Order and quantum phase transitions in the cuprate superconductors

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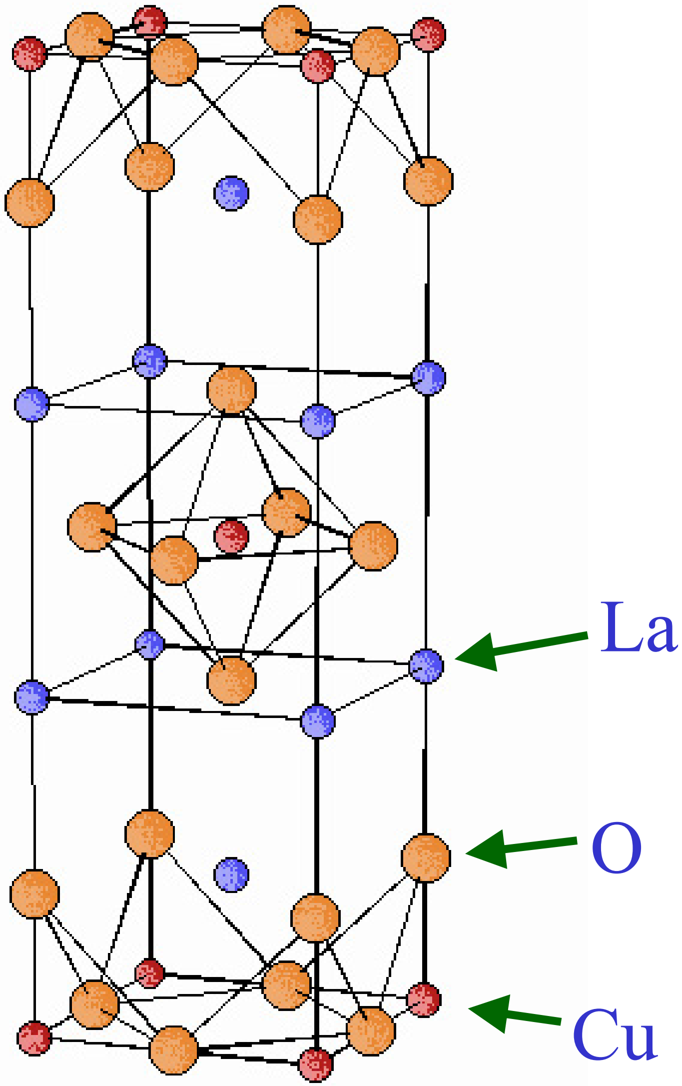
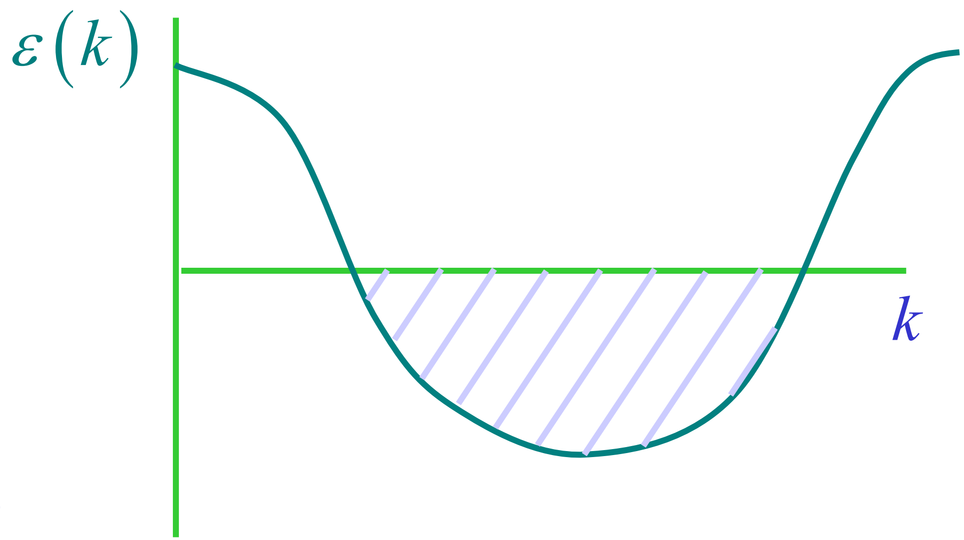
Talk online:
Google Sachdev

YALE UNIVERSITY TERCENTENNIAL 1701 2001

Parent compound of the high temperature superconductors: La_2CuO_4



Band theory

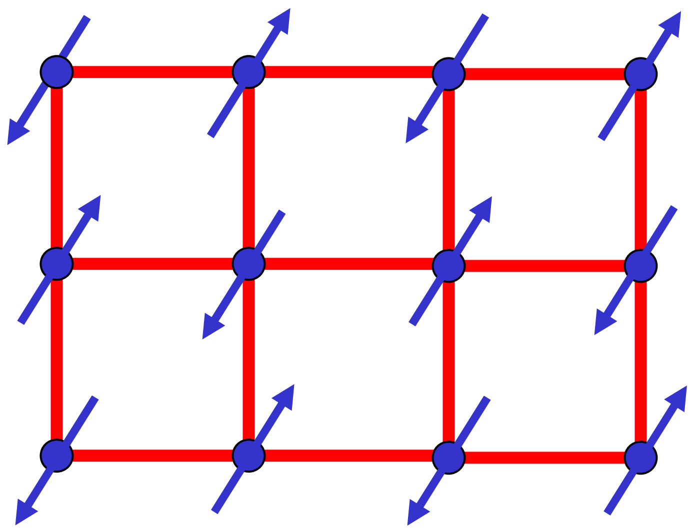


Half-filled band of Cu 3d orbitals – ground state is predicted to be a metal.

However, La_2CuO_4 is a very good insulator

Parent compound of the high temperature superconductors: La_2CuO_4

Mott insulator: square lattice antiferromagnet



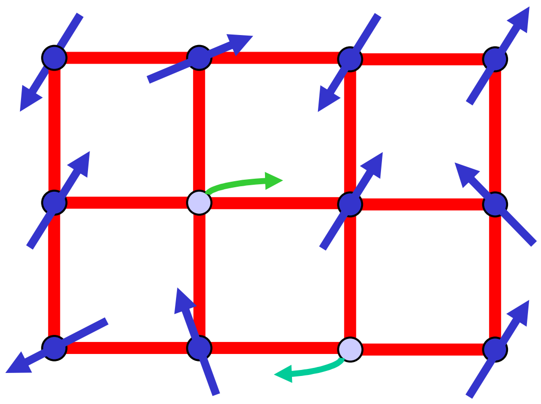
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic Néel order, or "collinear magnetic (CM) order"

Néel order parameter: $\vec{\phi} = (-1)^{i_x + i_y} \vec{S}_i$

$$\langle \vec{\phi} \rangle \neq 0 \quad ; \quad \langle \vec{S}_i \rangle \neq 0$$

Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$

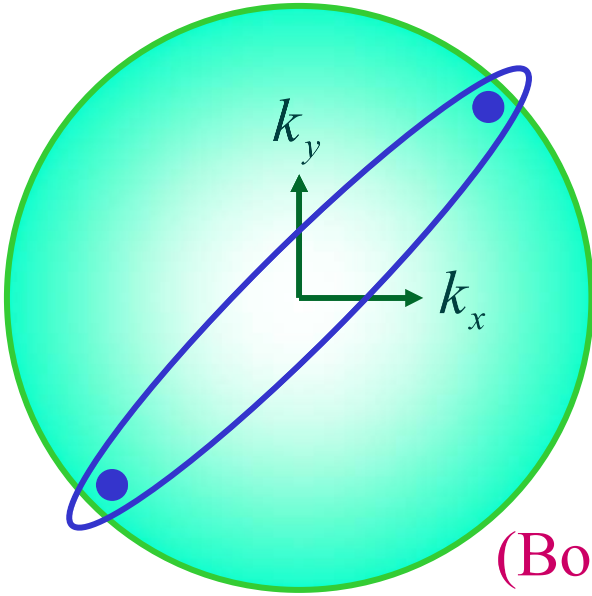


$$\left\langle \vec{S} \right\rangle = 0$$

Exhibits superconductivity below a high critical temperature T_c

Superconductivity in a doped Mott insulator

BCS superconductor obtained by the Cooper instability of a *metallic Fermi liquid*



Pair wavefunction

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

(Bose-Einstein) condensation of Cooper pairs

Many ***low*** temperature properties of the cuprate superconductors appear to be qualitatively similar to those predicted by BCS theory.

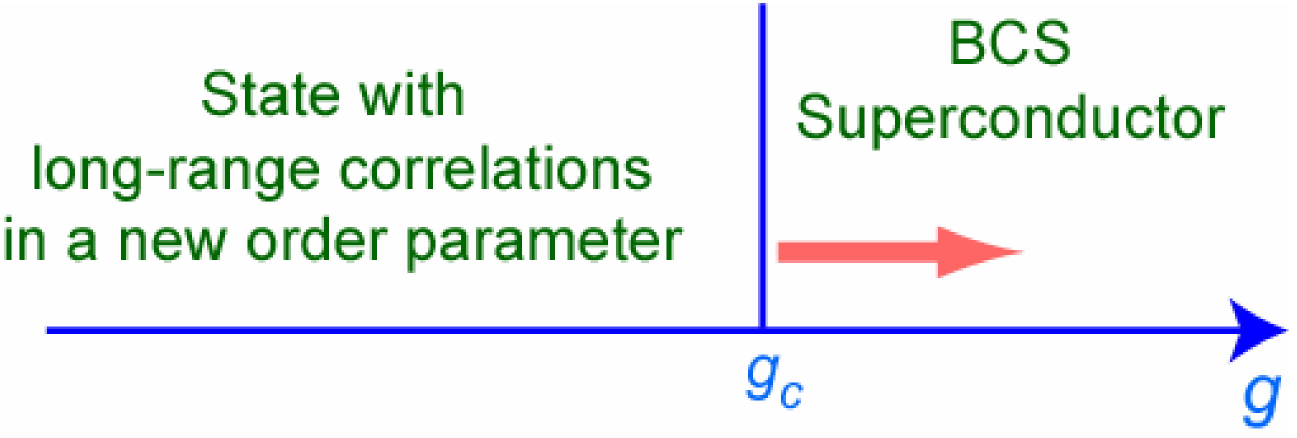
Superconductivity in a doped Mott insulator

Review: S. Sachdev, *Science* **286**, 2479 (1999).

Hypothesis: cuprate superconductors are characterized by additional order parameters (possibly fluctuating), associated with the proximate Mott insulator, along with the familiar order associated with the condensation of Cooper pairs in BCS theory. These orders lead to new low energy excitations.

Superconductivity in a doped Mott insulator

Review: S. Sachdev, *Science* **286**, 2479 (1999).



Study physics in a generalized phase diagram which includes new phases (which need not be experimentally accessible) with long-range correlations in the additional order parameters. Expansion away from quantum critical points provides a systematic and controlled theory of the low energy excitations (including their behavior near imperfections such as impurities and vortices and their response to applied fields) and of crossovers into "incoherent" regimes at finite temperature.

Outline

I. **Simple model of a quantum phase transition**
   Coupled ladder antiferromagnet

II. Interplay of CM and SC order in the cuprates: theory and neutron scattering experiments

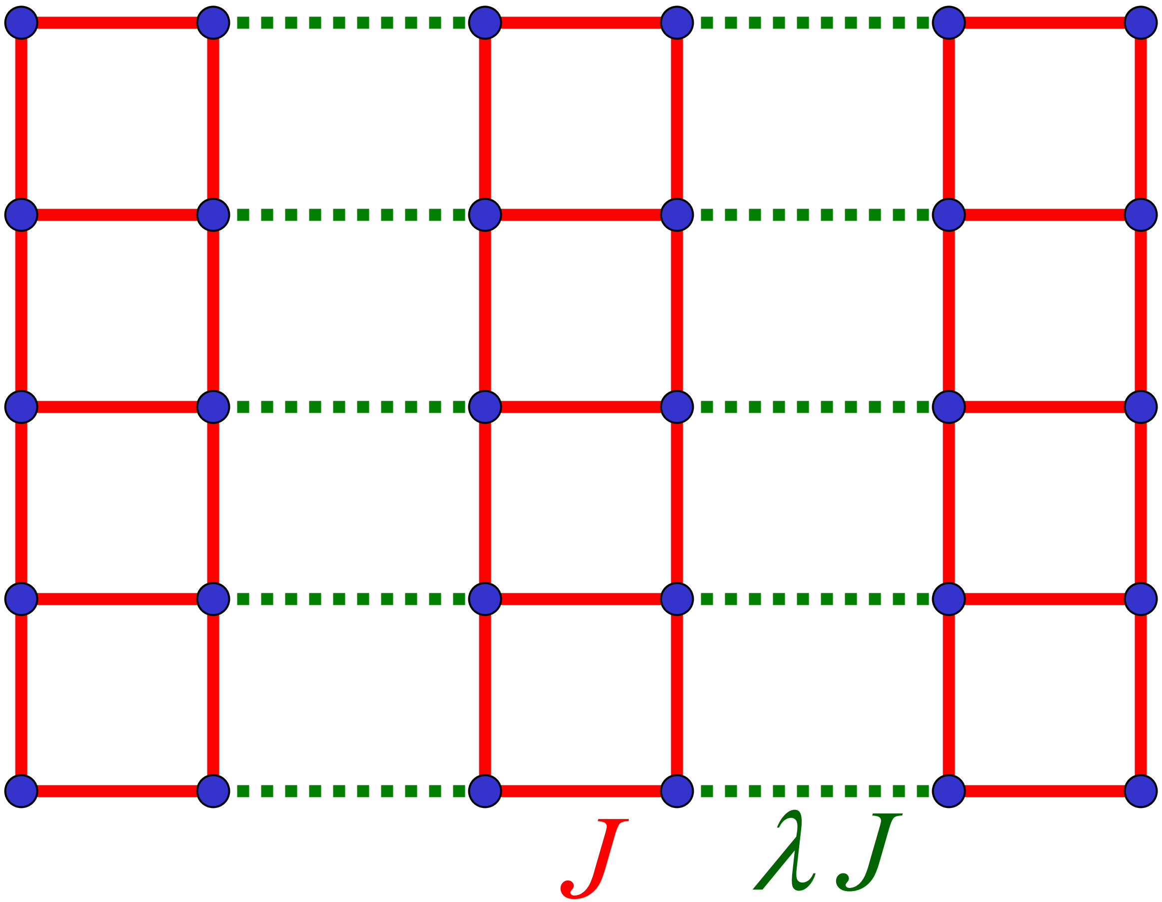
III. Microscopic theory: bond order and a global phase diagram (STM experiments)

