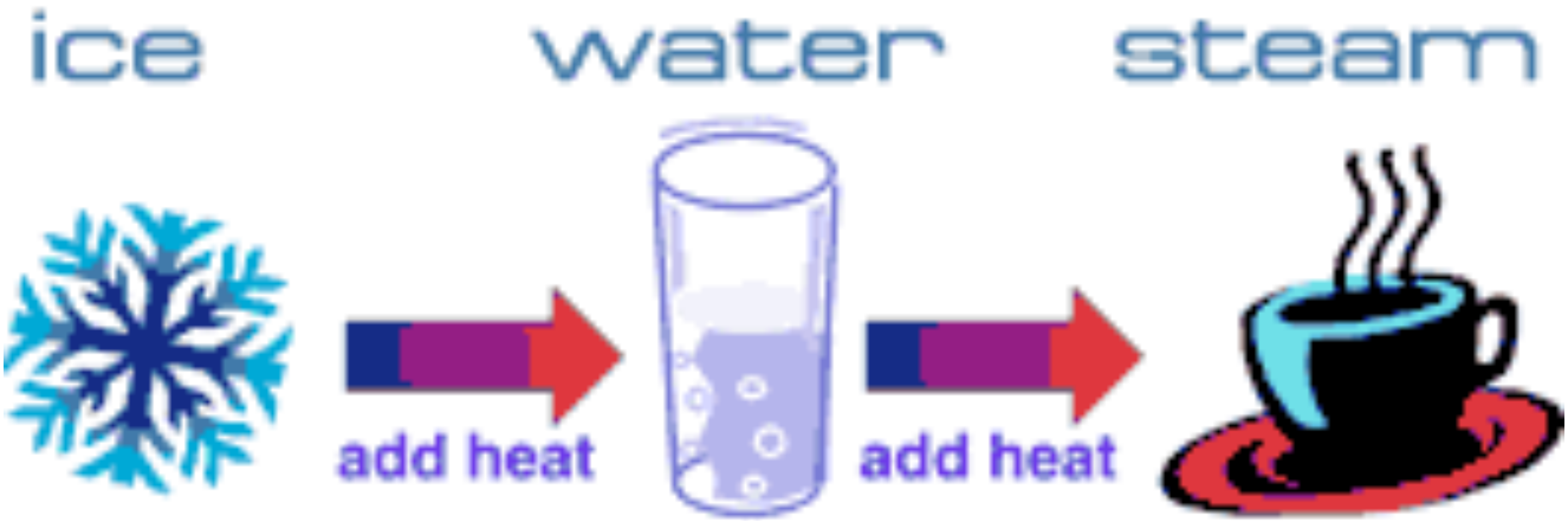
Quantum Criticality and Black Holes

Subir Sachdev

Talk online at http://sachdev.physics.harvard.edu

What is a “phase transition” ?



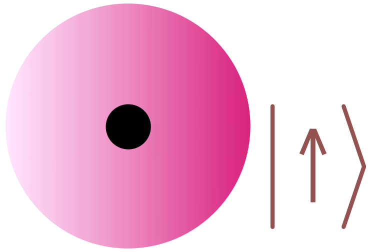
A change in the collective properties of a macroscopic number of atoms

What is a “*quantum phase transition*” ?

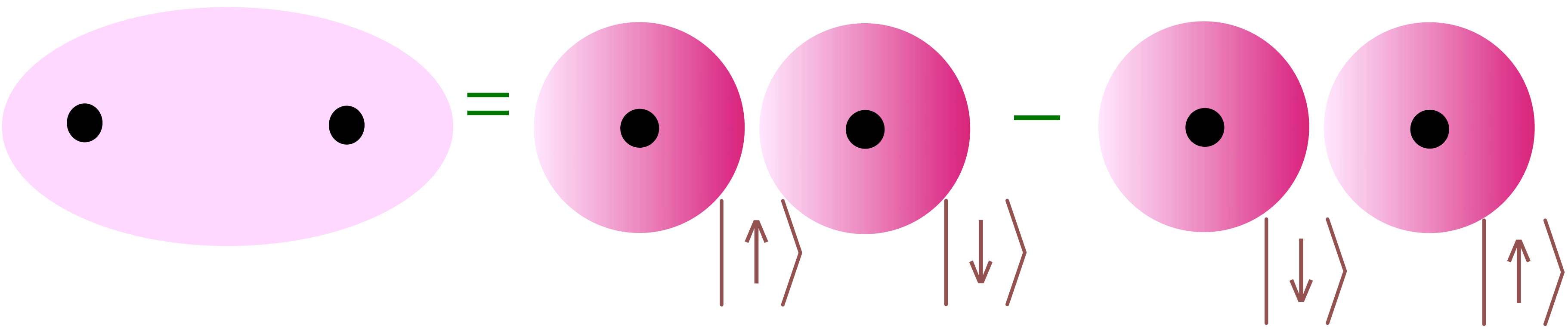
Change in the nature of entanglement in a macroscopic quantum system.

Entanglement

Hydrogen atom:



Hydrogen molecule:



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Superposition of two electron states leads to non-local correlations between spins

Outline

1. Superfluid-insulator quantum transitions

   Experiments on ultracold atoms

2. Theory of quantum-critical transport

   Collisionless-t0-hydrodynamic crossover of conformal field theories

3. Entanglement of valence bonds

   Deconfined criticality in antiferromagnets

4. Nernst effect in the cuprate superconductors

   Quantum criticality and dyonic black holes

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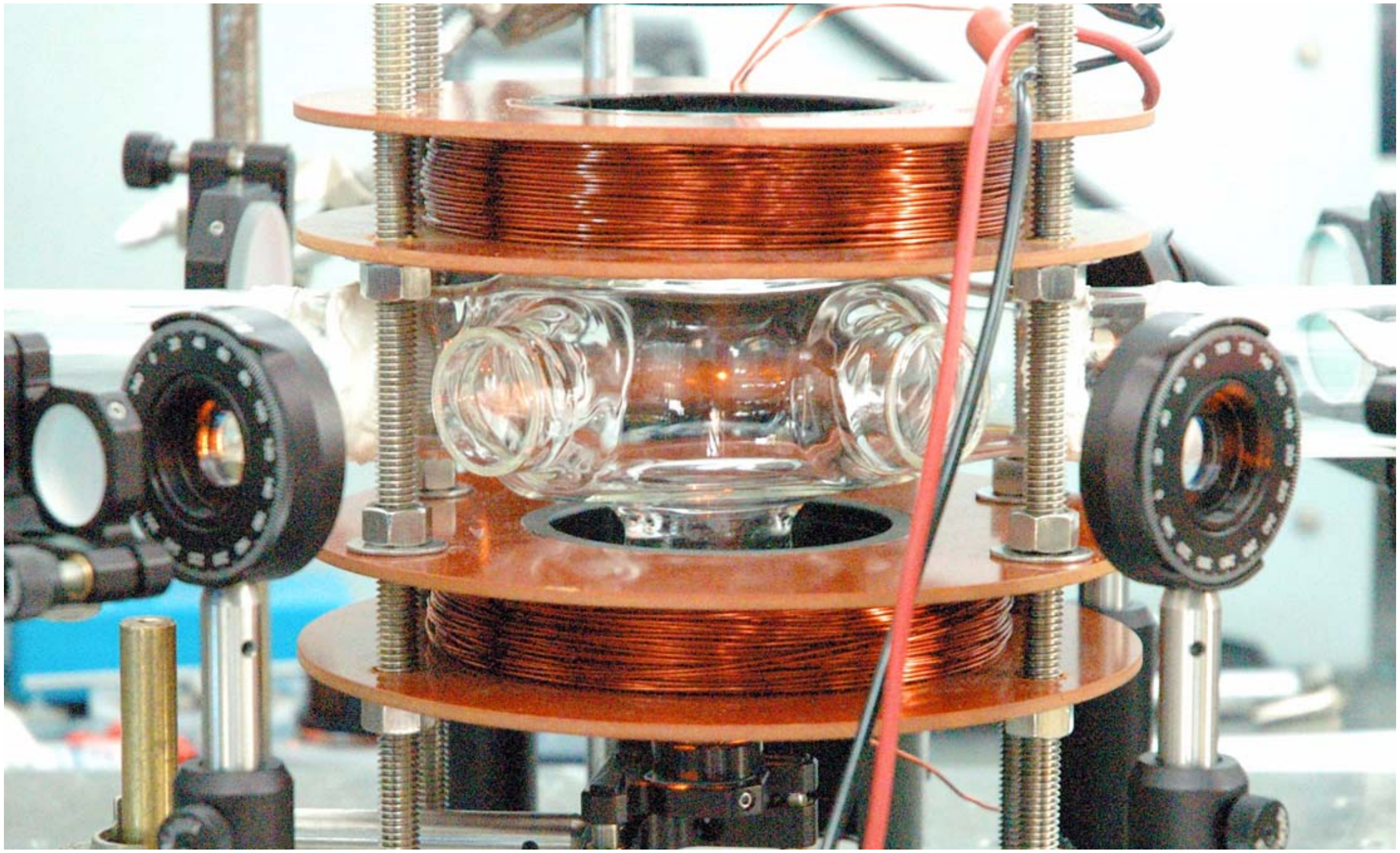
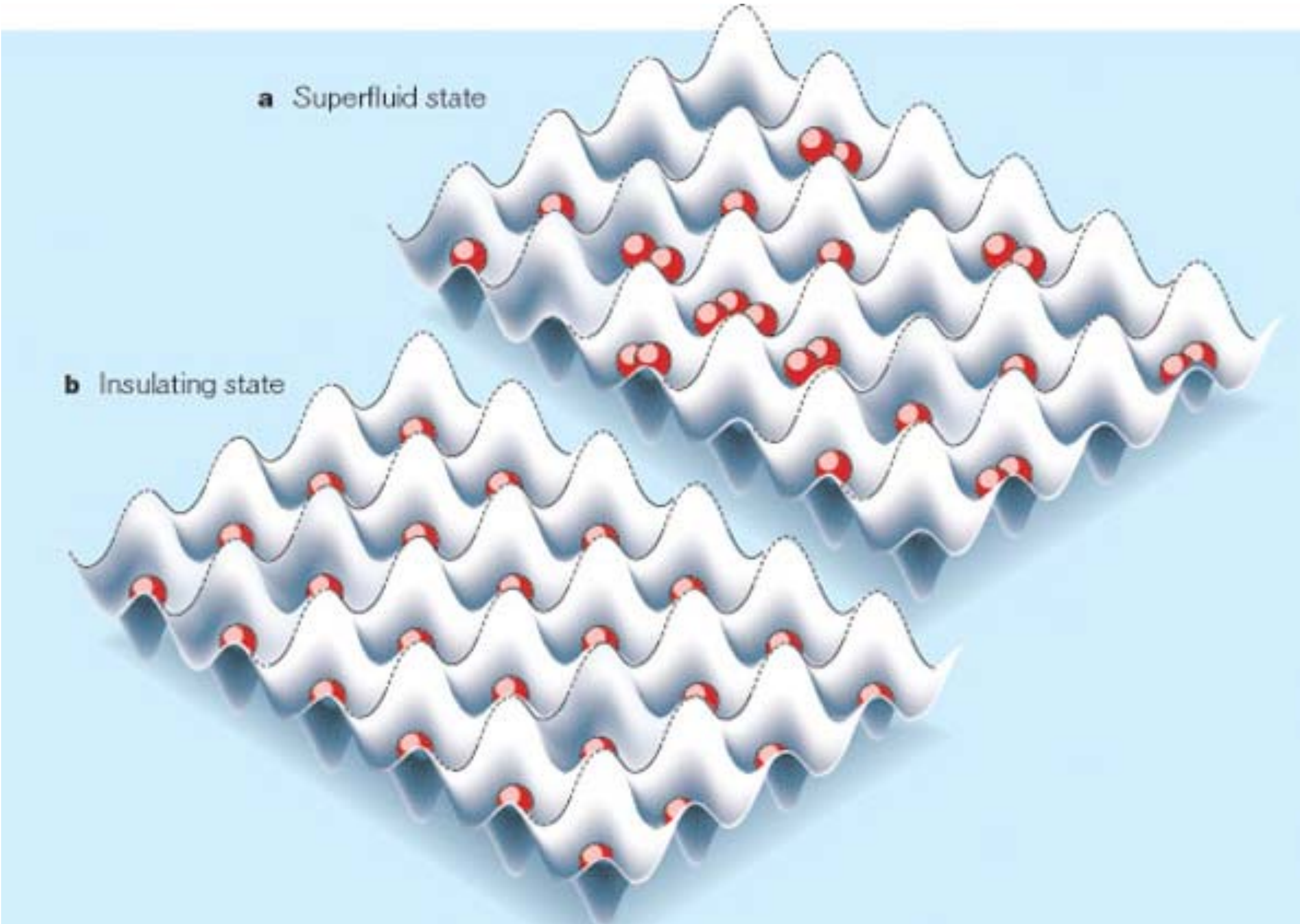
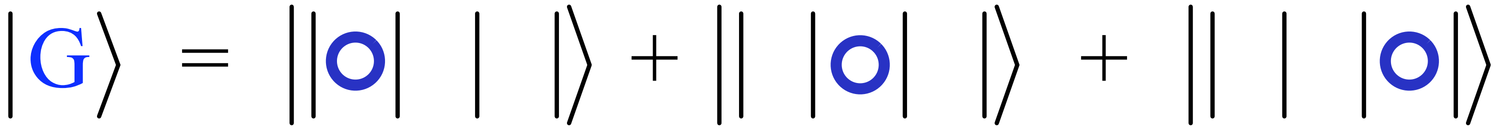


Trap for ultracold ^{87}Rb atoms



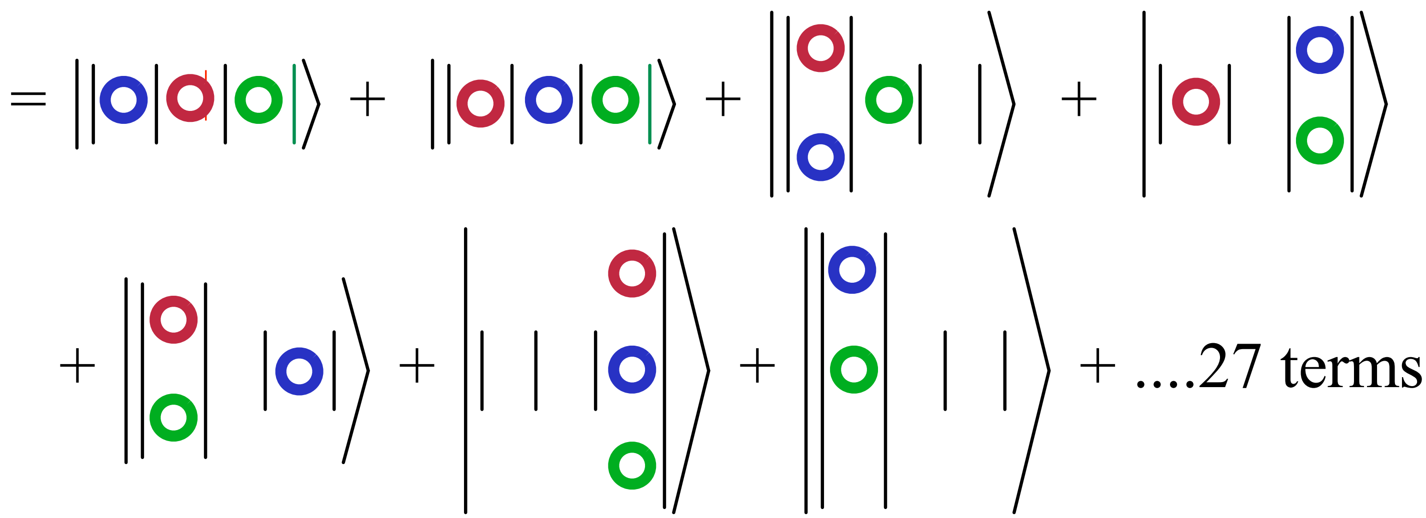
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Bose-Einstein condensate in a periodic potential



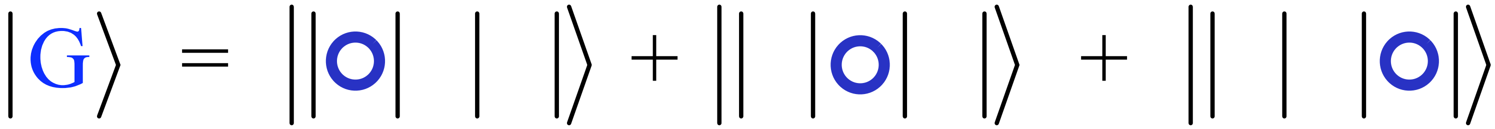
Lowest energy state for many atoms

$$|BEC\rangle = |G\rangle|G\rangle|G\rangle$$



Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)

Breaking up the Bose-Einstein condensate



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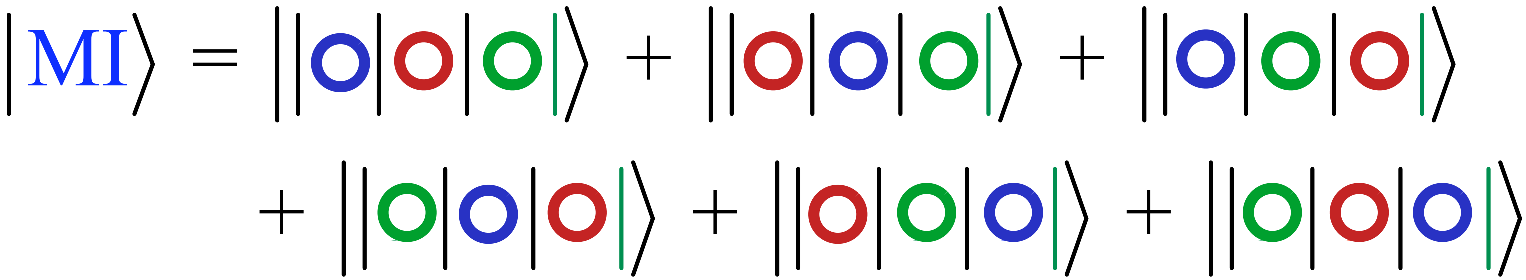
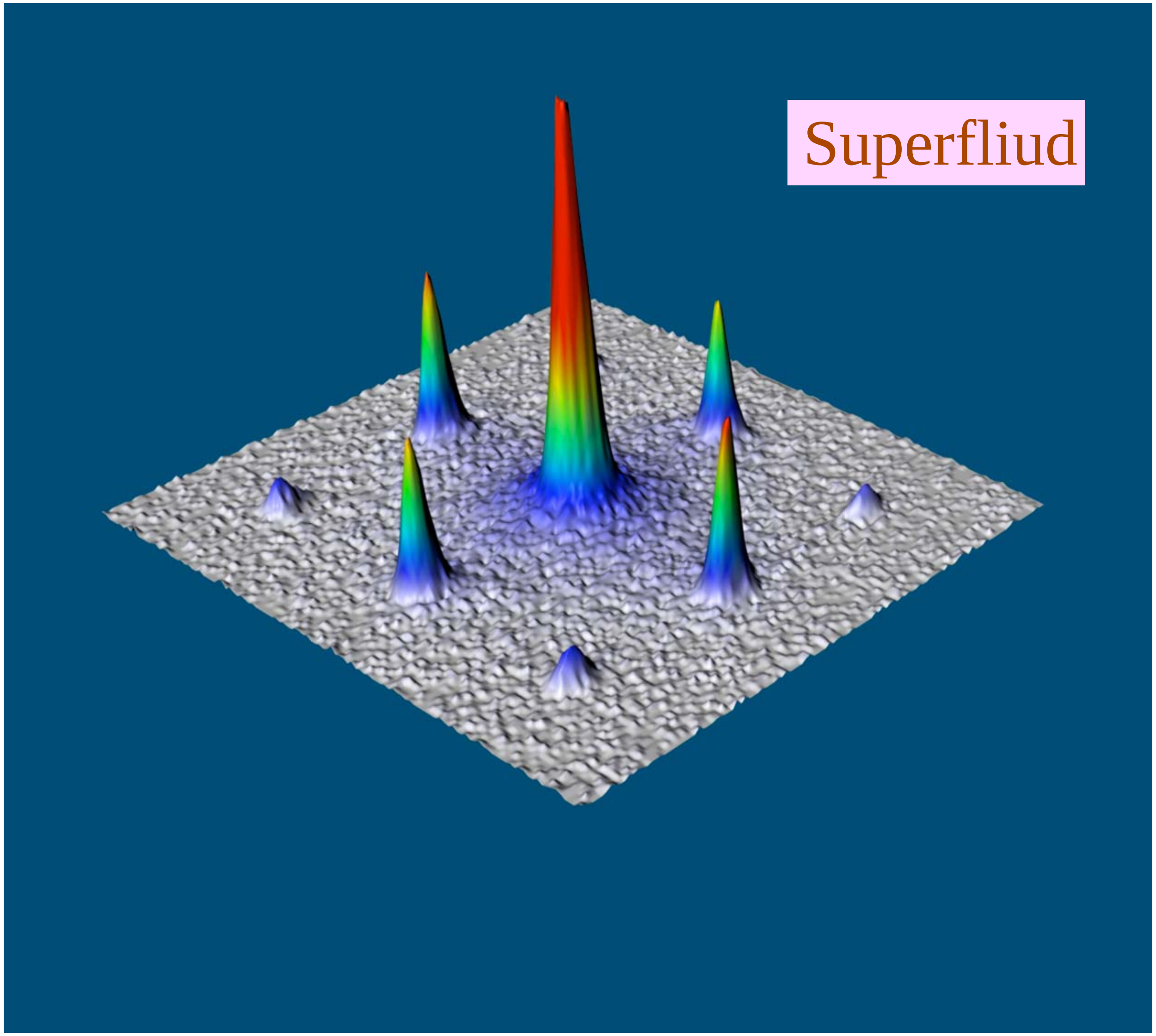


By tuning repulsive interactions between the atoms, states with multiple atoms in a potential well can be suppressed. The lowest energy state is then a *Mott insulator* – it has negligible number fluctuations, and atoms cannot “flow”

Velocity distribution of ^{87}Rb atoms



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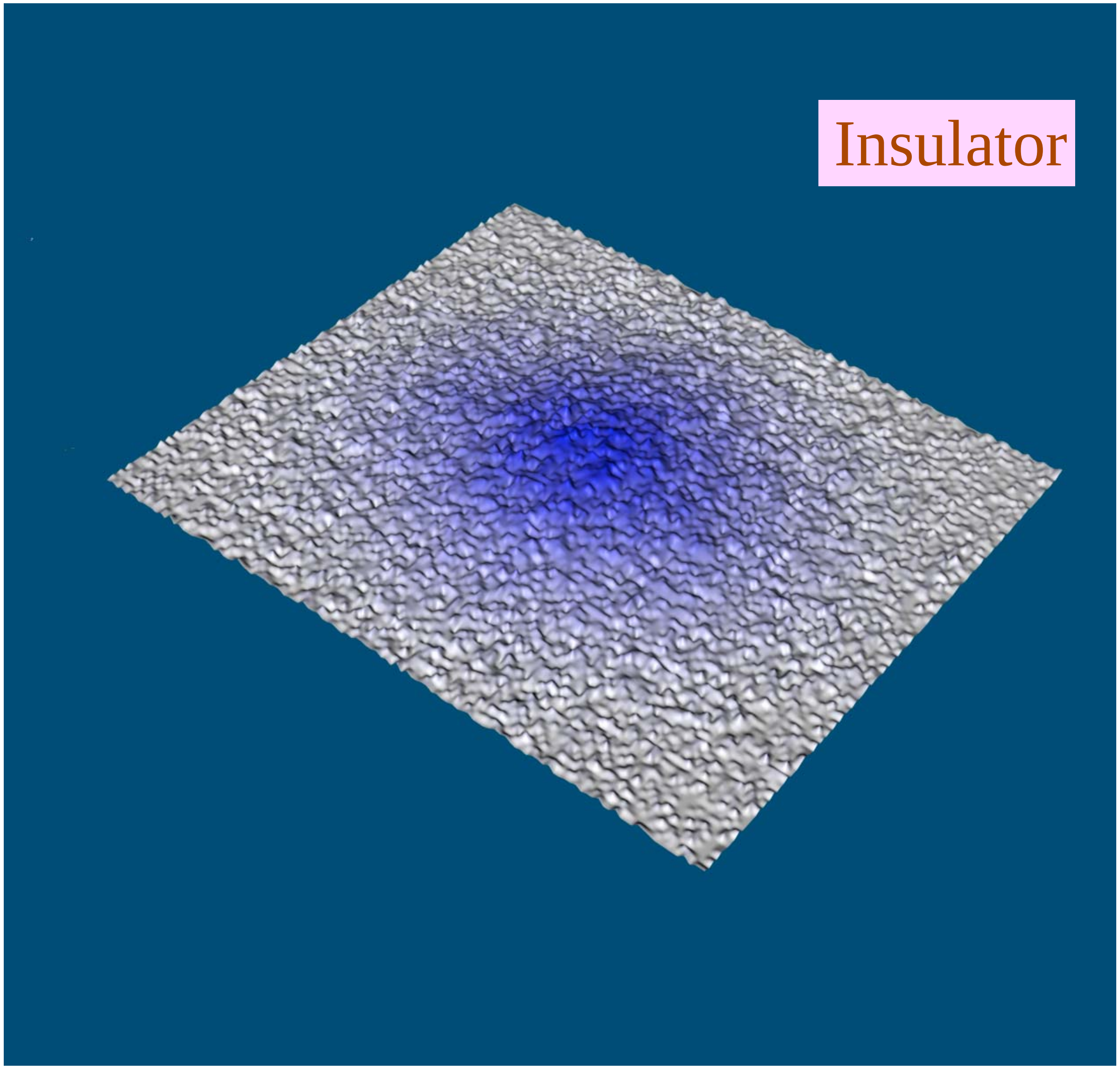

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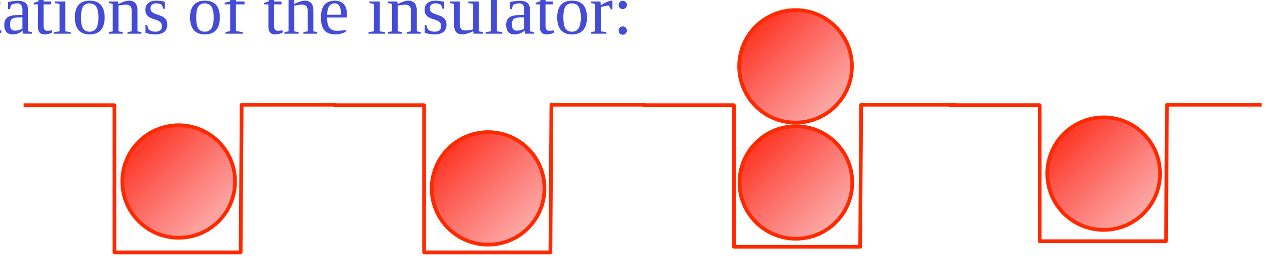
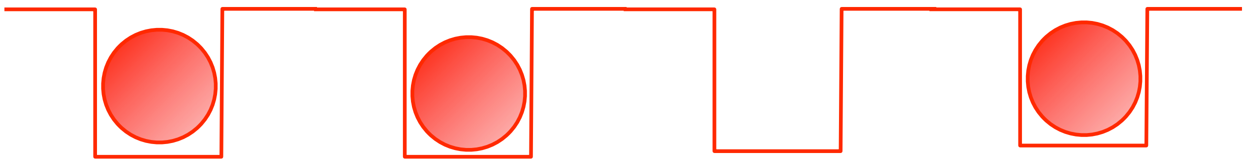
The insulator:

Excitations of the insulator:

