

Entanglement, holography, and the quantum phases of matter

Condensed Matter Physics in the City
London, June 20, 2012

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World
arXiv:1203.4565

sachdev.physics.harvard.edu





Liza Huijse



Max Metlitski



Brian Swingle

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement

“Complex entangled” states of quantum matter in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals in higher temperature superconductors, spin liquids

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topological field theory

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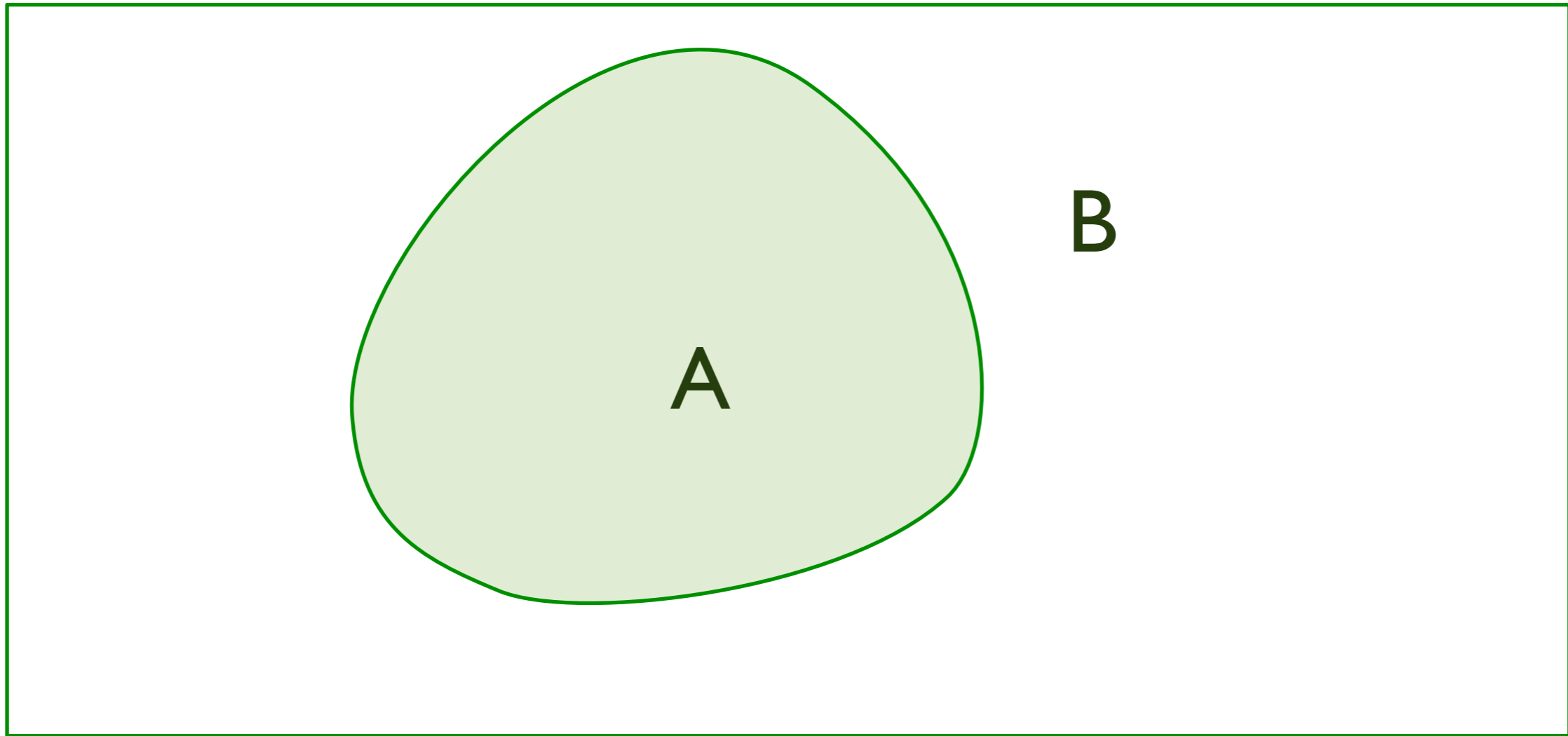
conformal field theory

Compressible quantum matter

Strange metals in higher temperature superconductors, spin liquids

?

Entanglement entropy



$|\Psi\rangle \Rightarrow$ Ground state of entire system,
 $\rho = |\Psi\rangle\langle\Psi|$

$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_E = -\text{Tr}(\rho_A \ln \rho_A)$

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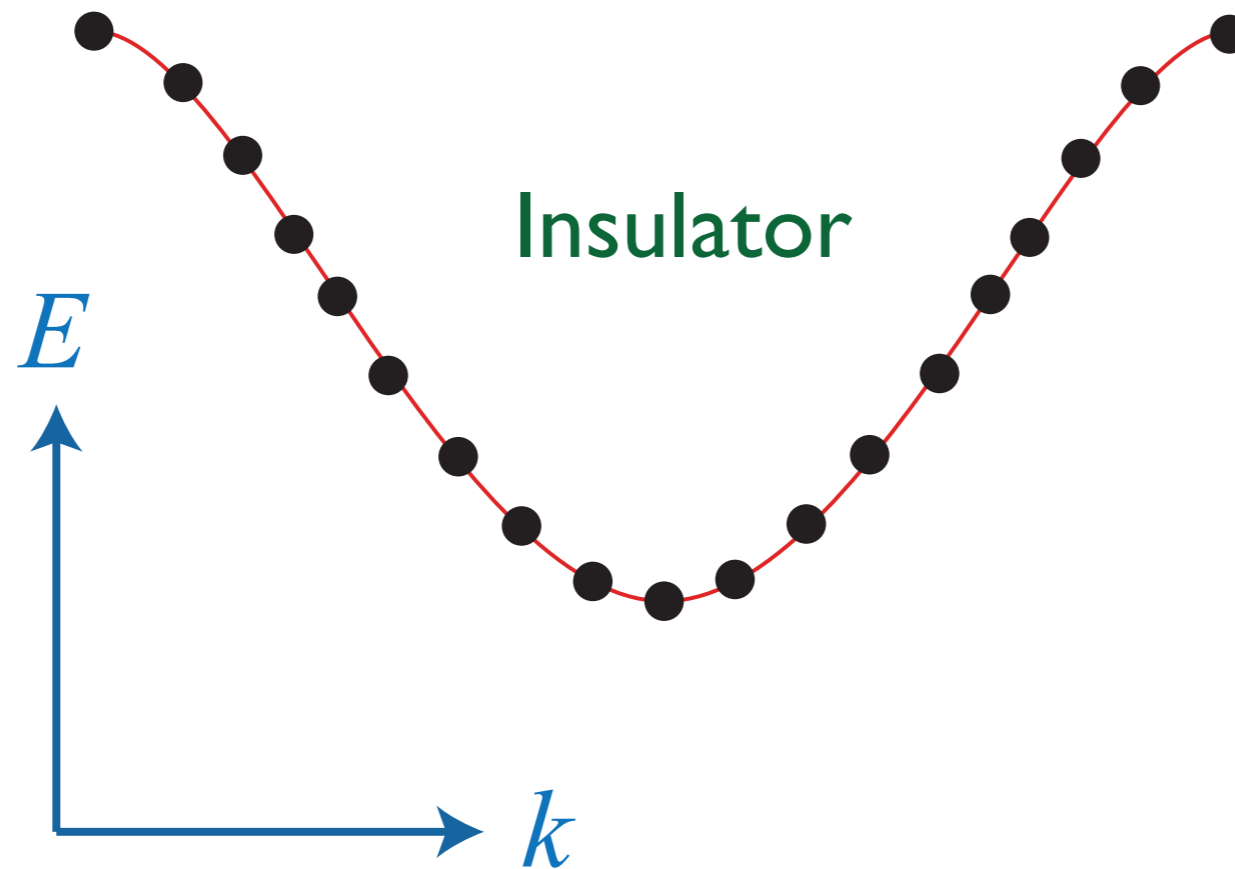
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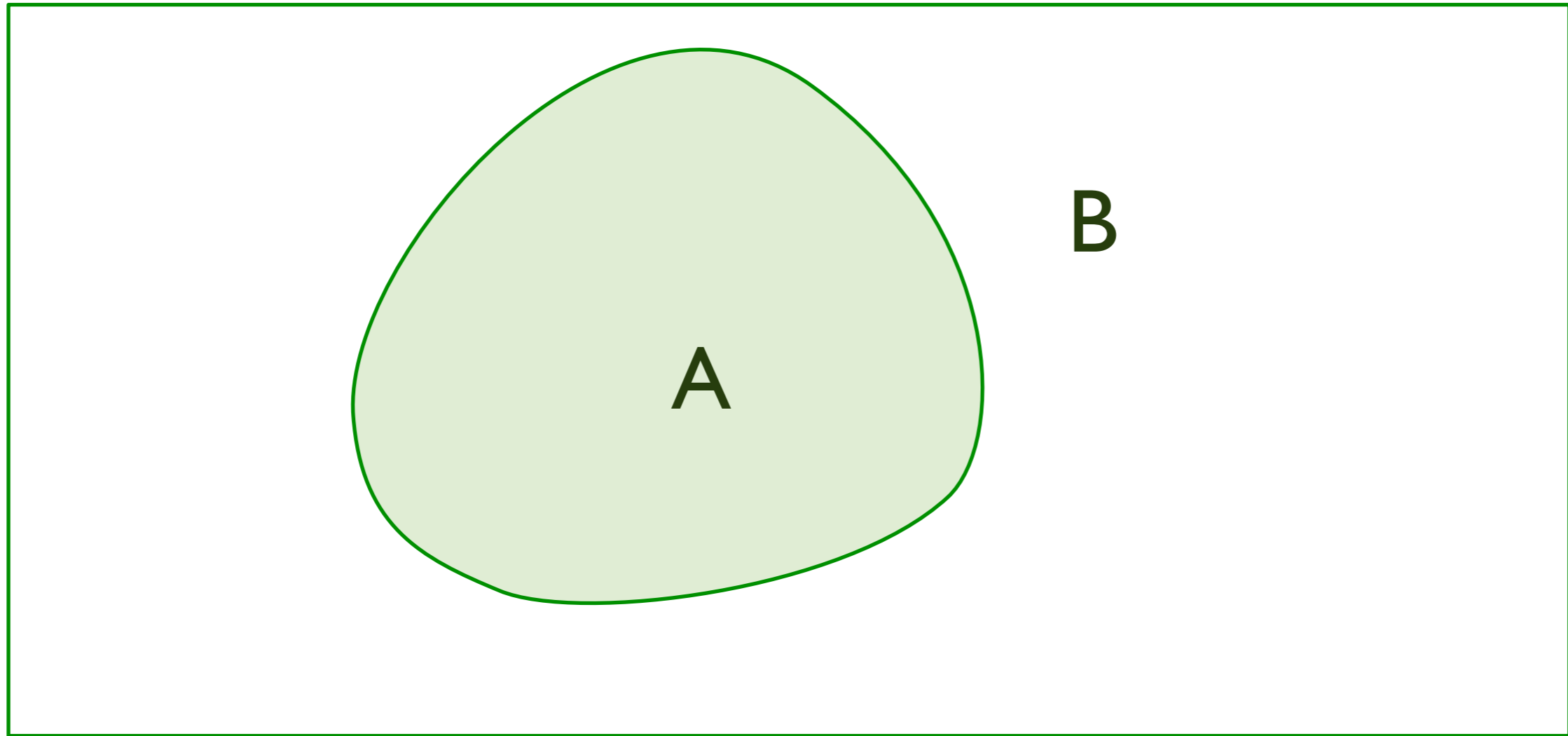
Strange metals in higher temperature superconductors, spin liquids

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator



$$S_E = aP - b \exp(-cP)$$

where P is the surface area (perimeter) of the boundary between A and B.

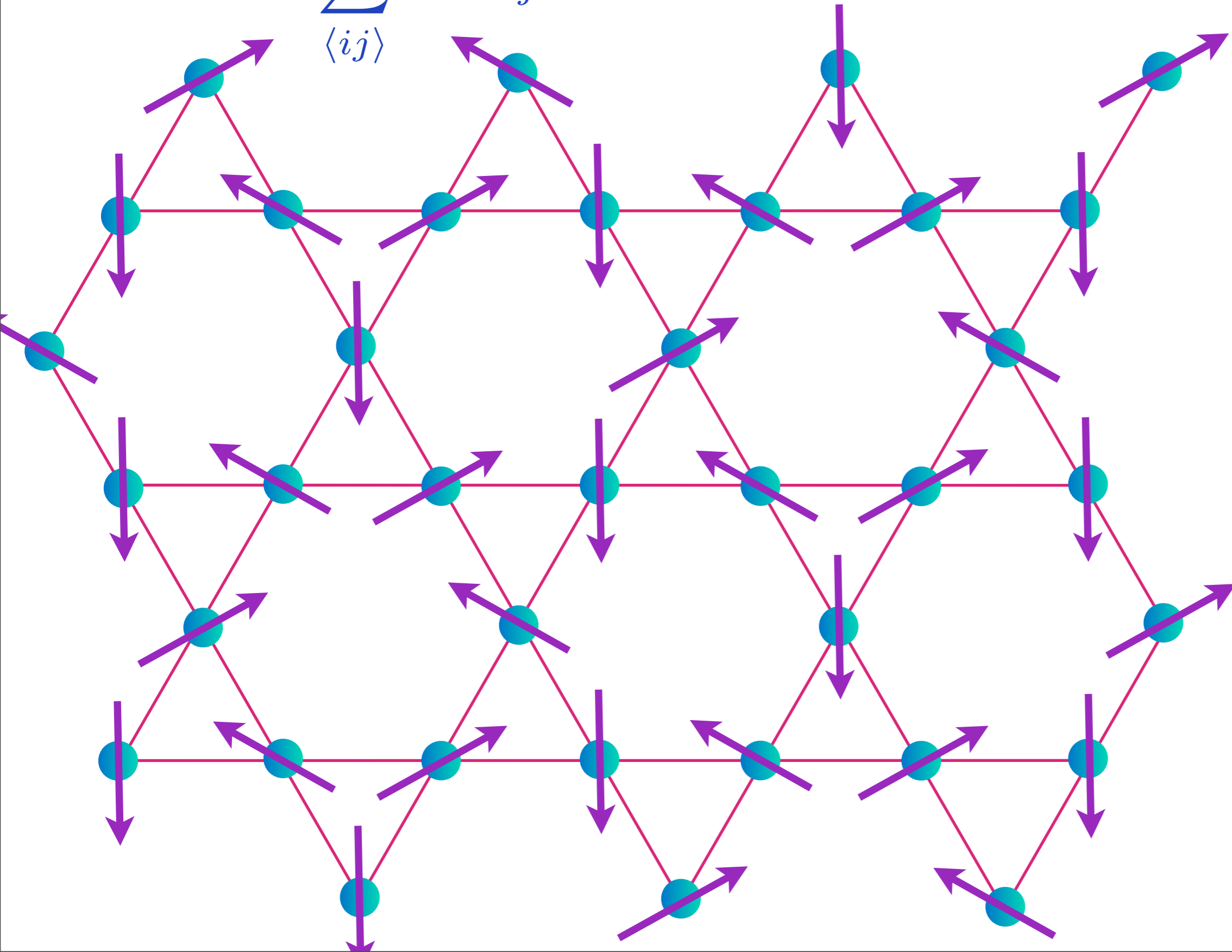
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Kagome antiferromagnet

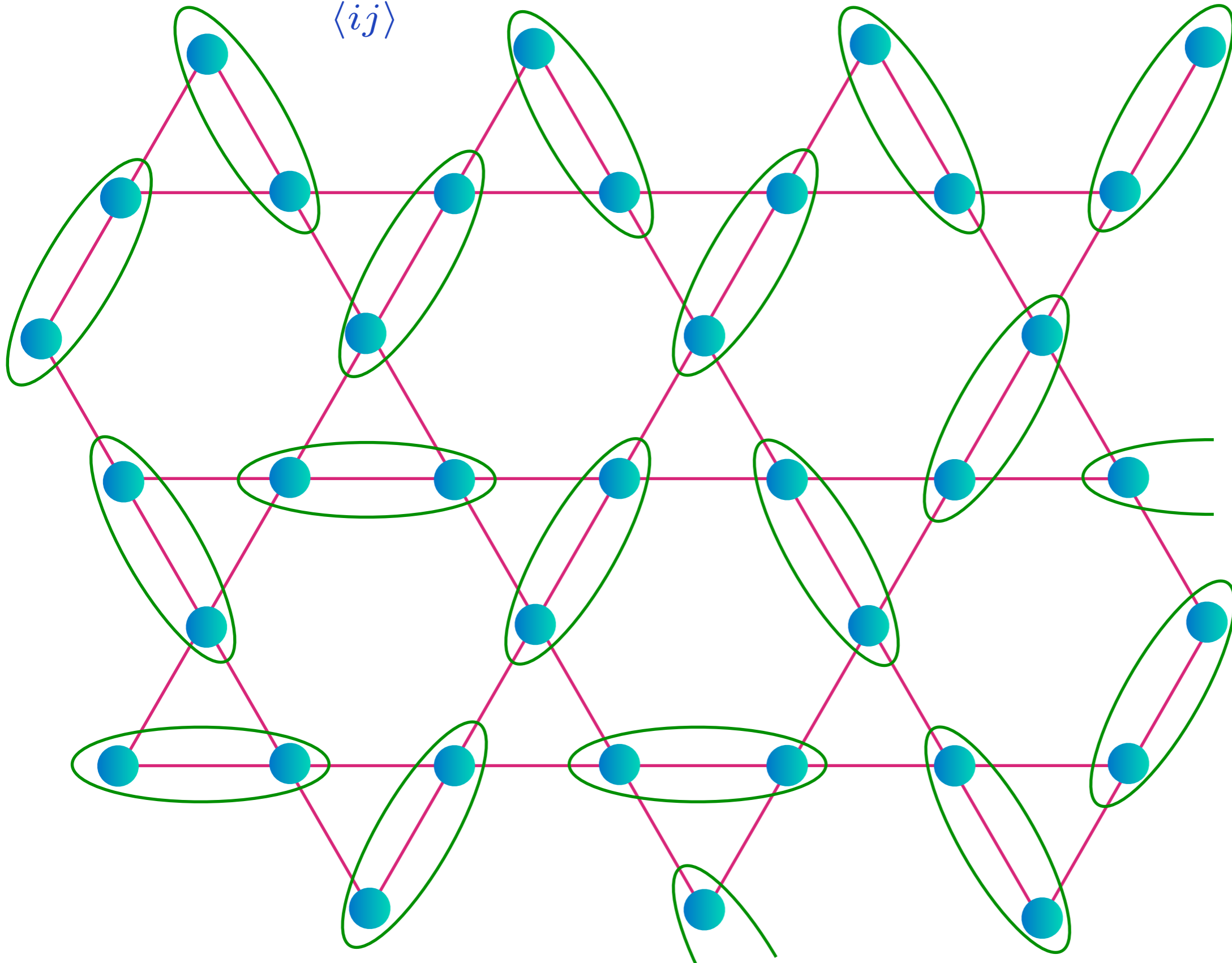
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$\left(\begin{array}{c} \bullet \\ \bullet \end{array} \right) = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

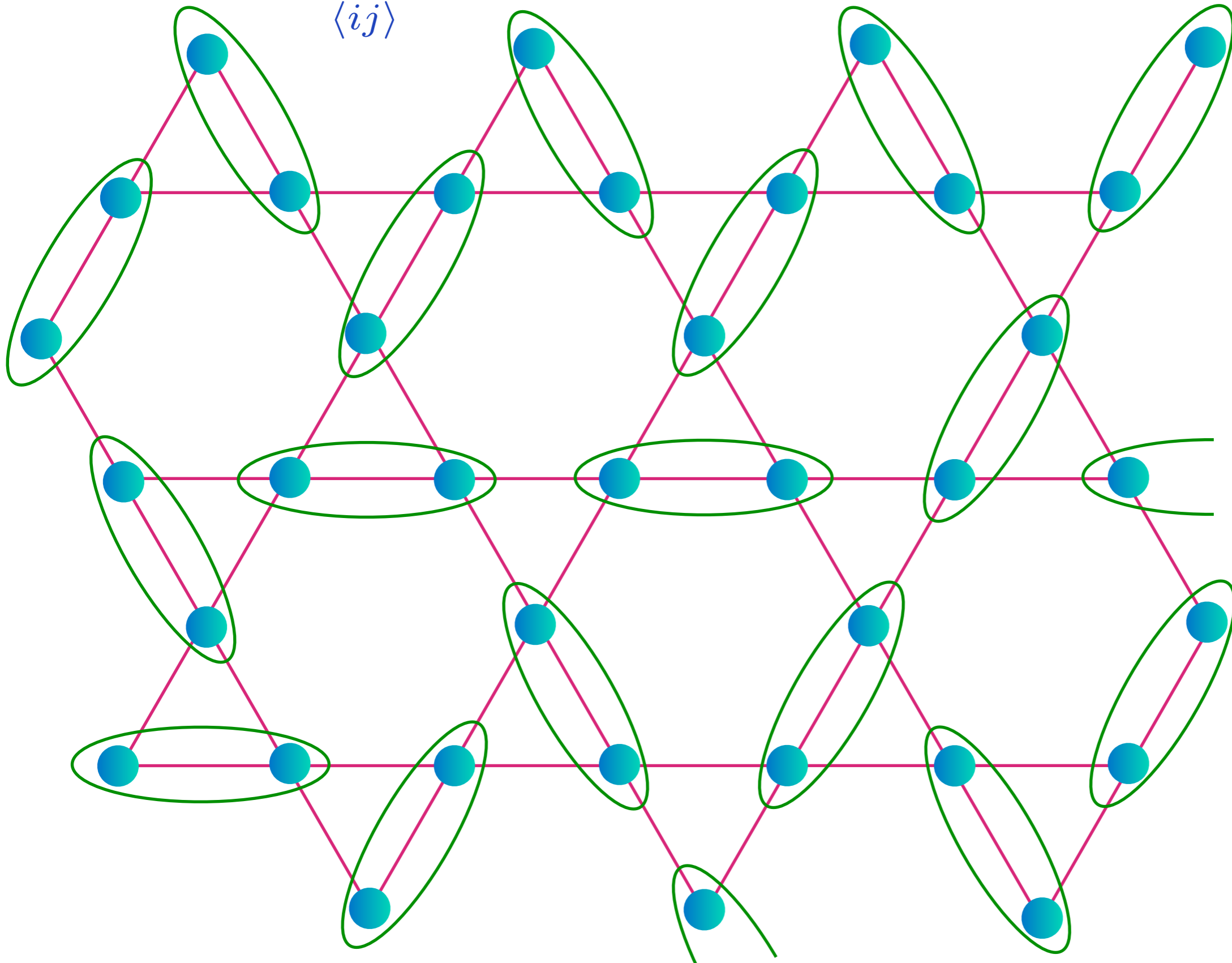


P. Fazekas and
P. W. Anderson,
Philos. Mag.
30, 23 (1974).

Mott insulator: Kagome antiferromagnet

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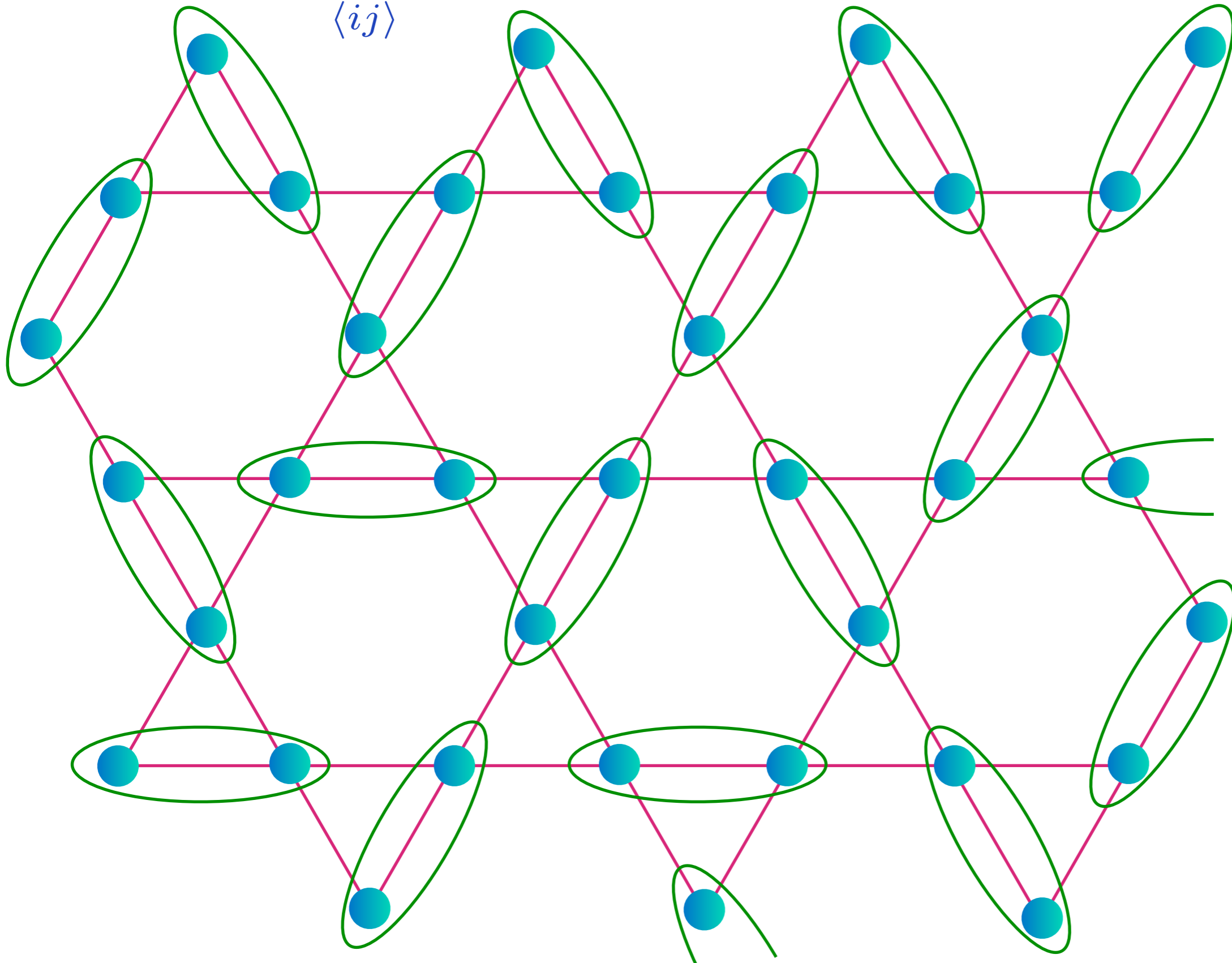


