

Stringing together the quantum phases of matter

Lorentz Lectures, Leiden
May 7, 14, 21, June 4, 2012

Subir Sachdev

See also lecture at the 2011 Solvay conference,
Theory of the Quantum World, chair D.J. Gross.
100th anniversary of the first Solvay conference,
Radiation and the Quanta, chair H.A. Lorentz.
[arXiv:1203.4565](https://arxiv.org/abs/1203.4565)

Talk online at sachdev.physics.harvard.edu





Wednesday, June 6, 2012

**Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states**

Modern phases of quantum matter
Not adiabatically connected
to independent electron states:

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

*many-particle, long-range
quantum entanglement*

States of quantum matter with long-range entanglement in d spatial dimensions

Useful classification is provided by nature of excitations with vanishing energy:

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I. Gapped systems without zero energy excitations

States of quantum matter with long-range entanglement in d spatial dimensions

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1. Gapped systems without zero energy excitations

2. “Relativistic” systems with zero energy excitations at isolated points in momentum space

States of quantum matter with long-range entanglement in d spatial dimensions

Useful classification is provided by nature of excitations with vanishing energy:

1. Gapped systems without zero energy excitations

2. “Relativistic” systems with zero energy excitations at isolated points in momentum space

3. “Compressible” systems with zero energy excitations on $d-1$ dimensional surfaces in momentum space.

States of quantum matter with long-range entanglement in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

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Conformal quantum matter

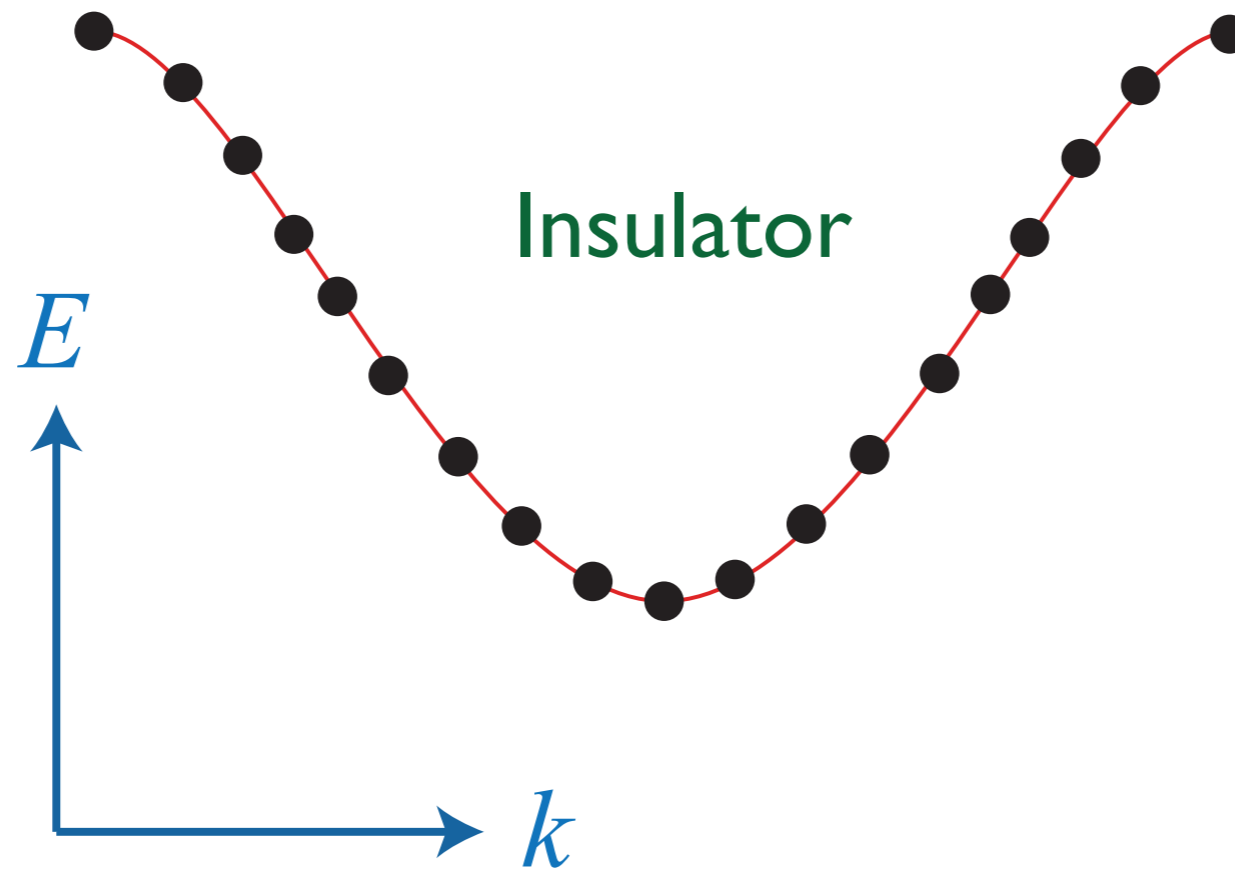
Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

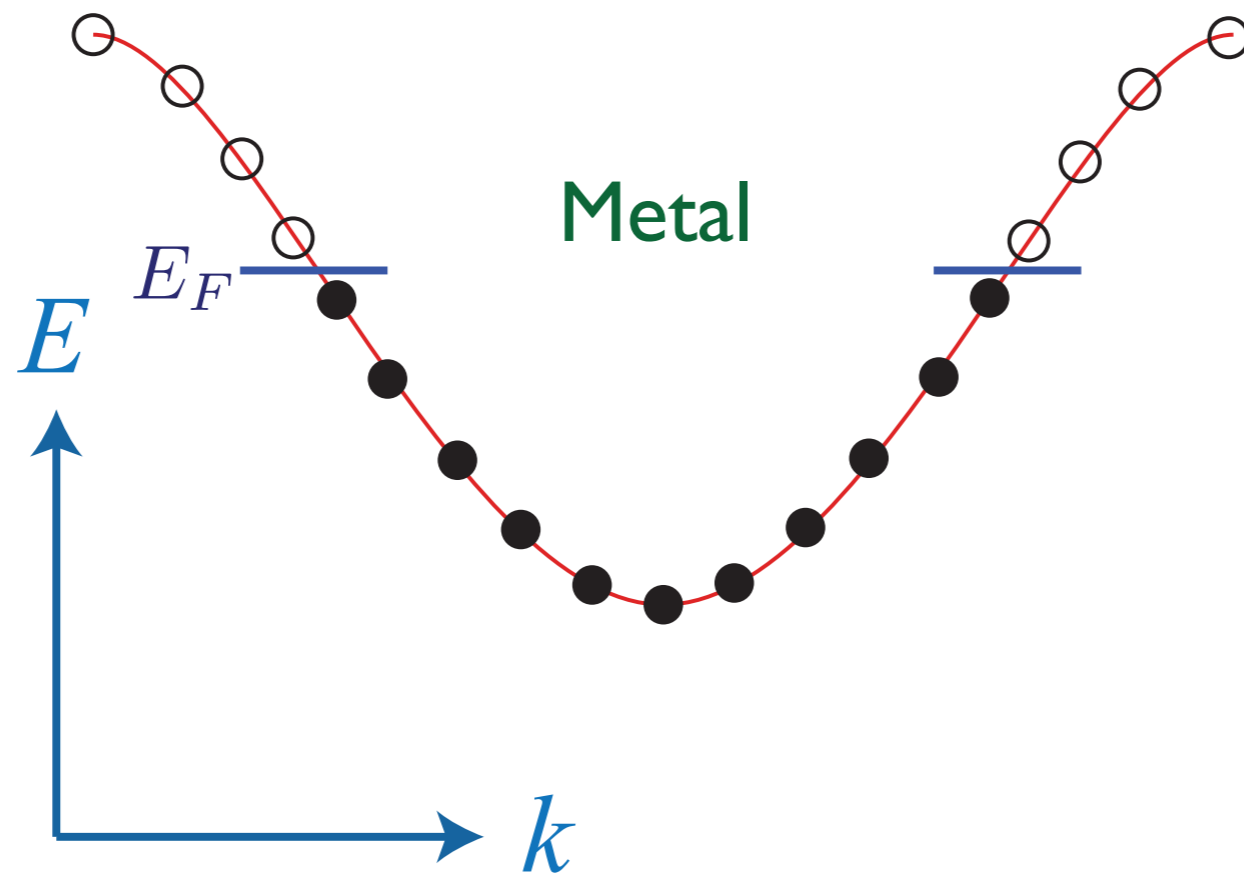
Gapped quantum matter

Band insulators



An even number of electrons per unit cell

Metals



An odd number of electrons per unit cell

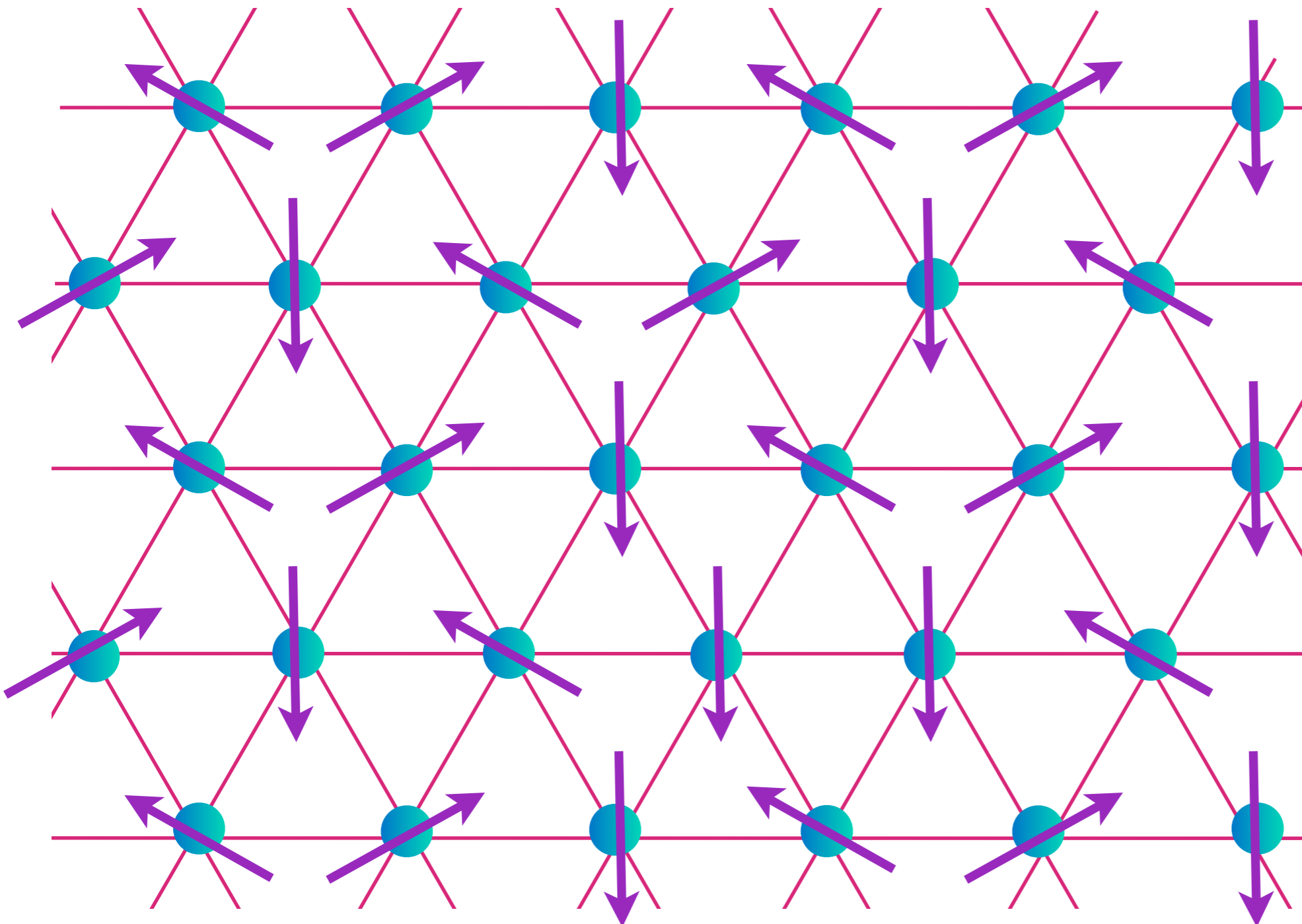
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Imagine quantum fluctuations are so strong that the Neel order does not have long-range correlations.

Naive “classical” picture: we obtain a quantum *disordered* state in which all spin-spin correlations decay exponentially over a short length scale.

Mott insulator: Triangular lattice antiferromagnet

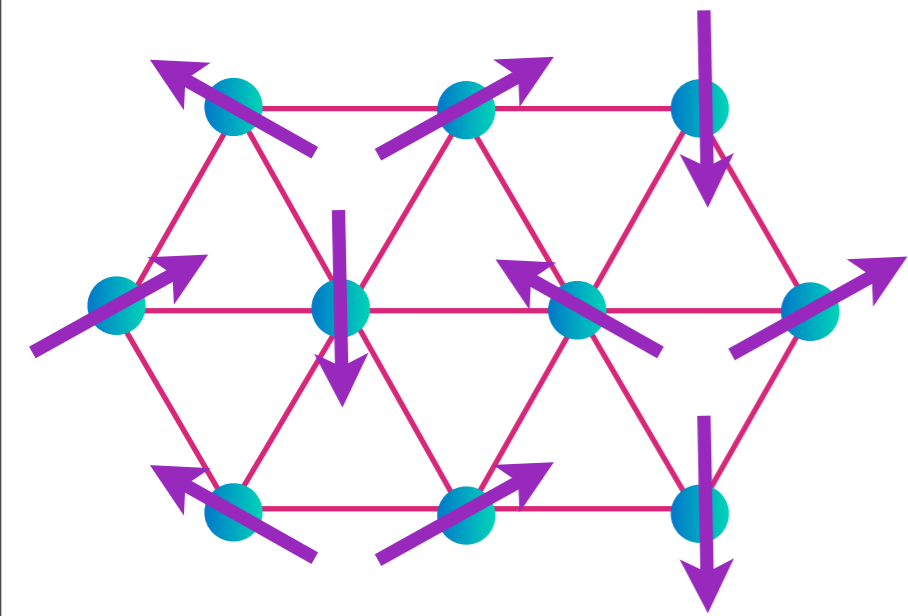
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Naive “classical” picture: we obtain a quantum *disordered* state in which all spin-spin correlations decay exponentially over a short length scale.

Modern “quantum” understanding: the discrete quantum degrees of freedom require a state with long-range entanglement.

Mott insulator: Triangular lattice antiferromagnet



non-collinear Néel state

Z_2 spin liquid
with neutral $S = 1/2$ spinons
and **vison** excitations

S_c

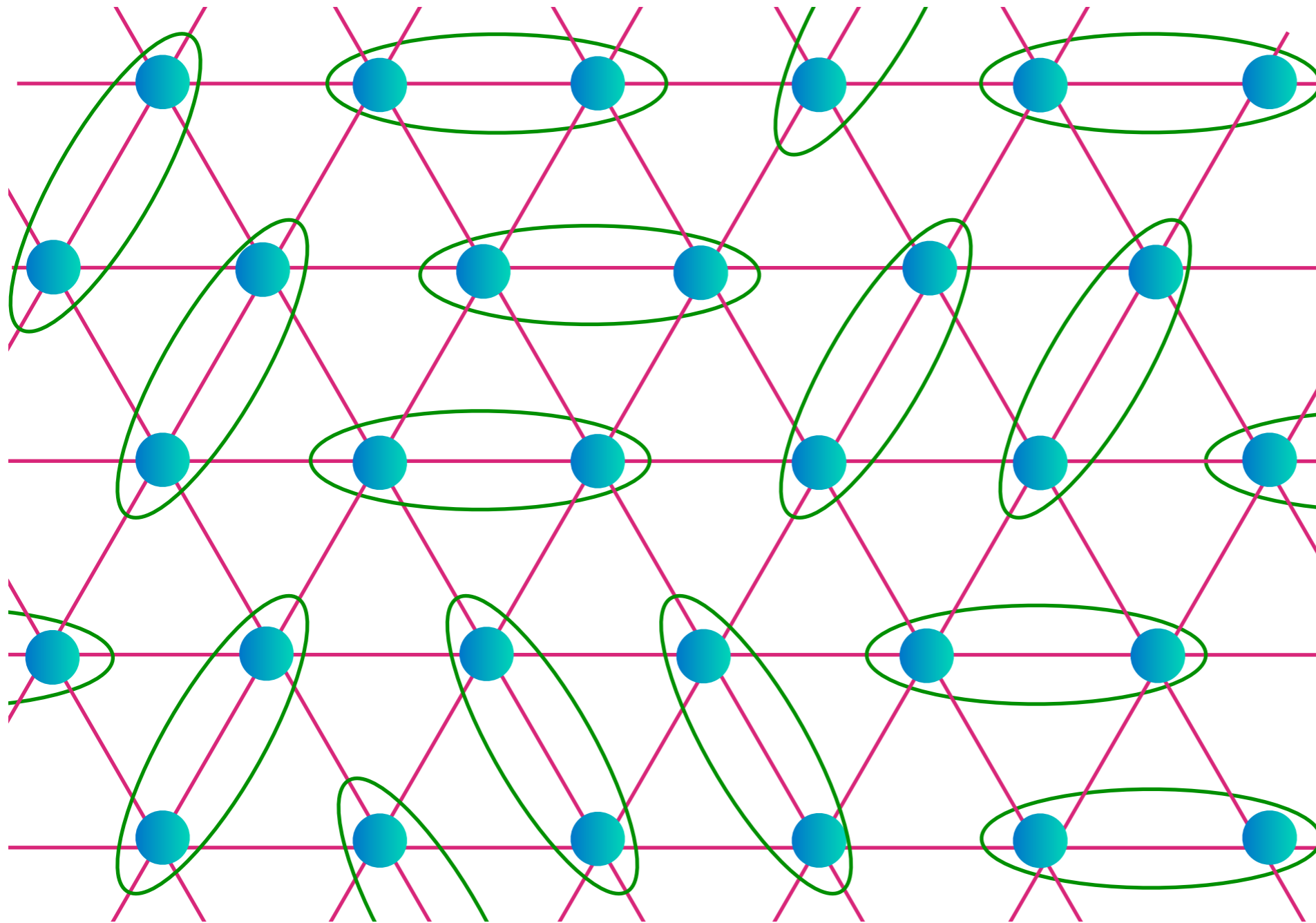
S

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell

$$\begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ \text{---} \end{array} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

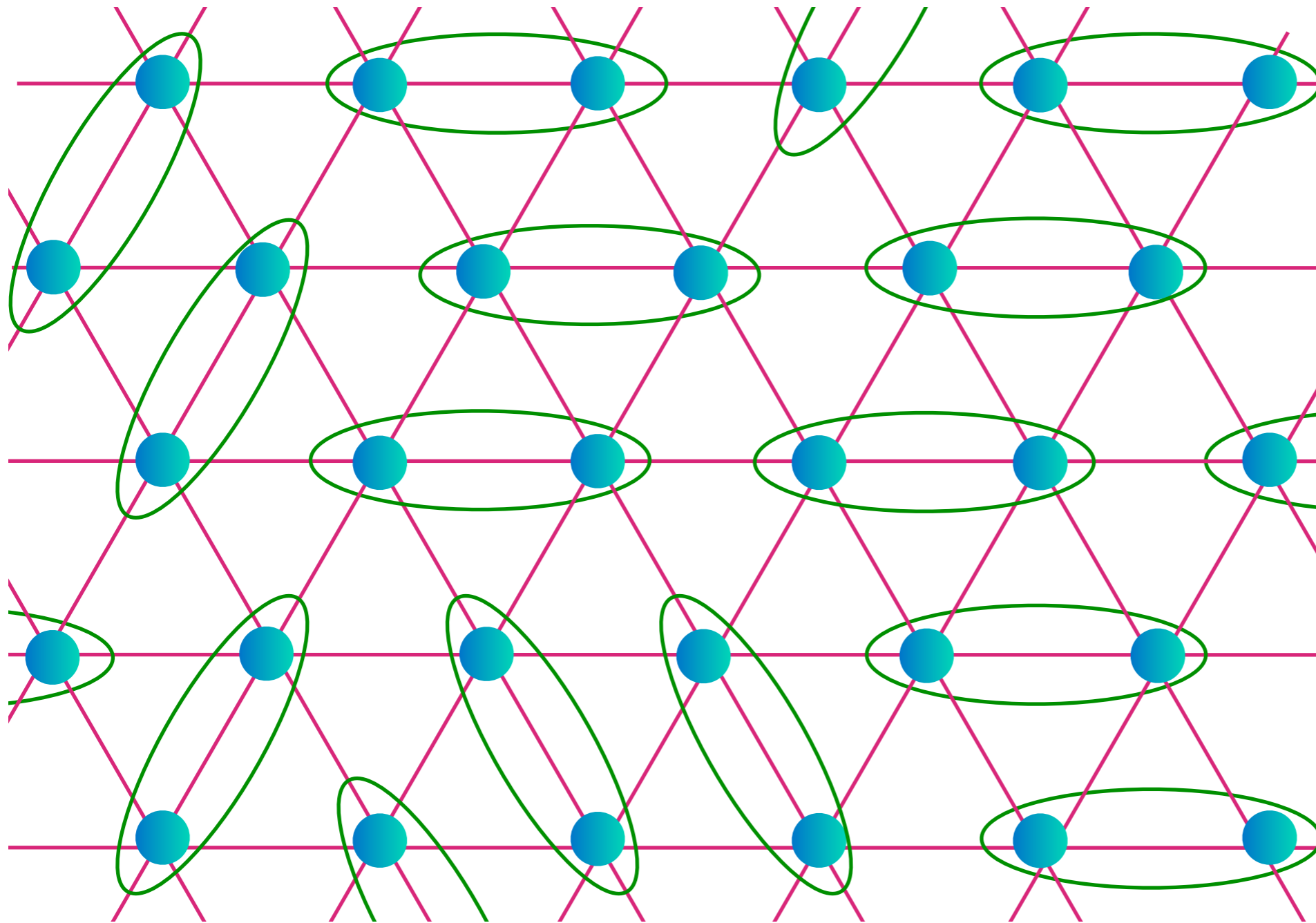


P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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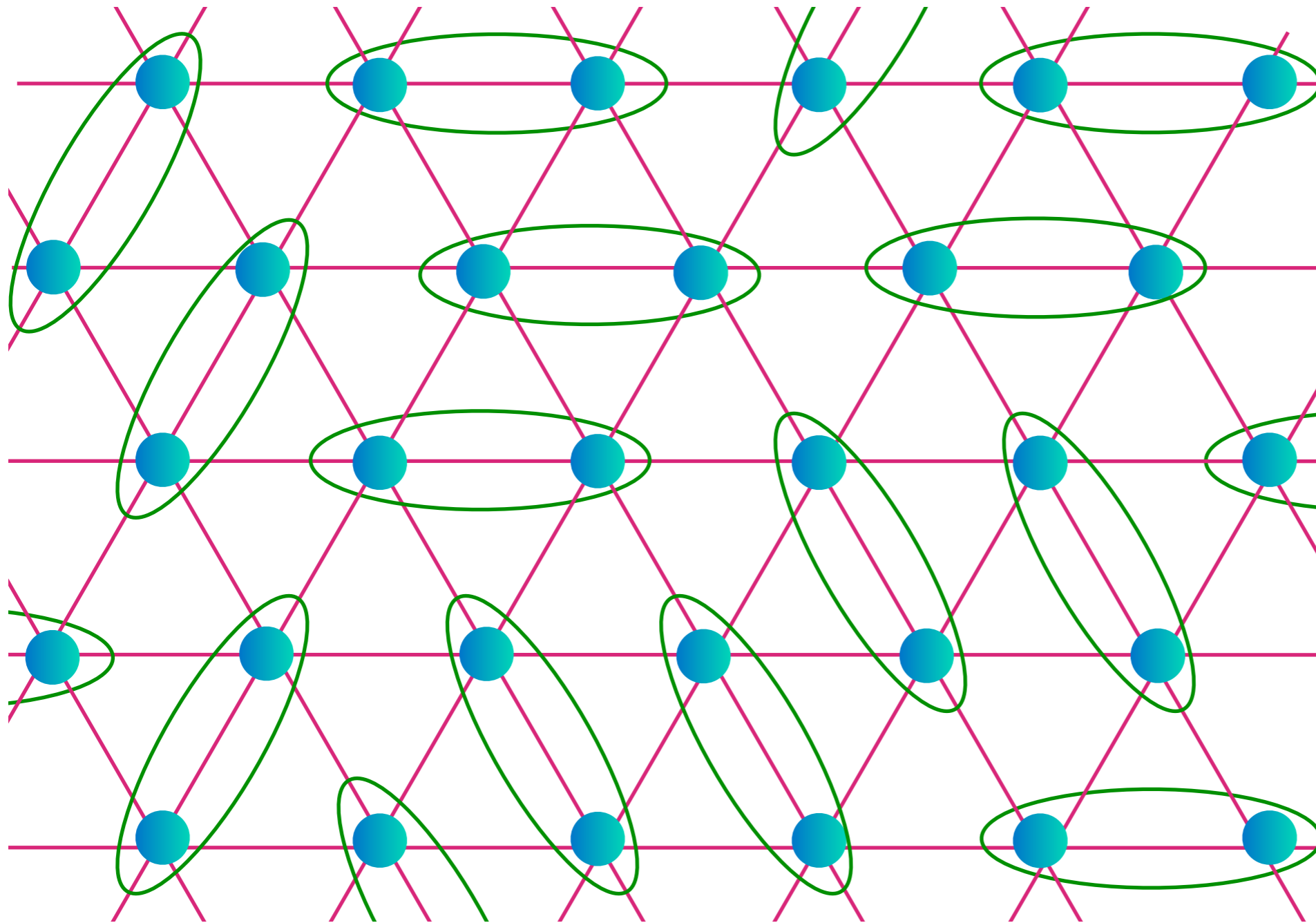


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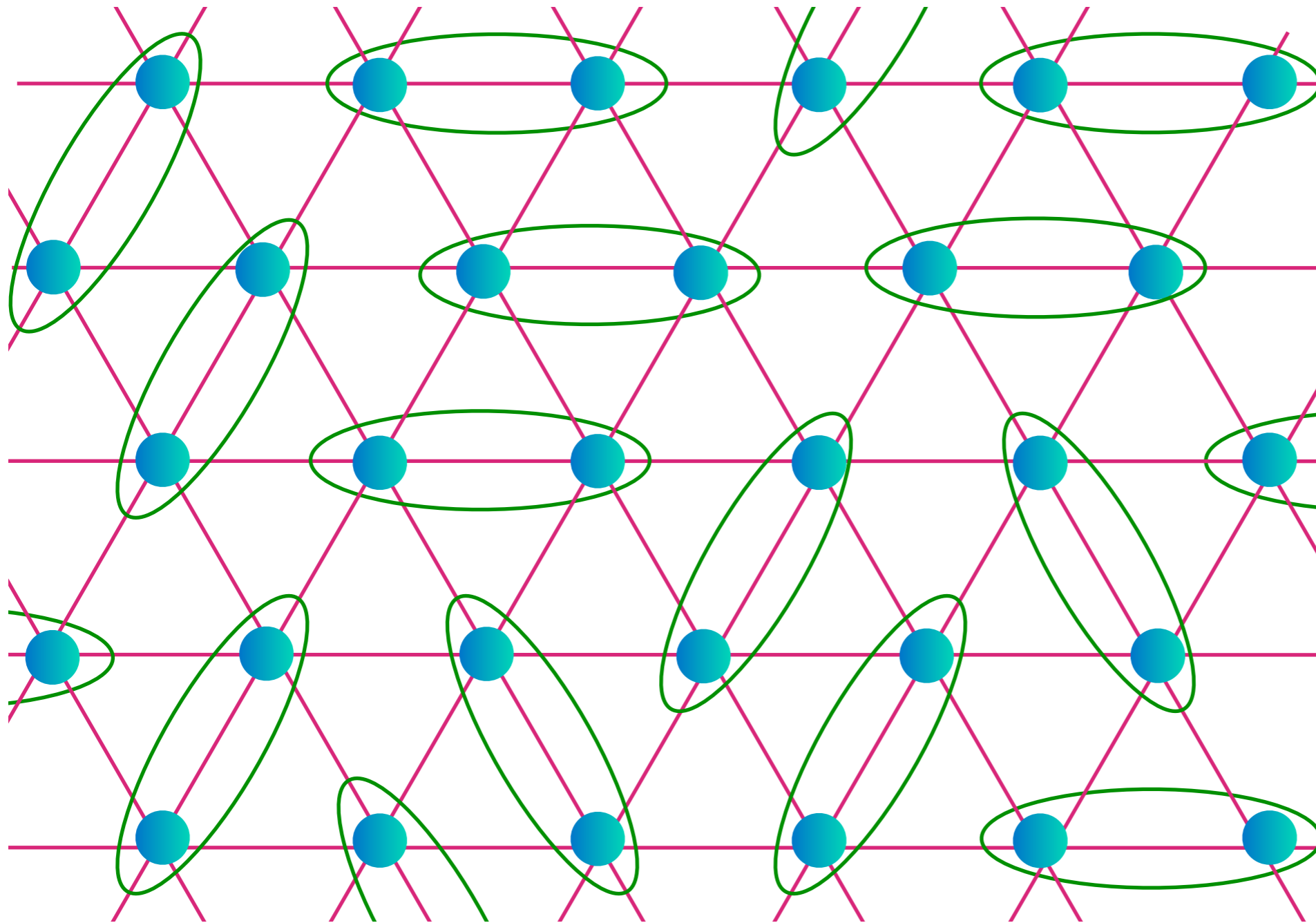


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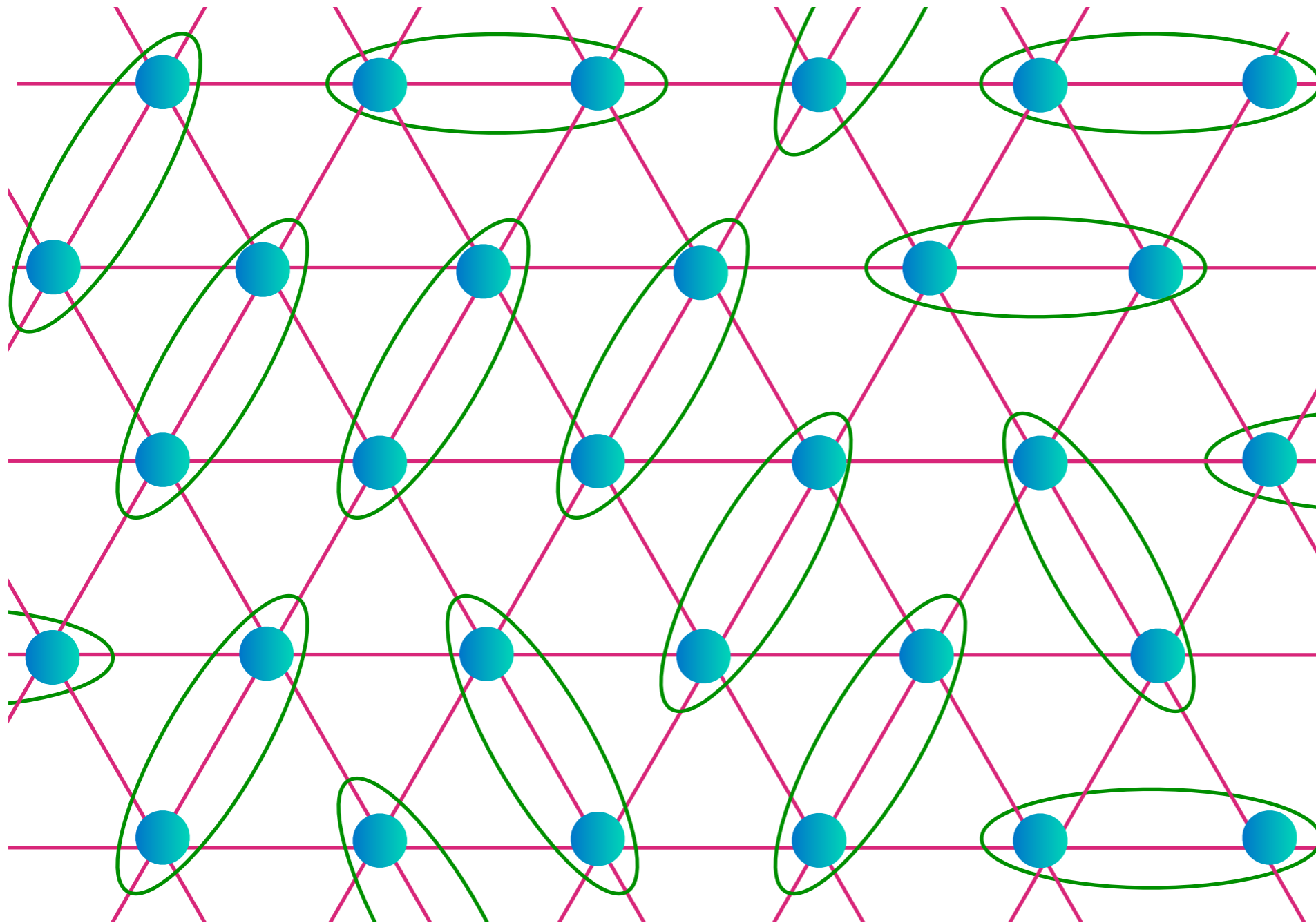


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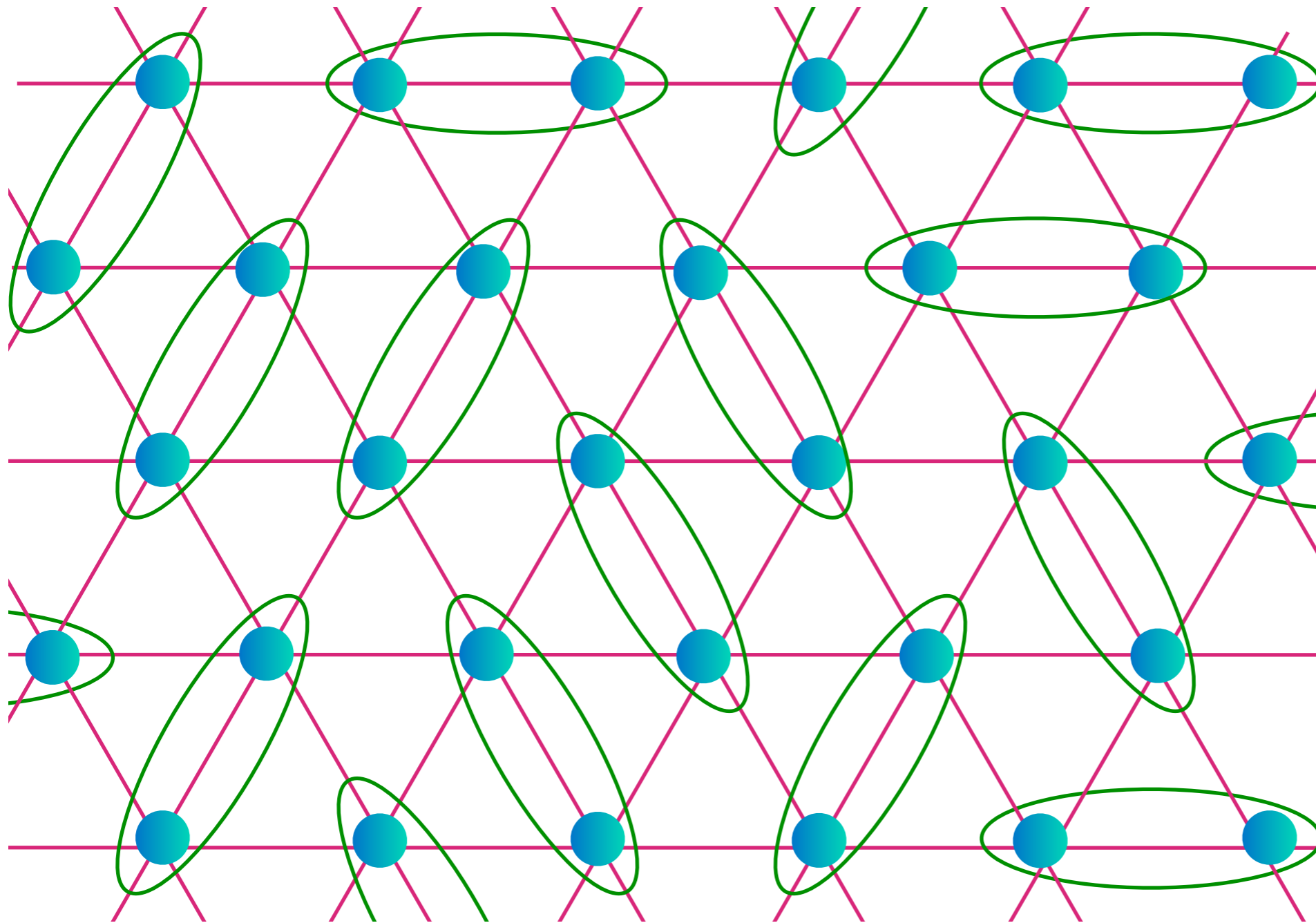


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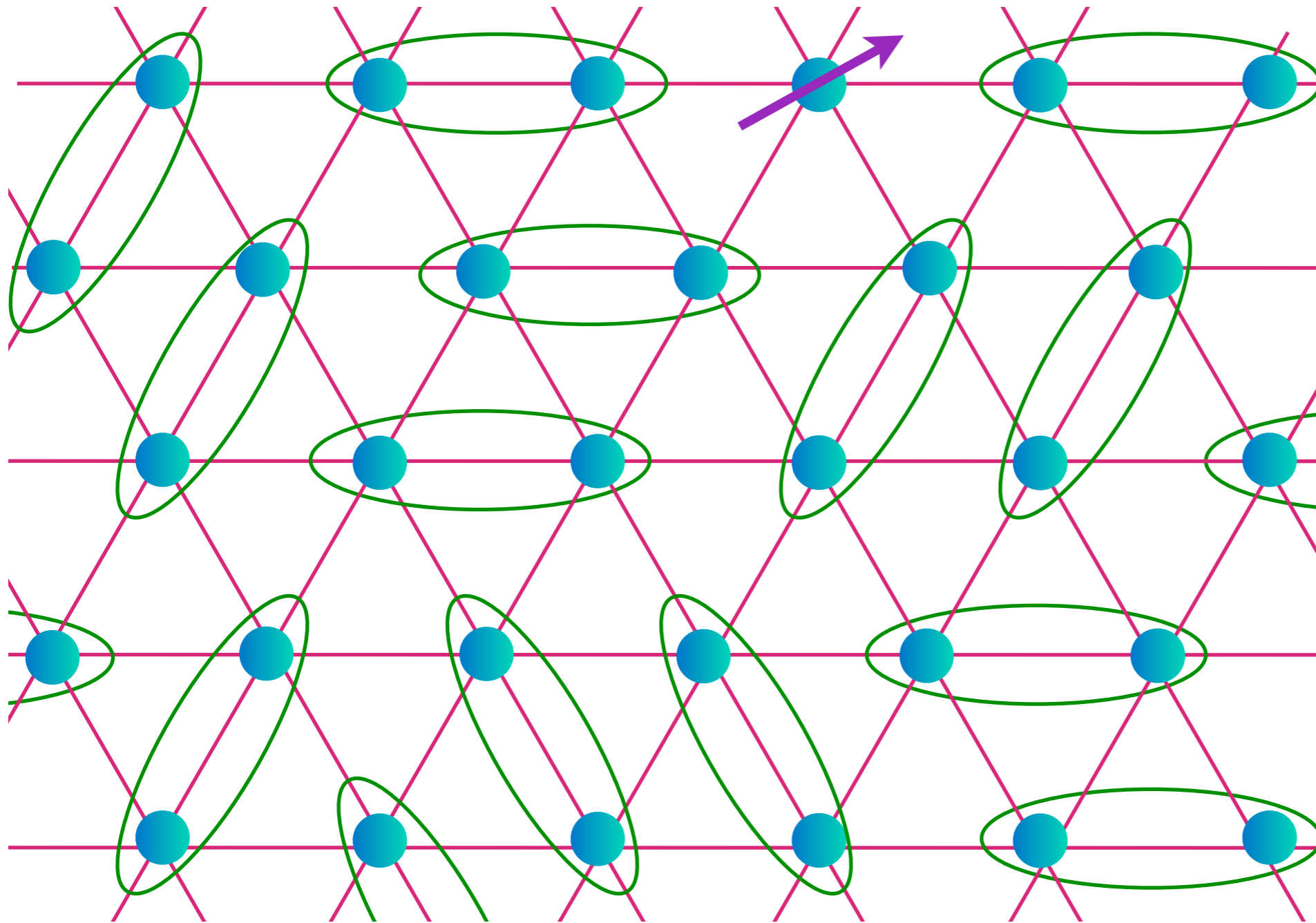


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Excitations of the Z_2 Spin liquid


Spinon: $S=1/2$, charge 0

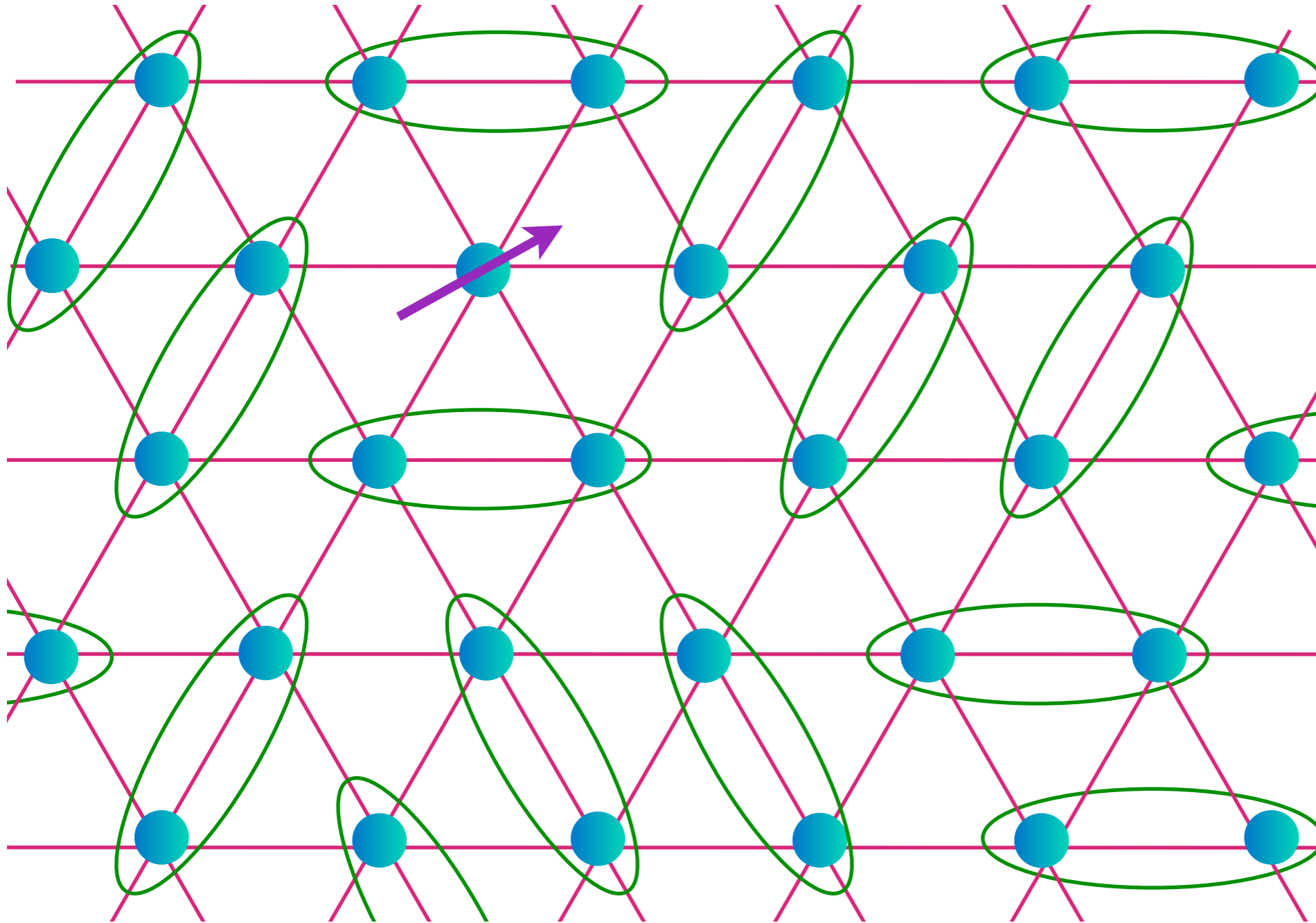

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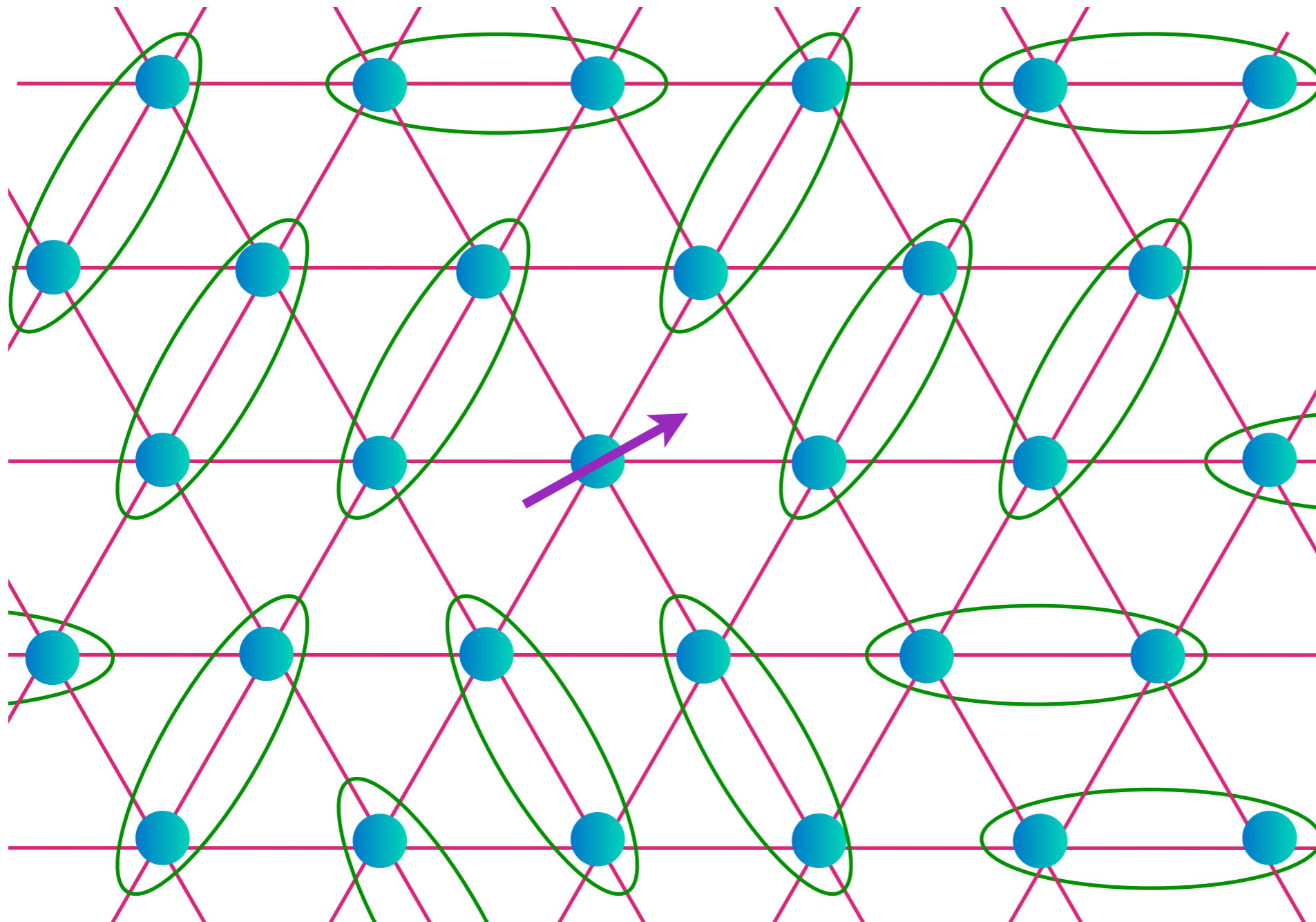

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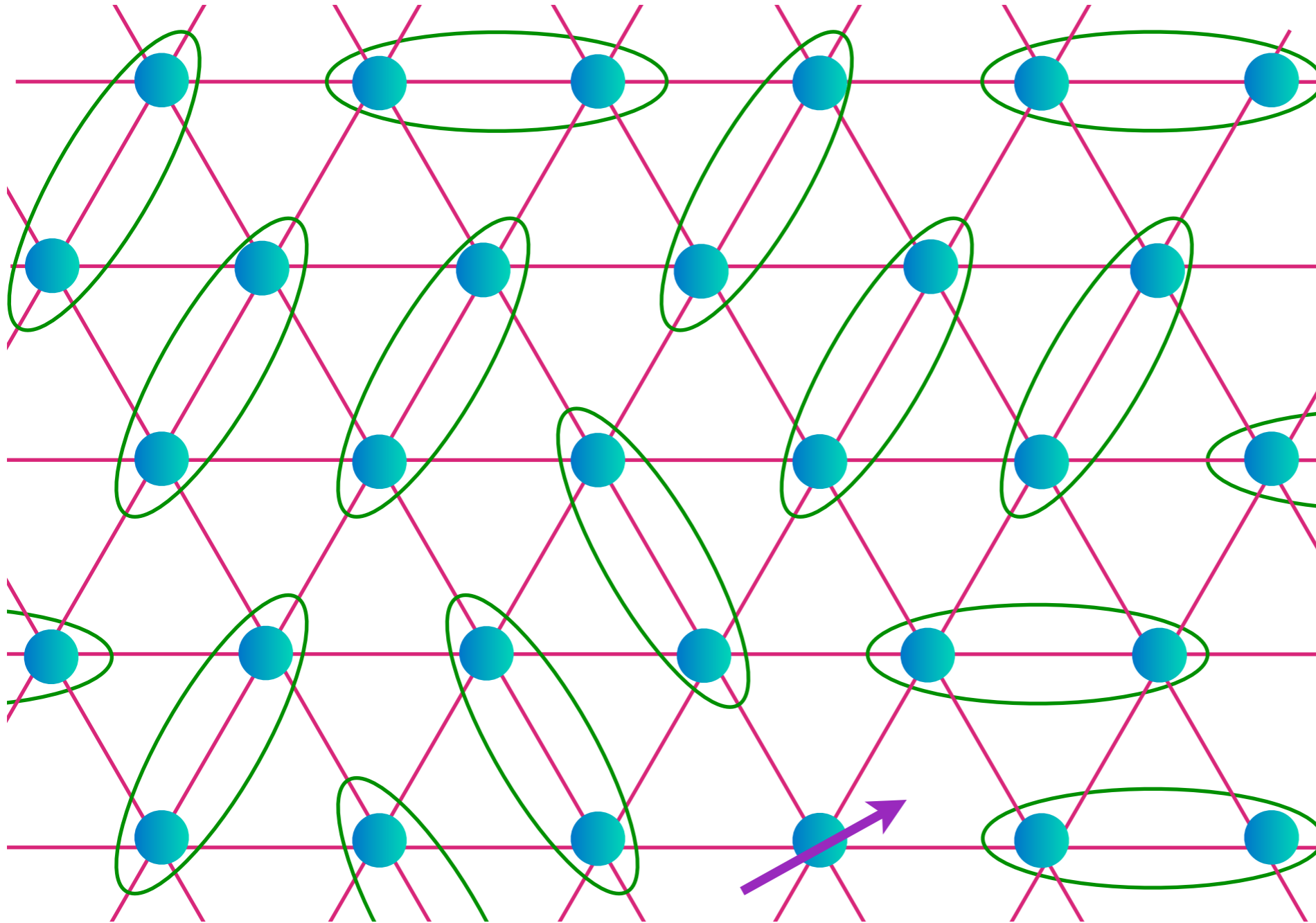

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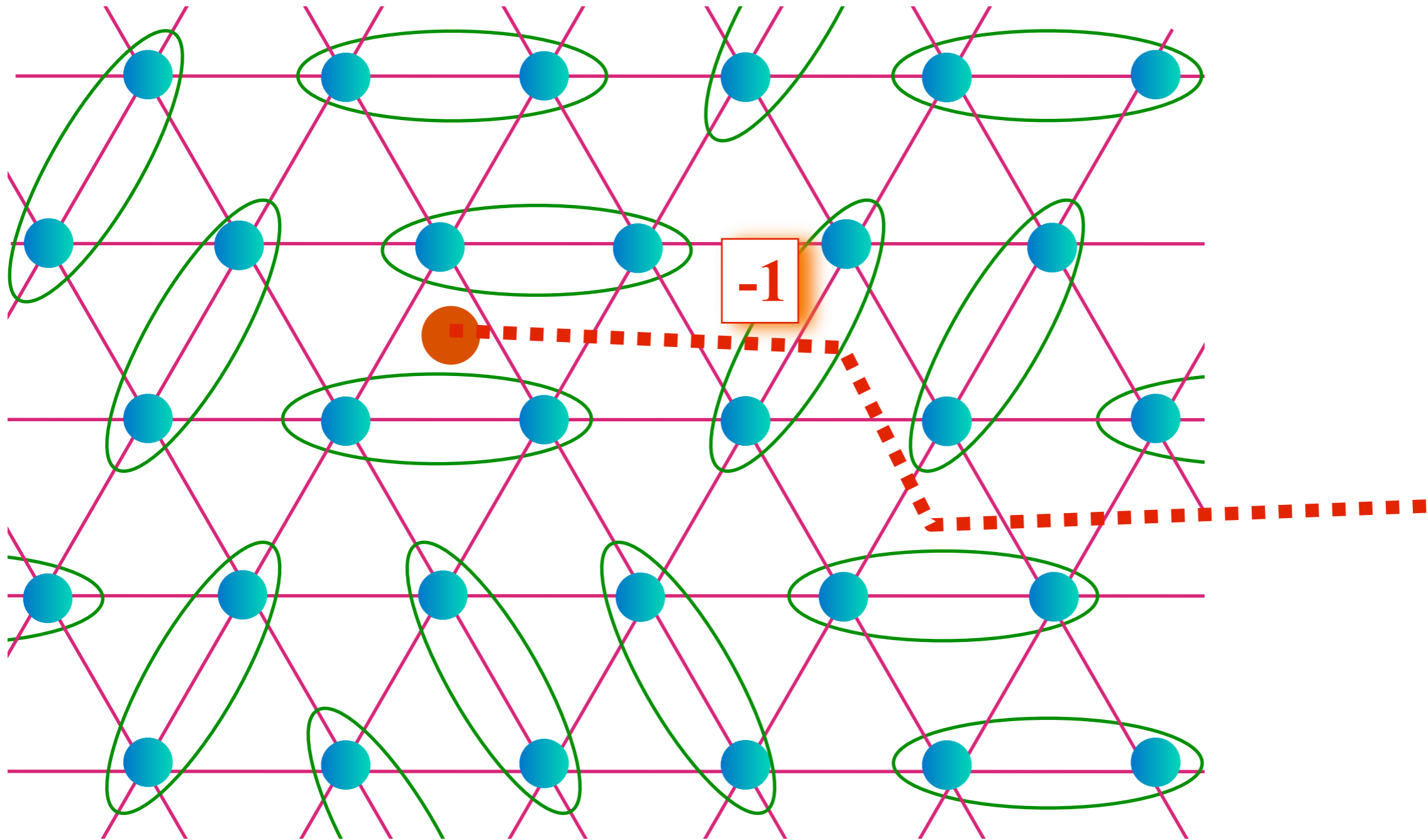

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Excitations of the Z_2 Spin liquid

A vison

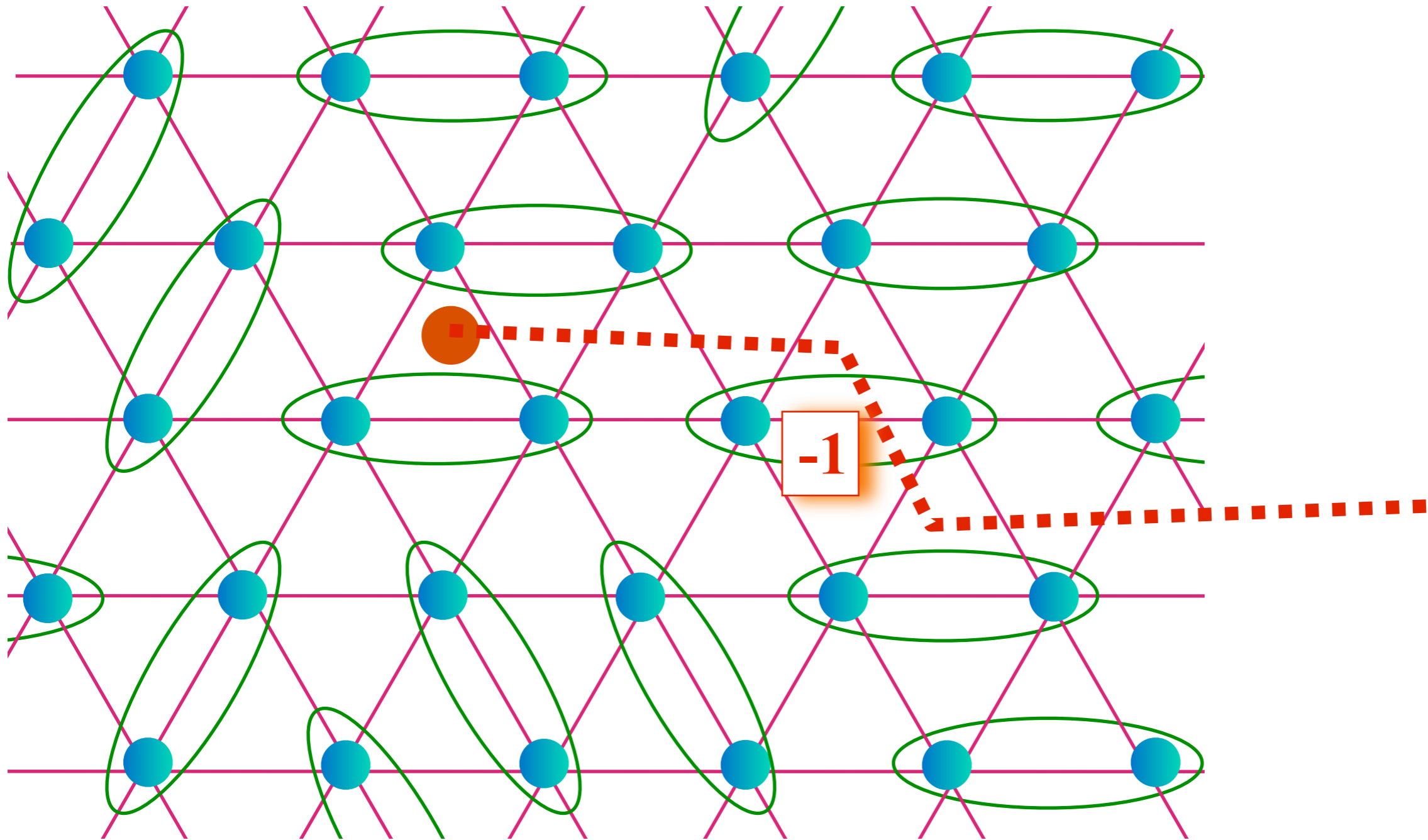

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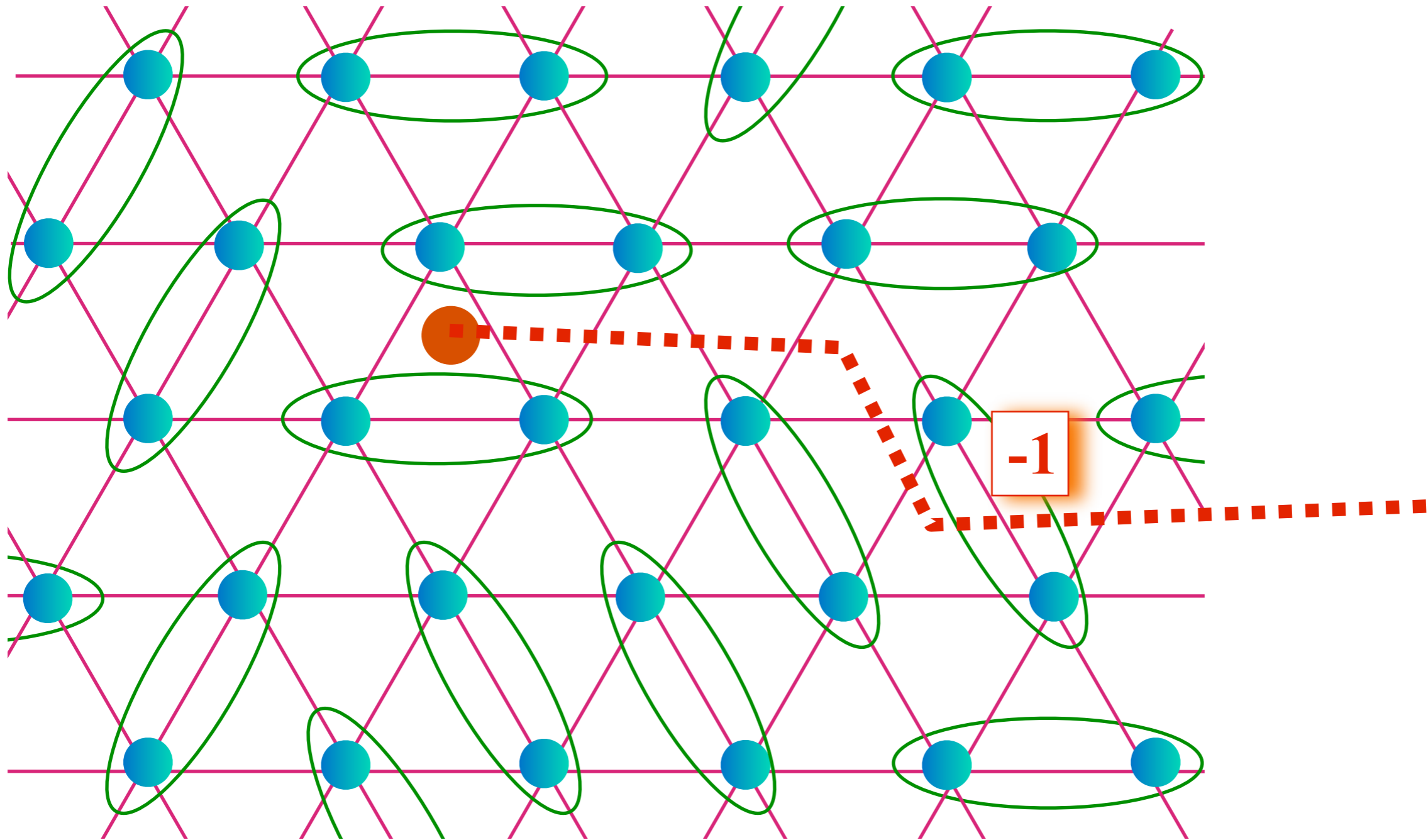
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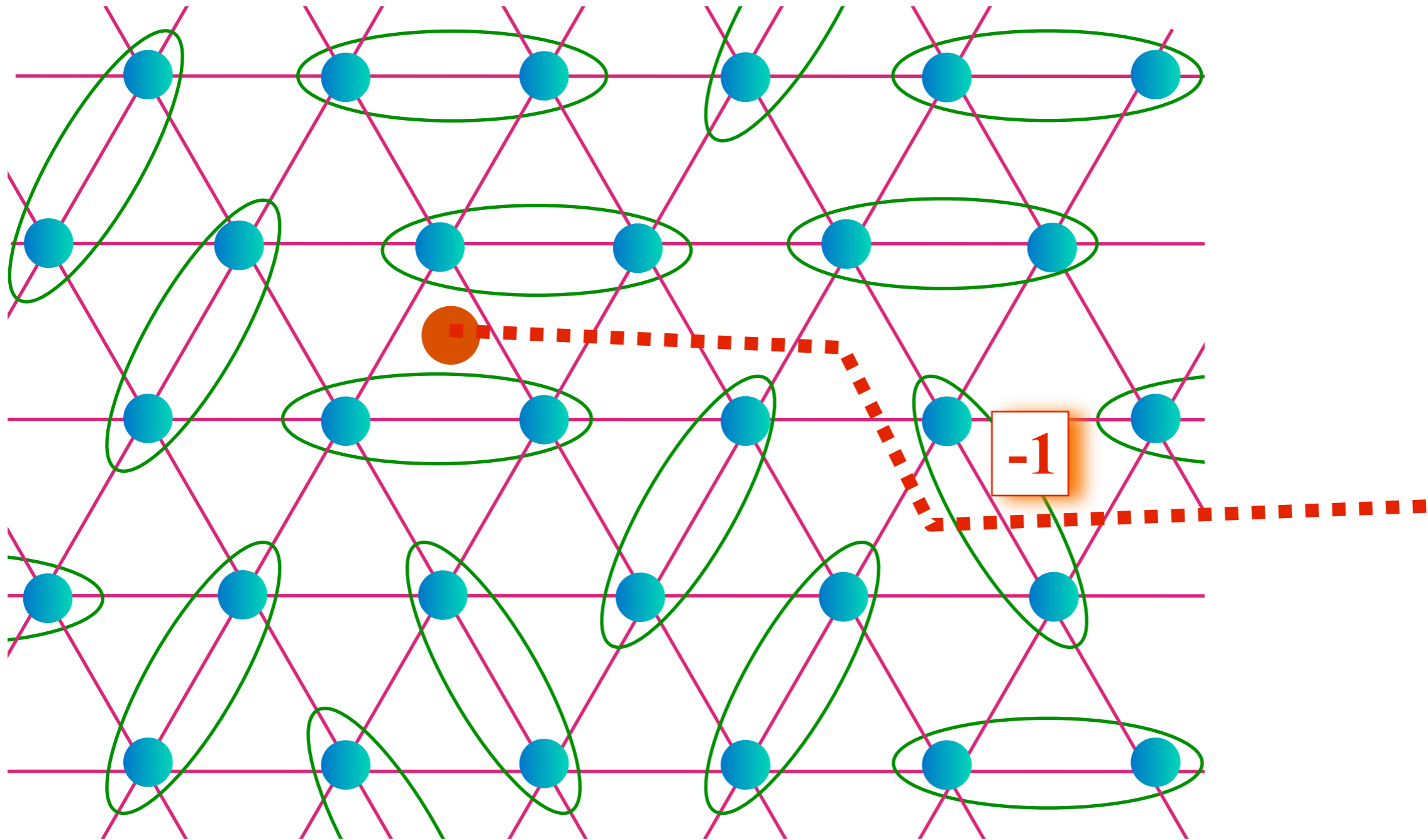
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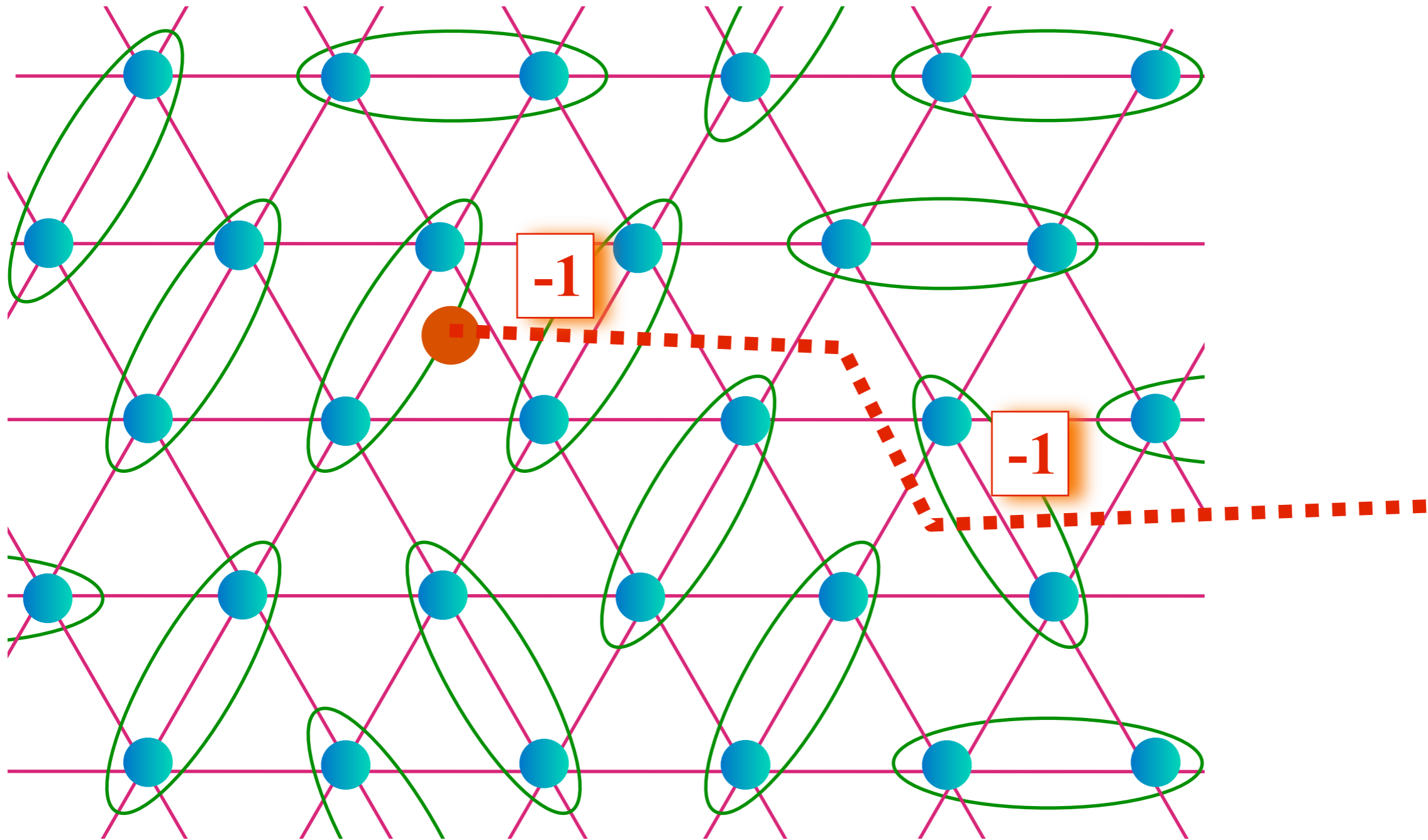
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Excitations of the Z_2 Spin liquid

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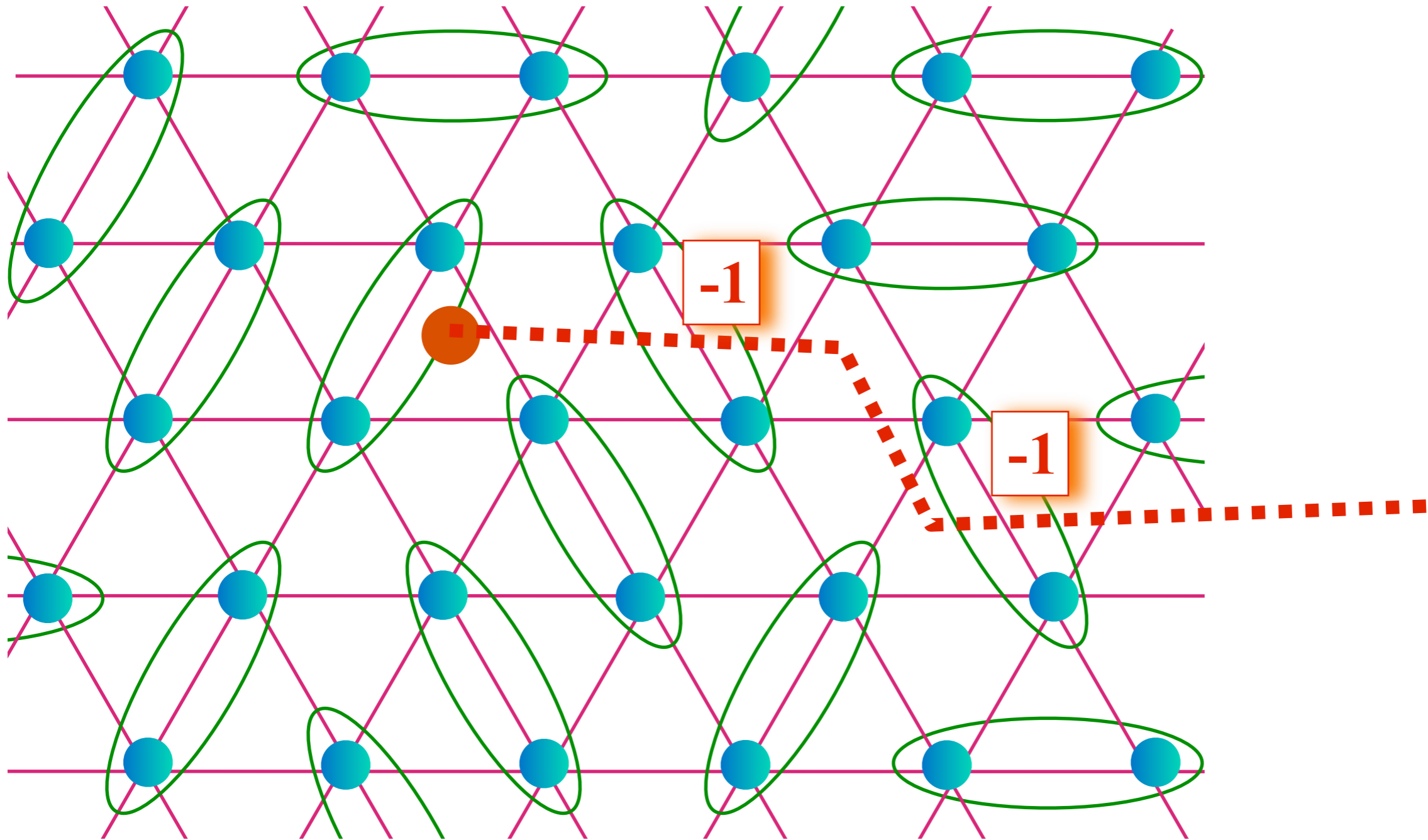
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Excitations of the Z_2 Spin liquid

A vison

- Visions are the dark matter of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

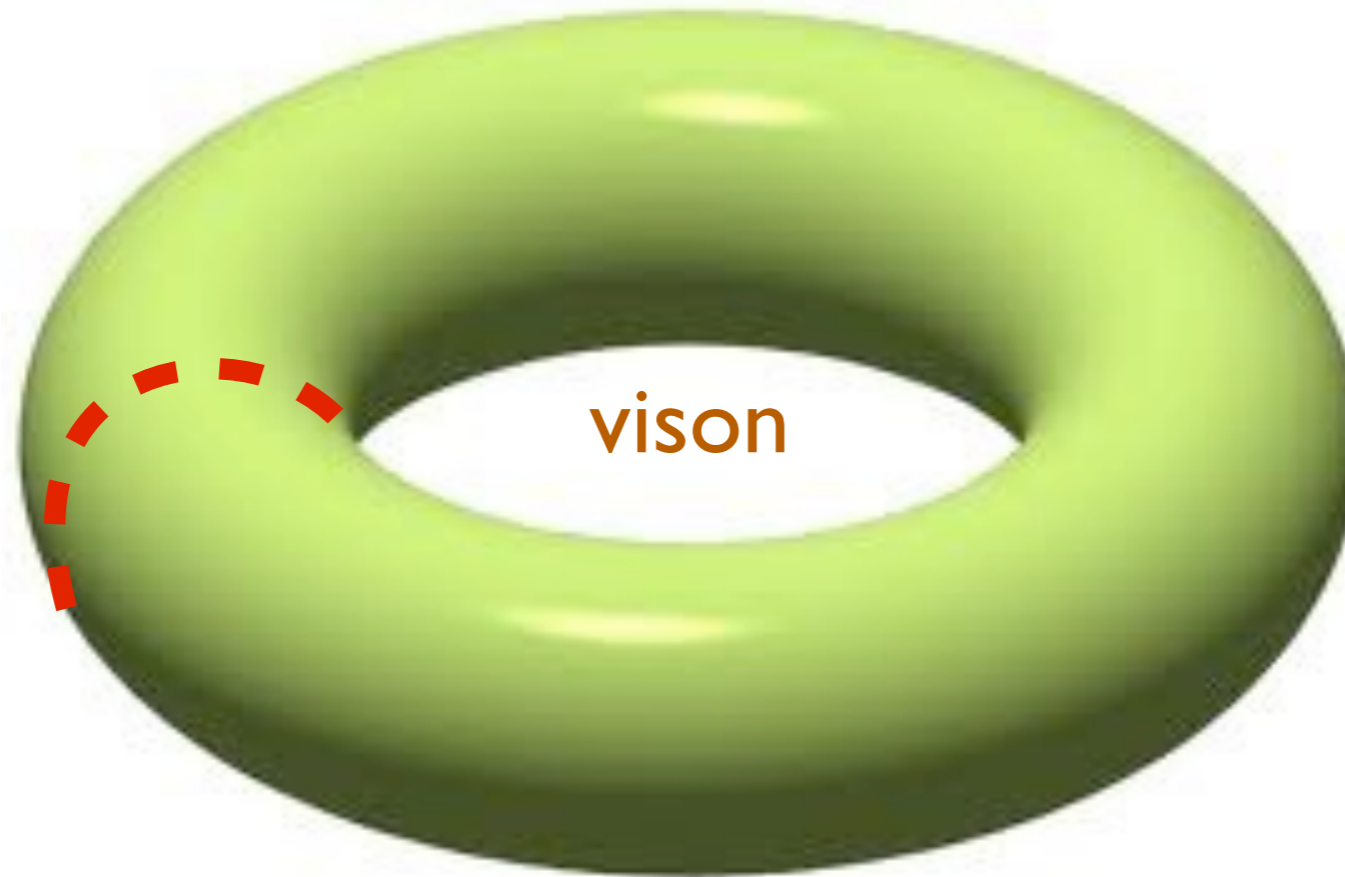
N. Read and B. Chakraborty, *Phys. Rev. B* **40**, 7133 (1989)
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

Topological order in the Z_2 spin liquid ground state



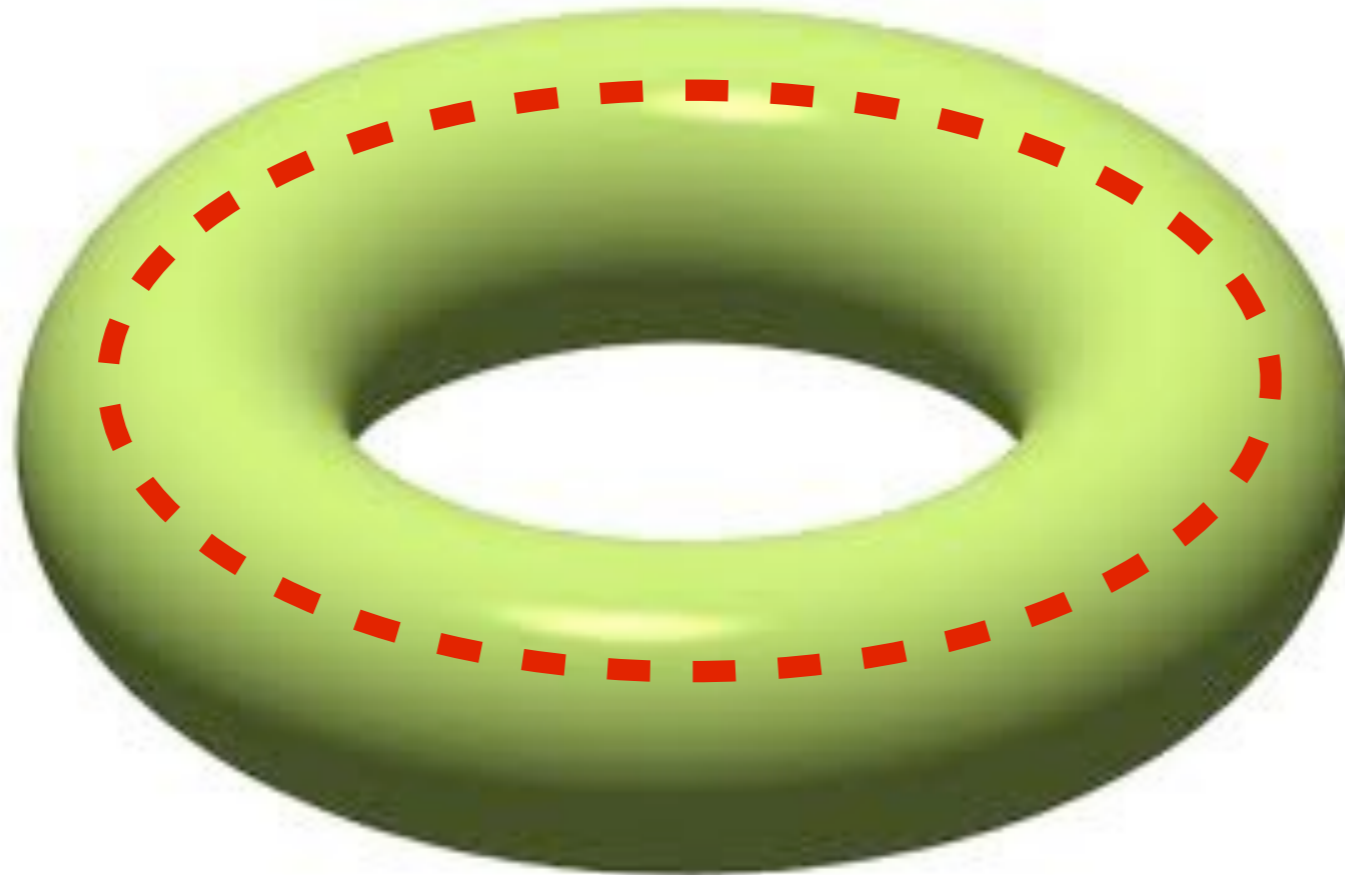
4-fold degeneracy on the torus

Topological order in the Z_2 spin liquid ground state



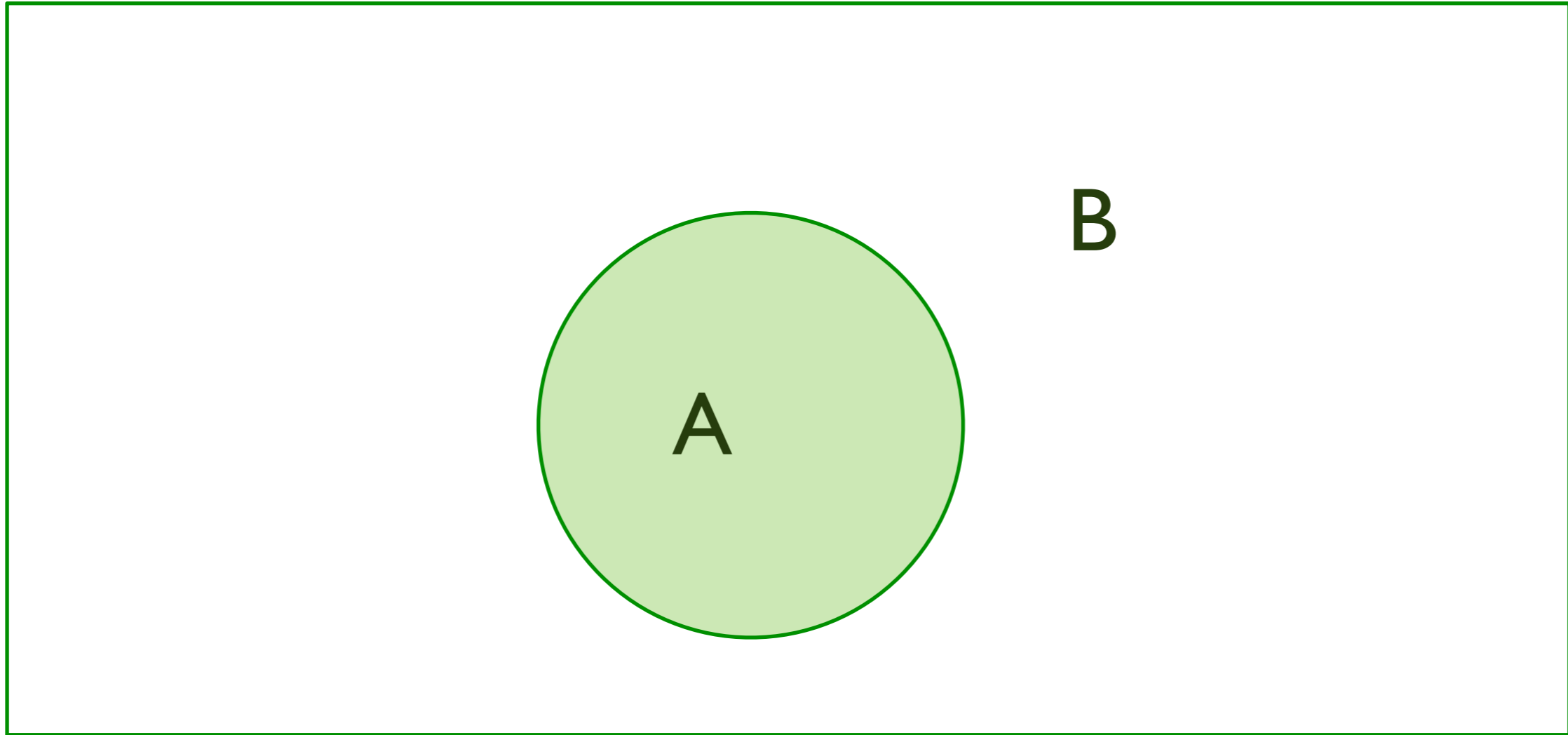
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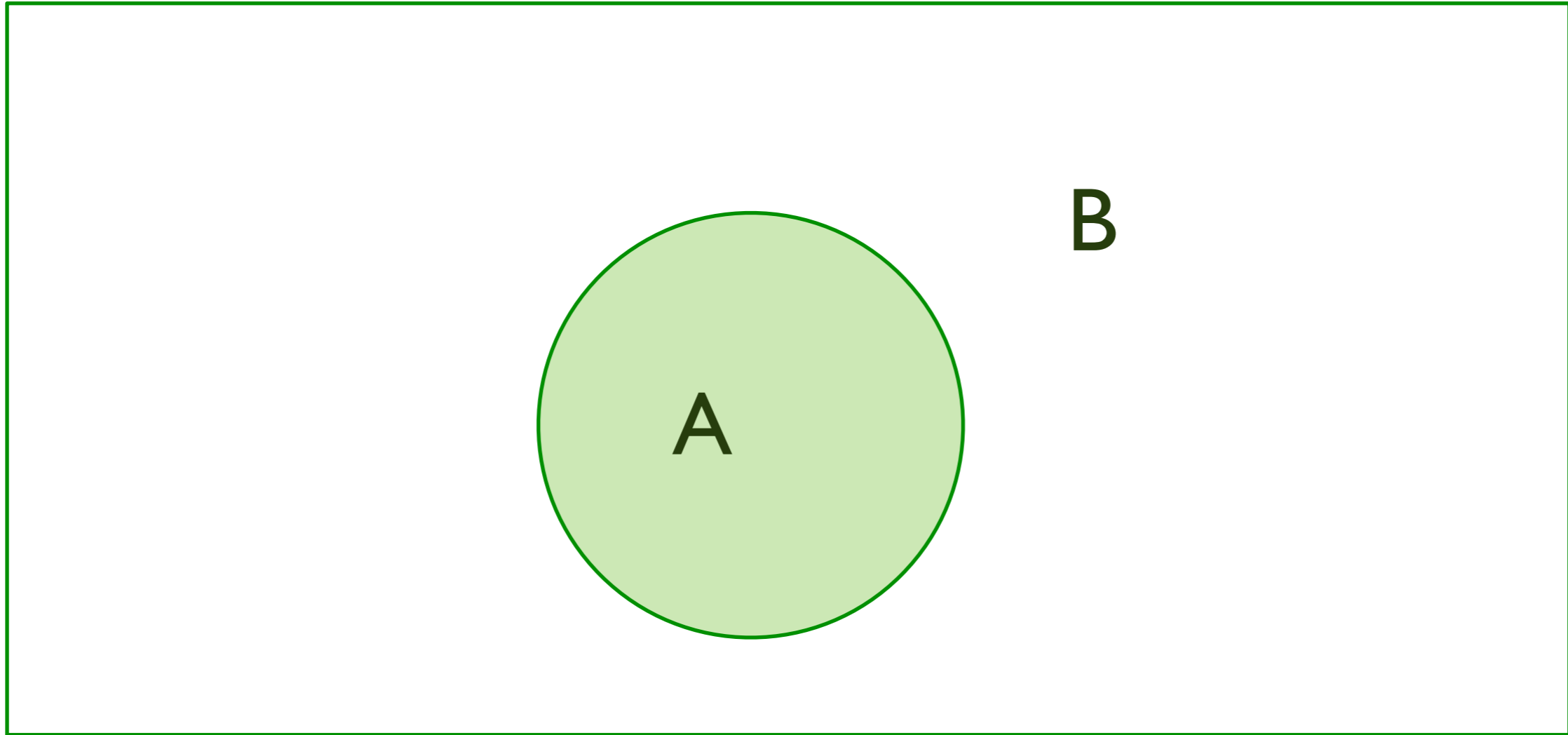
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$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_{EE} = -\text{Tr}(\rho_A \ln \rho_A)$

Topological order in the Z_2 spin liquid ground state

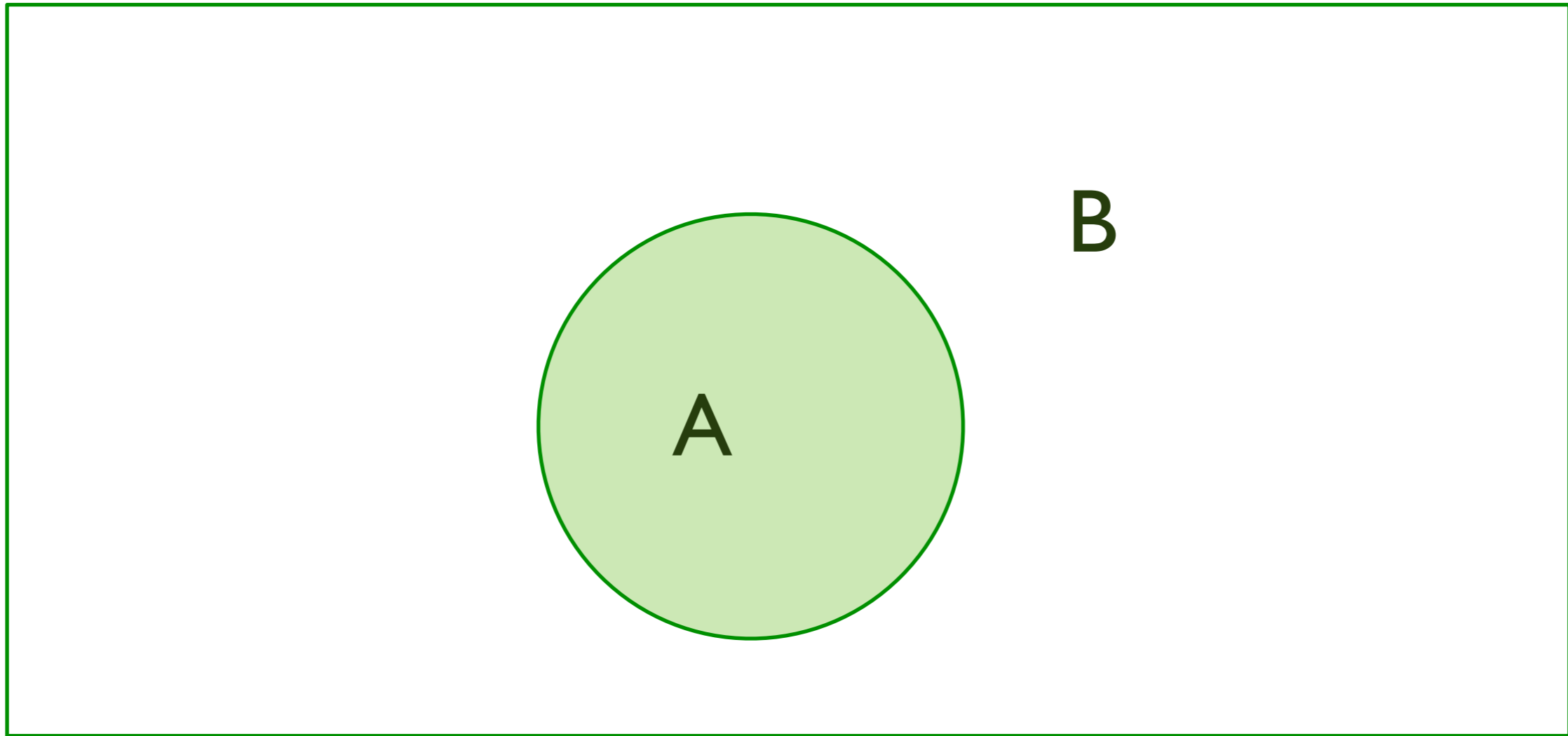


Entanglement entropy of a band insulator:

$$S_{EE} = aL - \exp(-bL)$$

where L is the perimeter of the boundary between A and B.

Topological order in the Z_2 spin liquid ground state



Entanglement entropy of a Z_2 spin liquid:

$$S_{EE} = aL - \ln(2)$$

where L is the perimeter of the boundary between A and B.
The $\ln(2)$ is a universal characteristic of the Z_2 spin liquid,
and implies *long-range* quantum entanglement.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Topological order in the Z_2 spin liquid ground state

These properties of the ground state can be described by effective theories:

deconfined phase of a Z_2 gauge theory

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

F. A. Bais, P. van Driel, and M. de Wild Propitius, *Phys. Lett. B* **280**, 63 (1992).

T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

topological doubled Chern-Simons gauge theory

J. Maldacena, G. Moore, and N. Seiberg, *JHEP* 0110:005 (2001).

M. Freedman, C. Nayak, K. Shtengel, K. Walker, and

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Z. Wang, *Annals of Physics* **310**, 428 (2004).

● Good recent numerical evidence of Z_2 spin liquid on kagome and square lattices

Simeng Yan, D.A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

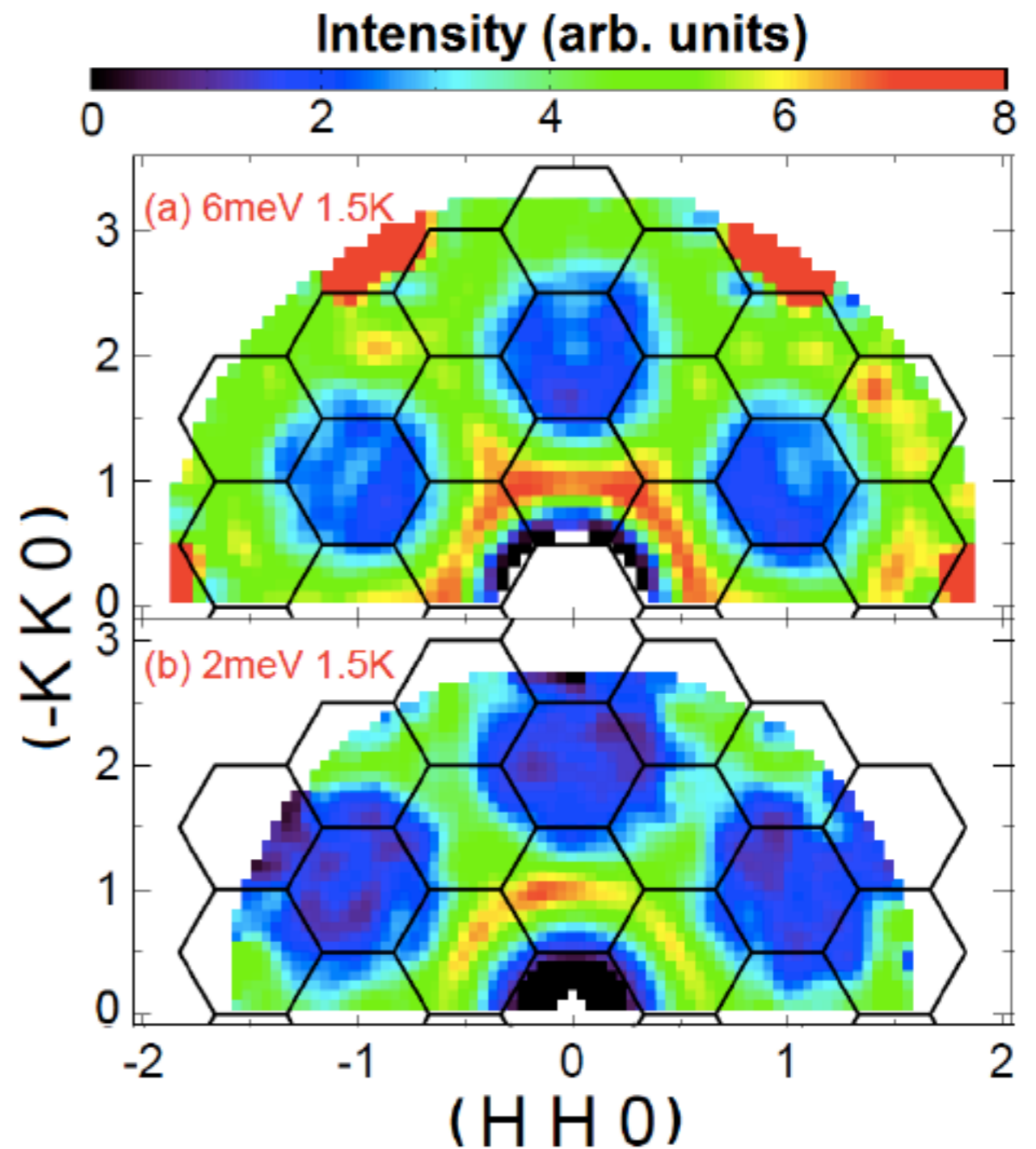
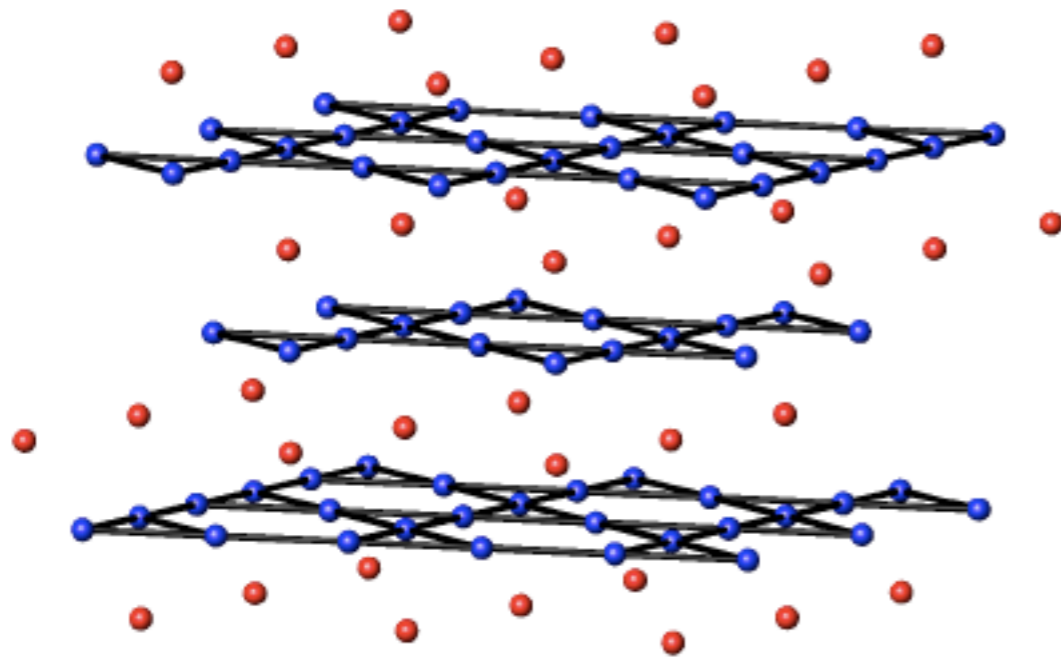
J. Hong-Chen Jiang, Hong Yao, and L. Balents, arXiv:1112.2241.

Ling Wang, Zheng-Cheng Gu, Xiao-Gang Wen, and F. Verstraete, arXiv:1112.3331

● Promising experimental candidate: the kagome antiferromagnet

Young Lee, APS meeting, March 2012

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$ (also called Herbertsmithite)



Quantum Hall states

Similar topological properties,
but no time-reversal symmetry:

- ground state degeneracy on a torus
- universal entanglement entropy
- gapless edge states on spaces with boundaries
(can also happen for some spin liquids)
- topological Chern-Simons gauge theories

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Compressible quantum matter

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Conformal quantum matter

A. Field theory: graphene

*B. Field theory: superfluid-
insulator transition*

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Conformal quantum matter

A. Field theory: graphene

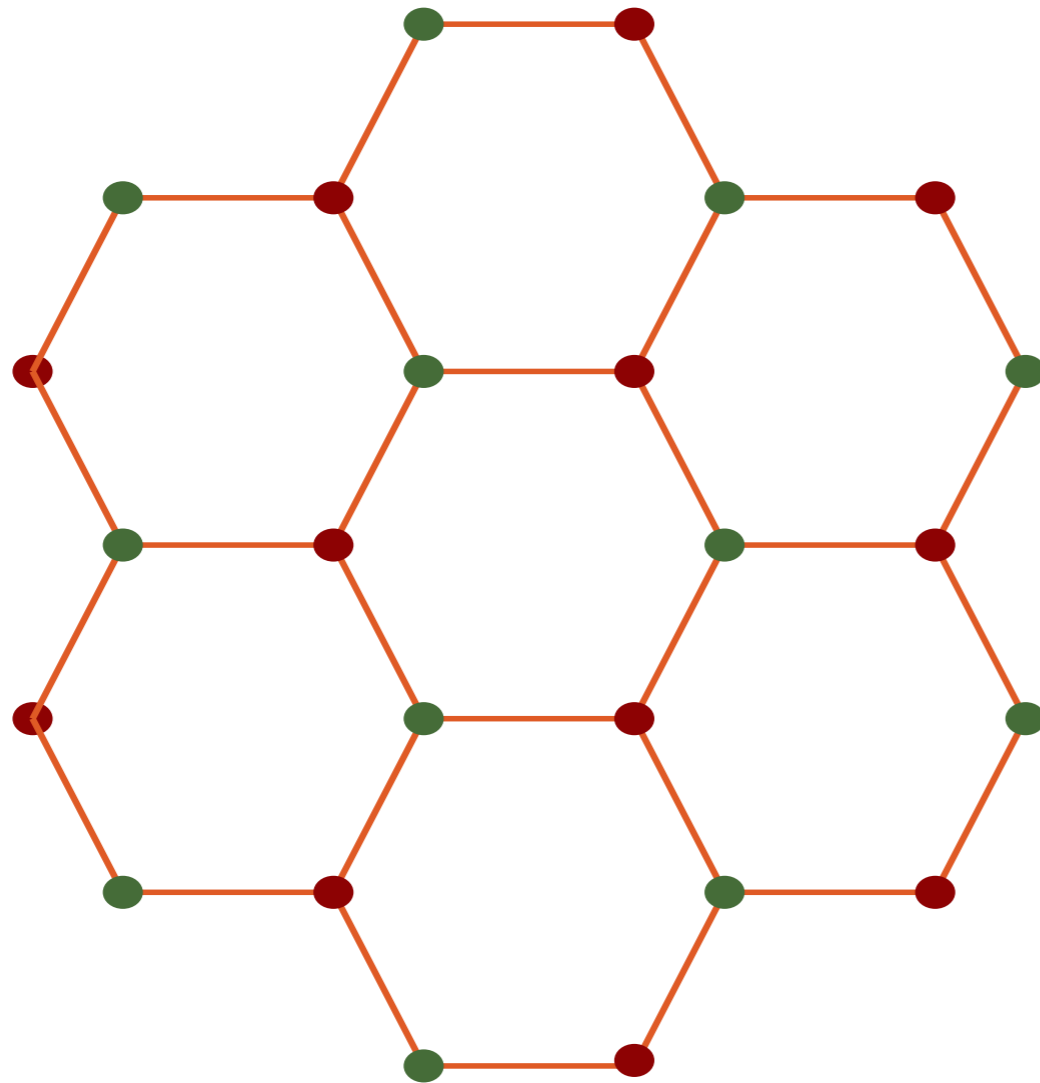
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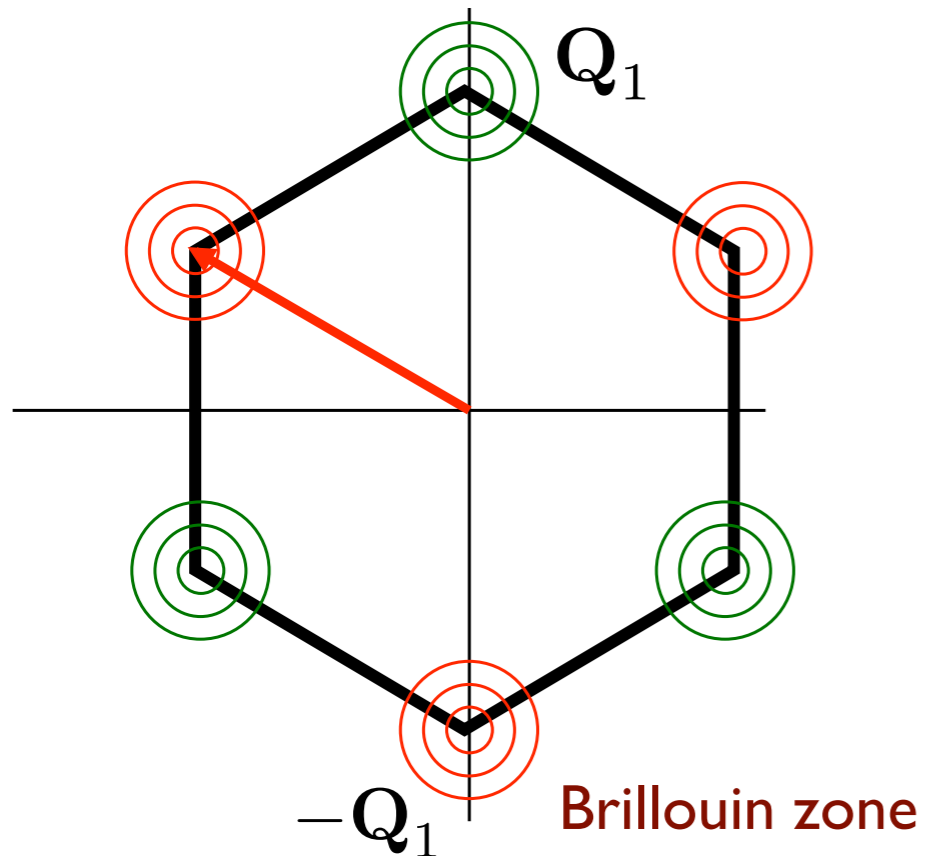
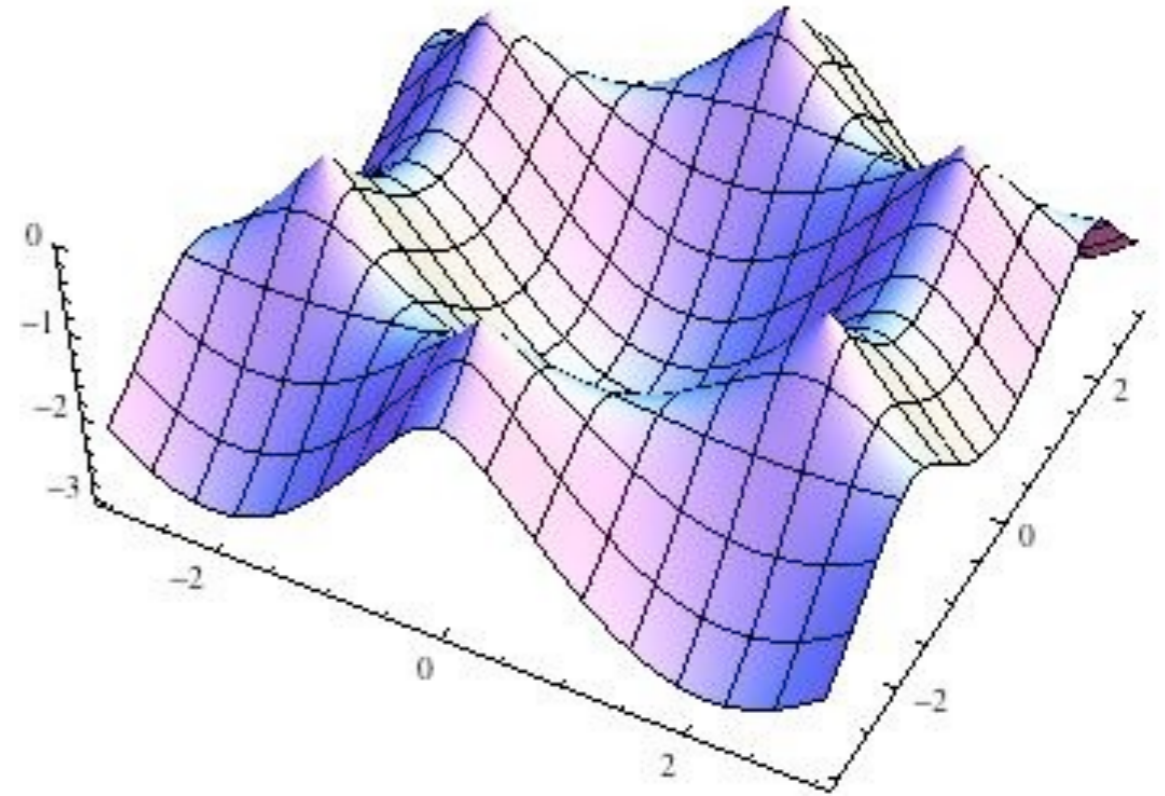
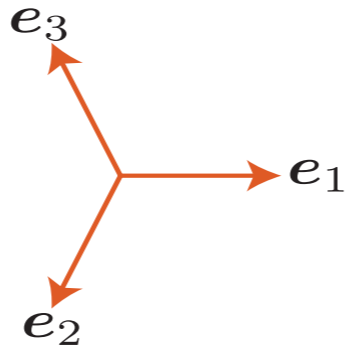
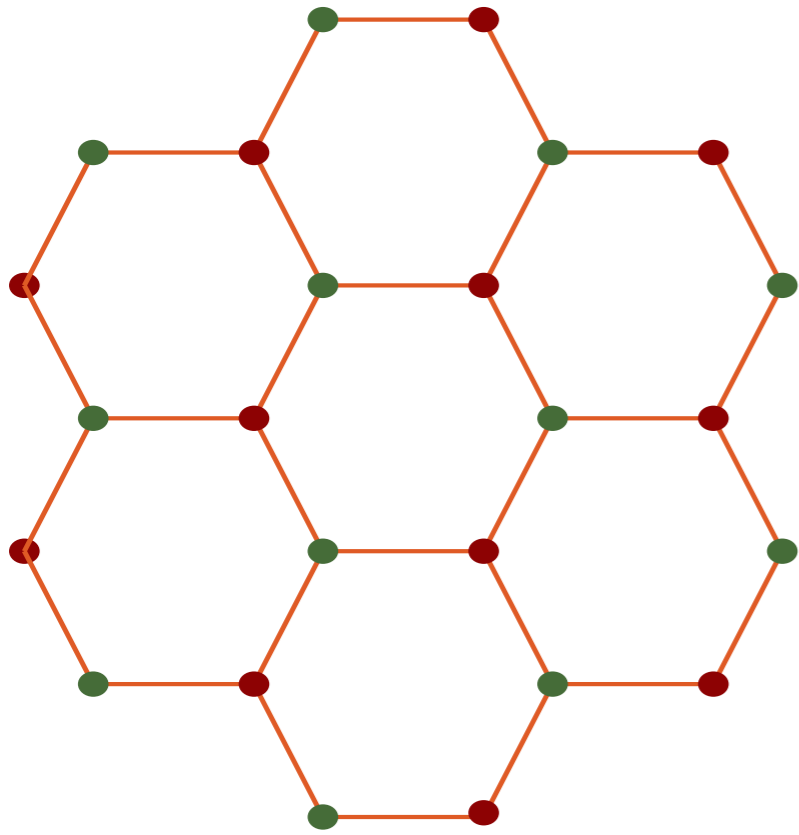
D. Gauge-gravity duality

Honeycomb lattice

(describes graphene after adding long-range Coulomb interactions)



$$H = -t \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)$$



**Semi-metal with
massless Dirac fermions
at small U/t**

We define the Fourier transform of the fermions by

$$c_A(\mathbf{k}) = \sum_{\mathbf{r}} c_A(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \quad (4)$$

and similarly for c_B . **A** and **B** are sublattice indices. The hopping Hamiltonian is

$$H_0 = -t \sum_{\langle ij \rangle} \left(c_{Ai\alpha}^\dagger c_{Bj\alpha} + c_{Bj\alpha}^\dagger c_{Ai\alpha} \right) \quad (5)$$

where α is a spin index. If we introduce Pauli matrices τ^a in sublattice space ($a = x, y, z$), this Hamiltonian can be written as

$$H_0 = \int \frac{d^2k}{4\pi^2} c^\dagger(\mathbf{k}) \left[-t \left(\cos(\mathbf{k} \cdot \mathbf{e}_1) + \cos(\mathbf{k} \cdot \mathbf{e}_2) + \cos(\mathbf{k} \cdot \mathbf{e}_3) \right) \tau^x + t \left(\sin(\mathbf{k} \cdot \mathbf{e}_1) + \sin(\mathbf{k} \cdot \mathbf{e}_2) + \sin(\mathbf{k} \cdot \mathbf{e}_3) \right) \tau^y \right] c(\mathbf{k}) \quad (6)$$

The low energy excitations of this Hamiltonian are near $\mathbf{k} \approx \pm \mathbf{Q}_1$.