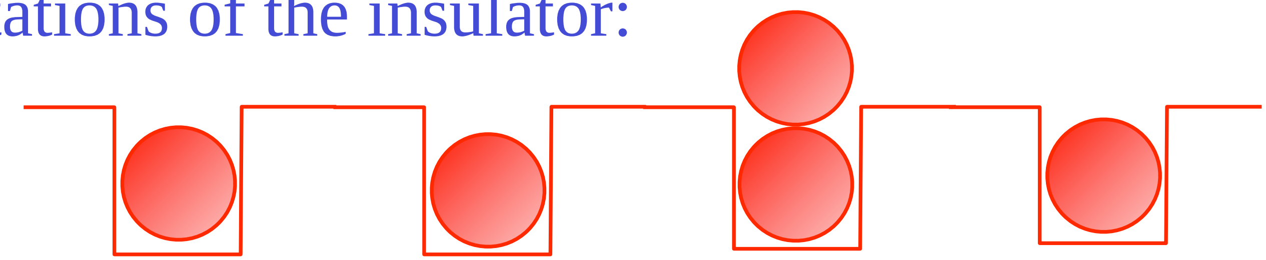


Particles $\sim \psi^\dagger$



Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$

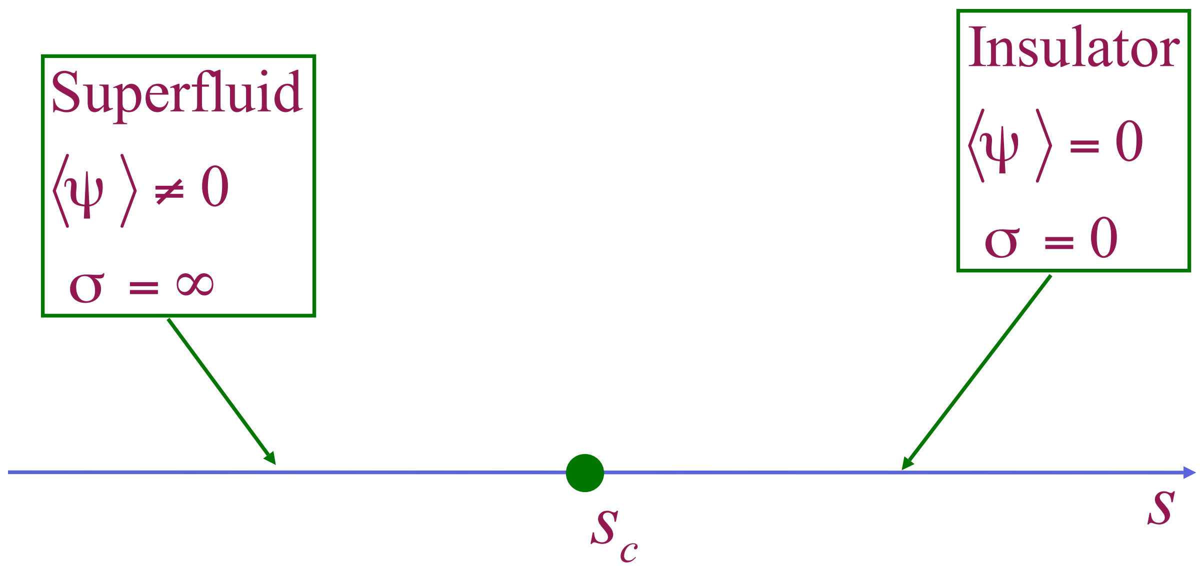
Holes ~ ψ

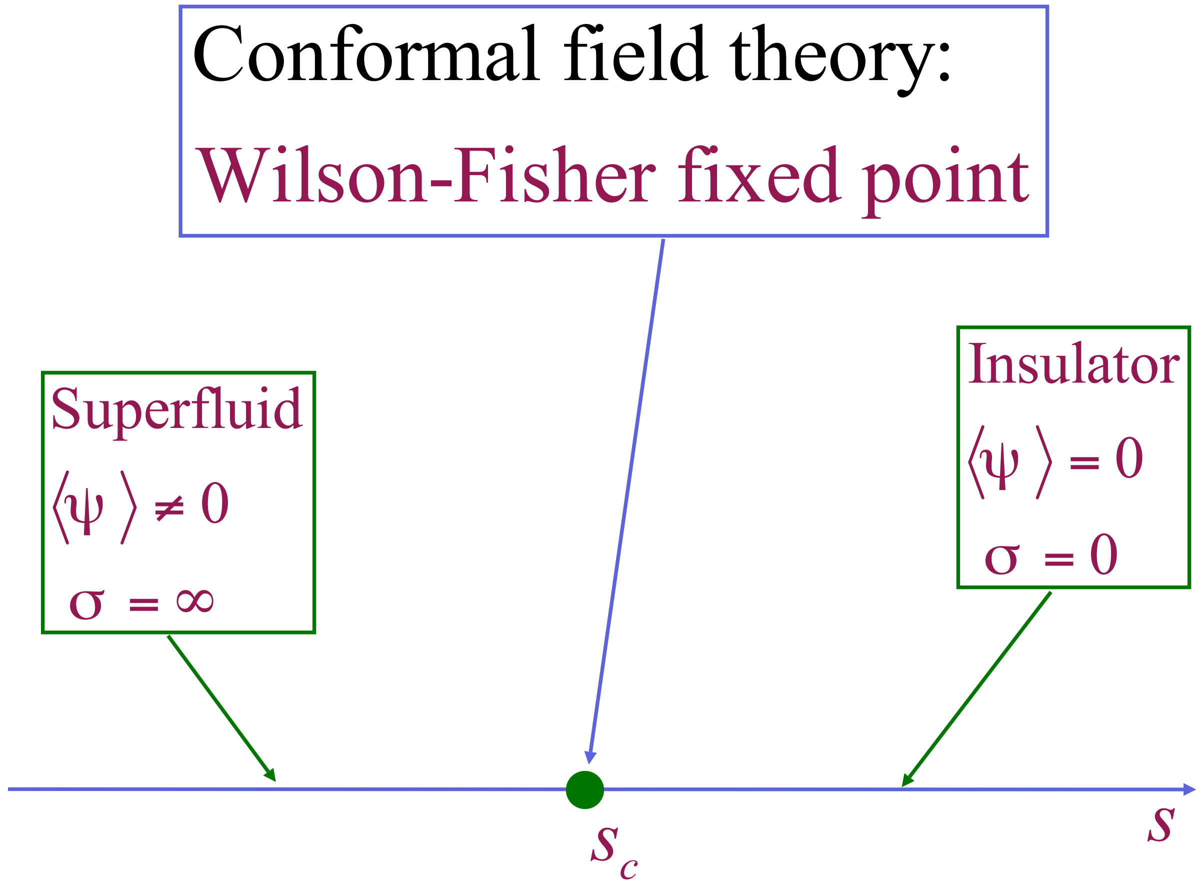
Density of particles = density of holes ⇒
"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2x d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla}\psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

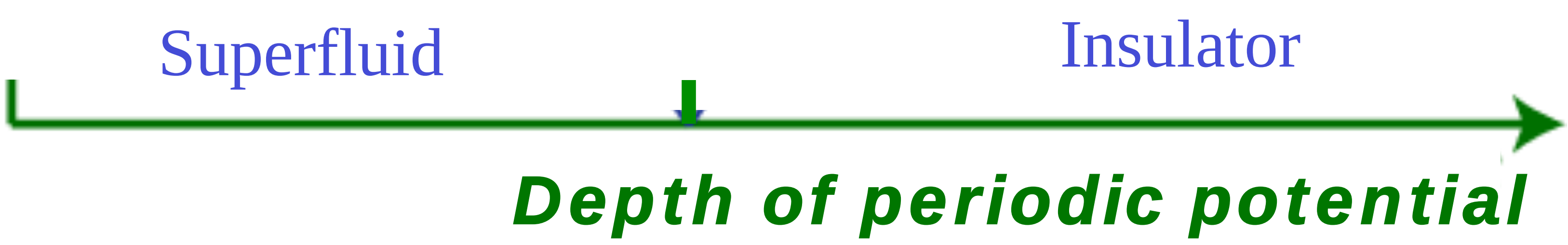
$$\text{Insulator} \quad \Leftrightarrow \quad \langle \psi \rangle = 0$$

$$\text{Superfluid} \quad \Leftrightarrow \quad \langle \psi \rangle \neq 0$$

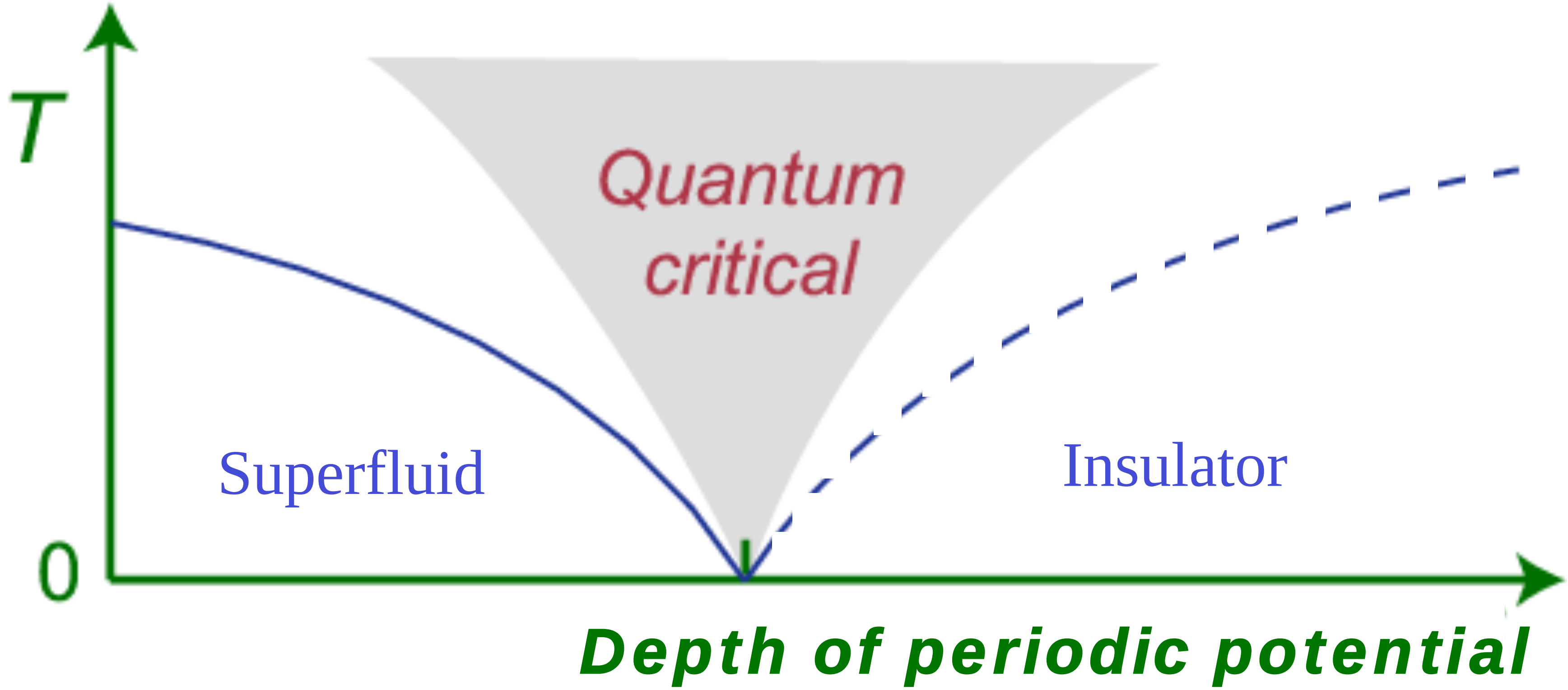

Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$
Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$
s_c
s



Conformal field theory:
Wilson-Fisher fixed point
Superfluid
$\langle \psi \rangle \neq 0$
$\sigma = \infty$
Insulator
$\langle \psi \rangle = 0$
$\sigma = 0$
s_c
s


Superfluid
Insulator
Depth of periodic potential

Non-zero temperature phase diagram



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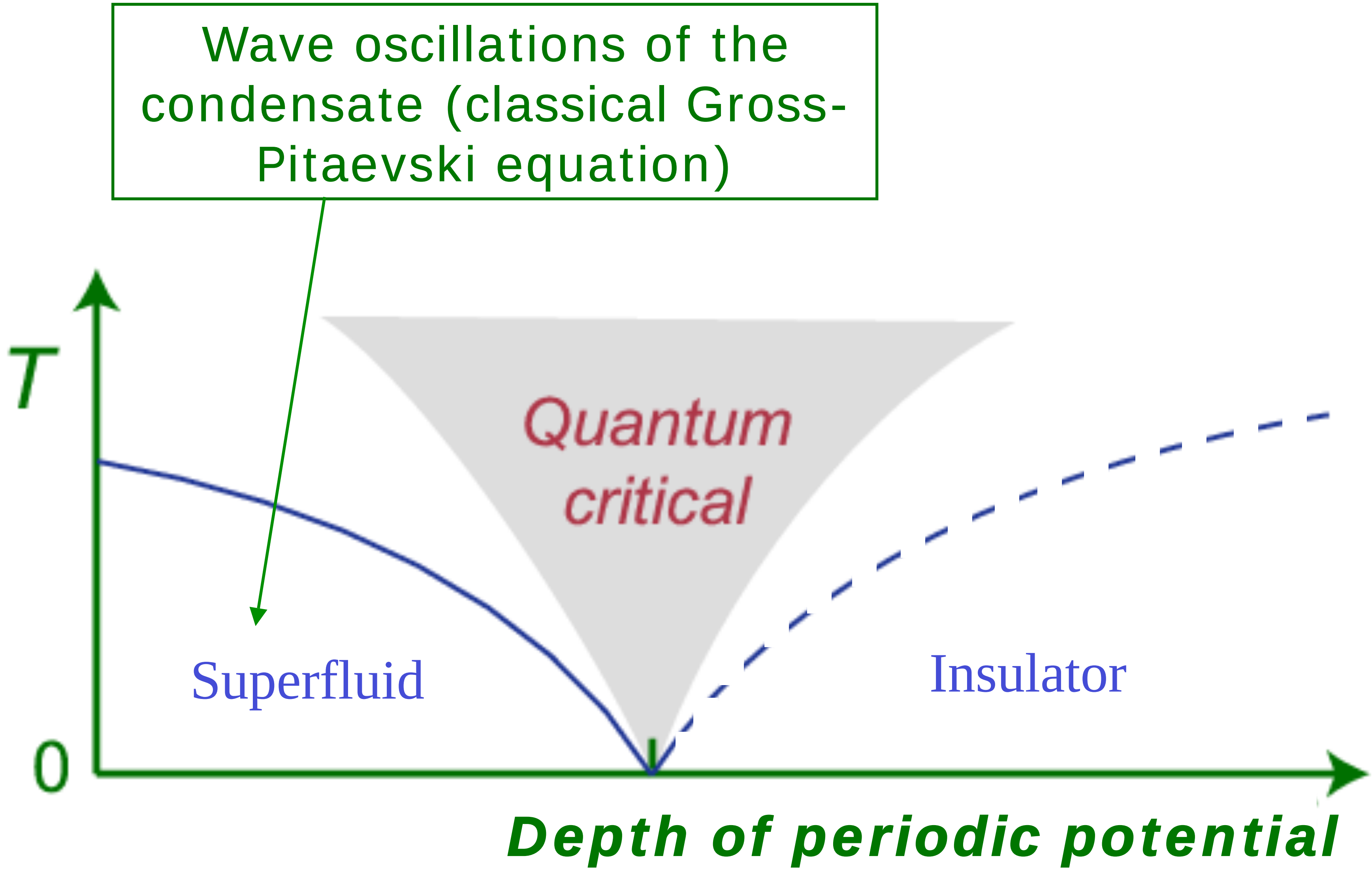

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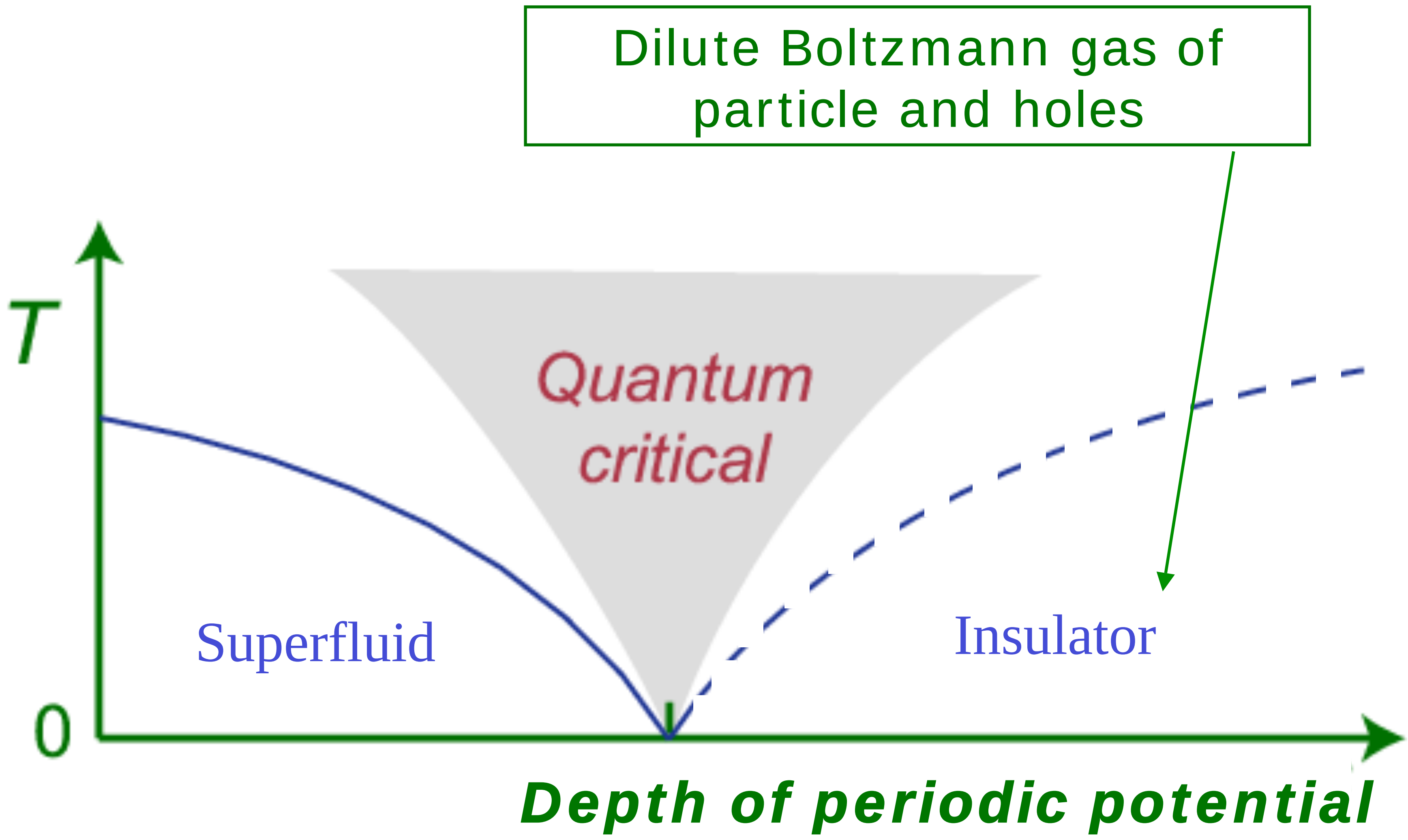

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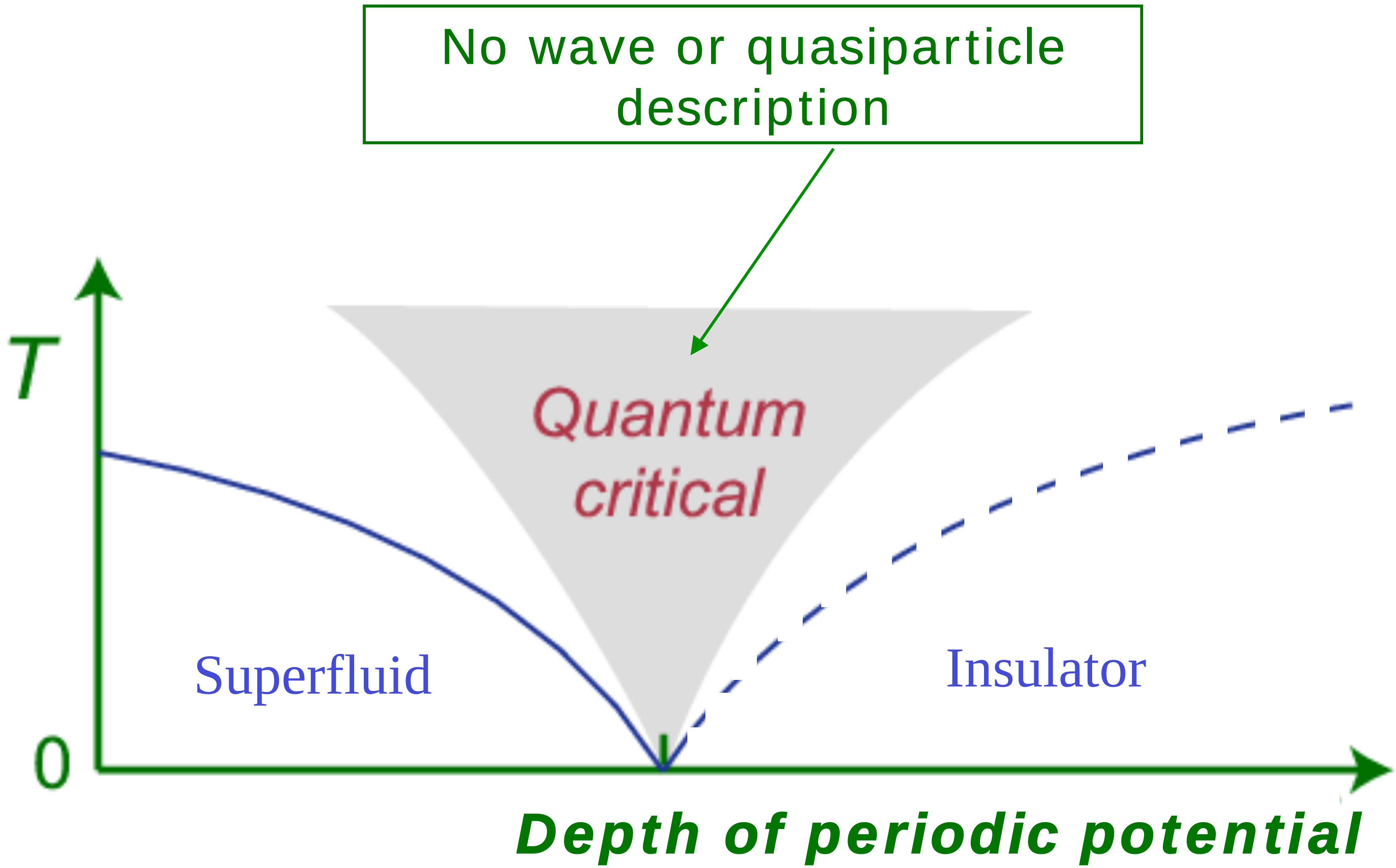


Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

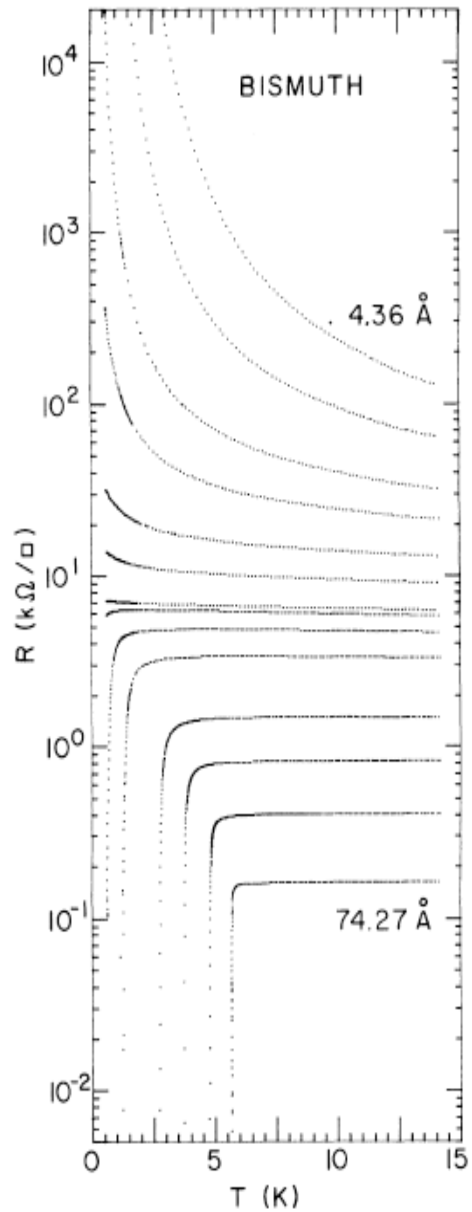
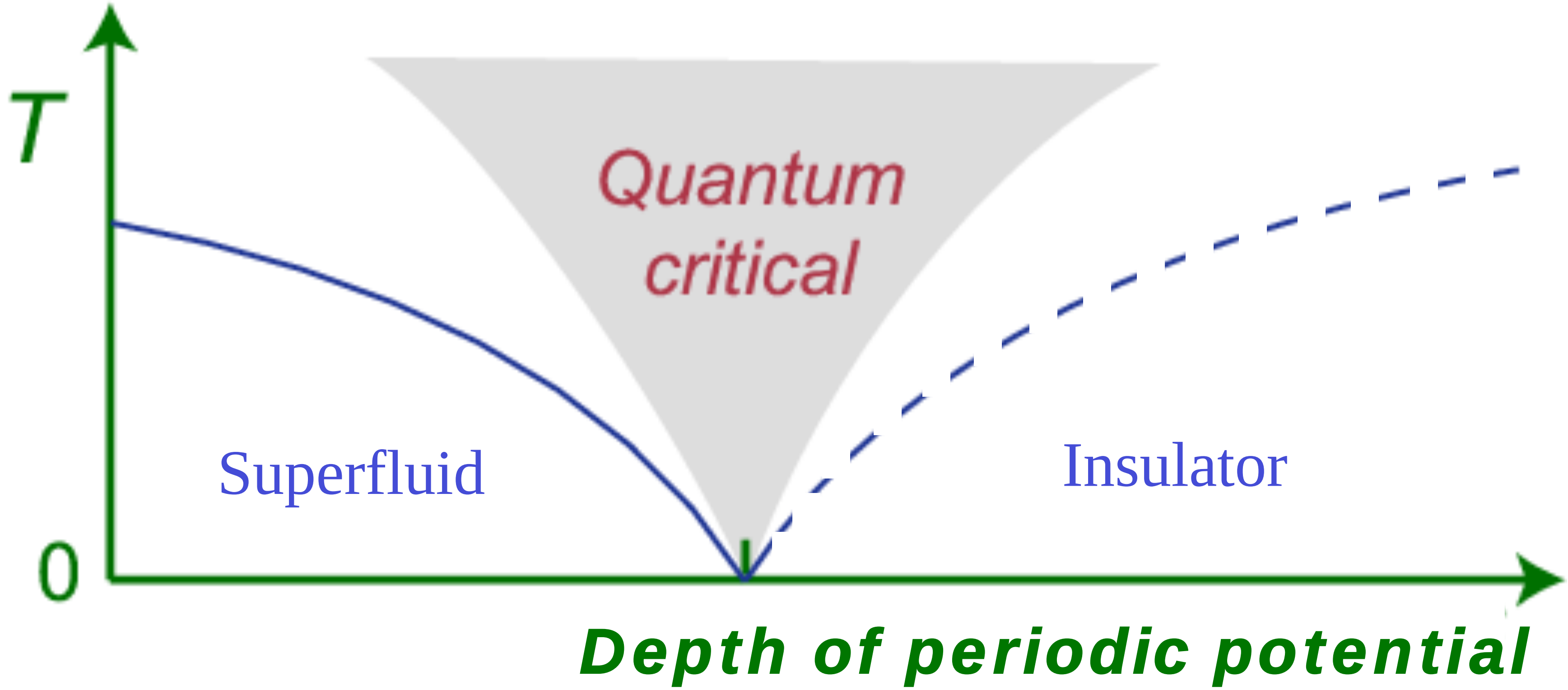
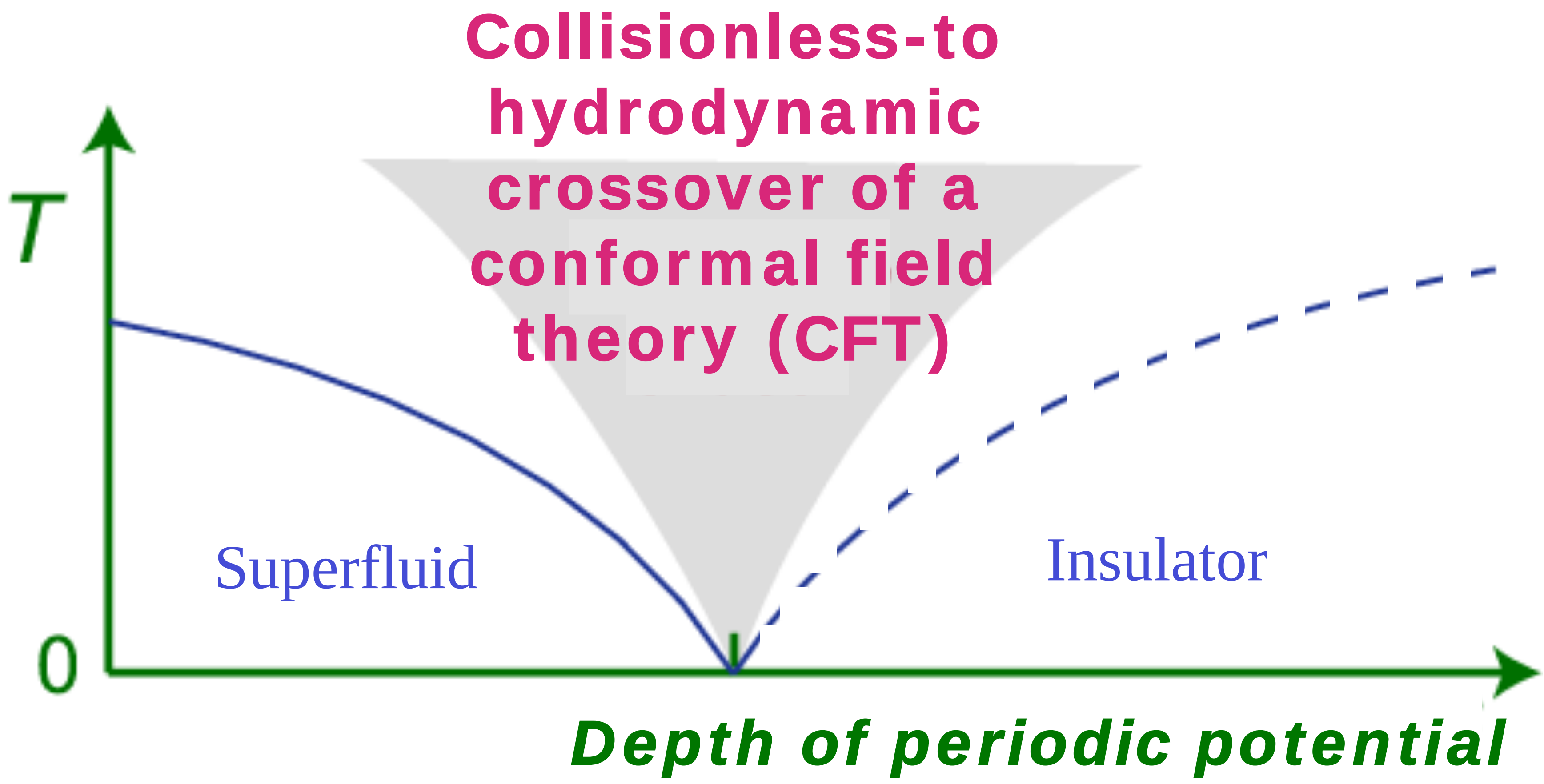


