

# Stripes, d-wave superconductivity, and quantum spin liquids

*Stripes, Planckian Dissipation and Quantum Supremacy*

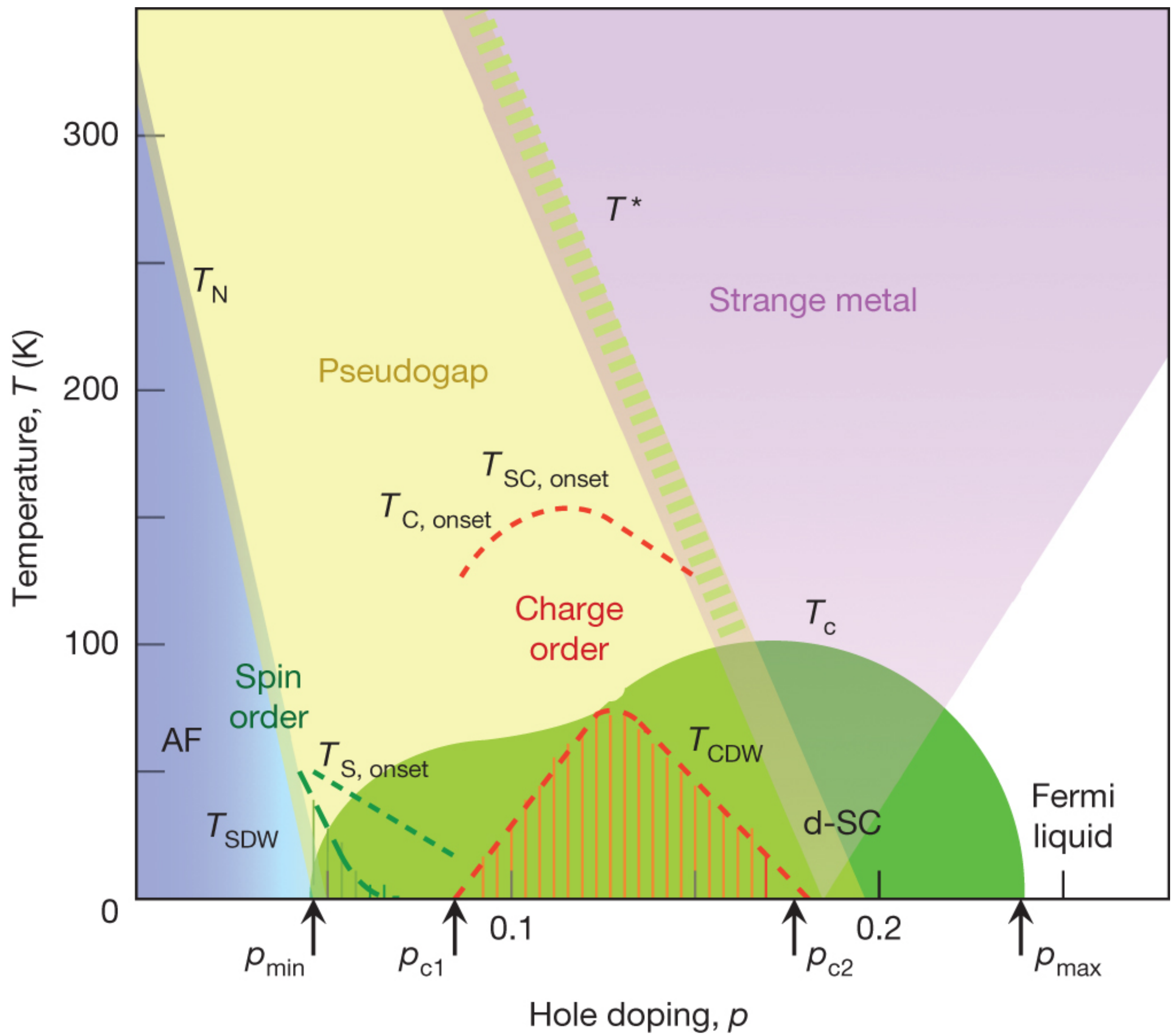
*Zaanen Fest*



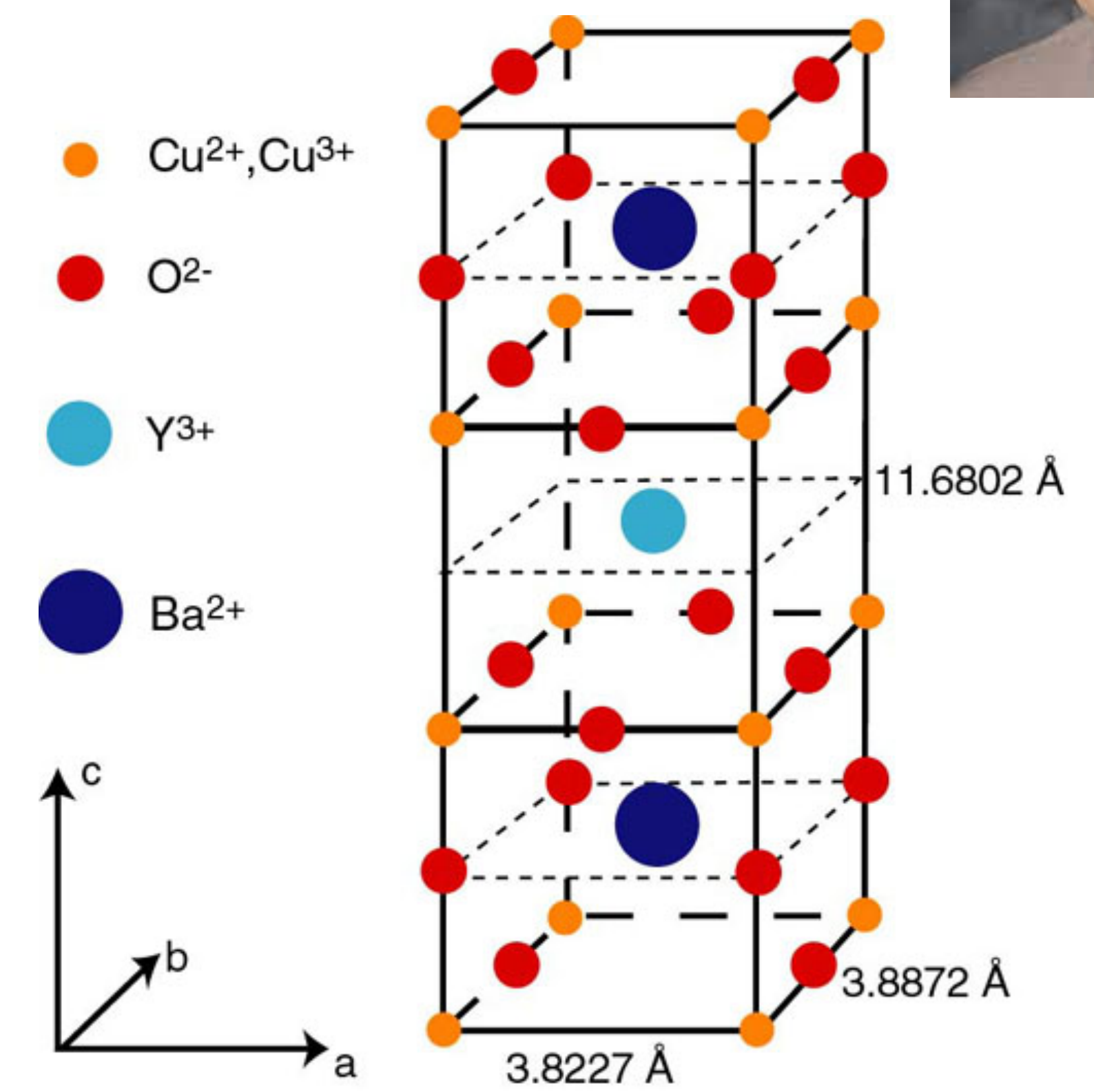
Leiden University  
July 17, 2023  
Subir Sachdev



Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)



- $\text{Cu}^{2+}, \text{Cu}^{3+}$
- $\text{O}^{2-}$
- $\text{Y}^{3+}$
- $\text{Ba}^{2+}$



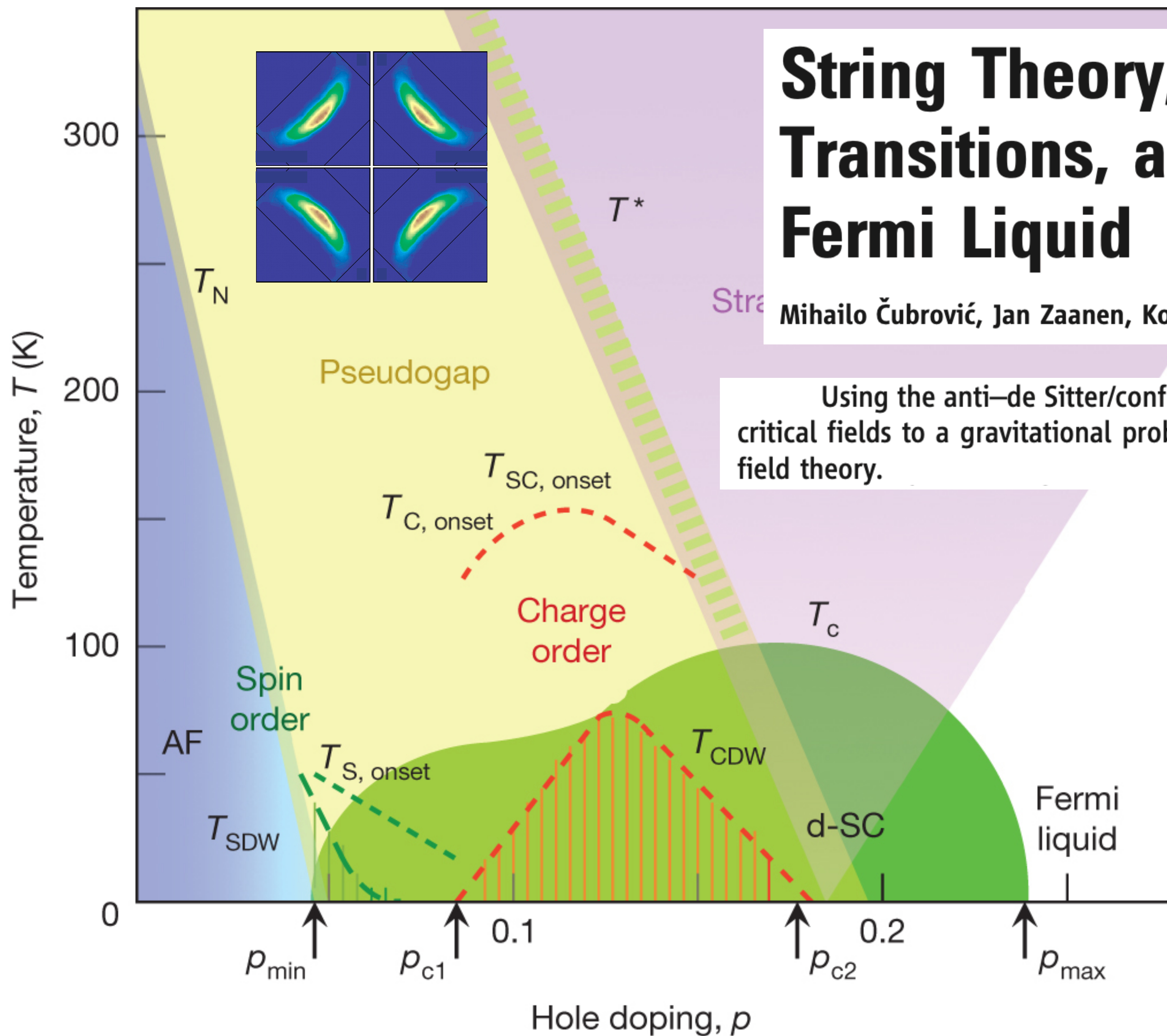


# String Theory, Quantum Phase Transitions, and the Emergent Fermi Liquid

SCIENCE VOL 325 24 JULY 2009 439

Mihailo Čubrović, Jan Zaanen, Koenraad Schalm\*

Using the anti-de Sitter/conformal field theory correspondence to relate fermionic quantum critical fields to a gravitational problem, we computed the spectral functions of fermions in the field theory.



“Probe” fermions form “small” Fermi surfaces in the presence of  $AdS_2$  geometry describing a background “spin liquid”.

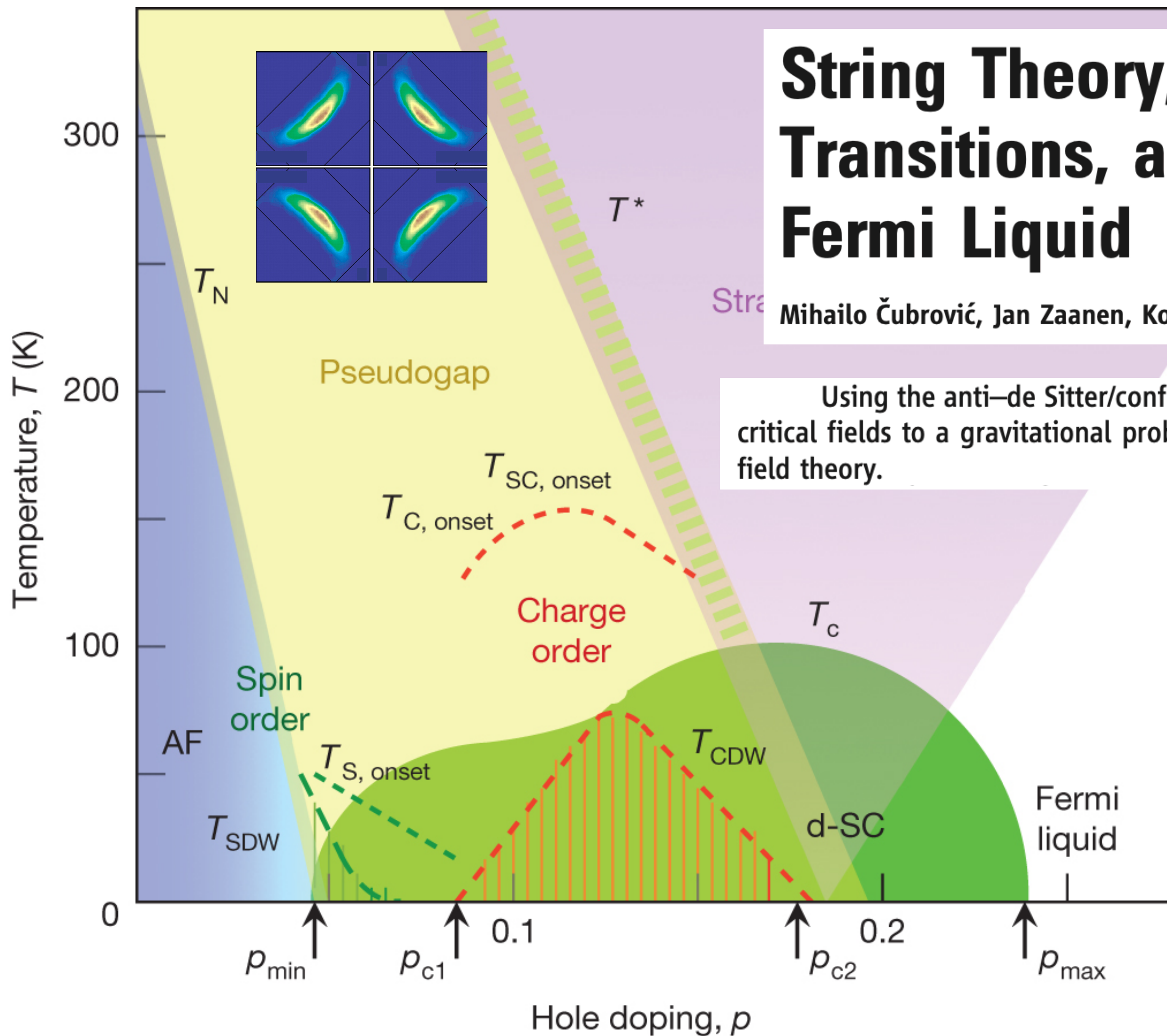


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SS (2010): The  $AdS_2$  spin liquid is described by the SY(K) model

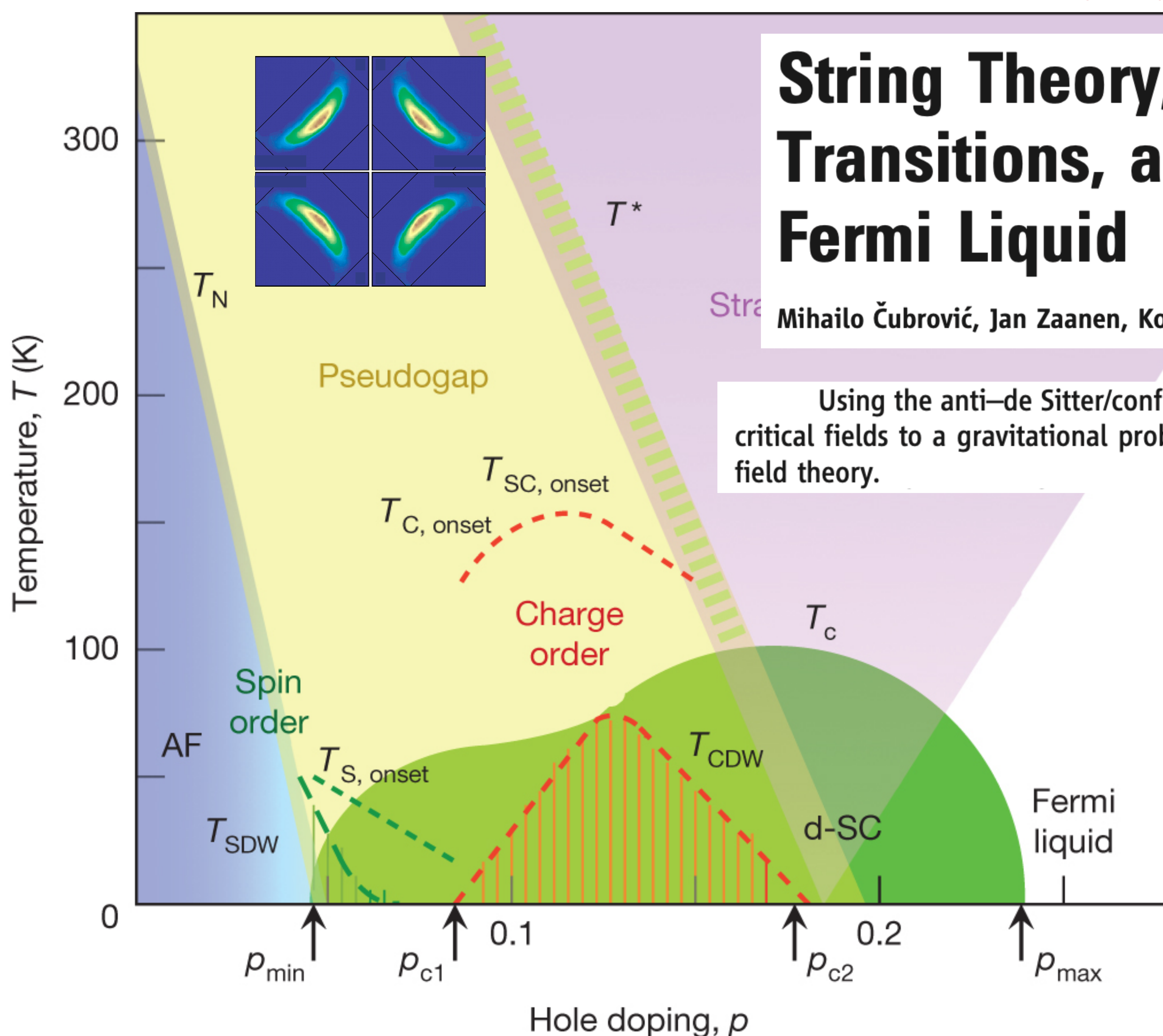


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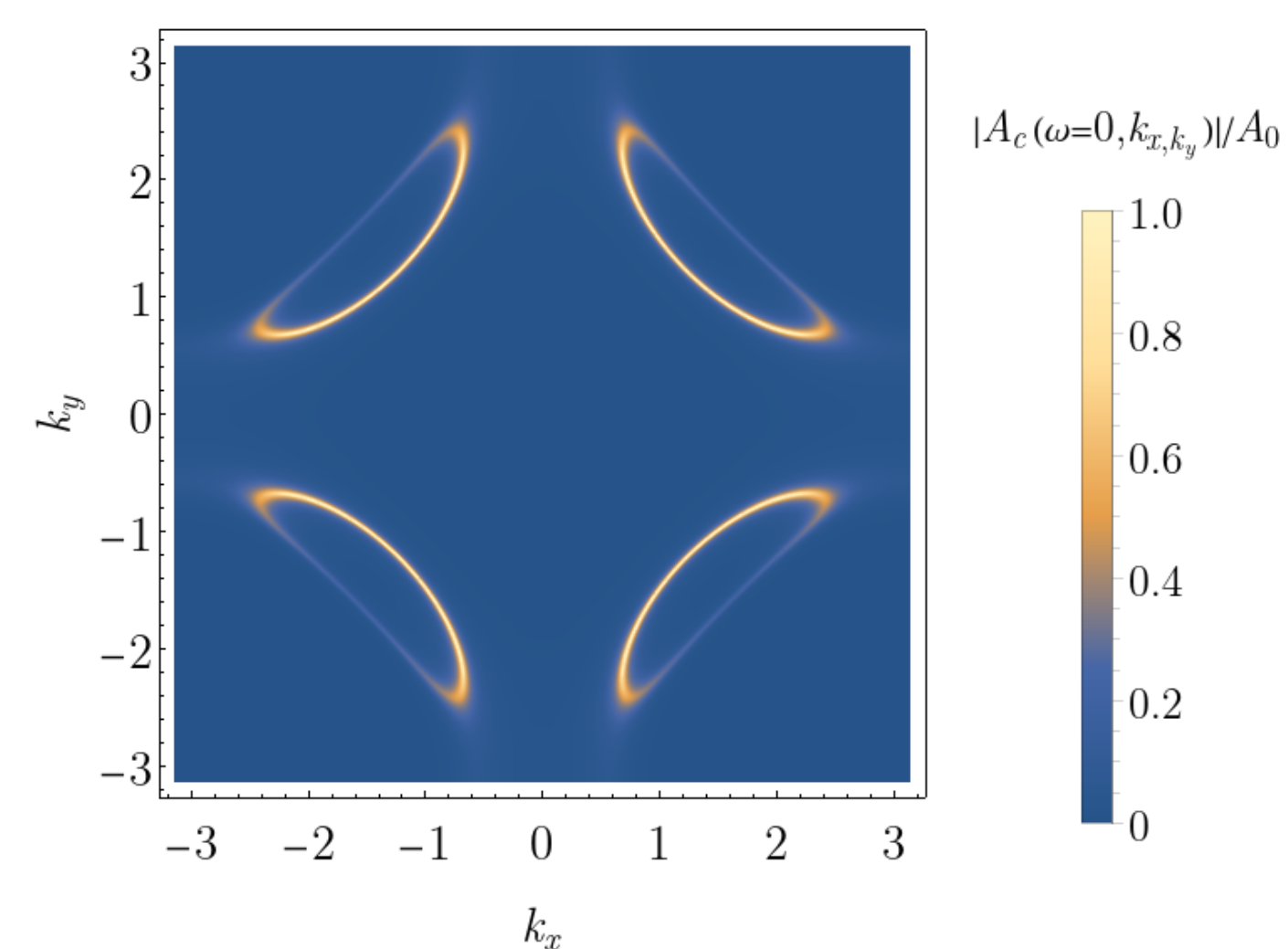
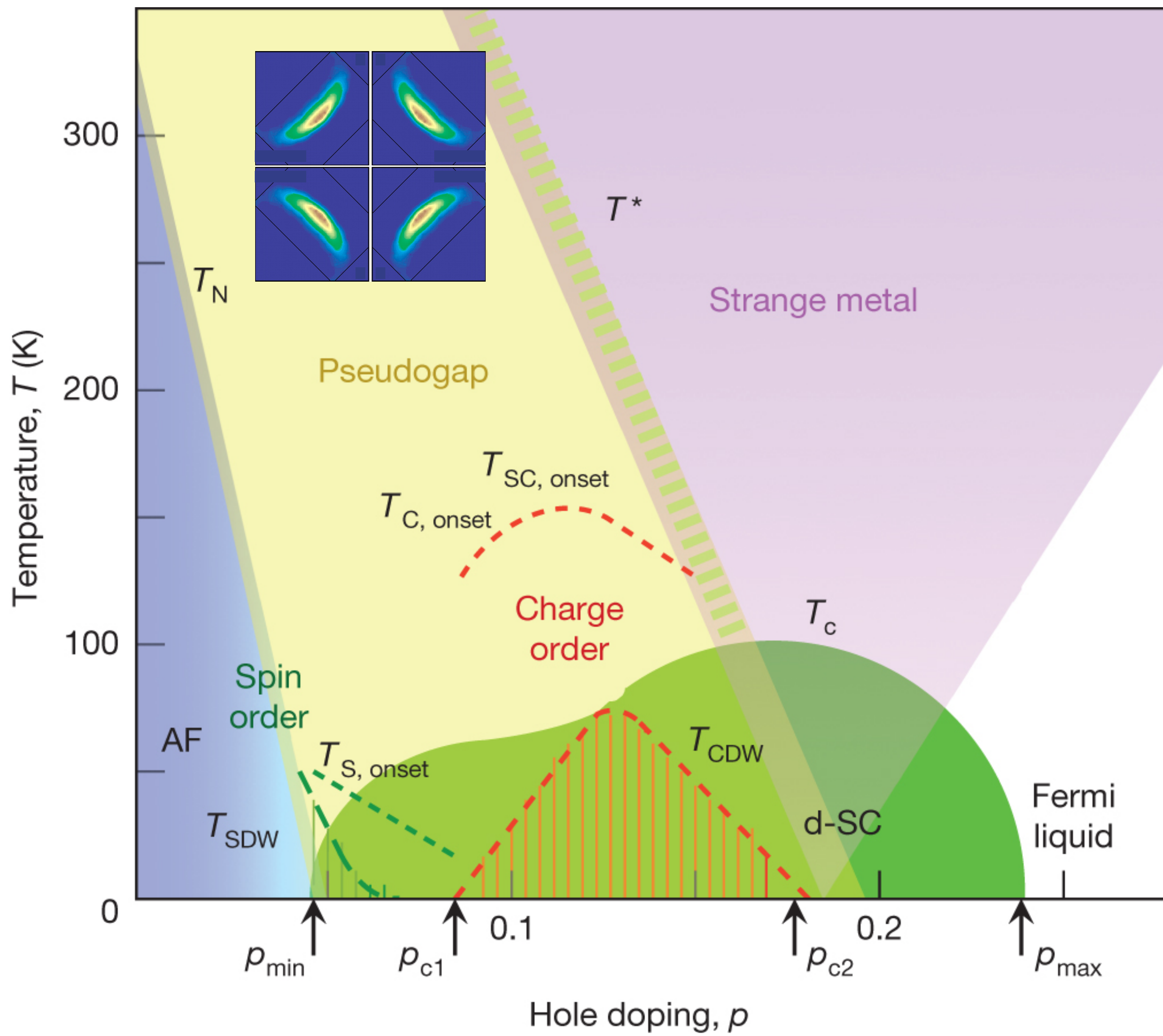
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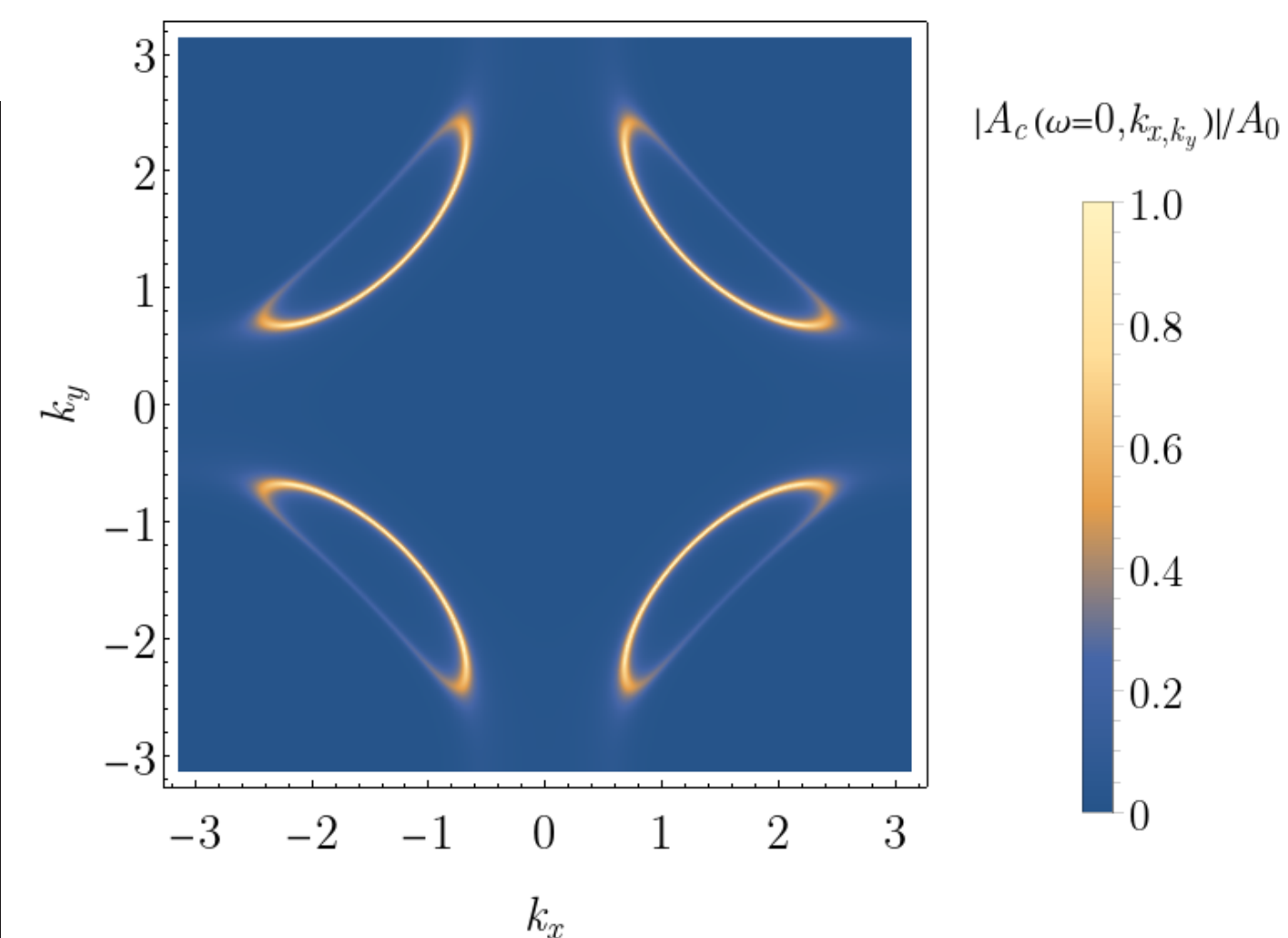
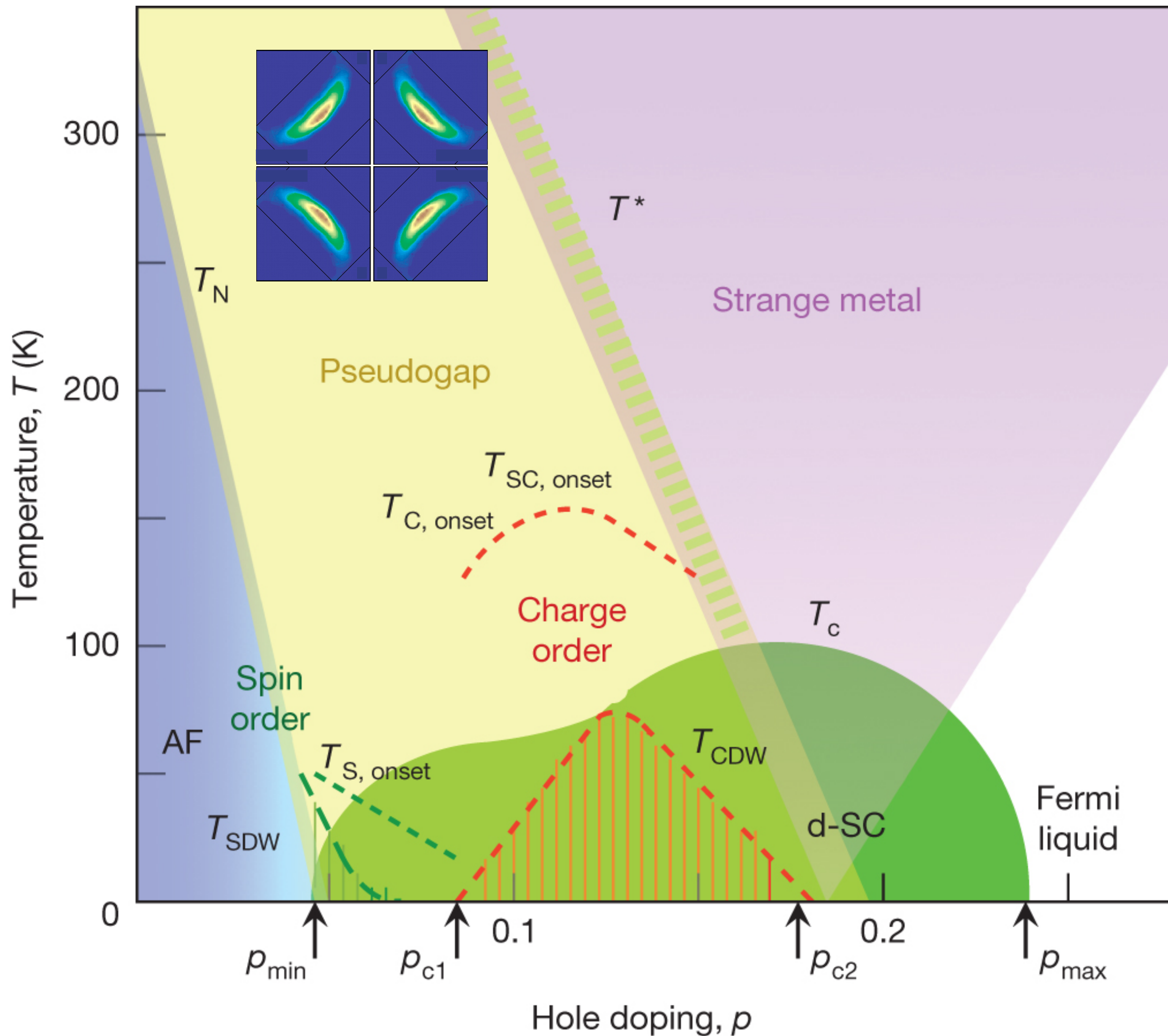
This has transformed quantum gravity studies of black holes, and has led to a (partial) resolution of Hawking's information paradox.



E. Mascot,  
A. Nikolaenko,  
M. Tikhanovskaya,  
Ya-Hui Zhang,  
D. K. Morr, and  
S. S., *PRB* **105**,  
075146 (2022)

Hole pocket Fermi surfaces  
of size  $p$  with  
charge  $e$ , spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,  
*PRB* **73**, 174501 (2006).  
T. D. Stanescu and G. Kotliar,  
*PRB* **74**, 125110 (2006).  
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges,  
*PRL* **97**, 136401 (2006).  
S. Sakai, Y. Motome, M. Imada,  
*PRL* **102**, 056404 (2009).  
J. Skolimowski and M. Fabrizio,  
*PRB* **106**, 045109 (2022).  
Jinchao Zhao, Gabriele La Nave, Philip Phillips,  
arXiv:2304.04787.

