States of quantum matter with long-range entanglement in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

Conformal quantum matter

A. Field theory:graphene

B. Field theory: superfluid-insulator transition

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Conformal quantum matter

A. Field theory:graphene

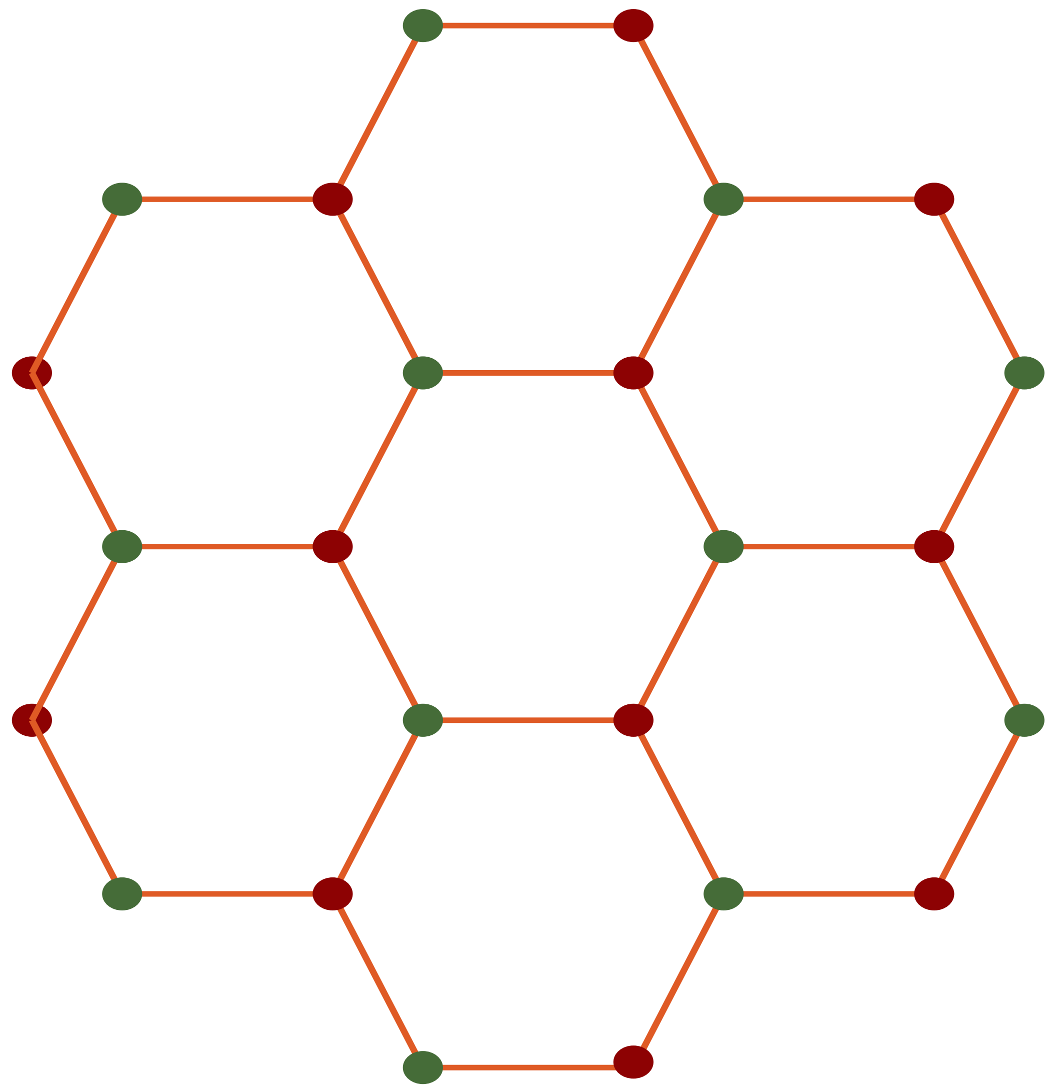
B. Field theory: superfluid-insulator transition

C. Field theory: antiferromagnets

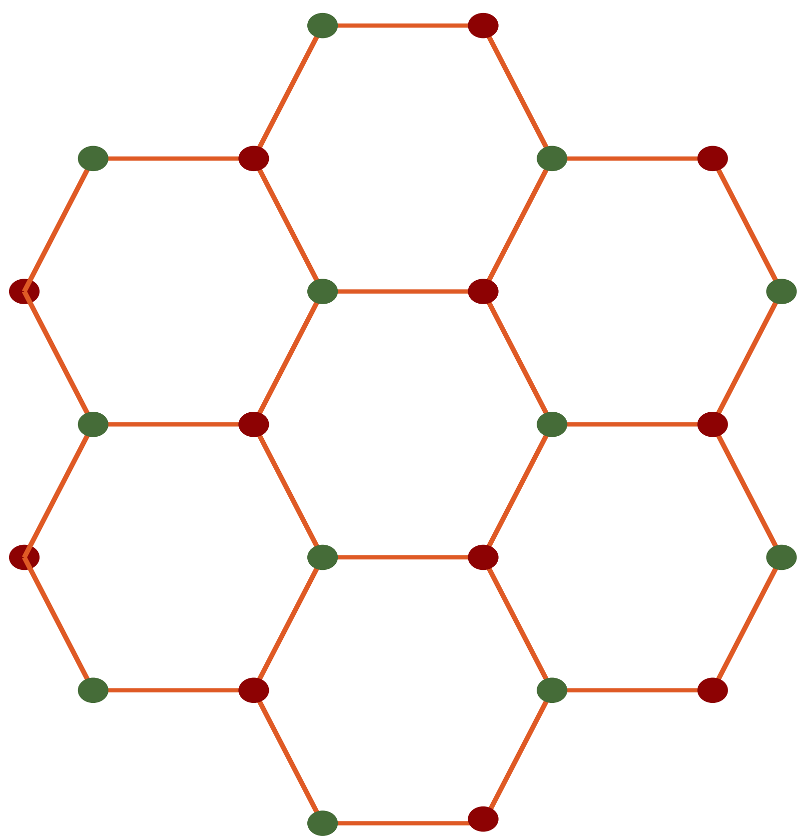
D. Gauge-gravity duality

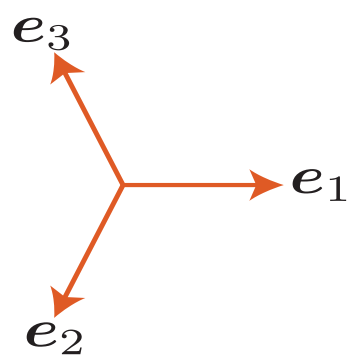
Honeycomb lattice

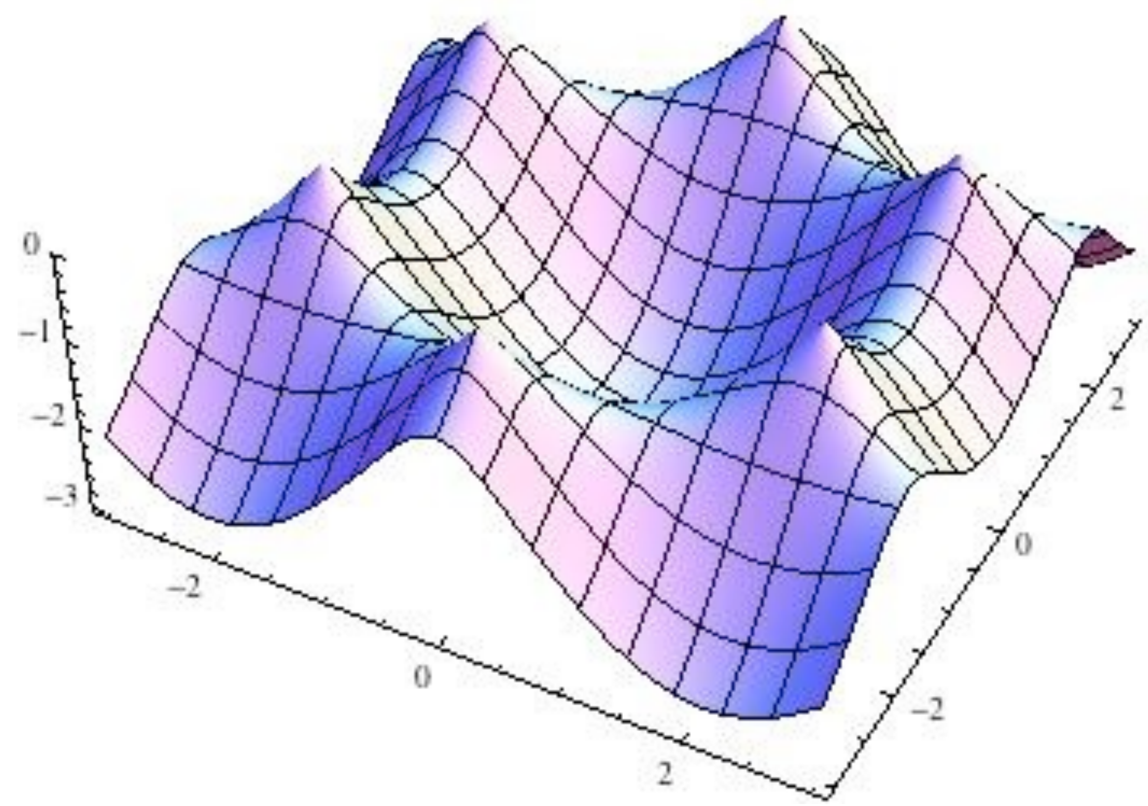
(describes graphene after adding long-range Coulomb interactions)

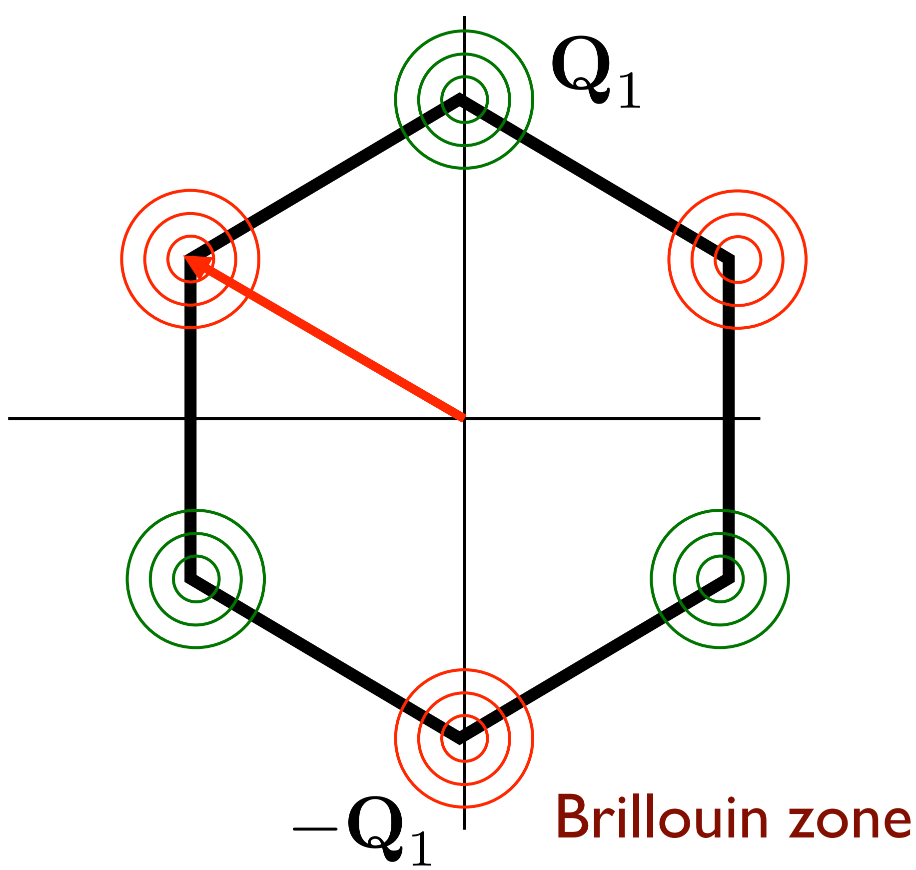


$$H = -t \sum_{\langle ij \rangle} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)$$









Semi-metal with
massless Dirac fermions
at small *U/t*

We define the Fourier transform of the fermions by

$$c_A(\mathbf{k}) = \sum_{\mathbf{r}} c_A(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \tag{4}$$

and similarly for c_B. A and B are sublattice indices.
The hopping Hamiltonian is

$$H_0 = -t \sum_{\langle ij \rangle} \left(c^\dagger_{Ai\alpha} c_{Bj\alpha} + c^\dagger_{Bj\alpha} c_{Ai\alpha} \right) \tag{5}$$

where α is a spin index. If we introduce Pauli matrices τ^a in sublattice space ($a = x, y, z$), this Hamiltonian can be written as

$$\begin{aligned} H_0 &= \int \frac{d^2k}{4\pi^2} c^\dagger(\mathbf{k}) \Big[-t \Big(\cos(\mathbf{k}\cdot\mathbf{e}_1) + \cos(\mathbf{k}\cdot\mathbf{e}_2) + \cos(\mathbf{k}\cdot\mathbf{e}_3) \Big) \tau^x \\ &\quad + t \Big(\sin(\mathbf{k}\cdot\mathbf{e}_1) + \sin(\mathbf{k}\cdot\mathbf{e}_2) + \sin(\mathbf{k}\cdot\mathbf{e}_3) \Big) \tau^y \Big] c(\mathbf{k}) \end{aligned} \tag{6}$$

The low energy excitations of this Hamiltonian are near $\mathbf{k} \approx \pm \mathbf{Q}_1$.

In terms of the fields near $\mathbf{Q}_1$ and $-\mathbf{Q}_1$, we define

$$\begin{aligned}
\Psi_{A1\alpha}(\mathbf{k}) &= c_{A\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
\Psi_{A2\alpha}(\mathbf{k}) &= c_{A\alpha}(-\mathbf{Q}_1 + \mathbf{k}) \\
\Psi_{B1\alpha}(\mathbf{k}) &= c_{B\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
\Psi_{B2\alpha}(\mathbf{k}) &= c_{B\alpha}(-\mathbf{Q}_1 + \mathbf{k})
\end{aligned} \tag{7}$$

We consider Ψ to be a 8 component vector, and introduce Pauli matrices ρ^a which act in the $1,2$ valley space. Then the Hamiltonian is

$$H_0 = \int \frac{d^2k}{4\pi^2} \Psi^\dagger(\mathbf{k}) \Big(v\tau^y k_x + v\tau^x \rho^z k_y \Big) \Psi(\mathbf{k}), \tag{8}$$

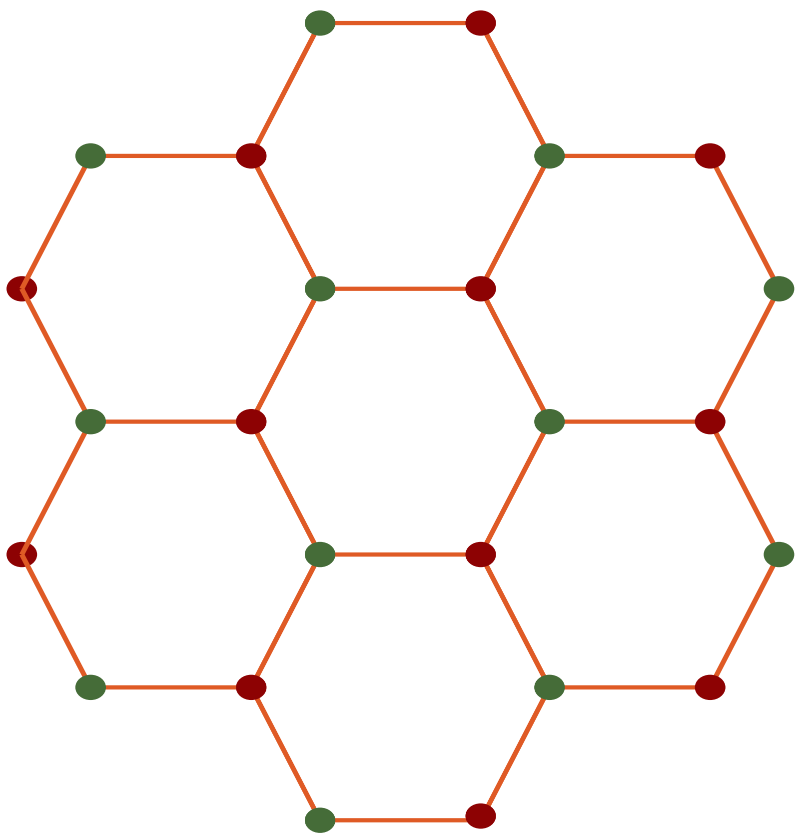
where $v = 3t/2$; below we set $v = 1$. Now define $\overline{\Psi} = \Psi^\dagger \rho^z \tau^z$. Then we can write the imaginary time Lagrangian as

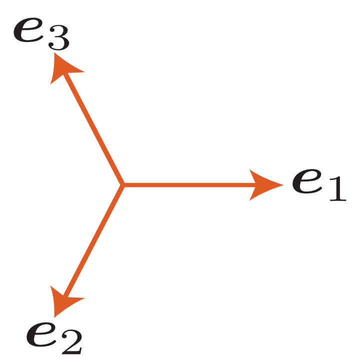
$$\mathcal{L}_0 = -i\overline{\Psi} \left(\omega\gamma_0 + k_x\gamma_1 + k_y\gamma_2 \right) \Psi \tag{9}$$

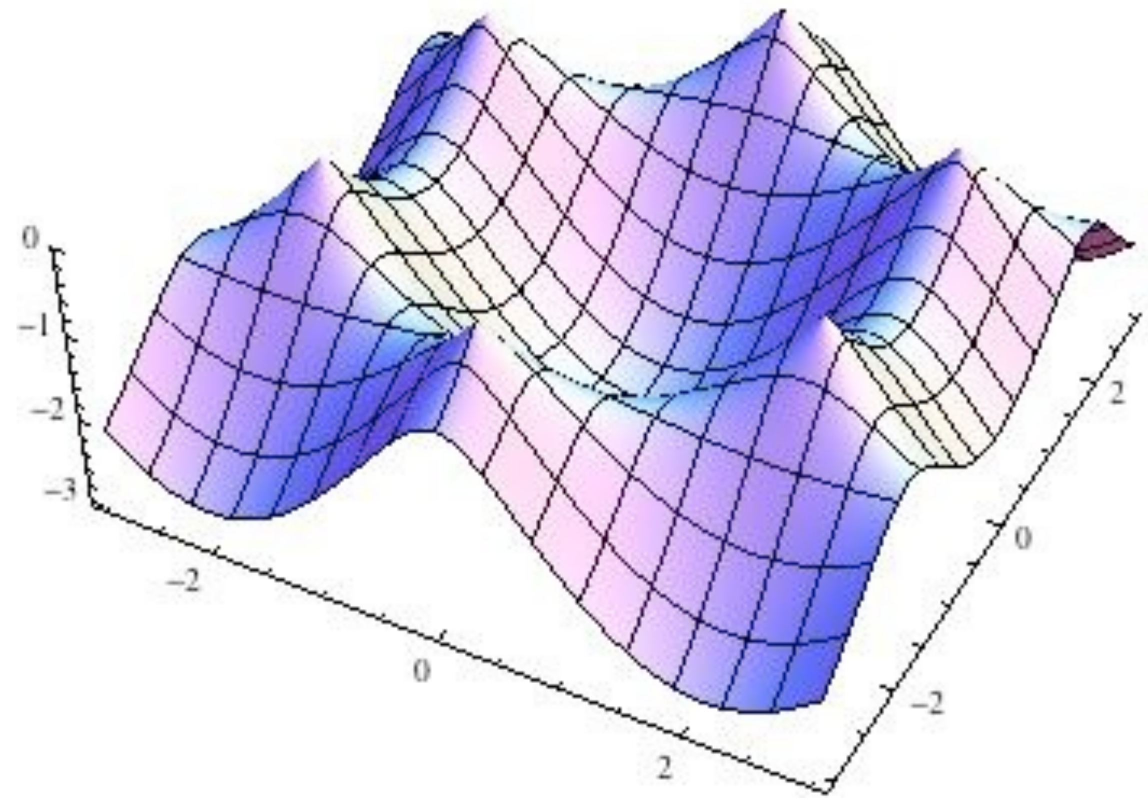
where

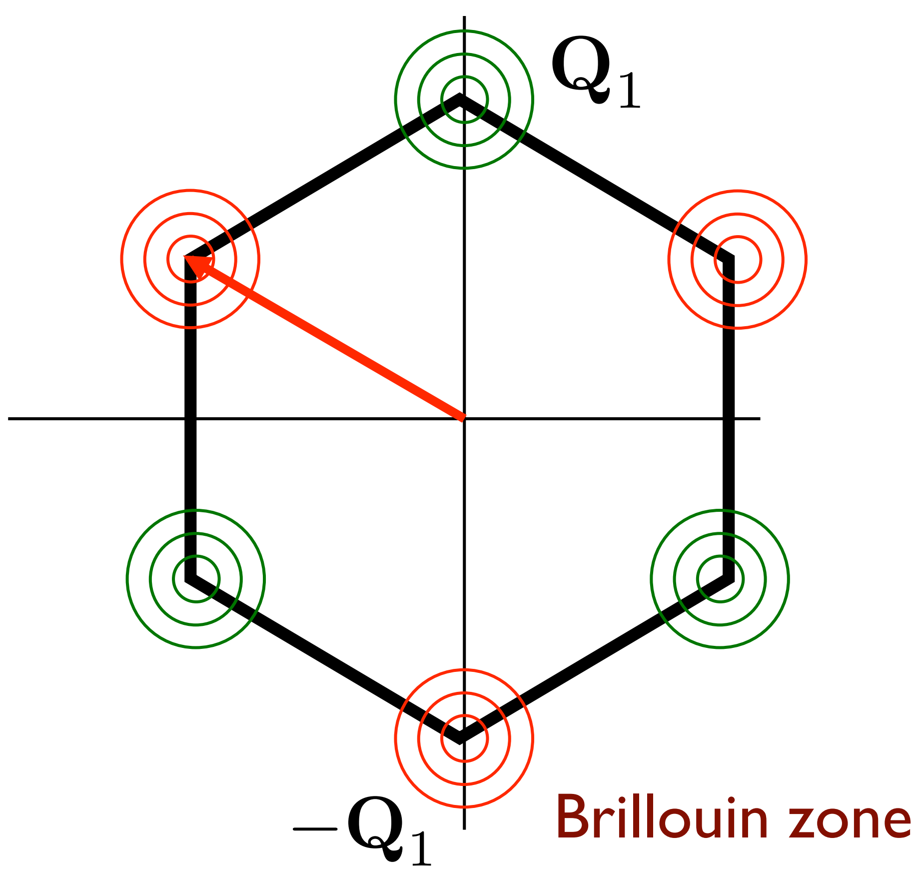
$$\gamma_0 = -\rho^z\tau^z \quad \gamma_1 = \rho^z\tau^x \quad \gamma_2 = -\tau^y \tag{10}$$

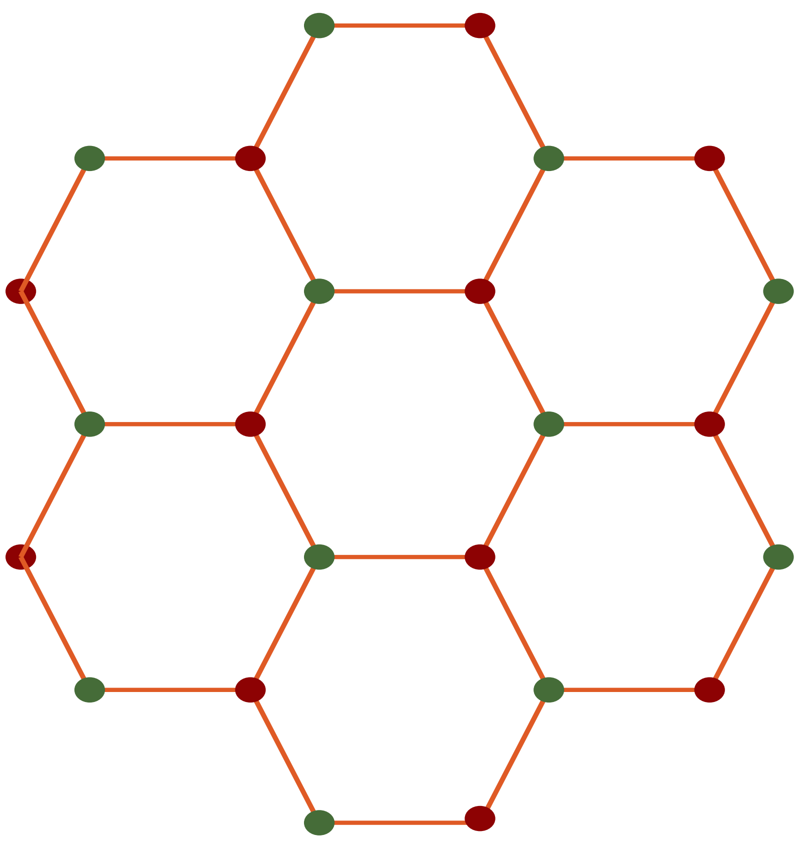
Exercise: Observe that $\mathcal{L}_0$ is invariant under the scaling transformation $x' = xe^{-\ell}$ and $\tau' = \tau e^{-\ell}$. Write the Hubbard interaction U in terms of the Dirac fermions, and show that it has the tree-level scaling transformation $U' = Ue^{-\ell}$. So argue that all short-range interactions are *irrelevant* in the Dirac semi-metal phase.

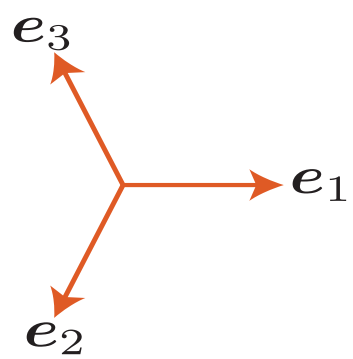


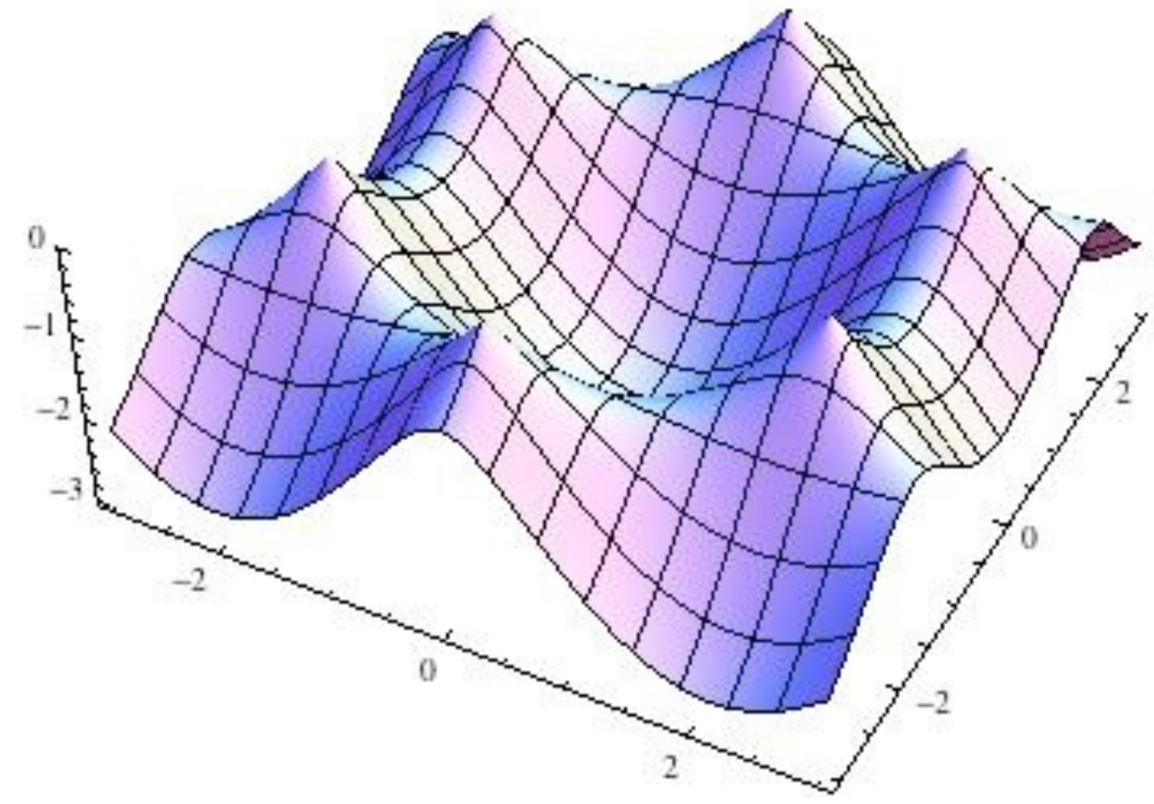

e_3
e_1
e_2

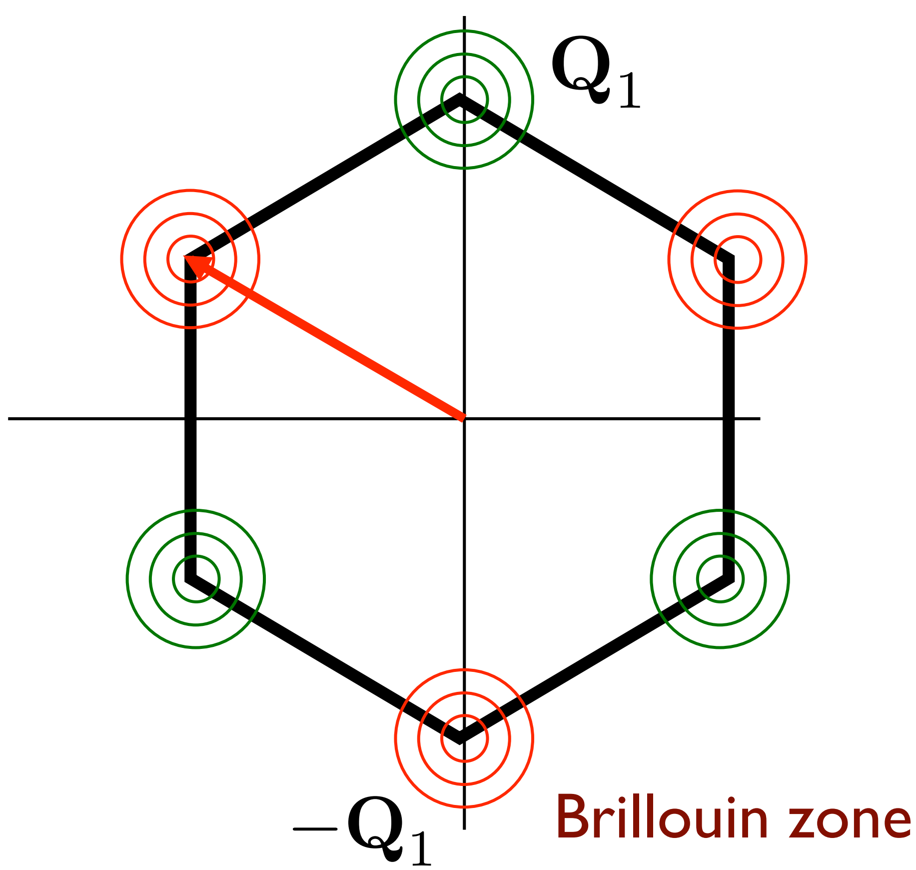



$\mathbf{Q}_1$
$-\mathbf{Q}_1$
Brillouin zone









The theory of free Dirac fermions is invariant under conformal transformations of spacetime. This is a realization of a simple conformal field theory in 2+1 dimensions: a <u>CFT3</u>

The Hubbard Model at large U

$$H = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

In the limit of large U, and at a density of one particle per site, this maps onto the Heisenberg antiferromagnet

