Quantum Criticality and Black Holes

Talk online: sachdev.physics.harvard.edu

Particle theorists

Sean Hartnoll, KITP
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

Condensed matter theorists

Markus Mueller, Harvard
Lars Fritz, Harvard
Subir Sachdev, Harvard

Three foci of modern physics

Quantum phase transitions

Three foci of modern physics

Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

Three foci of modern physics

Quantum phase transitions

Black holes

Three foci of modern physics

Quantum phase transitions

Black holes

Bekenstein and Hawking originated the quantum theory, which has found fruition in string theory

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Universal description of fluids based upon conservation laws and positivity of entropy production

Black holes

Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

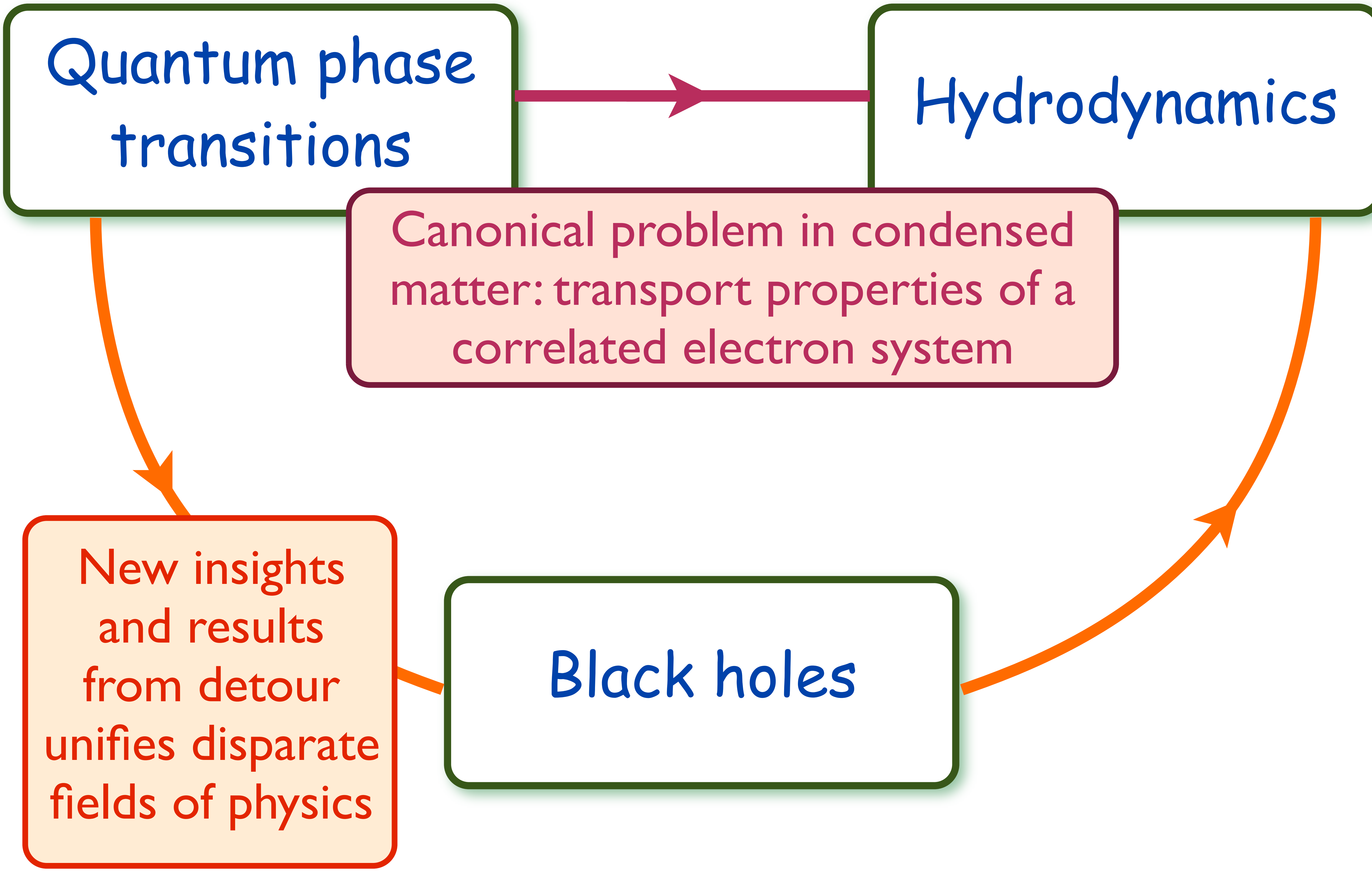
Three foci of modern physics

Quantum phase transitions → Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

Black holes

Three foci of modern physics



Three foci of modern physics

Quantum phase transitions

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Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

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Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

Hydrodynamics

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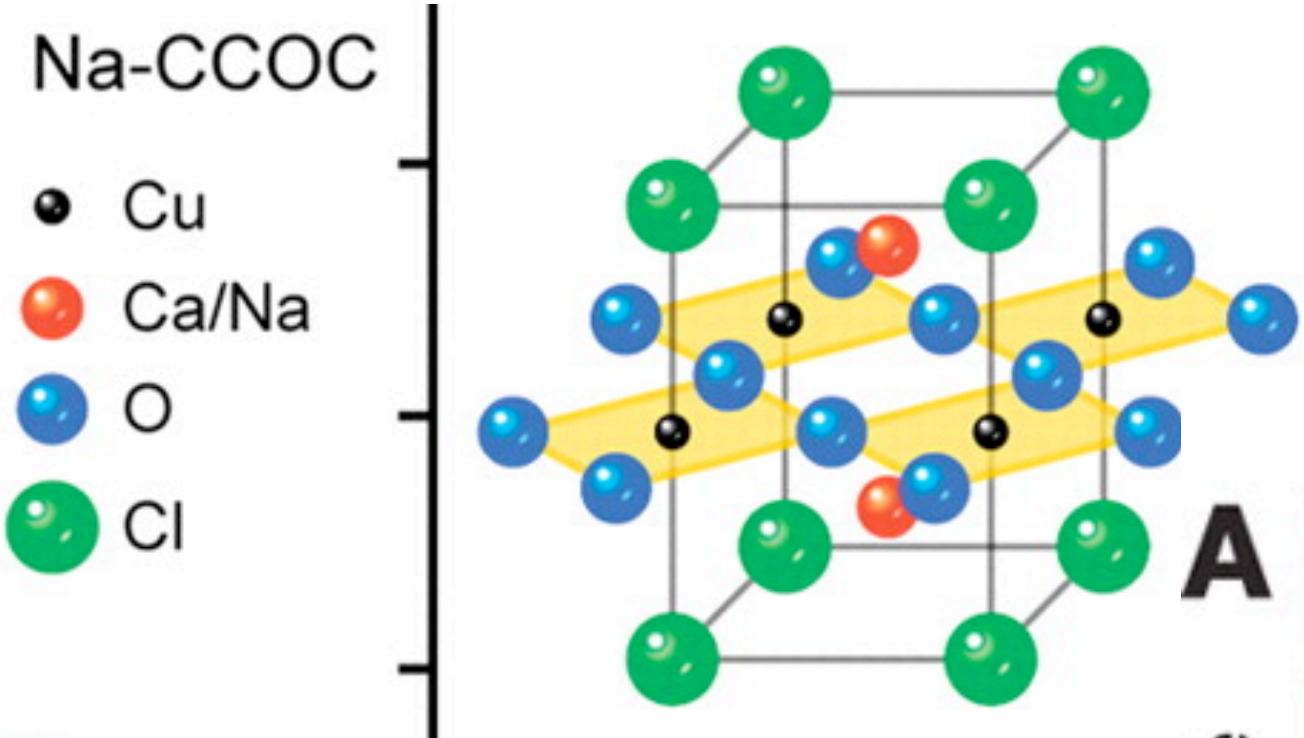
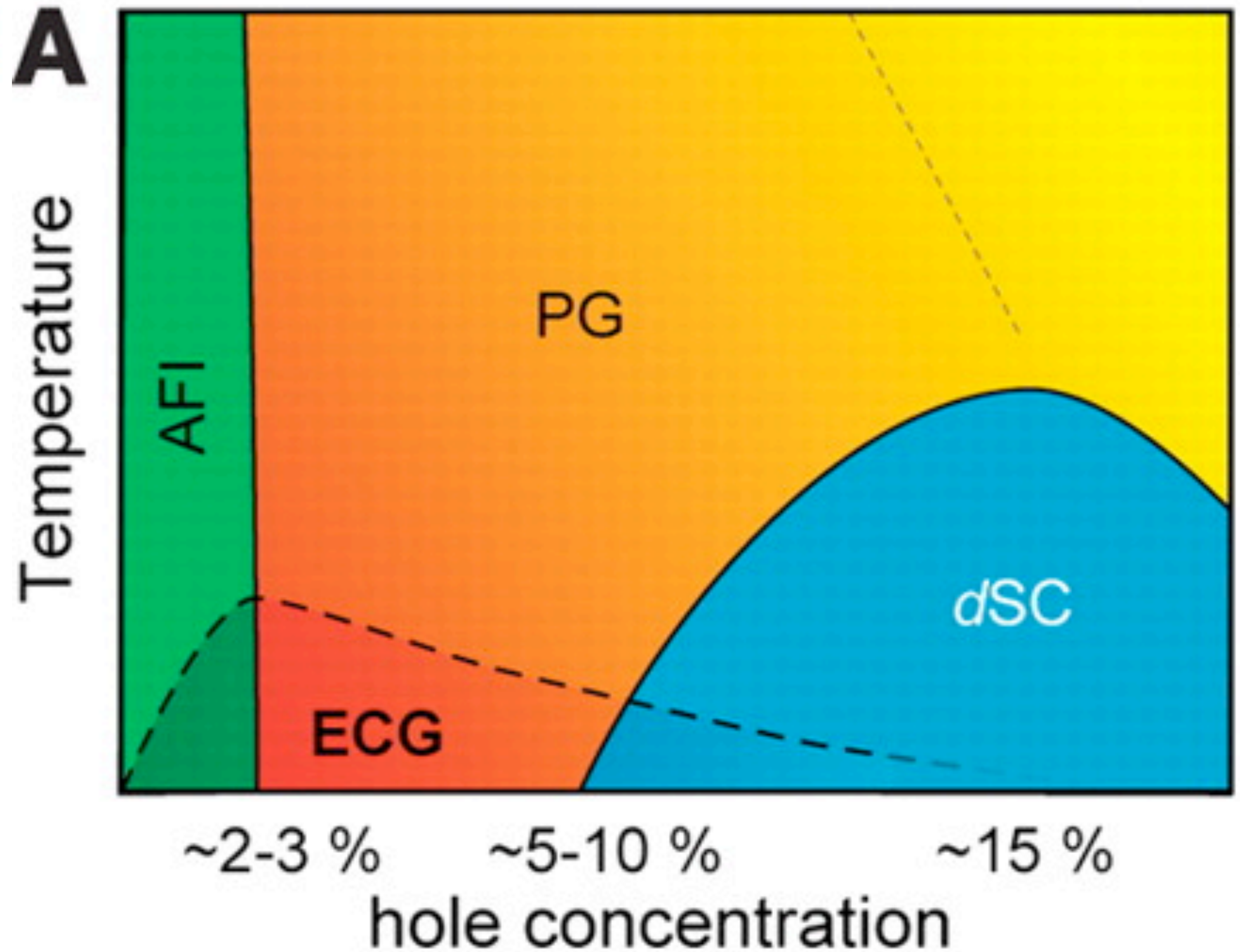
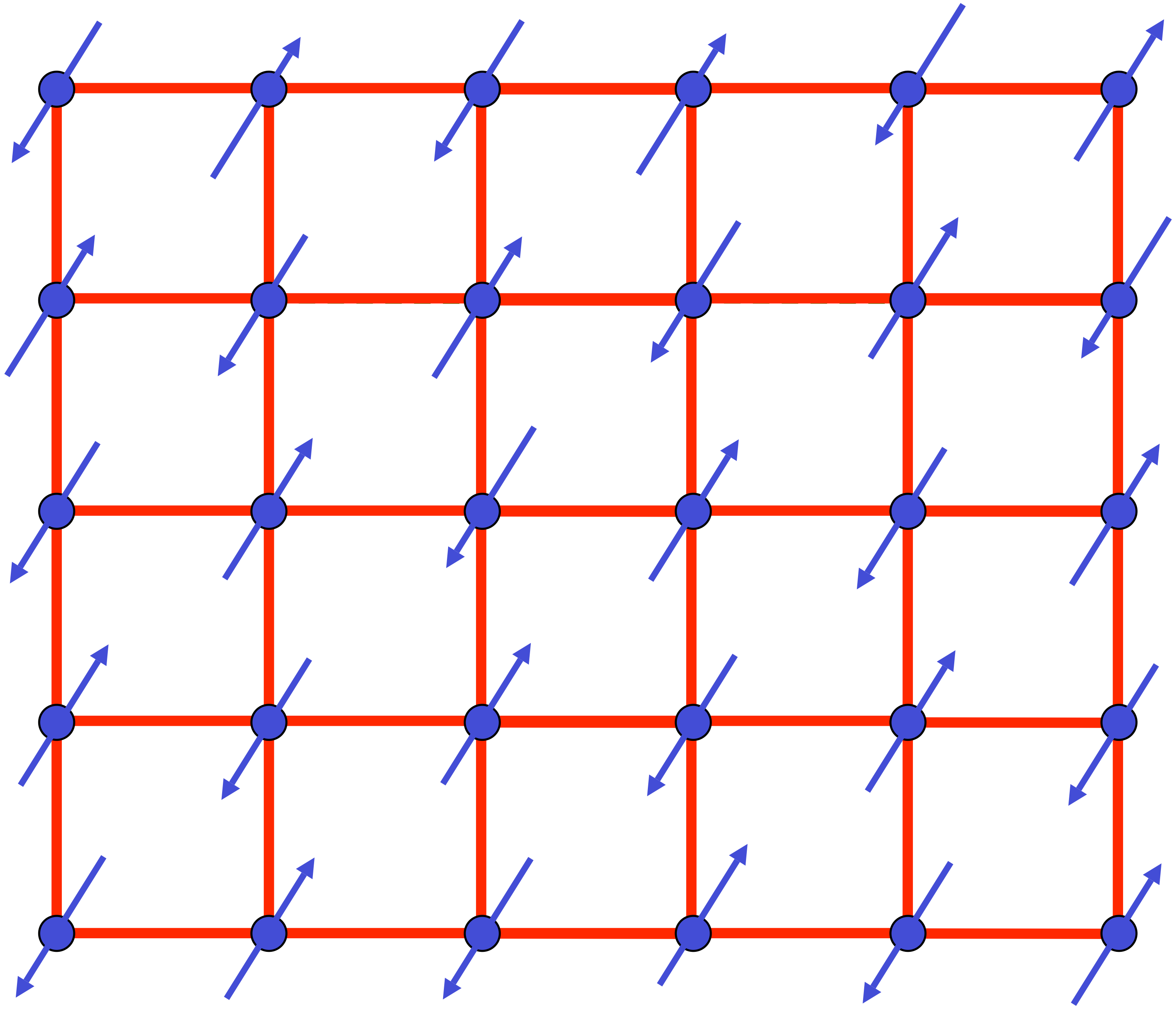
The cuprate superconductors





Antiferromagnetic (Neel) order in the insulator

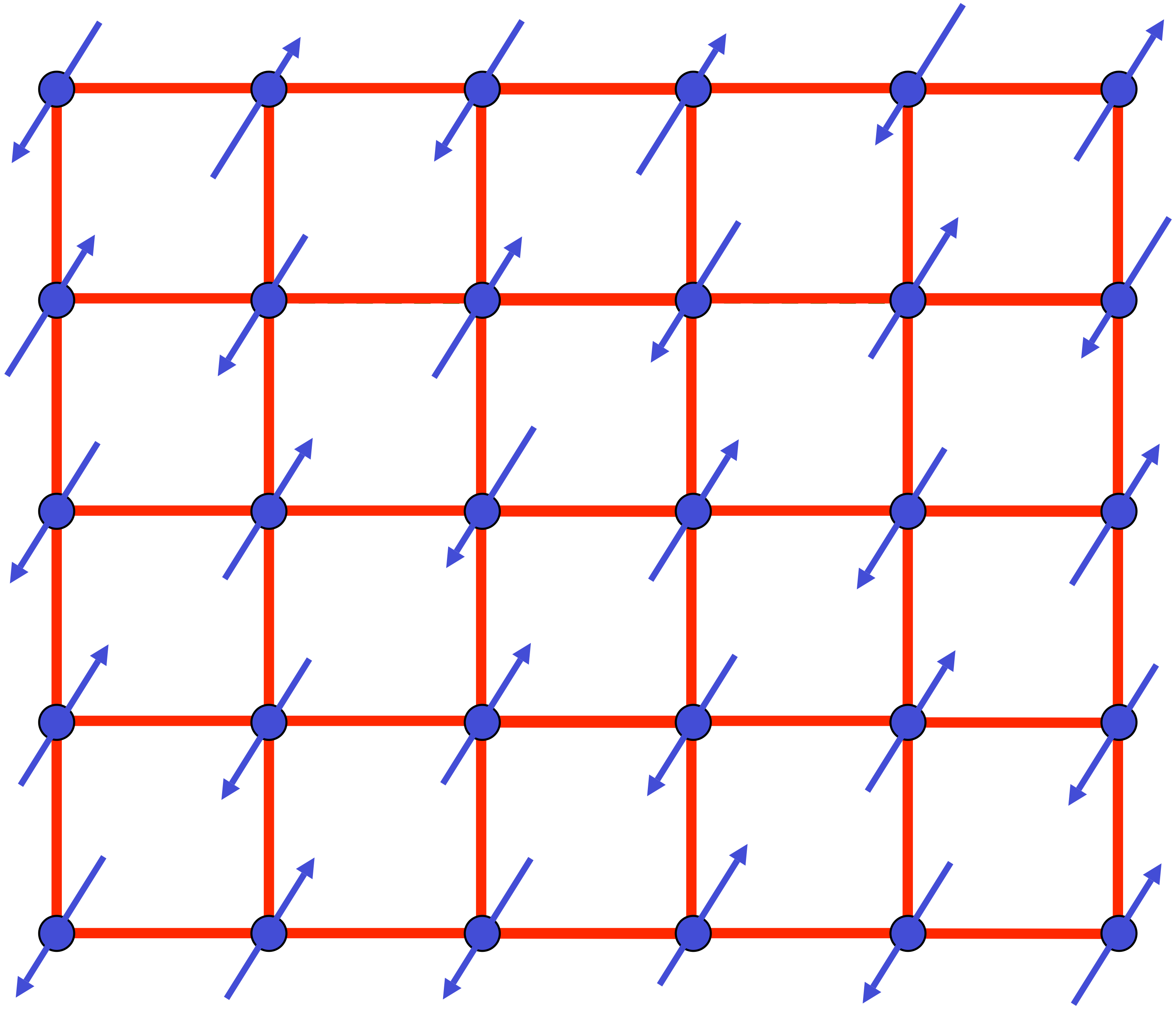


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

No entanglement of spins

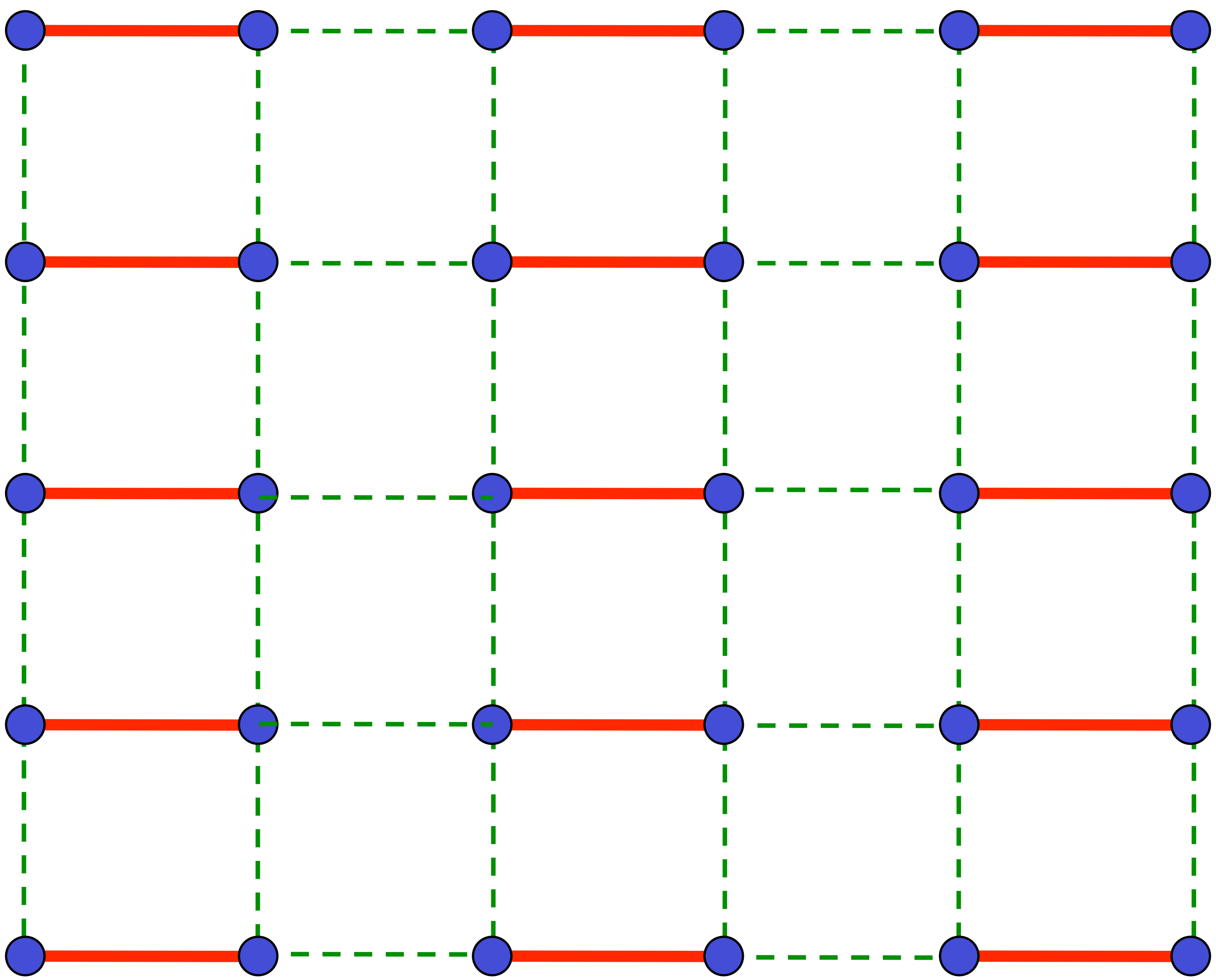
Antiferromagnetic (Neel) order in the insulator



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

Excitations: 2 spin waves (Goldstone modes)



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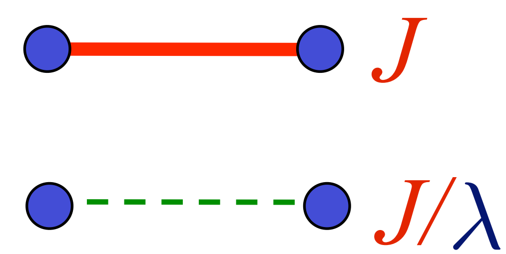
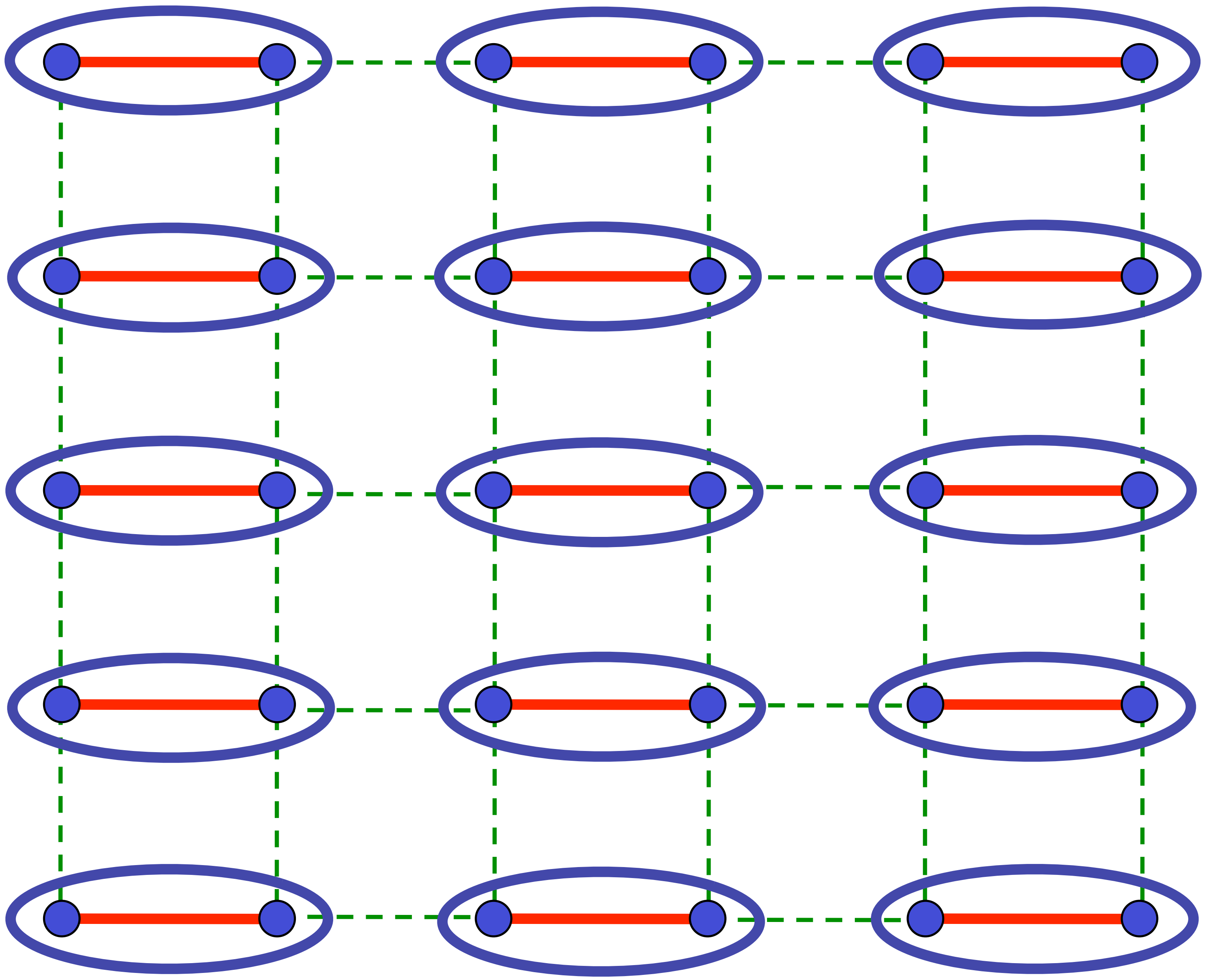


Weaken some bonds to induce spin entanglement in a new quantum phase



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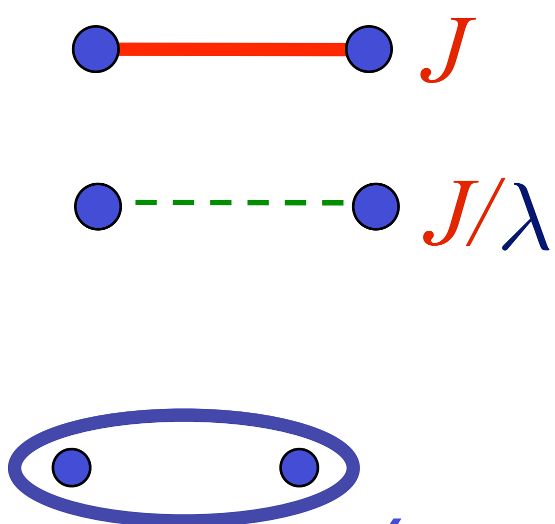
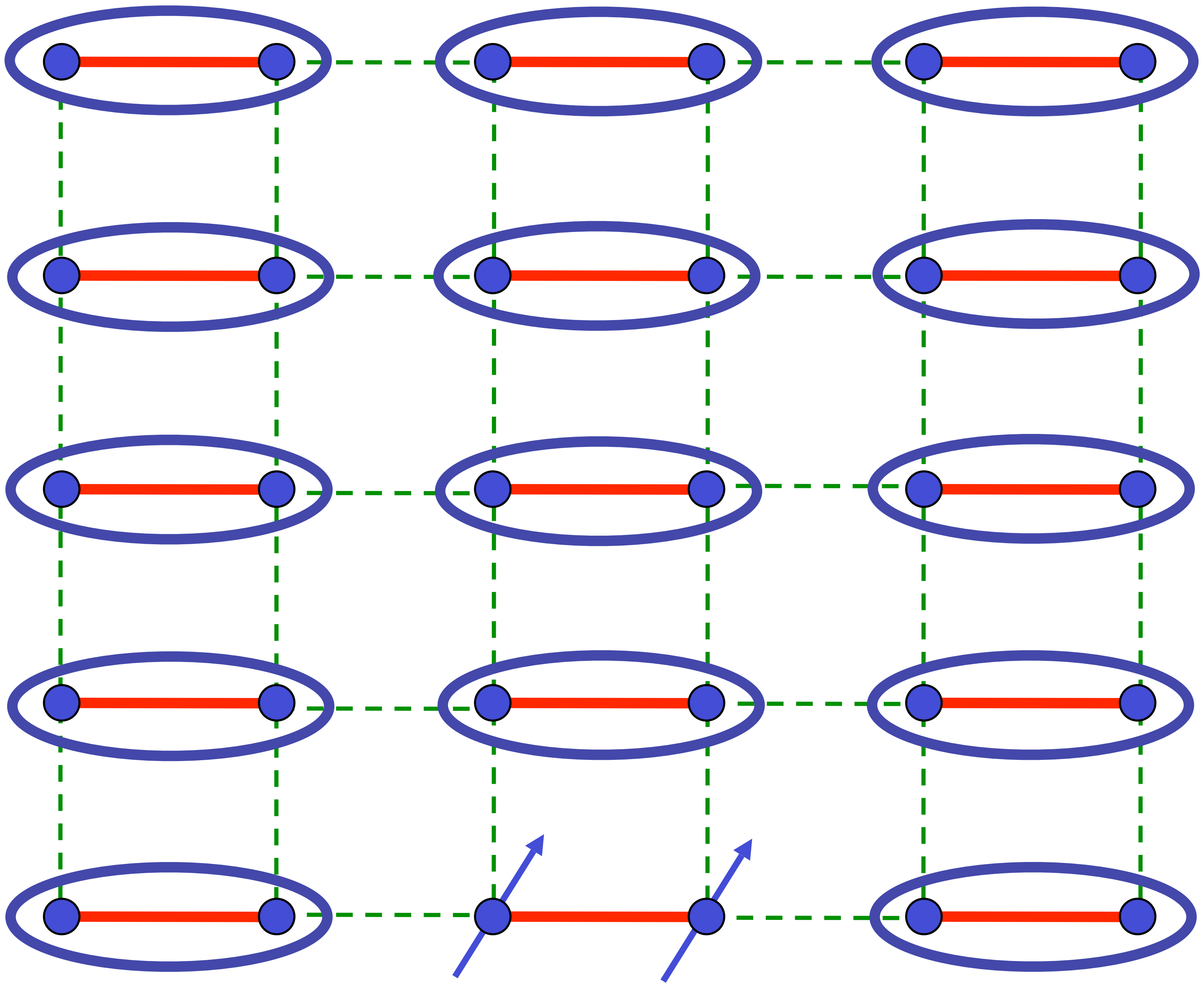


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Ground state is a product of pairs of entangled spins.