In terms of the fields near \mathbf{Q}_1 and $-\mathbf{Q}_1$, we define

$$\begin{aligned}
 \Psi_{A1\alpha}(\mathbf{k}) &= c_{A\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
 \Psi_{A2\alpha}(\mathbf{k}) &= c_{A\alpha}(-\mathbf{Q}_1 + \mathbf{k}) \\
 \Psi_{B1\alpha}(\mathbf{k}) &= c_{B\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
 \Psi_{B2\alpha}(\mathbf{k}) &= c_{B\alpha}(-\mathbf{Q}_1 + \mathbf{k})
 \end{aligned} \tag{7}$$

We consider Ψ to be a 8 component vector, and introduce Pauli matrices ρ^a which act in the 1, 2 valley space. Then the Hamiltonian is

$$H_0 = \int \frac{d^2k}{4\pi^2} \Psi^\dagger(\mathbf{k}) \left(v\tau^y k_x + v\tau^x \rho^z k_y \right) \Psi(\mathbf{k}), \tag{8}$$

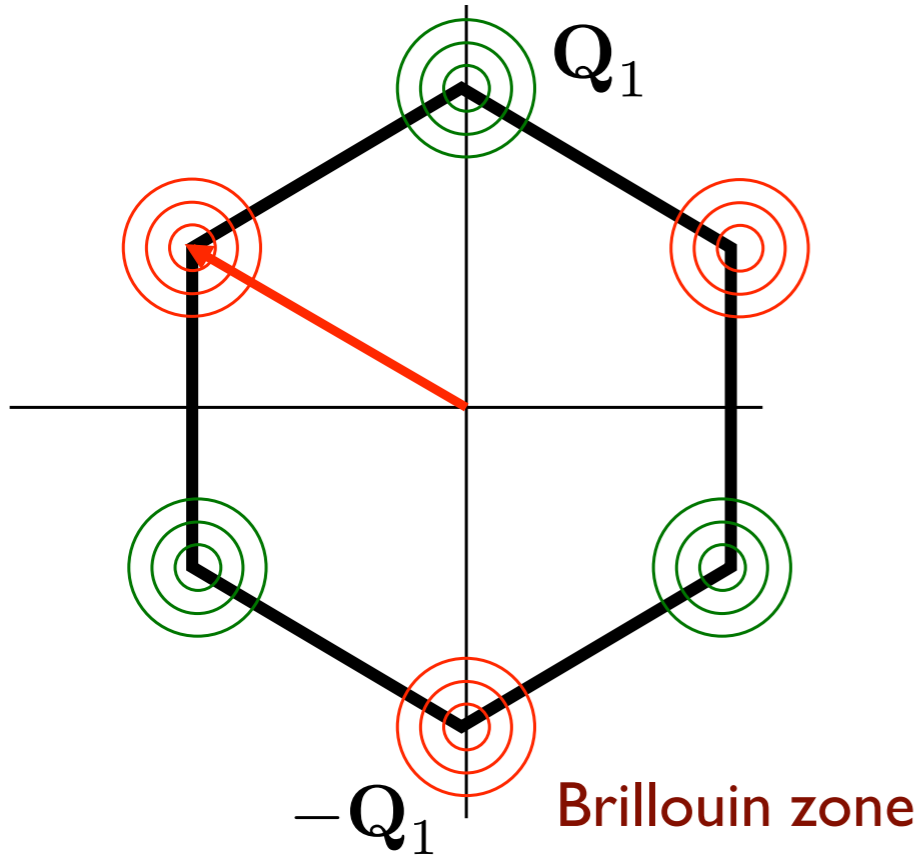
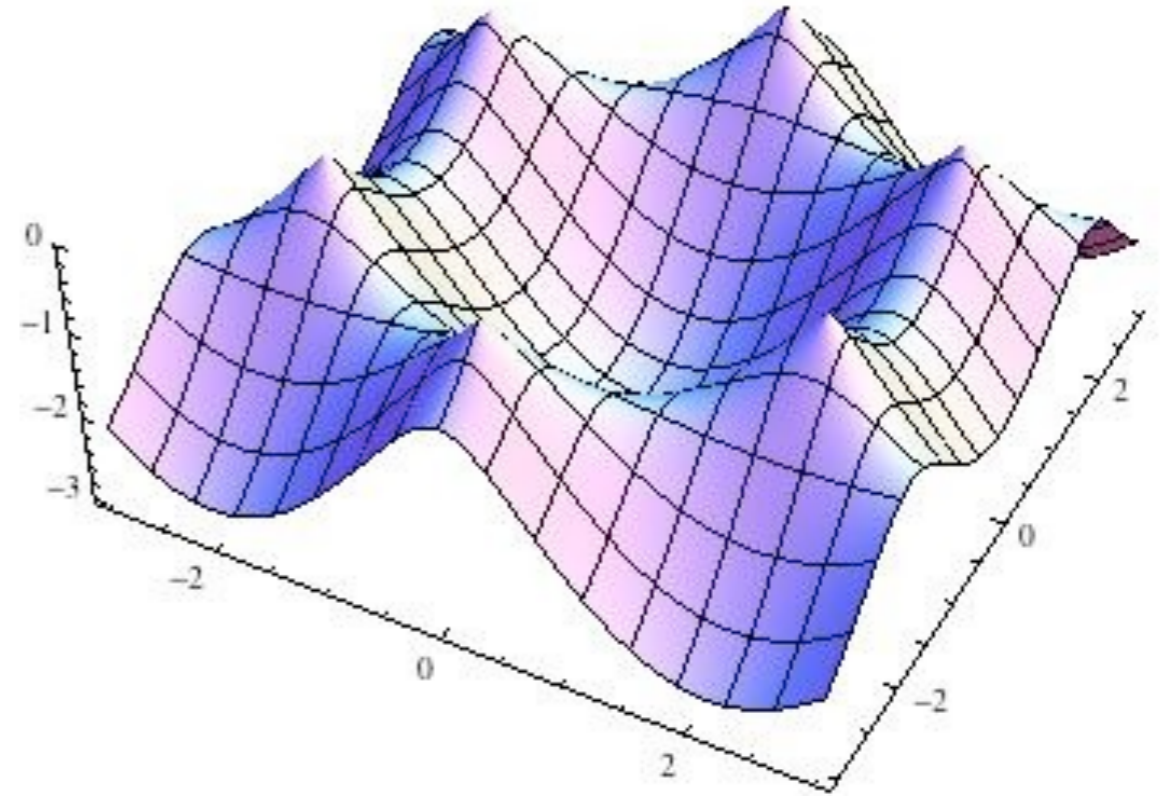
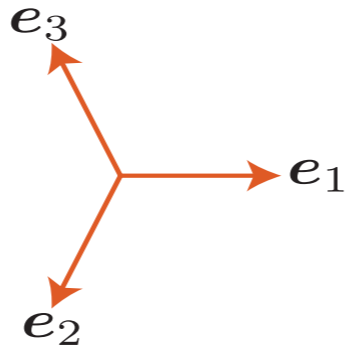
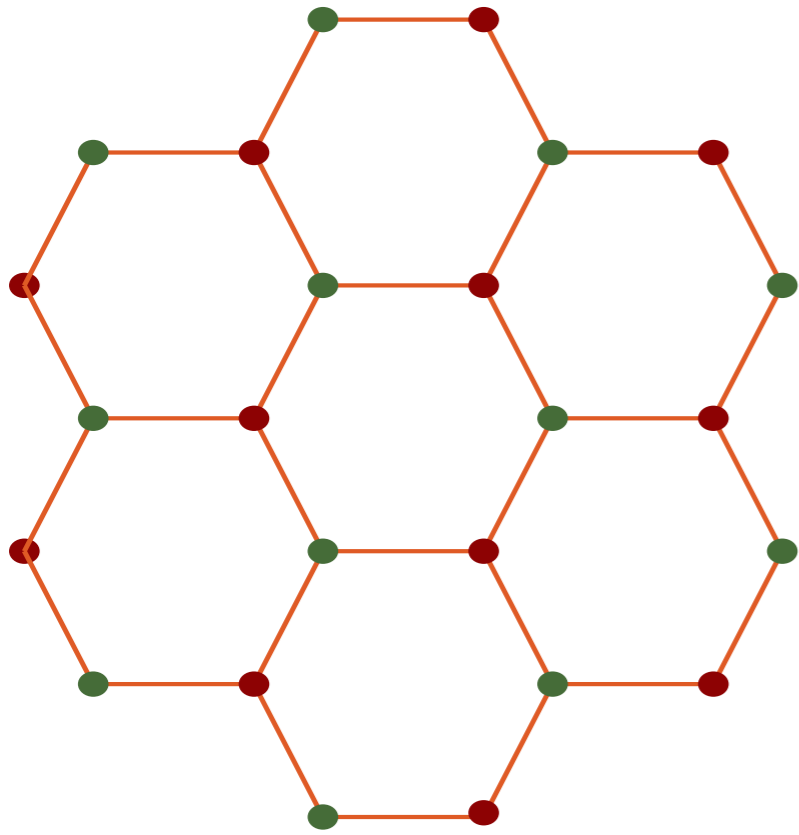
where $v = 3t/2$; below we set $v = 1$. Now define $\bar{\Psi} = \Psi^\dagger \rho^z \tau^z$. Then we can write the imaginary time Lagrangian as

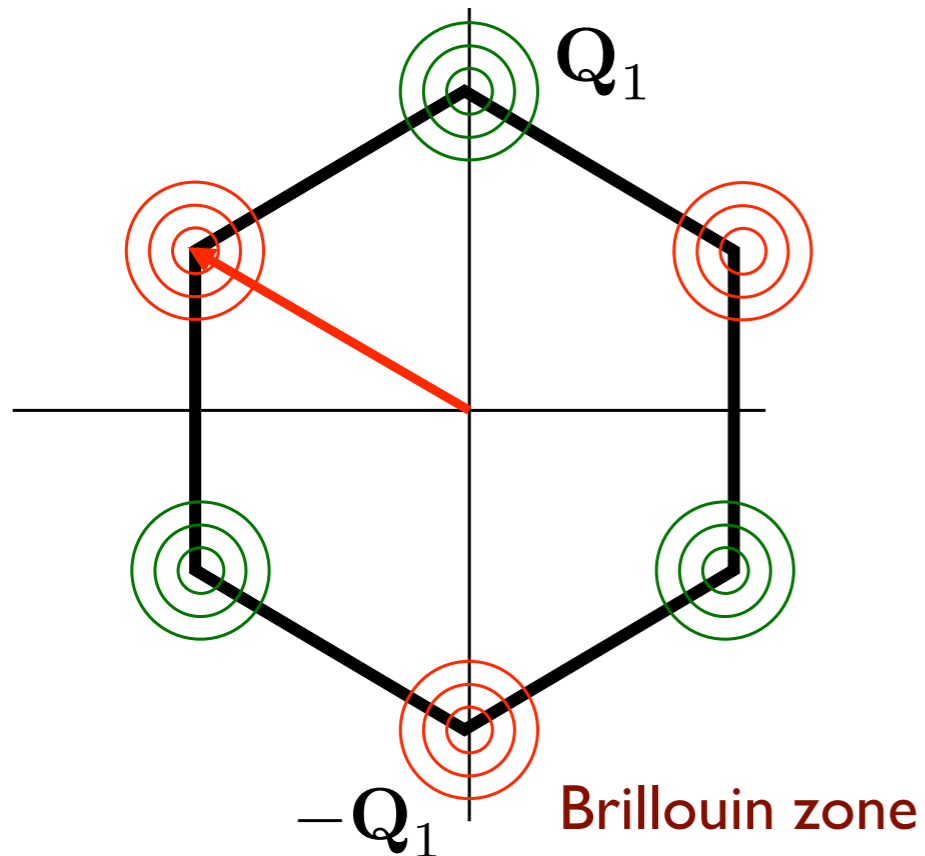
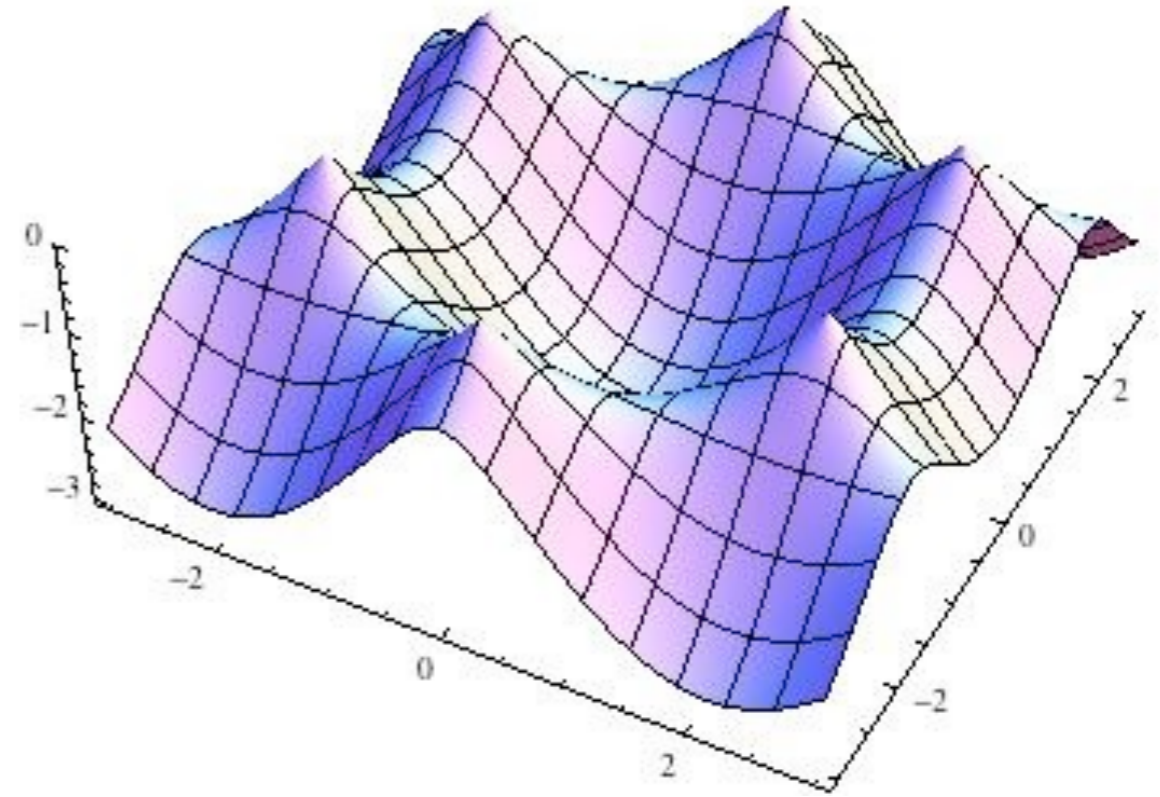
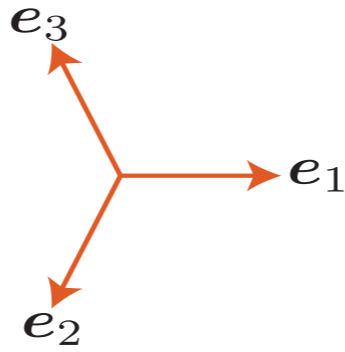
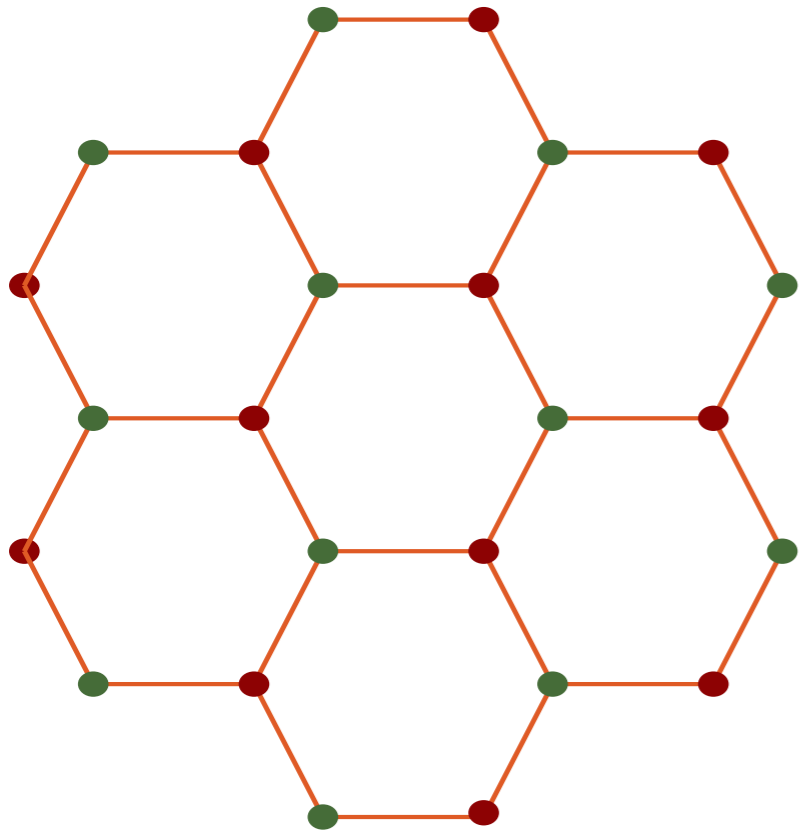
$$\mathcal{L}_0 = -i\bar{\Psi} (\omega\gamma_0 + k_x\gamma_1 + k_y\gamma_2) \Psi \tag{9}$$

where

$$\gamma_0 = -\rho^z \tau^z \quad \gamma_1 = \rho^z \tau^x \quad \gamma_2 = -\tau^y \tag{10}$$

Exercise: Observe that \mathcal{L}_0 is invariant under the scaling transformation $x' = xe^{-\ell}$ and $\tau' = \tau e^{-\ell}$. Write the Hubbard interaction U in terms of the Dirac fermions, and show that it has the tree-level scaling transformation $U' = Ue^{-\ell}$. So argue that all short-range interactions are *irrelevant* in the Dirac semi-metal phase.





The theory of free Dirac fermions is invariant under conformal transformations of spacetime. This is a realization of a simple conformal field theory in 2+1 dimensions: a CFT3

The Hubbard Model at large U

$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

In the limit of large U , and at a density of one particle per site, this maps onto the Heisenberg antiferromagnet

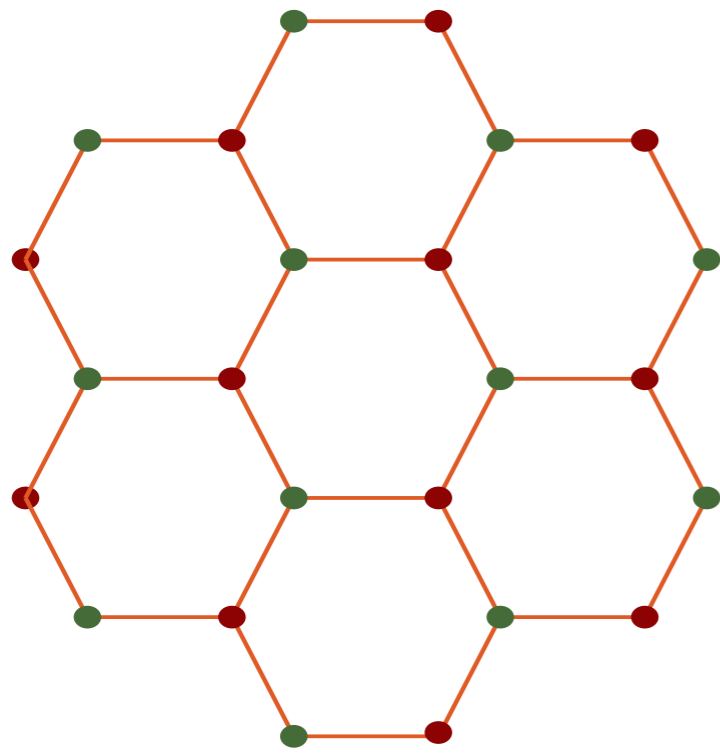
$$H_{AF} = \sum_{i < j} J_{ij} S_i^a S_j^a$$

where $a = x, y, z$,

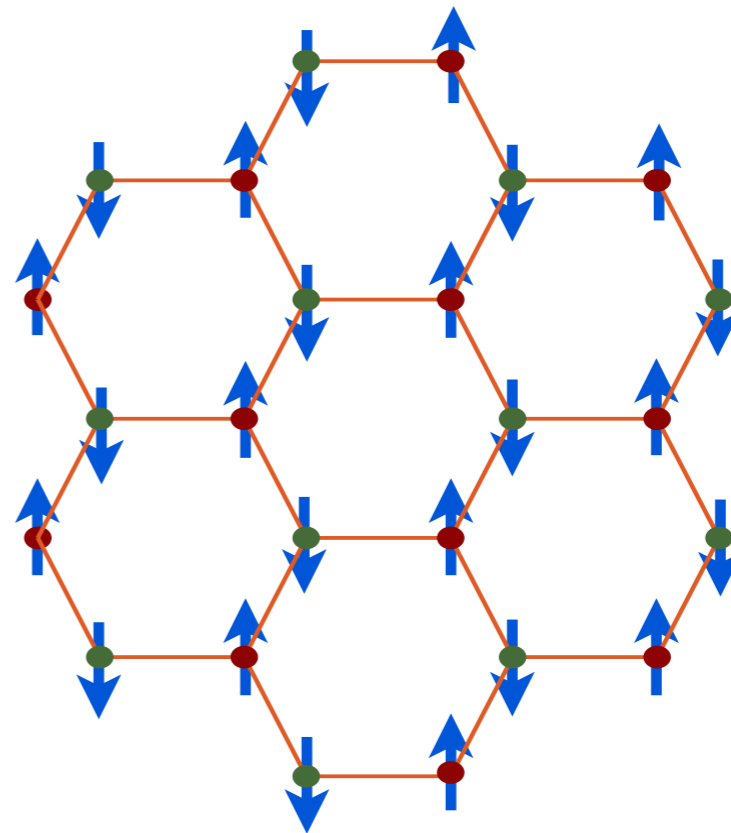
$$S_i^a = \frac{1}{2} c_{i\alpha}^{a\dagger} \sigma_{\alpha\beta}^a c_{i\beta},$$

with σ^a the Pauli matrices and

$$J_{ij} = \frac{4t_{ij}^2}{U}$$



Dirac
semi-metal



Insulating
antiferromagnet
with Neel order

U/t

Antiferromagnetism

We use the operator equation (valid on each site i):

$$U \left(n_{\uparrow} - \frac{1}{2} \right) \left(n_{\downarrow} - \frac{1}{2} \right) = -\frac{2U}{3} S_i^{a2} + \frac{U}{4} \quad (11)$$

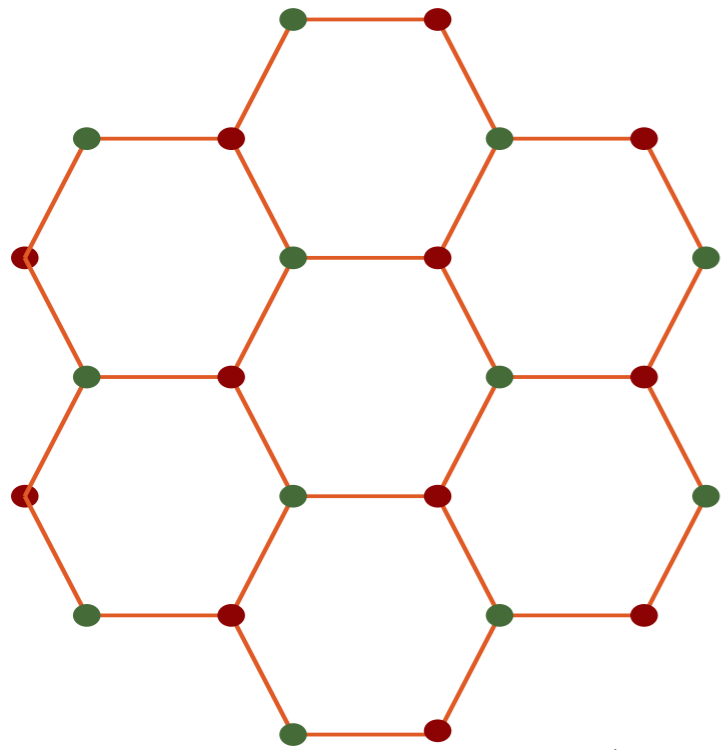
Then we decouple the interaction via

$$\exp \left(\frac{2U}{3} \sum_i \int d\tau S_i^{a2} \right) = \int \mathcal{D}J_i^a(\tau) \exp \left(- \sum_i \int d\tau \left[\frac{3}{8U} J_i^{a2} - J_i^a S_i^a \right] \right) \quad (12)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for J_i^a .

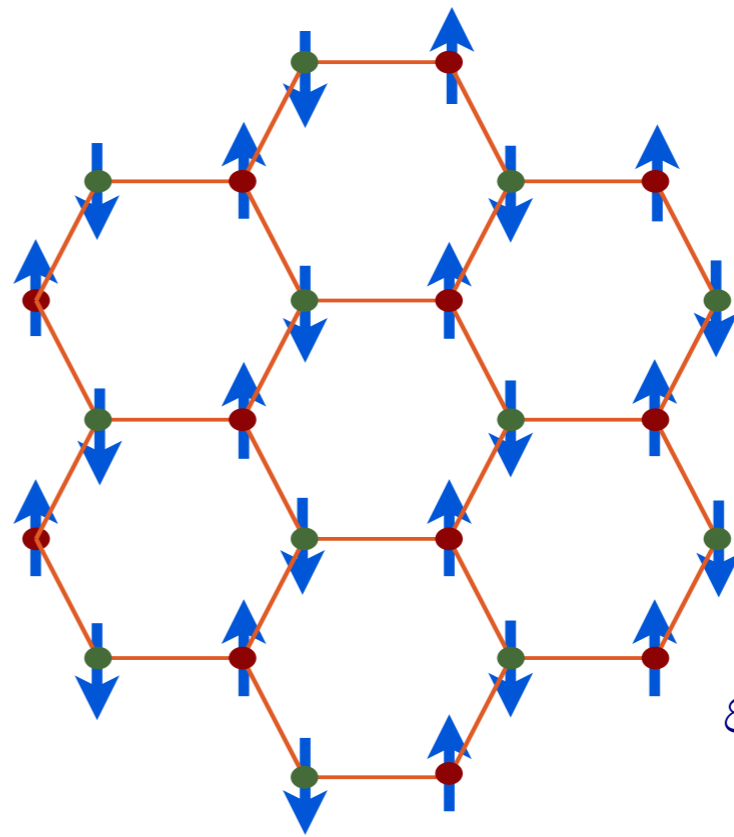
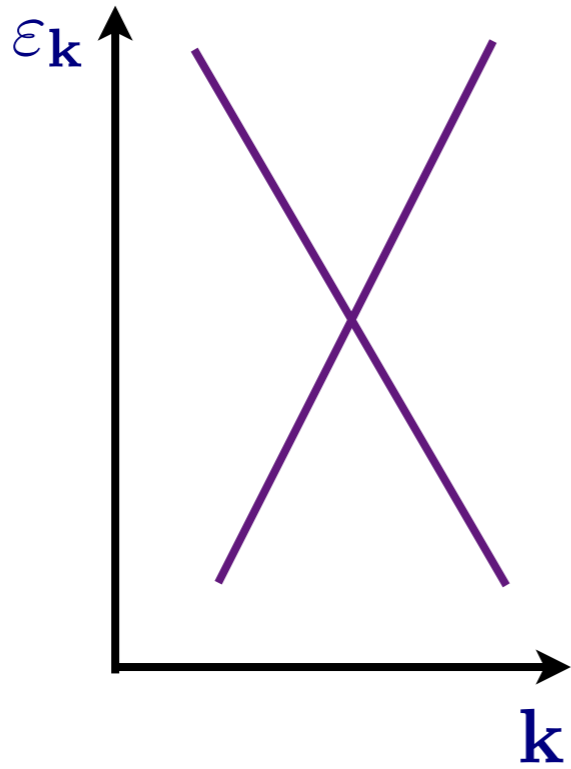
Long wavelength fluctuations about this saddle point are described by a field theory of the Néel order parameter, φ^a , coupled to the Dirac fermions in the **Gross-Neveu** model.

$$\mathcal{L} = \bar{\Psi} \gamma_{\mu} \partial_{\mu} \Psi + \frac{1}{2} \left[(\partial_{\mu} \varphi^a)^2 + s \varphi^{a2} \right] + \frac{u}{24} (\varphi^{a2})^2 - \lambda \varphi^a \bar{\Psi} \rho^z \sigma^a \Psi$$



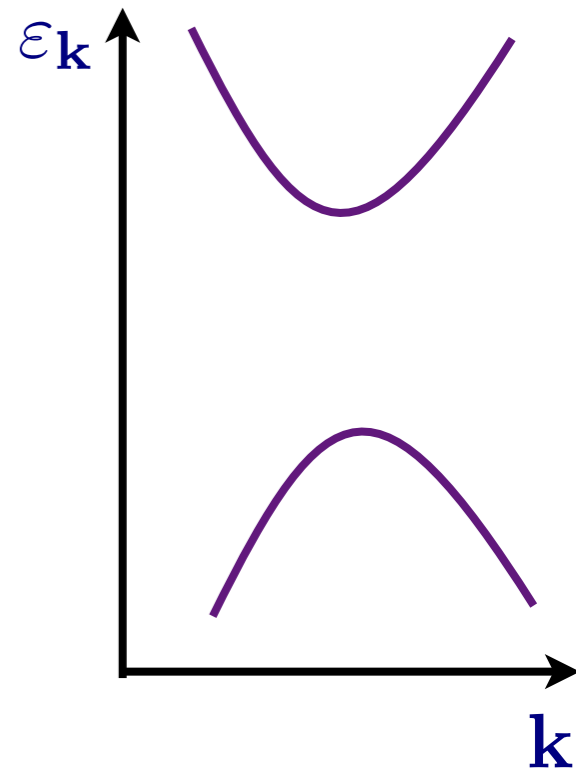
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$

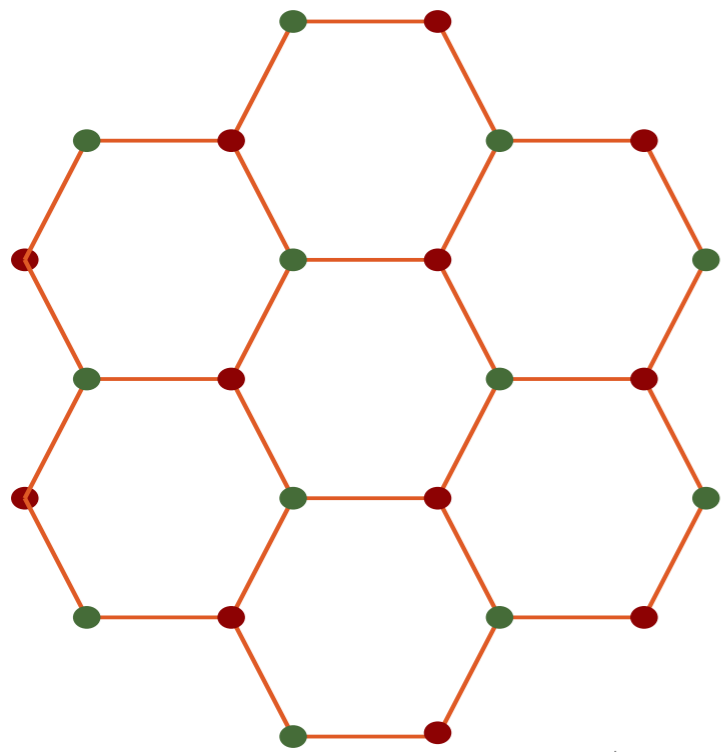


Insulating
antiferromagnet
with Neel order

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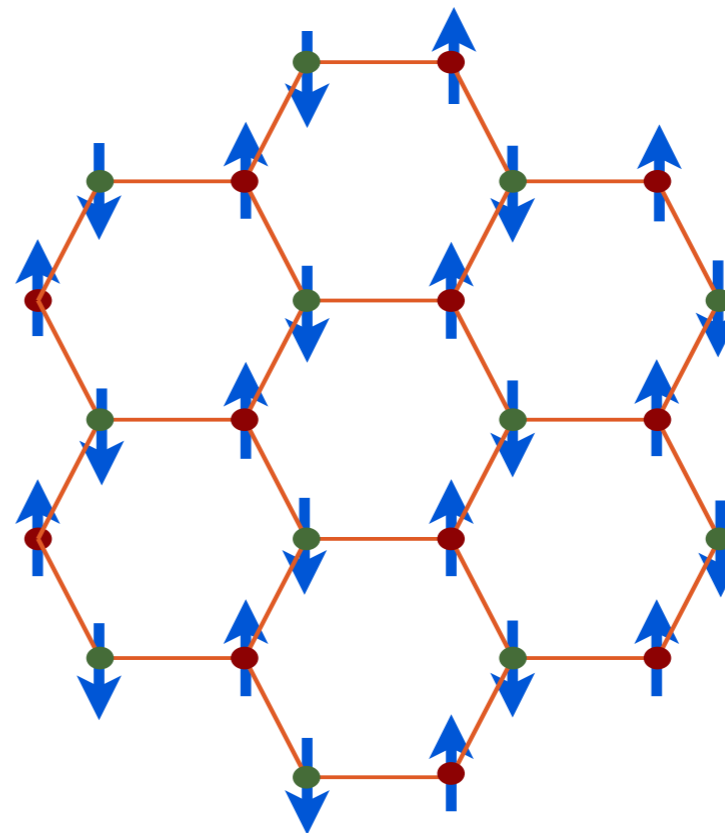
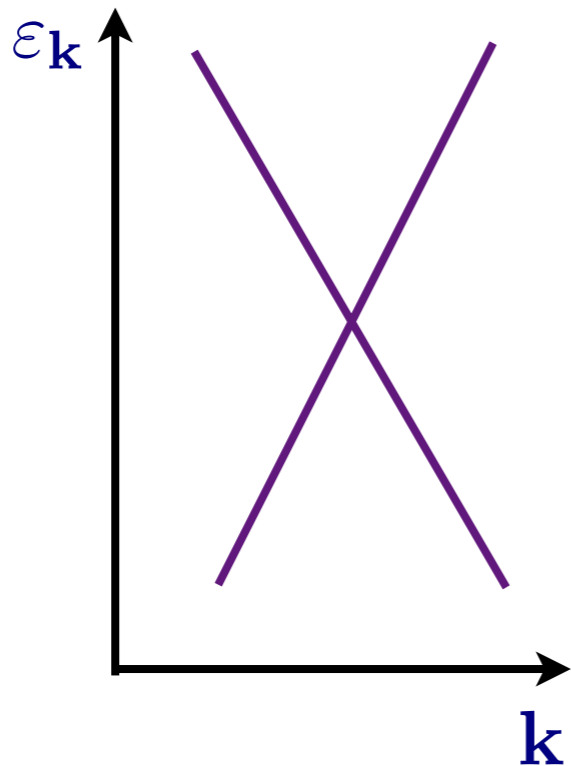


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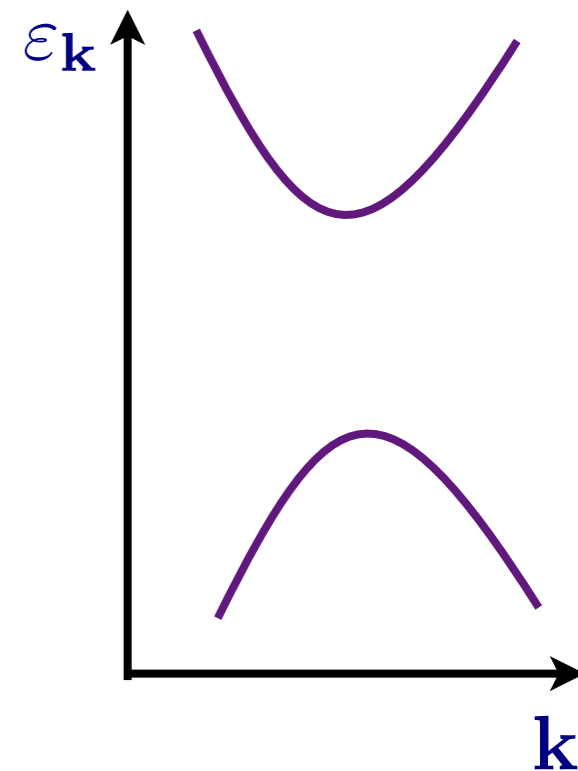
Dirac
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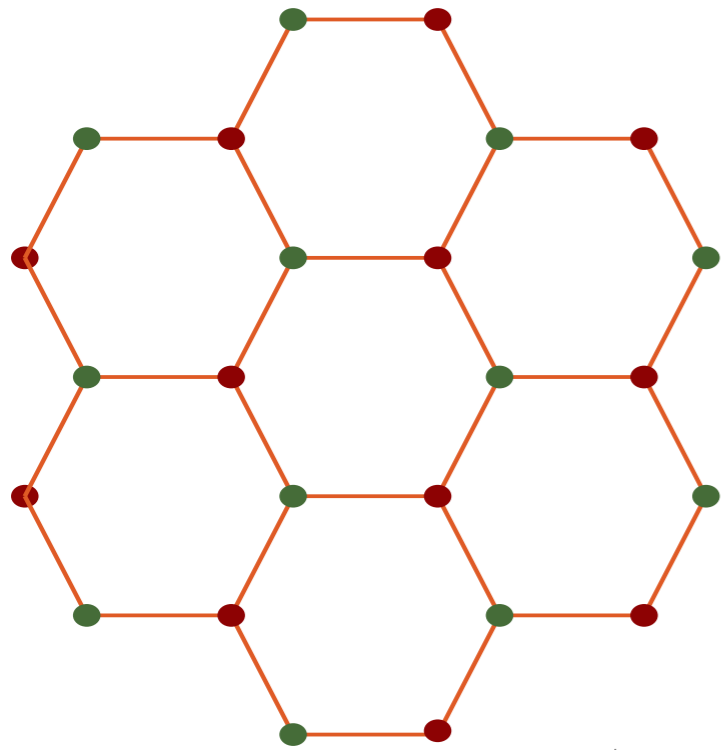
Insulating
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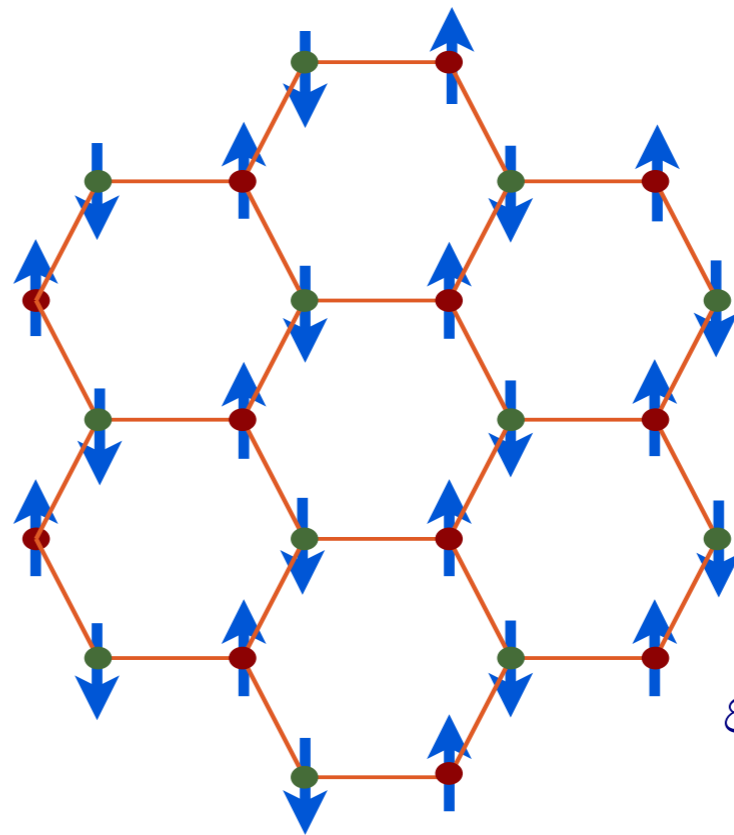
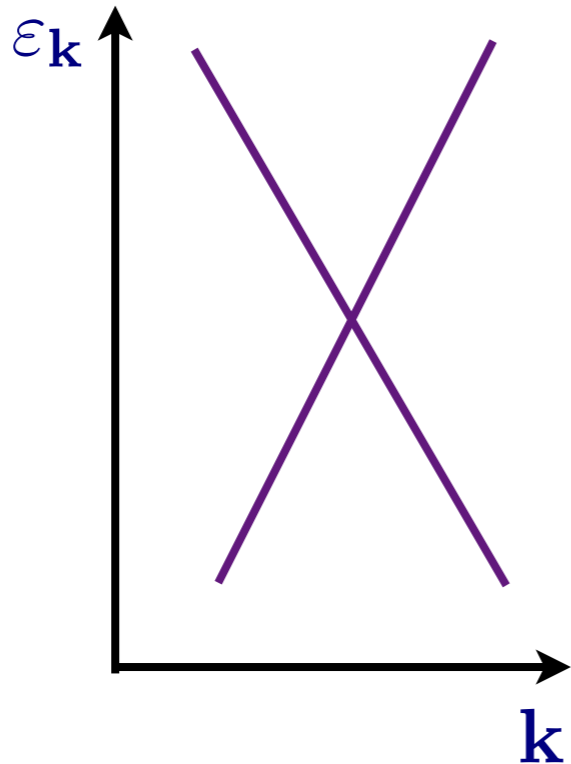
S

At the quantum critical point, the non-linear couplings λ and u in the Gross-Neveu model reach non-zero fixed-point values under the renormalization group flow. The critical theory is an *interacting* CFT₃



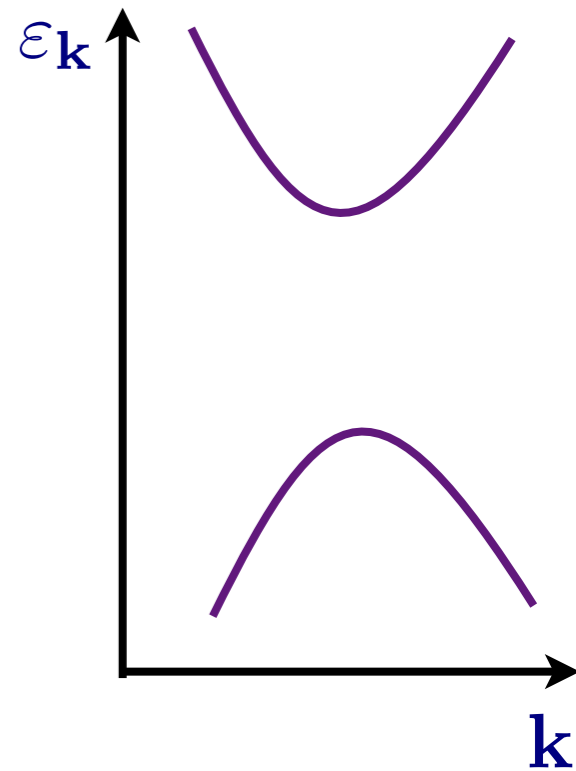
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Insulating
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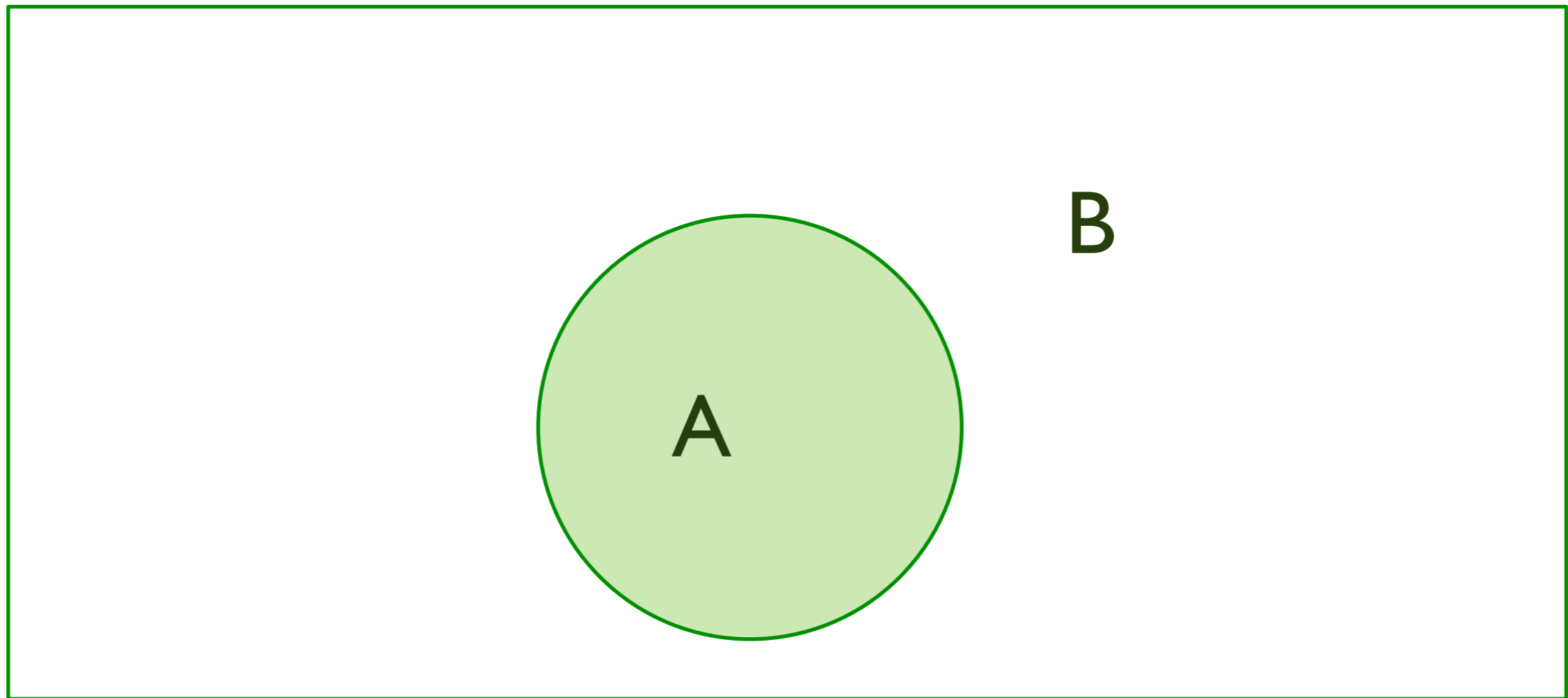
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Free CFT3

Interacting CFT3
with long-range entanglement

Long-range entanglement in a CFT3

- Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where γ is a universal number associated with the CFT3.



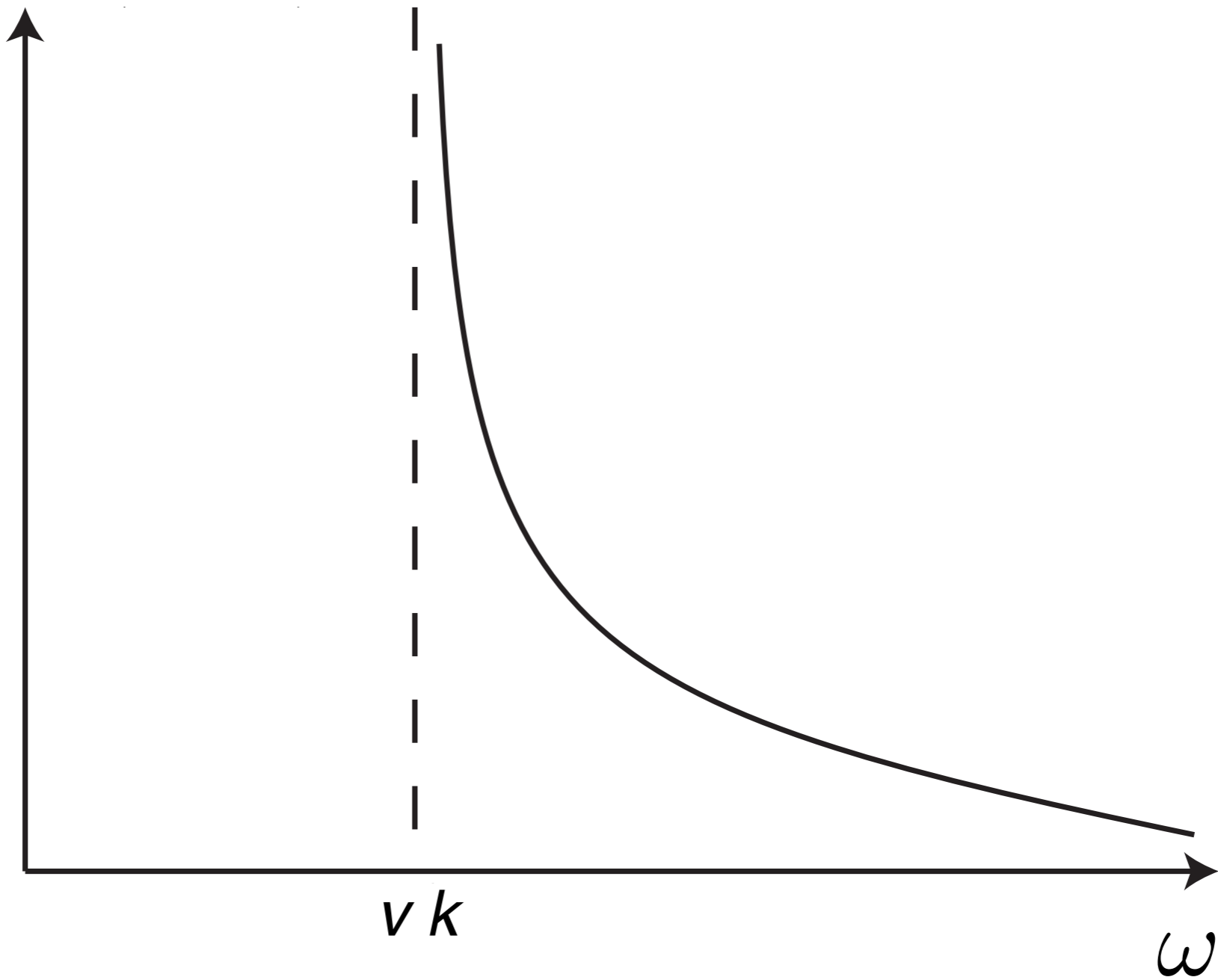
M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Electron Green's function for the interacting CFT3

$$G(k, \omega) = \langle \Psi(k, \omega); \Psi^\dagger(k, \omega) \rangle \sim \frac{i\omega + vk_x\tau^y + vk_y\tau^x\rho^z}{(\omega^2 + v^2k_x^2 + v^2k_y^2)^{1-\eta/2}}$$

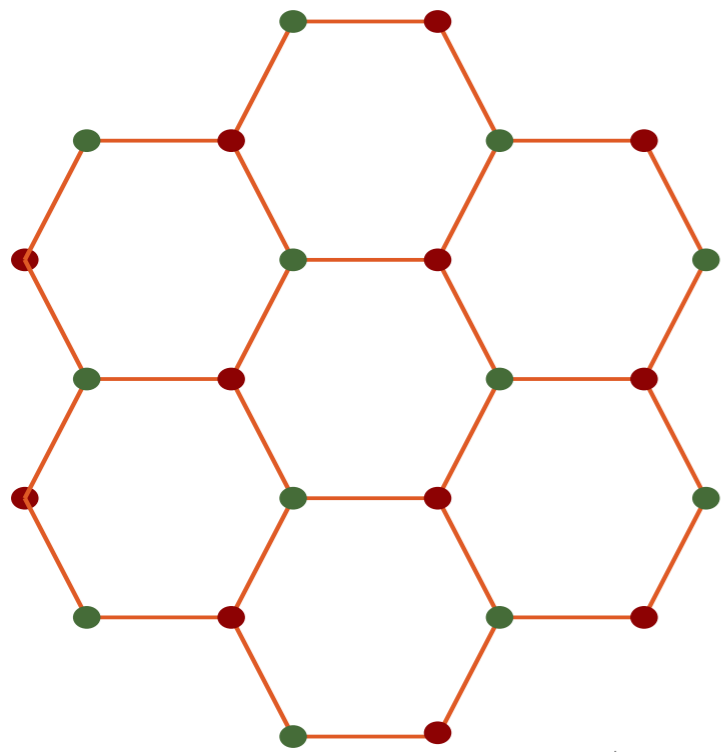
where $\eta > 0$ is the *anomalous dimension* of the fermion. Note that this leads to a fermion spectral density which has no quasiparticle pole: thus the quantum critical point has no well-defined quasiparticle excitations.

$\text{Im}G(k, \omega)$



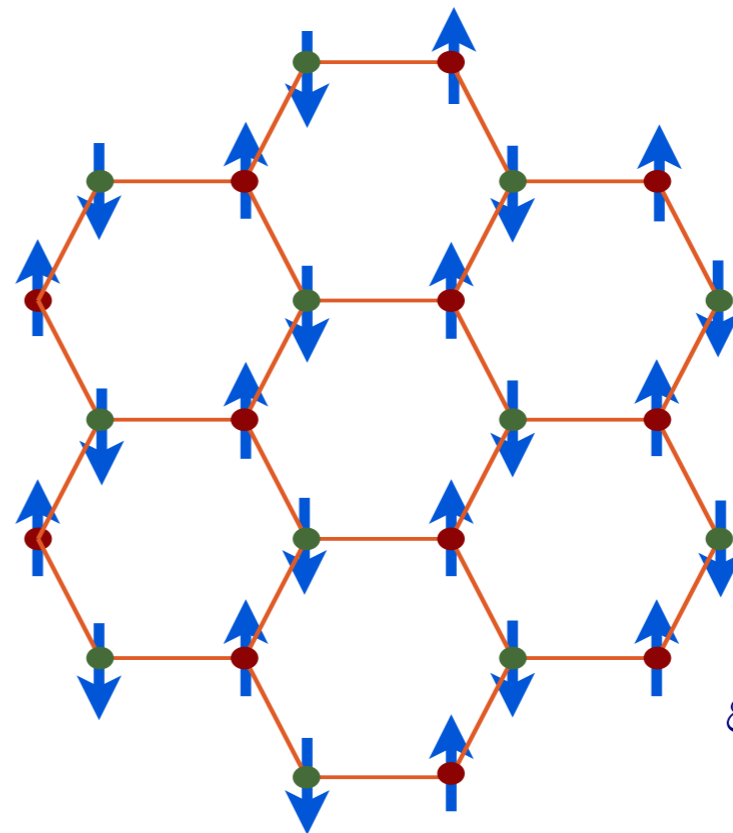
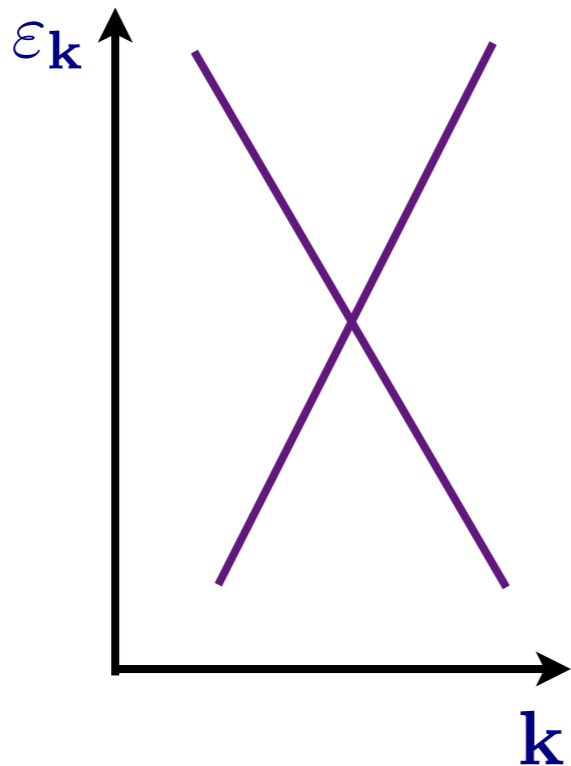
vk

ω



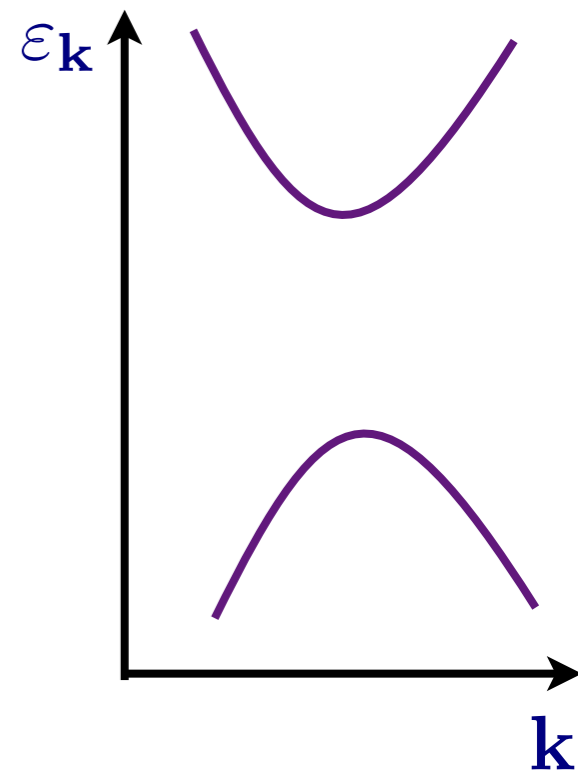
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

$$\langle \varphi^a \rangle \neq 0$$



S

Quantum phase transition described by a strongly-coupled conformal field theory without well-defined quasiparticles

Electrical transport

The conserved electrical current is

$$J_\mu = -i\bar{\Psi}\gamma_\mu\Psi. \quad (1)$$

Let us compute its two-point correlator, $K_{\mu\nu}(k)$ at a spacetime momentum k_μ at $T = 0$. At leading order, this is given by a one fermion loop diagram which evaluates to

$$\begin{aligned} K_{\mu\nu}(k) &= \int \frac{d^3p}{8\pi^3} \frac{\text{Tr} [\gamma_\mu (i\gamma_\lambda p_\lambda + m\rho^z \sigma^z) \gamma_\nu (i\gamma_\delta (k_\delta + p_\delta) + m\rho^z \sigma^z)]}{(p^2 + m^2)((p+k)^2 + m^2)} \\ &= -\frac{2}{\pi} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \int_0^1 dx \frac{k^2 x(1-x)}{\sqrt{m^2 + k^2 x(1-x)}}, \end{aligned} \quad (2)$$

where the mass $m = 0$ in the semi-metal and at the quantum critical point, while $m = |\lambda N_0|$ in the insulator. Note that the current correlation is purely transverse, and this follows from the requirement of current conservation

$$k_\mu K_{\mu\nu} = 0. \quad (3)$$

Of particular interest to us is the K_{00} component, after analytic continuation to Minkowski space where the spacetime momentum k_μ is replaced by (ω, k) . The conductivity is obtained from this correlator via the Kubo formula

$$\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} K_{00}(\omega, k). \quad (4)$$

In the insulator, where $m > 0$, analysis of the integrand in Eq. (2) shows that the spectral weight of the density correlator has a gap of $2m$ at $k = 0$, and the conductivity in Eq. (4) vanishes.

These properties are as expected in any insulator.

In the metal, and at the critical point, where $m = 0$, the fermionic spectrum is gapless, and so is that of the charge correlator. The density correlator in Eq. (2) and the conductivity in Eq. (4) evaluate to the simple universal results

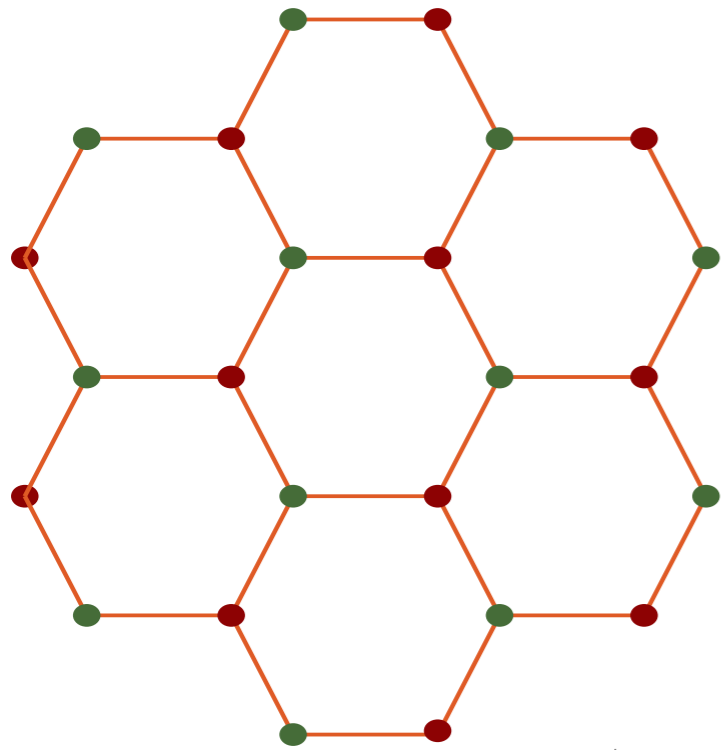
$$\begin{aligned} K_{00}(\omega, k) &= \frac{1}{4} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\ \sigma(\omega) &= 1/4. \end{aligned} \quad (5)$$

Going beyond one-loop, we find *no change* in these results in the

semi-metal to all orders in perturbation theory. At the quantum critical point, there are no anomalous dimensions for the conserved current, but the amplitude does change yielding

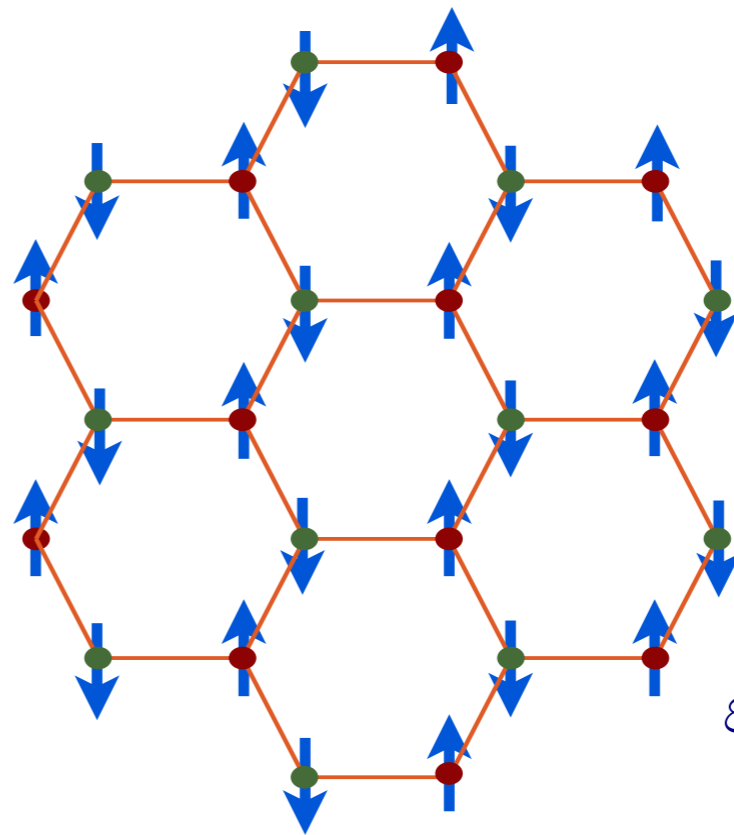
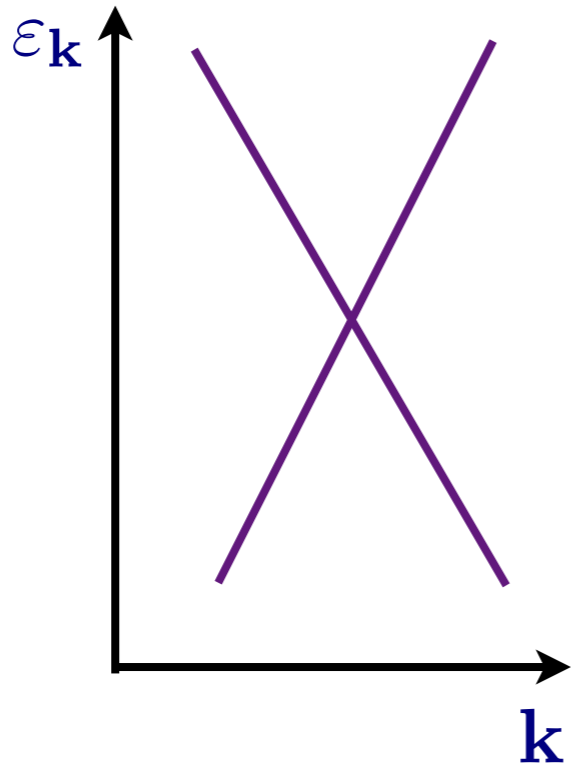
$$\begin{aligned} K_{00}(\omega, k) &= \mathcal{K} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\ \sigma(\omega) &= \mathcal{K}, \end{aligned} \tag{6}$$

where \mathcal{K} is a universal number dependent only upon the universality class of the quantum critical point. The value of the \mathcal{K} for the Gross-Neveu model is not known exactly, but can be estimated by computations in the $(3 - d)$ or $1/N$ expansions.



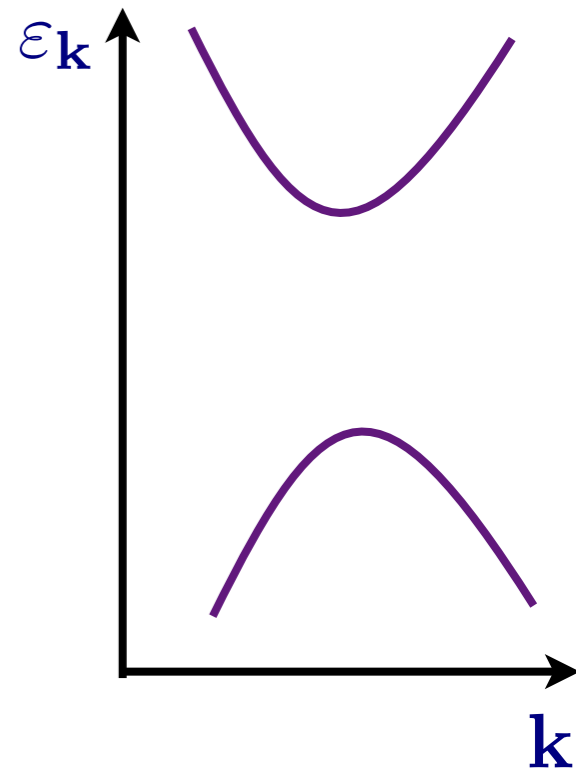
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

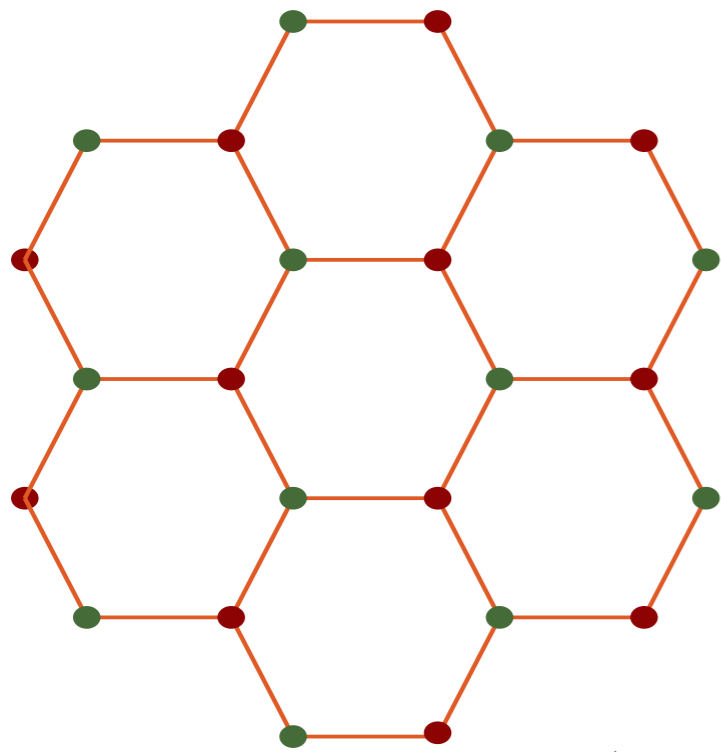
$$\langle \varphi^a \rangle \neq 0$$



S

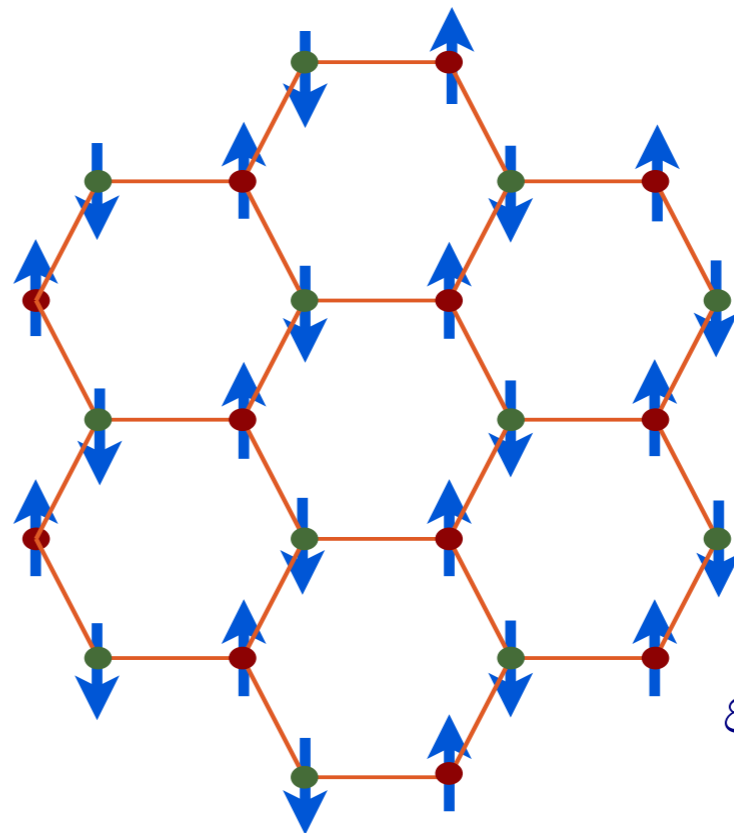
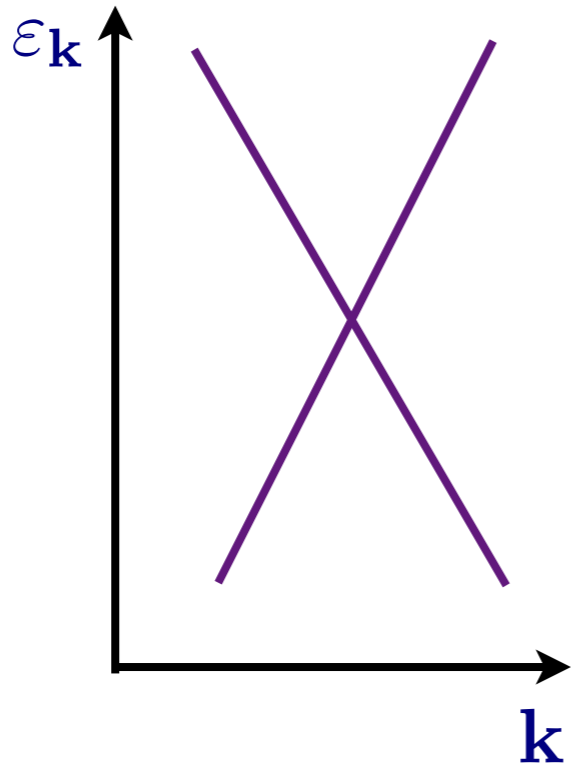
Free CFT3

Interacting CFT3
with long-range entanglement



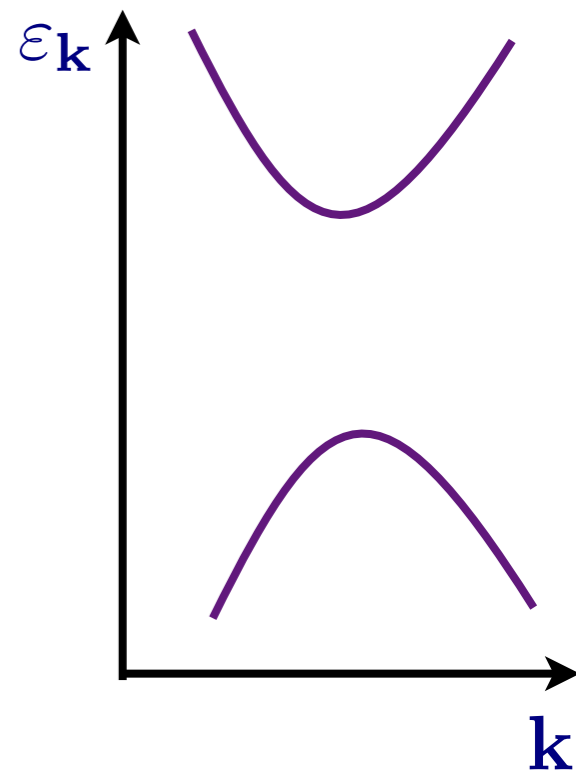
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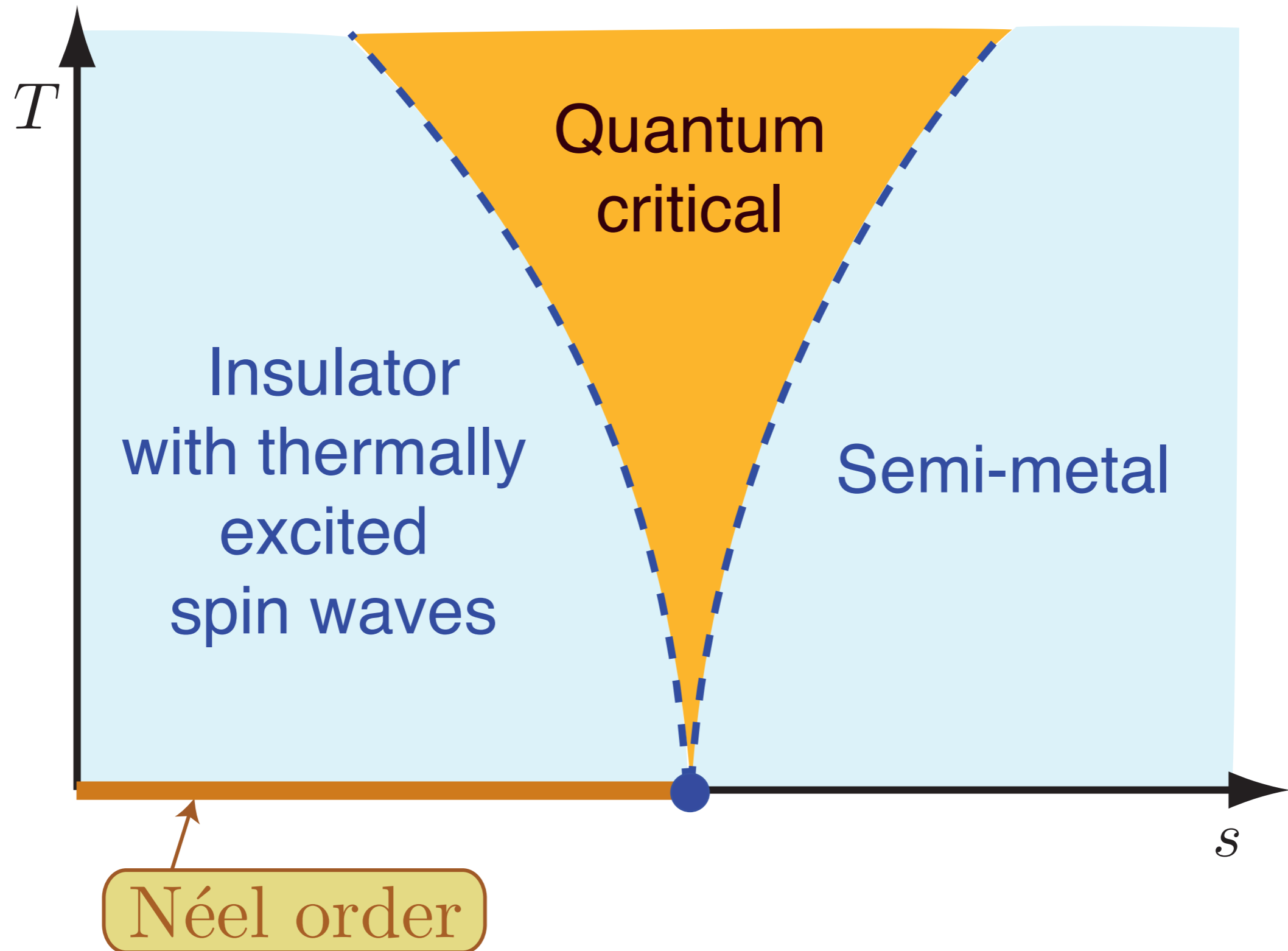


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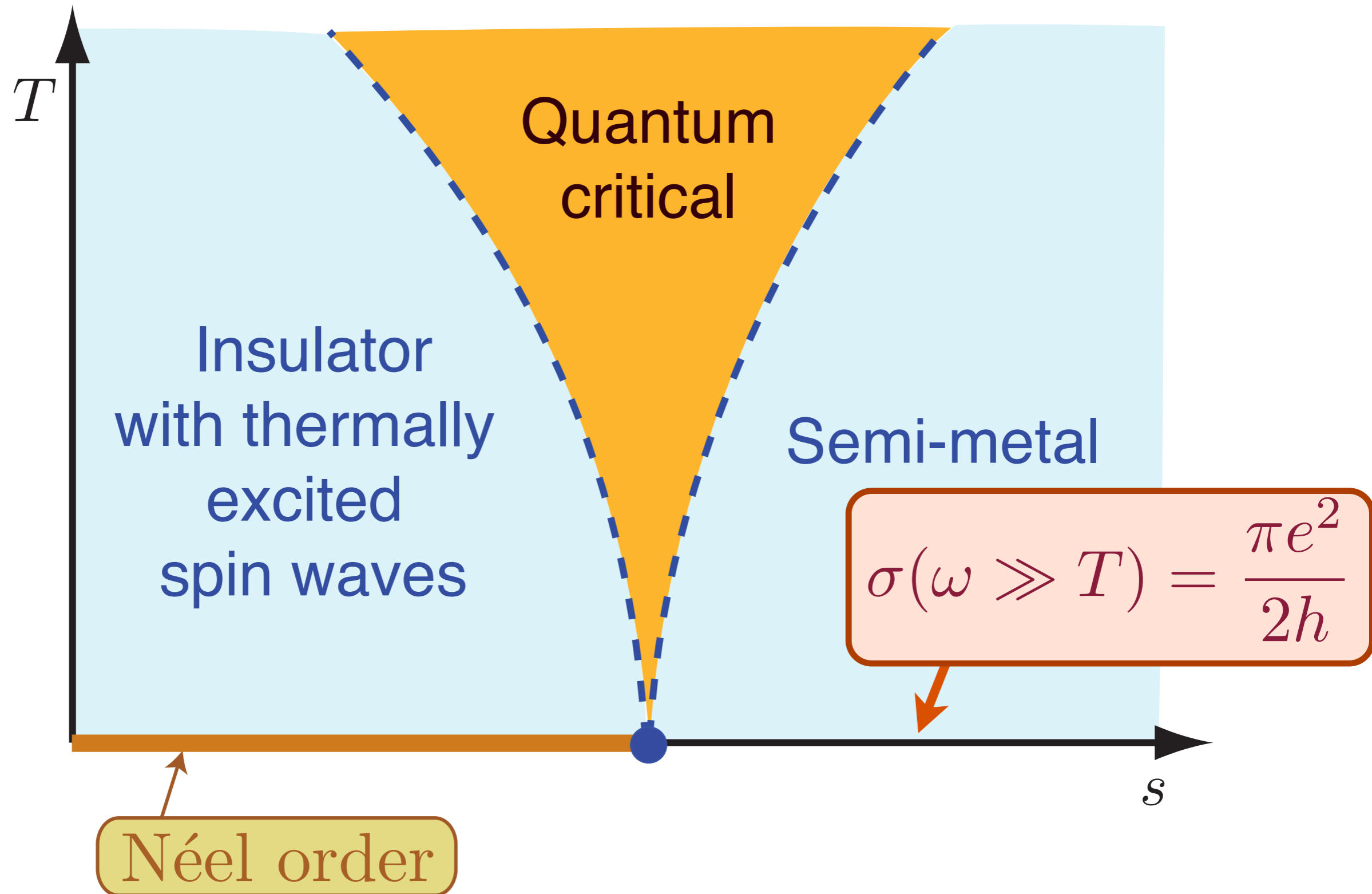
$$\sigma(\omega) = \frac{\pi e^2}{2h}$$

$$\sigma(\omega) = \frac{\mathcal{K} e^2}{h}$$

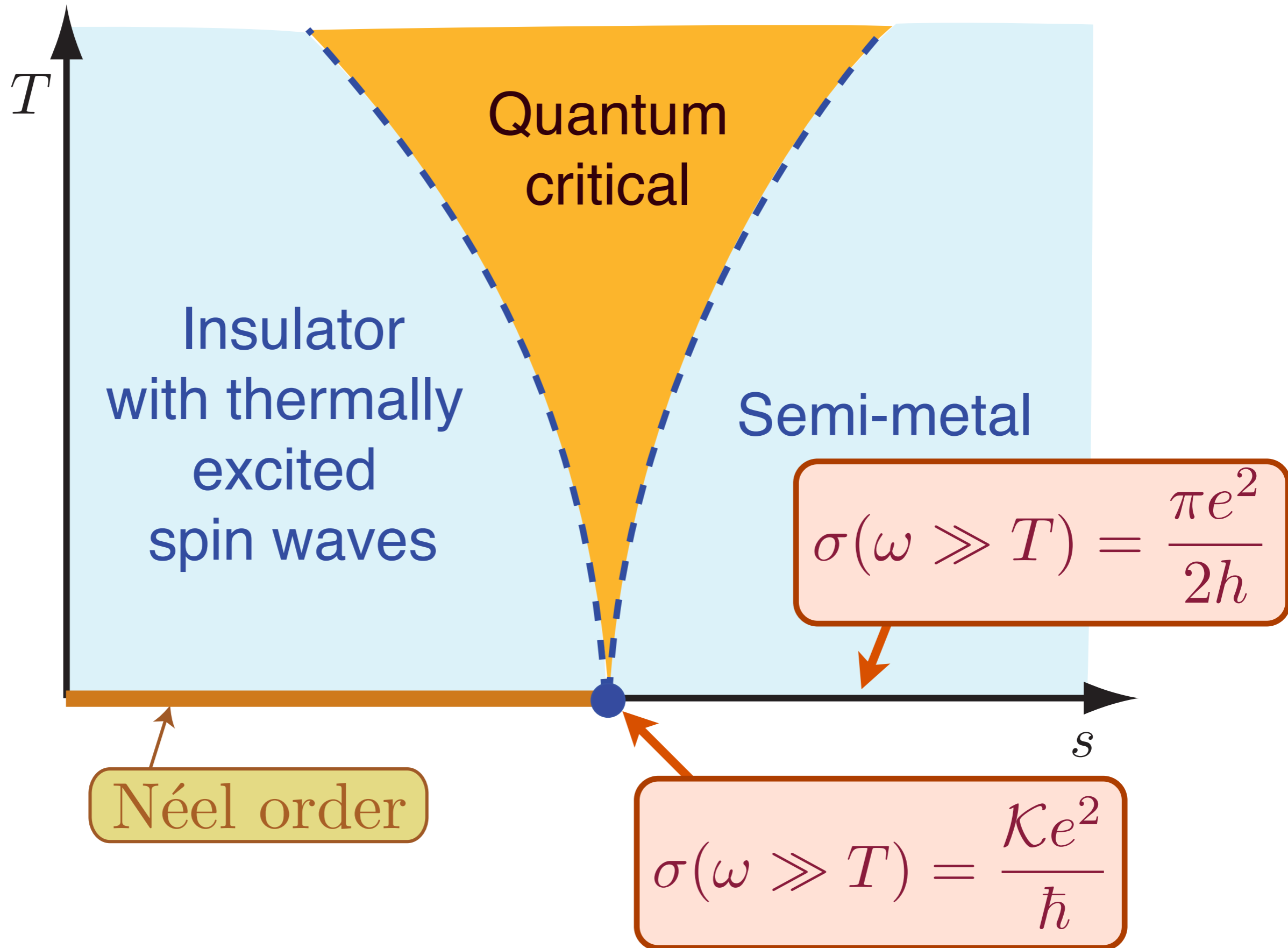
Phase diagram at non-zero temperatures



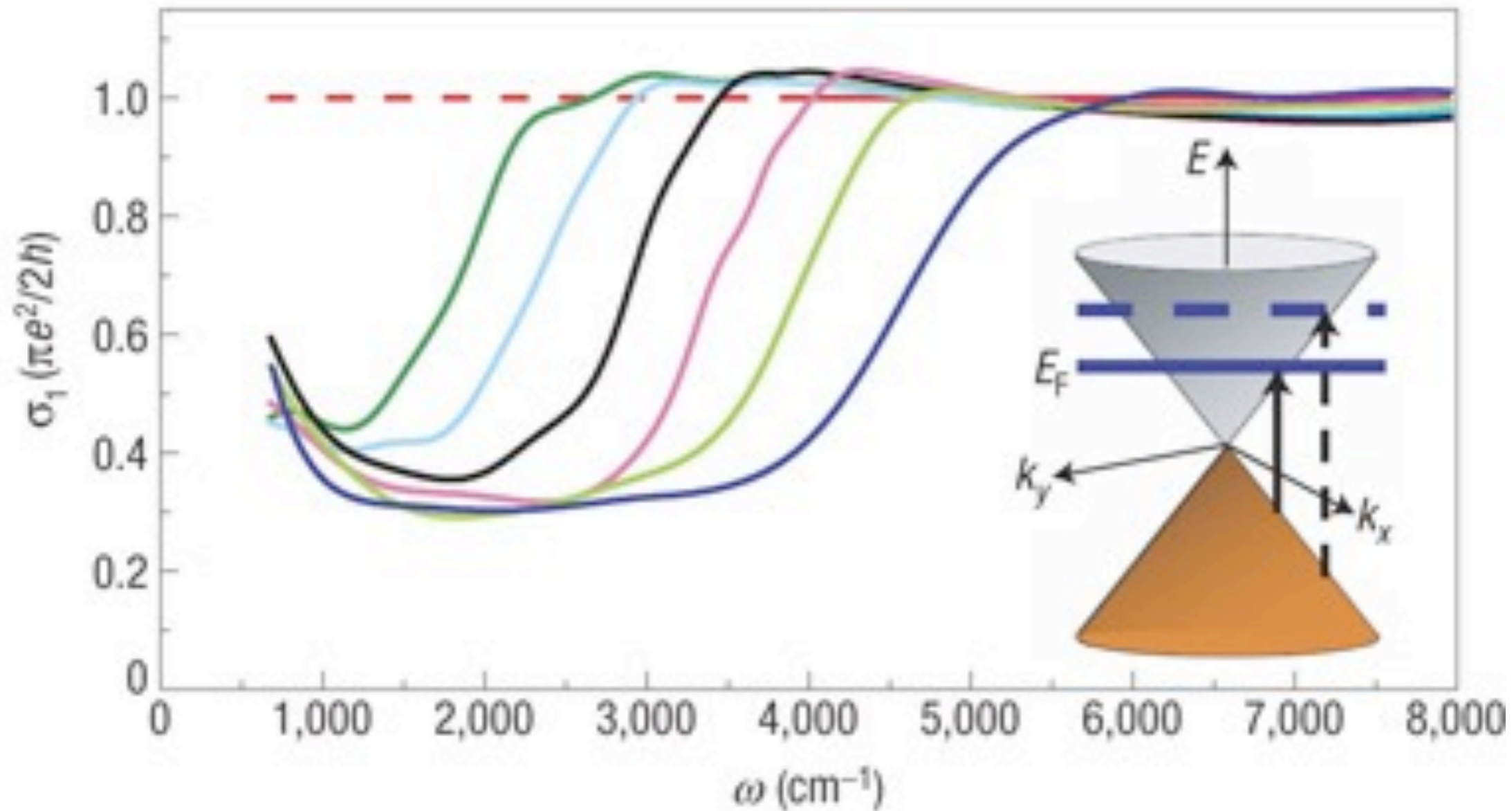
Phase diagram at non-zero temperatures



Phase diagram at non-zero temperatures



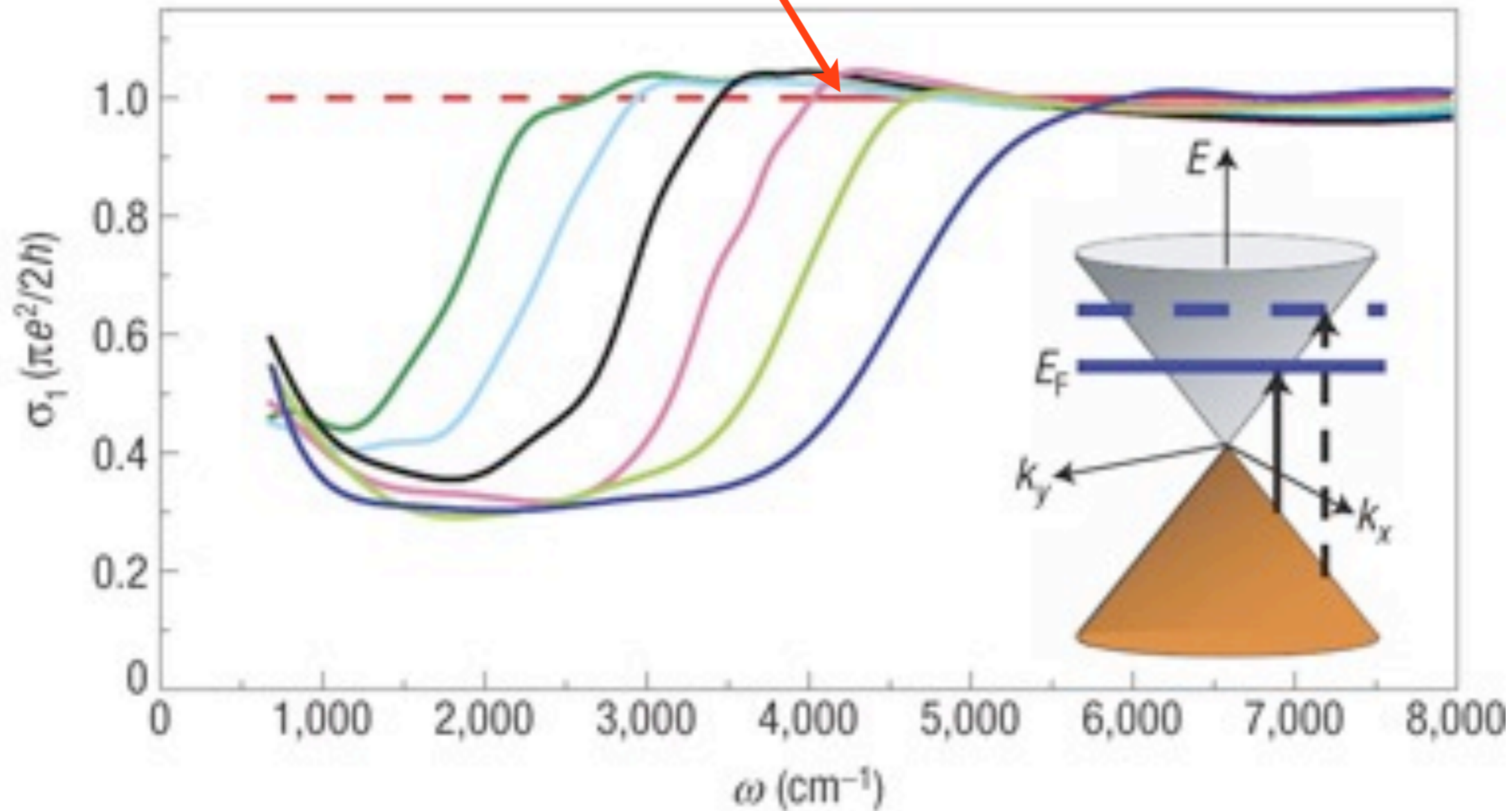
Optical conductivity of graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* 4, 532 (2008).

Optical conductivity of graphene

Undoped graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

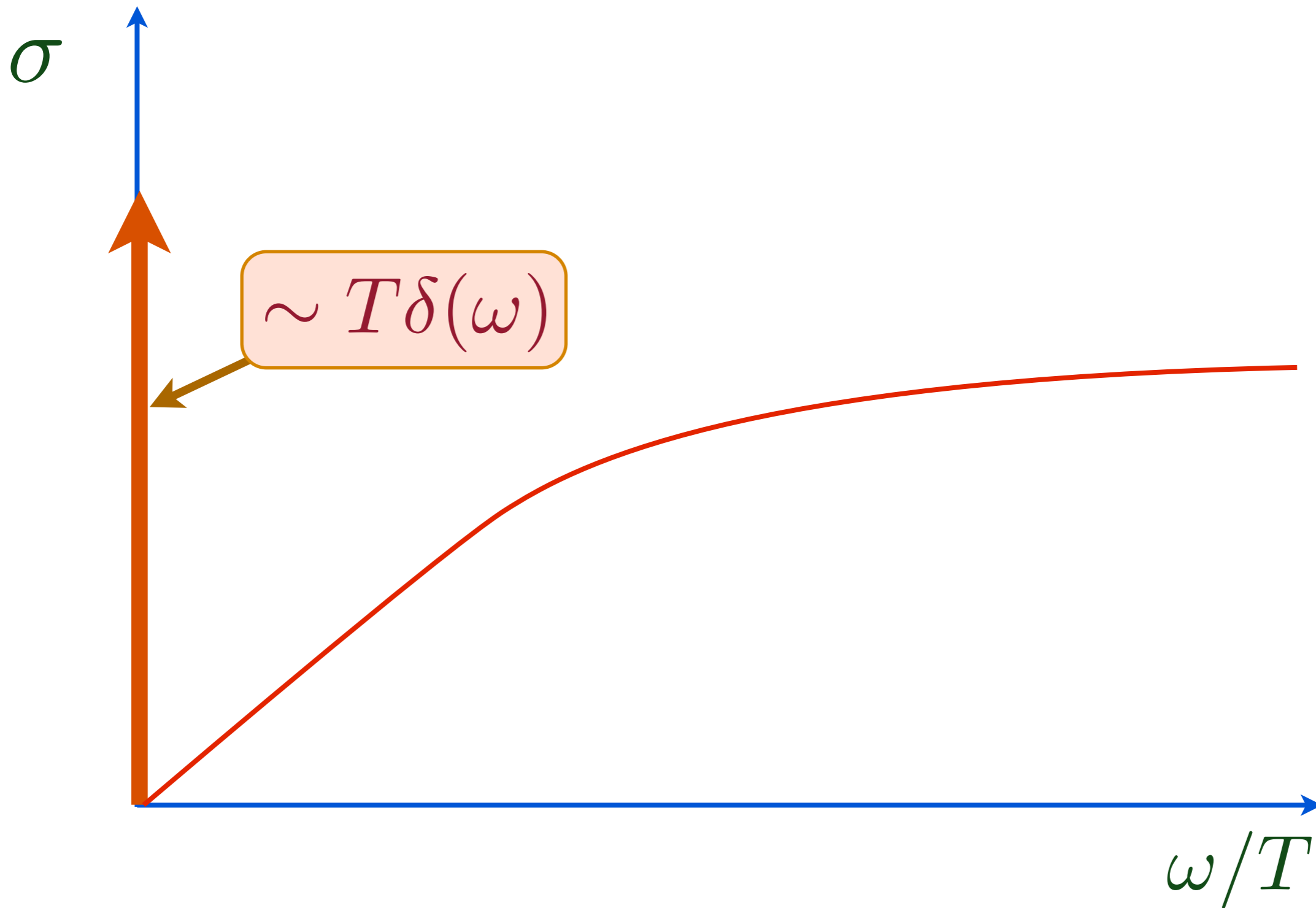
Non-zero temperatures

At the quantum-critical point at one-loop order, we can set $m = 0$, and then repeat the computation in Eq. (2) at $T > 0$. This only requires replacing the integral over the loop frequency by a summation over the Matsubara frequencies, which are quantized by odd multiples of πT . Such a computation, via Eq. (4) leads to the conductivity

$$\text{Re}[\sigma(\omega)] = (2T \ln 2) \delta(\omega) + \frac{1}{4} \tanh\left(\frac{|\omega|}{4T}\right); \quad (7)$$

the imaginary part of $\sigma(\omega)$ is the Hilbert transform of $\text{Re}[\sigma(\omega)] - 1/4$. Note that this reduces to Eq. (5) in the limit $\omega \gg T$. However, the most important new feature of Eq. (7) arises for $\omega \ll T$, where we find a delta function at zero frequency in the real part. Thus the d.c. conductivity is infinite at this order, arising from the collisionless transport of thermally excited carriers.

Electrical transport in a free CFT3 for $T > 0$



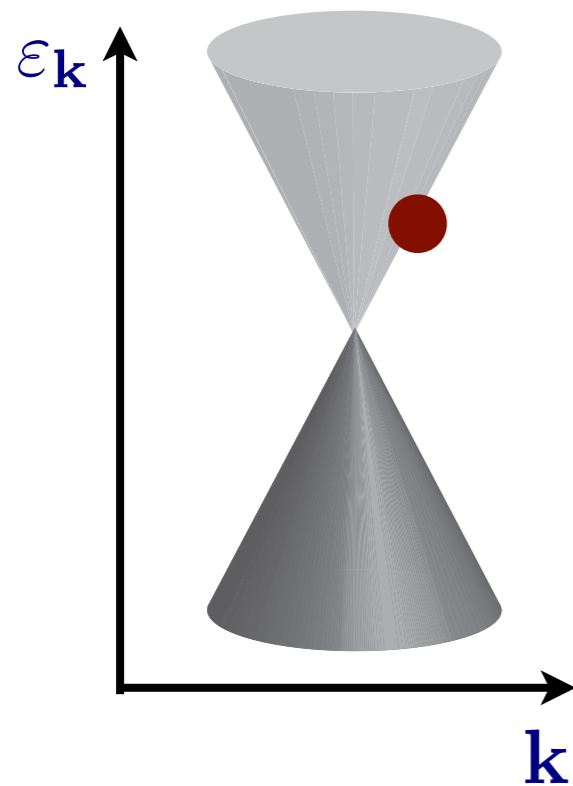
Particles



Momentum



Current



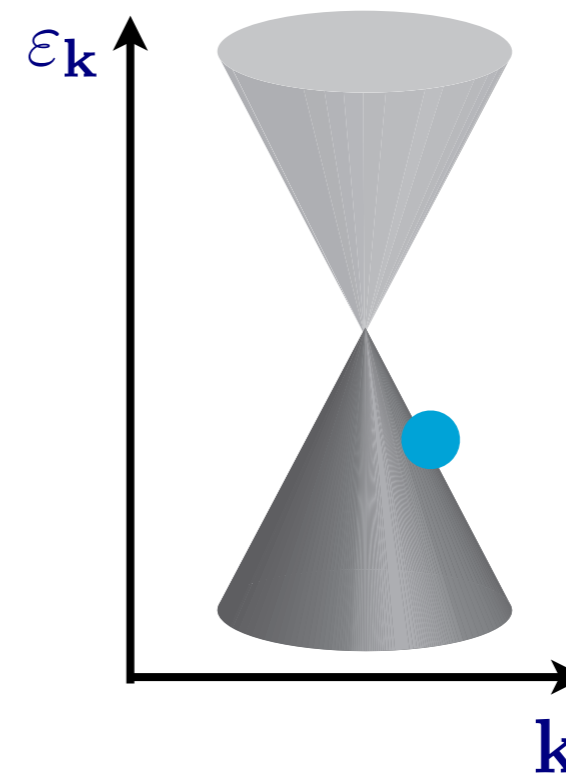
Holes



Momentum



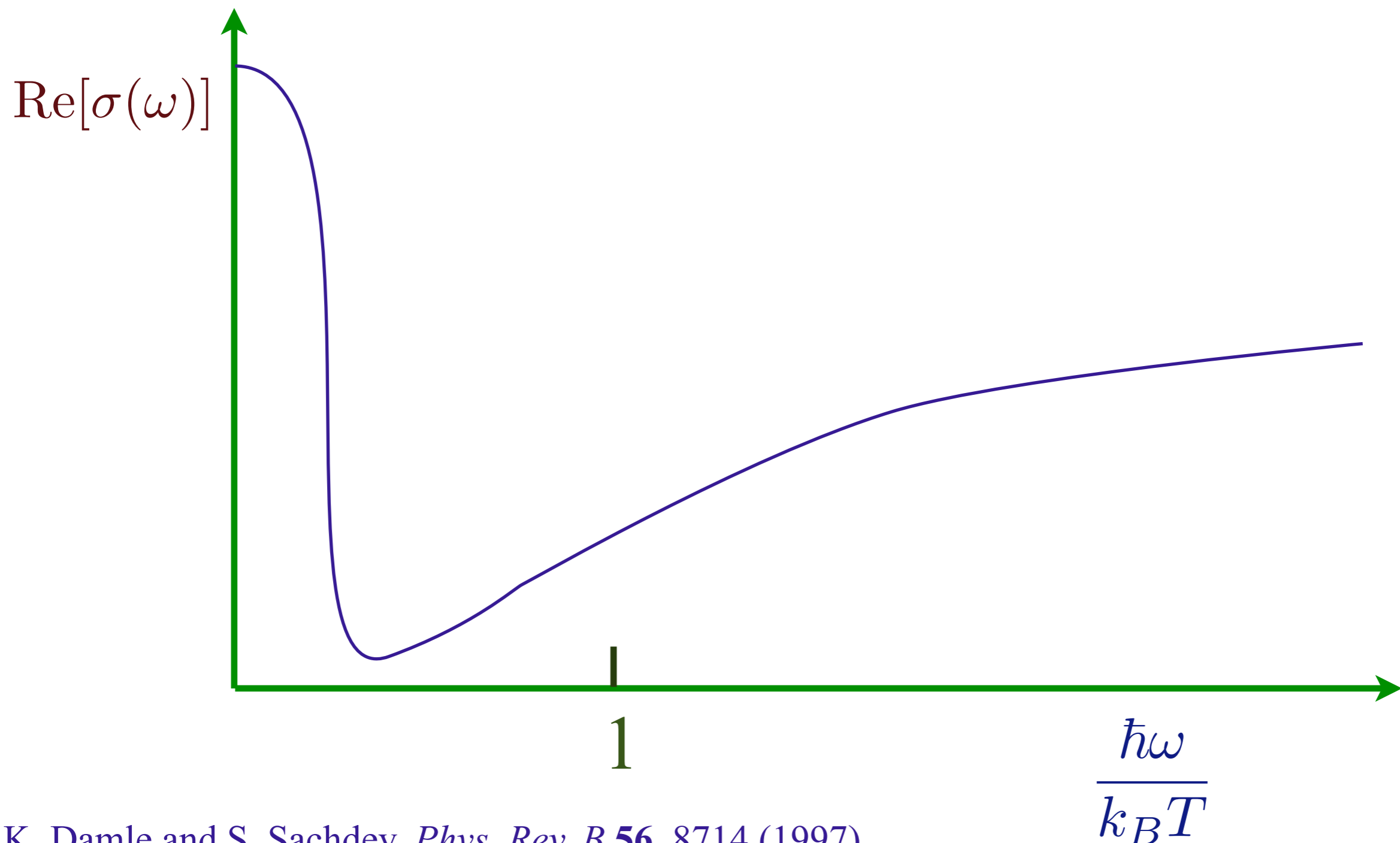
Current



Particle hole symmetry: current carrying state has zero momentum, and collisions can relax current to zero

Electrical transport for a (weakly) interacting CFT3

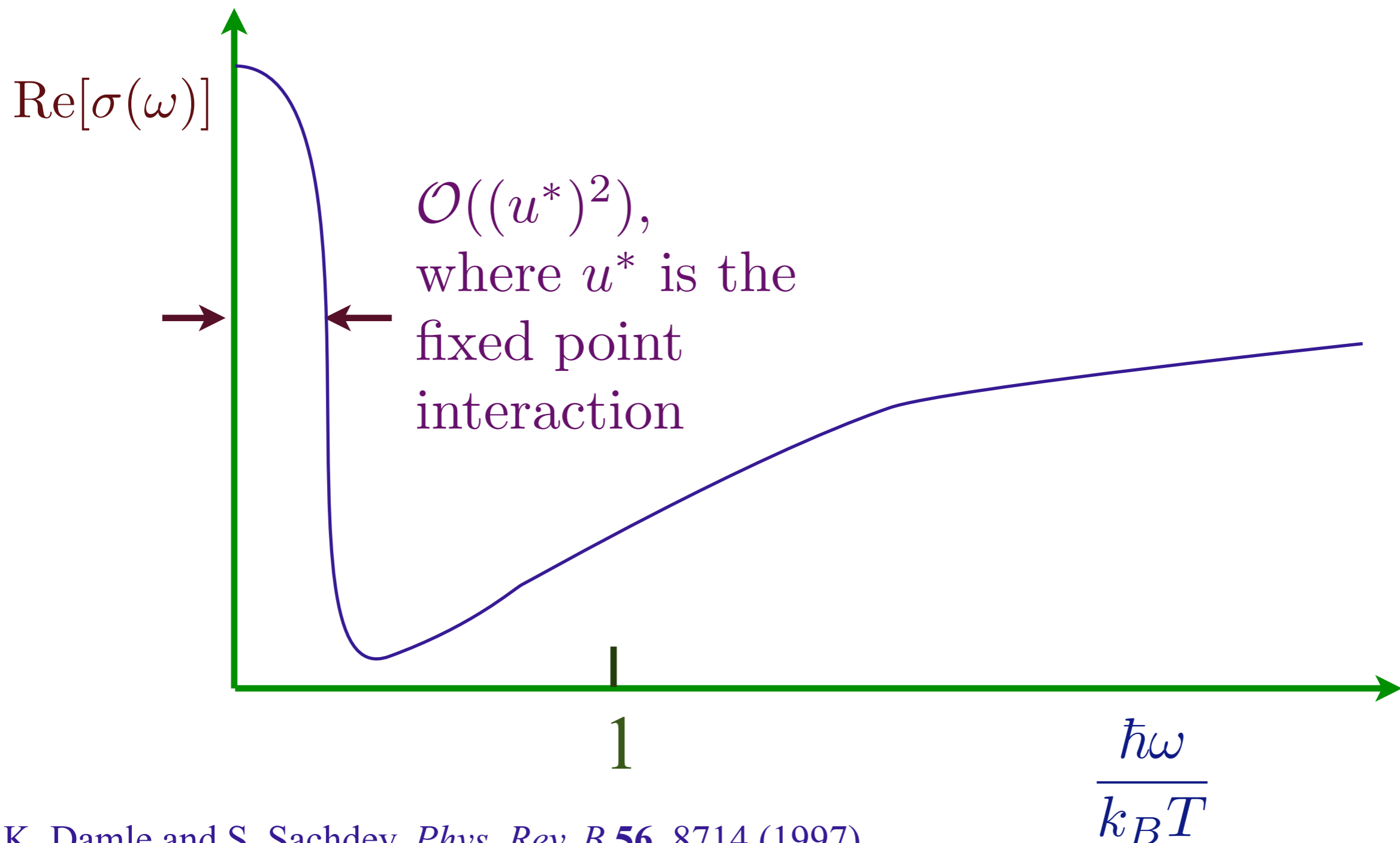
$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right) ; \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Electrical transport for a (weakly) interacting CFT3

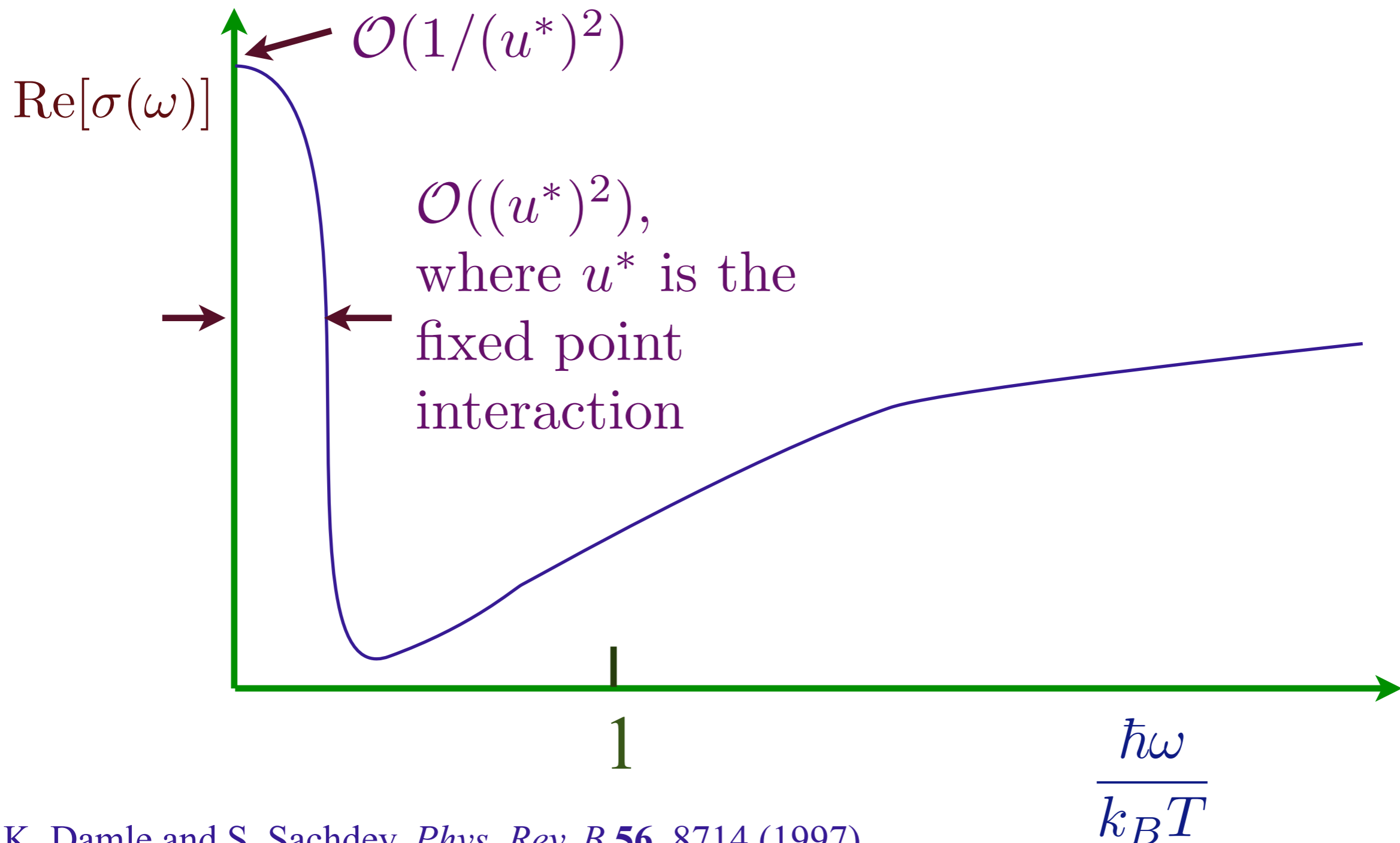
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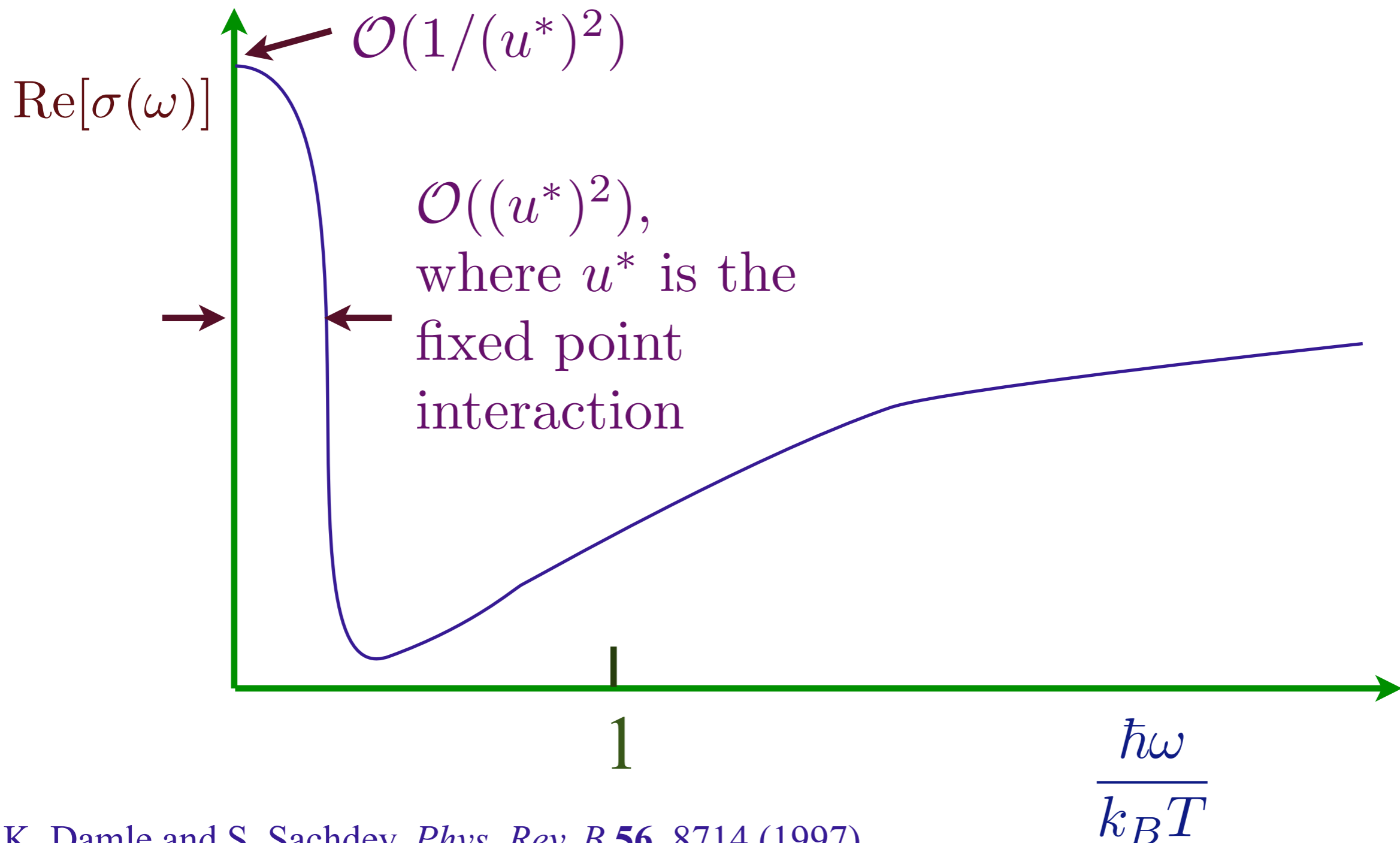
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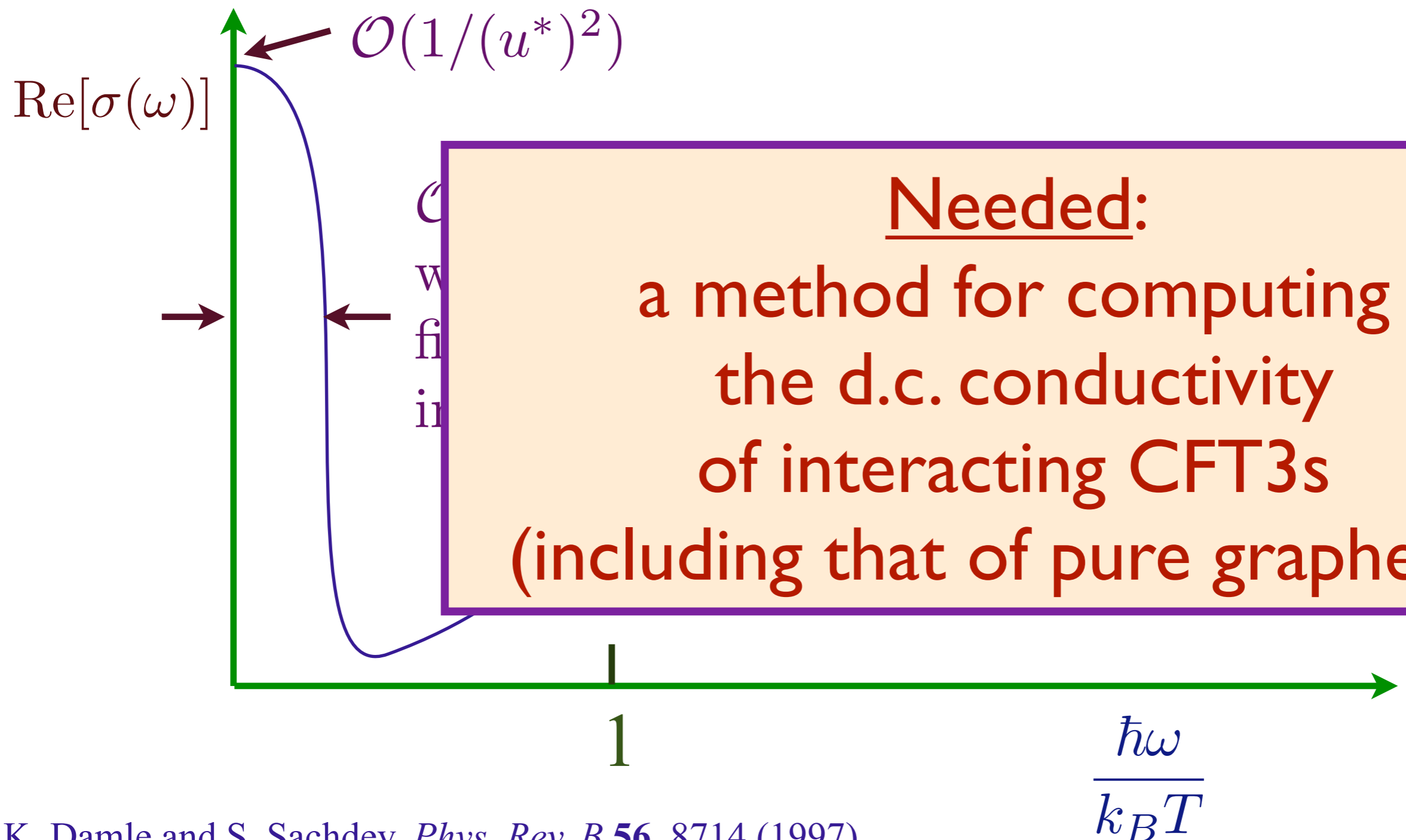
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K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Conformal quantum matter

A. Field theory: graphene

*B. Field theory: superfluid-
insulator transition*

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Conformal quantum matter

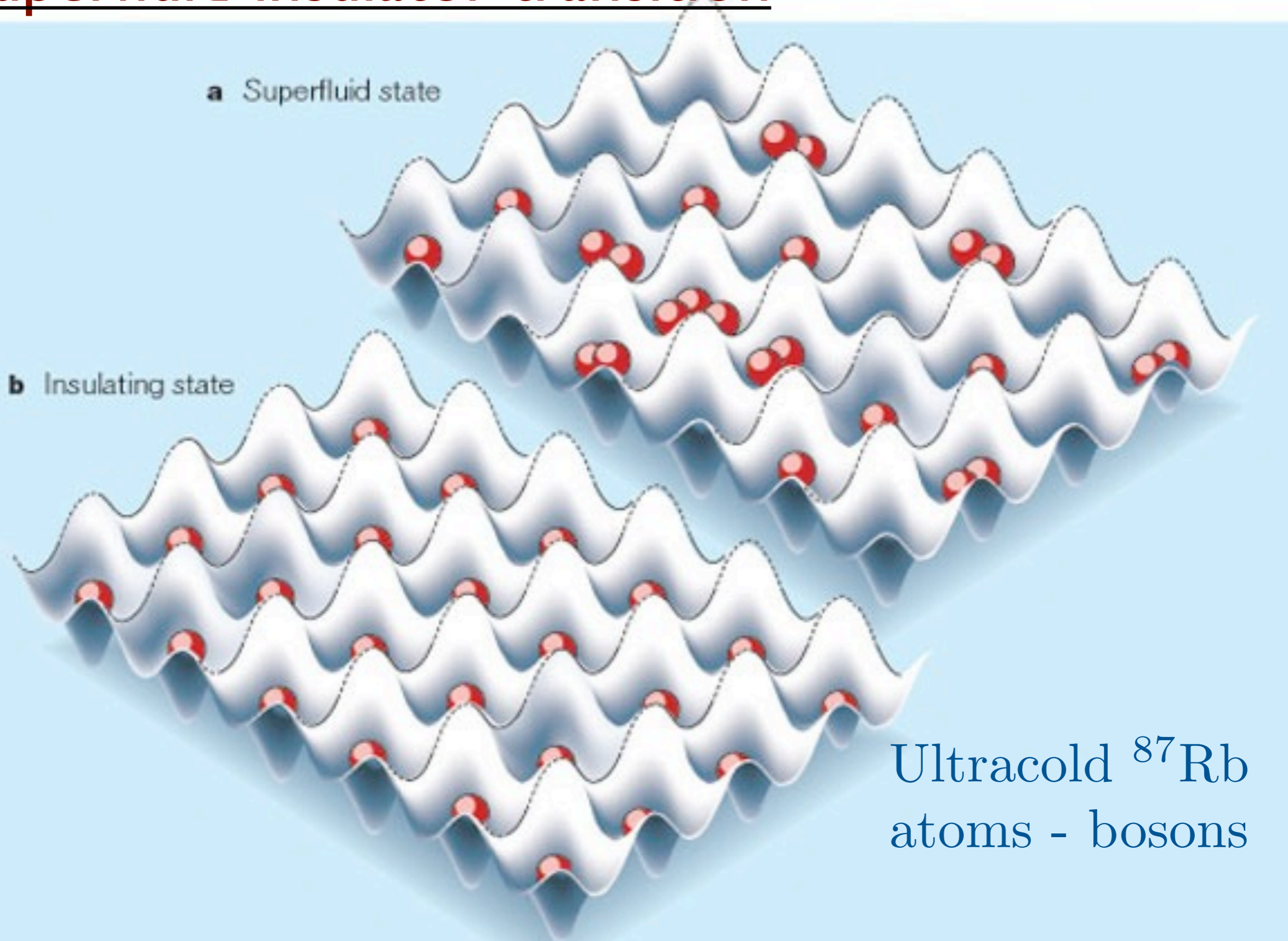
A. Field theory: graphene

*B. Field theory: superfluid-
insulator transition*

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, b_j^\dagger , hopping between the sites, j , of a lattice, with short-range repulsive interactions.