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Non-zero temperature phase diagram



Non-zero temperature phase diagram



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless-to-hydrodynamic crossover of a CFT in 2+1 dimensions

Consider the retarded density-density correlation function

$$C(k,\omega) = \langle \rho(k,\omega)\rho(-k,-\omega)\rangle_{\rm ret}$$

The characteristic collision time for excitations of the CFT is $\hbar/k_B T$. So, for $|\omega - k| \gg T$ we have the collisionless conformal behavior

$$C(k,\omega) = K \frac{k^2}{\sqrt{k^2-\omega^2}}$$

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$$C(k,\omega) = K \frac{k^2}{\sqrt{k^2-\omega^2}}$$

while for $\omega, k \ll T$, we have the hydrodynamic behavior

$$C(k,\omega) = \chi \frac{D_c k^2}{-i\omega + D_c k^2}.$$

So the high frequency conductivity $\sigma(\omega \gg T) = K$, and the hydrodynamic conductivity $\sigma(\omega \ll T) = D_c\chi$, and in general $K \neq D_c\chi$.

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport co-efficients are universally determined by $k_B T$

Charge diffusion constant $$D_c = \Theta \frac{c^2}{k_B T}$$

Conductivity $$\sigma_Q = D_c \chi = \overline{\Theta} \frac{4e^2}{h}$$

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The AdS/CFT correspondence (Maldacena, Polyakov, Witten, Son) relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

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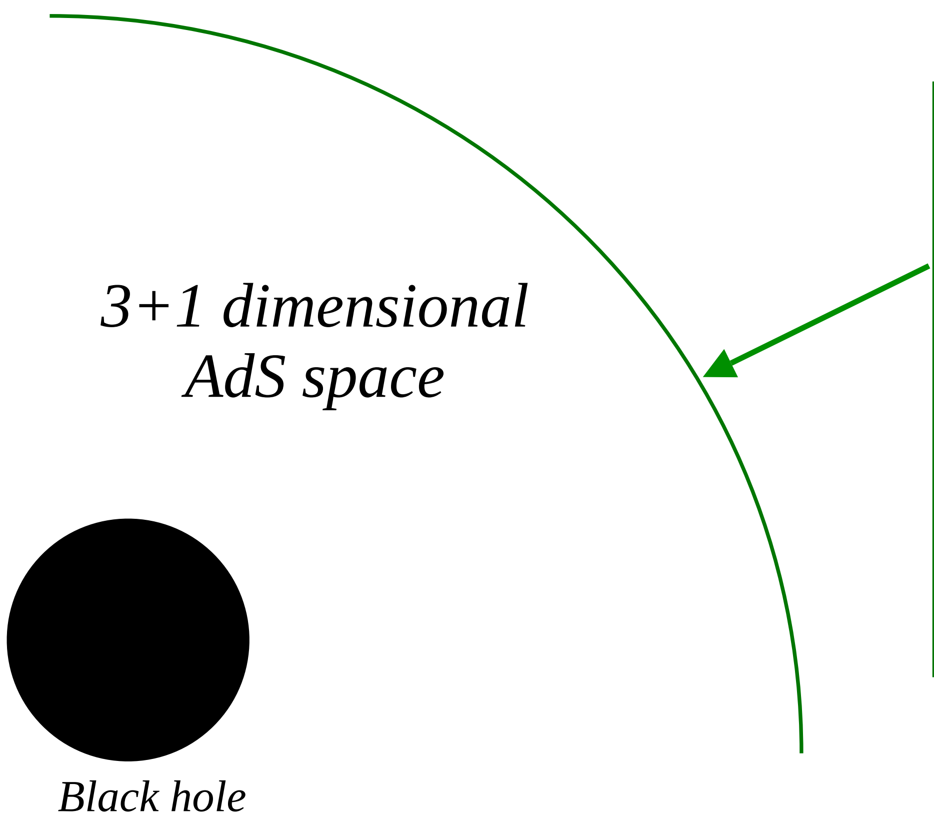
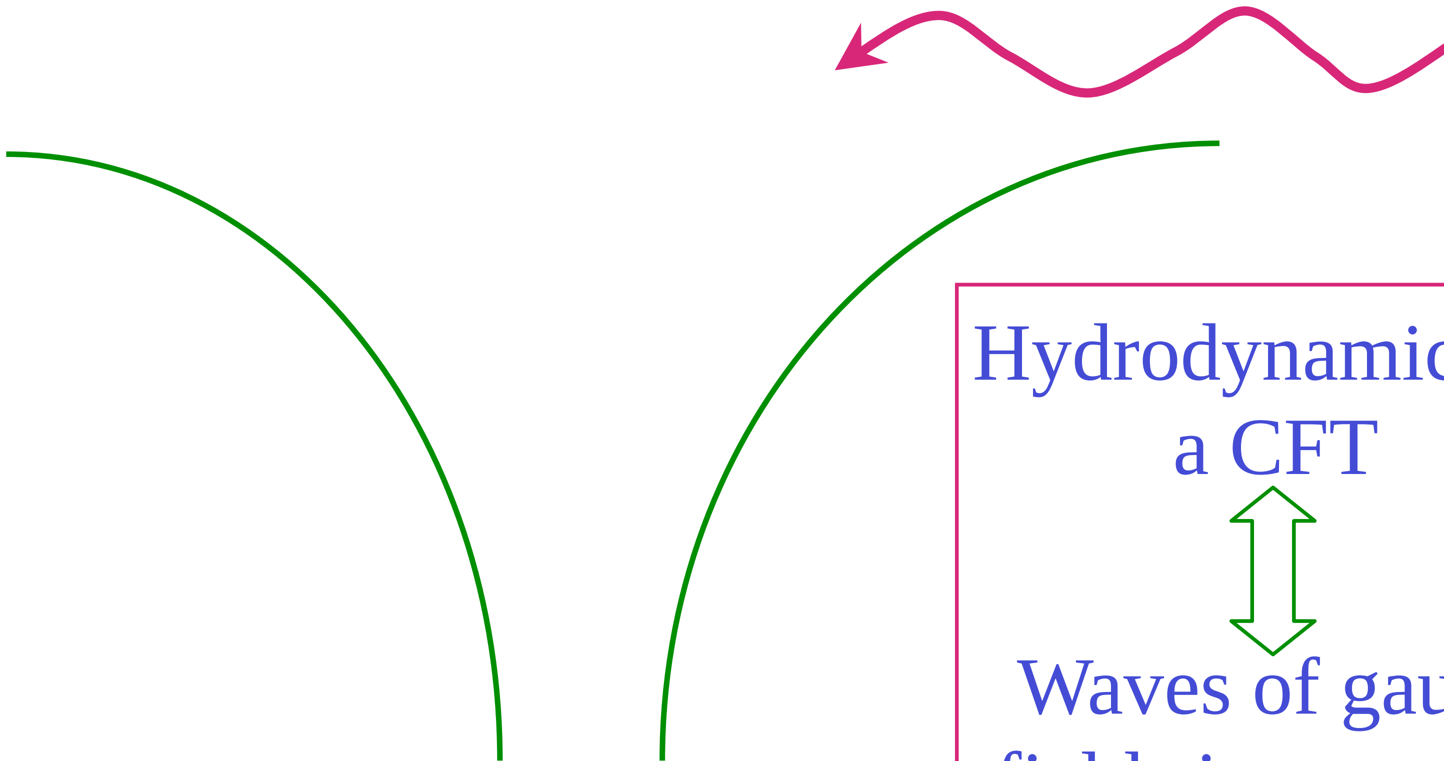


Holographic representation of black hole physics in a 2+1 dimensional CFT at a temperature equal to the Hawking temperature of the black hole.

Hydrodynamics of a conformal field theory (CFT)



Hydrodynamics of a CFT

⇕

Waves of gauge fields in a curved background

Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a SU(*N*) gauge field and 16 supercharges, we know the exact diffusion constant associated with a global SO(8) symmetry:

Spin diffusion constant $$D_c = \frac{3}{4\pi}\frac{c^2}{k_B T}$$

Spin conductivity $$\sigma_Q = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless-to-hydrodynamic crossover of solvable SYM_3

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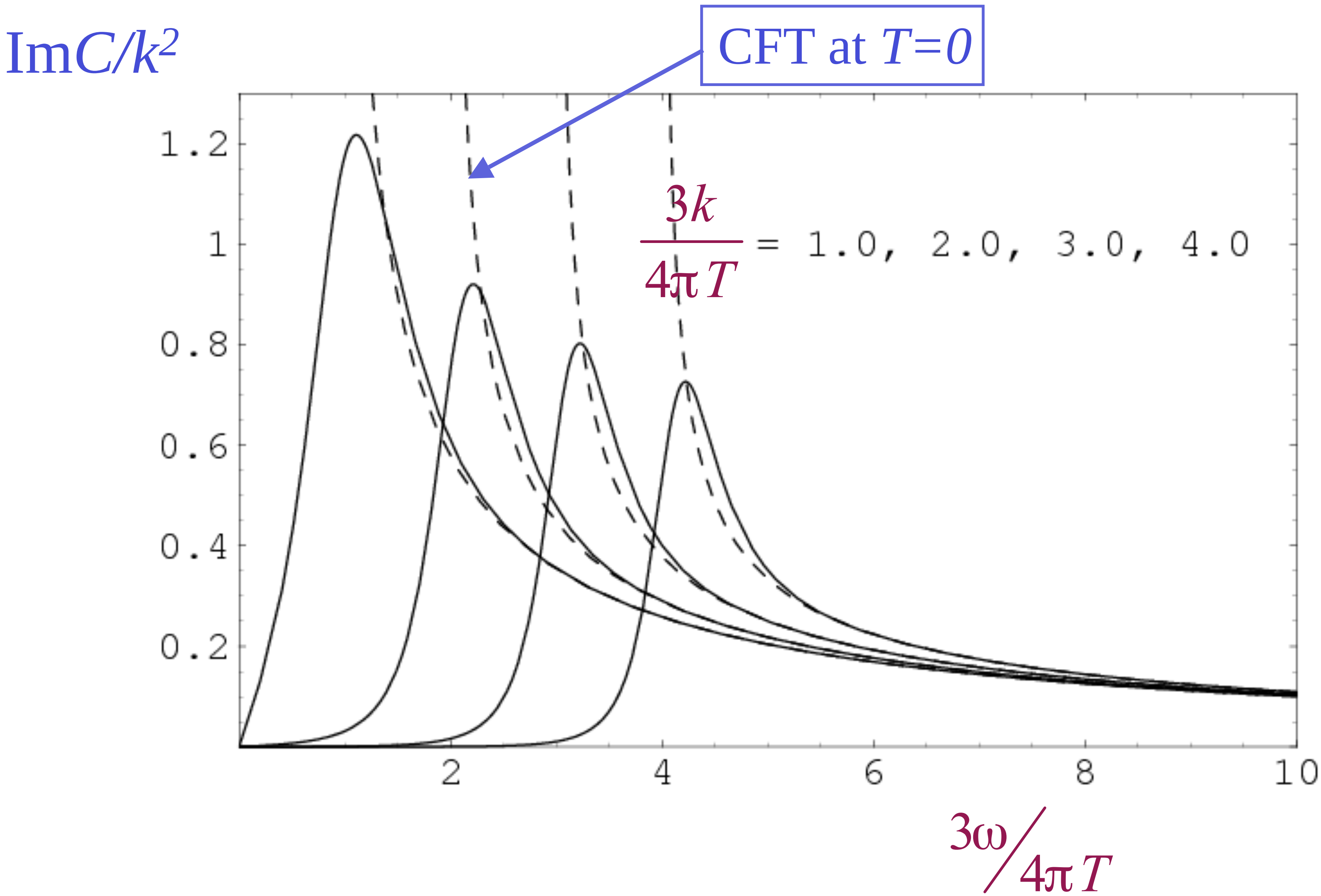

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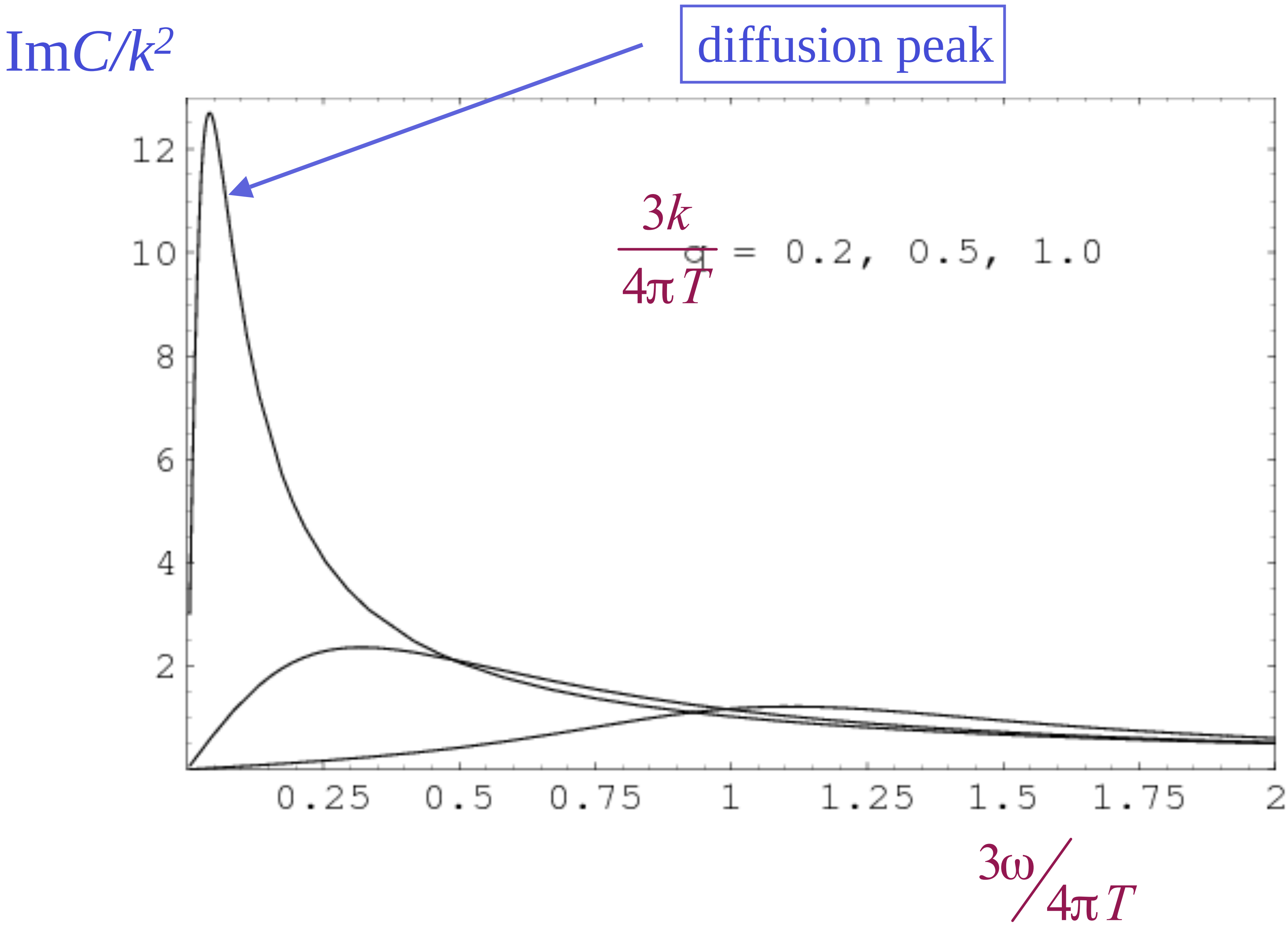

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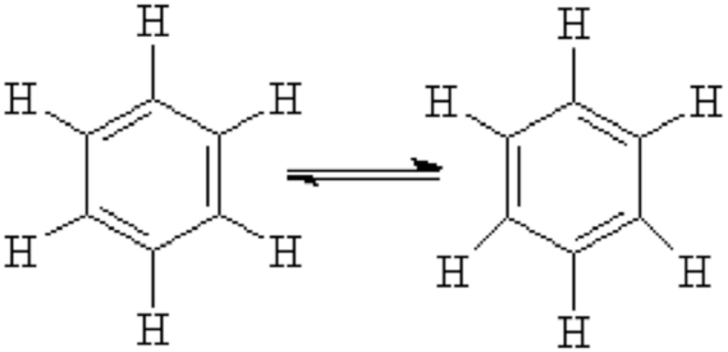
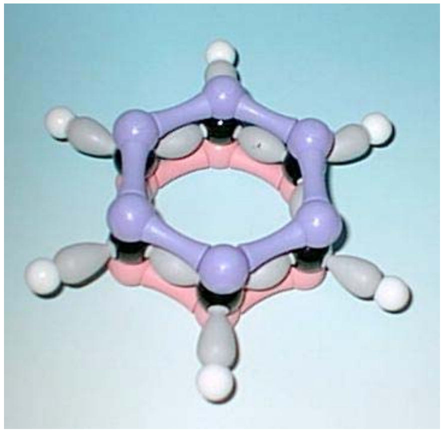
Valence bonds in benzene





Resonance in benzene leads to a symmetric configuration of valence bonds (*F. Kekulé*, *L. Pauling*)

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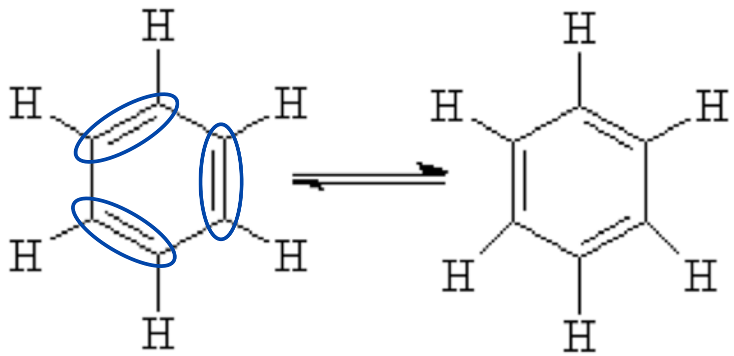

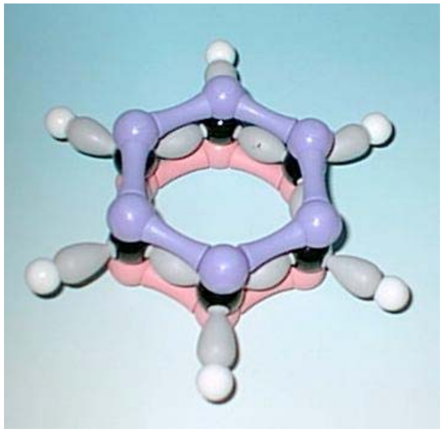

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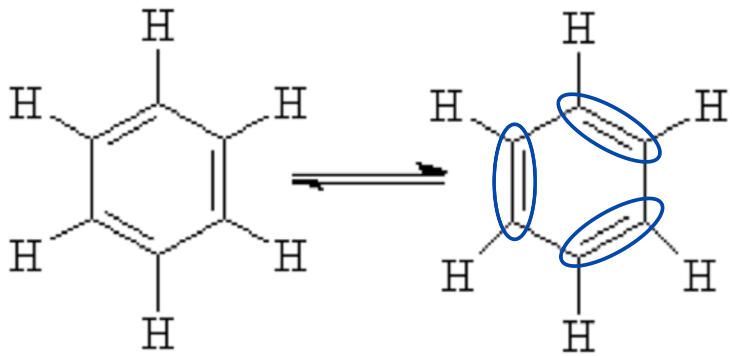

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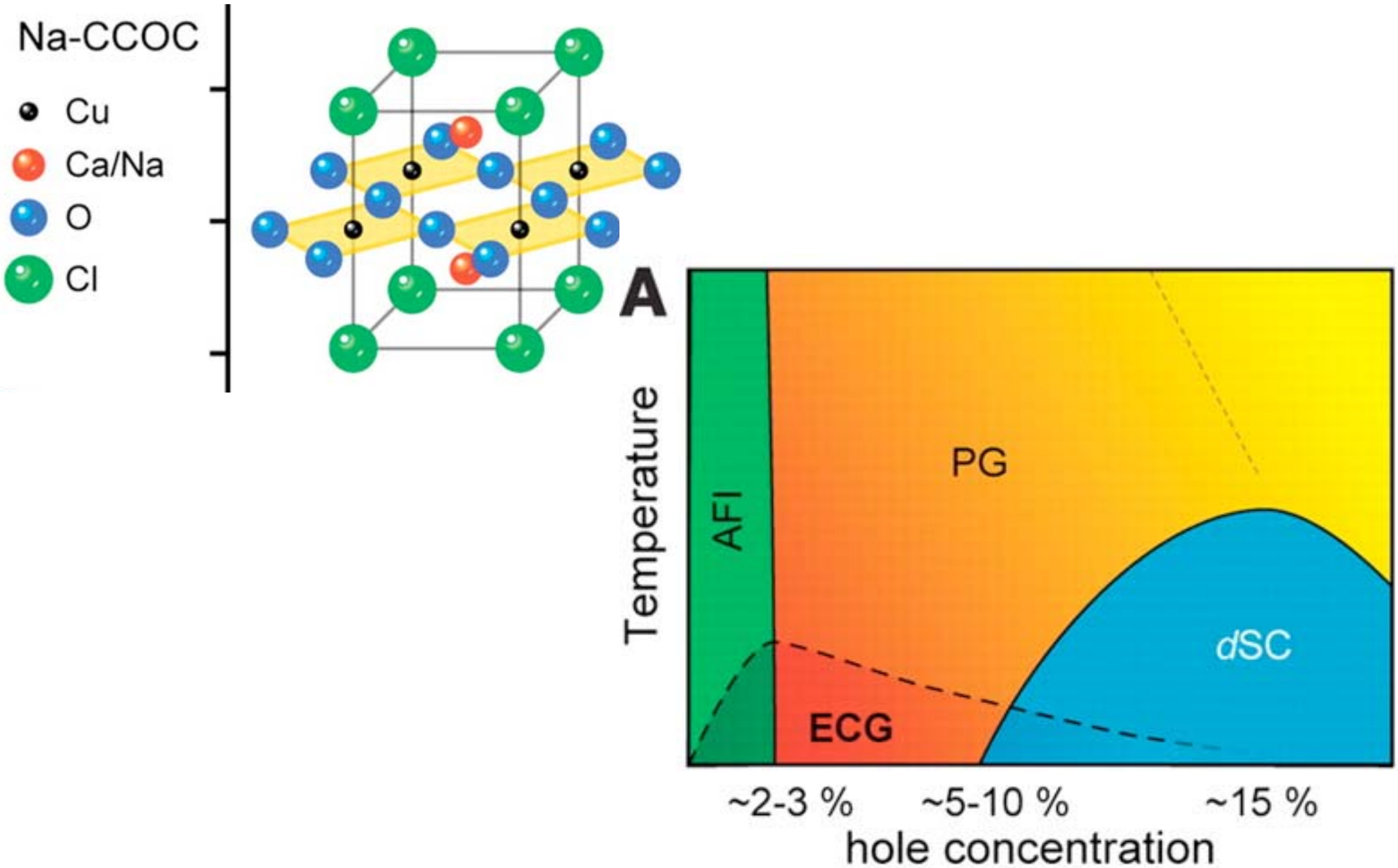
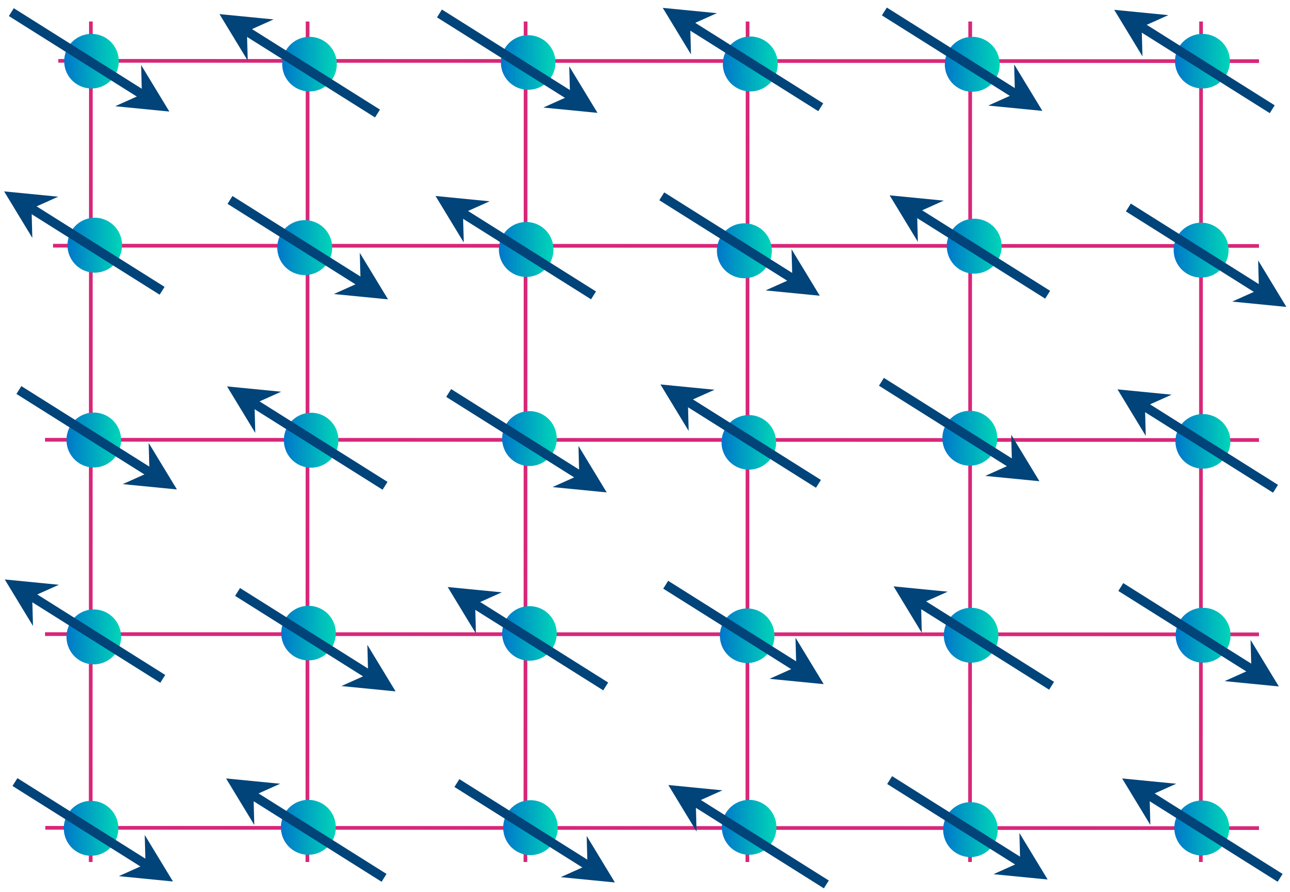
Temperature-doping phase diagram of the cuprate superconductors