IV. Conclusions

I. Coupled ladder antiferromagnet

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev*. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled 2-leg ladders



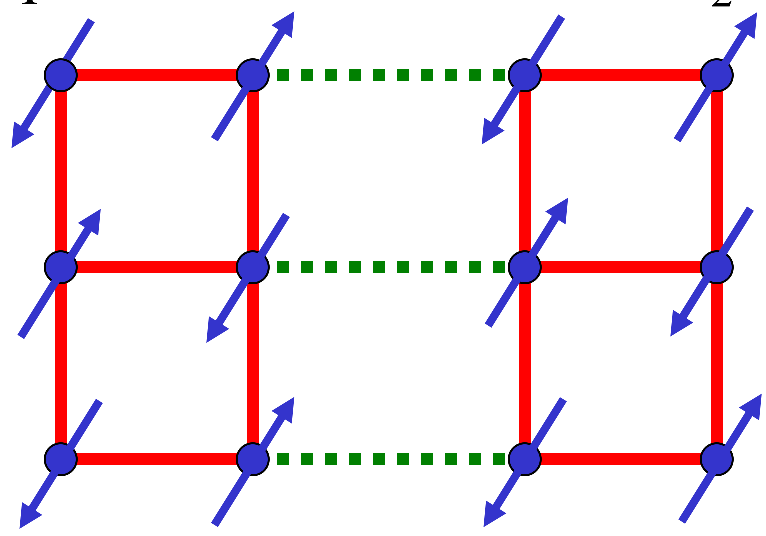
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



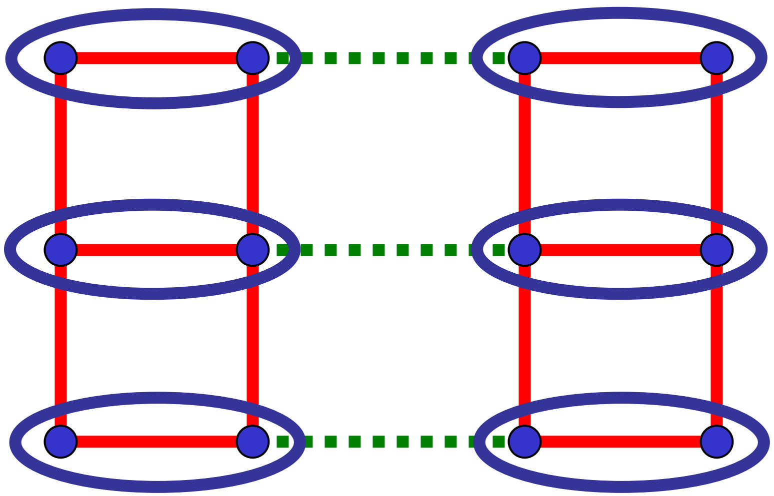
Ground state has long-range collinear magnetic (Neel) order

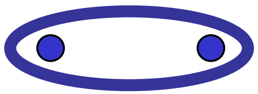
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

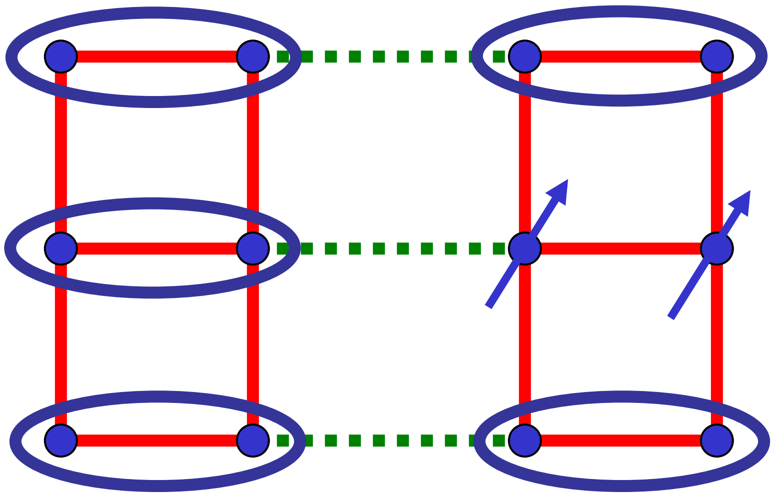
Real space Cooper pairs with their charge localized. Upon doping, motion and condensation of Cooper pairs leads to superconductivity

Paramagnetic ground state

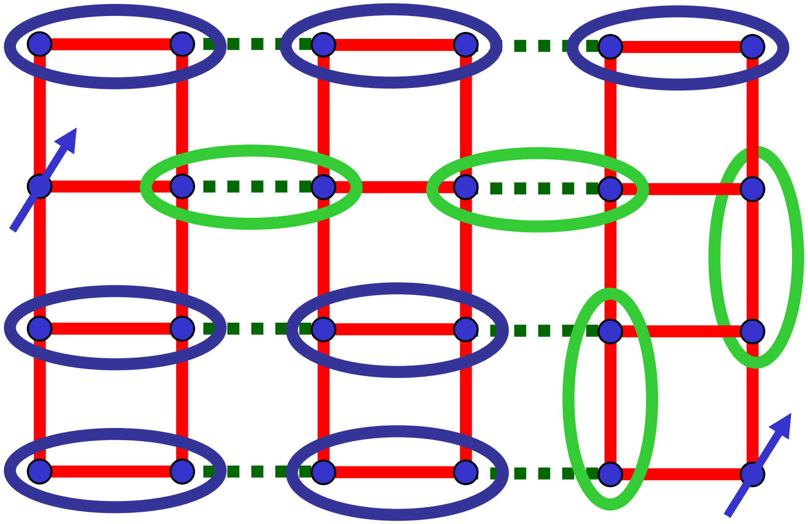
$$\left\langle \vec{S}_i \right\rangle = 0$$

λ close to 0

Excitations

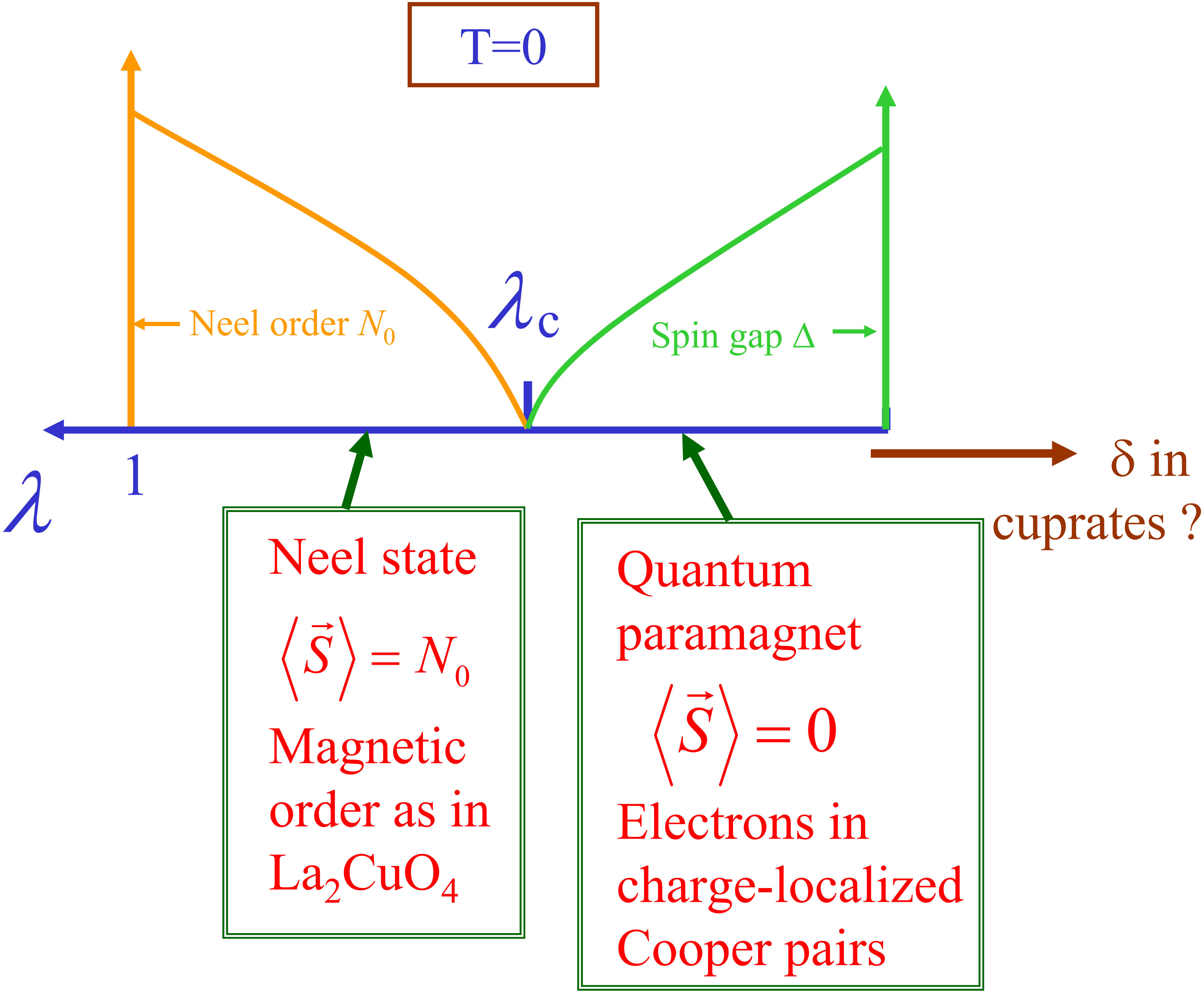


Excitation: S=1 ***exciton***
(spin collective mode)



S=1/2 spinons are ***confined*** by a linear potential.

Energy dispersion away from antiferromagnetic wavevector $\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$



T=0
Neel order N_0
λ_c
Spin gap Δ
λ
1
δ in cuprates ?
Neel state
$\langle \vec{S} \rangle = N_0$
Magnetic order as in La_2CuO_4
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Electrons in charge-localized Cooper pairs

Outline

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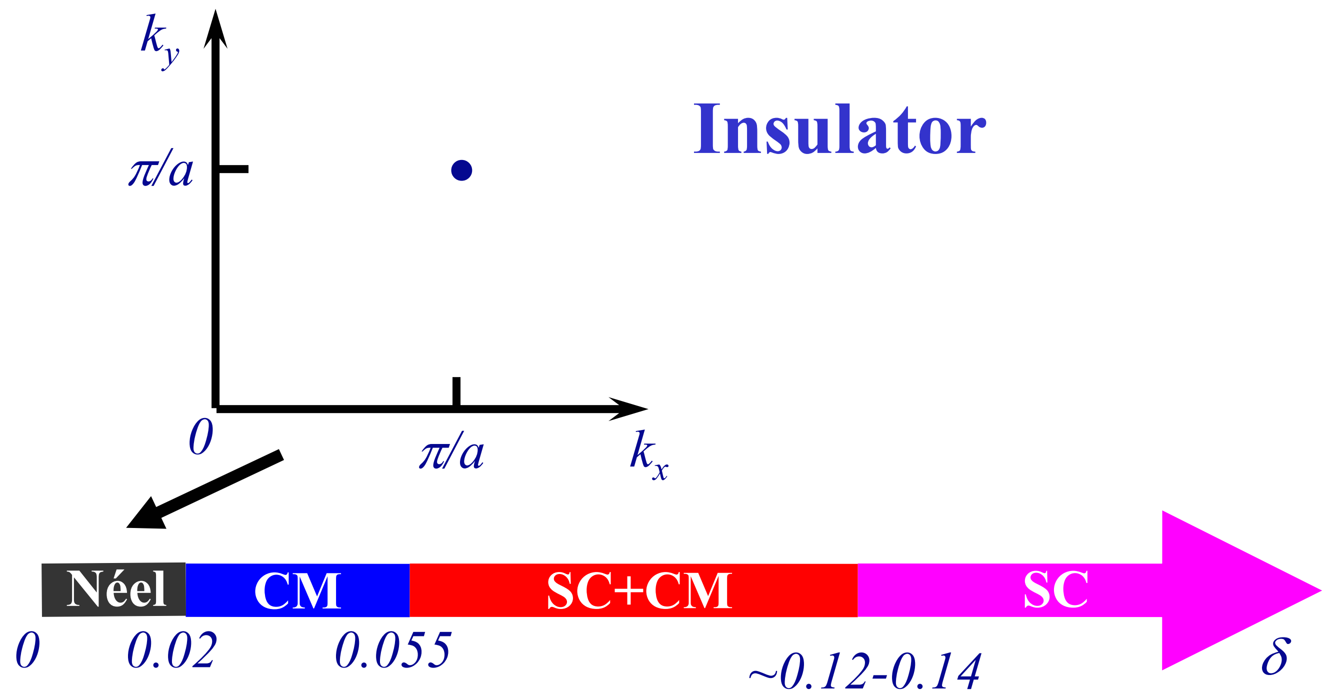
II. Interplay of CM and SC order in the cuprates

