Stringing together the quantum phases of matter

Lorentz Lectures, Leiden
May 7, 14, 21, June 4, 2012

Subir Sachdev

See also lecture at the 2011 Solvay conference,
Theory of the Quantum World, chair D.J. Gross.
100th anniversary of the first Solvay conference,
Radiation and the Quanta, chair H.A. Lorentz.
arXiv:1203.4565

Talk online at sachdev.physics.harvard.edu

PHYSICS
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Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

many-electron quantum states are adiabatically
connected to independent electron states

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle, long-range
quantum entanglement

States of quantum matter with long-range entanglement in *d* spatial dimensions

Useful classification is provided by
nature of excitations with vanishing energy:

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1. Gapped systems without zero energy excitations

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Useful classification is provided by
nature of excitations with vanishing energy:

1. Gapped systems without zero energy excitations

2. "Relativistic" systems with zero energy excitations at isolated points in momentum space

3. "Compressible" systems with zero energy excitations on d-1 dimensional surfaces in momentum space.

States of quantum matter with long-range entanglement in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

States of quantum matter with long-range entanglement in *d* spatial dimensions

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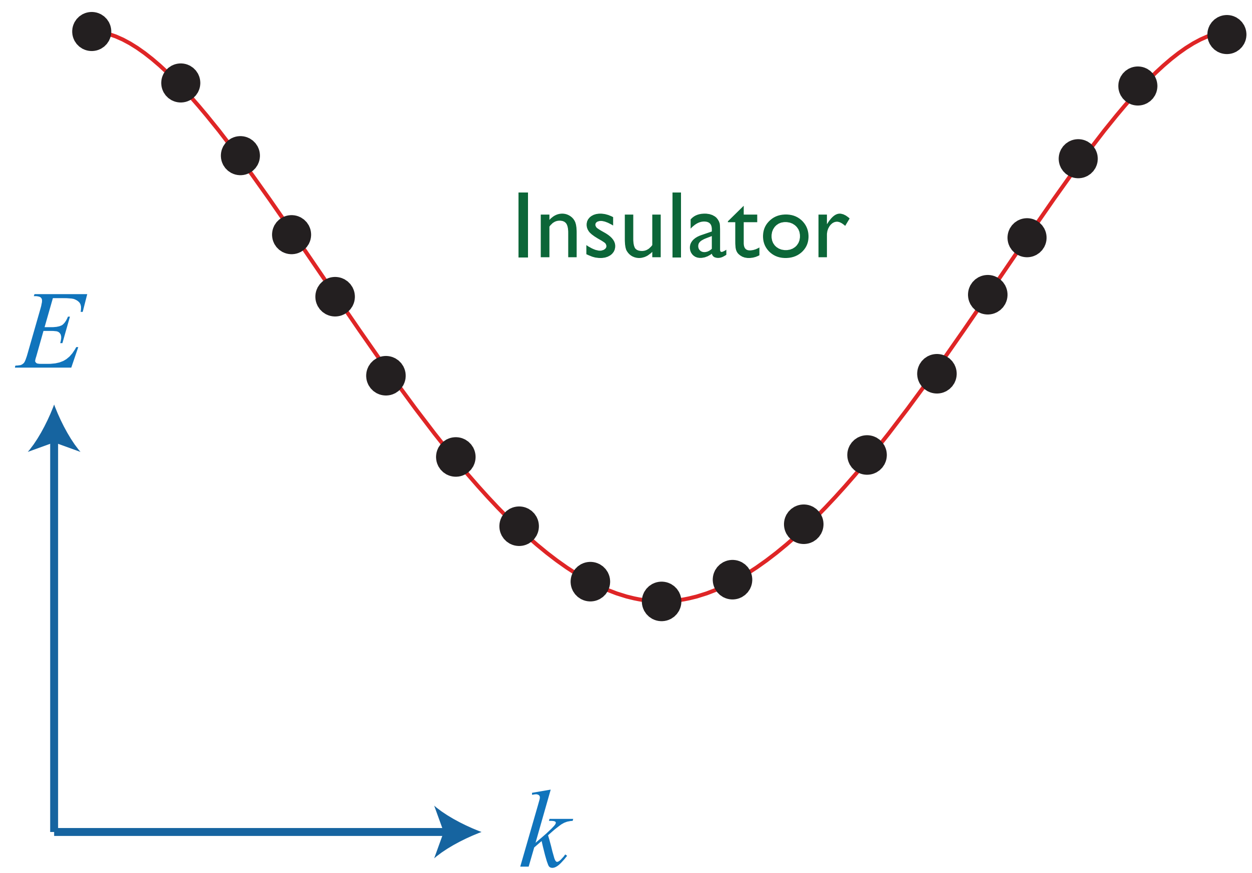
Graphene, strange metals in high temperature superconductors, spin liquids

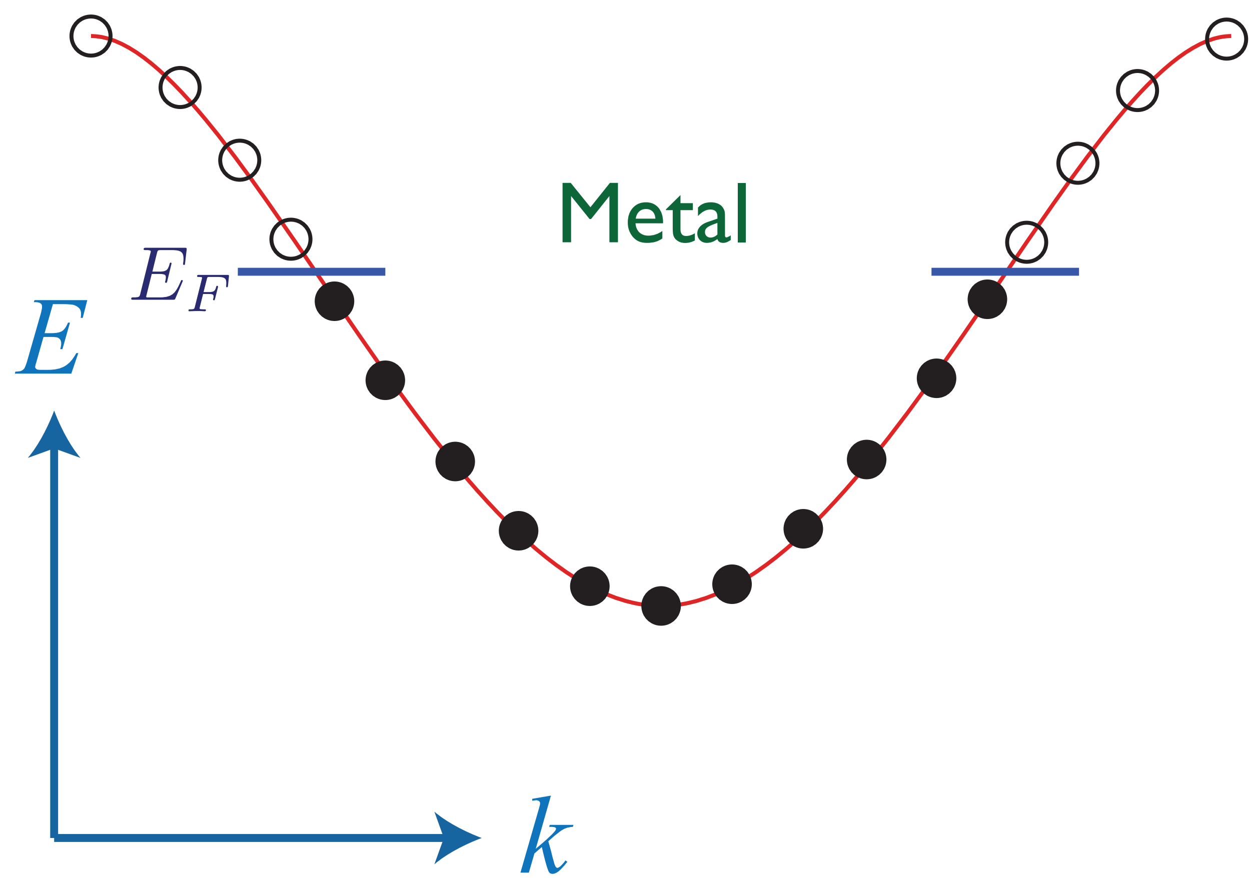
Gapped quantum matter

Band insulators



An even number of electrons per unit cell

Metals



An odd number of electrons per unit cell

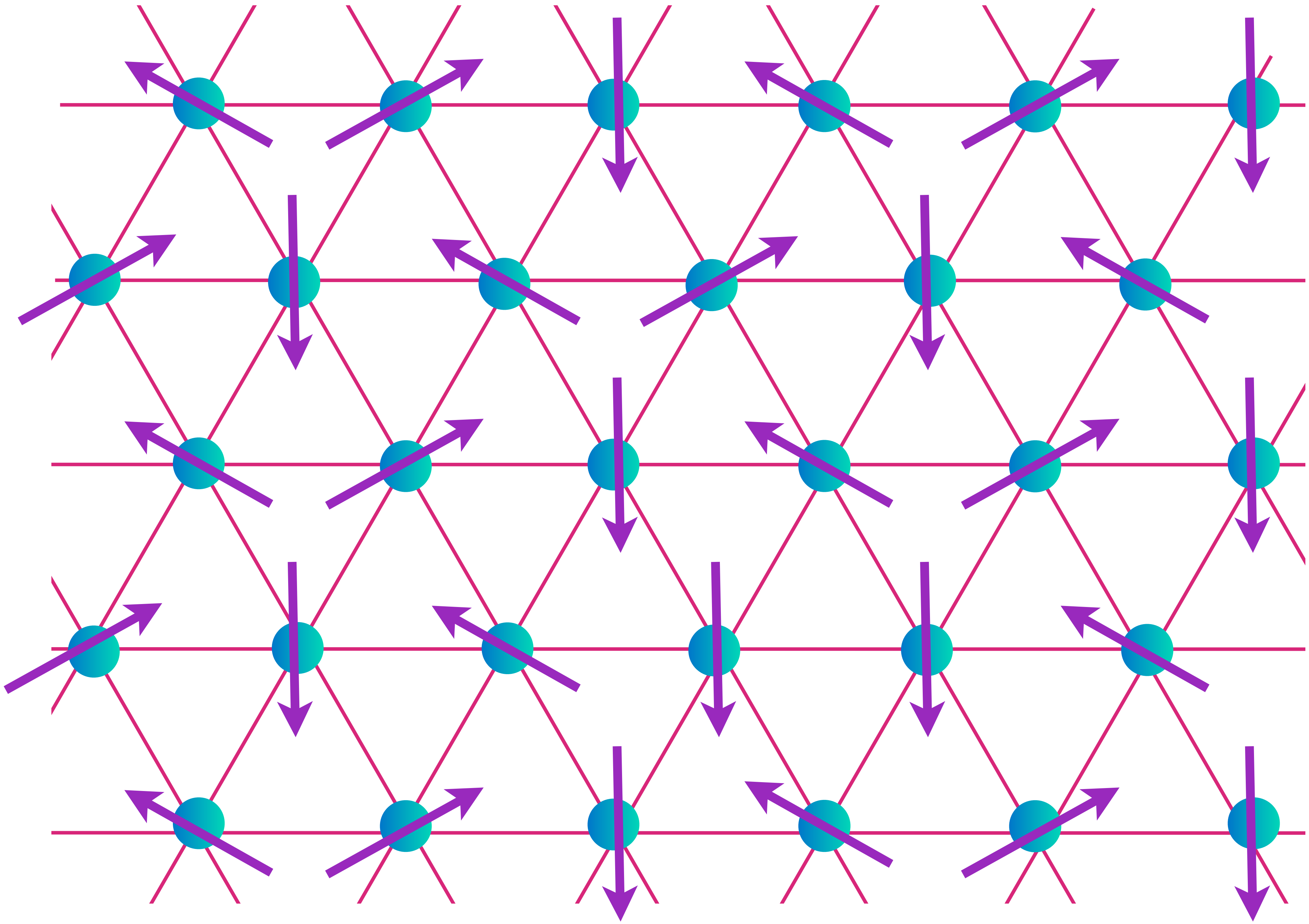
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mott insulator: Triangular lattice antiferromagnet

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Imagine quantum fluctuations are so strong that the Neel order does not have long-range correlations.

Naive "classical" picture: we obtain a quantum *disordered* state in which all spin-spin correlations decay exponentially over a short length scale.