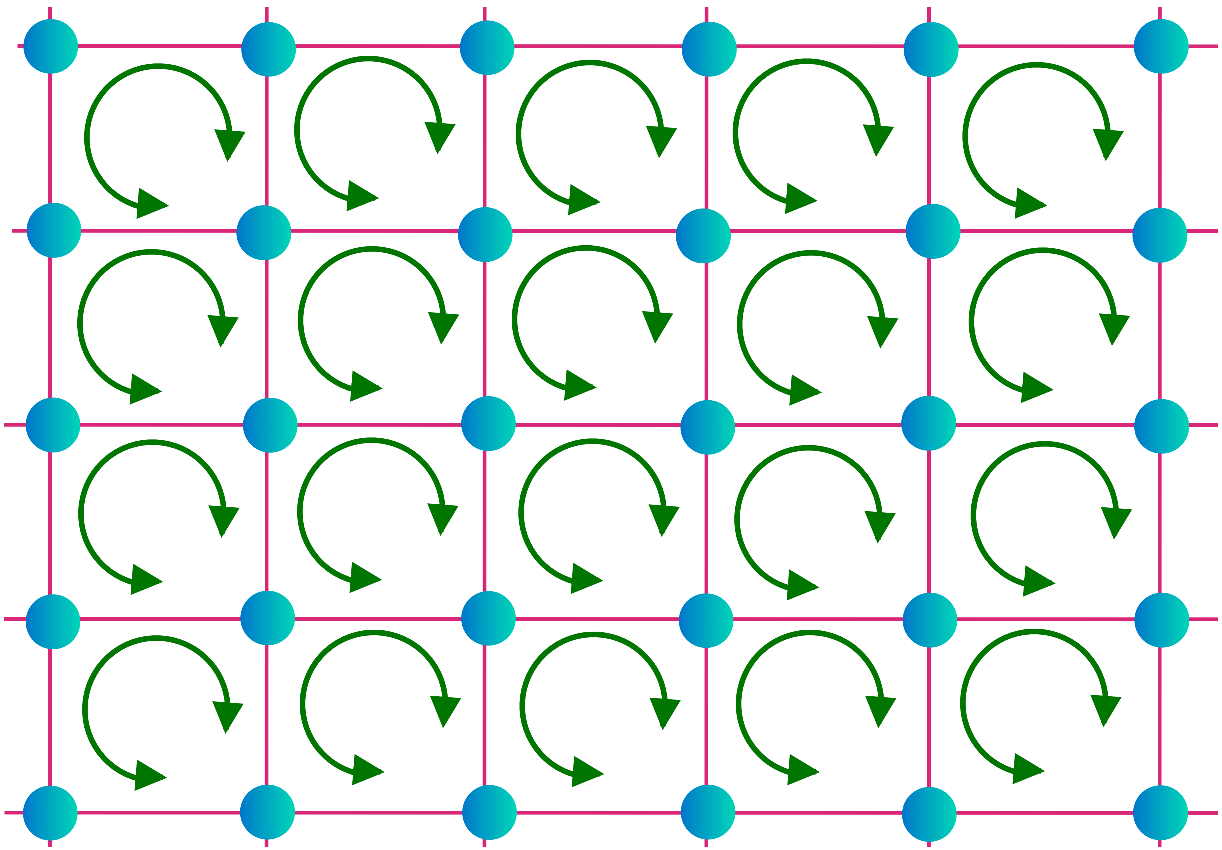


Antiferromagnetic (Neel) order in the insulator



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{ spin operator with } S = 1/2$$

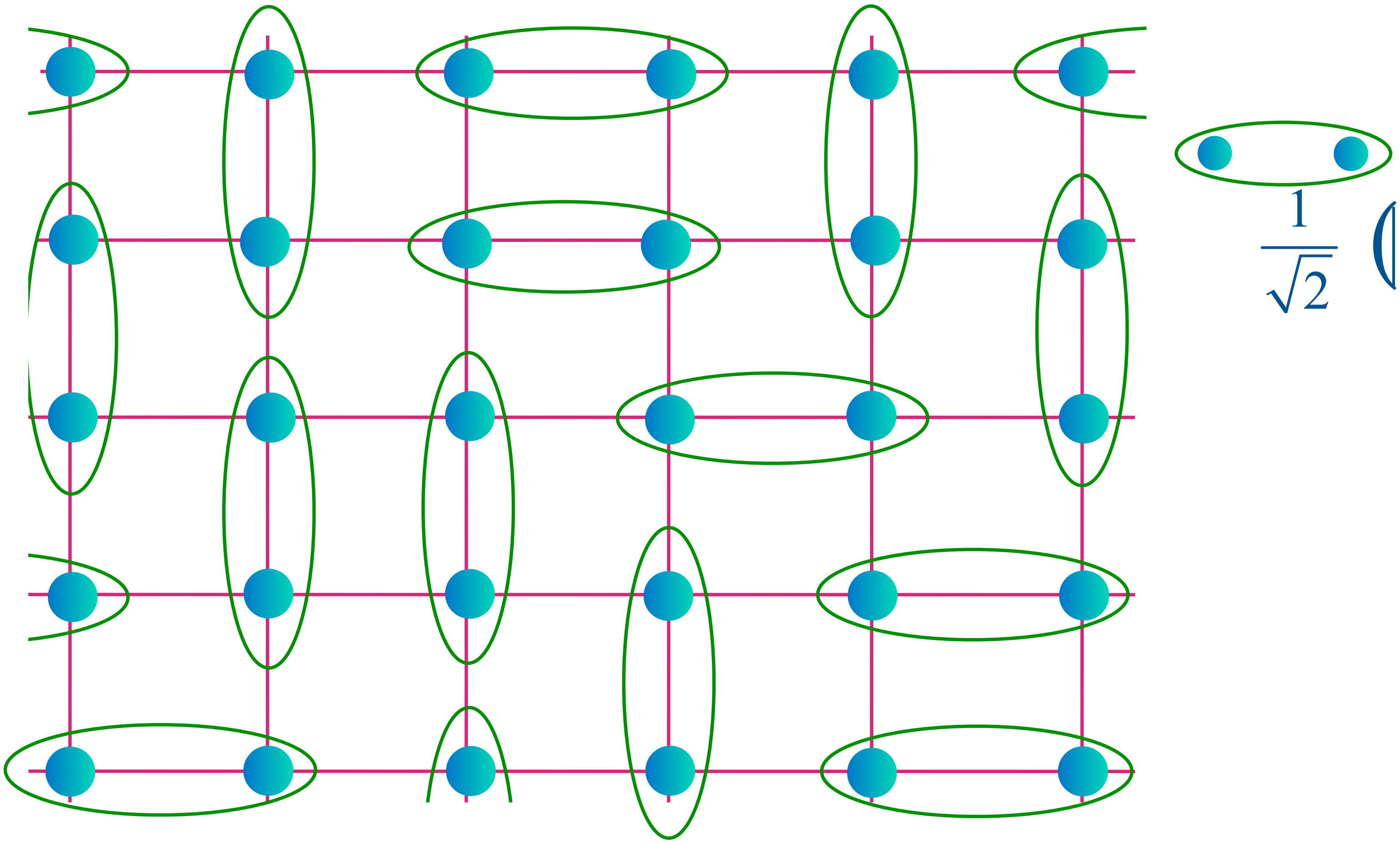
Induce formation of valence bonds by *e.g.* ring-exchange interactions



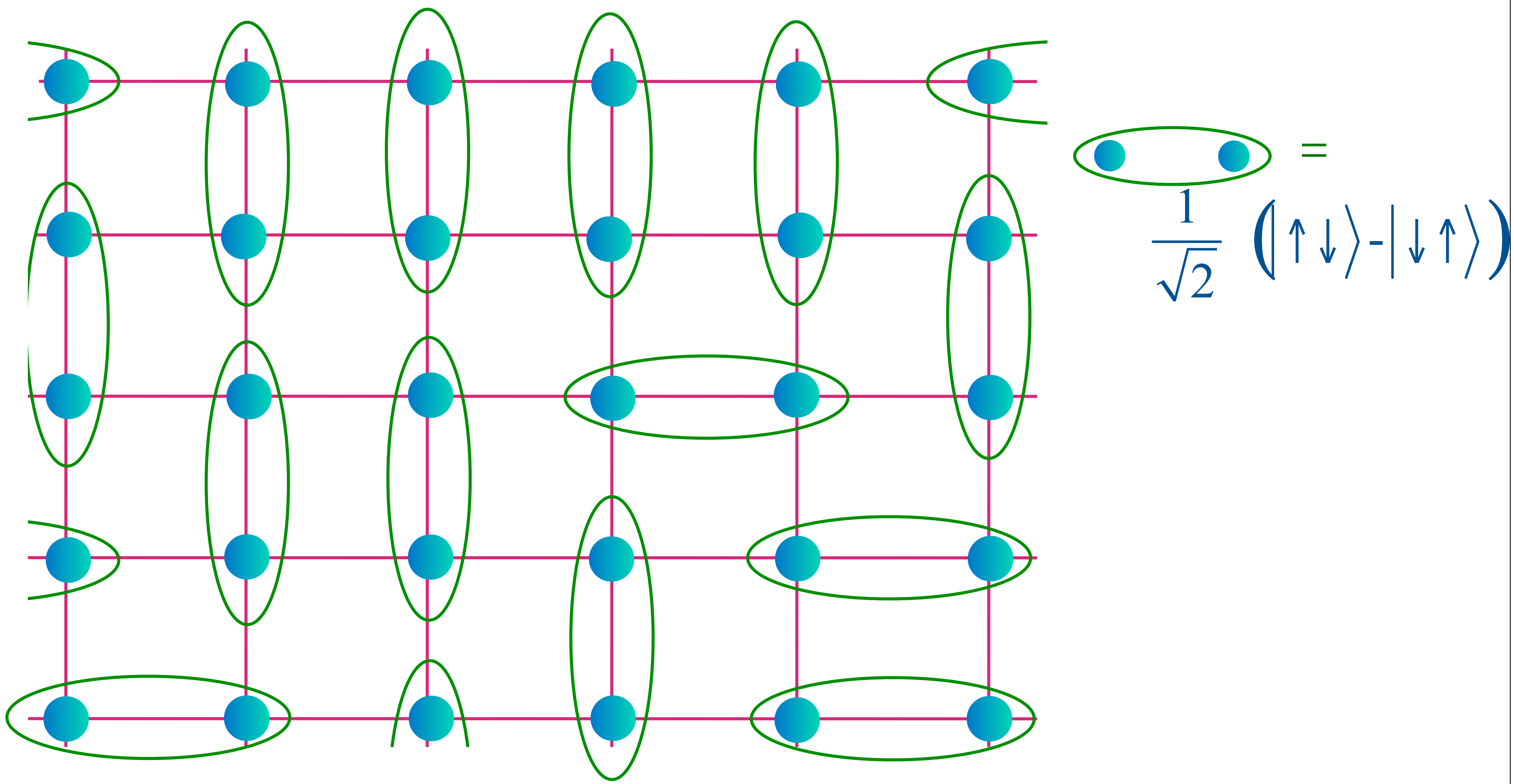
$$H = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j + K\sum_{\Box}\text{four spin exchange}$$

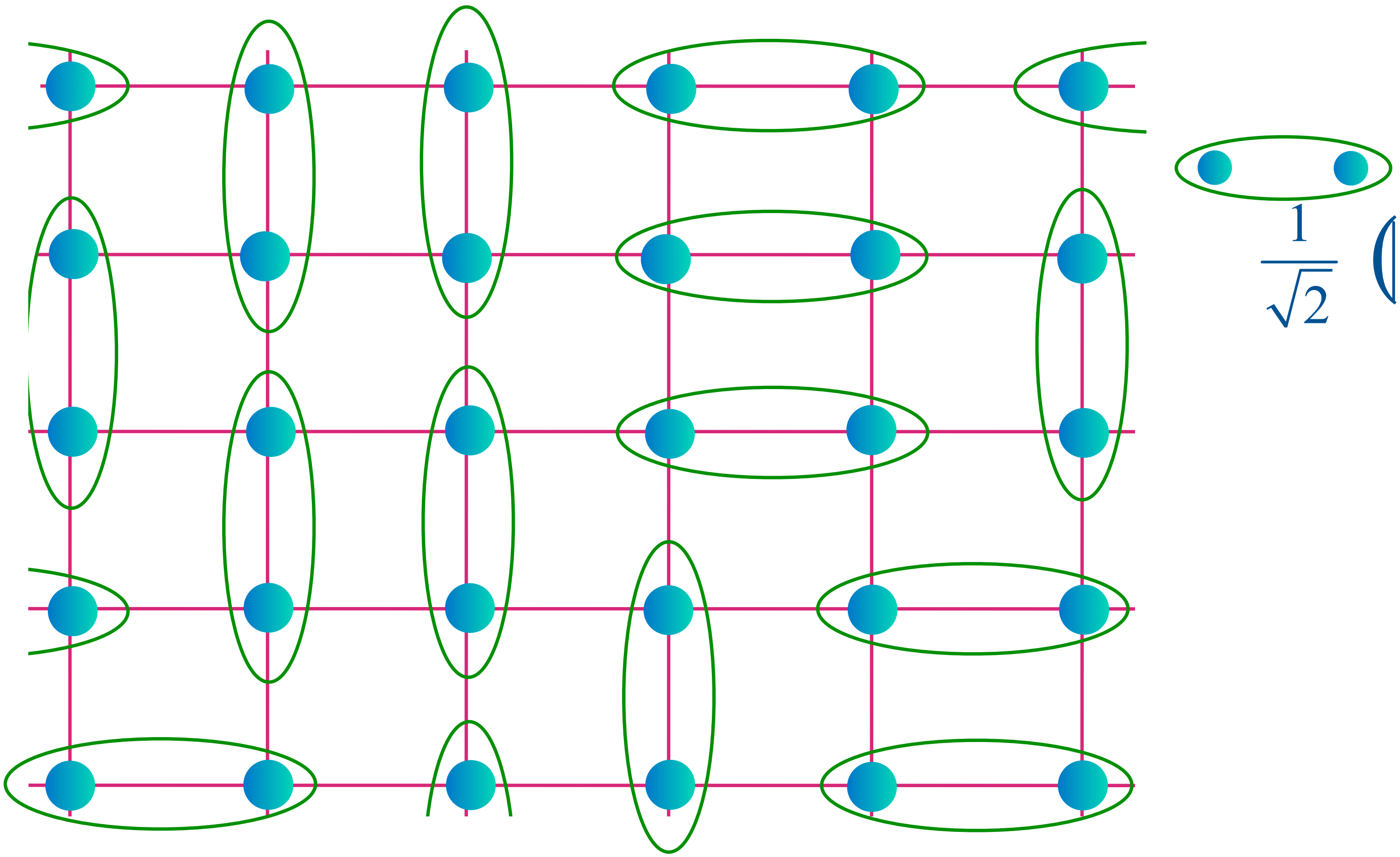
A. W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007)

As in H_2 and benzene, each electron wants to pair up with another electron and form a valence bond



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$


=
$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$



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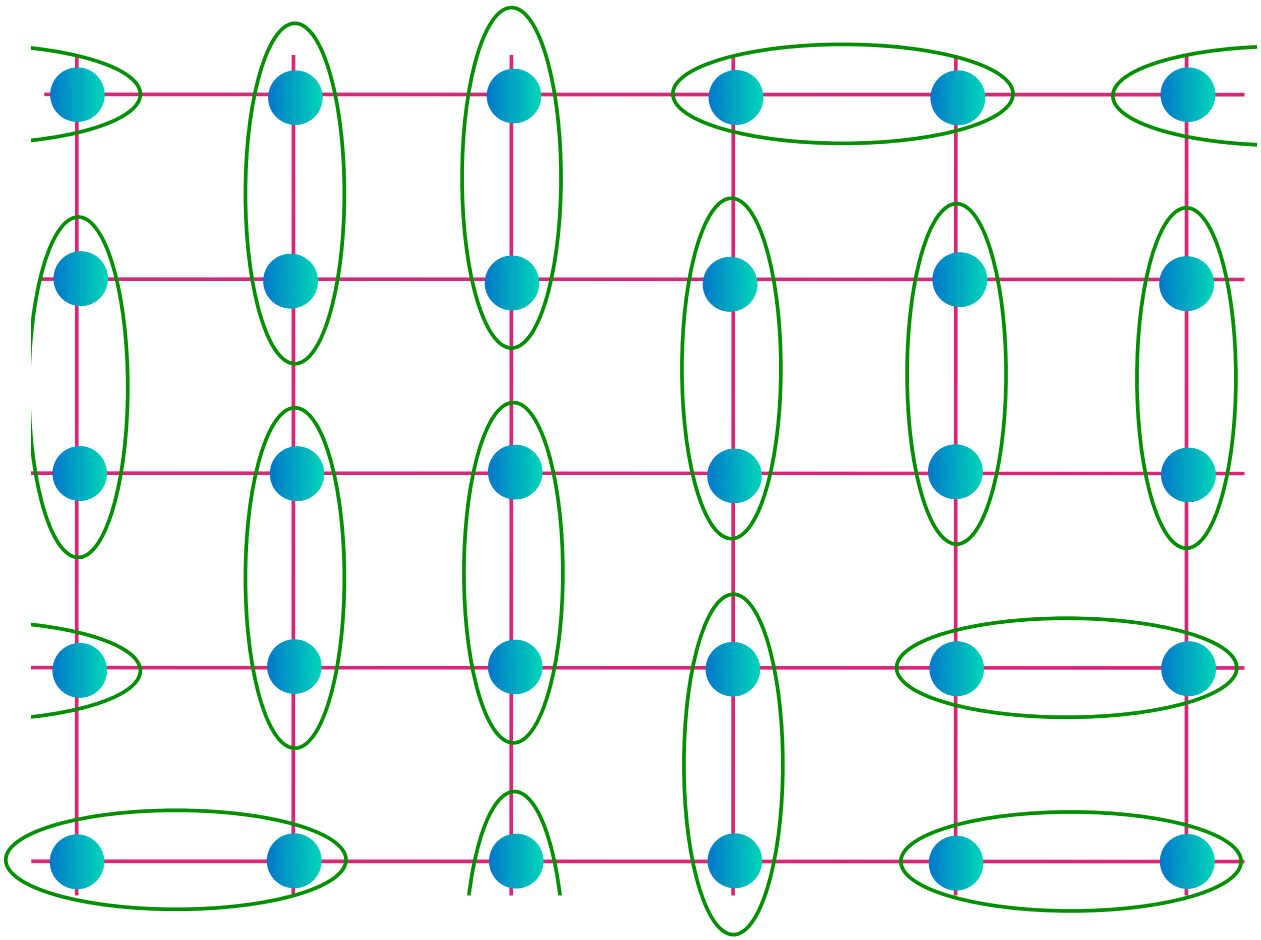

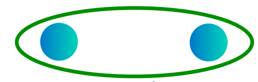

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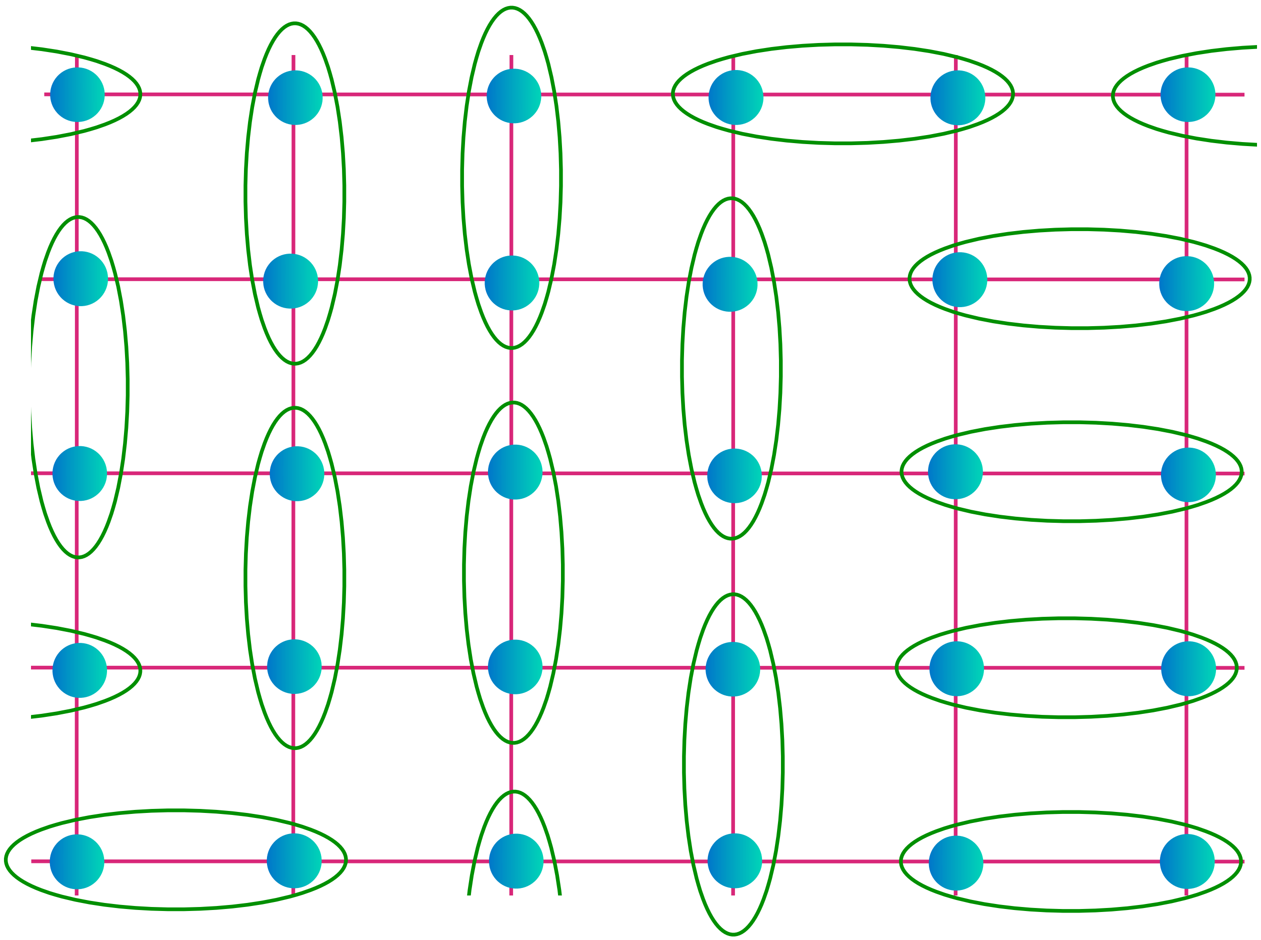
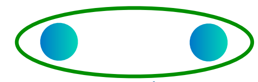
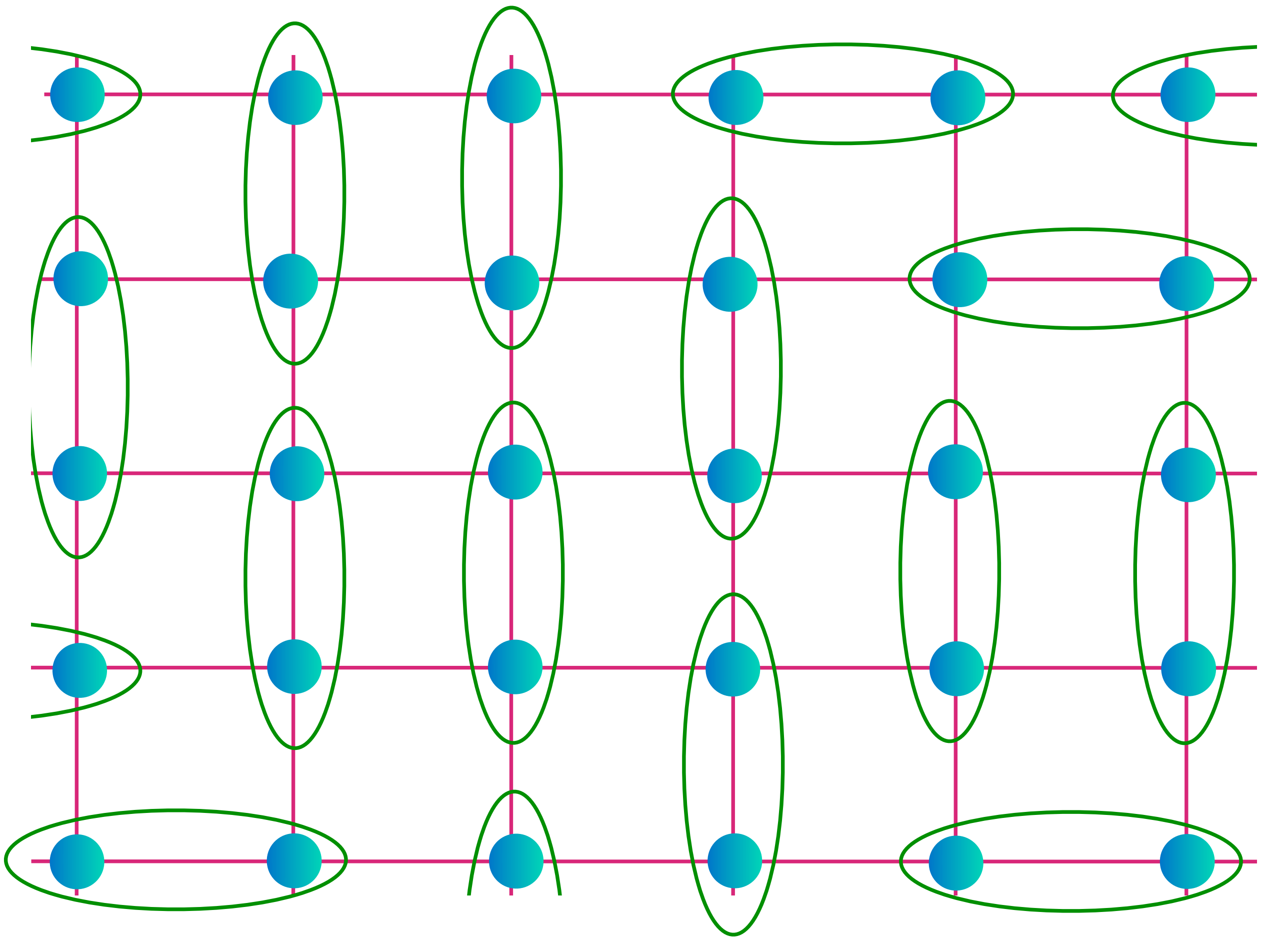
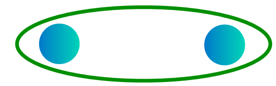