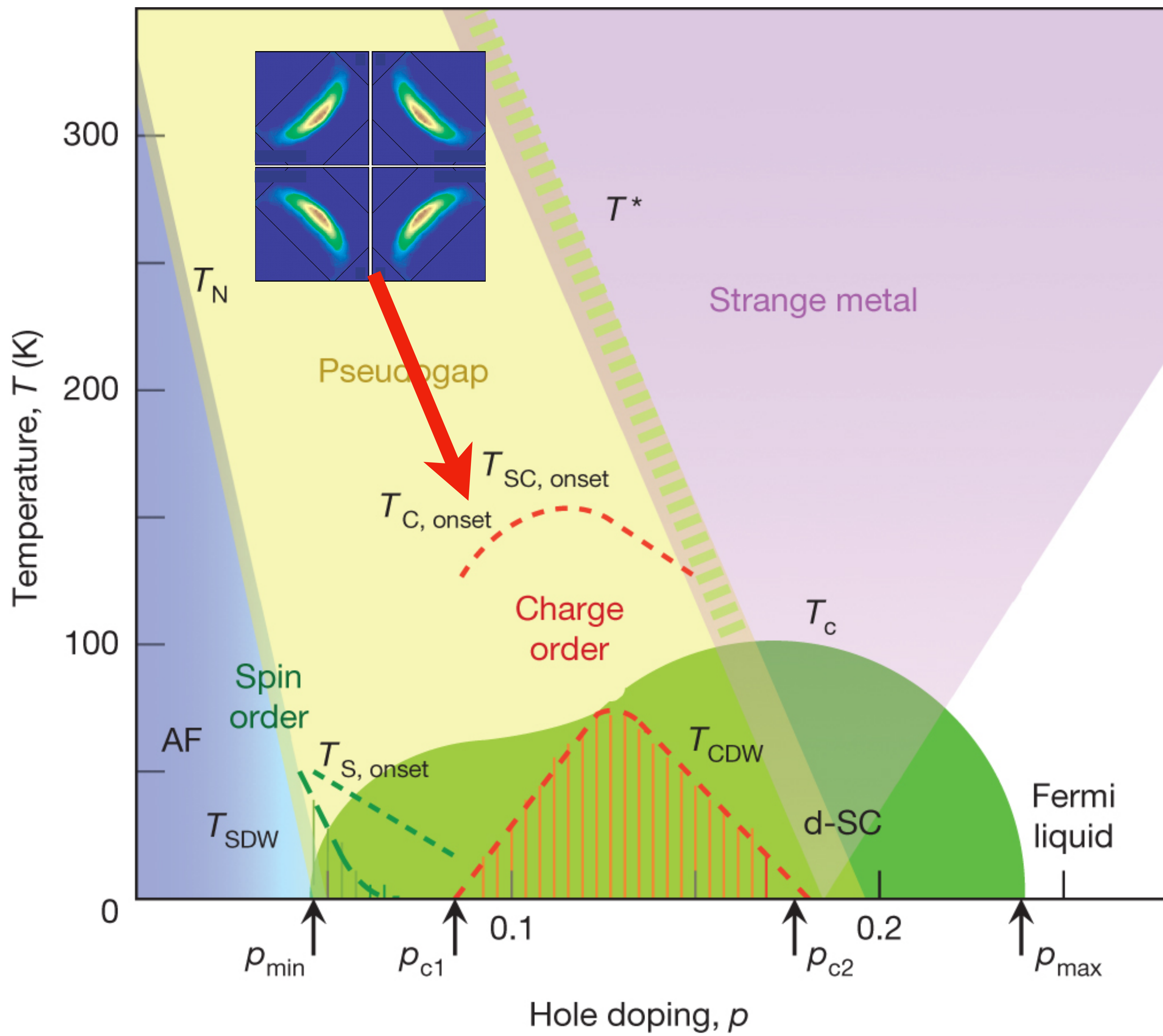


E. Mascot,  
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075146 (2022)

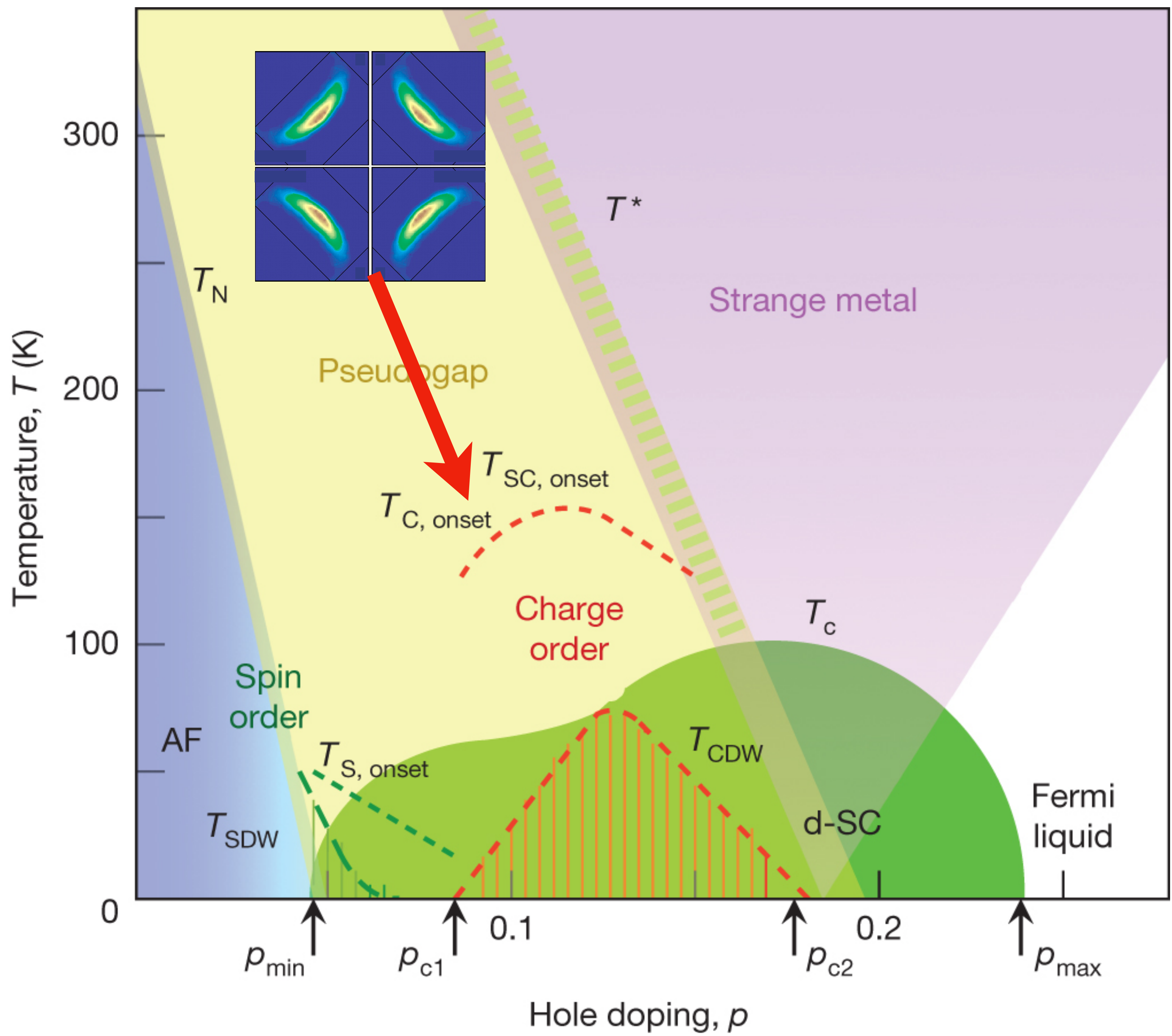
Hole pocket Fermi surfaces  
of size  $p$  with  
charge  $e$ , spin-1/2 quasiparticles  
+  
'spectator'  
square lattice spin liquid  
at half-filling.

FL\*: Spin liquid is *required* because  
the Fermi surface does not enclose  
the Luttinger volume  $(1 + p)$ .

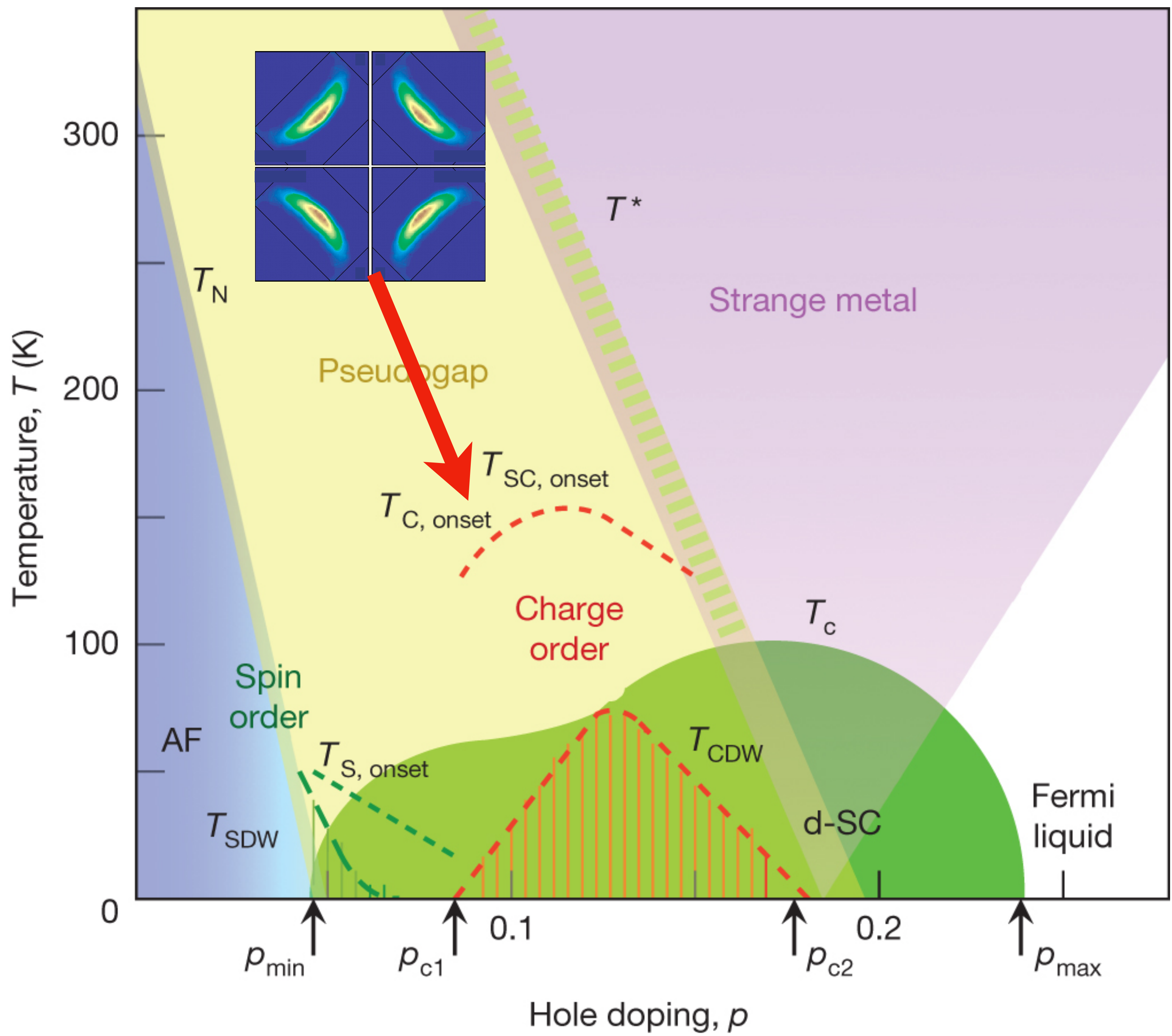
T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)



Use the pseudogap metal  
in place of the Fermi liquid  
as the ‘parent’ to  
*conventional*  
*d*-wave superconductor,  
charge density wave,  
spin density wave,  
pair density wave  
...



The onset of conventional order is a *confinement transition* for the emergent gauge theory describing the fractionalized excitations of the spin liquid in the FL\* state.



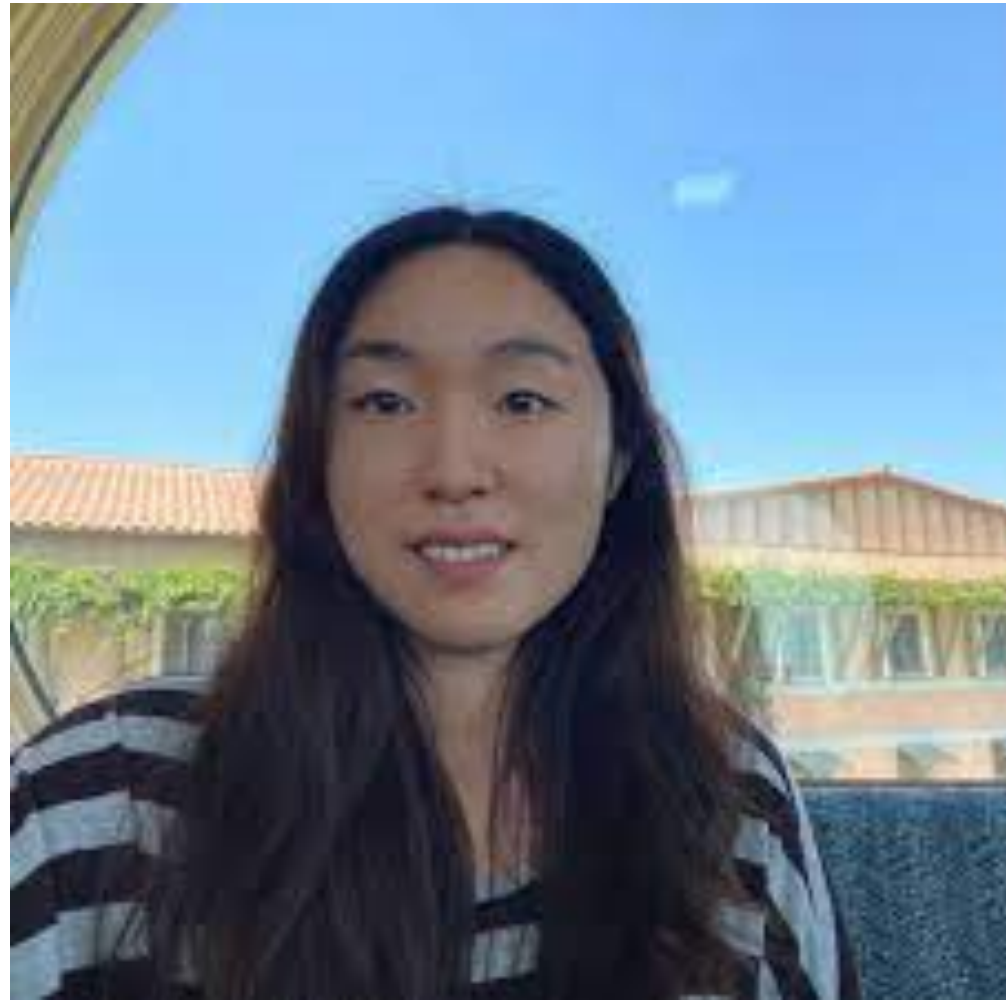
The onset of conventional order is a *confinement transition* for the emergent gauge theory describing the fractionalized excitations of the spin liquid in the FL\* state.

But which spin liquid?

A spin liquid which is ultimately IR-unstable to confinement



**Maine Christos**



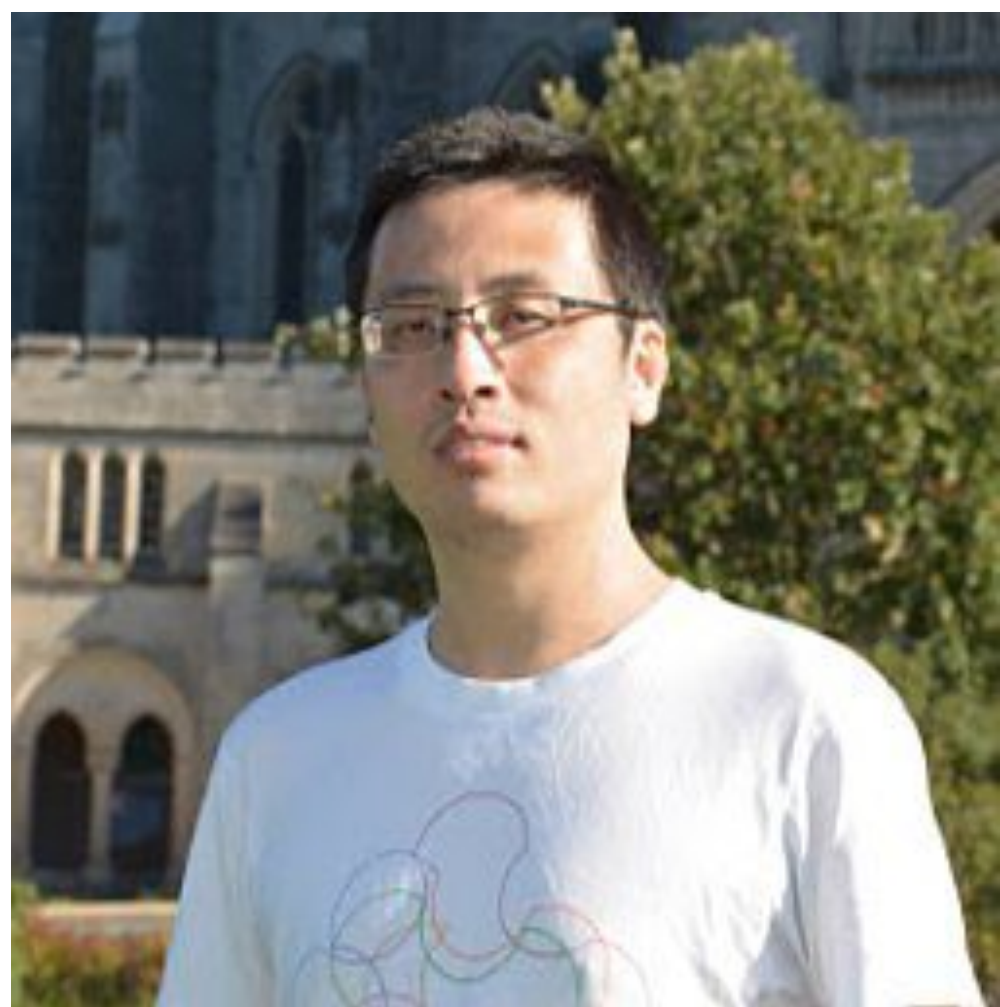
**Zhu-Xi Luo**  
→GA Tech



**Henry Shackleton**



**Mathias Scheurer**  
Innsbruck → Stuttgart



**Ya-Hui Zhang**  
Johns Hopkins



**Alexander Nikolaenko**



**Darshan Joshi**  
TIFR Hyderabad



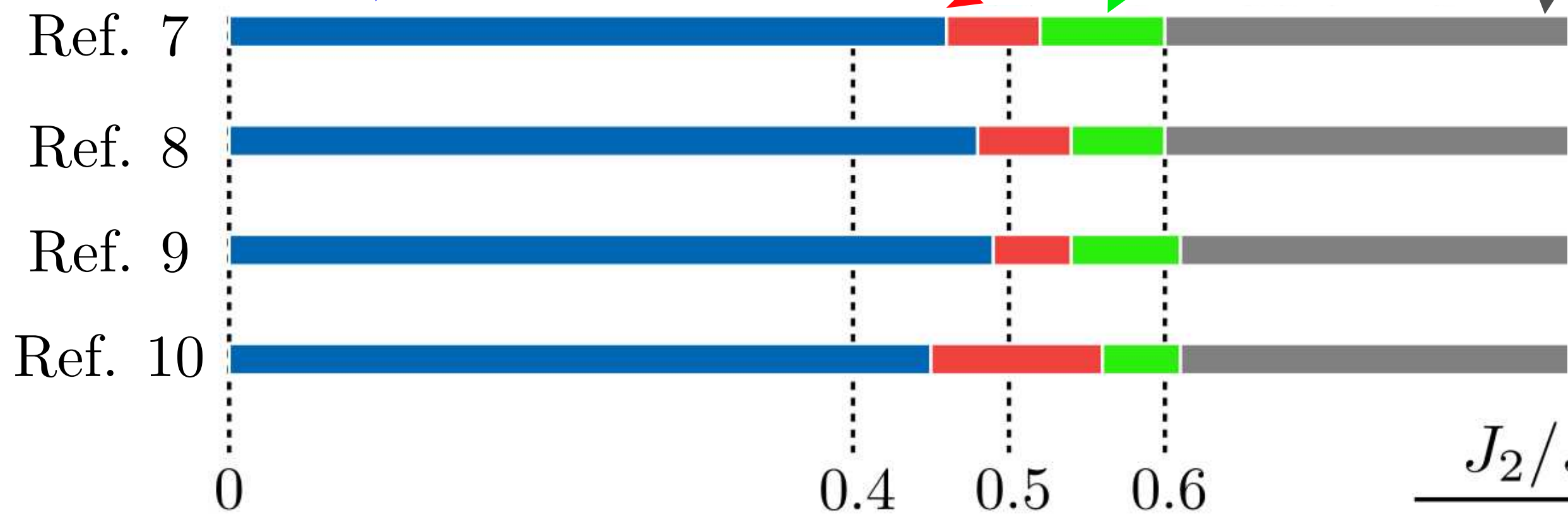
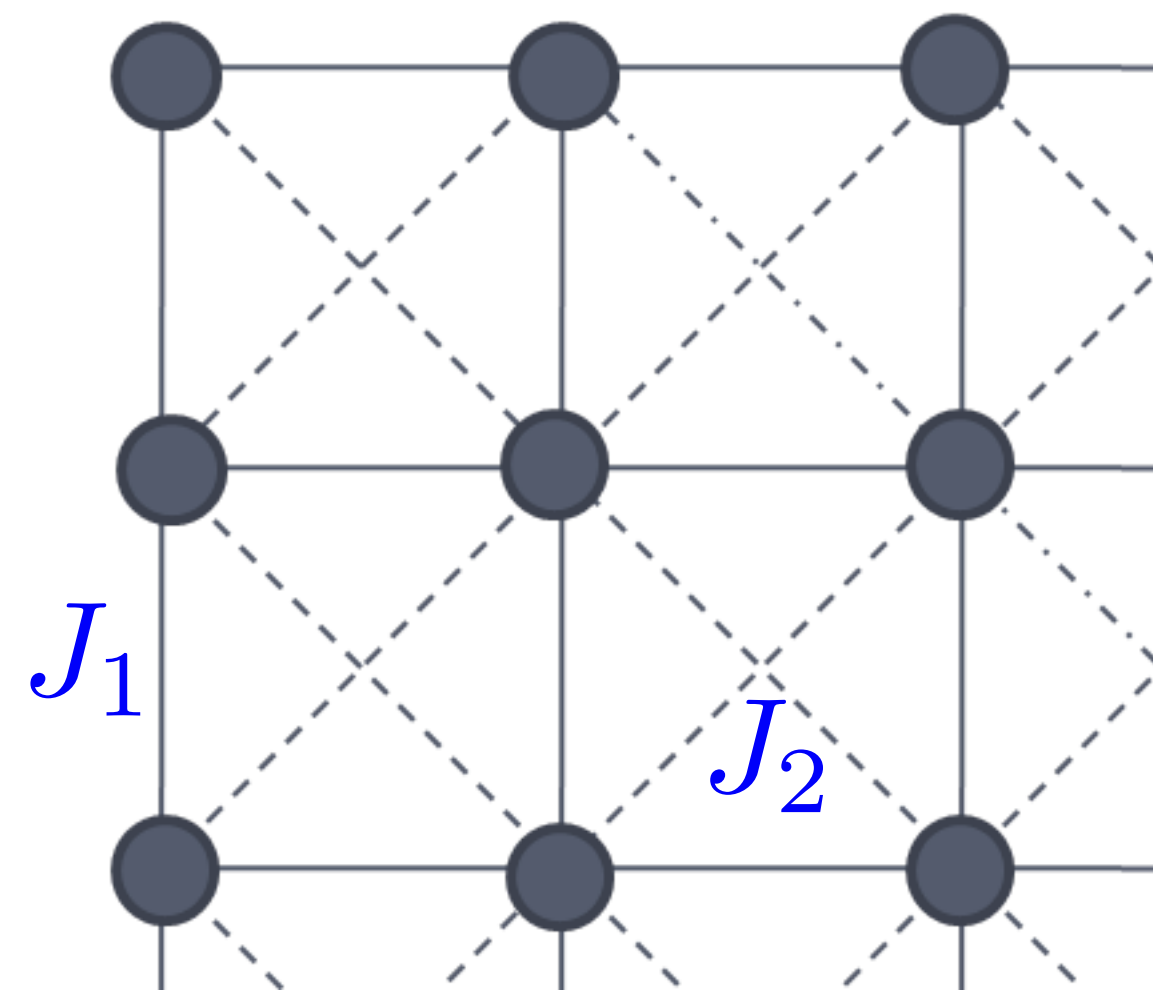
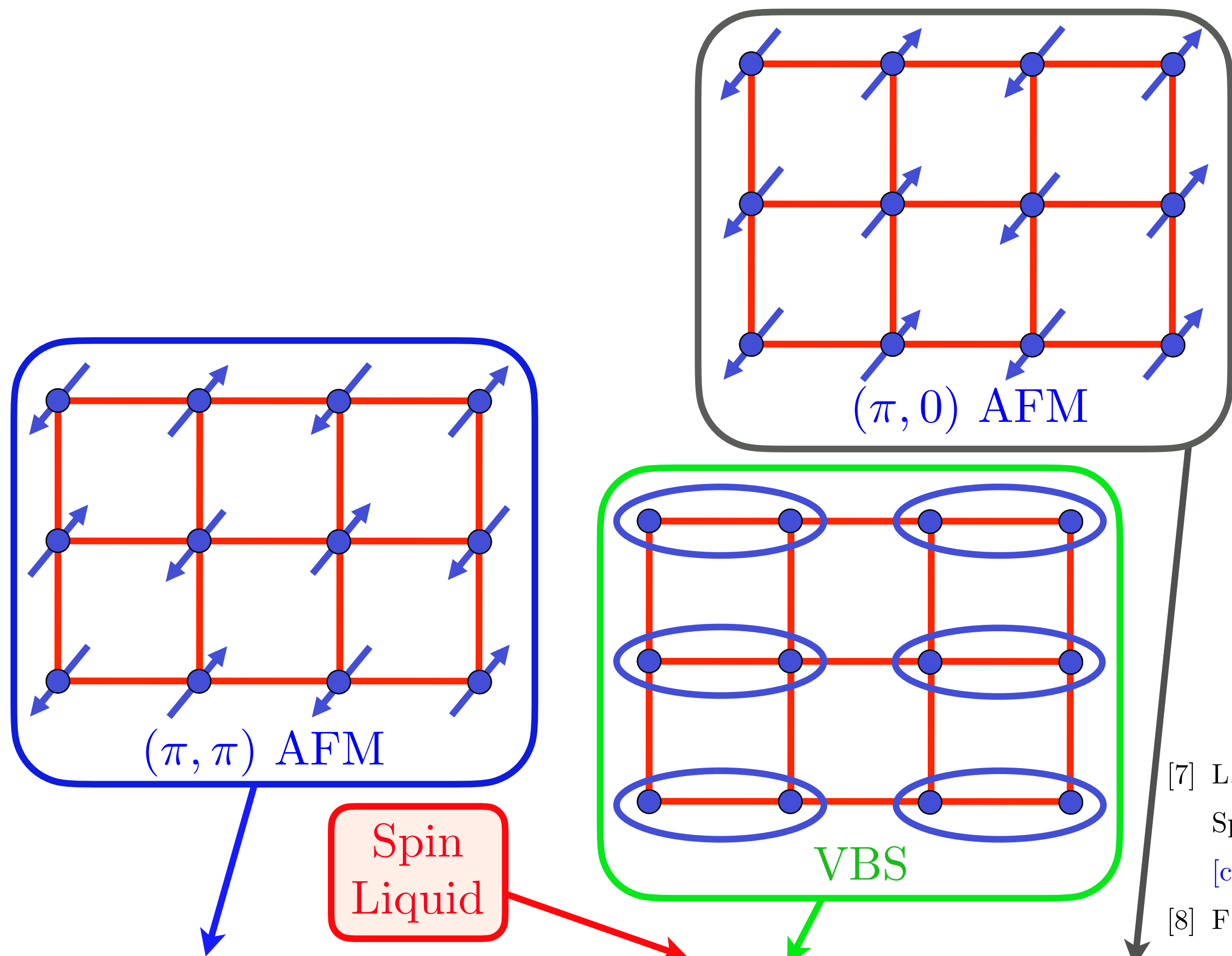
**Jonas von Milczewski**

1. The  $\pi$ -flux spin liquid

2. Doping the  $\pi$ -flux spin liquid: FL\*

3. Confinement transitions of  $\pi$ -flux-FL\*

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



[7] L. Wang and A. W. Sandvik, “Critical Level Crossings and Gapless Spin Liquid in the Square-Lattice Spin-1/2  $J_1$ - $J_2$  Heisenberg Antiferromagnet,” *Phys. Rev. Lett.* **121**, 107202 (2018), [arXiv:1702.08197 \[cond-mat.str-el\]](#).

[8] F. Ferrari and F. Becca, “Gapless spin liquid and valence-bond solid in the  $J_1$ - $J_2$  Heisenberg model on the square lattice: Insights from singlet and triplet excitations,” *Phys. Rev. B* **102**, 014417 (2020), [arXiv:2005.12941 \[cond-mat.str-el\]](#).

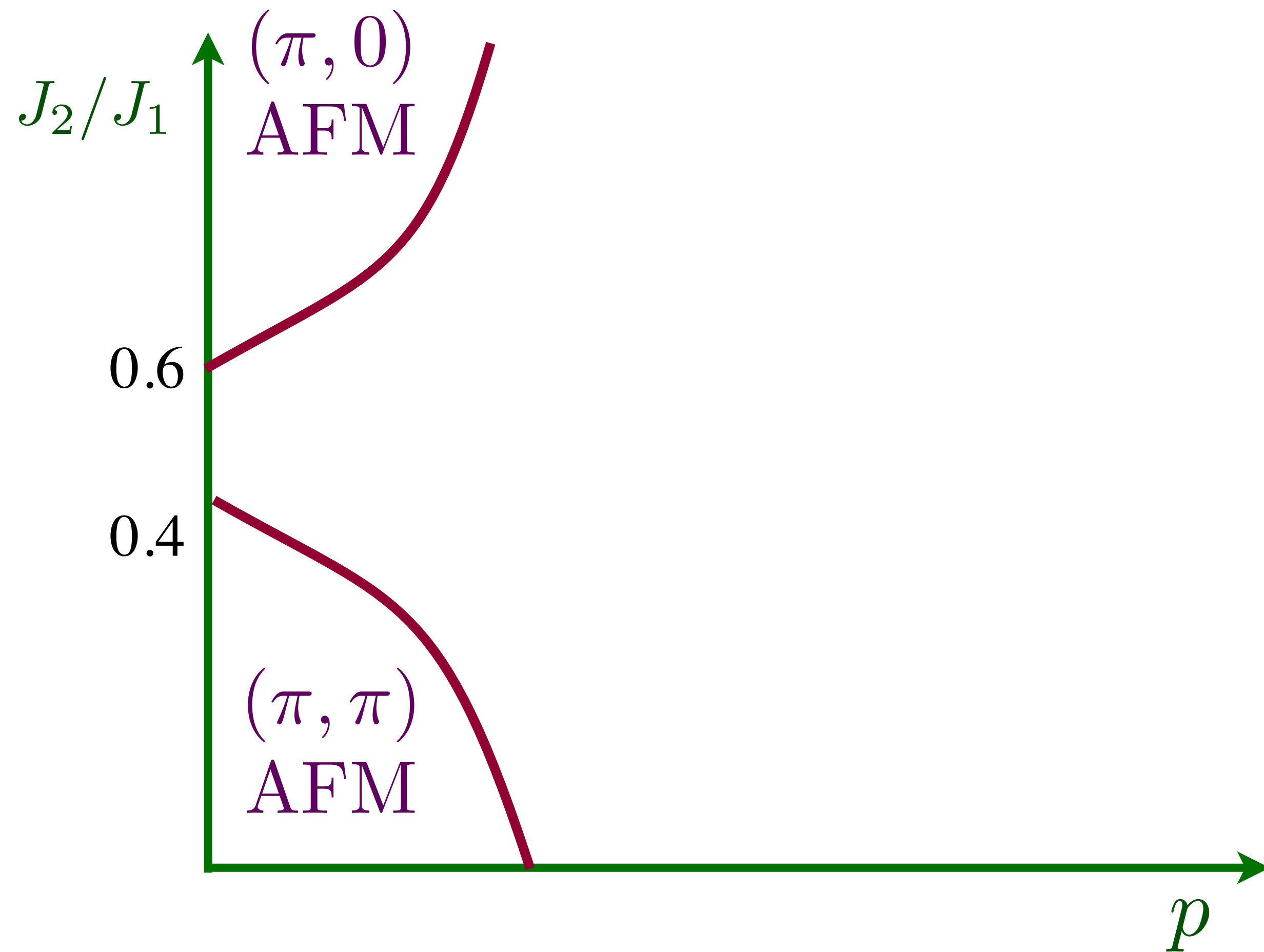
[9] Y. Nomura and M. Imada, “Dirac-Type Nodal Spin Liquid Revealed by Refined Quantum Many-Body Solver Using Neural-Network Wave Function, Correlation Ratio, and Level Spectroscopy,” *Phys. Rev. X* **11**, 031034 (2021), [arXiv:2005.14142 \[cond-mat.str-el\]](#).

[10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, “Gapless quantum spin liquid and global phase diagram of the spin-1/2  $J_1$ - $J_2$  square antiferromagnetic Heisenberg model,” (2020), [arXiv:2009.01821 \[cond-mat.str-el\]](#).

