The onset of antiferromagnetism in metals

Lorentz Lecture, Leiden
June 4, 2012

Subir Sachdev

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PHYSICS
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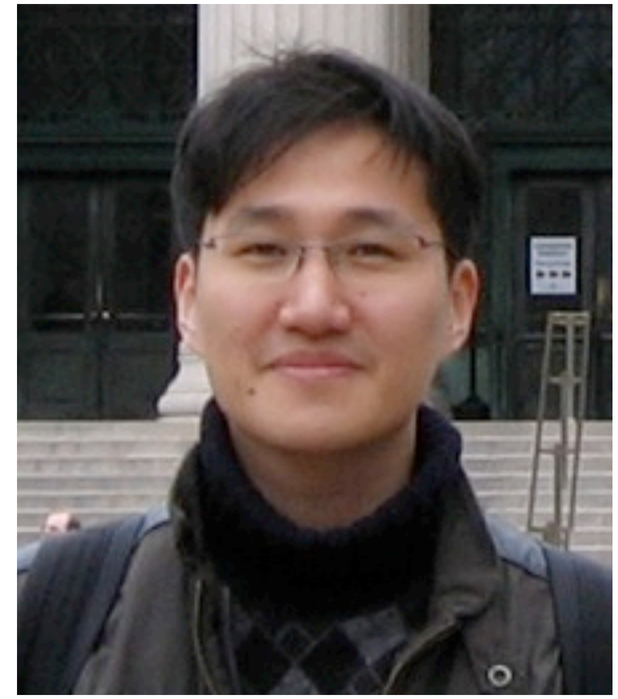
Max Metlitski



Eun Gook Moon

Matthias Punk

Erez Berg

PHYSICS

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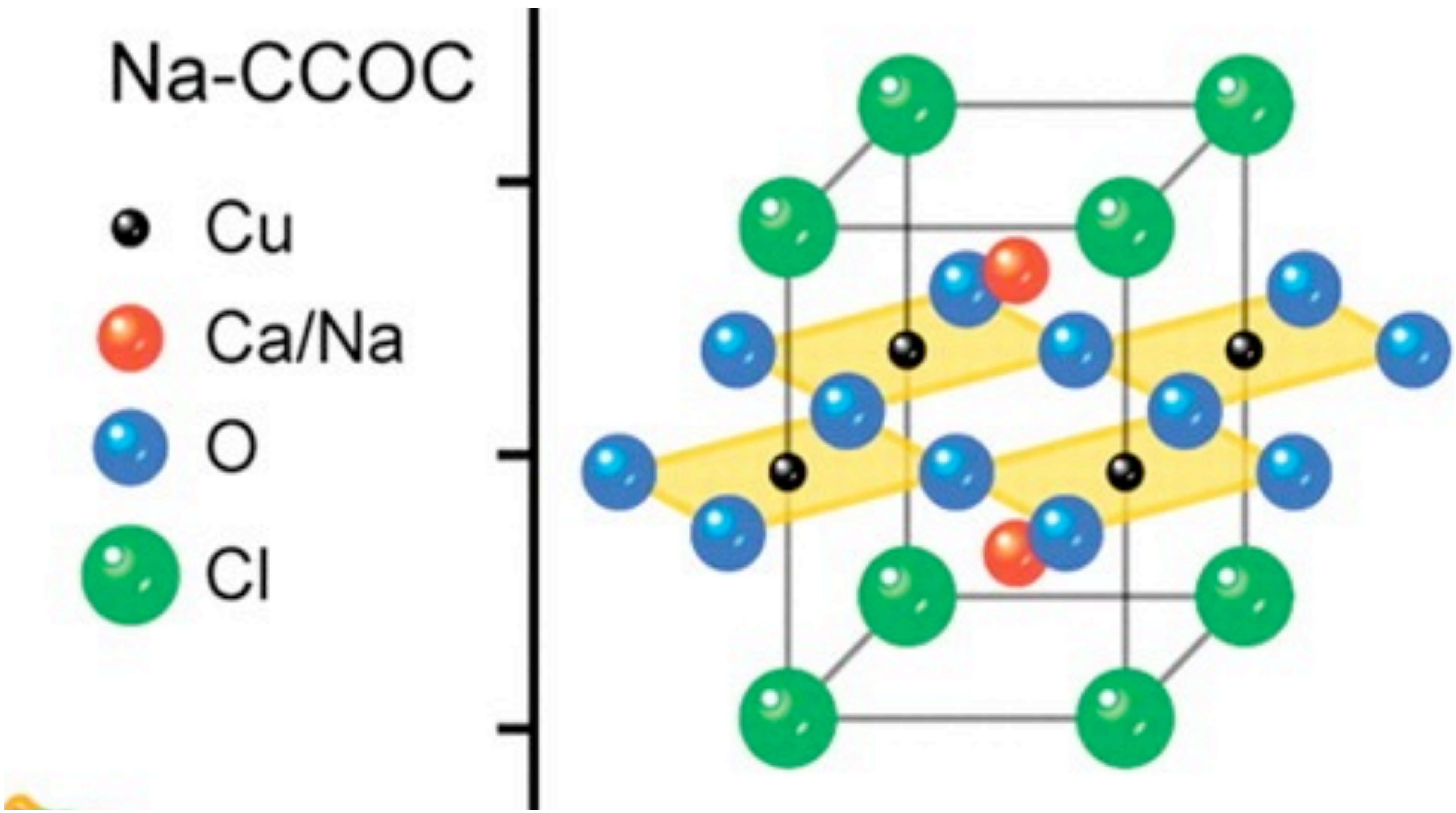
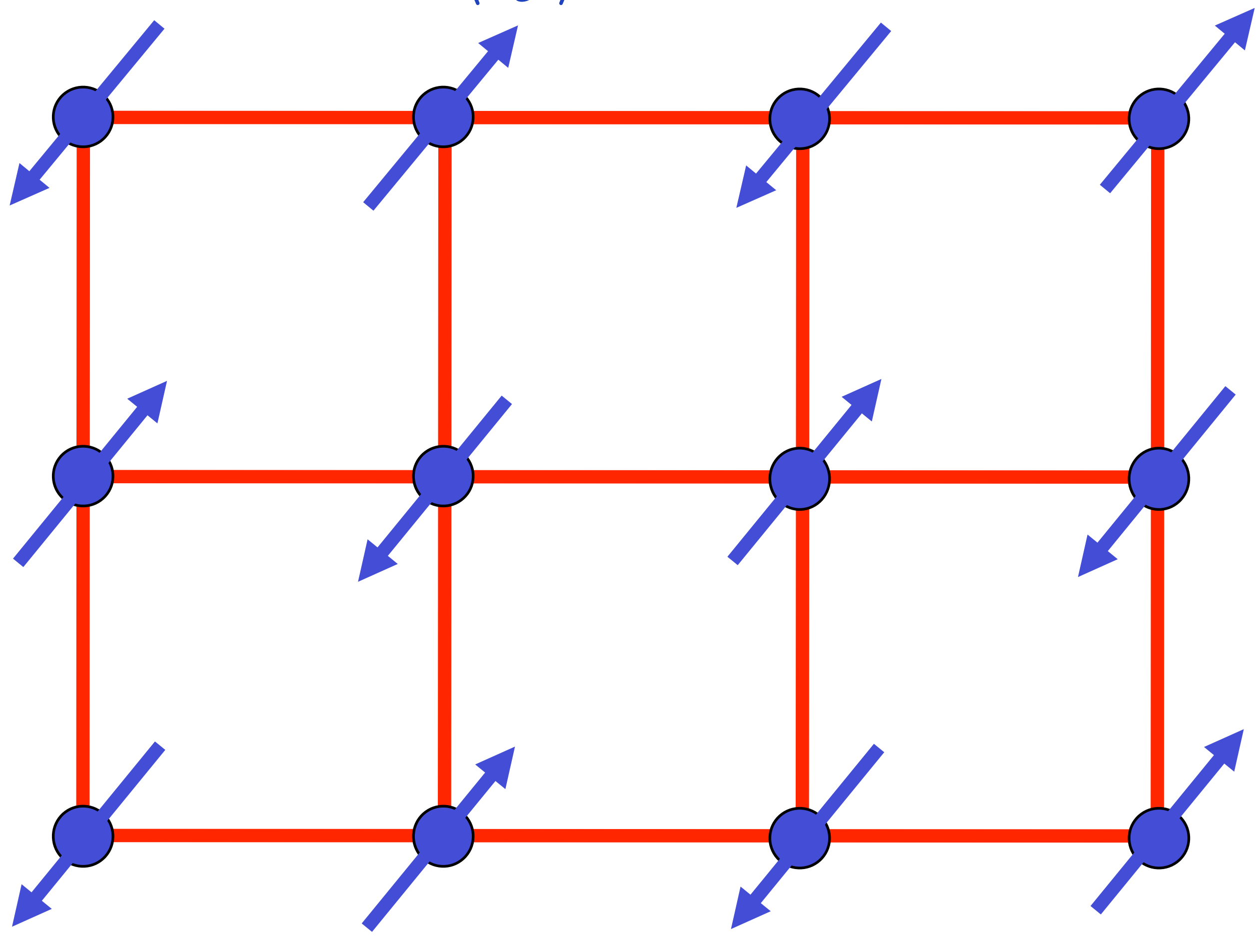
The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

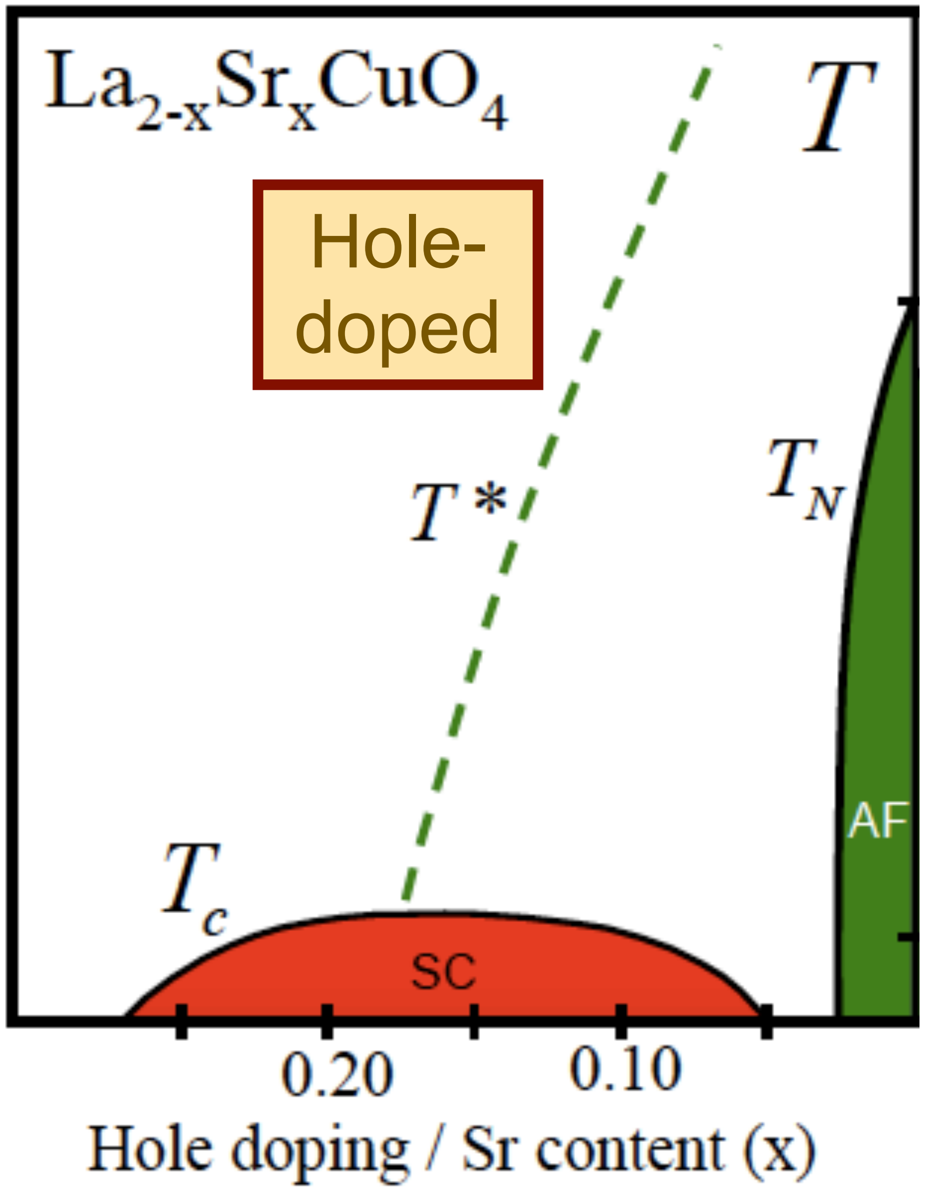


Ground state has long-range Néel order

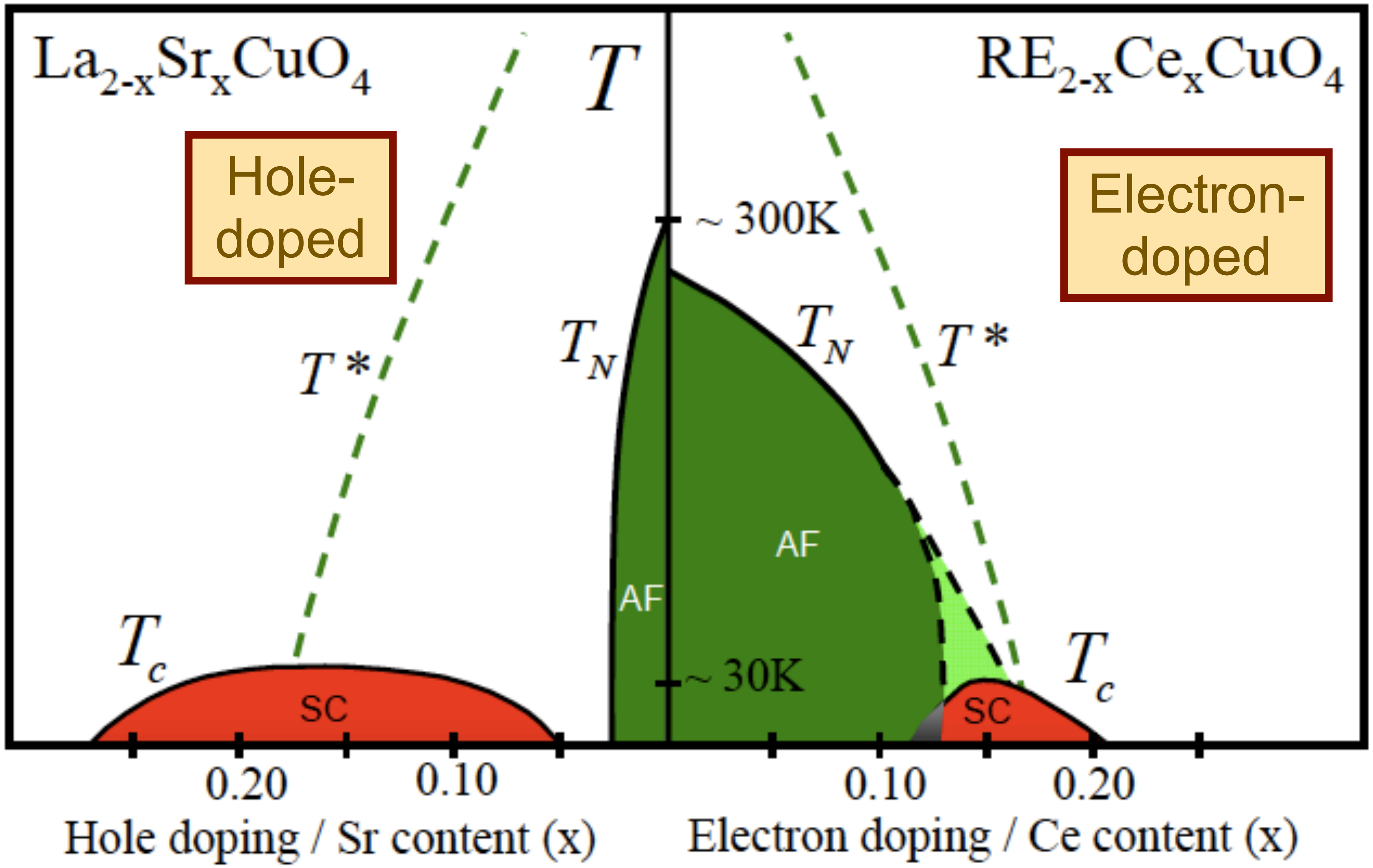
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.


$La_{2-x}Sr_xCuO_4$
T
Hole-doped
T^*
T_N
AF
T_c
SC
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

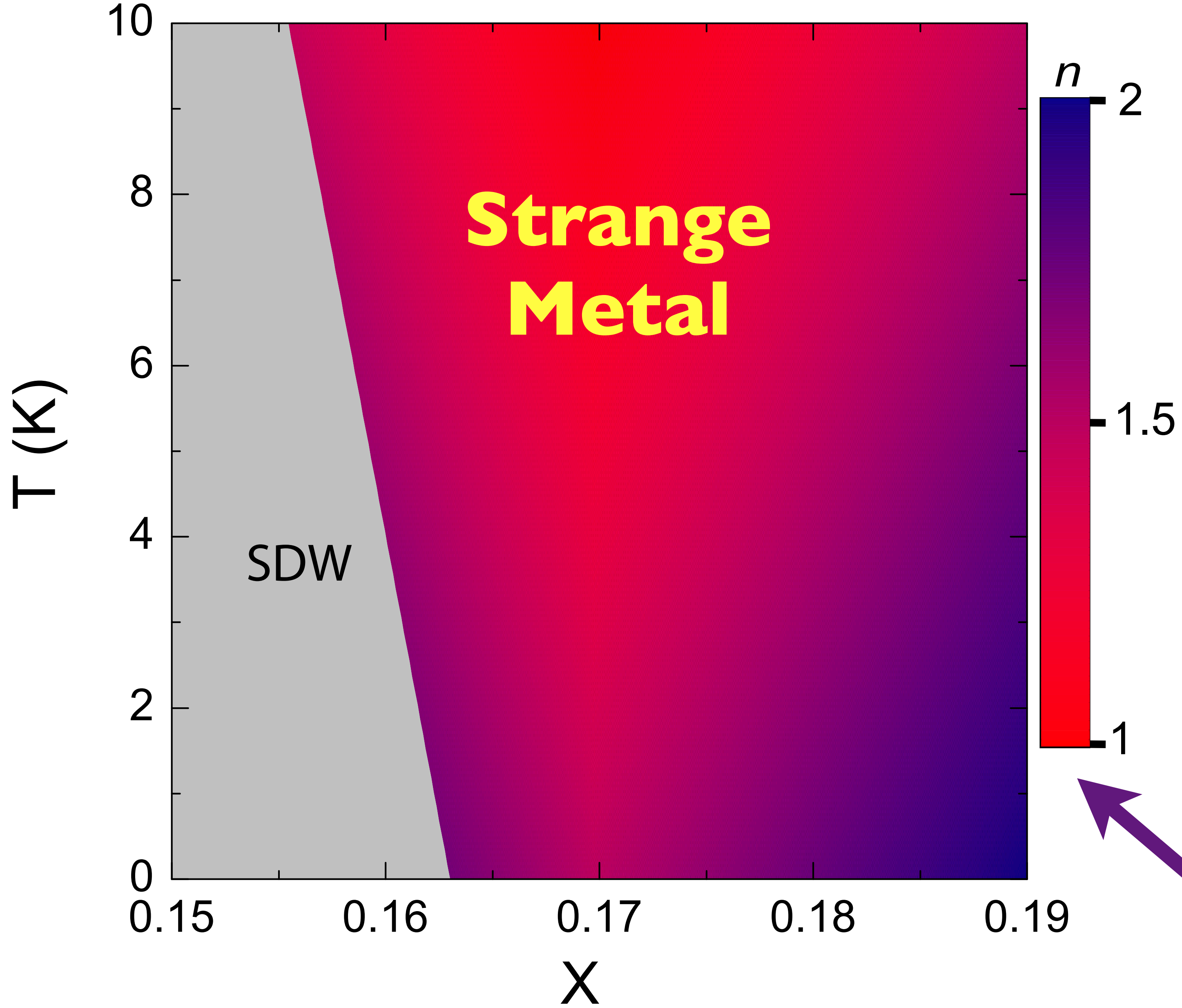
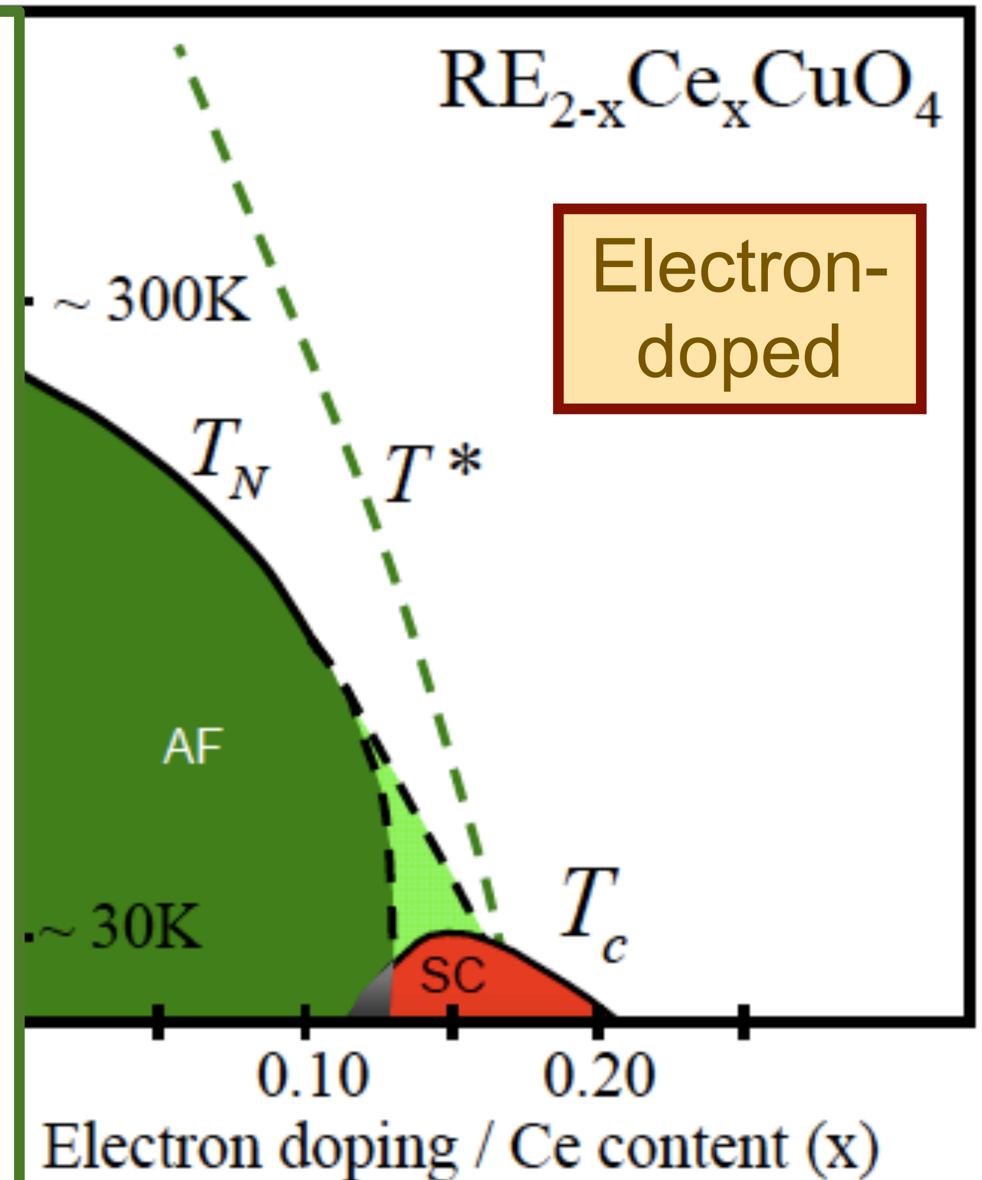


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

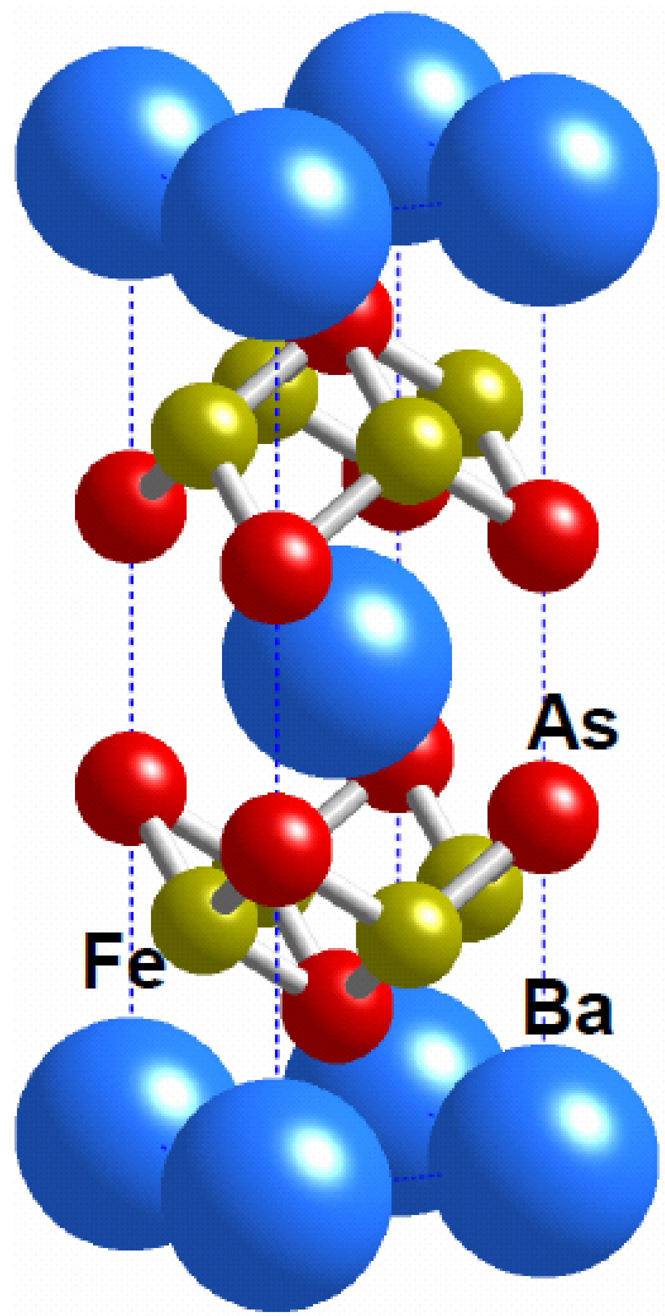


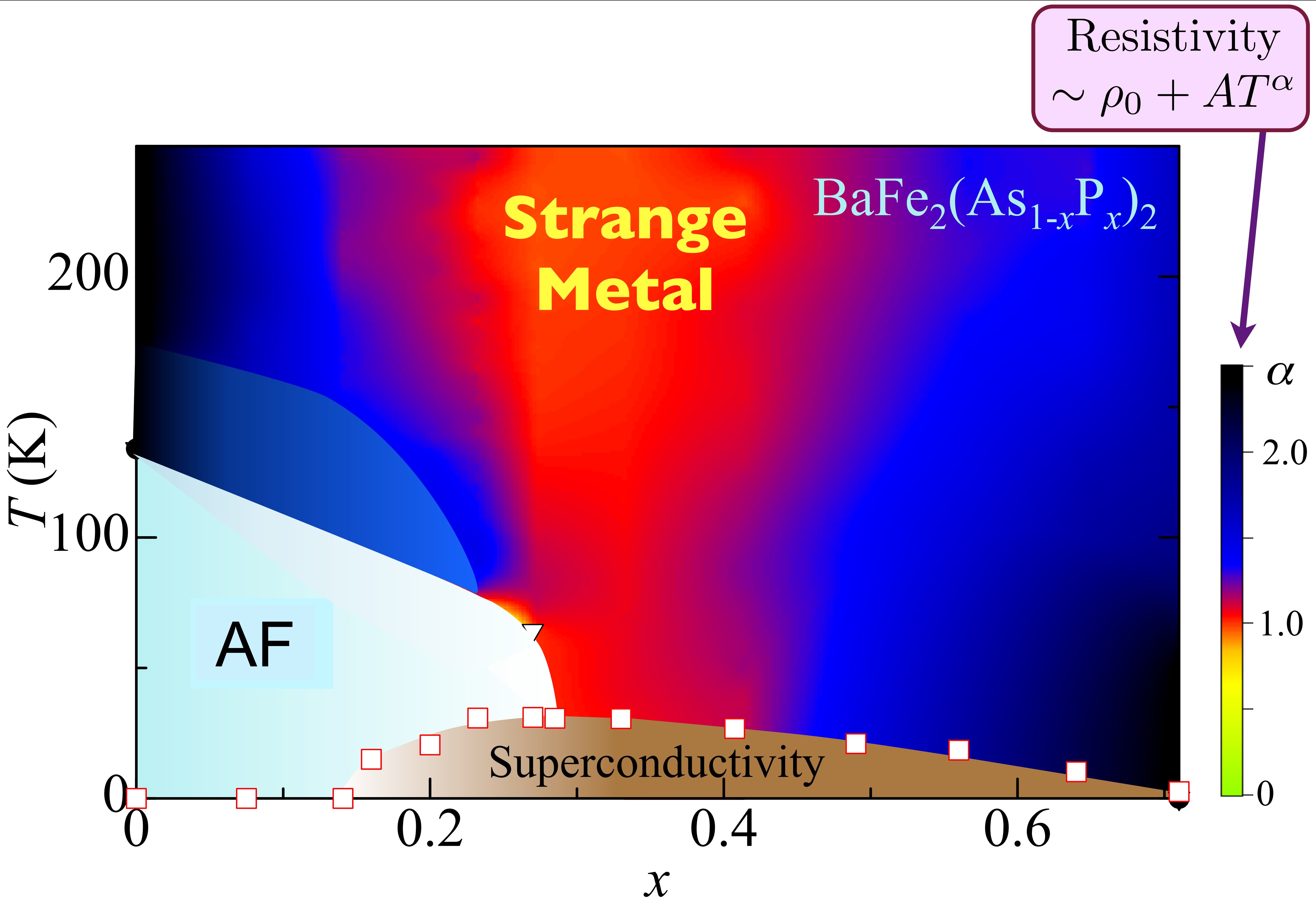
Resistivity

$\sim \rho_0 + AT^n$

Iron pnictides:

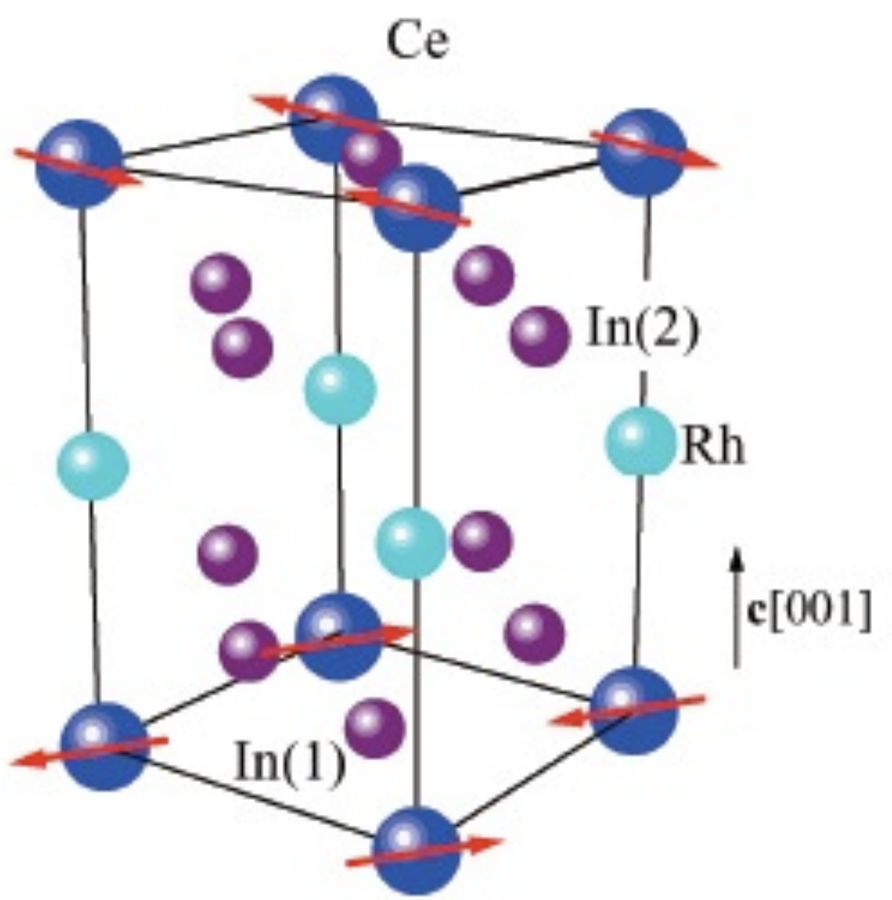
a new class of high temperature superconductors

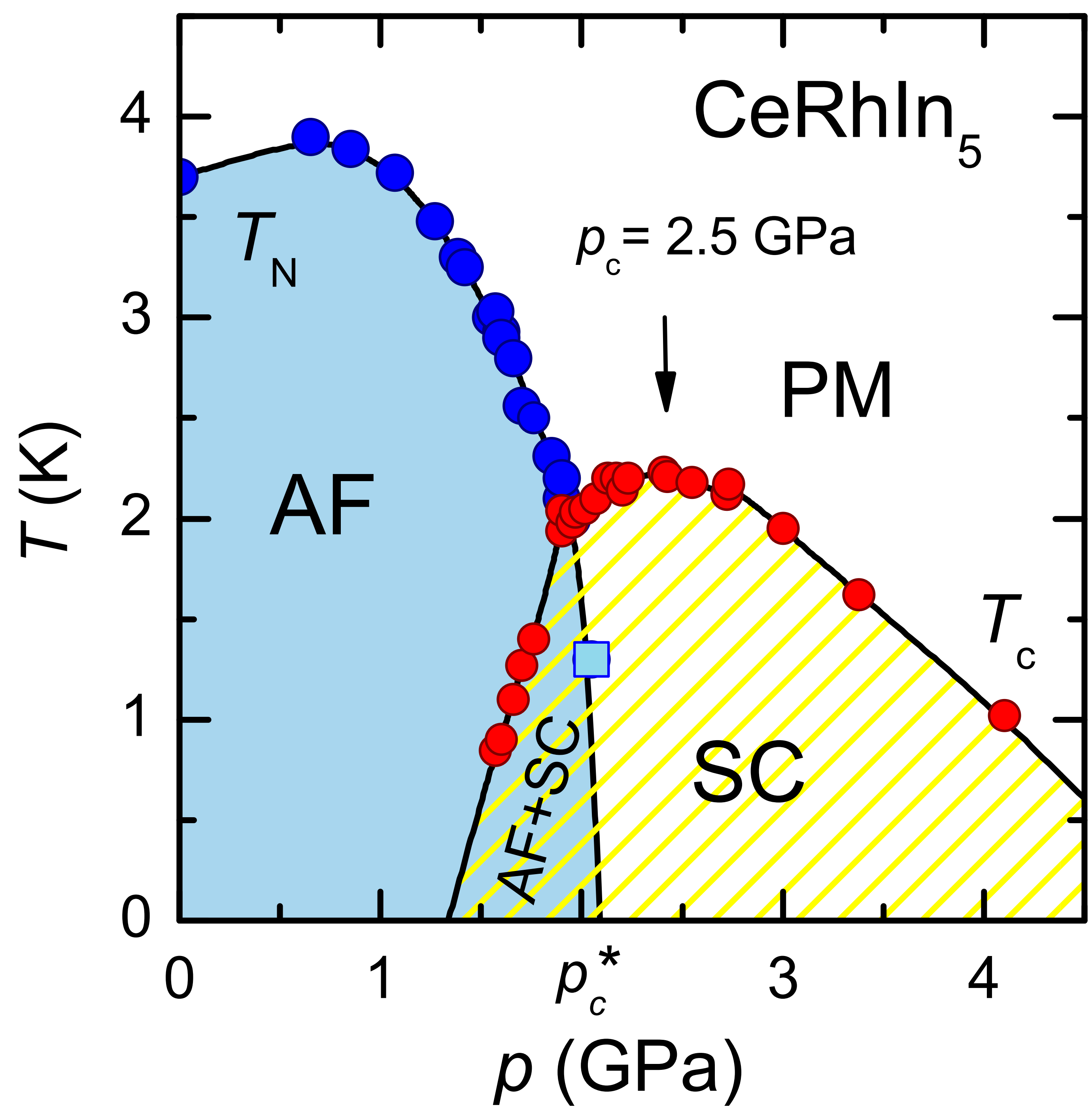




S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

Lower T_c superconductivity in the heavy fermion compounds





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Questions

- *Can quantum fluctuations near the onset of antiferromagnetism induce higher temperature superconductivity ?*

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- *How should such a theory be extended to apply to the hole-doped cuprates ?*

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- *How should such a theory be extended to apply to the hole-doped cuprates ?*

- *What is the physics of the strange metal ?*

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

2. Quantum field theory of the onset of antiferromagnetism in a metal

3. Quantum Monte Carlo without the sign problem

4. Fractionalization in metals, and the hole-doped cuprates

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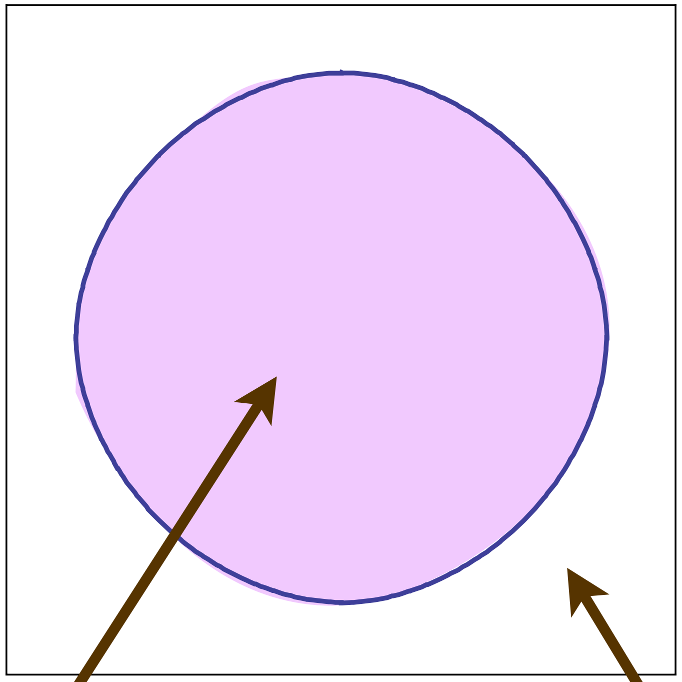
Fermi surface

