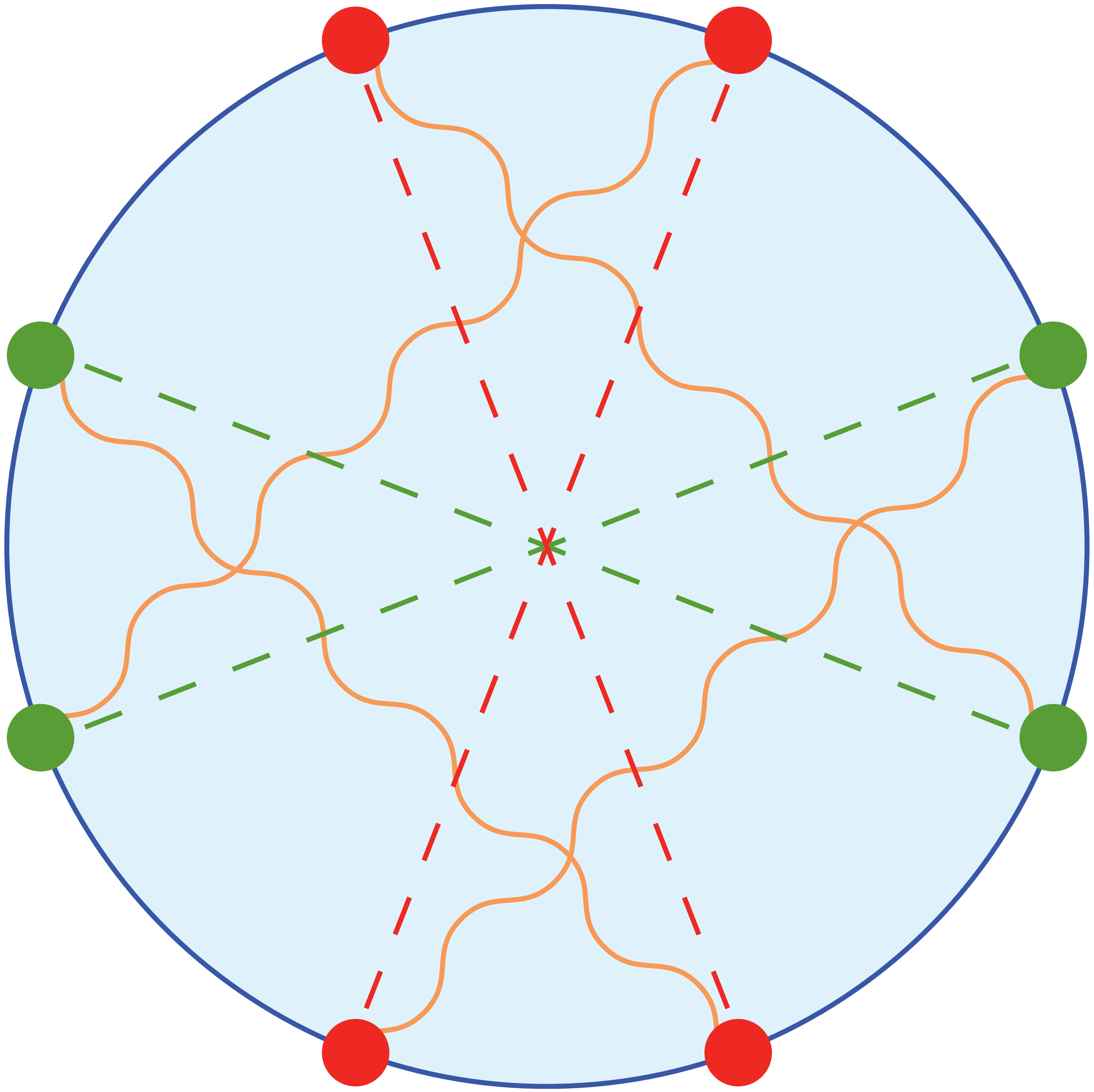
$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Pairing "glue" from antiferromagnetic fluctuations



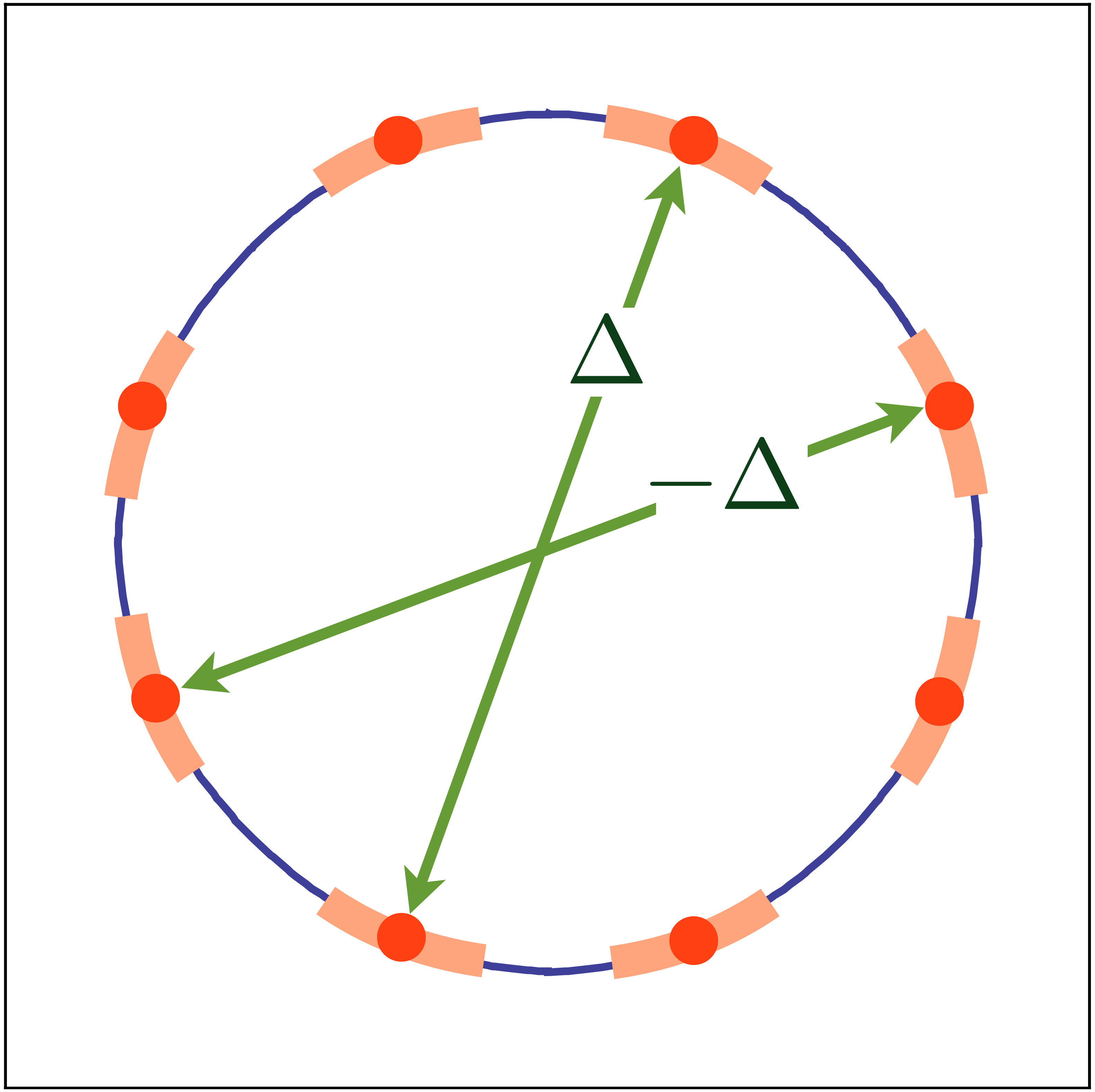
V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)

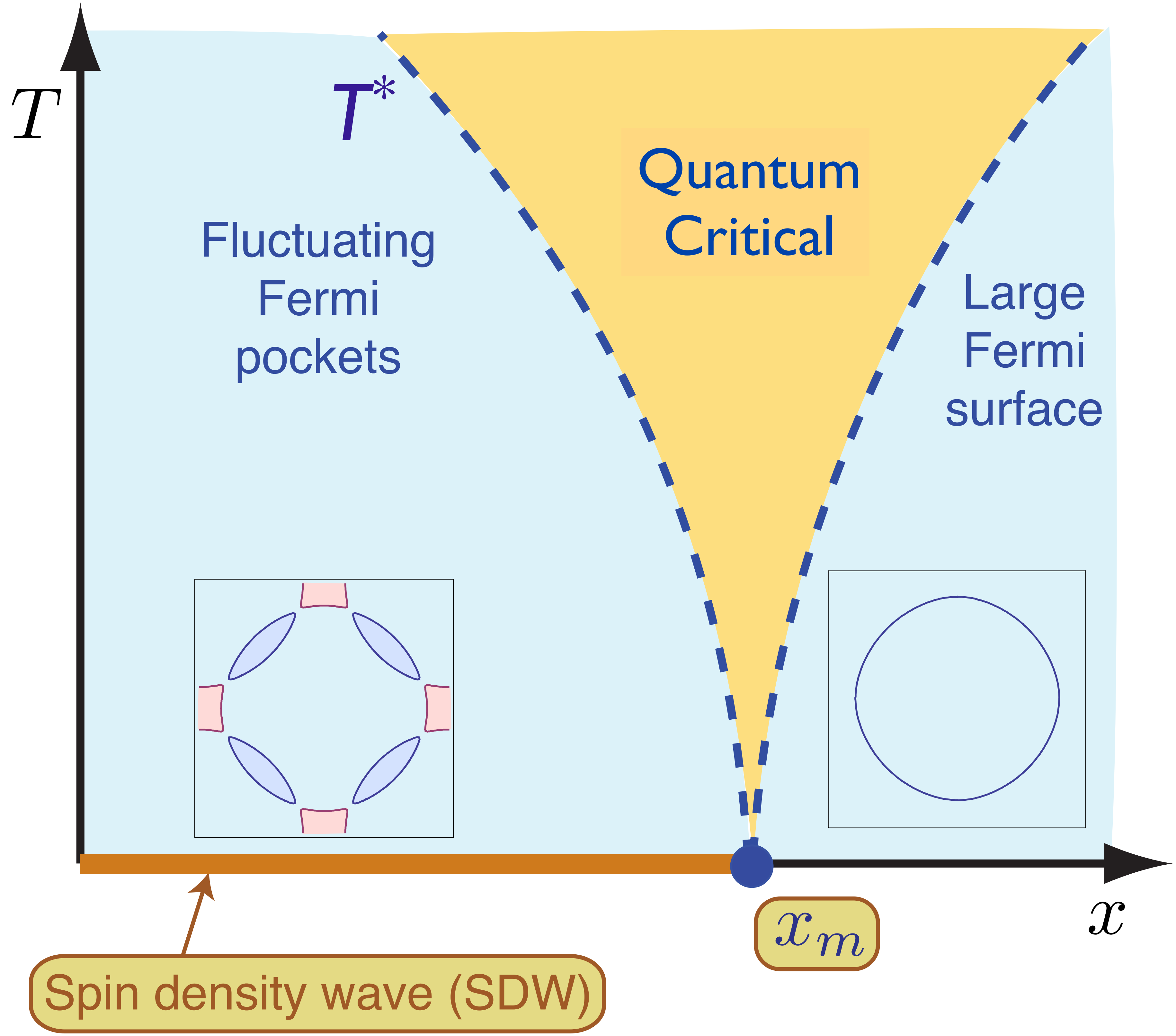
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



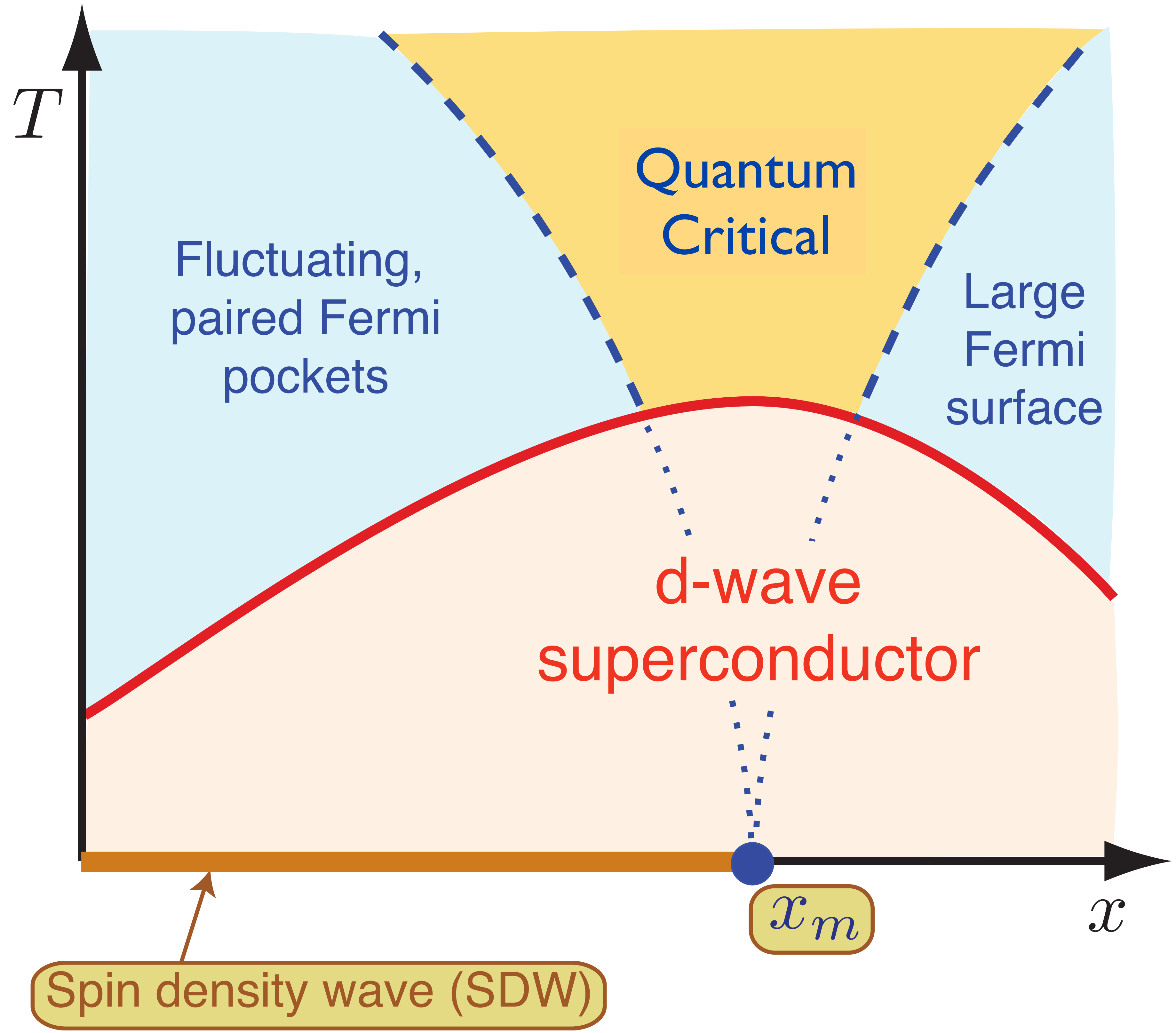
Unconventional pairing at *and near* hot spots

Fermi surface+antiferromagnetism



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor

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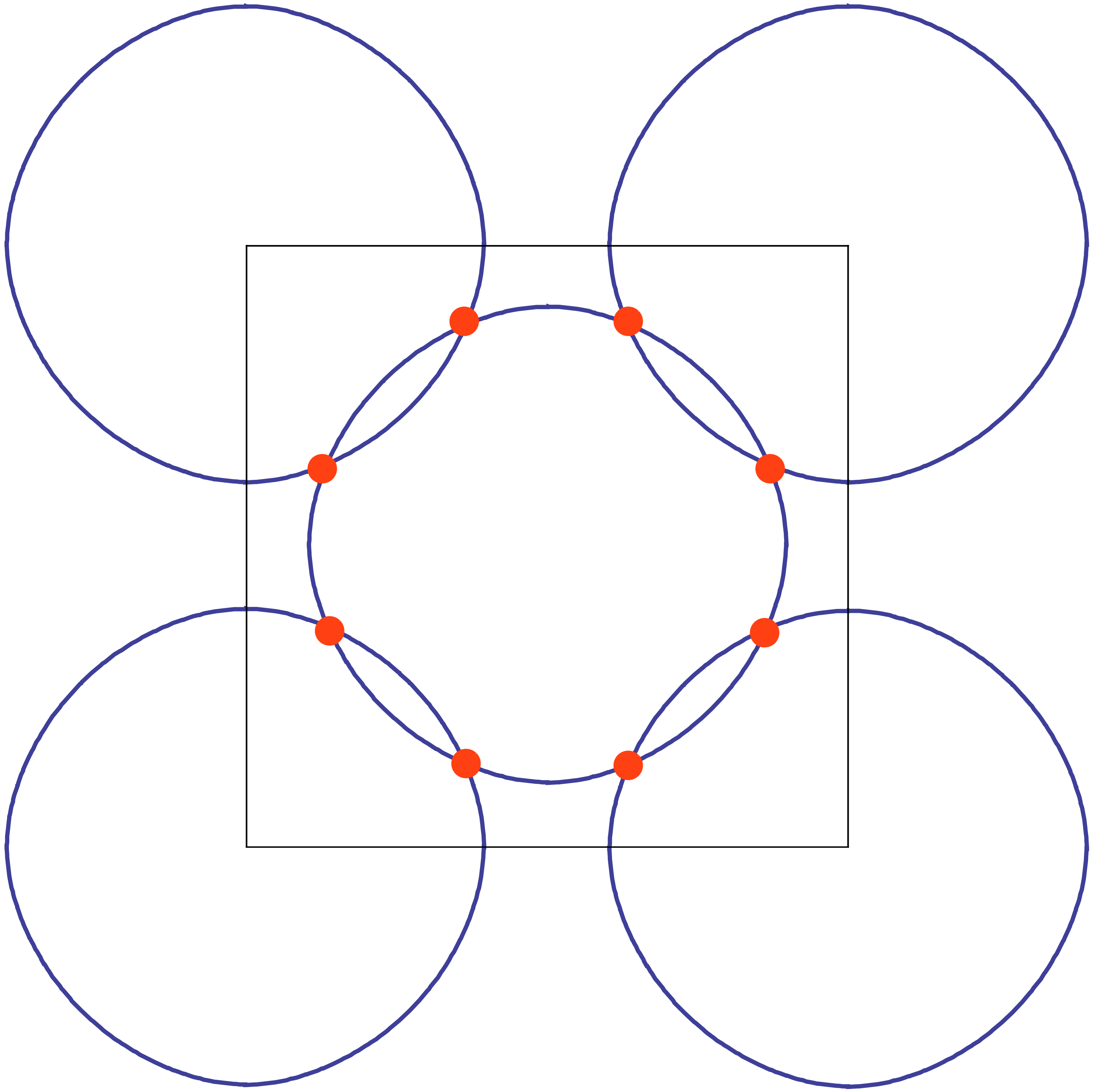
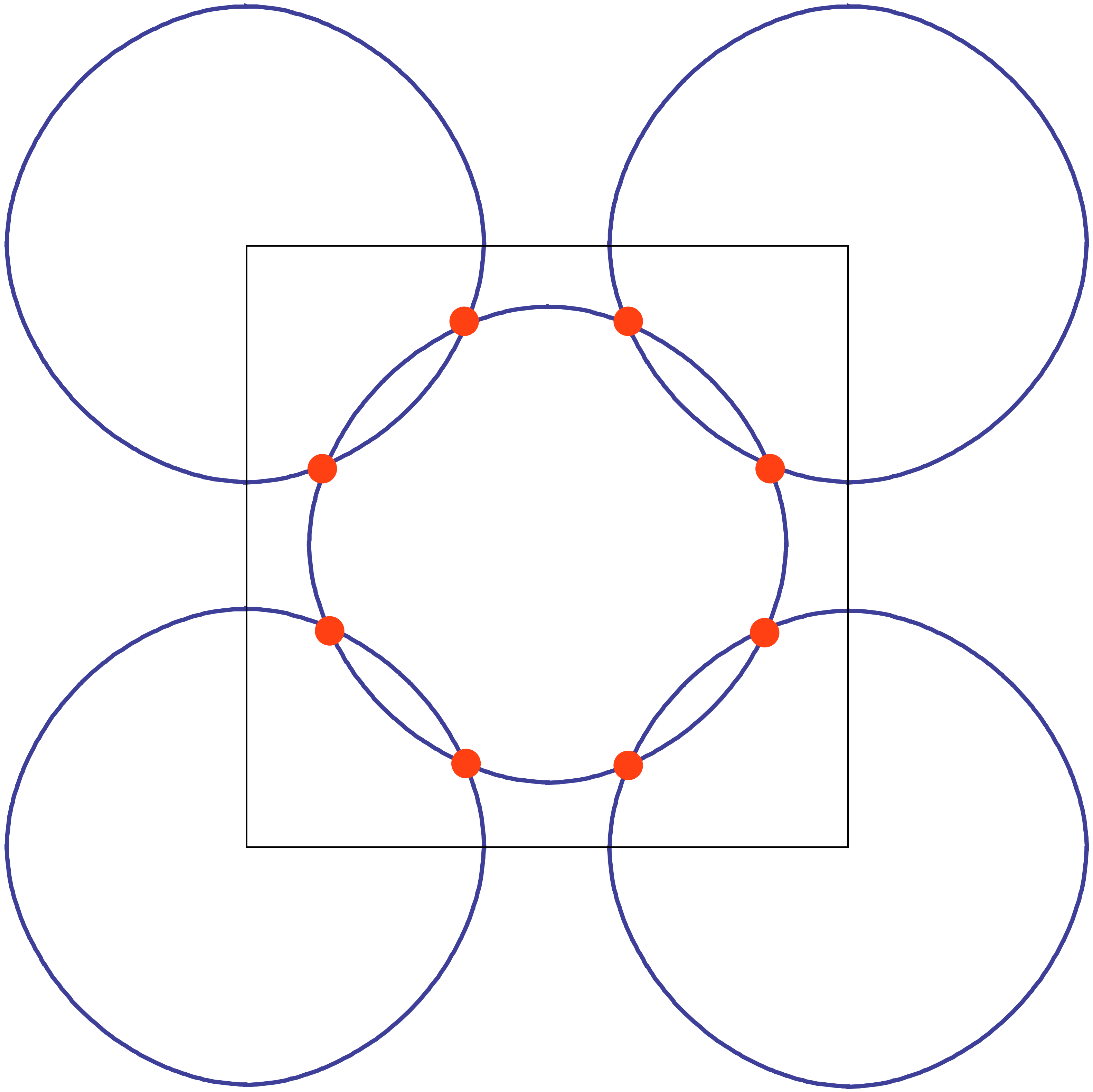
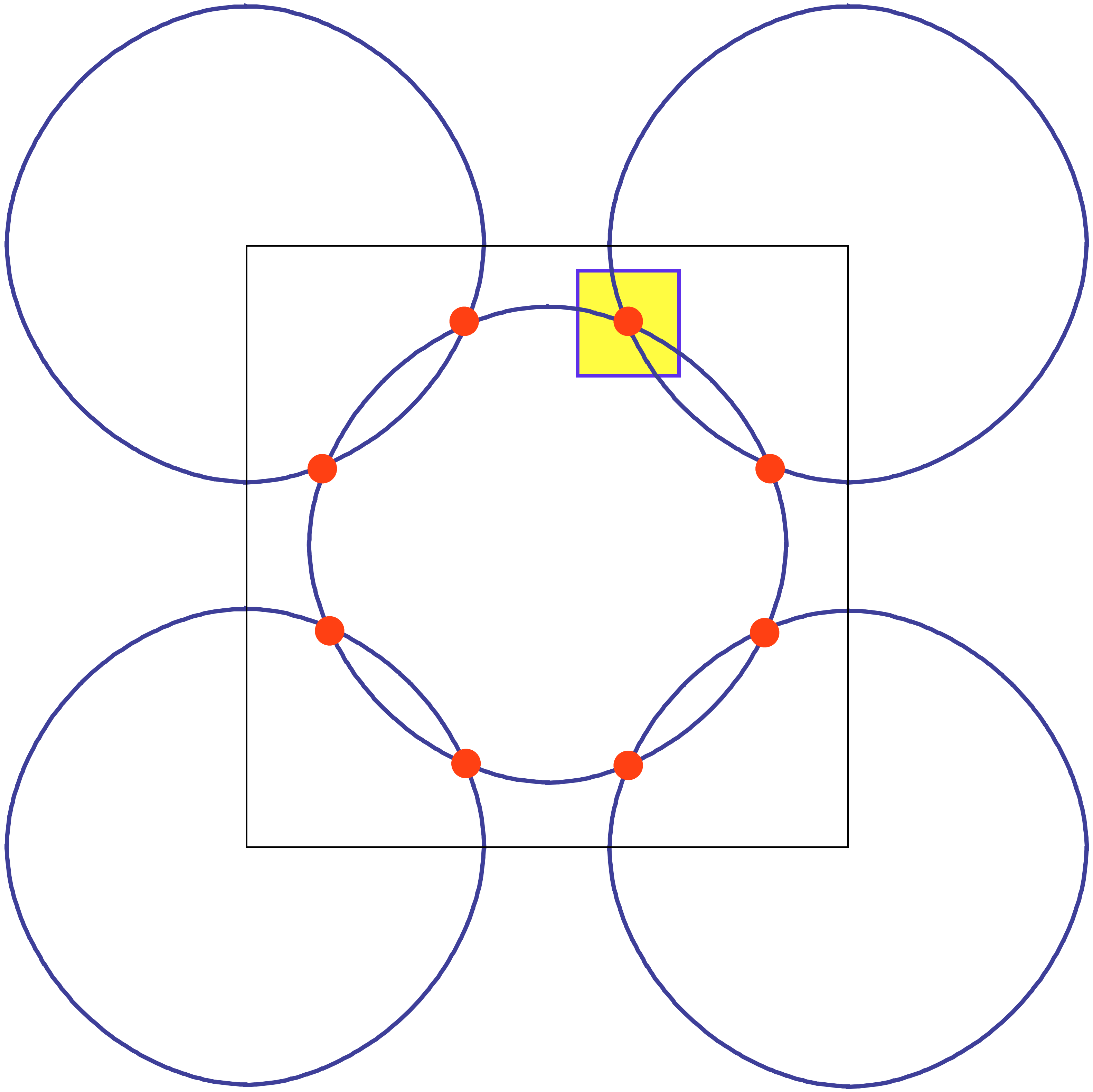