# High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang<sup>1,\*</sup> and Steven A. Kivelson<sup>2</sup>

PHYSICAL REVIEW LETTERS **127**, 097002 (2021)



Superconducting valence bond fluid in  
lightly doped 8-leg  $t$ - $J$  cylinders

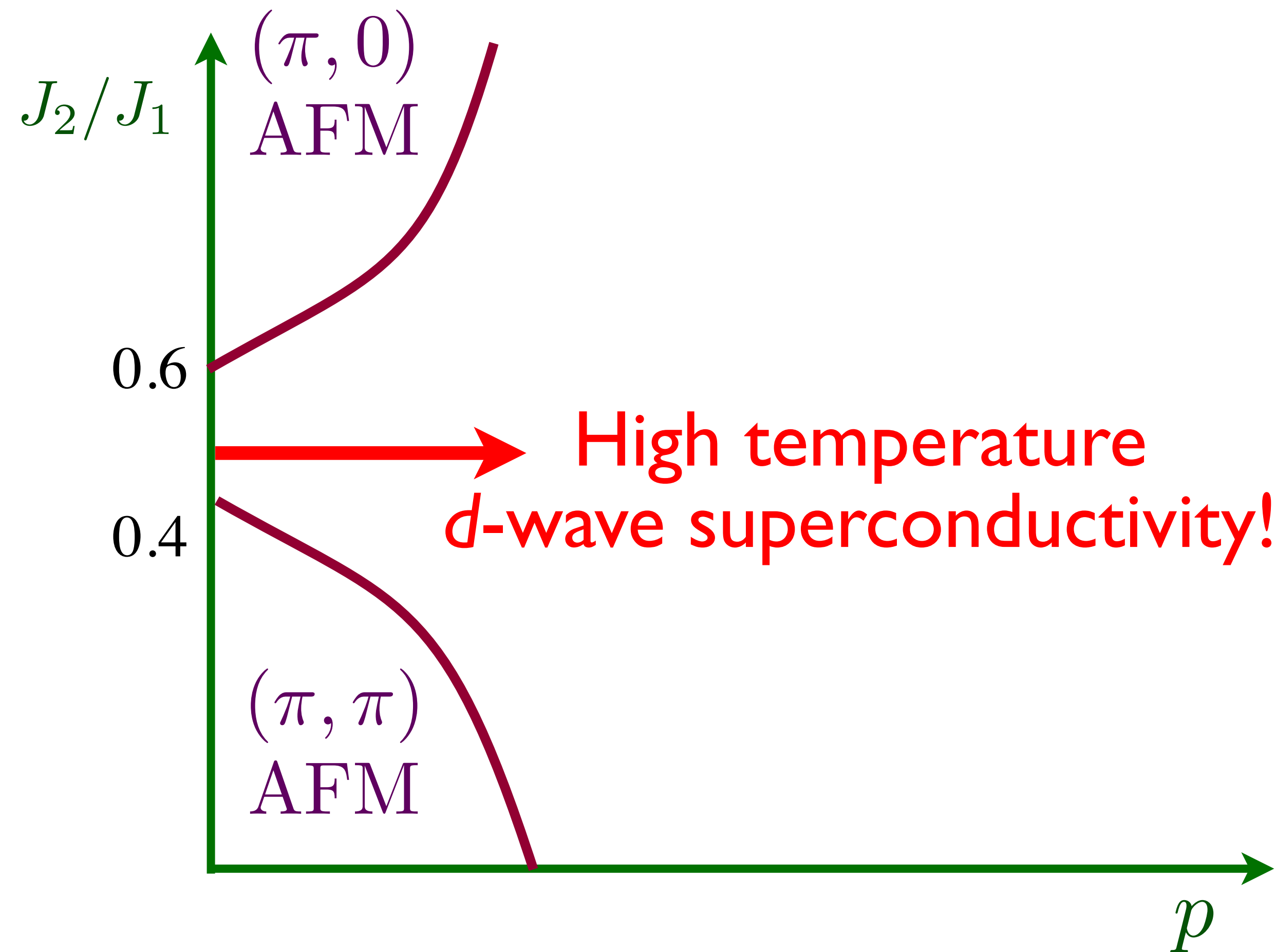
Hong-Chen Jiang, Steven A. Kivelson, and  
Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any “competing” charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

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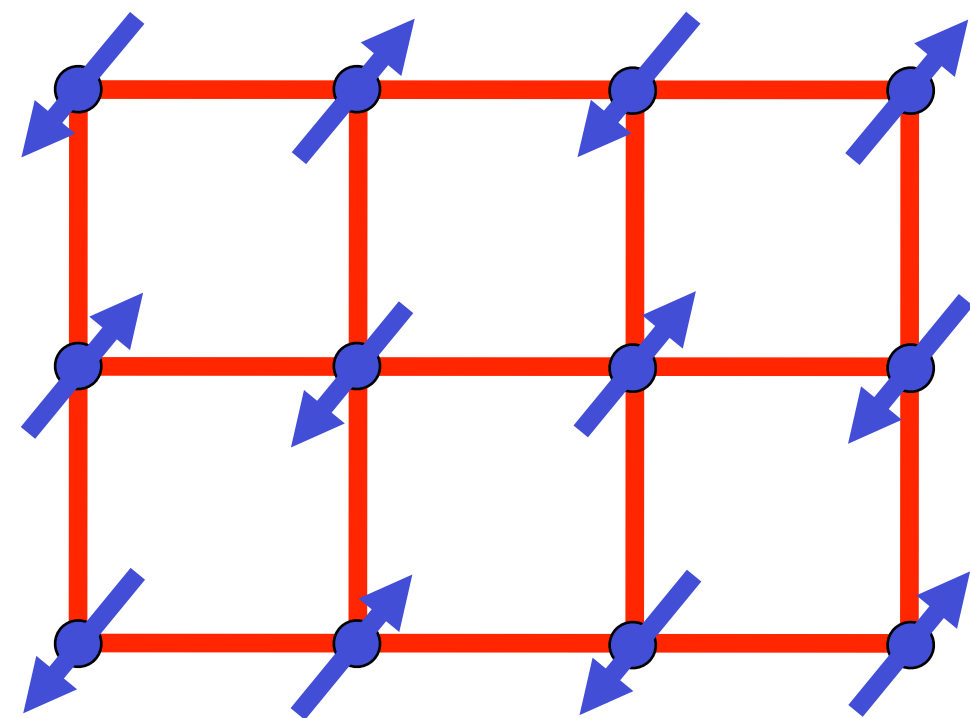


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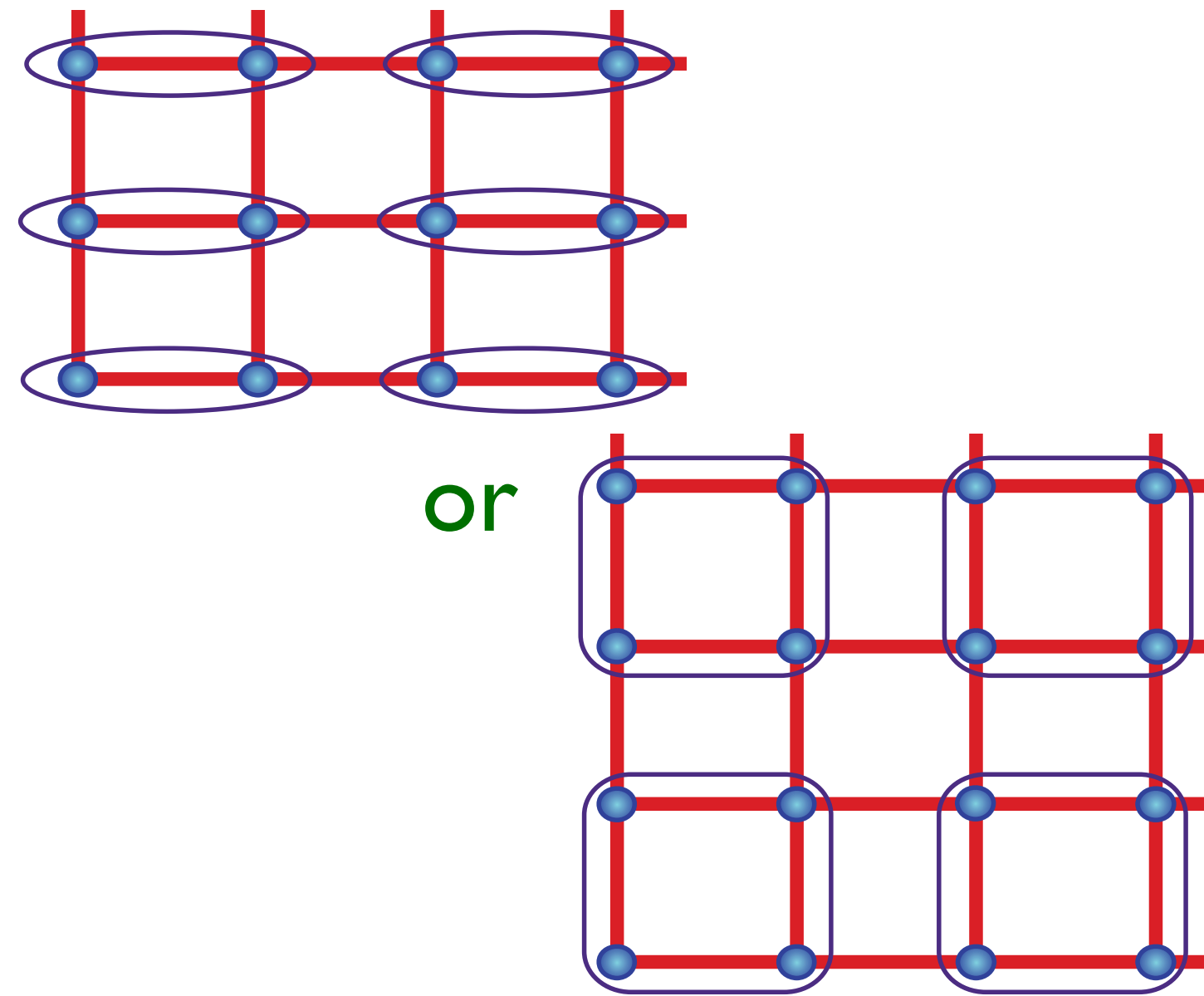
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# Insulating $S=1/2$ antiferromagnet



Higgs phase,  $\langle z_\alpha \rangle \neq 0$ :  
Néel order



Confining phase,  $\langle z_\alpha \rangle = 0$ :  
VBS order

$s$

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger bosons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

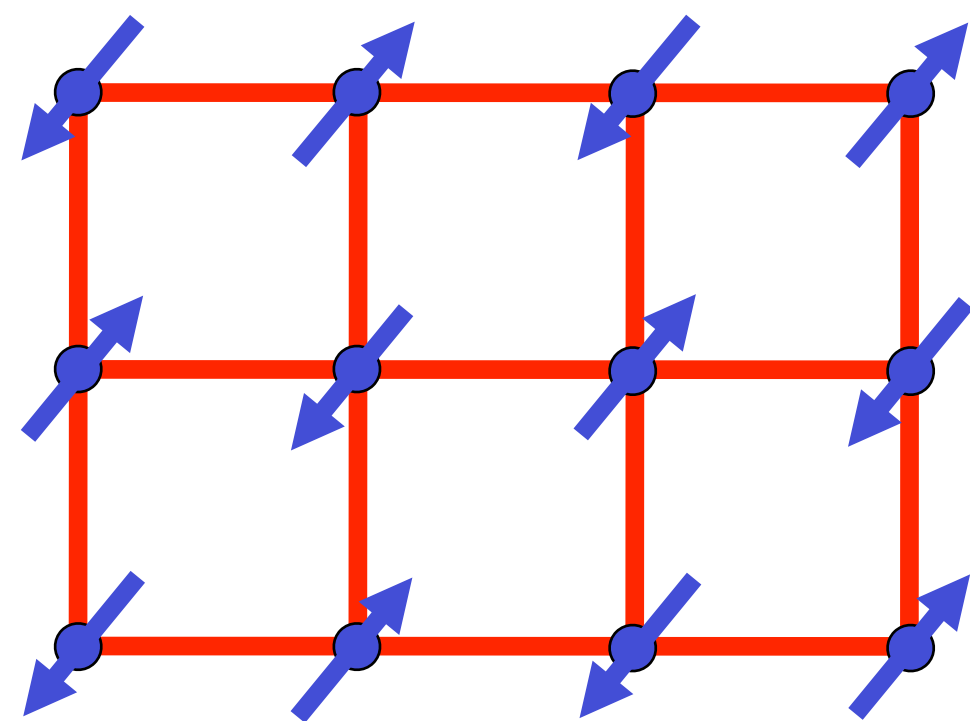
Mean-field spin liquid  
with gapped bosonic spinons.

Low energy  $\mathbb{C}\mathbb{P}^1$  U(1) gauge theory

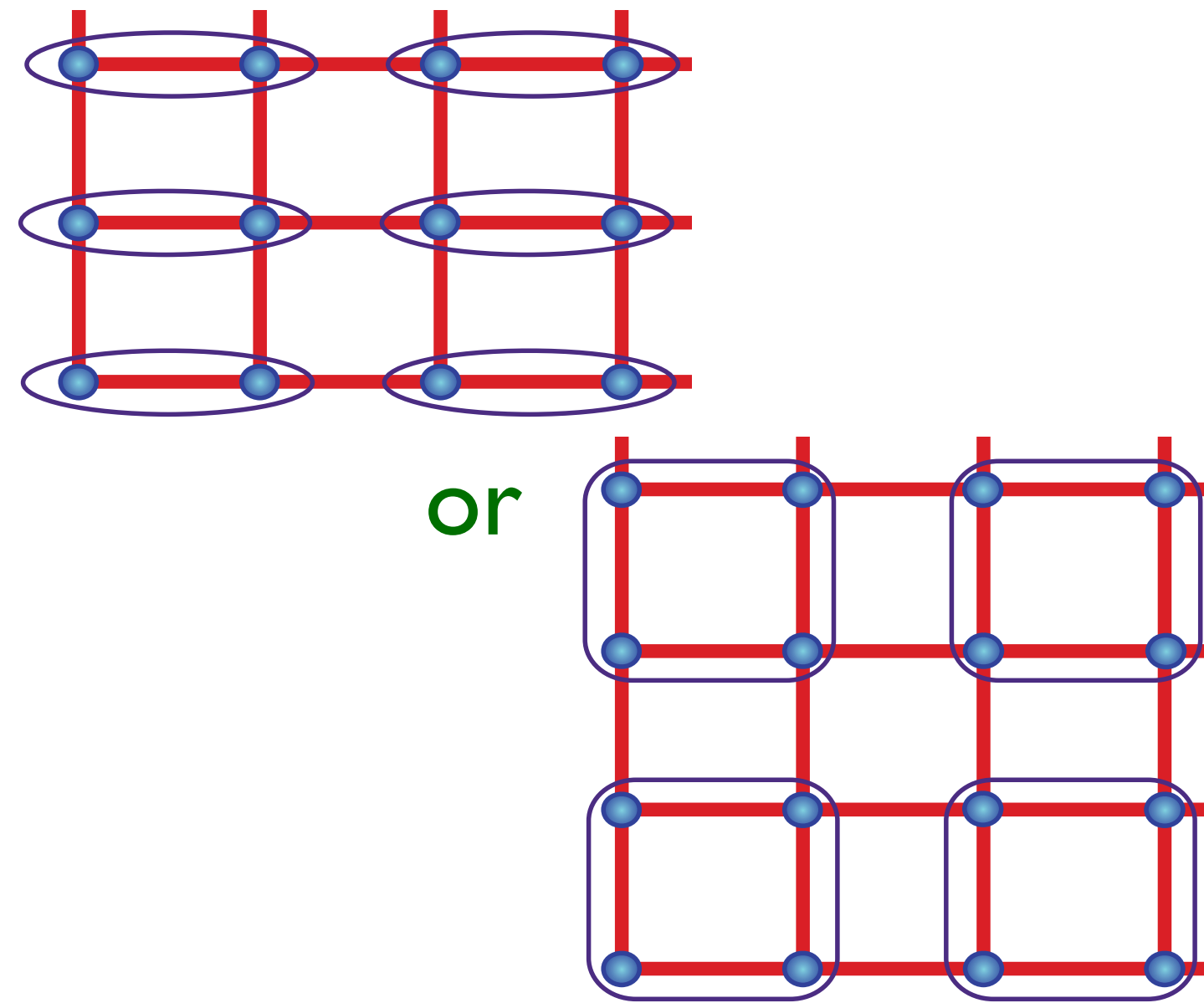
$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - ia_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

# Insulating $S=1/2$ antiferromagnet



Confining phase:  
Néel order



Confining phase:  
VBS order

$$H = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Schwinger fermions

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

$\pi$ -flux mean-field theory

with gapless spinons at 2 Dirac points.

Low energy theory of  $N_f = 2$

Dirac fermions  $\Psi_s$  coupled to an emergent  $SU(2)_N$  gauge field.

Confining order parameters are Néel and VBS states, with a global  $SO(5)_f$  symmetry!

$$\mathcal{L} = i\bar{\Psi}_s \gamma_\mu D_\mu \Psi_s + \dots$$

Dual to  $\mathbb{C}P^1$  U(1) gauge theory.

[Submitted on 28 Jun 2023]

# The $SO(5)$ Deconfined Phase Transition under the Fuzzy Sphere Microscope: Approximate Conformal Symmetry, Pseudo-Criticality, and Operator Spectrum

Zheng Zhou, Liangdong Hu, W. Zhu, Yin-Chen He

The deconfined quantum critical point (DQCP) is an example of phase transitions beyond the Landau symmetry breaking paradigm that attracts wide interest. However, its nature has not been settled after decades of study. In this paper, we apply the recently proposed fuzzy sphere regularization to study the  $SO(5)$  non-linear sigma model (NL $\sigma$ M) with a topological Wess-Zumino-Witten term, which serves as a dual description of the DQCP with an exact  $SO(5)$  symmetry. We demonstrate that the fuzzy sphere functions as a powerful microscope, magnifying and revealing a wealth of crucial information about the DQCP, ultimately paving the way towards its final answer. In particular, through exact diagonalization, we provide clear evidence that the DQCP exhibits approximate conformal symmetry. The evidence includes the existence of a conserved  $SO(5)$  symmetry

1. The  $\pi$ -flux spin liquid

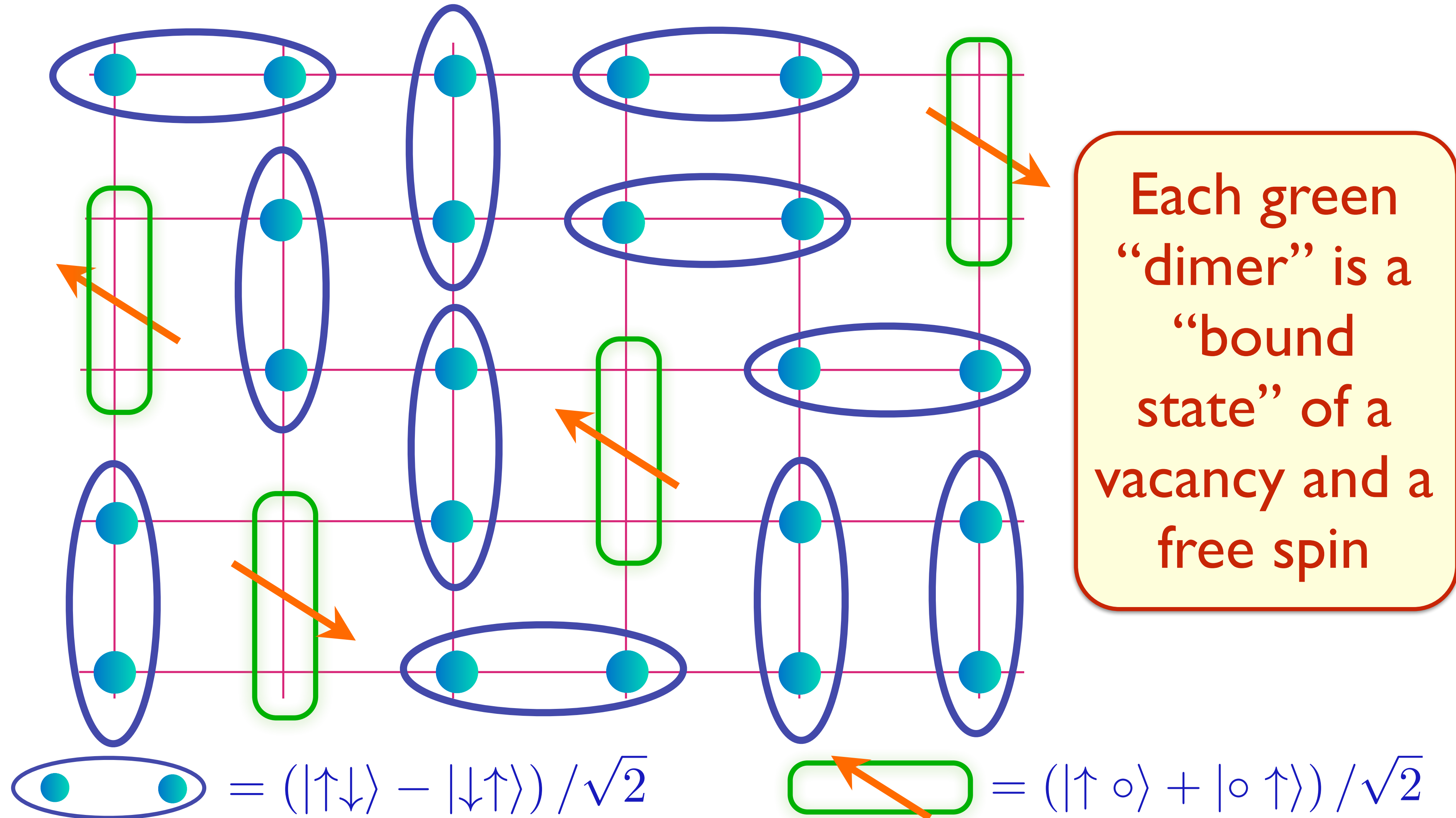
2. Doping the  $\pi$ -flux spin liquid: FL\*

3. Confinement transitions of  $\pi$ -flux-FL\*

# FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

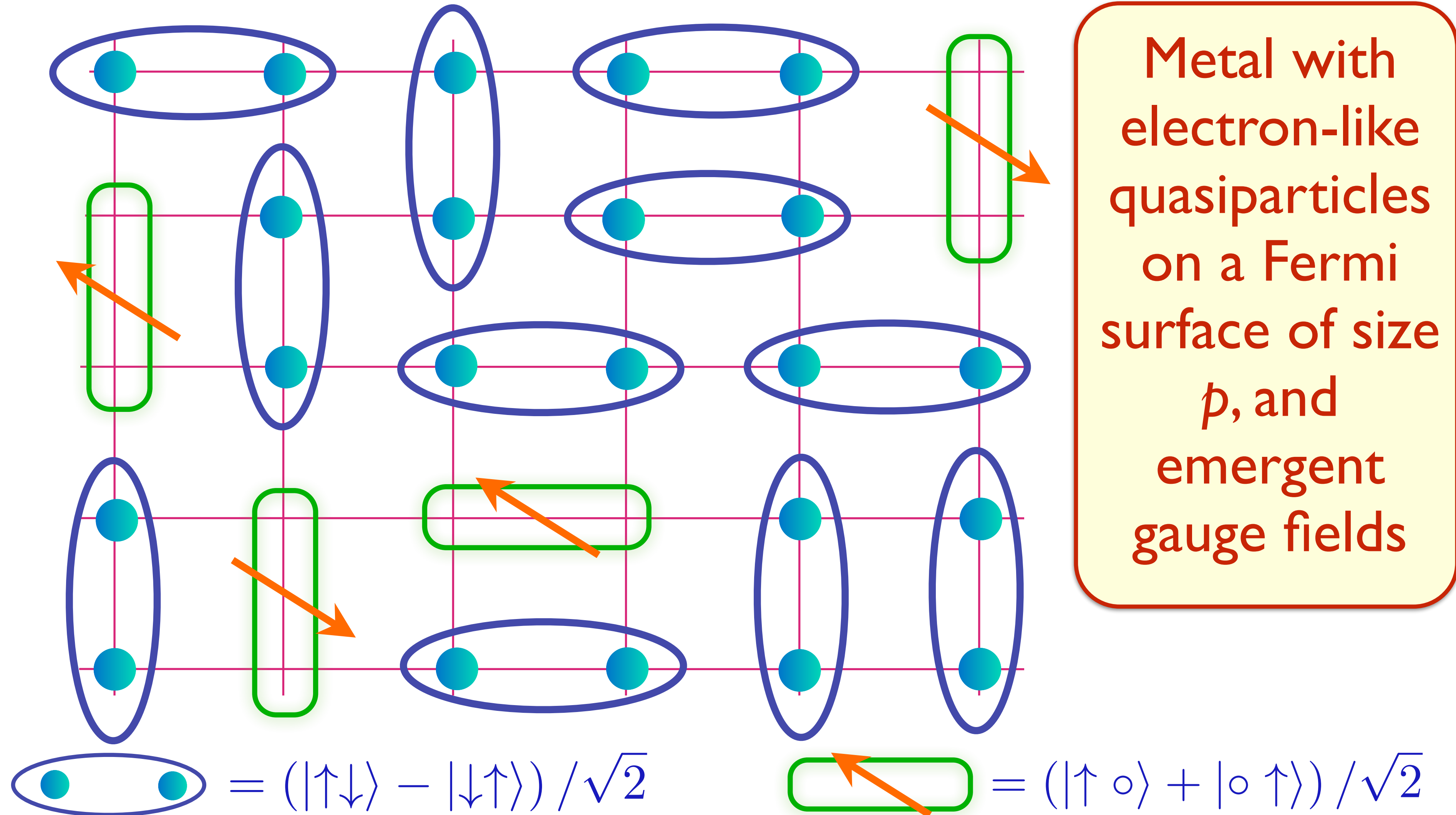
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



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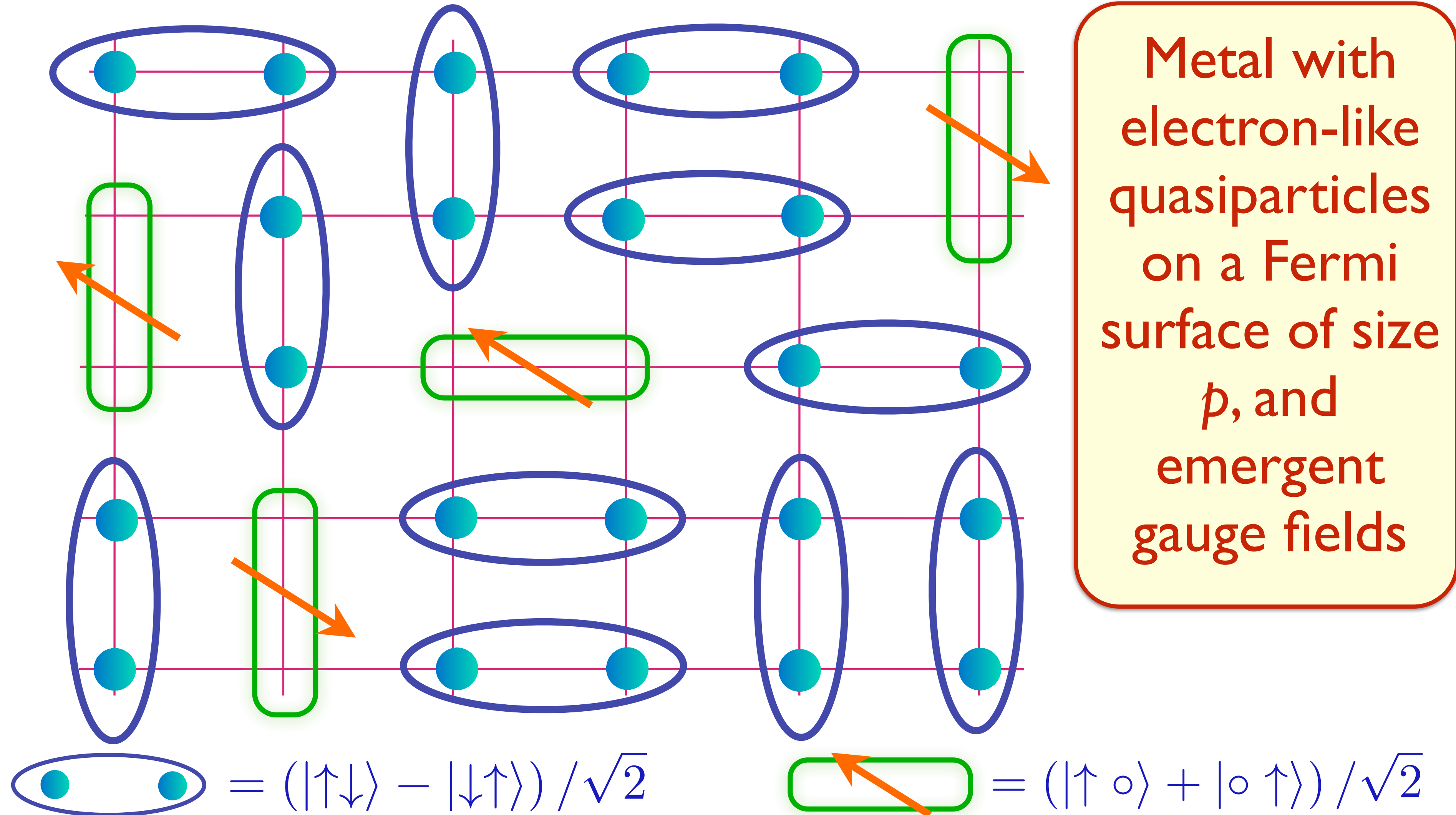


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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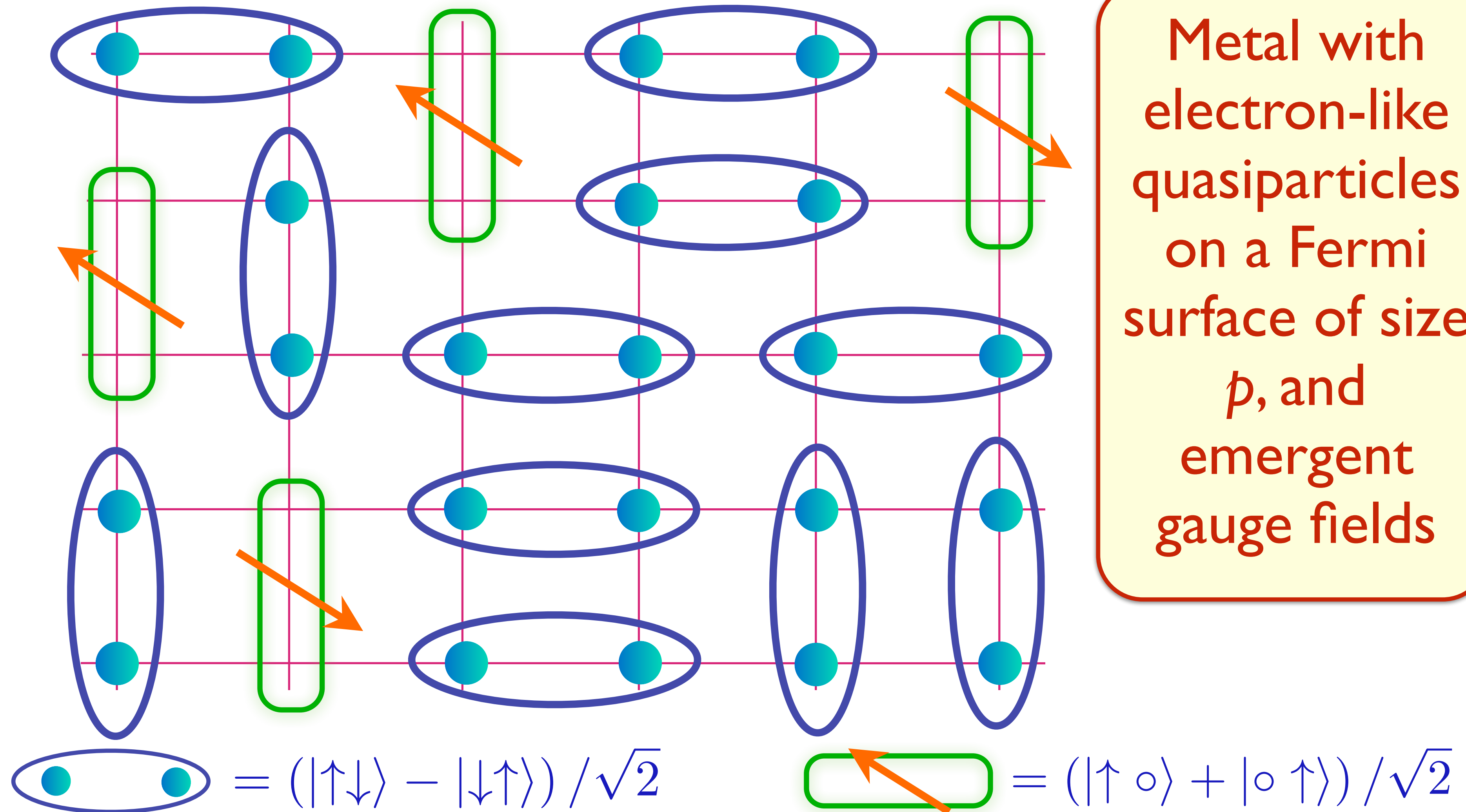


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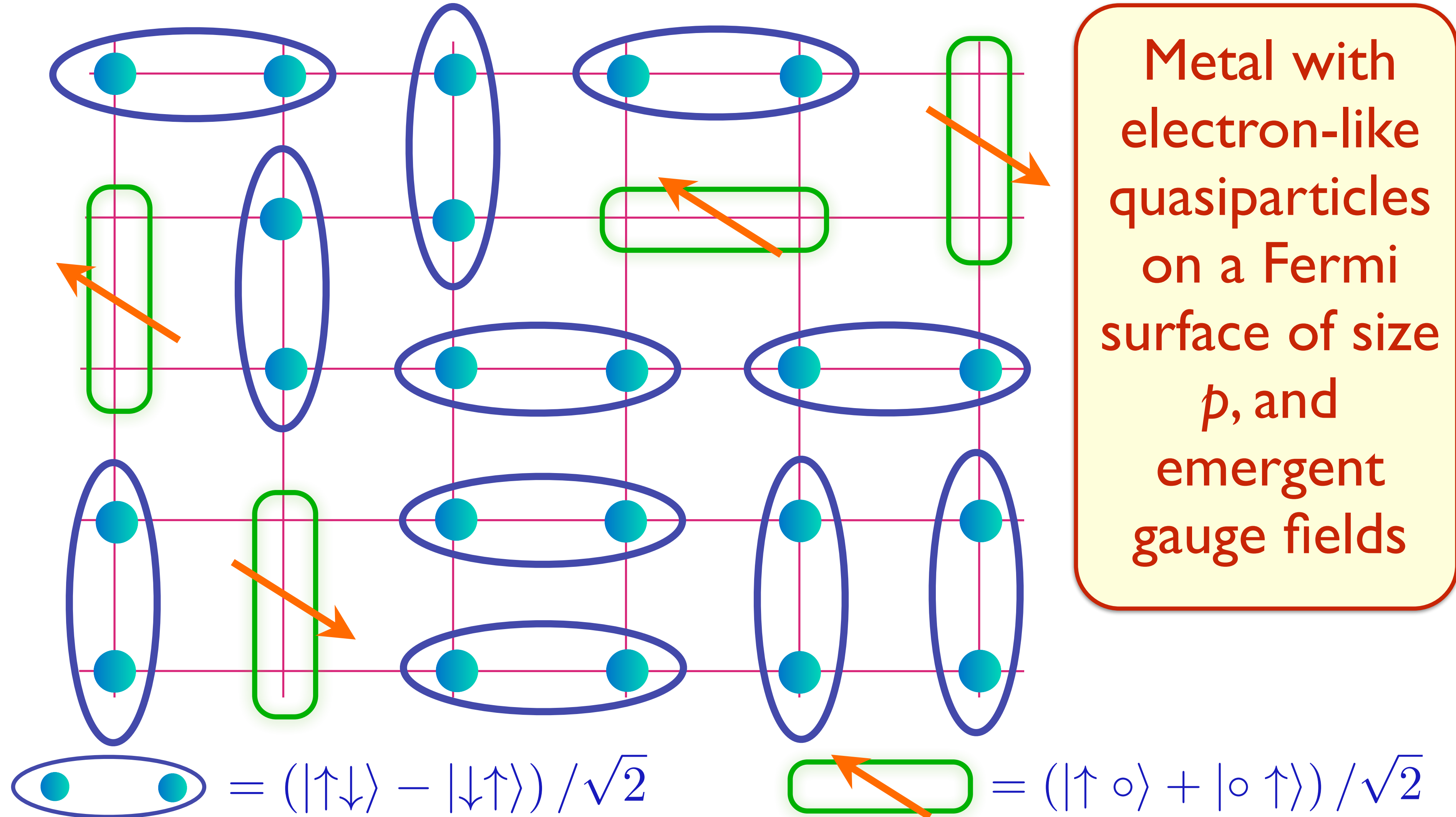


Metal with electron-like quasiparticles on a Fermi surface of size  $p$ , and emergent gauge fields

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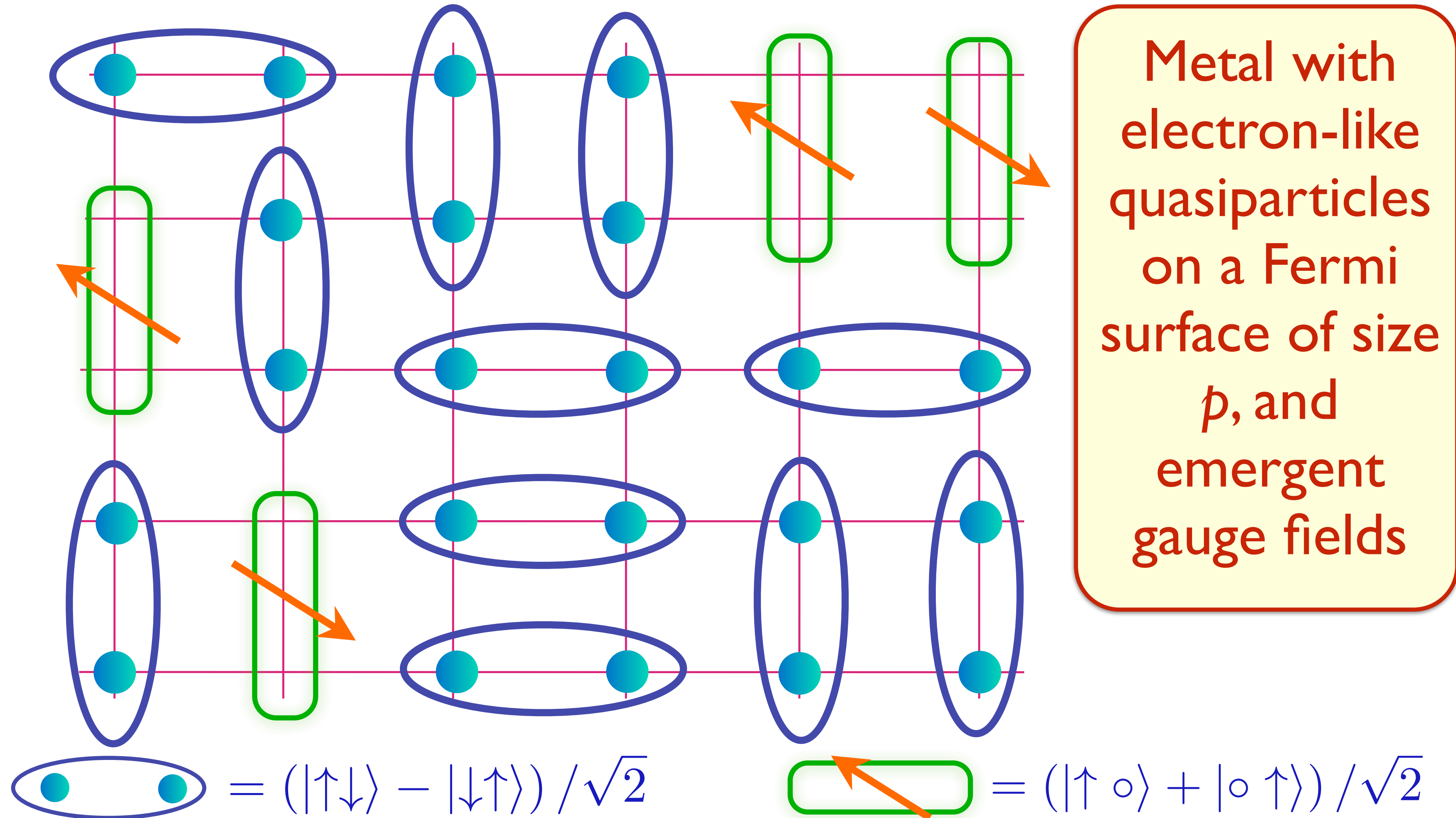