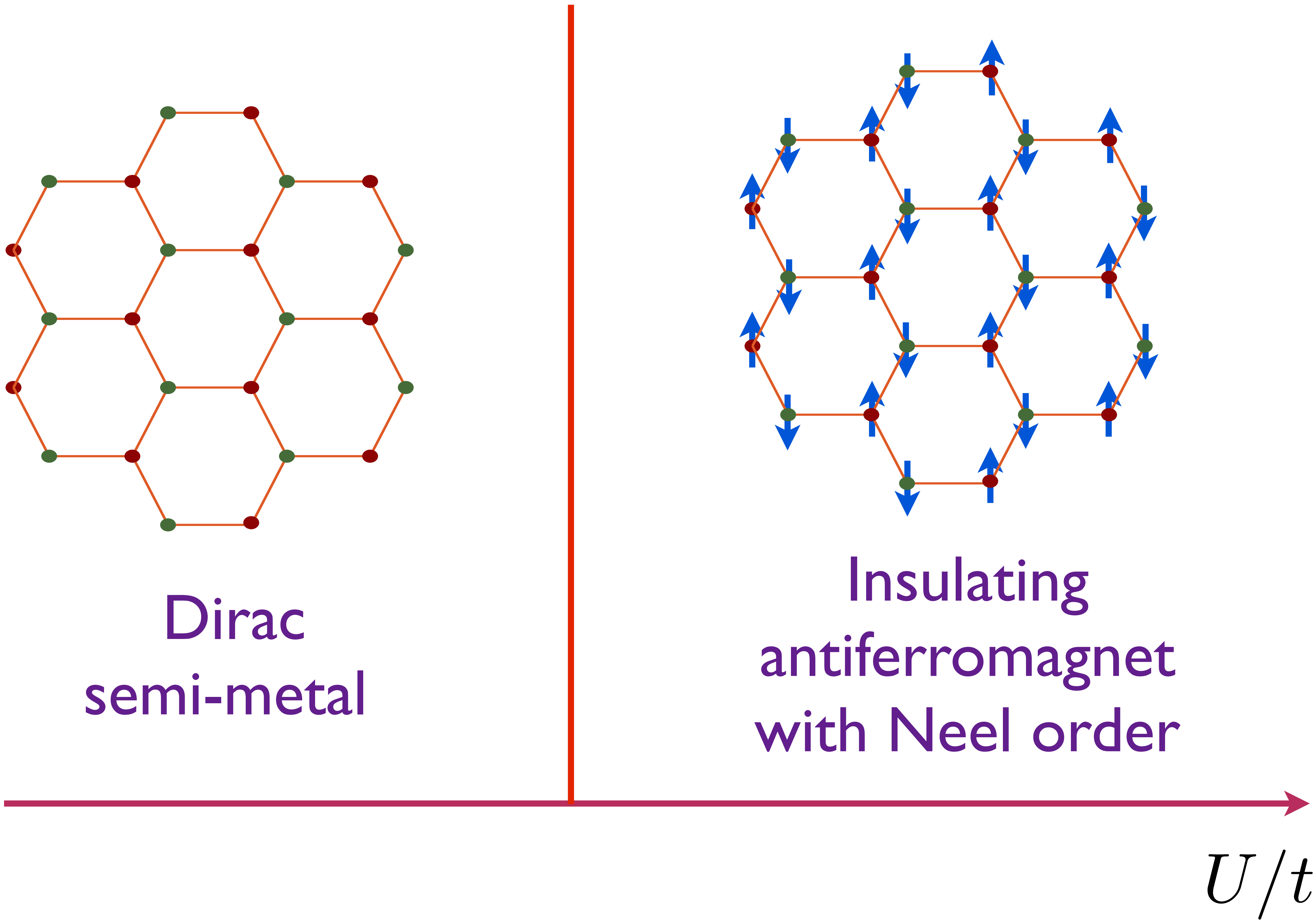
$$H_{AF} = \sum_{i<j} J_{ij} S_i^a S_j^a$$

where $a = x, y, z$,

$$S_i^a = \frac{1}{2} c_{i\alpha}^{a\dagger} \sigma_{\alpha\beta}^a c_{i\beta},$$

with σ^a the Pauli matrices and

$$J_{ij} = \frac{4t_{ij}^2}{U}$$


Dirac
semi-metal
Insulating
antiferromagnet
with Neel order
U/t

Antiferromagnetism

We use the operator equation (valid on each site i):

$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}S^{a2} + \frac{U}{4} \qquad (11)$$

Then we decouple the interaction via

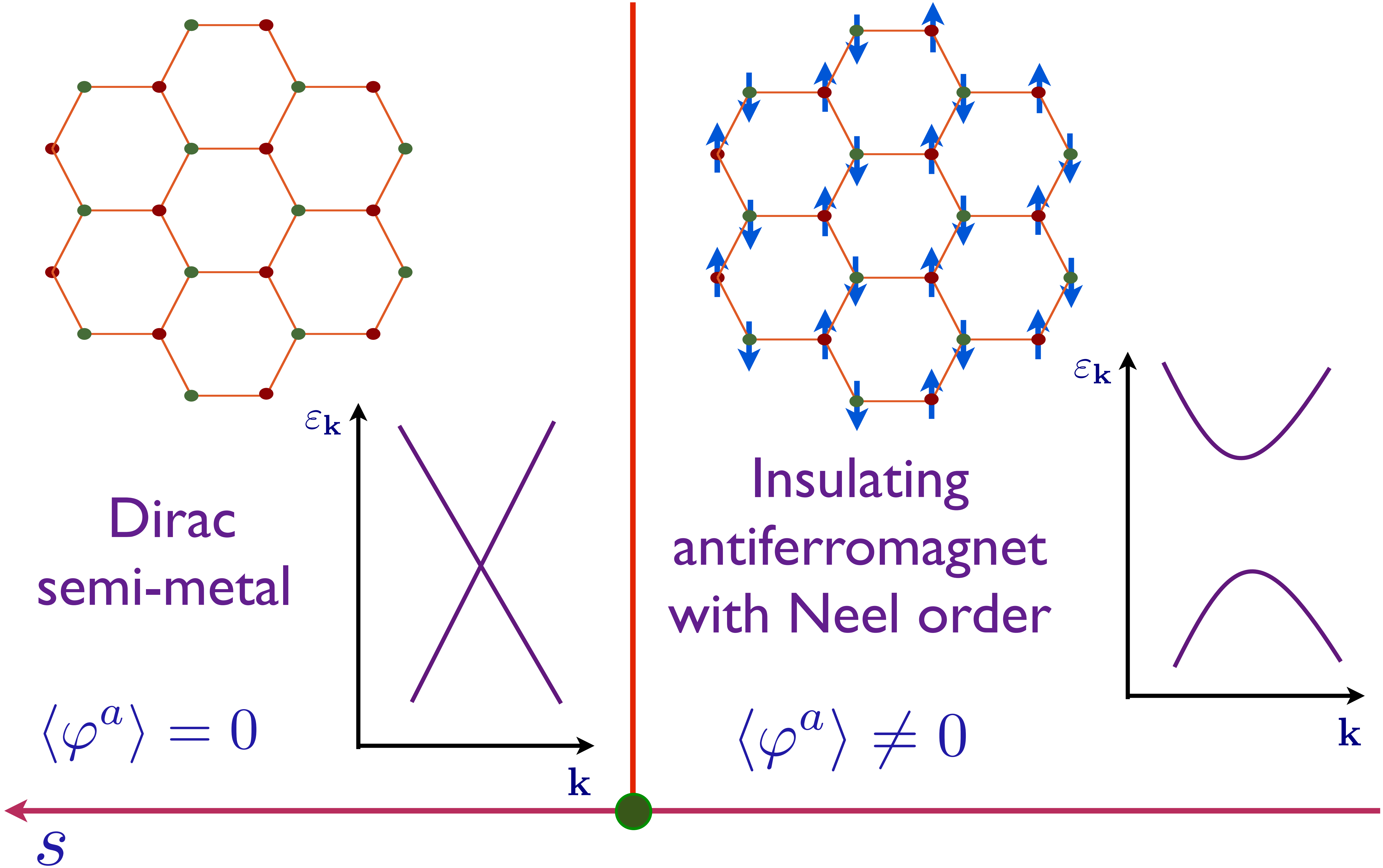
$$\exp\left(\frac{2U}{3}\sum_i \int d\tau S_i^{a2}\right) = \int \mathcal{D}J_i^a(\tau)\exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}J_i^{a2} - J_i^a S_i^a\right]\right) \qquad (12)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for J_i^a.

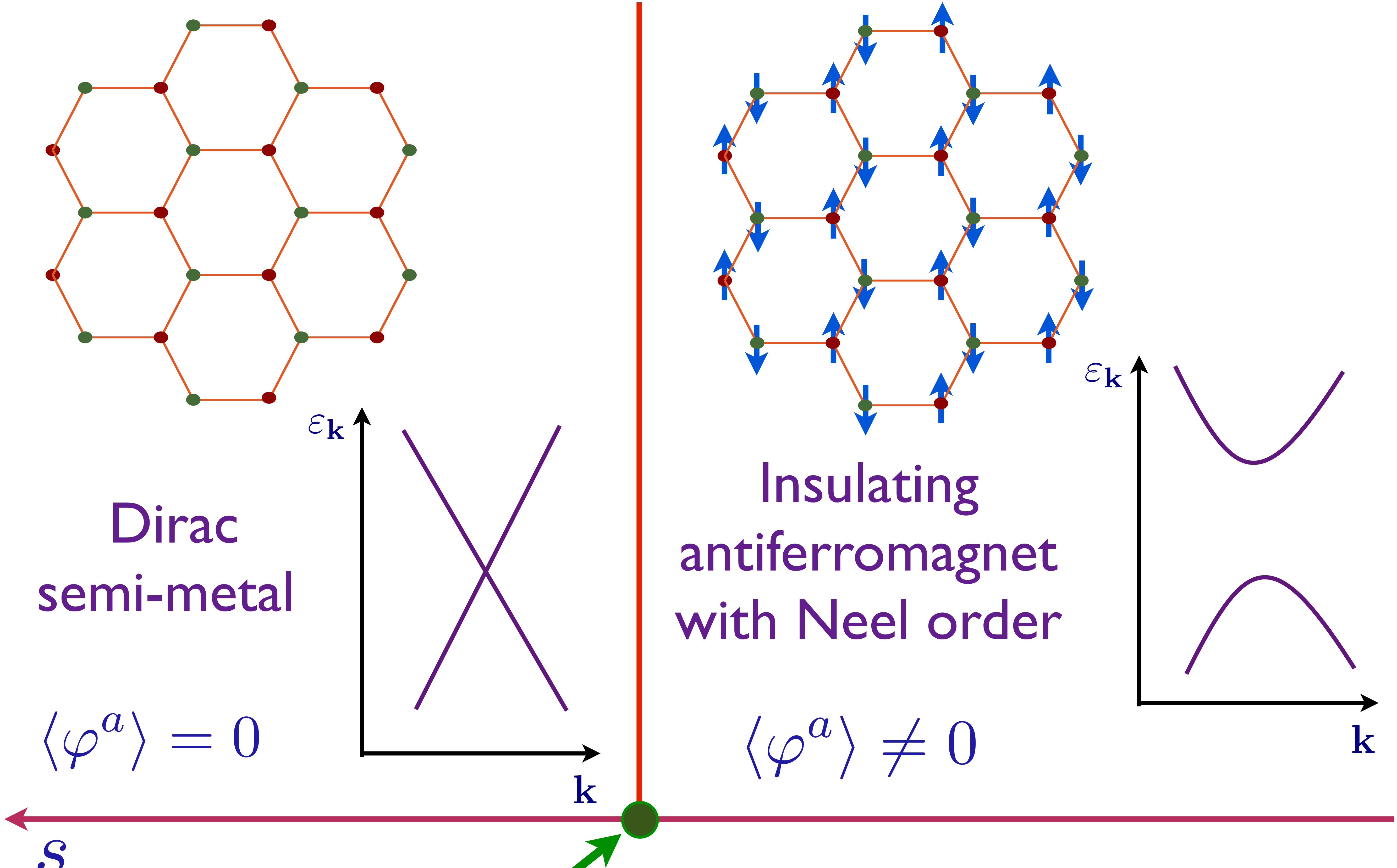
Long wavelength fluctuations about this saddle point are described by a field theory of the Néel order parameter, φ^a, coupled to the Dirac fermions in the **Gross-Neveu** model.

$$\mathcal{L} = \overline{\Psi}\gamma_\mu\partial_\mu\Psi + \frac{1}{2}\left[(\partial_\mu\varphi^a)^2 + s\varphi^{a2}\right] + \frac{u}{24}\left(\varphi^{a2}\right)^2 - \lambda\varphi^a\overline{\Psi}\rho^z\sigma^a\Psi$$

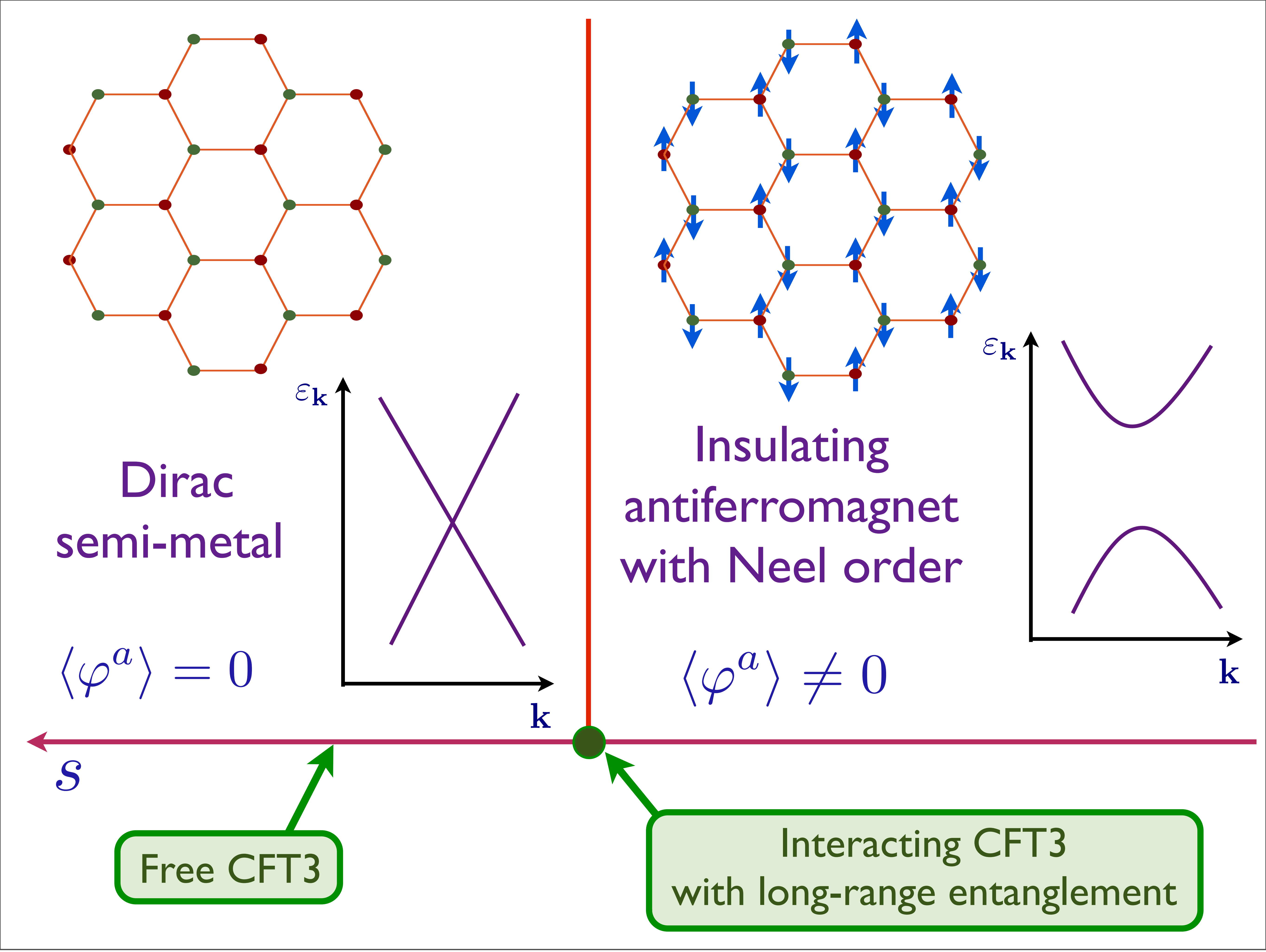
I.F. Herbut, V. Juricic, and B. Roy, *Phys. Rev. B* **79**, 085116 (2009)



Monday, June 4, 2012

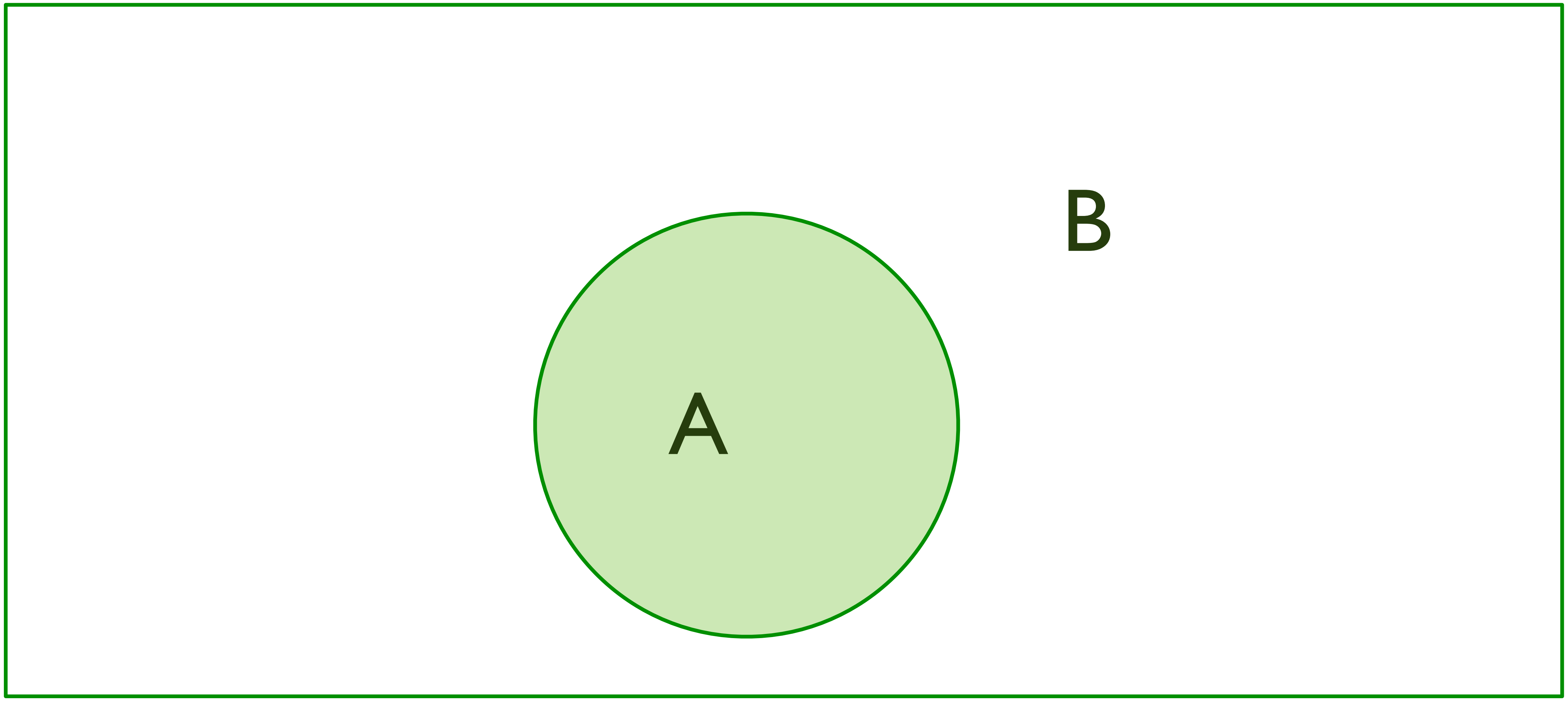


At the quantum critical point, the non-linear couplings λ and u in the Gross-Neveu model reach non-zero fixed-point values under the renormalization group flow. The critical theory is an *interacting* CFT3


Dirac
semi-metal
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
$\langle \varphi^a \rangle = 0$
Insulating
antiferromagnet
with Neel order
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
$\langle \varphi^a \rangle \neq 0$
s
Free CFT3
Interacting CFT3
with long-range entanglement

Long-range entanglement in a CFT3

- Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where γ is a universal number associated with the CFT3.

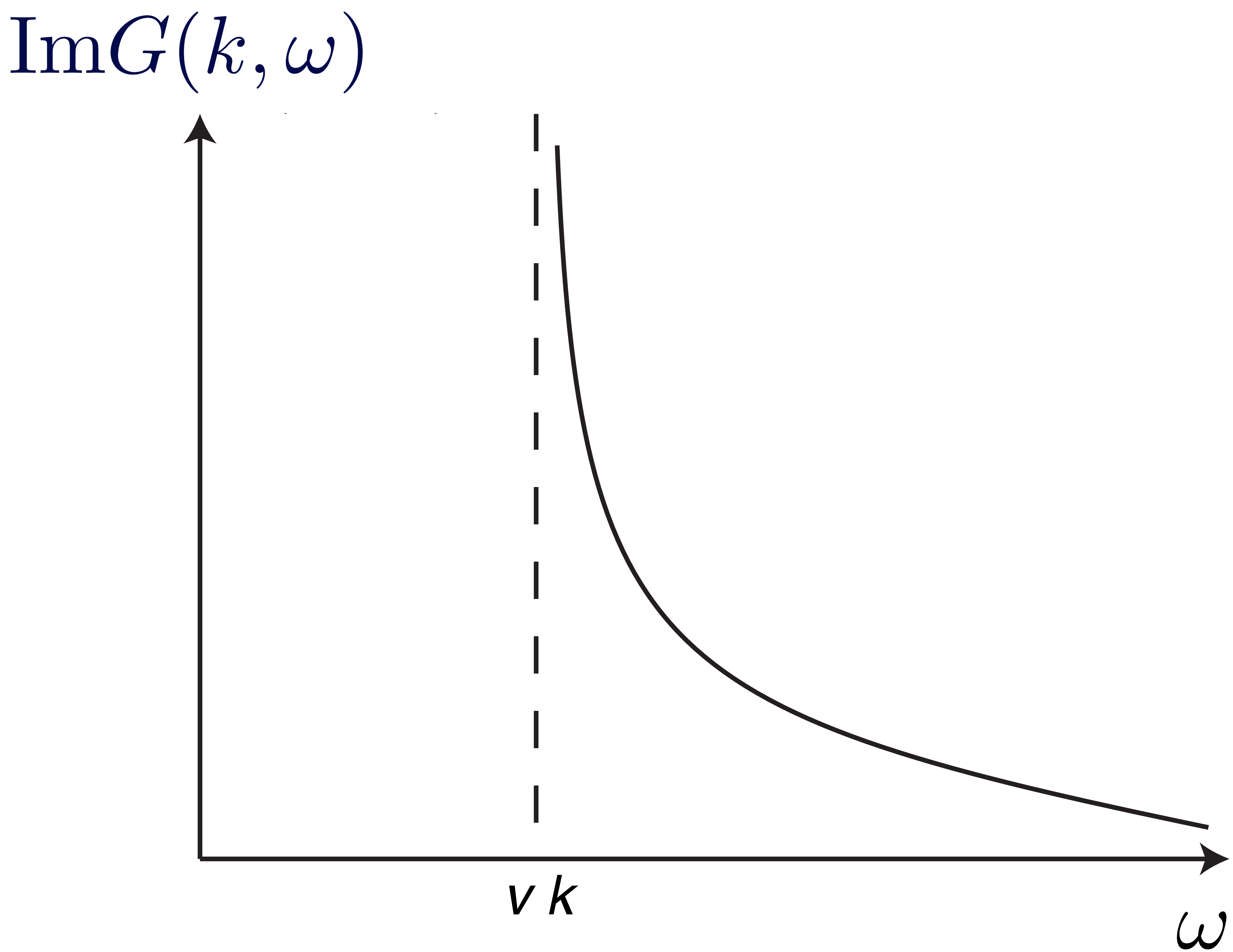


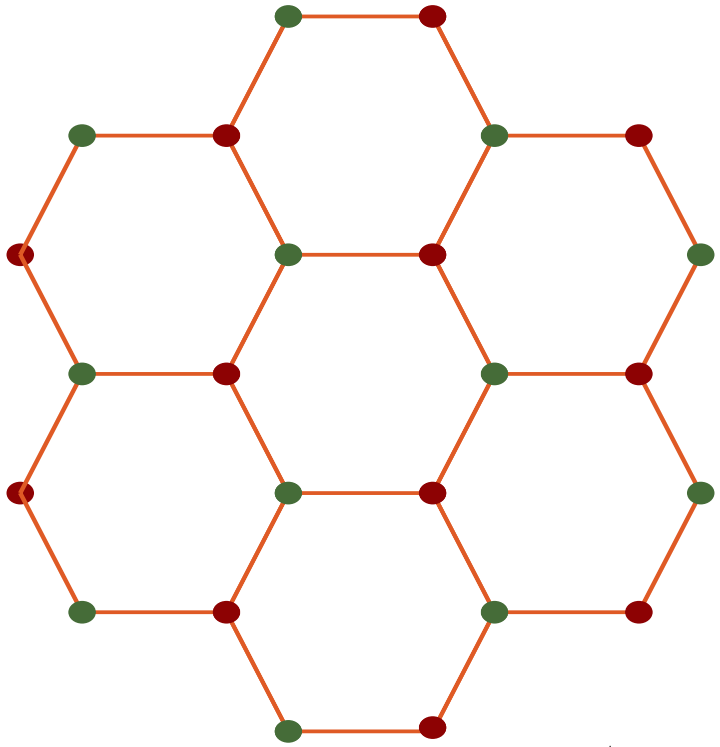
M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

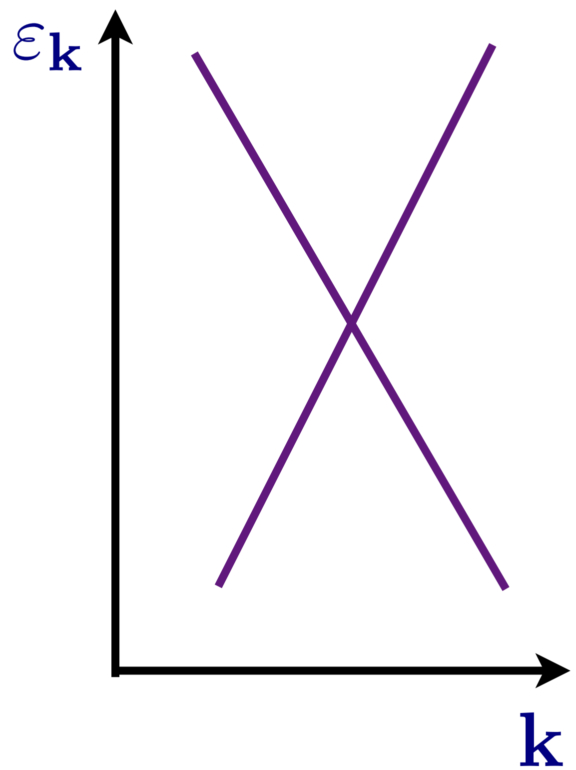
Electron Green's function for the interacting CFT3

$$G(k,\omega) = \left\langle \Psi(k,\omega); \Psi^\dagger(k,\omega) \right\rangle \sim \frac{i\omega + v k_x \tau^y + v k_y \tau^x \rho^z}{(\omega^2 + v^2 k_x^2 + v^2 k_y^2)^{1-\eta/2}}$$

where $\eta > 0$ is the *anomalous dimension* of the fermion. Note that this leads to a fermion spectral density which has no quasiparticle pole: thus the quantum critical point has no well-defined quasiparticle excitations.

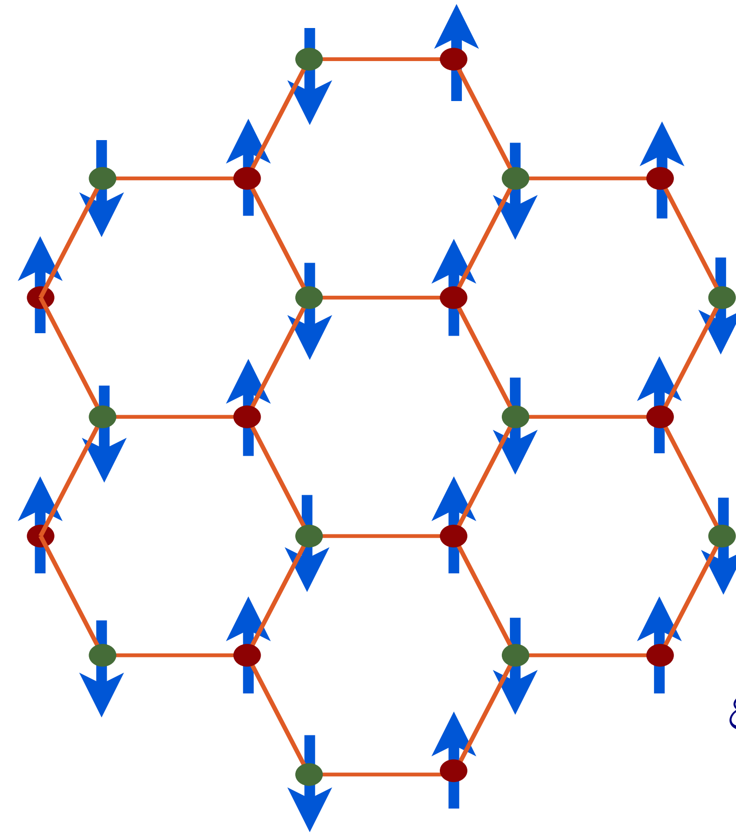

$\mathrm{Im}G(k,\omega)$
$v\,k$
ω



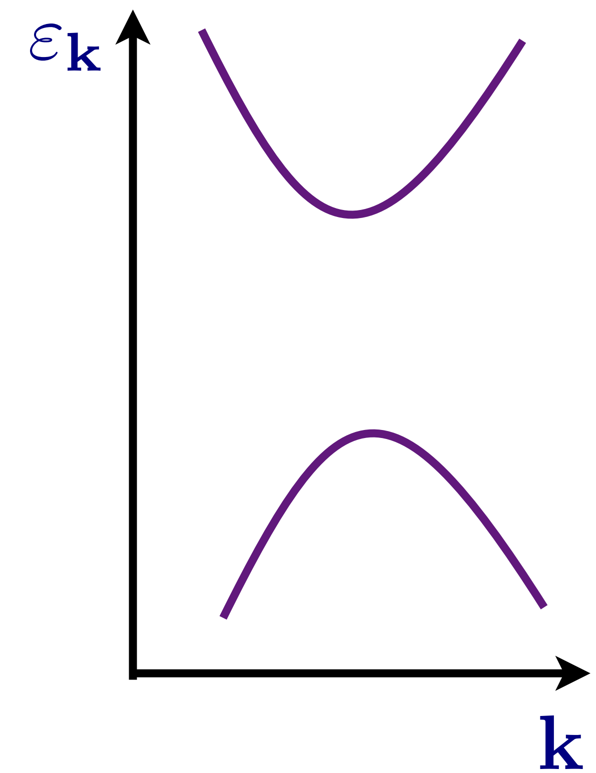


Dirac
semi-metal

$\langle \varphi^a \rangle = 0$



Insulating
antiferromagnet
with Neel order



$\langle \varphi^a \rangle \neq 0$

s

Quantum phase transition described by a strongly-coupled conformal field theory without well-defined quasiparticles

Electrical transport

The conserved electrical current is

$$J_\mu = -i\overline{\Psi}\gamma_\mu\Psi. \tag{1}$$

Let us compute its two-point correlator, $K_{\mu\nu}(k)$ at a spacetime momentum k_μ at $T = 0$. At leading order, this is given by a one fermion loop diagram which evaluates to

$$\begin{aligned} K_{\mu\nu}(k) &= \int \frac{d^3p}{8\pi^3} \frac{\mathrm{Tr}\left[\gamma_\mu(i\gamma_\lambda p_\lambda + m\rho^z\sigma^z)\gamma_\nu(i\gamma_\delta(k_\delta + p_\delta) + m\rho^z\sigma^z)\right]}{(p^2+m^2)((p+k)^2+m^2)} \\ &= -\frac{2}{\pi}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\int_0^1 dx \frac{k^2 x(1-x)}{\sqrt{m^2 + k^2 x(1-x)}}, \end{aligned} \tag{2}$$

where the mass $m = 0$ in the semi-metal and at the quantum critical point, while $m = |\lambda N_0|$ in the insulator. Note that the current correlation is purely transverse, and this follows from the requirement of current conservation

$$k_\mu K_{\mu\nu} = 0. \tag{3}$$

Of particular interest to us is the K_{00} component, after analytic continuation to Minkowski space where the spacetime momentum k_μ is replaced by (ω, k). The conductivity is obtained from this correlator via the Kubo formula

$$\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} K_{00}(\omega, k). \tag{4}$$

In the insulator, where $m > 0$, analysis of the integrand in Eq. (2) shows that that the spectral weight of the density correlator has a gap of $2m$ at $k = 0$, and the conductivity in Eq. (4) vanishes. These properties are as expected in any insulator.