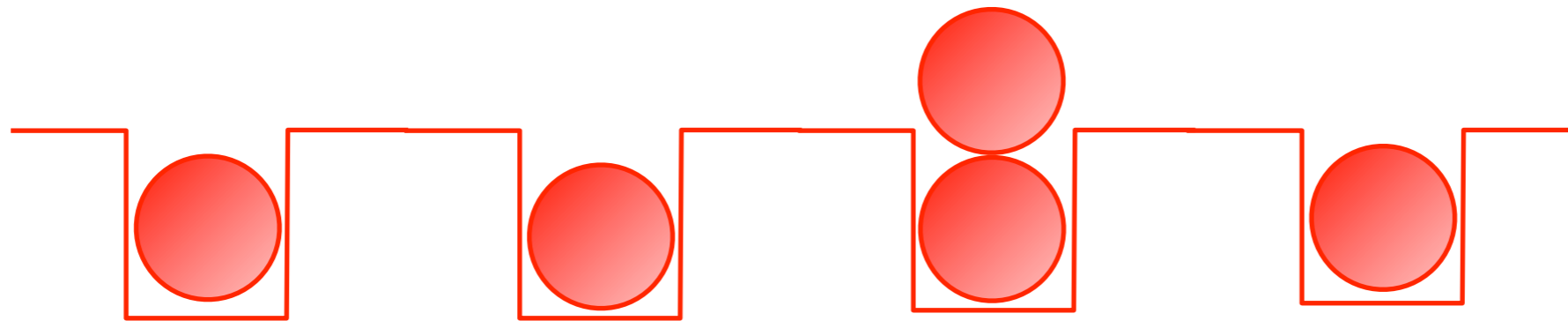
$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \dots$$

$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

Excitations of the insulator:



Particles $\sim \psi^\dagger$



Holes $\sim \psi$

Density of particles = density of holes \Rightarrow

“relativistic” field theory for ψ :

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c) |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

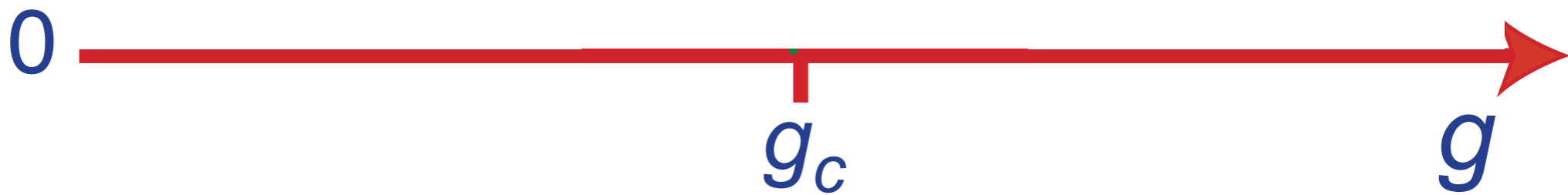
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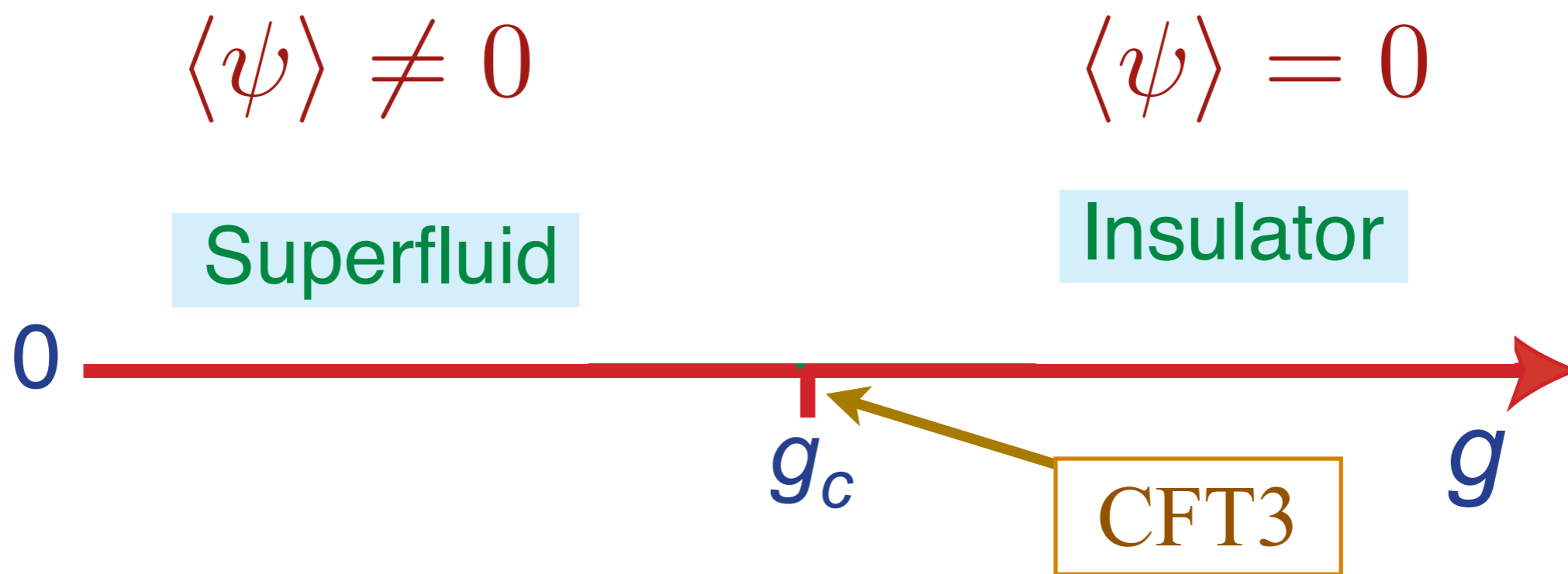
$$\langle \psi \rangle \neq 0$$

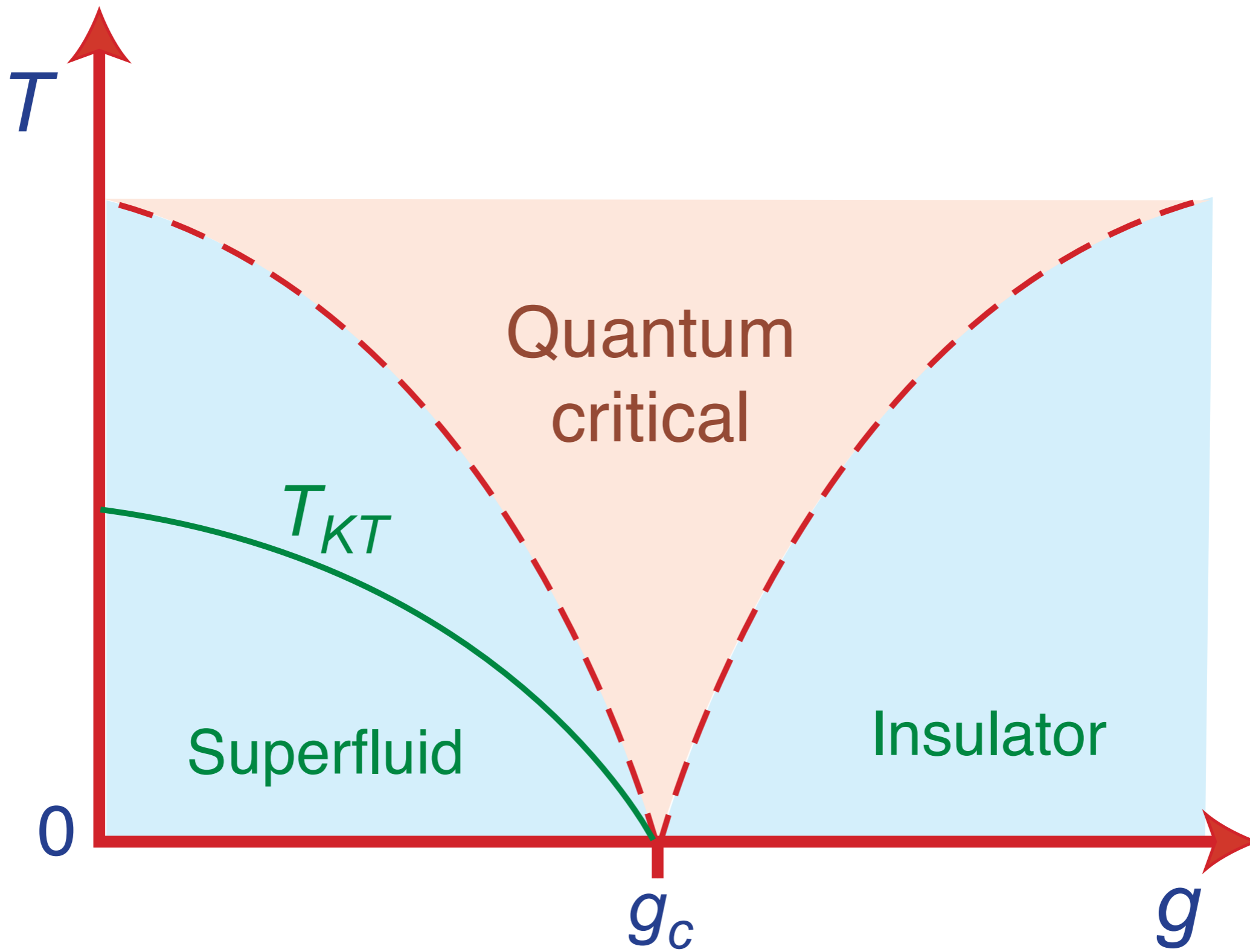
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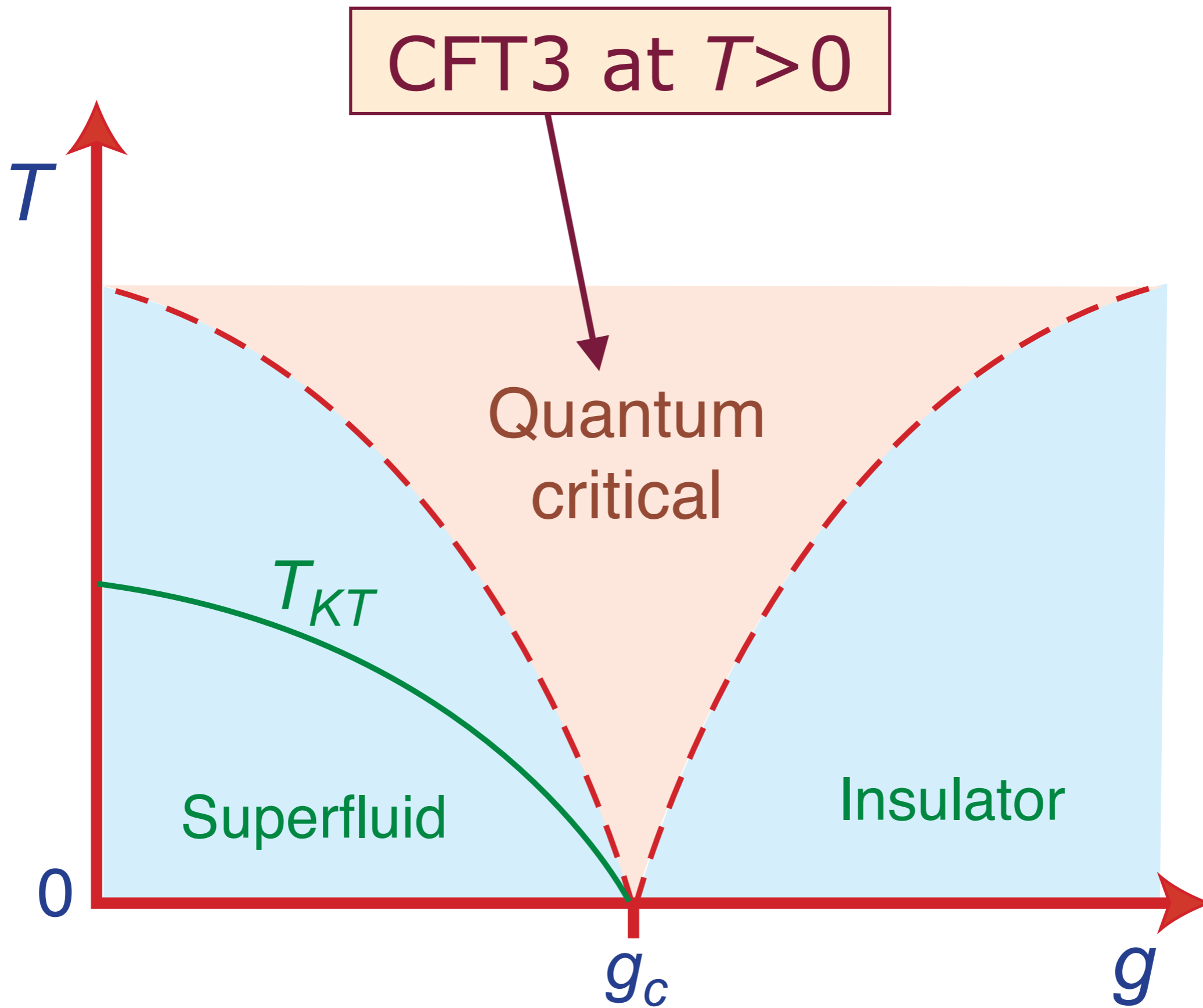
Superfluid

Insulator









Quantum critical transport

Quantum “*nearly perfect fluid*”
with shortest possible
equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where \mathcal{C} is a *universal* constant

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Zaanen: Planckian dissipation

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999).

Quantum critical transport

Transport co-efficients not determined
by collision rate, but by
universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

(Q is the “charge” of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-efficients not determined
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Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)]$$

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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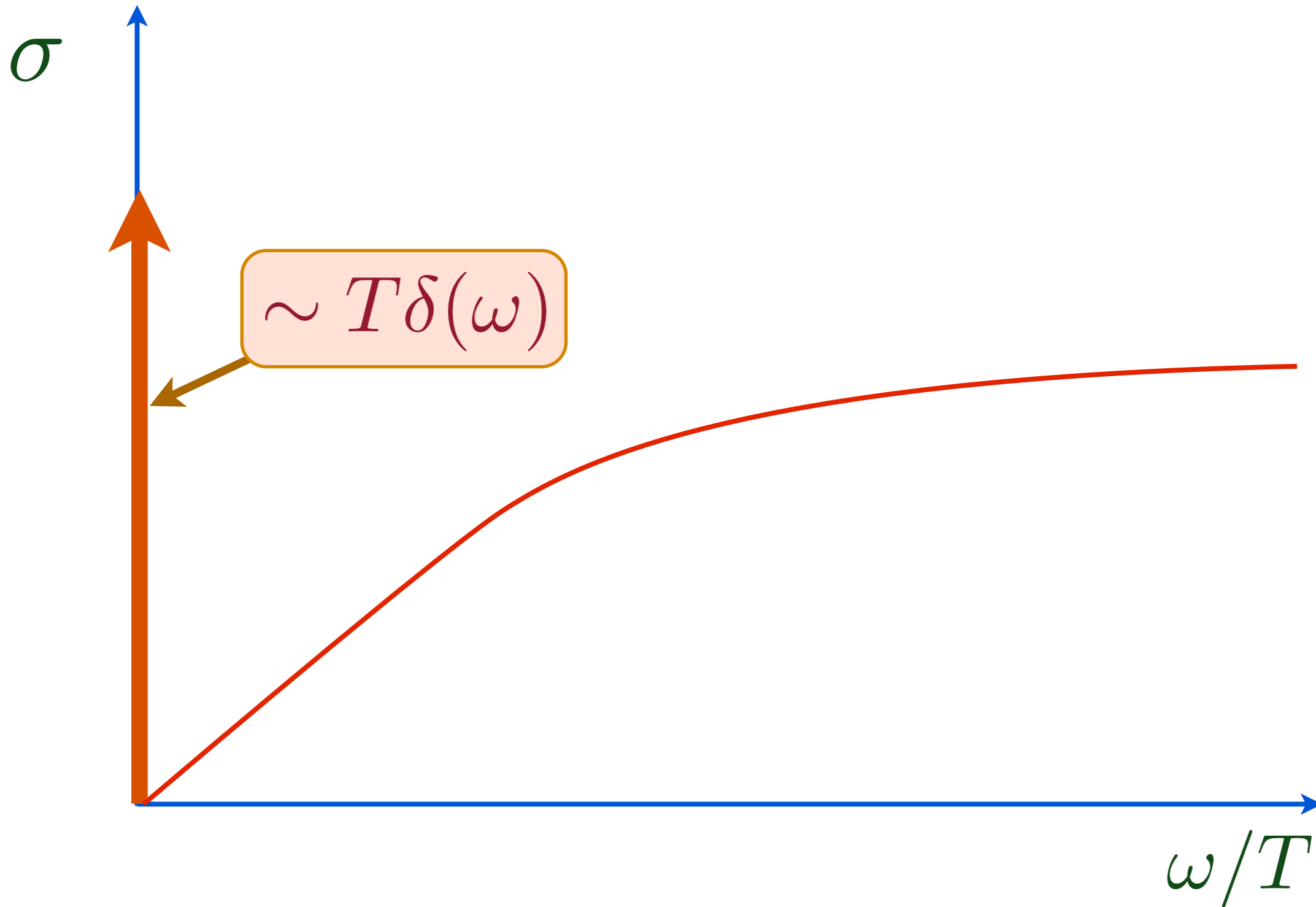
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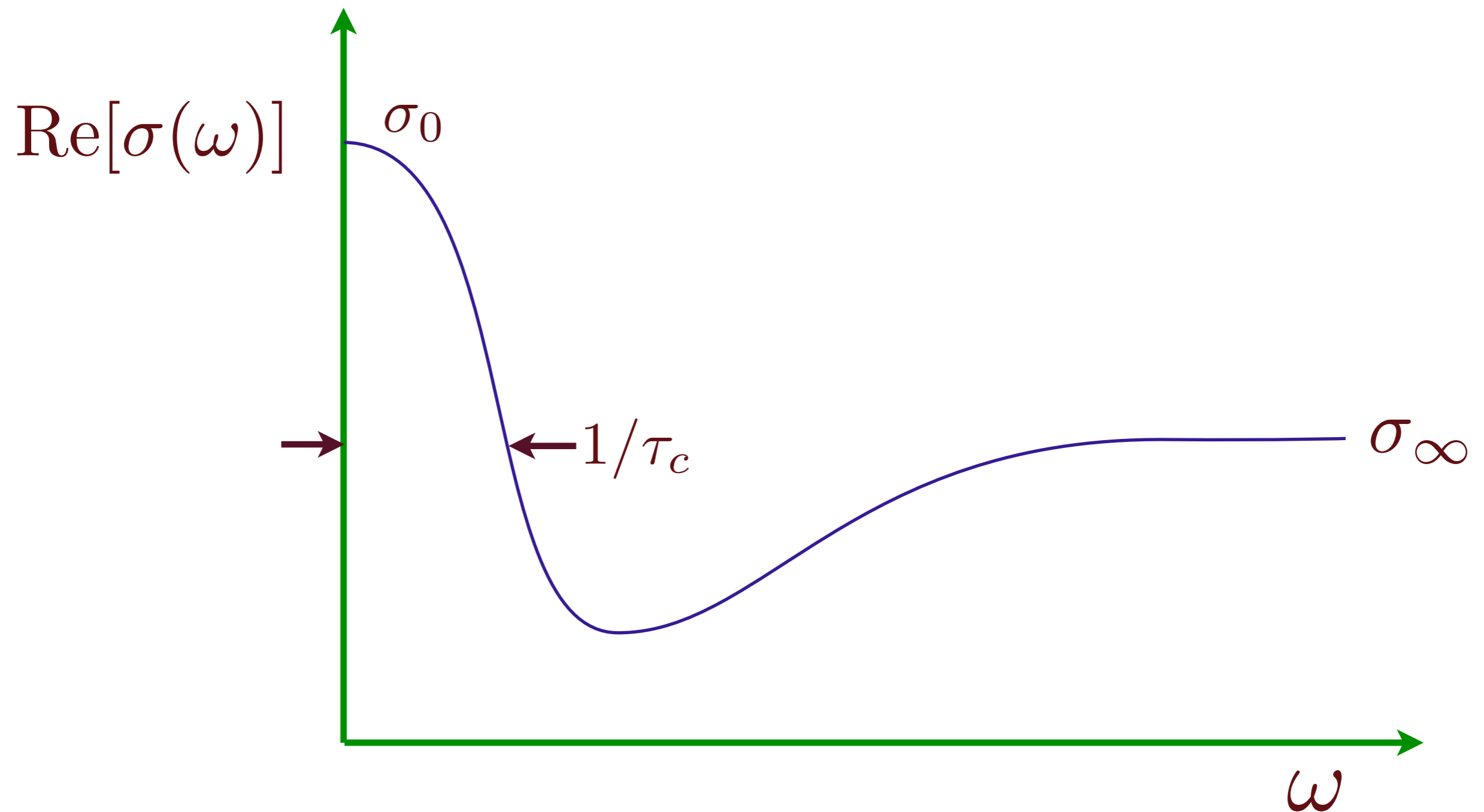
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \rightarrow \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the “optical conductivity”) in the CFT3.

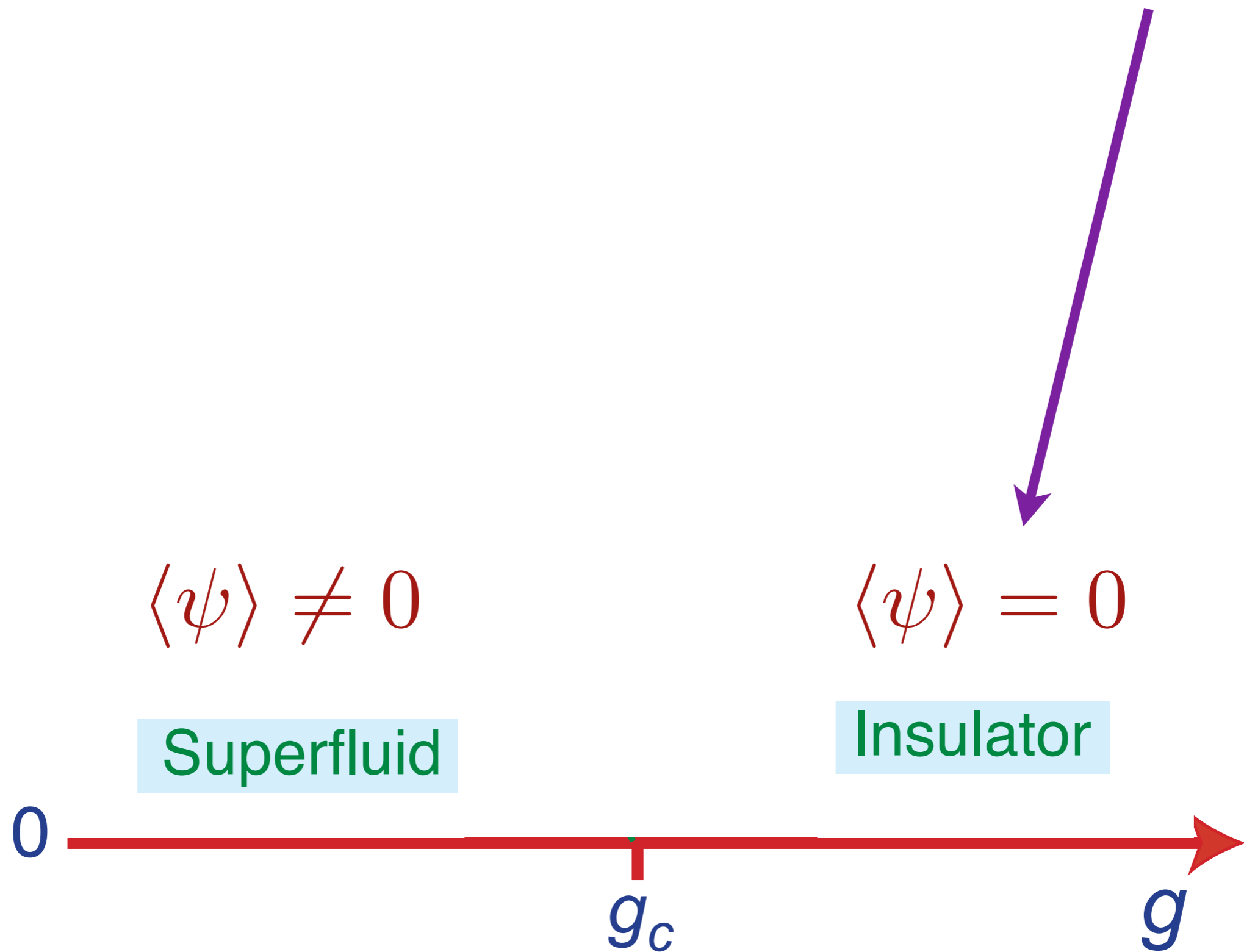
Electrical transport in a free-field theory for $T > 0$



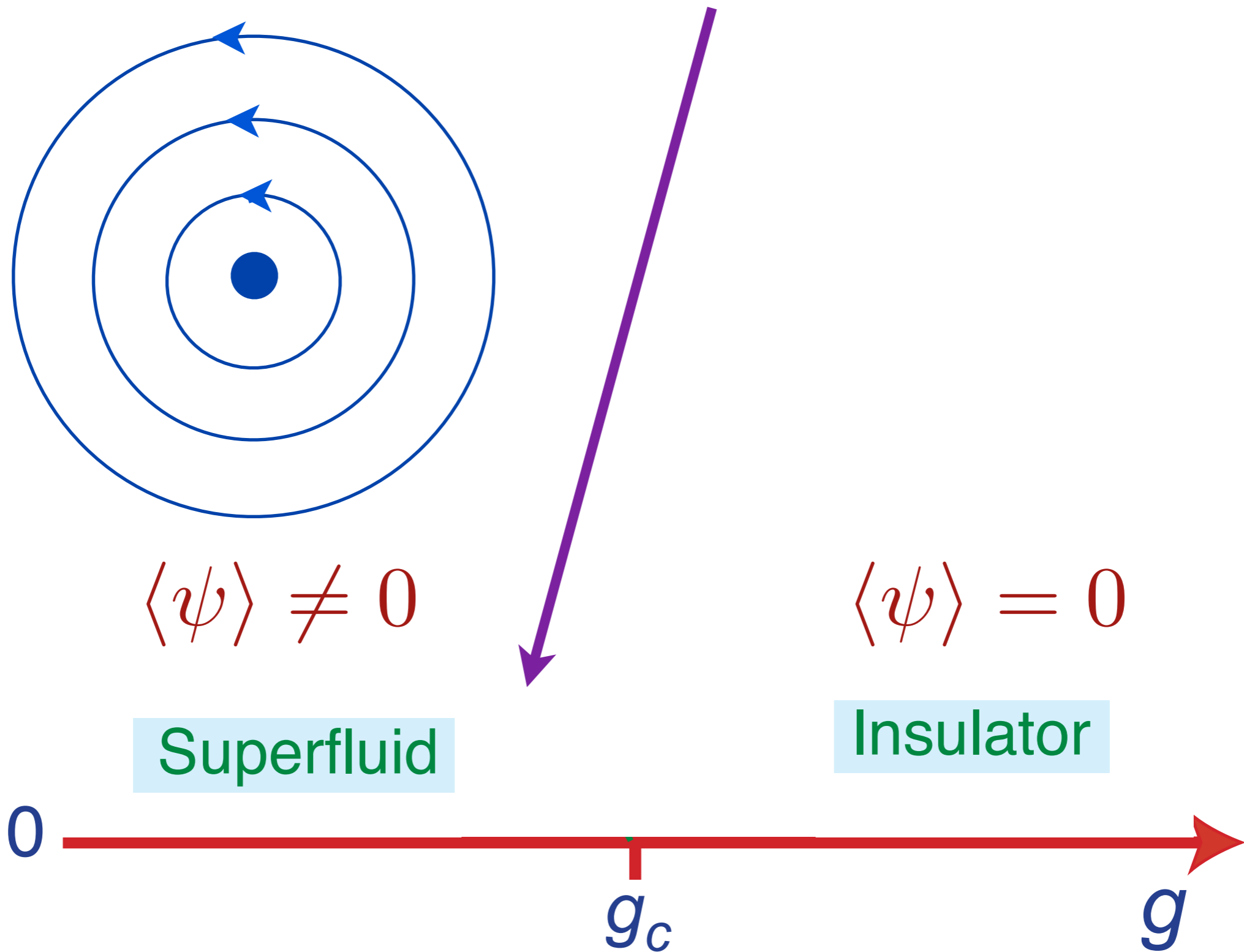
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



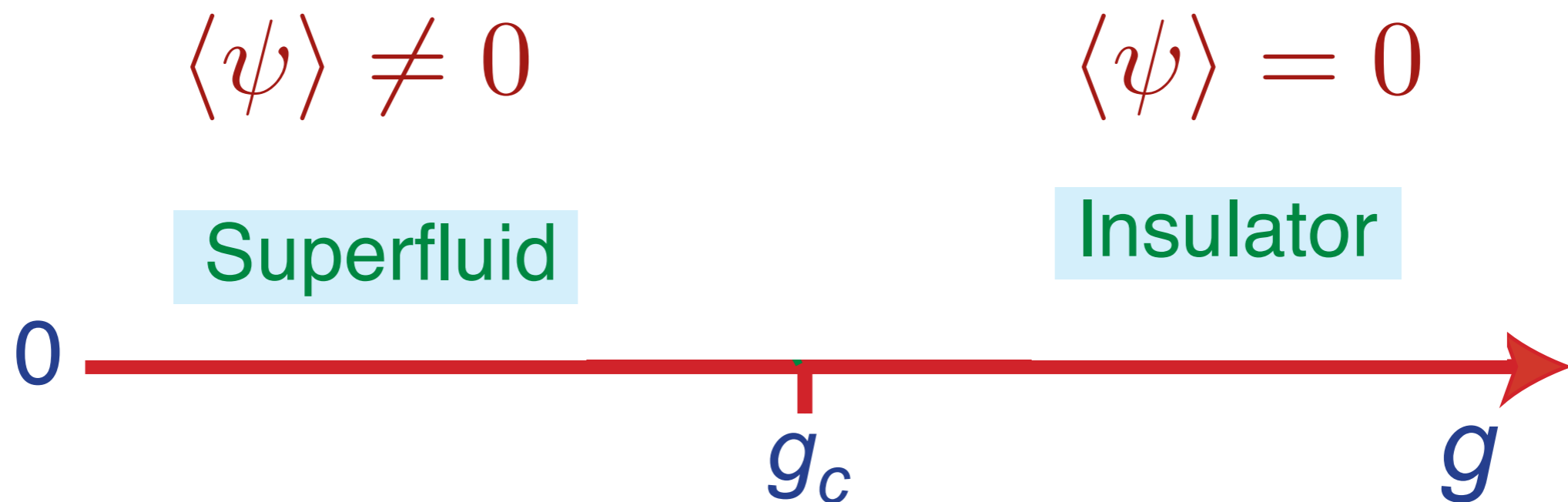
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



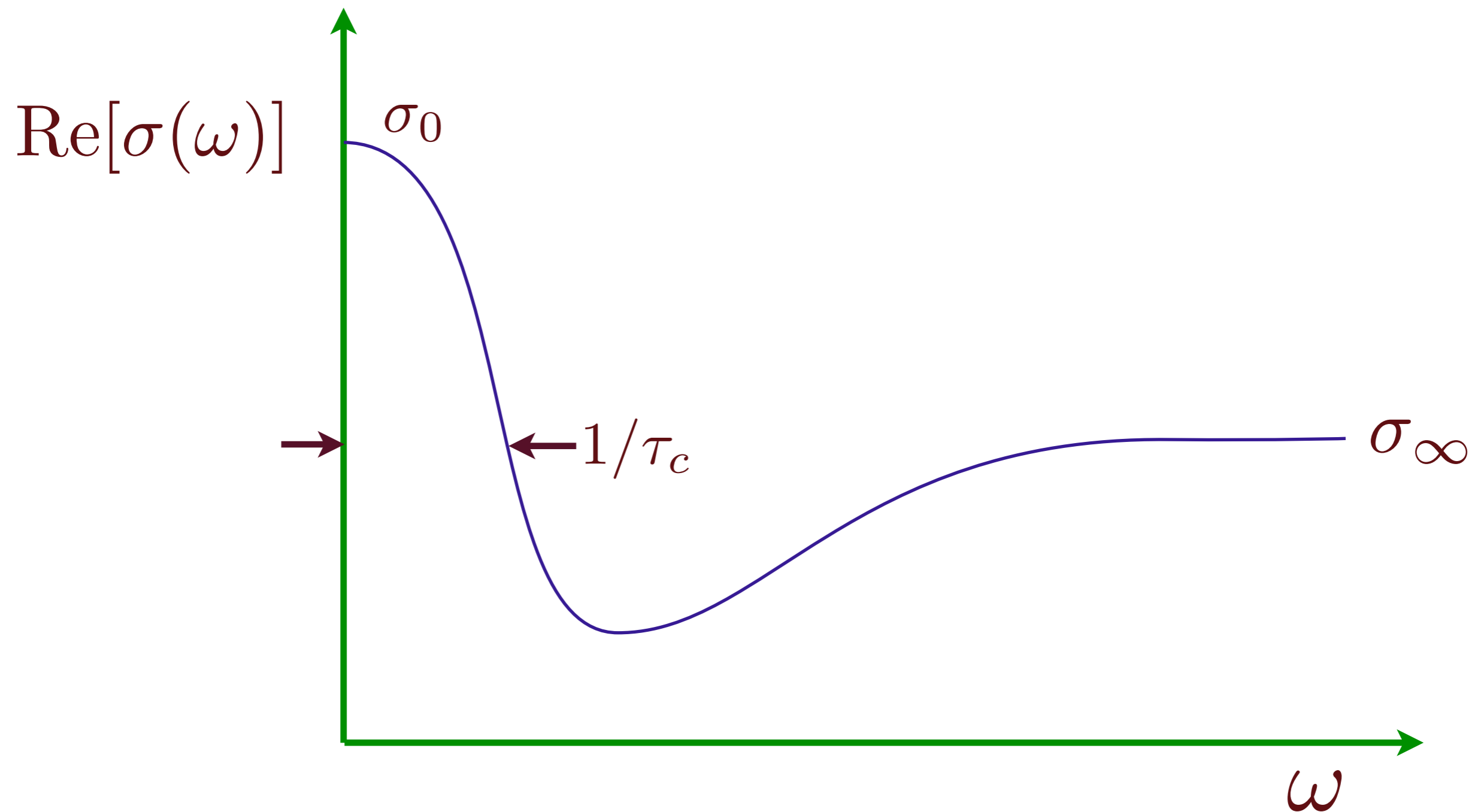
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

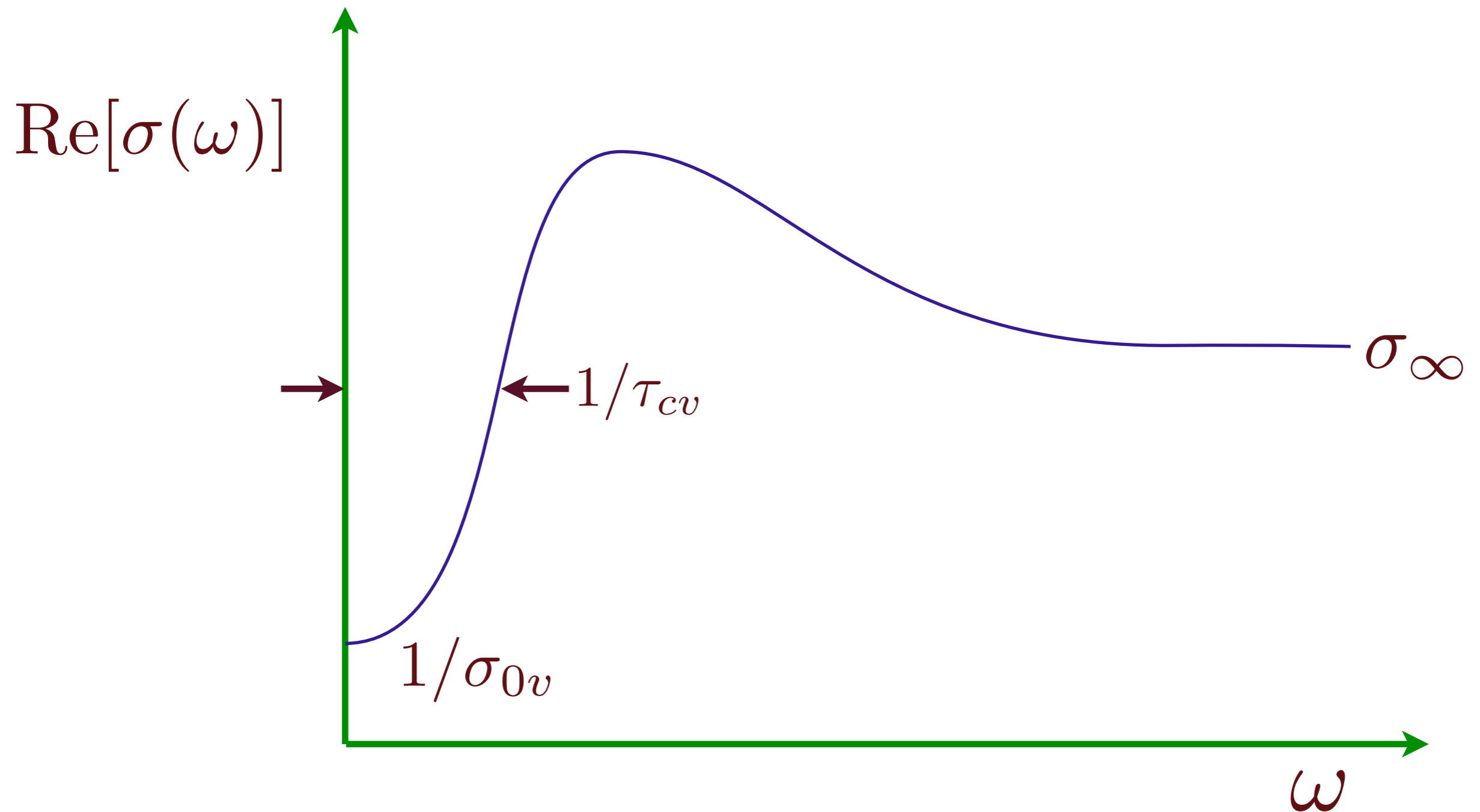
Conductivity = Resistivity of vortices



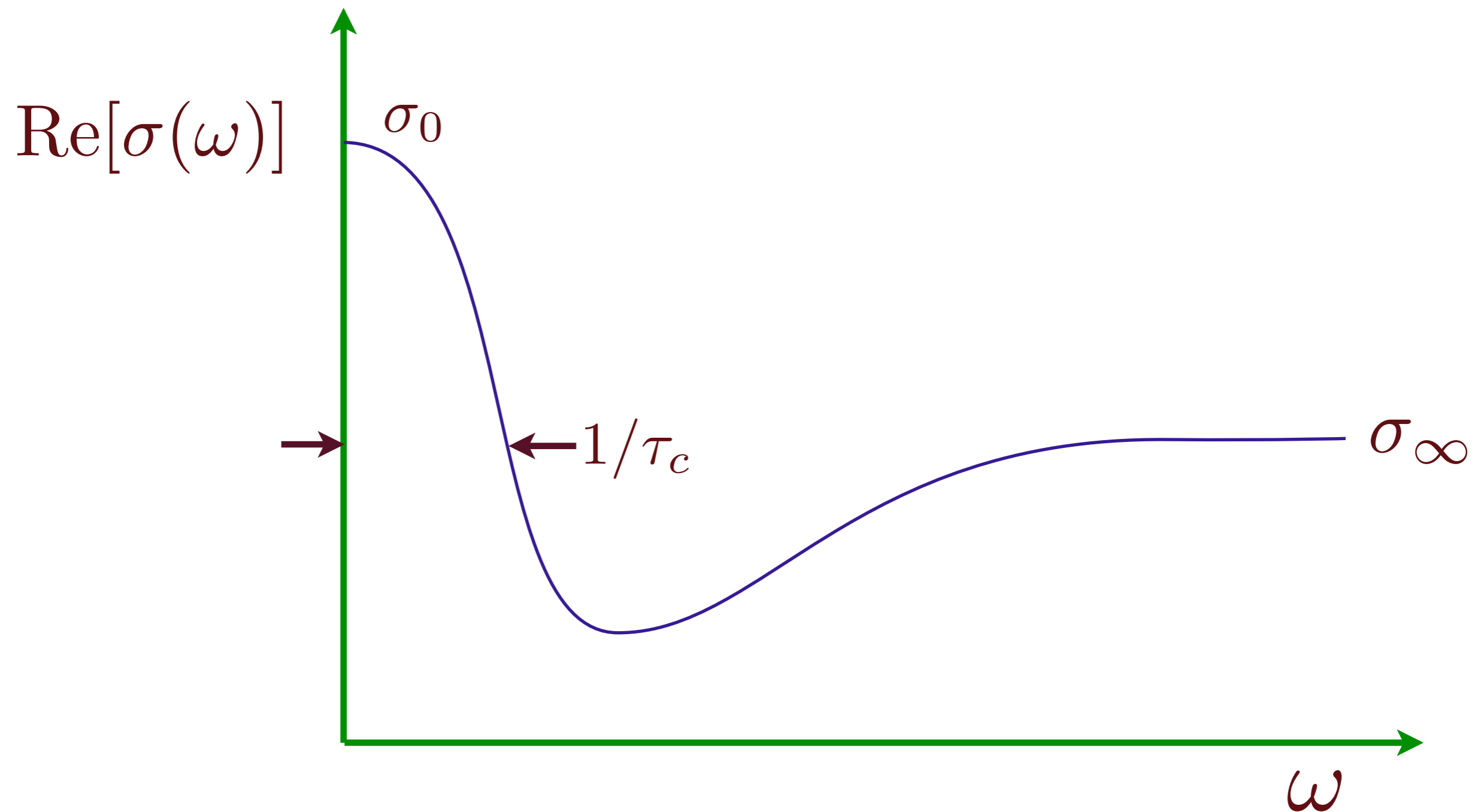
Boltzmann theory of bosons



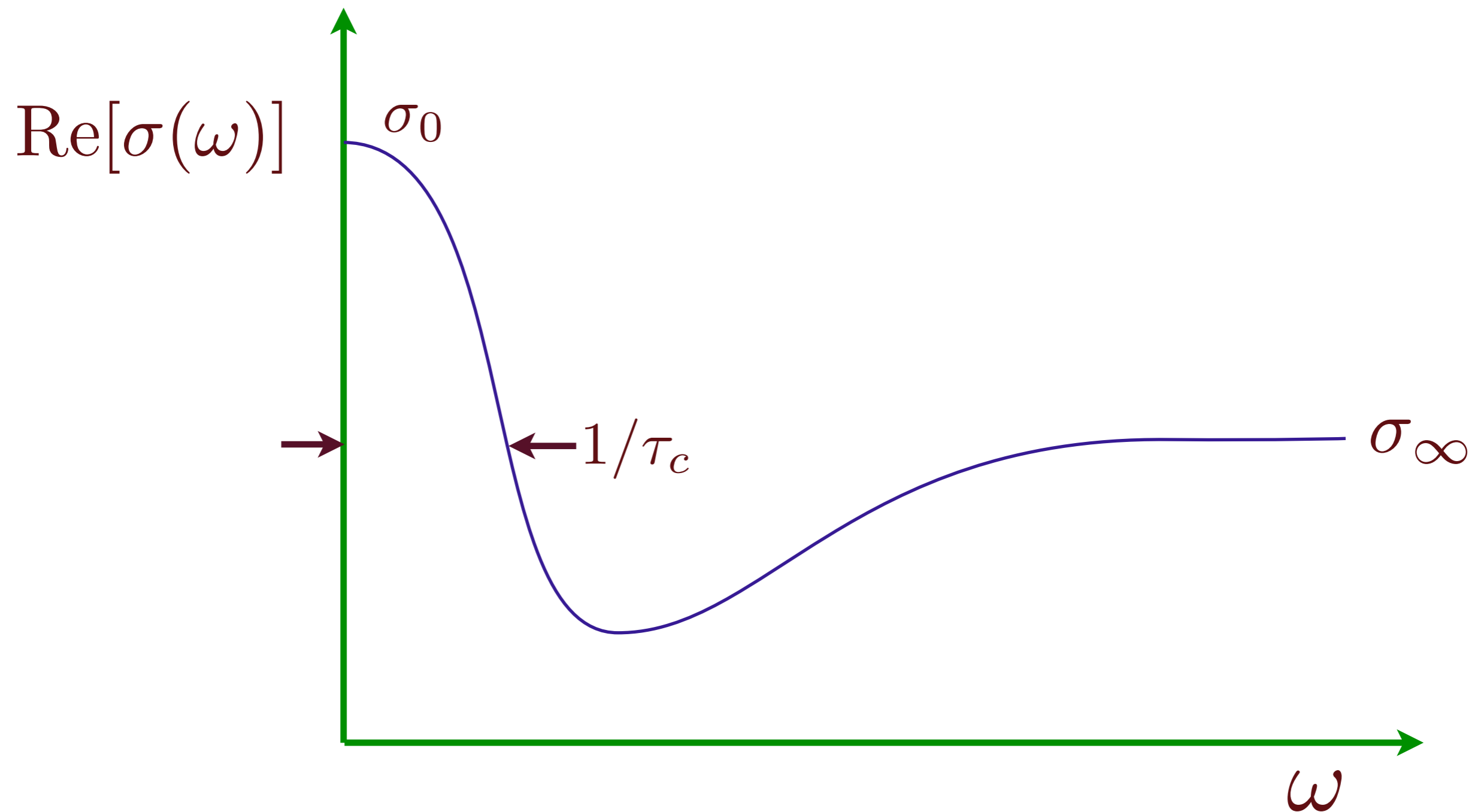
Boltzmann theory of vortices



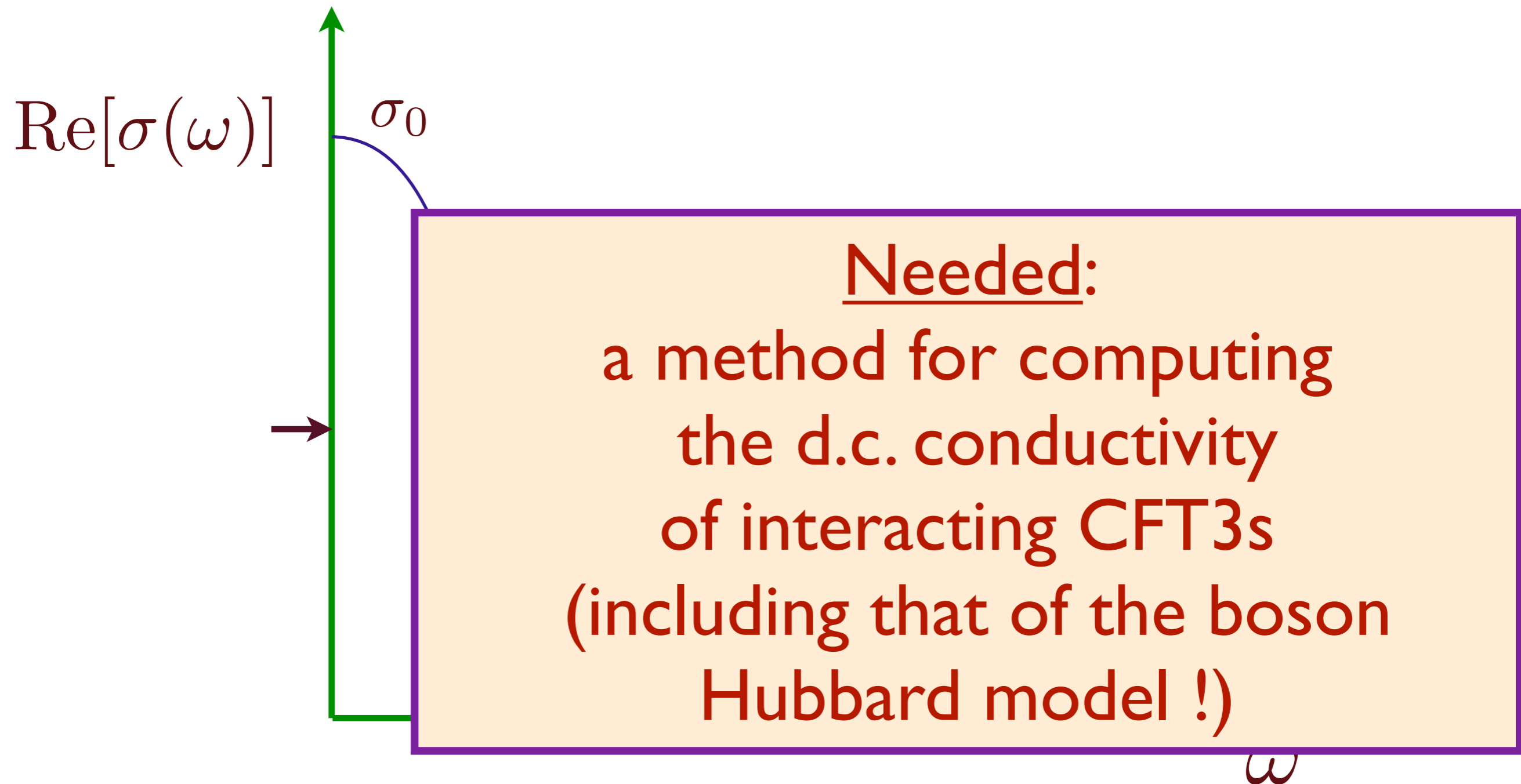
Boltzmann theory of bosons



Boltzmann theory of bosons



Boltzmann theory of bosons



Conformal quantum matter

A. Field theory: graphene

*B. Field theory: superfluid-
insulator transition*

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Conformal quantum matter

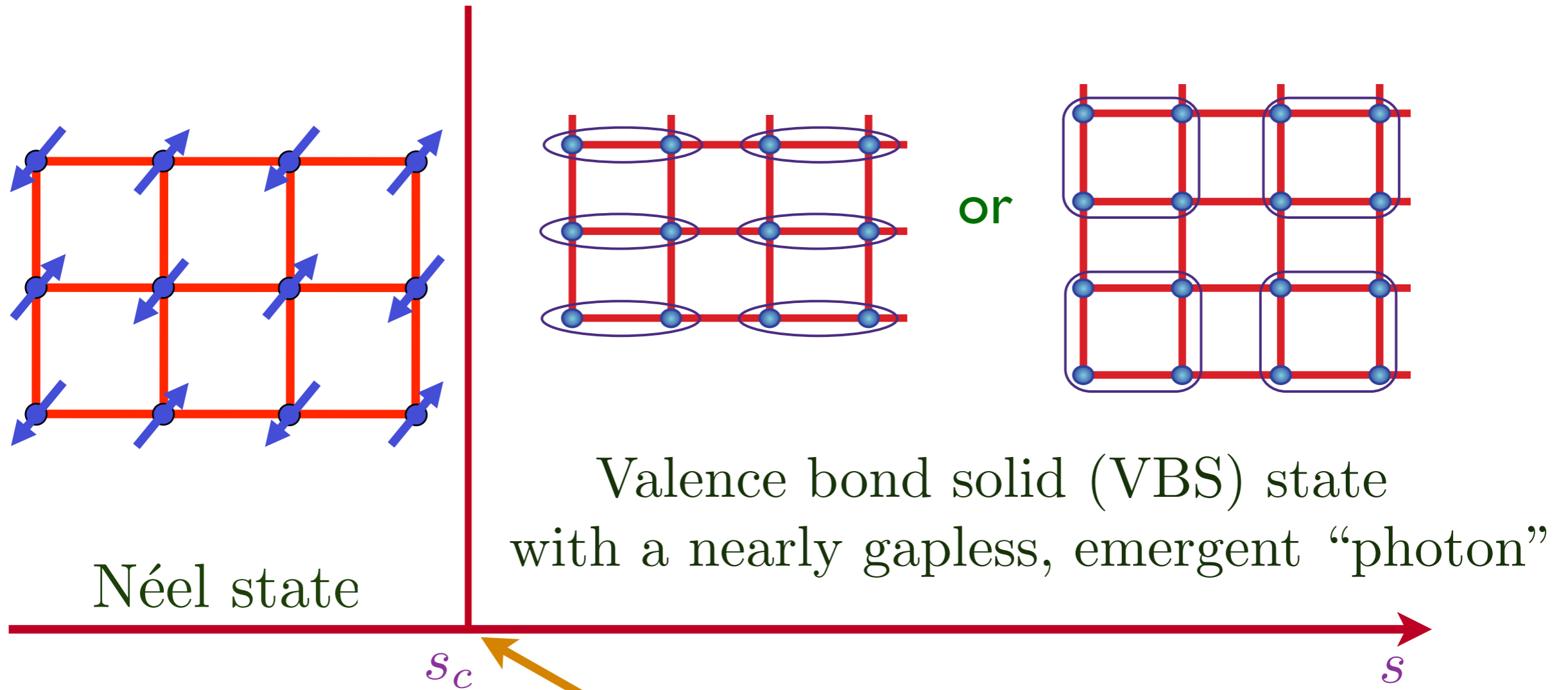
A. Field theory: graphene

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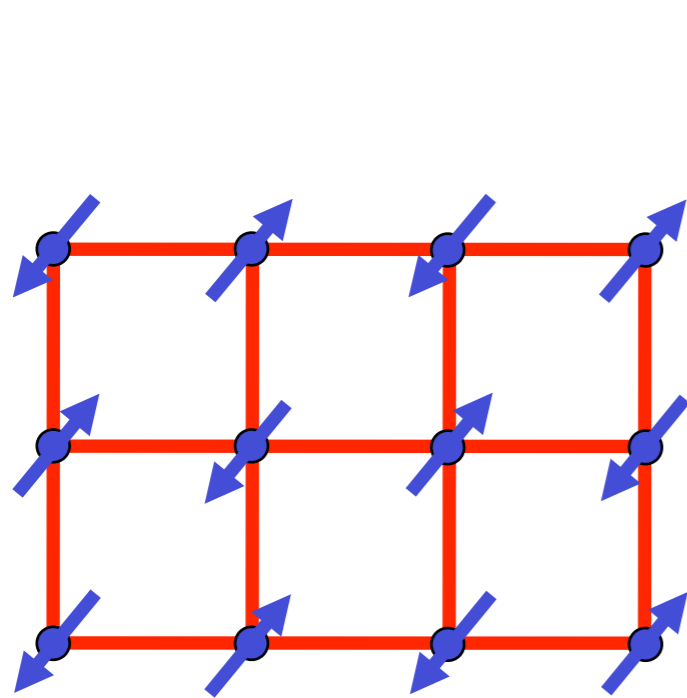
D. Gauge-gravity duality

Quantum critical point in a frustrated square lattice antiferromagnet

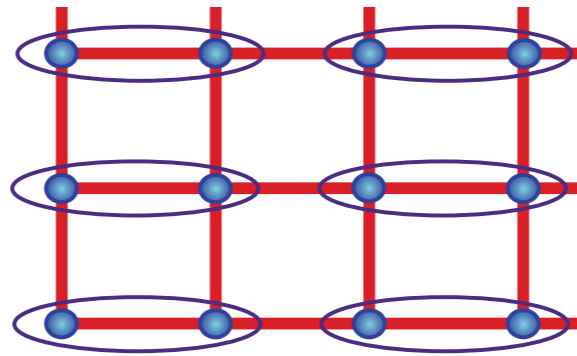


Long-range entanglement described by a CFT3
with an emergent U(1) “photon”

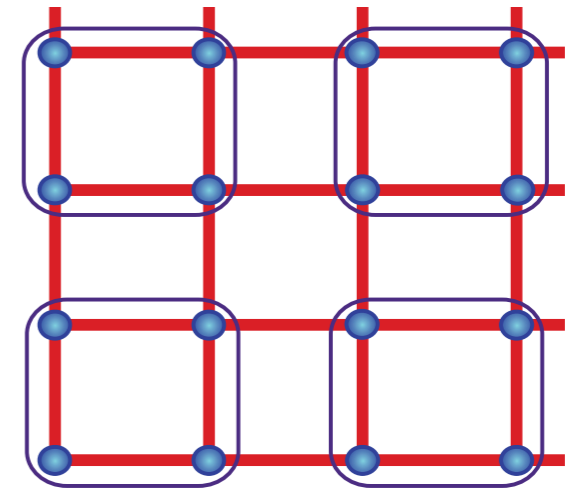
Quantum critical point in a frustrated square lattice antiferromagnet



Néel state



or



Valence bond solid (VBS) state
with a nearly gapless, emergent “photon”

s_c

s

Critical theory for photons and deconfined spinons:

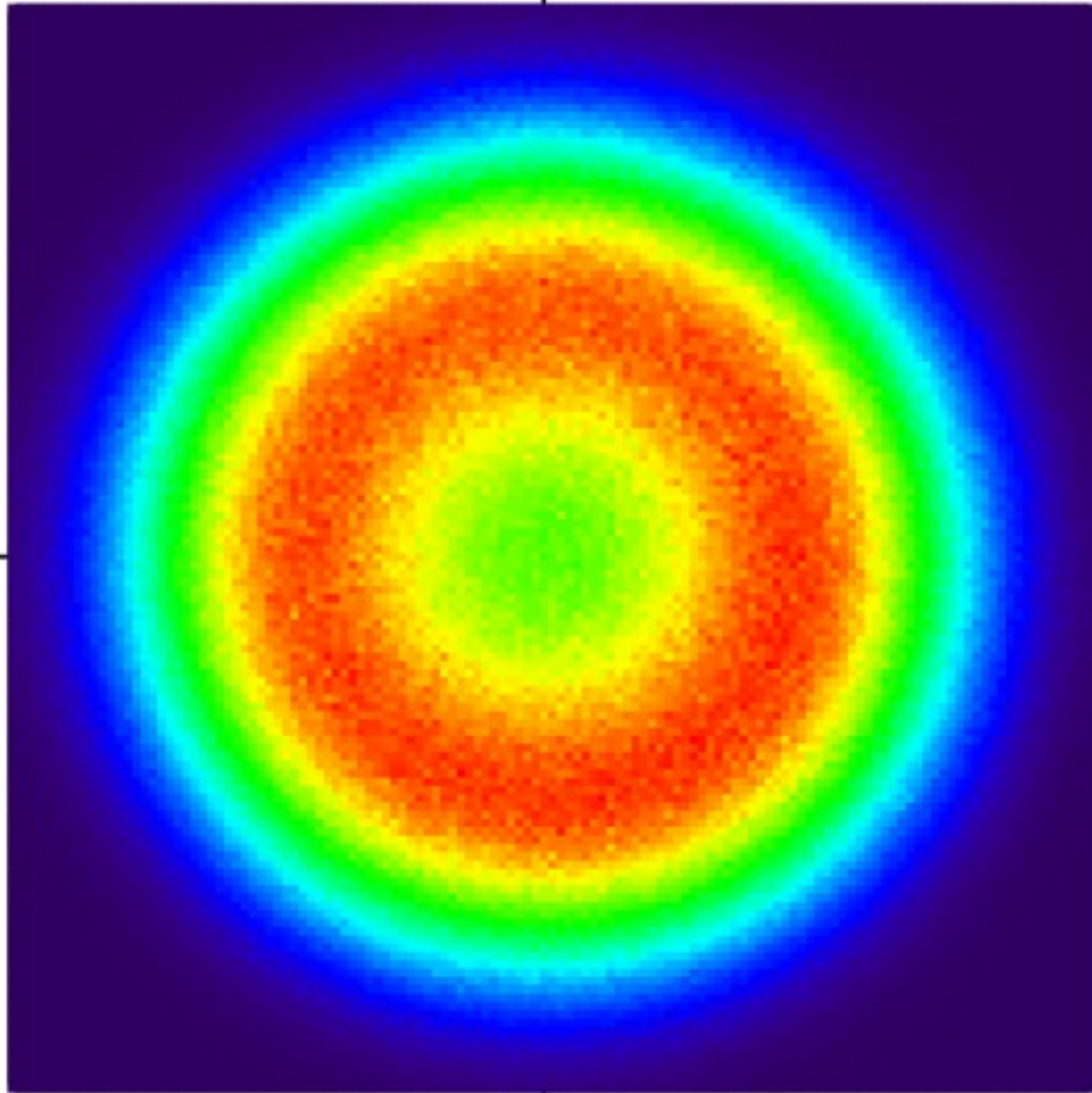
$$\mathcal{S}_z = \int d^2r d\tau \left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

$$\mathcal{H}_{\text{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

$|\text{Im}[\Psi_{\text{vbs}}]$



Distribution of VBS
order Ψ_{vbs} at large Q

$\text{Re}[\Psi_{\text{vbs}}]$

*Circular symmetry is
evidence for
emergent $U(1)$
photon*

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Conformal quantum matter

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C. Field theory: antiferromagnets

D. Gauge-gravity duality

Talks online: <http://qpt.physics.harvard.edu/talks/lorentz12.pdf>

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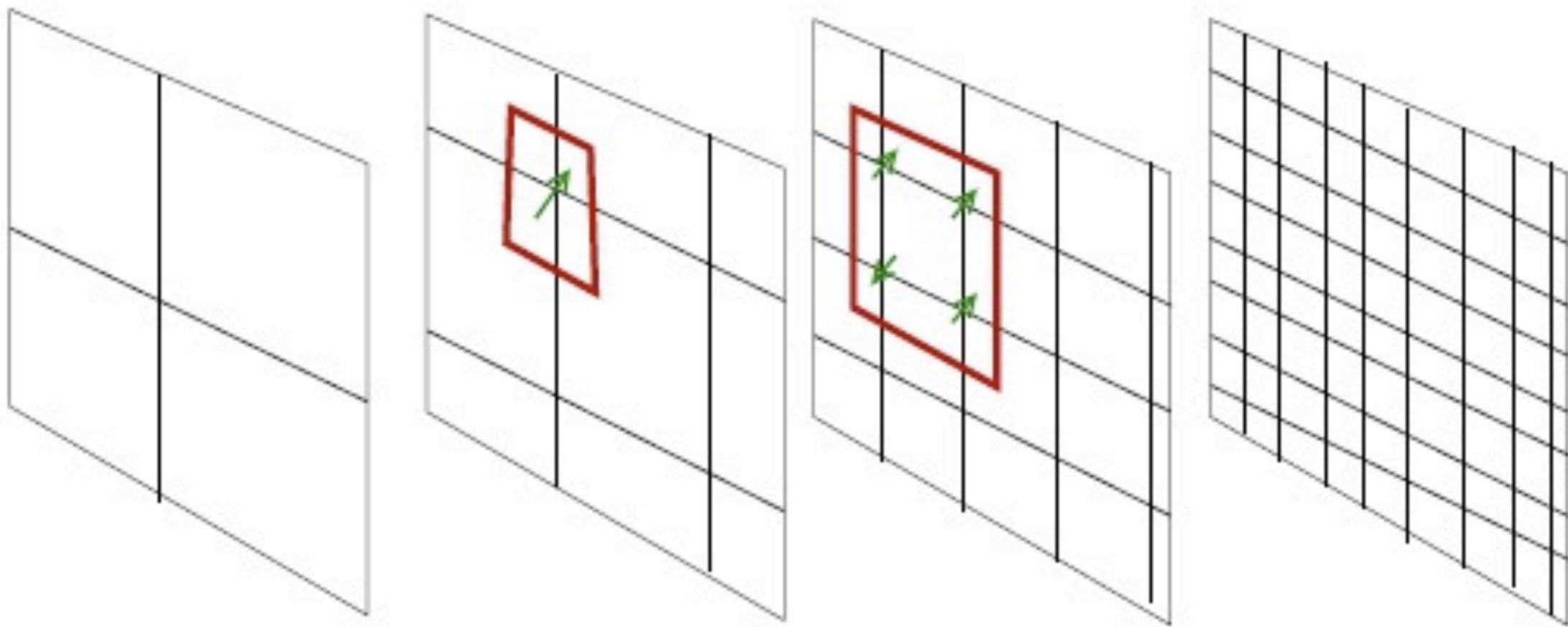
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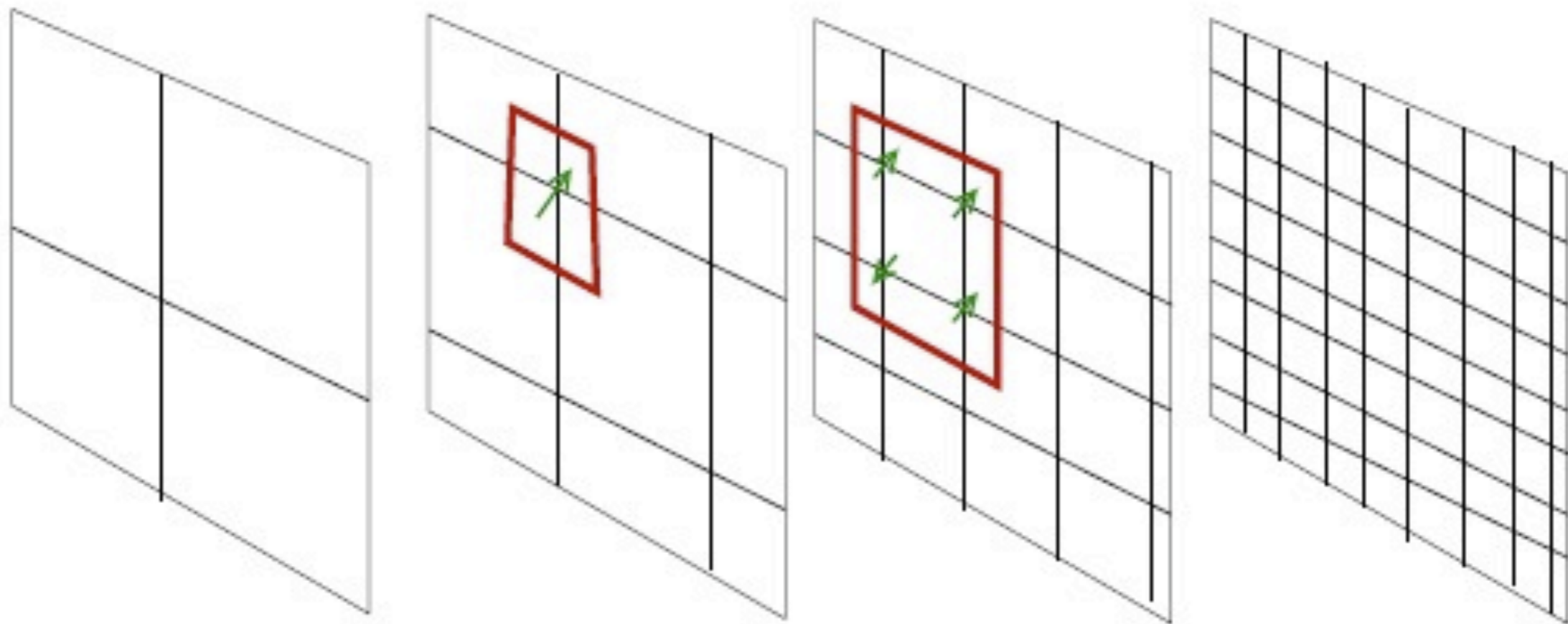
Field theories in $d + 1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

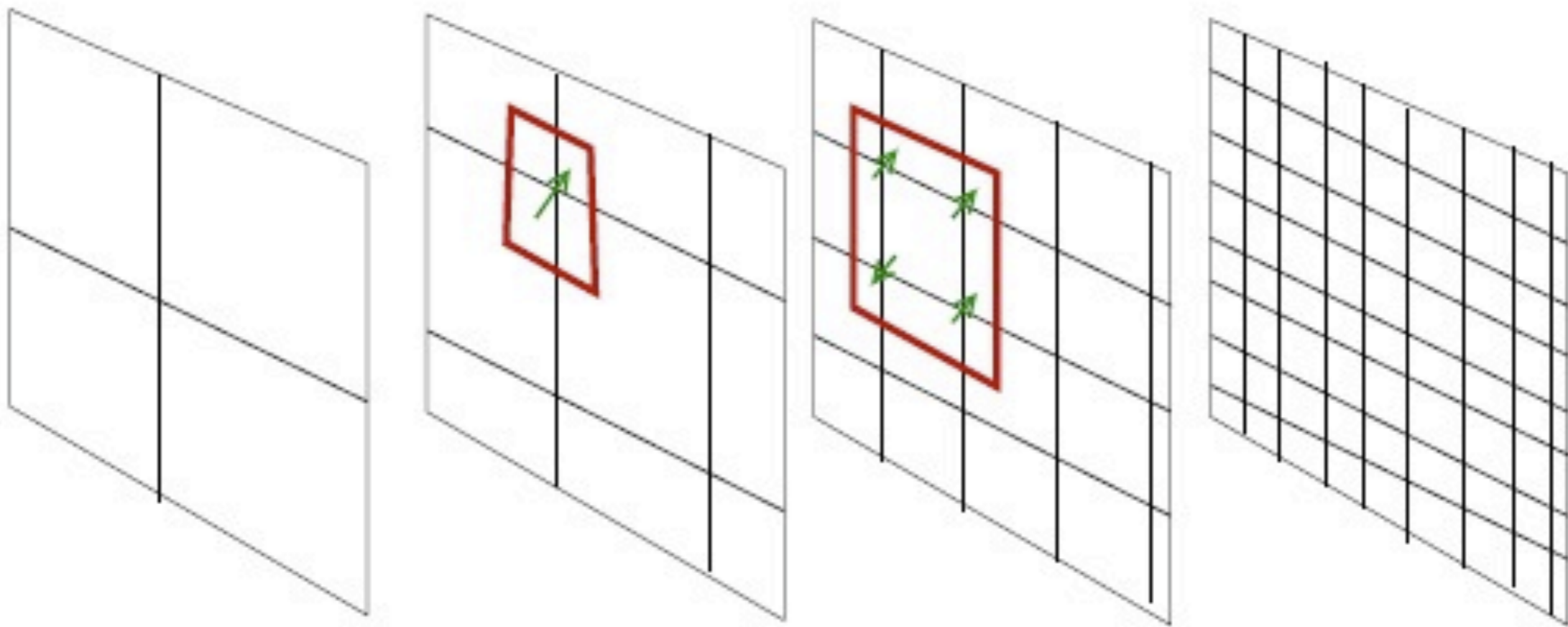
where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u .



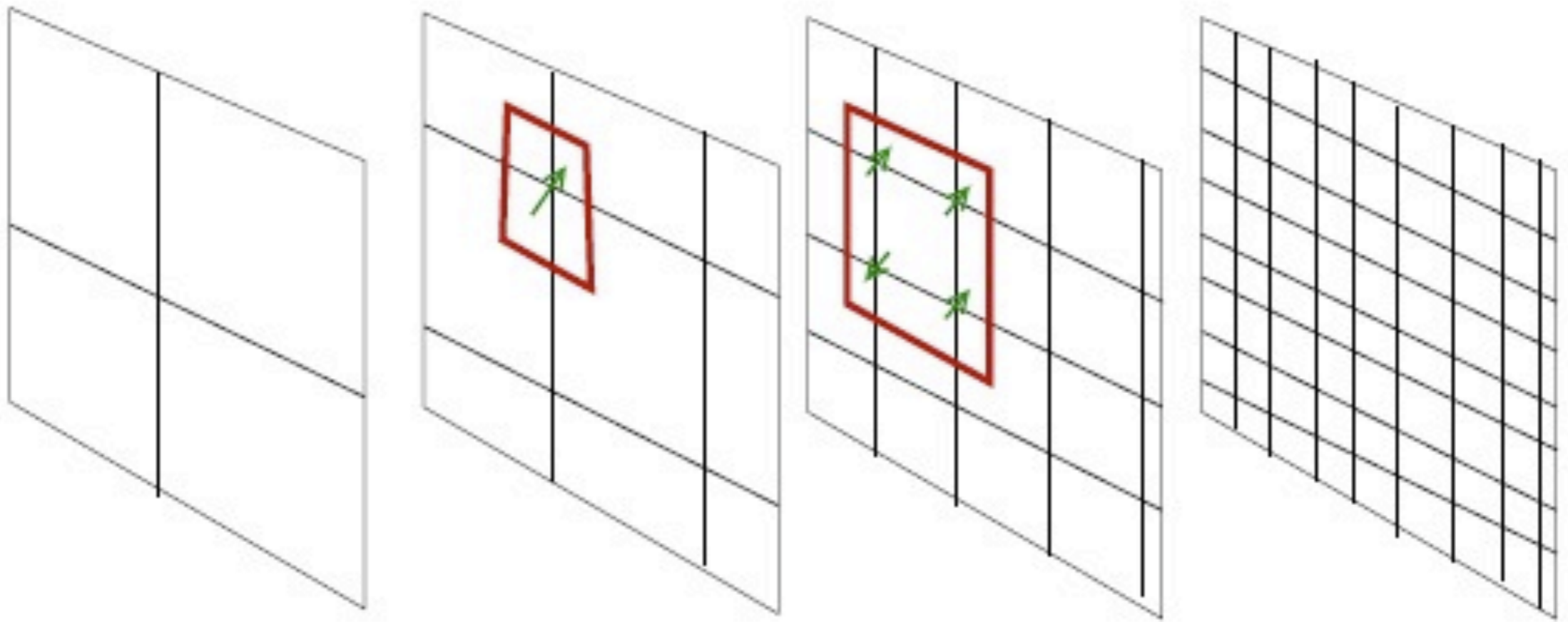
—————→ u



r ←



r ←



r ←

Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

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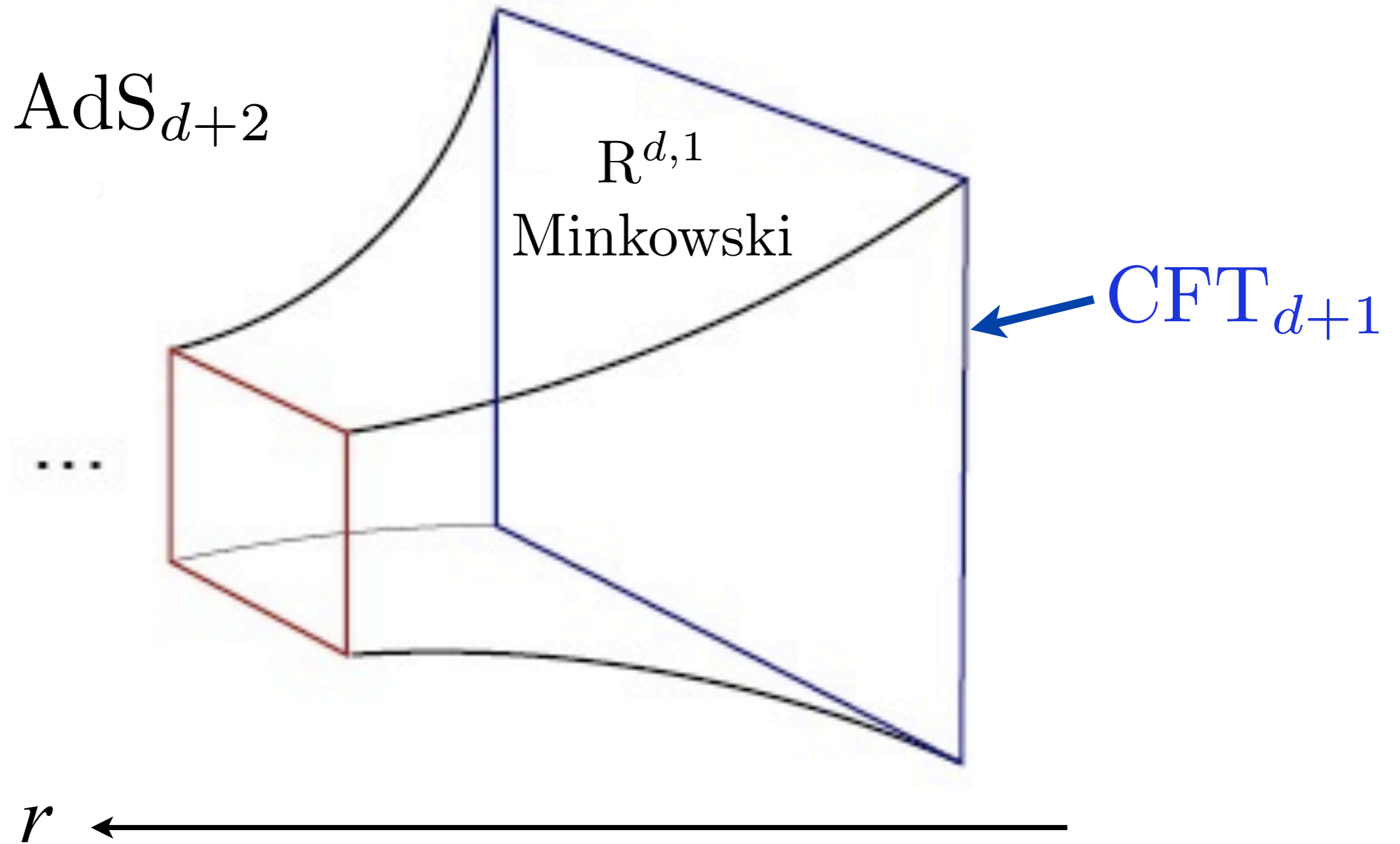
$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

This gives the unique metric

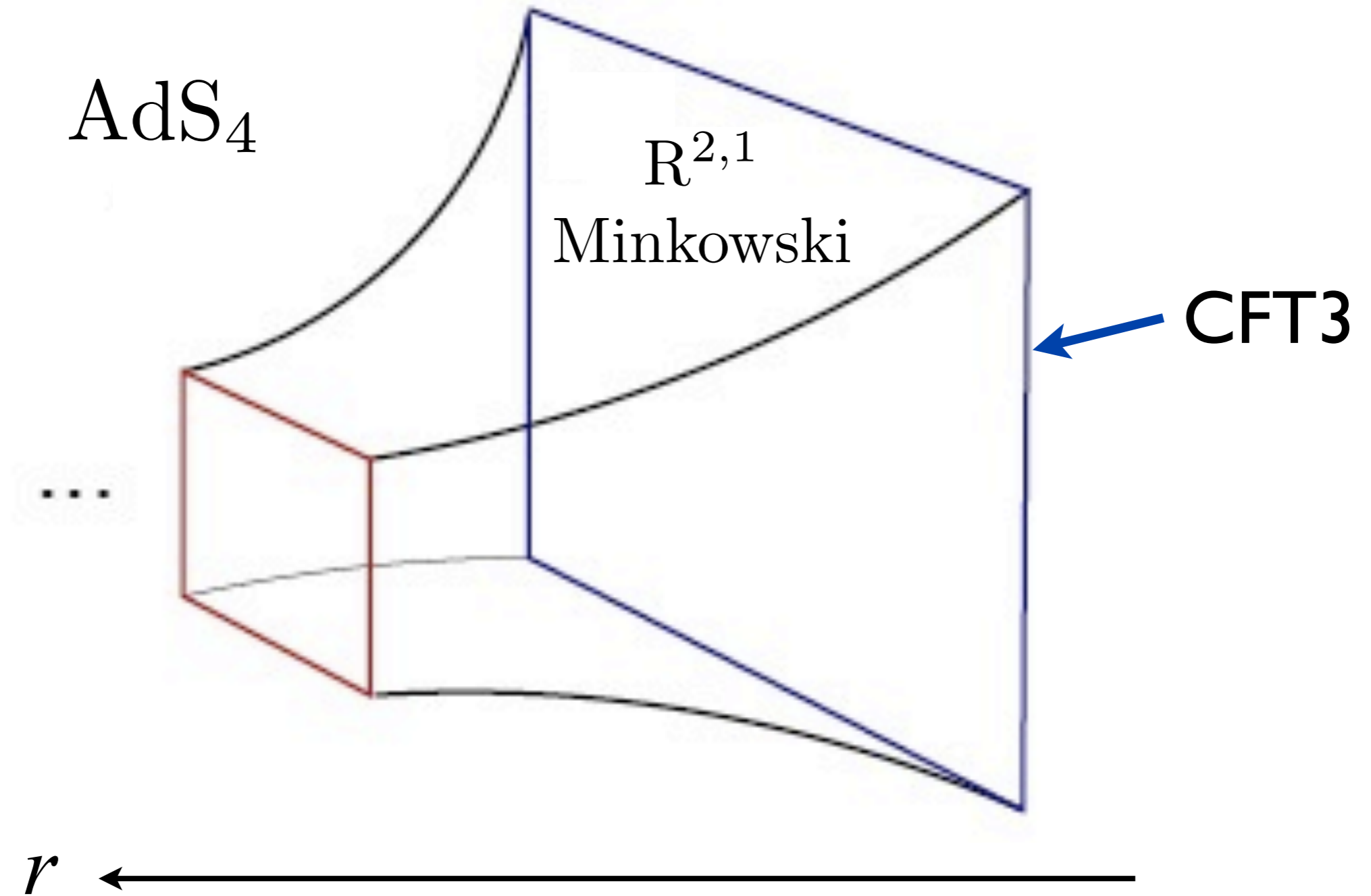
$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2} .

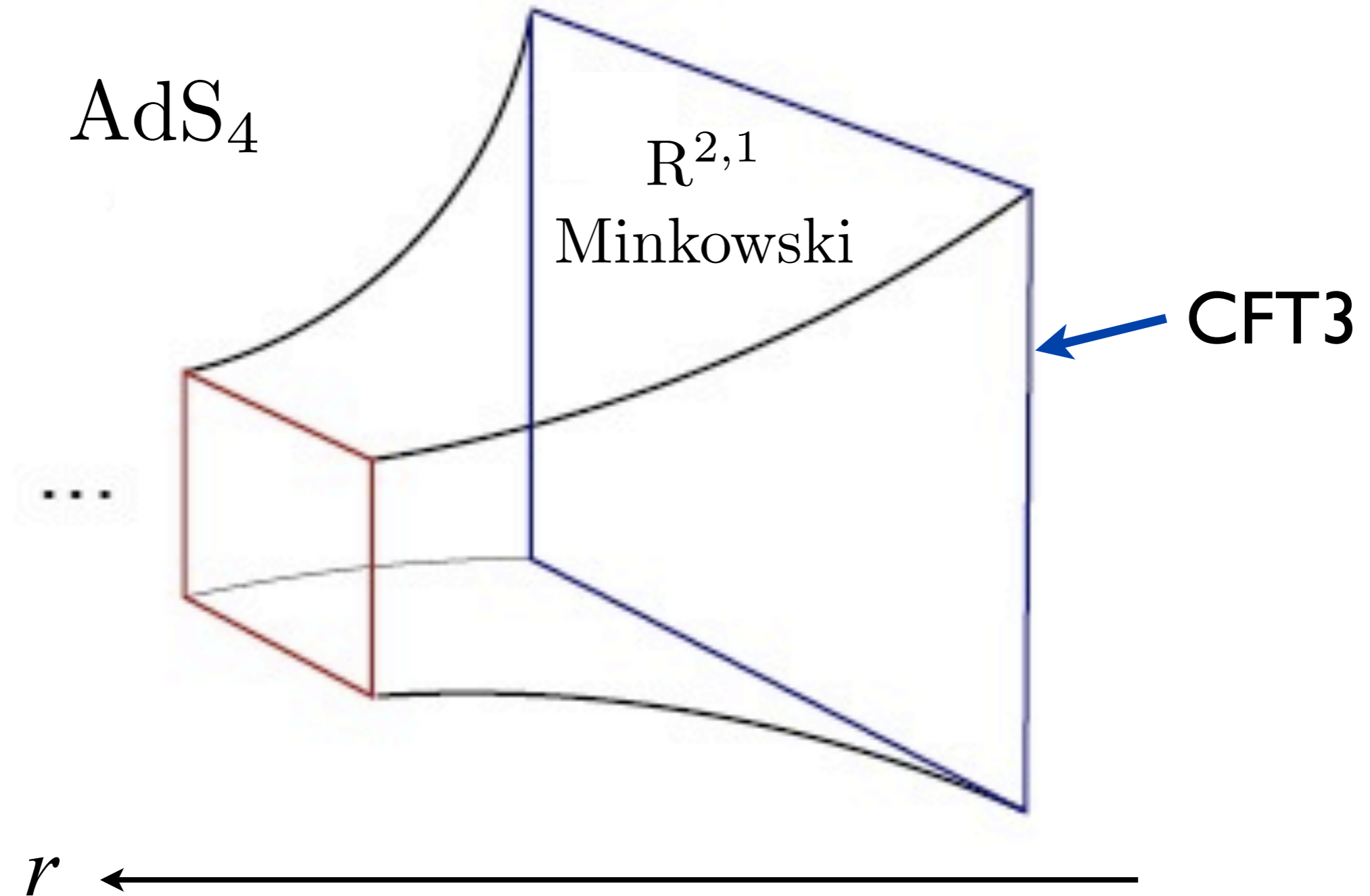
AdS/CFT correspondence



AdS/CFT correspondence



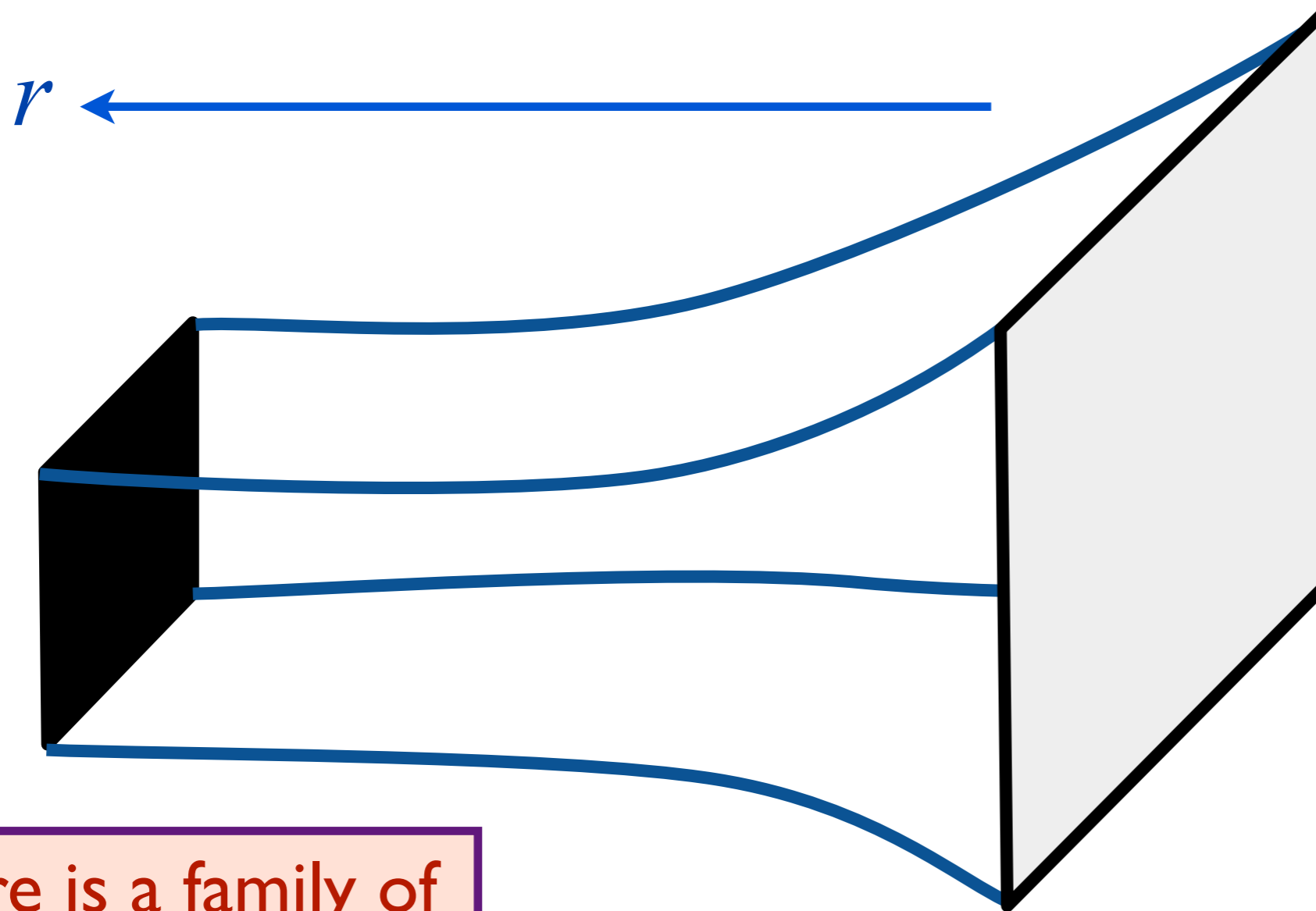
AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

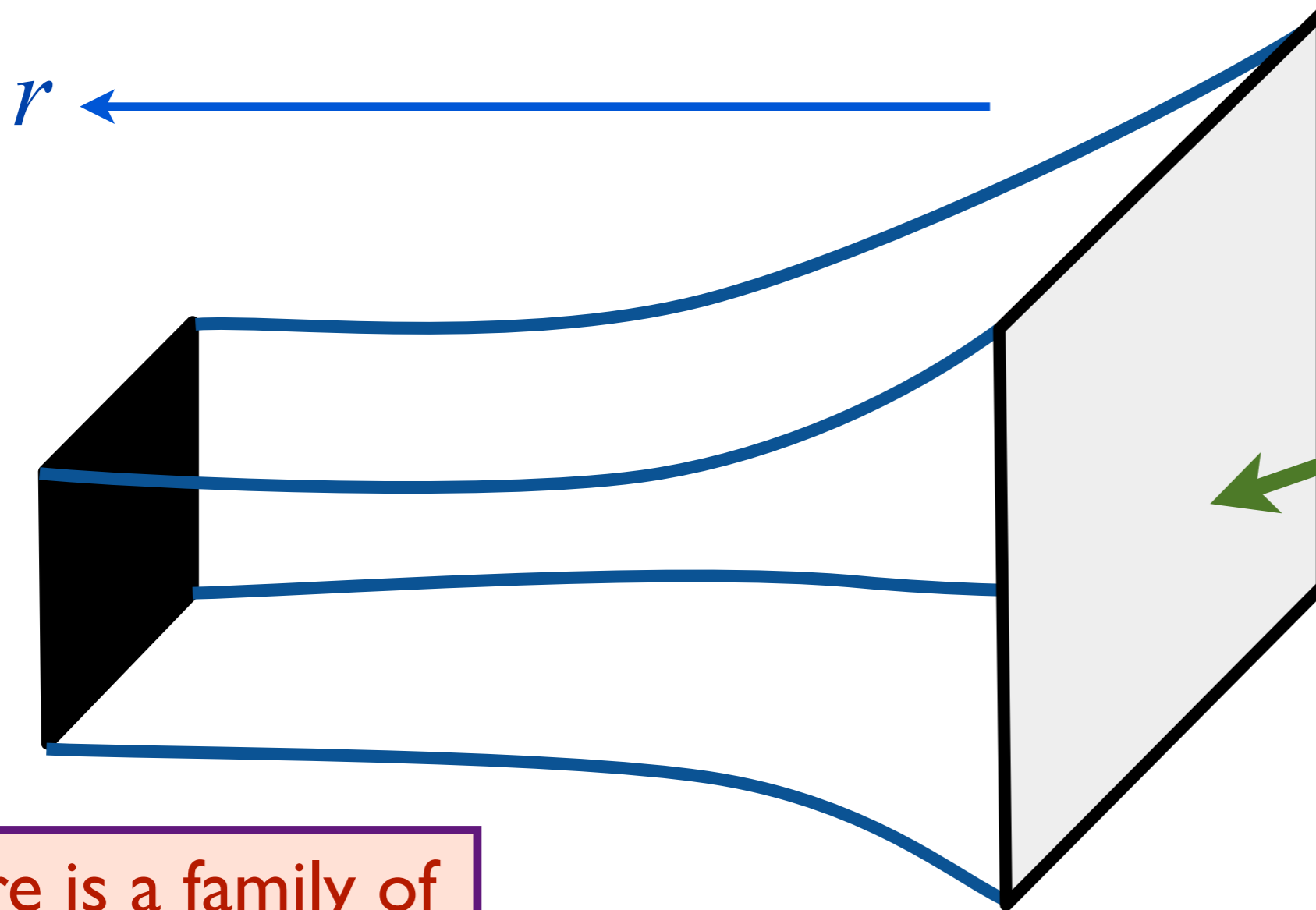
AdS₄-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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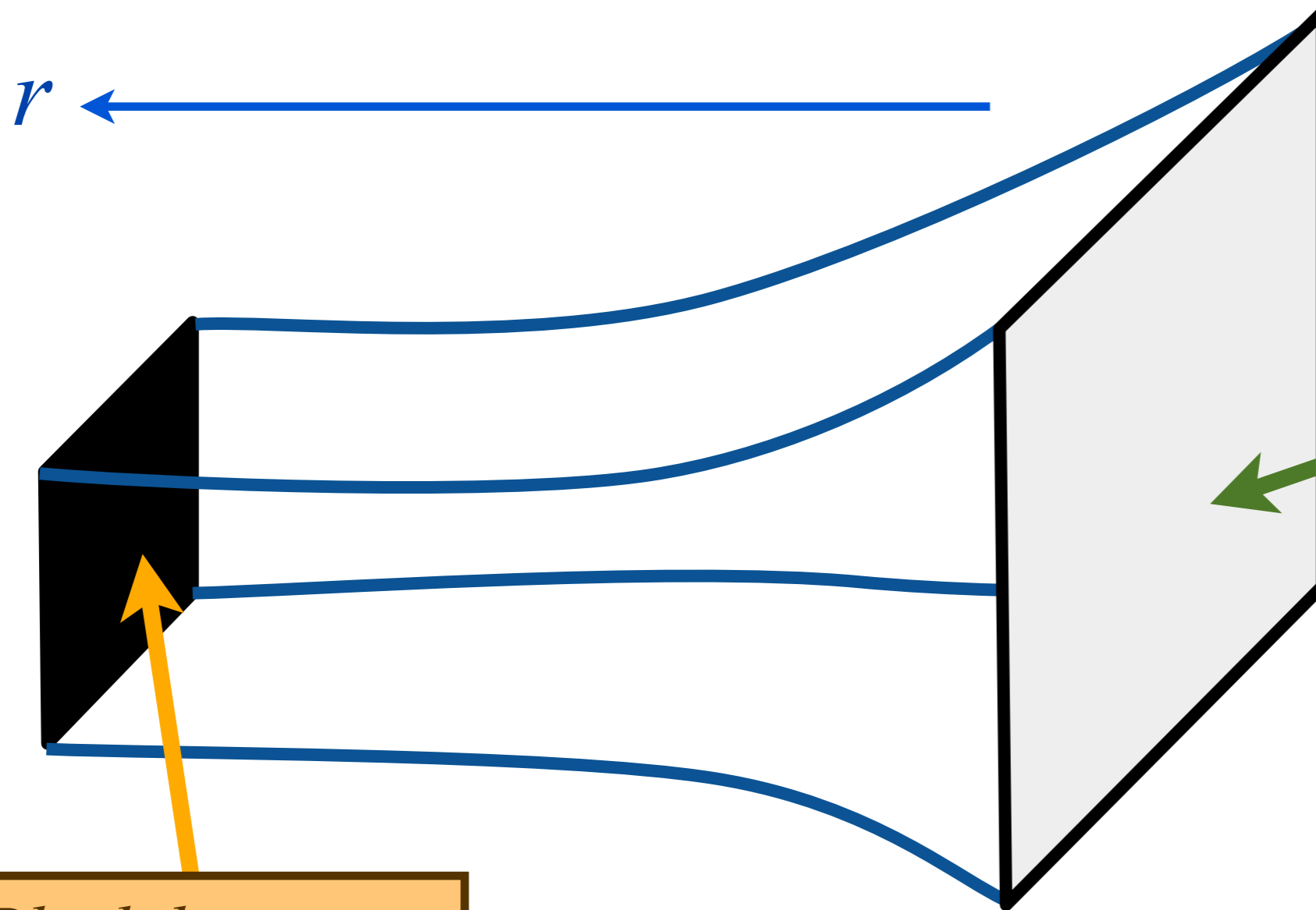
A 2+1 dimensional system at its quantum critical point:
 $k_B T = \frac{3\hbar}{4\pi R}$

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$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1 - (r/R)^3$

AdS₄-Schwarzschild black-brane



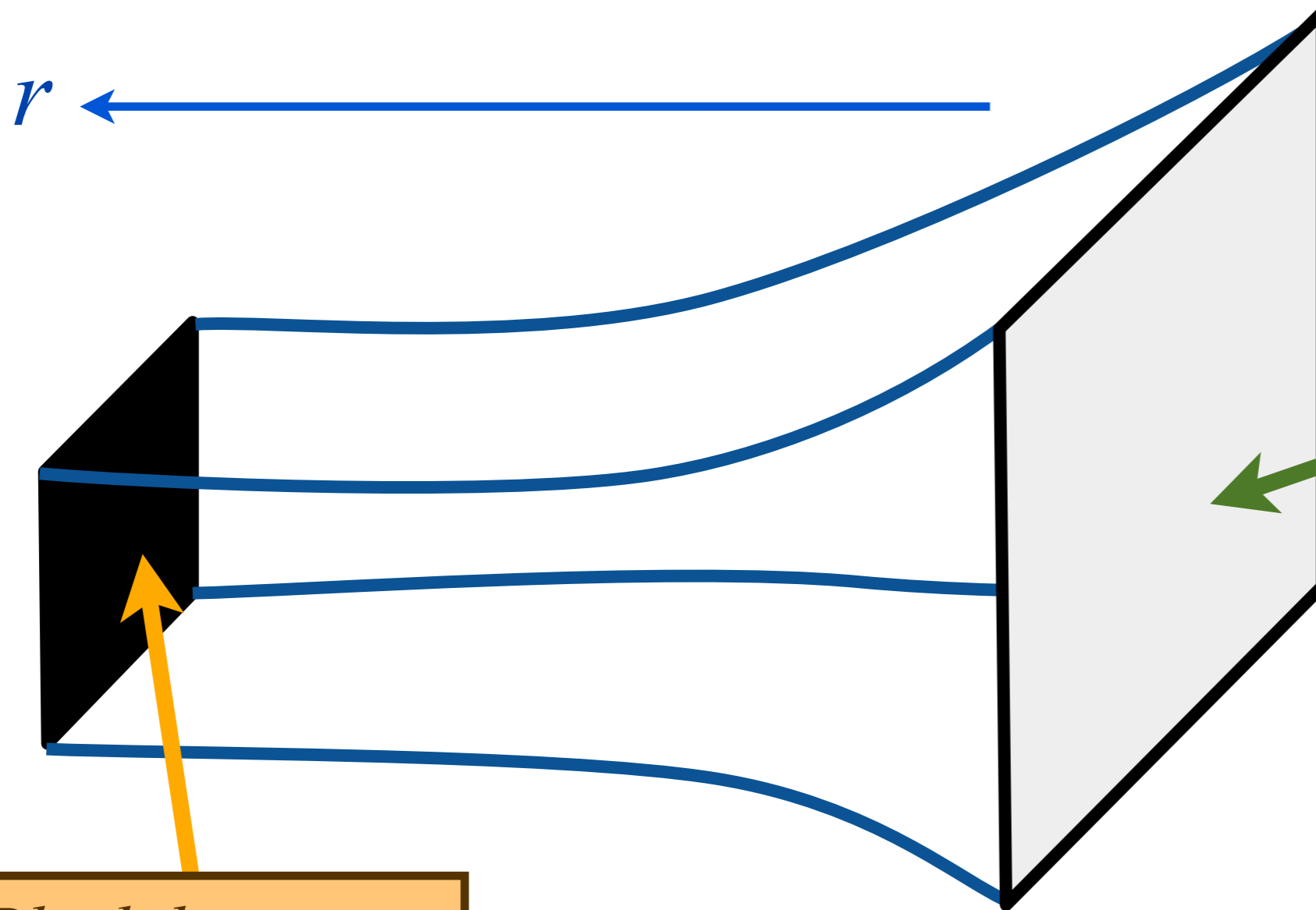
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Black-brane at temperature of 2+1 dimensional quantum critical system

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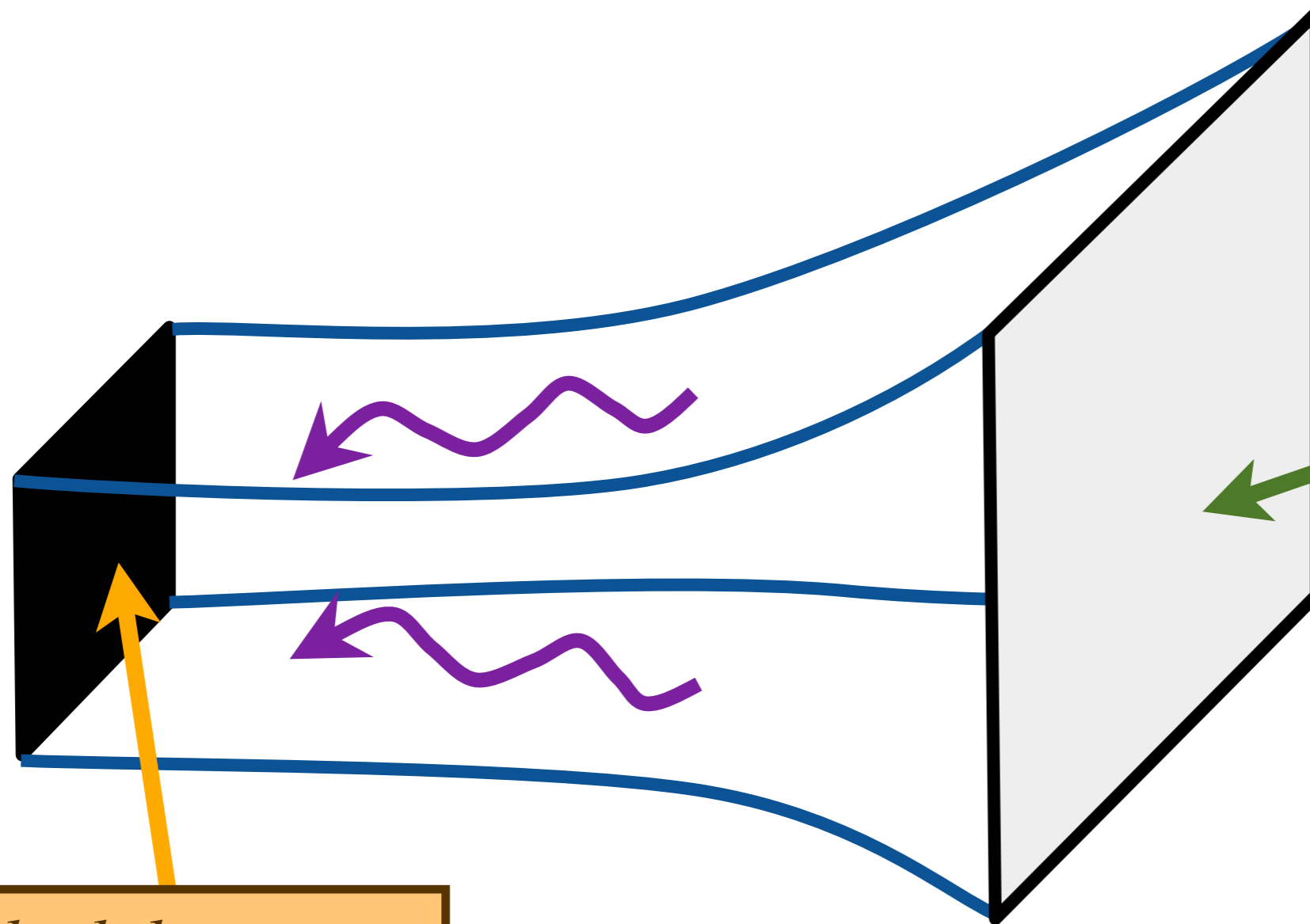


A 2+1 dimensional system at its quantum critical point:
$$k_B T = \frac{3\hbar}{4\pi R}$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS₄-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point:
$$k_B T = \frac{3\hbar}{4\pi R}$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right].$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

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This is to be solved subject to the constraint

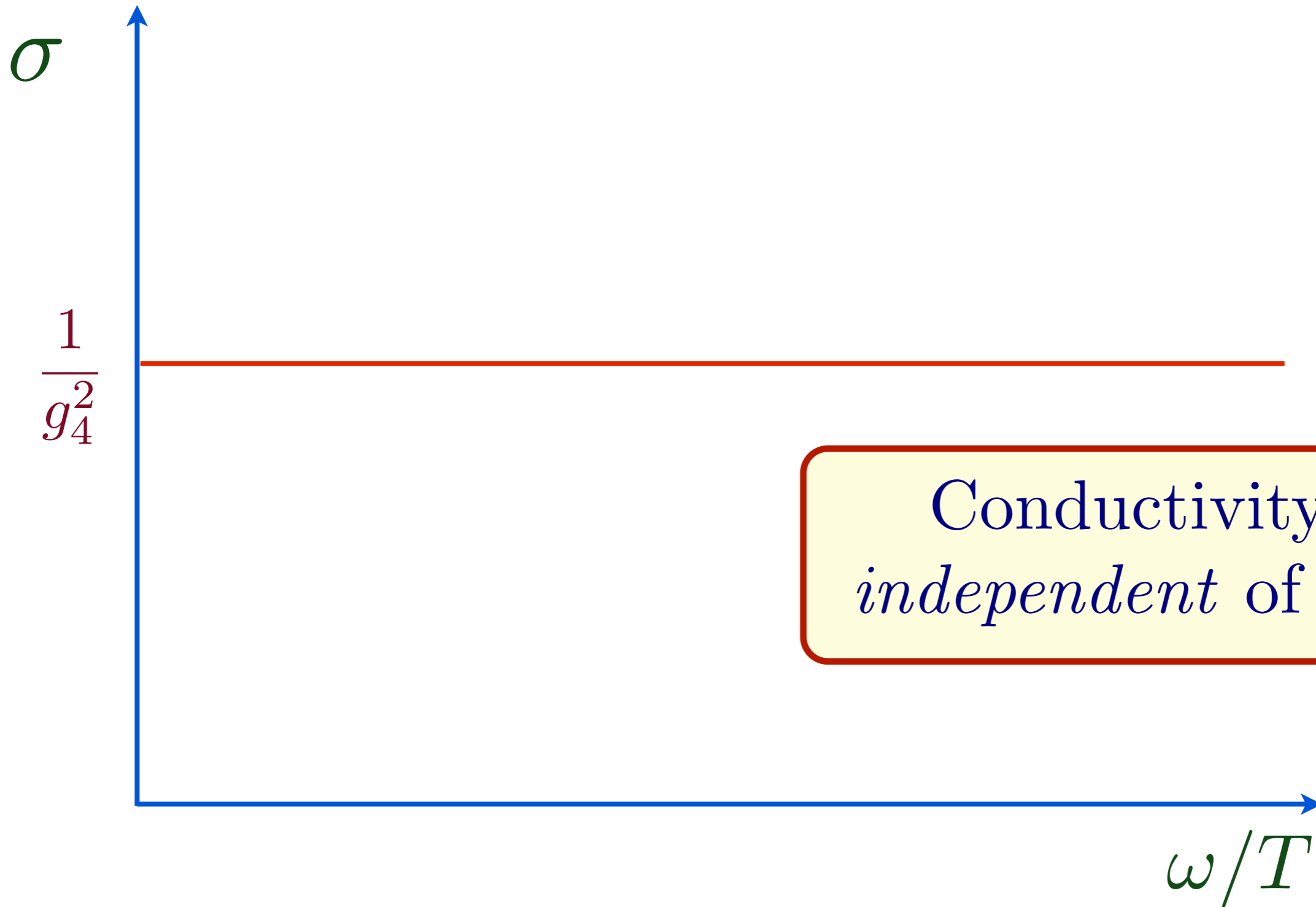
$$A_\mu(r \rightarrow 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where \mathcal{A}_μ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

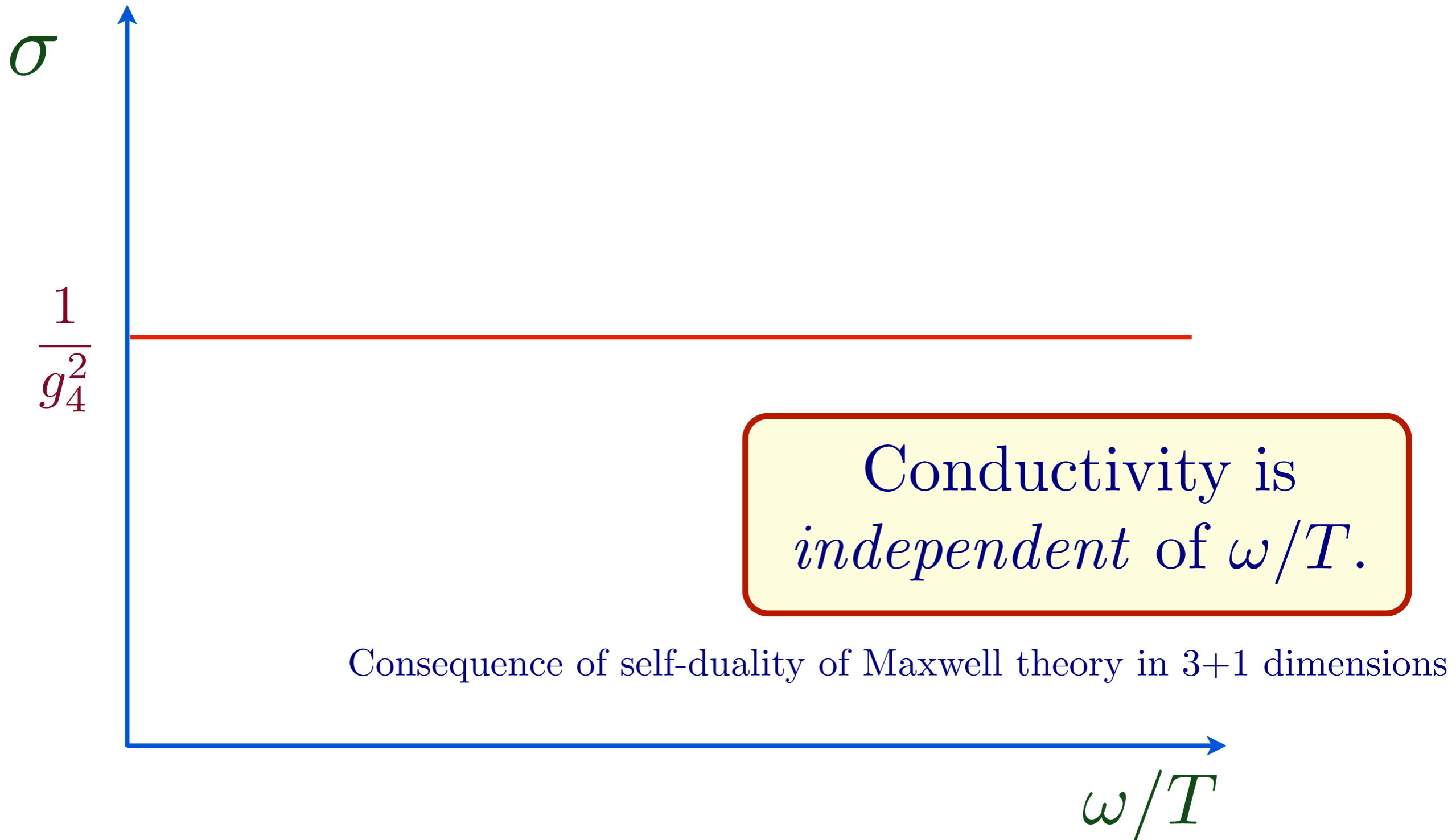
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AdS4 theory of electrical transport in a strongly interacting CFT3 for $T > 0$



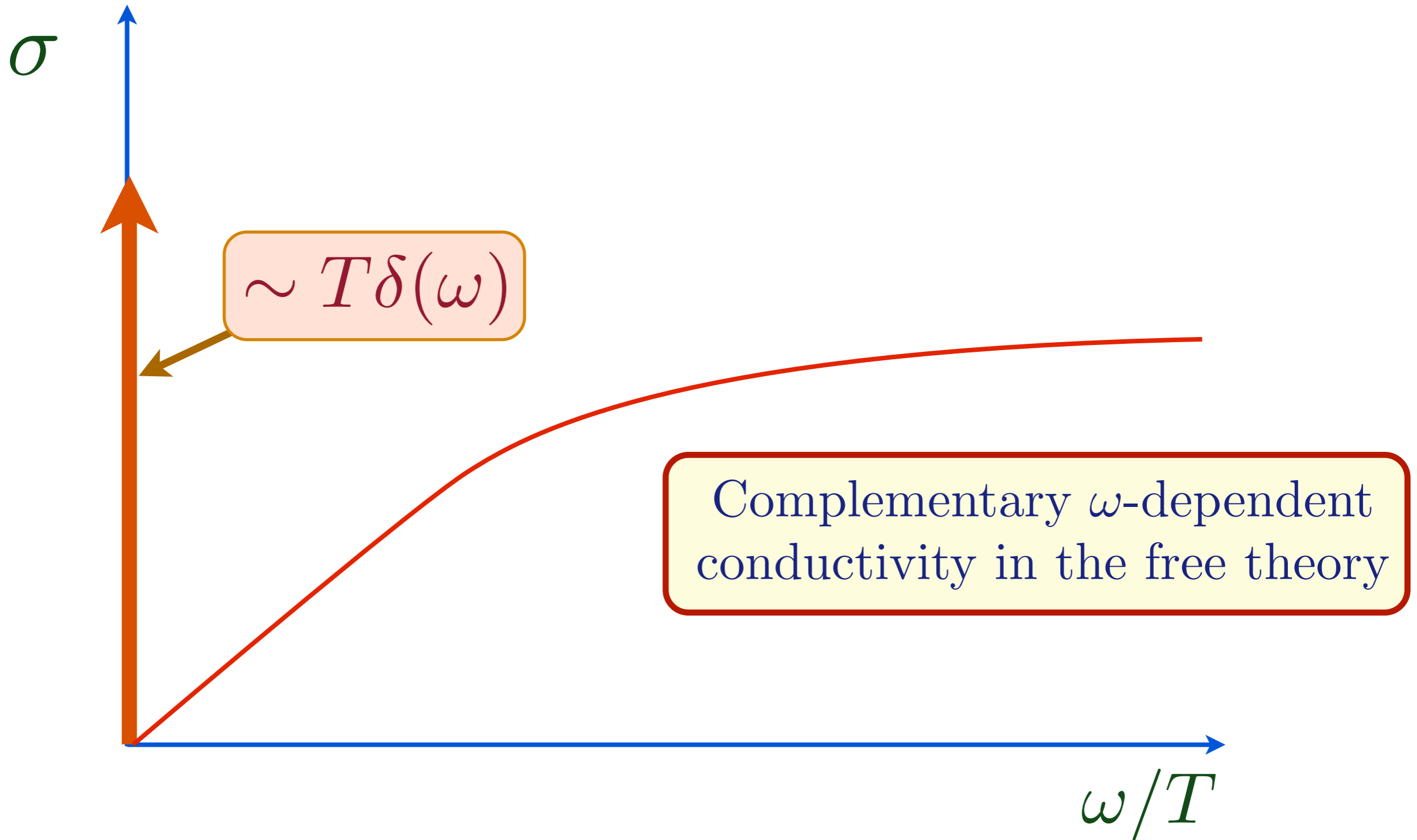
Conductivity is independent of ω/T .

AdS4 theory of electrical transport in a strongly interacting CFT3 for $T > 0$



C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

Electrical transport in a free CFT3 for $T > 0$



Improving the AdS₄ theory of “nearly perfect fluids”

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R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS₄):

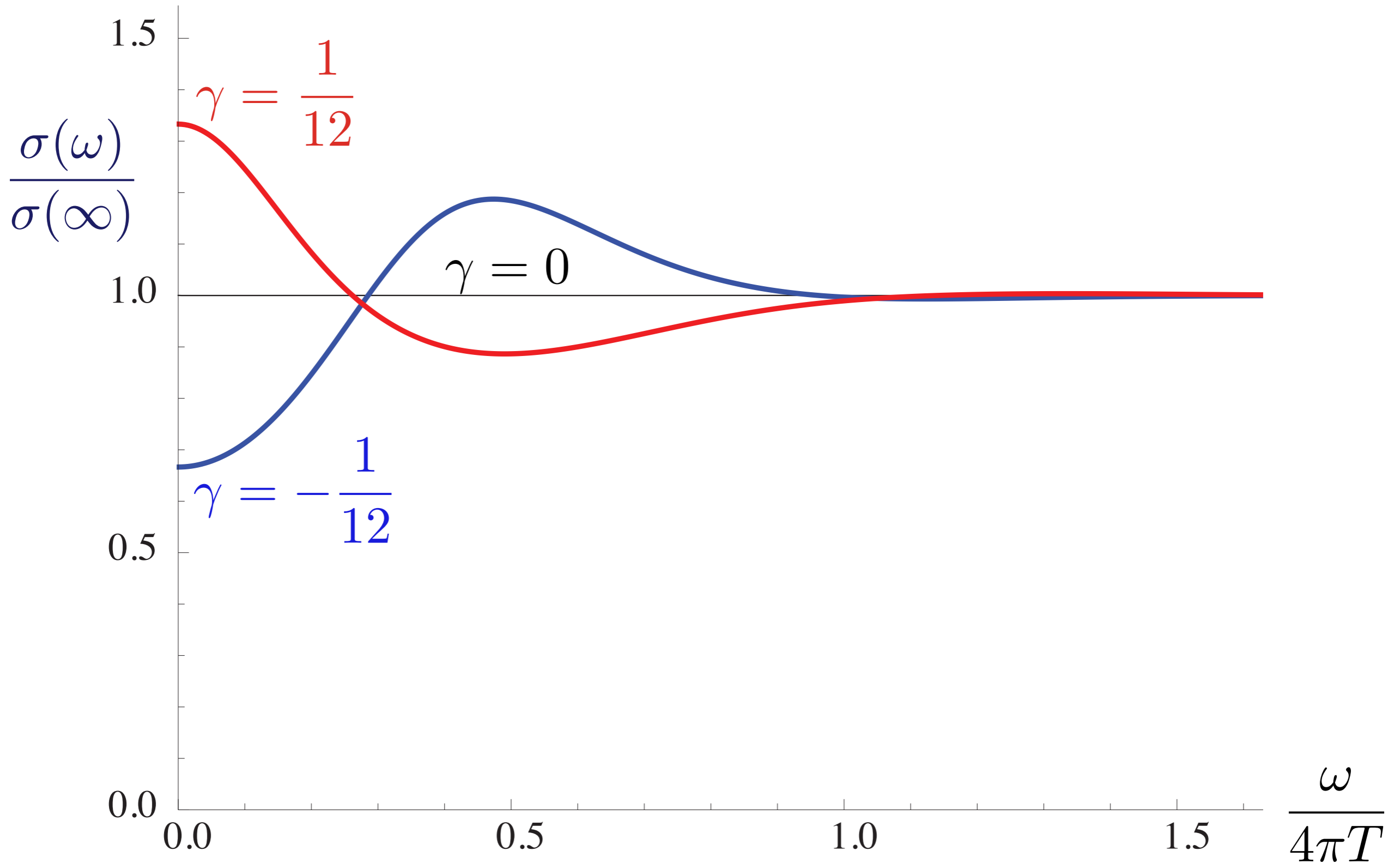
$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} + \frac{\gamma L^2}{g_4^2} C_{abcd} F^{ab} F^{cd} \right],$$

where C_{abcd} is the Weyl curvature tensor.