Metal with "large" Fermi surface

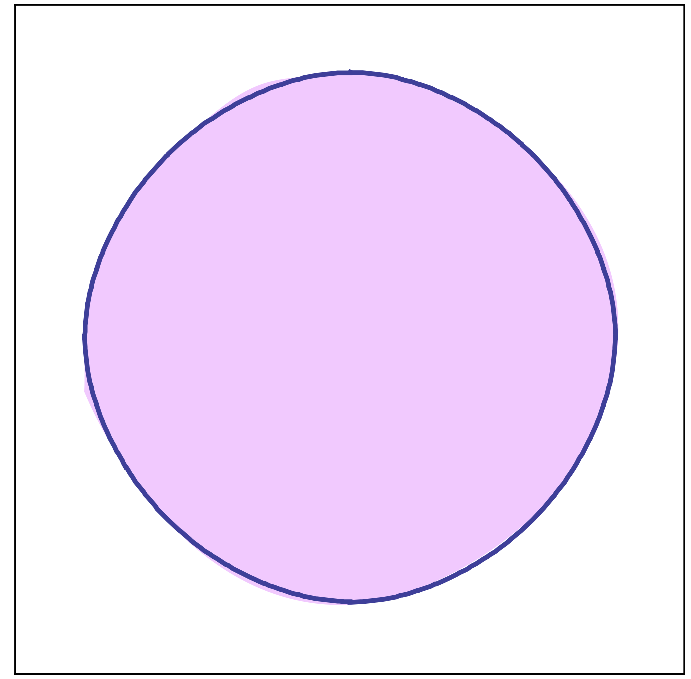


Momenta with electronic states empty

Momenta with electronic states occupied

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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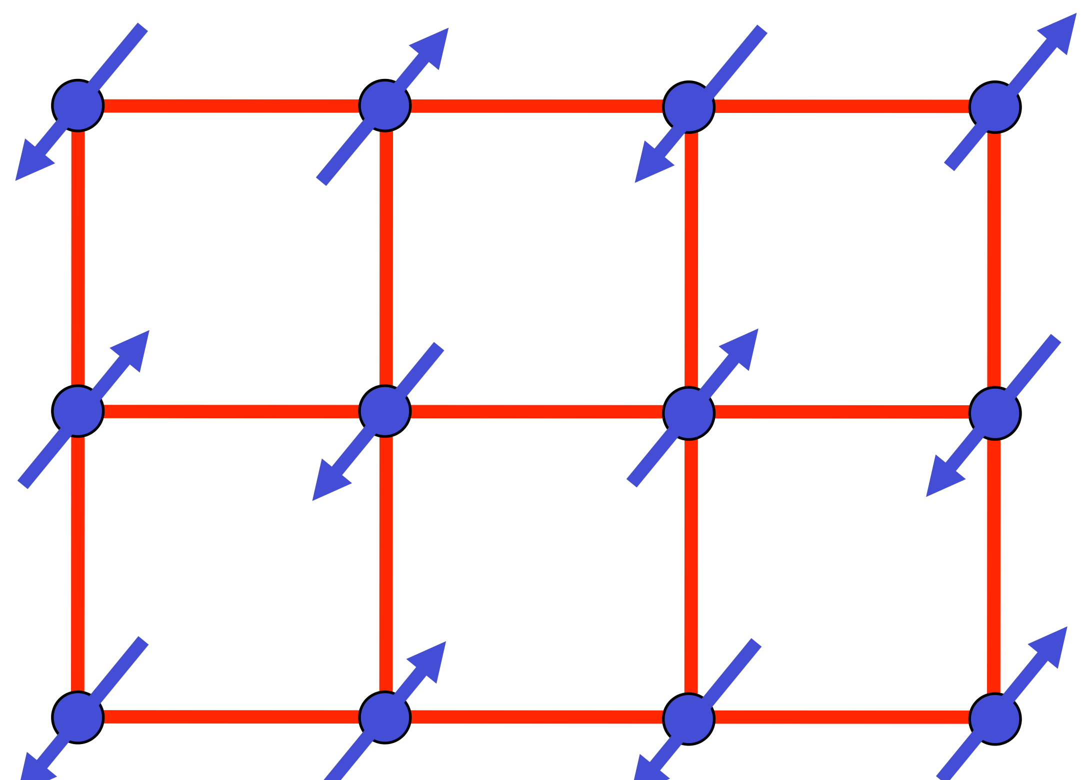
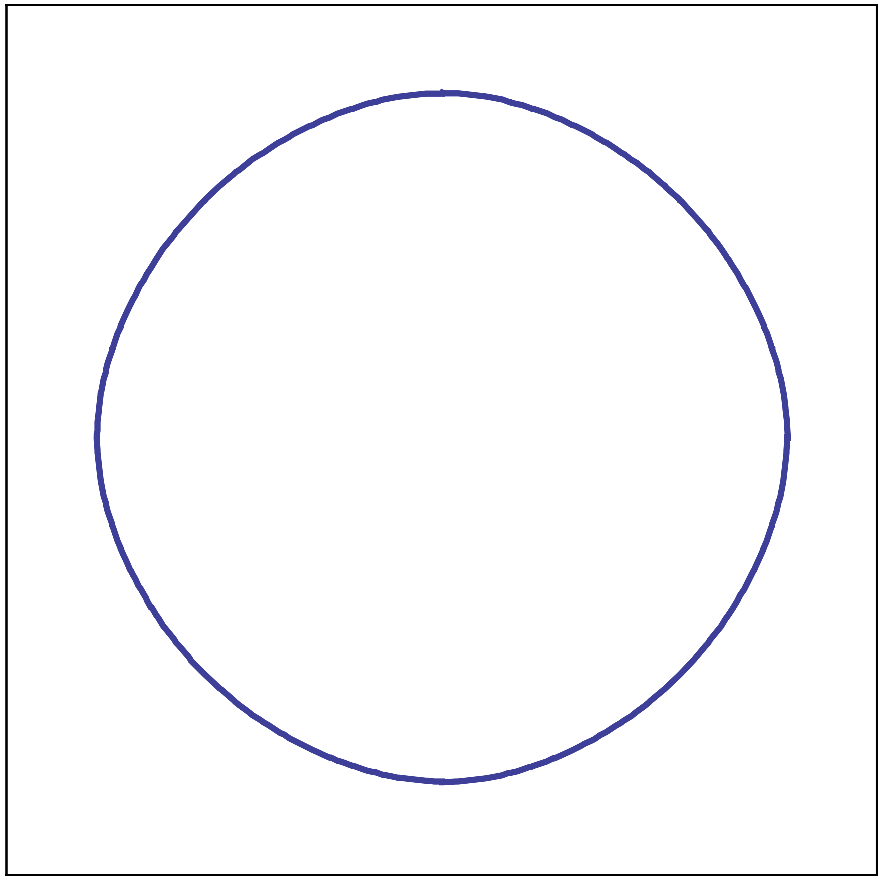


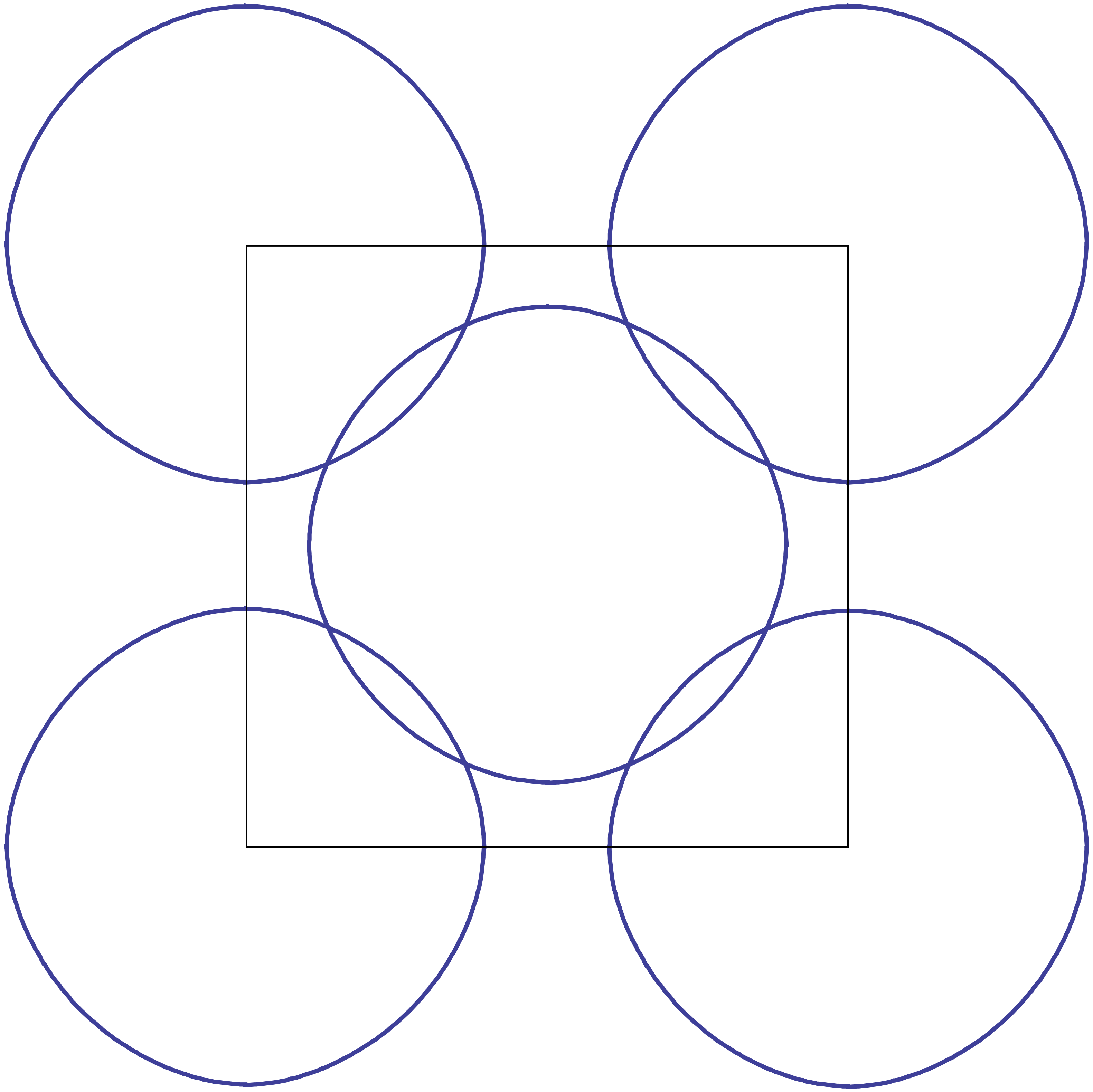
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

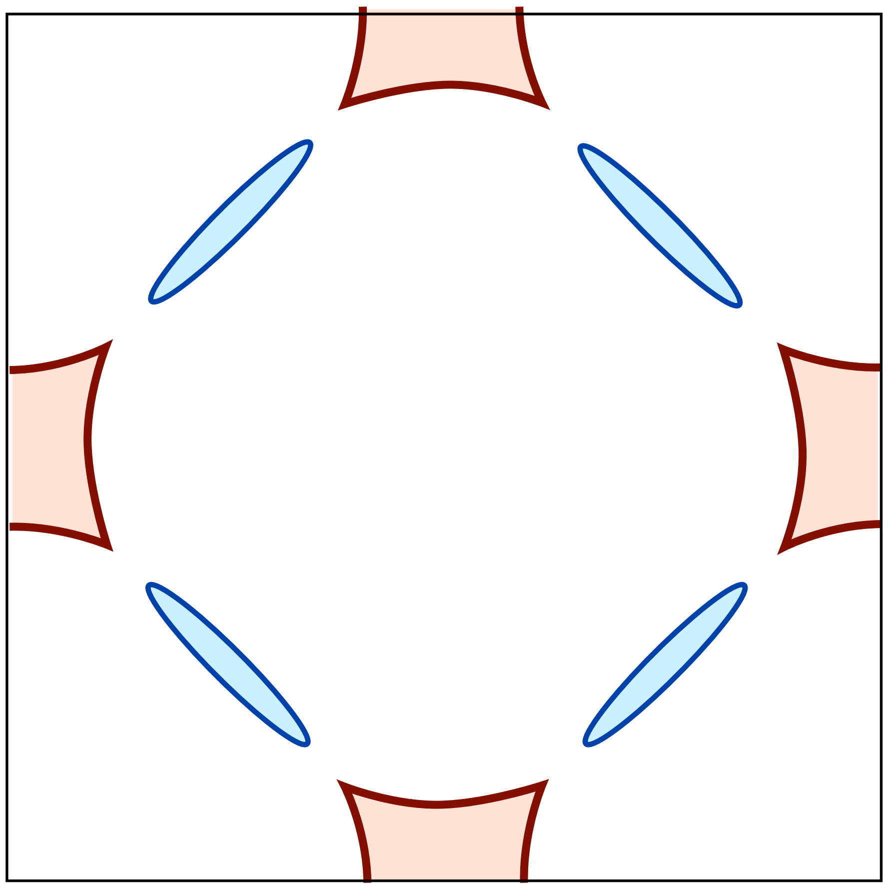
where $\mathbf{K}$ is the ordering wavevector.



Metal with “large” Fermi surface

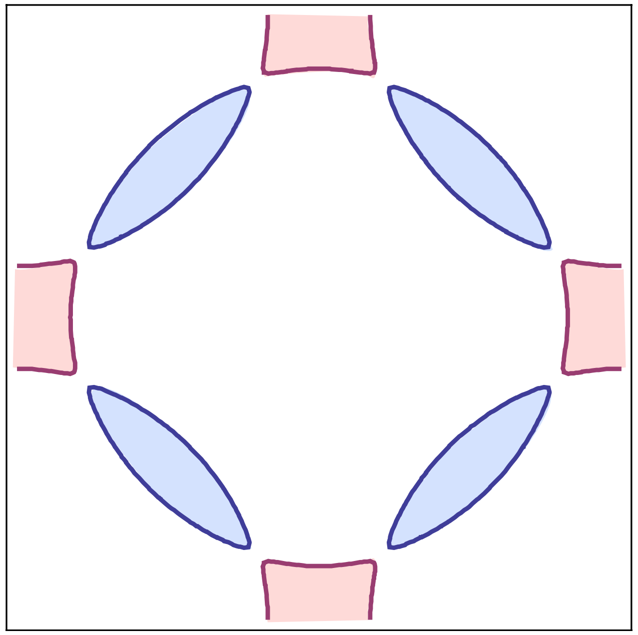


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



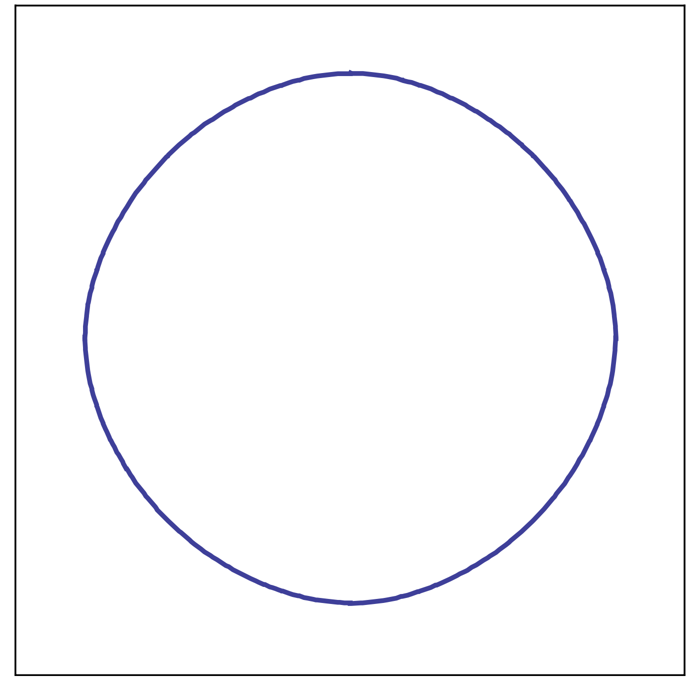
Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets



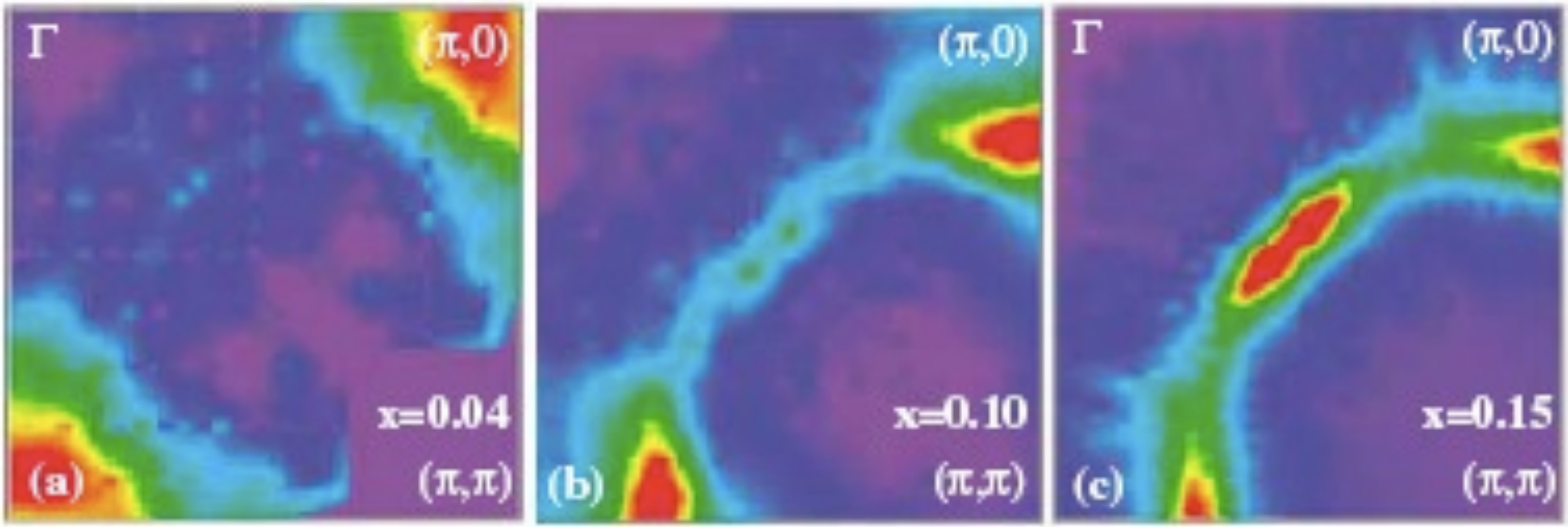
$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

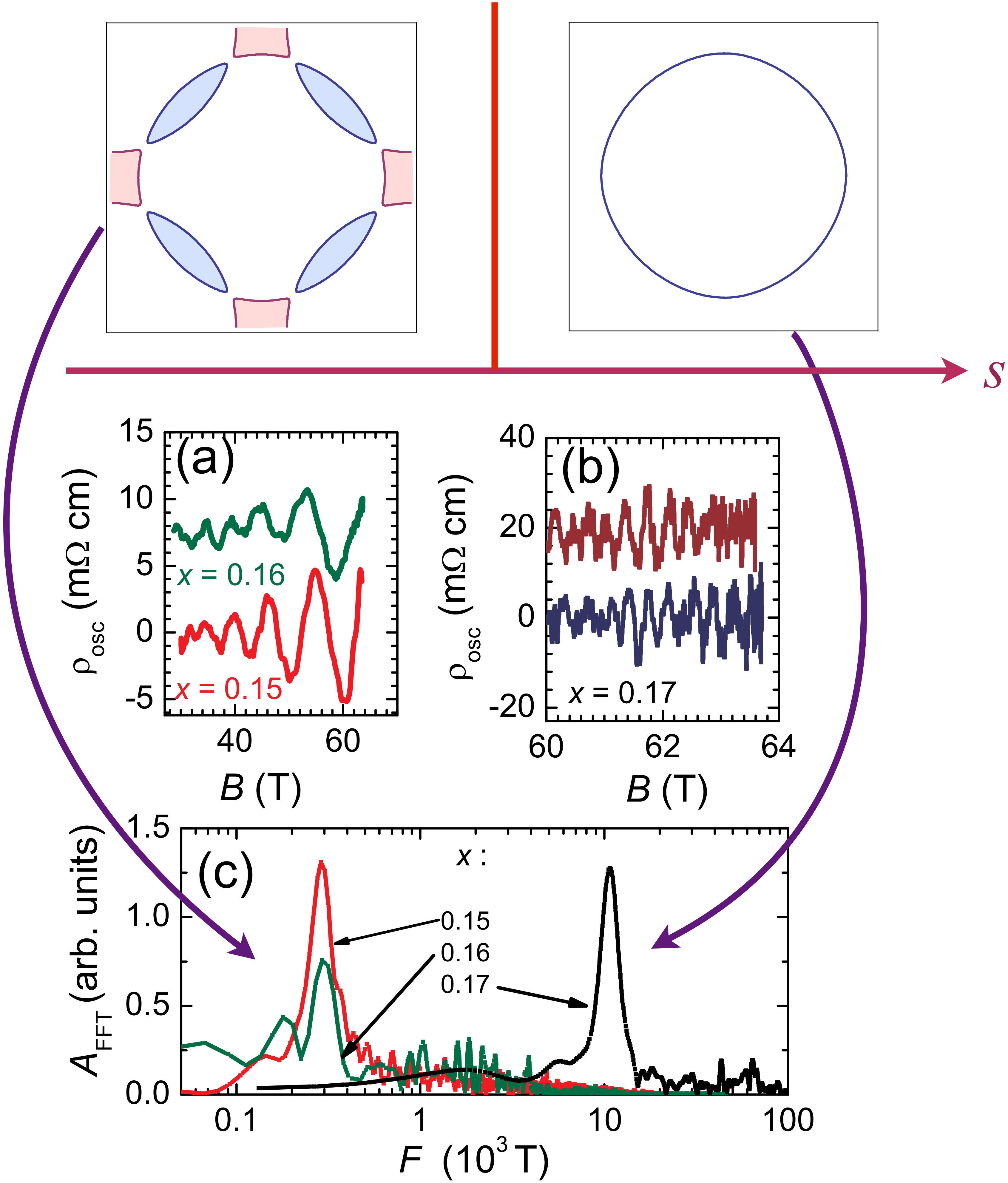
Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Photoemission in $Nd_{2-x}Ce_xCuO_4$



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

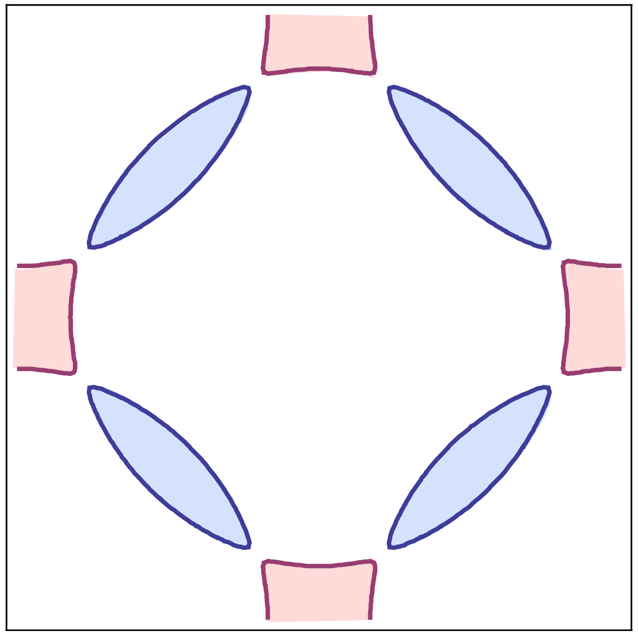


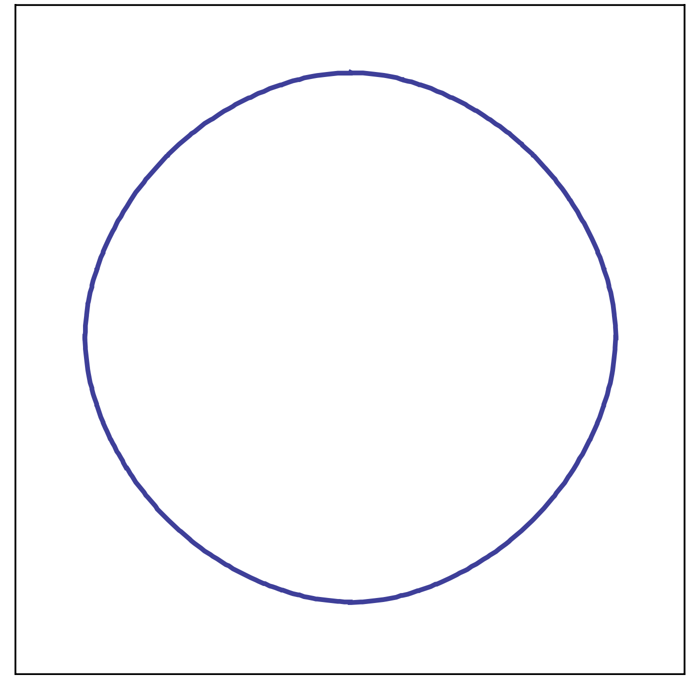
Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

Fermi surface+antiferromagnetism





$\langle \vec{\varphi} \rangle \neq 0$

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Metal with electron and hole pockets

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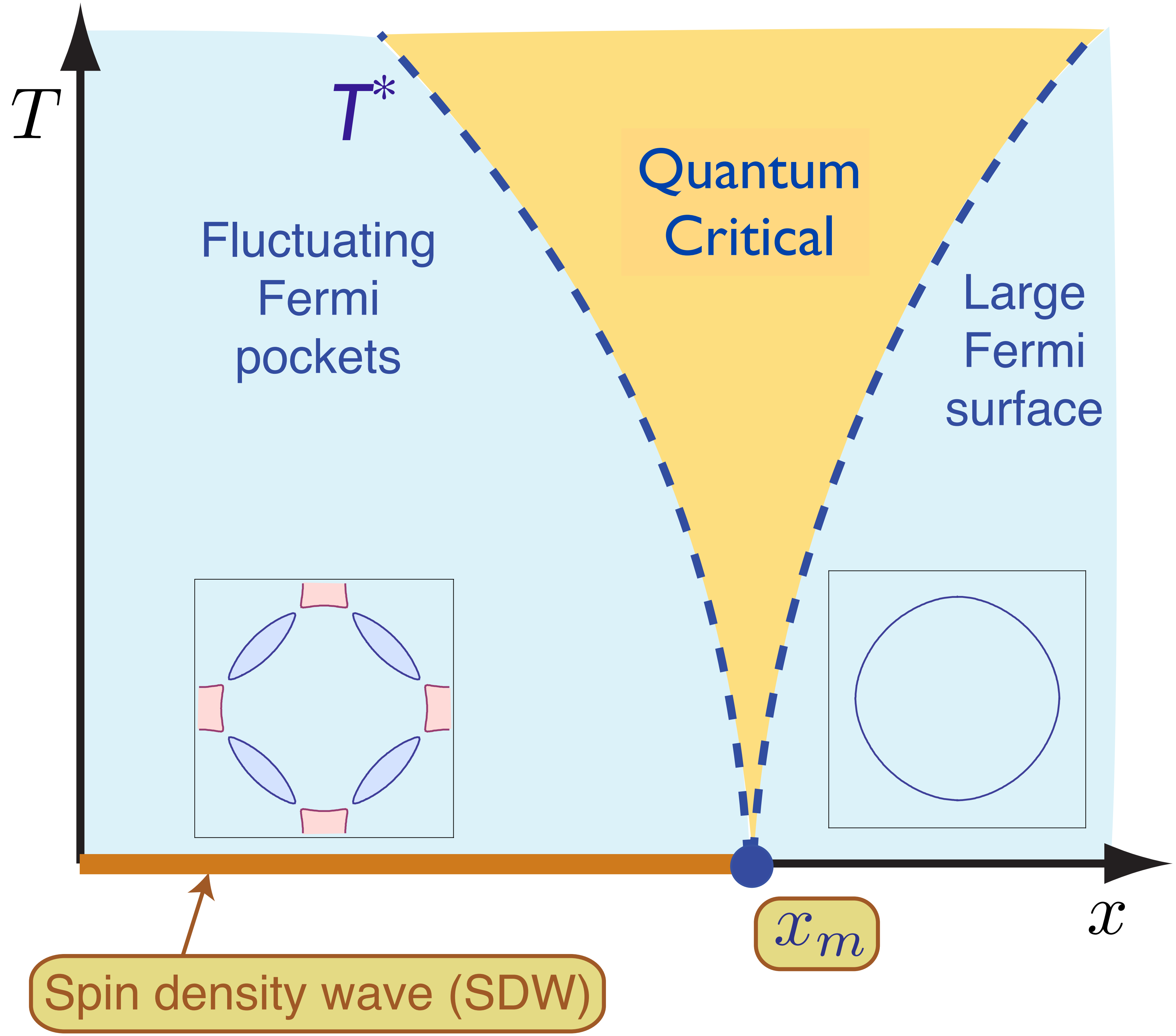
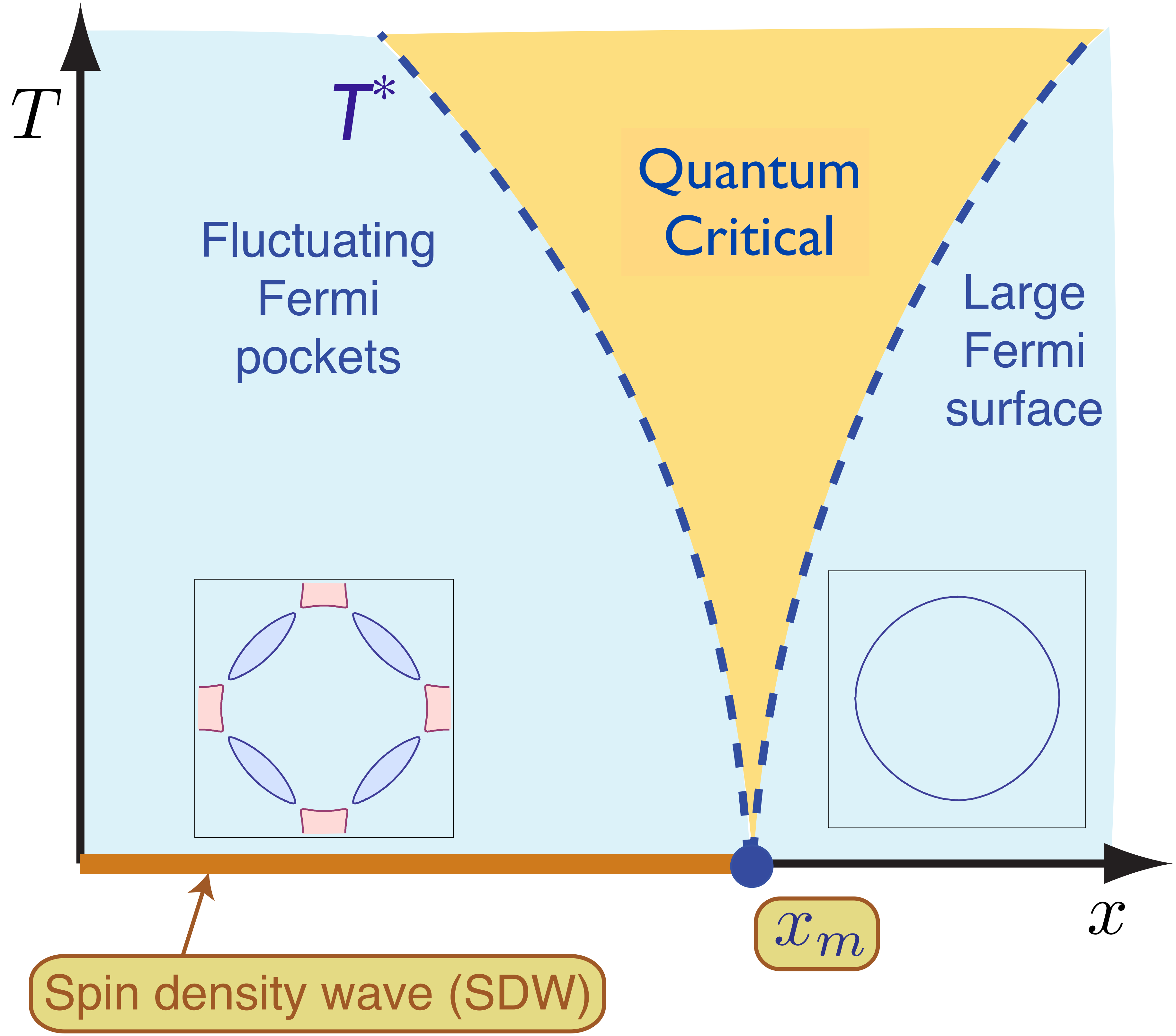
Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

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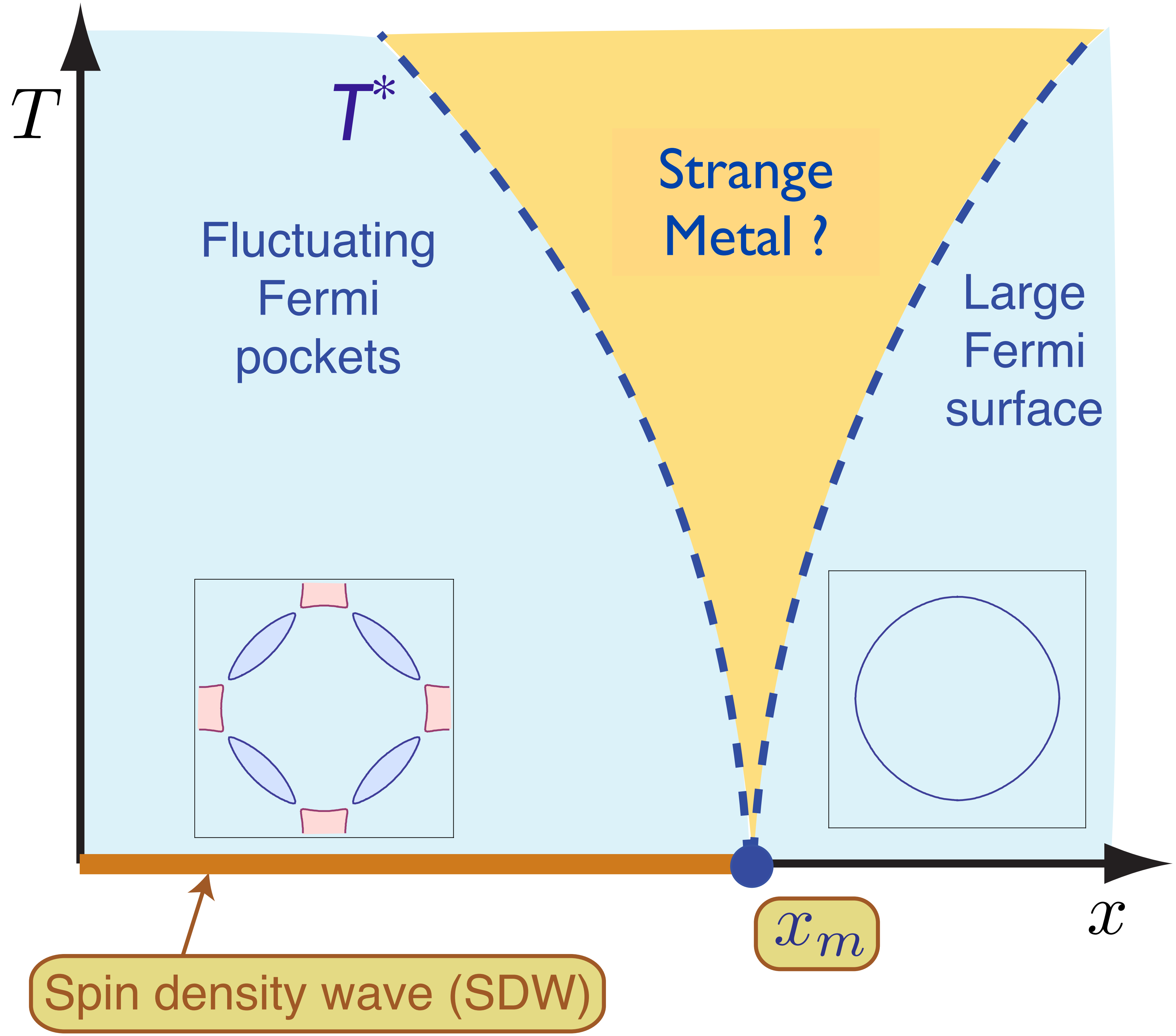

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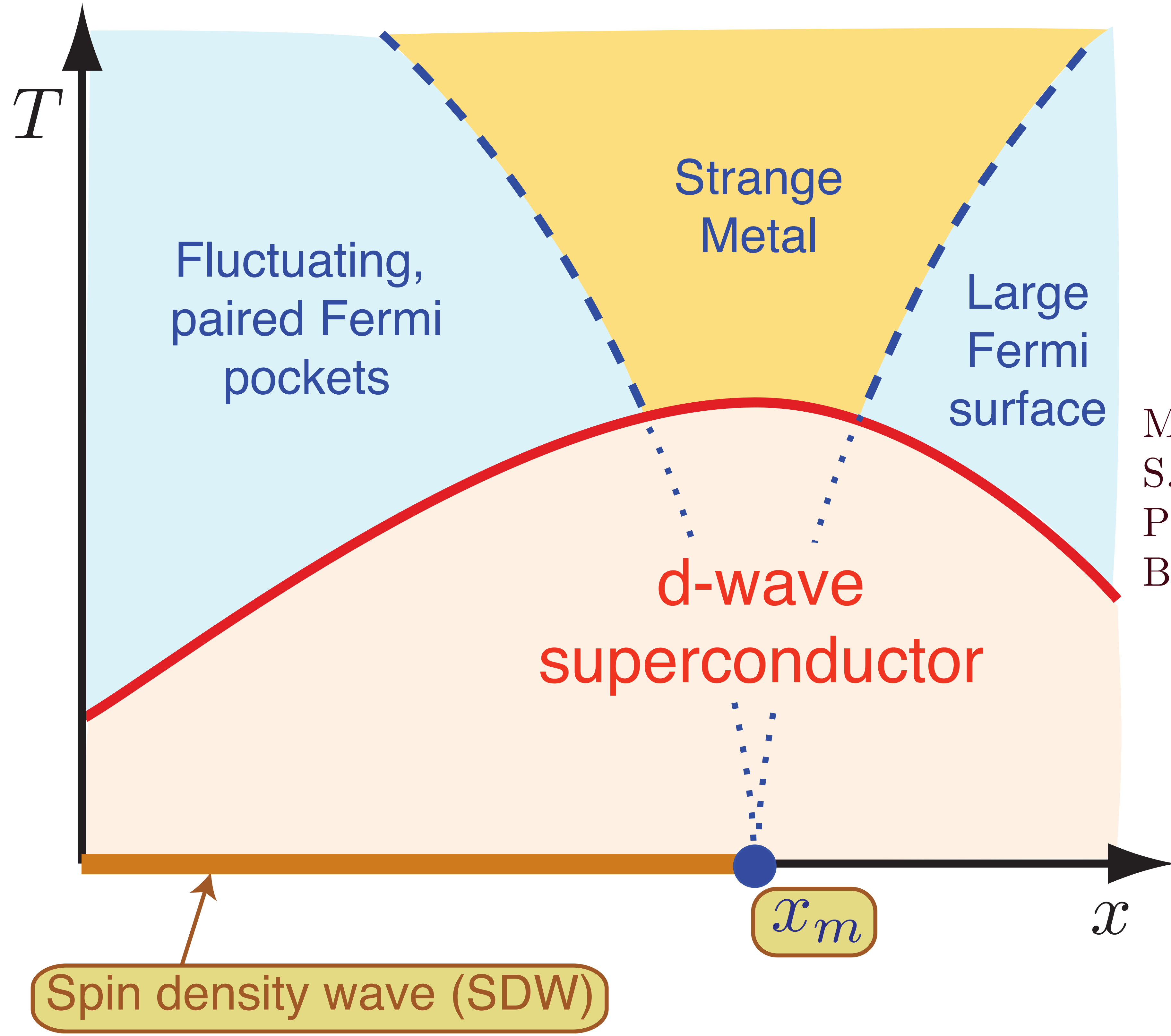
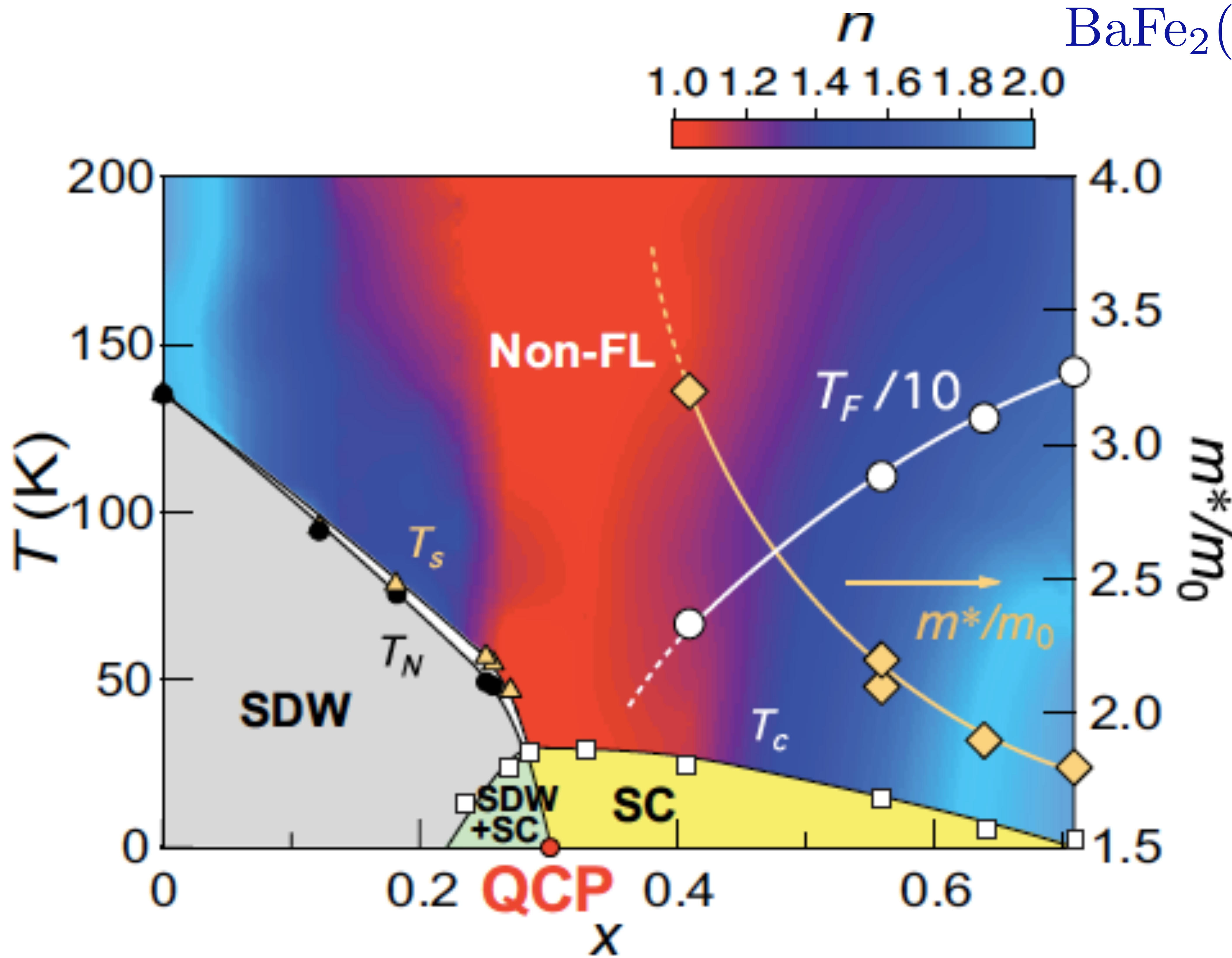