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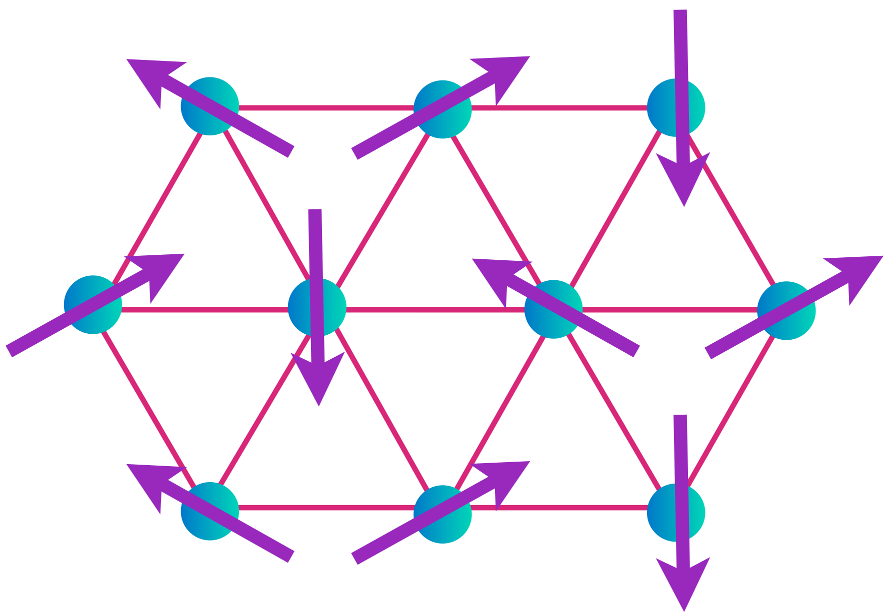
Modern "quantum" understanding: the discrete quantum degrees of freedom require a state with *long-range* entanglement.

Mott insulator: Triangular lattice antiferromagnet



non-collinear Néel state

Z_2 spin liquid
with neutral $S = 1/2$ spinons
and **vison** excitations

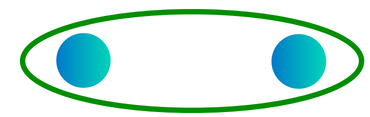
s_c

s

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



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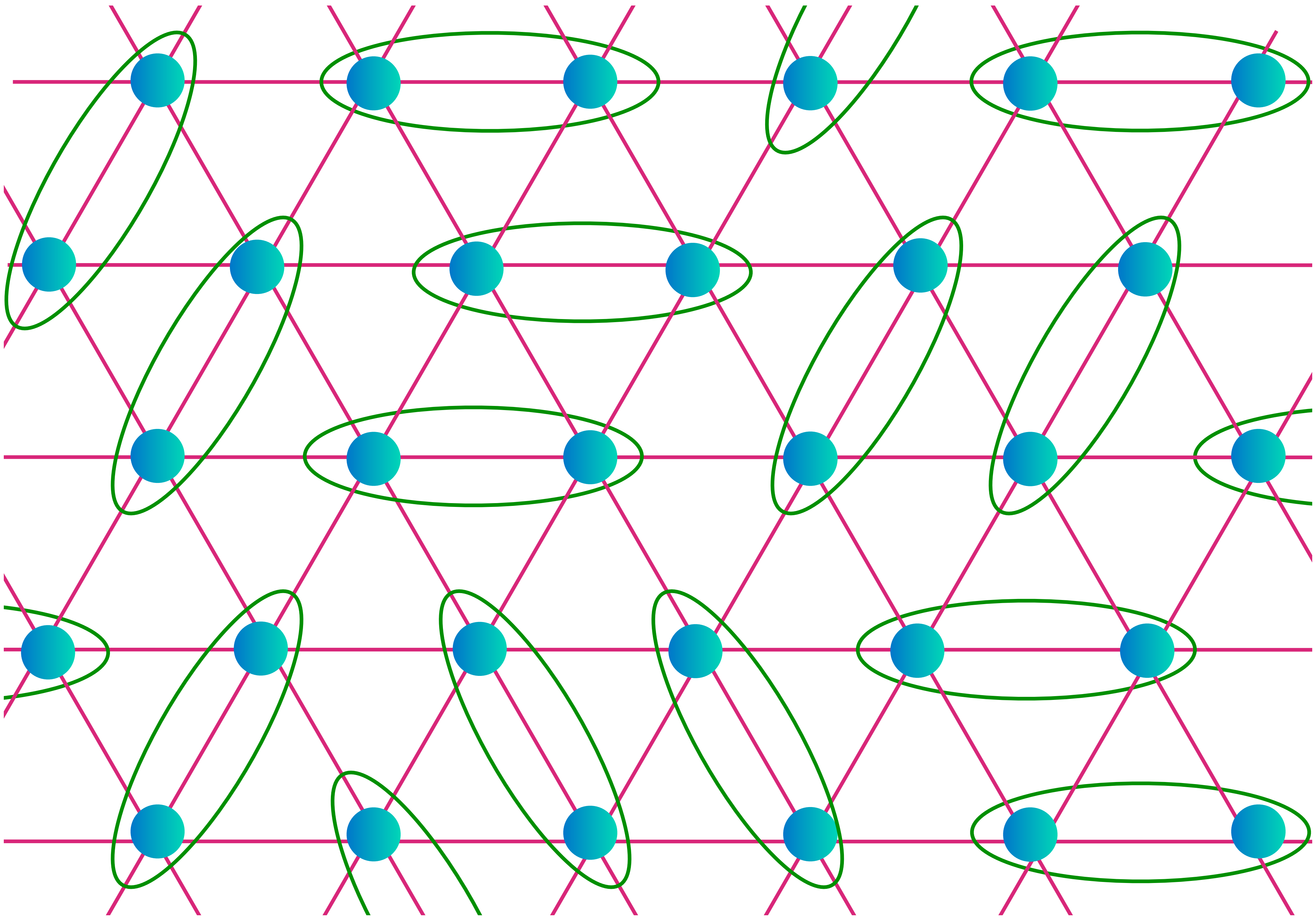

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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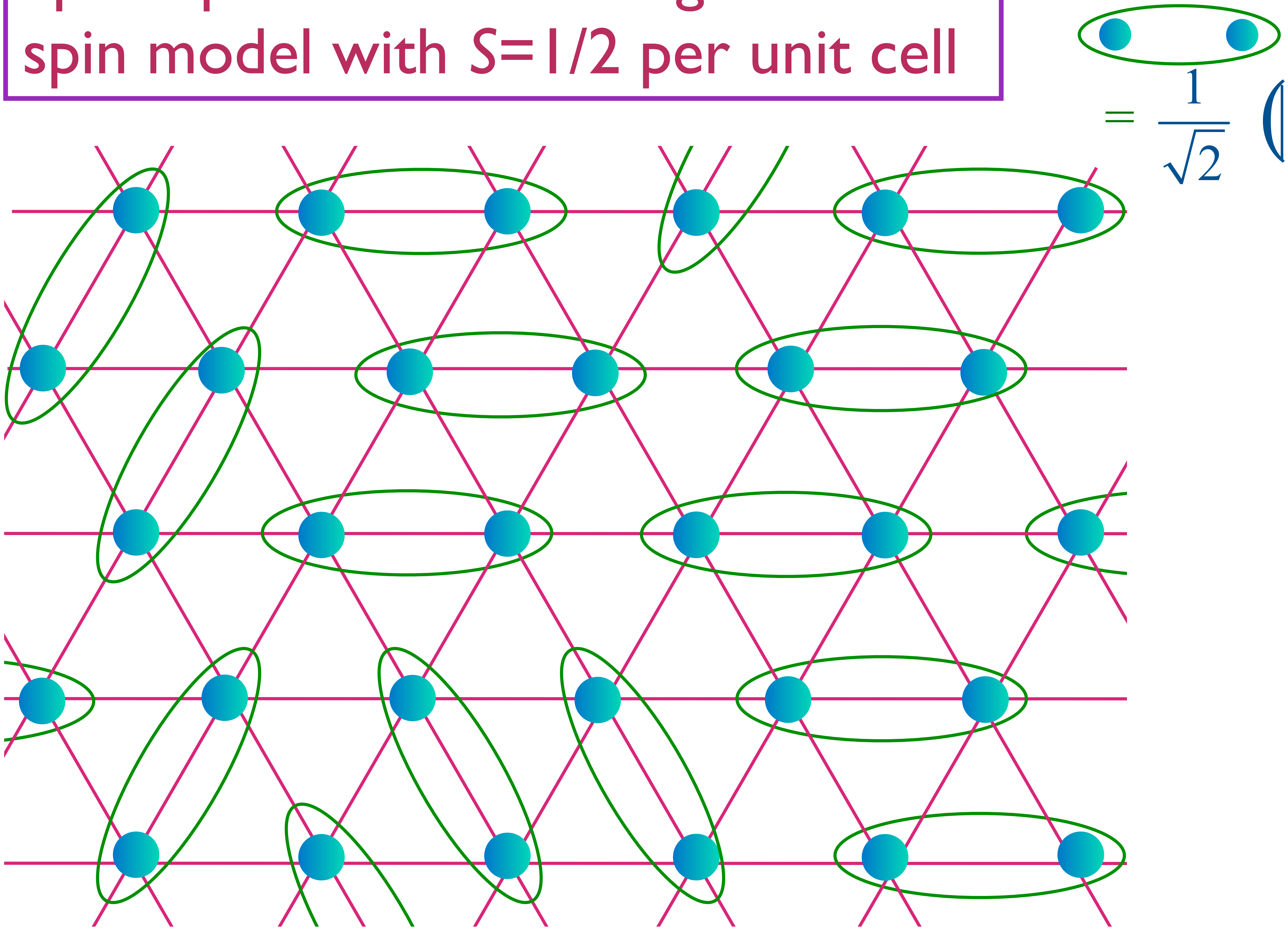

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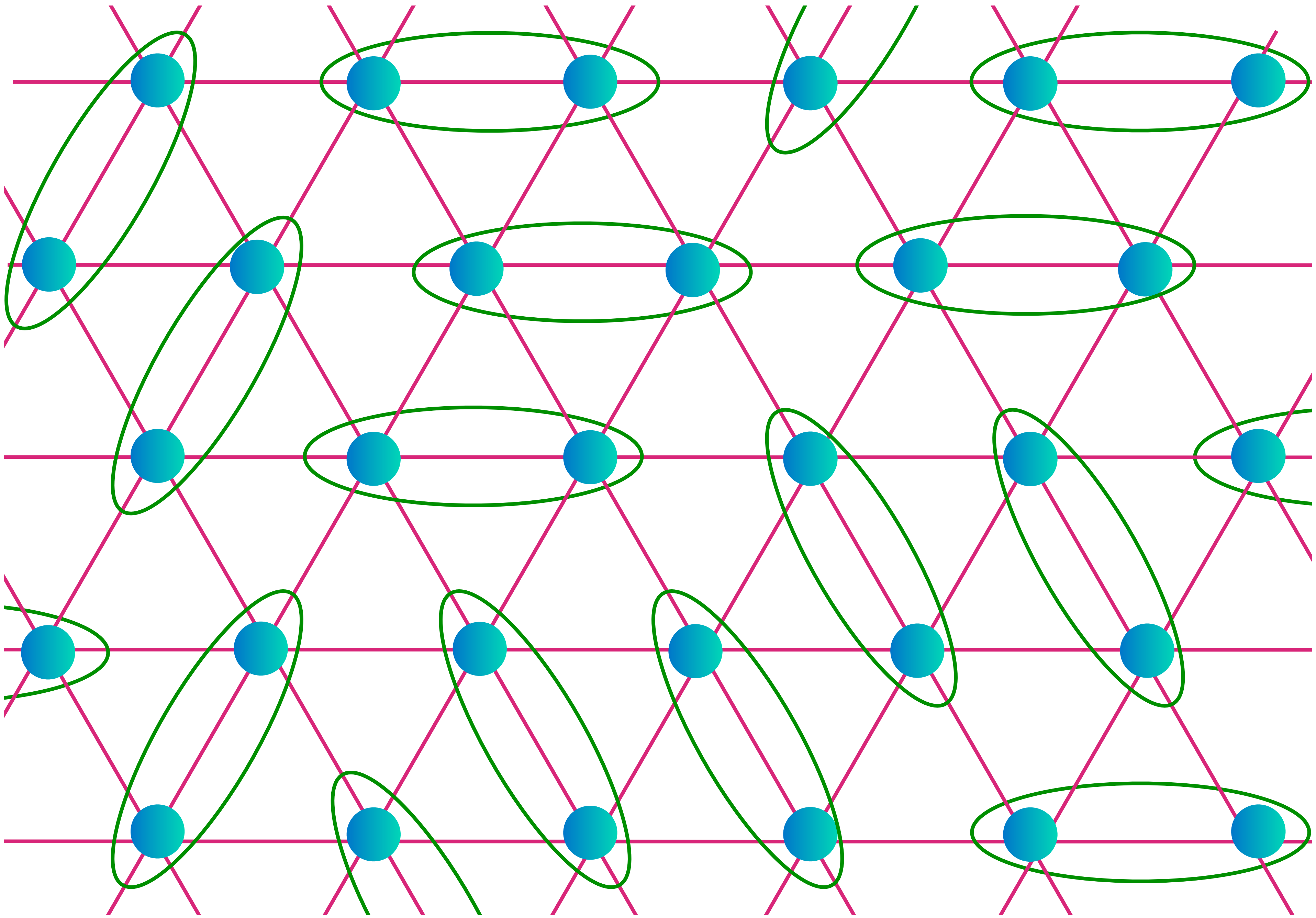