Stability and causality constraints restrict $|\gamma| < 1/12$.

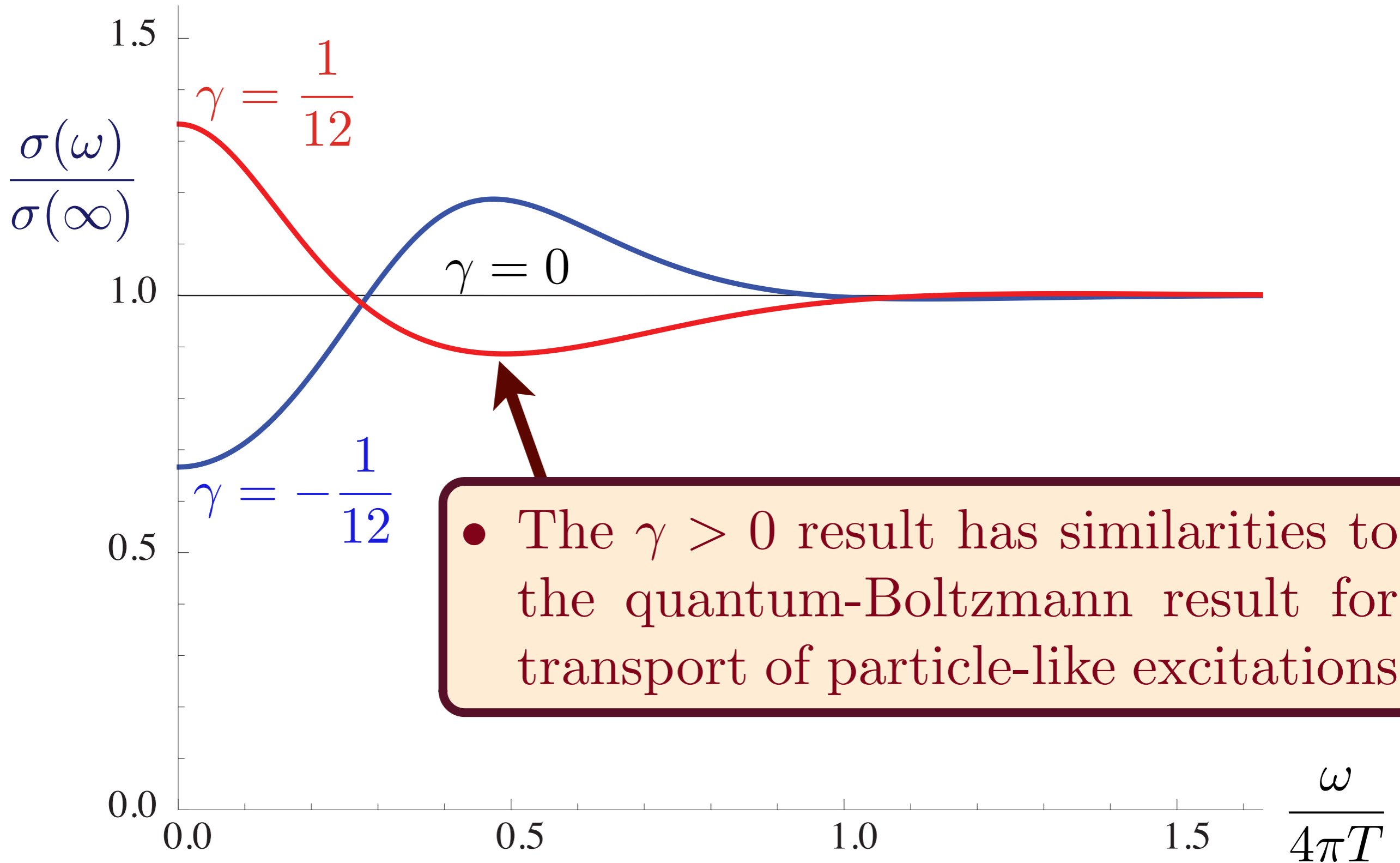
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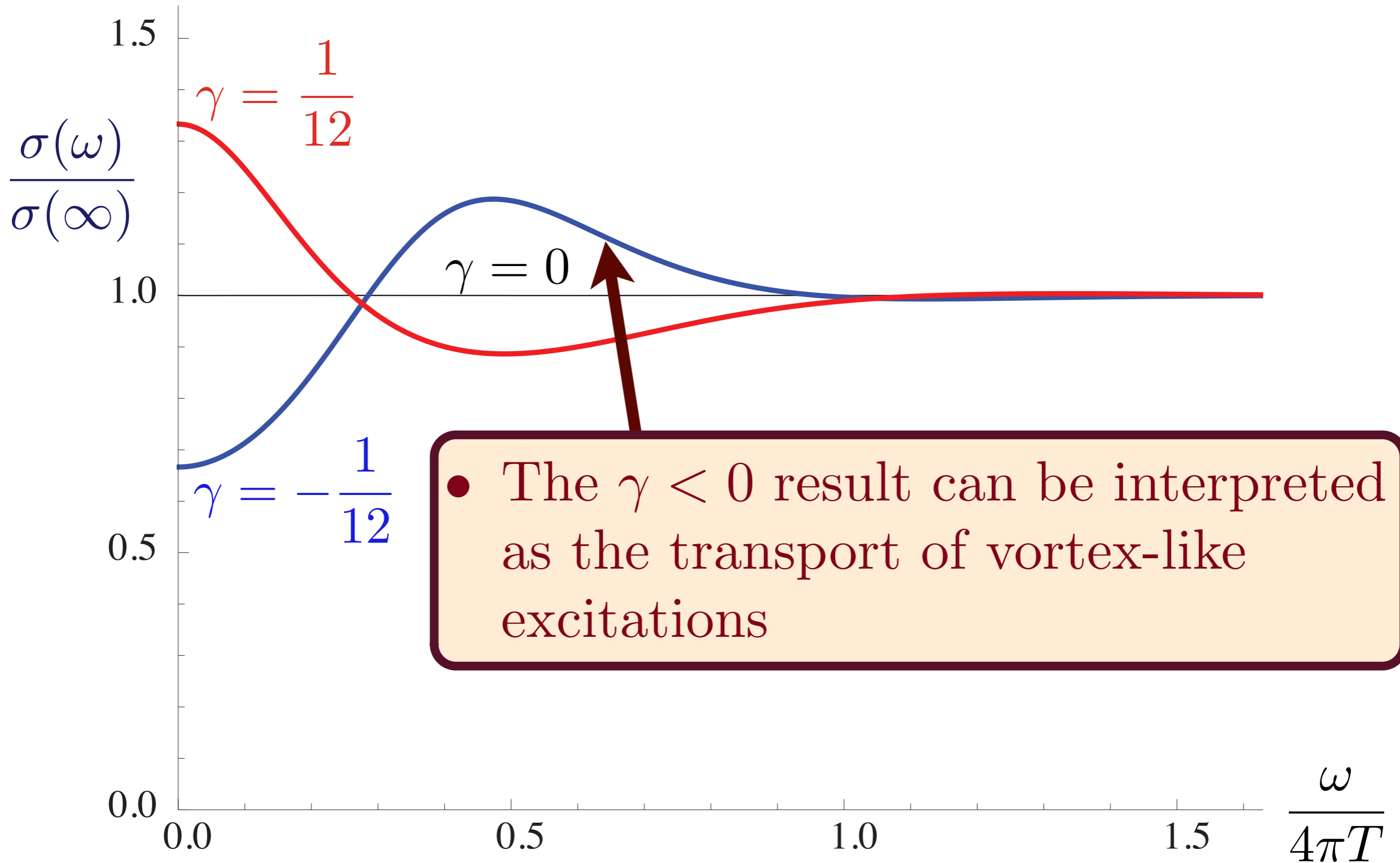
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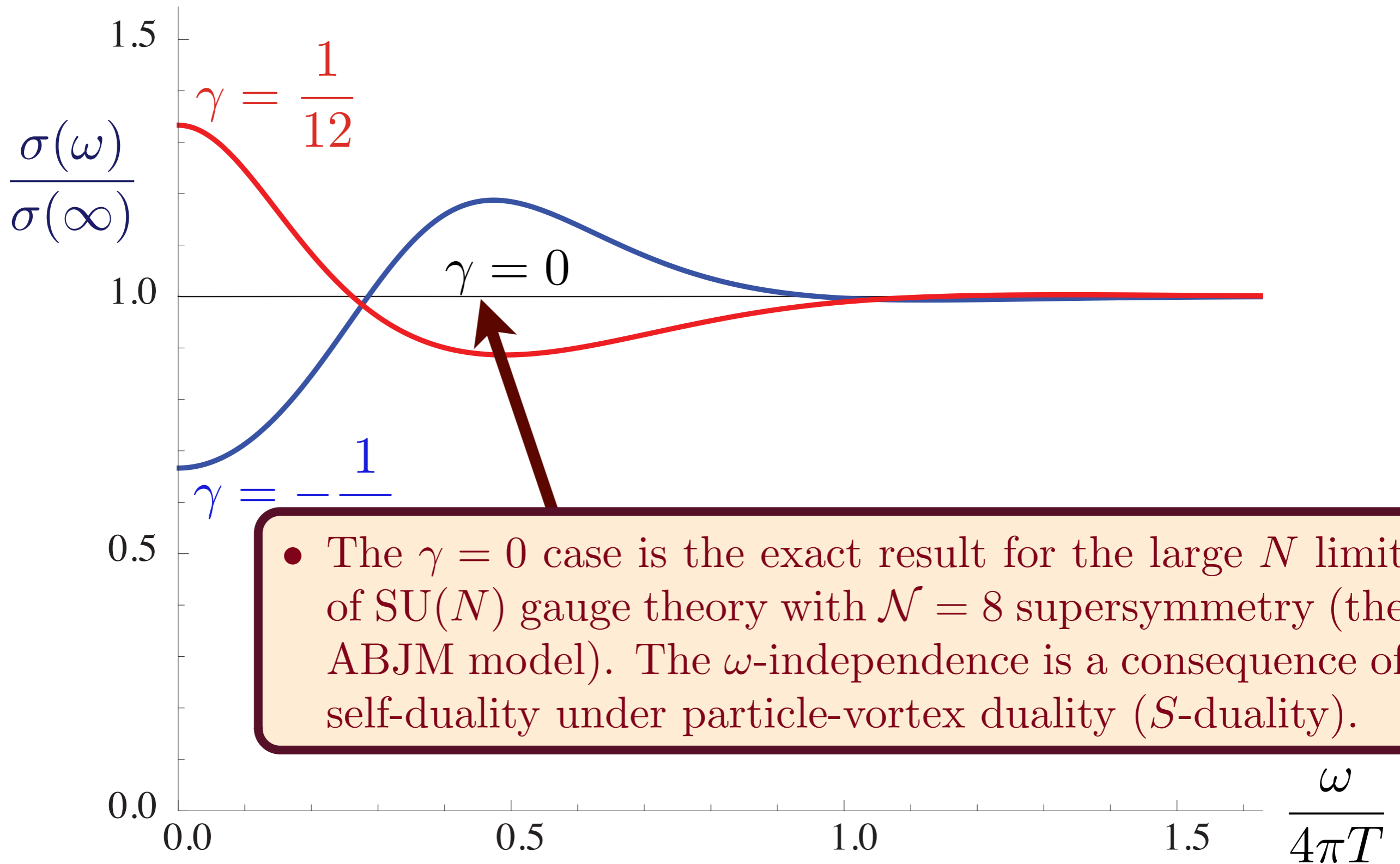
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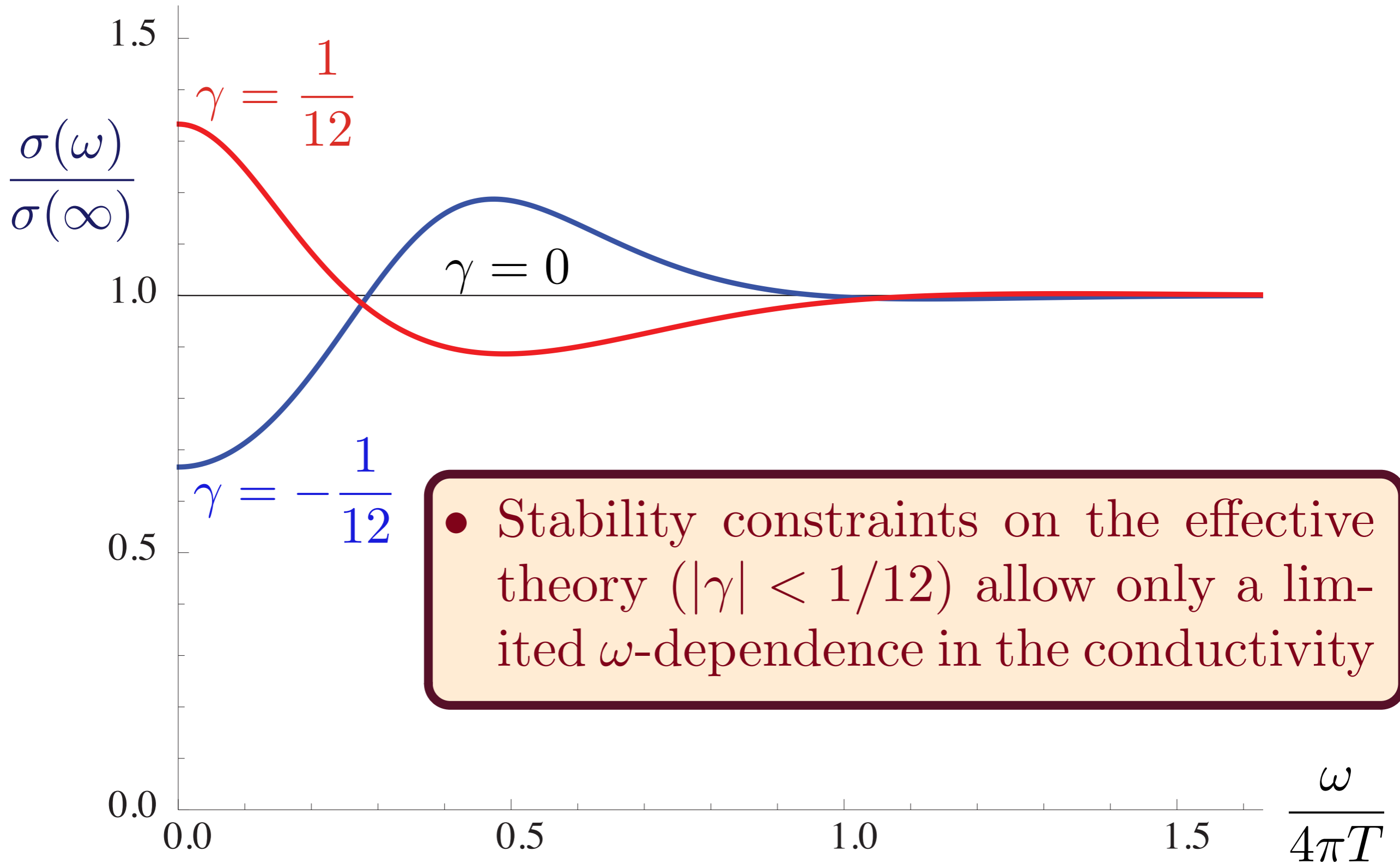
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Conclusions

Conformal quantum matter

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 New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

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Conclusions

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

States of quantum matter with long-range entanglement in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

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Liza Huijse



Max Metlitski



Brian Swingle

Compressible quantum matter

A. Fermi liquids: graphene

B. Holography: Reissner - Nördstrom solution

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

D. Holography: scaling arguments for entropy and entanglement entropy

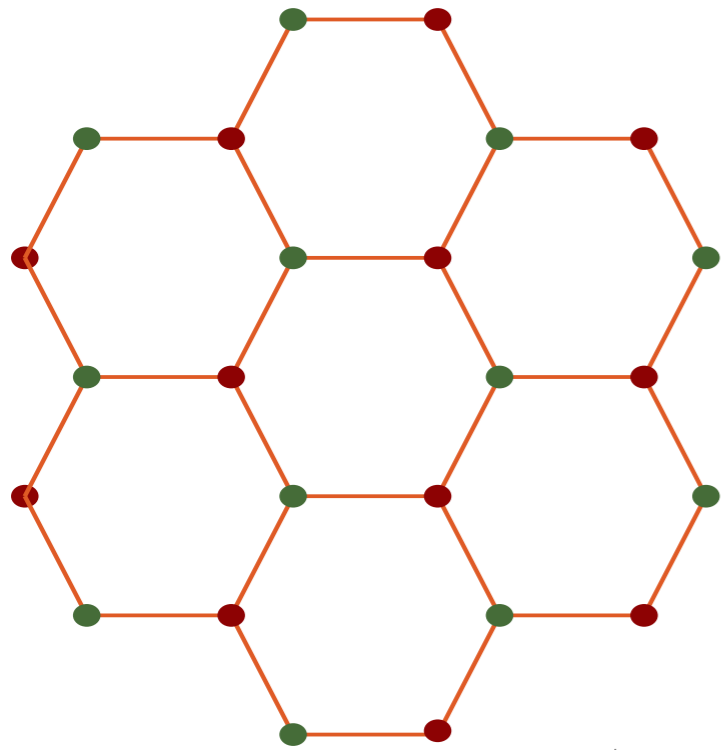
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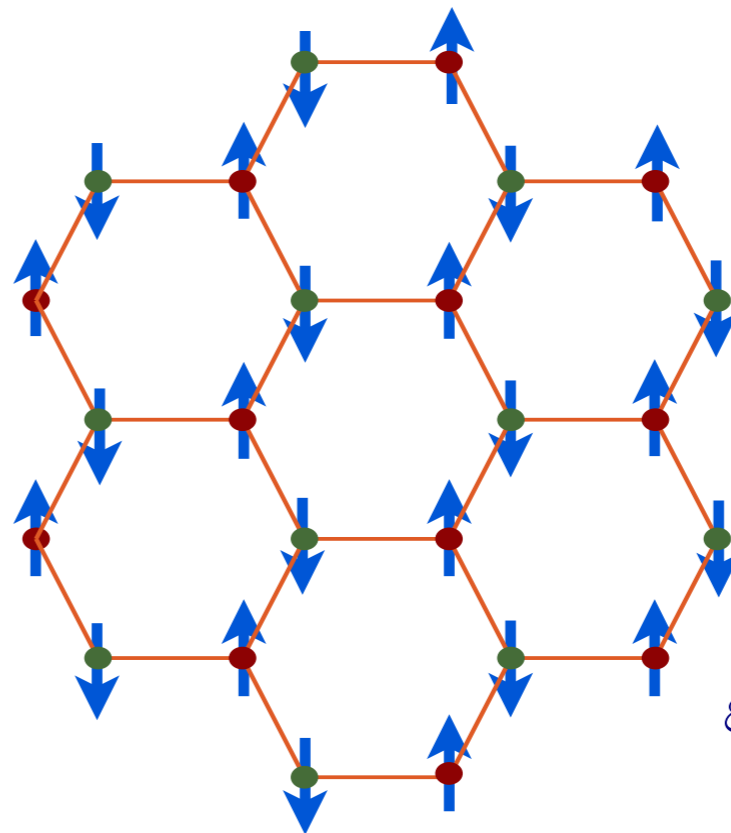
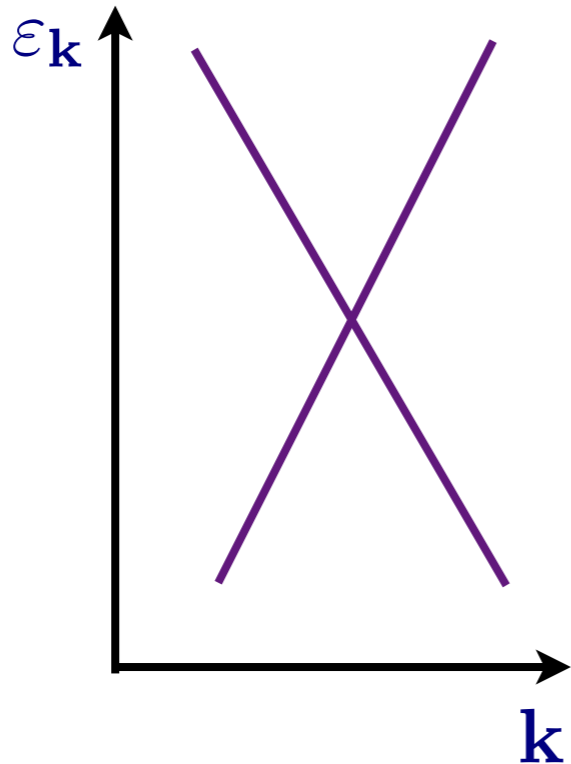
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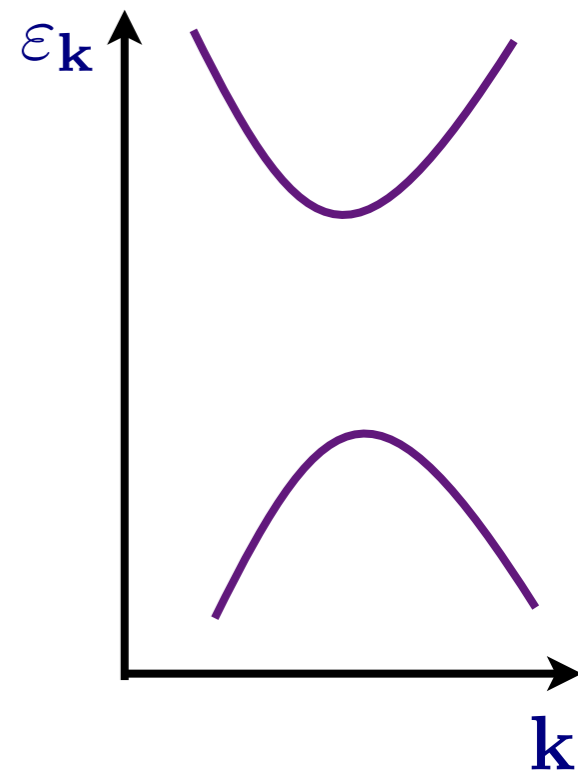
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

$$\langle \varphi^a \rangle \neq 0$$

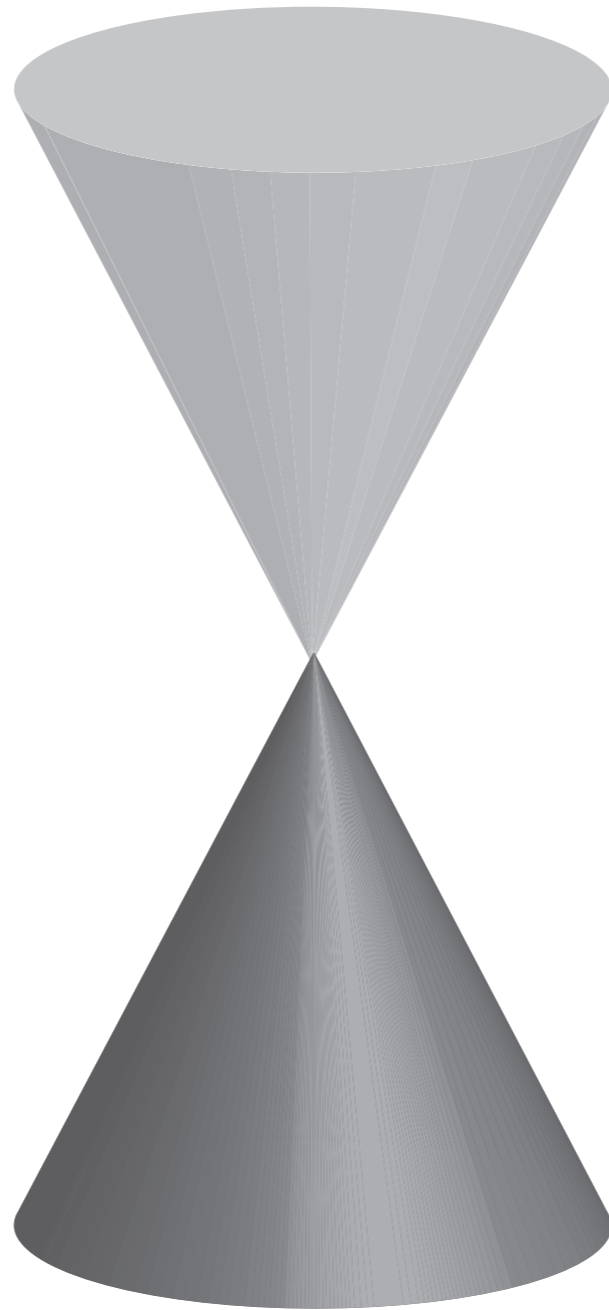


S

Free CFT3

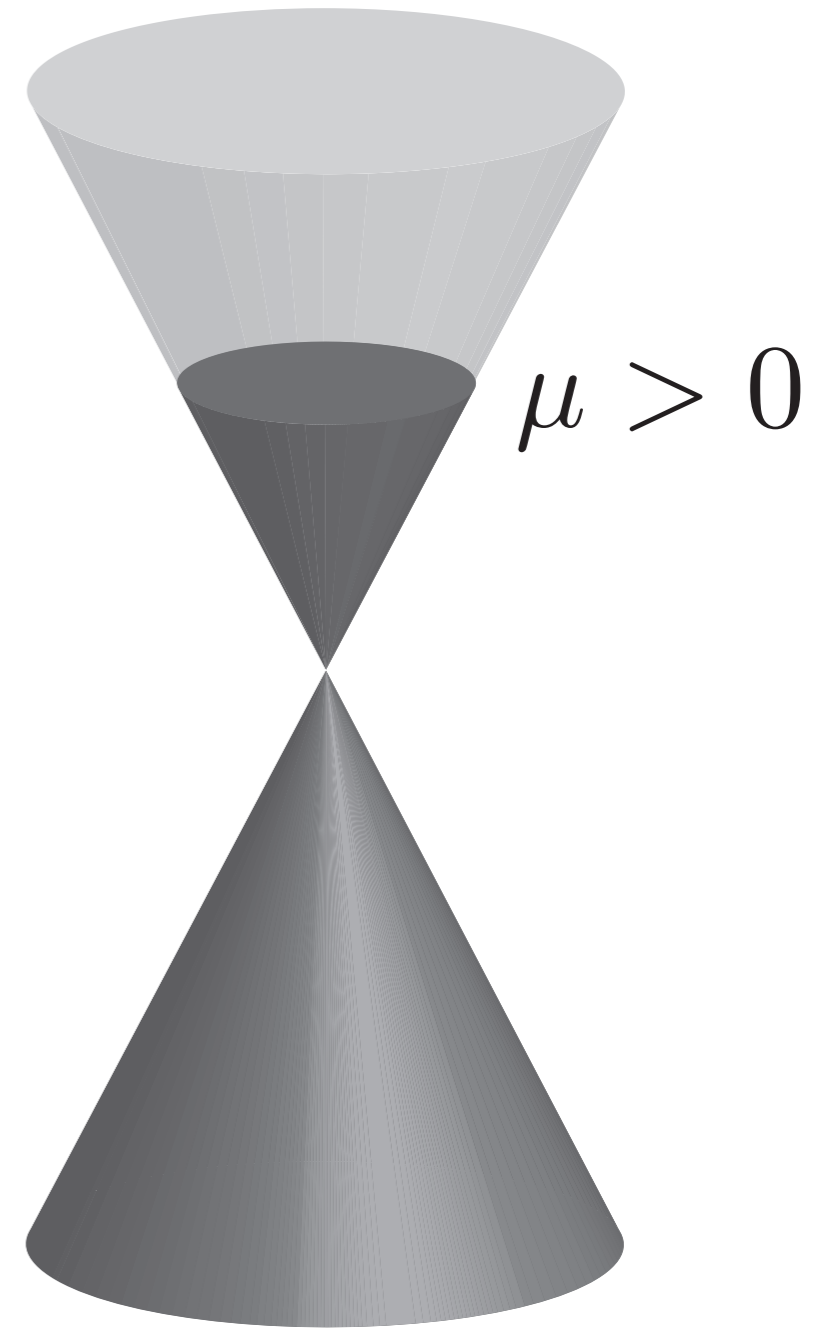
Interacting CFT3
with long-range entanglement

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



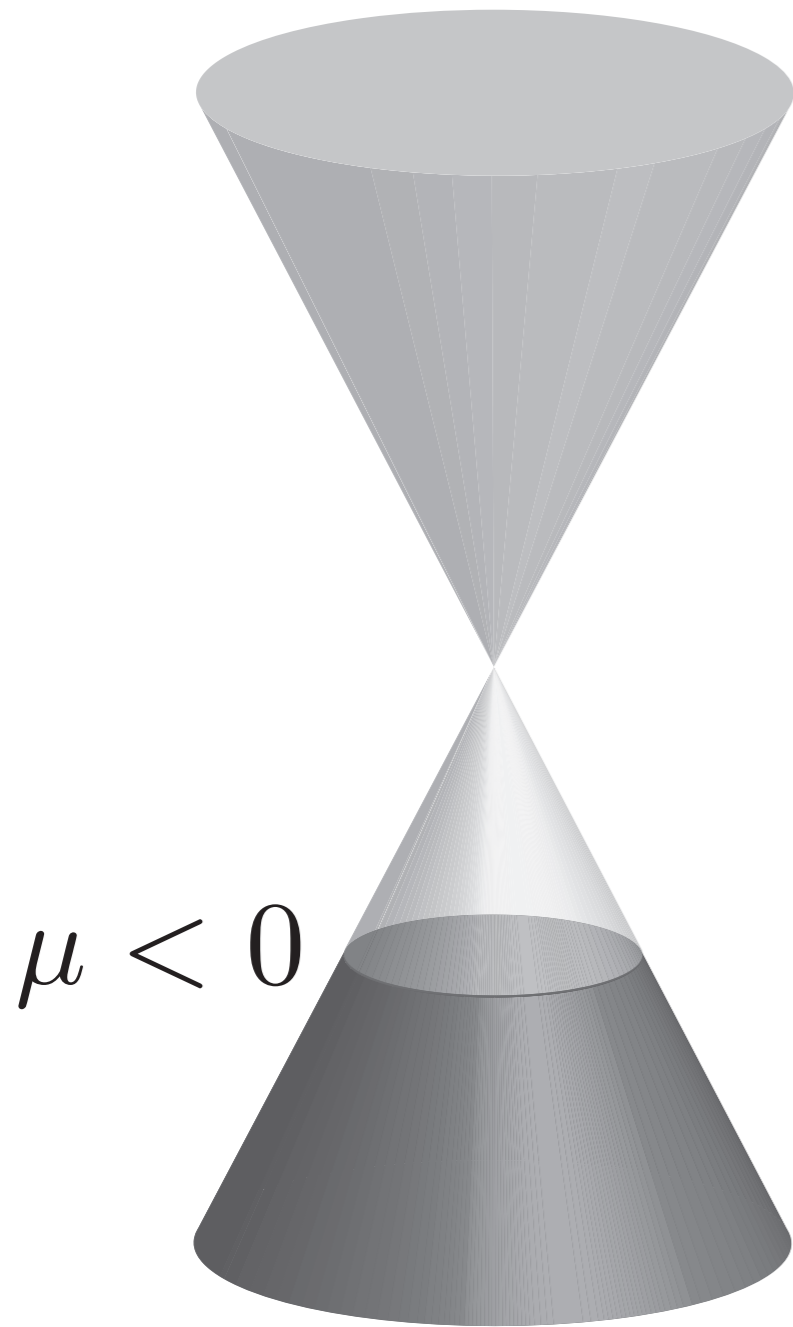
**Dirac
semi-metal**

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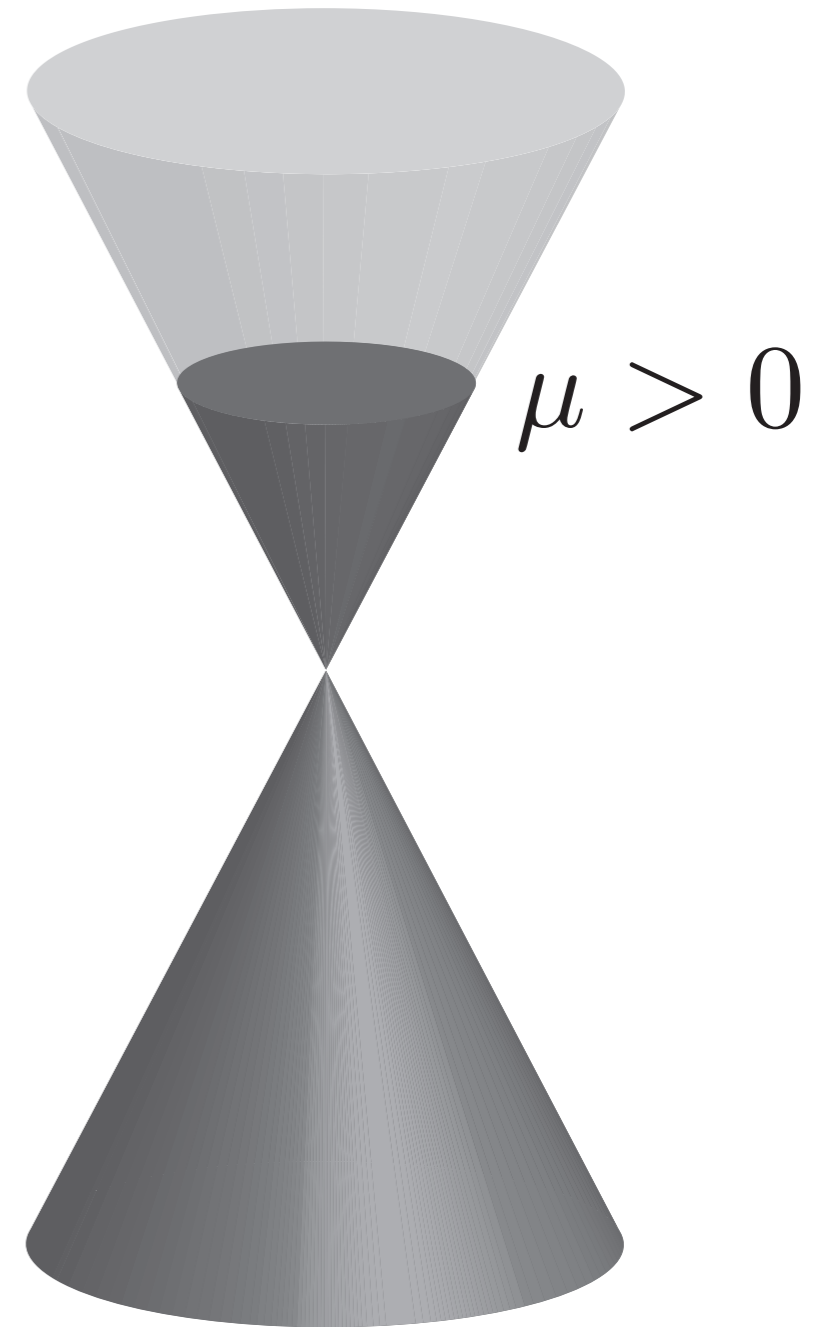
**Electron
Fermi surface**

Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



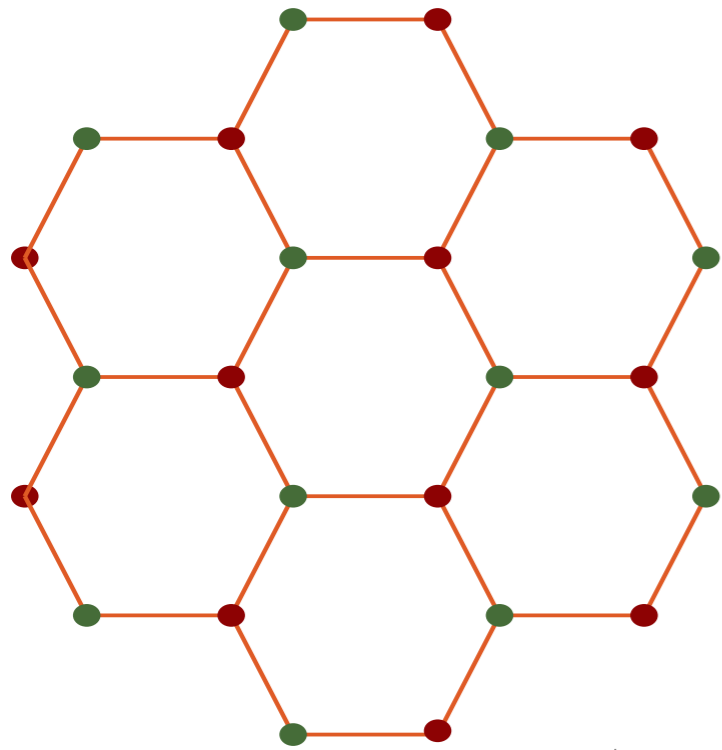
$$\mu < 0$$

**Hole
Fermi surface**



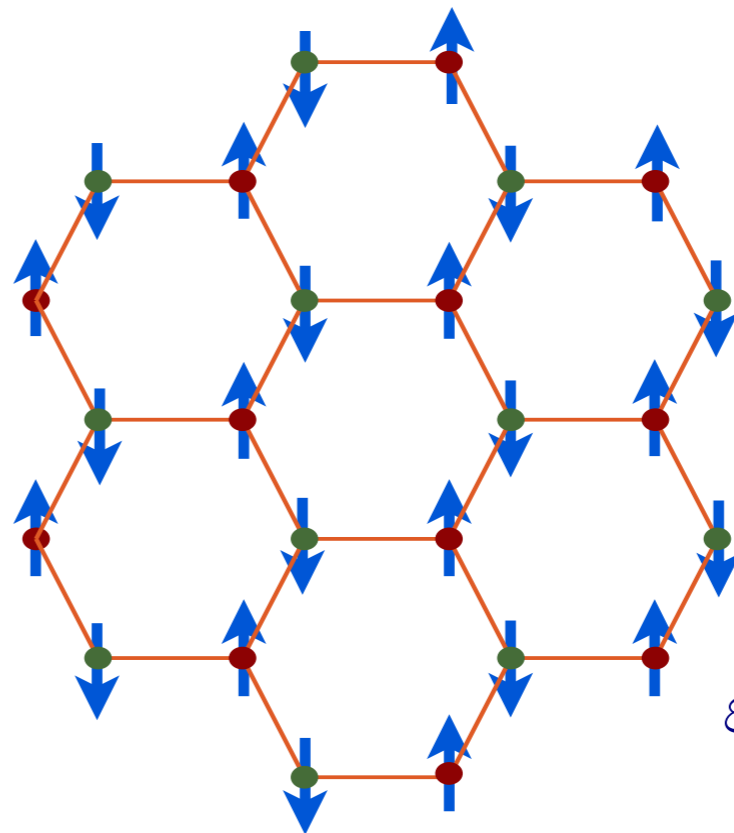
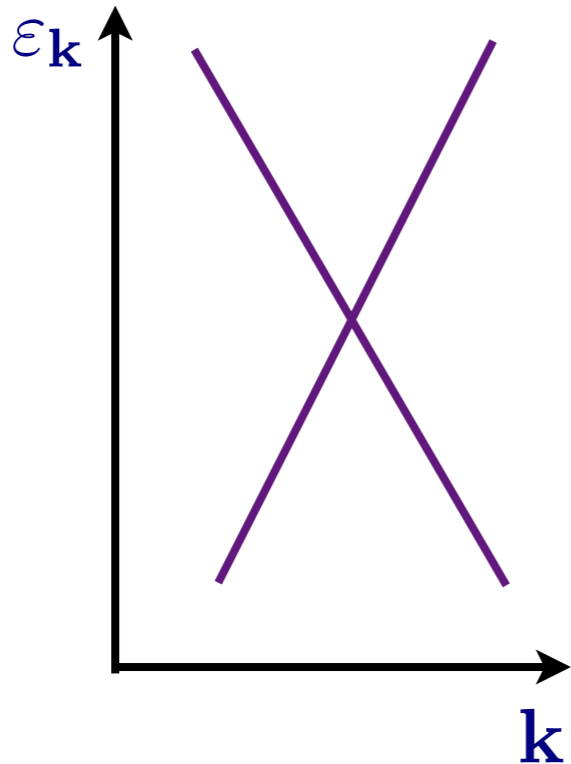
$$\mu > 0$$

**Electron
Fermi surface**



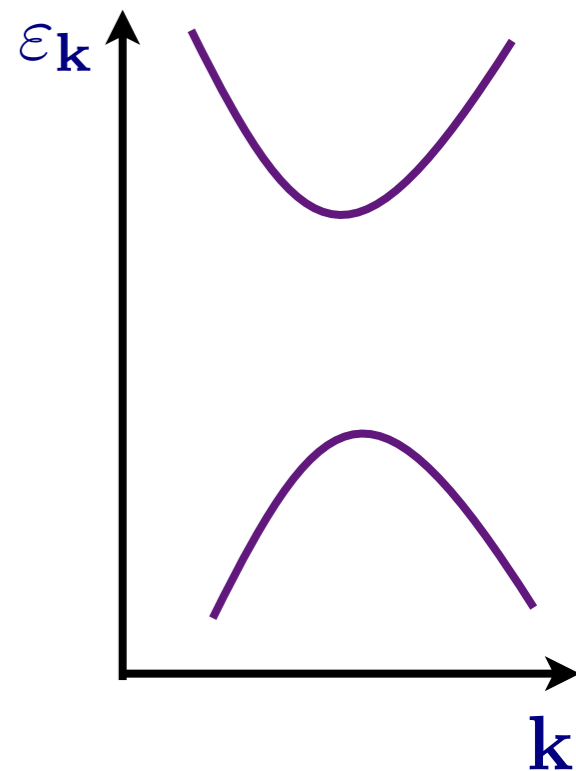
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

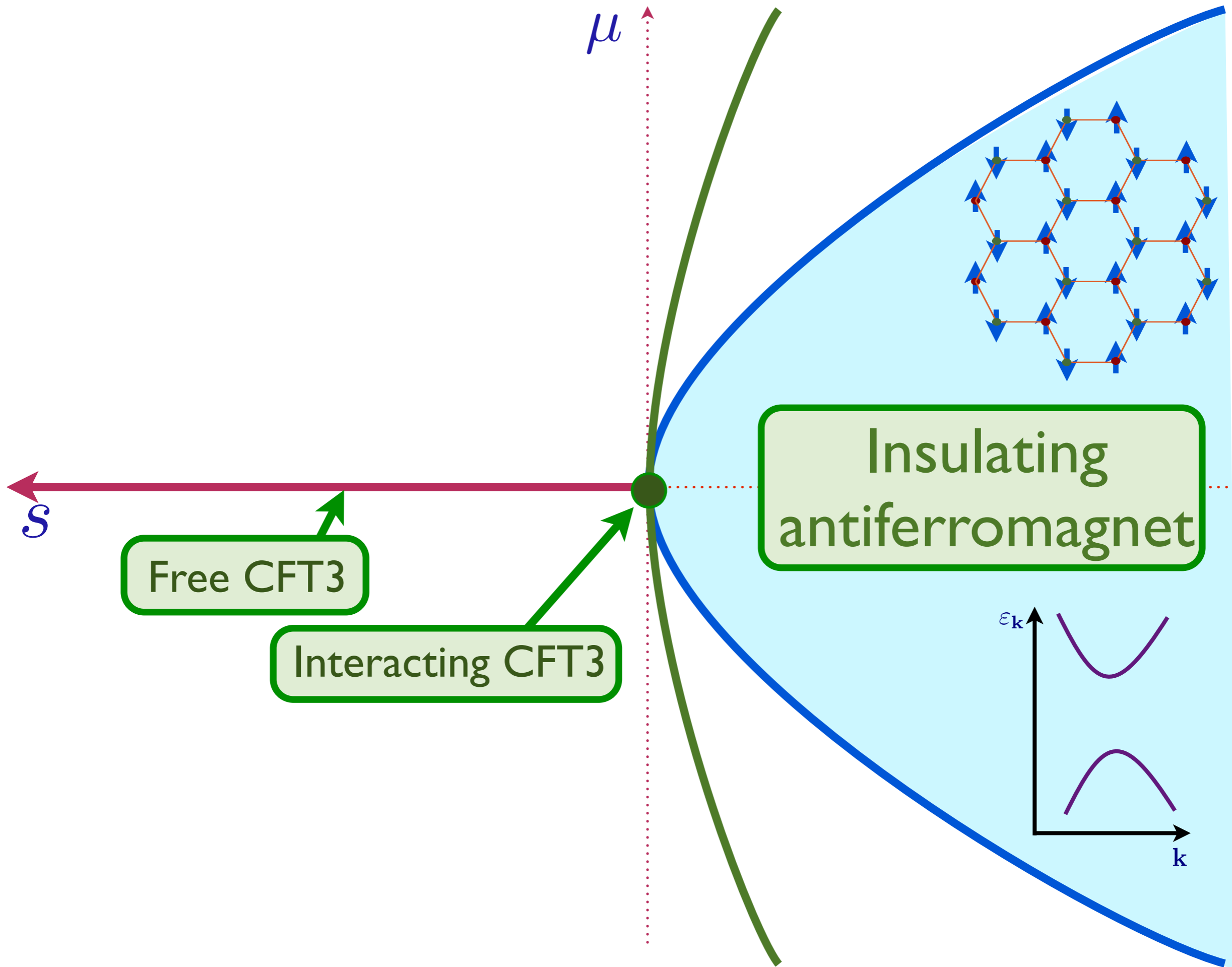
$$\langle \varphi^a \rangle \neq 0$$

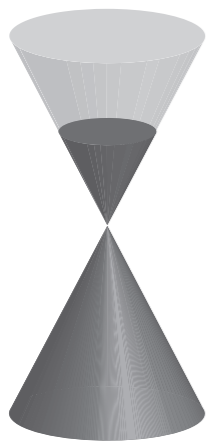


S

Free CFT3

Interacting CFT3
with long-range entanglement





Electron metal

μ

S

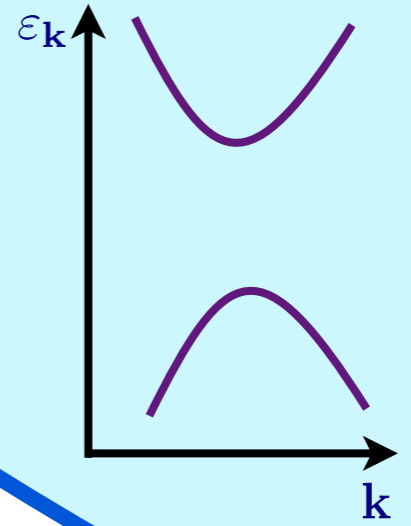
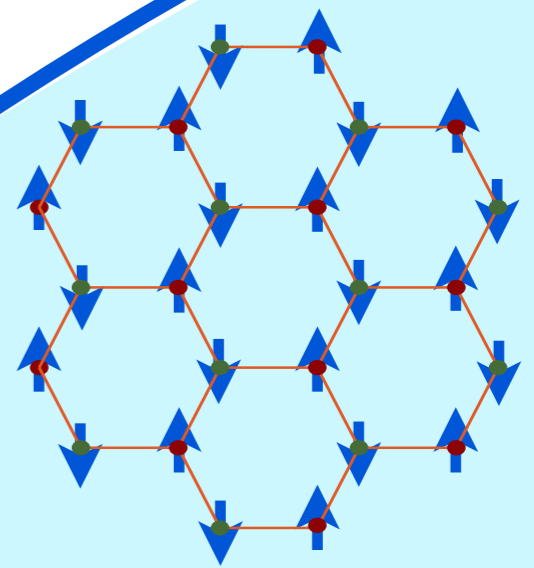
Free CFT3

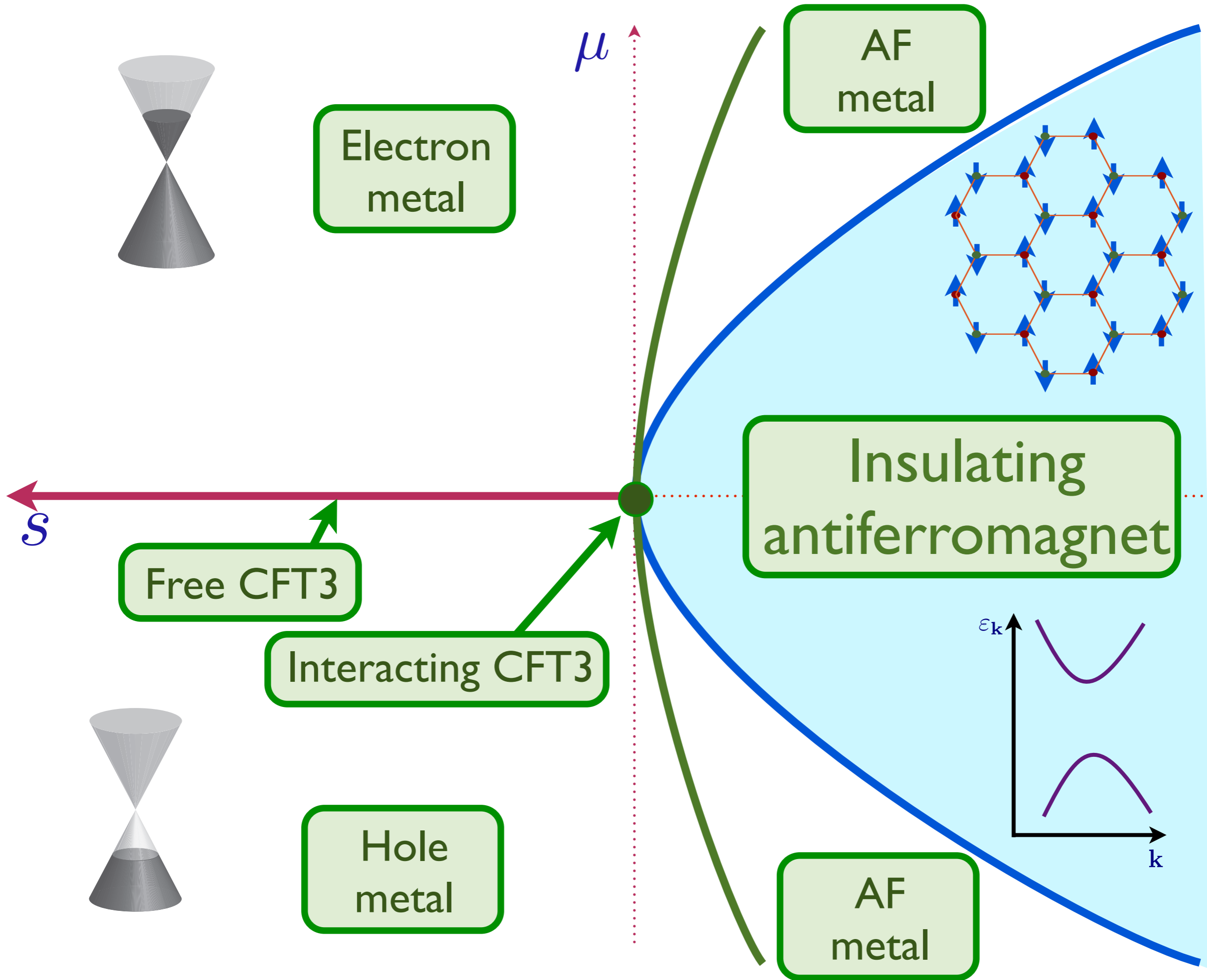
Interacting CFT3



Hole metal

Insulating antiferromagnet





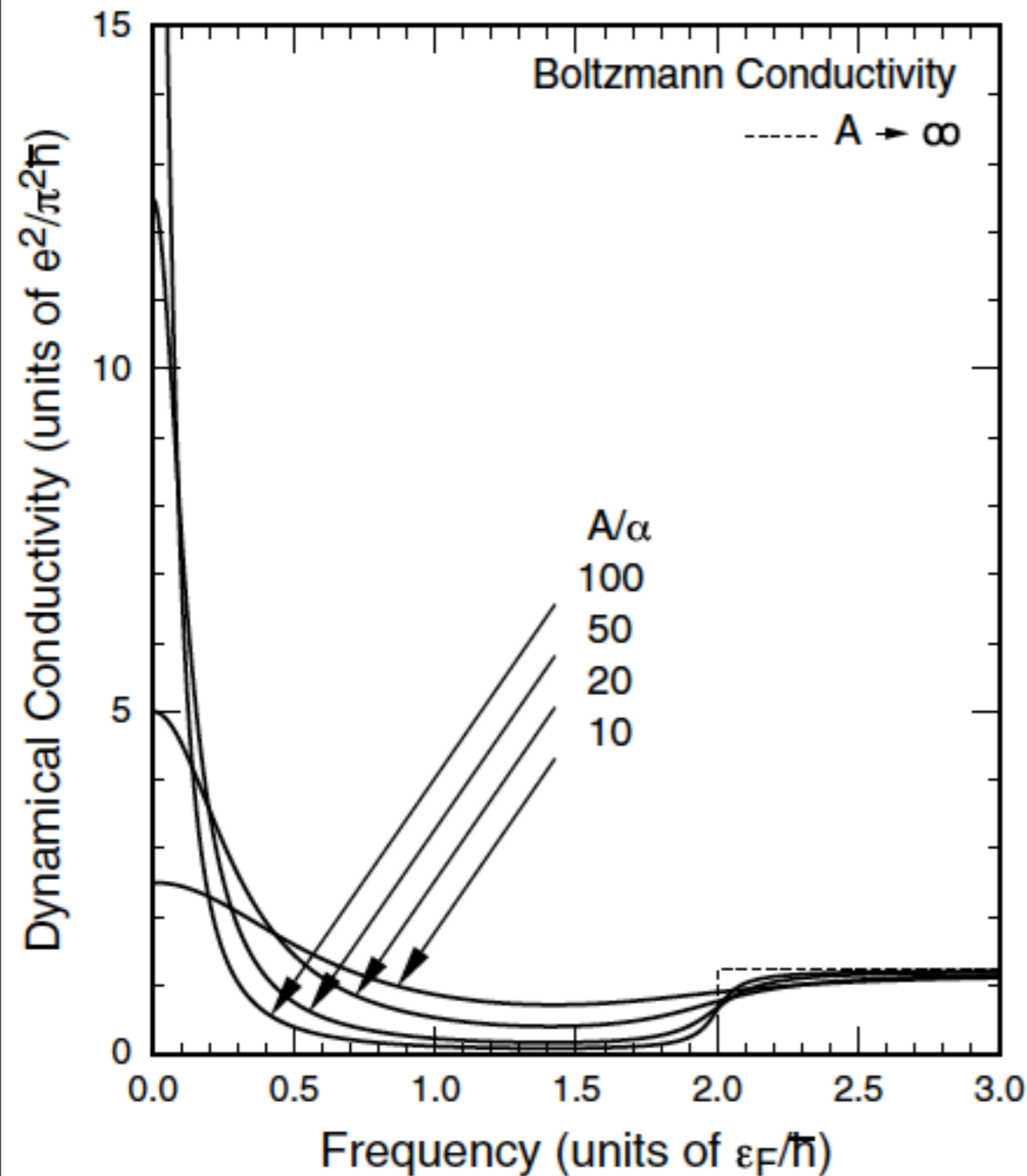
Transport in graphene at non-zero μ

From the Kubo formula

$$\sigma(\omega) = 2 (ev_F)^2 \frac{\hbar}{i} \sum_{ss'} \int \frac{d^2k}{4\pi^2} \frac{f(\varepsilon_s(\mathbf{k})) - f(\varepsilon_{s'}(\mathbf{k}))}{(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}))(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}) + \hbar\omega + i\eta)}$$

where $\varepsilon_s(\mathbf{k}) = s\hbar v_F |\mathbf{k}|$ and $s, s' = \pm 1$ for the valence and conduction bands.

Transport in graphene at non-zero μ

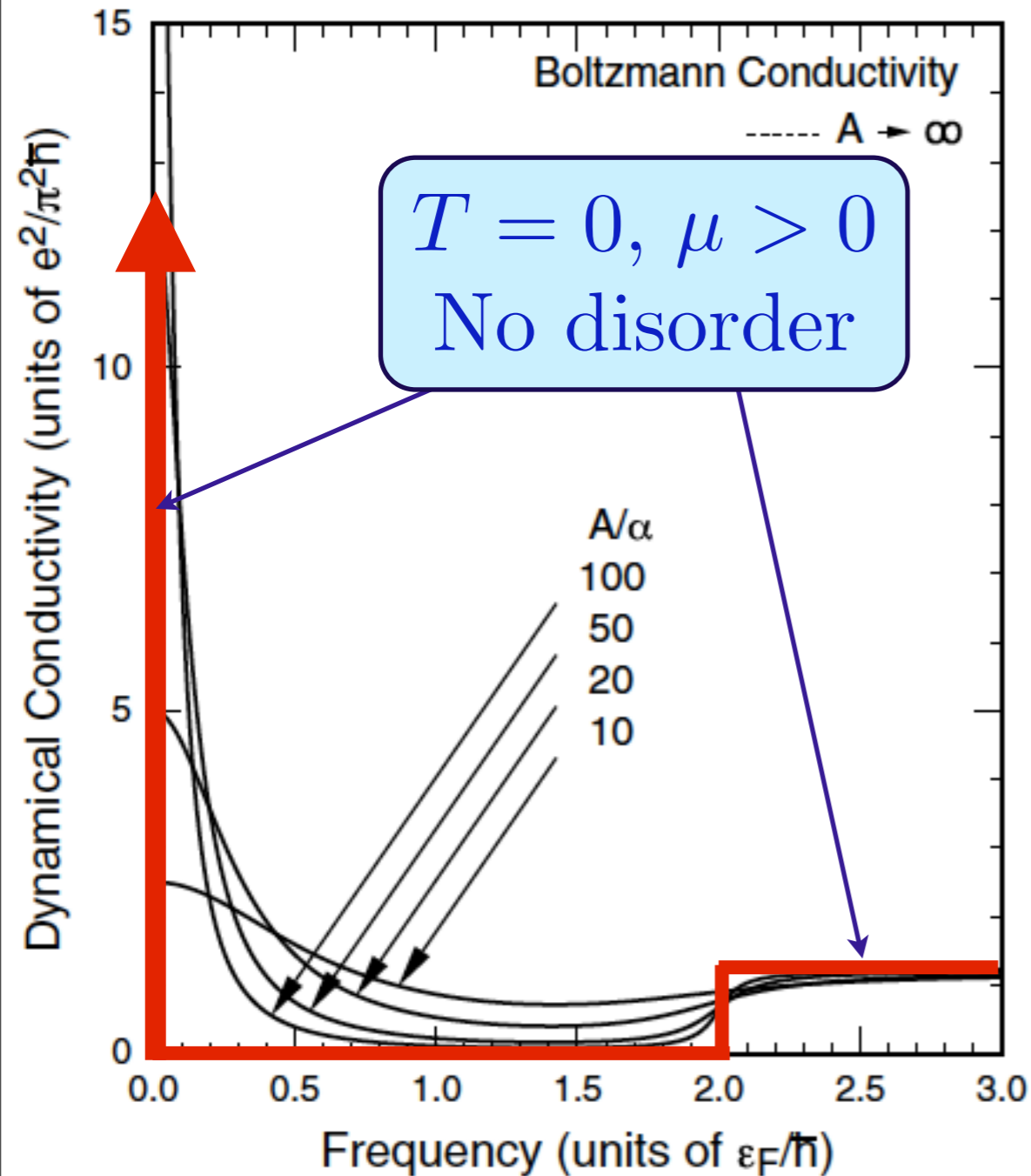


A is inversely proportional to disorder. In the clean limit $A \rightarrow \infty$, at $T = 0$

$$\text{Re}[\sigma(\omega)] = \frac{e^2}{\hbar} \left[\frac{\epsilon_F}{\hbar} \delta(\omega) + \frac{1}{4} \theta(|\omega| - 2\epsilon_F) \right]$$

Notice delta function is present even at $T = 0$ at non-zero density: this is a generic consequence of the conservation of momentum in any clean interacting Fermi liquid. Only “umklapp” scattering can broaden this delta function.

Transport in graphene at non-zero μ



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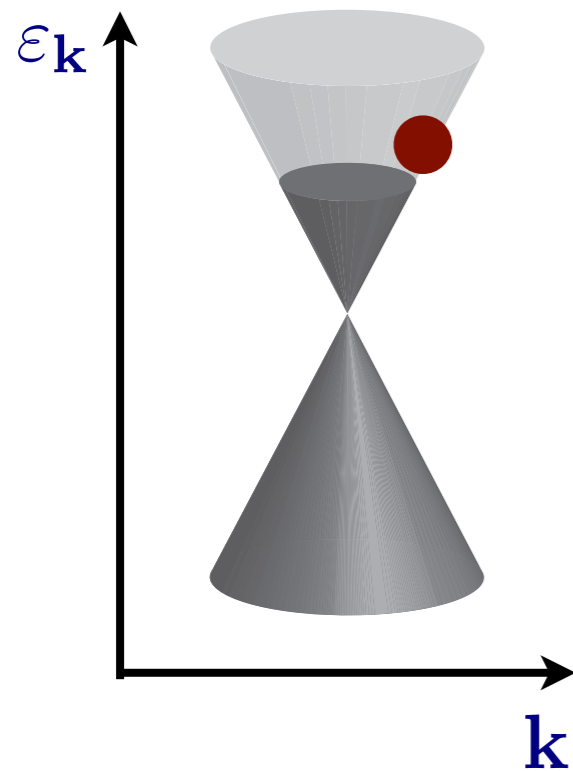
Particles



Momentum



Current



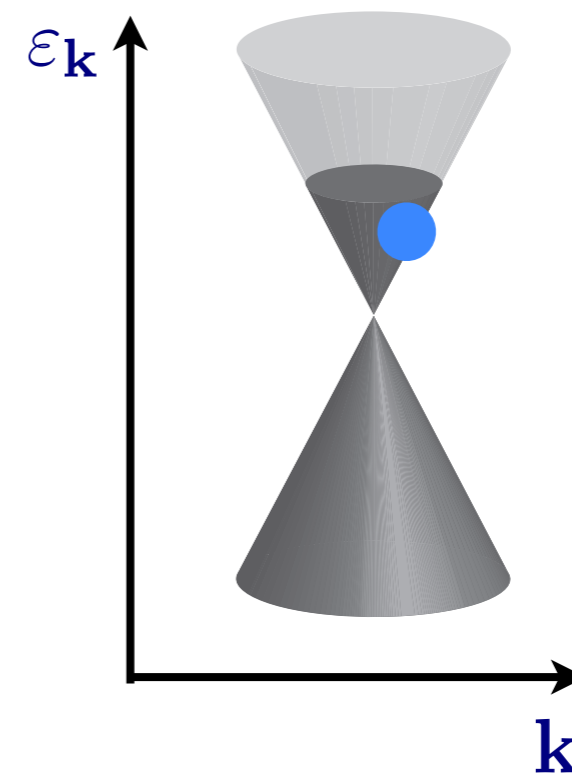
Holes



Momentum



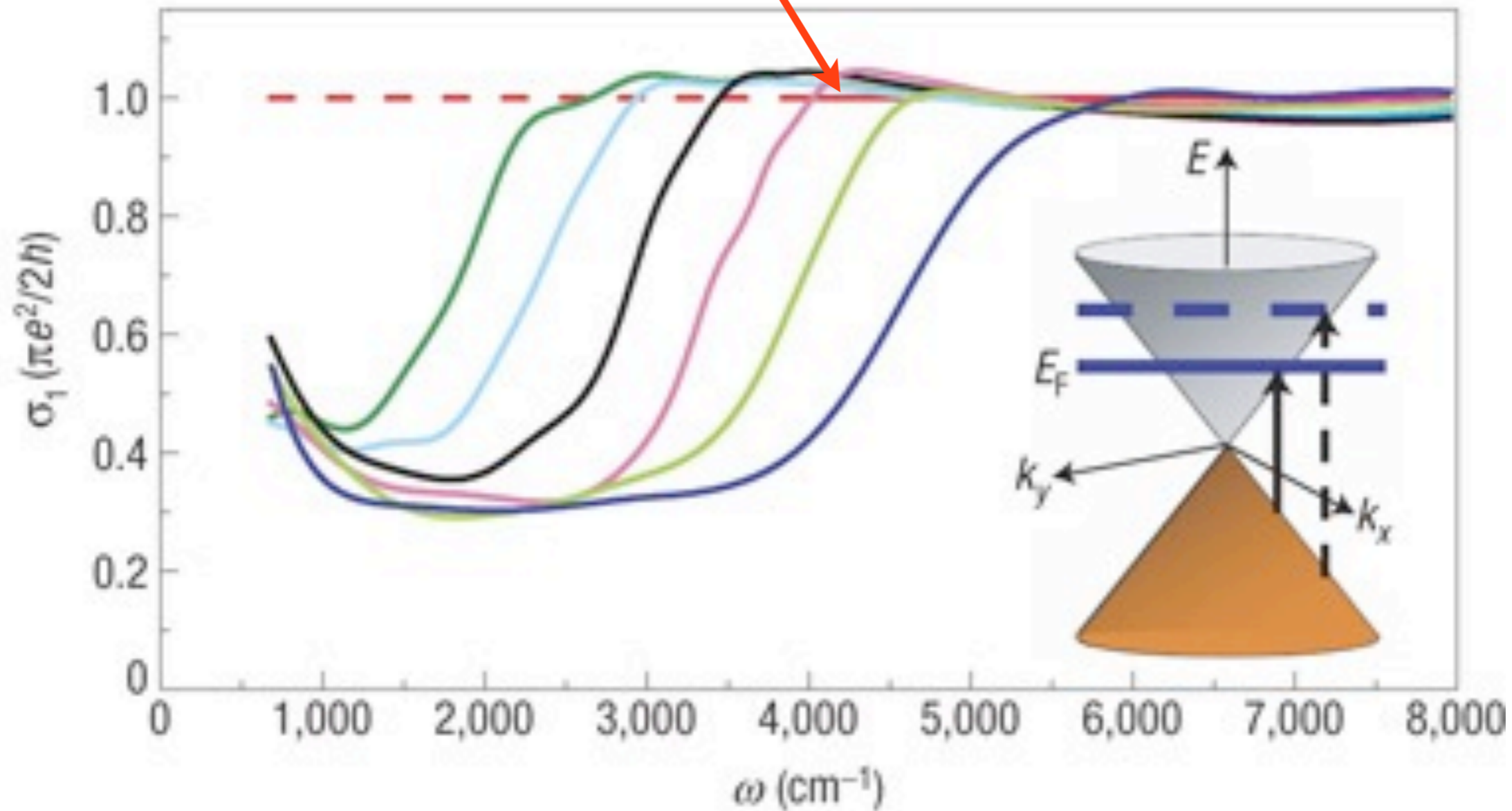
Current



Current carrying state has non-zero momentum, and collisions cannot relax current to zero

Optical conductivity of graphene

Undoped graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

Compressible quantum matter

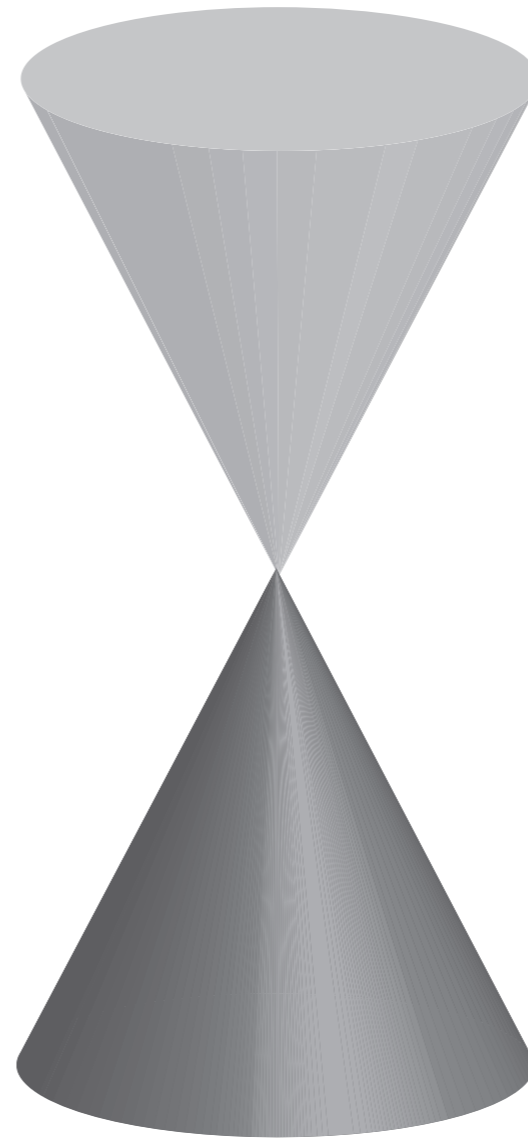
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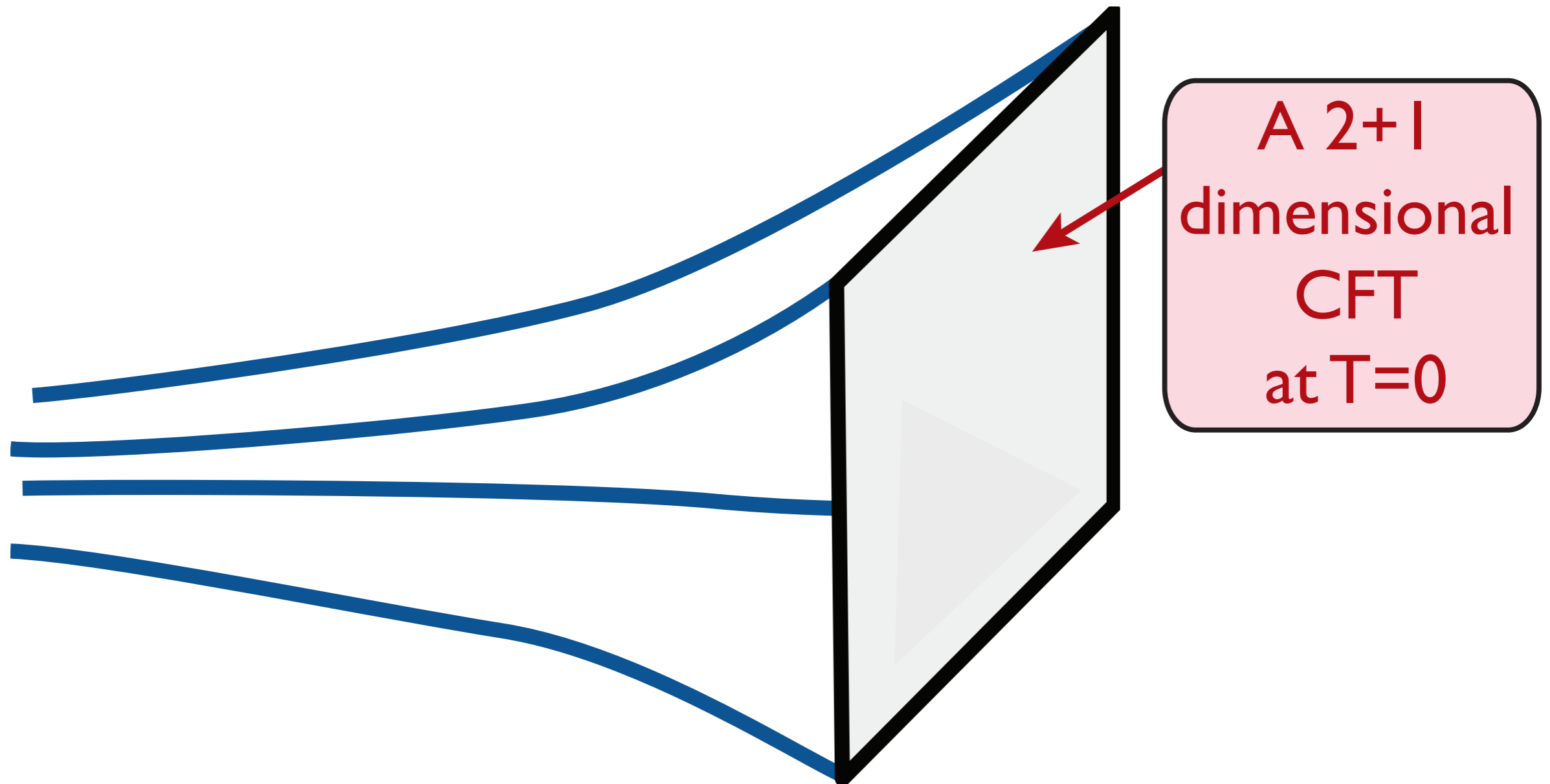
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Begin with a CFT

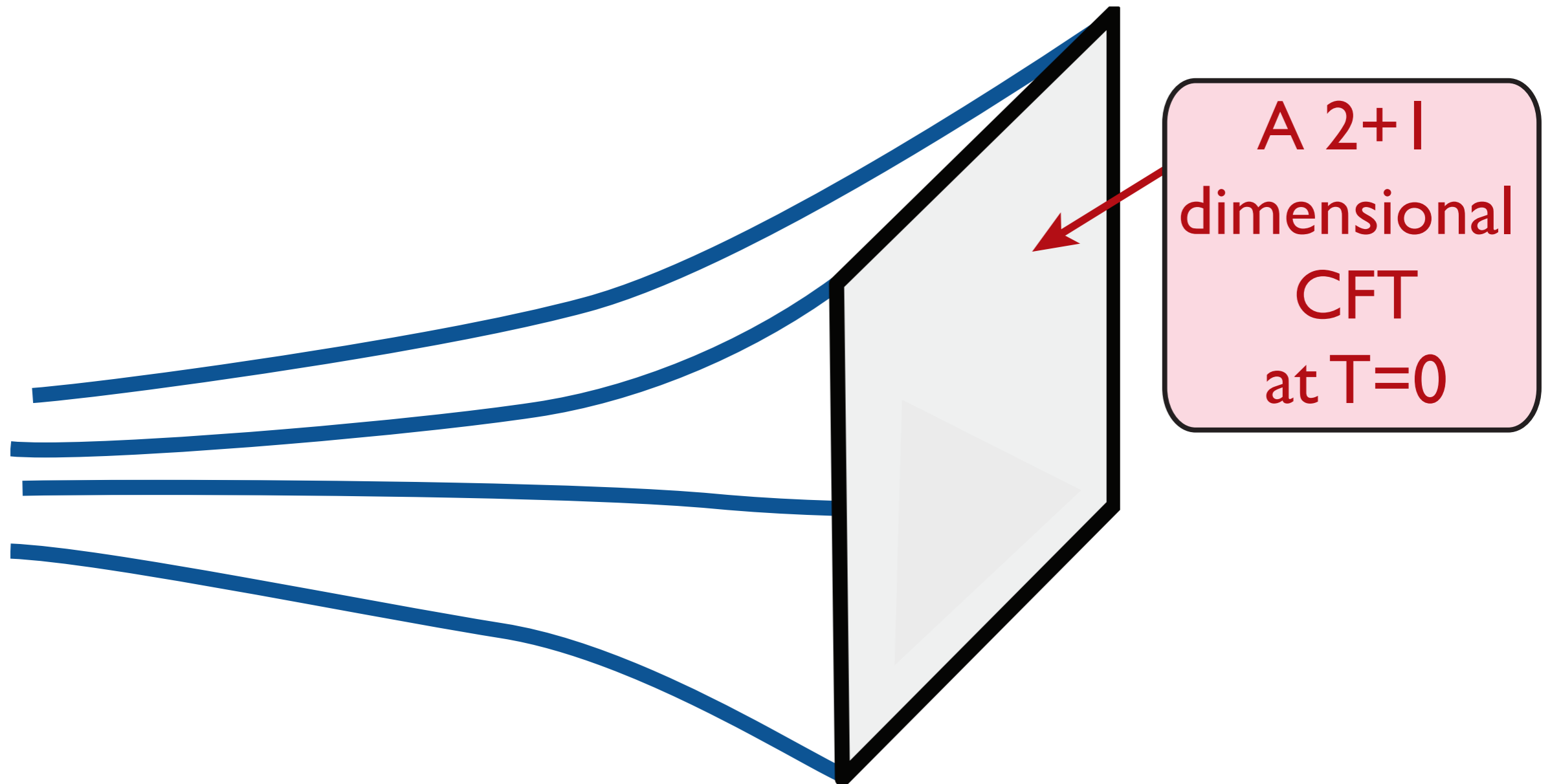


Holographic representation: AdS₄



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

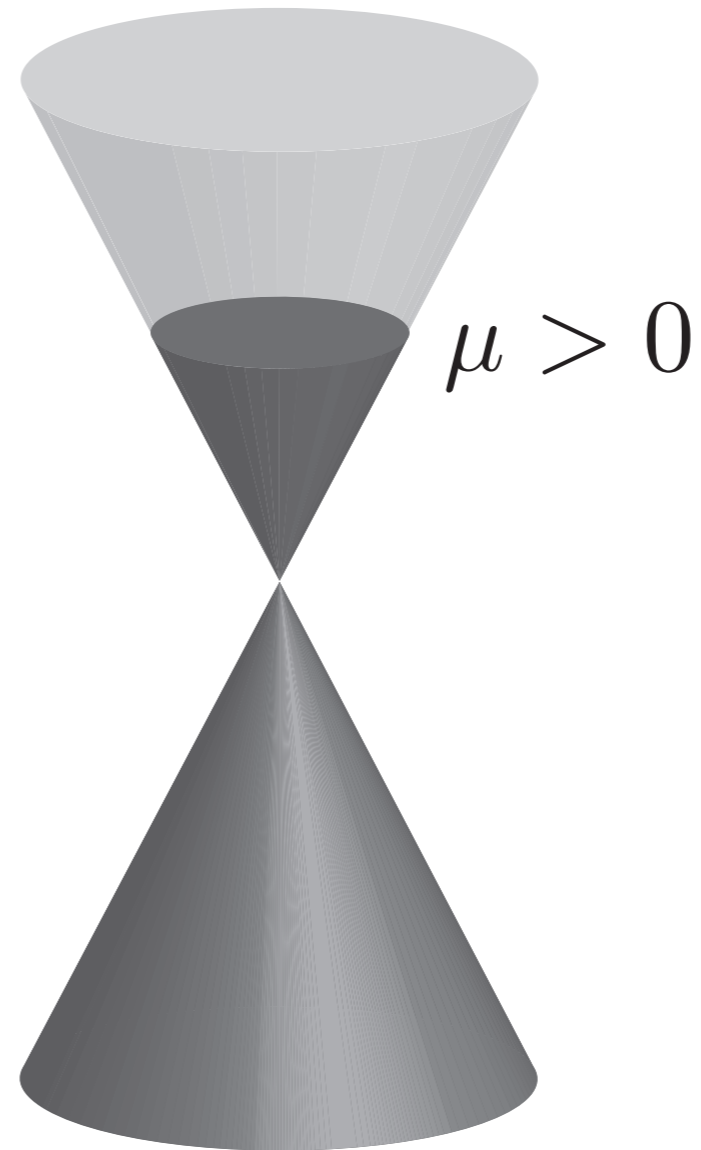
Holographic representation: AdS₄



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1$

Apply a chemical potential



AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right].$$

This is to be solved subject to the constraint

$$A_\mu(r \rightarrow 0, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where \mathcal{A}_μ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

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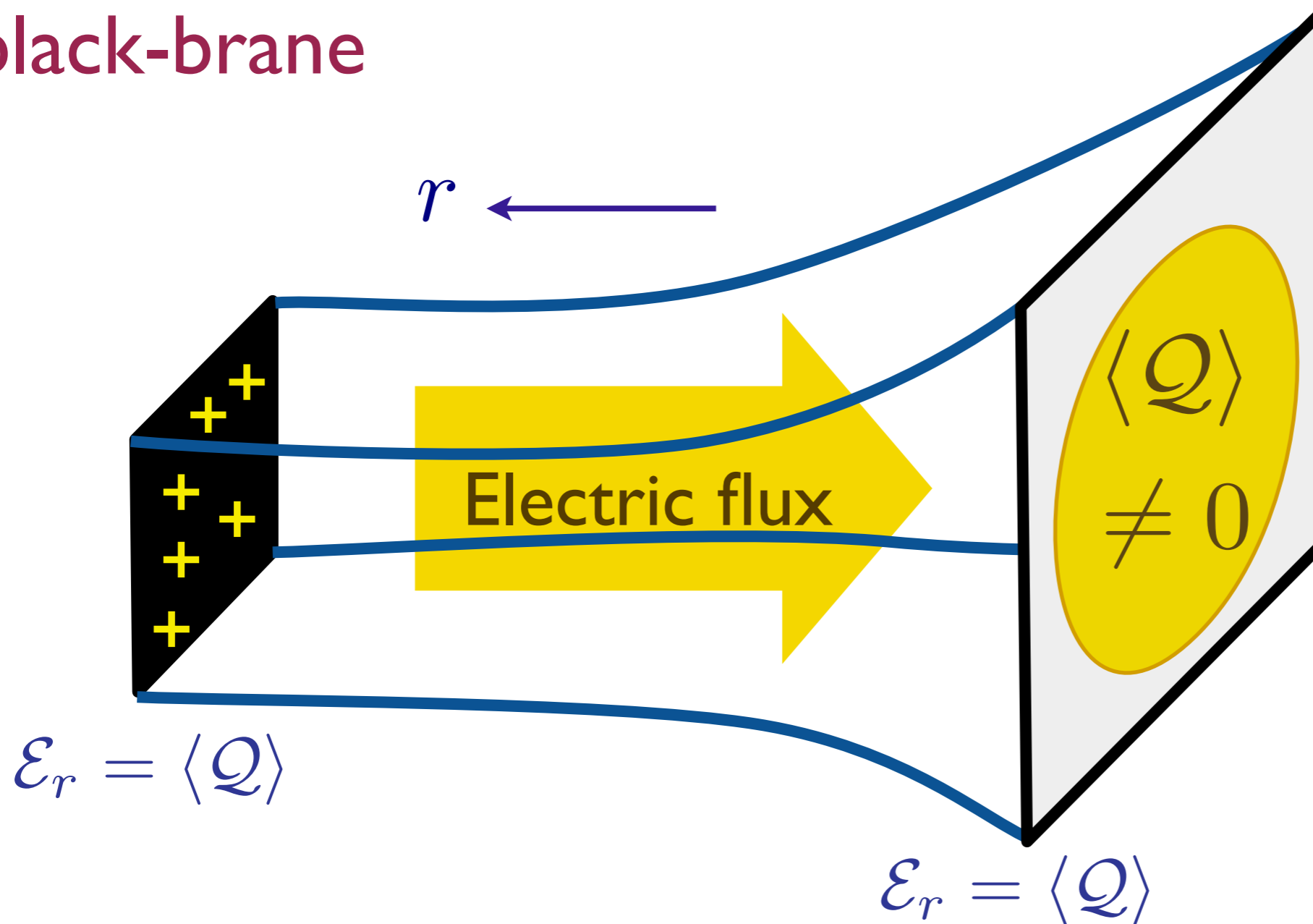
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$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

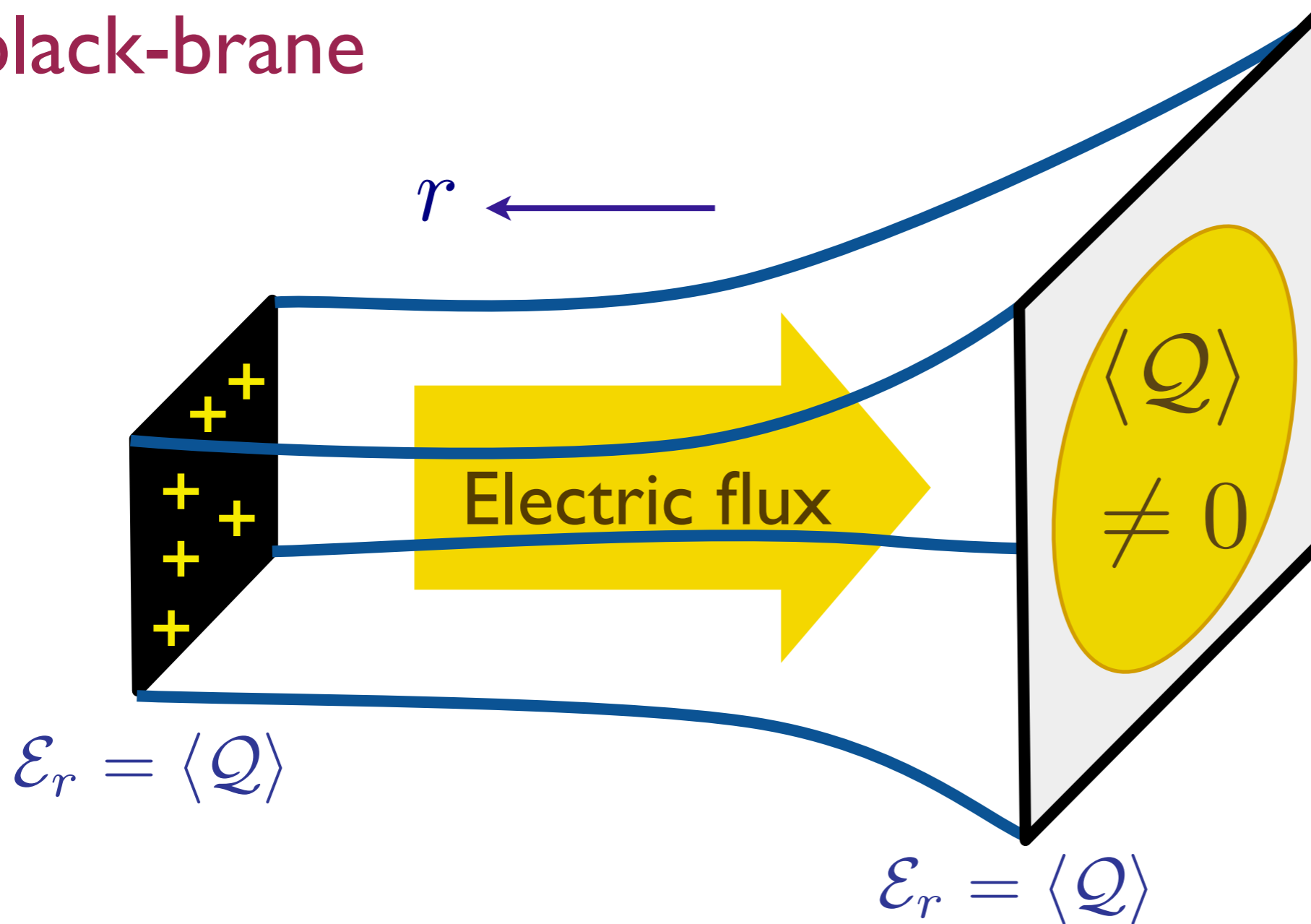
The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

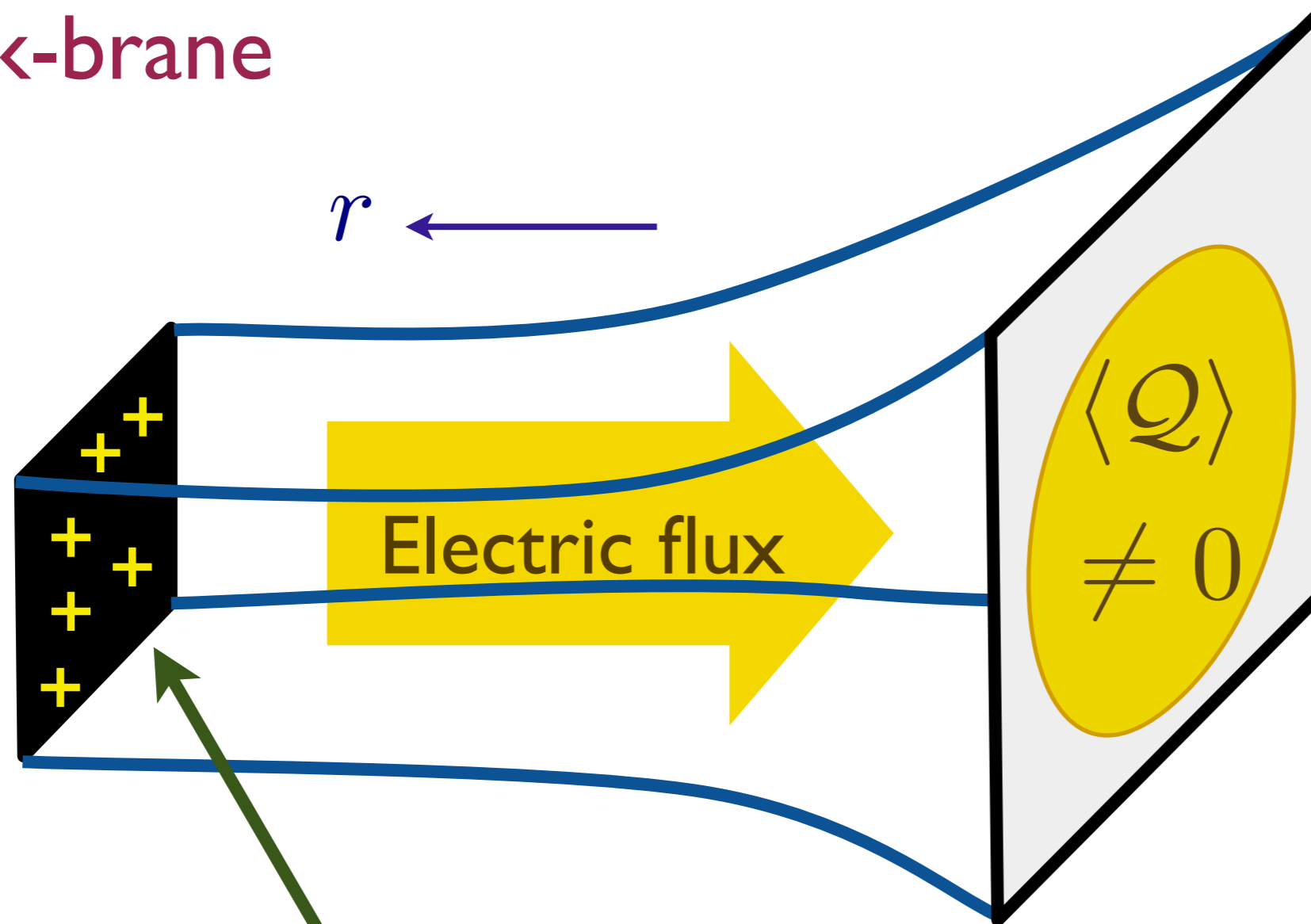
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$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

$$\text{with } f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right) \text{ and } R = \frac{\sqrt{6}Lg_4}{\kappa\mu}, \text{ and } A_\tau = \mu \left(1 - \frac{r}{R}\right)$$

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane

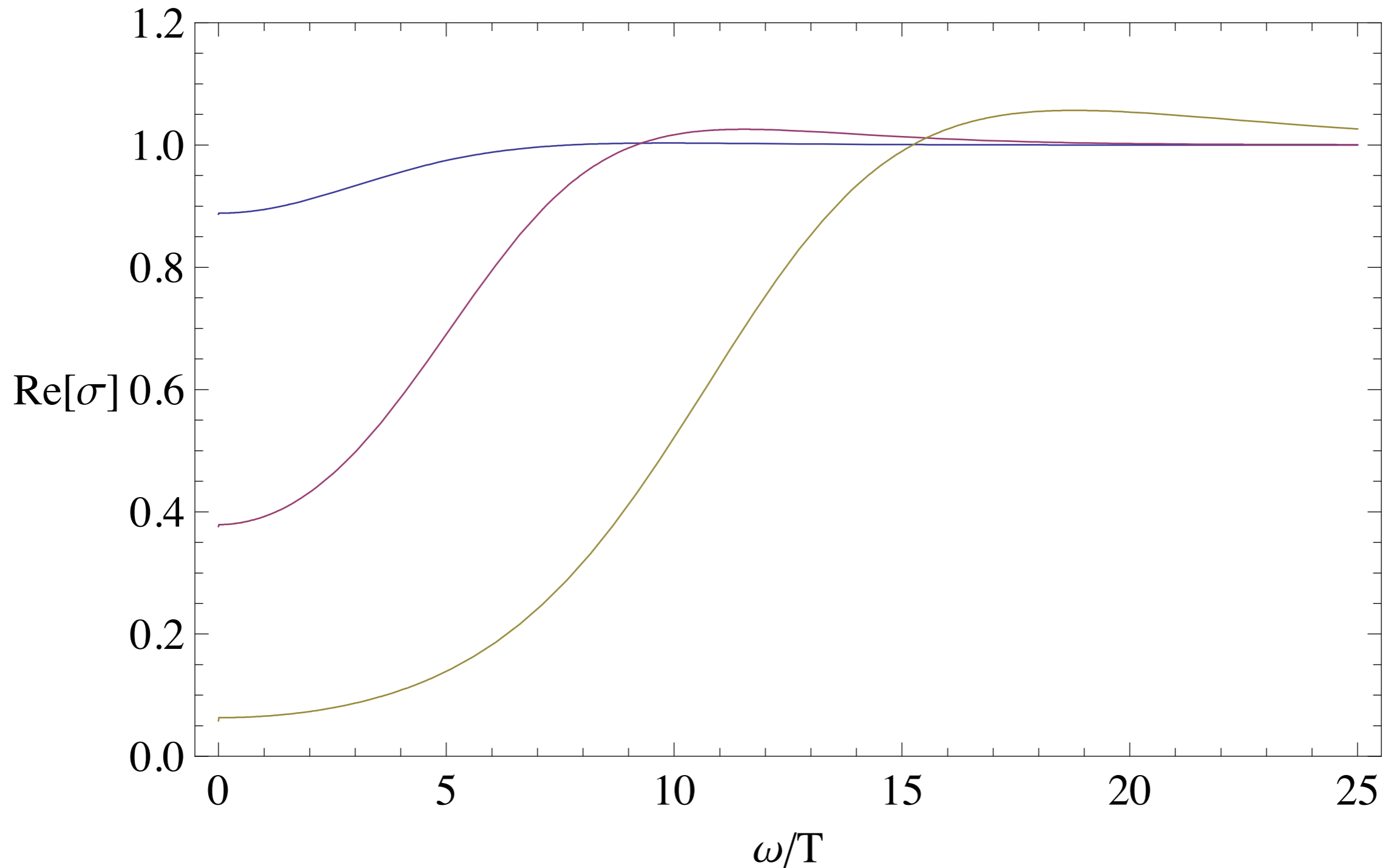


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $AdS_2 \times R^2$

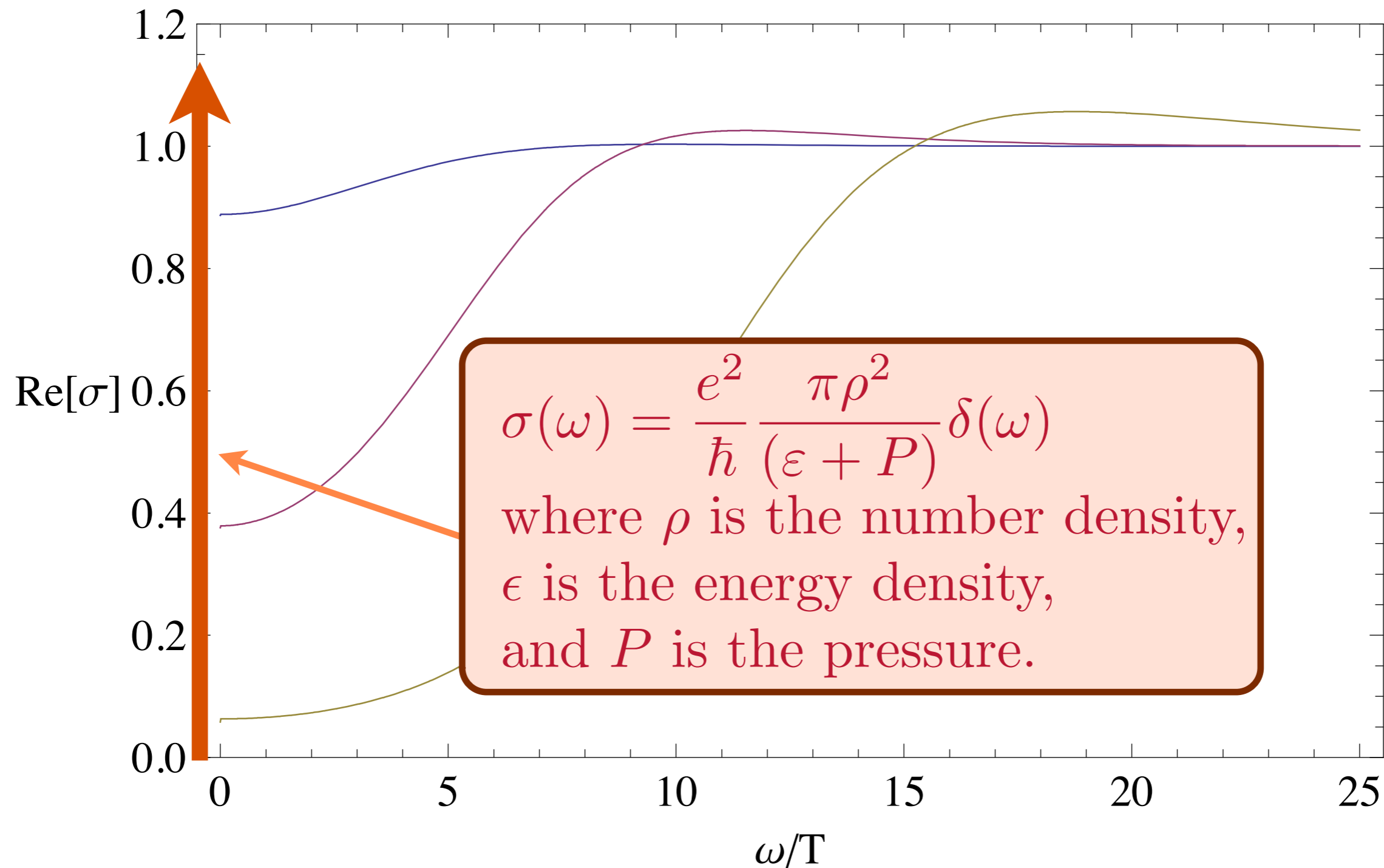
$$ds^2 = \frac{L^2}{6} \left(\frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$

T. Faulkner, H. Liu,
J. McGreevy,
and D. Vegh,
arXiv:0907.2694

Compute conductivity using response to a time-dependent vector potential as a function of ω/T and μ/T



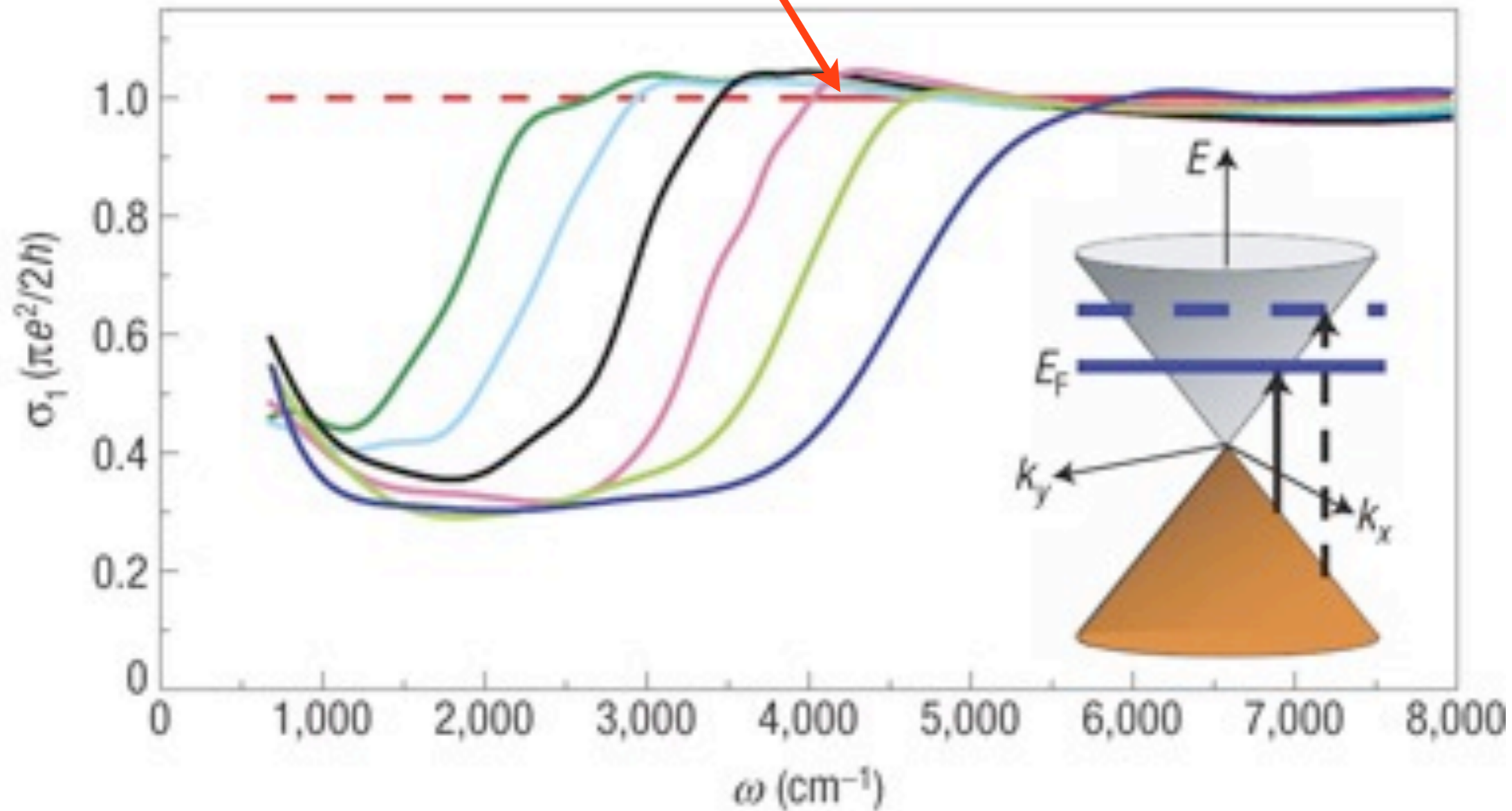
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Features of $\text{AdS}_2 \times R^2$

- Has non-zero entropy density at $T = 0$, and “volume” law for entanglement entropy.
- Green’s function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface
- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with “hidden Fermi surfaces” of gauge-charged particles (the *quarks*).

S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);

M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

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Compressible quantum matter

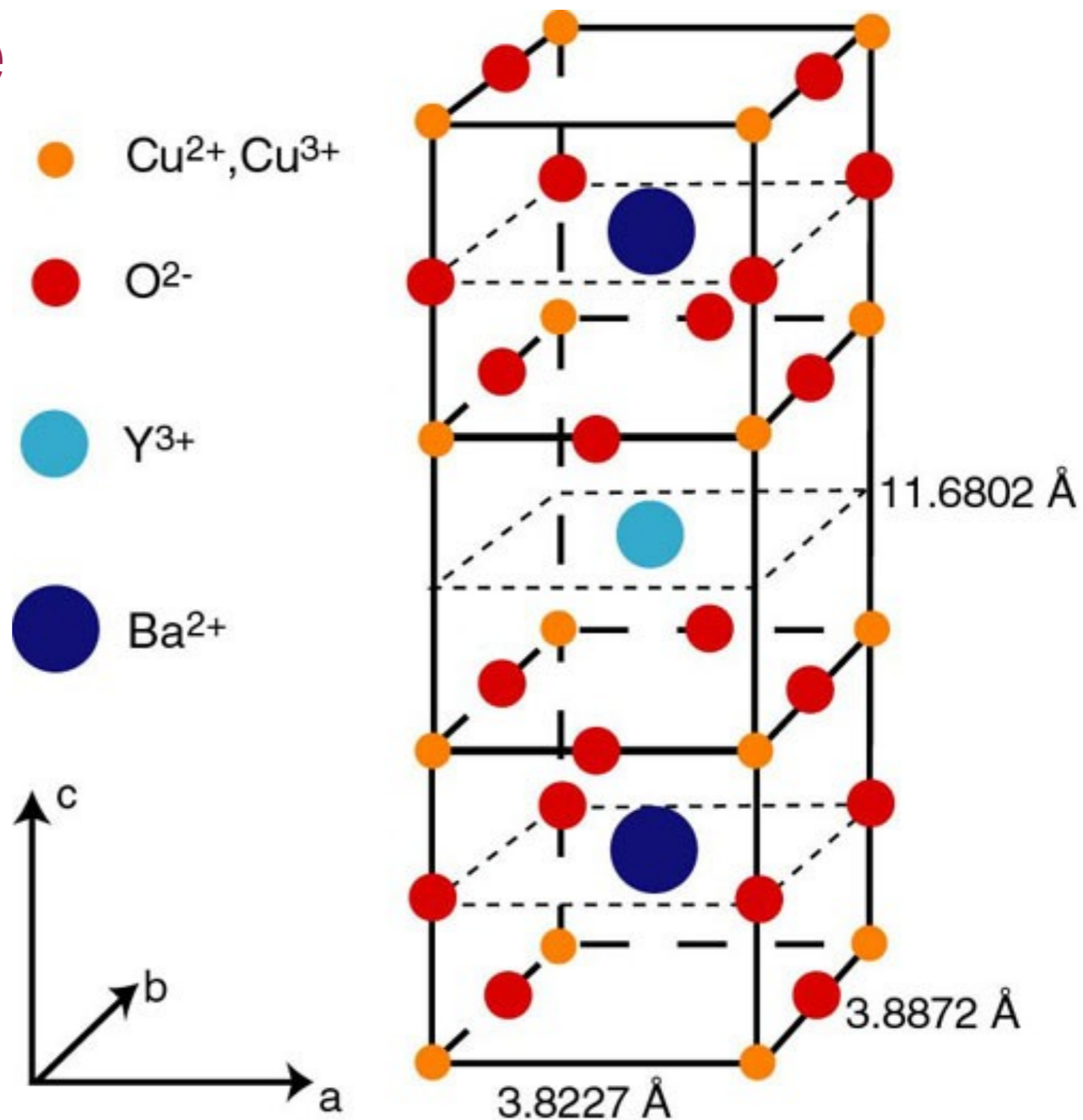
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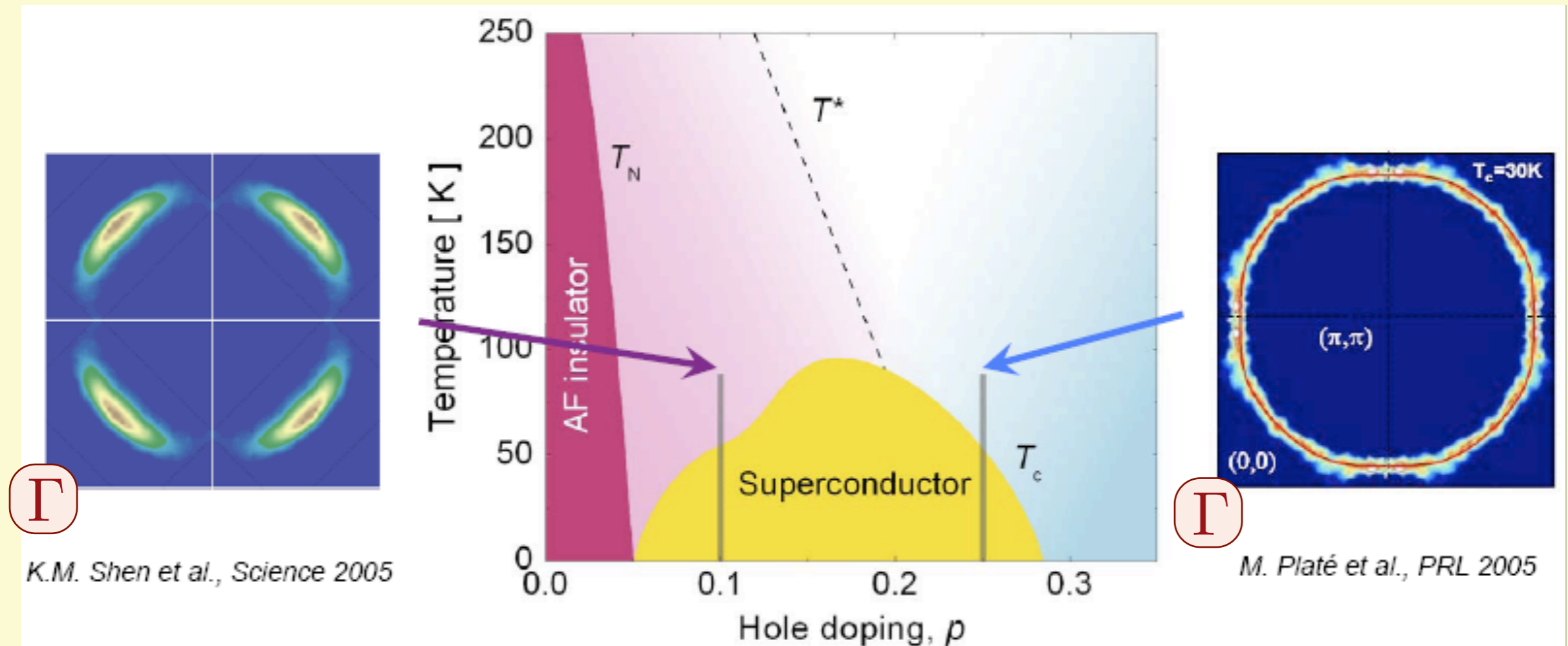
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High temperature superconductors



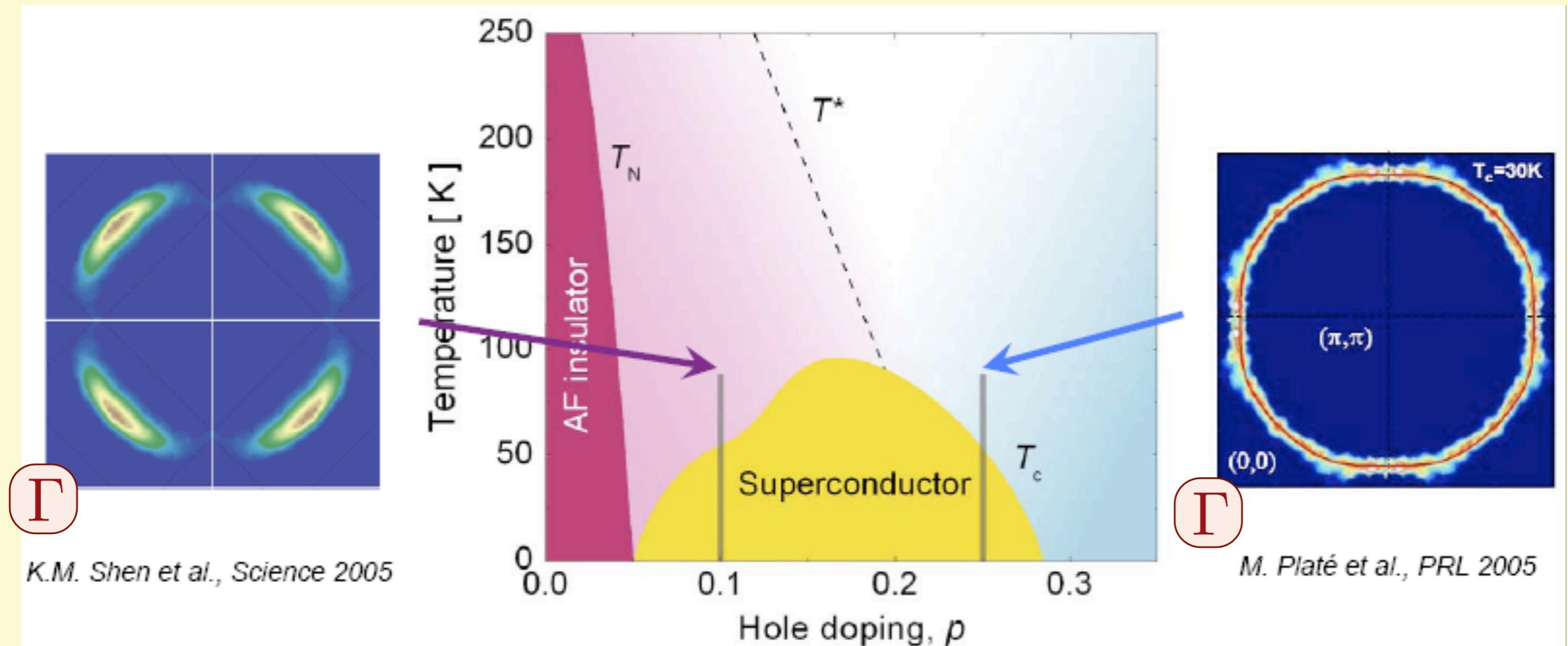
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole
Fermi-pockets

Large hole
Fermi surface

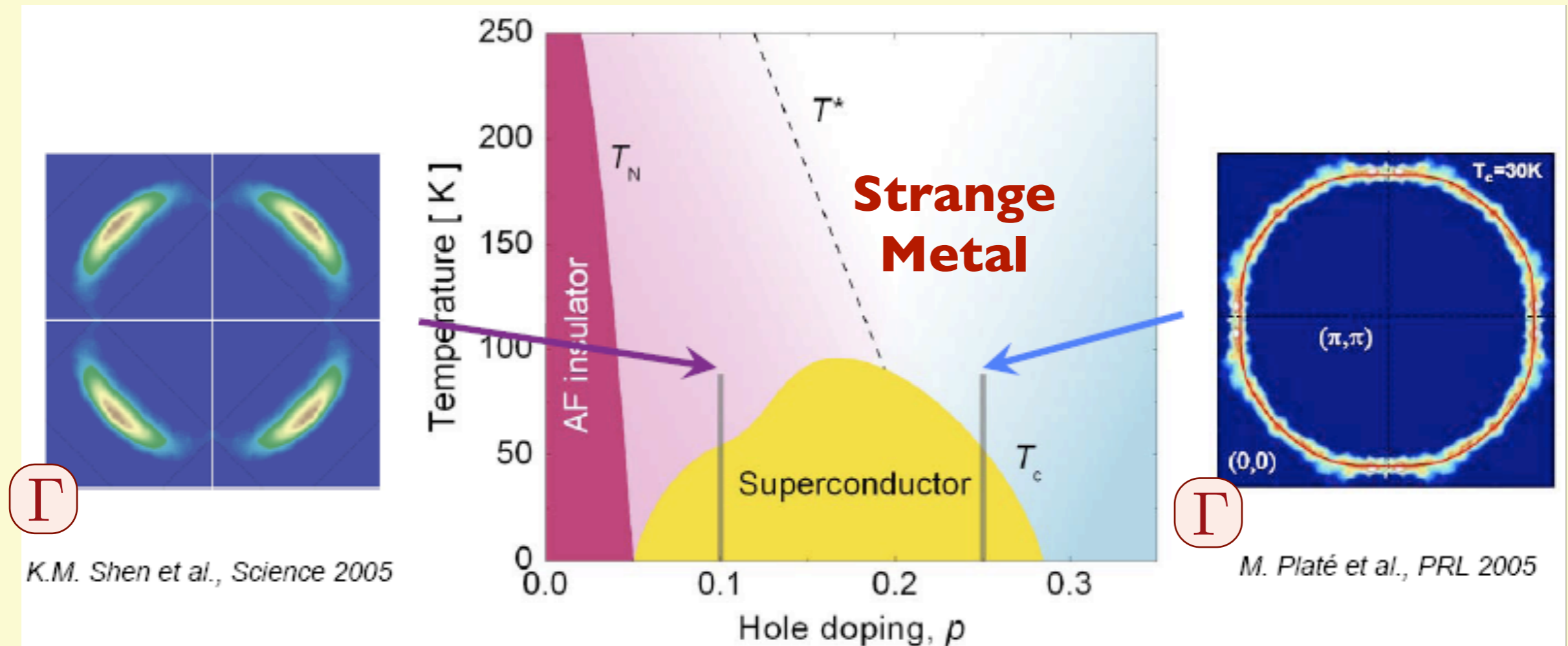
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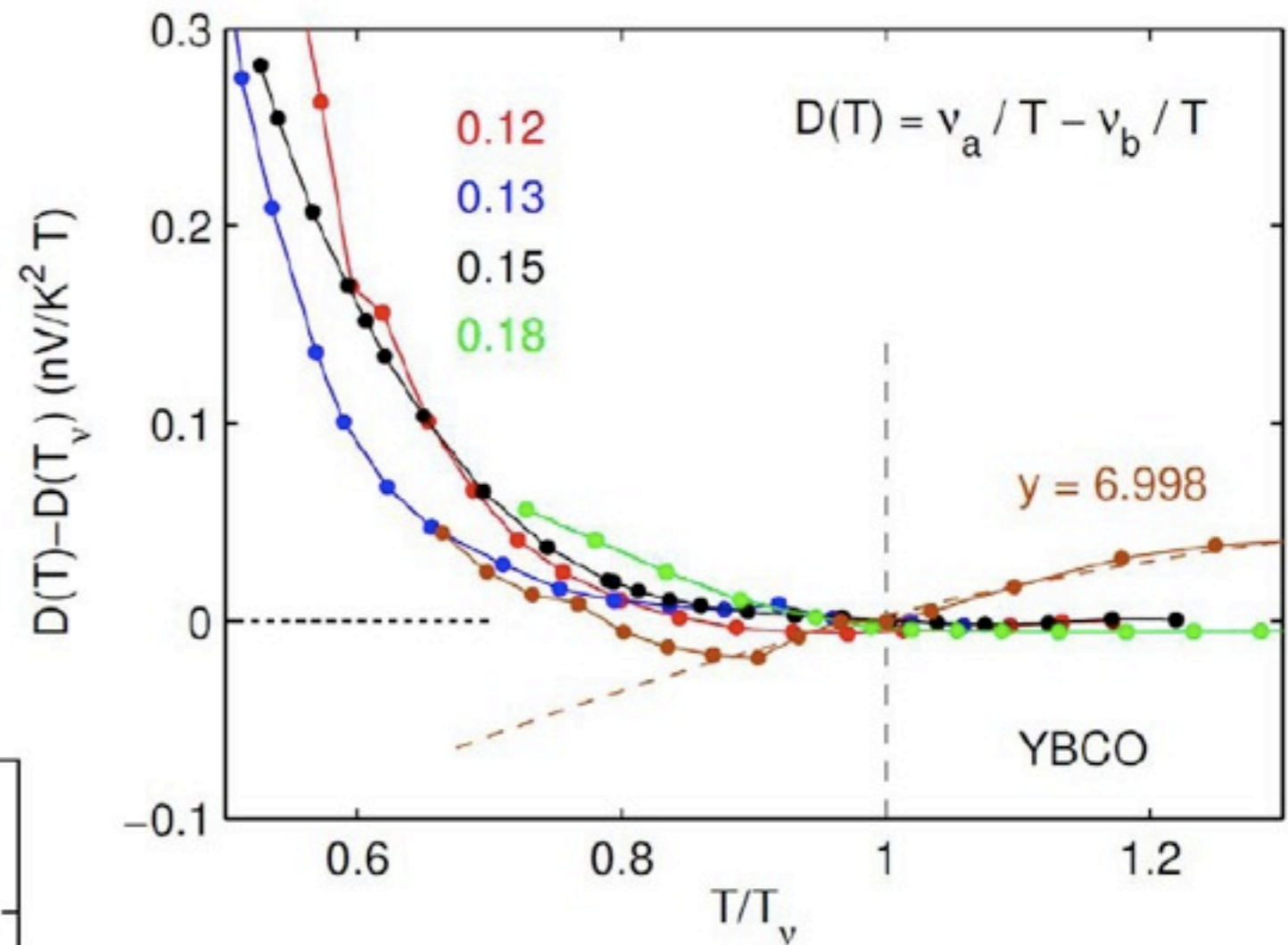
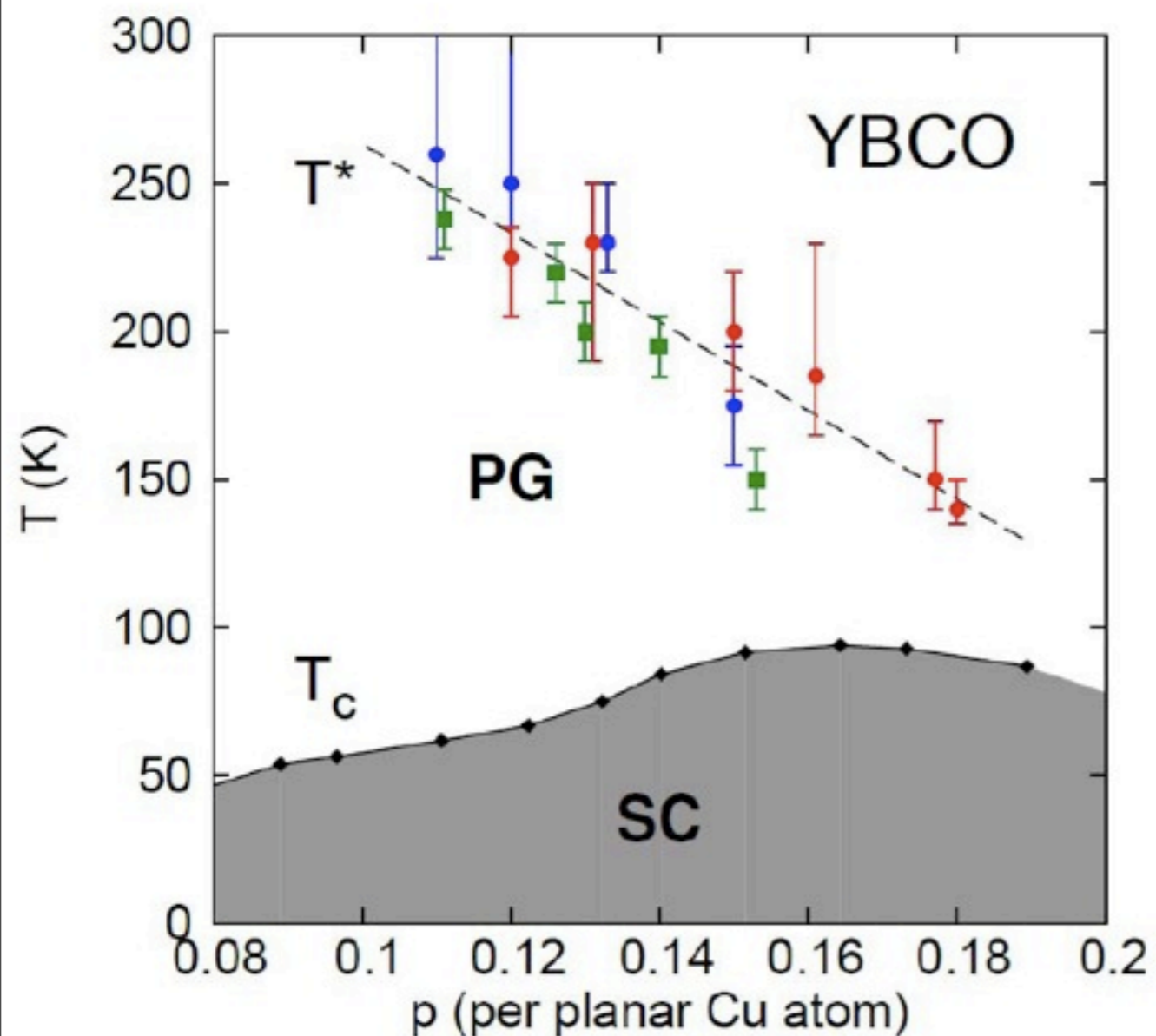


Smaller hole Fermi-pockets

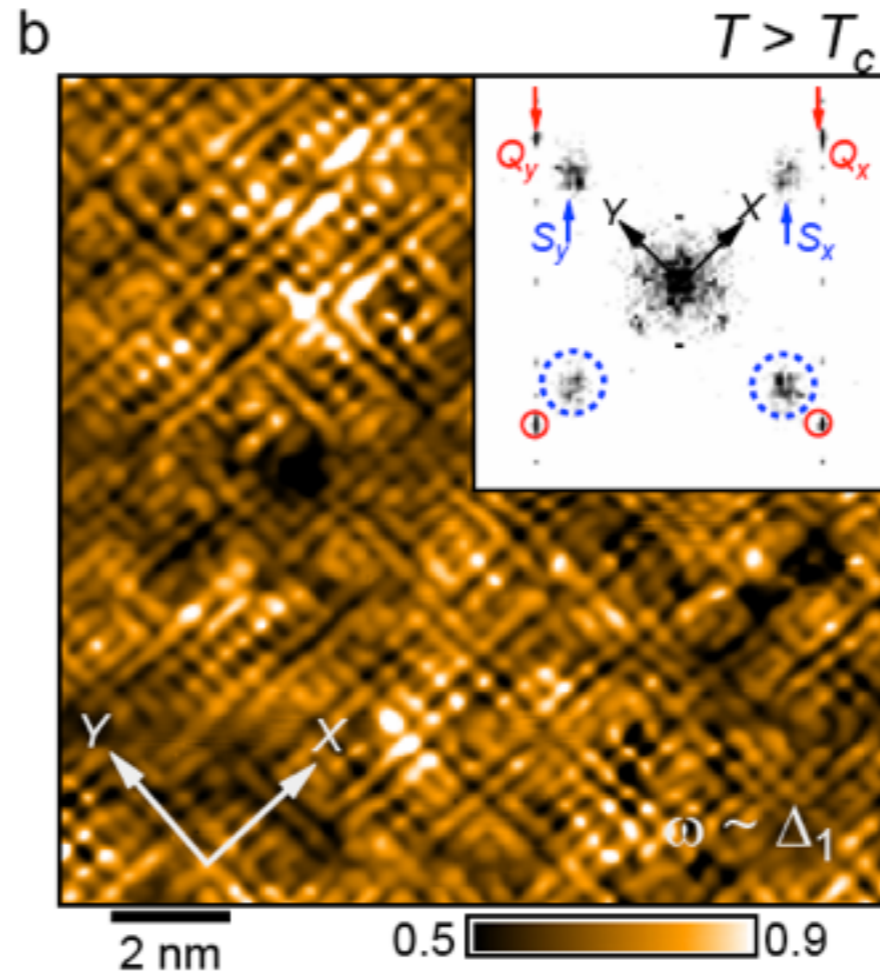
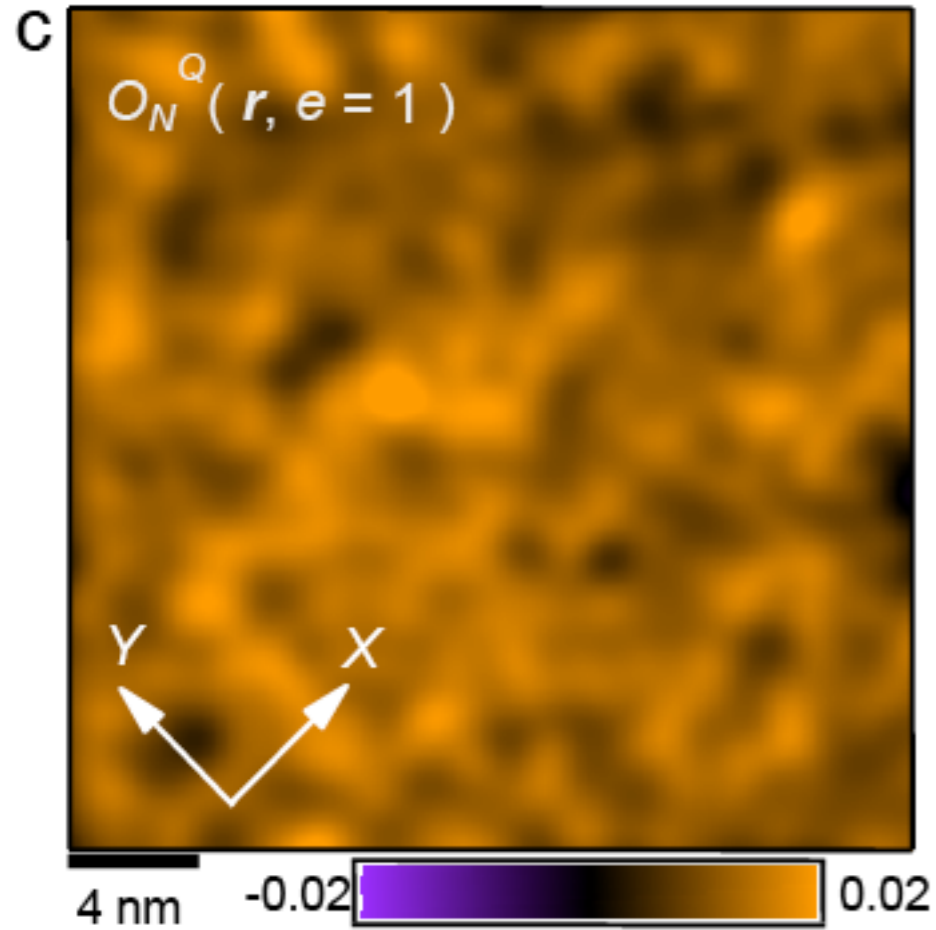
Large hole Fermi surface

Broken rotational symmetry in the pseudogap phase of a high- T_c superconductor

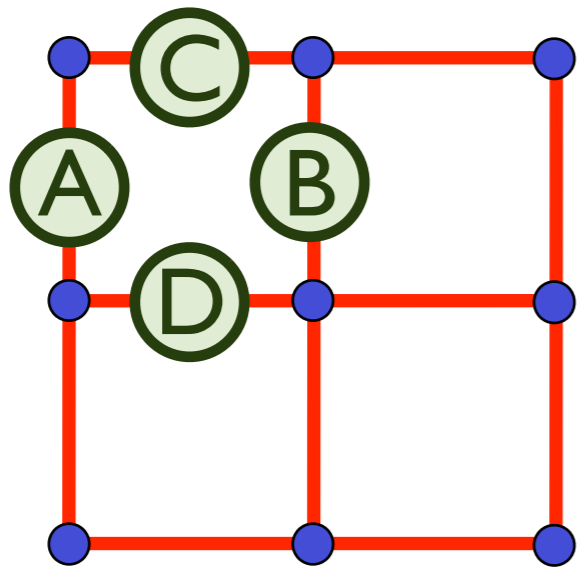
R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).