“Hot” spots

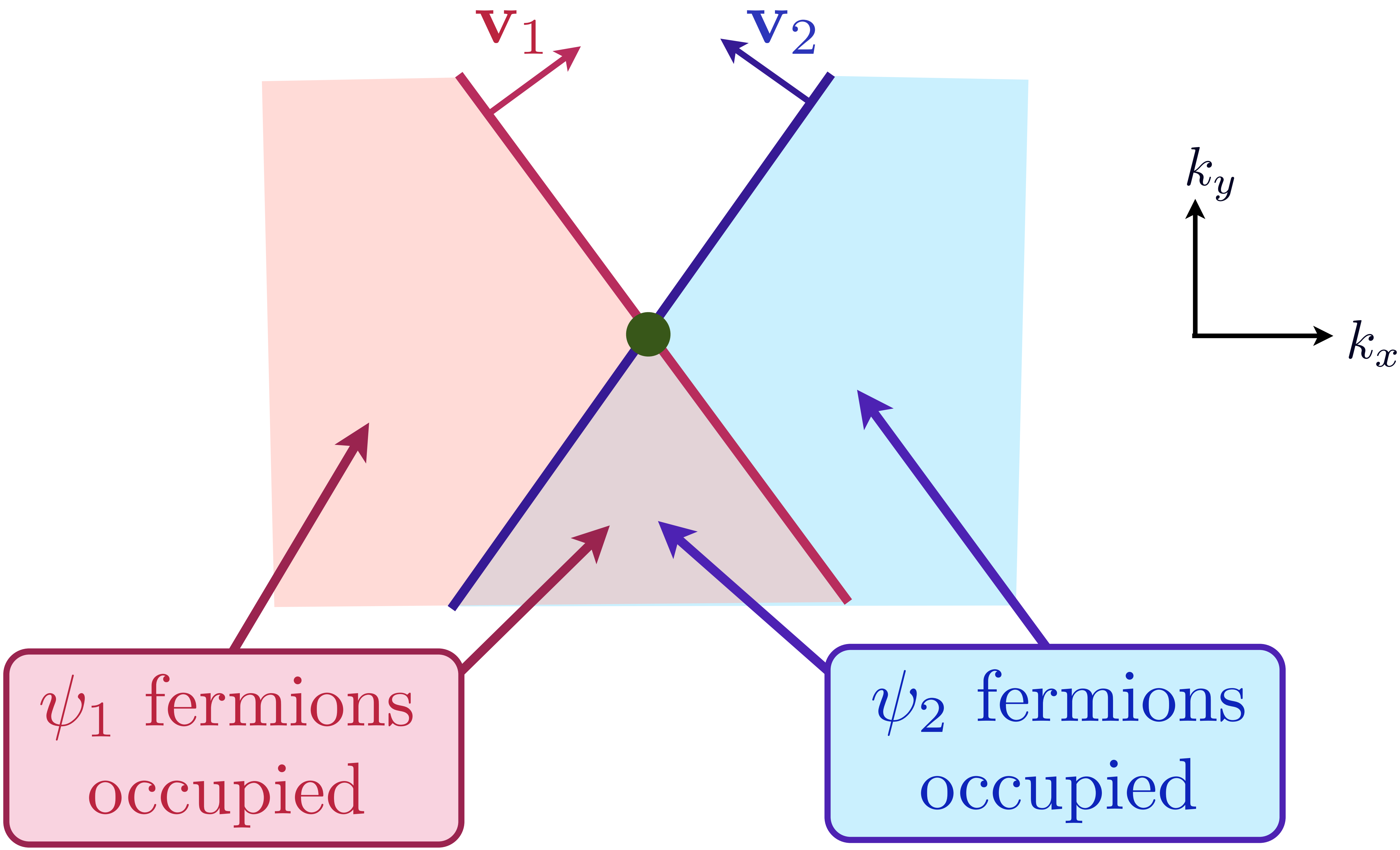


Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$)
and boson order parameter $\vec{\varphi}$,
interacting with coupling λ



Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ

$$
\begin{aligned}
\mathcal{L} \;=\;& \psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha} \\
&+ \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 \\
&- \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta}\right)
\end{aligned}
$$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ

$$
\begin{aligned}
\mathcal{L} \;=\;& \psi_{1\alpha}^{\dagger}\left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha} + \psi_{2\alpha}^{\dagger}\left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha} \\
& + \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 \\
& - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^{\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta} + \psi_{2\alpha}^{\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}\right)
\end{aligned}
$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)
S.A. Hartnoll, D.M. Hofman, M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **84**, 125115 (2011)

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha} \\
& + \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \\
& - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right)
\end{aligned}
$$

Theory flows to strong-coupling in $d = 2$.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)
S.A. Hartnoll, D.M. Hofman, M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **84**, 125115 (2011)

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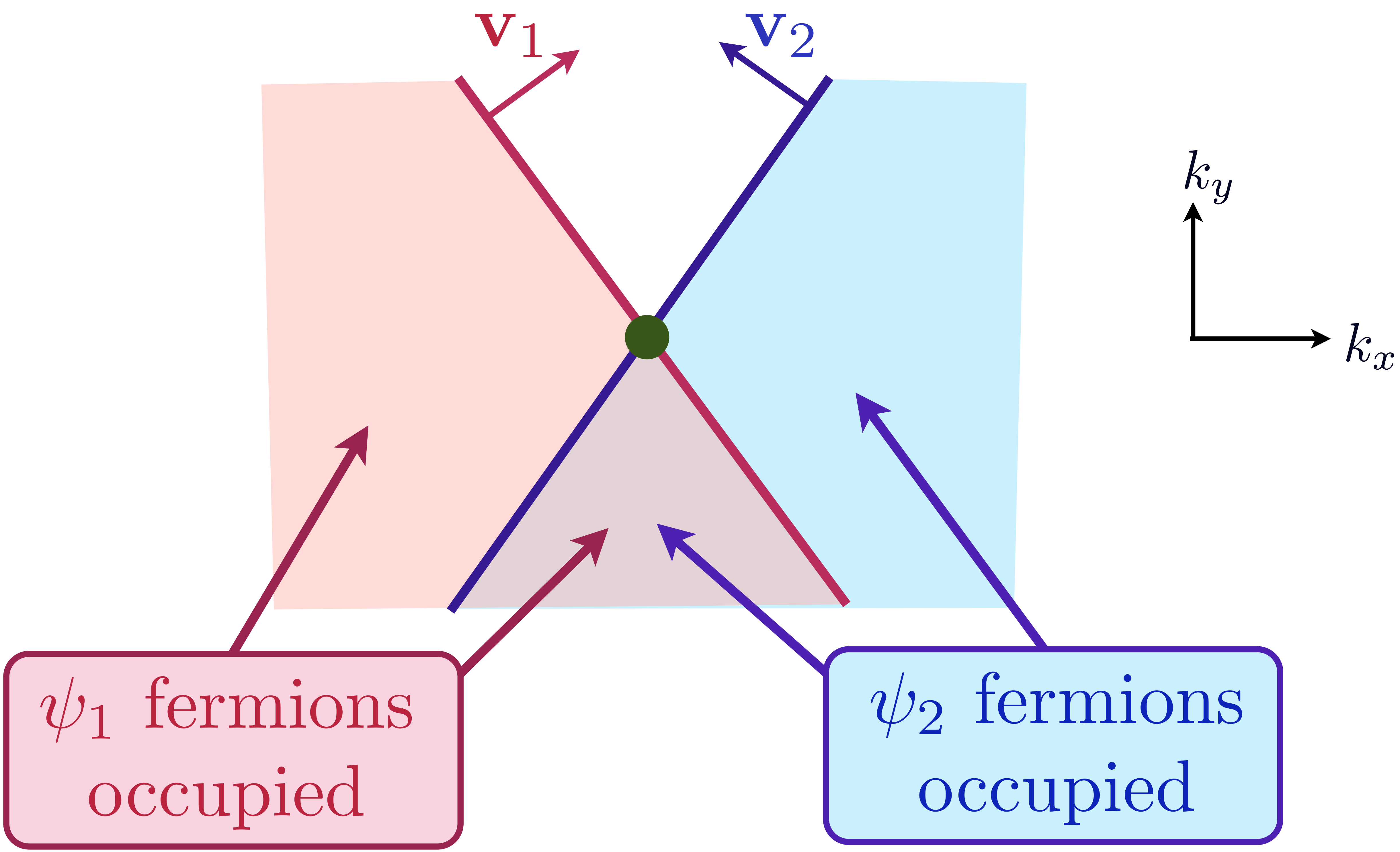
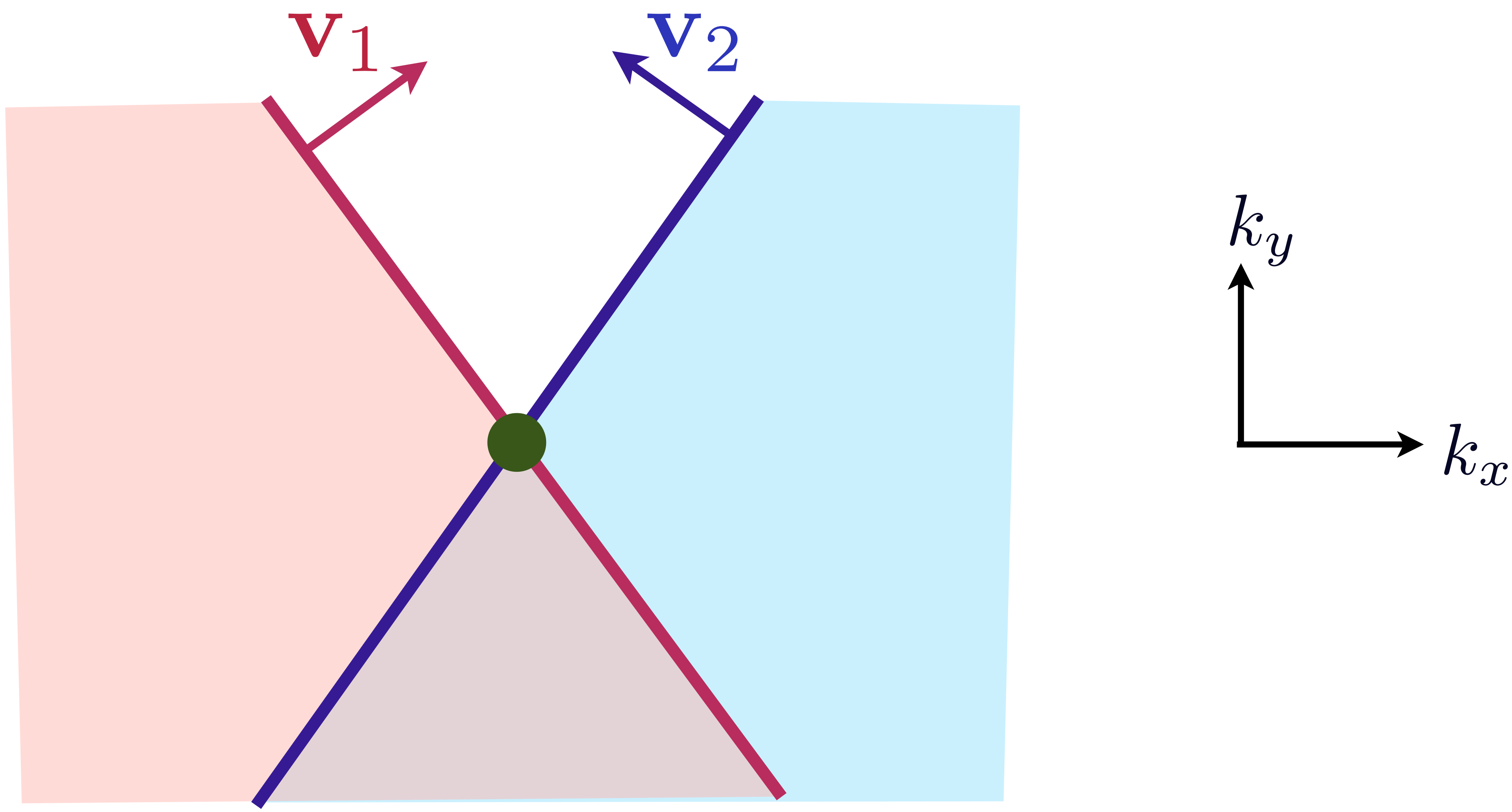
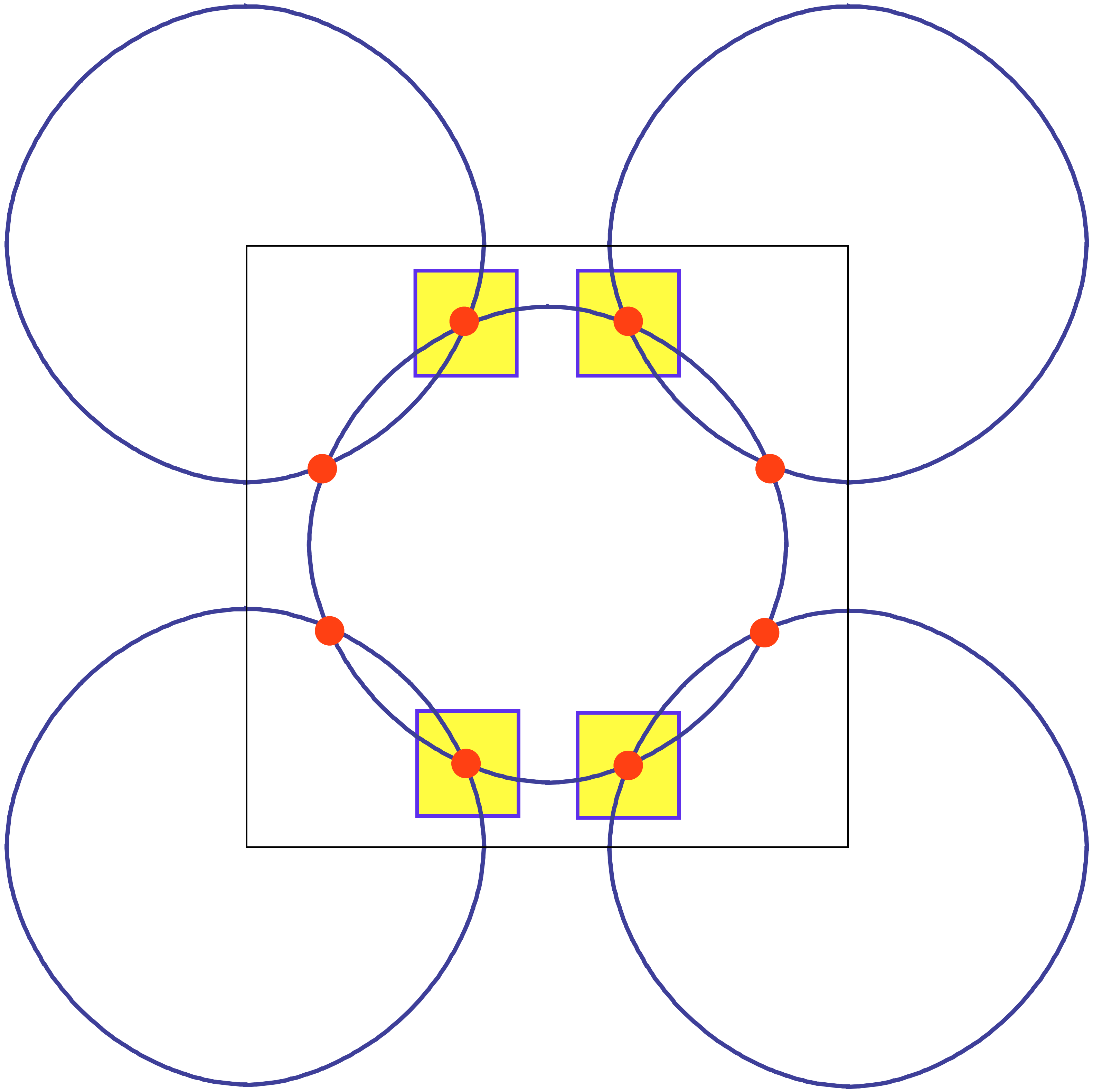


Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ

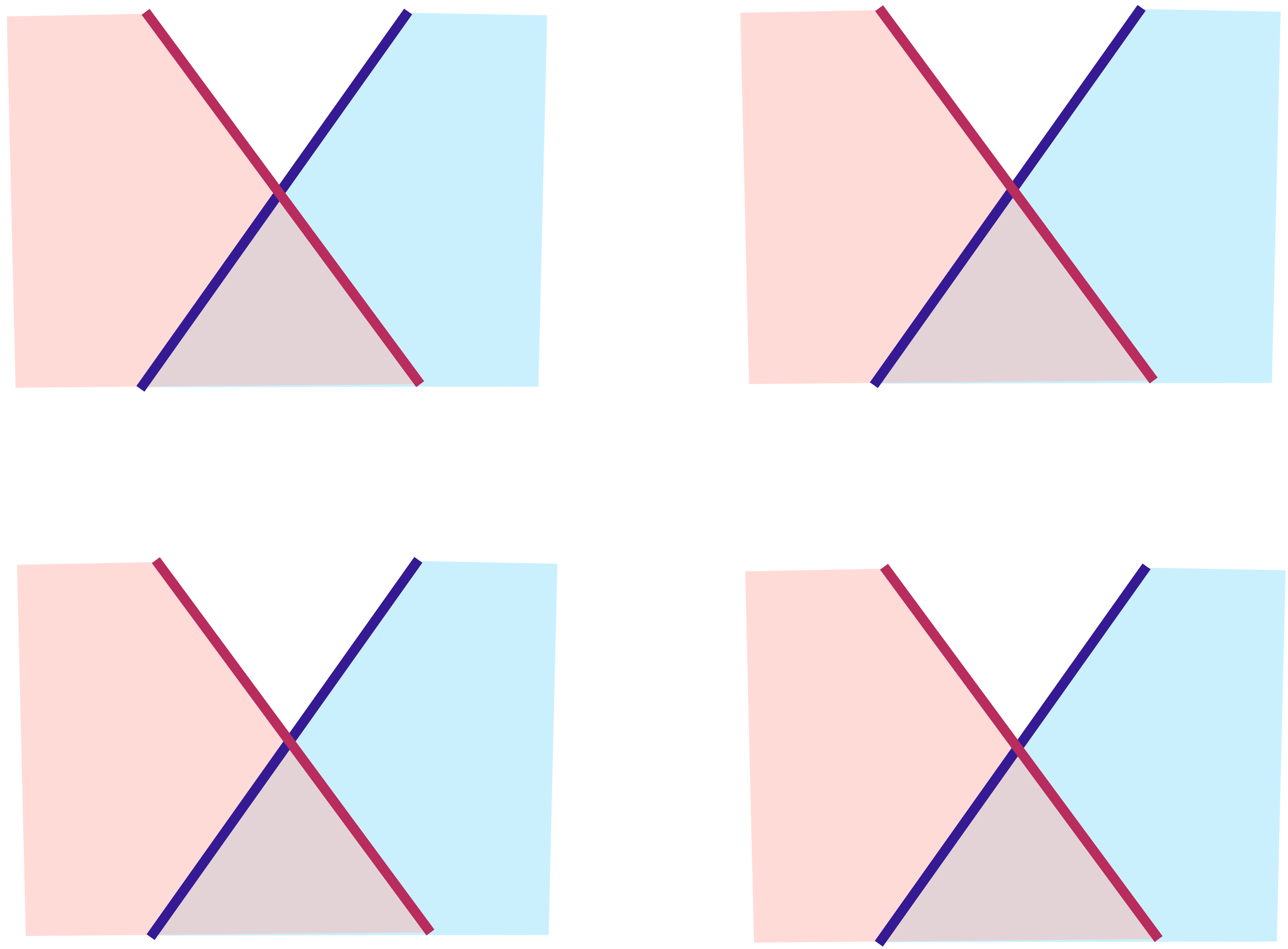


To faithfully realize low energy theory in quantum Monte Carlo, we need a UV completion in which Fermi lines don't end and all weights are positive.