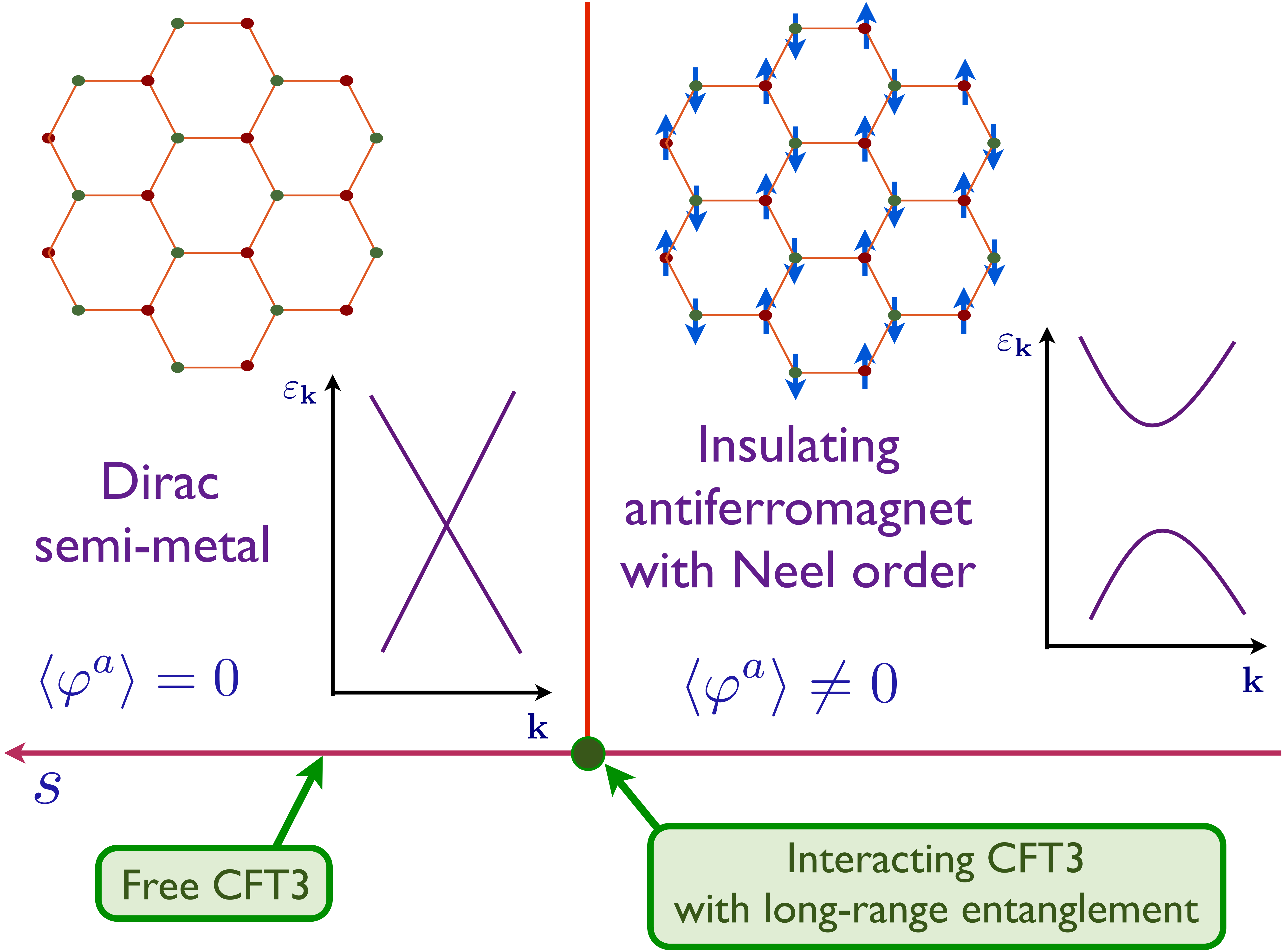
In the metal, and at the critical point, where $m = 0$, the fermionic spectrum is gapless, and so is that of the charge correlator. The density correlator in Eq. (2) and the conductivity in Eq. (4) evaluate to the simple universal results

$$\begin{aligned} K_{00}(\omega, k) &= \frac{1}{4} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\ \sigma(\omega) &= 1/4. \end{aligned} \tag{5}$$

Going beyond one-loop, we find *no change* in these results in the semi-metal to all orders in perturbation theory. At the quantum critical point, there are no anomalous dimensions for the conserved current, but the amplitude does change yielding

$$\begin{aligned} K_{00}(\omega, k) &= \mathcal{K}\frac{k^2}{\sqrt{k^2-\omega^2}} \\ \sigma(\omega) &= \mathcal{K}, \end{aligned} \tag{6}$$

where $\mathcal{K}$ is a universal number dependent only upon the universality class of the quantum critical point. The value of the $\mathcal{K}$ for the Gross-Neveu model is not known exactly, but can be estimated by computations in the $(3-d)$ or $1/N$ expansions.



$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Dirac semi-metal
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Insulating antiferromagnet with Neel order
$\varepsilon_{\mathbf{k}}$
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Free CFT3
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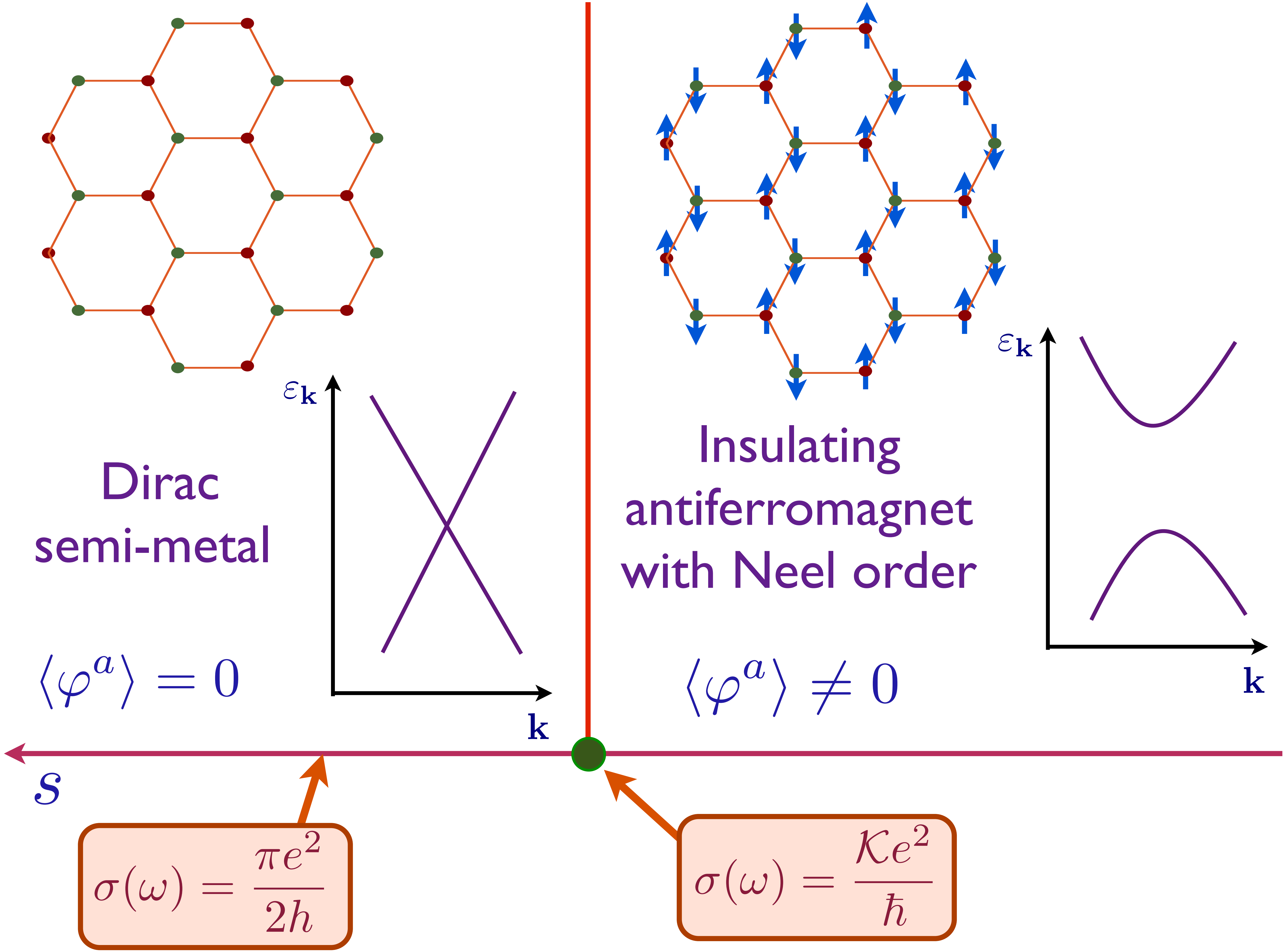
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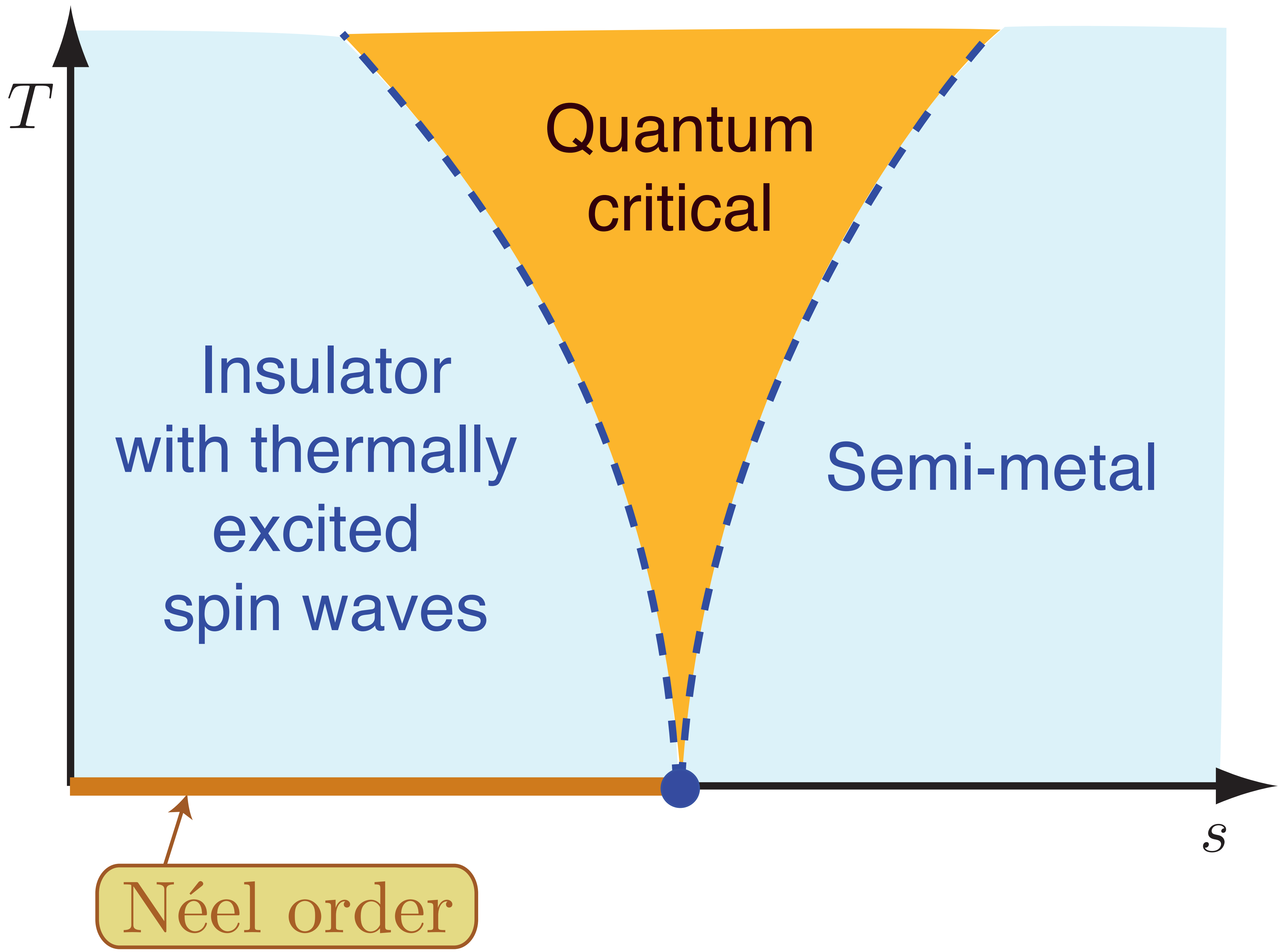
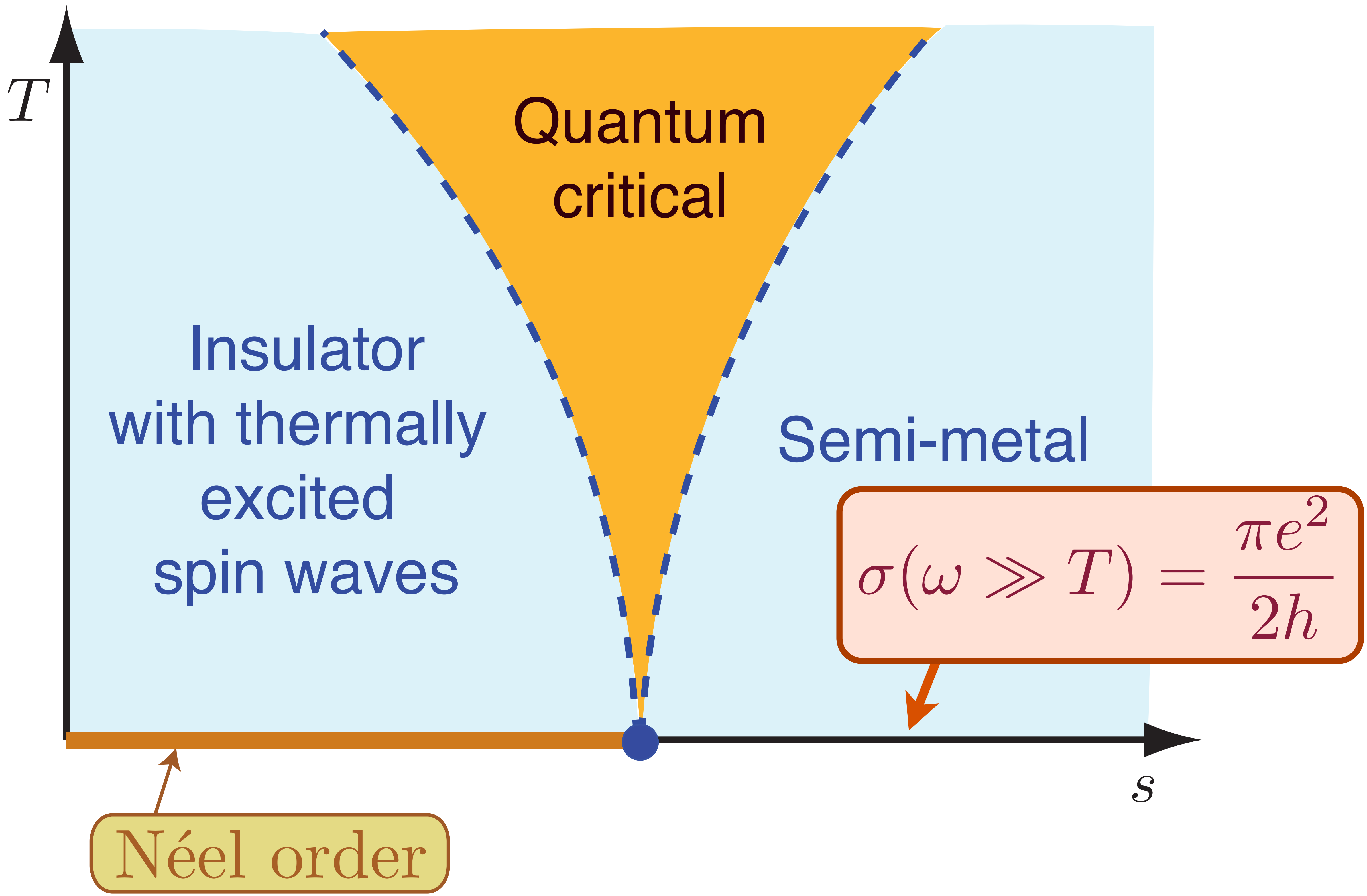
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
s
$\sigma(\omega) = \frac{\pi e^2}{2h}$
$\sigma(\omega) = \frac{\mathcal{K} e^2}{\hbar}$

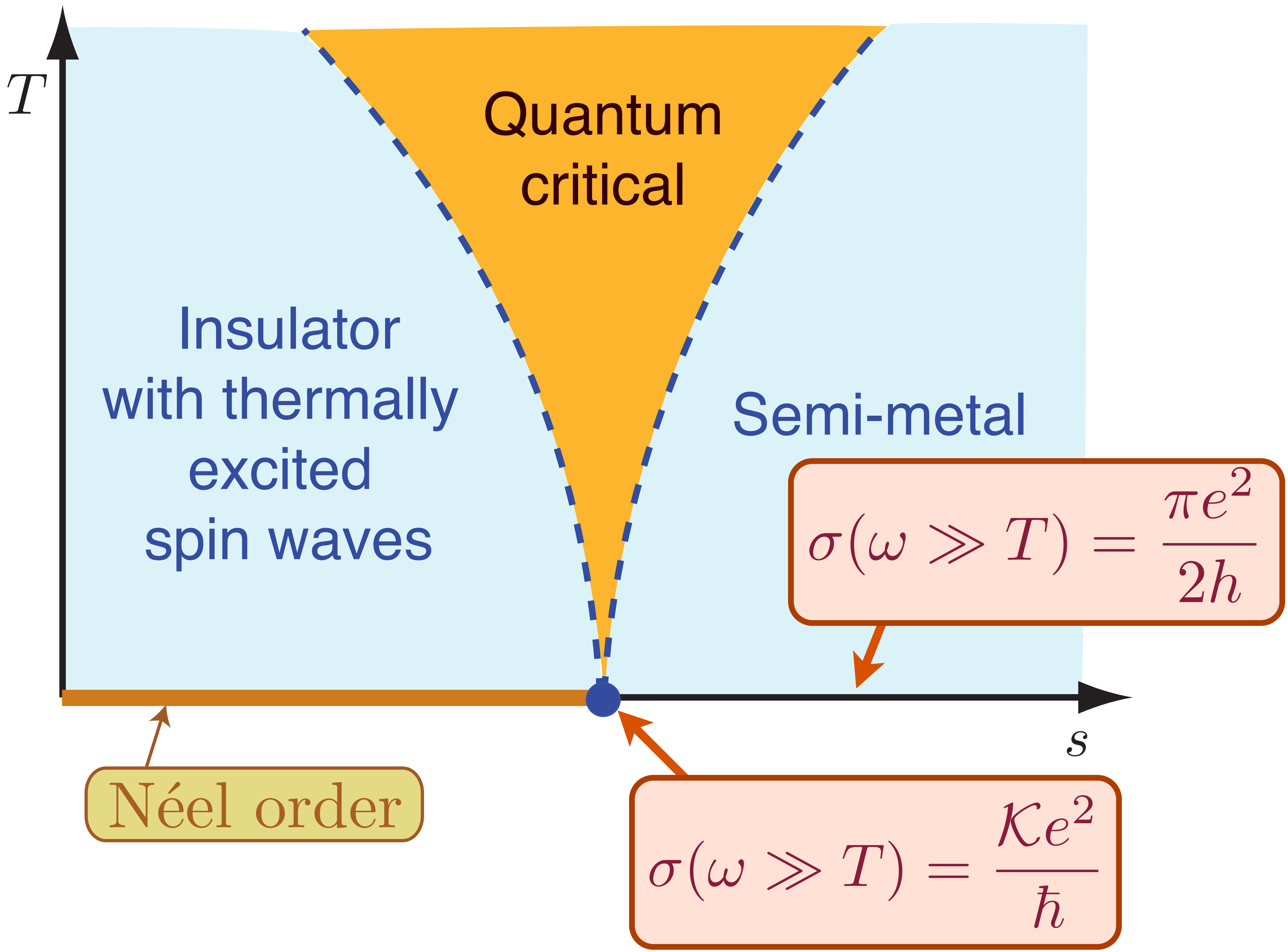
Phase diagram at non-zero temperatures



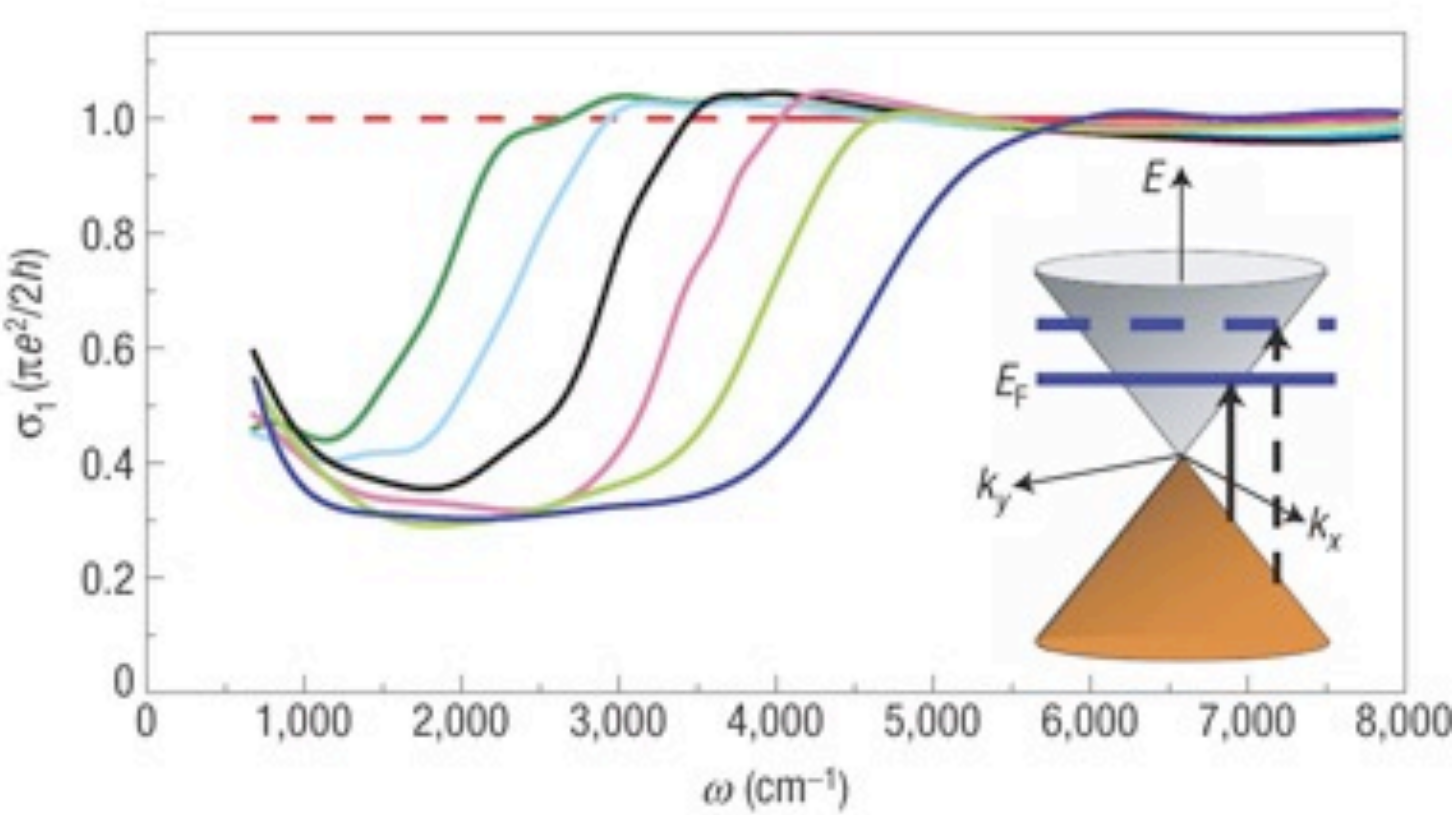
Phase diagram at non-zero temperatures



Phase diagram at non-zero temperatures



Optical conductivity of graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, Nature Physics 4, 532 (2008).

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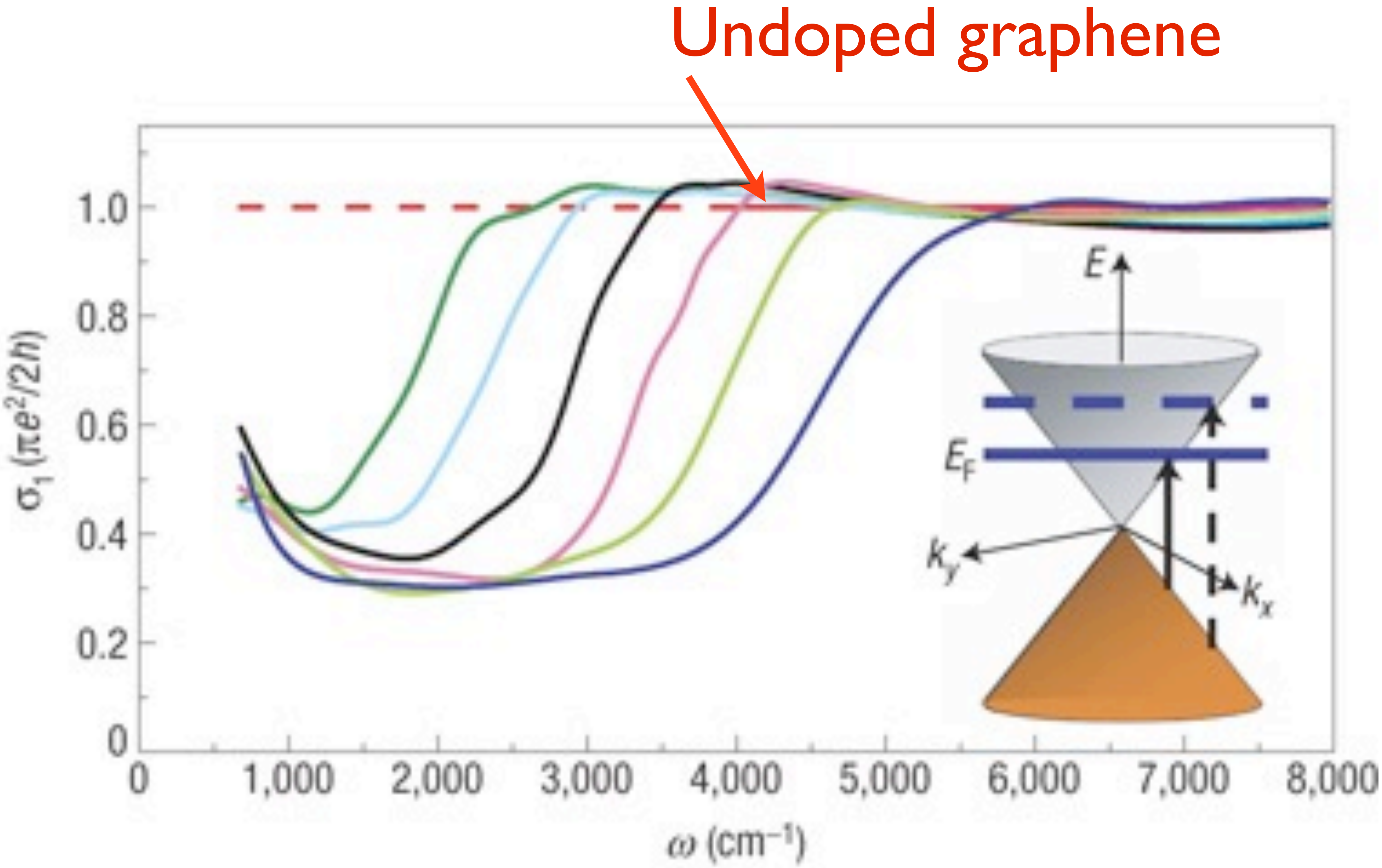


Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

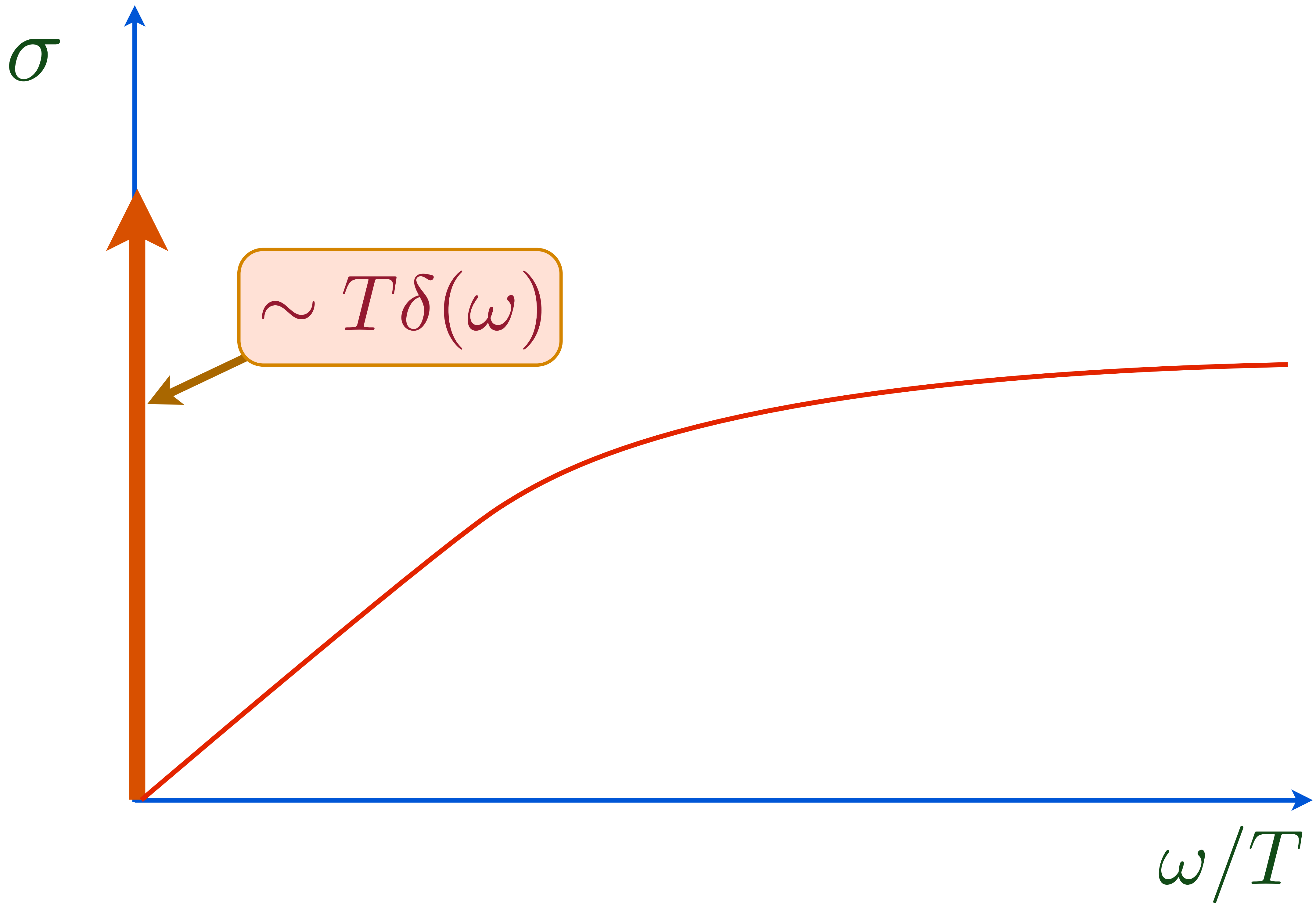
Non-zero temperatures

At the quantum-critical point at one-loop order, we can set $m = 0$, and then repeat the computation in Eq. (2) at $T > 0$. This only requires replacing the integral over the loop frequency by a summation over the Matsubara frequencies, which are quantized by odd multiples of πT. Such a computation, via Eq. (4) leads to the conductivity

$$\mathrm{Re}[\sigma(\omega)] = (2T\ln 2)\,\delta(\omega) + \frac{1}{4}\tanh\left(\frac{|\omega|}{4T}\right); \qquad (7)$$

the imaginary part of $\sigma(\omega)$ is the Hilbert transform of $\mathrm{Re}[\sigma(\omega)] - 1/4$. Note that this reduces to Eq. (5) in the limit $\omega \gg T$. However, the most important new feature of Eq. (7) arises for $\omega \ll T$, where we find a delta function at zero frequency in the real part. Thus the d.c. conductivity is infinite at this order, arising from the collisionless transport of thermally excited carriers.