$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Entangled liquid of valence bonds (Resonating valence bonds – RVB)

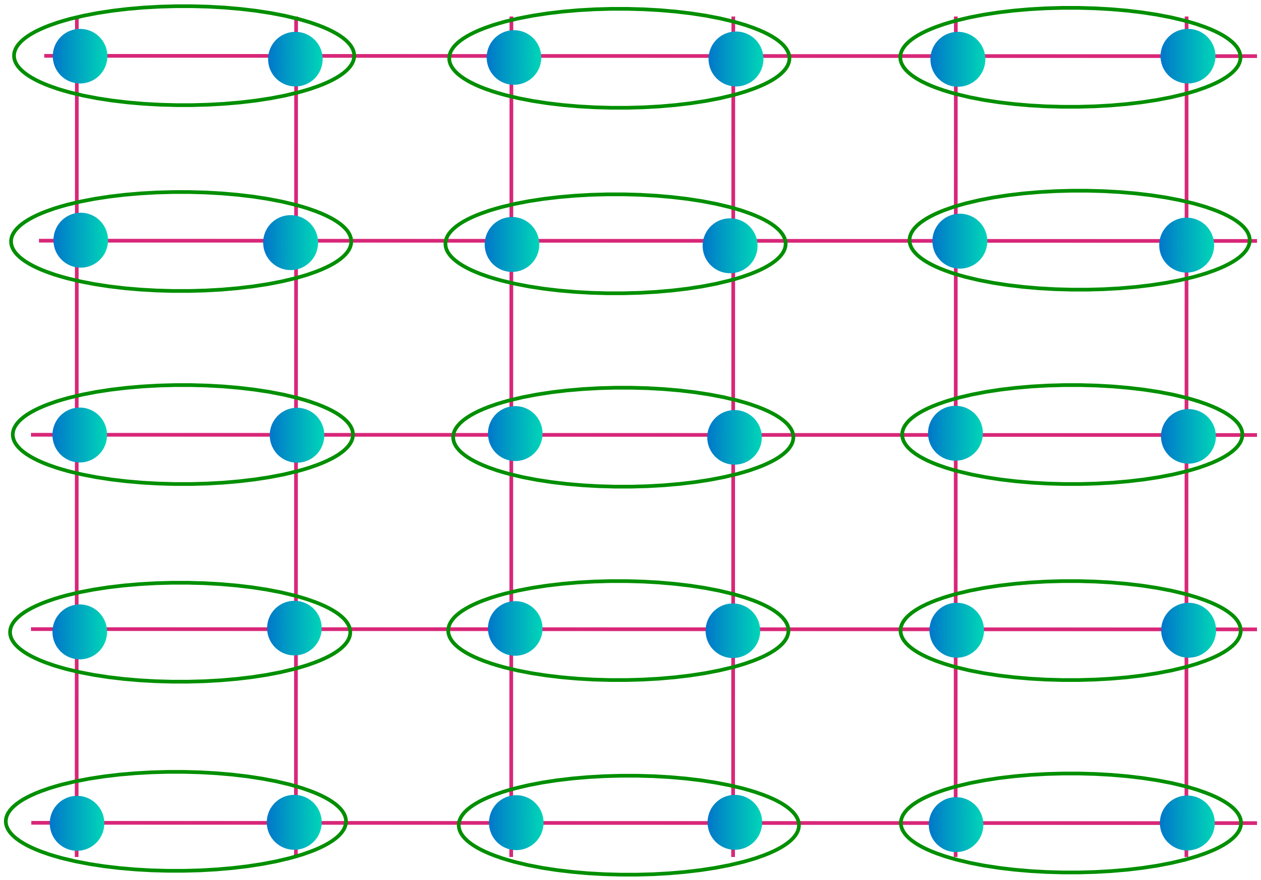


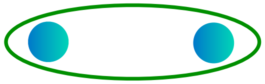
 =

$$\frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$$

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

Valence bond solid (VBS)





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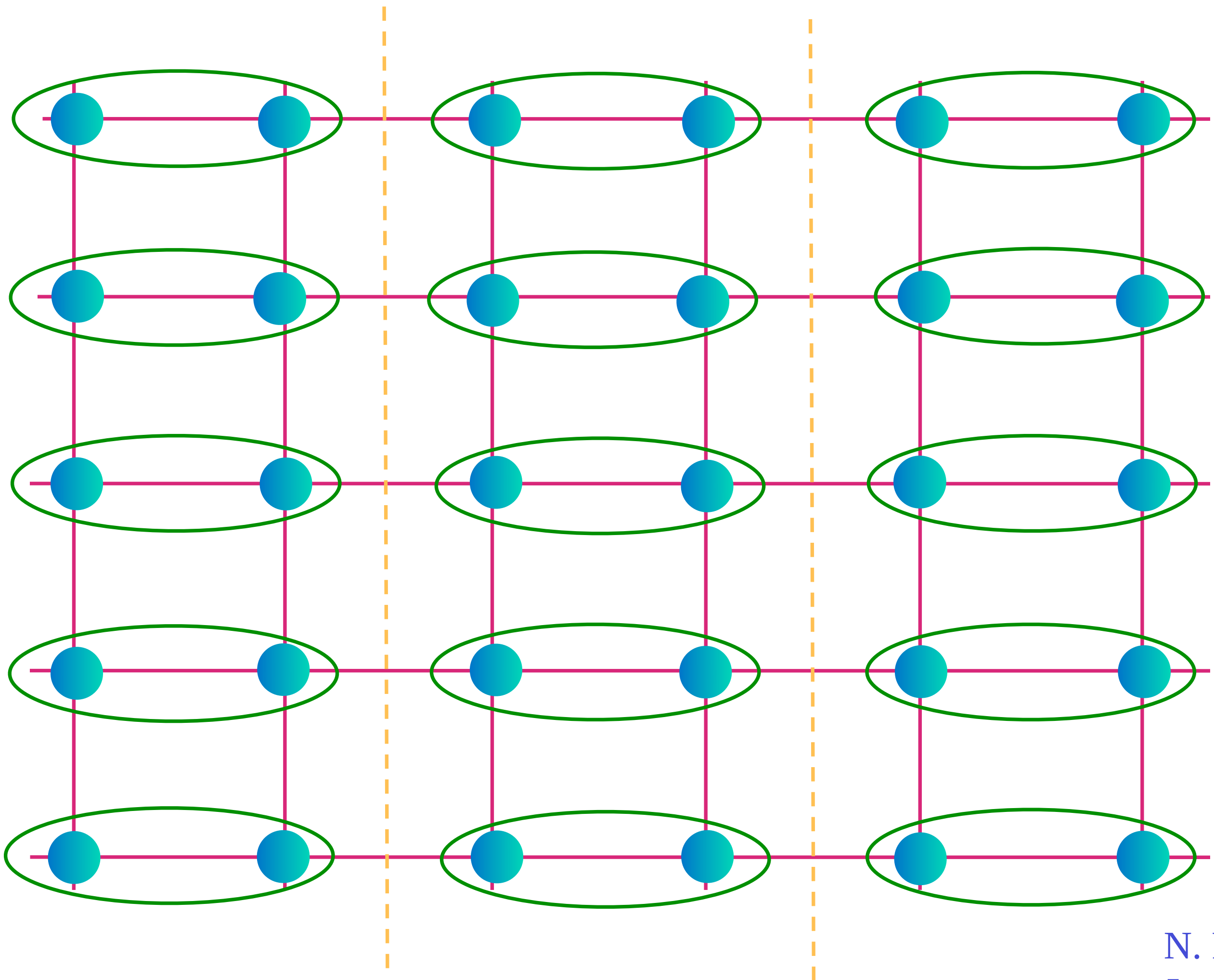

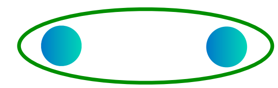 =

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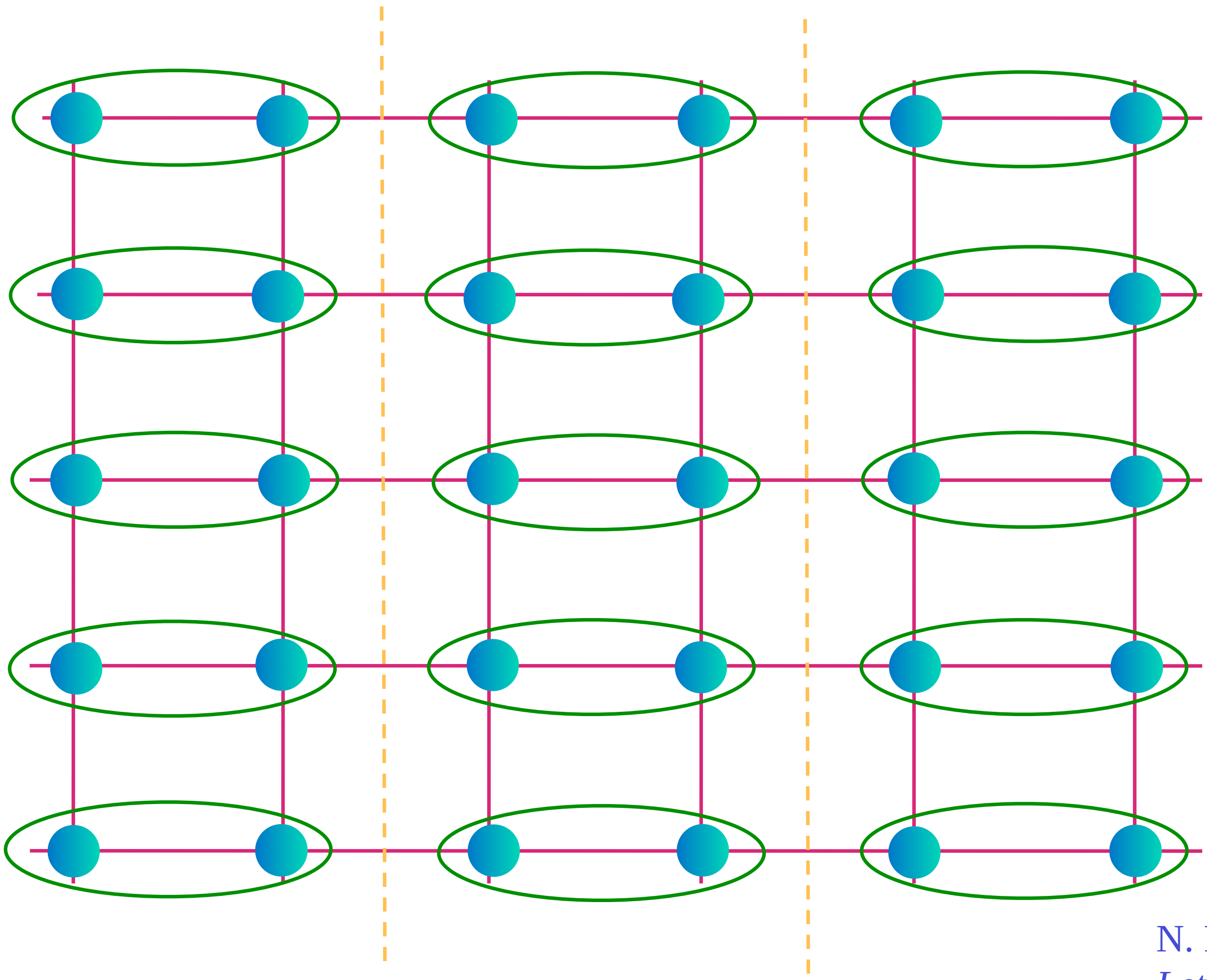
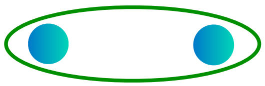
More possibilities for entanglement with nearby states





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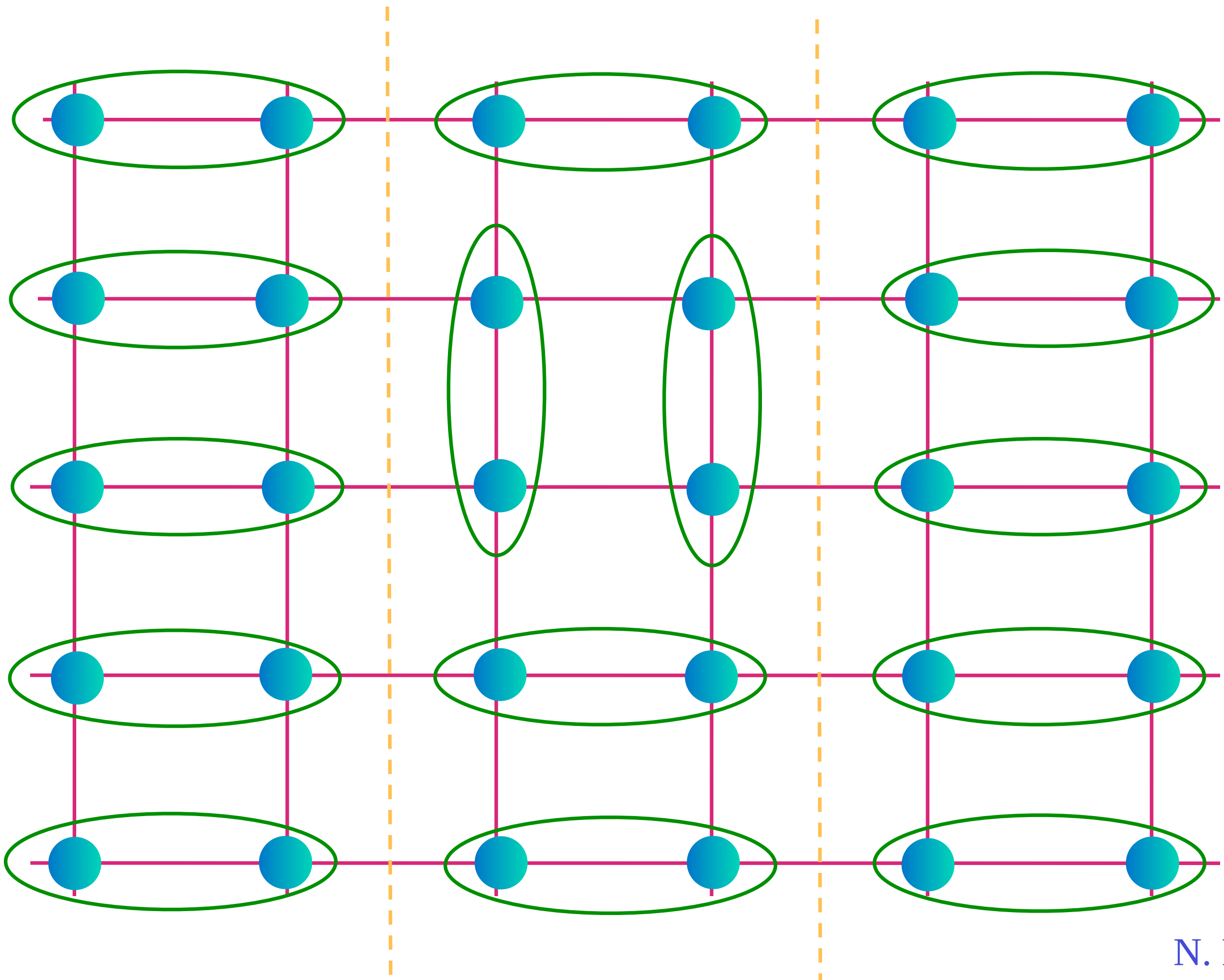

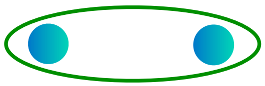

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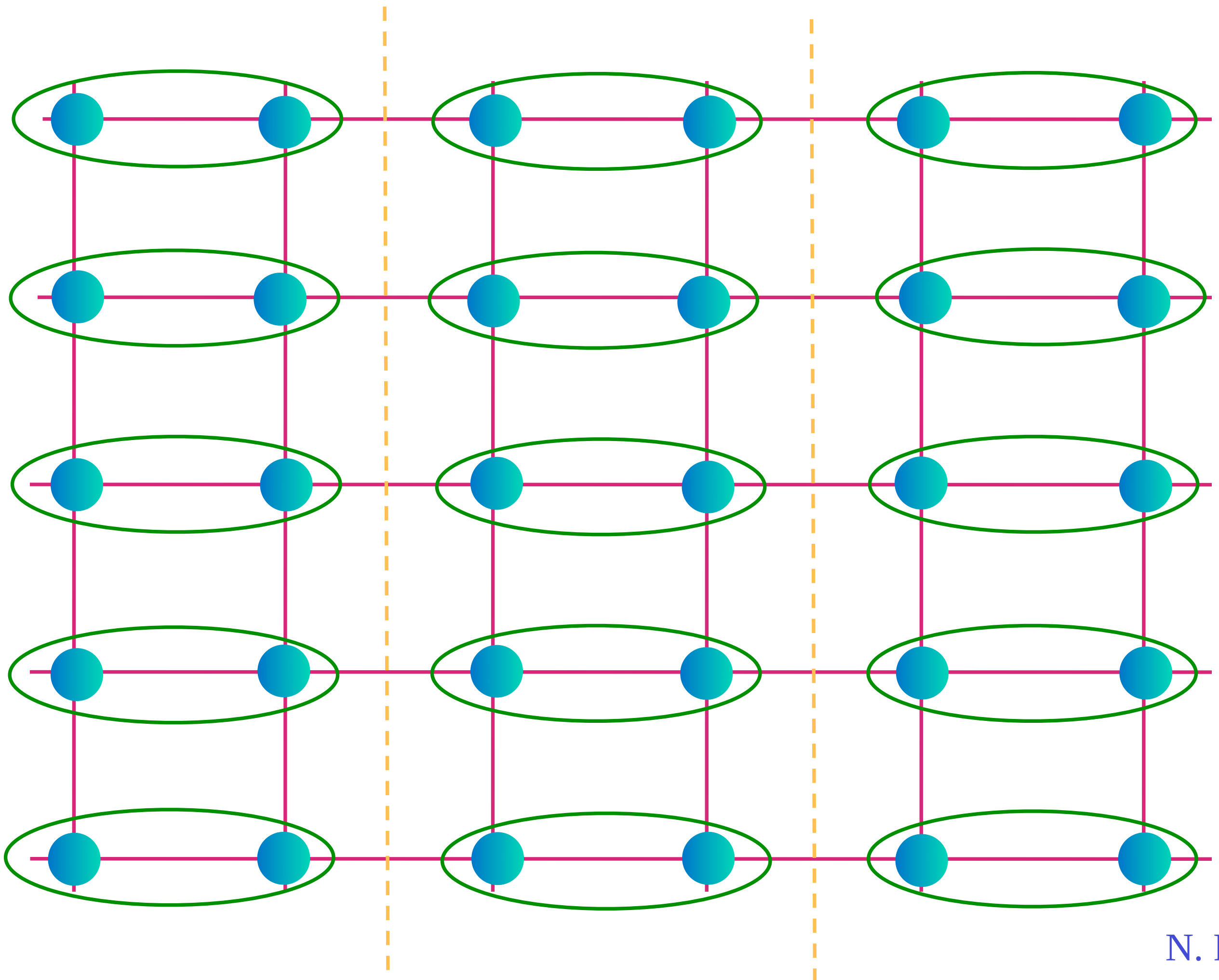

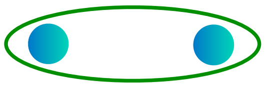

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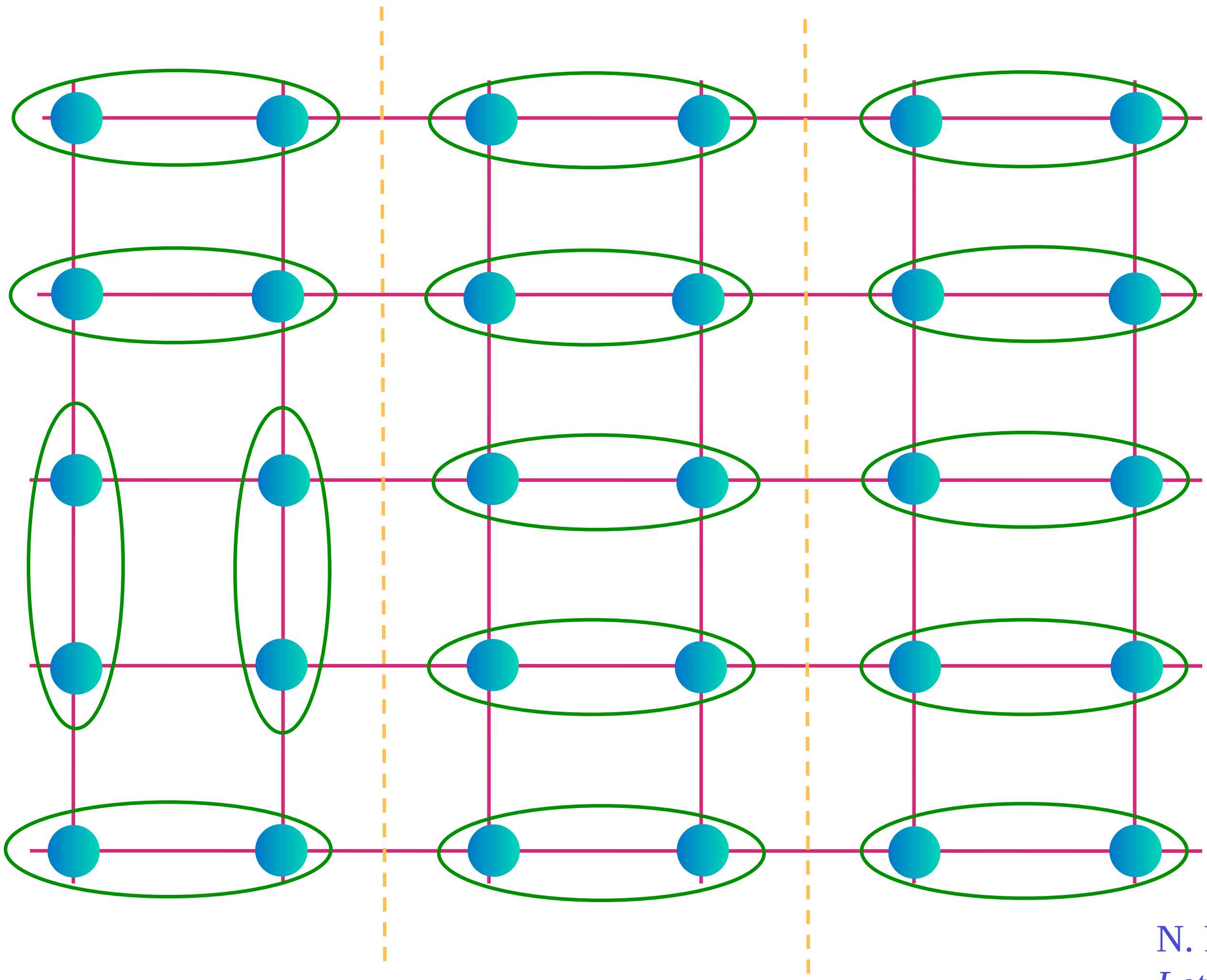

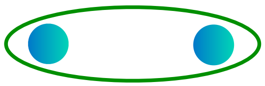

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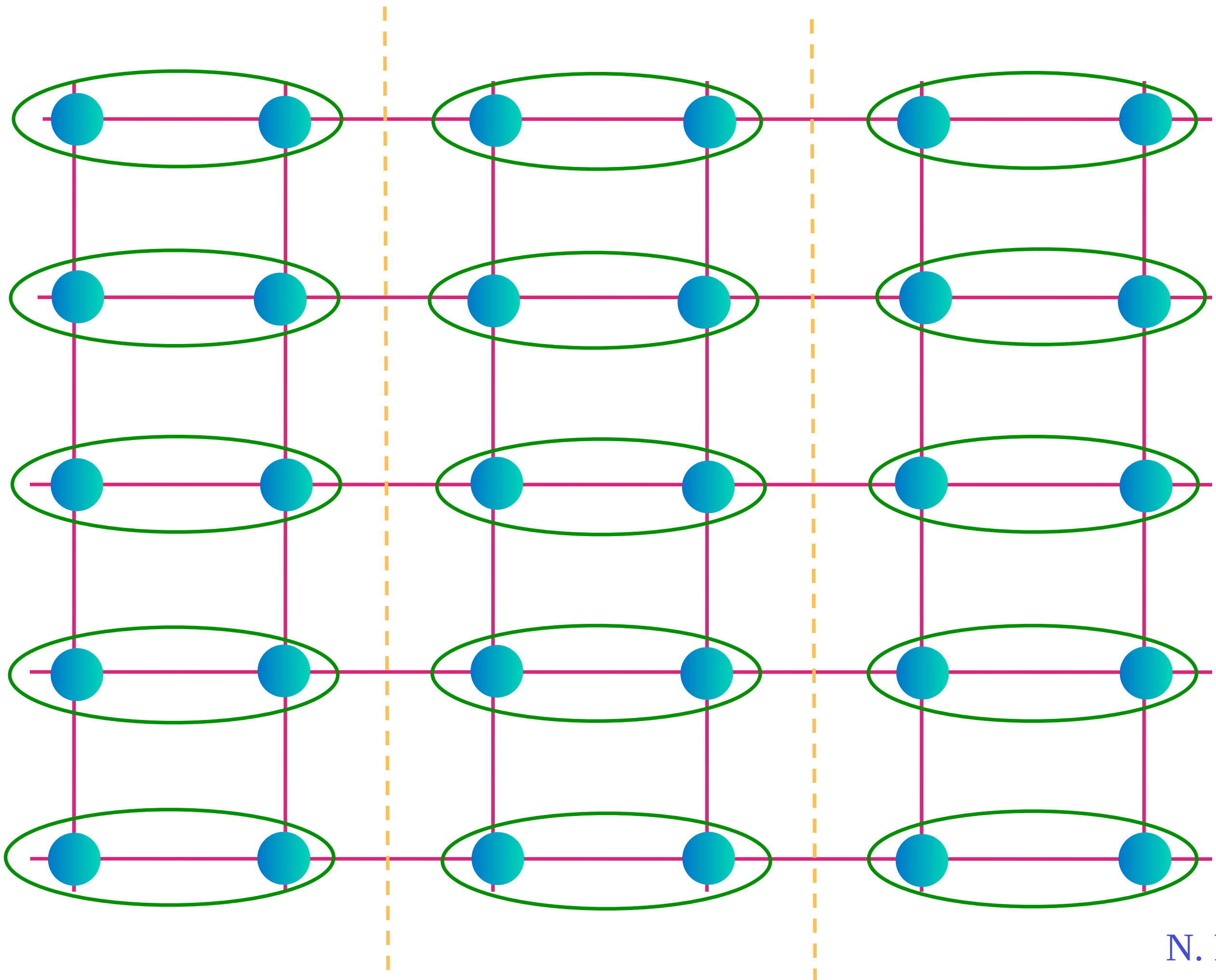

