

Quantum statistical mechanics of charged black holes and strange metals

Review articles:
arXiv:2304.13744, 2305.01001

Colloquium Ehrenfestii
Instituut Lorentz, Leiden University
July 5, 2023

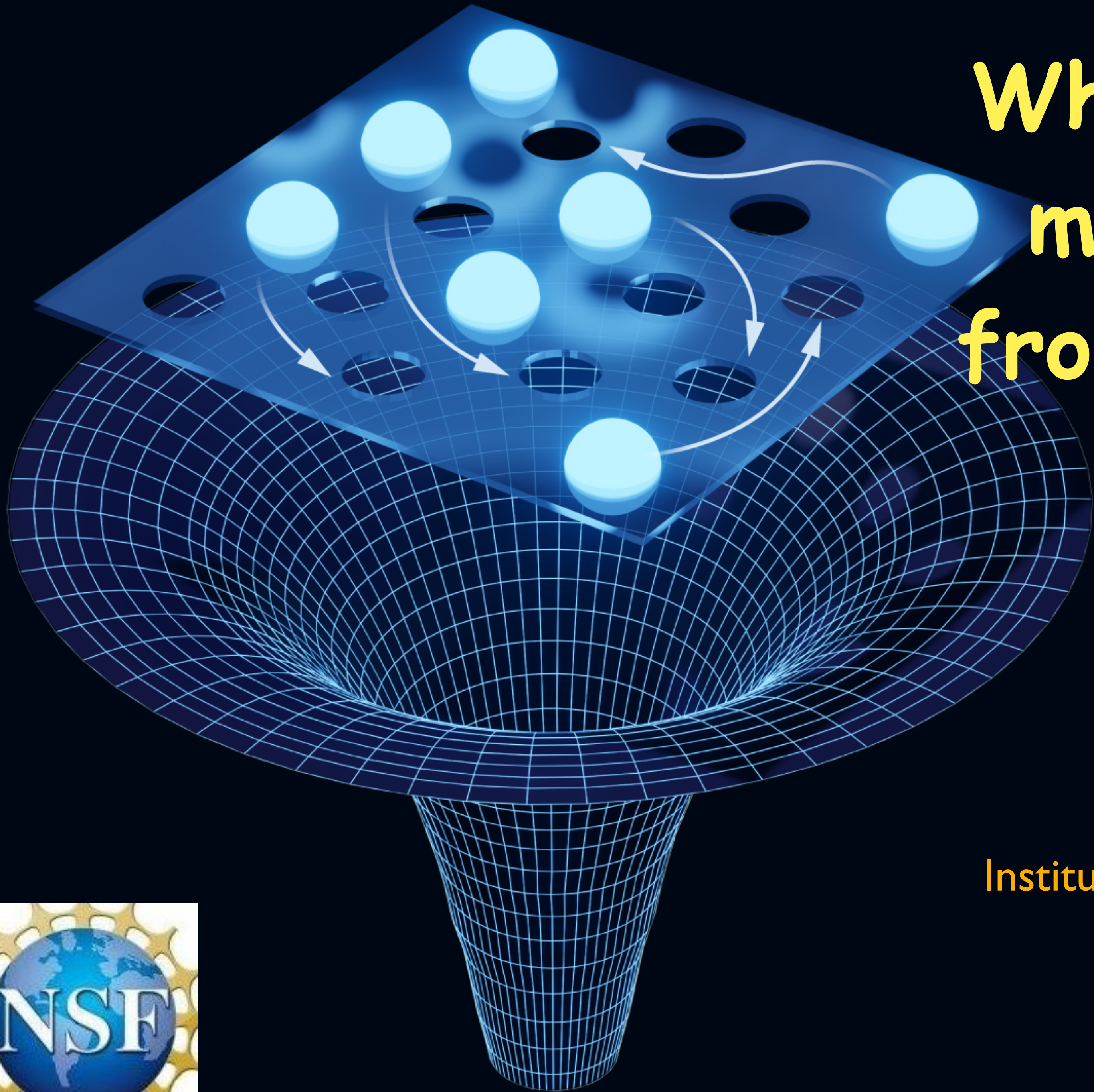
Subir Sachdev



Talk online: sachdev.physics.harvard.edu



When nature entangles millions of particles: from quantum materials to black holes



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Foundations

by

Boltzmann

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

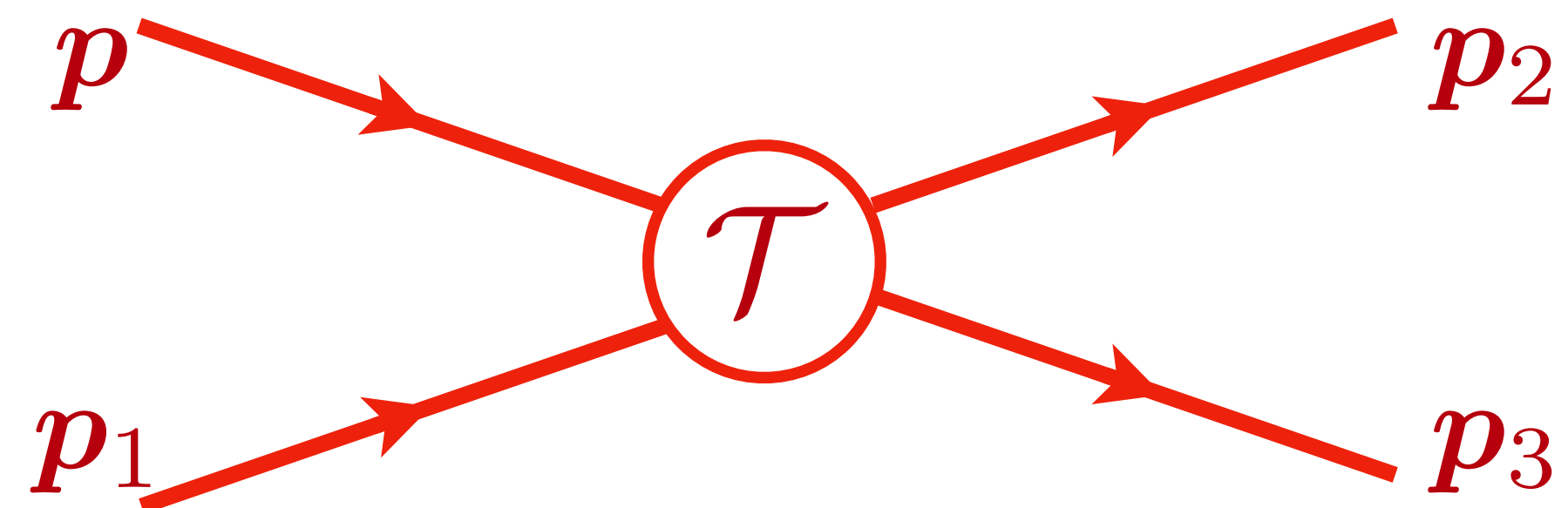
Vienna, Austria

Boltzmann equation (1872)

Dilute classical gas

Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
$$- 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$
$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} - f_{\mathbf{p}_2} f_{\mathbf{p}_3}]$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

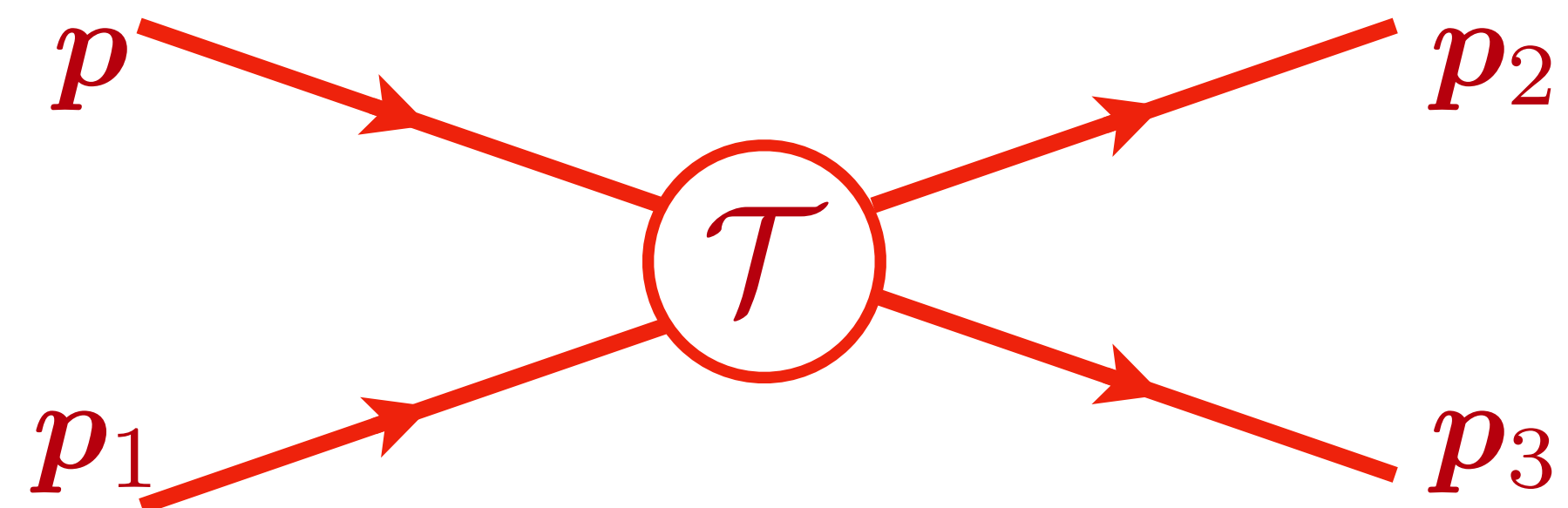
Vienna, Austria

Quantum Boltzmann equation (Landau)

Dense gas of electrons

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
$$- 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$
$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} (1 - f_{\mathbf{p}_2}) (1 - f_{\mathbf{p}_3}) - f_{\mathbf{p}_2} f_{\mathbf{p}_3} (1 - f_{\mathbf{p}}) (1 - f_{\mathbf{p}_1})]$$

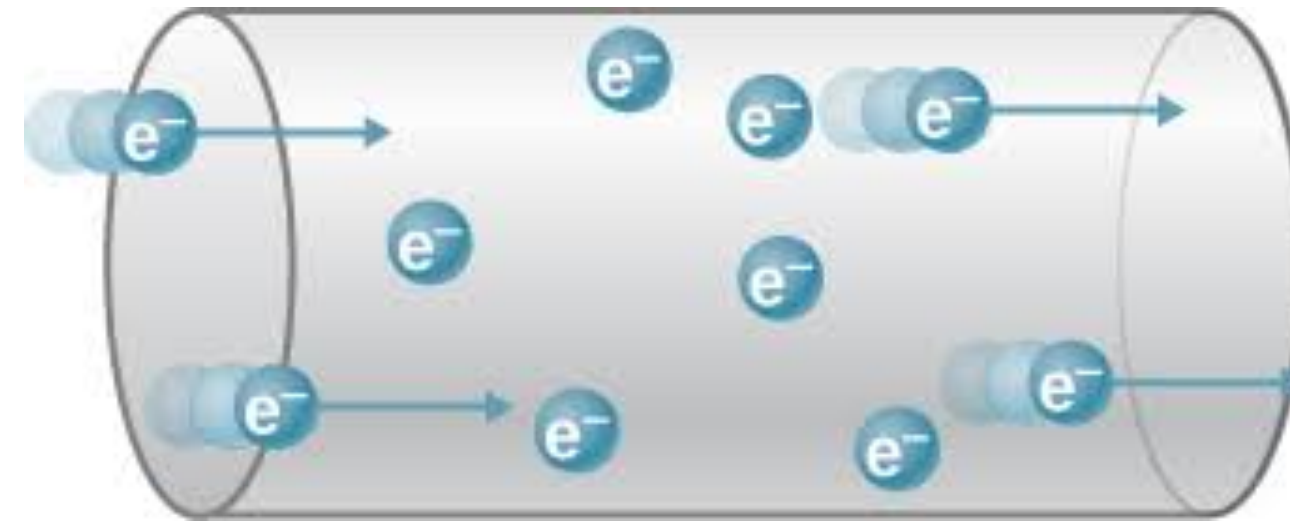


Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Current flow with electrons in Copper

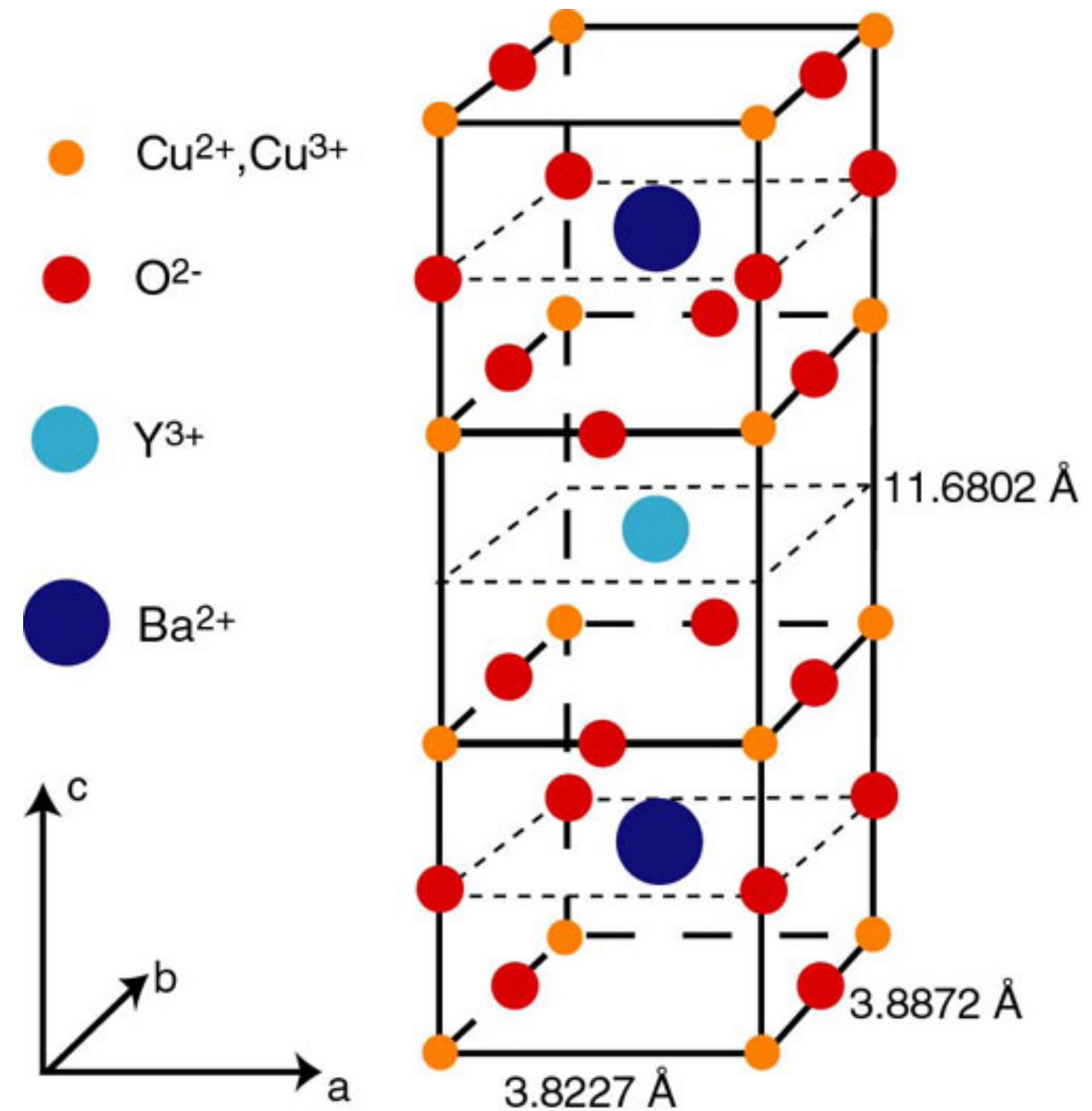


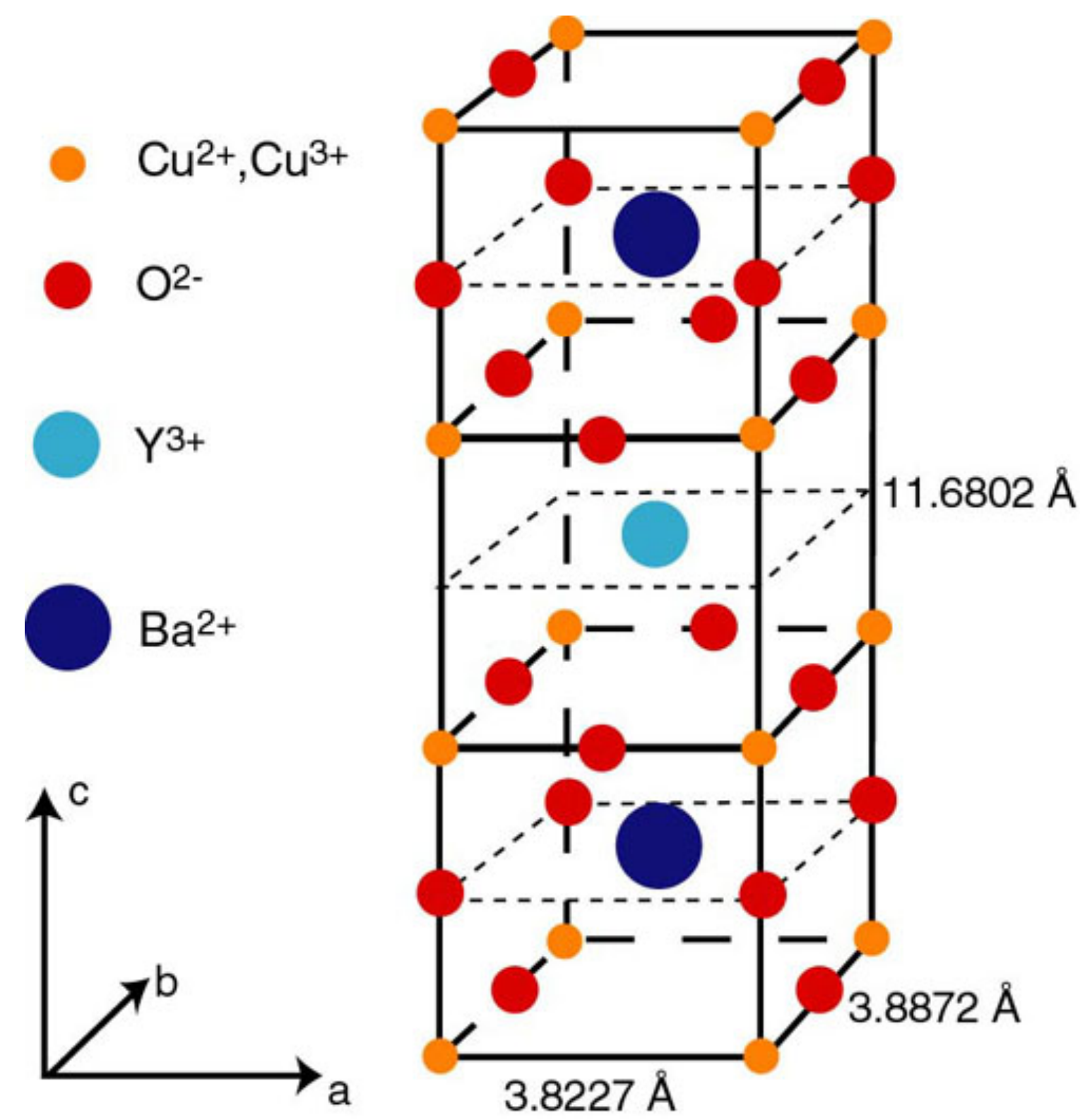
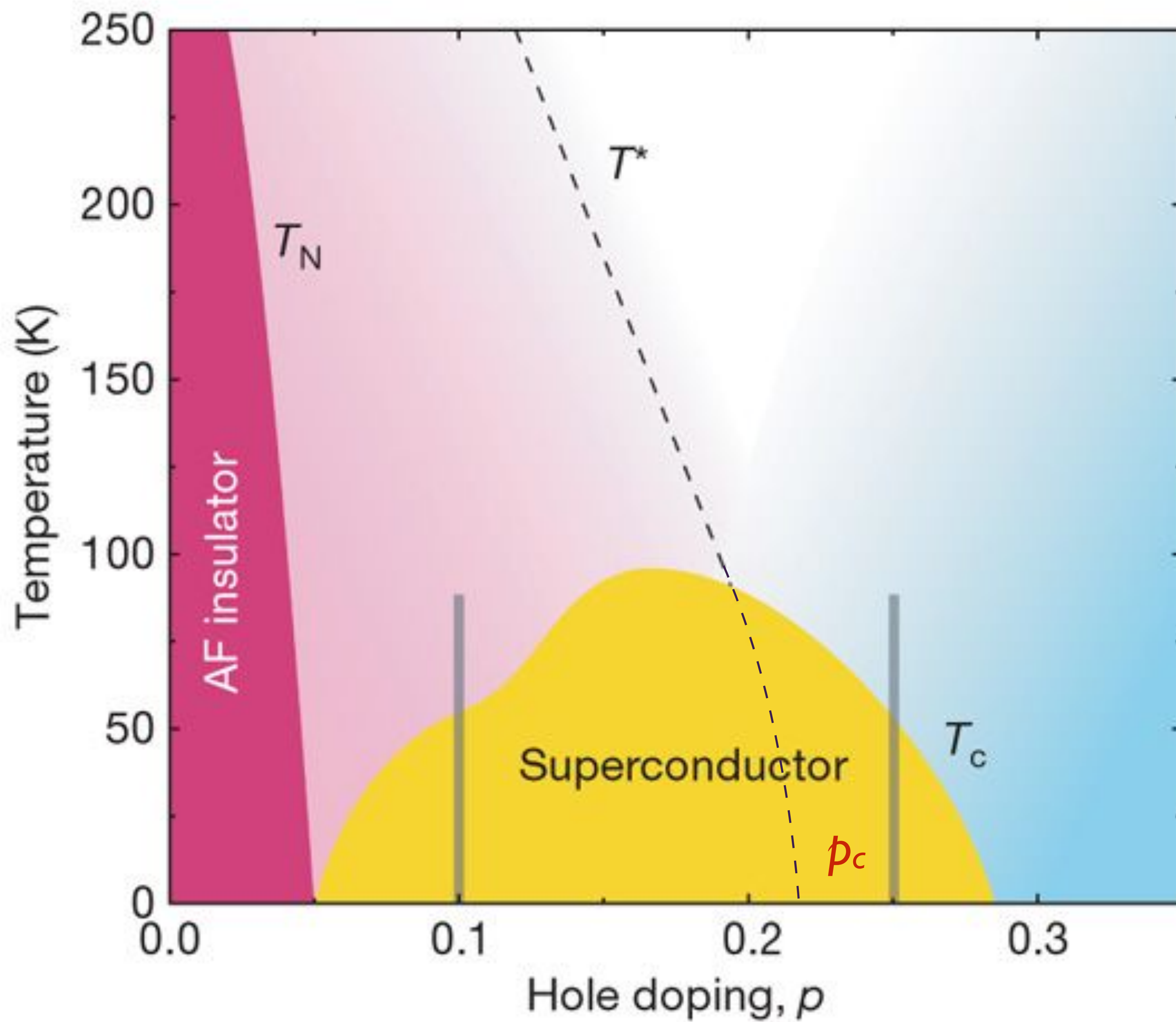
Flow of electrons described by Boltzmann equation \Rightarrow
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

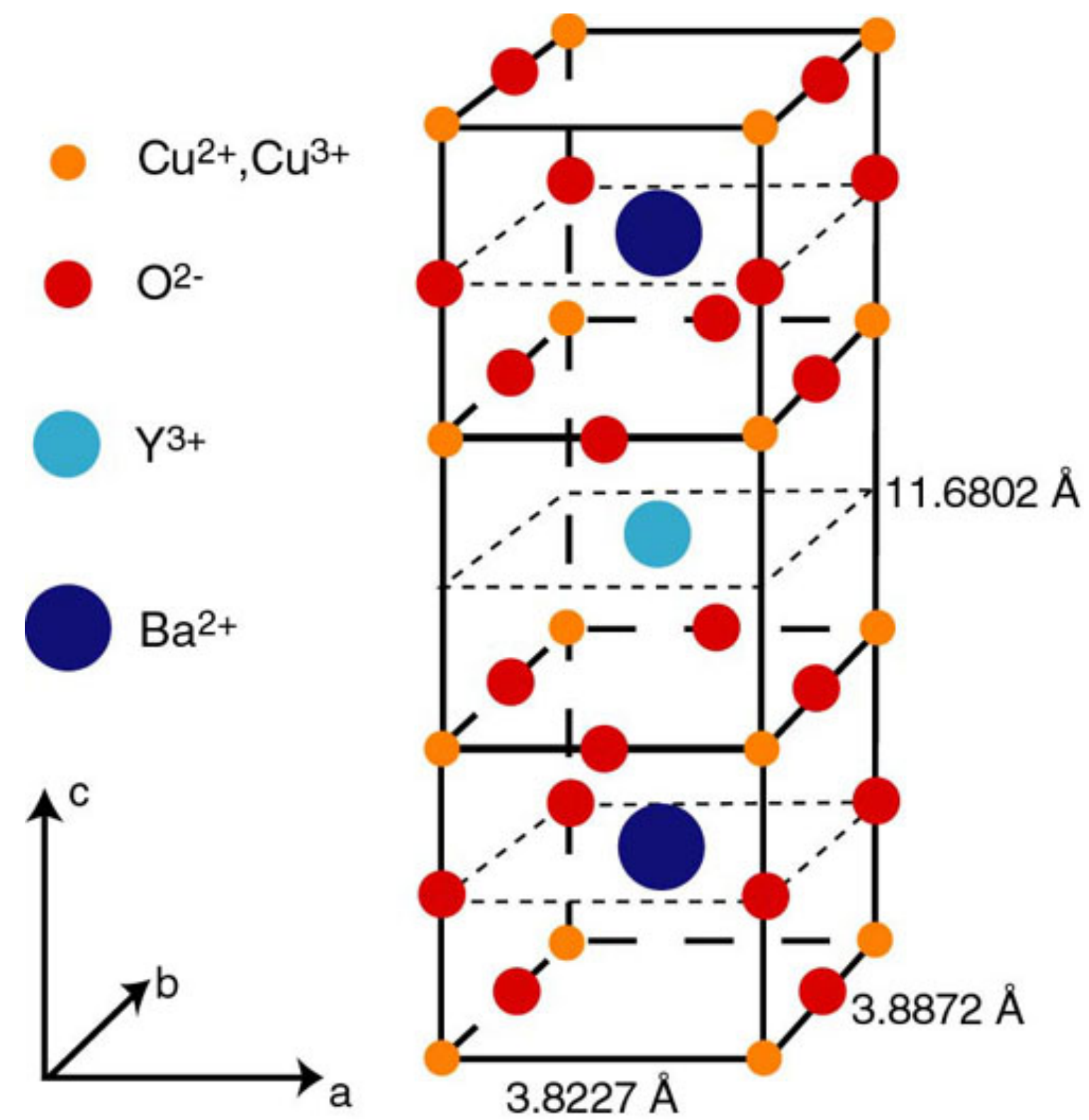
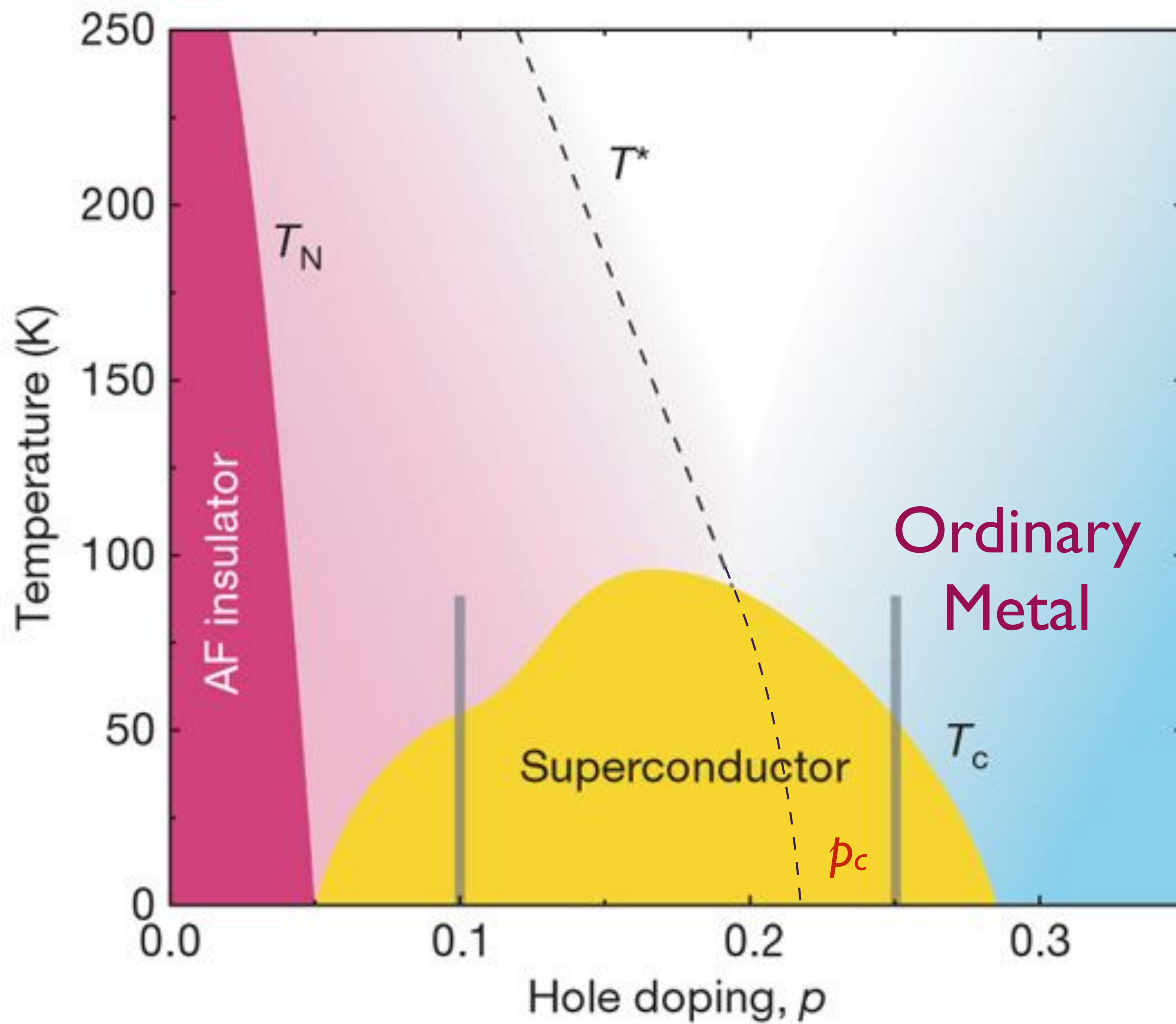
The time τ is much longer than a limiting 'Planckian time' $\frac{\hbar}{k_B T}$.

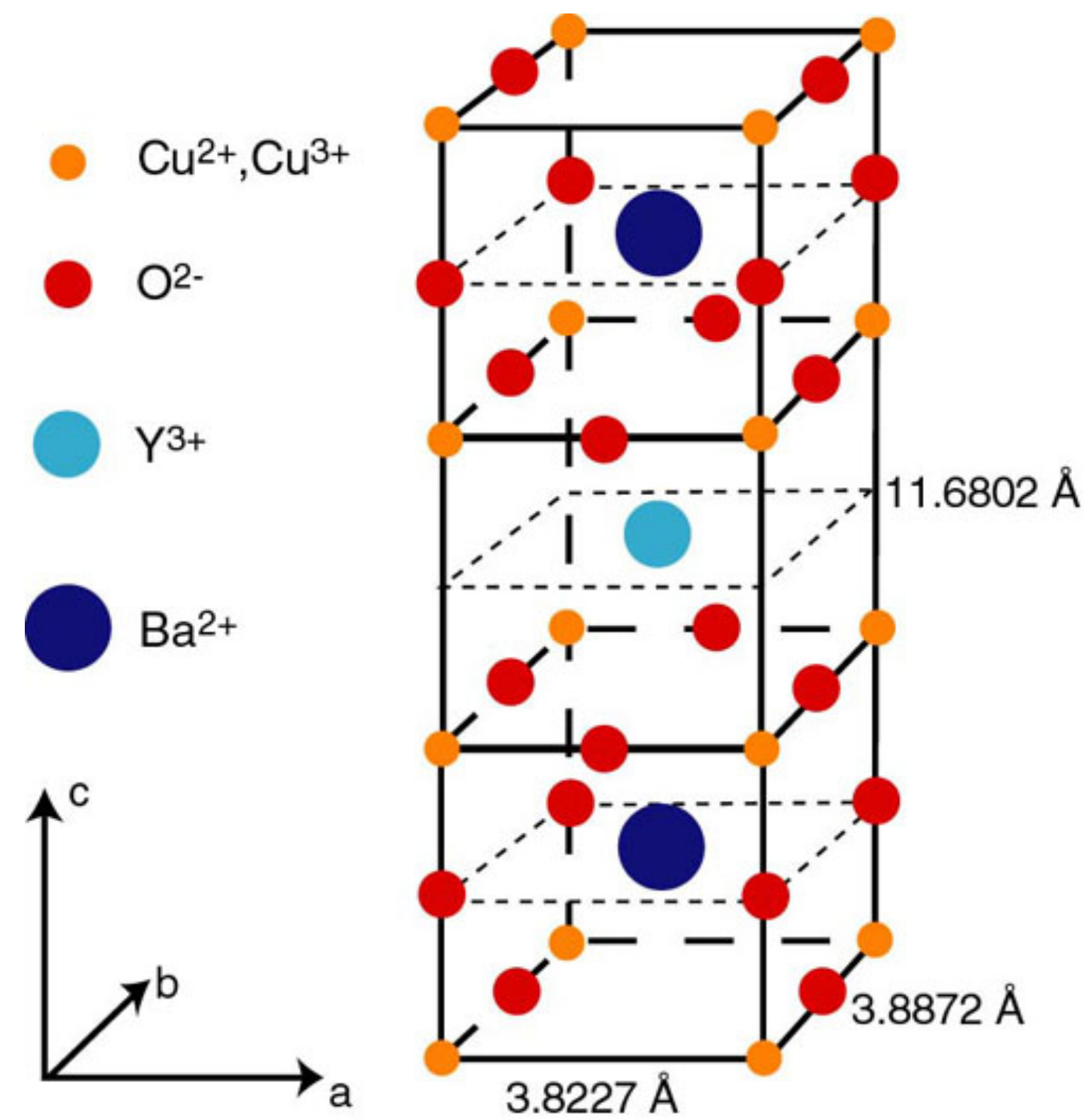
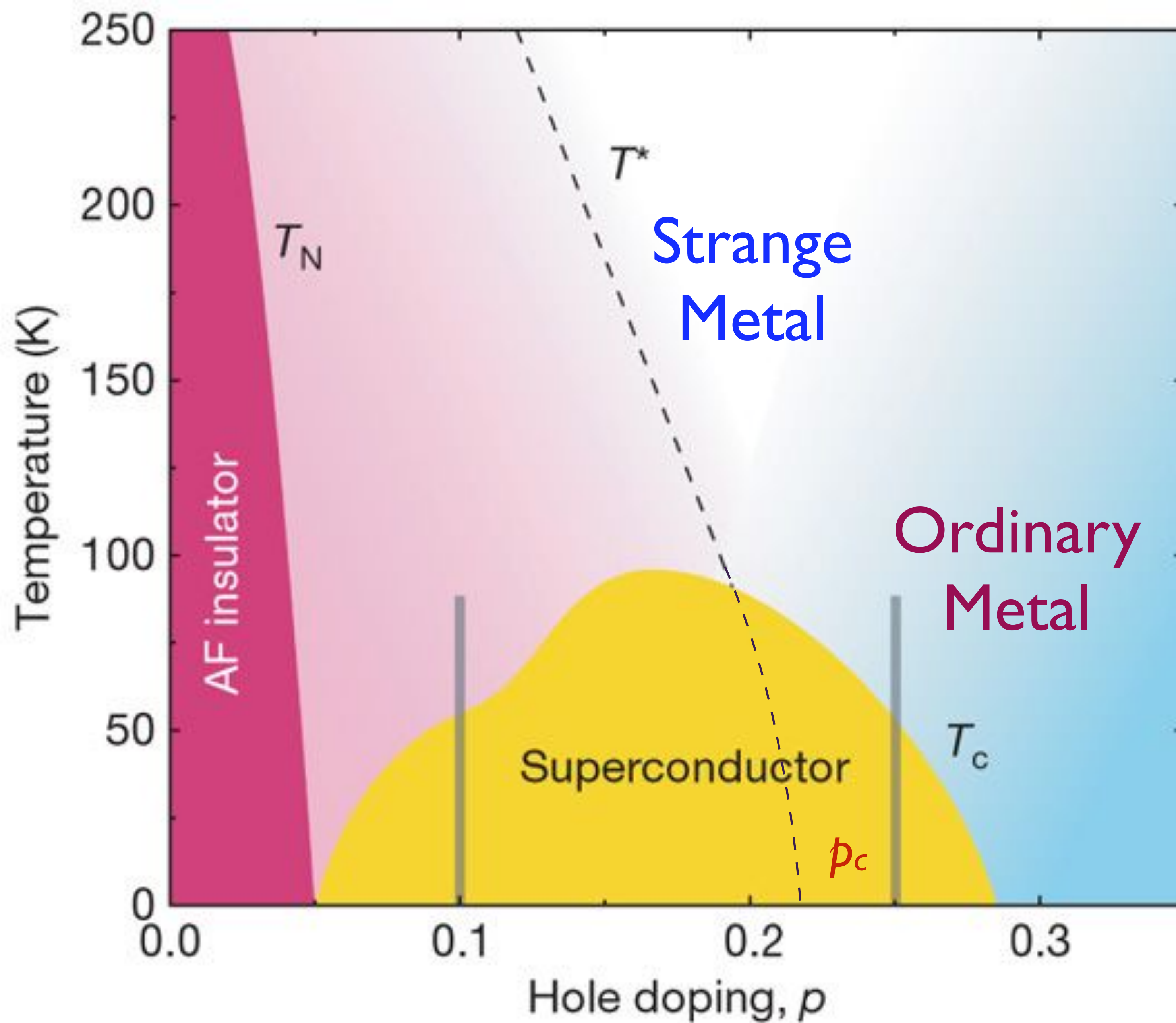
The long scattering time implies that individual electrons are well-defined.

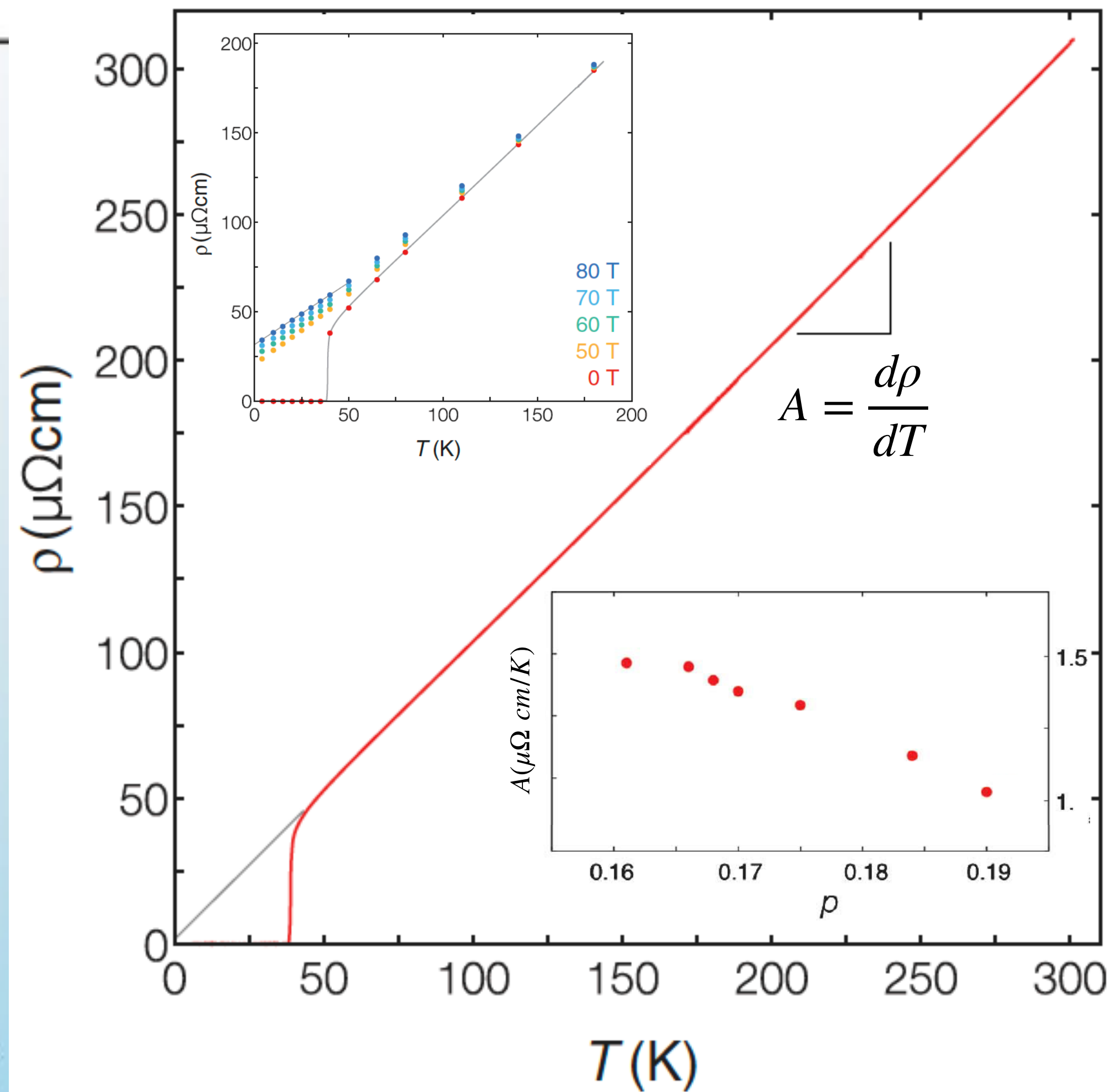
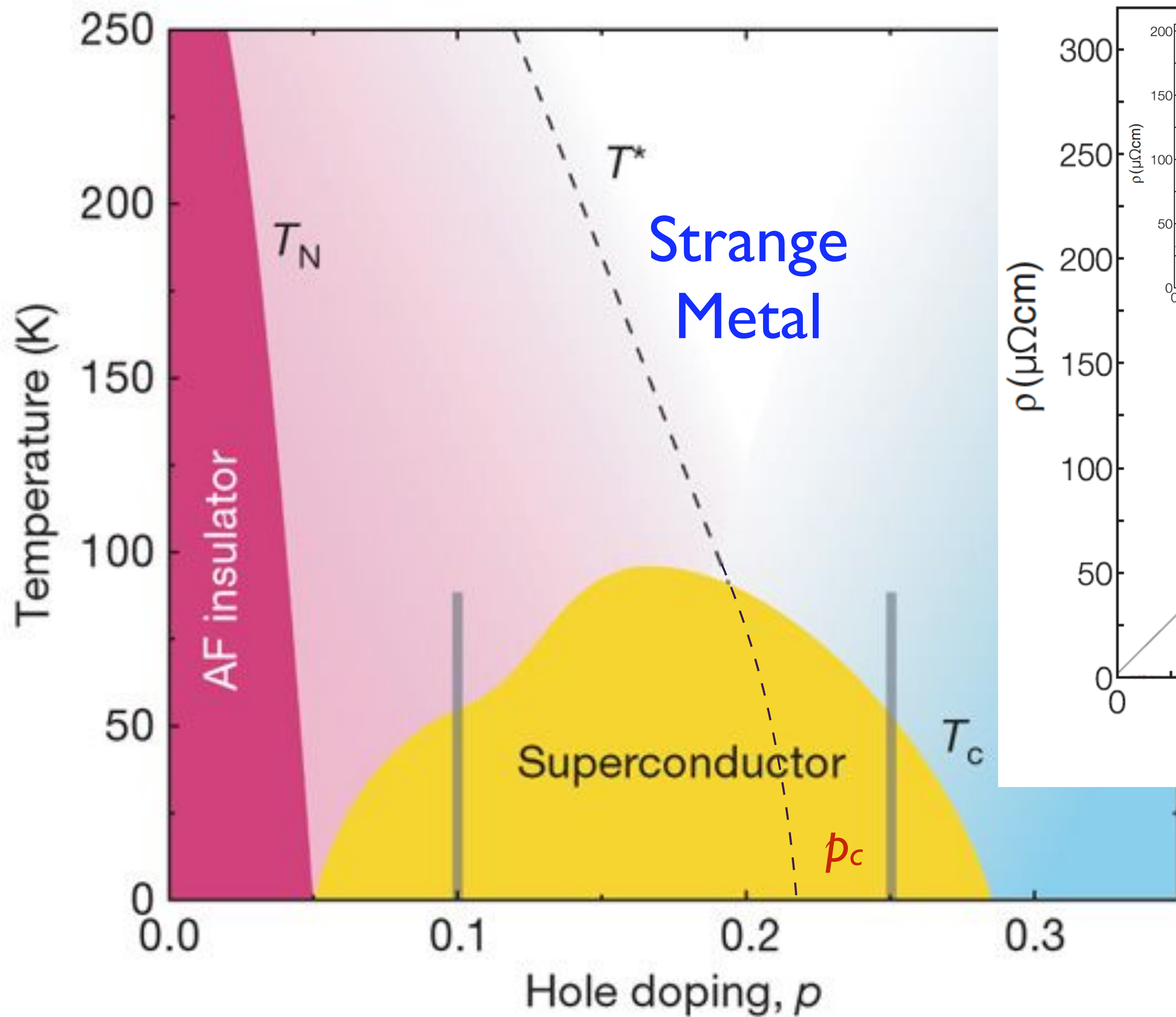
Cuprate high temperature superconductors