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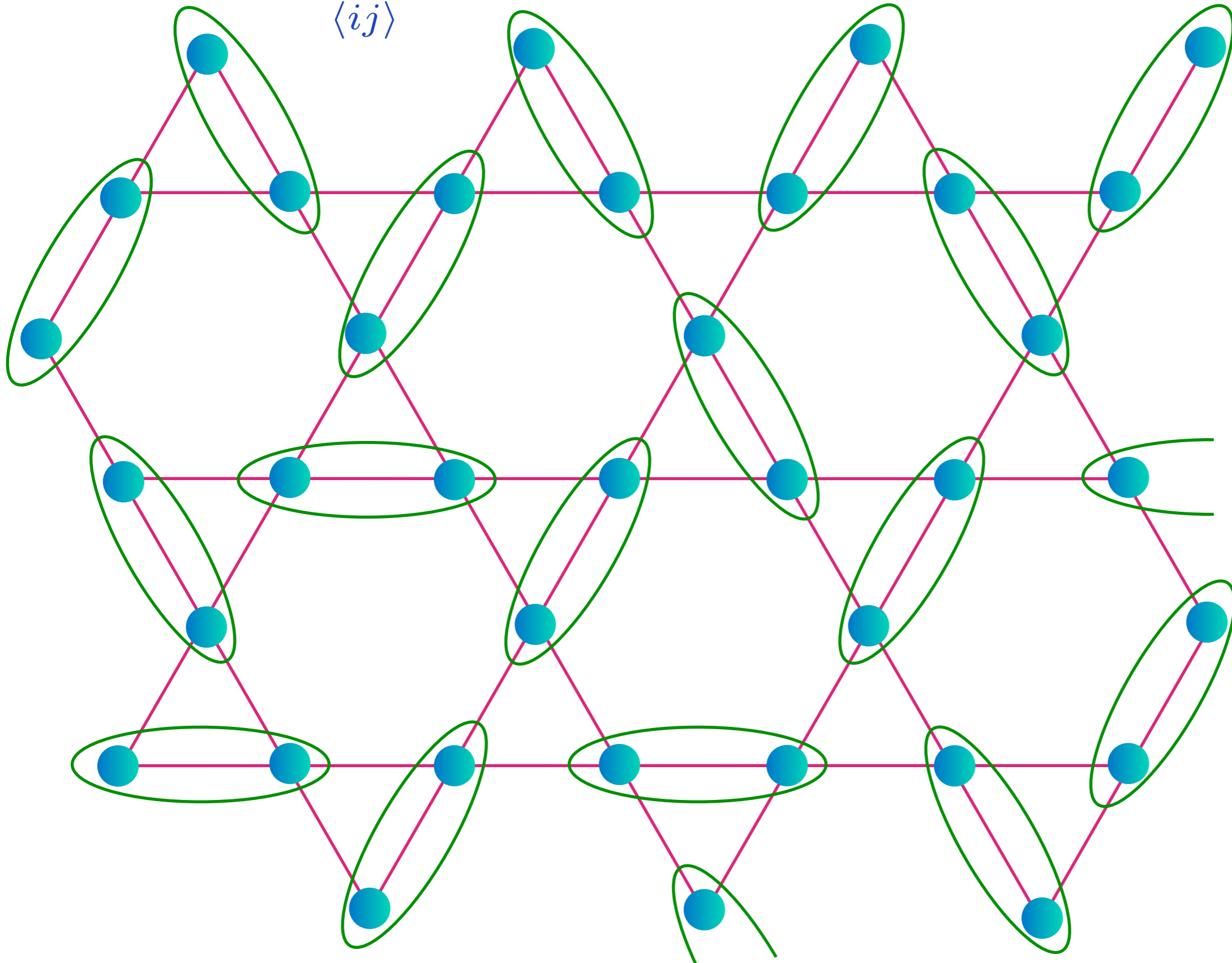


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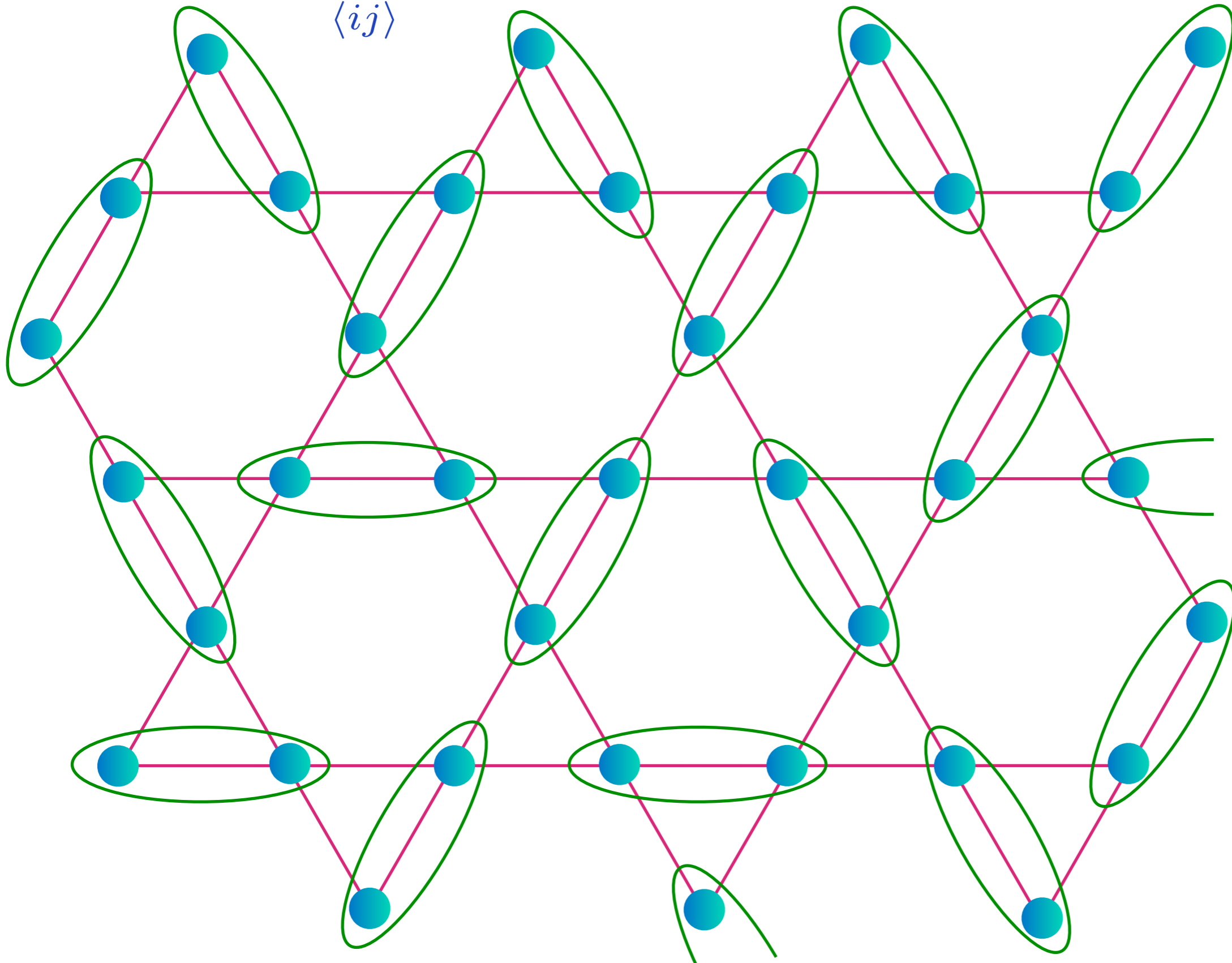


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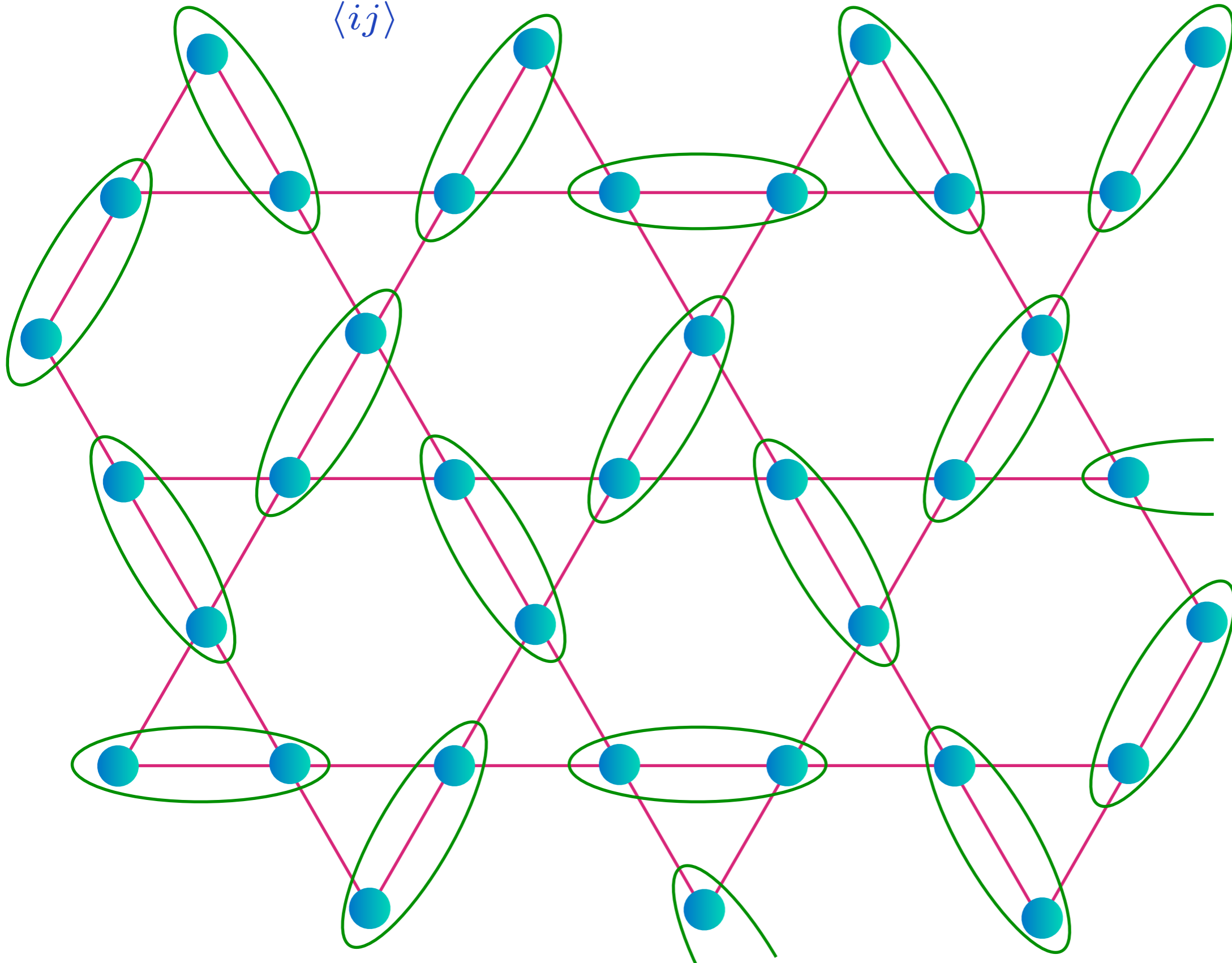


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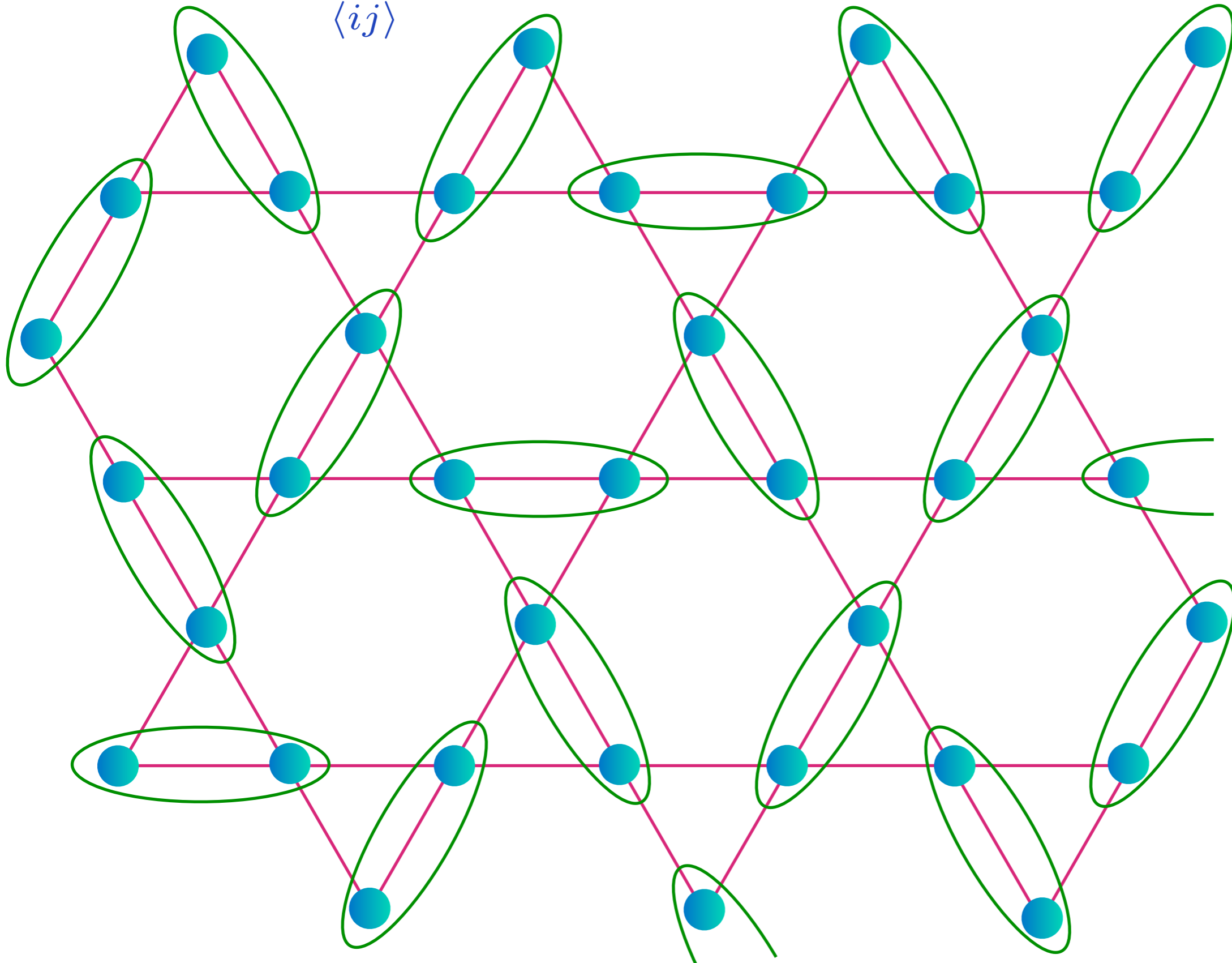


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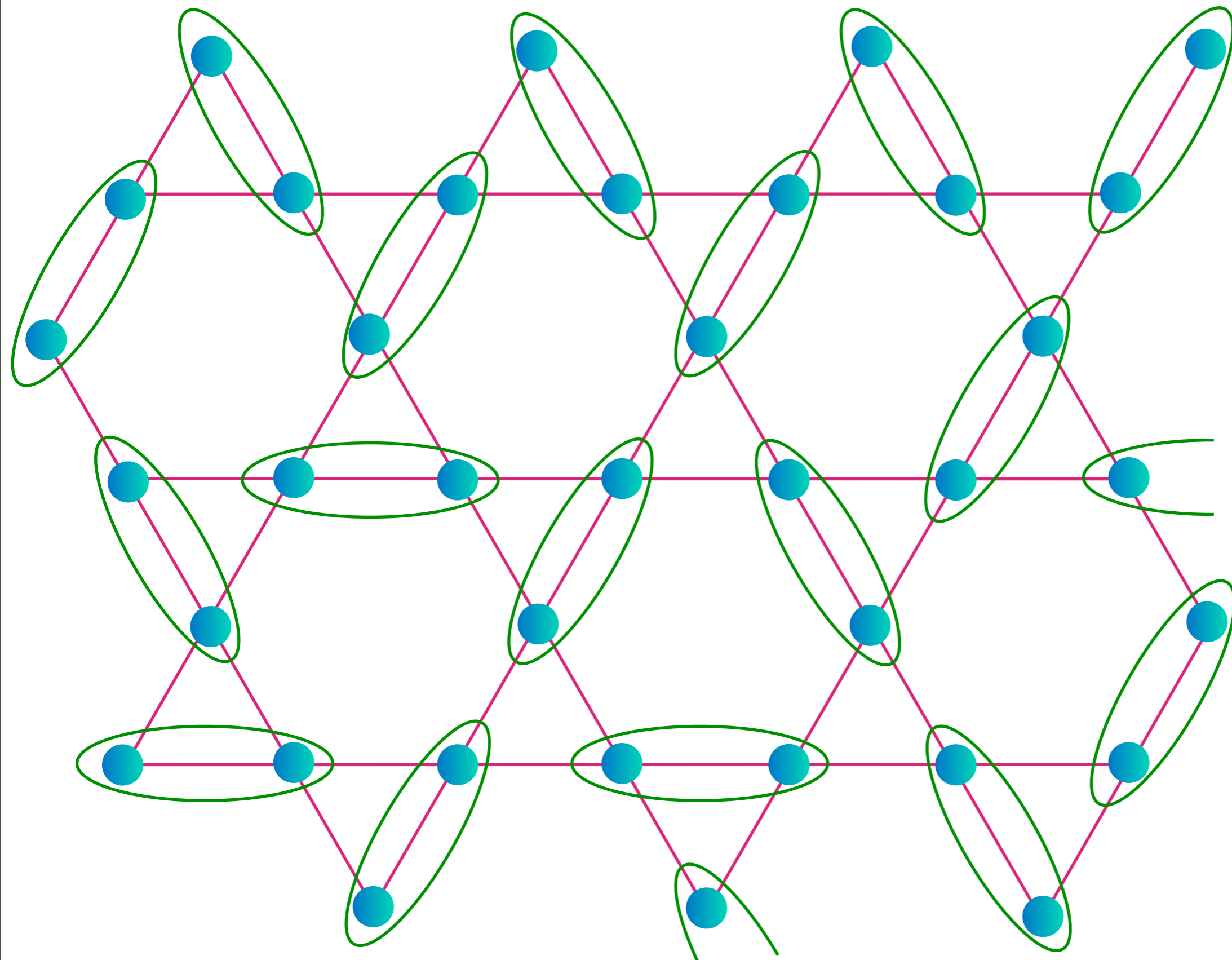


P. Fazekas and
P. W. Anderson,
Philos. Mag.
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Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration

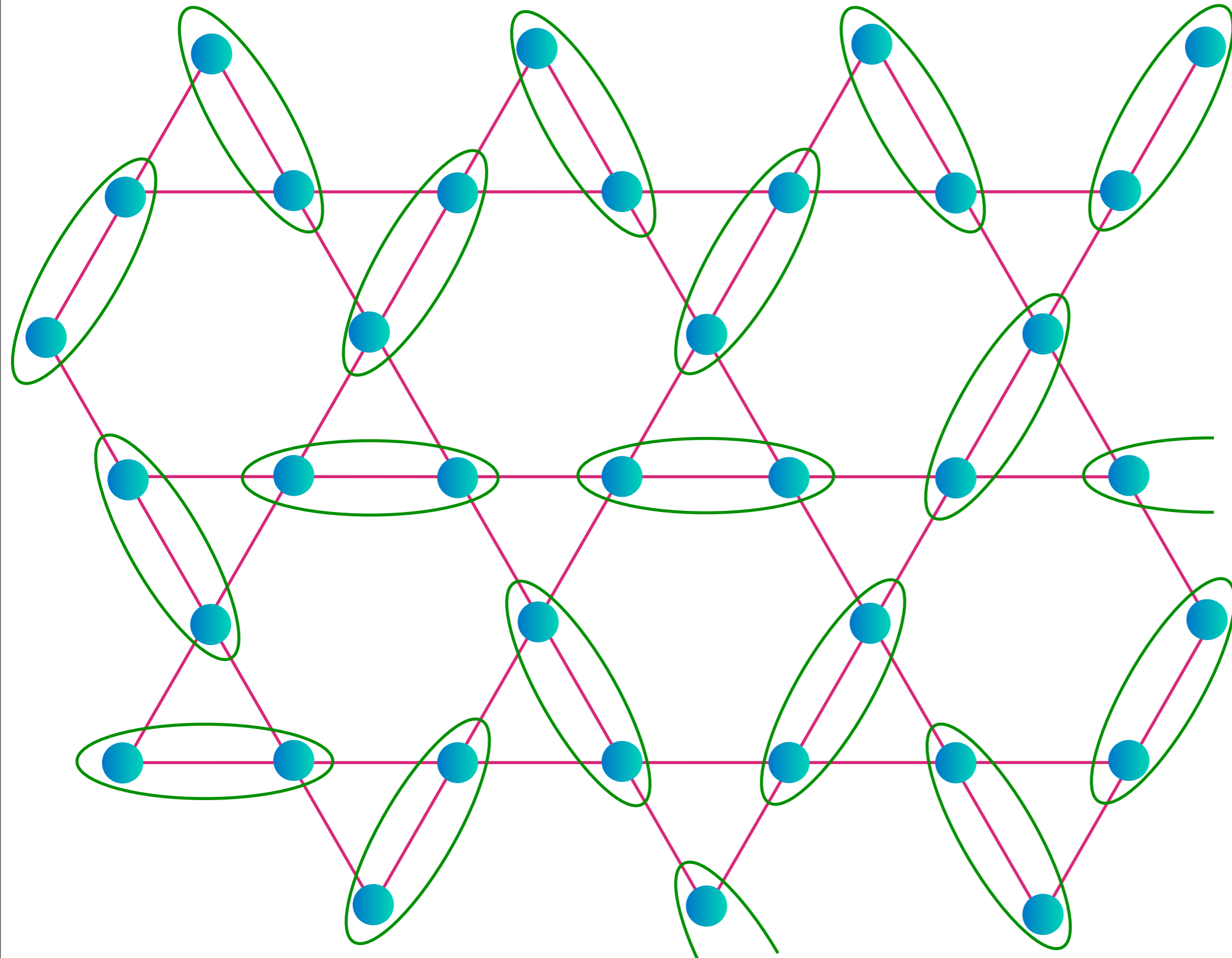


D. Rokhsar and
S. Kivelson,
Phys. Rev. Lett.
61, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

A nearby configuration

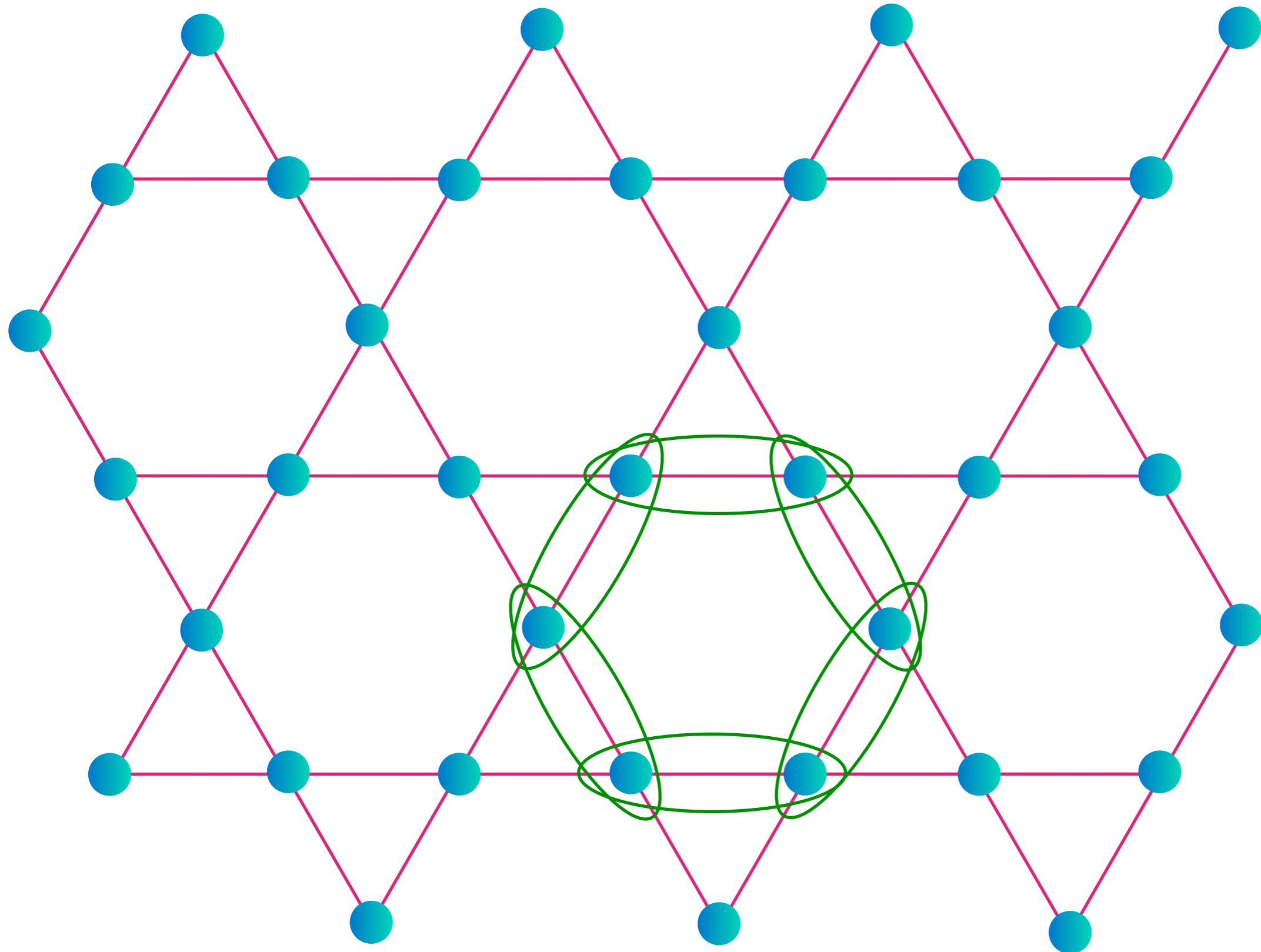


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61, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Difference: a closed loop