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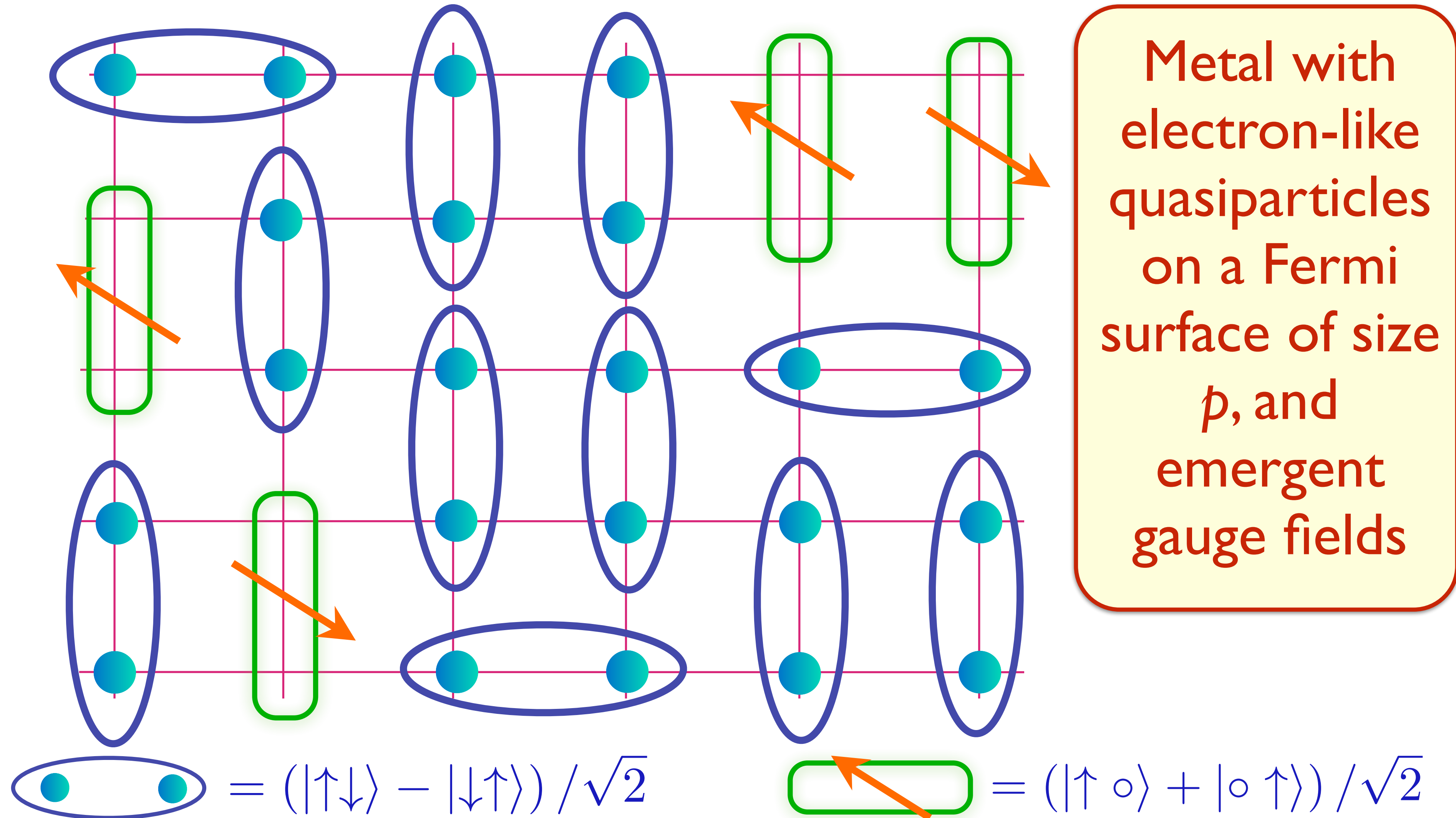


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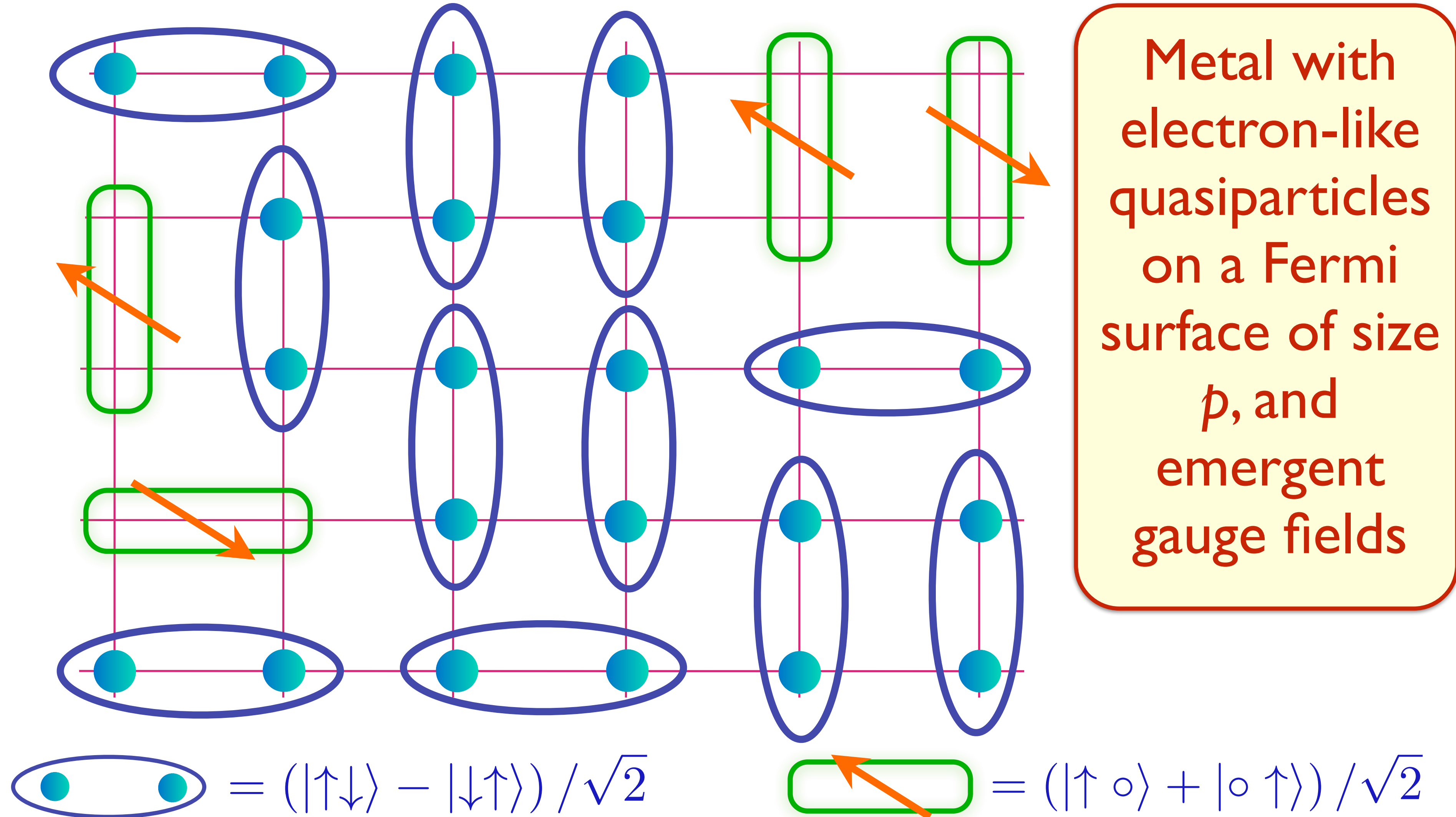


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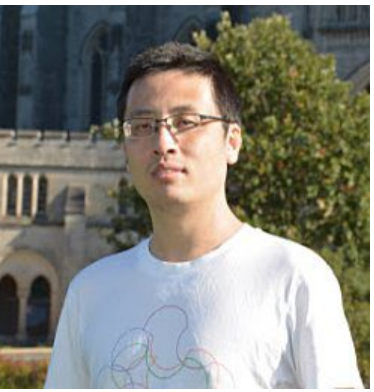
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Metal with  
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Ya-Hui Zhang

# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,  
PRR **2**, 023172 (2020)

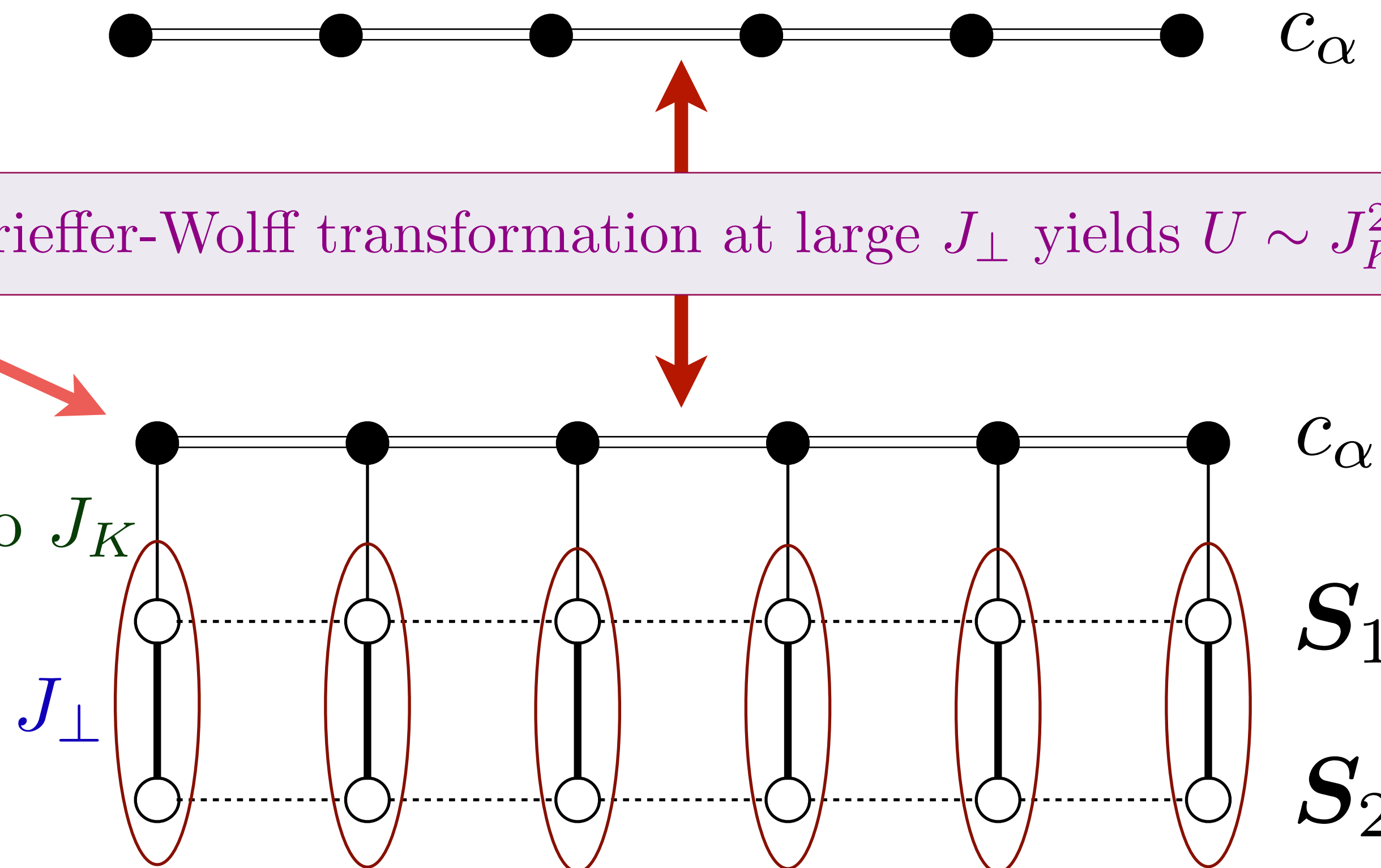
Free holes of density  $1+p$

Schrieffer-Wolff transformation at large  $J_{\perp}$  yields  $U \sim J_K^2/J_{\perp}$

Hubbard model of hole density  $1+p$

Antiferromagnetic Kondo  $J_K$

Ancilla qubits



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

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Ya-Hui Zhang and S. S.,  
PRR **2**, 023172 (2020)

Ya-Hui  
Zhang

Free  
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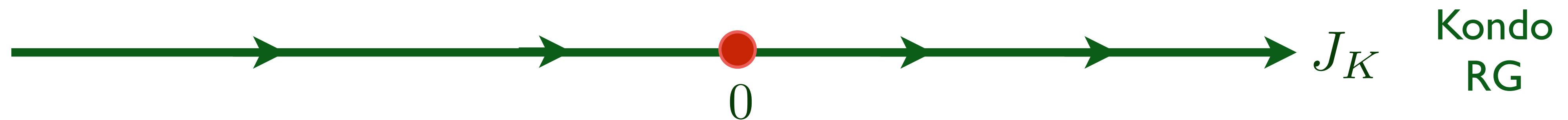
Hubbard  
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hole density  
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Antiferromagnetic Kondo  $J_K$

Ancilla  
qubits

$J_{\perp}$

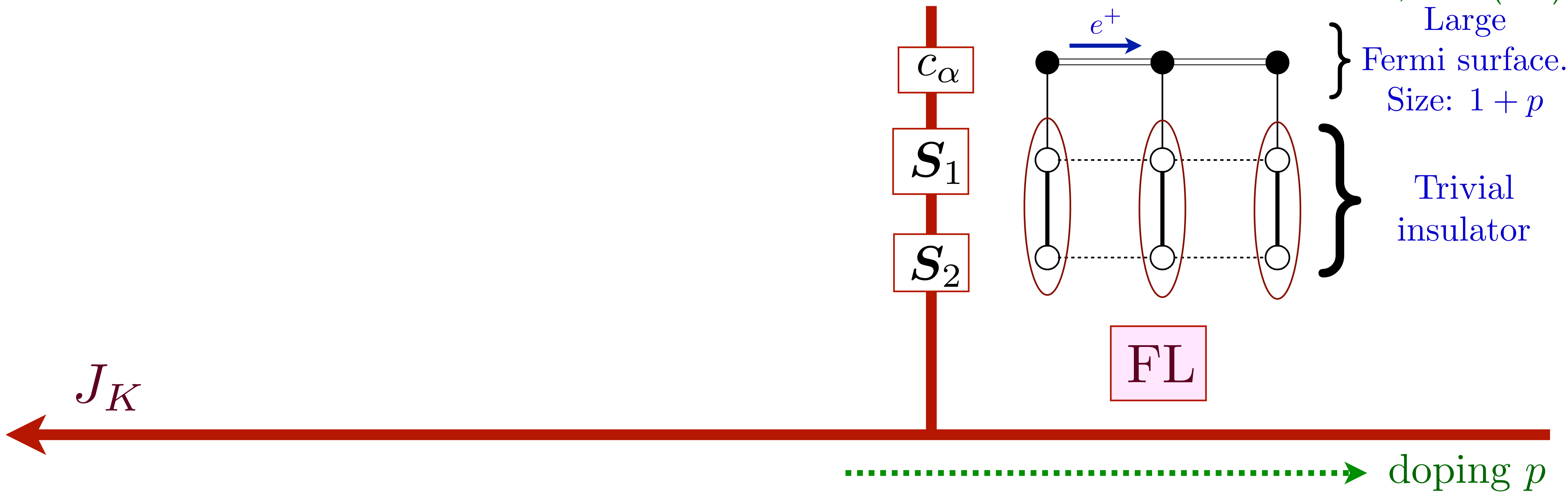
Ferromagnetic  
Kondo  $\tilde{J}_K$



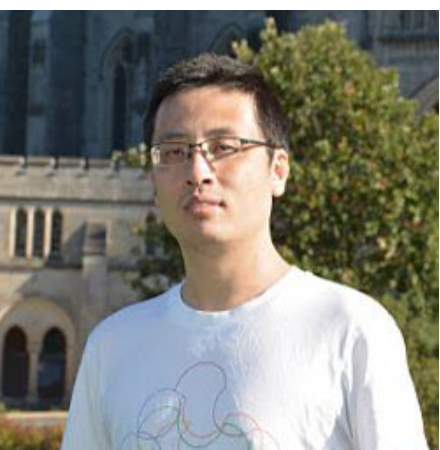
$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\sigma_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{S}_{1i} + J_{\perp} \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$

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Ya-Hui  
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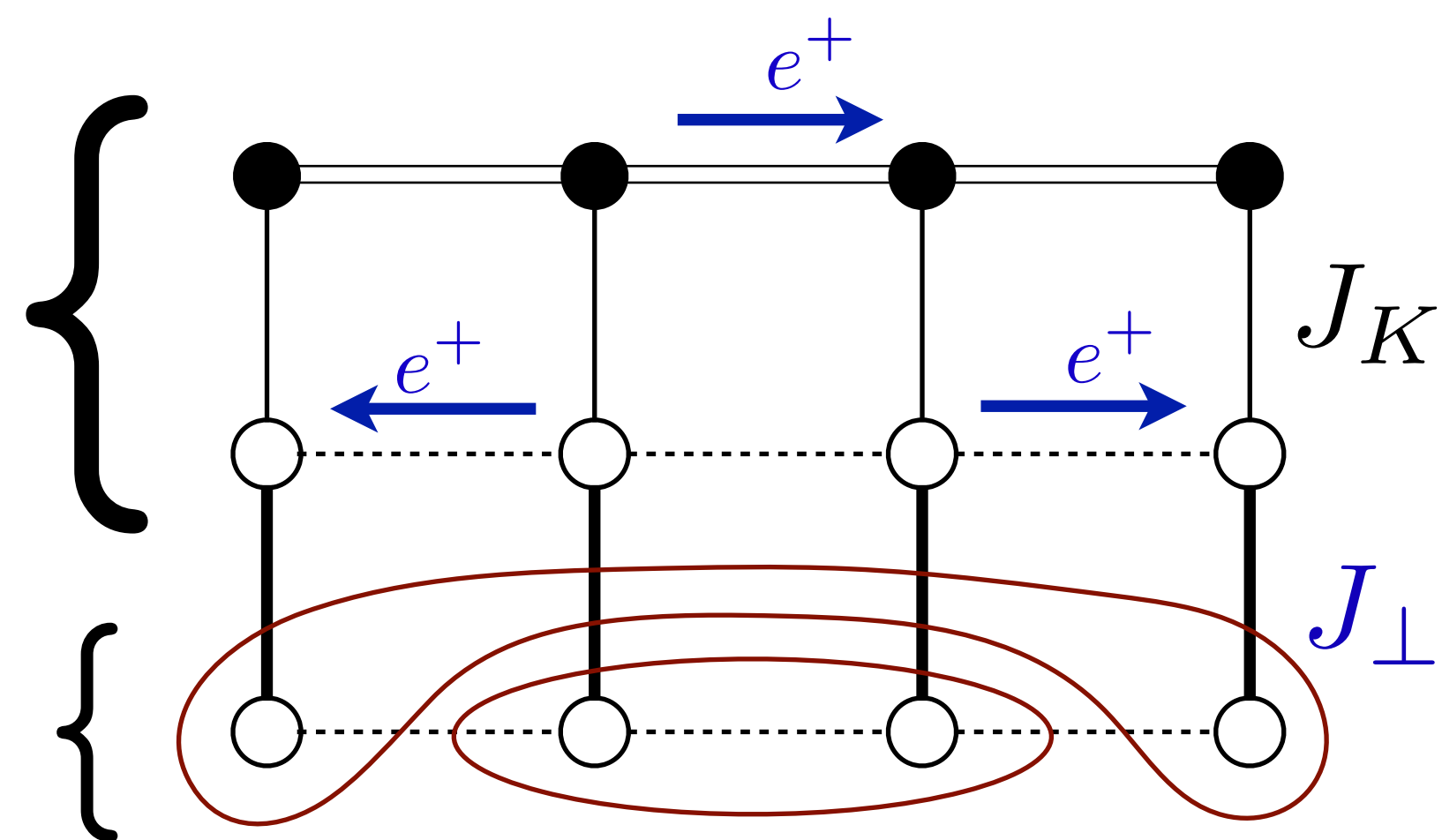


# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,  
PRR **2**, 023172 (2020)

Kondo lattice  
heavy Fermi liquid.  
Size  $1 + p + 1$   
 $= p \pmod{2}$ .  
*Small* Fermi surface!

Spin liquid



Large  
Fermi surface.  
Size:  $1 + p$

Trivial  
insulator

FL\*

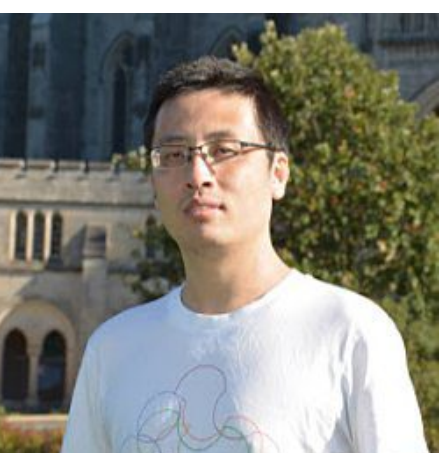
FL

$J_K$

doping  $p$

Pseudogap metal =  
Kondo Lattice Heavy  
Fermi Liquid  
 $\oplus$   
Spin Liquid

Ya-Hui  
Zhang

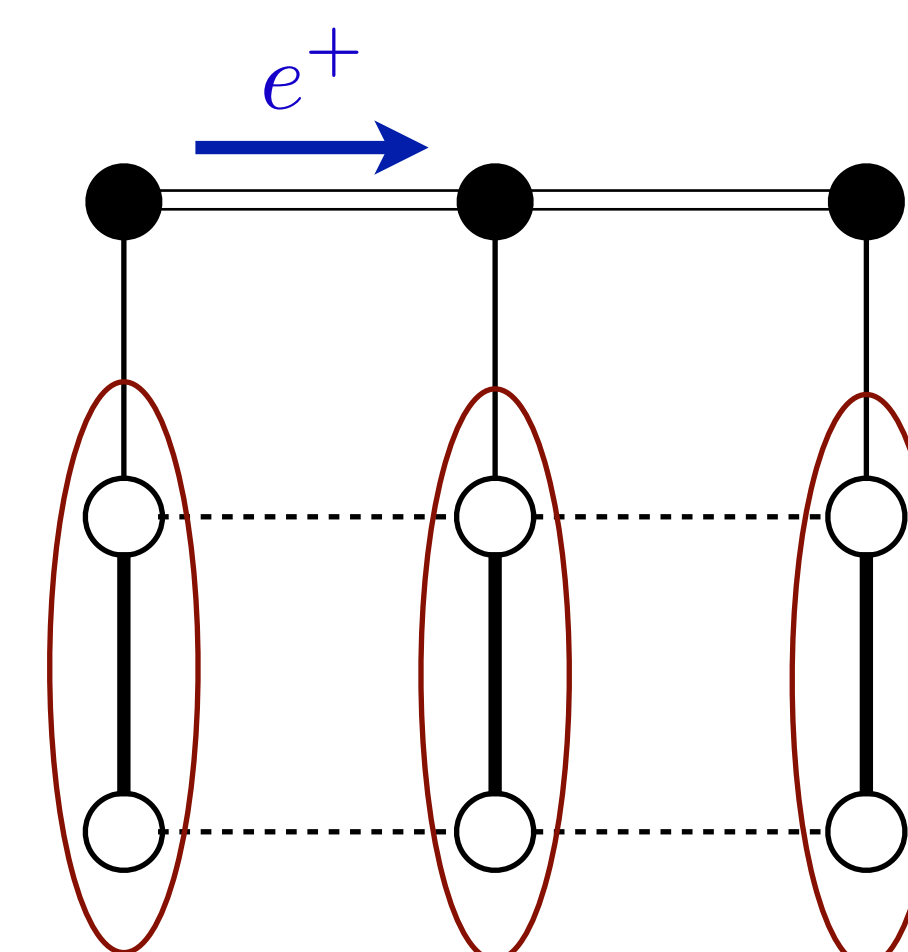
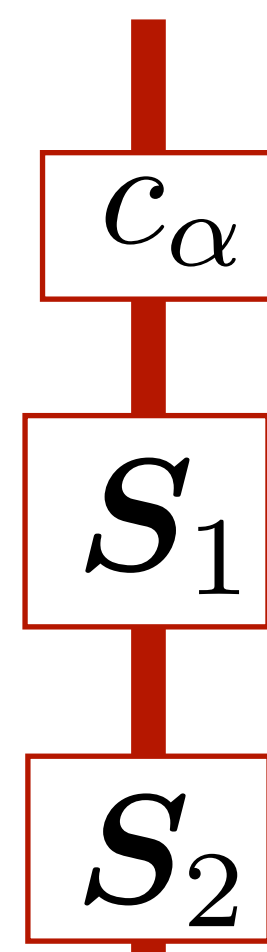
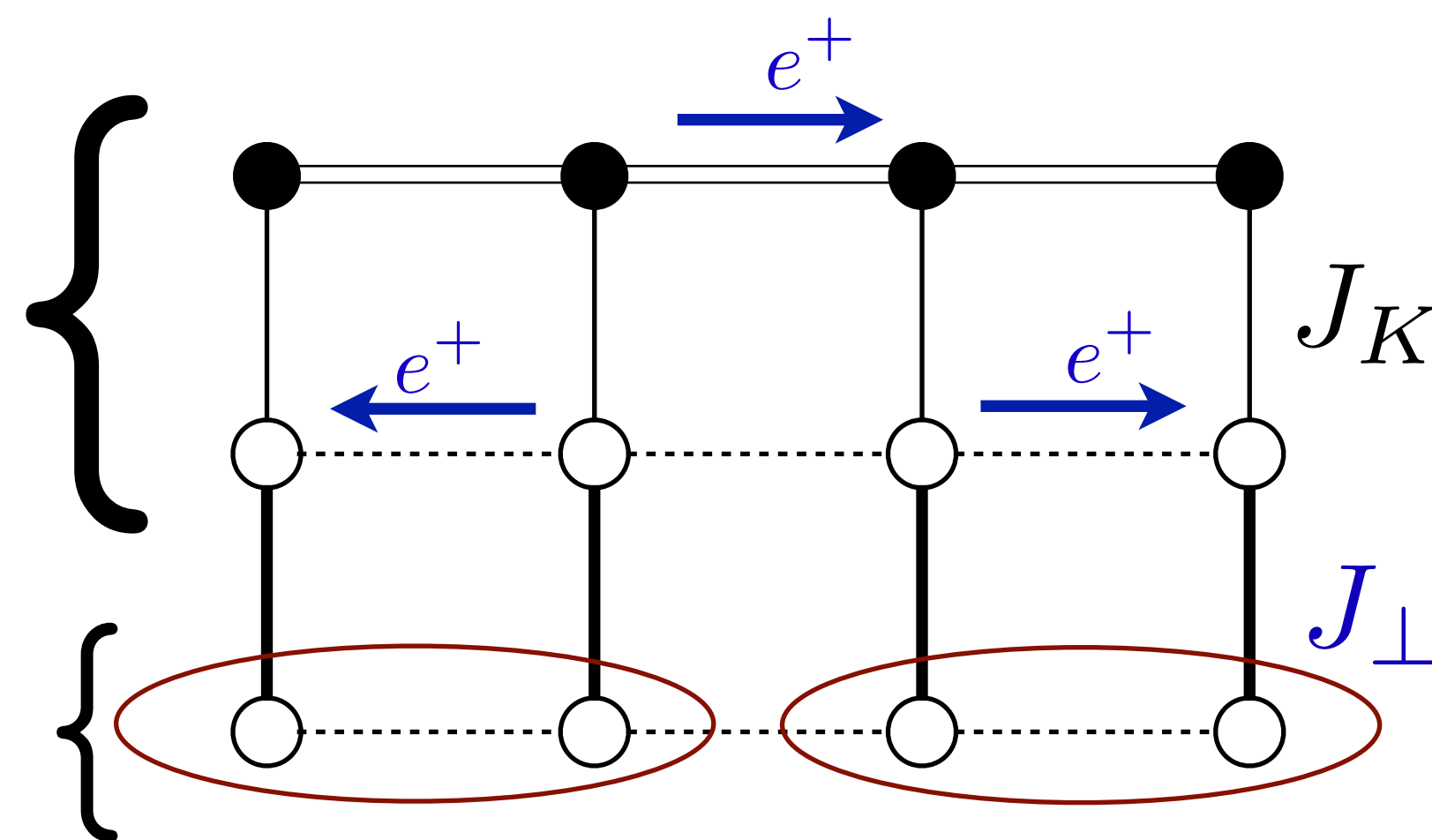


# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,  
PRR **2**, 023172 (2020)

Kondo lattice  
heavy Fermi liquid.  
Size  $1 + p + 1$   
 $= p \pmod{2}$ .  
*Small* Fermi surface!

Spin liquid



Large  
Fermi surface.  
Size:  $1 + p$

Trivial  
insulator

FL\*

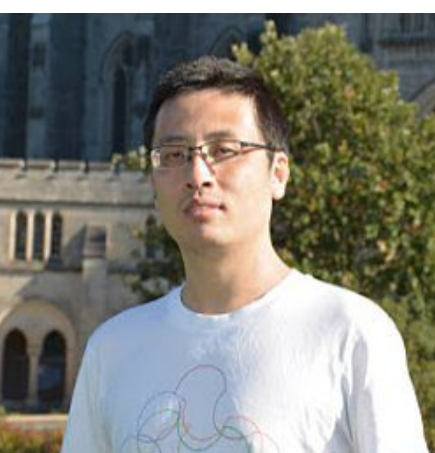
FL

$J_K$

doping  $p$

Pseudogap metal =  
Kondo Lattice Heavy  
Fermi Liquid  
 $\oplus$   
Spin Liquid

Ya-Hui  
Zhang

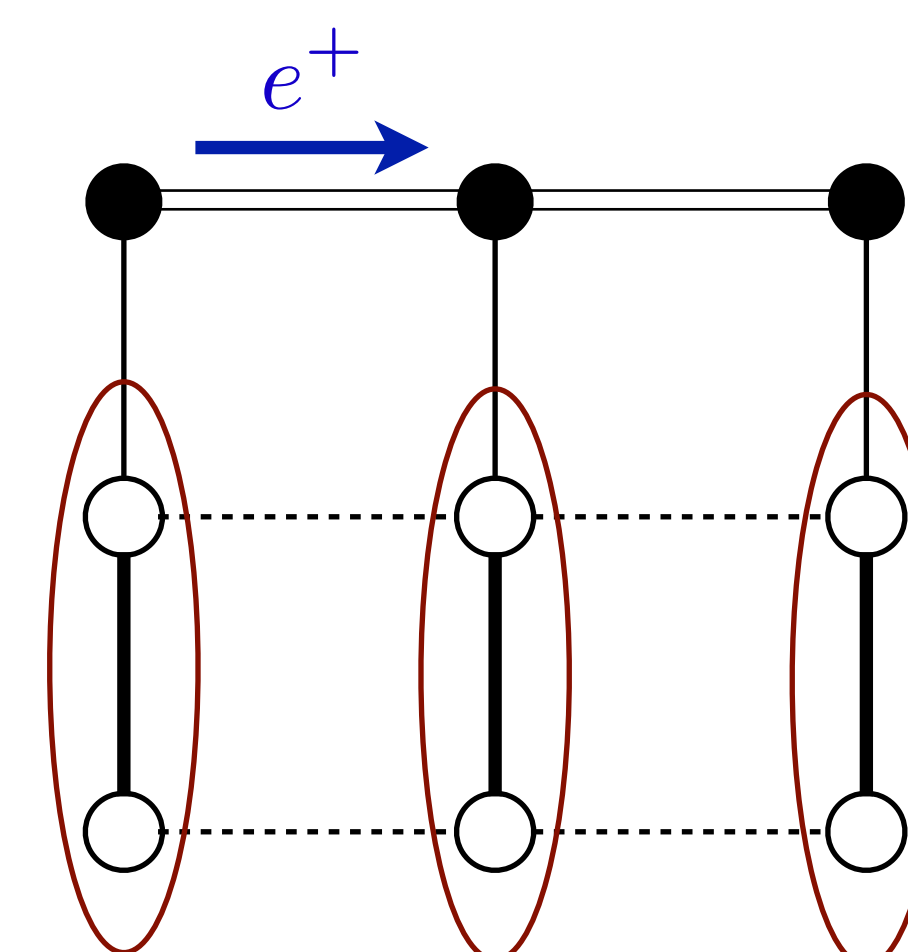
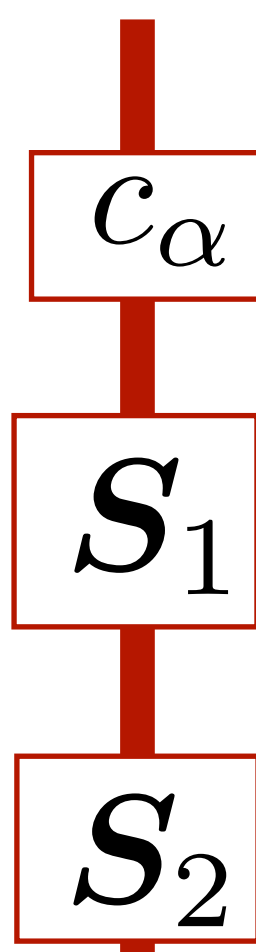
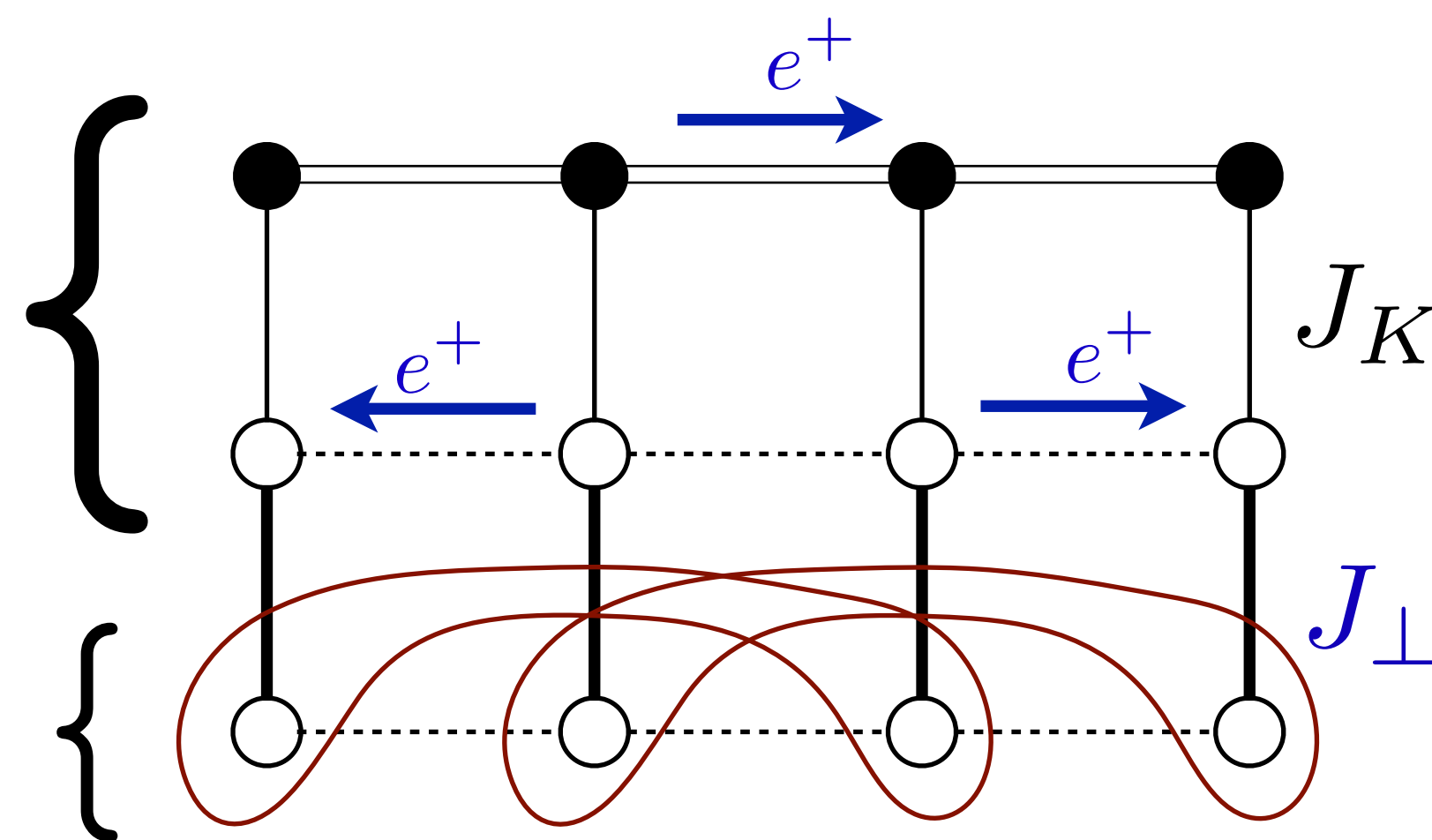