M. A. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)

SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory

$BaFe_2(As_{1-x}\ P_x)_2$



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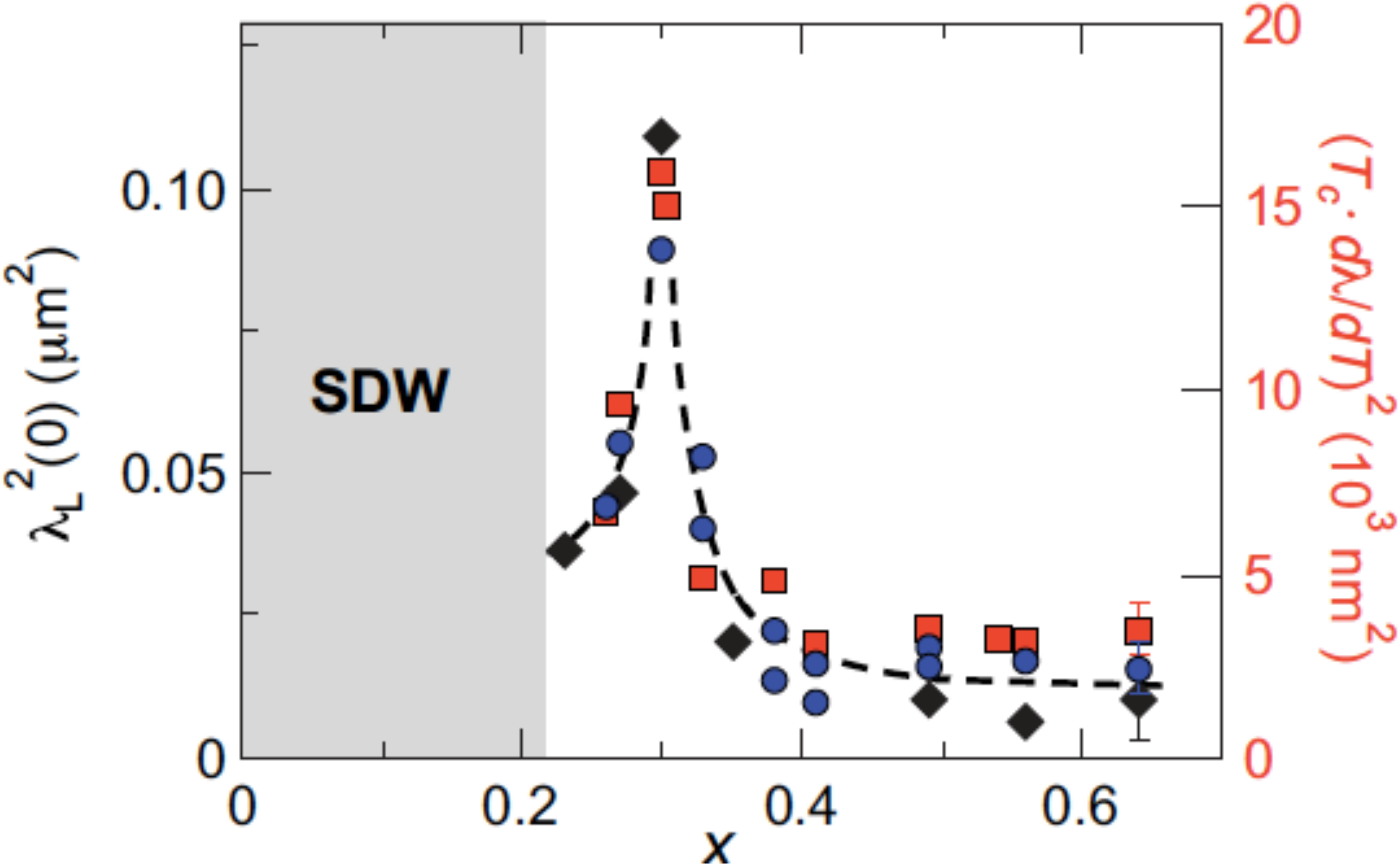

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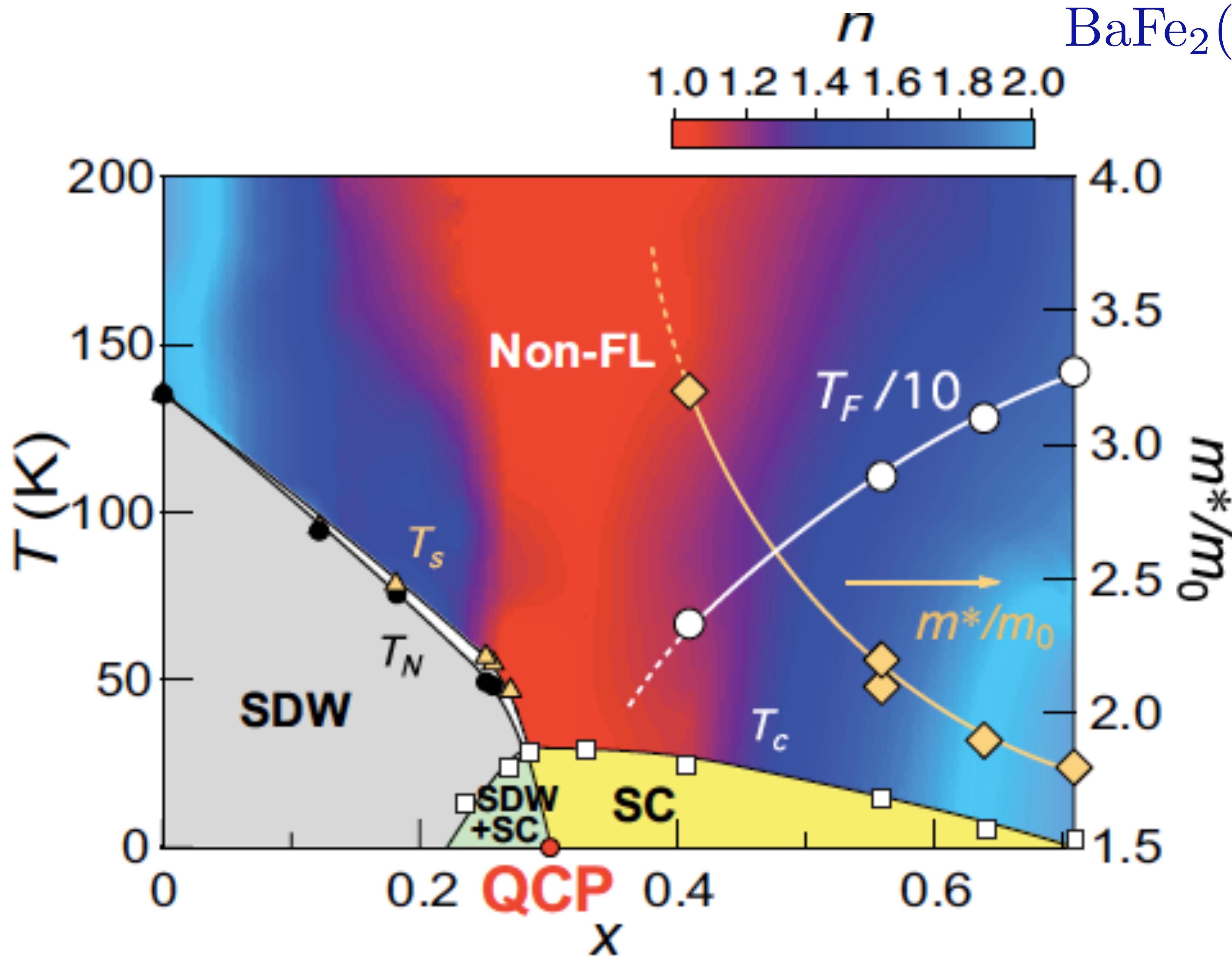

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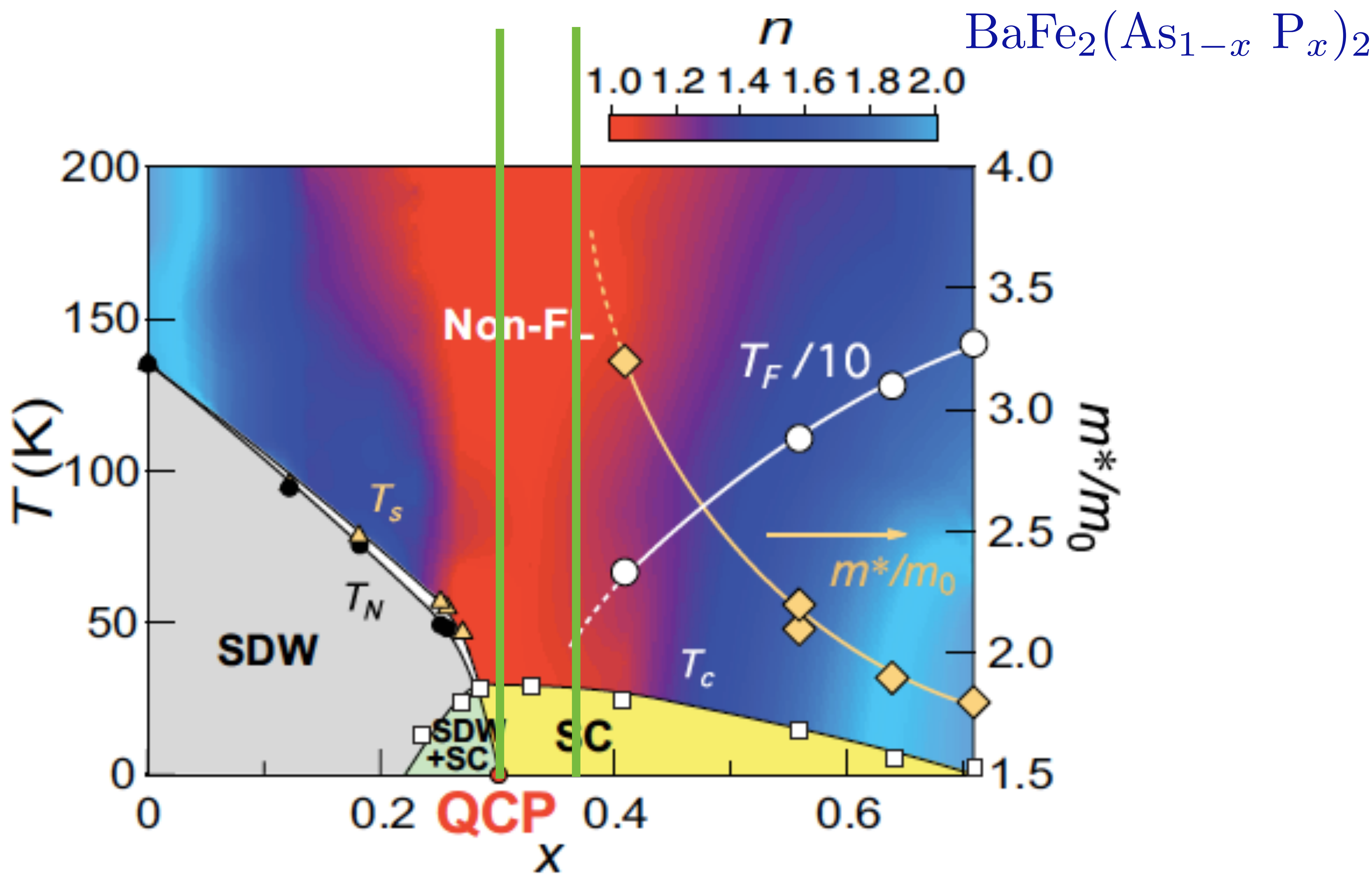
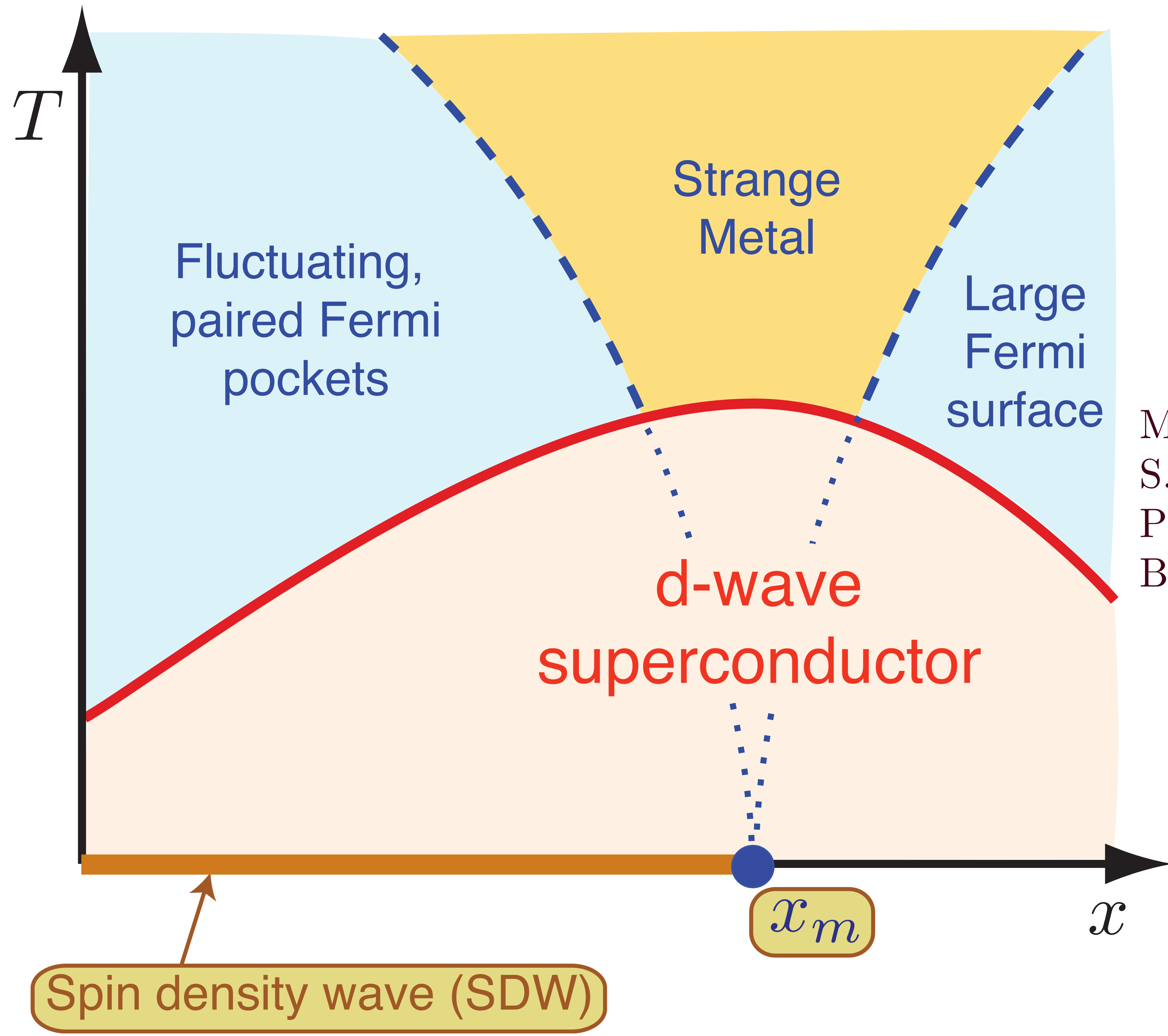


Notice shift between the position of the QCP in the superconductor, and the divergence in effective mass in the metal measured at high magnetic fields

Theory of quantum criticality in the cuprates



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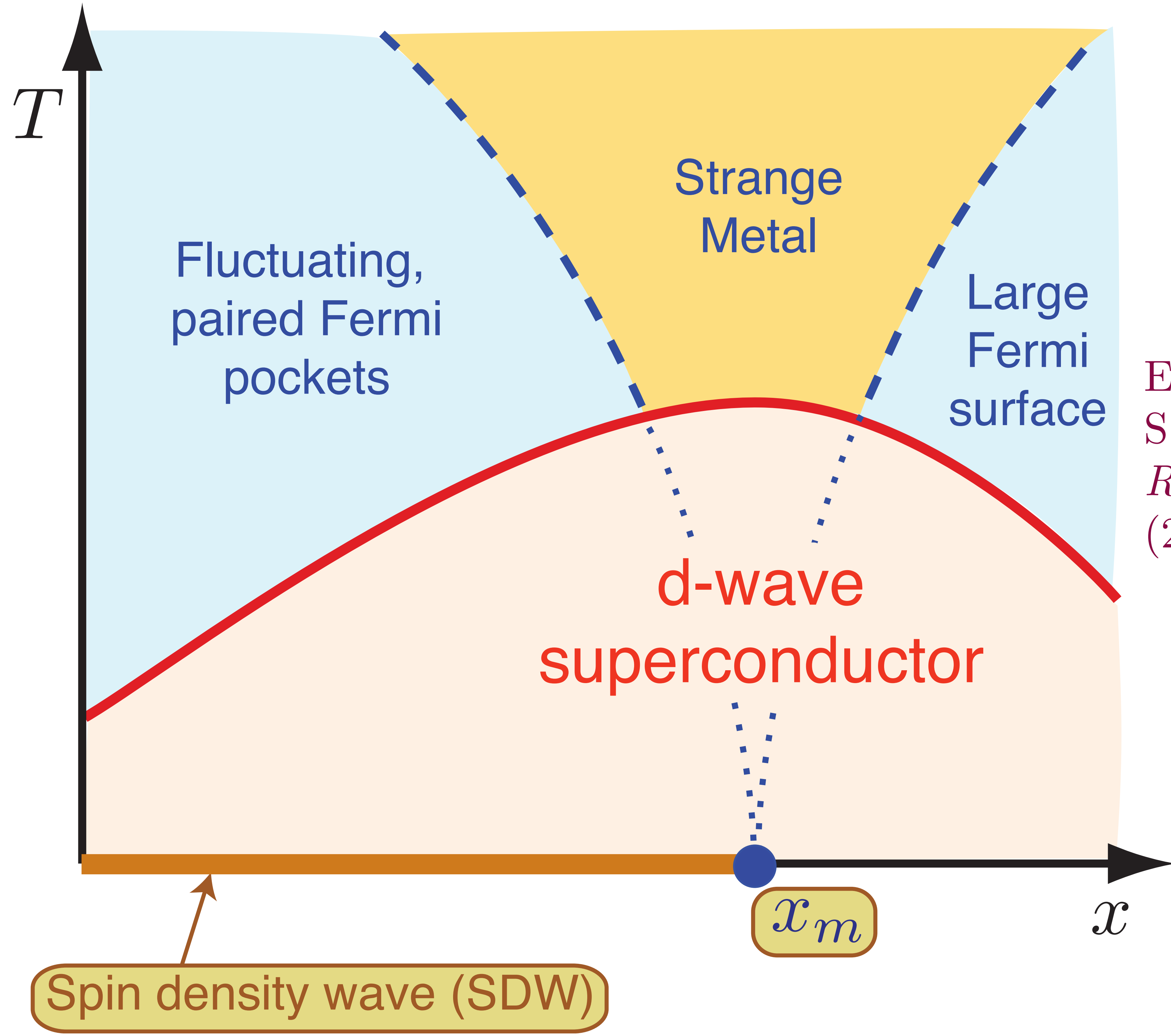

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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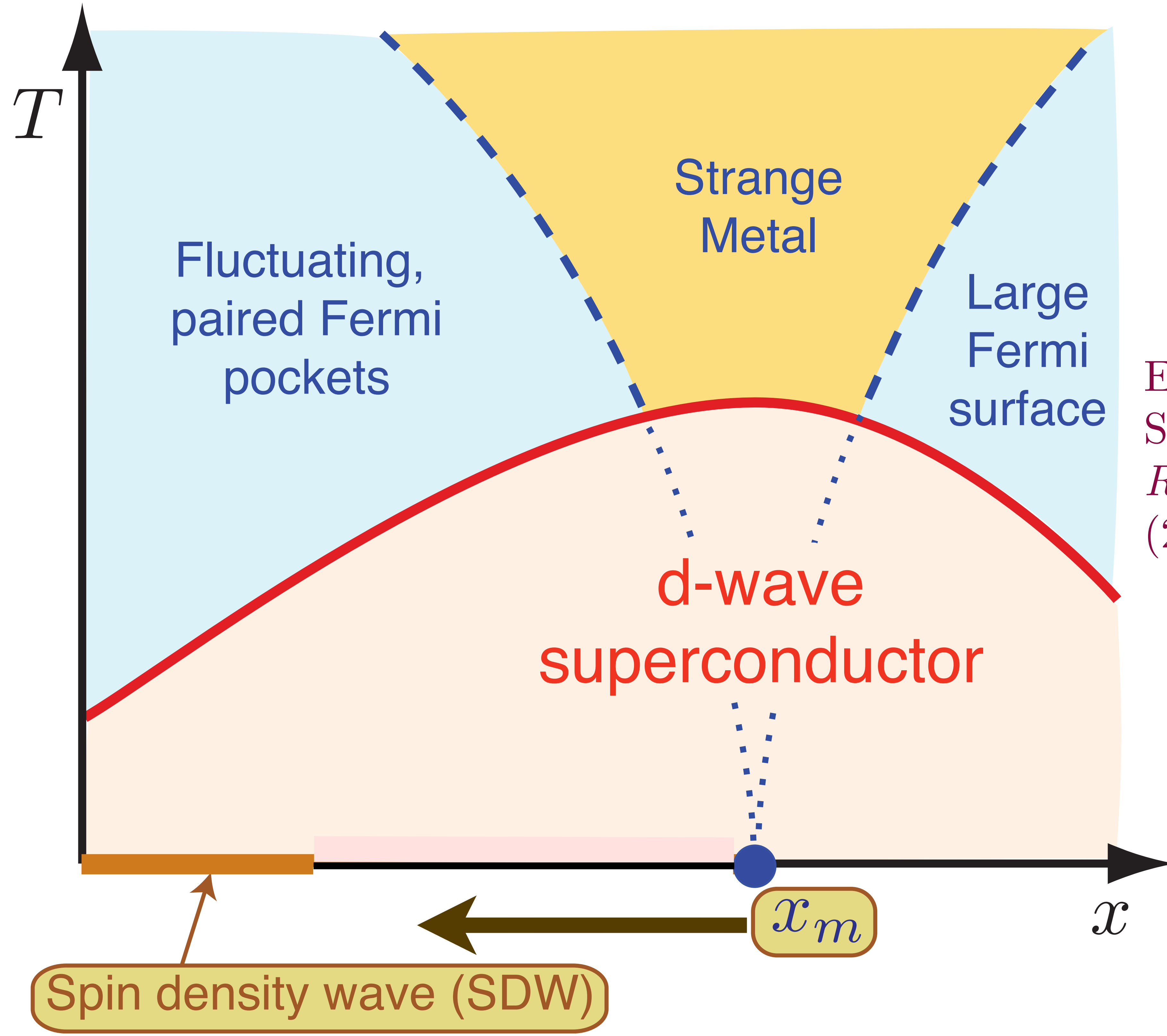

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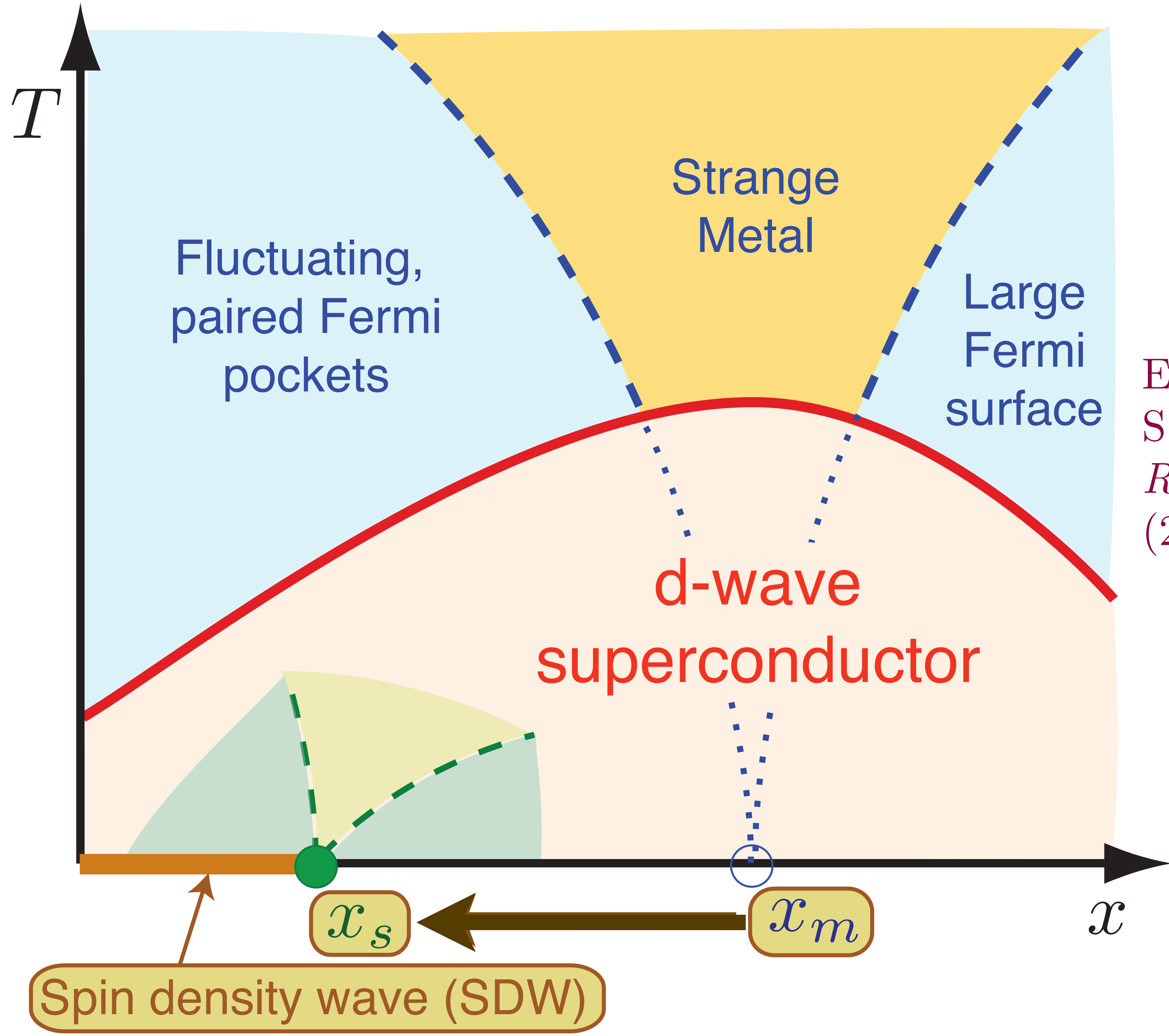

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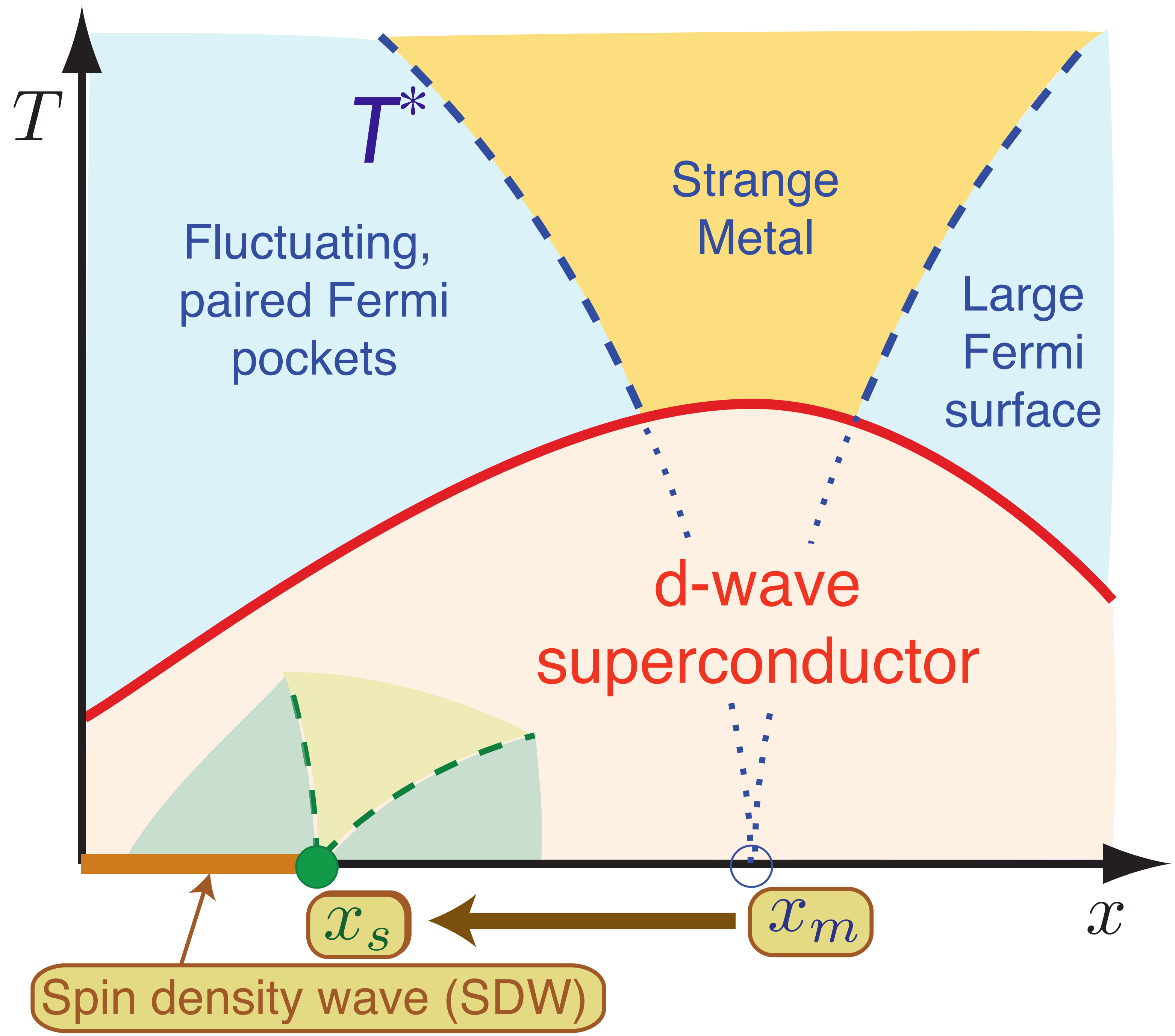
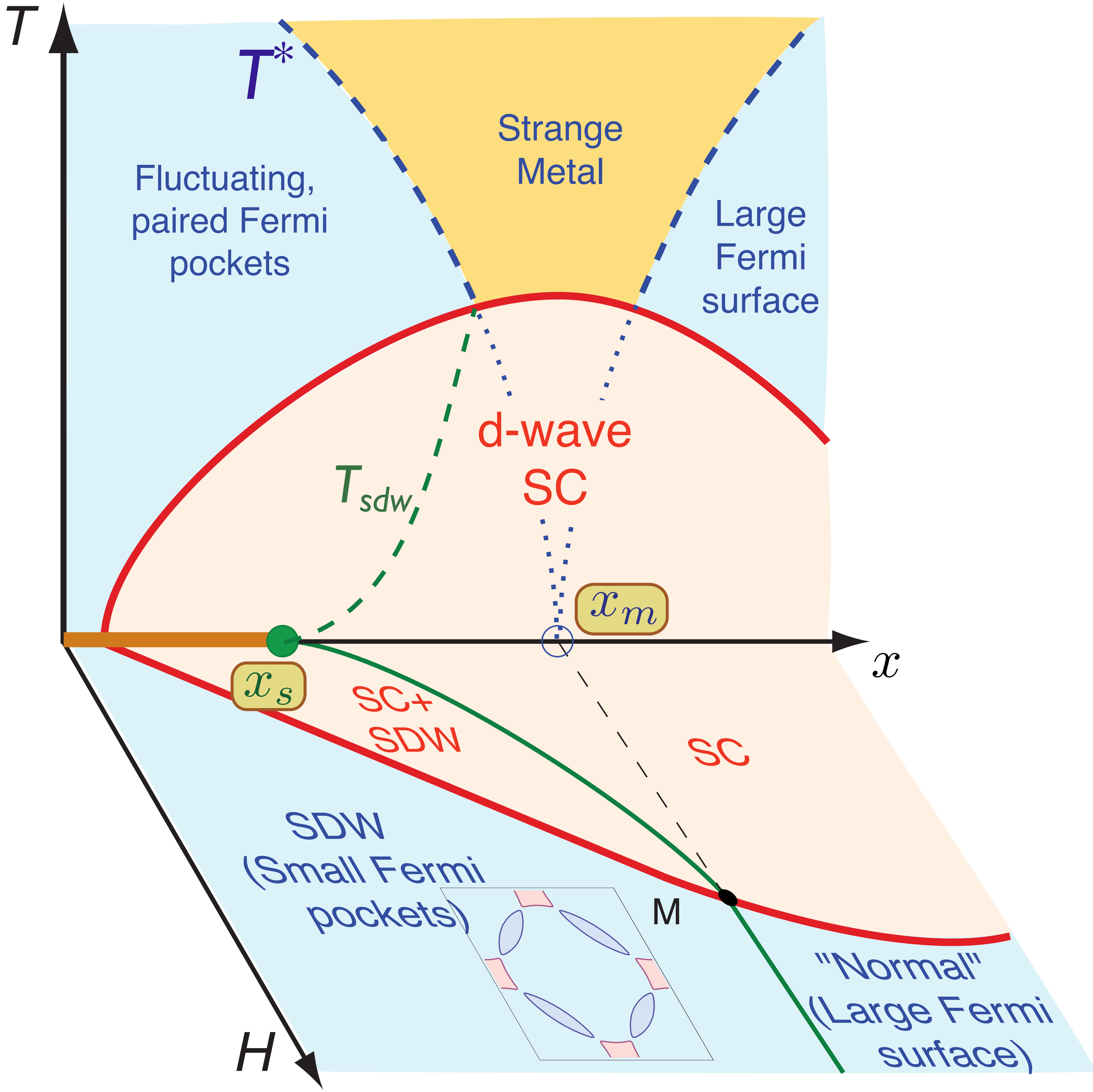


T
T^*
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
d-wave superconductor
x_s
x_m
x
Spin density wave (SDW)

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



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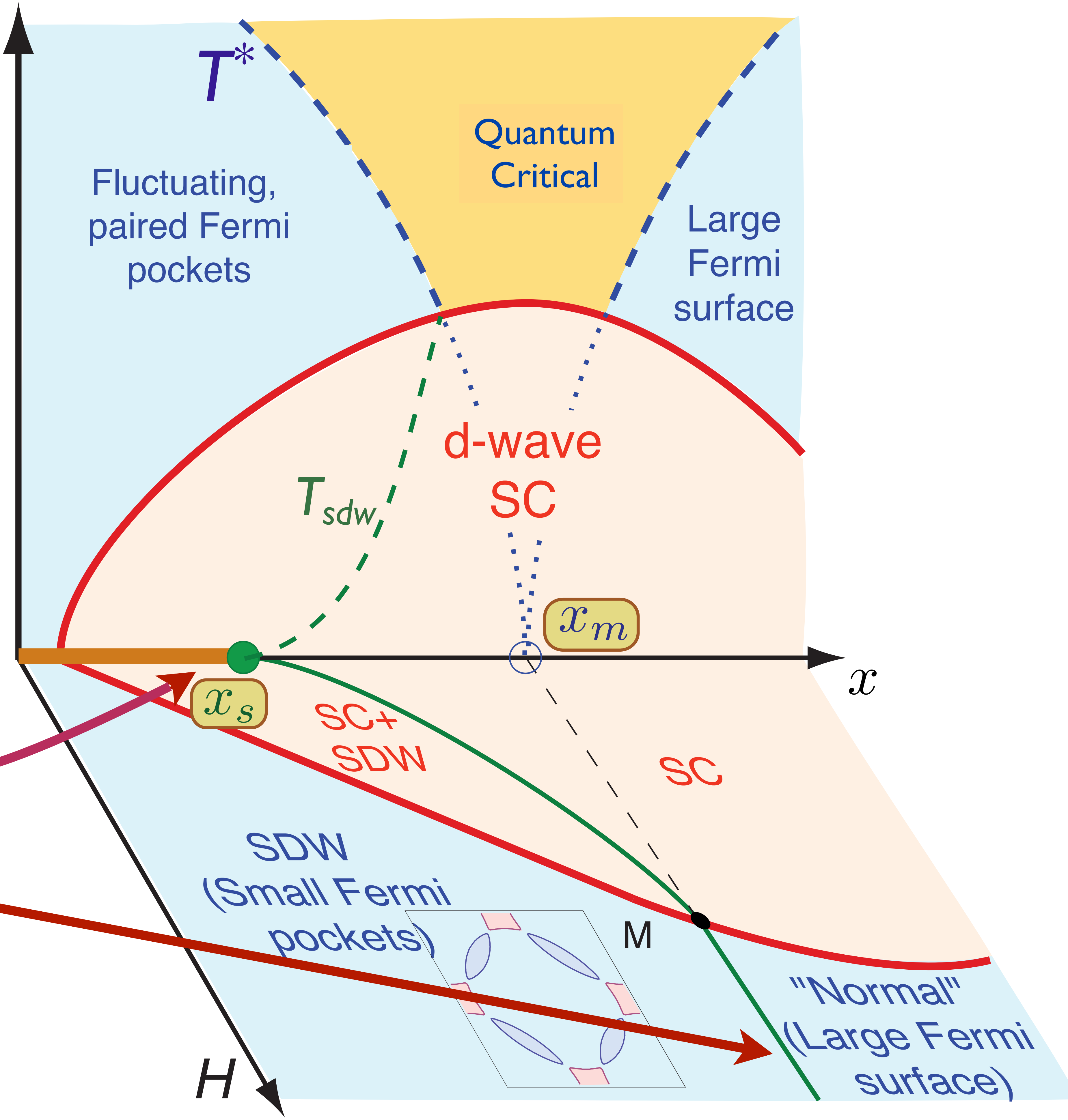