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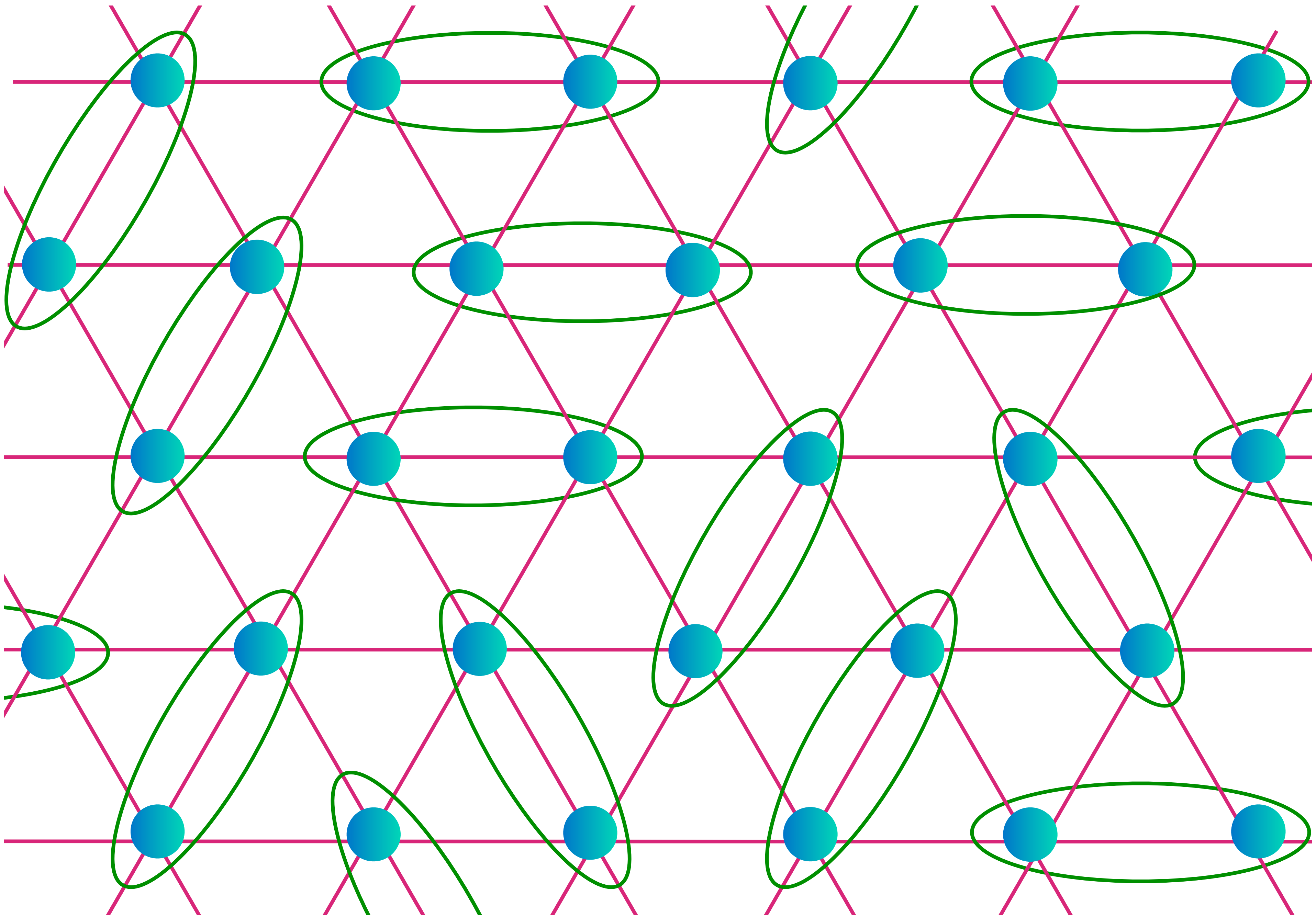

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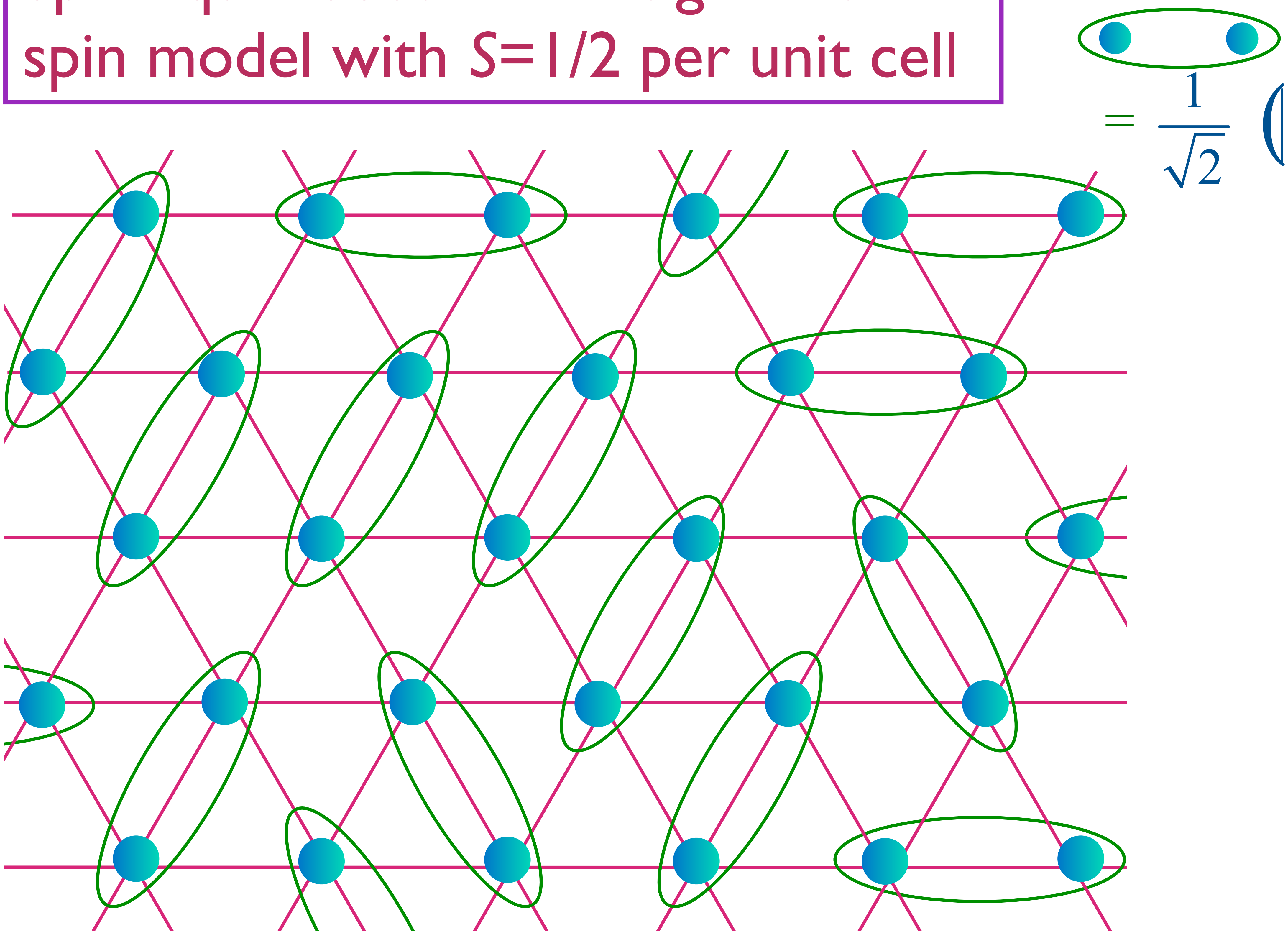

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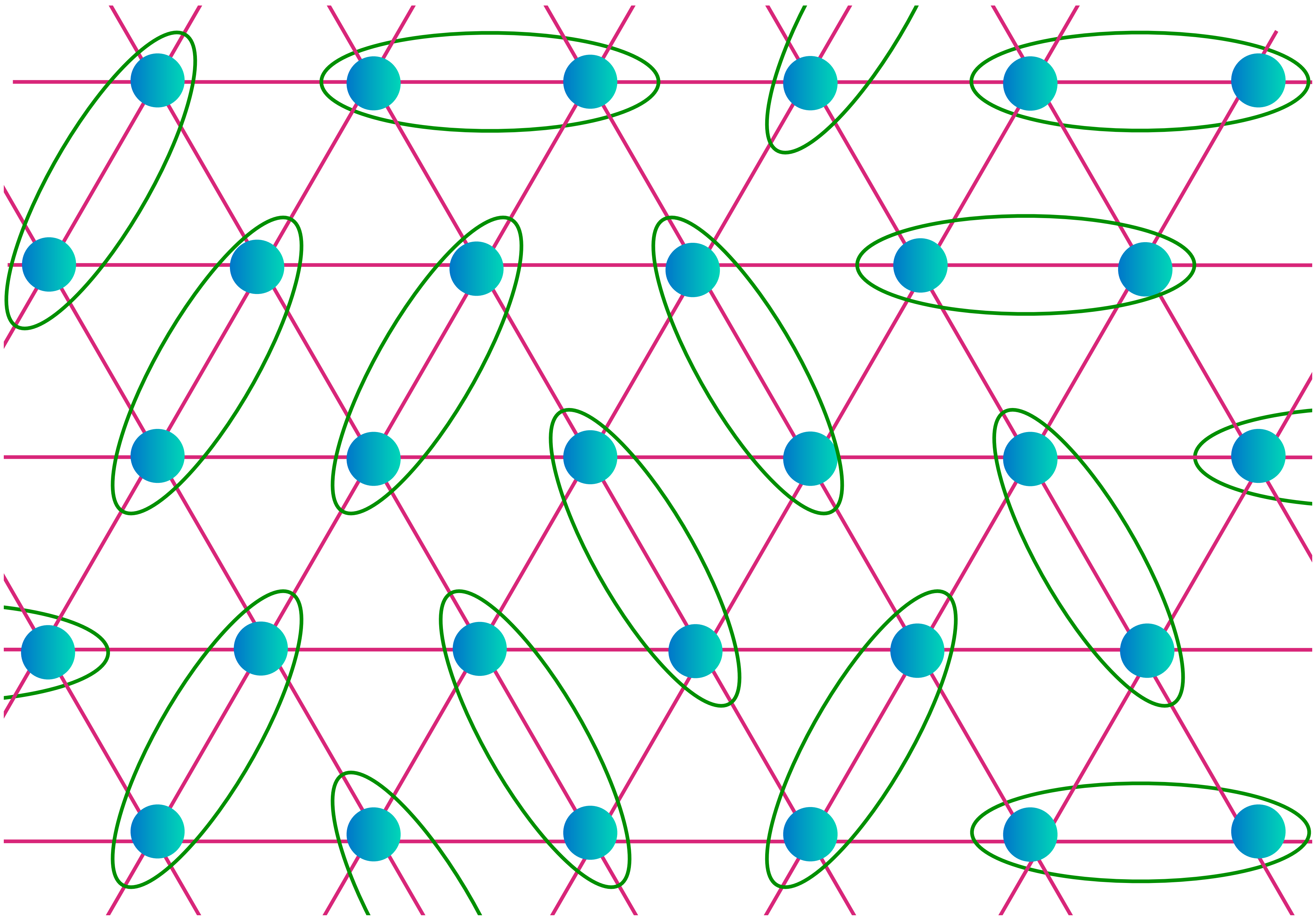

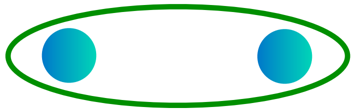

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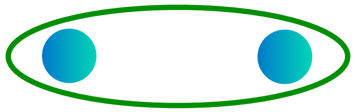
Excitations of the Z_2 Spin liquid