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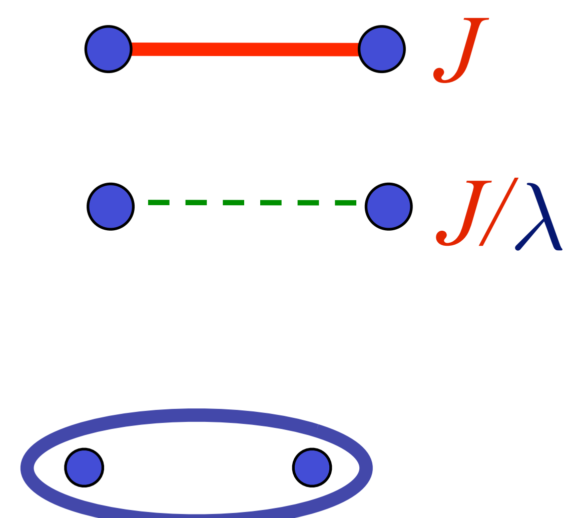
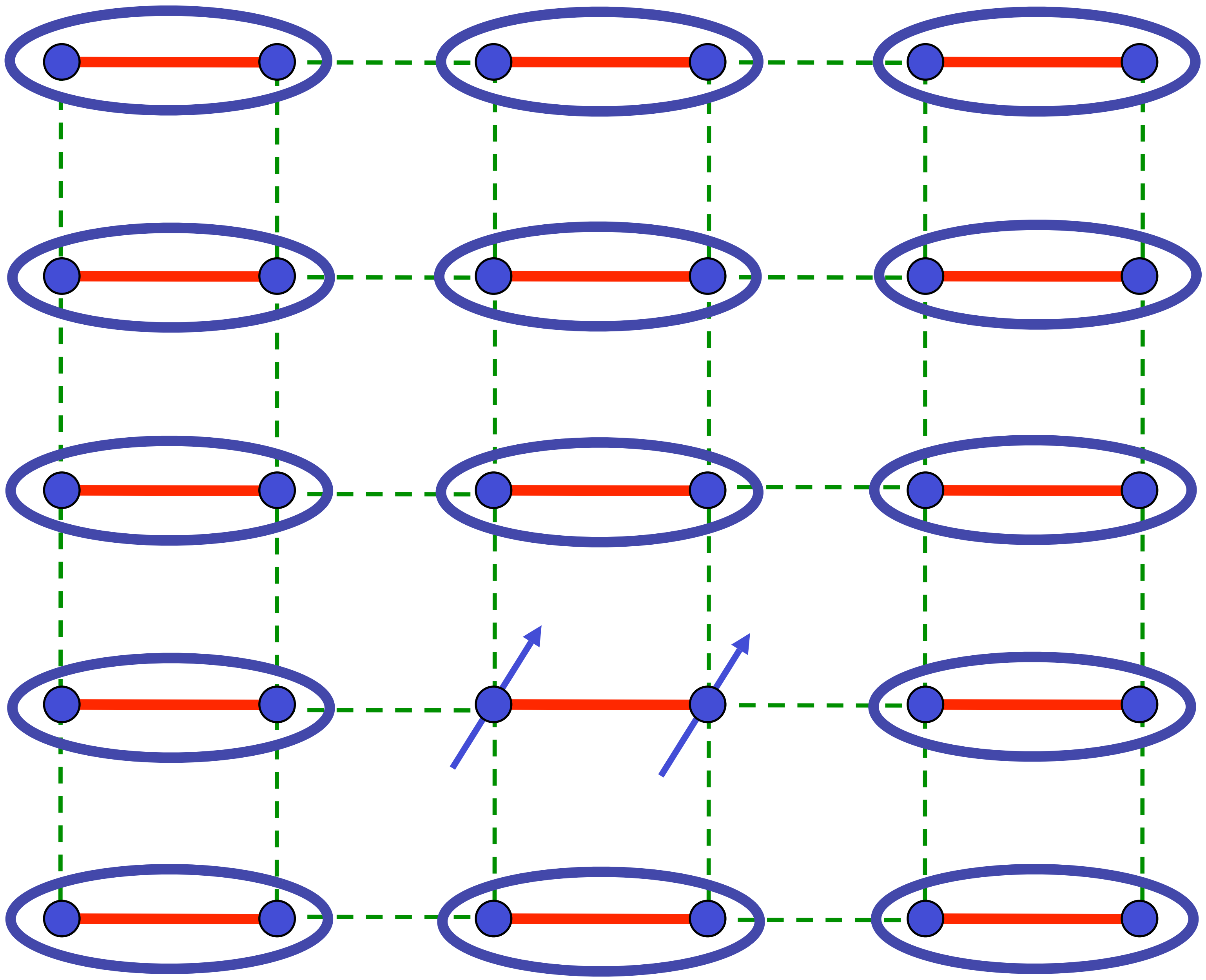


$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \Big)$$

Excitations: 3 *S=1* triplons



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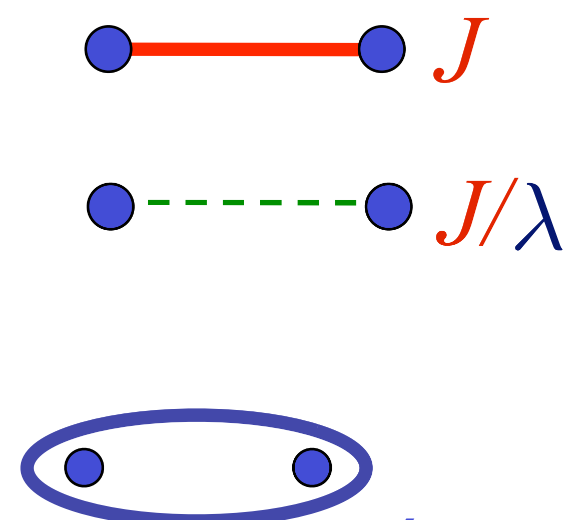

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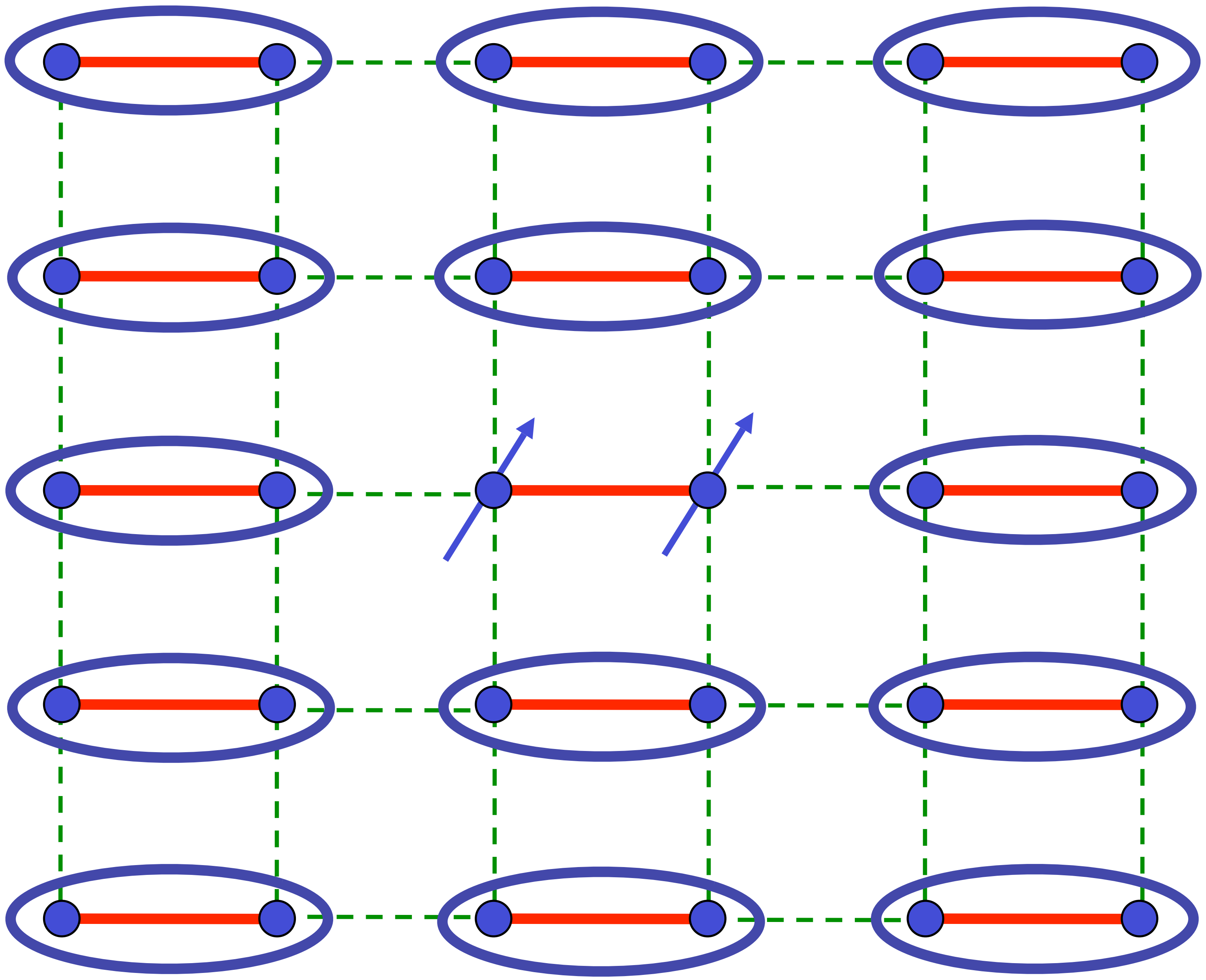
Excitations: 3 $S=1$ triplons



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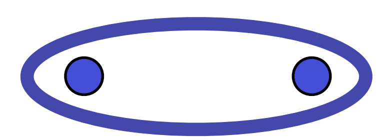
J

J/λ



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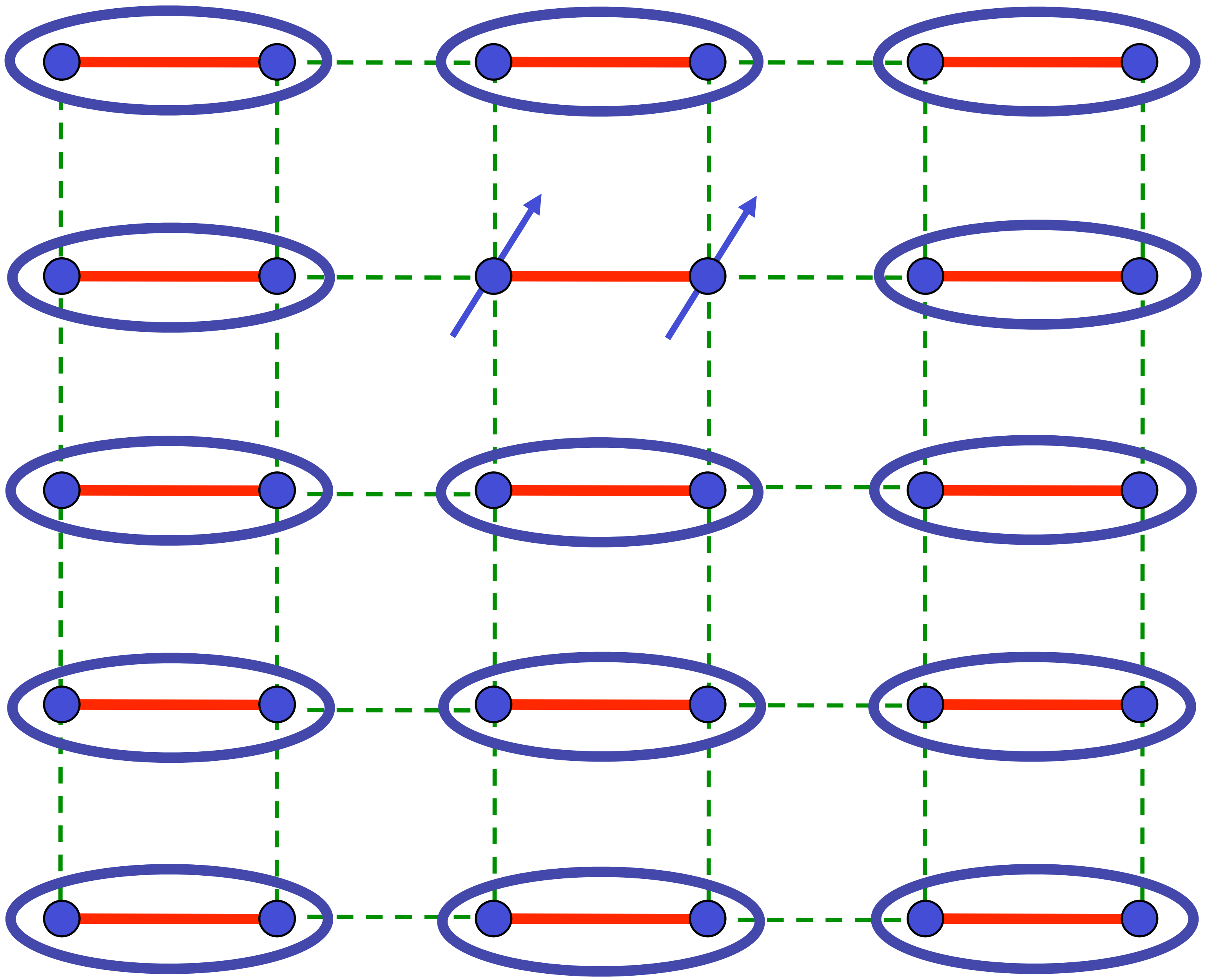

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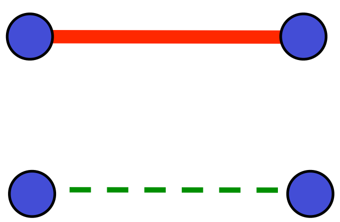
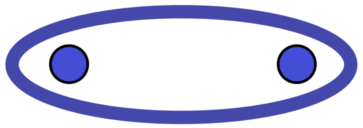
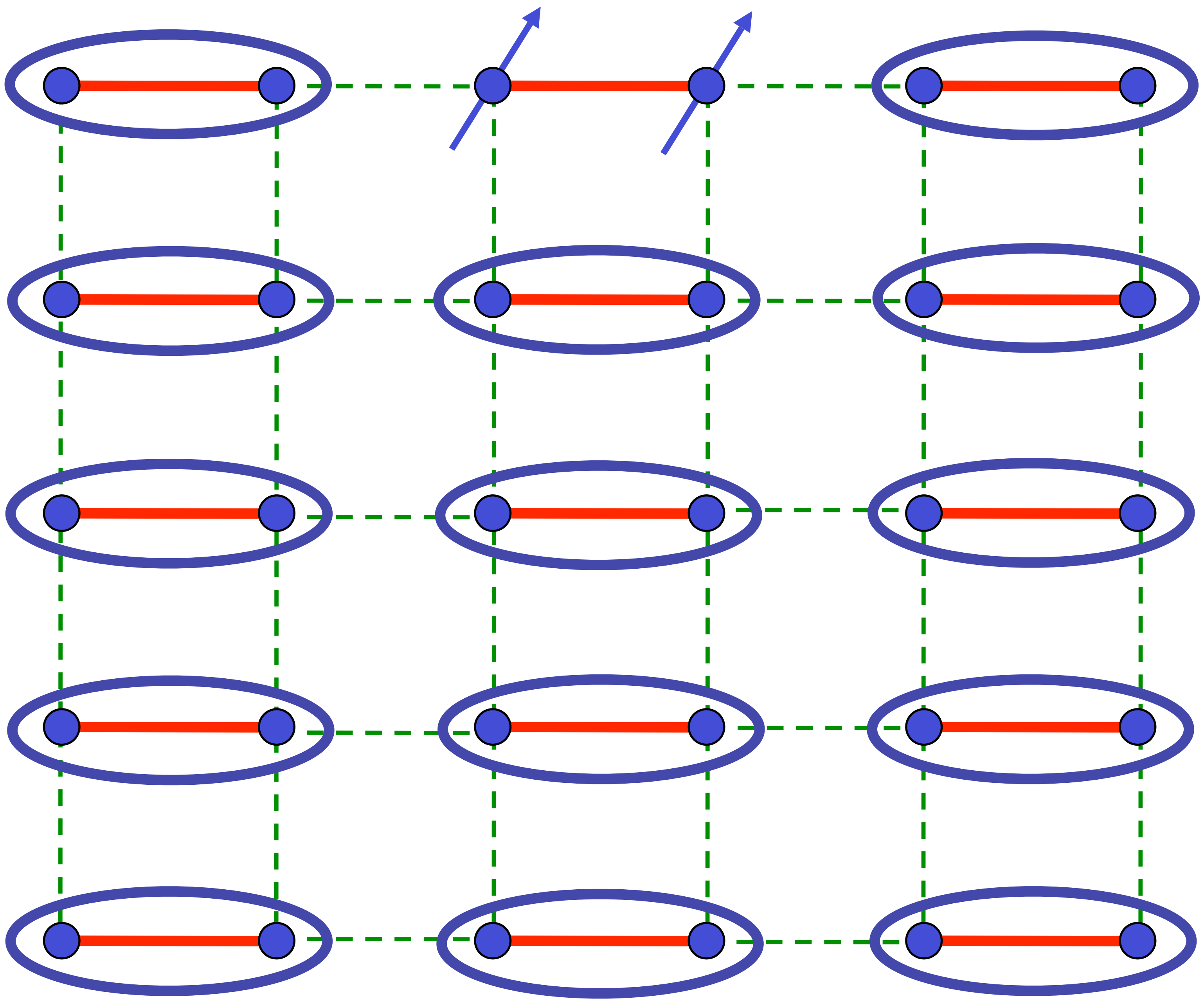




$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$

Excitations: 3 $S=1$ triplons



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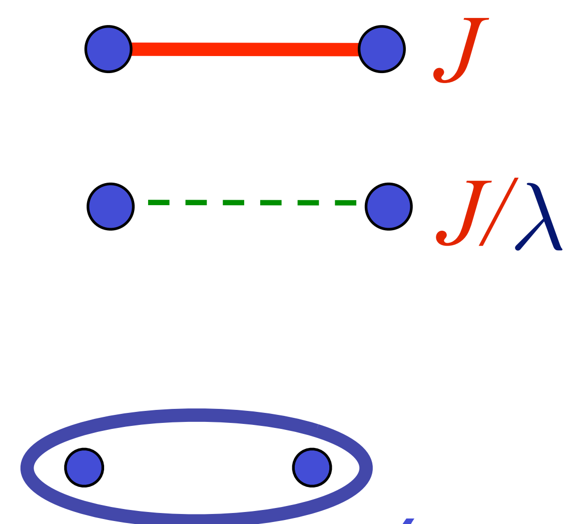
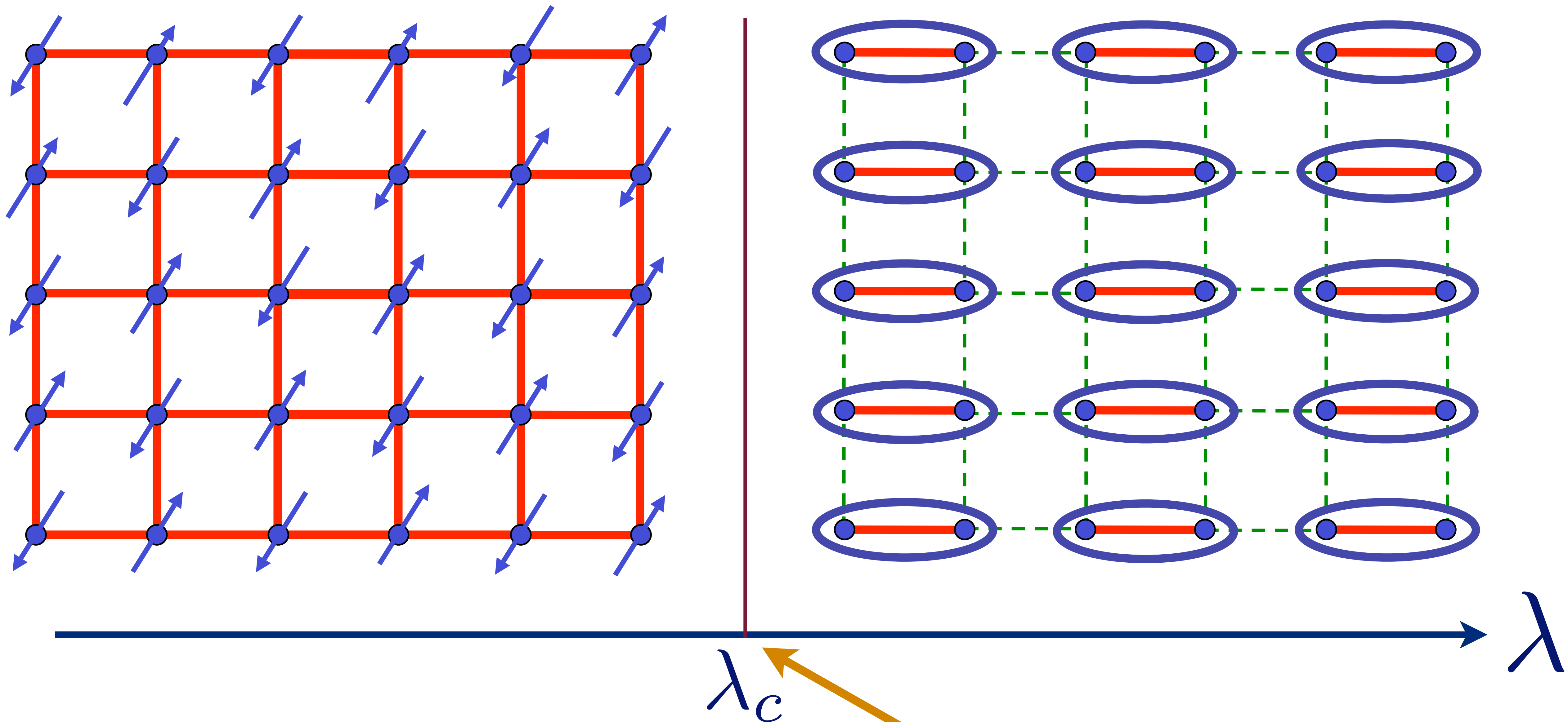


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

Excitations: 3 $S=1$ triplons

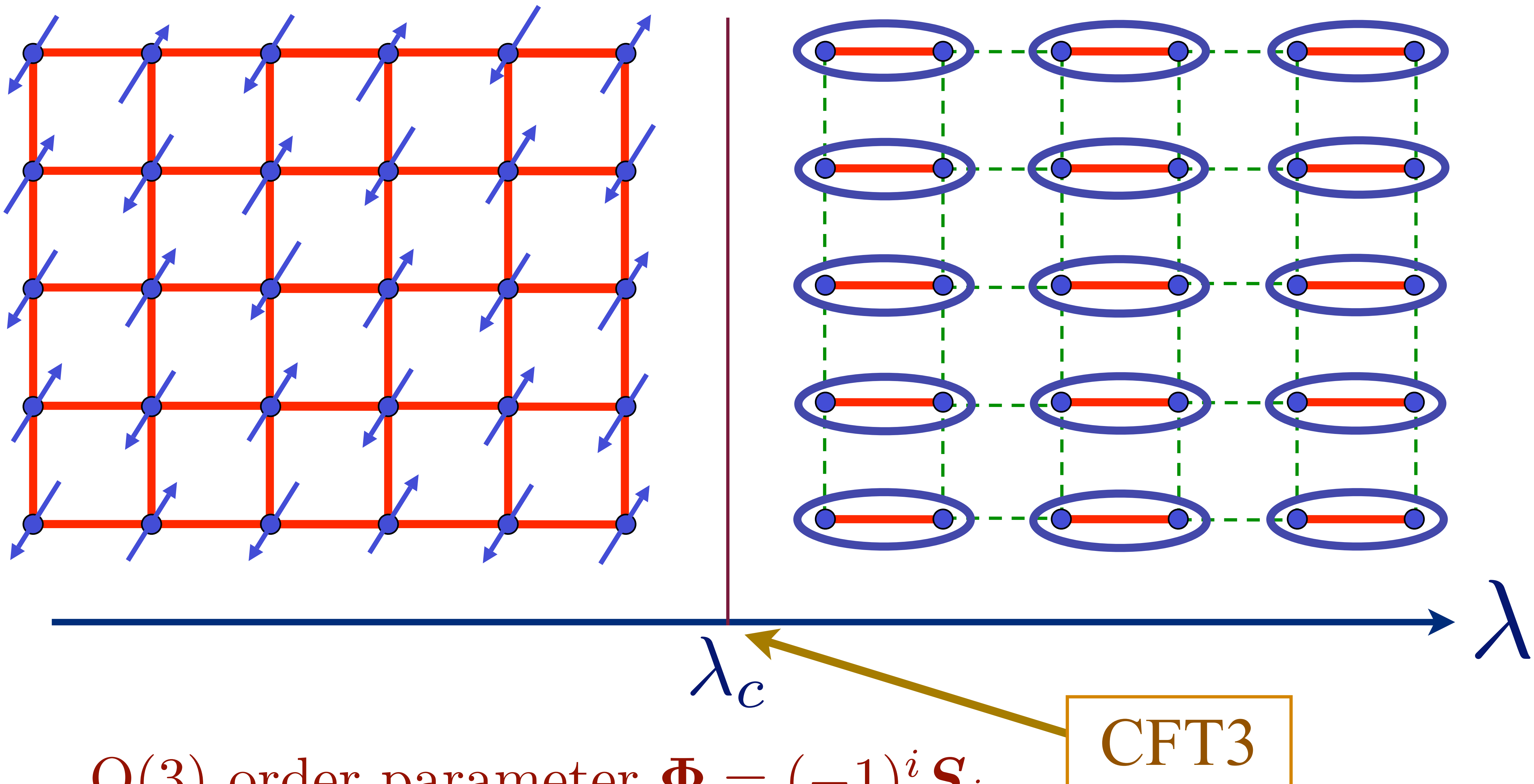
Phase diagram as a function of the ratio of exchange interactions, λ



Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

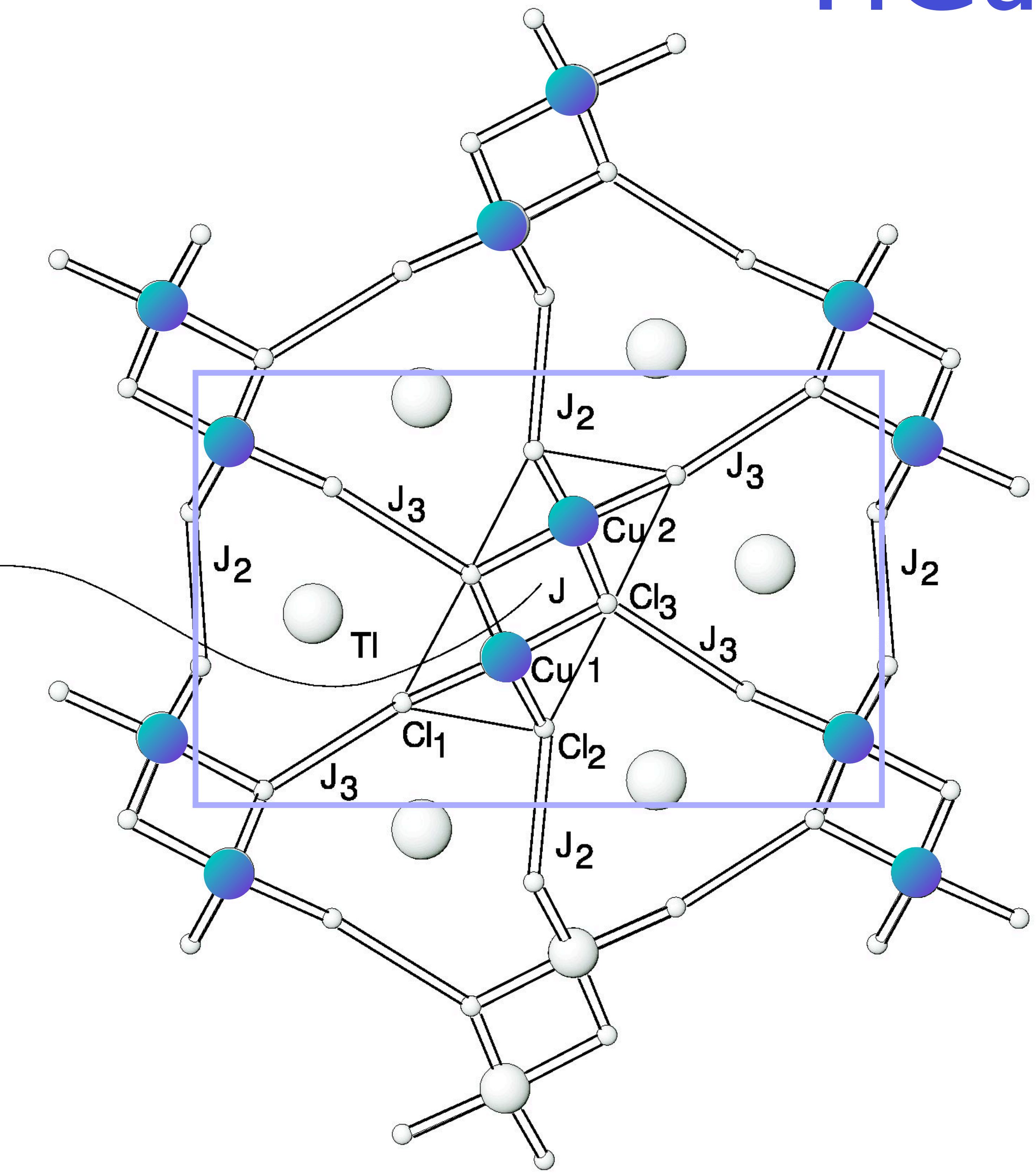
Phase diagram as a function of the ratio of exchange interactions, λ



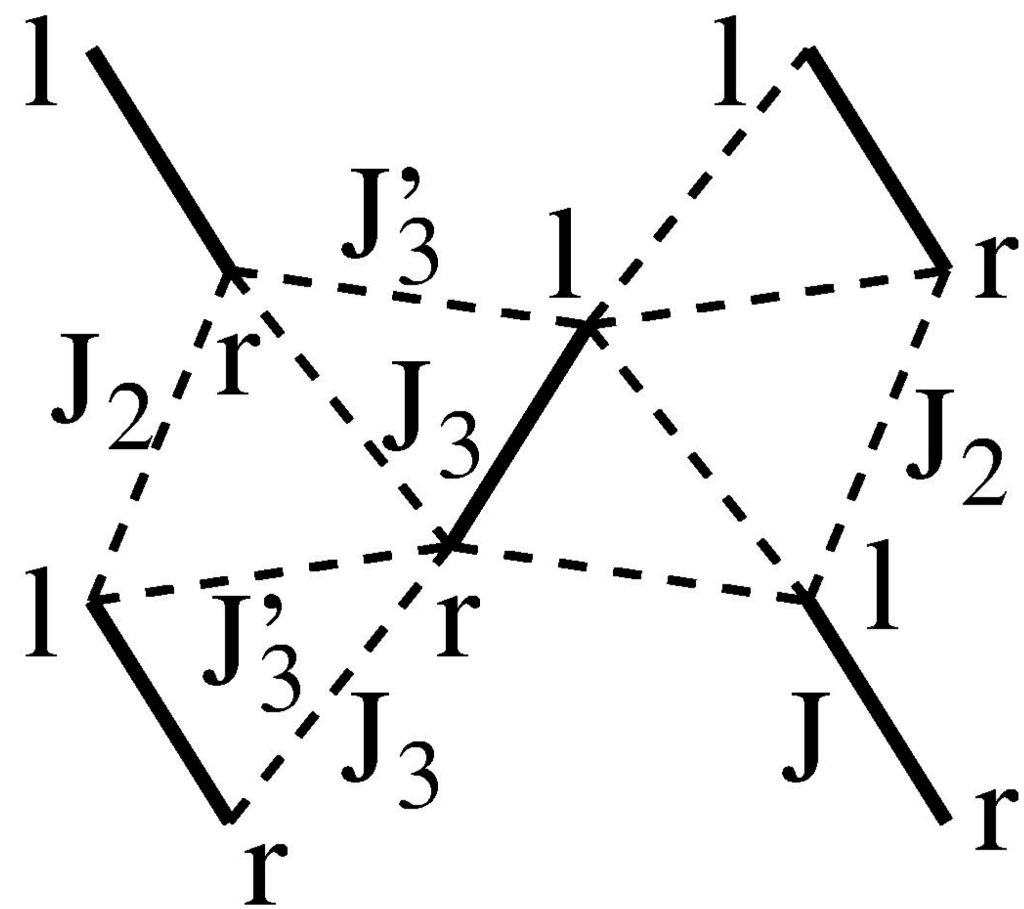
O(3) order parameter $\boldsymbol{\Phi} = (-1)^i \boldsymbol{S}_i$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \boldsymbol{\Phi})^2 + c^2 (\vec{\nabla} \boldsymbol{\Phi})^2 + s \boldsymbol{\Phi}^2 + u \left(\boldsymbol{\Phi}^2 \right)^2 \right]$$