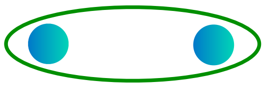

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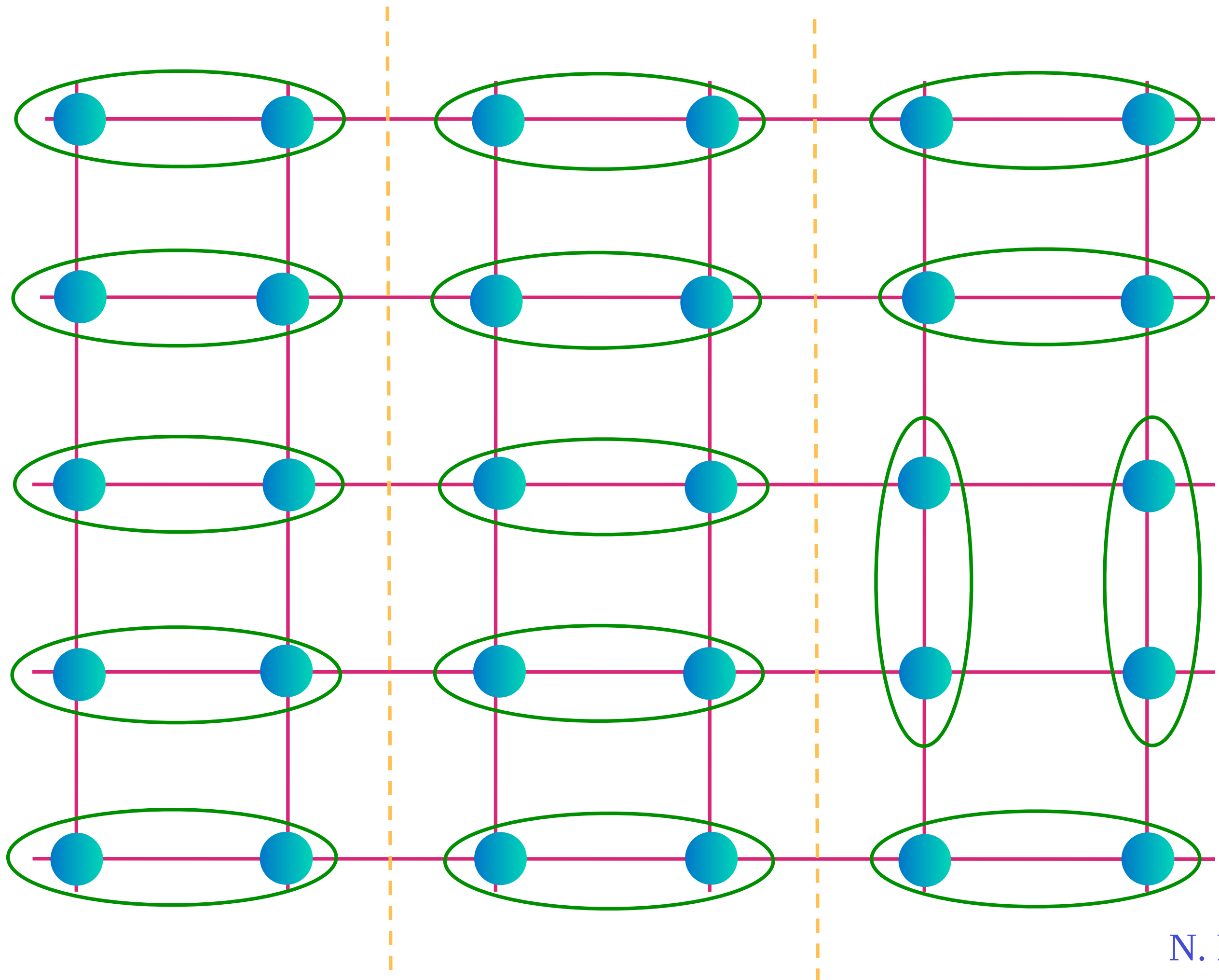

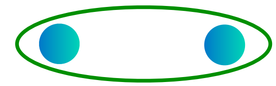 =

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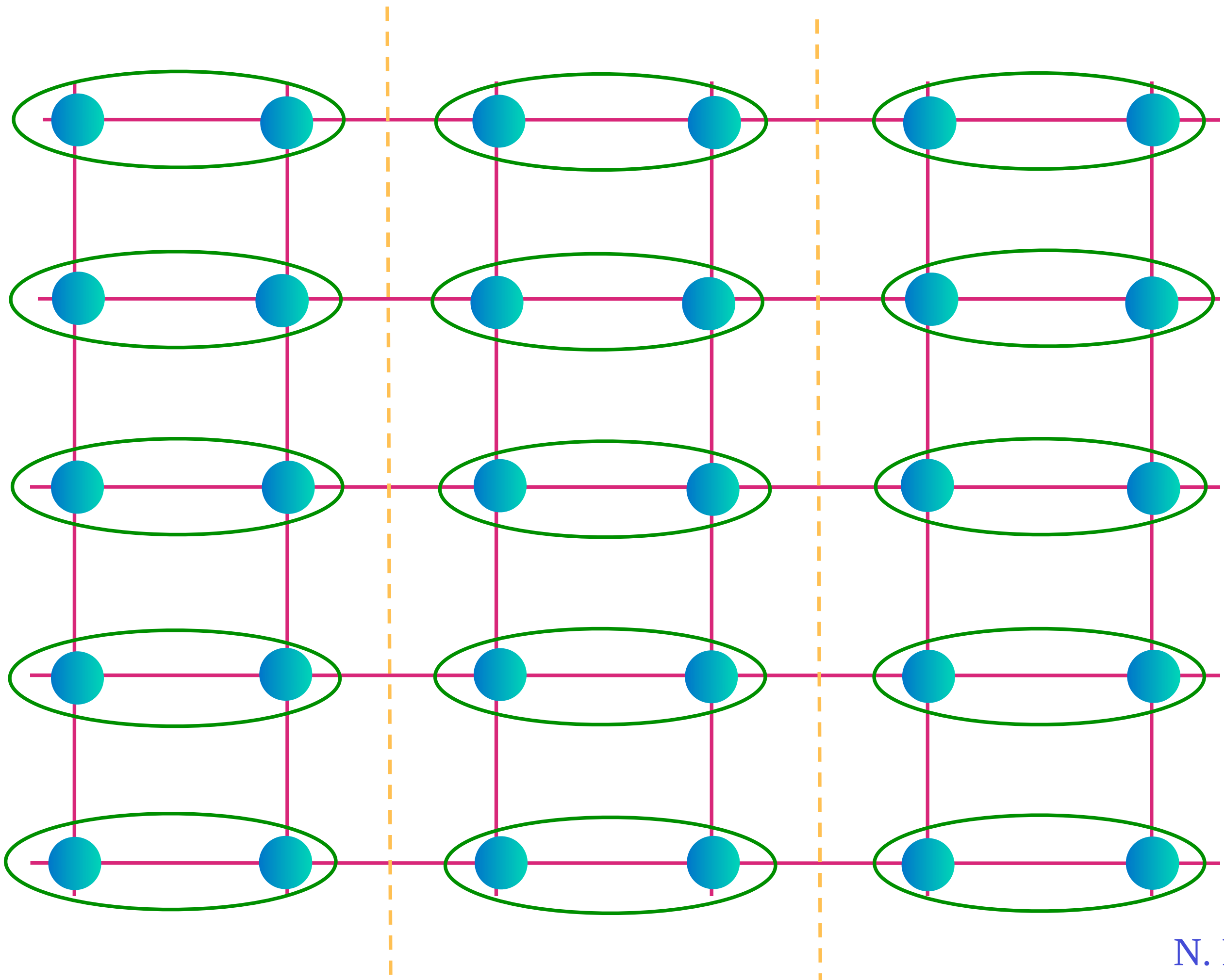

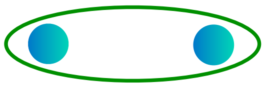

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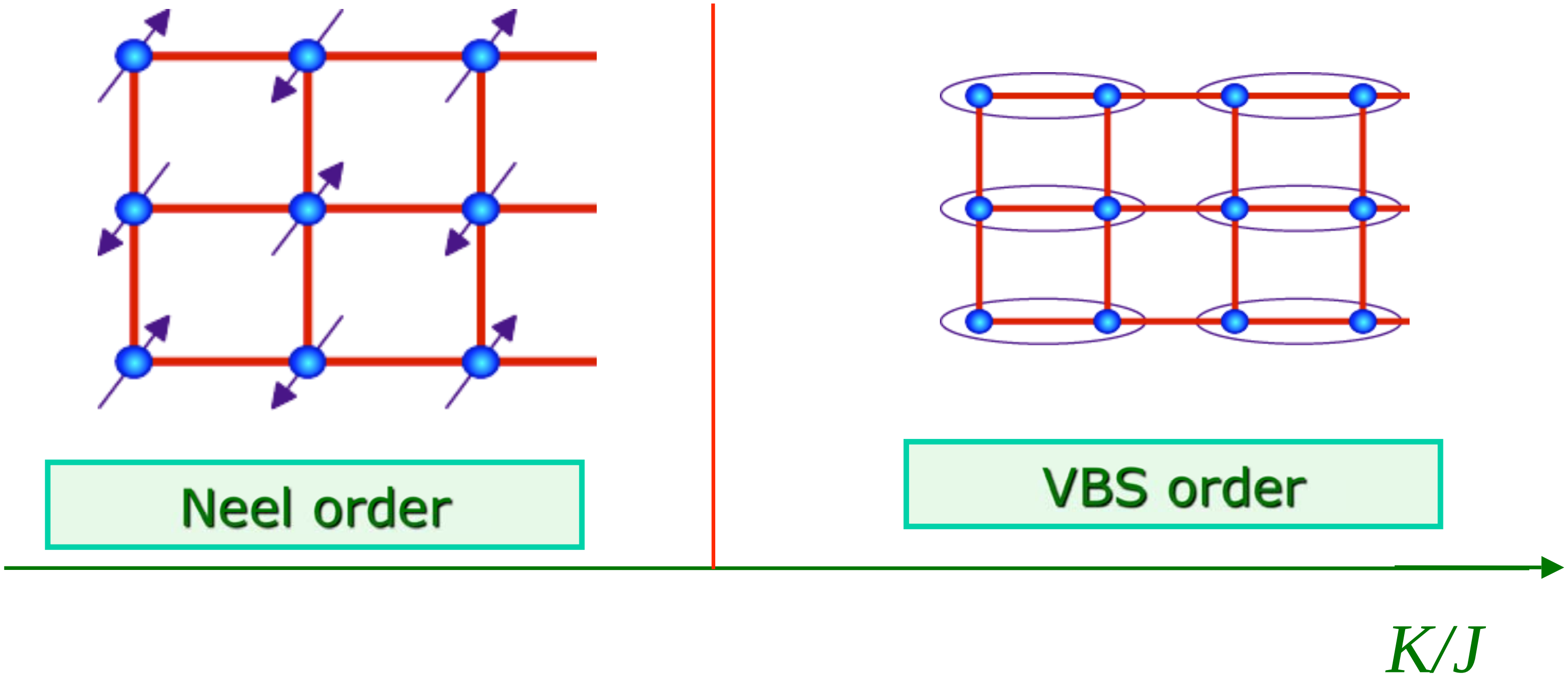
Phase diagram of square lattice antiferromagnet



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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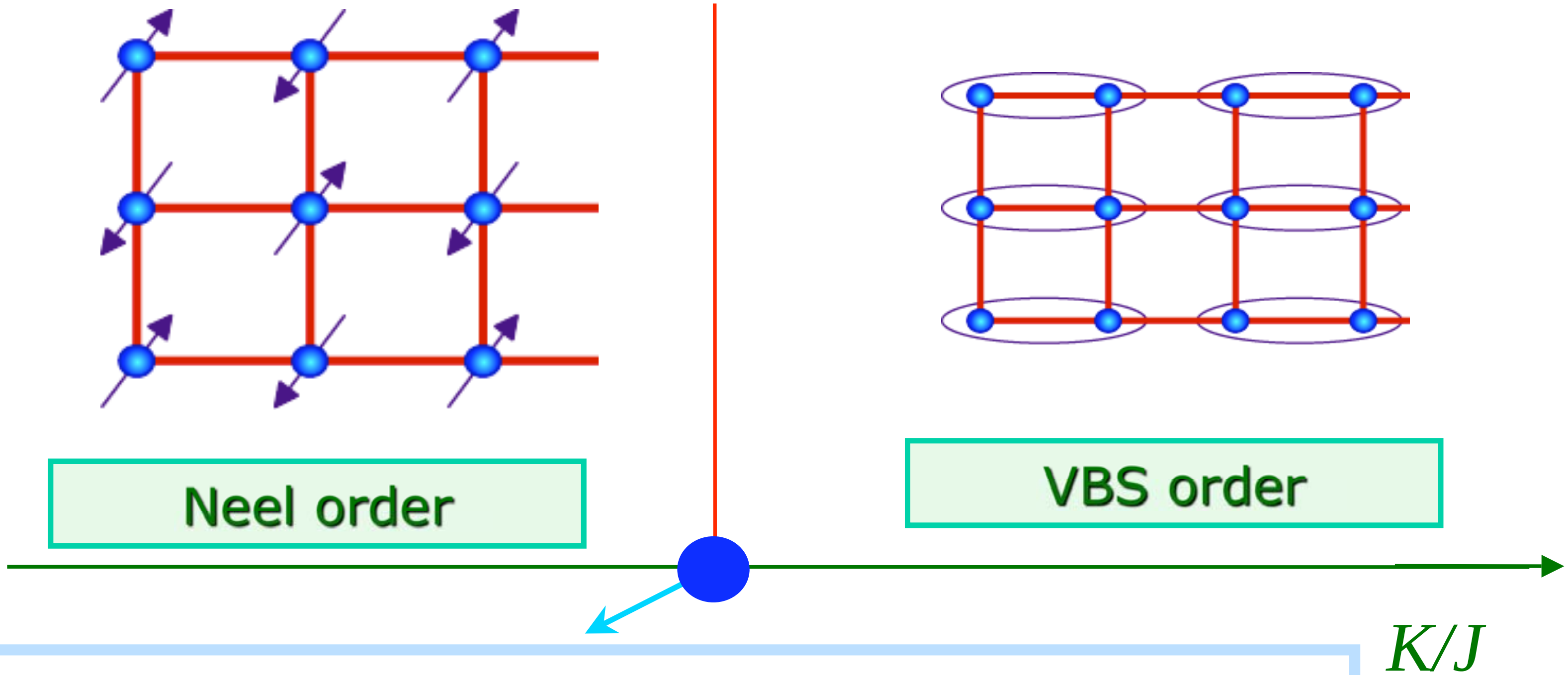

RVB physics appears at the quantum critical point which has fractionalized excitations: "*deconfined criticality*"

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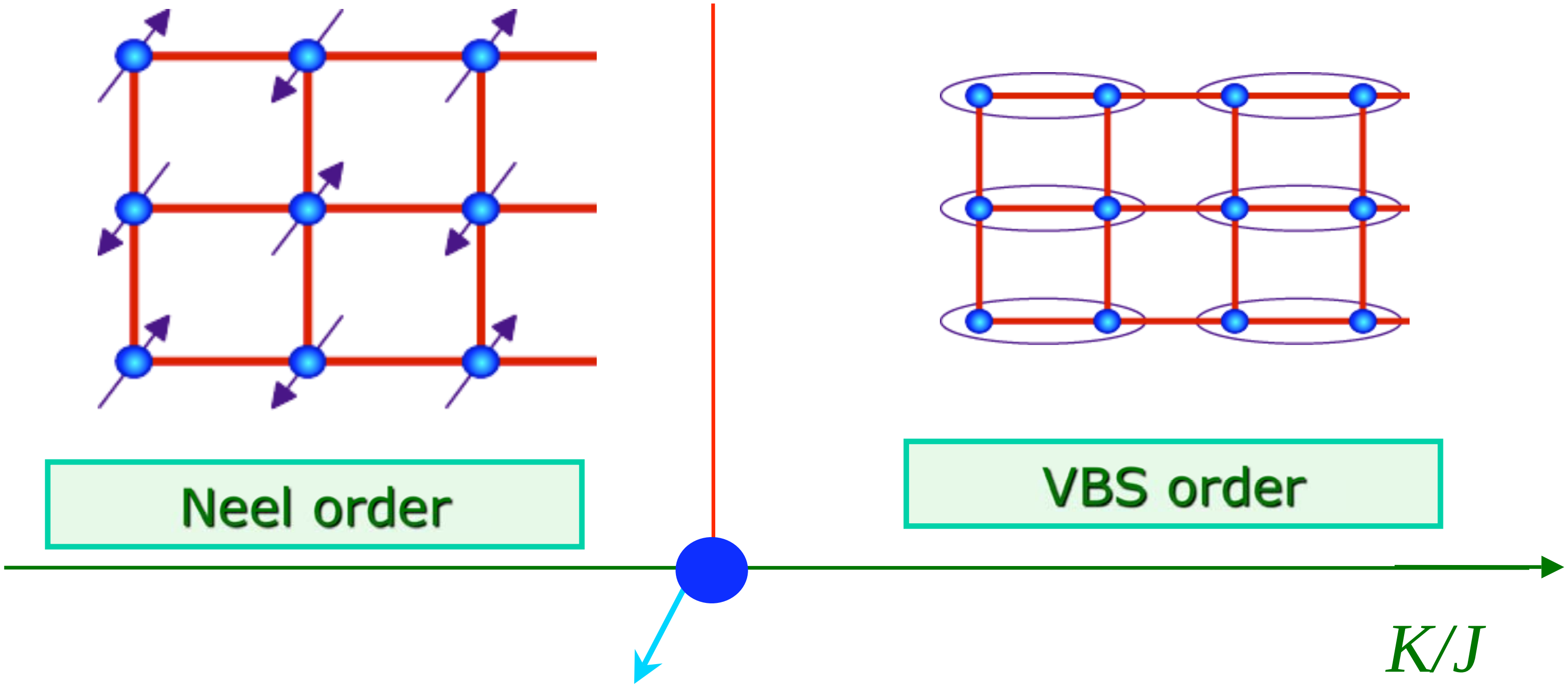

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + r|z_\alpha|^2 + \frac{u}{2}\left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

at its critical point $r = r_c$, where z_α are the neutral $S = 1/2$ spinons and A_μ is a *non-compact* U(1) gauge field.

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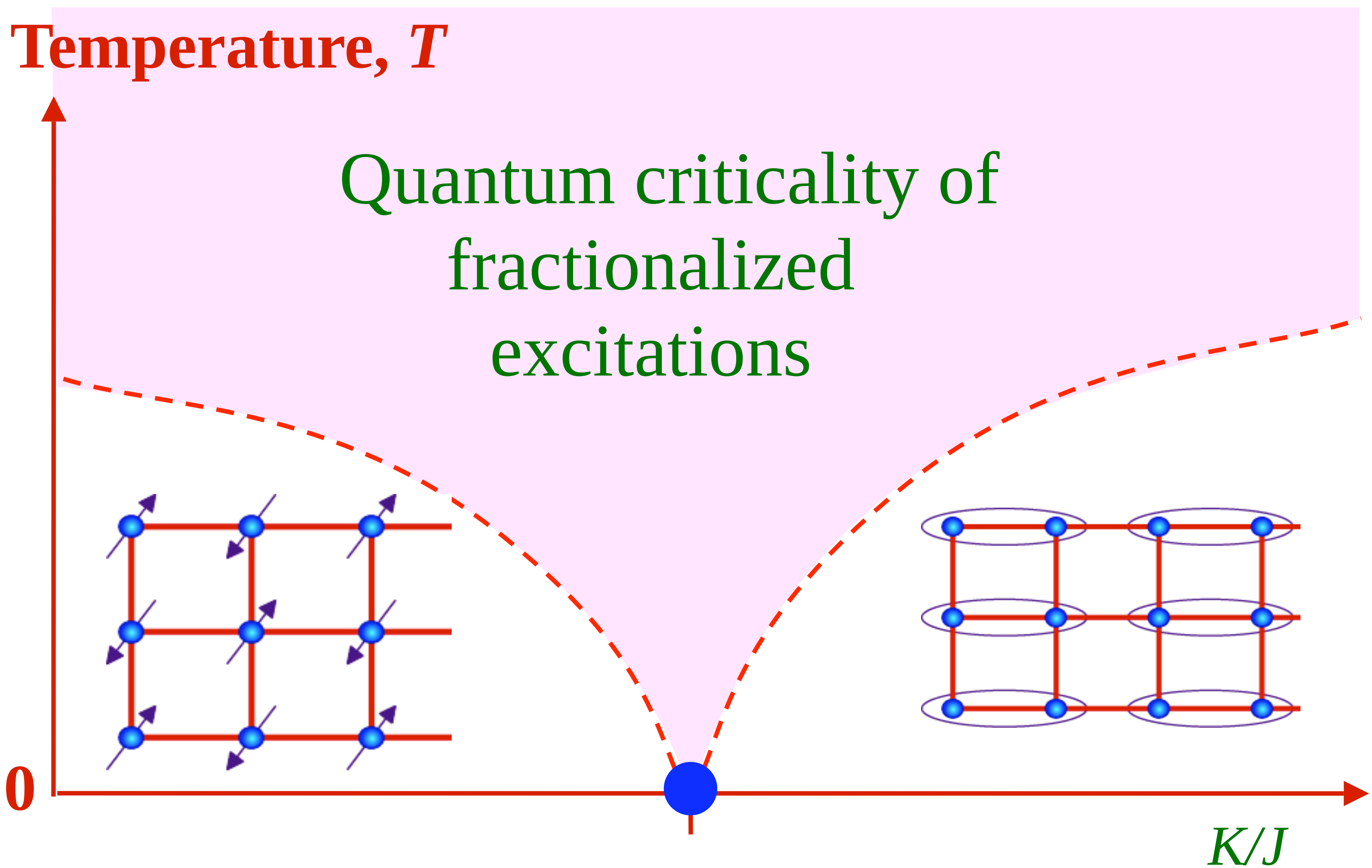
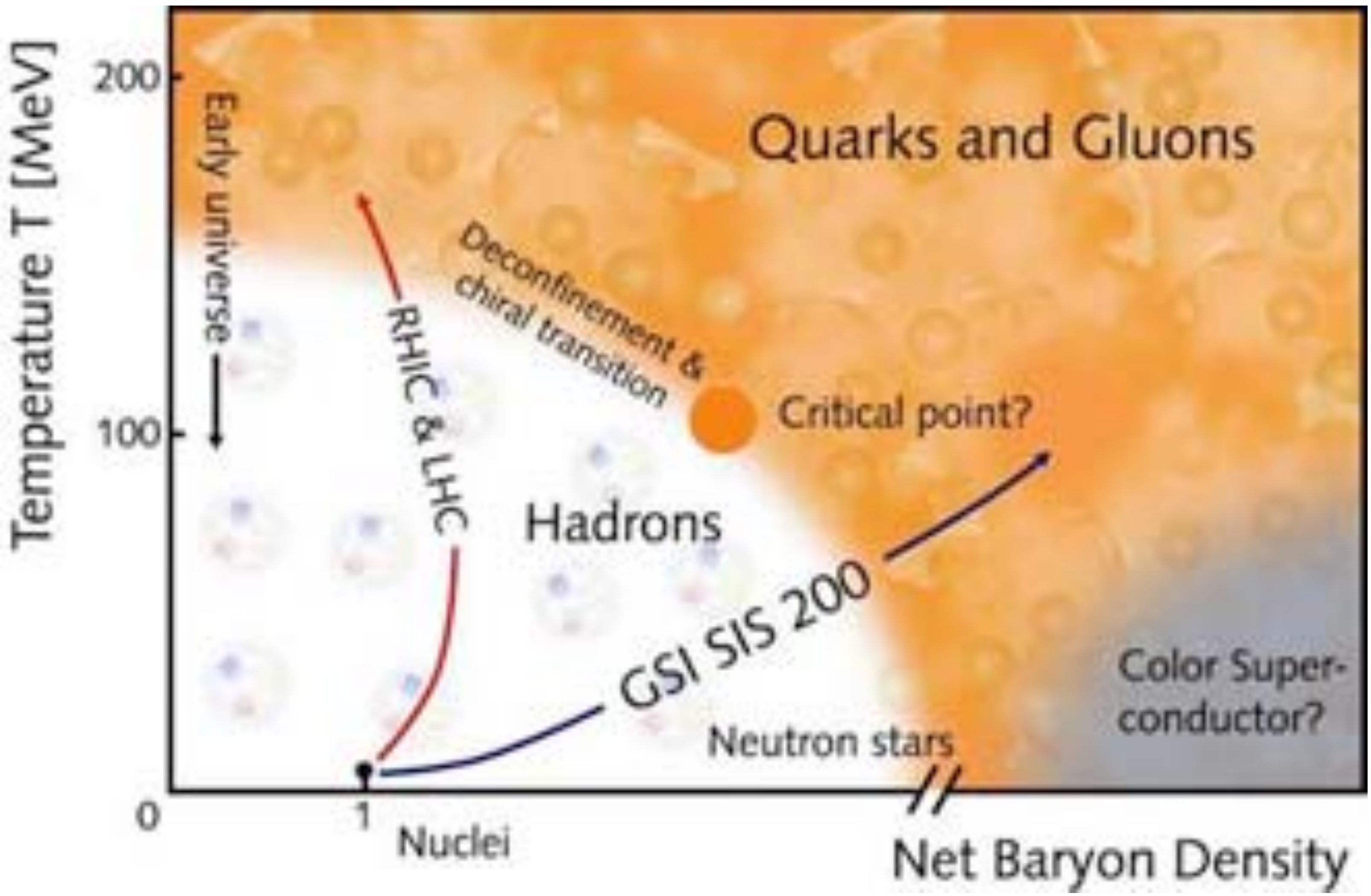

Temperature, *T*
Quantum criticality of
fractionalized
excitations
0
K/J

Phases of nuclear matter



Outline

1. Superfluid-insulator quantum transitions

    Experiments on ultracold atoms

2. Theory of quantum-critical transport

    Collisionless-t0-hydrodynamic crossover of conformal field theories

3. Entanglement of valence bonds

    Deconfined criticality in antiferromagnets

4. Nernst effect in the cuprate superconductors

    Quantum criticality and dyonic black holes

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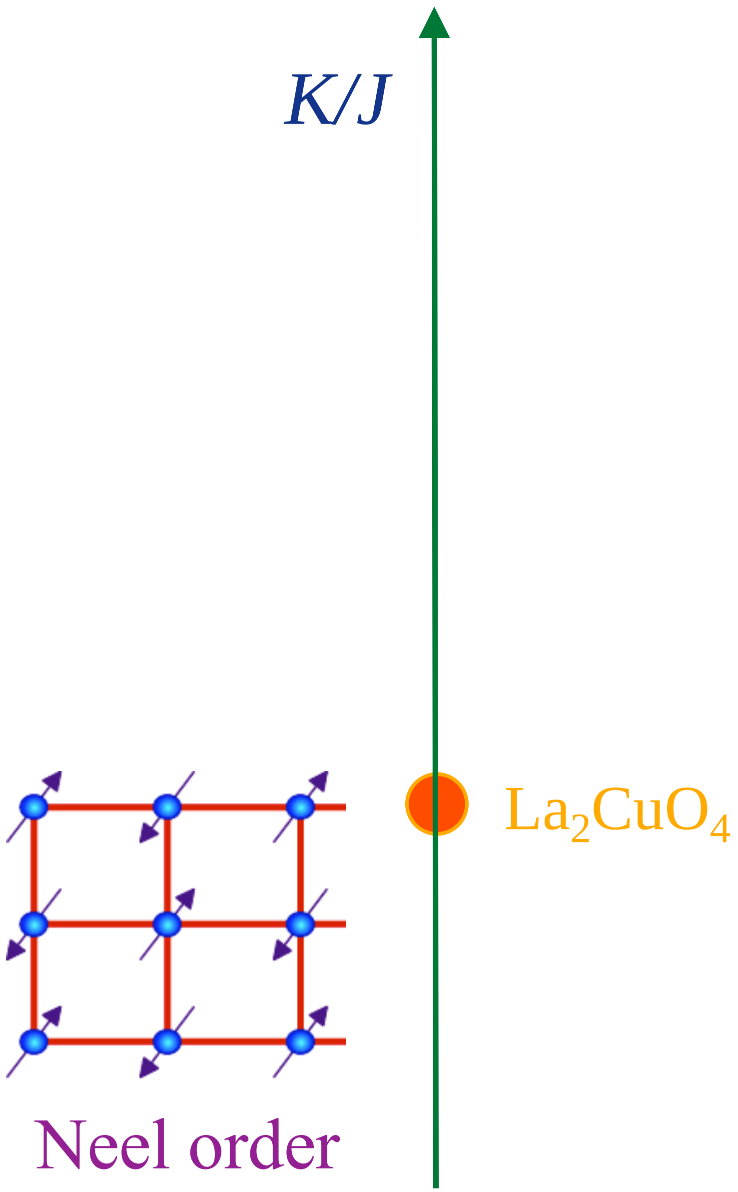
Phase diagram of doped antiferromagnets