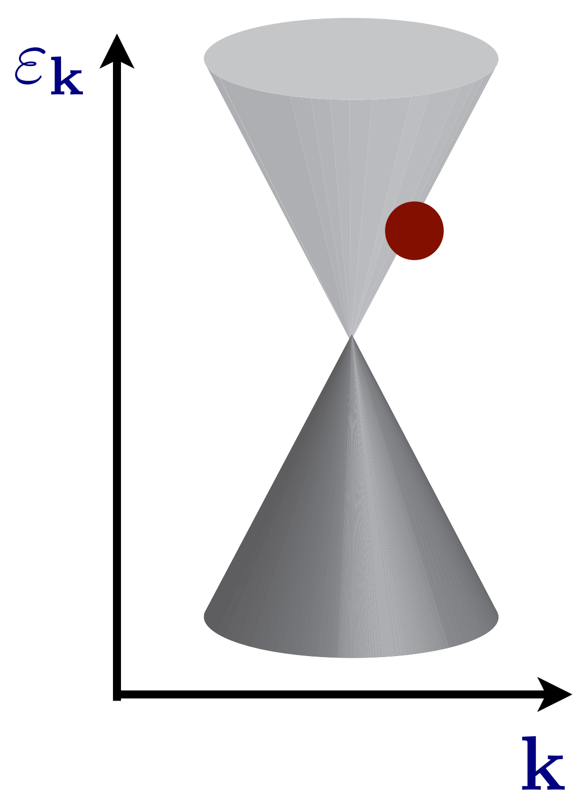
Electrical transport in a free CFT3 for *T* > *0*



Particles

Momentum

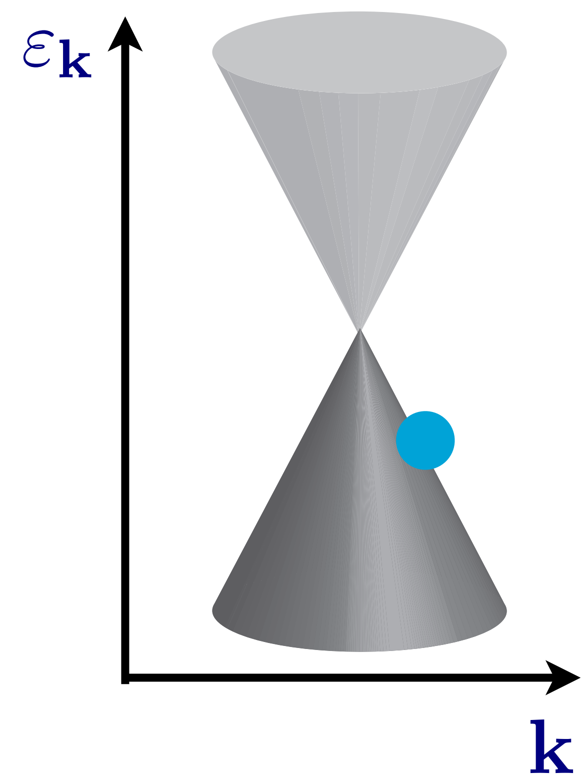
Current



Holes

Momentum

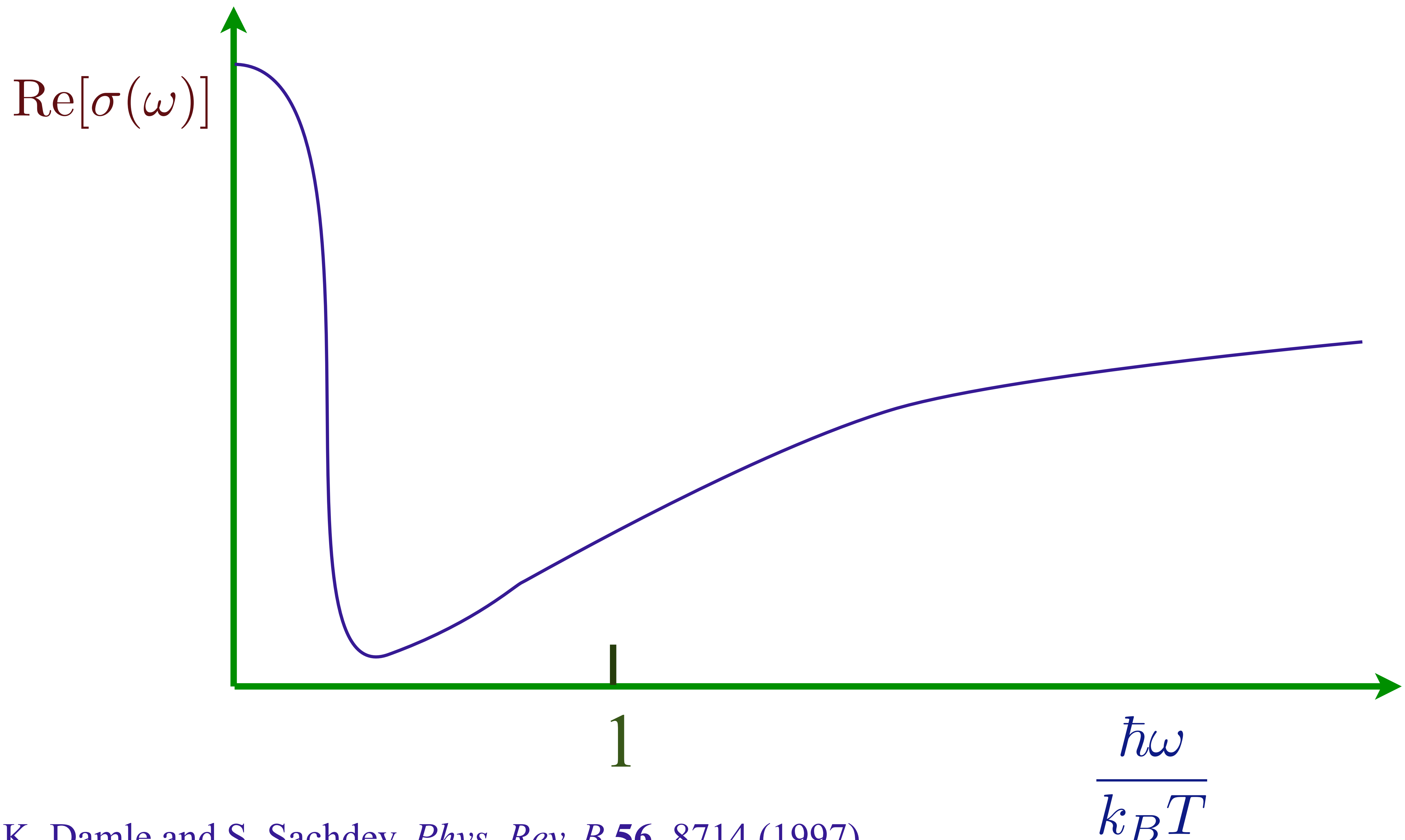
Current



Particle hole symmetry: current carrying state has zero momentum, and collisions can relax current to zero

Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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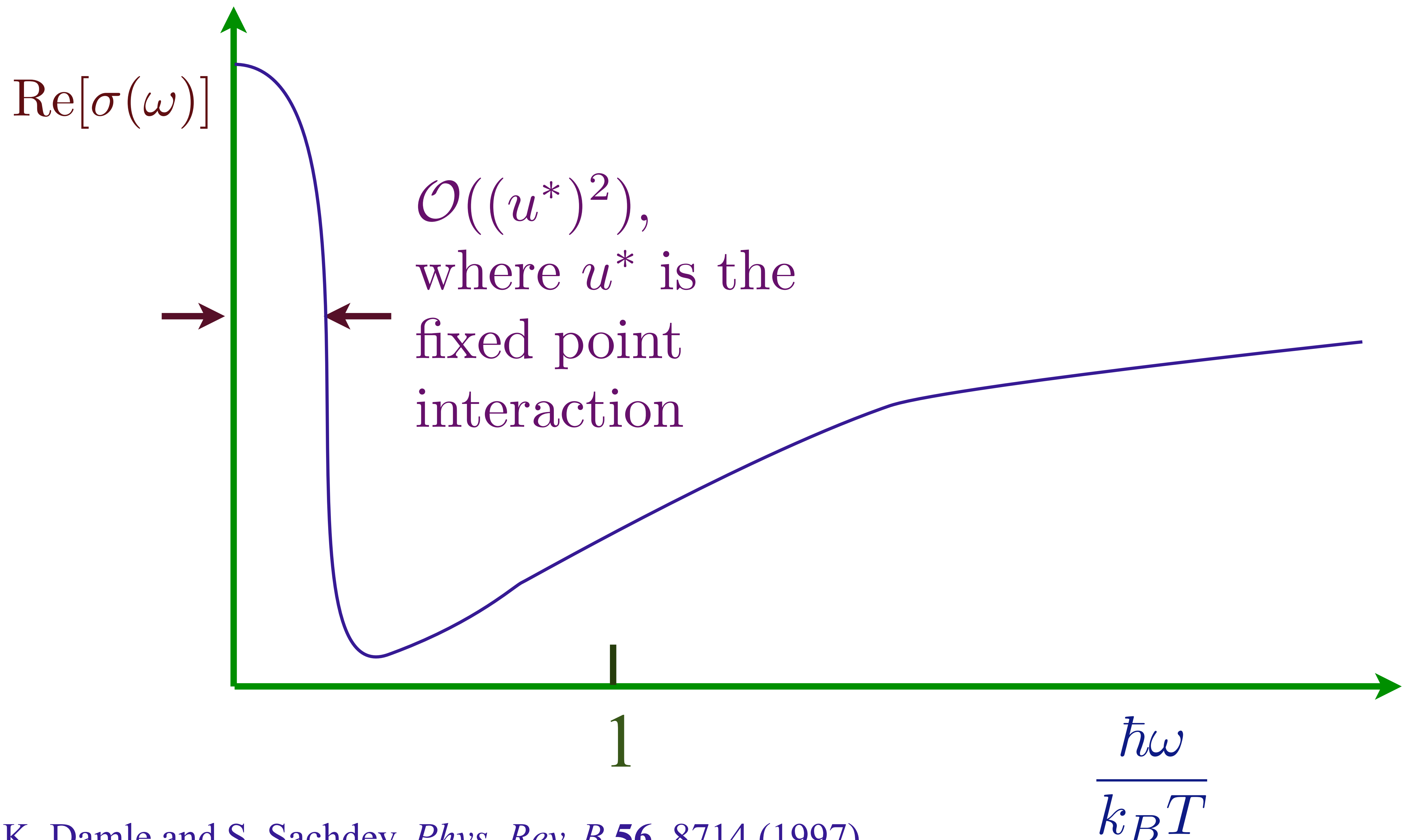

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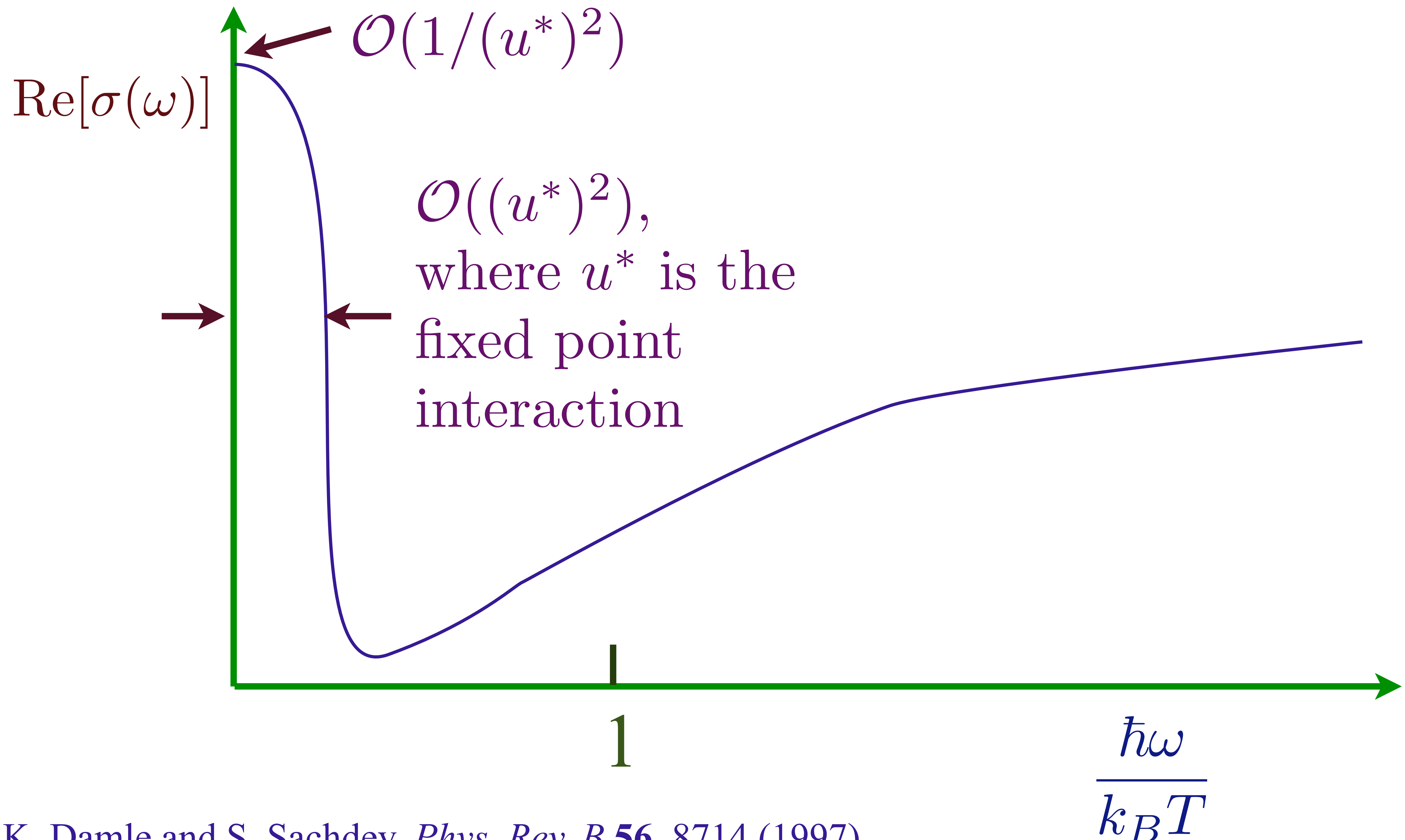

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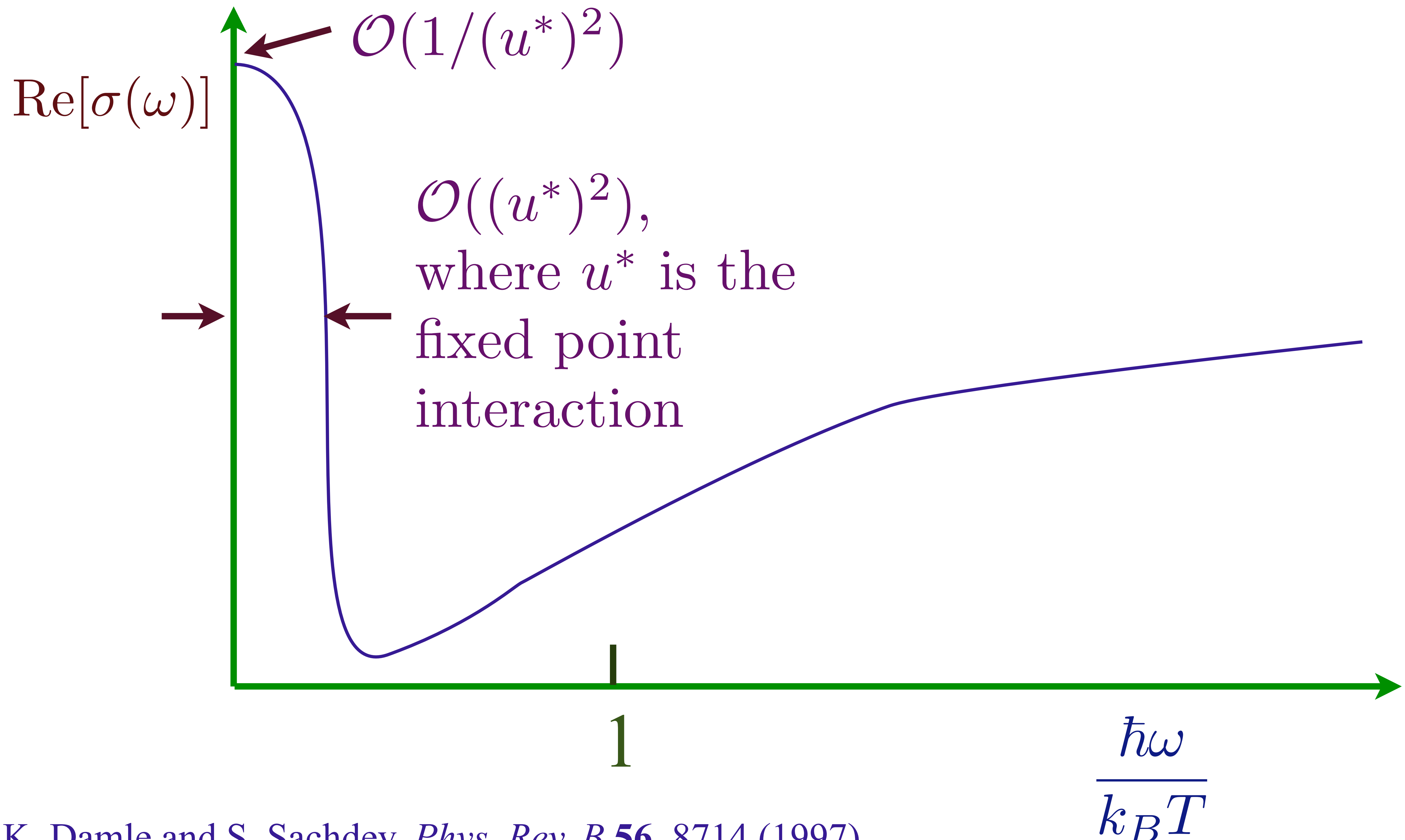

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Electrical transport for a (weakly) interacting CFT3

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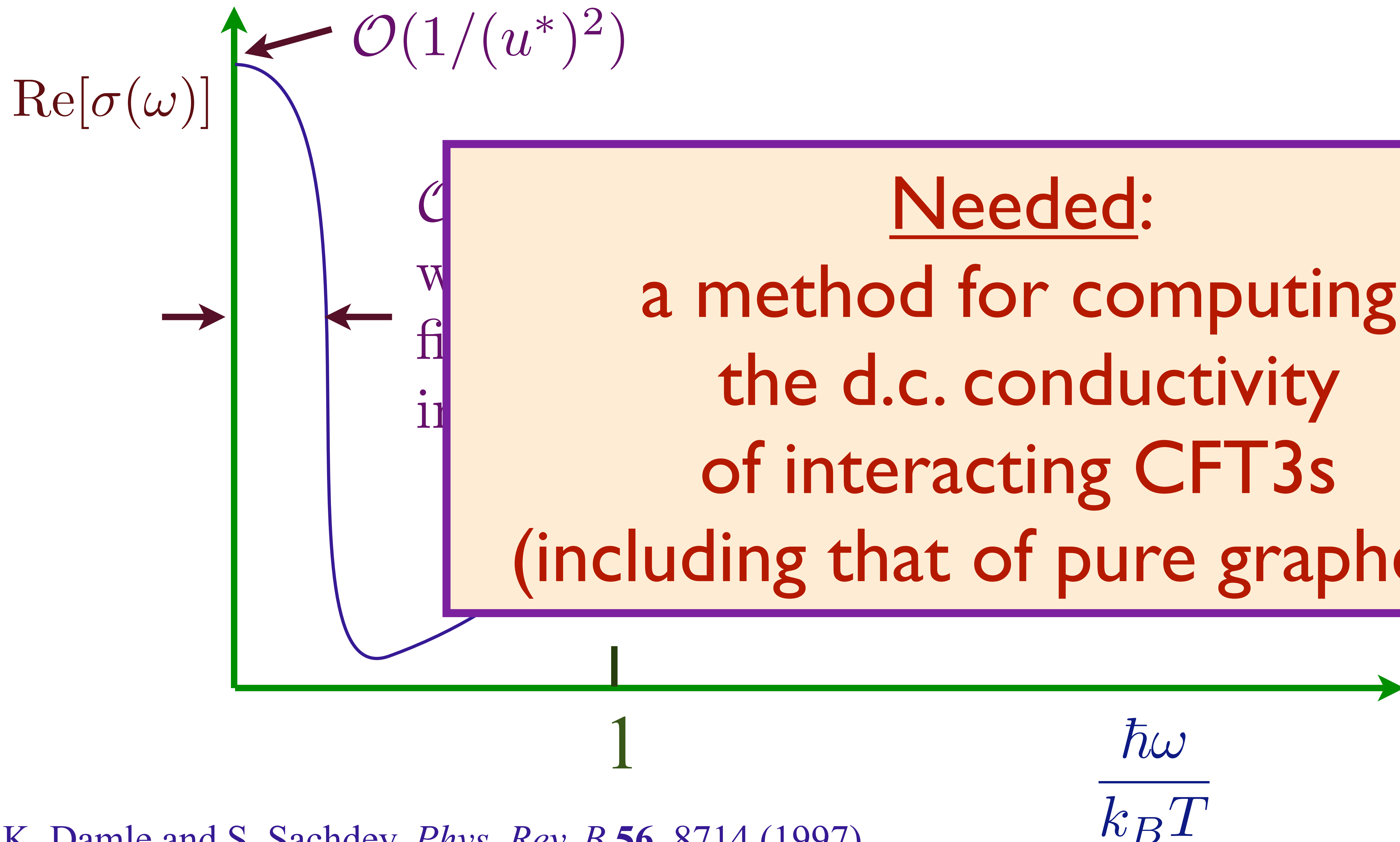

Needed:
a method for computing
the d.c. conductivity
of interacting CFT3s
(including that of pure graphene!)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Conformal quantum matter

A. Field theory:graphene

B. Field theory: superfluid-insulator transition

C. Field theory: antiferromagnets

D. Gauge-gravity duality

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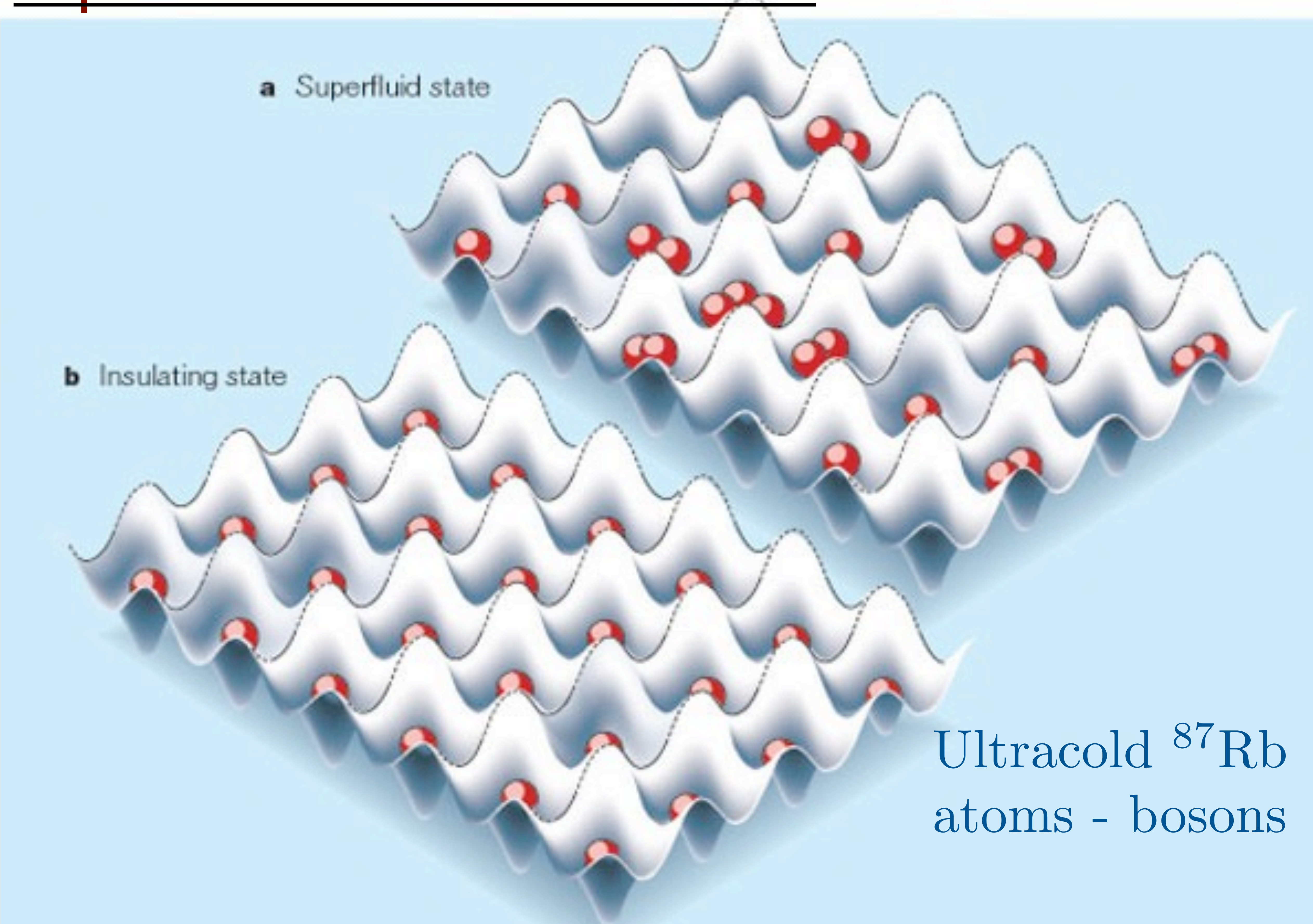
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

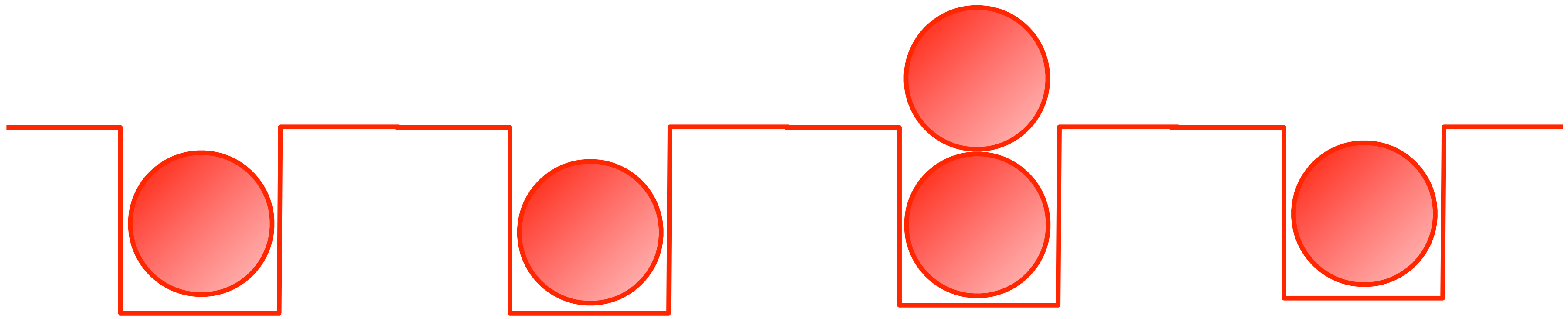
$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

Excitations of the insulator:



Particles ~ $\psi^\dagger$

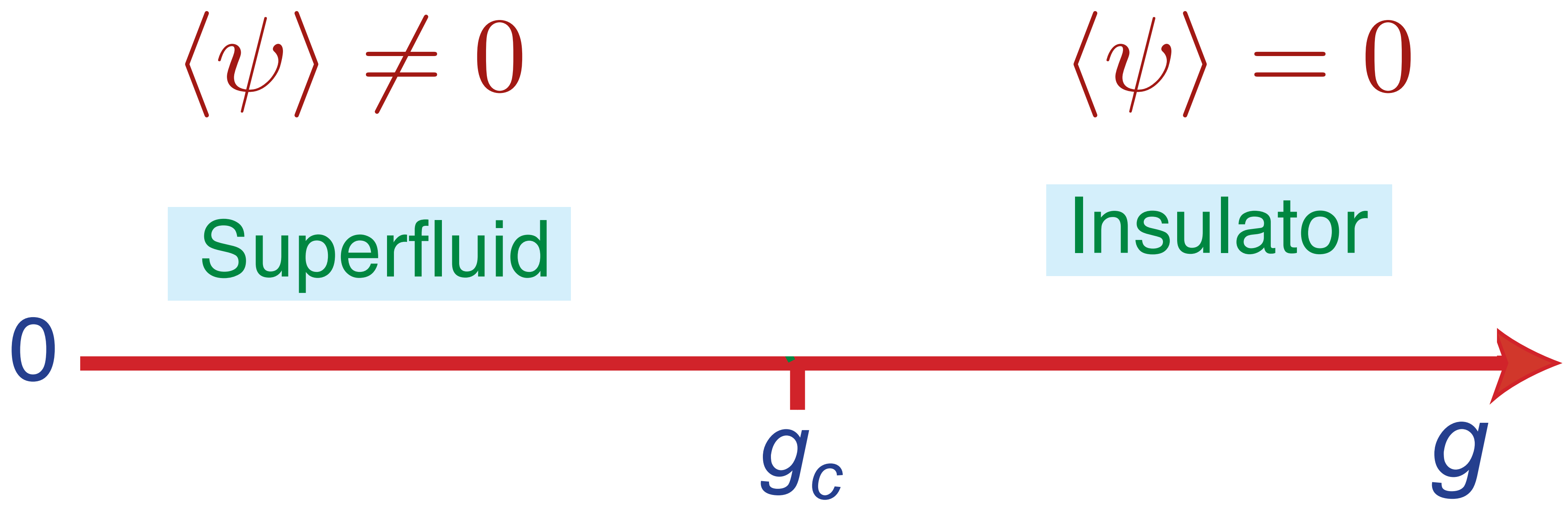
Holes ~ ψ

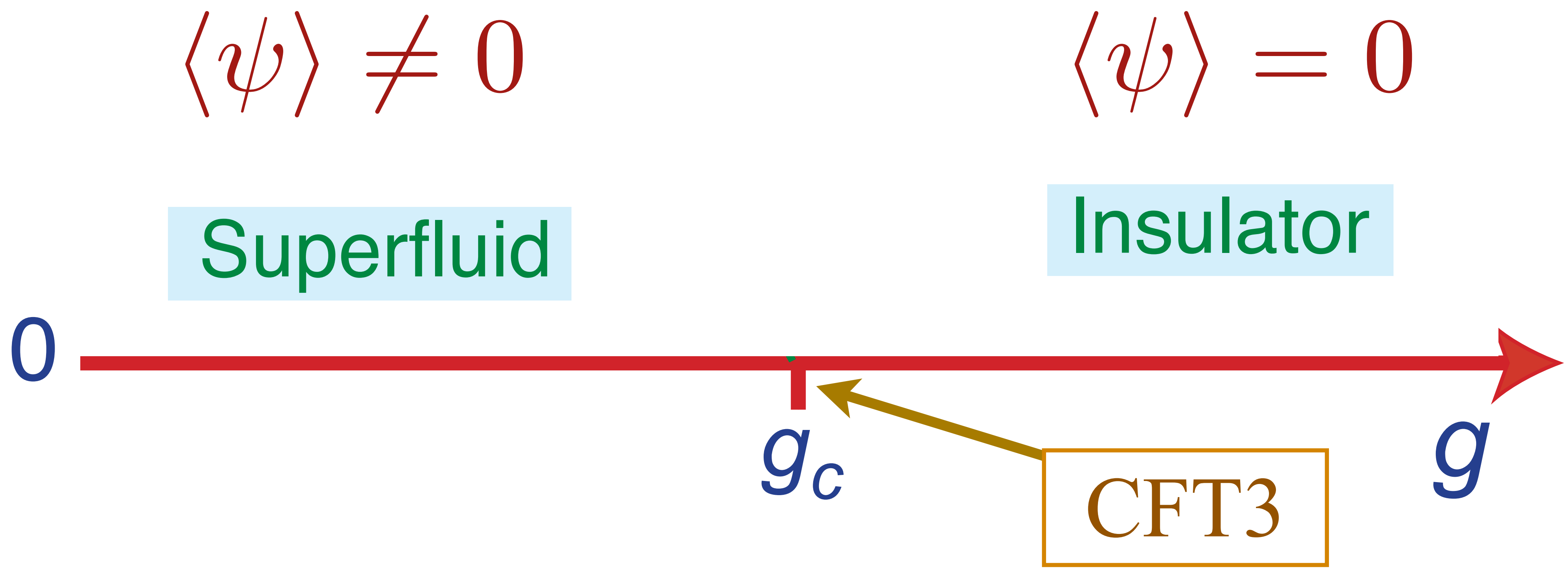
Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

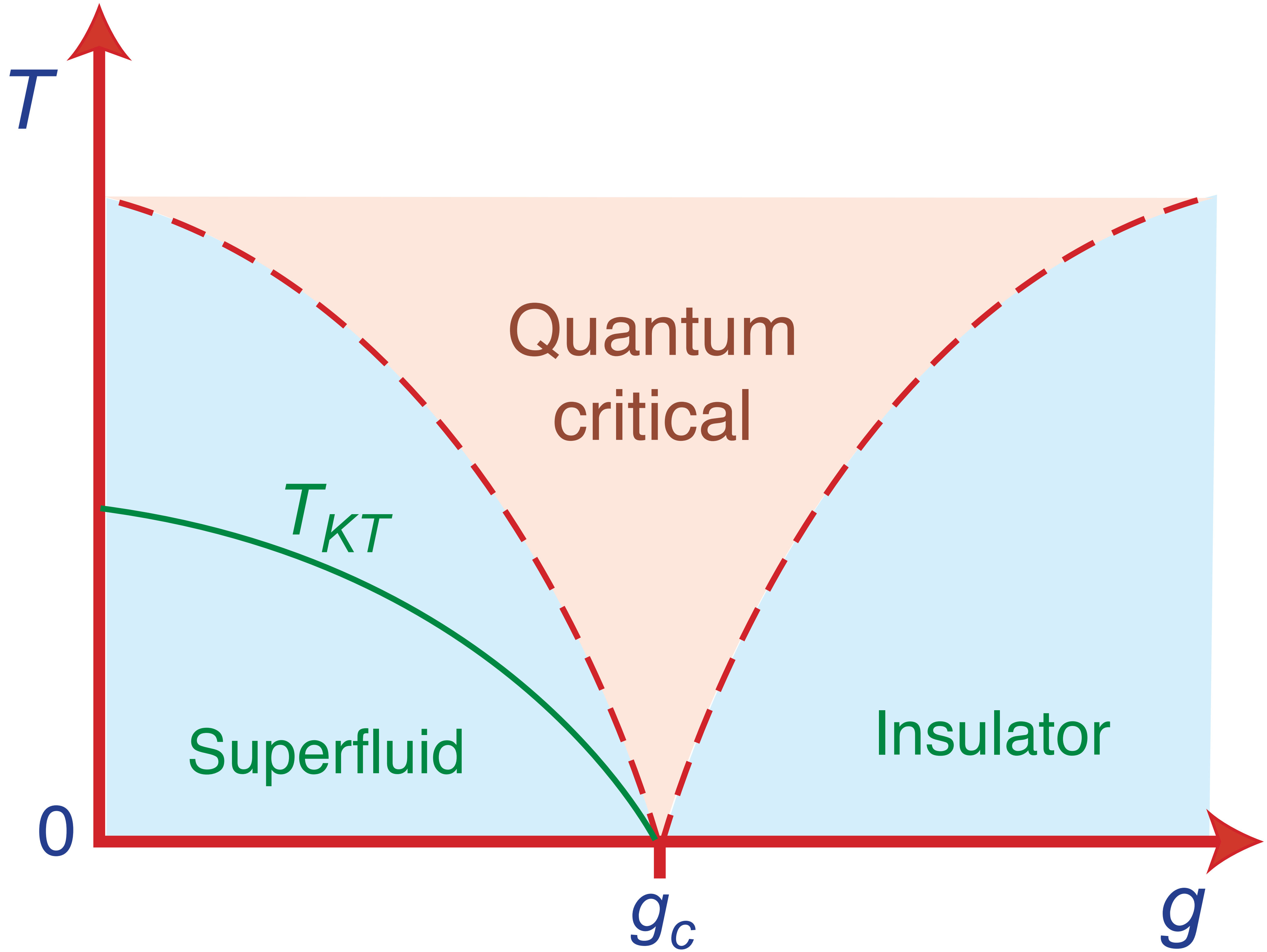
.