STM measurements of $Z(r)$, the energy asymmetry in density of states in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$.



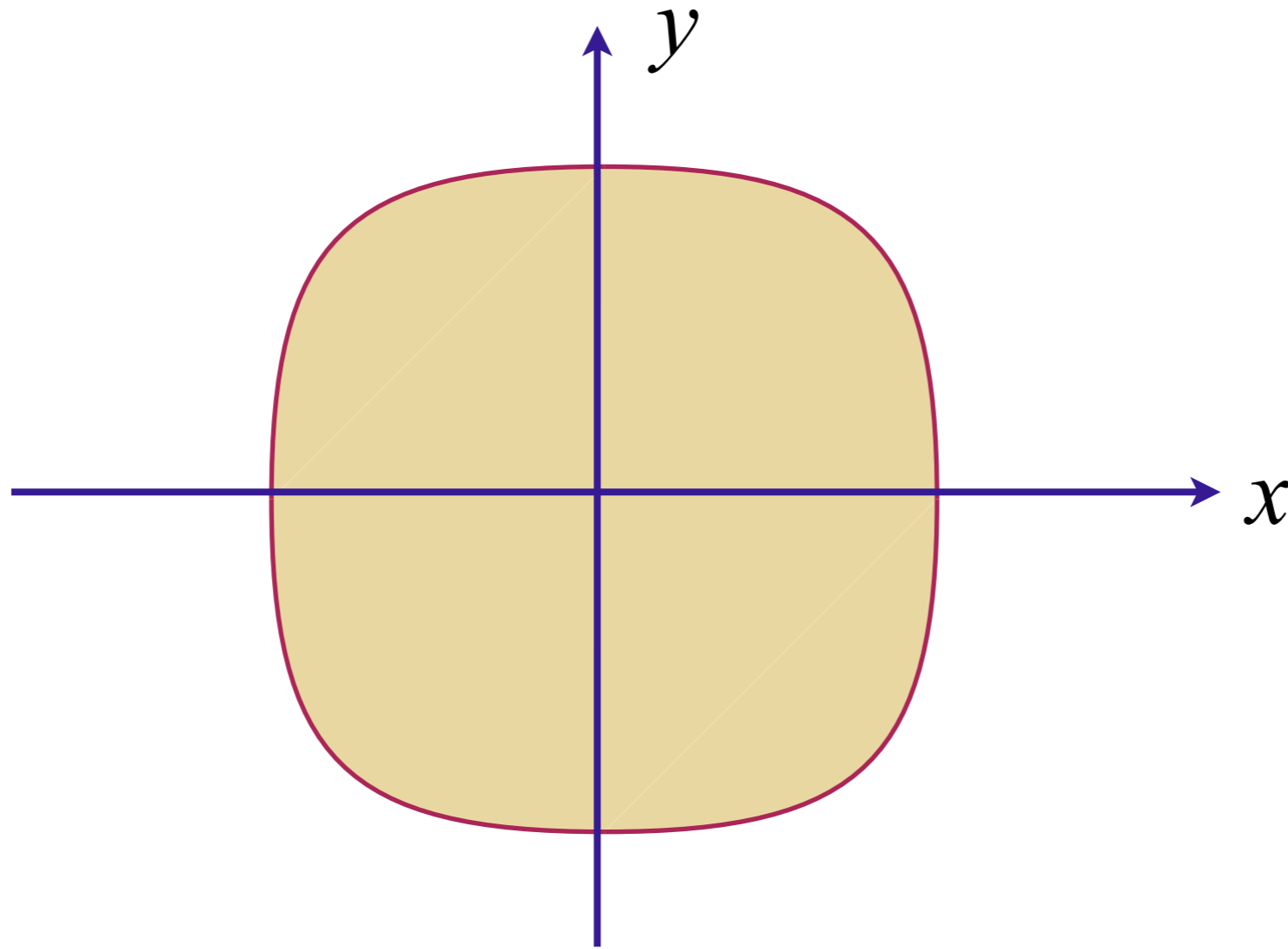
M. J. Lawler, K. Fujita, Jinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, *Nature* **466**, 347 (2010)



$$O_N = Z_A + Z_B - Z_C - Z_D$$

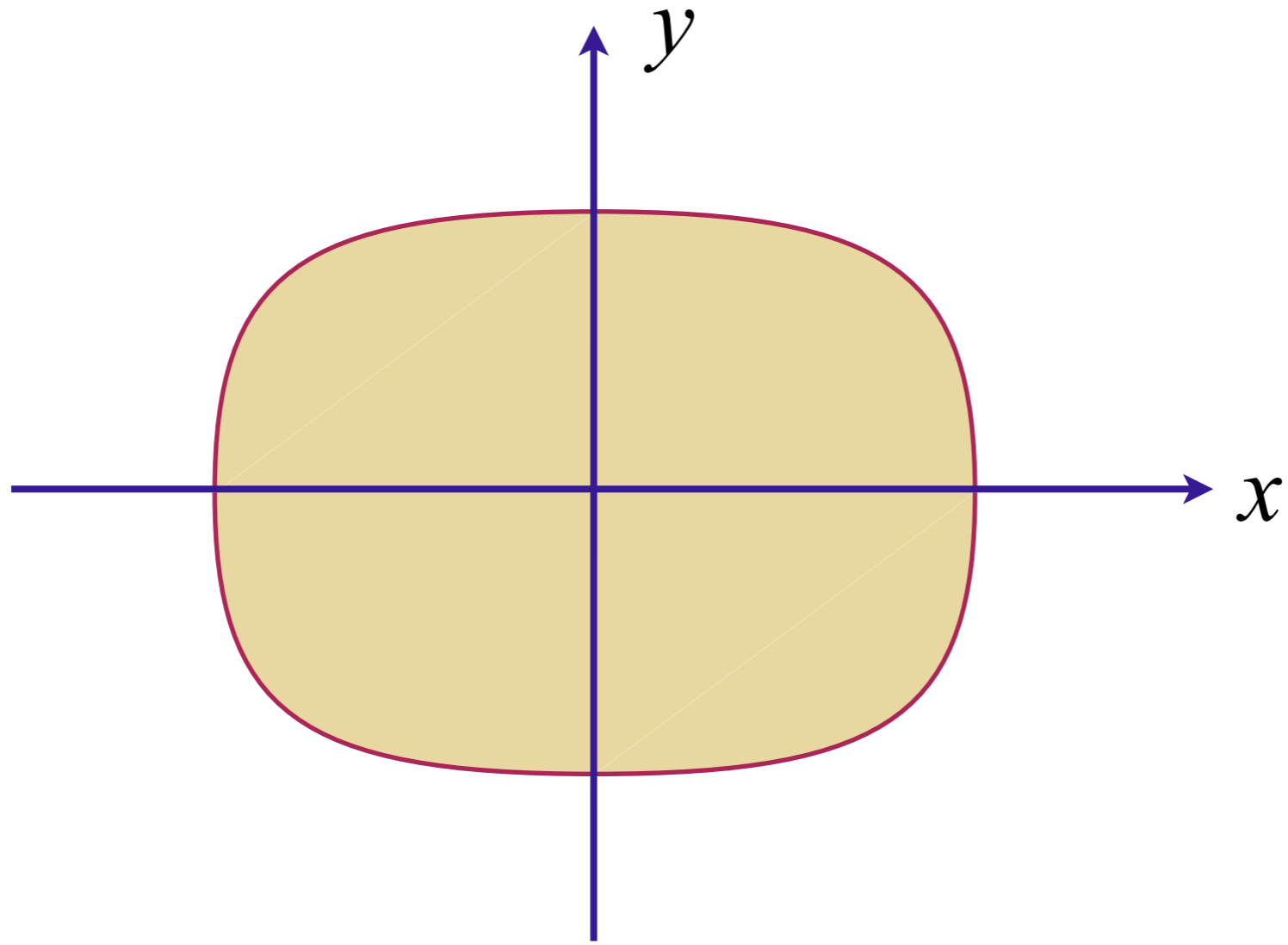
Strong anisotropy of electronic states between x and y directions:
Electronic “Ising-nematic” order

Quantum criticality of Ising-nematic ordering



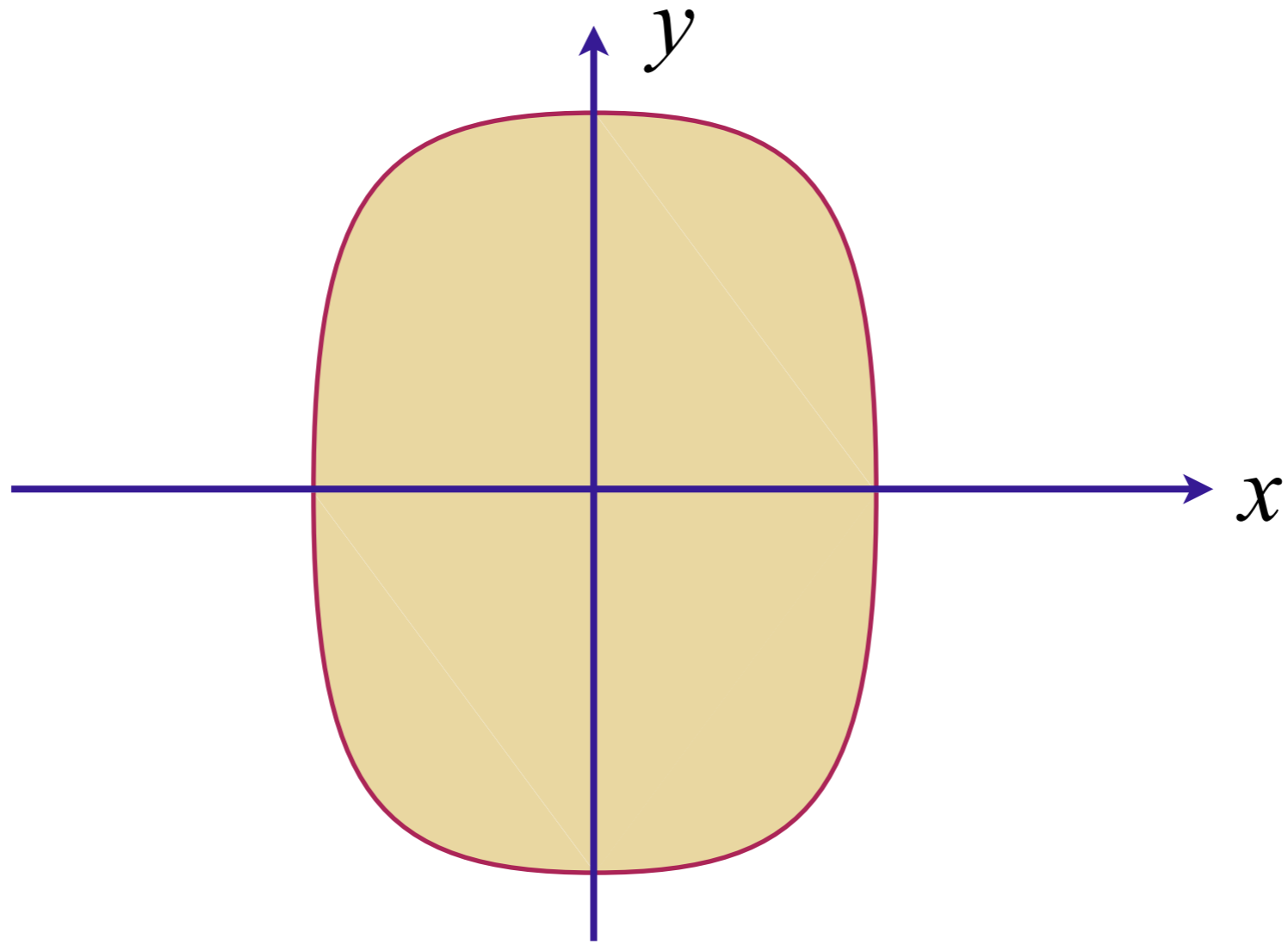
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



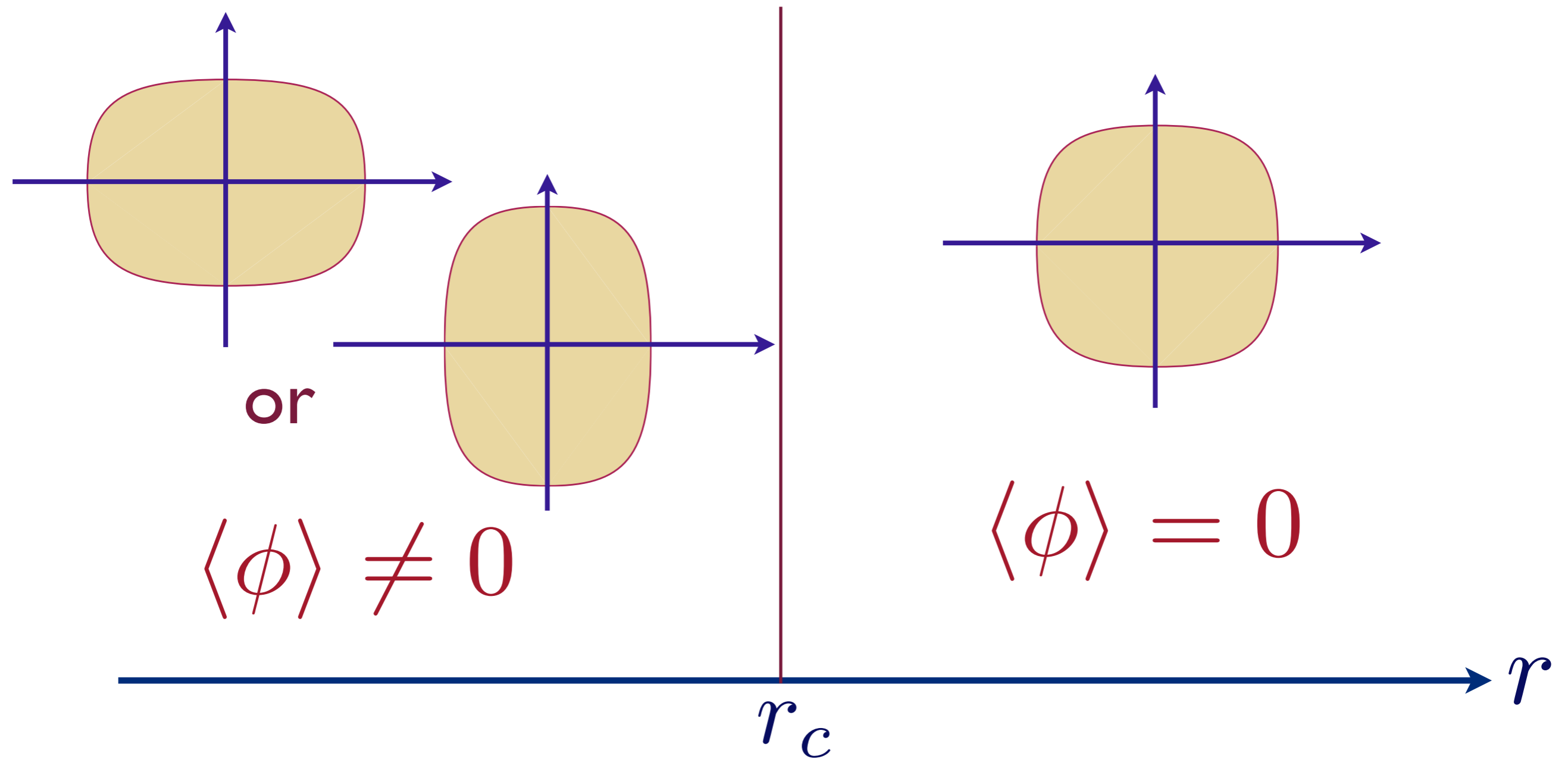
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2 k (\cos k_x - \cos k_y) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

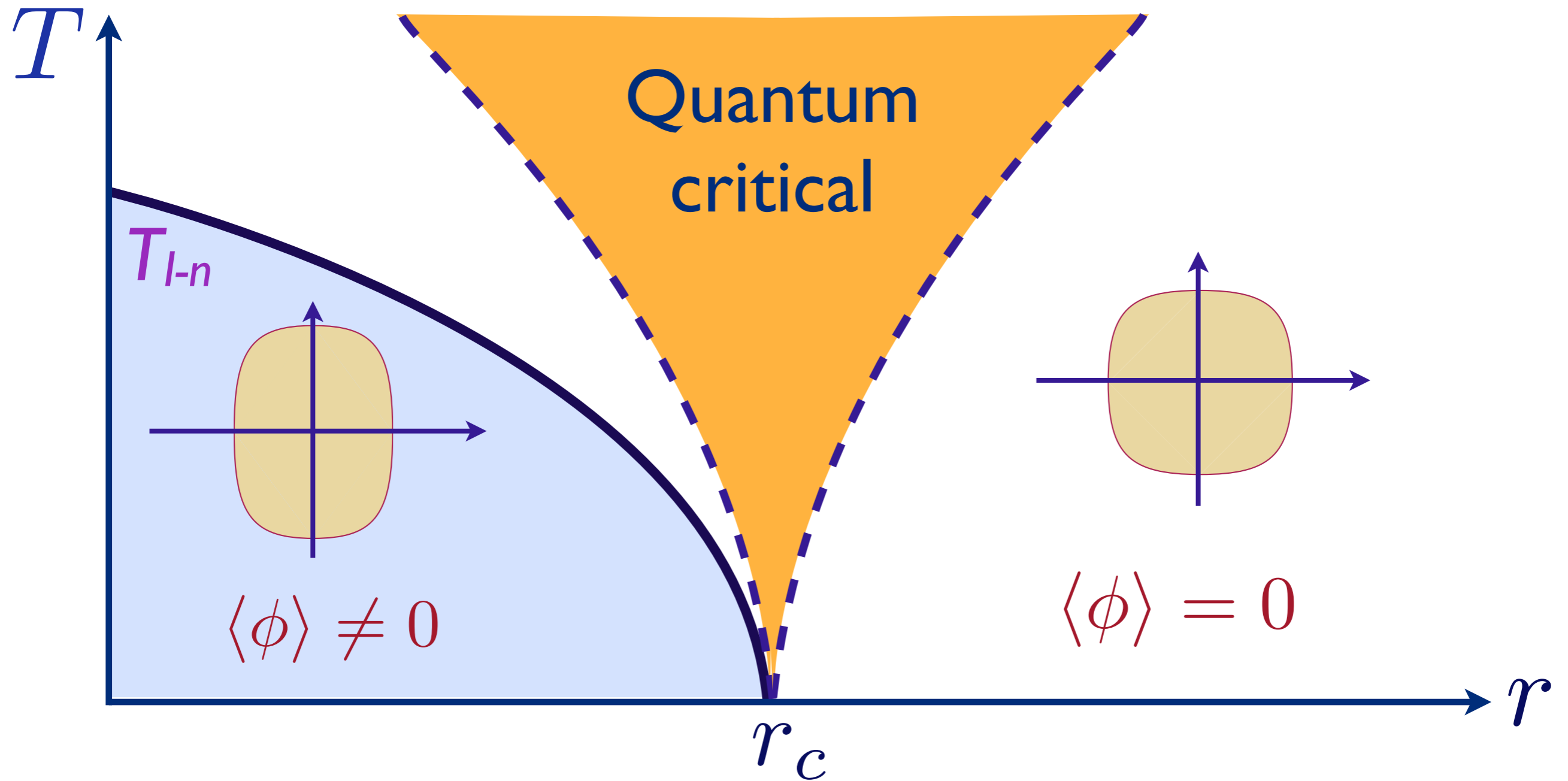
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



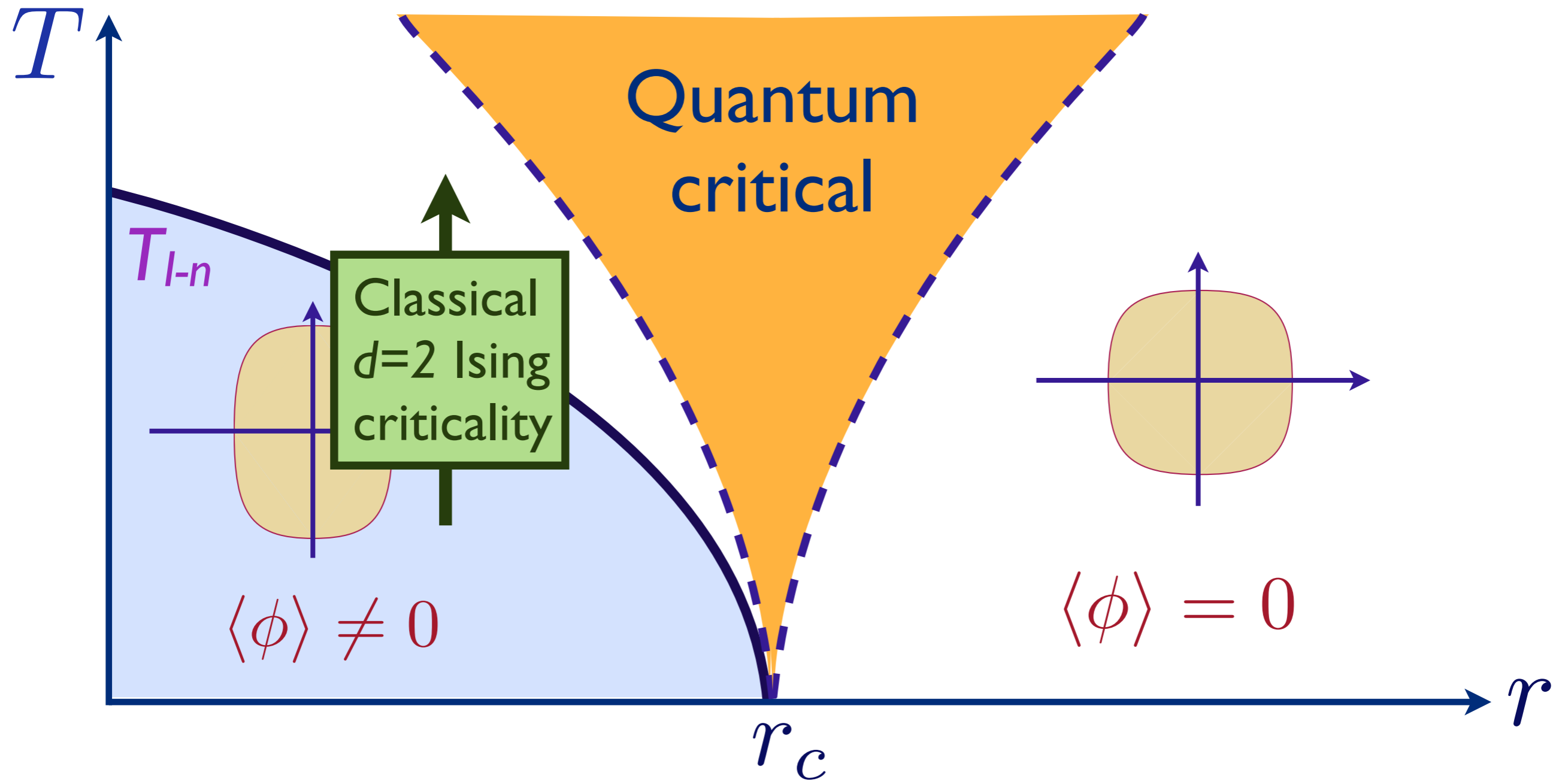
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



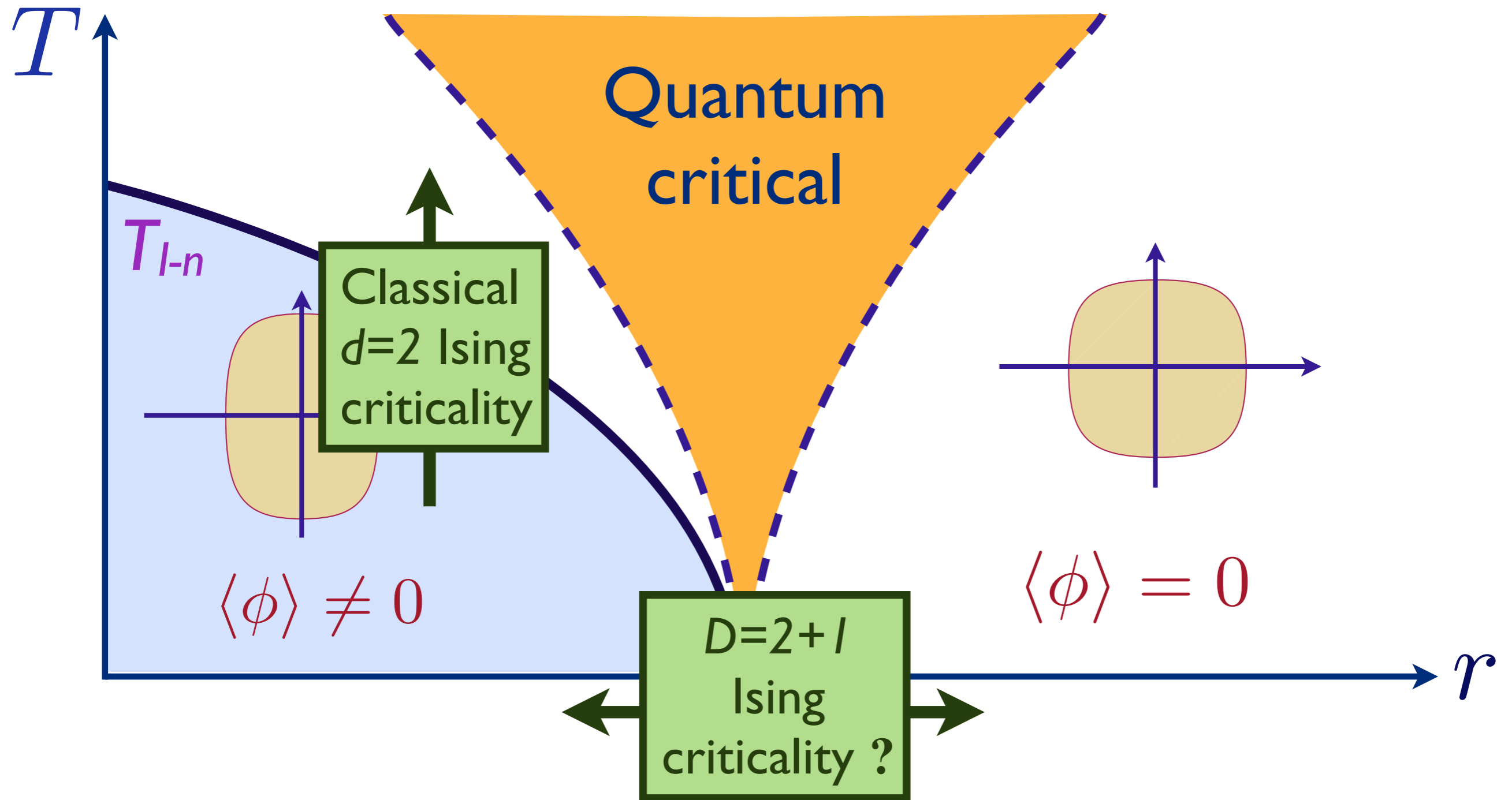
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



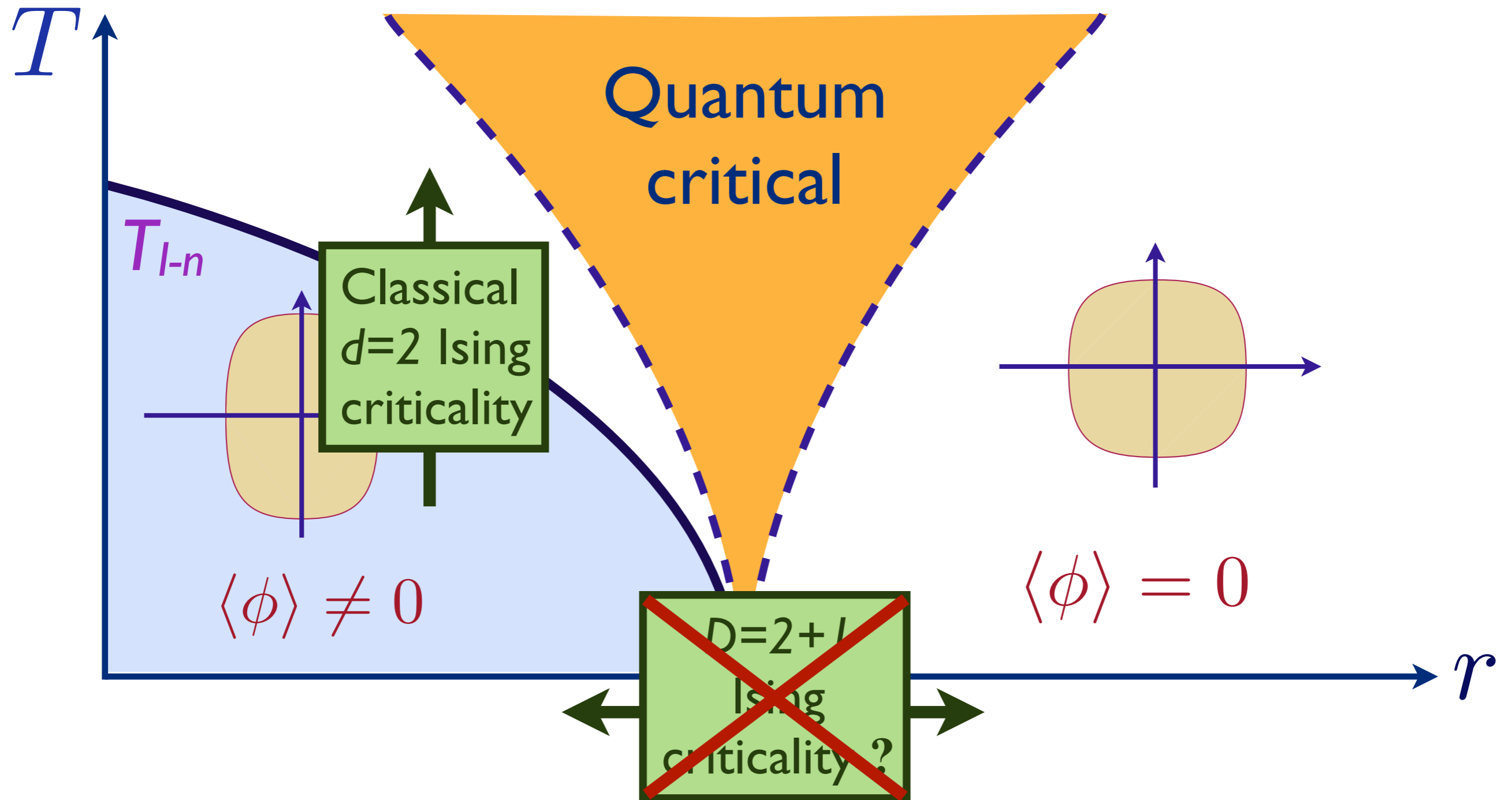
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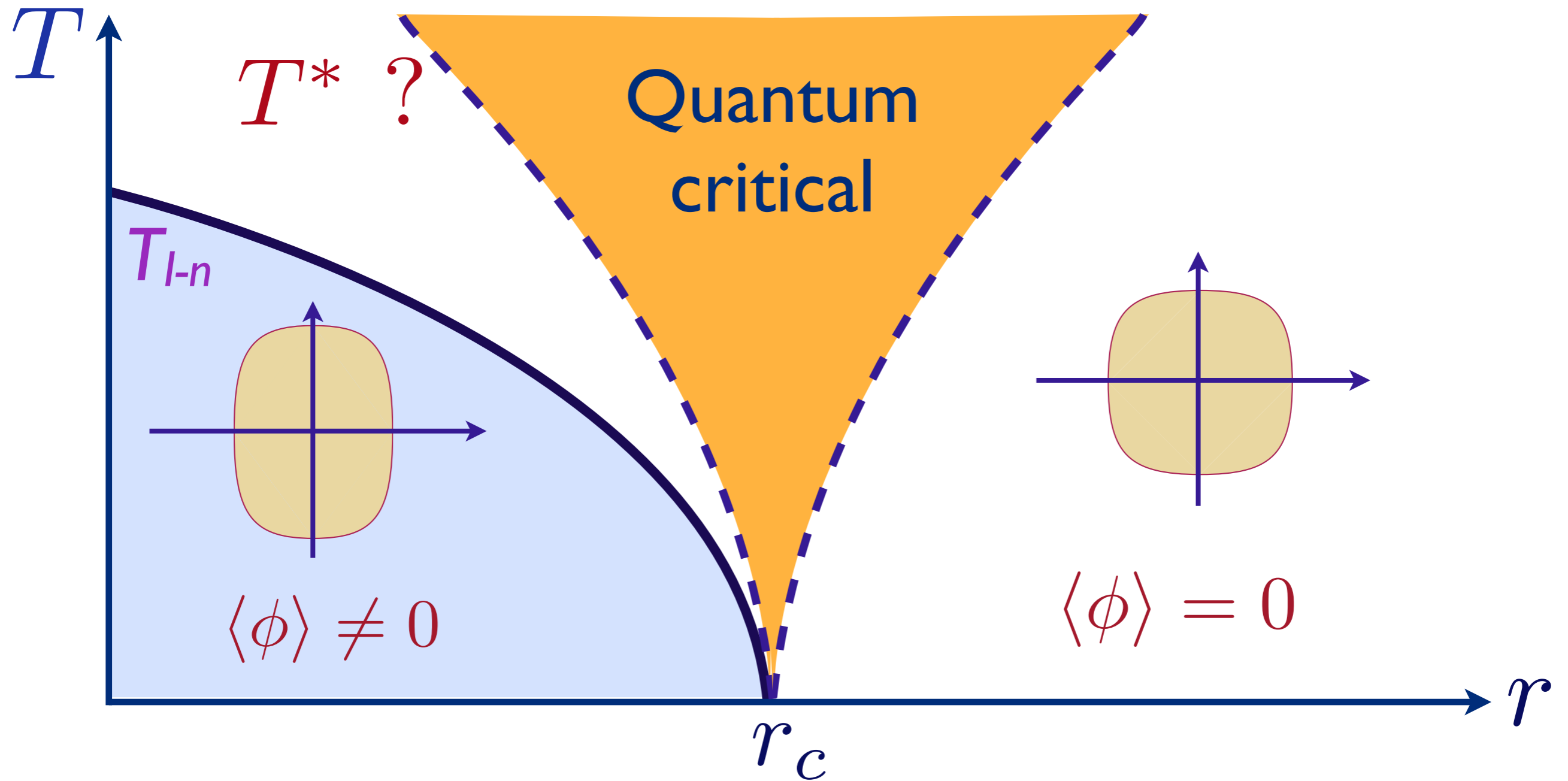
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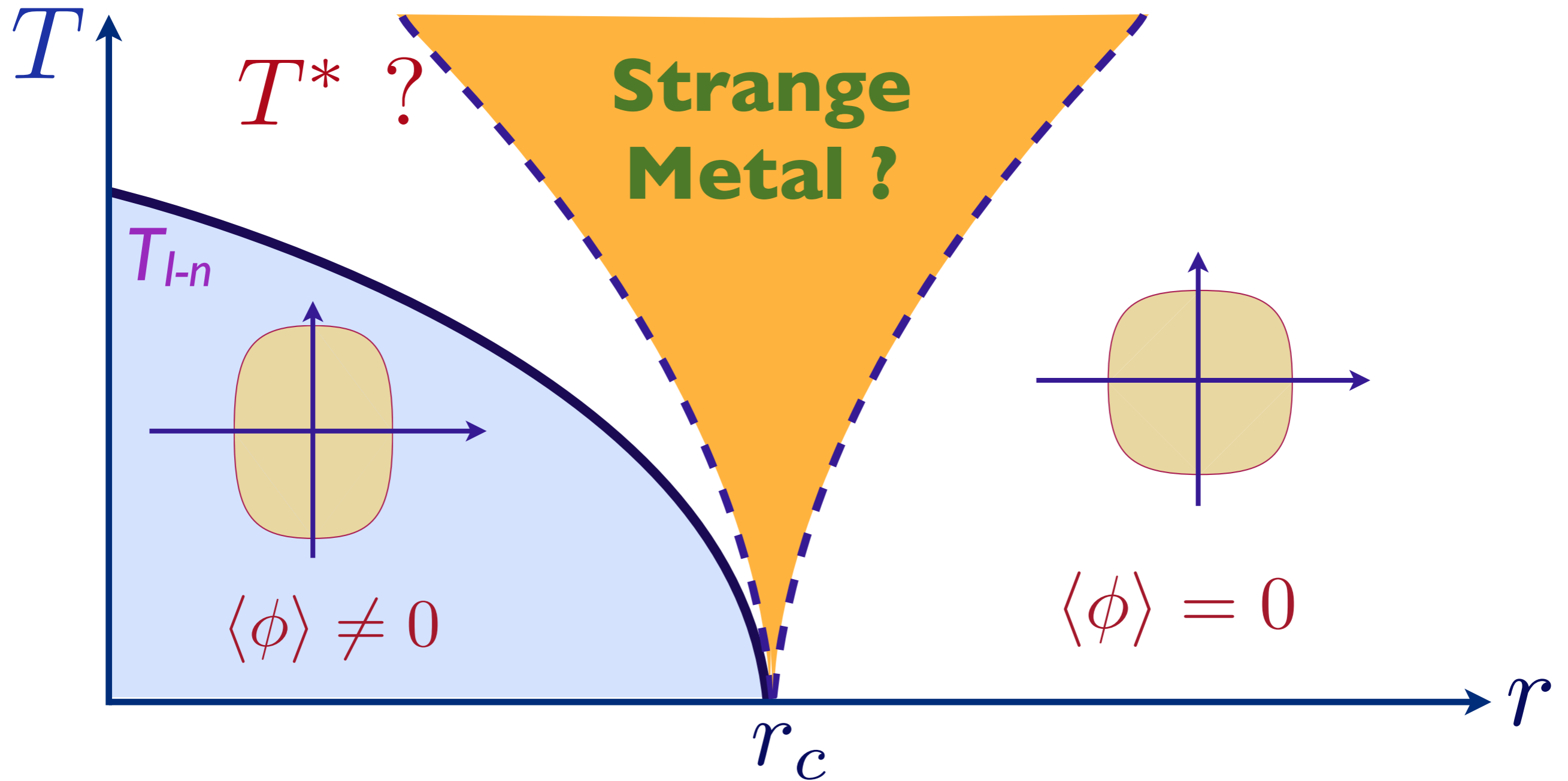
Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

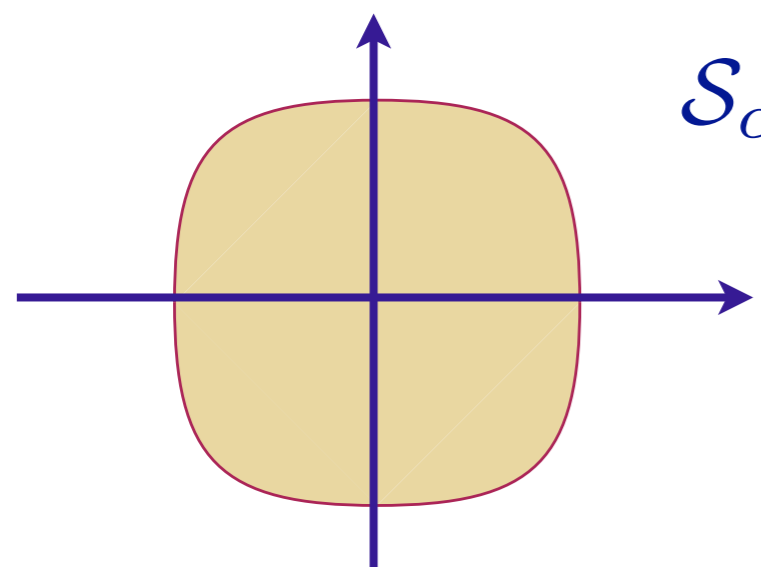
$$\mathcal{S}_\phi = \int d^2r d\tau [(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4]$$

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right]$$

Effective action for electrons:

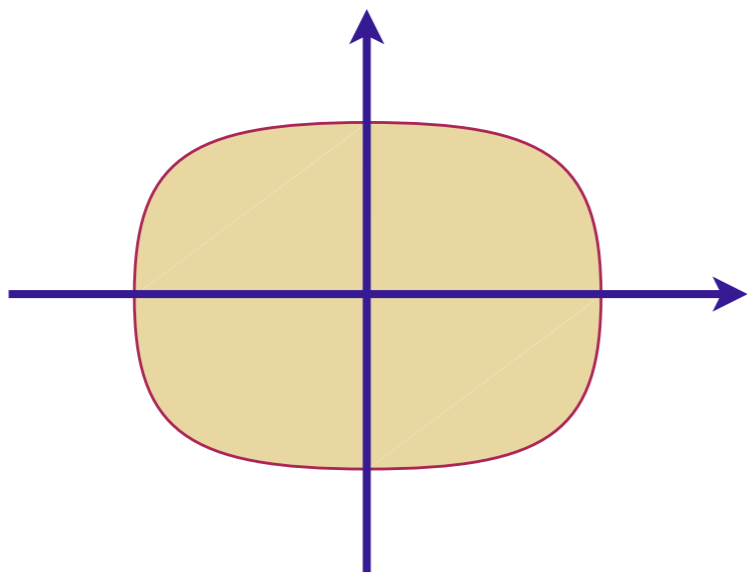

$$\begin{aligned} \mathcal{S}_c &= \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \right] \\ &\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha} \end{aligned}$$

Quantum criticality of Ising-nematic ordering

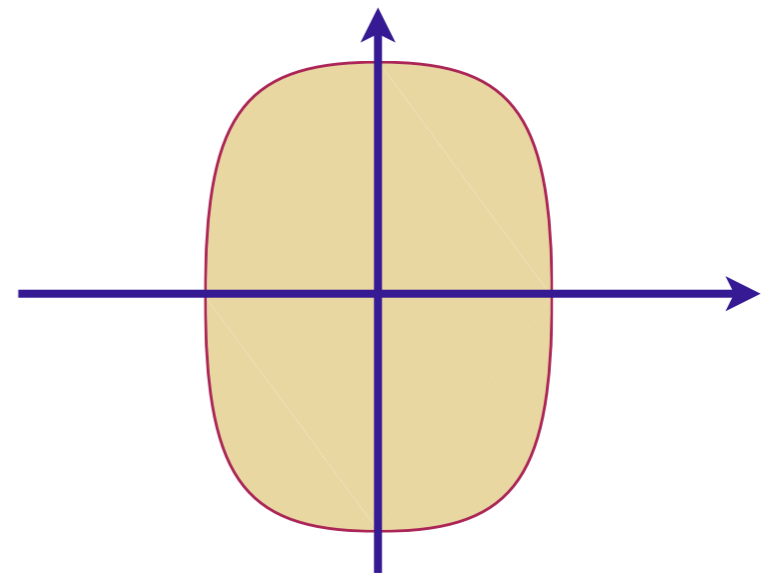
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{q}} (\cos k_x - \cos k_y) c_{\mathbf{k}+\mathbf{q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2, \alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0$$



$$\langle \phi \rangle < 0$$

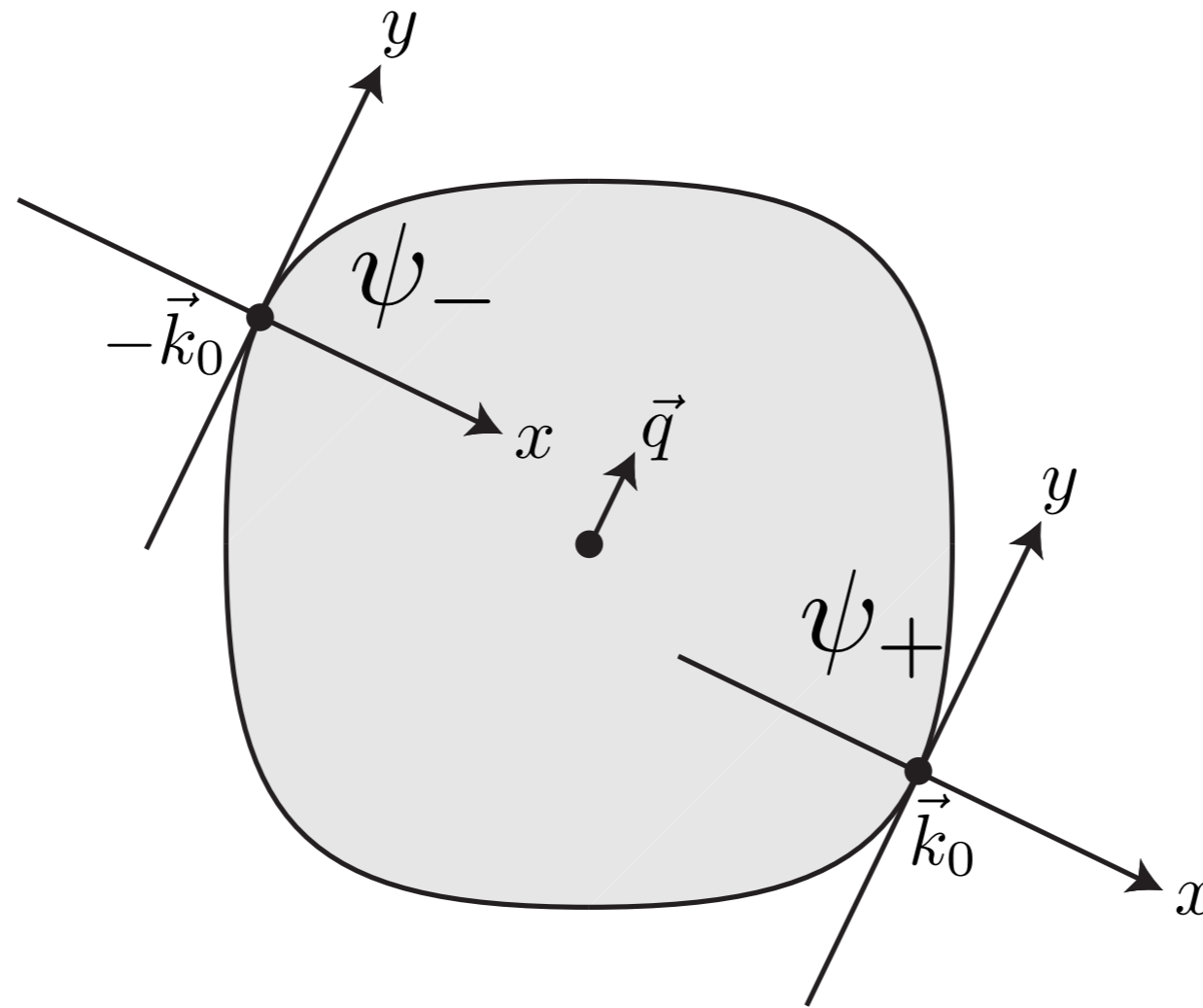
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$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger (\partial_\tau + \varepsilon_{\mathbf{k}}) c_{\mathbf{k}\alpha}$$

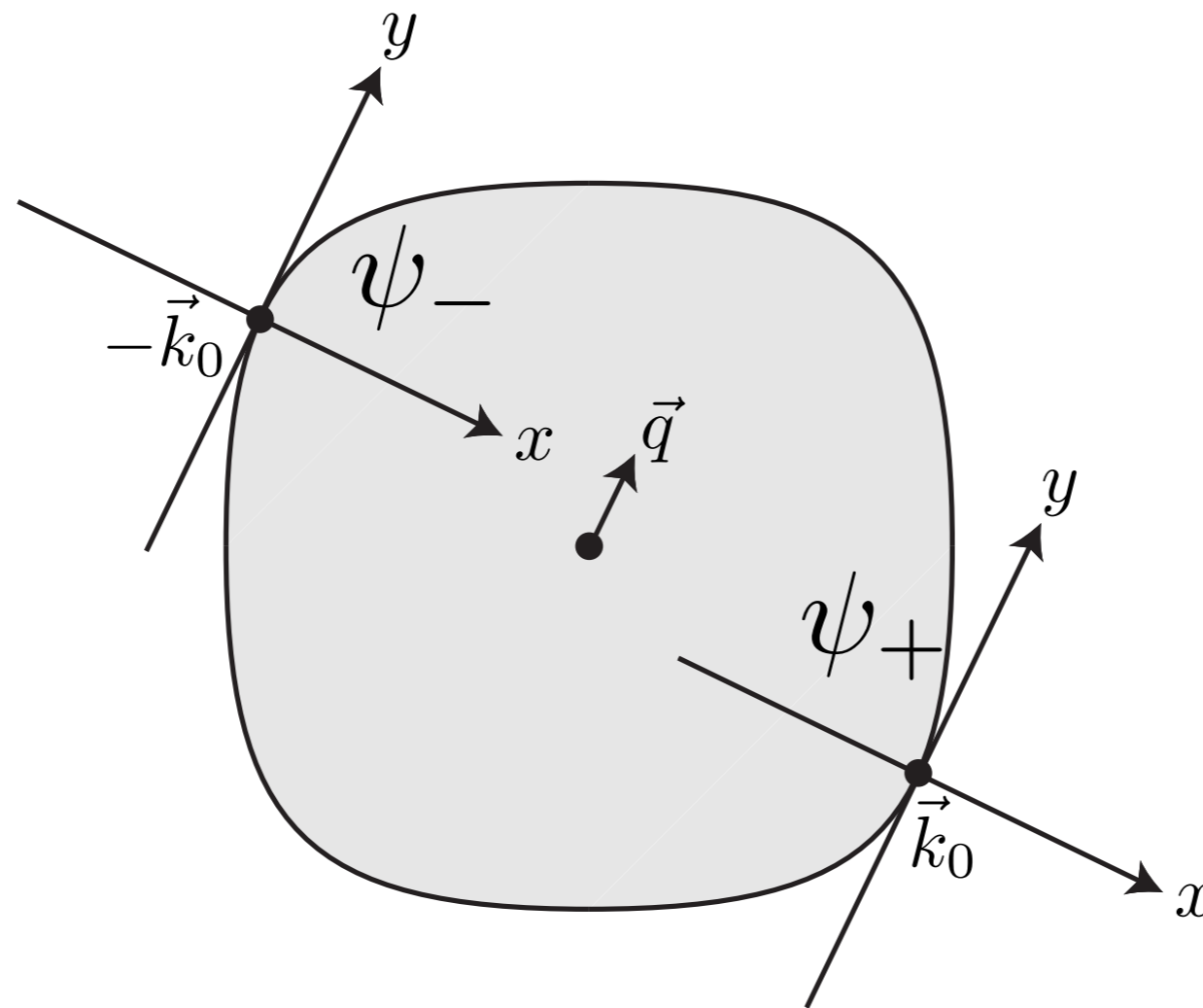
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Quantum criticality of Ising-nematic ordering



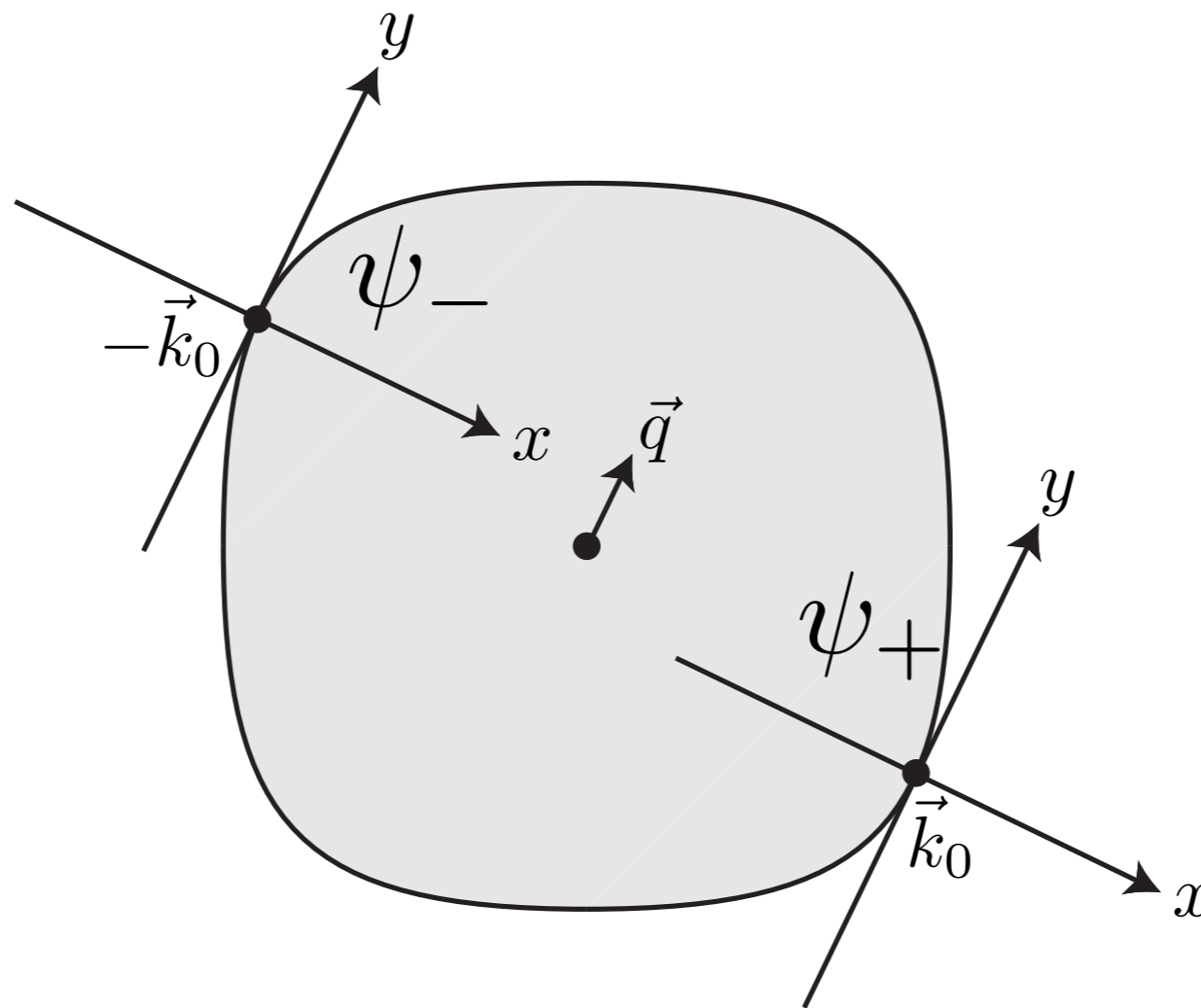
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Quantum criticality of Ising-nematic ordering



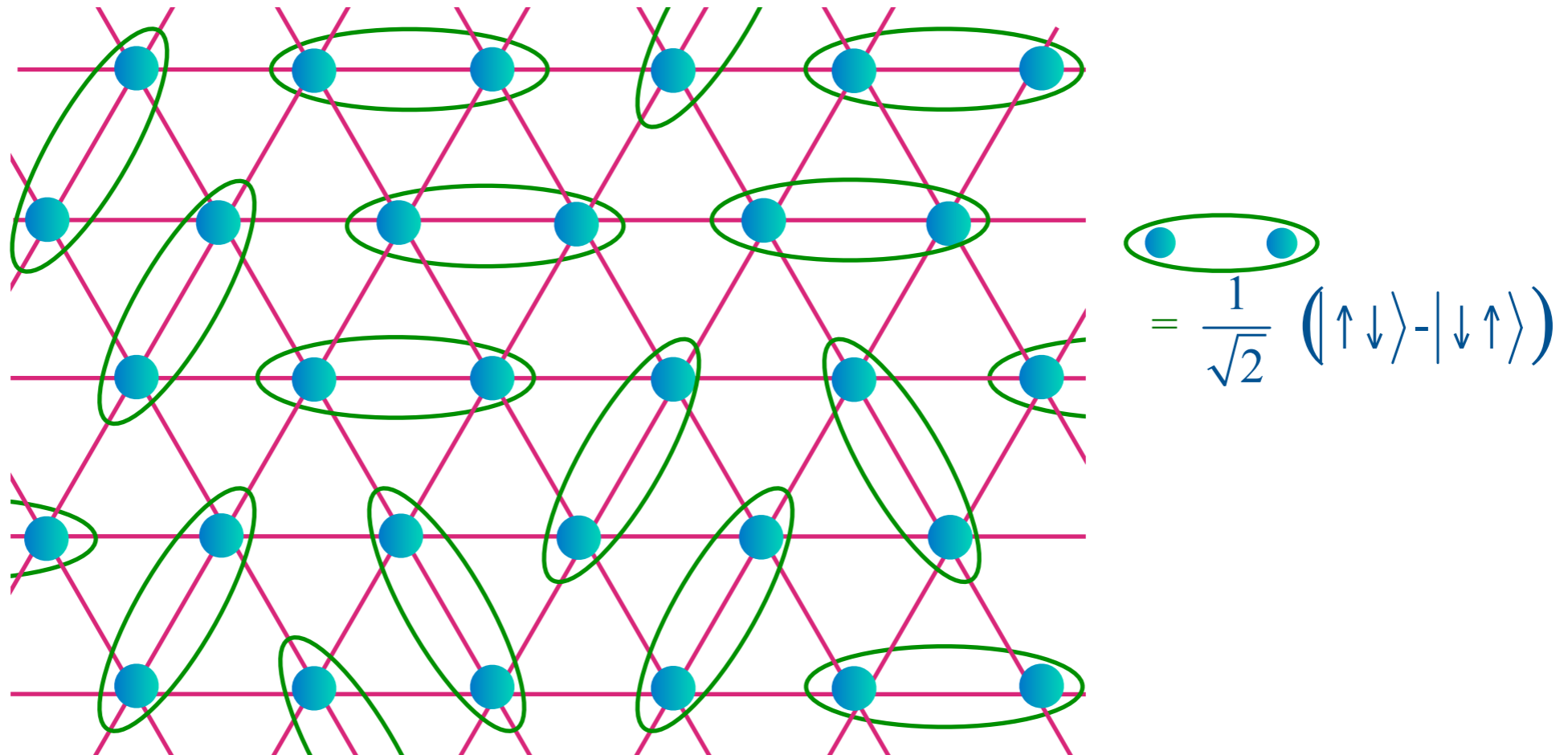
- ϕ fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering



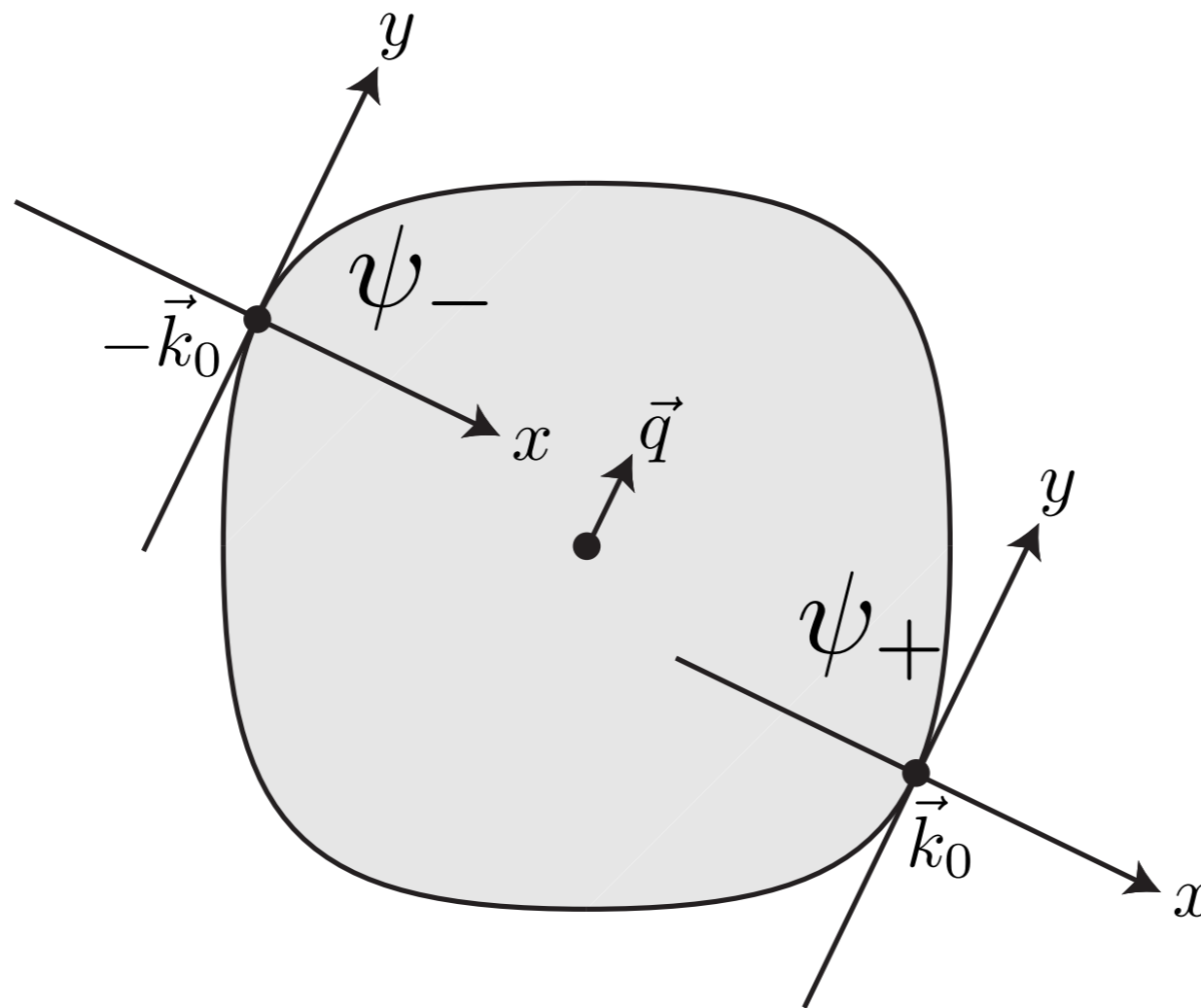
$$\begin{aligned} \mathcal{L}[\psi_{\pm}, \phi] = & \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ & - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2 \end{aligned}$$

- Model of a spin liquid (“Bose metal”): couple fermions to a dynamical gauge field A_μ .



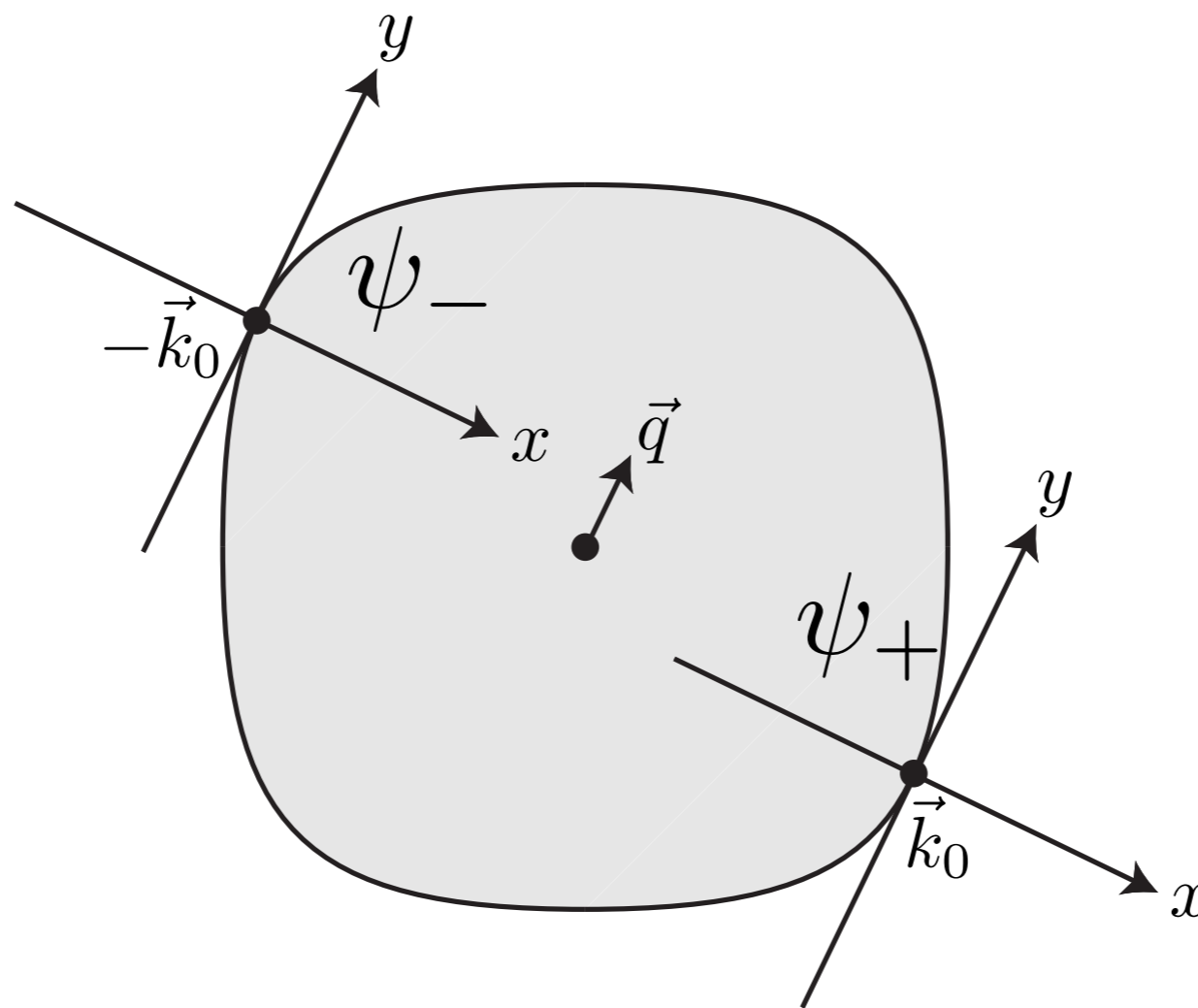
$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

Quantum criticality of Ising-nematic ordering



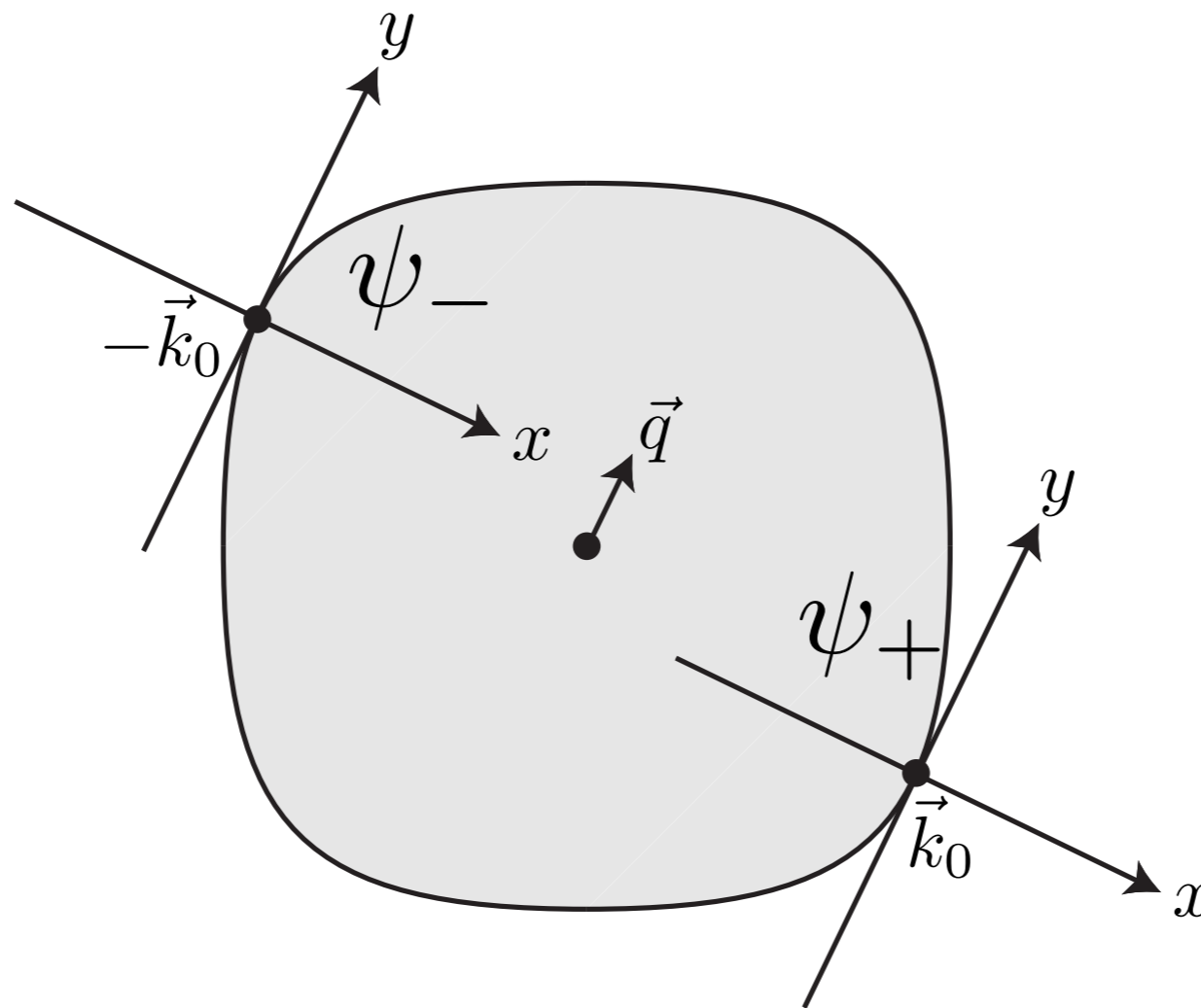
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Field theory of U(1) spin liquid



$$\mathcal{L}[\psi_{\pm}, a] = \psi_{+}^{\dagger} (\partial_{\tau} - i\partial_x - \partial_y^2) \psi_{+} + \psi_{-}^{\dagger} (\partial_{\tau} + i\partial_x - \partial_y^2) \psi_{-} - a (\psi_{+}^{\dagger} \psi_{+} - \psi_{-}^{\dagger} \psi_{-}) + \frac{1}{2g^2} (\partial_y a)^2$$

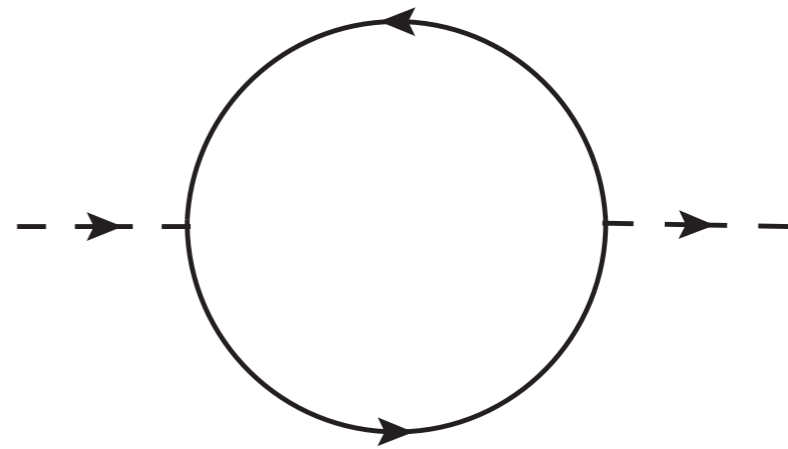
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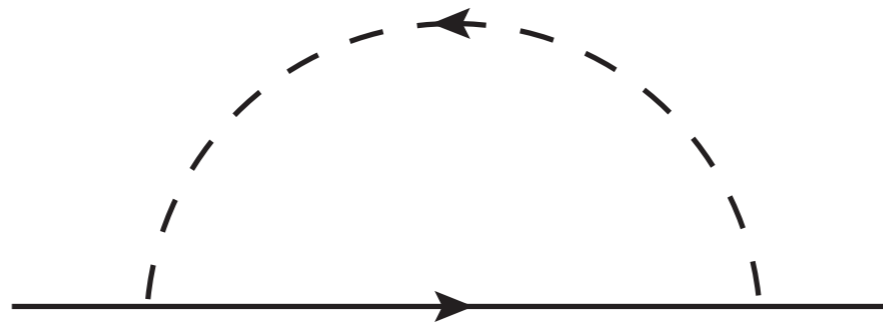
One loop ϕ self-energy with N_f fermion flavors:

$$D(\vec{q}, \omega) = N_f \int \frac{d^2 k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{[-i(\Omega + \omega) + k_x + q_x + (k_y + q_y)^2] [-i\Omega - k_x + k_y^2]}$$
$$= \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$



Electron self-energy at order $1/N_f$:

$$\begin{aligned} \Sigma(\vec{k}, \Omega) &= -\frac{1}{N_f} \int \frac{d^2q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2] \left[\frac{q_y^2}{g^2} + \frac{|\omega|}{|q_y|} \right]} \\ &= -i \frac{2}{\sqrt{3}N_f} \left(\frac{g^2}{4\pi} \right)^{2/3} \text{sgn}(\Omega) |\Omega|^{2/3} \end{aligned}$$

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y \phi)^2$$

Schematic form of ϕ and fermion Green's functions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \frac{|\omega|}{|q_y|}}, \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i \text{sgn}(\omega) |\omega|^{2/3} / N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering

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Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned} \mathcal{L}_{\text{scaling}} = & \psi_+^\dagger (-i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i\partial_x - \partial_y^2) \psi_- \\ & - g\phi (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + (\partial_y \phi)^2 \end{aligned}$$

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Simple scaling argument for $z = 3/2$.

Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

$$\phi \rightarrow \phi s^{(2z+1)/4}$$

$$\psi \rightarrow \psi s^{(2z+1)/4}$$

$$g \rightarrow g s^{(3-2z)/4}$$

So the action is invariant provided $z = 3/2$.

Quantum criticality of Ising-nematic ordering

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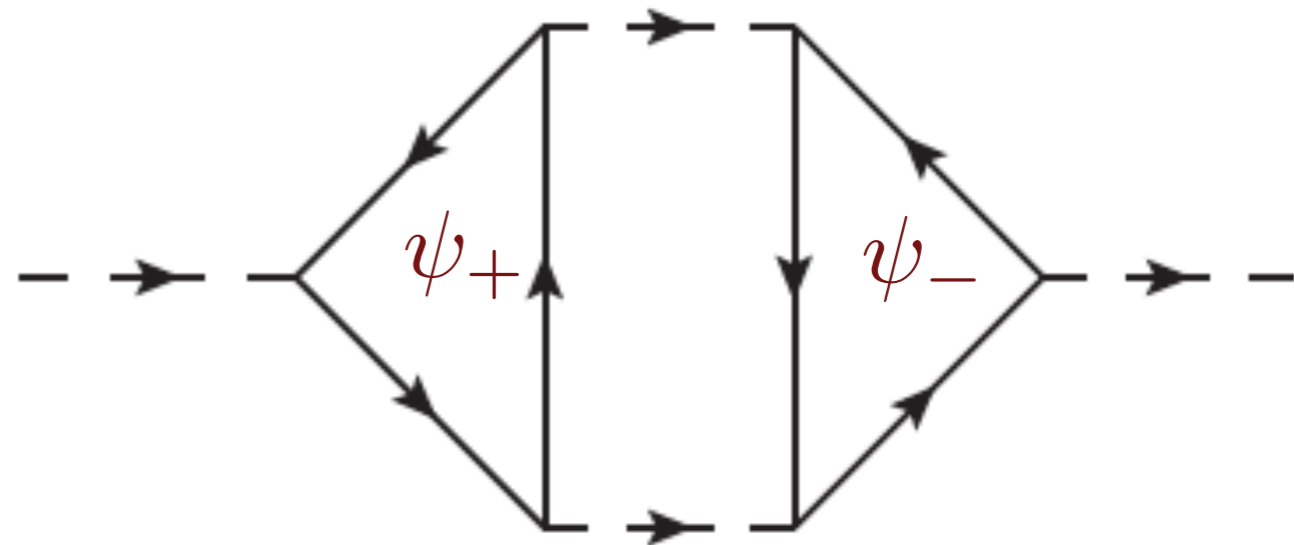
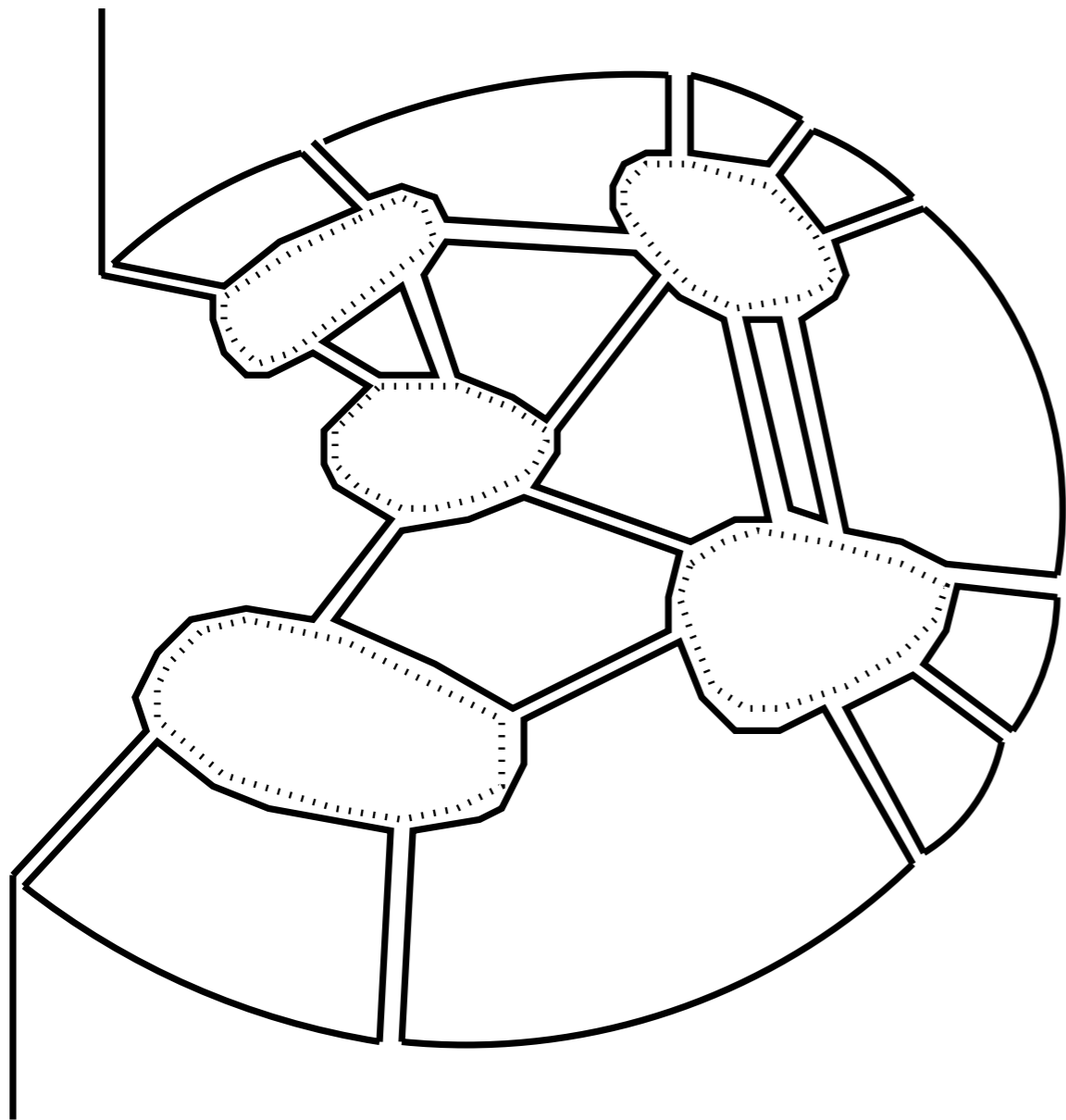
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The $1/N_f$ expansion is *not* determined by counting fermion loops, because of infrared singularities created by the Fermi surface. The $|\omega|^{2/3}/N_f$ fermion self-energy leads to additional powers of N_f , and a breakdown in the loop expansion.

Computations in the $1/N$ expansion



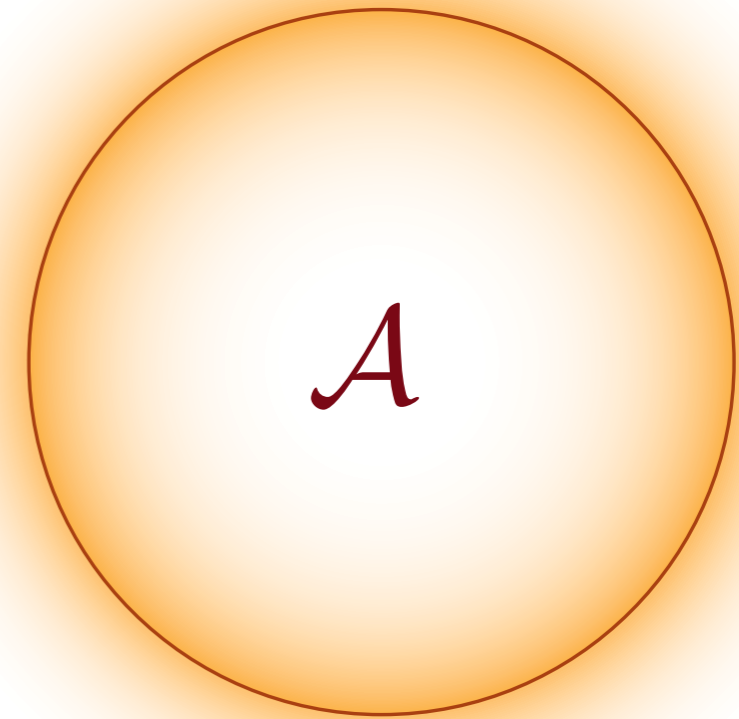
Graph mixing ψ_+ and ψ_- is $\mathcal{O}(N^{3/2})$ (instead of $\mathcal{O}(N)$), violating genus expansion

All planar graphs of ψ_+ alone are as important as the leading term

M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review B* **80**, 165102 (2009)

Properties of the strange metal at the Ising-nematic critical point



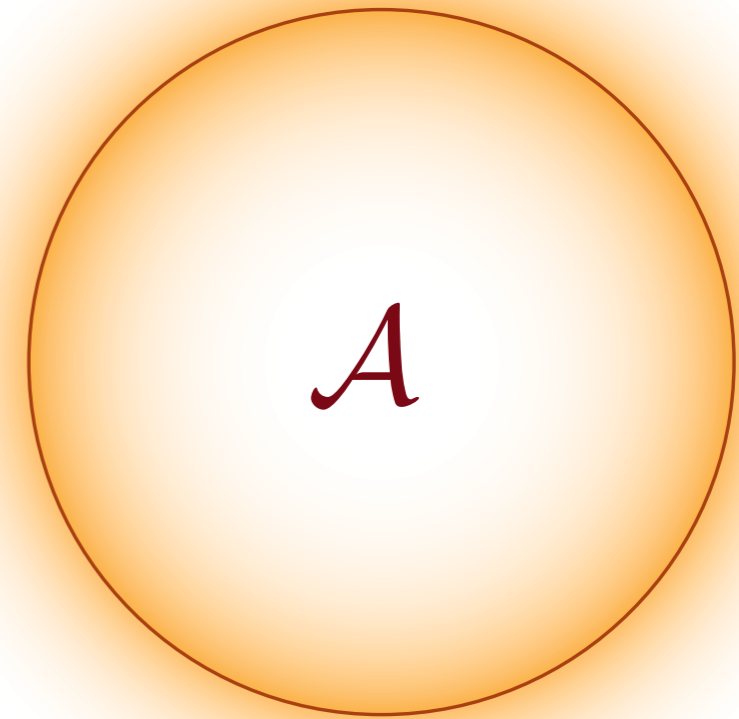
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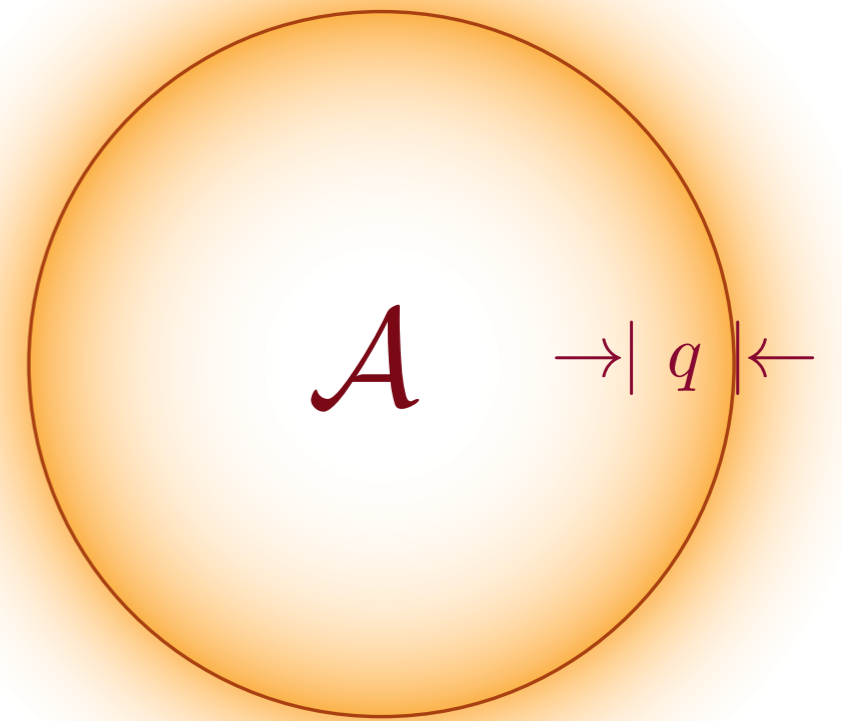
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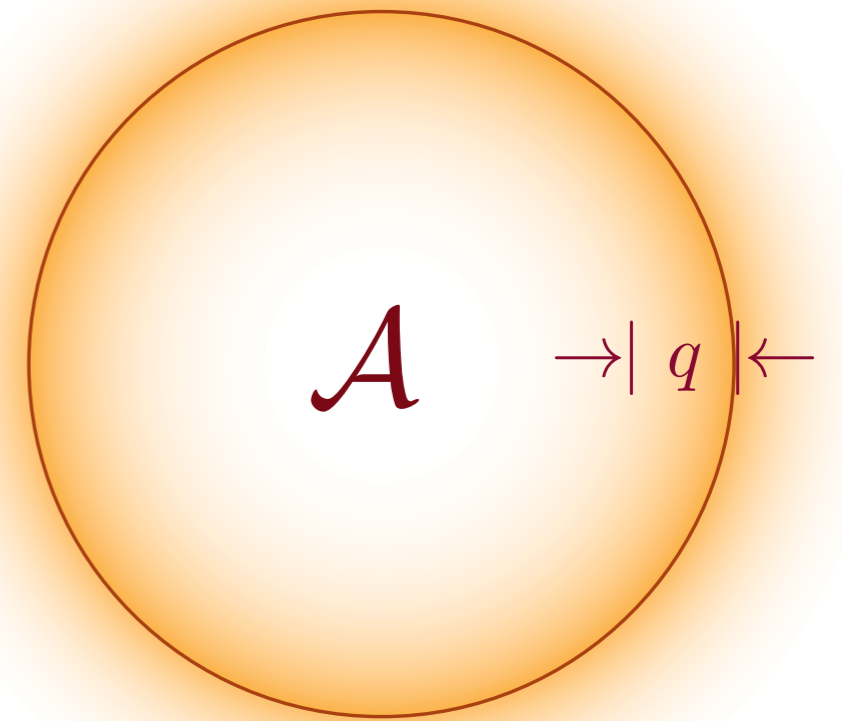
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- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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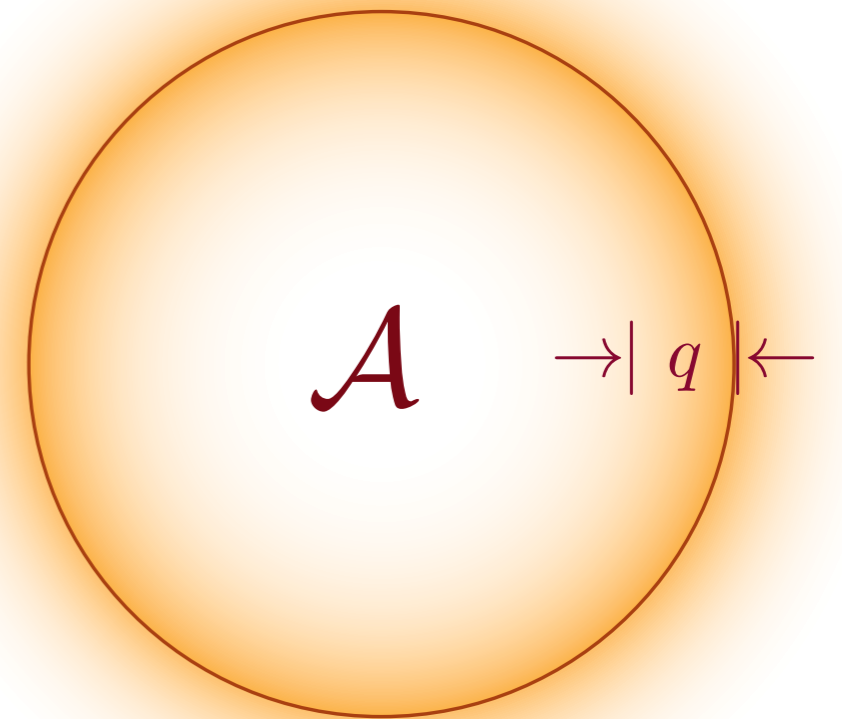
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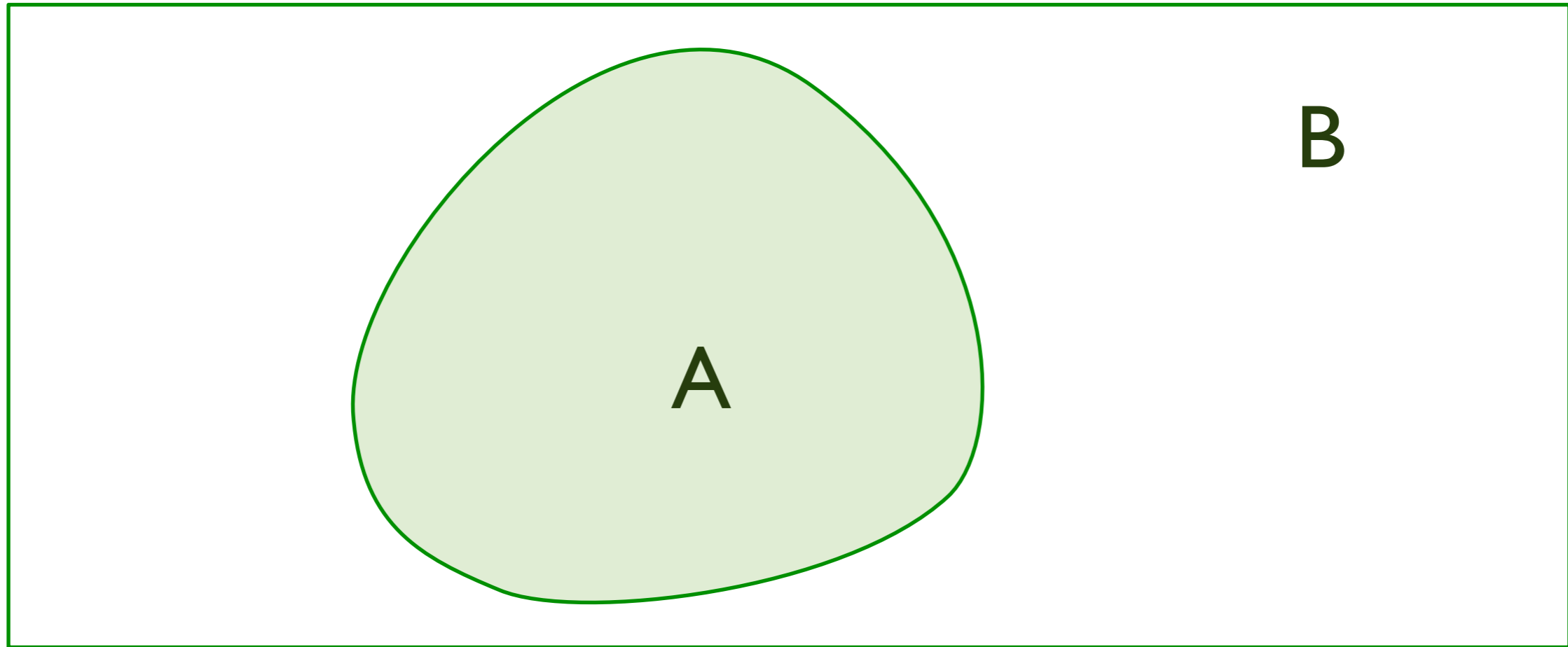
- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}/z}$ with $d_{\text{eff}} = 1$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

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D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

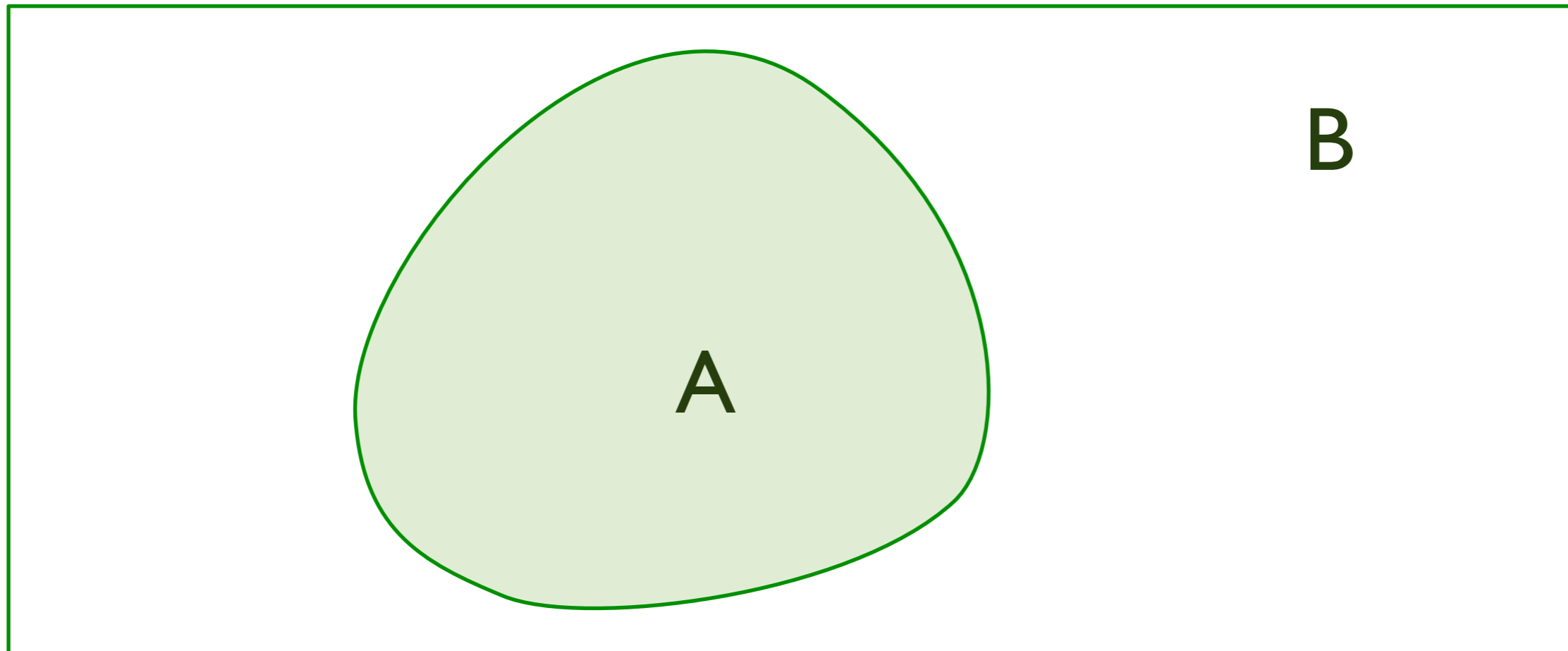
Entanglement entropy



Measure strength of quantum entanglement of region A with region B .

$\rho_A = \text{Tr}_B \rho =$ density matrix of region A
Entanglement entropy $S_{EE} = -\text{Tr}(\rho_A \ln \rho_A)$

Entanglement entropy of Fermi surfaces



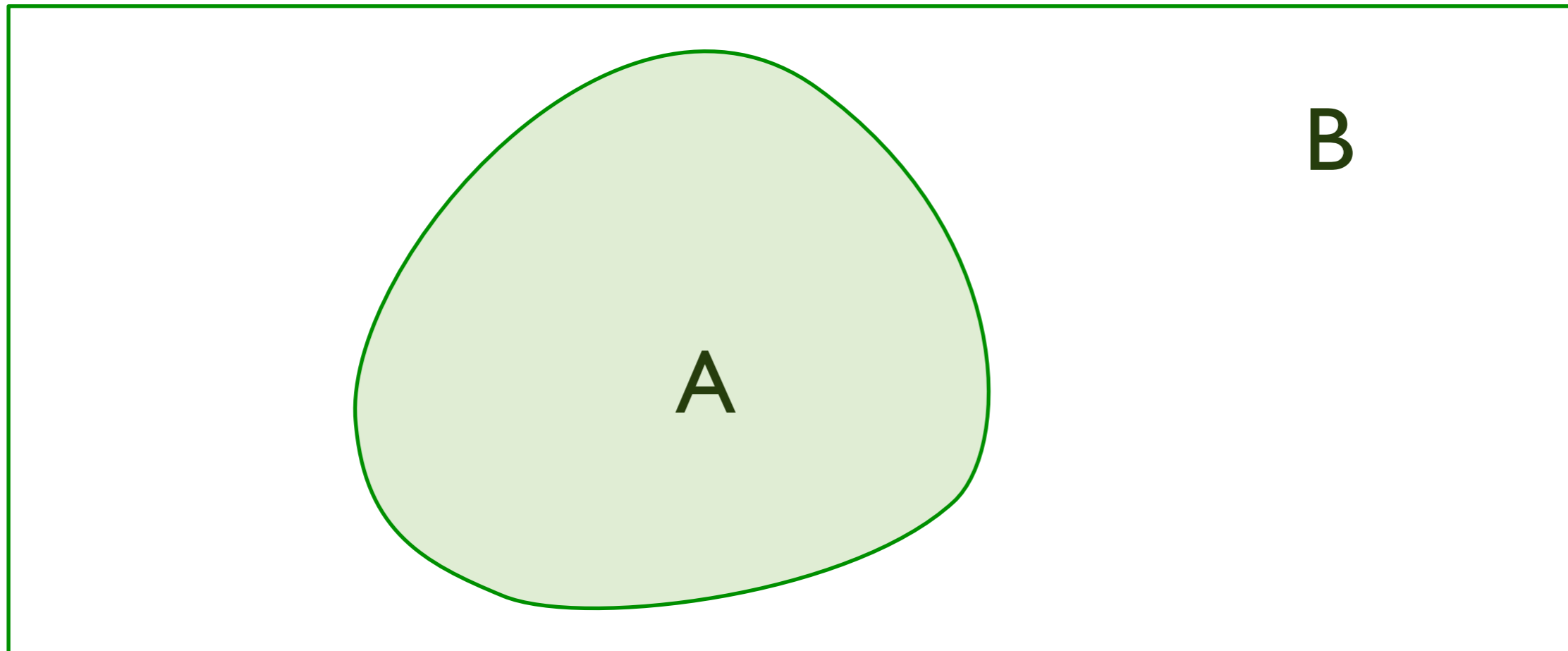
Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F ,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of Fermi surfaces



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Non-Fermi liquids have, at most, the “1/12” prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Compressible quantum matter

A. Fermi liquids: graphene

*B. Holography: Reissner - Nördstrom
solution*

*C. Non-Fermi liquids:
nematic critical point (and $U(1)$ spin liquids)*

*D. Holography: scaling arguments for
entropy and entanglement entropy*

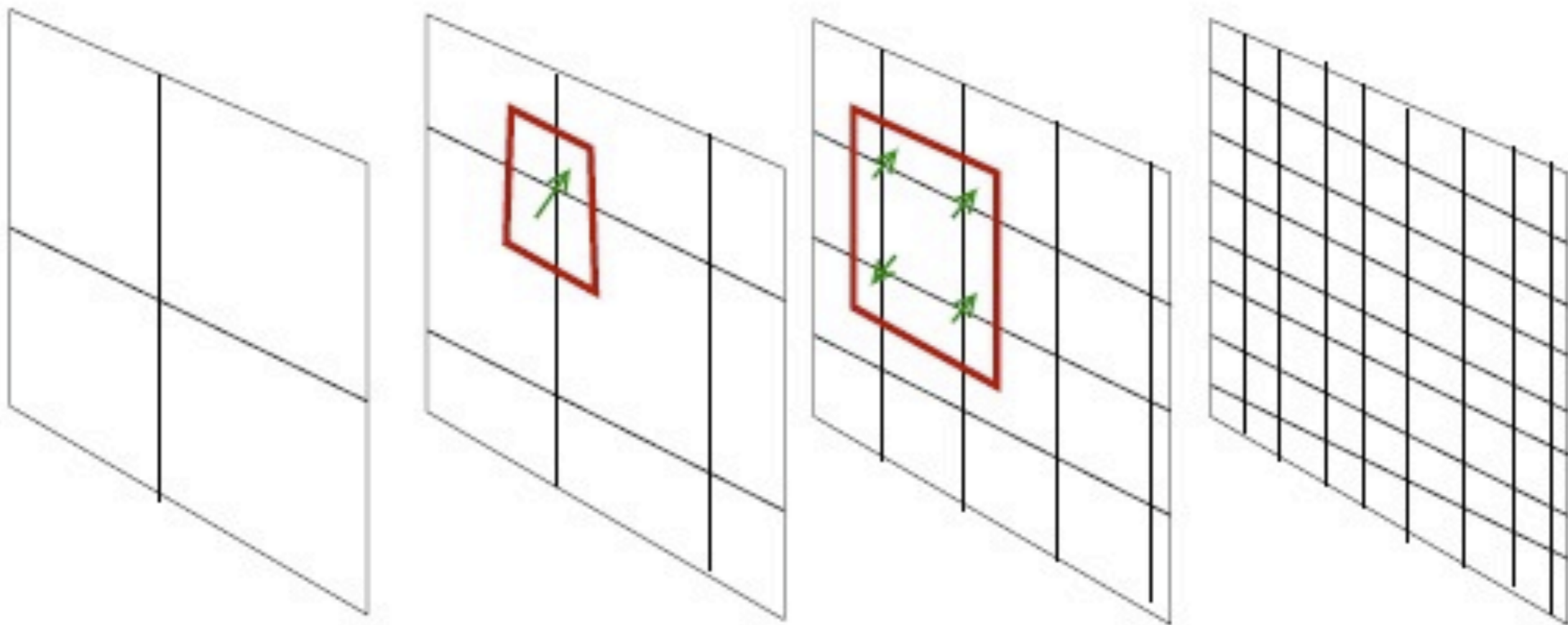
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r ←

Consider the metric which transforms under rescaling as

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\t &\rightarrow \zeta^z t \\ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

θ is the violation of hyperscaling exponent.

