The SYK model: a window into non-Fermi liquids

Distinguished Lectures on Topological Materials
Institute for Materials Science
Los Alamos National Laboratory
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PHYSICS

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What is a non-Fermi liquid ?

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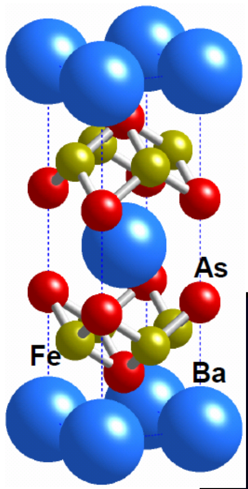
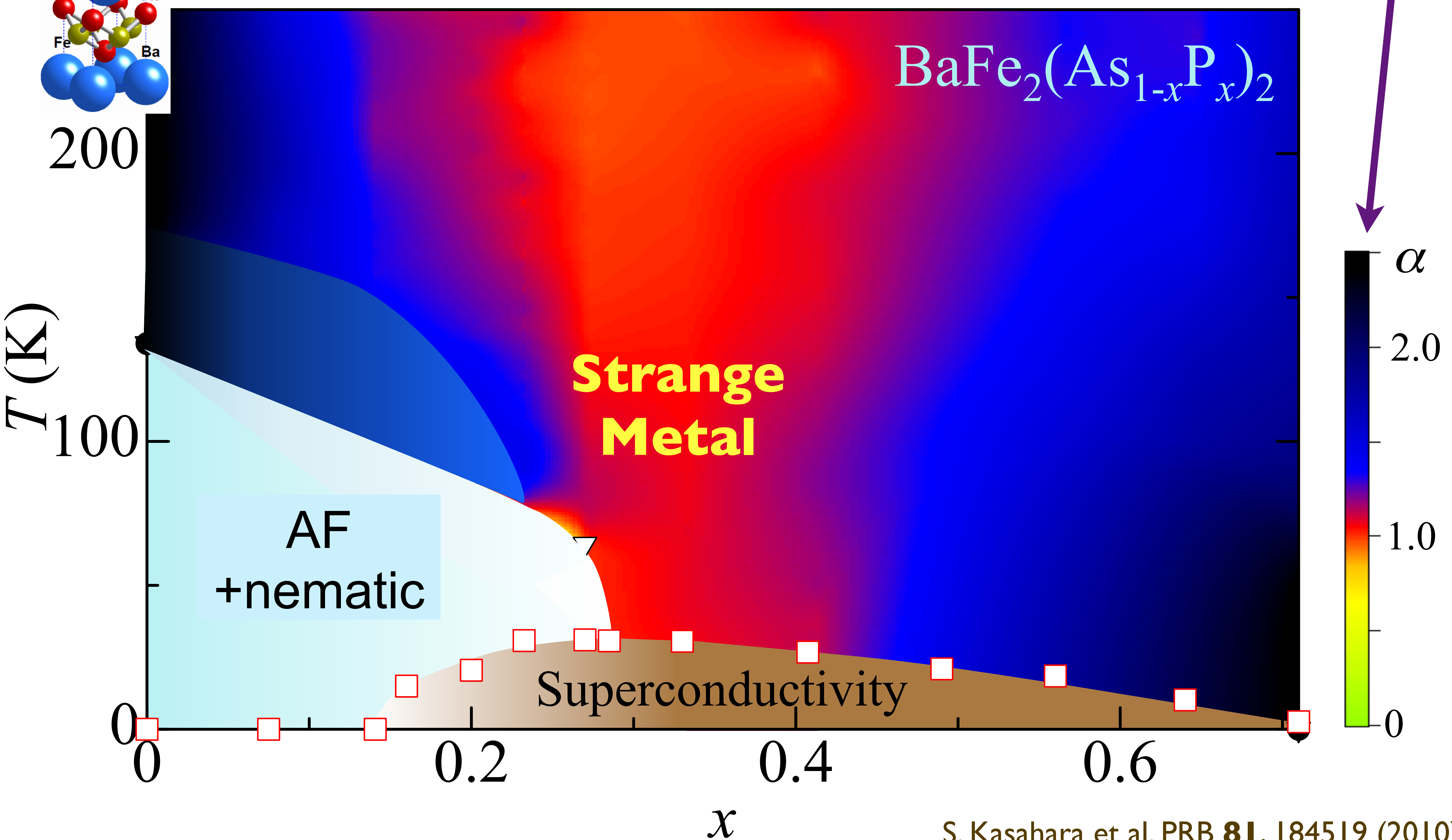
Experiments:

Resistivity
$\sim \rho_0 + AT^\alpha$





S. Kasahara et al. PRB **81**, 184519 (2010)

What is a non-Fermi liquid ?

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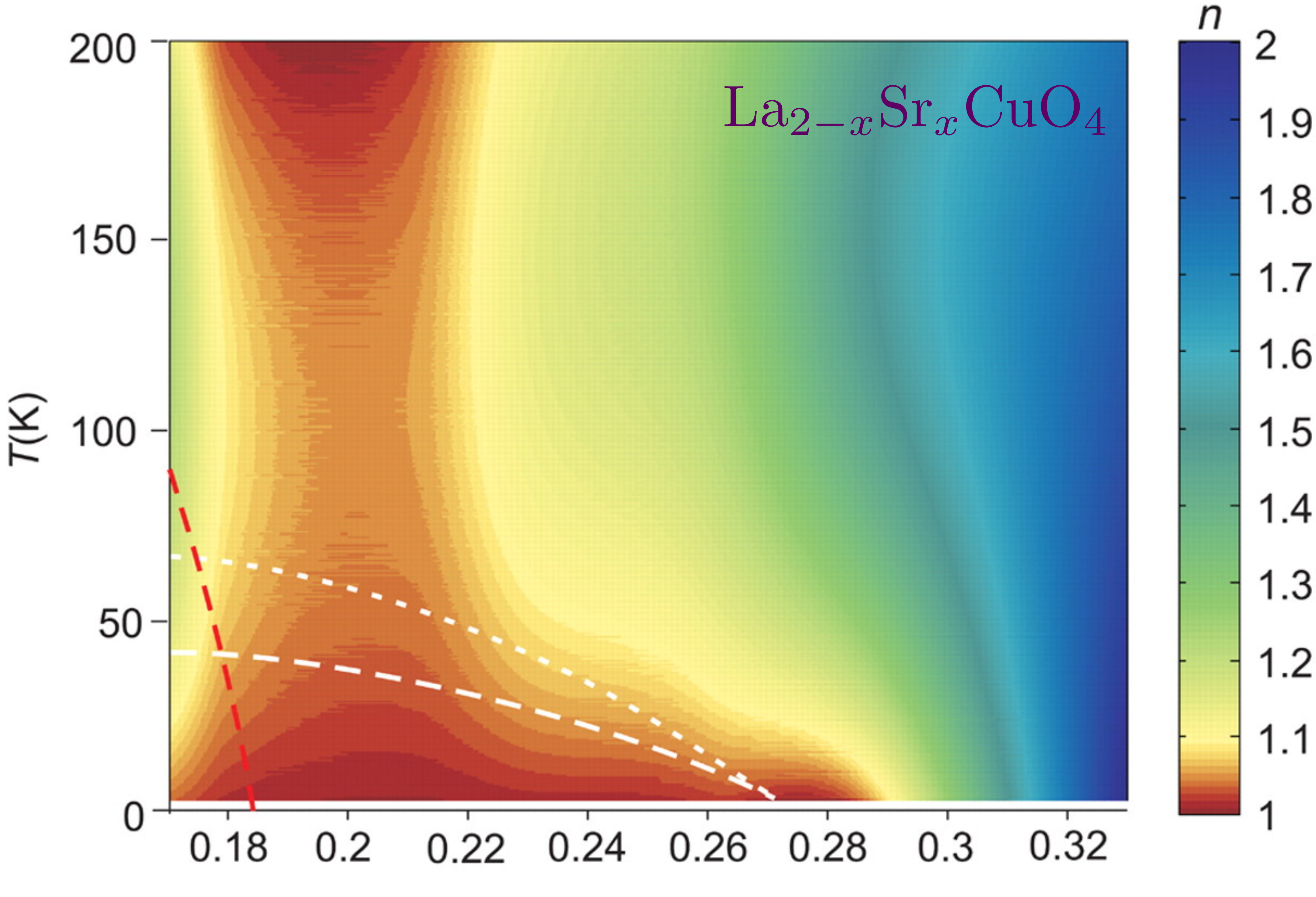

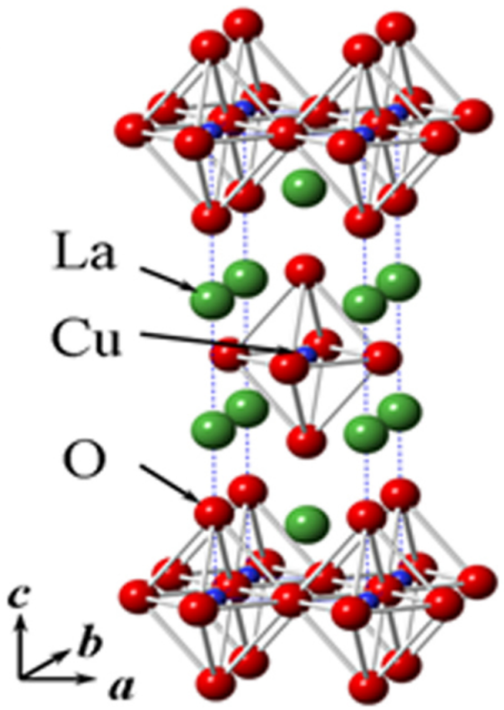

R.A. Cooper et al. Science **323**, 603 (2009)

What is a non-Fermi liquid ?

- **A compressible state of quantum matter:**
  There is a global U(1) charge $\mathcal{Q}$ with commutes with Hamiltonian, and upon applying a chemical potential change $\delta\mu$

$$\begin{aligned} \mathcal{H} &\Rightarrow \mathcal{H} - \delta\mu\,\mathcal{Q} \\ \frac{\delta\langle\mathcal{Q}\rangle}{\delta\mu} &\neq 0 \quad \text{as } T \to 0 \text{ in the thermodynamic limit} \end{aligned}$$

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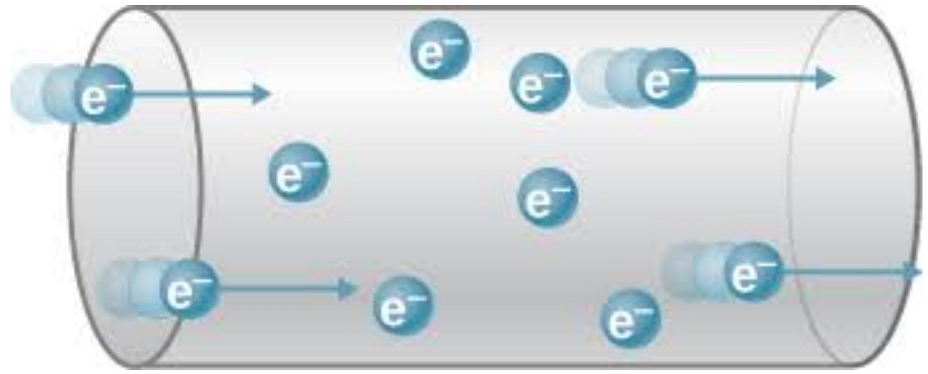
  Dirac/Weyl fermions, CFTs in spatial dimension $d > 1$ are *not* compressible.

- **No quasiparticle excitations:**
  In the presence of quasiparticles, the low-lying many-body energies E can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \dots$$

  Luttinger liquids, systems with an energy gap (TQFTs), do have quasiparticles.

Current flow with quasiparticles



Flowing quasiparticles scatter off each other in a typical scattering time τ

This time is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

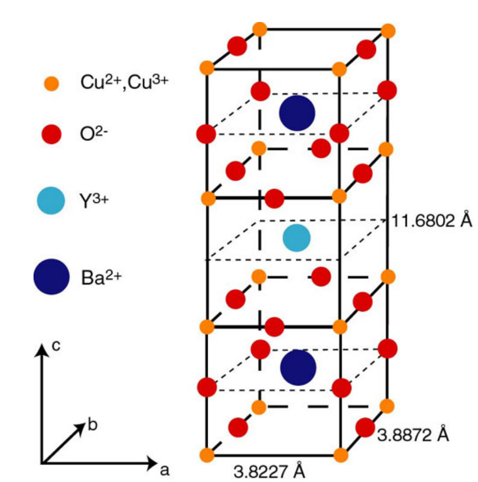
A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Electron scattering time τ in 7 different strange metals

$$\frac{1}{\tau} = \alpha \, \frac{k_B T}{\hbar}$$

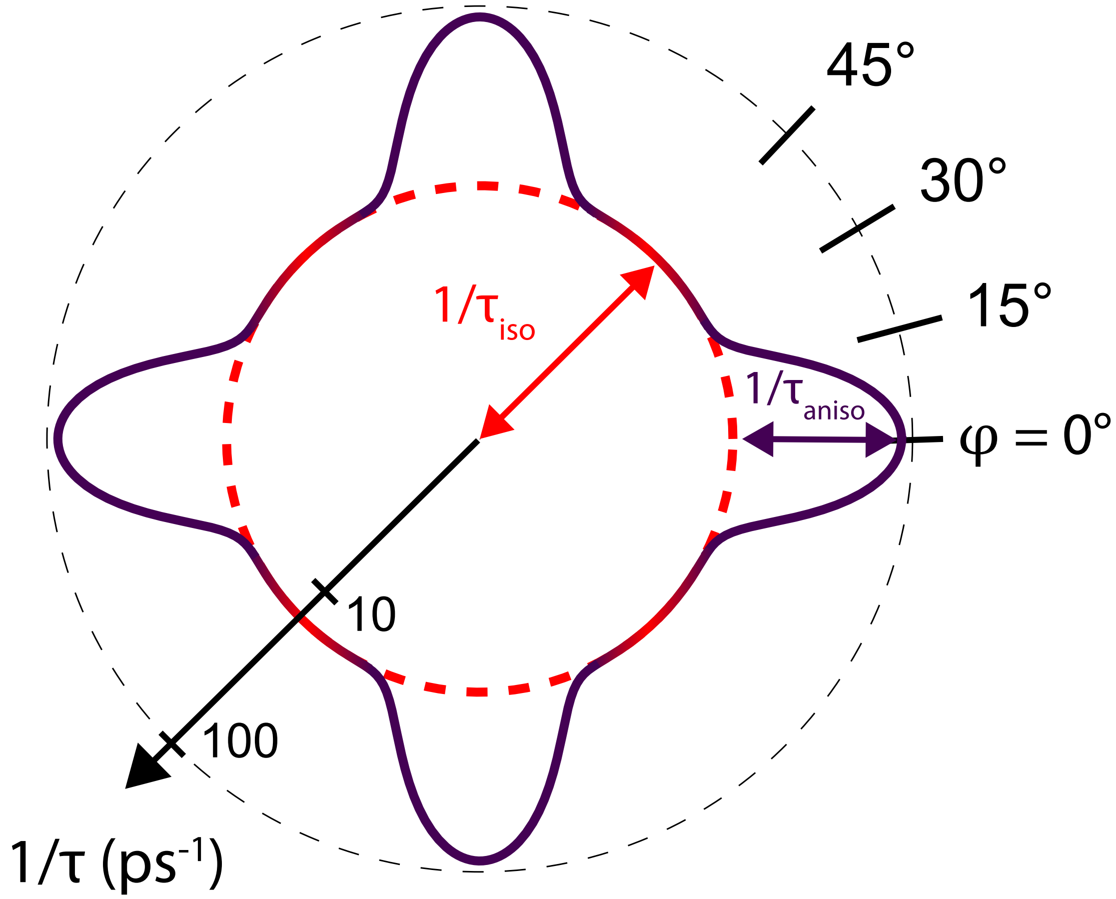
Current flow without quasiparticles

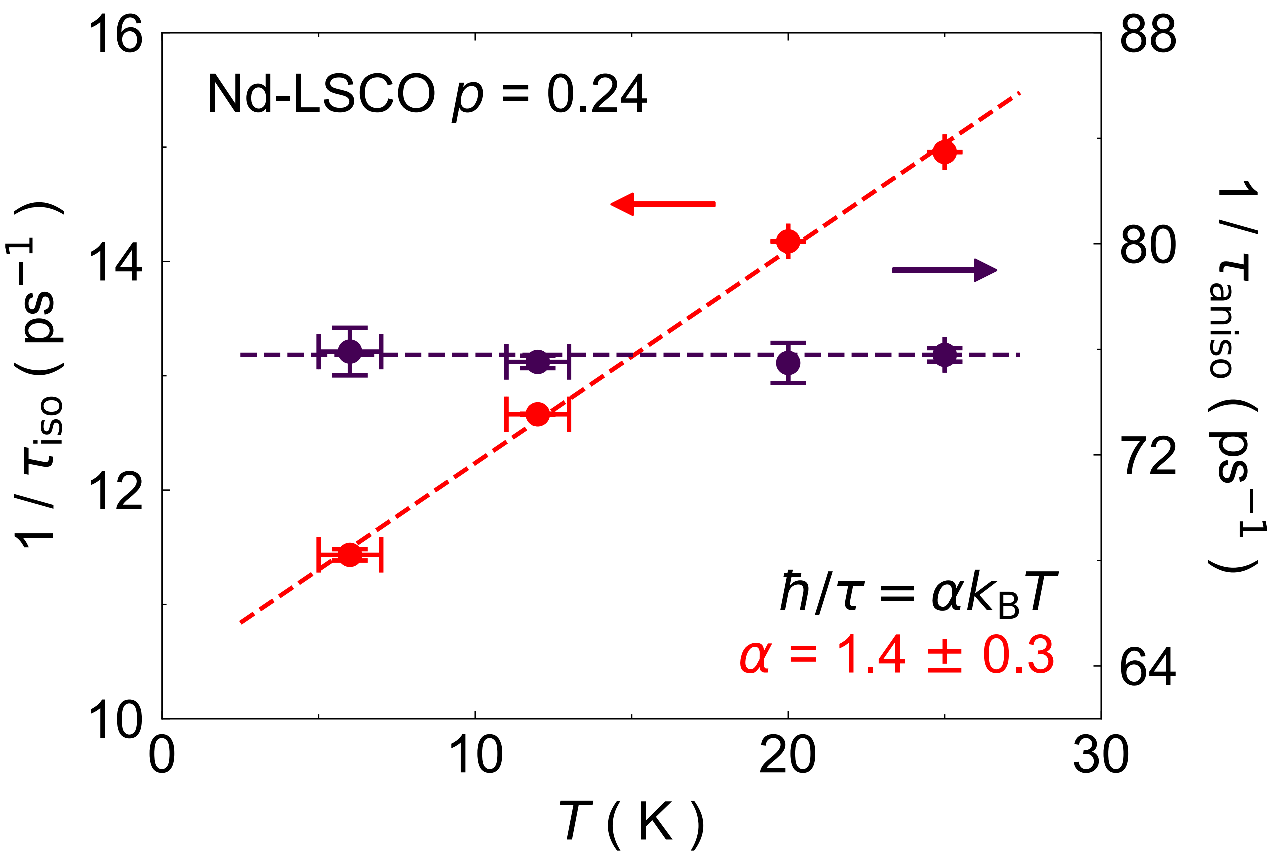


Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.





$$\frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

1. Random matrix and SYK models

2. Time reparameterization soft mode

3. Random t-J model

4. Charged black holes

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

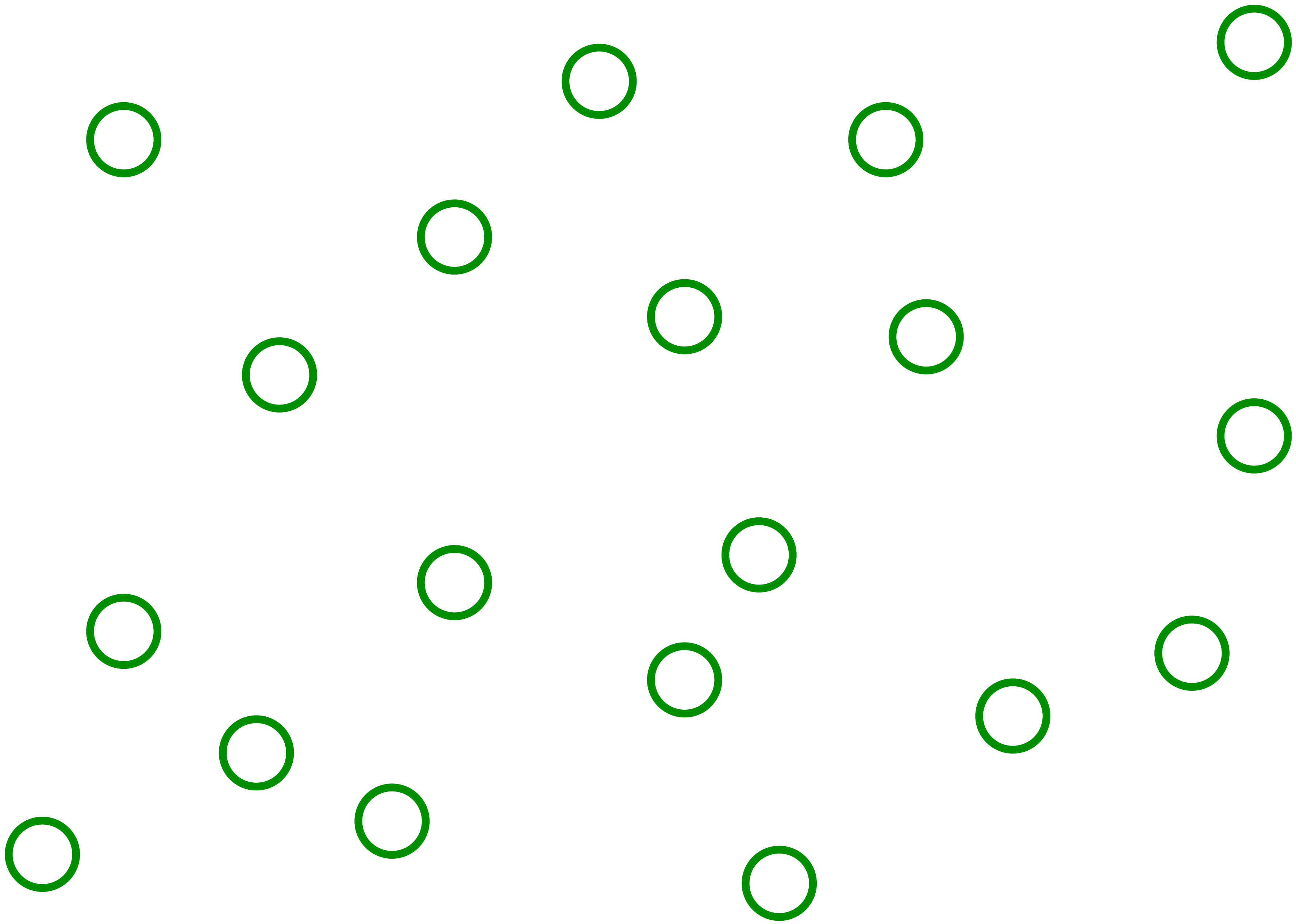
$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