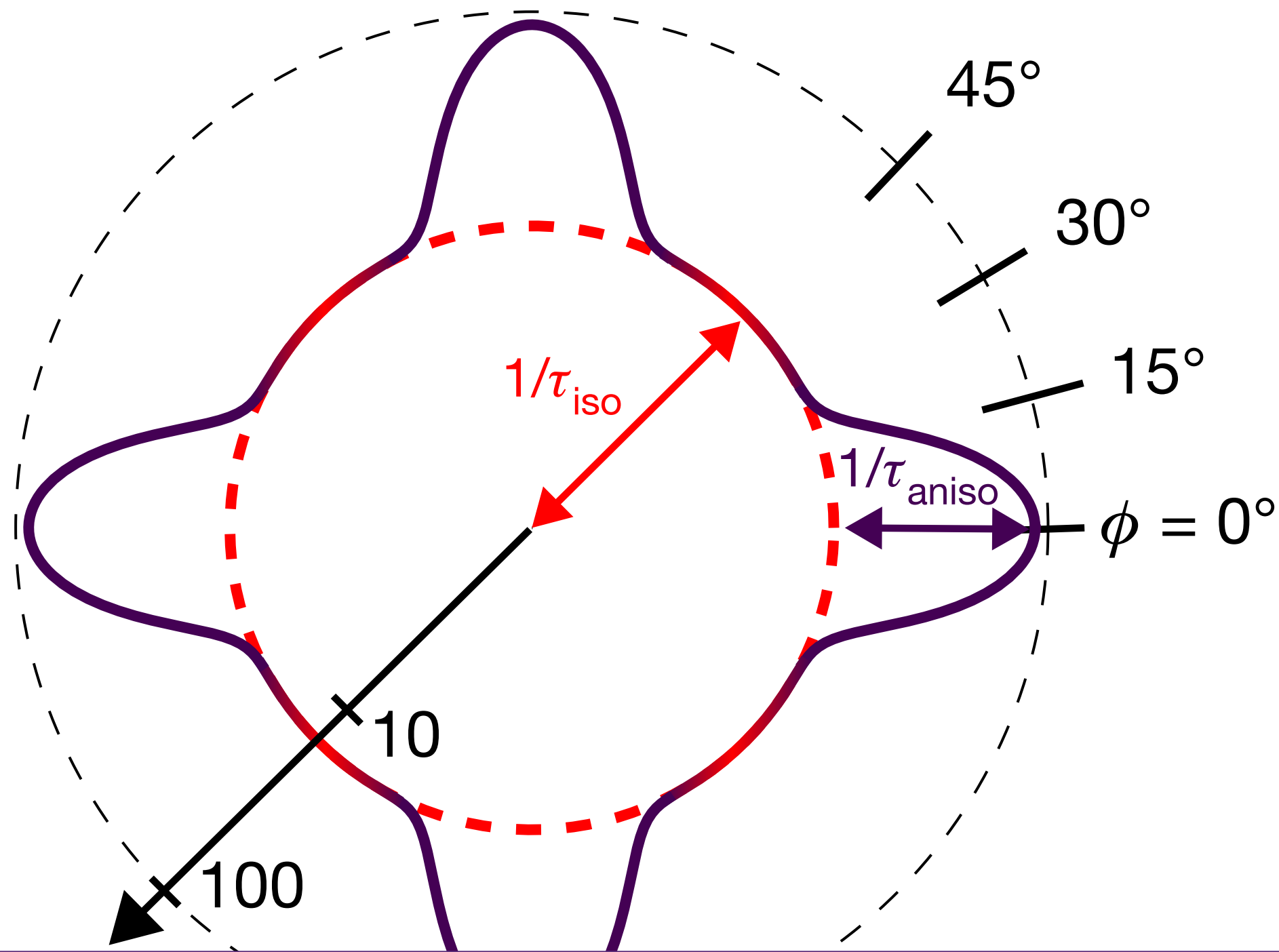


LSCO: Giraldo-Gallo et al. 2018

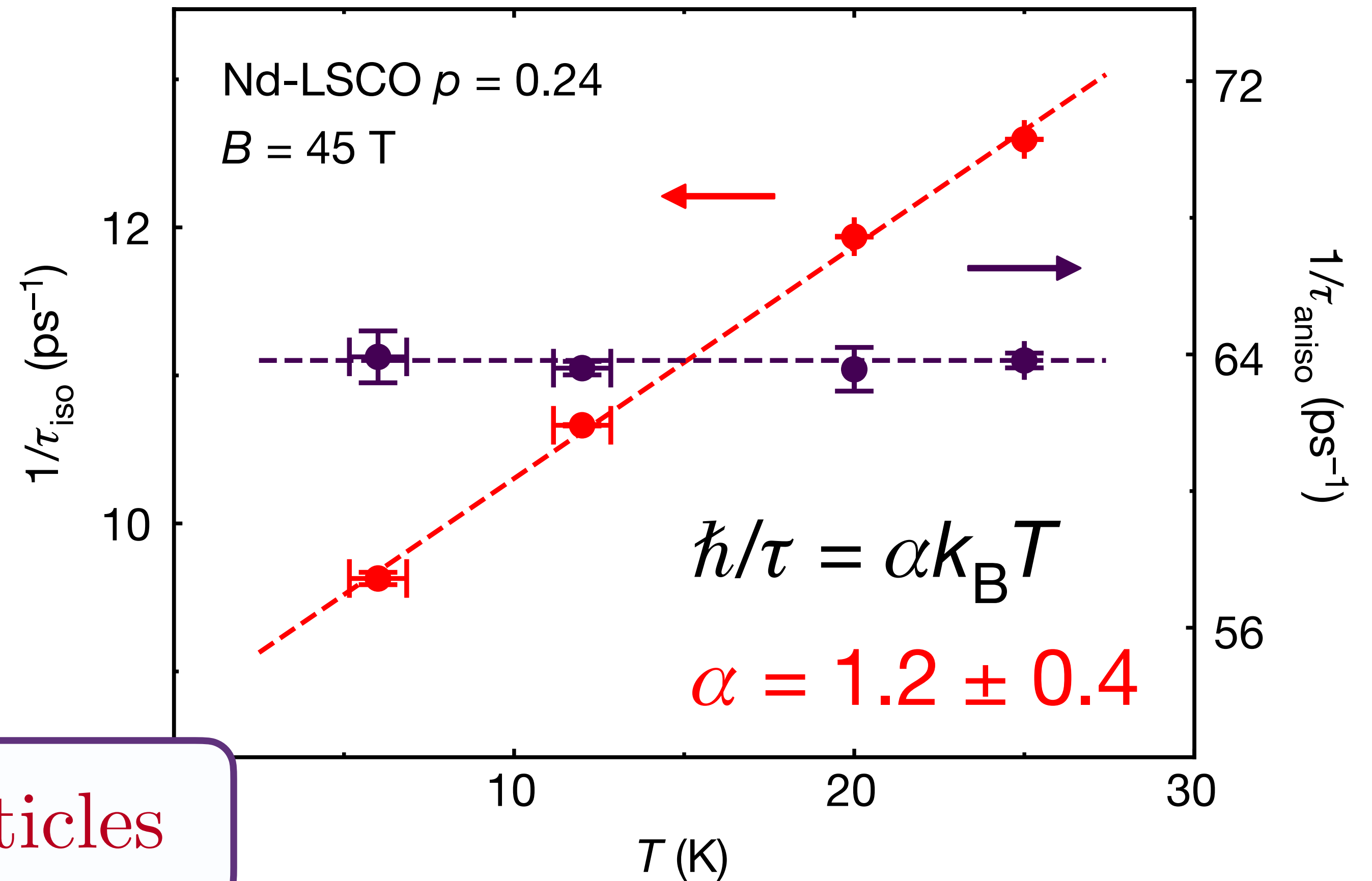
Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw



Current flow without quasiparticles



No Boltzmann-Landau quasiparticle description \Rightarrow
Many particle quantum entanglement
from quantum interference between “collisions”

Sachdev-Ye-Kitaev Model

The SYK model

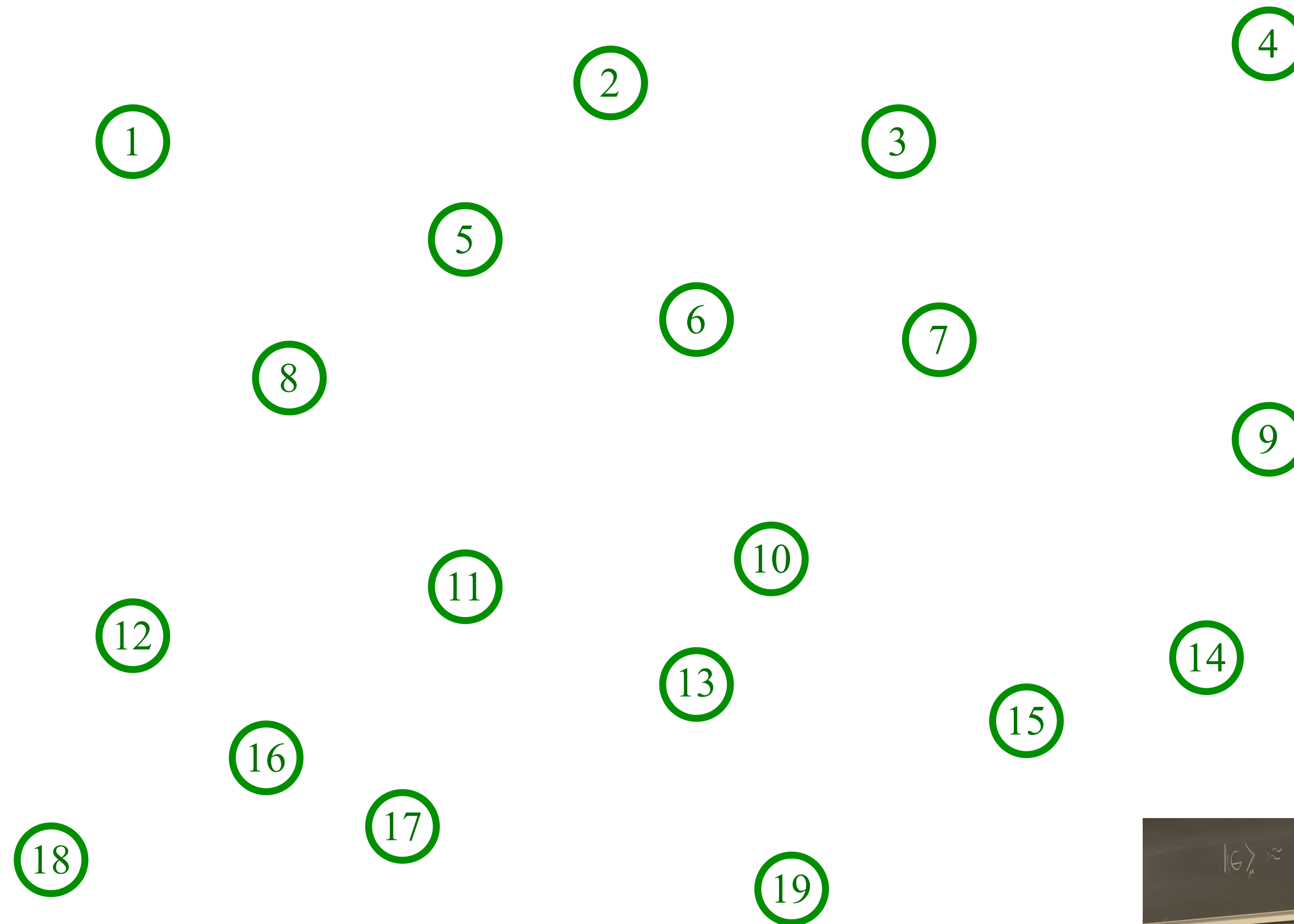
Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle
quantum entanglement.

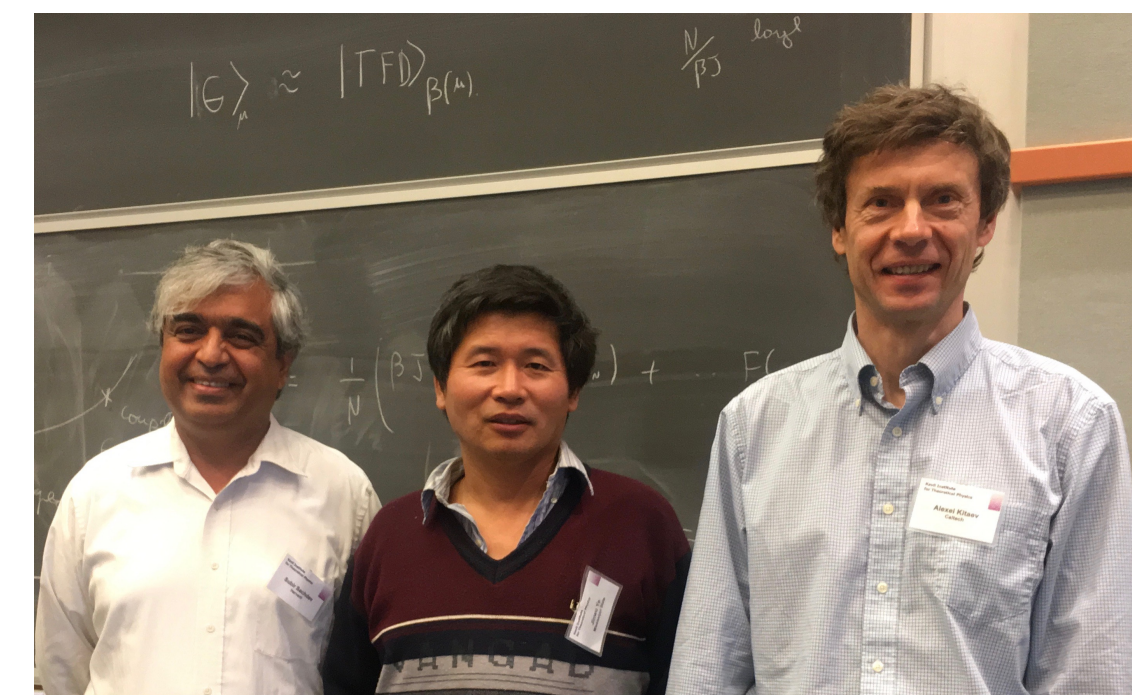
Yields a metal in which current is carried
not by individual electrons,
but by an entangled “quantum soup”

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

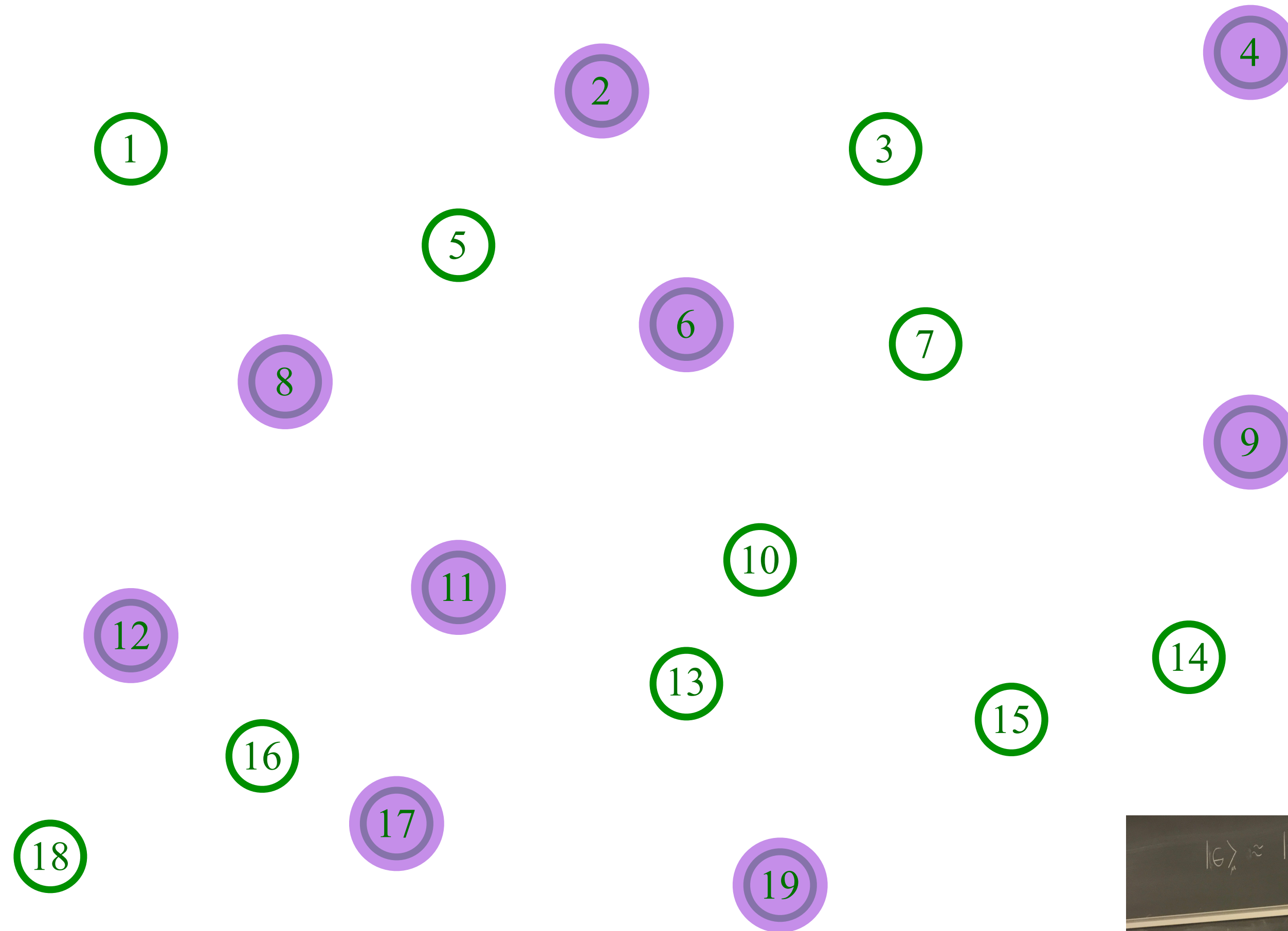


Pick a set of random positions

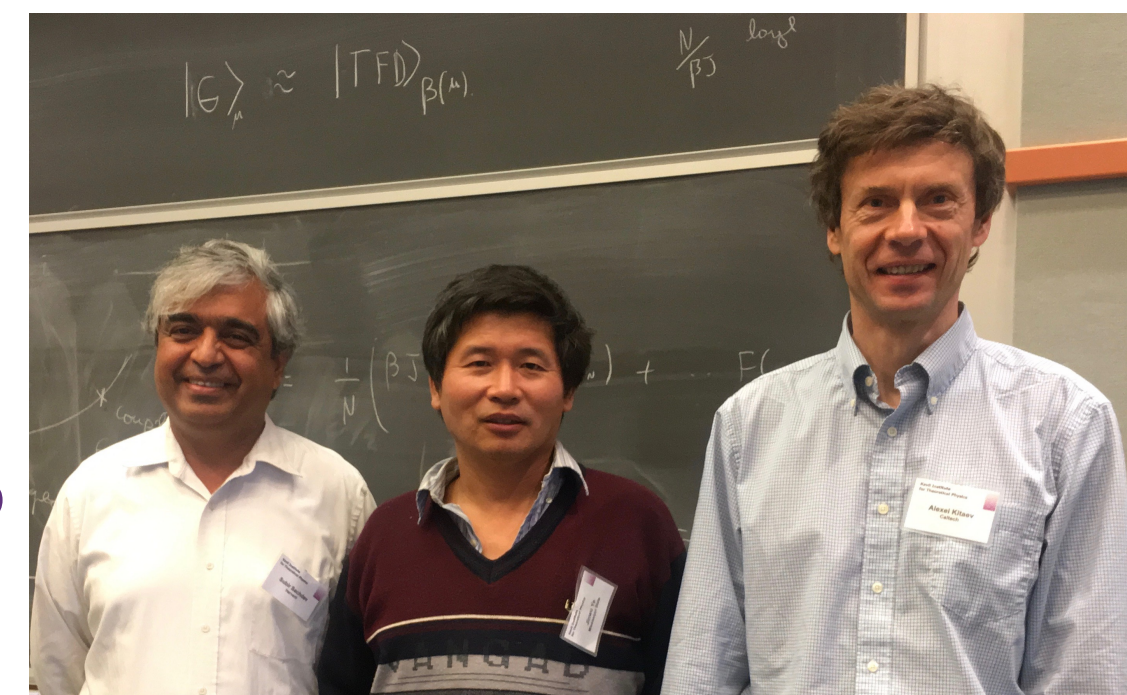


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



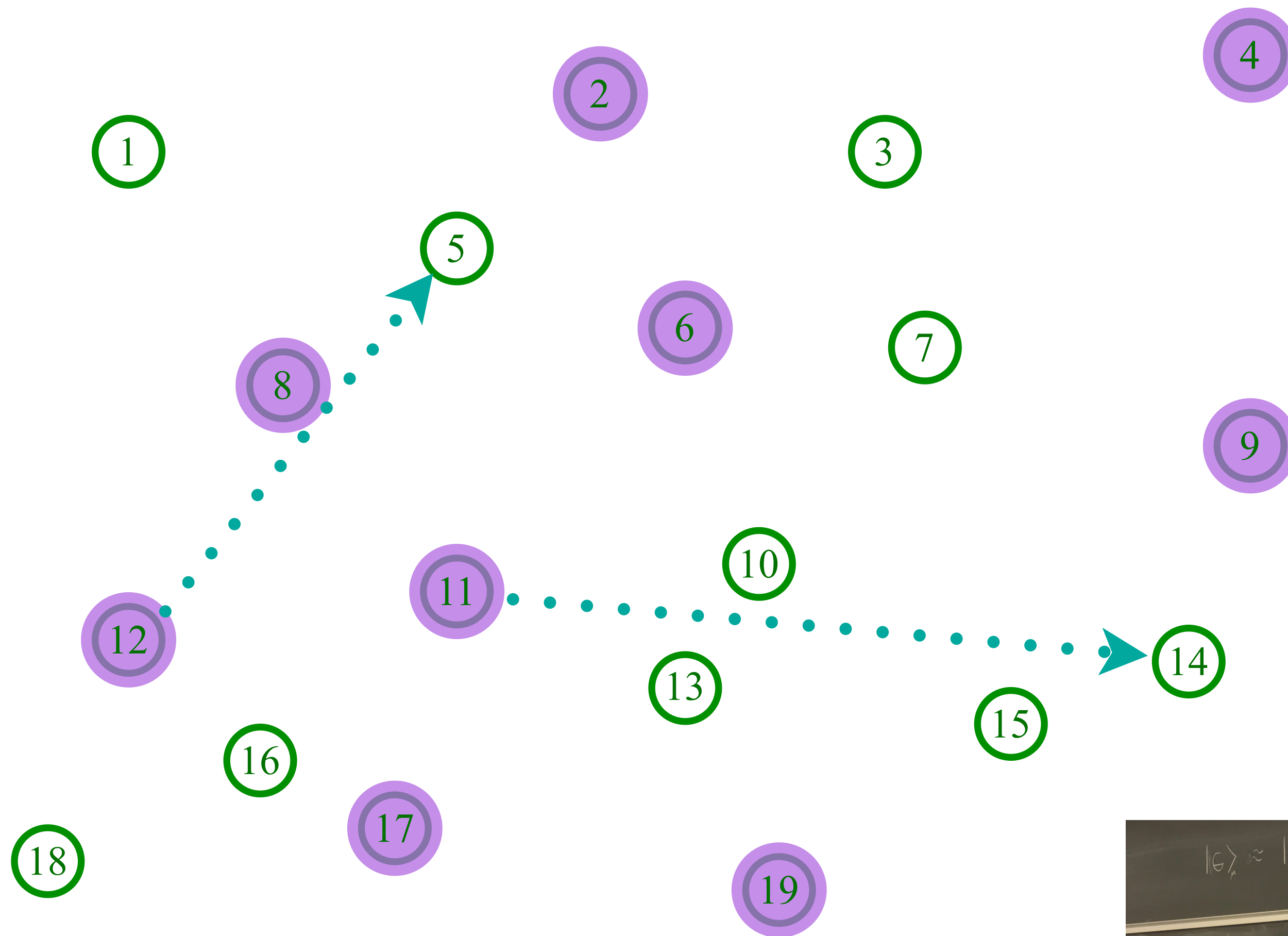
Place electrons randomly on some sites



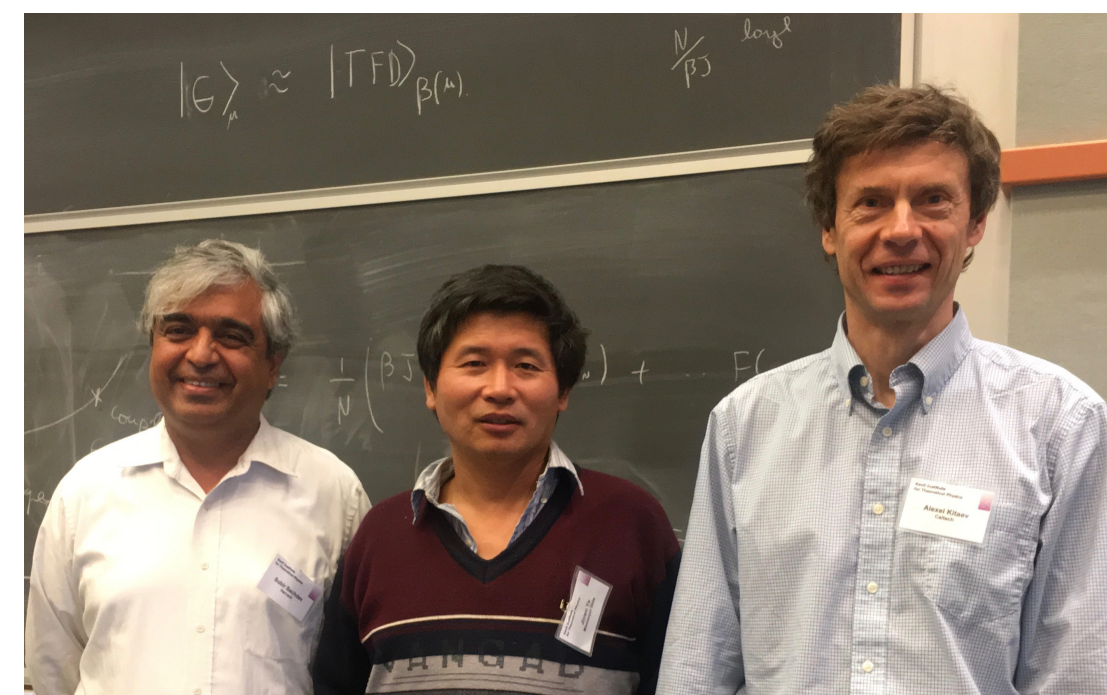
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



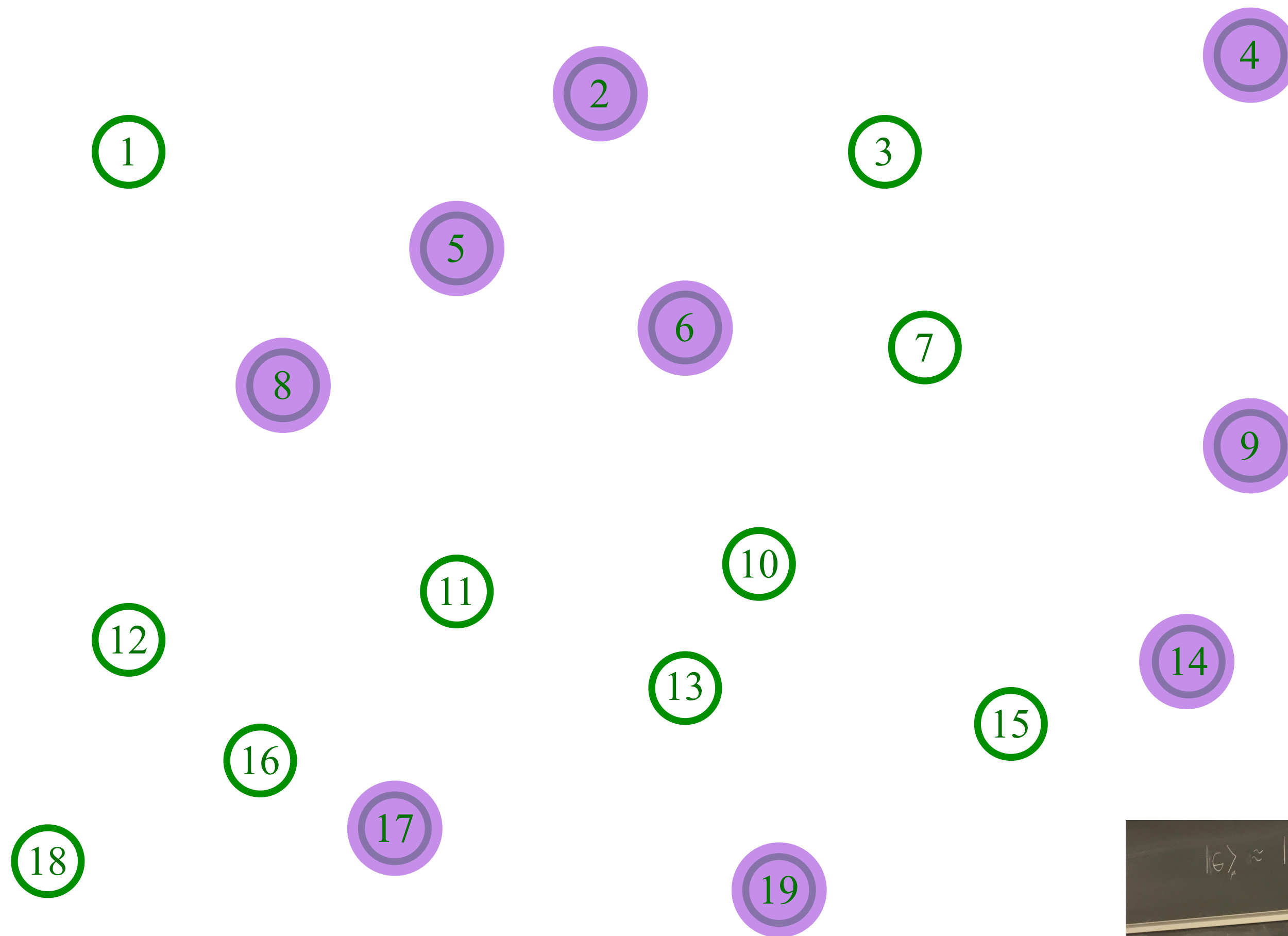
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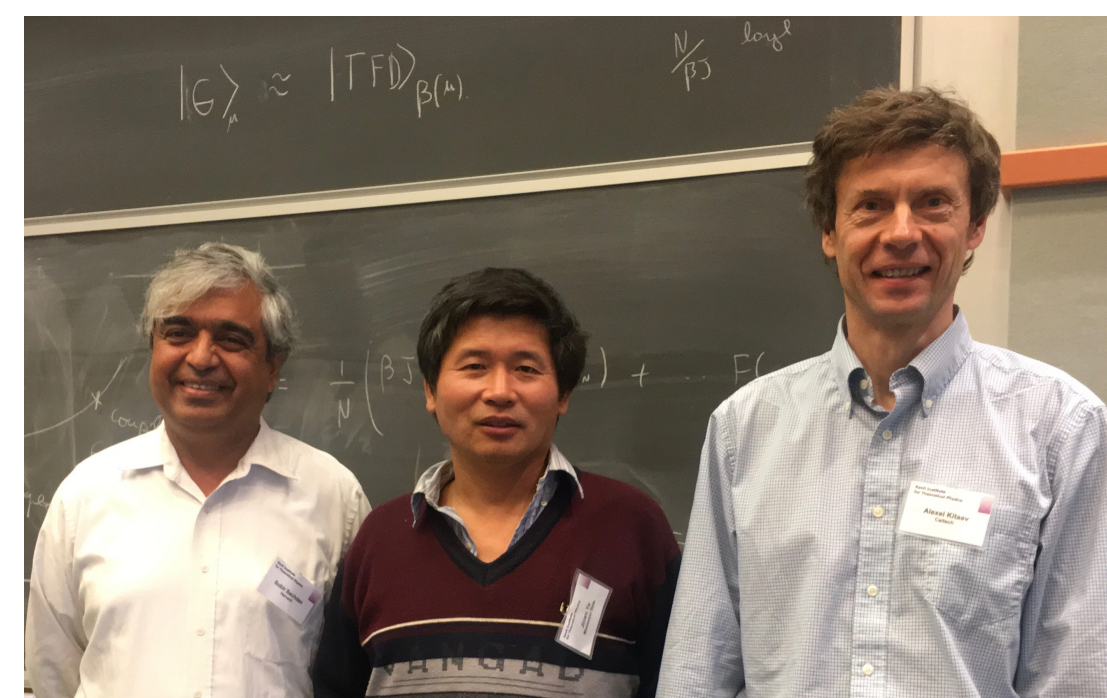
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



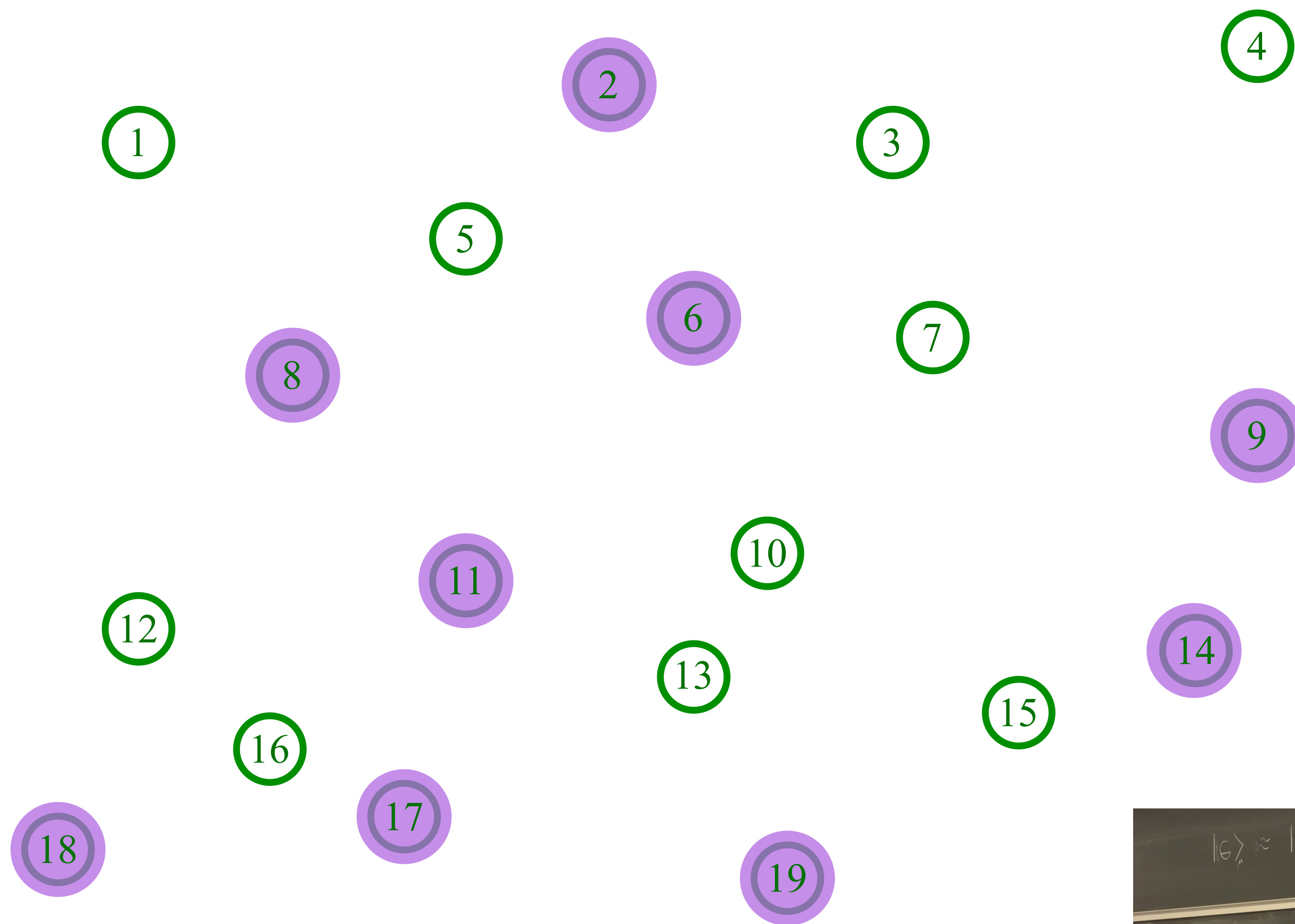
Entangle electrons pairwise randomly



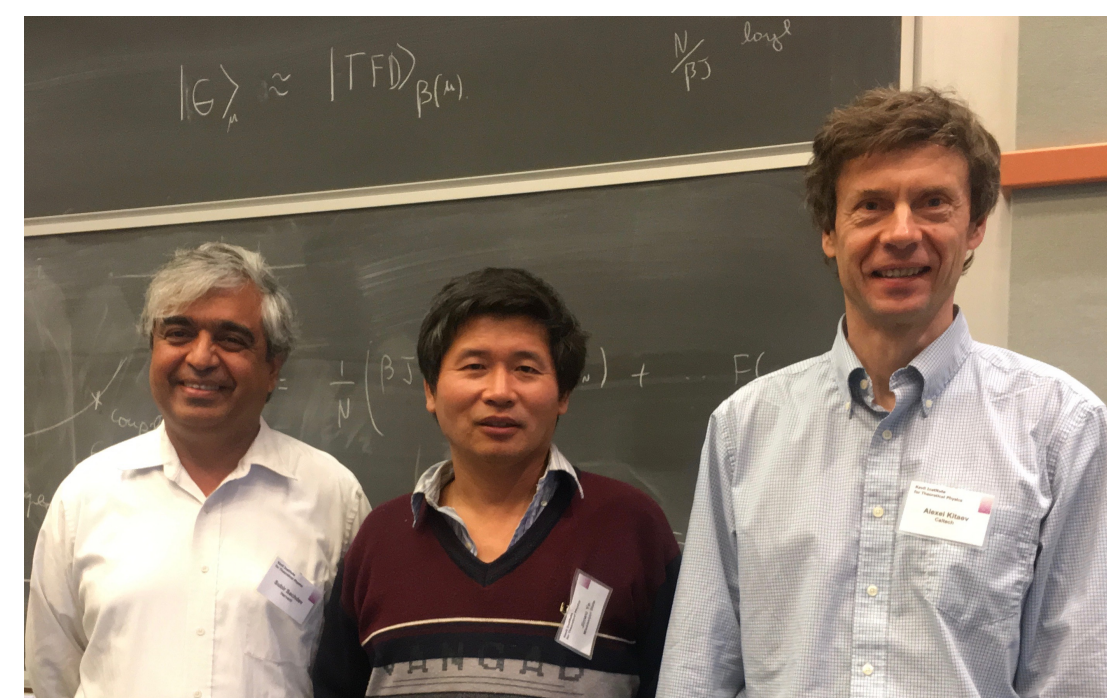
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



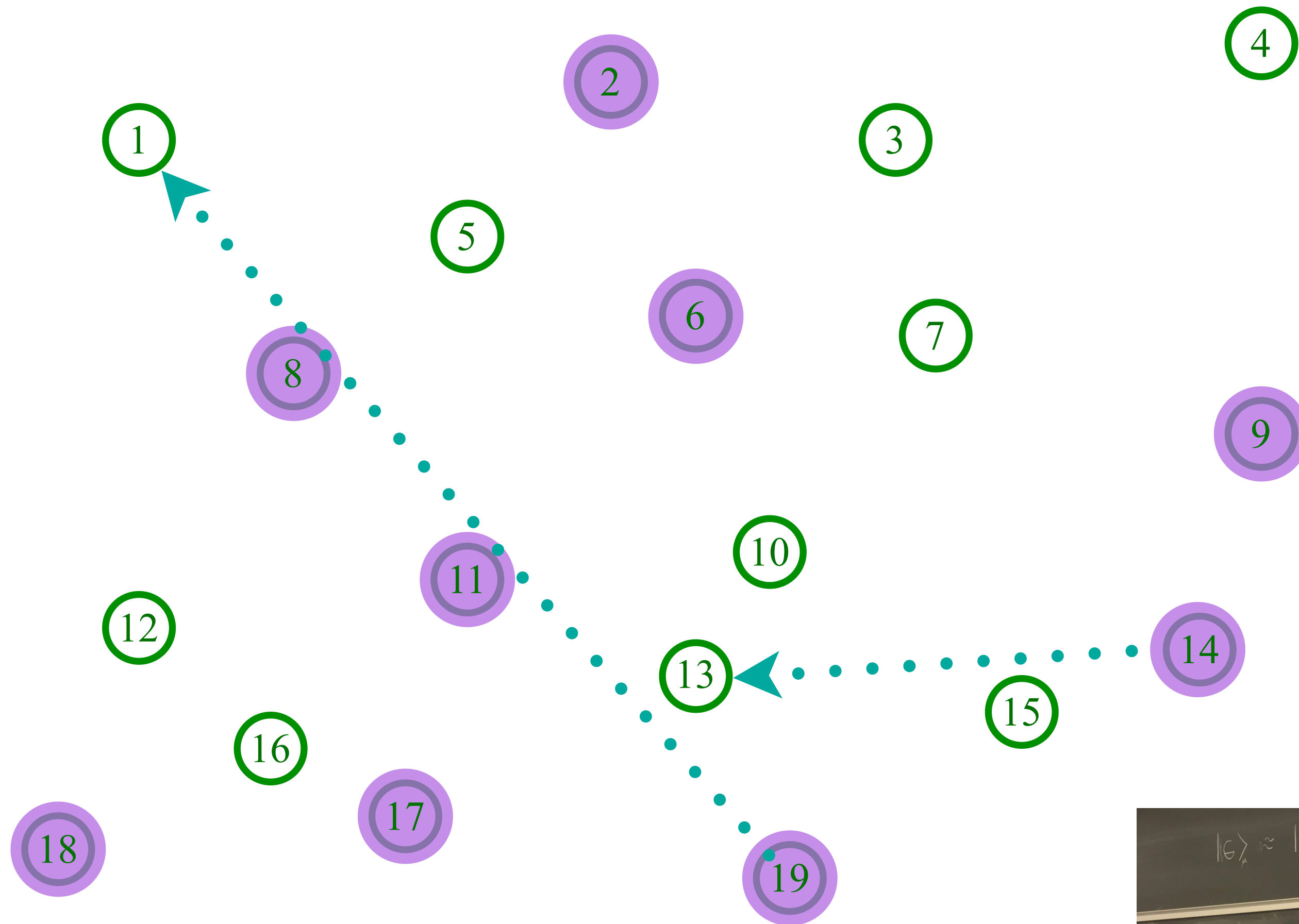
Entangle electrons pairwise randomly



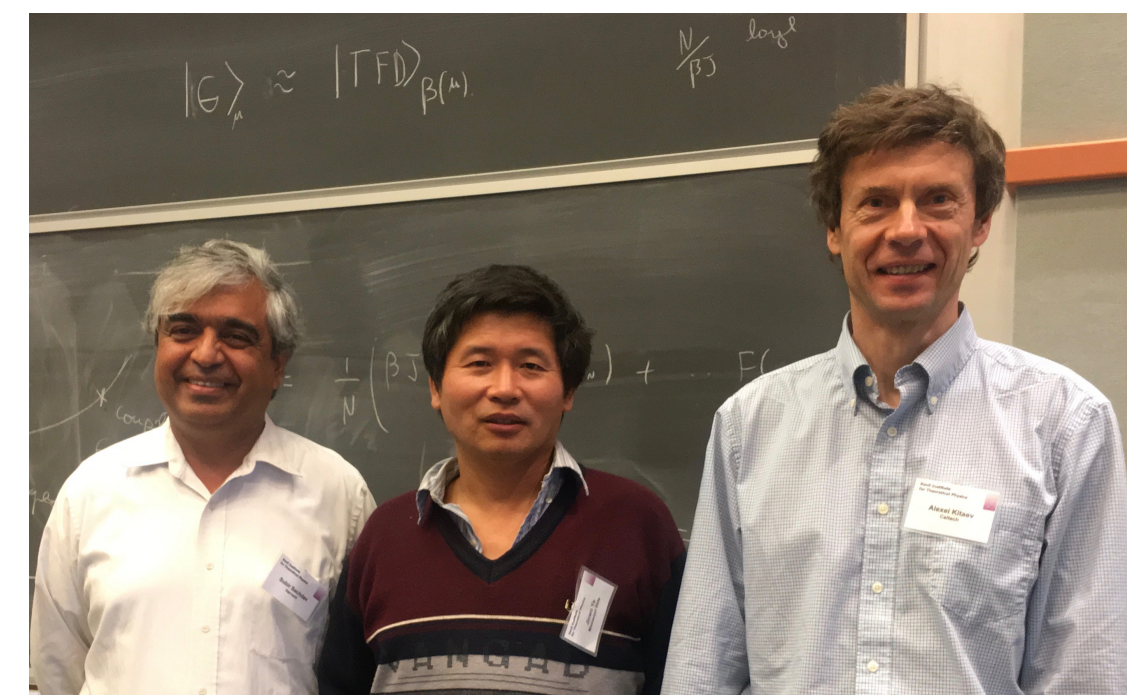
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