Spinon: S=1/2, charge 0



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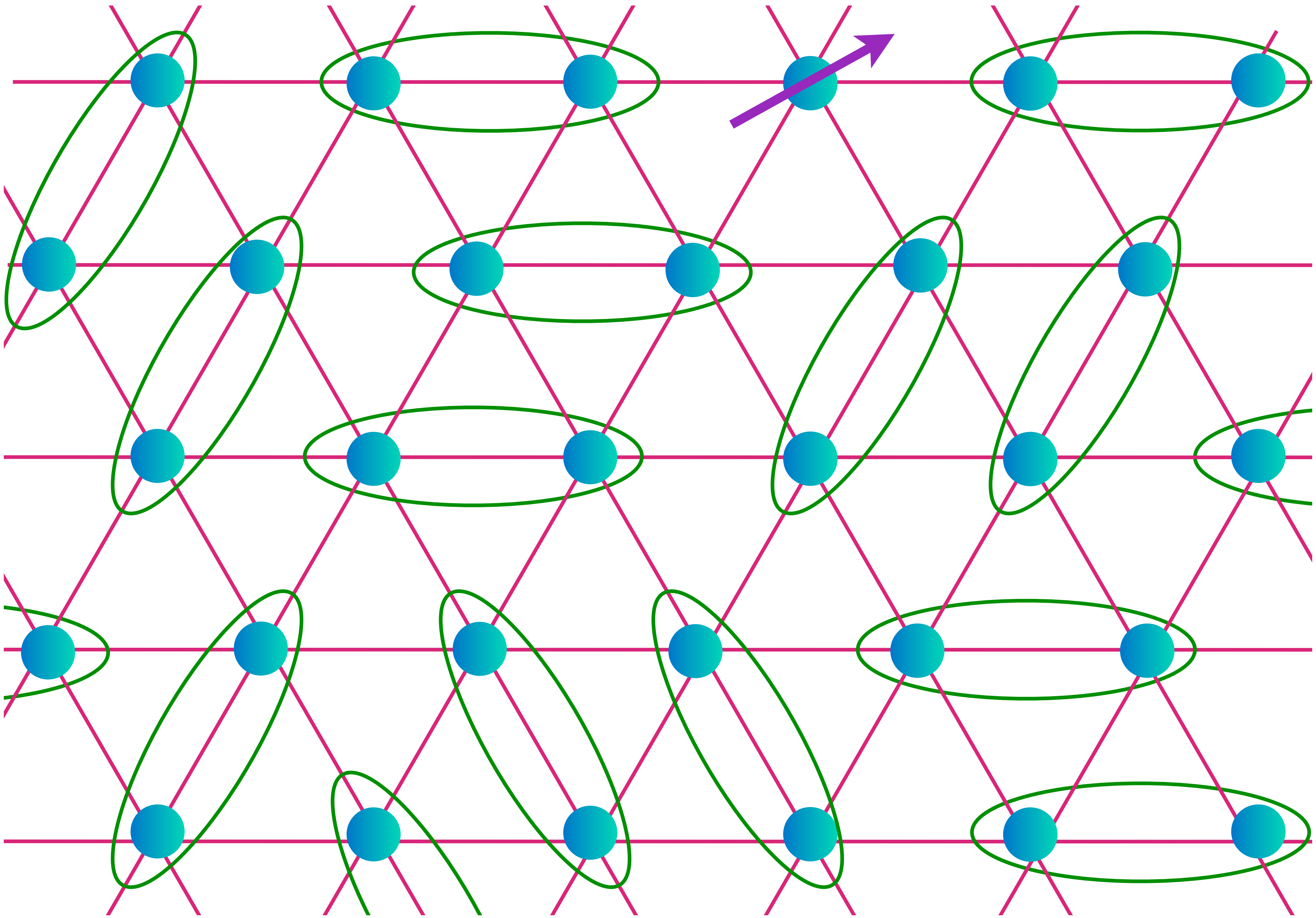

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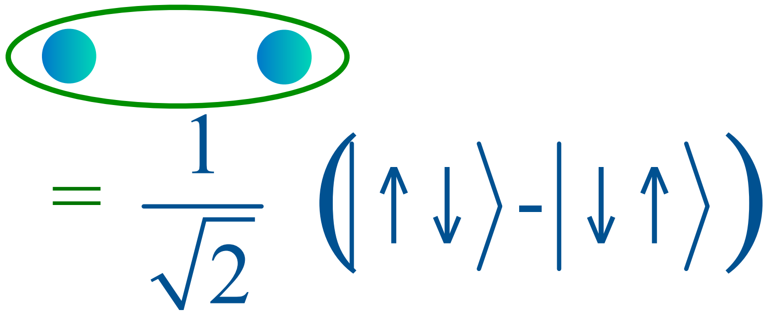

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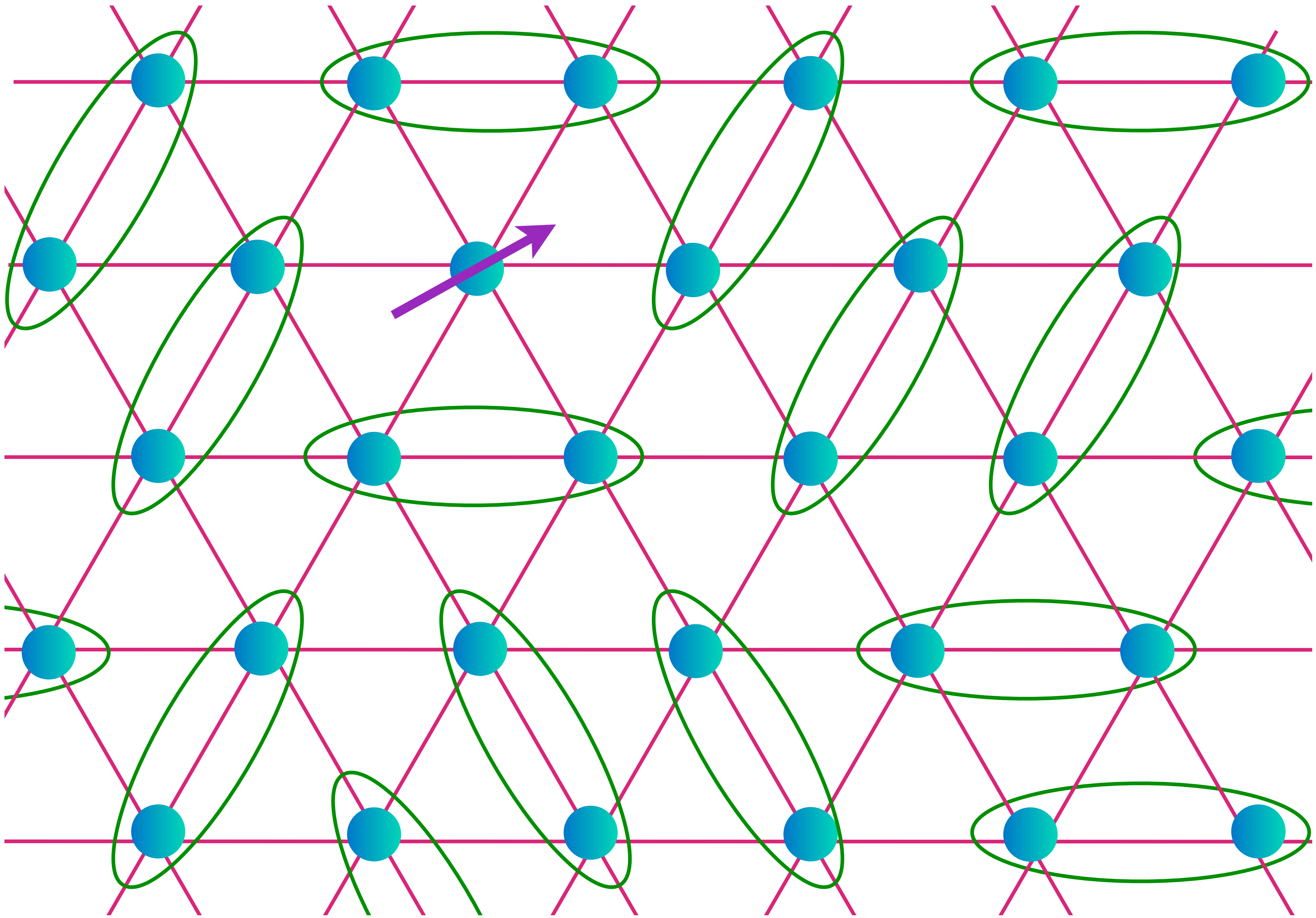

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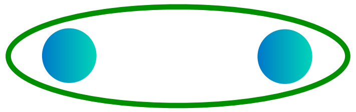

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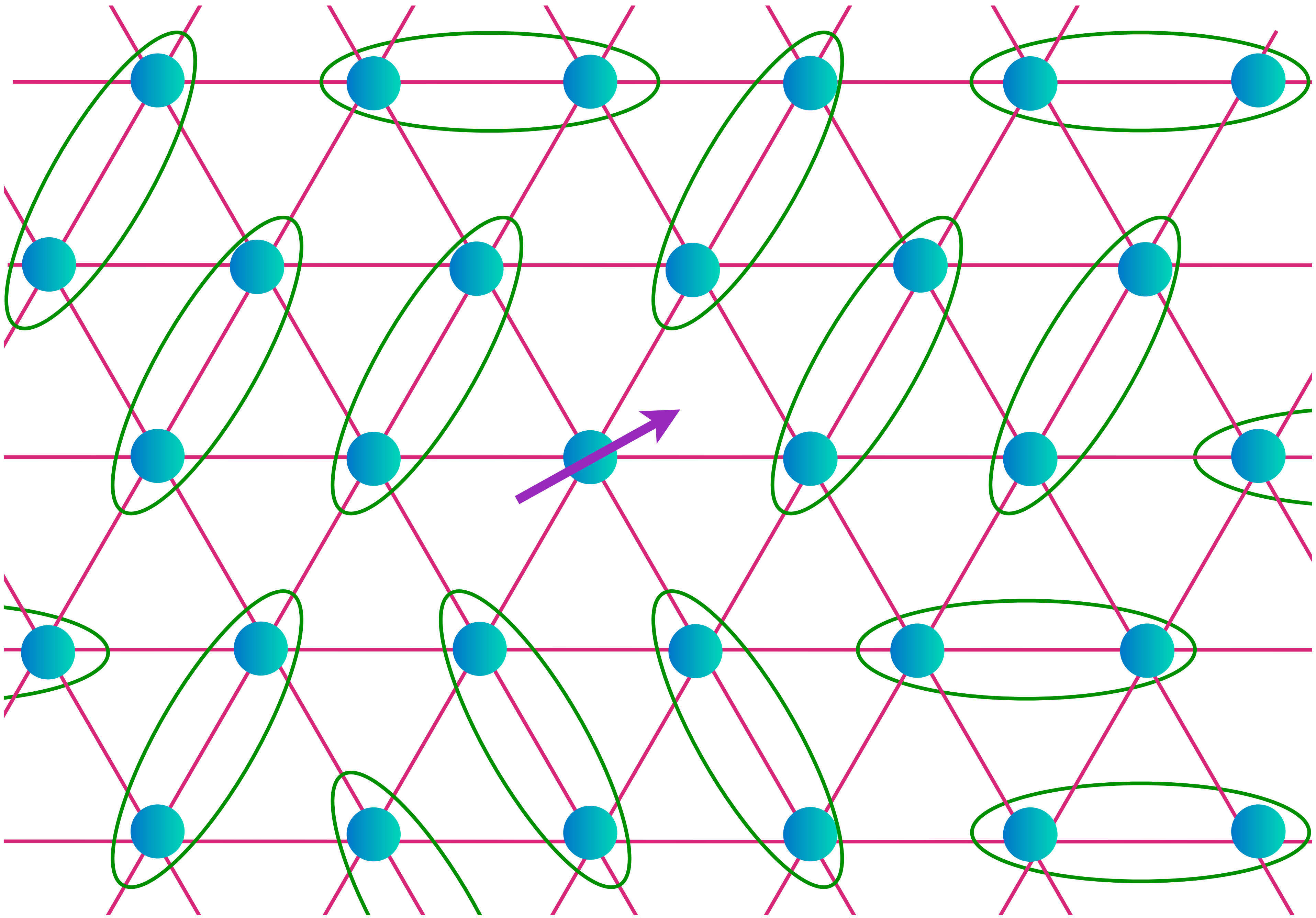

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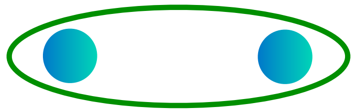