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The most general choice of such a metric is

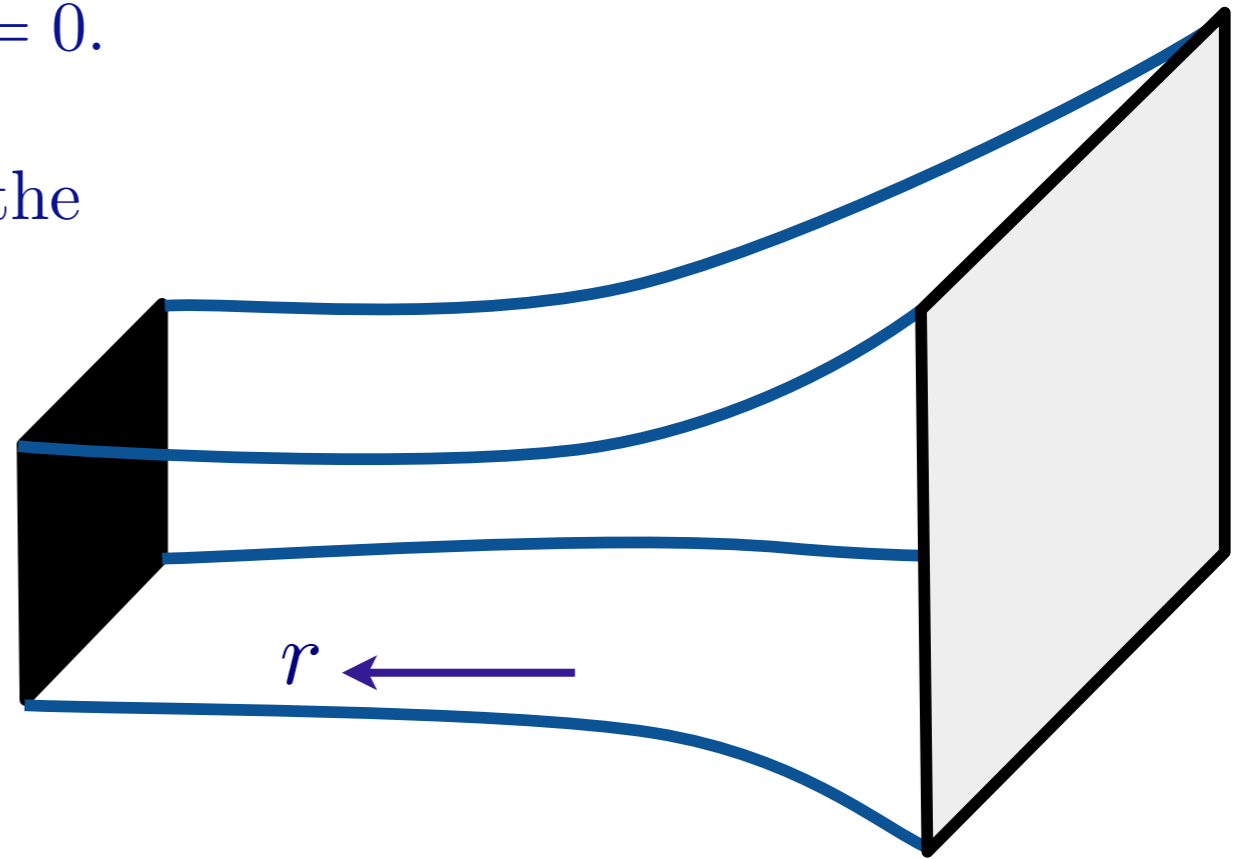
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

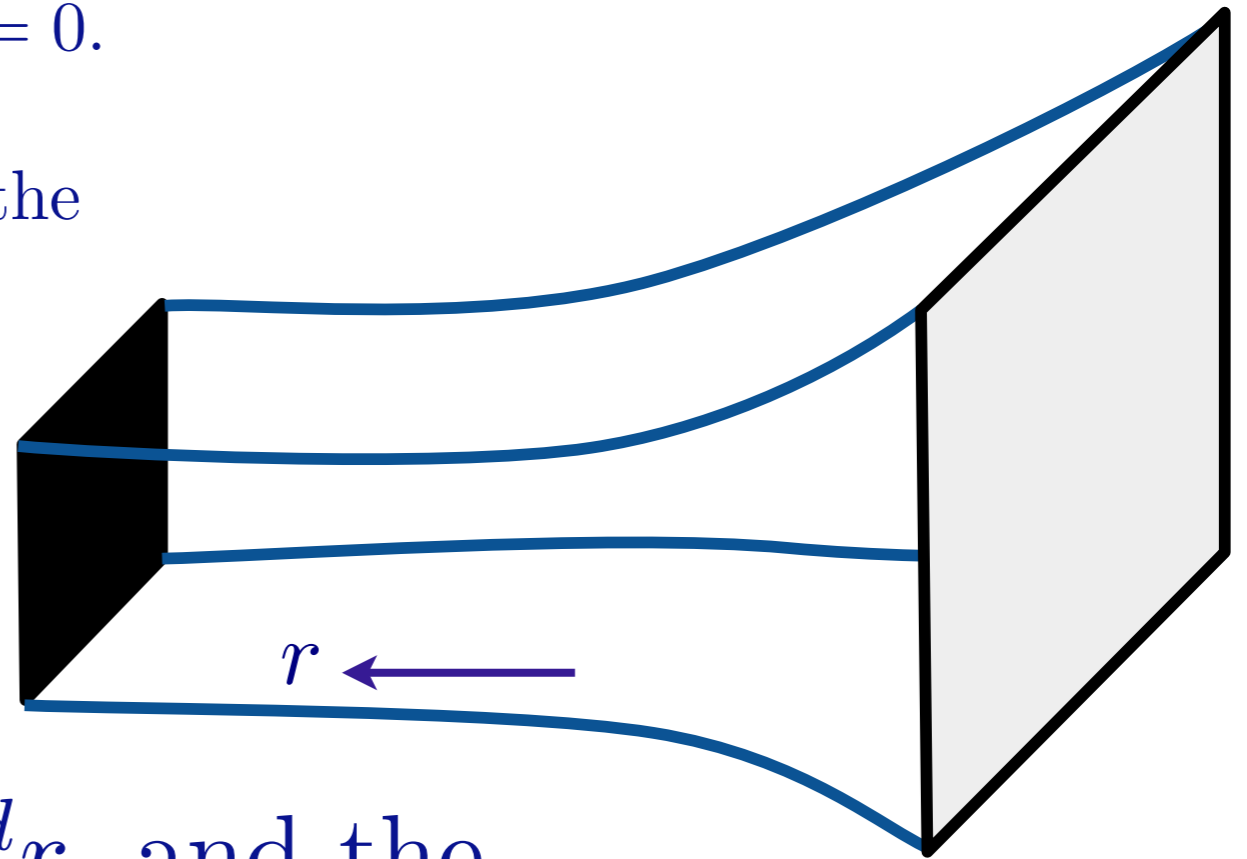
The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



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Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.

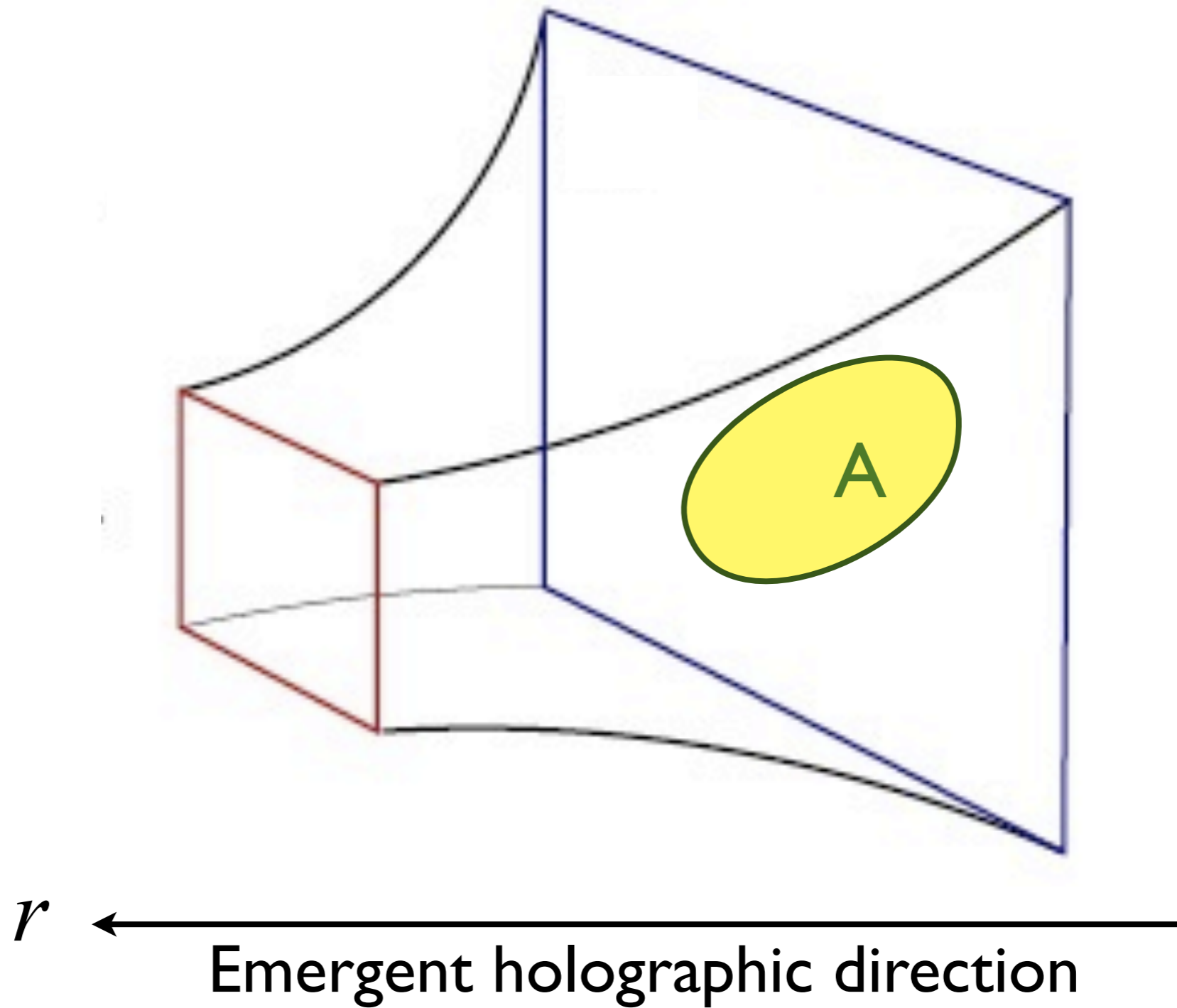
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- The thermal entropy density scales as

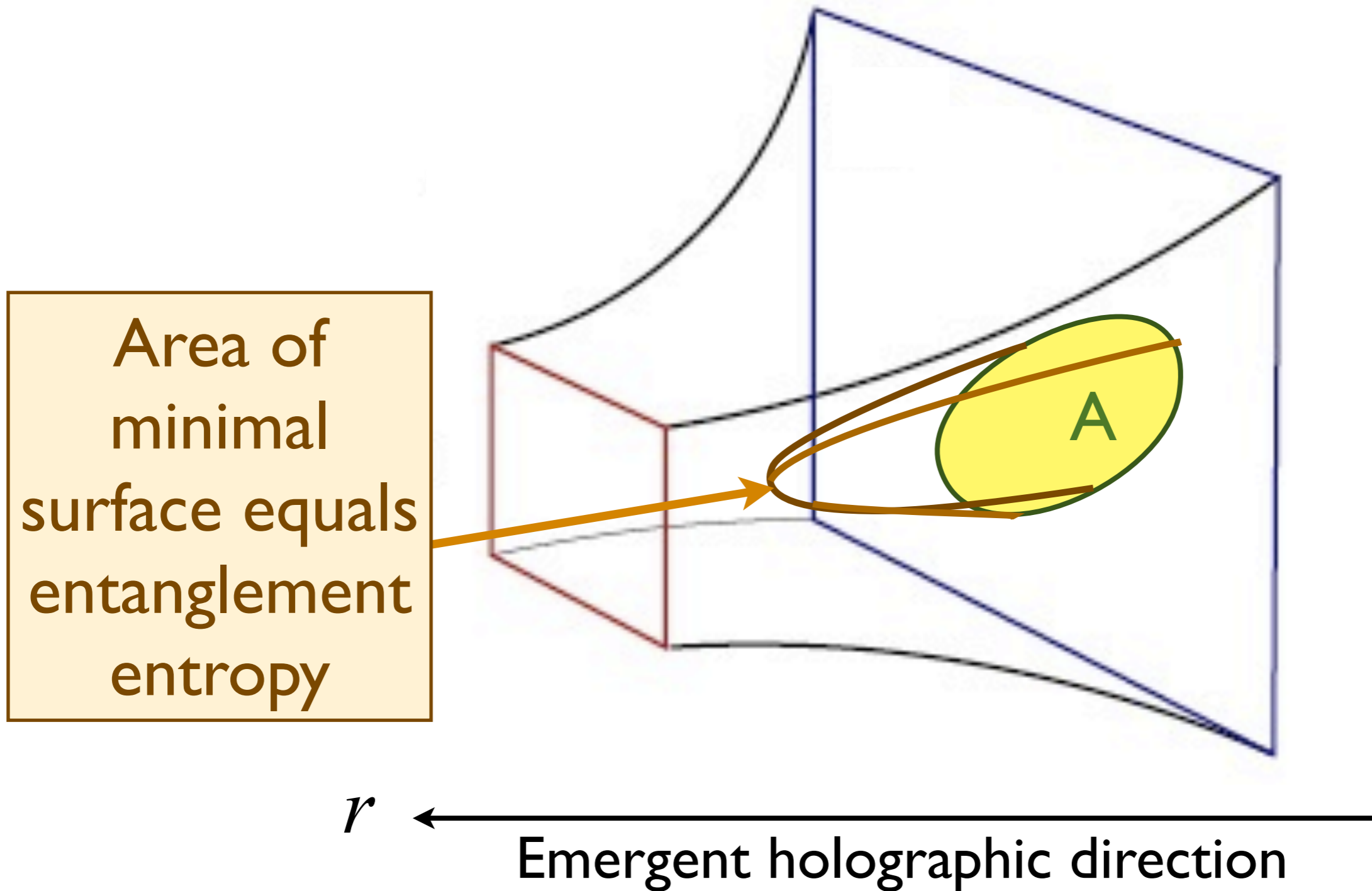
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The third law of thermodynamics requires $\theta < d$.

Holographic entanglement entropy



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S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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- The entanglement entropy, S_E , of an entangling region with boundary surface ‘area’ P scales as

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases}$$

All local quantum field theories obey the “area law” (upto log violations) and so $\theta \leq d - 1$.

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Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

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Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

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- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of non-Fermi liquids

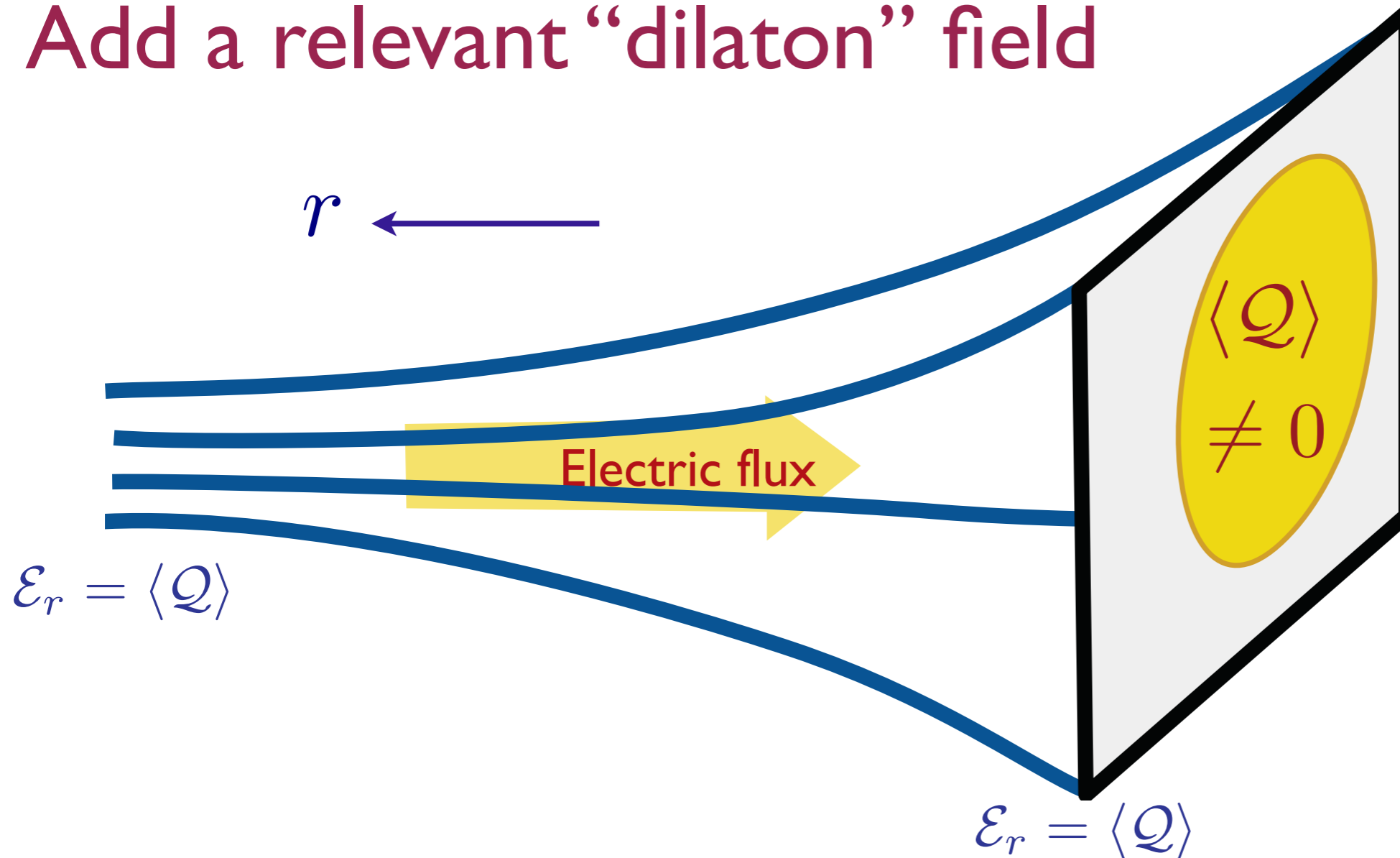
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- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



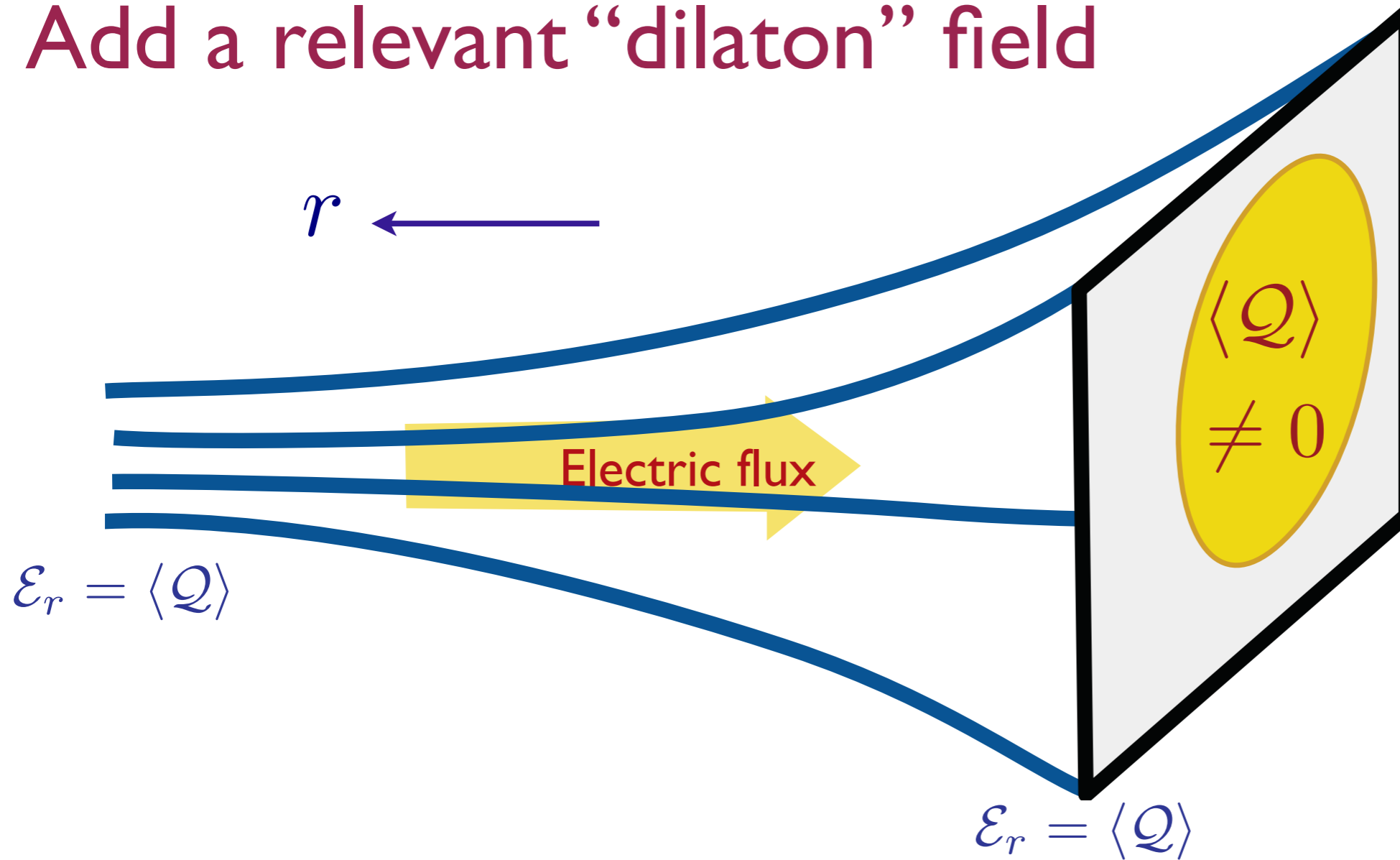
$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holographic theory of a non-Fermi liquid (NFL)

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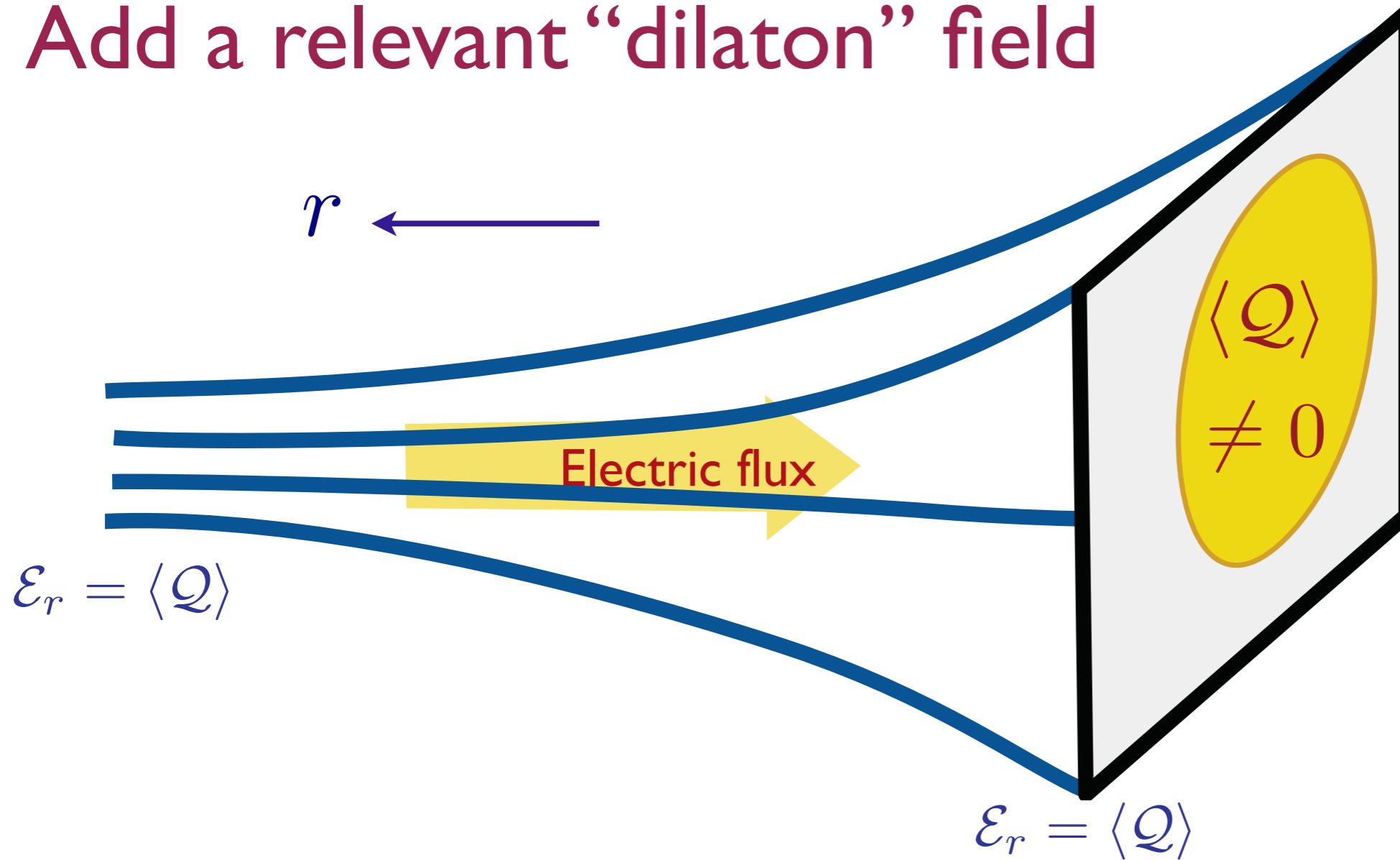
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This is a “bosonization” of the Fermi surface

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$
with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^\delta$, $\Phi(r) \sim \ln(r)$ as $r \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
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Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

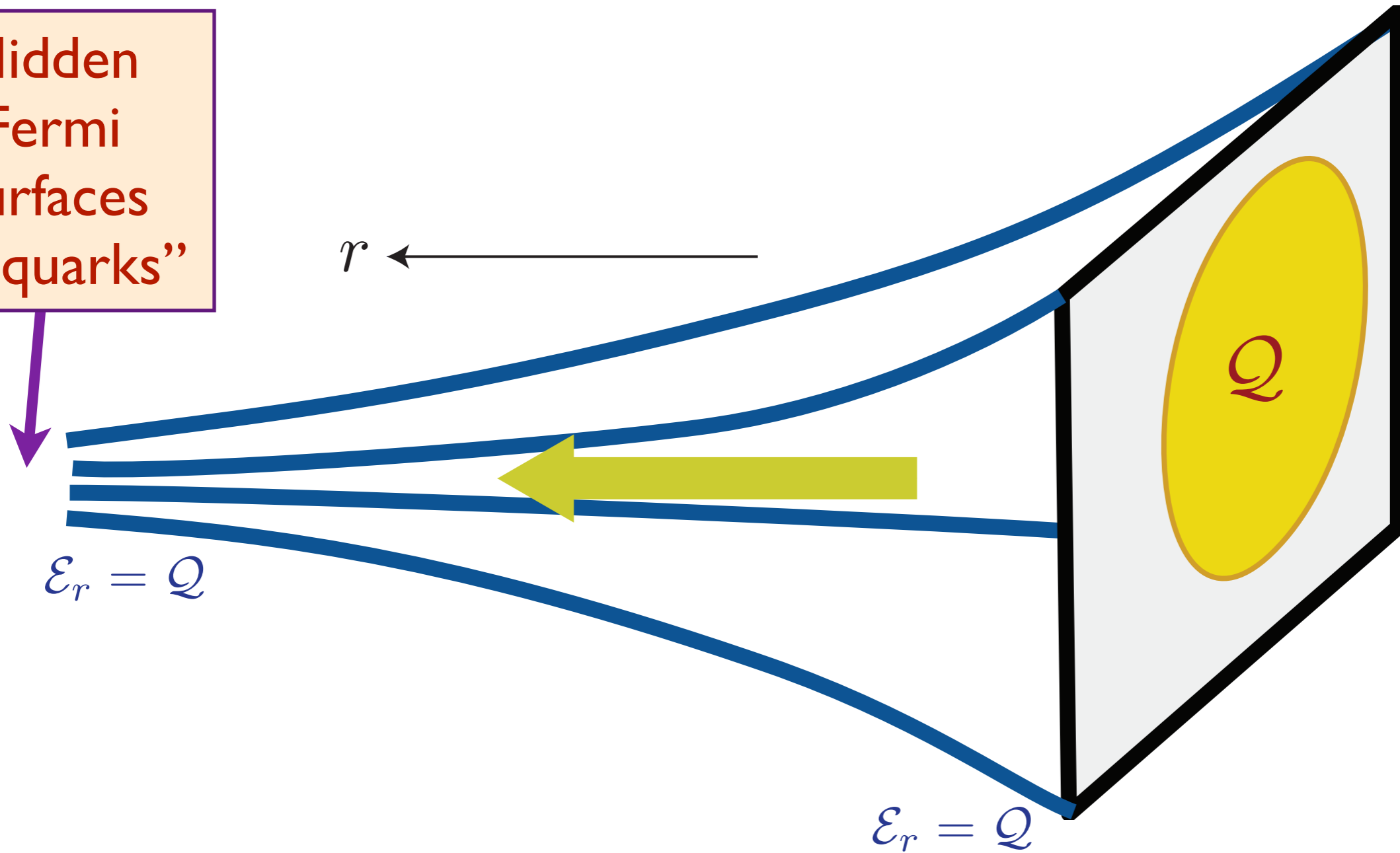
- The entanglement entropy has log-violation of the area law

$$S_E = \Xi Q^{(d-1)/d} P \ln P.$$

where P is surface area of the entangling region, and Ξ is a dimensionless constant which is **independent of all UV details**, of Q , and of any property of the entangling region. Note $Q^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

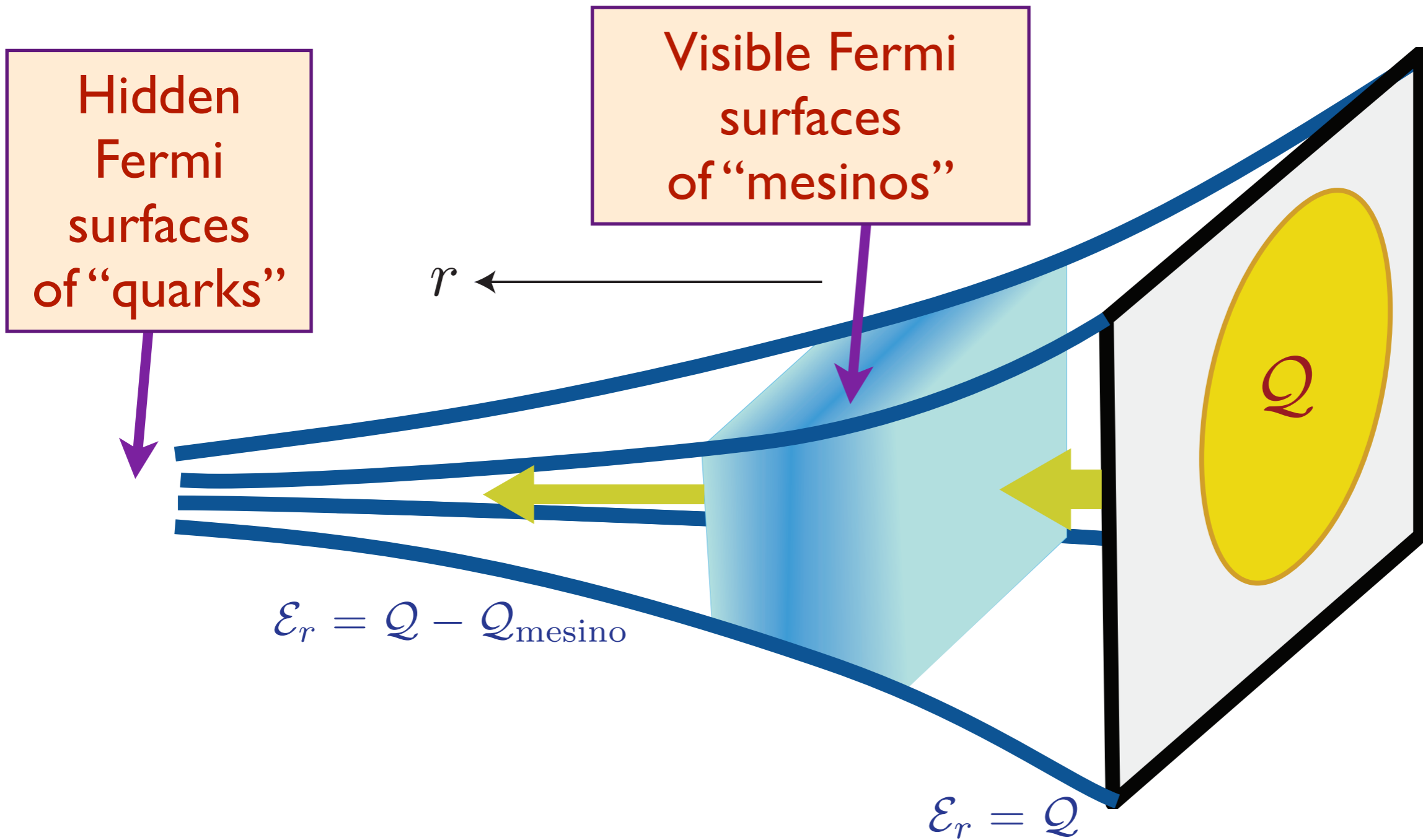
Holographic theory of a non-Fermi liquid (NFL)

Hidden Fermi surfaces of “quarks”



Gauss Law and the “attractor” mechanism
 \Leftrightarrow Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)

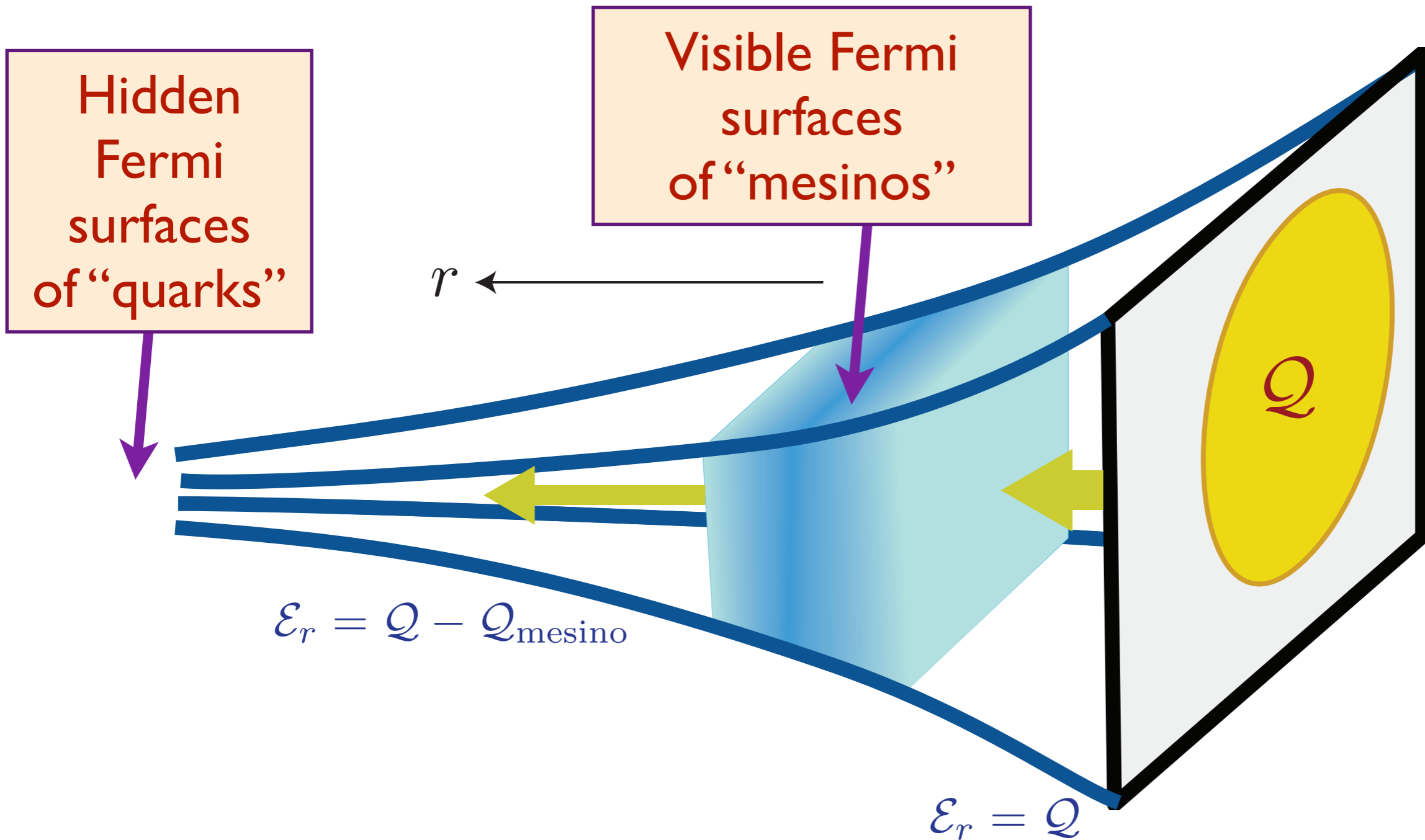


A state with *partial* confinement

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

S. Sachdev, *Physical Review D* **84**, 066009 (2011)

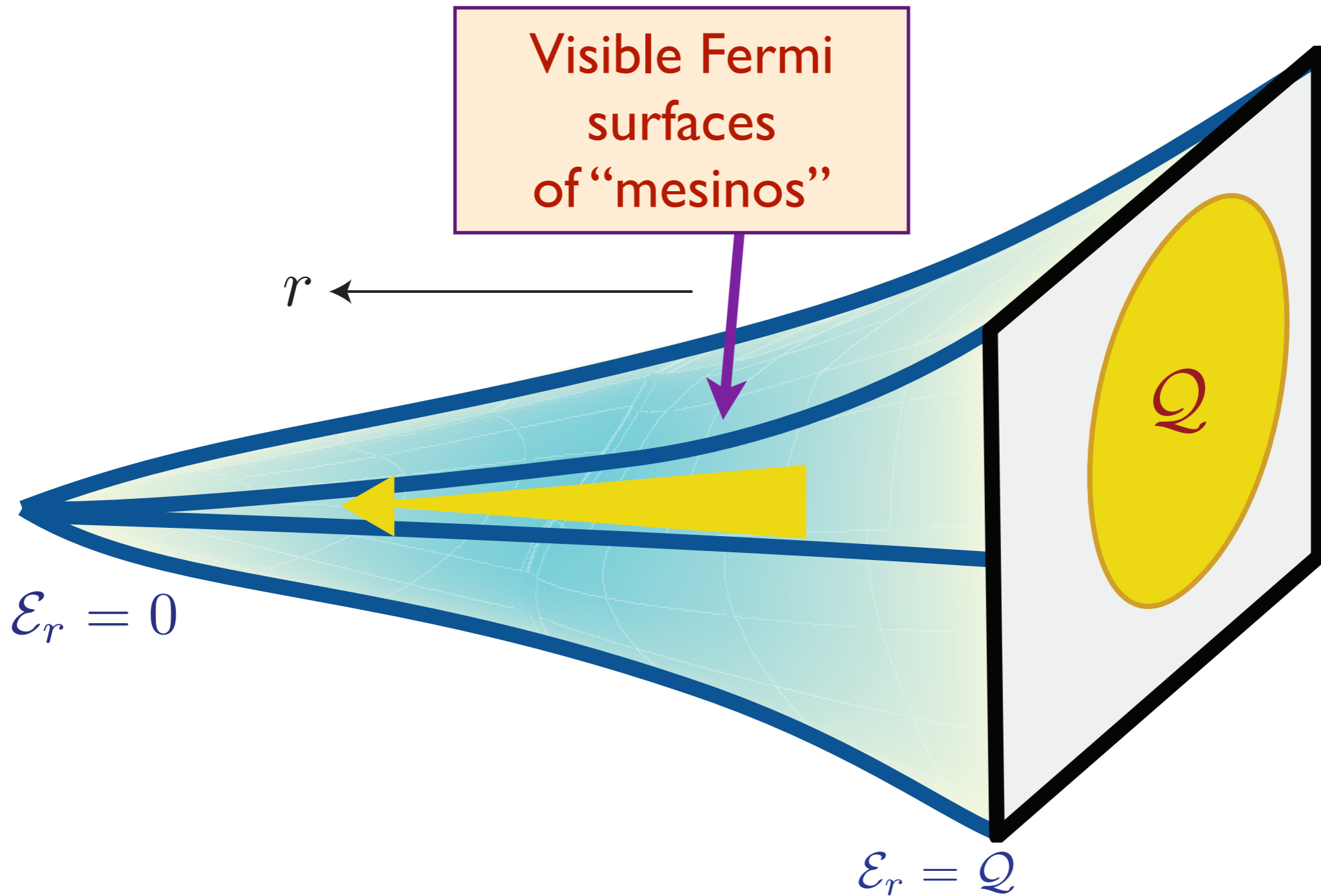
Holographic theory of a fractionalized-Fermi liquid (FL*)



The “mesinos” corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

These are spectators, and are expected to have well-defined quasiparticle excitations.

Holographic theory of a Fermi liquid (FL)



- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

Conclusions

Compressible quantum matter

● Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

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- Fermi liquid (FL) state described by a confining holographic geometry
- Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a state with partial confinement: the fractionalized Fermi liquid (FL*)

The onset of antiferromagnetism in metals

Lorentz Lecture, Leiden
June 4, 2012

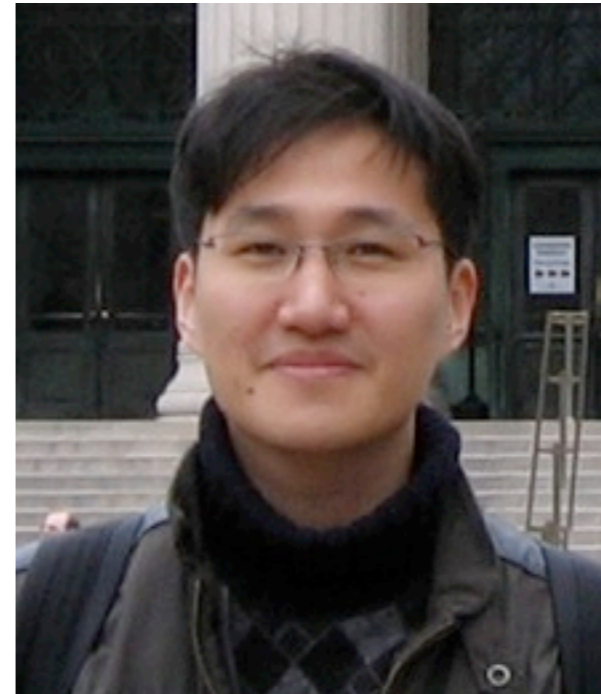
Subir Sachdev

sachdev.physics.harvard.edu





Max Metlitski



Eun Gook Moon



Matthias Punk



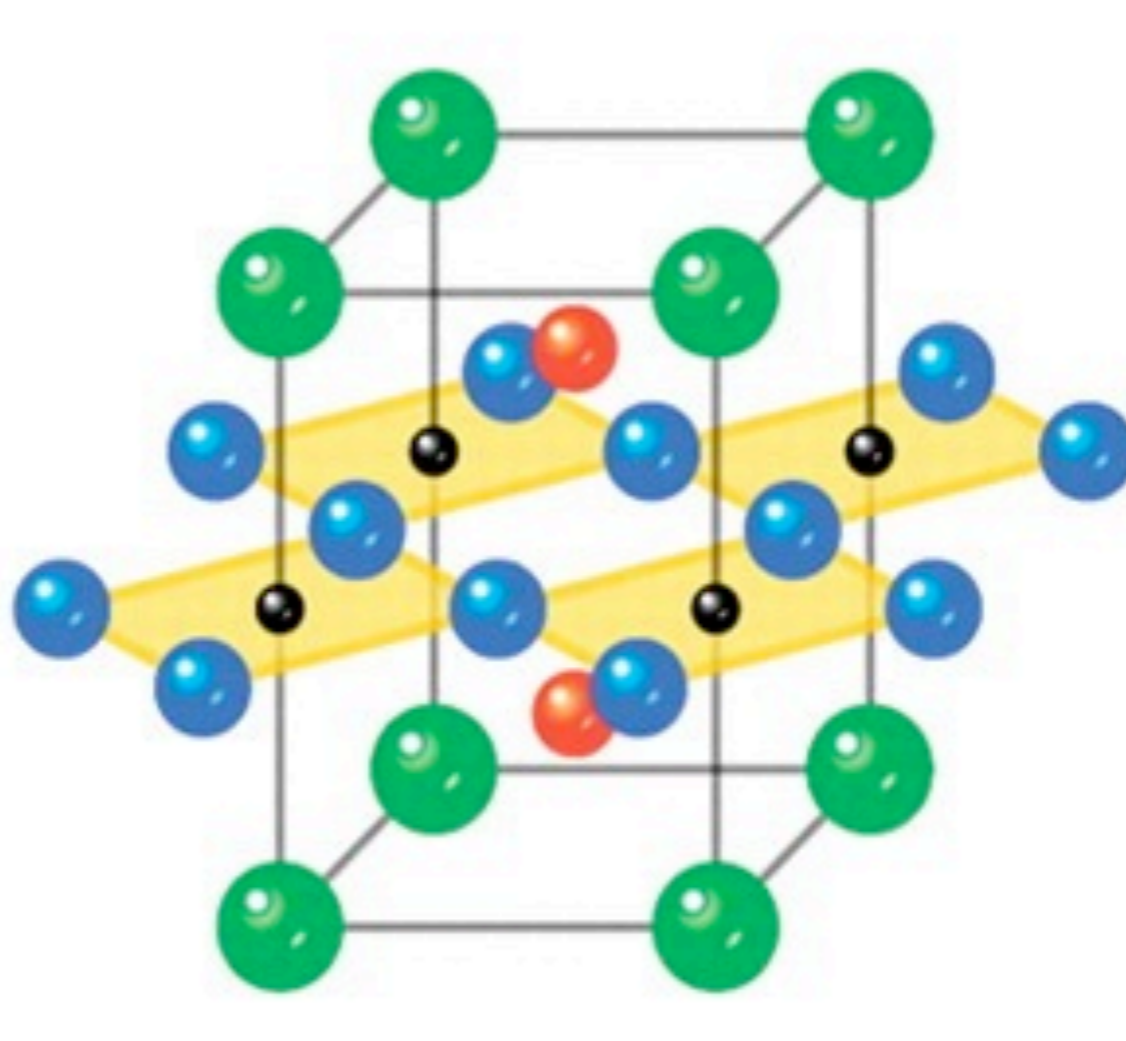
Erez Berg



The cuprate superconductors

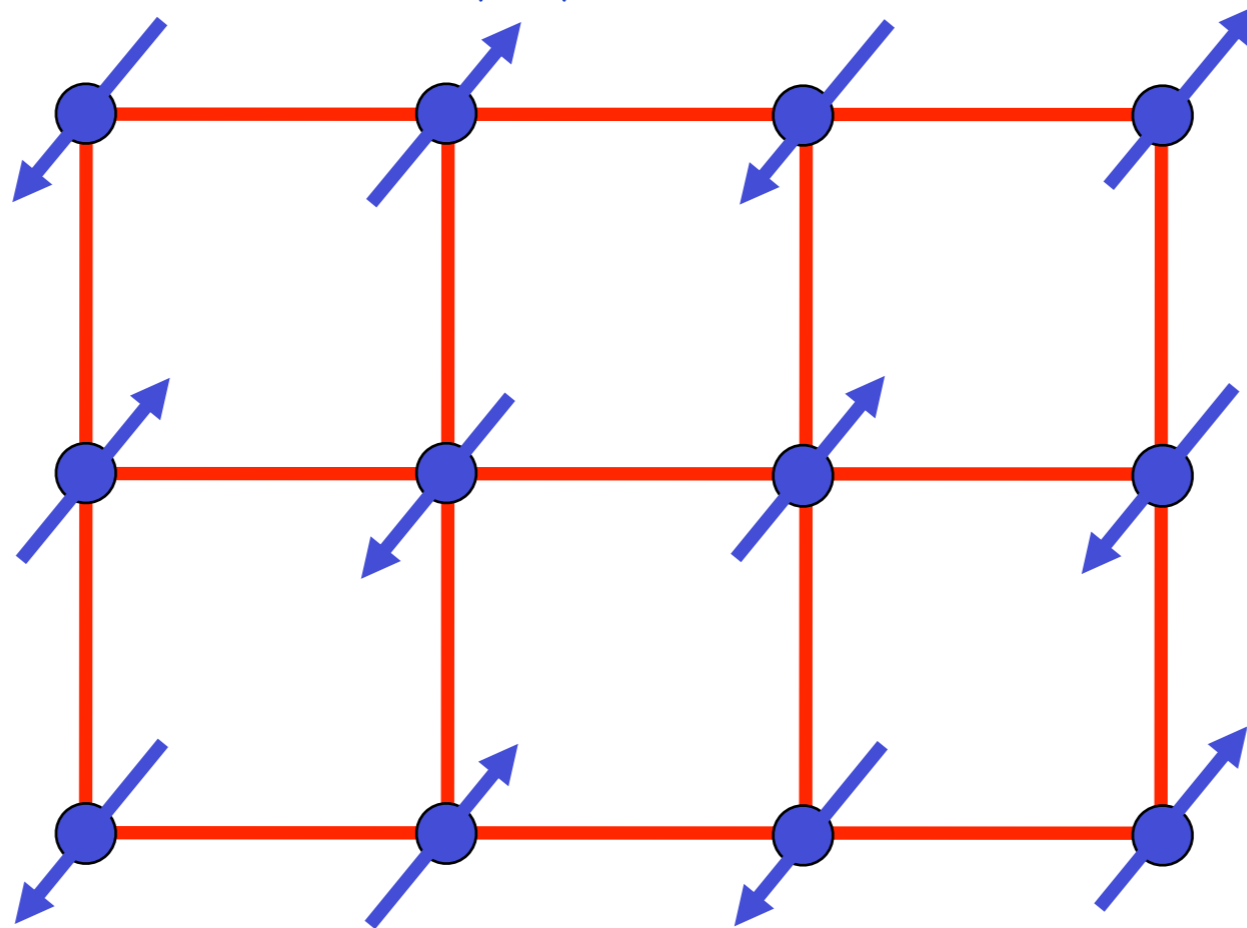
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

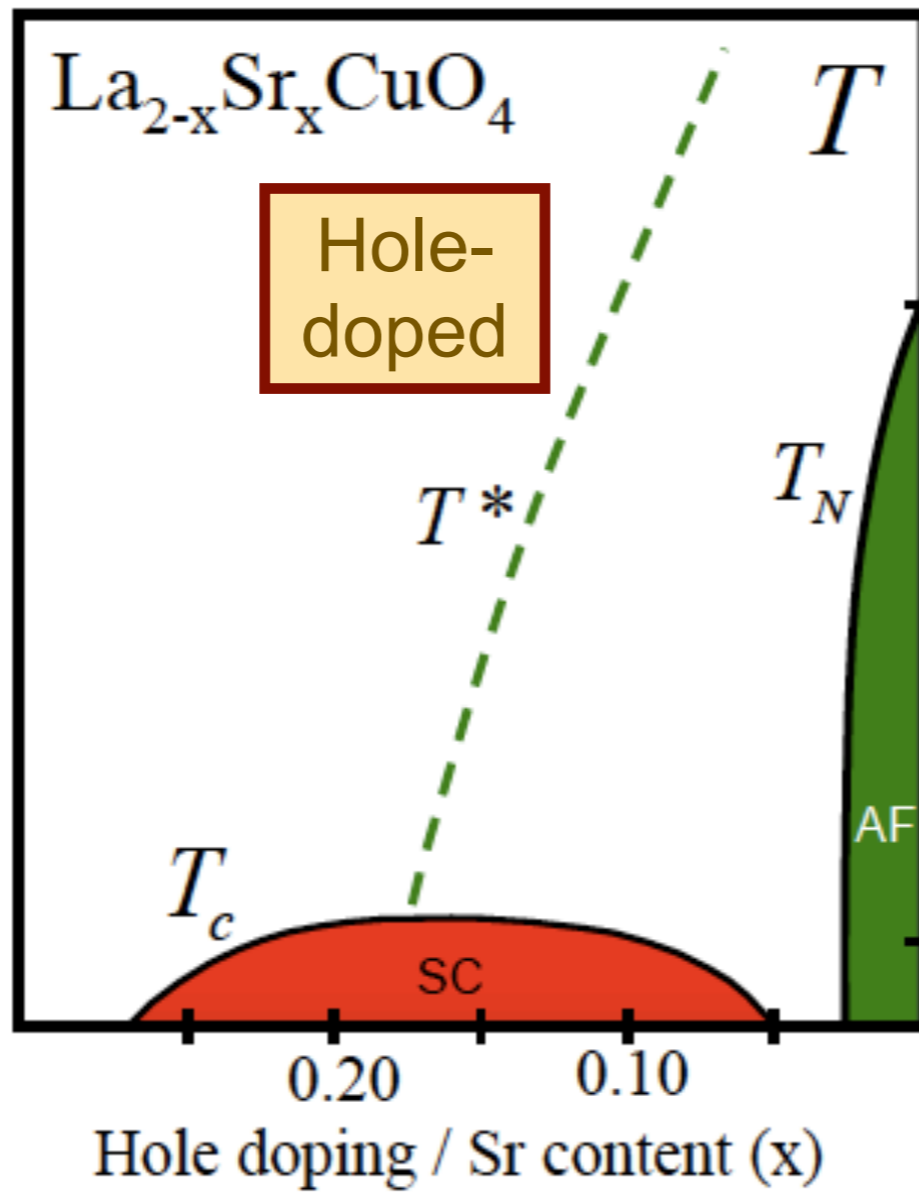


Ground state has long-range Néel order

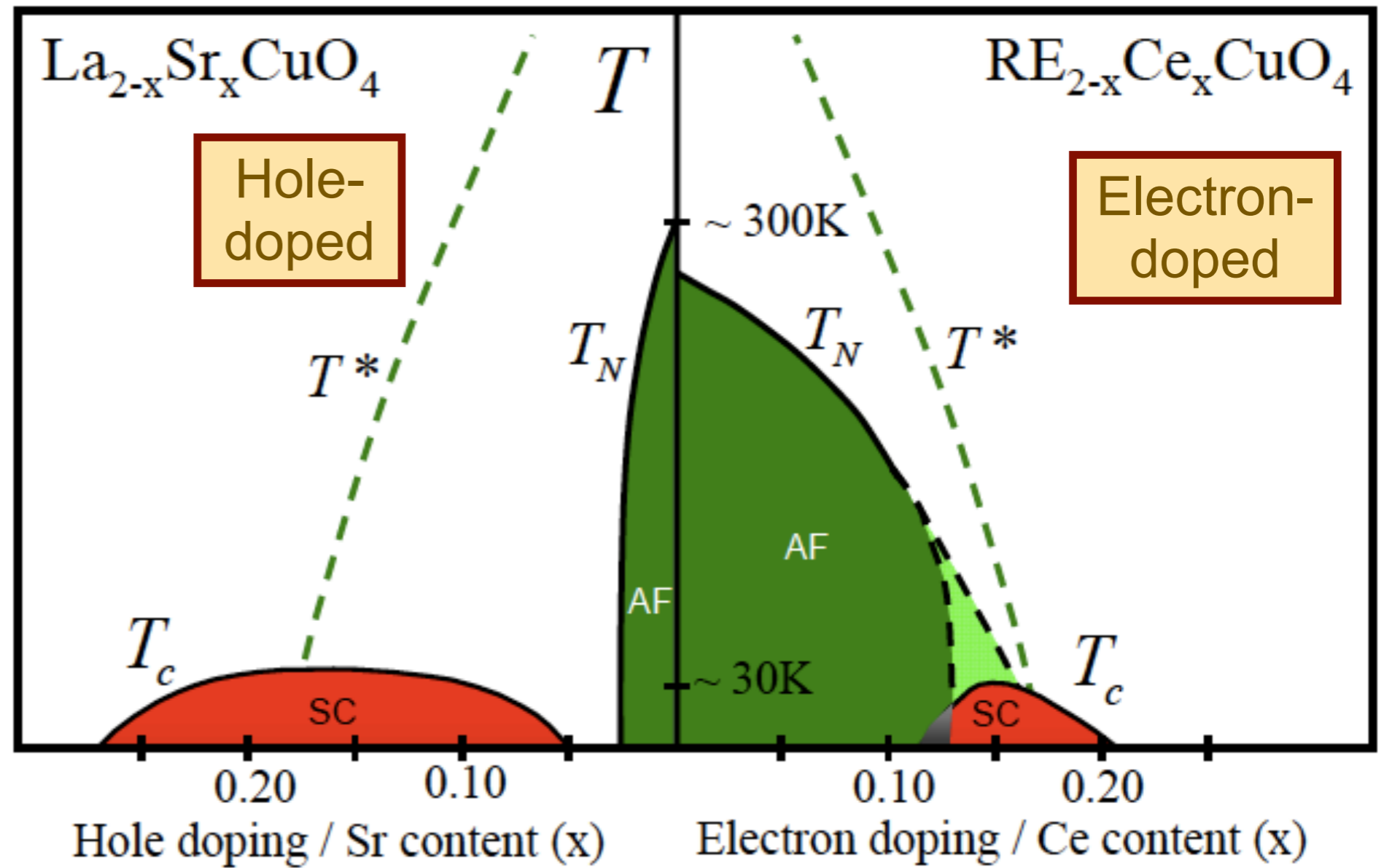
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.



Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

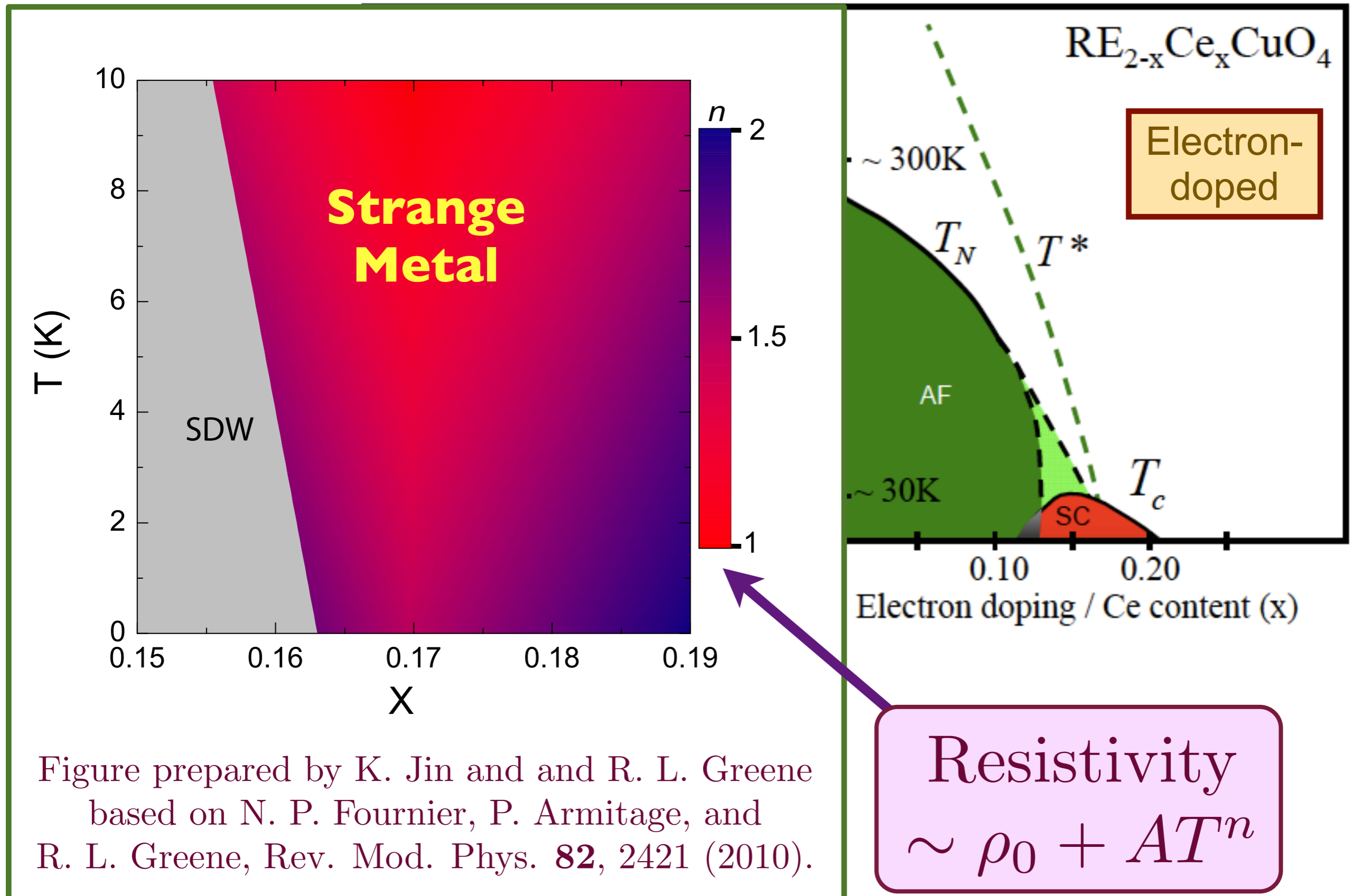
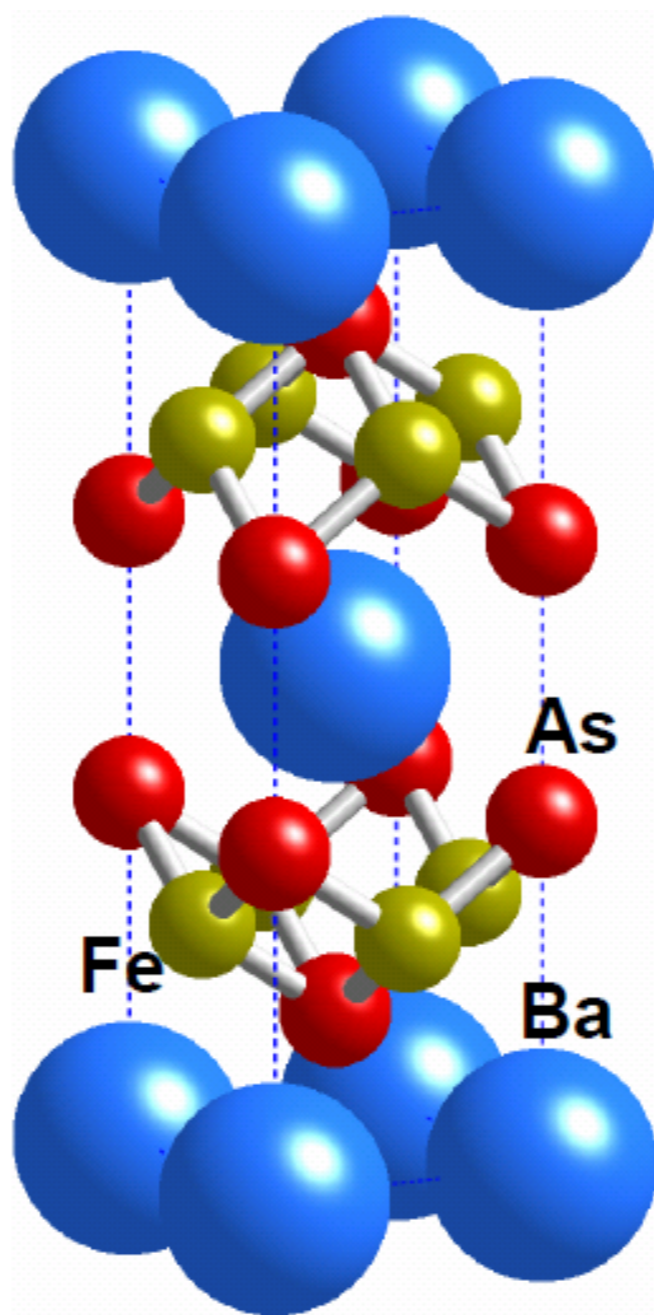


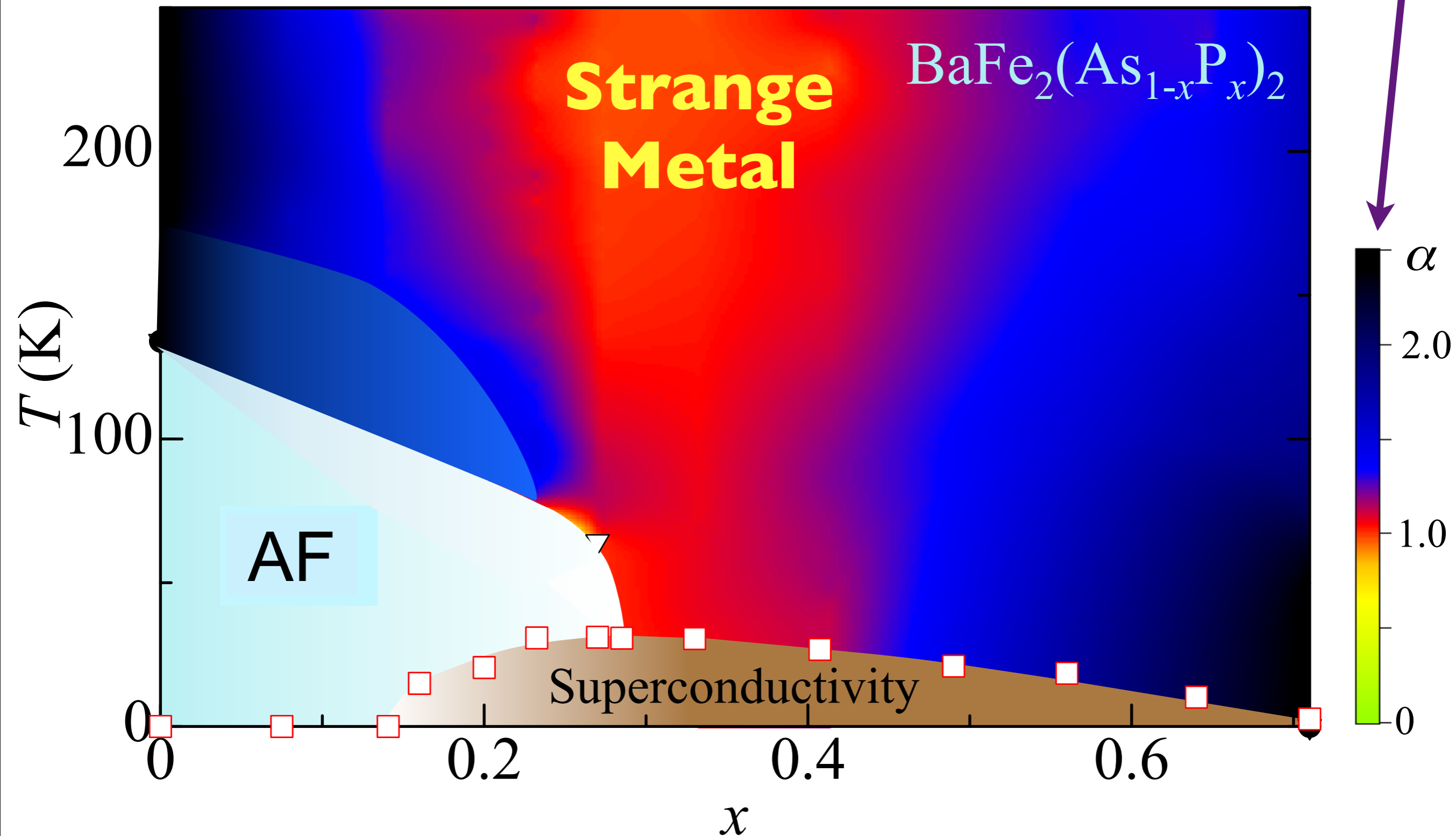
Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

Iron pnictides:

a new class of high temperature superconductors

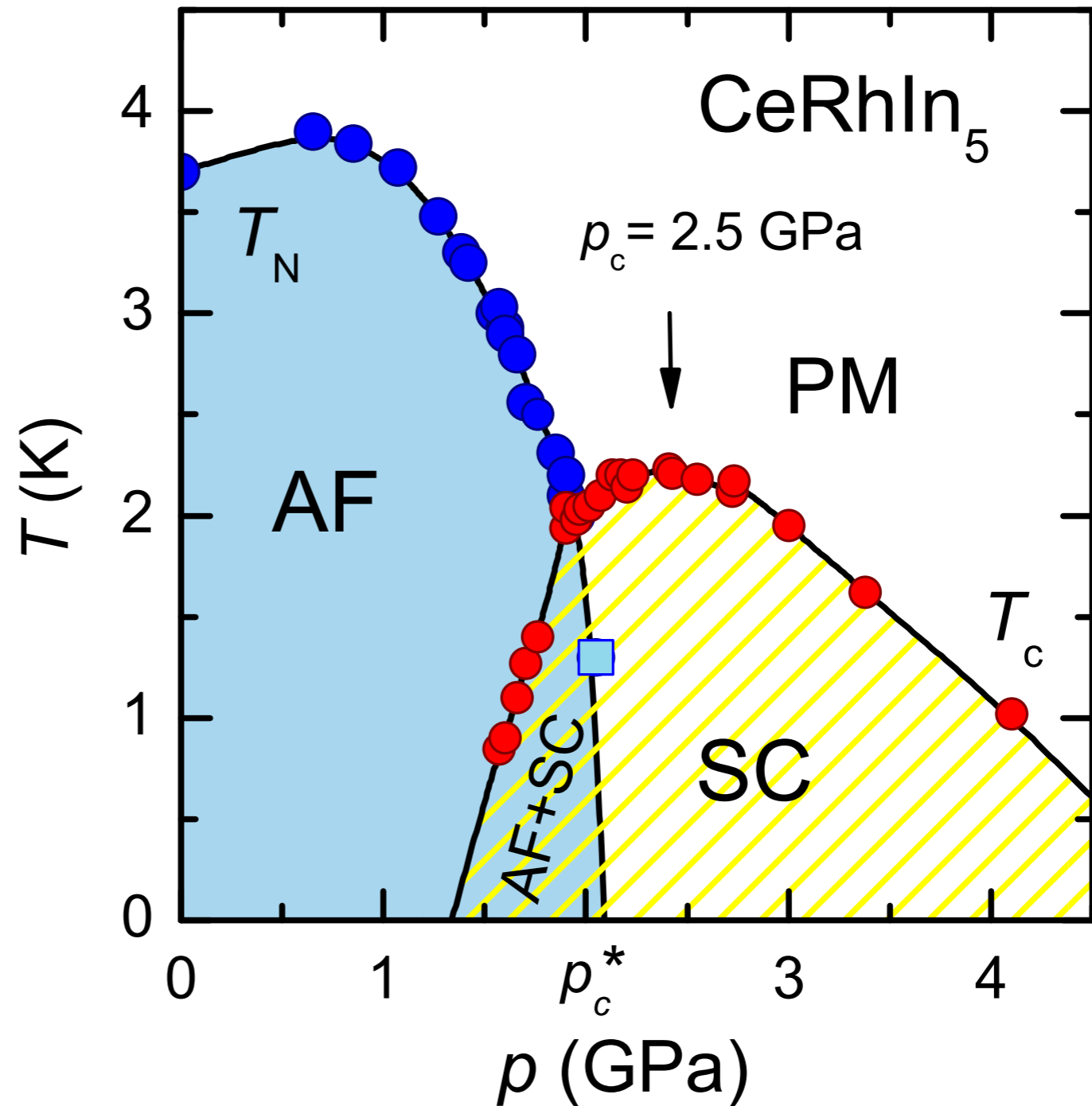
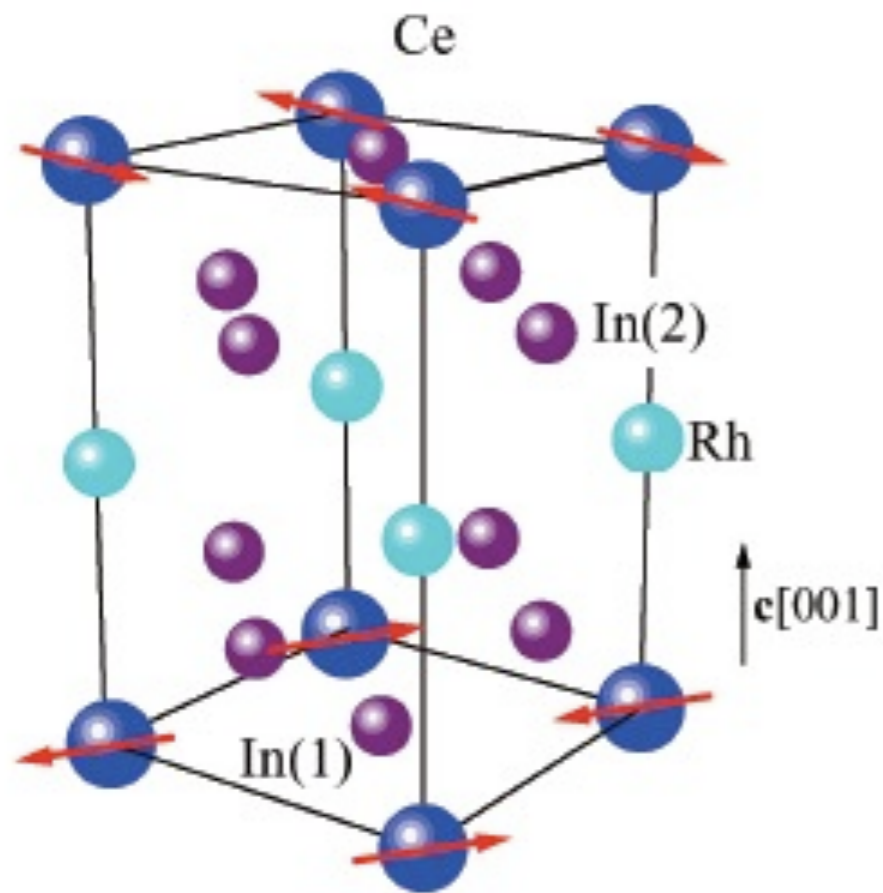


Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

Lower T_c superconductivity in the heavy fermion compounds



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Questions

- *Can quantum fluctuations near the onset of antiferromagnetism induce higher temperature superconductivity ?*

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- *How should such a theory be extended to apply to the hole-doped cuprates ?*
- *What is the physics of the strange metal ?*

Outline

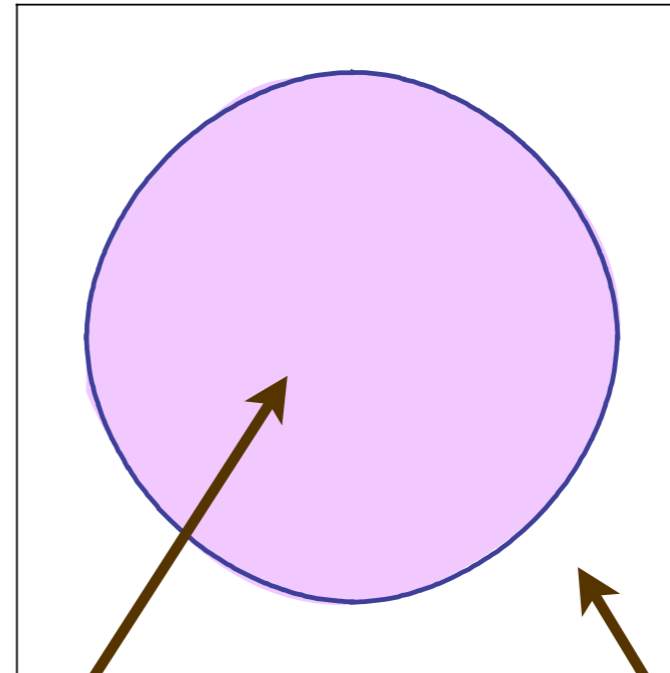
1. Phenomenology of the onset of antiferromagnetism in a metal
2. Quantum field theory of the onset of antiferromagnetism in a metal
3. Quantum Monte Carlo without the sign problem
4. Fractionalization in metals,
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Fermi surface

Metal with “large”
Fermi surface

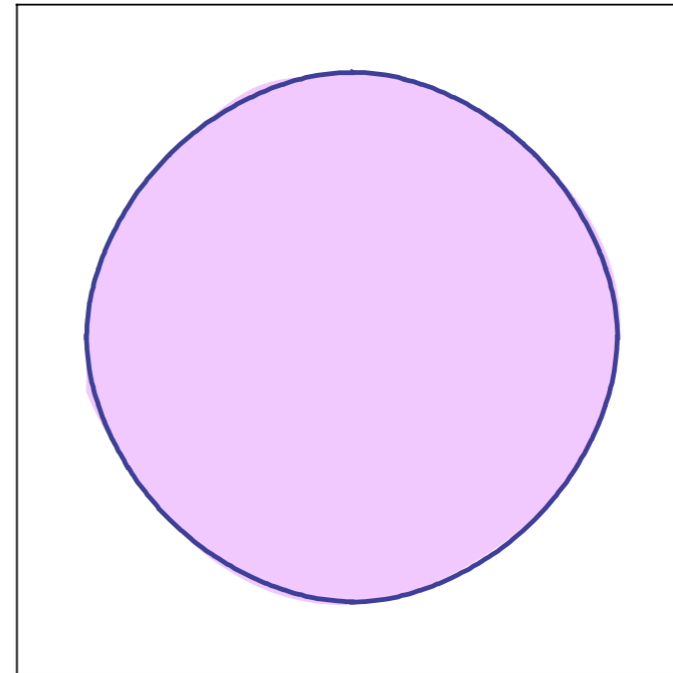


Momenta with
electronic
states empty

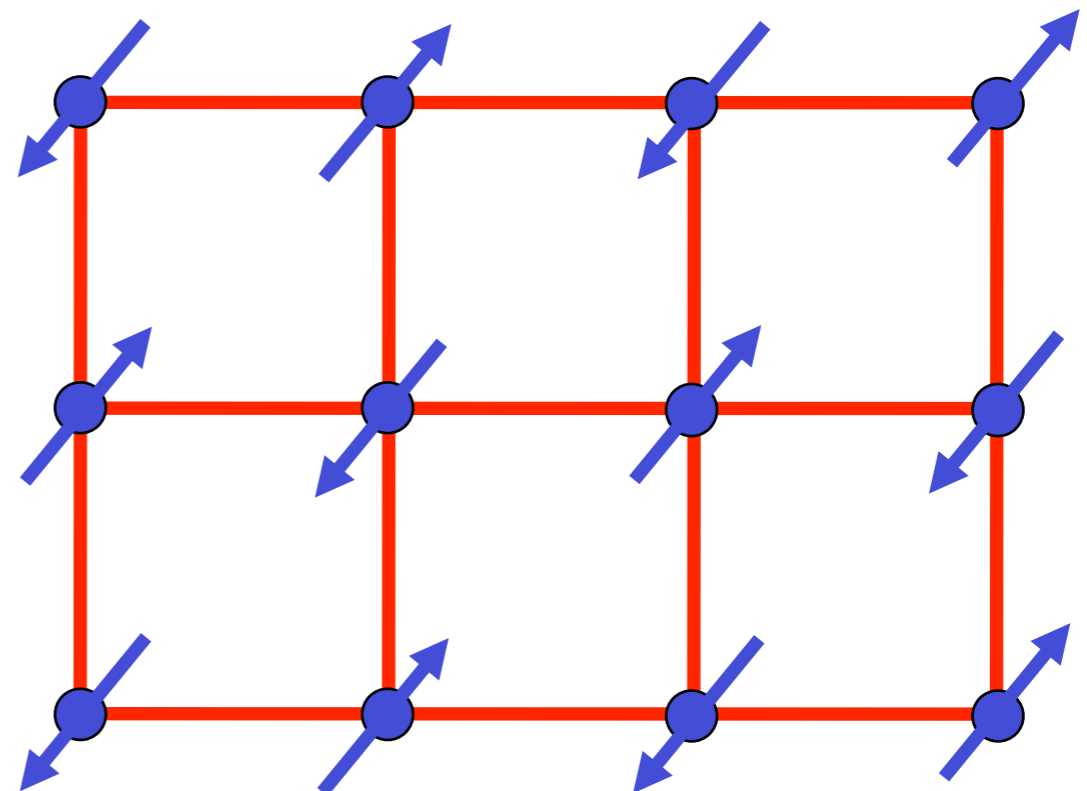
Momenta with
electronic
states
occupied

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



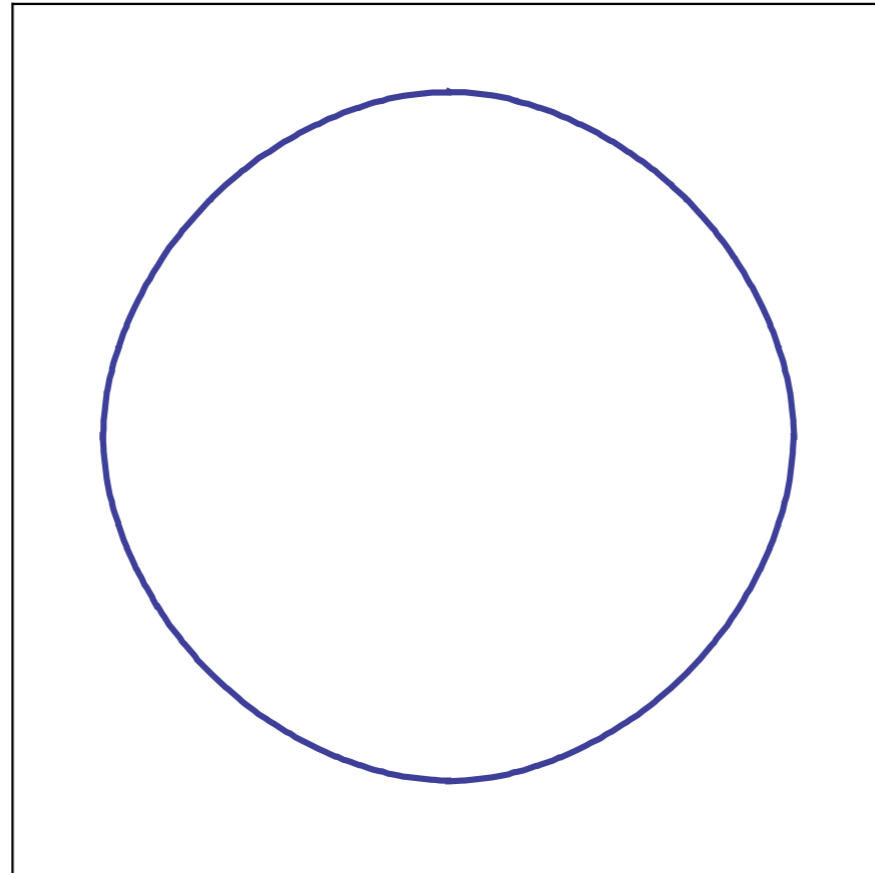
+



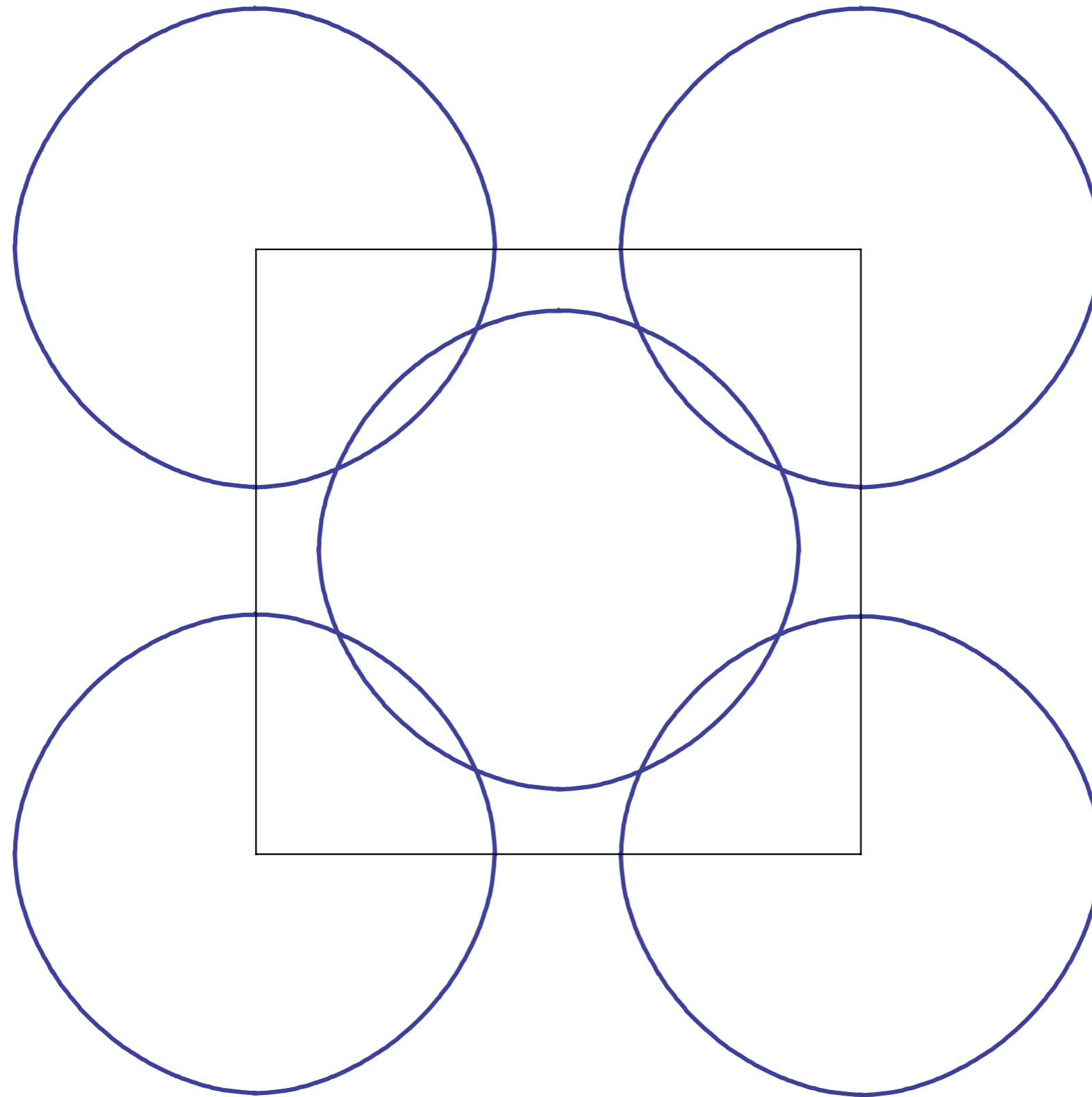
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

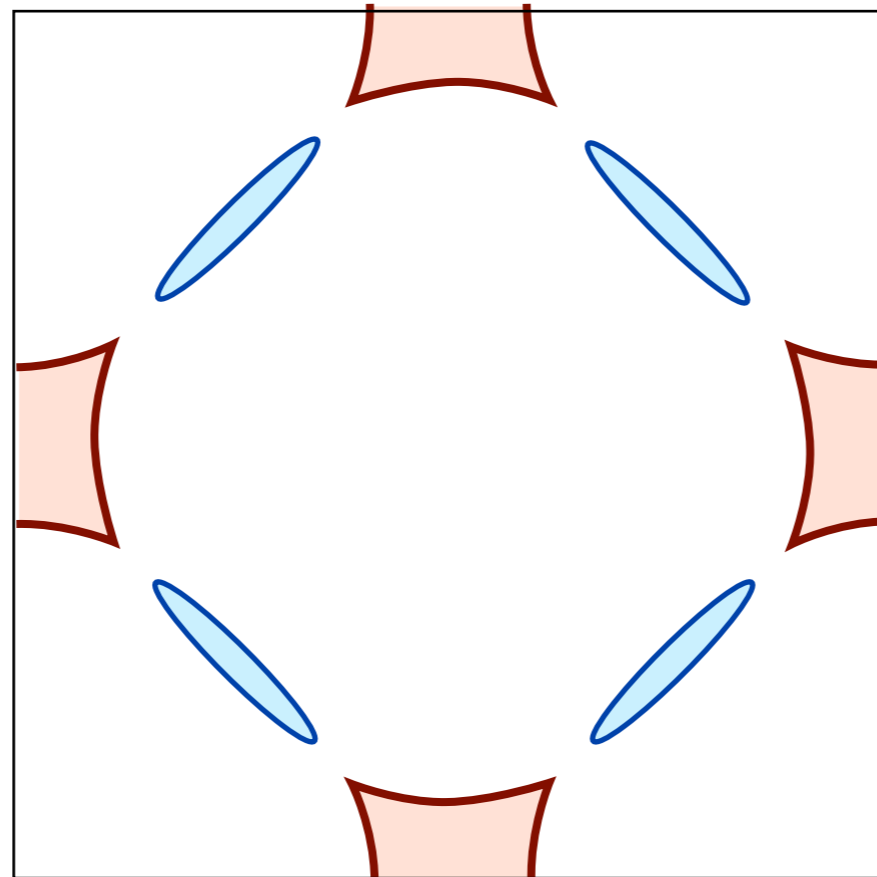
where \mathbf{K} is the ordering wavevector.



Metal with “large” Fermi surface

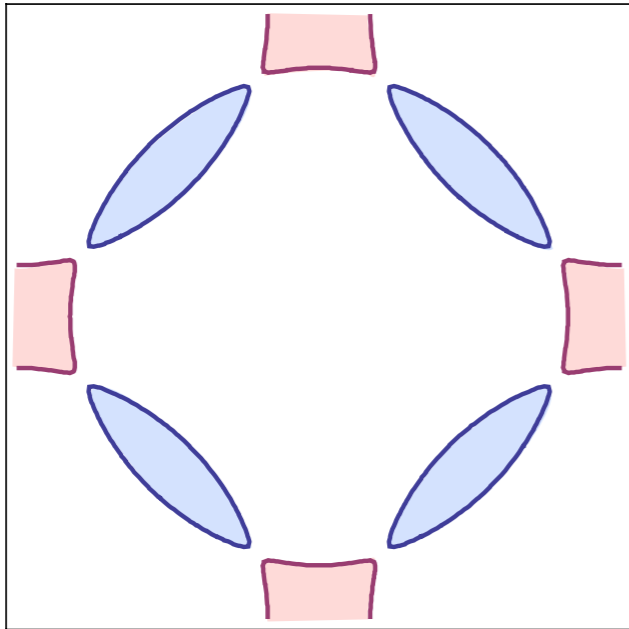


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



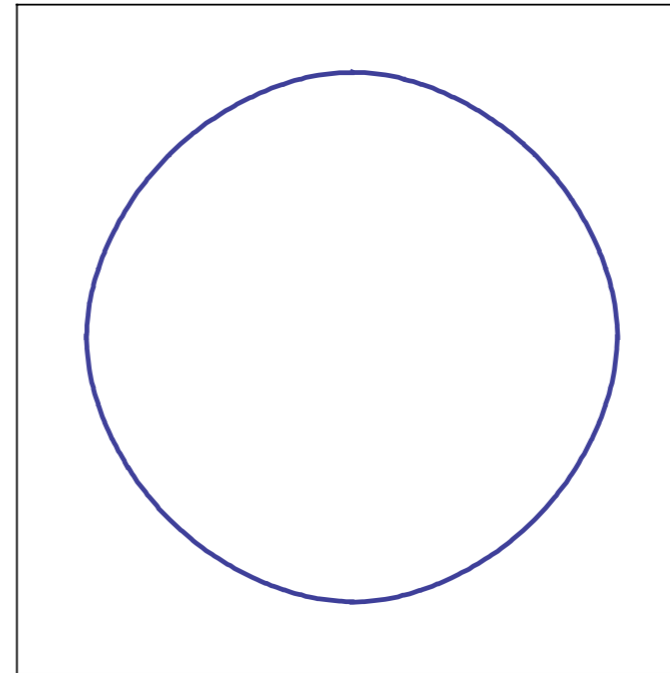
Electron and hole pockets in
antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



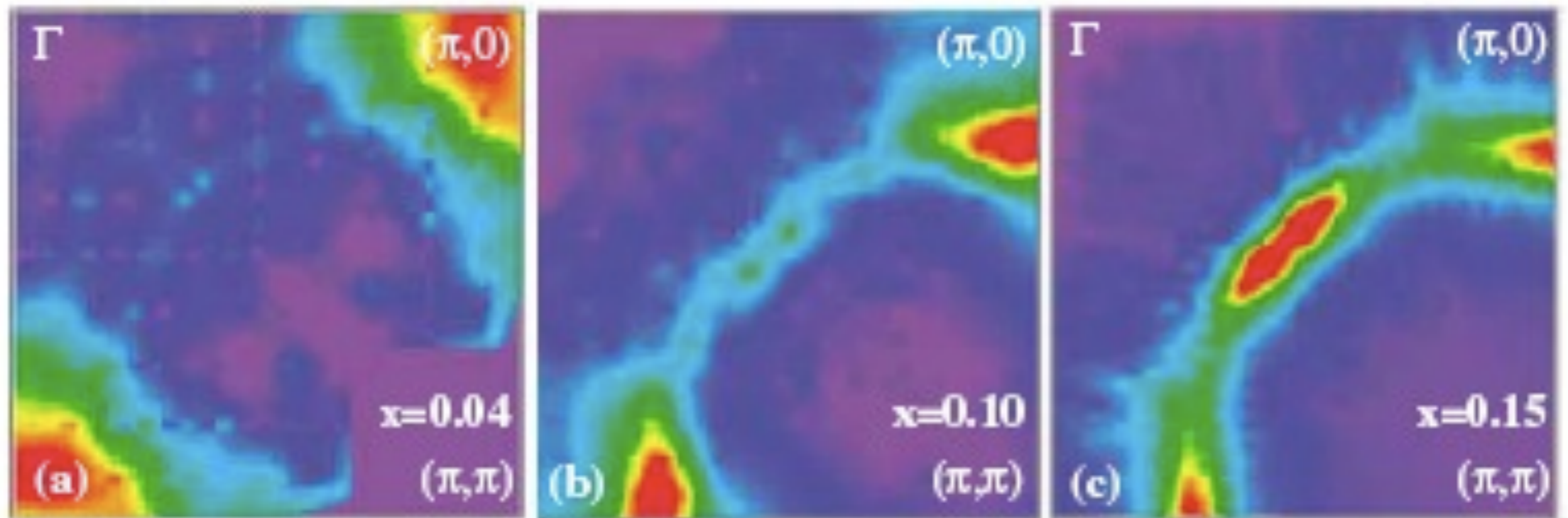
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

← Increasing interaction

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Photoemission in $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$

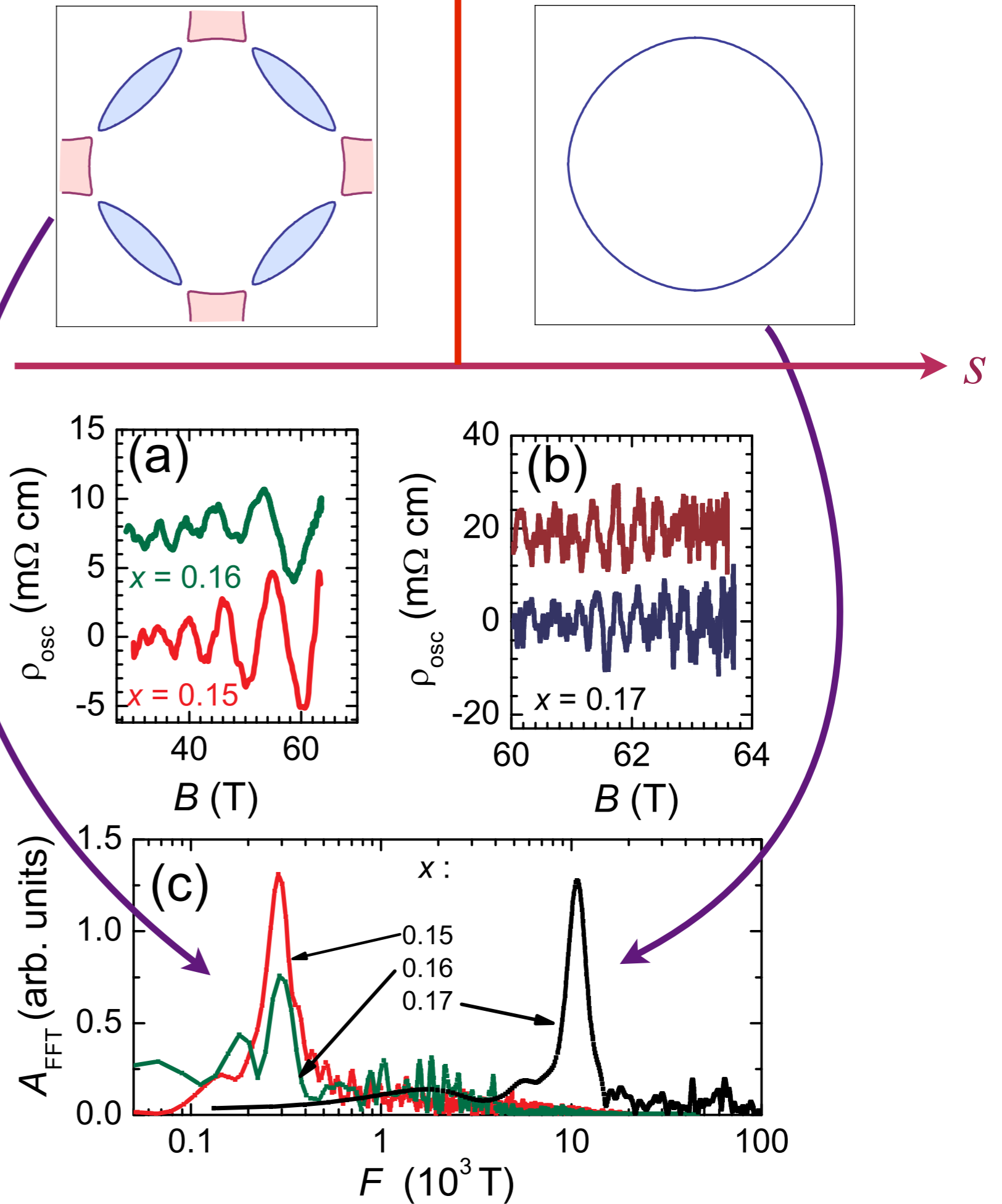


N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

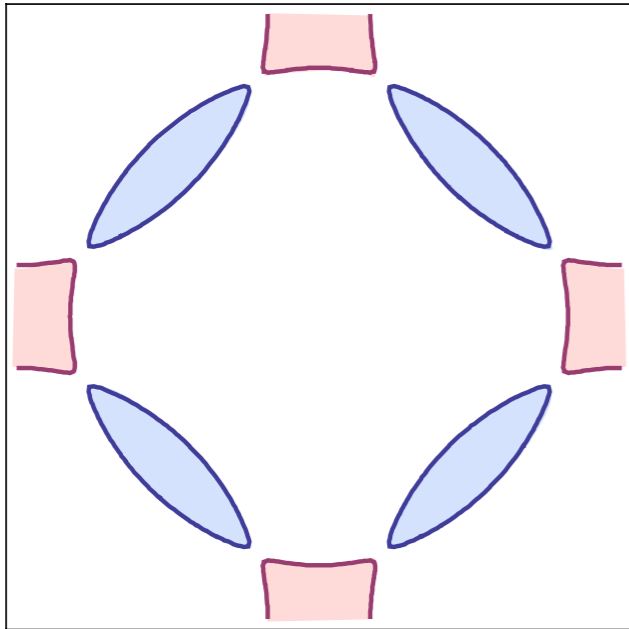
Quantum oscillations



T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

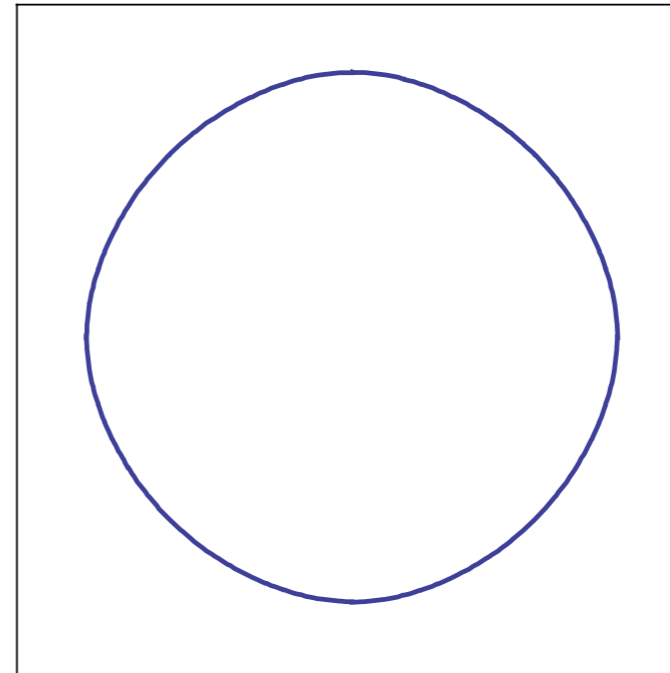


Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



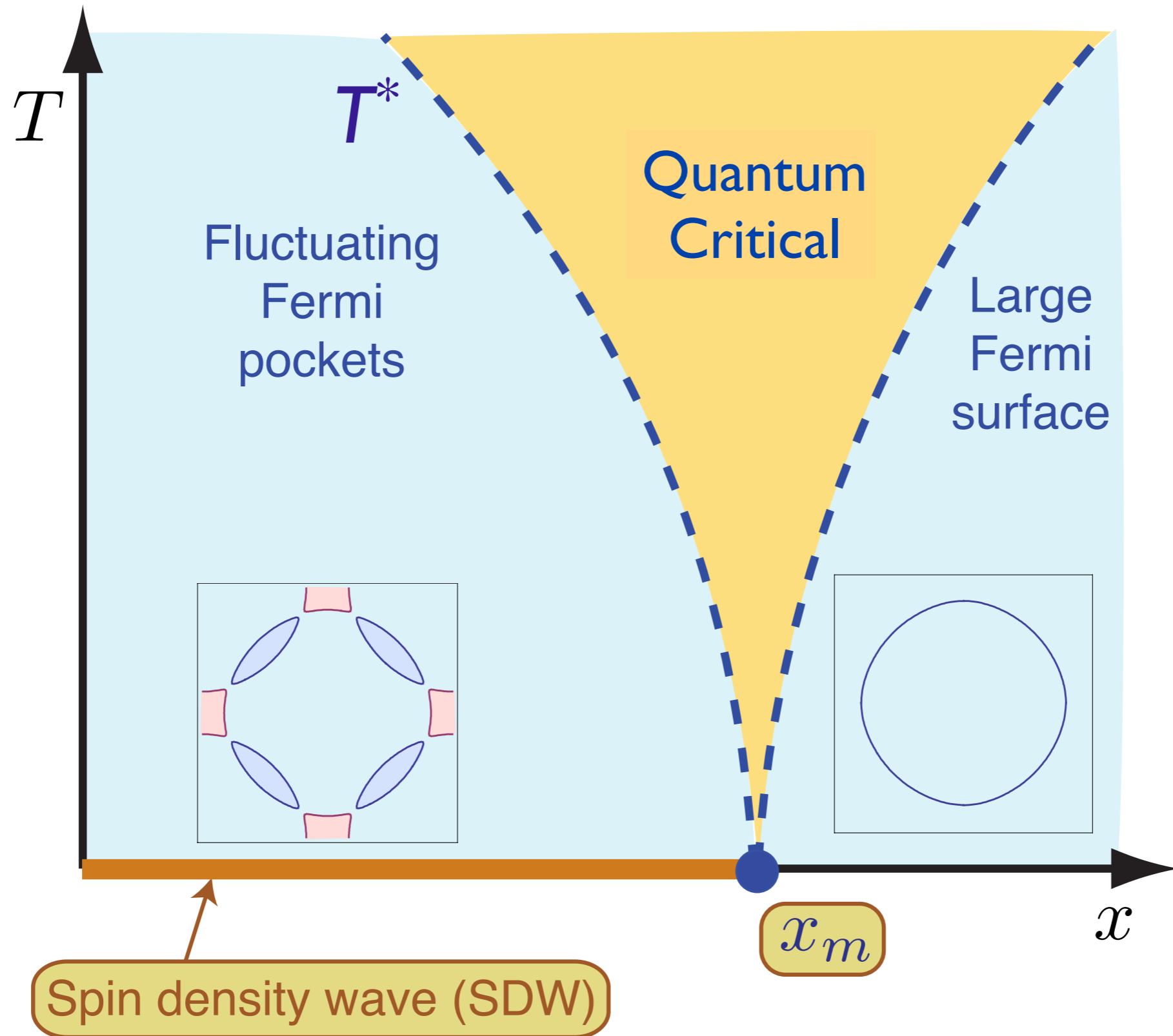
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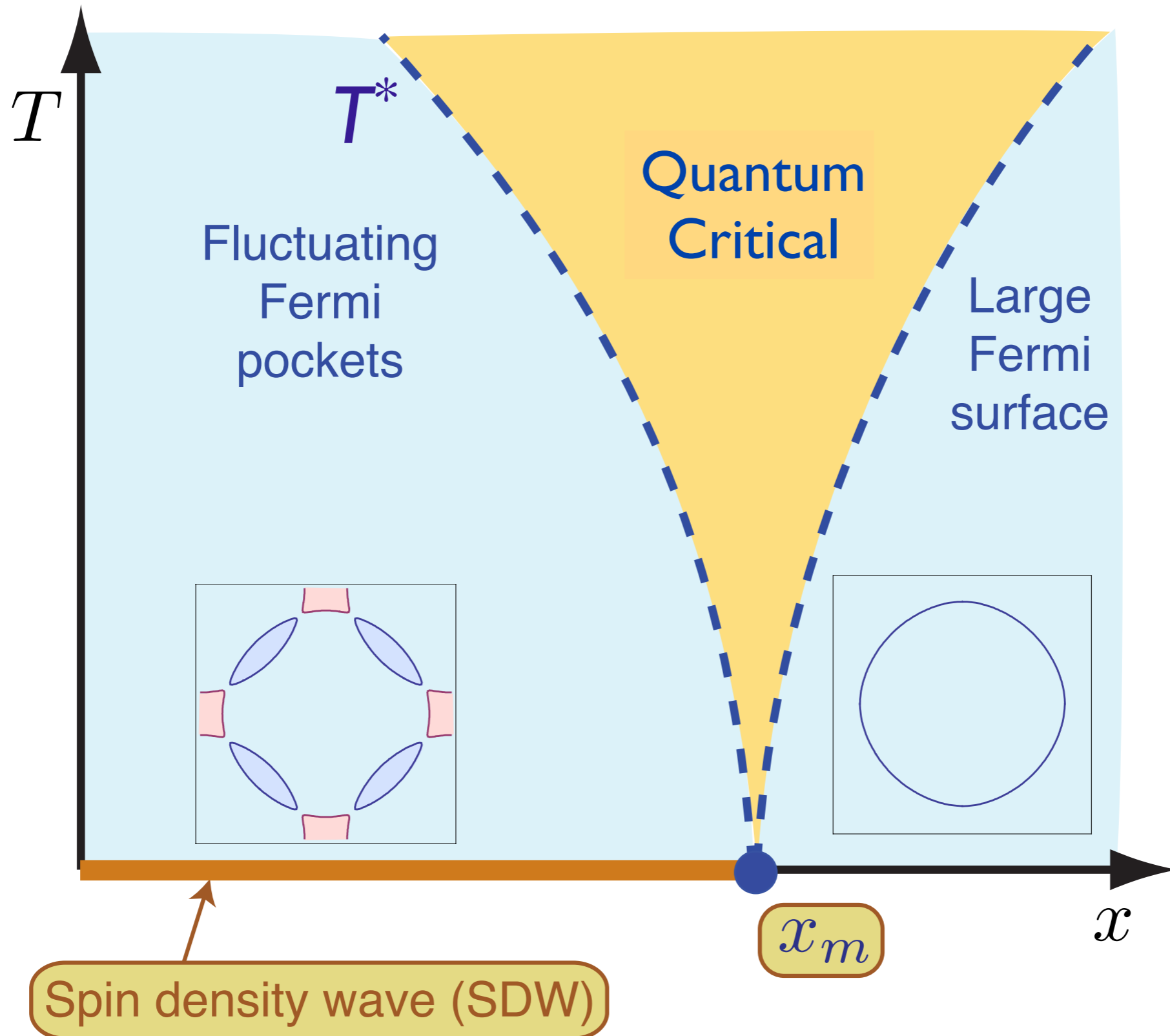
S

Theory of quantum criticality in the cuprates



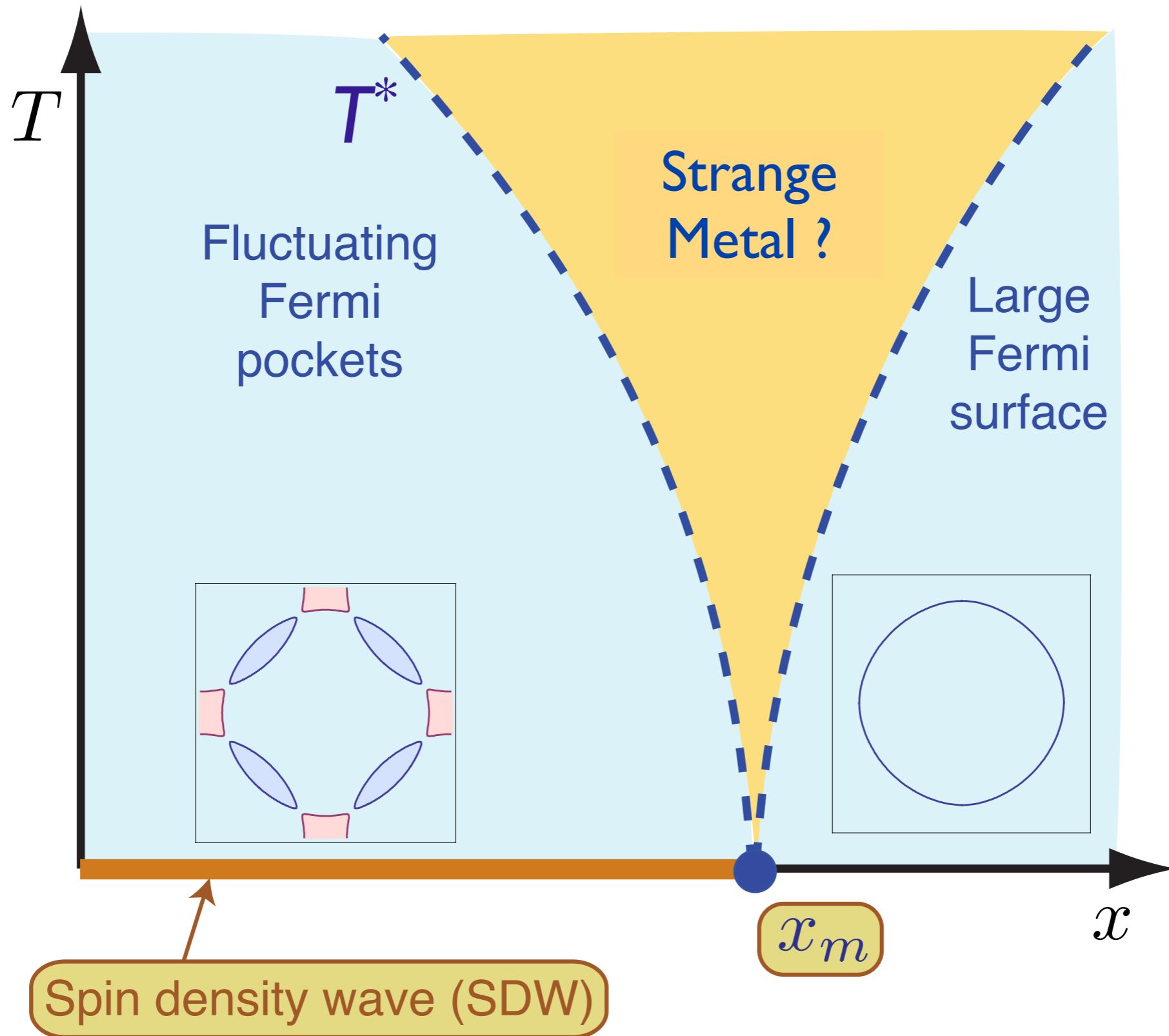
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



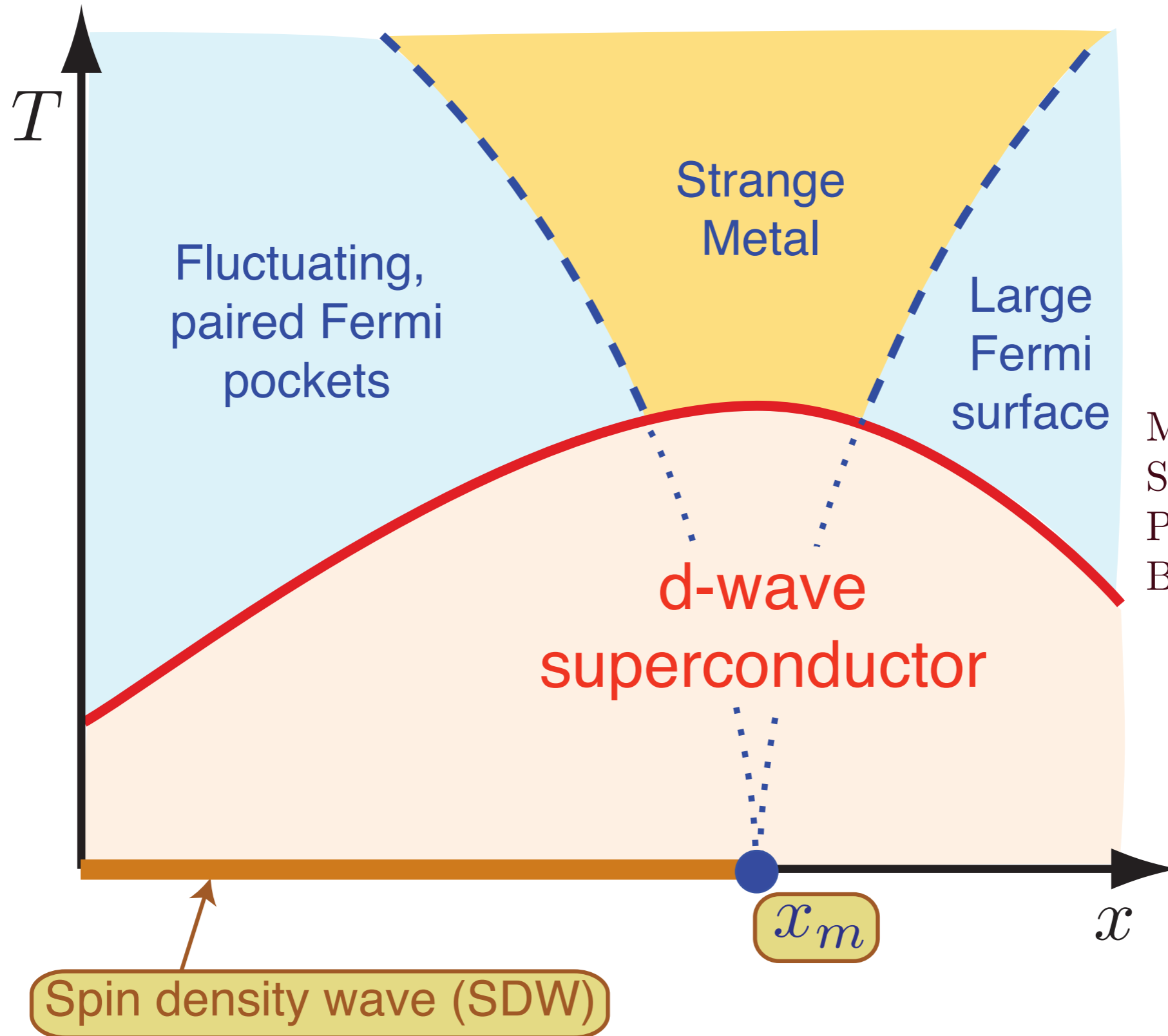
Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