T=0 phases of LSCO

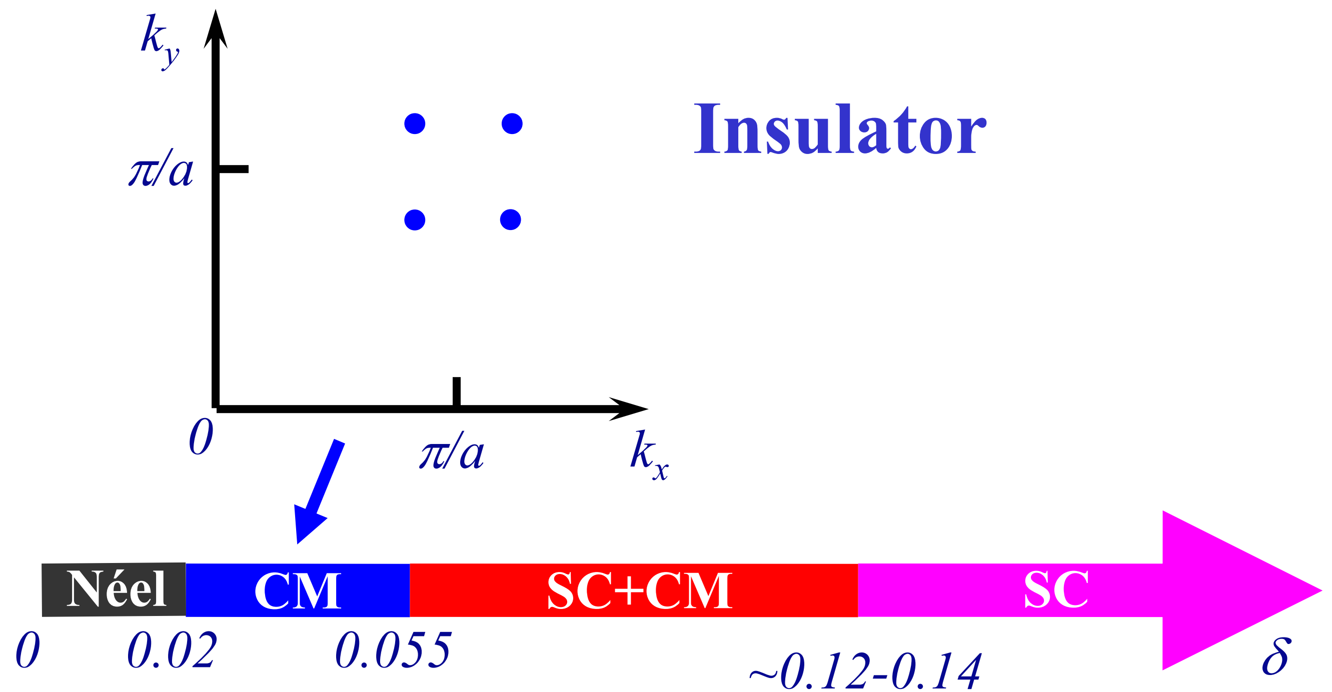


(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al., Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

II. Interplay of CM and SC order in the cuprates

T=0 phases of LSCO



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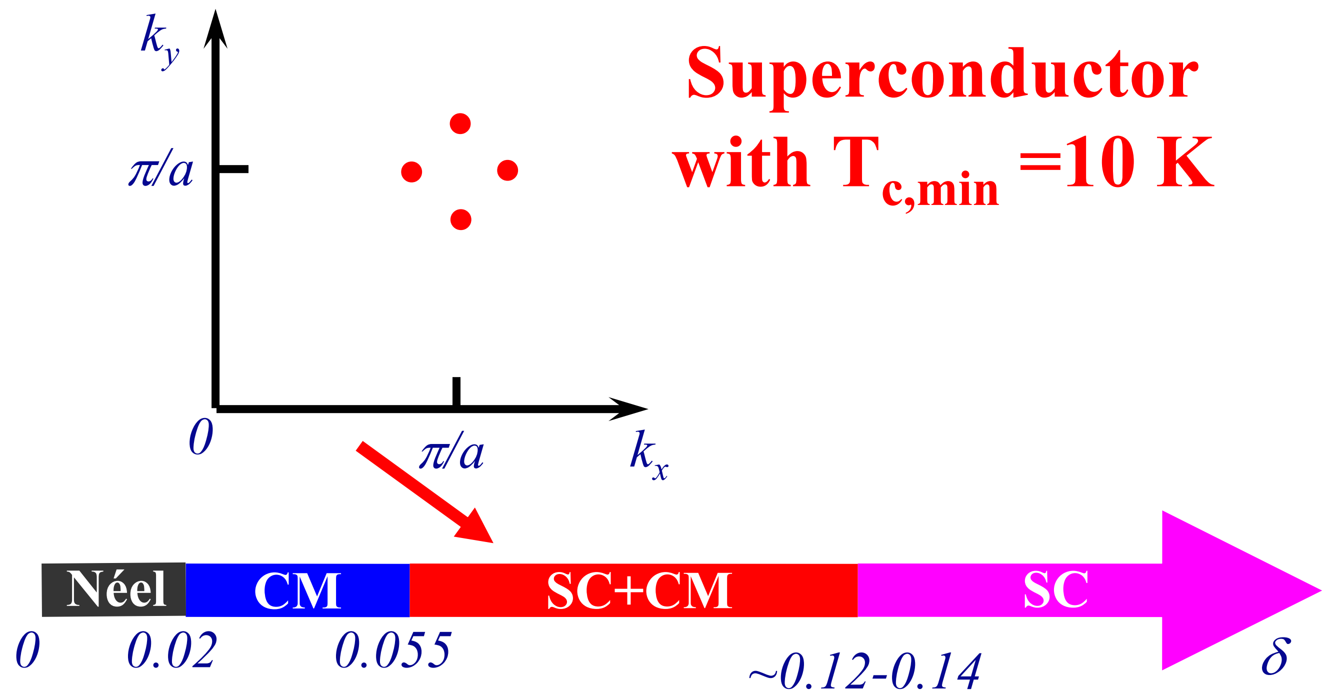

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G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
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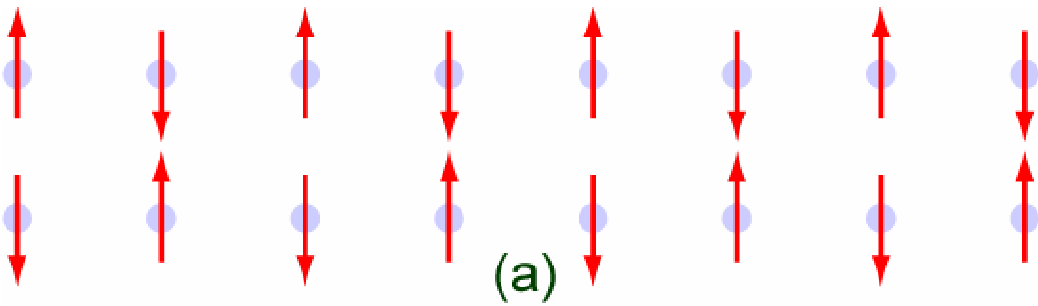
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

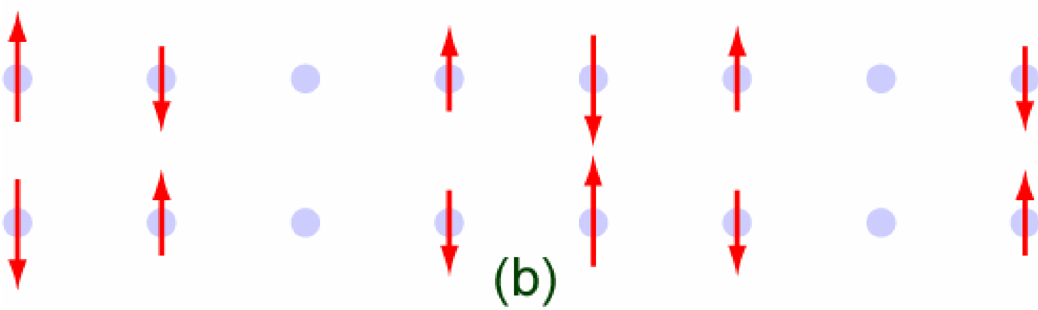
Collinear magnetic (spin density wave) order

$$\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K}.\vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K}.\vec{r}_j\right)$$

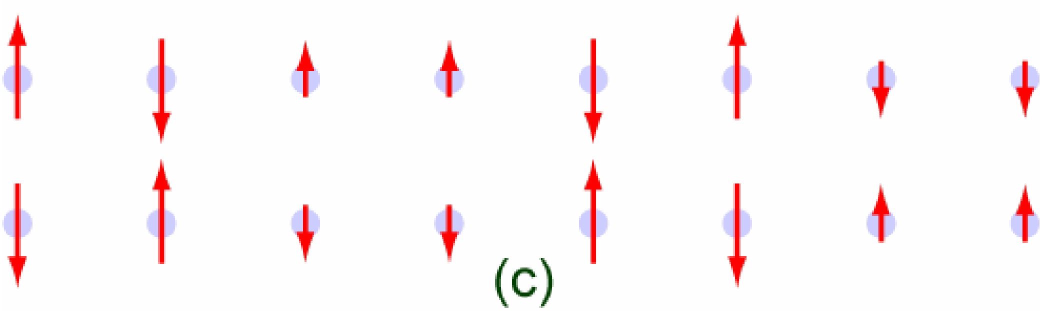
Collinear spins



$$\vec{K} = (\pi, \pi) \; ; \; \boldsymbol{N}_2 = 0$$



$$\vec{K} = (3\pi/4, \pi) \; ; \; \boldsymbol{N}_2 = 0$$

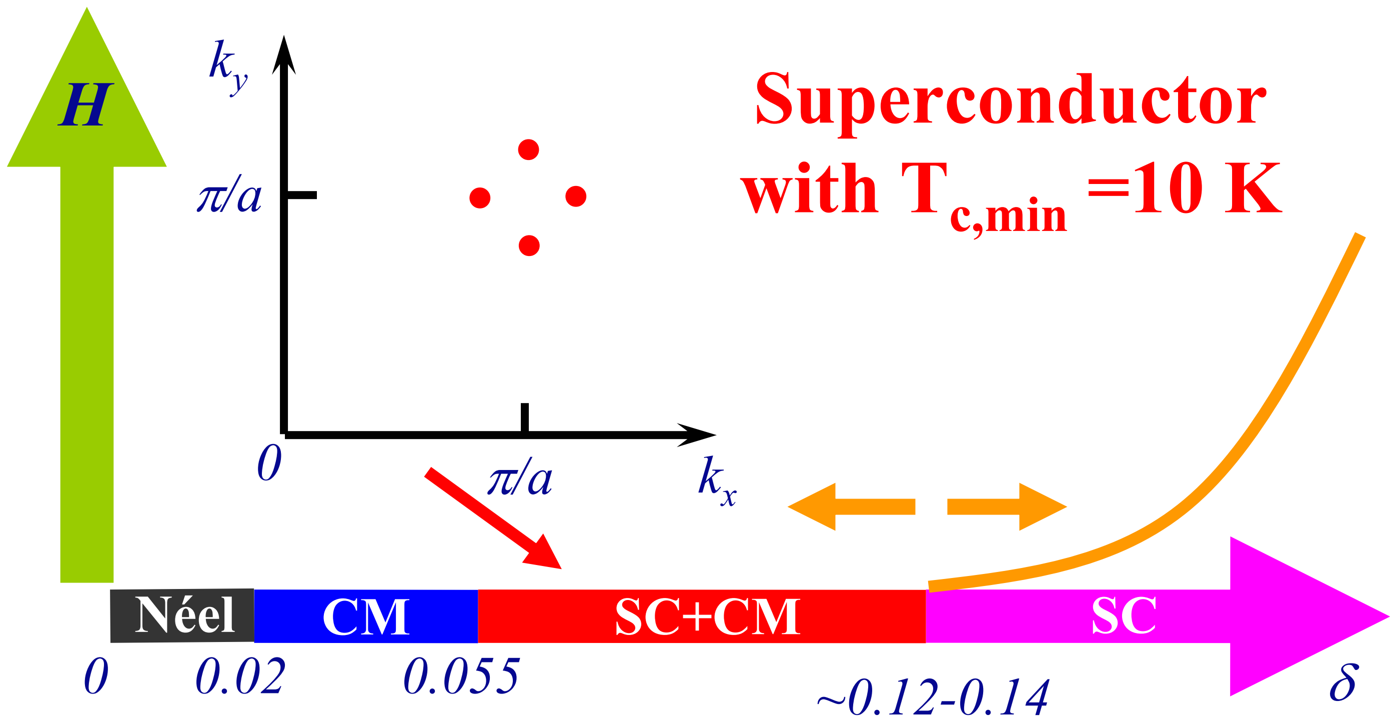


$$\vec{K} = (3\pi/4, \pi) \; ;$$

$$\boldsymbol{N}_2 = \left(\sqrt{2} - 1\right) \boldsymbol{N}_1$$

II. Interplay of CM and SC order in the cuprates

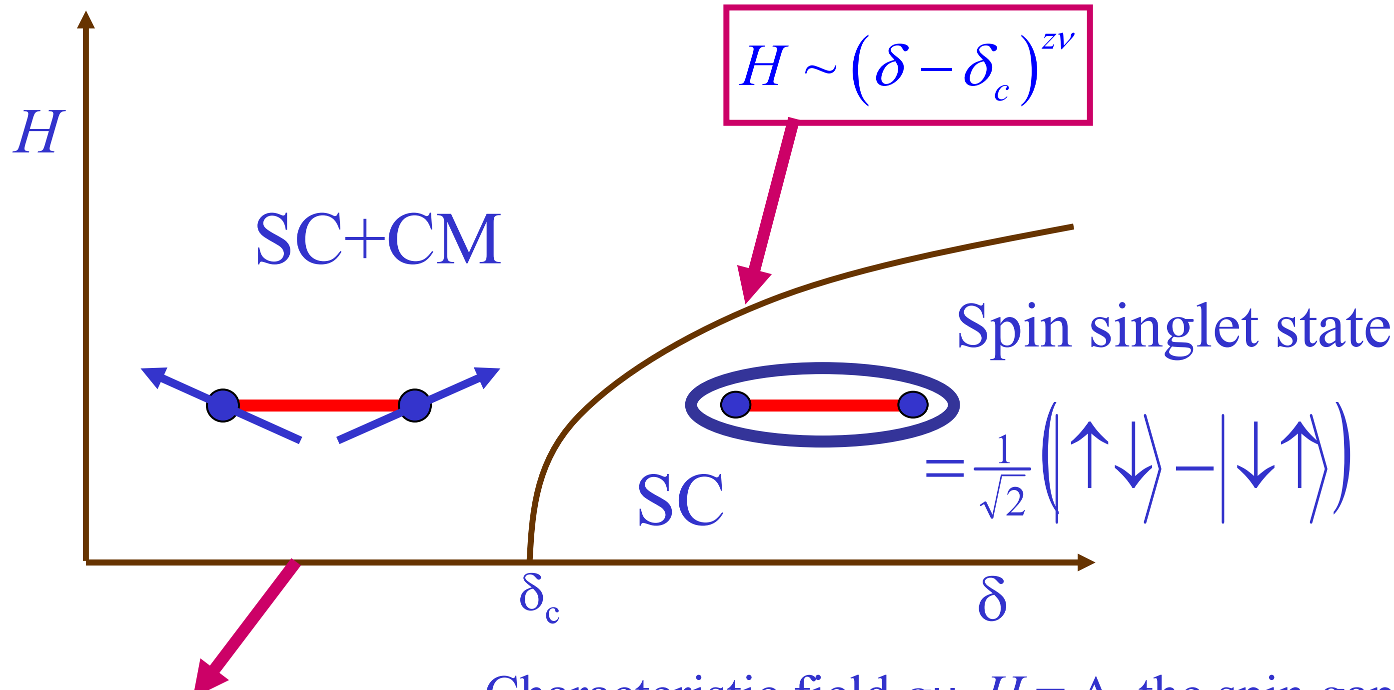
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+CM phases