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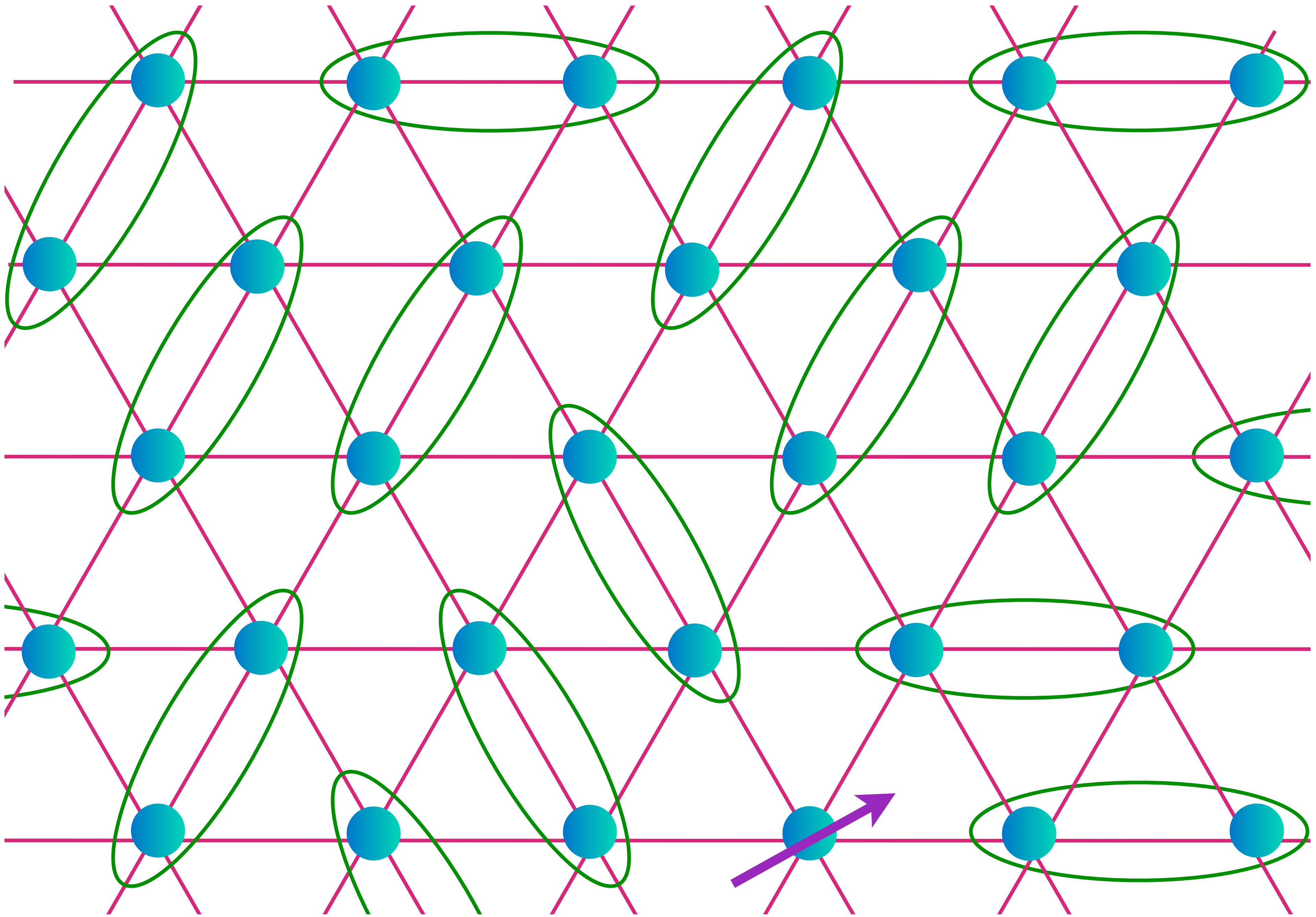
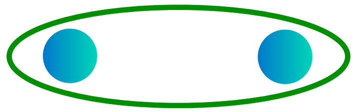


Excitations of the Z_2 Spin liquid

A vison



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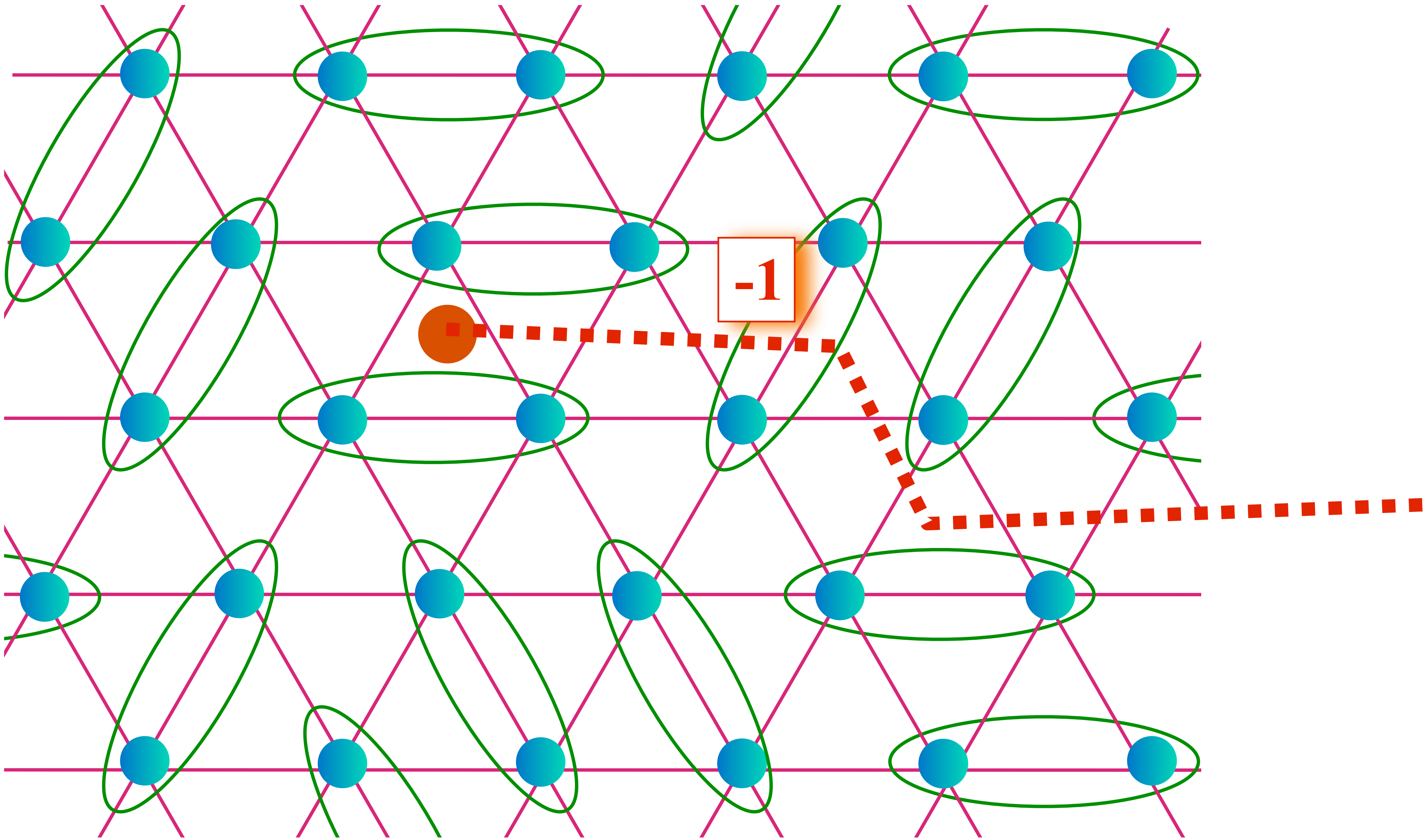

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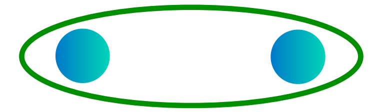

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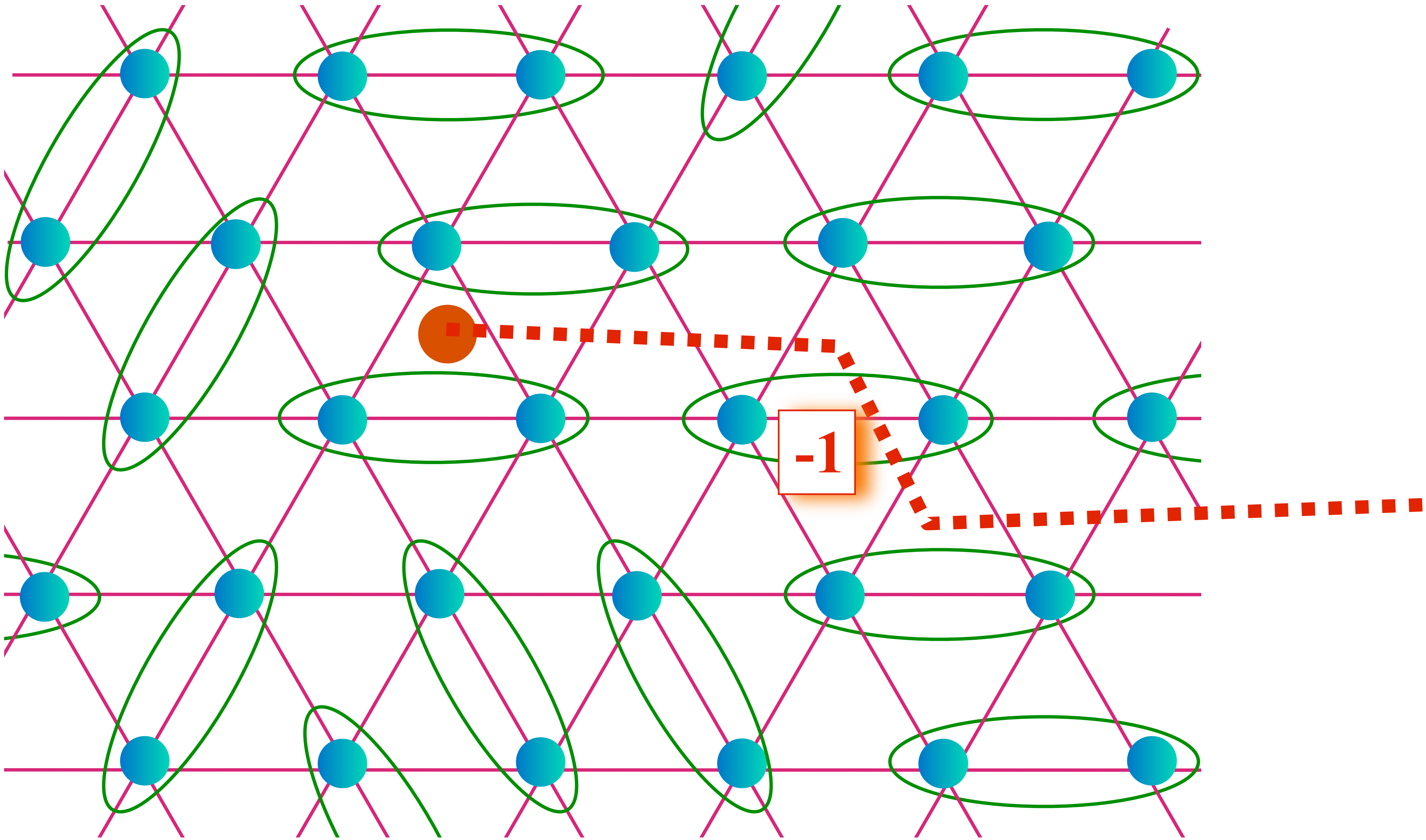

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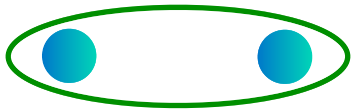

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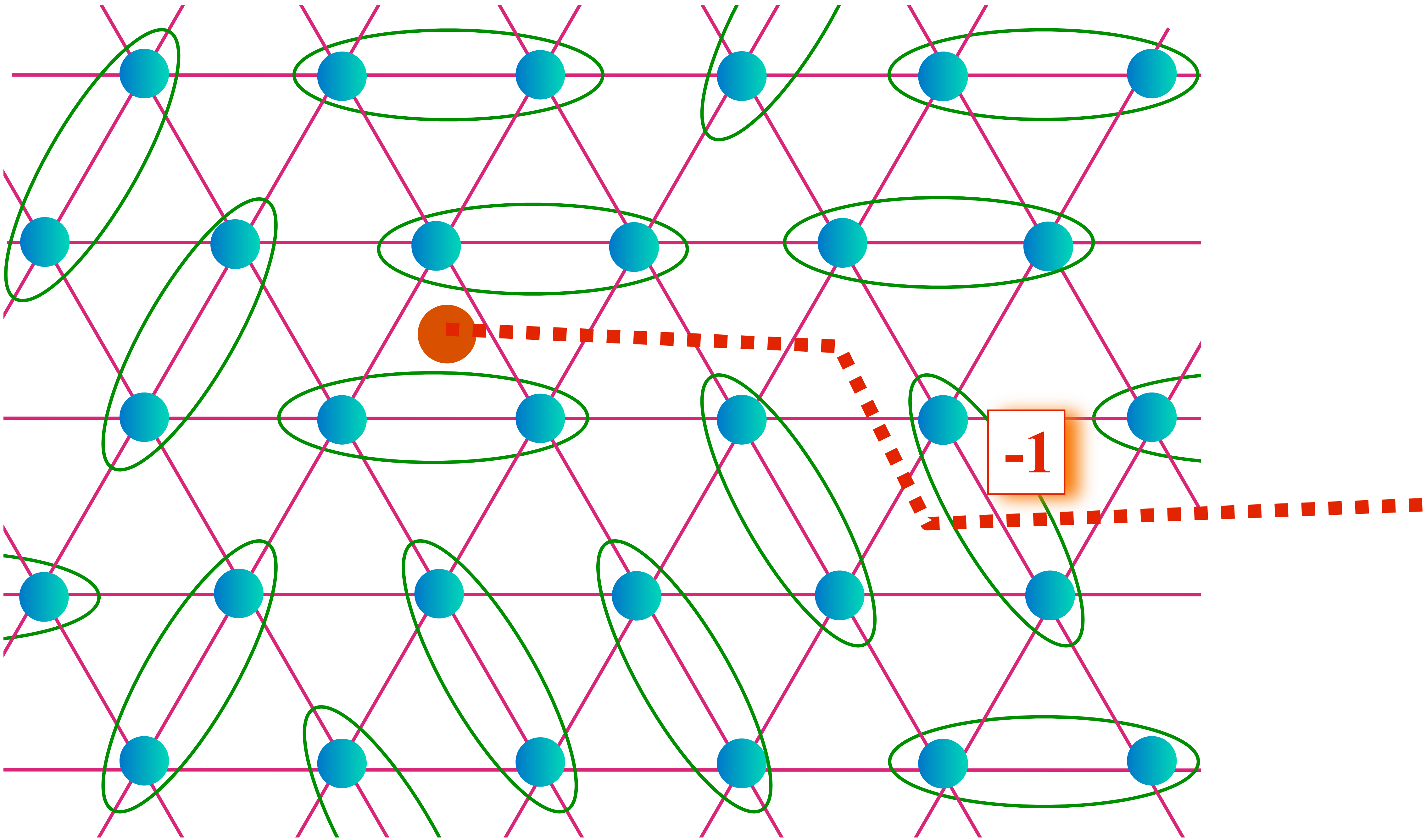

