# Yukawa-SYK models and a universal theory of strange metals

Localisation 2022
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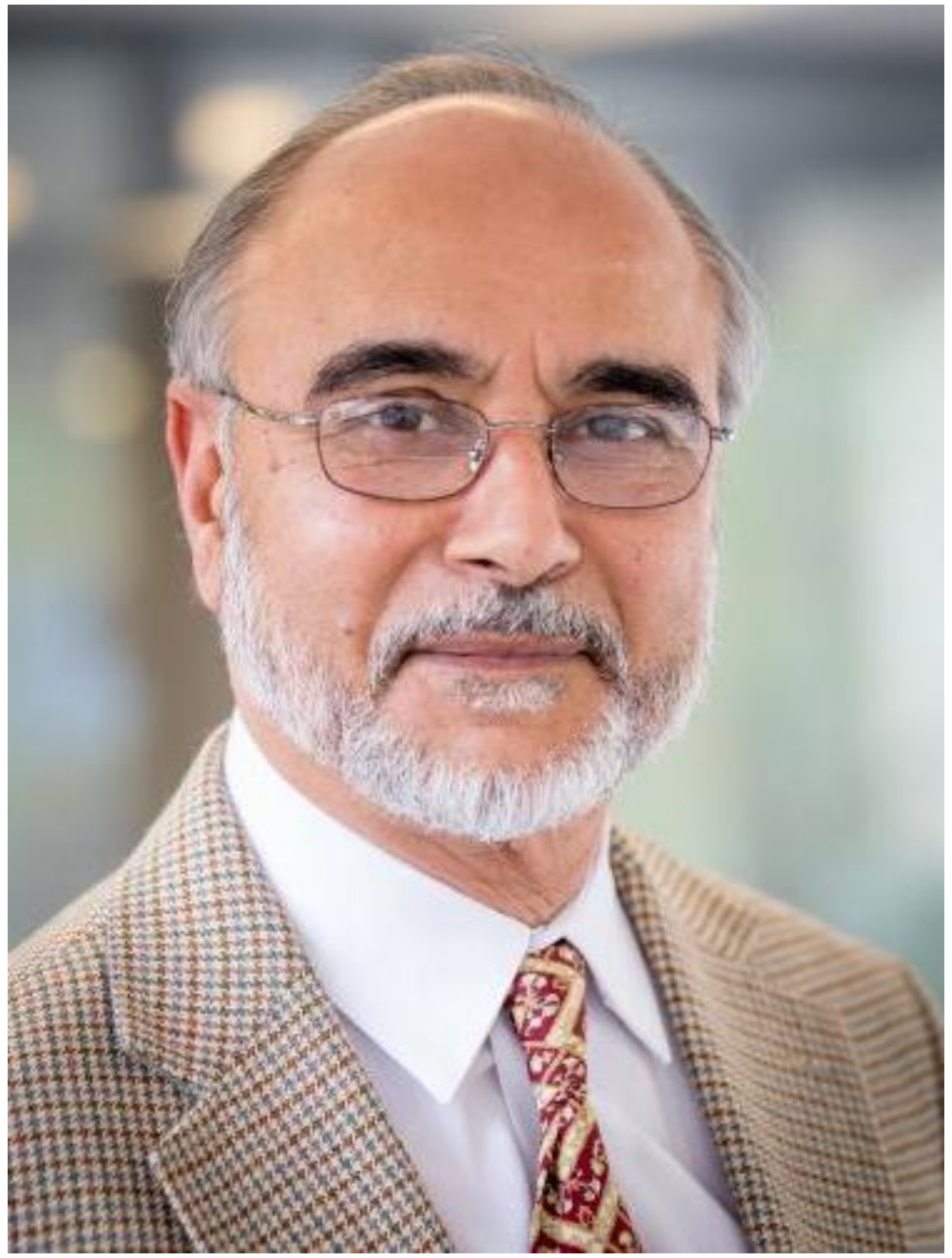
Subir Sachdev

Talk online: sachdev.physics.harvard.edu



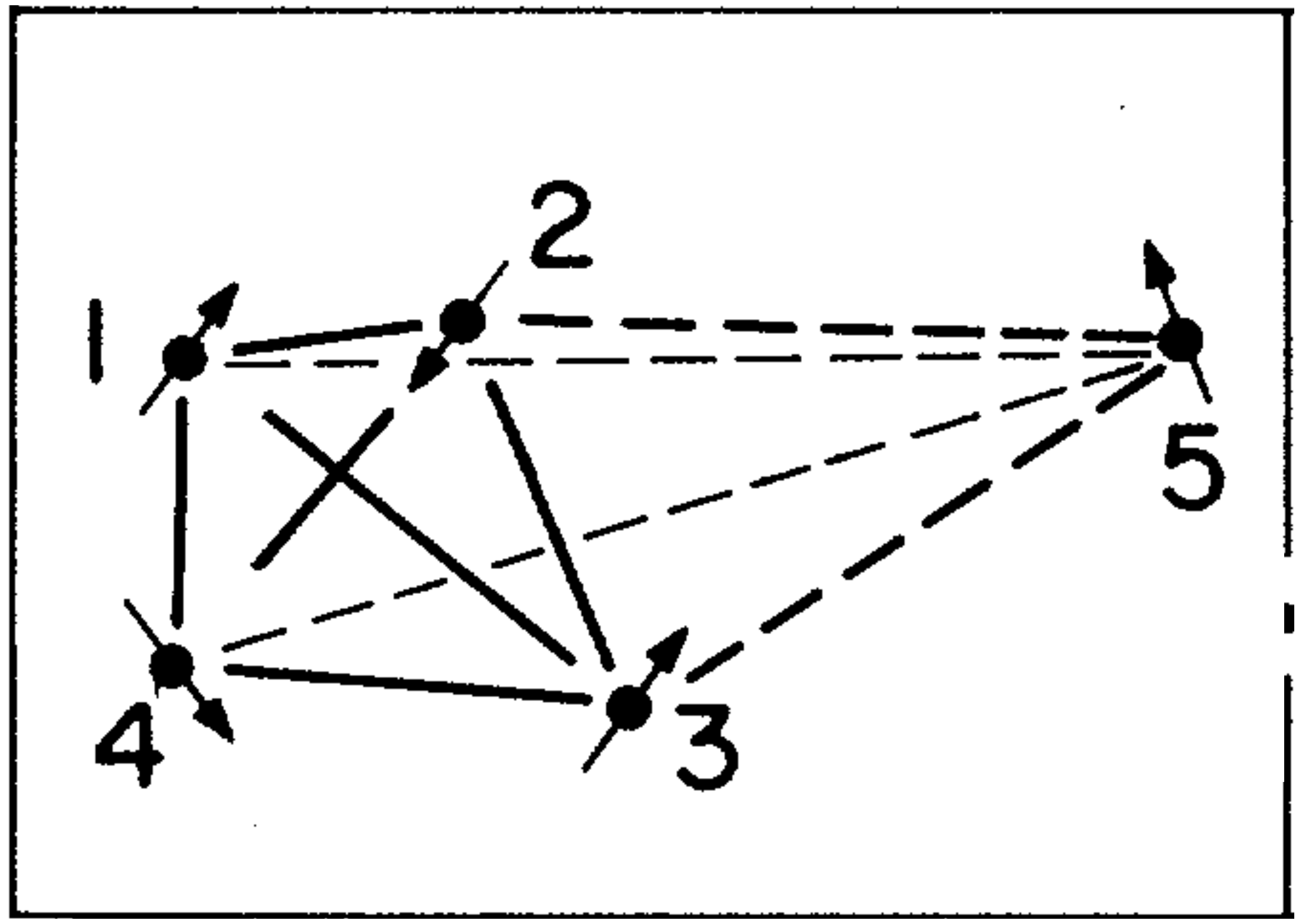
**Ravindra N. Bhatt**

## Scaling Studies of Highly Disordered Spin-$\frac{1}{2}$ Antiferromagnetic Systems

R. N. Bhatt and P. A. Lee
*Bell Laboratories, Murray Hill, New Jersey 07974*

(Received 6 May 1981; revised manuscript received 16 November 1981)
Physical Review Letters **48**, 344 (1982)

$$H = \frac{1}{2}\sum_{i \neq j} J(\vec{r}_i - \vec{r}_j)\vec{S}_i \cdot \vec{S}_j\,, \qquad J(\vec{r}) \sim \exp\left(-2\left|\vec{r}\right|/a_0\right)$$



## Spin Dynamics of Nearly Localized Electrons

M. A. Paalanen, S. Sachdev, and R. N. Bhatt
*AT&T Bell Laboratories, Murray Hill, New Jersey 07974*

and

A. E. Ruckenstein
*Department of Physics, University of California, San Diego, La Jolla, California 92037*

(Received 16 June 1986)
Physical Review Letters **57**, 2061 (1986)