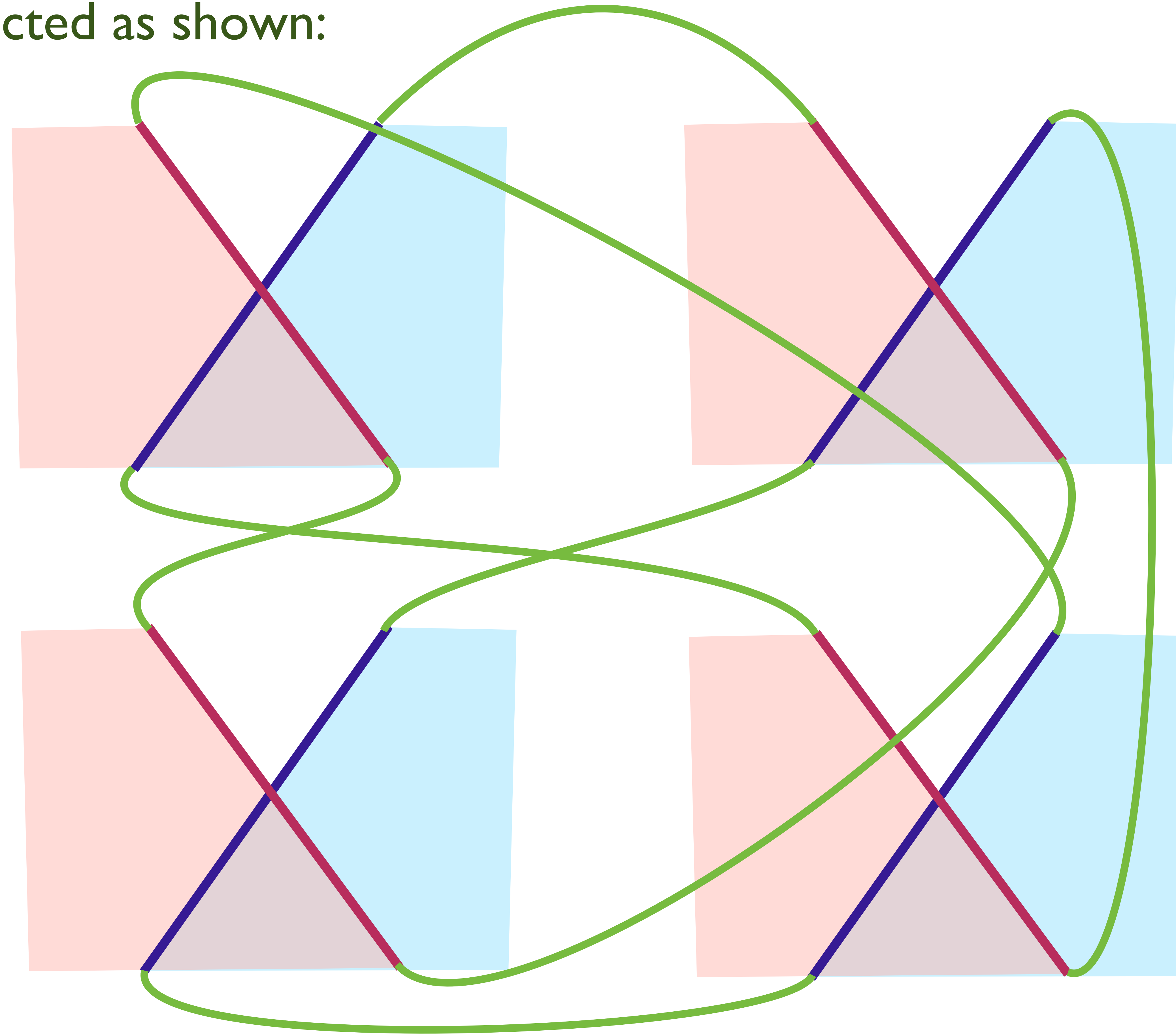


Low energy theory for critical point near hot spots

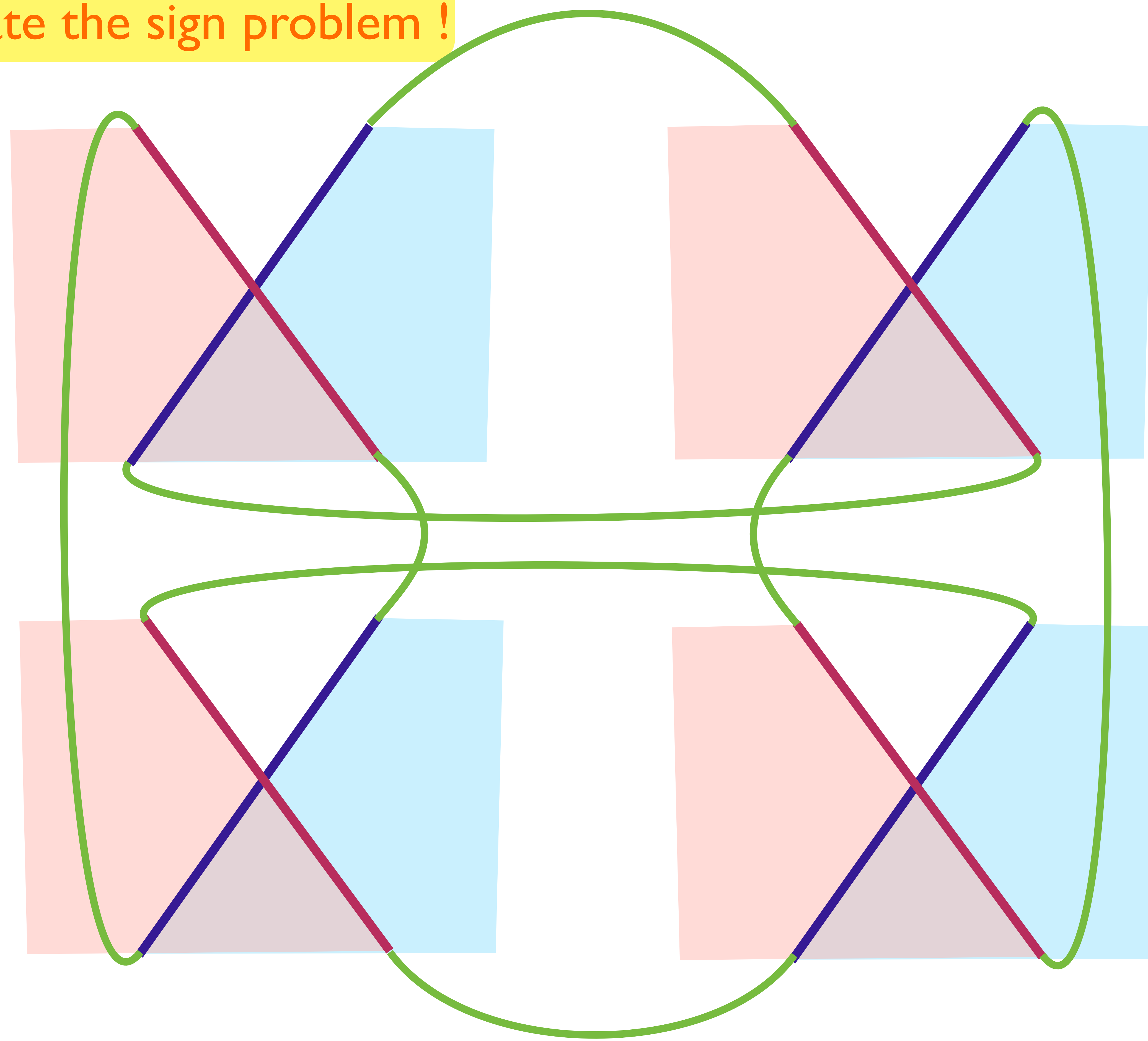
We have 4 copies
of the hot spot theory......



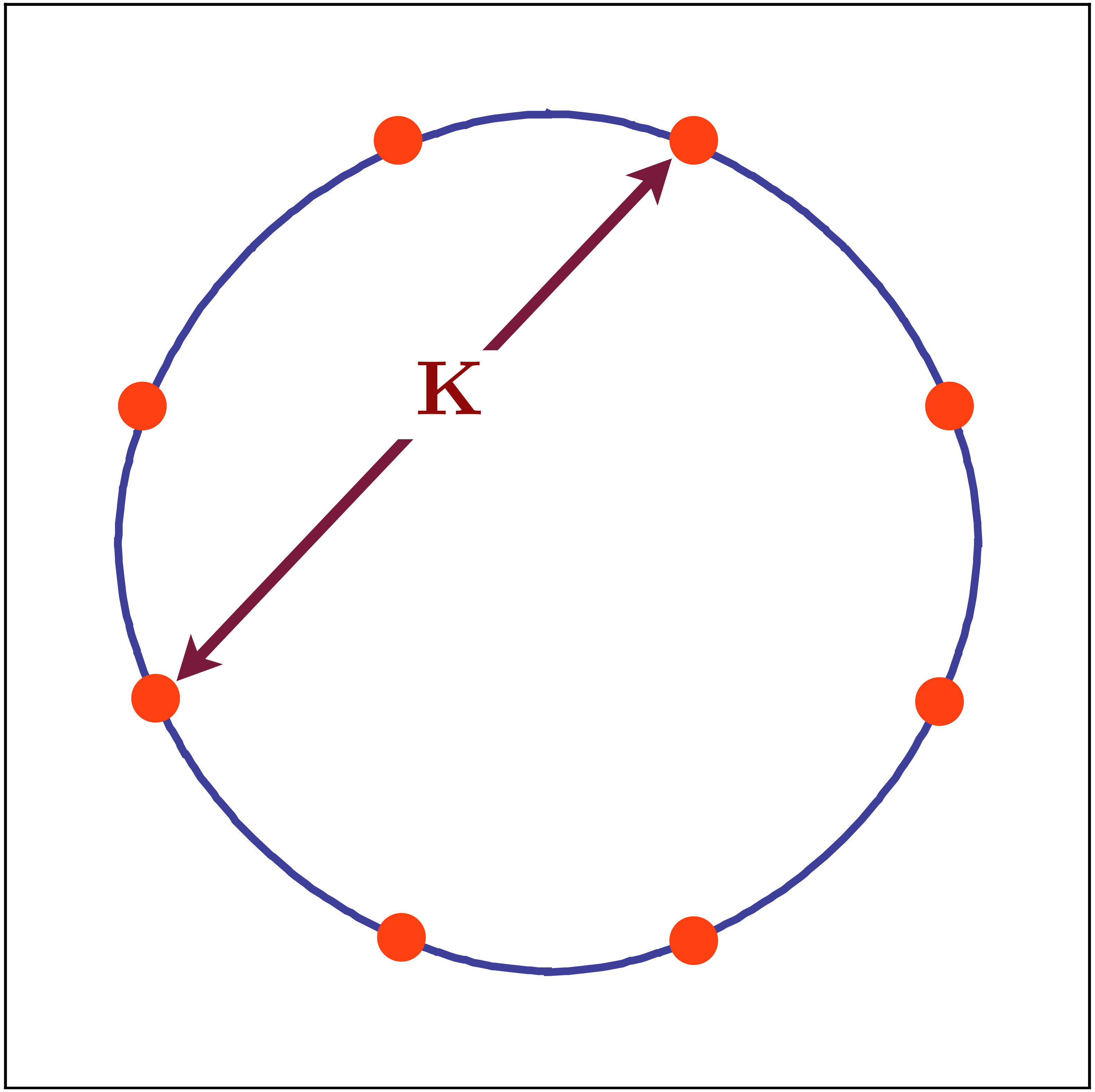
and their Fermi lines are connected as shown:



Reconnect Fermi lines and eliminate the sign problem !

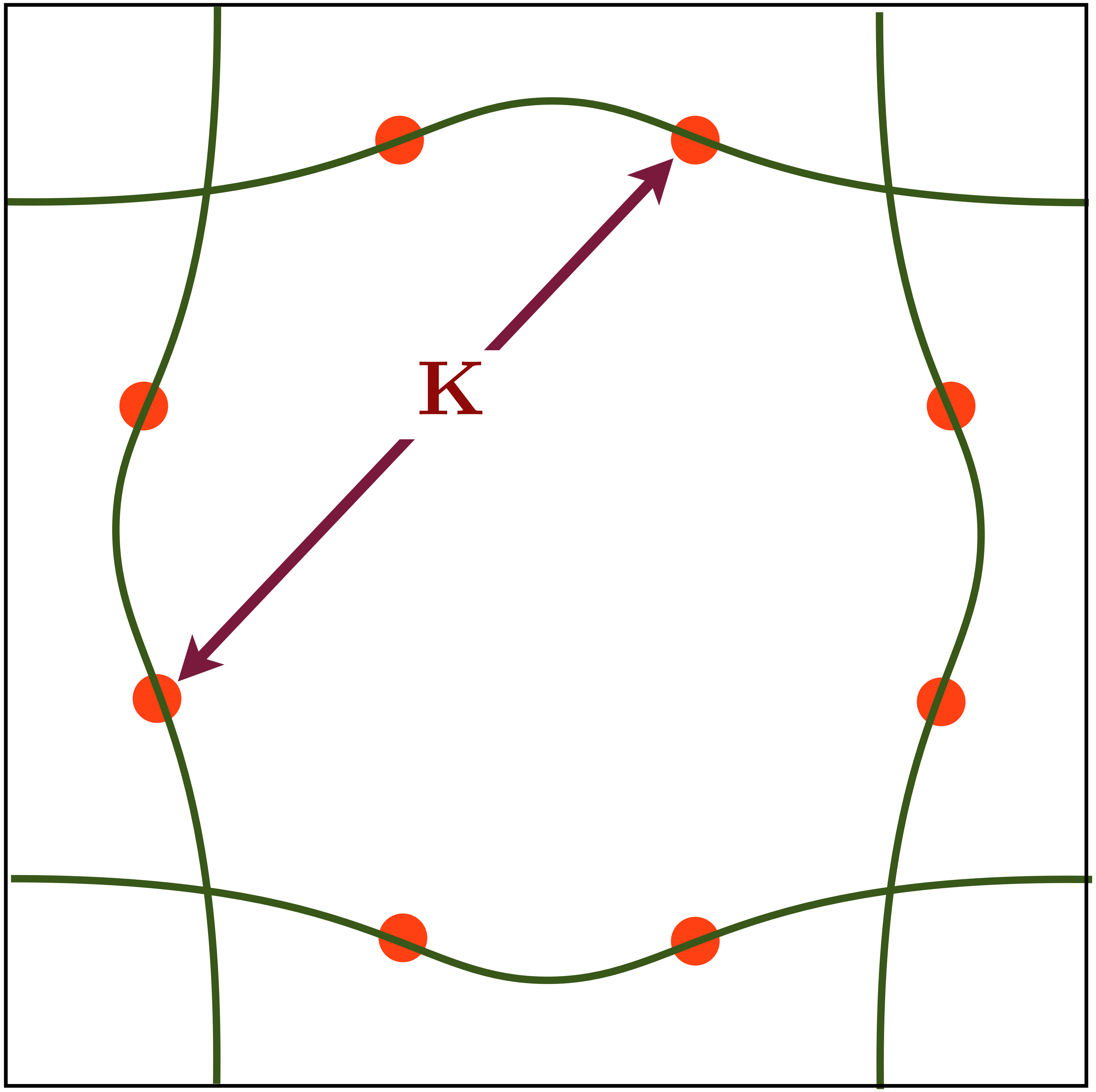


QMC for the onset of antiferromagnetism



Hot spots in a single band model

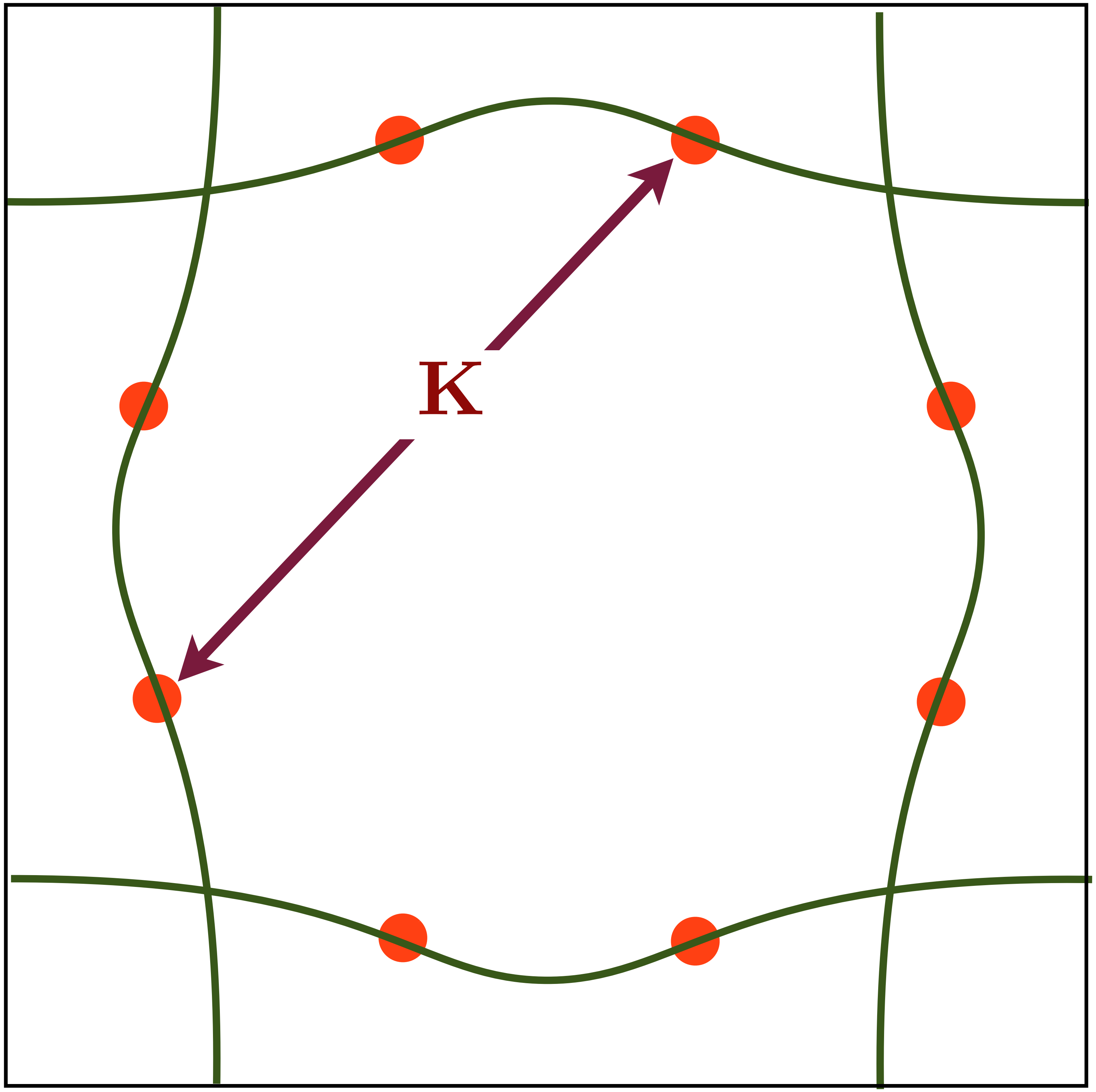
QMC for the onset of antiferromagnetism



Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

QMC for the onset of antiferromagnetism



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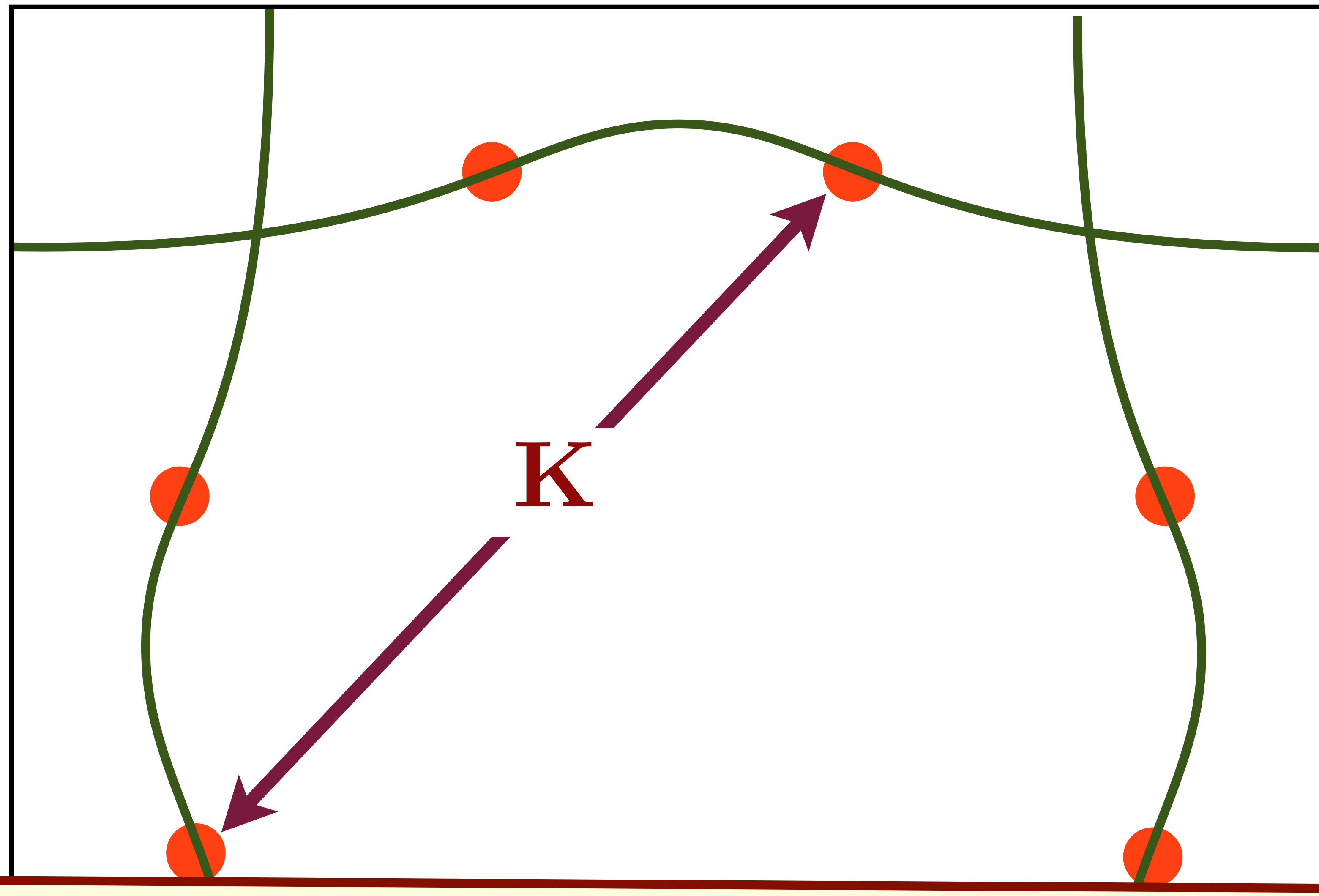

E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

No sign problem in
fermion determinant Monte Carlo !