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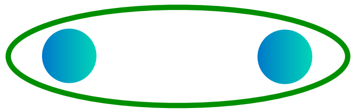

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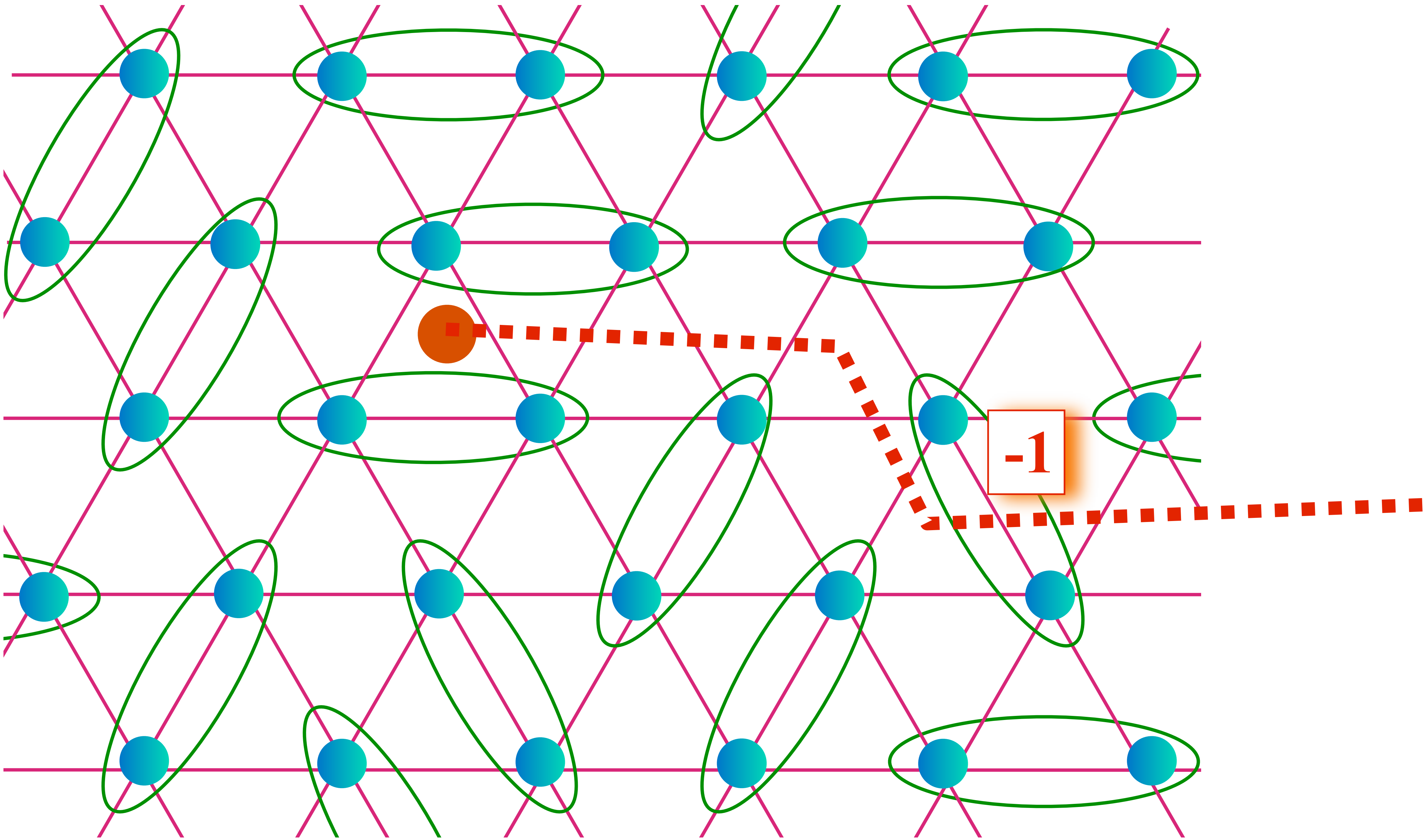

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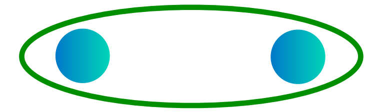

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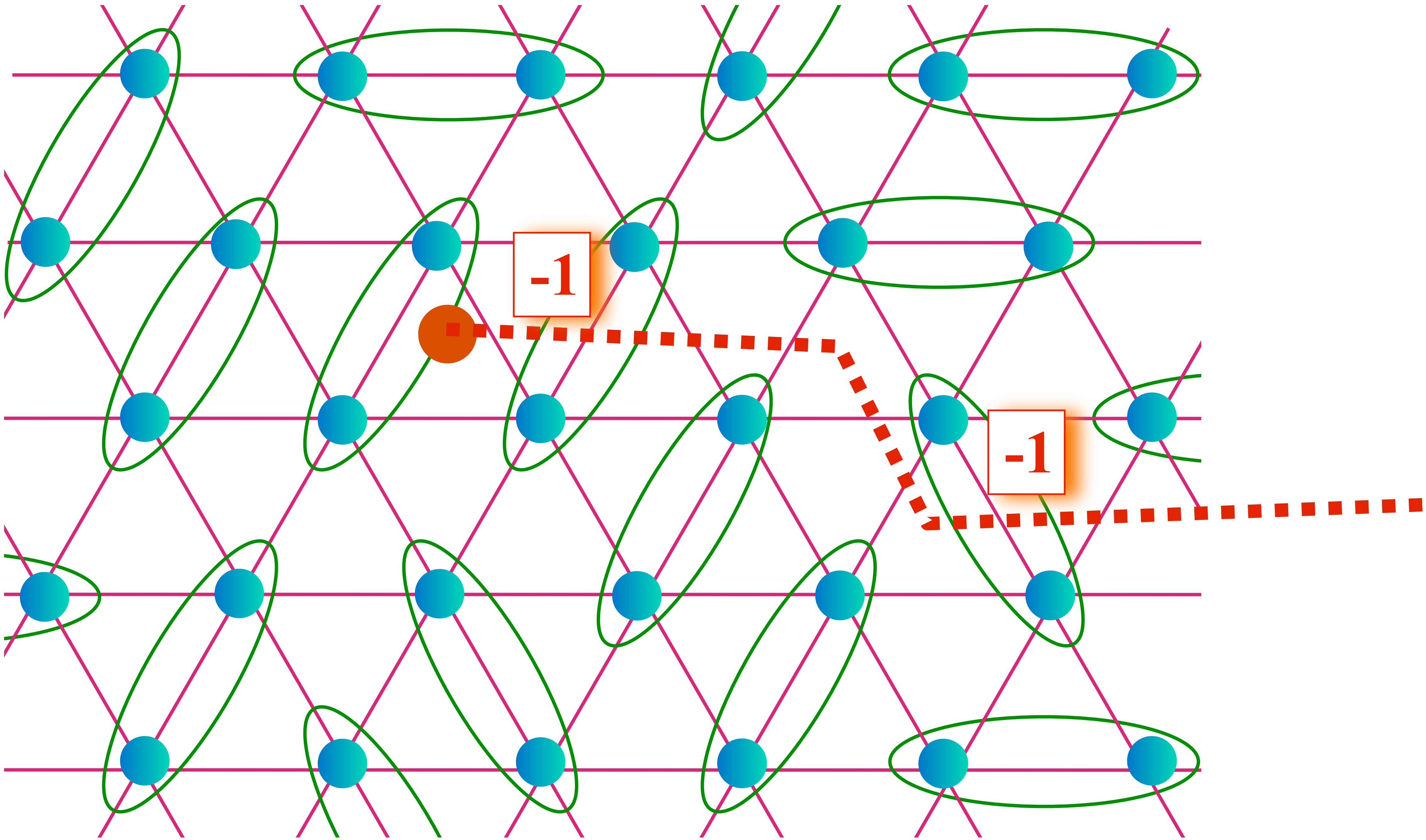

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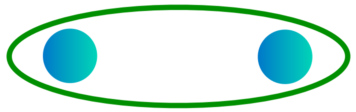

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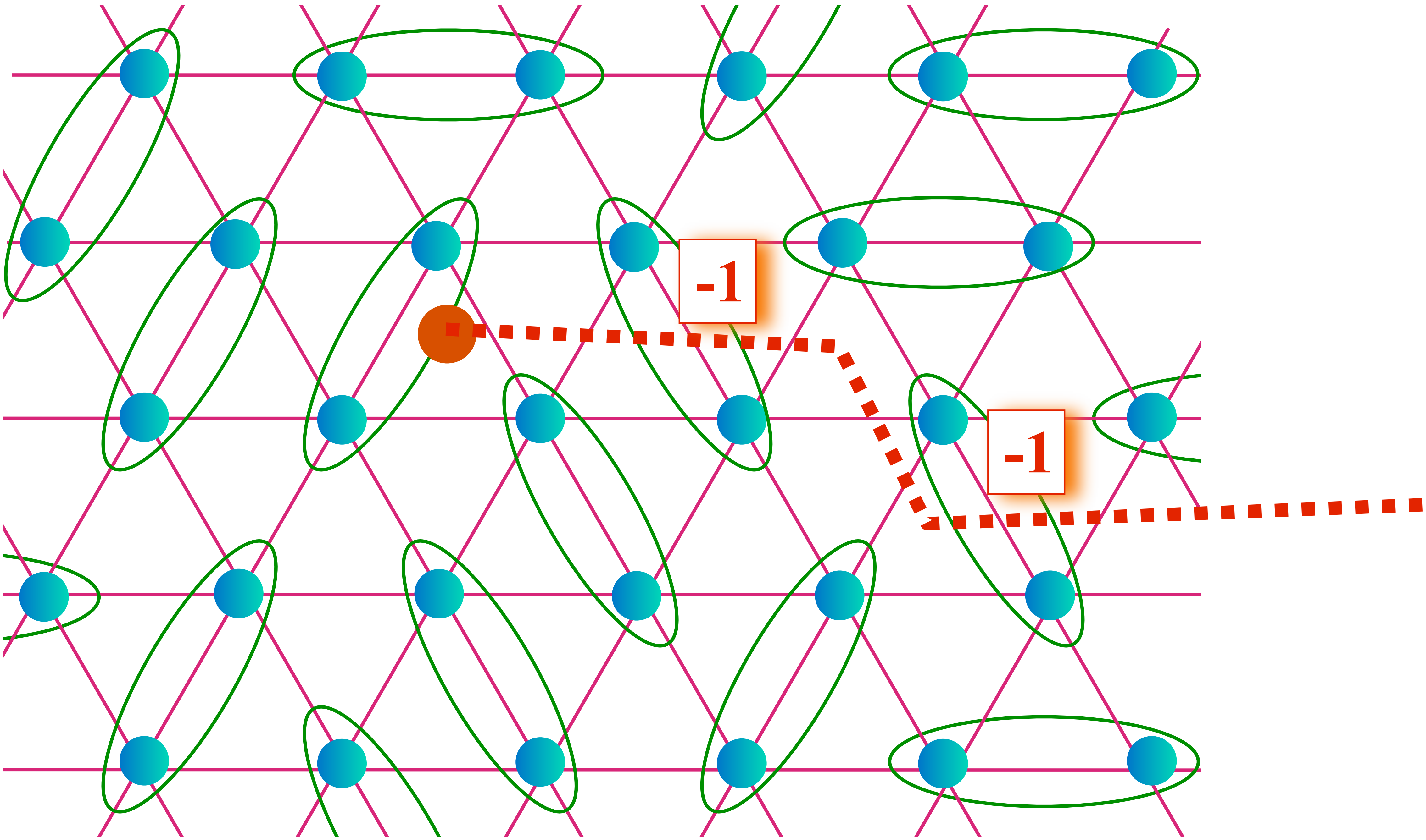

Excitations of the Z_2 Spin liquid

A vison

- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and B. Chakraborty, *Phys. Rev. B* **40,** 7133 (1989)
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