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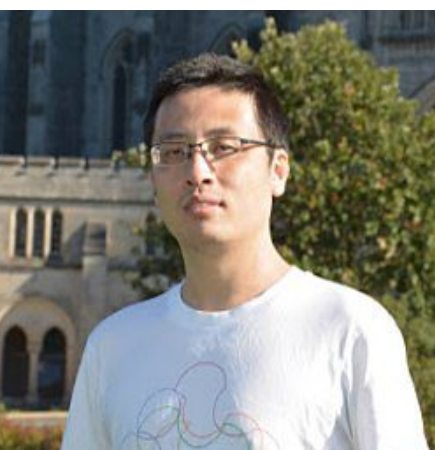
FL

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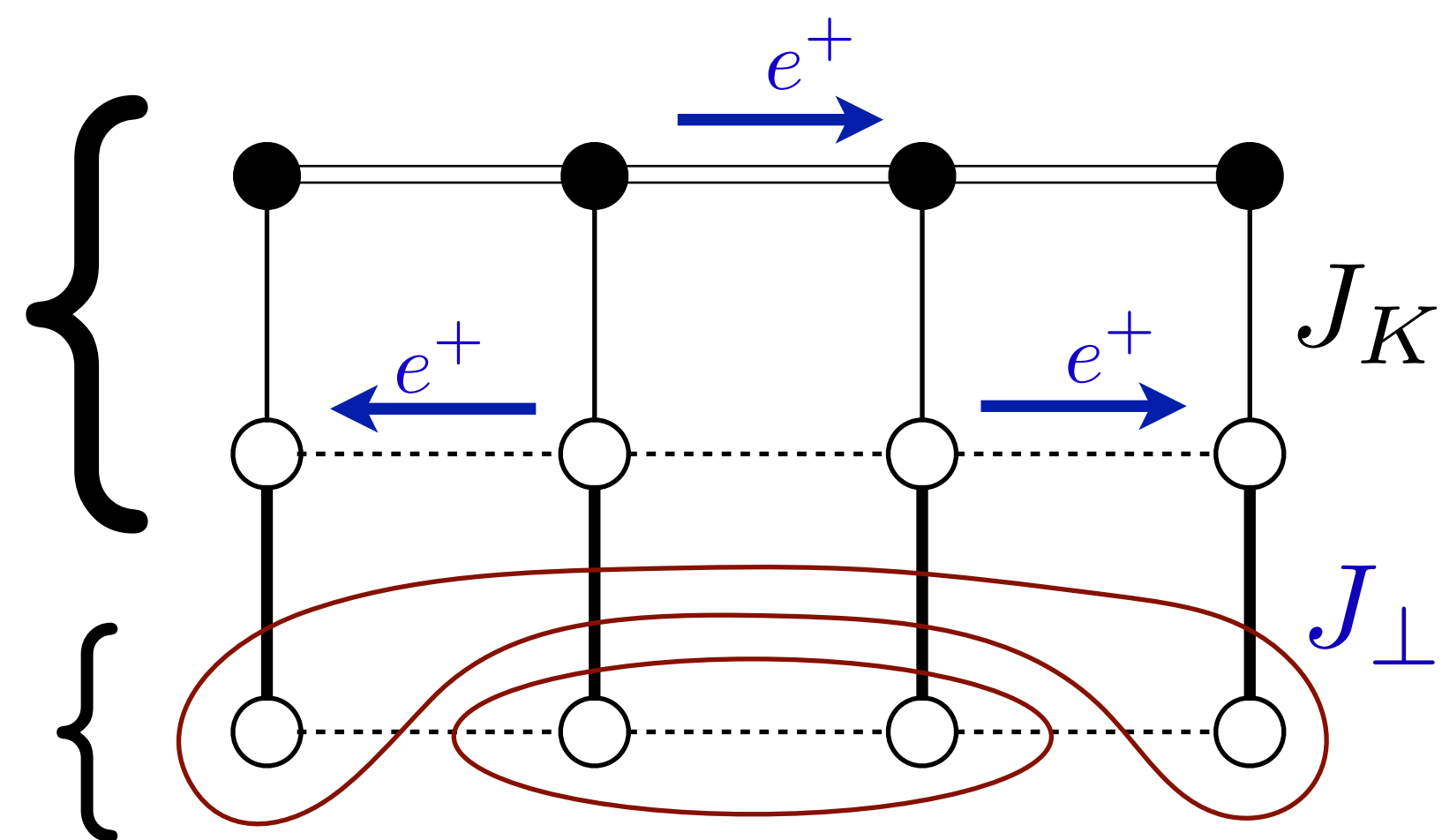


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Ya-Hui Zhang and S. S.,  
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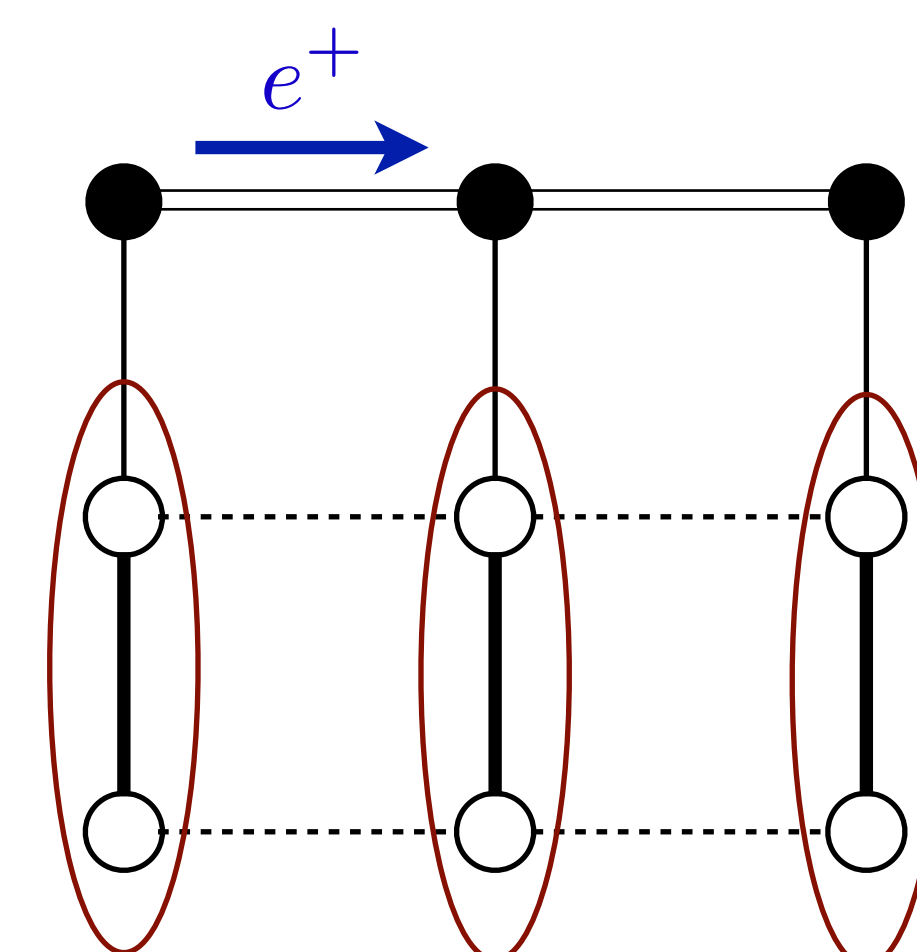


FL\*

$c_\alpha$

$S_1$

$S_2$



FL

Large  
Fermi surface.  
Size:  $1 + p$

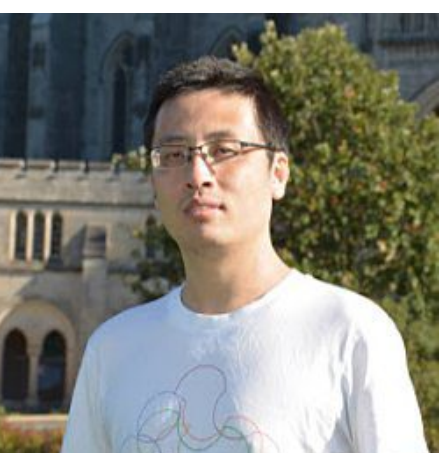
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Ya-Hui  
Zhang



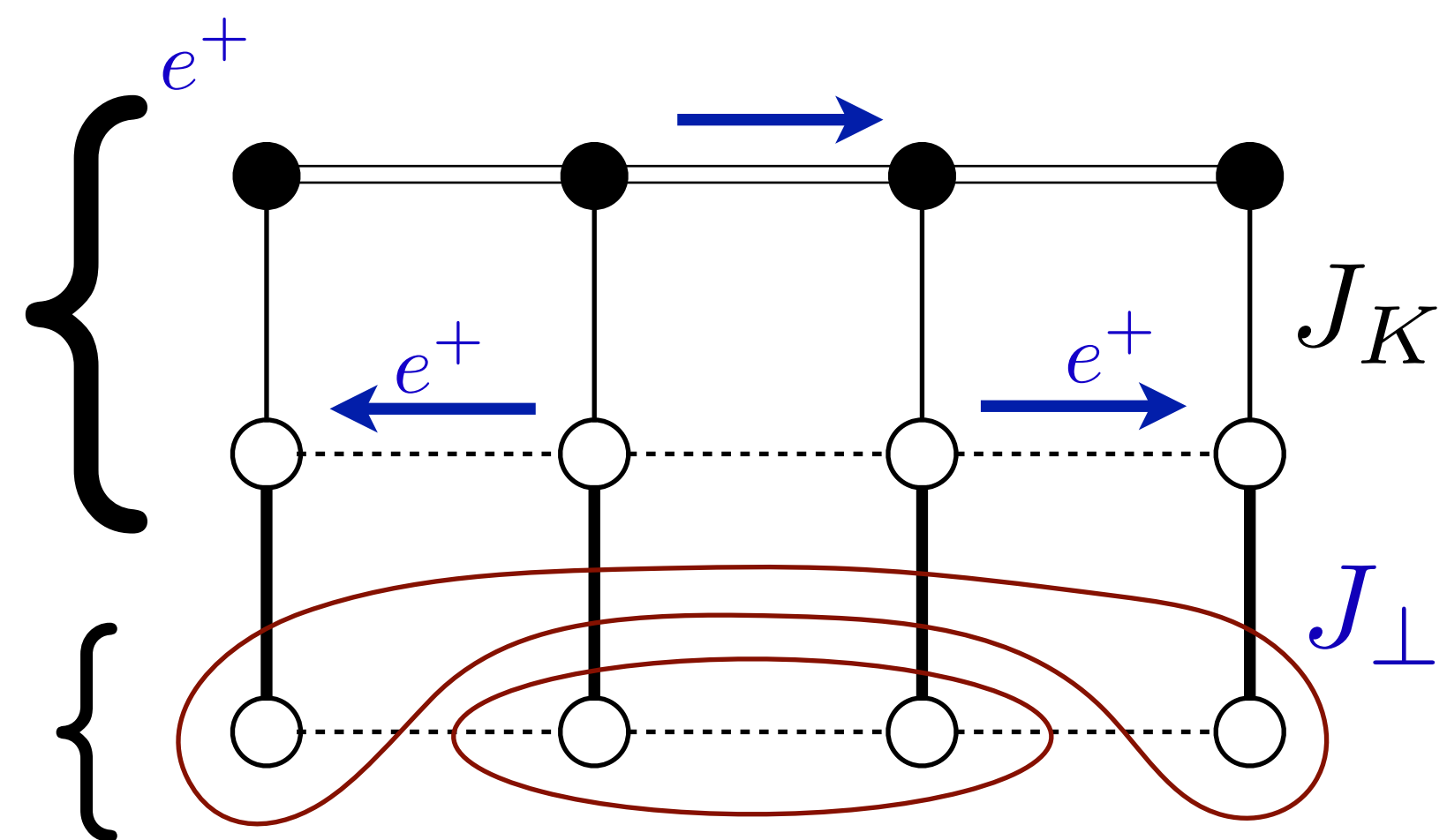
1. The  $\pi$ -flux spin liquid
2. Doping the  $\pi$ -flux spin liquid: FL\*
3. Confinement transitions of  $\pi$ -flux-FL\*

# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,  
PRR **2**, 023172 (2020)

Kondo lattice  
heavy Fermi liquid.

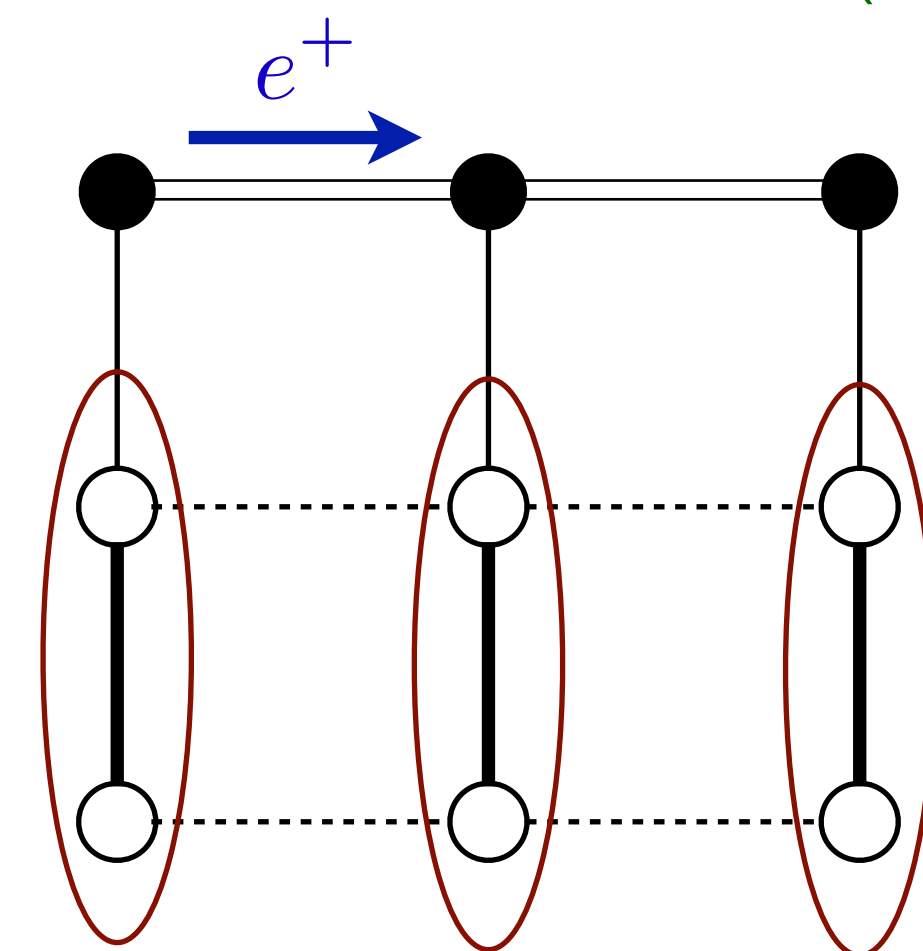
Spin liquid



$c_\alpha$

$S_1$

$S_2$



FL\*

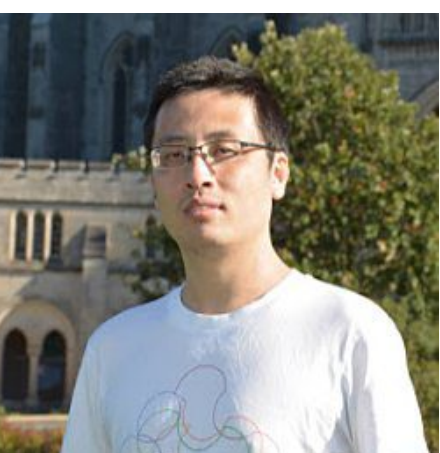
FL

Pseudogap metal =  
Kondo Lattice Heavy  
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$J_K$

Ya-Hui Zhang

.....  $\rightarrow$  doping  $p$

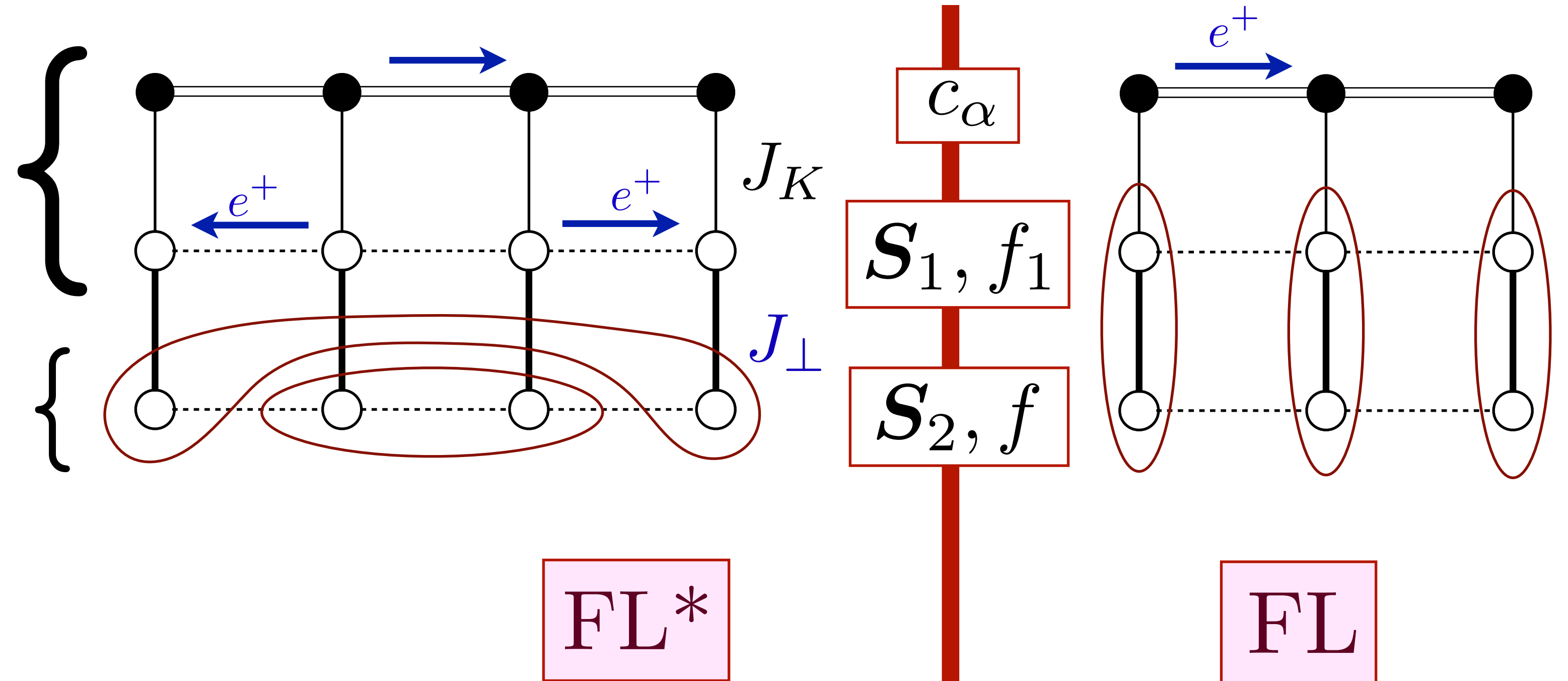


# Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S.,  
PRR **2**, 023172 (2020)

Higgs field 1  
 $\Phi \sim c_\alpha^\dagger f_{1\alpha}$   
 $\langle \Phi \rangle \neq 0$

$\pi$ -flux spin liquid  
of  $f_\alpha$  with  $SU(2)_N$   
gauge field



$J_K$   
Ya-Hui Zhang

.....  $\rightarrow$  doping  $p$

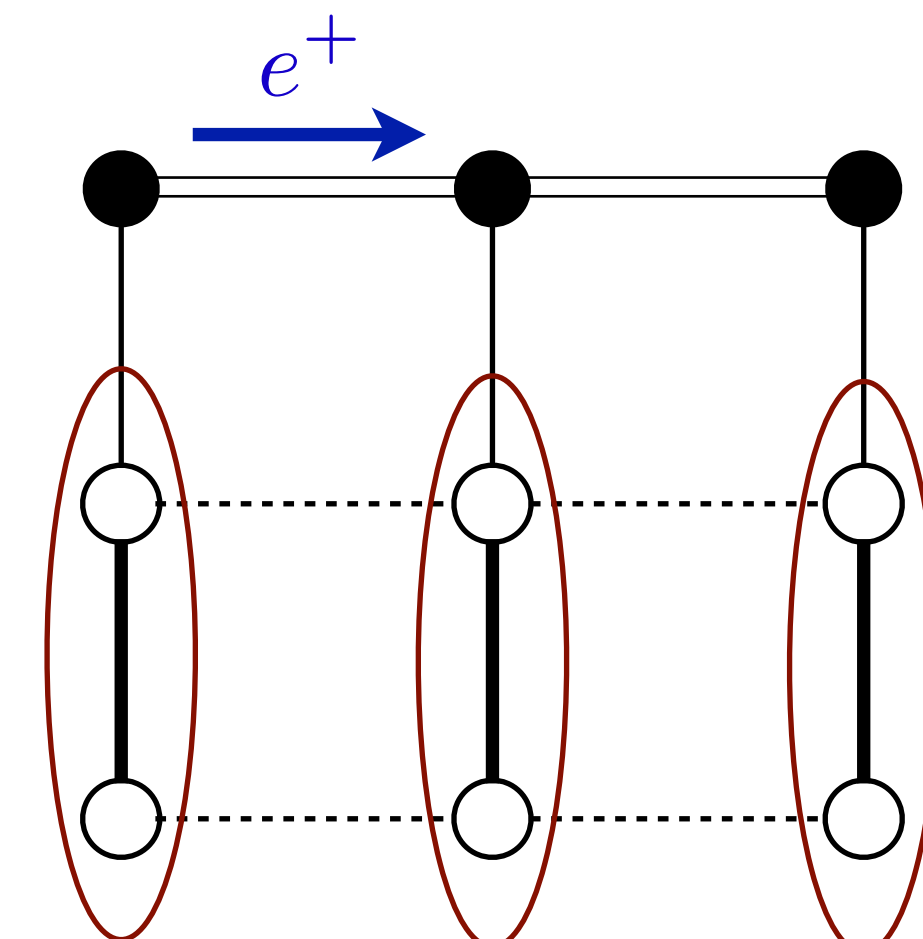
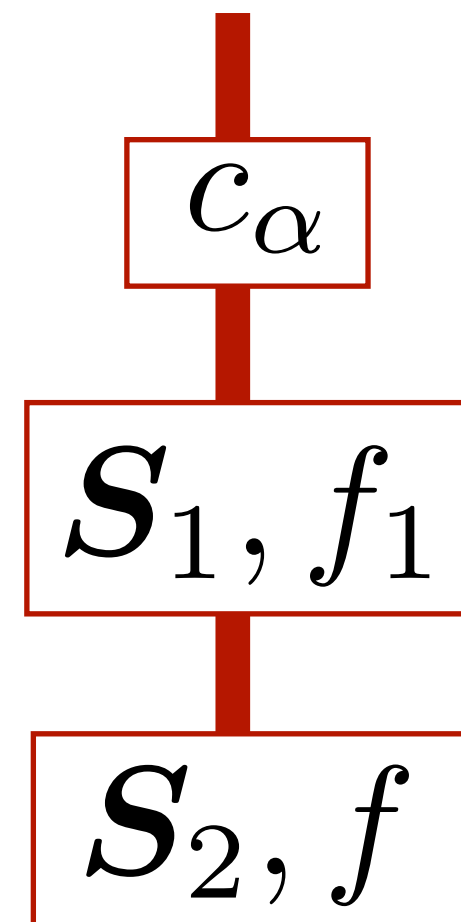
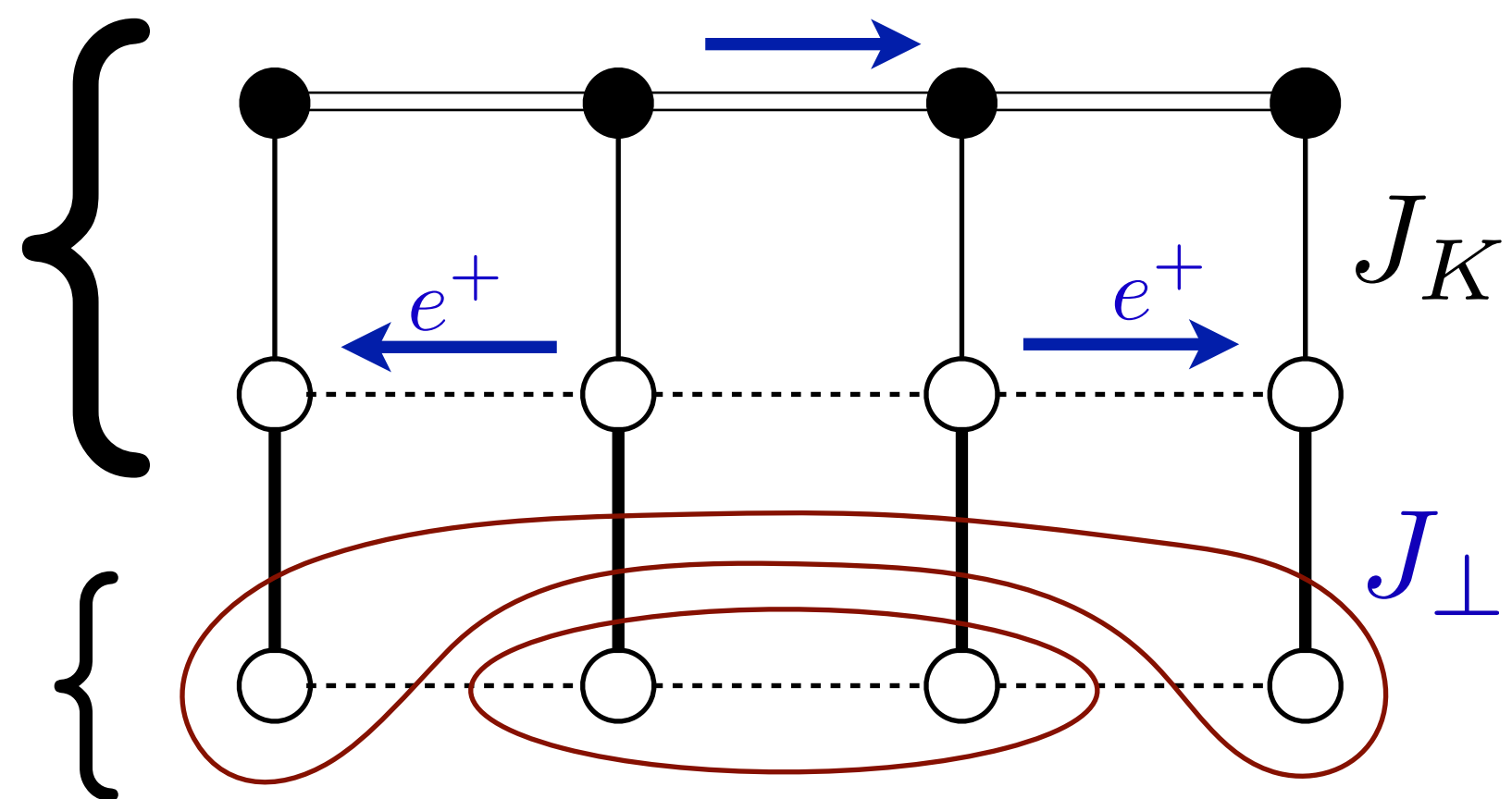
# Ancilla theory of the Hubbard model

Higgs field 1

$$\Phi \sim c_\alpha^\dagger f_{1\alpha}$$

$$\langle \Phi \rangle \neq 0$$

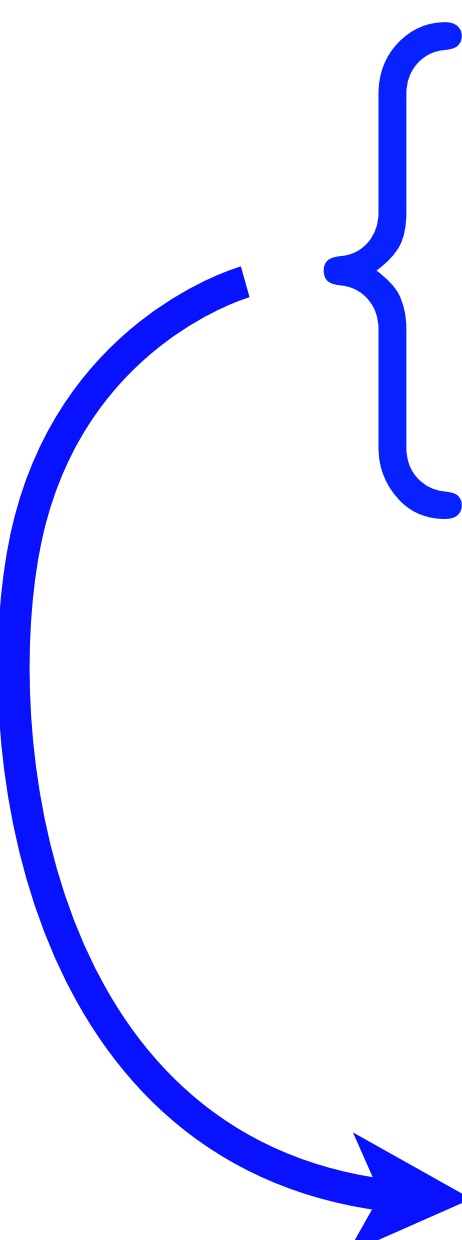
$\pi$ -flux spin liquid  
of  $f_\alpha$  with  $SU(2)_N$   
gauge field



FL\*

FL

Higgs field 2  
Charge  $e$ ,  $SU(2)_N$  fundamental

$$B \sim \begin{pmatrix} f_{1\alpha}^\dagger f_\alpha \\ \varepsilon_{\alpha\beta} f_{1\alpha}^\dagger f_\beta^\dagger \end{pmatrix}$$


$J_K$

doping  $p$



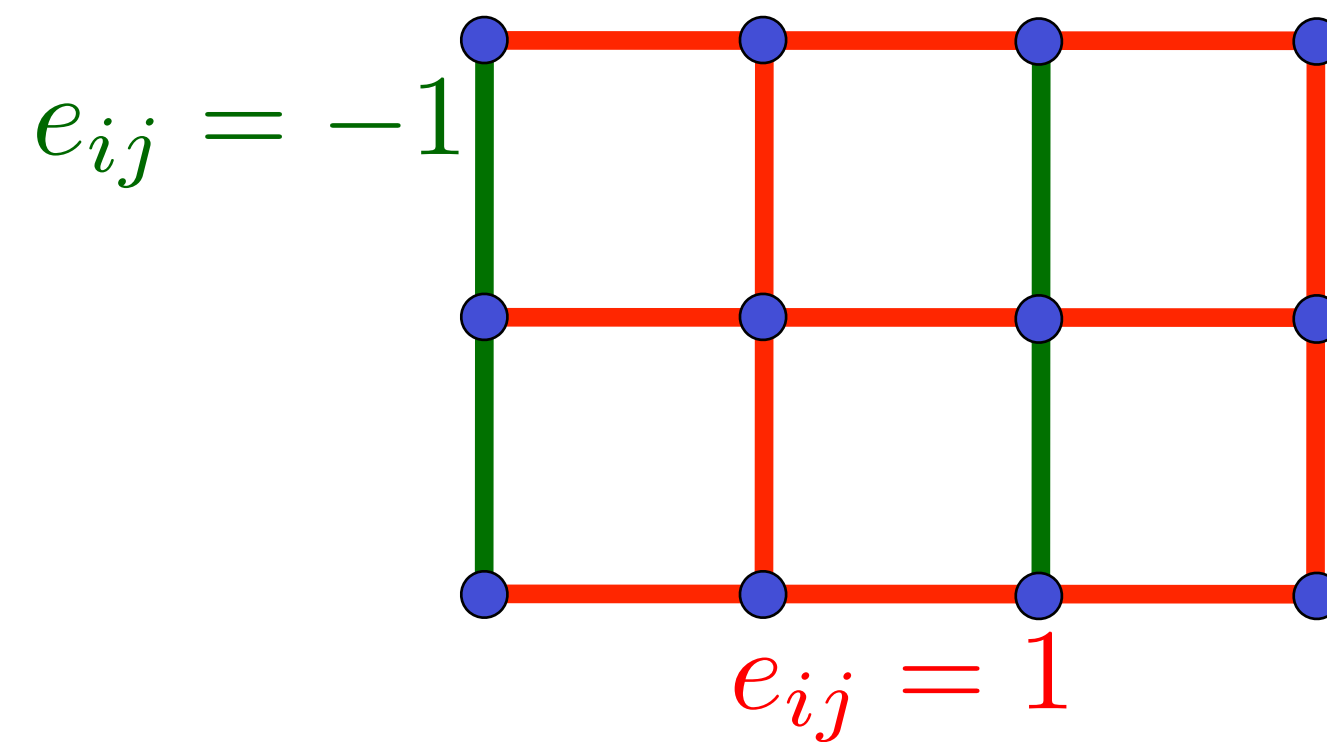
Ya-Hui Zhang

Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

# Confinement of $SU(2)_N$ gauge theory by charge fluctuations

- Begin with the  $\pi$ -flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left( f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right)$$

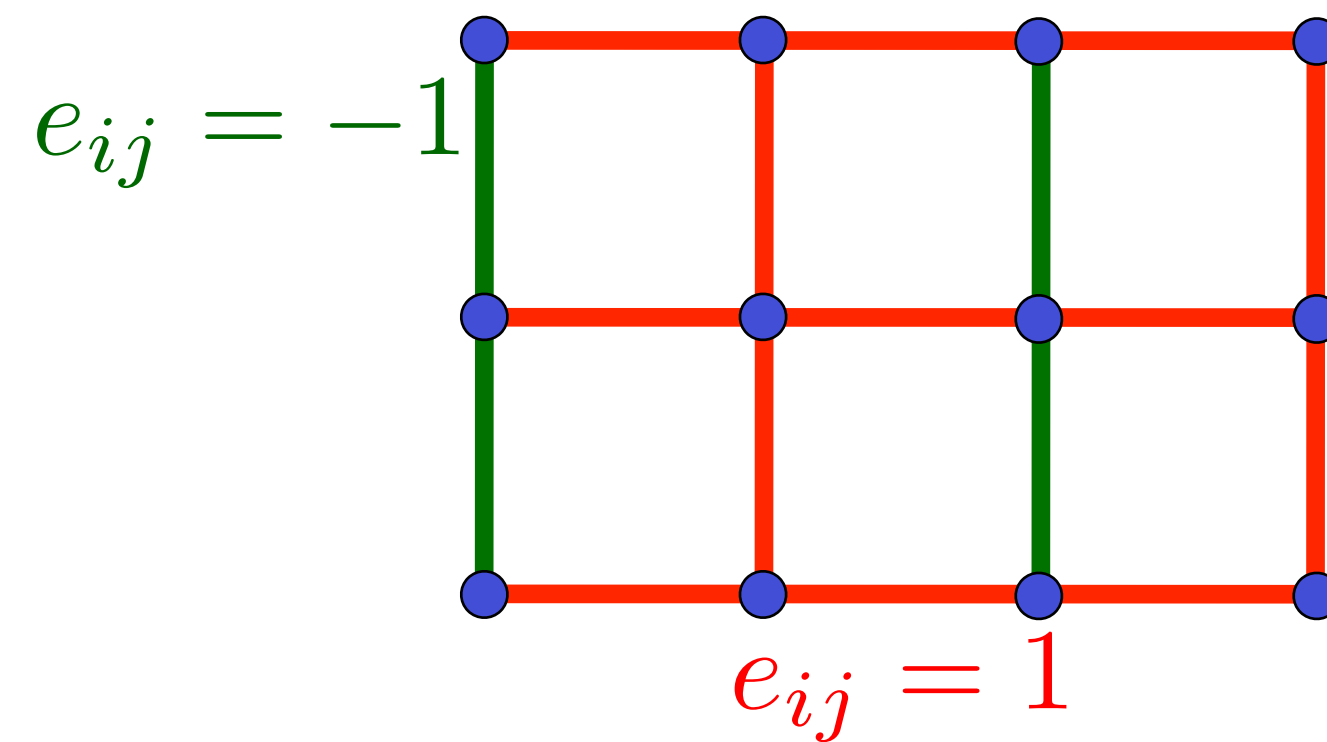


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$H_f$  is invariant under  $SU(2)$  rotations in spin and  $SU(2)_N$  rotations in Nambu space;  $U_{ij}$  is the  $SU(2)_N$  gauge field.