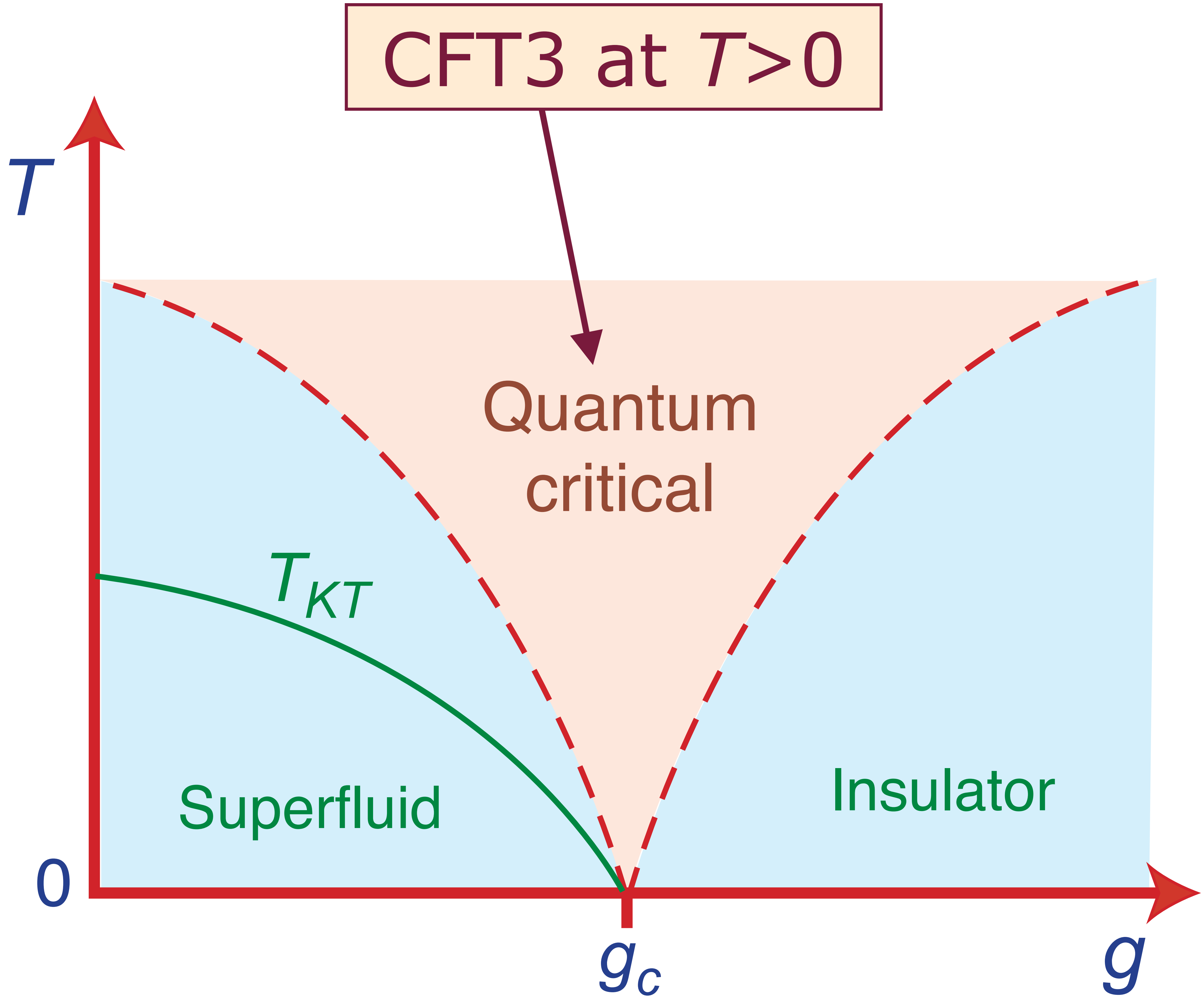
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


$\langle\psi\rangle \neq 0$
$\langle\psi\rangle = 0$
Superfluid
Insulator
0
g_c
g


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
CFT3
g


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT3 at $T>0$
Quantum critical
T
T_{KT}
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C}\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

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where $\mathcal{C}$ is a *universal* constant

Zaanen: Planckian dissipation

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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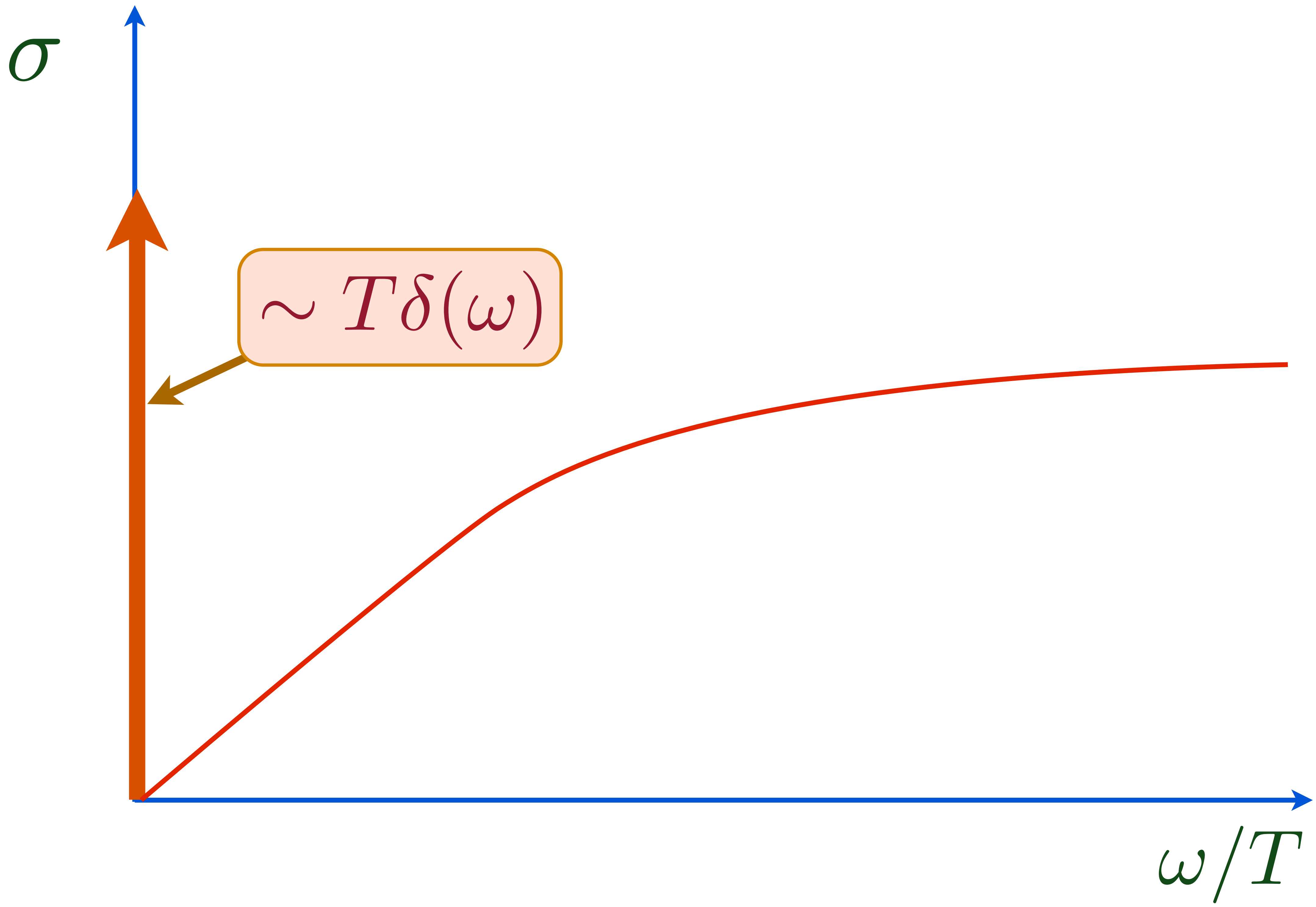
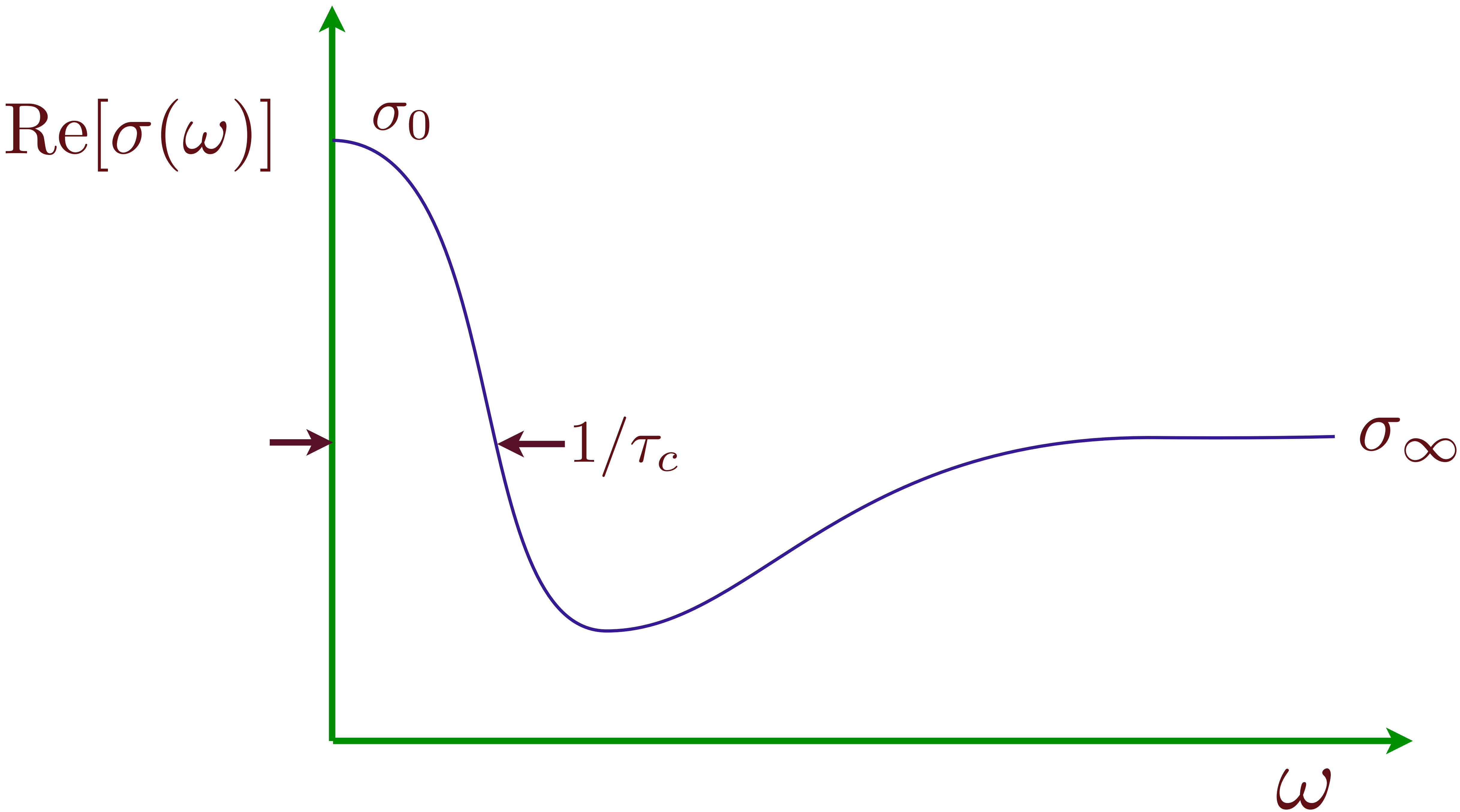
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

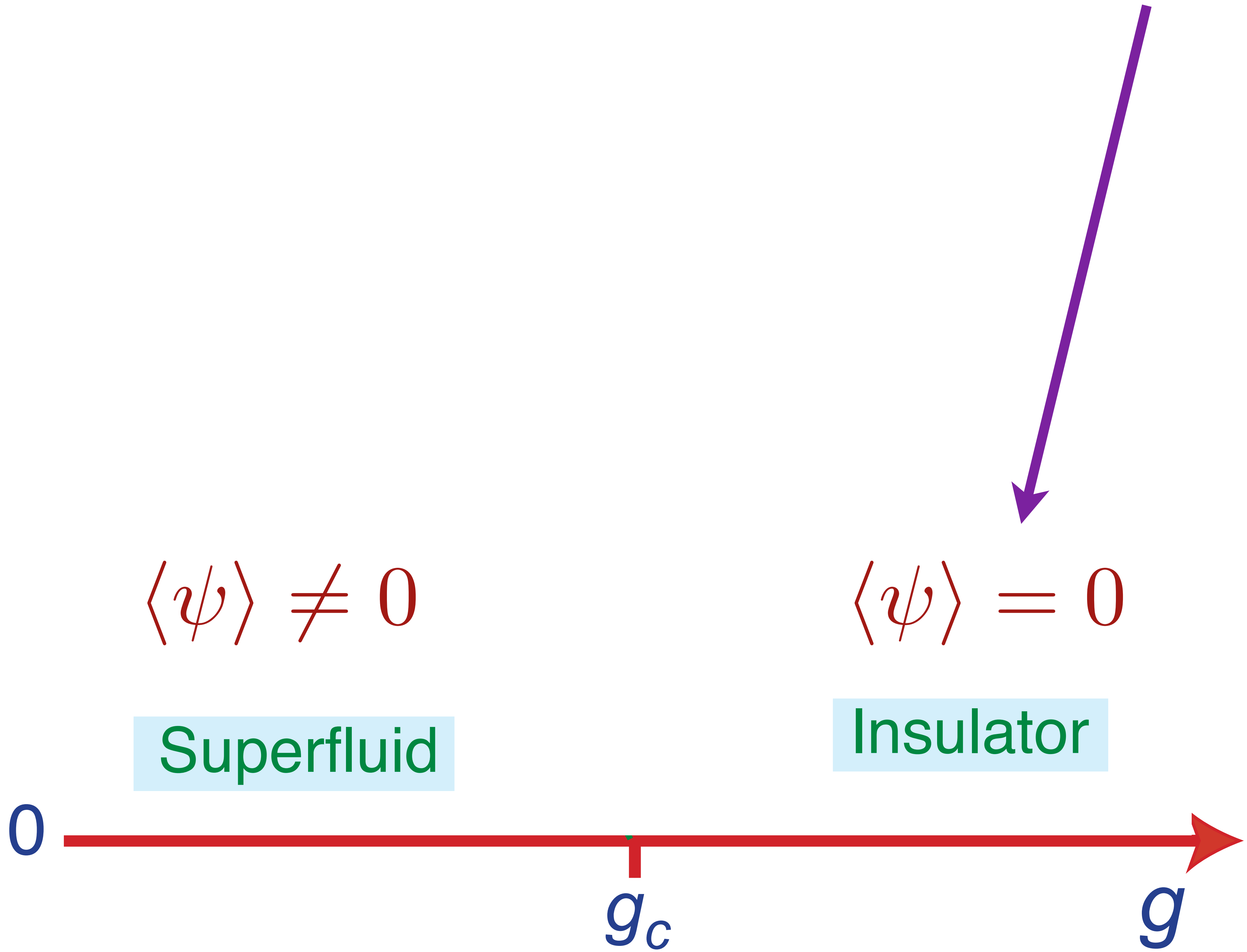
Electrical transport in a free-field theory for *T* > *0*



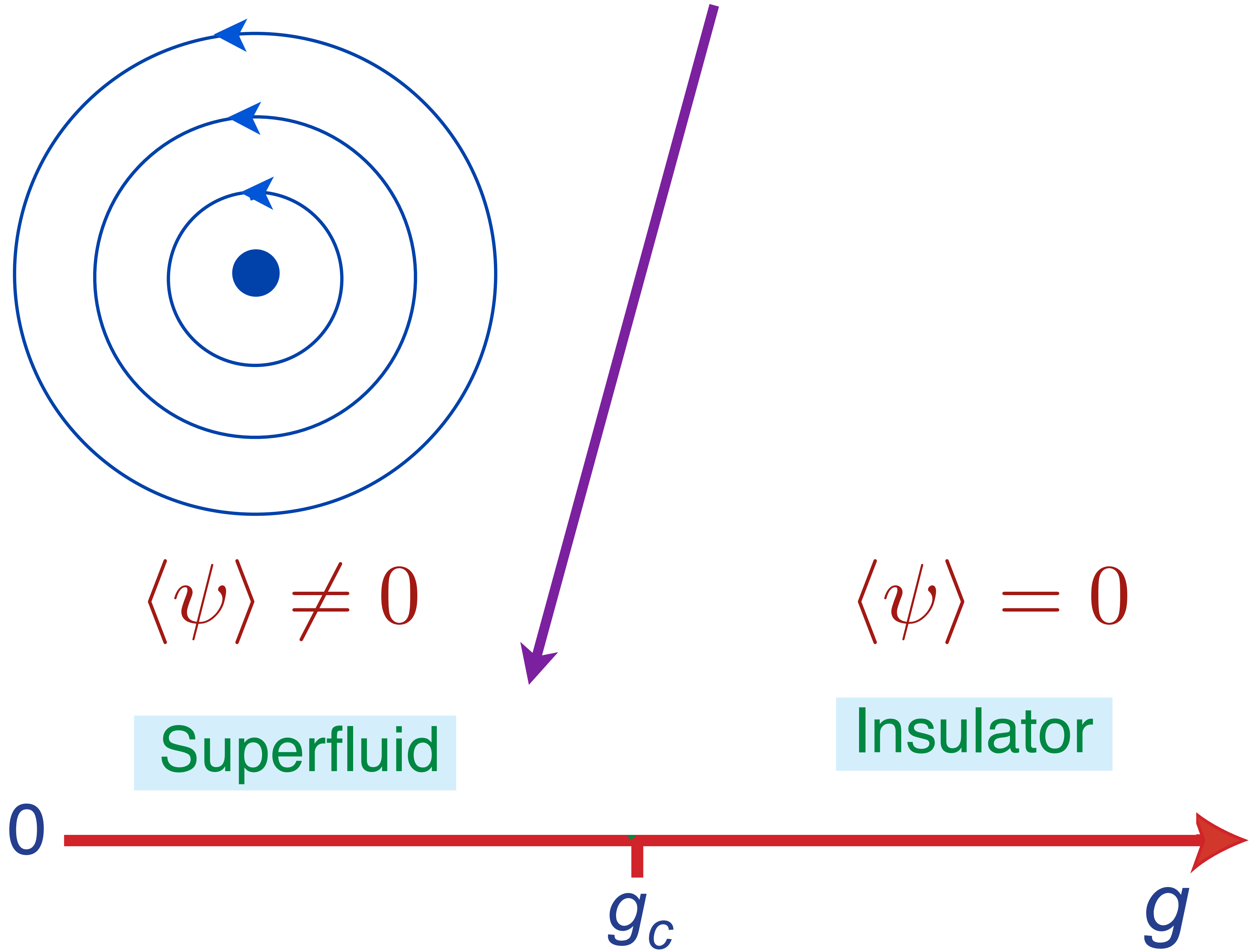
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



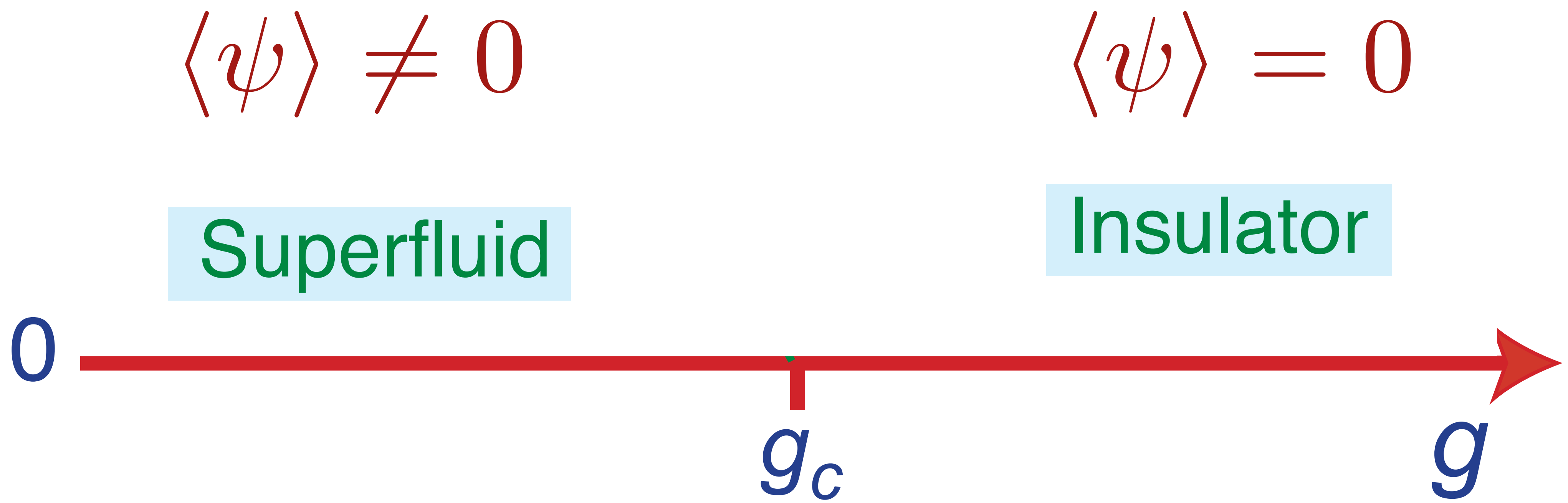
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



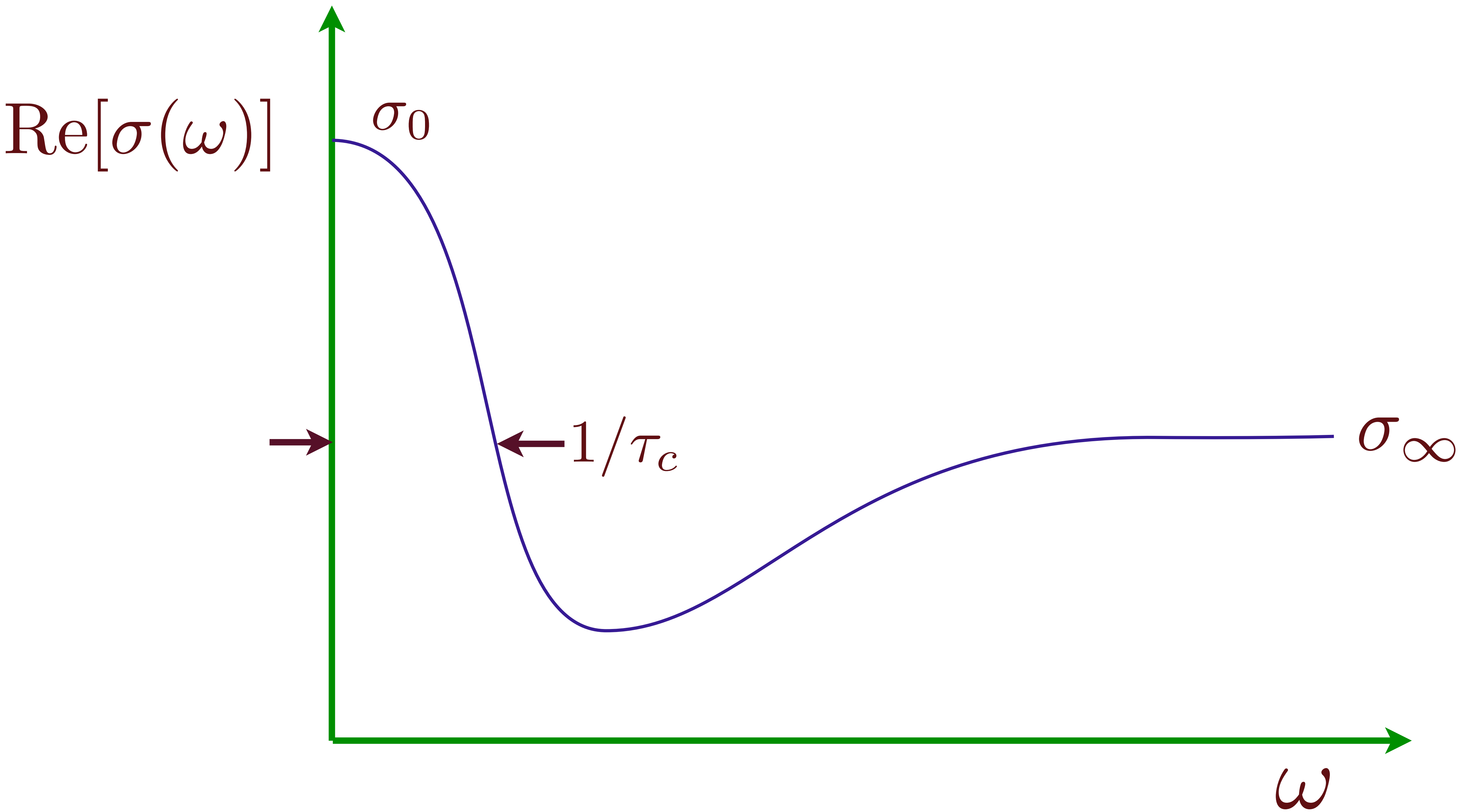
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) "dual" CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

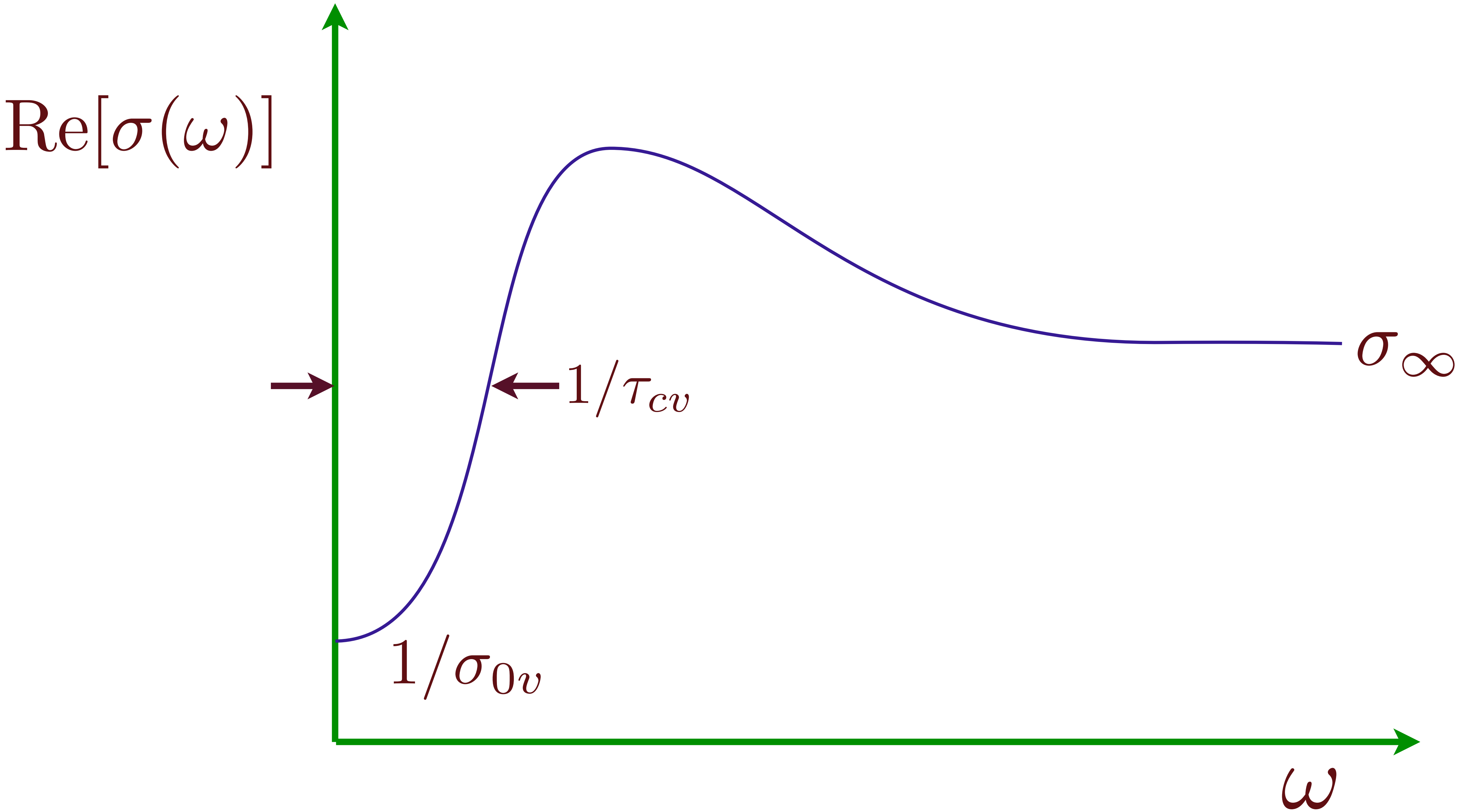
$$\text{Conductivity} = \text{Resistivity of vortices}$$