Theory of quantum criticality in the cuprates



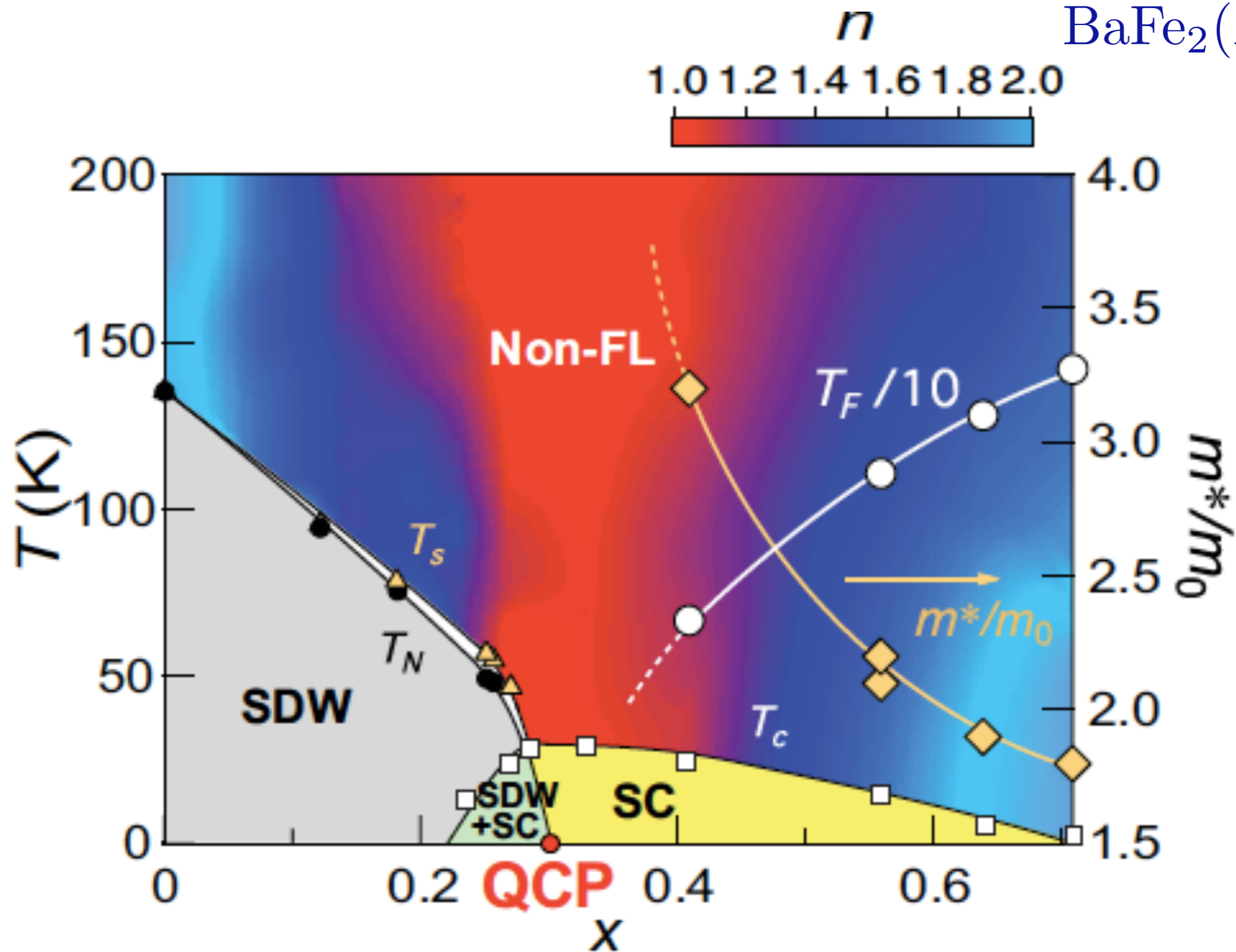
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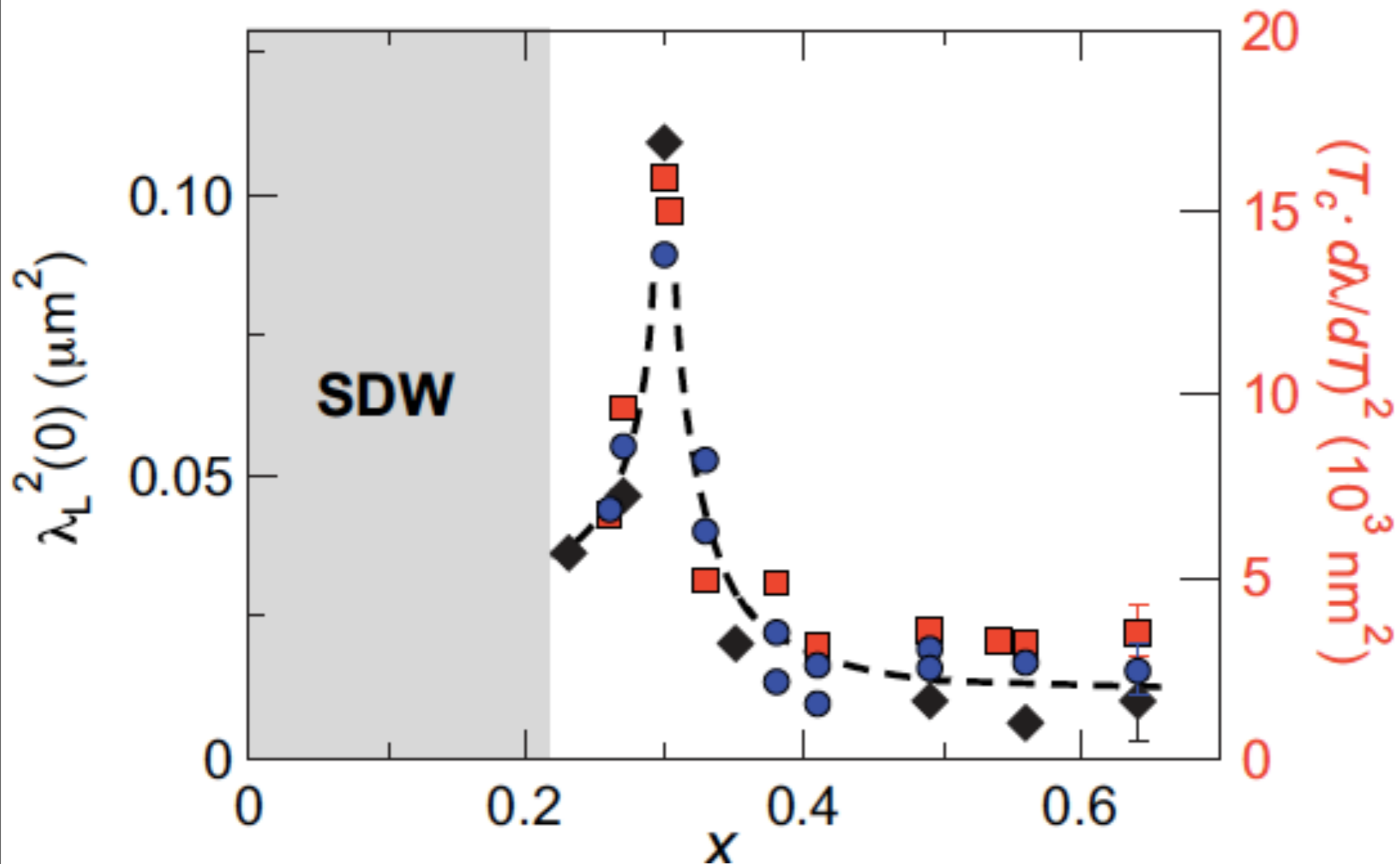


M. A. Metlitski and
S. Sachdev,
Physical Review
B **82**, 075128 (2010)

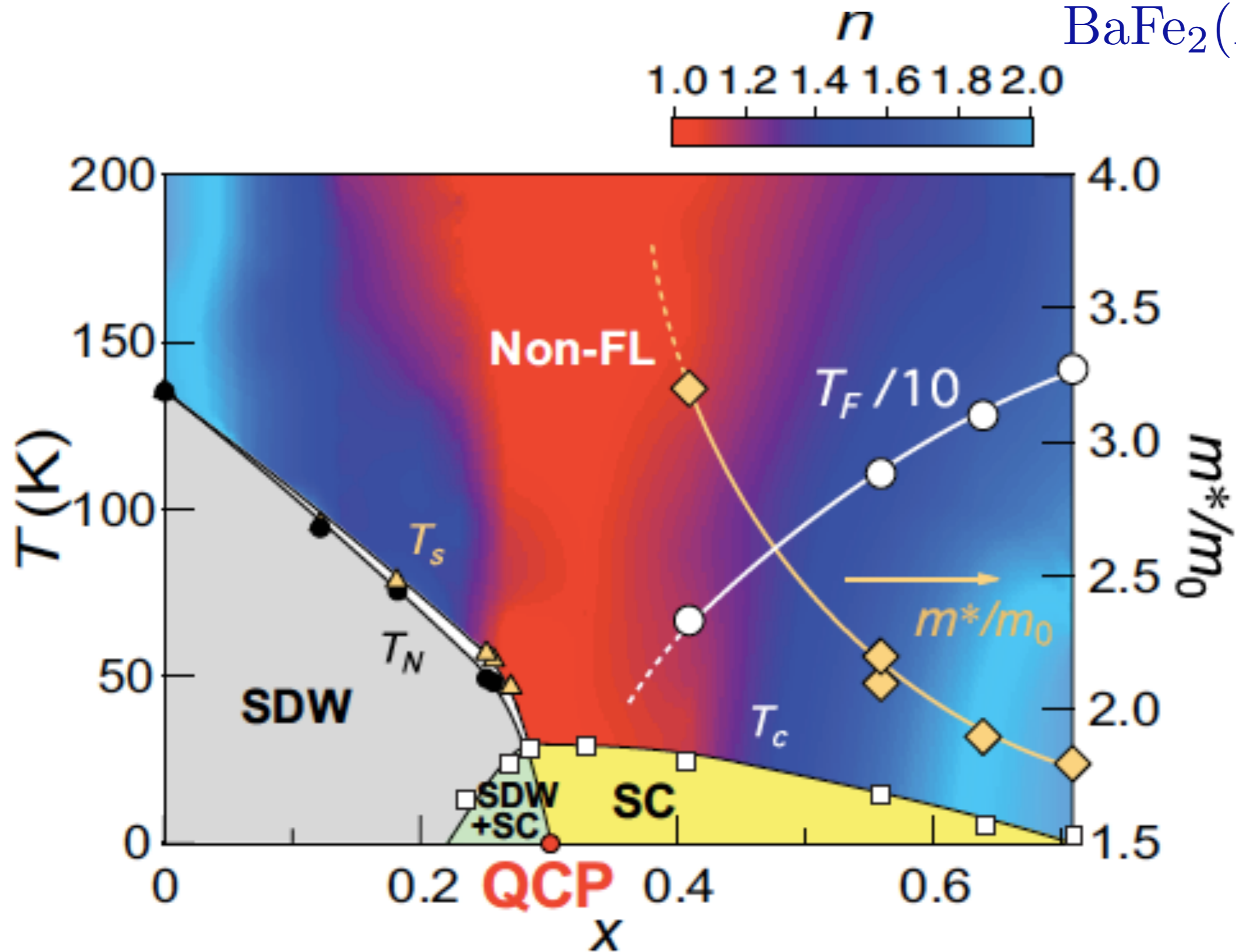
SDW quantum critical point is unstable to *d*-wave superconductivity
This instability is stronger than that in the BCS theory



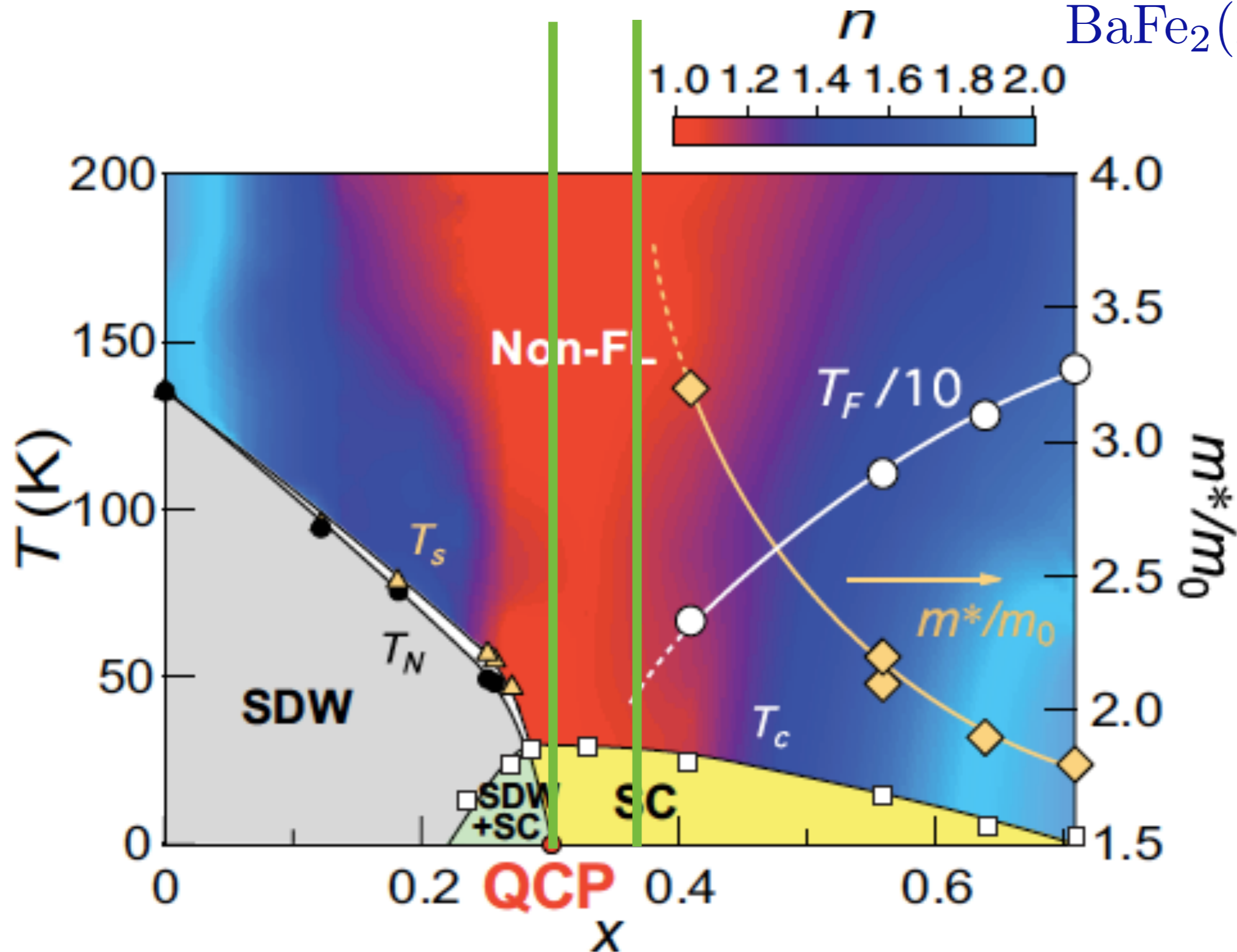
K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M.A. Tanatar, H. Kitano, N. Salovich, R.W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, Science in press



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M.A. Tanatar, H. Kitano, N. Salovich, R.W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, Science in press

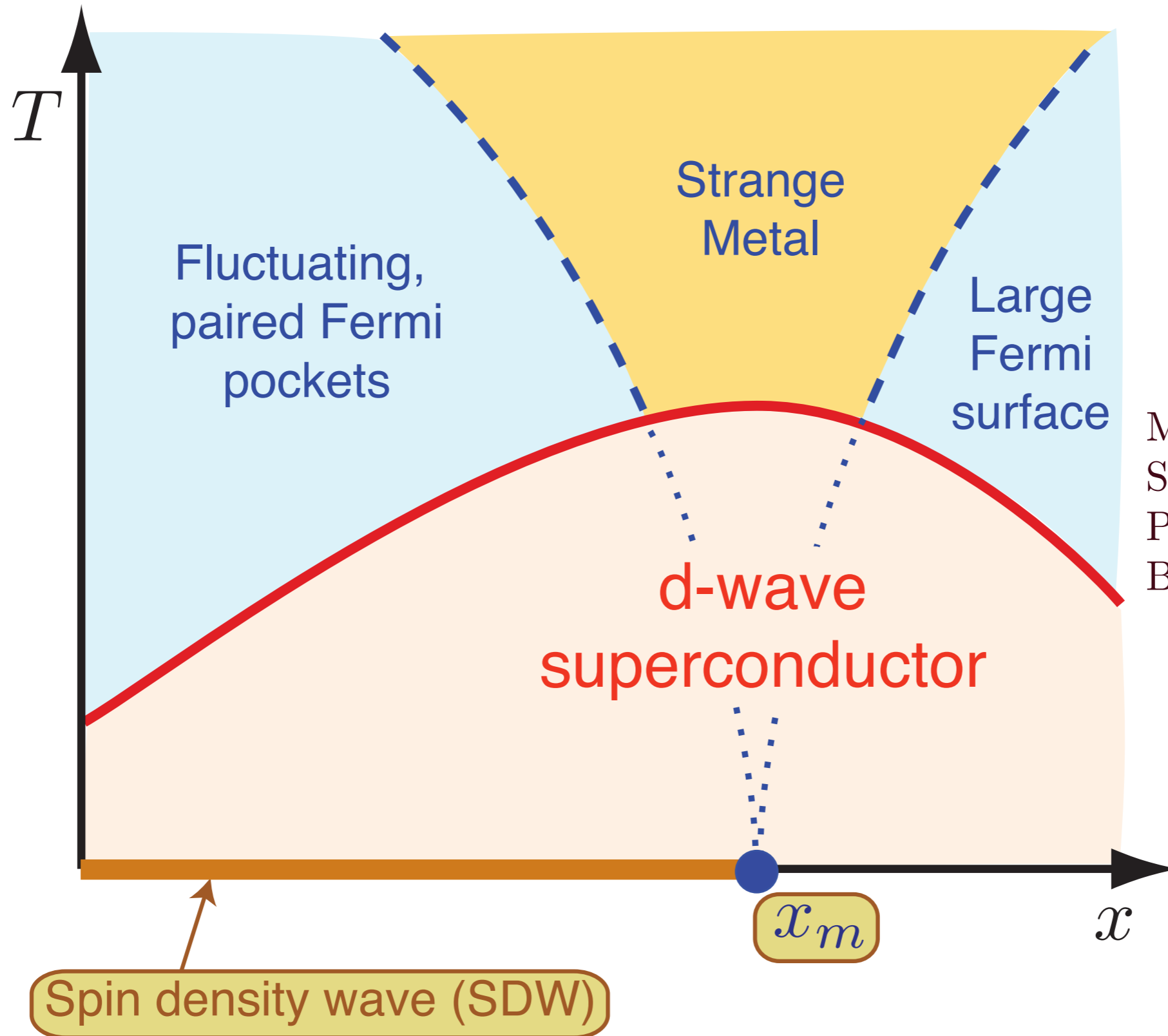


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Notice shift between the position of the QCP in the superconductor, and the divergence in effective mass in the metal measured at high magnetic fields

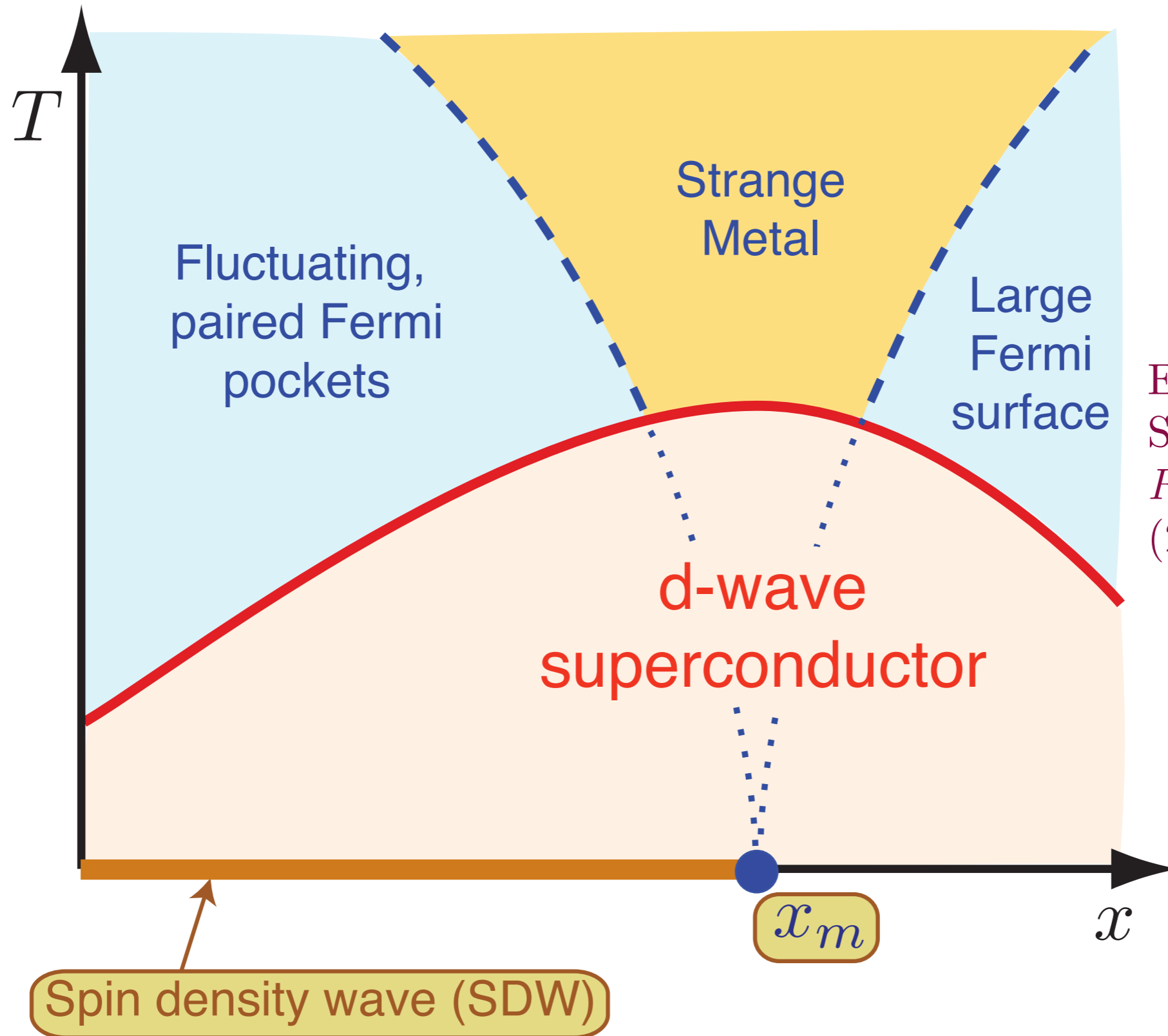
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M. A. Metlitski and
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Physical Review
B **82**, 075128 (2010)

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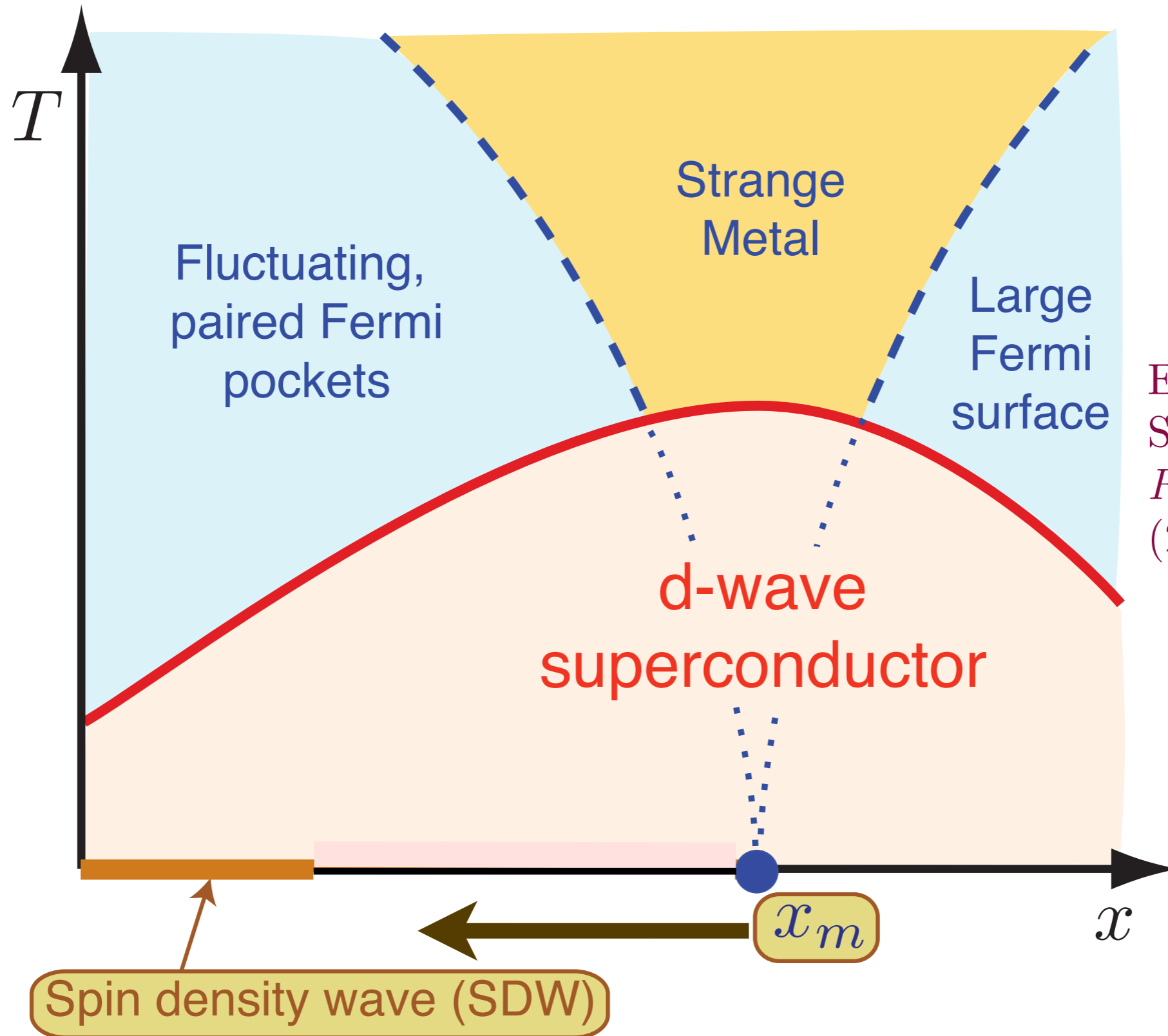
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

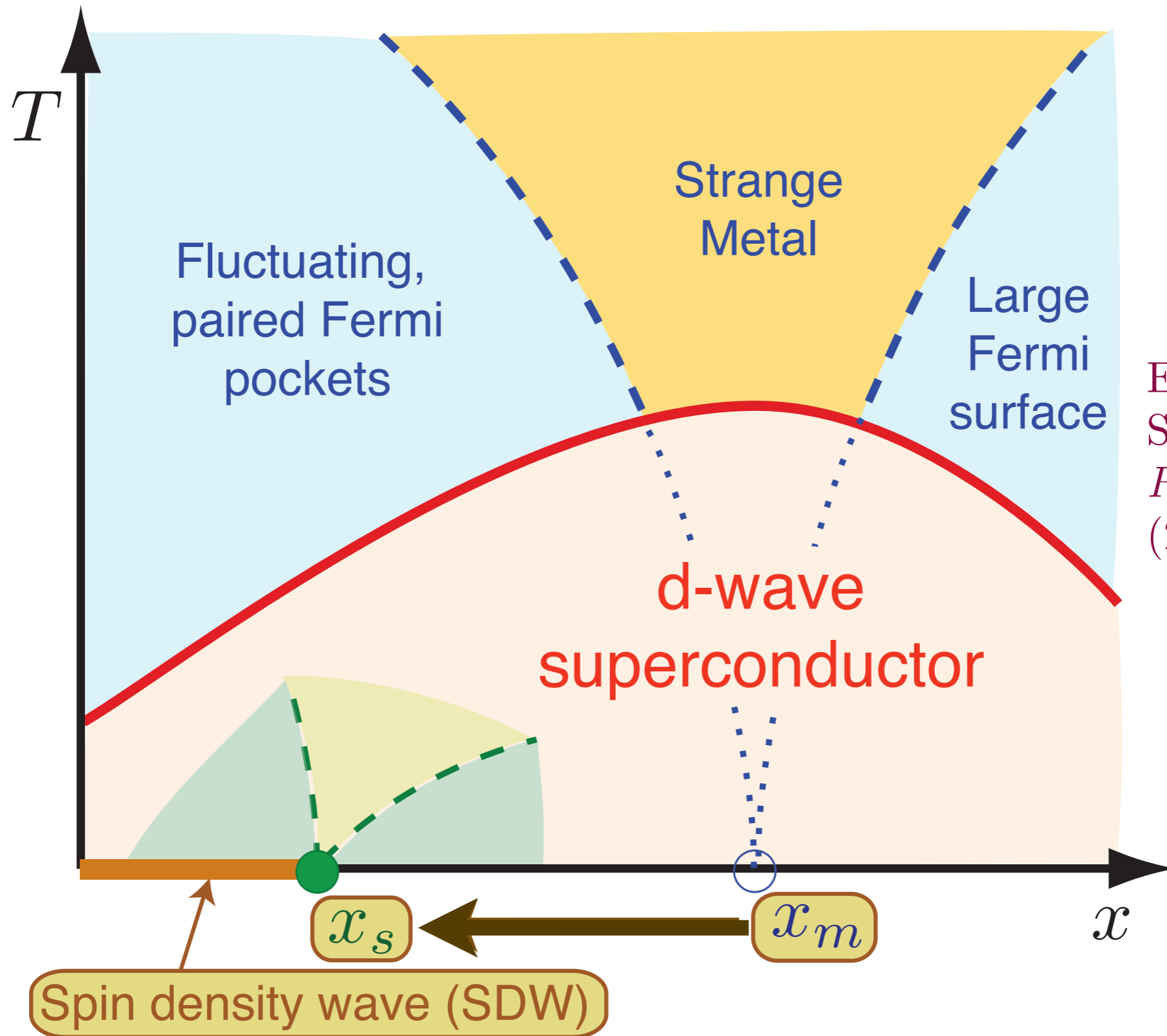
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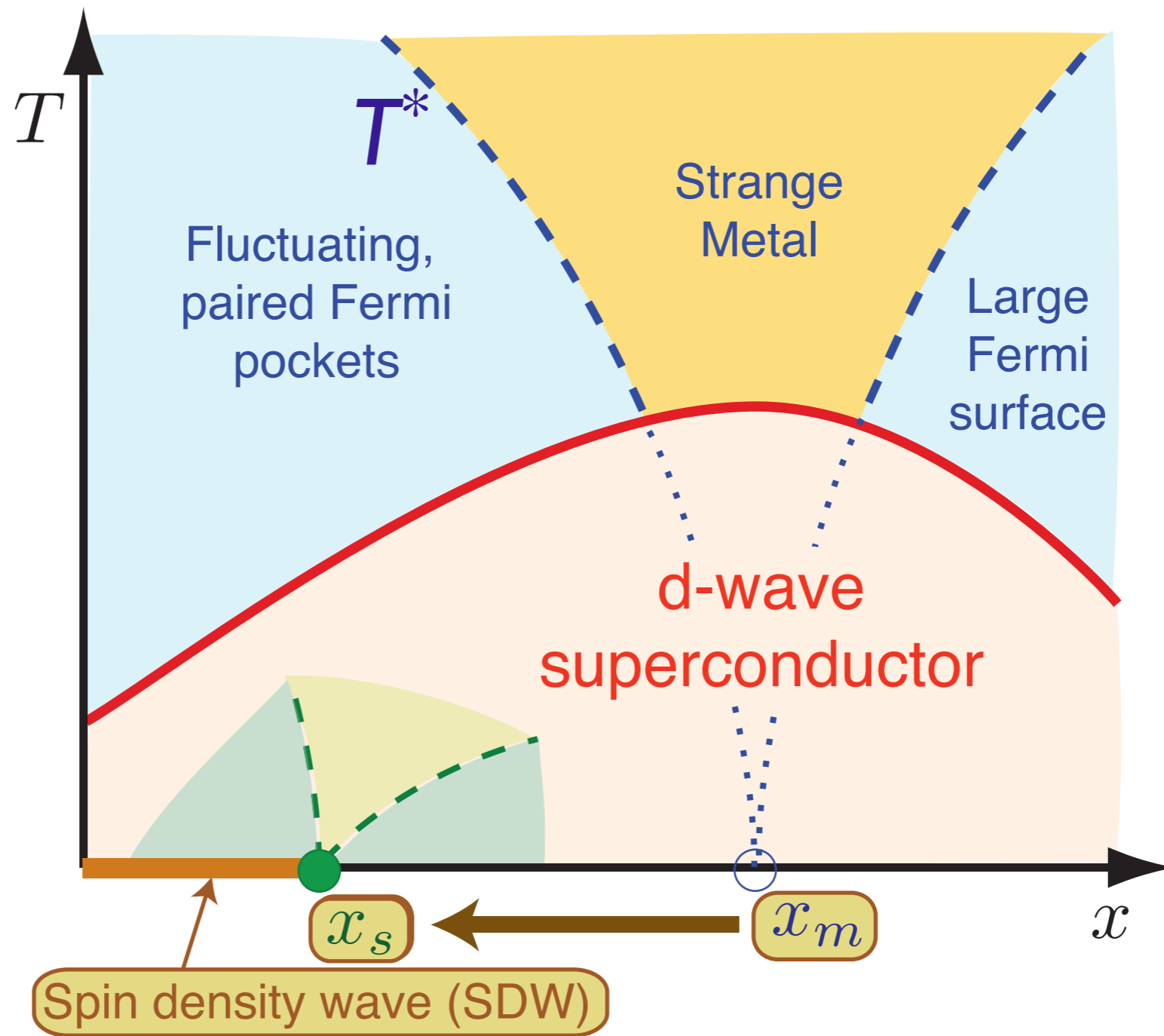
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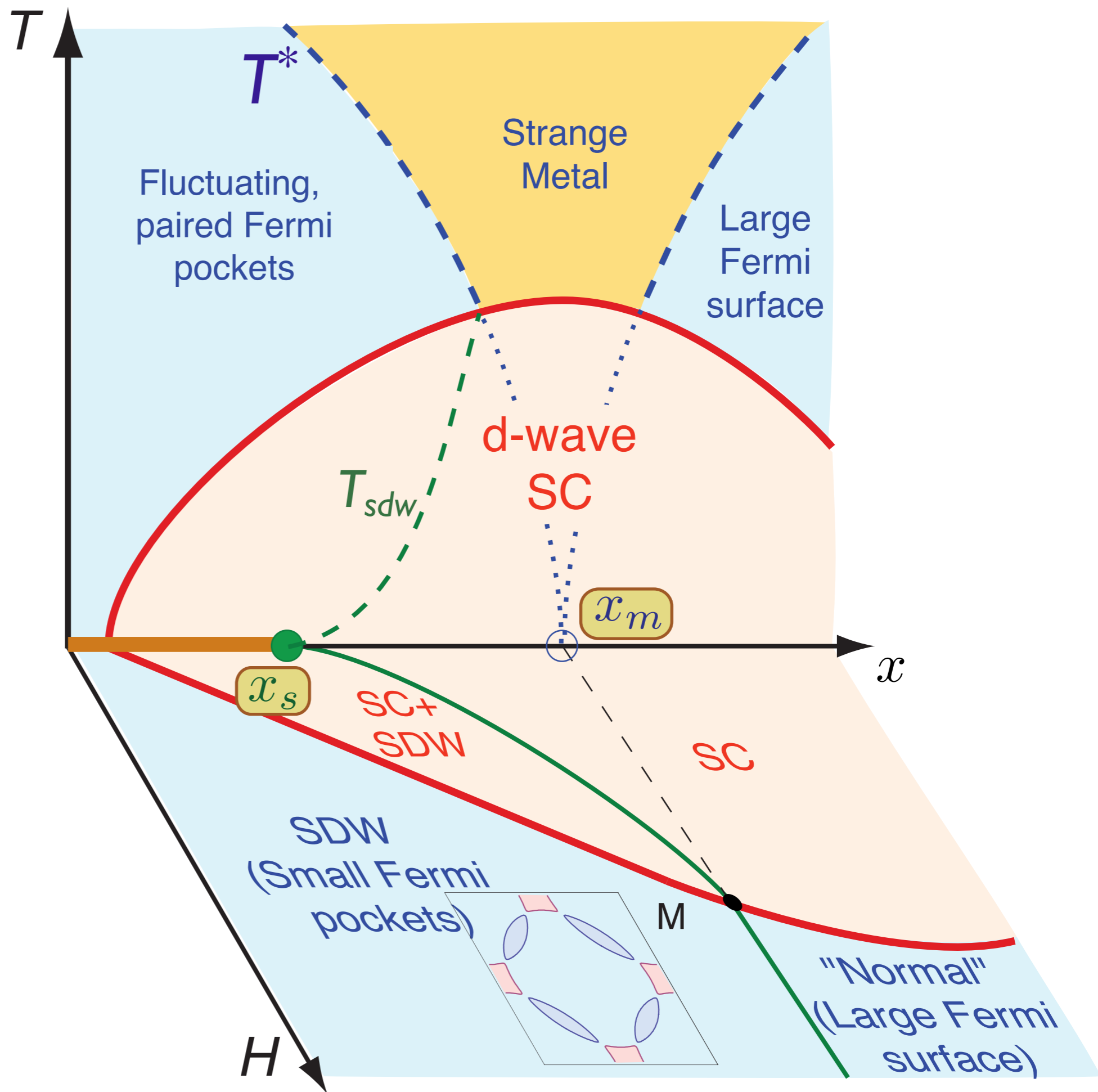


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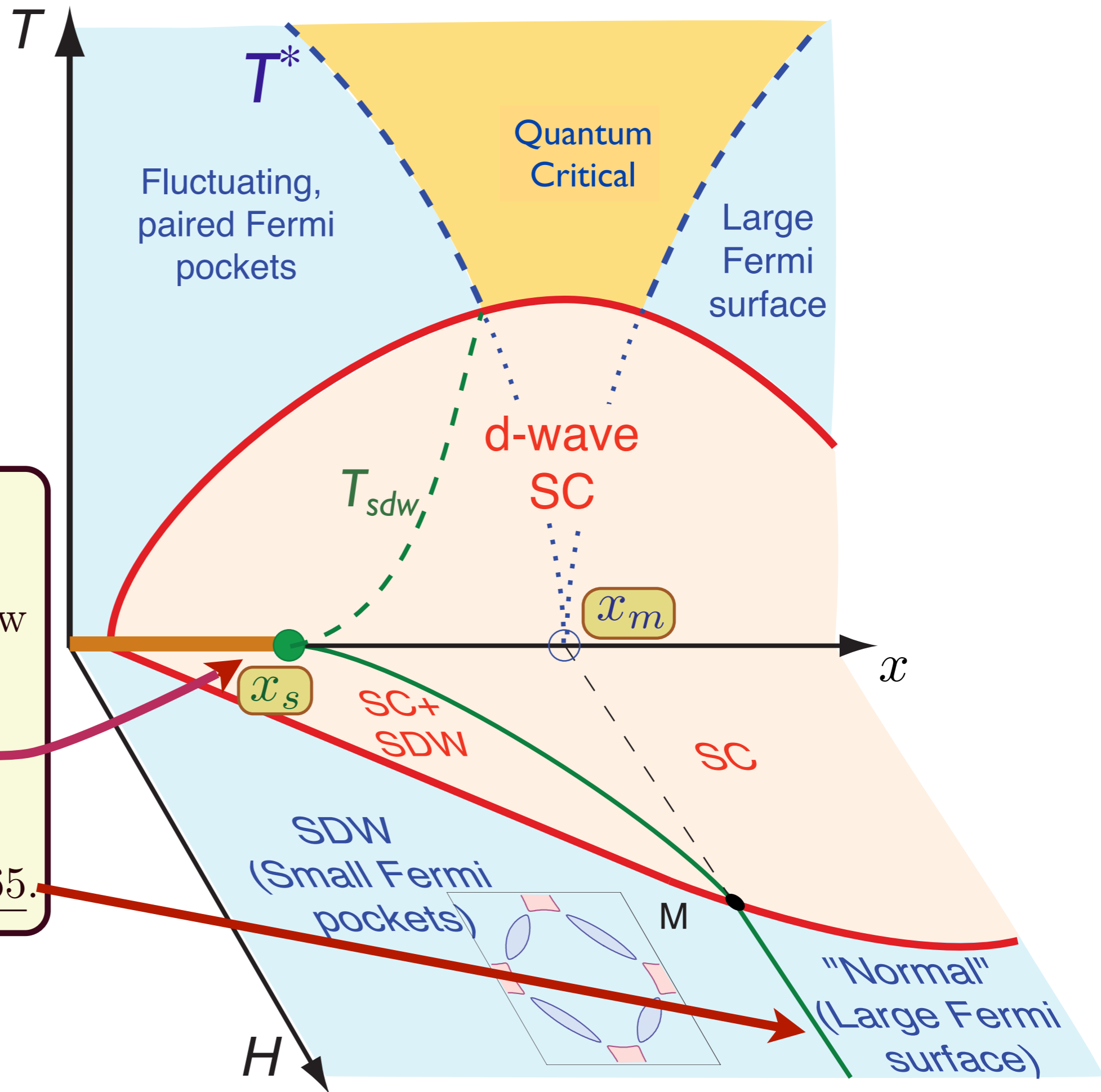


E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).



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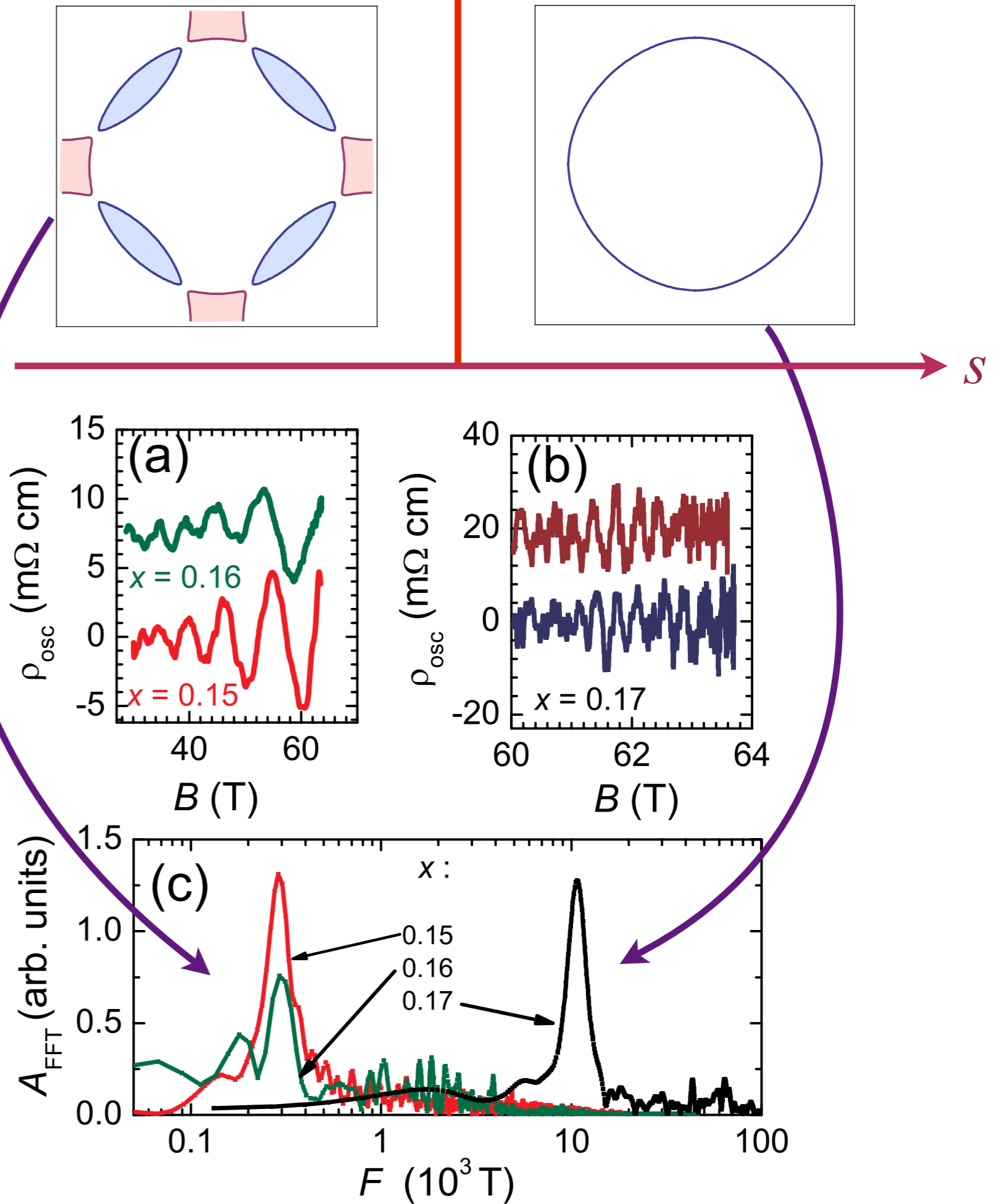
Neutron scattering experiments on $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.

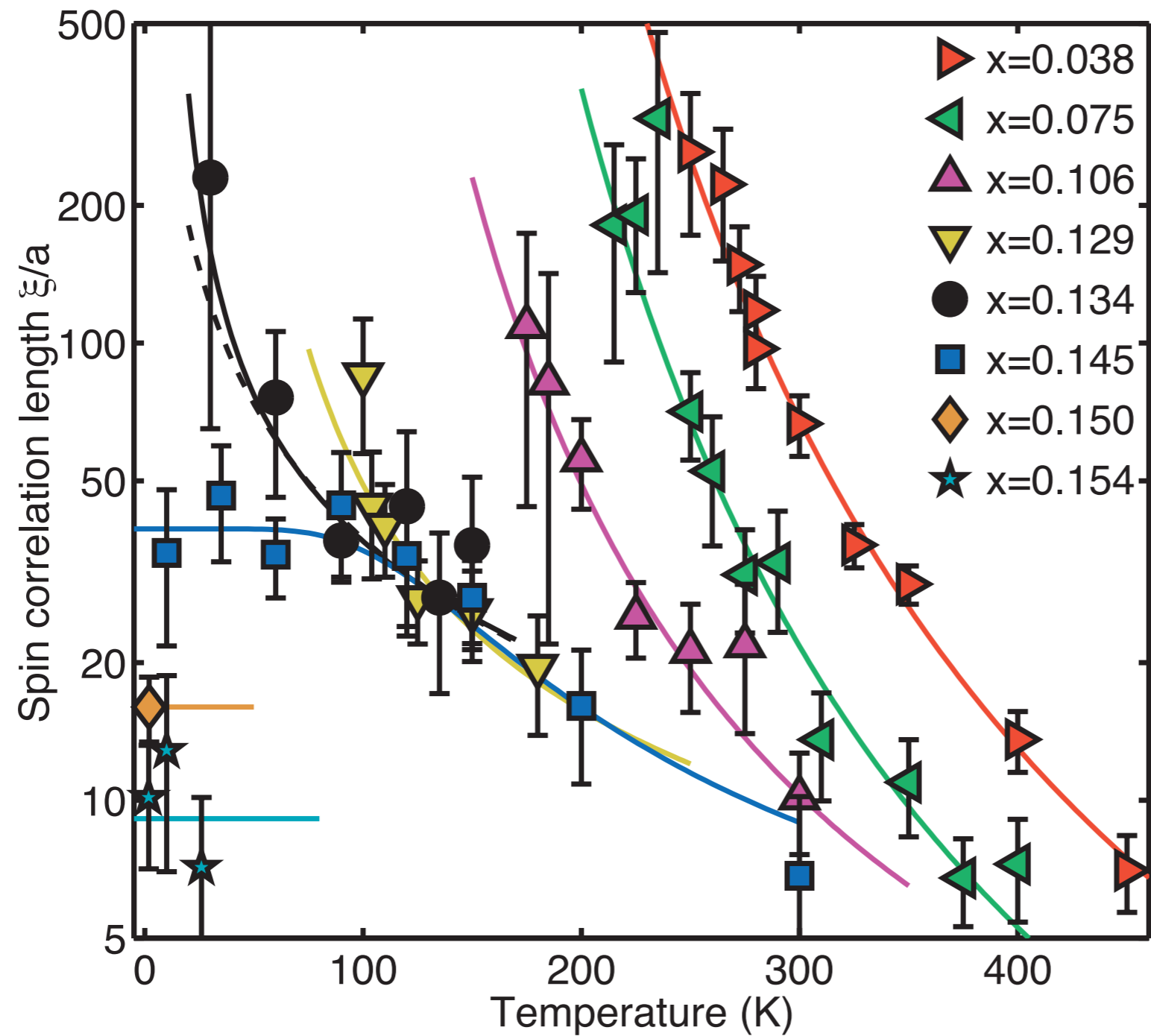


Quantum oscillations



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M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

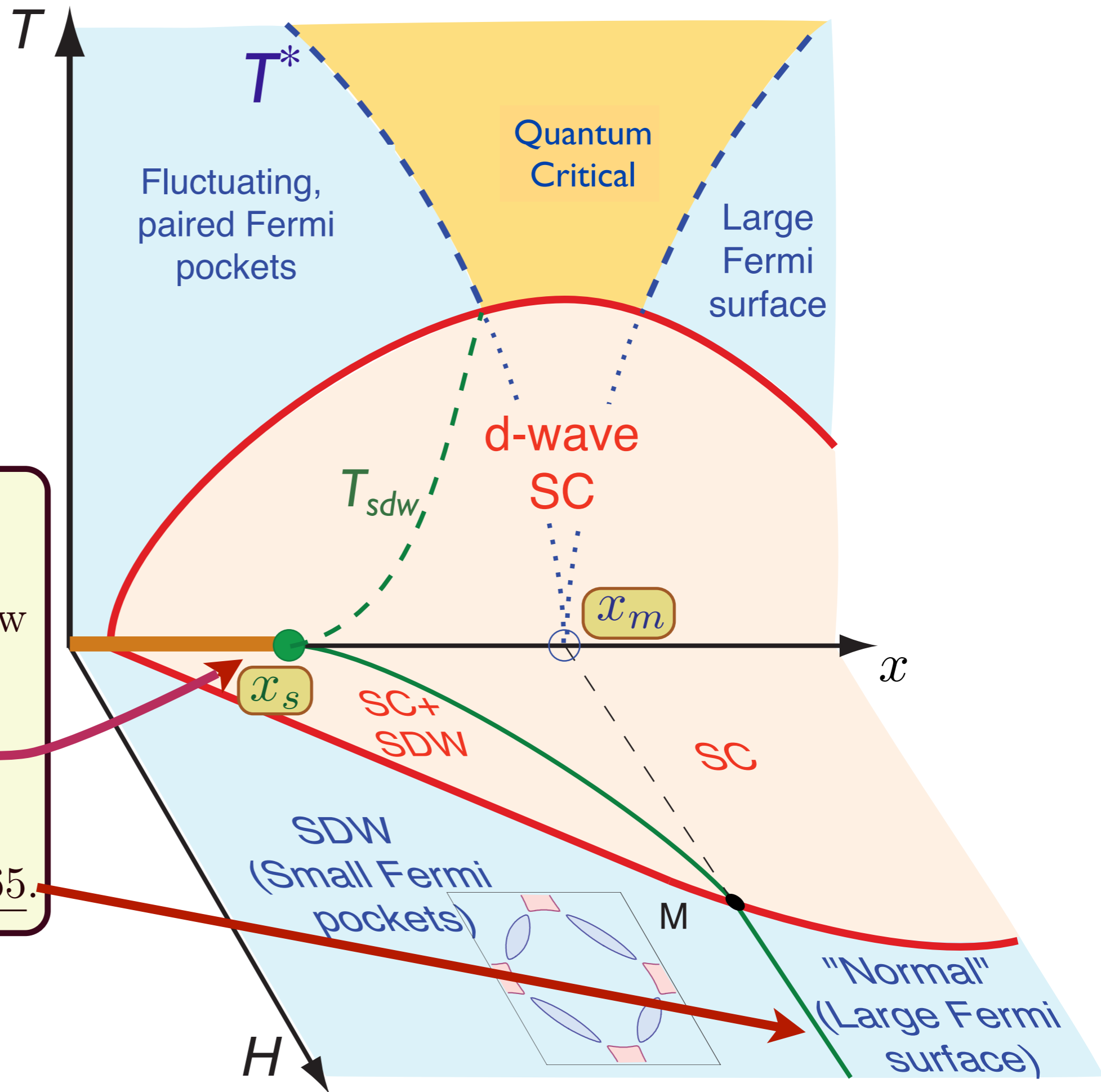




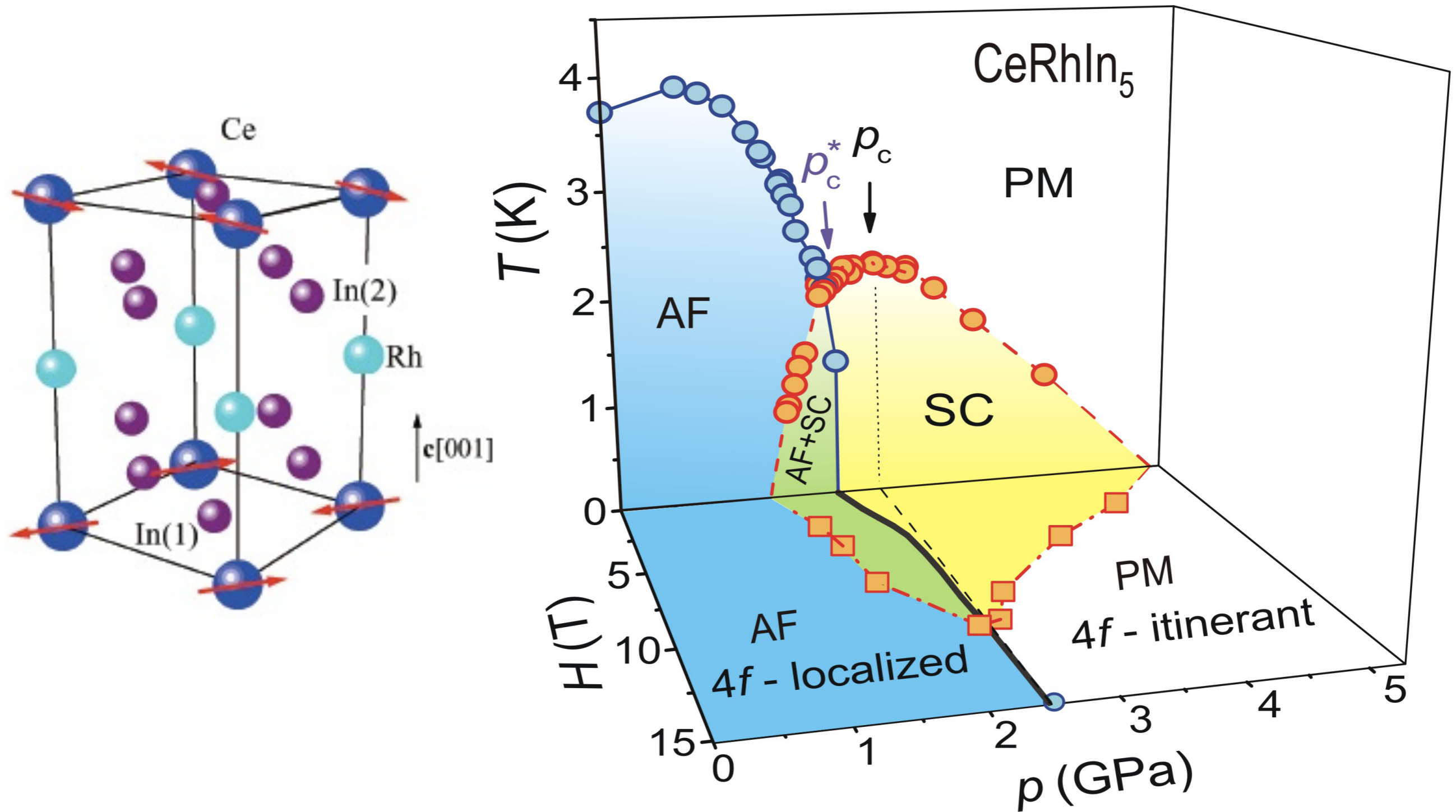
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).

E. Demler, S. Sachdev
and Y. Zhang, *Phys. Rev. Lett.* **87**,
067202 (2001).

Neutron scattering experiments on $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.



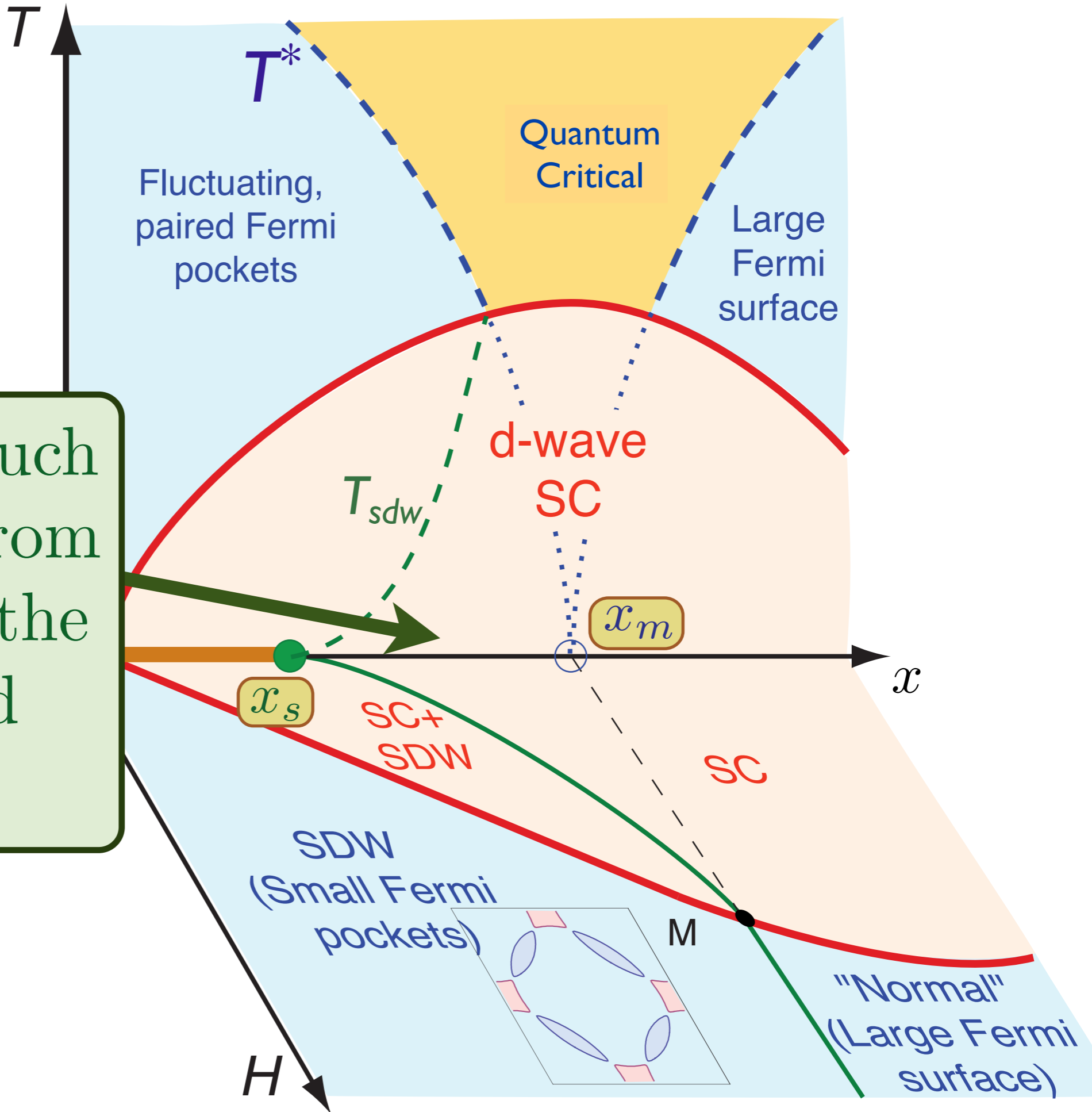
Similar phase diagram for CeRhIn₅



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.

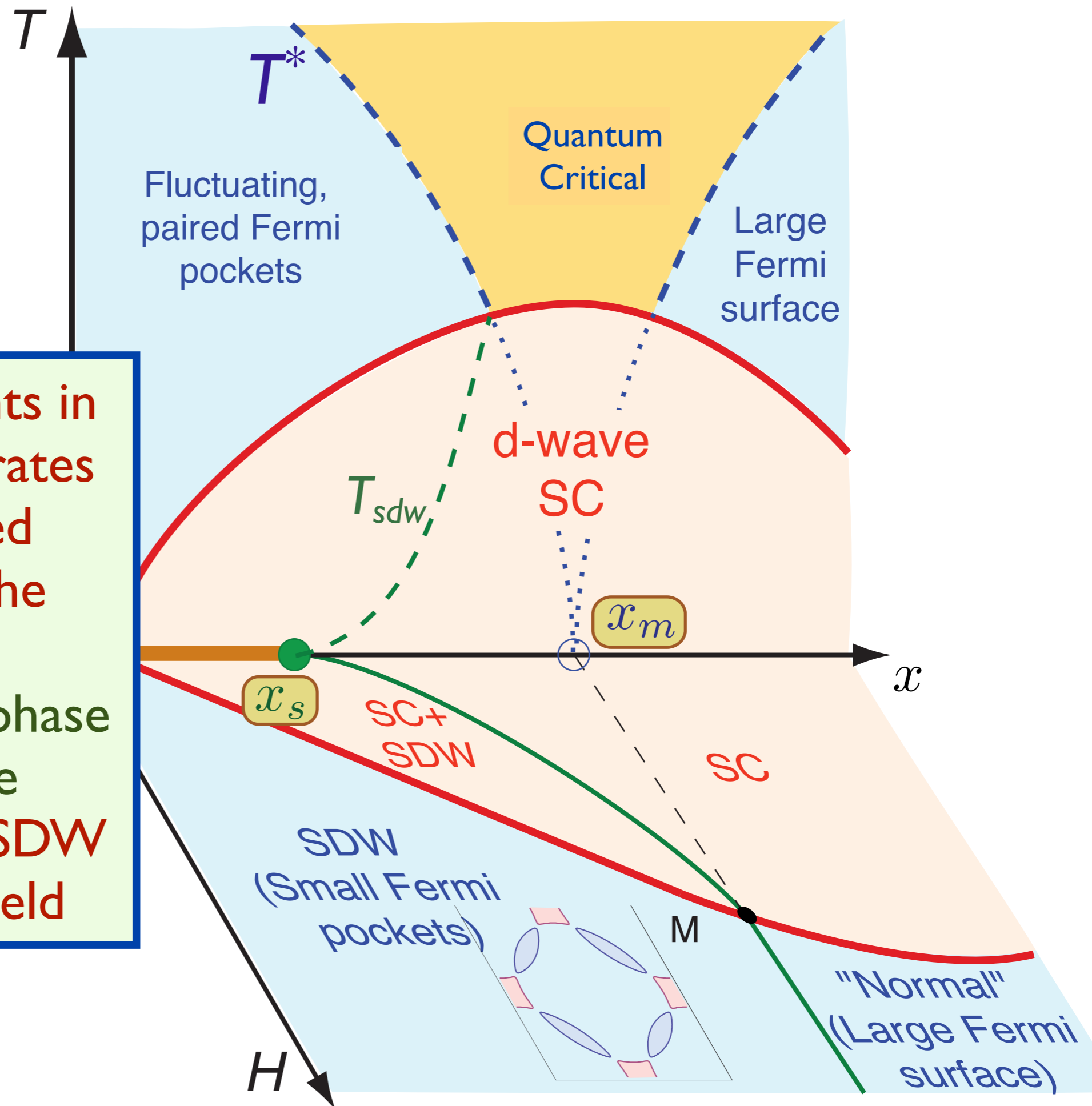
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

There is a much larger shift from x_m to x_s in the hole-doped cuprates.



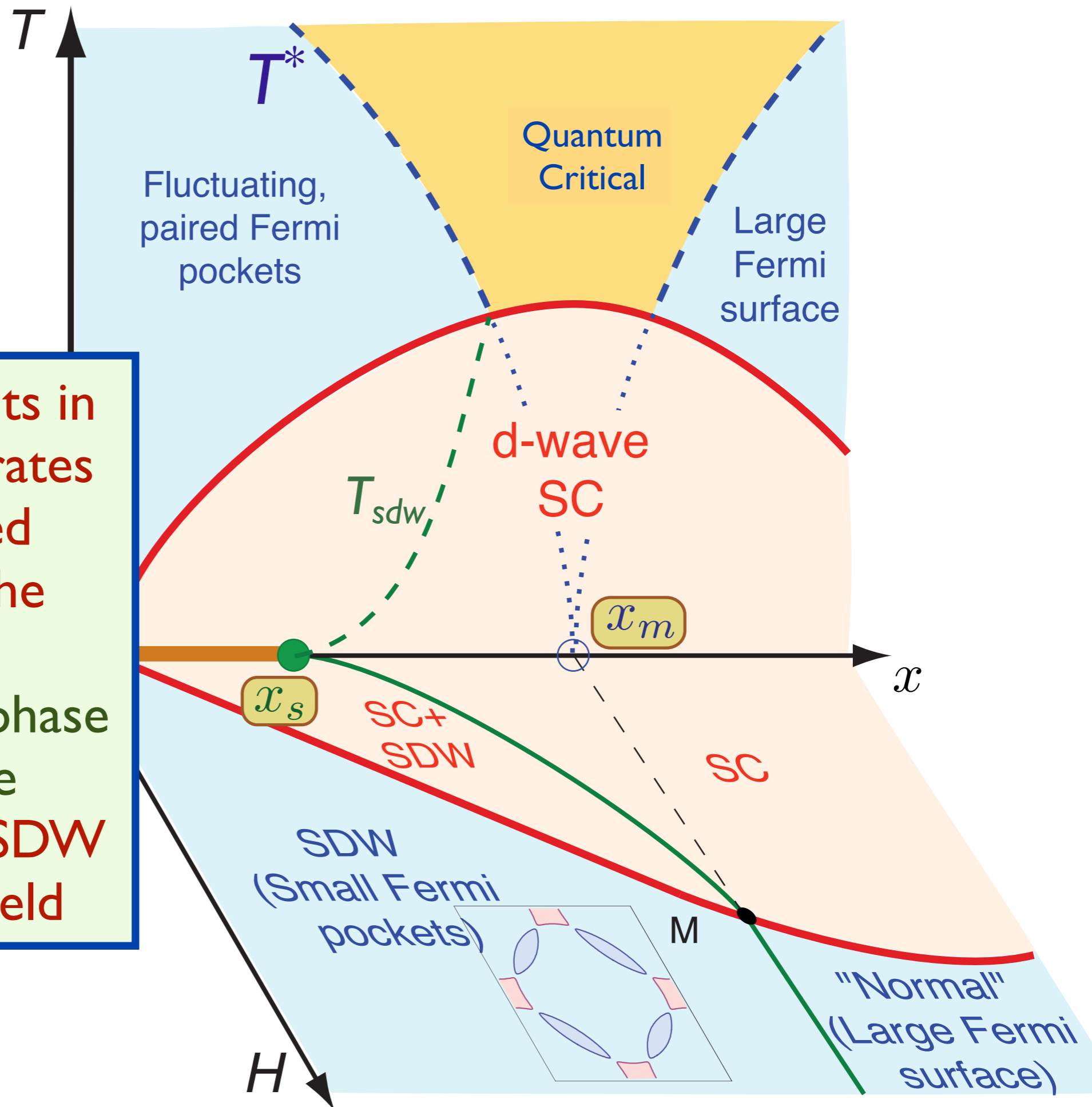
E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).

Many experiments in
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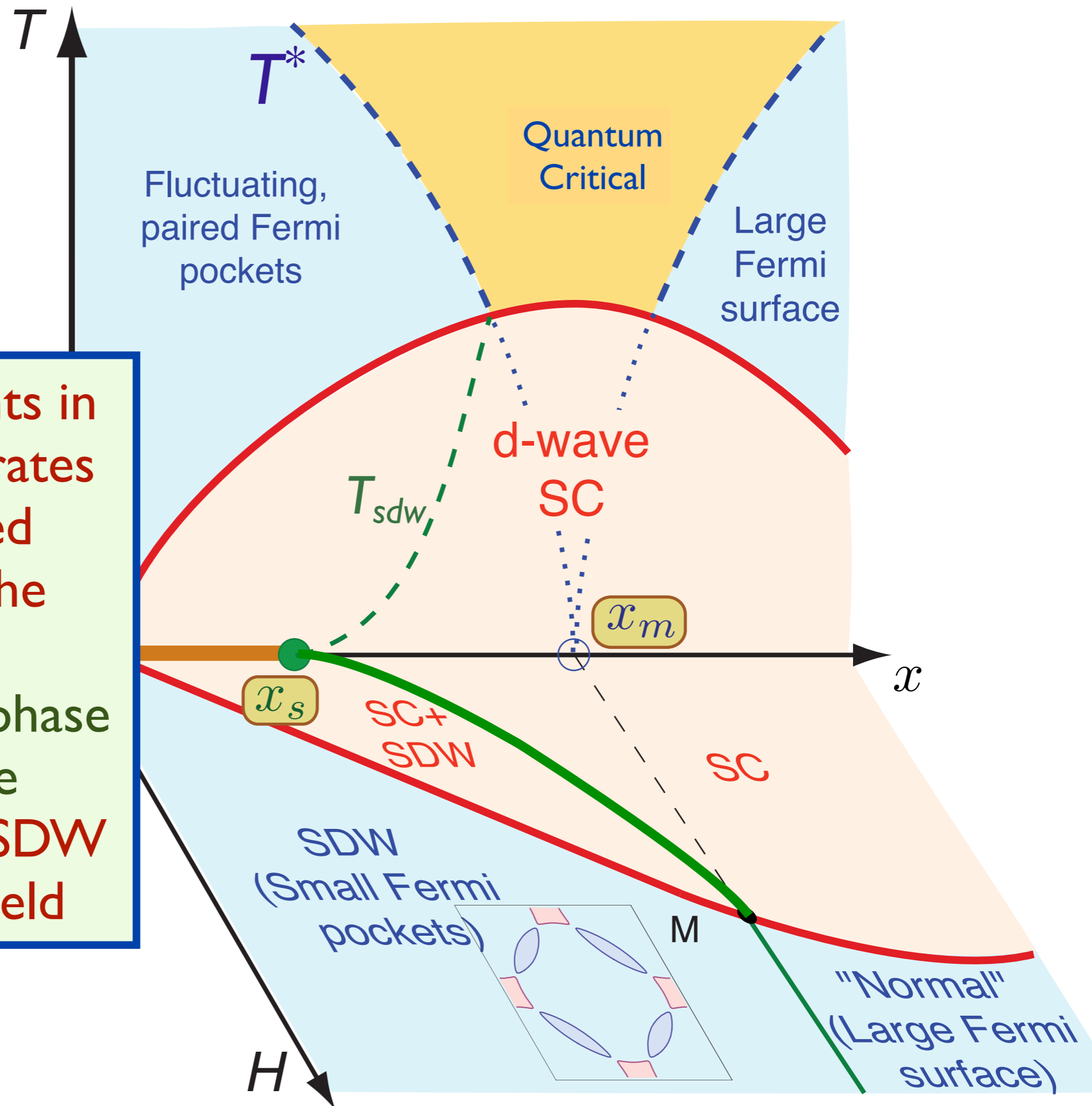
E. Demler, S. Sachdev
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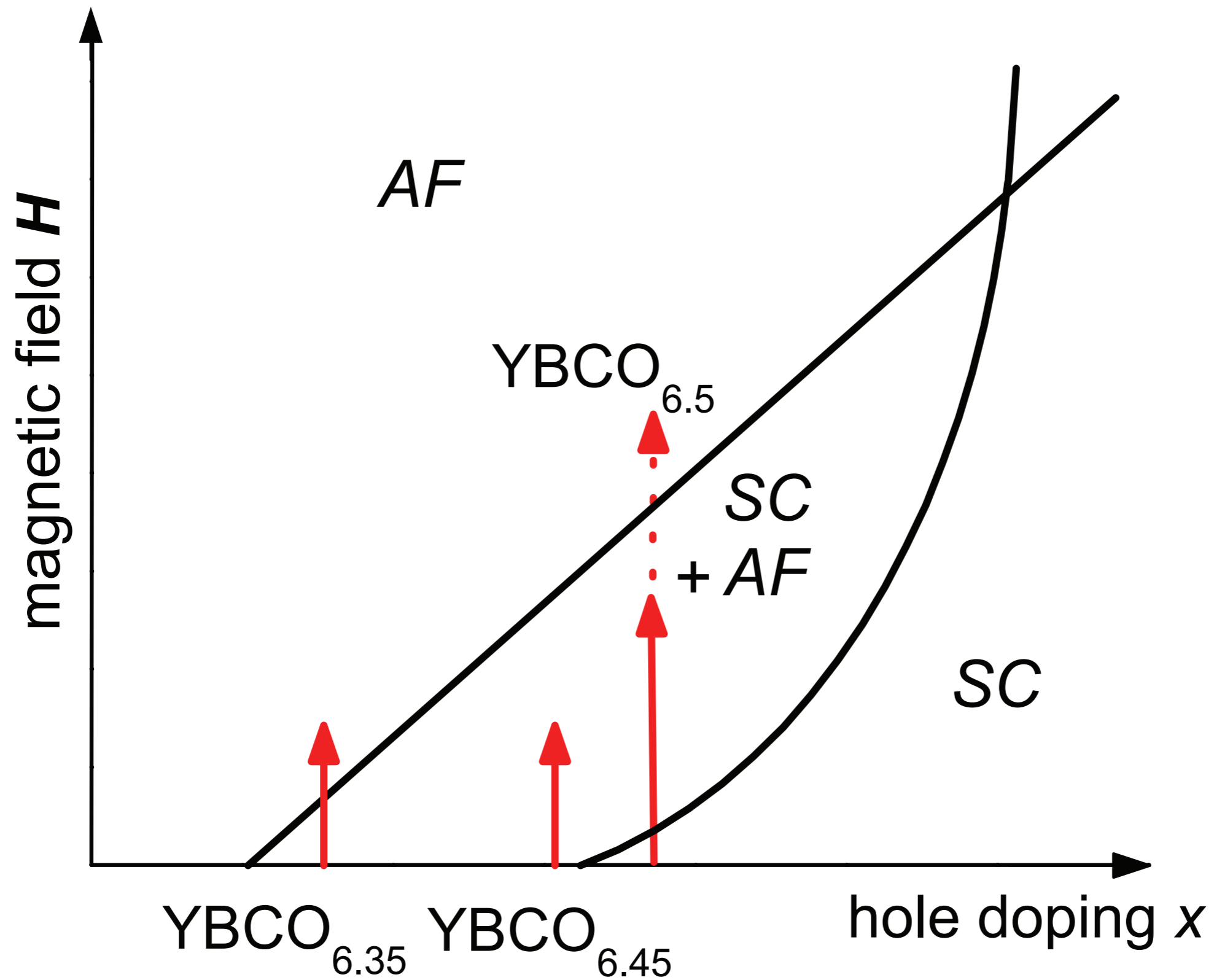
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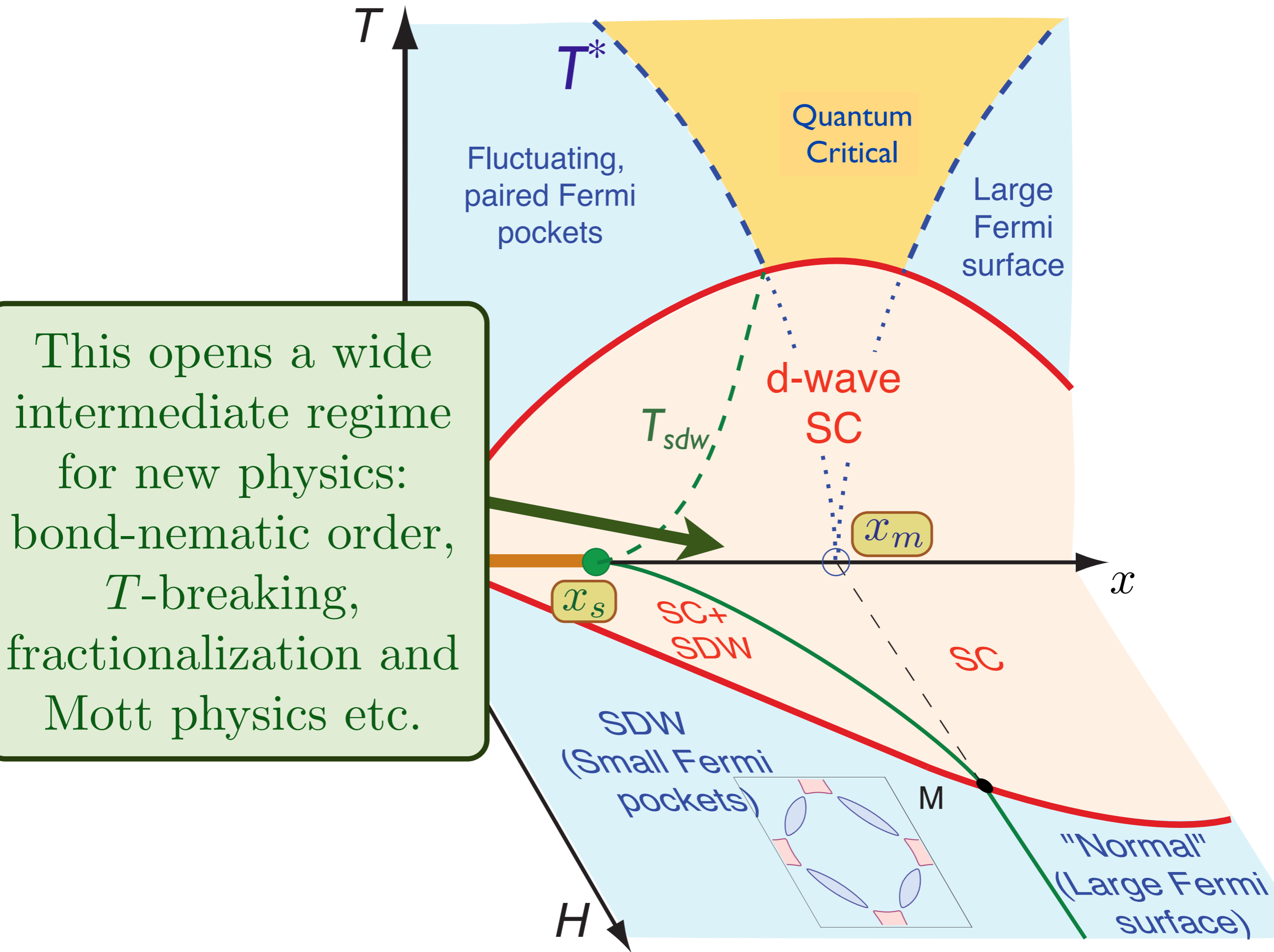
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D. Haug, V. Hinkov, Y. Sidis, P. Bourges, N. B. Christensen, A. Ivanov, T. Keller, C. T. Lin, and B. Keimer, *New J. Phys.* **12**, 105006 (2010)



This opens a wide intermediate regime for new physics: bond-nematic order, T -breaking, fractionalization and Mott physics etc.

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal
2. Quantum field theory of the onset of antiferromagnetism in a metal
3. Quantum Monte Carlo without the sign problem
4. Fractionalization in metals,
and the hole-doped cuprates

Outline

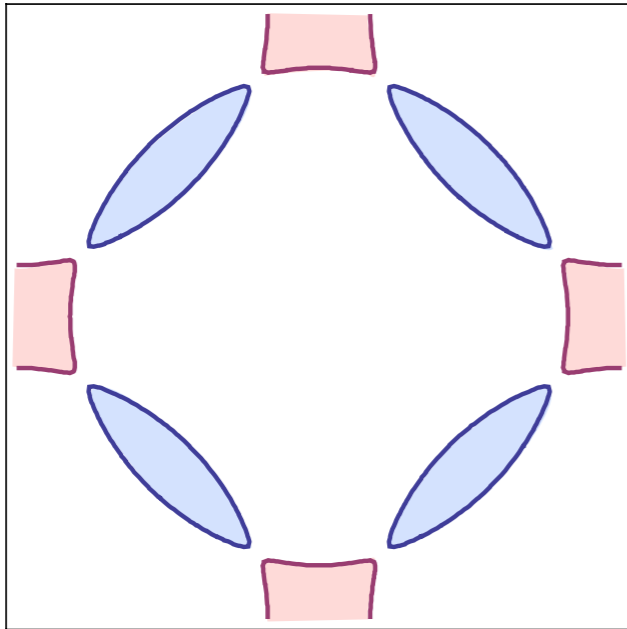
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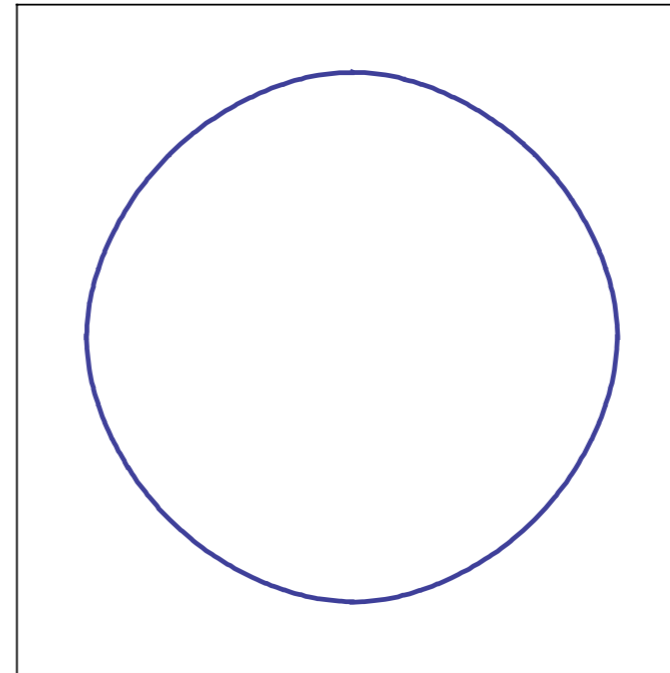
4. Fractionalization in metals,
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Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

← Increasing interaction

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Boson-fermion theory for both phases

$$\mathcal{S} = \int d^2r d\tau [\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}]$$

$$\mathcal{L}_c = c_a^\dagger \varepsilon (-i \nabla) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla \varphi_\alpha)^2 + \frac{r}{2} \varphi_\alpha^2 + \frac{u}{4} (\varphi_\alpha^2)^2$$

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$$\mathcal{L}_{c\varphi} = \lambda \varphi_\alpha e^{i\mathbf{K}\cdot\mathbf{r}} c_a^\dagger \sigma_{ab}^\alpha c_b.$$

“Yukawa” coupling between fermions and antiferromagnetic order:

$\lambda^2 \sim U$, the Hubbard repulsion

Hertz-Moriya-Millis theory

- Integrate out Fermi surface quasiparticles and obtain an effective theory for the order parameter $\vec{\varphi}$ alone.

Hertz-Moriya-Millis theory

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- This is dangerous, and will lead to non-local in the $\vec{\varphi}$ theory. Hertz focused on only the simplest such non-local term.

Hertz-Moriya-Millis theory

- Integrate out Fermi surface quasiparticles and obtain an effective theory for the order parameter $\vec{\varphi}$ alone.
- This is dangerous, and will lead to non-local in the $\vec{\varphi}$ theory. Hertz focused on only the simplest such non-local term.
- However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in two spatial dimensions.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

- In $d = 2$, we *must* work in local theories which keeps both the order parameter and the Fermi surface quasiparticles “alive”.

Sung-Sik Lee, *Phys. Rev. B* **80**, 165102 (2009)

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075128 (2010)

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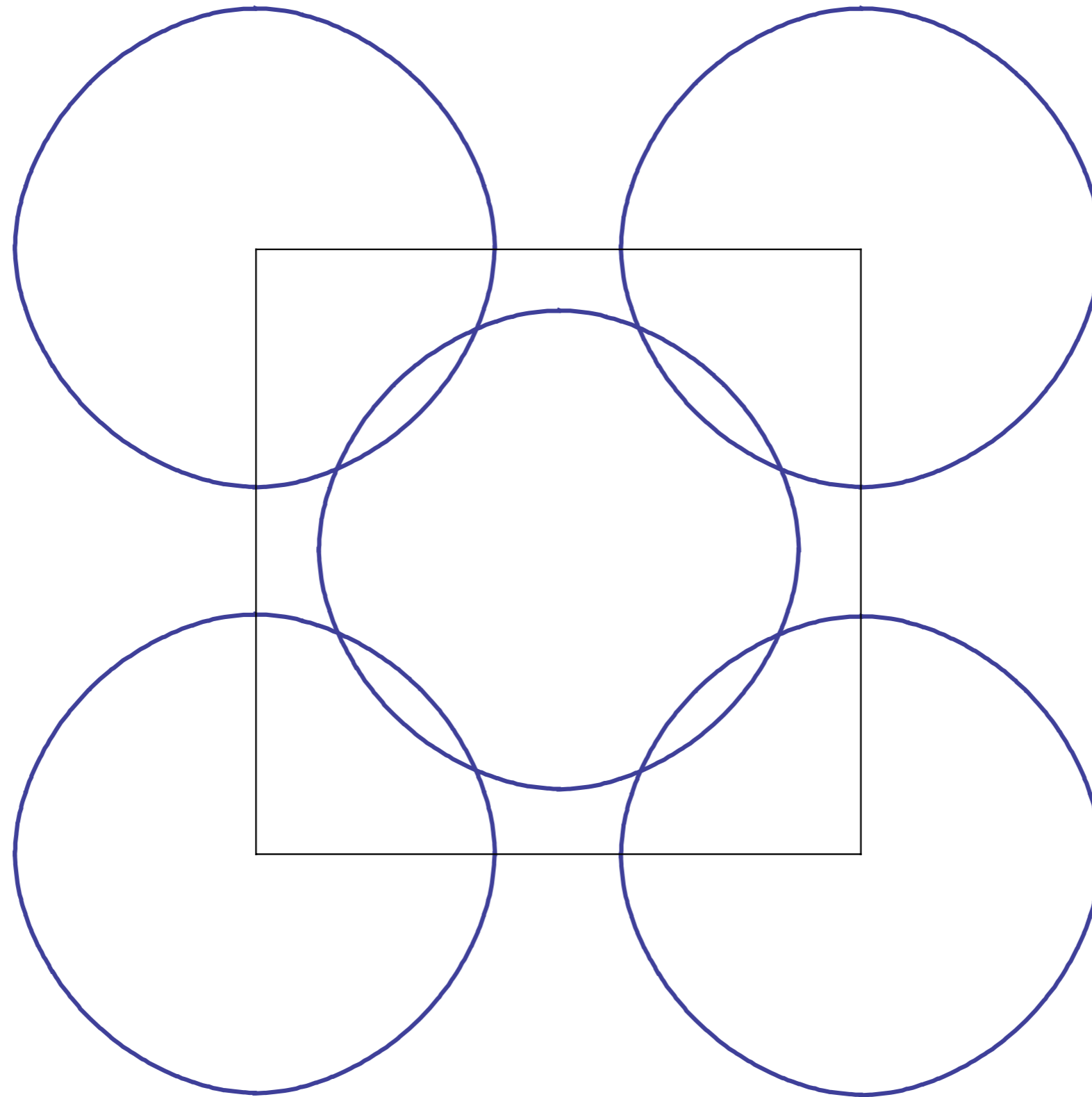
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075128 (2010)

- In $d = 2$, we *must* work in local theories which keeps both the order parameter and the Fermi surface quasiparticles “alive”.
- The theories can be organized in a $1/N$ expansion, where N is the number of fermion “flavors”.
- At subleading order, resummation of all “planar” graphics is required (at least): this theory is even more complicated than QCD.

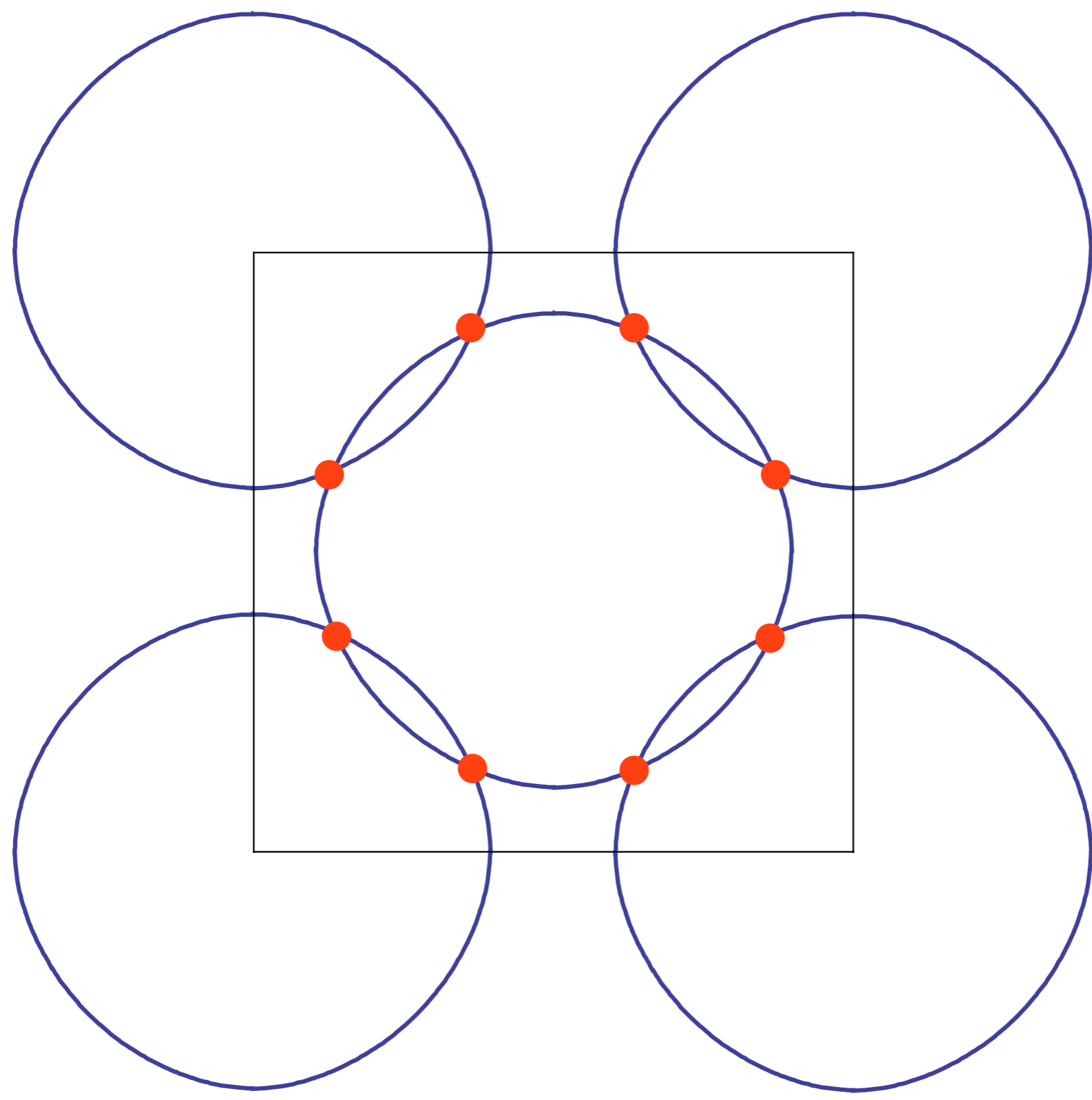
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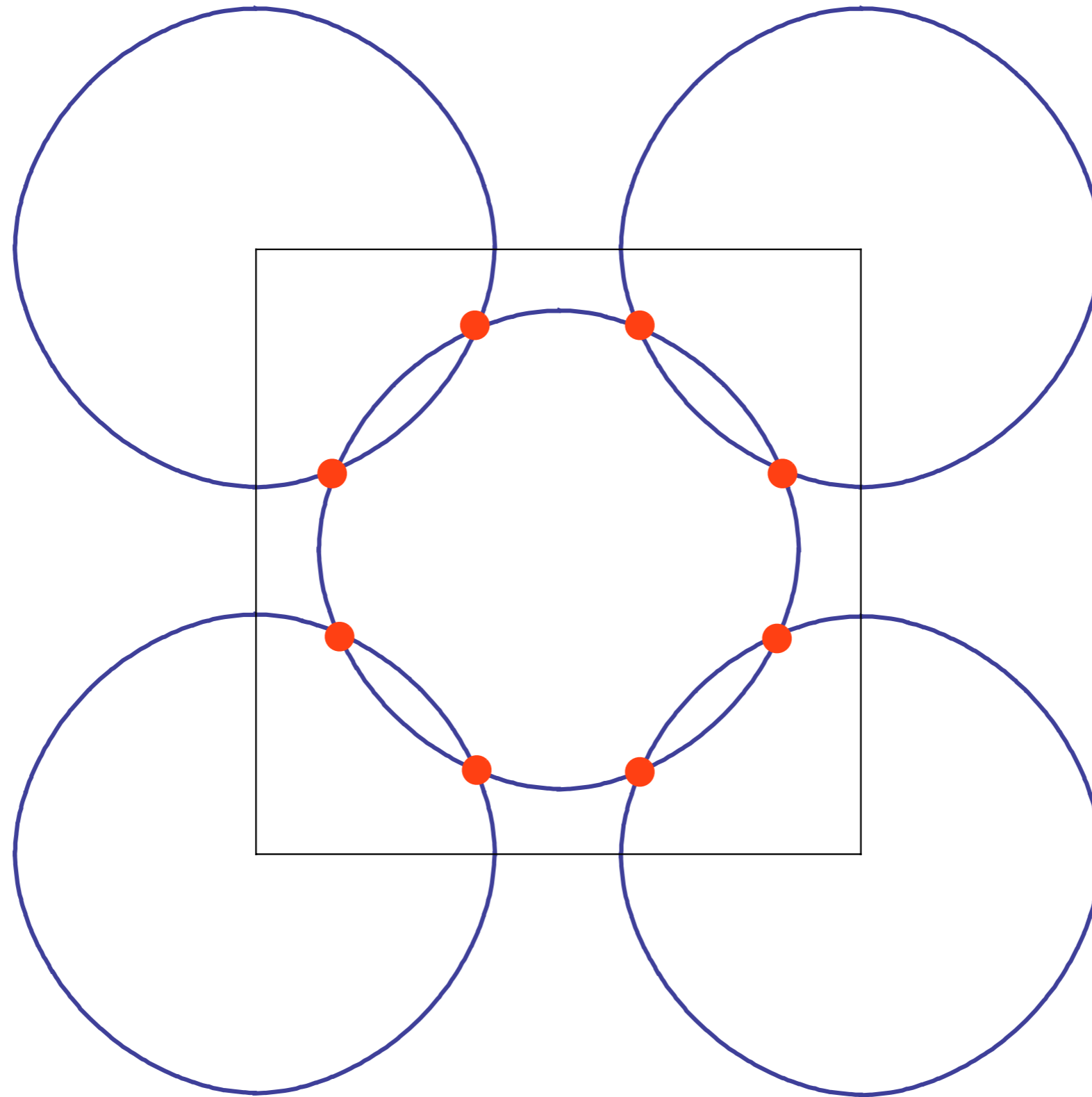
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075128 (2010)



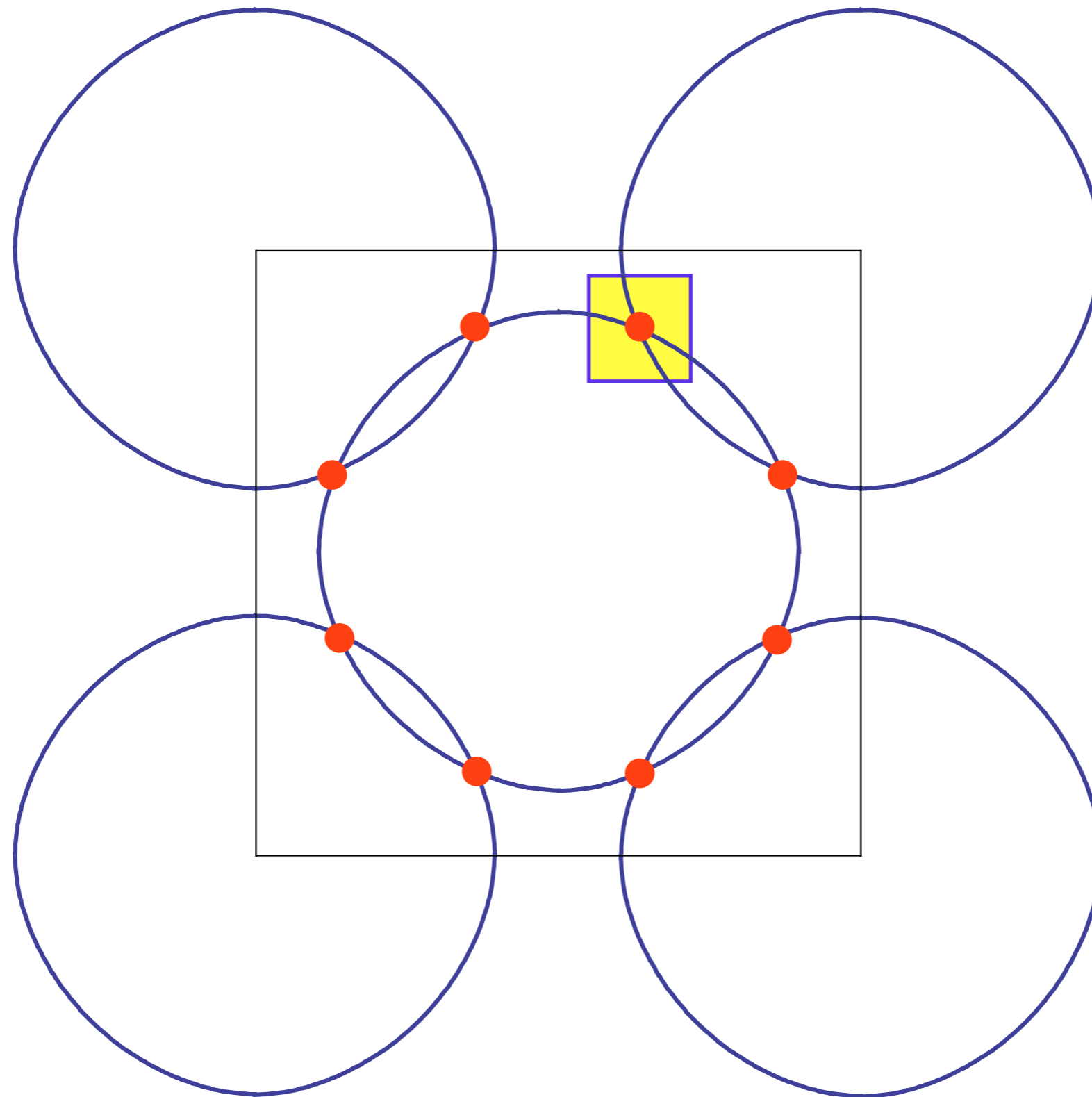
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



“Hot” spots

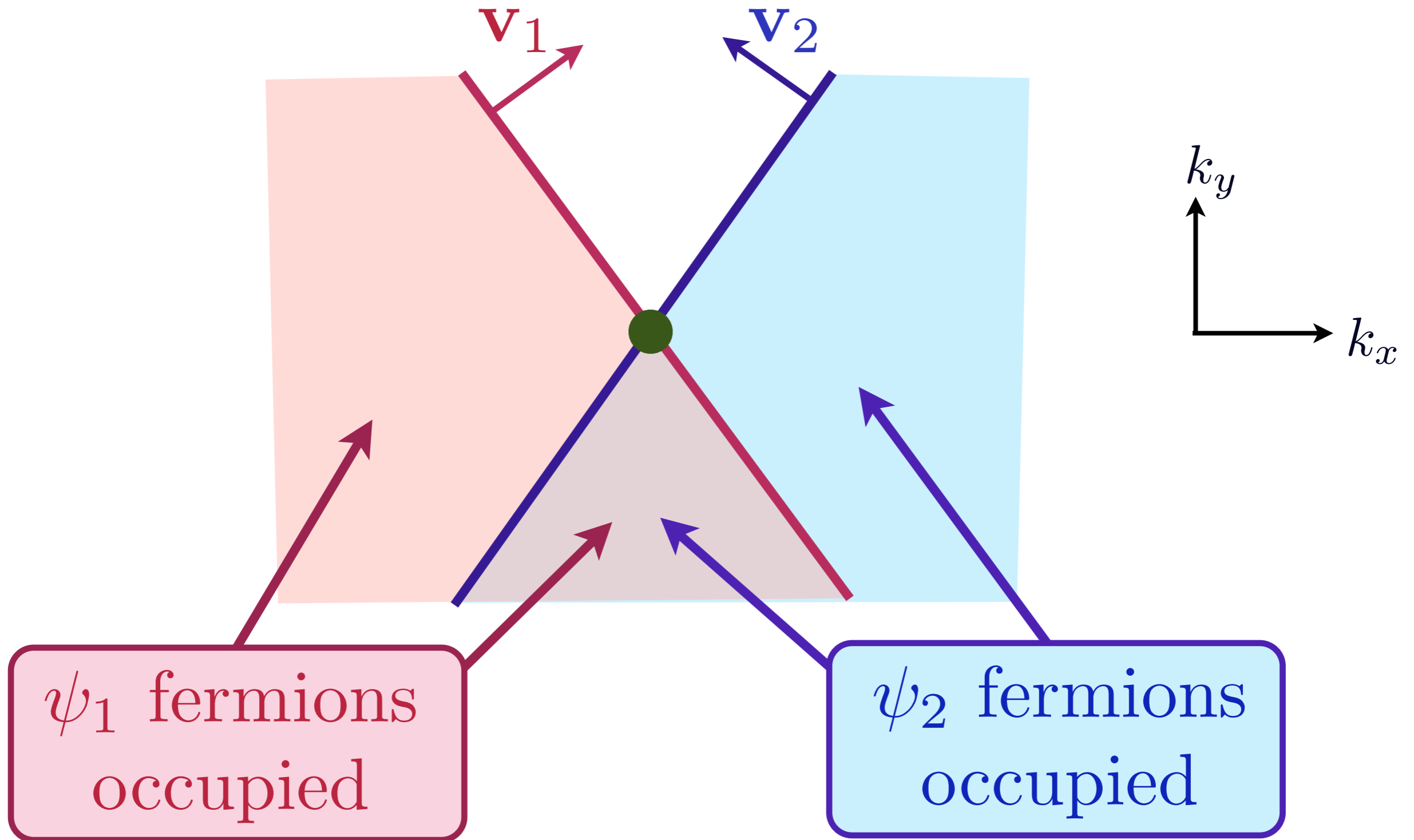


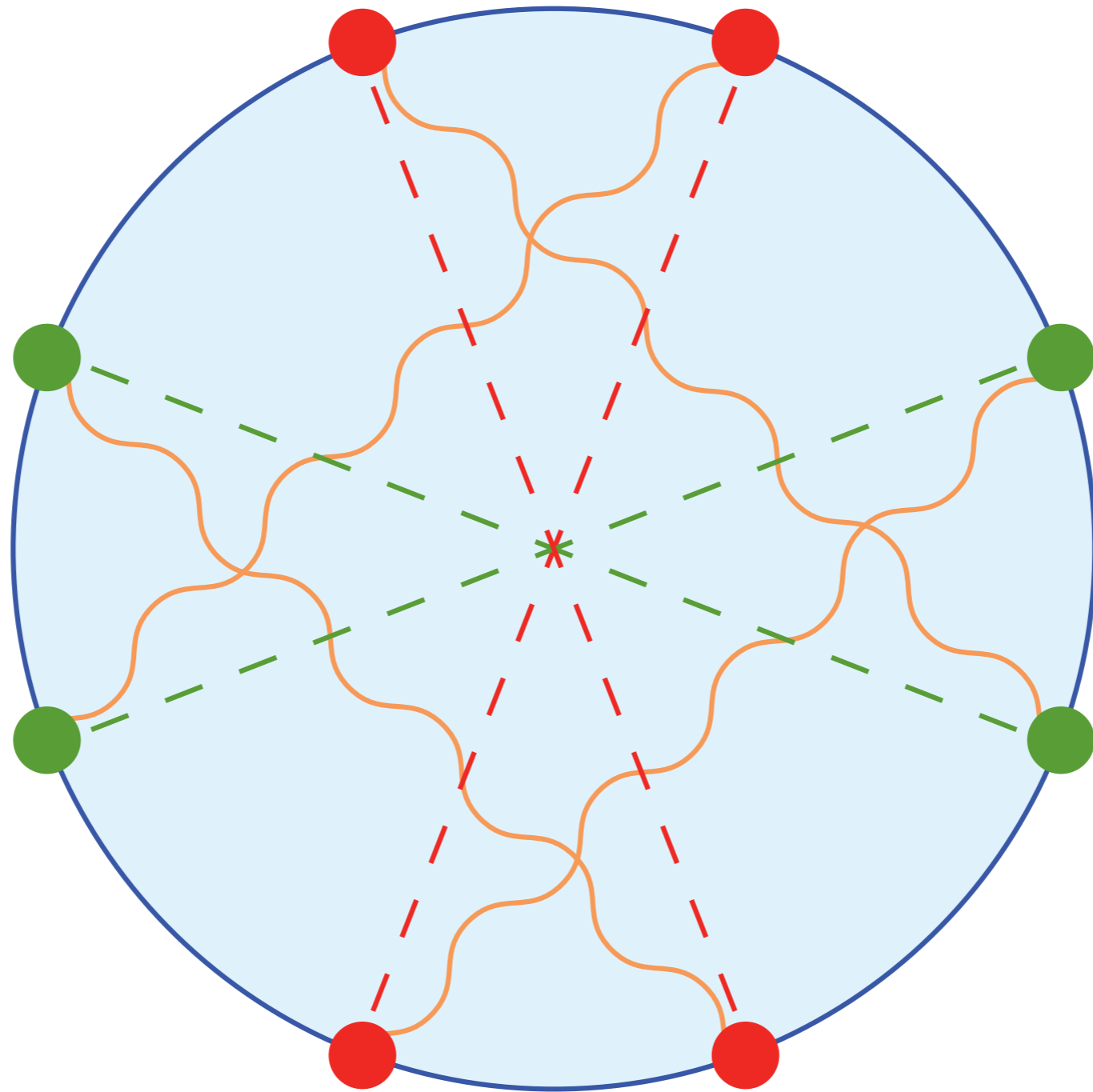
Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

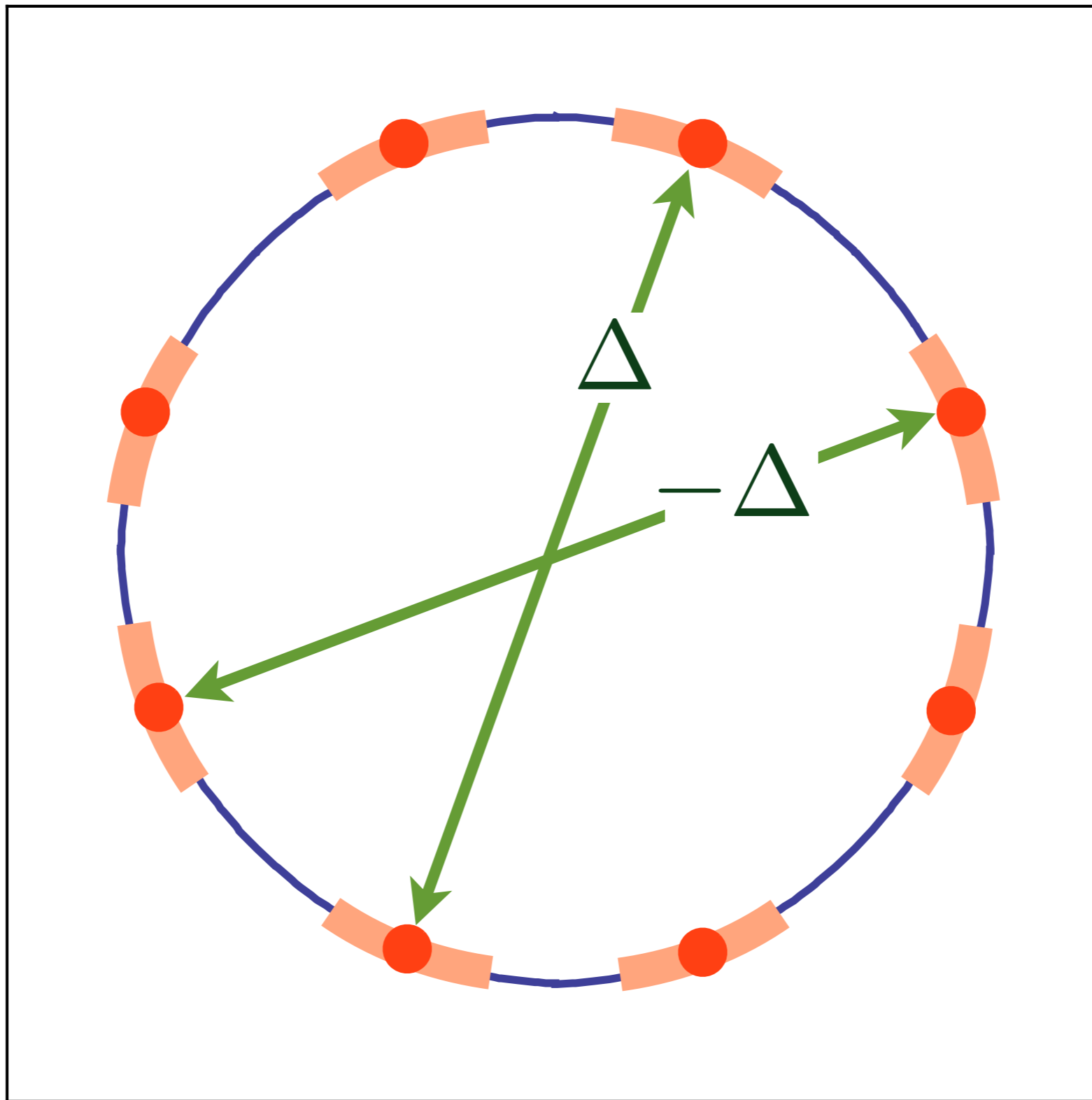
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ





Pairing “glue” from antiferromagnetic fluctuations

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at and near hot spots

At stronger coupling,
different effects compete:

- Pairing glue becomes stronger.



At stronger coupling,
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At stronger coupling,
different effects compete:

- Pairing glue becomes stronger.
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.
- Other instabilities can appear *e.g.* to charge density waves/stripe order.

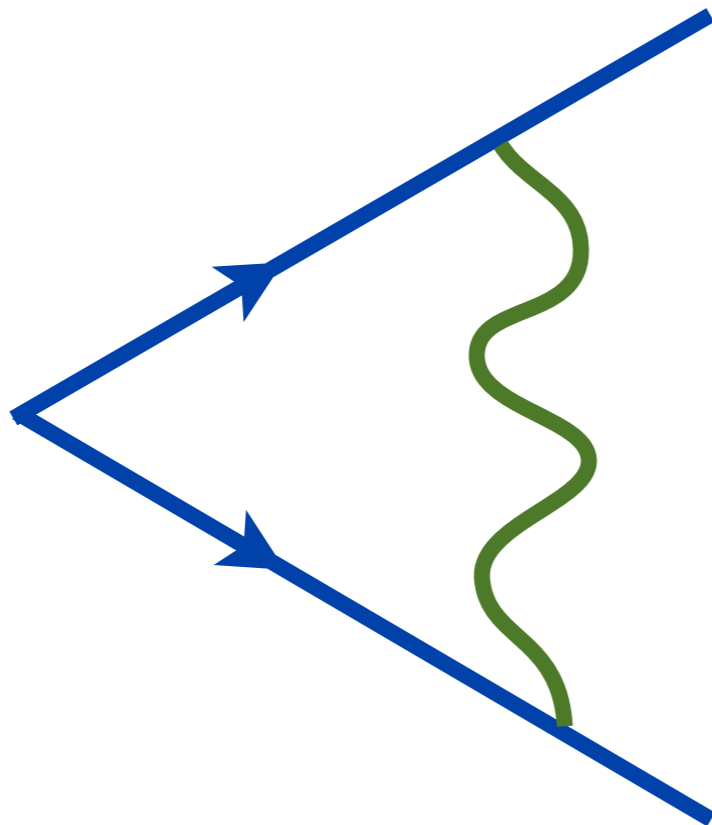


BCS theory

Electron-phonon coupling

$$1 + \lambda_{\text{e-ph}} \log \left(\frac{\omega_D}{\omega} \right)$$

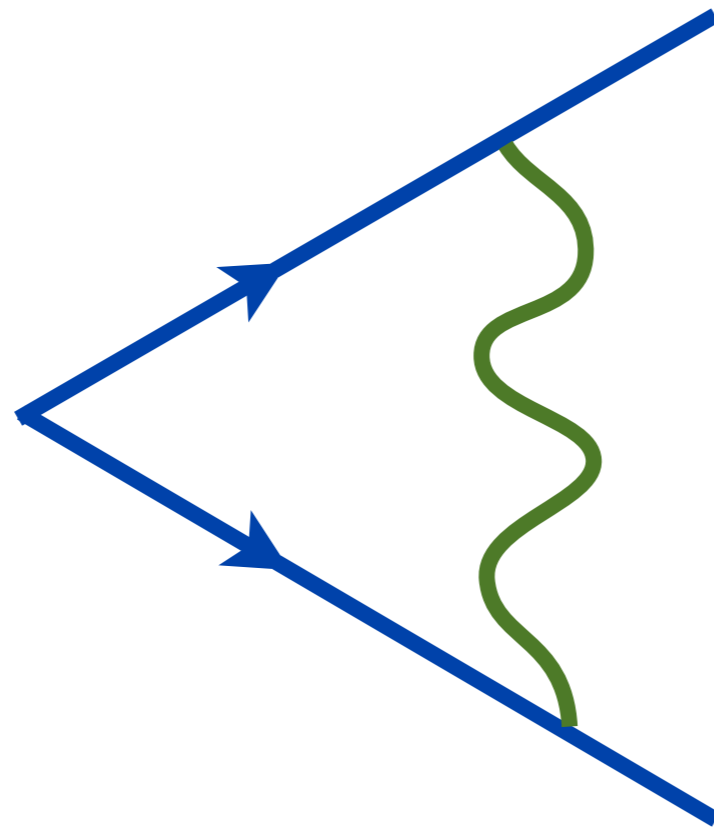
Debye frequency



Enhancement of pairing susceptibility by interactions

Antiferromagnetic critical point

$$1 + \frac{\sin \theta}{2\pi} \log^2 \left(\frac{E_F}{\omega} \right)$$



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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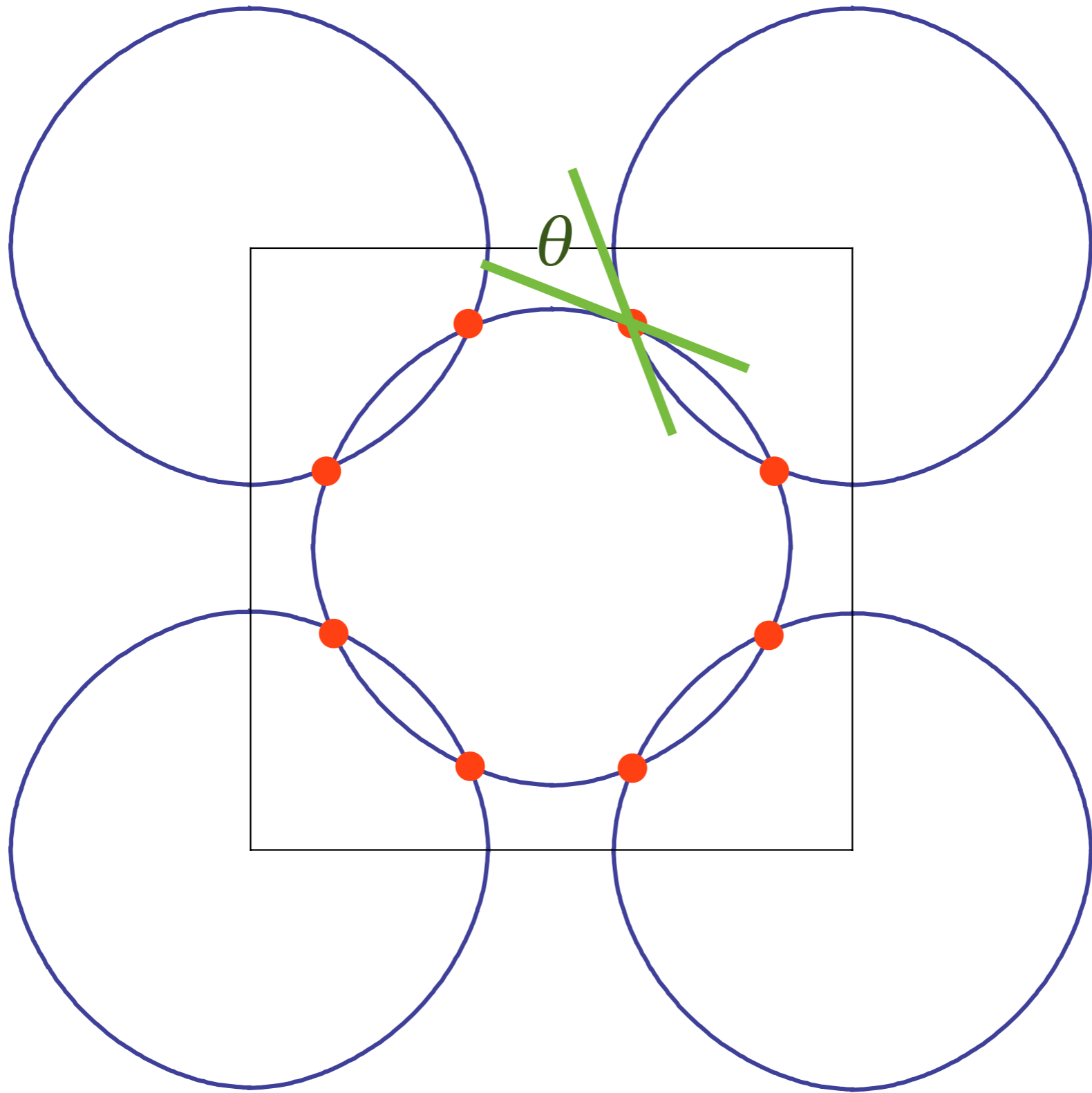
$$1 + \frac{\sin \theta}{2\pi} \log^2 \left(\frac{E_F}{\omega} \right)$$



Fermi
energy

θ is the angle between Fermi lines.
Independent of interaction strength
 U in 2 dimensions.