Determinant is positive because of Kramer's degeneracy, and no additional symmetries are needed; holds for arbitrary band structure and band filling, provided ***K*** only connects hot spots in distinct bands

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)}$$

$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

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$$+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)}$$

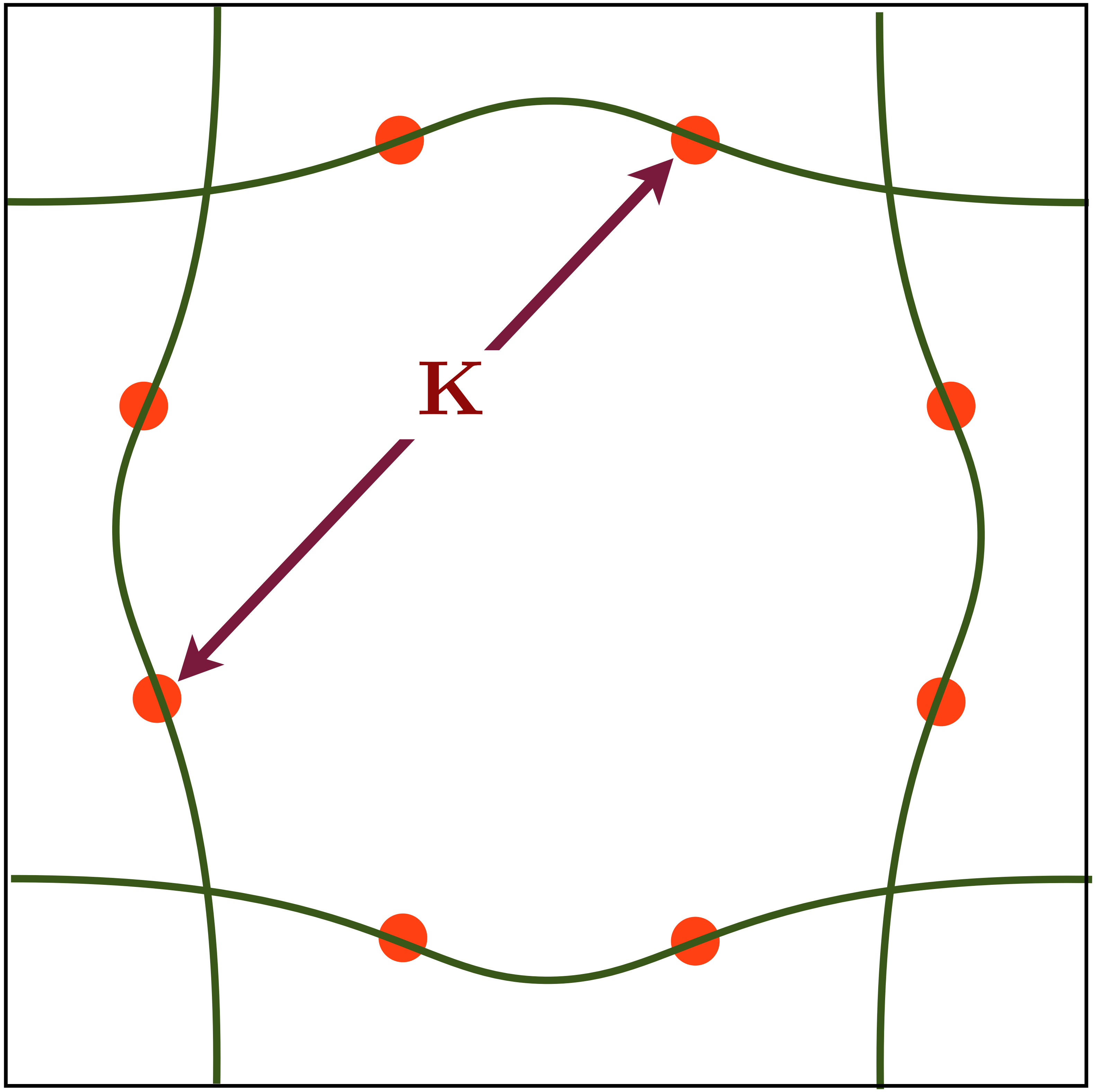
$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.}$$

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

No sign problem !

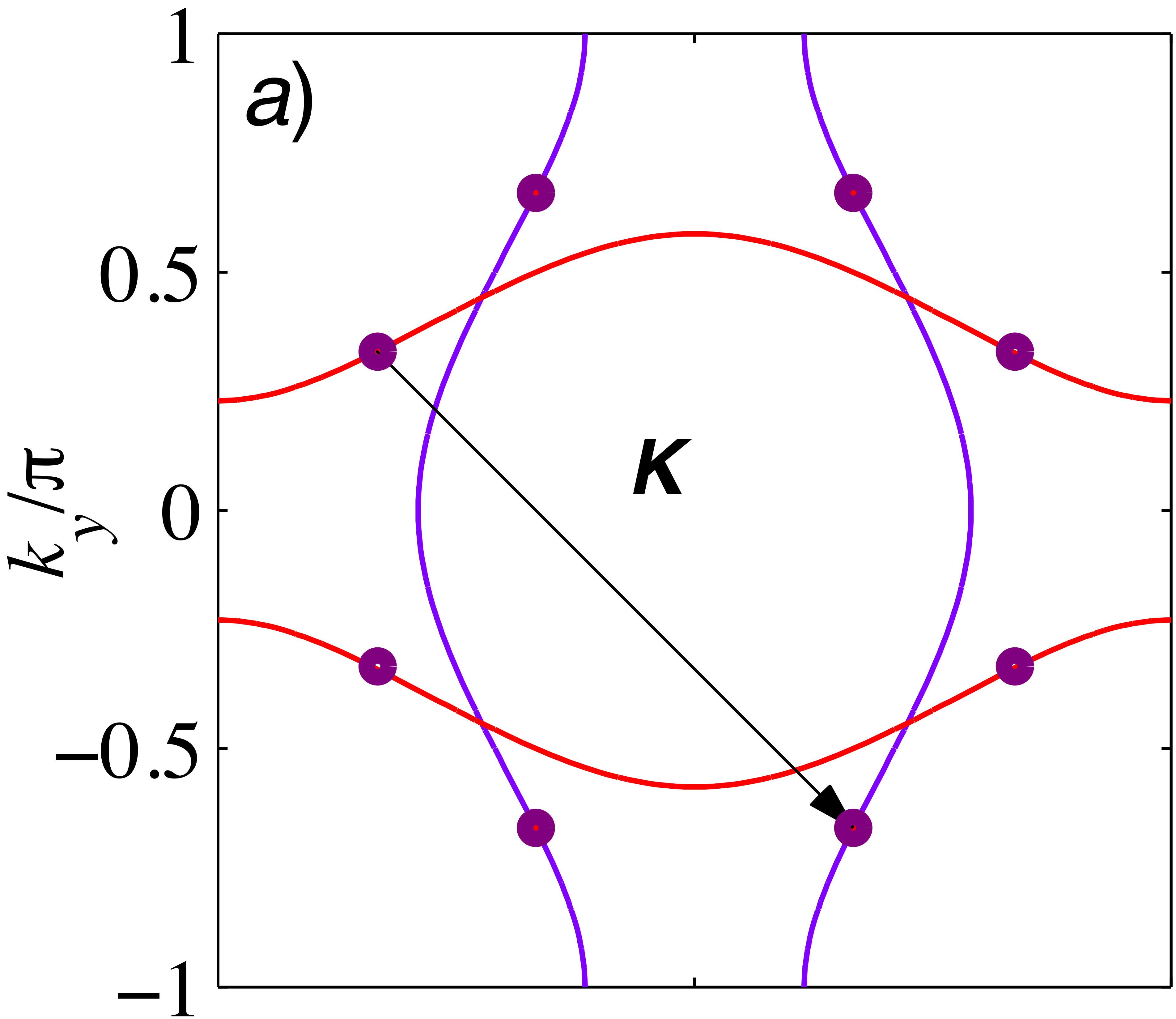
QMC for the onset of antiferromagnetism



Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

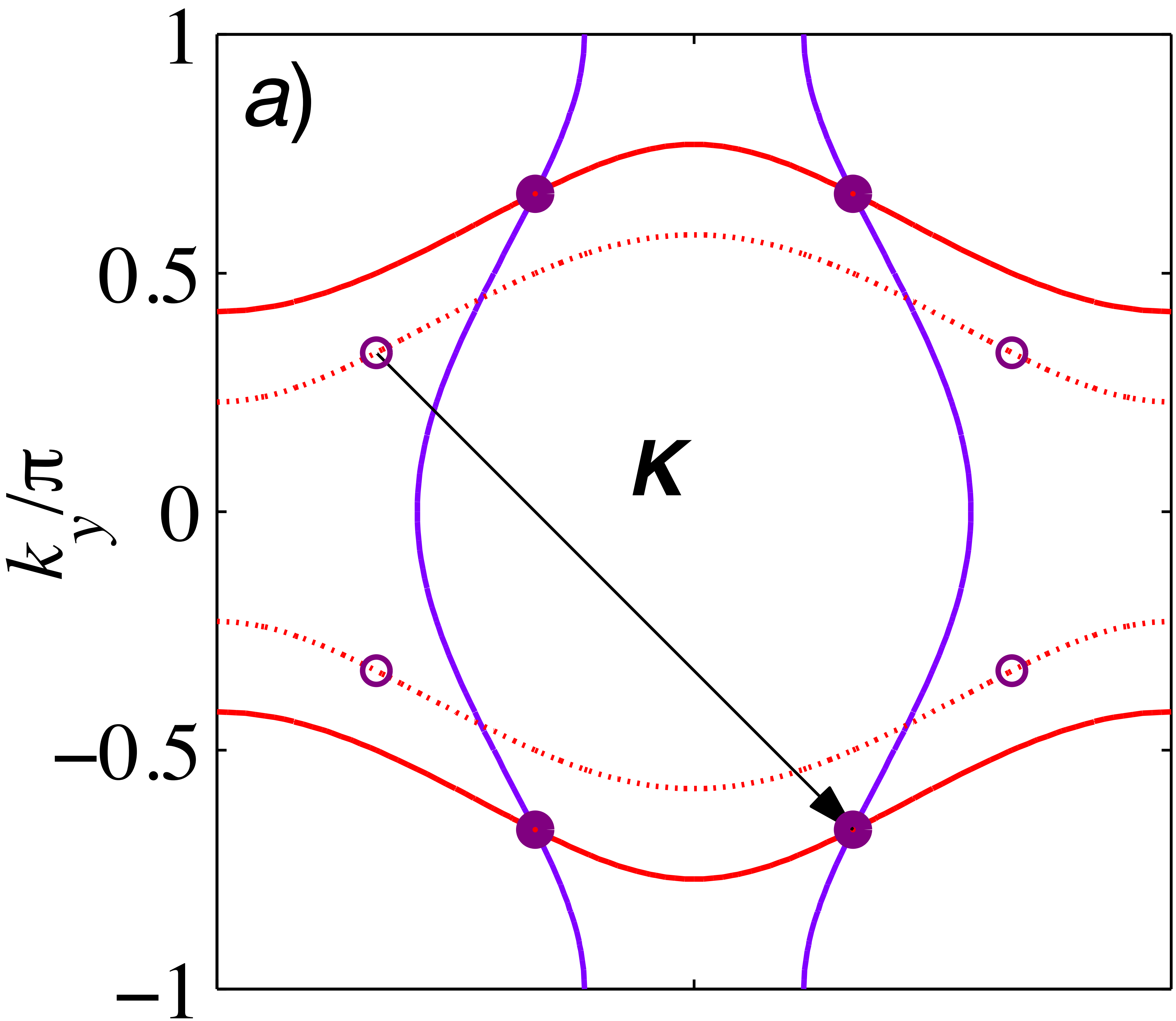
QMC for the onset of antiferromagnetism



Center Brillouin zone at $(\pi,\pi,)$

E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
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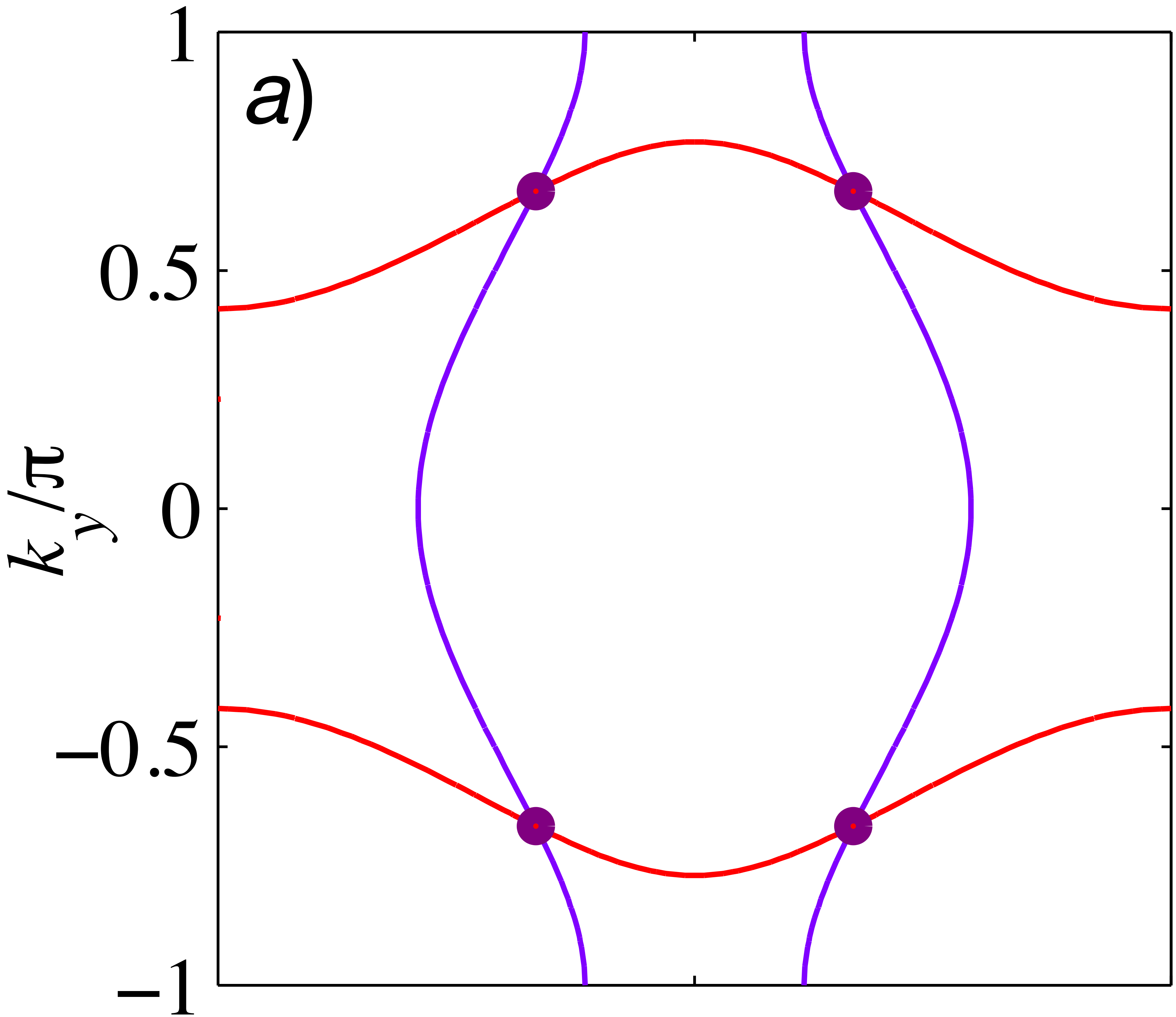
Move one of the Fermi surface by (π,π,)

QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
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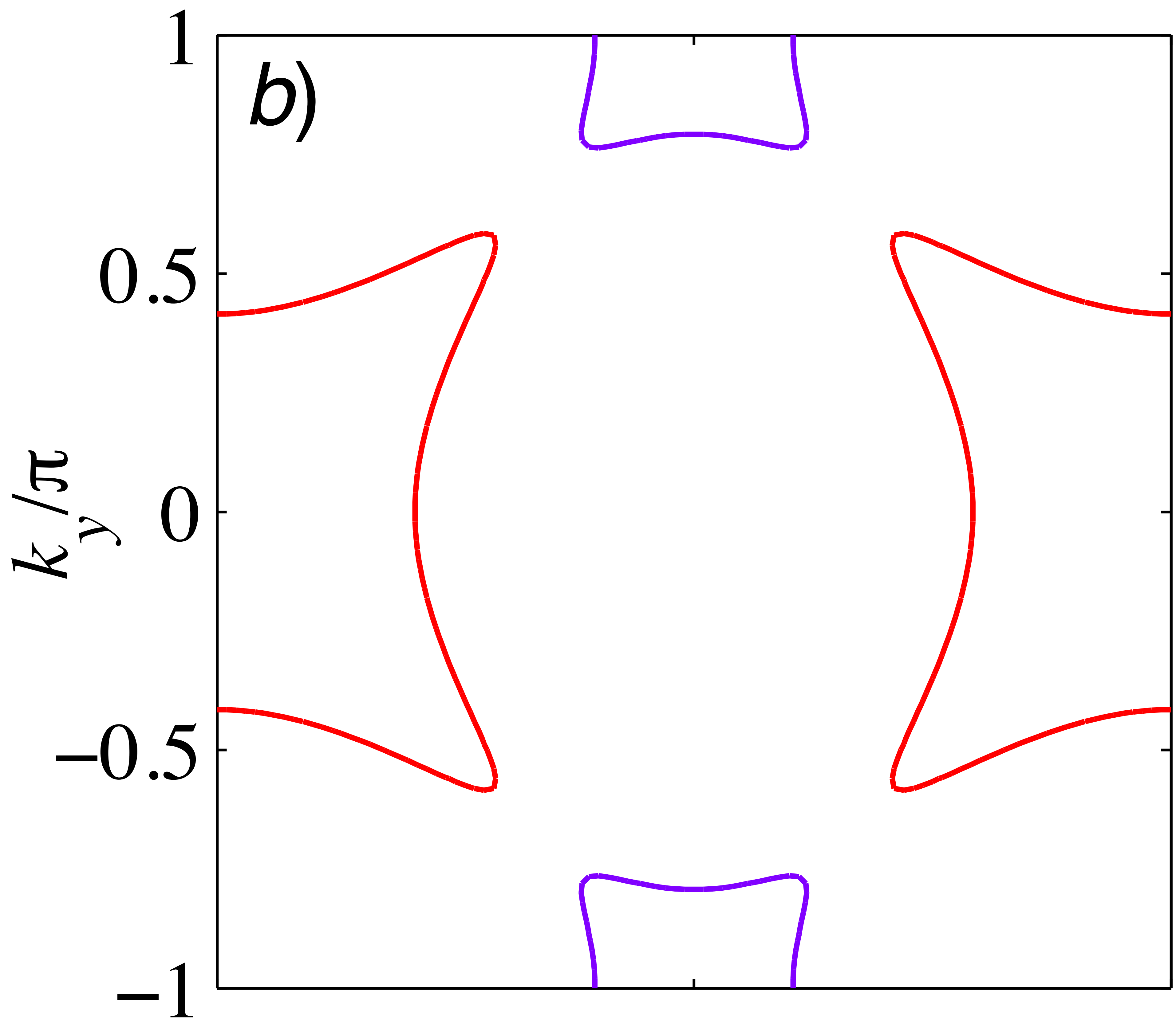
arXiv:1206.0742

Now hot spots are at Fermi surface intersections

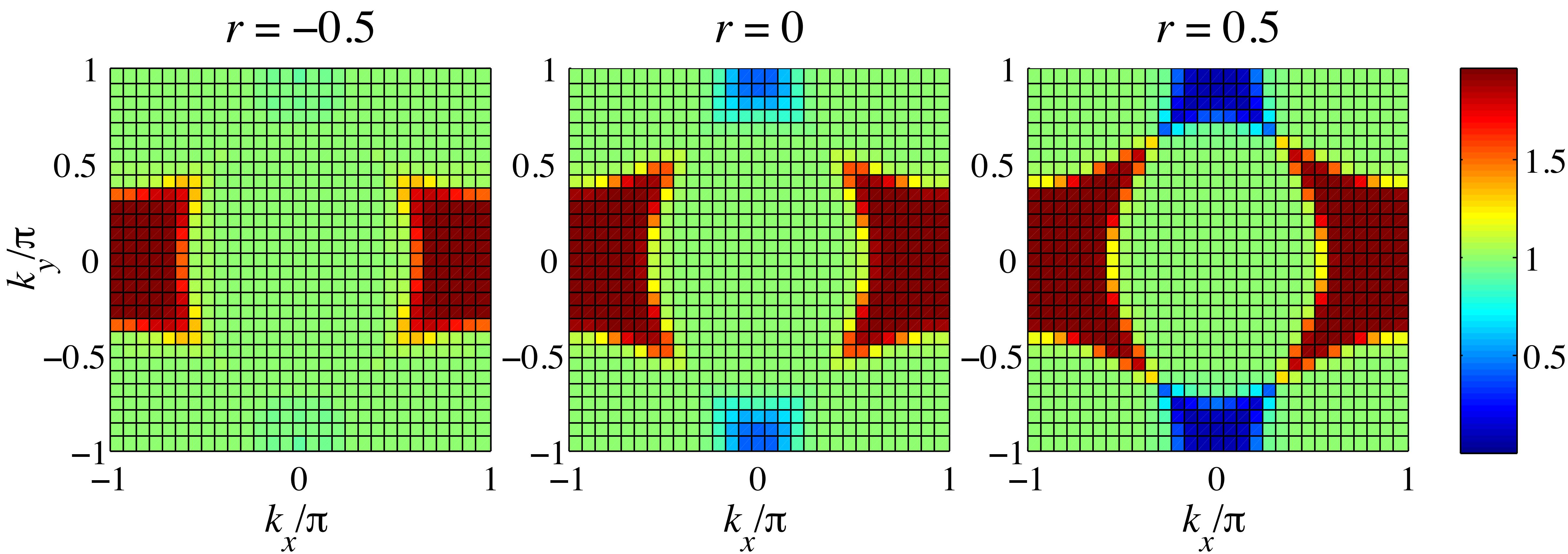
QMC for the onset of antiferromagnetism



E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Expected Fermi surfaces in the AFM ordered phase