$TlCuCl_3$



J2
J3
J3
Cu 2
J2
J
Cl3
J2
Tl
J3
Cu 1
Cl1
Cl2
J3
J2



l
l
J'3
l
r
r
J2
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J3
J2
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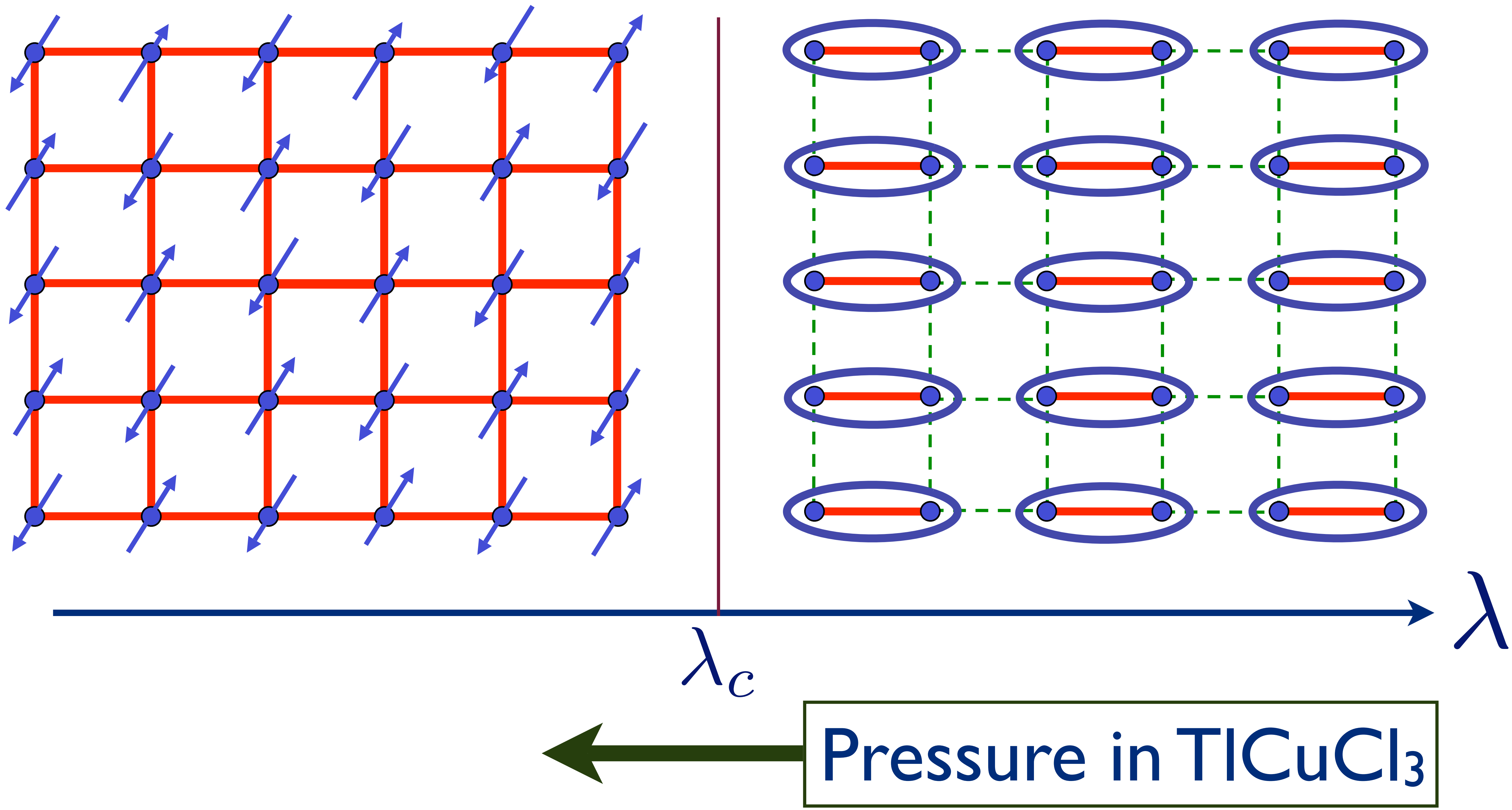
J'3
J3
J
r
r

Phase diagram as a function of the ratio of exchange interactions, λ



$TlCuCl_3$ at ambient pressure

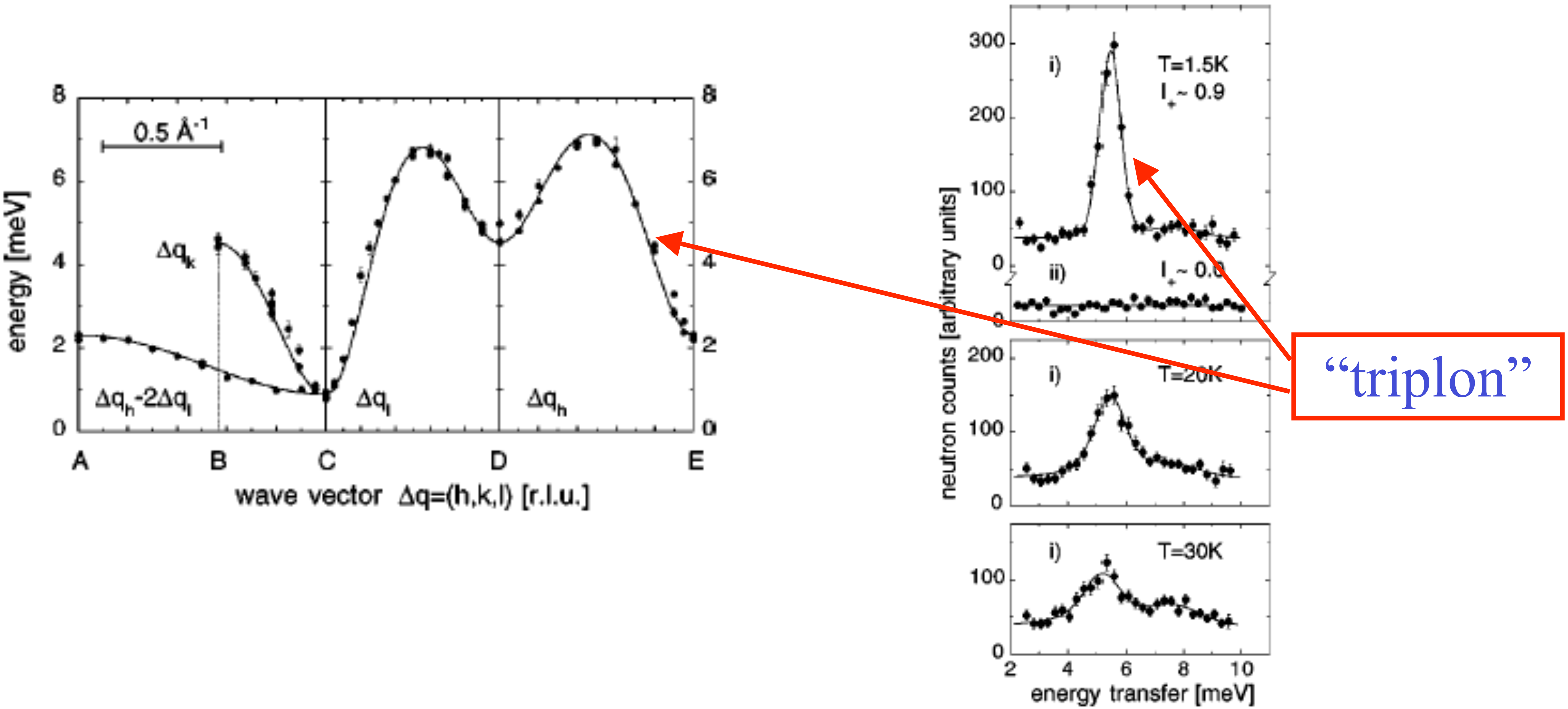
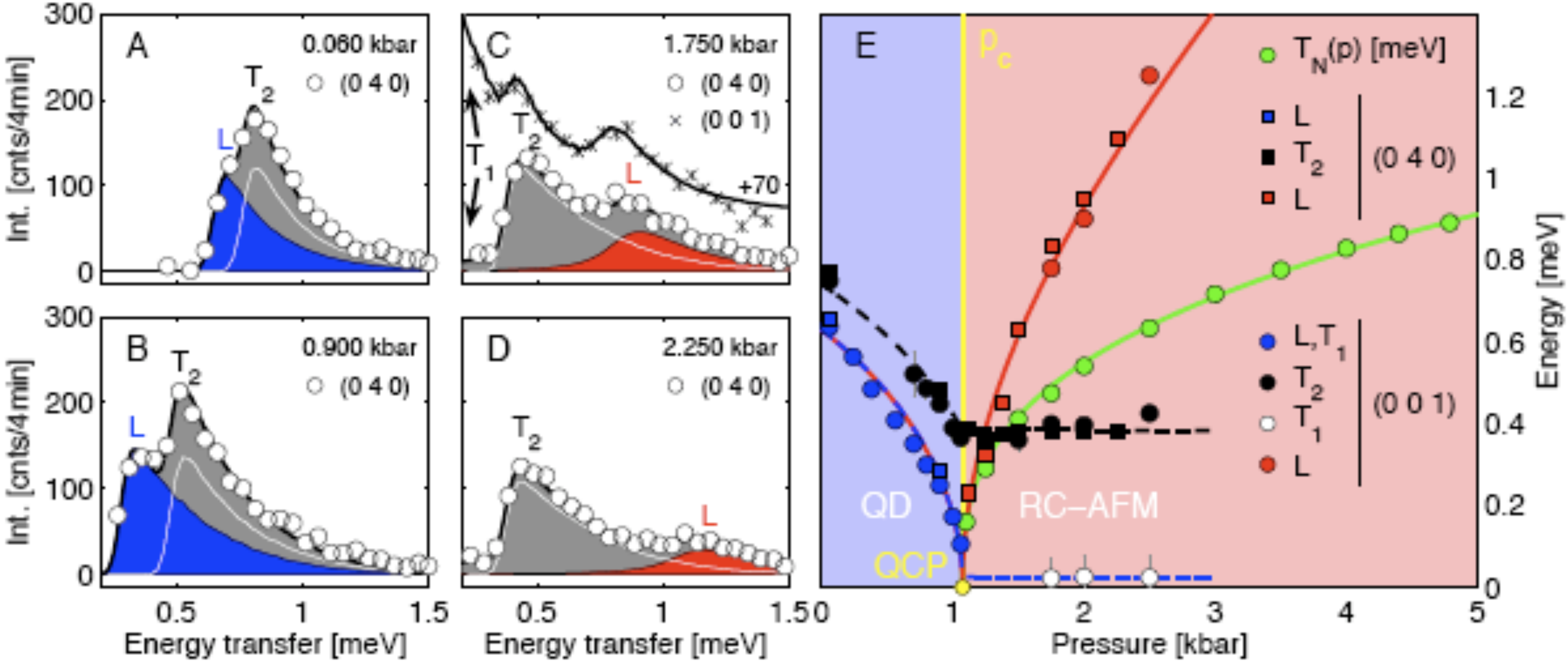


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

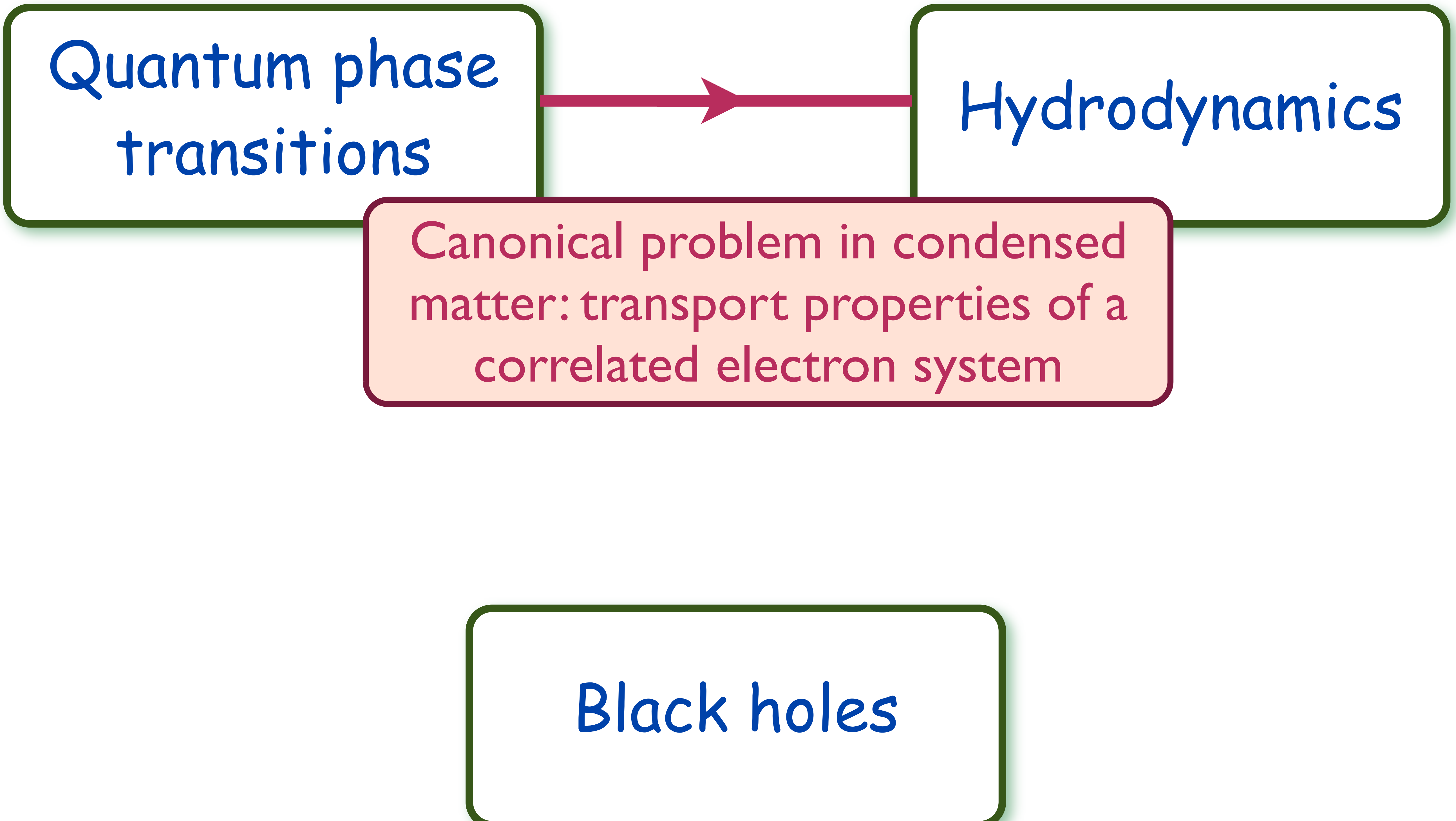
Three foci of modern physics

Quantum phase transitions

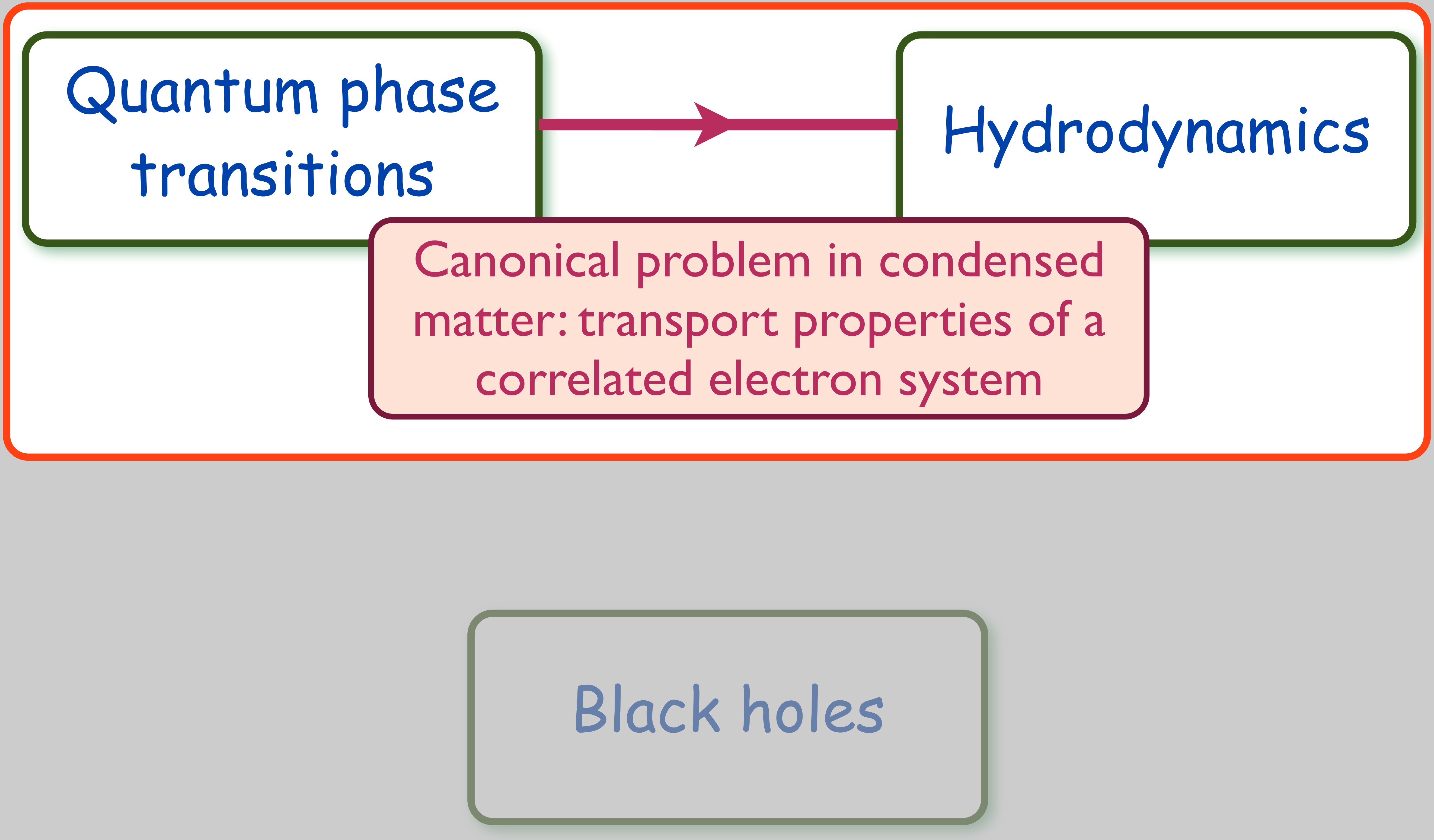
Hydrodynamics

Black holes

Three foci of modern physics

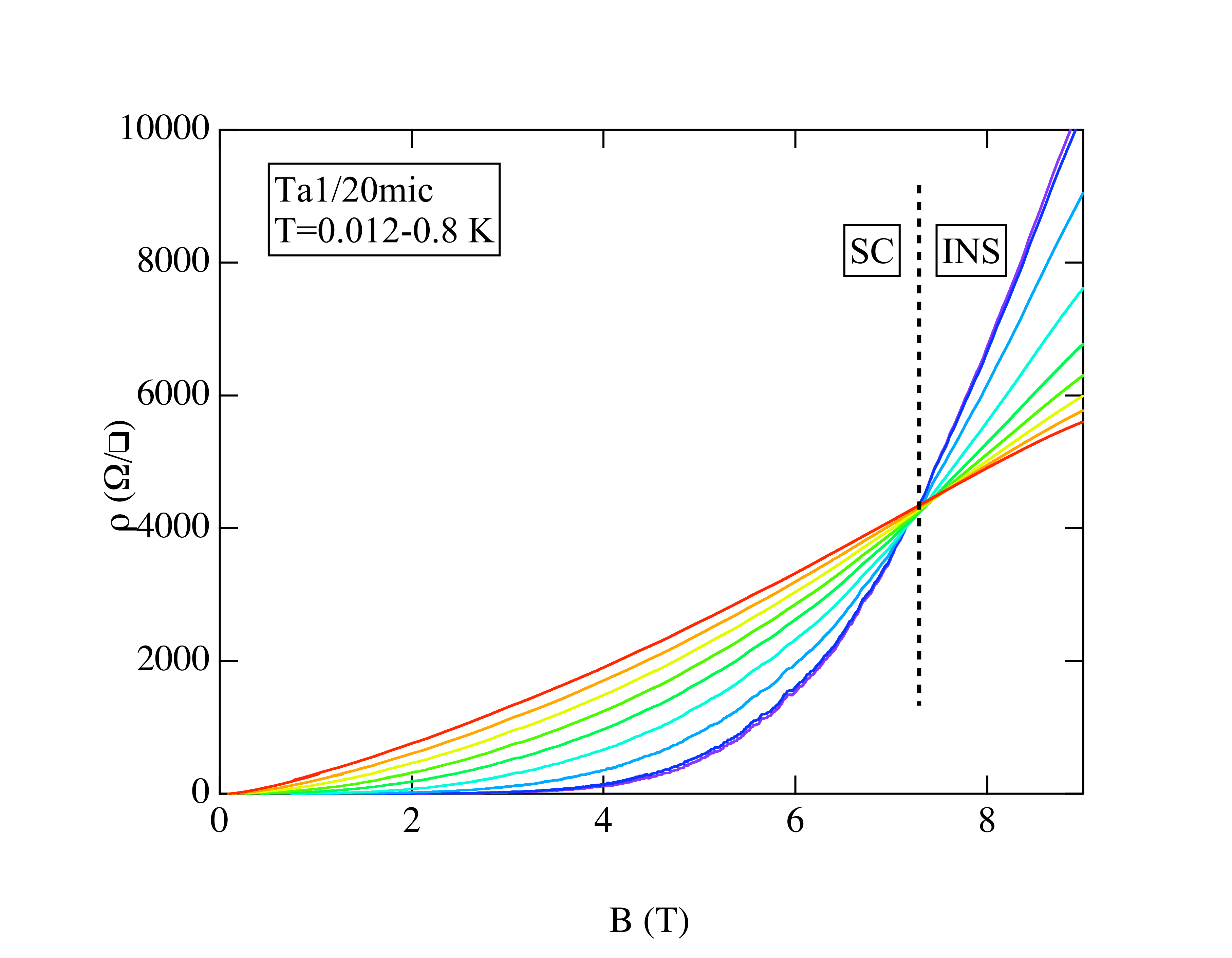


Three foci of modern physics



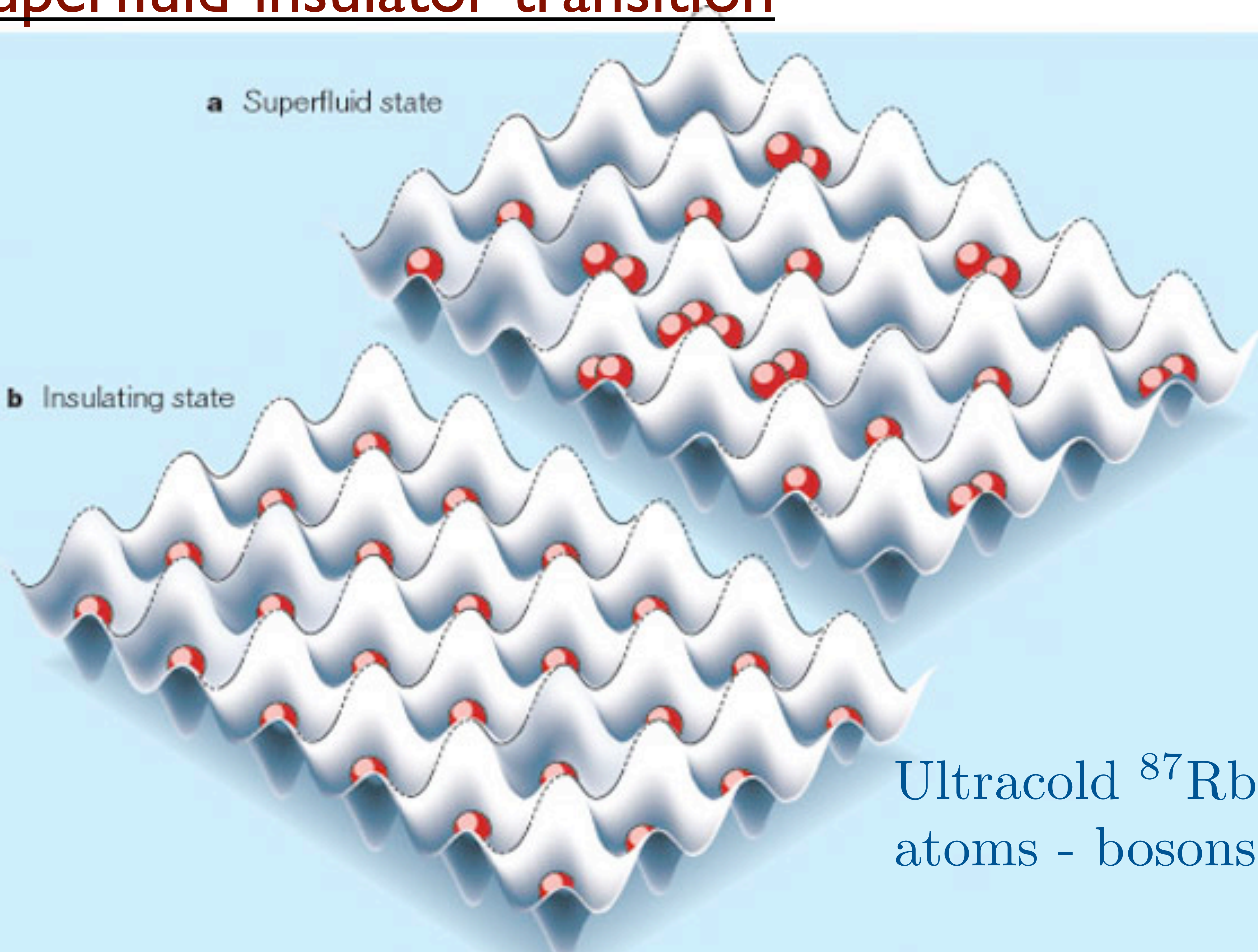
Superfluid-insulator transition

Indium Oxide films



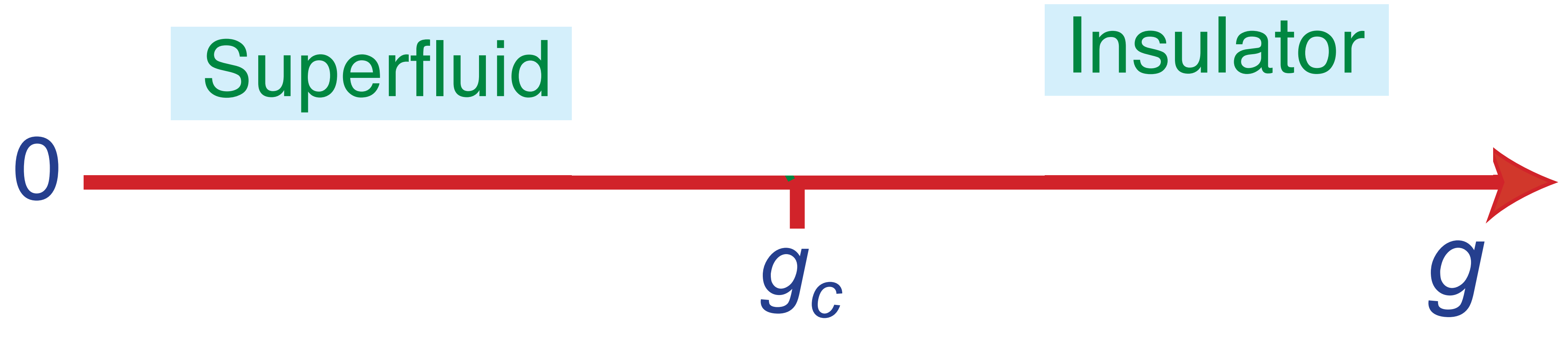
G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Bjornsson, and K. A. Moler, *Europhys. Lett.* **75**, 611 (2006).

Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



Superfluid
Insulator
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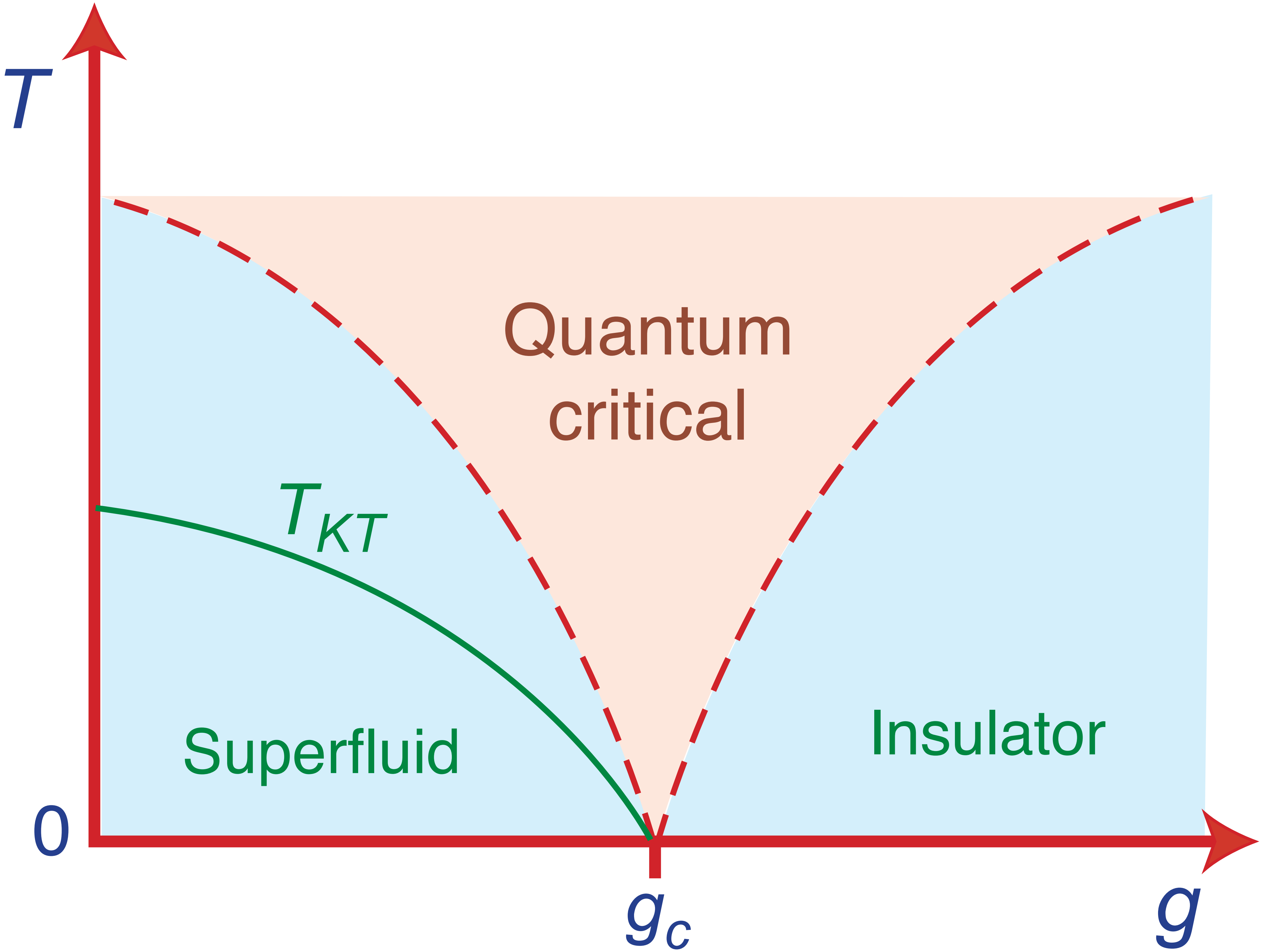
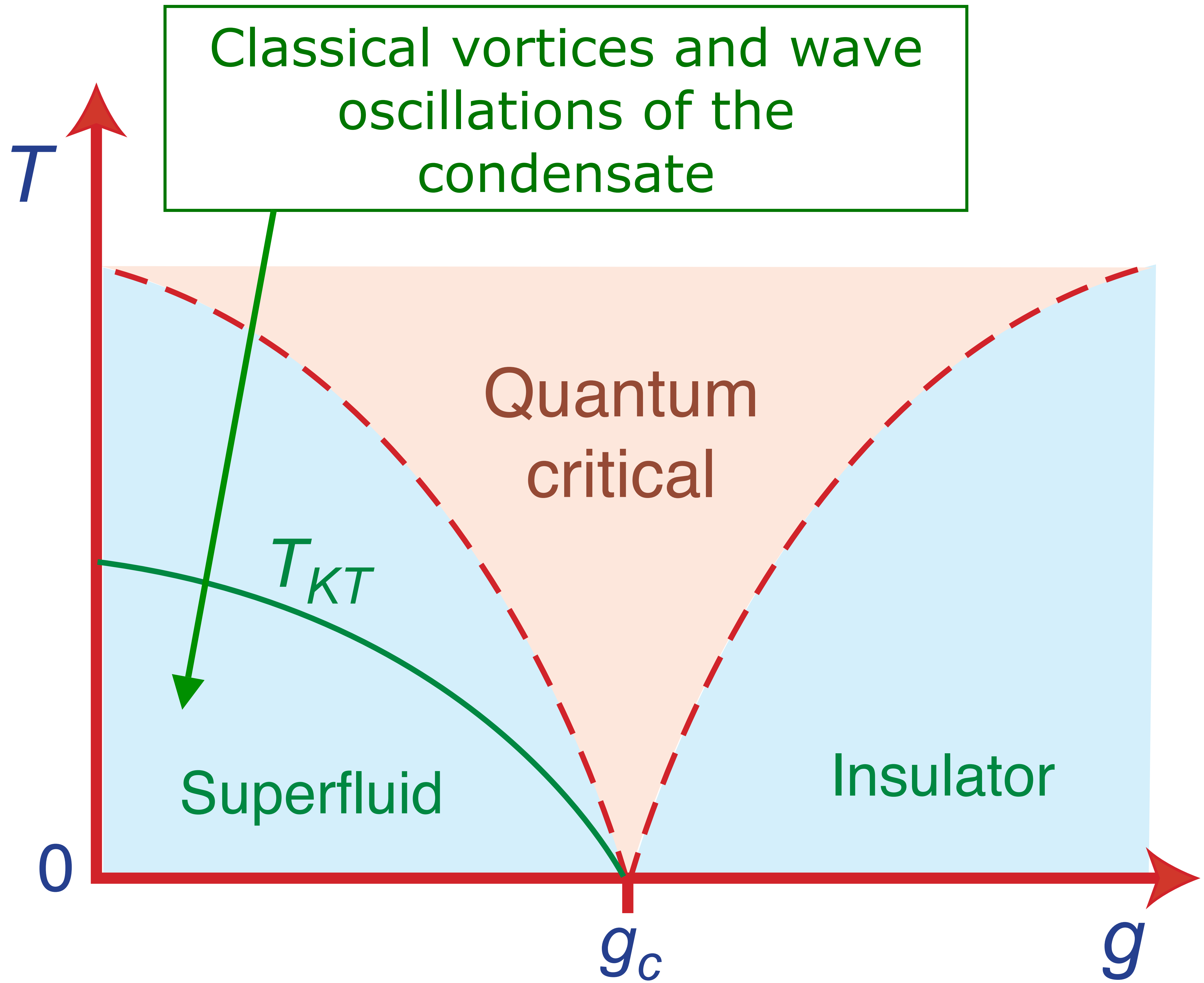
g

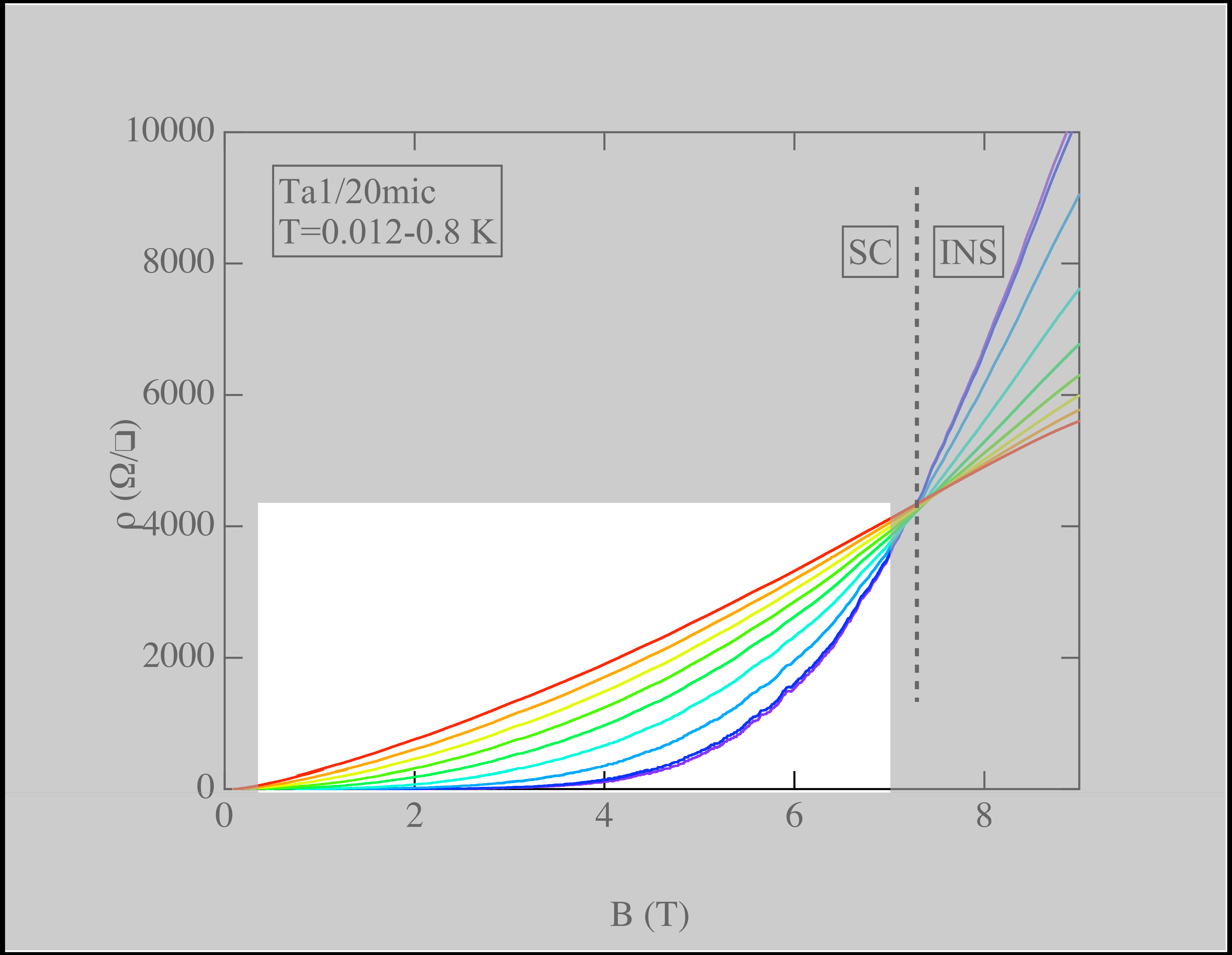


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


Classical vortices and wave oscillations of the condensate
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Superfluid-insulator transition

Indium Oxide films



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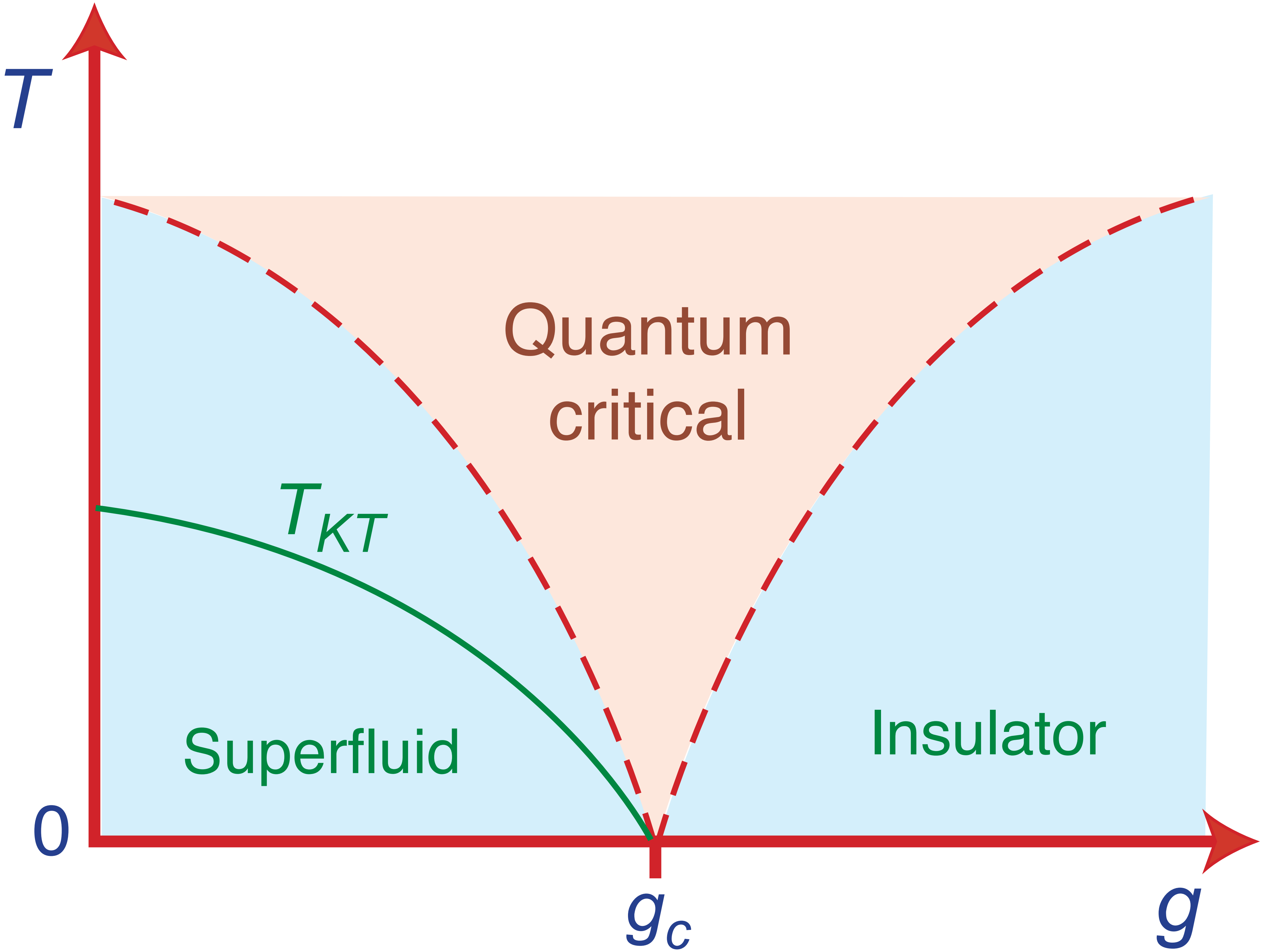
T
Quantum
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Superfluid
Insulator
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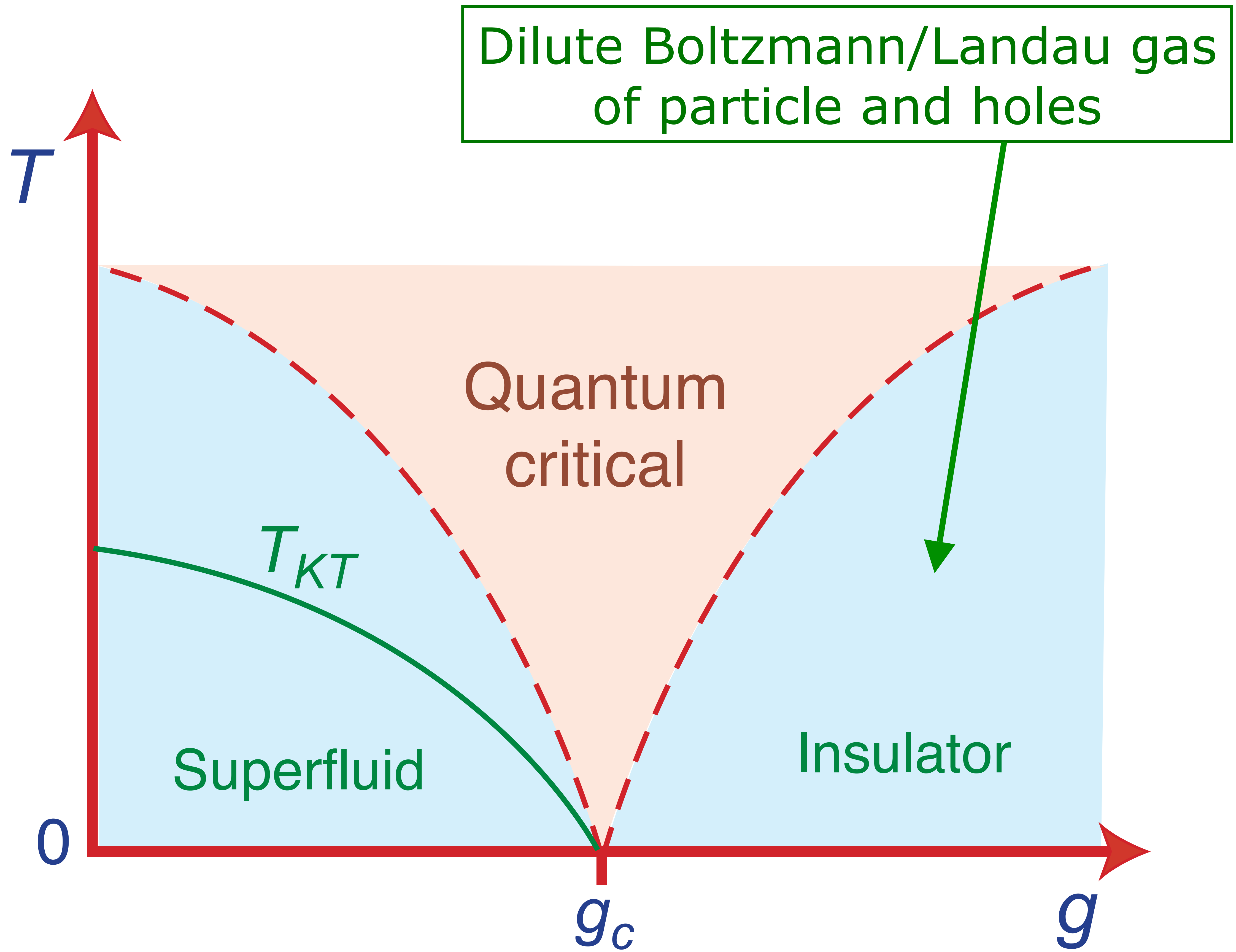
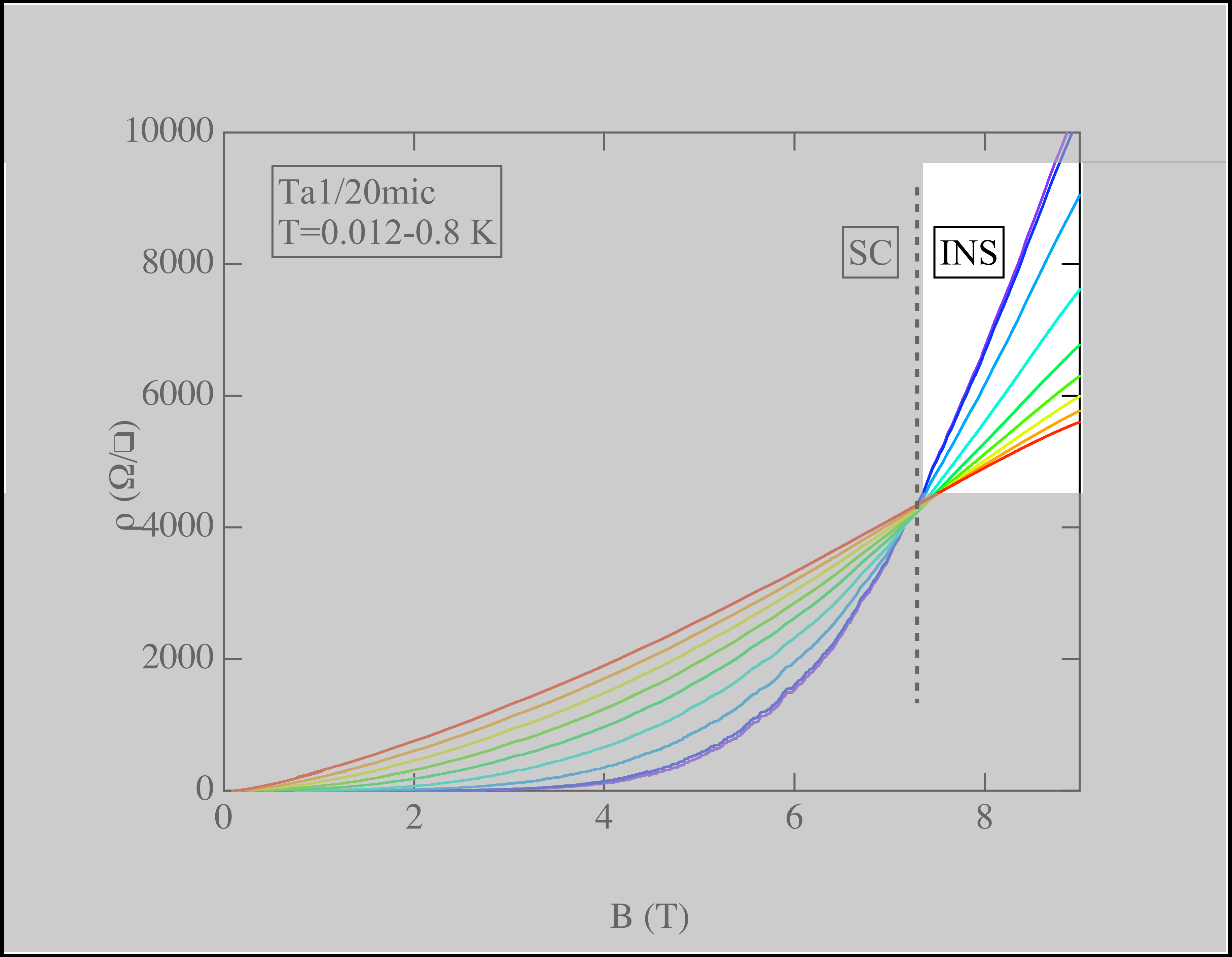


Dilute Boltzmann/Landau gas
of particle and holes
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Superfluid-insulator transition

Indium Oxide films



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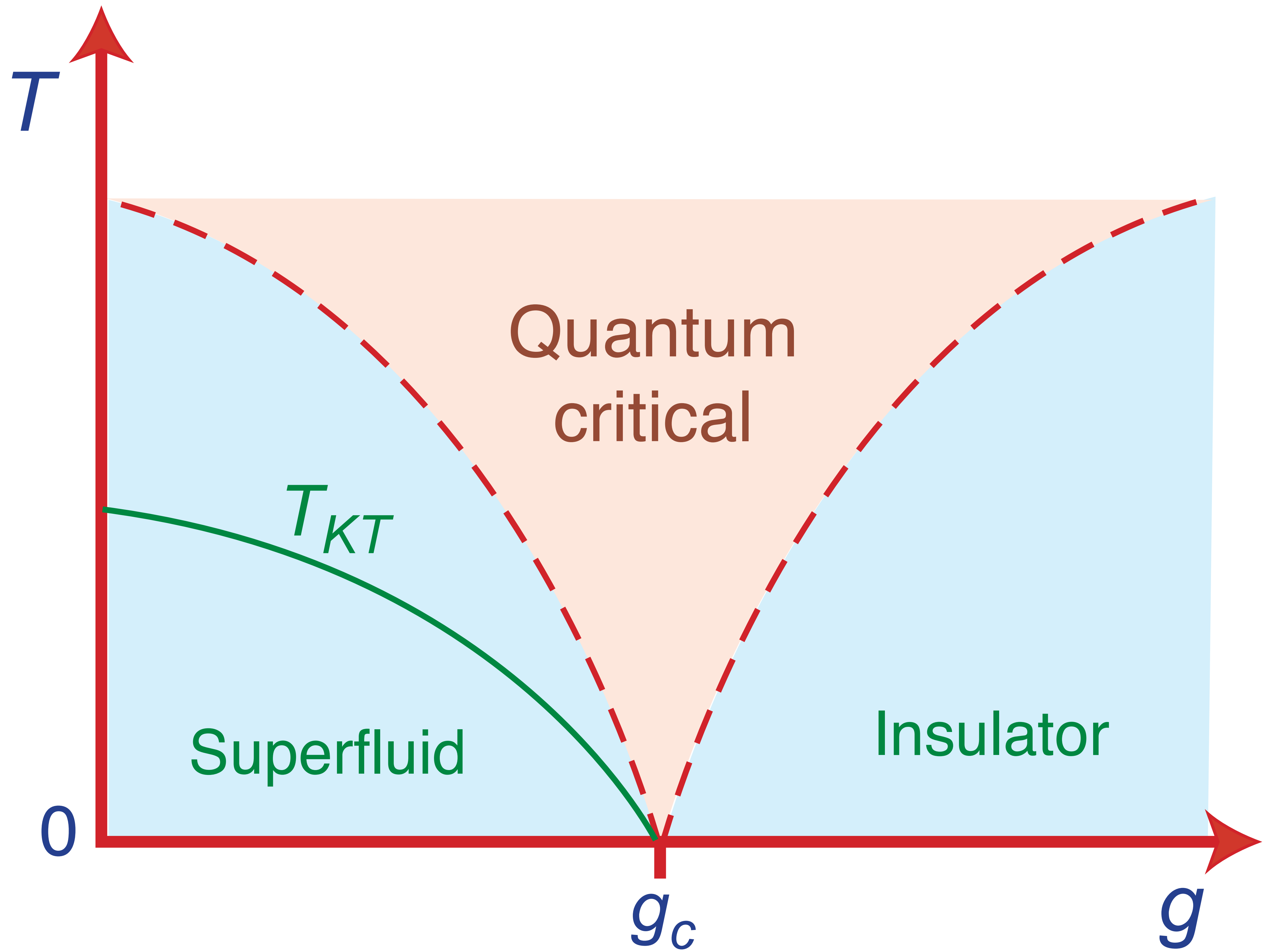
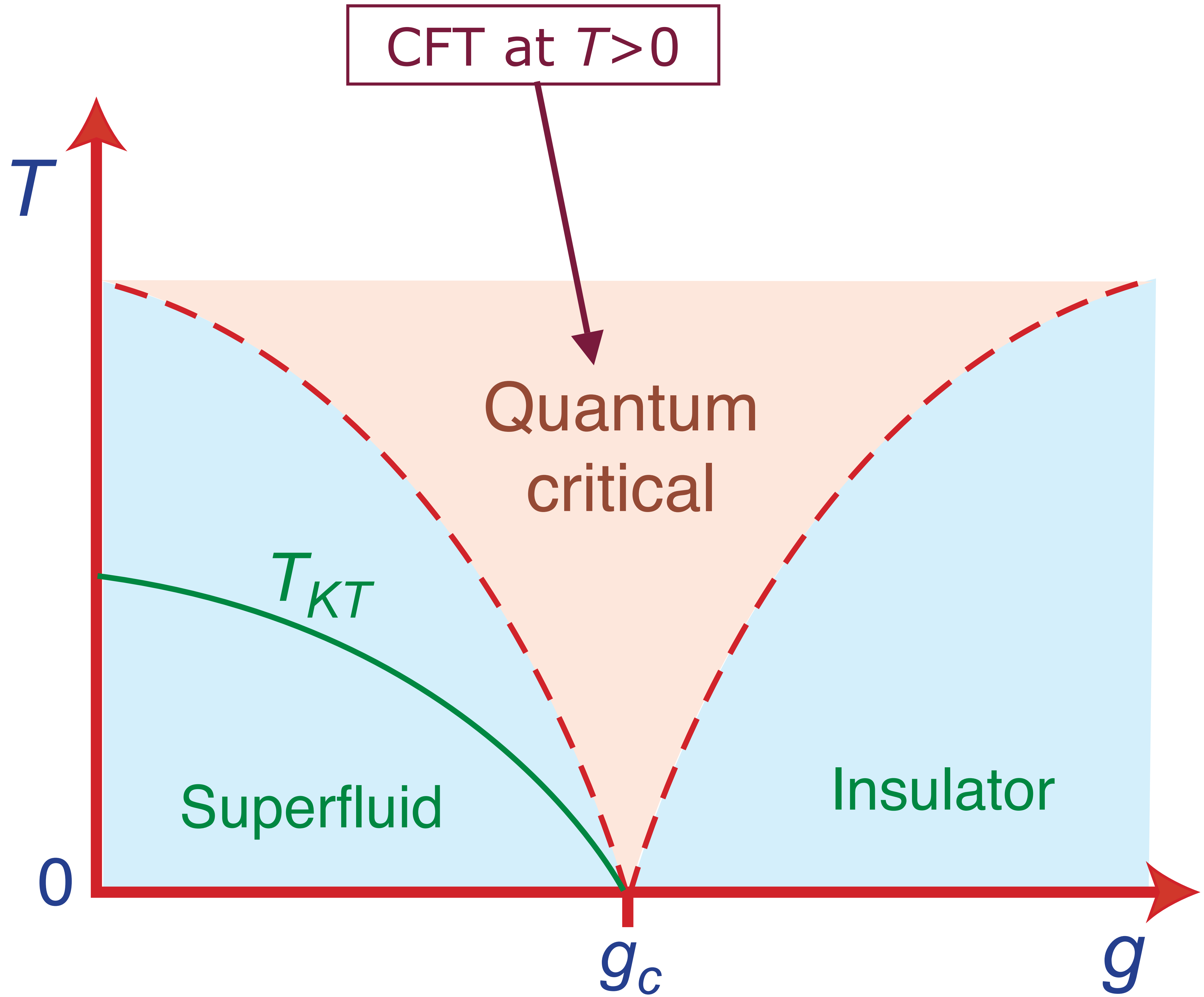

T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

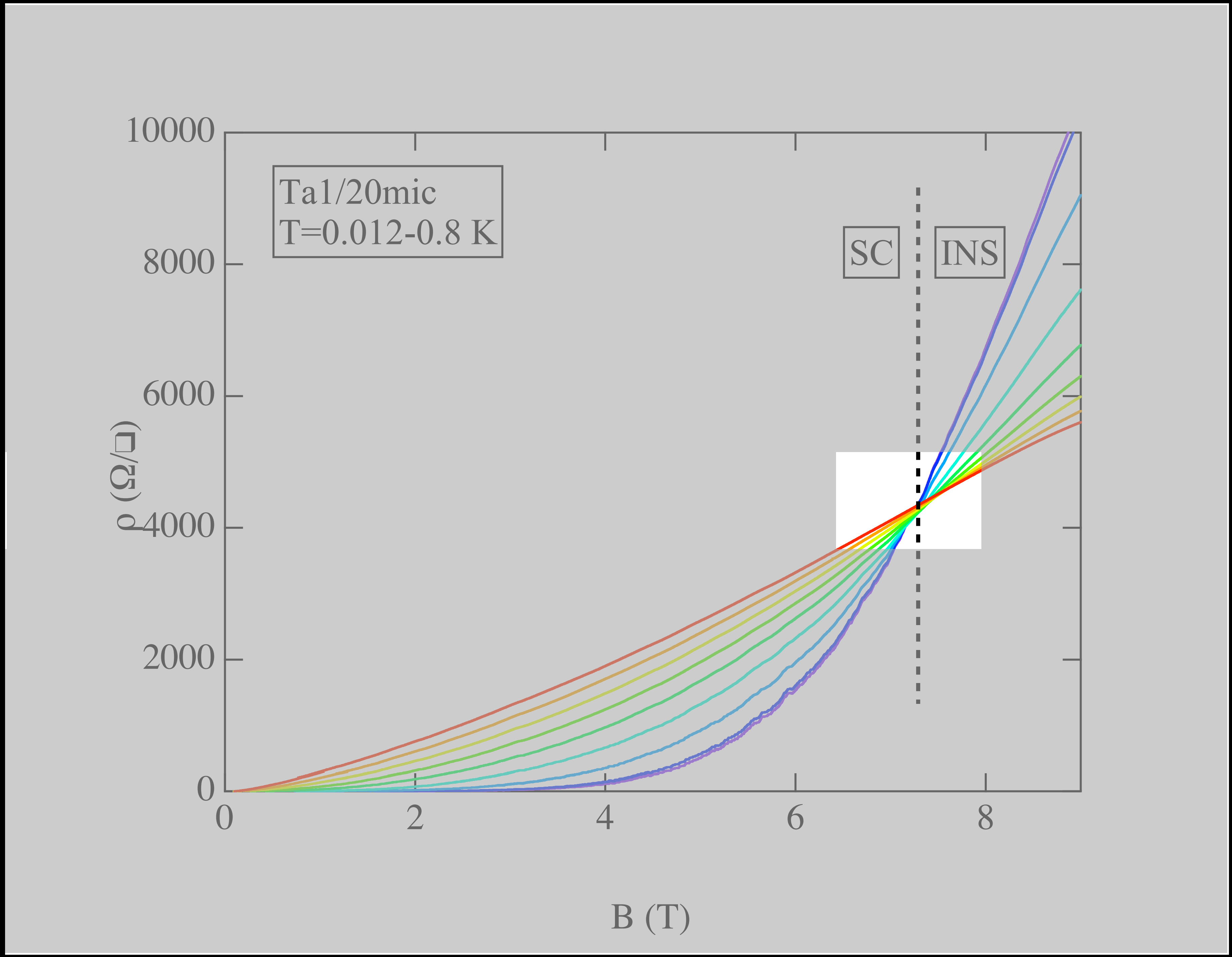
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1) \,]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Superfluid-insulator transition

Indium Oxide films



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Three foci of modern physics