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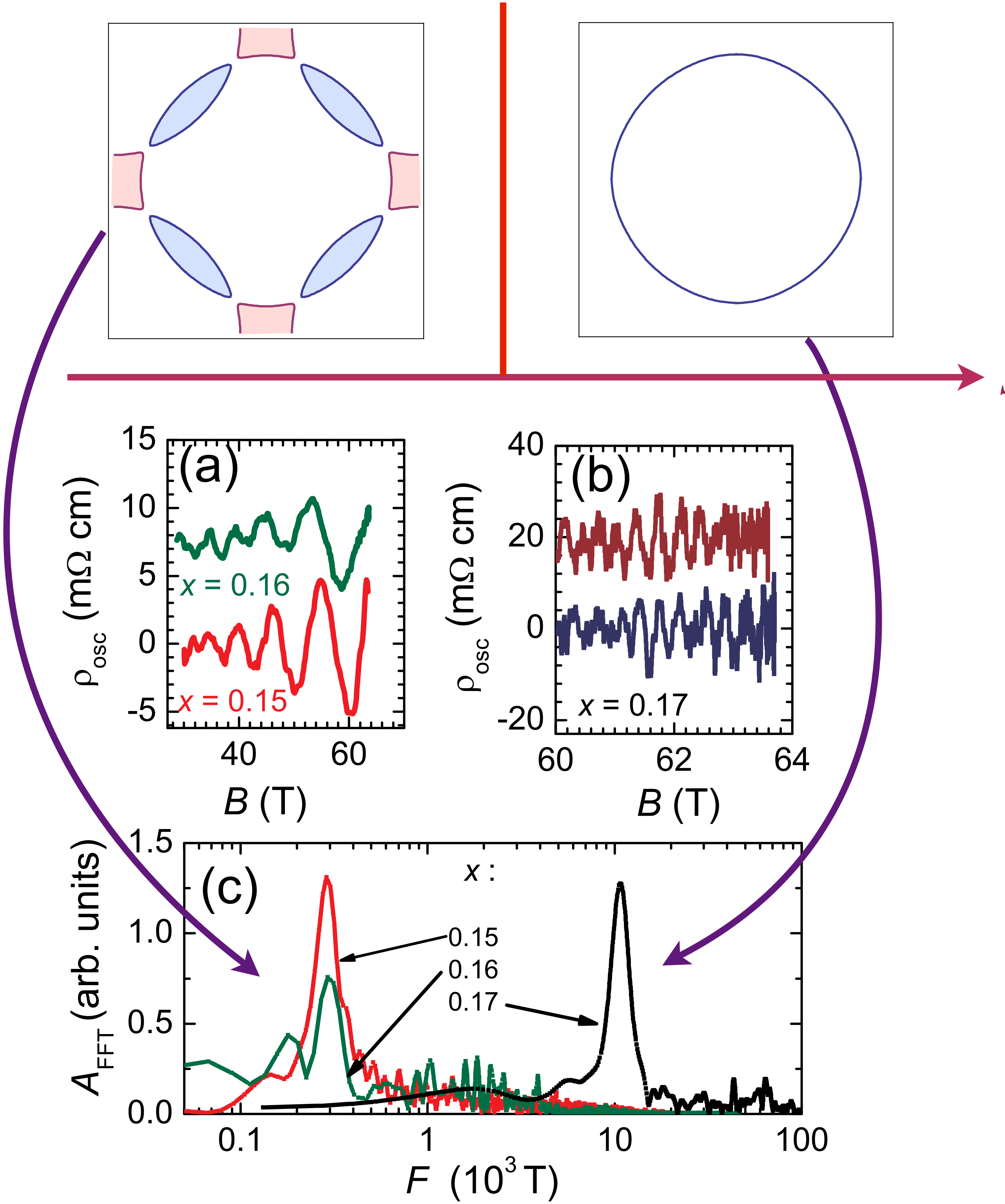

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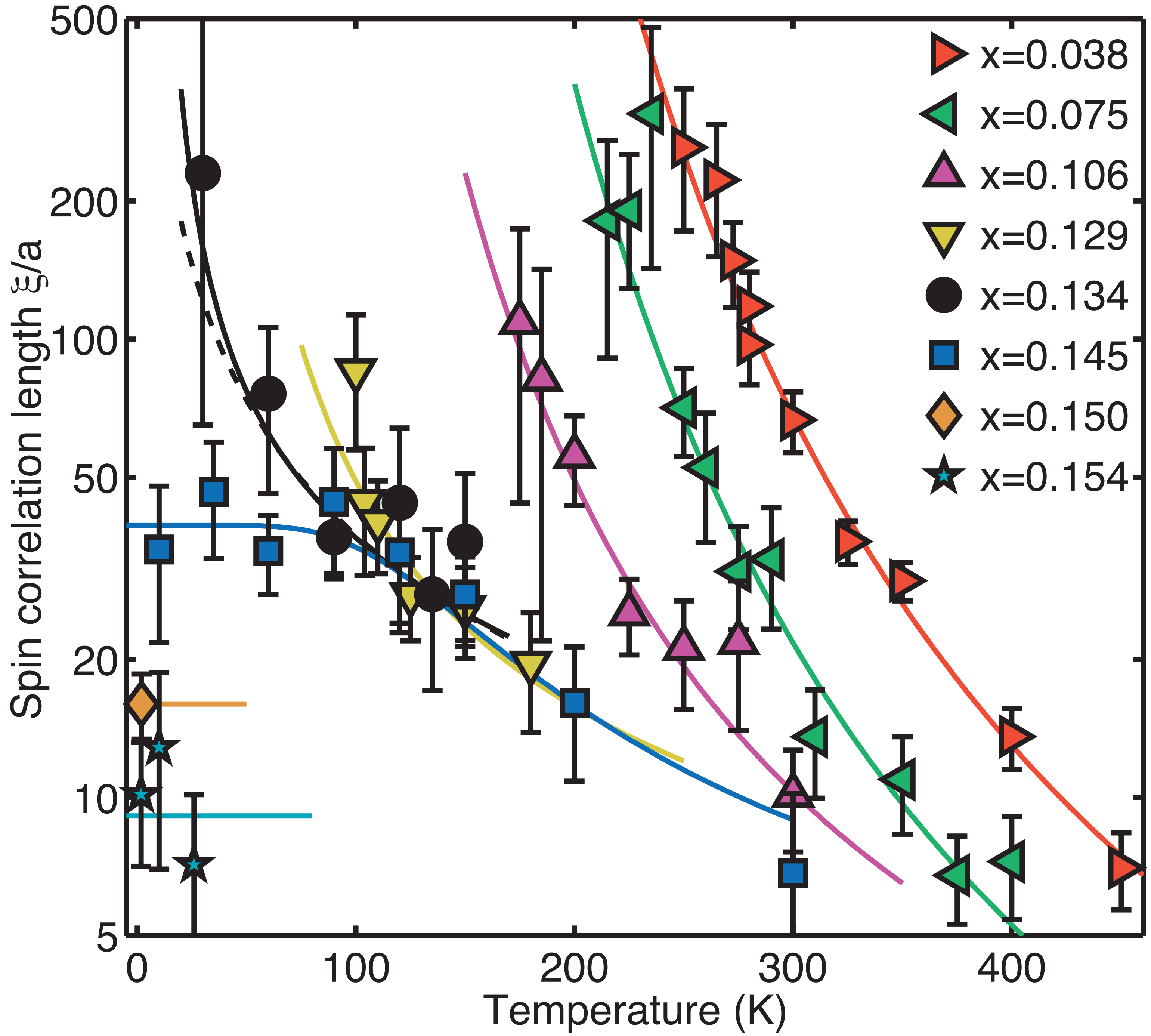

E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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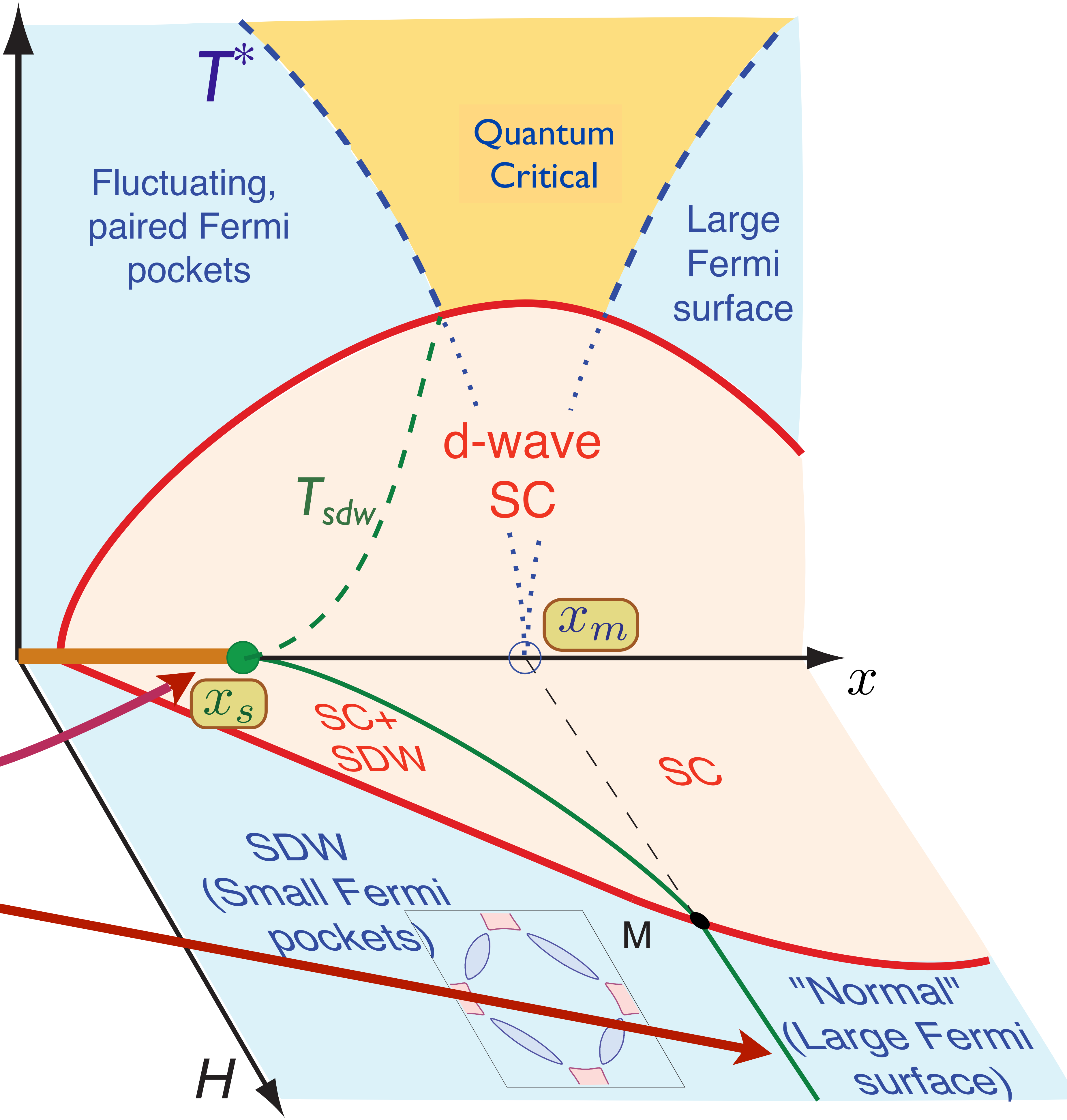
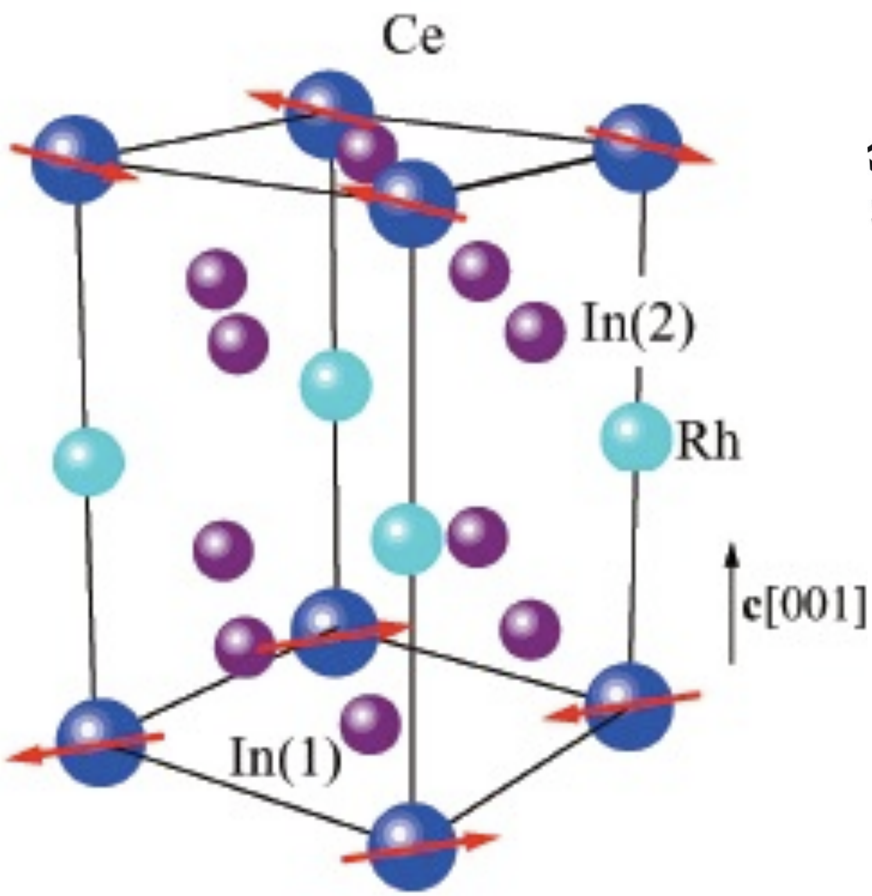
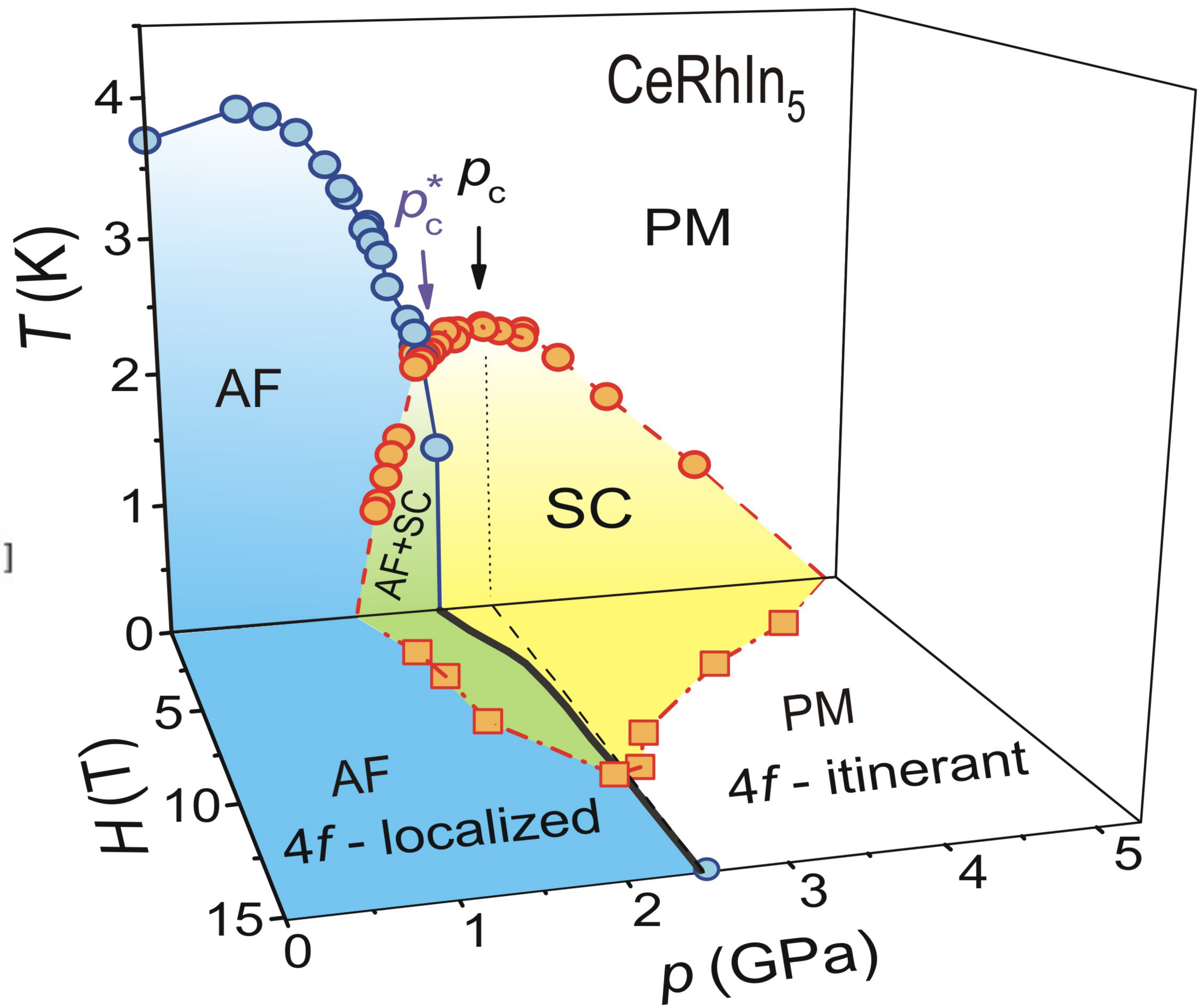
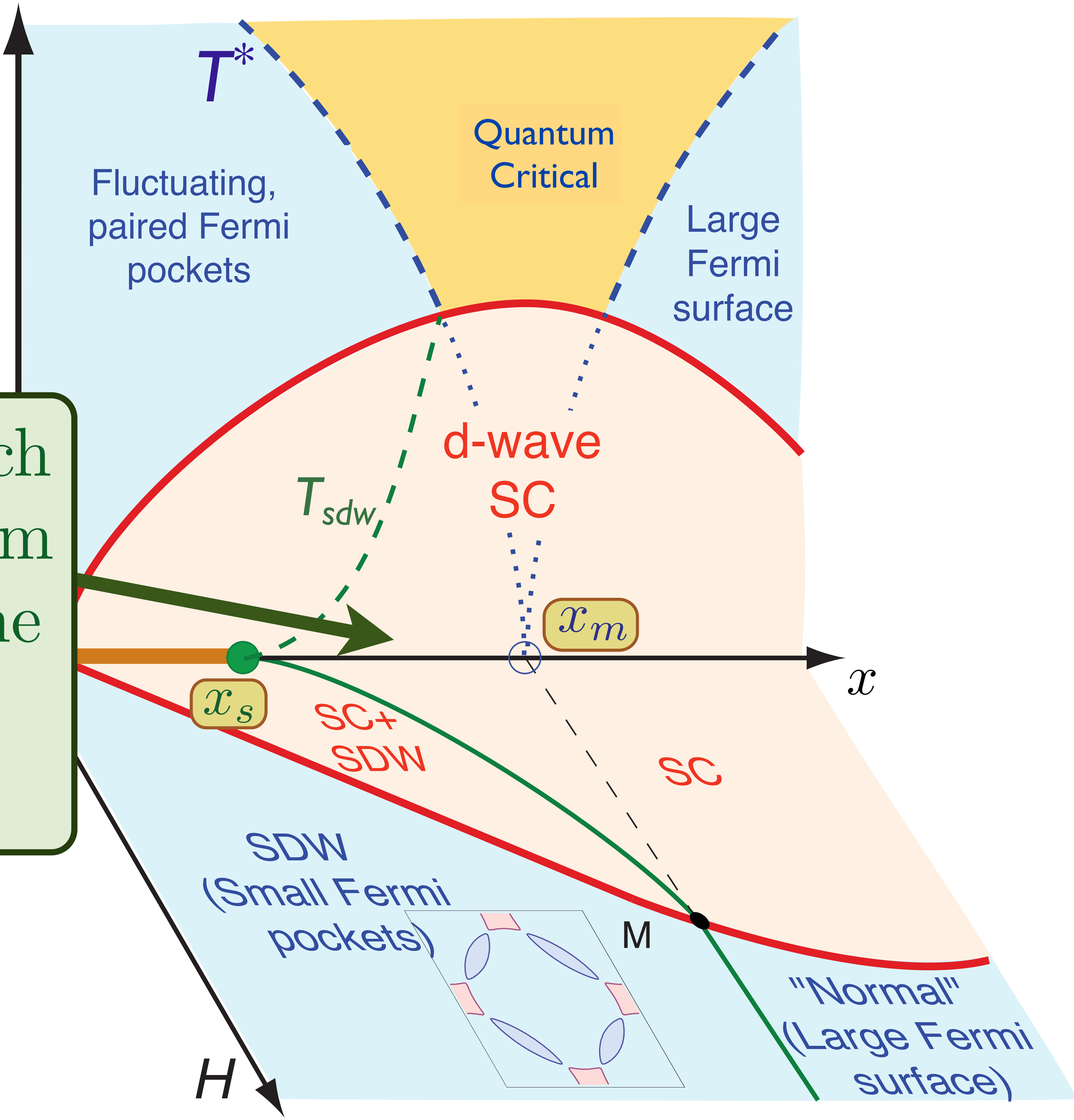


Similar phase diagram for $CeRhIn_5$



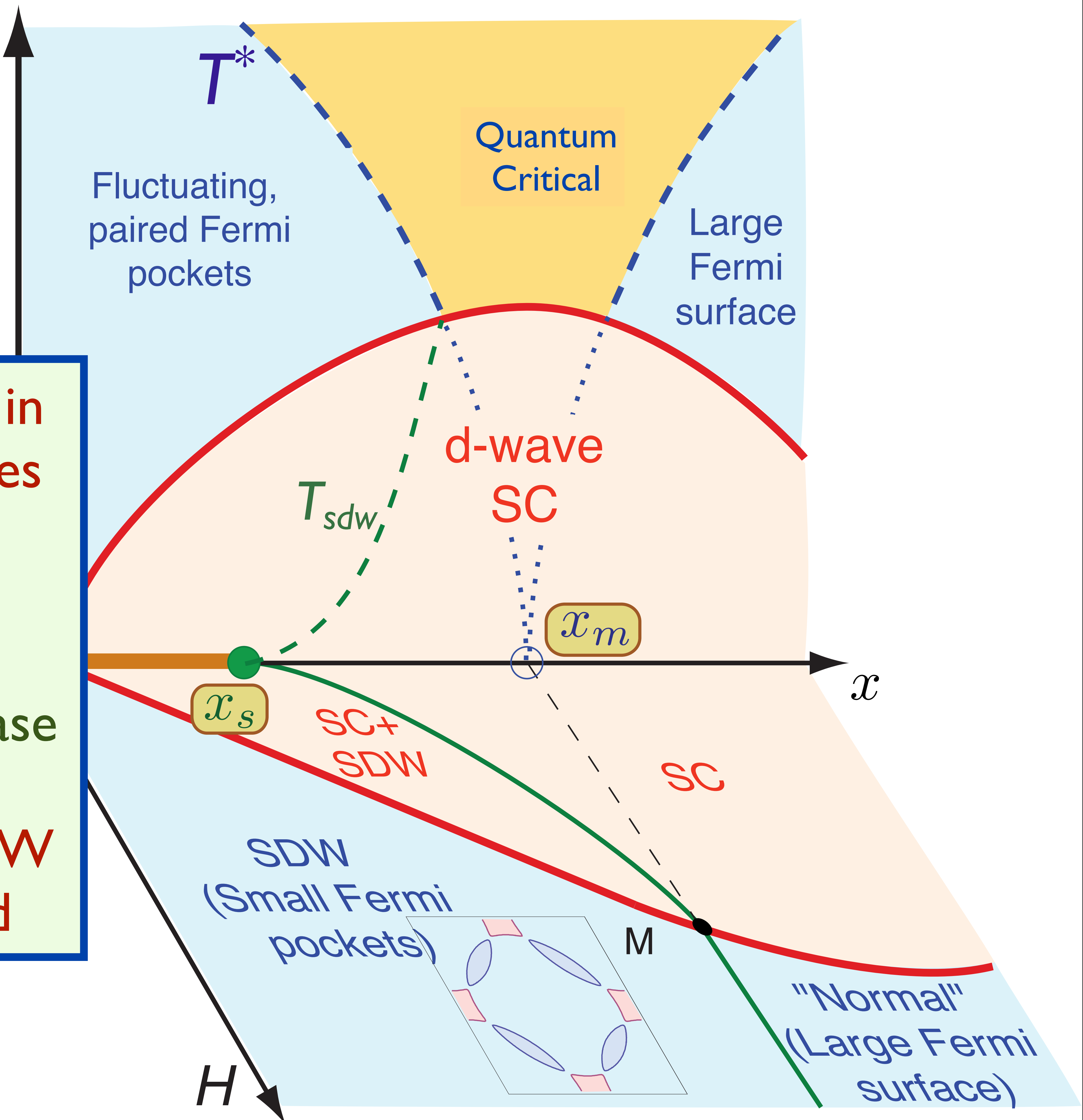


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)



There is a much larger shift from x_m to x_s in the hole-doped cuprates.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Many experiments in hole-doped cuprates have presented evidence for the predicted green quantum phase transition line from SC to SC+SDW in a magnetic field

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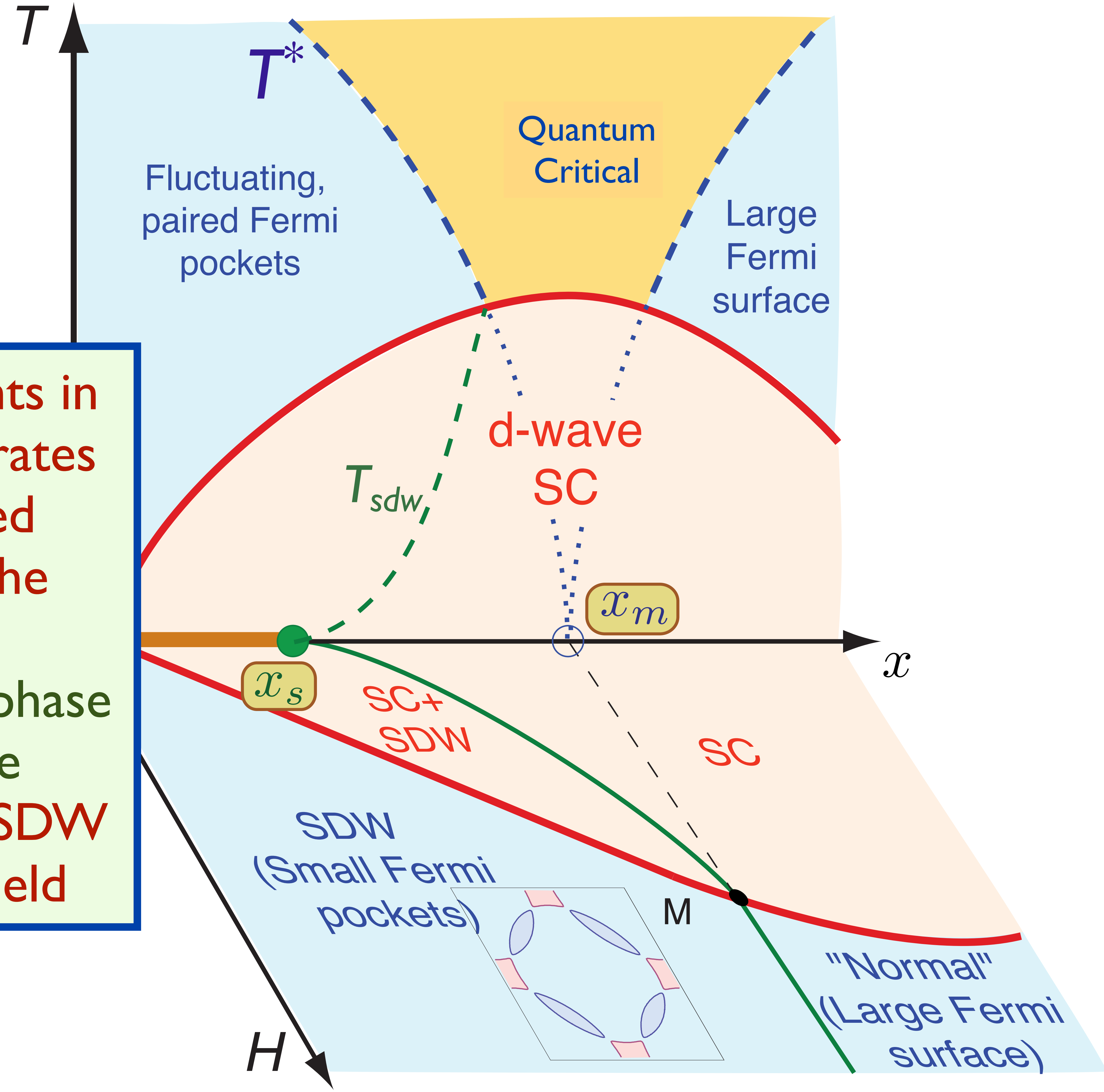

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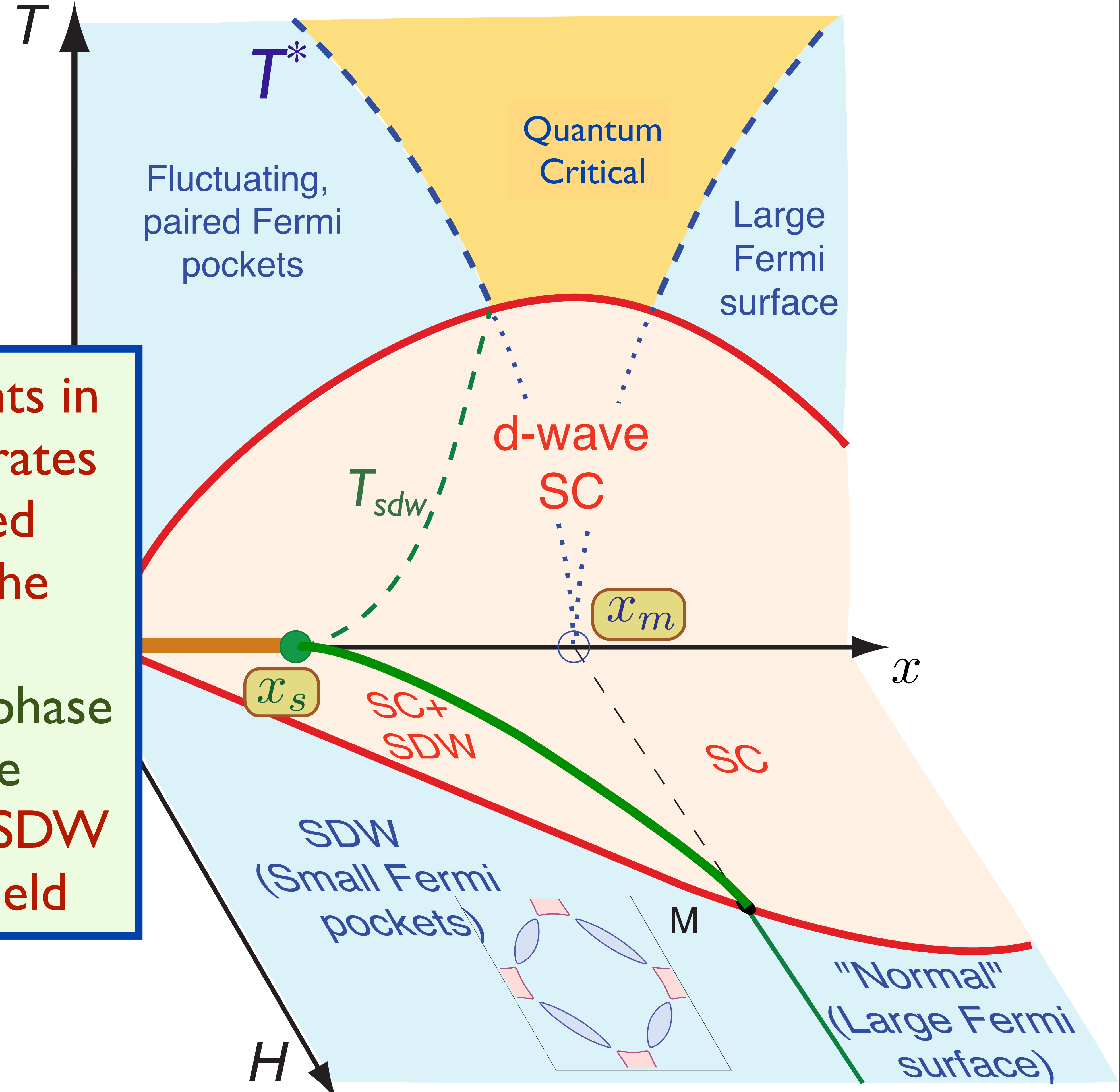

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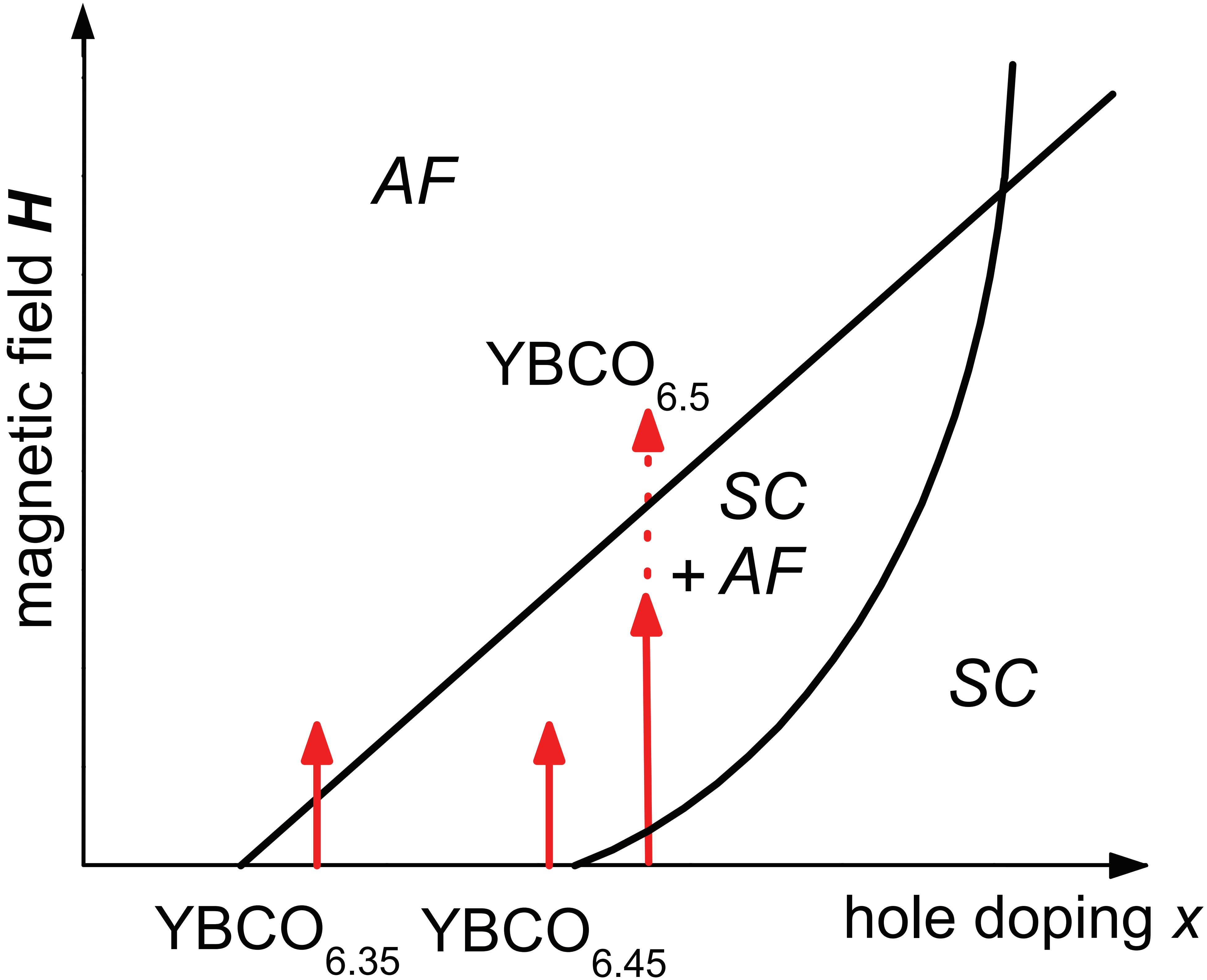
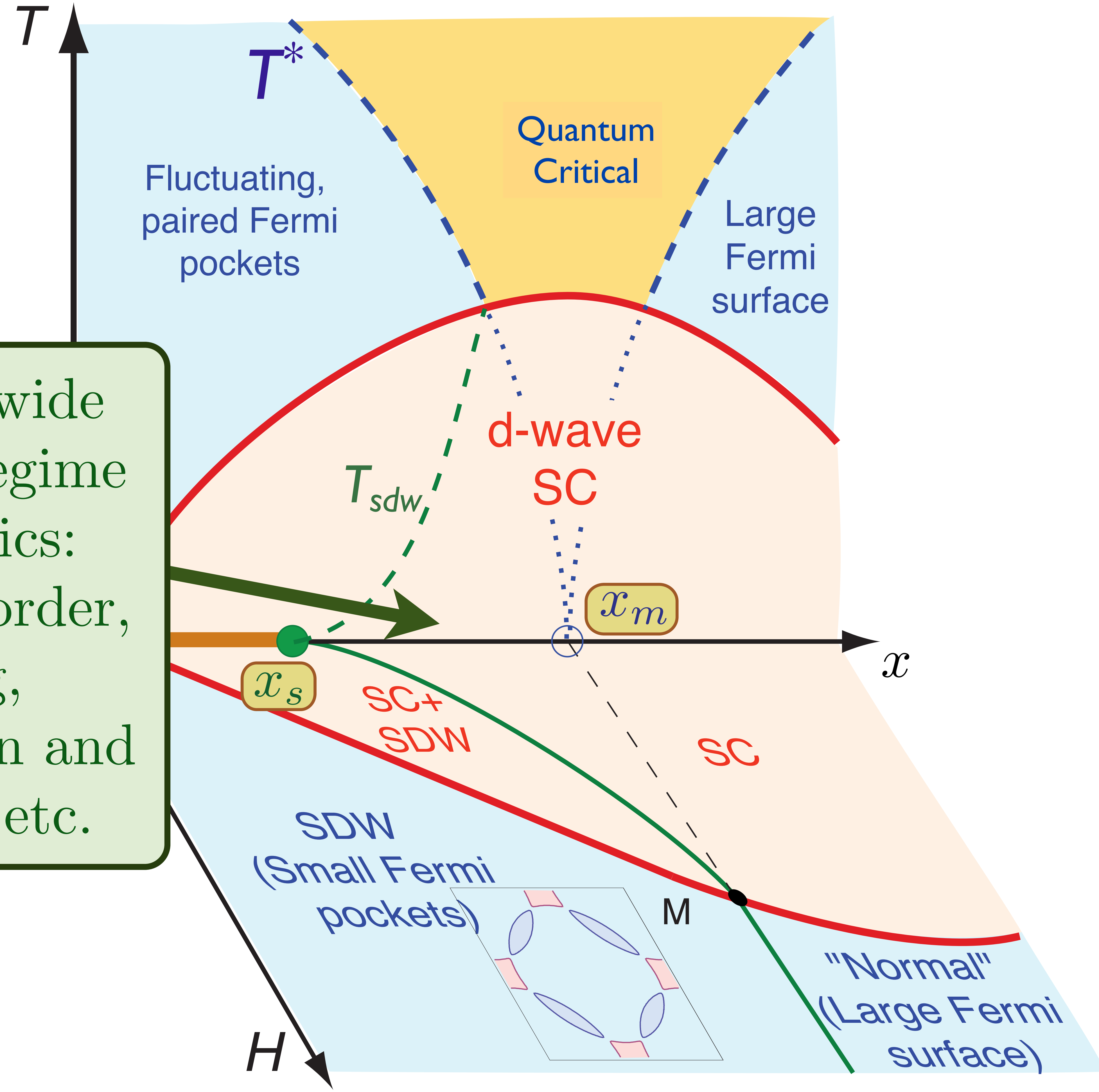


D. Haug, V. Hinkov, Y. Sidis, P. Bourges, N. B. Christensen, A. Ivanov, T. Keller, C. T. Lin, and B. Keimer, *New J. Phys.* **12**, 105006 (2010)



This opens a wide intermediate regime for new physics: bond-nematic order, T-breaking, fractionalization and Mott physics etc.