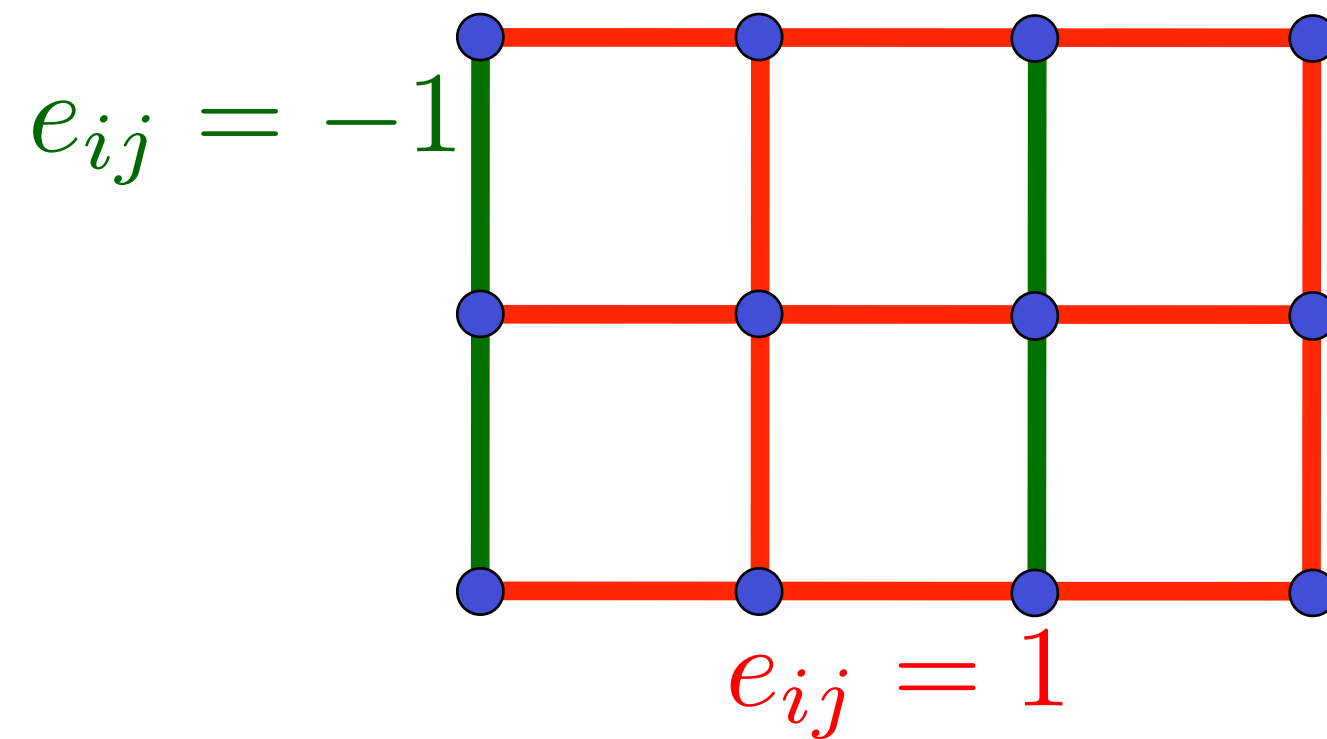


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- The nearest-neighbor effective Hamiltonian for charge  $e$ ,  $SU(2)_N$  fundamental boson  $B_i$  is constrained by the fact that the composite of  $B_i$  and  $\Psi_i$  is an electron:

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left( B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \dots$$

# Confinement of $SU(2)_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2} \sum_i \rho_i^2 + V_1 \sum_i \rho_i (\rho_{i+\hat{x}} + \rho_{i+\hat{y}}) + g \sum_{\langle ij \rangle} |\Delta_{ij}|^2$$

$$+ J_1 \sum_{\langle ij \rangle} Q_{ij}^2 + K_1 \sum_{\langle ij \rangle} J_{ij}^2.$$

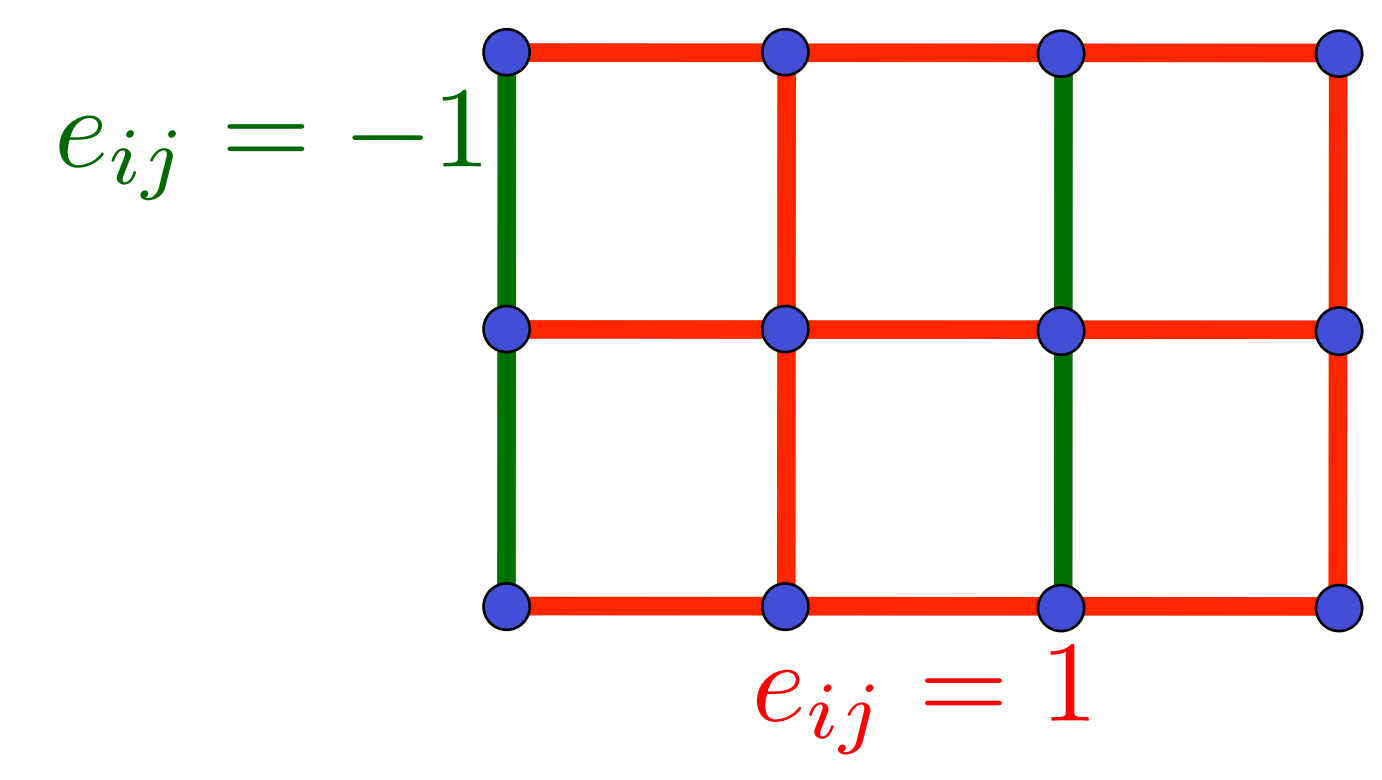
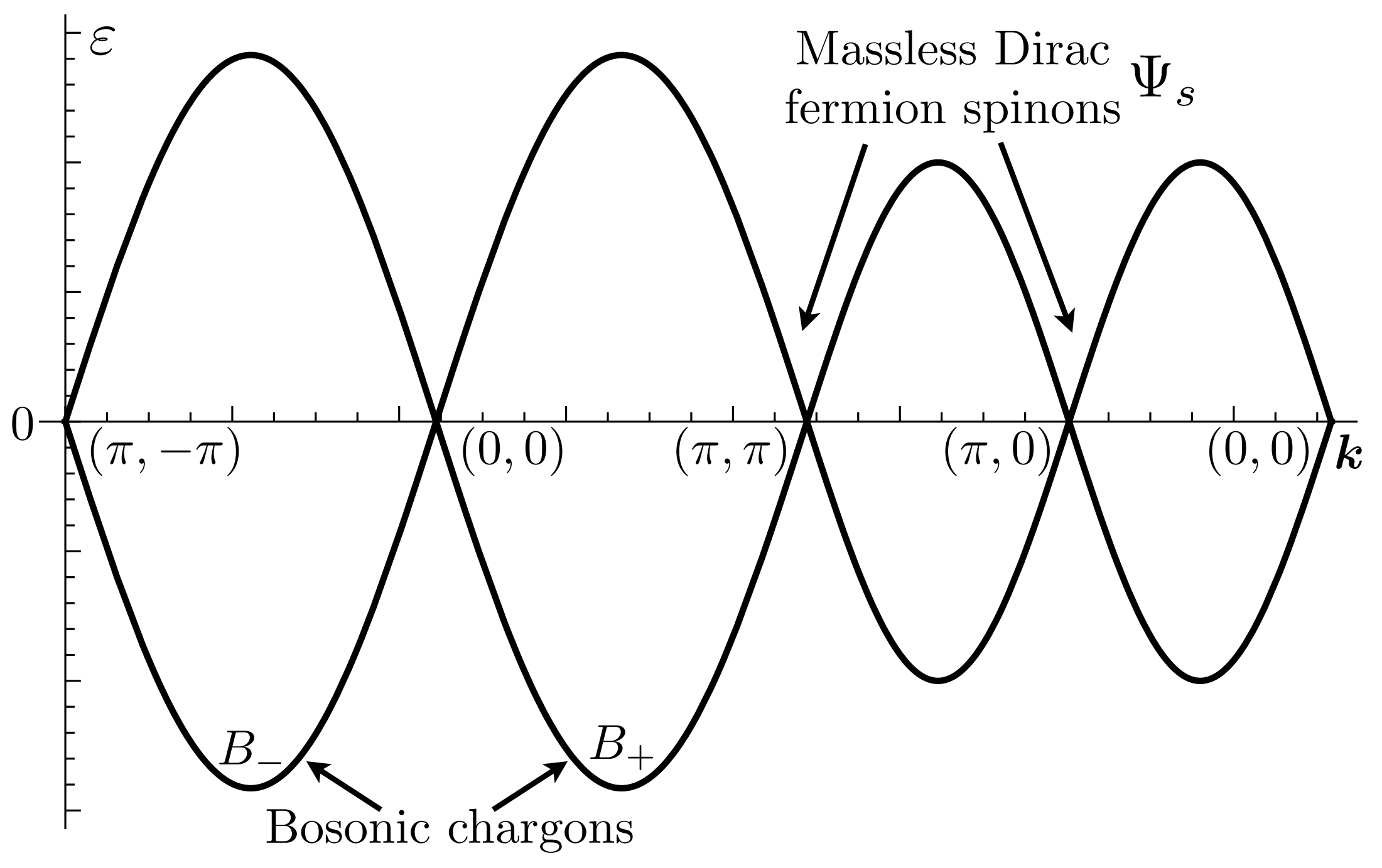
site charge density:  $\langle c_{i\alpha}^\dagger c_{i\alpha} \rangle \sim \rho_i = B_i^\dagger B_i$

bond density:  $\langle c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim Q_{ij} = Q_{ji} = \text{Im} \left( B_i^\dagger e_{ij} U_{ij} B_j \right)$

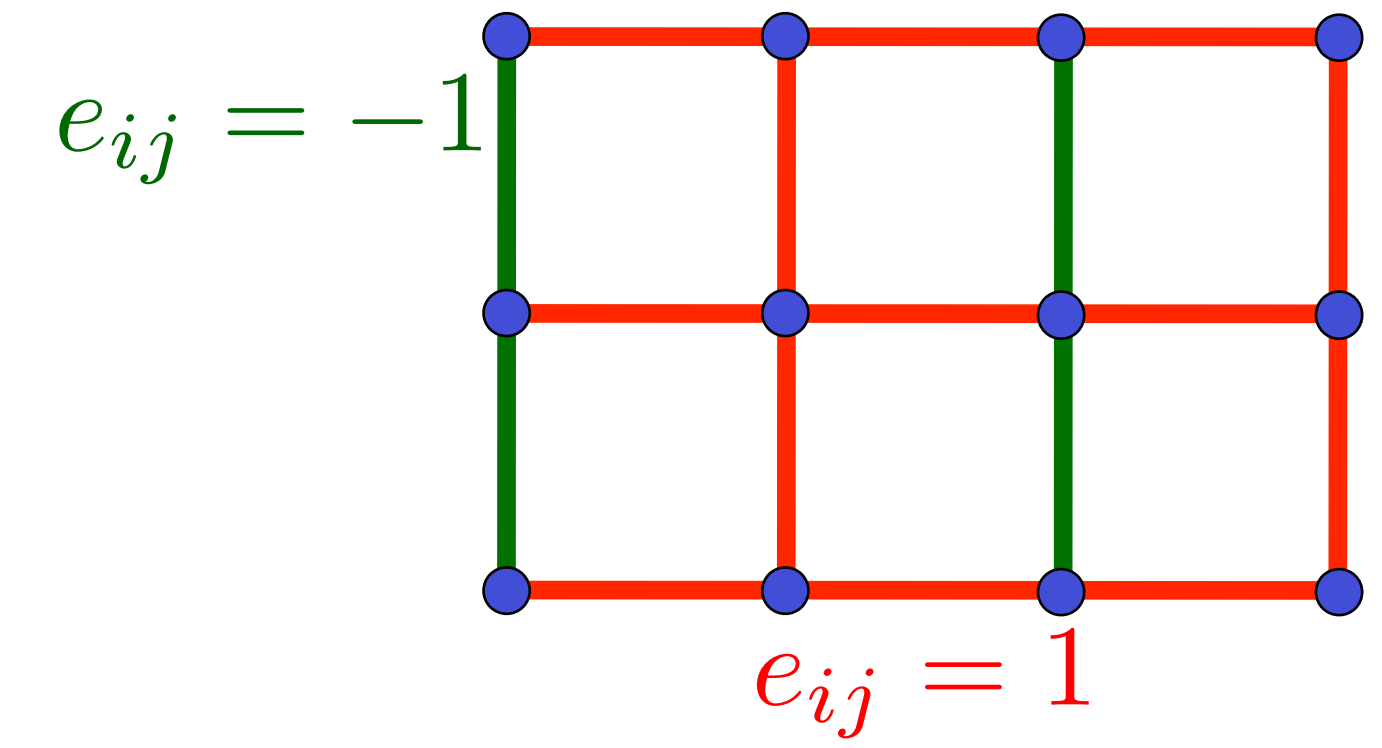
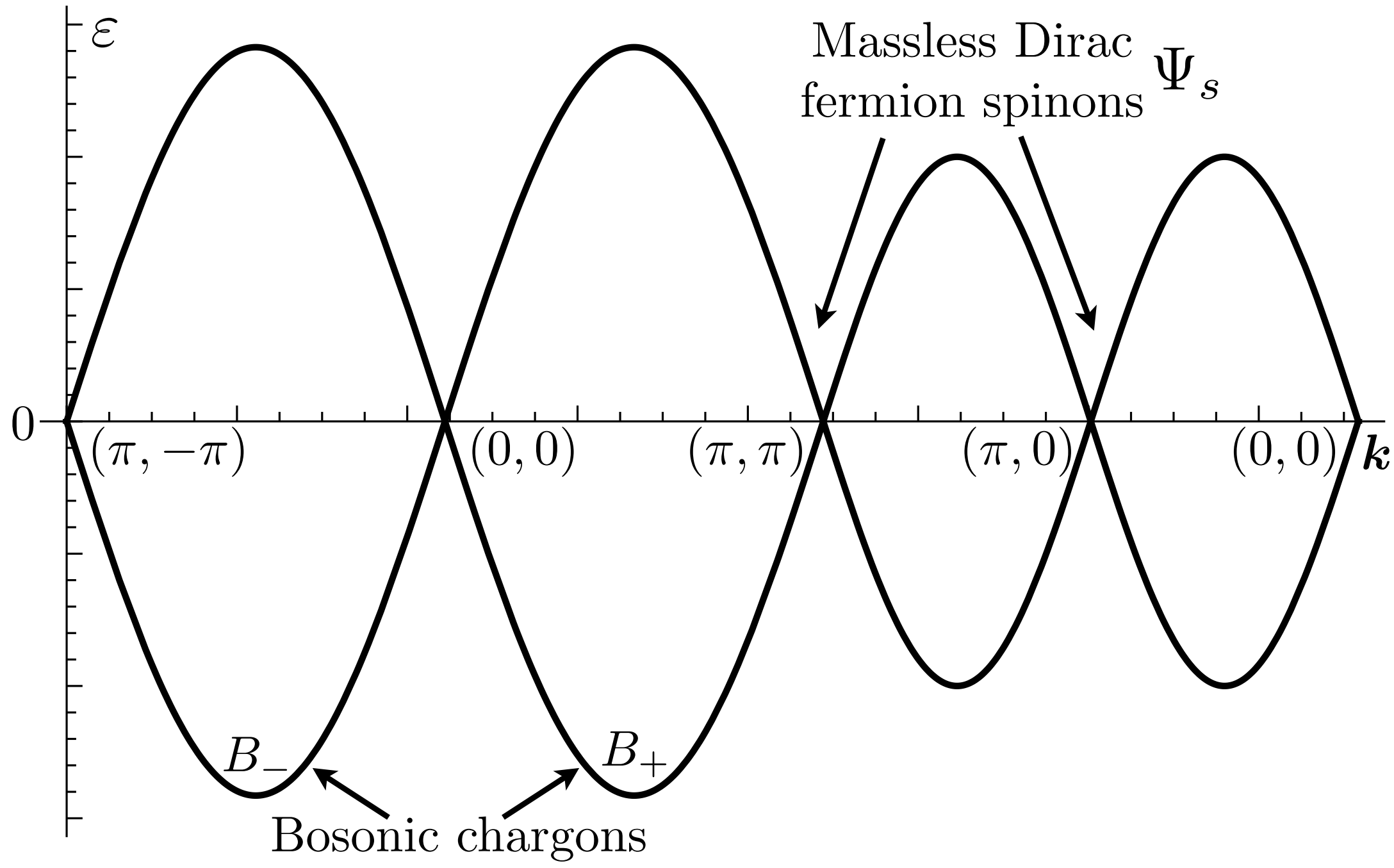
bond current:  $i \langle c_{i\alpha}^\dagger c_{j\alpha} - c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim J_{ij} = -J_{ji} = \text{Re} \left( B_i^\dagger e_{ij} U_{ij} B_j \right)$

Pairing:  $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{ij} = \Delta_{ji} = \varepsilon_{ab} B_{ai} e_{ij} U_{ij} B_{bj}.$

# Confinement of $SU(2)_N$ gauge theory by charge fluctuations



# Confinement of $SU(2)_N$ gauge theory by charge fluctuations



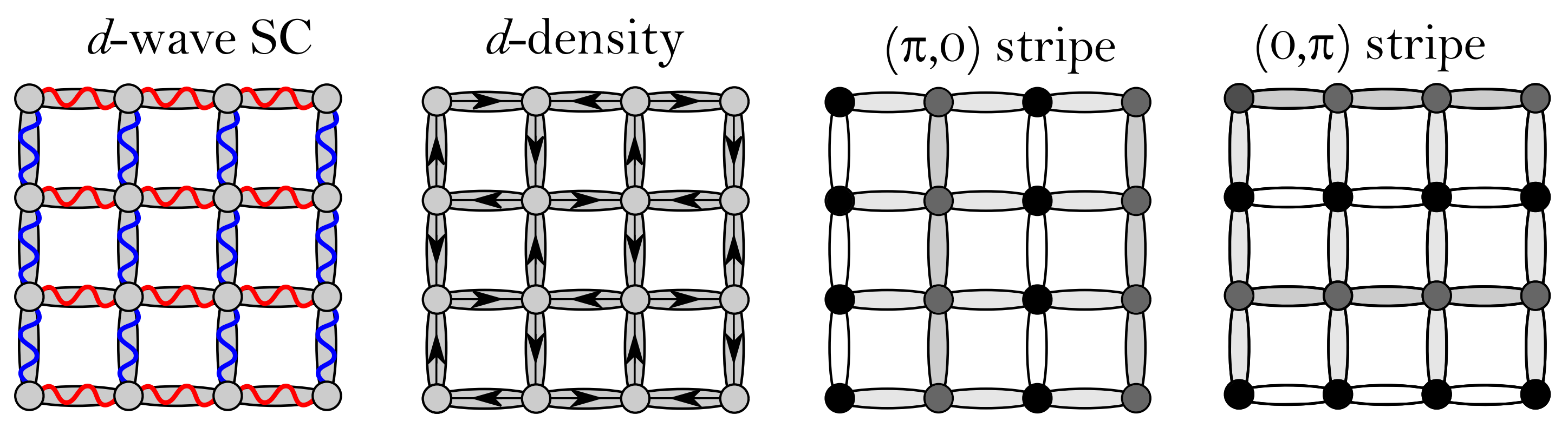
$SU(2)_N$  gauge-invariant and  $SU(2)$  spin invariant order parameters of Higgs phases:

$x$ -CDW :  $\rho_{(\pi,0)} = B_{a+}^* B_{a+} - B_{a-}^* B_{a-}$

$y$ -CDW :  $\rho_{(0,\pi)} = B_{a+}^* B_{a-} + B_{a-}^* B_{a+}$

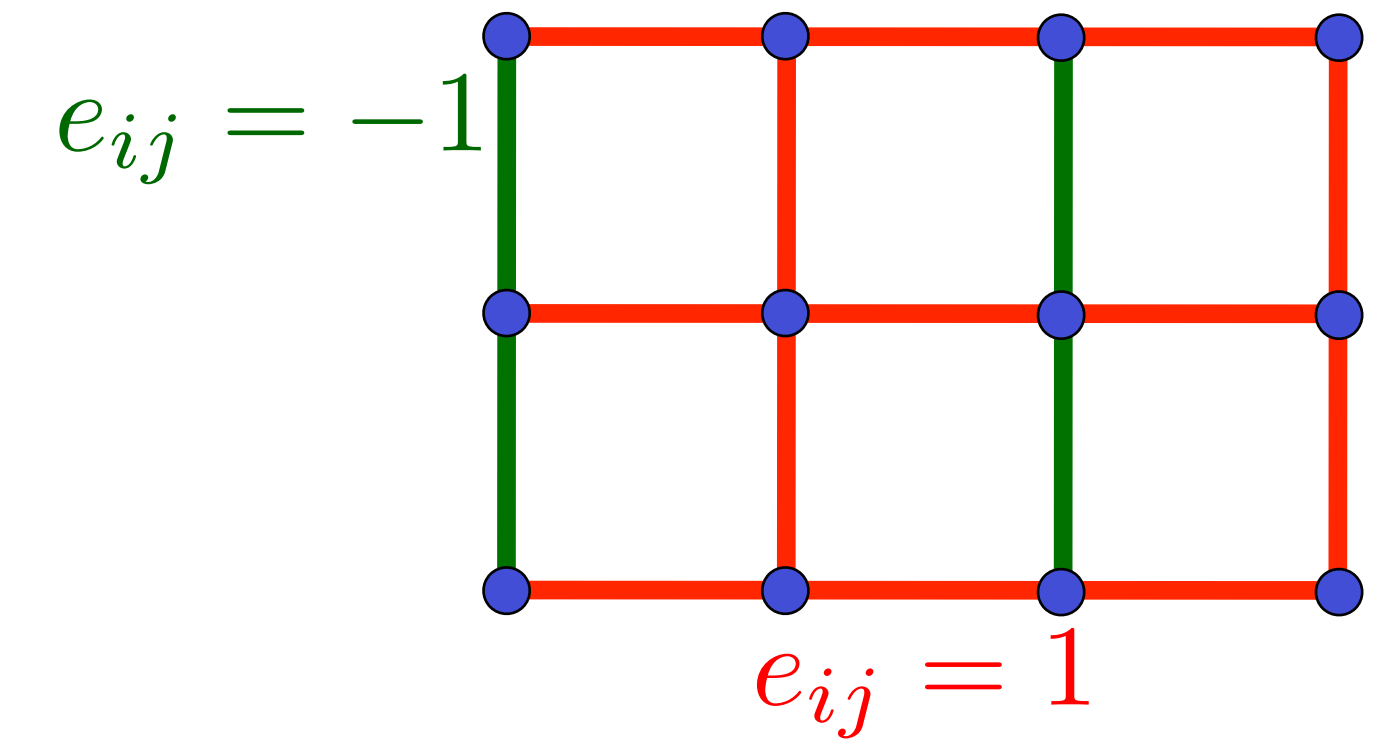
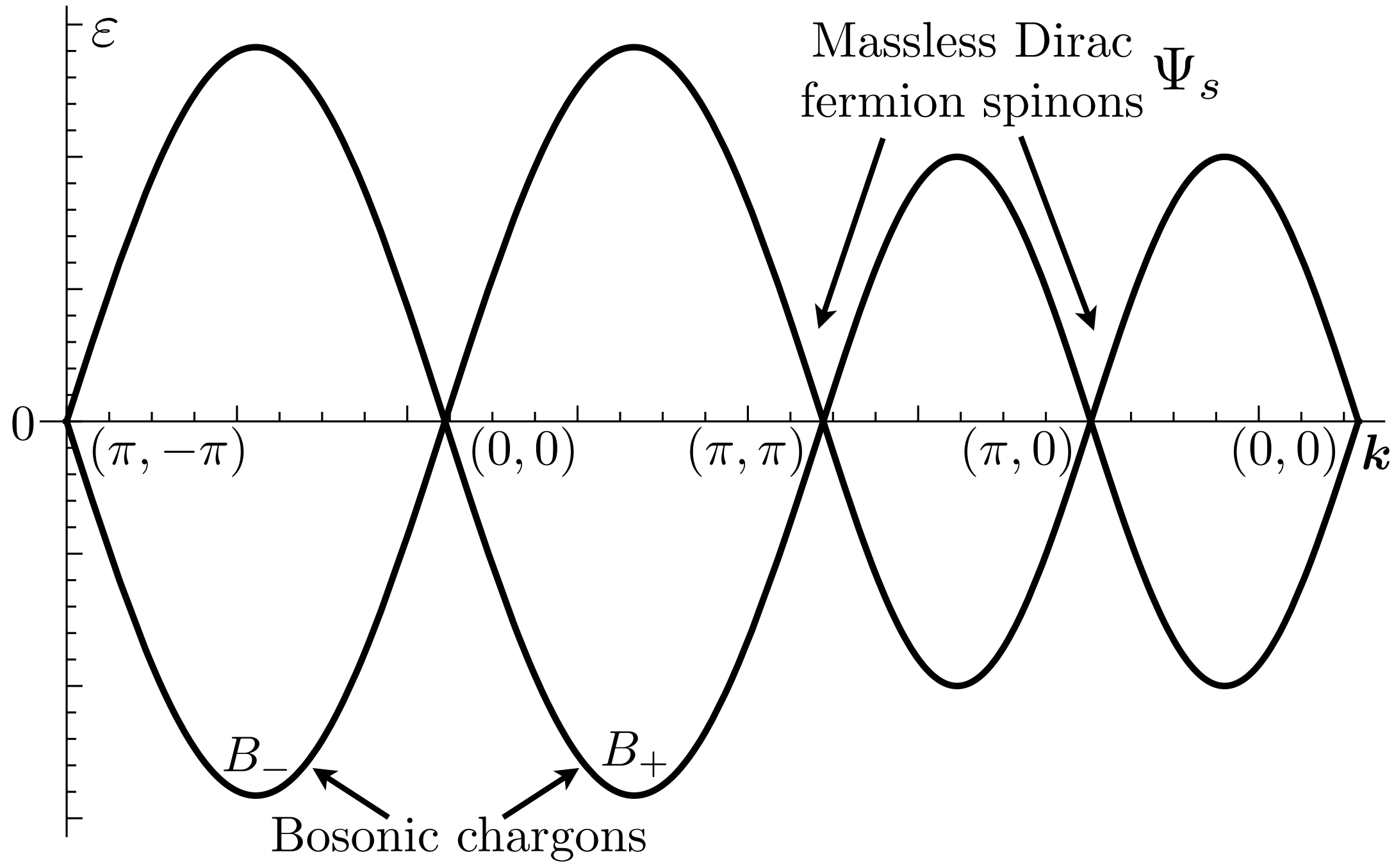
$d$ -density wave :  $D = i (B_{a+}^* B_{a-} - B_{a-}^* B_{a+})$

$d$ -wave superconductor :  $\Delta = \varepsilon_{ab} B_{a+} B_{b-}$

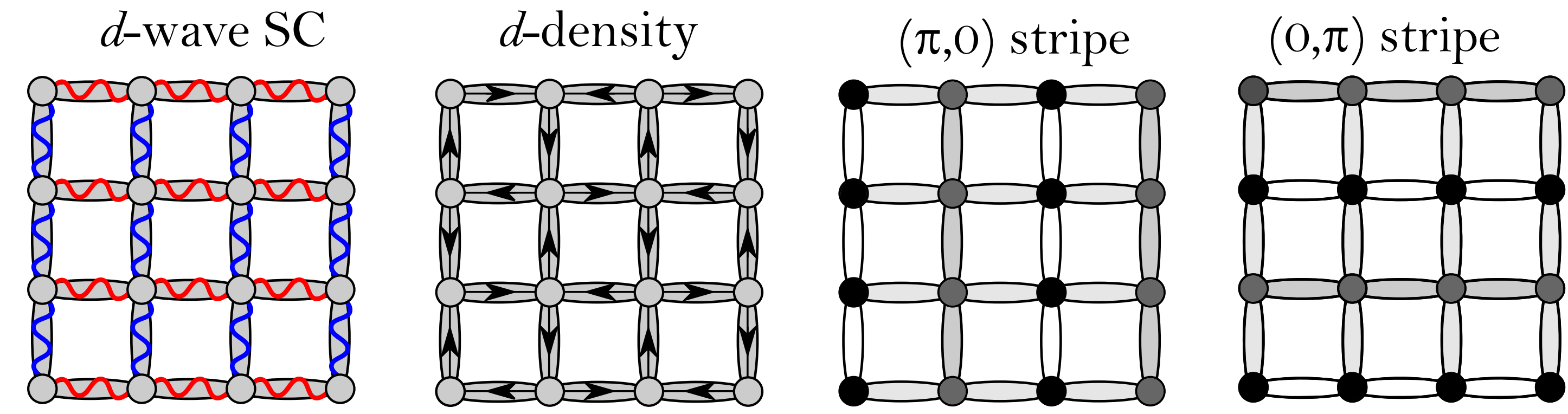


The  $\mathcal{O}(B_{a\pm}^2)$  terms in the energy have a  $SO(5)_b$  rotation symmetry between these orders.

# Confinement of $SU(2)_N$ gauge theory by charge fluctuations

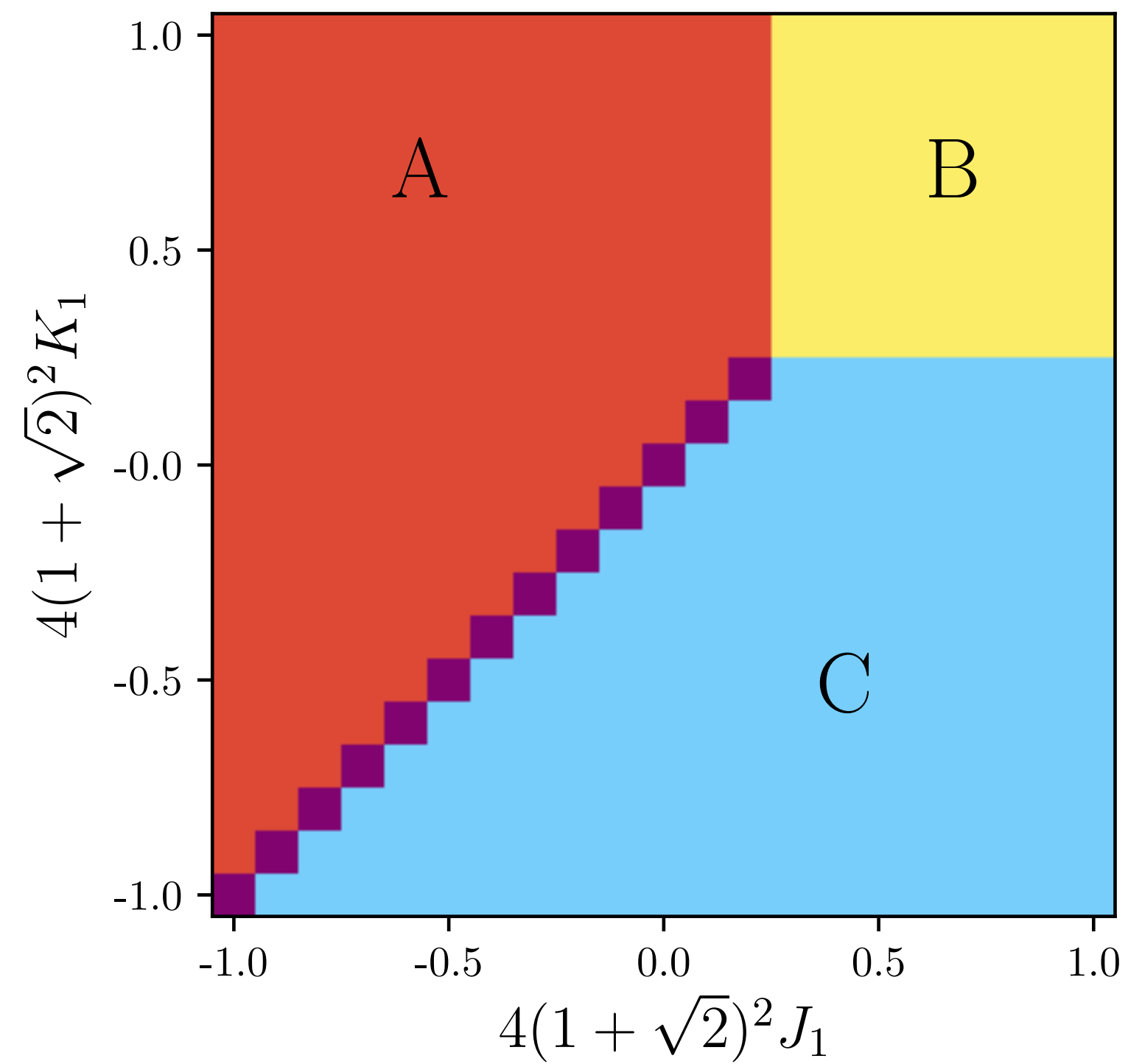


The  $B_{av}$  ( $a \rightarrow SU(2)_N$  gauge,  $v \rightarrow$  valley) are the “square roots” of conventional *d*-wave superconductor, charge density wave, pair density wave  
 ...