Follow intensity of elastic Bragg spots in a magnetic field

Zeeman term: only effect in coupled ladder system



Elastic scattering intensity

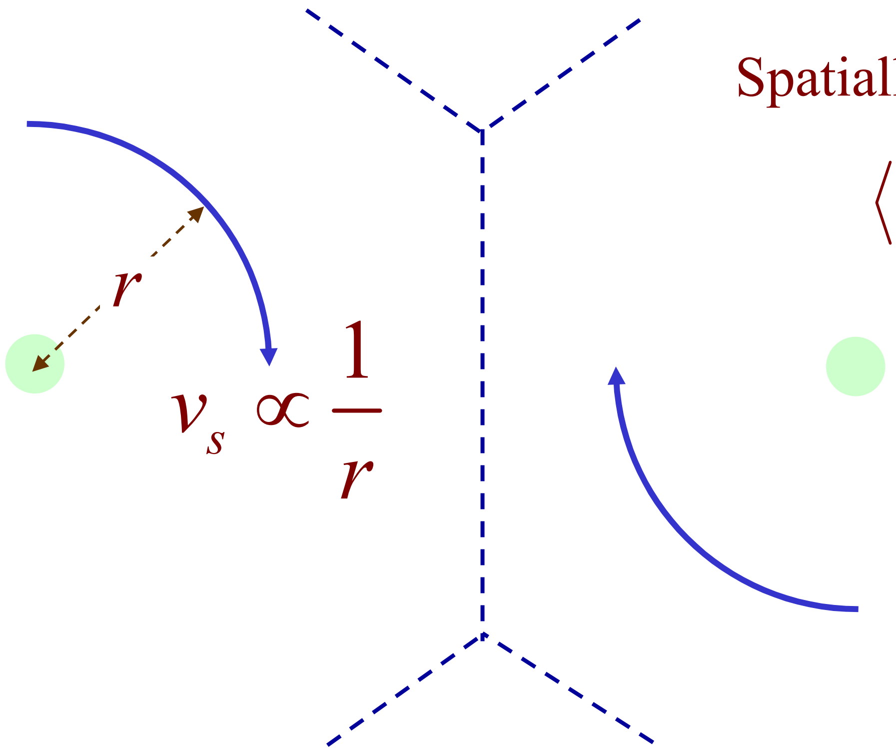
$$I(H) = I(0) + a\left(\frac{H}{J}\right)^2$$

Characteristic field $g\mu_B H = \Delta$, the spin gap

1 Tesla = 0.116 meV

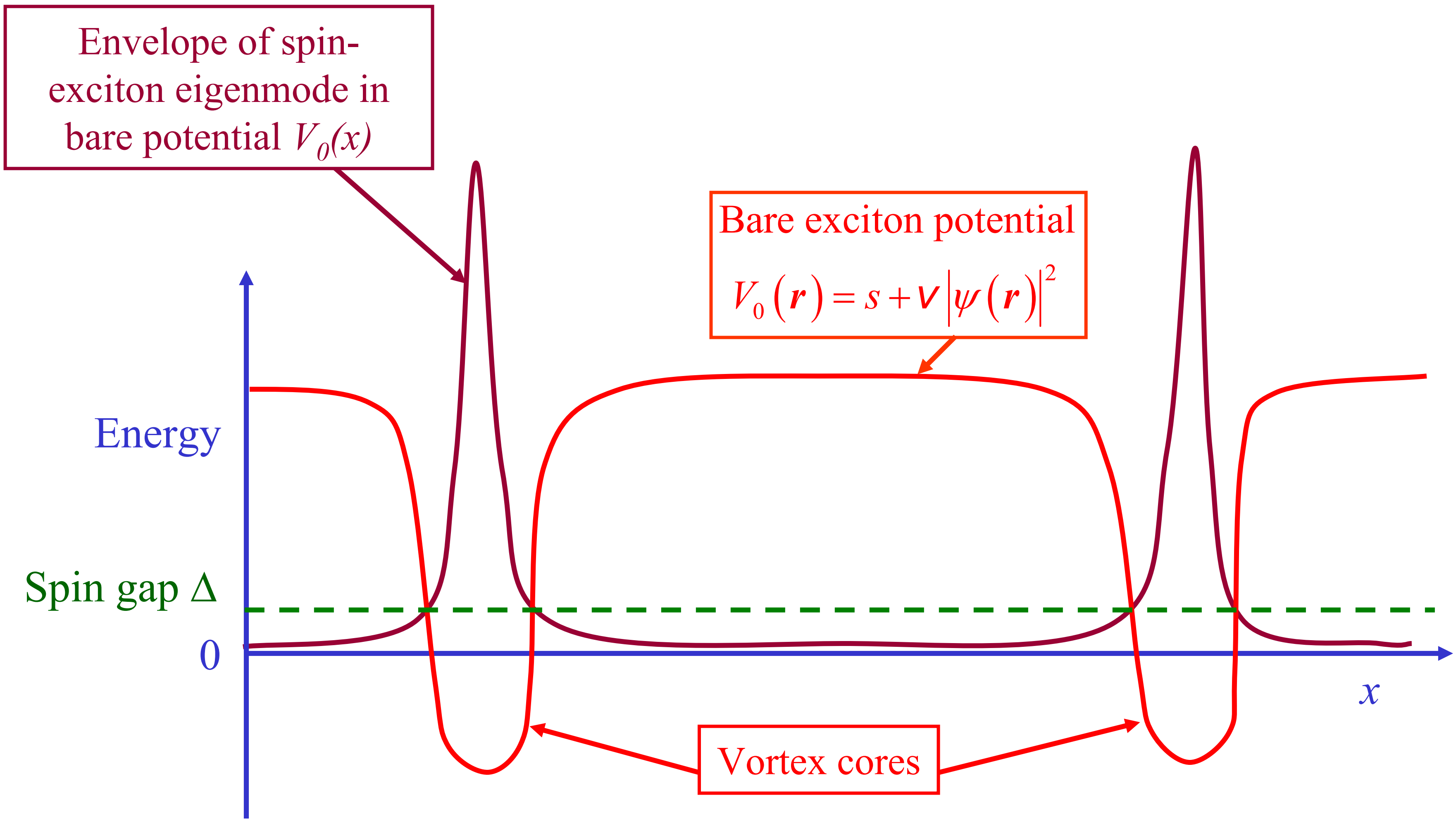
Effect is negligible over experimental field scales

A magnetic field applied to a superconductor induces a lattice of vortices in superflow

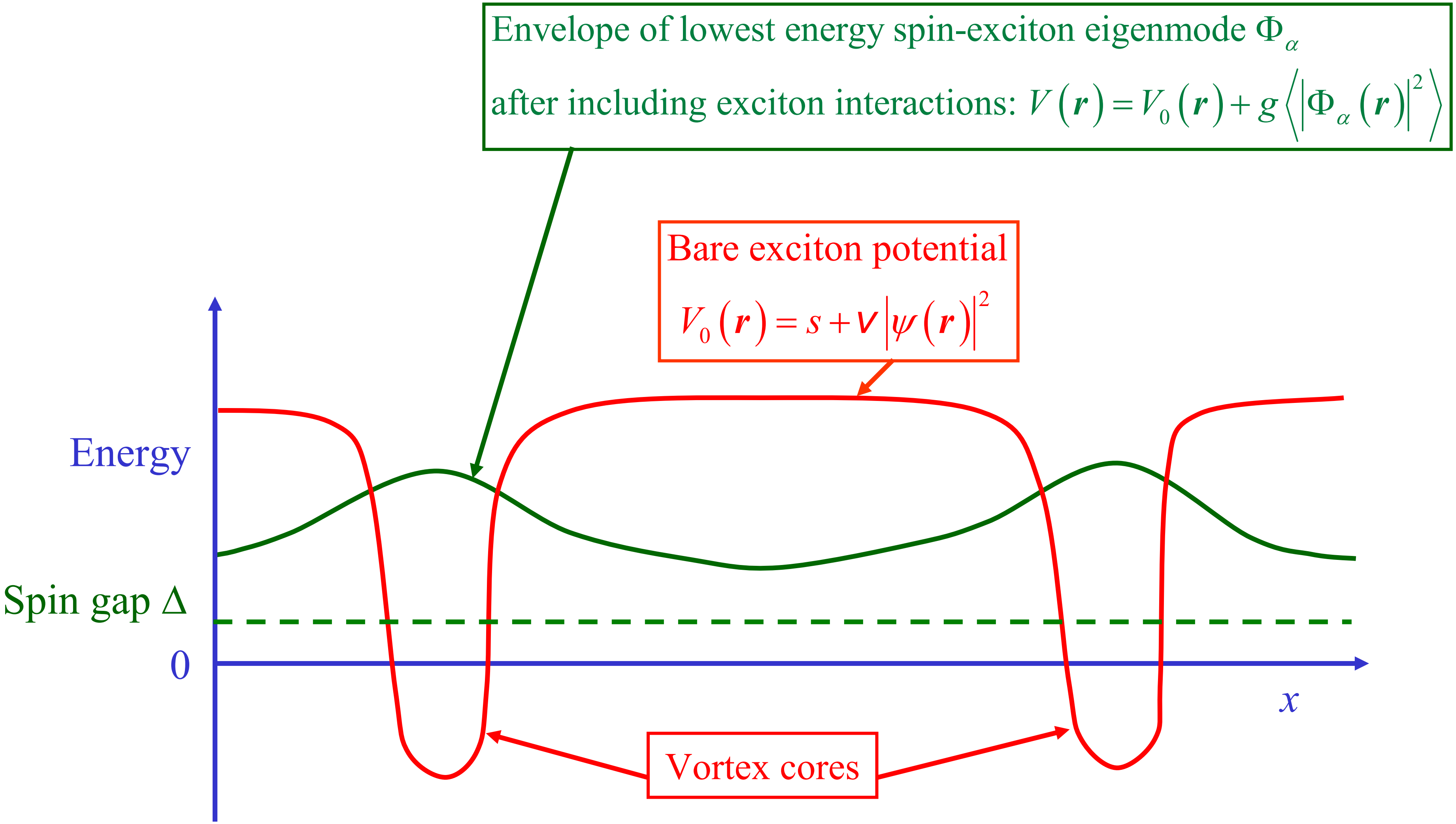


Spatially averaged superflow kinetic energy

$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$



D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) proposed **static** magnetism (with Δ=0) localized **within** vortex cores



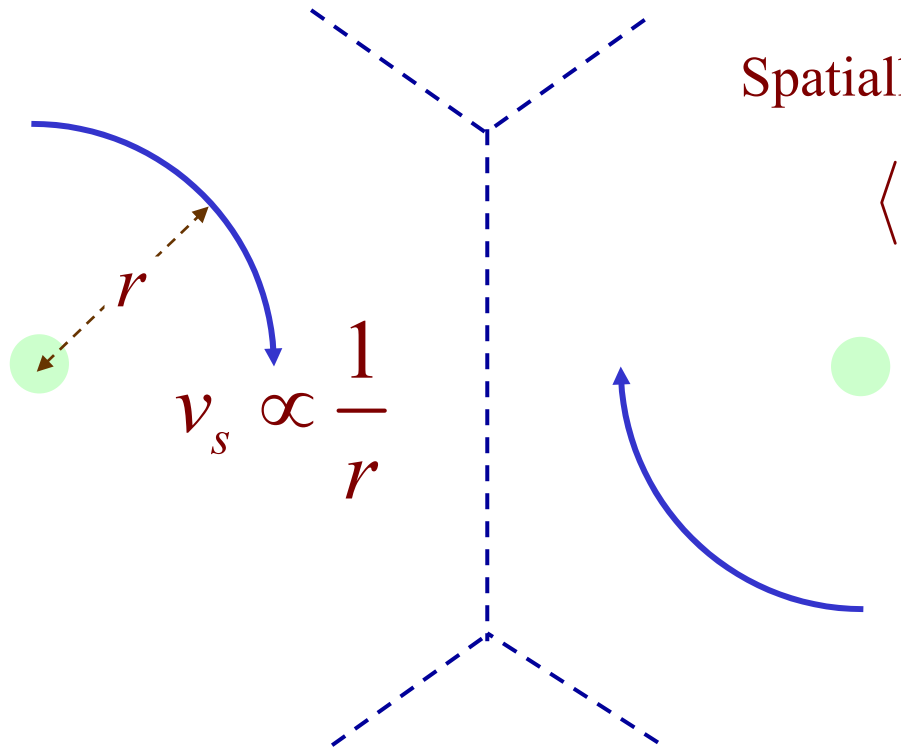
Strongly relevant repulsive interactions between excitons imply that excitons must be extended as $\Delta \rightarrow 0$.

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

A.J. Bray and M.A. Moore, *J. Phys.* C **15**, L7 65 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, *Phys. Rev. Lett.* **43**, 942 (1979).