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

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3. Quantum Monte Carlo without the sign problem

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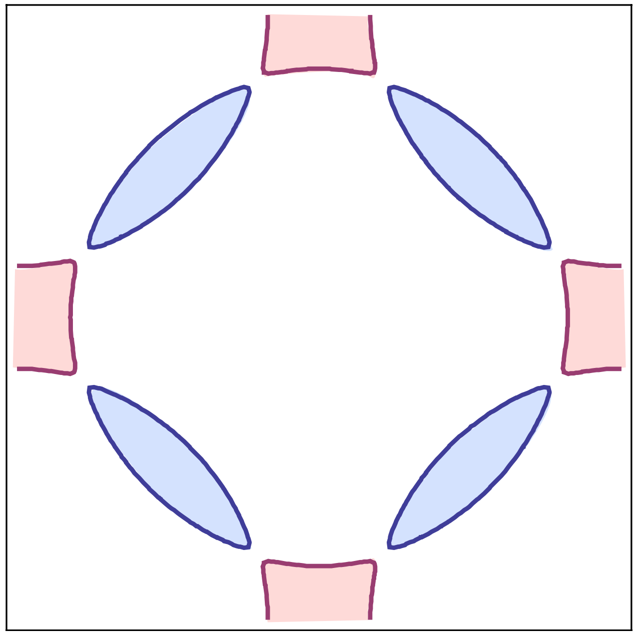
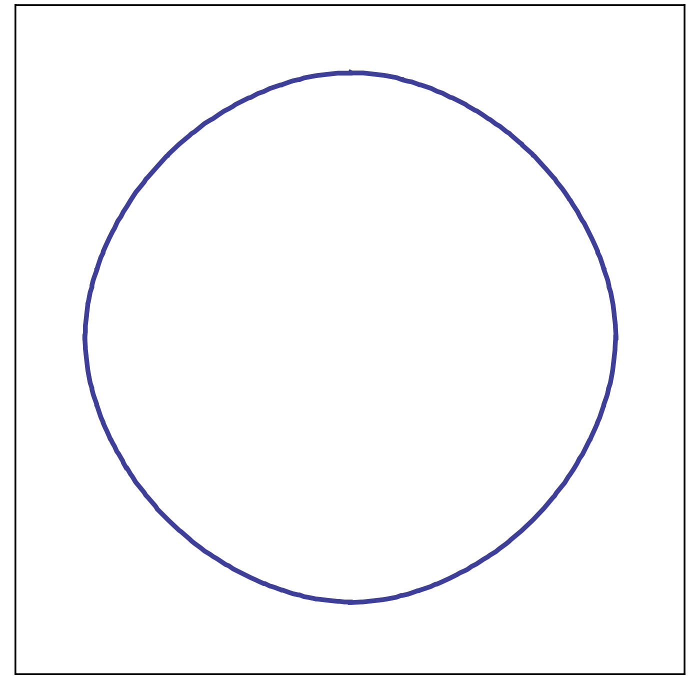
Quantum phase transition with Fermi surface reconstruction





$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

Increasing interaction

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Boson-fermion theory for both phases

$$\mathcal{S} = \int d^2 r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi} \right]$$

$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

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$$\mathcal{L}_{c\varphi} = \lambda\, \varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b.$$

"Yukawa" coupling between fermions and antiferromagnetic order:
$\lambda^2 \sim U$, the Hubbard repulsion

Hertz-Moriya-Millis theory

- Integrate out Fermi surface quasiparticles and obtain an effective theory for the order parameter $\vec{\varphi}$ alone.

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- This is dangerous, and will lead to non-local in the $\vec{\varphi}$ theory. Hertz focused on only the simplest such non-local term.

Hertz-Moriya-Millis theory

- Integrate out Fermi surface quasiparticles and obtain an effective theory for the order parameter $\vec{\varphi}$ alone.

- This is dangerous, and will lead to non-local in the $\vec{\varphi}$ theory. Hertz focused on only the simplest such non-local term.

- However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in two spatial dimensions.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

- In $d = 2$, we *must* work in local theores which keeps both the order parameter and the Fermi surface quasiparticles "alive".

Sung-Sik Lee, *Phys. Rev. B* **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075128 (2010)

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- In $d = 2$, we *must* work in local theores which keeps both the order parameter and the Fermi surface quasiparticles "alive".
- The theories can be organized in a $1/N$ expansion, where N is the number of fermion "flavors".
- At subleading order, resummation of all "planar" graphics is required (at least): this theory is even more complicated than QCD.

Sung-Sik Lee, *Phys. Rev. B* **80**, 165102 (2009)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)
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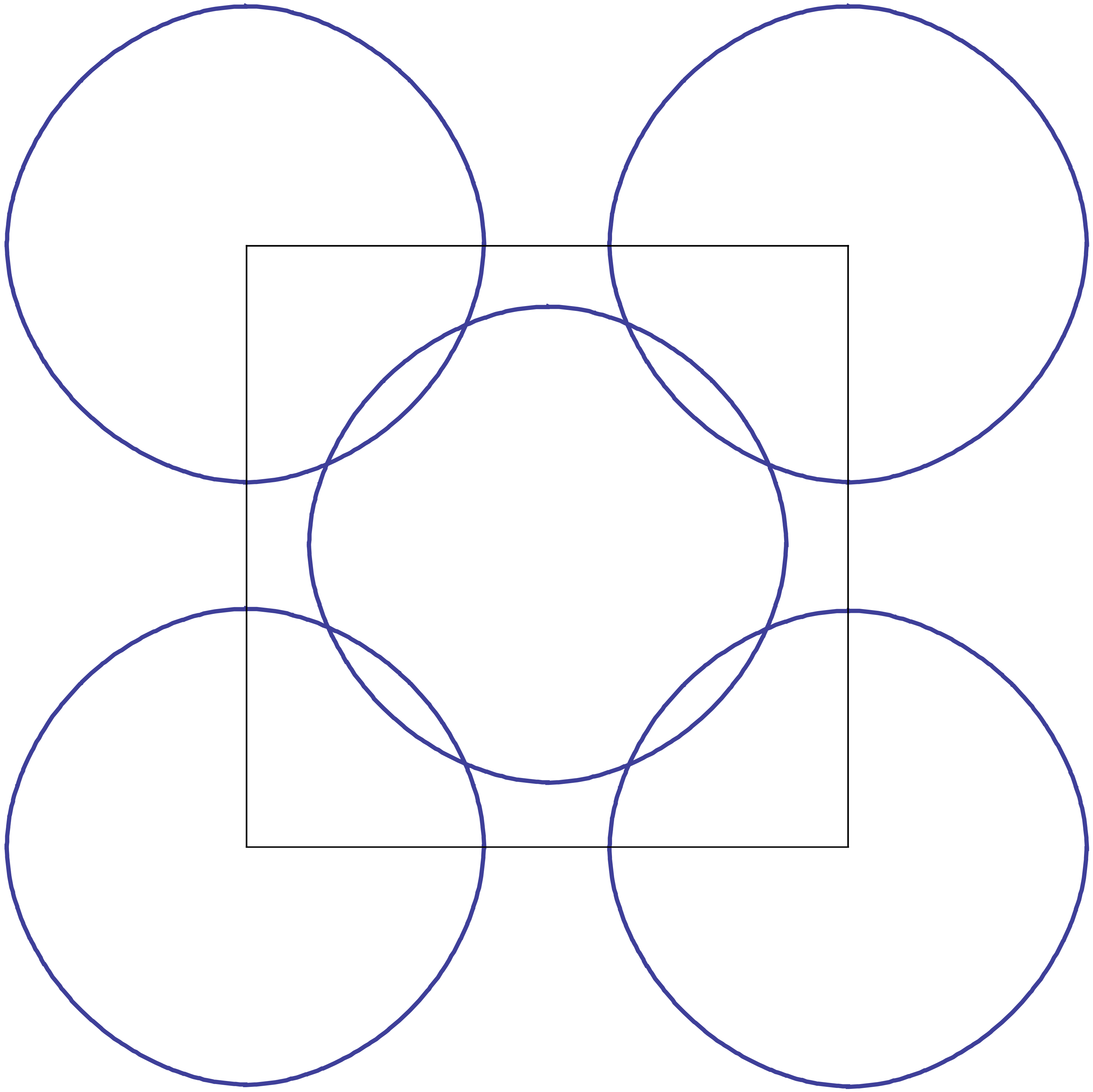
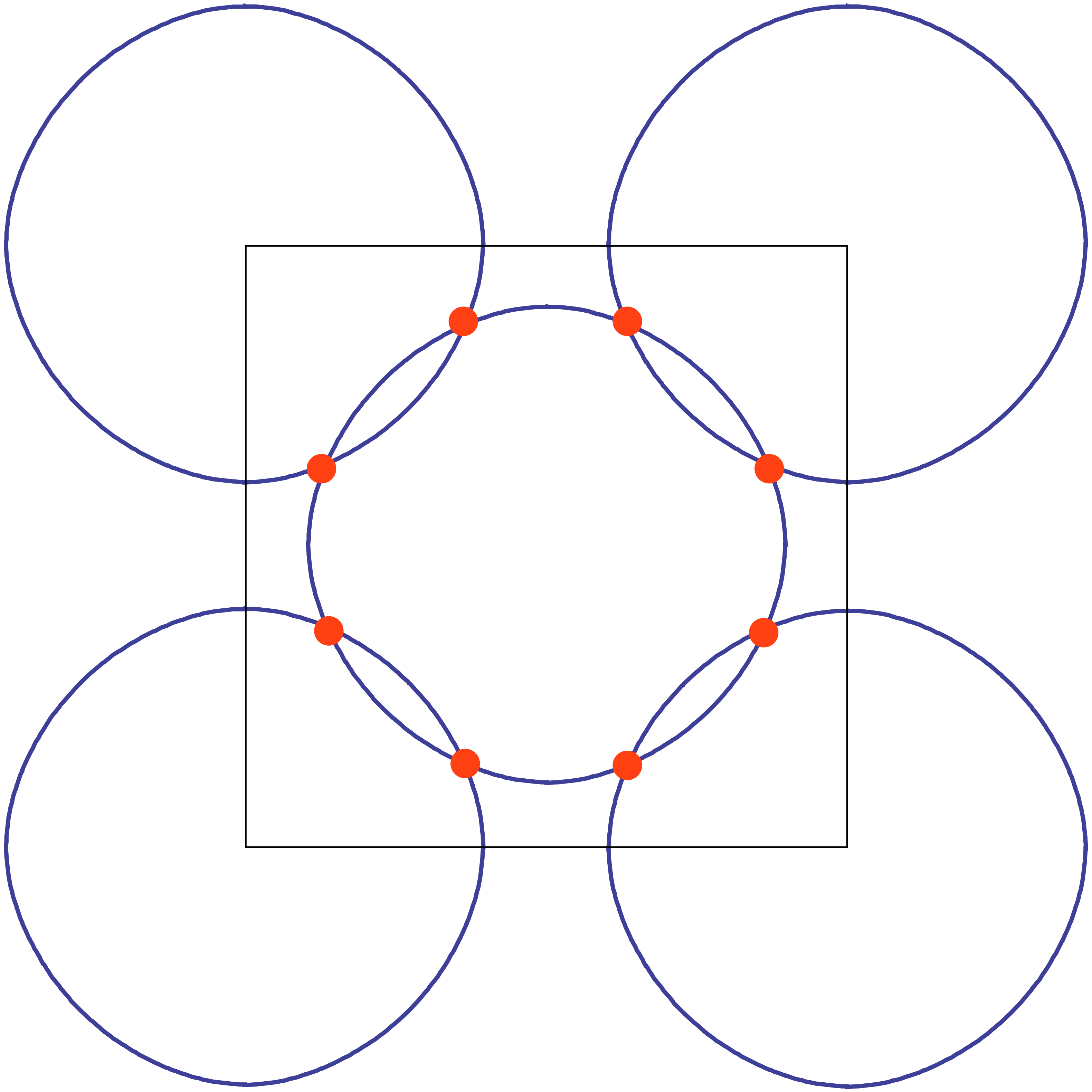
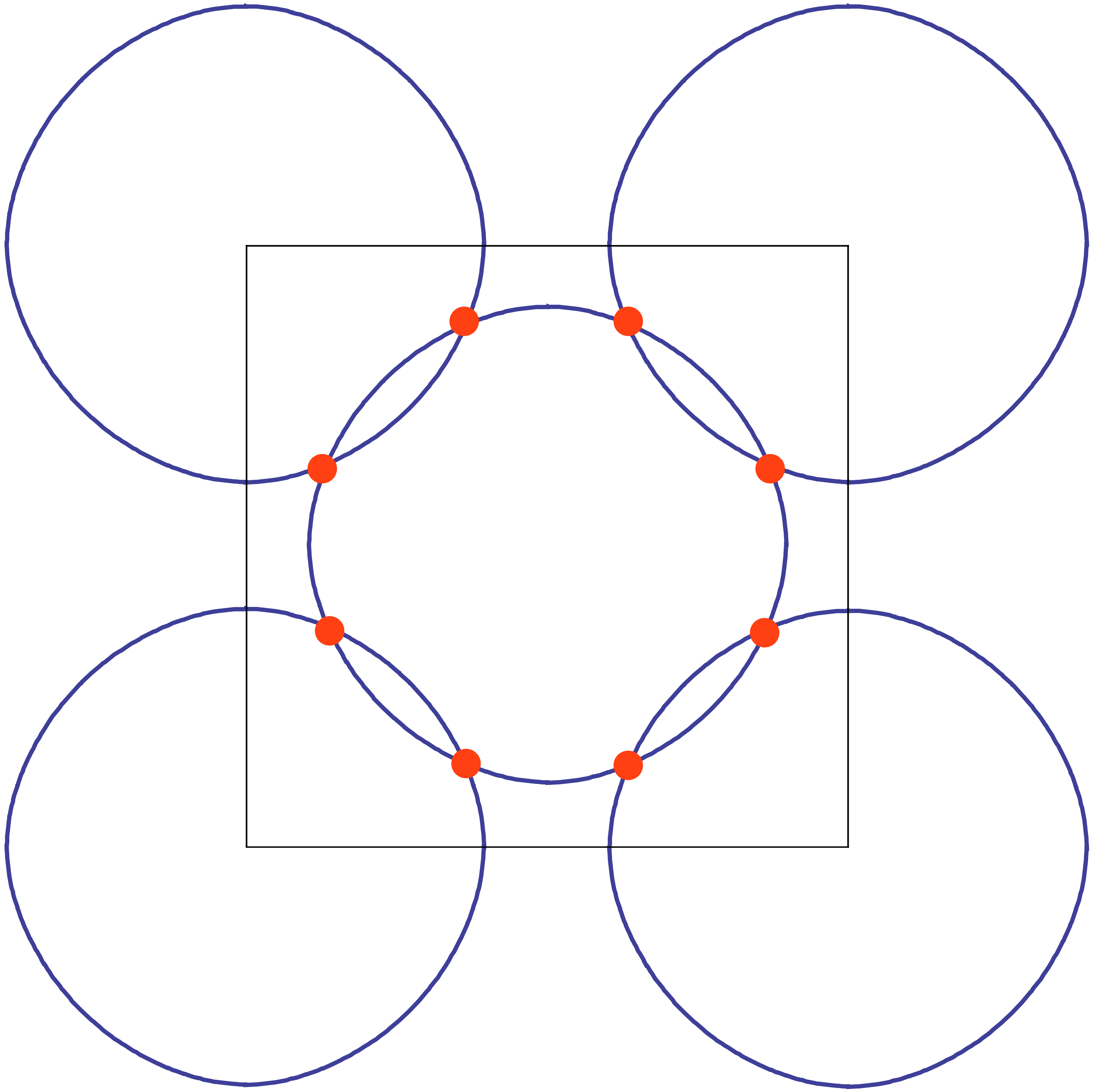


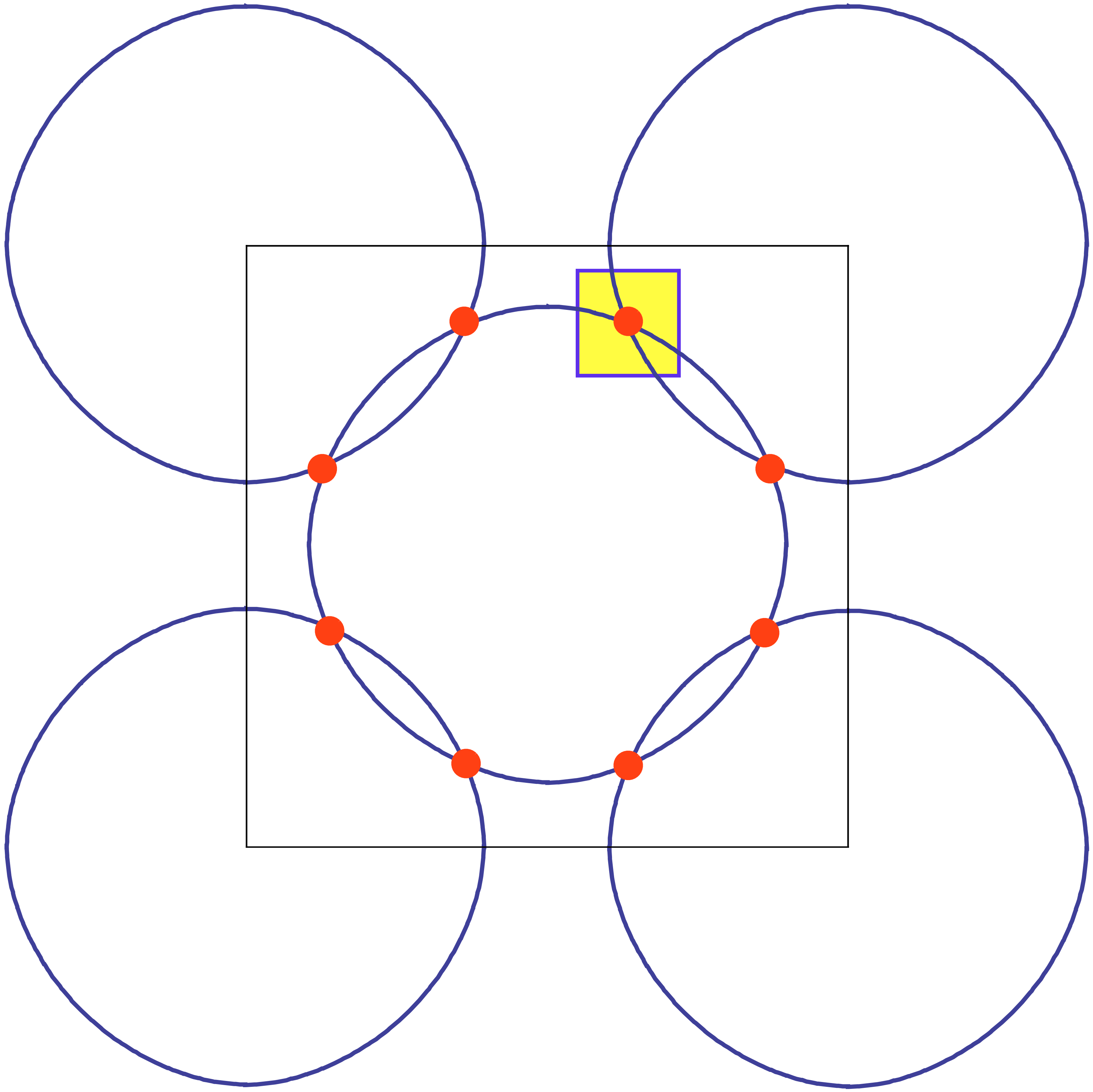
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



“Hot” spots

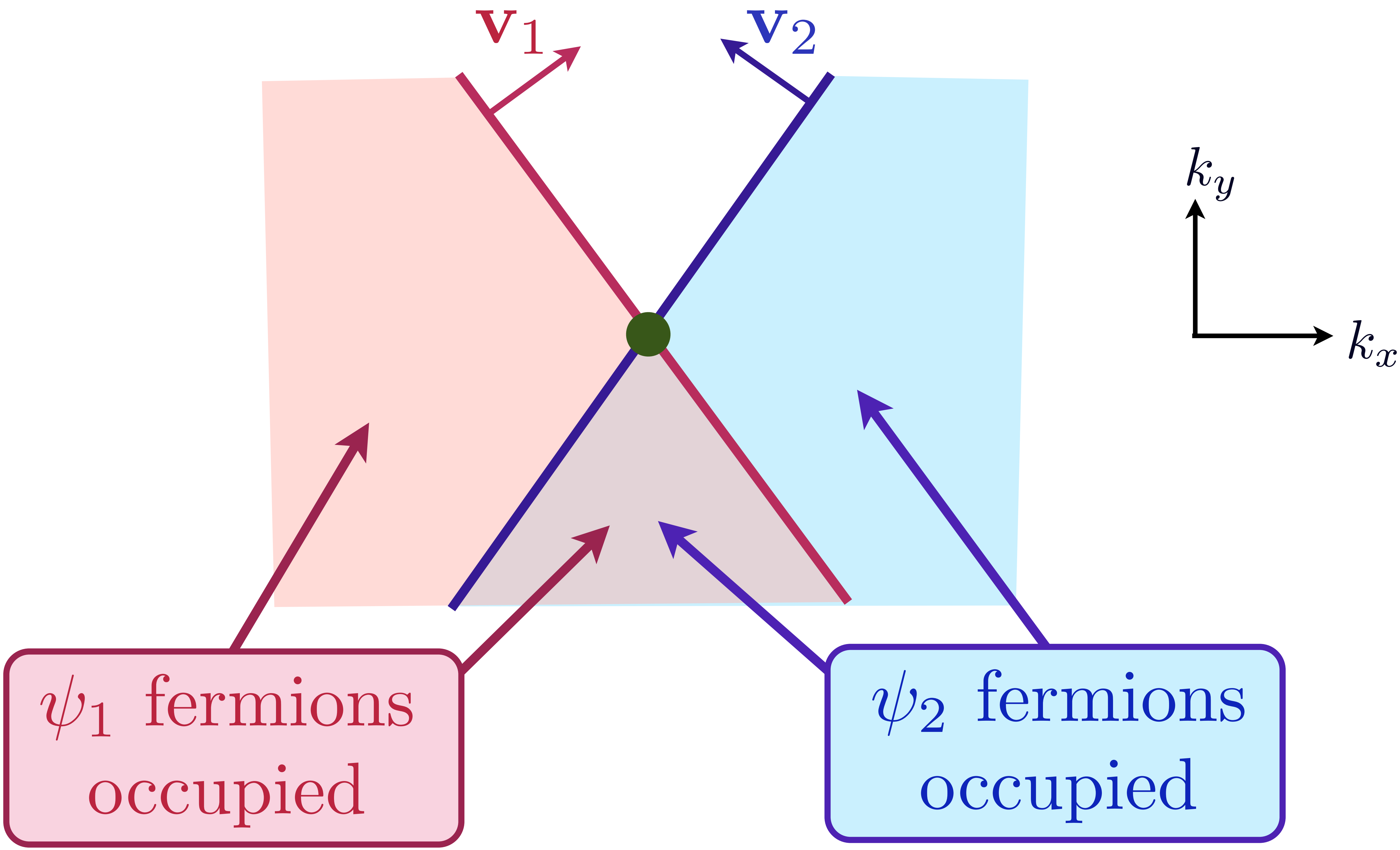


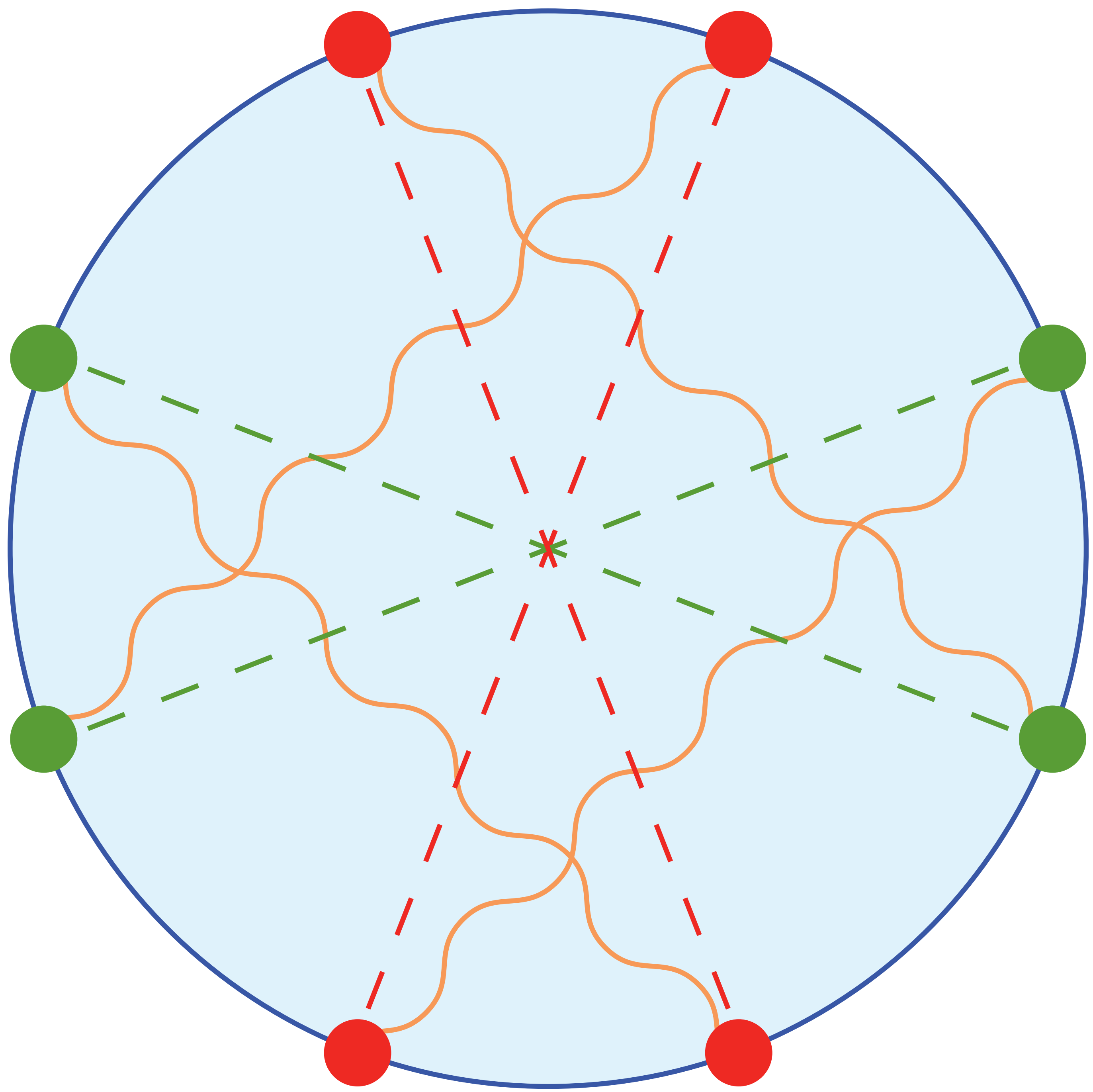
Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

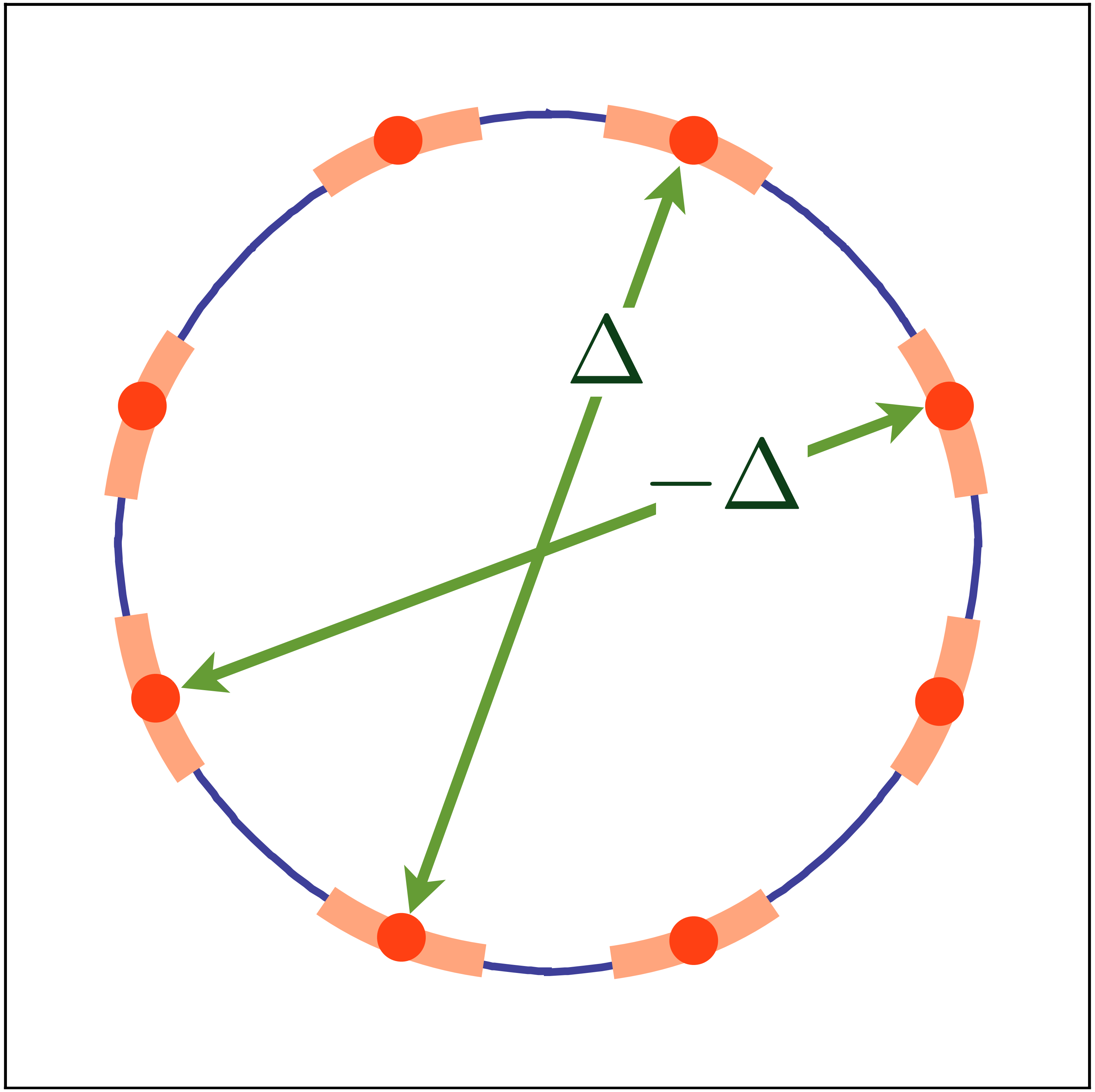
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ





Pairing "glue" from antiferromagnetic fluctuations

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

At stronger coupling, different effects compete:

- Pairing glue becomes stronger.

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- Pairing glue becomes stronger.

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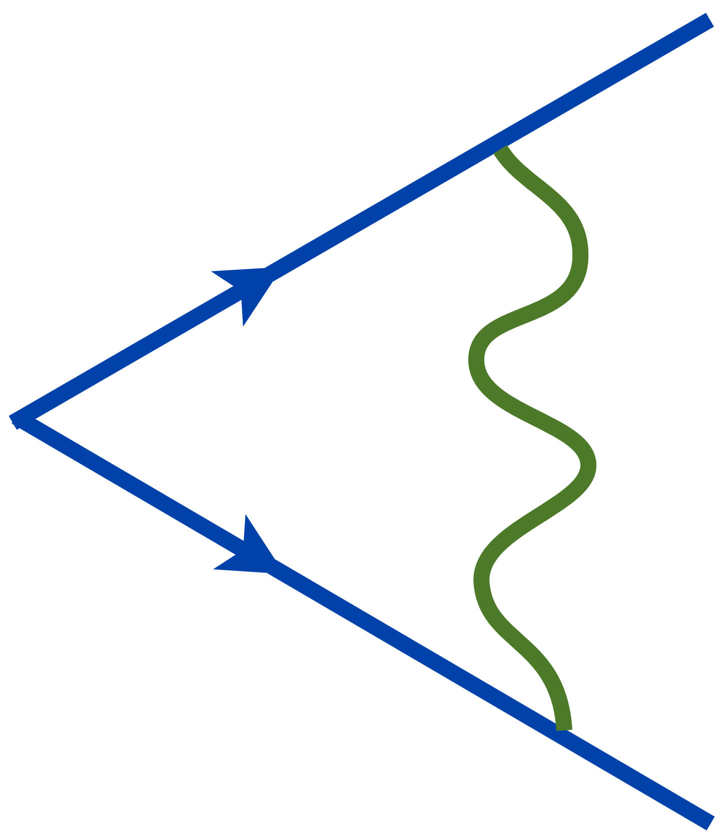
- Other instabilities can appear *e.g.* to charge density waves/stripe order.

BCS theory

Electron-phonon coupling

$$1 + \lambda_{\text{e-ph}} \log\left(\frac{\omega_D}{\omega}\right)$$

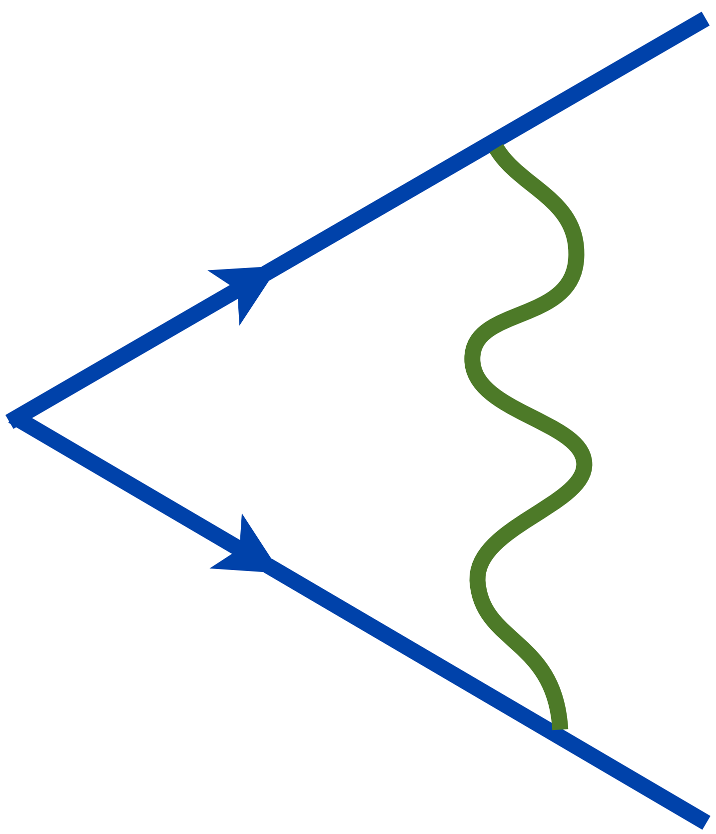
Debye frequency



Enhancement of pairing susceptibility by interactions

Antiferromagnetic critical point

$$1 + \frac{\sin\theta}{2\pi} \log^2\left(\frac{E_F}{\omega}\right)$$



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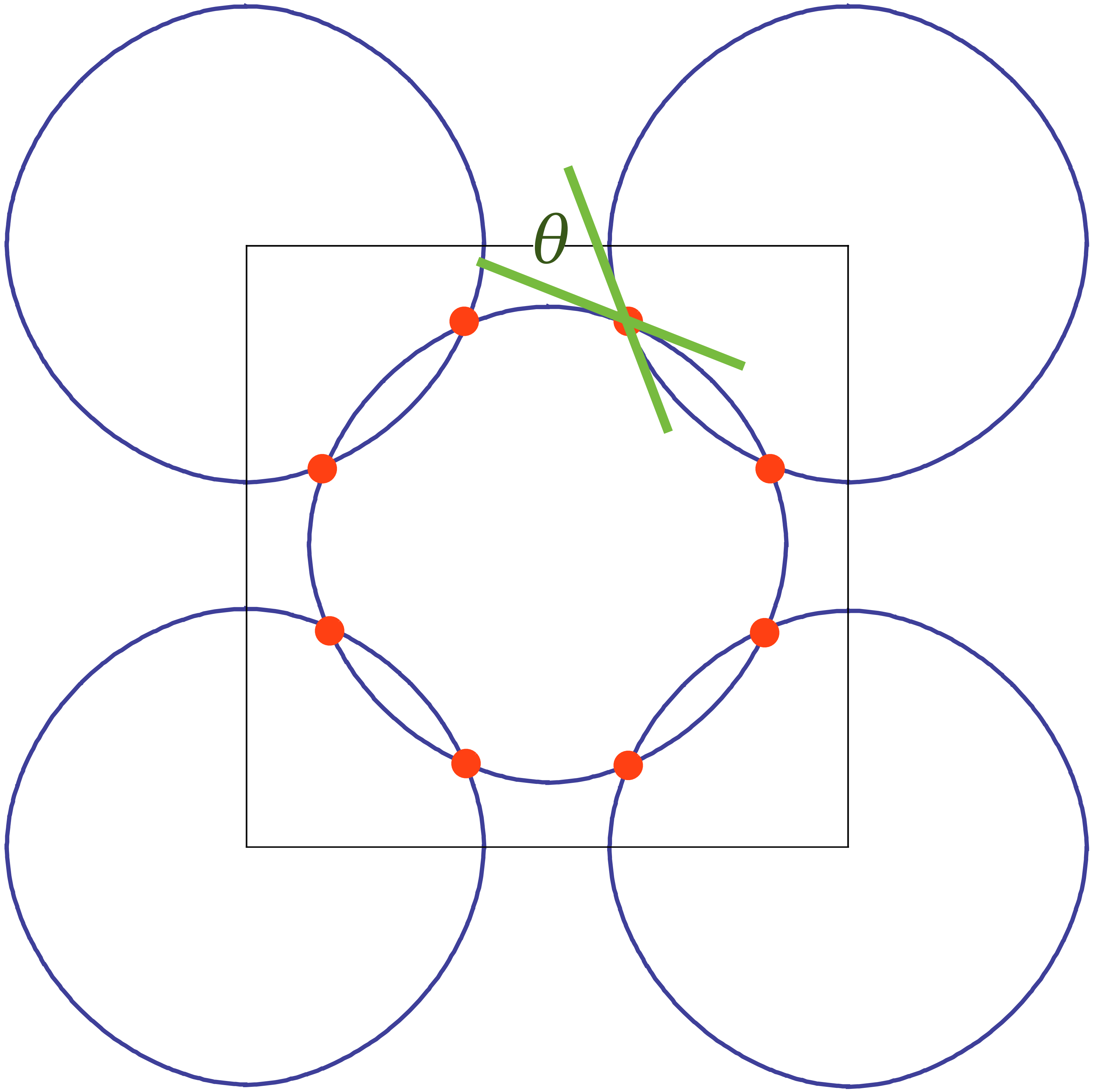
Fermi energy

θ is the angle between Fermi lines. *Independent* of interaction strength U in 2 dimensions.