Topological order in the Z_2 spin liquid ground state

4-fold degeneracy on the torus

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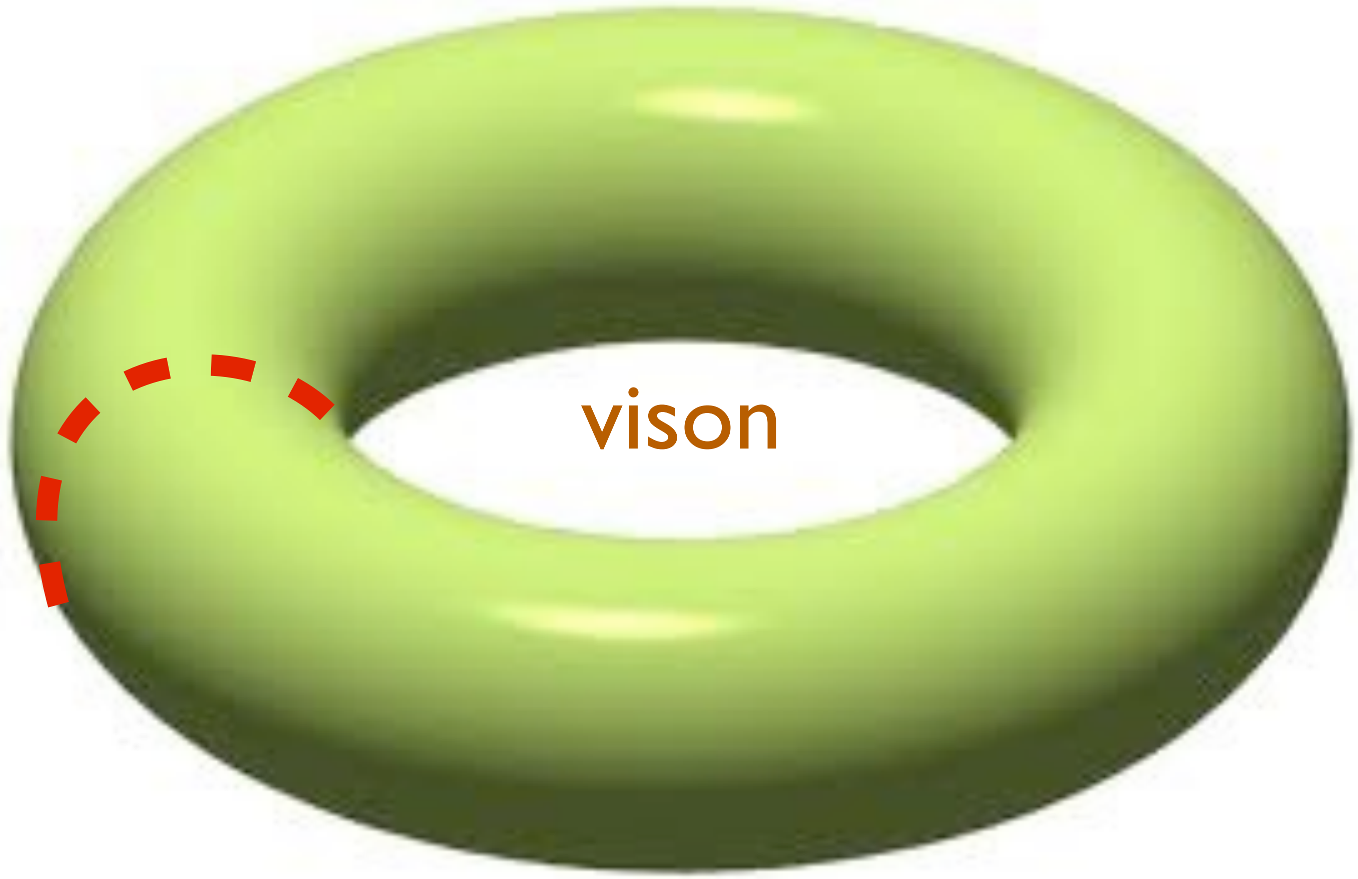

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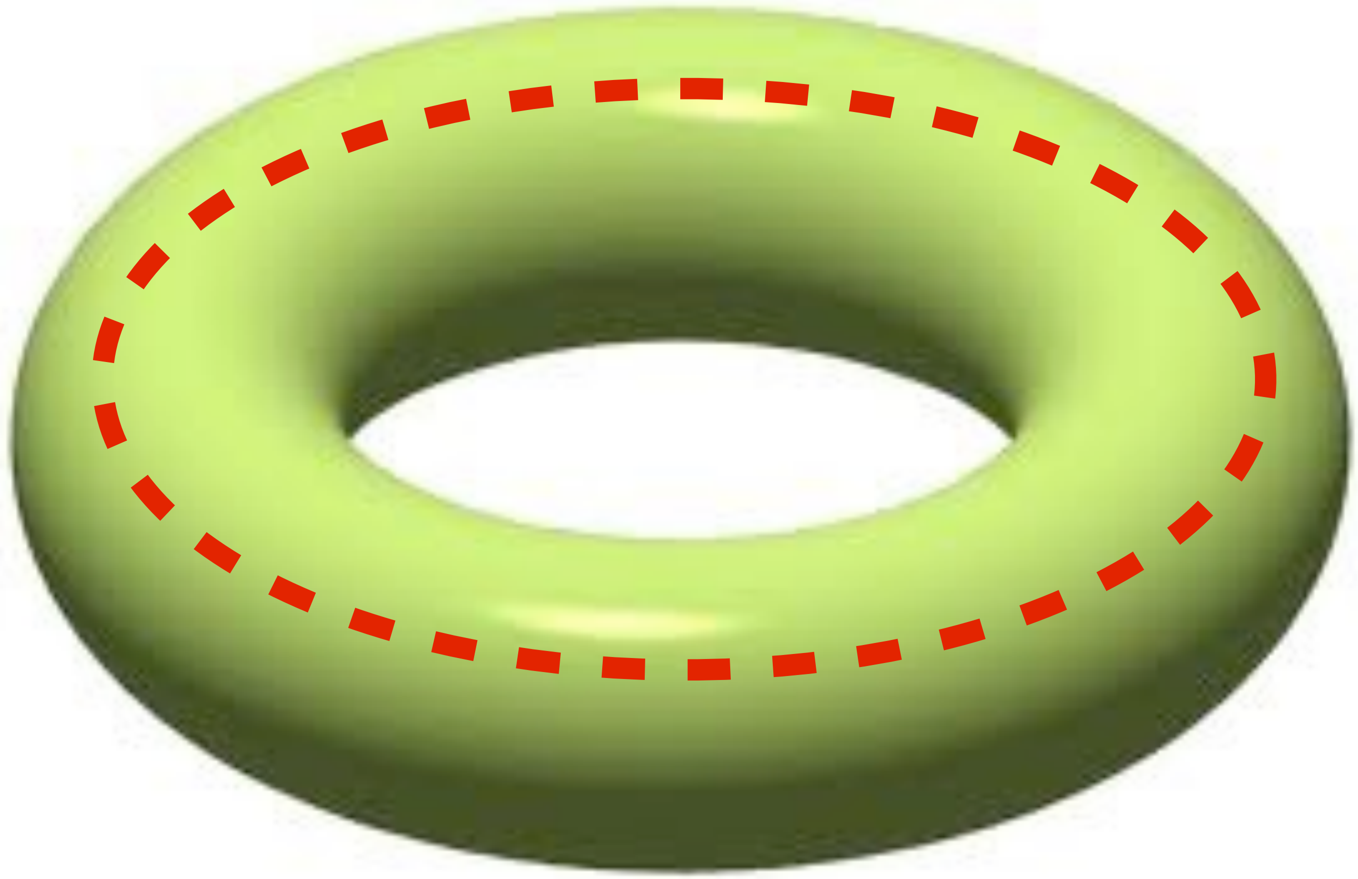

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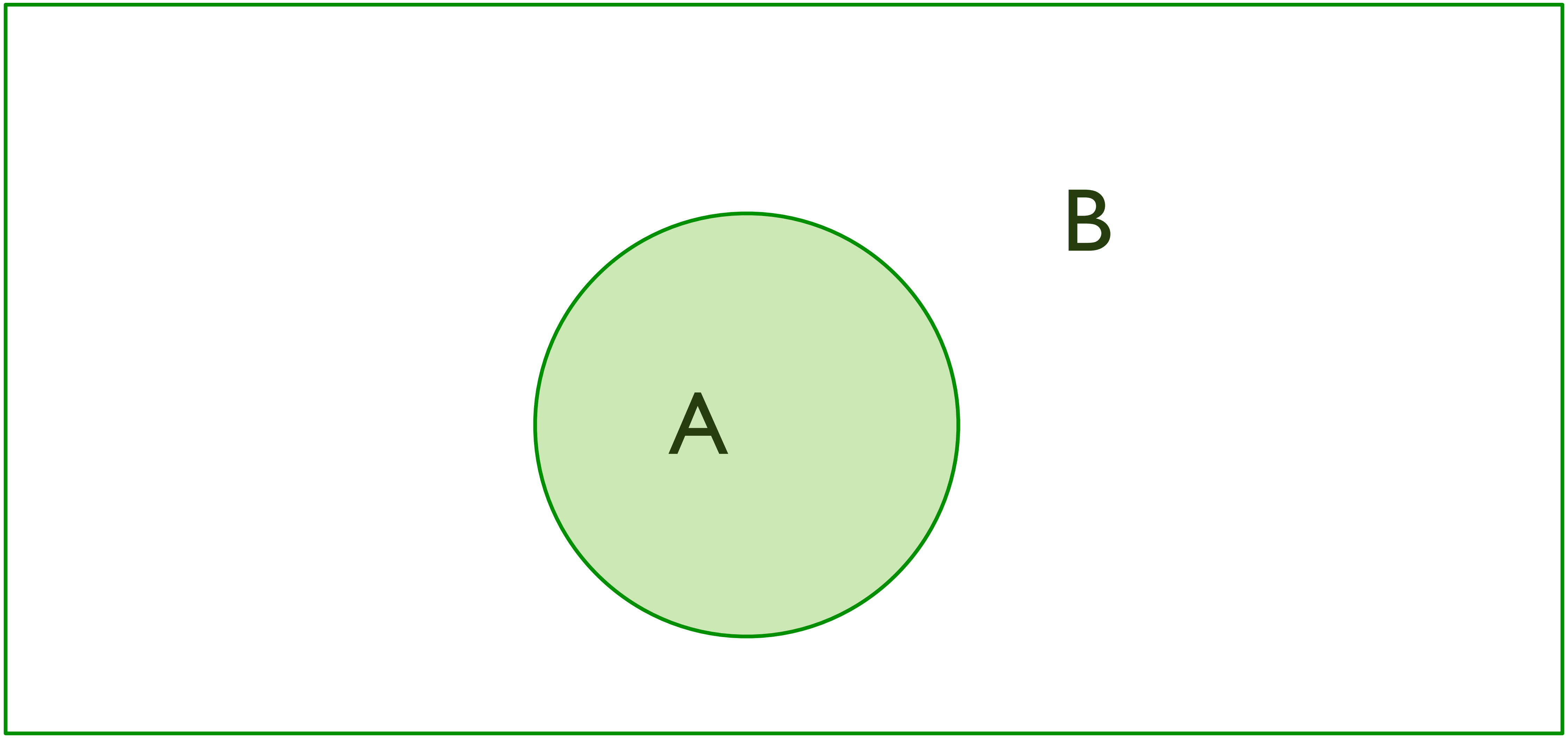
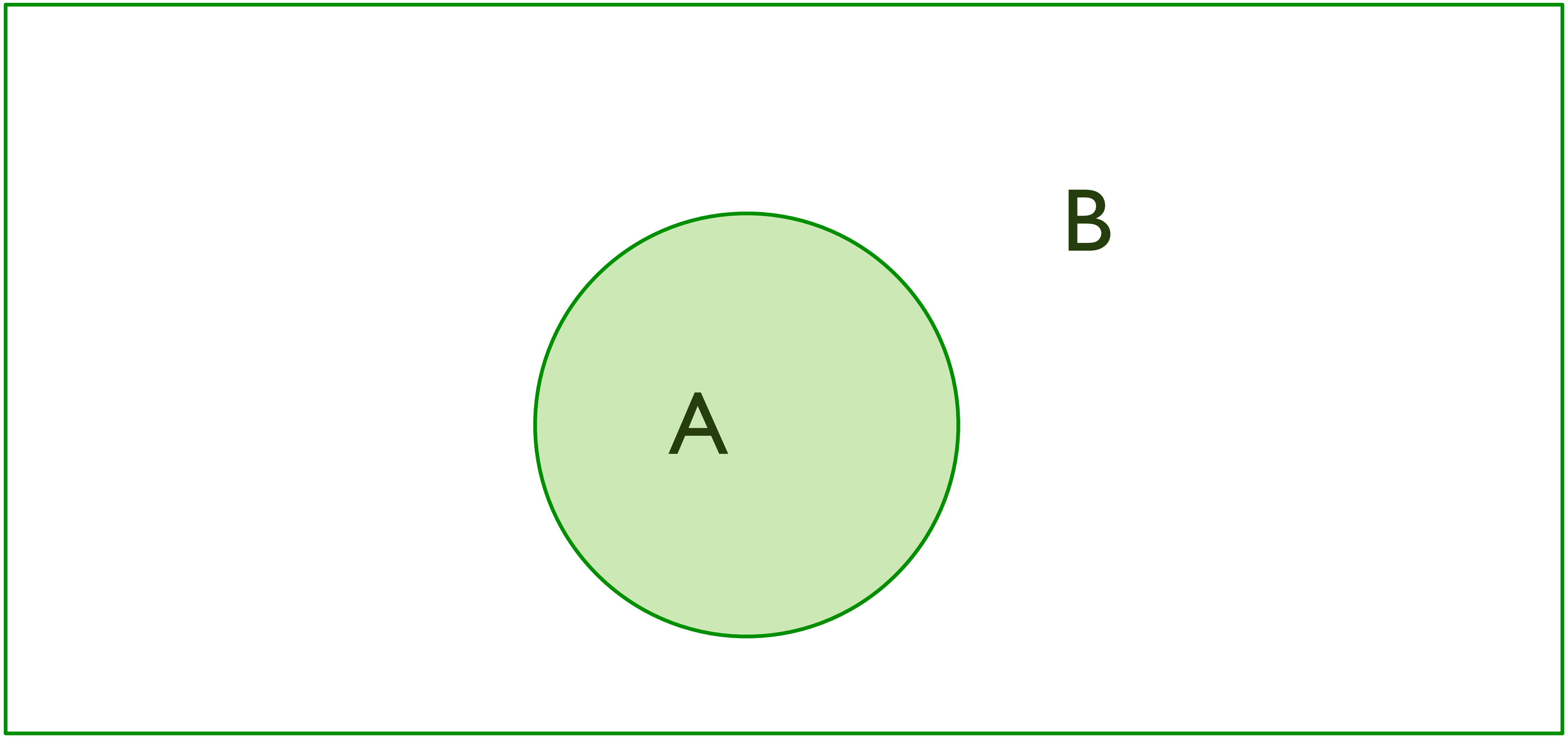