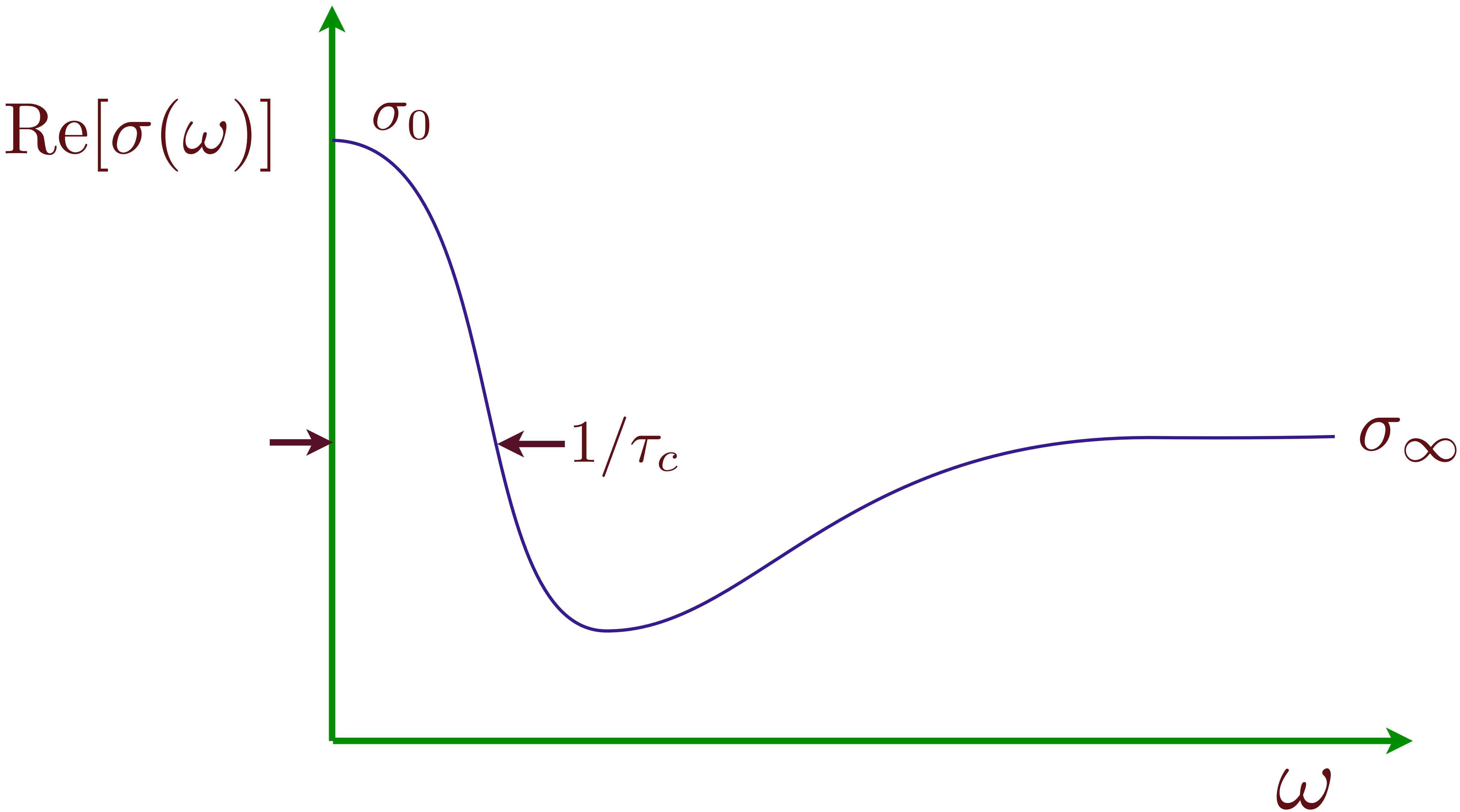
Boltzmann theory of bosons



Boltzmann theory of vortices



Boltzmann theory of bosons



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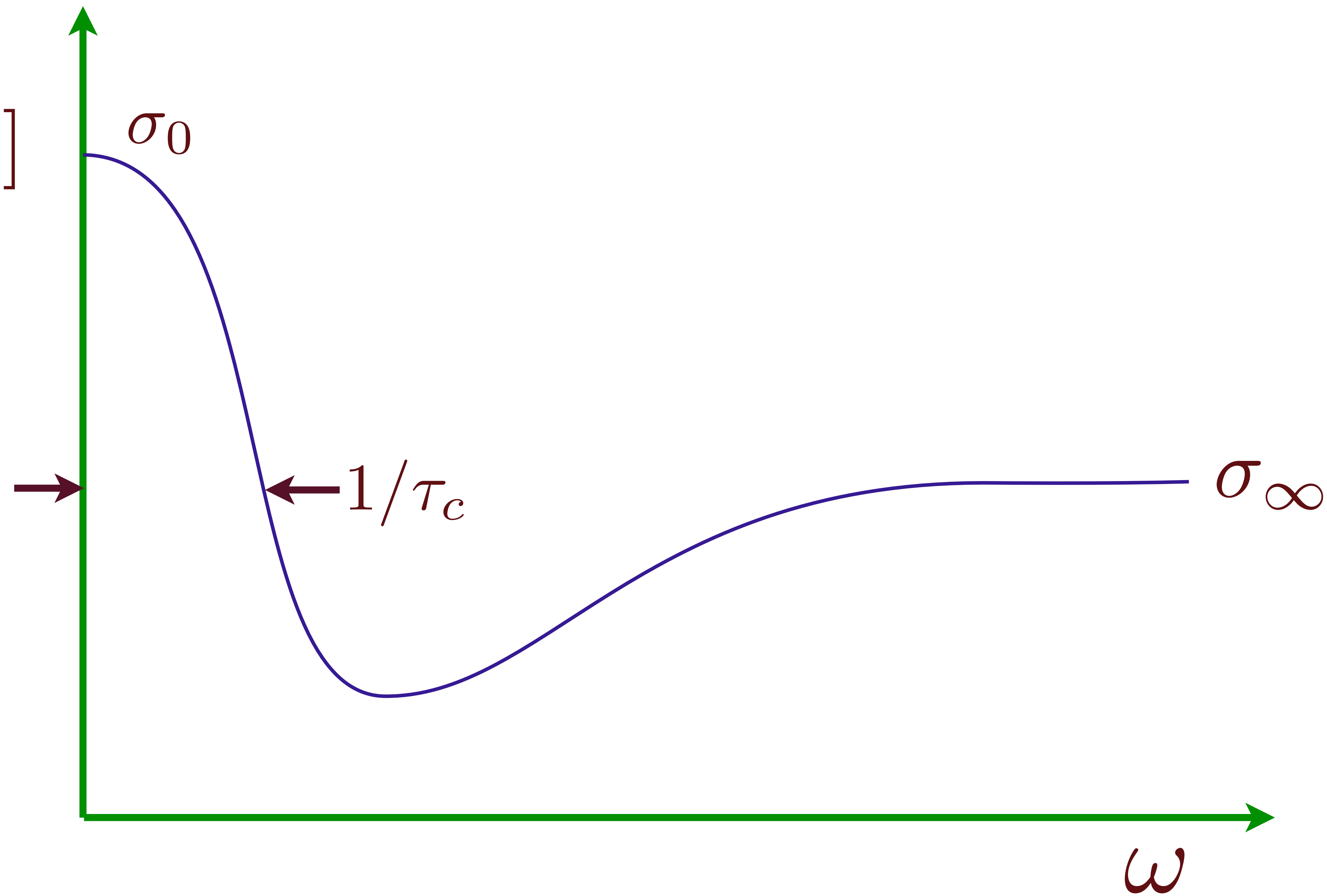

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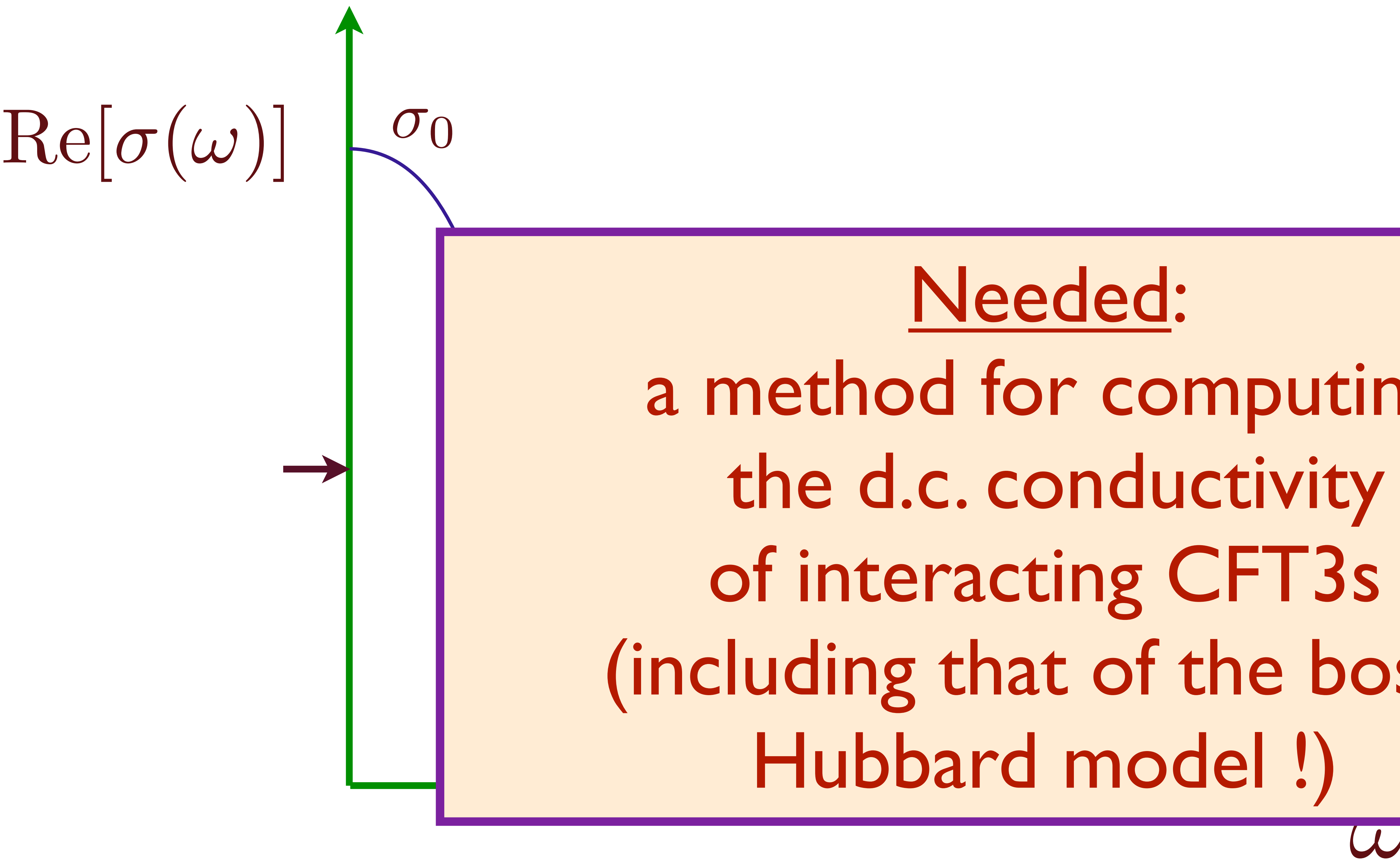

<u>Needed</u>:
a method for computing
the d.c. conductivity
of interacting CFT3s
(including that of the boson
Hubbard model !)

Conformal quantum matter

A. Field theory:graphene

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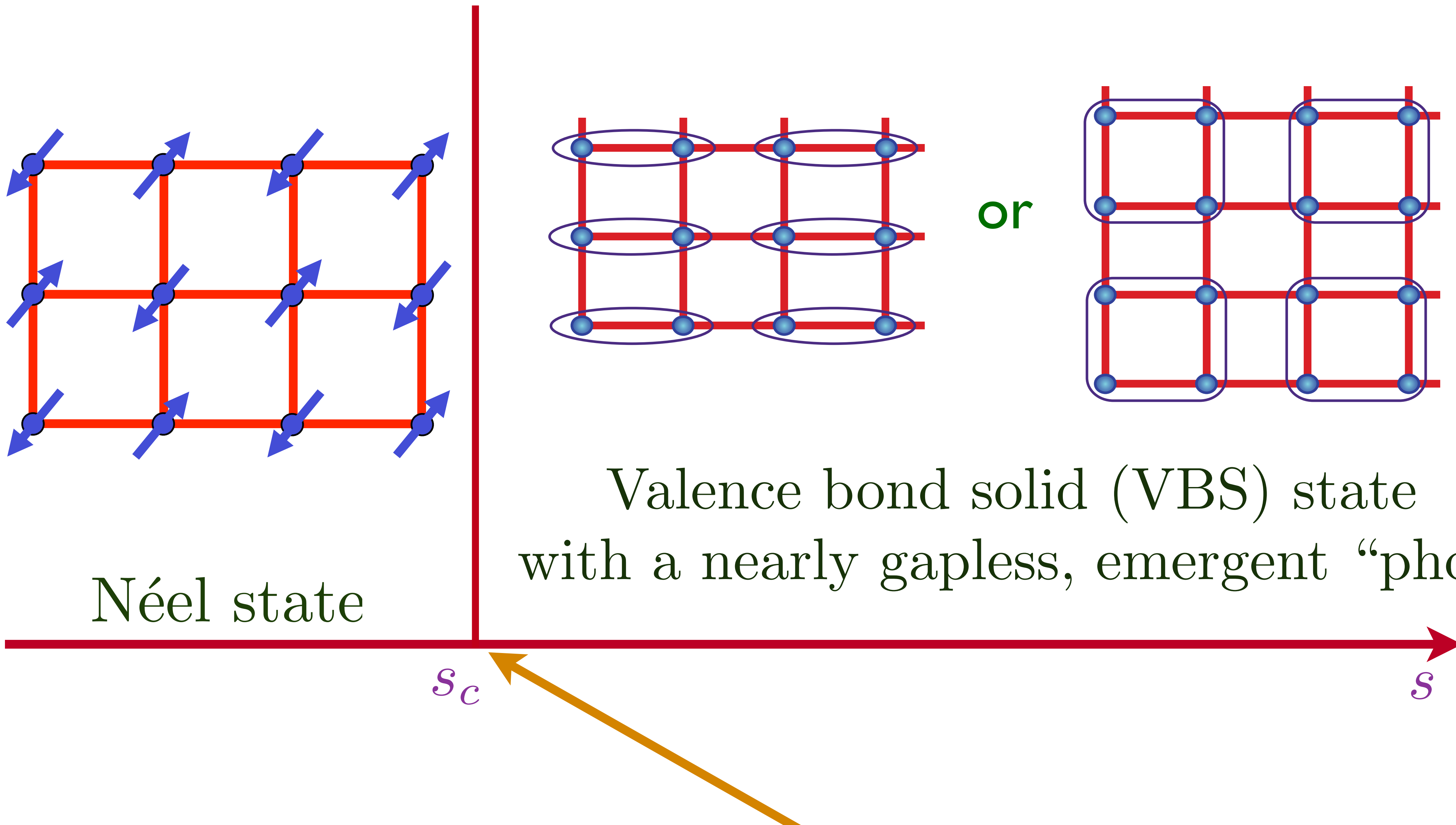
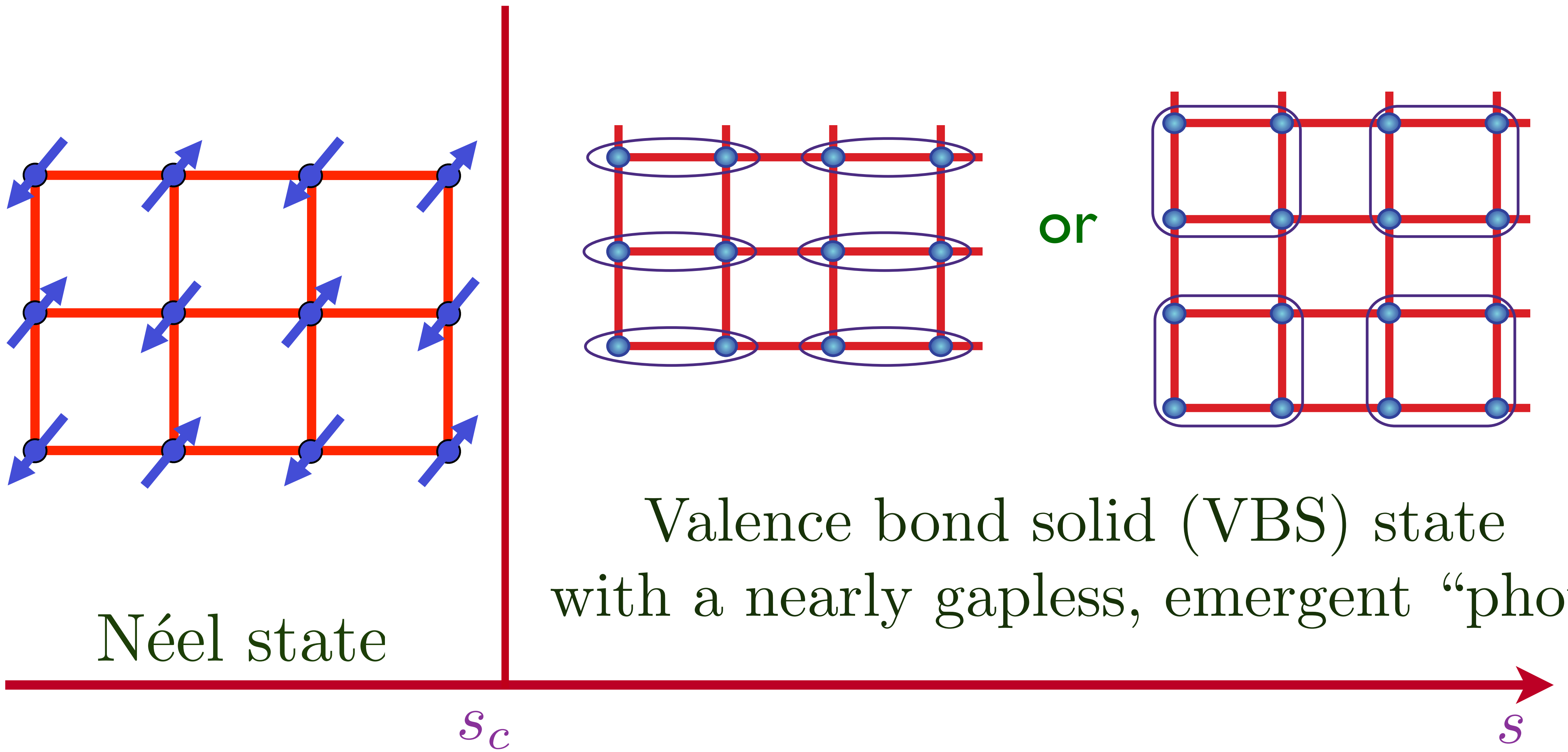
Quantum critical point in a frustrated square lattice antiferromagnet



Long-range entanglement described by a CFT3
with an emergent U(1) "photon"

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Quantum critical point in a frustrated square lattice antiferromagnet

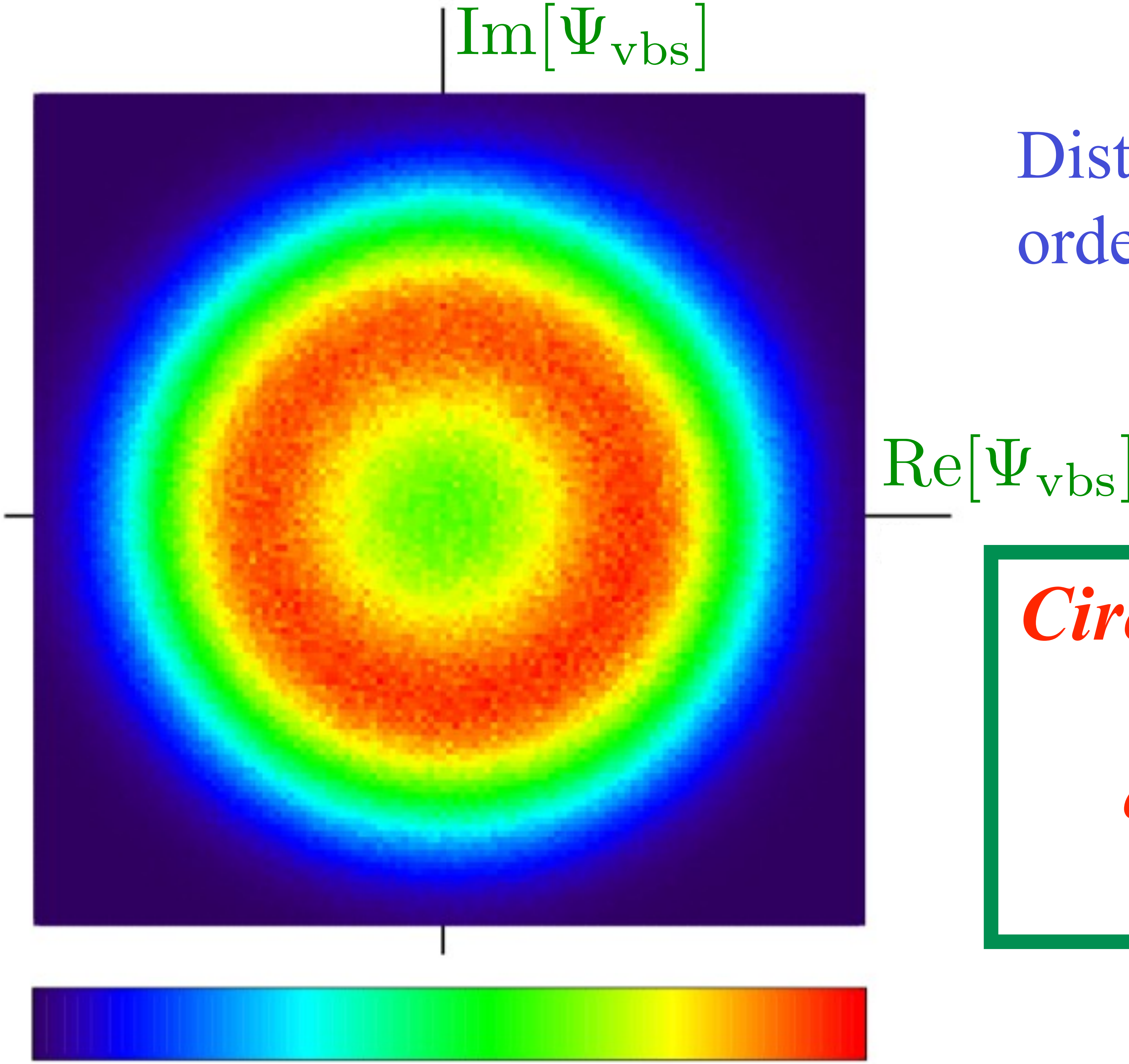


Critical theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2r d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$



Distribution of VBS order Ψ_{vbs} at large Q

Circular symmetry is evidence for emergent U(1) photon

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Conformal quantum matter

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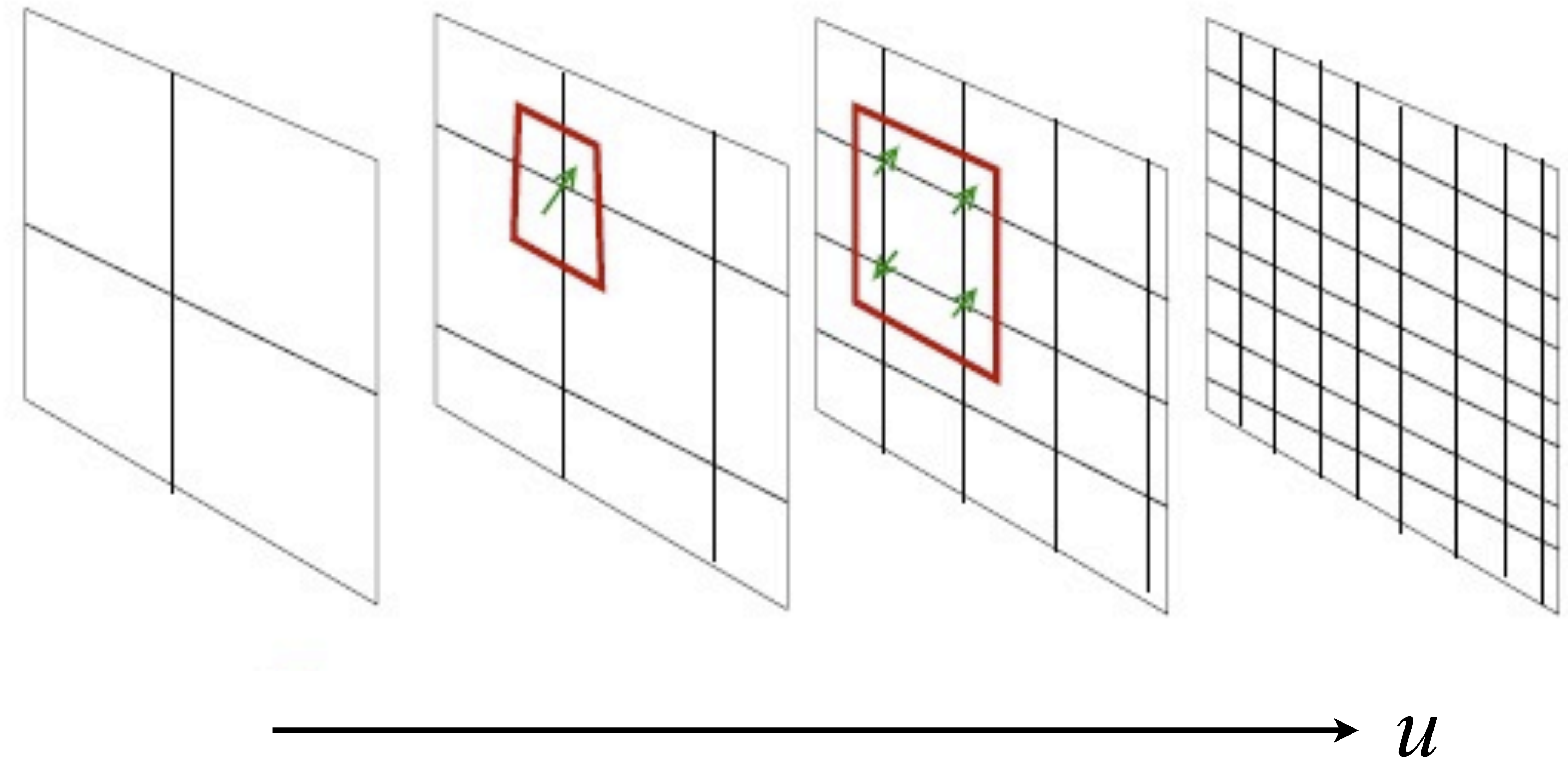
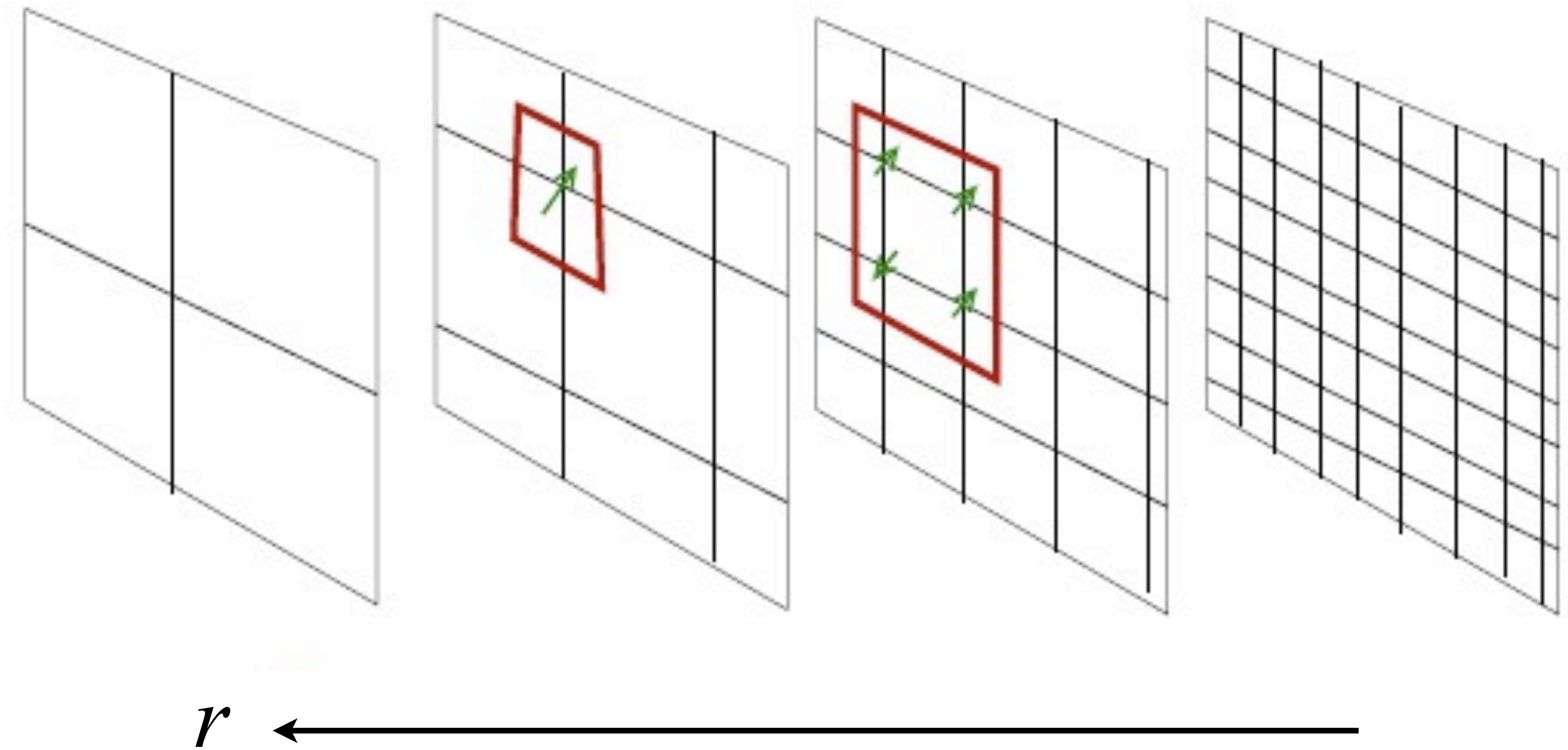
Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.