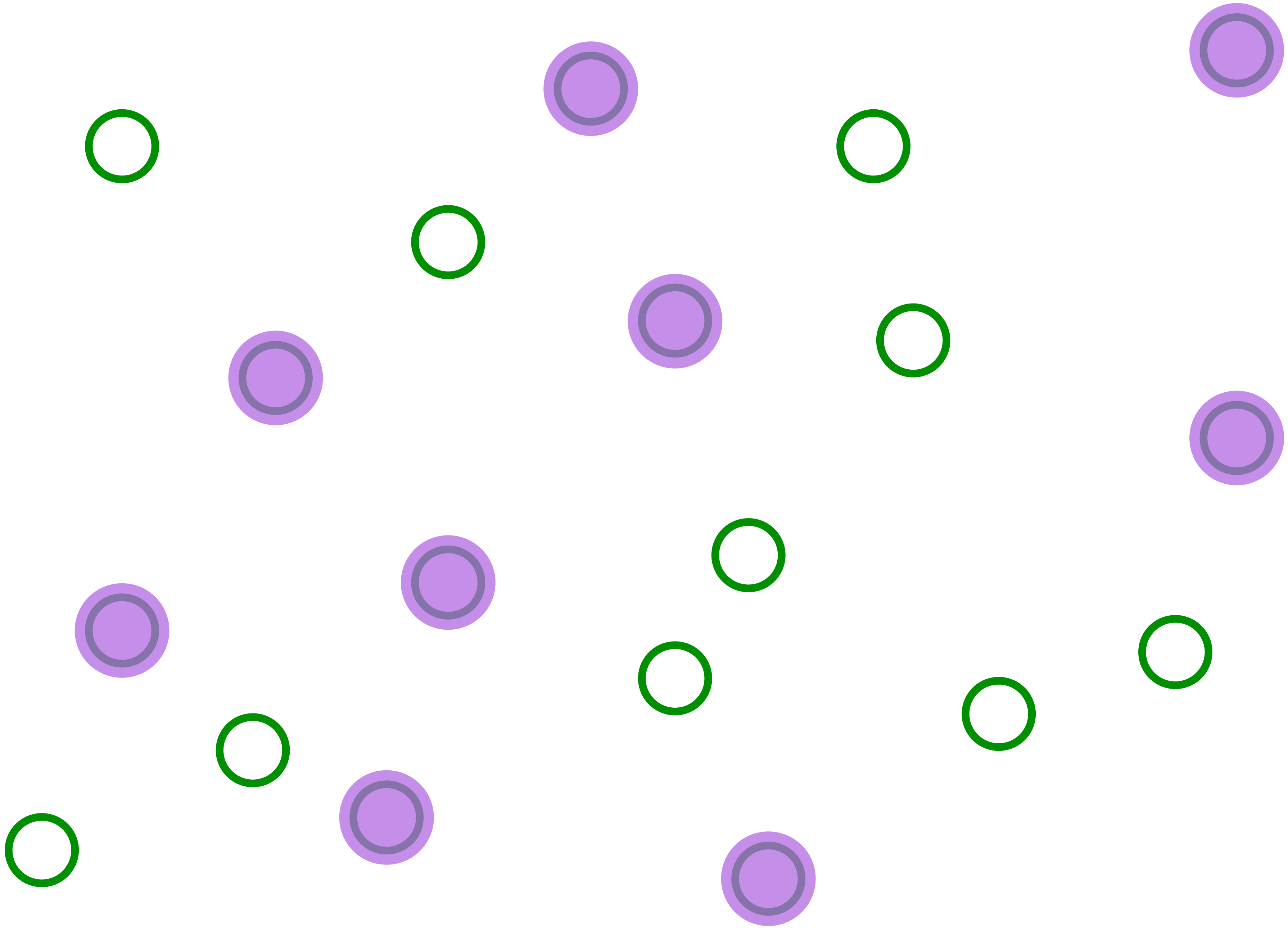
Fermions occupying the eigenstates of a
N x *N* random matrix

The random matrix model



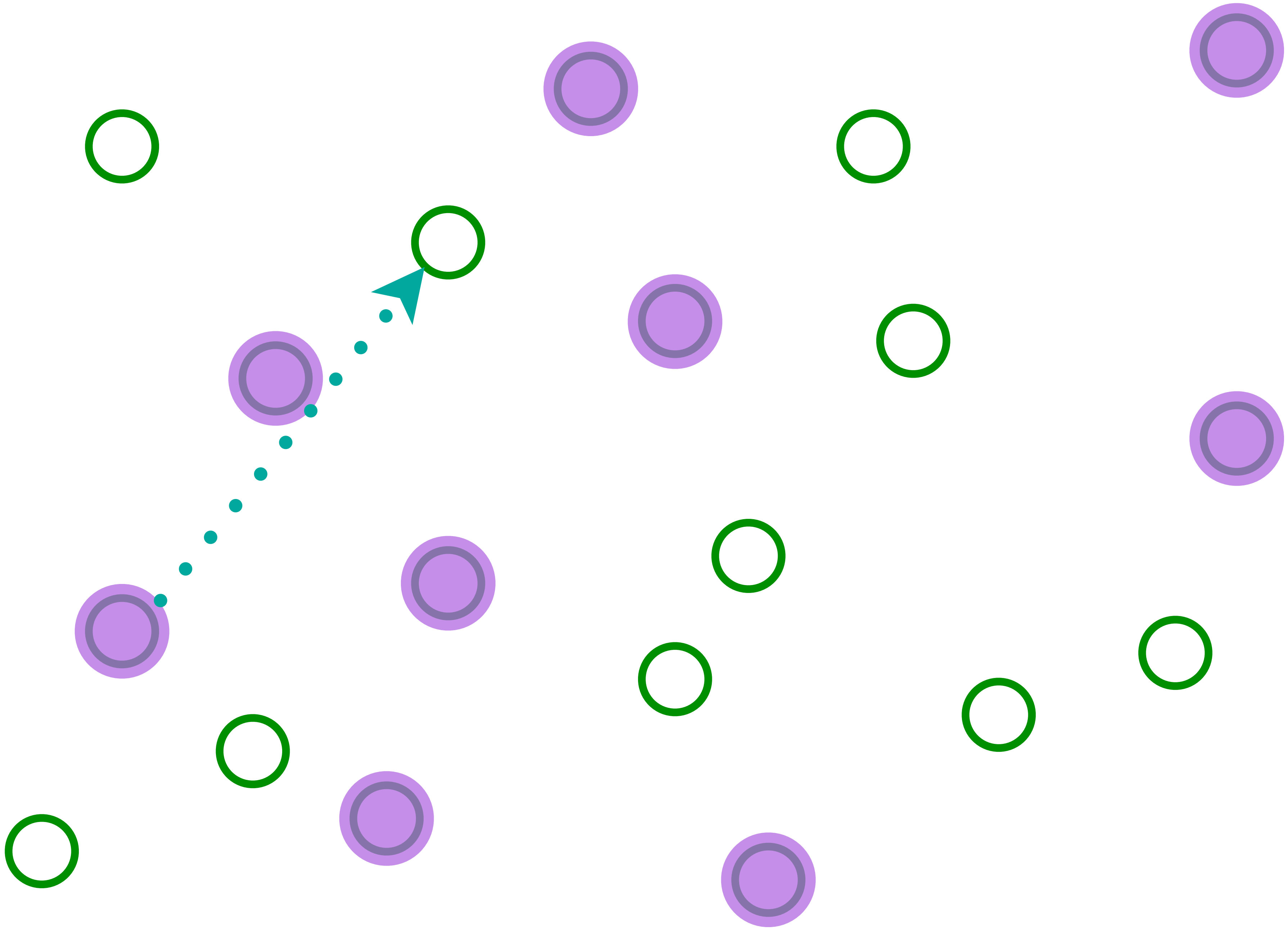
Pick a set of random positions

The random matrix model



Place electrons randomly on some sites

The random matrix model



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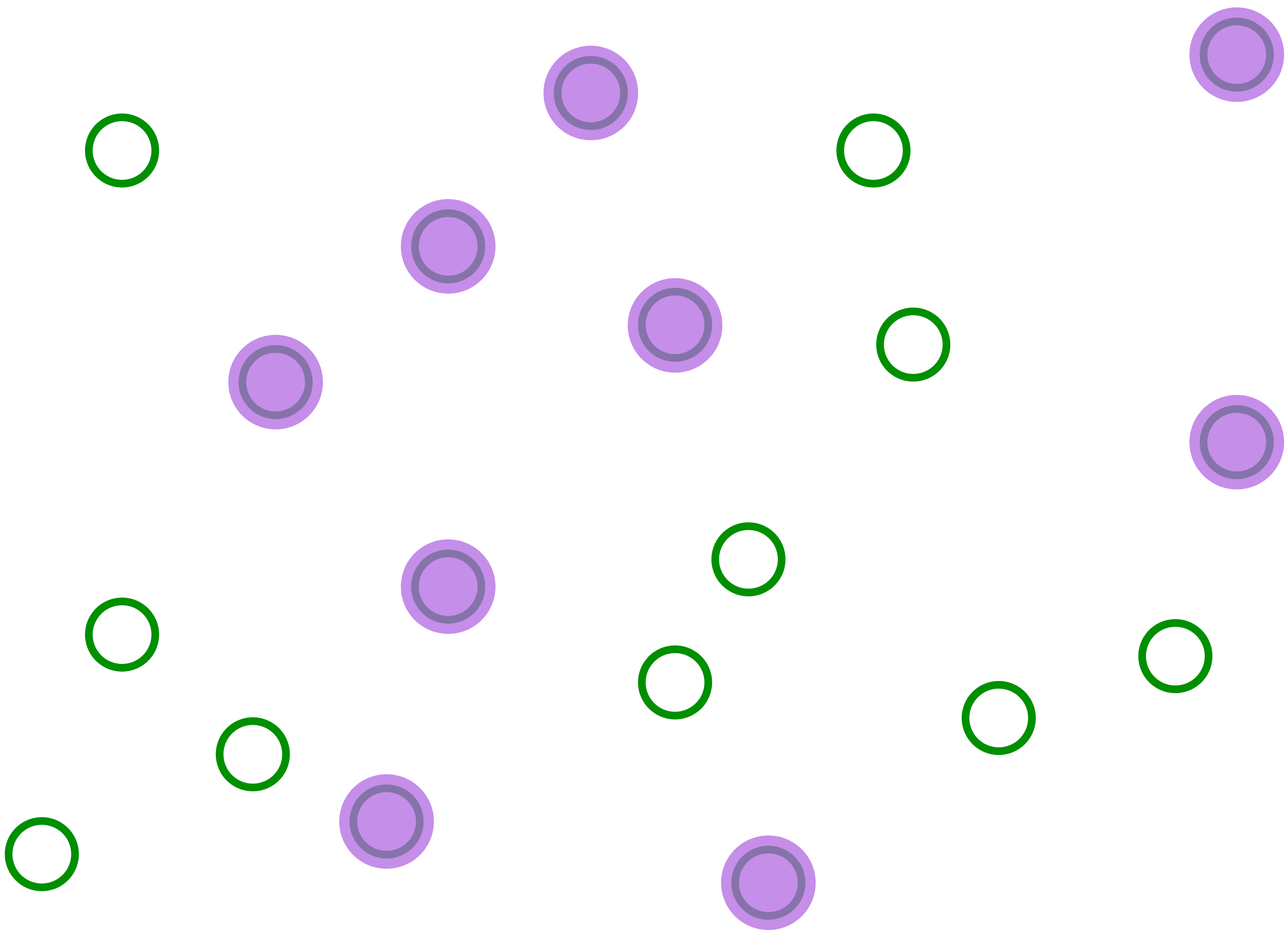
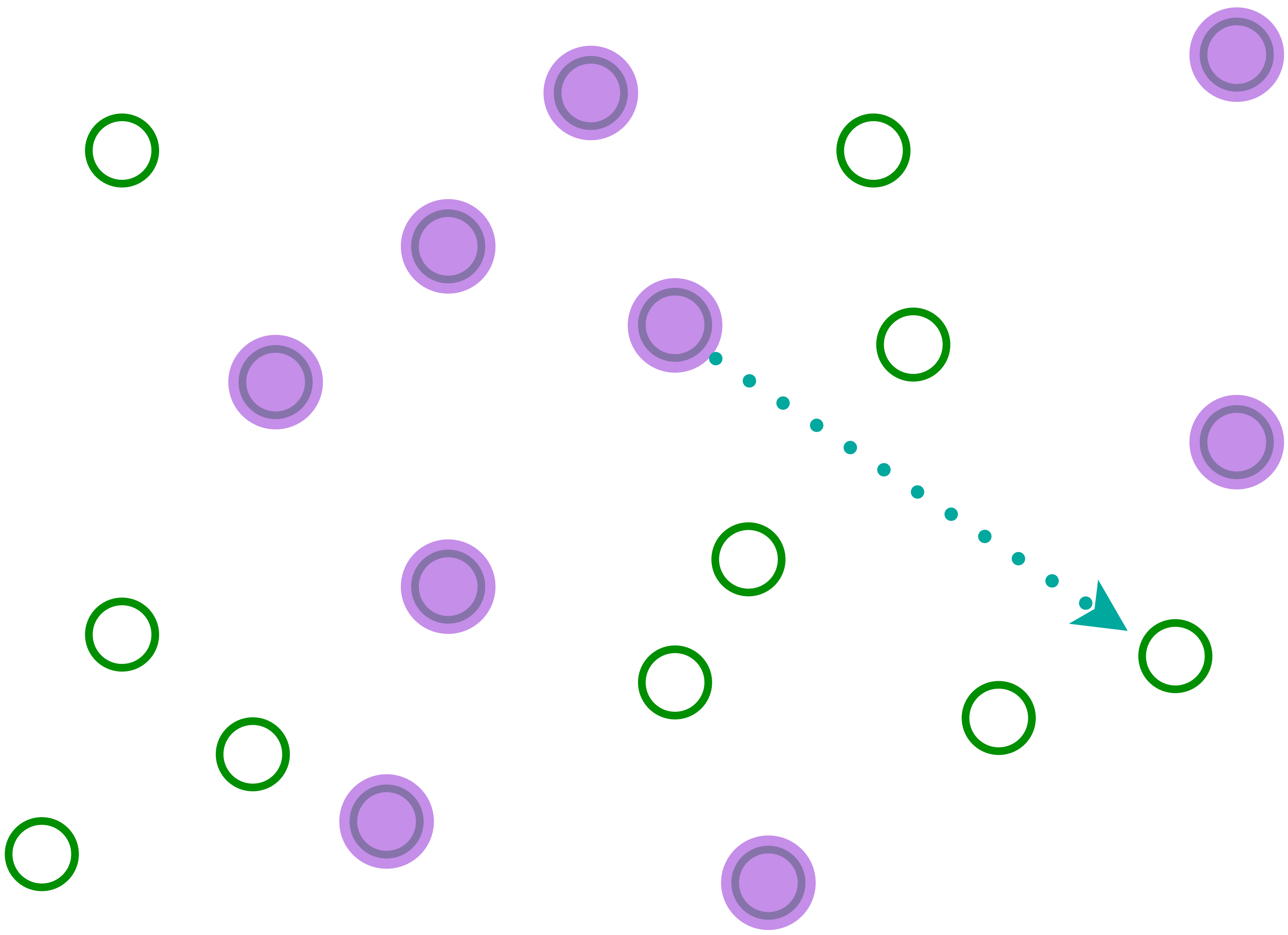


Electrons can hop anywhere with a random amplitude

The random matrix model



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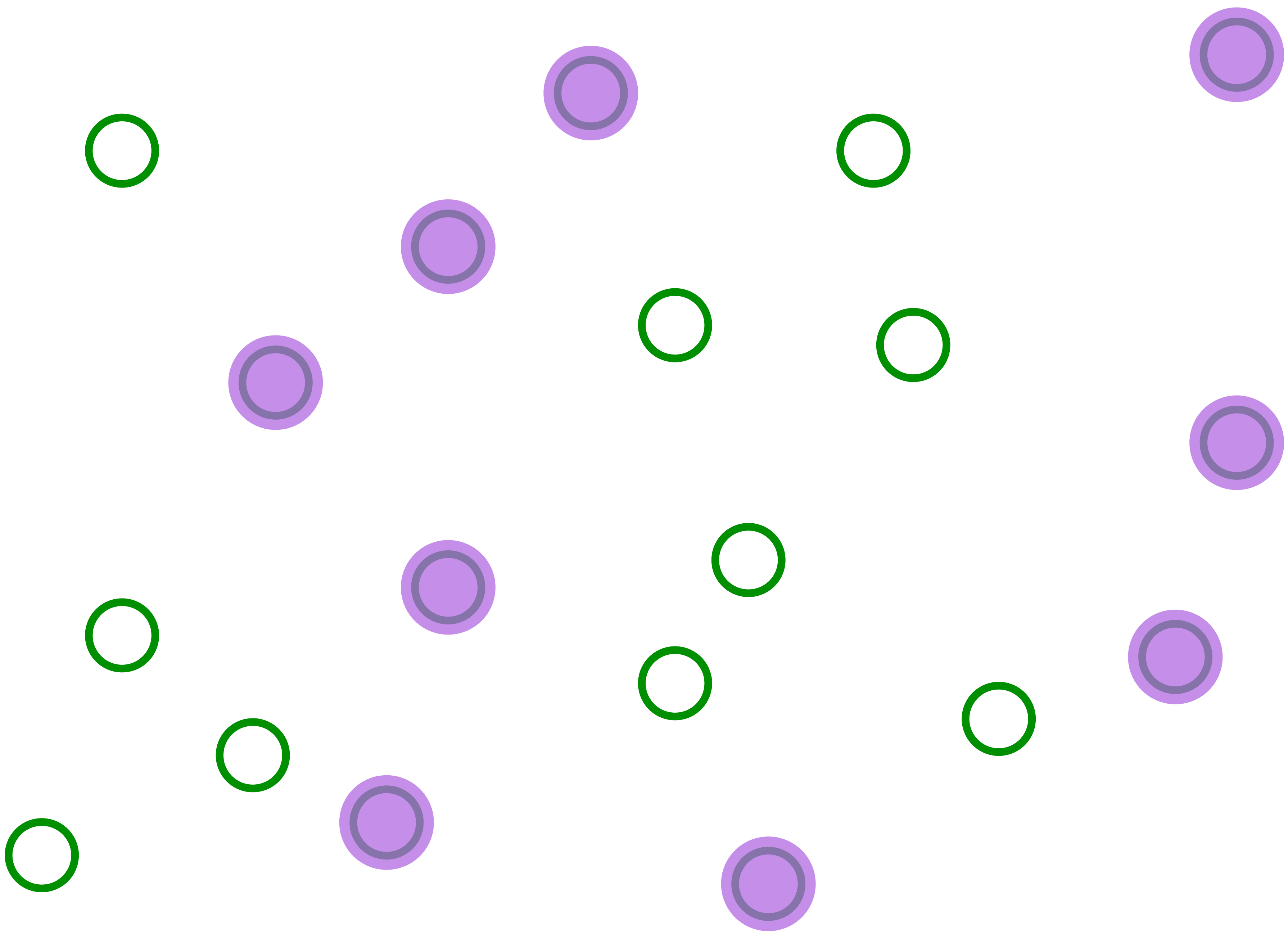

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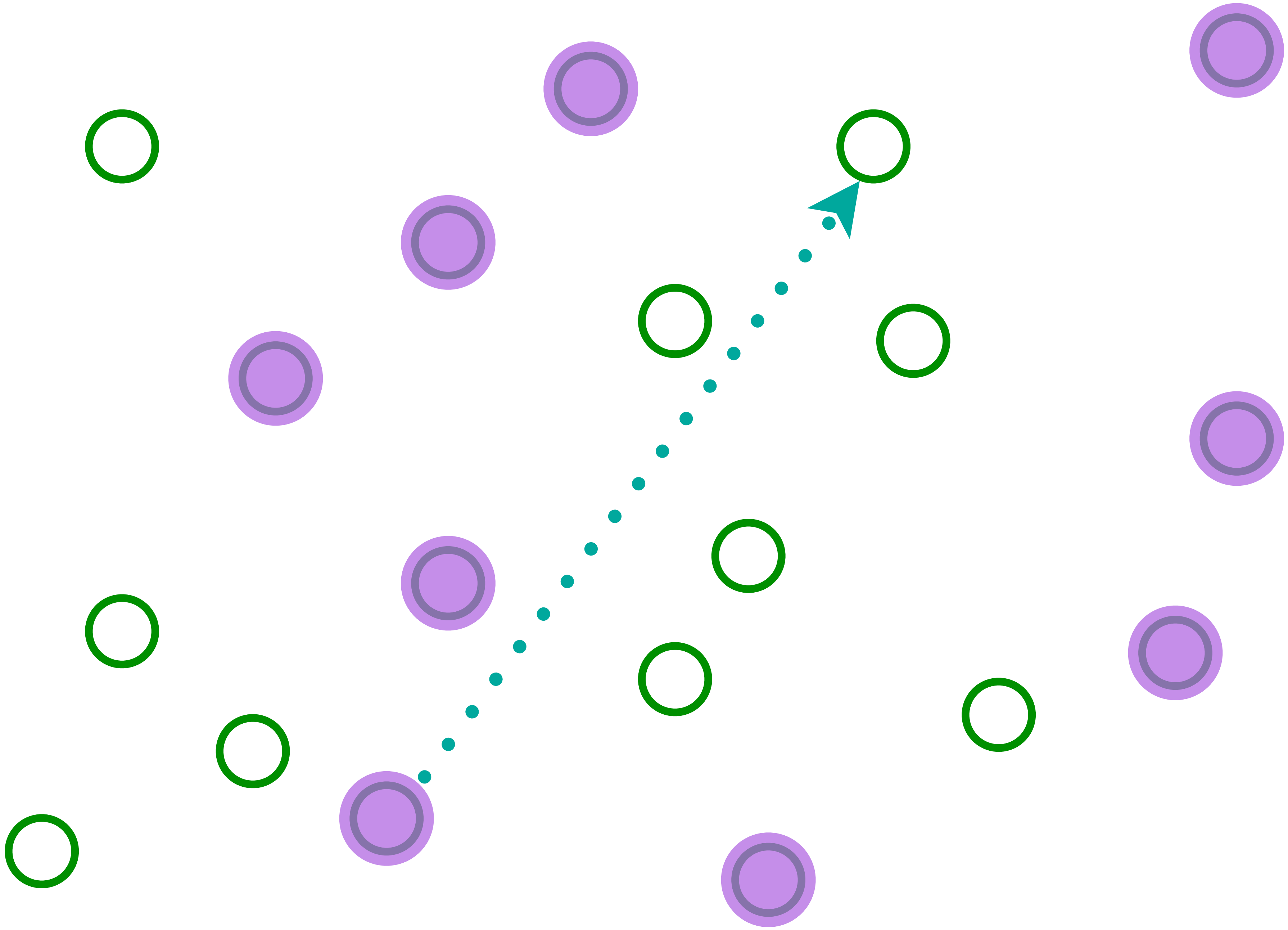