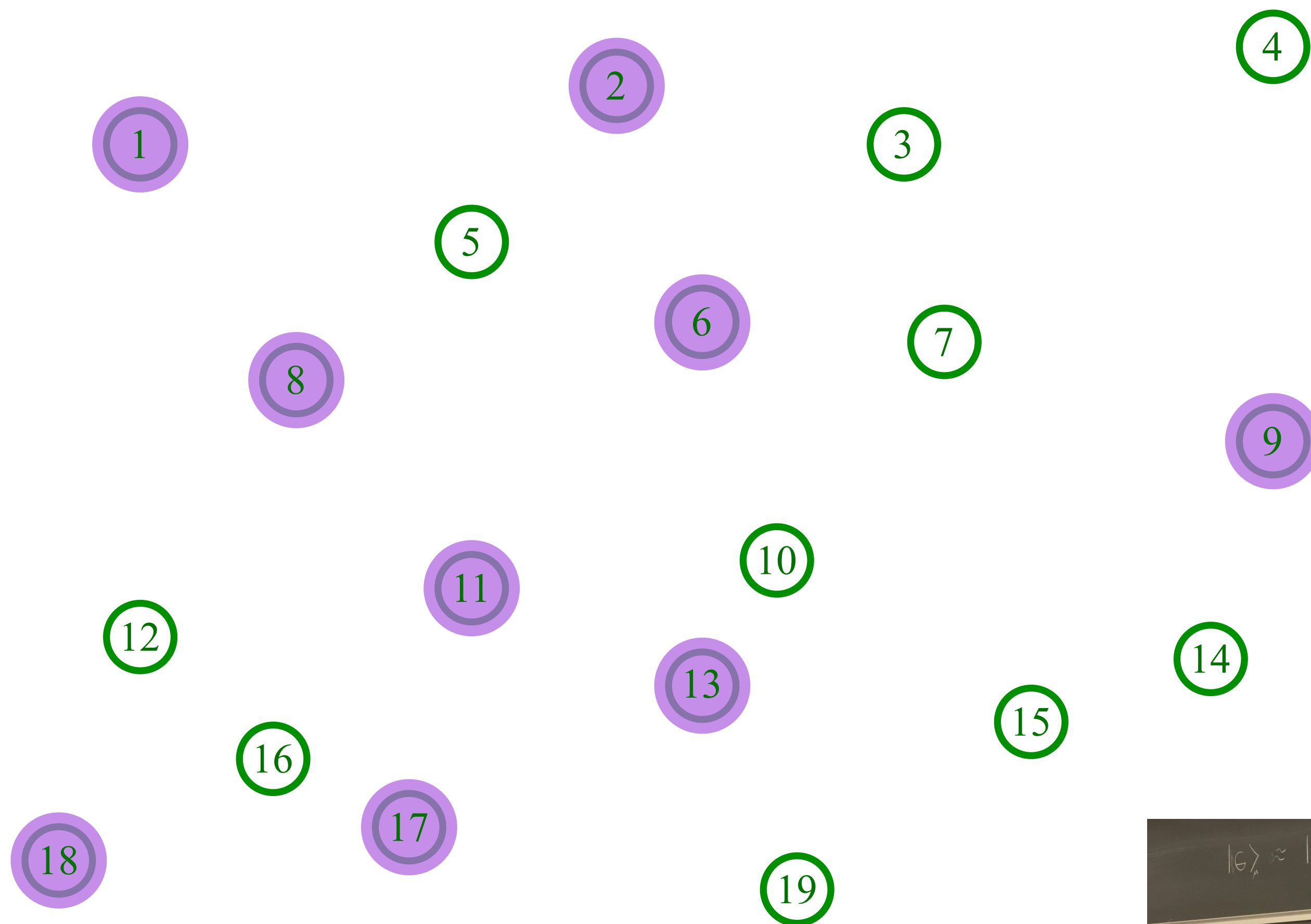
Entangle electrons pairwise randomly



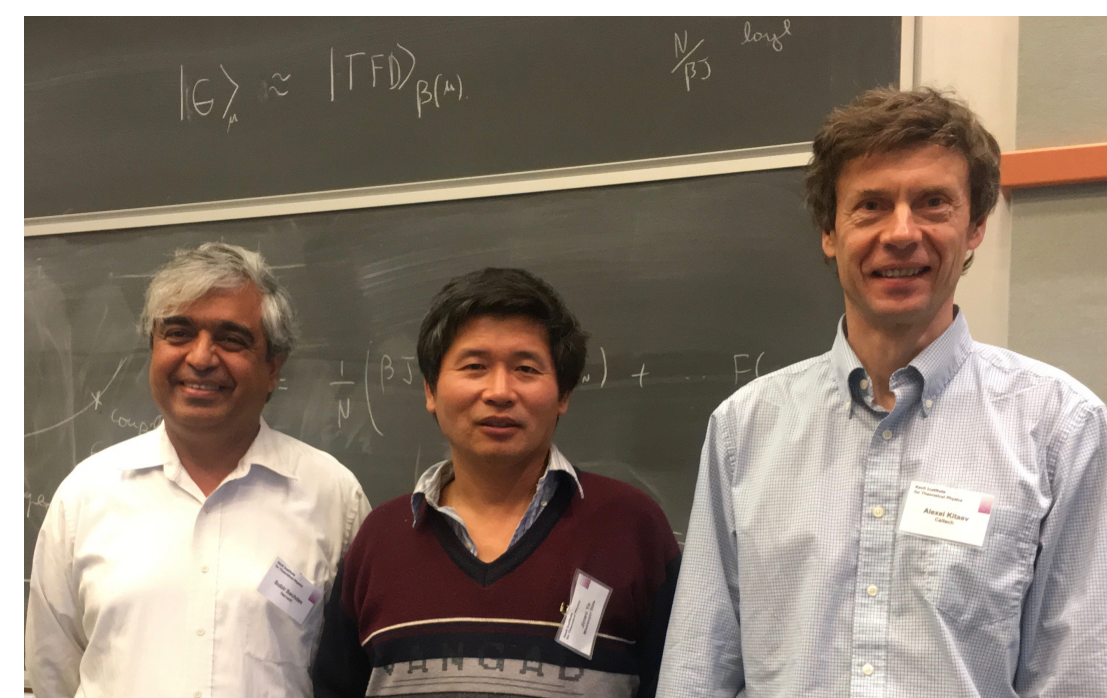
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

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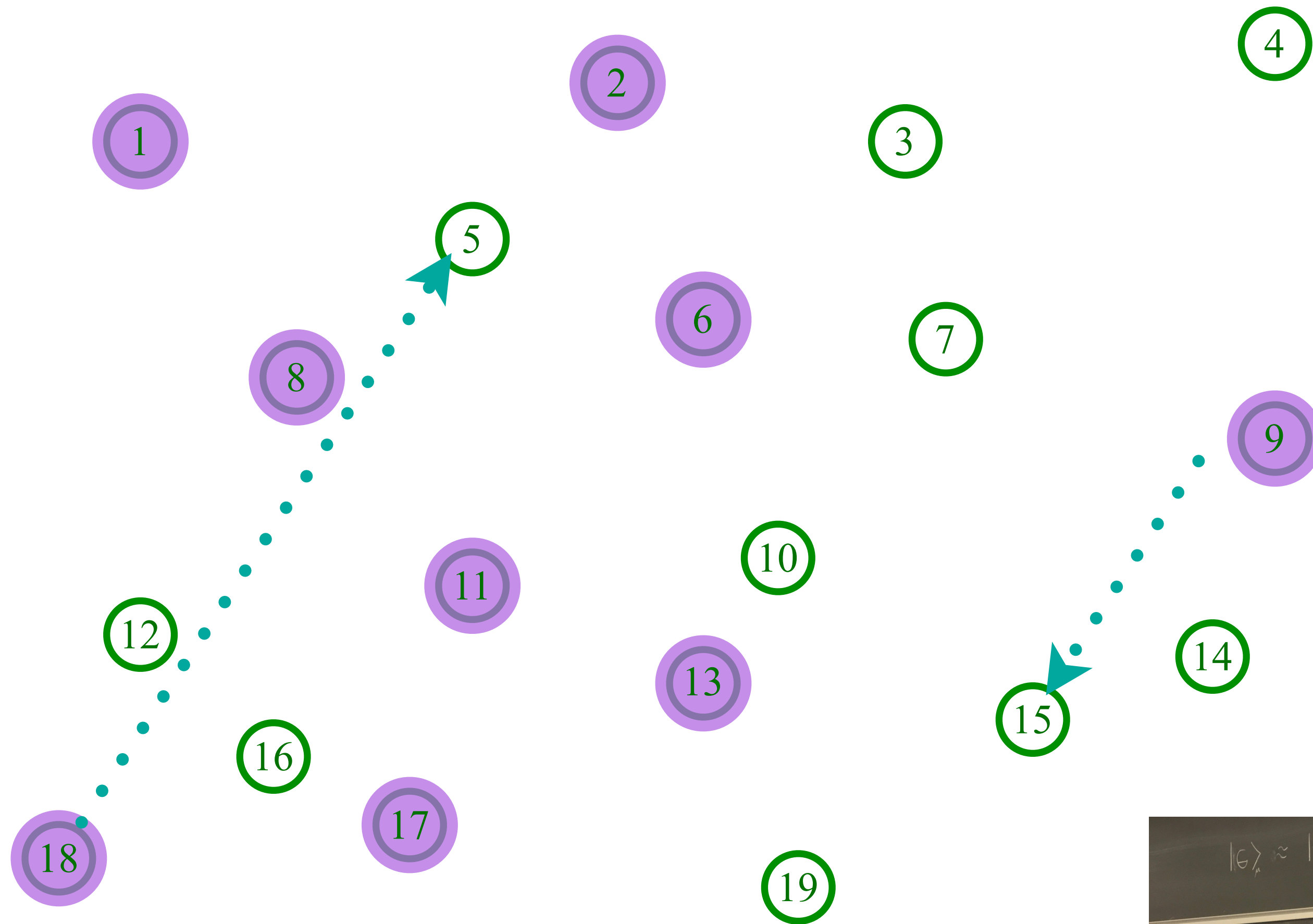
Entangle electrons pairwise randomly



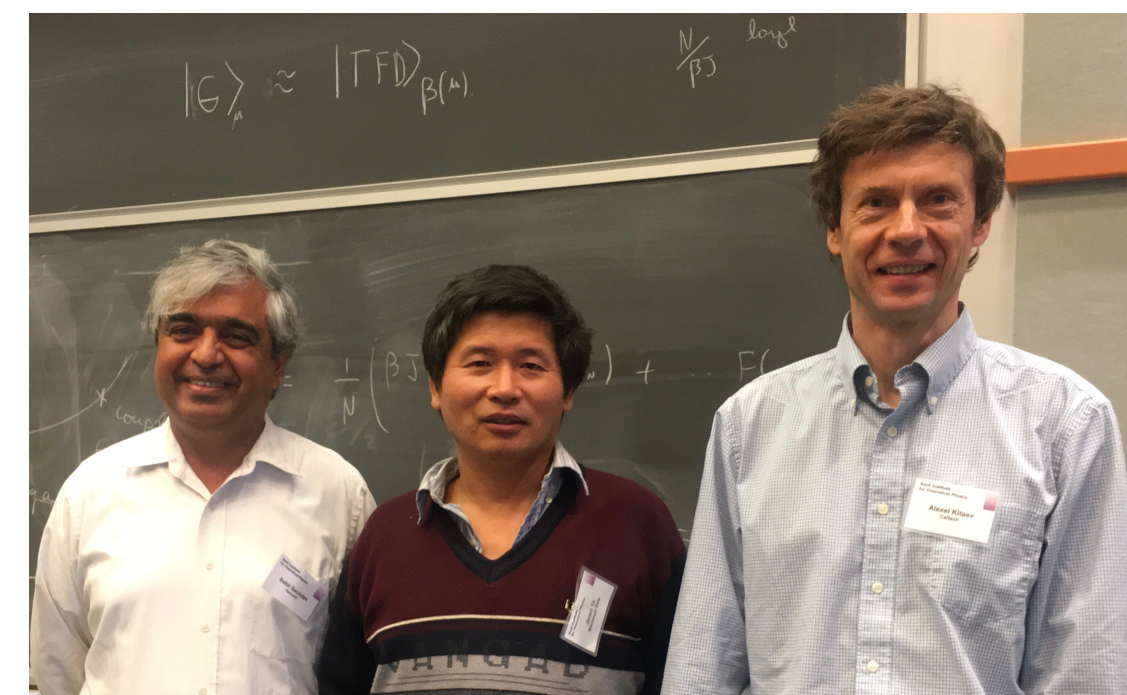
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



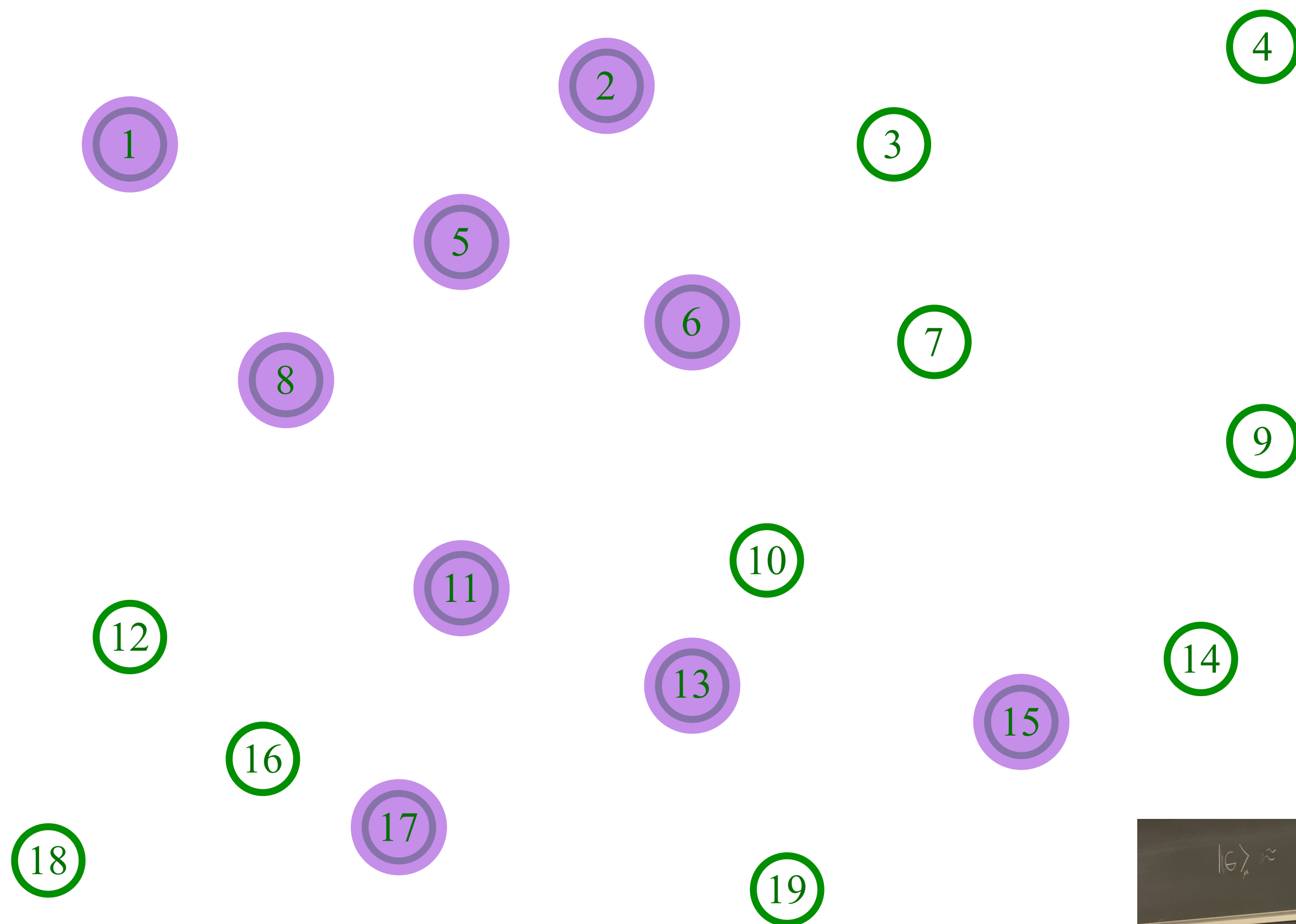
Entangle electrons pairwise randomly



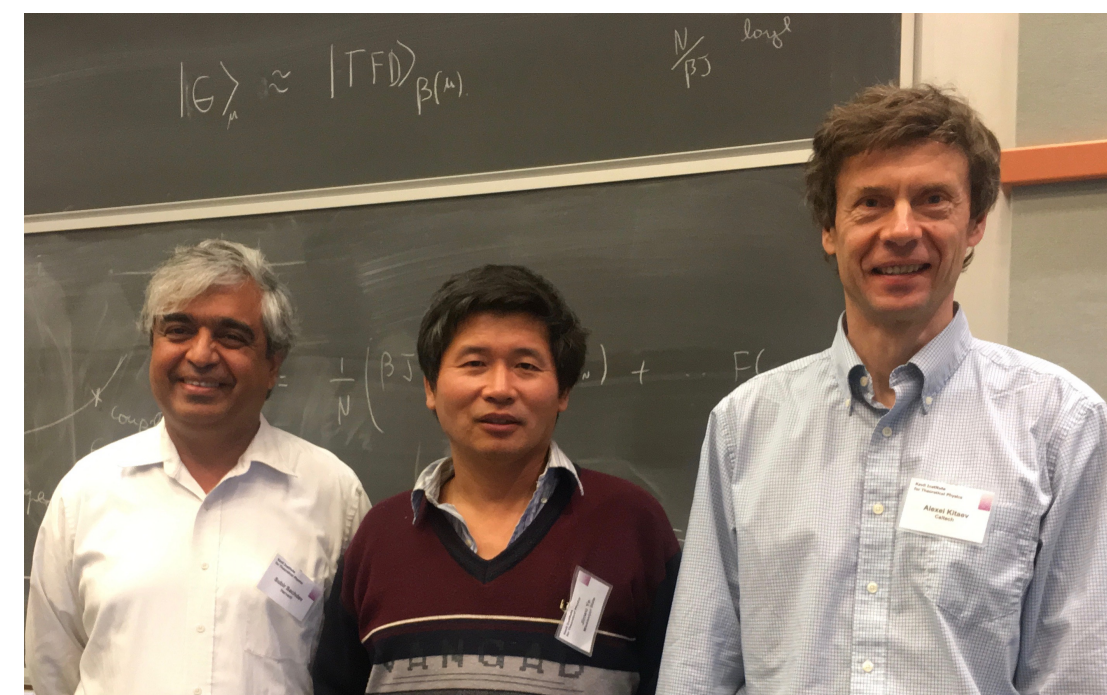
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

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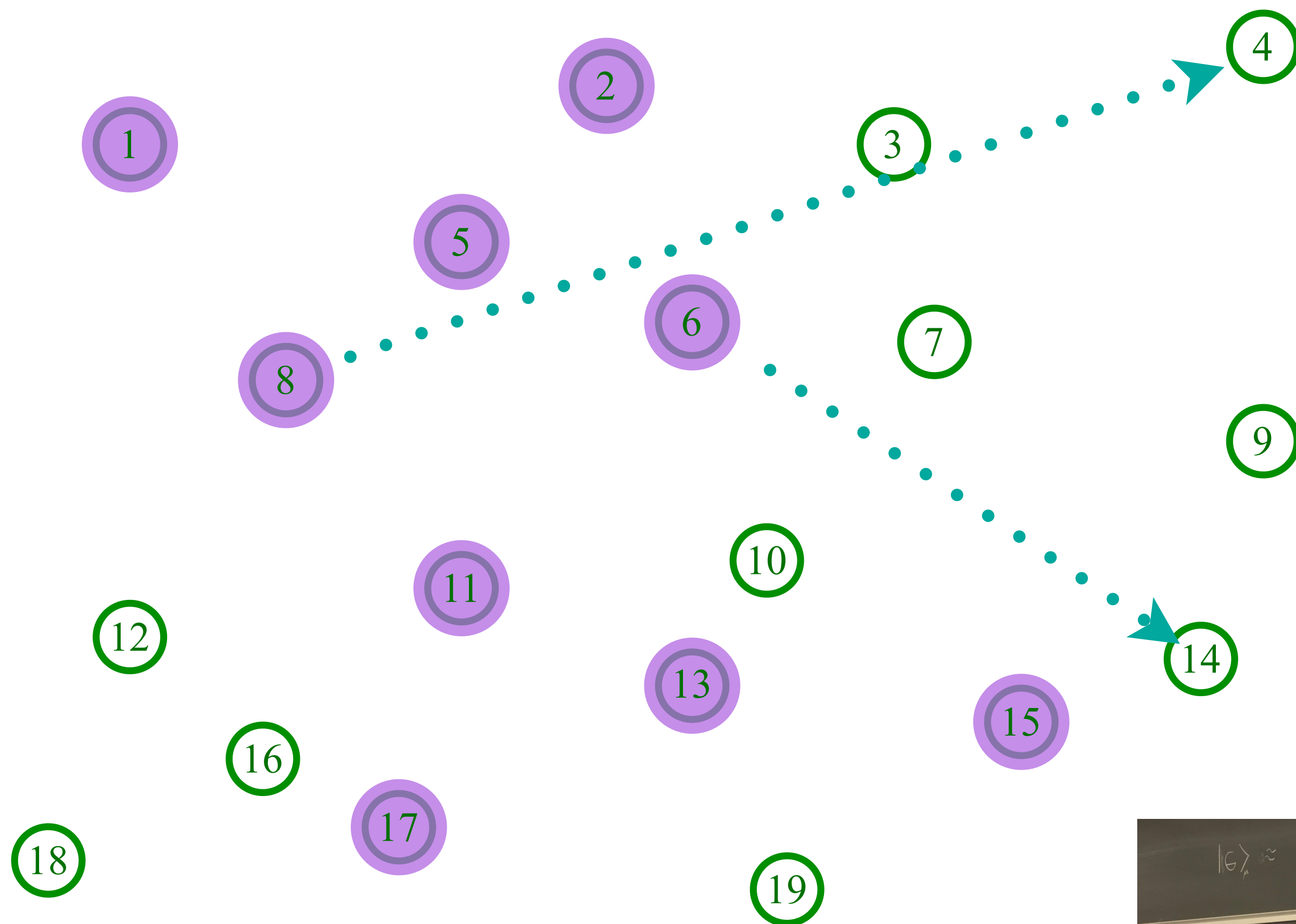
Entangle electrons pairwise randomly



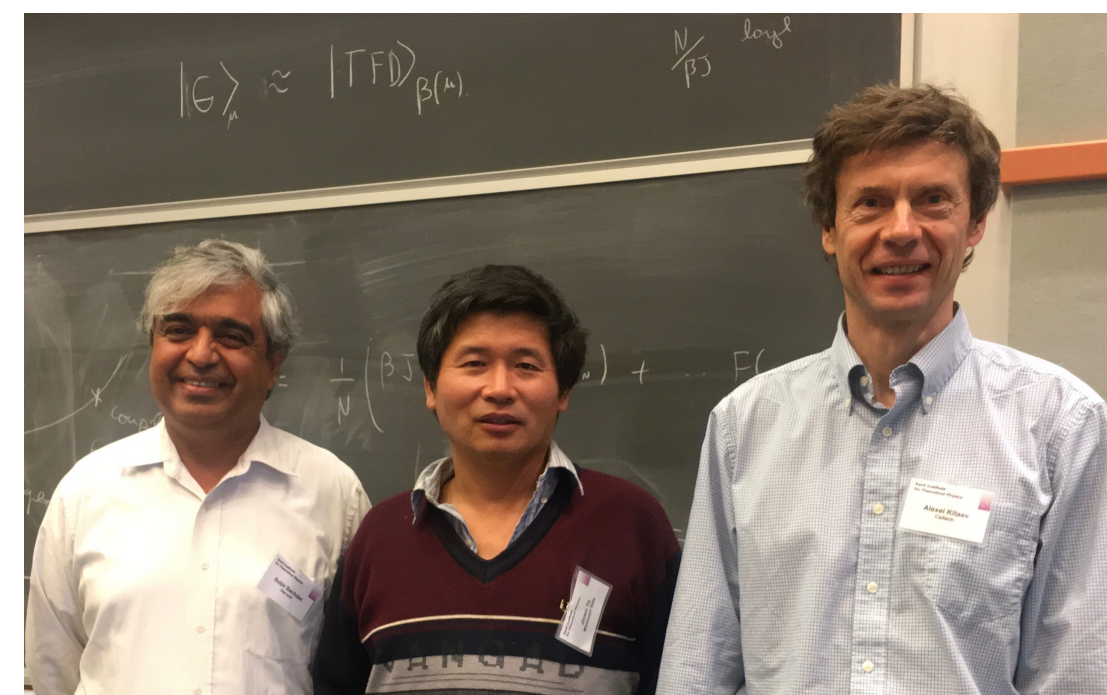
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



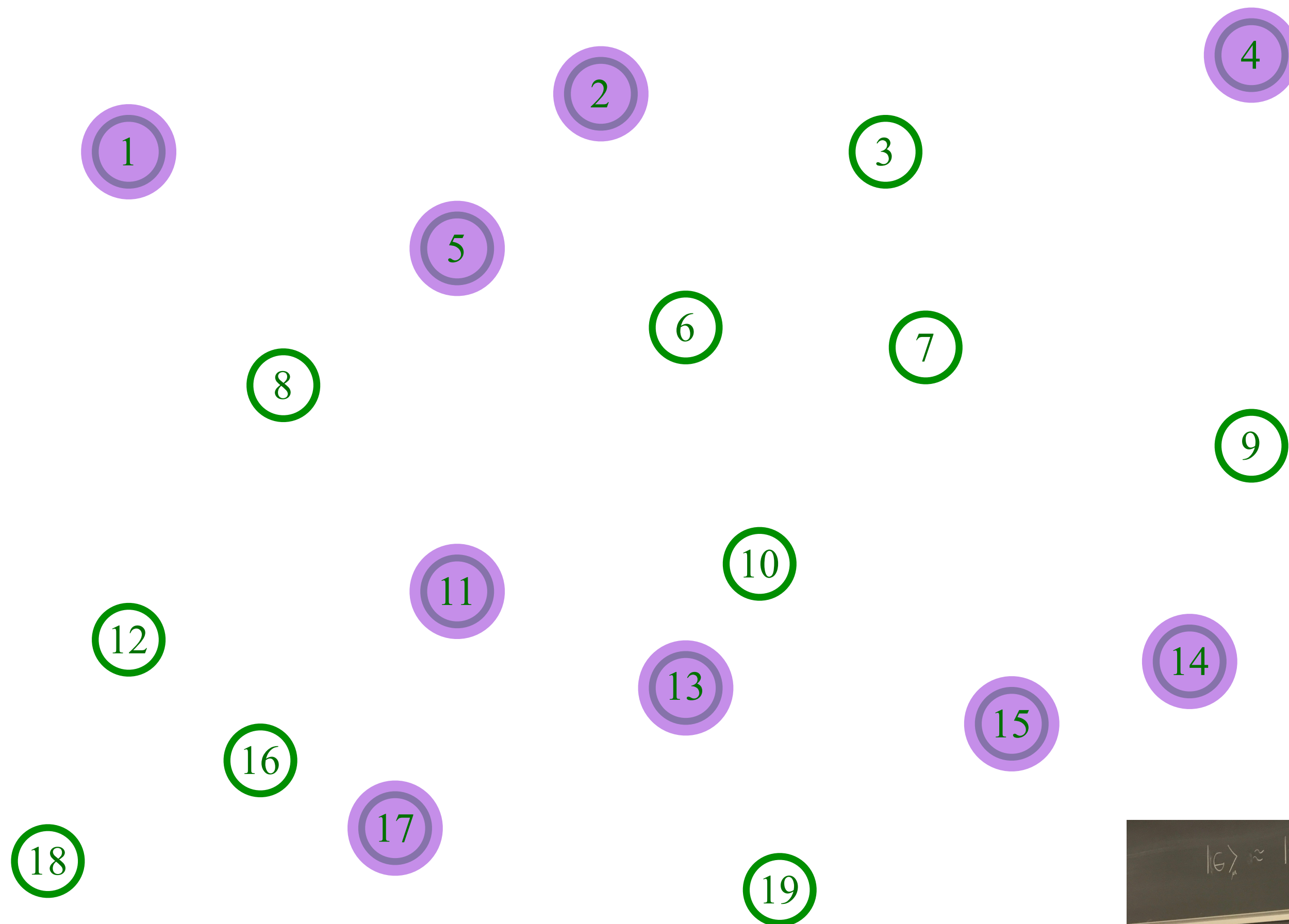
Entangle electrons pairwise randomly



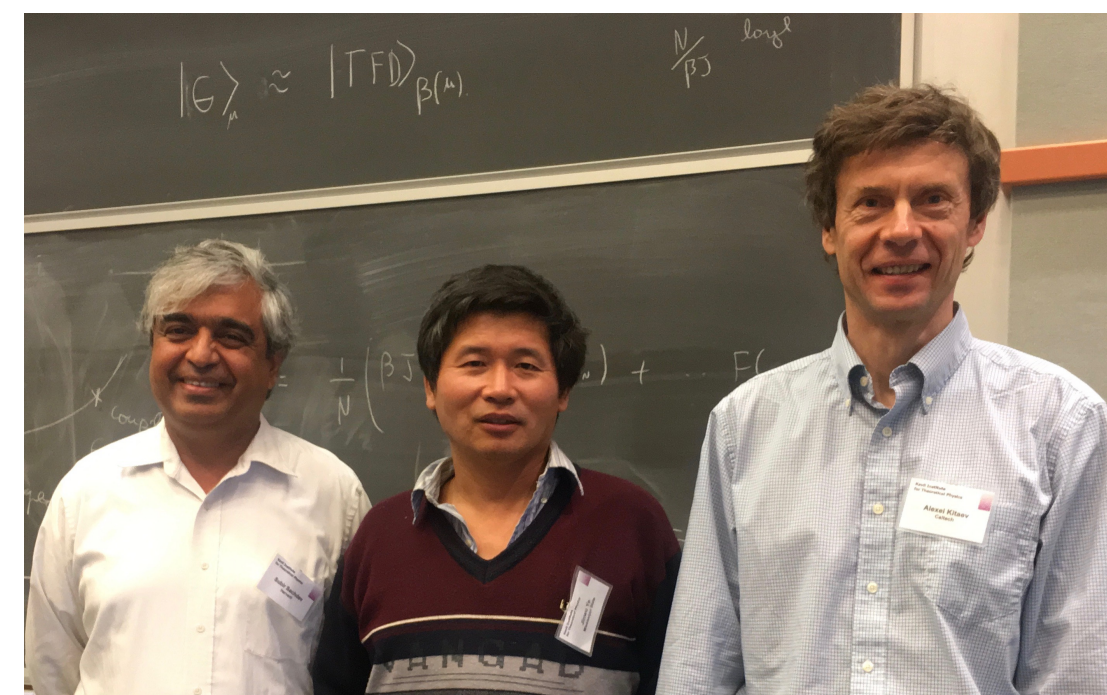
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

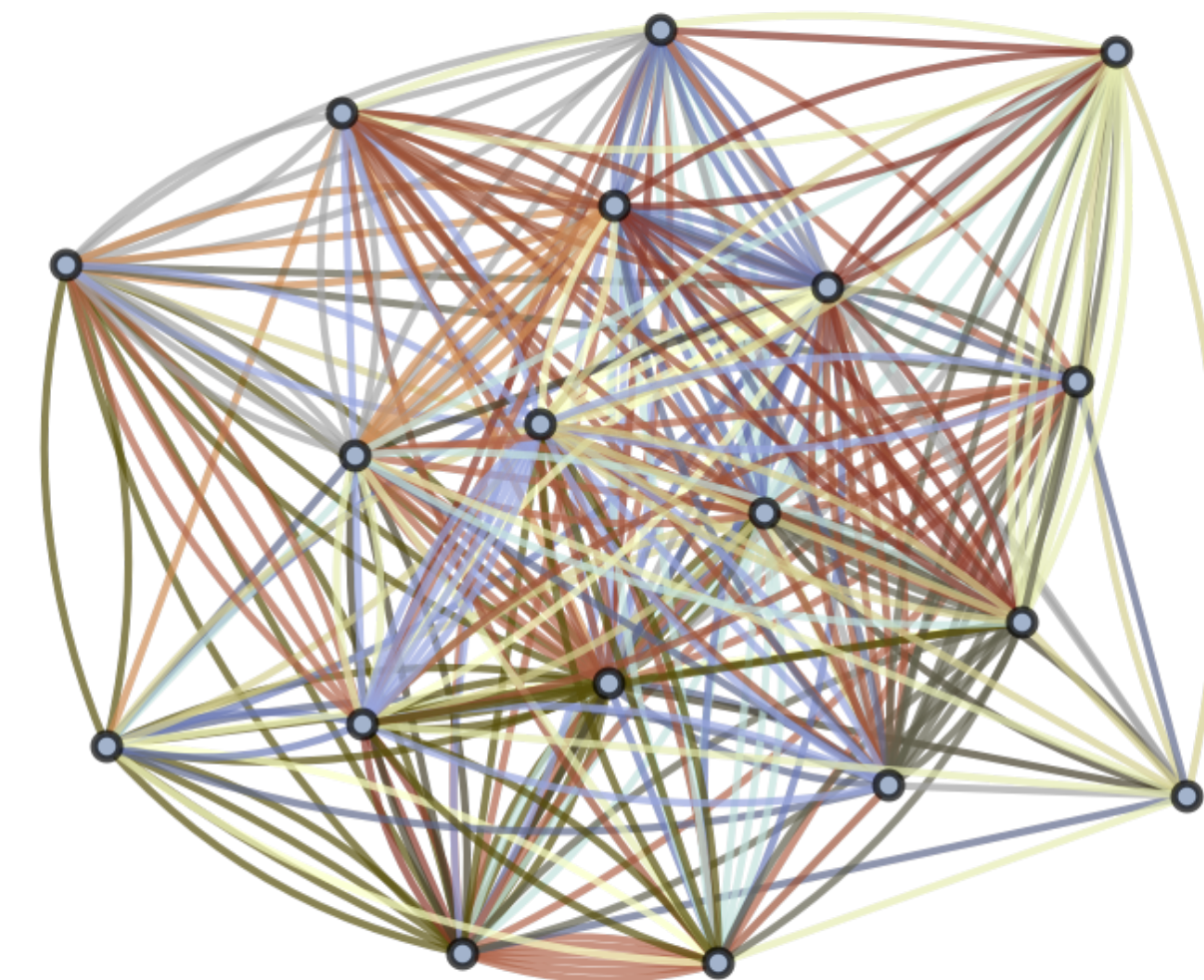
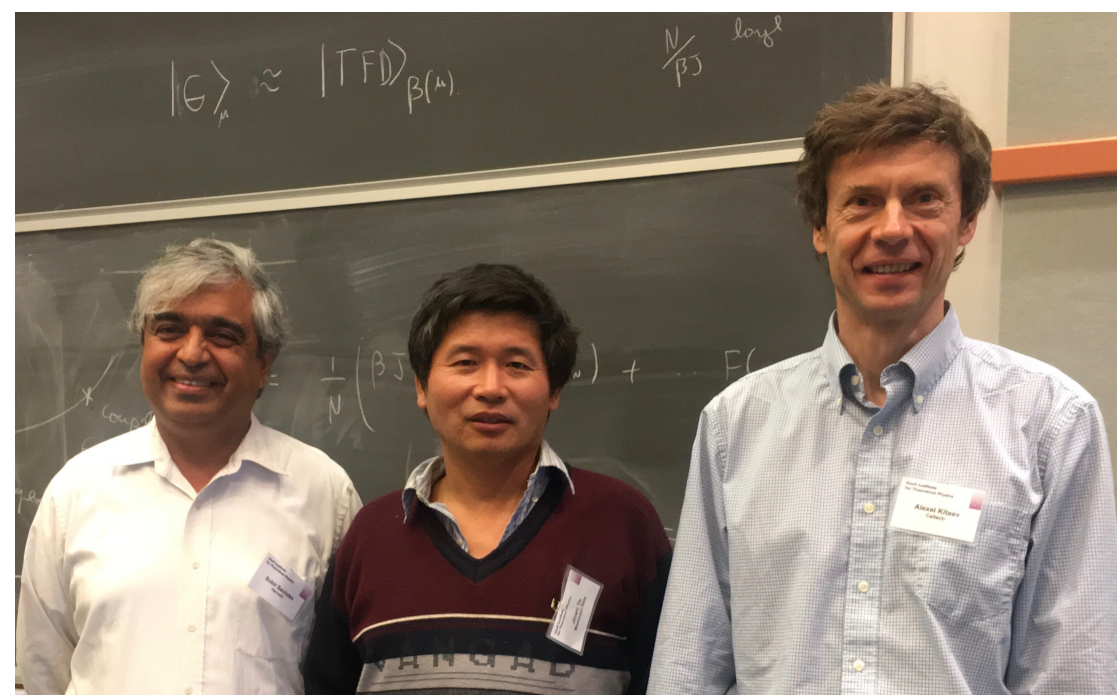
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta; \gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta; \gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta; \gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

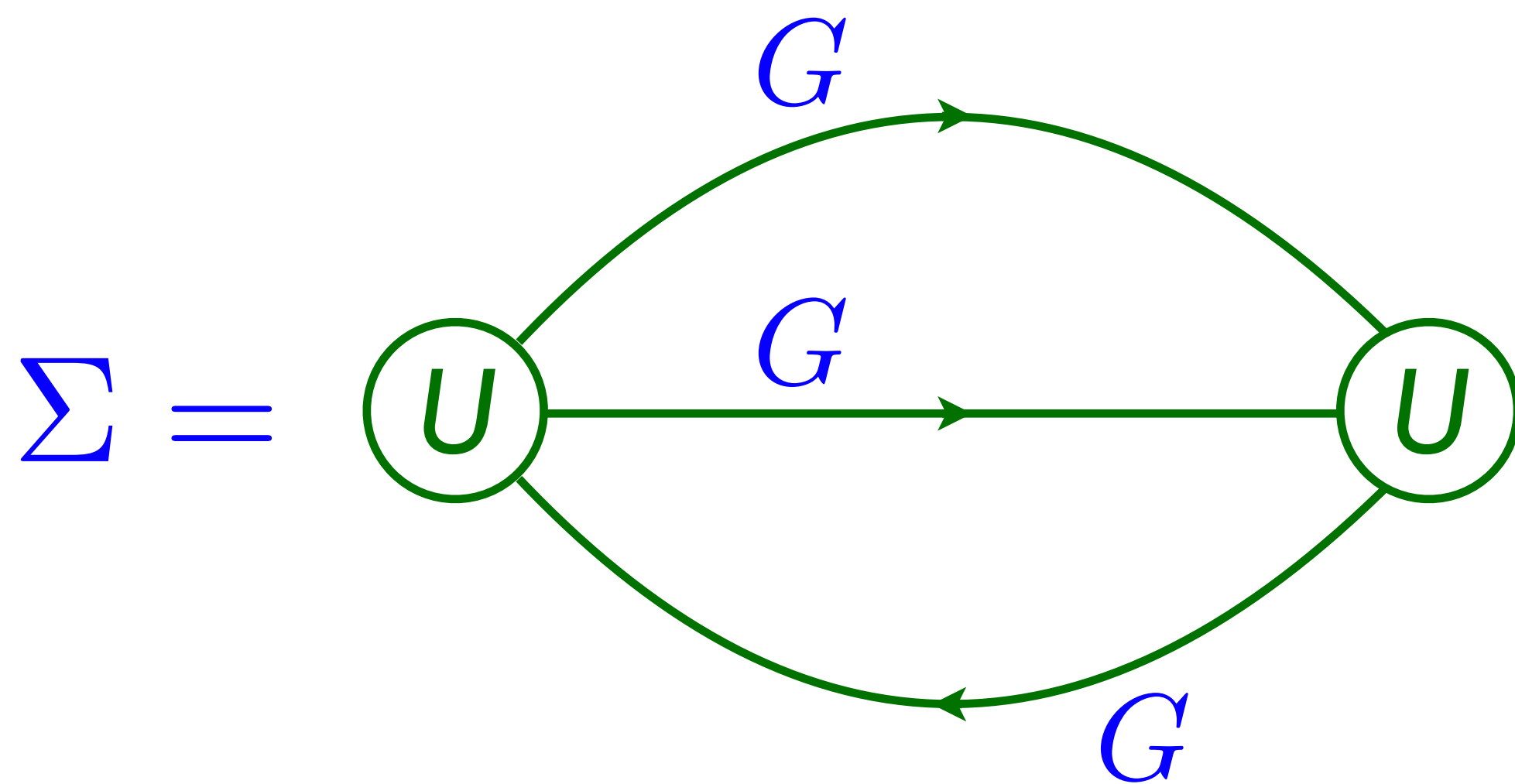


The SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function

$G(\tau) = -\sum_{\alpha} \langle c_{\alpha}(\tau) c_{\alpha}^{\dagger}(\tau) \rangle / N$ in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$

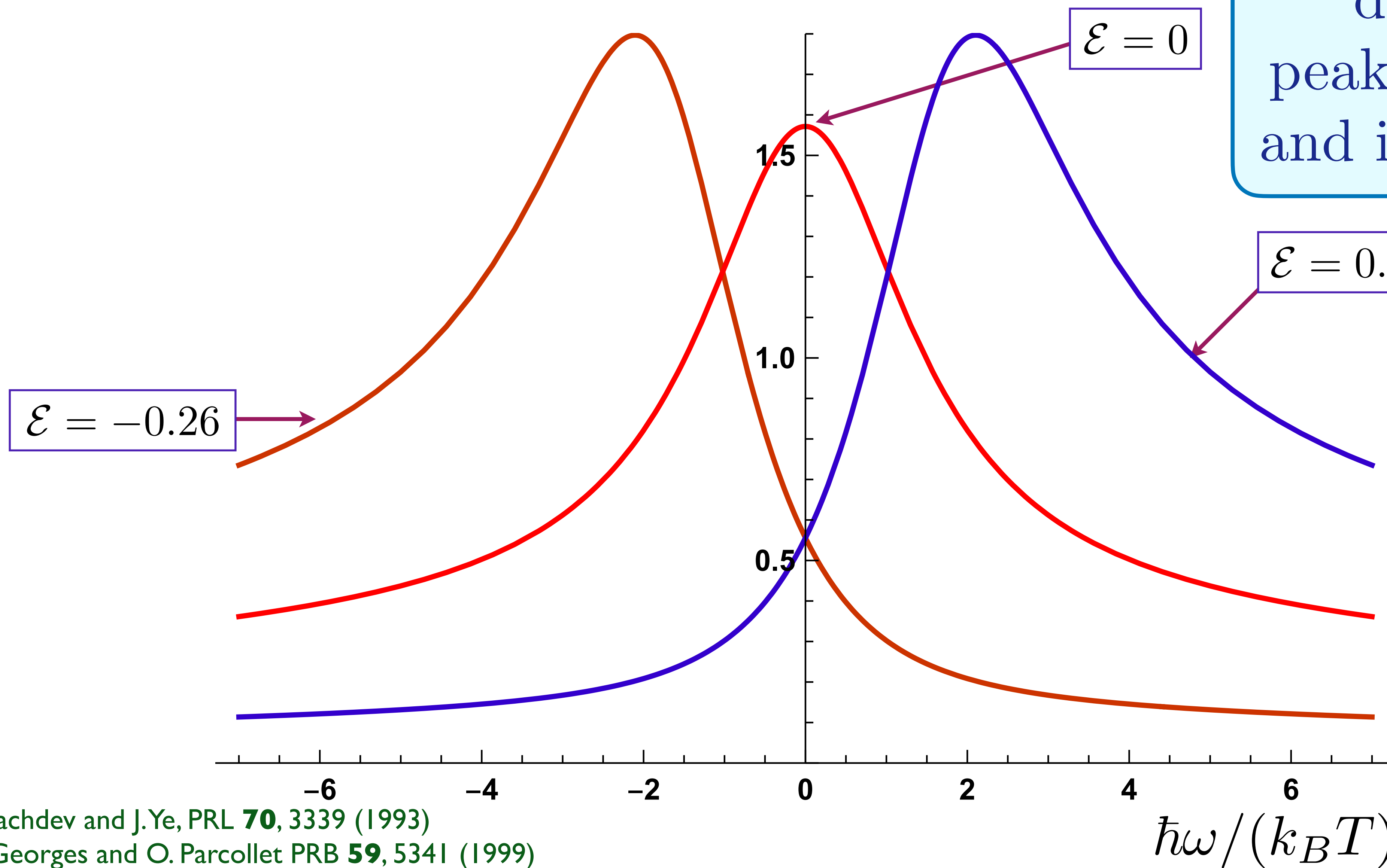


S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)



The SYK model

$$-\text{Im}G^R(\omega)$$



Conformal 'Planckian' dynamics with peak width $\sim k_B T / \hbar$ and independent of U

The SYK model

The (averaged) partition function can be written as path integral over the bilocal fermion Green's function $G(\tau_1, \tau_2) \sim \frac{1}{N} \sum_{\alpha} c_{\alpha}(\tau_1) c_{\alpha}^{\dagger}(\tau_2)$

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(\tau_1, \tau_2) \exp(-N S_{\text{eff}}[G])$$