J. McGreevy, arXiv0909.0518



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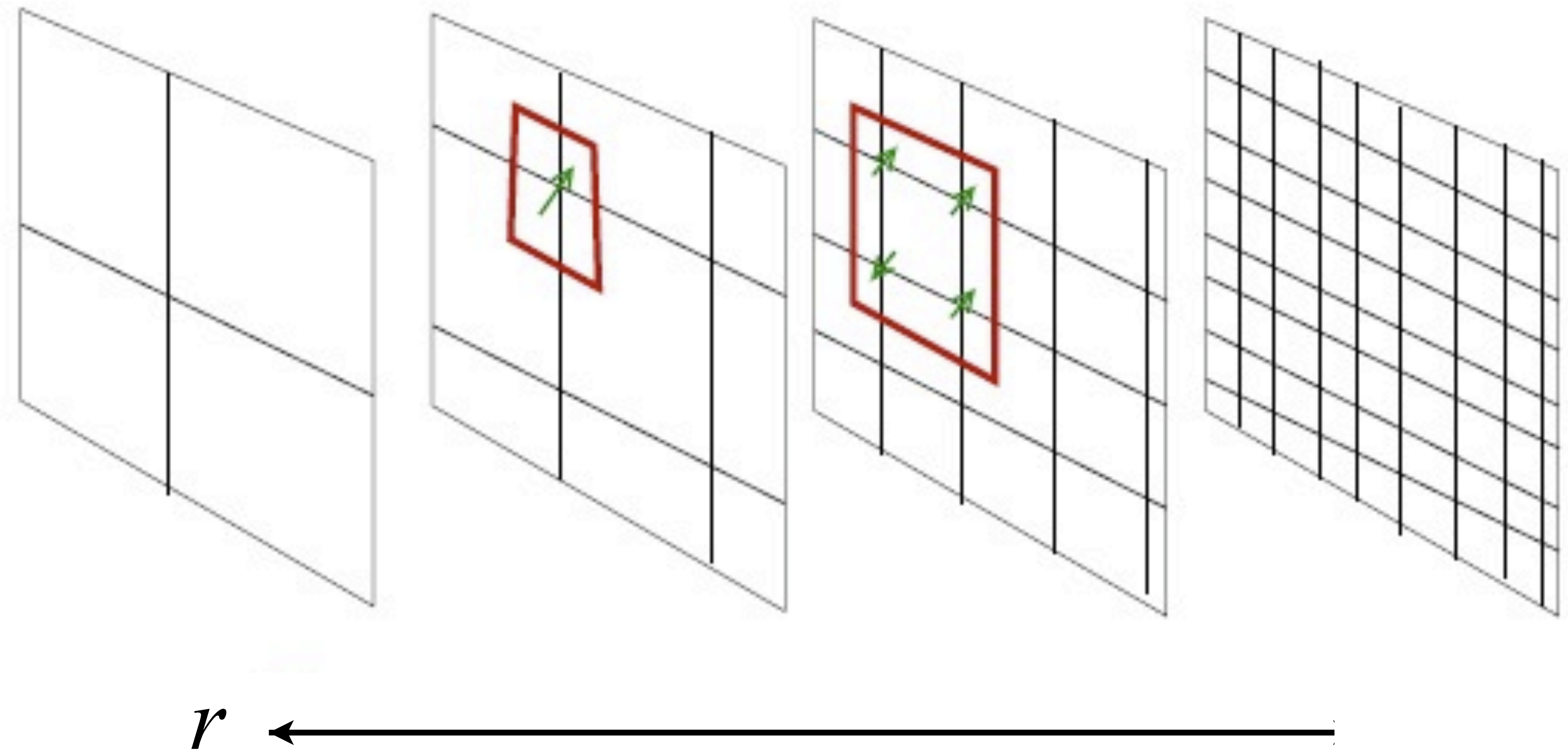
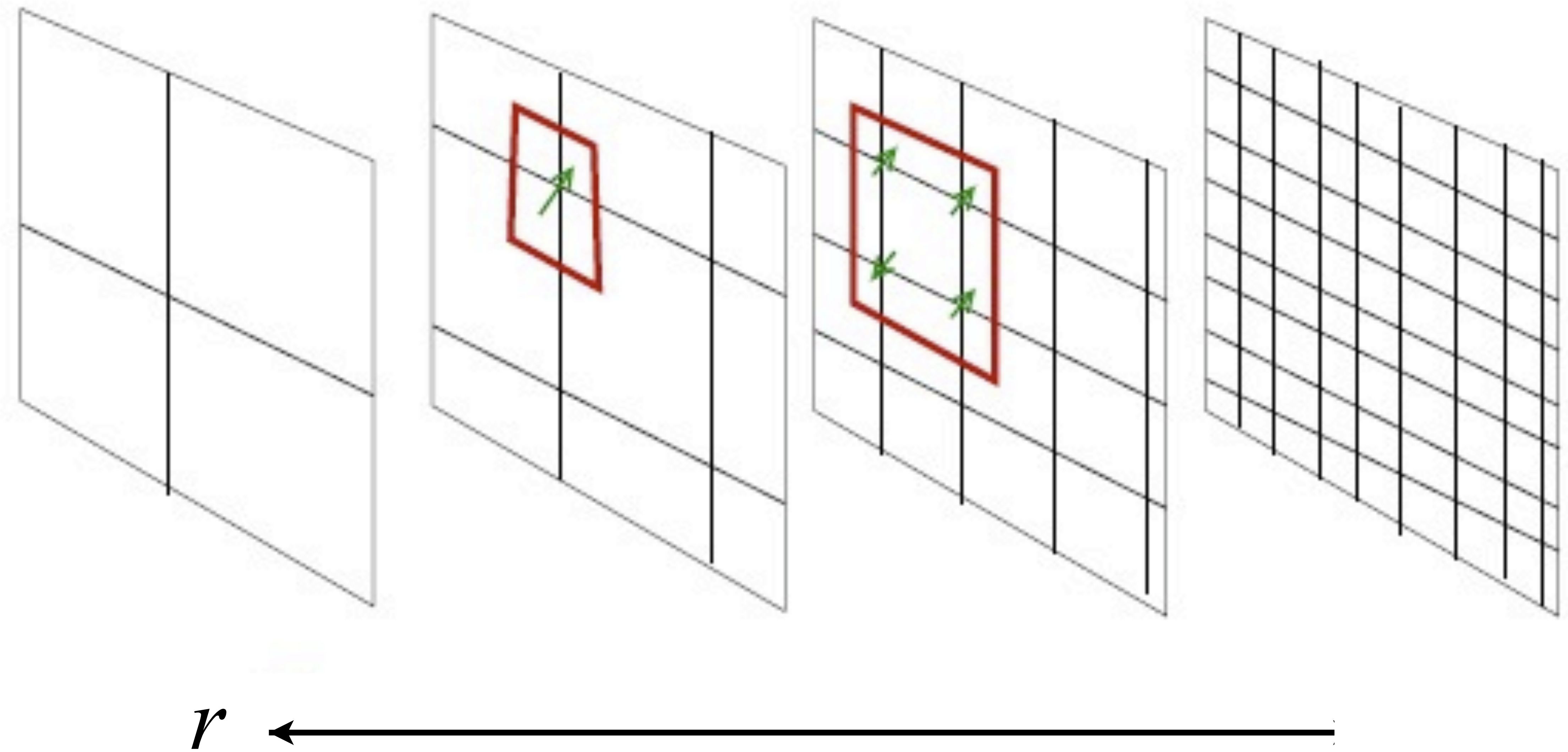


r



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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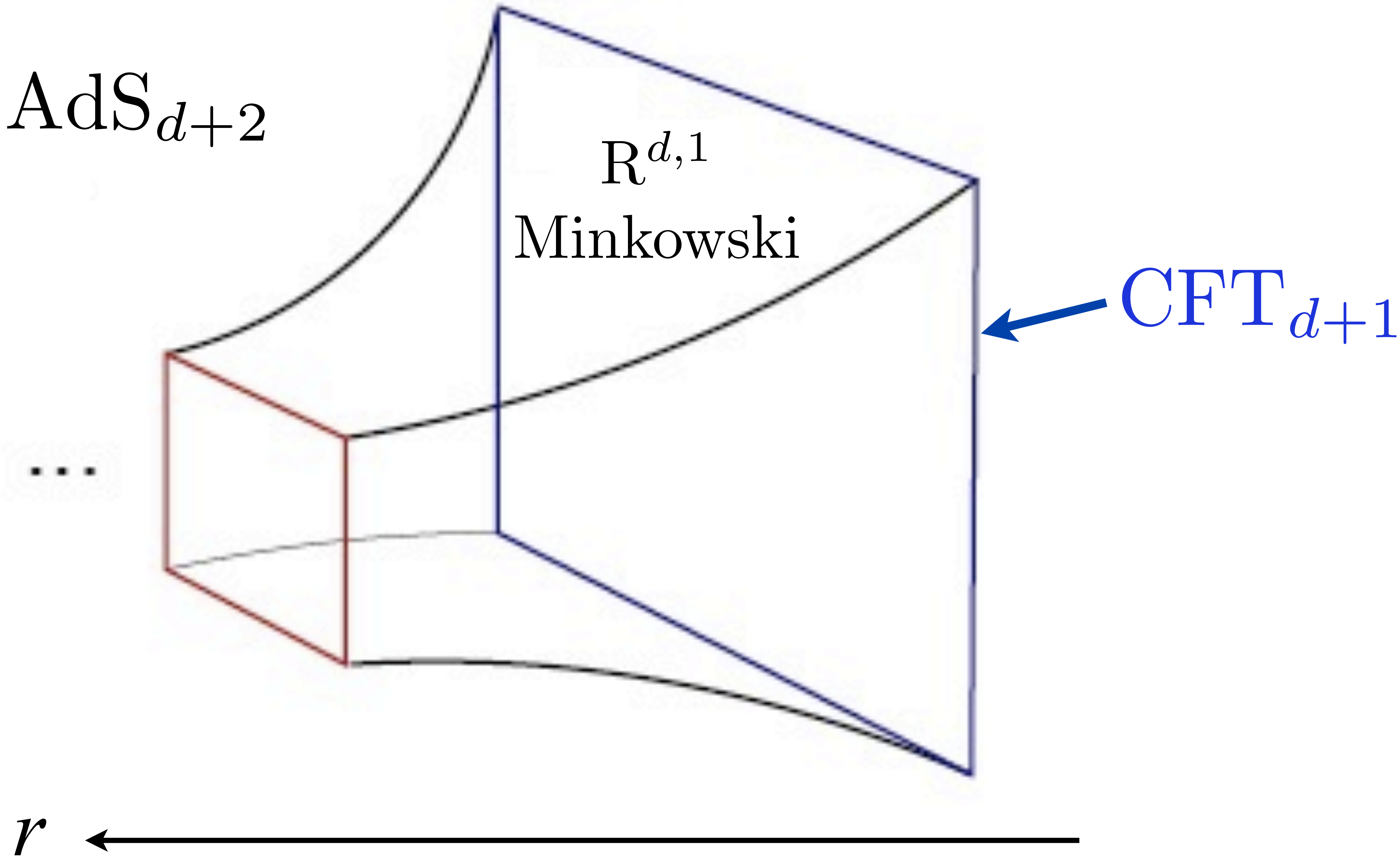
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS/CFT correspondence



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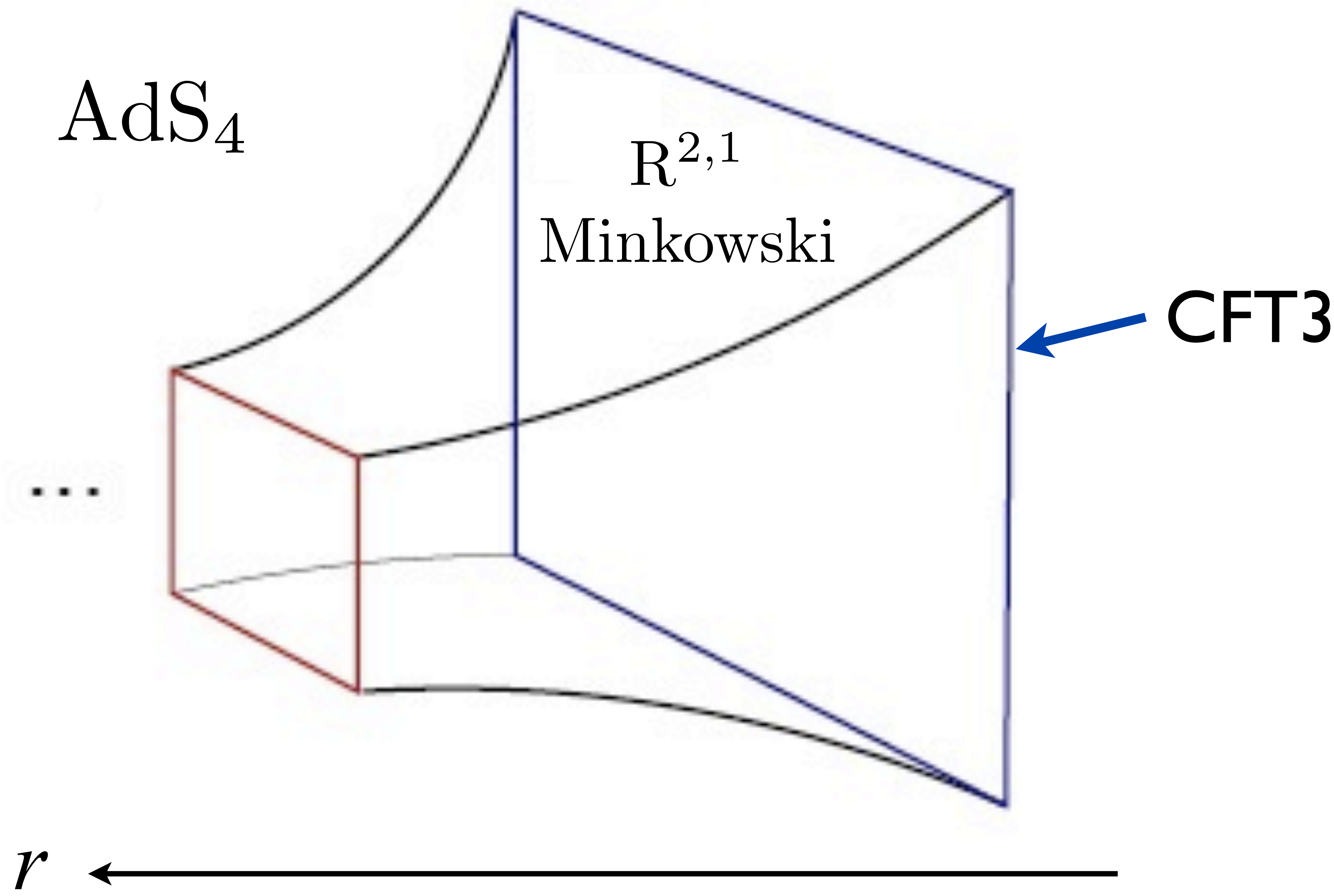

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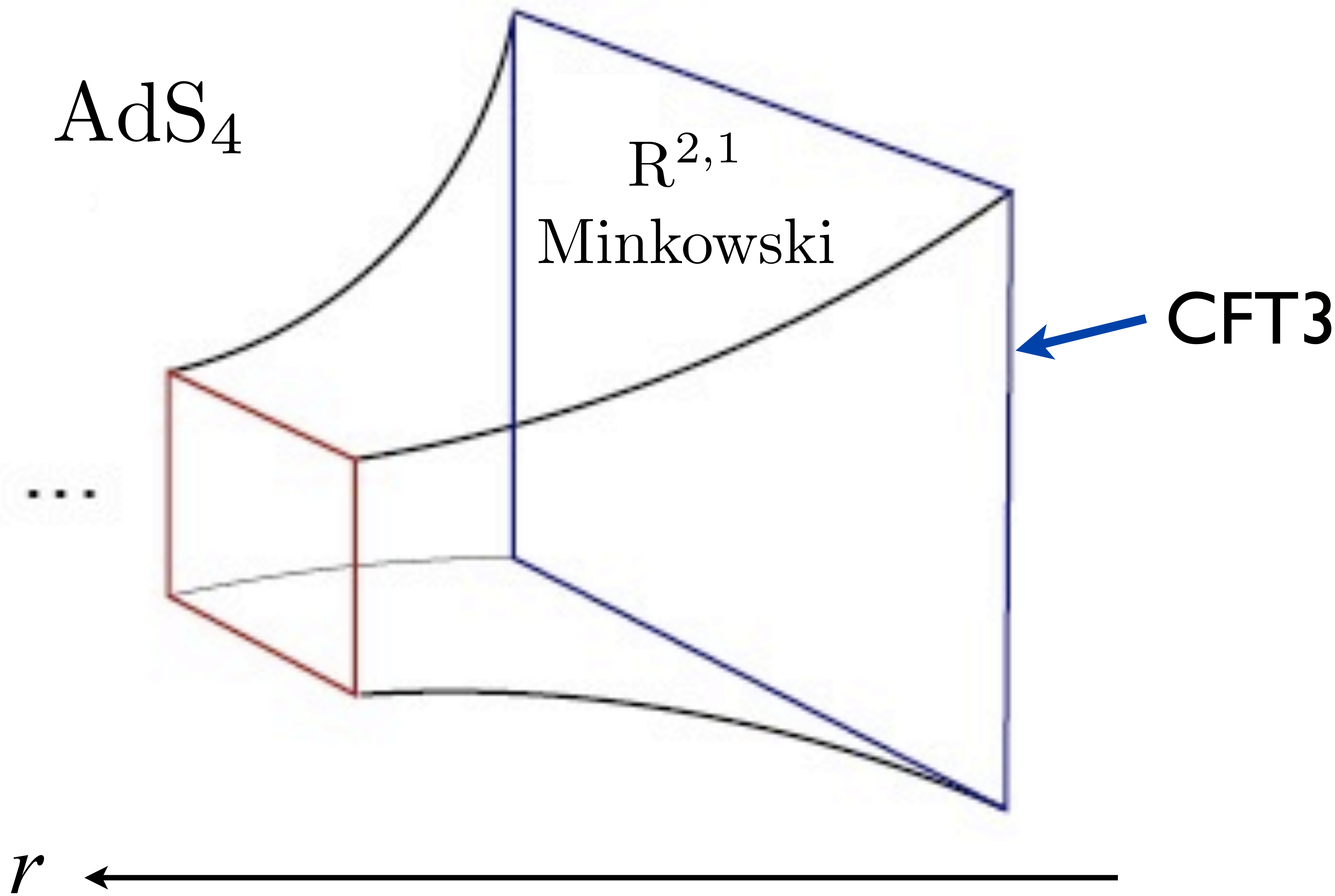
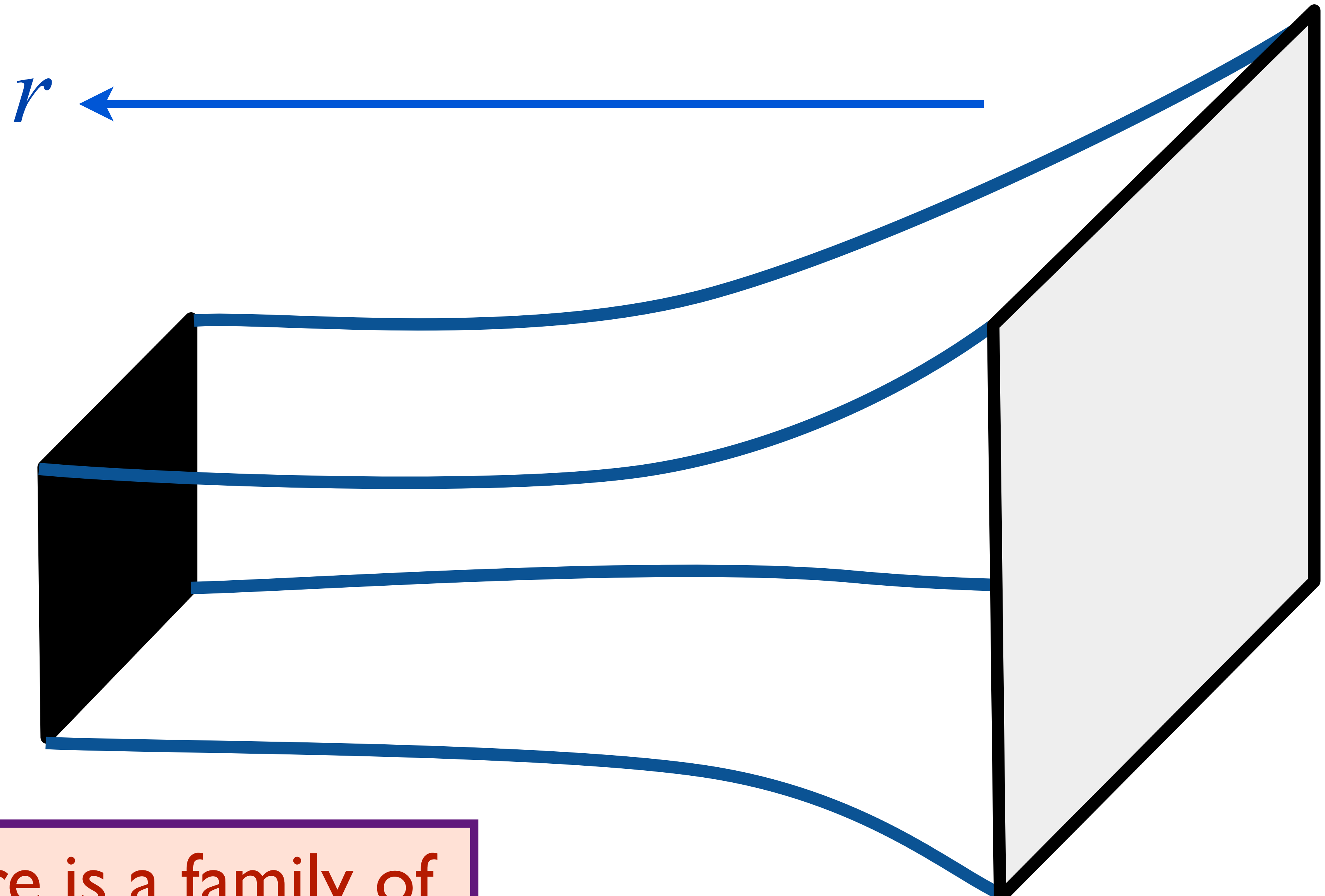


This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS$_4$-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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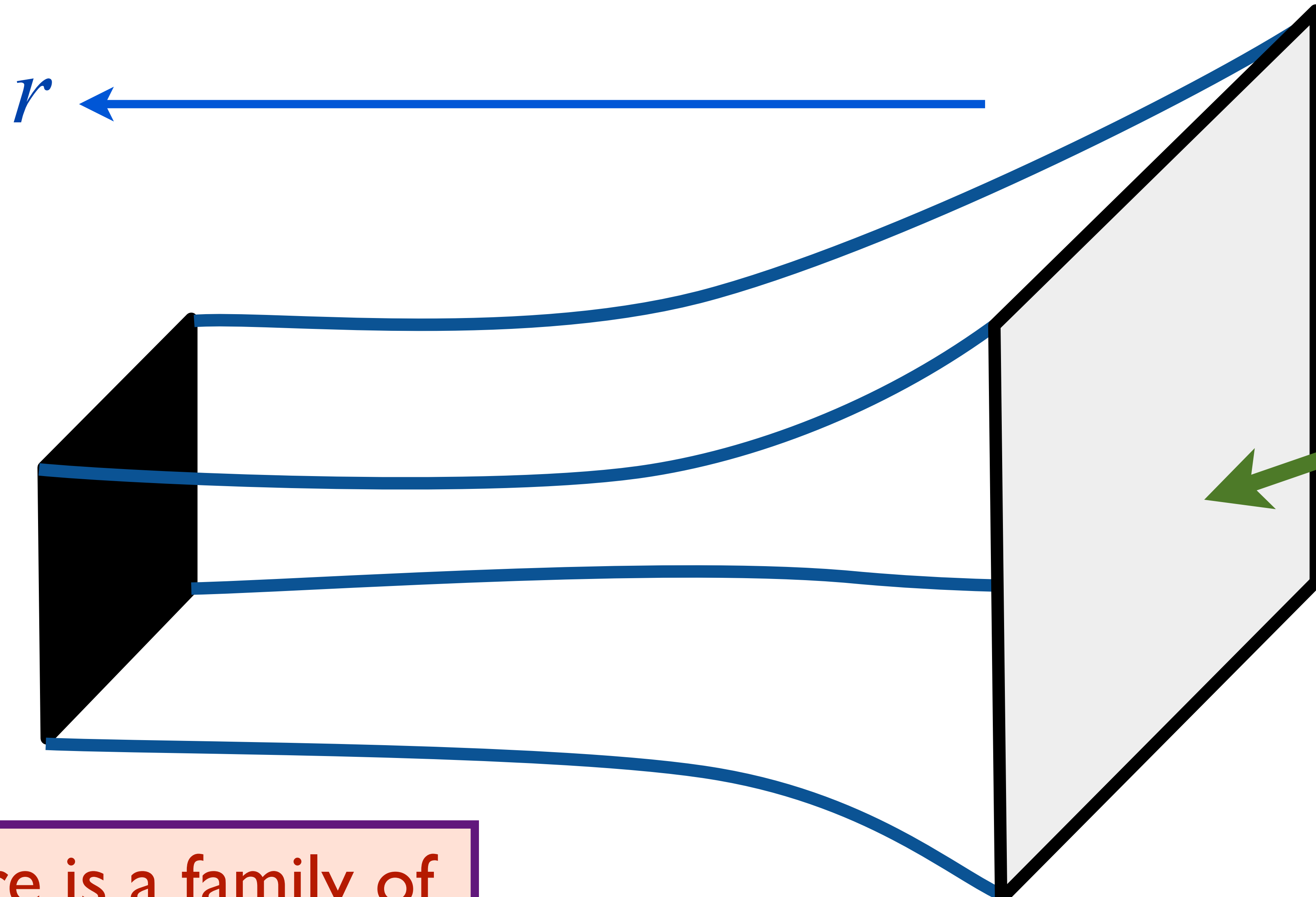

A 2+1 dimensional system at its quantum critical point: $k_B T = \frac{3\hbar}{4\pi R}$.

There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

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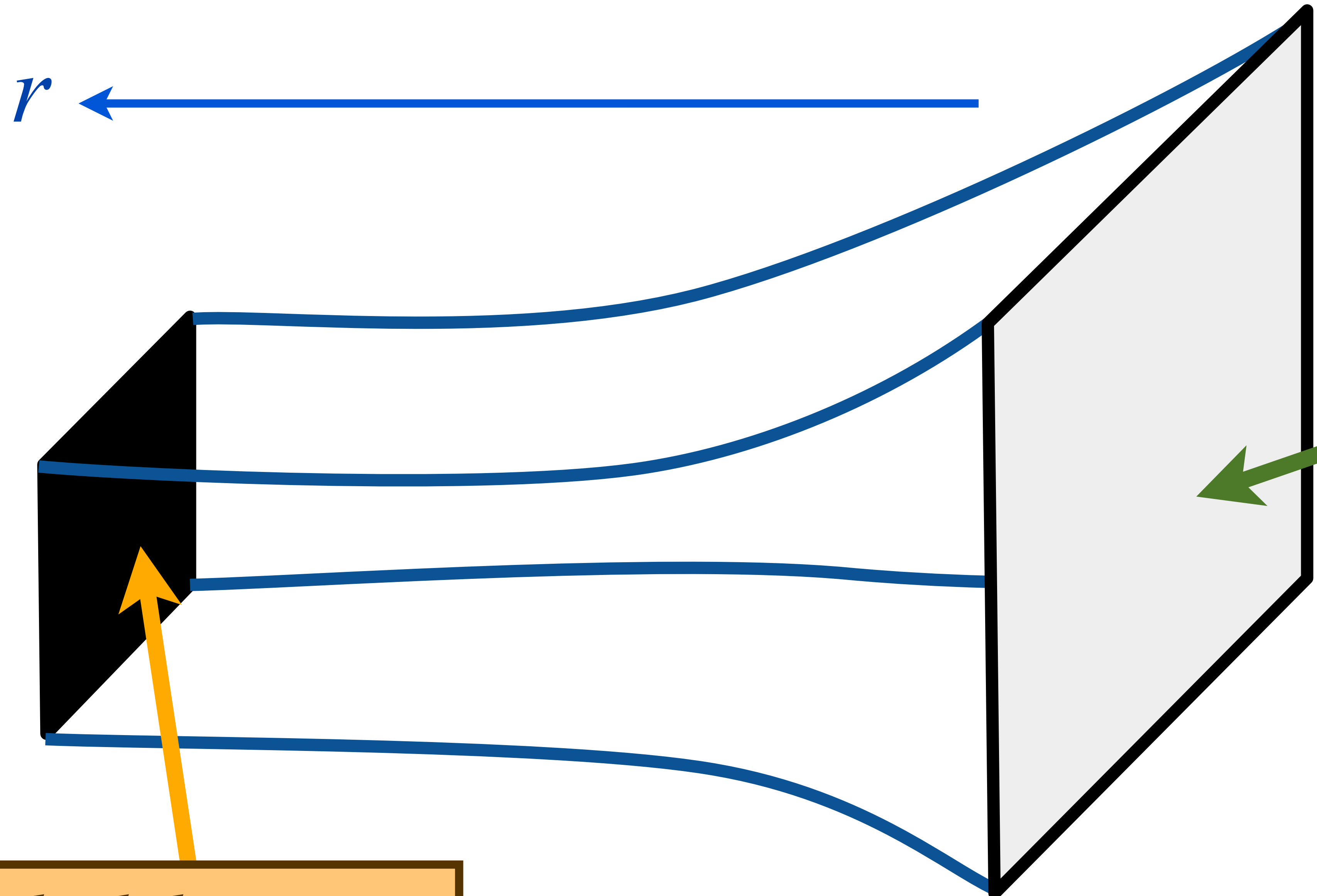

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Black-brane at temperature of 2+1 dimensional quantum critical system

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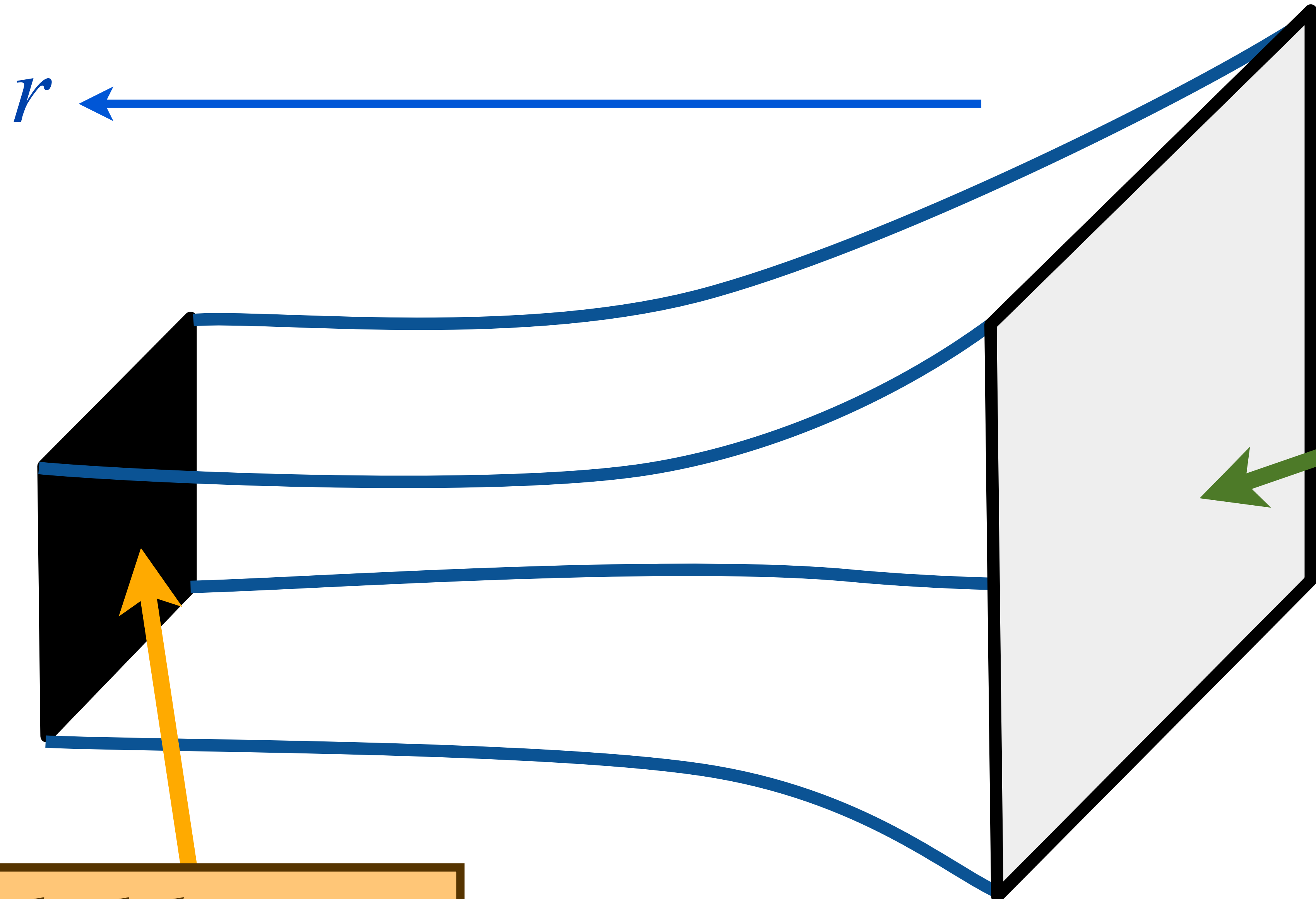