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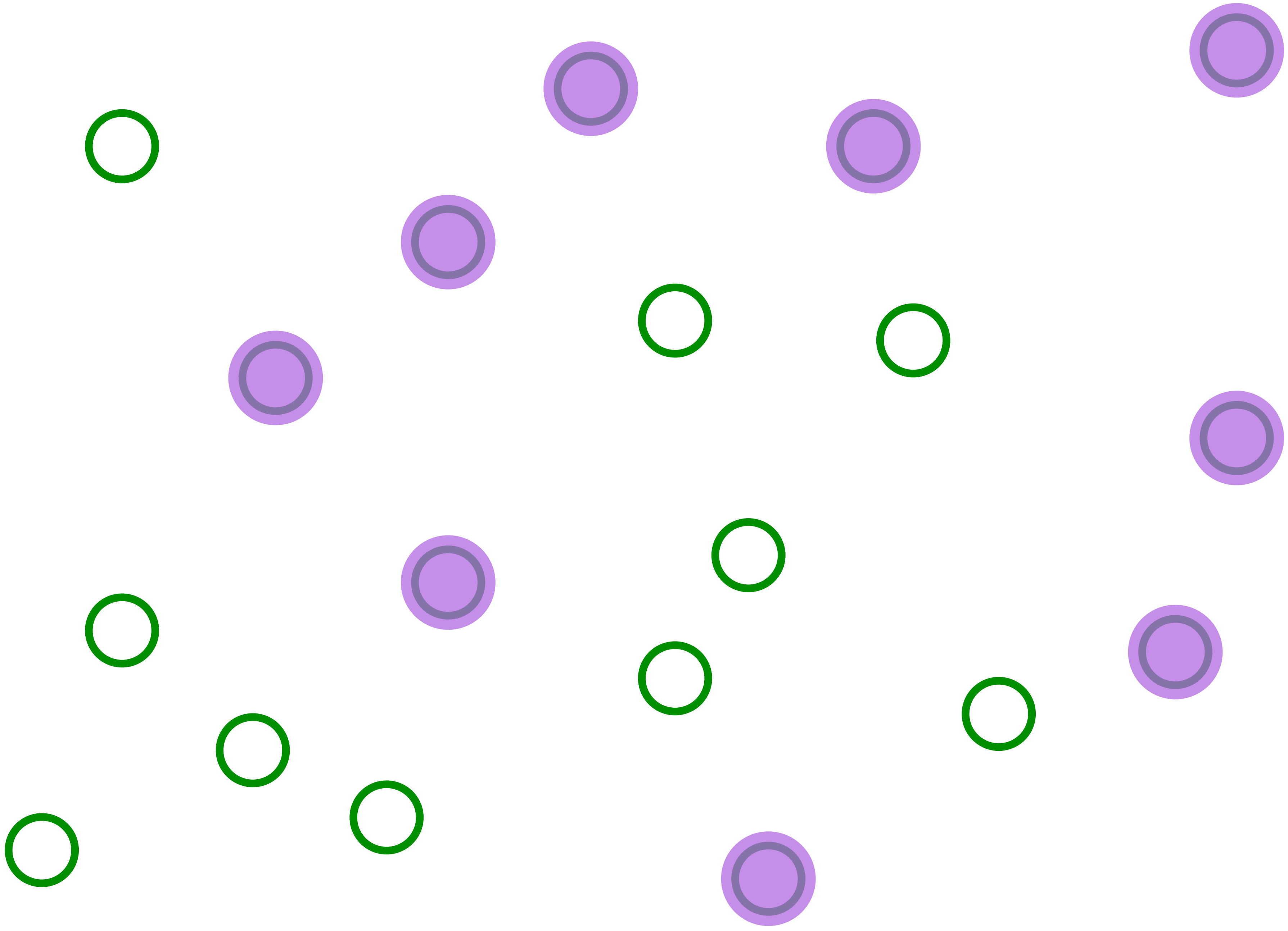


Electrons can hop anywhere with a random amplitude

A simple model of a metal with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

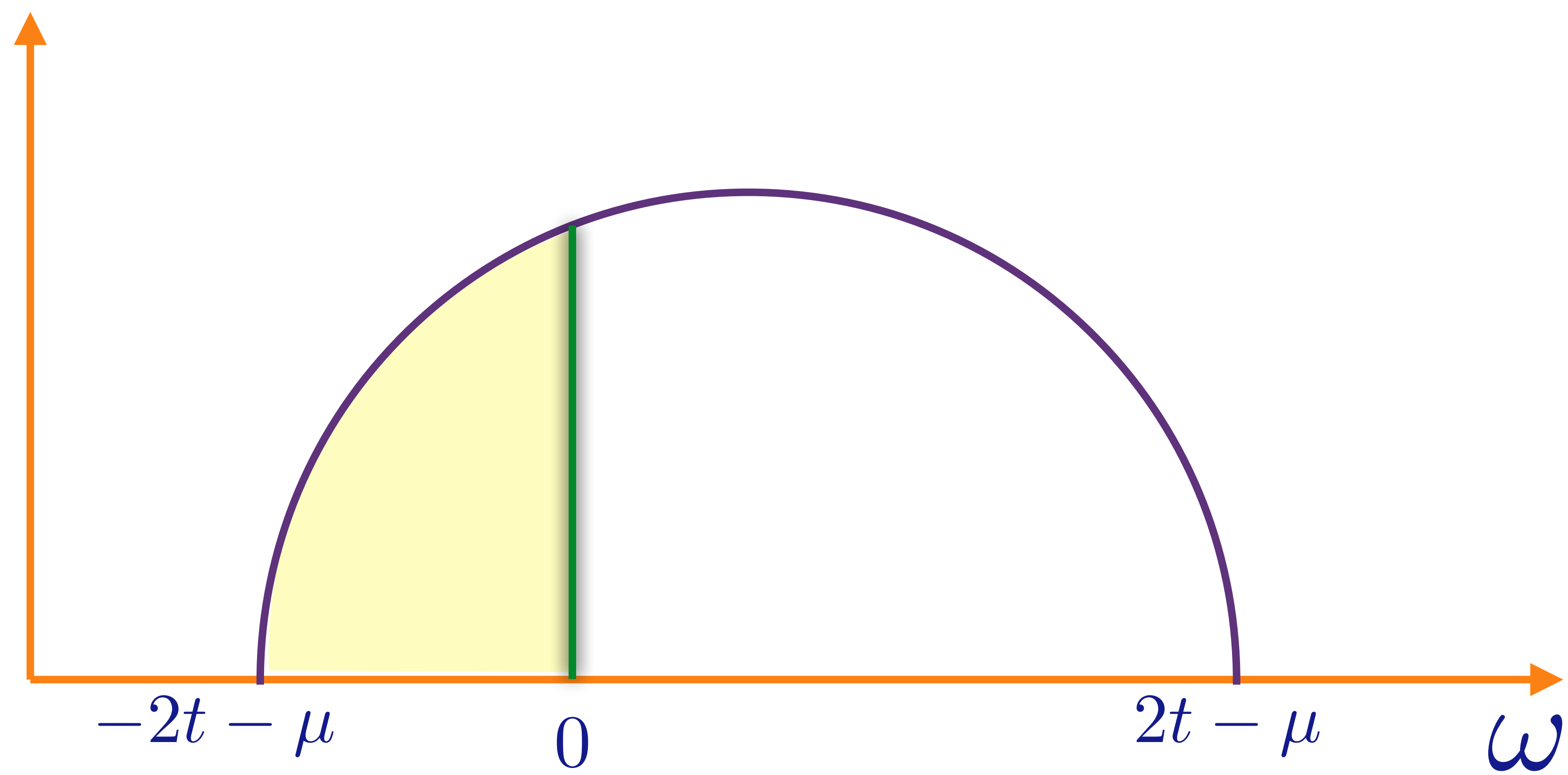
$$G(\tau) \equiv -T_\tau \left\langle c_i(\tau) c_i^\dagger(0) \right\rangle$$

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

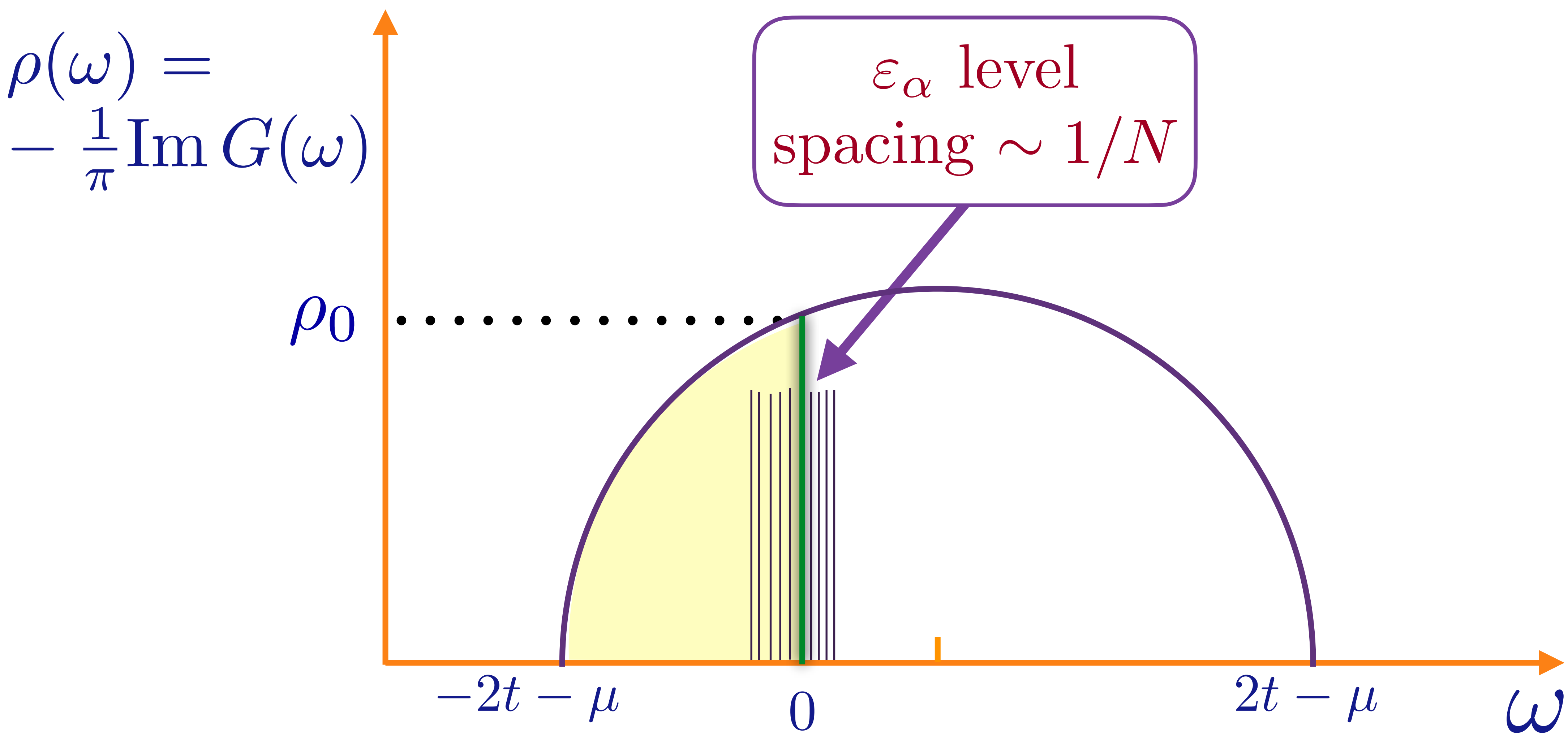
$G(\omega)$ can be determined by solving a quadratic equation.

$\rho(\omega) = -\frac{1}{\pi}\mathrm{Im}\, G(\omega)$



A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The single-particle density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$, and $\rho_0 \equiv \rho(\omega = 0)$.



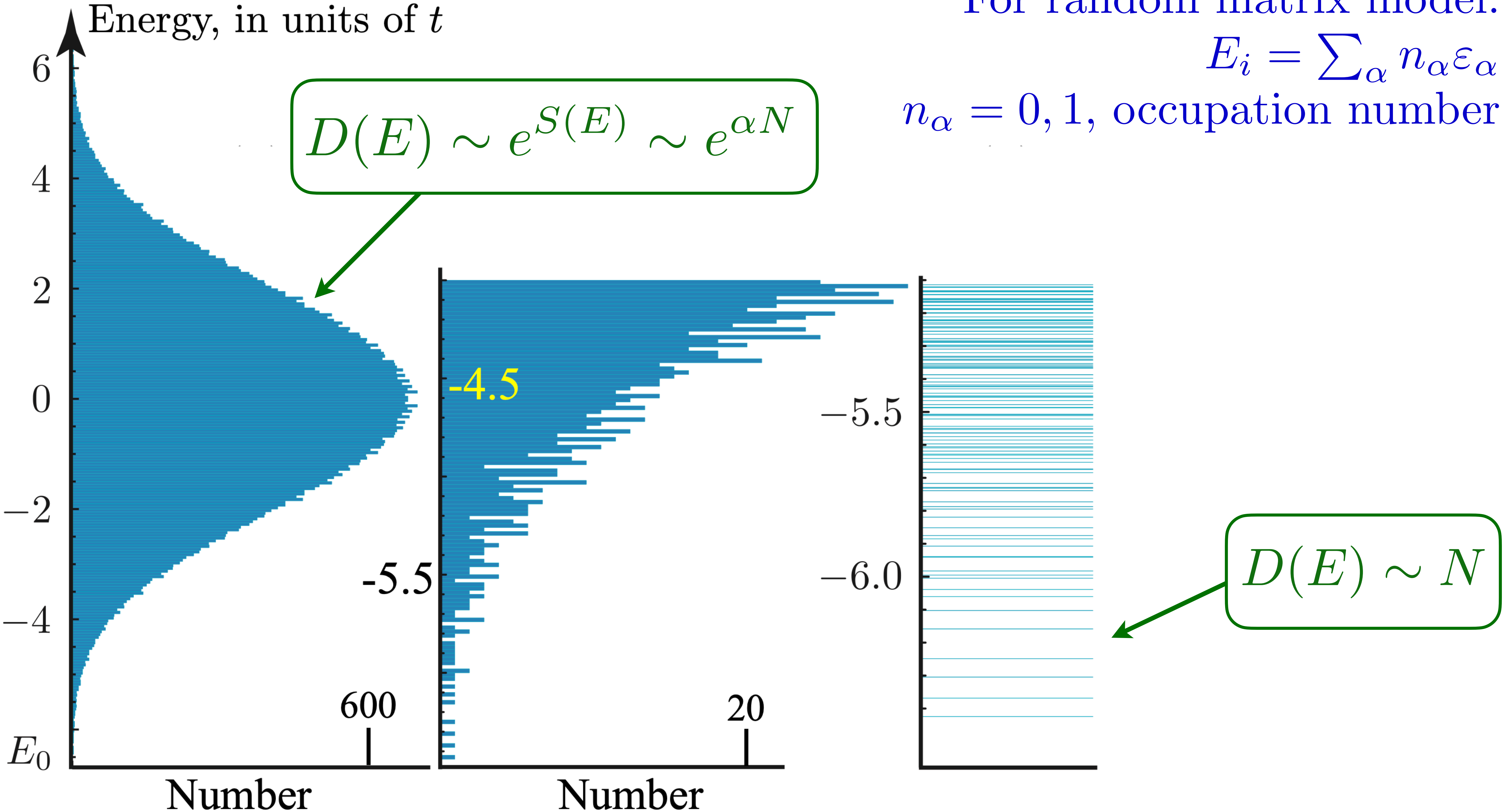
Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_i \Rightarrow \text{Many body eigenvalue}$$

For random matrix model:

$$E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$$

$n_\alpha = 0, 1$, occupation number



Random matrix model

The Sachdev-Ye-Kitaev (SYK) model

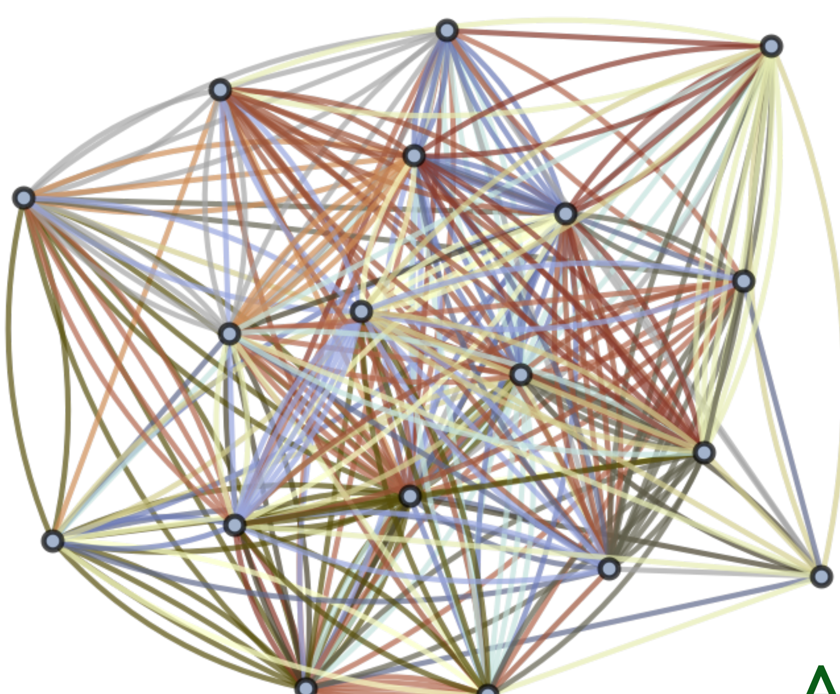
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

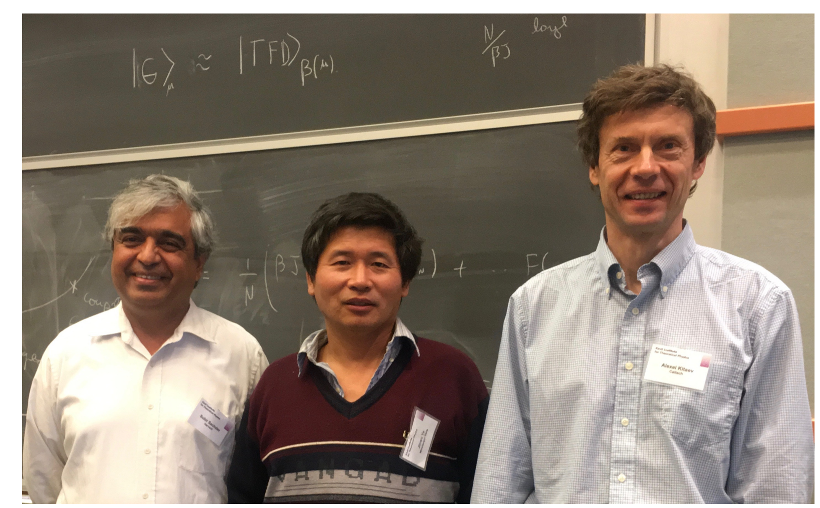
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