Dominant effect with coexisting superconductivity: **uniform** softening of spin excitations by superflow kinetic energy



$$v_s \propto \frac{1}{r}$$

Spatially averaged superflow kinetic energy

$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Competing order is enhanced in a "halo" around each vortex

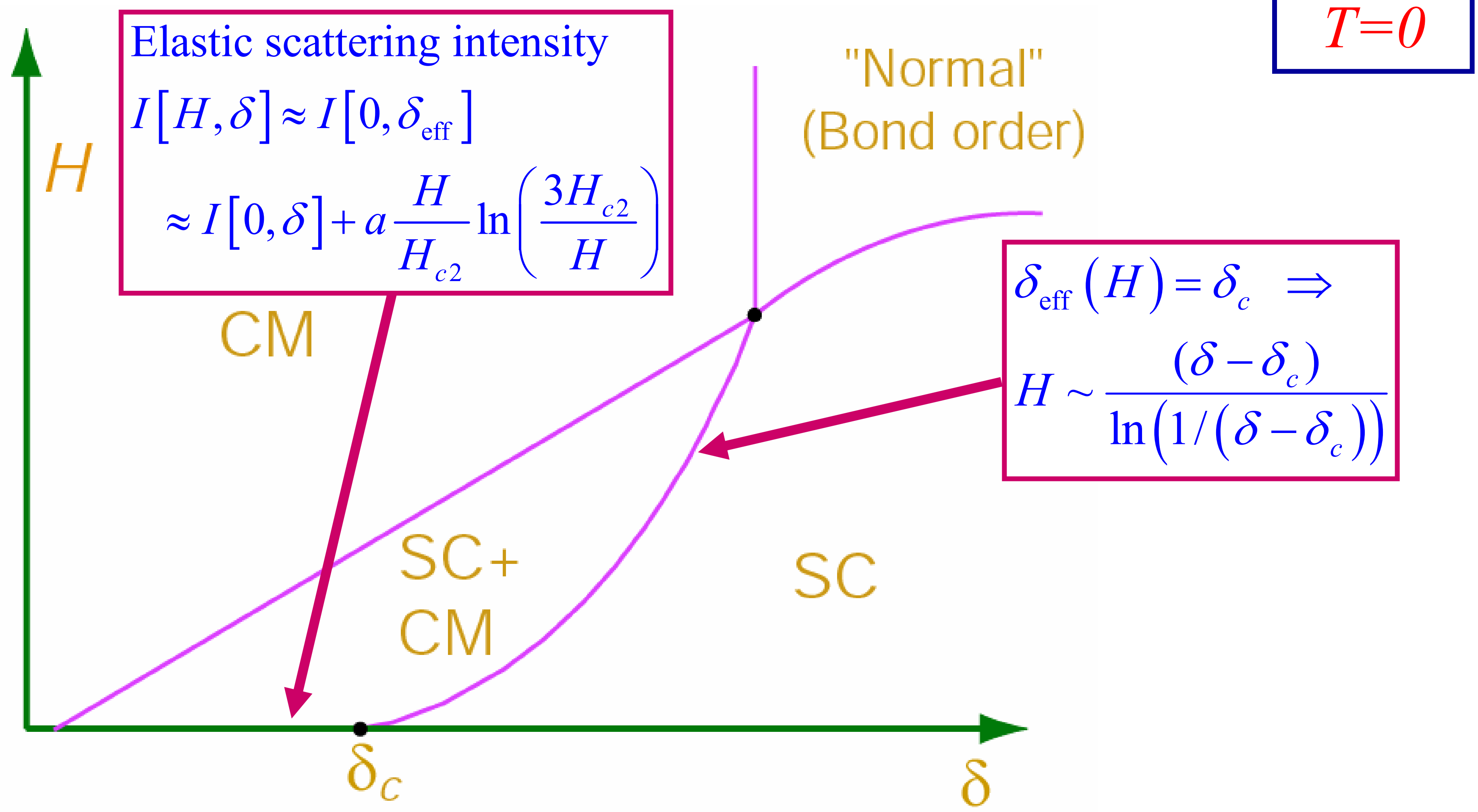
The suppression of SC order appears to the SDW order as an effective δ:

$$\delta_{\text{eff}}\left(H\right) = \delta - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Main results

T=0



Elastic scattering intensity

$$I[H,\delta] \approx I[0,\delta_{\text{eff}}]$$

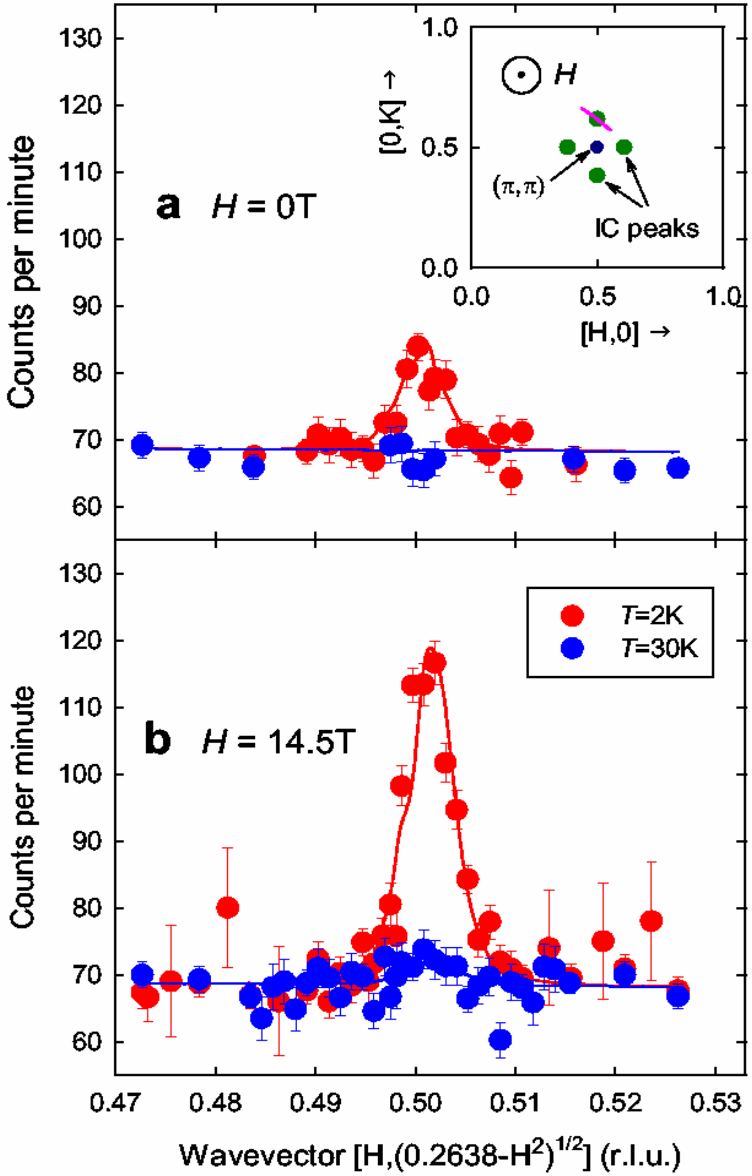
$$\approx I[0,\delta] + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$\delta_{\text{eff}}(H) = \delta_c \quad \Rightarrow$$

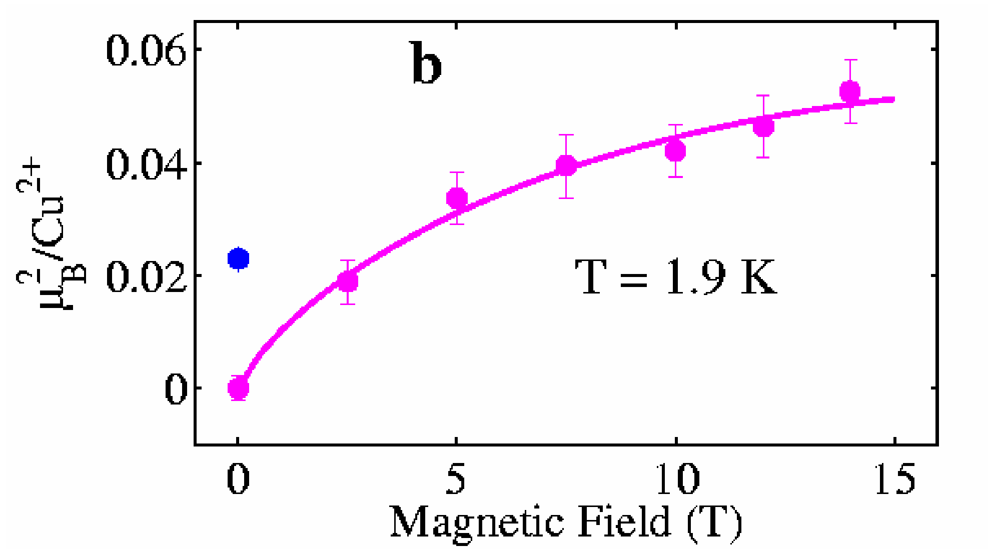
$$H \sim \frac{(\delta - \delta_c)}{\ln(1/(\delta - \delta_c))}$$

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



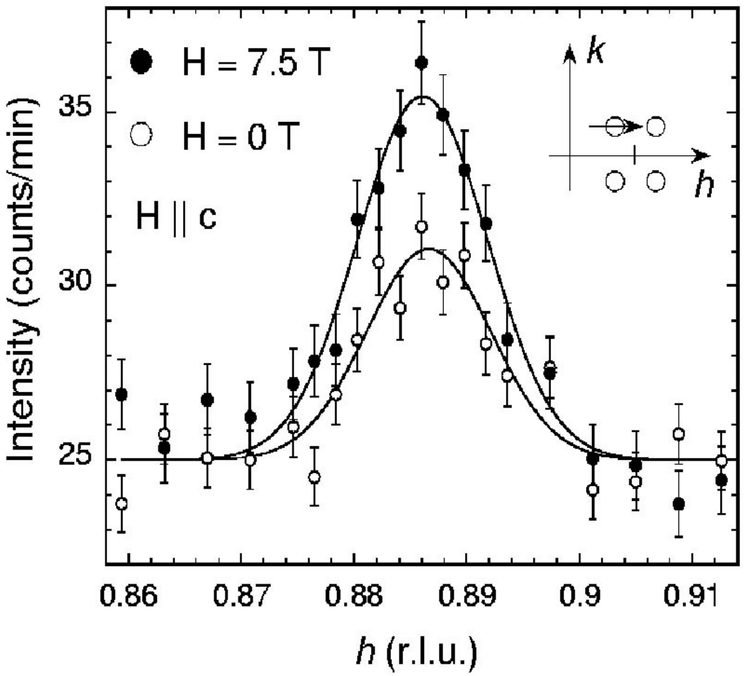
Solid line - fit to : $I(H) = a\dfrac{H}{H_{c2}}\ln\left(\dfrac{H_{c2}}{H}\right)$

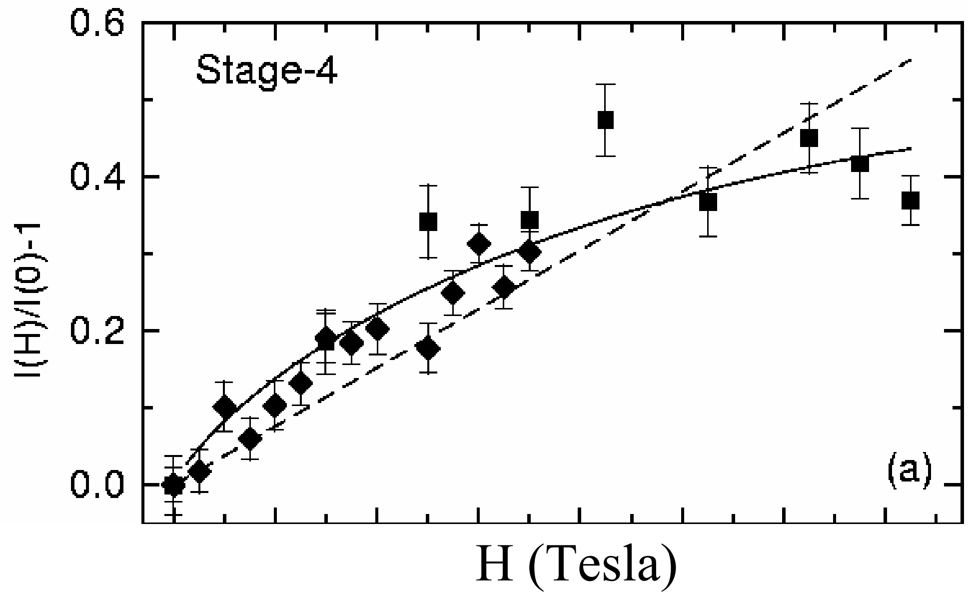
See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev*. B **62**, R14677 (2000).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+CM) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, *Phys. Rev.* B **66**, 014528 (2002).