# Confinement of $SU(2)_N$ gauge theory by charge fluctuations

$$\langle B \rangle \neq 0$$

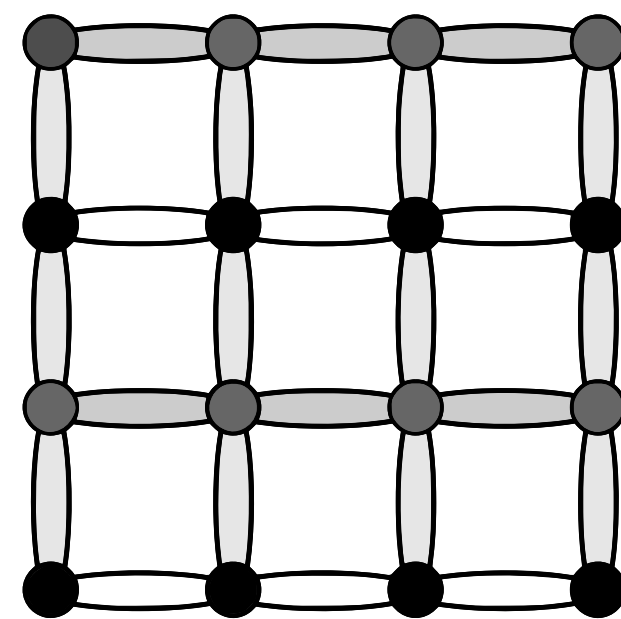
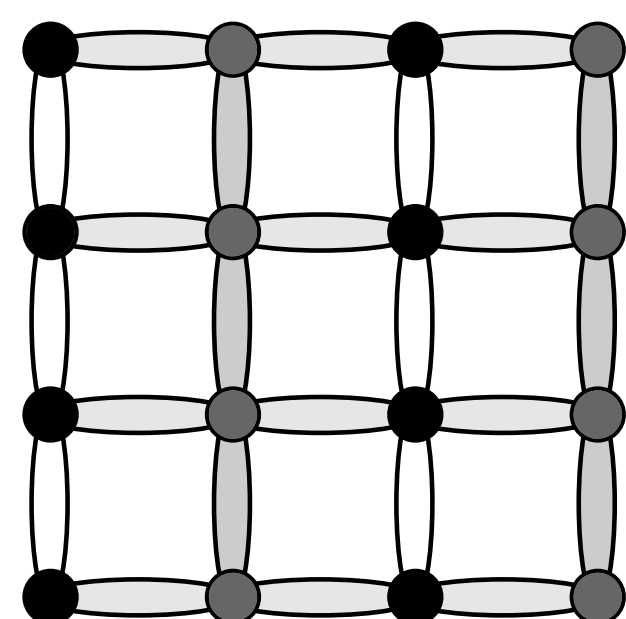
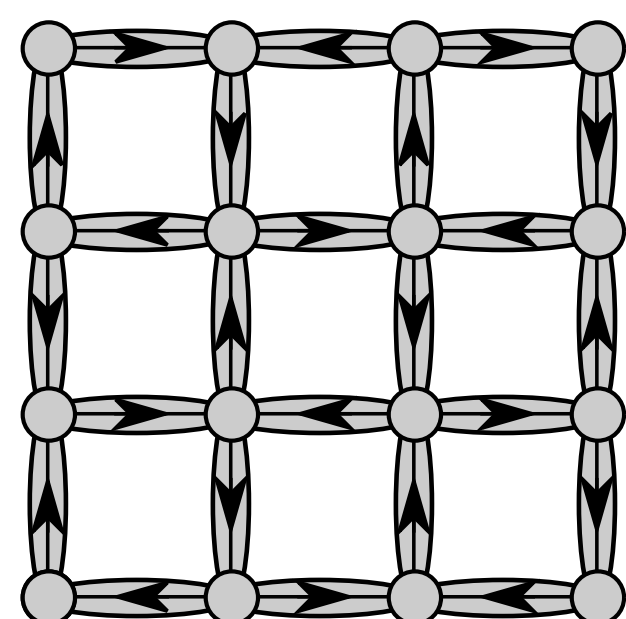
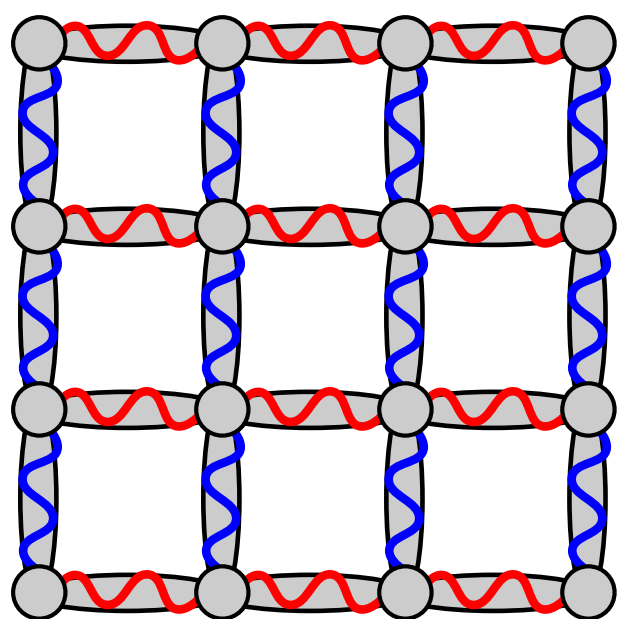


**Phase B**  
*d*-wave SC

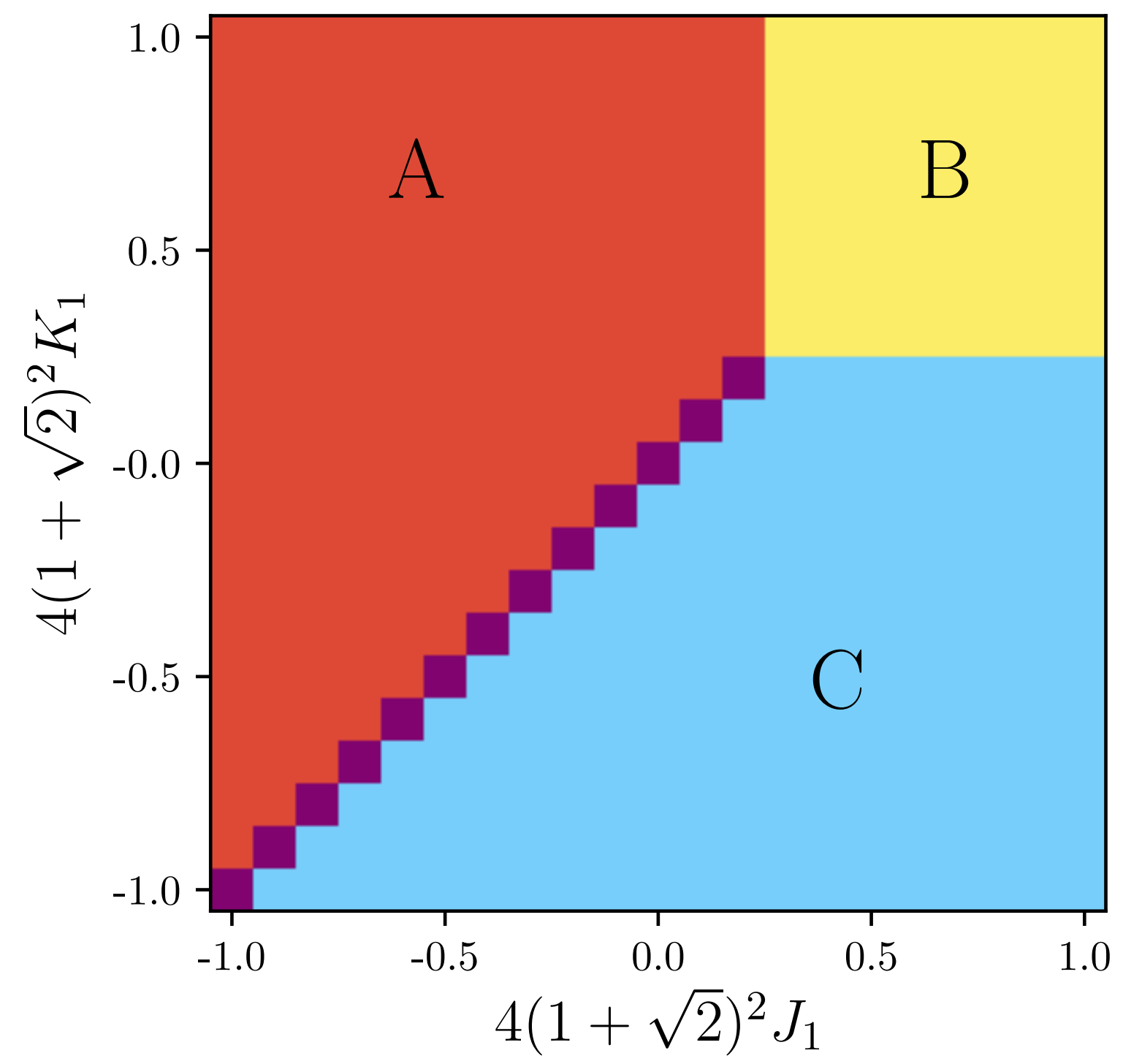
**Phase C**  
*d*-density

**Phase A**  
 $(\pi, 0)$  stripe

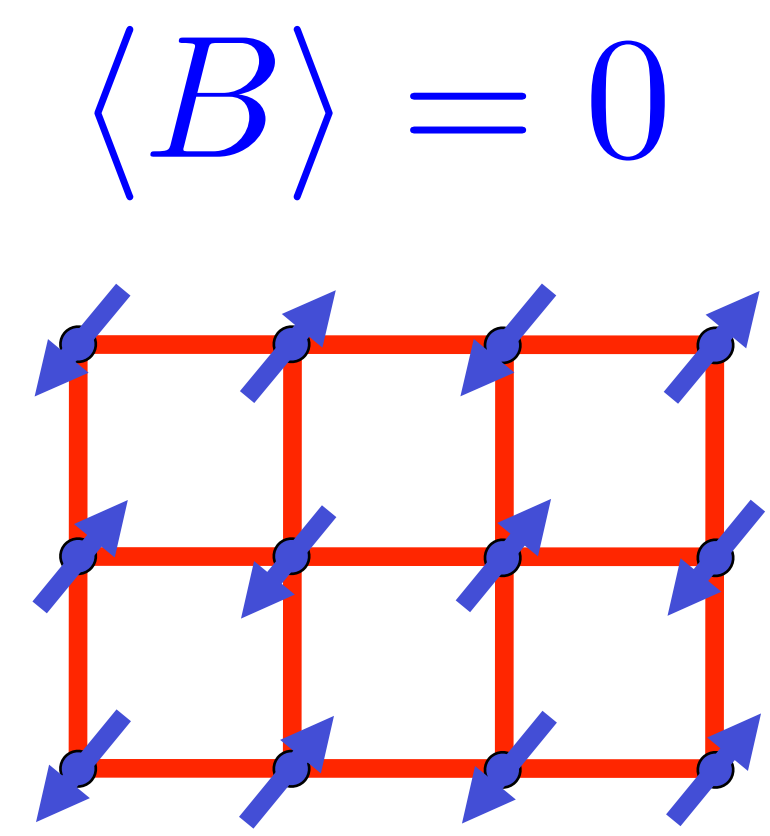
**Phase A**  
 $(0, \pi)$  stripe



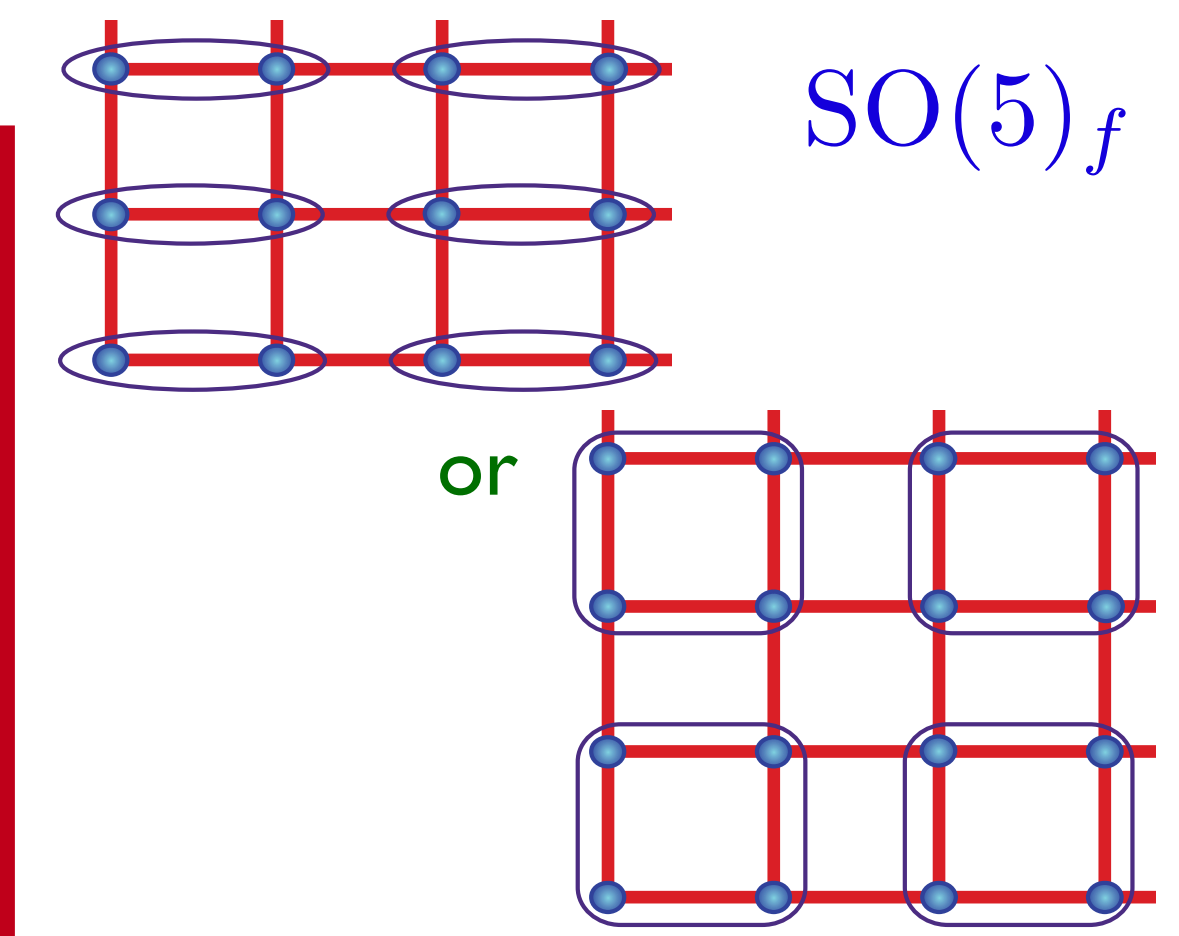
# Global phase diagram of $SU(2)_N$ gauge theory



$\langle B \rangle \neq 0$   
 $SO(5)_b$



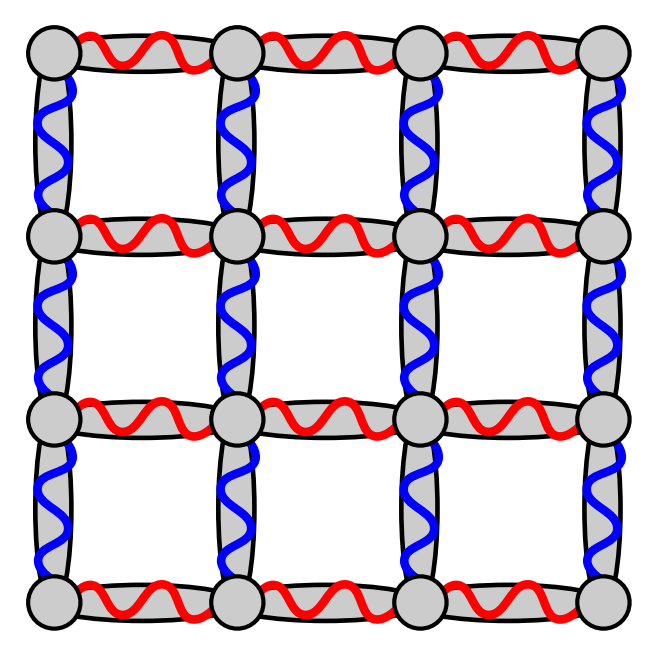
Confining phase:  
 Néel order



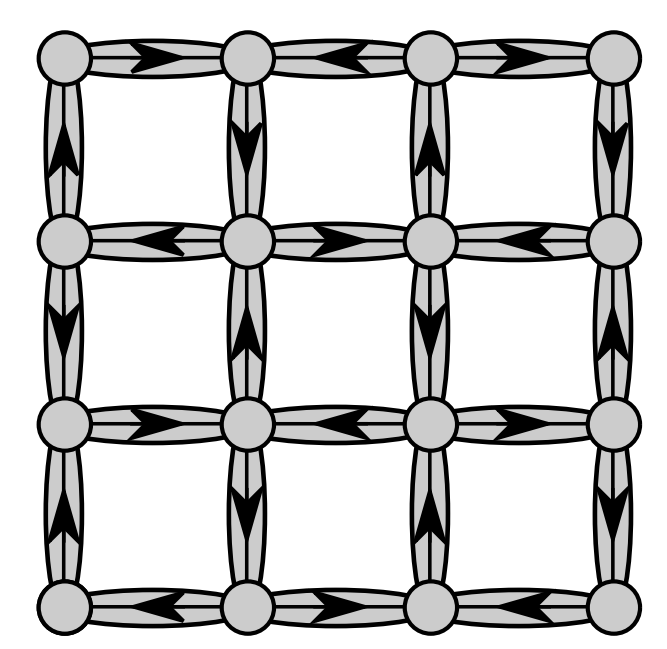
Confining phase:  
 VBS order



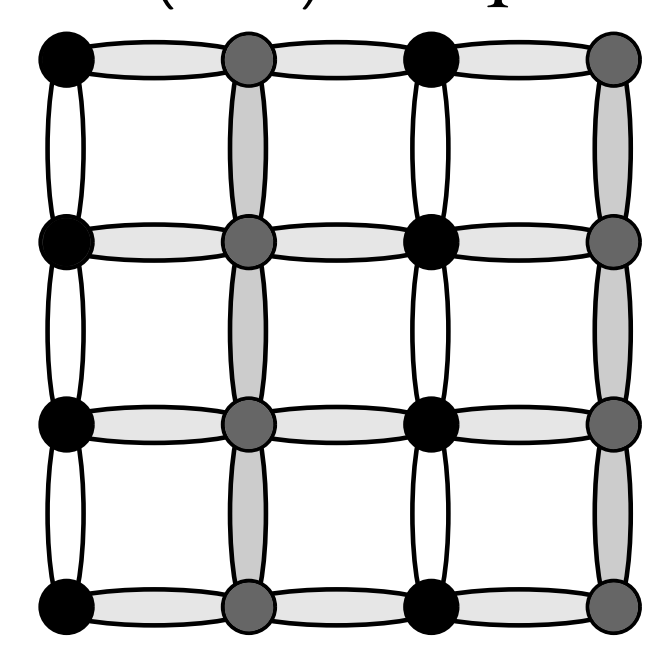
**Phase B**  
*d*-wave SC



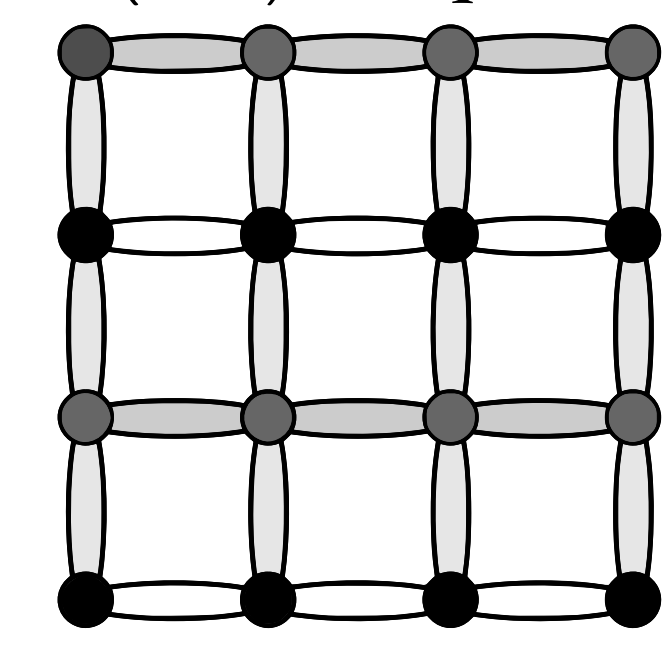
**Phase C**  
*d*-density



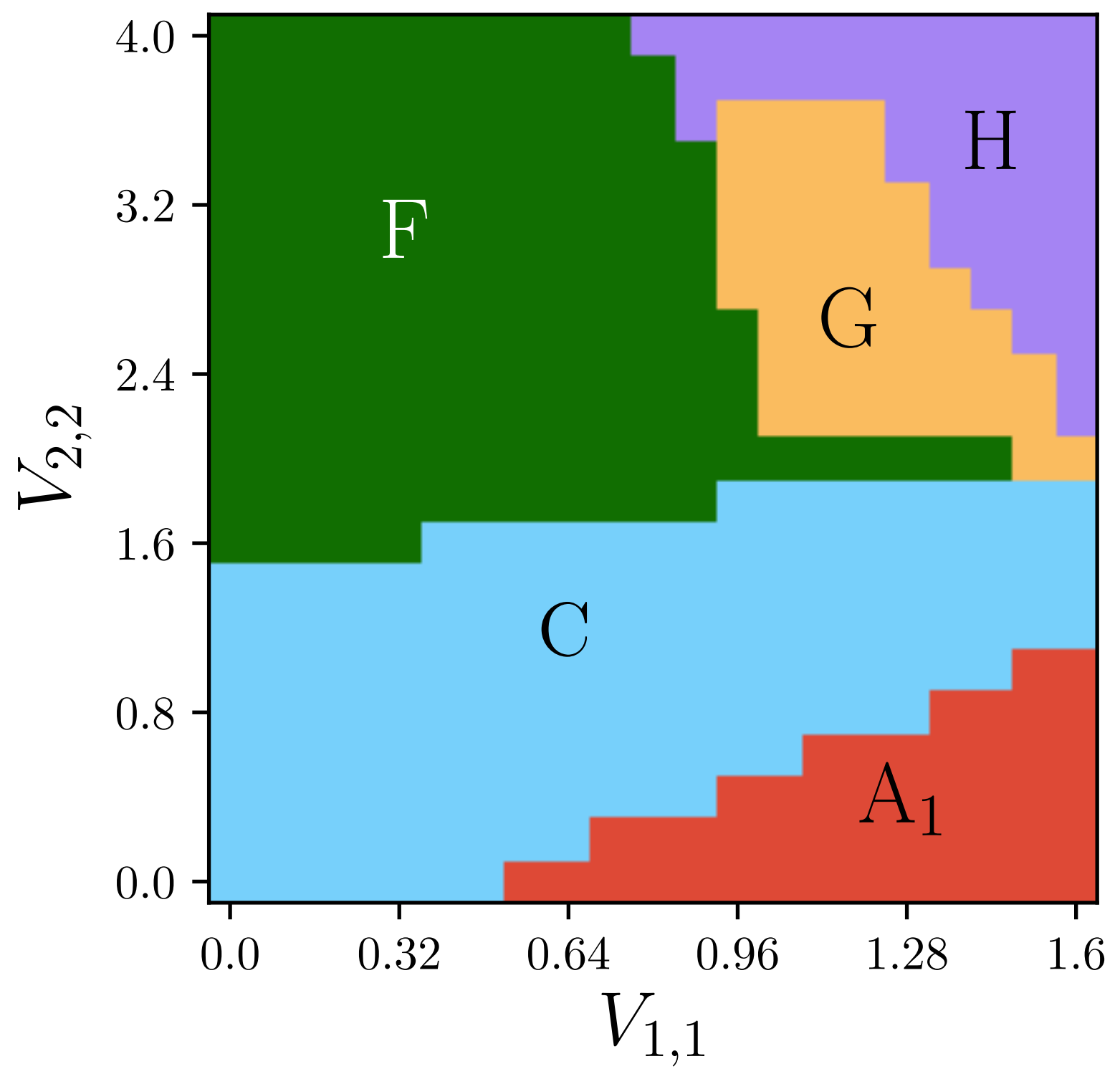
**Phase A**  
 $(\pi, 0)$  stripe



**Phase A**  
 $(0, \pi)$  stripe

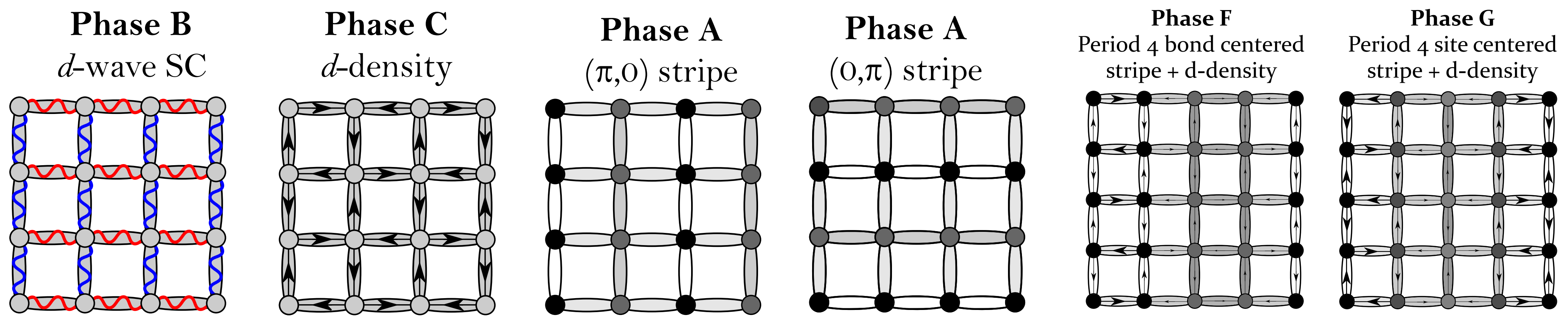
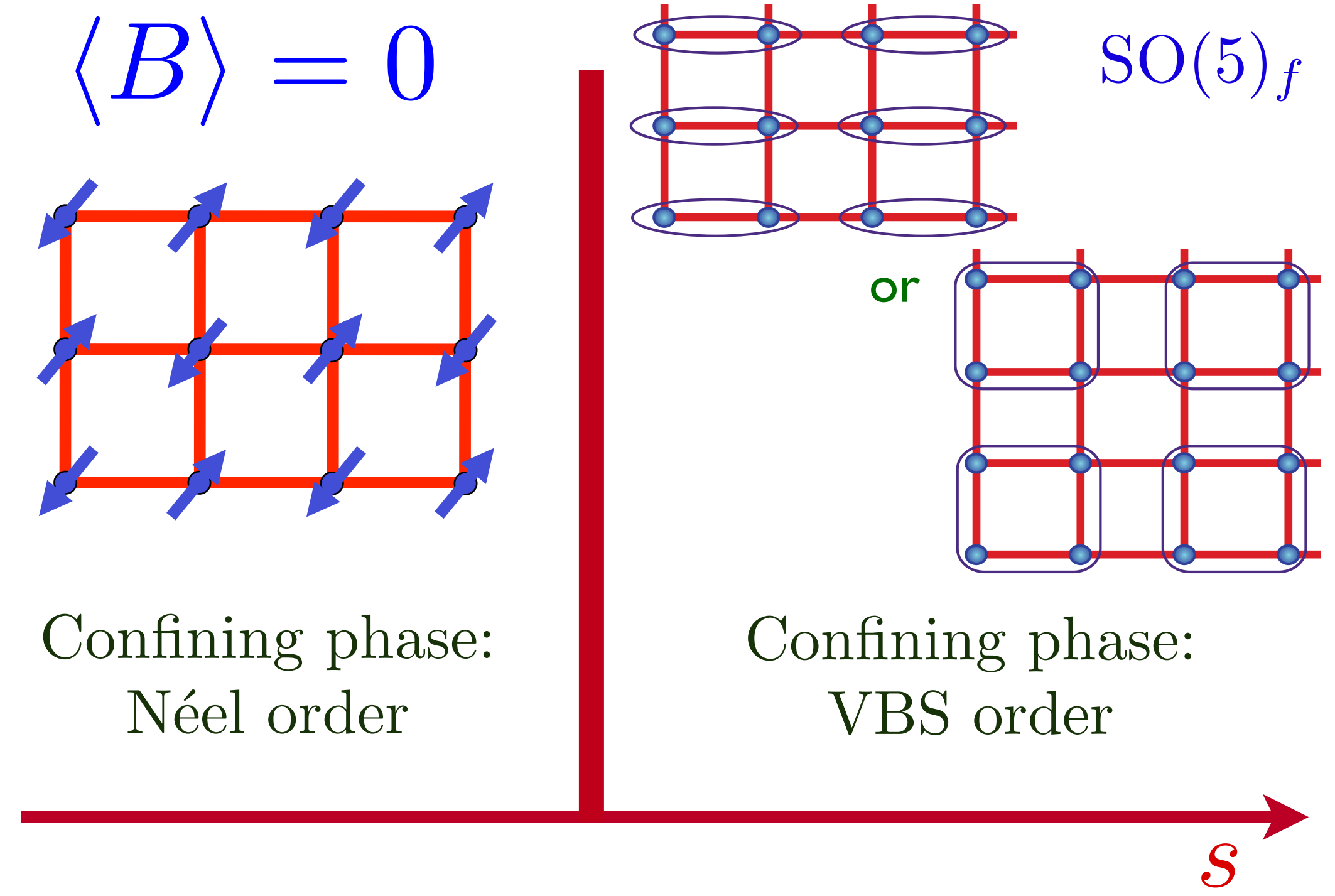


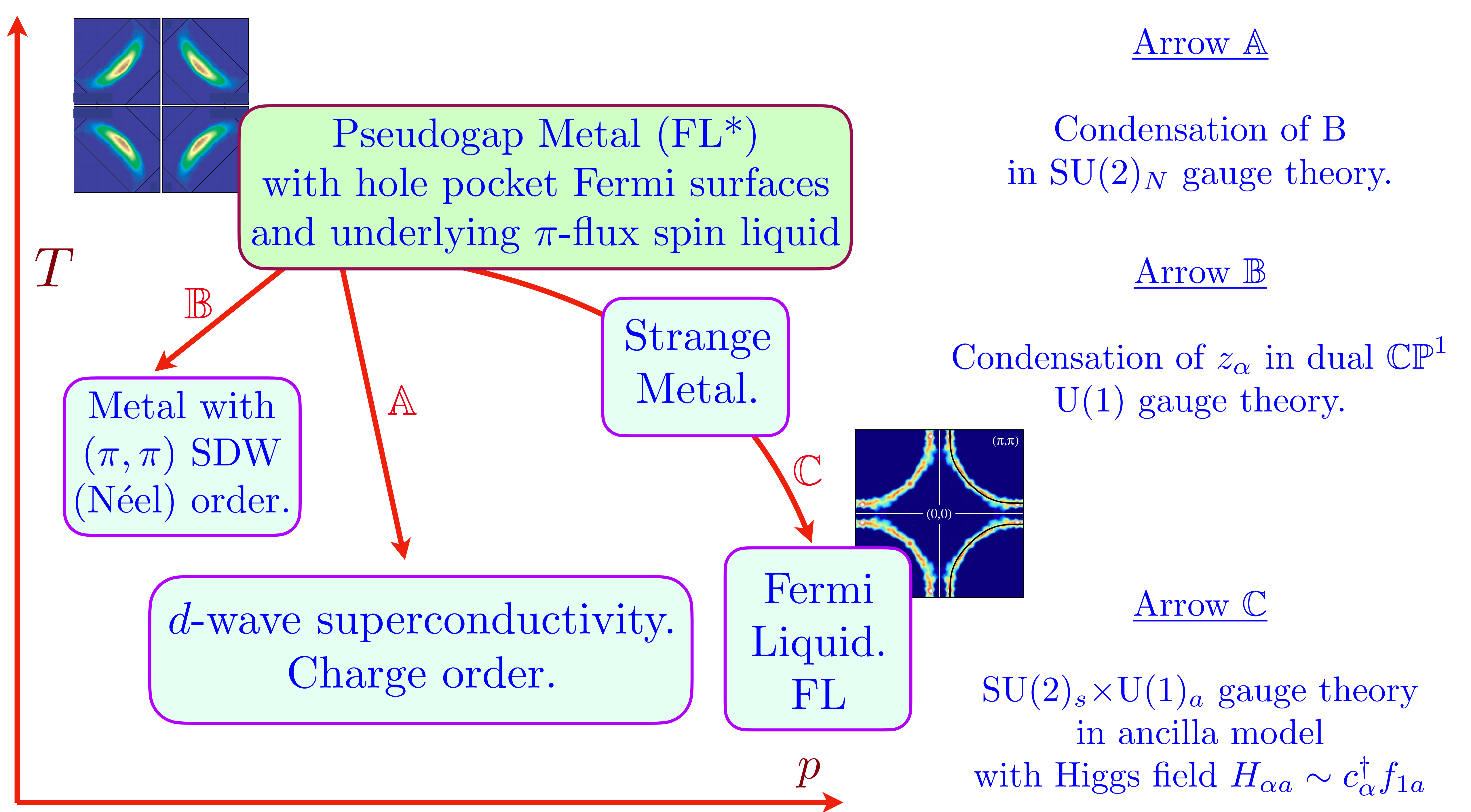
# Global phase diagram of $SU(2)_N$ gauge theory