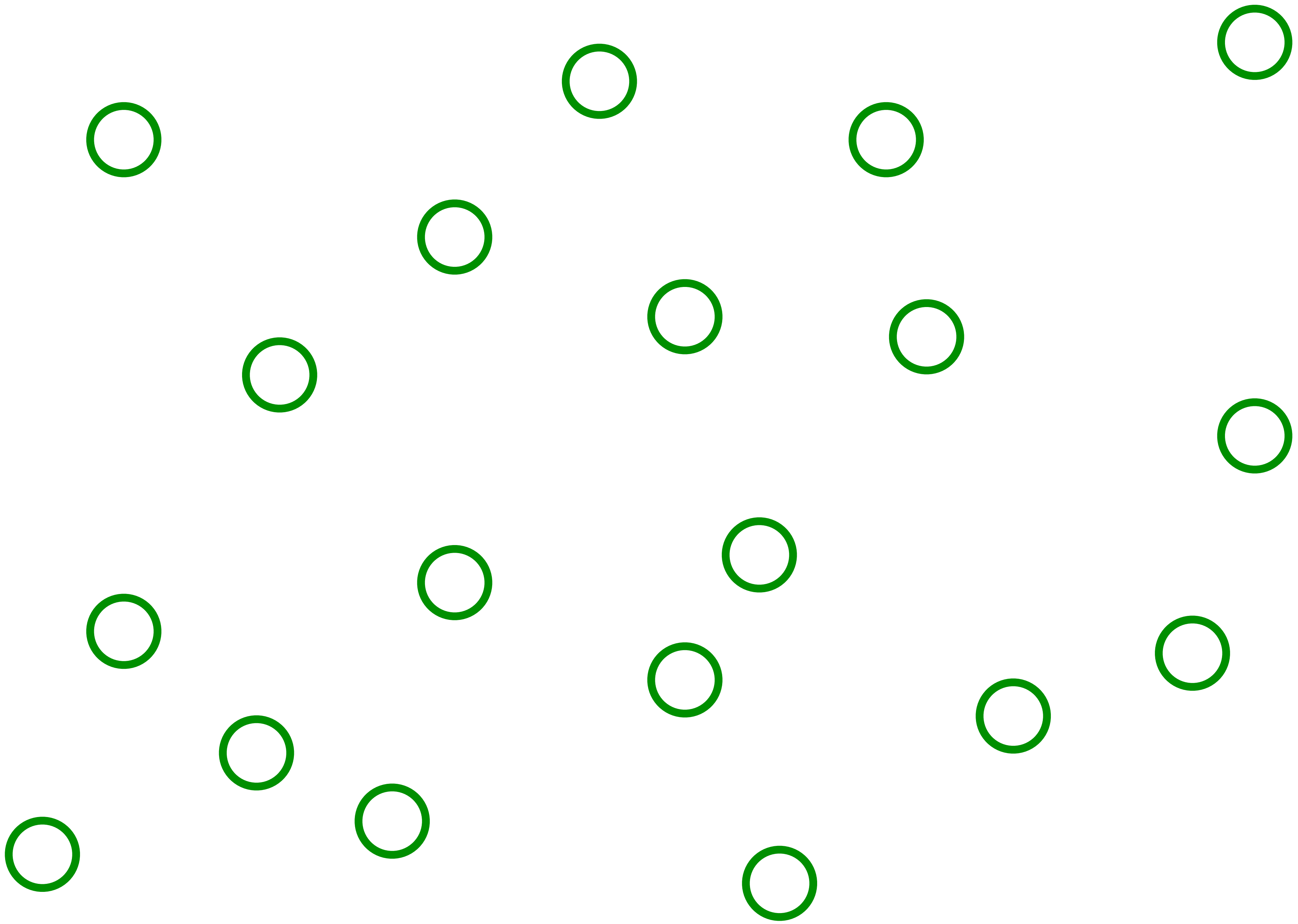


S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions

The SYK model

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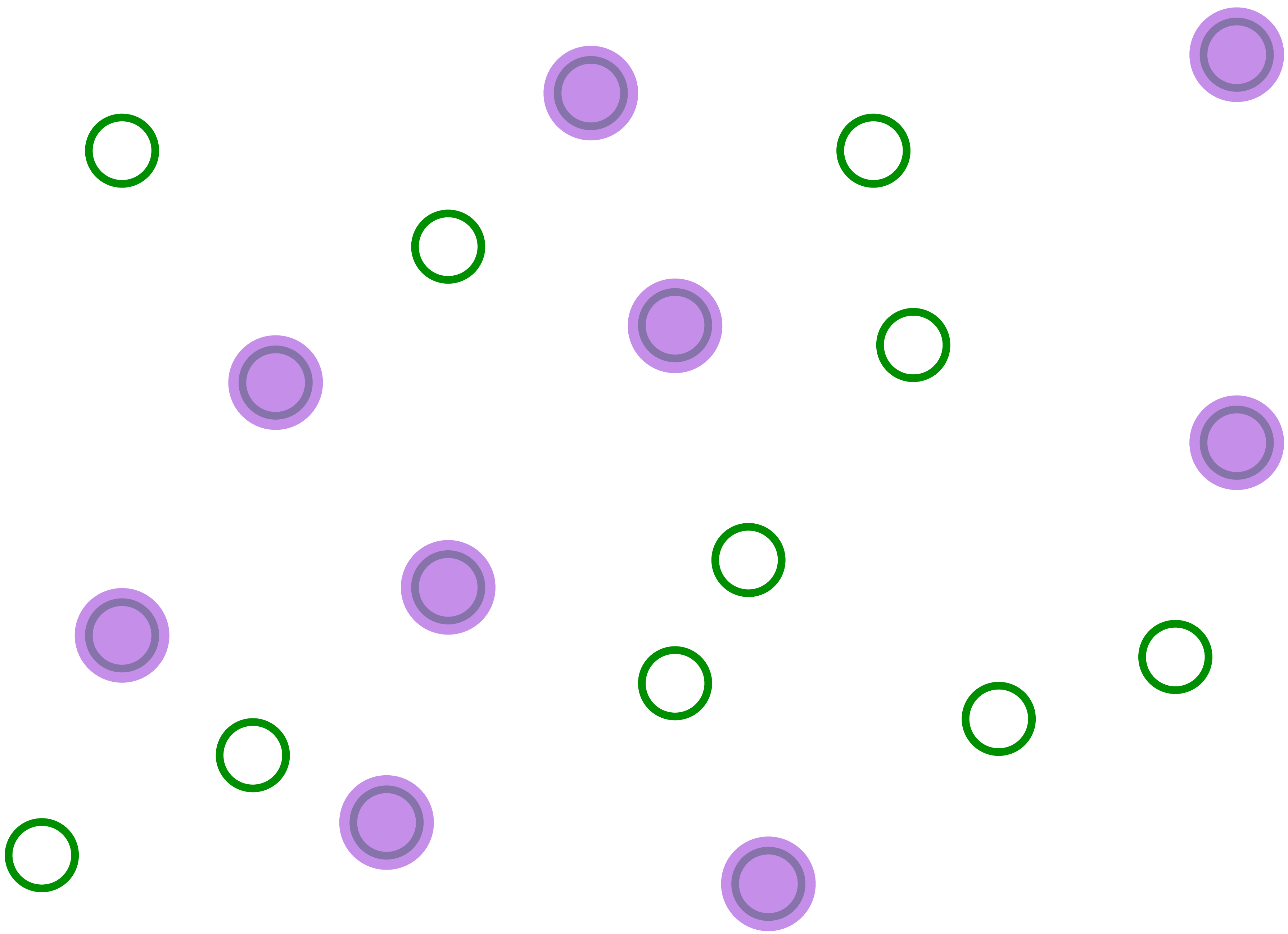

Place electrons randomly on some sites

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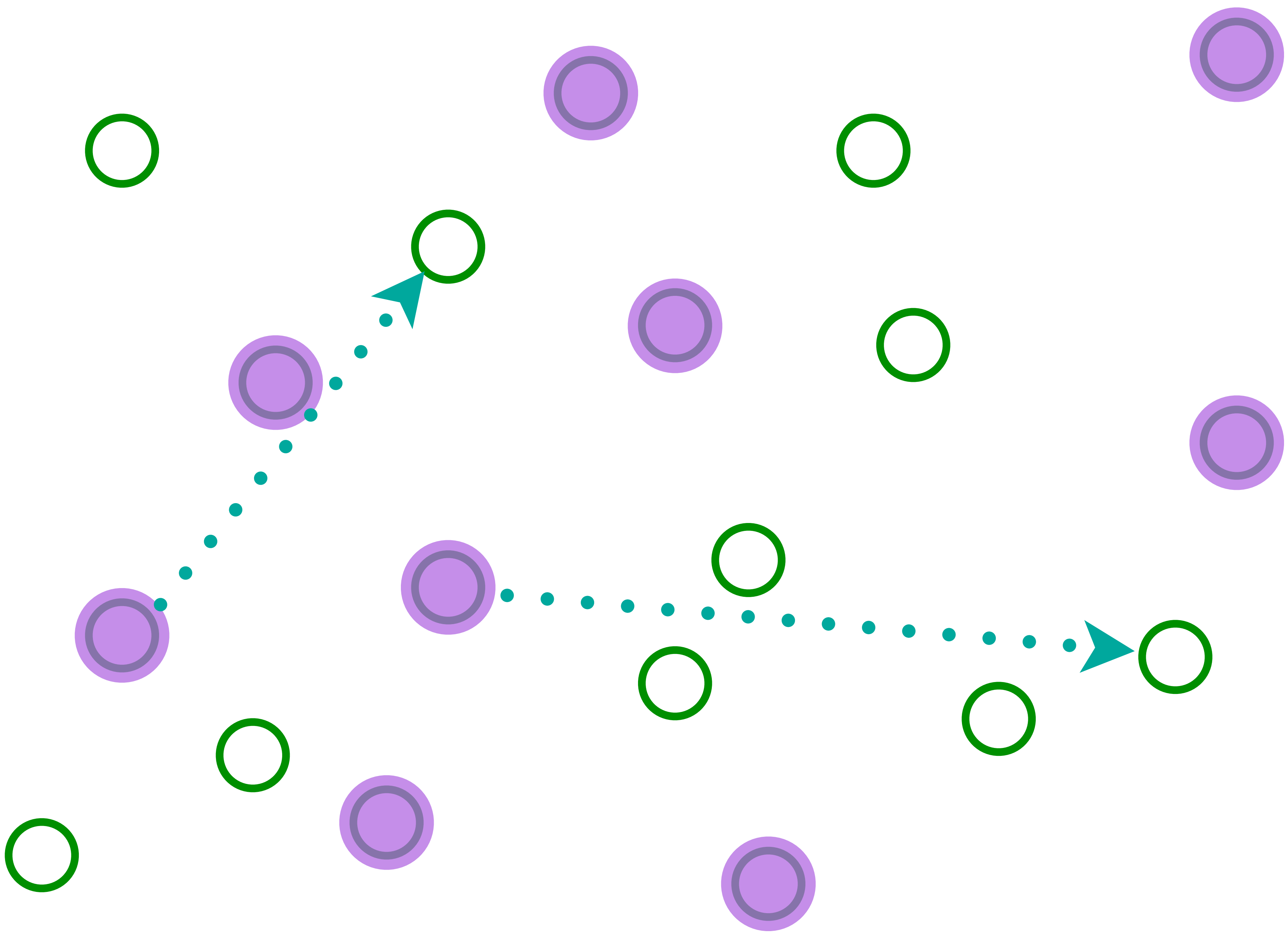

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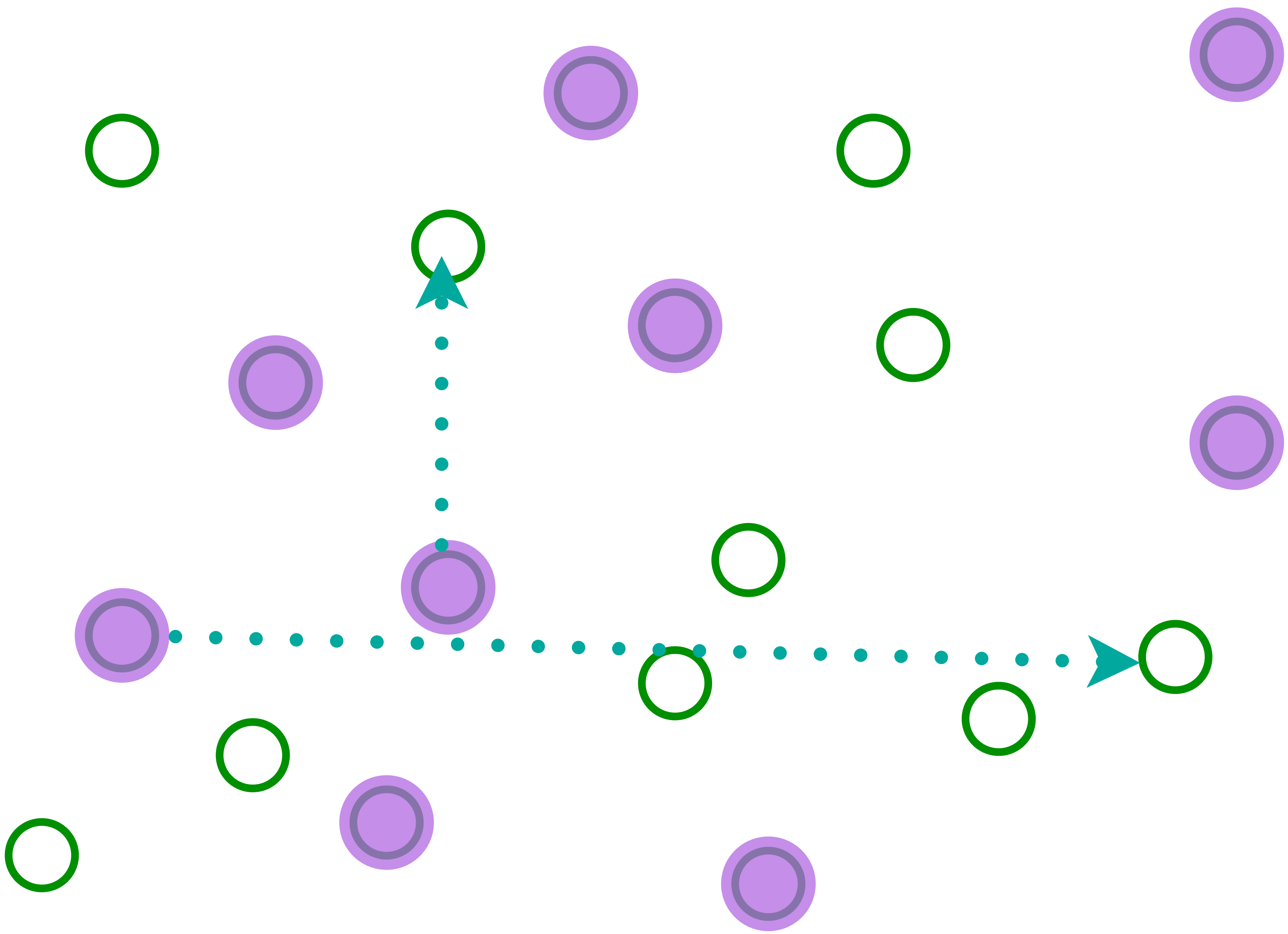

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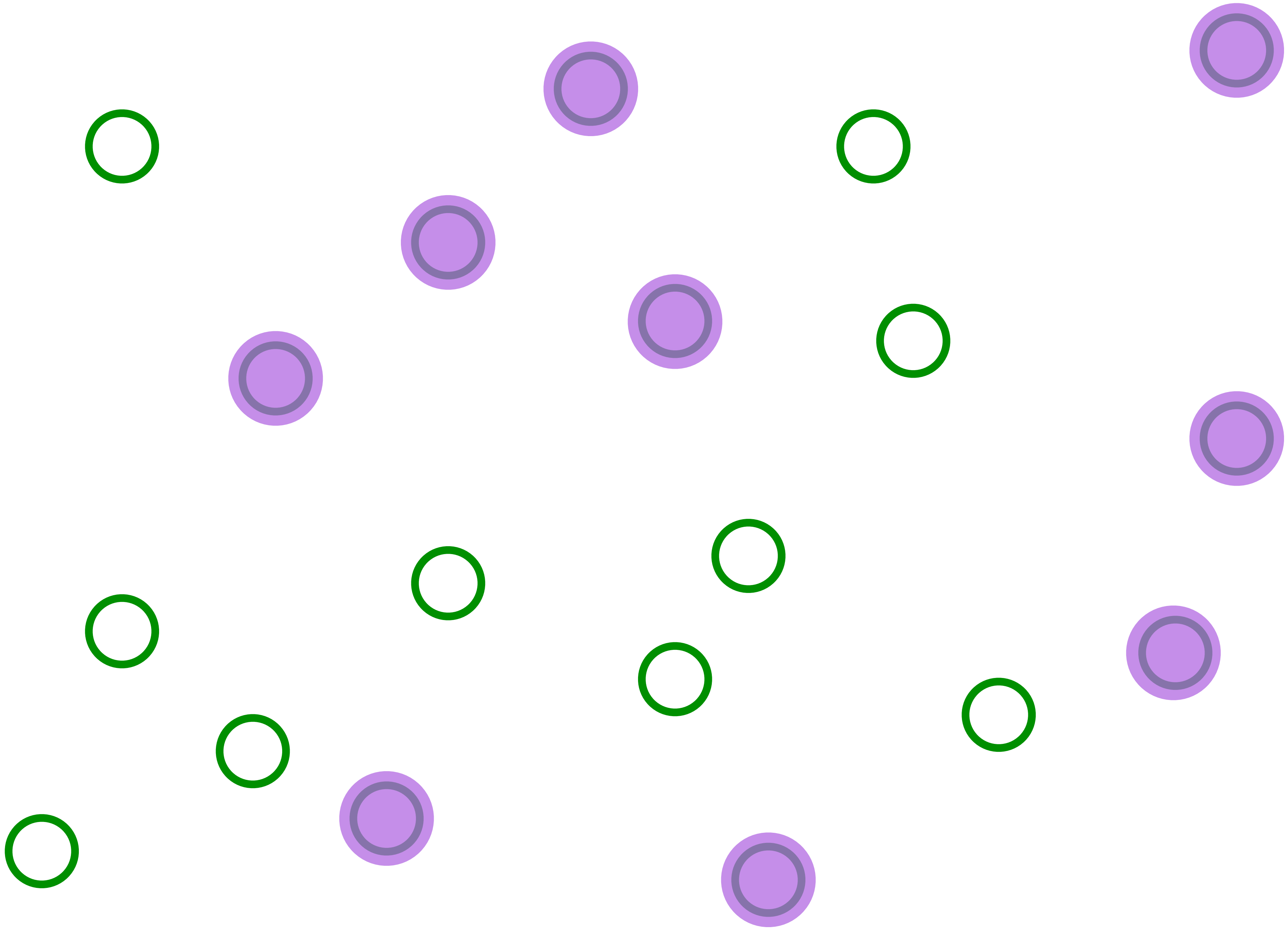
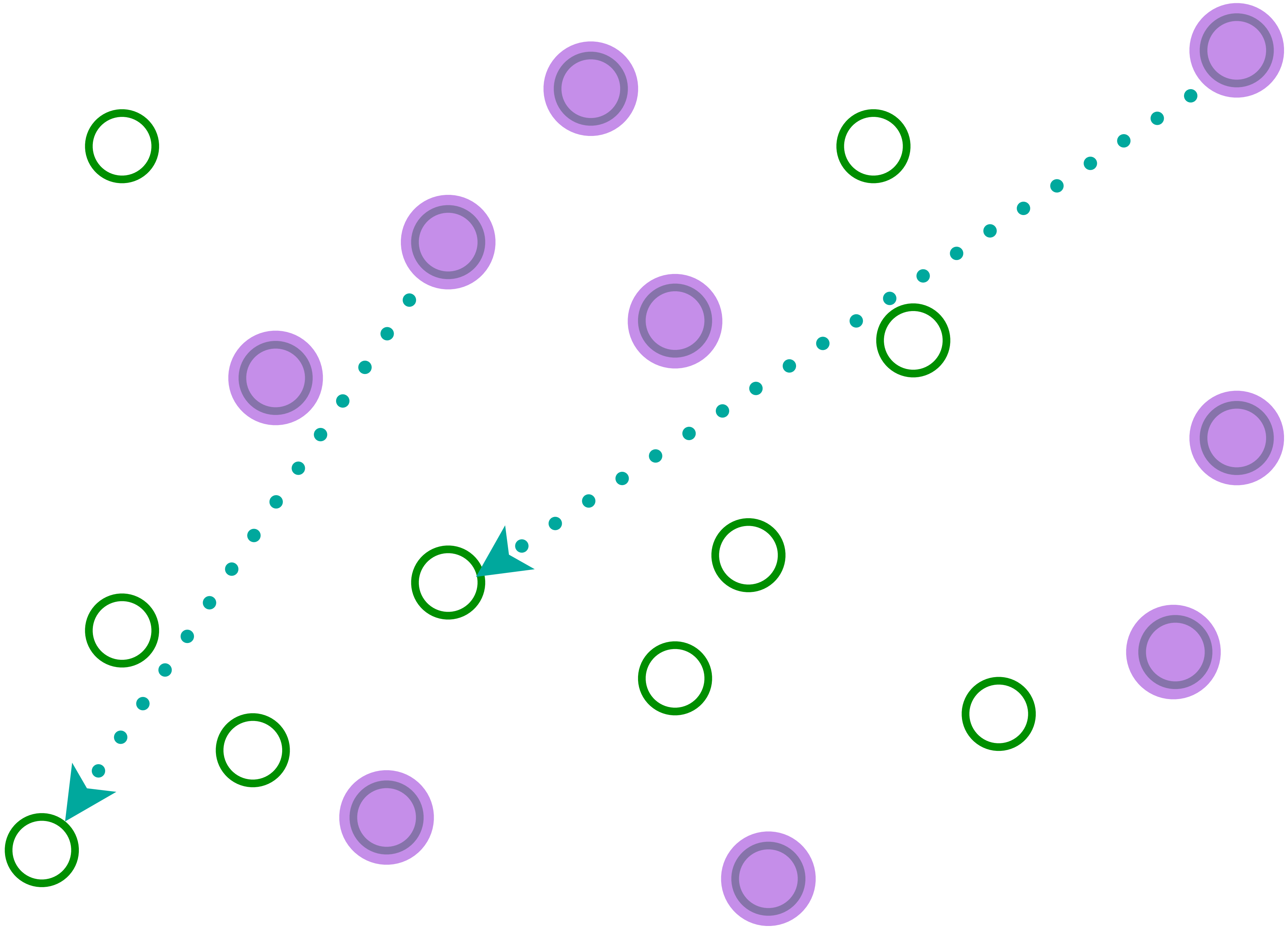


Entangle electrons pairwise randomly

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



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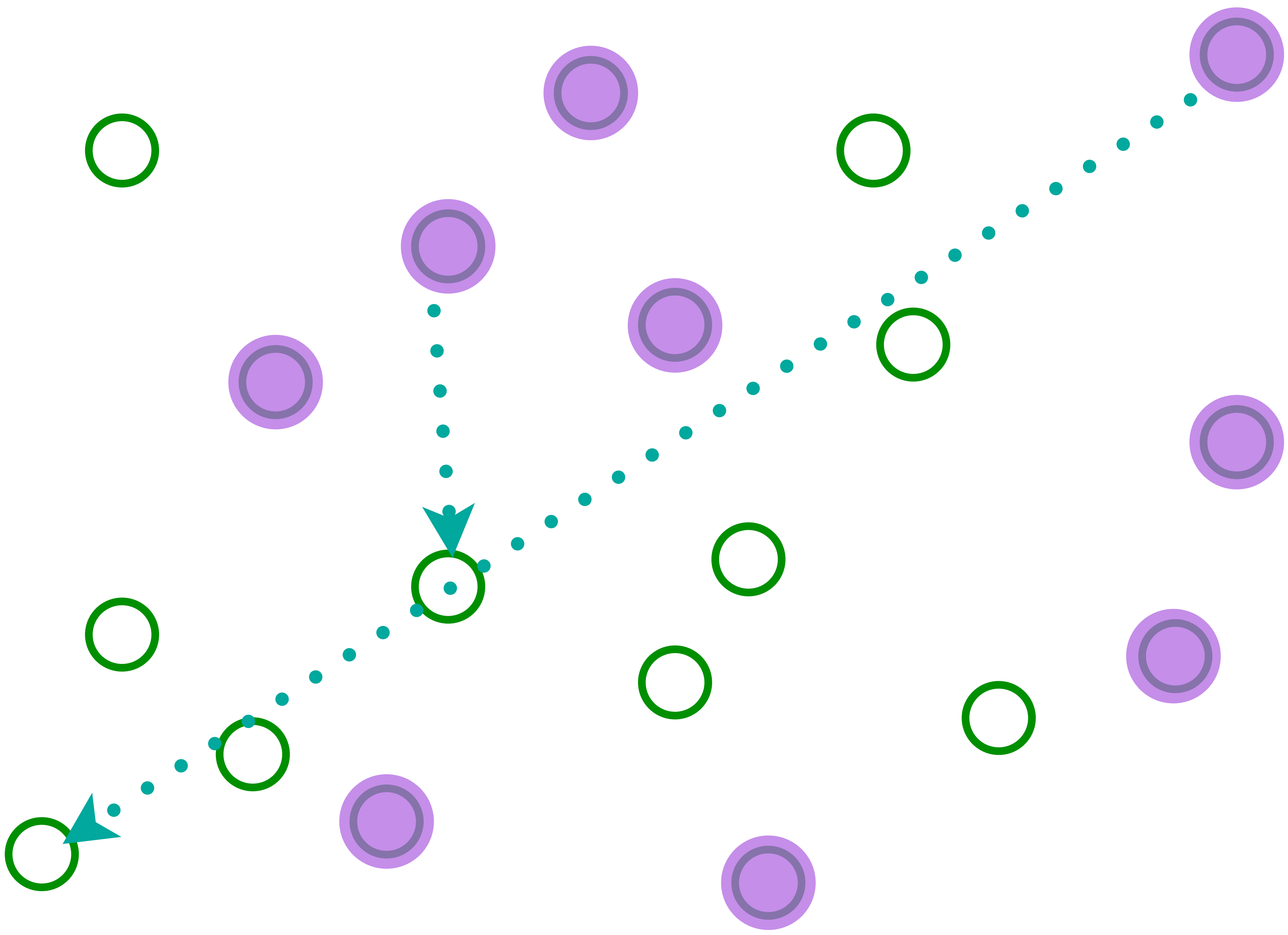