The large N saddle point equation $\delta S_{\text{eff}}/\delta G = 0$ for $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ is

$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

Time reparameterization symmetry:

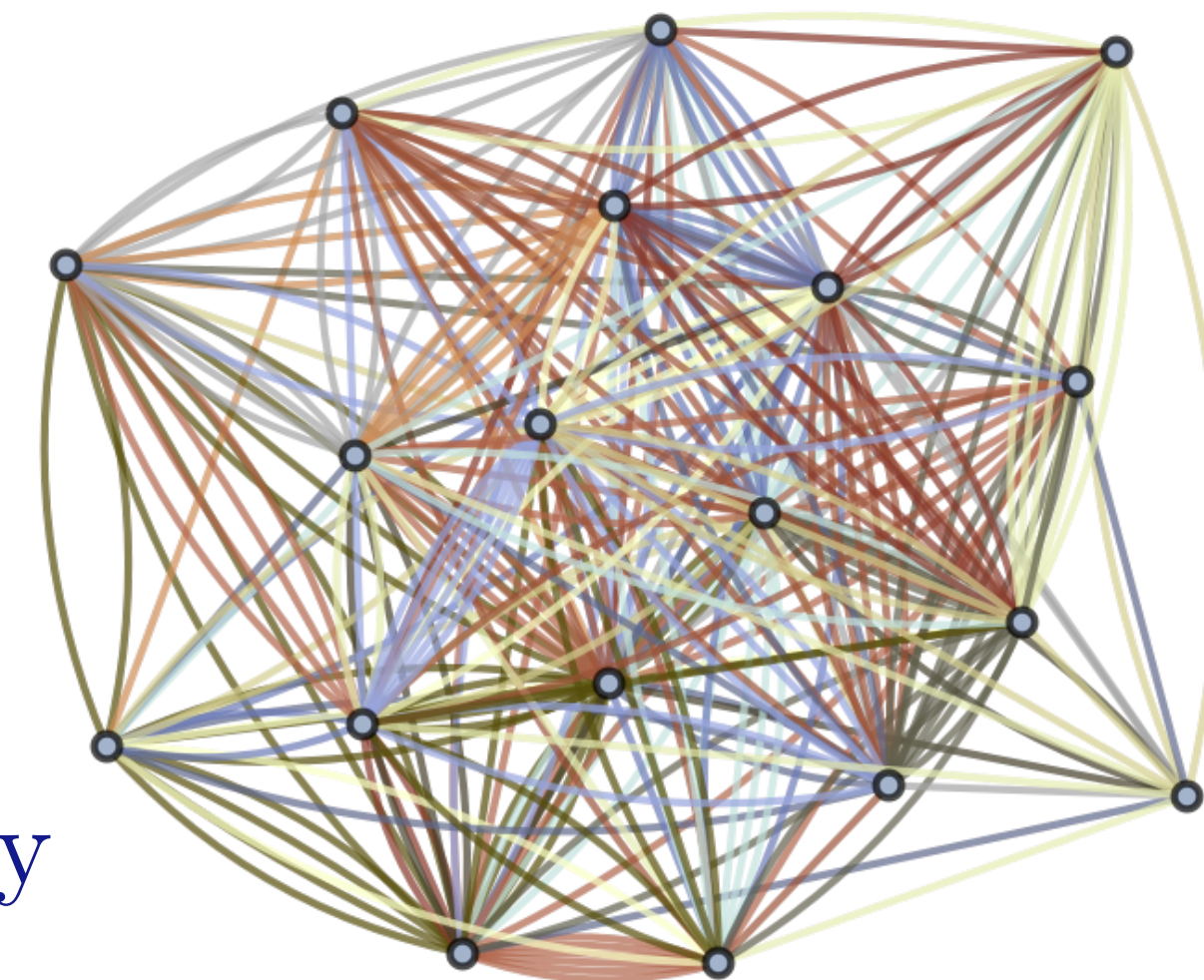
At frequencies $\ll U$, the path integral for is invariant under time reparameterization $f(\sigma)$

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \tilde{G}(\sigma_1, \sigma_2)$$

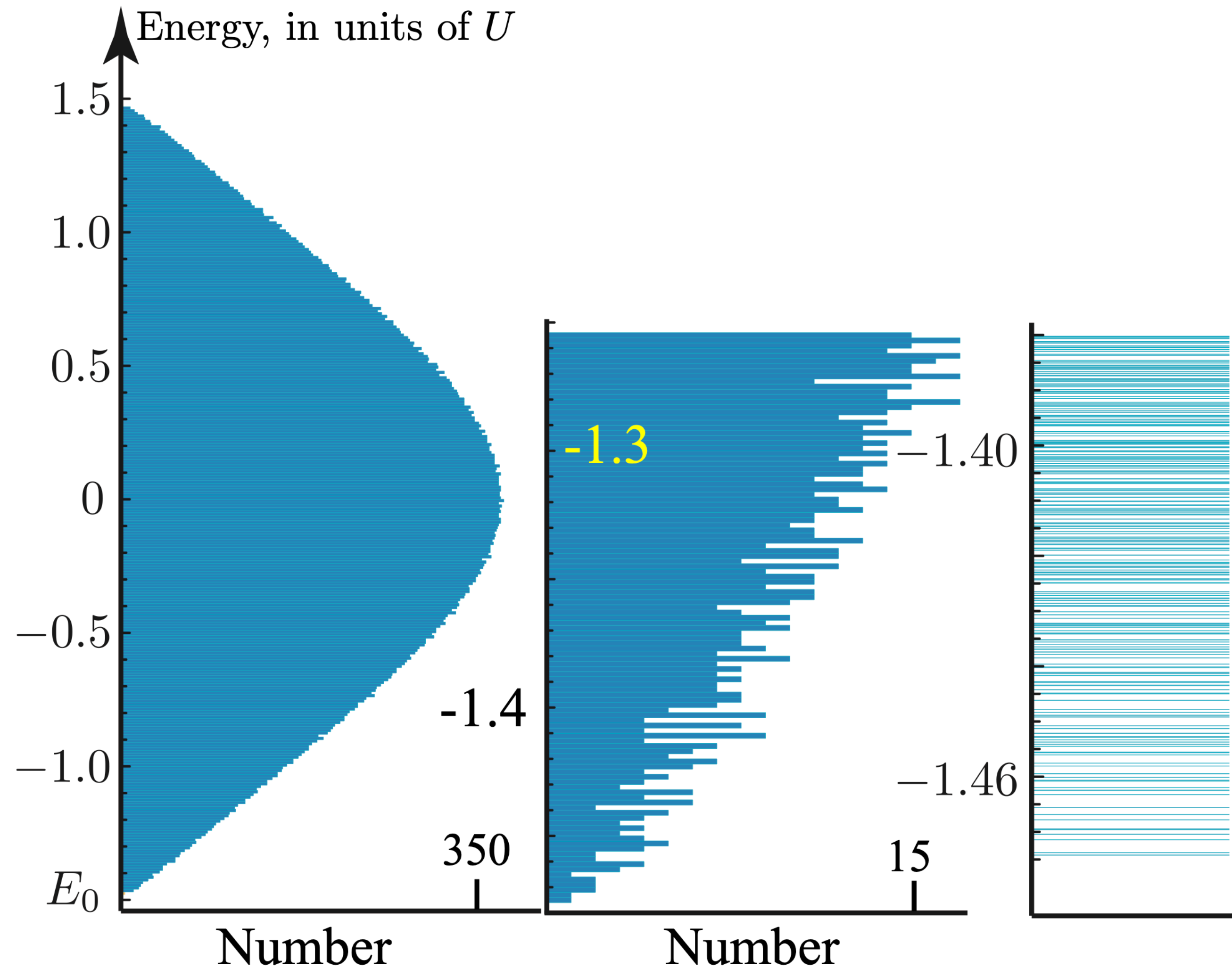
There is also an emergent U(1) gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5**, 041025 (2015)



Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

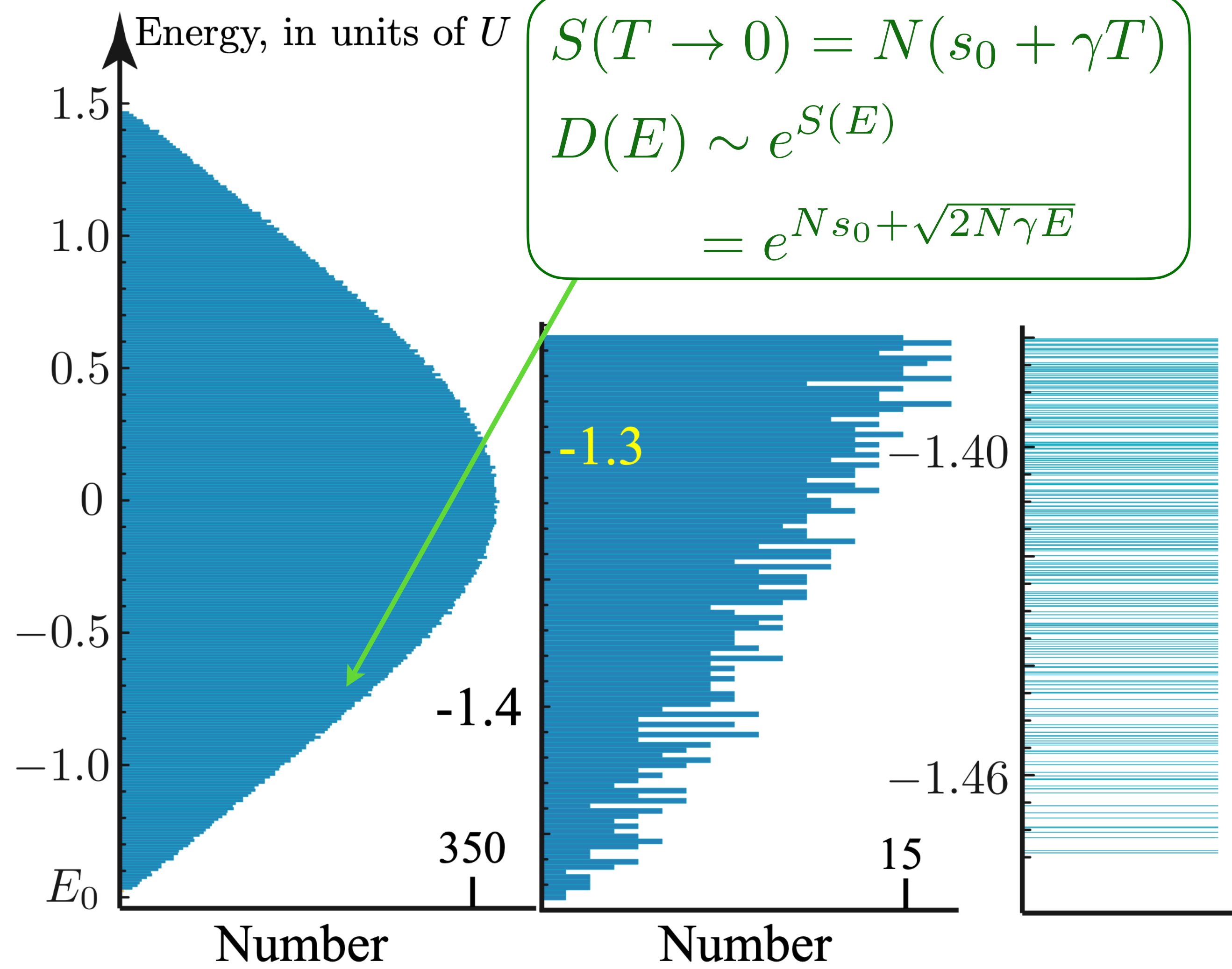


Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

At $Q = 1/2$



$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$

$$= 0.46484769917 \dots$$

A. Georges, O. Parcollet, and S. Sachdev,
PRB **63**, 134406 (2001)

Complex SYK model

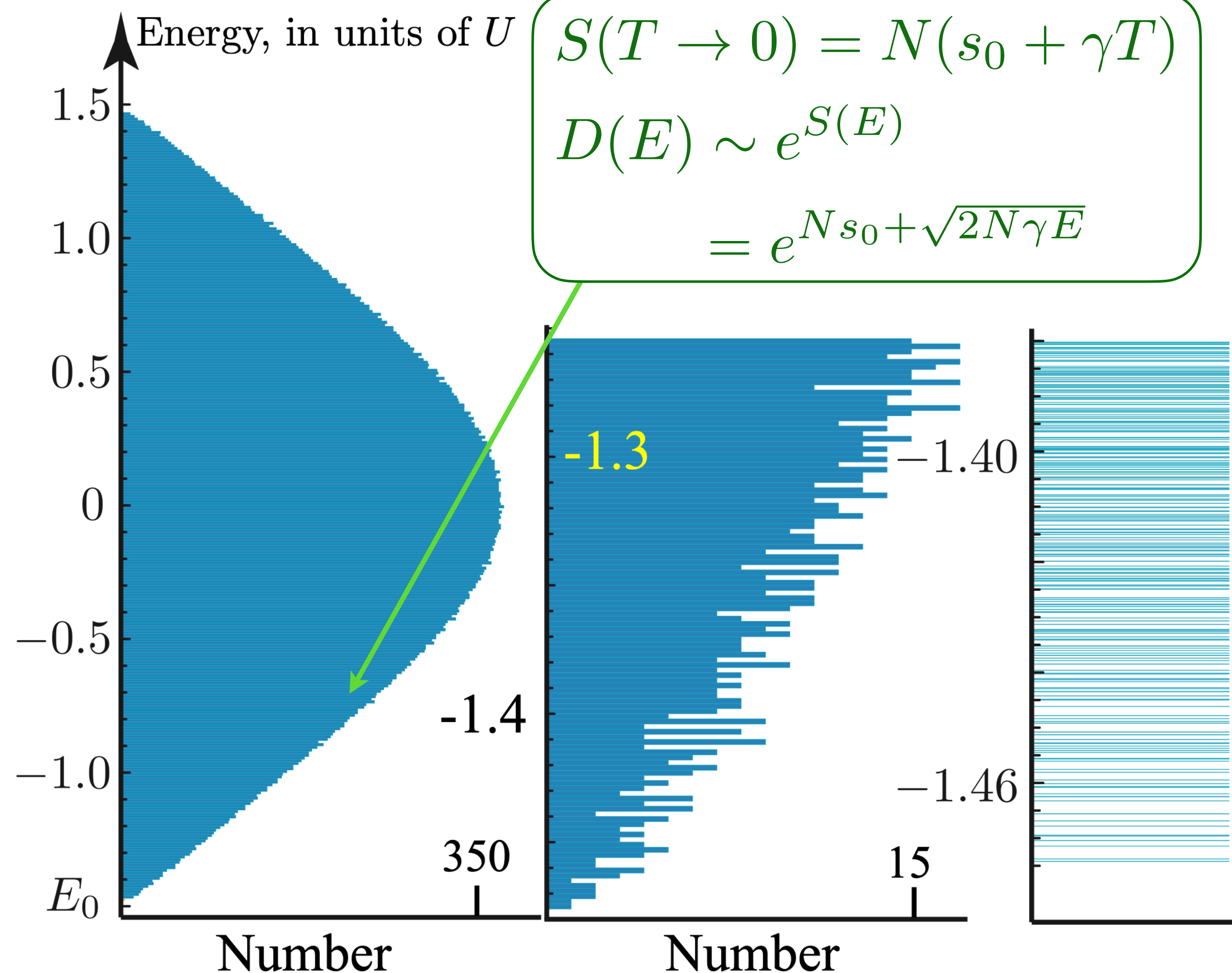
Many-body density of states

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A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



Energy level spacing $\sim e^{-N s_0}$!

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

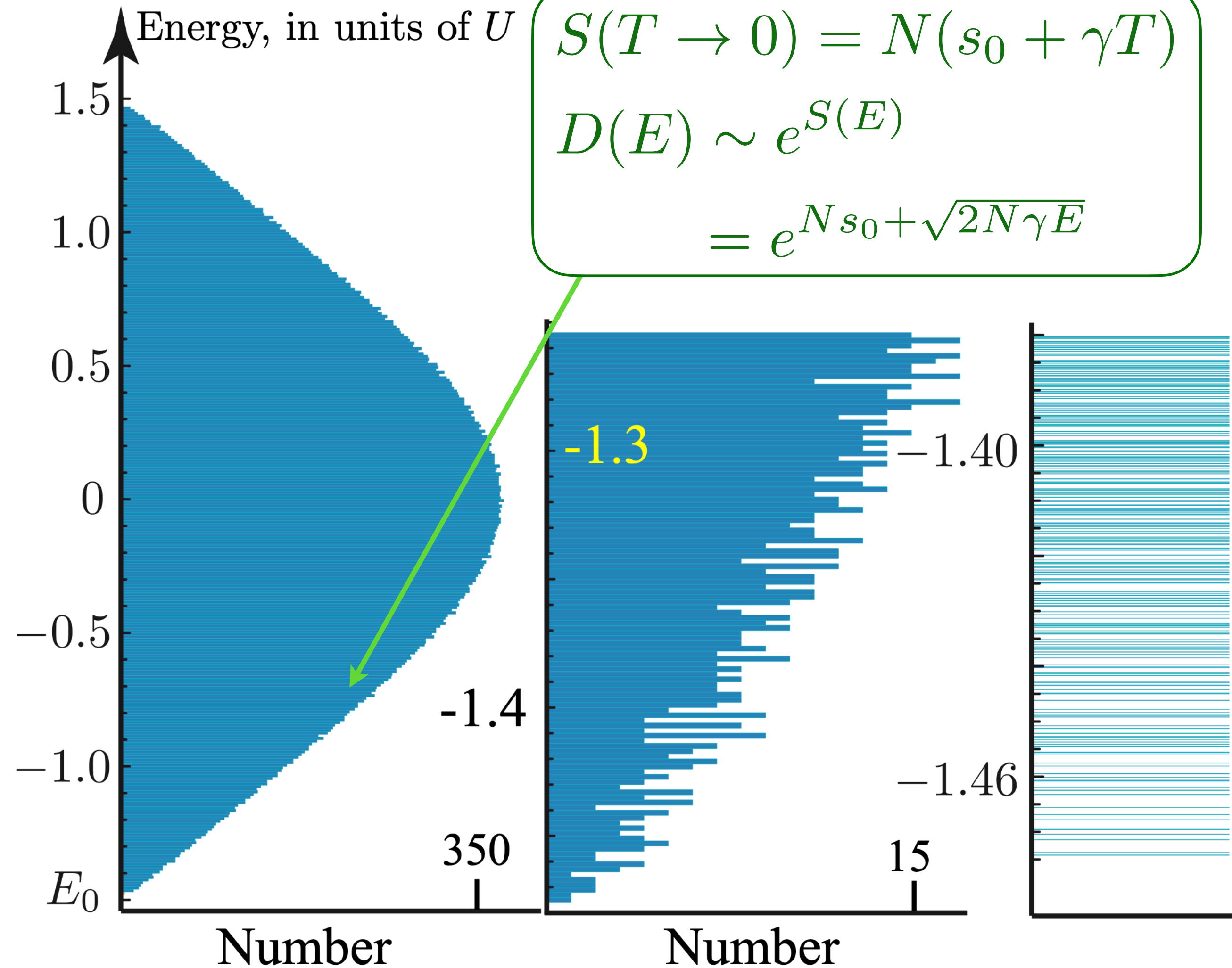
Many-body density of states

Beyond Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

At $Q = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.46484769917 \dots$$



$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$$D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E})$$

J. S. Cotler et al., JHEP 05 (2017) 118

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

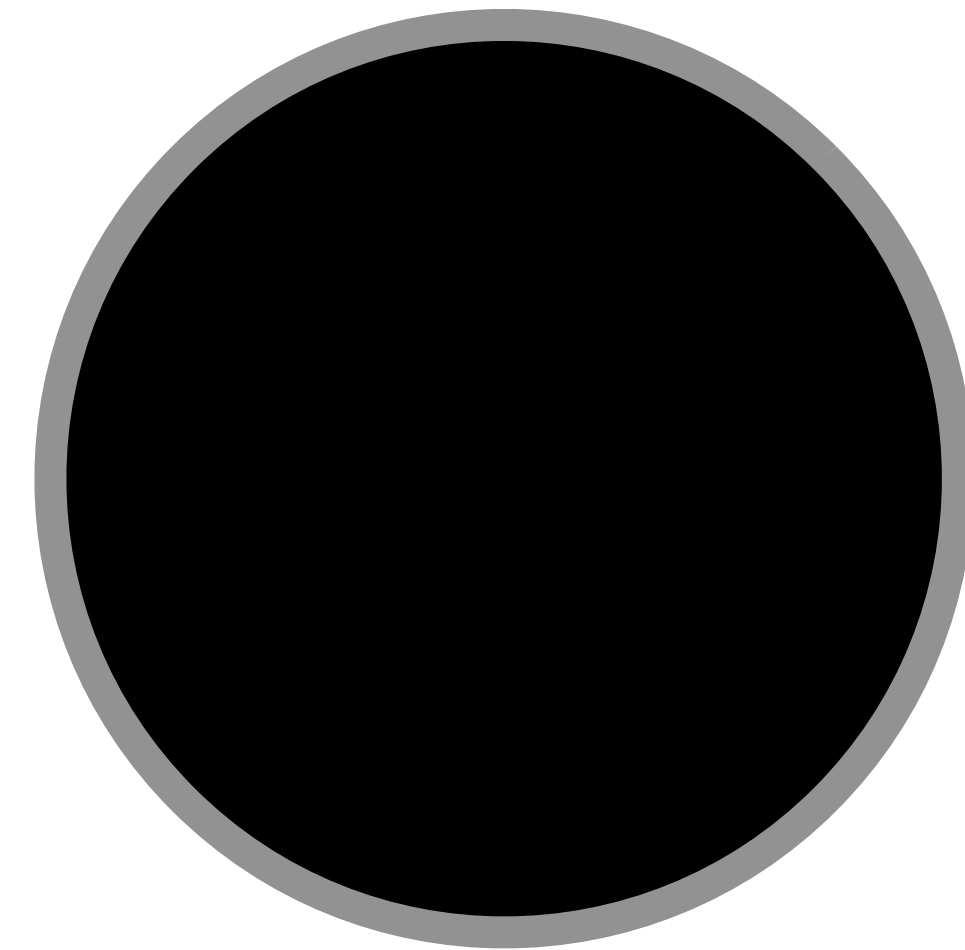
Complex SYK model

Quantum black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

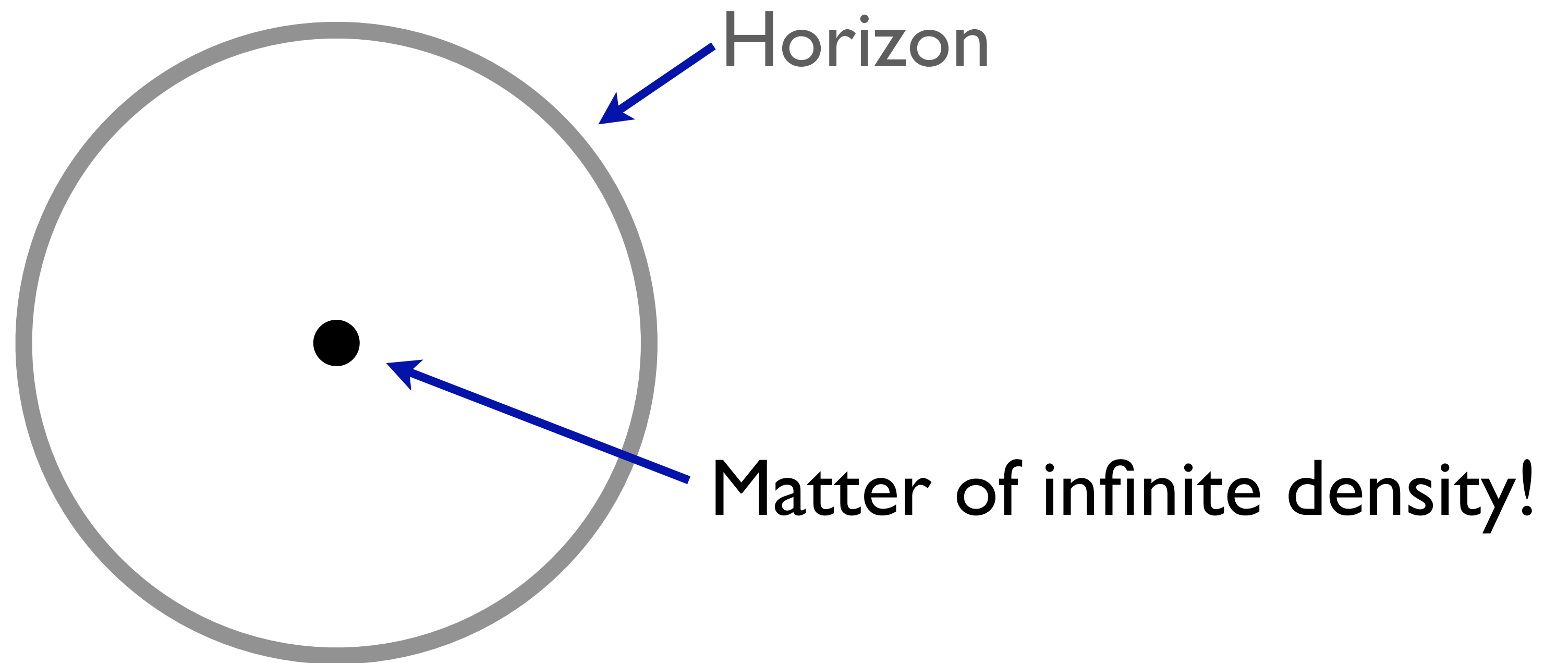
Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm}$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

It was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

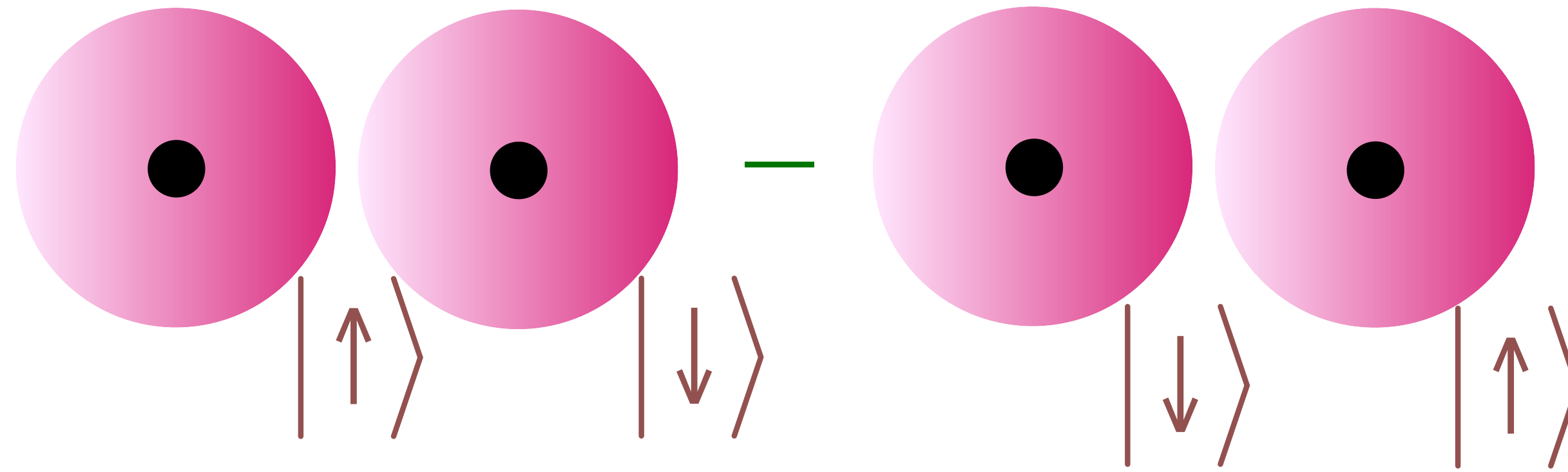
What is inside a black hole ???

natürliche
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt, daß es wirklich mit der Theorie

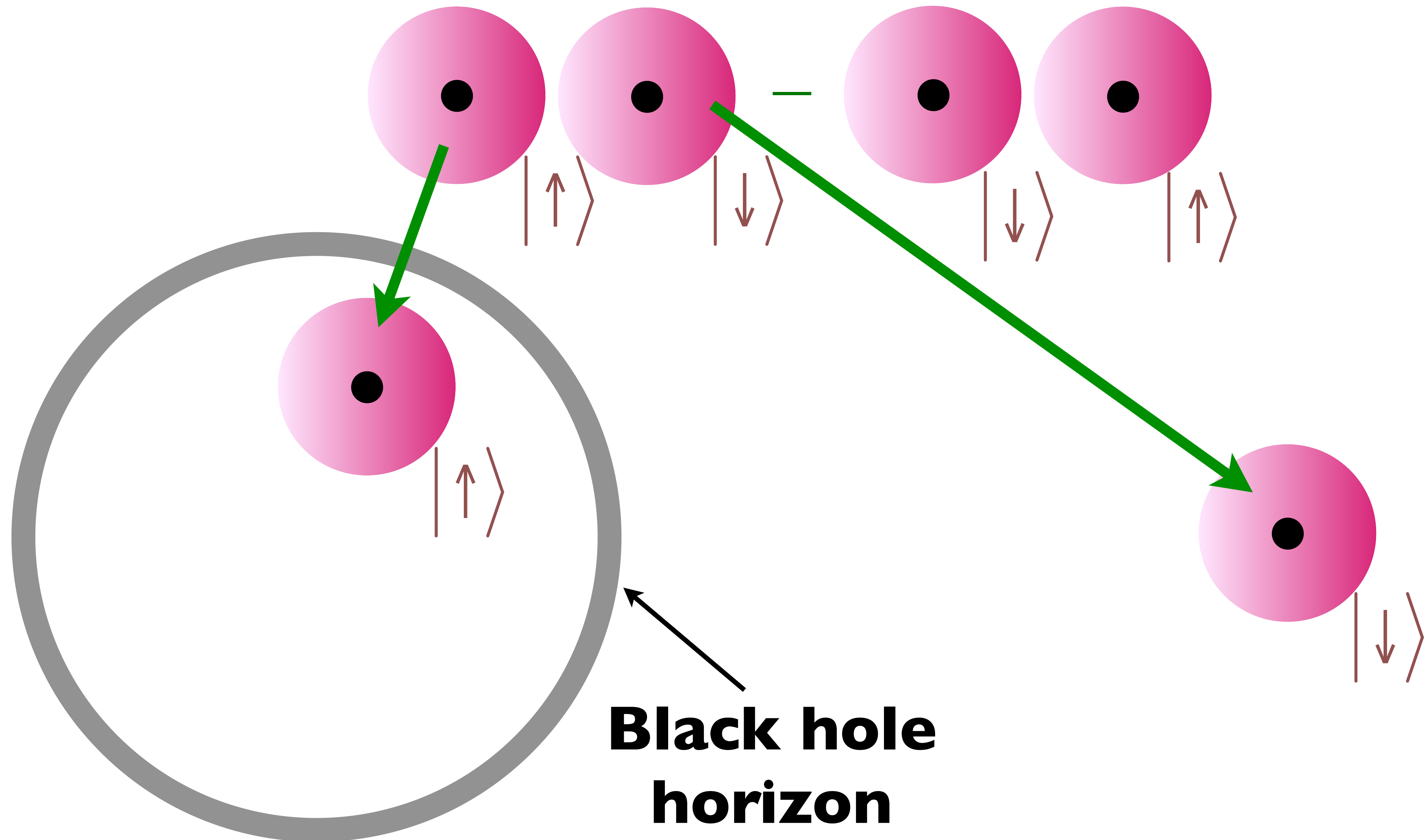
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

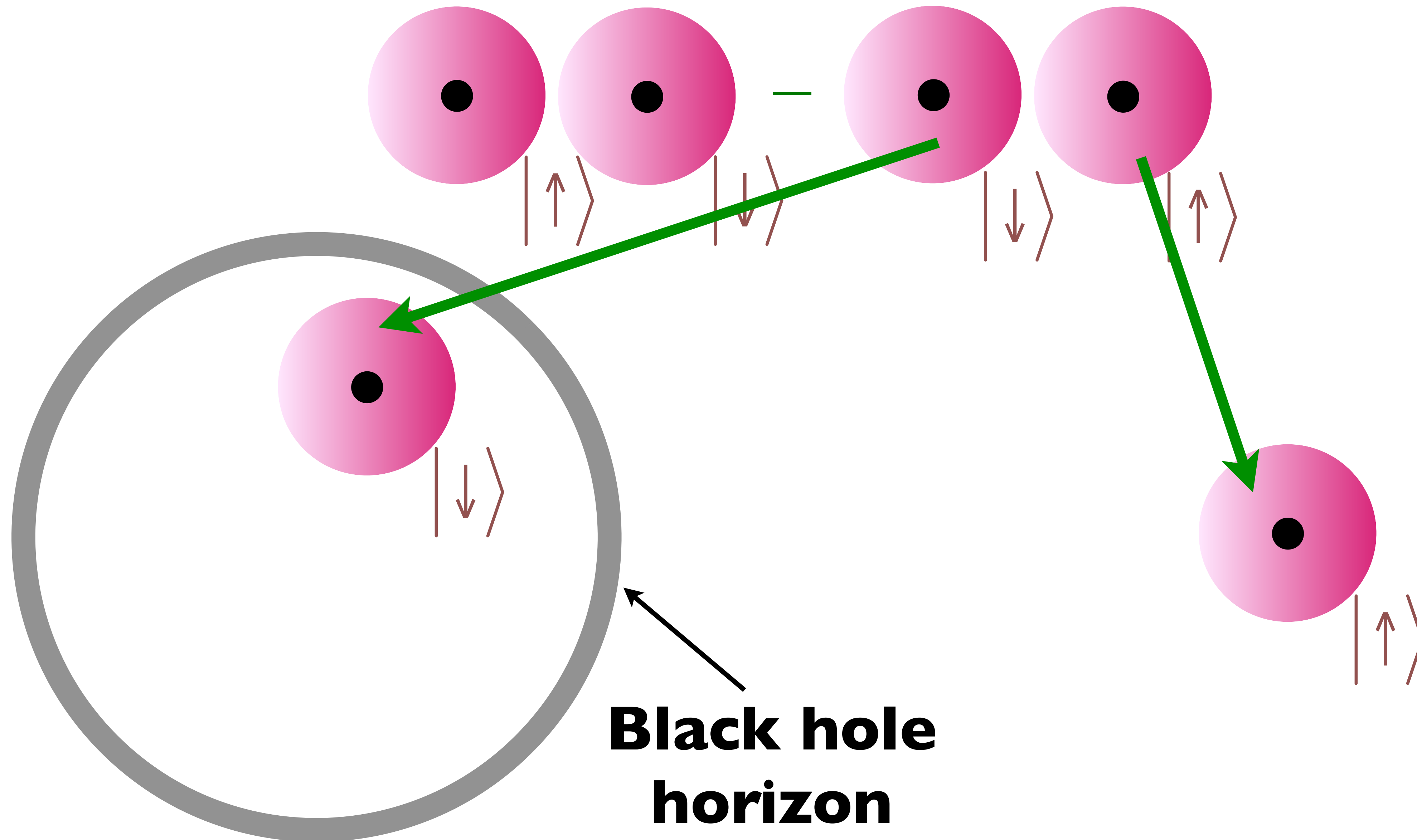
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



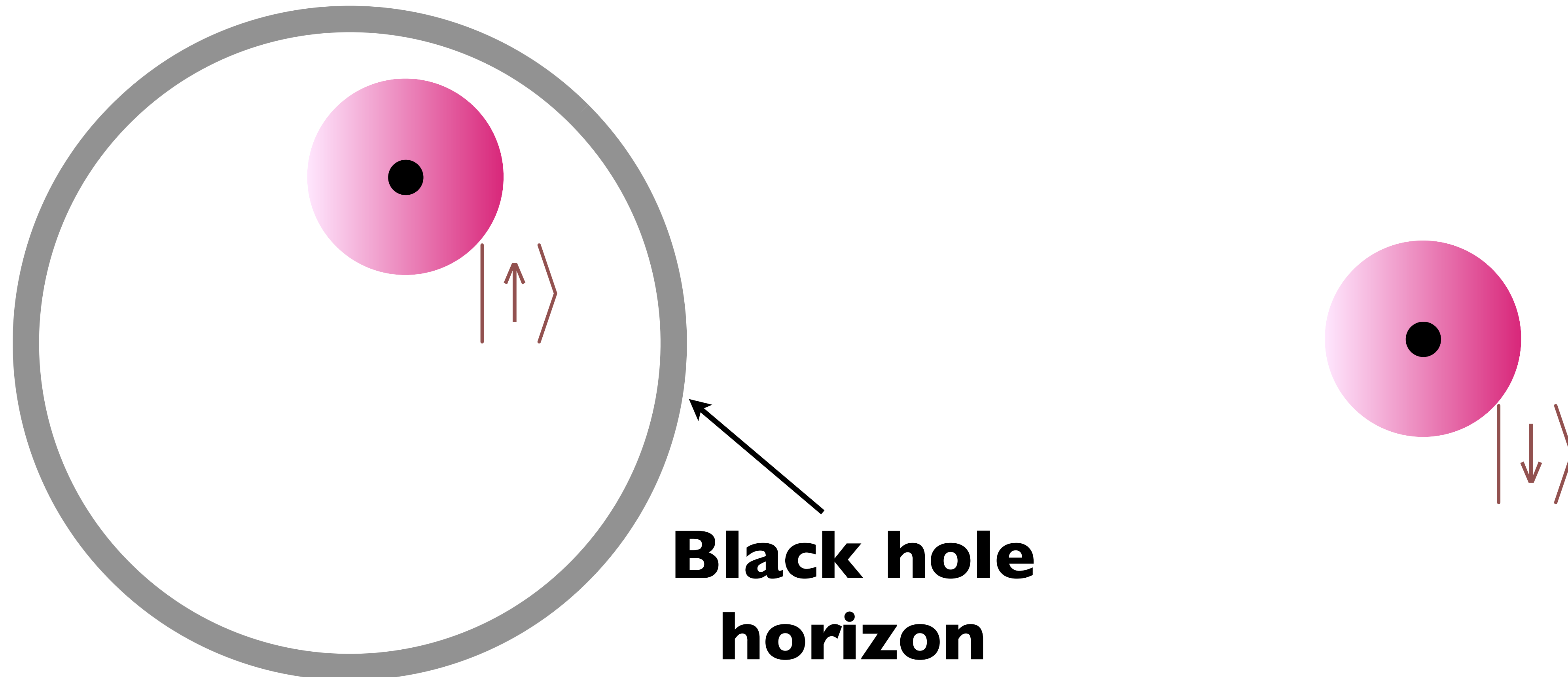
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

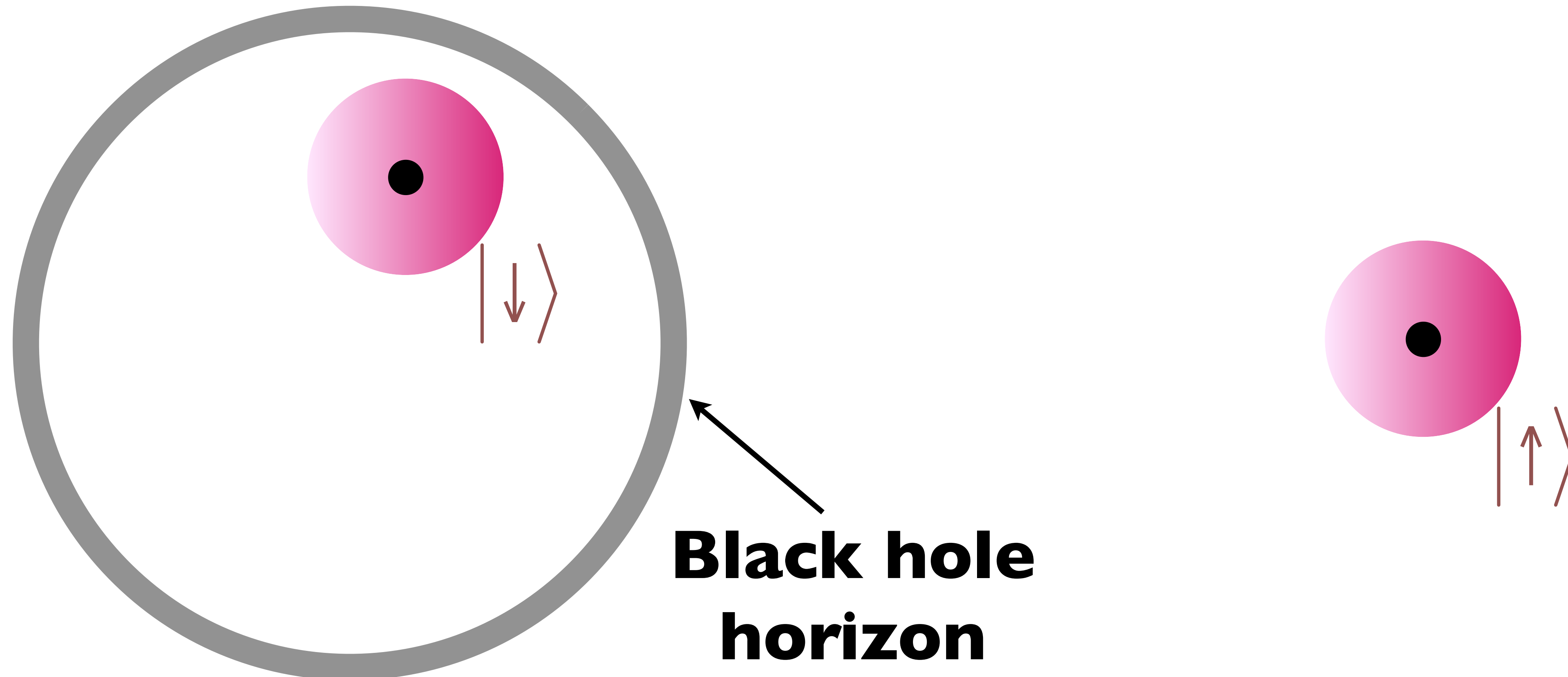
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

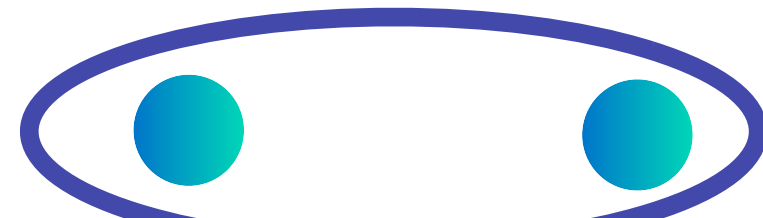
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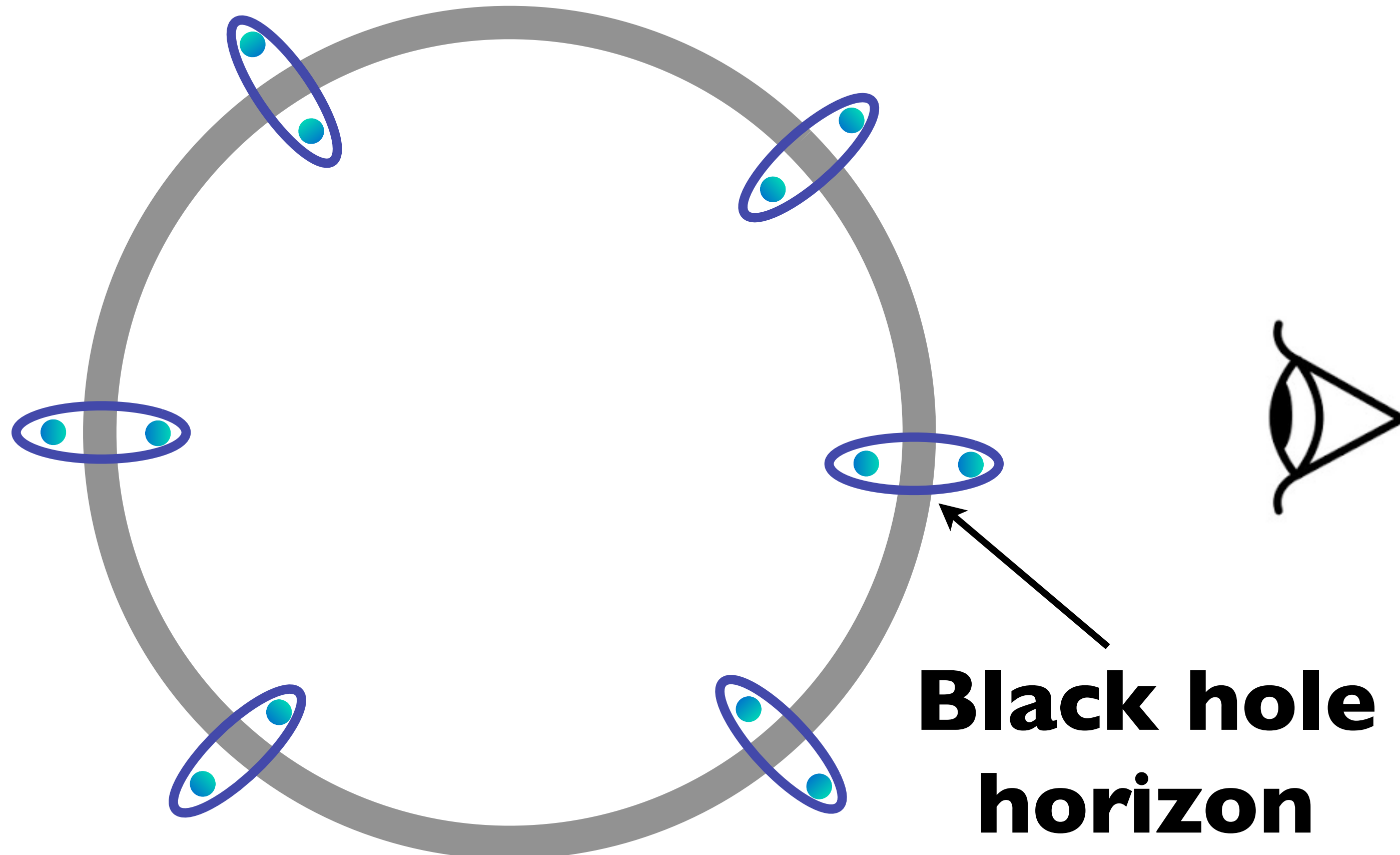
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Quantum Entanglement across a black hole horizon

Quantum entanglement
on the surface


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside*
the black hole,
Hawking obtained

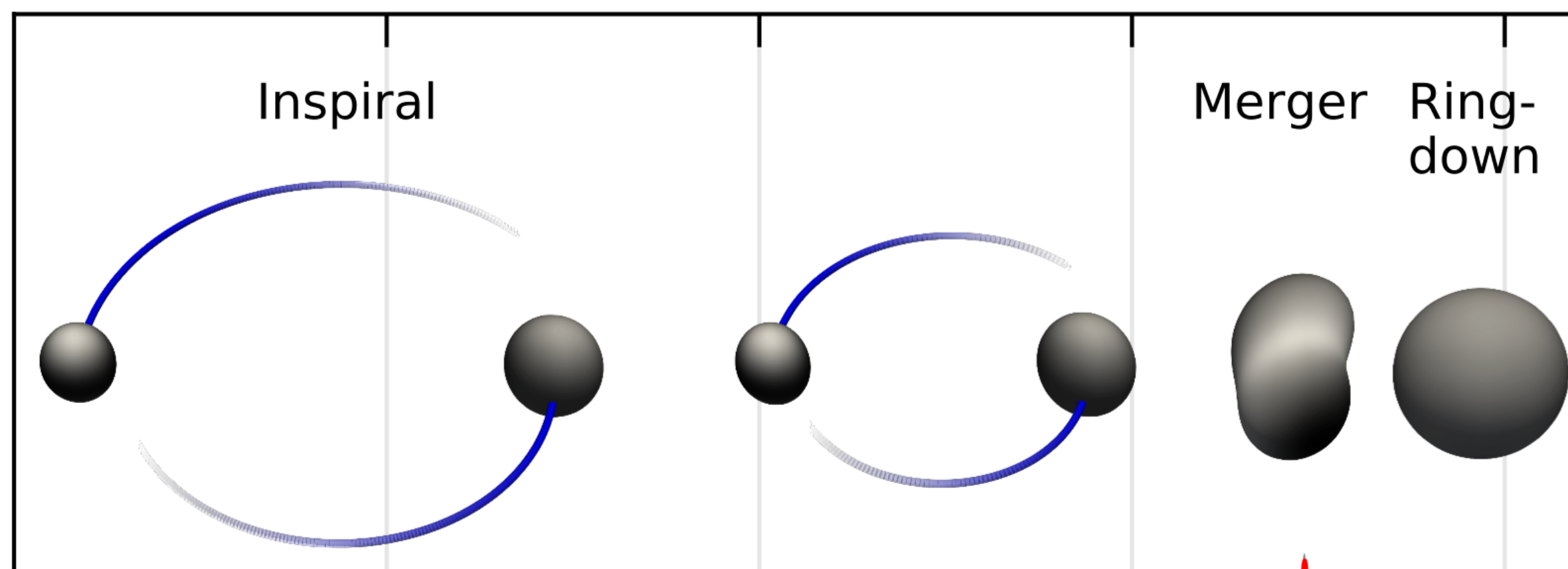
$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the
black hole horizon.