(see also Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)


θ

Enhancement of pairing susceptibility by interactions

Antiferromagnetic critical point

$$1 + \frac{\sin\theta}{2\pi} \log^2\left(\frac{E_F}{\omega}\right)$$

- **Universal** $\log^2$ singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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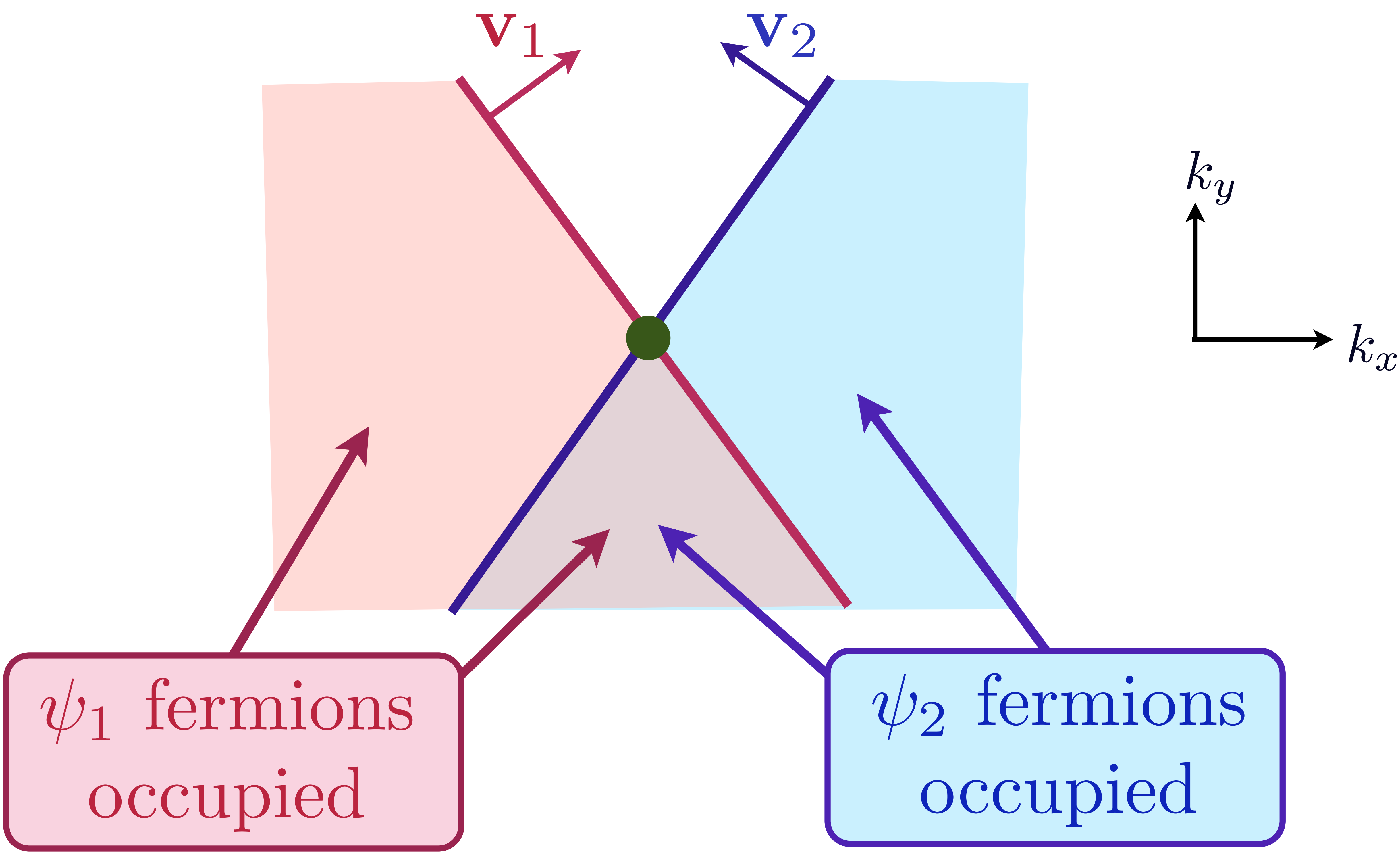

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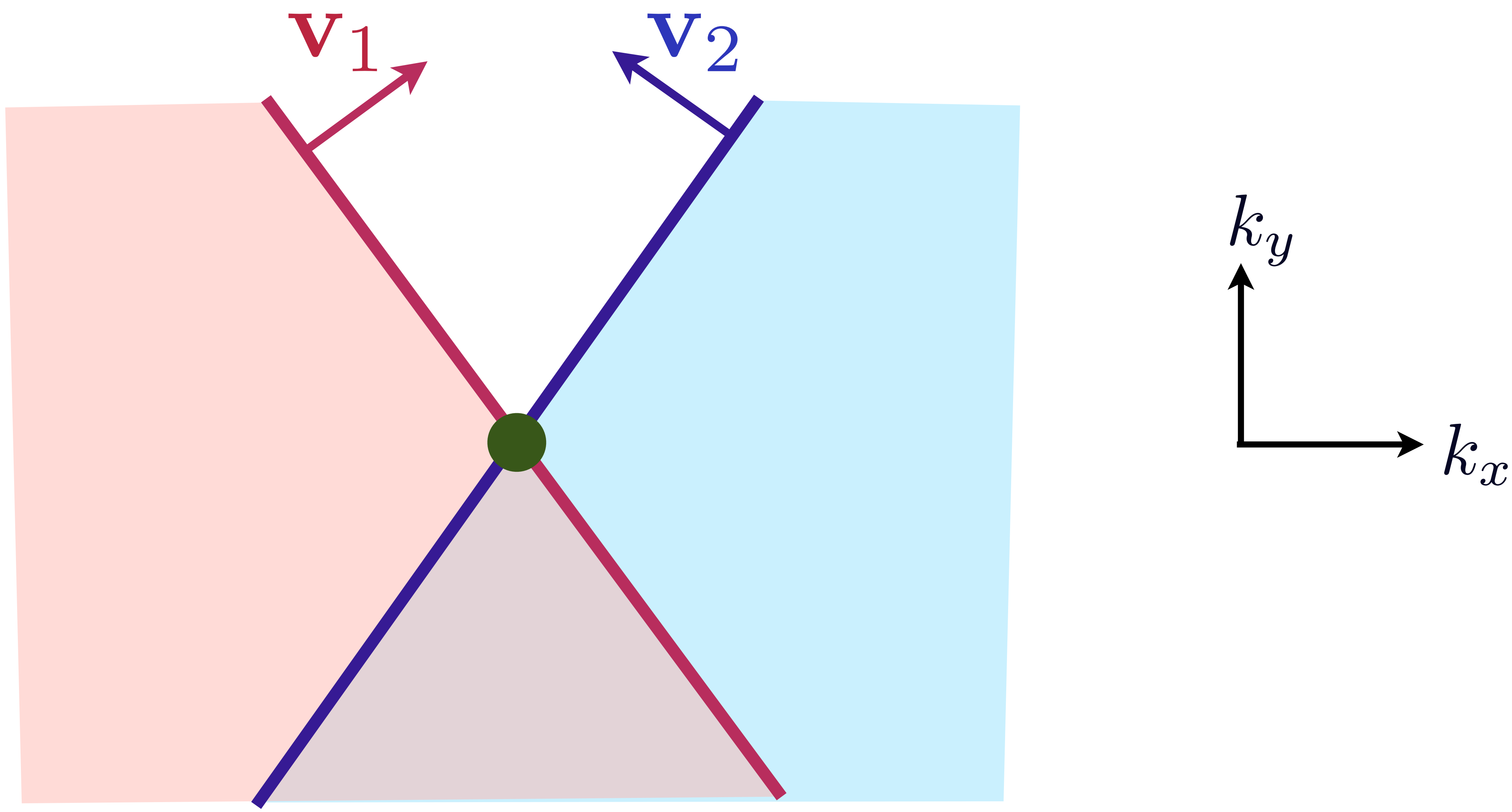
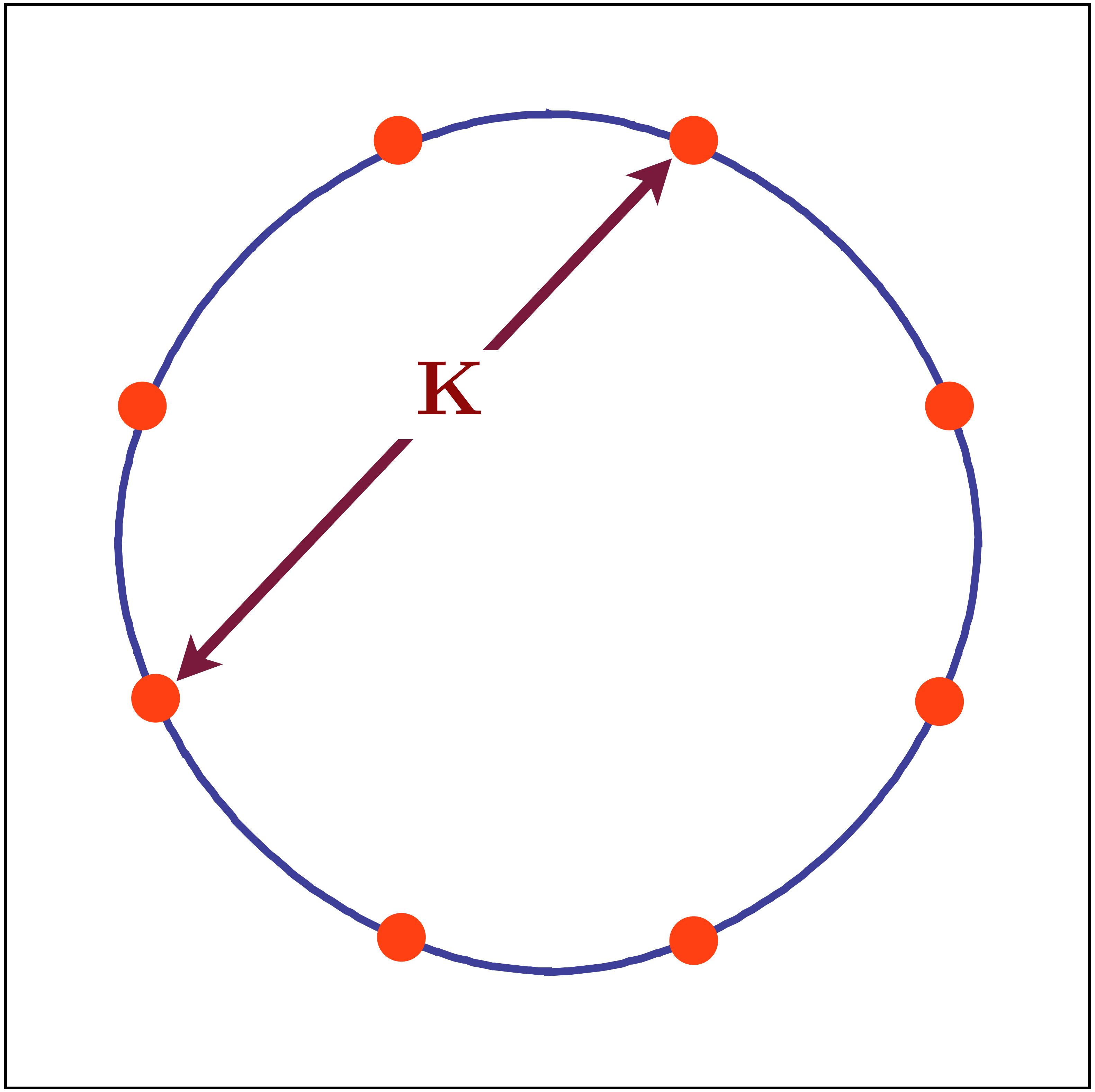


To faithfully realize low energy theory in quantum Monte Carlo, we need a UV completion in which Fermi lines don't end and all weights are positive.

QMC for the onset of antiferromagnetism



Hot spots in a single band model

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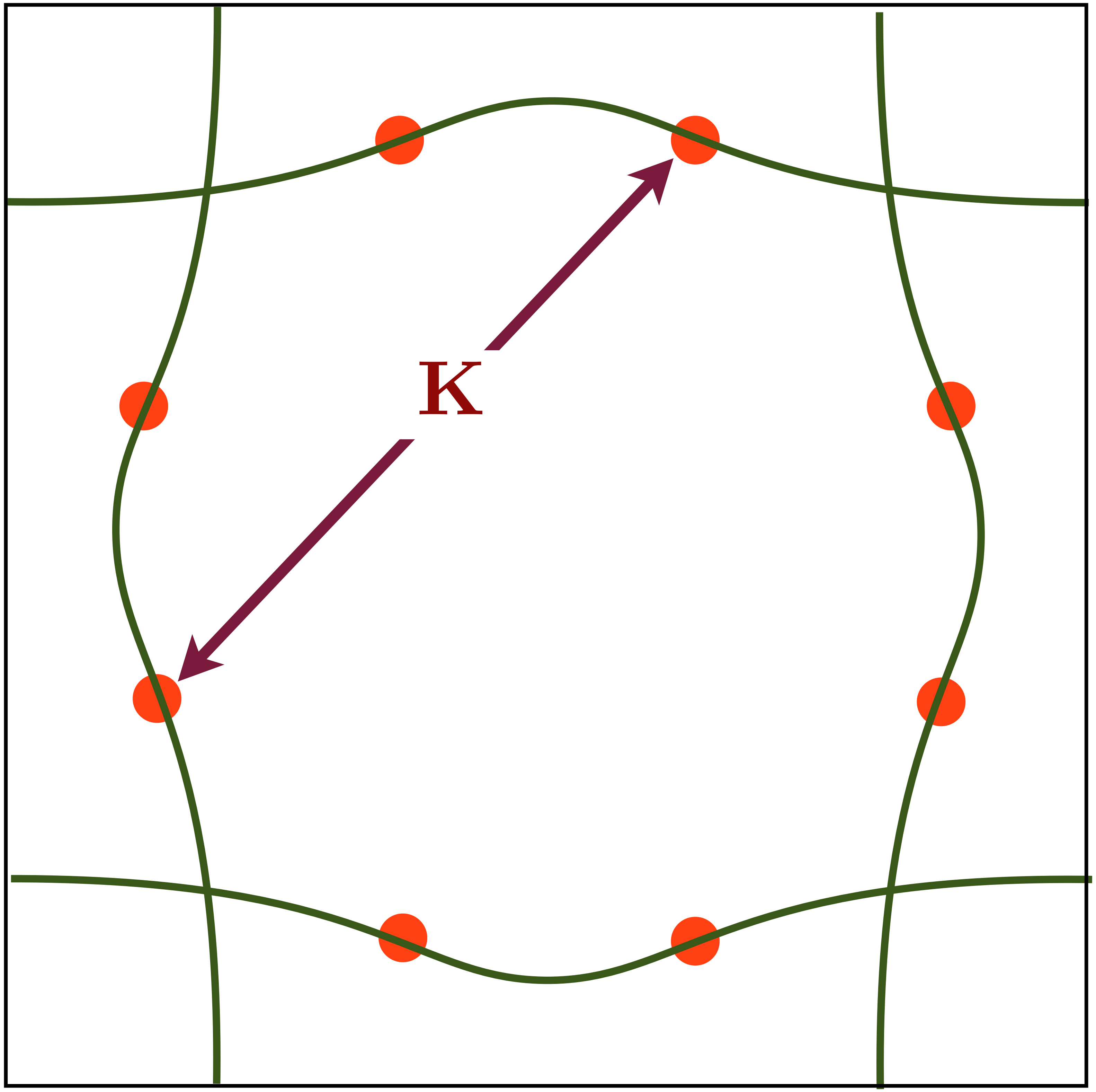

Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

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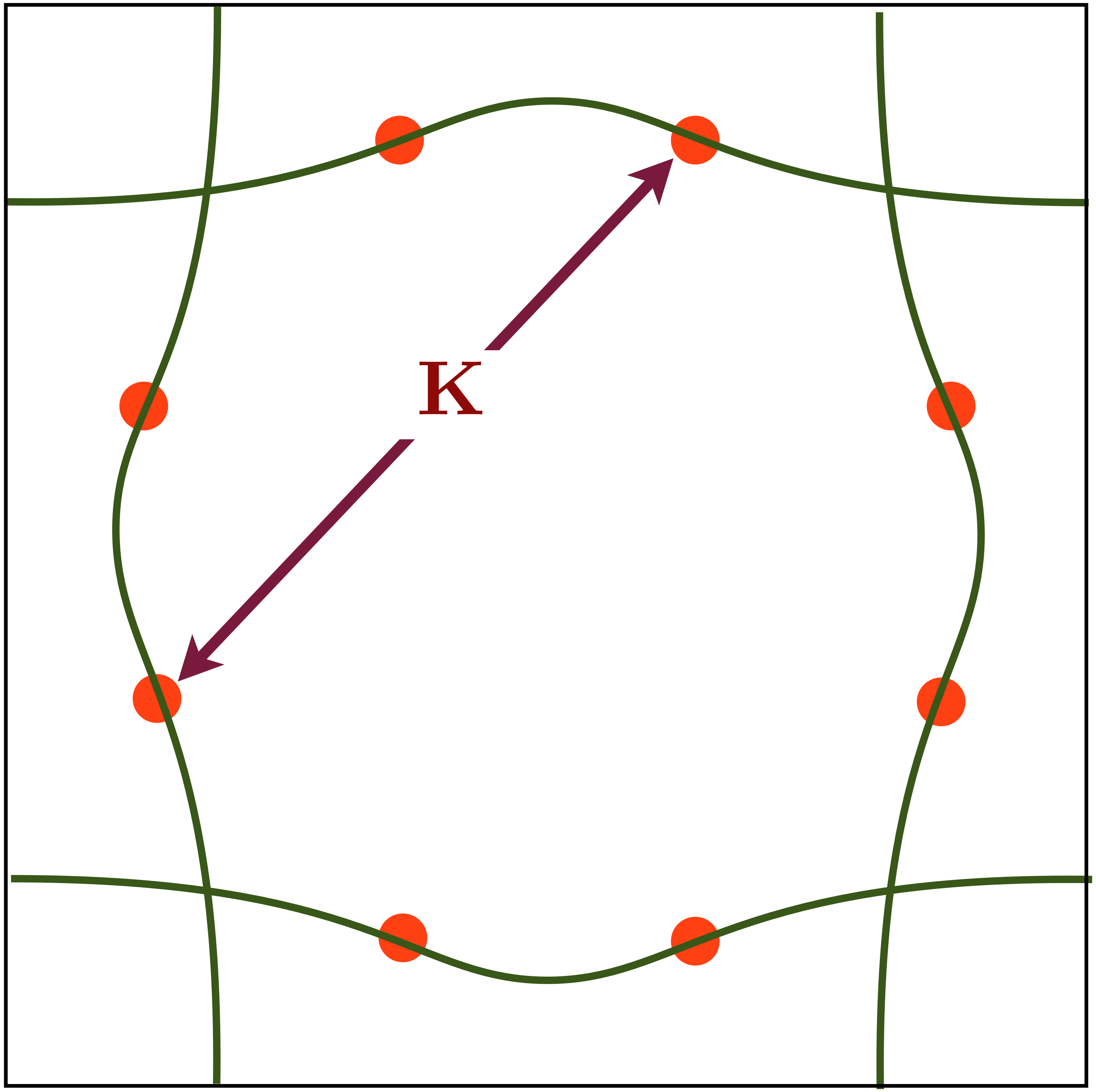

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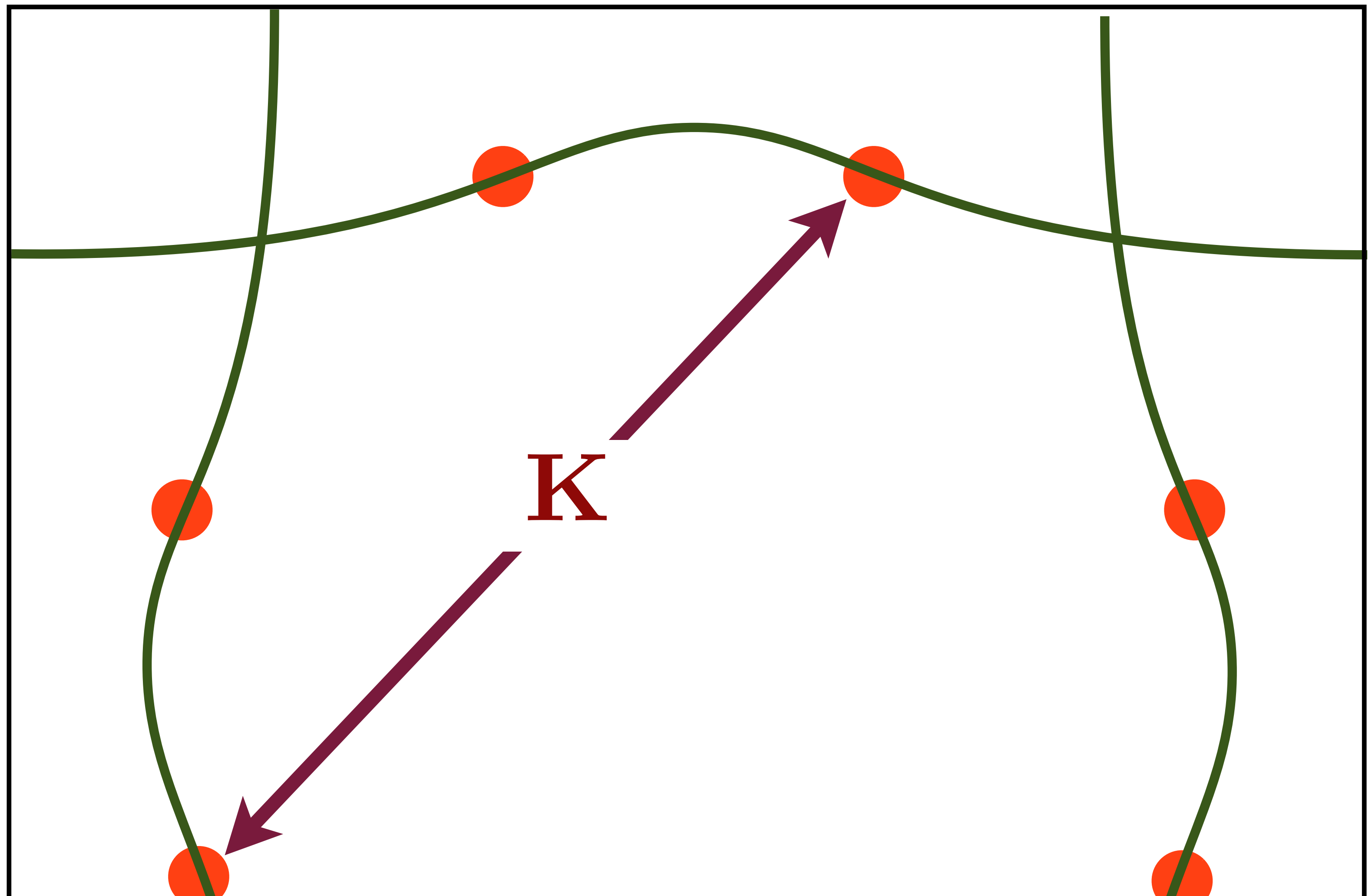

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No sign problem in
fermion determinant Monte Carlo !

Determinant is positive because of Kramer's degeneracy, and no additional symmetries are needed; holds for arbitrary band structure and band filling, provided ***K*** only connects hot spots in distinct bands

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha^{(x)} \mathcal{D}c_\alpha^{(y)} \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

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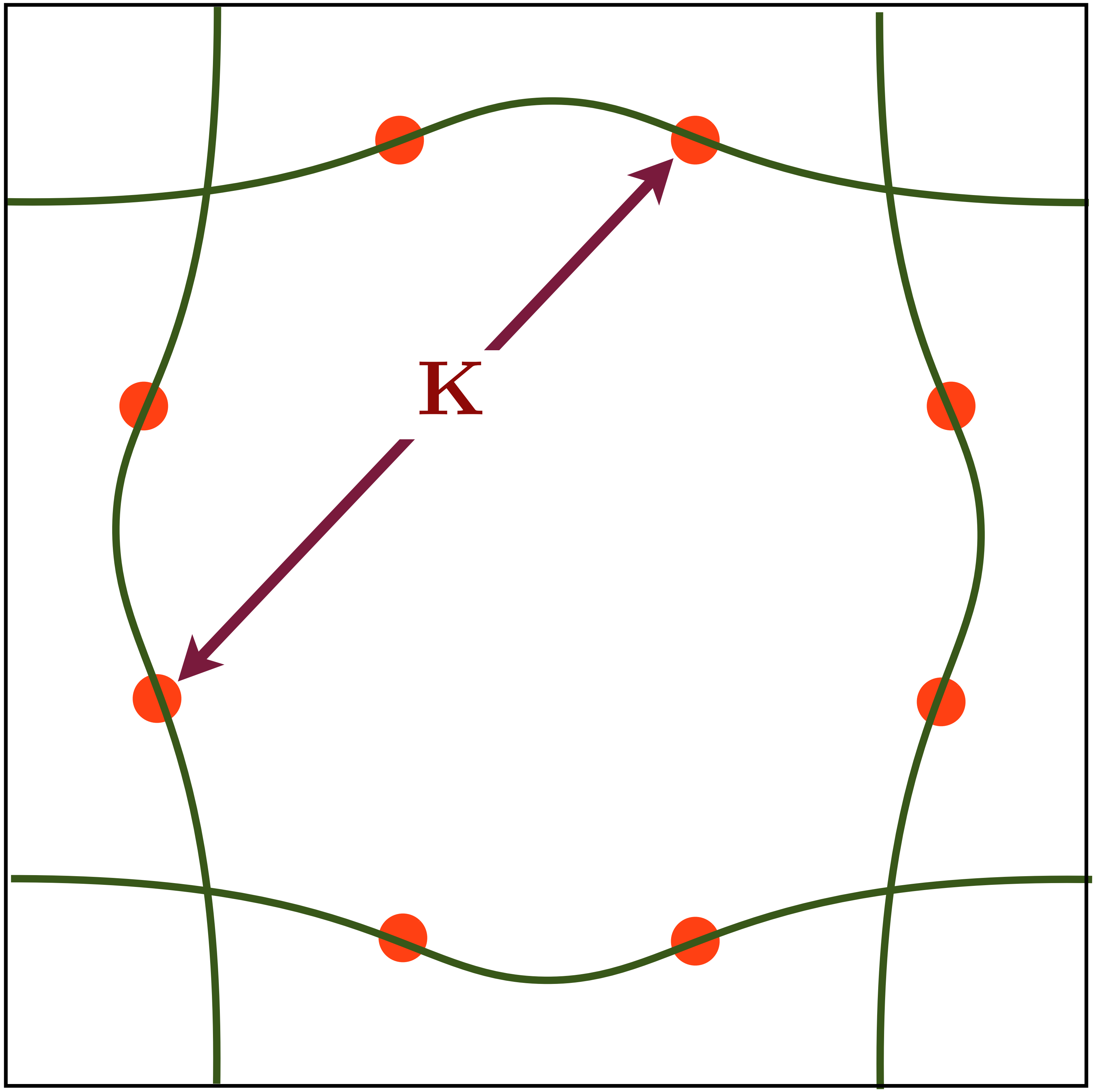
E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

No sign problem !

QMC for the onset of antiferromagnetism



Hot spots in a two band model

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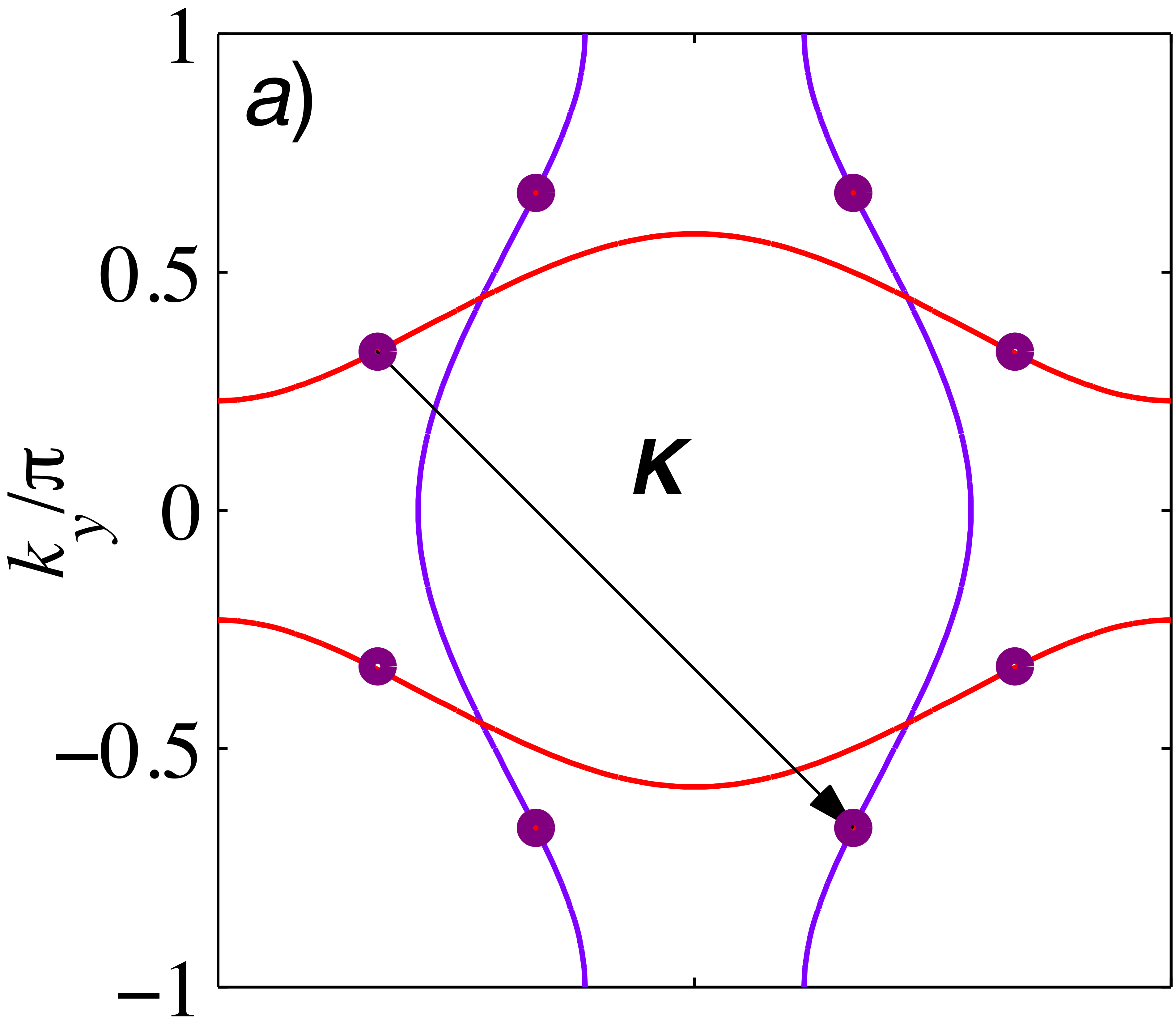

Center Brillouin zone at (π,π,)

E. Berg,
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arXiv:1206.0742

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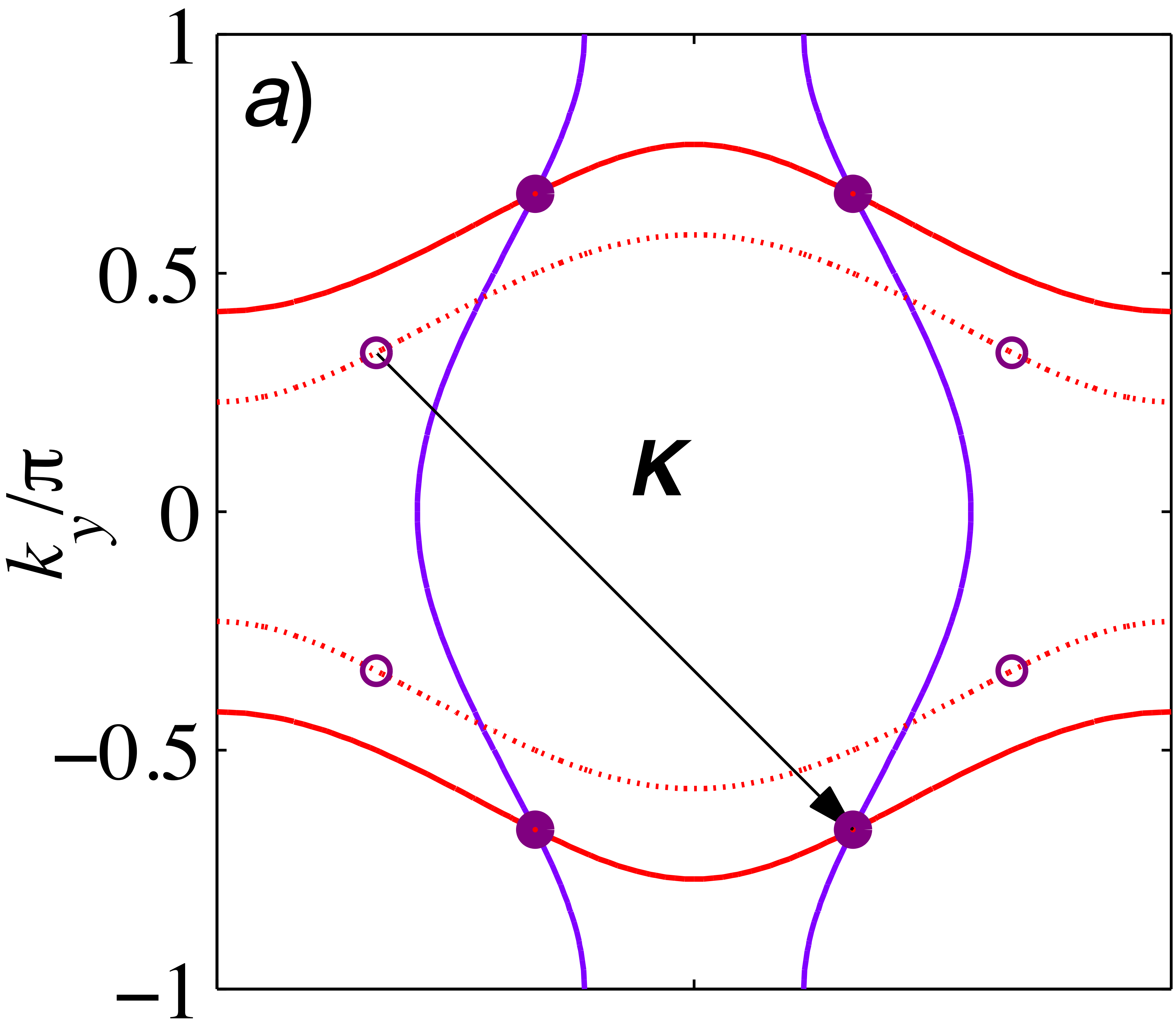

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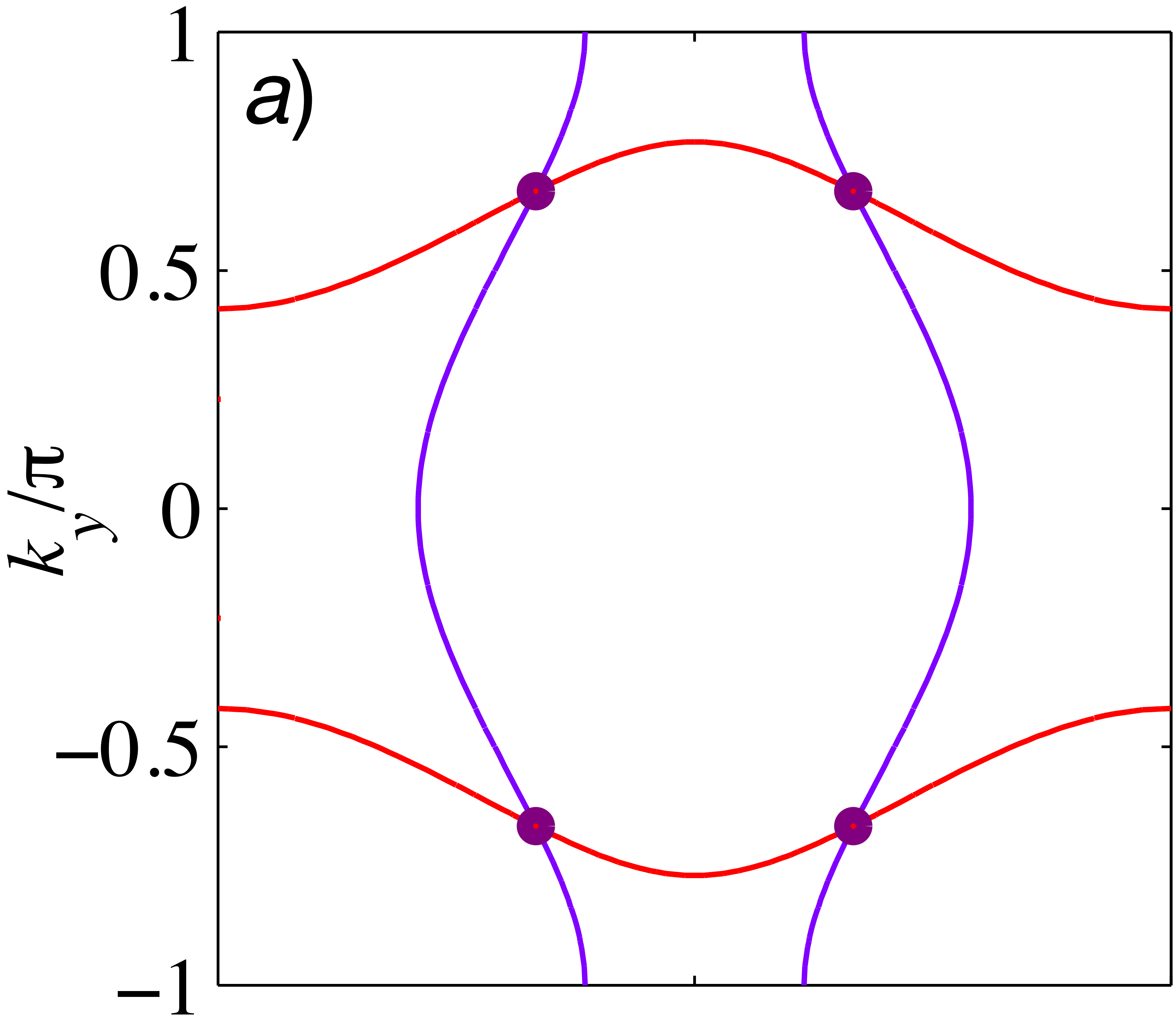
Move one of the Fermi surface by (π,π,)

QMC for the onset of antiferromagnetism



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Now hot spots are at Fermi surface intersections

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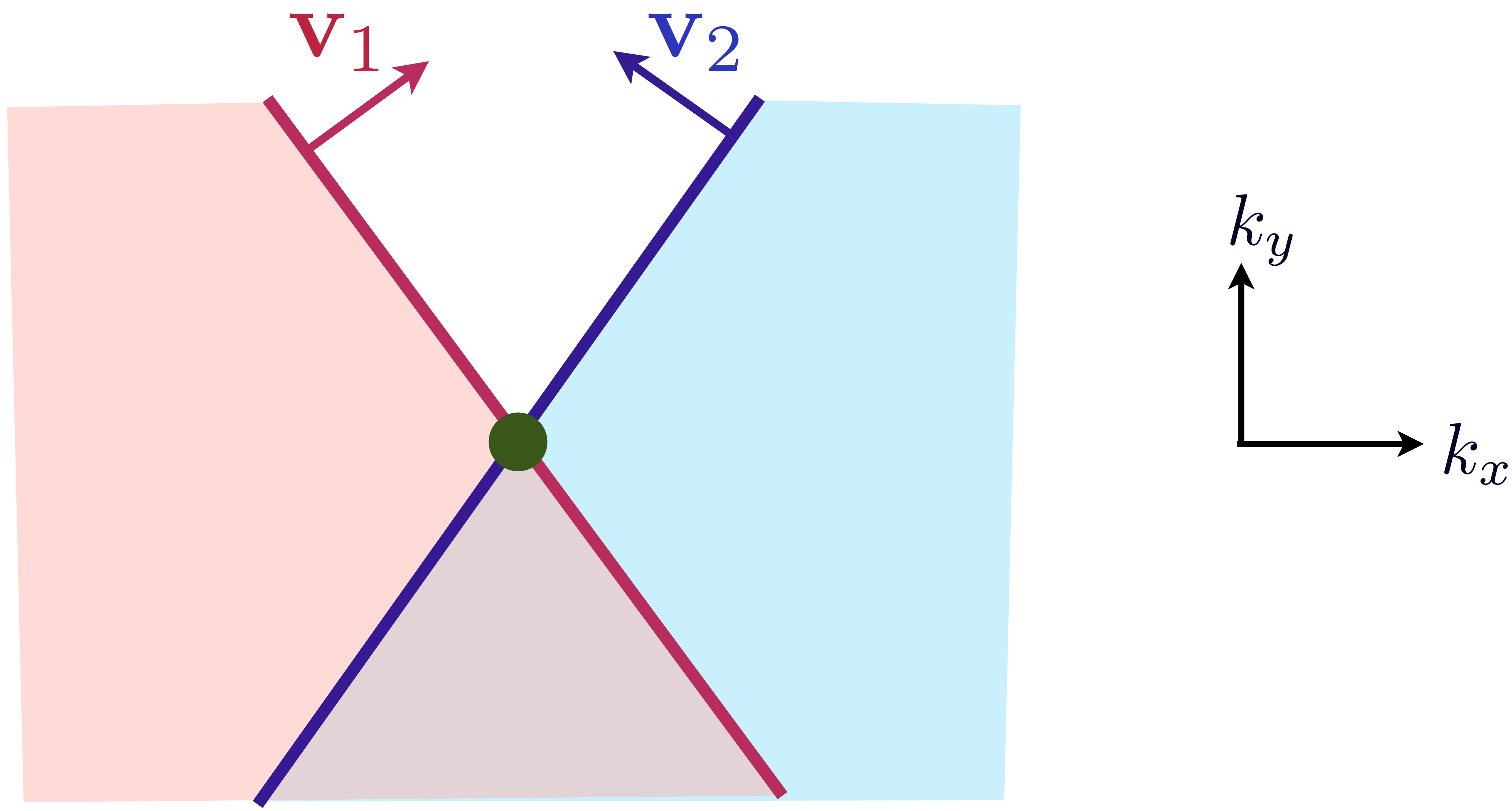