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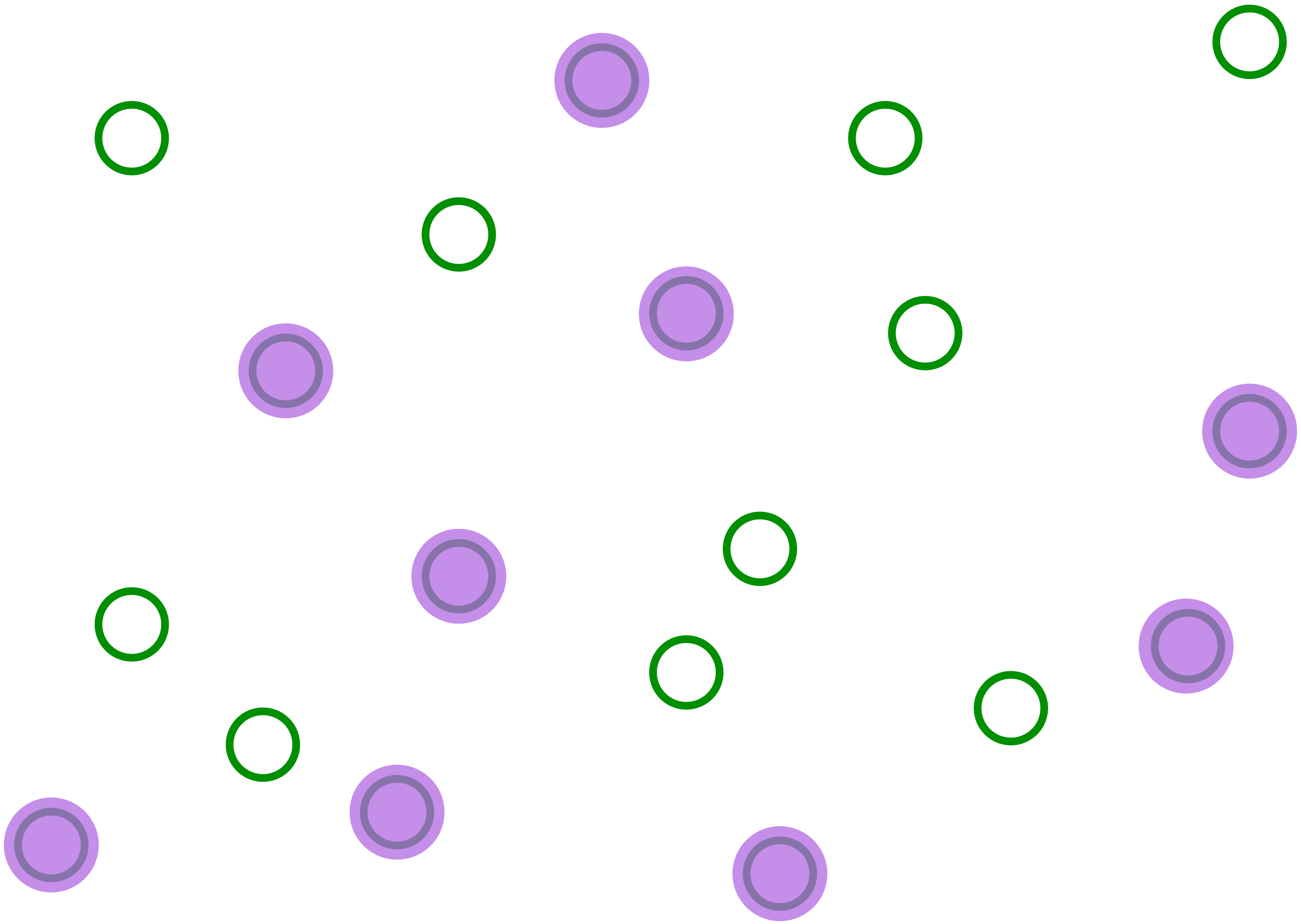

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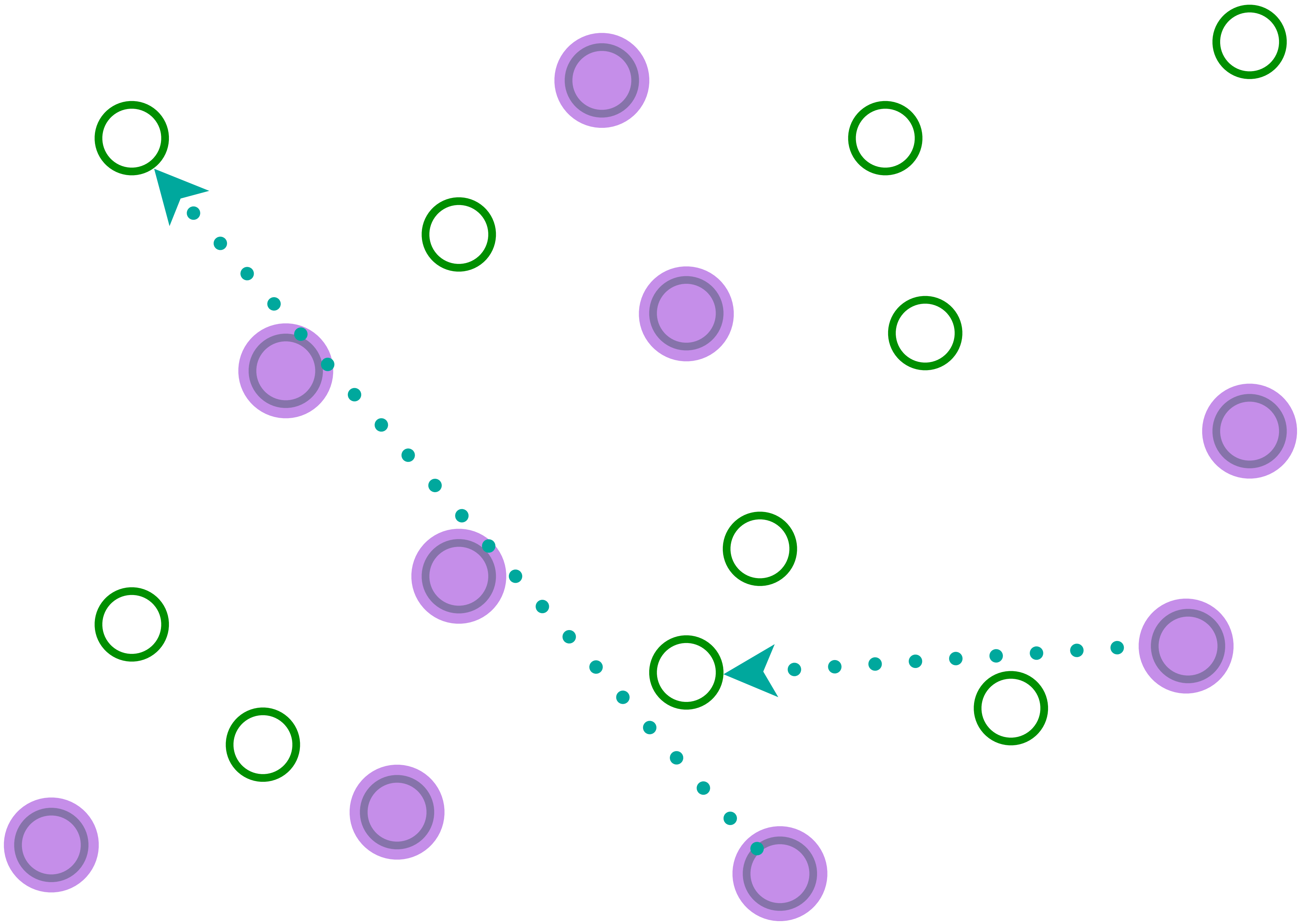

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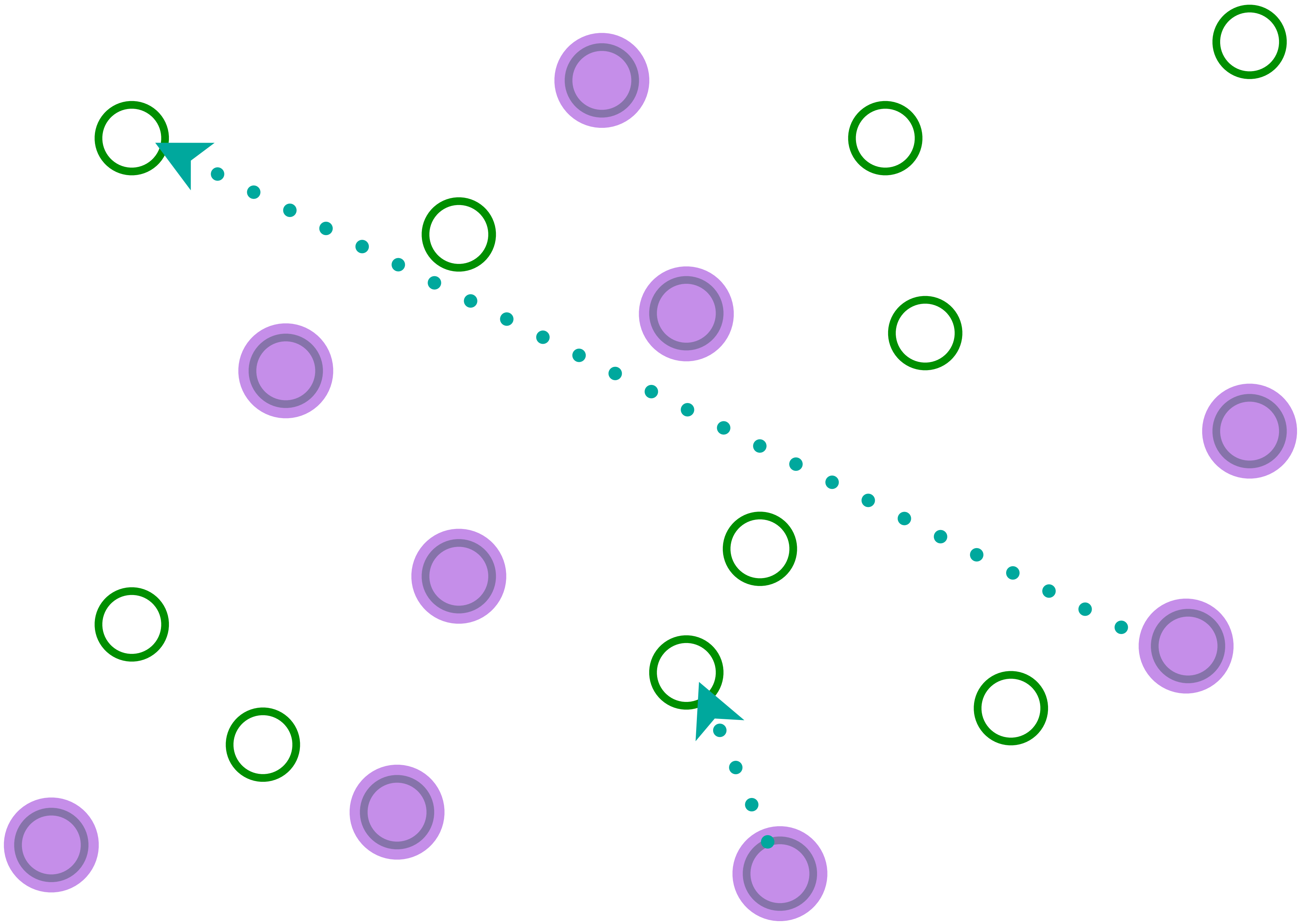

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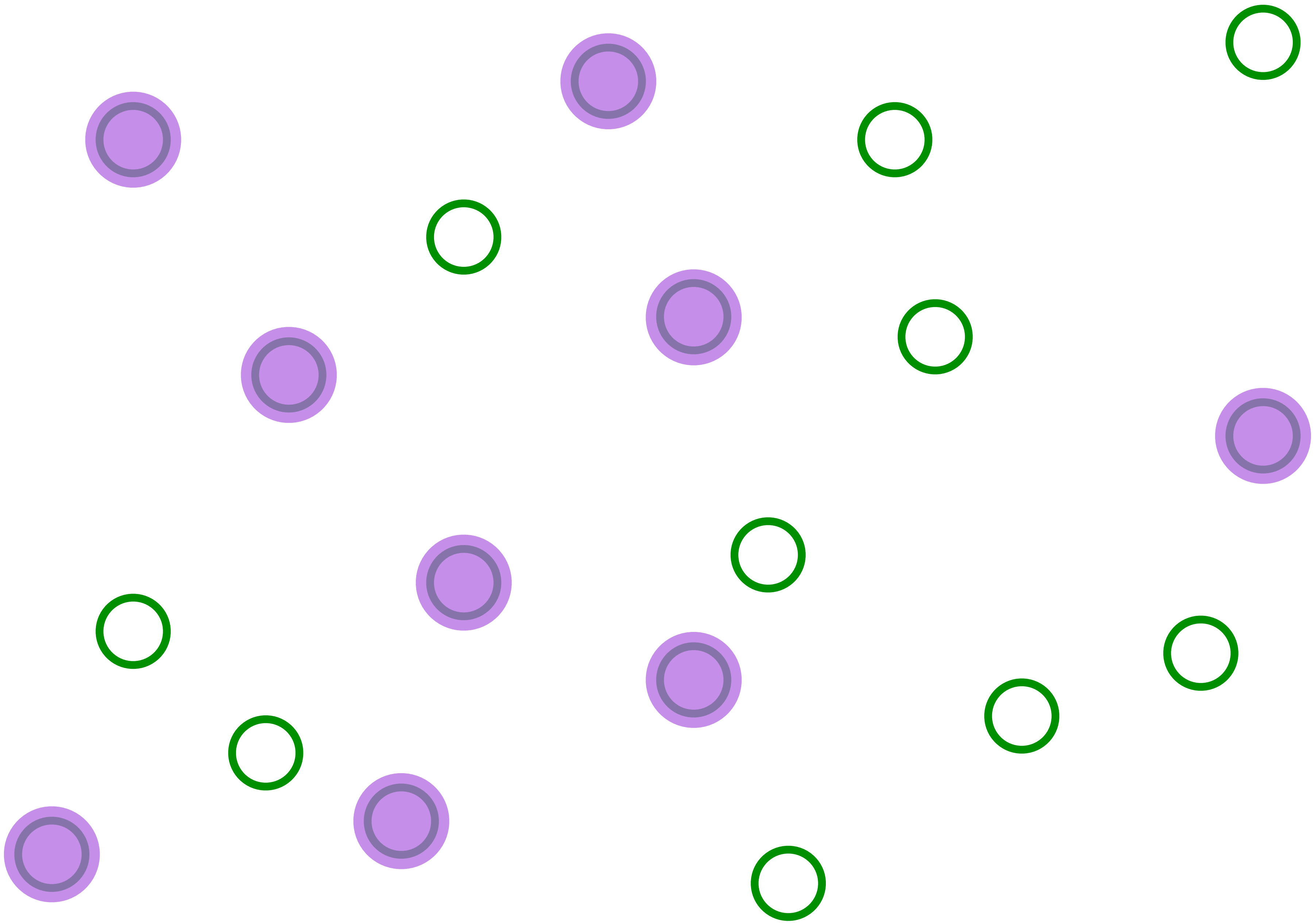

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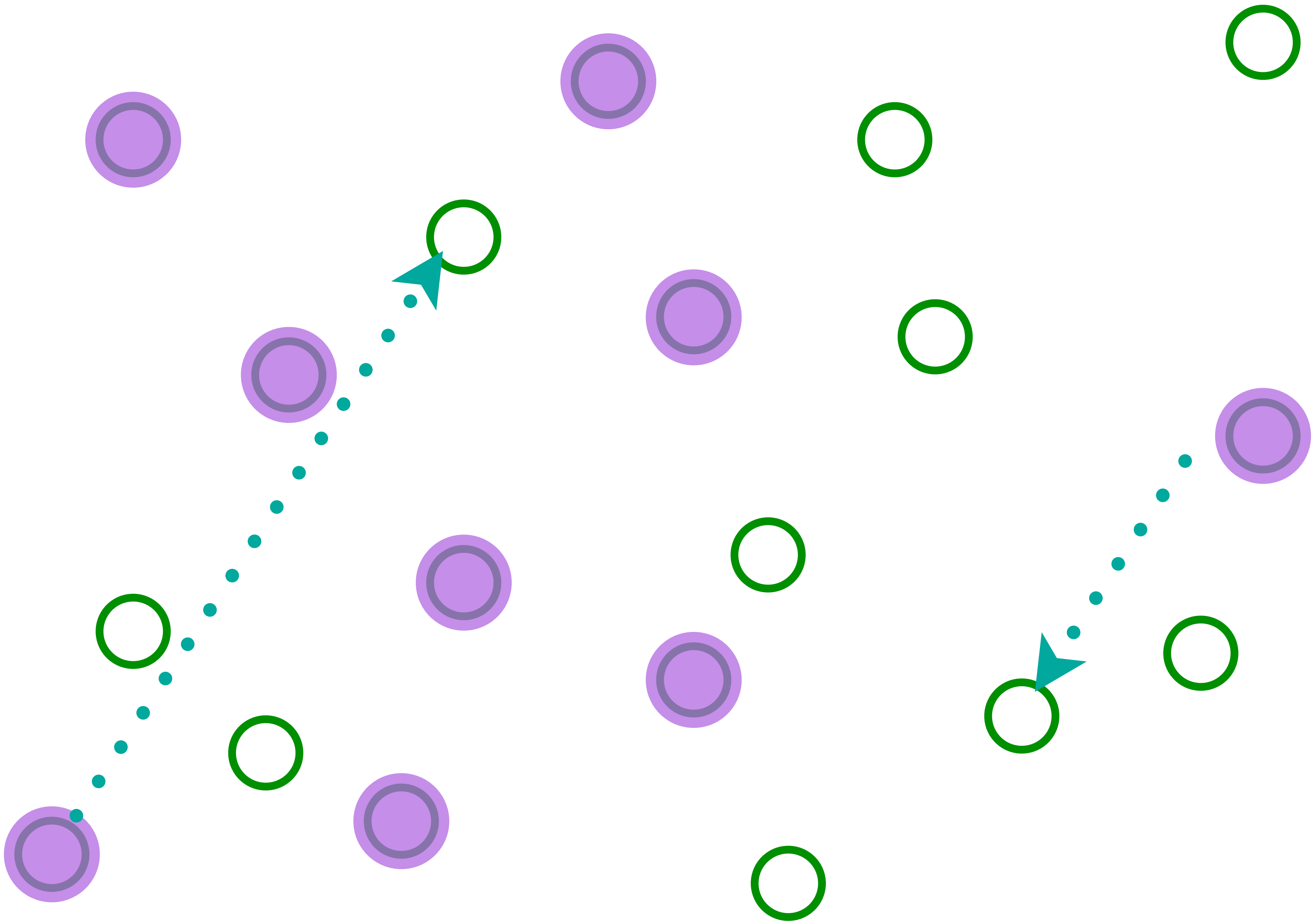

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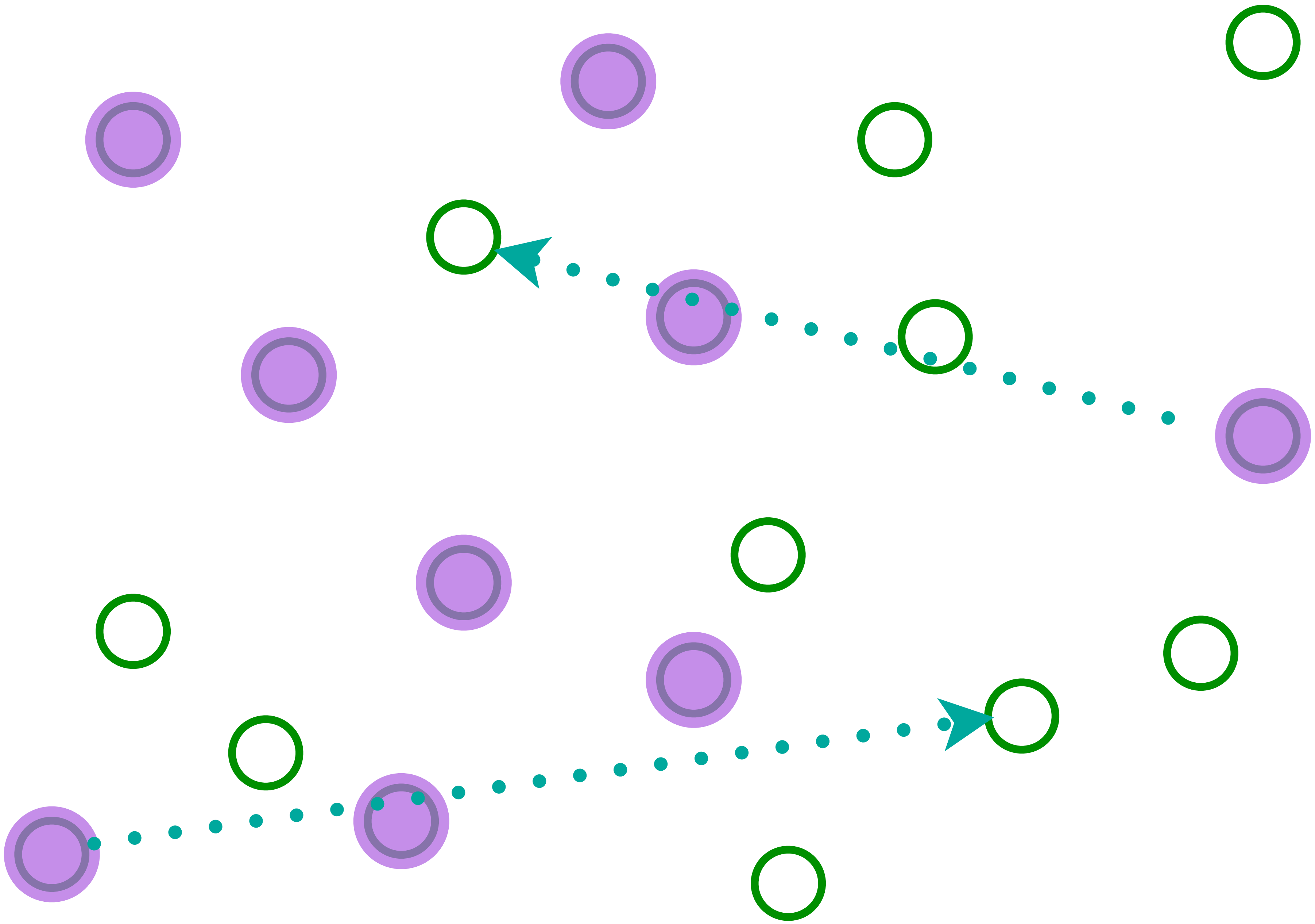