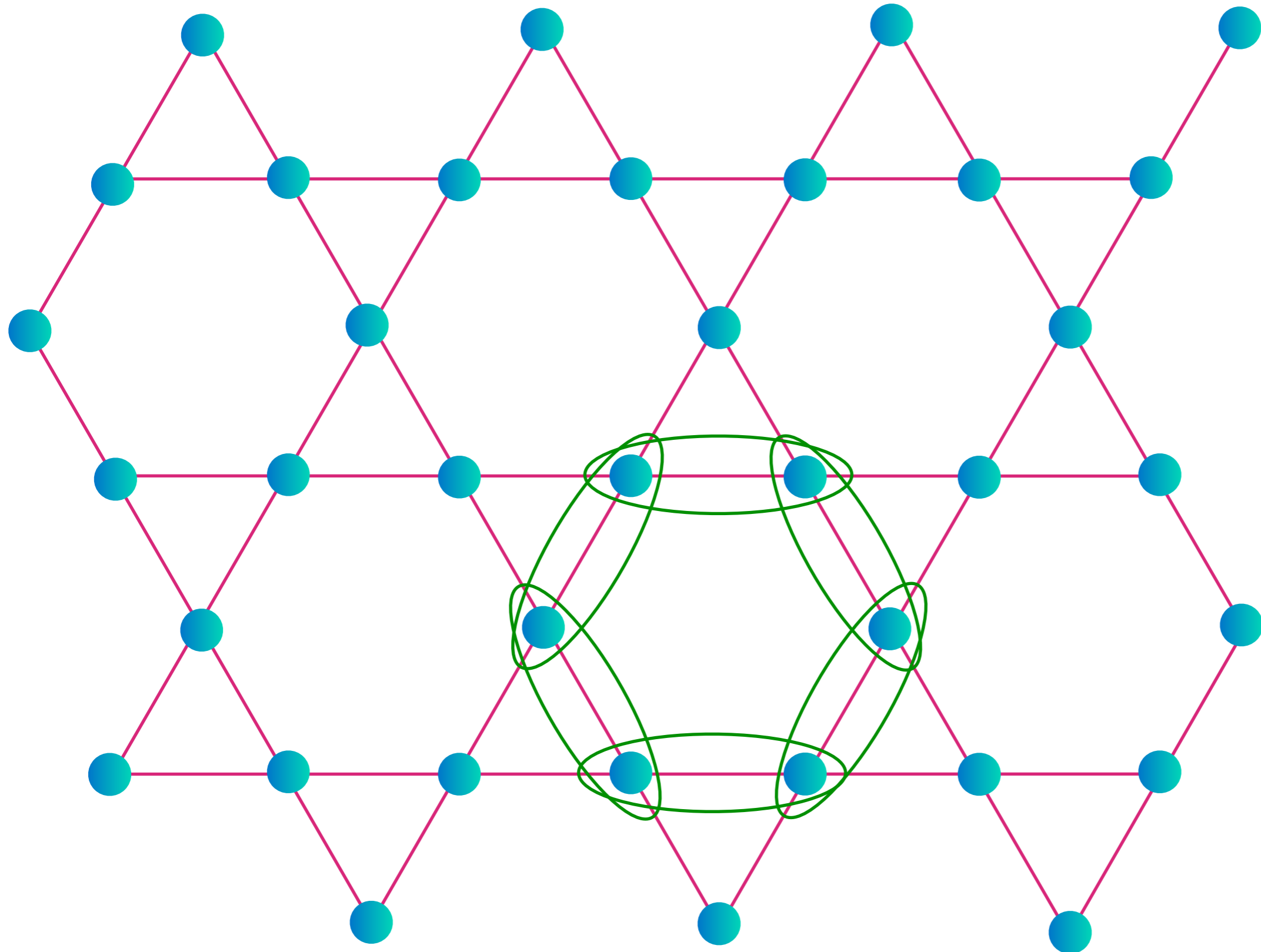


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Alternative view

Ground state: sum over closed loops

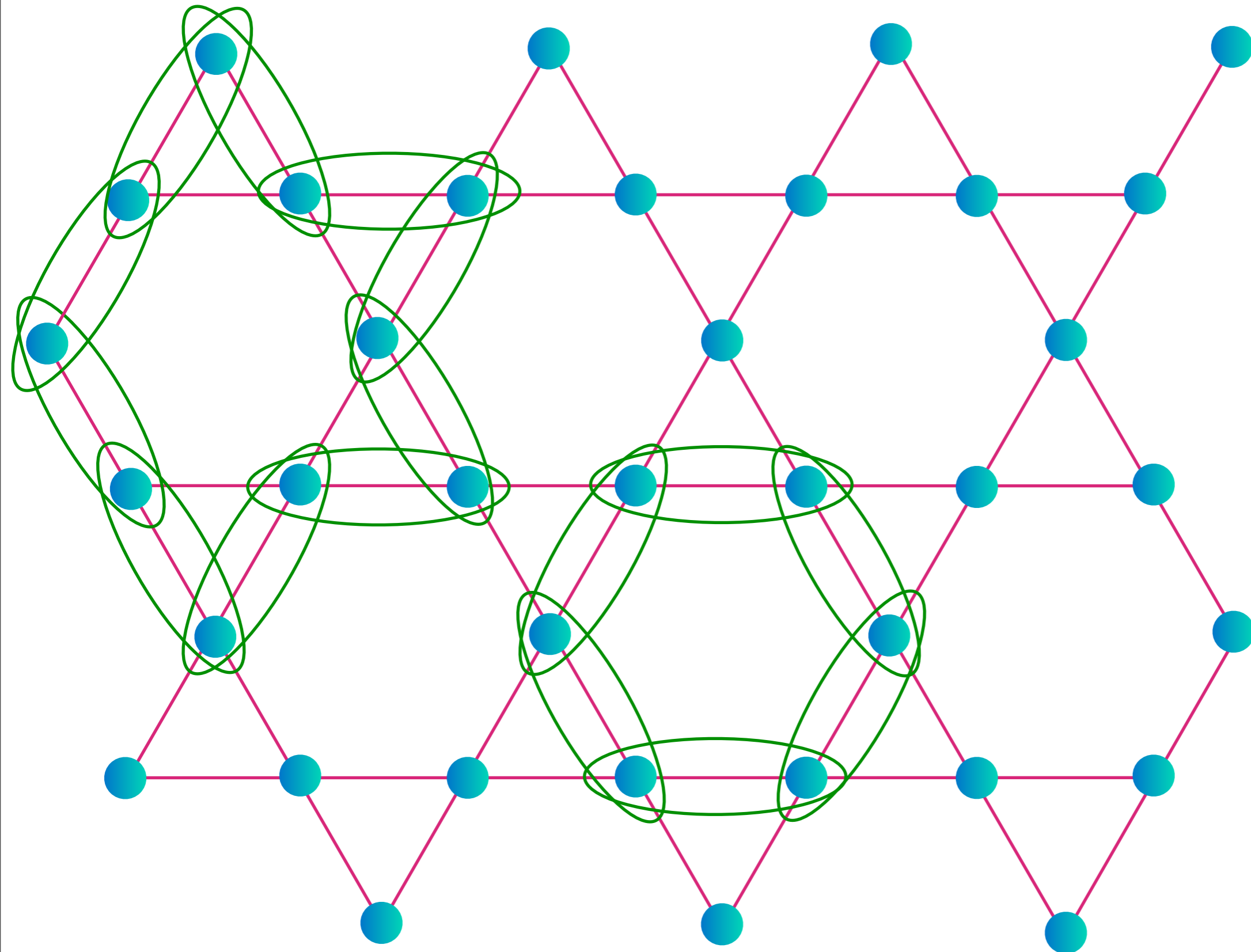


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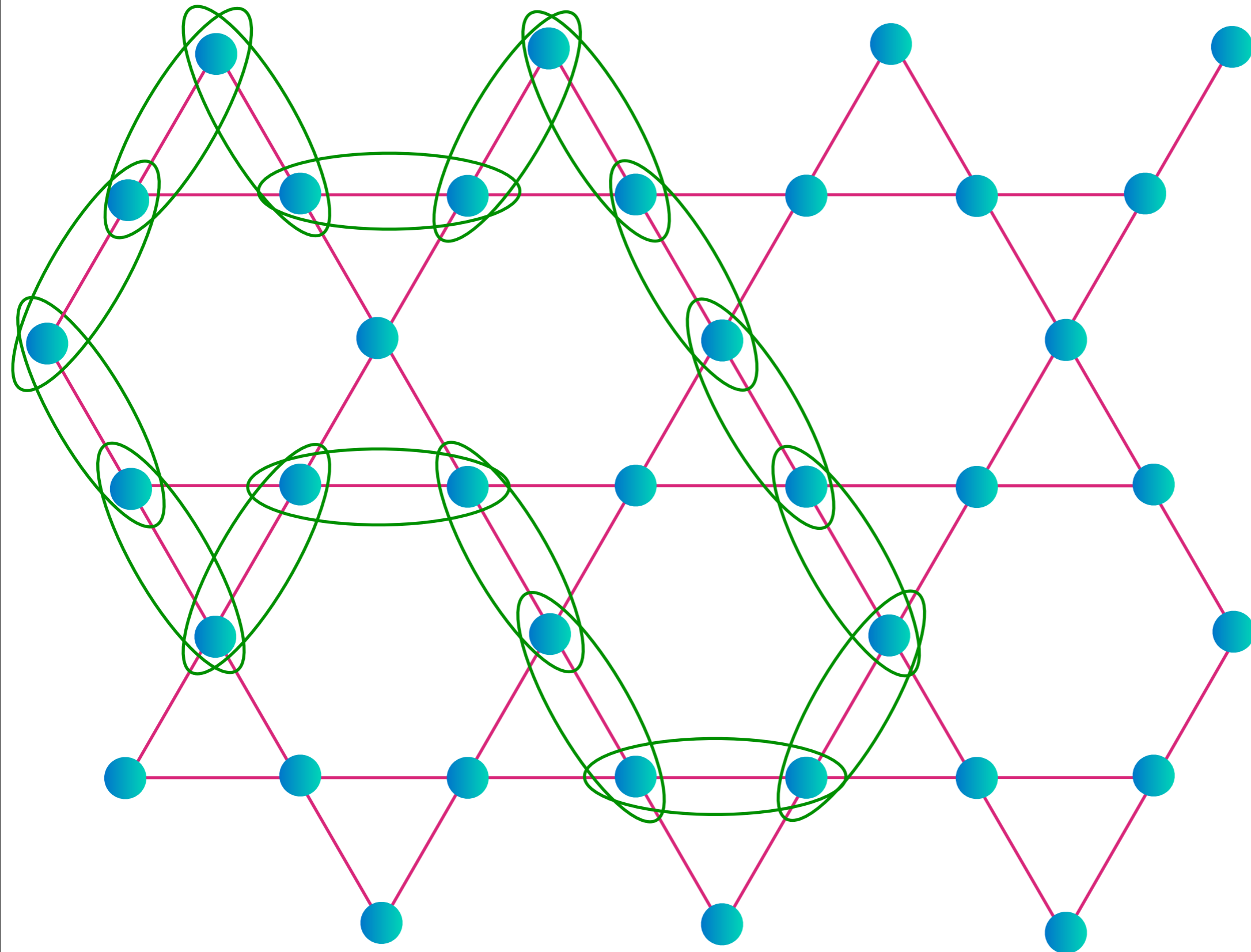


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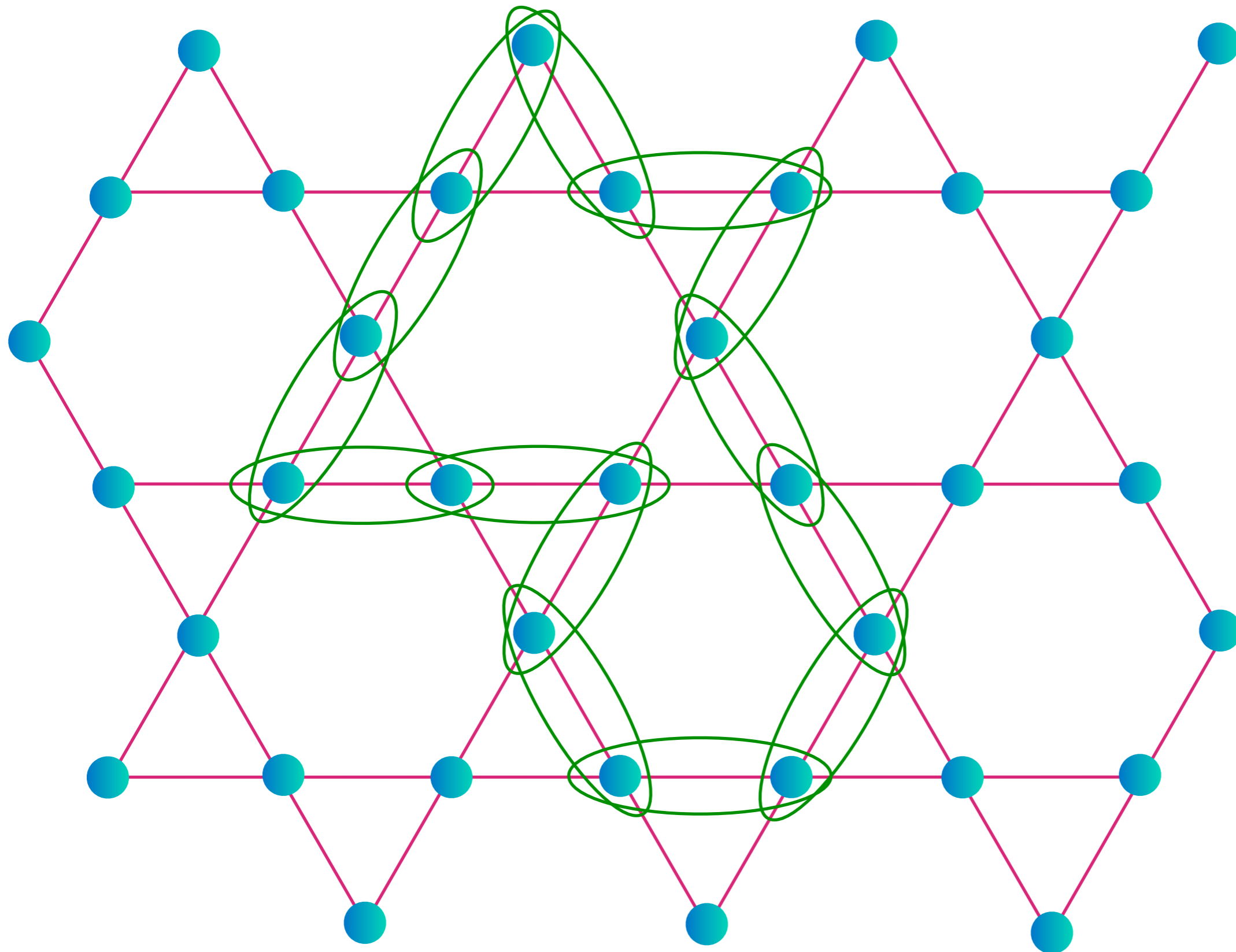


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Mott insulator: Kagome antiferromagnet

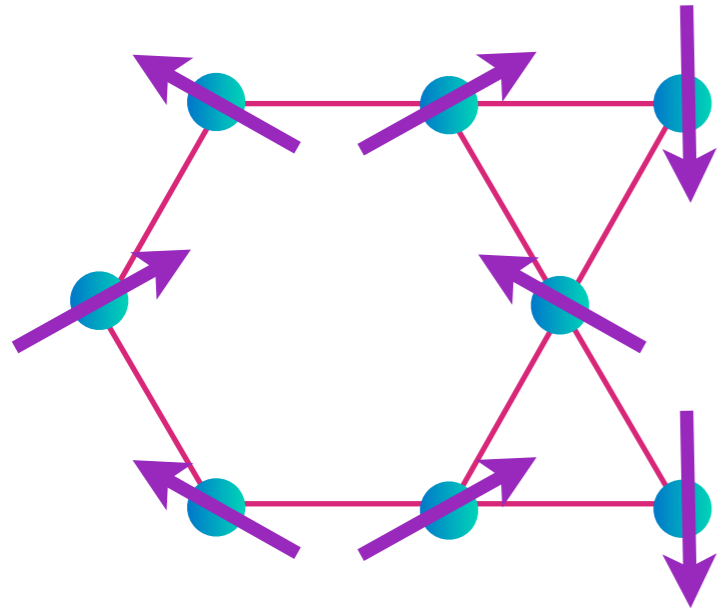
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Mott insulator: Kagome antiferromagnet



non-collinear Néel state

Quantum “disordered” state with exponentially decaying spin correlations.

S_c

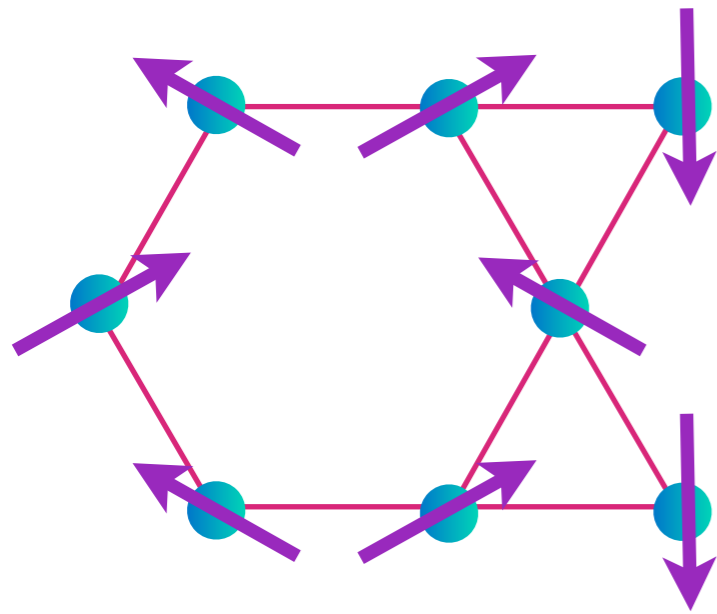
S

Mott insulator: Kagome antiferromagnet

Entangled quantum state:

A “spin liquid” described by the deconfined state of a \mathbb{Z}_2 gauge theory, and shown to be stable to all fluctuations.

The universal properties are also described by a doubled Chern-Simons topological field theory.



non-collinear Néel state

S_c

S

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)

M. Freedman, C. Nayak, K. Shtengel, K. Walker, and

Z. Wang, *Annals of Physics* **310**, 428 (2004)

Mott insulator: Kagome antiferromagnet

\mathbb{Z}_2 spin liquid: parton construction

Write spin operators in terms of $S = 1/2$ ‘partons’ $\vec{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} b_{i\beta}$.
The ground state is

$$|\Psi\rangle = \mathcal{P} \exp \left(f(i-j) \varepsilon^{\alpha\beta} b_{i\alpha}^\dagger b_{j\beta}^\dagger \right) |0\rangle$$

where \mathcal{P} projects onto one parton per site.

The excitations are

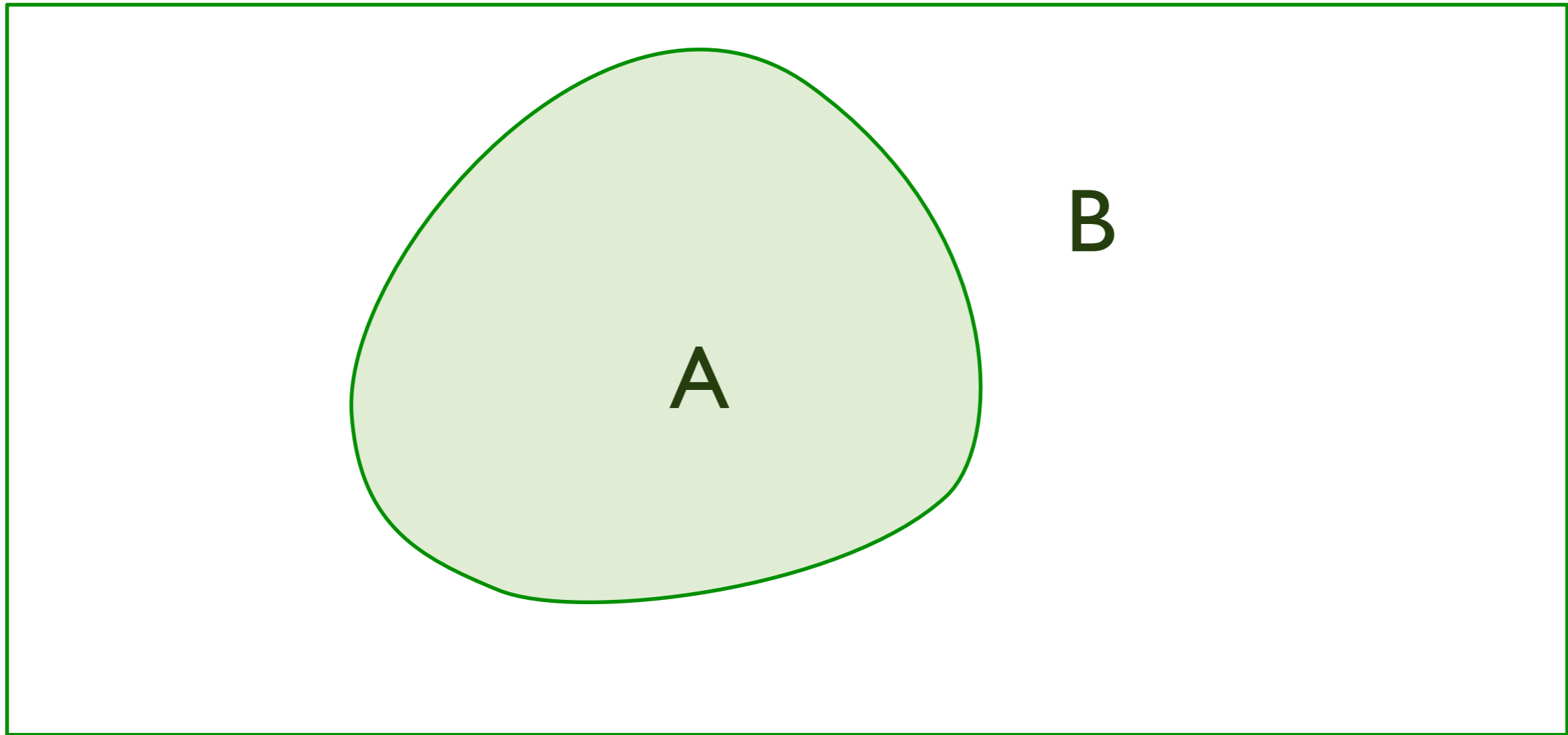
- Neutral, $S = 1/2$ spinons, which are quanta of b_α
- Spinless, neutral ‘visons’, which are the analogs of Abrikosov vortices in the pairing wavefunction f (because the emergent gauge field compact, vortices and anti-vortices are equivalent).

The spinons and visons carry ‘electric’ and ‘magnetic’ charges of a \mathbb{Z}_2 gauge field.

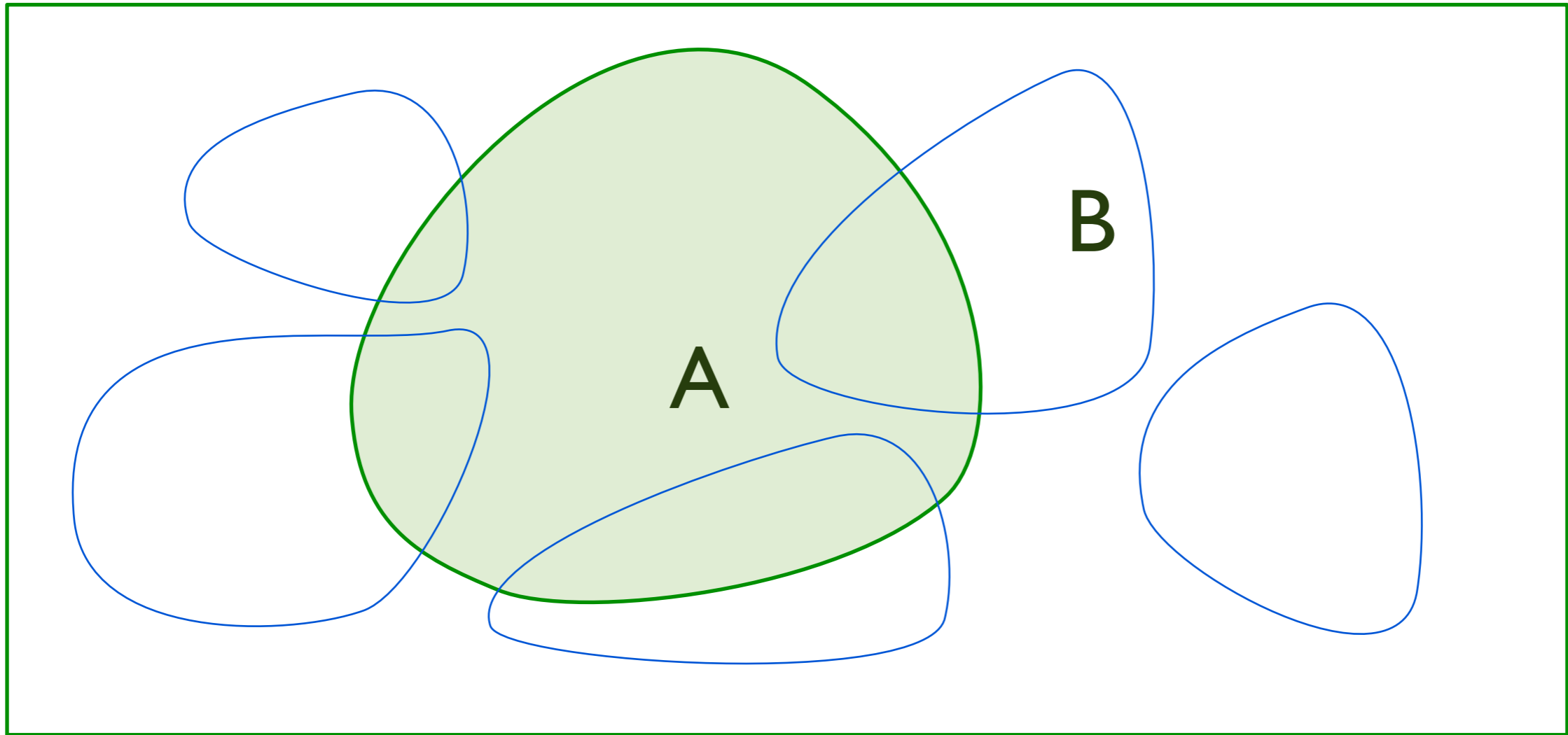
S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)