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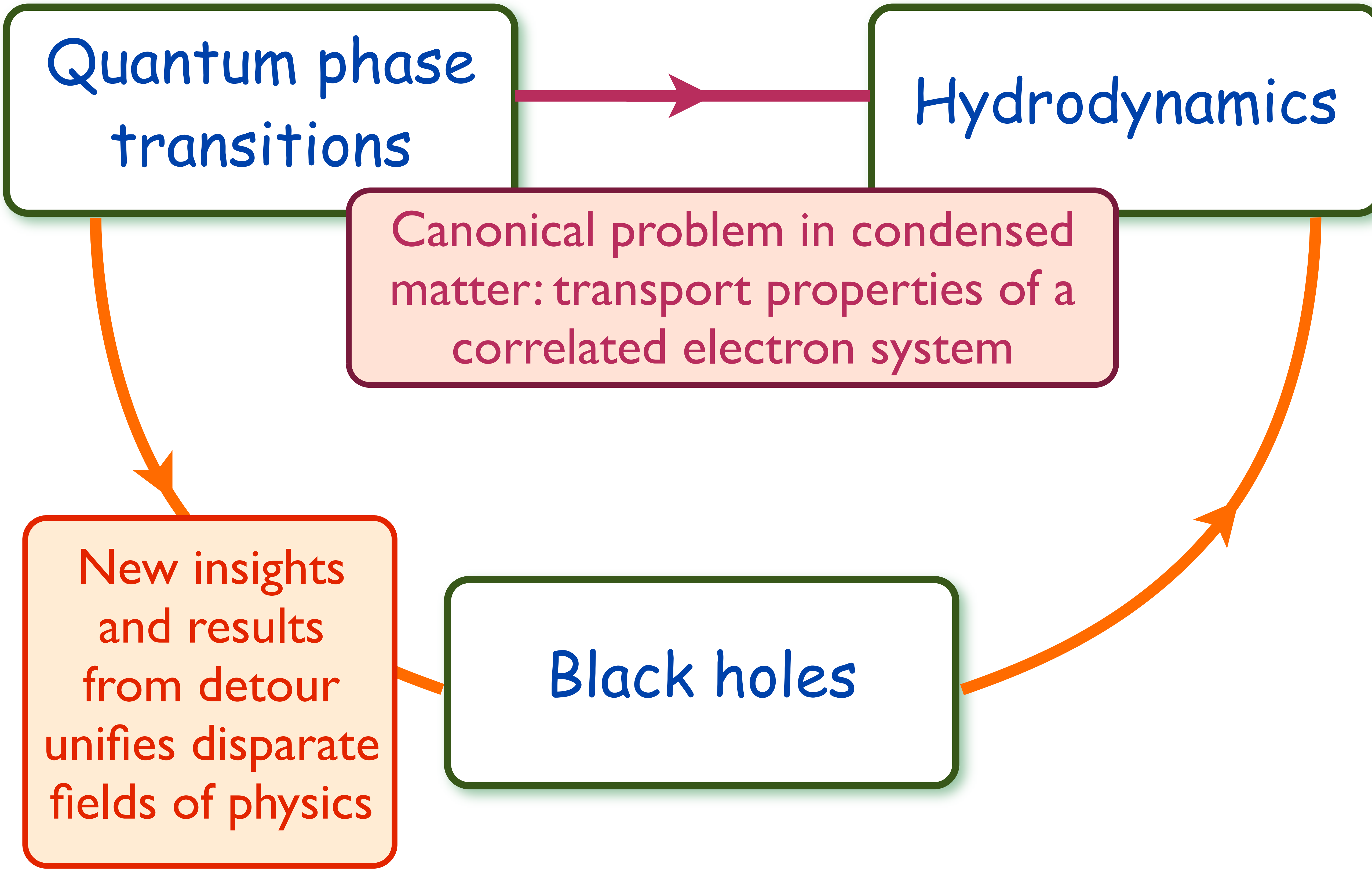
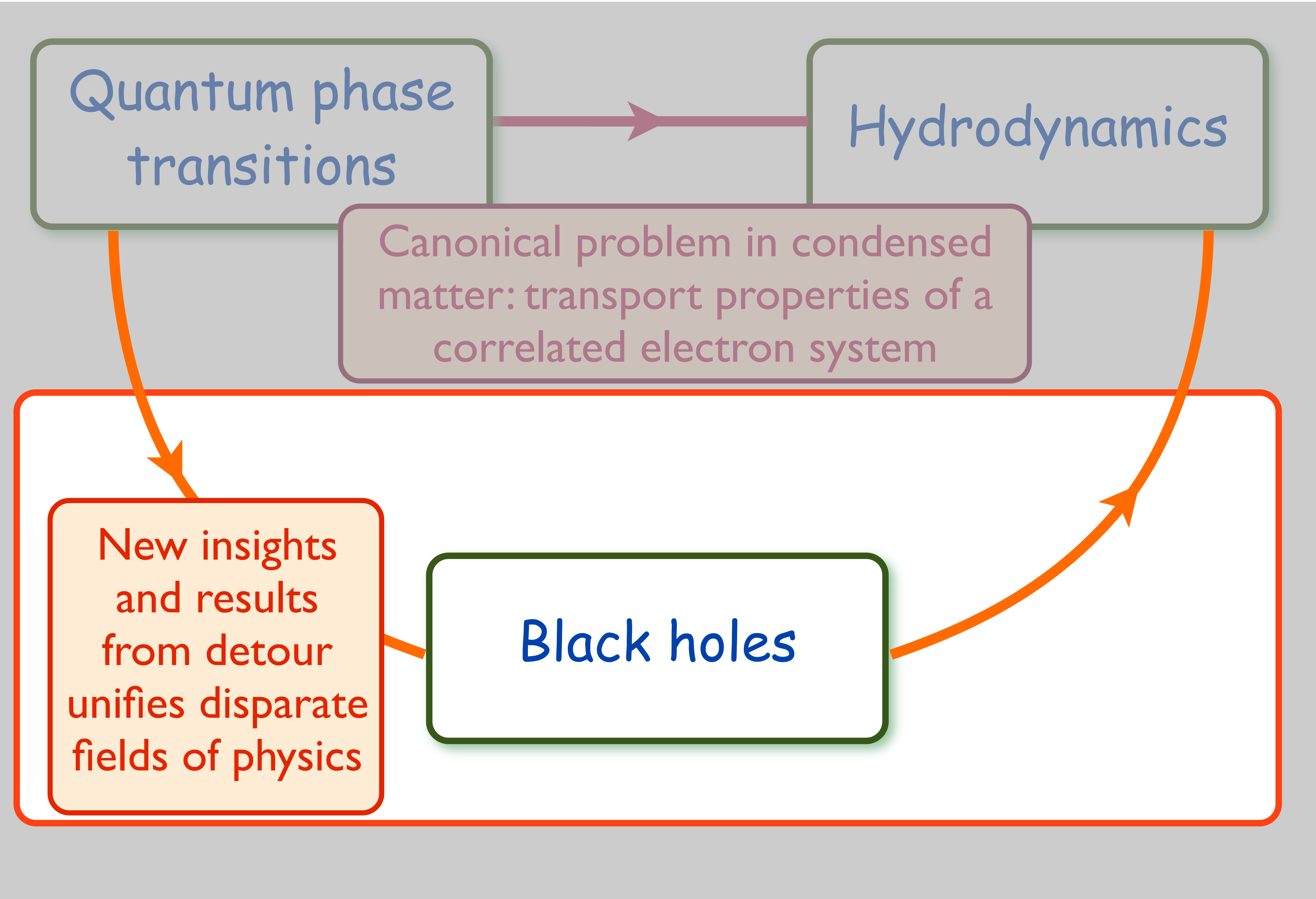
Three foci of modern physics



Three foci of modern physics



Black Holes

Objects so massive that light is gravitationally bound to them.

Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \frac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

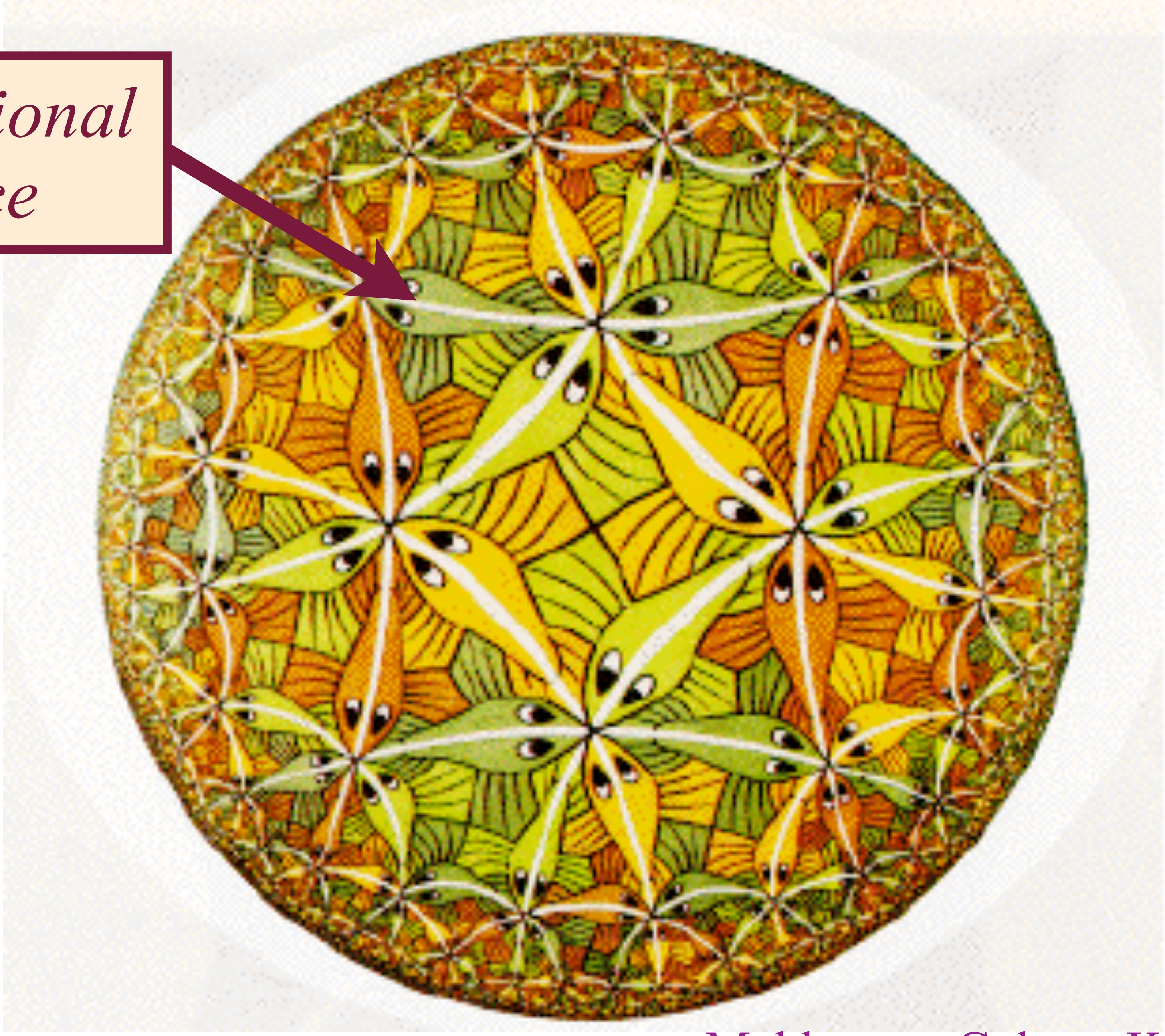
Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space



Maldacena, Gubser, Klebanov, Polyakov, Witten

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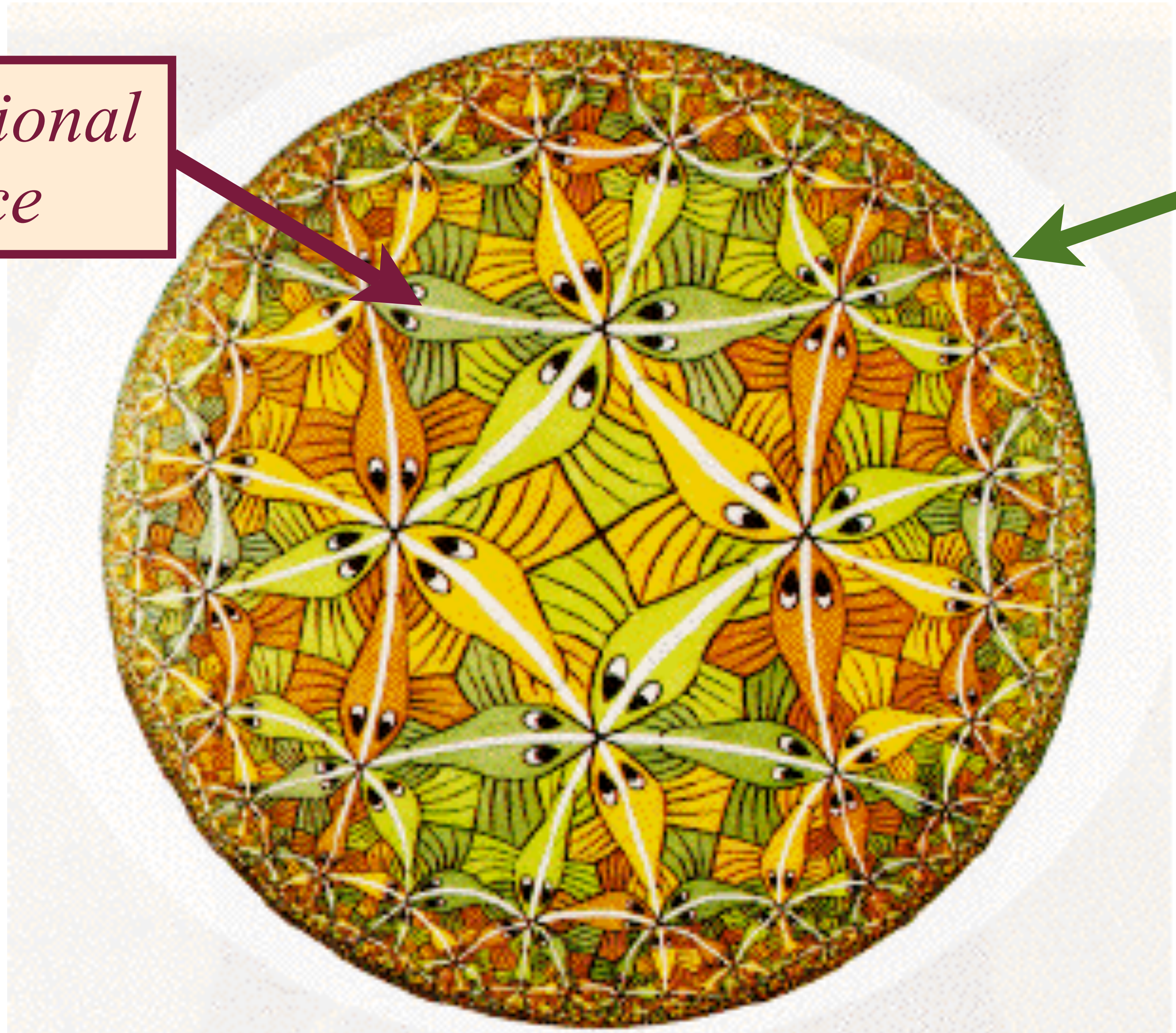
3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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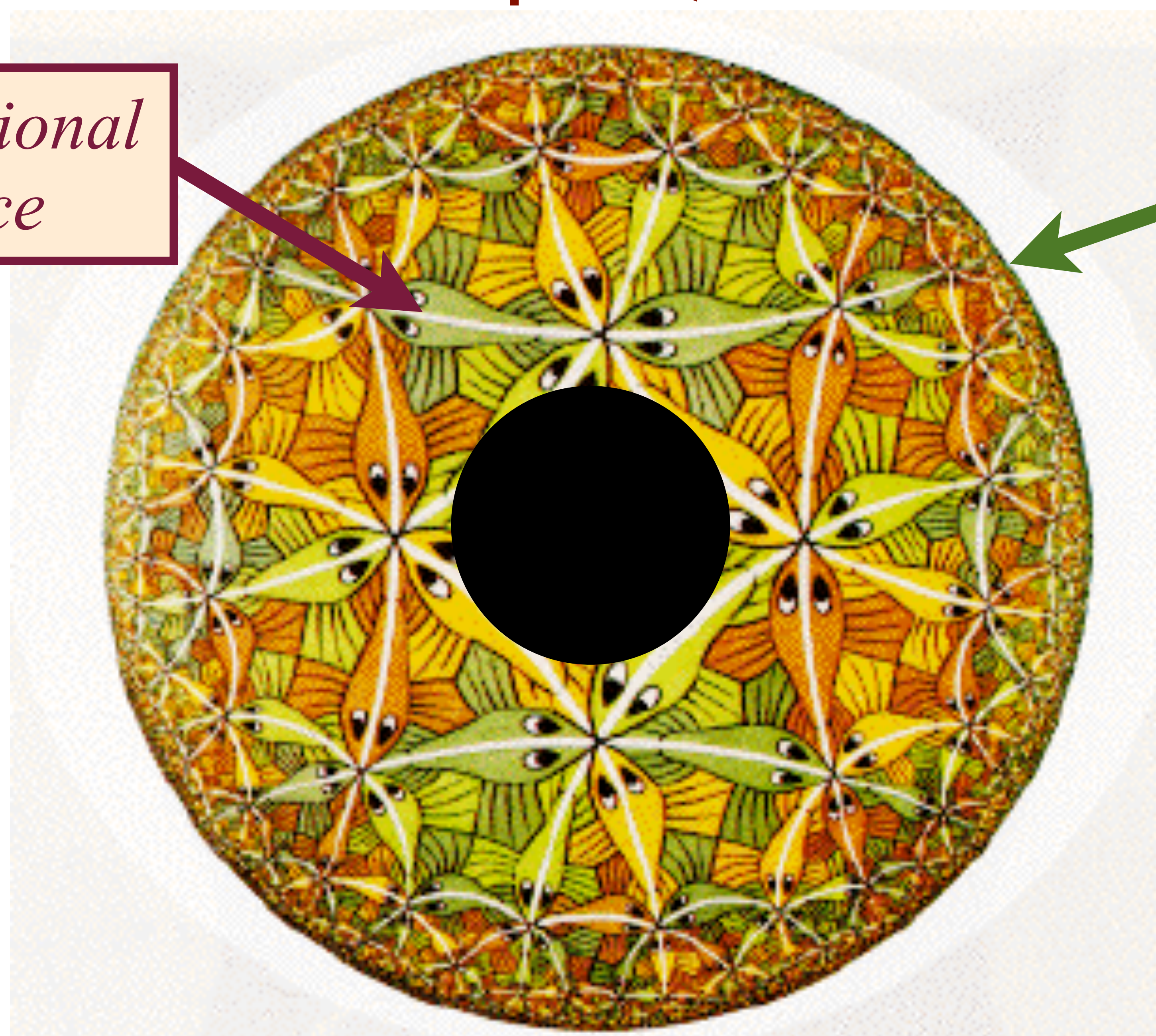

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AdS/CFT correspondence

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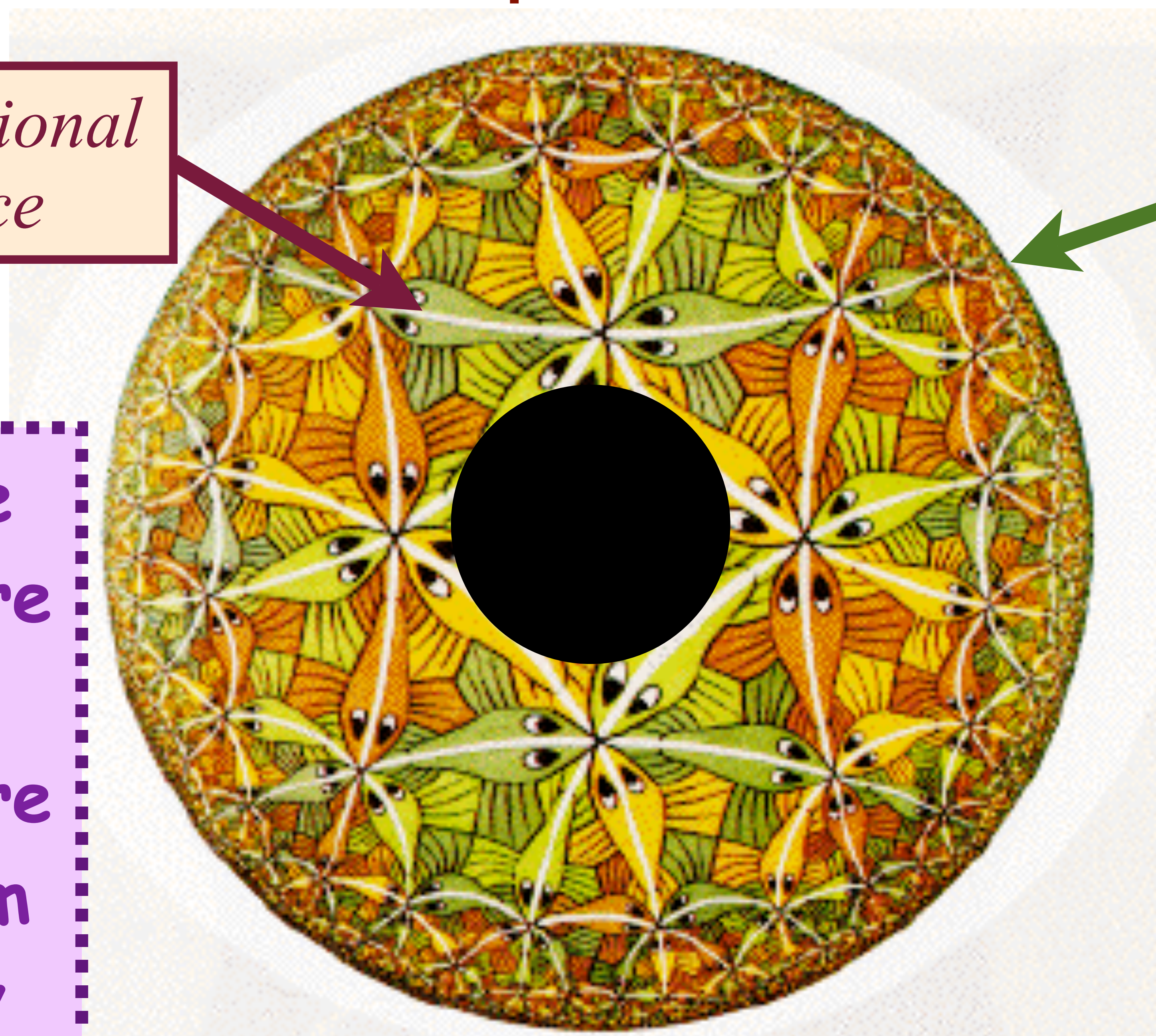

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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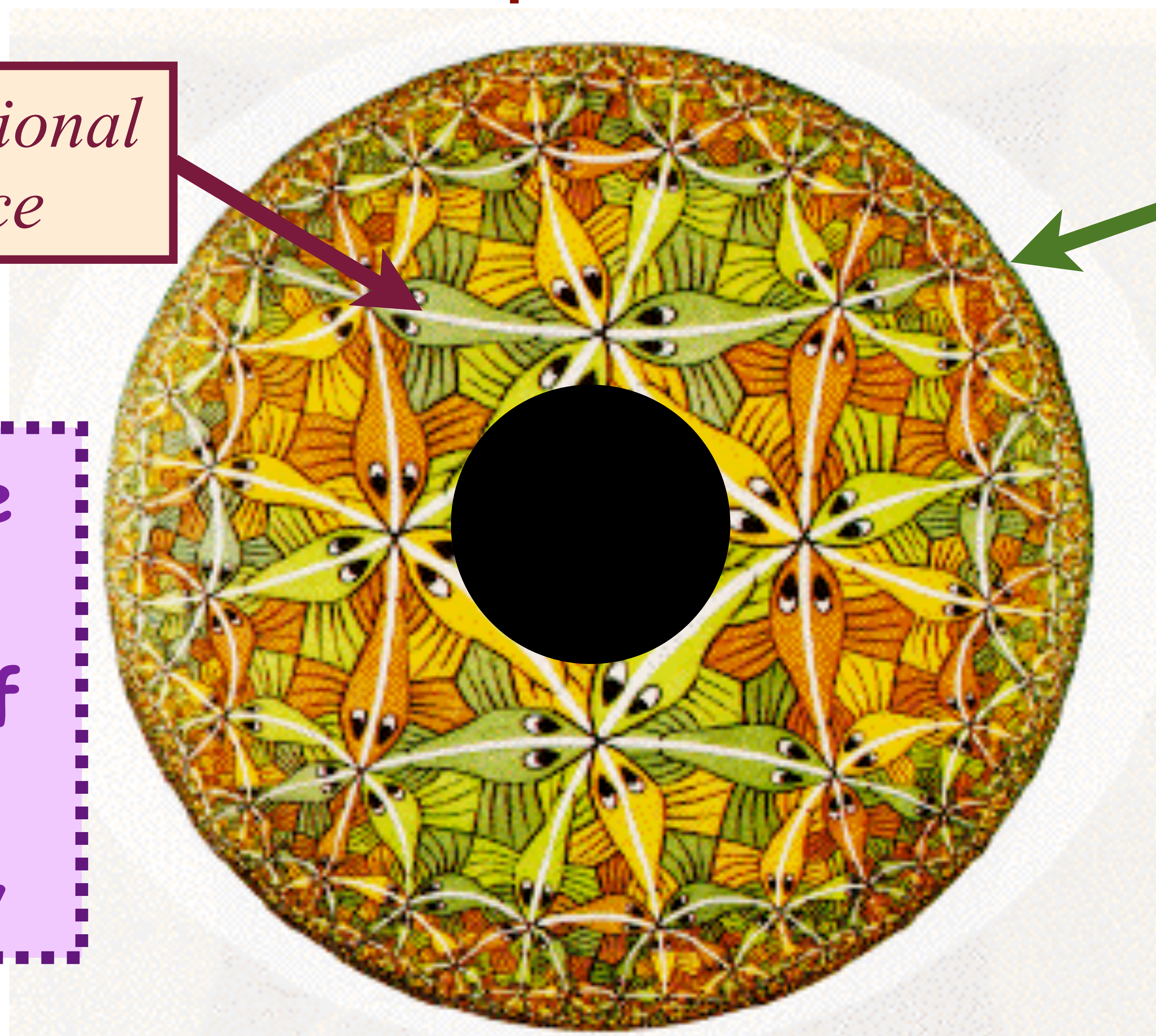

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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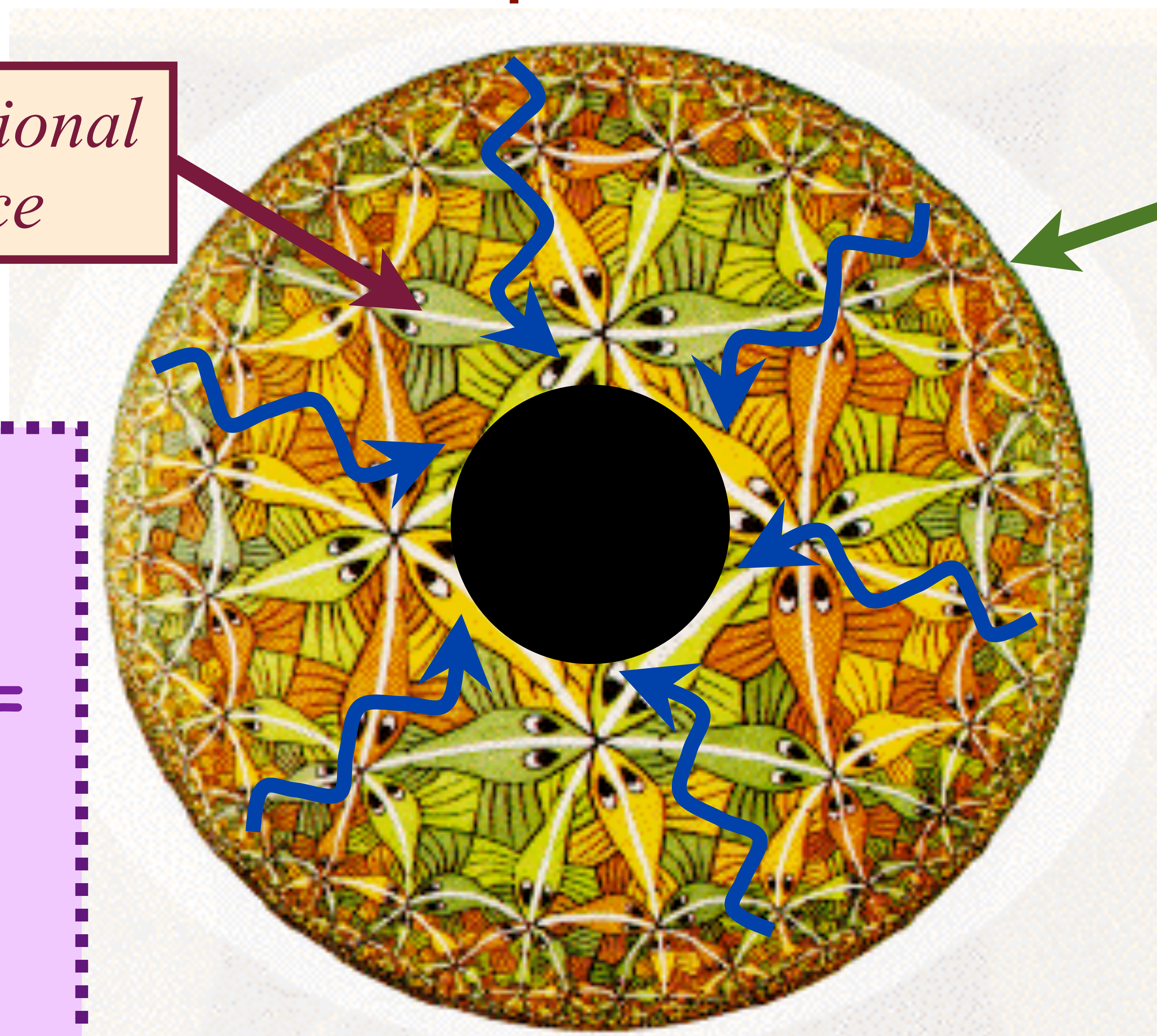