Strong disorder fixed point ⇒ Random singlet phase

# Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

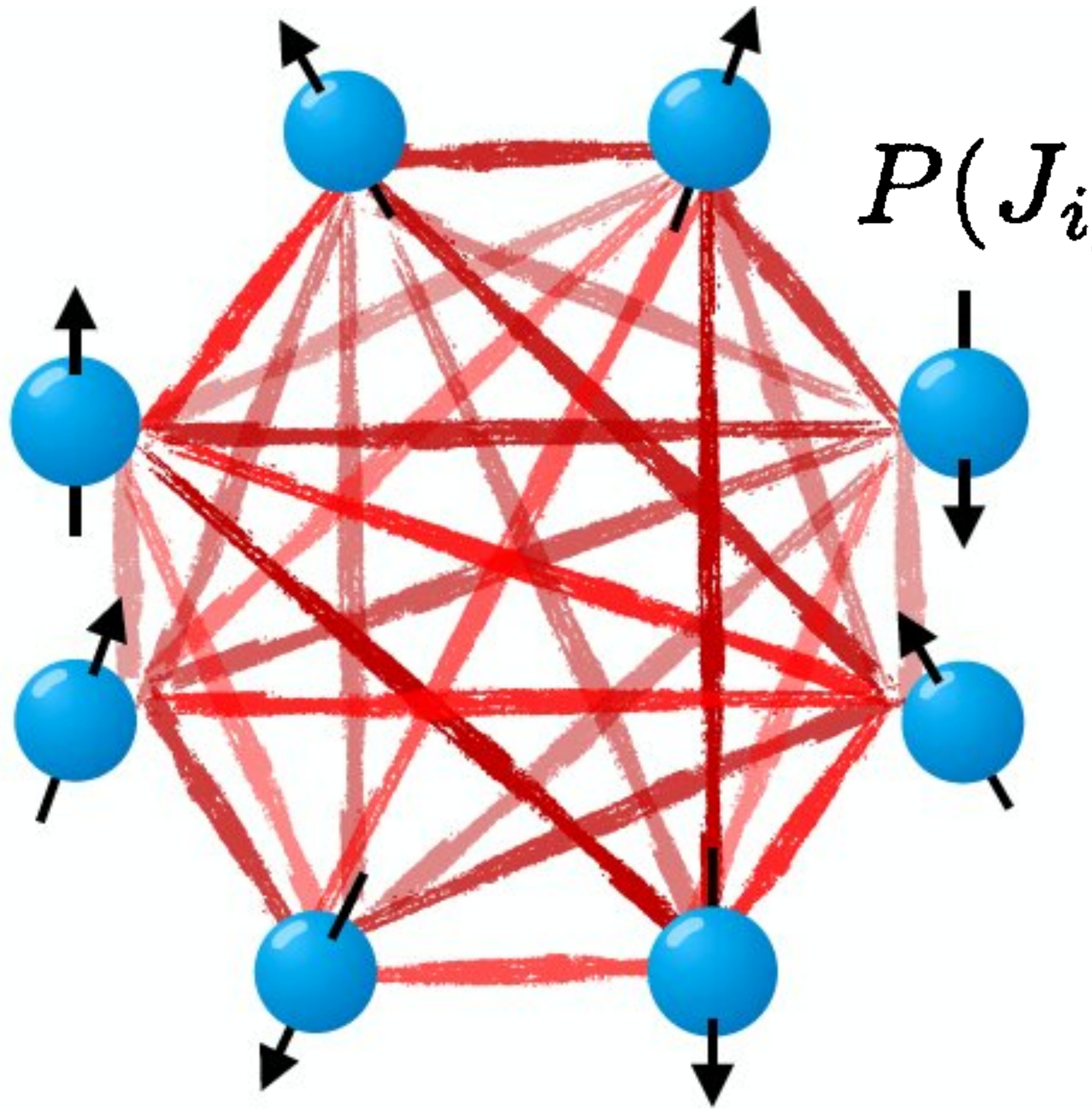
Subir Sachdev and Jinwu Ye

*Departments of Physics and Applied Physics, P.O. Box 2157, Yale University, New Haven, Connecticut 06520*

(Received 22 December 1992)

Physical Review Letters **70**, 3339 (1993)

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{\mathcal{S}}_i \cdot \hat{\mathcal{S}}_j ,$$



$$P(J_{ij}) \sim \exp[-J_{ij}^2/(2J^2)]$$

Disorder self-averages $\Rightarrow$ Gapless quantum spin liquid

Spin auto-correlation on every site in a single sample: $\left\langle \hat{\mathcal{S}}_i(\tau) \cdot \hat{\mathcal{S}}_i(0) \right\rangle = \dfrac{C}{|\tau|} + \ldots$

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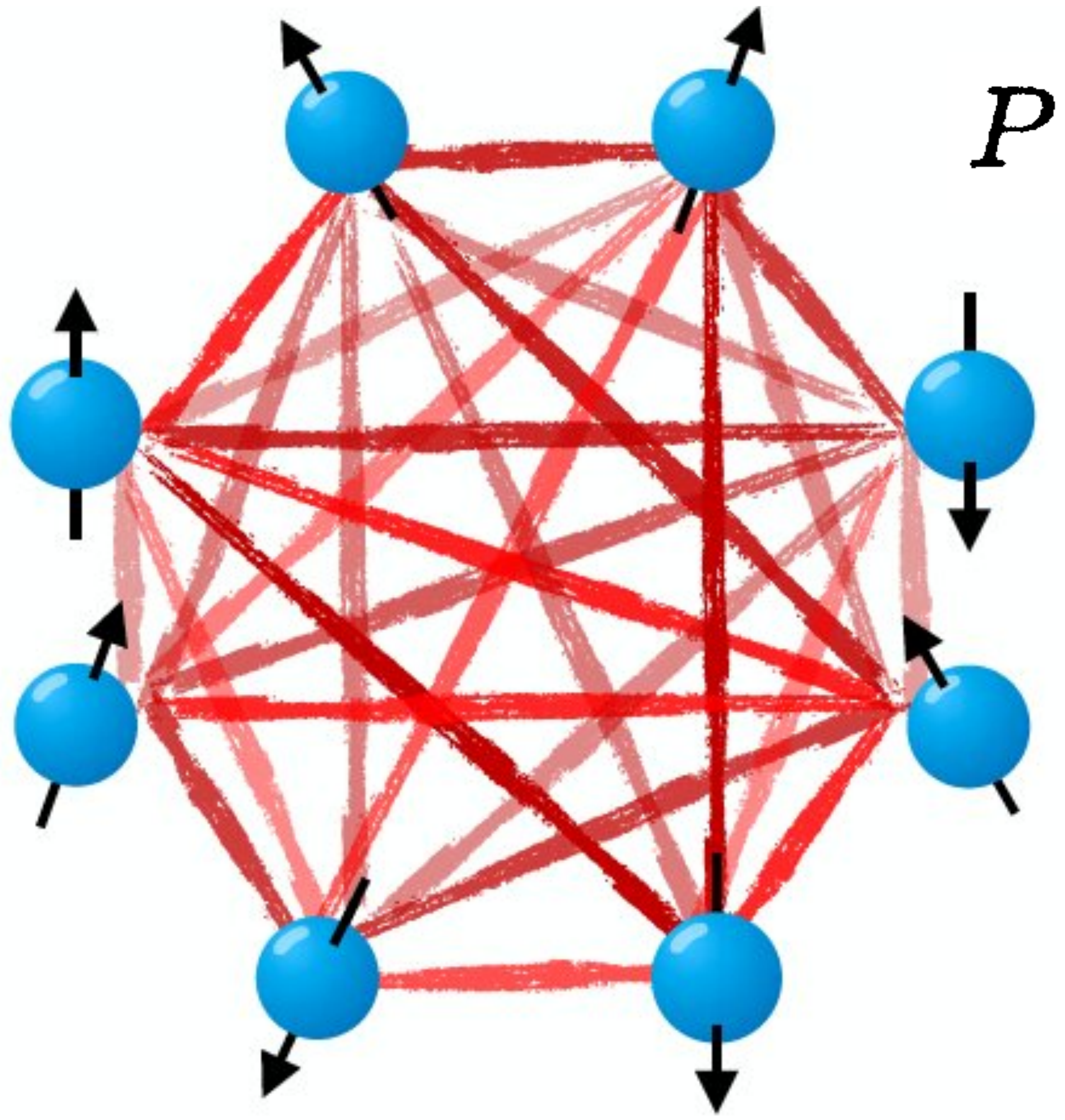

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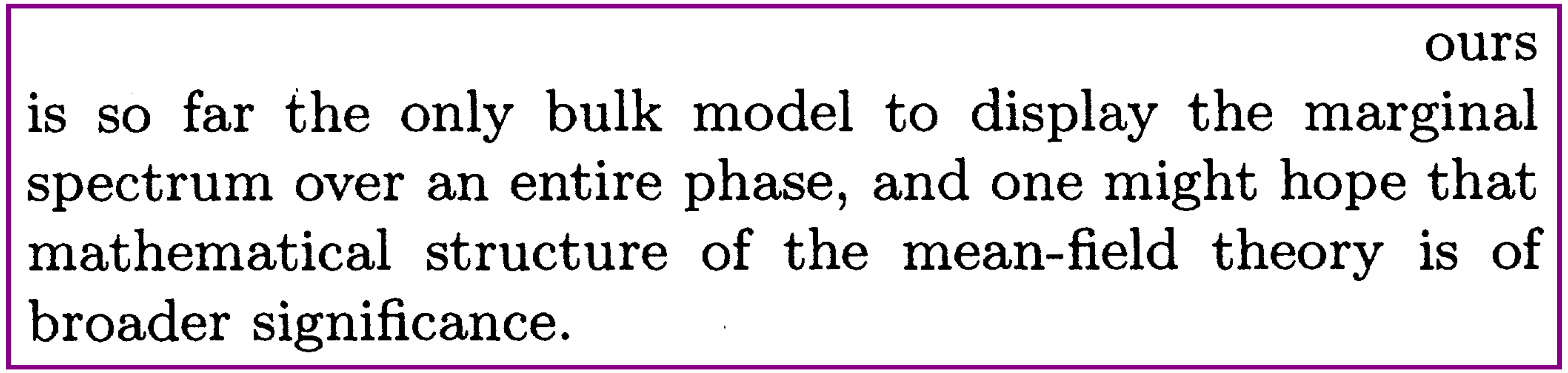

ours is so far the only bulk model to display the marginal spectrum over an entire phase, and one might hope that mathematical structure of the mean-field theory is of broader significance.