$\langle B \rangle \neq 0$

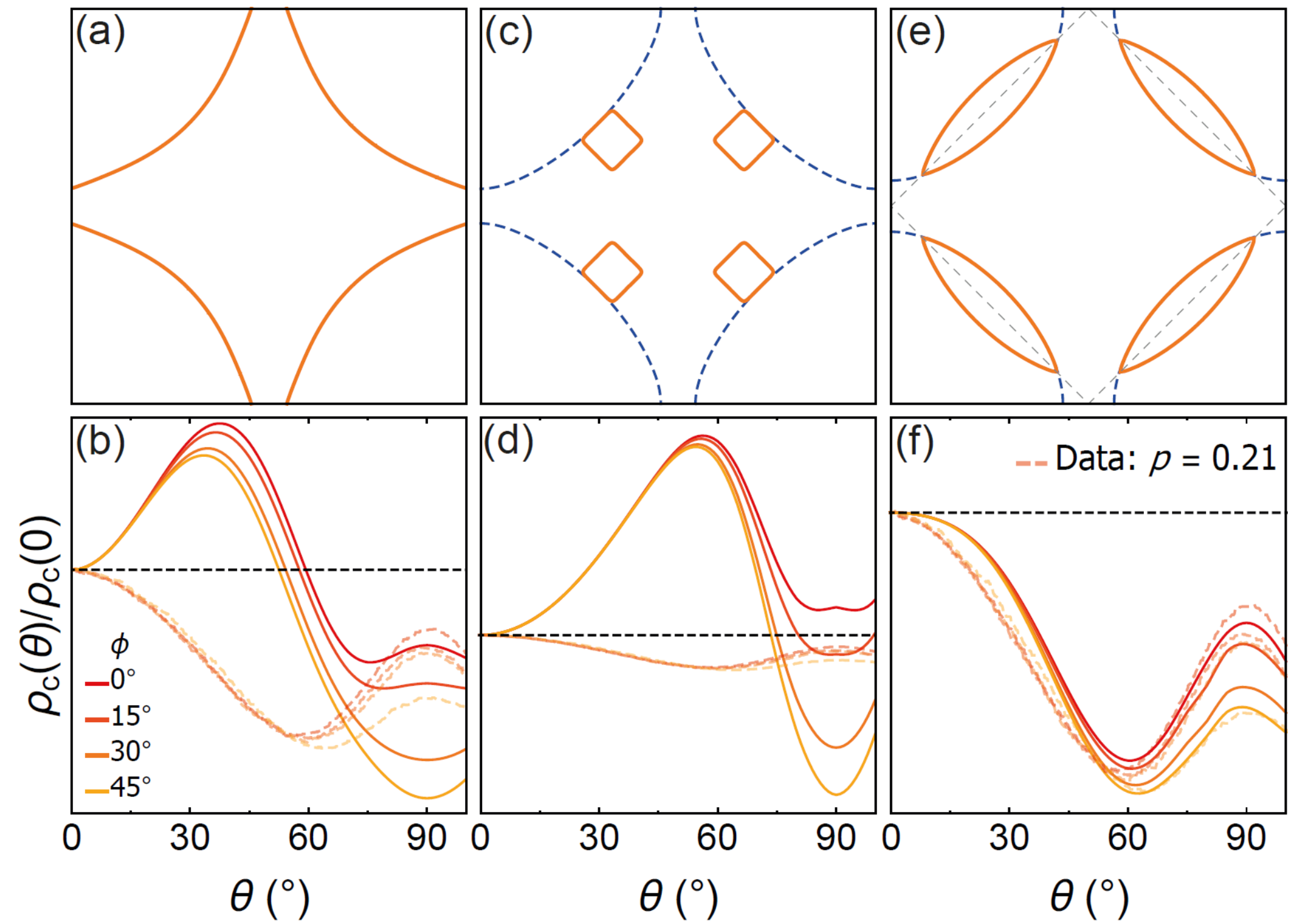
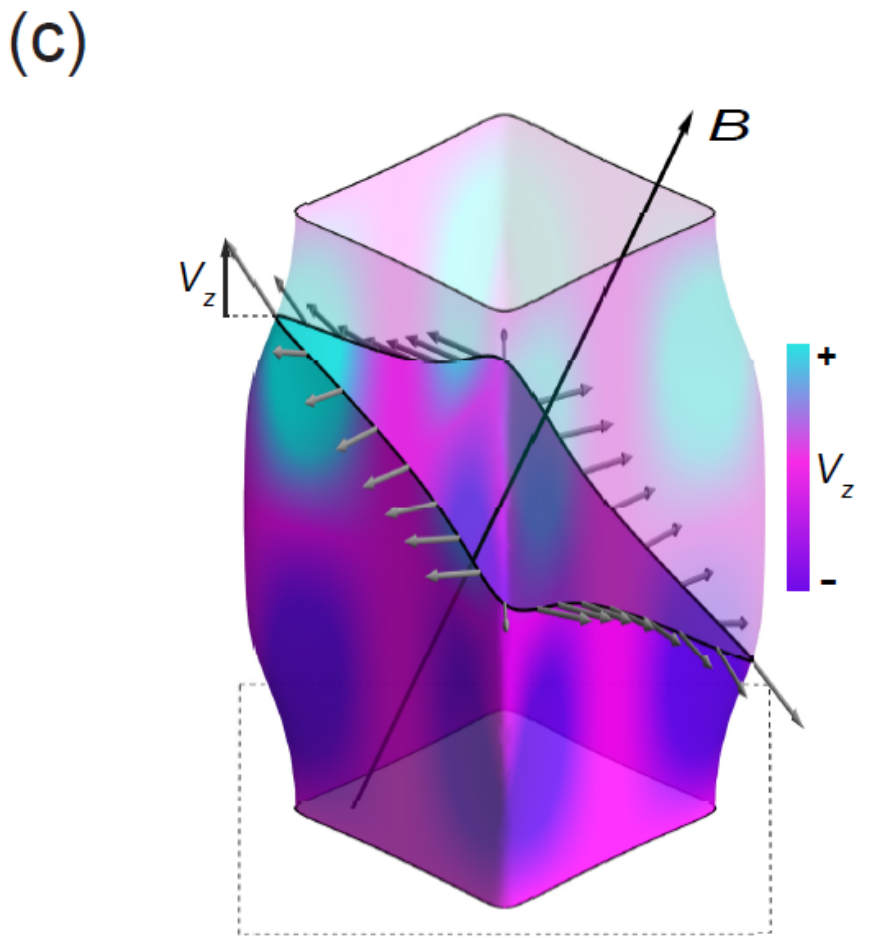
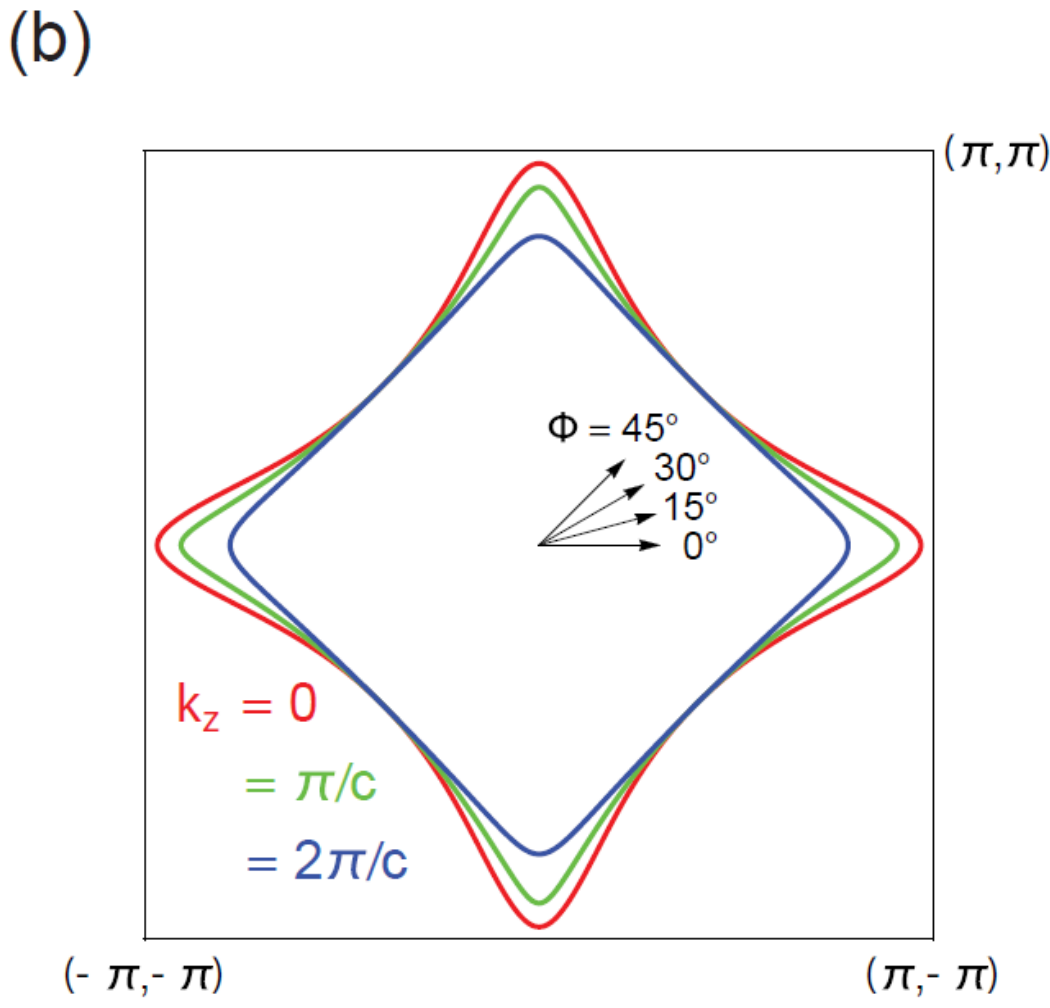
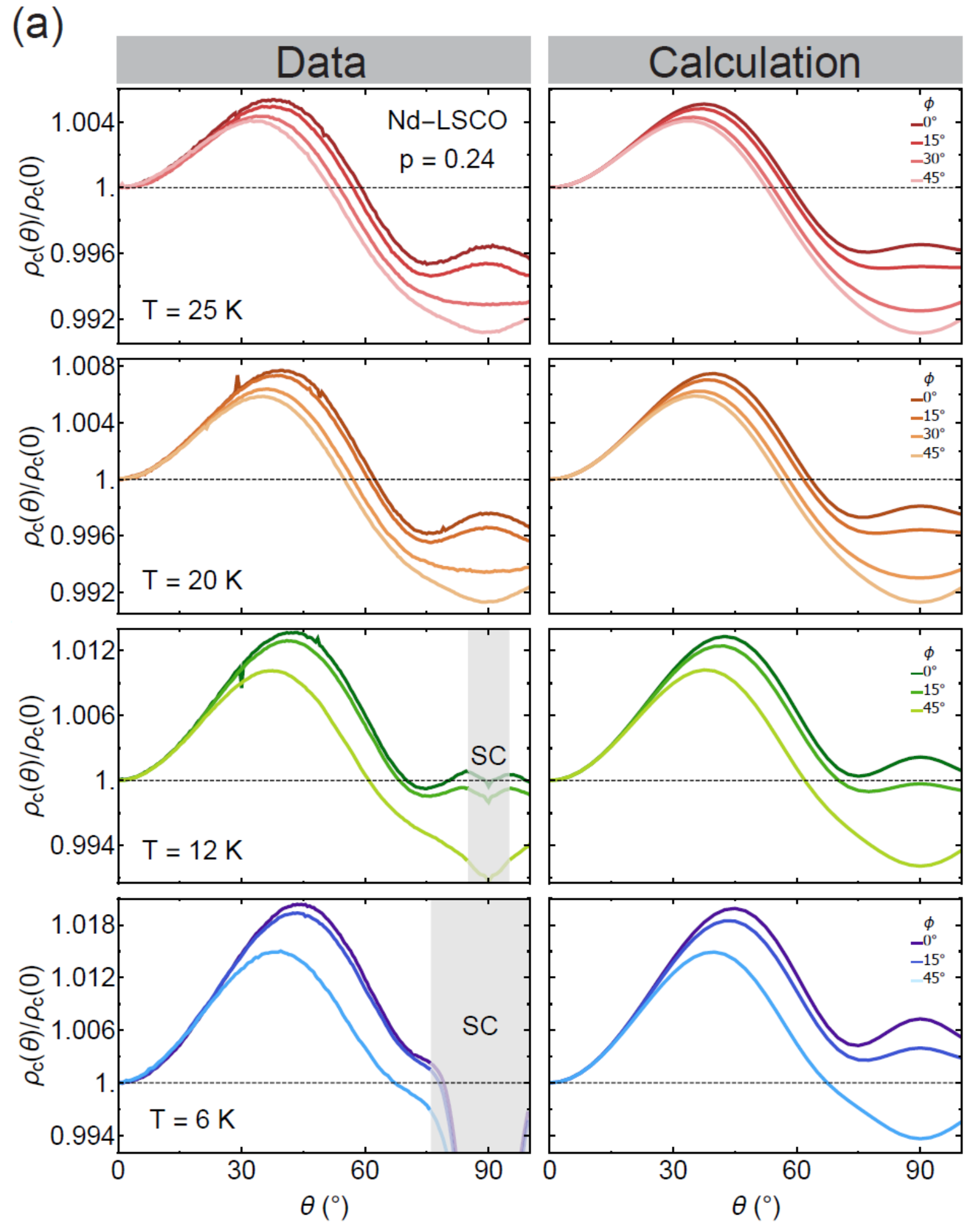
Including further-neighbor couplings in  $B$





# Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics **18**, 558 (2022)

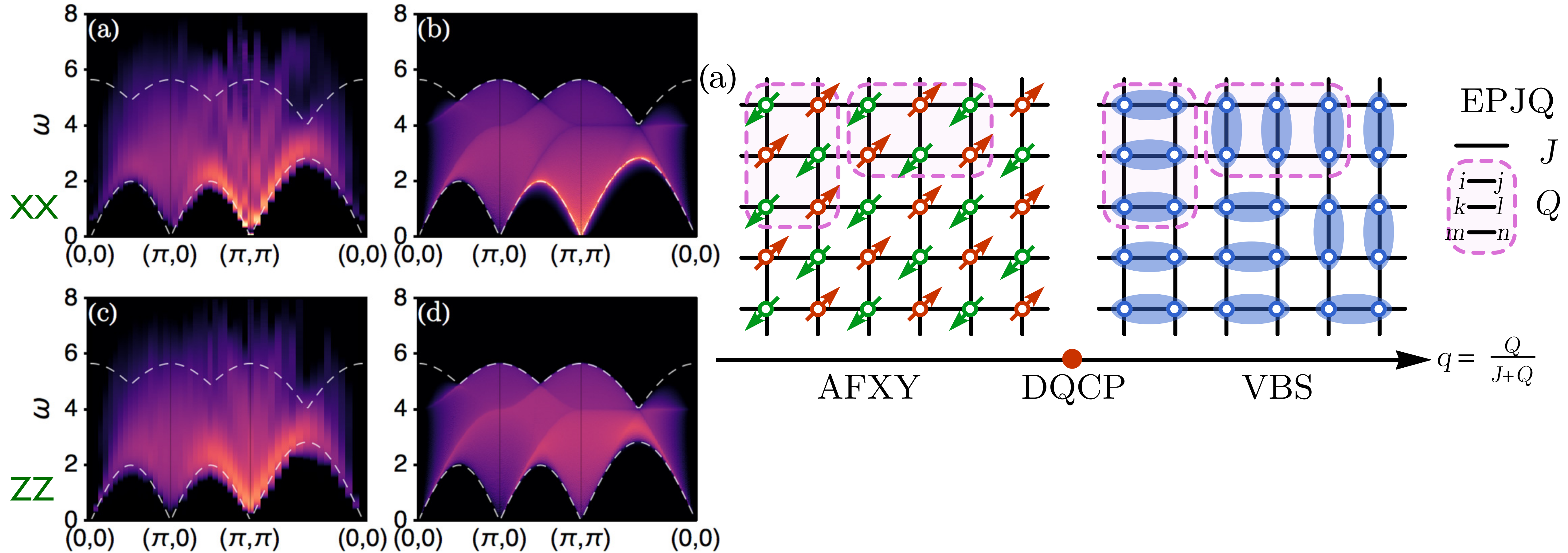


$p < p_c$  Reconstructed Fermi surface

$p > p_c$  Large Fermi surface

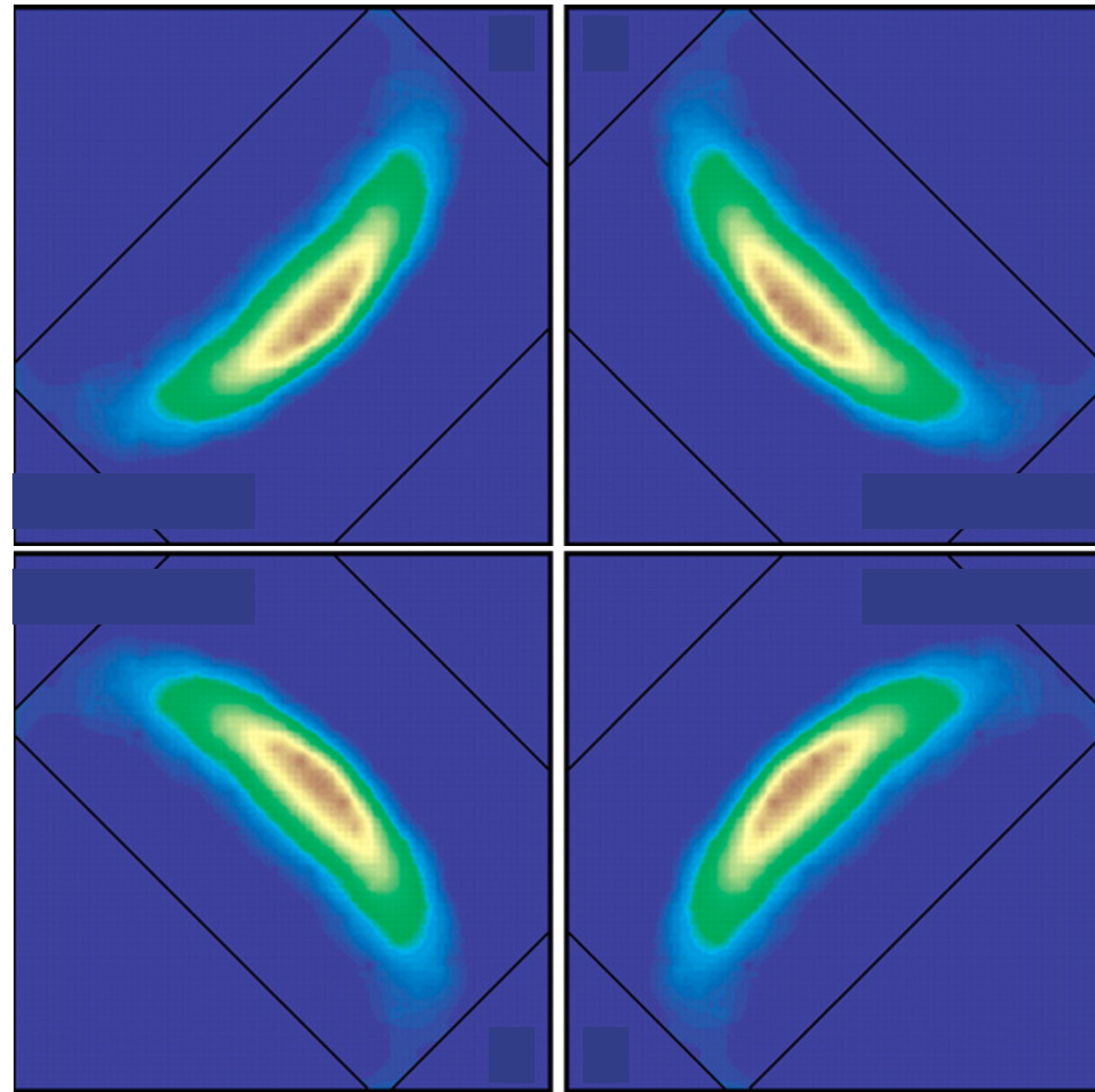
QMC

Free fermion spinons in  $\pi$ -flux

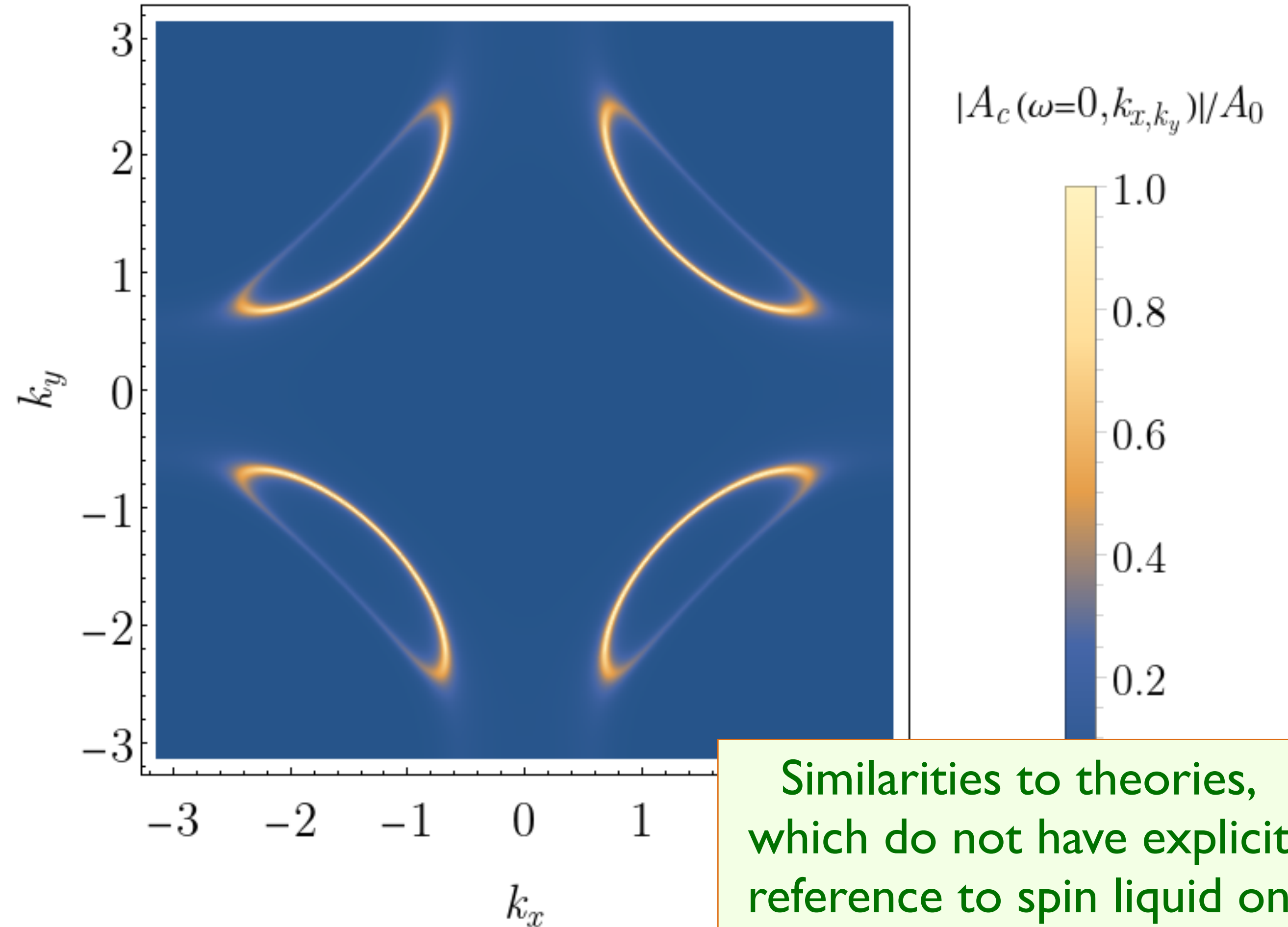


Nvsen Ma, Guang-Yu Sun, Yi-Zhuang You, Cenke Xu, Ashvin Vishwanath, Anders W. Sandvik, and Zi Yang Meng, PRB **98**, 174421 (2018)

# Photoemission at small $p$



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$   
at  $x = 0.10$



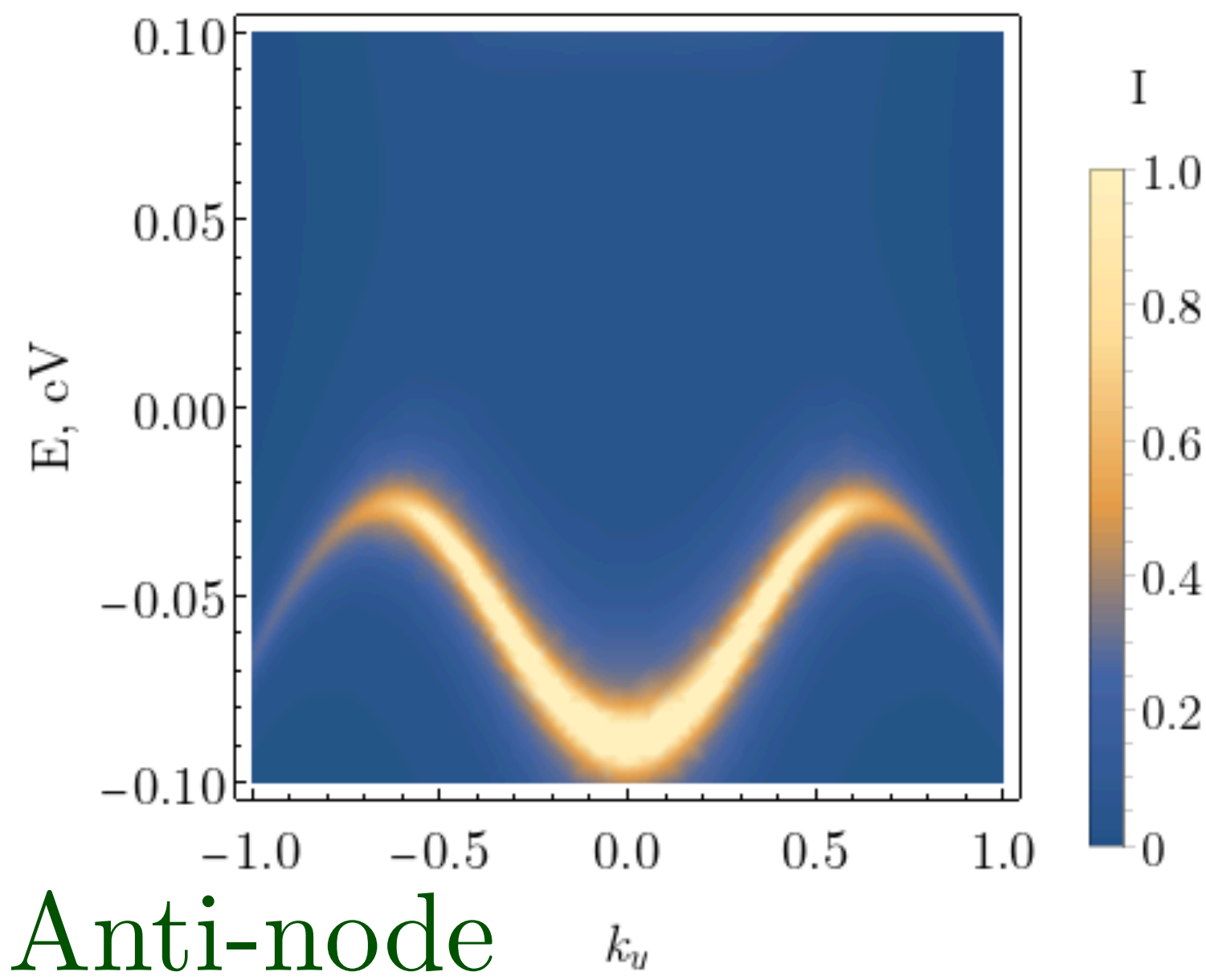
“*Fermi arcs*”

Similarities to theories,  
which do not have explicit  
reference to spin liquid on  
second ancilla layer

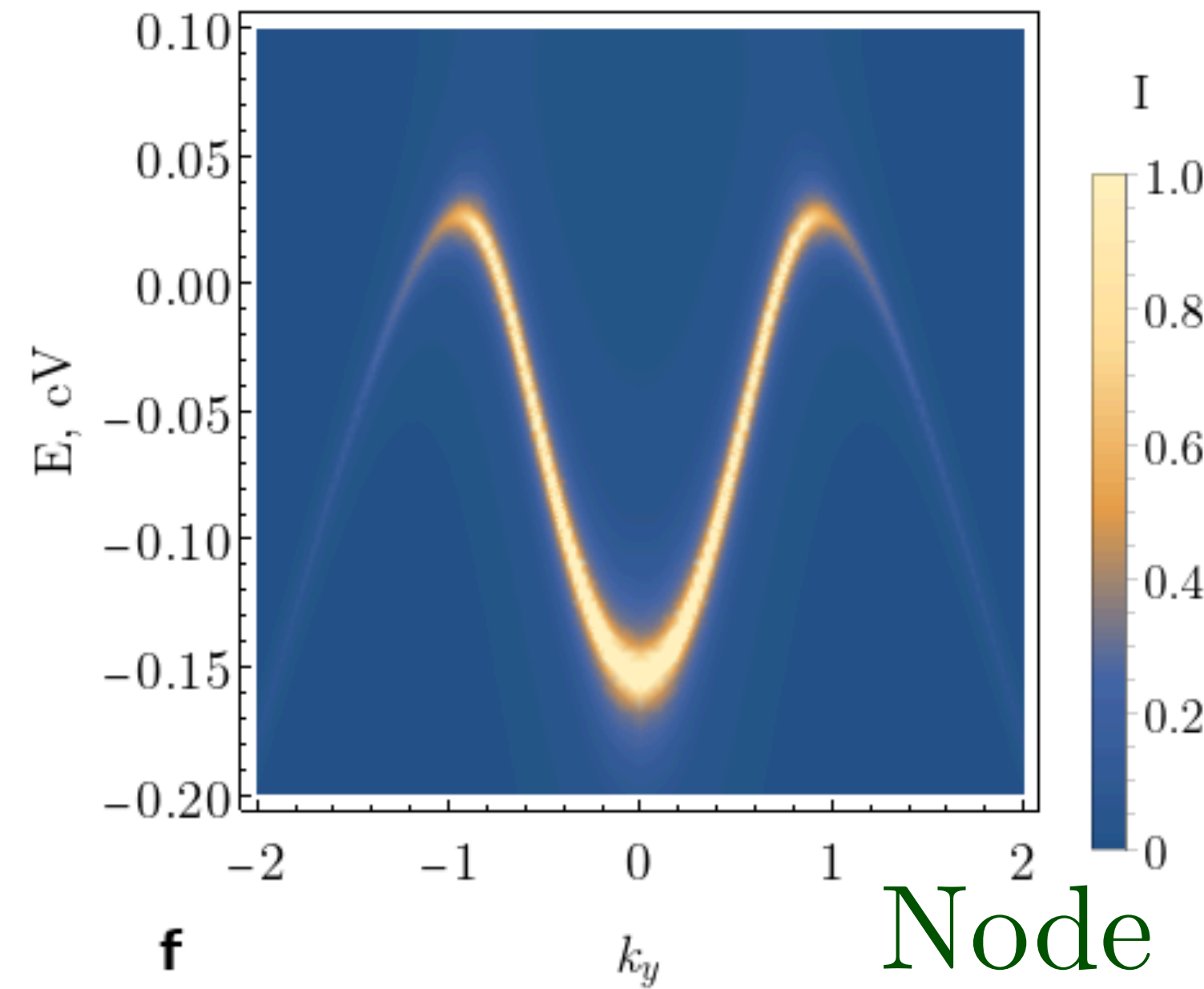
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,  
PRB **73**, 174501 (2006)  
S. Sakai, Y. Motome, M. Imada,  
PRL **102**, 056404 (2009)

FL\* in a  
**one-band** model

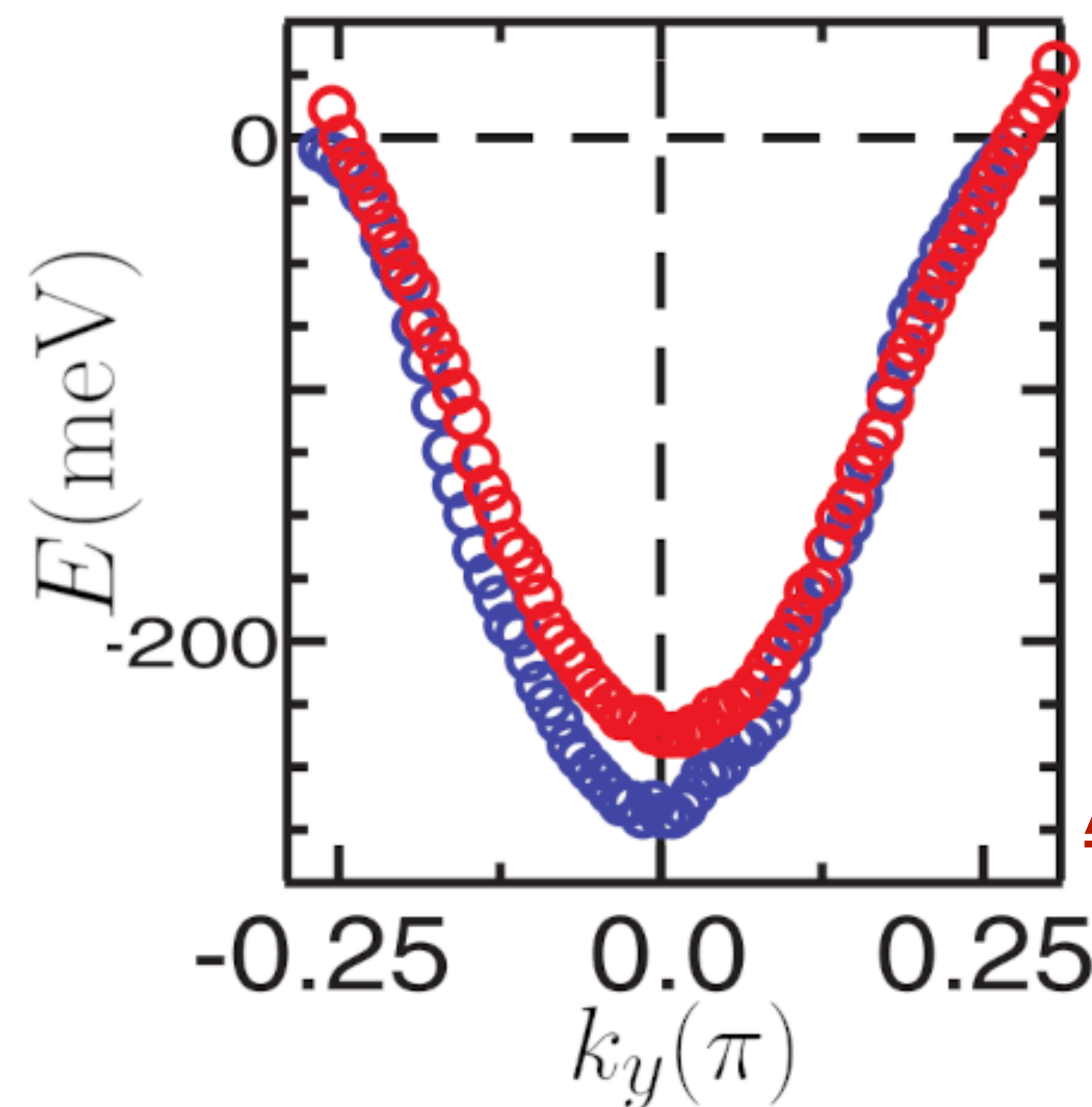
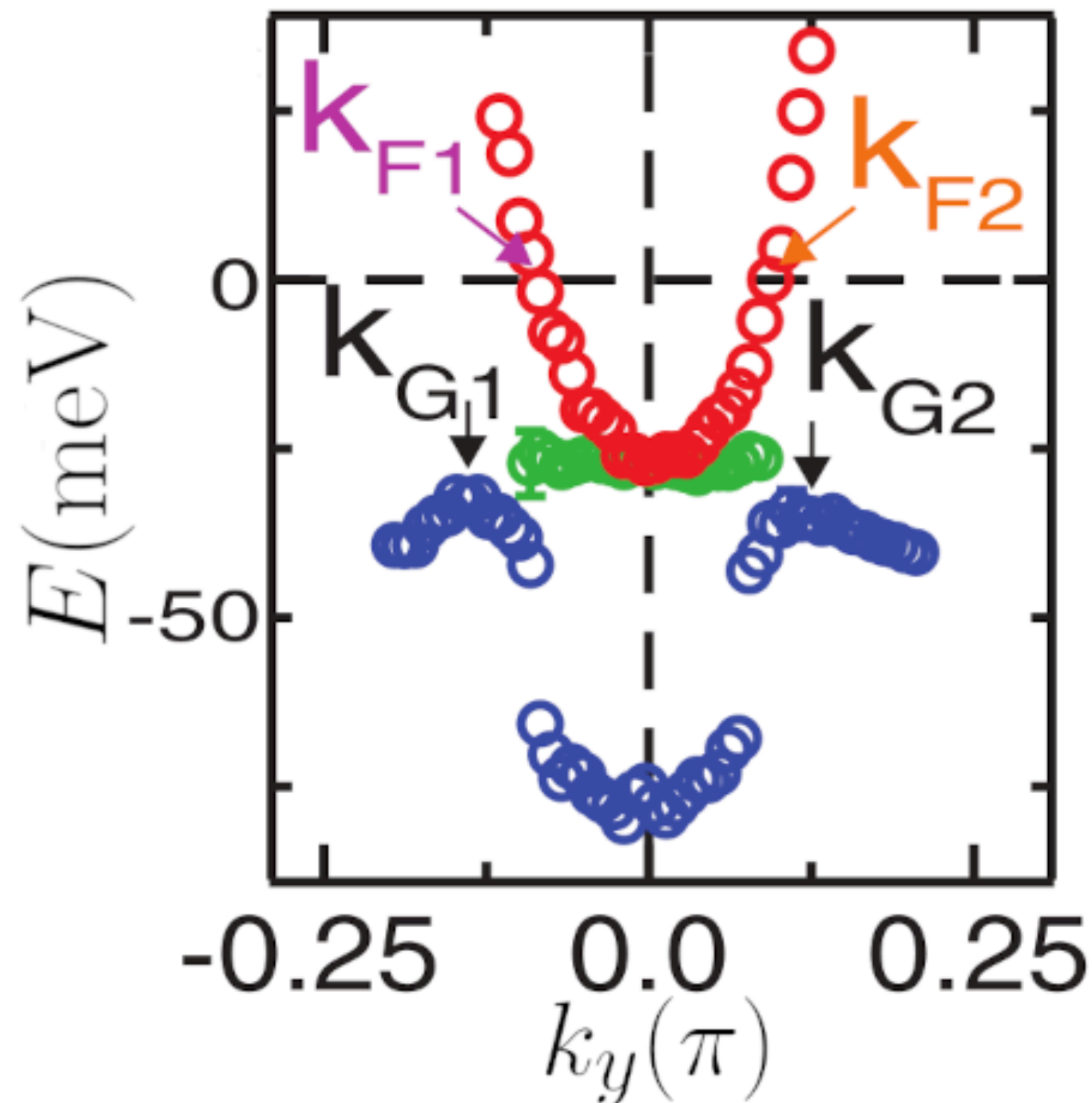
Second ancilla layer is needed  
to describe MDC and EDC



Anti-node



Node



ARPES on  
**Bi2201**

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Similarities to theories,  
which do not have explicit  
reference to spin liquid on  
second ancilla layer

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# Unified $SU(2) \times U(1)$ gauge theory of spinons, electrons and Higgs bosons: uncanny similarities to the Salam-Weinberg-Glashow theory of weak interactions

- The electromagnetic  $U(1)$  is effectively global, because  $\alpha \ll 1$ .
- The fermionic spinons transform as a fundamental of gauge  $SU(2)$ , with a massless Dirac spectrum

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left( \Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right),$$

where  $U_{ij}$  is the (lattice)  $SU(2)$  gauge field. The spinons are the analog of the neutrinos

- The Higgs sector has a boson  $B_i$  which is fundamental of  $SU(2)$

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left( B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \mathcal{O}(B_i^4)$$

- The hole pockets in the nodal region of the Brillouin zone are described by electron  $\bar{c}_{i\alpha}$  which have a Yukawa coupling to the spinons and the Higgs field  $B_i = (B_{1i}, B_{2i})$ :

$$H_Y = \sum_{ij} \bar{t}_{ij} \bar{c}_{i\alpha}^\dagger \bar{c}_{j\alpha} + i \sum_i \left( B_{1i} f_{i\alpha}^\dagger \bar{c}_{i\alpha} - B_{2i} \varepsilon_{\alpha\beta} f_{i\alpha} \bar{c}_{i\beta} \right) + \text{H.c.}$$