Quantum criticality in 2+1 dimensions

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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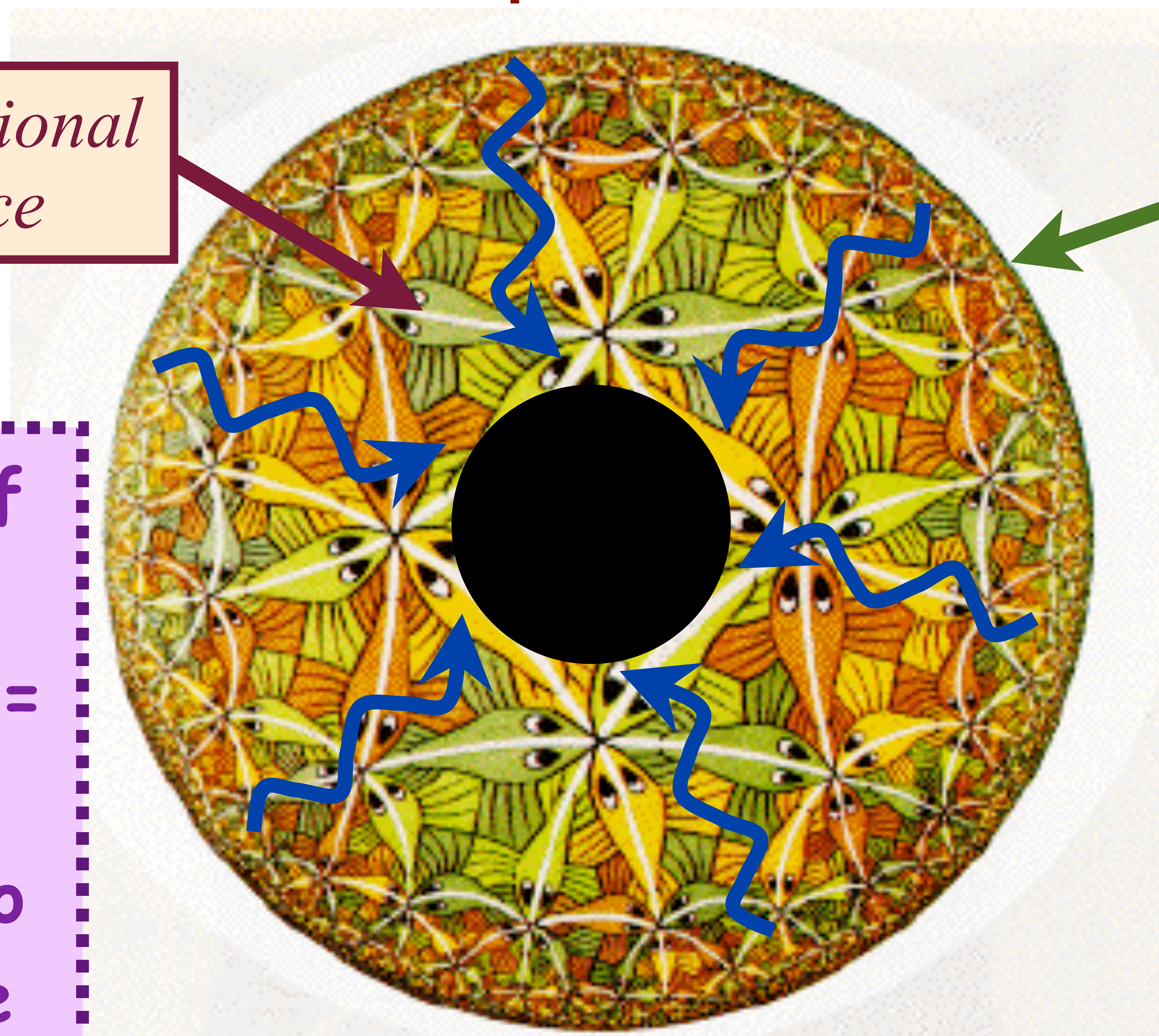
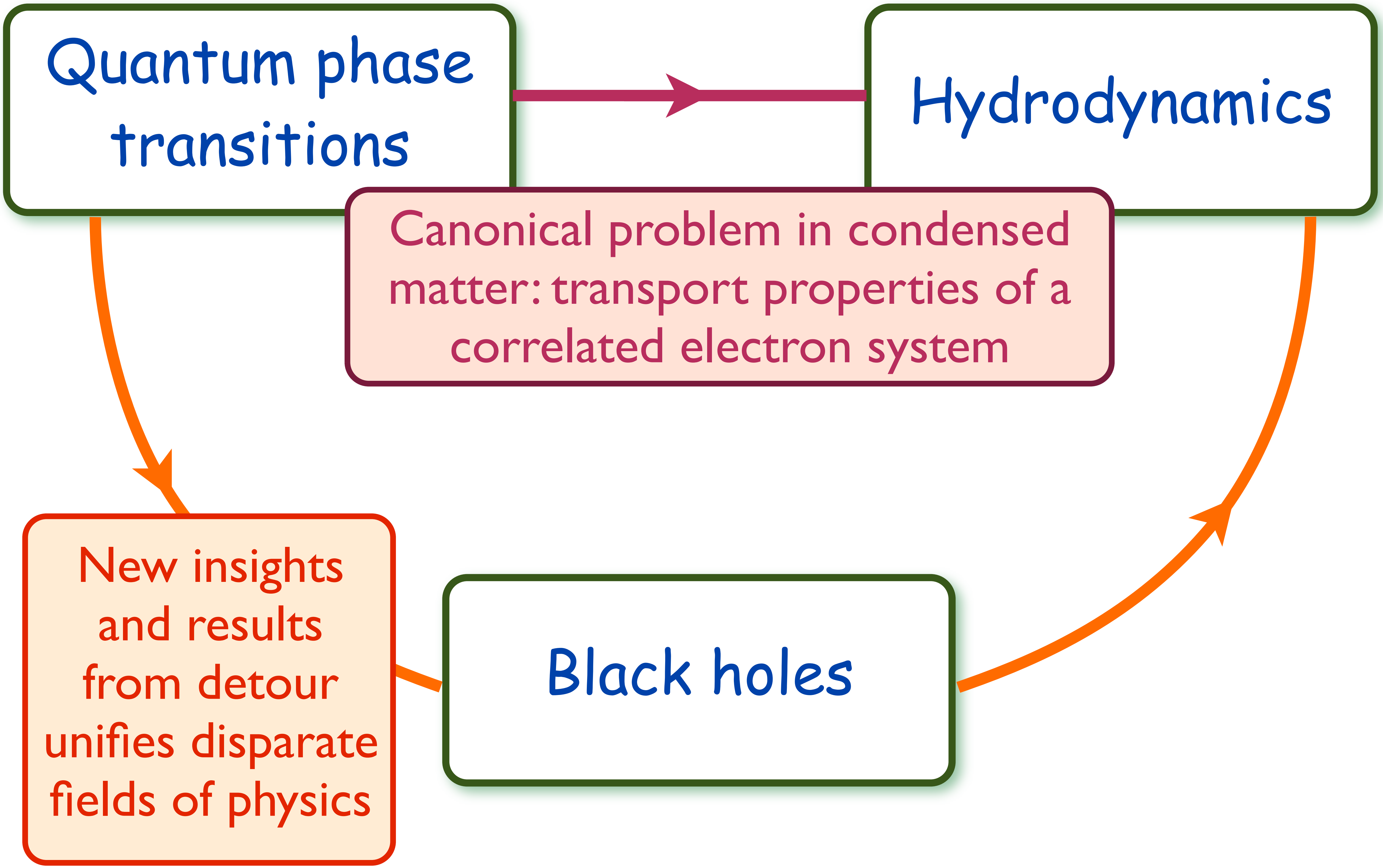
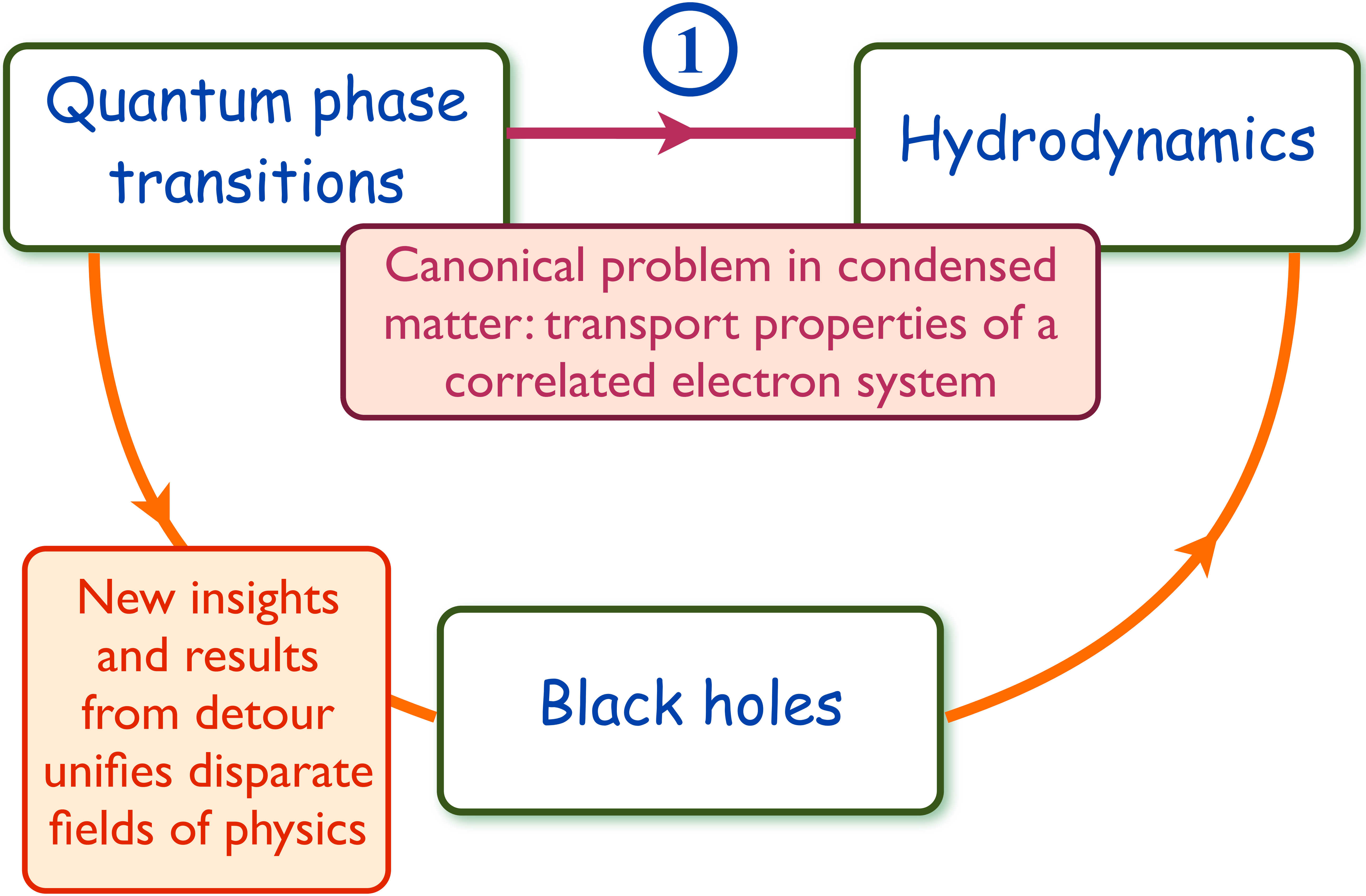


Kovtun, Policastro, Son

Three foci of modern physics



Three foci of modern physics



Hydrodynamics of quantum critical systems

1. Use quantum field theory + quantum transport equations + classical hydrodynamics

   Uses physical model but strong-coupling makes explicit solution difficult

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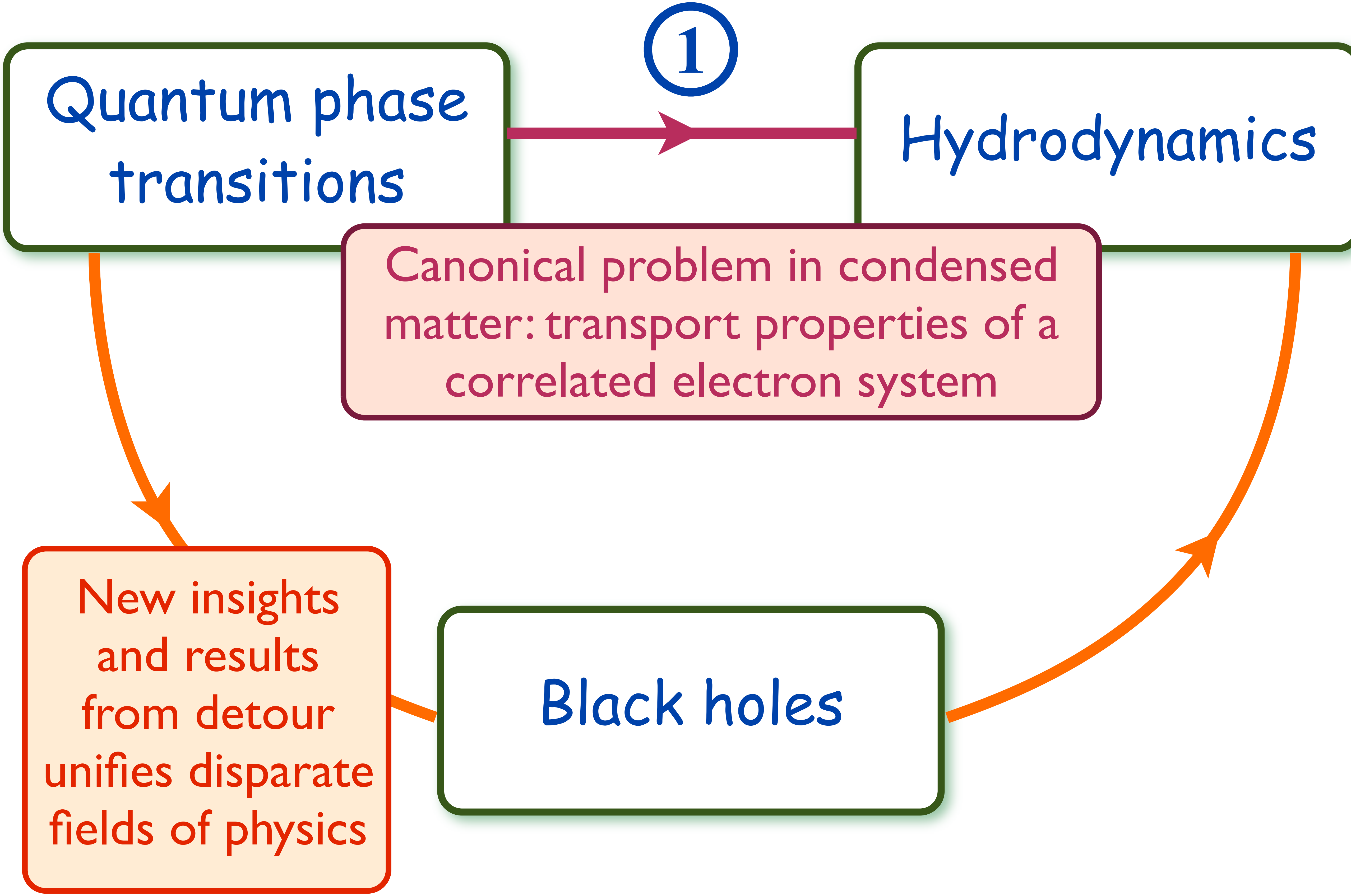

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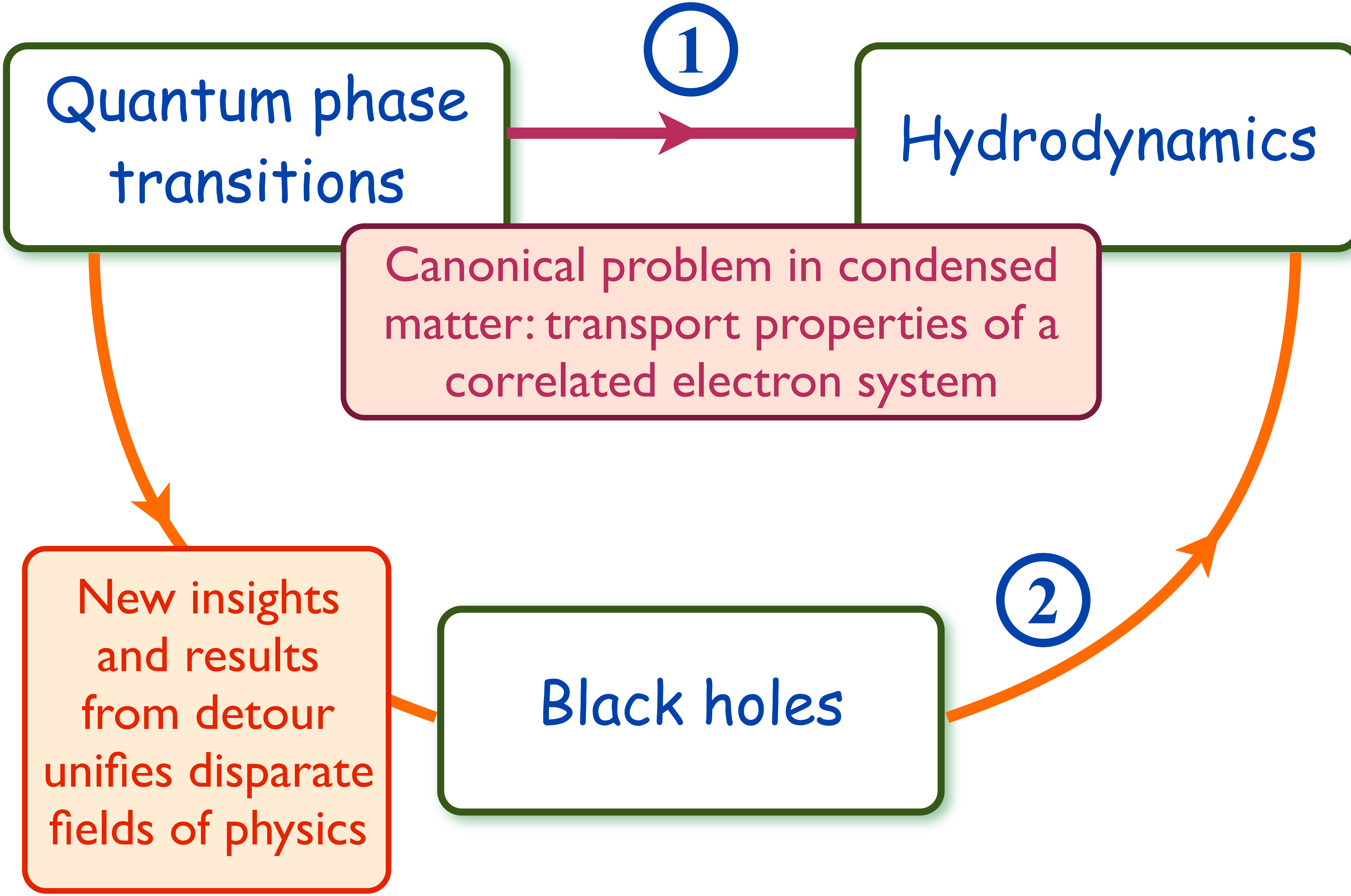

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Find perfect agreement between 1. and 2.
In some cases, results were obtained by 2. earlier !!

Applications:

1. Magneto-thermo-electric transport in graphene and near the superconductor-insulator transition

   Hydrodynamic cyclotron resonance

   Nernst effect

2. Quark-gluon plasma

   Low viscosity fluid

3. Fermi gas at unitarity

   Non-relativistic AdS/CFT

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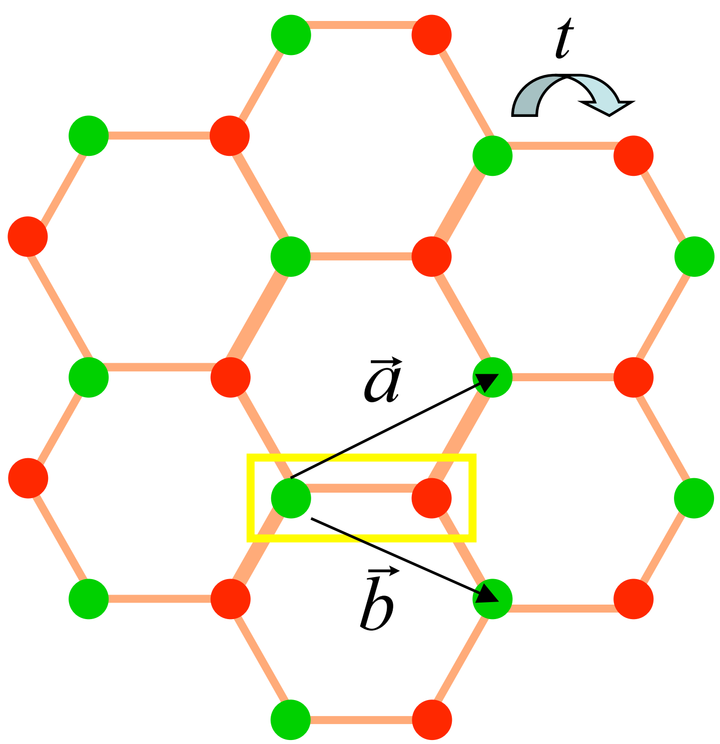
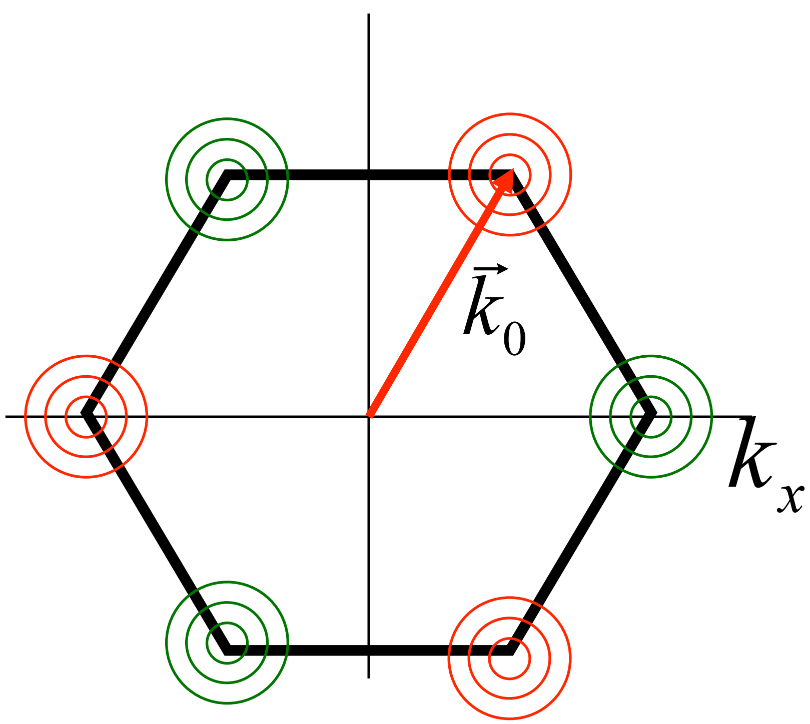
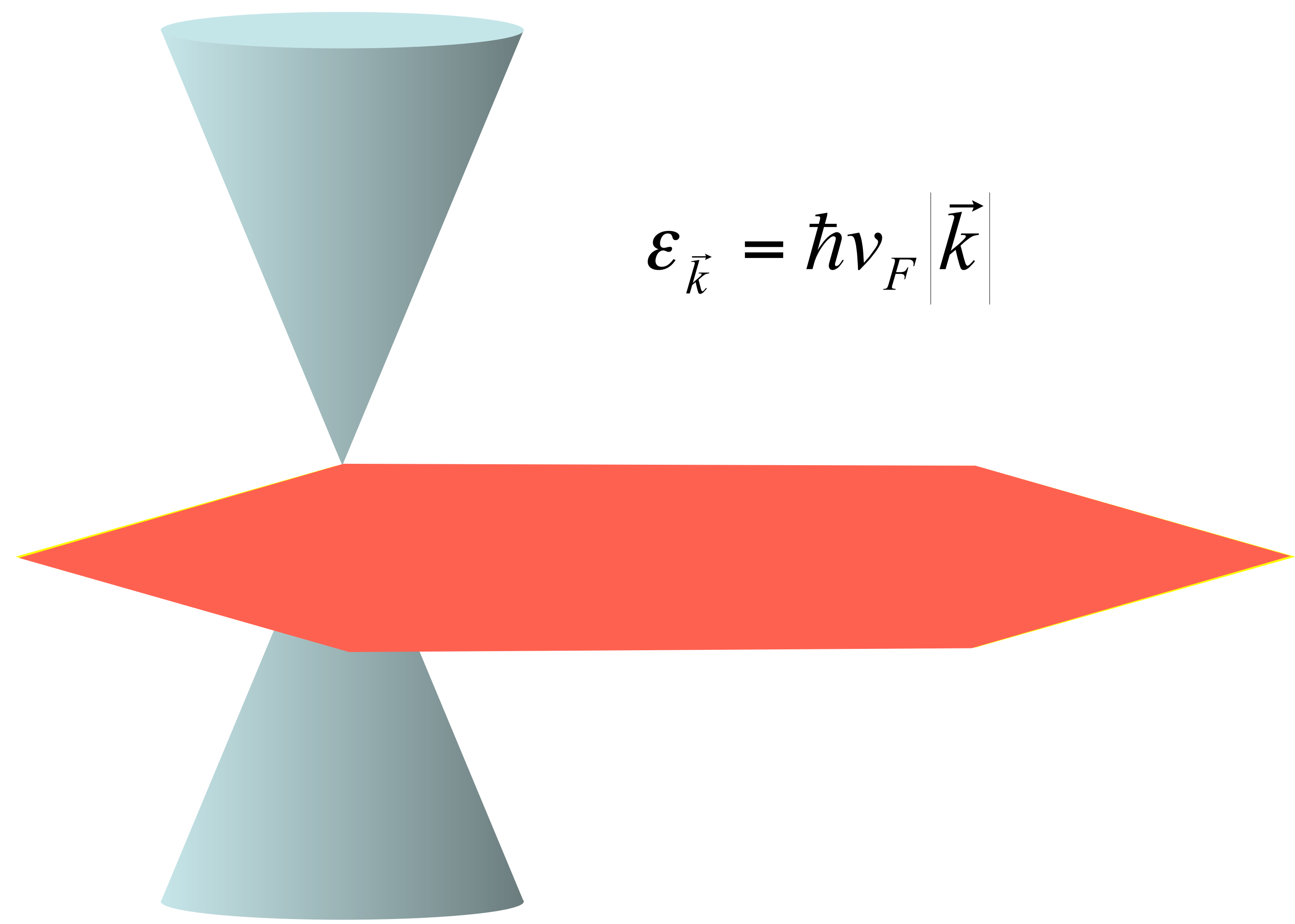
Graphene

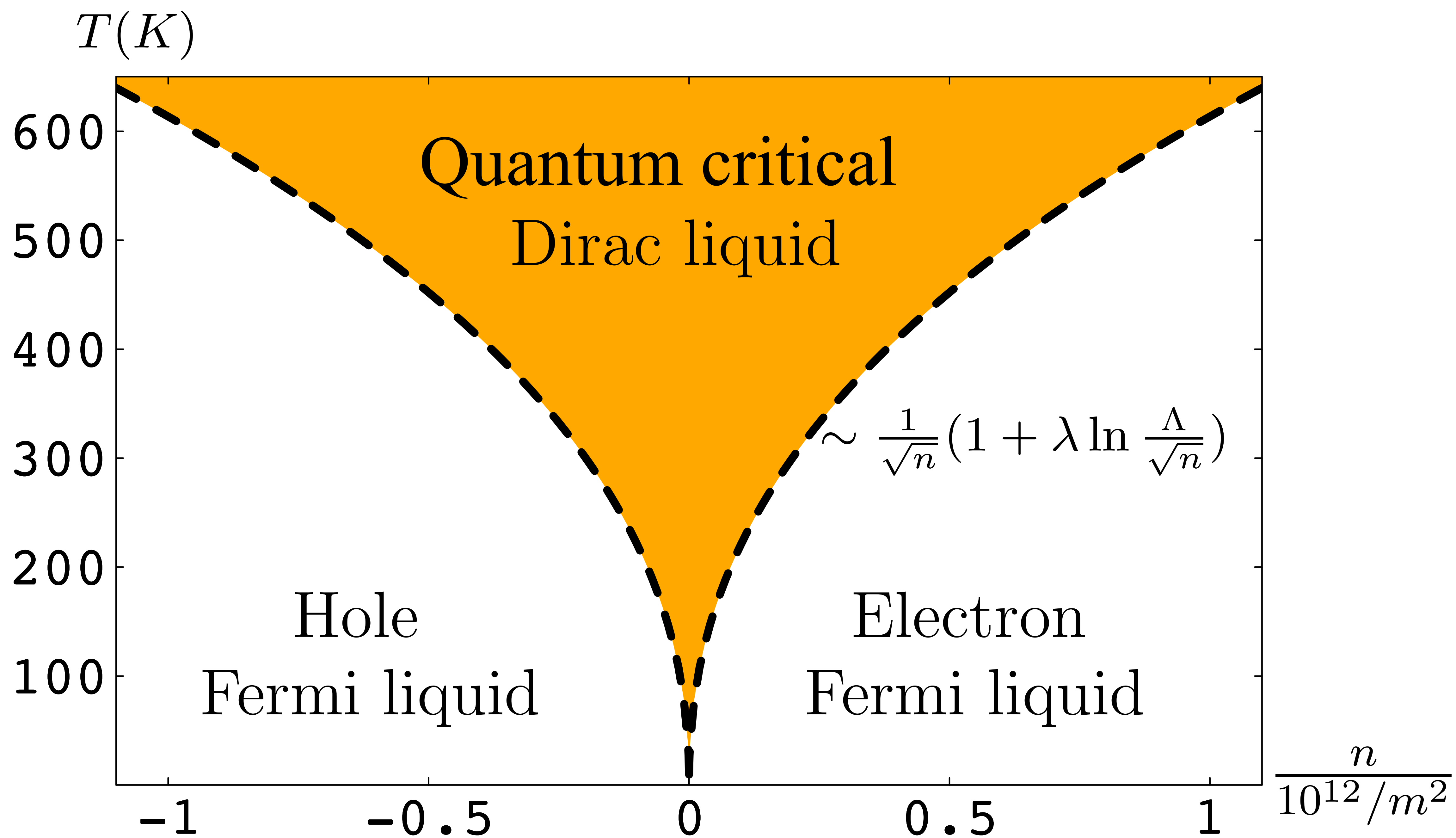






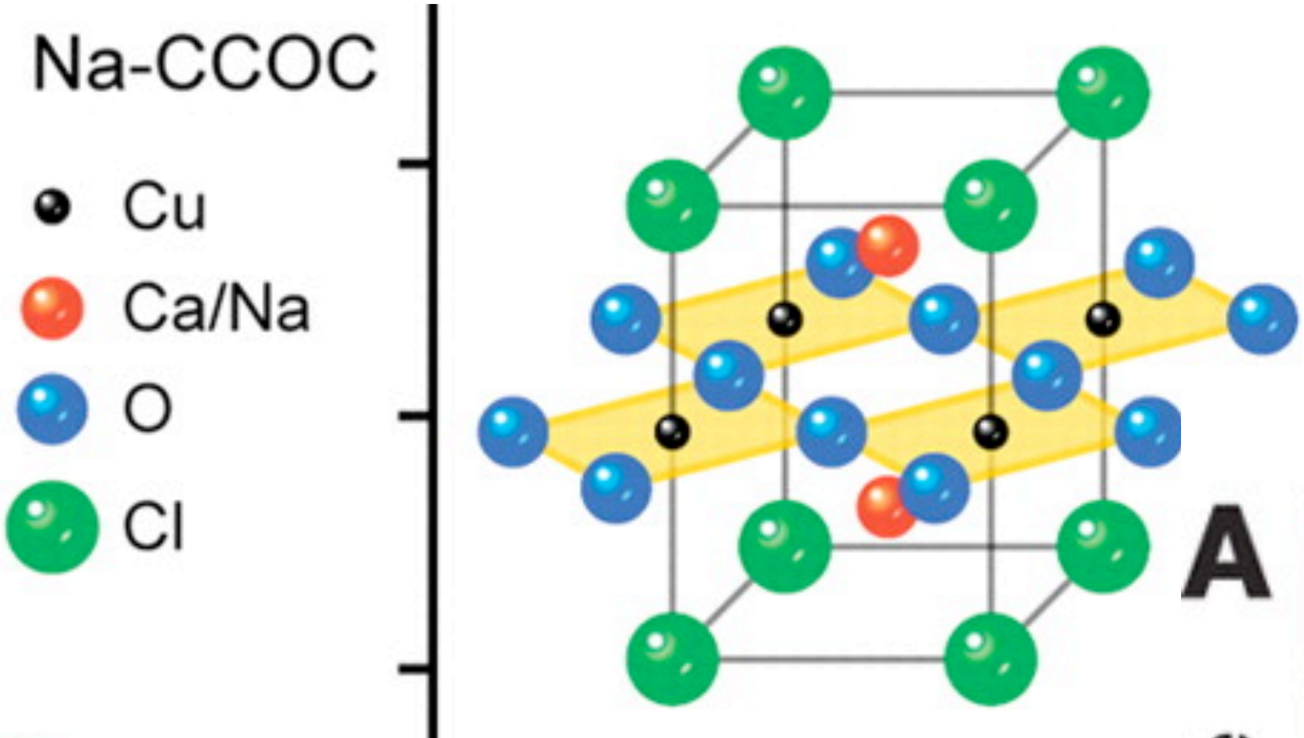
$$\varepsilon_{\vec{k}} = \hbar v_F \left| \vec{k} \right|$$