Needed: a realistic theory for self-averaging disorder in an incoherent metal

Disorder self-averages $\Rightarrow$ Gapless quantum spin liquid

Spin auto-correlation on every site in a single sample: $\left\langle \hat{\mathcal{S}}_i(\tau) \cdot \hat{\mathcal{S}}_i(0) \right\rangle = \frac{C}{|\tau|} + \ldots$

# Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large $N$ limit.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

# Yukawa-SYK models

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with $g_{ij\ell}$ independent random numbers with zero mean. The large $N$ saddle point equations are

$$\boxed{\begin{aligned} G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) &= g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{aligned}}$$

Make the low frequency ansatz

$$G(i\omega) \sim -i\operatorname{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

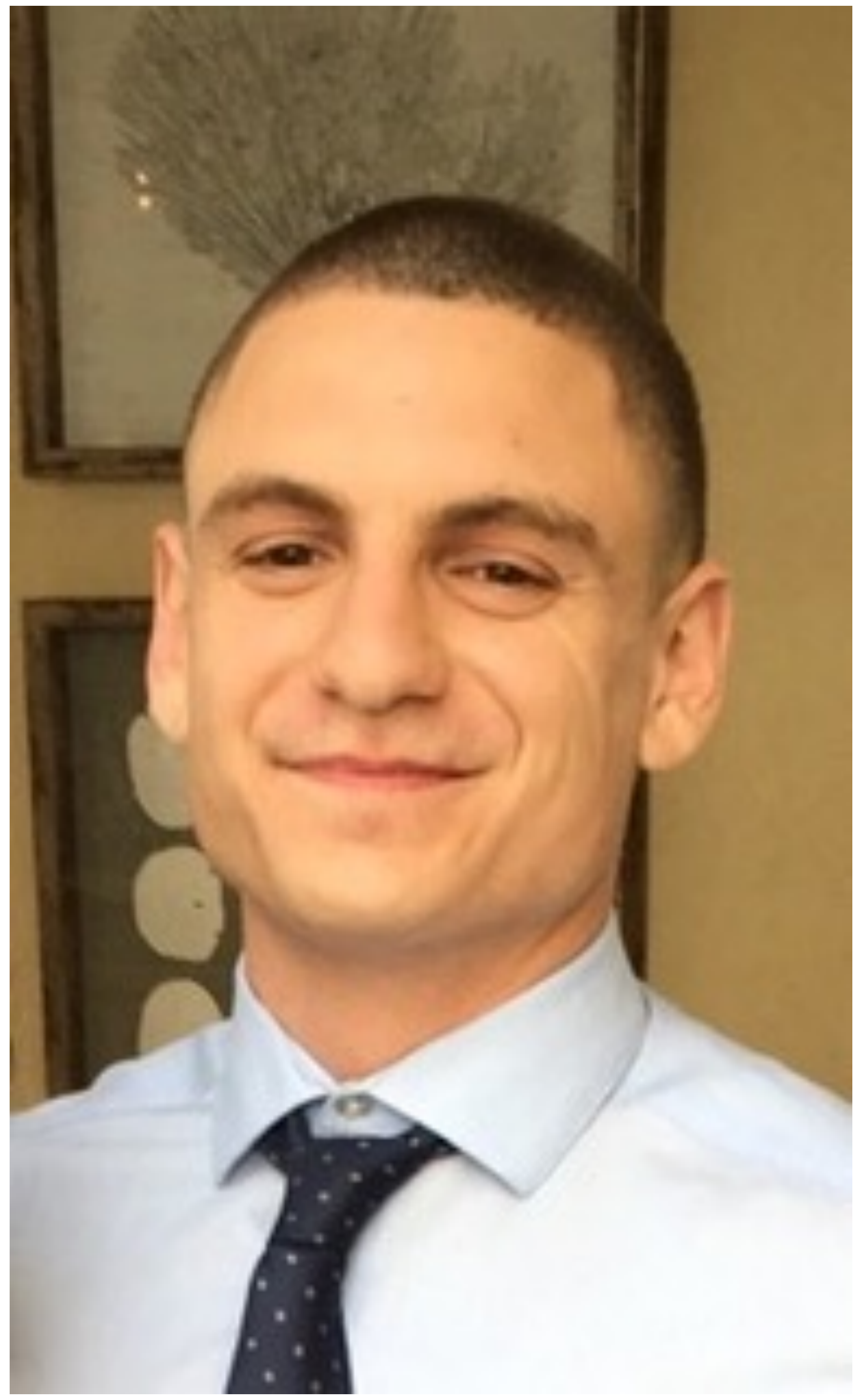
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