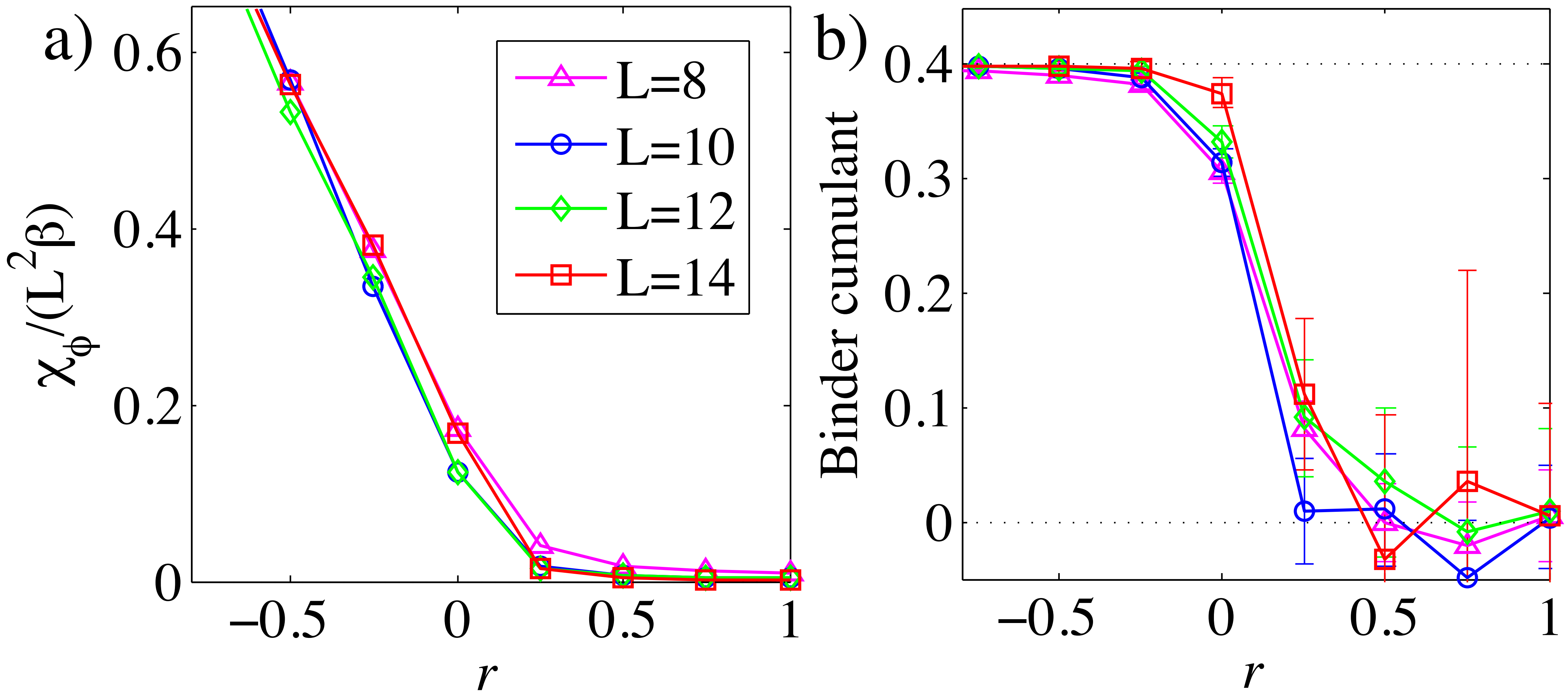
QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

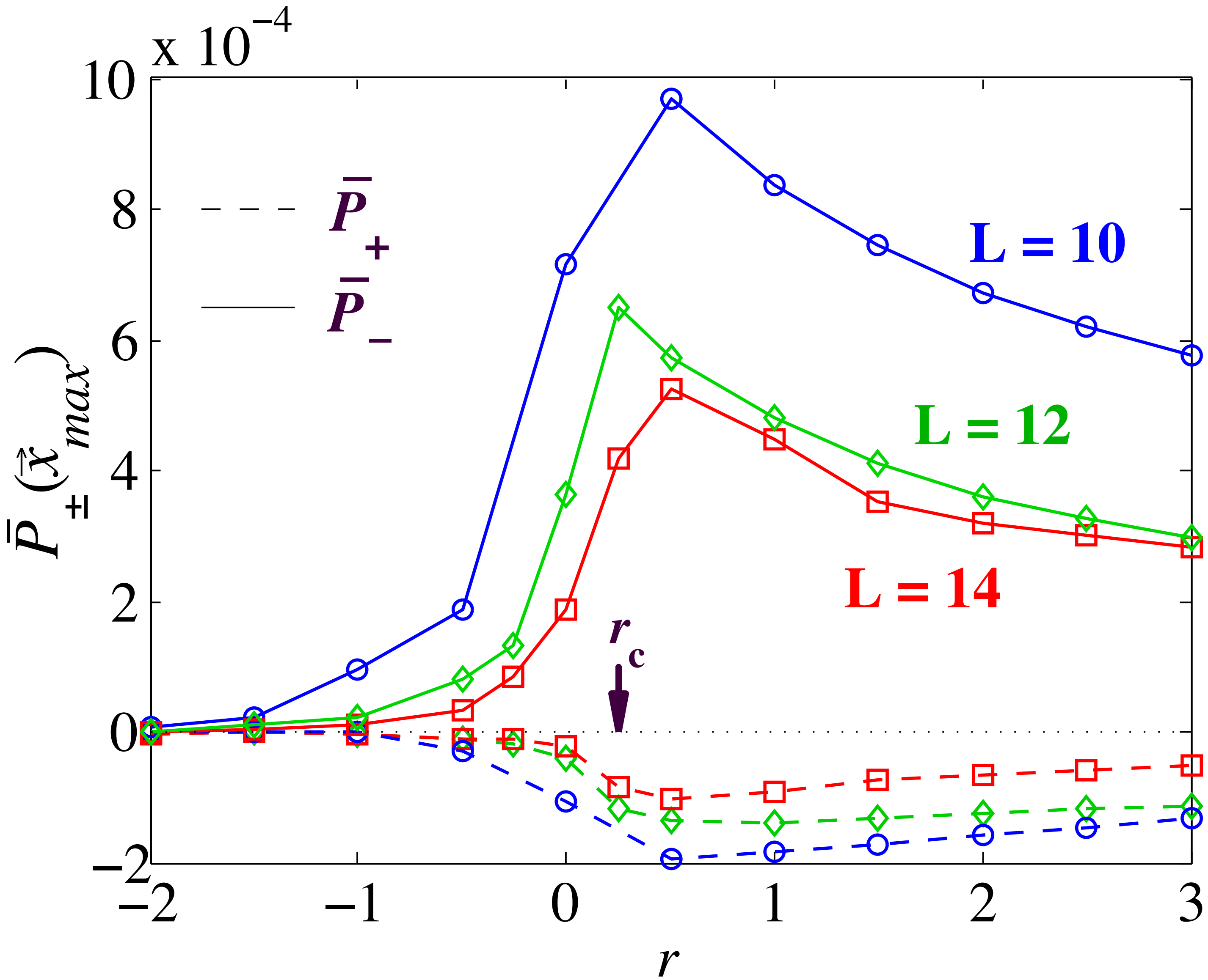
QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

Friday, August 3, 2012

Outline

1. Weak coupling theory

2. Universal critical theory

3. Quantum Monte Carlo
   without the sign problem

4. Features of strong coupling

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Features of strong coupling

- Shift in QCP due to superconductivity: "backbending" of SDW order.
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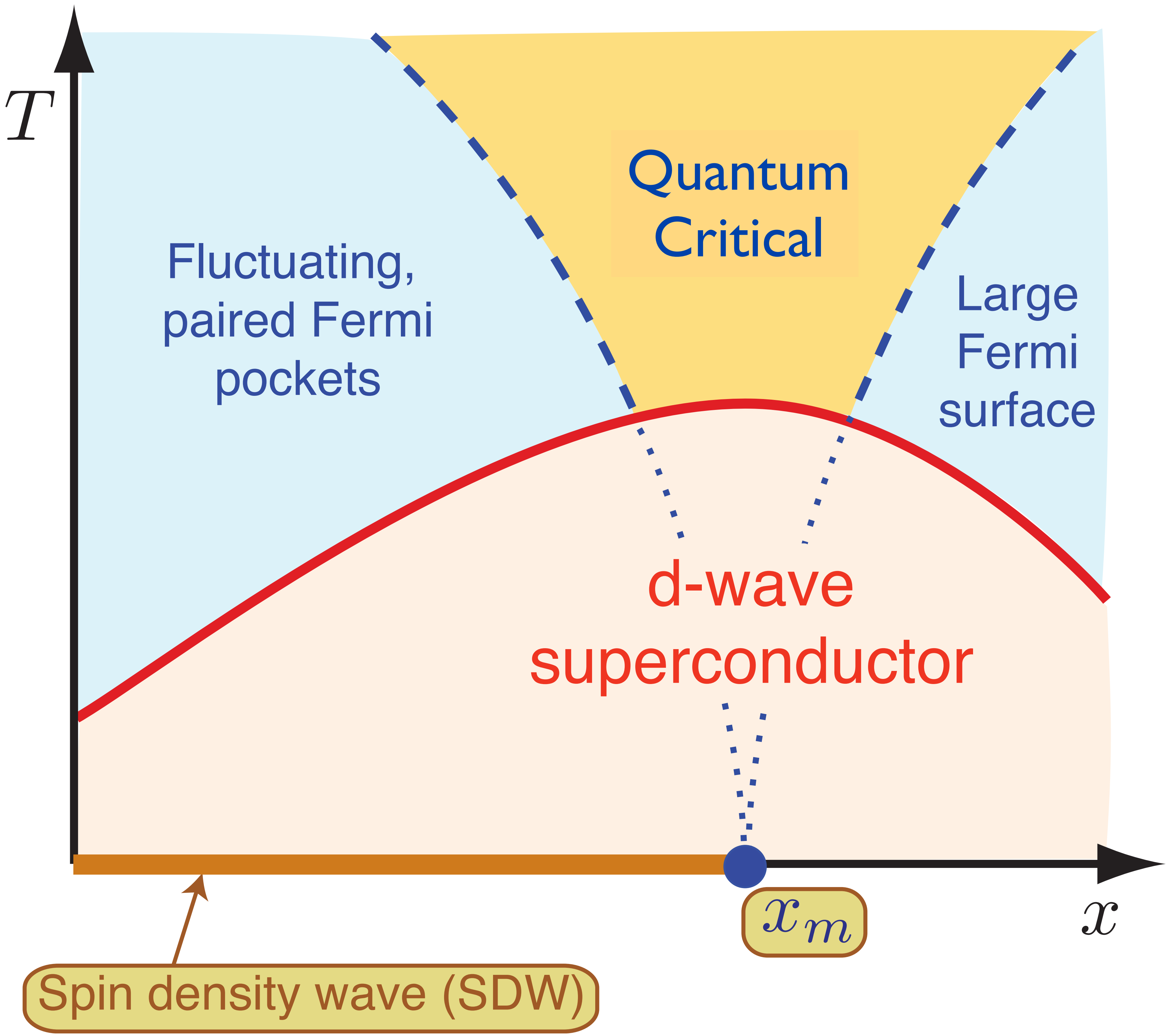
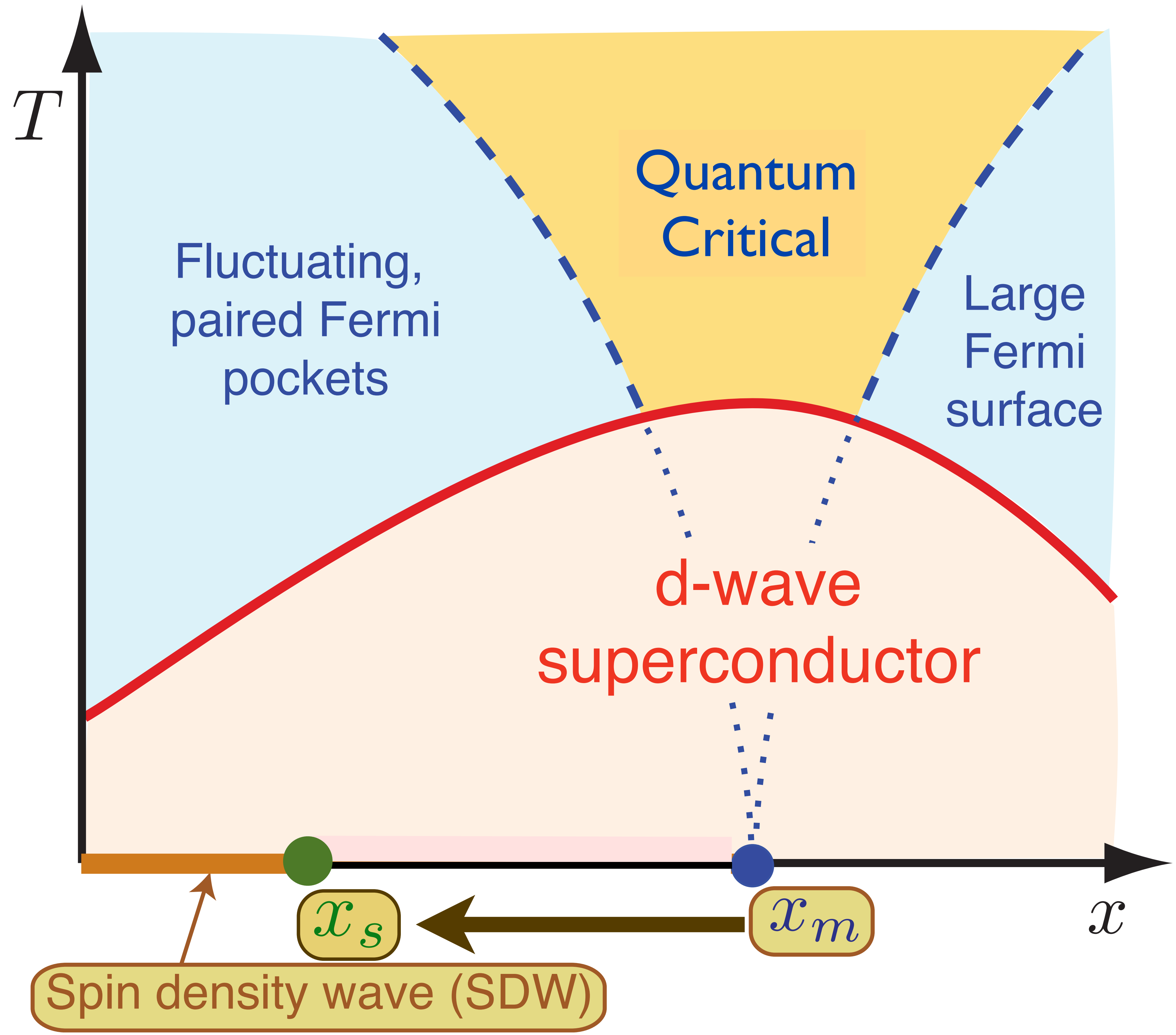


QCP for the onset of SDW order is actually within a superconductor

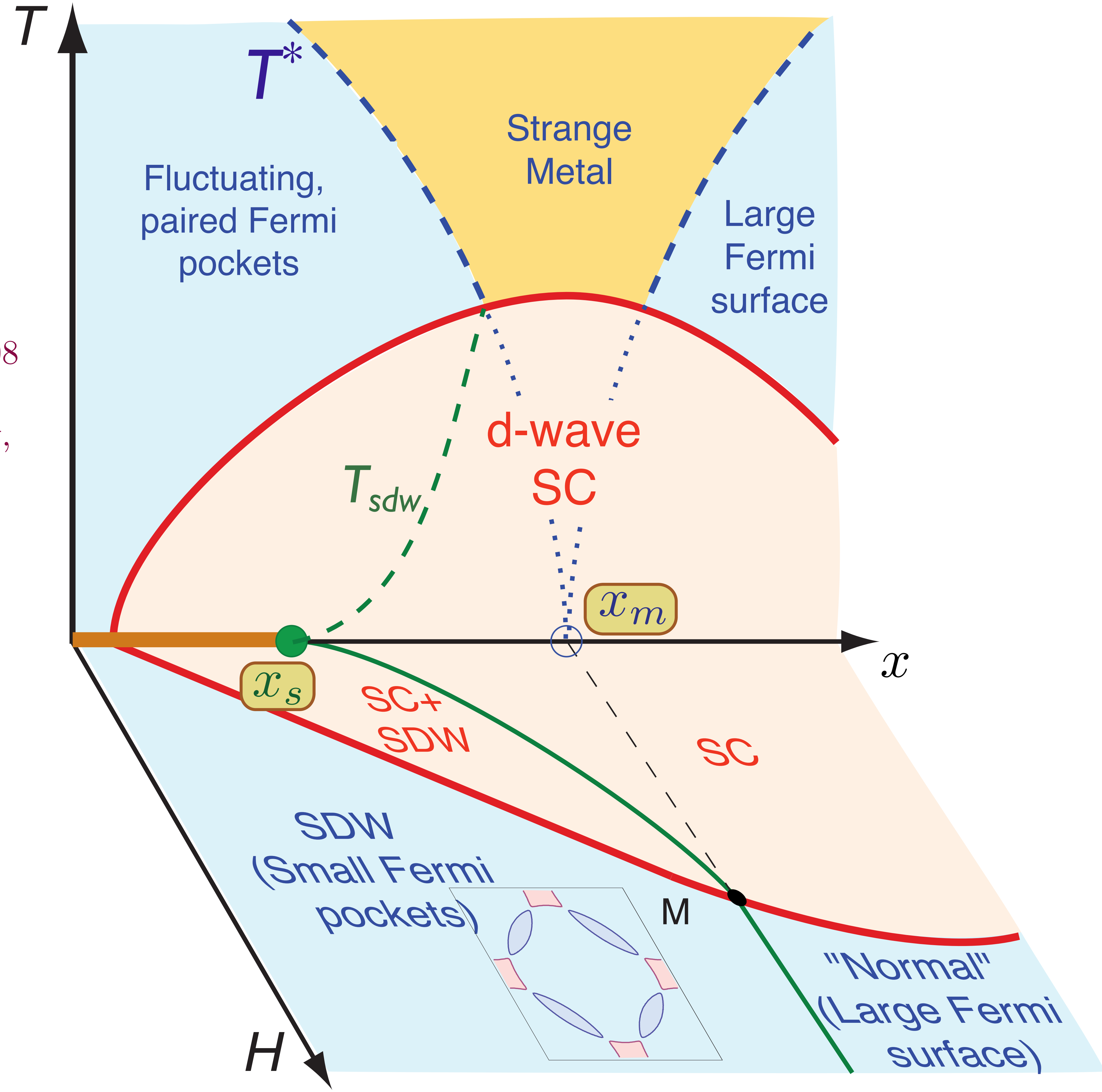


Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

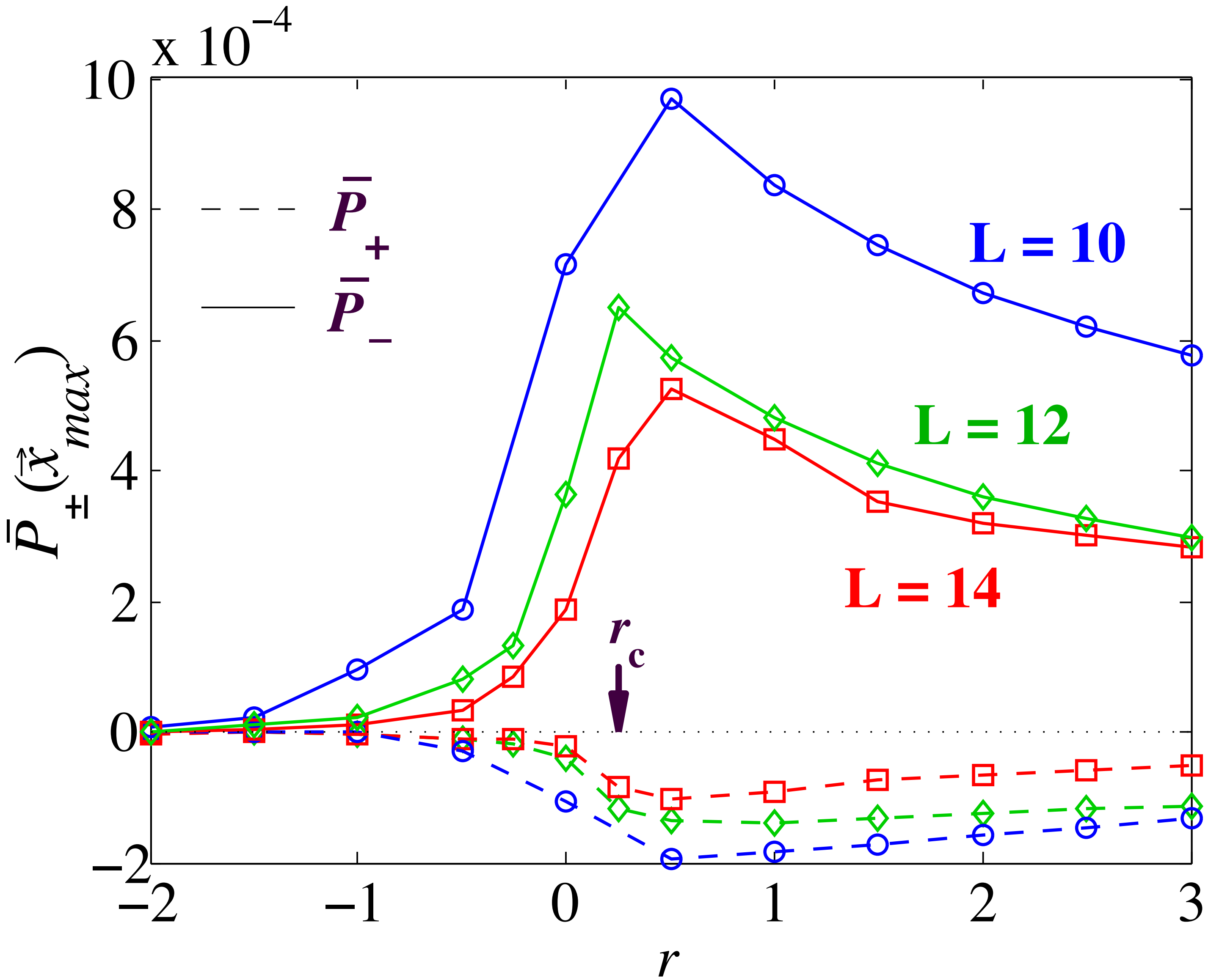
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