All other systems have
entropy proportional to
their volume.

Quantum black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3 / (8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3 / (4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M / c^3 = \hbar / (k_B T_H)$ which is Planckian!

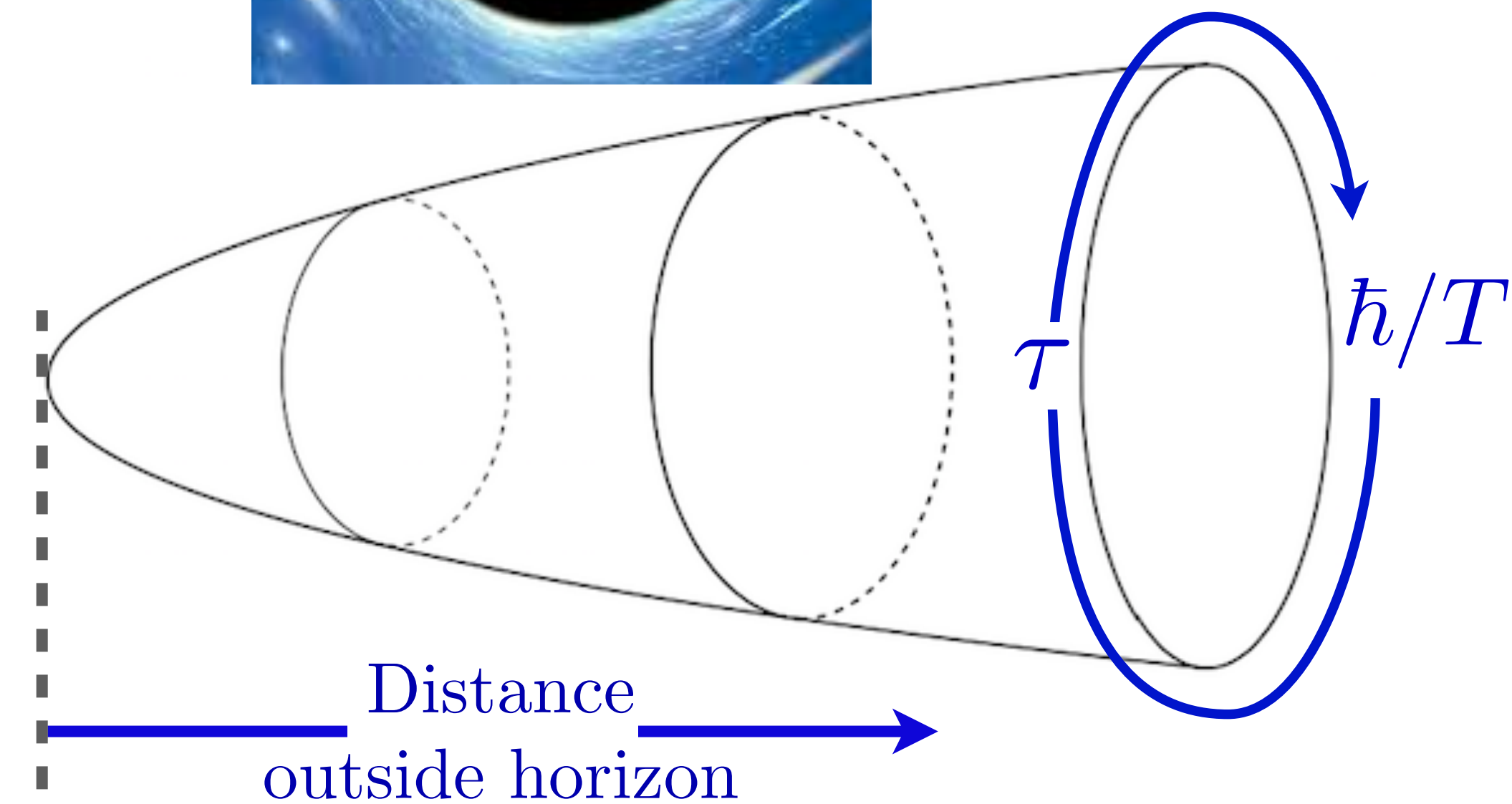


J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$



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$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

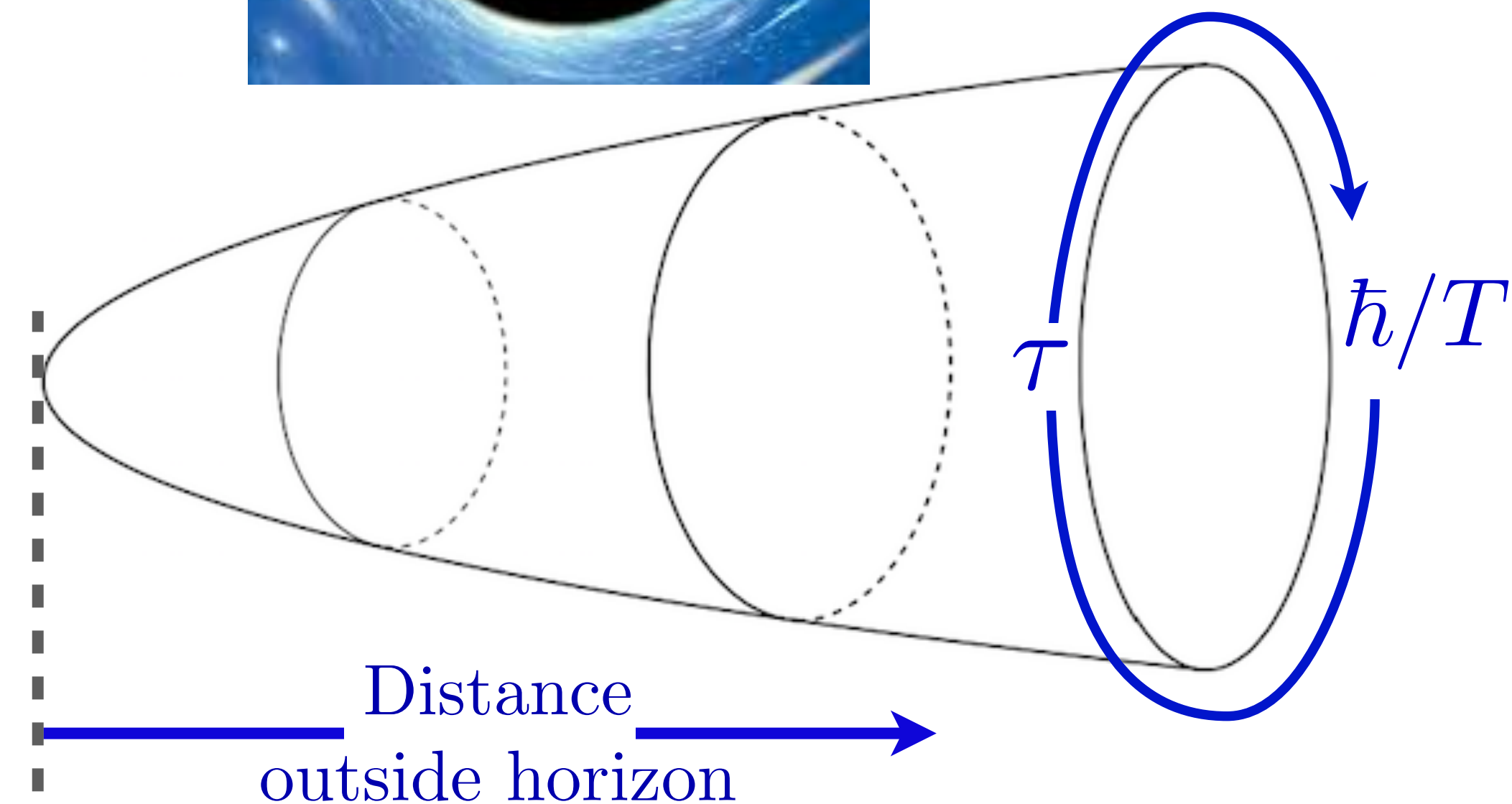
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



Quantum Black Holes

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose density of quantum states matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model
to a quantum theory of
charged black holes

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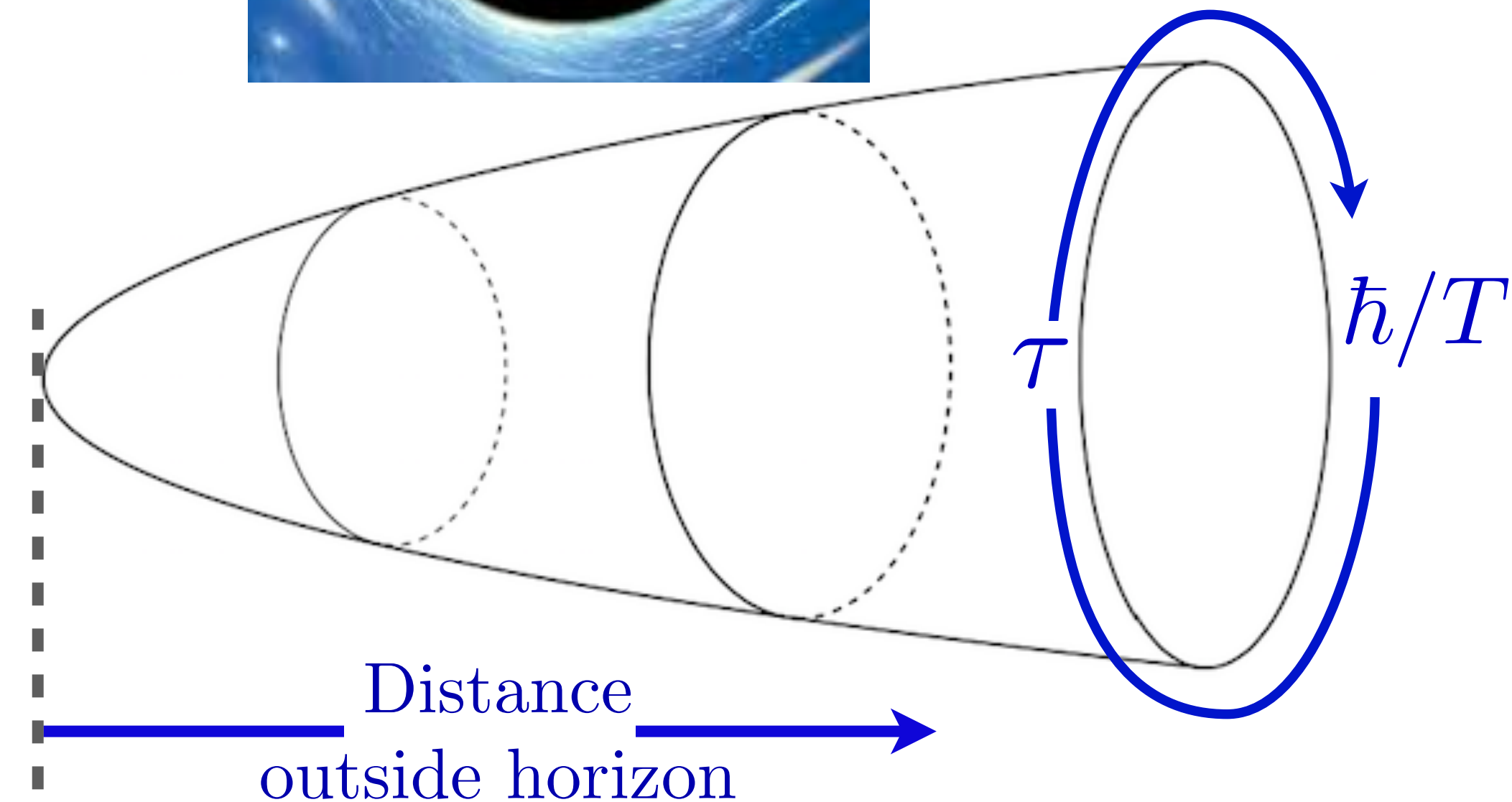
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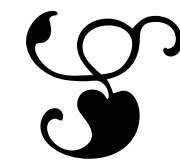


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PHYSICAL REVIEW LETTERS **105, 151602 (2010)**



Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

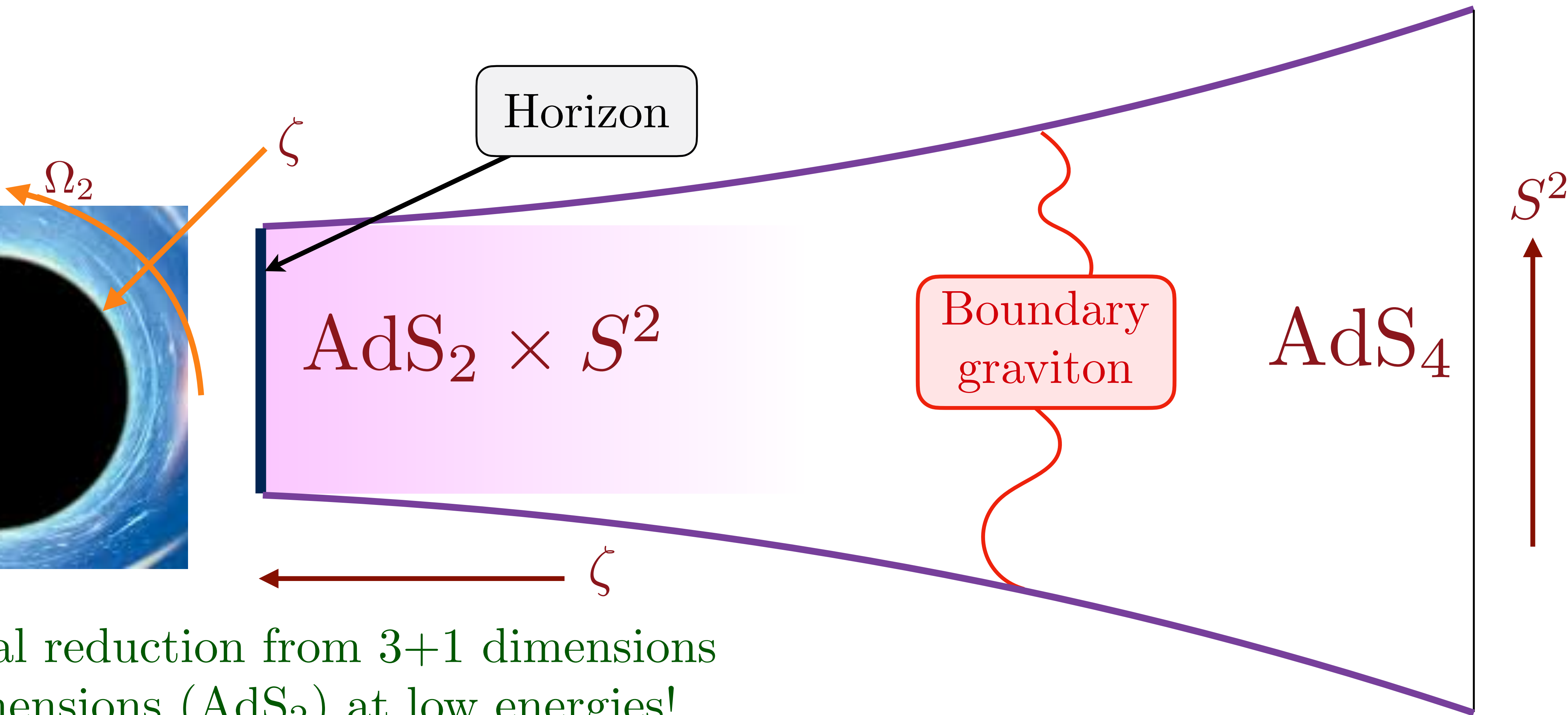
We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti-de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a “small” Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $\text{AdS}_2 \times \mathbb{R}^2$ physics of Reissner-Nordström black holes.

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions
to 1+1 dimensions (AdS_2) at low energies!

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group $SL(2,R)$.

Thermodynamics of quantum black holes with charge Q :



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Saddle-point:

$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} + \dots \right)$$

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Thermodynamics of quantum black holes with charge Q :

$$\begin{aligned} \mathcal{Z}(Q, T) &= \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_\mu] \right) \\ &\approx \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{JT gravity of AdS}_2+\text{boundary graviton}}^{(1+1)}[g_{\mu\nu}, A_\mu] \right) \end{aligned}$$

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Saddle-point:

$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} + \dots \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Quantum simulation of charged black holes by the SYK model

- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

Iliesiu, Murthy, Turiaci (2022)

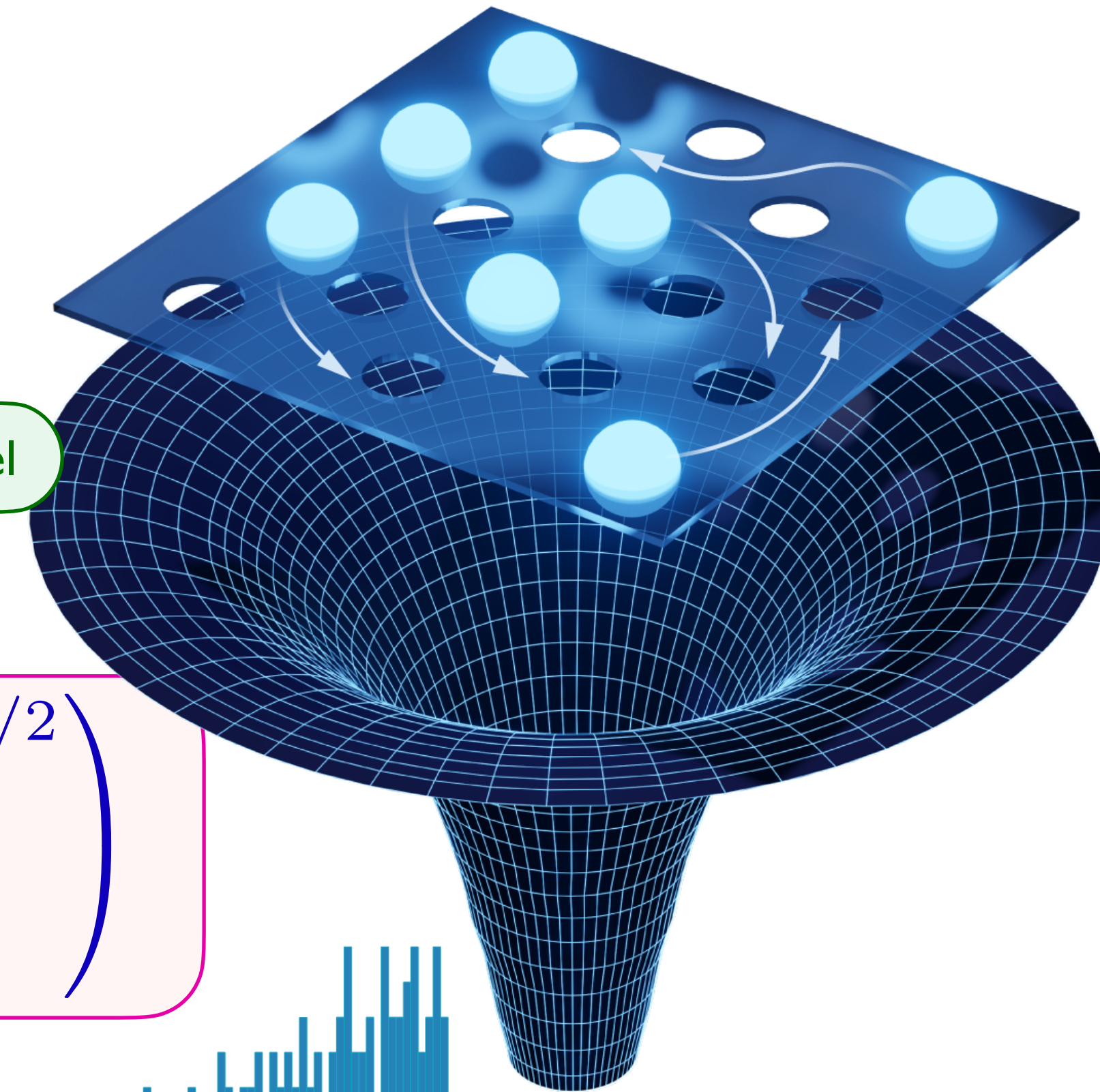
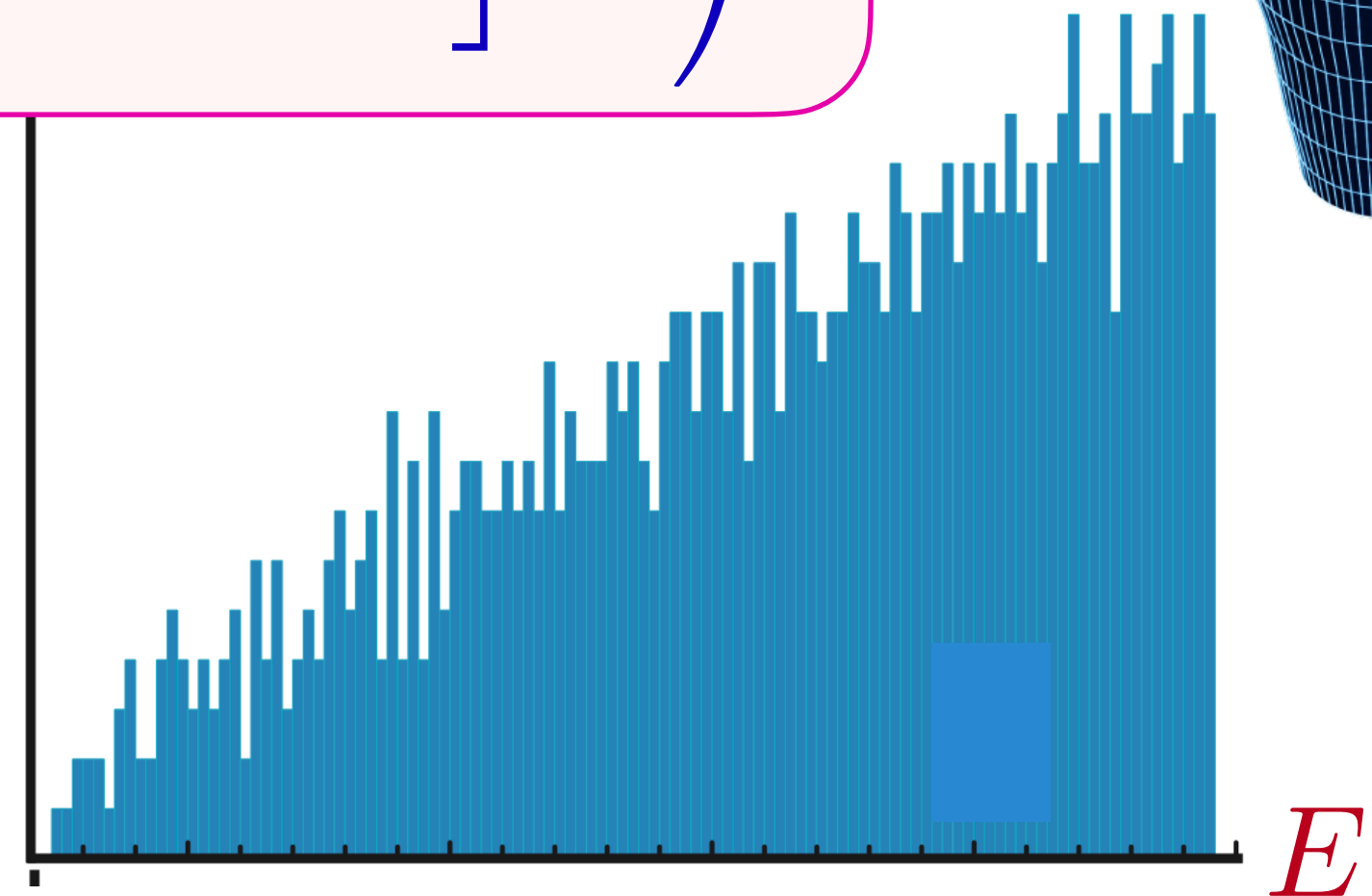
Bekenstein-Hawking

Developments from the SYK model

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.

$D(E)$

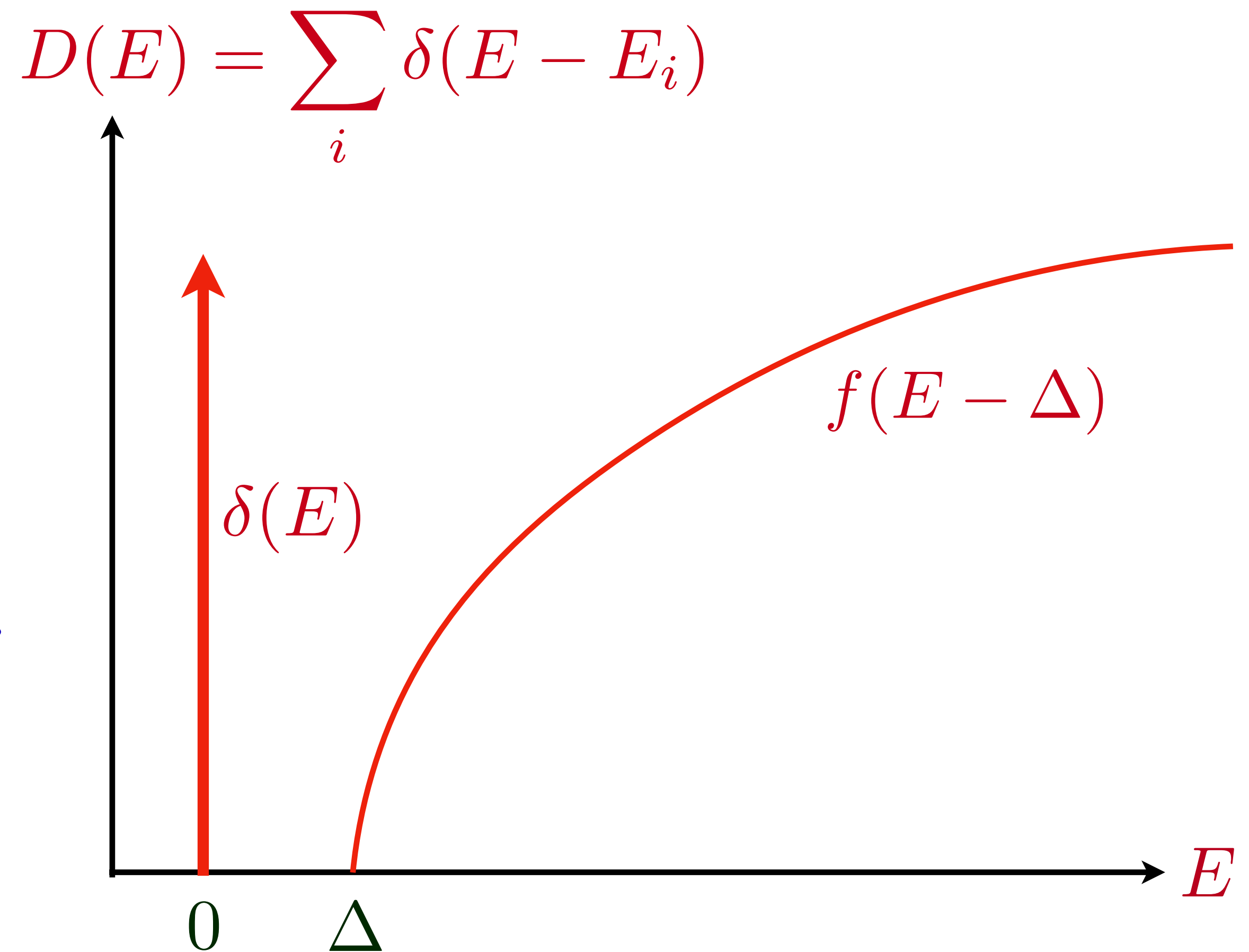


String theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

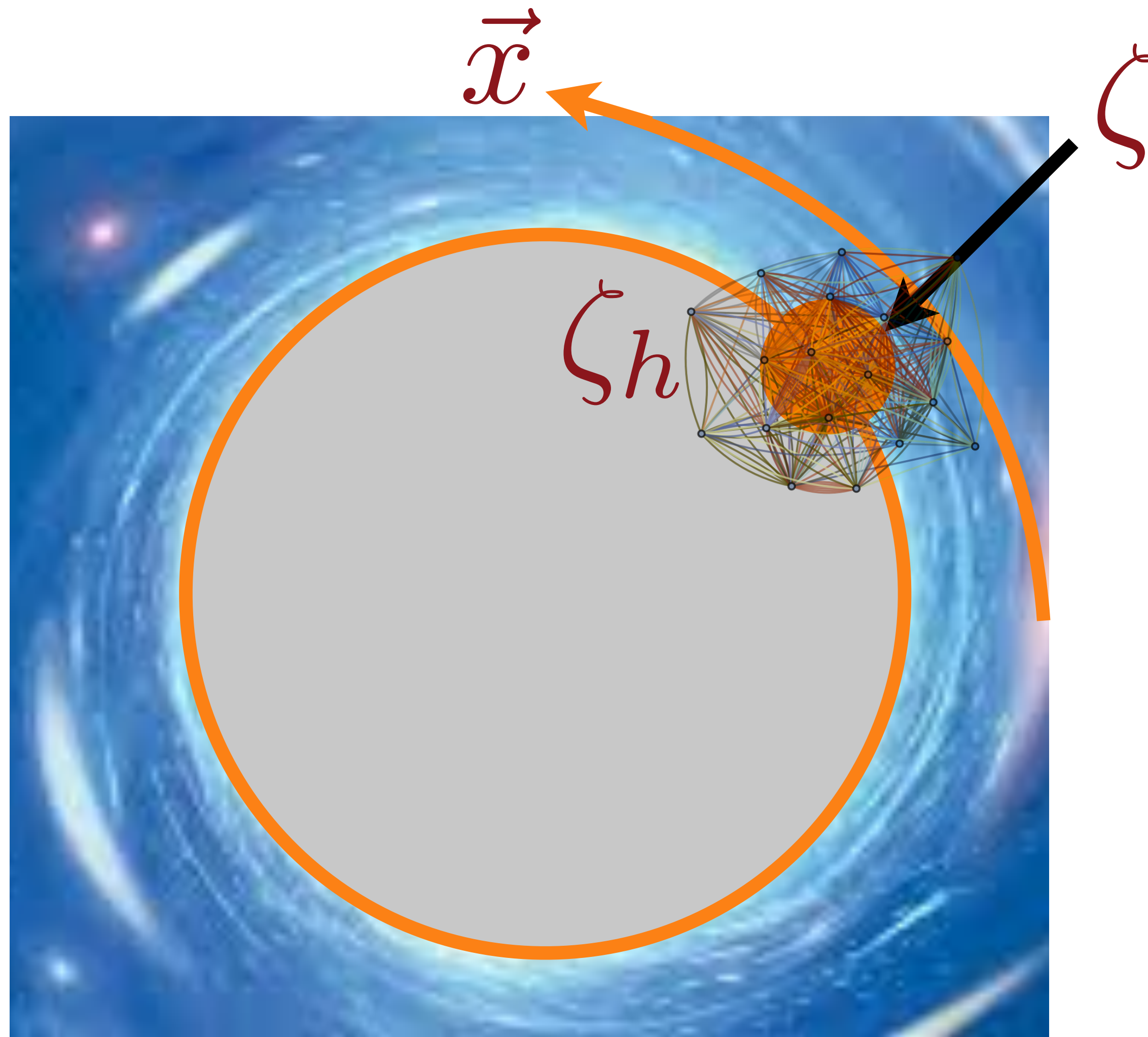
$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \delta(E) + \theta(E - \Delta) f(E - \Delta) + \dots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.



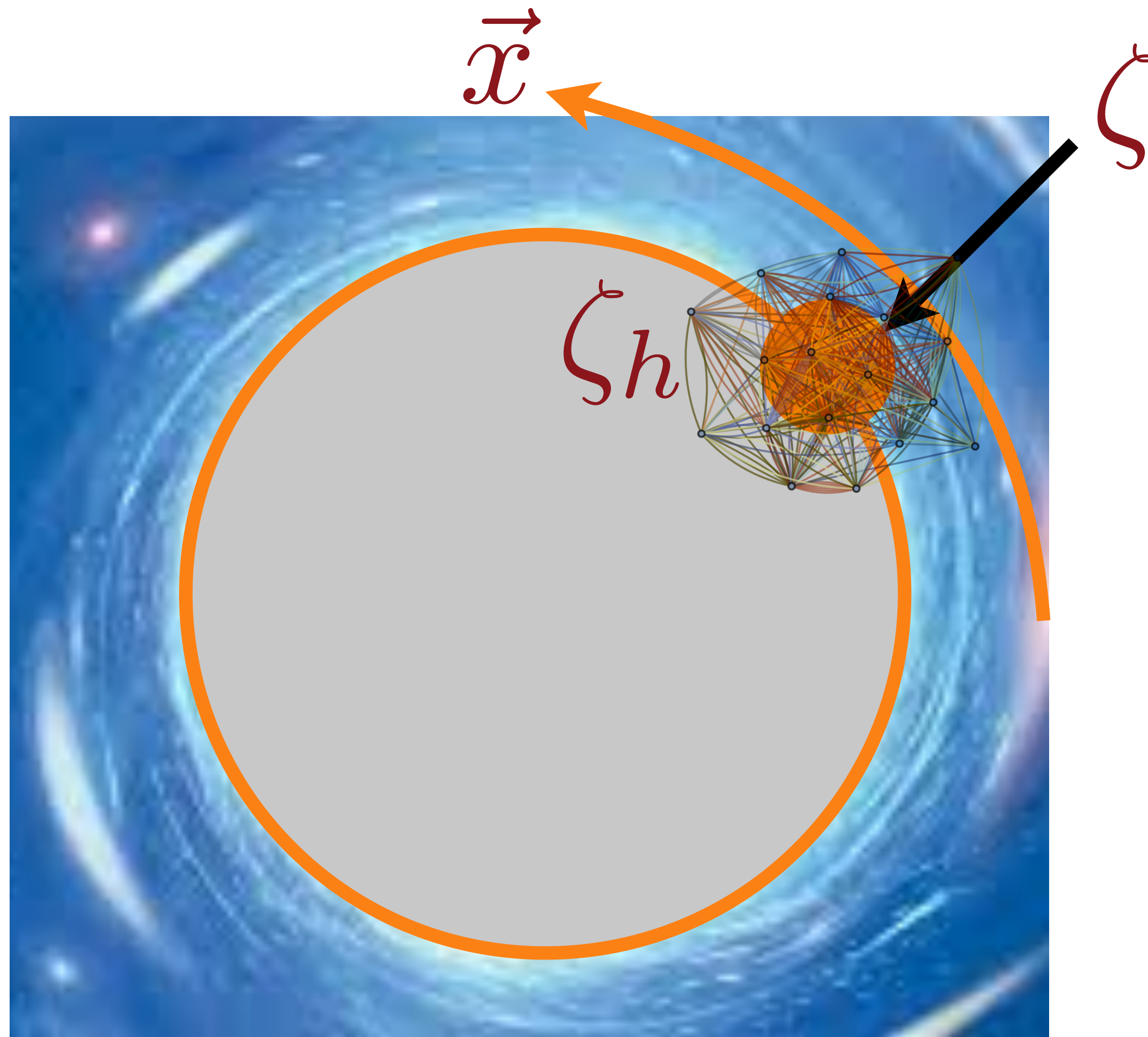
M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

Quantum simulation of charged black holes by the SYK model



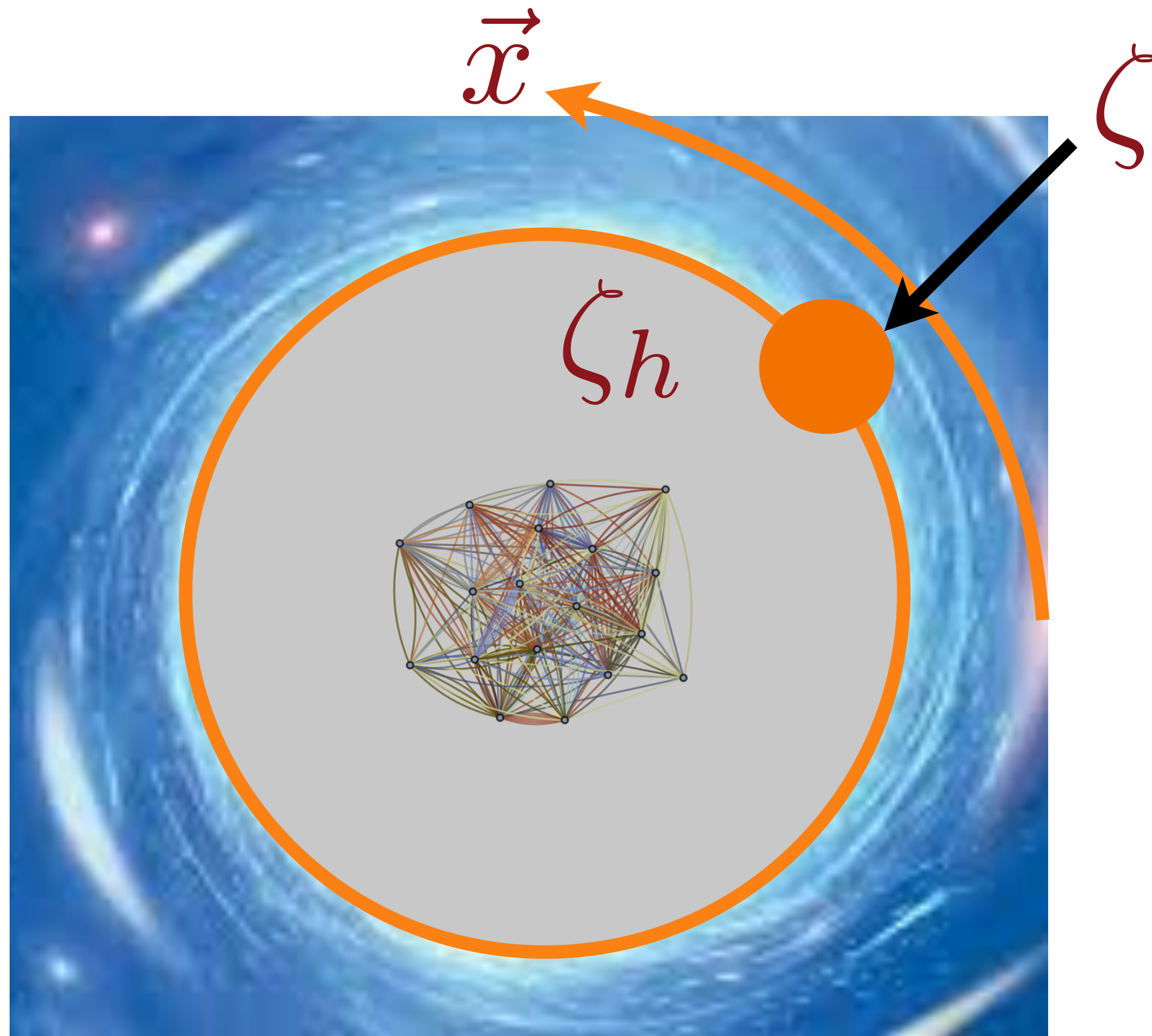
The SYK provides the needed realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