Y. Huh, M. Punk, and S. Sachdev, *Phys. Rev. B* **84**, 094419 (2011)

Entanglement in the Z_2 spin liquid

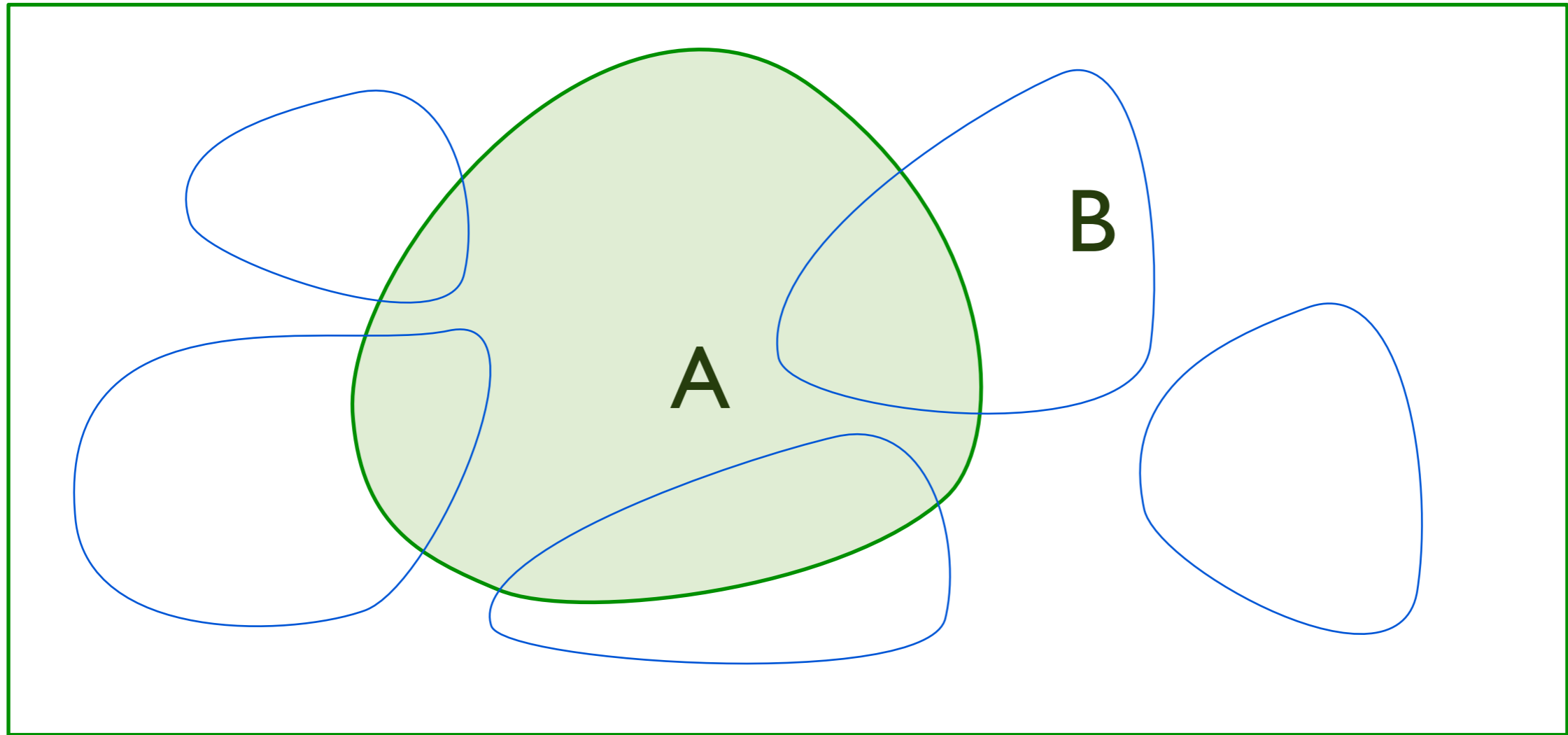


Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

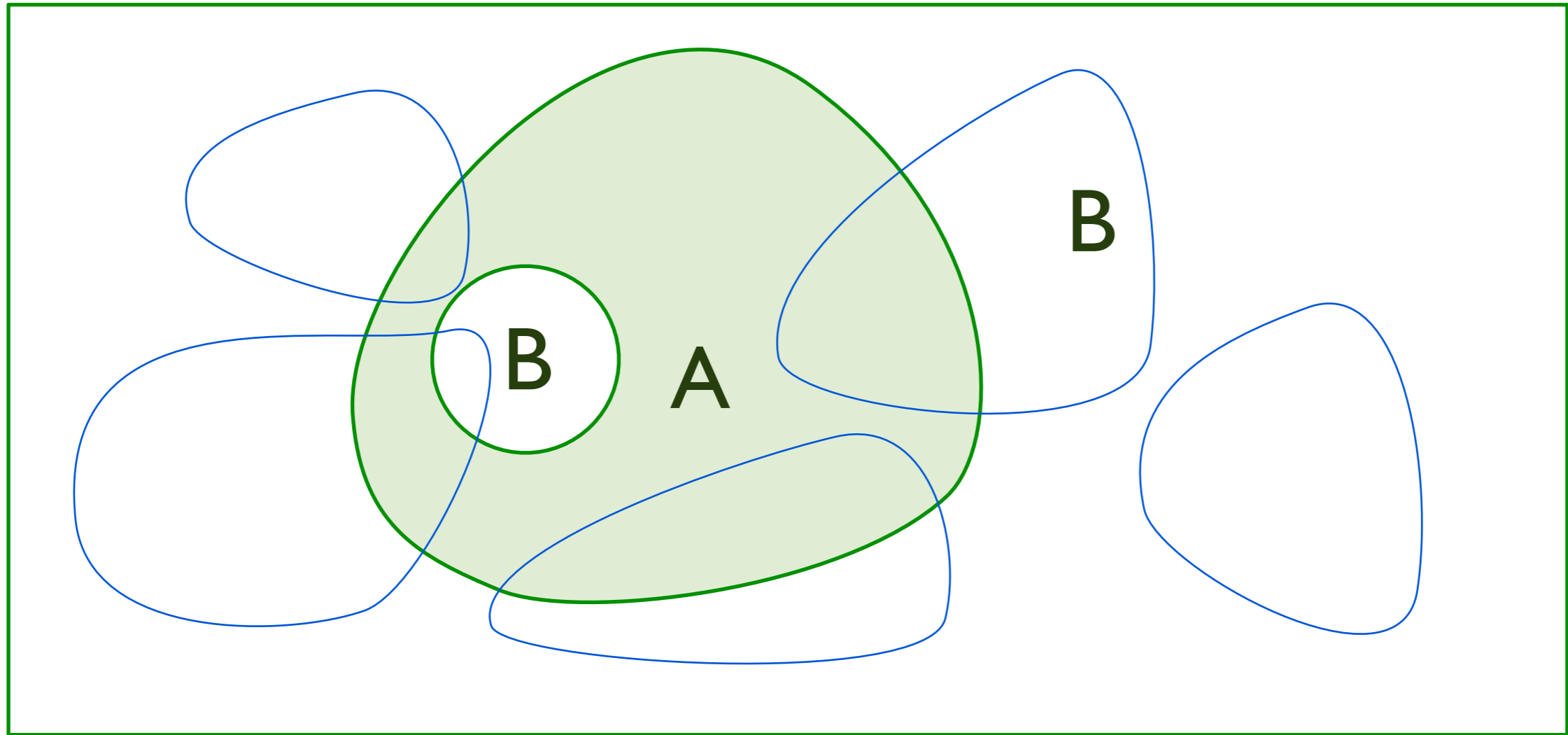
Entanglement in the Z_2 spin liquid



$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

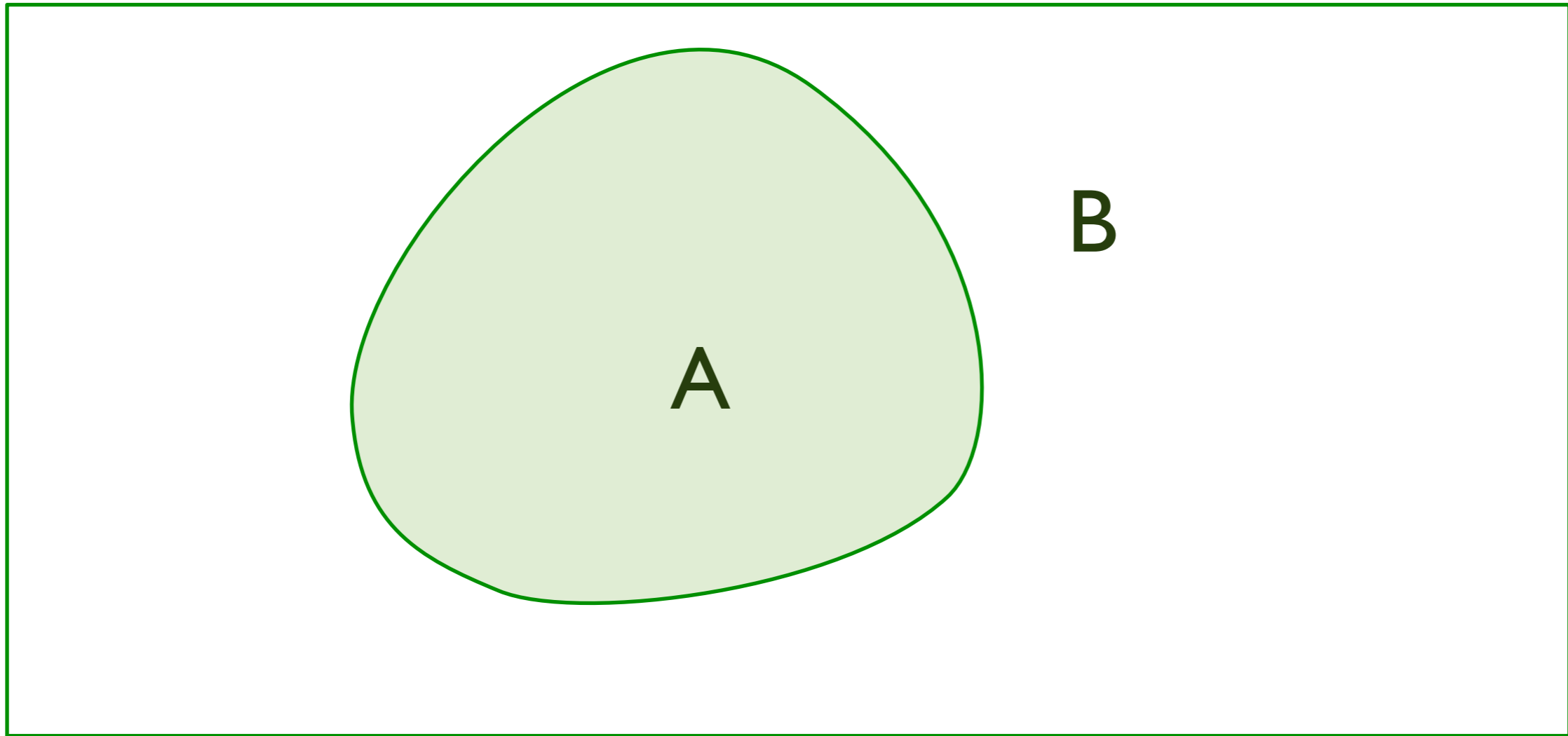
Entanglement in the Z_2 spin liquid



$$S_E = aP - \ln(4)$$

where P is the surface area (perimeter) of the boundary between A and B.

Entanglement in the Z_2 spin liquid



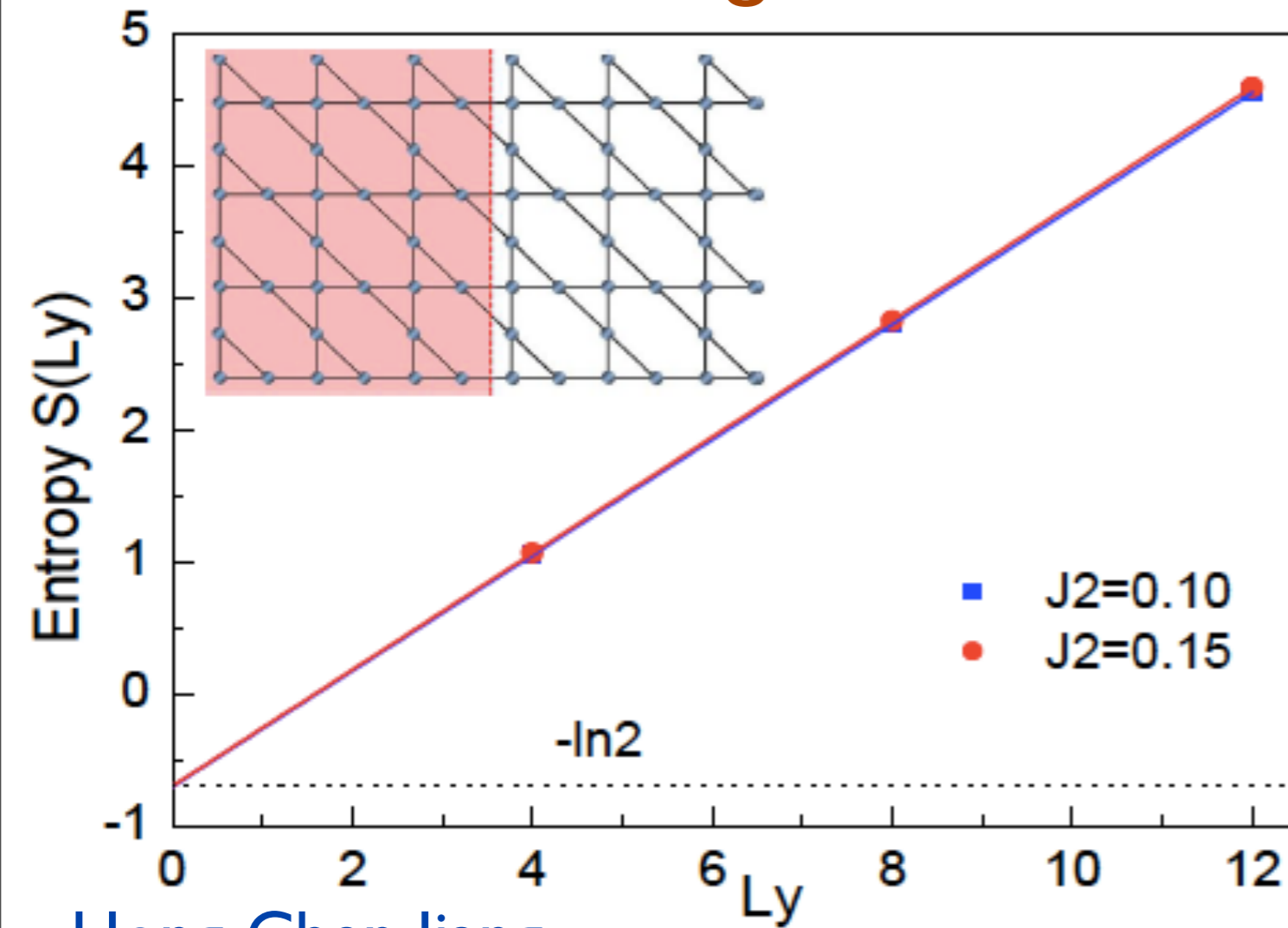
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Mott insulator: Kagome antiferromagnet

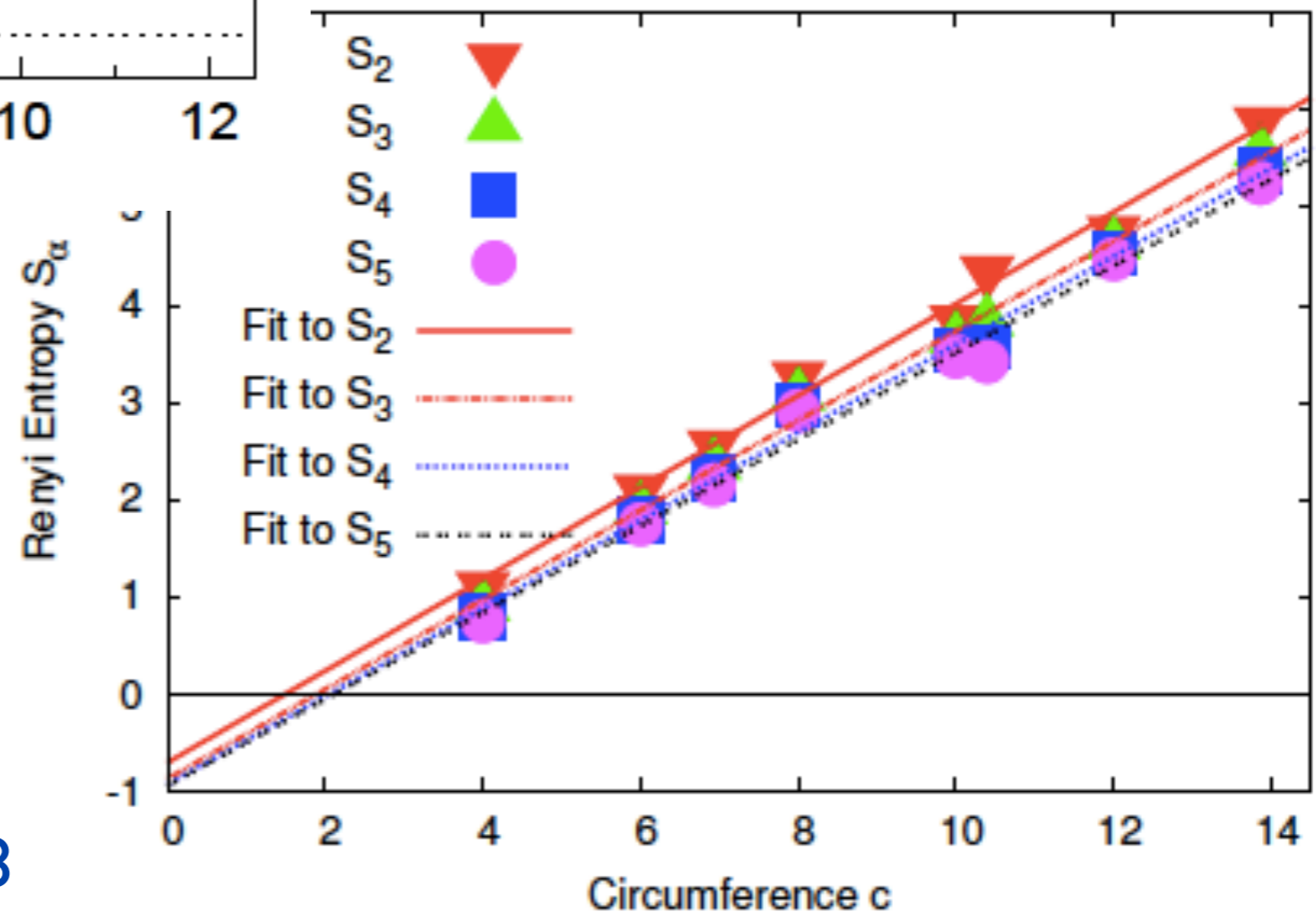
Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D.A. Huse, and S. R. White, *Science* **332**, 1173 (2011).



Hong-Chen Jiang,
Z. Wang,
and L. Balents,
arXiv:1205.4289

S. Depenbrock,
I. P. McCulloch,
and
U. Schollwoeck,
arXiv:1205.4858



Mott insulator: Kagome antiferromagnet

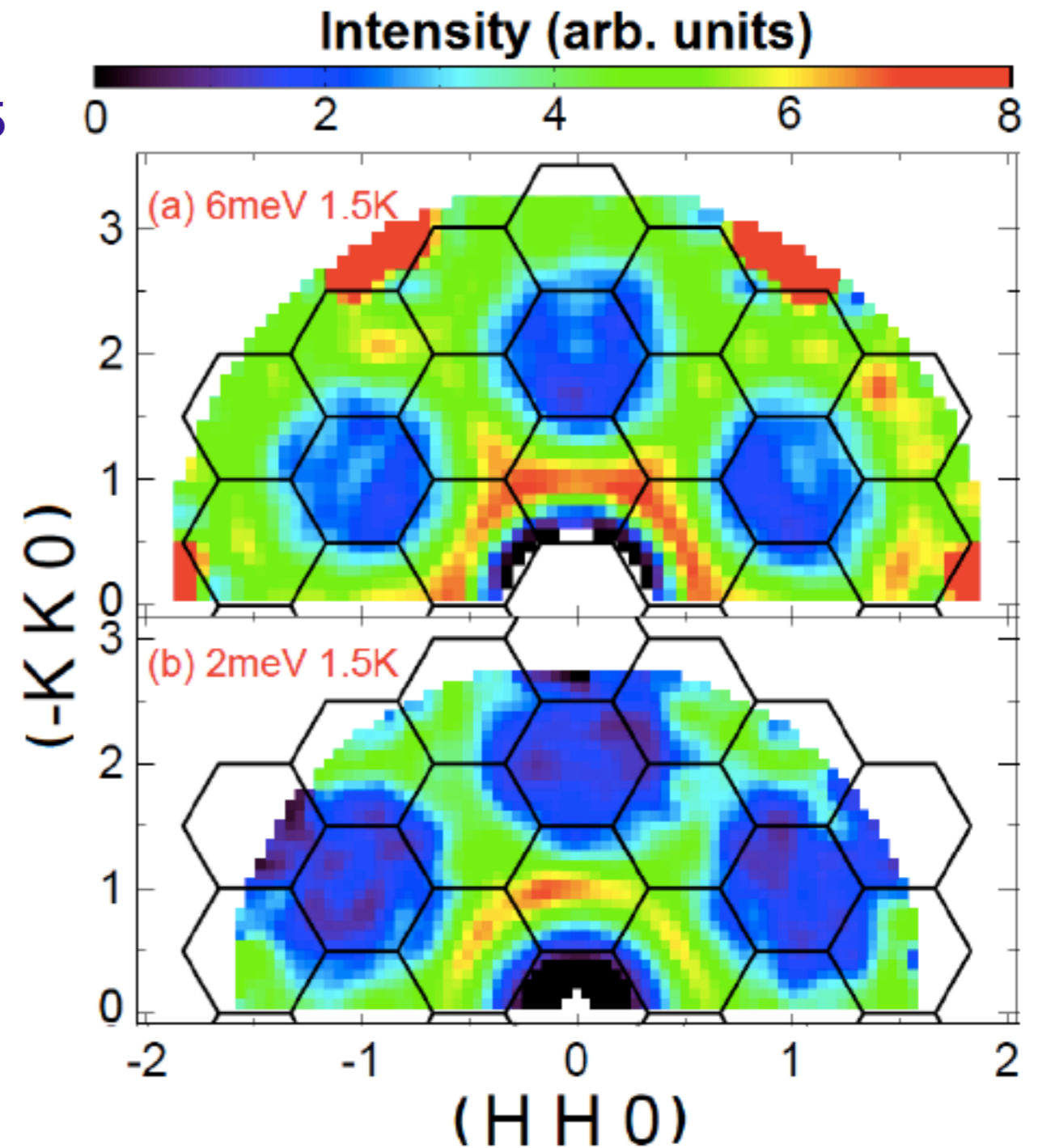
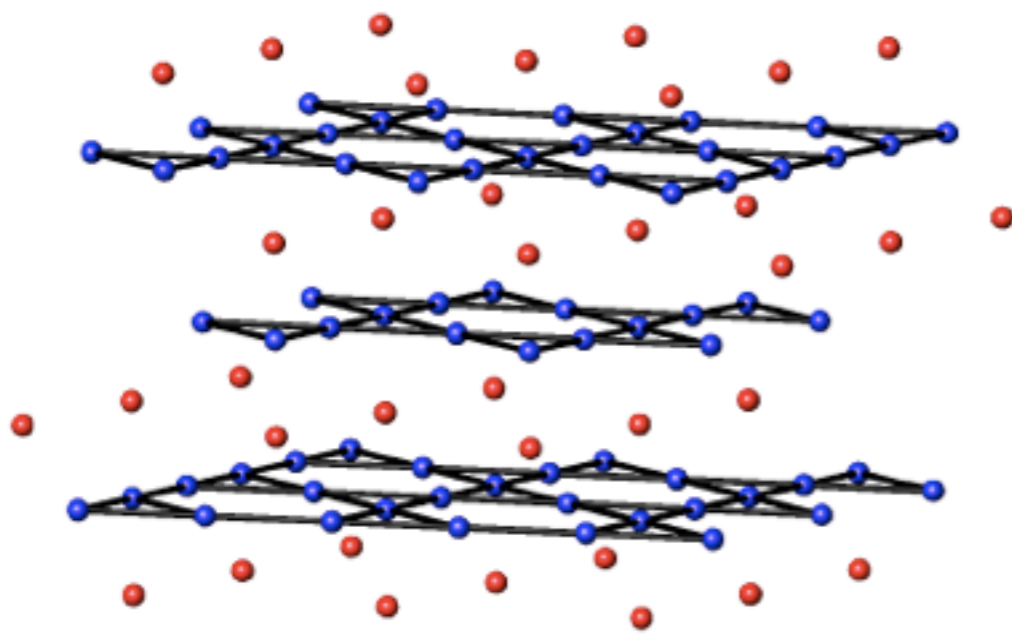
Evidence for spinons

Young Lee,

APS meeting, March 2012

<http://meetings.aps.org/link/BAPS.2012.MAR.H8.5>

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$ (also called Herbertsmithite)



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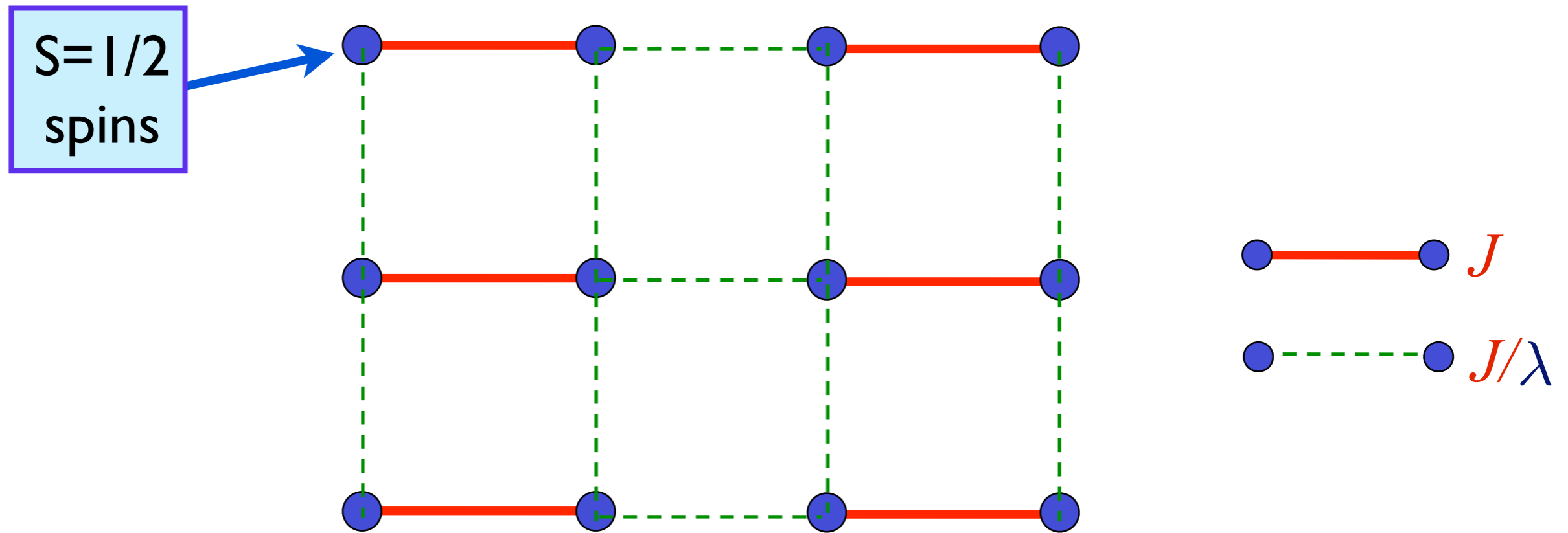
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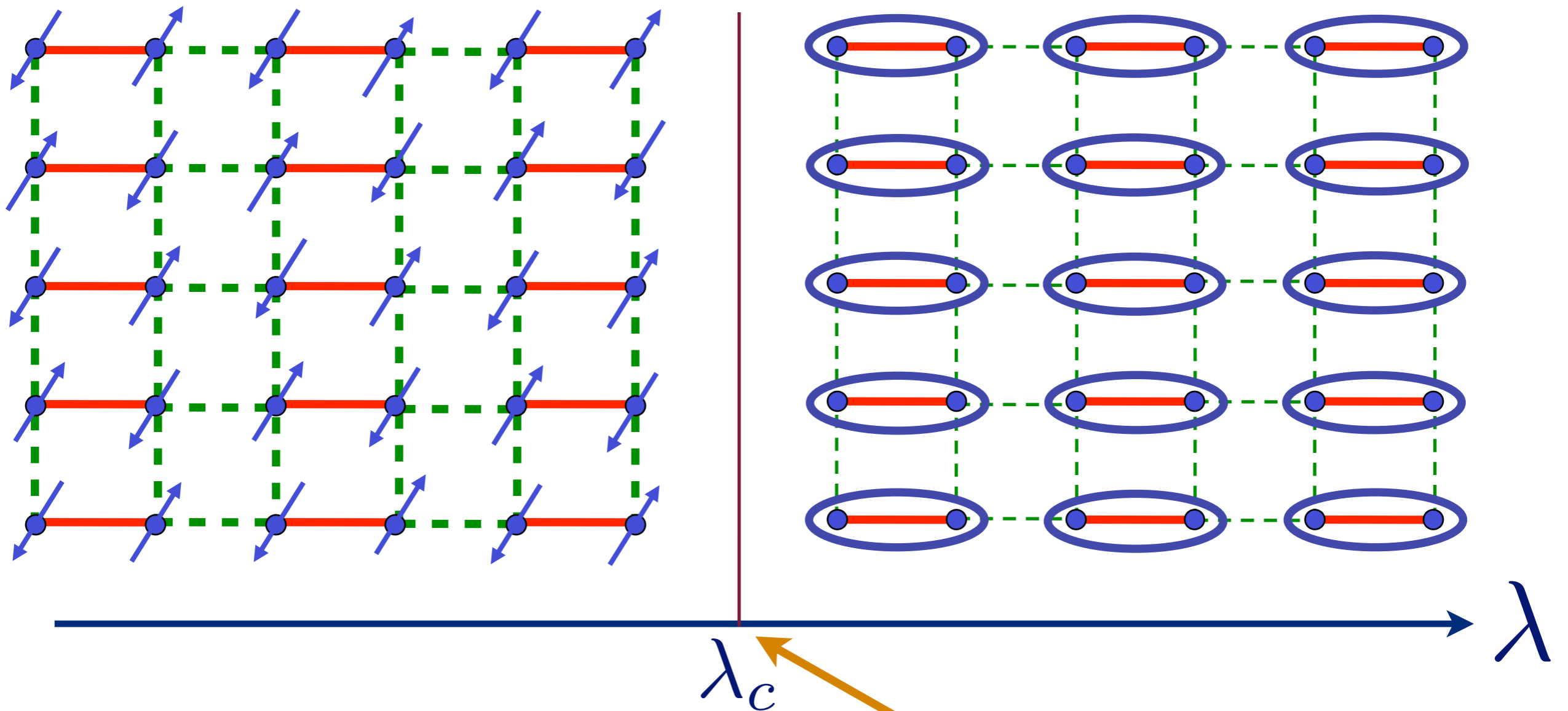
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