S. Sachdev, arXiv:0907.0008

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)



QMC for the onset of antiferromagnetism



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E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

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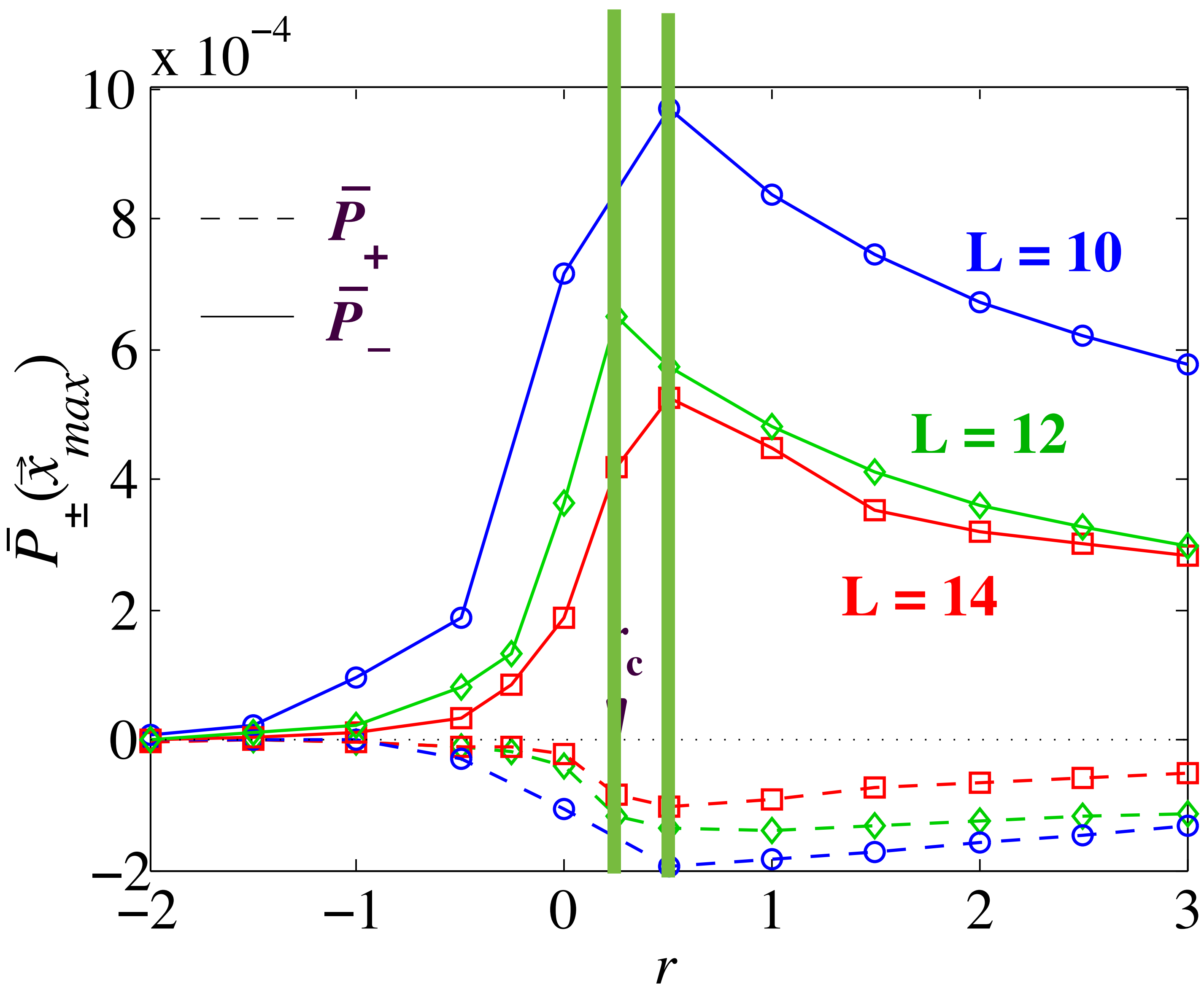

Notice shift between the position of the QCP in the superconductor, and the position of maximum pairing. This is found in numerous experiments.

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

Features of strong coupling

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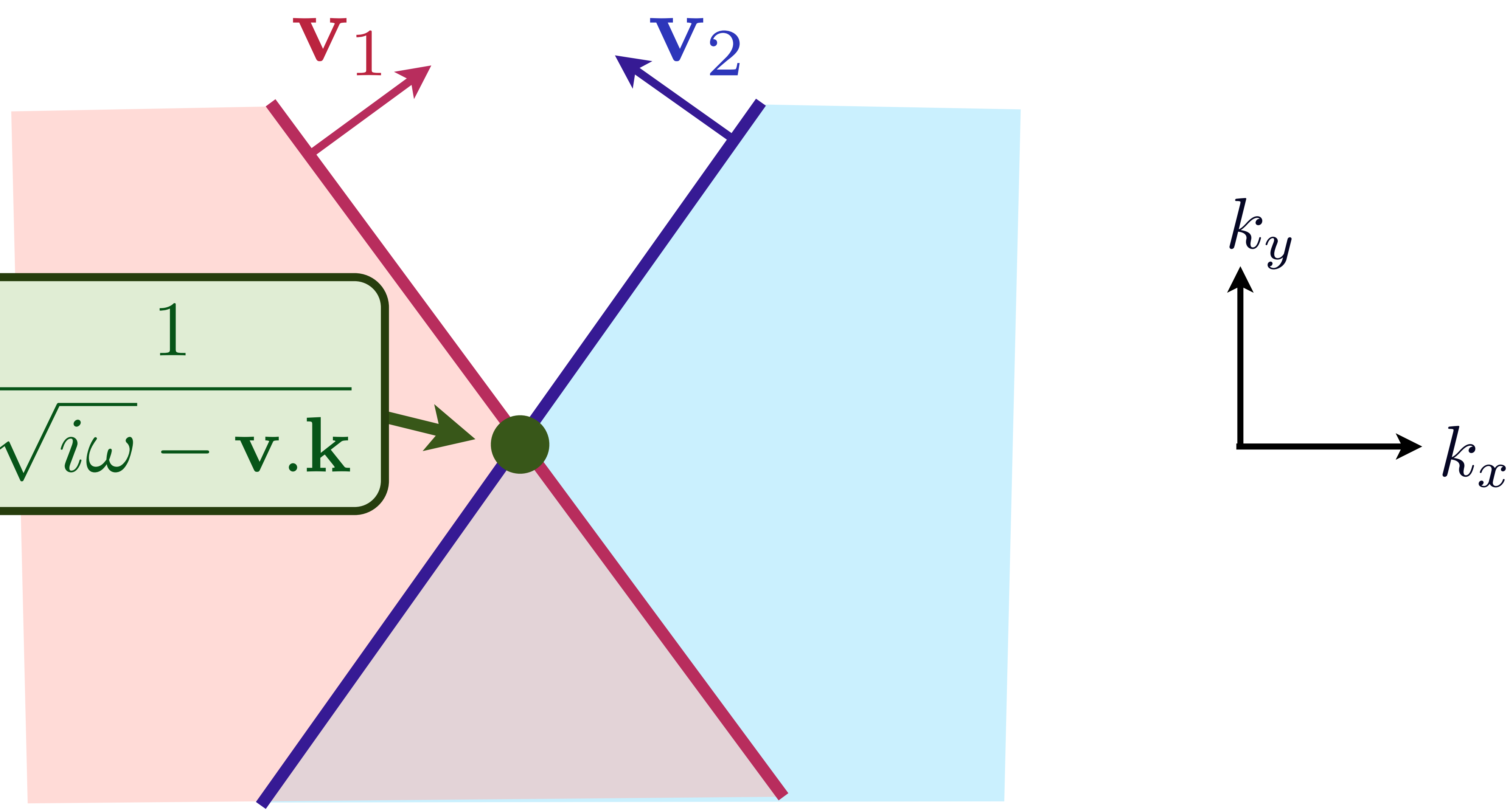
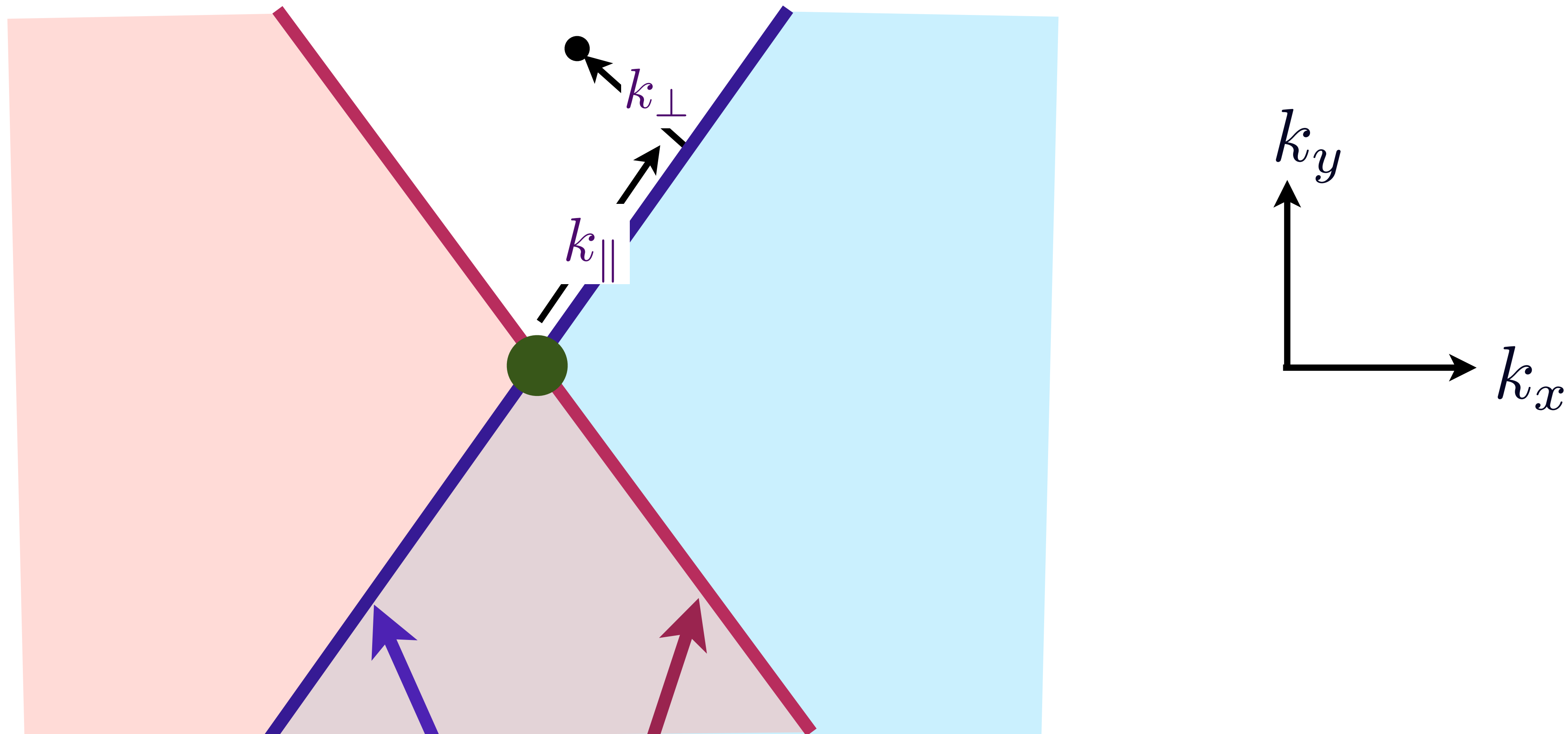
Two loop results: Non-Fermi liquid spectrum at hot spots



$$G_{\text{fermion}} \sim \frac{1}{\sqrt{i\omega} - \mathbf{v}.\mathbf{k}}$$

A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004)

Two loop results: Quasiparticle weight vanishes upon approaching hot spots



$$G_{\text{fermion}} = \frac{Z(k_\parallel)}{\omega - v_F(k_\parallel) k_\perp}, \quad Z(k_\parallel) \sim v_F(k_\parallel) \sim k_\parallel$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)

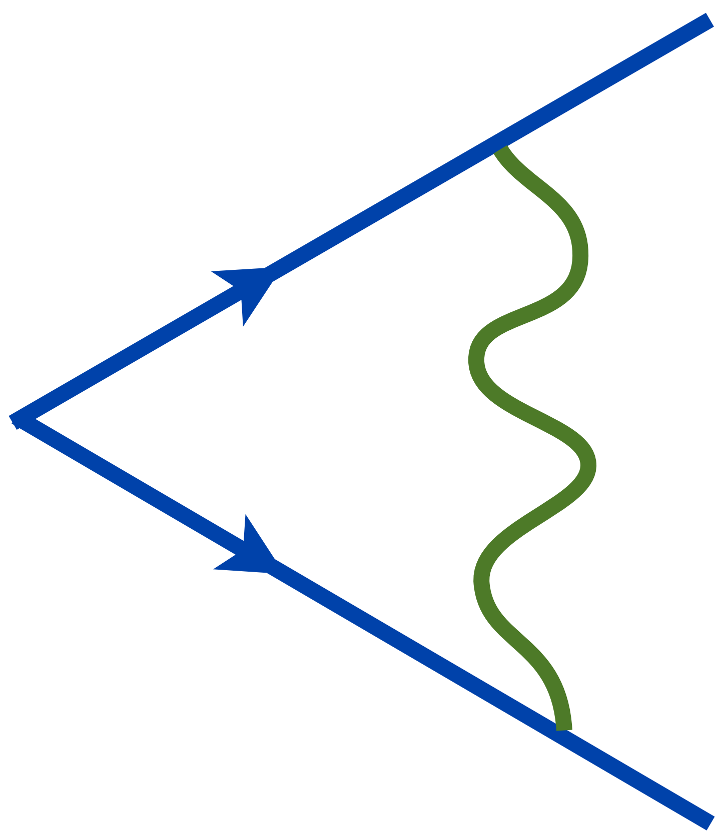
Pairing by SDW fluctuation exchange

Weak-coupling theory

Fermi energy

$$1 + \lambda^2 \rho(E_F) \log \left(\frac{E_F}{\omega} \right)$$

Density of states at Fermi energy



Pairing by SDW fluctuation exchange

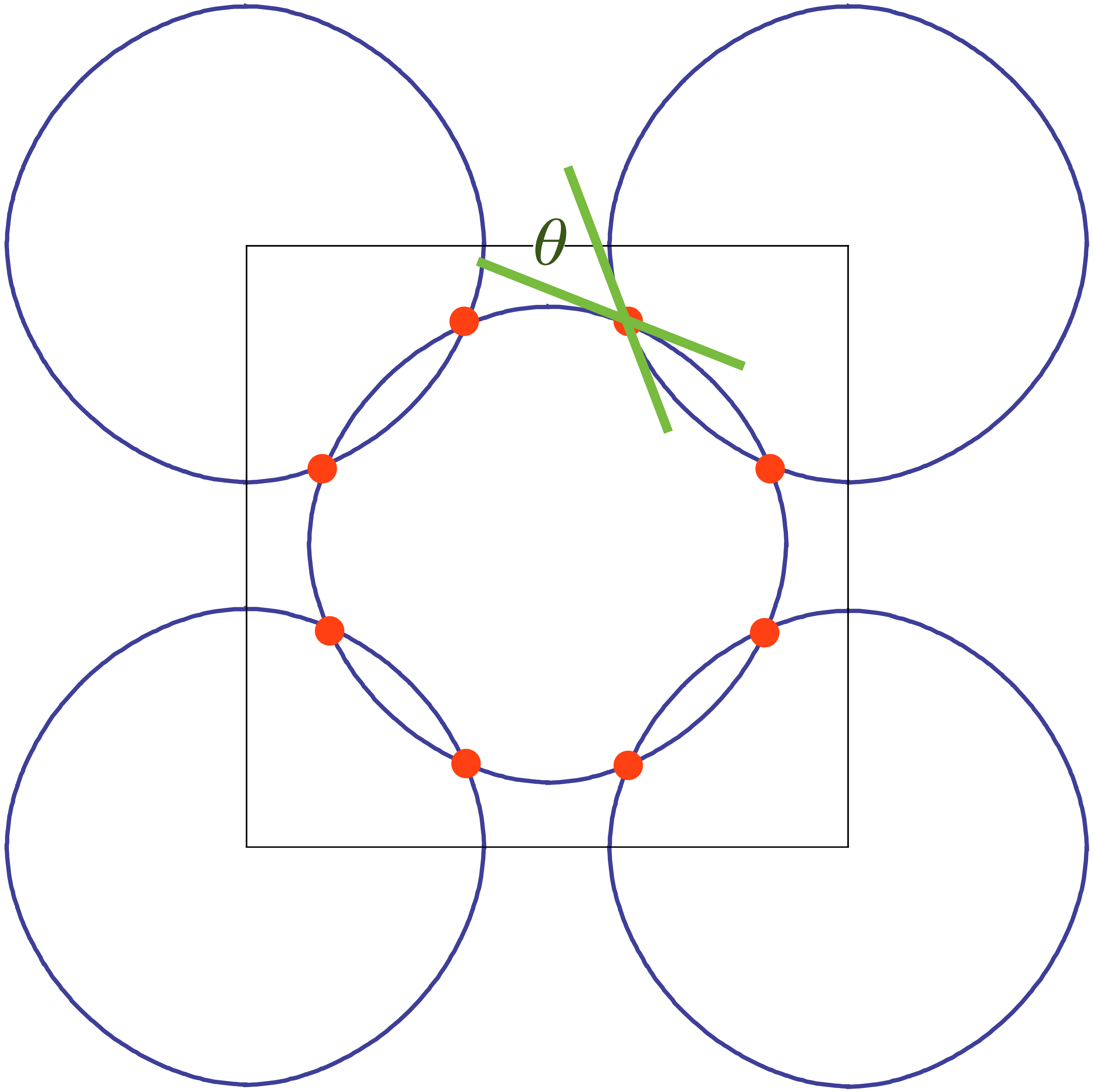
Antiferromagnetic critical point

Fermi energy

$$1 + \frac{\sin\theta}{2\pi} \log^2\left(\frac{E_F}{\omega}\right)$$

θ is the angle between Fermi lines.
Independent of interaction strength
U in 2 dimensions.

(see also Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)


θ

Pairing by SDW fluctuation exchange

Antiferromagnetic critical point

$$1 + \frac{\sin\theta}{2\pi} \log^2 \left(\frac{E_F}{\omega} \right)$$

- **Universal** $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)

Features of strong coupling

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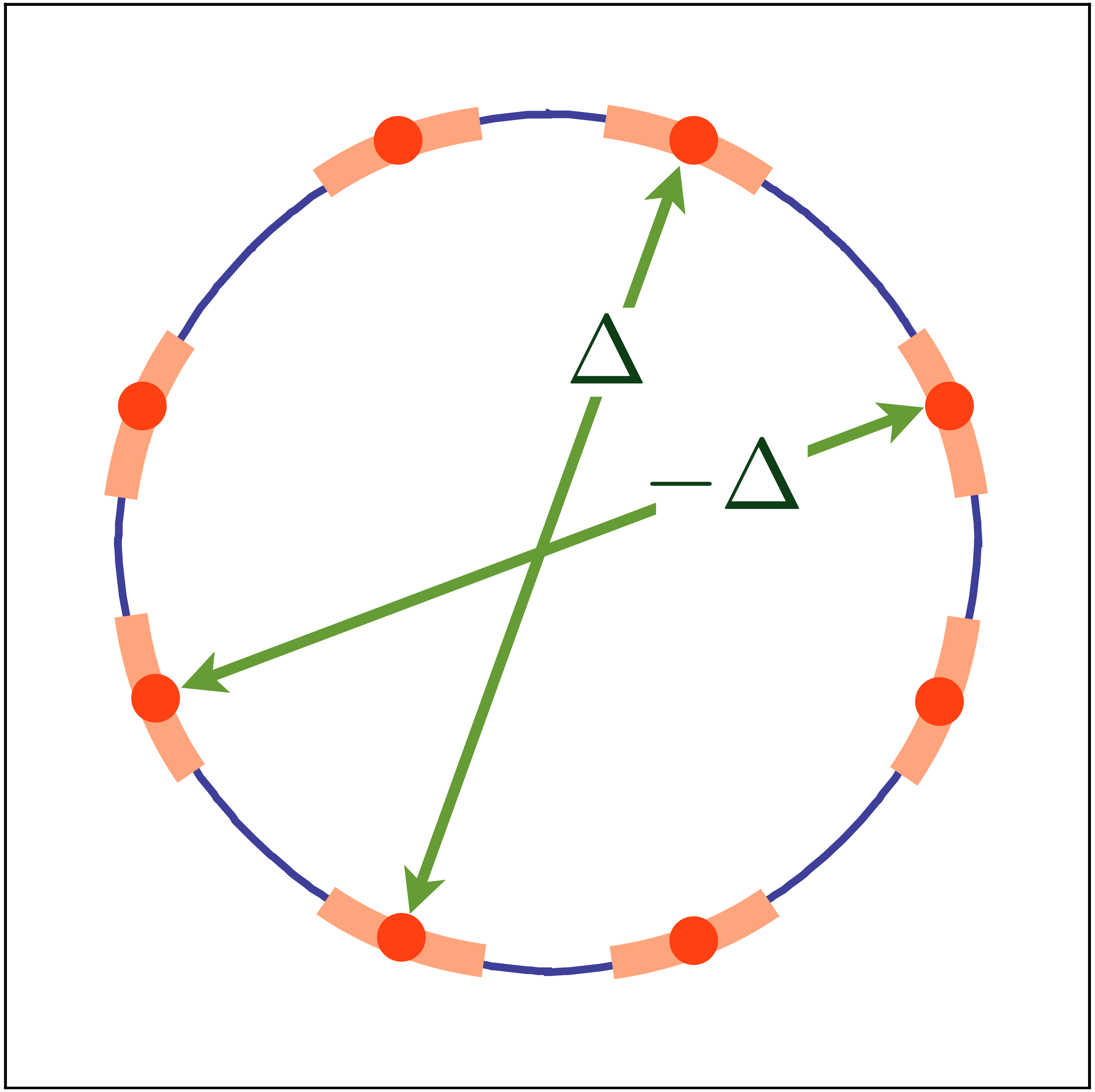
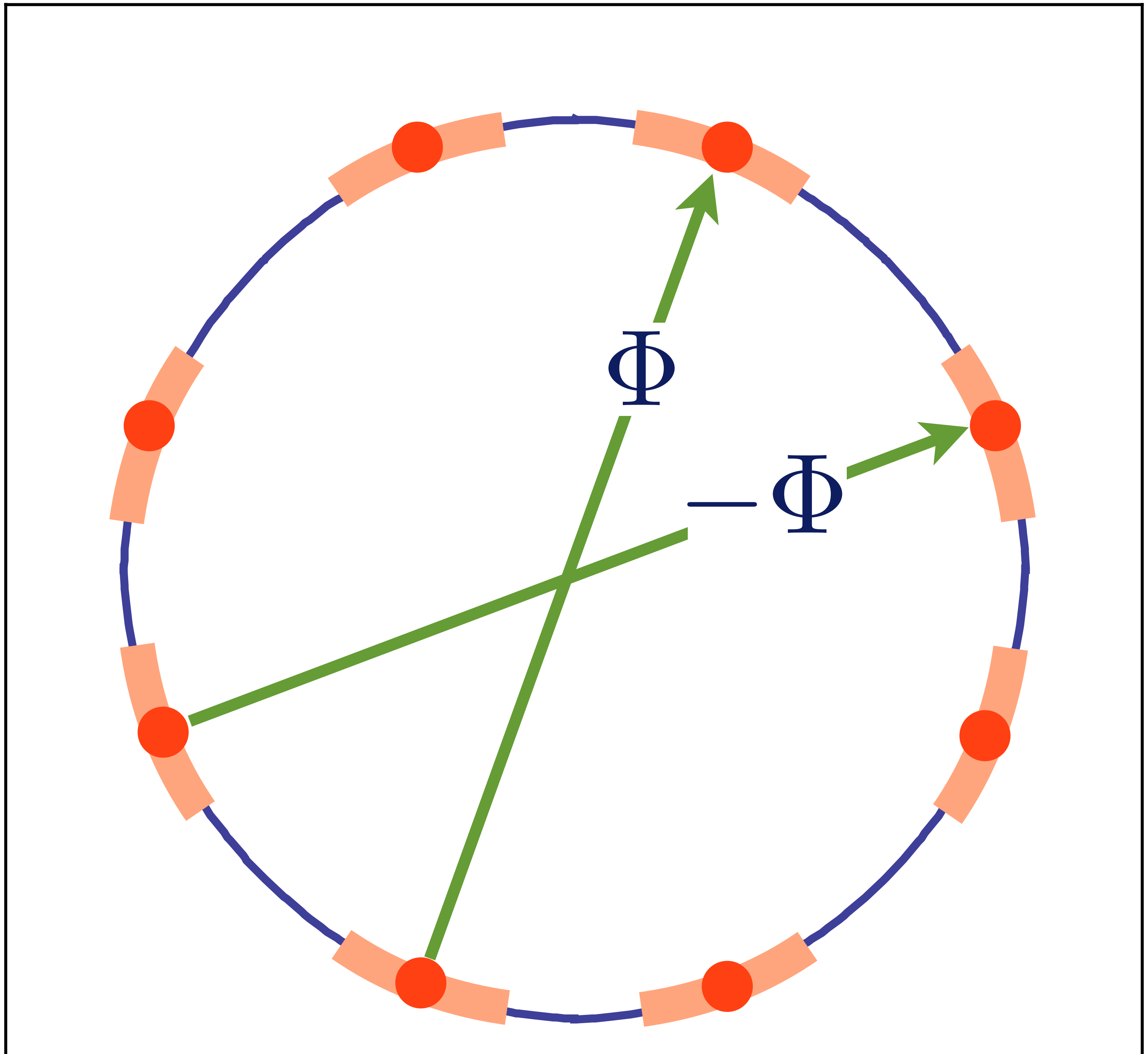
$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Log-squared instability to d-wave pairing