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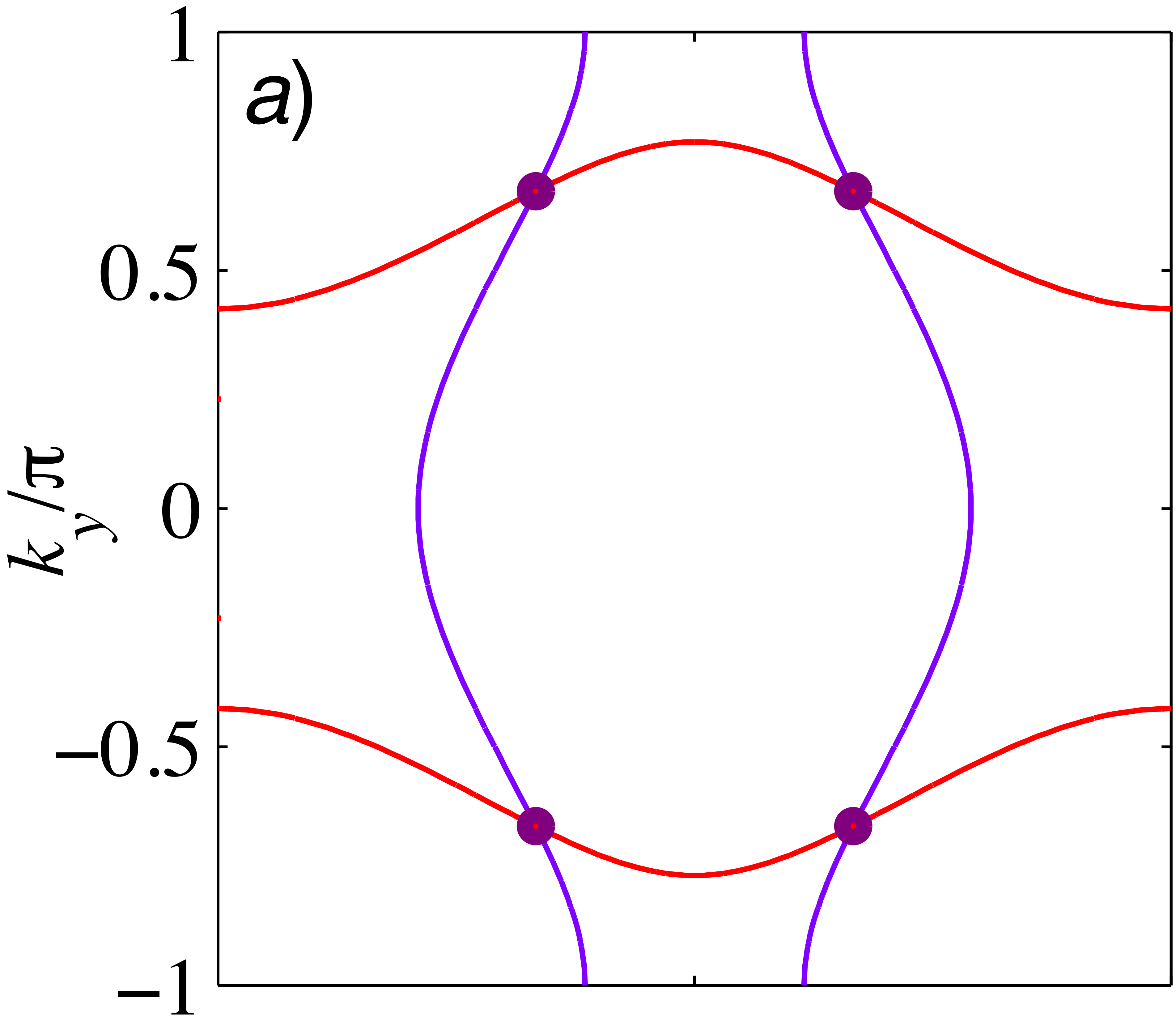

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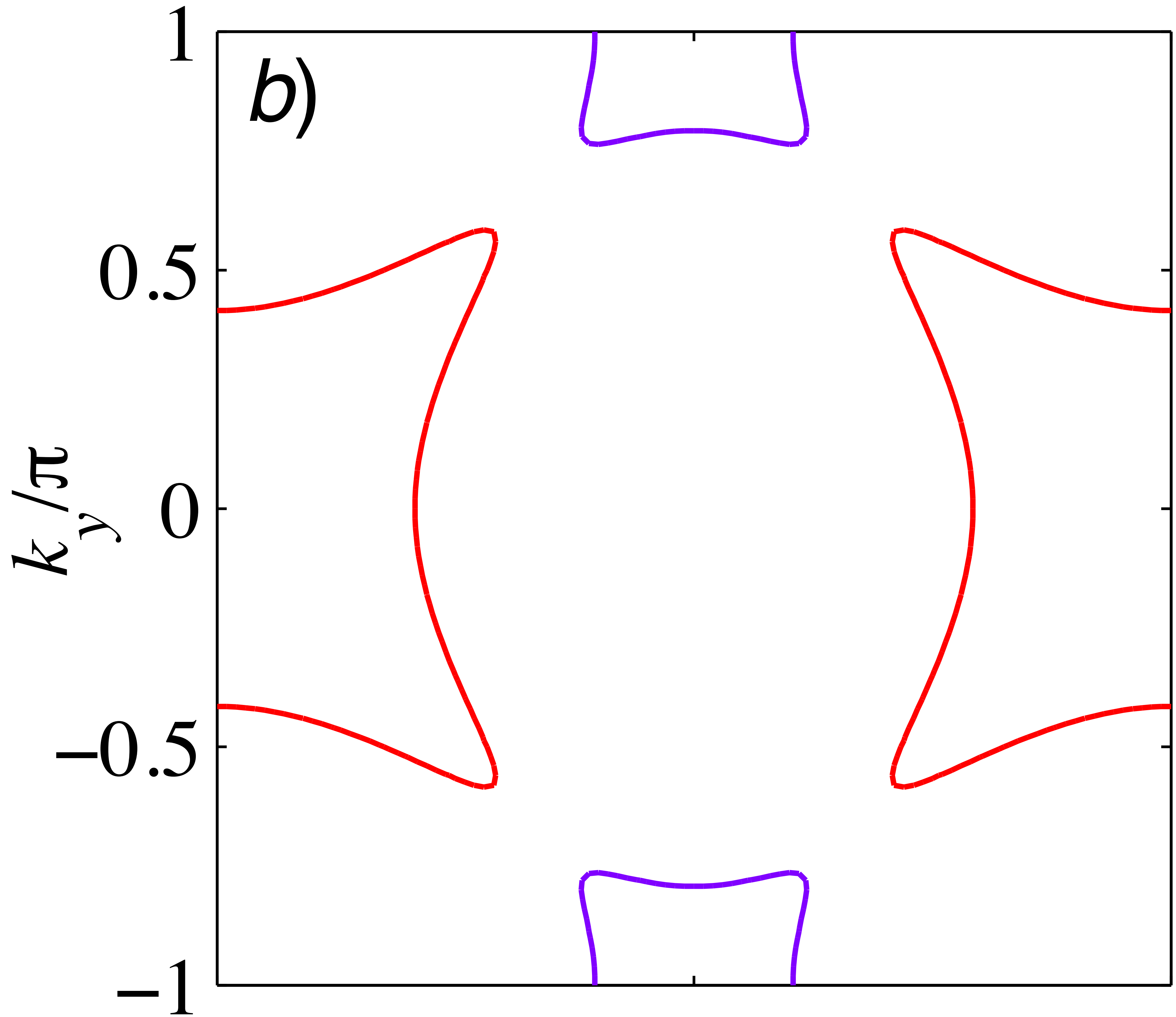

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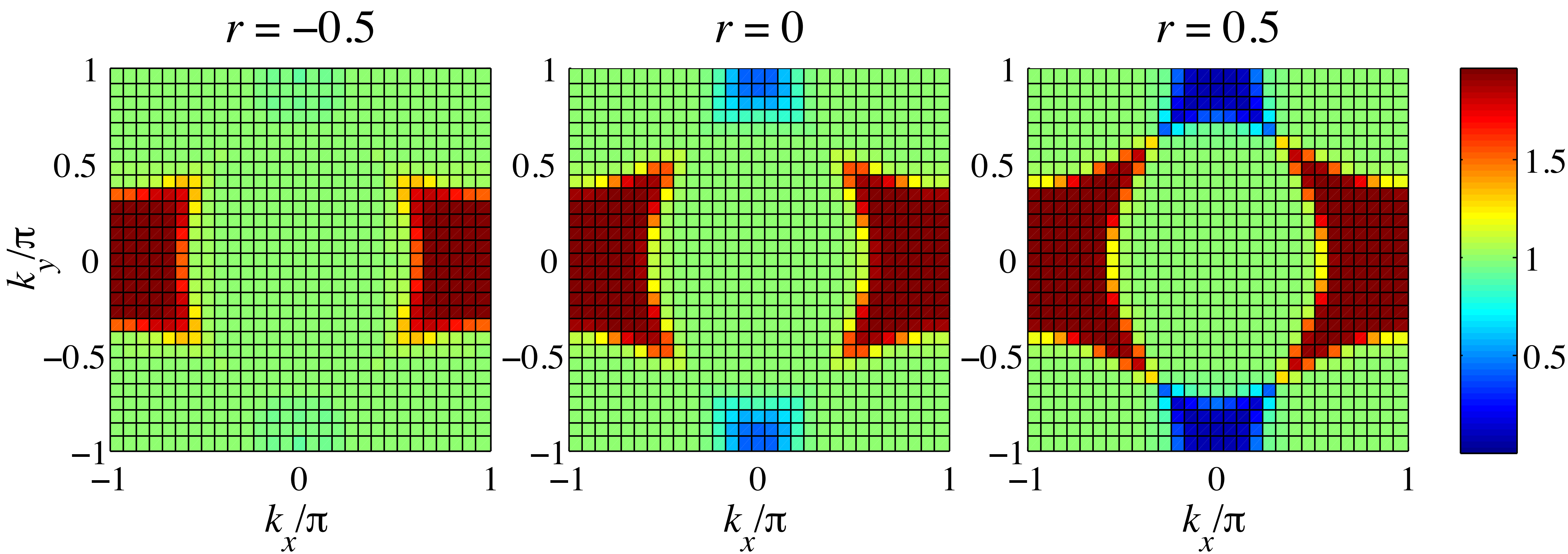
Expected Fermi surfaces in the AFM ordered phase

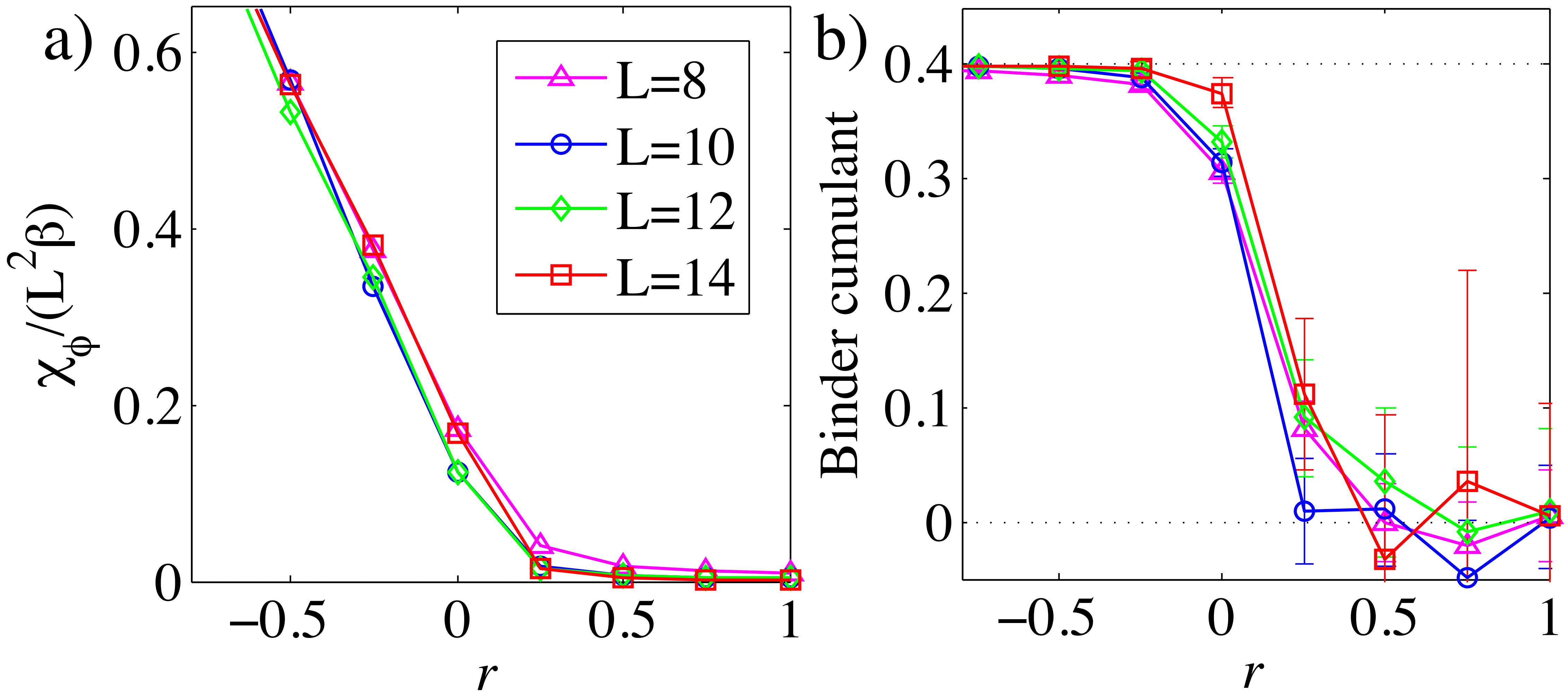
QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

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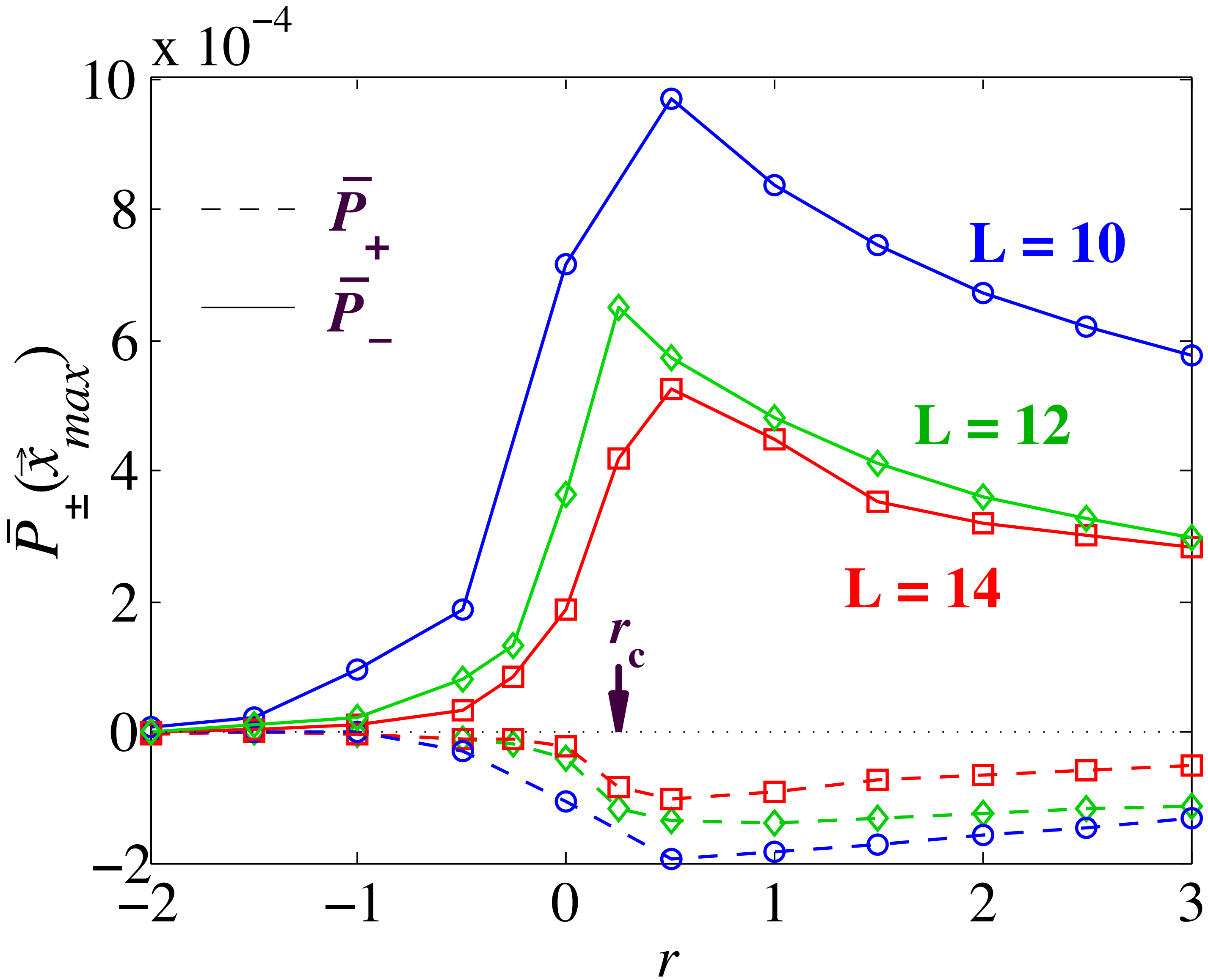
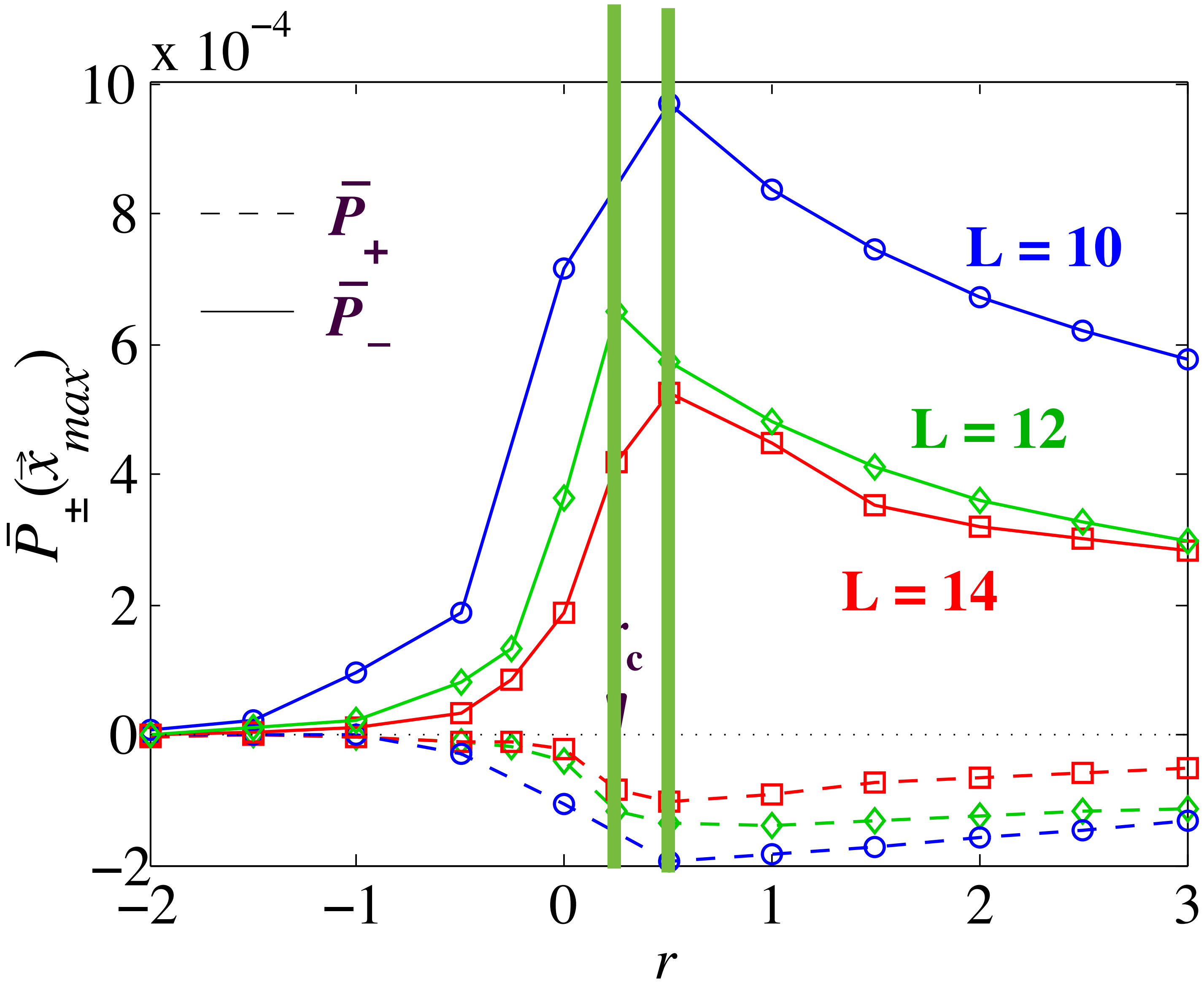


s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

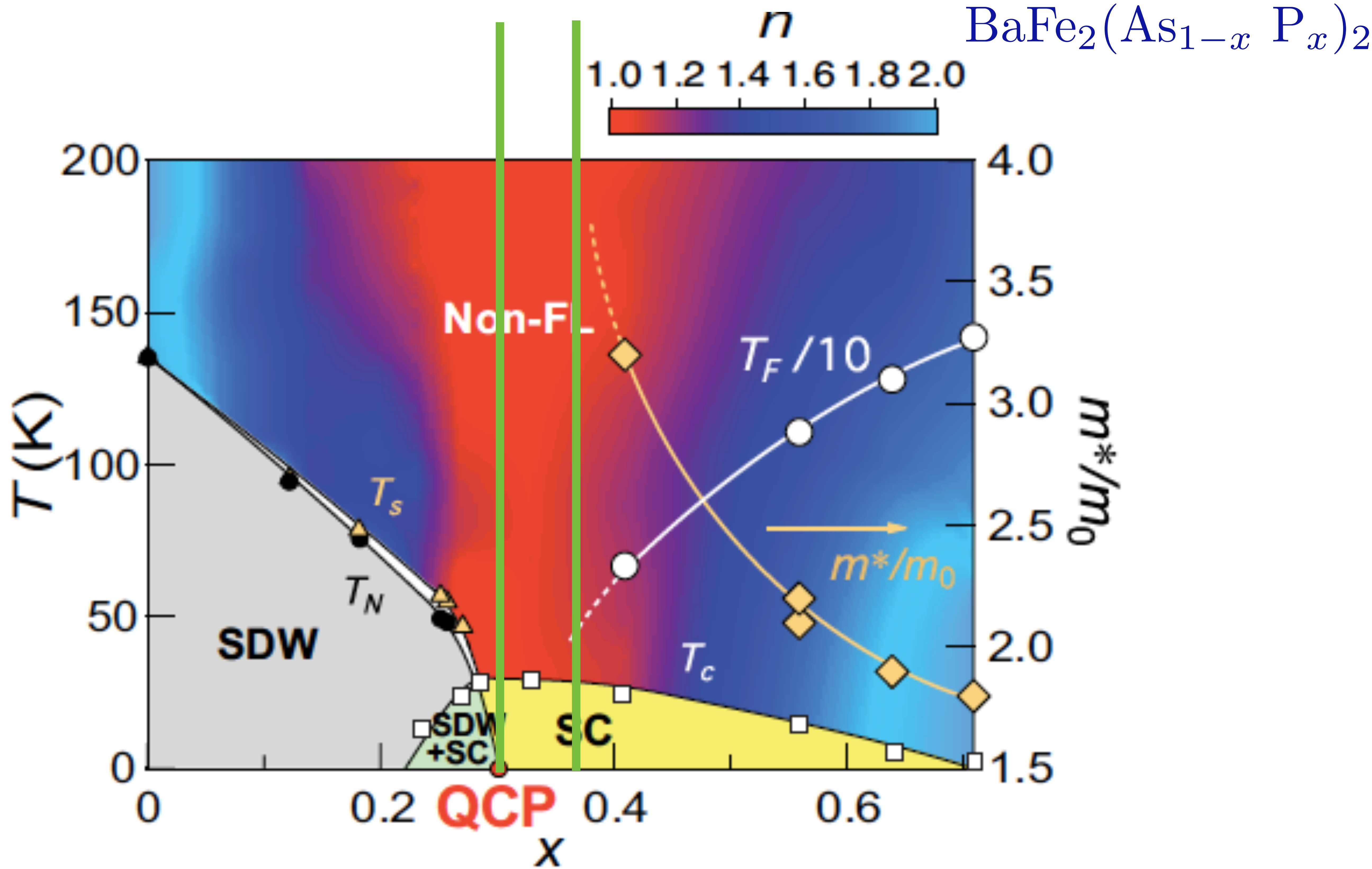
E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism



Notice shift between the position of the QCP in the superconductor, and the position of maximum pairing

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742



Notice shift between the position of the QCP in the superconductor, and the divergence in effective mass in the metal measured at high magnetic fields

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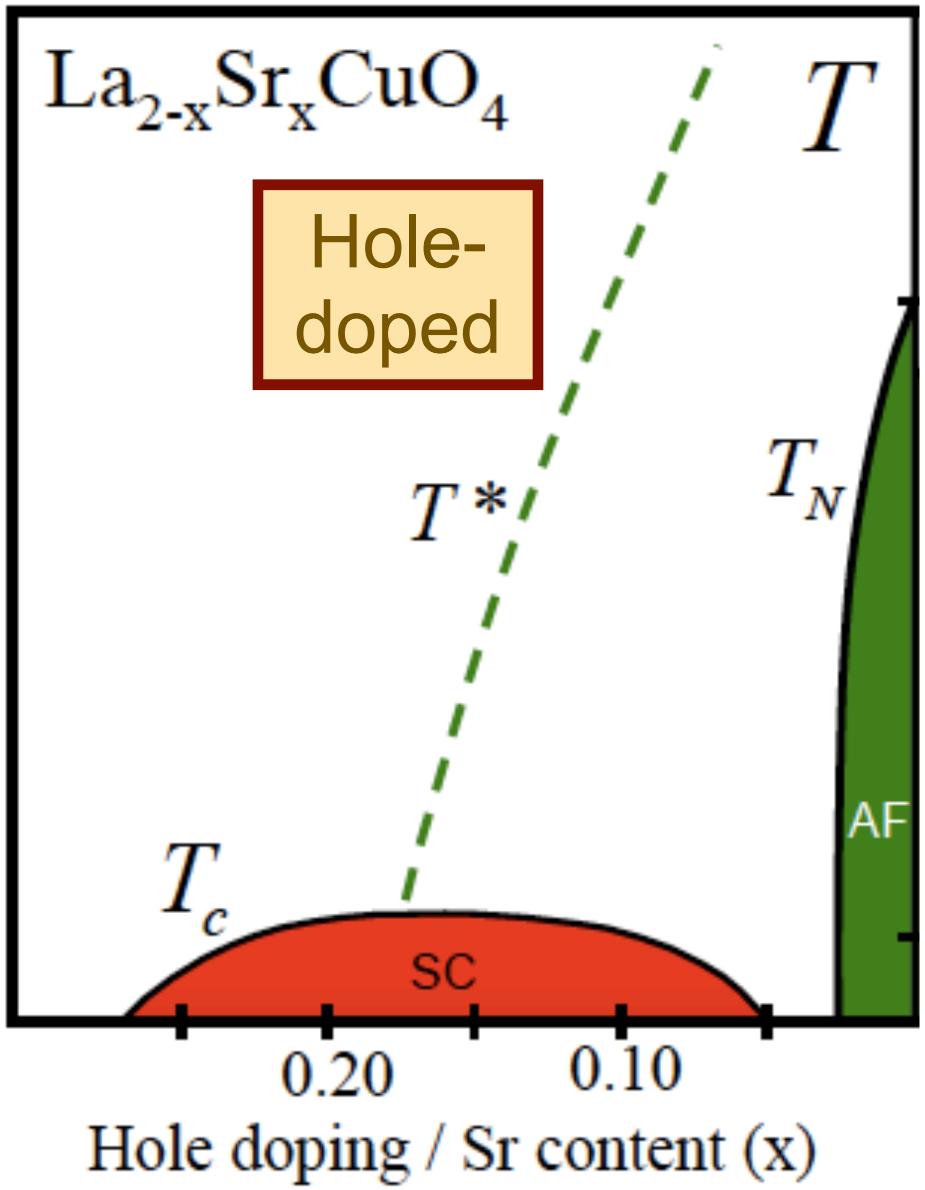
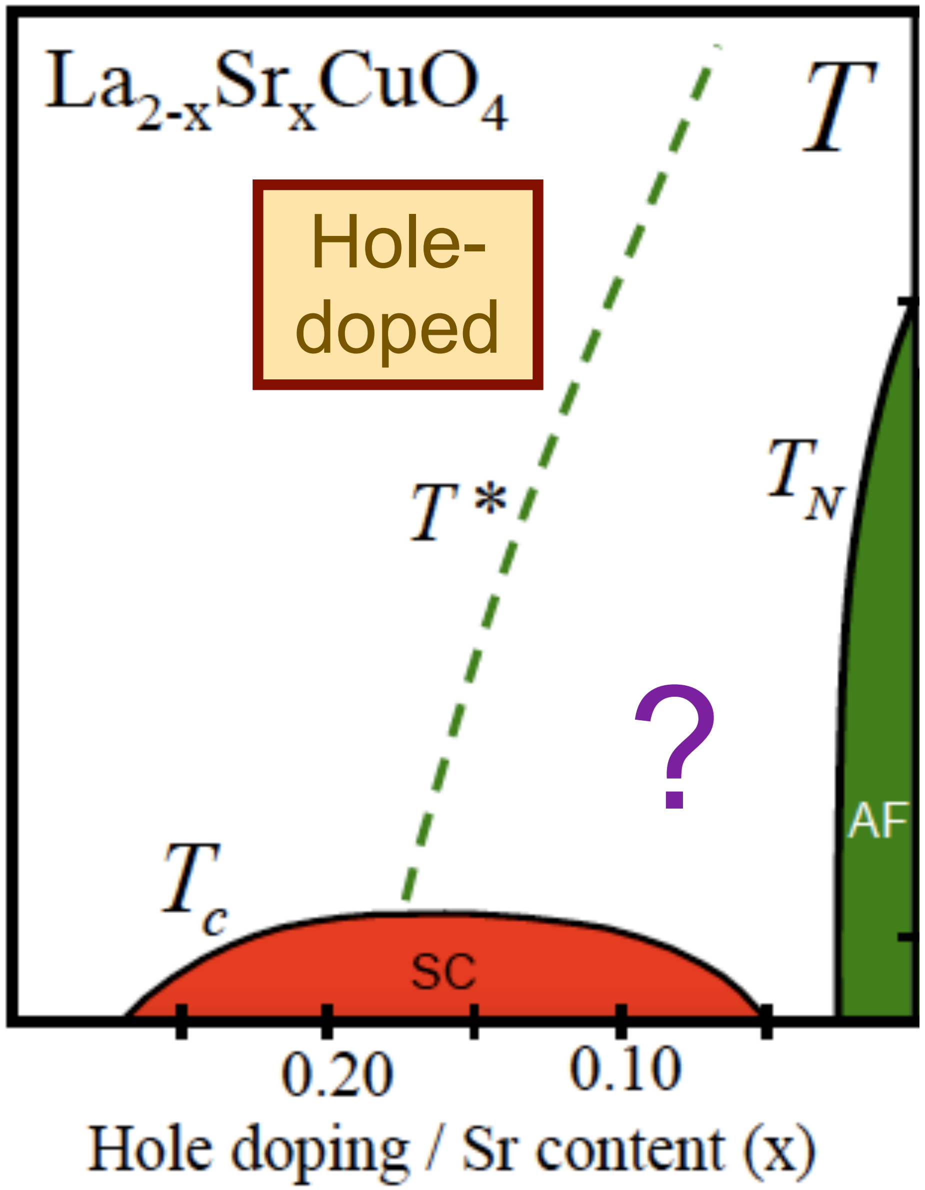

La2-xSrxCuO4
T
Hole-doped
T*
TN
AF
Tc
SC
0.20
0.10
Hole doping / Sr content (x)


$La_{2-x}Sr_xCuO_4$
Hole-doped
T
T^*
T_N
?
AF
T_c
SC
0.20
0.10
Hole doping / Sr content (x)

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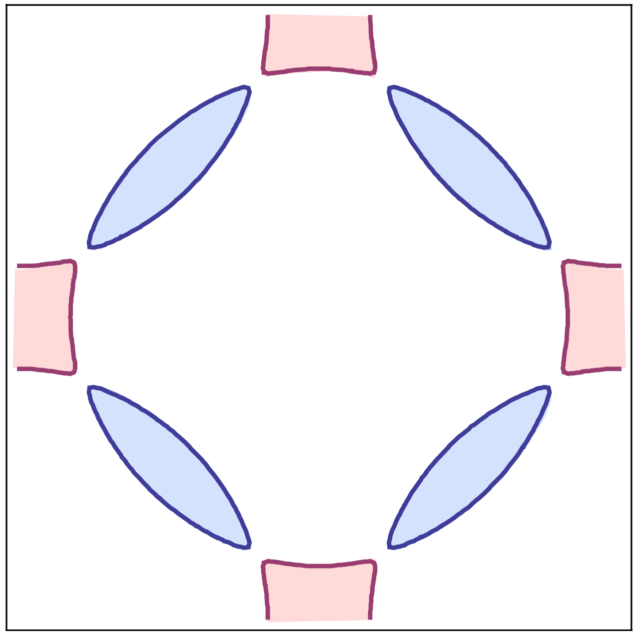
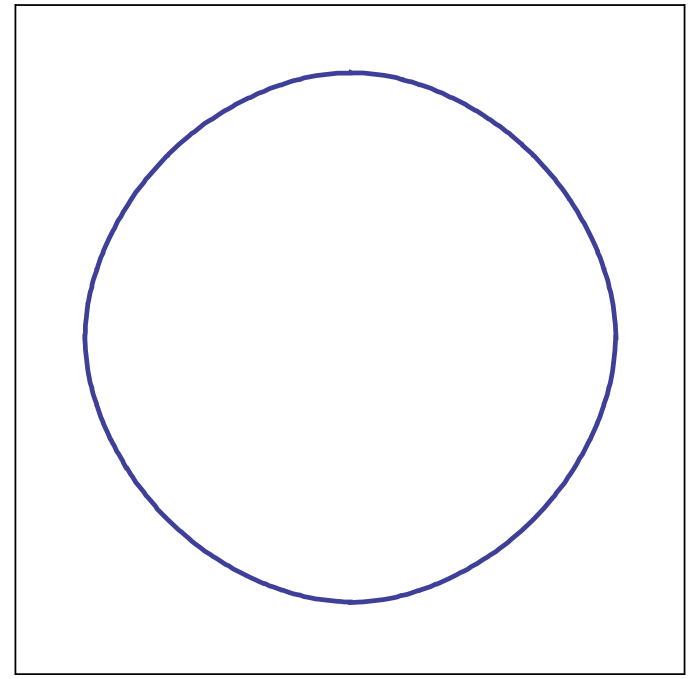


$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



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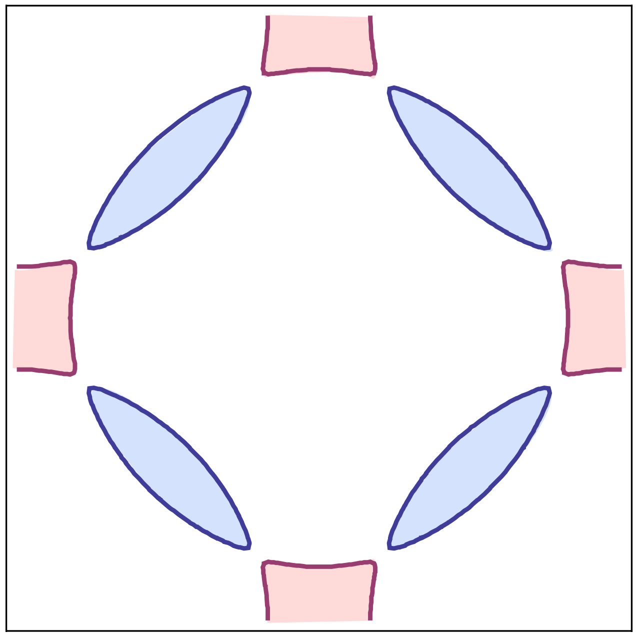
Metal with "large" Fermi surface

Separating onset of SDW order and Fermi surface reconstruction



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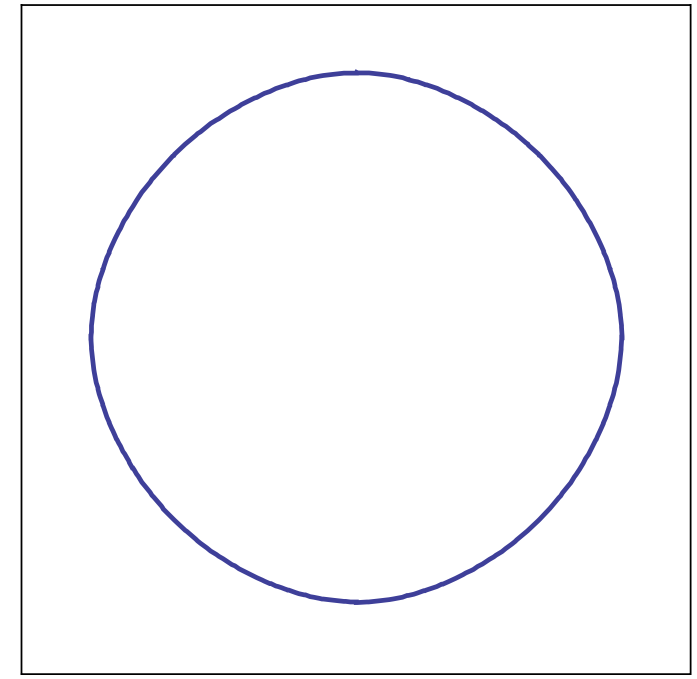

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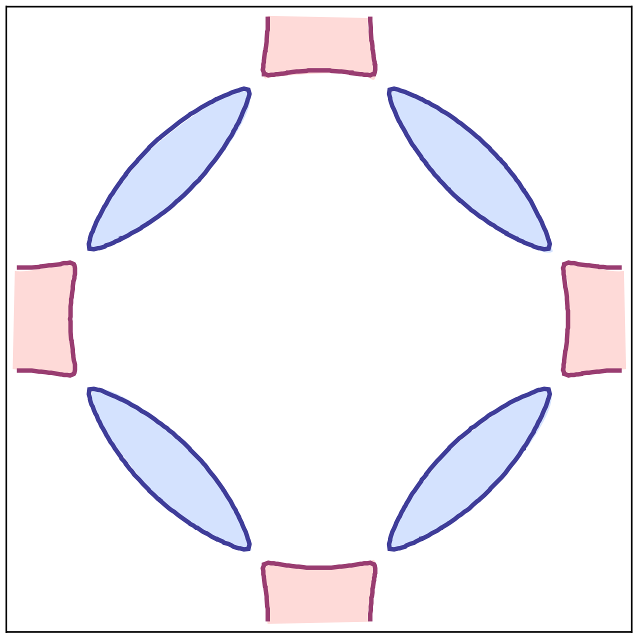