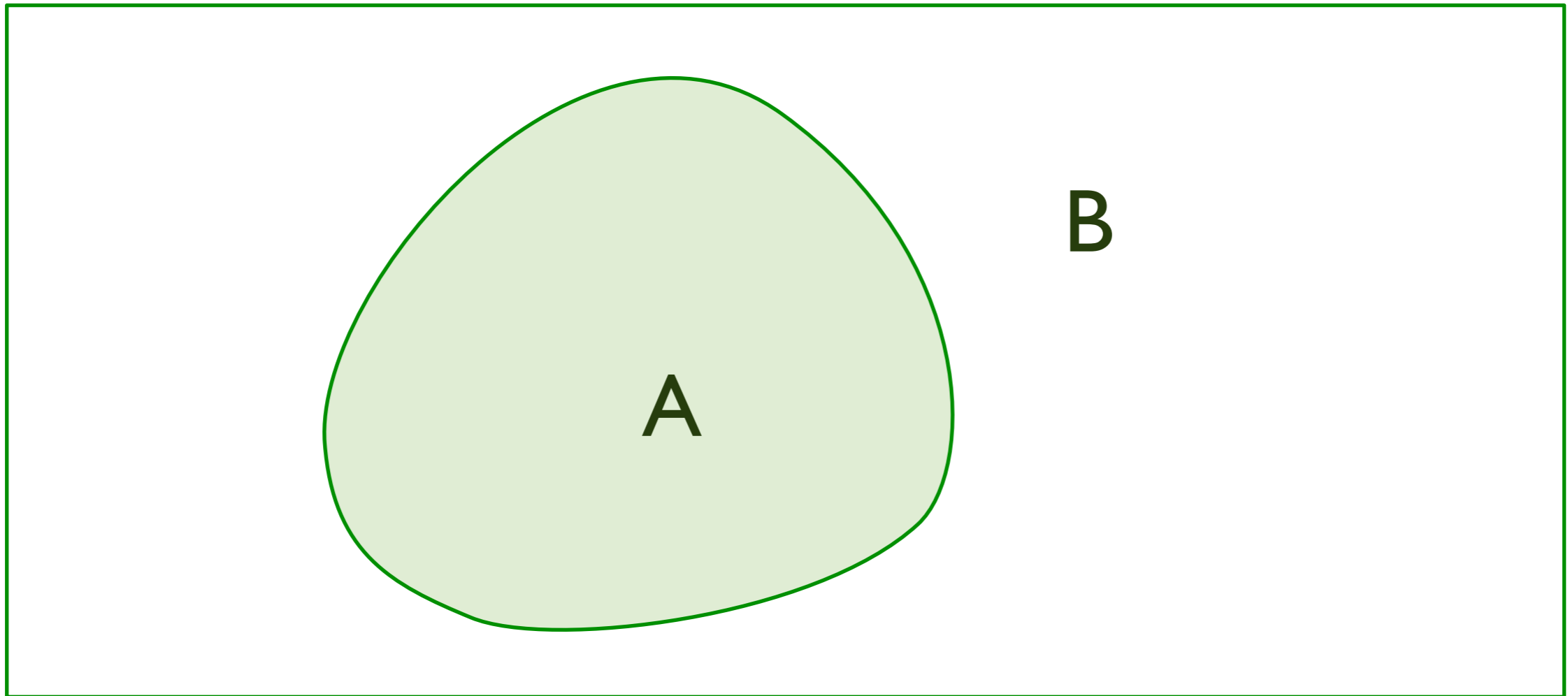
$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Quantum critical point described by
a CFT3 (O(3) Wilson-Fisher)

Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



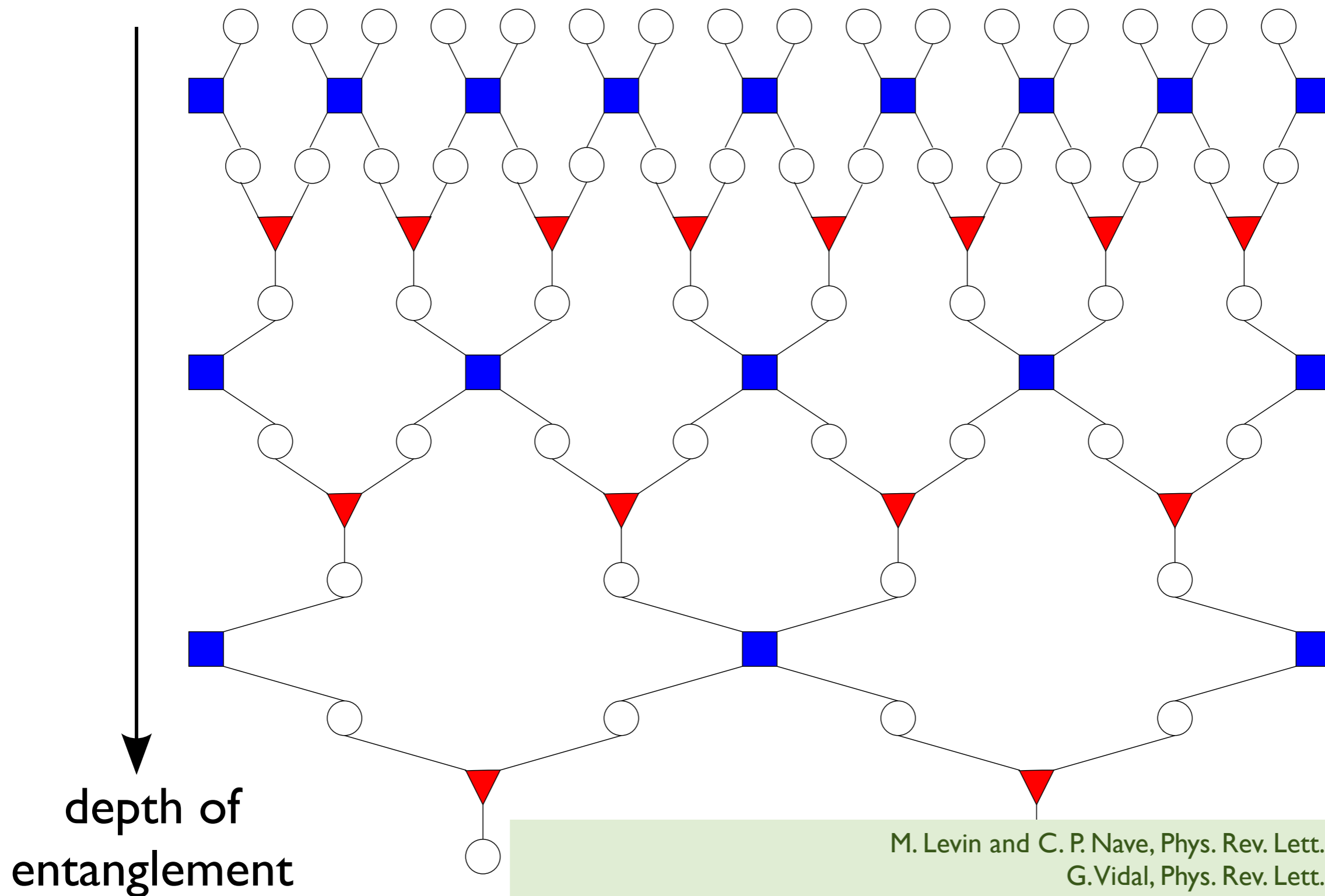
M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).

H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)

I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

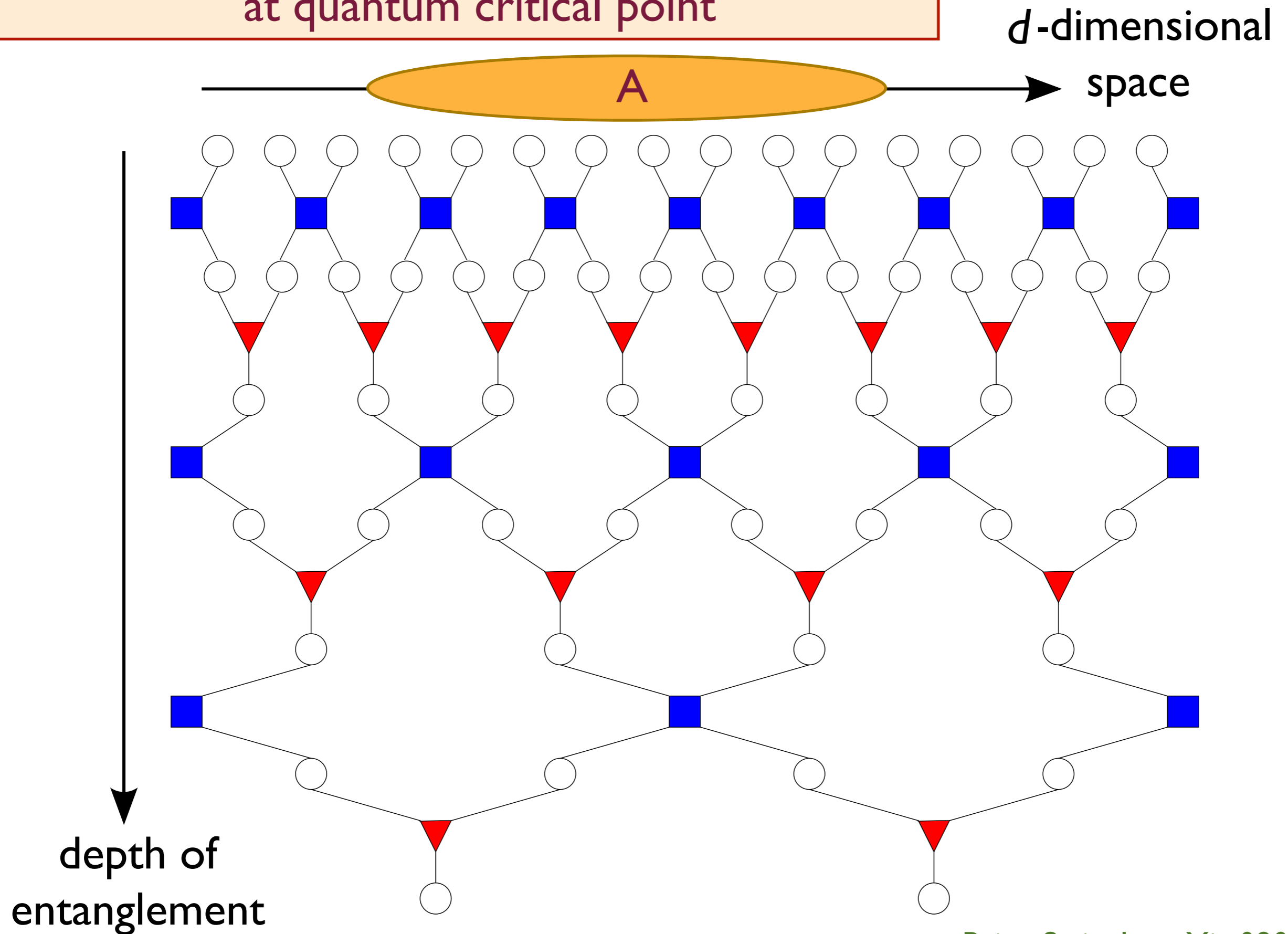
Tensor network representation of entanglement at quantum critical point

d -dimensional
space



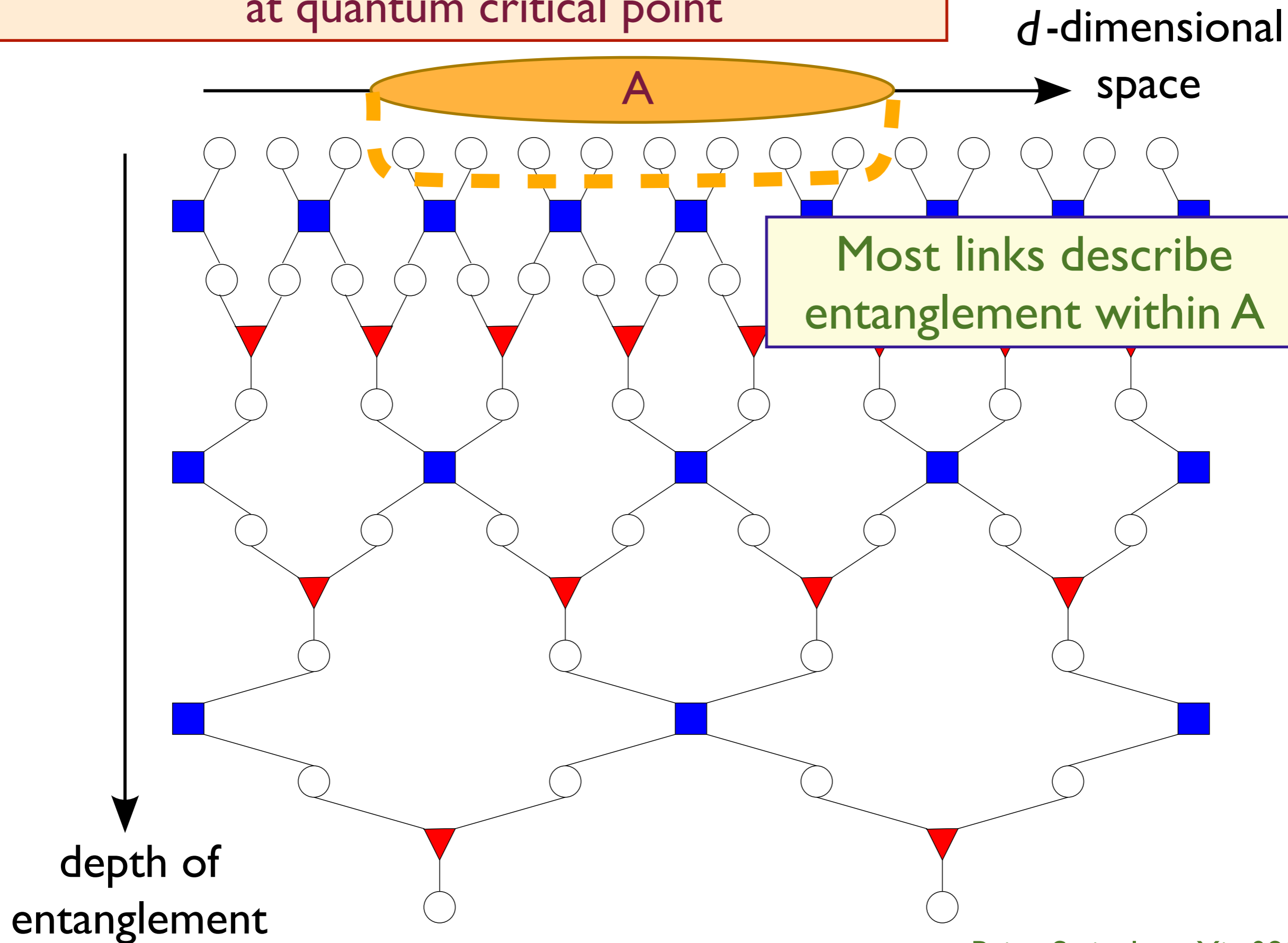
M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

Tensor network representation of entanglement at quantum critical point



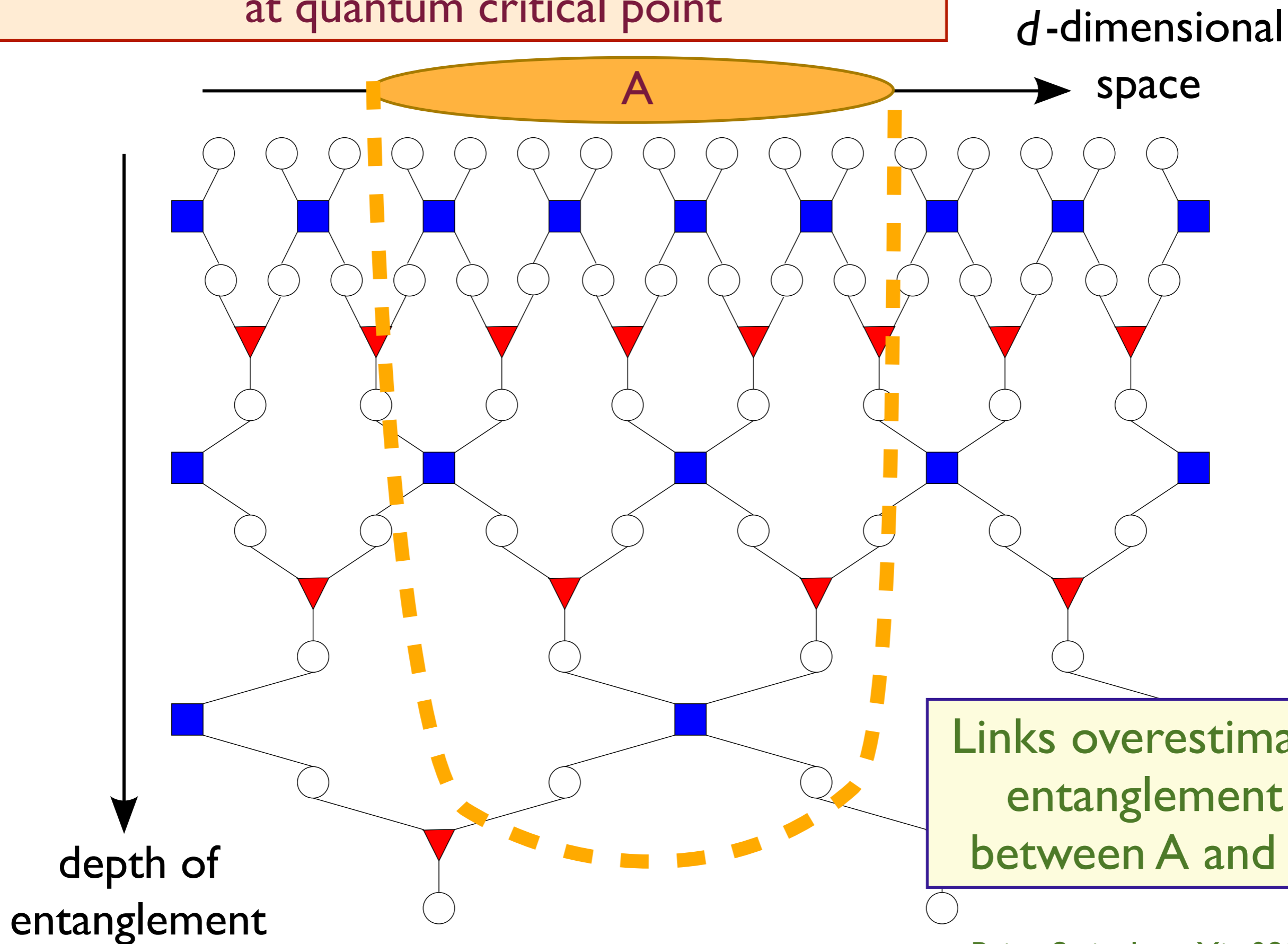
Brian Swingle, arXiv:0905.1317

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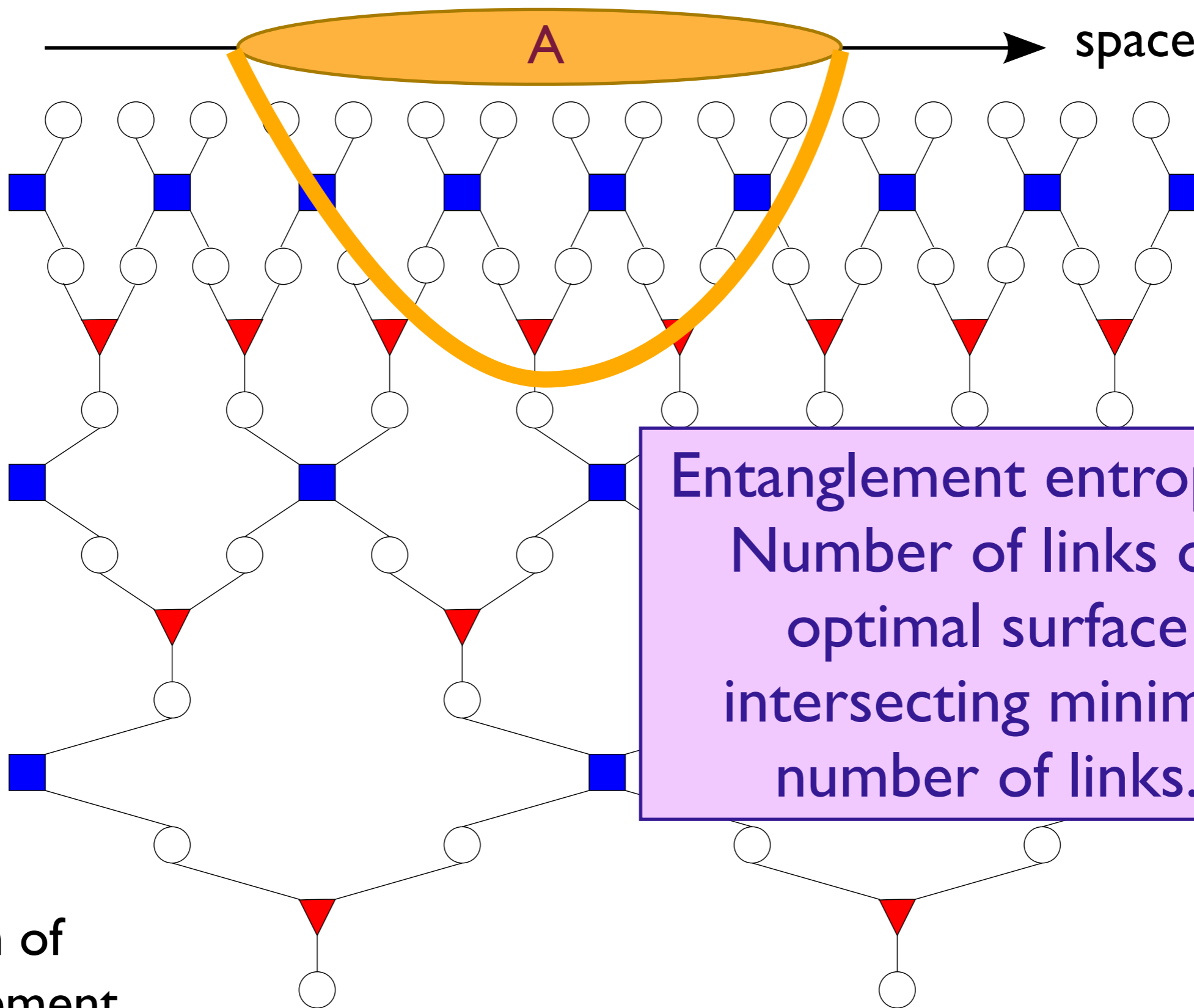
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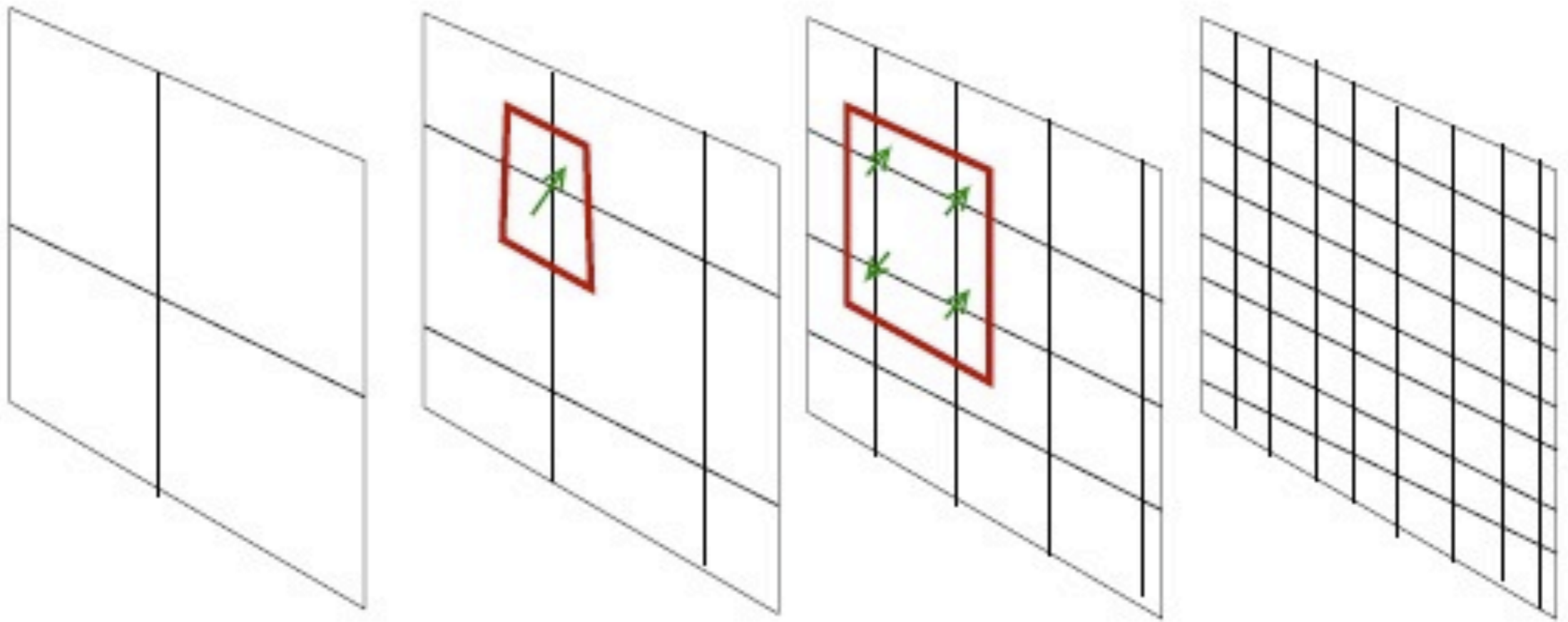
d -dimensional
space



Entanglement entropy =
Number of links on
optimal surface
intersecting minimal
number of links.

depth of
entanglement

Holography



r ←

Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

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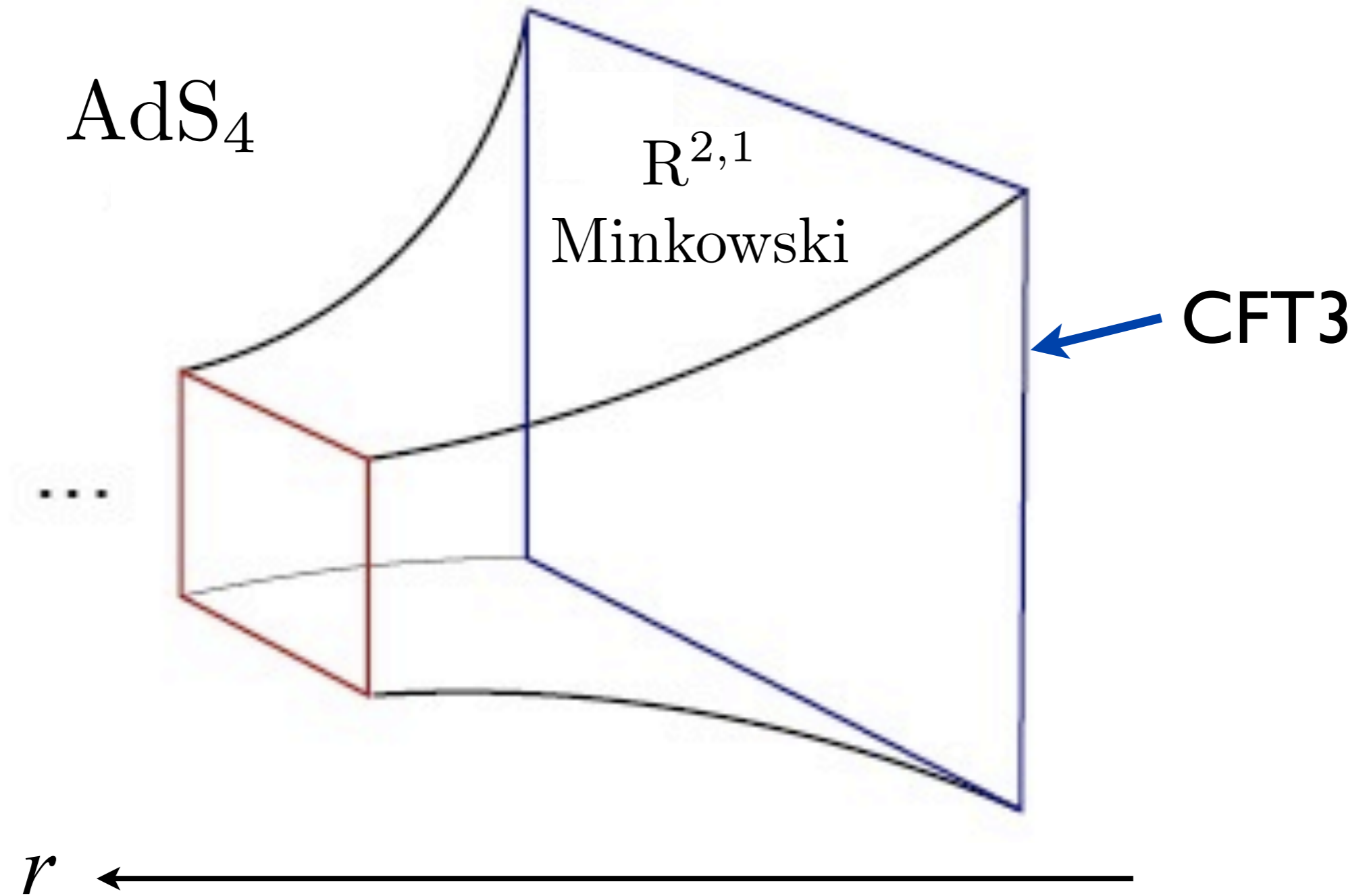
$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