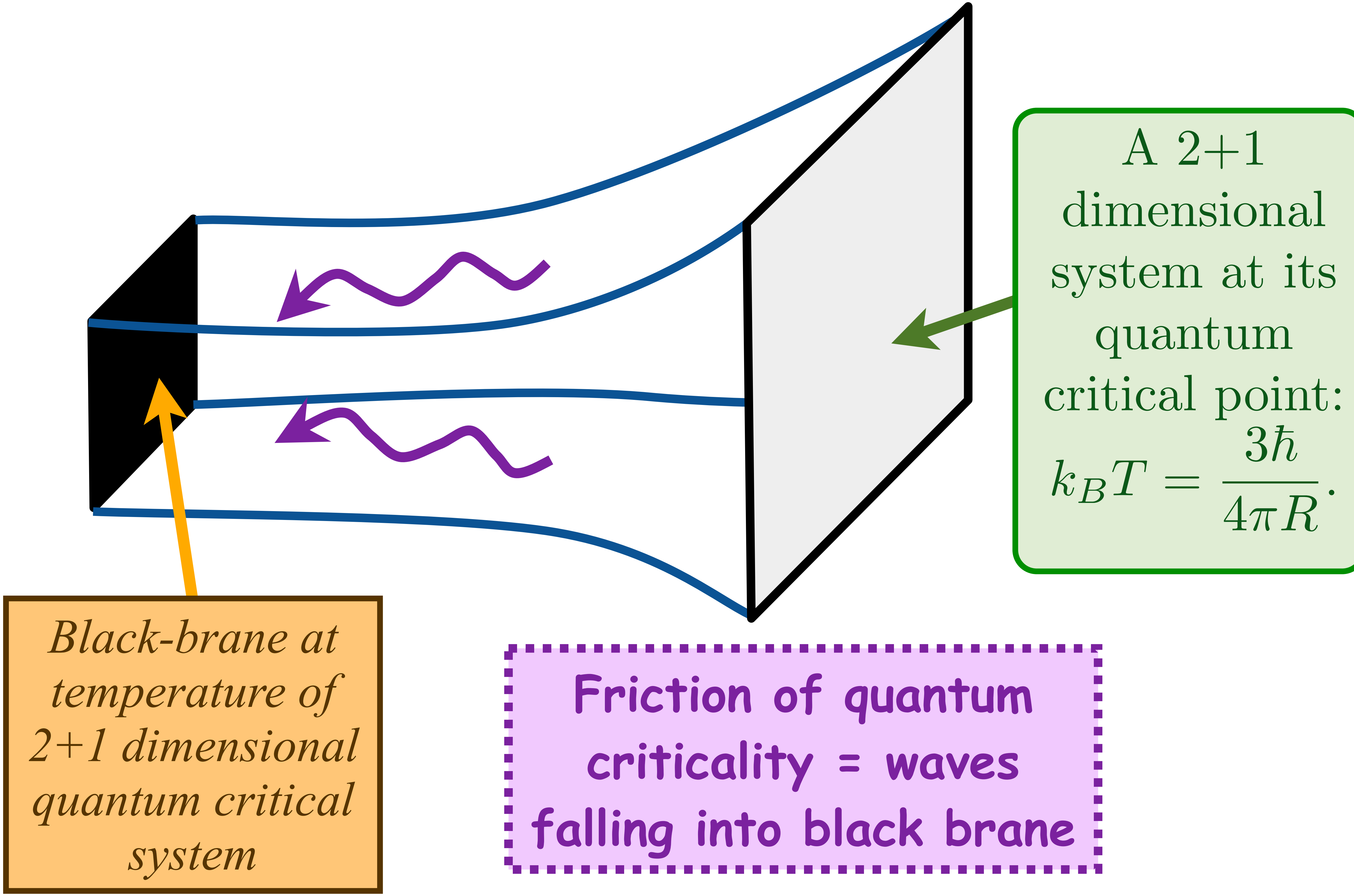
A 2+1 dimensional system at its quantum critical point:
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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This is to be solved subject to the constraint

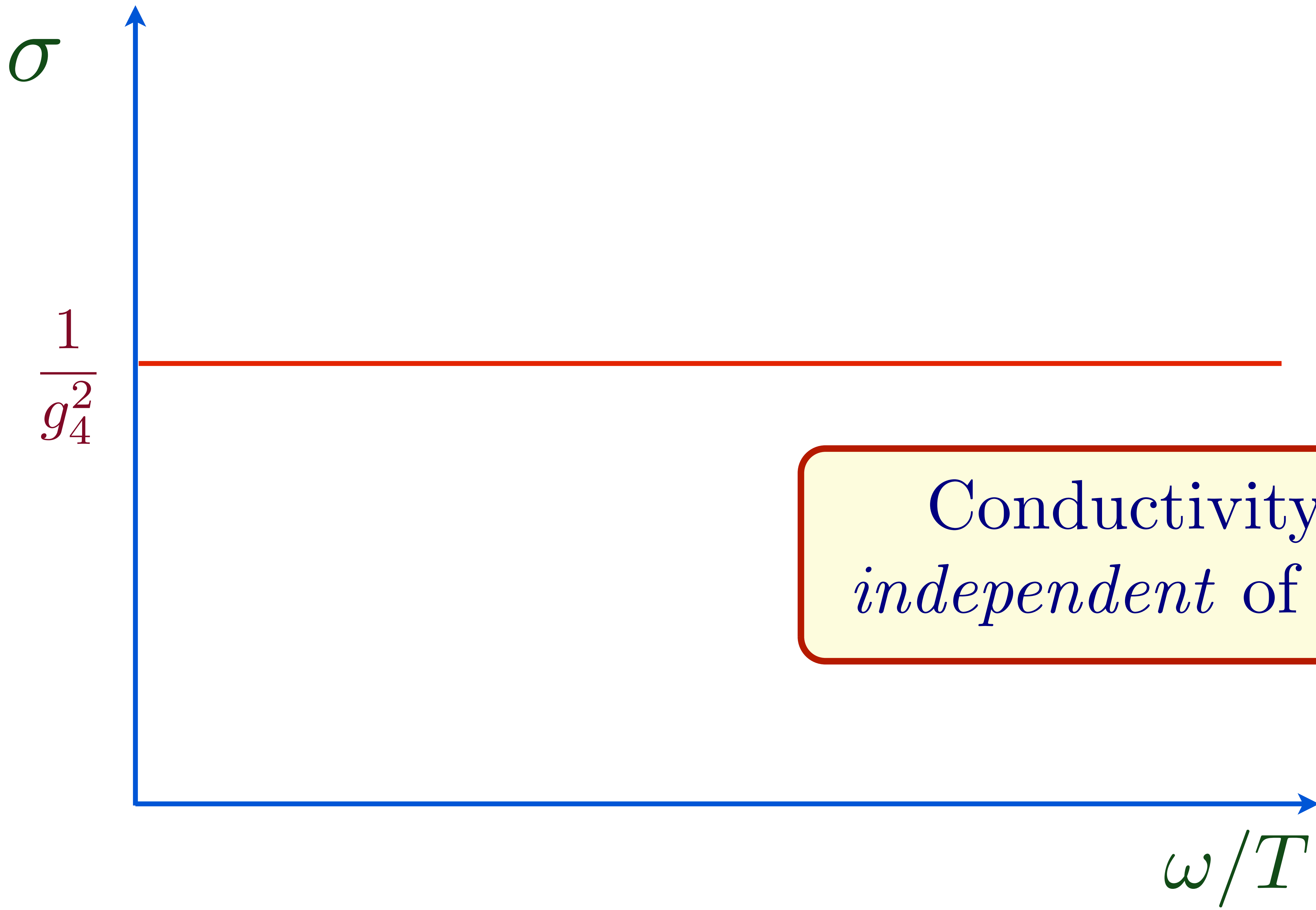
$$A_\mu(r \to 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where $\mathcal{A}_\mu$ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

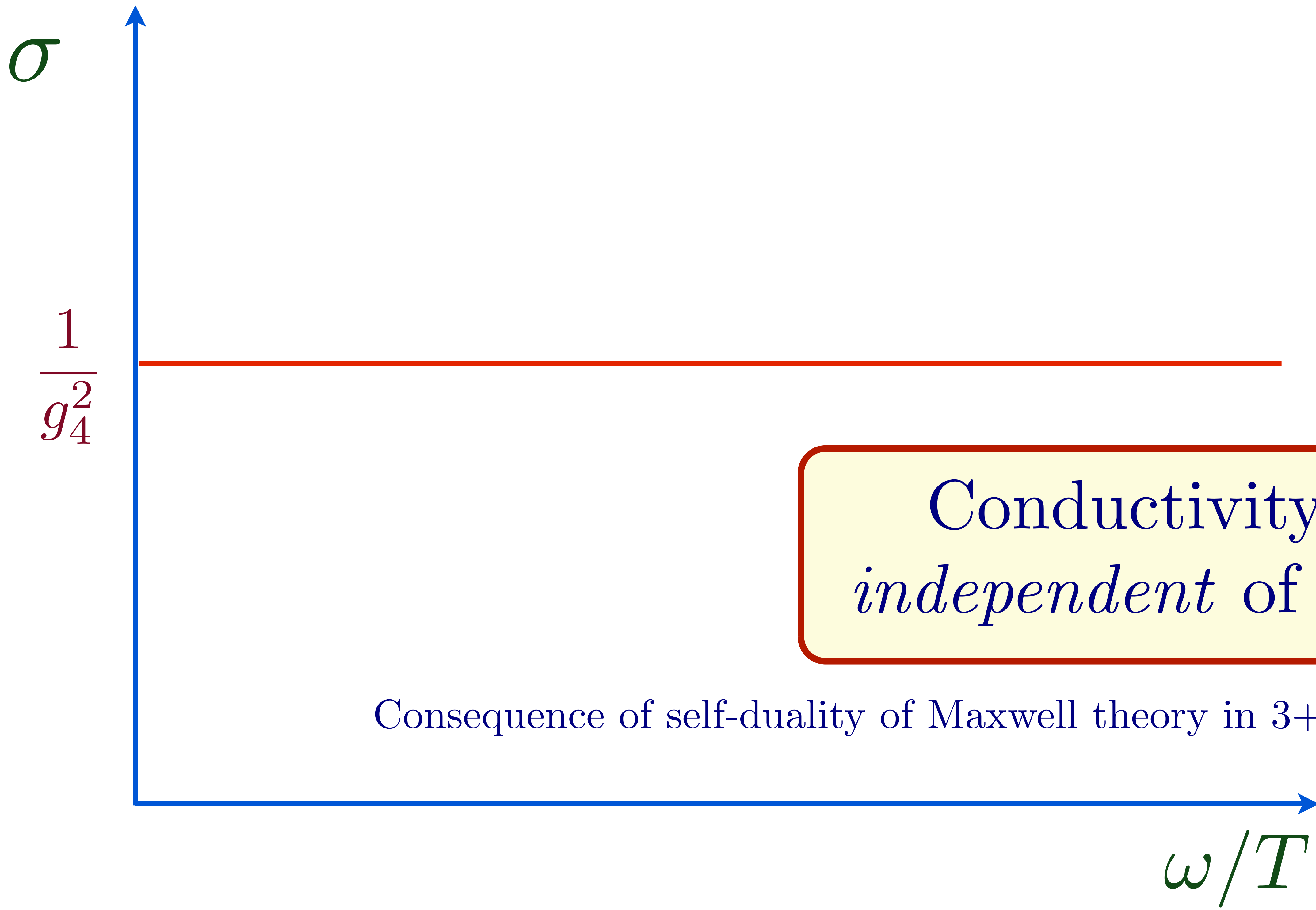
C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T* > *0*



Conductivity is *independent* of ω/T.

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*

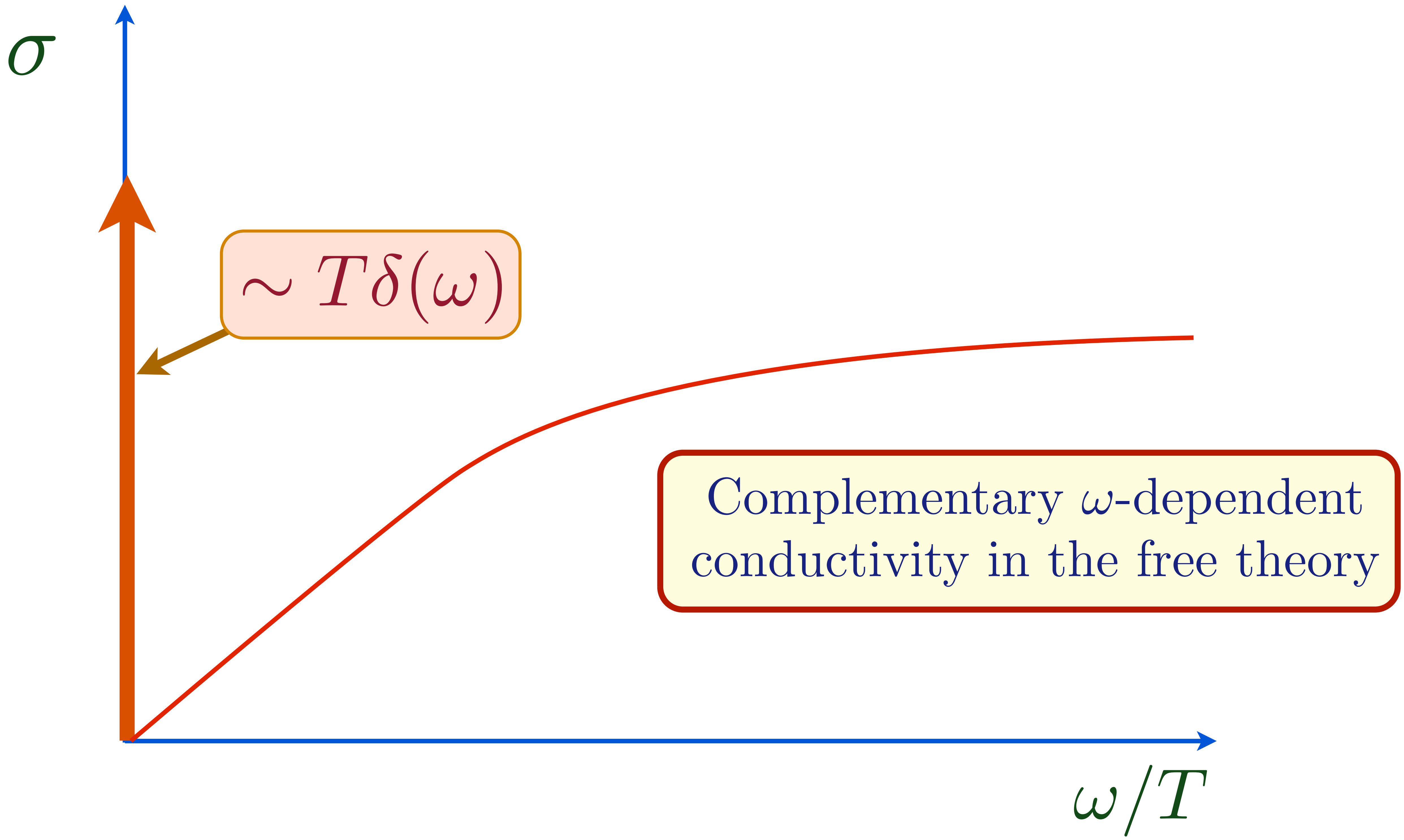


Conductivity is *independent* of ω/T.

Consequence of self-duality of Maxwell theory in 3+1 dimensions

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

Electrical transport in a free CFT3 for *T* > *0*



Improving the AdS_4 theory of “nearly perfect fluids”

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R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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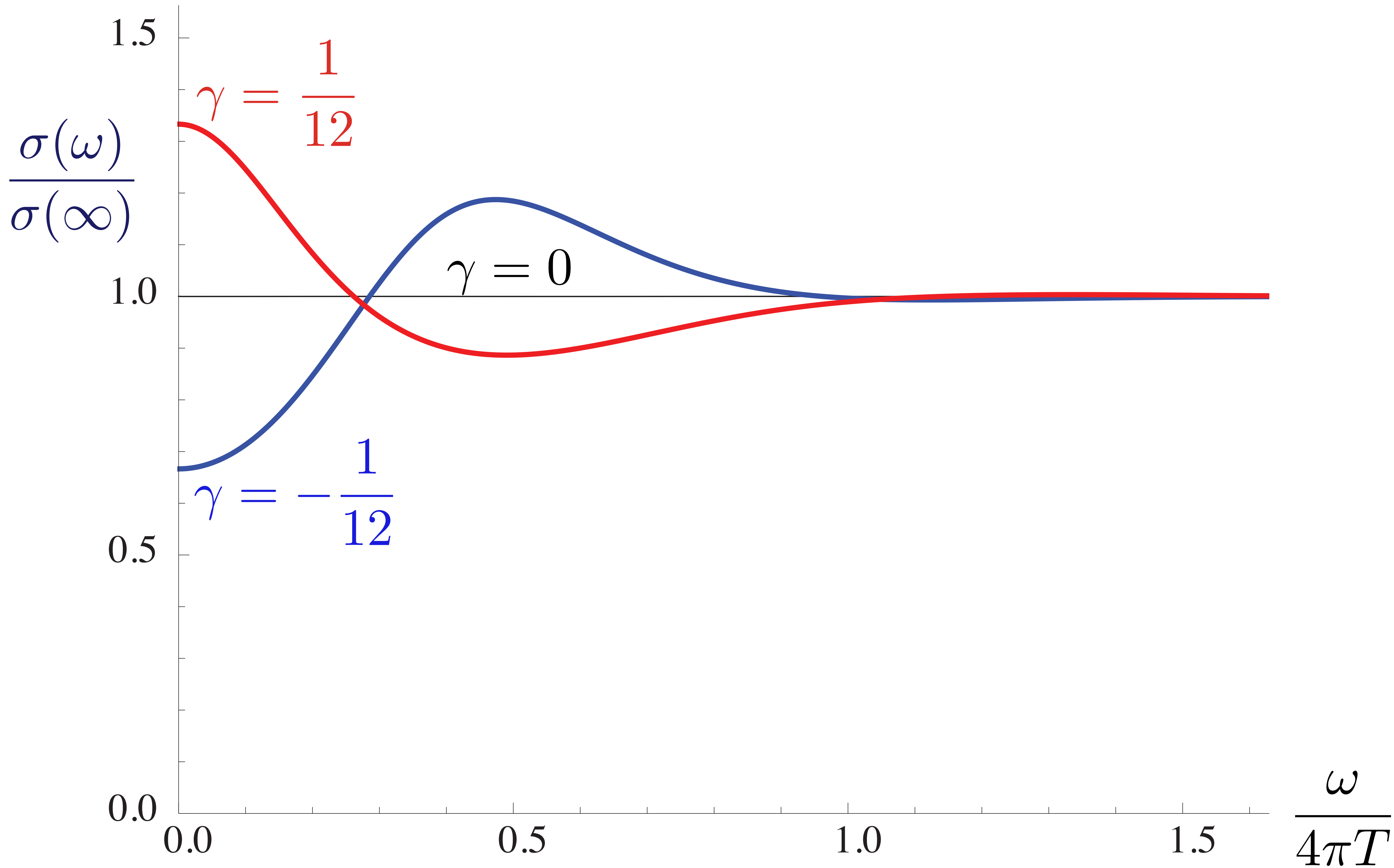
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab}F^{ab} + \frac{\gamma L^2}{g_4^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Improving the AdS$_4$ theory of "nearly perfect fluids"



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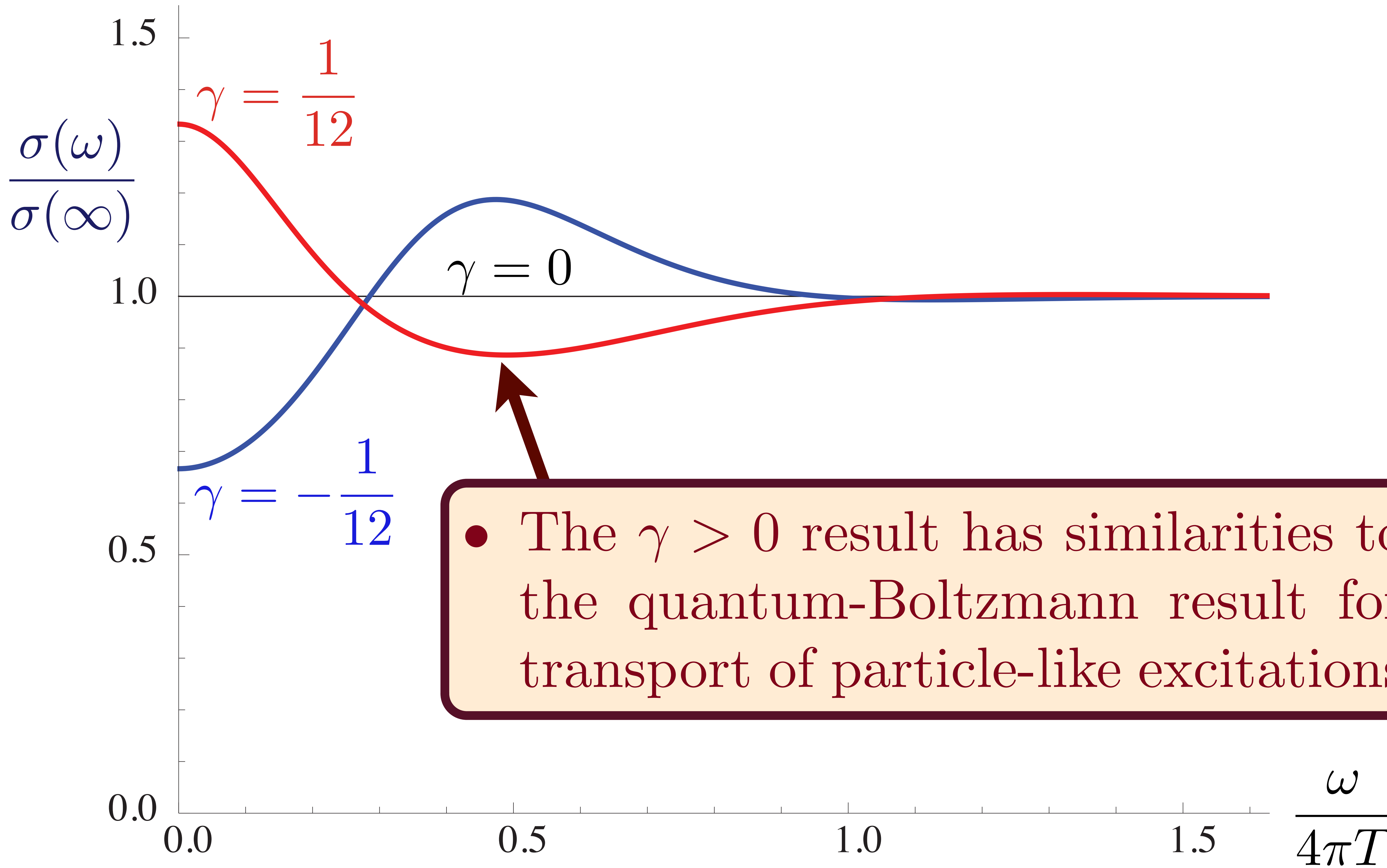

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

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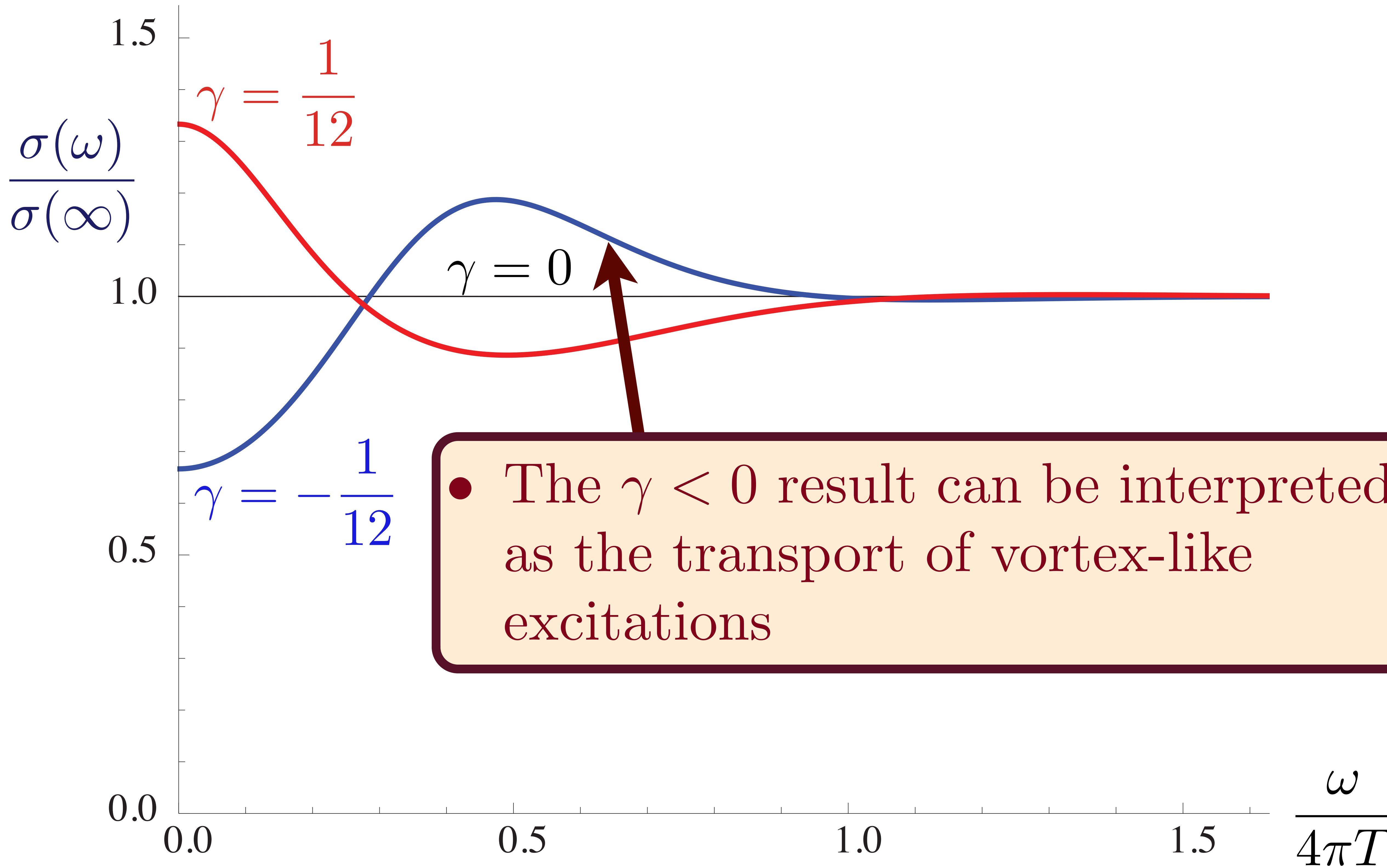

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

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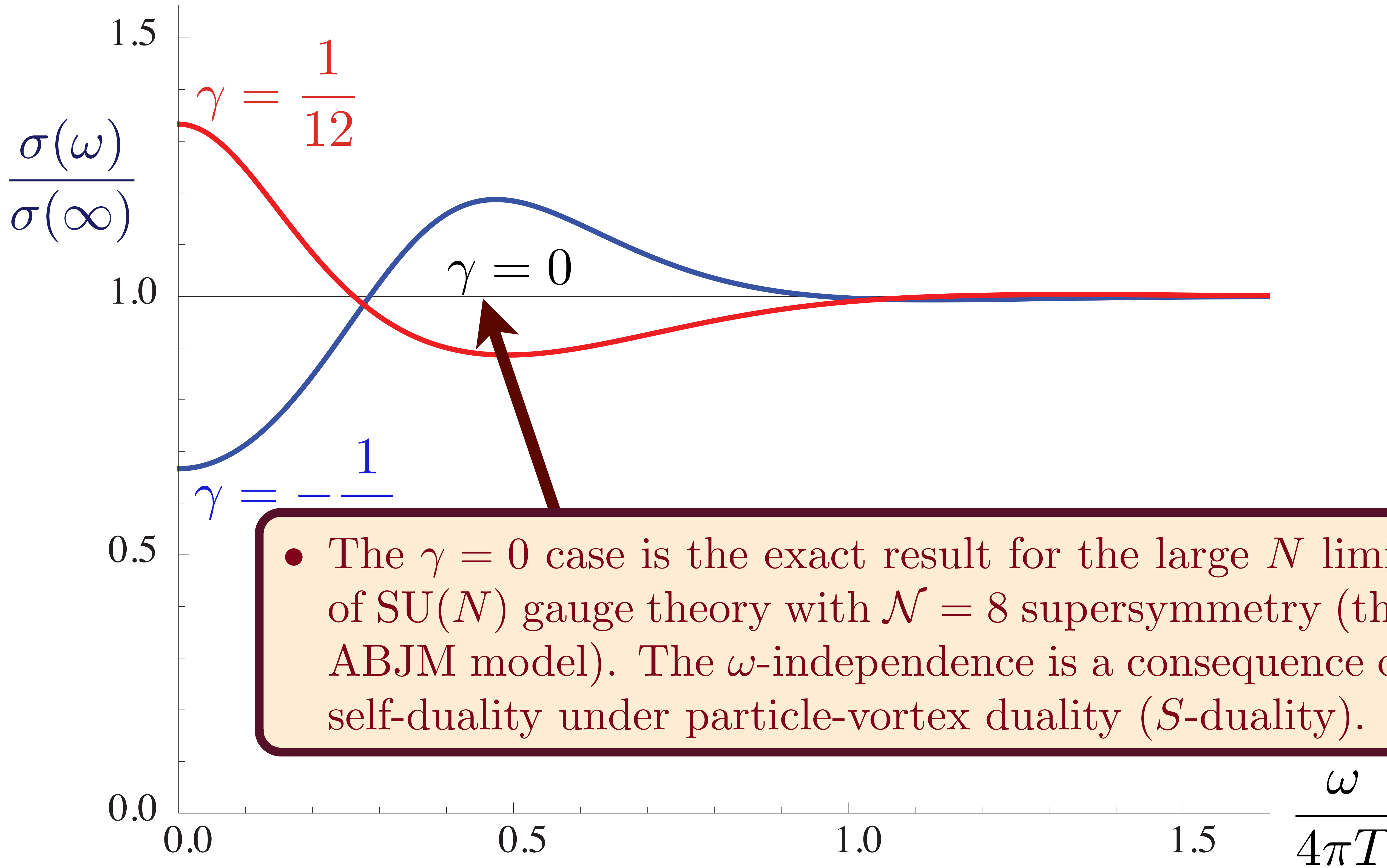

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

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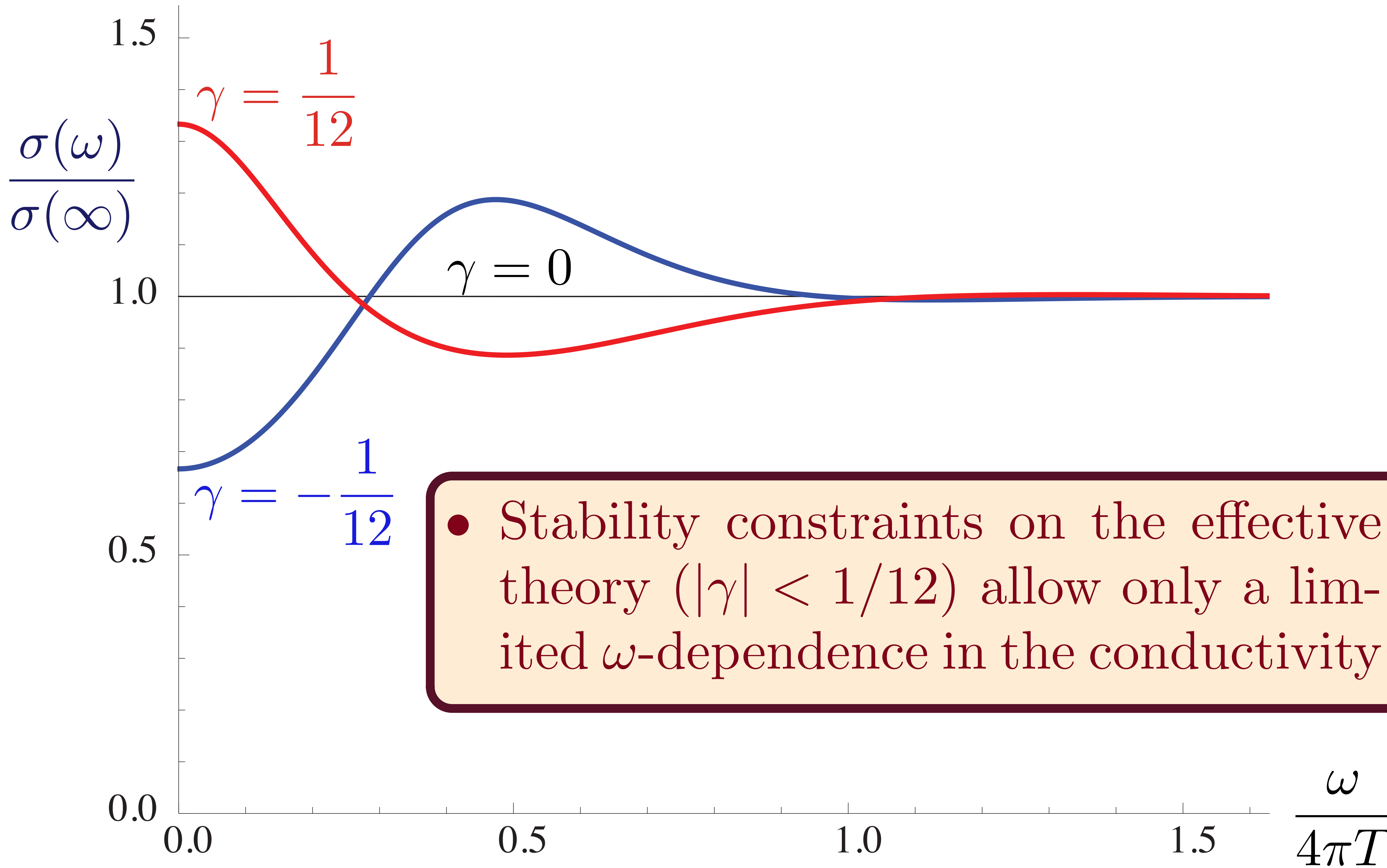

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Conclusions

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