**Aavishkar Patel**

Flatiron Institute, NYC

**Haoyu Guo**

Harvard

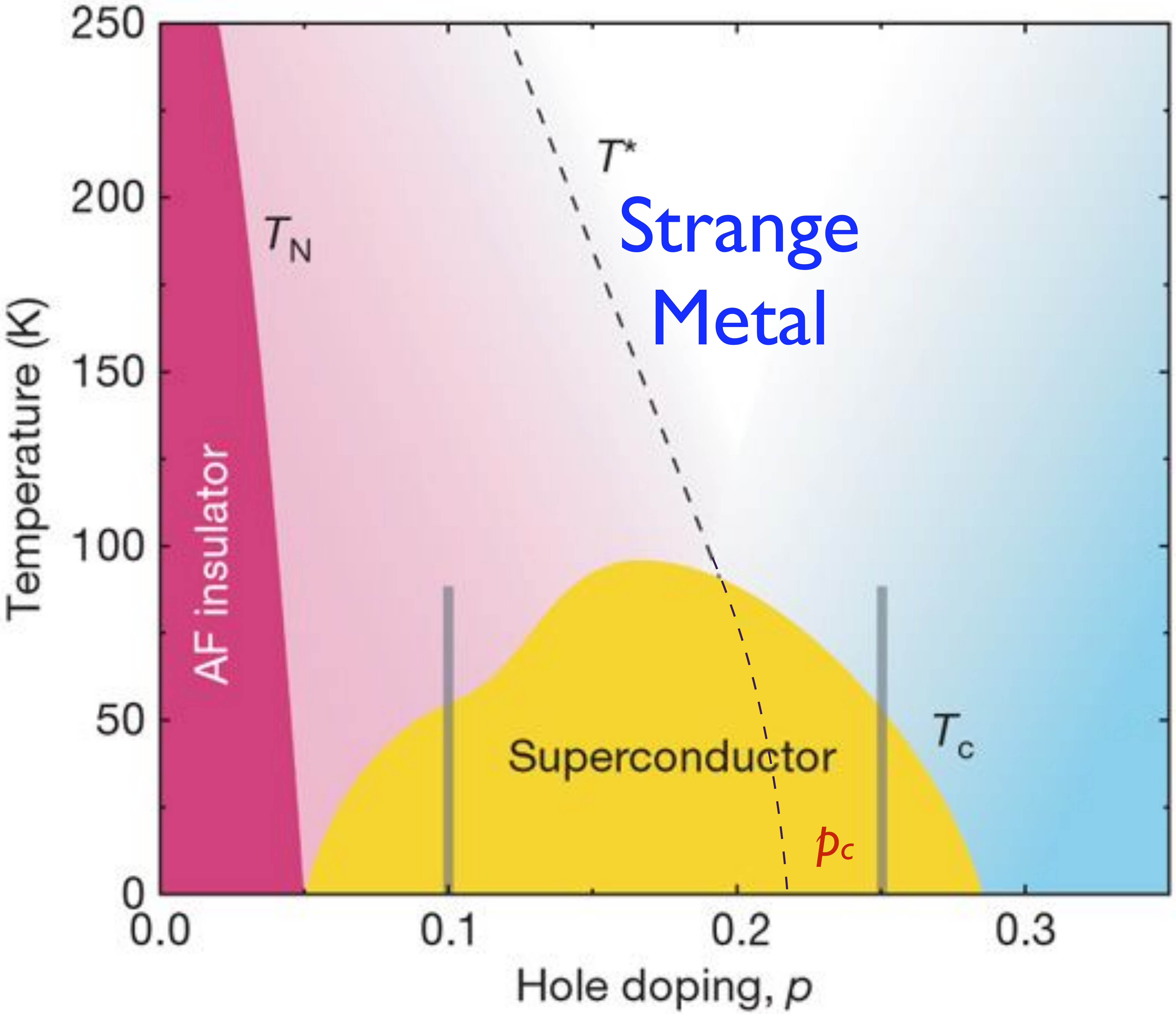


**Ilya Esterlis**

Harvard ➔ Wisconsin

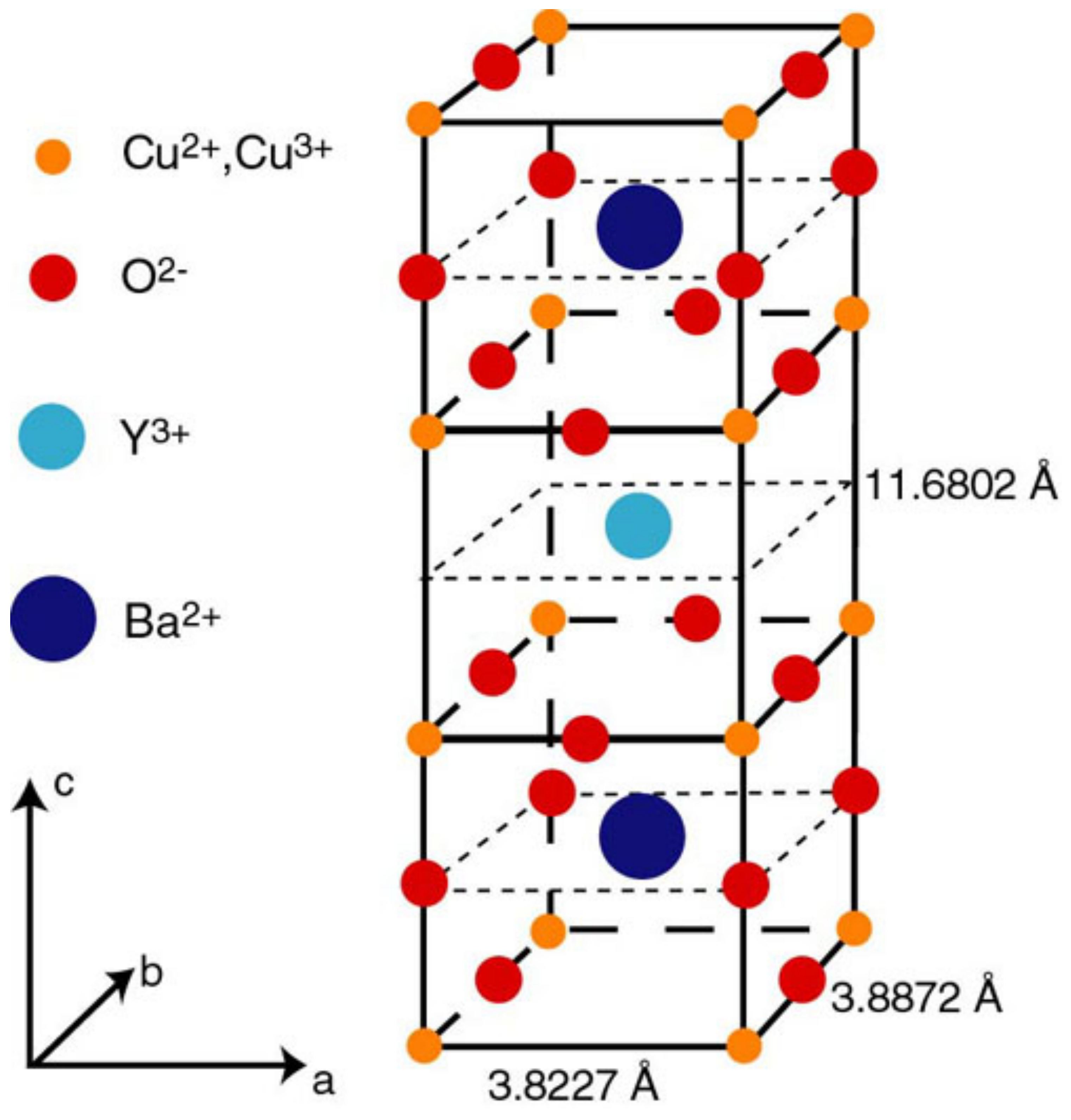
arXiv: 2103.08615, 2203.04990, 2207.08841

# Strange metals,
# Non-Fermi liquids,
# and
# Marginal Fermi liquids



Strange Metal

$p_c$



# Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
  and $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are bad metals.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

- Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

- Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim \text{Max}\left[\frac{k_B T}{\hbar}, \omega\right]$$

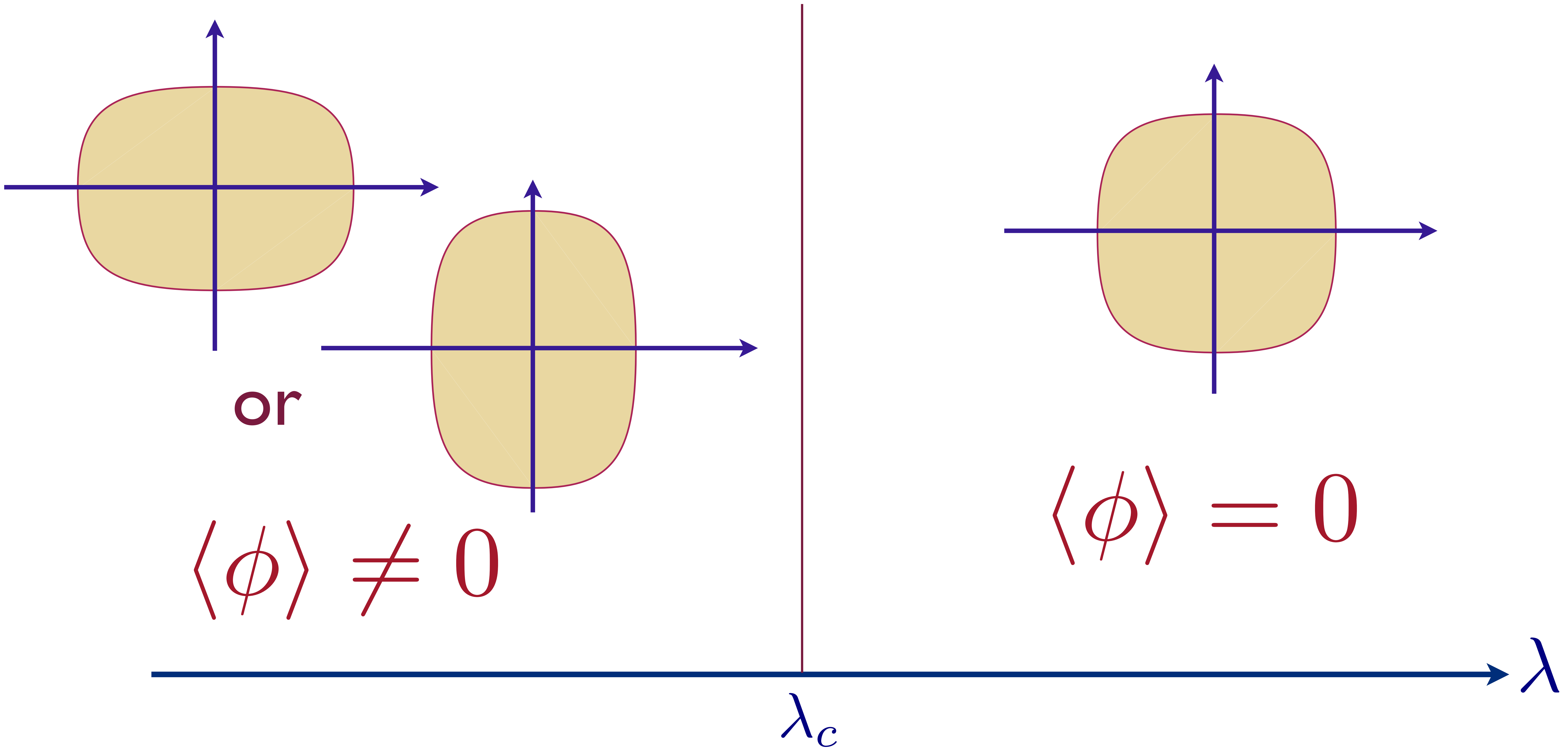
B. Michon……A. Georges, arXiv:2205.04030

- Photoemission: nearly "marginal" electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|$$

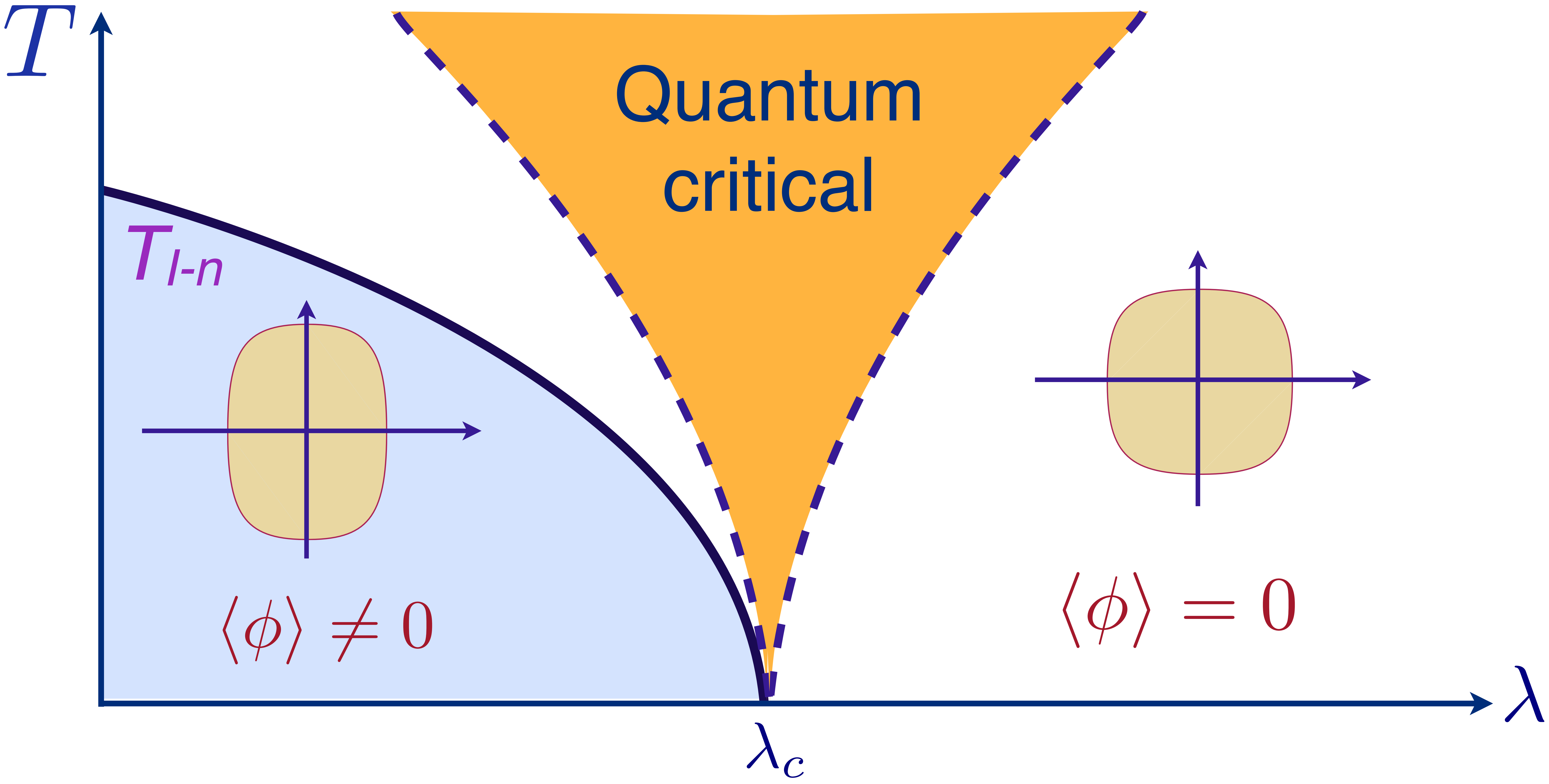
T.J. Reber….D. Dessau,
Nature Communications **10**, 5737 (2019)

# Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling $\lambda$

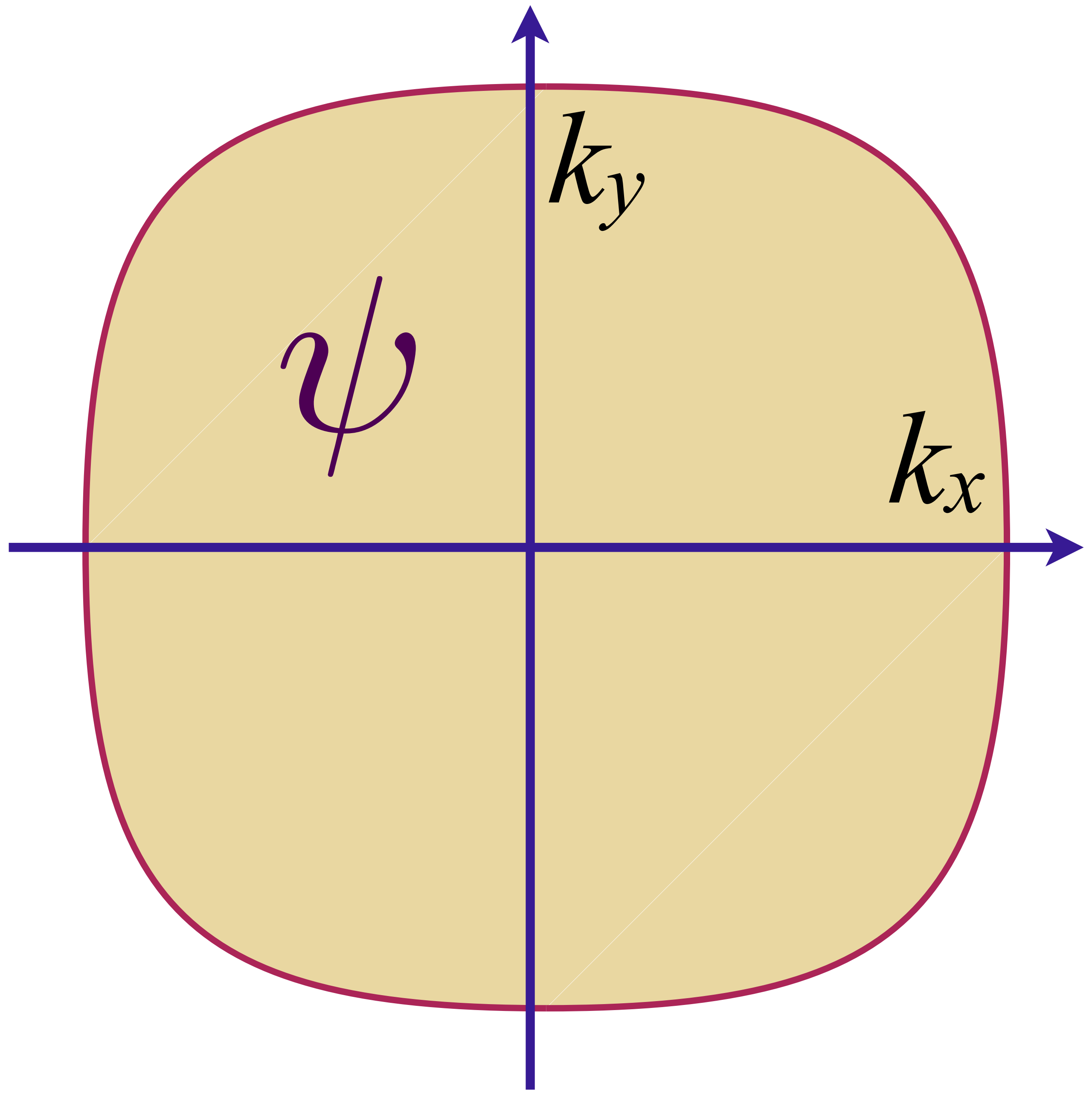
# Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of $T$ and $\lambda$

# Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson $\phi$
*e.g.* Ising-nematic order

$$+ \quad \mathcal{L}_\phi = \frac{1}{2}\left[ (\partial_\tau \phi)^2 + (\boldsymbol{\nabla}\phi)^2 + (\lambda - \lambda_c)\phi^2 \right]$$

"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

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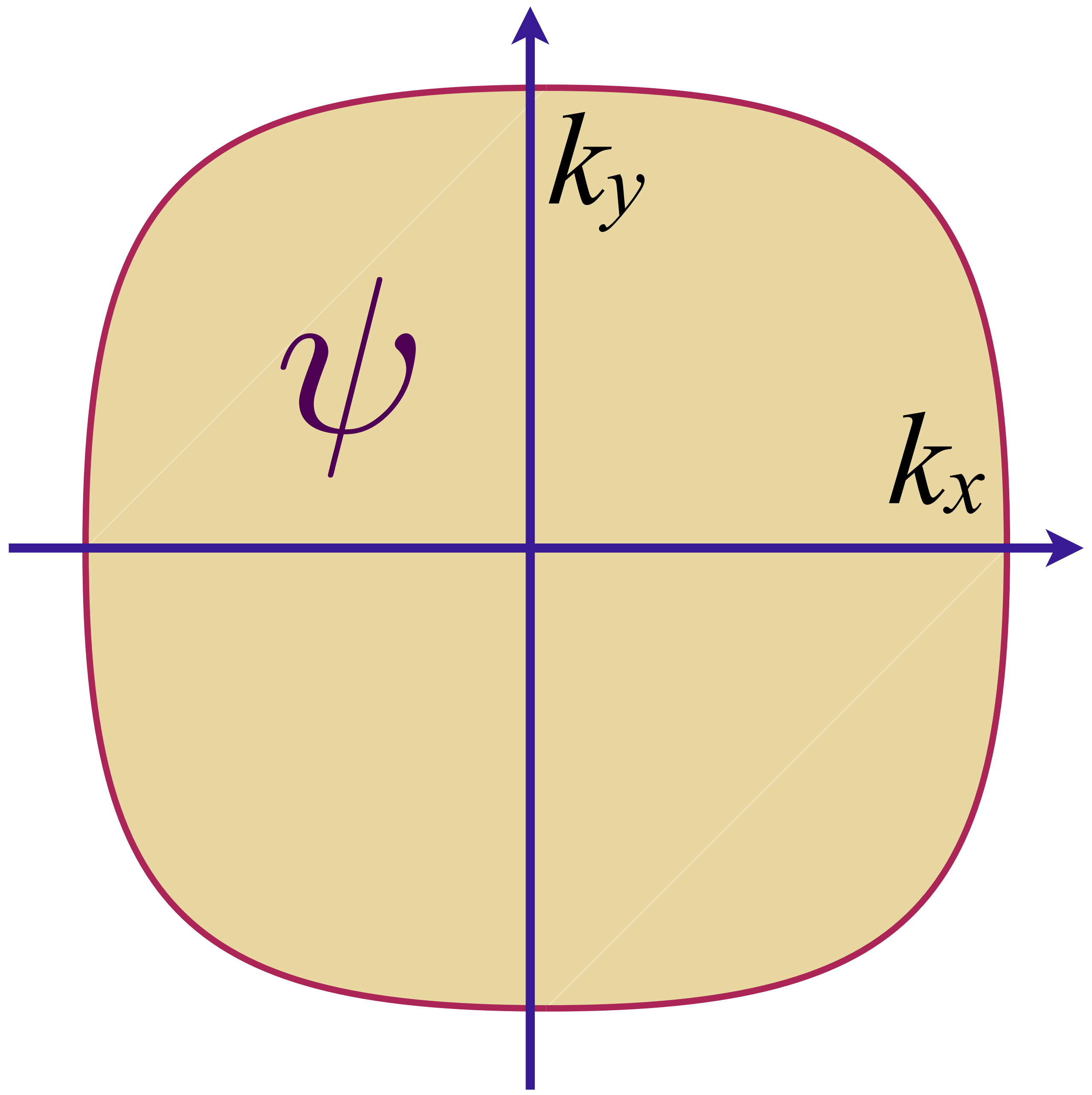

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Eliashberg solution for electron ($G$) and boson ($D$) Green's functions at small $\omega$:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i \mathrm{sgn}(\omega) |\omega|^{2/3} \,, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)} \,, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma |\Omega| / q}$$

P.A. Lee (1989)