This gives the unique metric

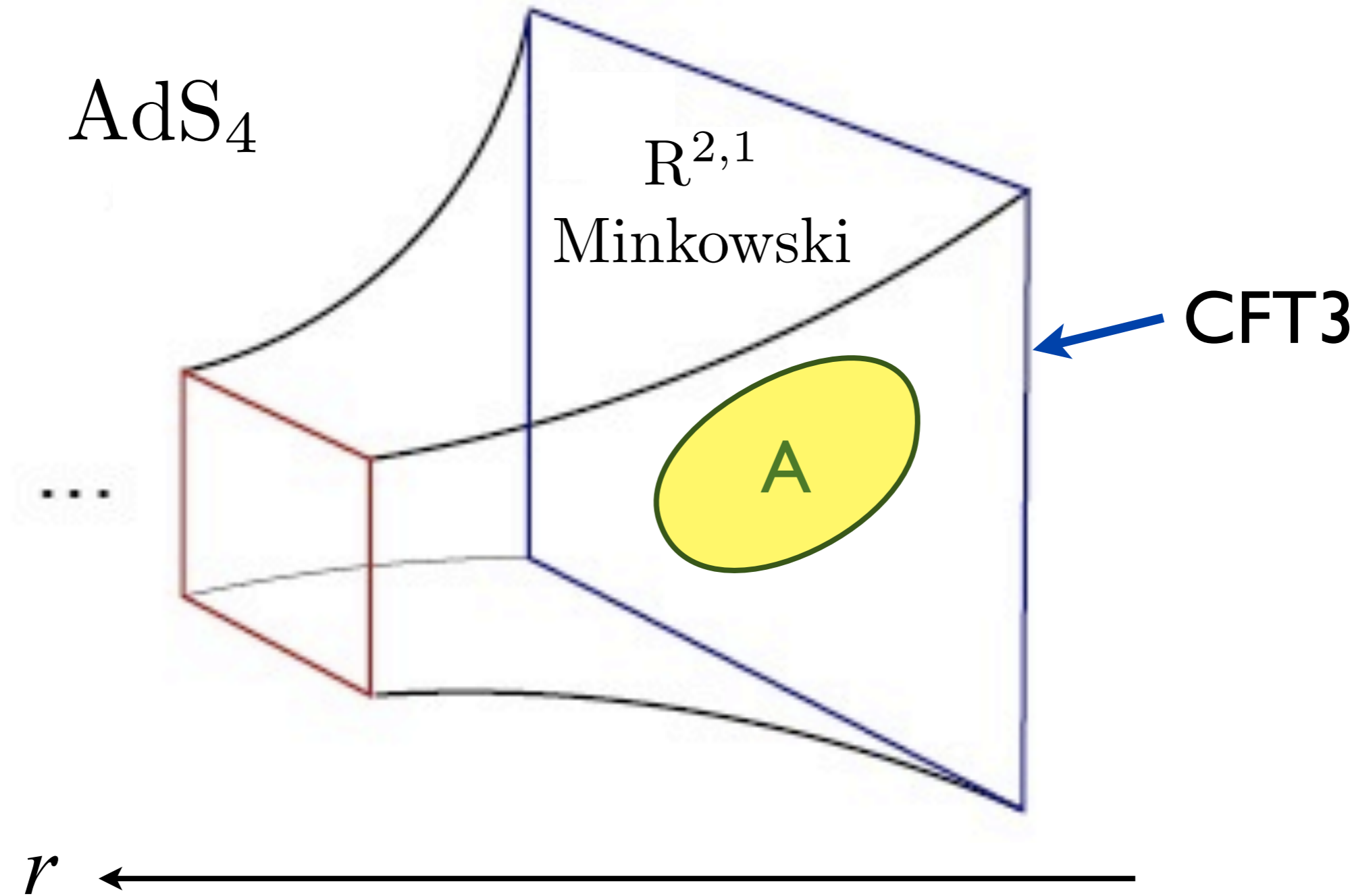
$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2} .

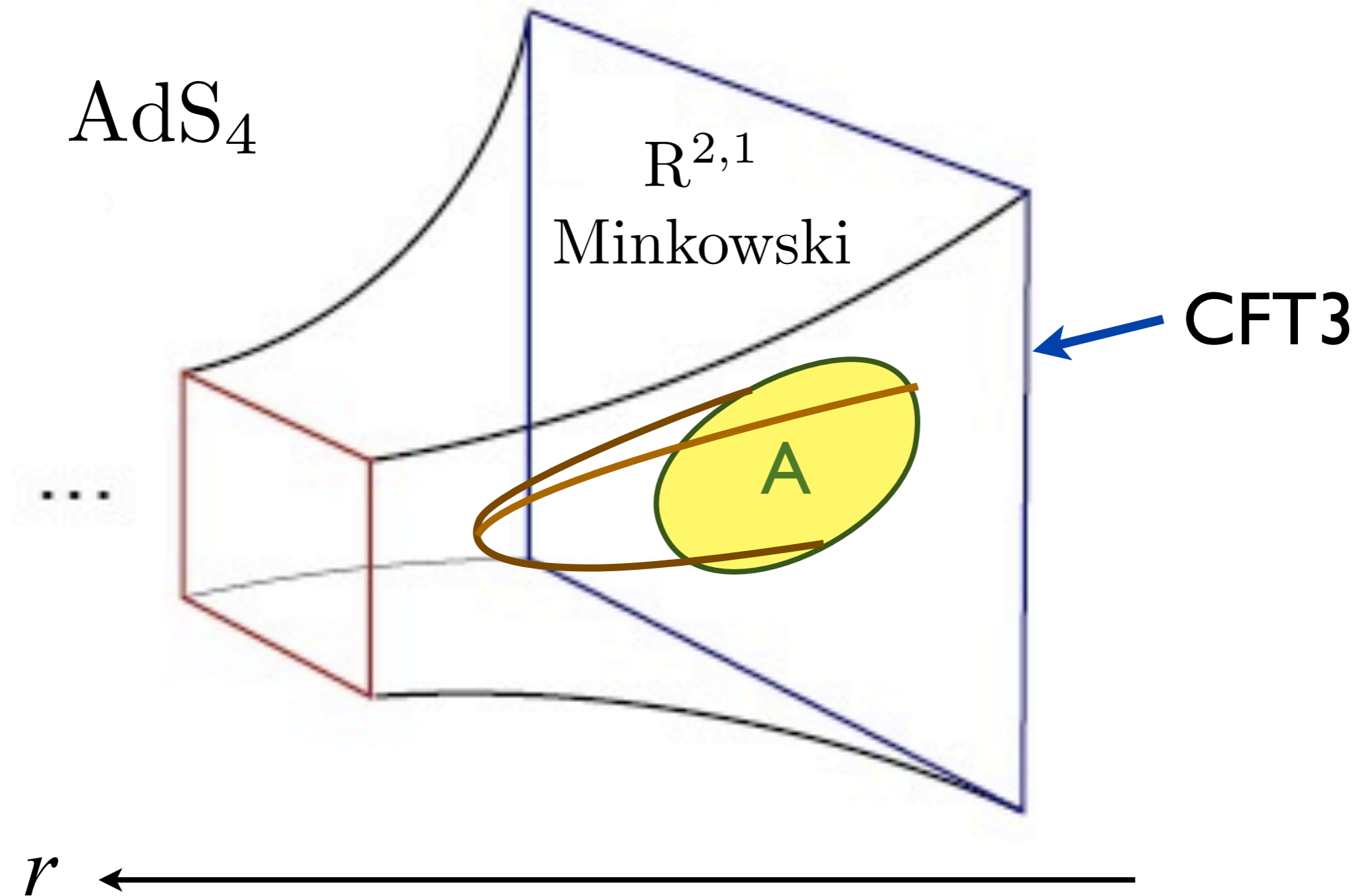
AdS/CFT correspondence



AdS/CFT correspondence



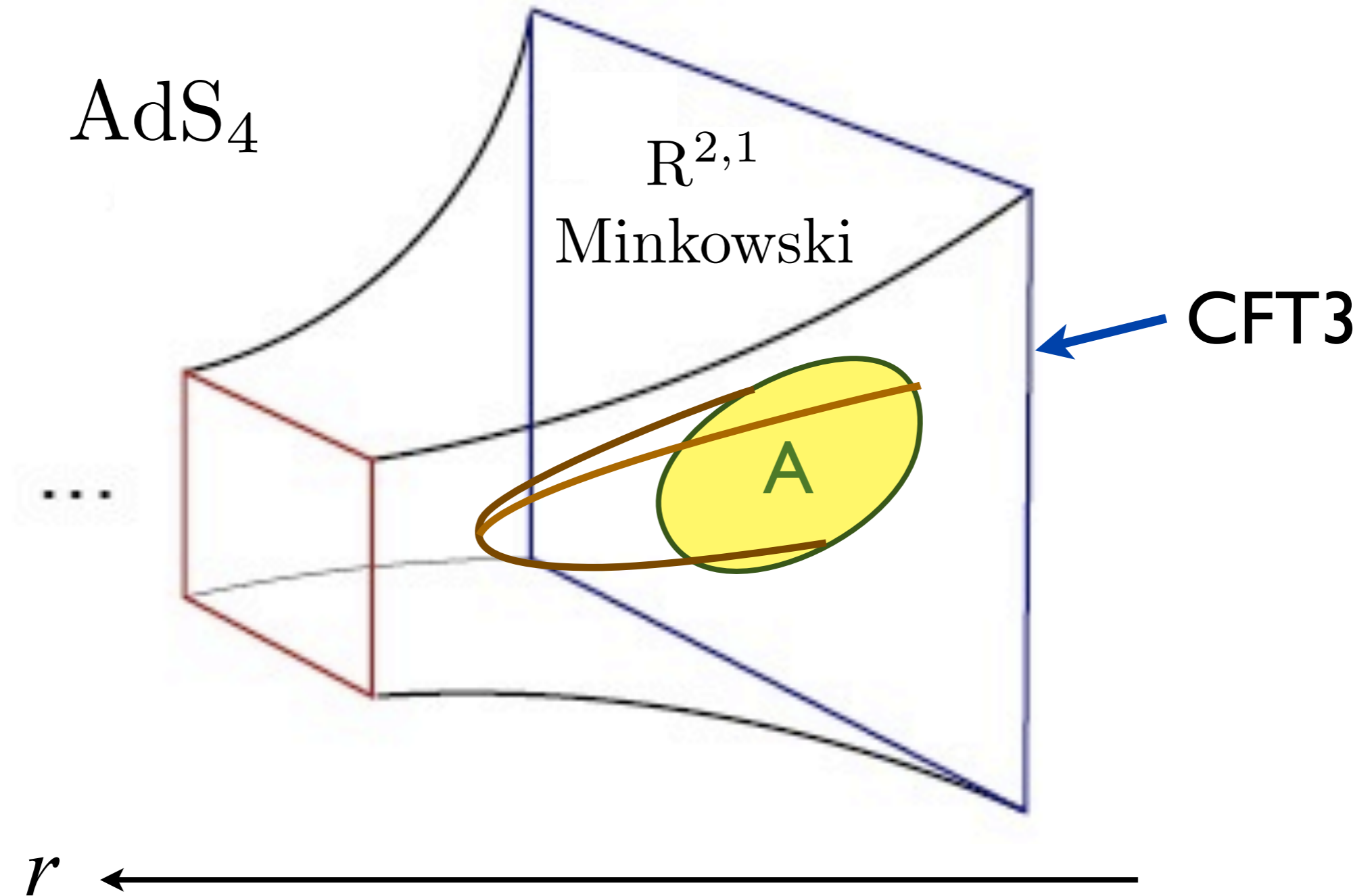
AdS/CFT correspondence



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe “outside” : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

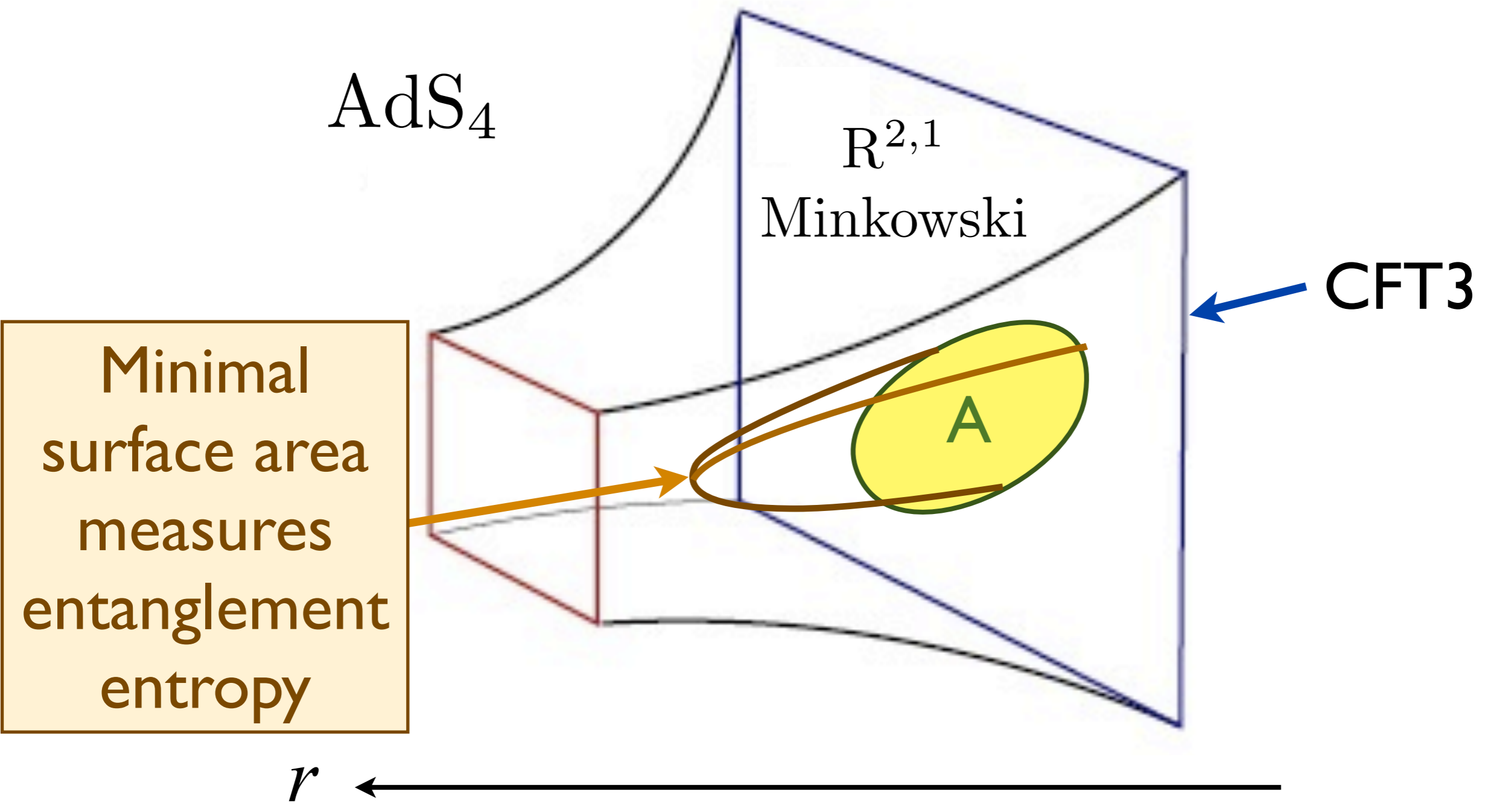
AdS/CFT correspondence



The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

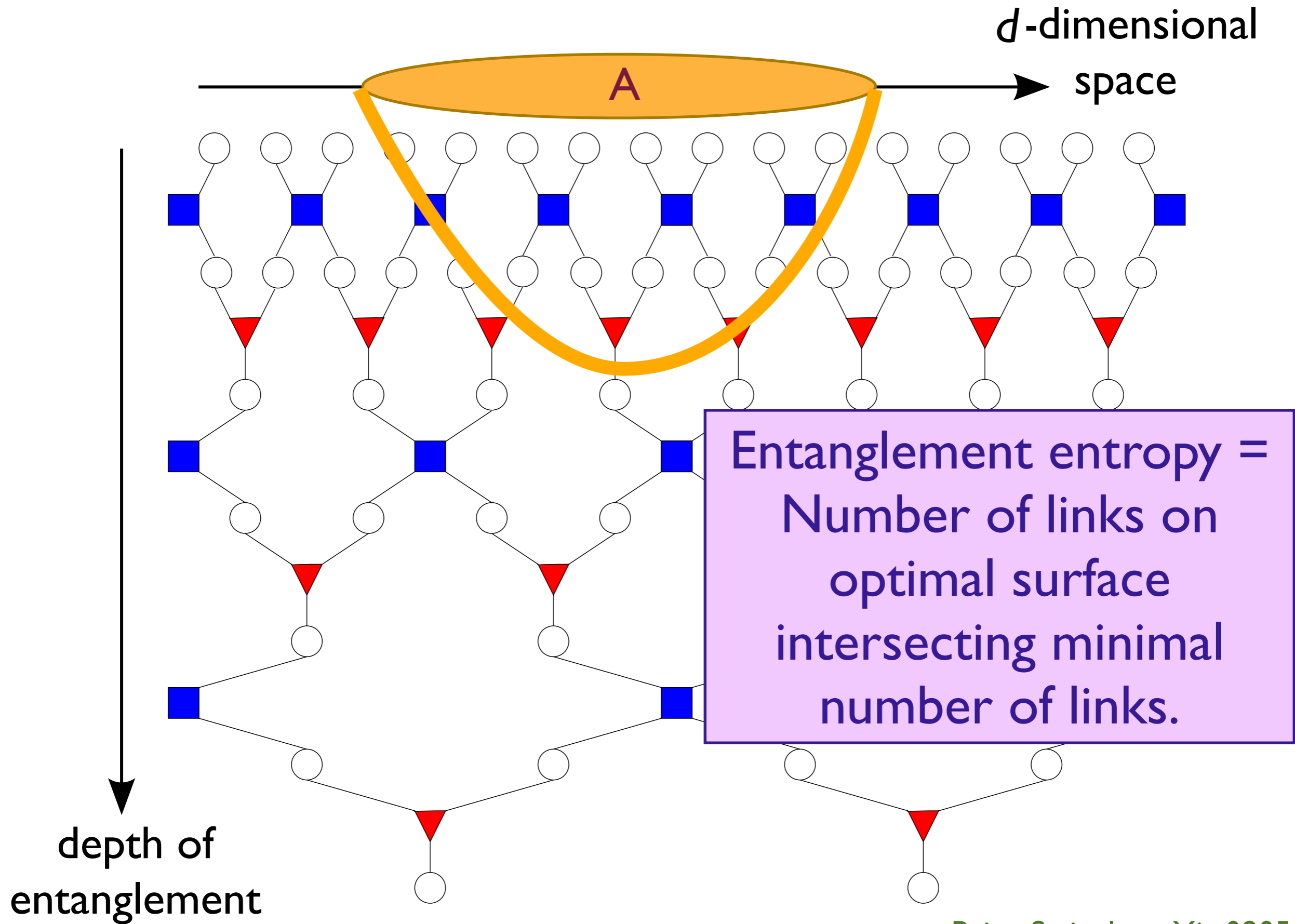
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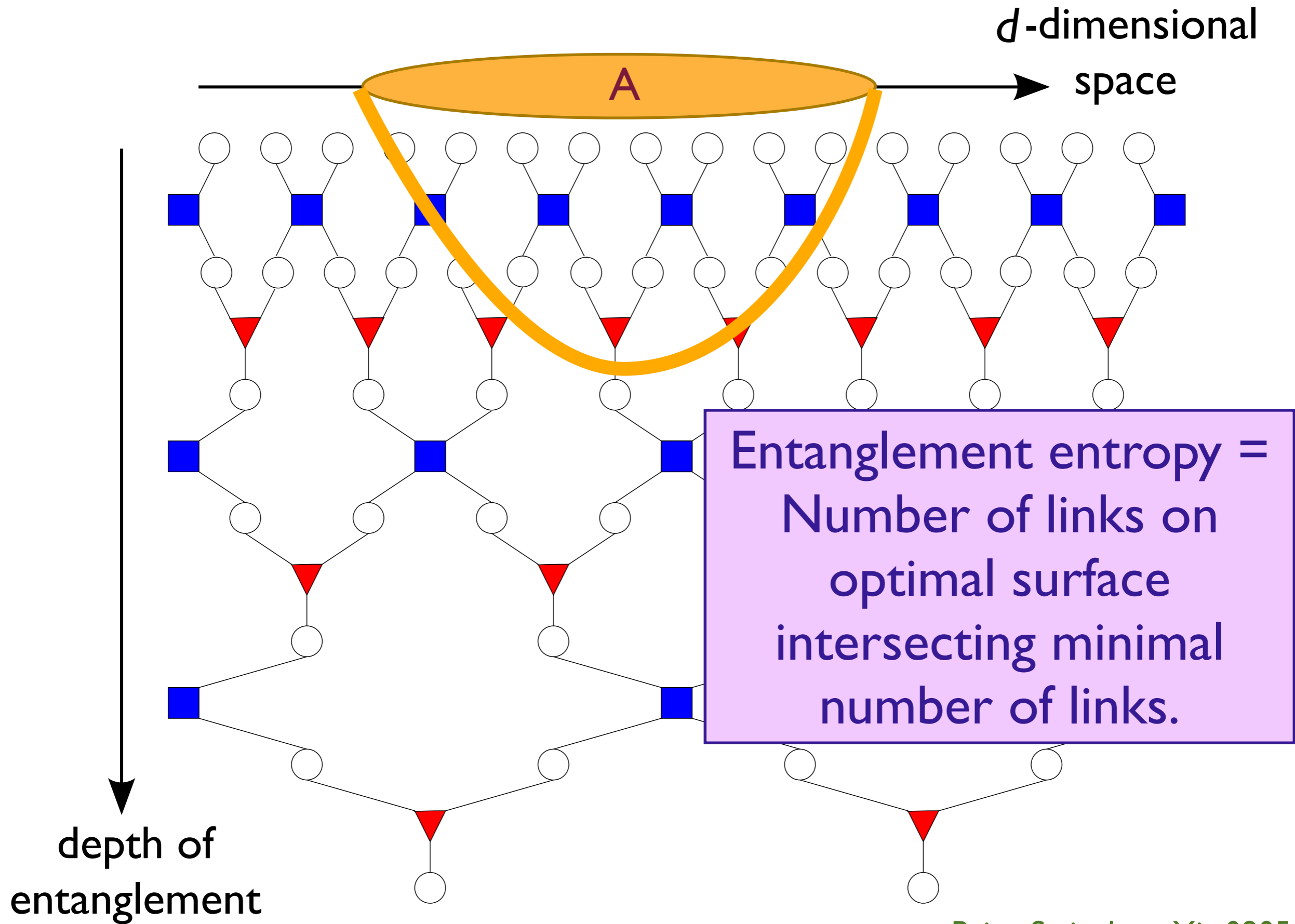


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Entanglement entropy

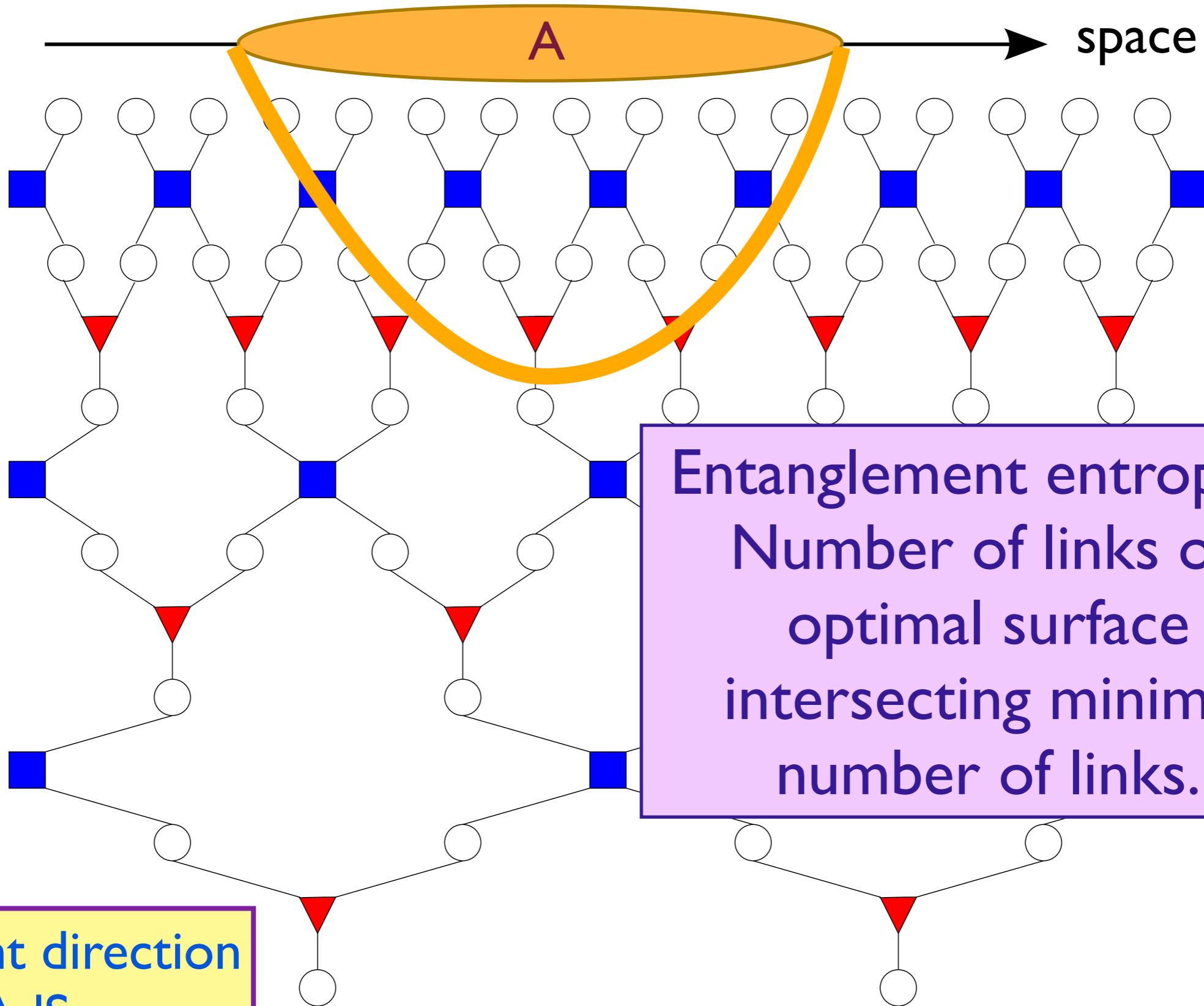


Entanglement entropy



Entanglement entropy

d -dimensional
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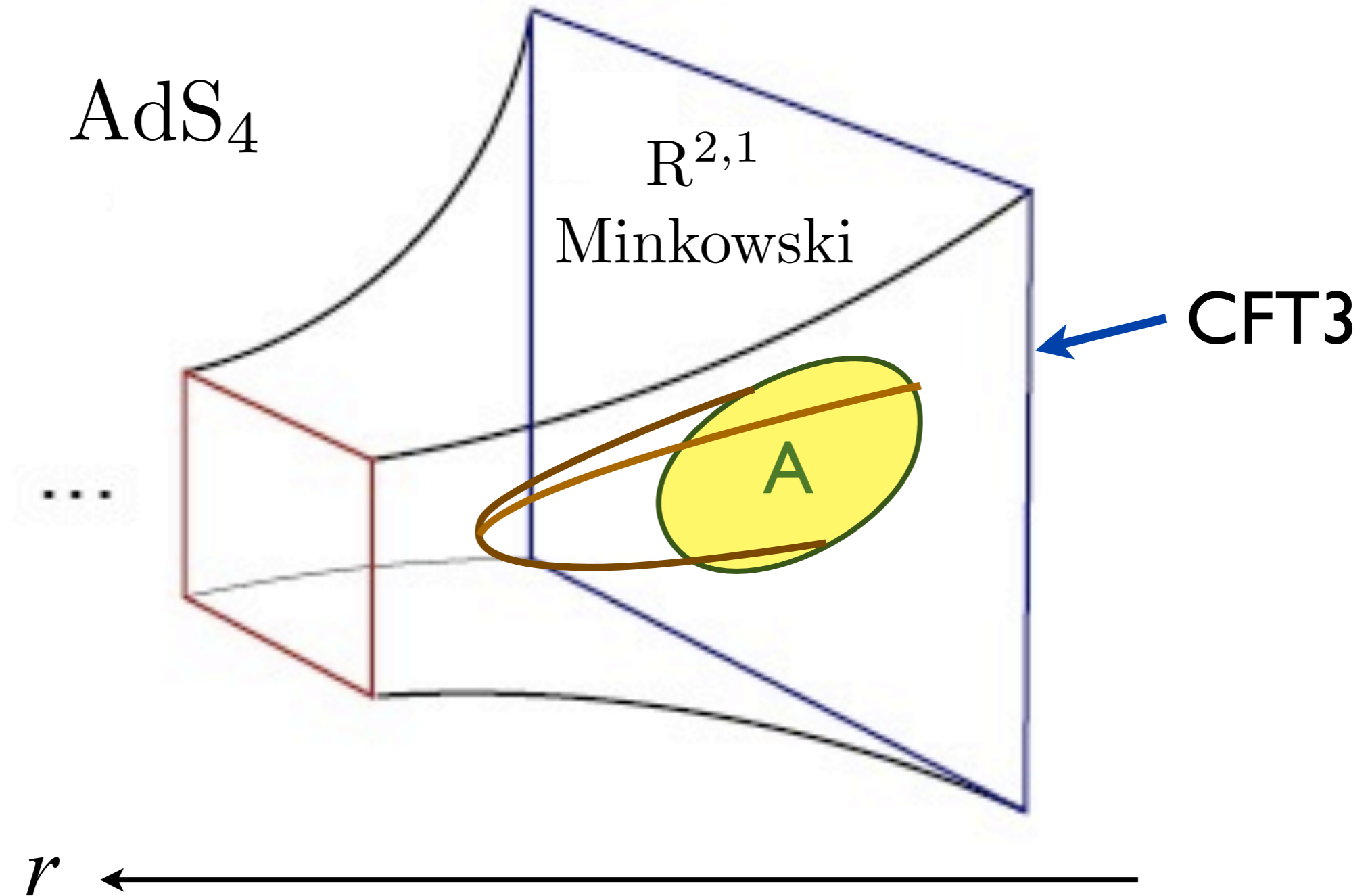


Entanglement entropy =
Number of links on
optimal surface
intersecting minimal
number of links.