Quantum simulation of charged black holes by the SYK model



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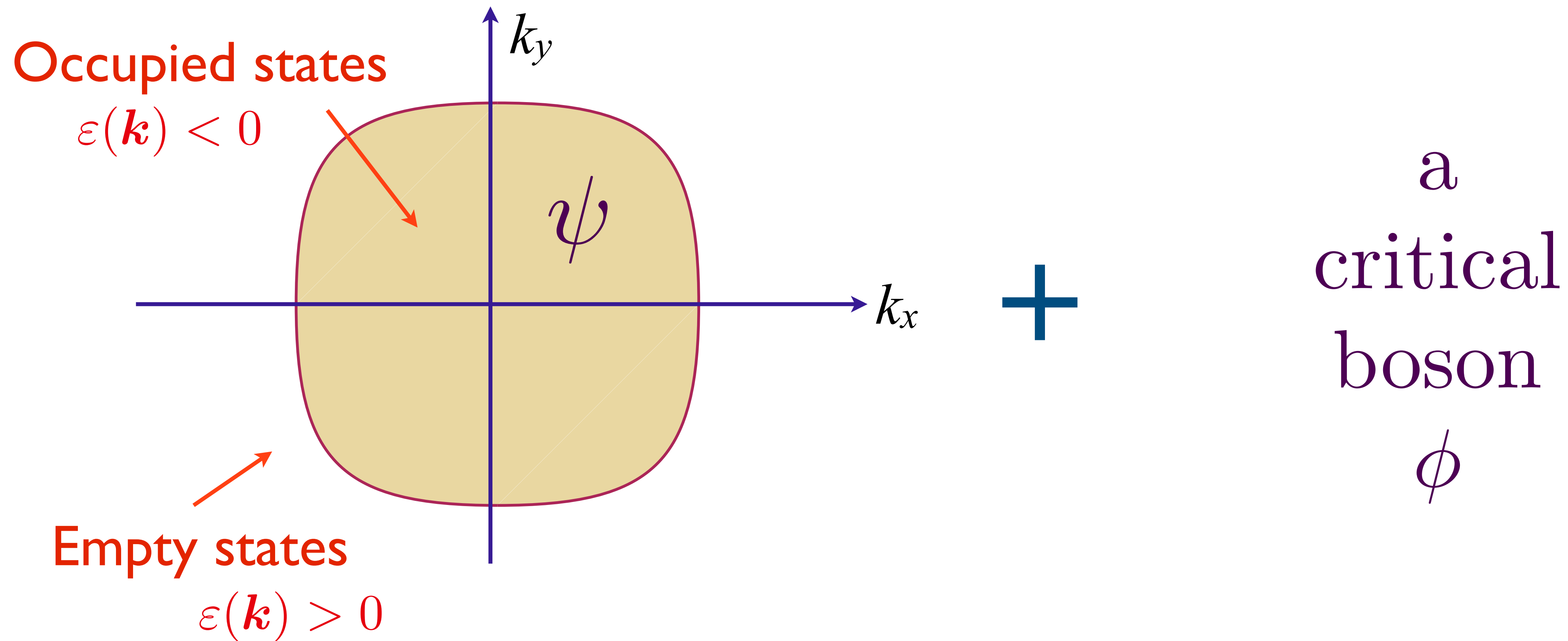
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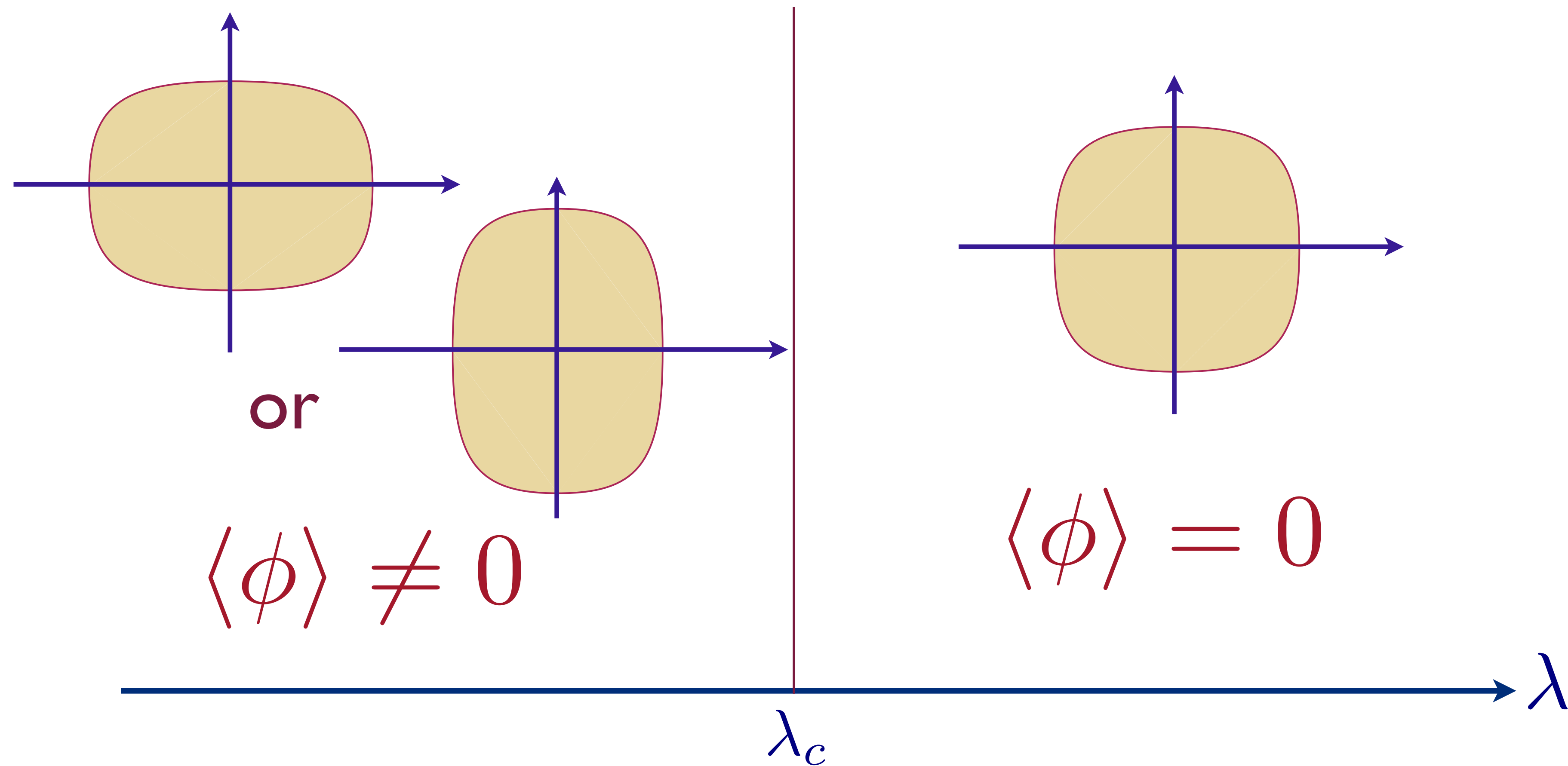
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From the SYK model to
a universal theory of
strange metals

Fermi surface coupled to a critical boson

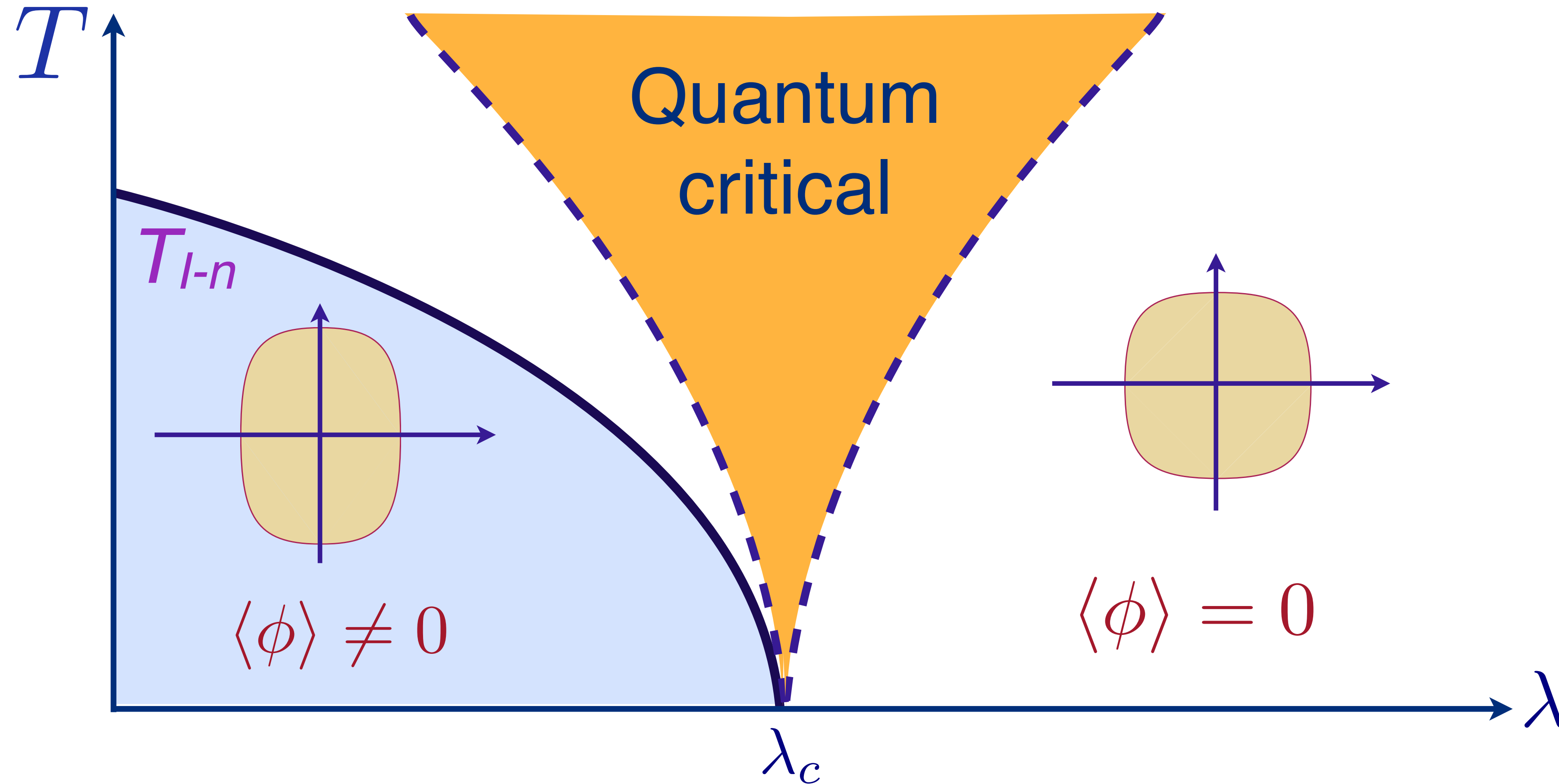


Quantum criticality of Ising-nematic ordering in a metal



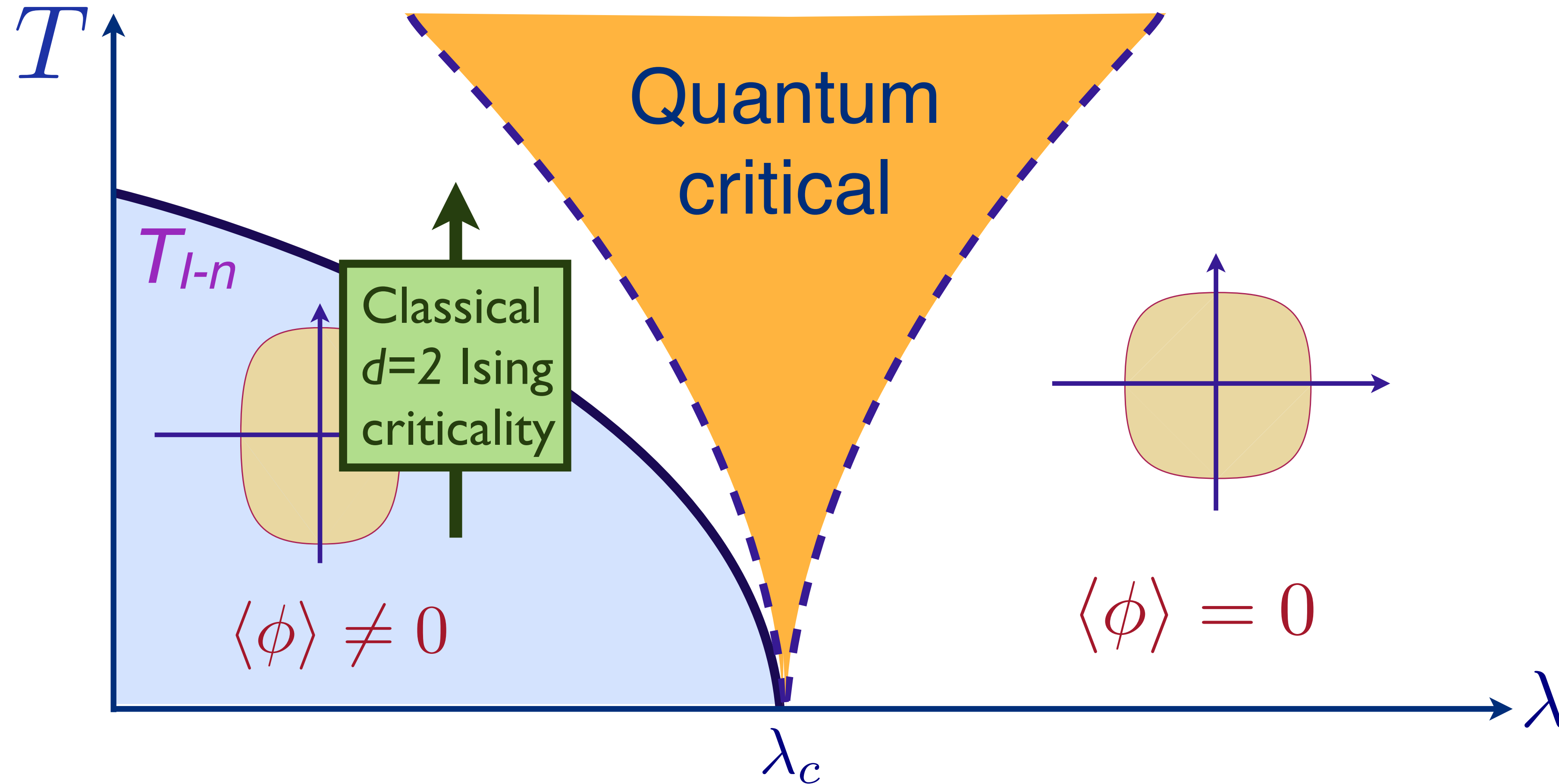
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



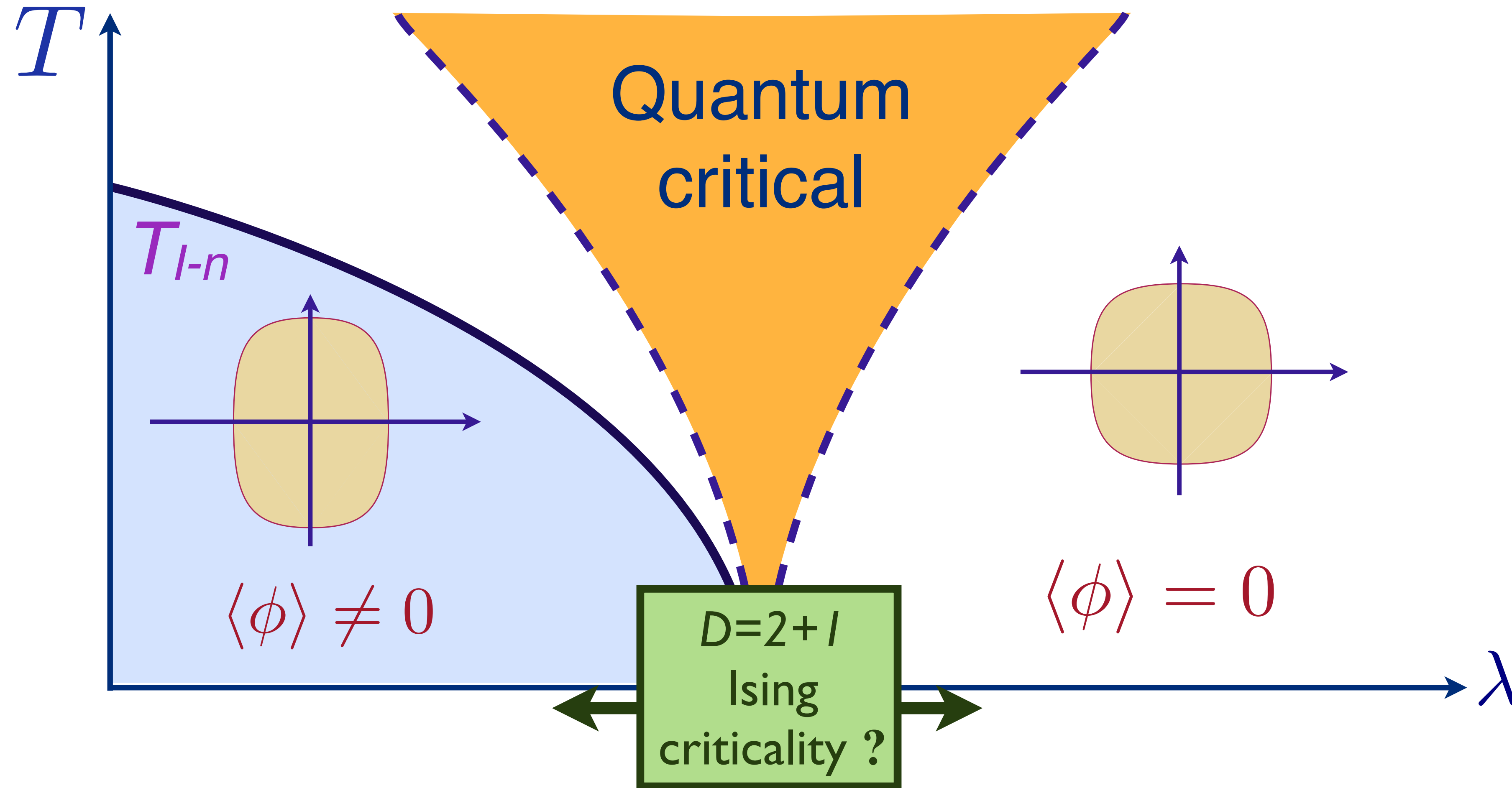
Phase diagram as a function of T and λ

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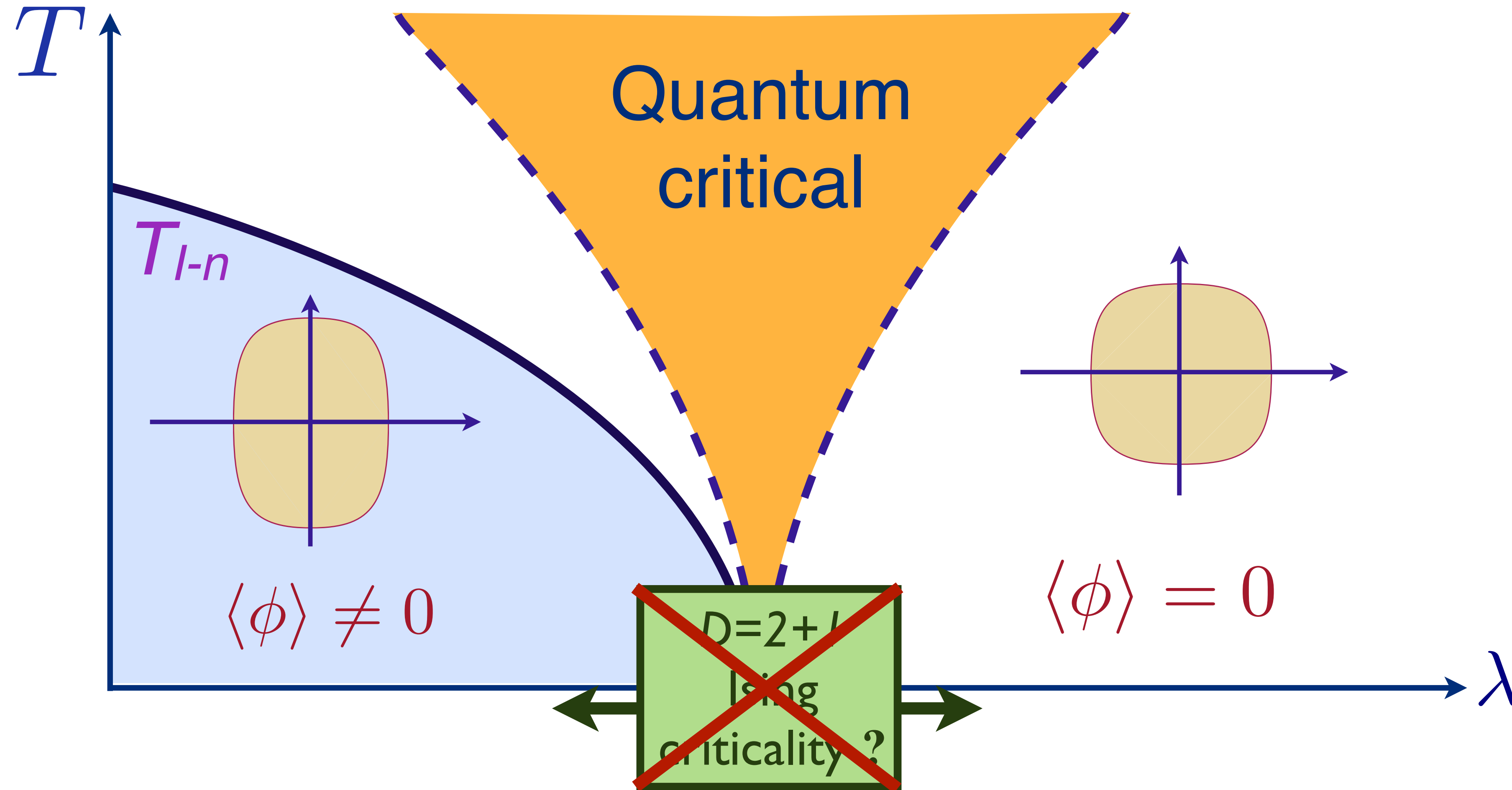
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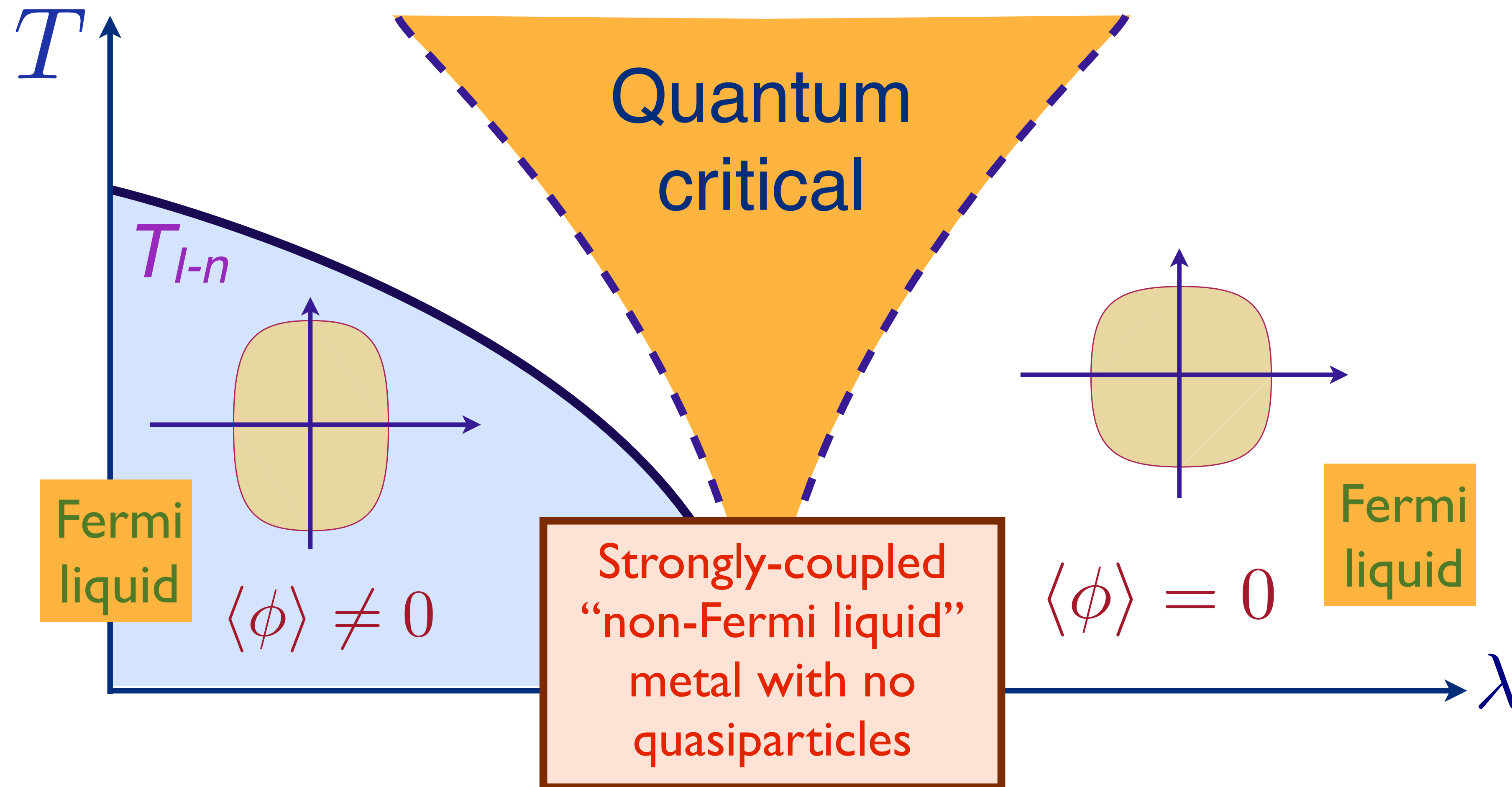
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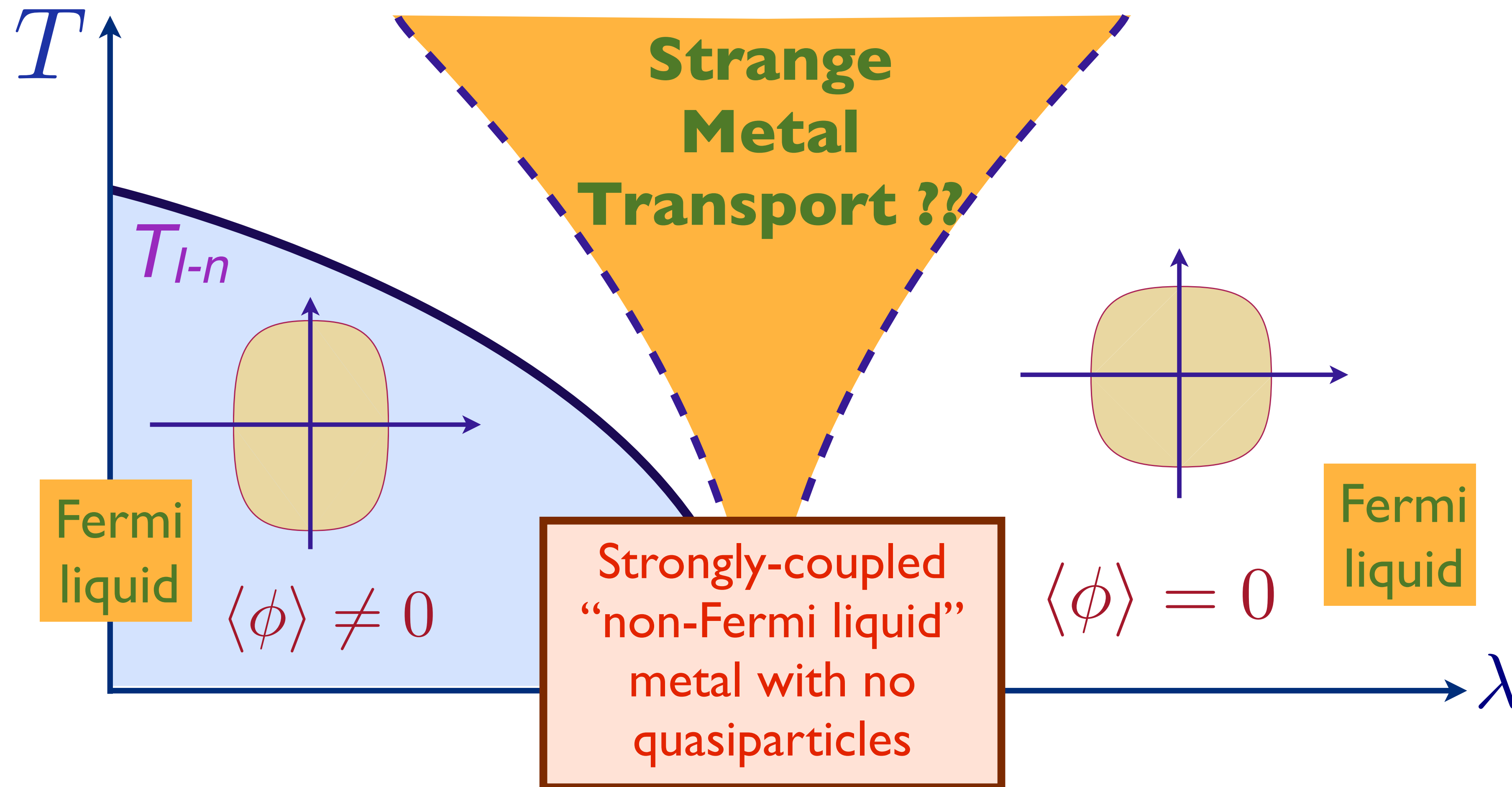
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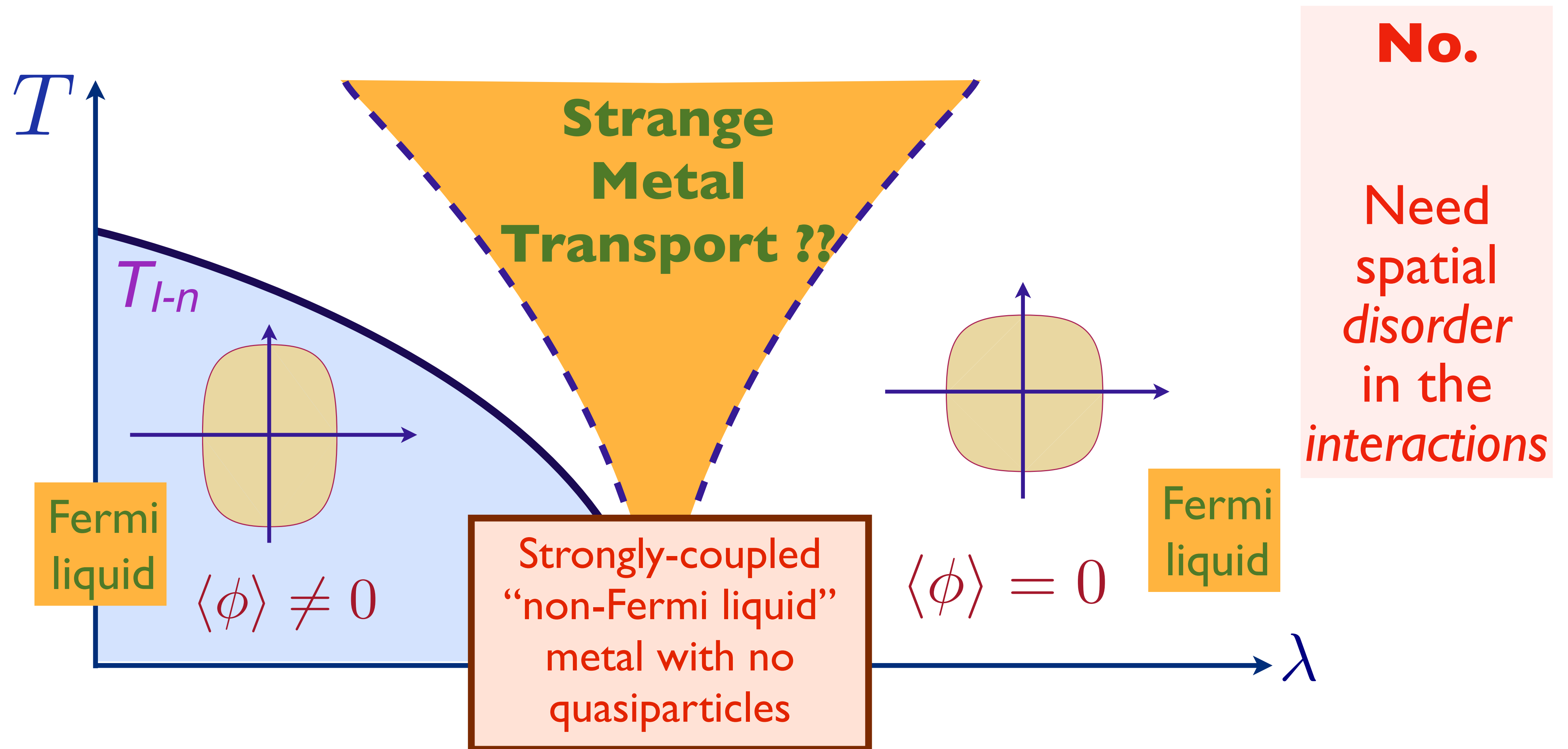
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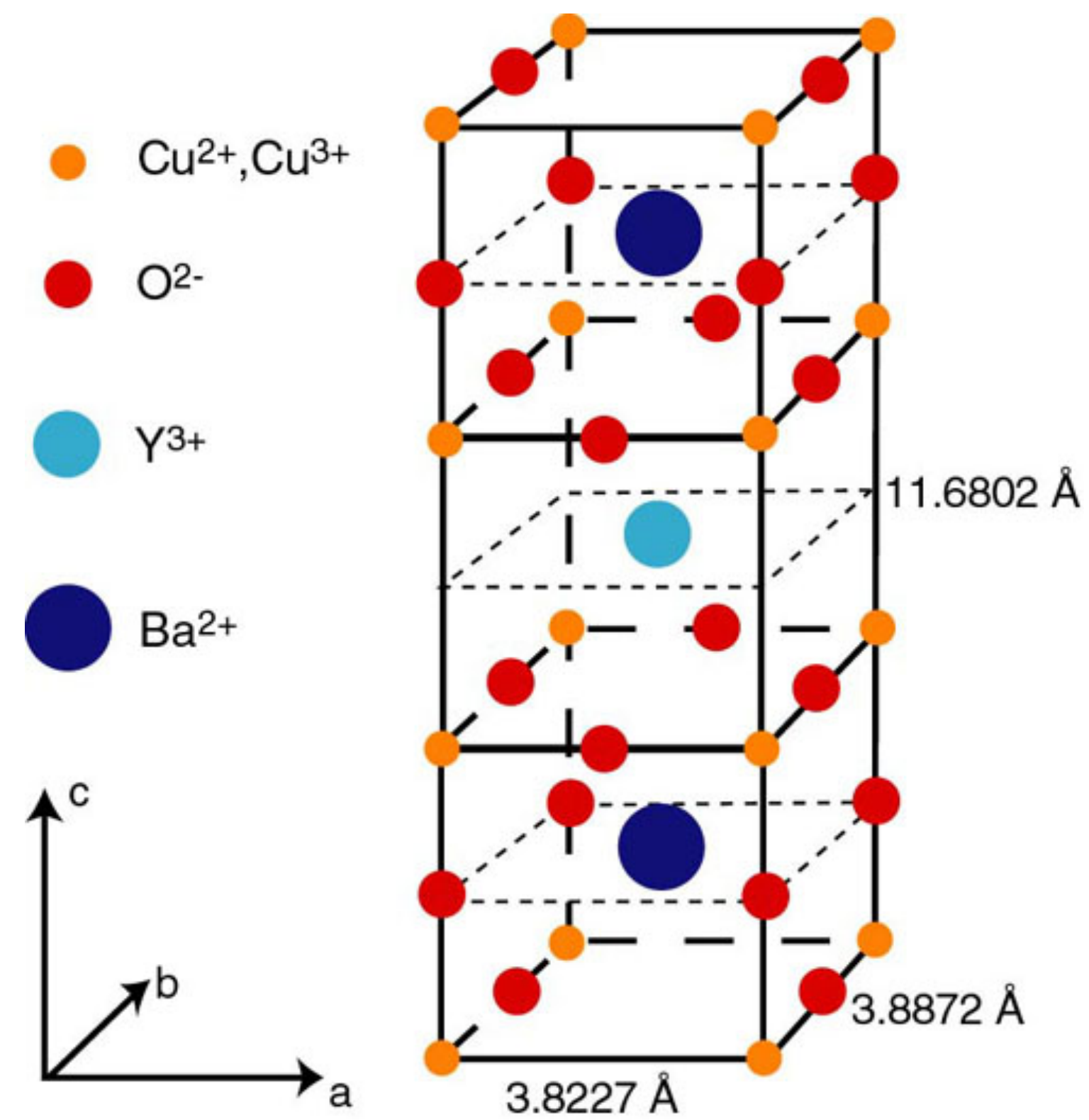
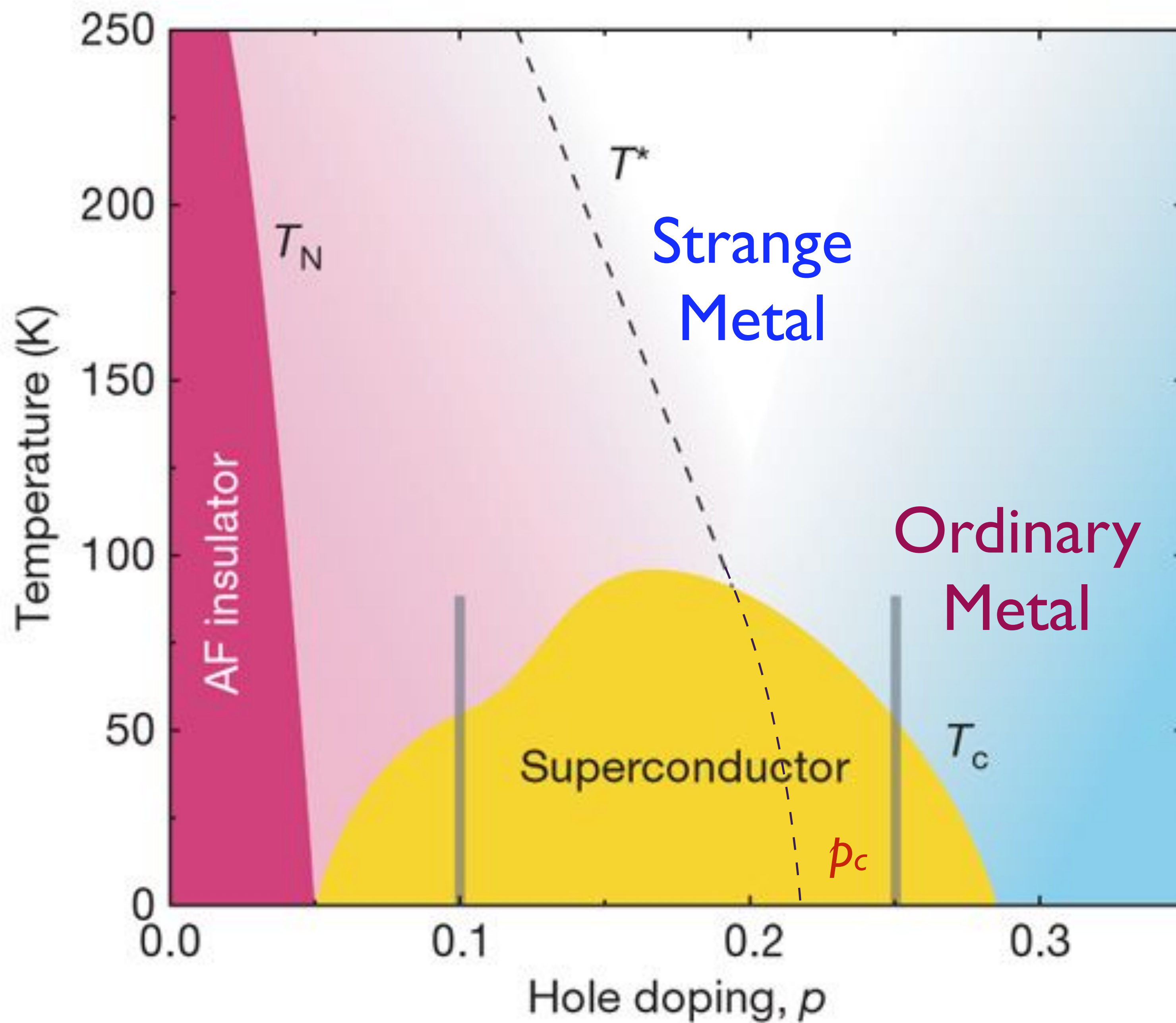


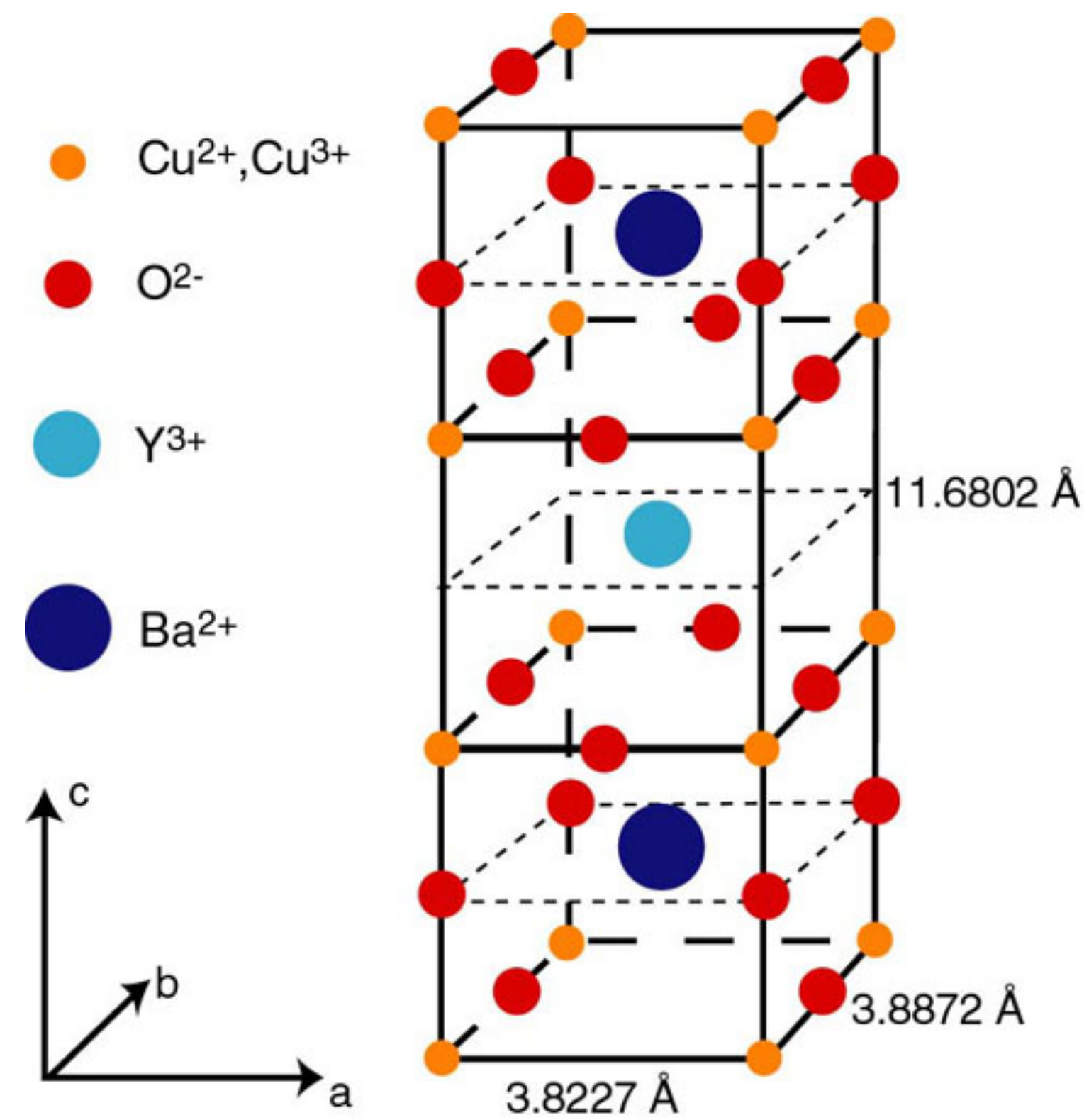
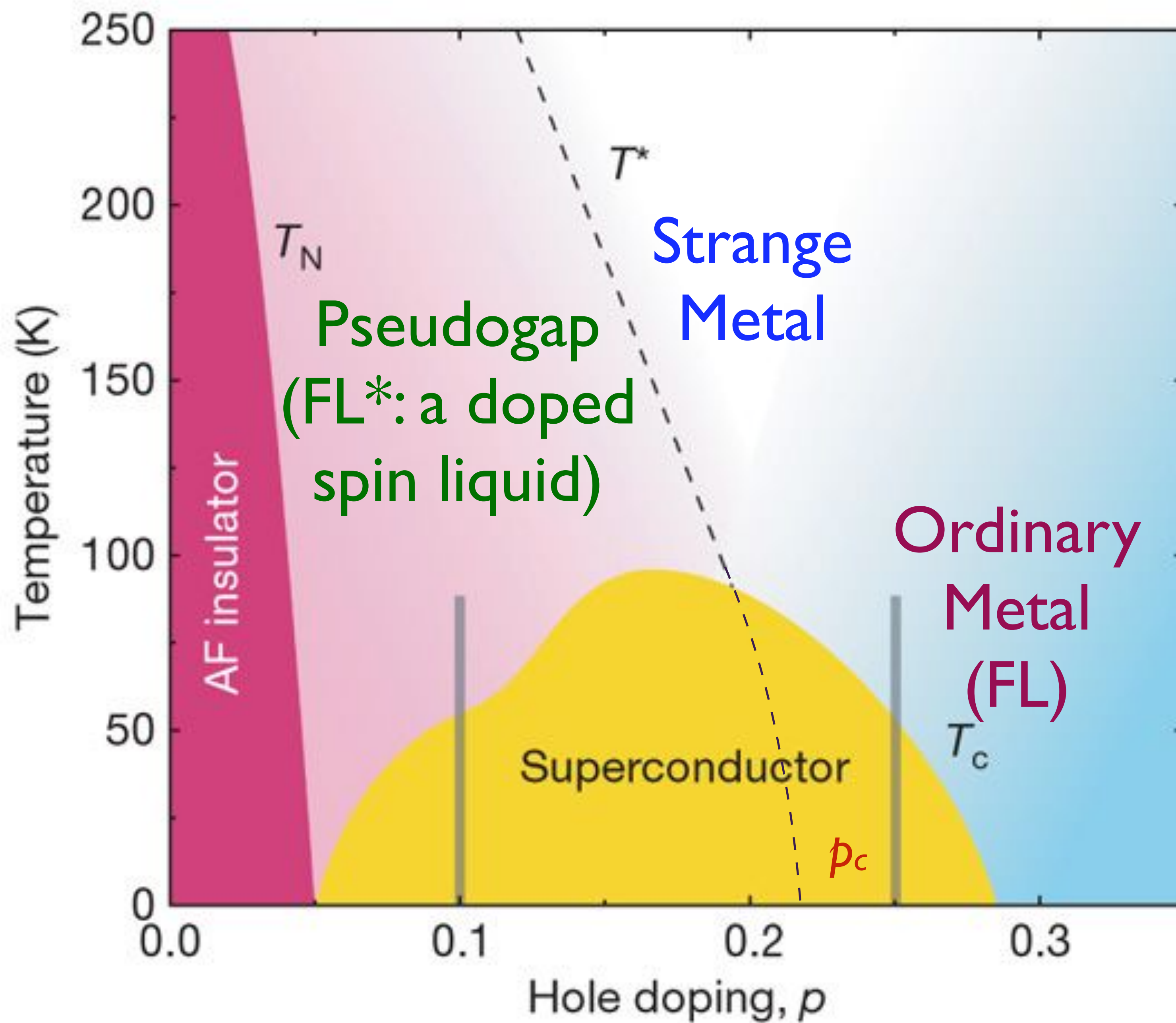
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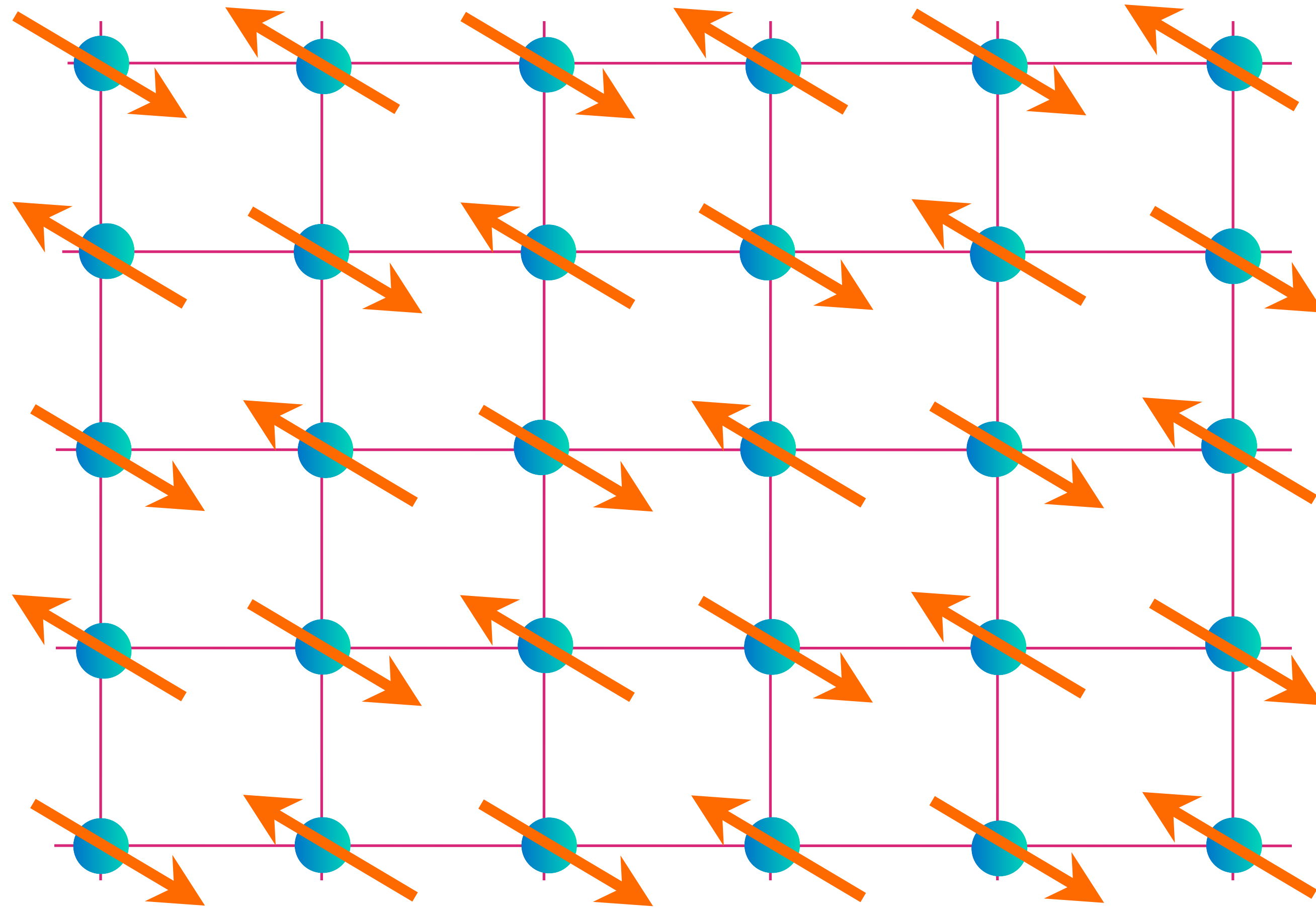