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

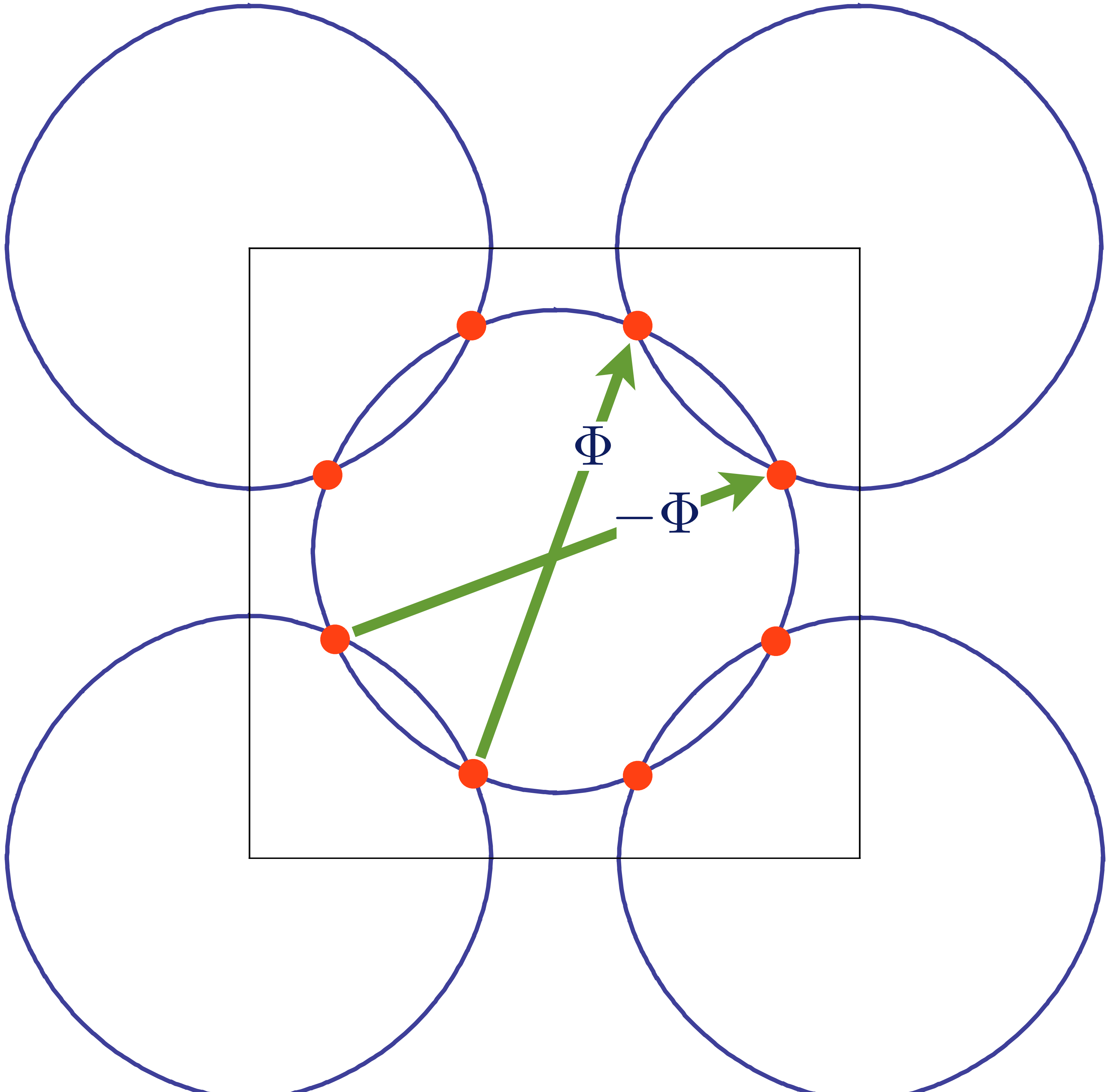


$\mathbf{Q}$ is '$2k_F$' wavevector

Similar log-squared instability in particle-hole channel to bond-modulated charge order

Bond-modulated charge order

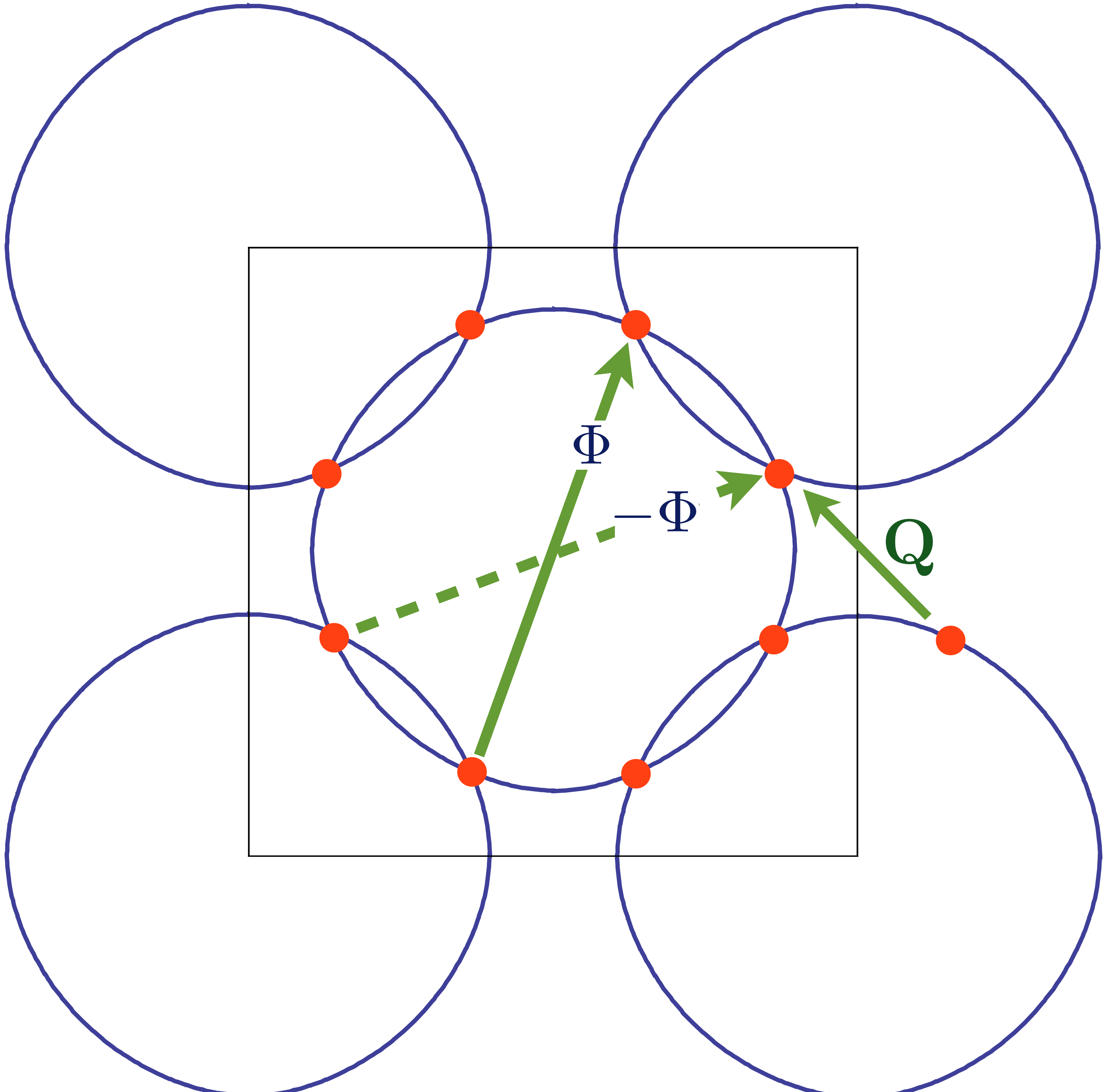
M.A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075128 (2010)



$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

Bond-modulated charge order

M.A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075128 (2010)



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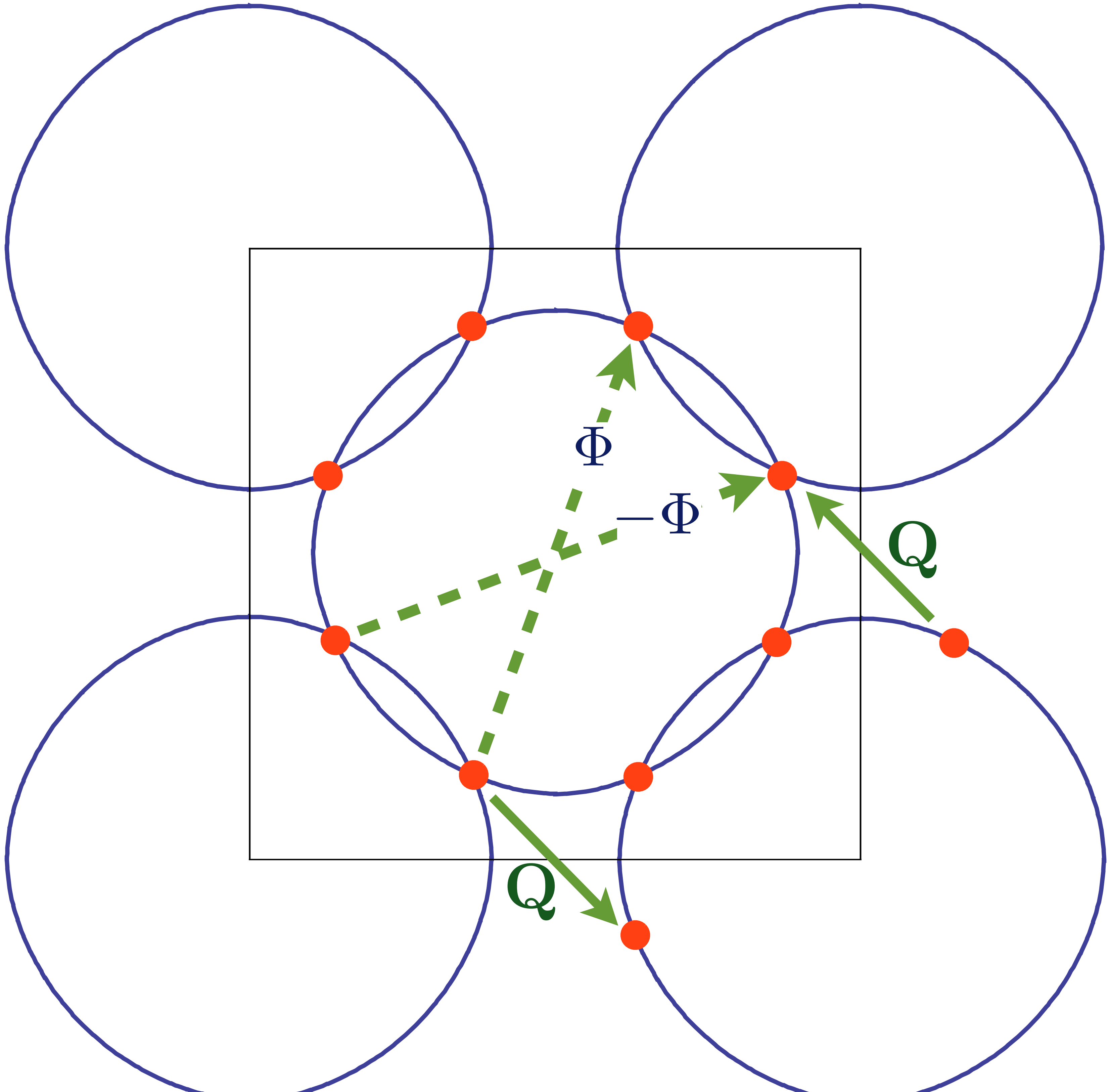
Bond-modulated charge order

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)



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M.A. Metlitski and S. Sachdev,
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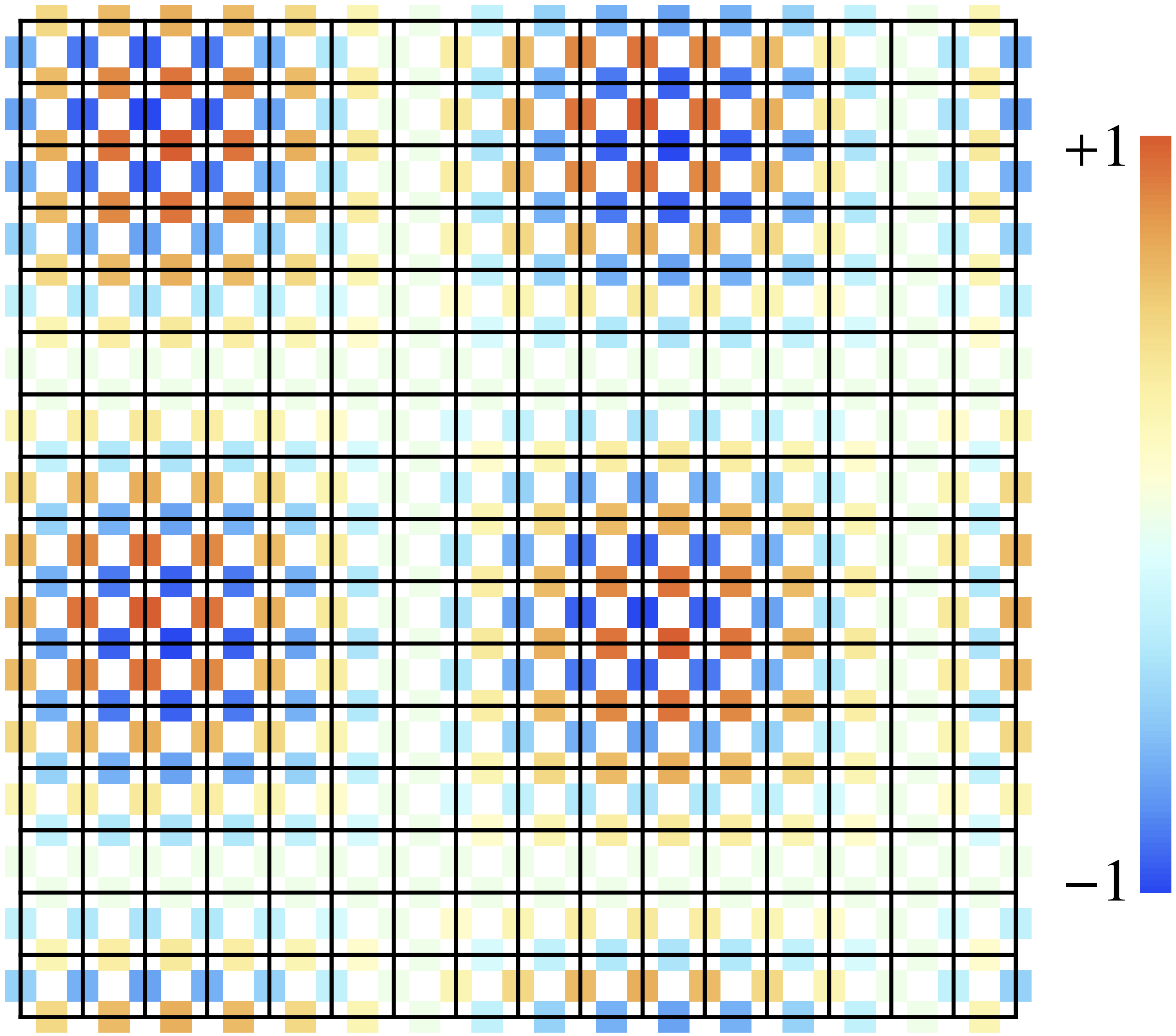

"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

No modulations on sites, $\langle c_{\mathbf{r}\alpha}^{\dagger} c_{\mathbf{s}\alpha} \rangle$ is modulated only for $\mathbf{r} \neq \mathbf{s}$.