Emergent direction
of AdS_{d+2}

Brian Swingle, arXiv:0905.1317

AdS/CFT correspondence



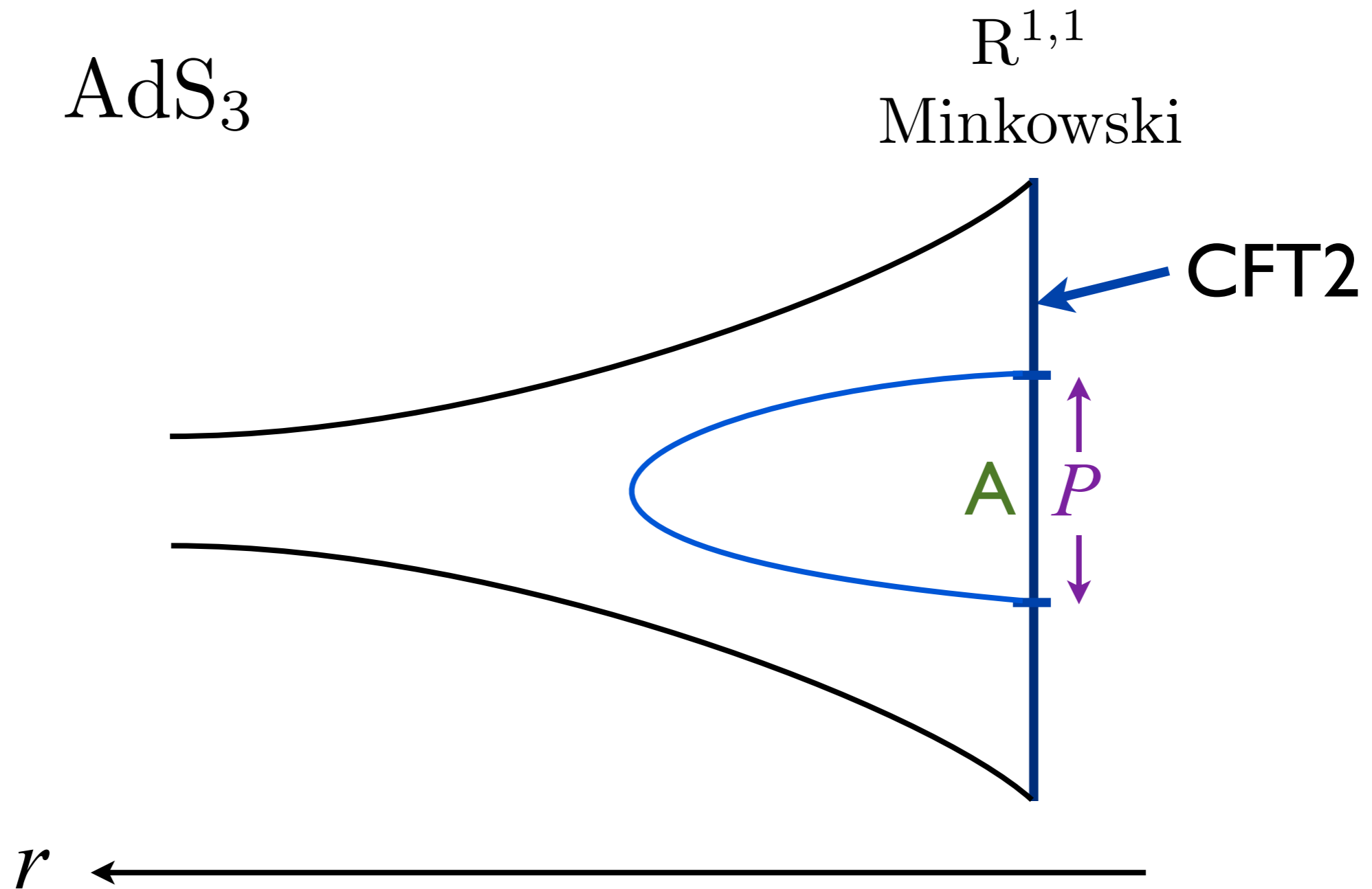
- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



- Computation of minimal surface area, or direct computation in CFT_2 , yield $S_E = (c/6) \ln P$, where c is the central charge.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

“Complex entangled” states of quantum matter in d spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

“Complex entangled” states of quantum matter in d spatial dimensions

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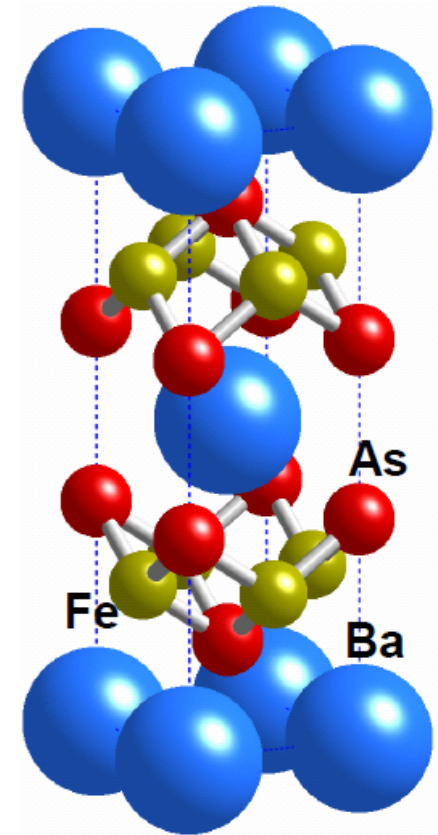
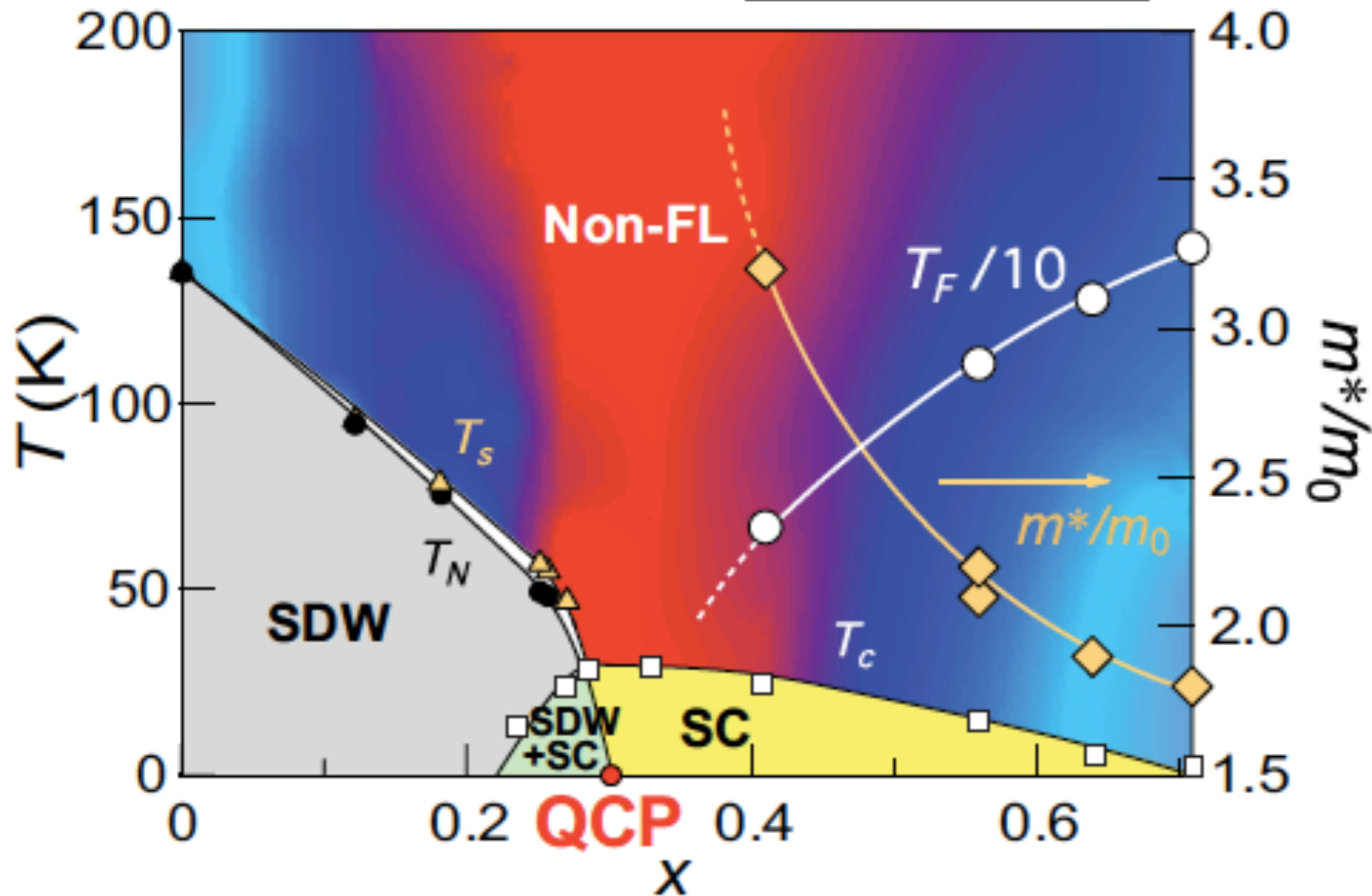
Conformal quantum matter

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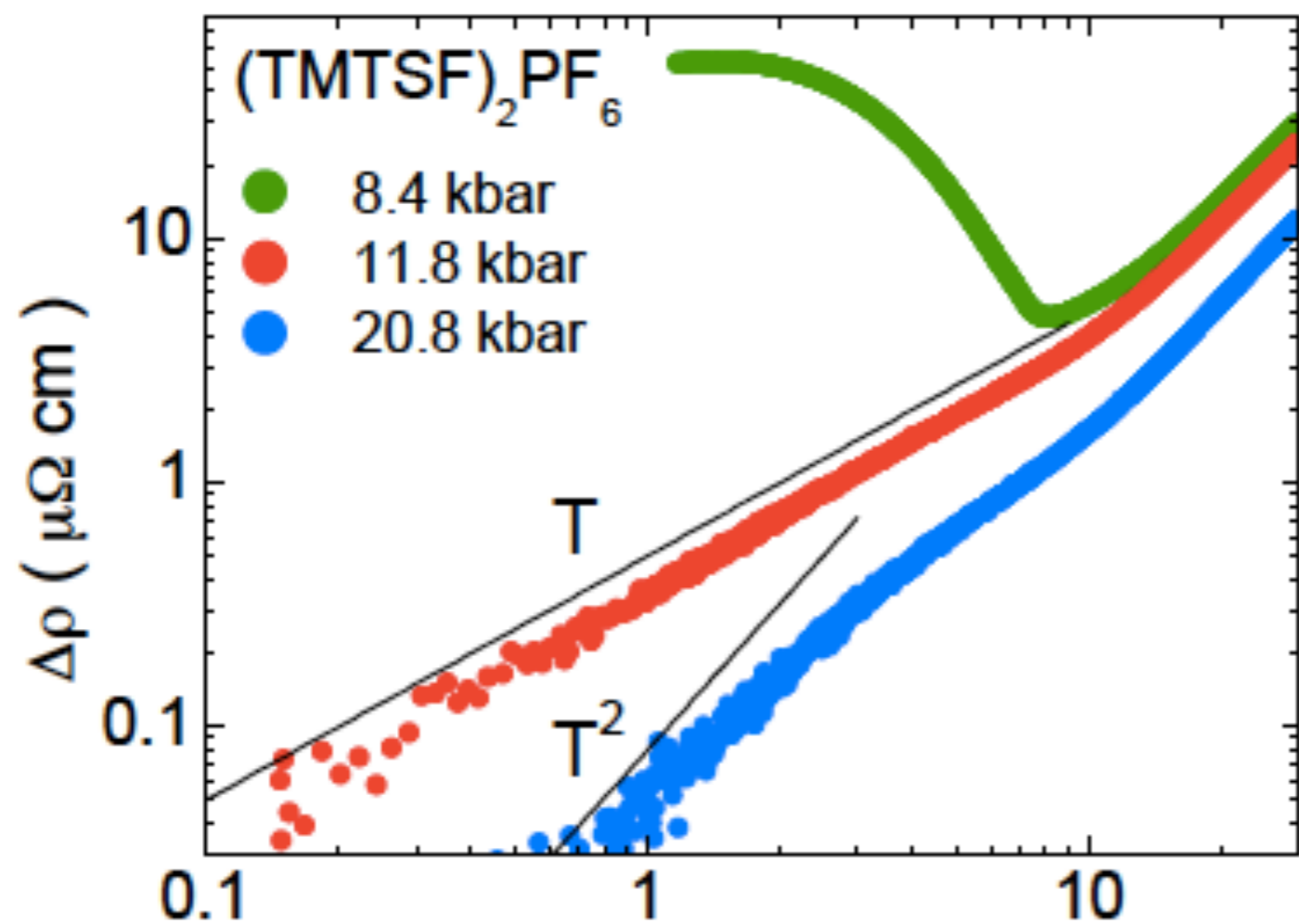
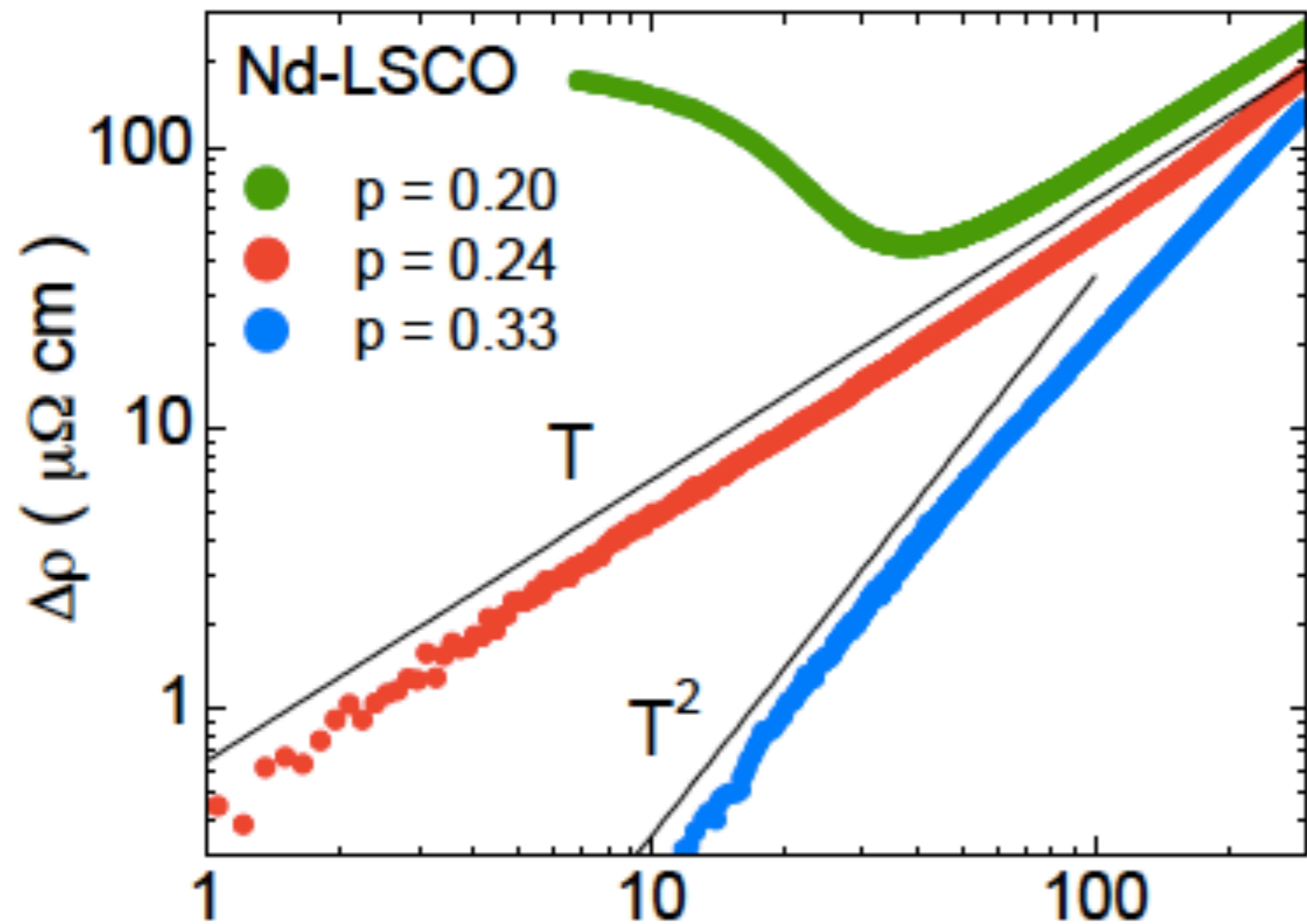
Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

Resistivity
 $\sim \rho_0 + AT^n$

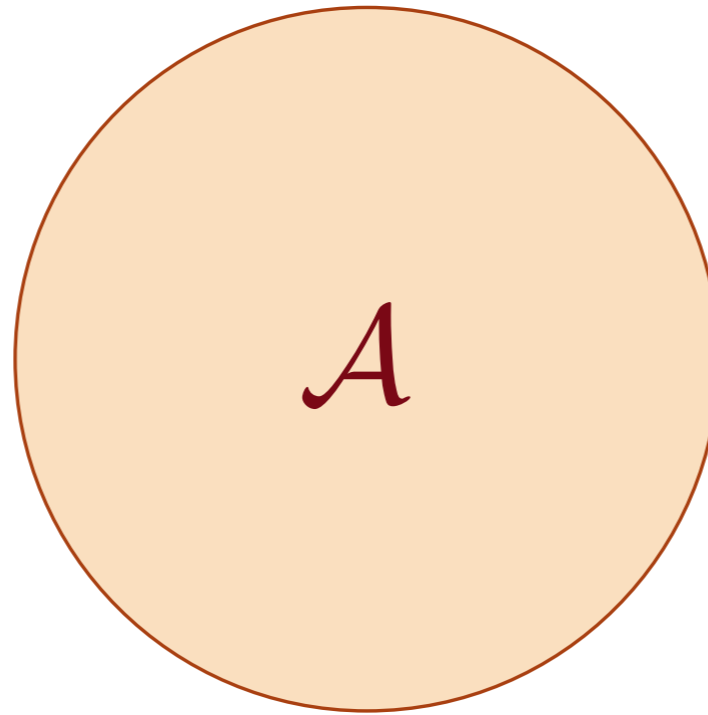


K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M.A. Tanatar, H. Kitano, N. Salovich, R.W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, Science in press

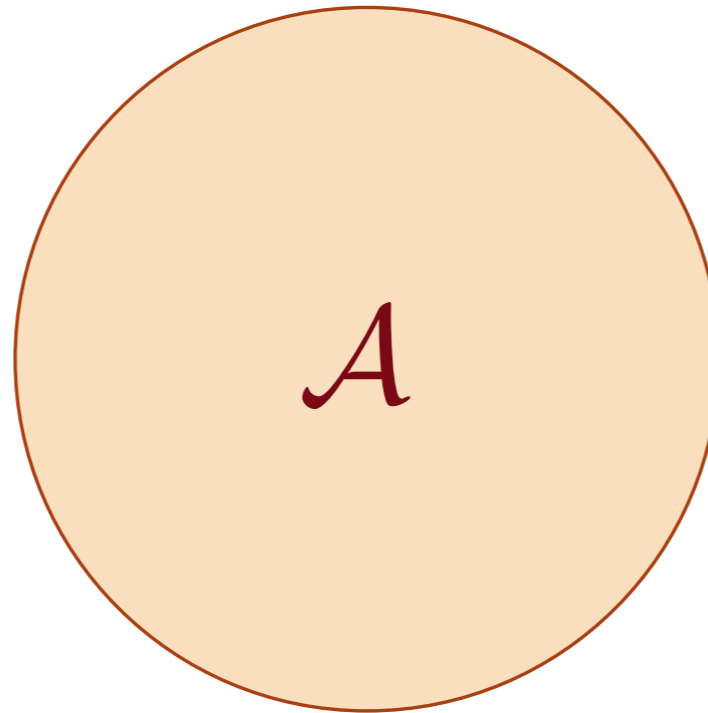


N. Doiron-Leyraud
and L. Taillefer,
arXiv:1204.0490

The Fermi liquid

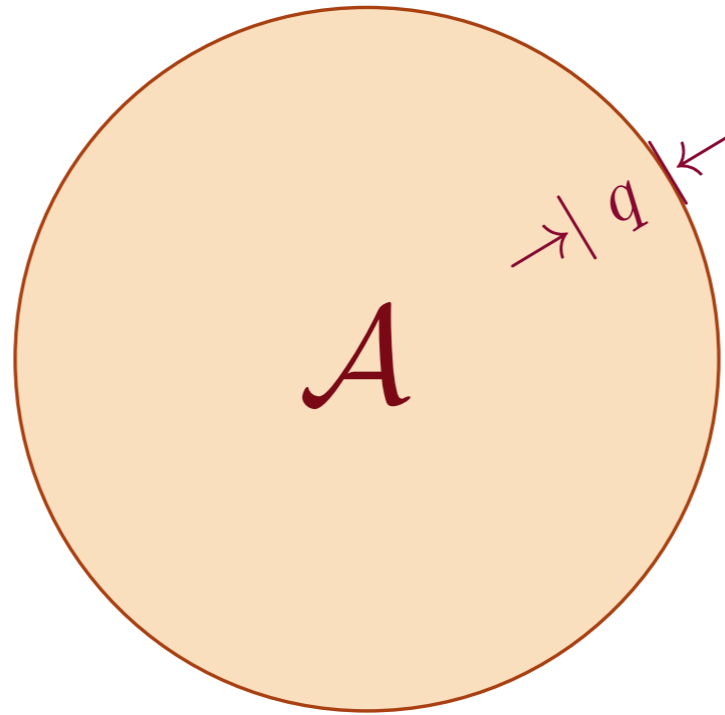


The Fermi liquid



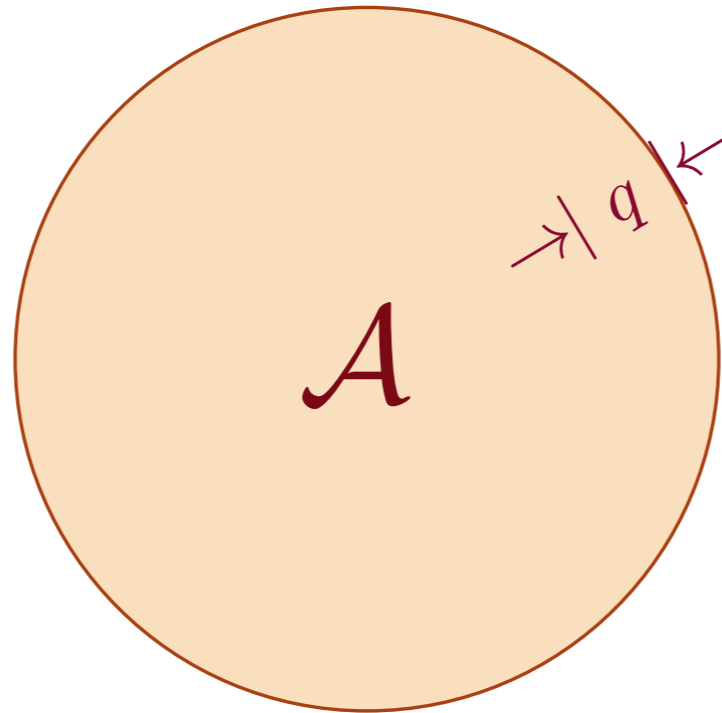
- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

The Fermi liquid



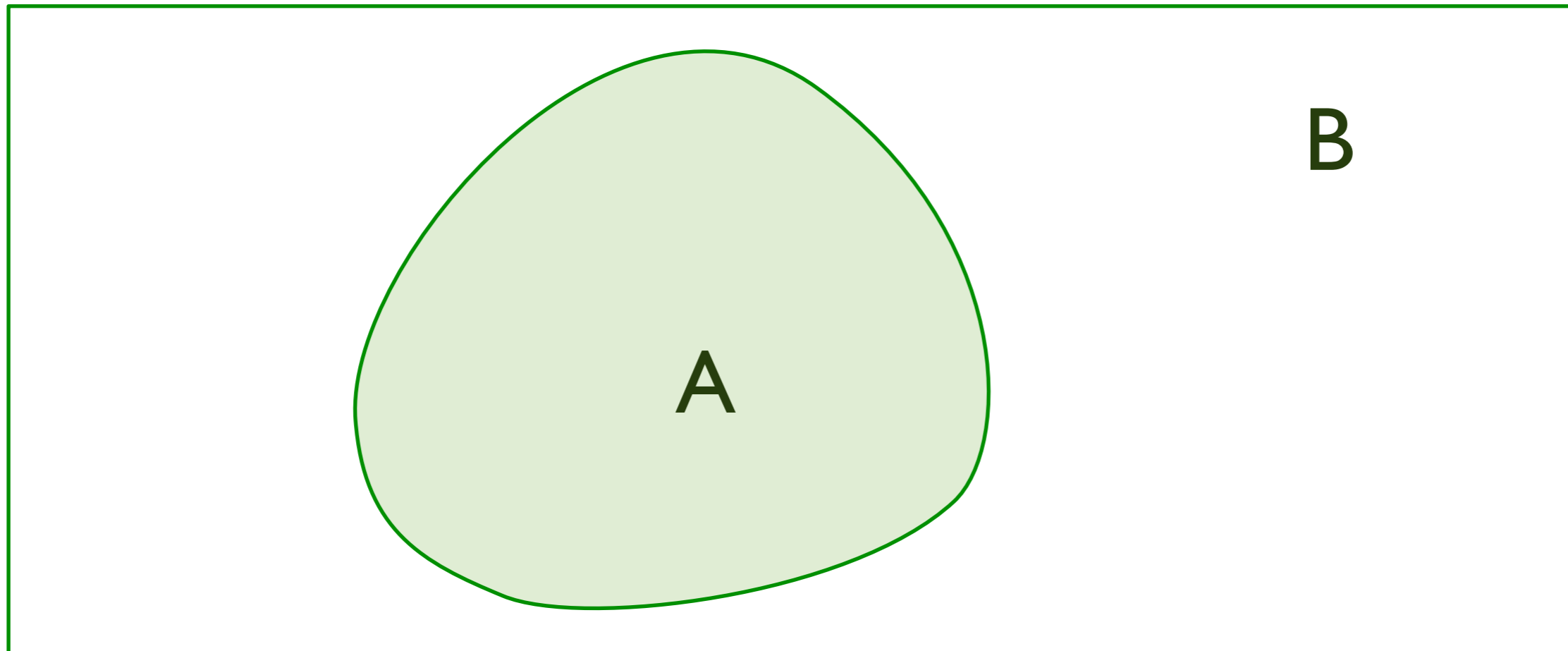
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The Fermi liquid



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- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}}$ with $d_{\text{eff}} = 1$.