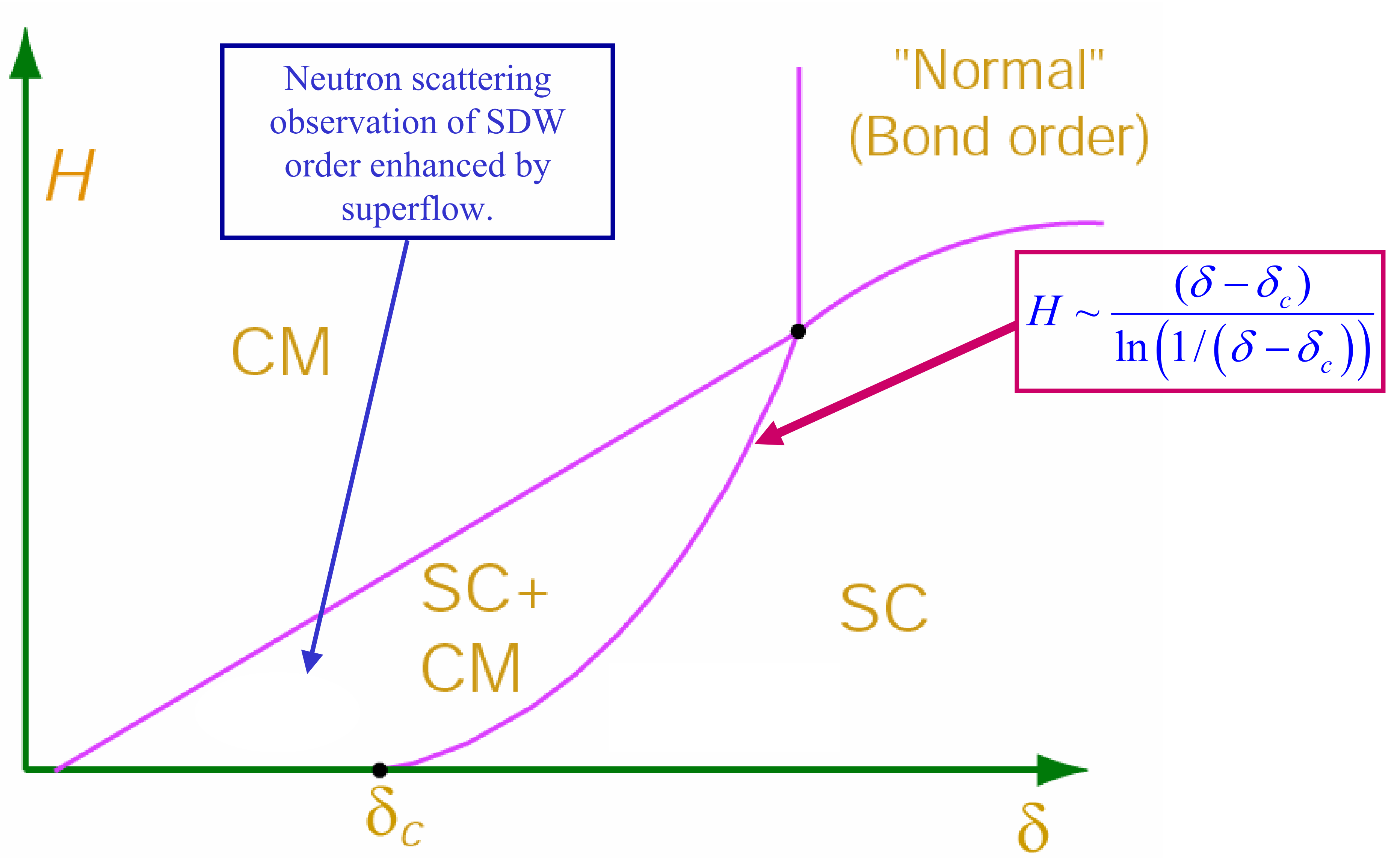




Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T



E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Outline

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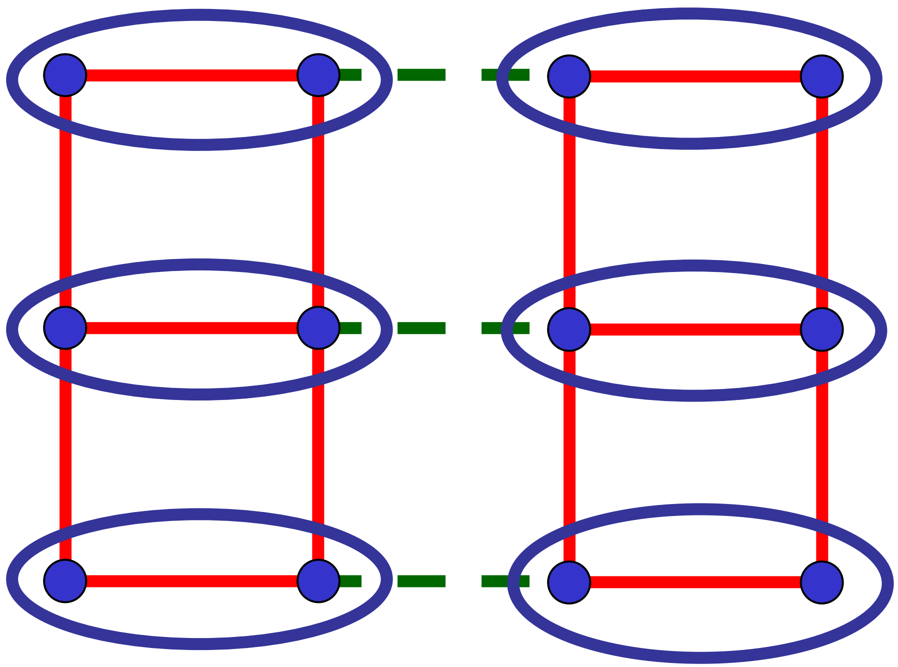
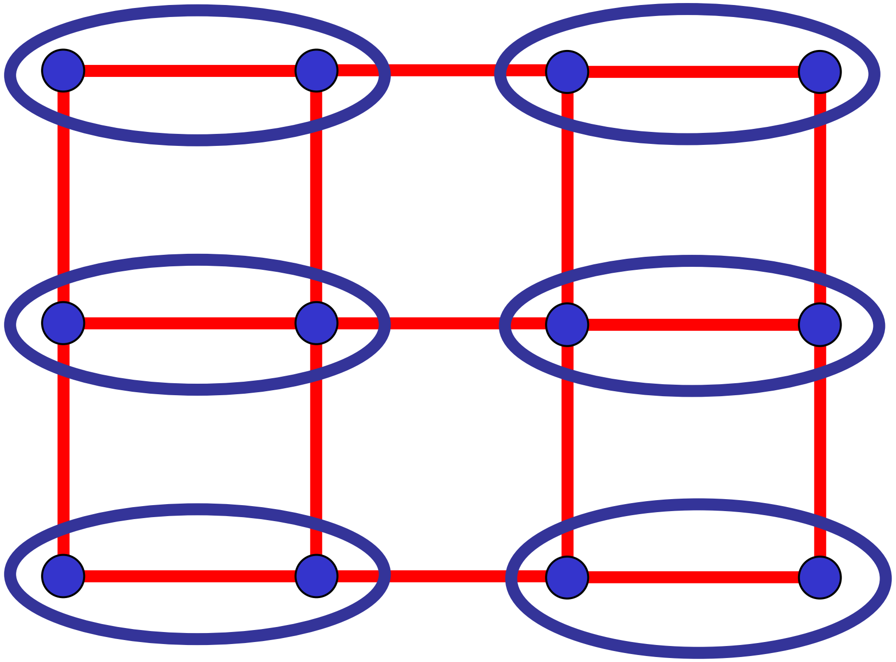
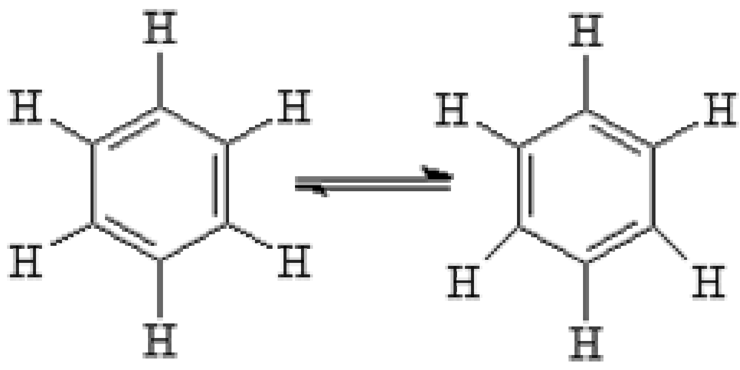
Paramagnetic ground state of coupled ladder model

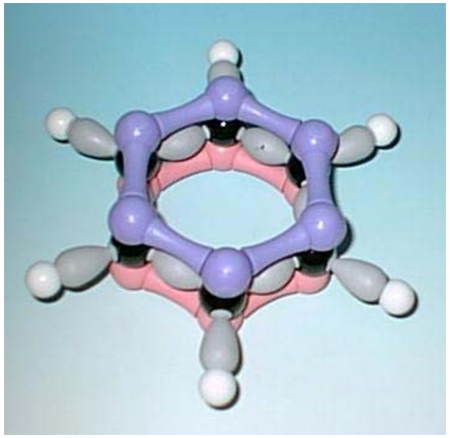


Can such a state with ***bond order*** be the ground state of a system with full square lattice symmetry ?

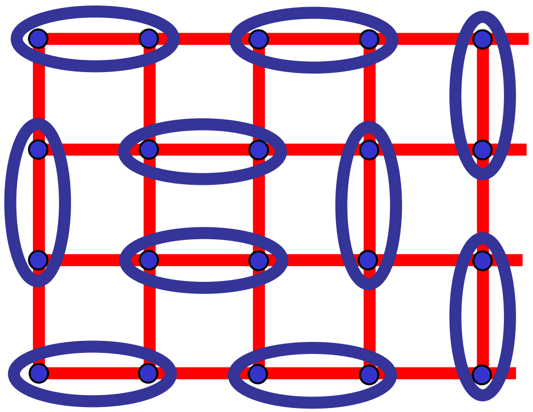


Resonating valence bonds



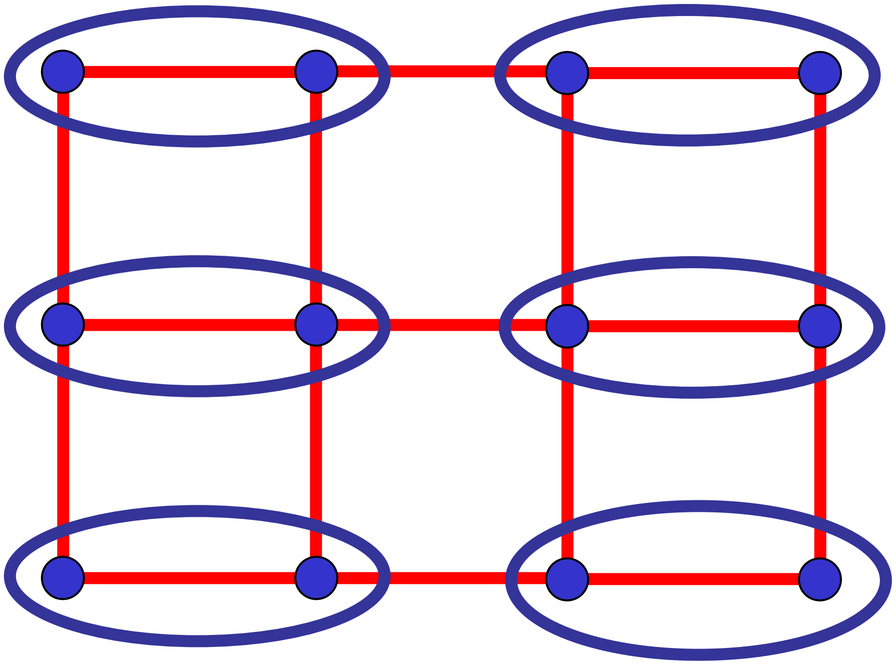


Resonance in benzene leads to a symmetric configuration of valence bonds
(*F. Kekulé, L. Pauling*)



The paramagnet on the square lattice should also allow other valence bond pairings, and this leads to a "resonating valence bond liquid"
(*P.W. Anderson, 1987*)

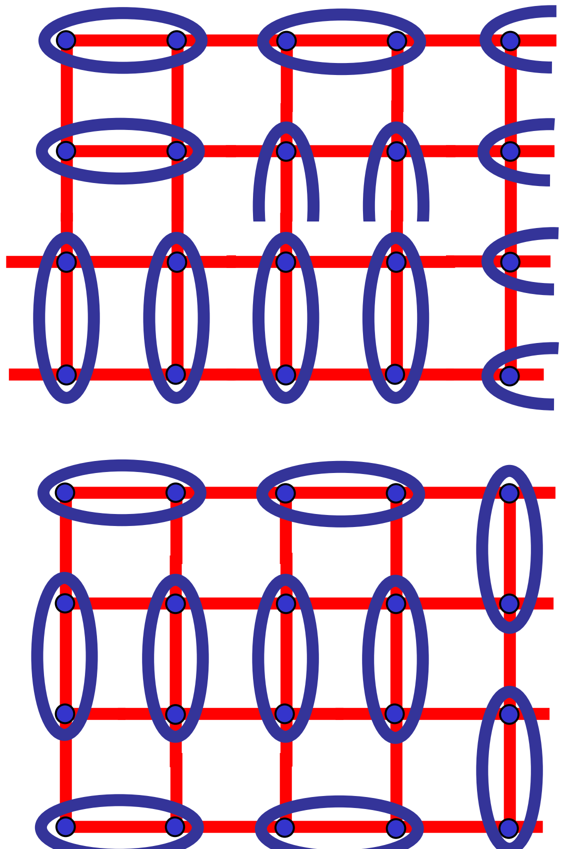
Can such a state with ***bond order*** be the ground state of a system with full square lattice symmetry ?



Surprising answer: **Yes** ! Here resonance acts to produce a state which breaks lattice symmetry by the appearance of ***bond order***

Such ***bond order*** is generic in paramagnetic states proximate to a magnetic state with collinear spins

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).



Origin of bond order

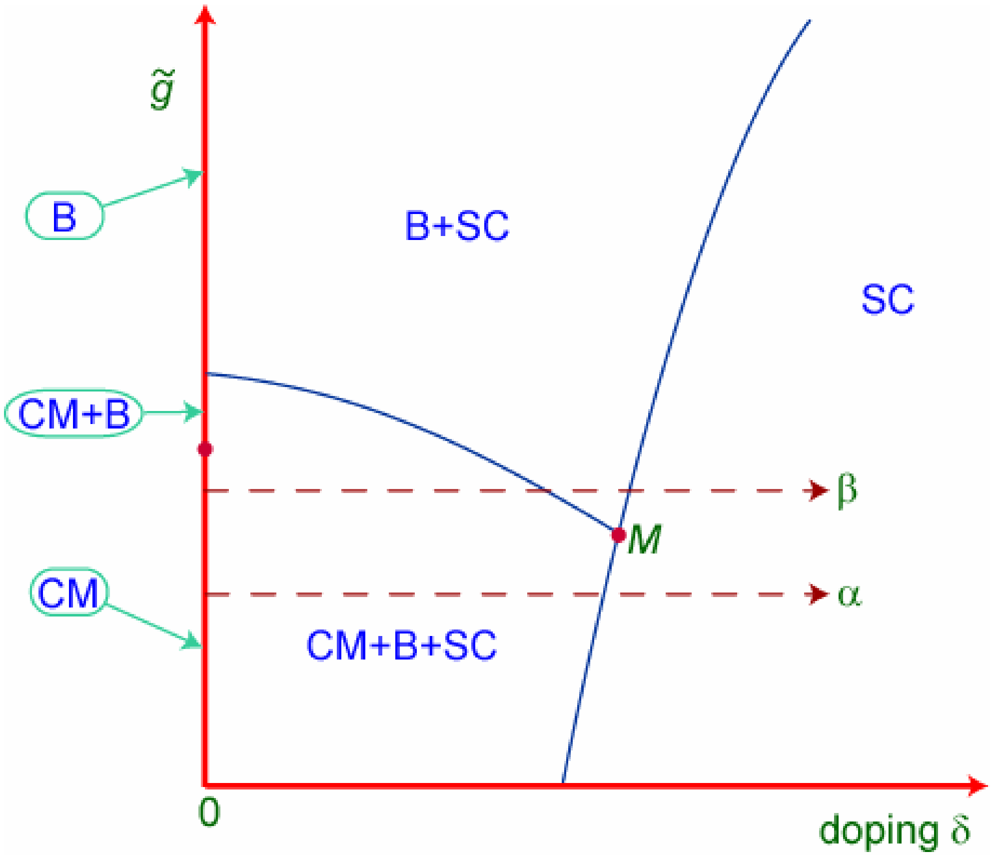
Quantum "entropic" effects prefer bond-ordered configurations in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.
These effects lead to a broken square lattice symmetry near the transition to the magnetically ordered states with collinear spins.