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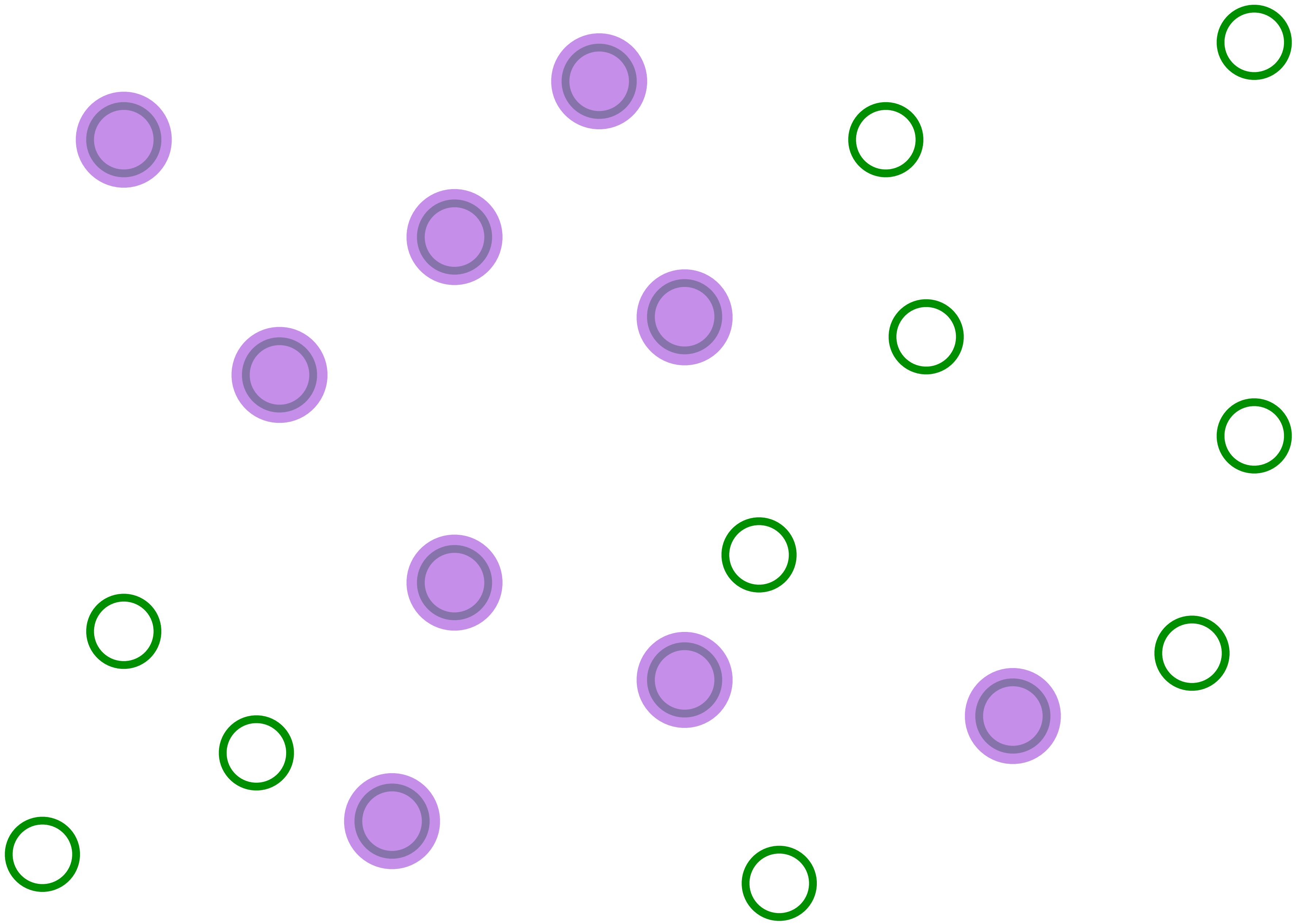

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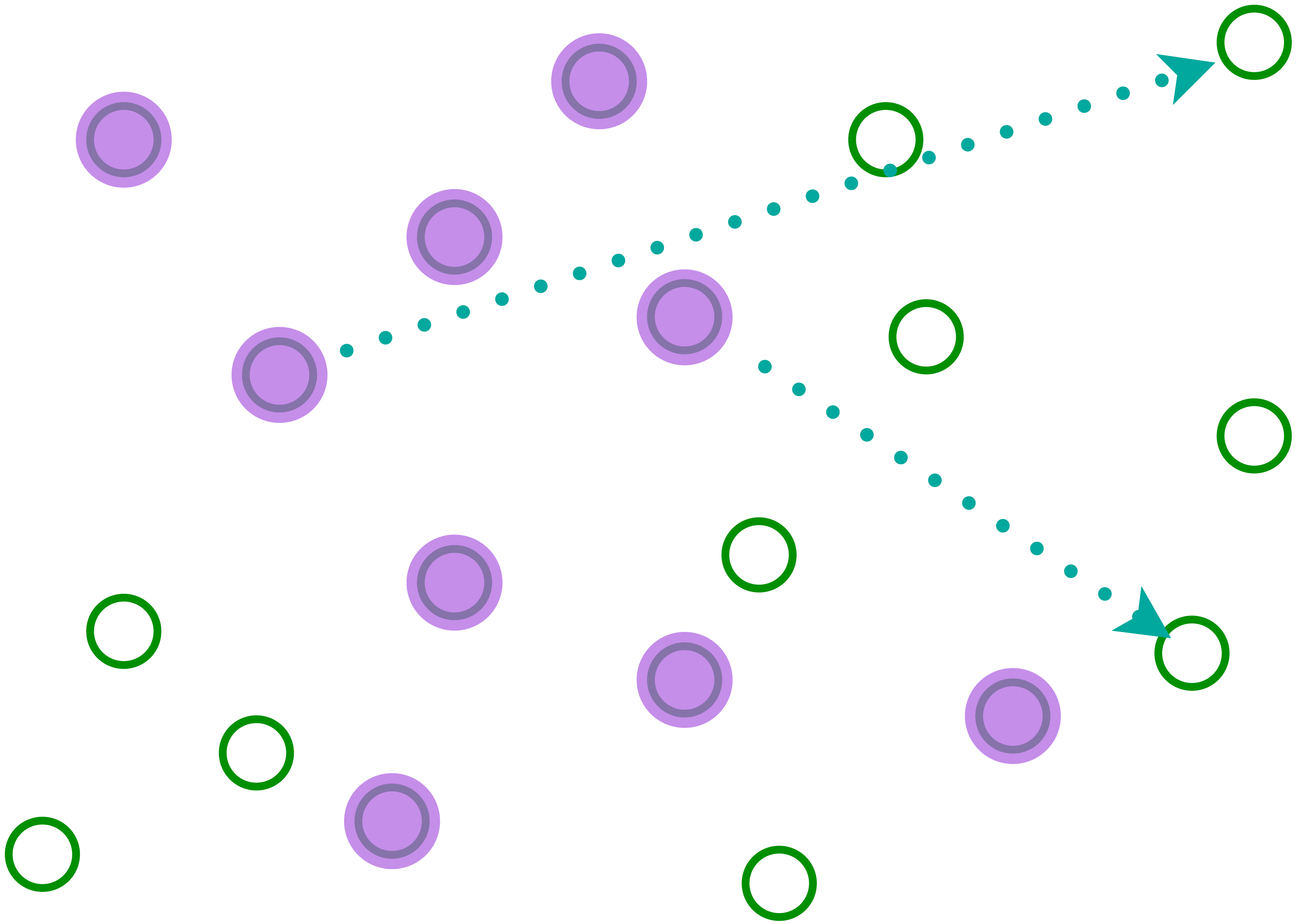

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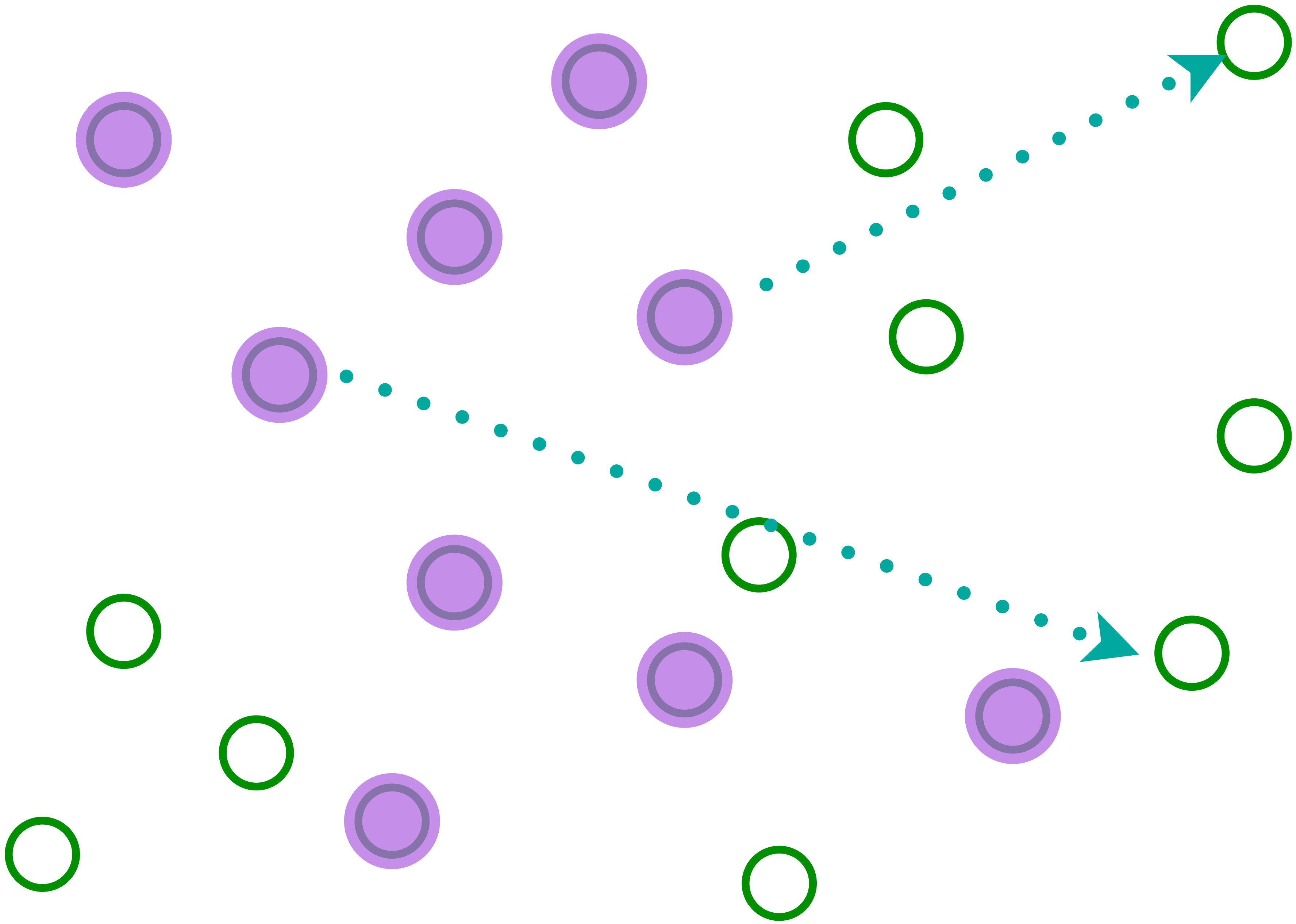

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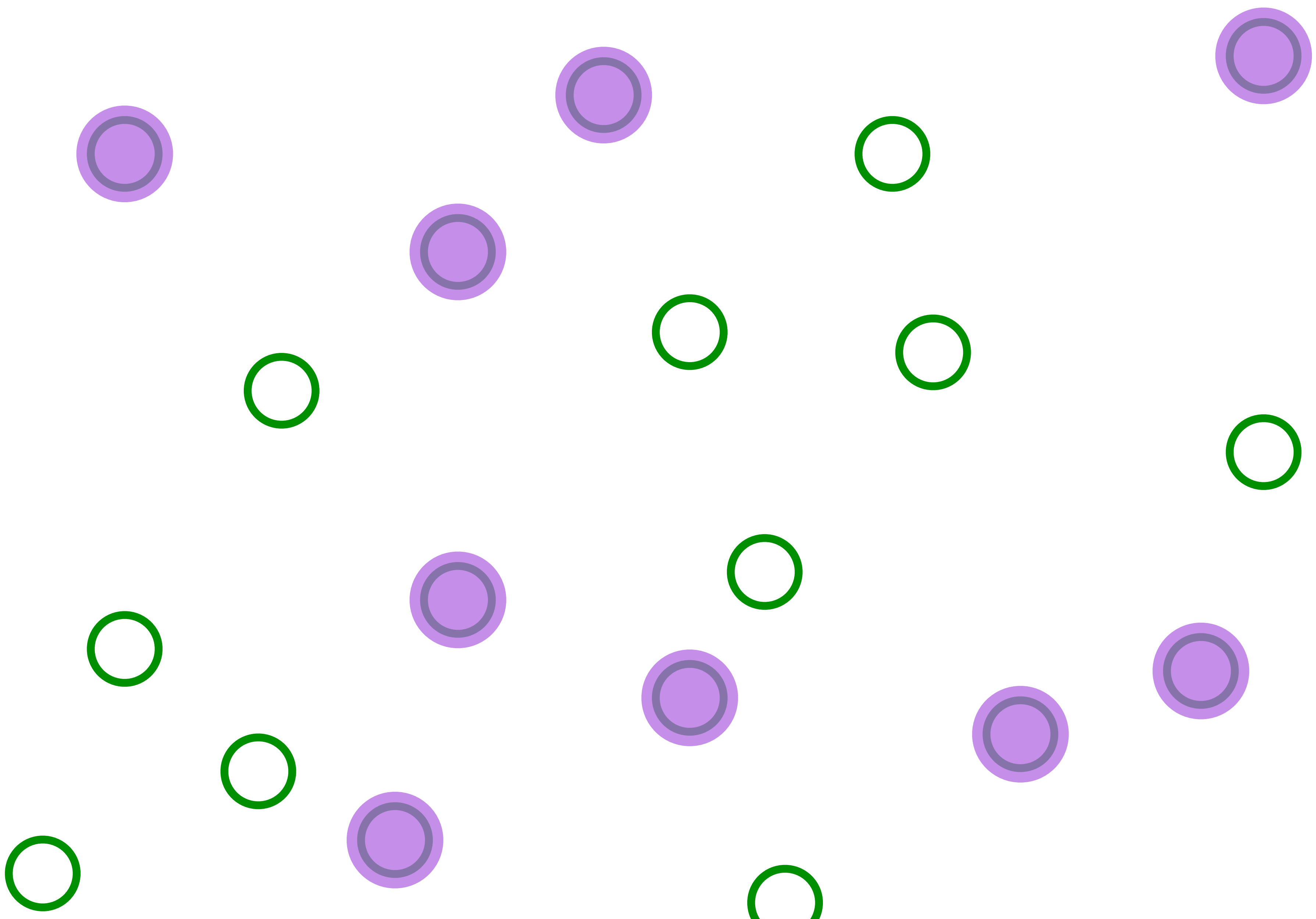

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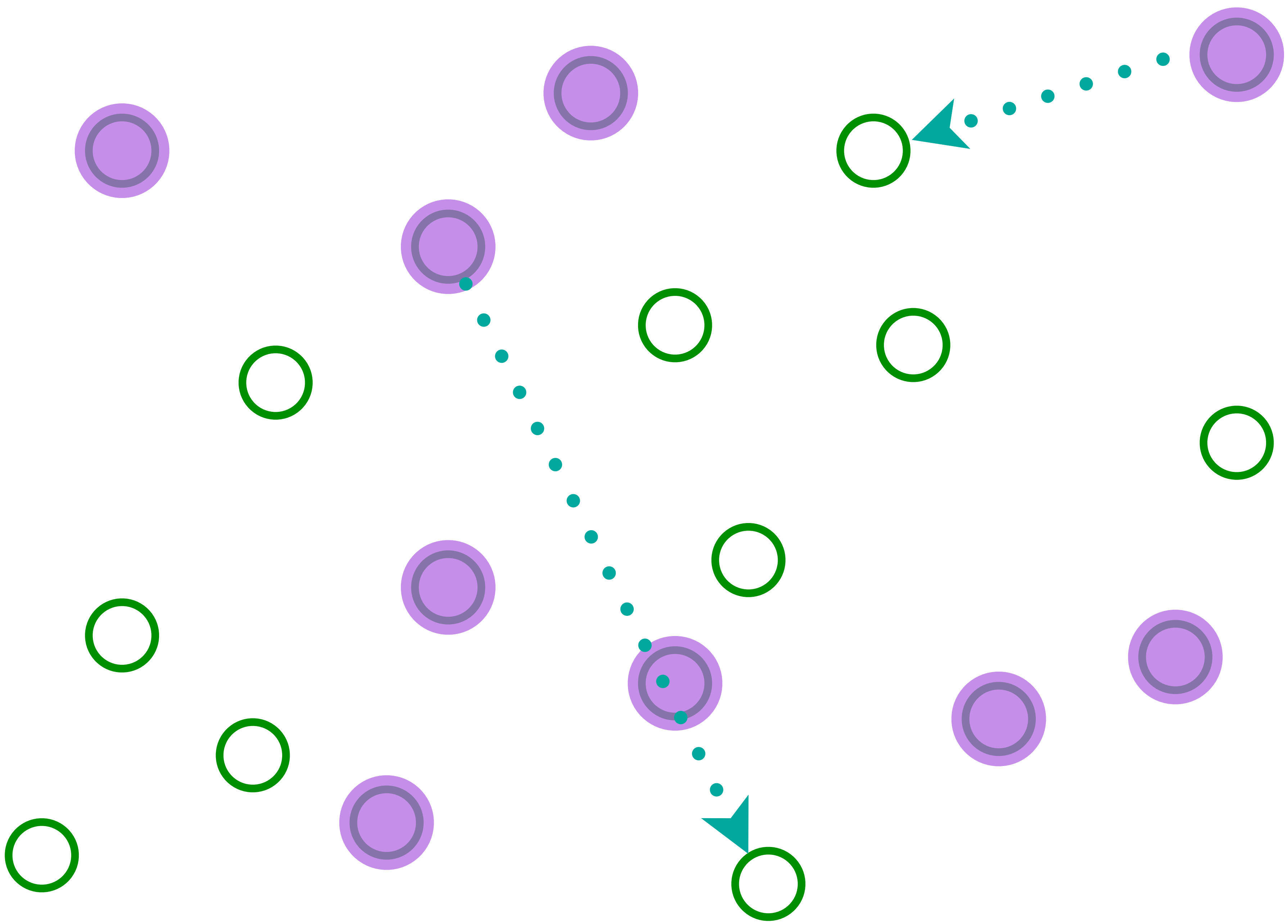

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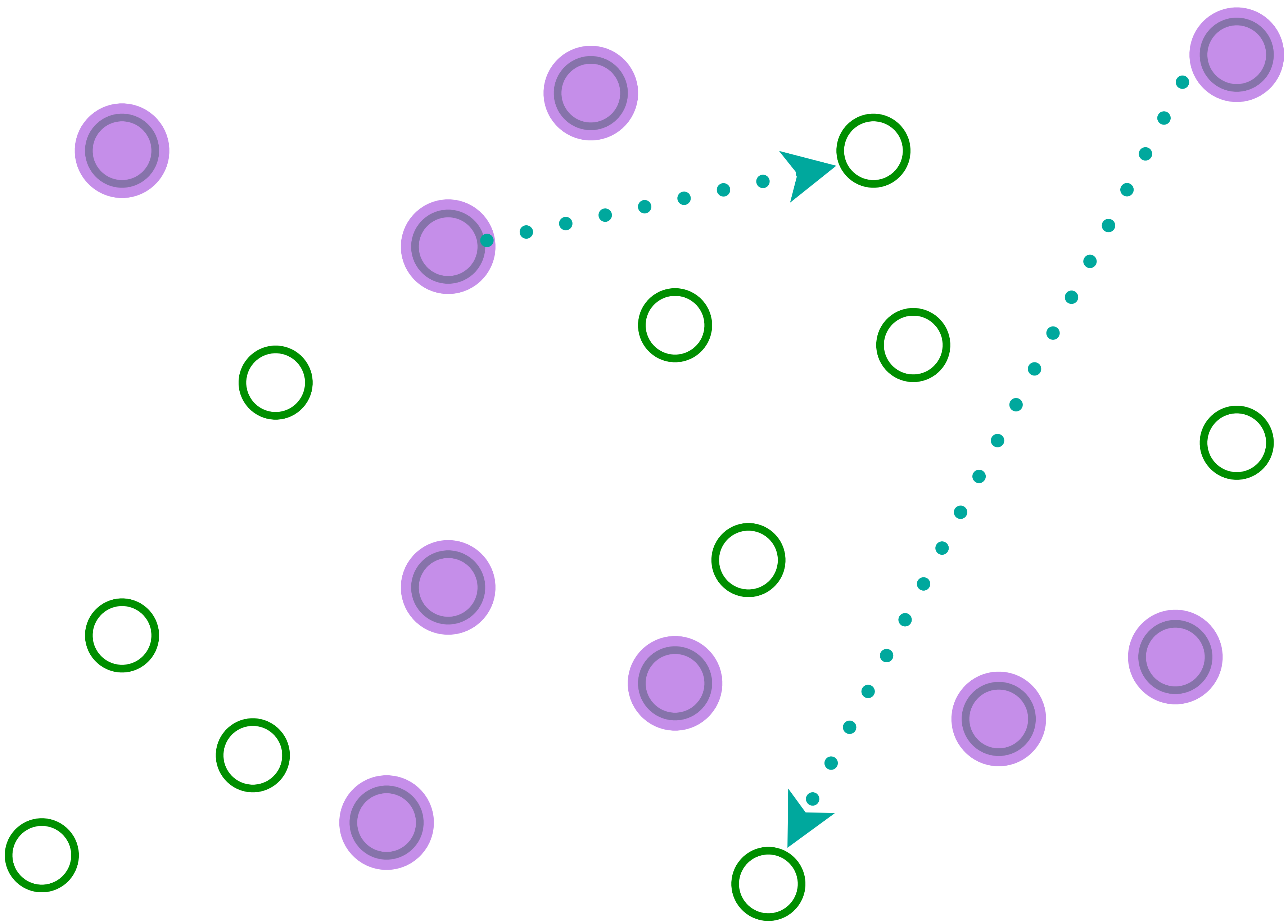