Entanglement entropy of Fermi surfaces

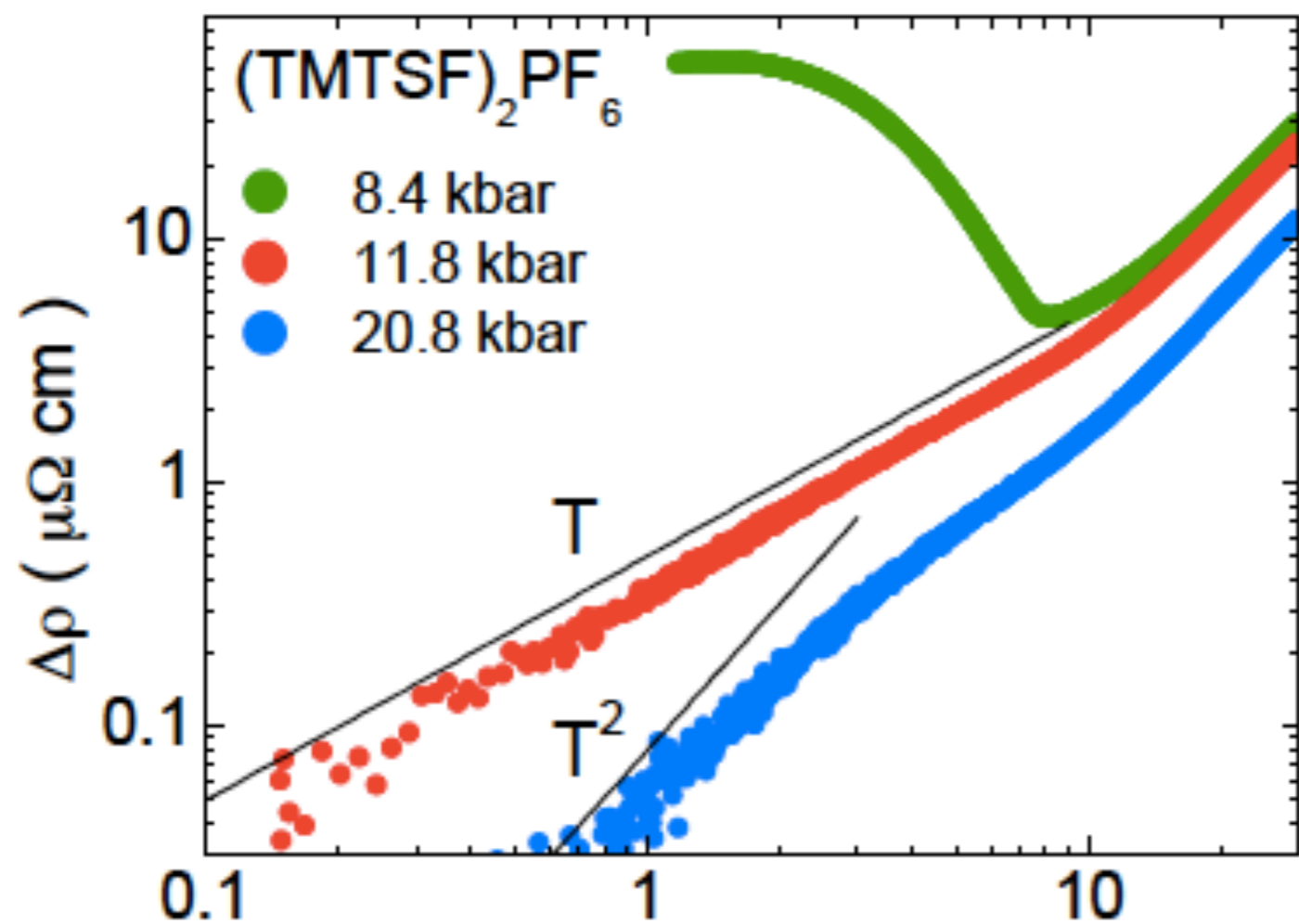
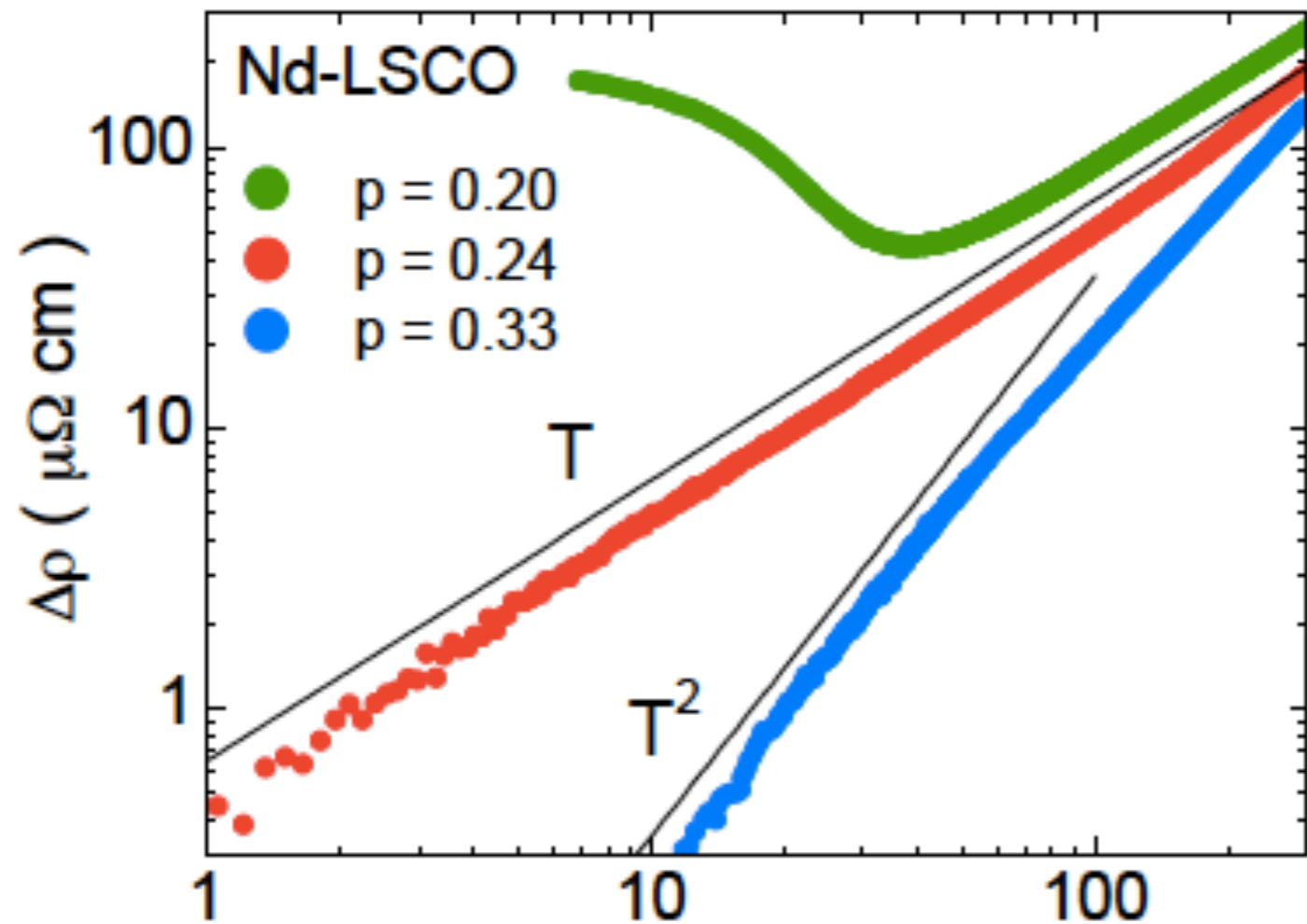


Logarithmic violation of “area law”: $S_E = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)



N. Doiron-Leyraud
and L. Taillefer,
arXiv:1204.0490

Strange metals

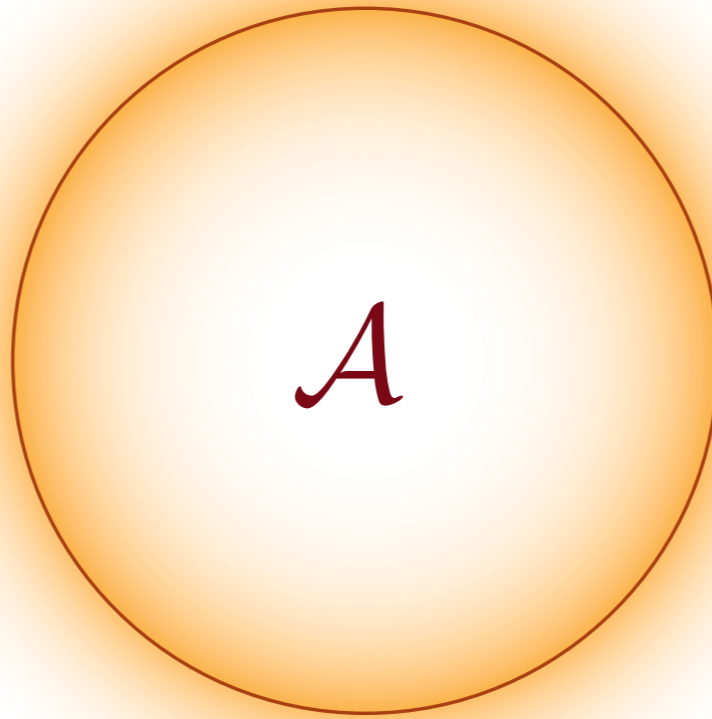
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Strange metals

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless scalar field, ϕ , which has low energy excitations near $\mathbf{q} = 0$. The field ϕ could represent

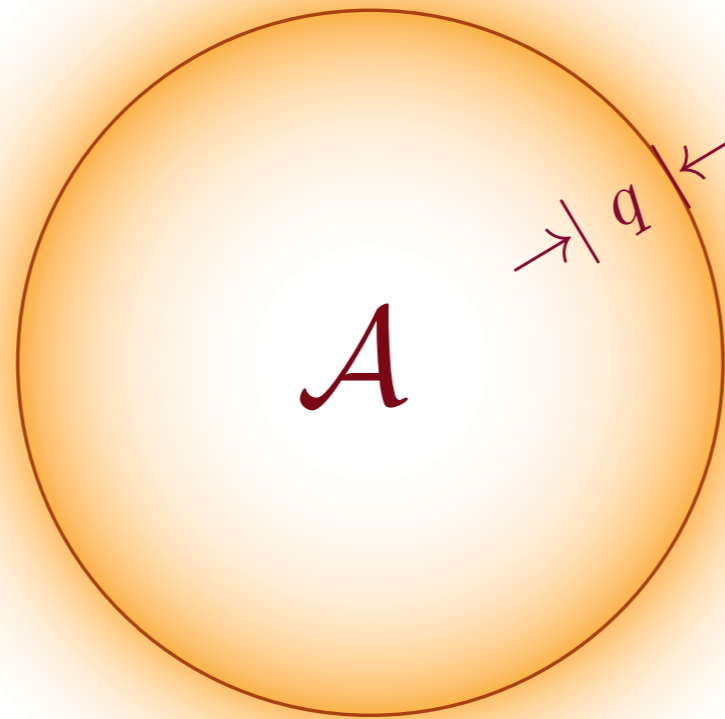
- ferromagnetic order
- breaking of point-group symmetry (Ising-nematic order)
- breaking of time-reversal symmetry
- circulating currents
- transverse component of an Abelian or non-Abelian gauge field.
- ...

Strange metals



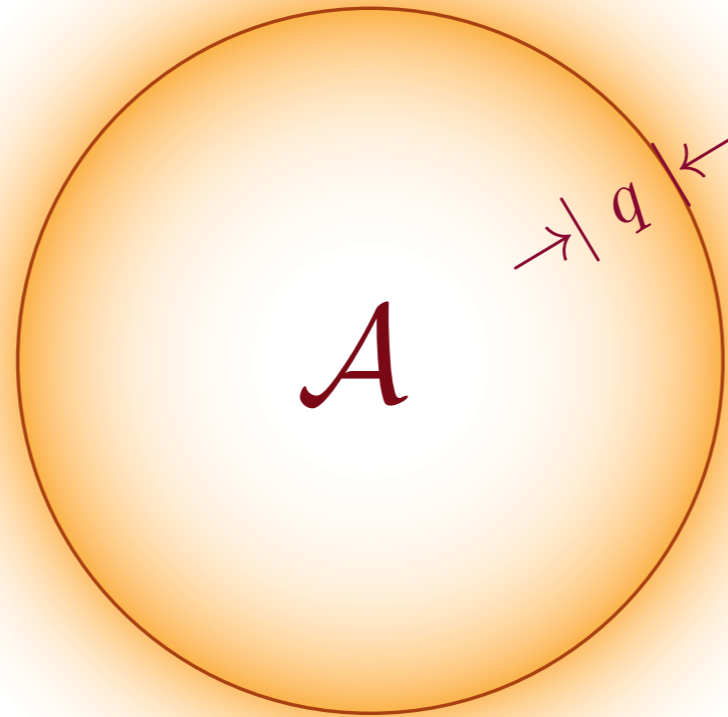
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Strange metals



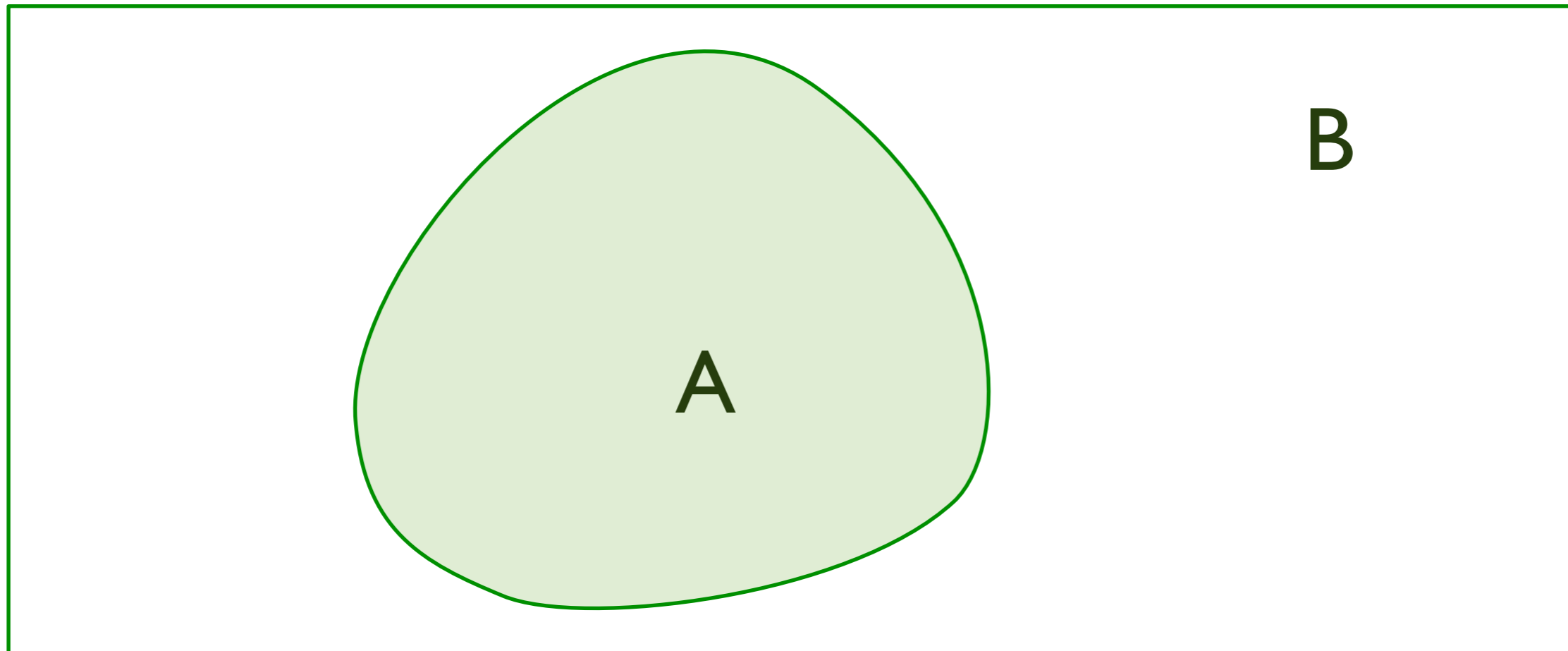
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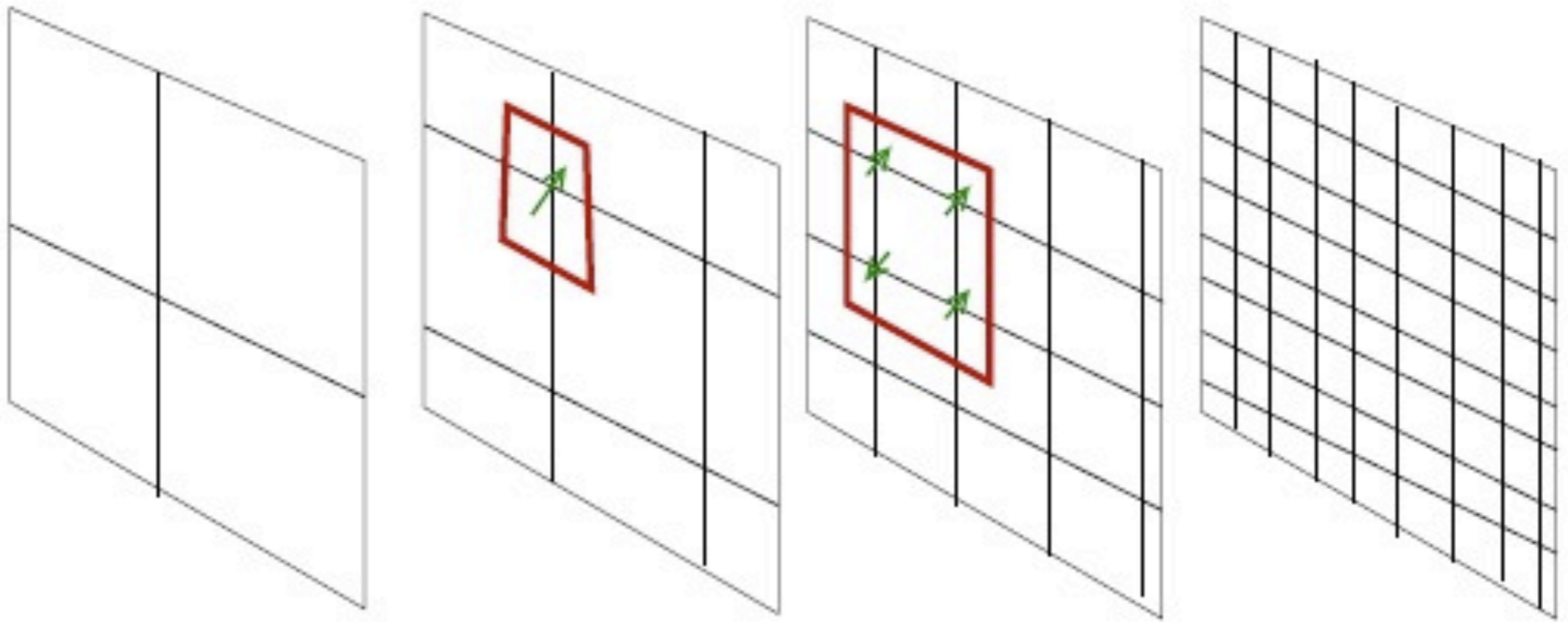
Logarithmic violation of “area law”: $S_E = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the “1/12” prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holography



r ←

Consider the metric which transforms under rescaling as

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\t &\rightarrow \zeta^z t \\ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

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θ is the violation of hyperscaling exponent.

The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

- The entanglement entropy, S_E , of an entangling region with boundary surface ‘area’ P scales as

$$S_E \sim \begin{cases} P & , \text{ for } \theta < d - 1 \\ P \ln P & , \text{ for } \theta = d - 1 \\ P^{\theta/(d-1)} & , \text{ for } \theta > d - 1 \end{cases}$$

All local quantum field theories obey the “area law” (upto log violations) and so $\theta \leq d - 1$.

- The null energy condition implies $z \geq 1 + \frac{\theta}{d}$.

Holography of strange metals

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d .

Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

Holography of strange metals

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Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

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$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

Holography of strange metals

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

- This metric can be realized in a Maxwell-Einstein-dilaton theory, which may be viewed as a “bosonization” of the non-Fermi liquid state. The entanglement entropy of this theory has log-violation of the area law with

$$S_E = \Xi Q^{(d-1)/d} P \ln P.$$

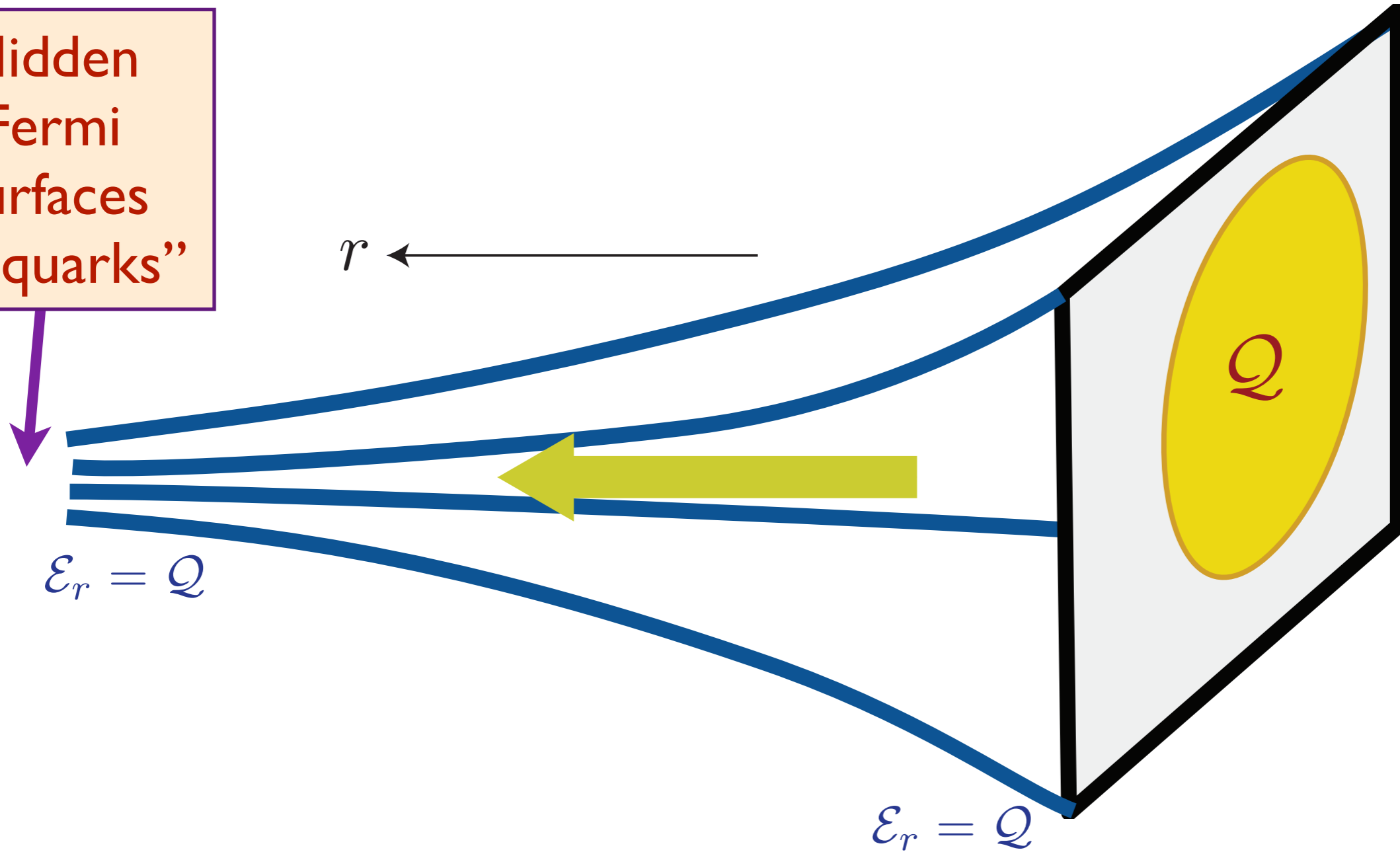
where P is surface area of the entangling region, and Ξ is a dimensionless constant which is **independent of all UV details**, of Q , and of any property of the entangling region.

Note $Q^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

Hidden Fermi surfaces of “quarks”



Gauss Law and the “attractor” mechanism
 \Leftrightarrow Luttinger theorem on the boundary


Conclusions

Gapped quantum matter

- Numerical and experimental observation of a spin liquid on the kagome lattice. Likely a Z_2 spin liquid.


Conclusions

Conformal quantum matter

 Numerical and experimental observation in coupled-dimer antiferromagnets, and at the superfluid-insulator transition of bosons in optical lattices.

Conclusions

Compressible quantum matter

 Field theory of a non-Fermi liquid obtained by coupling a Fermi surface to a gapless scalar field with low energy excitations near zero wavevector. Obtained promising holographic dual of this theory.

Conclusions

Compressible quantum matter

● Evidence for hidden Fermi surfaces in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a non-Fermi liquid (NFL) state of gauge theories at non-zero density.

After fixing $\theta = d - 1$ to obtain thermal entropy density $S \sim T^{1/z}$, we found

- Log violation of the area law in entanglement entropy, S_E .
- Leading-log S_E independent of shape of entangling region.
- The $d = 2$ bound $z \geq 3/2$, compared to $z = 3/2$ in three-loop field theory.
- Evidence for Luttinger theorem in prefactor of S_E .