(see also Ar. Abanov, A. V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



Enhancement of pairing susceptibility by interactions

Antiferromagnetic critical point

$$1 + \frac{\sin \theta}{2\pi} \log^2 \left(\frac{E_F}{\omega} \right)$$



- **Universal** \log^2 singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by $1/N$ factor in $1/N$ expansion.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

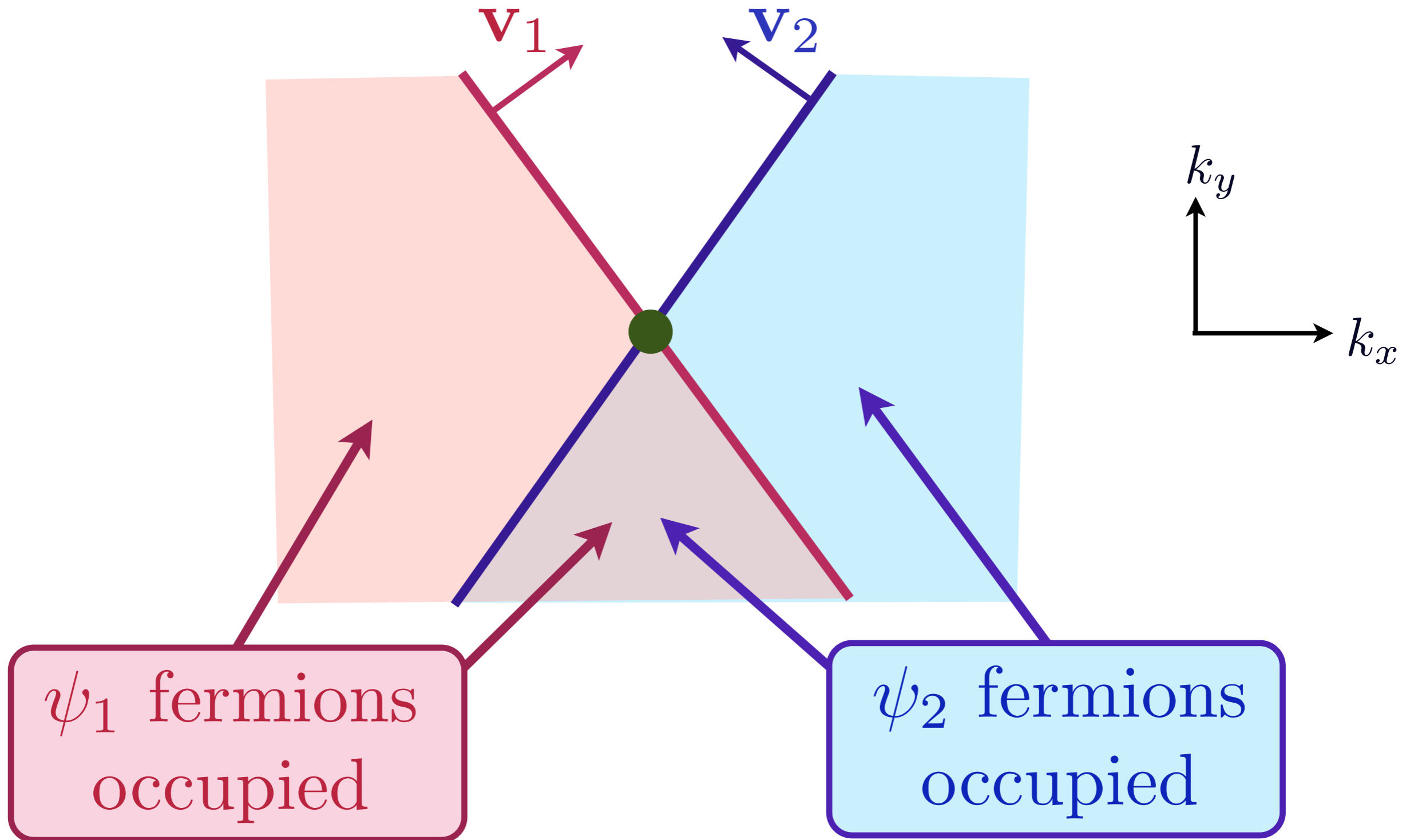
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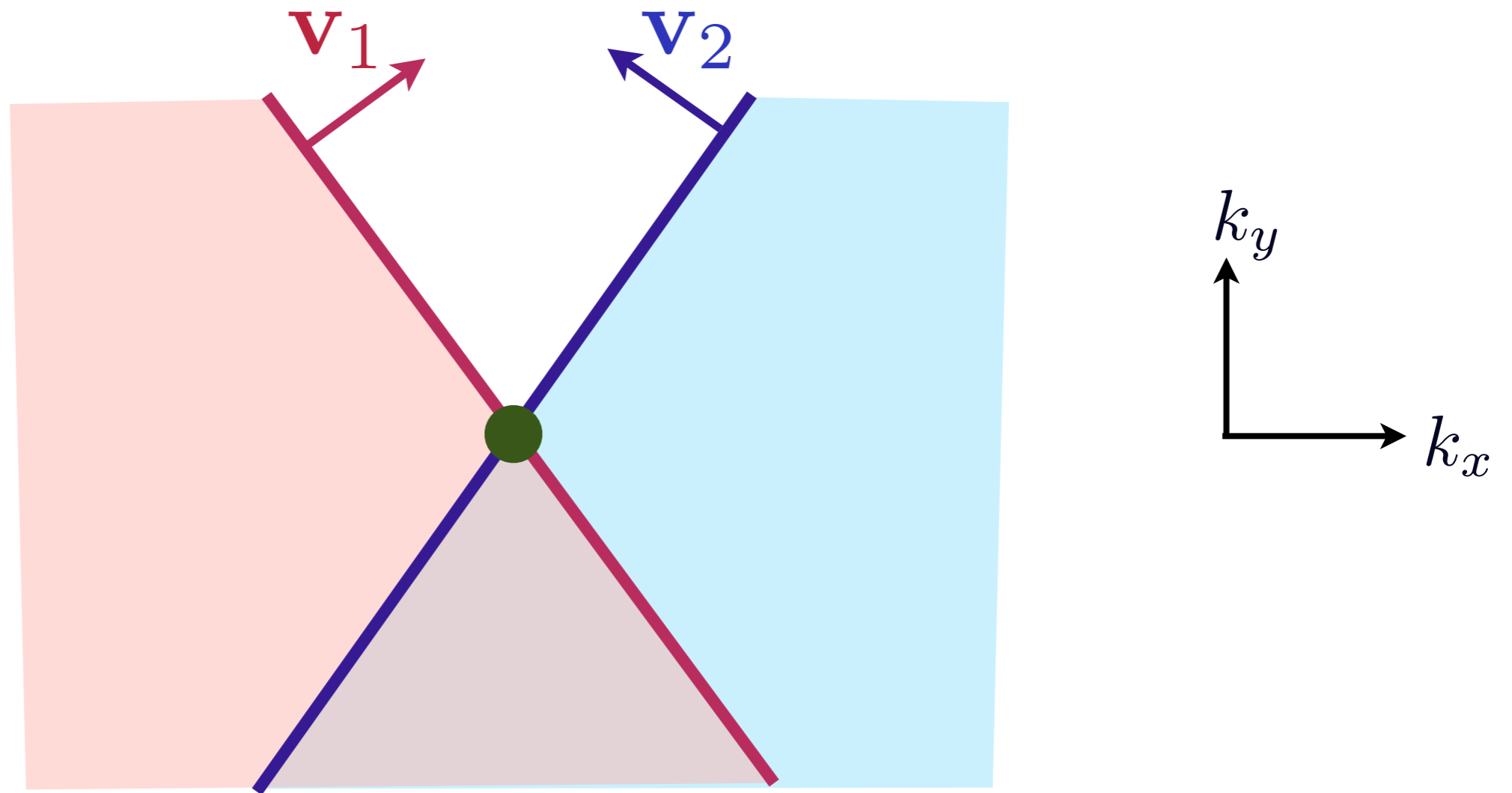
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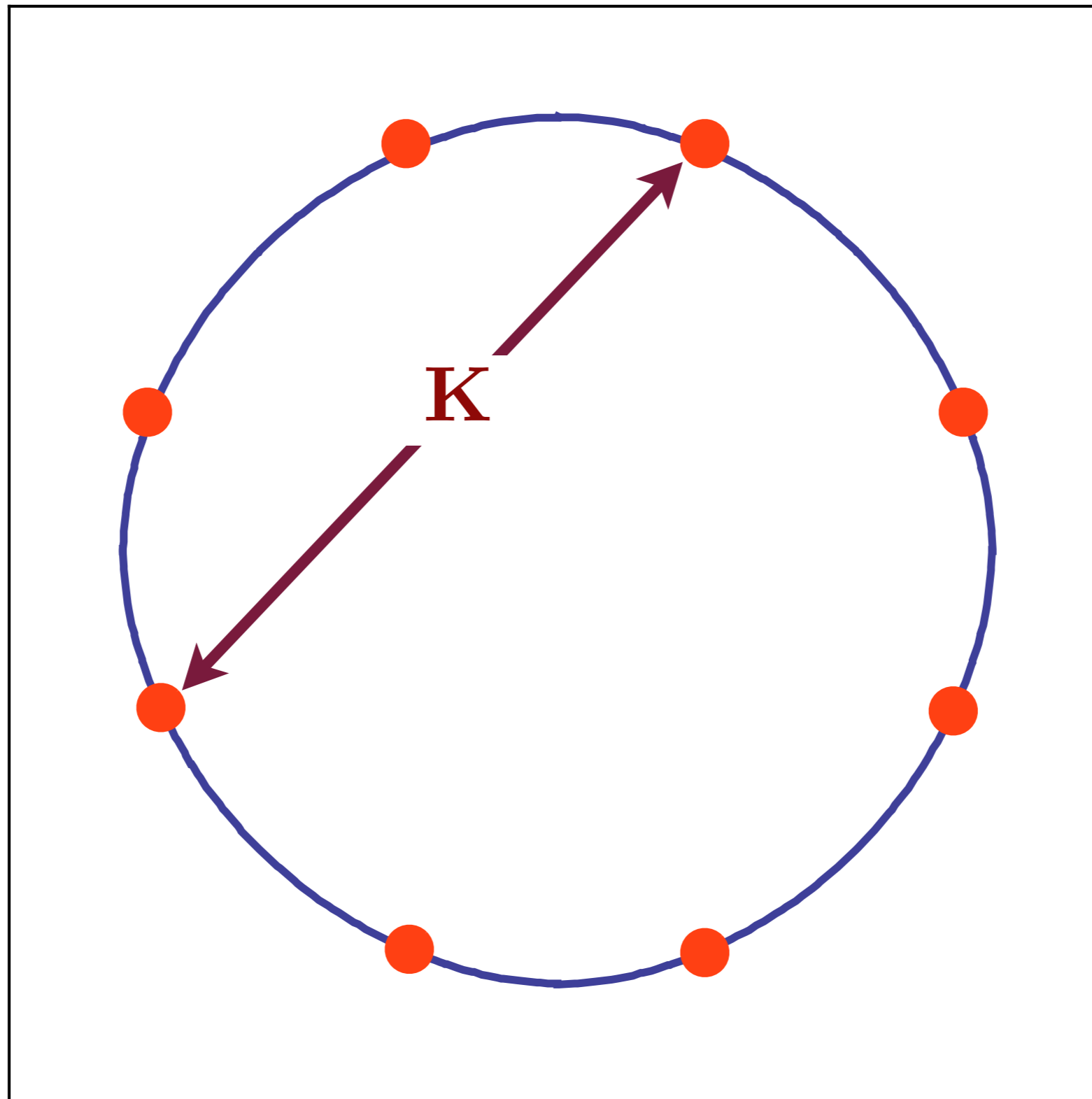


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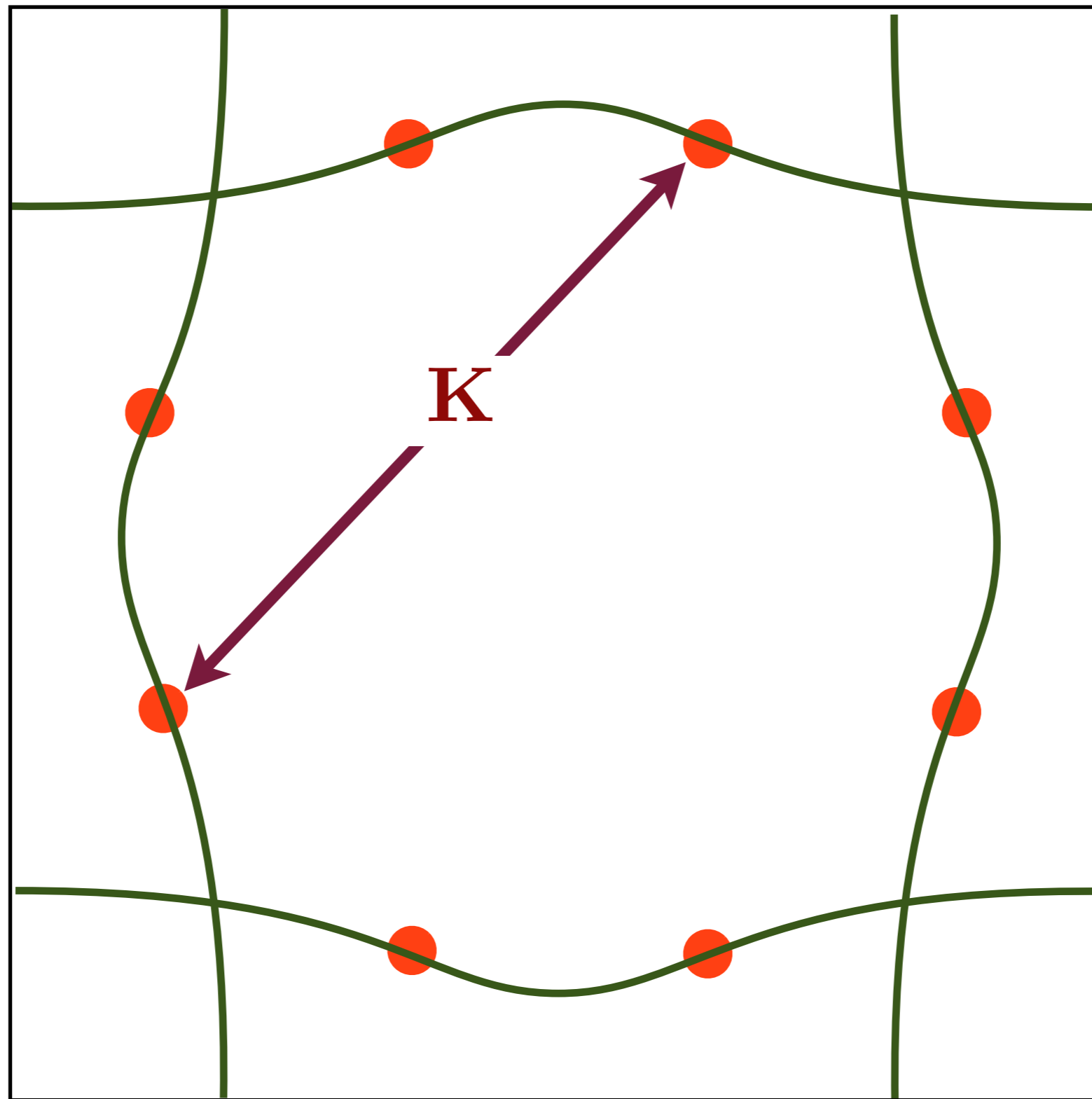
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QMC for the onset of antiferromagnetism



Hot spots in a single band model

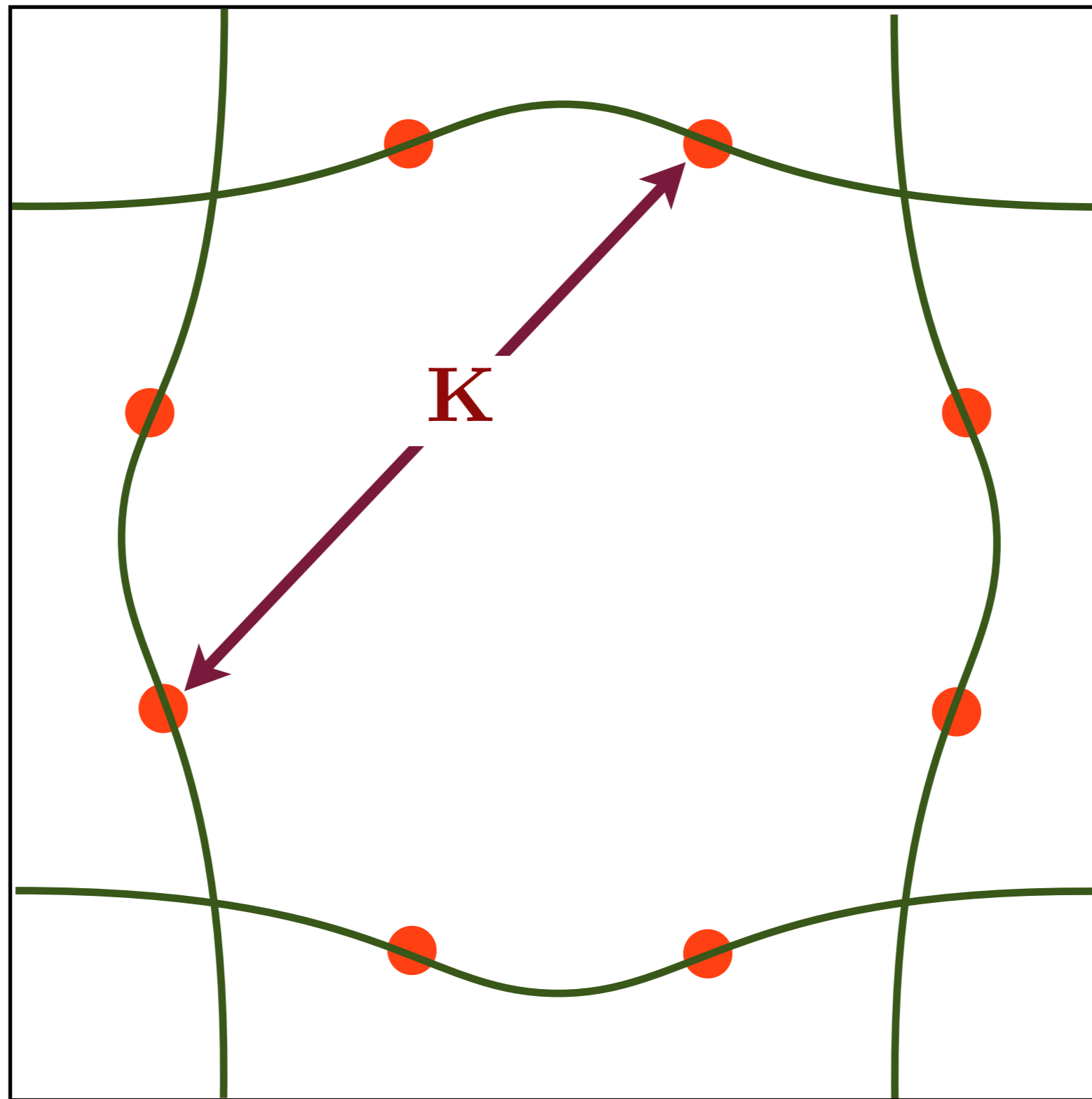
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E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Hot spots in a two band model

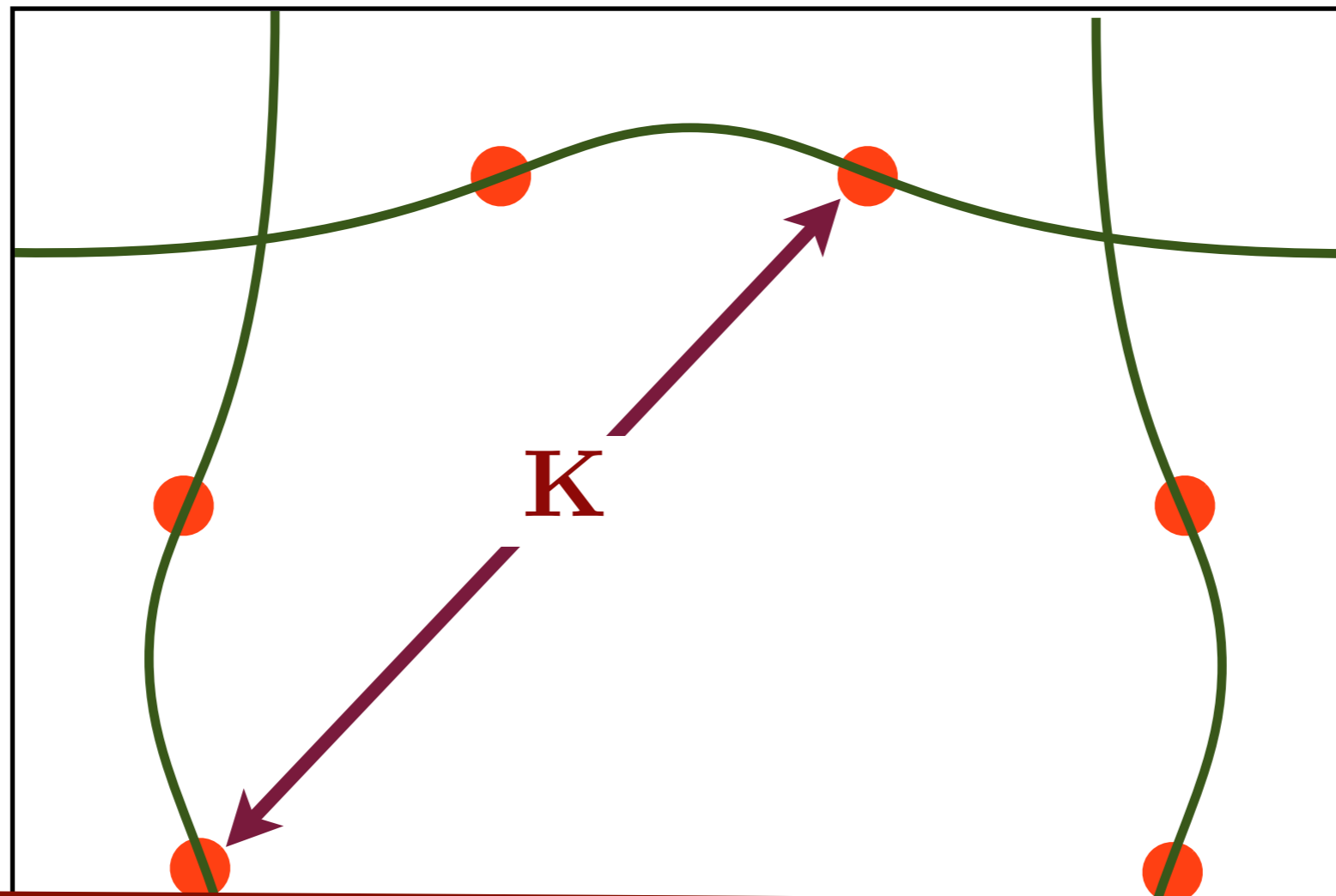
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No sign problem in
fermion determinant Monte Carlo !
Determinant is positive because of Kramer's
degeneracy, and no additional symmetries are needed; holds for
arbitrary band structure and band filling, provided **K** only
connects hot spots in distinct bands

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$
interacting with fluctuations of the
antiferromagnetic order parameter $\vec{\varphi}$.

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}c_{\alpha} \mathcal{D}\vec{\varphi} \exp(-\mathcal{S}) \\ \mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\ &+ \int d\tau d^2x \left[\frac{1}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \dots \right] \\ &- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta} \end{aligned}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}c_{\alpha}^{(x)} \mathcal{D}c_{\alpha}^{(y)} \mathcal{D}\vec{\varphi} \exp(-\mathcal{S}) \\ \mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)} \\ &+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)} \\ &+ \int d\tau d^2x \left[\frac{1}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \dots \right] \\ &- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.} \end{aligned}$$

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QMC for the onset of antiferromagnetism

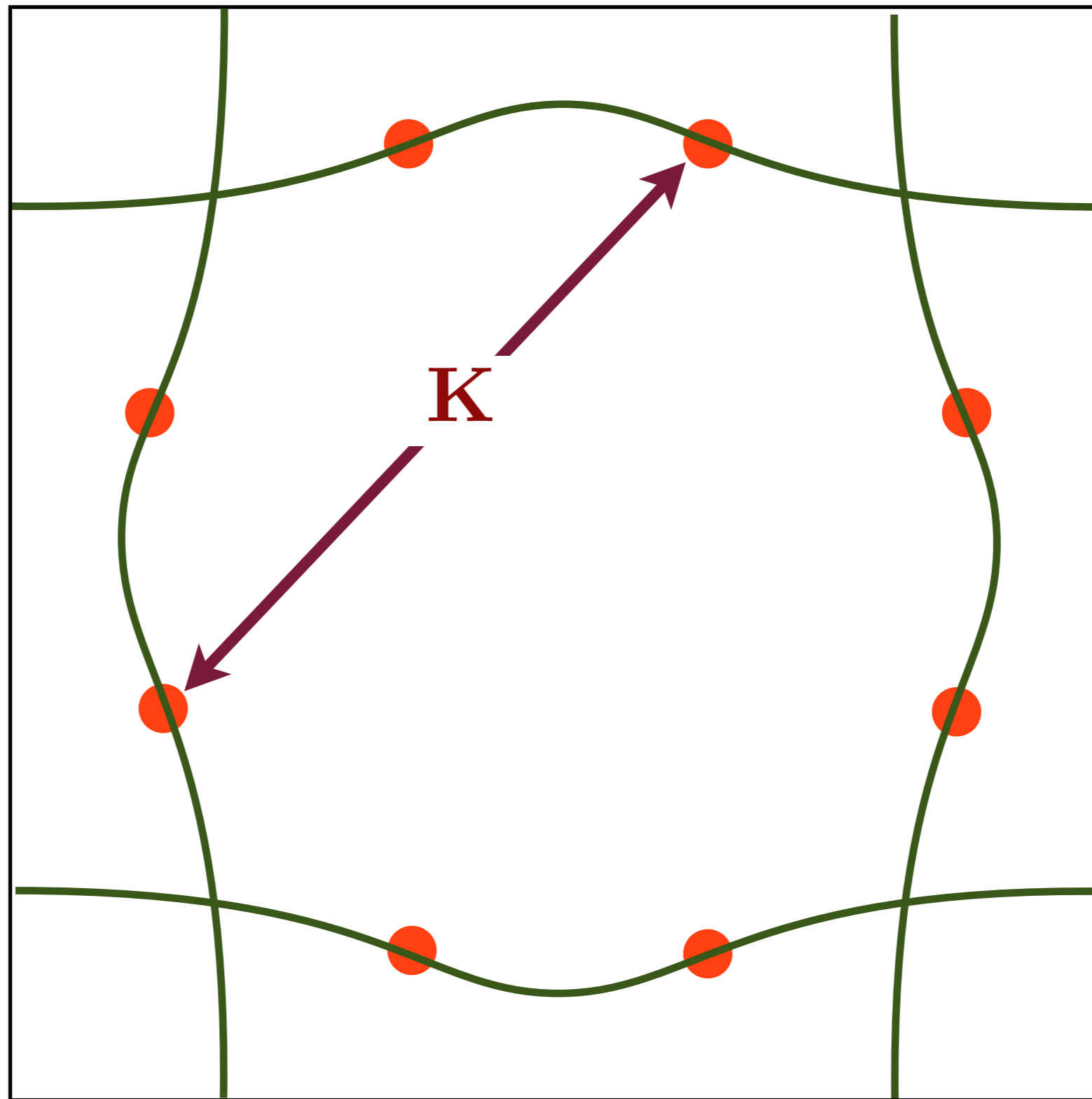
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No sign problem !

QMC for the onset of antiferromagnetism

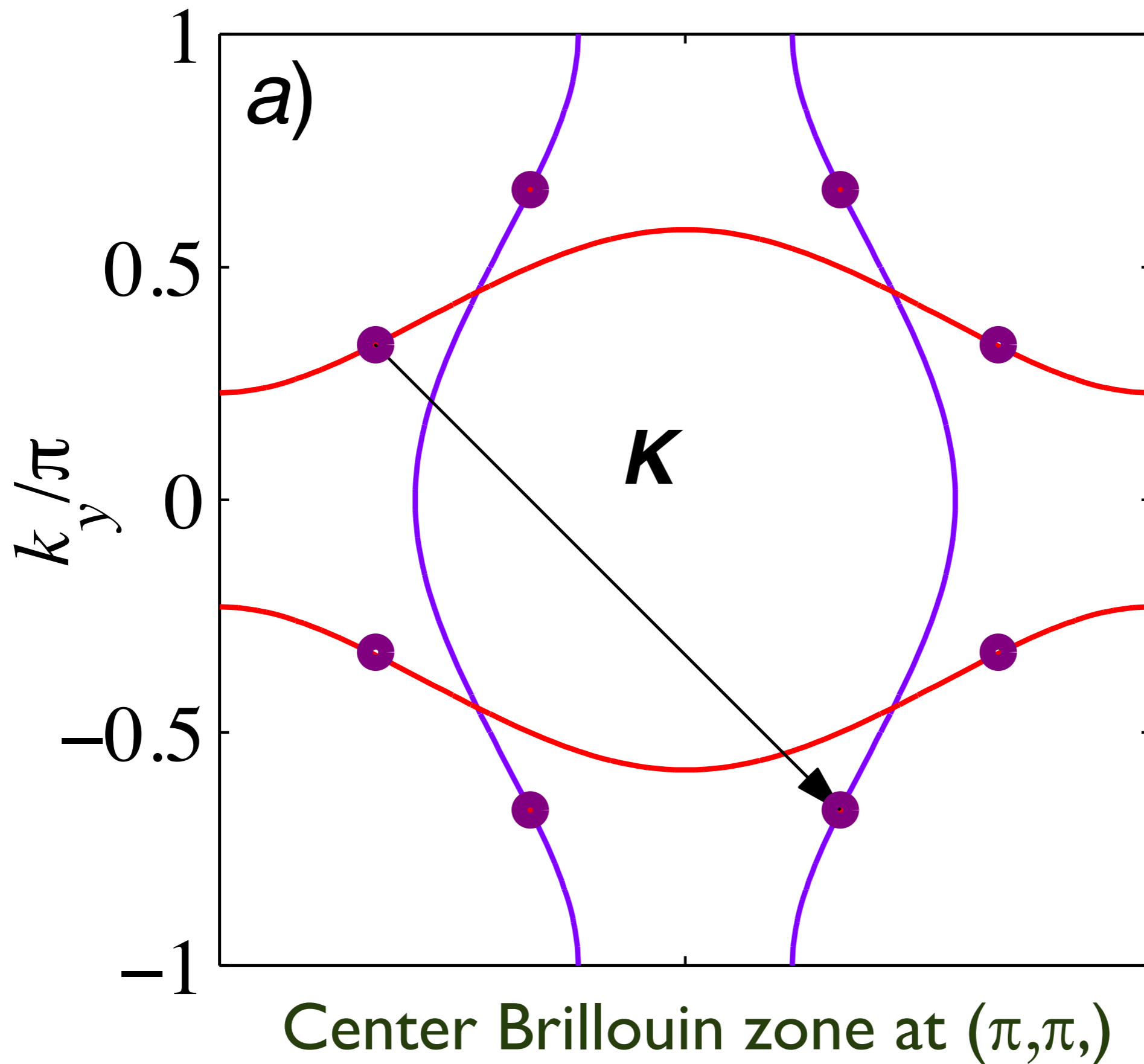


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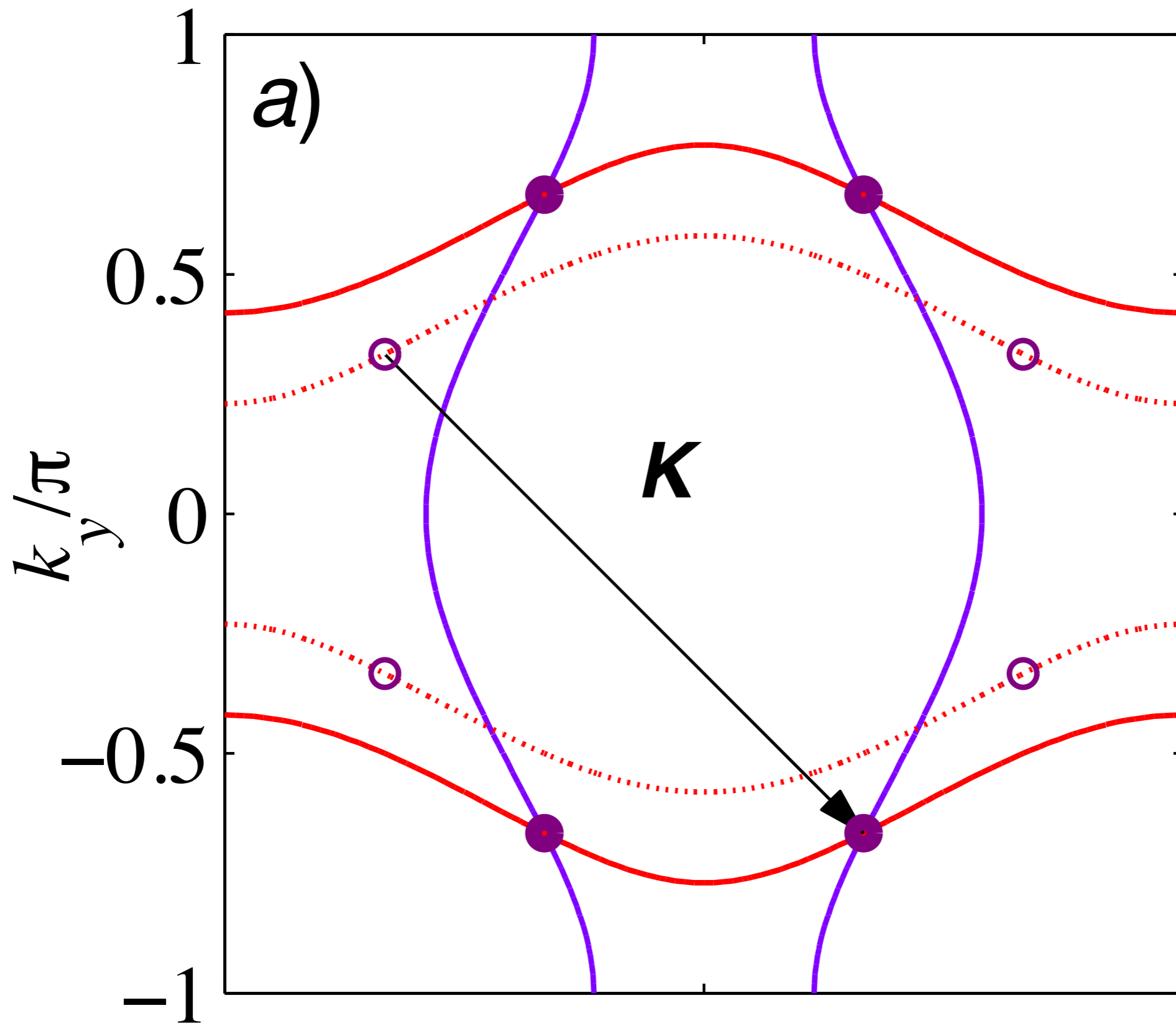
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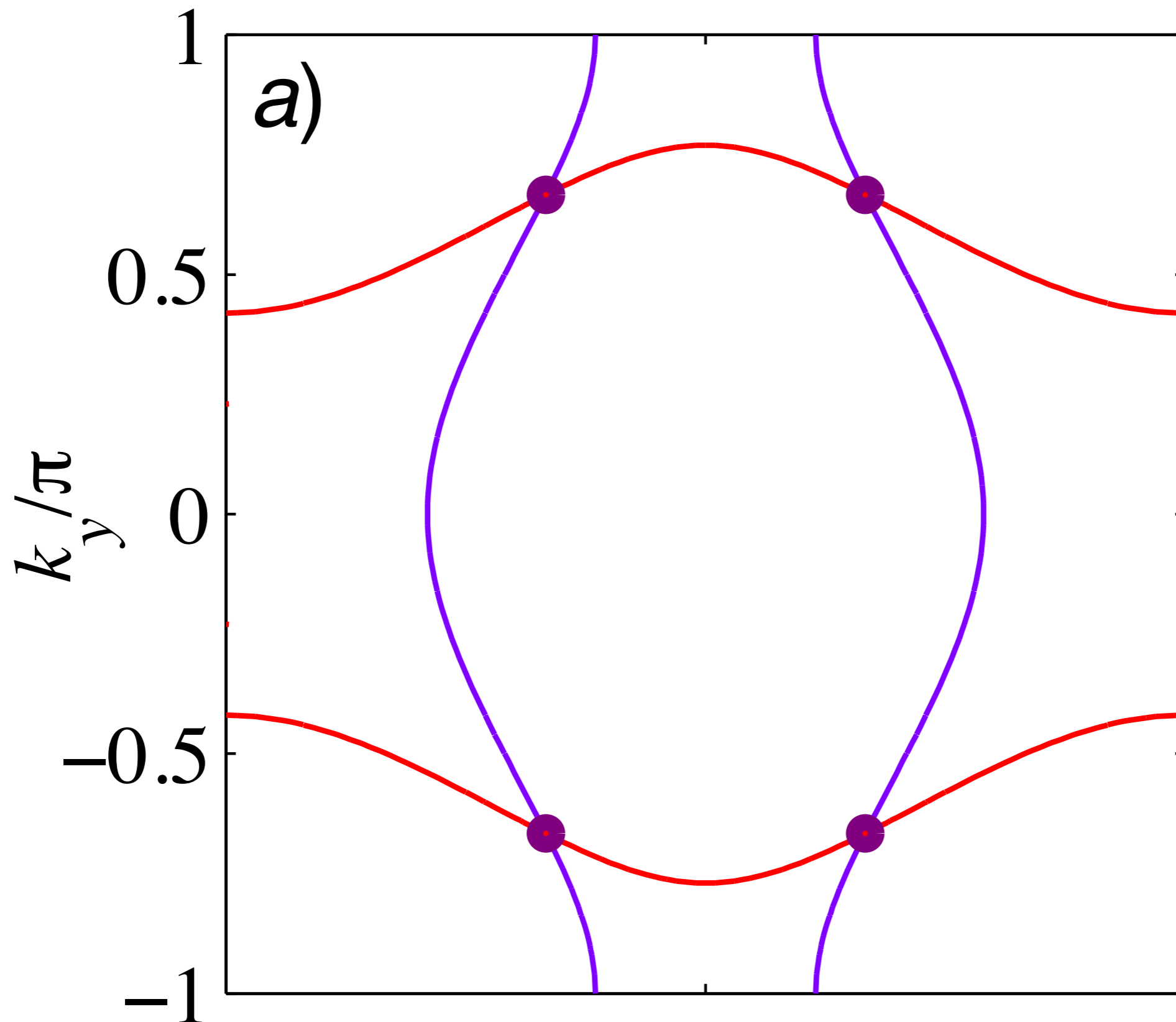
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Move one of the Fermi surface by (π, π)

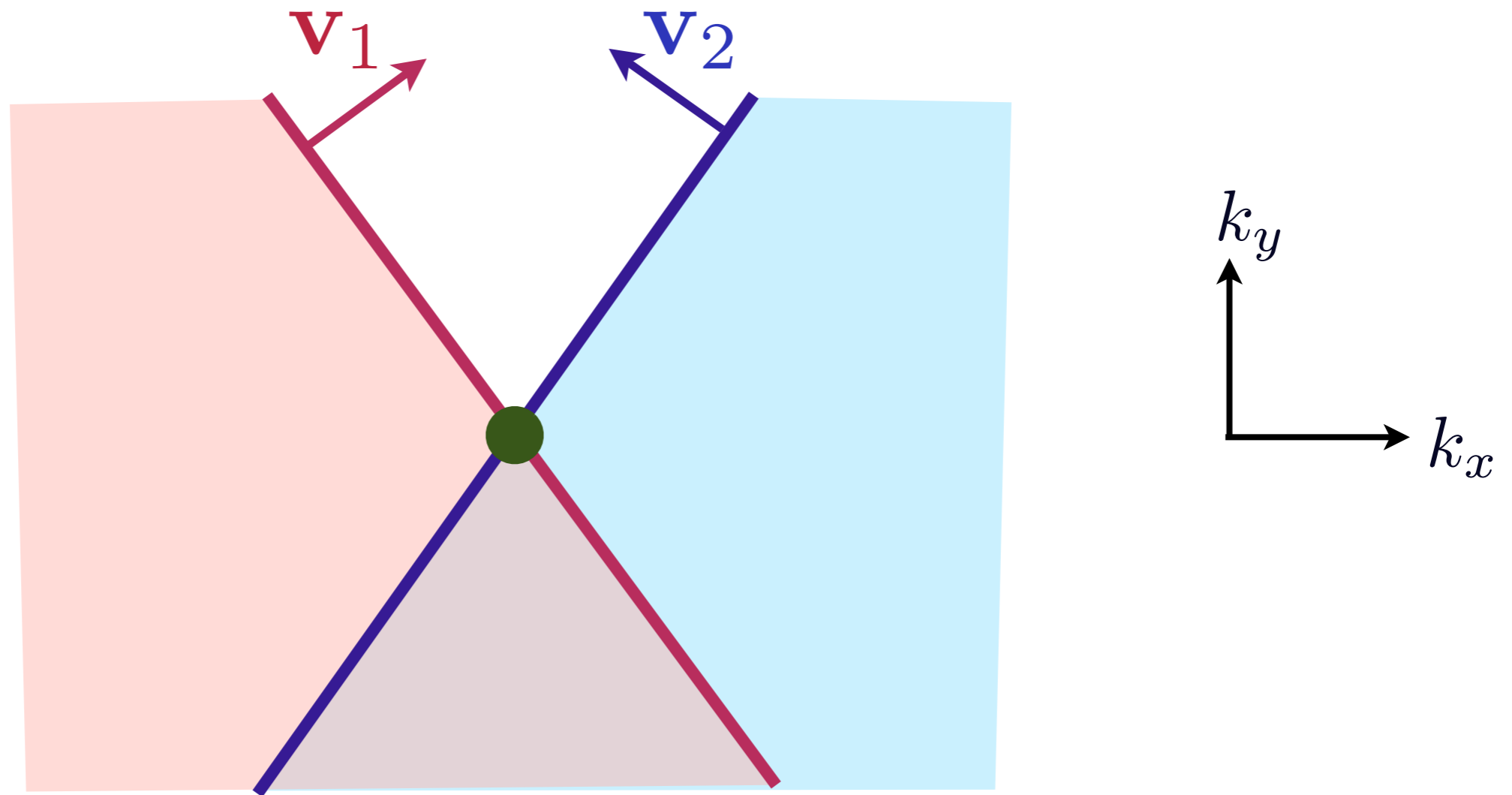
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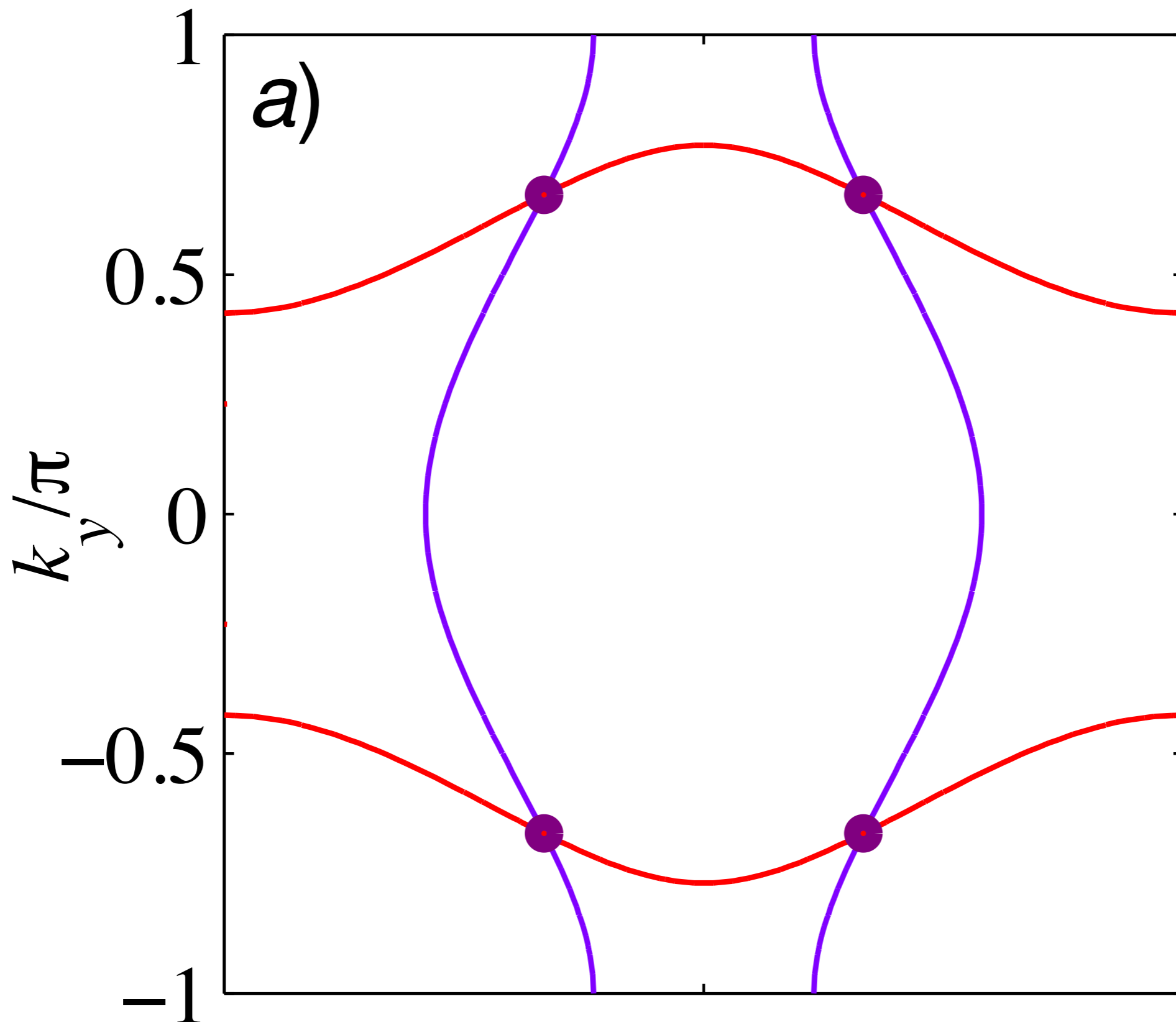
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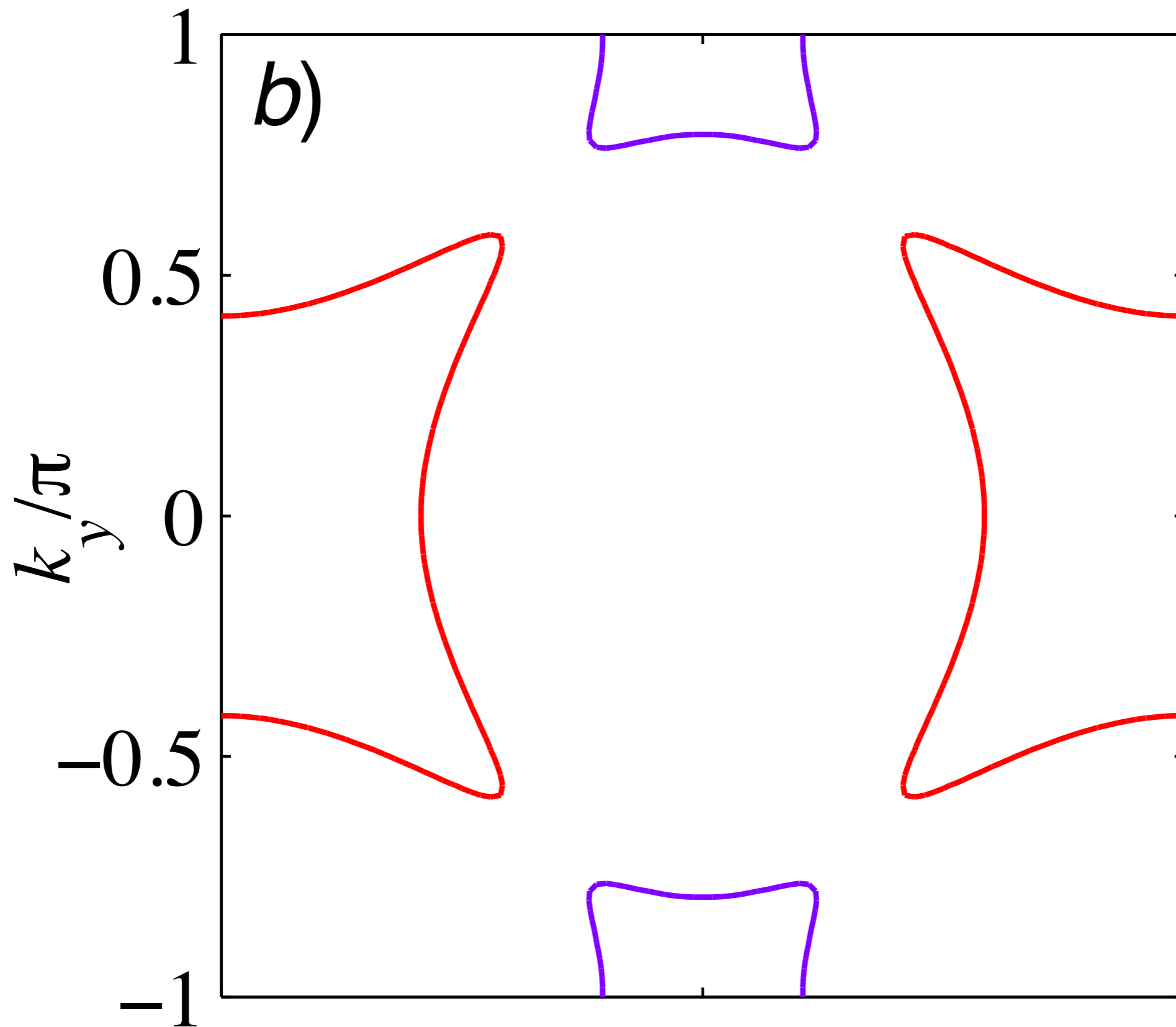


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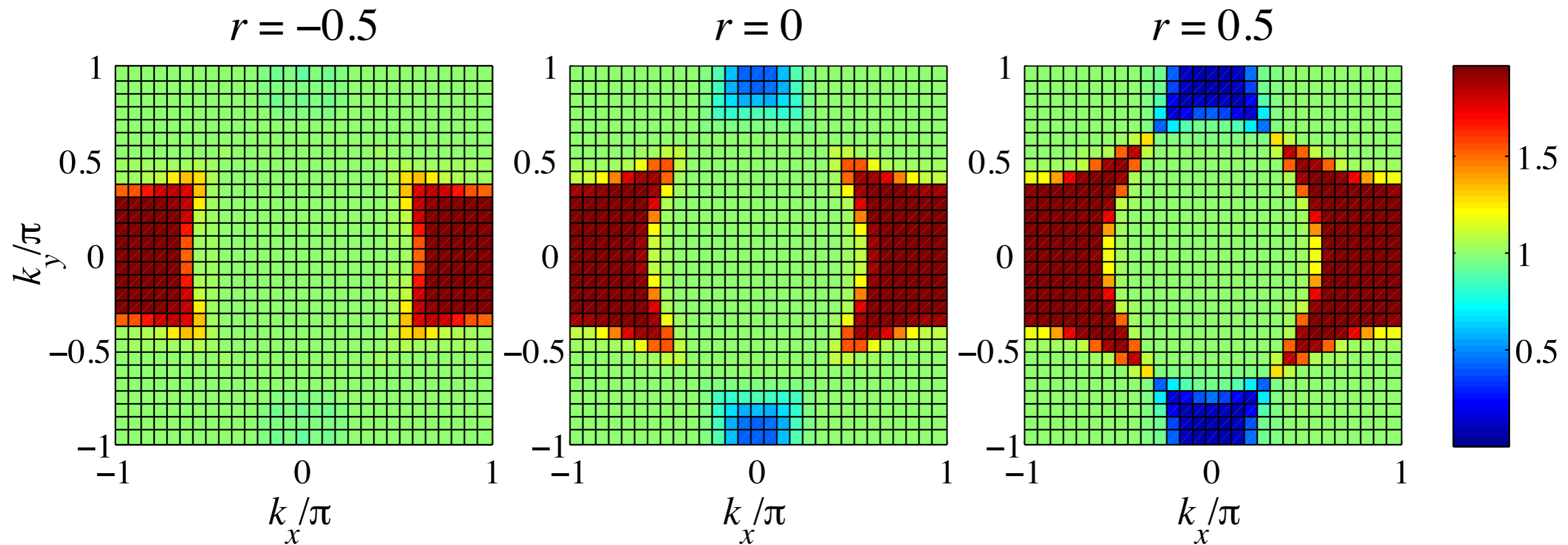
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Expected Fermi surfaces in the AFM ordered phase

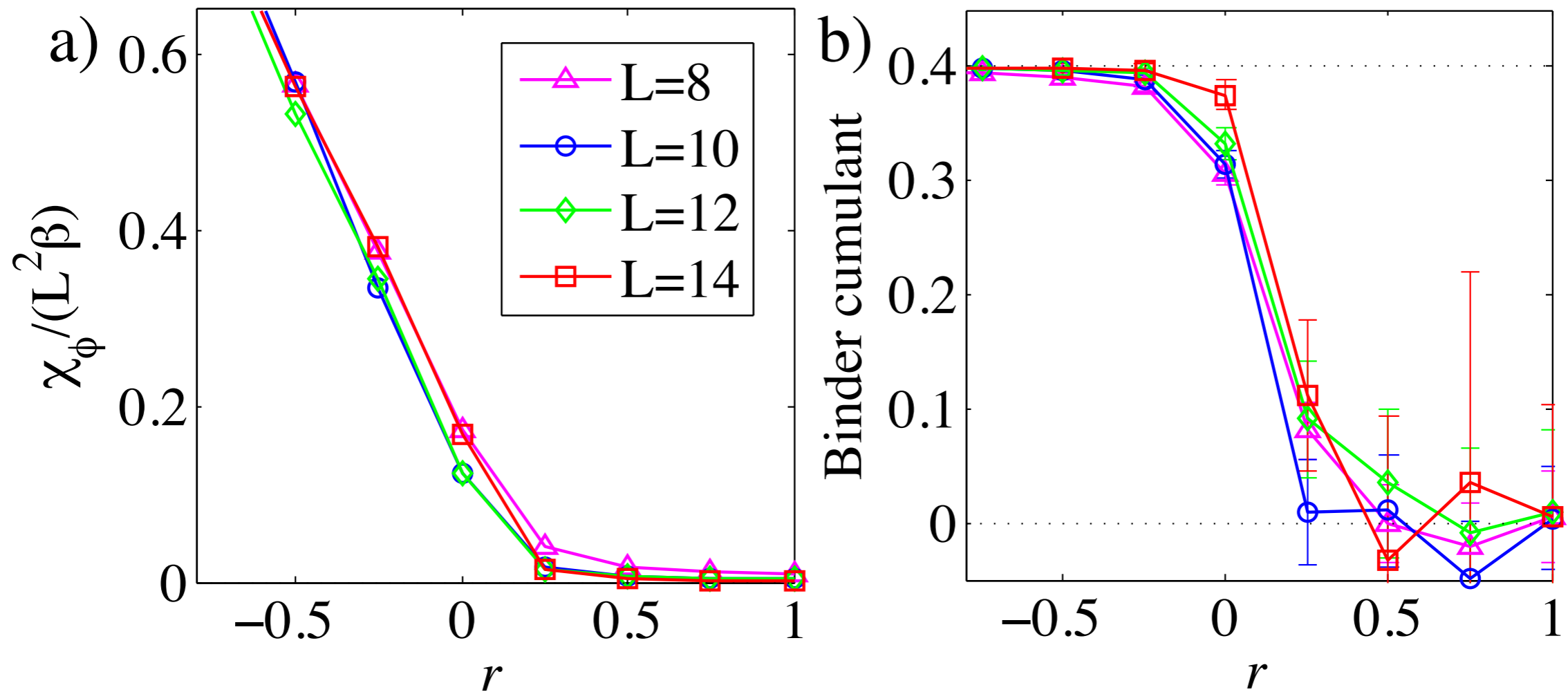
QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

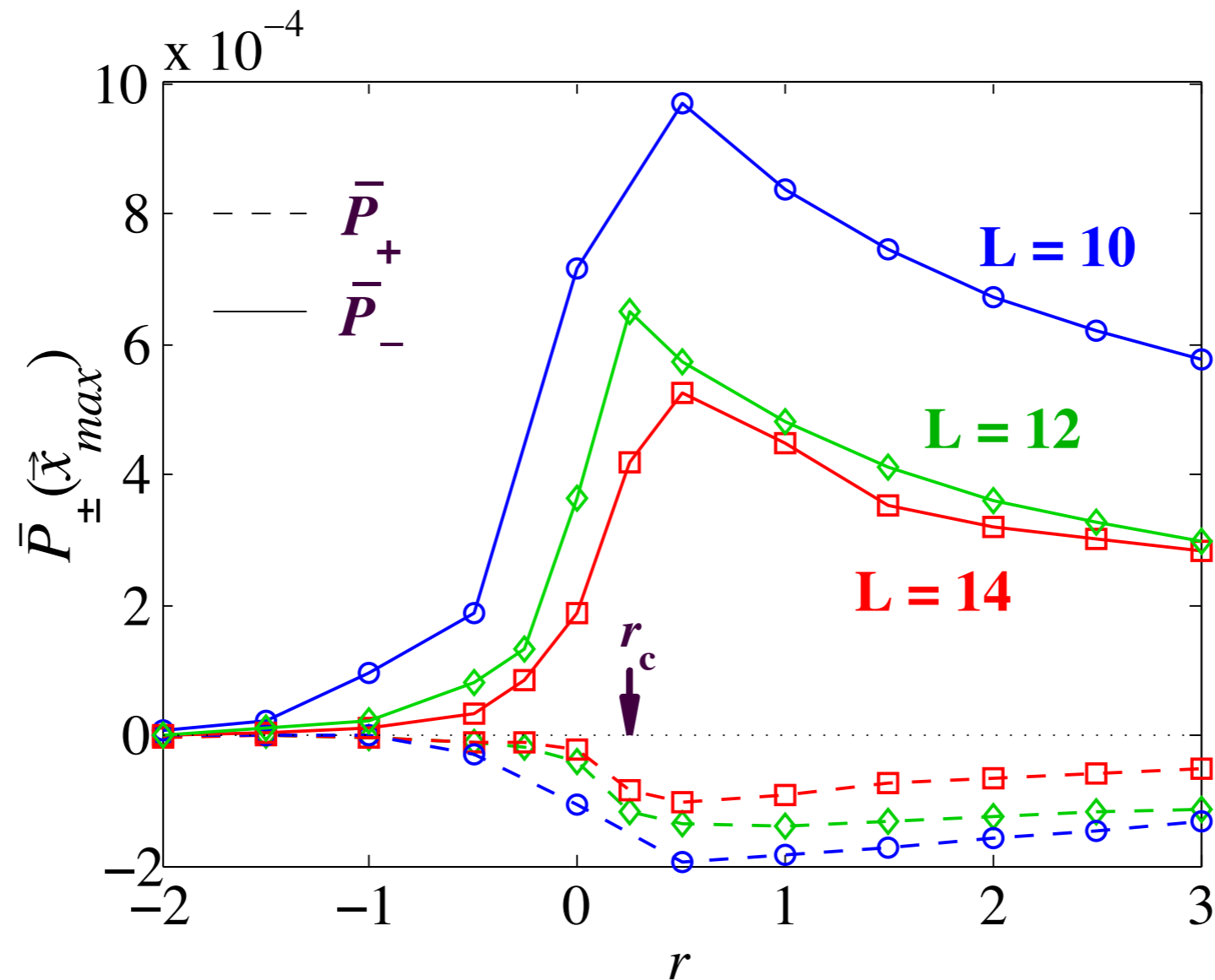
QMC for the onset of antiferromagnetism



AF susceptibility, χ_ϕ , and Binder cumulant
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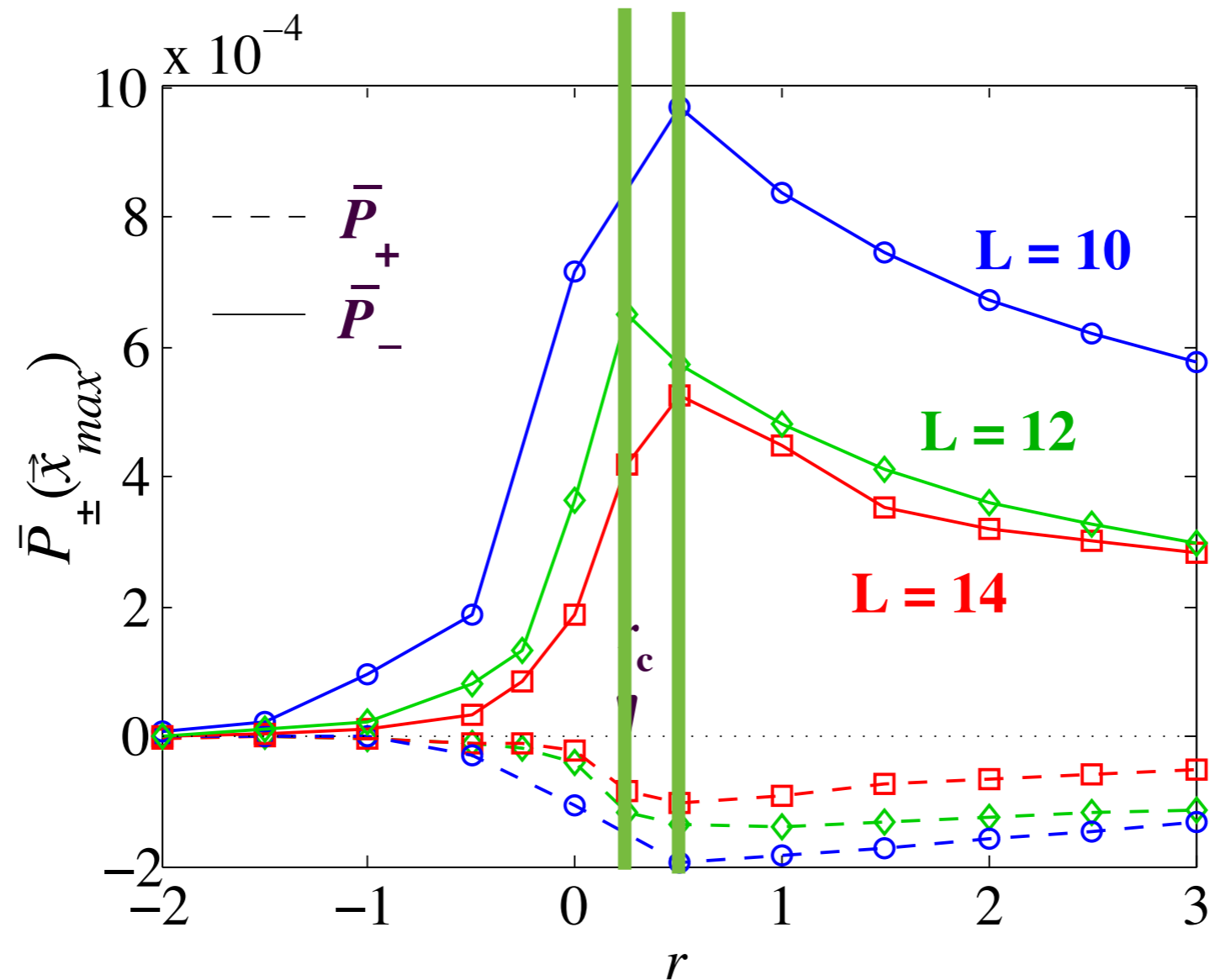
QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

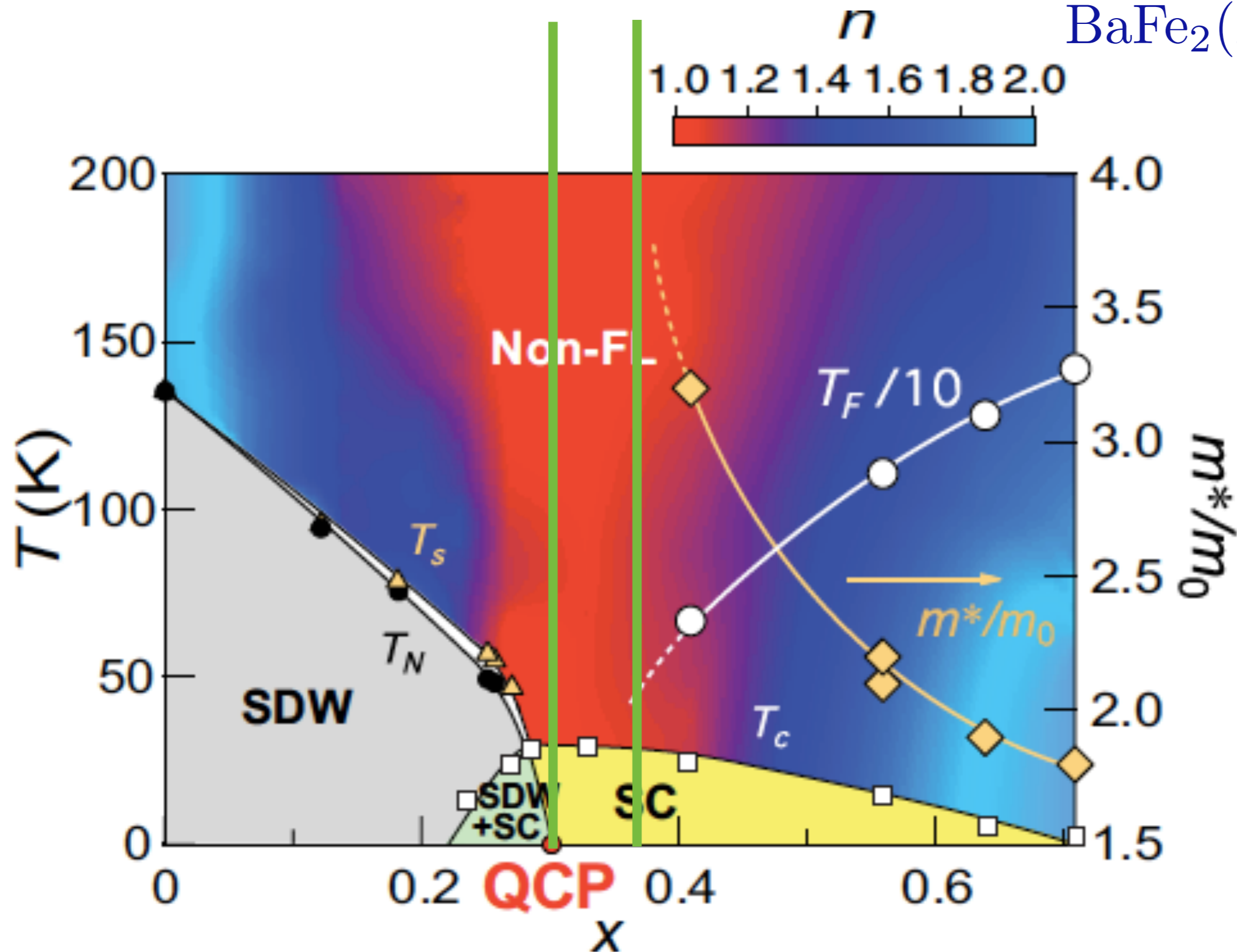
E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism



Notice shift between the position of the QCP in the superconductor, and the position of maximum pairing

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742



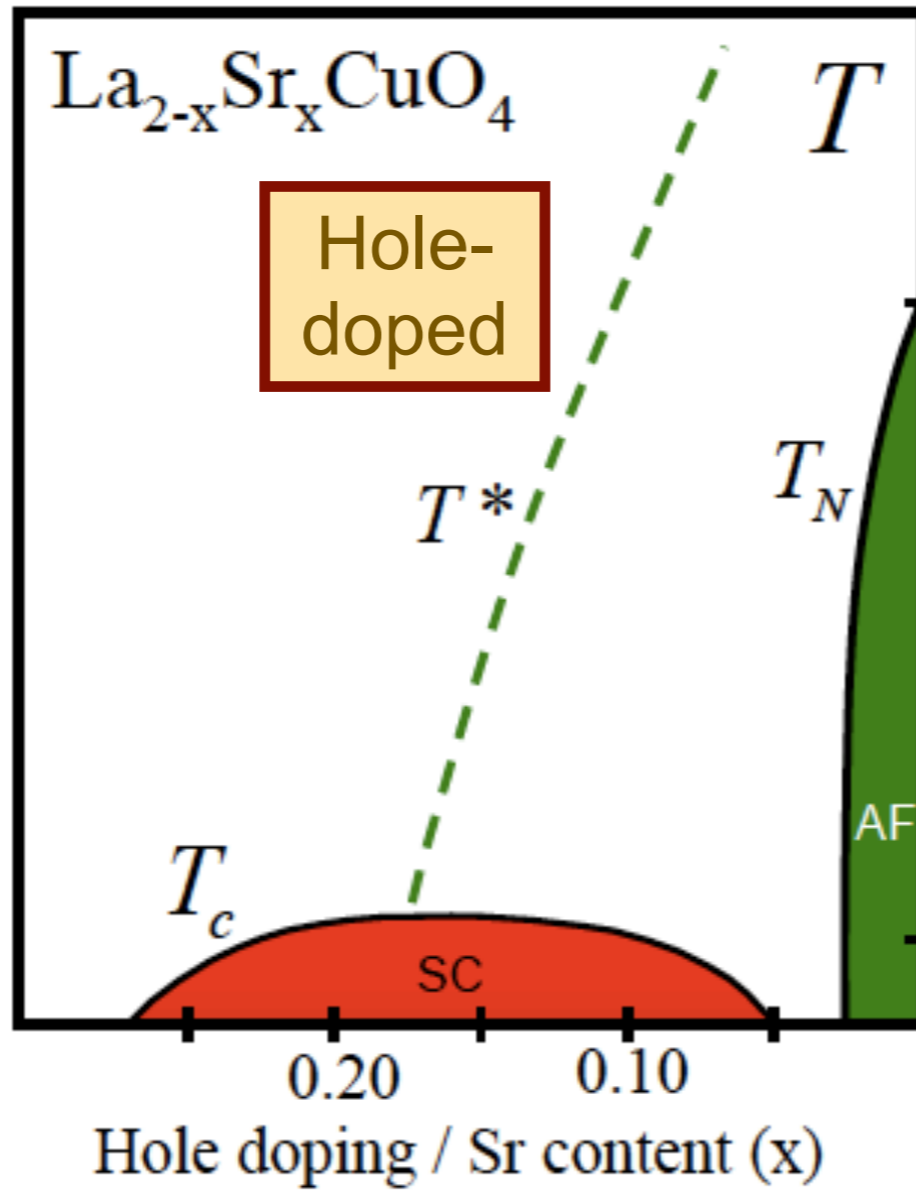
Notice shift between the position of the QCP in the superconductor, and the divergence in effective mass in the metal measured at high magnetic fields

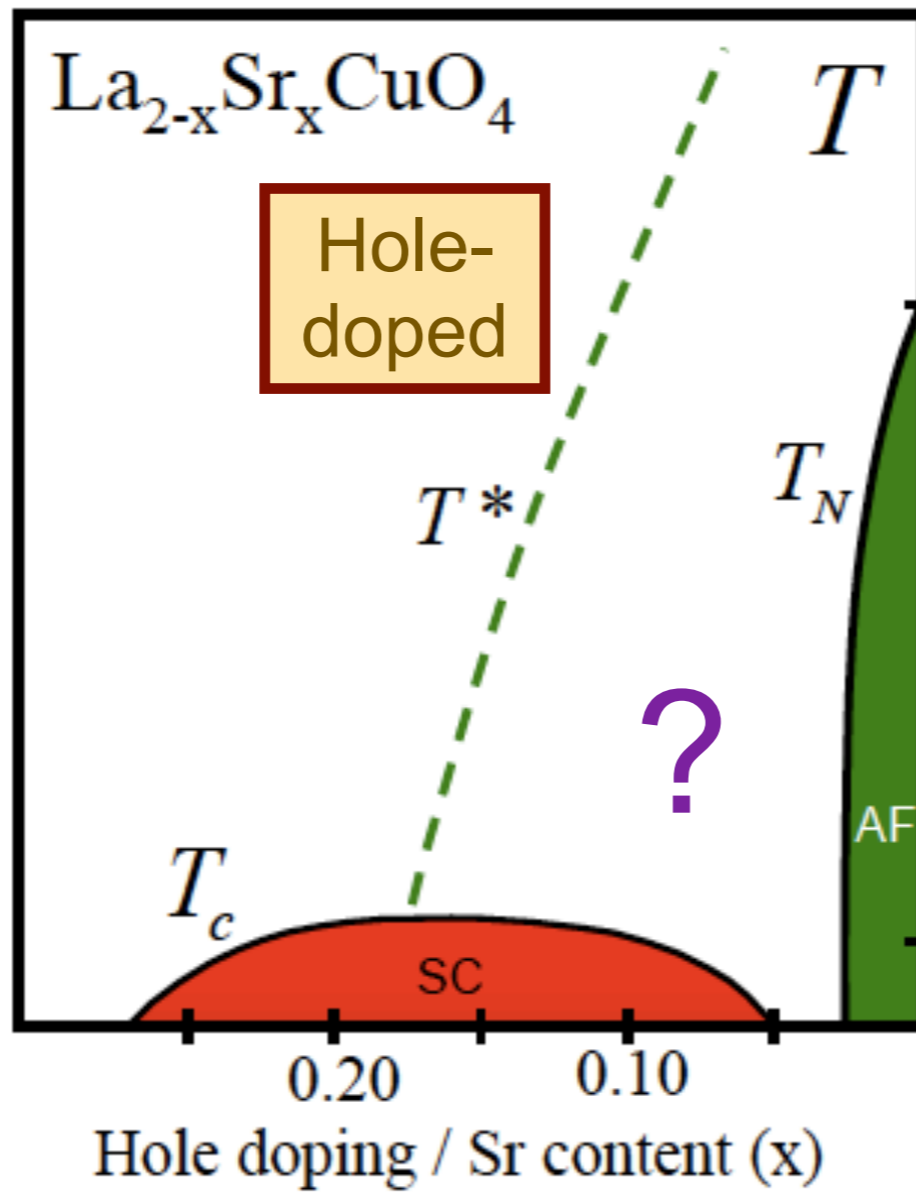
Outline

1. Phenomenology of the onset of antiferromagnetism in a metal
2. Quantum field theory of the onset of antiferromagnetism in a metal
3. Quantum Monte Carlo without the sign problem
4. Fractionalization in metals,
and the hole-doped cuprates

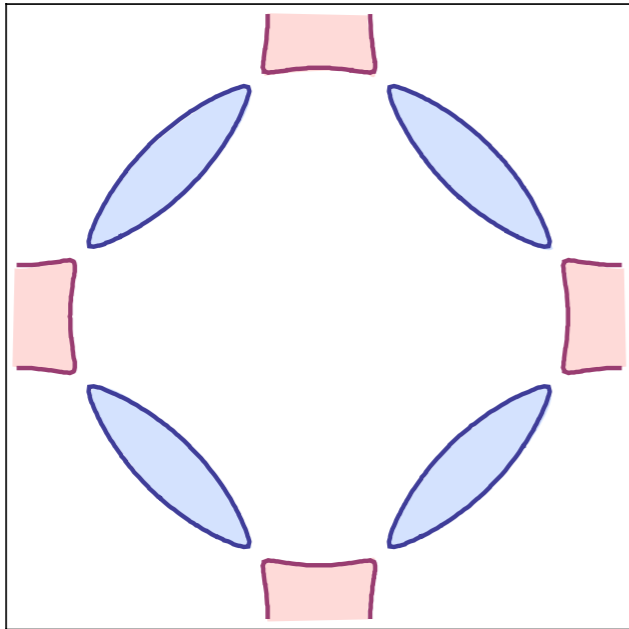
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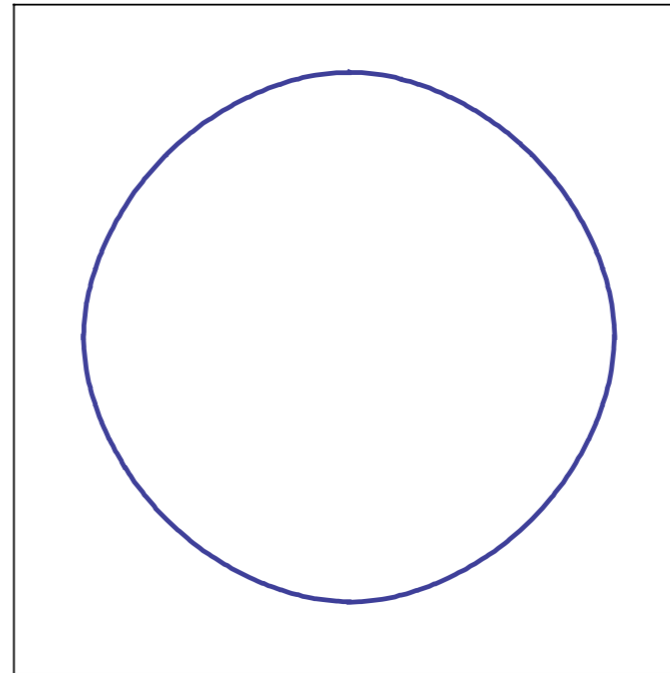


Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

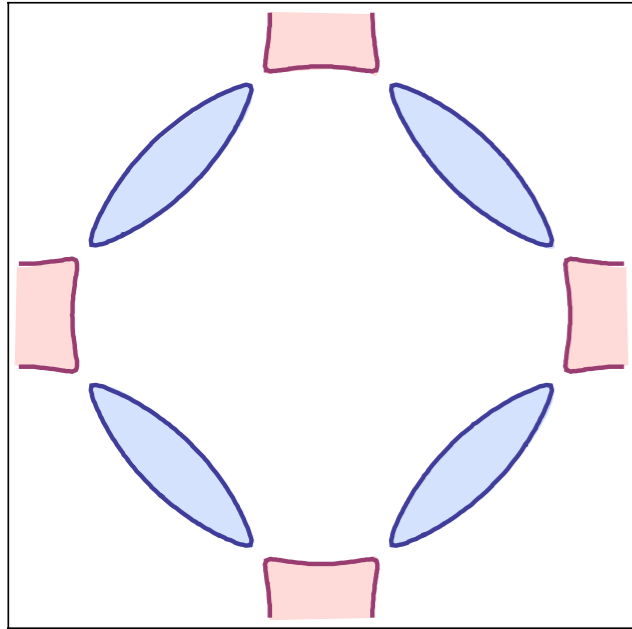


$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large"
Fermi surface

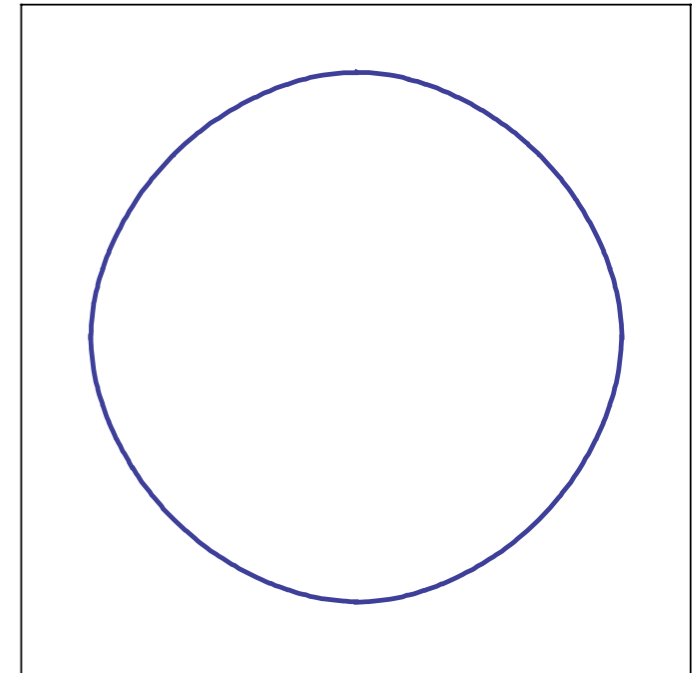


Separating onset of SDW order and Fermi surface reconstruction



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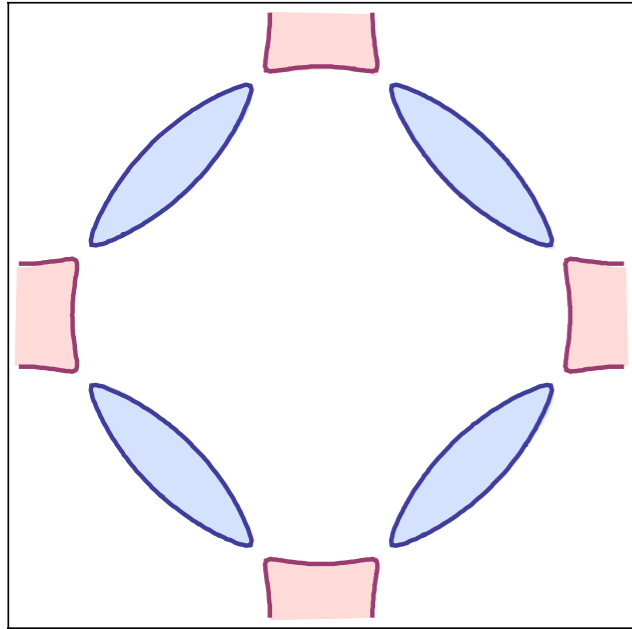


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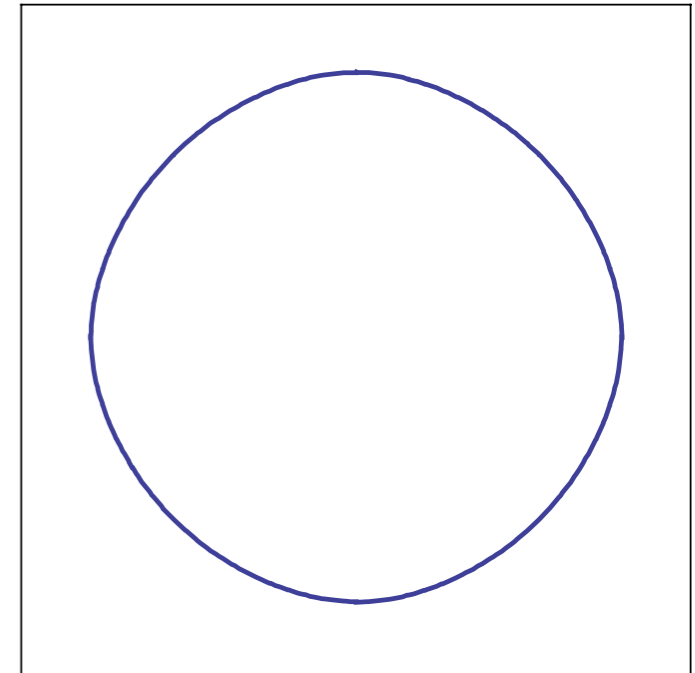


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Electron and/or hole
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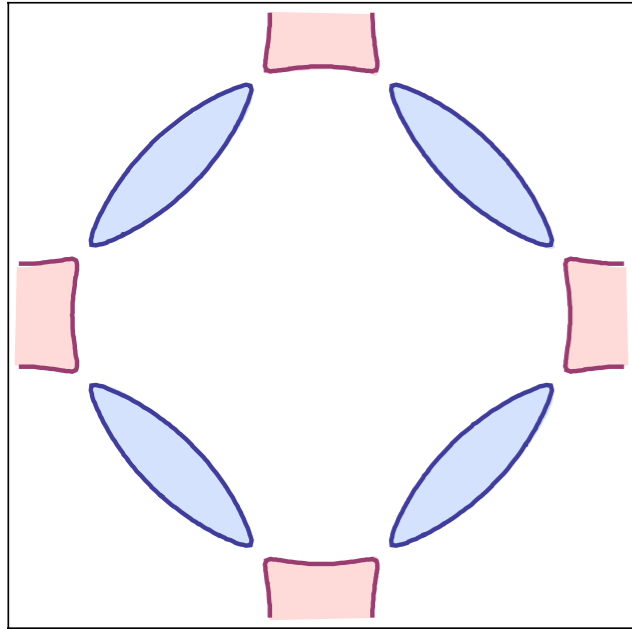


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T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

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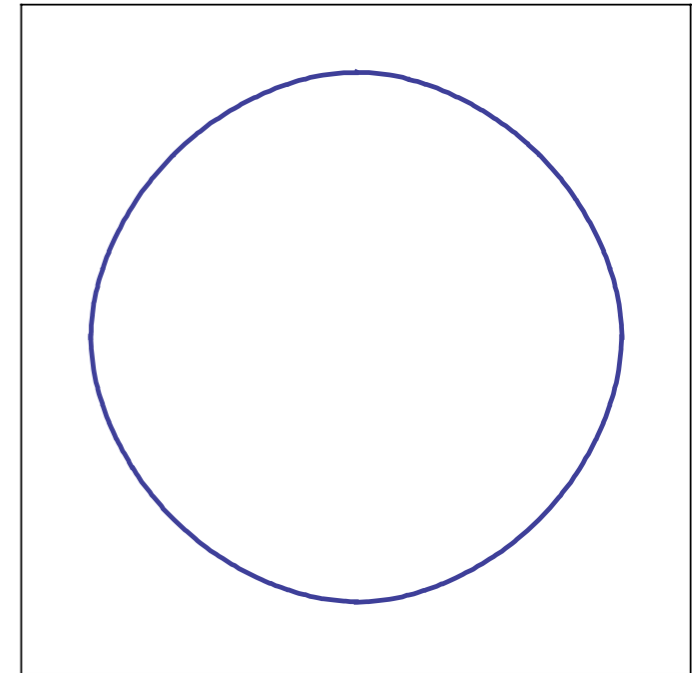
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Fractionalized Fermi
liquid (FL*) phase
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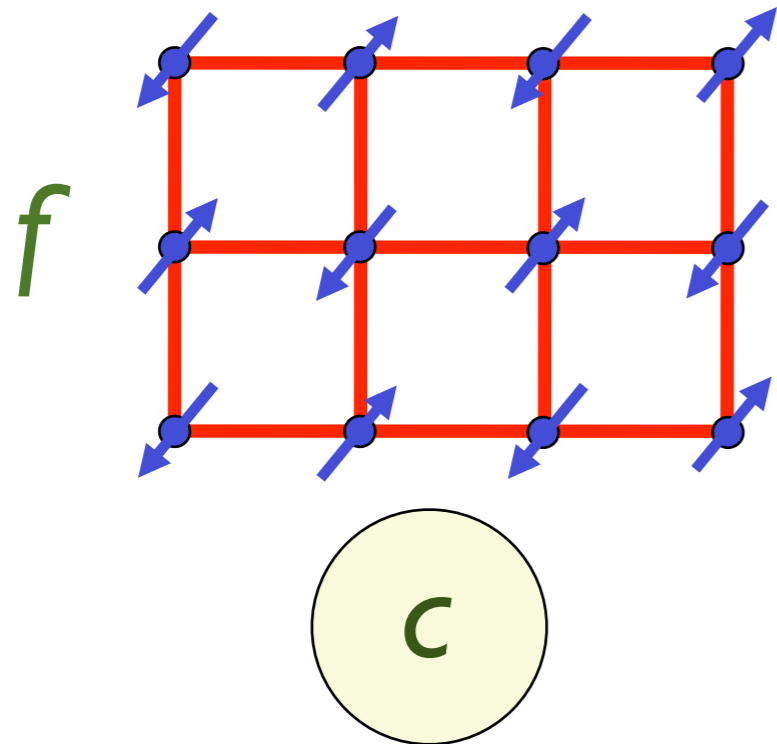


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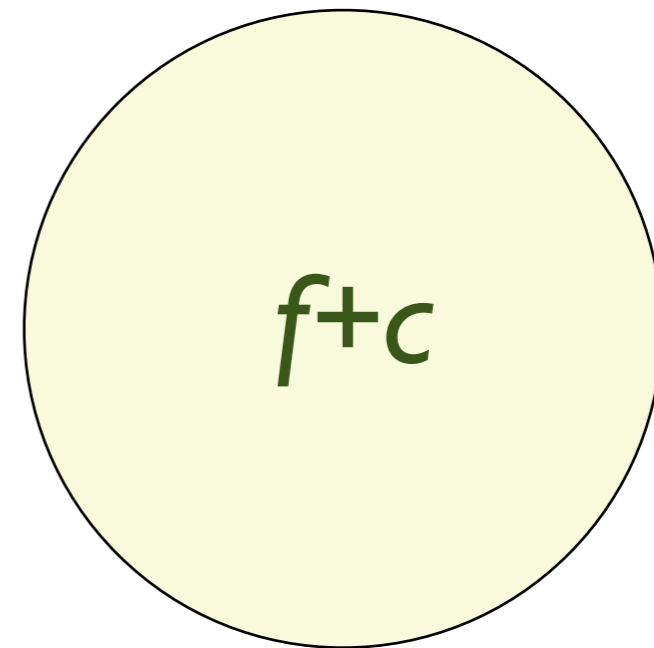
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Magnetic order and the heavy Fermi liquid in the Kondo lattice



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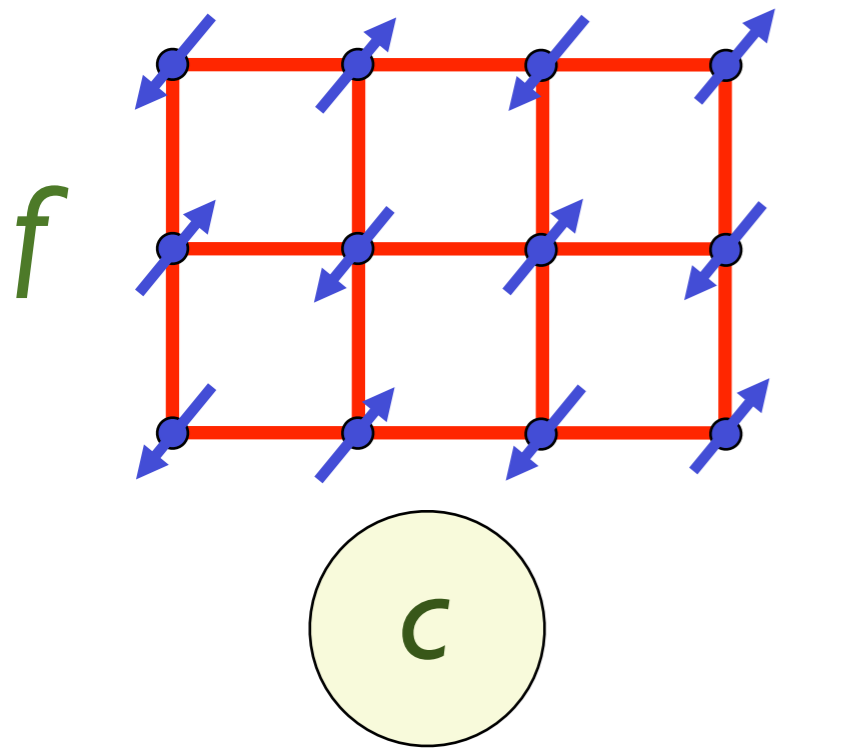
Magnetic Metal:
f-electron moments
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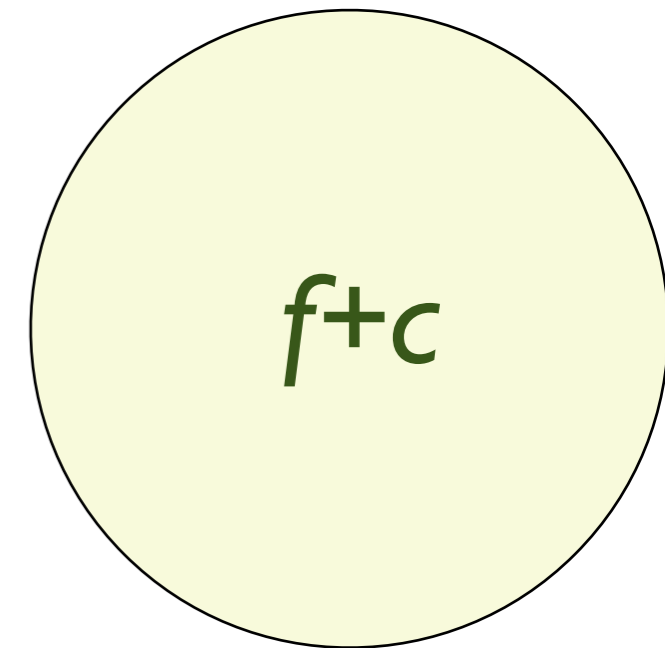
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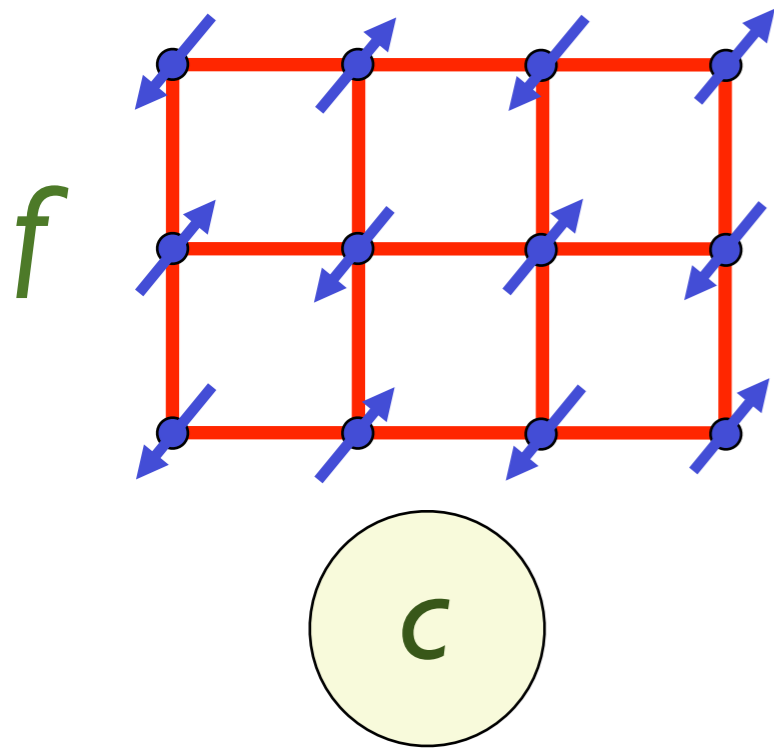


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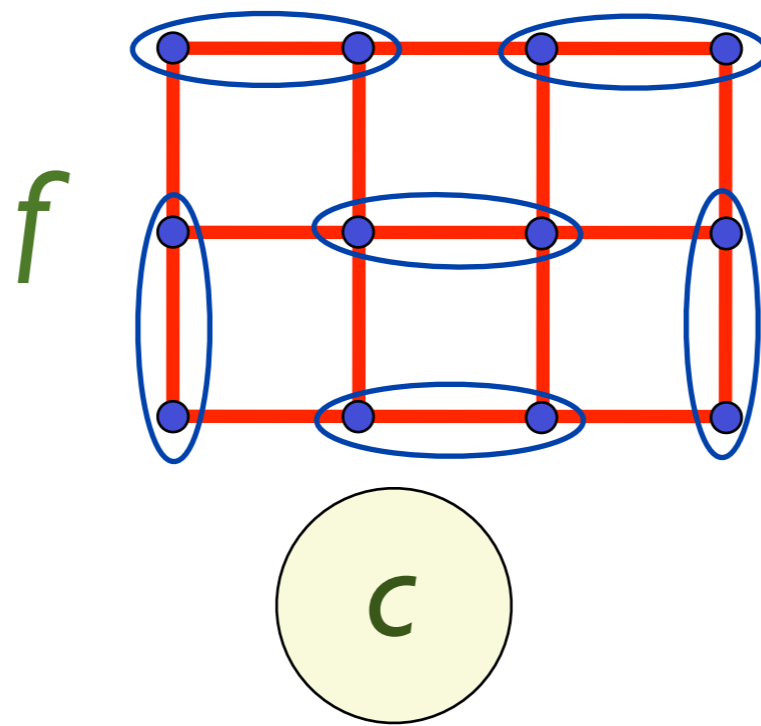
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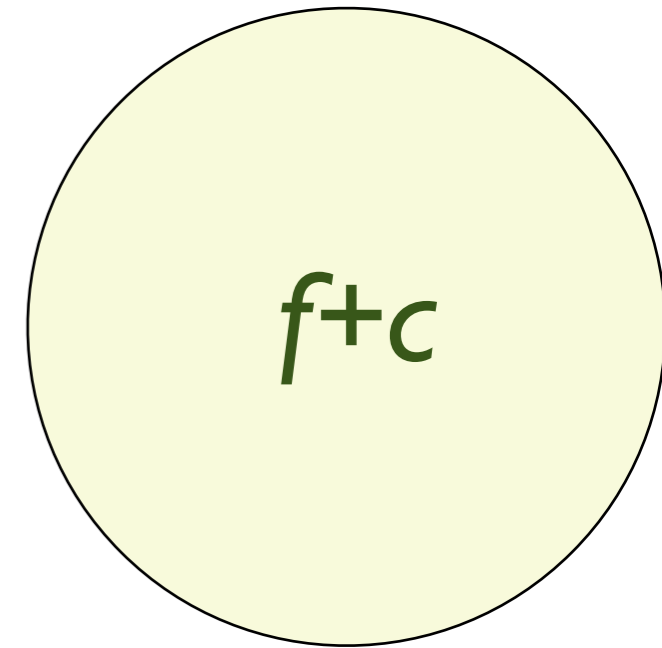
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Conduction electron
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and
spin-liquid of
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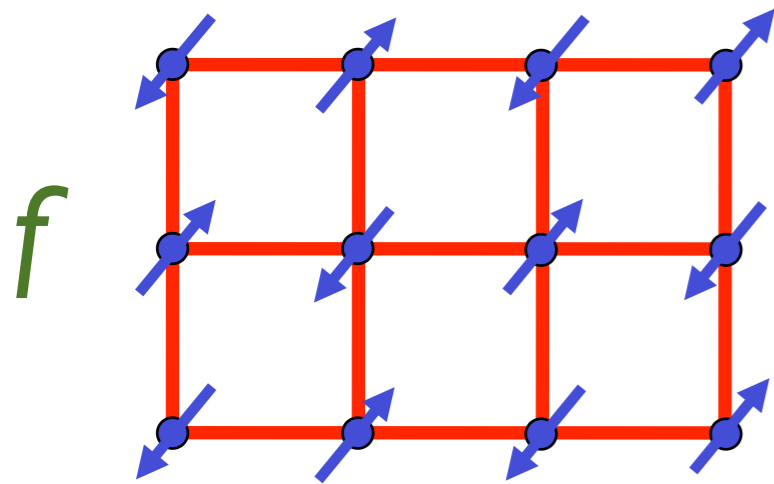


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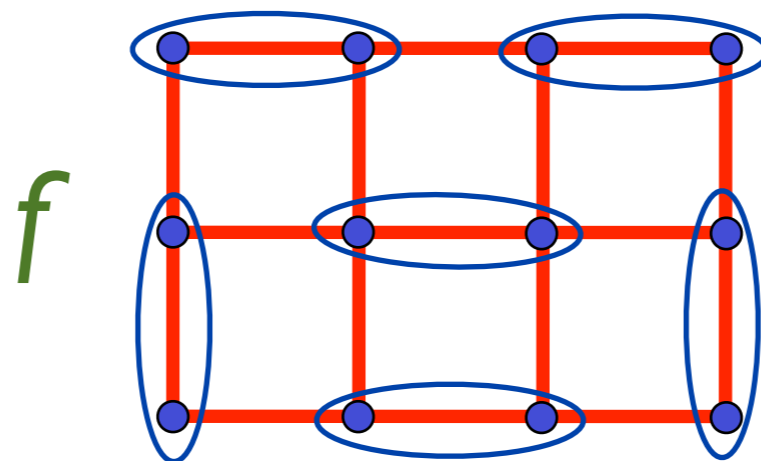
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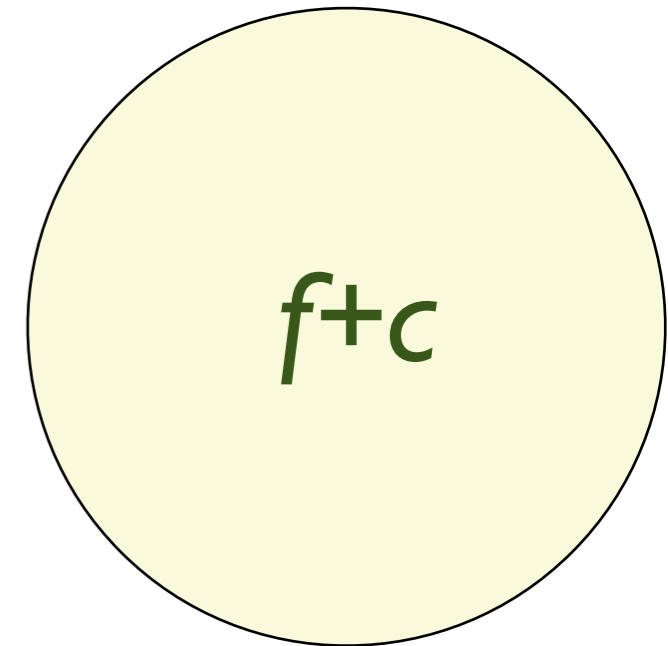
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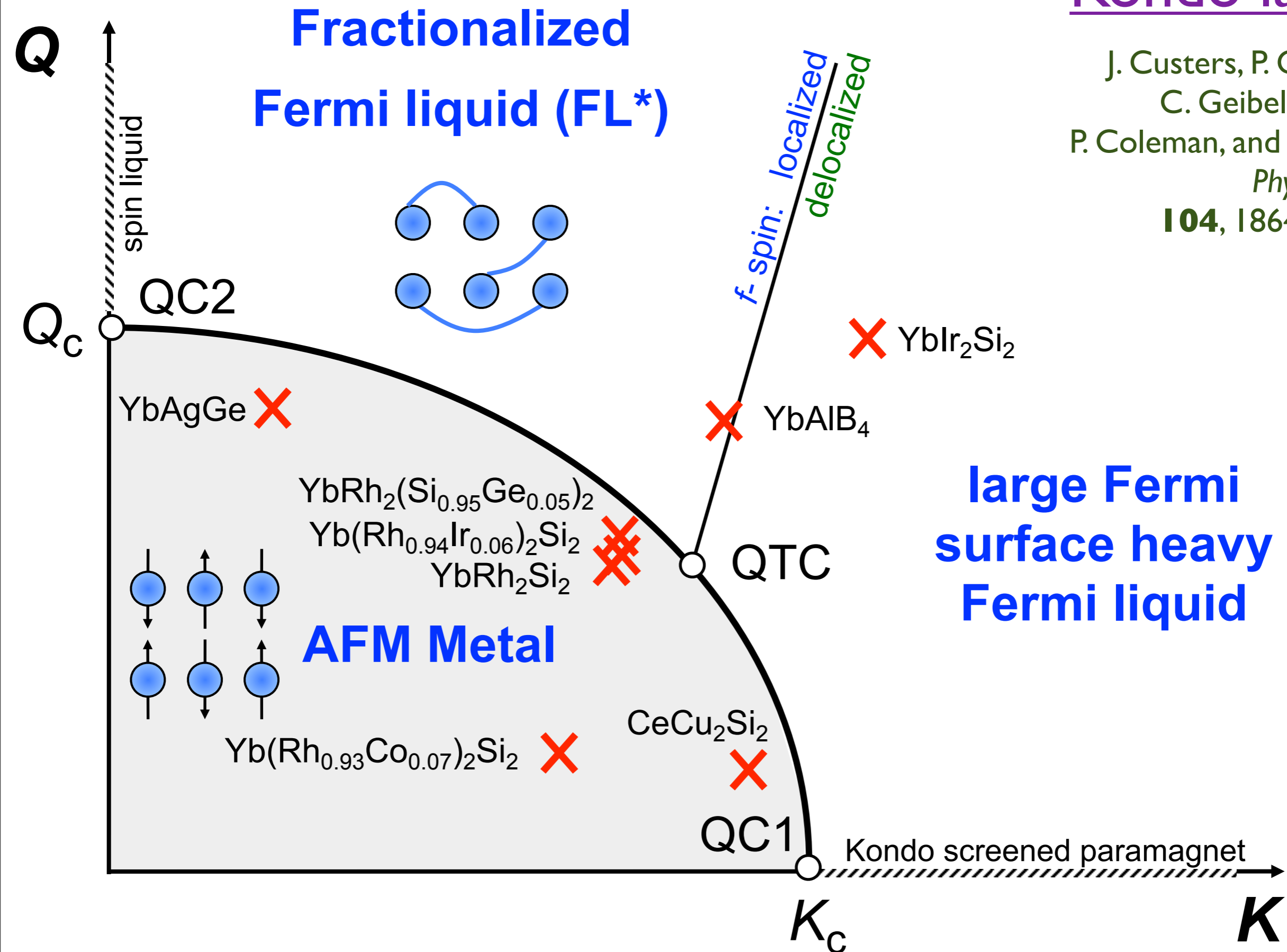
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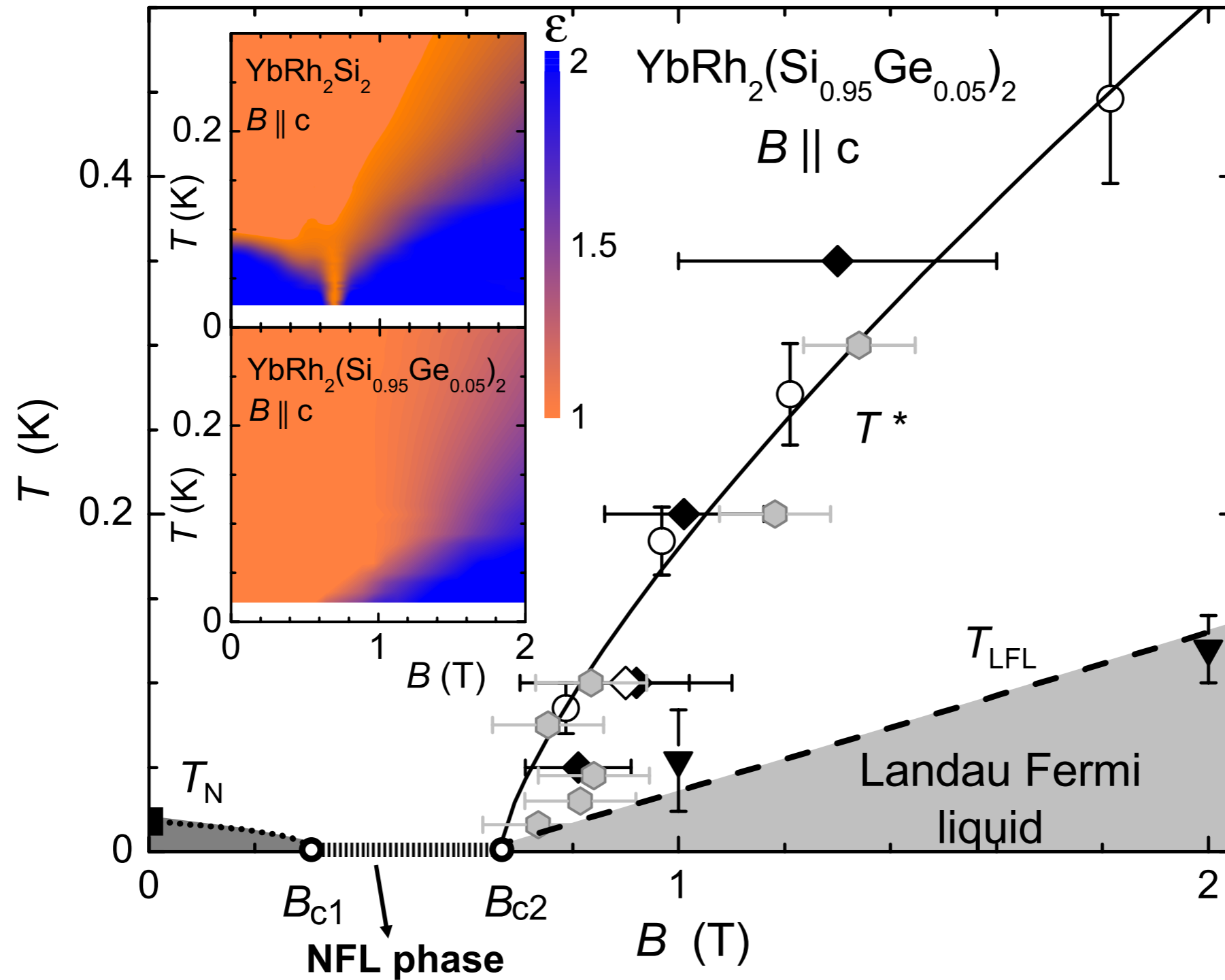
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Experimental perspective on same phase diagrams of Kondo lattice

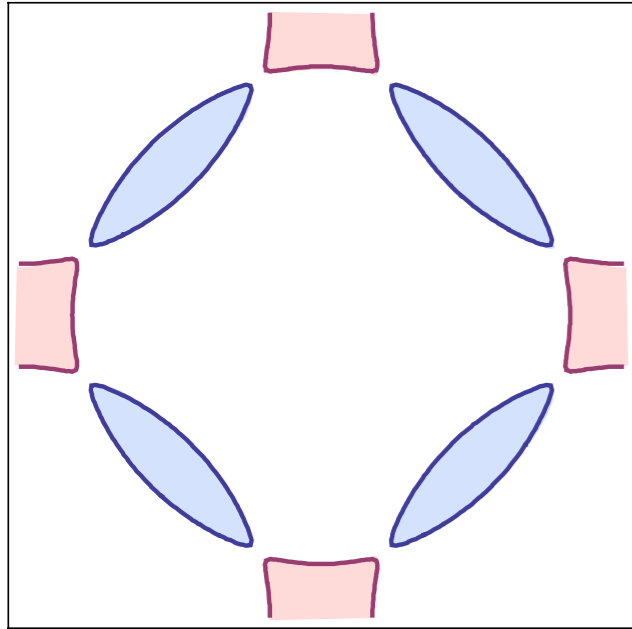
J. Custers, P. Gegenwart,
C. Geibel, F. Steglich,
P. Coleman, and S. Paschen,
Phys. Rev. Lett.
104, 186402 (2010)





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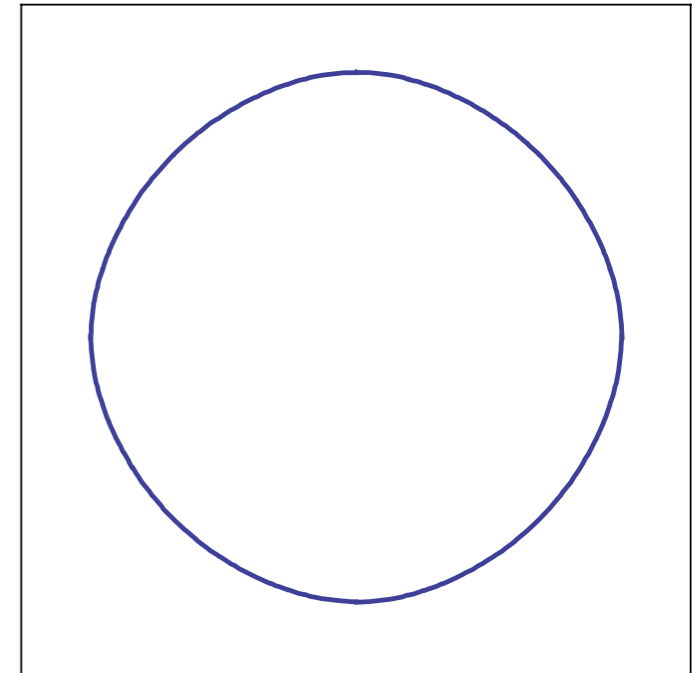
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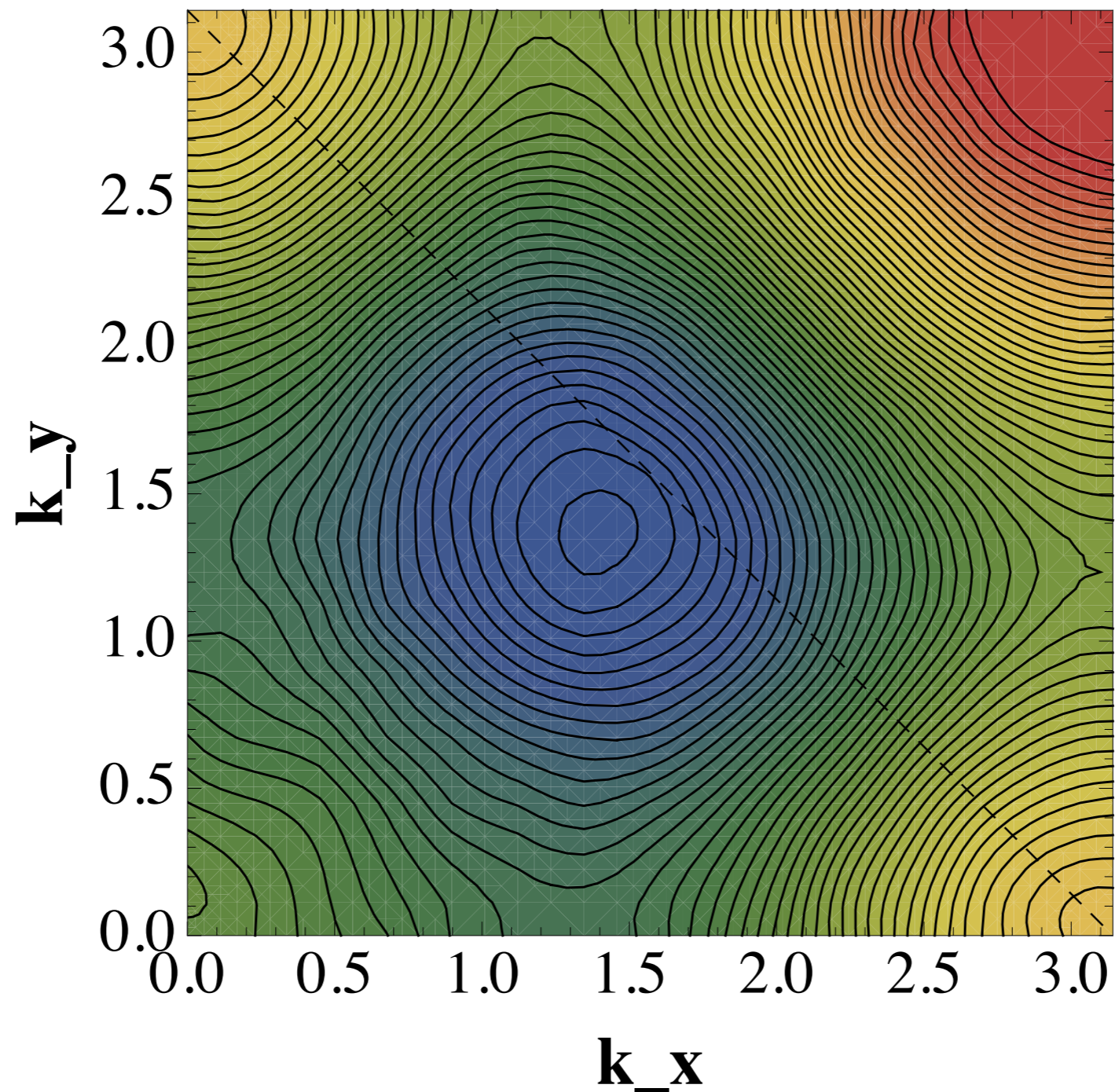
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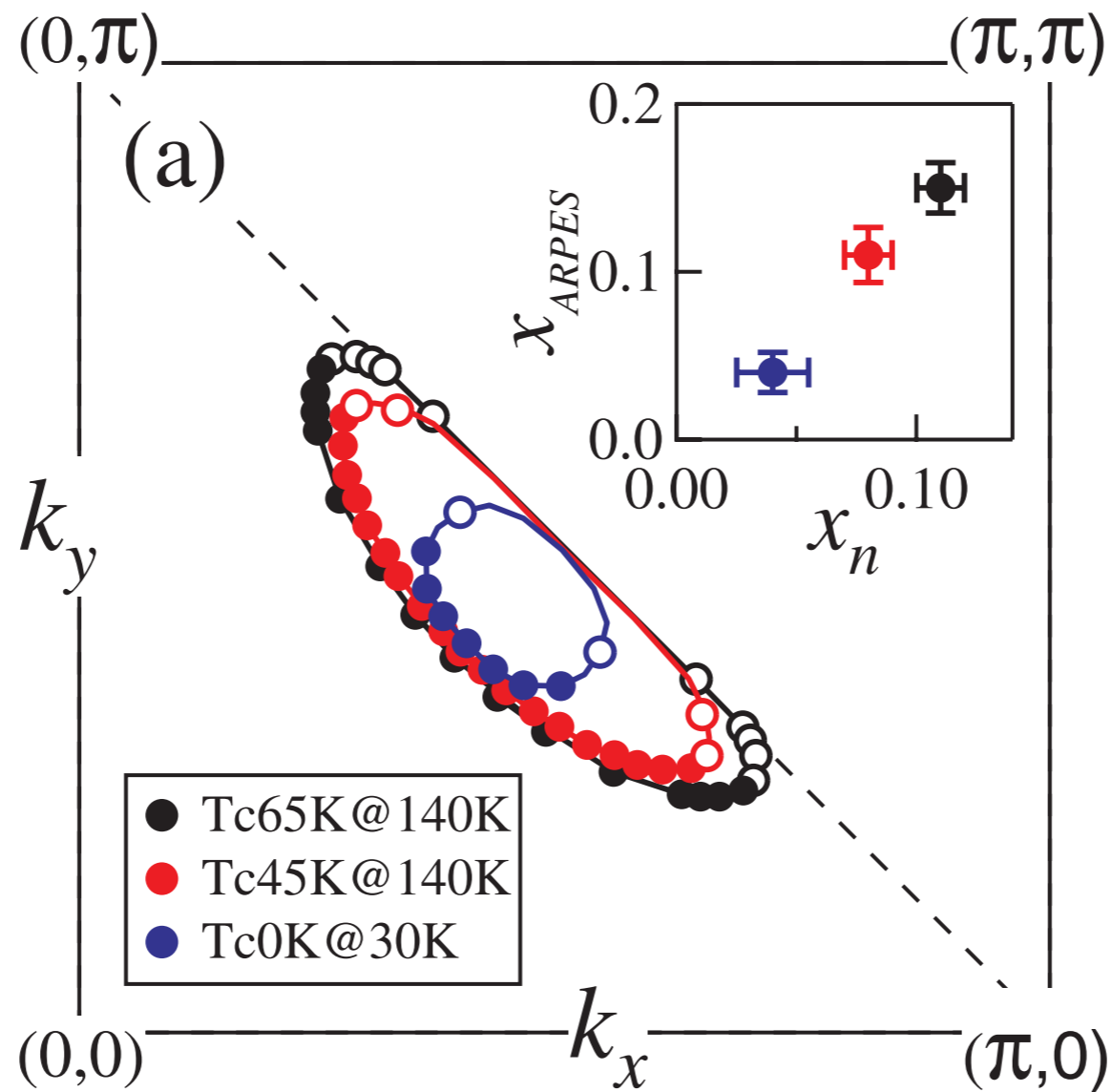
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T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)



Hole pocket of a \mathbb{Z}_2 -FL* phase
in a *single-band* t - J model

M. Punk and S. Sachdev, *Phys. Rev. B* **85**, 195123 (2012)



Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors

H.-B. Yang,¹ J. D. Rameau,¹ Z.-H. Pan,¹ G. D. Gu,¹ P. D. Johnson,¹ H. Claus,² D. G. Hinks,² and T. E. Kidd³

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

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T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

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- These topological excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa’s proof of the Luttinger theorem.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Questions

- *Can quantum fluctuations near the onset of antiferromagnetism induce higher temperature superconductivity ?*
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- *What is the physics of the strange metal ?*

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Strongly-coupled quantum criticality of Fermi surface change in a metal

Thank you !