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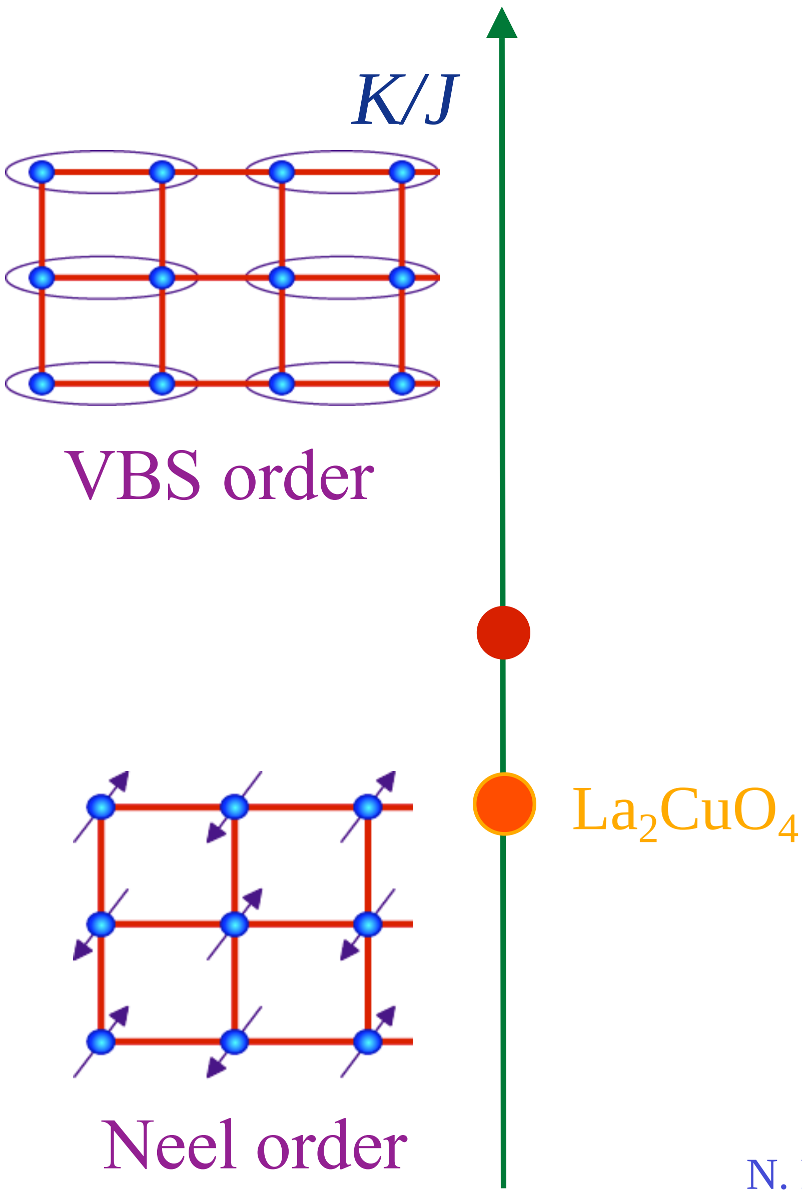

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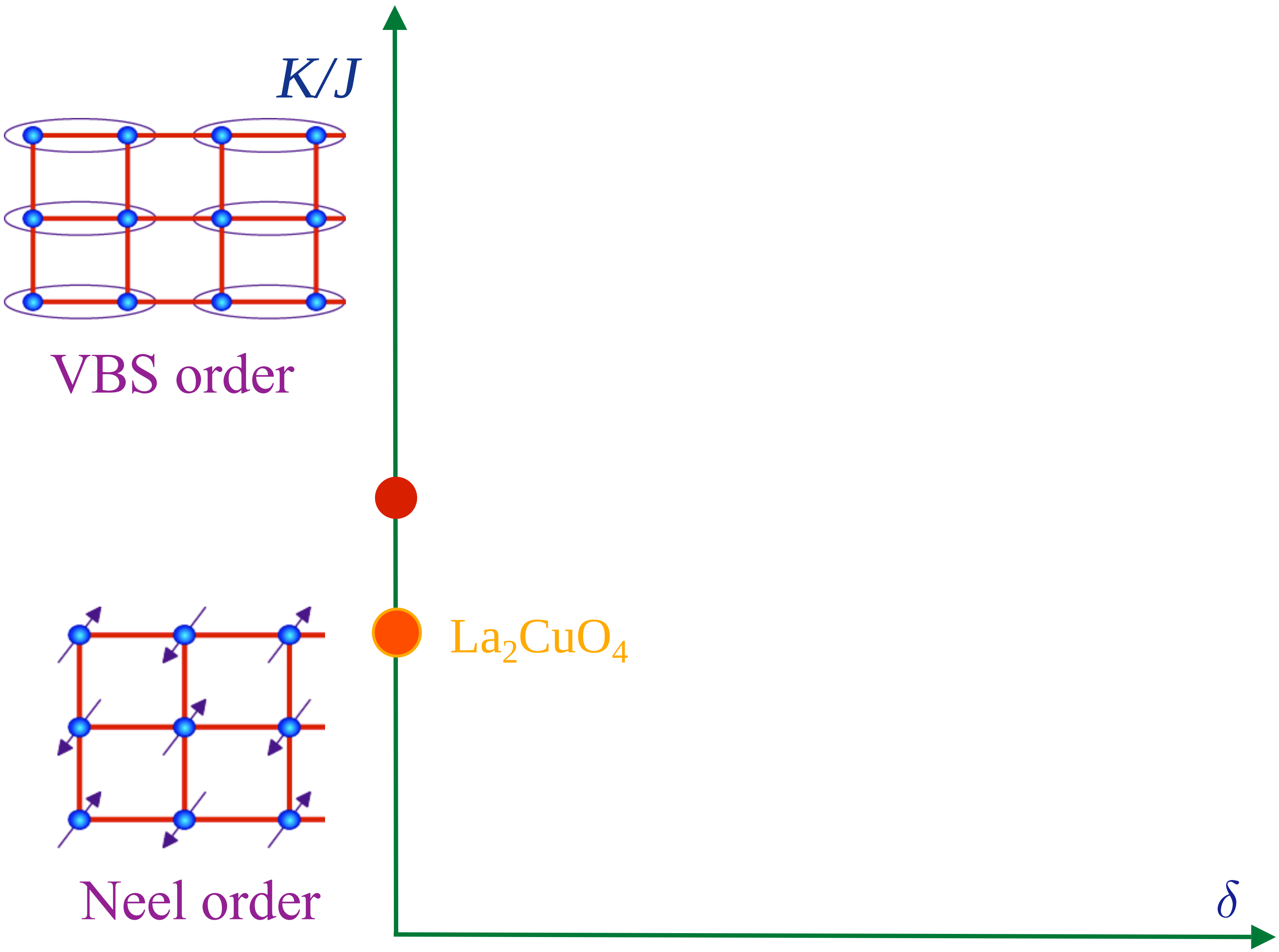
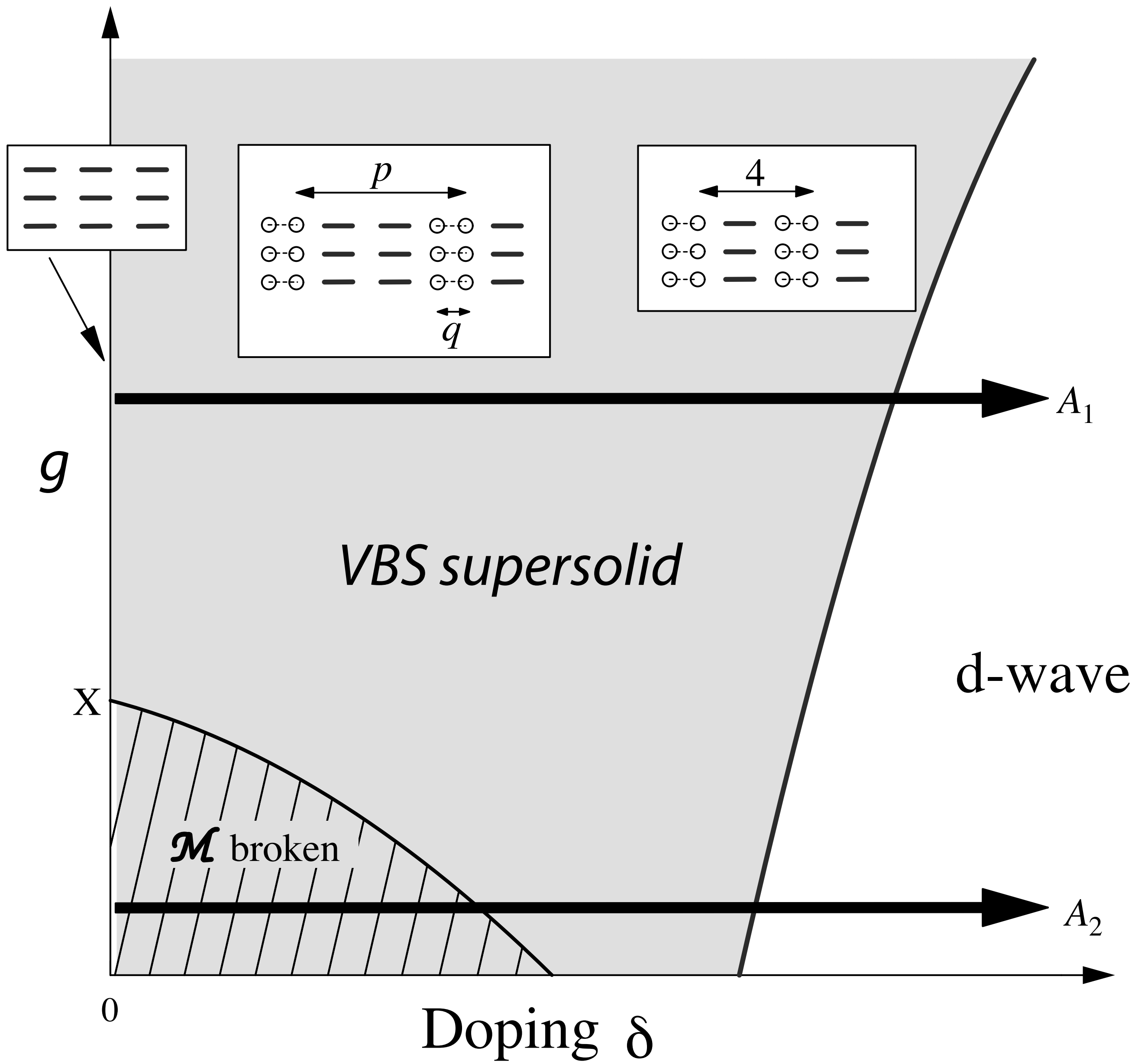
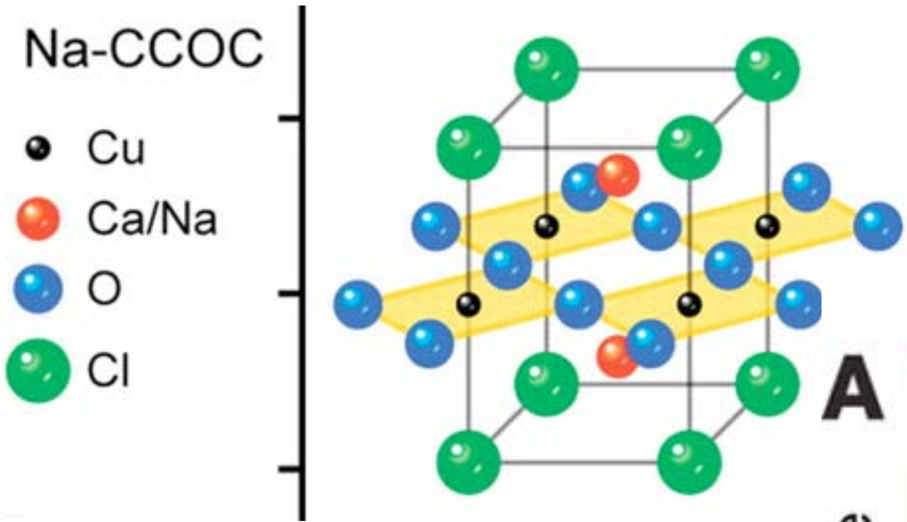
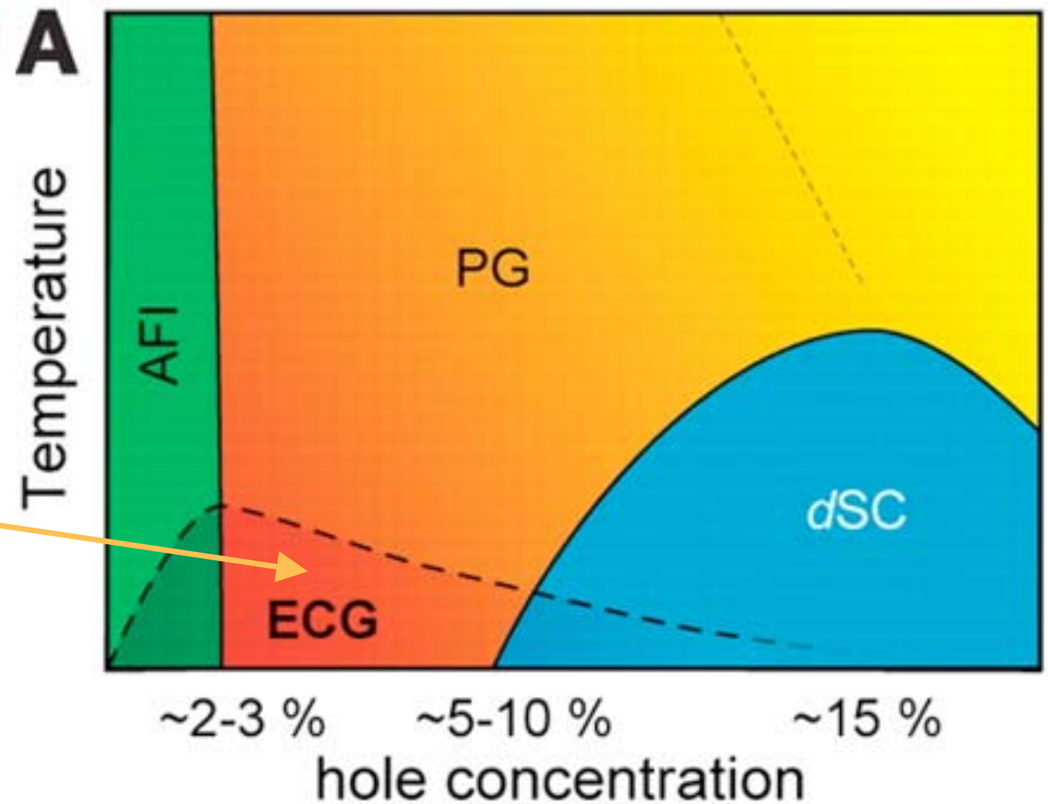




M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

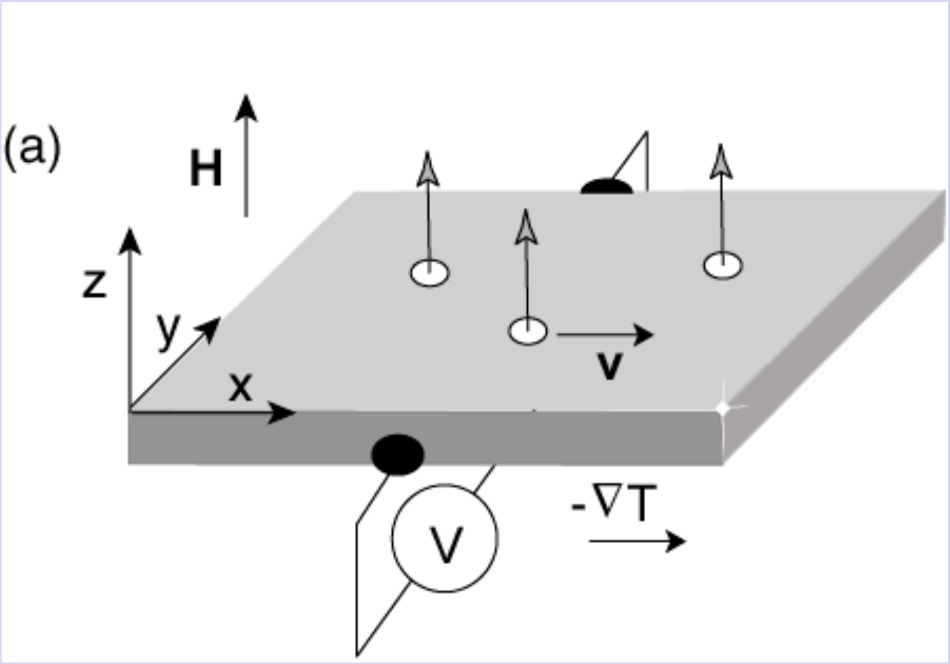
Temperature-doping phase diagram of the cuprate superconductors

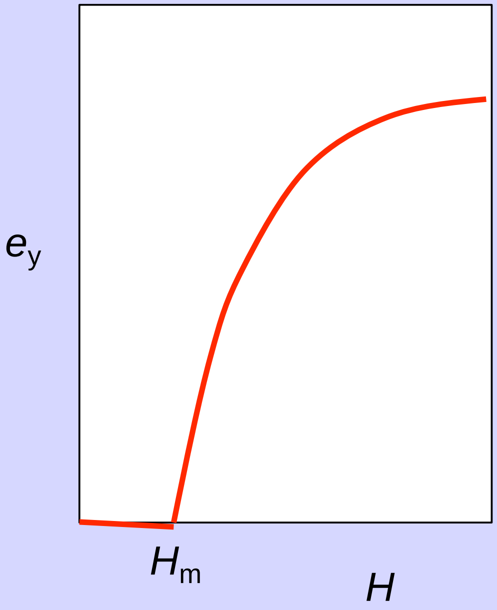




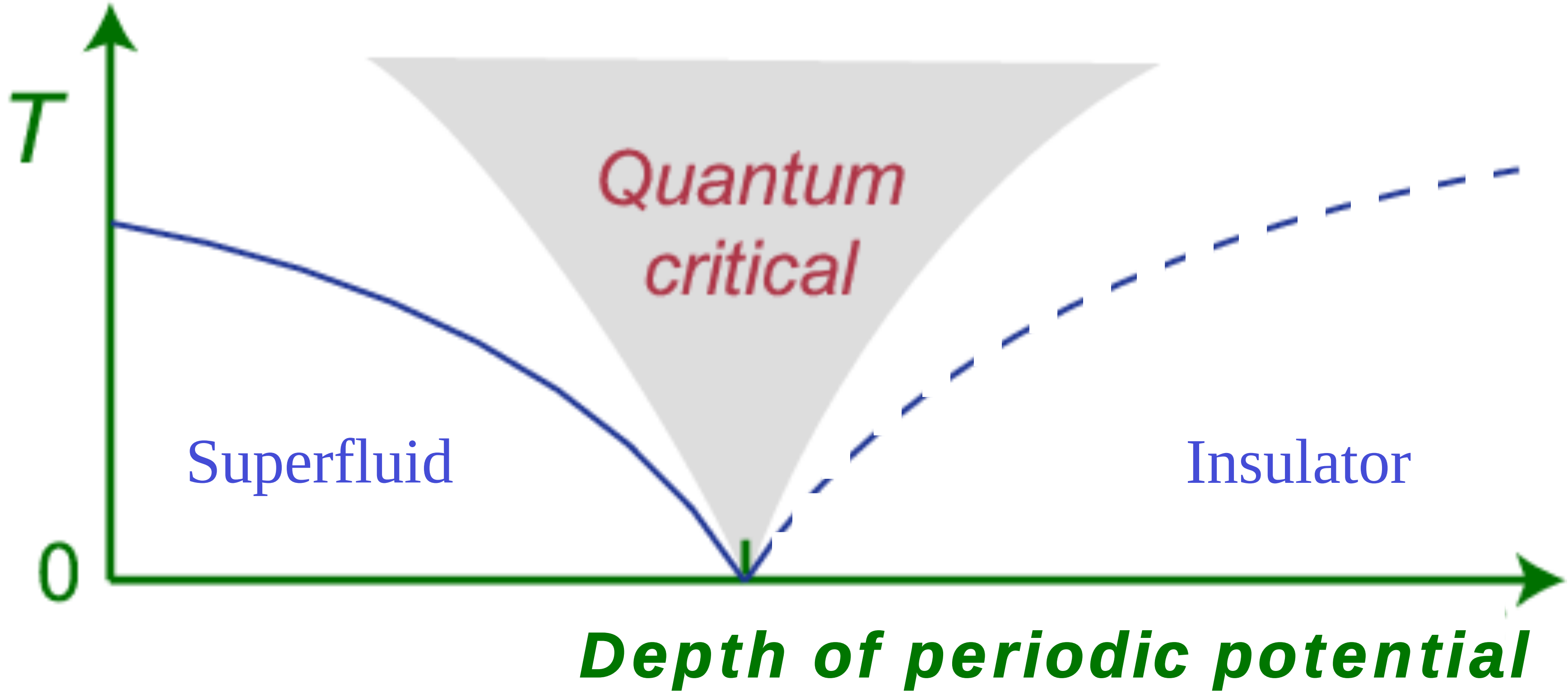
“Glassy” Valence Bond Solid (VBS)

Nernst experiment





Non-zero temperature phase diagram



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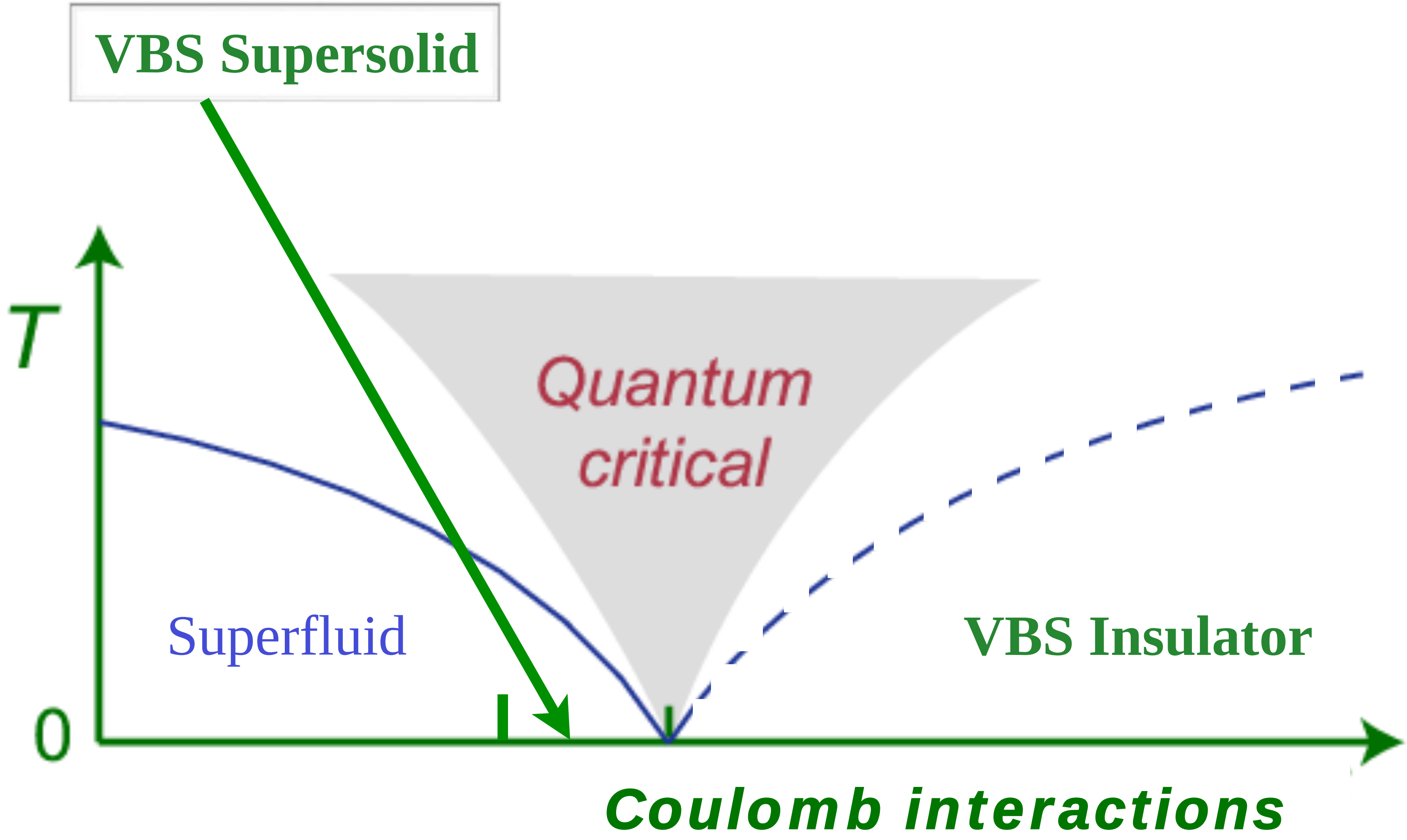