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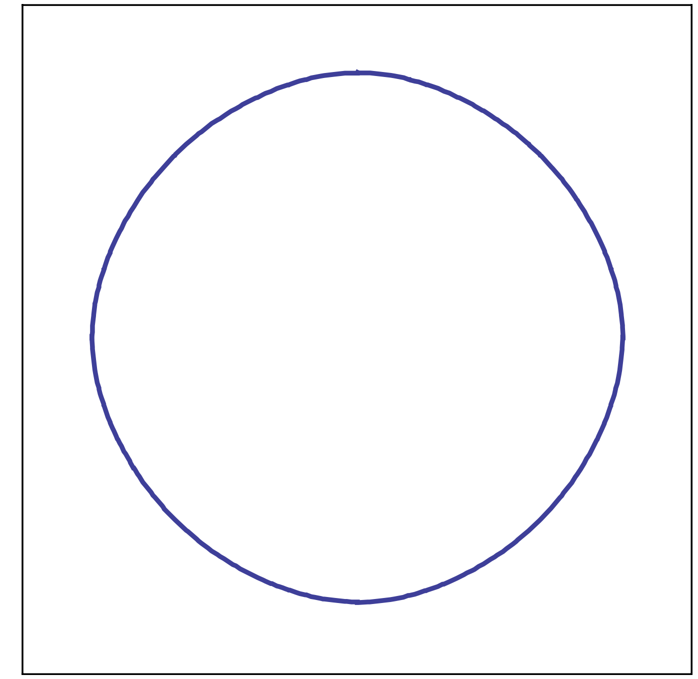

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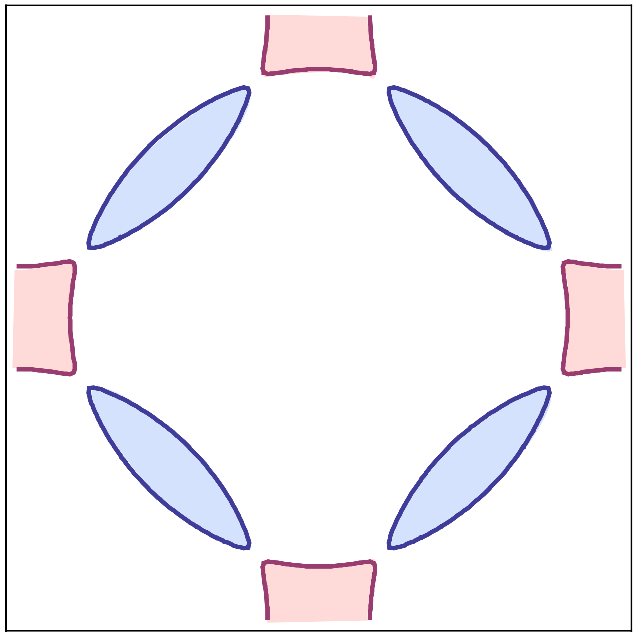

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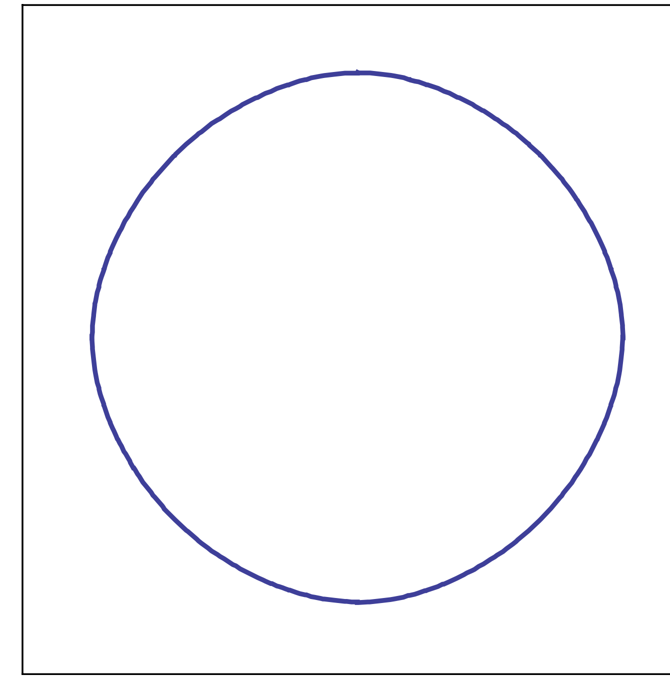

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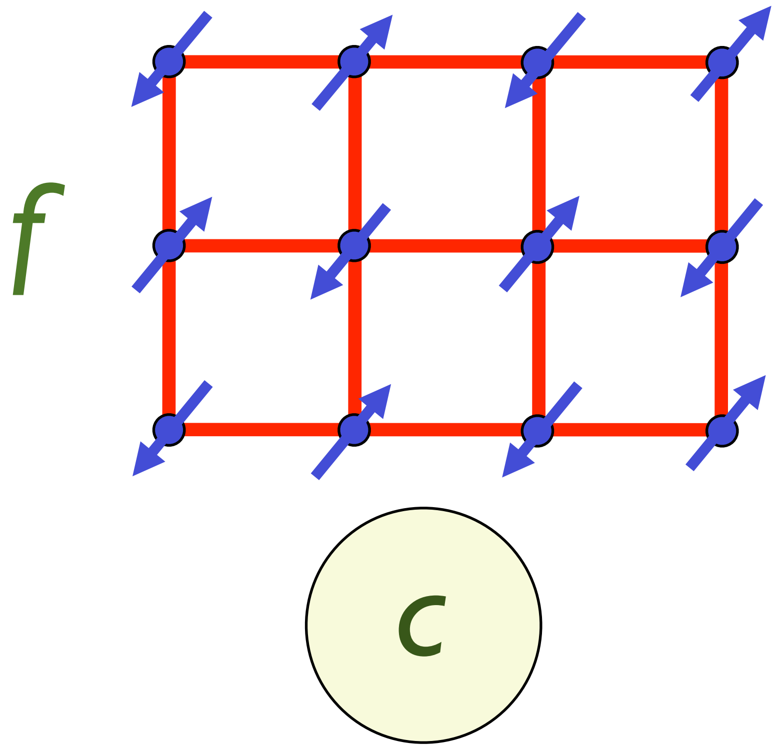
Magnetic order and the heavy Fermi liquid in the Kondo lattice



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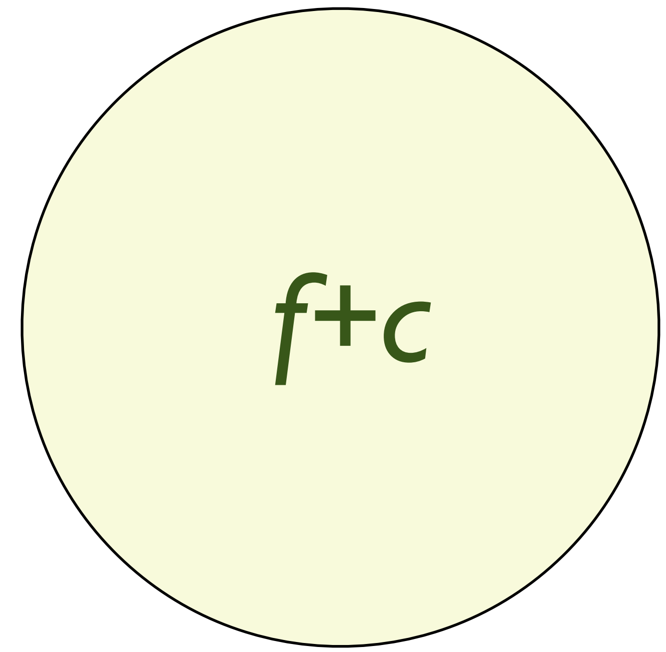
Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface



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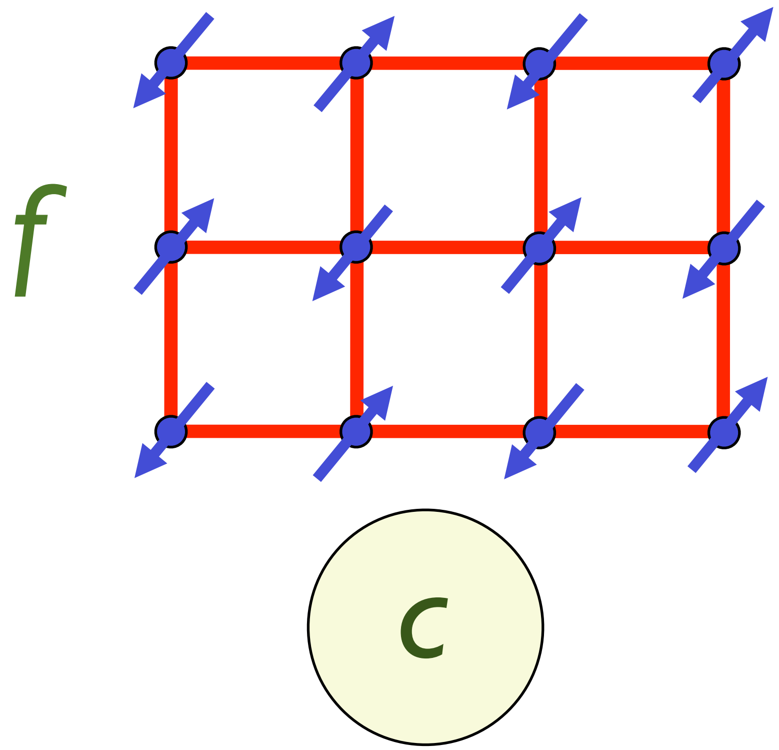
Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



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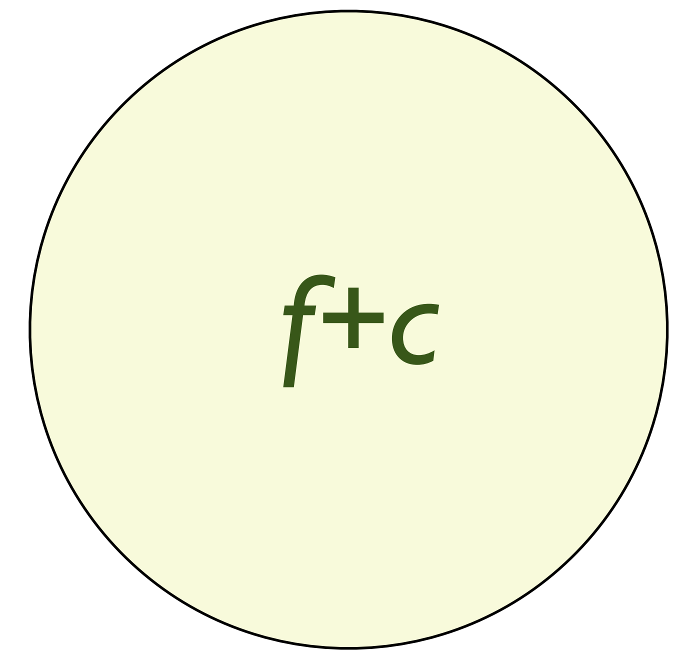

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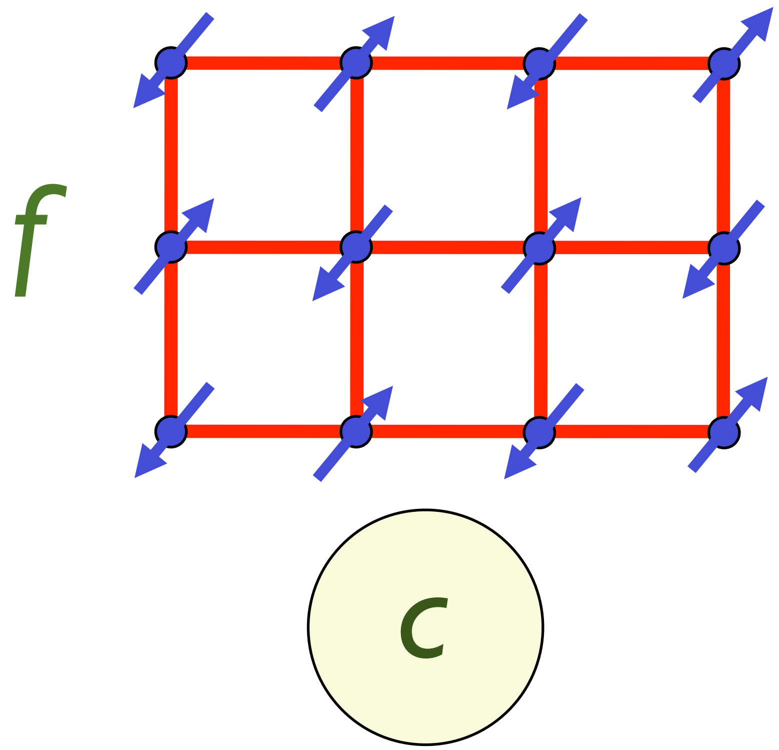

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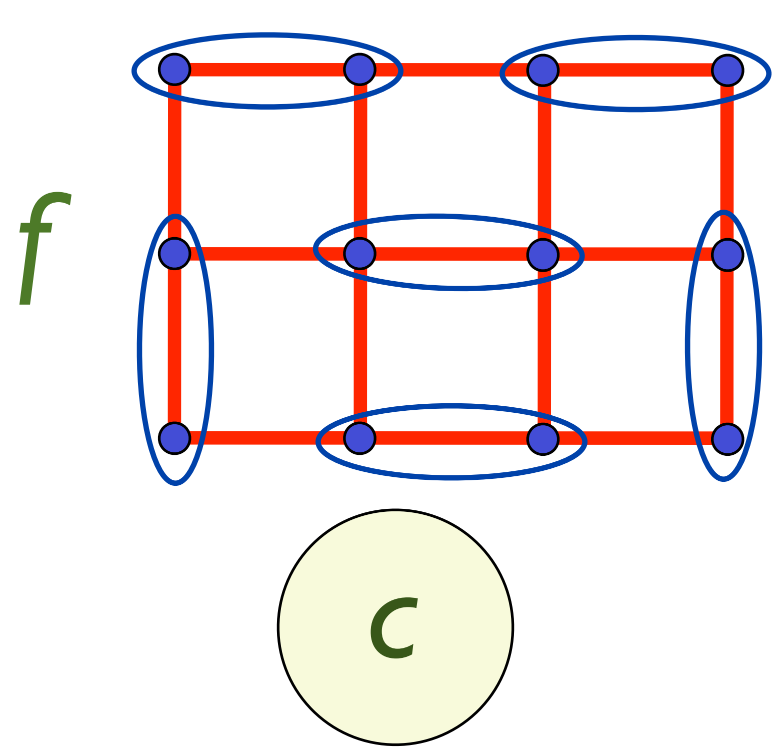

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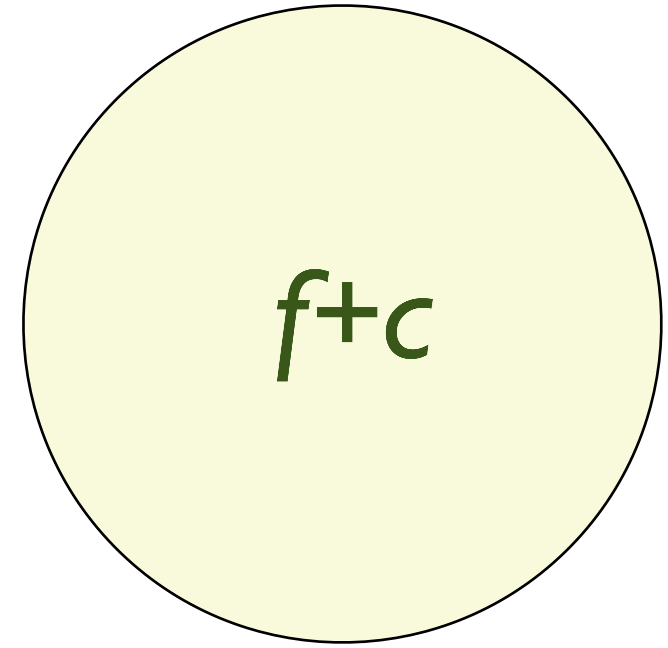
Conduction electron
Fermi surface
and
spin-liquid of
f-electrons



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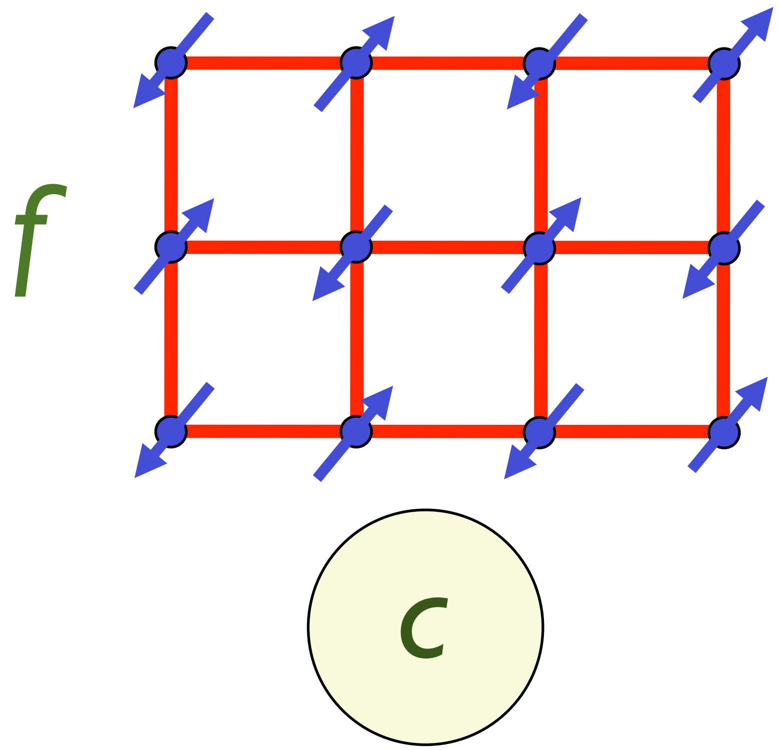

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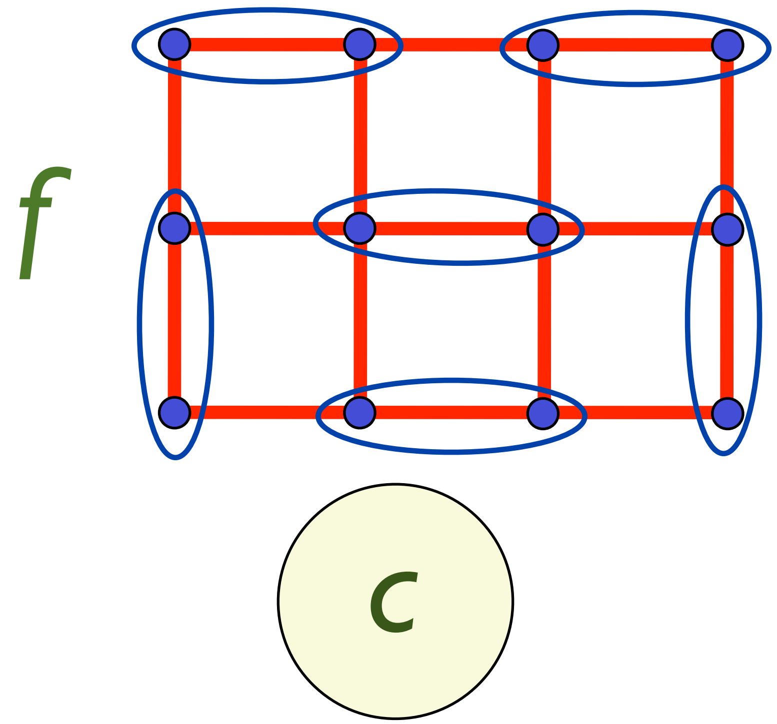

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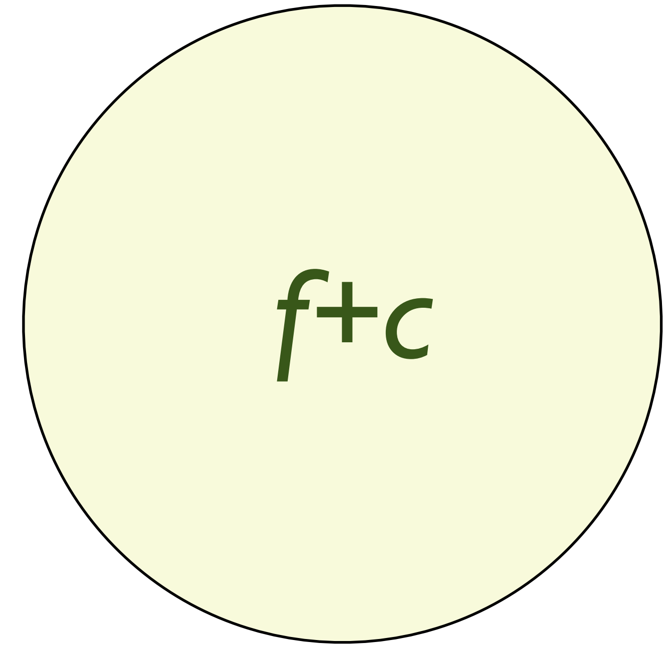

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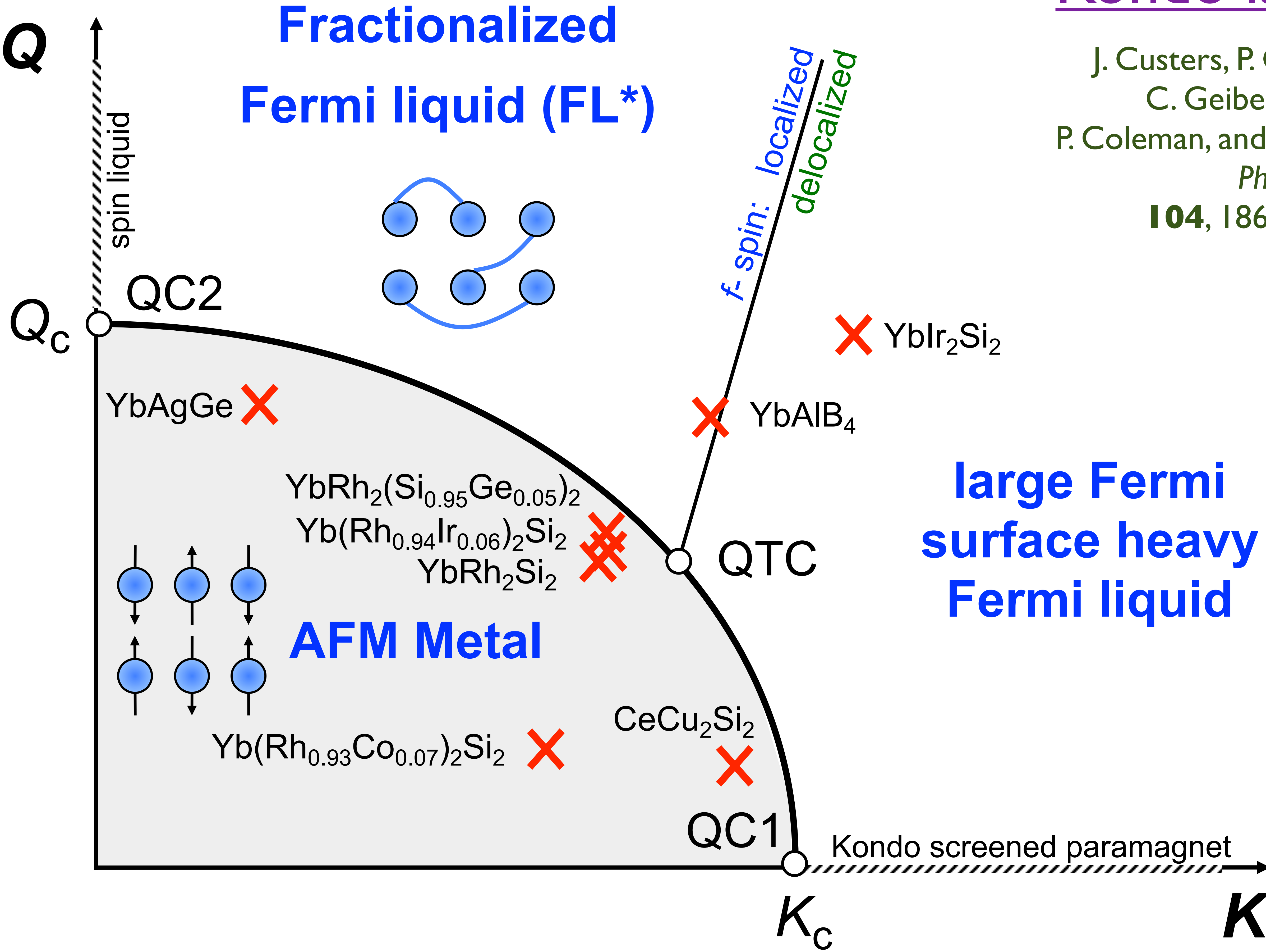
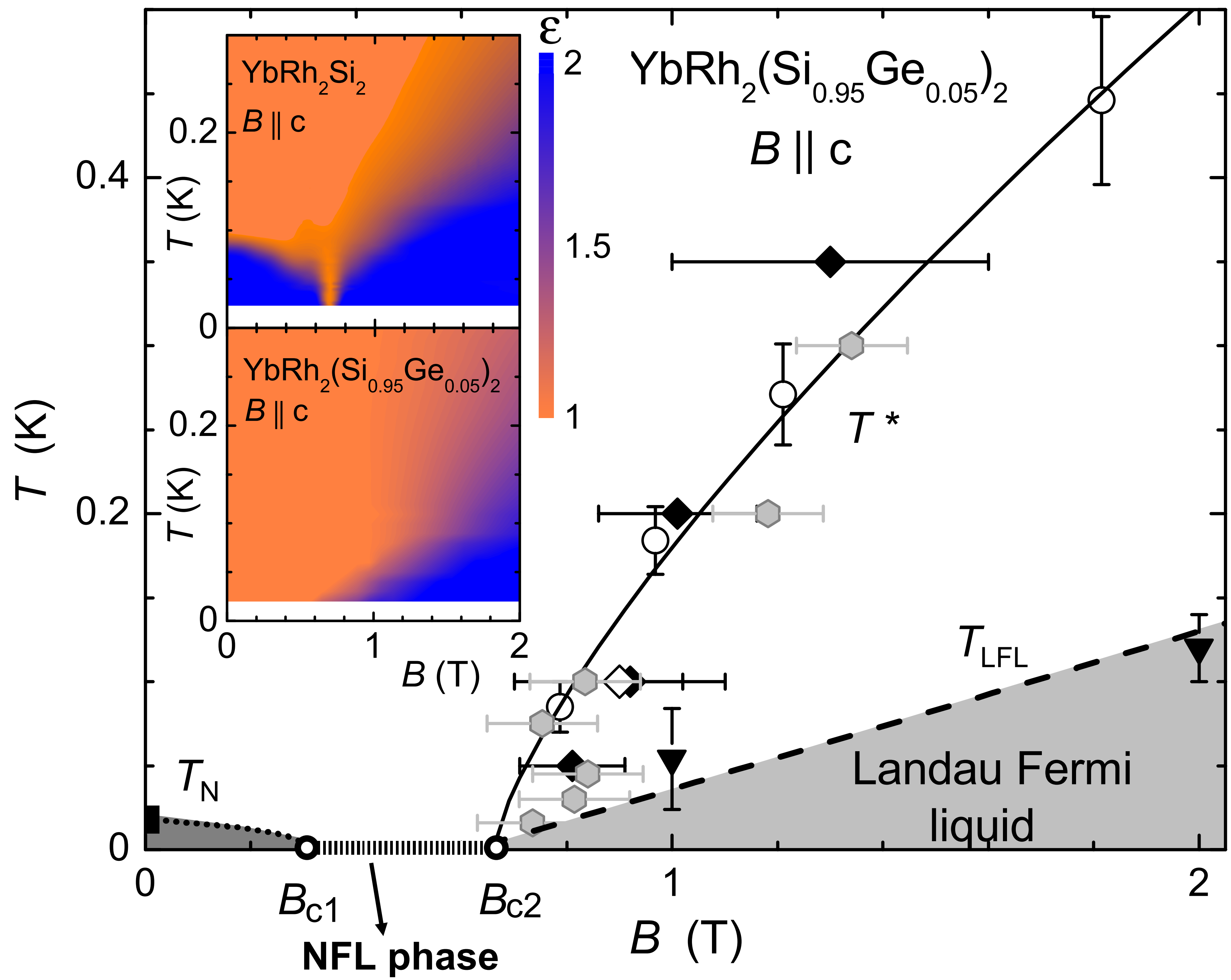
Experimental perpective on same phase diagrams of Kondo lattice

J. Custers, P. Gegenwart, C. Geibel, F. Steglich, P. Coleman, and S. Paschen, *Phys. Rev. Lett.* **104**, 186402 (2010)





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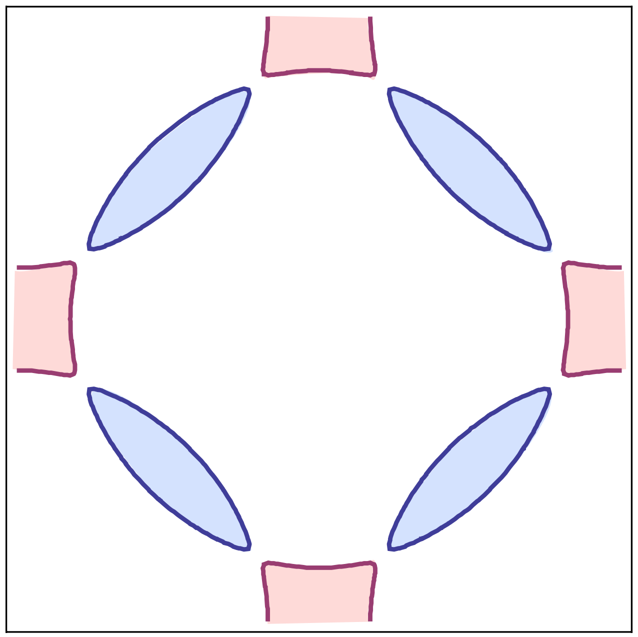

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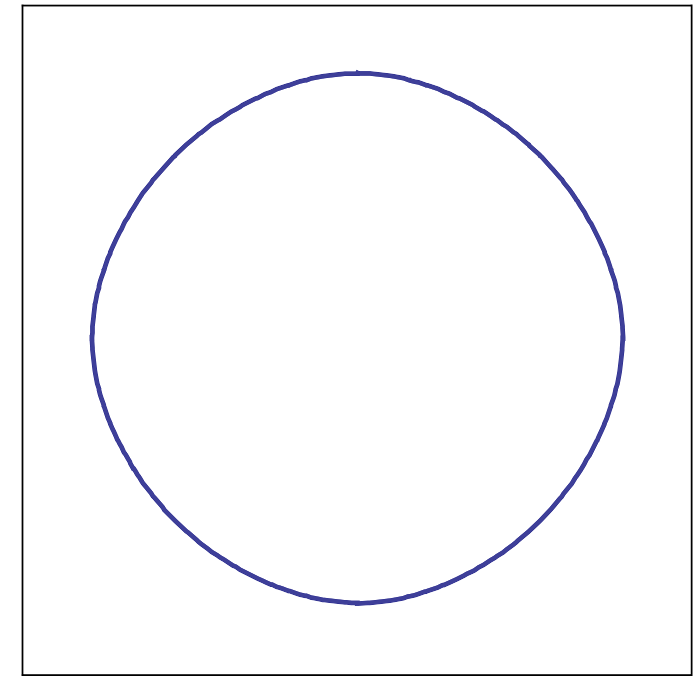

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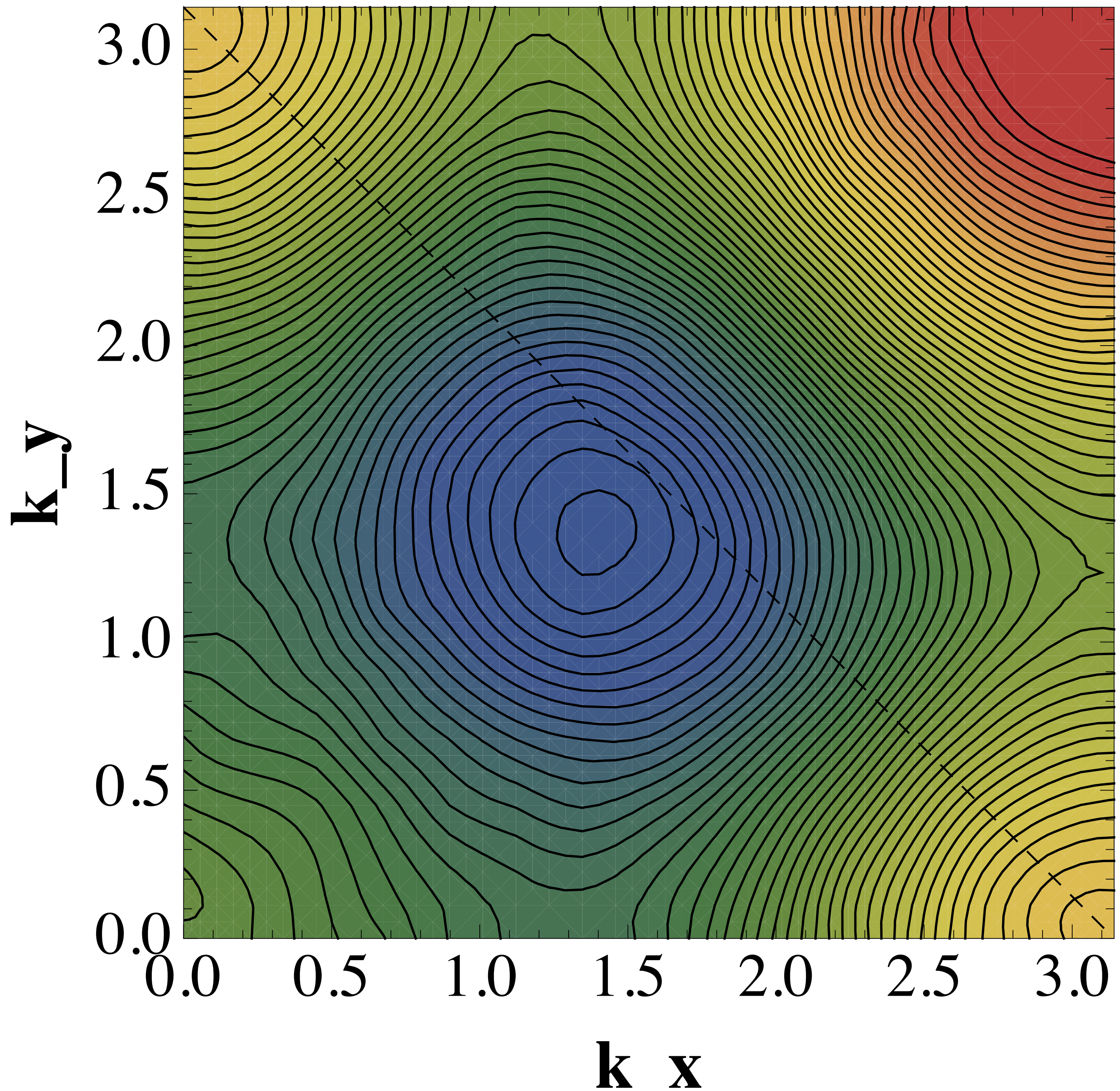
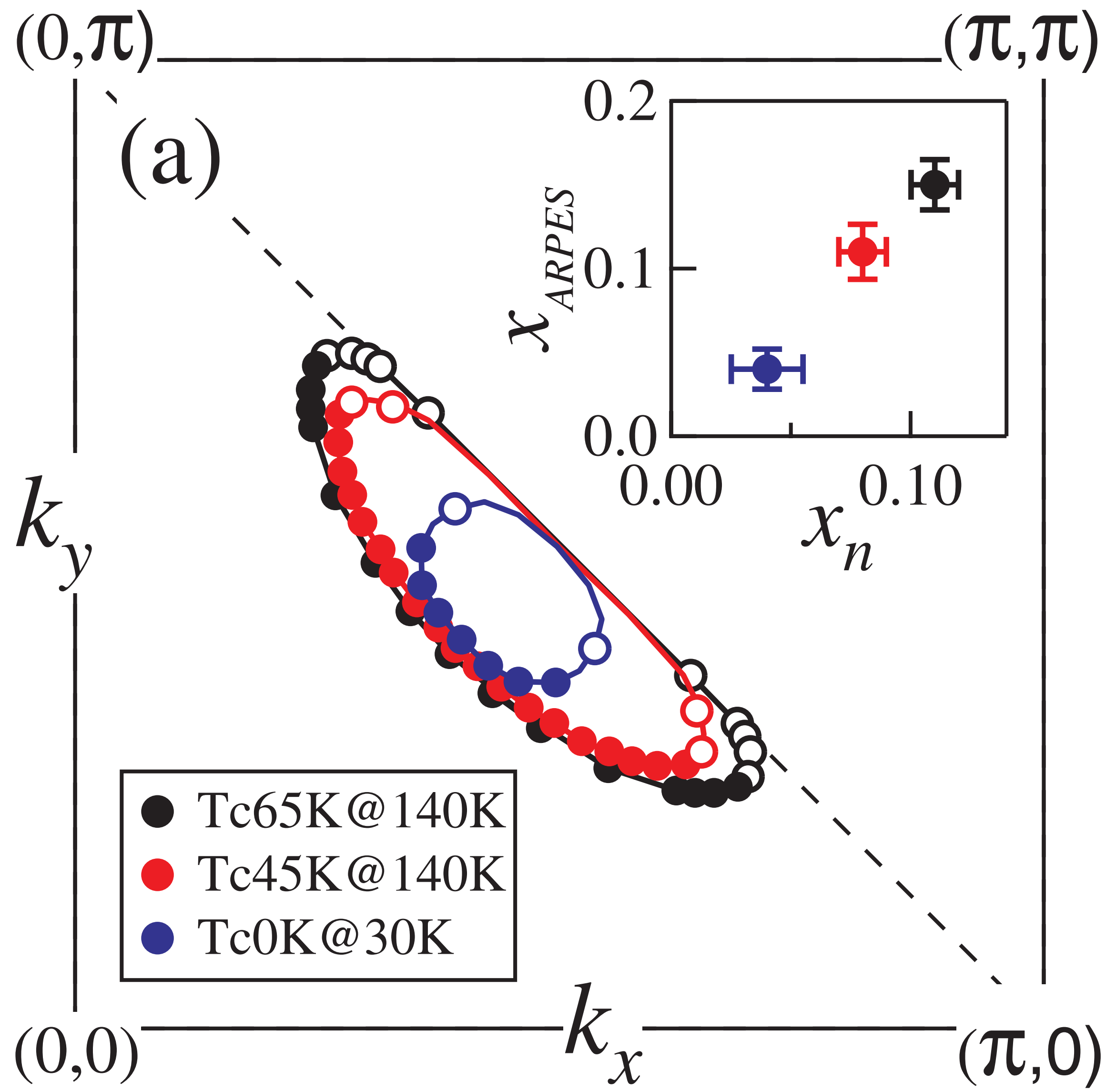


Hole pocket of a $\mathbb{Z}_2$-FL* phase in a *single*-band t-J model

M. Punk and S. Sachdev, *Phys. Rev. B* **85**, 195123 (2012)



PRL **107,** 047003 (2011) PHYSICAL REVIEW LETTERS week ending 22 JULY 2011

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors

H.-B. Yang,[1] J. D. Rameau,[1] Z.-H. Pan,[1] G. D. Gu,[1] P. D. Johnson,[1] H. Claus,[2] D. G. Hinks,[2] and T. E. Kidd[3]

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

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- Such a phase *must* have neutral $S = 1/2$ excitations (”spinons”), and collective spinless gauge excitations (“topological” order).

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Characteristics of FL* phase

- Fermi surface volume does not count all electrons.

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- These topological excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa’s proof of the Luttinger theorem.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Questions

- *Can quantum fluctuations near the onset of antiferromagnetism induce higher temperature superconductivity ?*

- *How should such a theory be extended to apply to the hole-doped cuprates ?*

- *What is the physics of the strange metal ?*

Questions and Answers

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  Strongly-coupled quantum criticality of Fermi surface change in a metal


Thank you !