Phase diagram as a function of T and λ





The dance of electrons on Cu atoms in YBCO



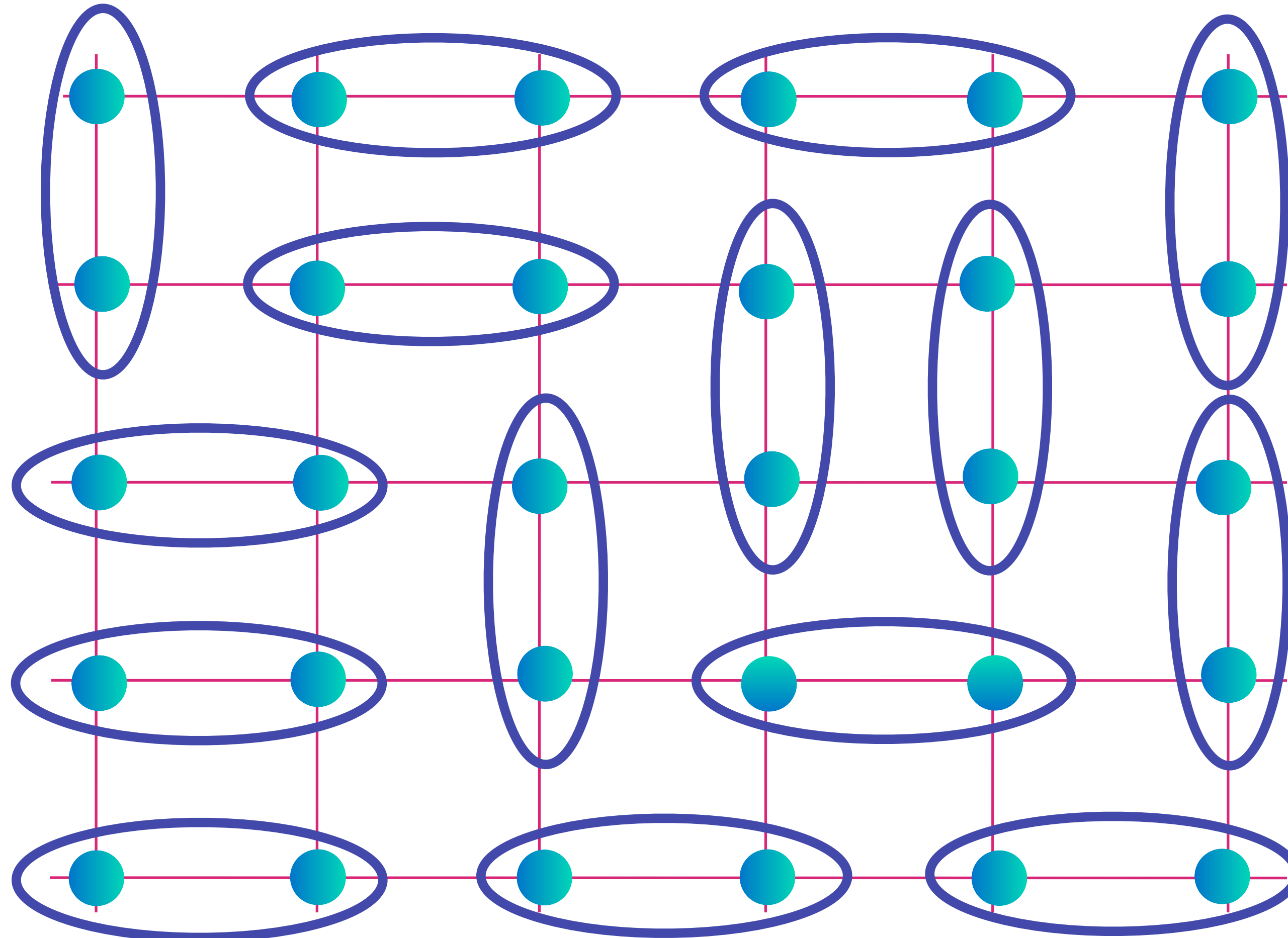
Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample

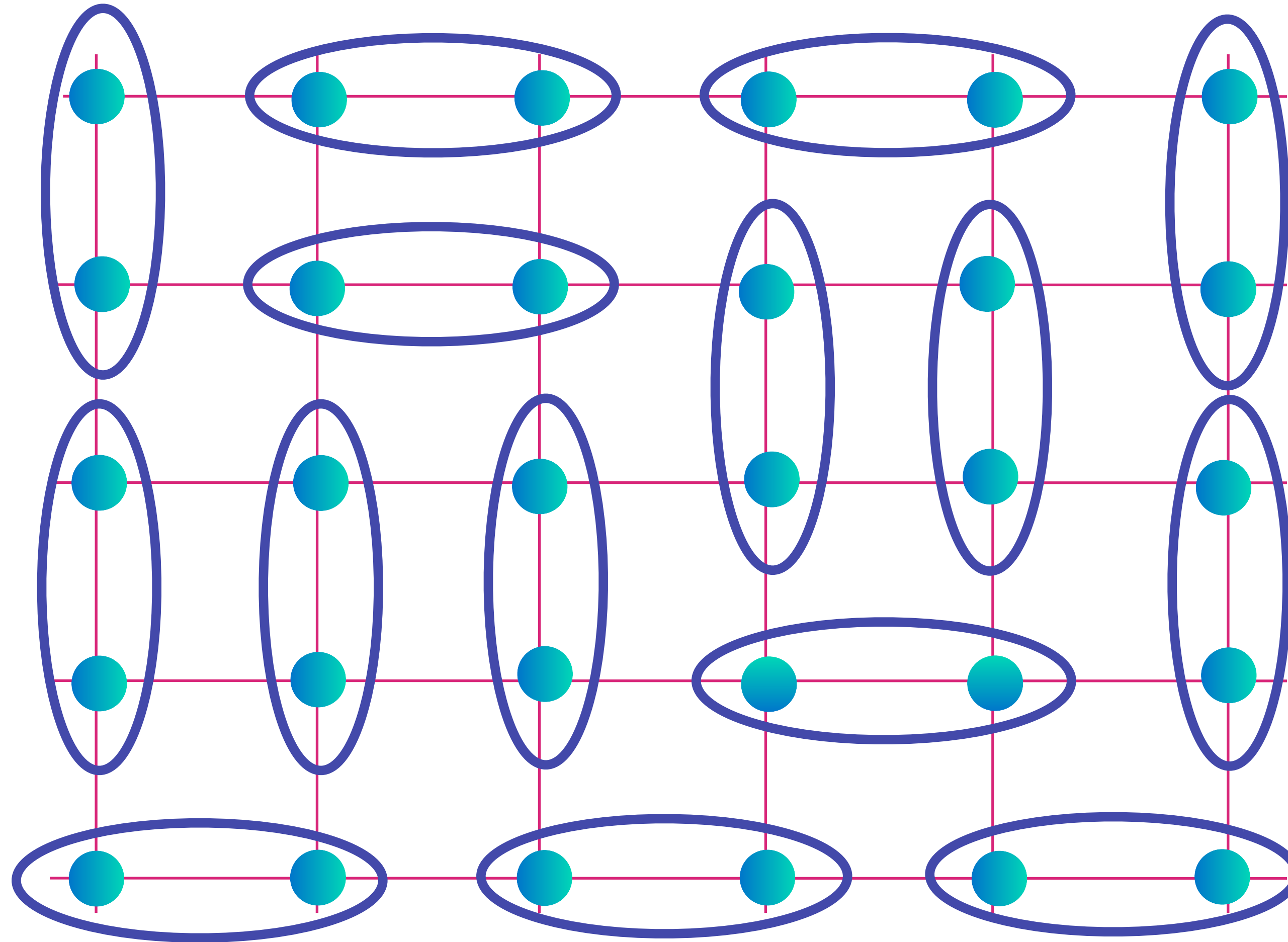
$$\text{[Diagram of two electrons in an oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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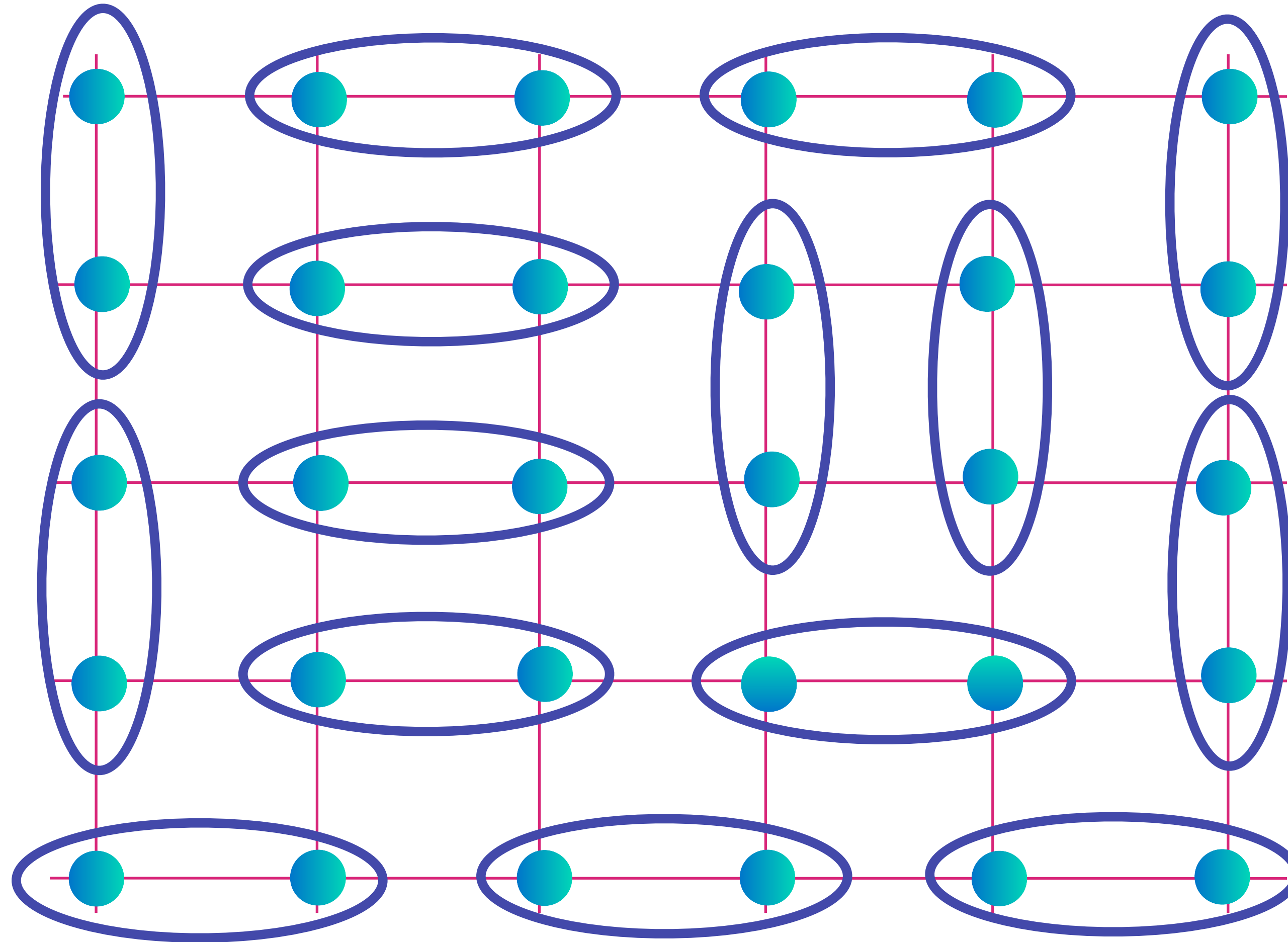


$$\text{[Diagram of two teal dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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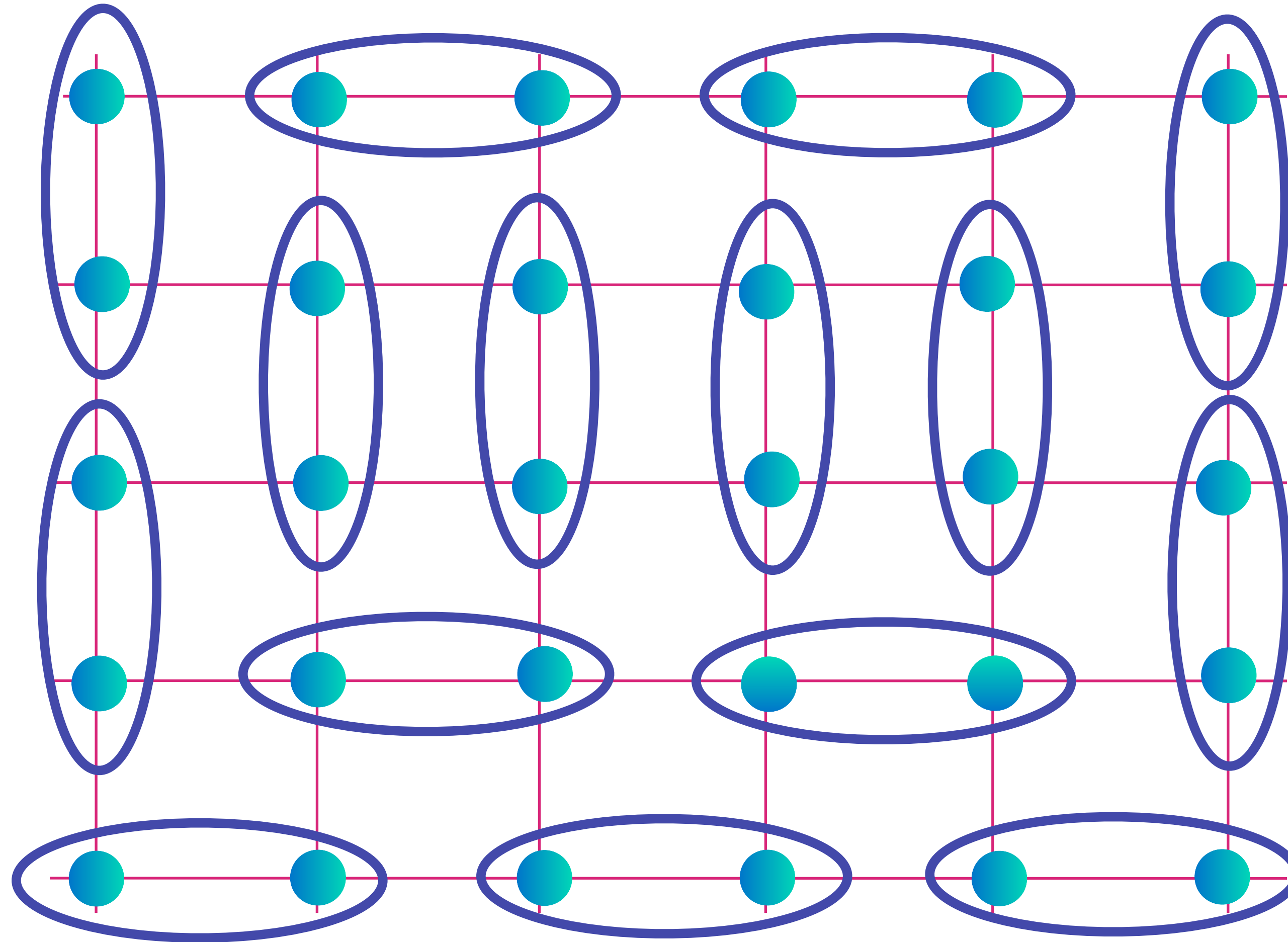
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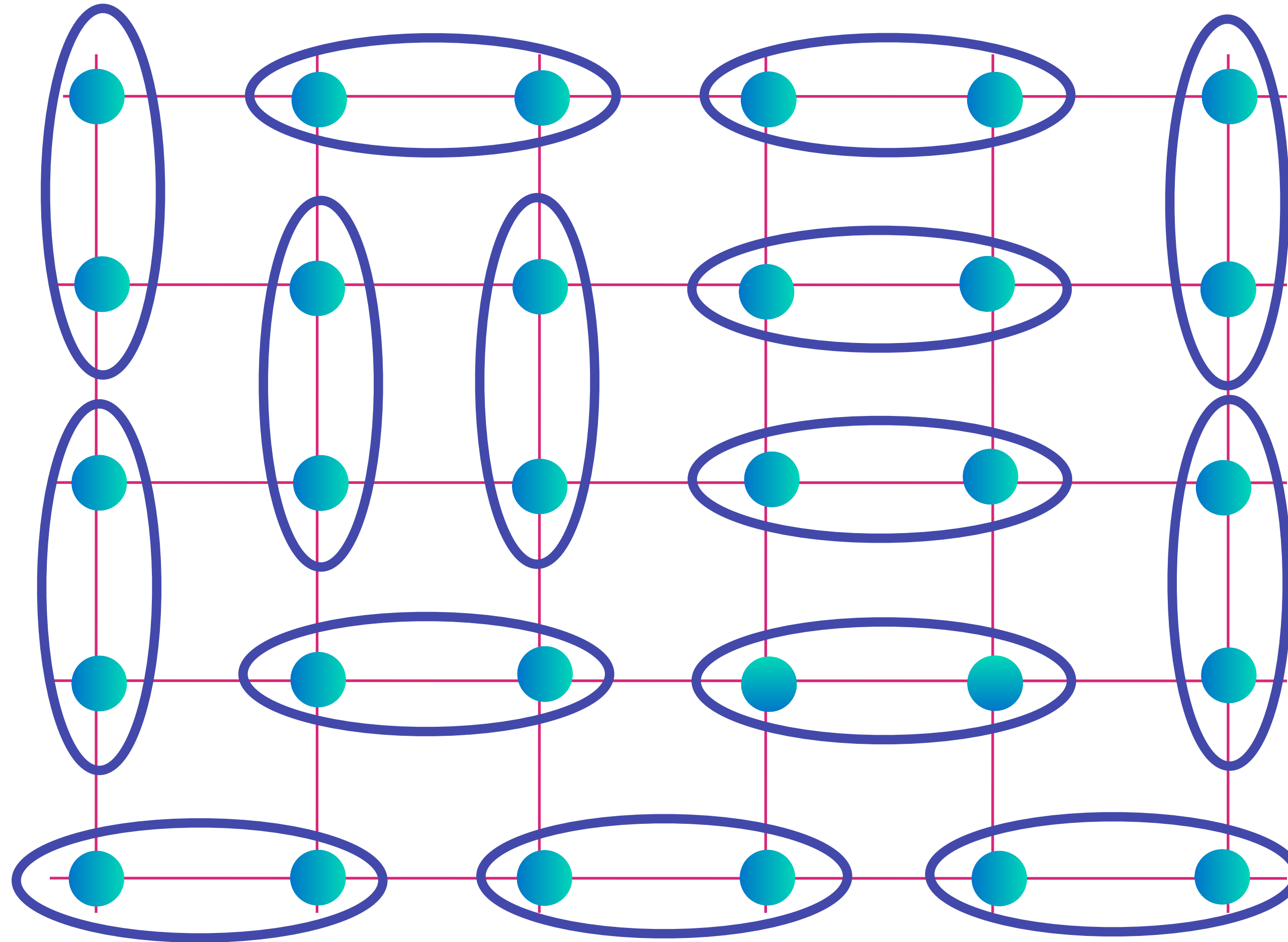
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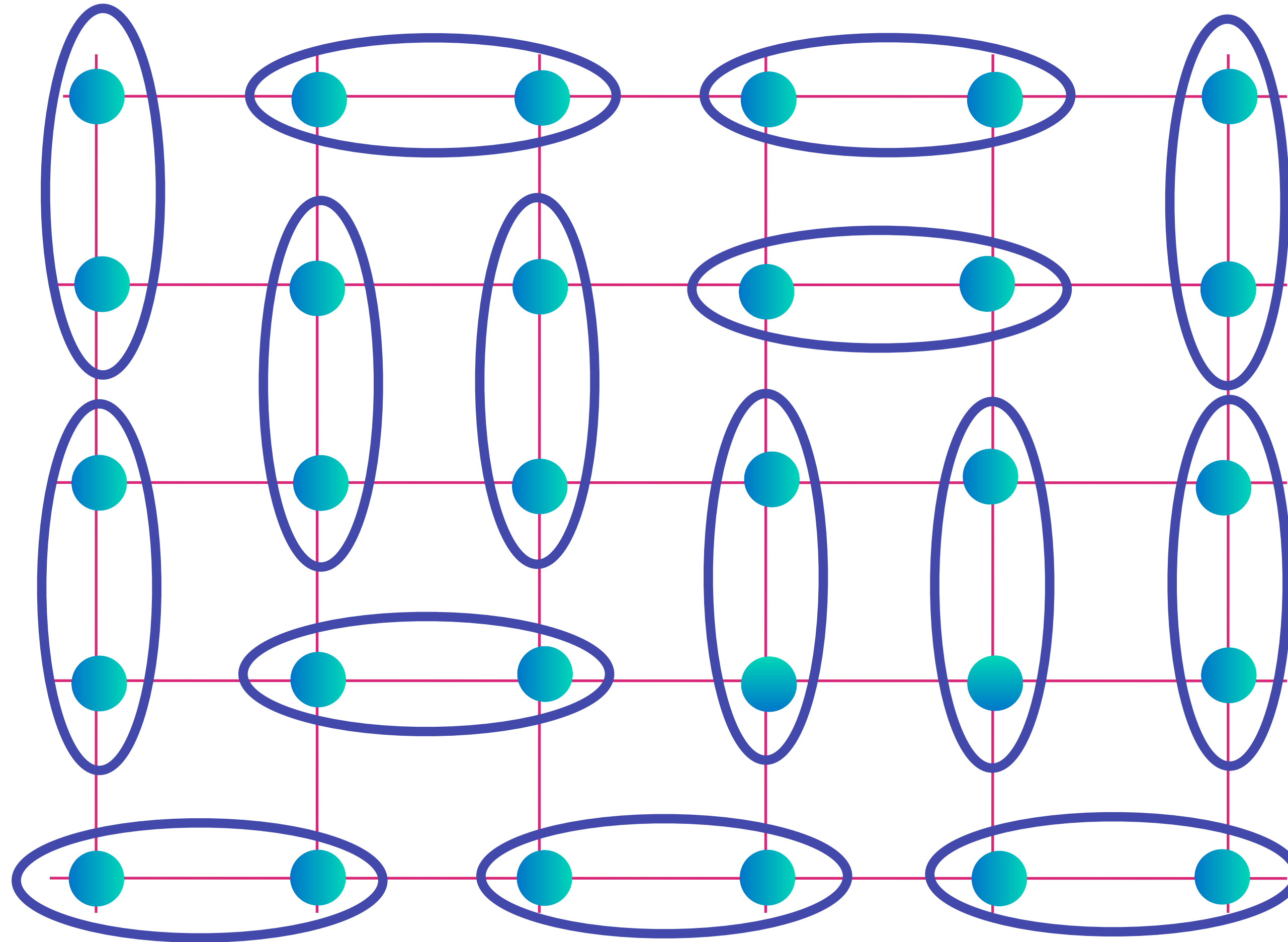
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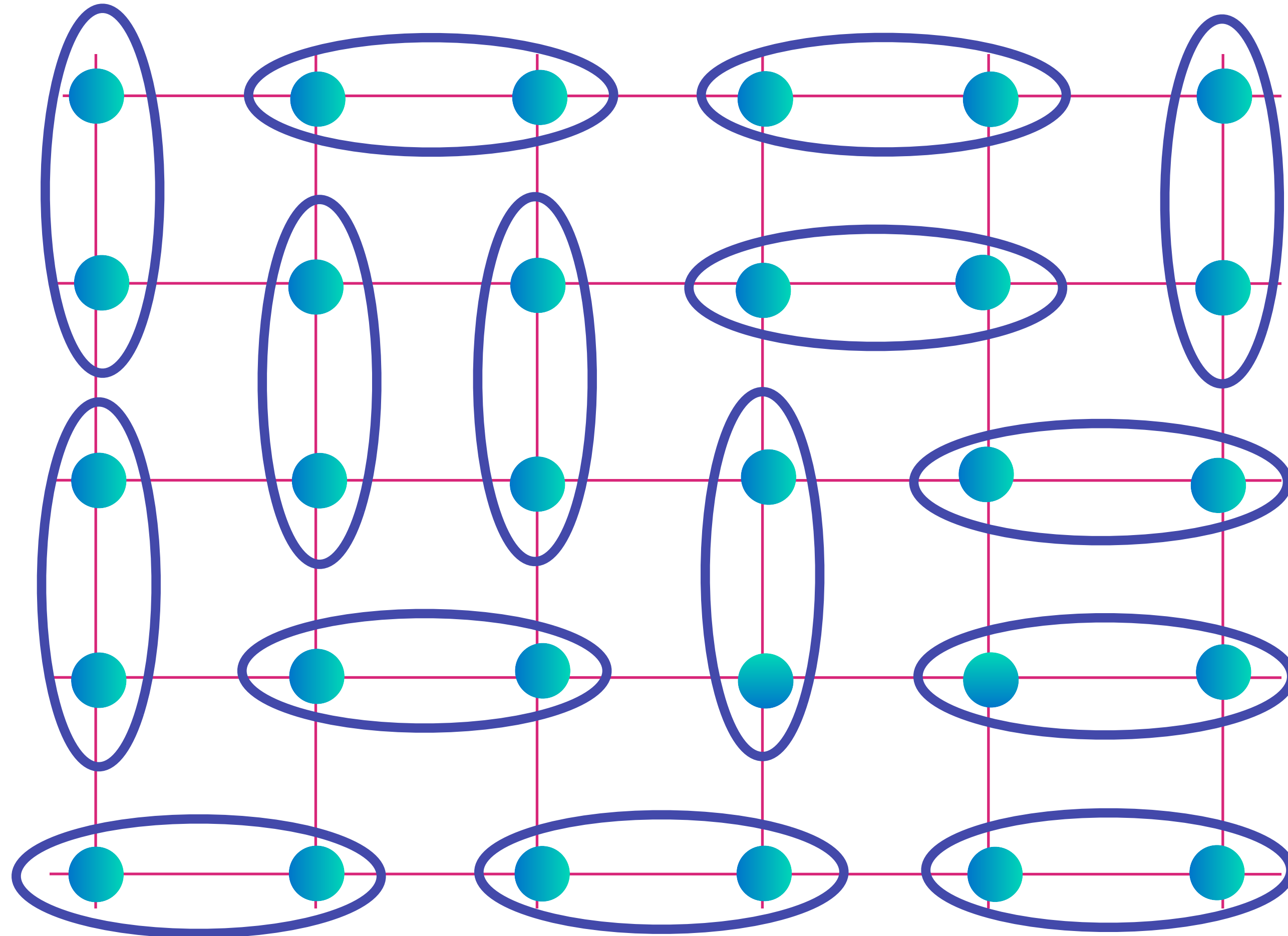
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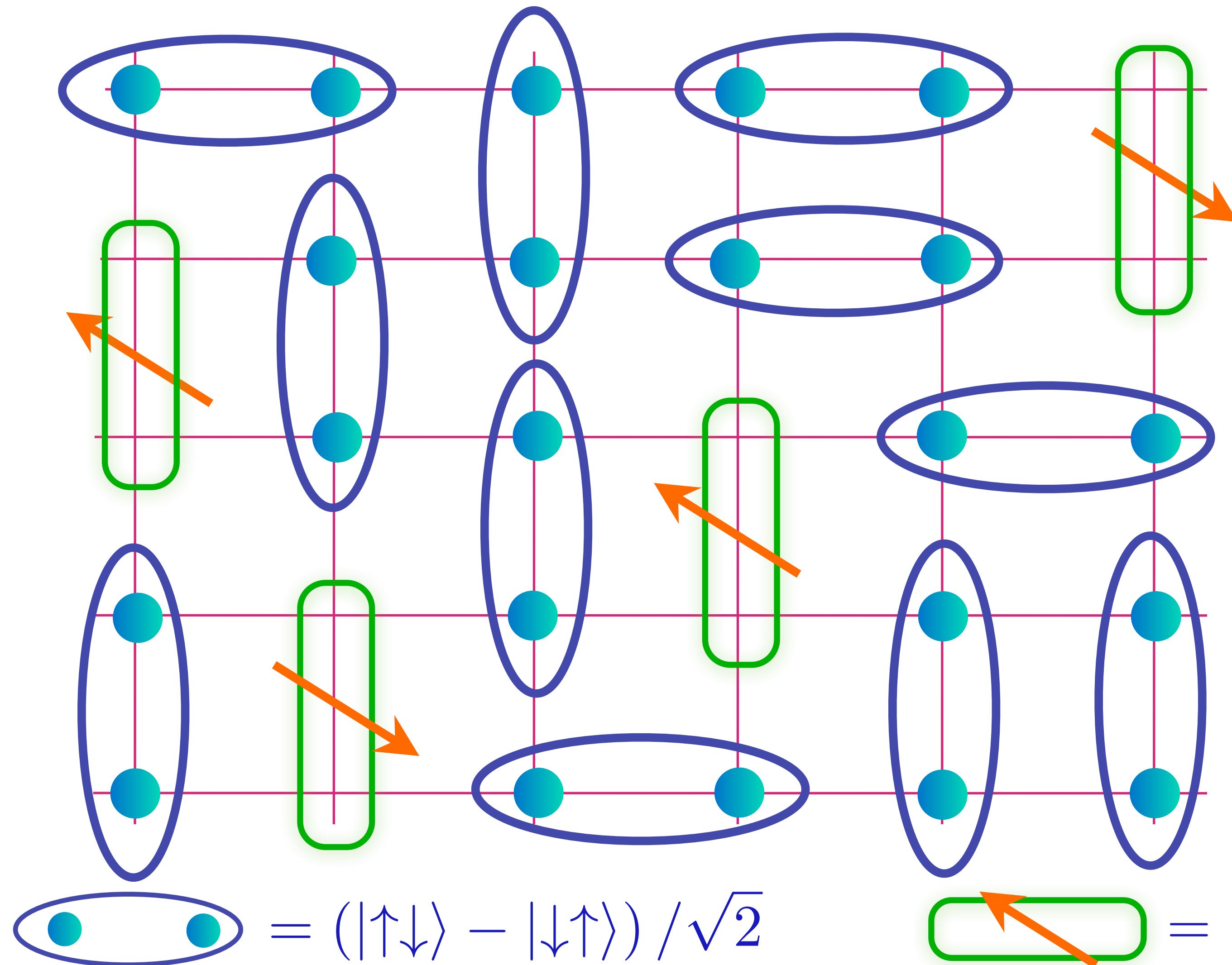
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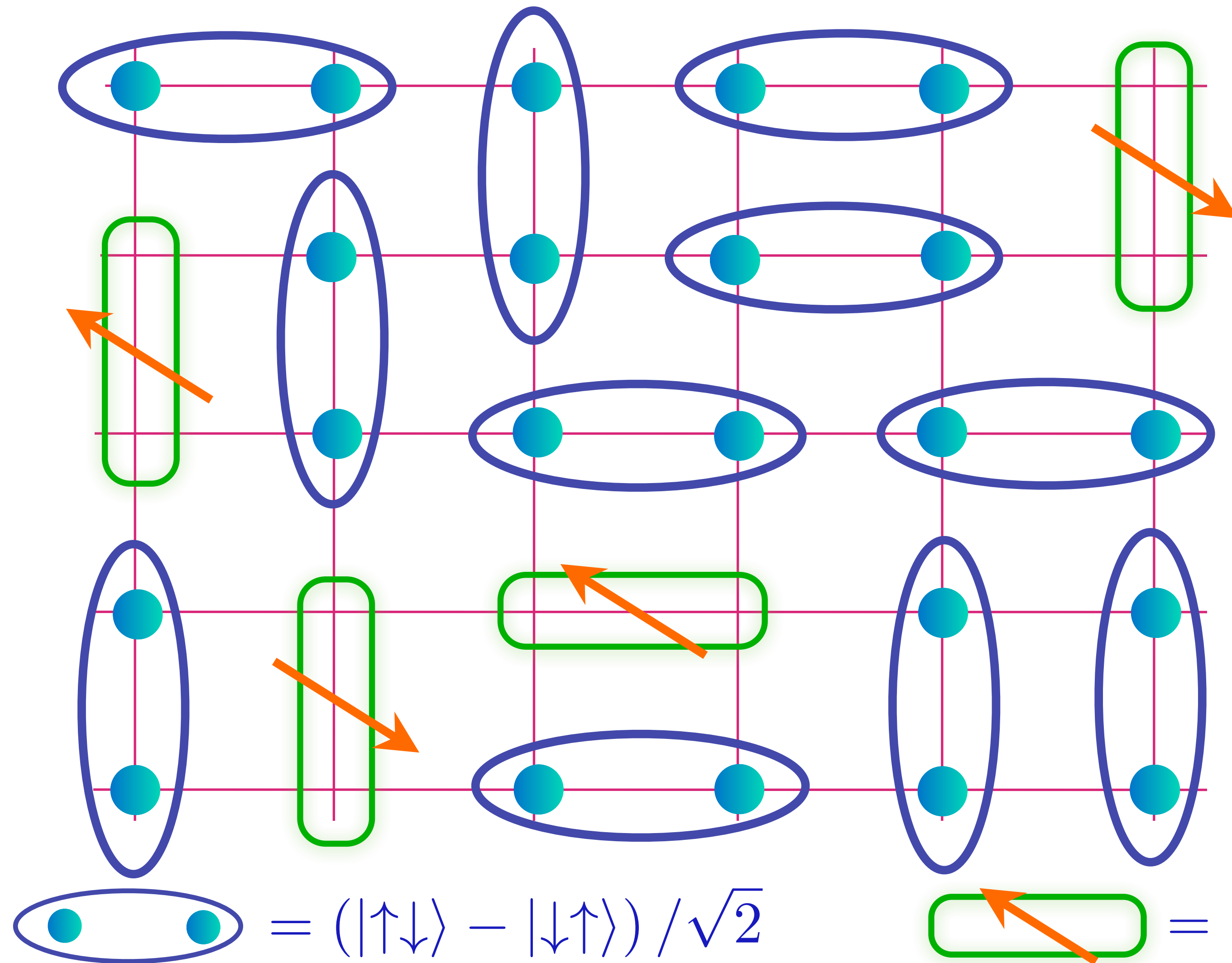


FL*

Metal with electron-like quasiparticles on a Fermi surface of size p , and emergent gauge fields

S.S. PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. S., M.Vojta, PRL **90**, 216403 (2003); E. G. Moon and S. S., PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

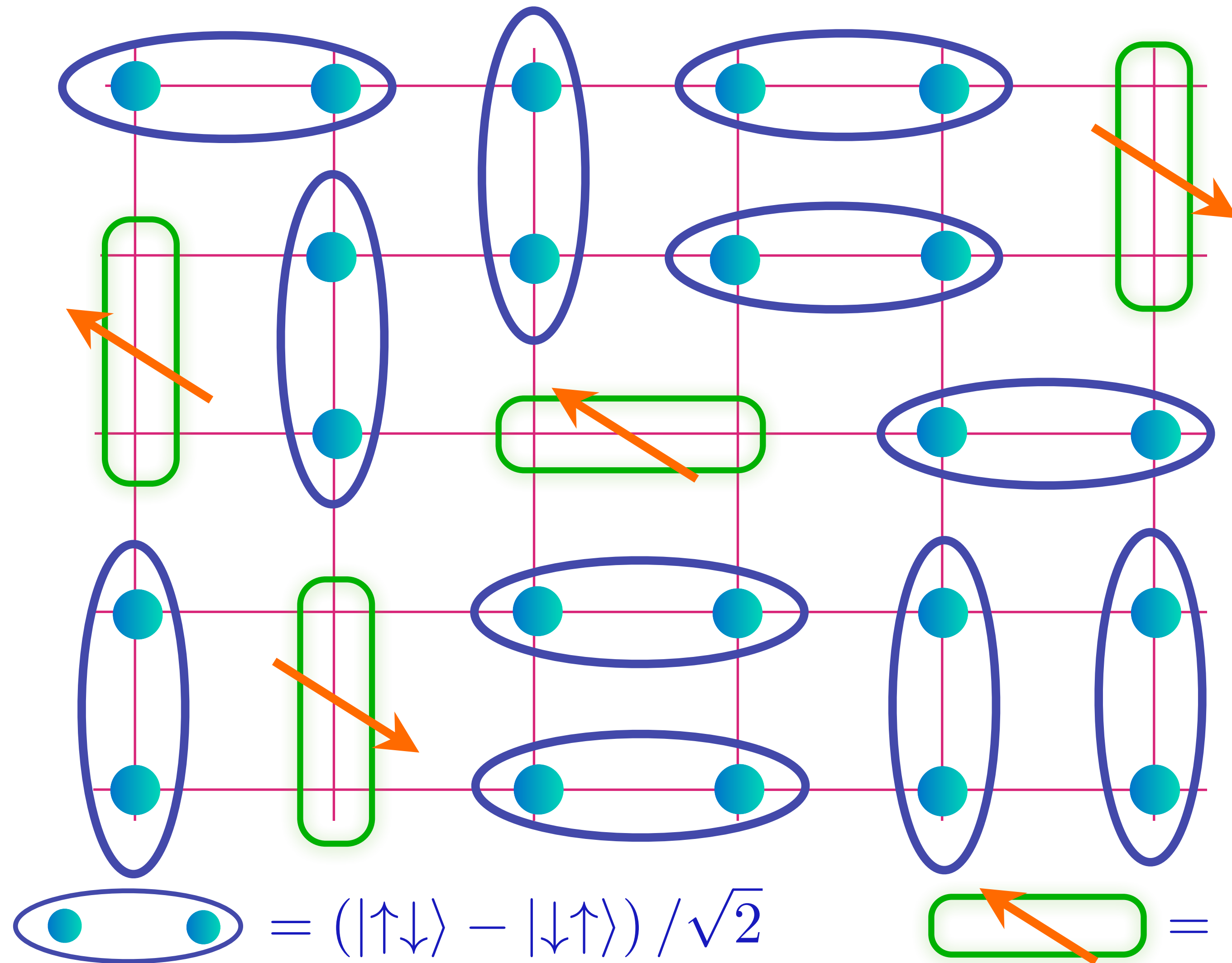
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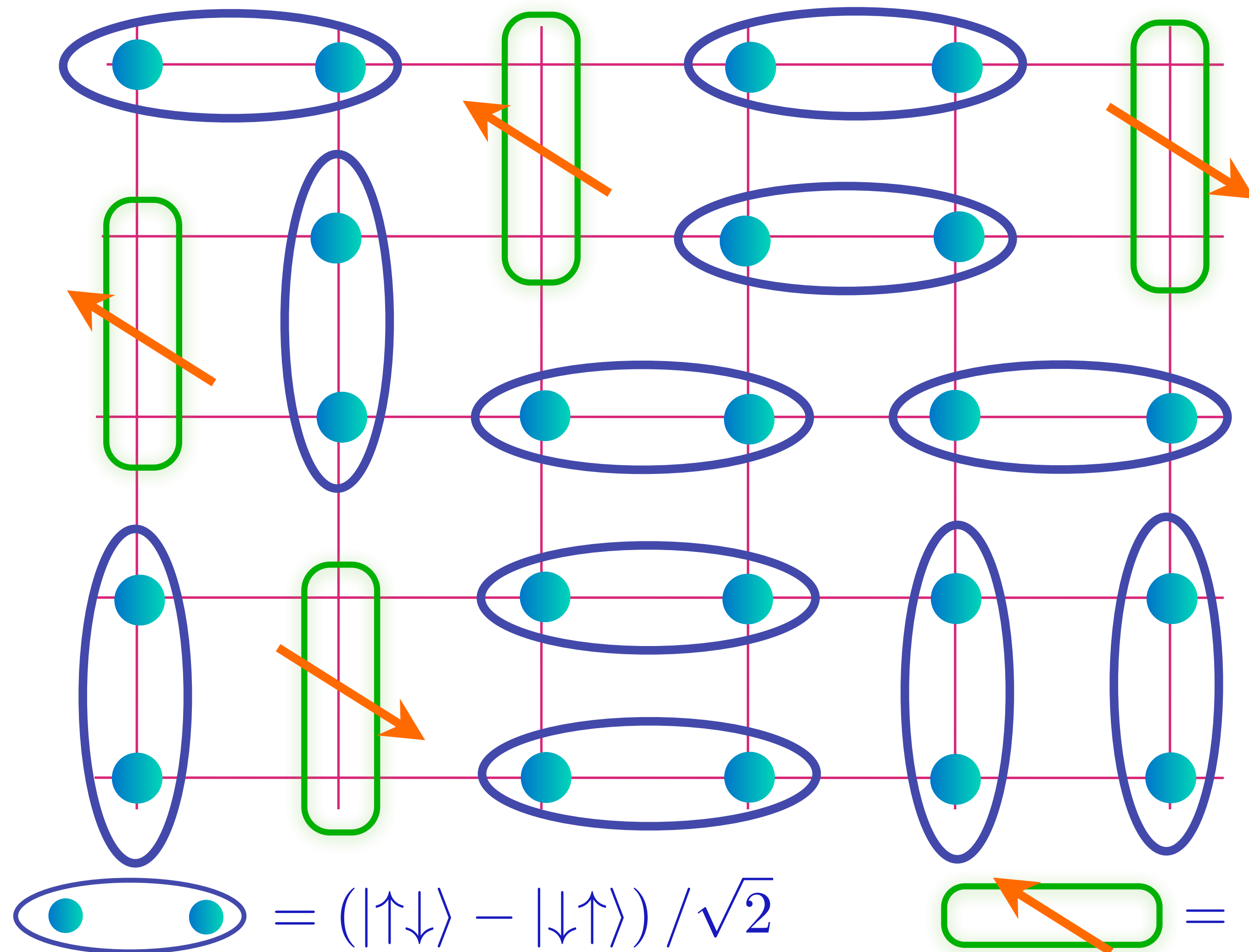
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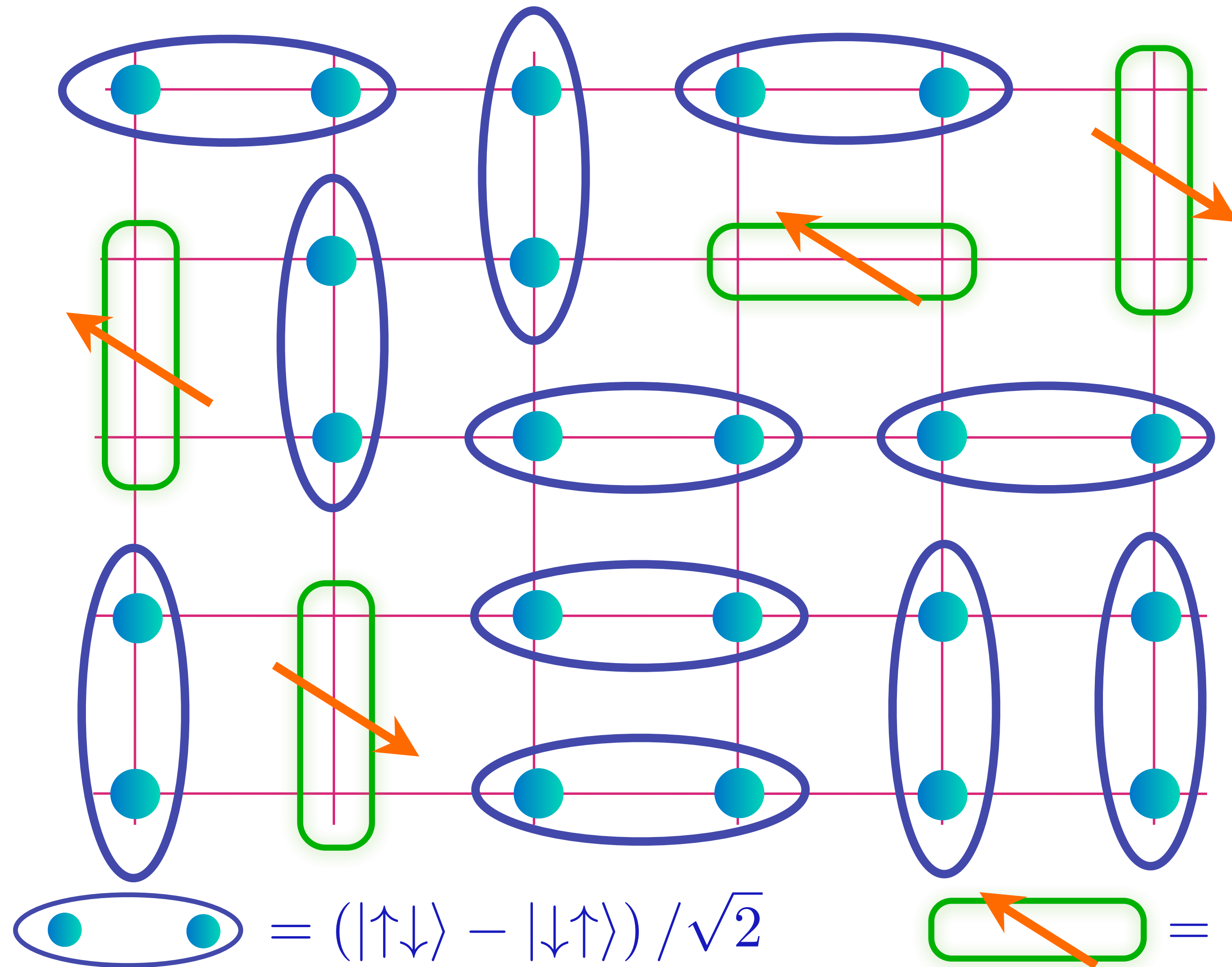