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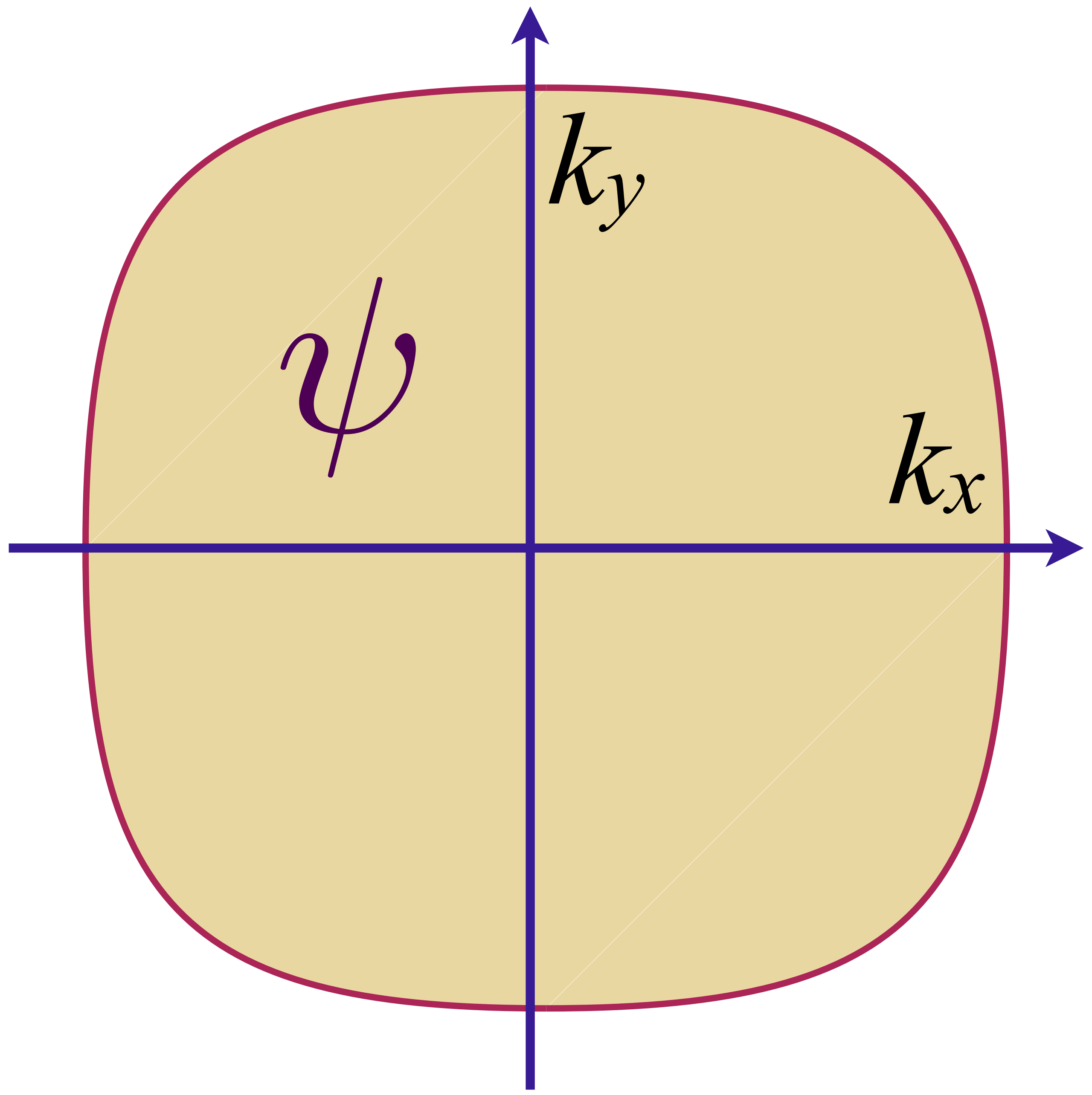

$+$

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"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau) \psi(r,\tau) \phi(r,\tau)$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990
Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Transport—a perfect metal!
Conservation of momentum and fermion-boson drag imply:

$\mathrm{Re}\left[\sigma(\omega)\right] = D\delta(\omega)$; Claims of anomalous transport and optical conductivity are incorrect

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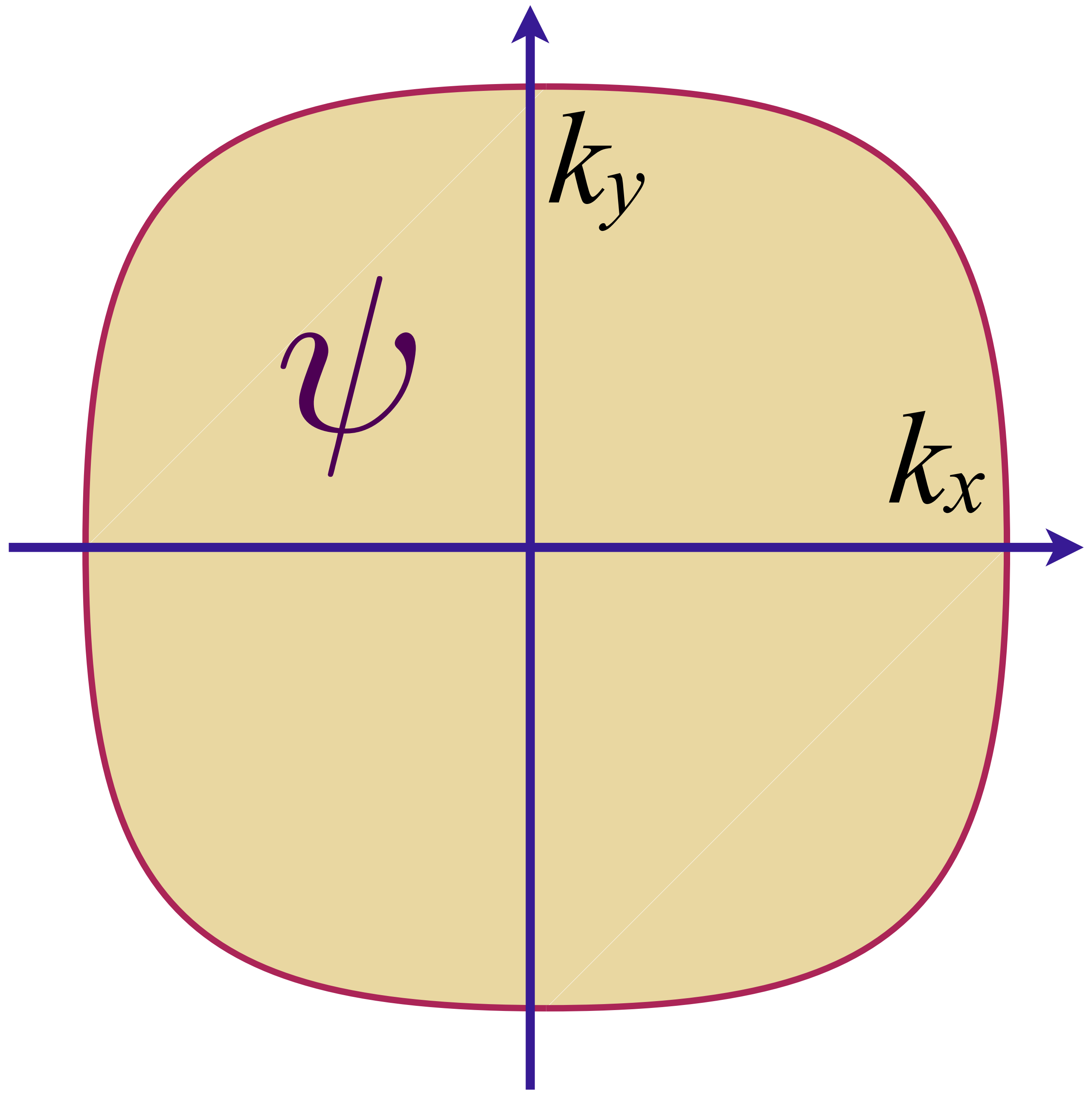
a critical boson $\phi$
*e.g.* Ising-nematic order



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Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\mathrm{Re}\left[\sigma(\omega)\right] = D\delta(\omega);$$

**A non-Fermi liquid but NOT a strange metal**

# Fermi liquids and their cousins: (defined by single-particle properties)

- Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \ll |\varepsilon|$ and a local density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

- Non-Fermi liquids: No quasiparticles.
Would-be fermionic quasiparticles have $1/\tau(\varepsilon) \gg |\varepsilon|$ and a local density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

- Marginal Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \sim |\varepsilon|$ and a local density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

# Fermi surface coupled to a critical boson:

## No spatial disorder

*A non-Fermi liquid but NOT a strange metal*

# Adding spatial disorder:
# Potential disorder - $v$
# and
# Interaction disorder - $g'$

# Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \qquad \mathcal{L}_\phi = \frac{1}{2} \left[ (\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + s\phi^2 \right]$$

“Yukawa” coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Random potential $\int d^2 r d\tau \, v(r) \, \psi^\dagger(r,\tau)\psi(r,\tau)$

Spatially random potential $v(r)$ with $\overline{v(r)} = 0$, $\overline{v(r)v(r')} = v^2 \delta(r - r')$

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Boson self energy: $\Pi \sim -\frac{g^2}{v^2}|\Omega|, \qquad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$

Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|); \qquad \frac{1}{\tau(\varepsilon)} \sim |\varepsilon|$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

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Random potential $\int d^2 r d\tau \, v(r) \, \psi^\dagger(r,\tau) \psi(r,\tau)$

The $g^2$log term does not contribute to transport:
With $g$ and $v$ non-zero, we obtain a non-zero residual resistivity
and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \ldots.$$

with $1/\rho(0) \sim 1/\tau_{\text{trans}} \sim v^2$.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990; T. C. Wu, Y. Liao, and M. S. Foster, arXiv:2206.01762

Fermi surface coupled to a critical boson:

No spatial disorder

*A non-Fermi liquid but NOT a strange metal*

---

Fermi surface coupled to a critical boson:

Potential disorder $v$

*A marginal Fermi liquid but NOT a strange metal*

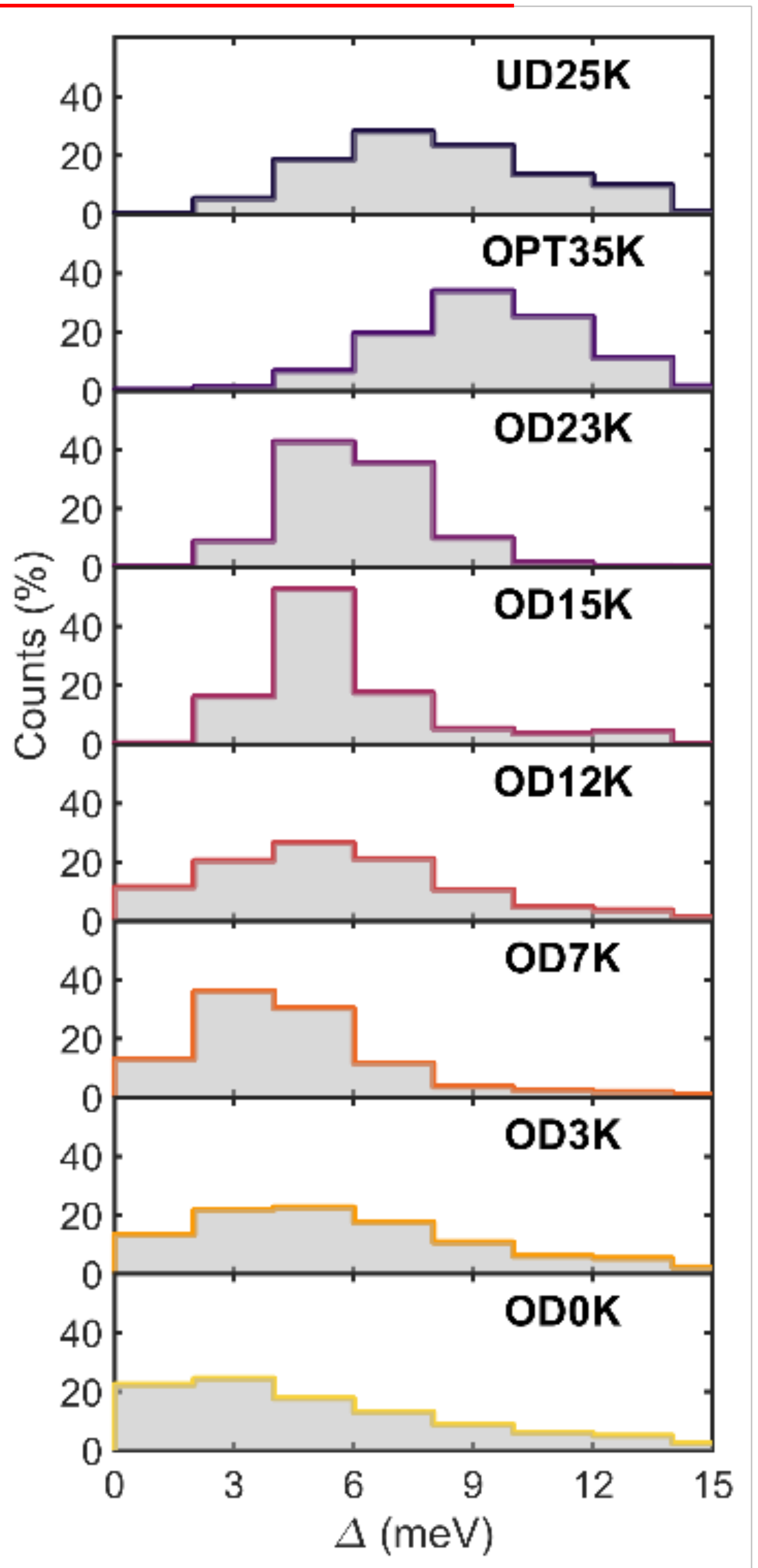
# Spatially random interactions!

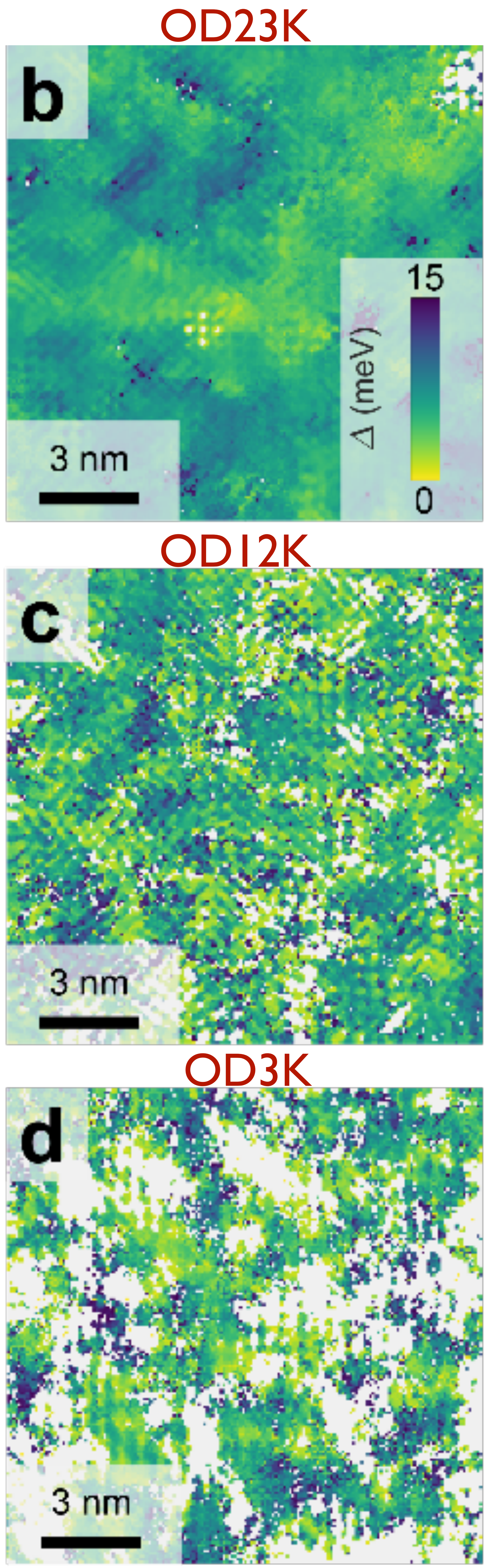
## Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge,
Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos,
Amber Vervloet, Steef Smit, Erik van Heumen,
Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin,
Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen,
Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740





# Spatially random interactions!

Randomness in hopping $t_{ij}$ leads to randomness in exchange interactions $t_{ij}^2/U$. The interaction associated with the $\phi$ collective mode has the schematic form

$$-\int d^2 r d\tau \, J(r) \psi^\dagger \psi^\dagger \psi \psi$$

where we have omitted a local 'form factor' for the interaction, and the random strength of the overall interaction is determined by the coupling $J(r)$. Upon decoupling

$$\int d^2 r d\tau \left[ \frac{\phi^2}{2J(r)} - \phi \psi^\dagger \psi \right]$$

This as a random 'mass' in the boson and is strongly relevant.

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This as a random 'mass' in the boson and is strongly relevant. A key idea is that we should account for the relevant disorder exactly by rescaling the field $\phi$ in a $r$-dependent manner so that

$$\boxed{\int d^2r d\tau \left[\frac{\phi^2}{2} - \sqrt{J(r)}\phi\psi^\dagger\psi\right]}$$

The disorder is in the boson-fermion coupling, and can be accounted for systematically.

# Key ingredient of our universal theory of strange metals:

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \qquad \mathcal{L}_\phi = \frac{1}{2} \left[ (\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + s\phi^2 \right]$$

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Spatially random Yukawa coupling $g'(r)$ with $\overline{g'(r)} = 0$, $\overline{g'(r)g'(r')} = g'^2\delta(r-r')$

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Fermion self energy: $\Sigma = \Sigma_v + \Sigma_g + \Sigma_{g'}$

$$\Sigma_v(i\omega) \sim -iv^2 \mathrm{sgn}(\omega), \quad \Sigma_g(i\omega) \sim -i\frac{g^2}{v^2}\omega \ln(1/|\omega|), \quad \Sigma_{g'}(i\omega) \sim -ig'^2\omega\ln(1/|\omega|)$$

Boson self energy: $\Pi = \Pi_g + \Pi_{g'}$

$$\Pi_g(i\Omega) \sim -\frac{g^2}{v^2}|\Omega|, \qquad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|, \qquad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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$$\Sigma_v(i\omega) \sim -iv^2 \mathrm{sgn}(\omega), \quad \Sigma_g(i\omega) \sim -i\frac{g^2}{v^2}\omega \ln(1/|\omega|), \quad \Sigma_{g'}(i\omega) \sim -ig'^2\omega \ln(1/|\omega|)$$

**Conductivity:**

The $g^2$ log term does not contribute to transport
but the $g'^2$ log term does!