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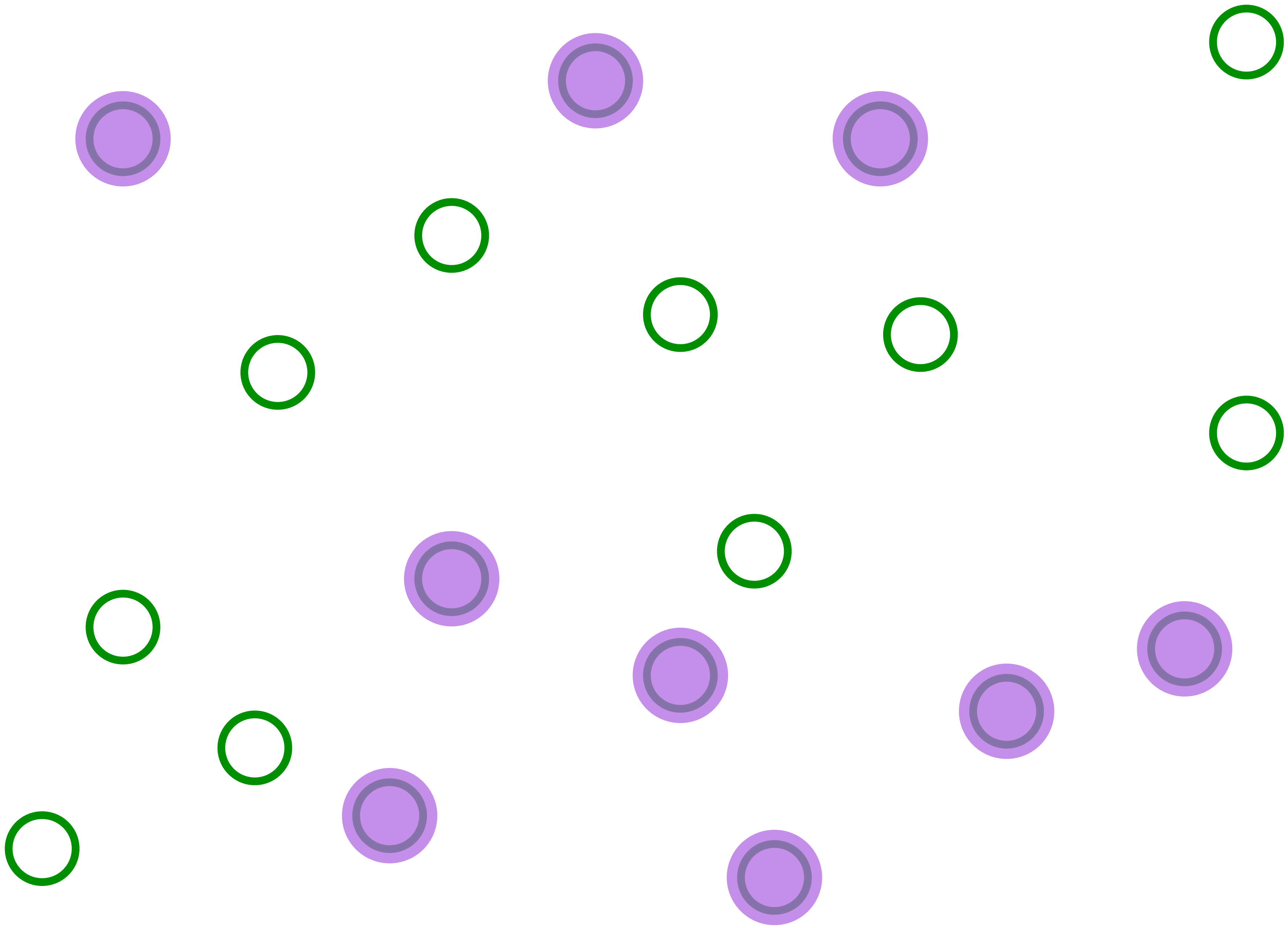


Entangle electrons pairwise randomly

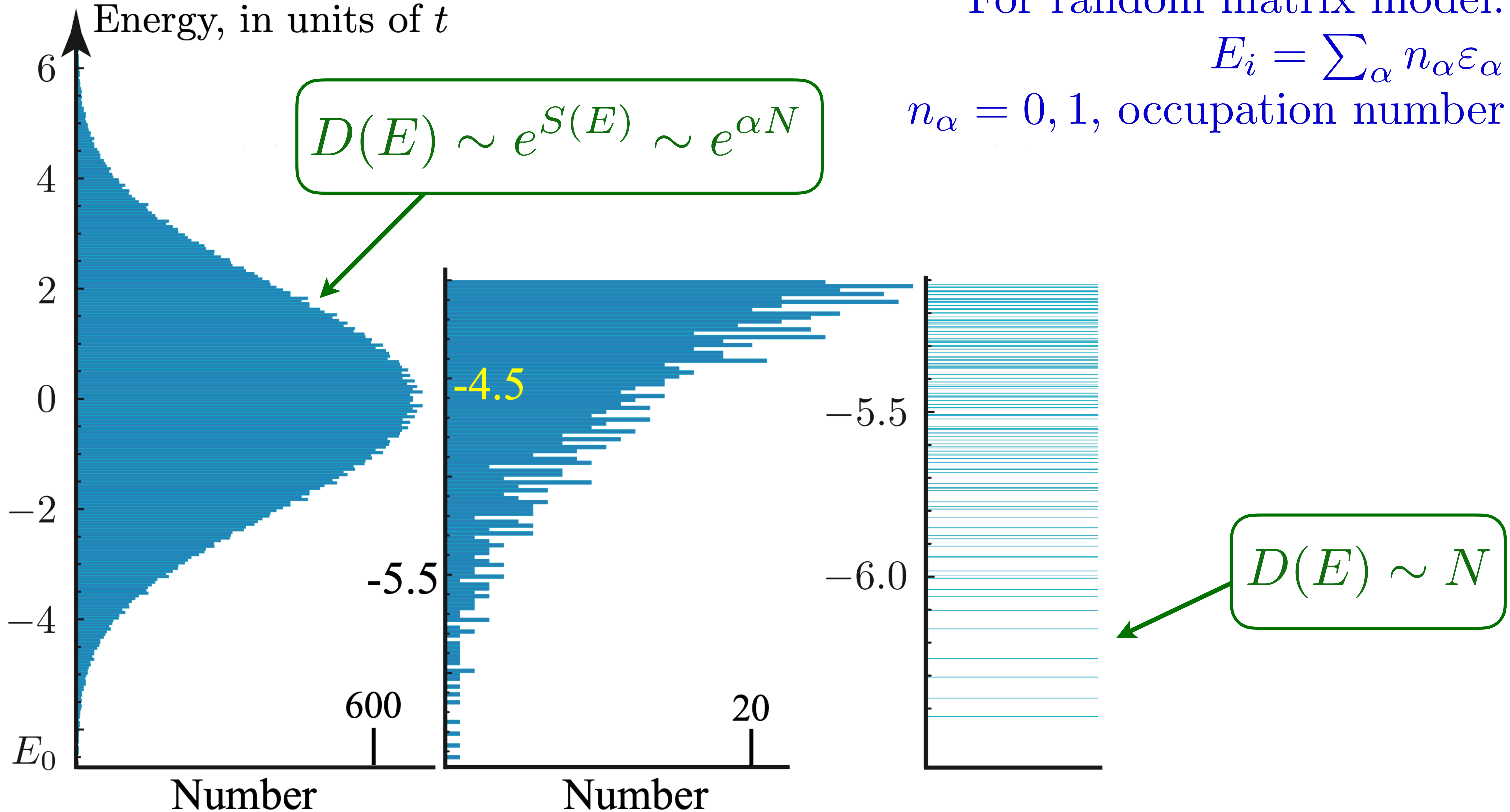
Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_i \Rightarrow \text{Many body eigenvalue}$$

For random matrix model:

$E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$

$n_\alpha = 0, 1$, occupation number



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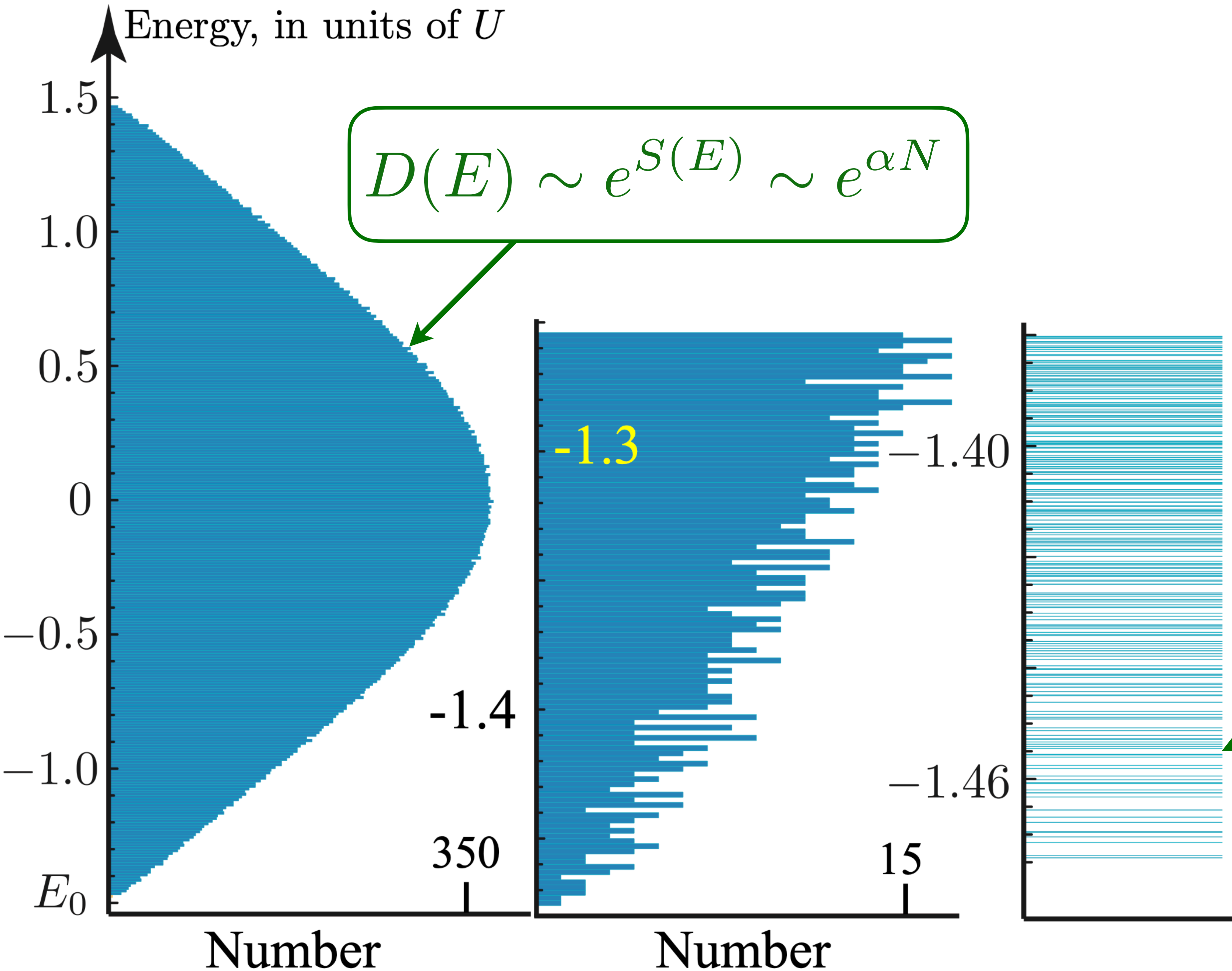


$$D(E) \sim e^{S(E)} \sim e^{\alpha N}$$

No quasiparticle decomposition of many-body states

$$D(E) \sim e^{S(E_0)} \sim e^{0.464848\ldots N}$$

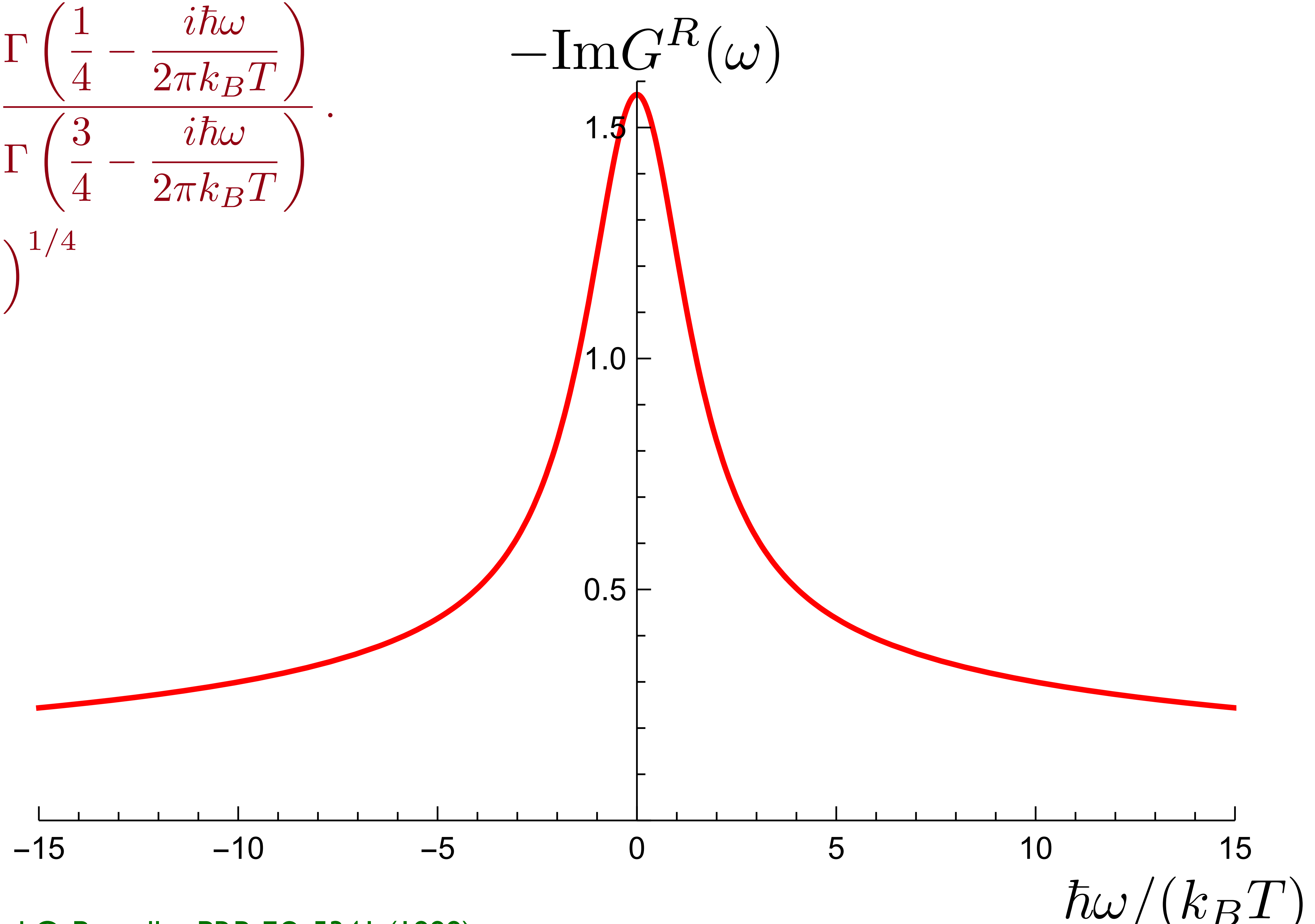
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Complex SYK model

The SYK model

$$G^R_{\rm SYK}(\omega) = \frac{-iC}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}.$$

$$C = \left(\frac{\pi}{U^2}\right)^{1/4}$$



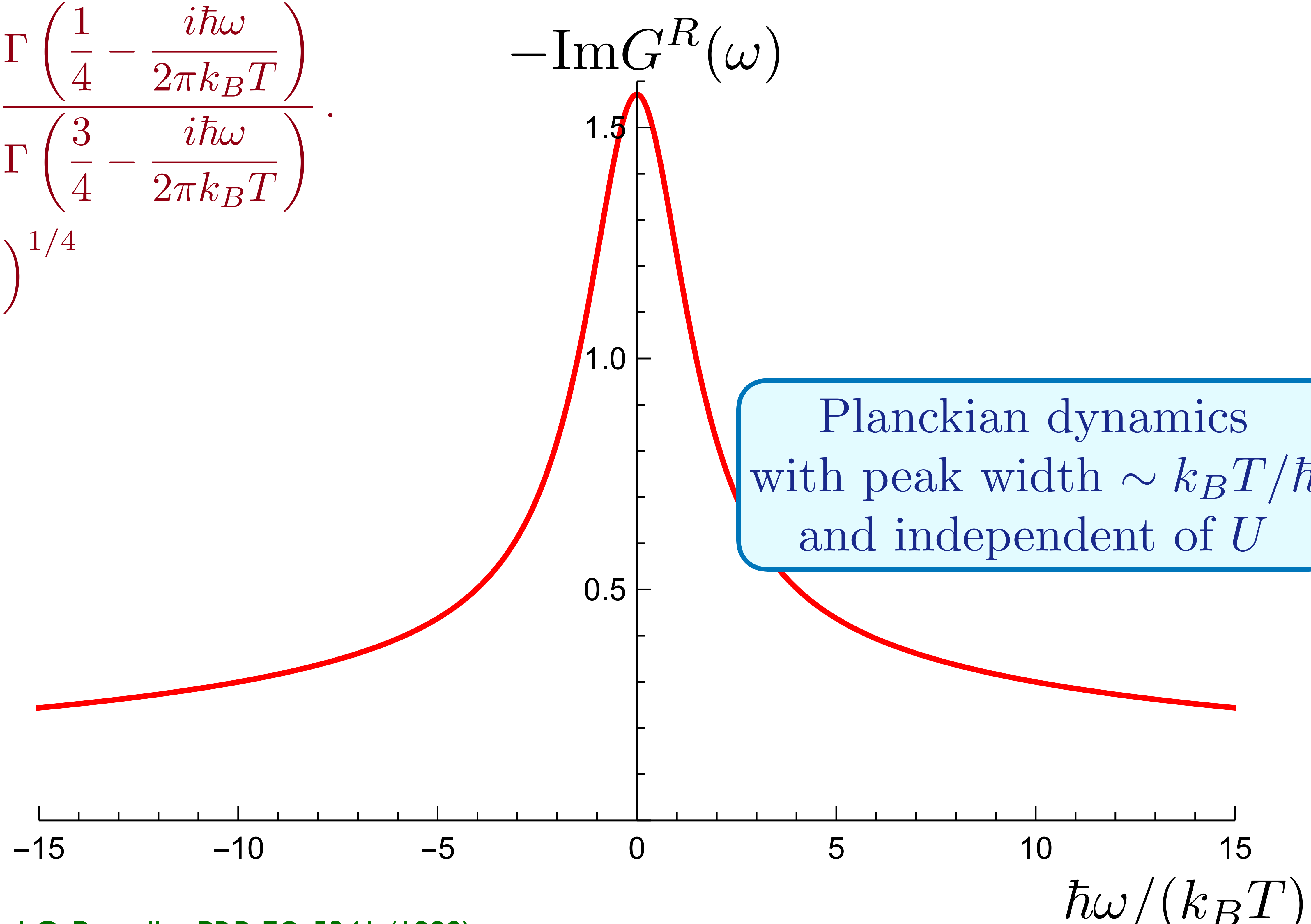
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

$$G^R_{\rm SYK}(\omega) =$$

$$\frac{-iC}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}\,.$$

$$C = \left(\frac{\pi}{U^2}\right)^{1/4}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

1. Random matrix and SYK models

2. Time reparameterization soft mode

3. Random t-J model

4. Charged black holes

Time reparameterization symmetry and 2D gravity

After introducing replicas $a = 1 \ldots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{ia}(\tau) \exp\left[-\sum_{ia}\int_0^\beta d\tau\, c_{ia}^\dagger \left(\frac{\partial}{\partial\tau} - \mu\right) c_{ia}\right.$$
$$\left. - \frac{U^2}{4N^3}\sum_{ab}\int_0^\beta d\tau d\tau' \left|\sum_i c_{ia}^\dagger(\tau) c_{ib}(\tau')\right|^4\right] .$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}G(\tau_1,\tau_2)\mathcal{D}\Sigma(\tau_1,\tau_2) \exp\left[-N\int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1,\tau_2)\left(G(\tau_2,\tau_1)\right.\right.$$
$$\left.\left. + \frac{1}{N}\sum_i c_i(\tau_2) c_i^\dagger(\tau_1)\right)\right] .$$

Time reparameterization symmetry and 2D gravity

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln\det\left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

Time reparameterization symmetry and 2D gravity

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$$+ \int d\tau_1 d\tau_2 \left[\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

Time reparameterization symmetry and 2D gravity

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4}G_s(f(\tau_1) - f(\tau_2))e^{i\phi(\tau_1)-i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-E_0/T+NS(E_0)-NS_{\text{eff}}[f,\phi]},$$

where $E_0 \propto N$ is the ground state energy.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Time reparameterization symmetry and 2D gravity

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\rm eff}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau \, \{\tan(\pi T f(\tau)), \tau\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\rm eff}\left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

The same effective action is obtained for the boundary graviton of 2D gravity on AdS_2.

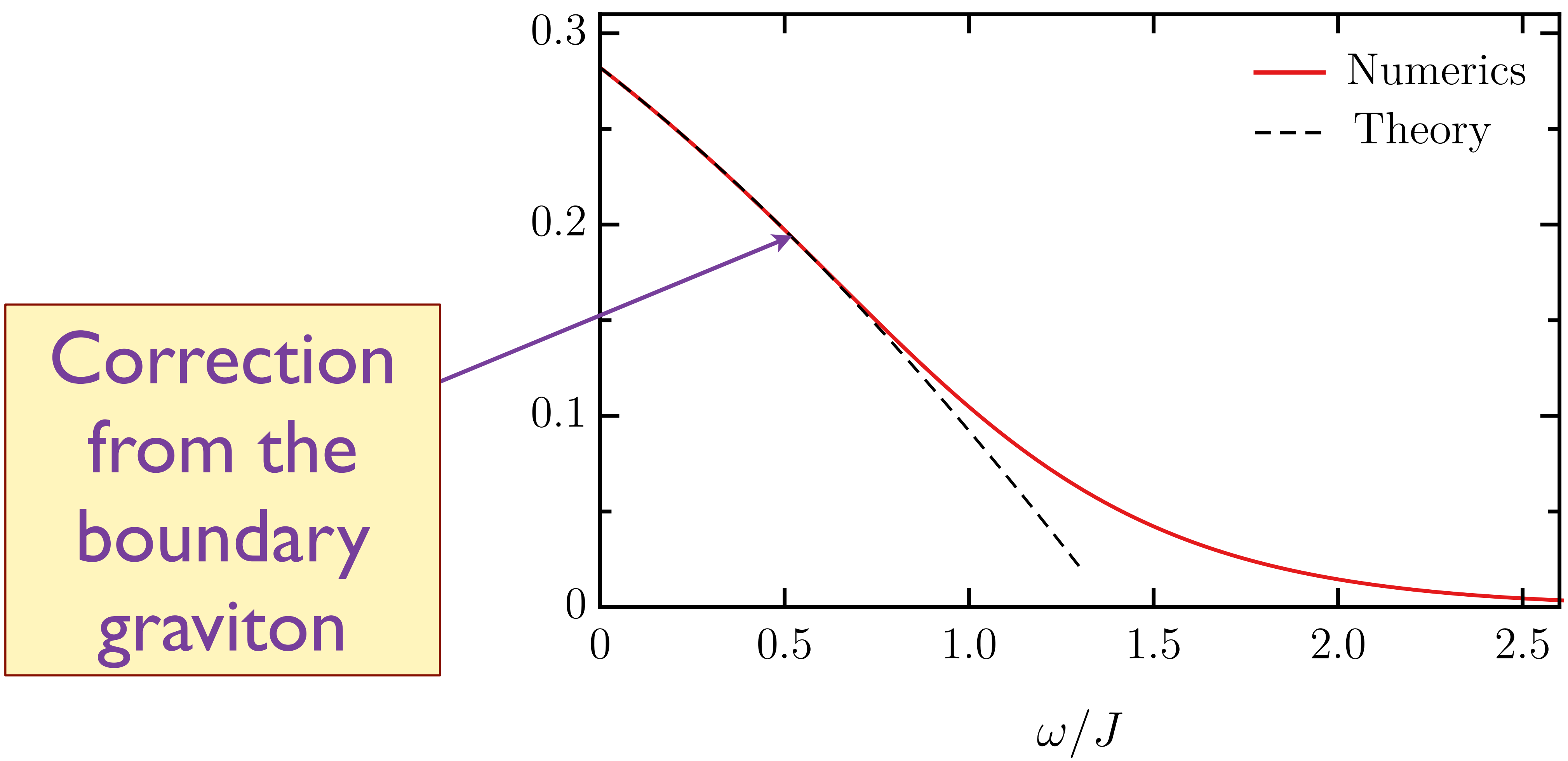
J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0| S_{+i} |n\rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\text{Im}\chi_L(\omega) \sim \text{sgn}(\omega)\Big[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots\Big]$$

Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory.
$\mathcal{C}$ is the co-efficient of the action for the 'boundary graviton' in holographic dual.