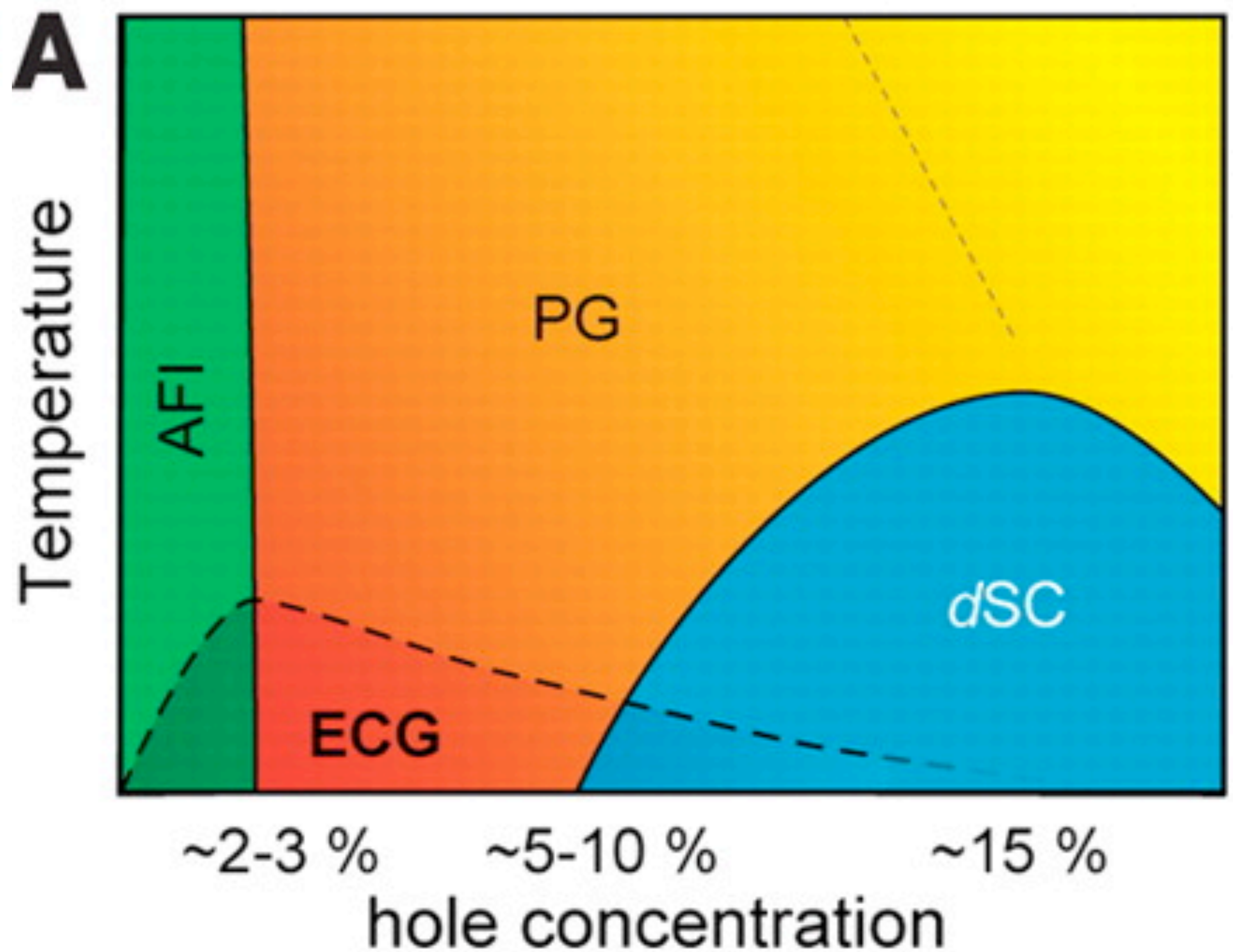
Graphene



$T(K)$
600
500
400
300
200
100
Quantum critical
Dirac liquid
$\sim \frac{1}{\sqrt{n}}(1 + \lambda \ln \frac{\Lambda}{\sqrt{n}})$
Hole
Fermi liquid
Electron
Fermi liquid
-1
-0.5
0
0.5
1
$\frac{n}{10^{12}/m^2}$

The cuprate superconductors





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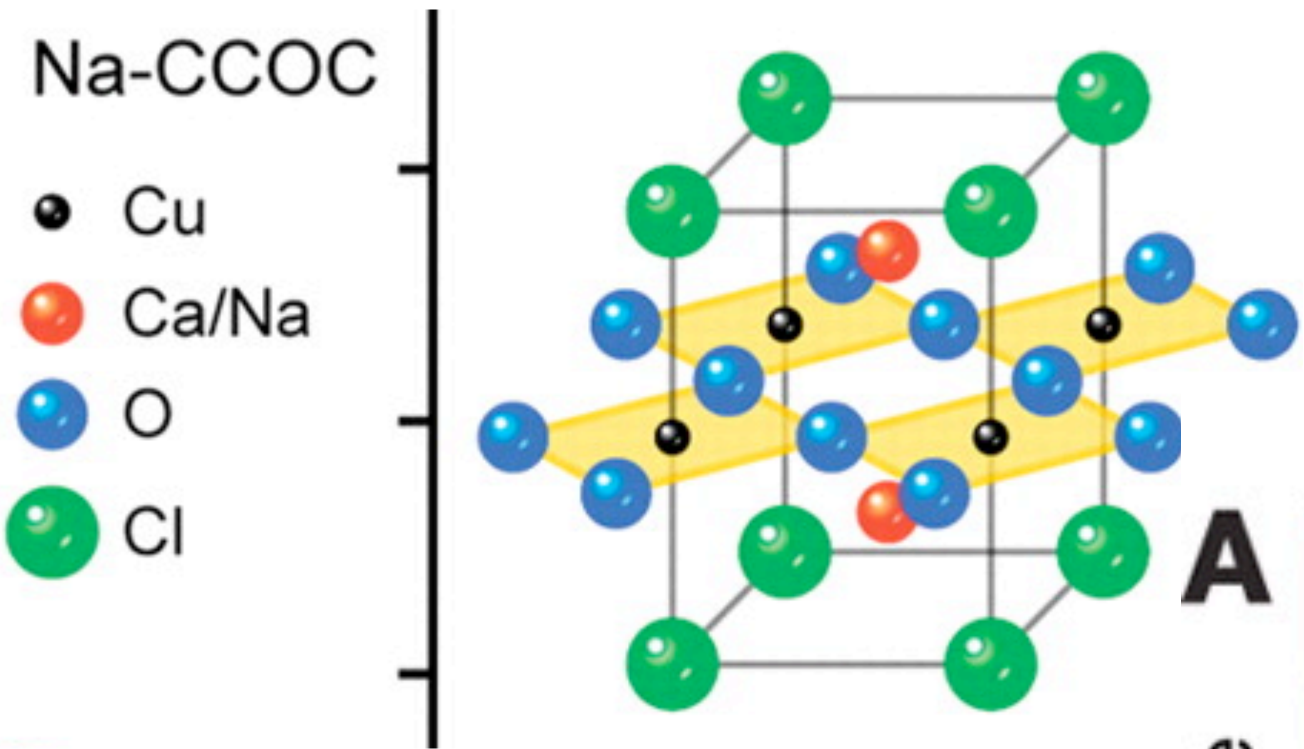
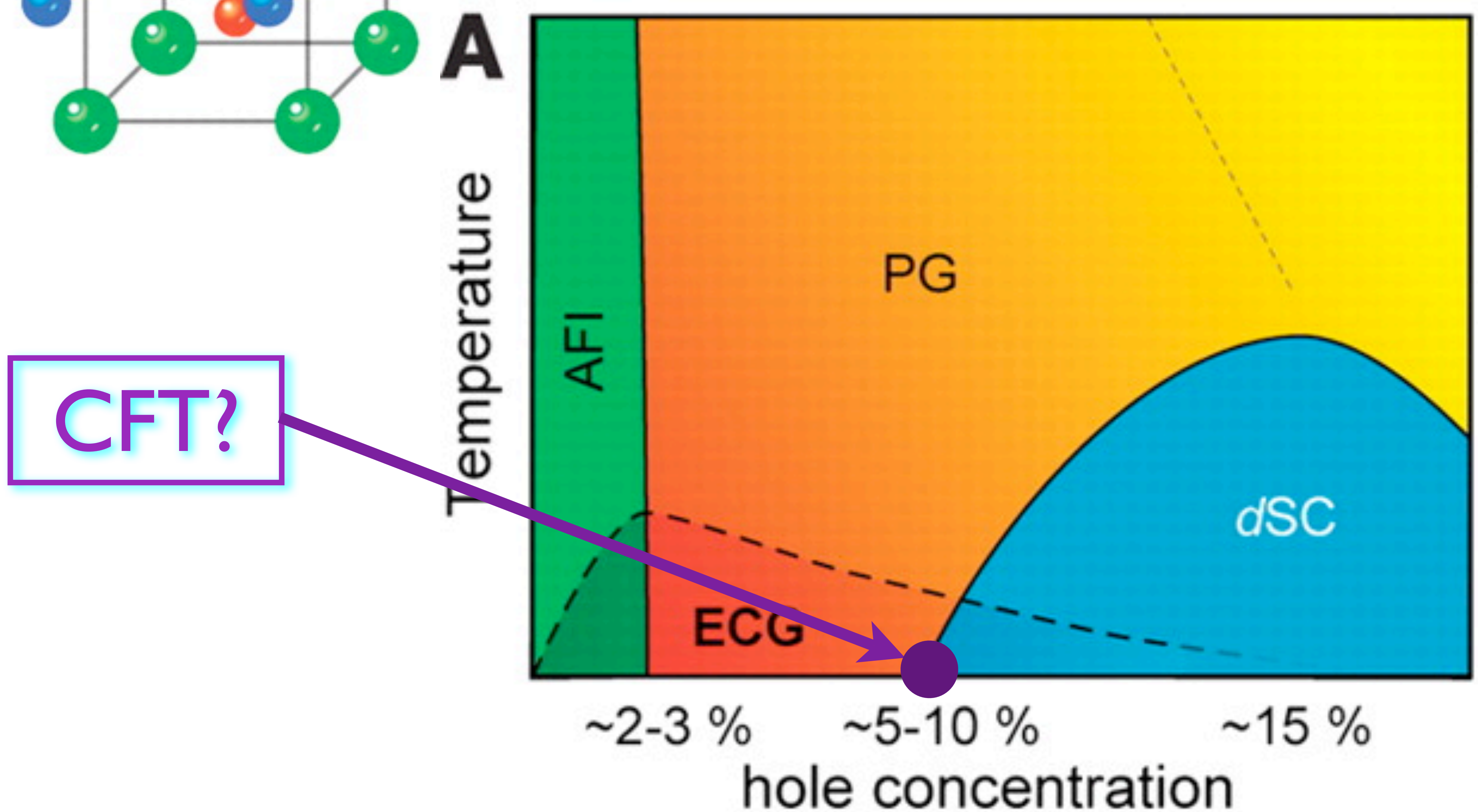
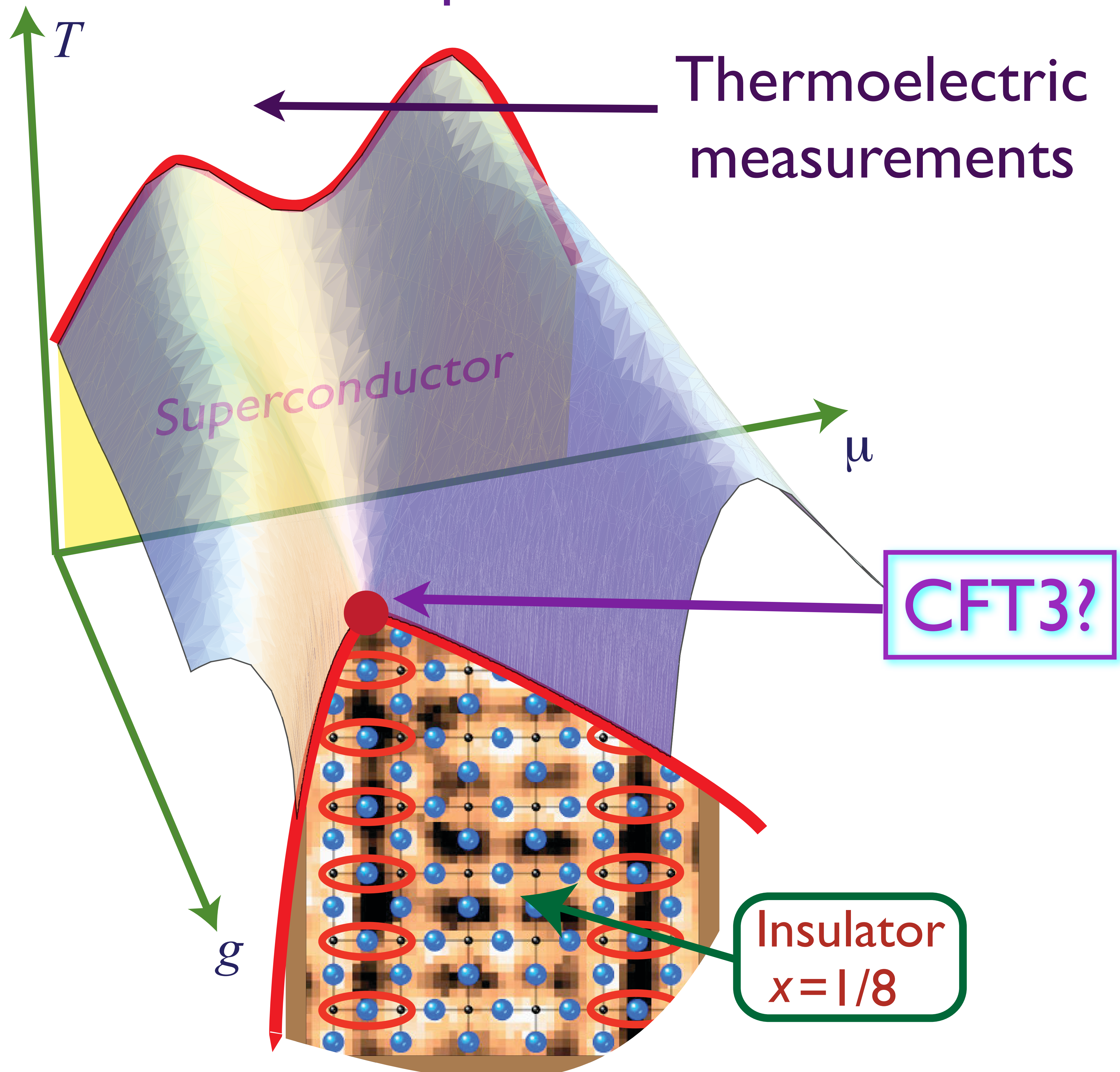




Cuprates


T
Thermoelectric measurements
Superconductor
μ
CFT3?
g
Insulator
x=1/8

Hydrodynamic cyclotron resonance at a frequency

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and with width

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

where B = magnetic field, ρ = charge density away from density of CFT, ε = energy density, P = pressure, v = velocity of "light" in CFT, and $\sigma_Q e^2/h$ is the universal conductivity of the CFT.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

“Wiedemann-Franz”-like relation for thermal conductivity, κ at $B = 0$

$$\kappa = \sigma_Q \left(\frac{k_B^2 T}{e^{*2}} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 .$$

At $B \neq 0$ and $\rho = 0$ we have a “Wiedemann-Franz” relation for “vortices”

$$\kappa = \frac{1}{\sigma_Q} k_B^2 T \left(\frac{v(\varepsilon + P)}{k_B T B} \right)^2 .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

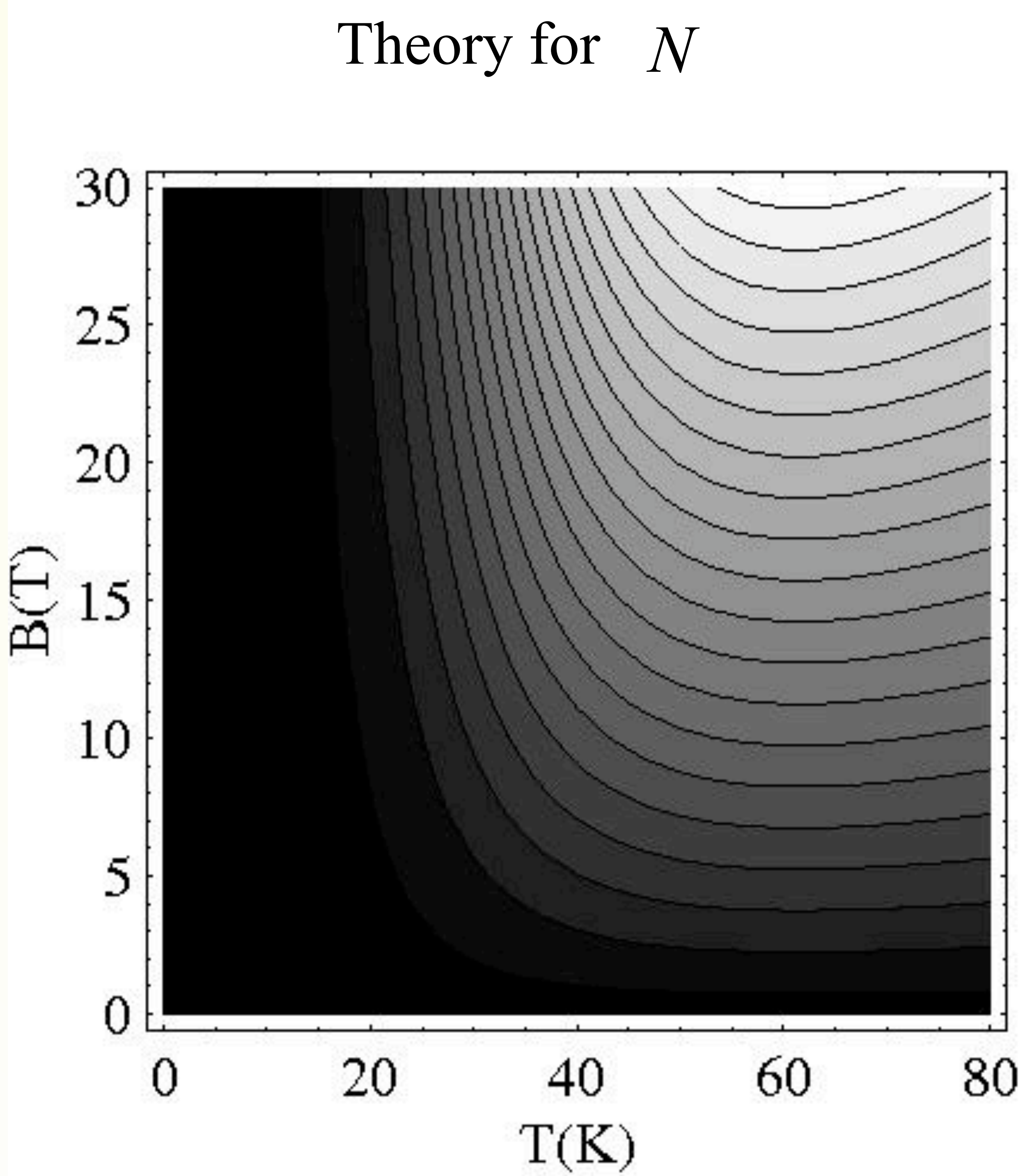
Nernst signal (transverse thermoelectric response)

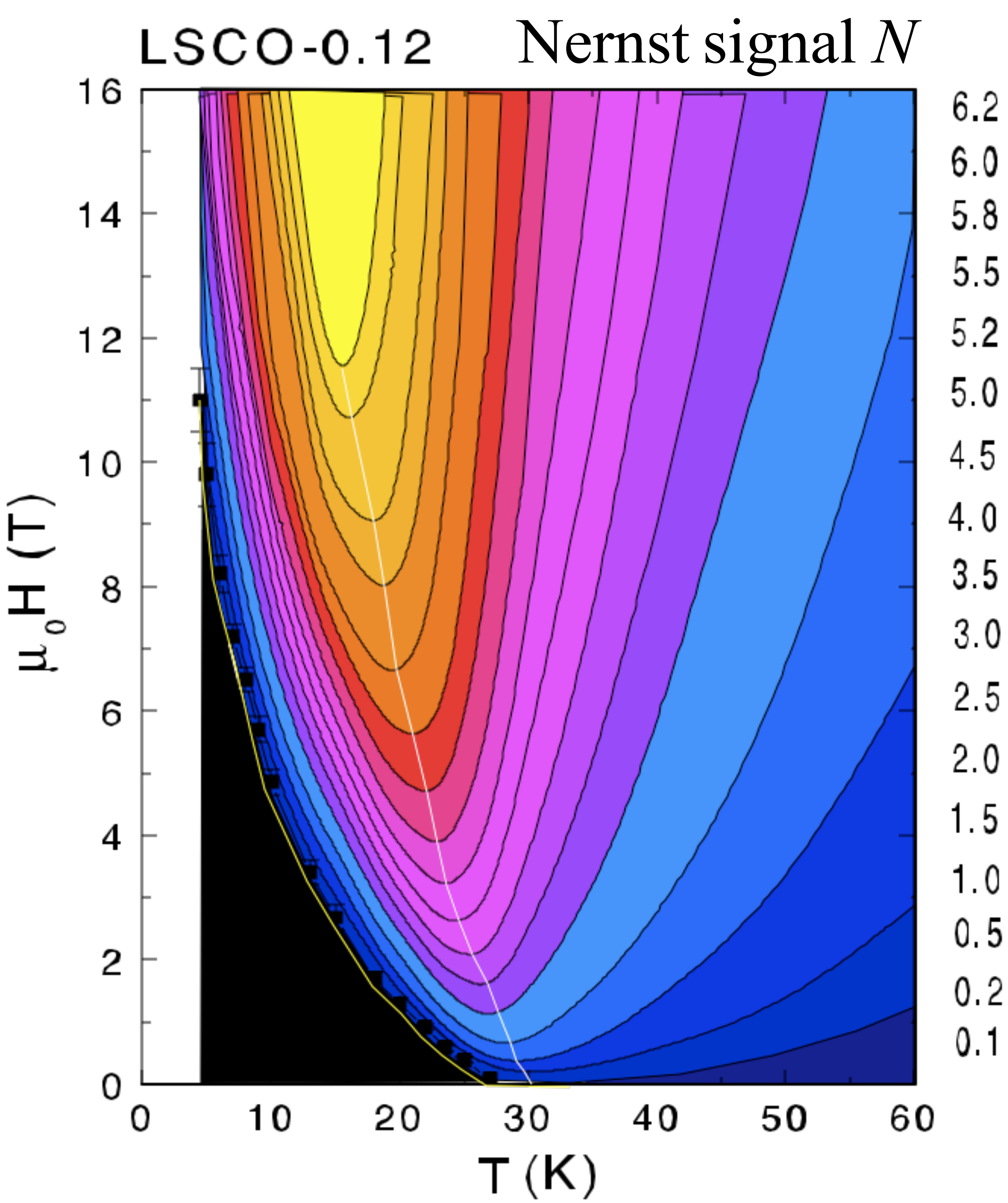
$$e_N = \left(\frac{k_B}{e^*}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

where $\tau_{\rm imp}$ is the momentum relaxation time due to impurities or umklapp scattering.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

LSCO Experiments





Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B 73, 024510 (2006).

B and T dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, τ_{imp} and v.

Specific quantitative predictions for THz experiments on graphene at room temperature.

Applications:

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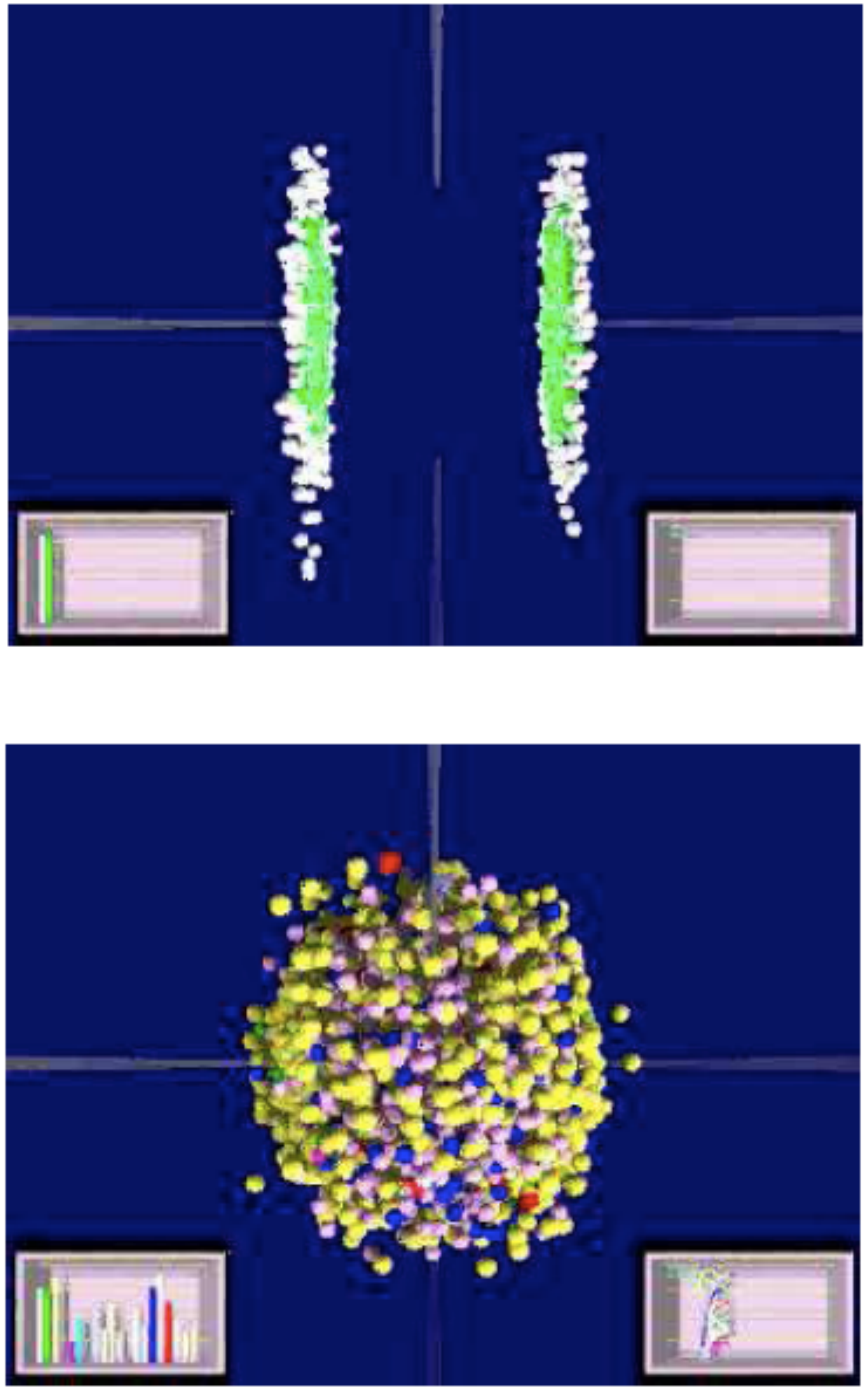
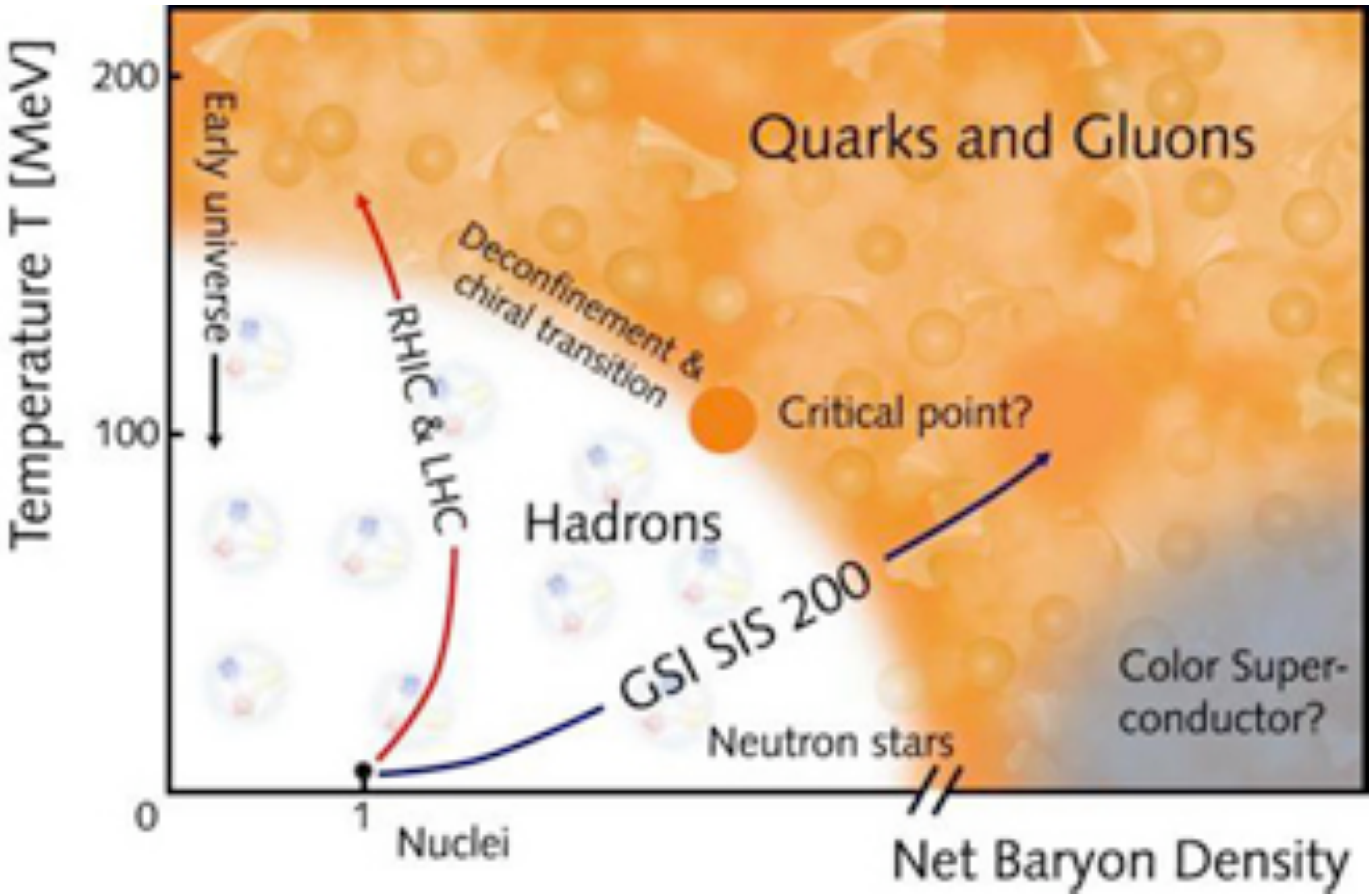
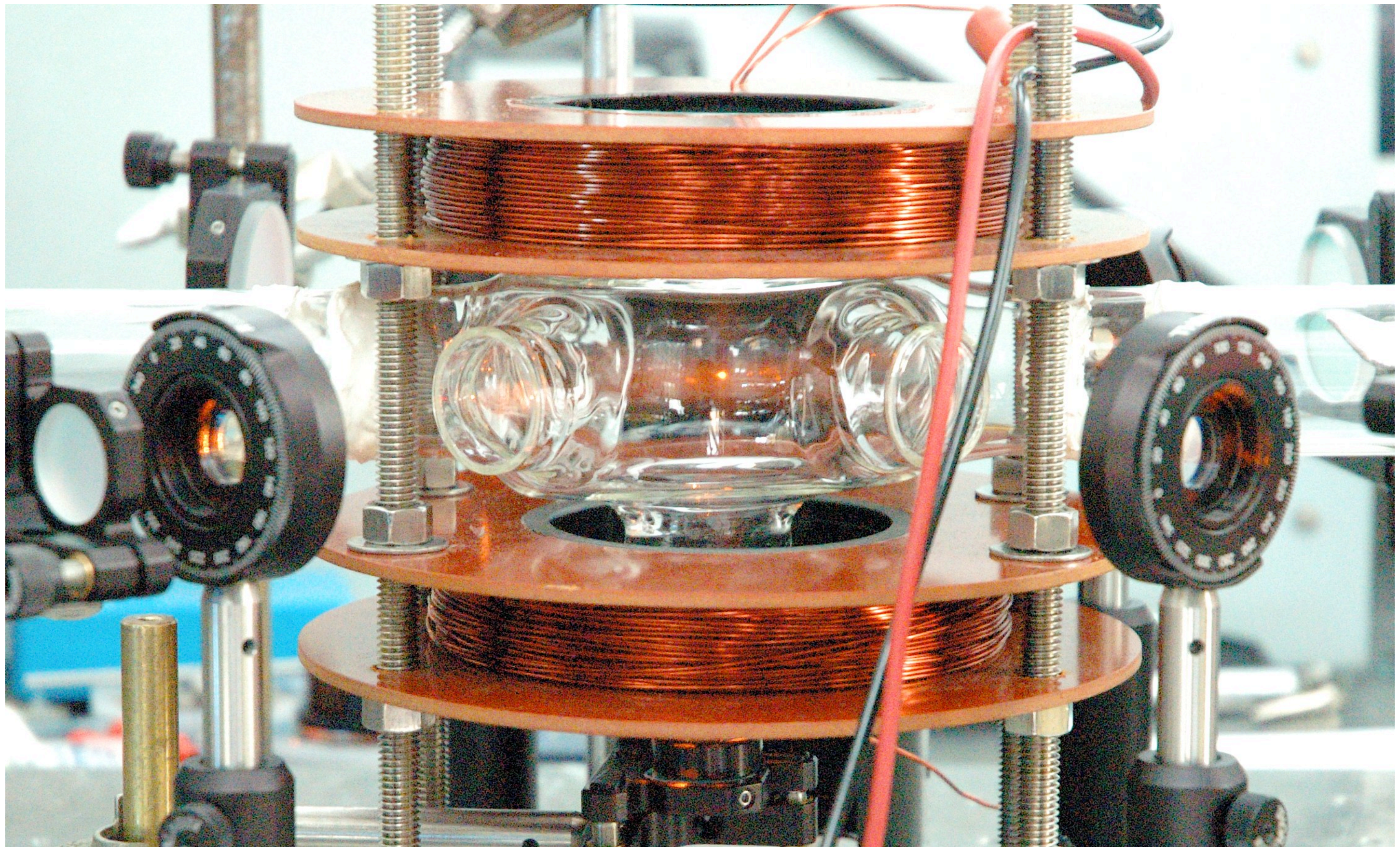
Au+Au collisions at RHIC