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022); Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

# Key ingredient of our universal theory of strange metals:

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left( \frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}} \qquad \mathcal{L}_\phi = \frac{1}{2} \left[ (\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + s \phi^2 \right]$$

"Yukawa" coupling: $\displaystyle \int d^2 r d\tau \; \underline{[g + g'(r)]} \, \psi^\dagger(r,\tau) \psi(r,\tau) \phi(r,\tau)$

Random potential $\displaystyle \int d^2 r d\tau \, v(r) \, \psi^\dagger(r,\tau) \psi(r,\tau)$

Conductivity: $\sigma(\omega) \sim \left[ 1/\tau_{\text{trans}}(\omega) - i\omega \, m^*(\omega)/m \right]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by $v^2$; Linear-in-$T$ resistivity determined by $g'^2$.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson:

No spatial disorder

*A non-Fermi liquid but NOT a strange metal*

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Potential disorder *v*

*A marginal Fermi liquid but NOT a strange metal*

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Fermi surface coupled to a critical boson:

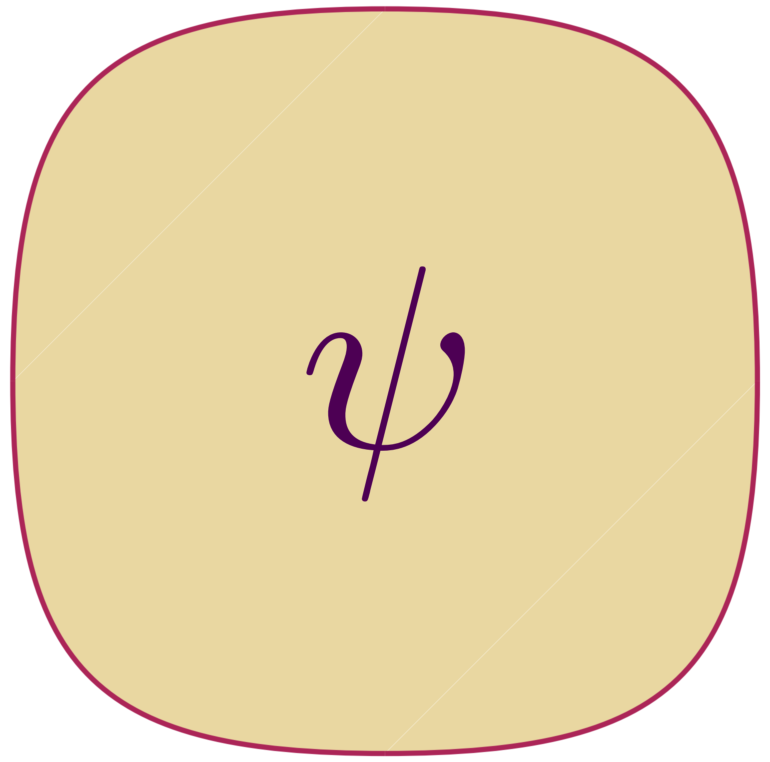
Interaction disorder *g'*

*A marginal Fermi liquid AND a strange metal*

Beyond 0-d "toy" models….

Universal large-N theory of strange metals in 2-d

# Strange metals from a Yukawa-SYK model



$+$

a critical boson $\phi$
*e.g.* Ising-nematic order

$$\text{“Yukawa” coupling:} \quad \frac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\text{Random potential:} \quad +\frac{1}{\sqrt{N}}\int d^2r d\tau\, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

$$\text{Random interactions:} \quad +\frac{1}{N}\int d^2r d\tau\, g'_{ijl}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g^*_{ijl} g_{abc}} = g^2\, \delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v^*_{ij}(r) v_{lm}(r')} = v^2\, \delta(r-r')\delta_{il}\delta_{jm}$$

$$\overline{g'_{ijl}(r)} = 0 \quad , \quad \overline{{g'}^*_{ijl}(r) g'_{abc}(r')} = {g'}^2\, \delta(r-r')\delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad i,j,\ldots = 1\ldots N$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

# Universal large-*N* theory of strange metals

$$\mathcal{Z} = \int \mathcal{D}G\,\mathcal{D}\Sigma\,\mathcal{D}D\,\mathcal{D}\Pi\,\exp(-NS_{\rm all})$$

$$\begin{aligned}
S_{\rm all} = &-\ln\det(\partial_\tau + \varepsilon_{\mathbf{k}} - \mu + \Sigma) + \frac{1}{2}\ln\det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\
&+ \int d\tau d^2r \int d\tau' d^2r' \Big[-\Sigma(\tau',\mathbf{r}';\tau,\mathbf{r})G(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{1}{2}\Pi(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') \\
&+ \frac{g^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}') + \frac{v^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') \\
&+ \frac{g'^2}{2}G(\tau,\mathbf{r};\tau',\mathbf{r}')G(\tau',\mathbf{r}';\tau,\mathbf{r})D(\tau,\mathbf{r};\tau',\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}')\Big]
\end{aligned}$$

Omitted terms associated with $\phi^4$ boson self-interaction;
Large $N$ saddle point yields Migdal-Eliashberg equations;
Large $N$ response functions generate ladders of MT and AL diagrams.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

# Key ingredient of our universal theory of strange metals:

## Fermion-boson drag:

- For electron-phonon scattering in metals, we have “Bloch’s law” (1931): a resistivity $\rho(T) \sim T^5$.

  However, Bloch’s law ignores conservation of total momentum, or **phonon drag**. Peierls (1932) pointed out that the conservation of total momentum implies that an electrical current cannot decay, and so the resistance is practically zero in a pure sample. But because of the weak electron-phonon coupling, Bloch’s law applies except in ultrapure crystals.

# Key ingredient of our universal theory of strange metals:

## Fermion-boson drag:

In a non-Fermi liquid, we cannot separate the momenta carried by the fermions and the bosons, because neither of them exists at low energies! We must treat the combined system together: *extreme drag*. The analog of Bloch's law does not apply.

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# Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

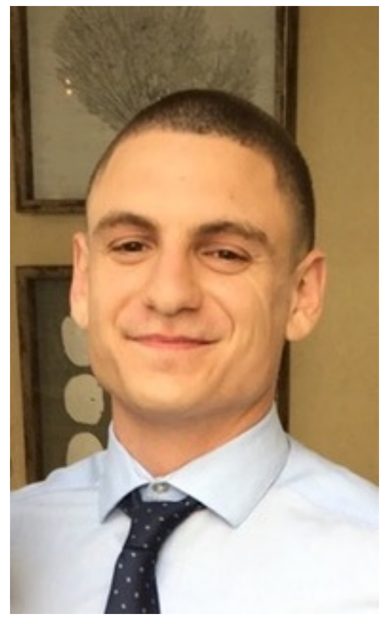
D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev,
arXiv: 2109.05037, Reviews of Modern Physics

# Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.
- Universal theory of a marginal Fermi liquid and a strange metal (including linear-$T$ resistivity): spatially random interactions in a two-dimensional quantum-critical metal, solvable in a Yukawa-SYK-like large $N$ limit.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev,
arXiv: 2109.05037, Reviews of Modern Physics

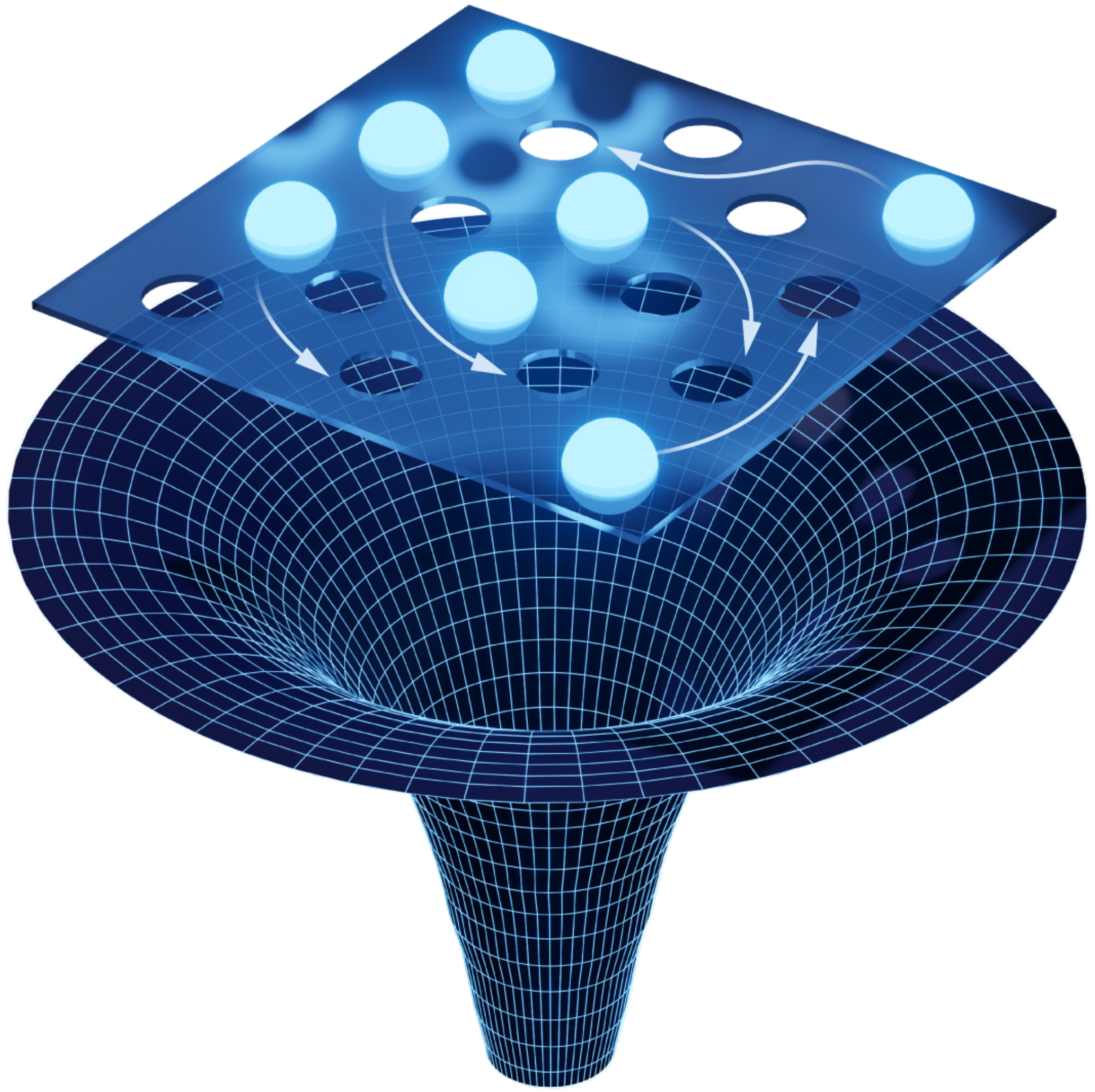




Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

# Summary

- Black holes with a net charge in asymptotically Minkowski space have a near horizon $\mathrm{AdS}_2 \times S^2$ geometry: this geometry has an emergemt time-reparameterization soft mode with an action identical to that of the SYK model. In other words, the SYK model is a quantum simulation of the low energy physics of charged black holes in Einstein-Maxwell theory.



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