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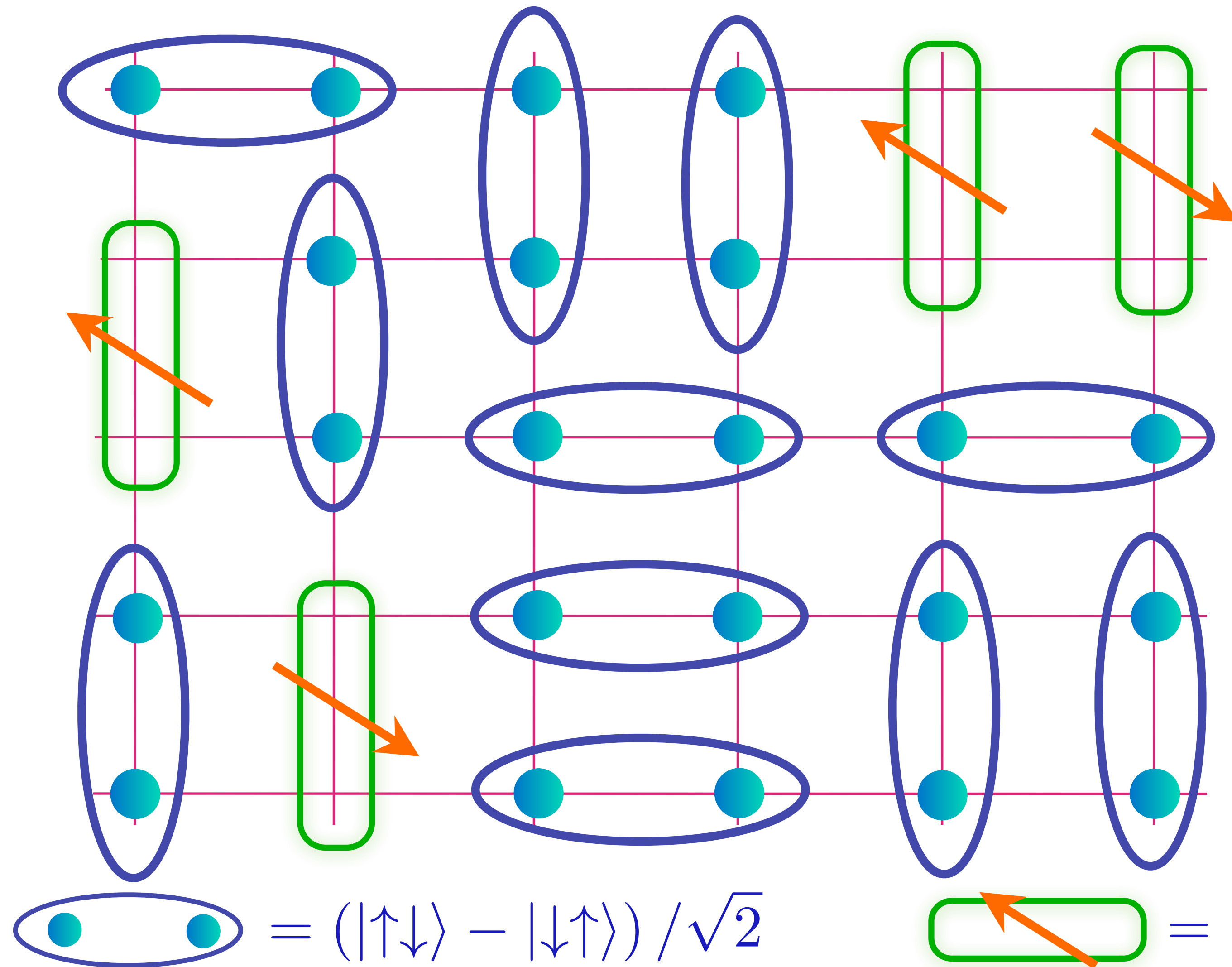
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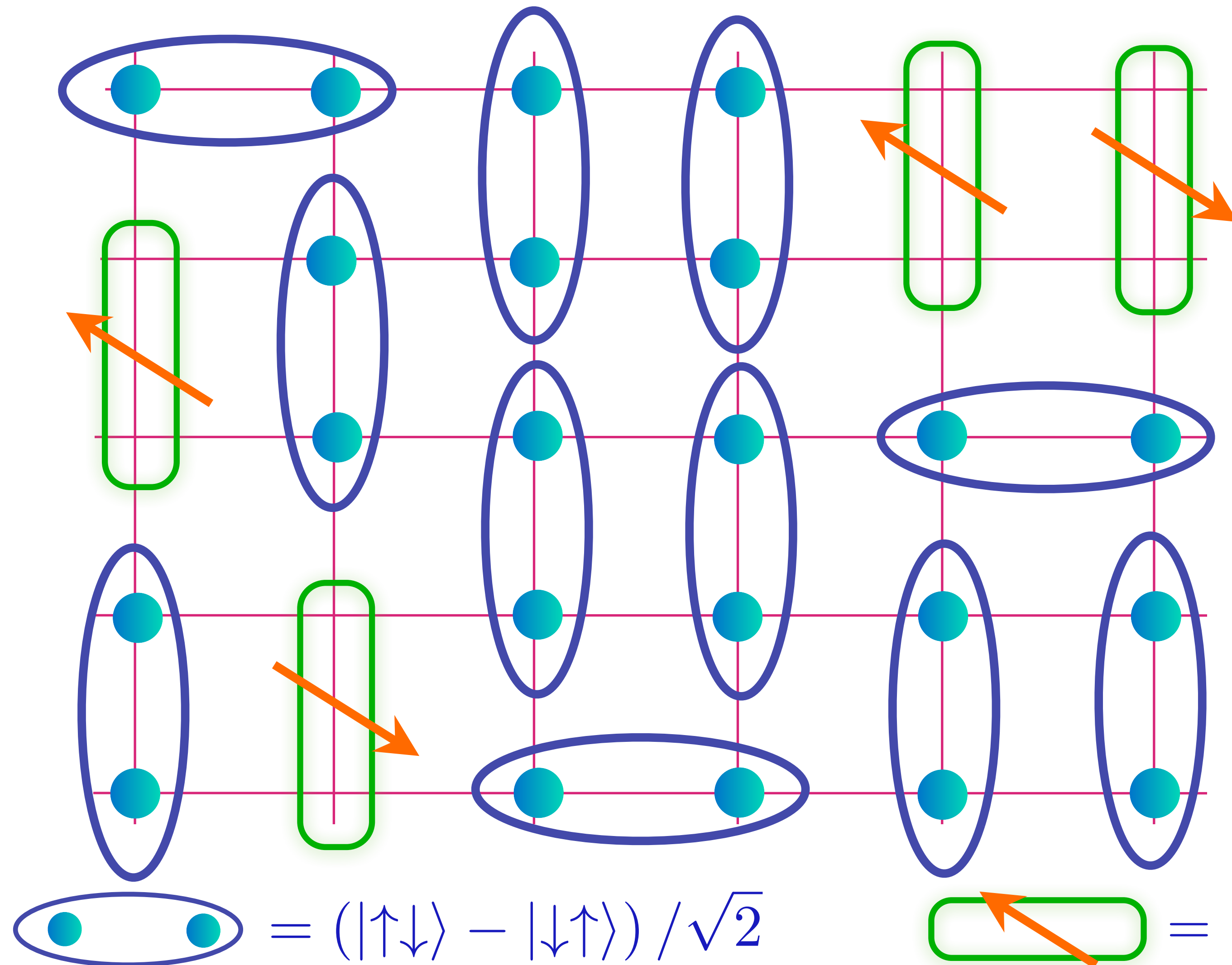


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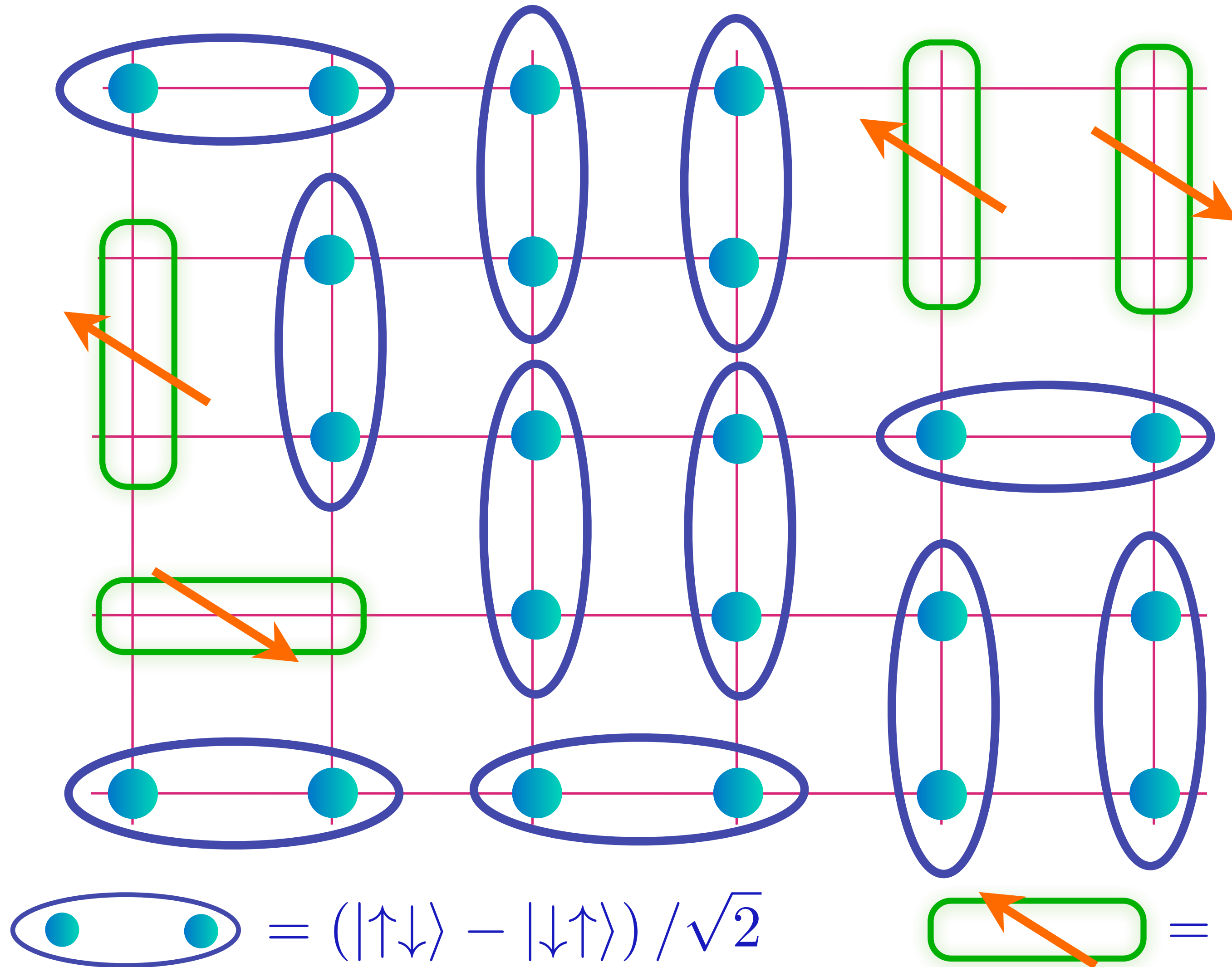


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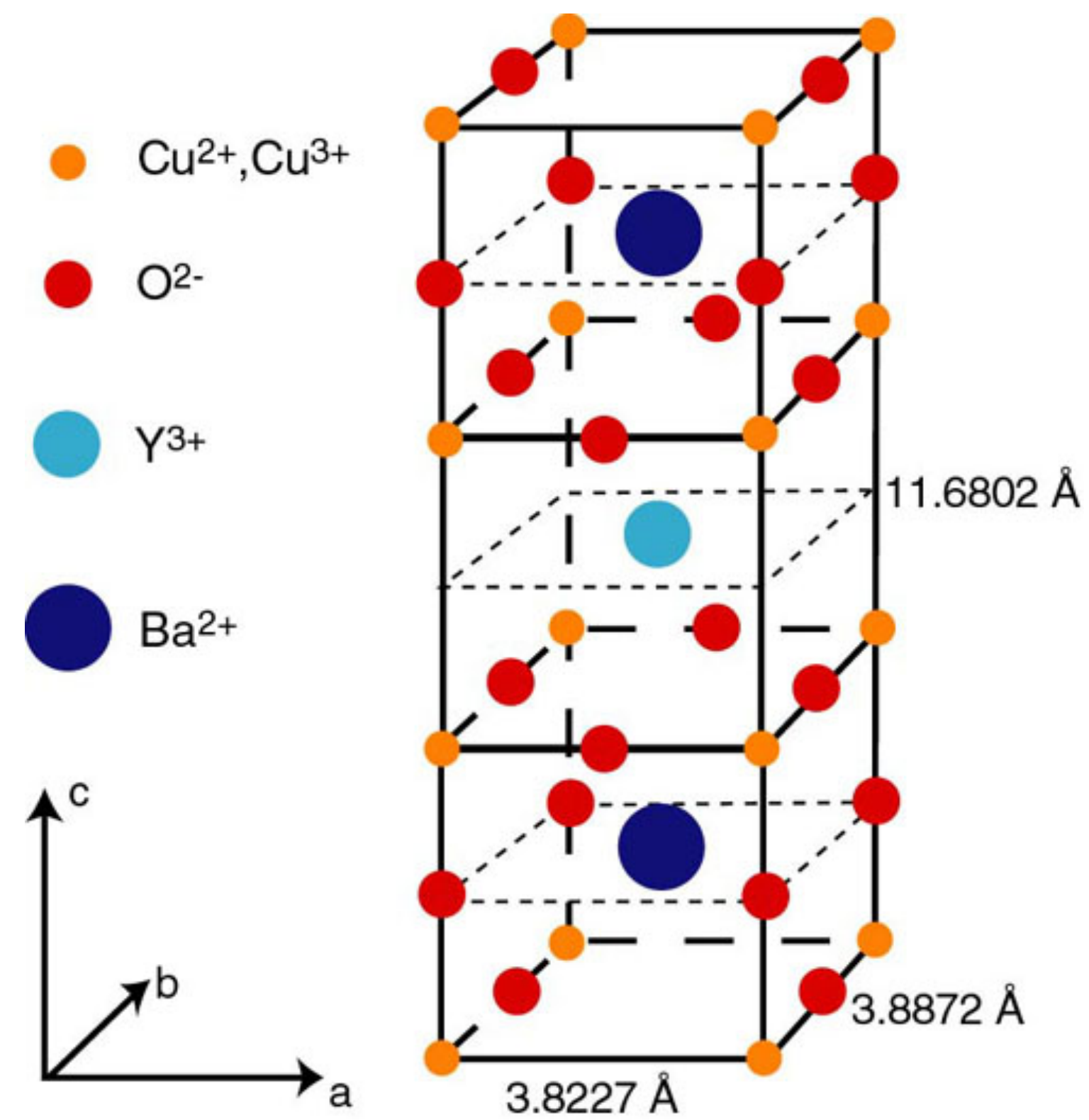
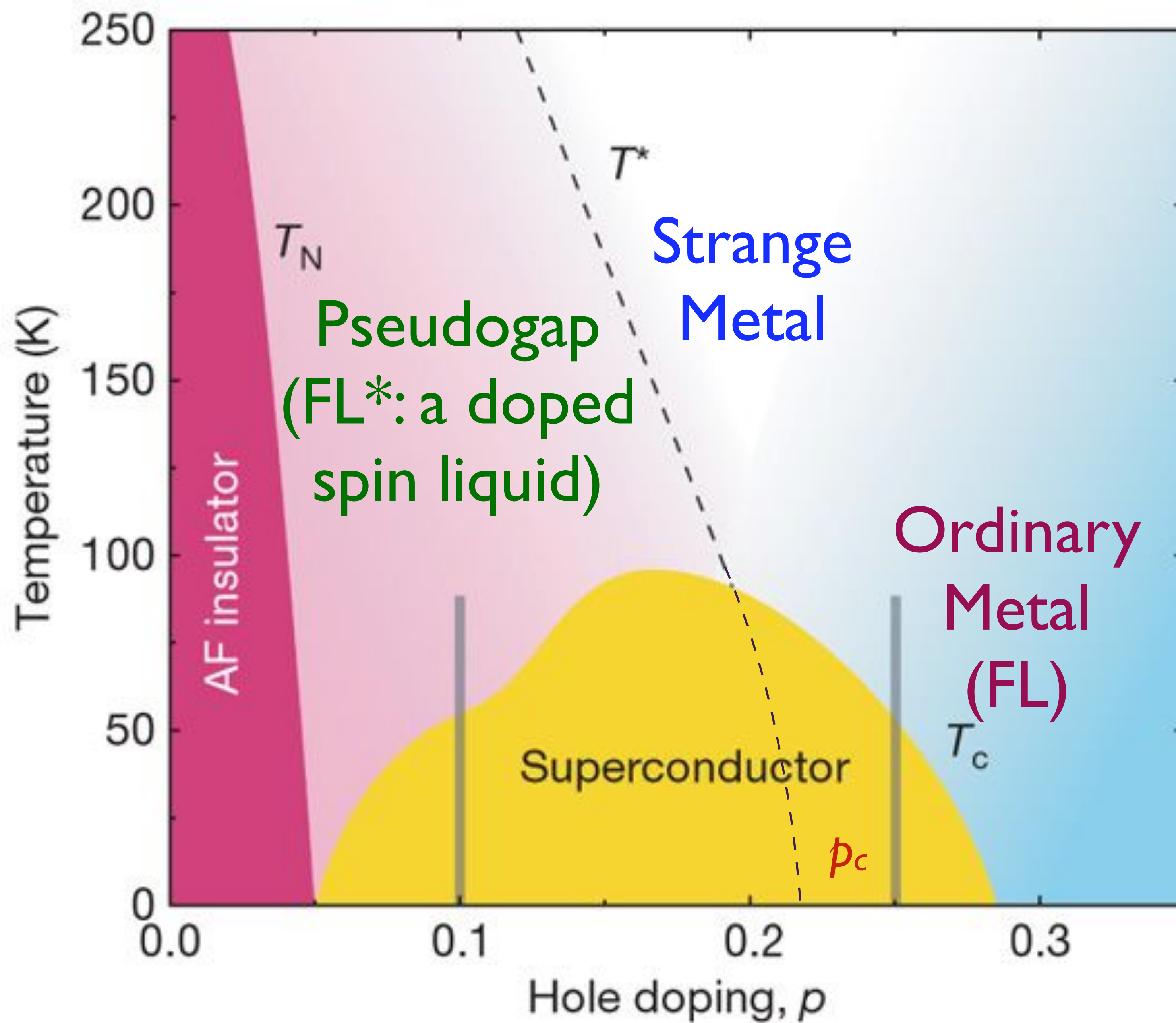
The dance of electrons on Cu atoms in YBCO

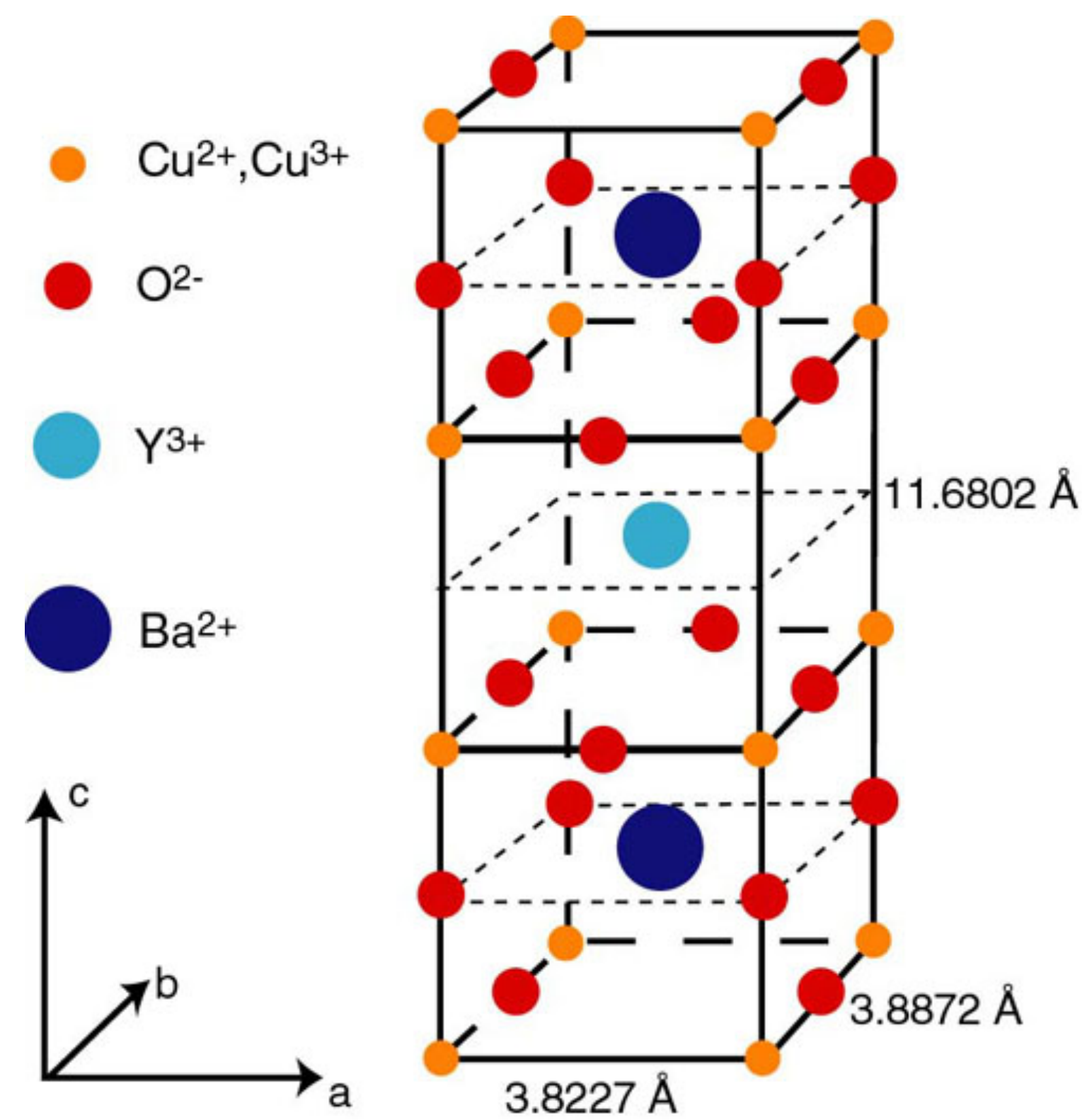
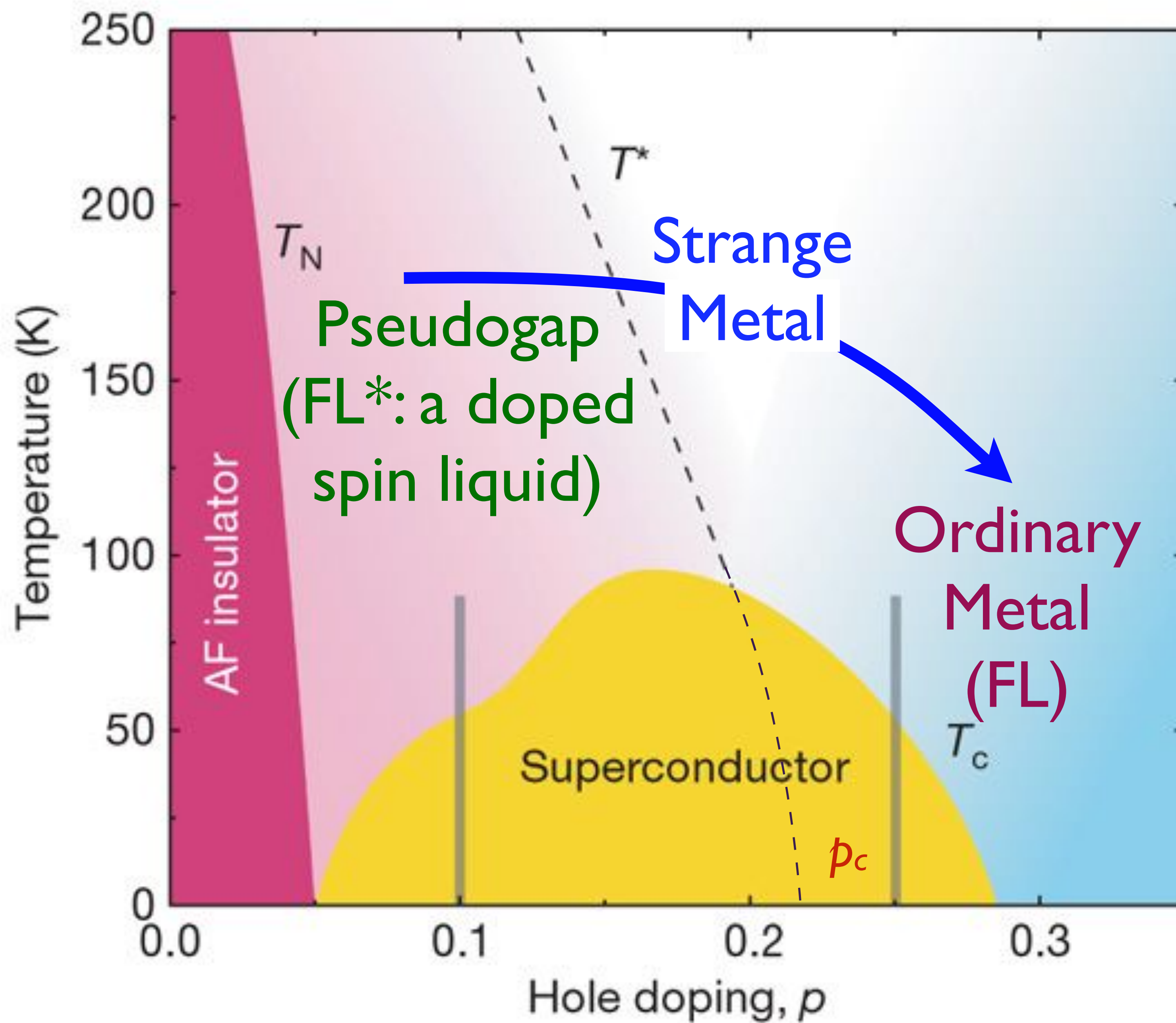


FL*

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S.S. PRB **49**, 6770 (1994); X.-G.Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. S., M.Vojta, PRL **90**, 216403 (2003); E. G. Moon and S. S., PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)





Small Fermi surface
of size p
+
spin liquid.

FL*

Large Fermi surface
of size $1 + p$

FL

$$\langle \Phi \rangle \neq 0$$

$$\langle \Phi \rangle = 0$$

doping p

Φ

Higgs boson mediating
electron fractionalization

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

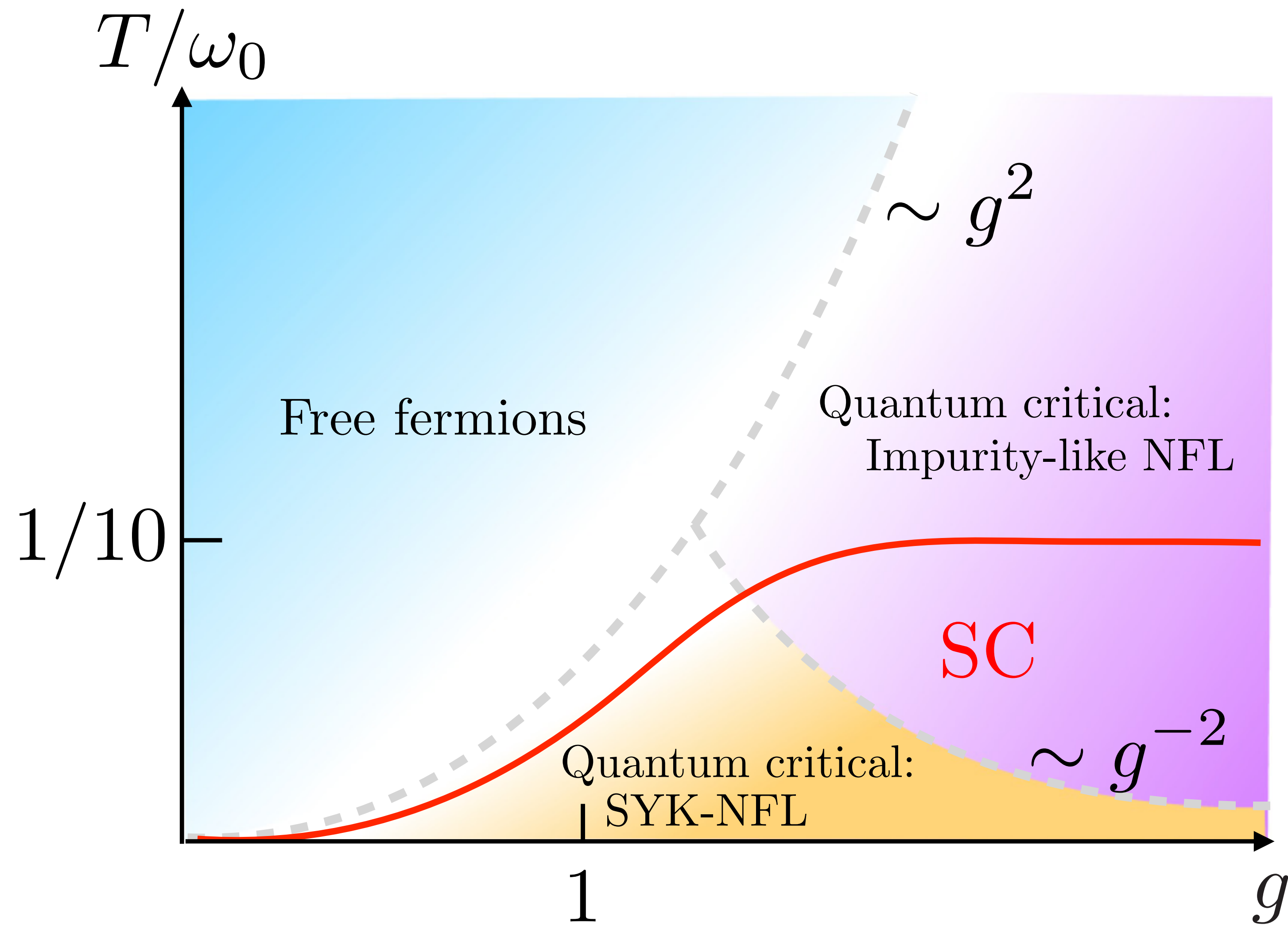
$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037 \dots$$

I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

Yukawa-SYK model



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

See also Yuxuan Wang, PRL **124**, 017002 (2020)

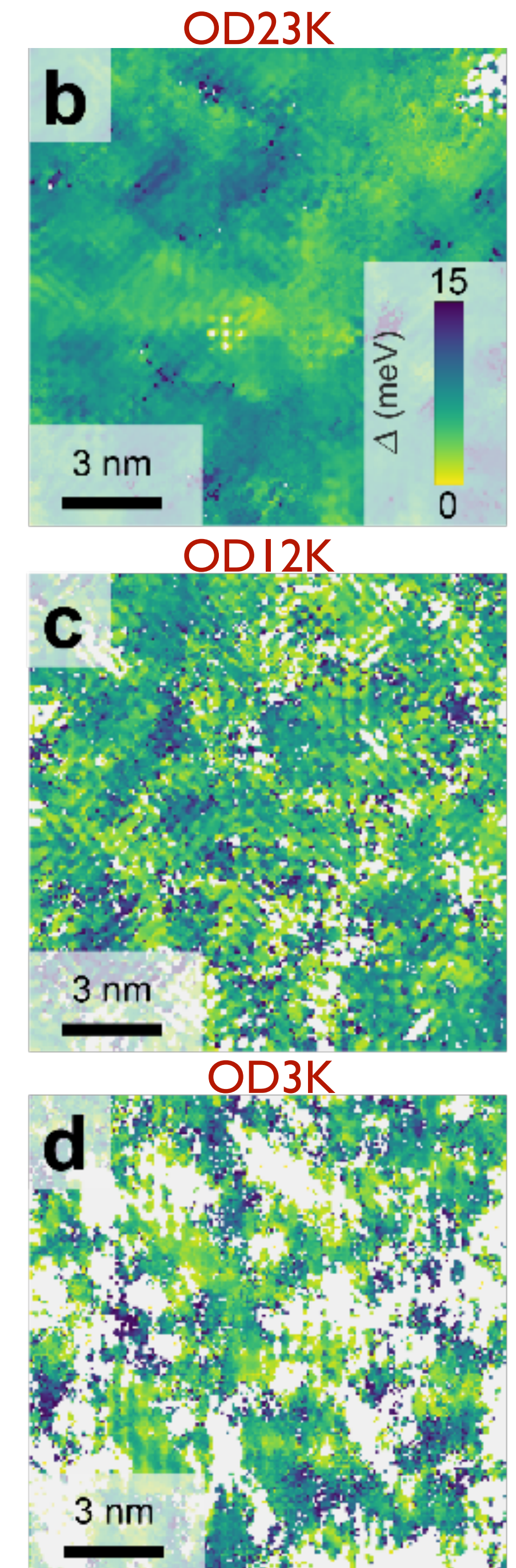
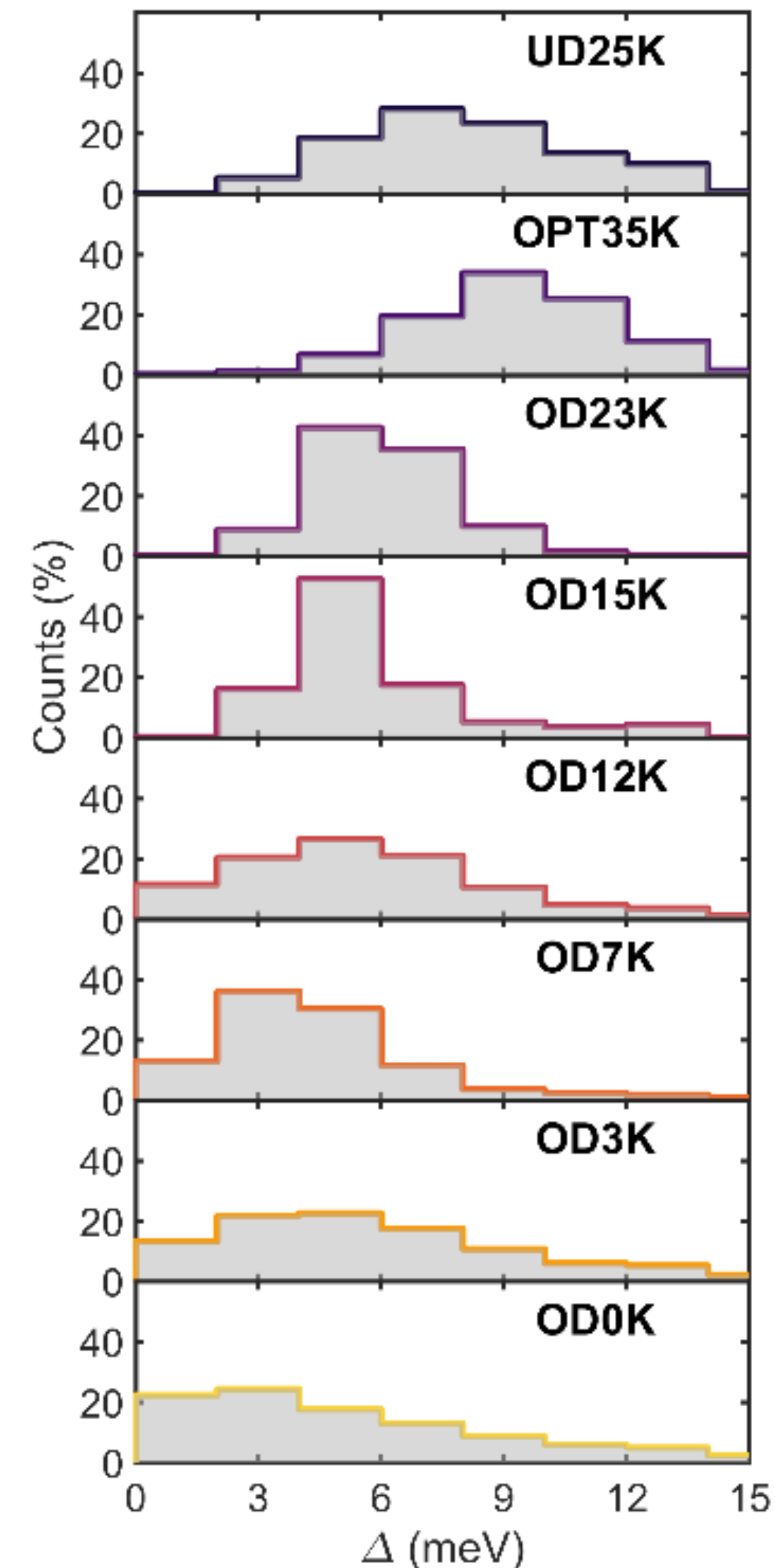
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(\text{Pb,Bi})_2\text{Sr}_2\text{CuO}_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(\text{Pb,Bi})_2\text{Sr}_2\text{CuO}_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740



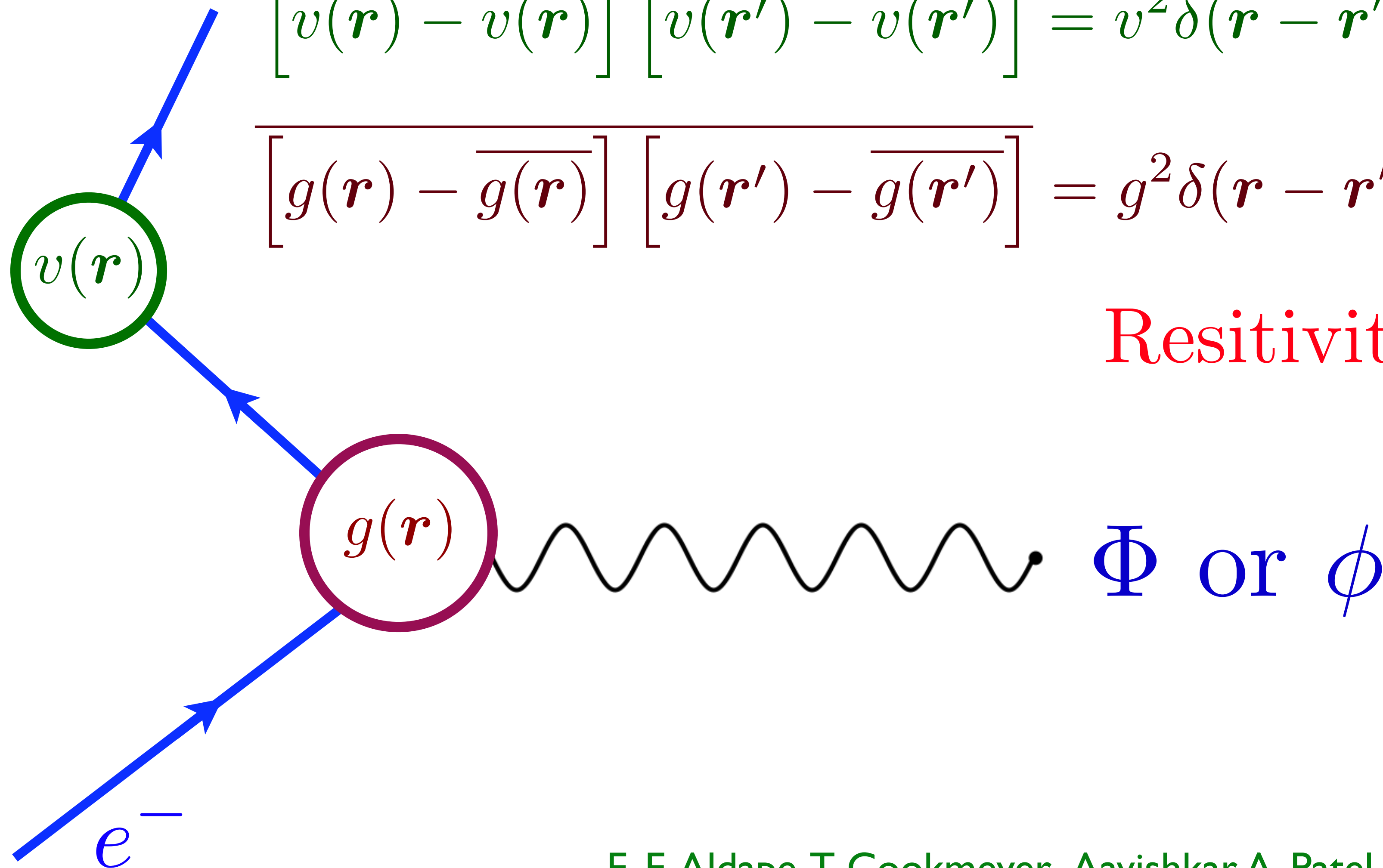
Universal theory of strange metals

$$\overline{[v(\mathbf{r}) - \overline{v(\mathbf{r})}] [v(\mathbf{r}') - \overline{v(\mathbf{r}')}] = v^2 \delta(\mathbf{r} - \mathbf{r}')$$

$$\overline{[g(\mathbf{r}) - \overline{g(\mathbf{r})}] [g(\mathbf{r}') - \overline{g(\mathbf{r}')}] = g^2 \delta(\mathbf{r} - \mathbf{r}')$$

Random interactions as
in Yukawa-SYK model

$$\text{Resistivity } \rho(T) \sim v^2 + g^2 T$$



E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, PRB **105**, 235111 (2022)

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990, Science, to appear

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).
Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP **94**, 041002 (2002)

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

Erik van Heumen.....Jan Zaanen, PRB **106**, 054515 (2022)

B. Michon.....A. Georges, Nat. Commun. **14**, 3033 (2023)

4. Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber....D. Dessau, Nature Communications **10**, 5737 (2019)

The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and the strange metal
- Fractionalization and new emergent particles, as in spin liquids.
- Higher temperature superconductivity (?)
- A quantum theory of the interior of a black hole.