$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Topological order in the Z_2 spin liquid ground state

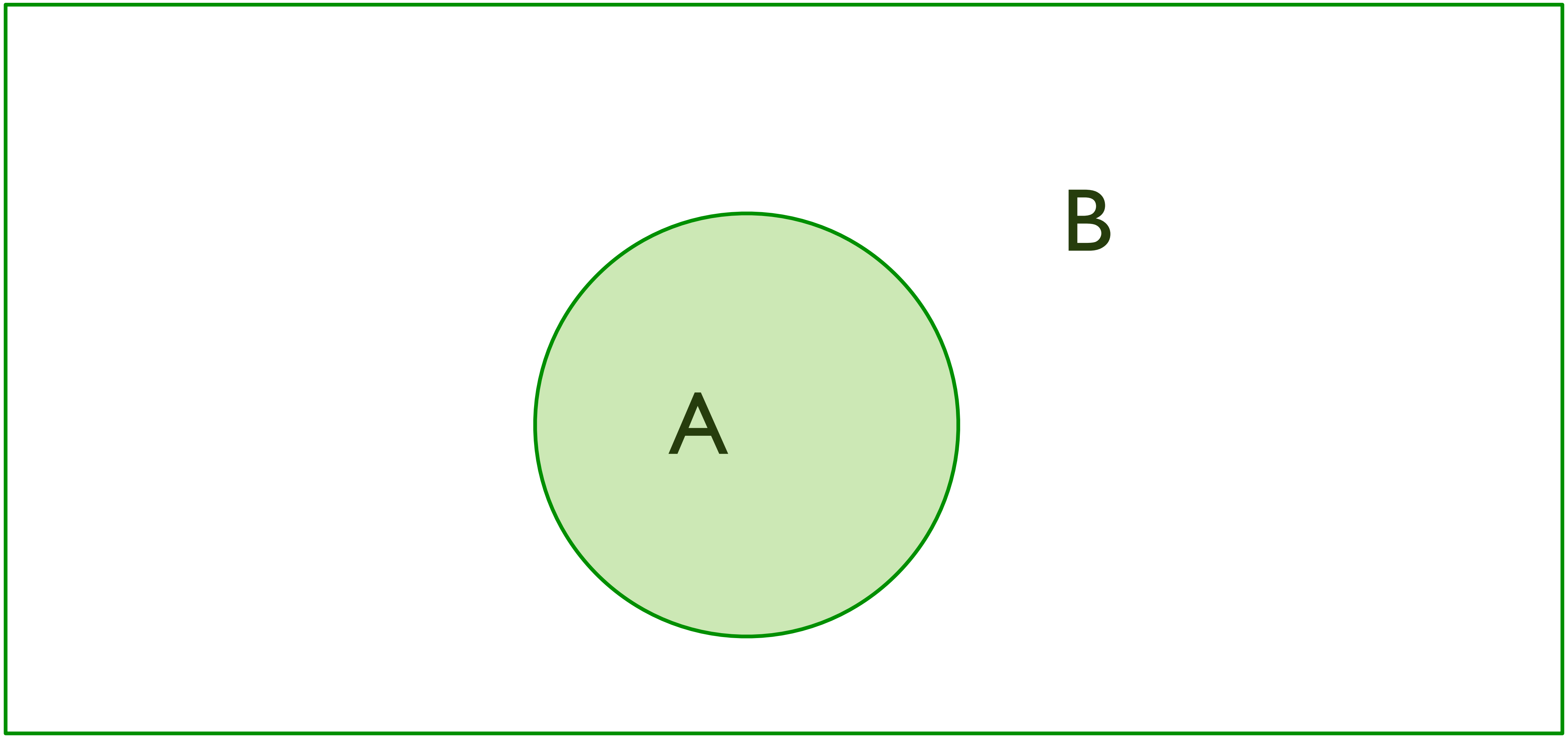


Entanglement entropy of a band insulator:

$$S_{EE} = aL - exp(-bL)$$

where L is the perimeter of the boundary between A and B.

Topological order in the Z_2 spin liquid ground state



Entanglement entropy of a Z_2 spin liquid:

$$S_{EE} = aL - \ln(2)$$

where L is the perimeter of the boundary between A and B.
The $\ln(2)$ is a universal characteristic of the Z_2 spin liquid, and implies *long-range* quantum entanglement.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006); Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Topological order in the Z_2 spin liquid ground state

These properties of the ground state can be described by effective theories:

- deconfined phase of a Z_2 gauge theory

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
F. A. Bais, P. van Driel, and M. de Wild Propitius, `Phys. Lett. B **280**, 63 (1992).
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

- topological doubled Chern-Simons gauge theory

J. Maldacena, G. Moore, and N. Seiberg, JHEP 0110:005 (2001).
M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, *Annals of Physics* **310**, 428 (2004).

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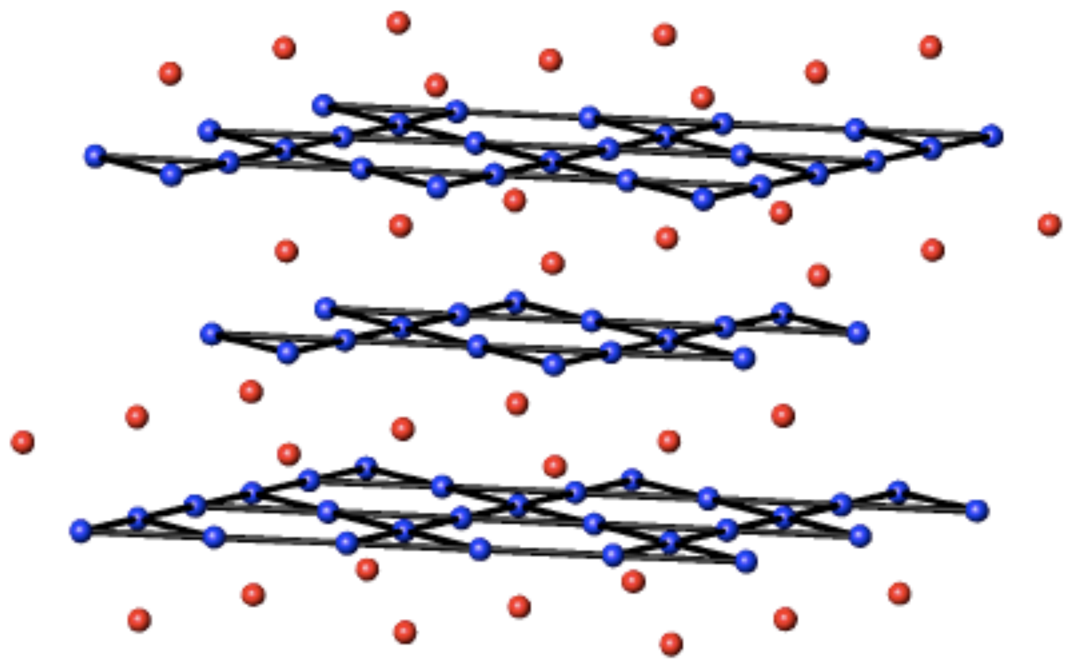
- Good recent numerical evidence of Z_2 spin liquid on kagome and square lattices

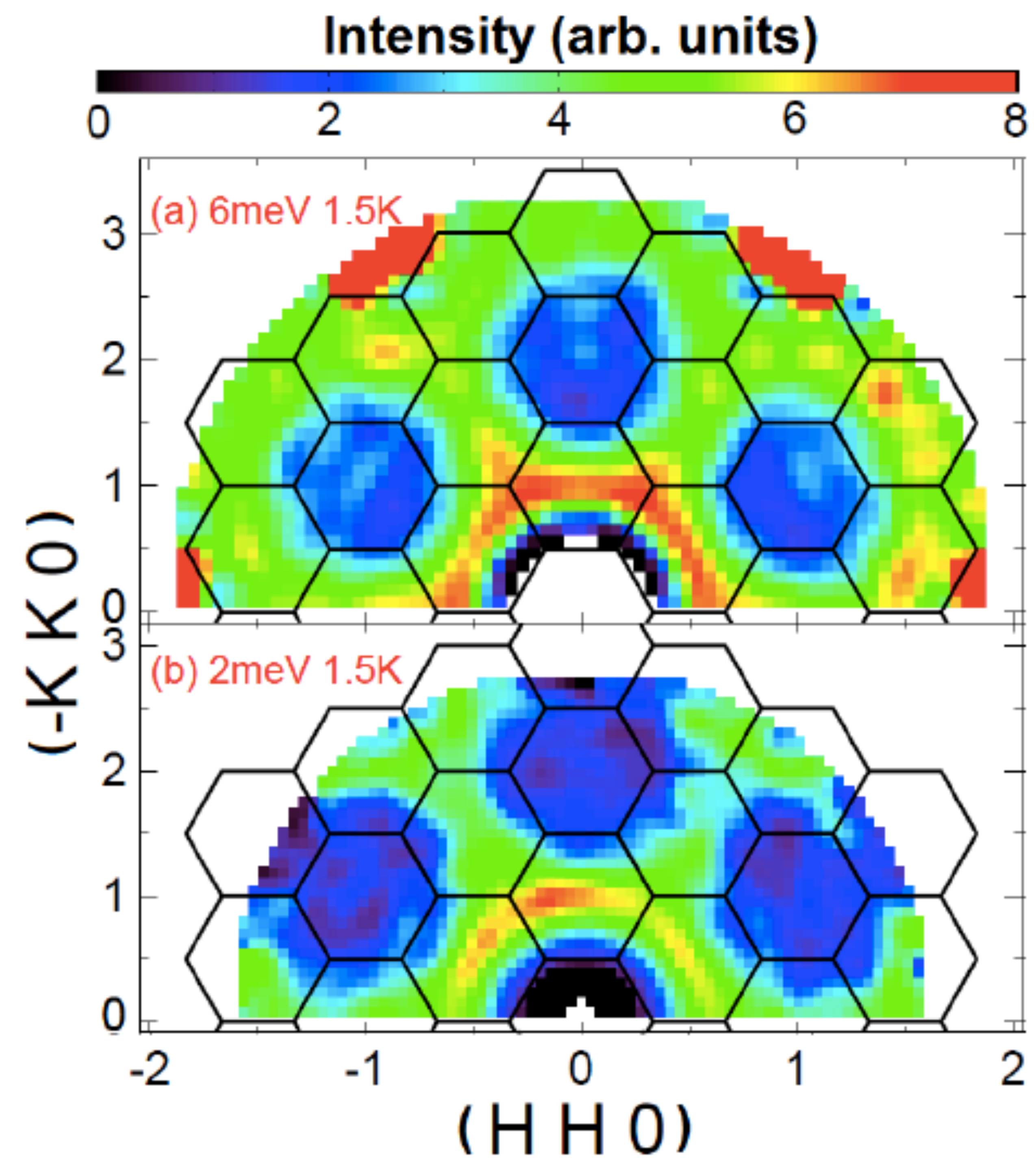
Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).
J. Hong-Chen Jiang, Hong Yao, and L. Balents, arXiv:1112.2241.
Ling Wang, Zheng-Cheng Gu, Xiao-Gang Wen, and F. Verstraete, arXiv:1112.3331

Promising experimental candidate: the kagome antiferromagnet

Young Lee, APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





Quantum Hall states

Similar topological properties,
but no time-reversal symmetry:

- ground state degeneracy on a torus
- universal entanglement entropy
- gapless edge states on spaces with boundaries
(can also happen for some spin liquids)
- topological Chern-Simons gauge theories