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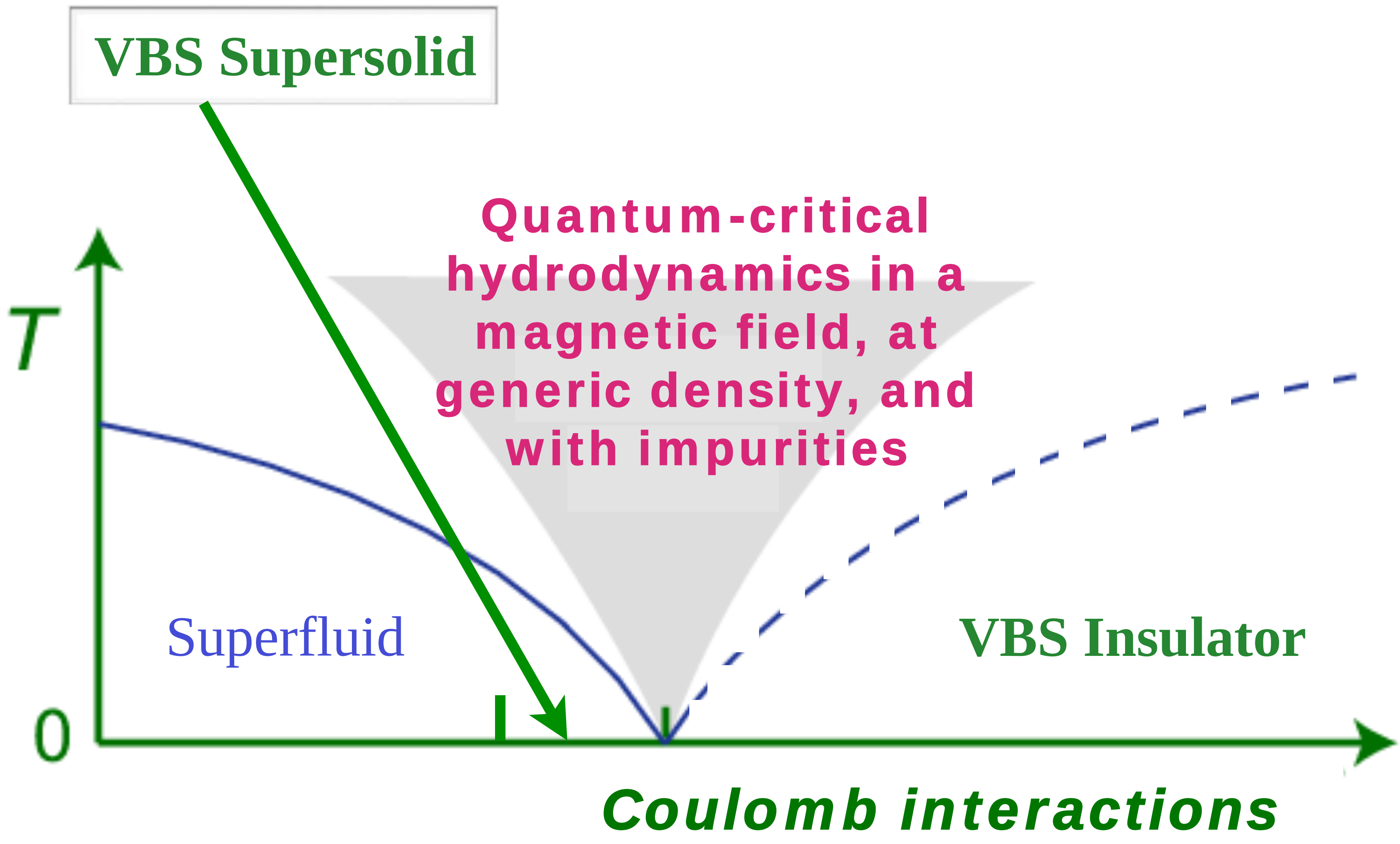
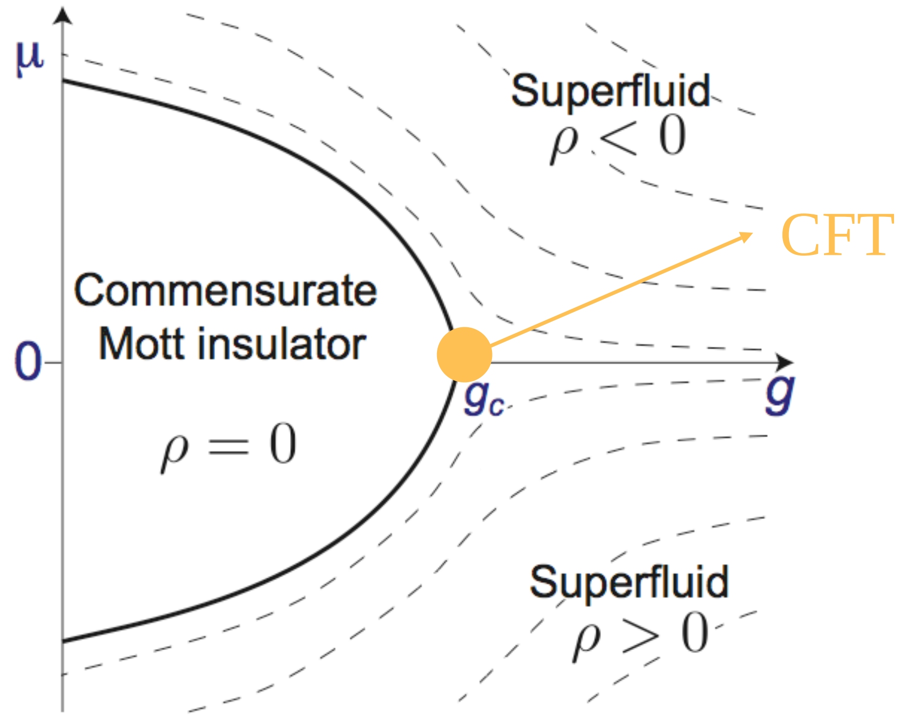


For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field B



e.g.

$$\mathcal{S} = \int d^2 r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|(\vec{\nabla} - i\vec{A})\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the hydrodynamic regime, $\hbar\omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.
The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the local number density,
- ε, the local energy density,
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \end{aligned}$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu \end{aligned}$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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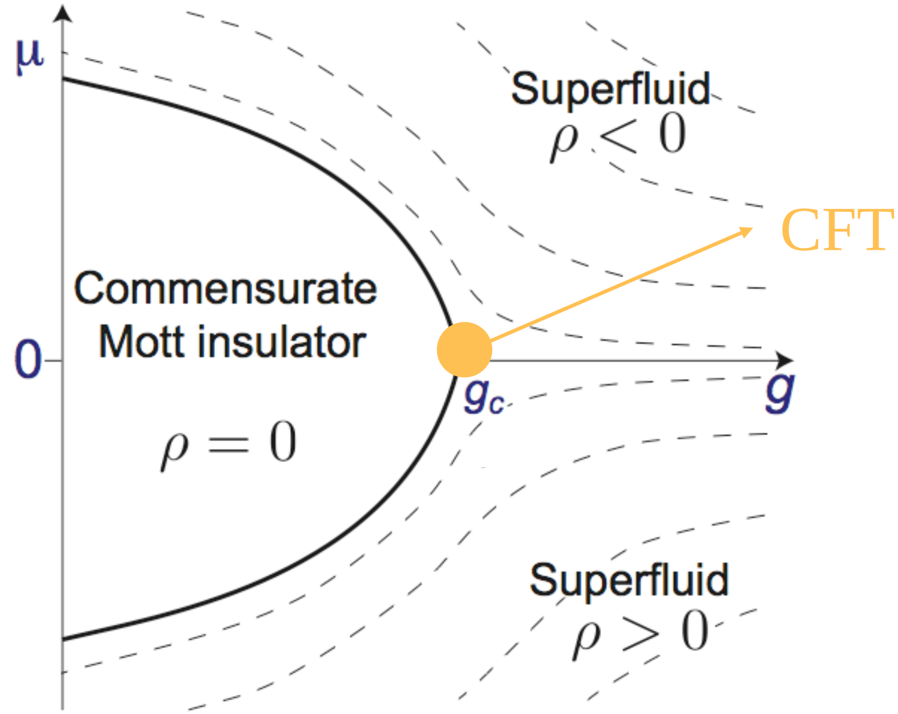
Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field *B*



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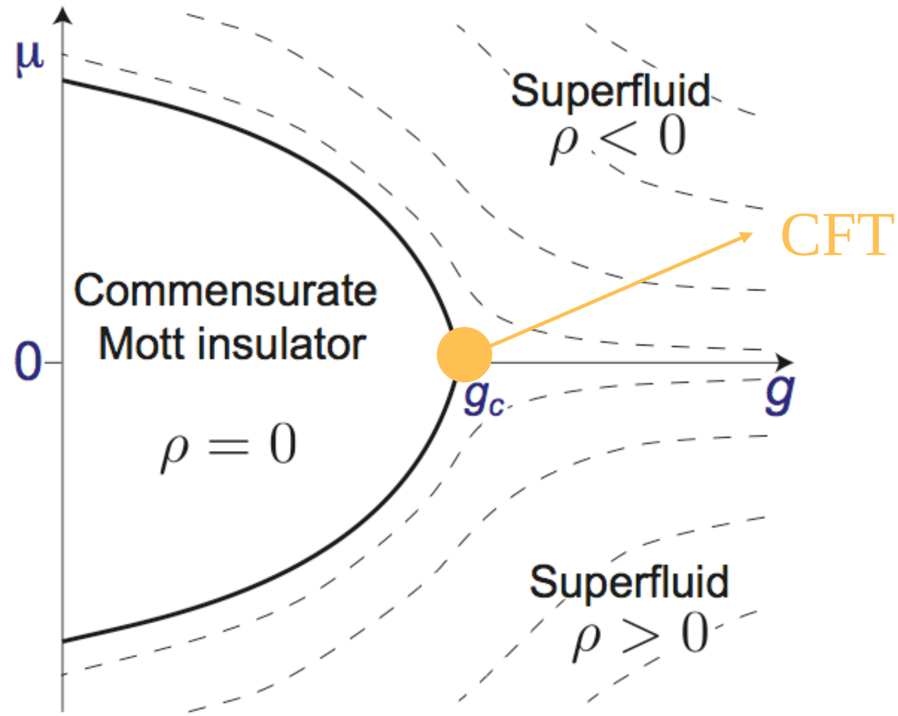
For experimental applications, we must move away from the ideal CFT

- A chemical potential μ
- A magnetic field B
- An impurity scattering rate $1/\tau_{imp}$ (its T dependence follows from scaling arguments)



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$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\rm imp}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} \left(\delta^\mu_\nu + u^\mu u_\nu\right) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda\right) + \mu \frac{\partial_\mu T}{T} \right]
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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\rm imp})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\rm imp})}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2} \right] .$$

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$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Hall conductivity

$$\begin{aligned} \sigma_{xy} &= -\frac{2e\rho c}{B}\left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\rm imp}}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2}\right] \\ &= B\left[\sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\rm imp} - i\omega)} + \frac{8e^3\rho^3 v^4}{(\varepsilon + P)^2(1/\tau_{\rm imp} - i\omega)^2}\right] \\ &\qquad \text{as } B \to 0 \end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Thermal conductivity

$$\begin{aligned}
\kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right) \left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right] \\
&= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]
\end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 \quad \boxed{\rightarrow 1 \text{ as } B \rightarrow 0} \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B} \right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \end{aligned}$$

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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086 \mu{\rm V/K}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

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Transverse thermoelectric co-efficient

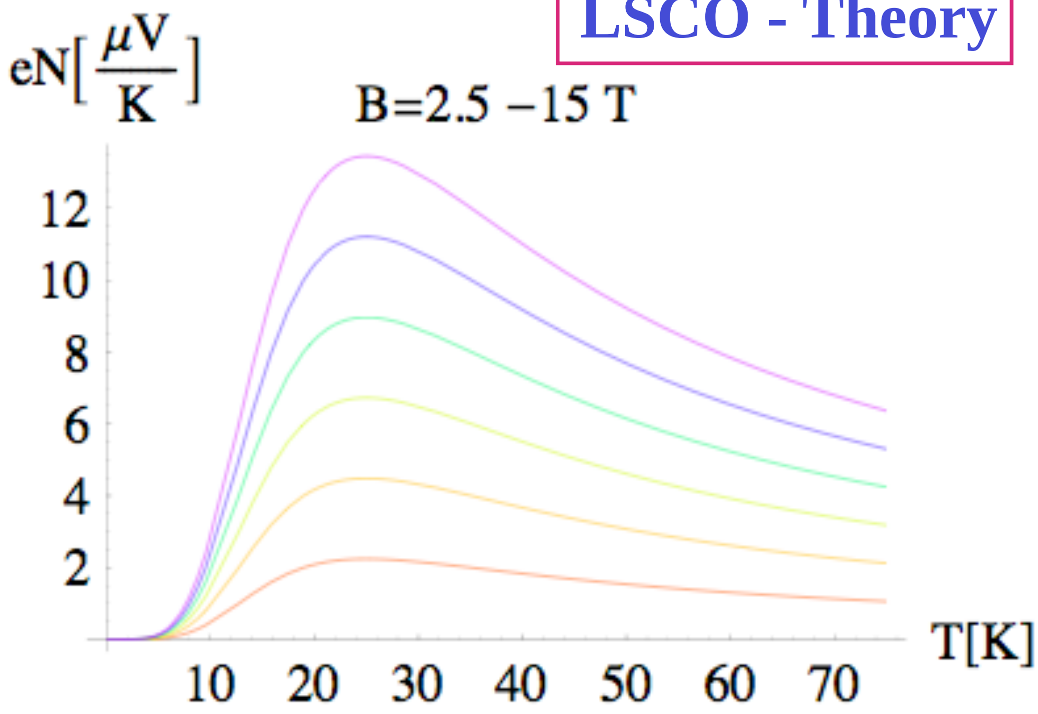
$$\left(\frac{h}{2ek_B}\right)\alpha_{xy} = \Phi_s \overline{B}\,(k_B T)^2 \left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P}(k_B T)^3\,\hbar/2\pi\tau_{\rm imp}}{\Phi_{\varepsilon+P}^2 (k_B T)^6 + \overline{B}^2 \overline{\rho}^2 (2\pi\tau_{\rm imp}/\hbar)^2},$$

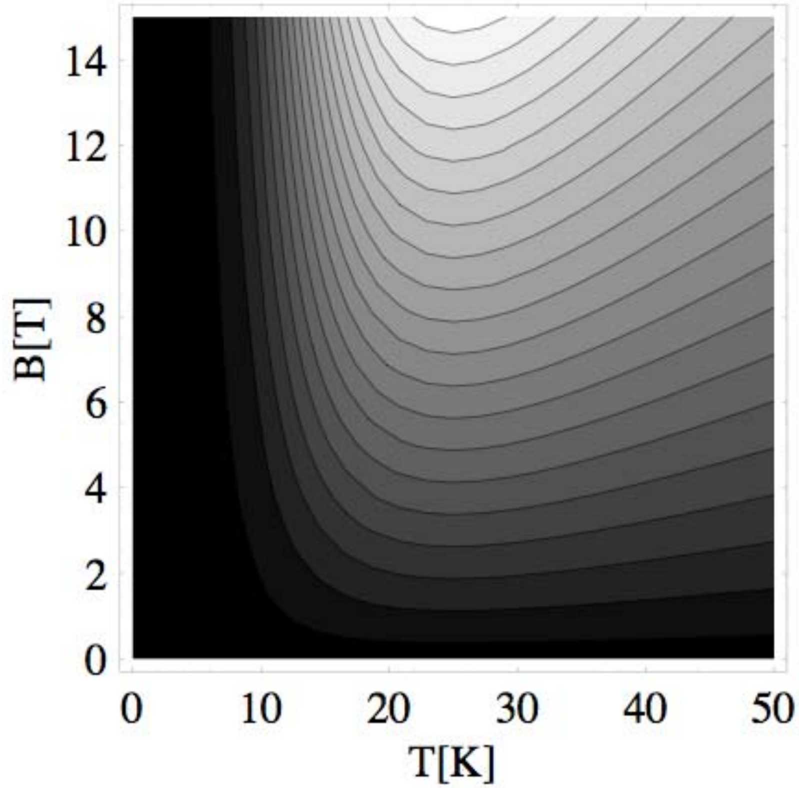
where

$$B = \overline{B}\phi_0/(\hbar v)^2 \quad ; \quad \rho = \overline{\rho}/(\hbar v)^2.$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

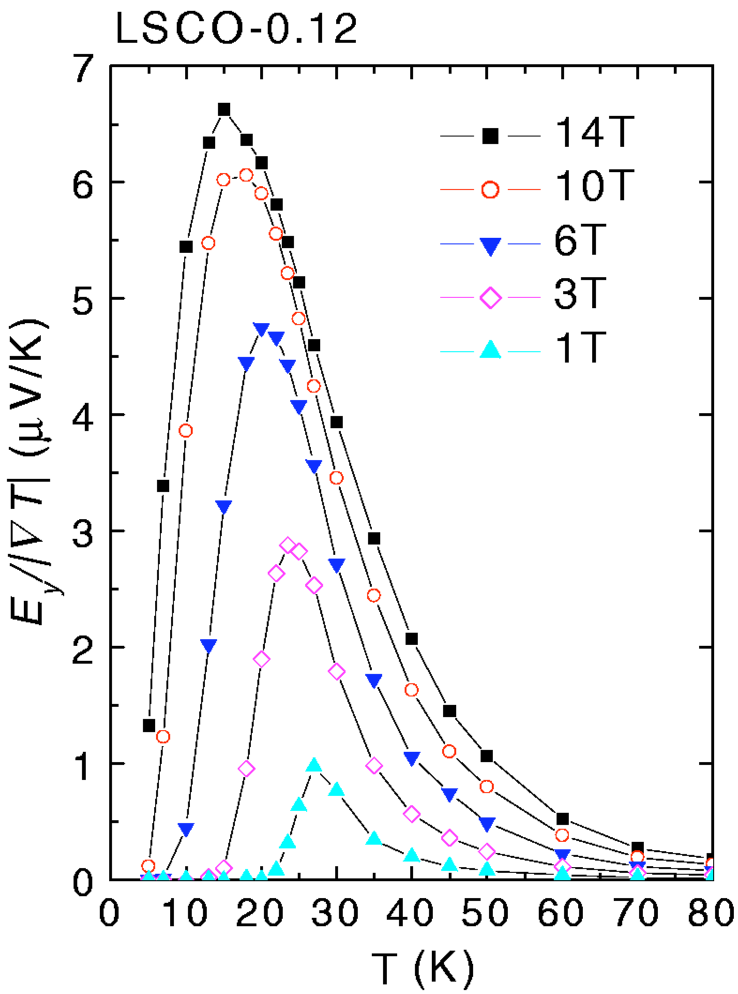
LSCO - Theory

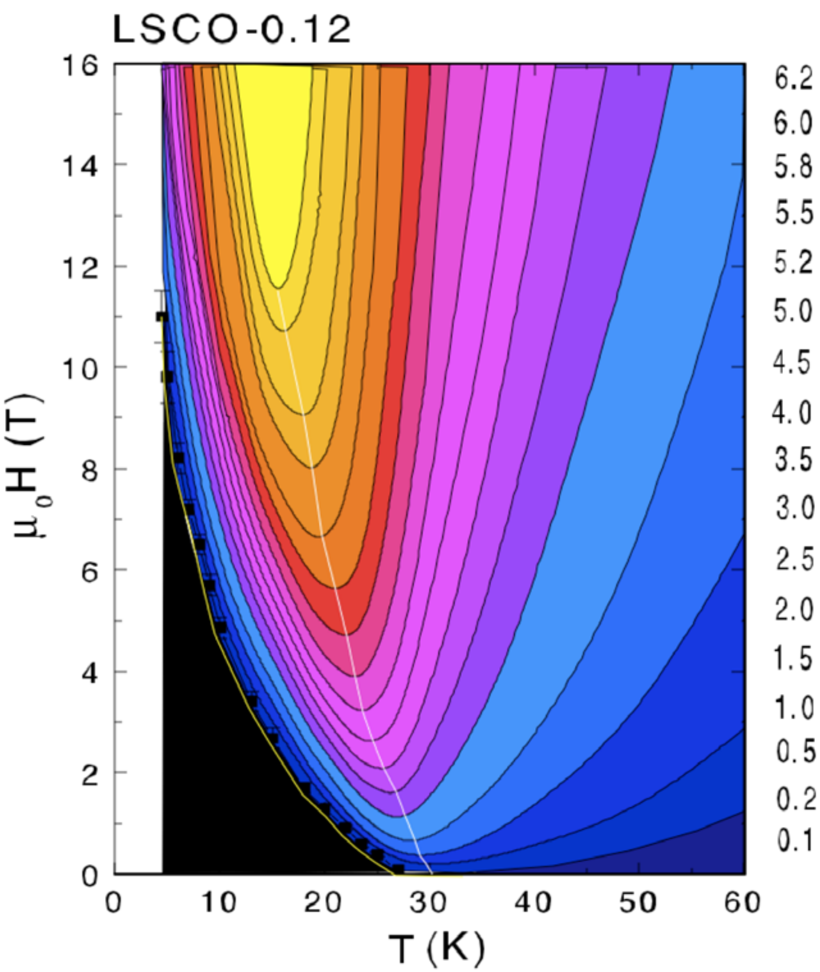




S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

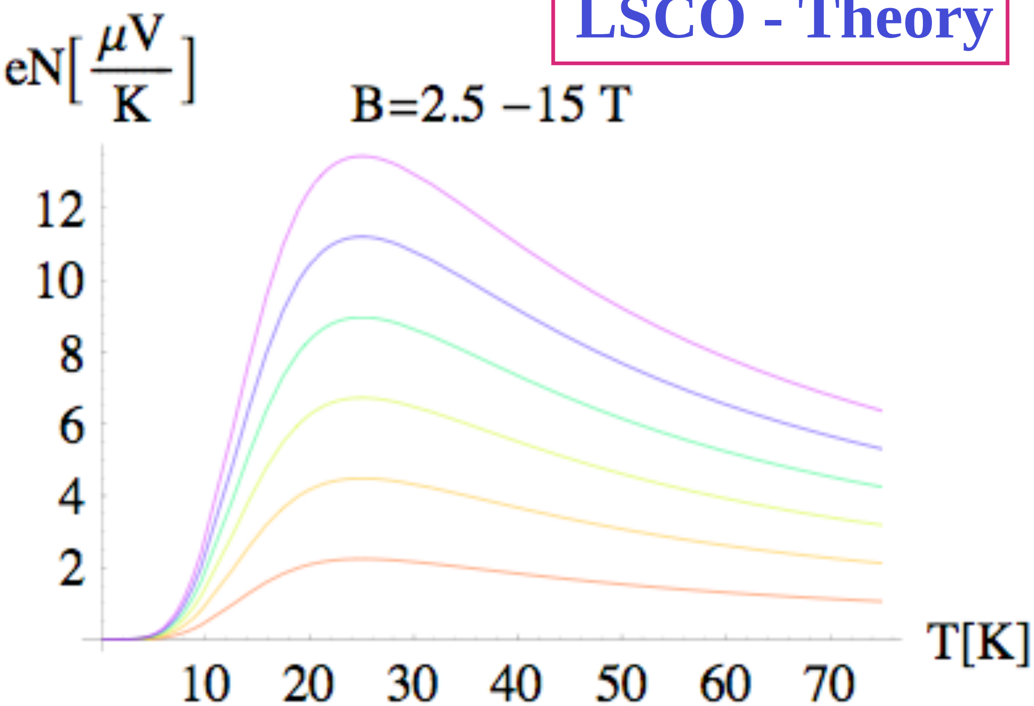
LSCO - Experiments

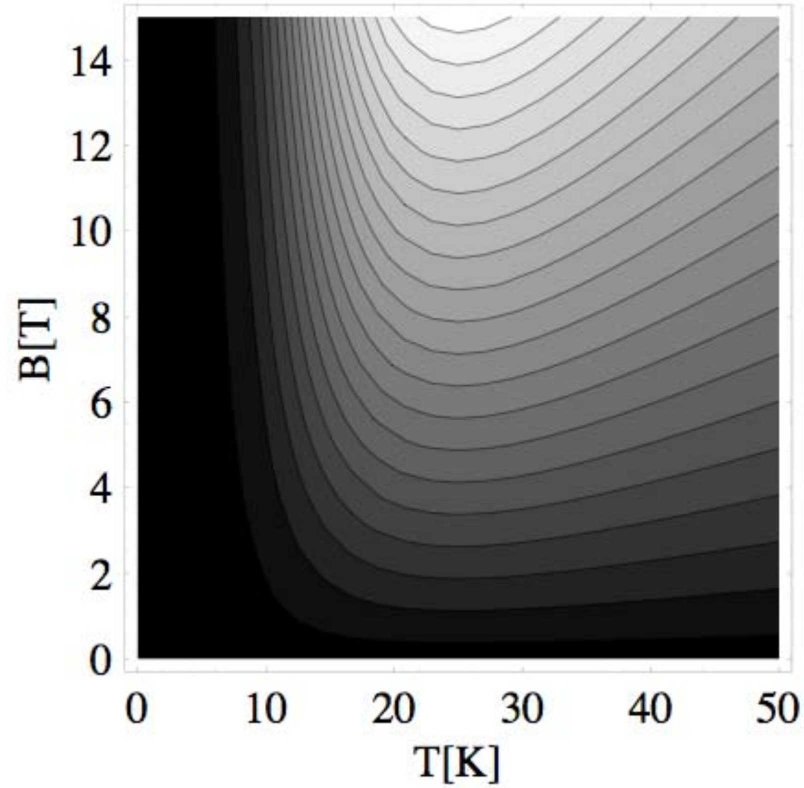




N. P. Ong *et al.*

LSCO - Theory





Only input parameters

$$\hbar v = 47 \text{ meV Å}$$

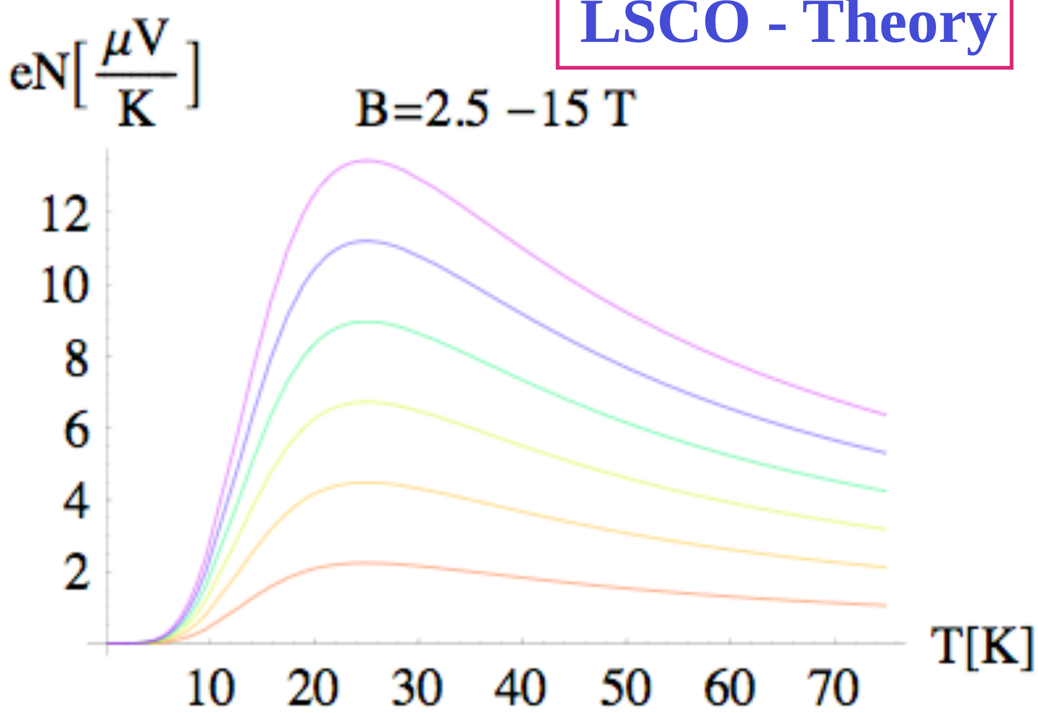
$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

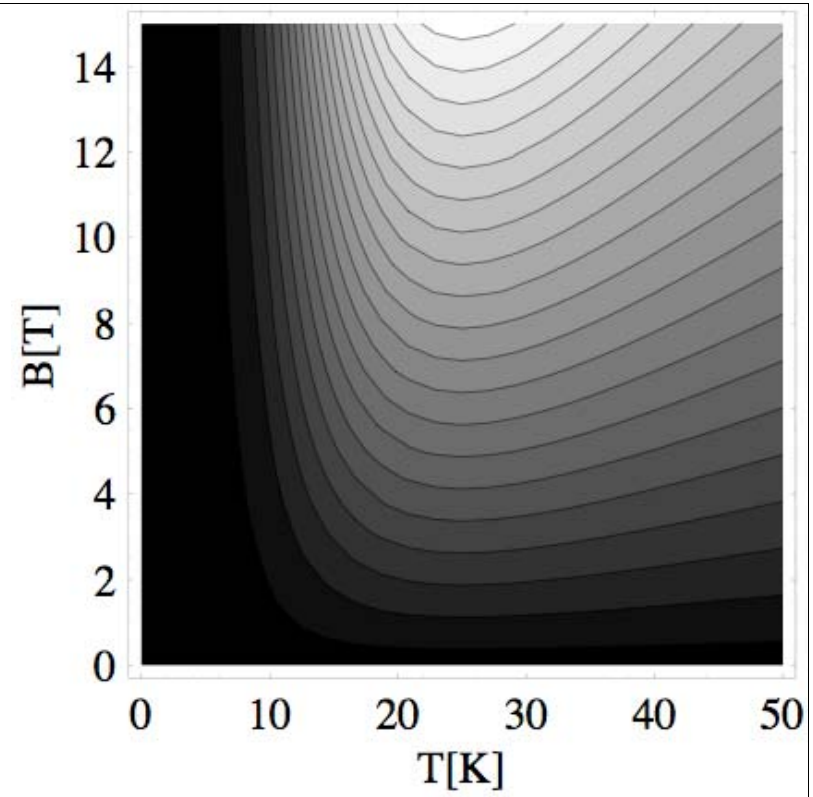
Output

$$\omega_c = 6.2\text{GHz} \cdot \frac{B}{1\text{T}} \left(\frac{35\text{K}}{T}\right)^3$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

LSCO - Theory





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Similar to velocity estimates by
A.V. Balatsky and Z-X. Shen, *Science* **284**, 1137 (1999).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

Conclusions

- Studies of new materials and trapped ultracold atoms are yielding new quantum phases, with novel forms of quantum entanglement.
- Some materials are of technological importance: e.g. high temperature superconductors.
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.
- Tabletop “laboratories for the entire universe”: quantum mechanics of black holes, quark-gluon plasma, neutrons stars, and big-bang physics.