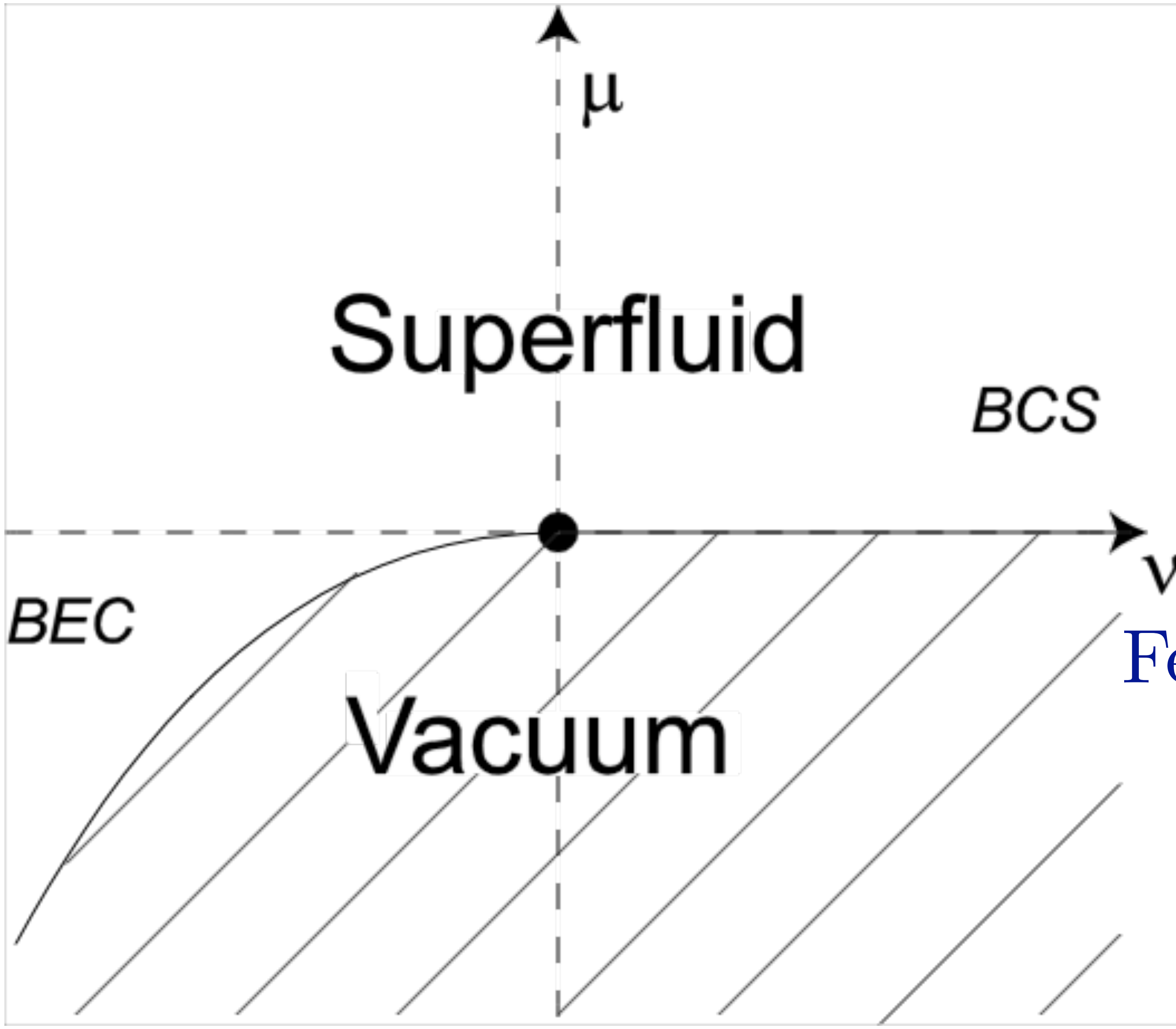
Quark-gluon plasma can be described as "quantum critical QCD"

Phases of nuclear matter

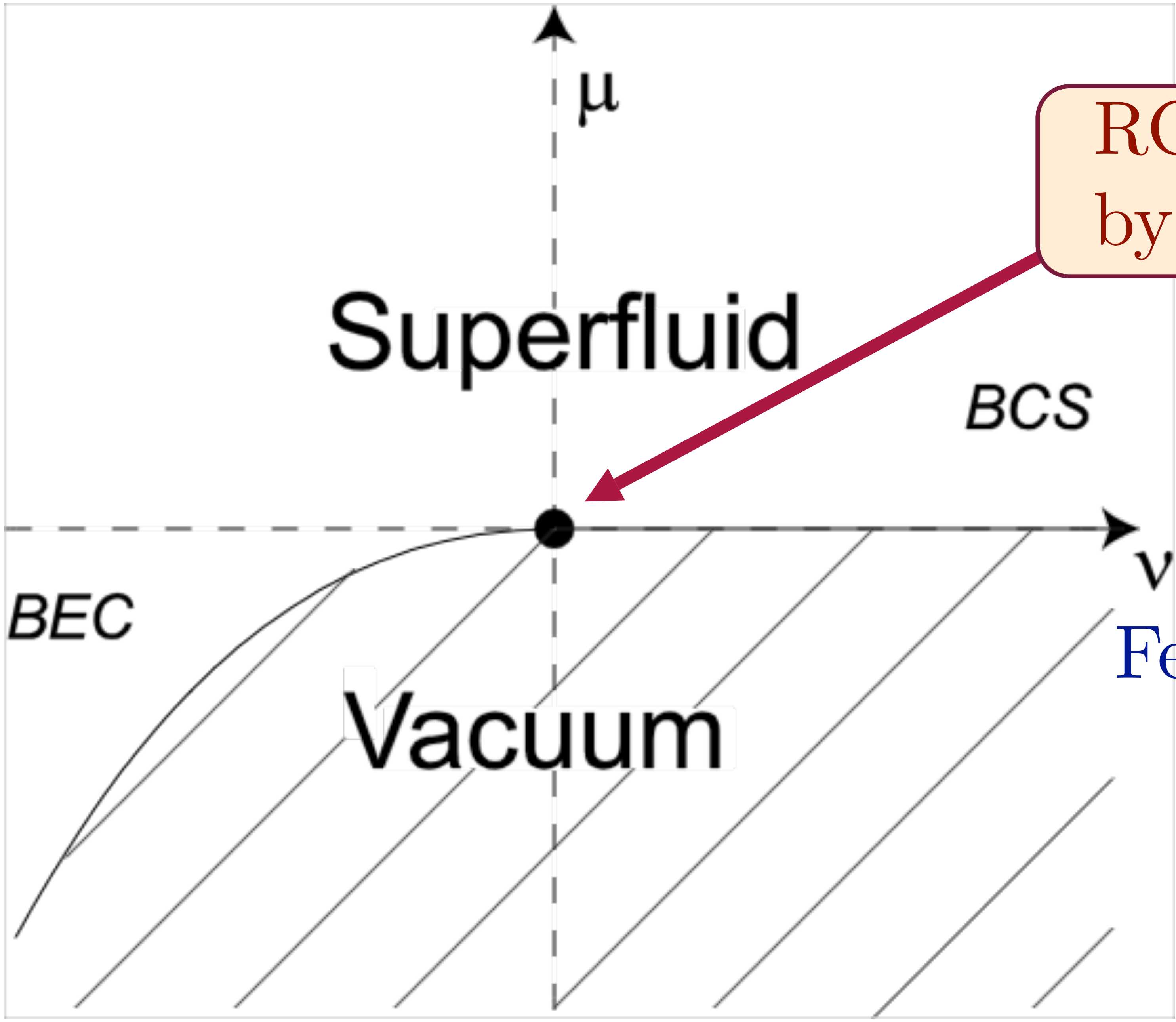




S=1/2 Fermi gas
at a Feshbach
resonance

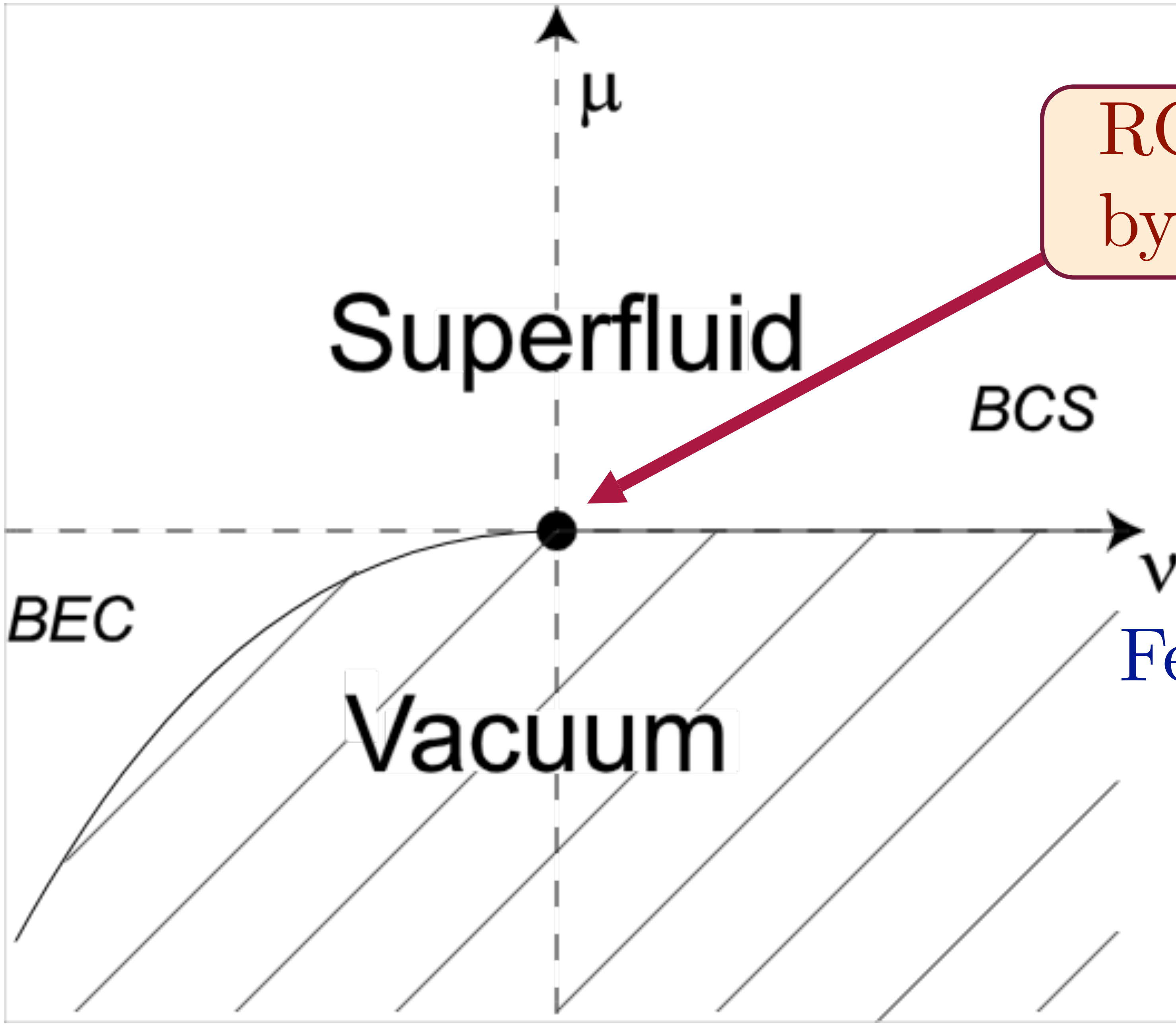


= detuning from Feshbach resonance



RG fixed point described by a "non-relativistic" CFT

= detuning from Feshbach resonance



RG fixed point described by a "non-relativistic" CFT

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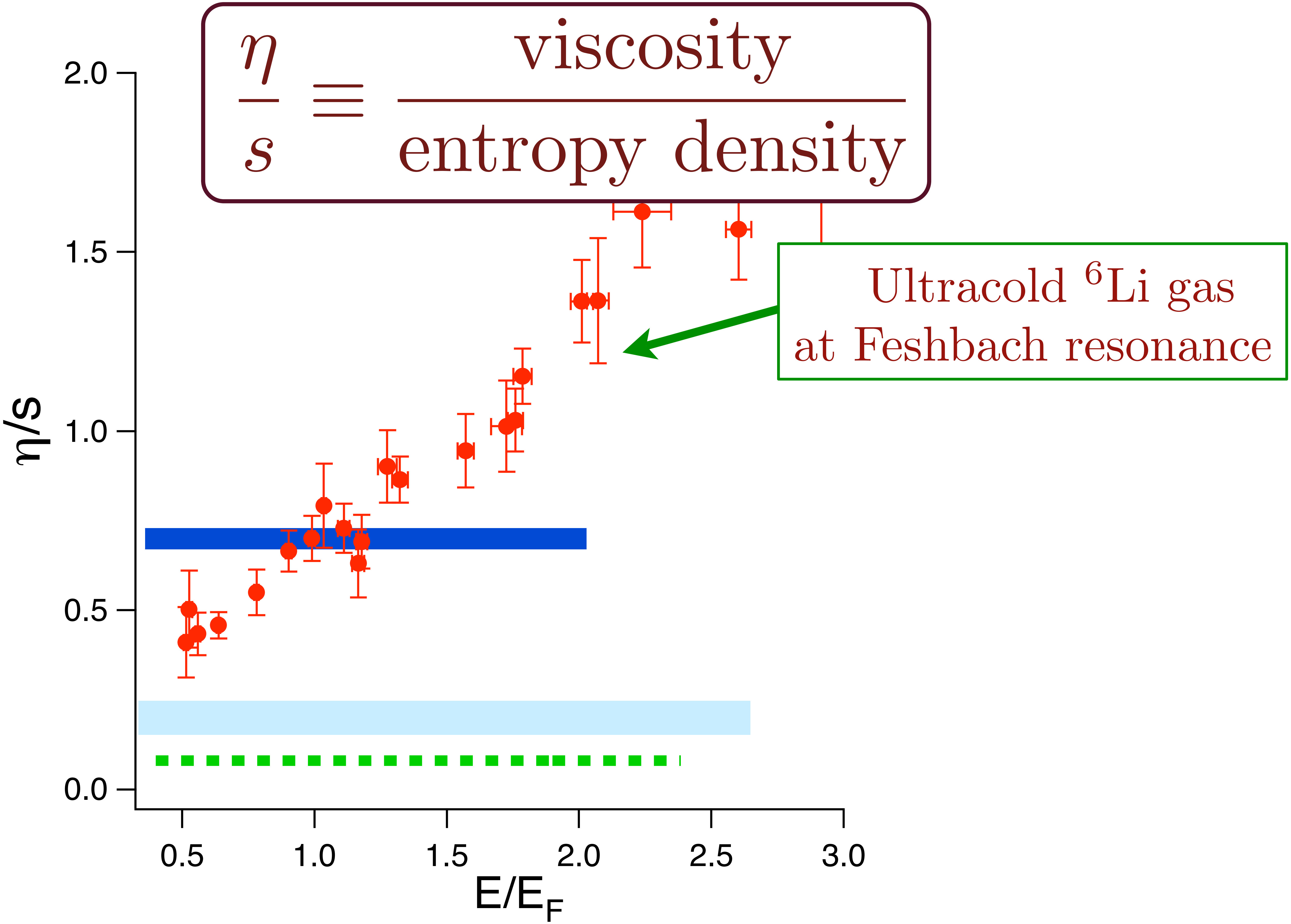
CFT is dual to quantum gravity models on AdS space. Explicit solutions of such gravity models with supersymmetry have been obtained

P. Nikolic and S. Sachdev, *Phys. Rev. A* **75**, 033608 (2007); D. T. Son, arXiv:0804.3972; K. Balasubramaniam and J. McGreevy, arXiv:0804.4053; W. D. Goldberger, arXiv:0806.2867; J. L. F. Barbón and C. A. Fuertes, arXiv:0806.3244; J. Maldacena, D. Martelli, and Y. Tachikawa, arXiv:0807.1100; A. Adams, K. Balasubramanian, and J. McGreevy, arXiv:0807.1111.



T. Schafer, *Phys. Rev. A* **76**, 063618 (2007).

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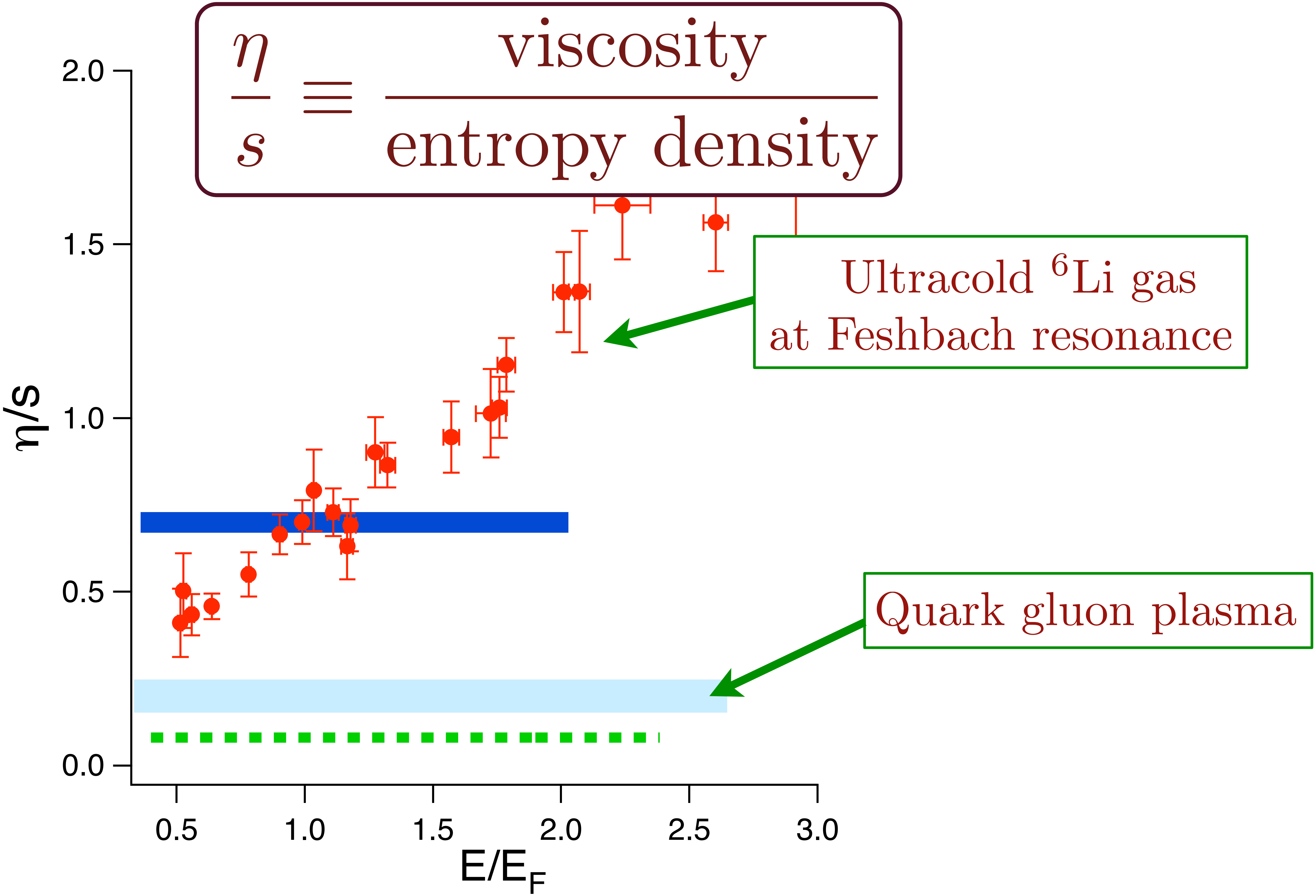

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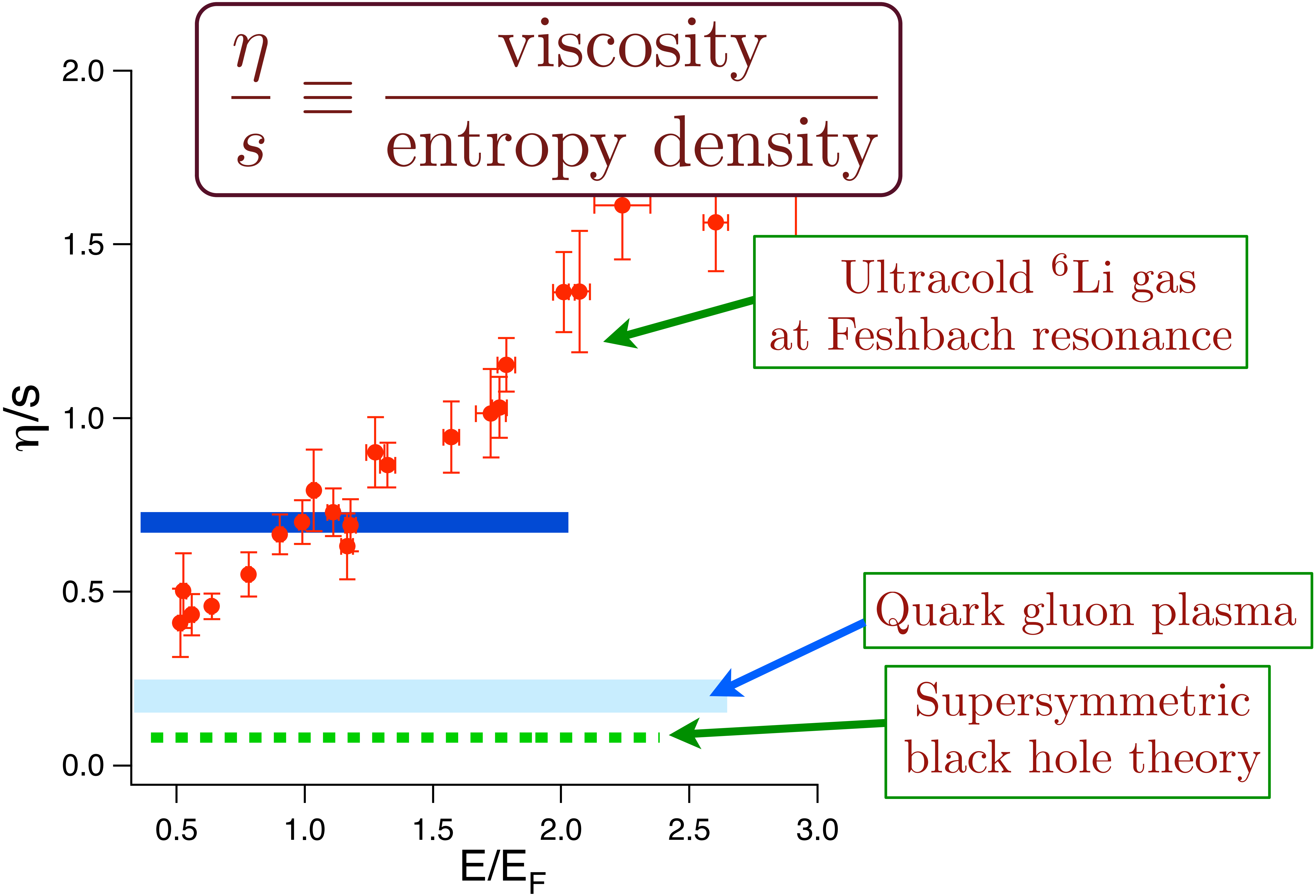

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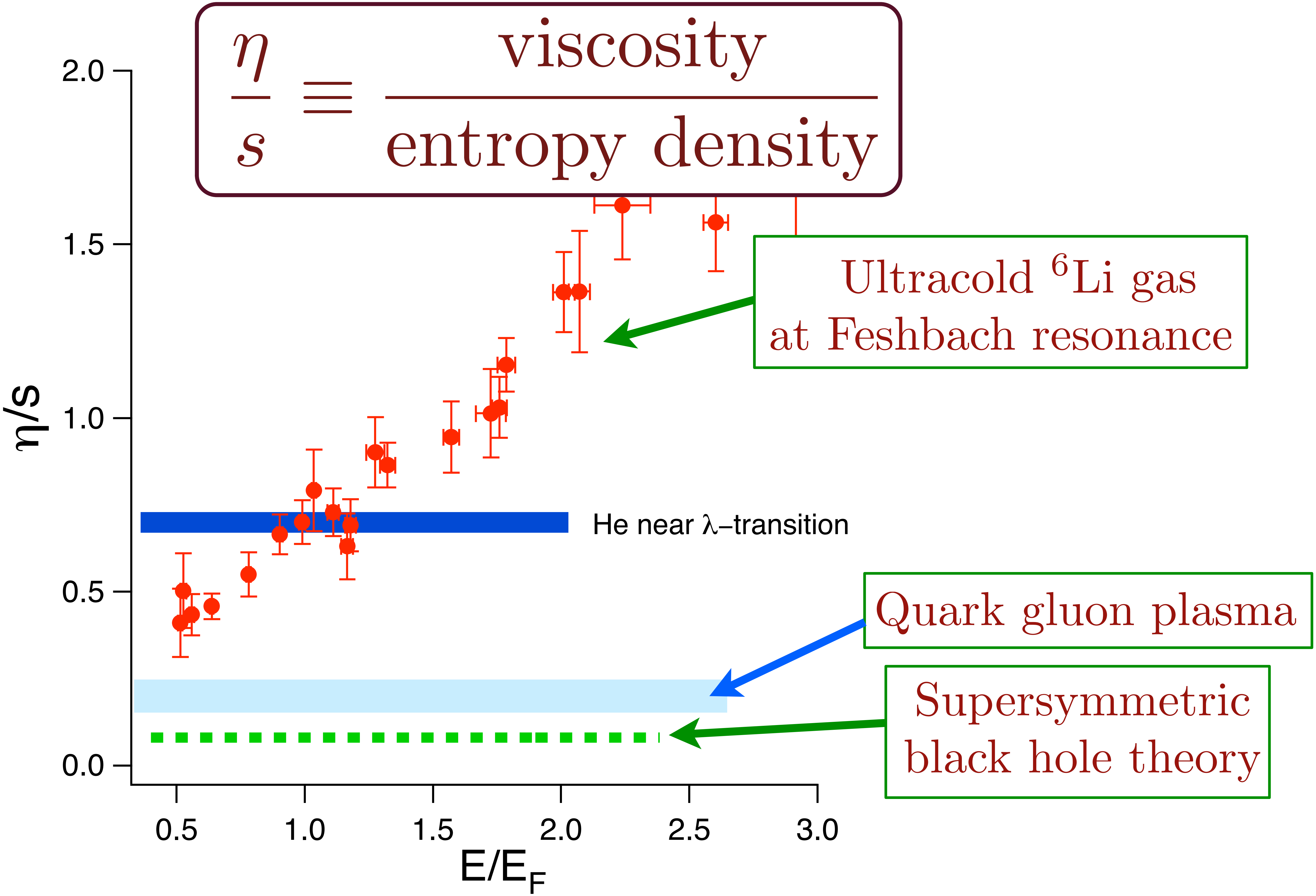

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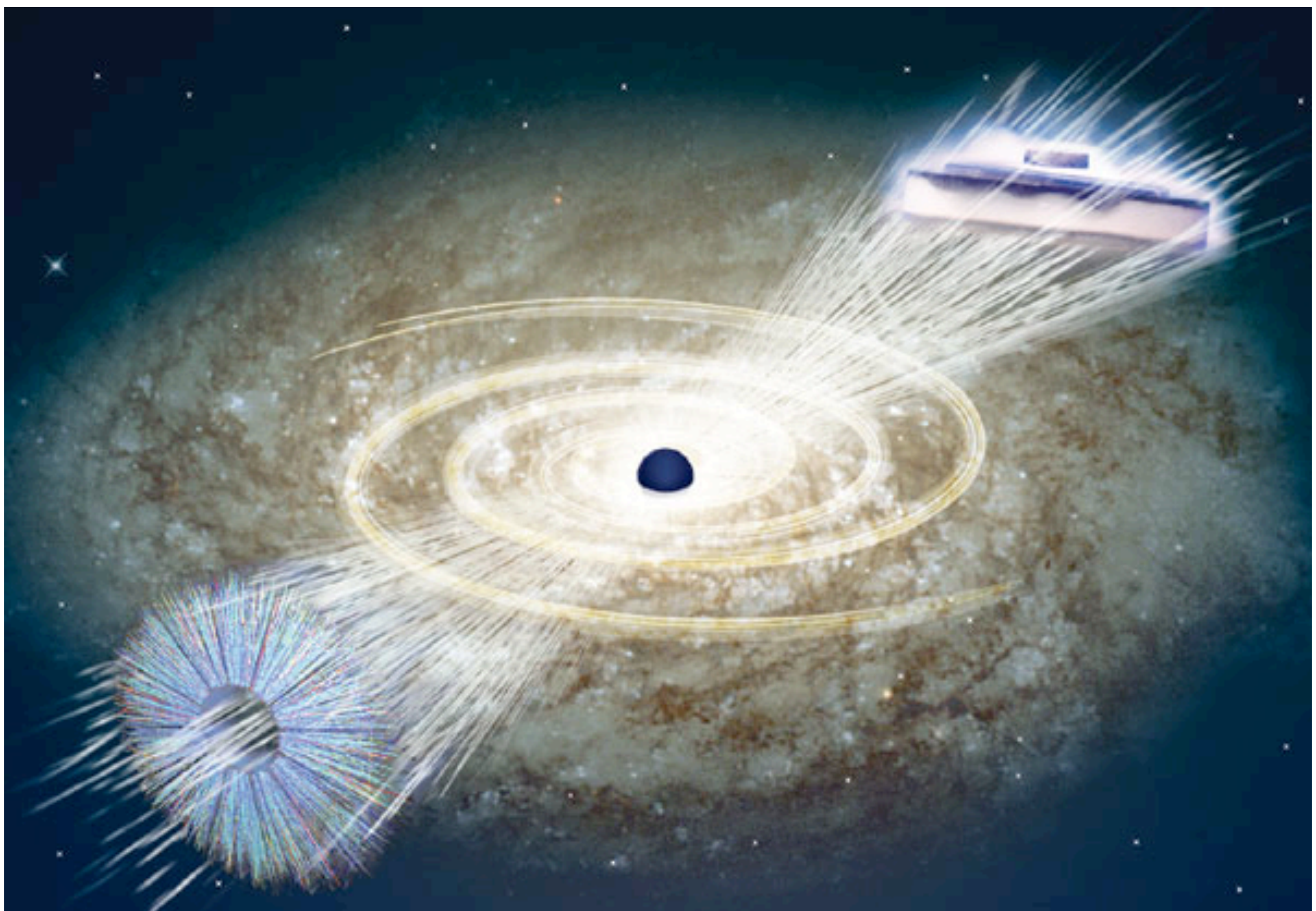
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Conclusions

- Theory for transport near quantum phase transitions in superfluids and antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of Nernst effect near the superfluid-insulator transition, and connection to cuprates.
- Quantum-critical magnetotransport in graphene.