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^{\dagger} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$

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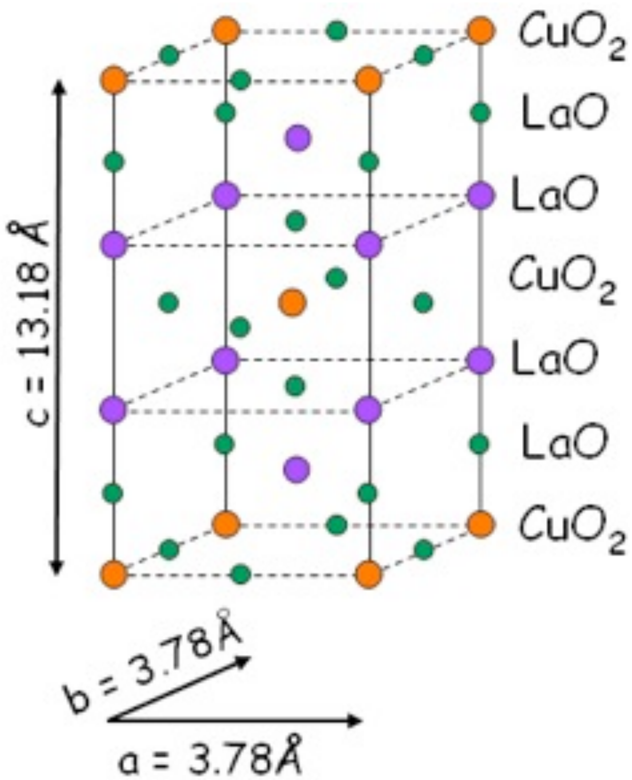
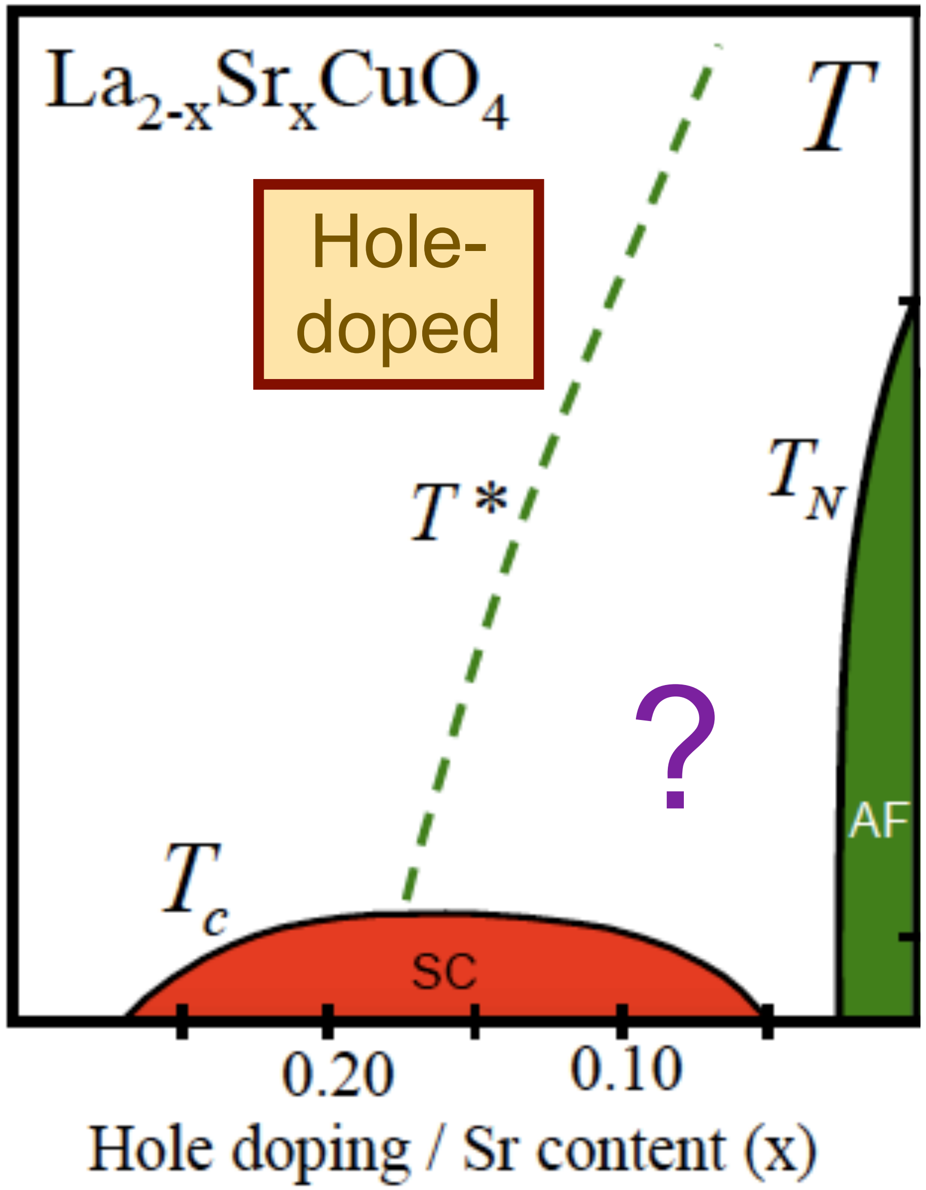
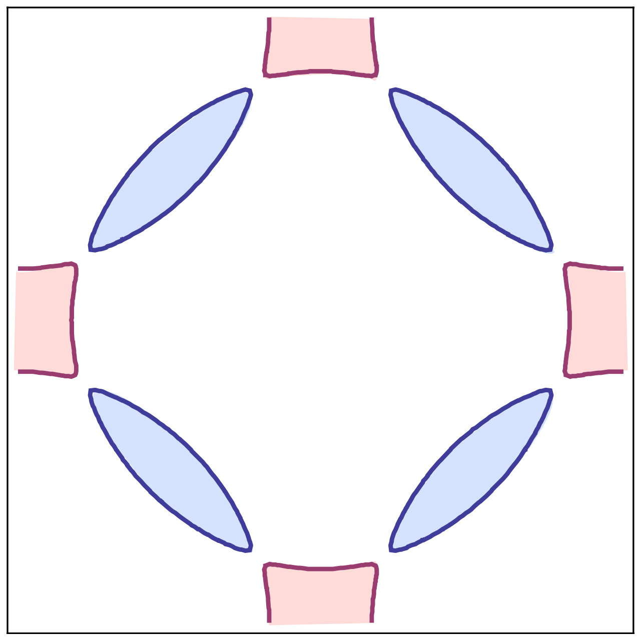

CuO2
LaO
LaO
CuO2
LaO
LaO
CuO2
c = 13.18 Å
b = 3.78 Å
a = 3.78 Å

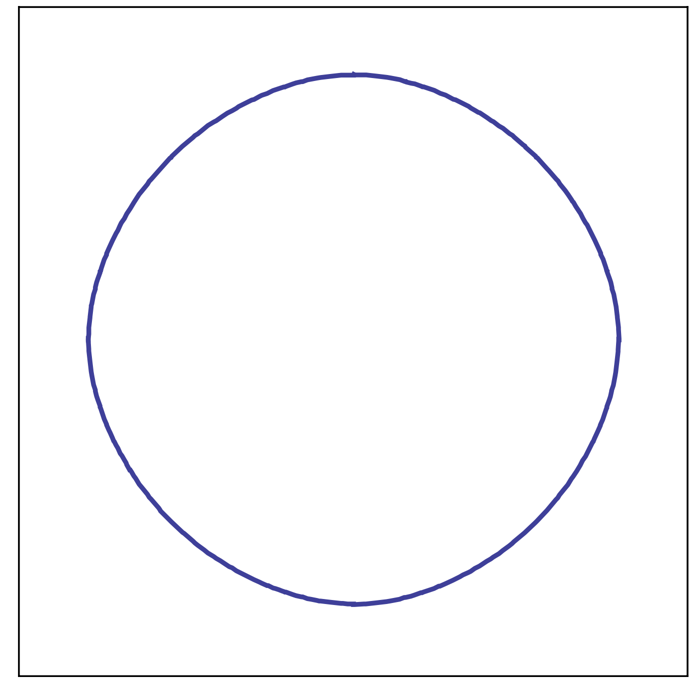

La2-xSrxCuO4
Hole-doped
T
T*
TN
?
AF
Tc
SC
0.20
0.10
Hole doping / Sr content (x)

Quantum phase transition with Fermi surface reconstruction



$\langle \vec{\varphi} \rangle \neq 0$

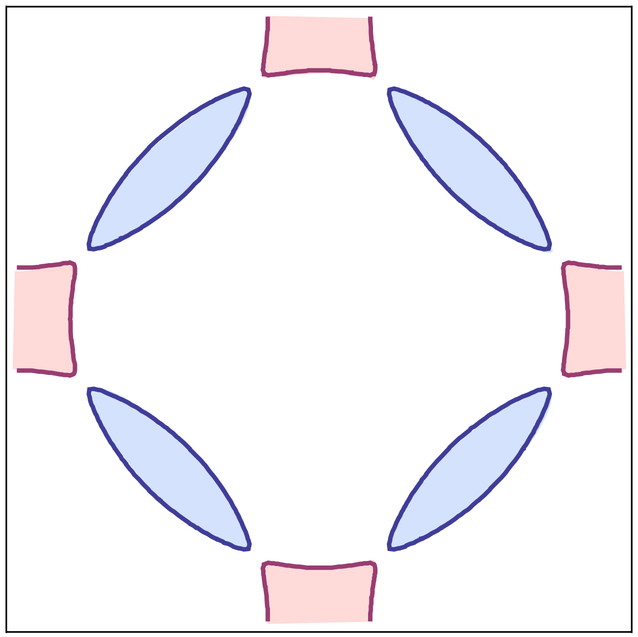
Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

Separating onset of SDW order and Fermi surface reconstruction



$\langle \vec{\varphi} \rangle \neq 0$

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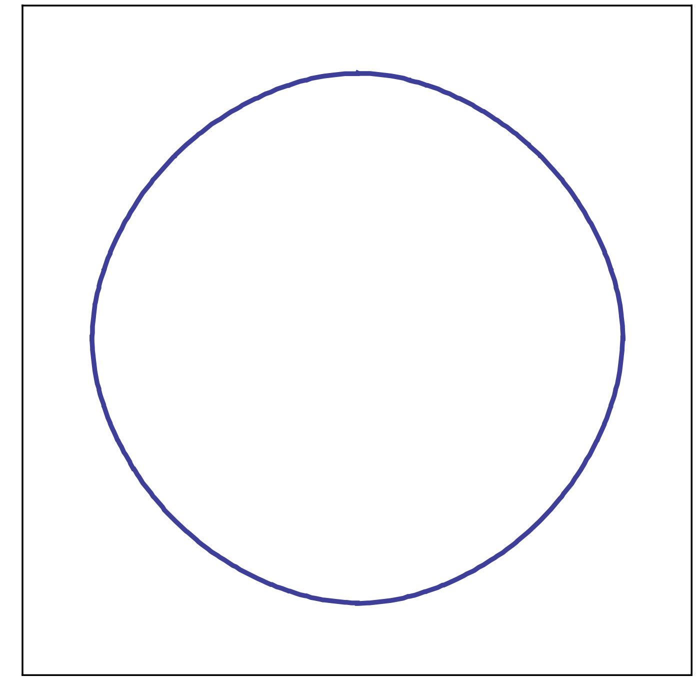

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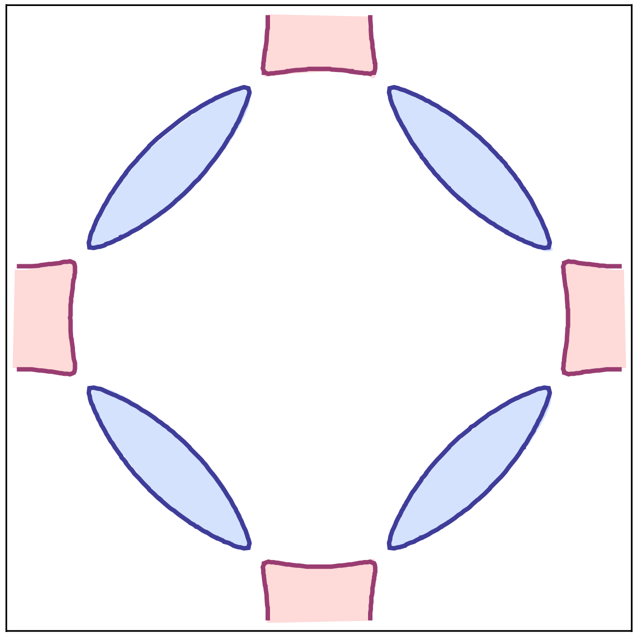
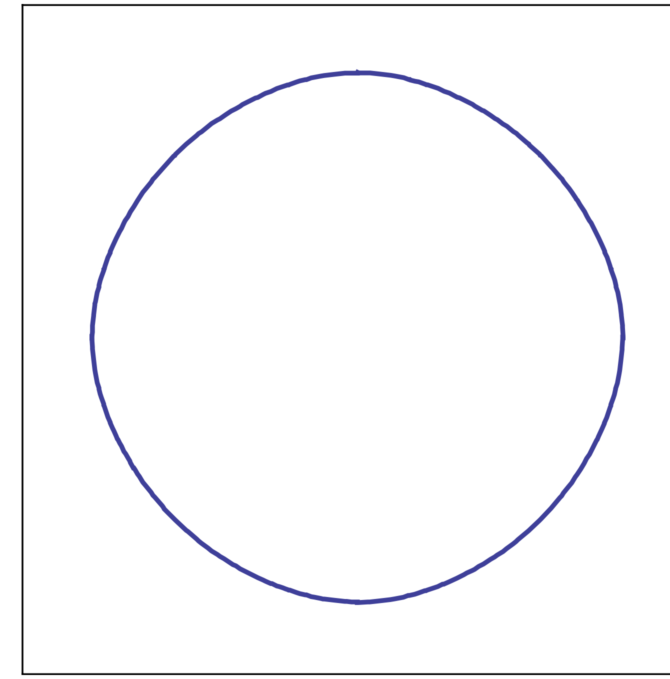


Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order



$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

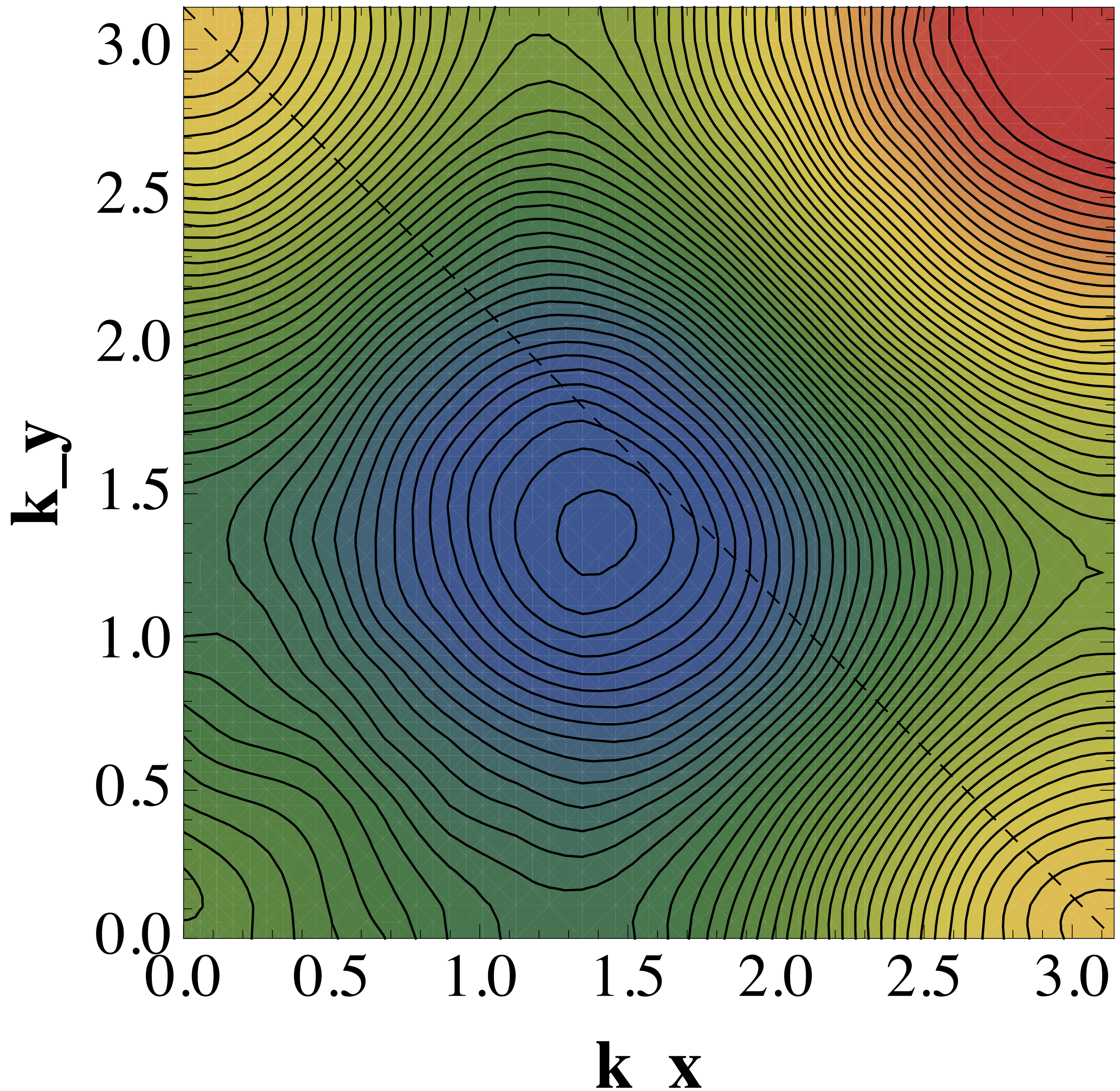
$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface

Metal with "large" Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)



Hole pocket of a $\mathbb{Z}_2$-FL* phase in a *single*-band t-J model

M. Punk and S. Sachdev, *Phys. Rev. B* **85**, 195123 (2012)

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.
- Such a phase *must* have neutral $S = 1/2$ excitations ("spinons"), and collective spinless gauge excitations ("topological" order).
- These topological excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa's proof of the Luttinger theorem.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

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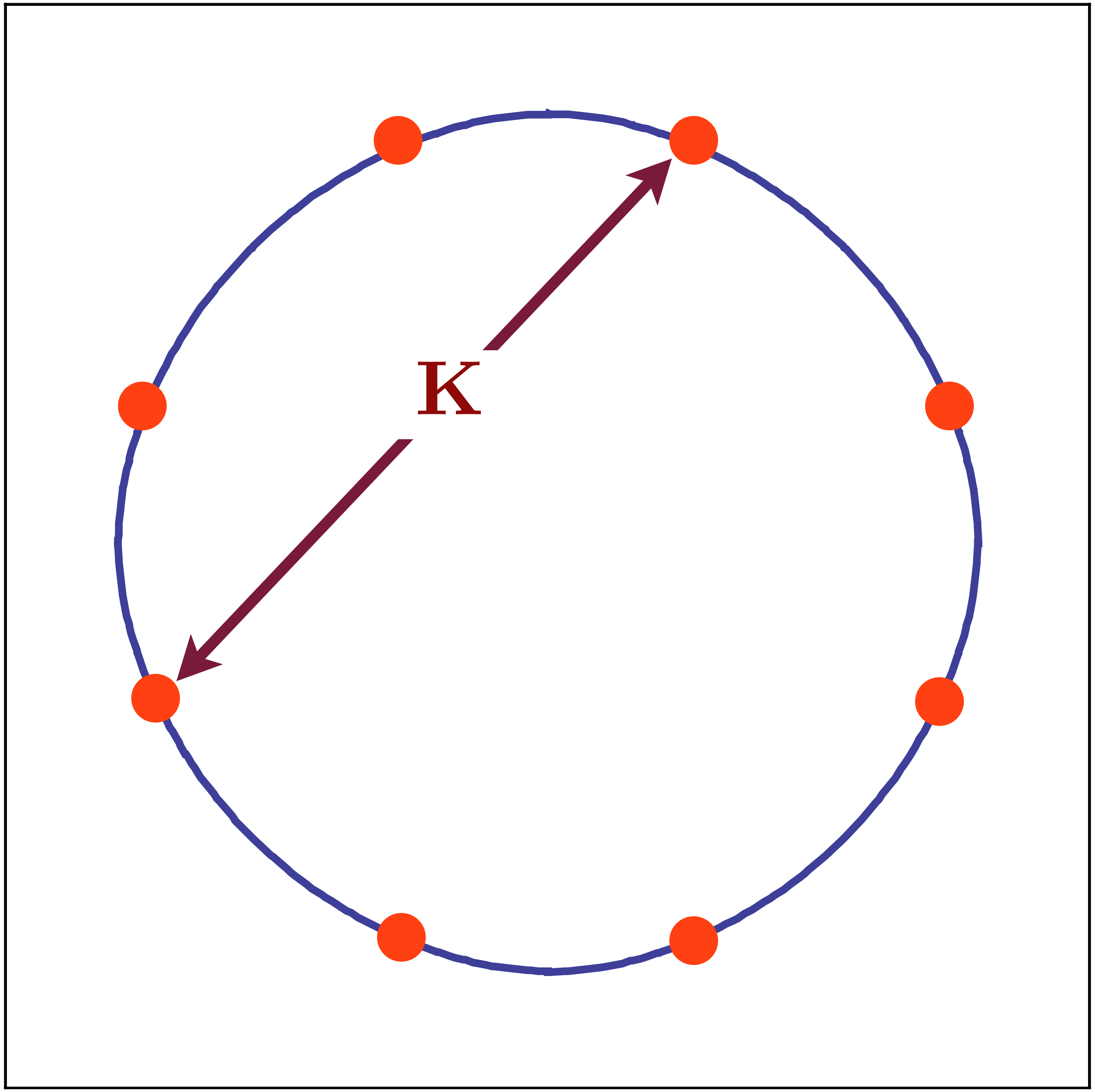
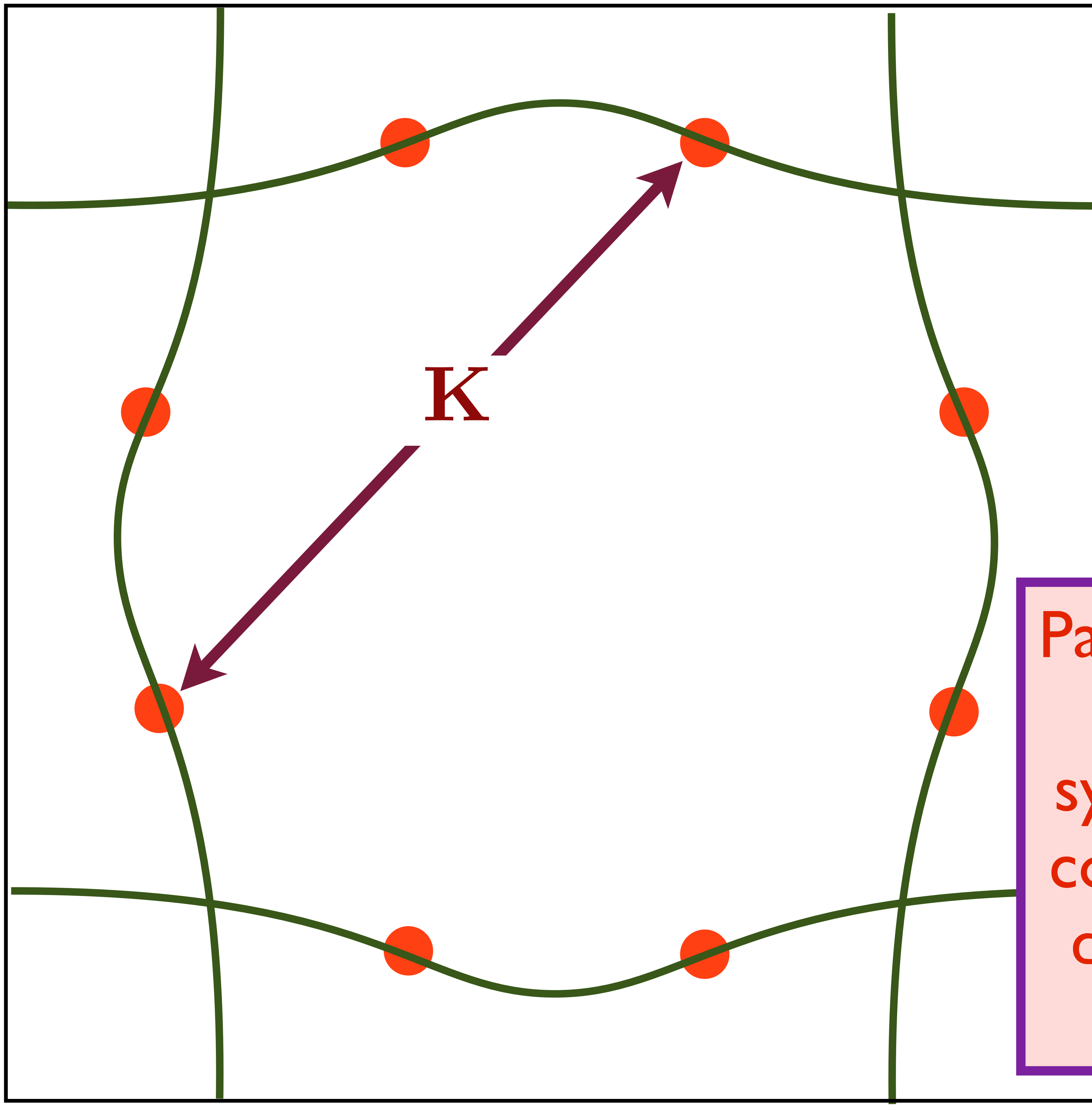


Hot spots in a single band model

QMC for the onset of antiferromagnetism

Faithful realization of low energy theory. Sign problem is absent as long as ***K*** connects hotspots in distinct bands



Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

Particle-hole or point-group symmetries or commensurate densities ***not*** required !