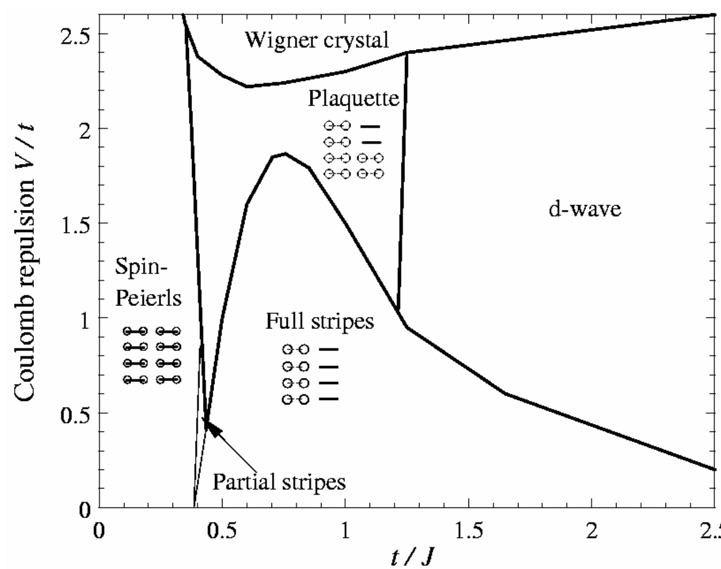
The quantum dimer model (D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett*. **61**, 2376 (1988); E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett*. B **4**, 225 (1990)) and semiclassical theories provide dual descriptions of this physics

N. Read and S. Sachdev, *Phys. Rev.* B **42**, 4568 (1990).

A global phase diagram

Vertical axis is any microscopic parameter which suppresses CM order

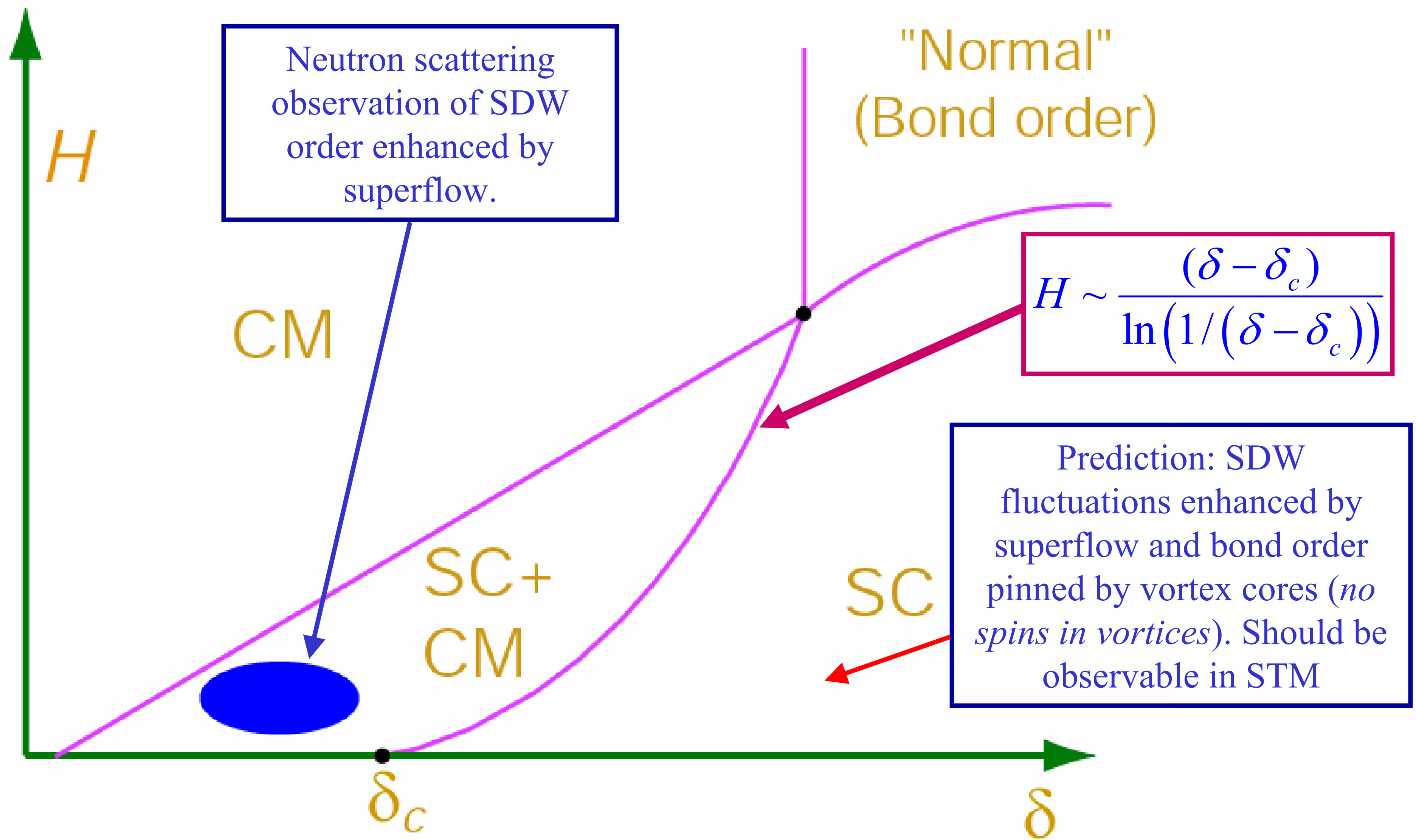




Microscopic theory for the interplay of bond (B) and *d*-wave superconducting (SC) order

- **Pairing order of BCS theory (SC)**
- **Collinear magnetic order (CM)**
- **Bond order (B)**

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999);
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000); M. Vojta, Phys. Rev. B **66**, 104505 (2002).

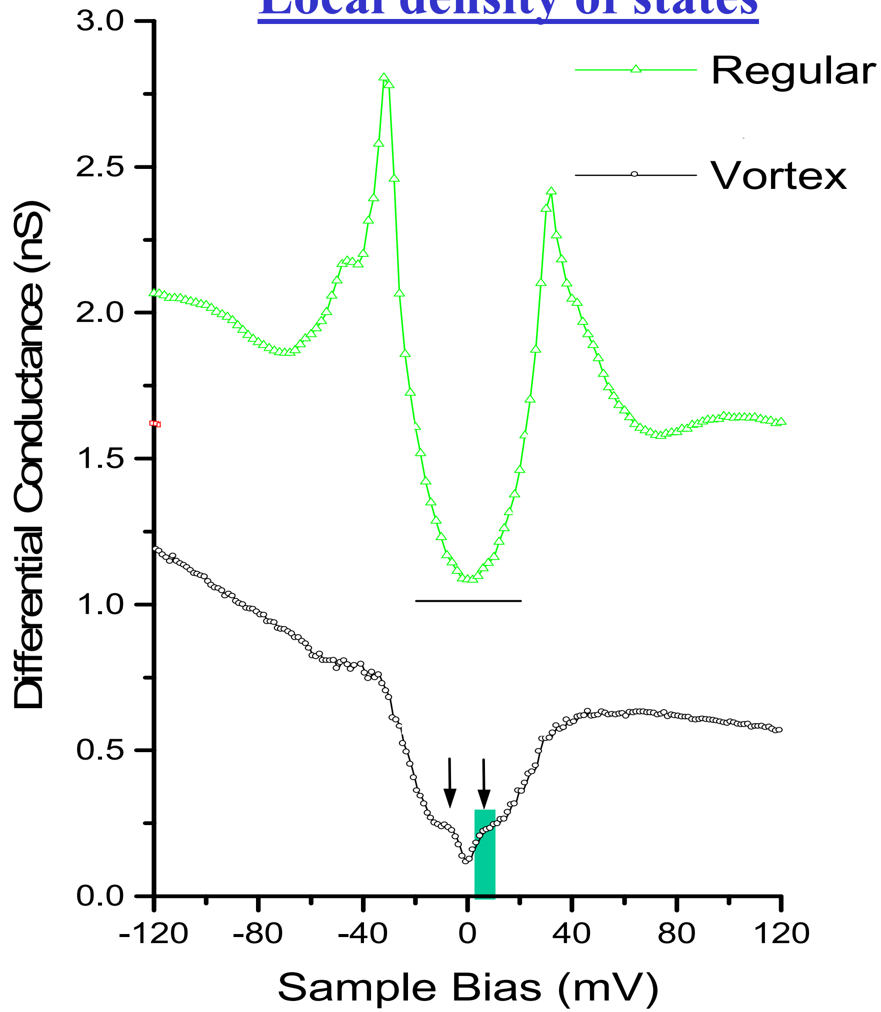


K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).
Y. Zhang, E. Demler and S. Sachdev, *Phys. Rev.* B **66,** 094501 (2002).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

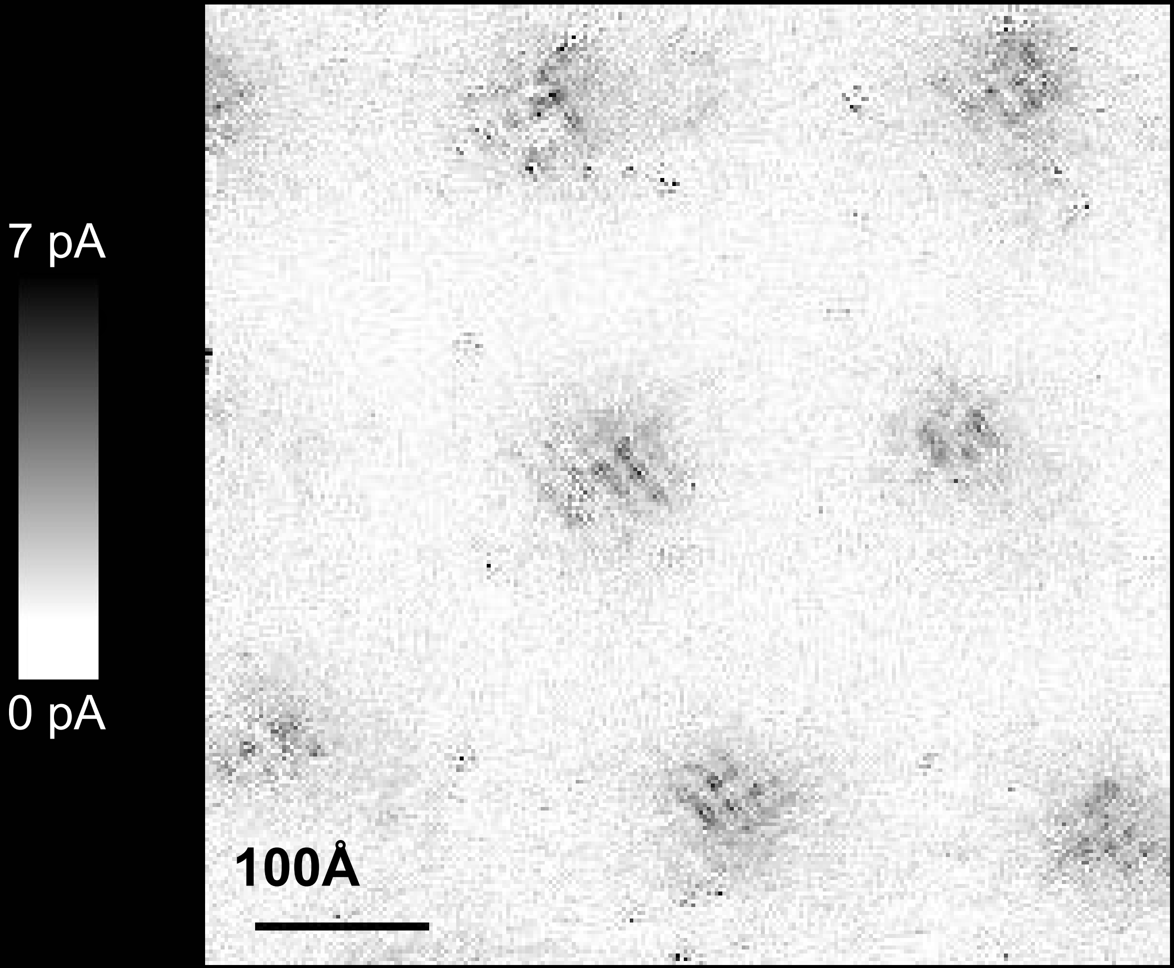


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

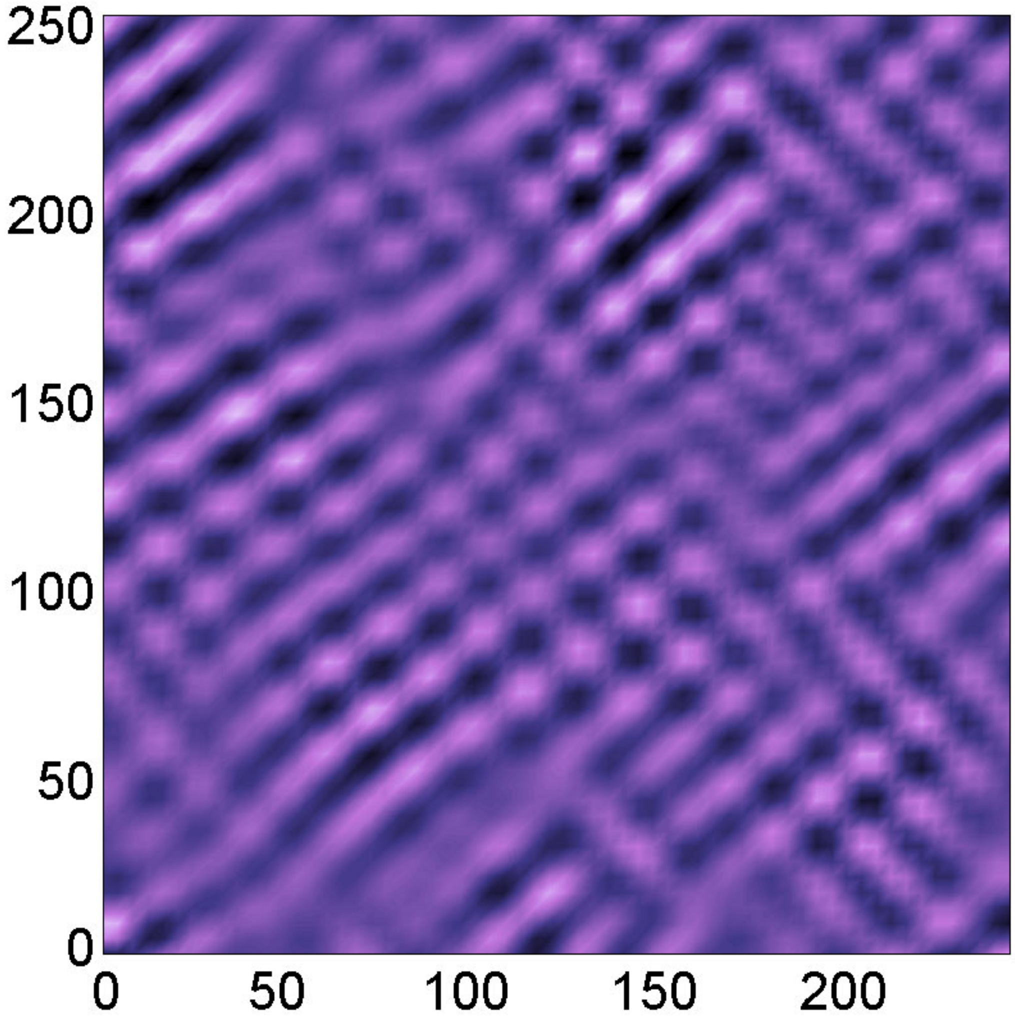
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

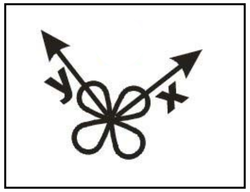


J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Our interpretation: LDOS modulations are signals of bond order of period 4 revealed in vortex halo

III. STM image of LDOS modulations in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

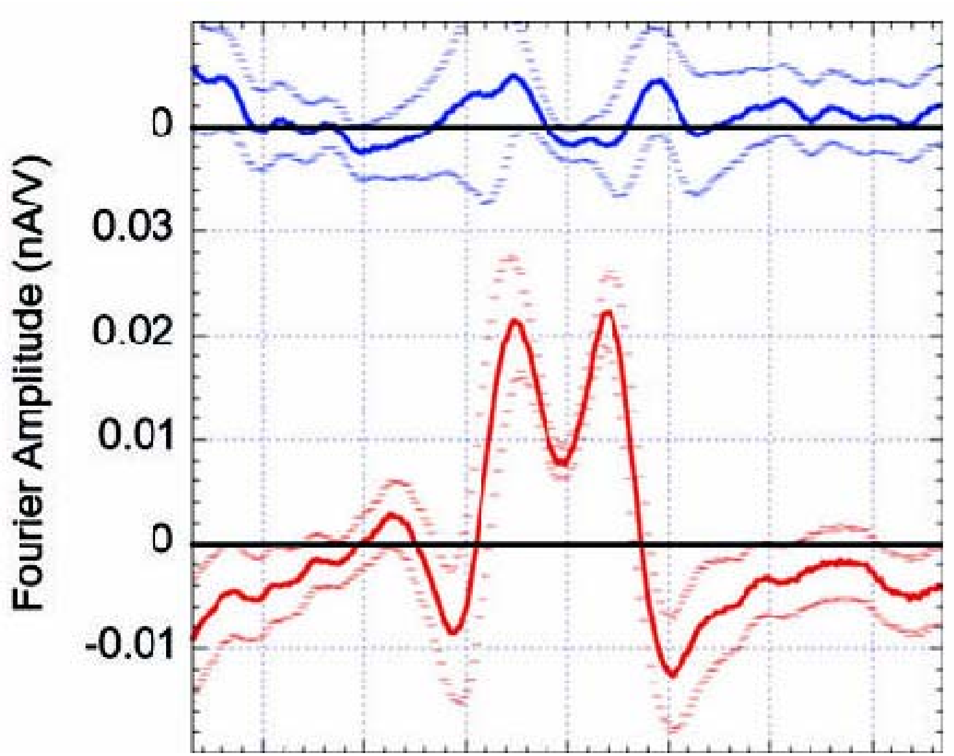




Period = 4 lattice spacings

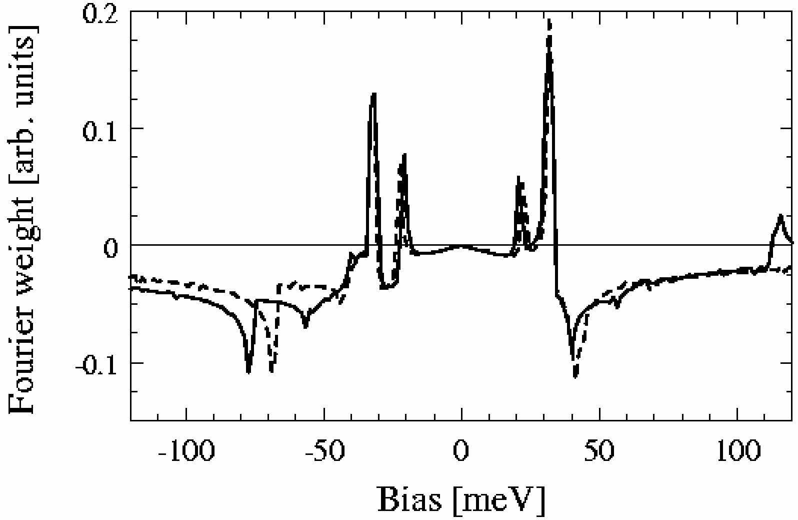
C. Howald, H. Eisaki, N. Kaneko, M. Greven,and A. Kapitulnik, *Phys. Rev.* B **67**, 014533 (2003).

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



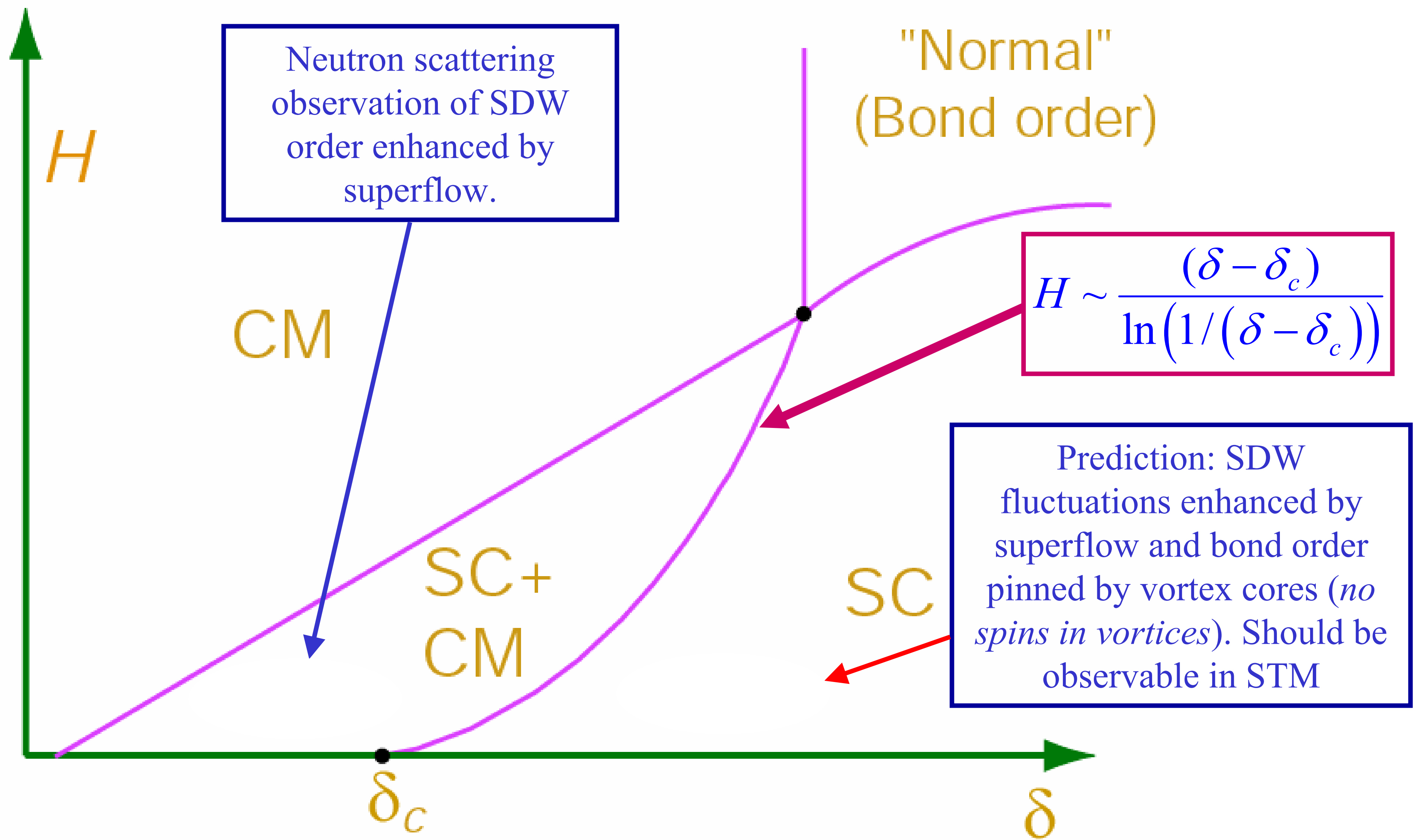
Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, *Phys. Rev.* B **67**, 014533 (2003).



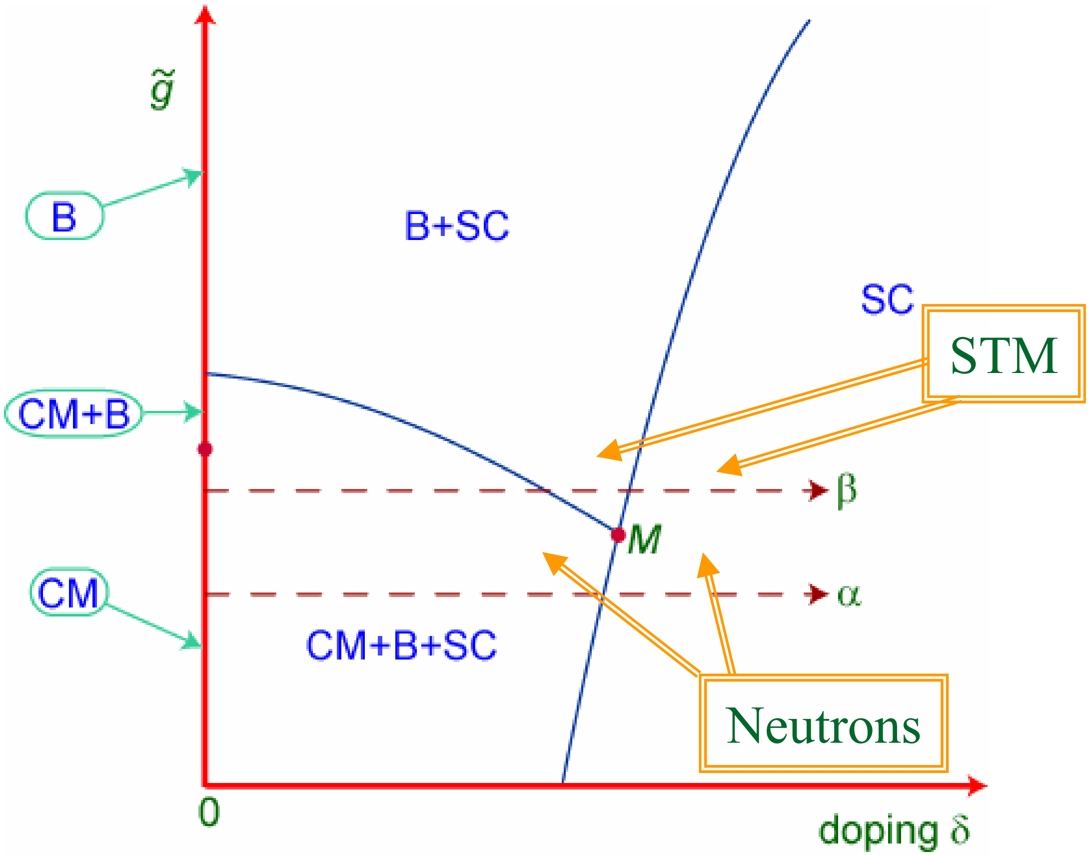
Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

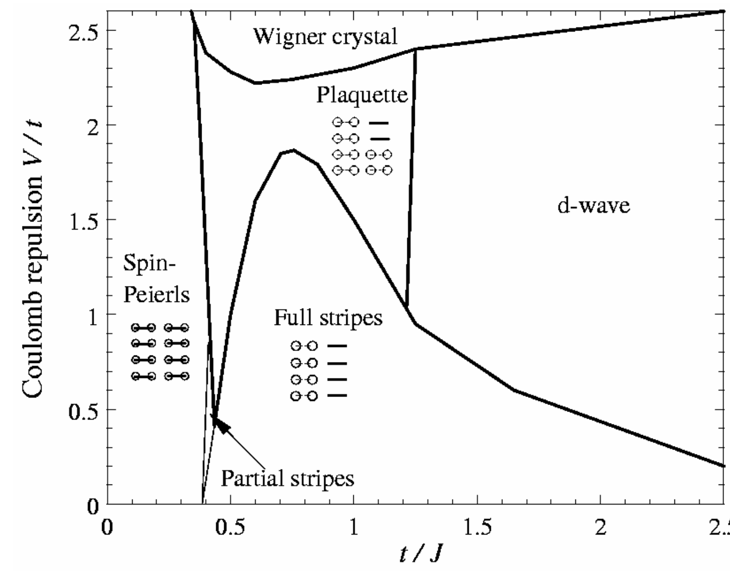
M. Vojta, *Phys. Rev.* B **66**, 104505 (2002); D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, *Phys. Rev.* B in press, cond-mat/0204011



K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001).
Y. Zhang, E. Demler and S. Sachdev, *Phys. Rev.* B **66,** 094501 (2002).

Global phase diagram





S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991). M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999); M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000); M. Vojta, Phys. Rev. B **66**, 104505 (2002).

See also S. Mazumdar, R.T. Clay, and D.K. Campbell, Phys. Rev. B **62**, 13400 (2000).
J. Zaanen, *Physica* C **217**, 317 (1999).
S.A. Kivelson, E. Fradkin, and V. Emery, *Nature* **393**, 550 (1998).
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).
C. Castellani, C. Di Castro, and M. Grilli, *Phys.Rev. Lett*. **75**, 4650 (1995).

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers.

II. Order parameters characterizing the Mott insulator compete with the order associated with the (Bose-Einstein) condensation of Cooper pairs.

III. Classification of Mott insulators shows that the appropriate order parameters are collinear magnetism and bond order.

IV. Theory of quantum phase transitions provides semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.

V. Future experiments should search for SC+CM to SC quantum transition driven by a magnetic field.