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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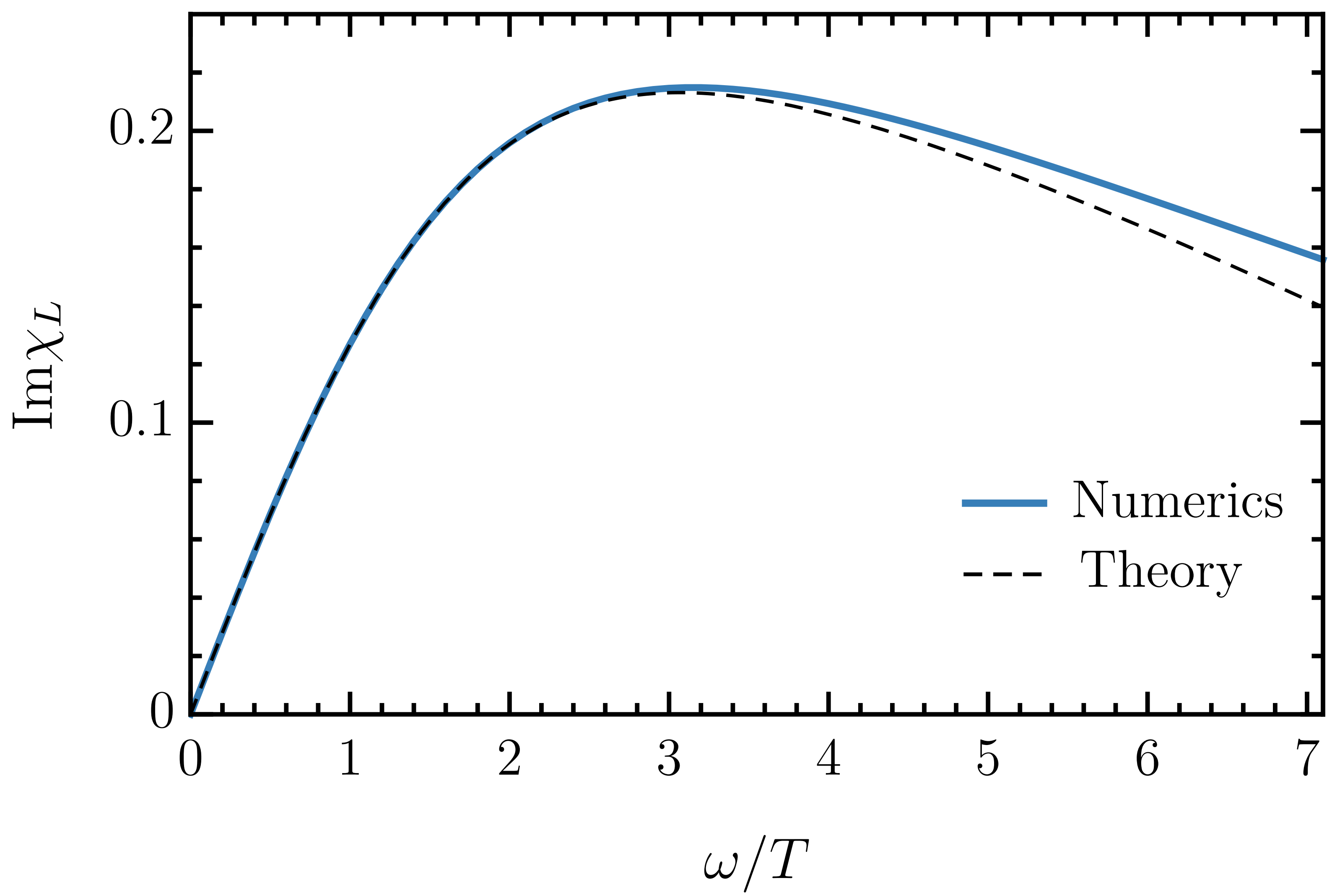

Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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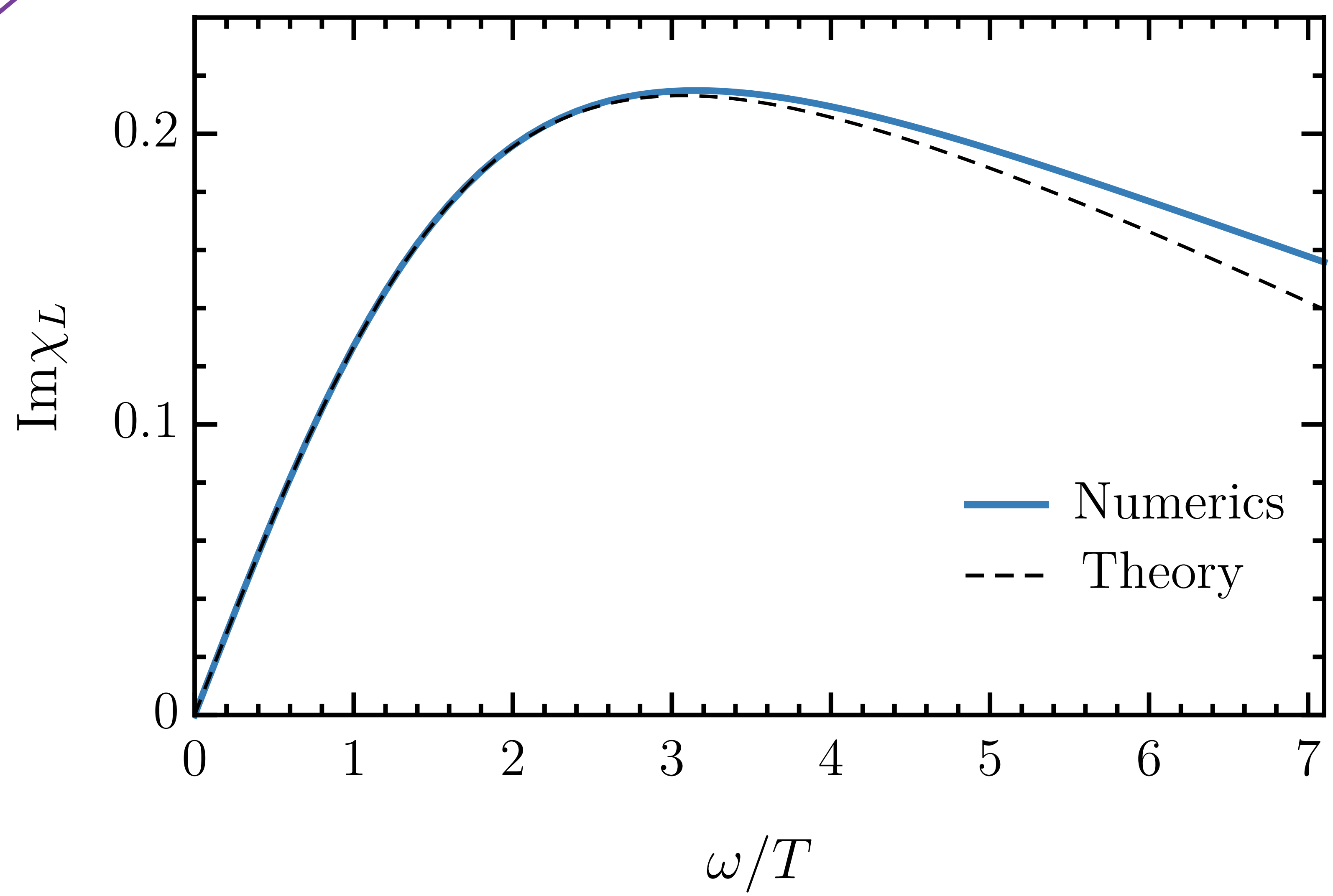
Conformally (SL(2,R)) invariant result with characteristic dissipative time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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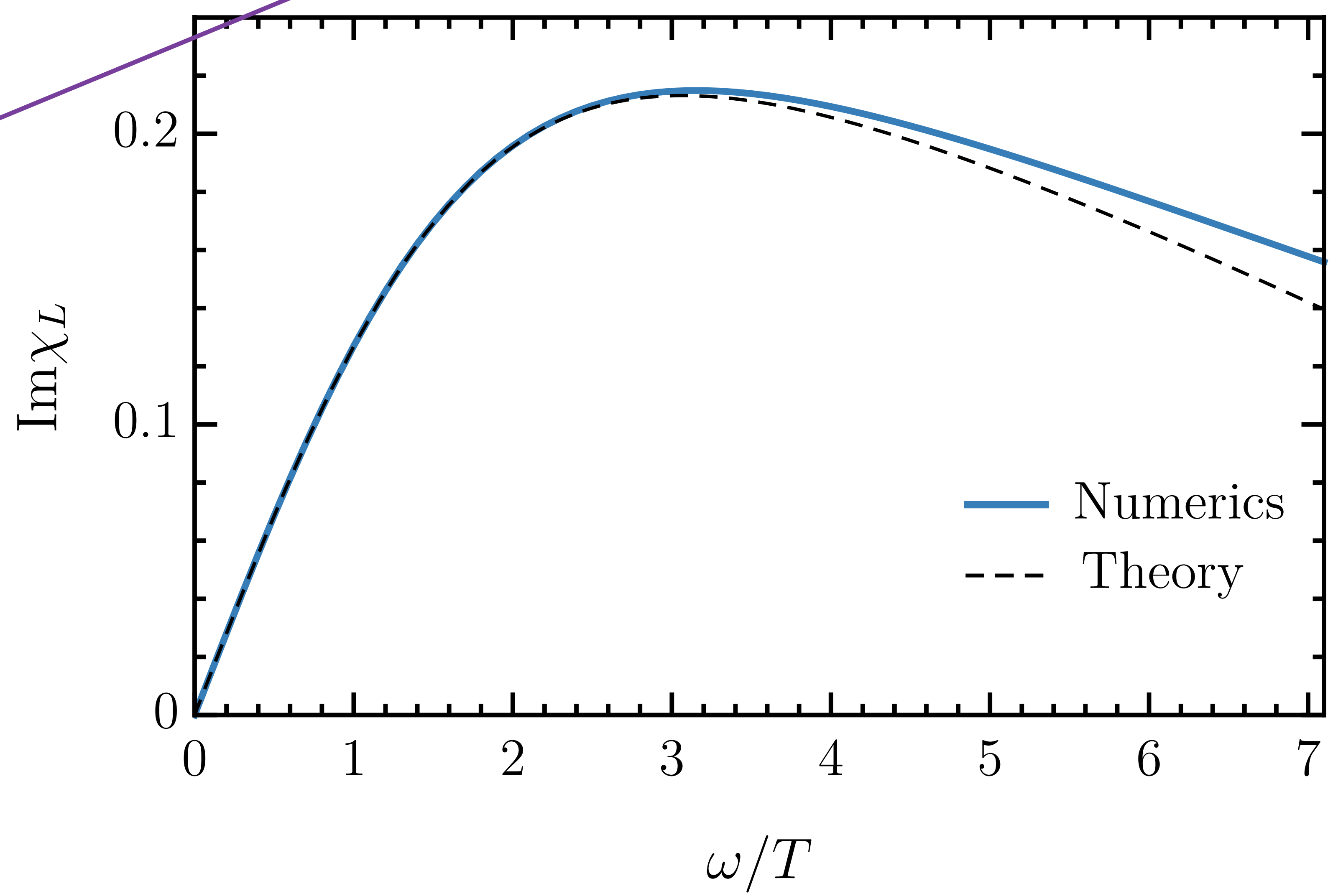
Correction from the boundary graviton



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

1. Random matrix and SYK models
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4. Charged black holes

Henry Shackleton

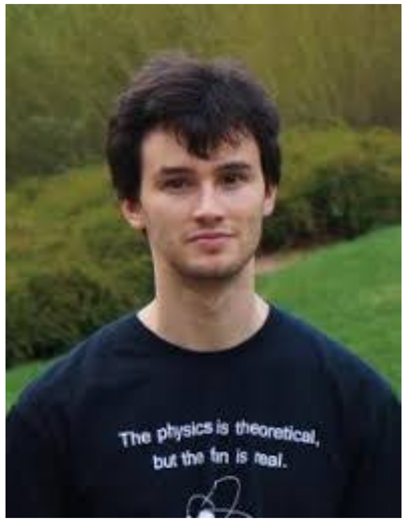
arXiv:2012.06589

Alexander Wietek

Antoine Georges

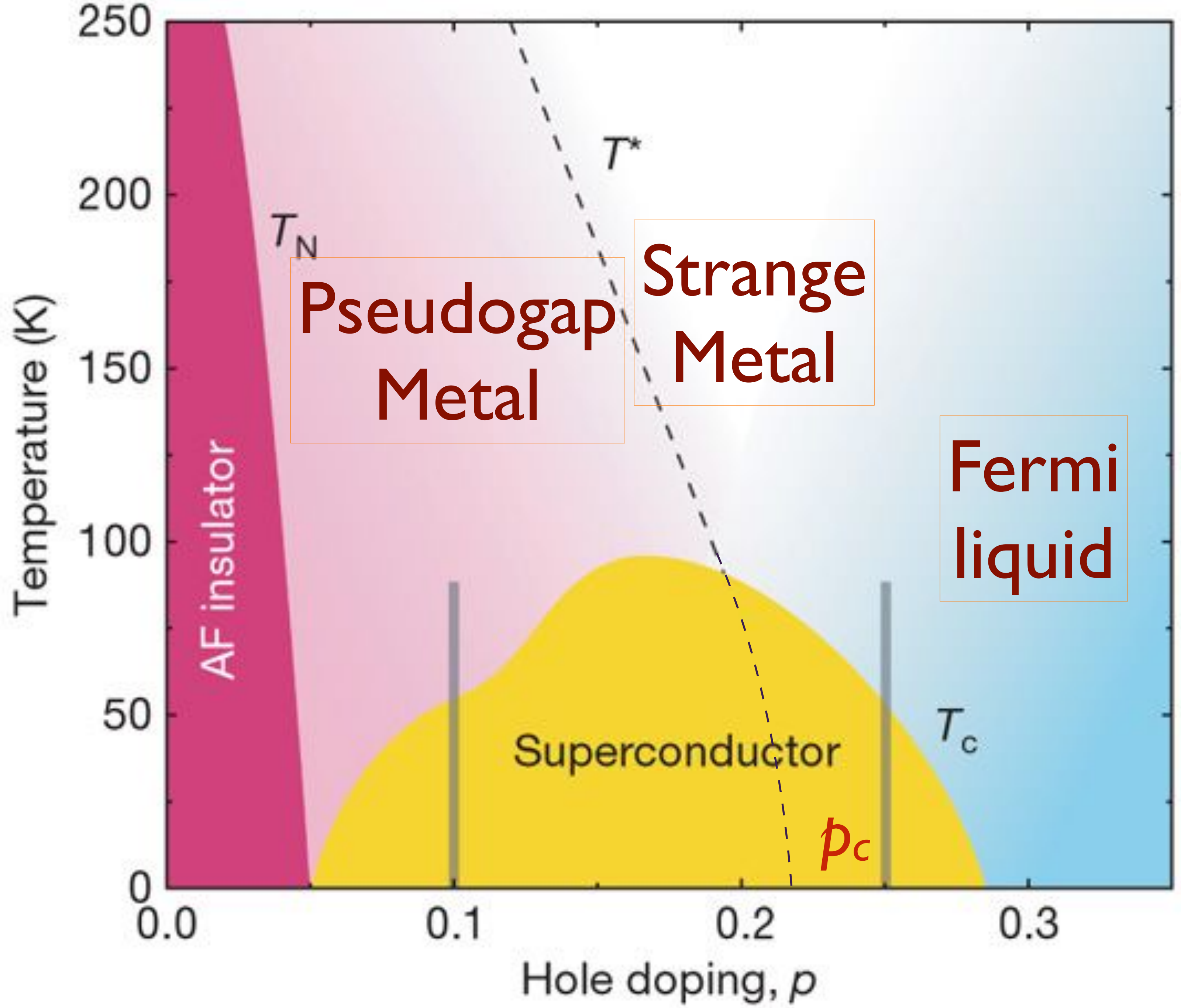
Maria Tikhanovskaya

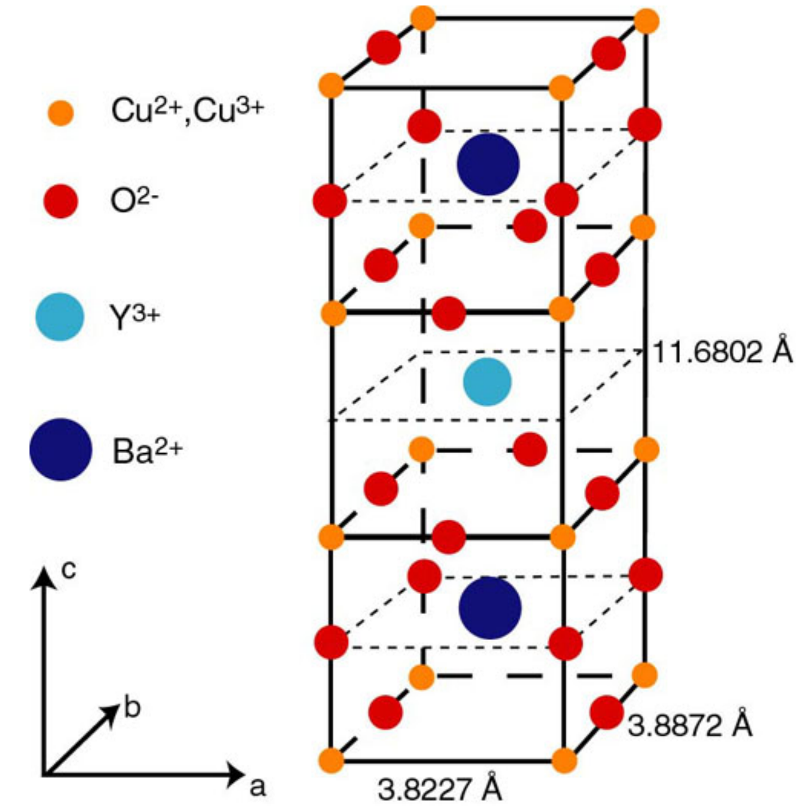
Haoyu Guo



Grigory Tarnopolsky

arXiv:2010.09742
arXiv:2012.14449

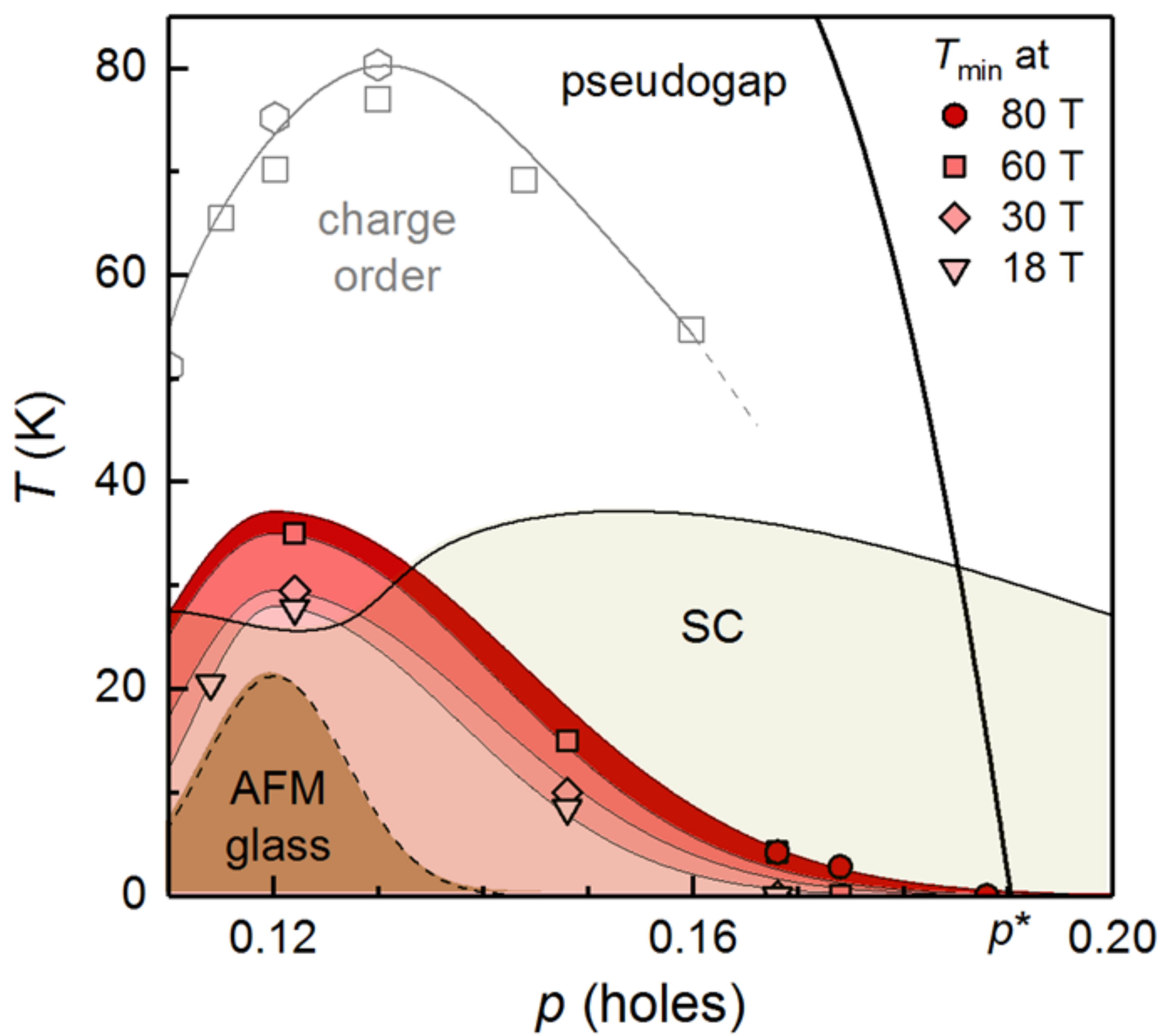

250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics **16**, 1064 (2020)

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



Random *t-J* model

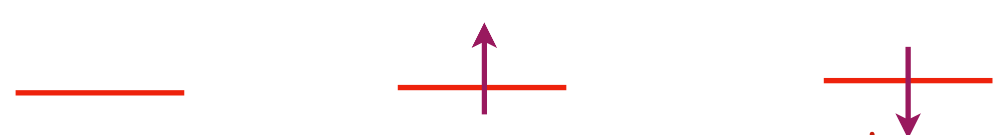
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N}\sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$



$|0\rangle$ $\quad c_\uparrow^\dagger |0\rangle$ $\quad c_\downarrow^\dagger |0\rangle$

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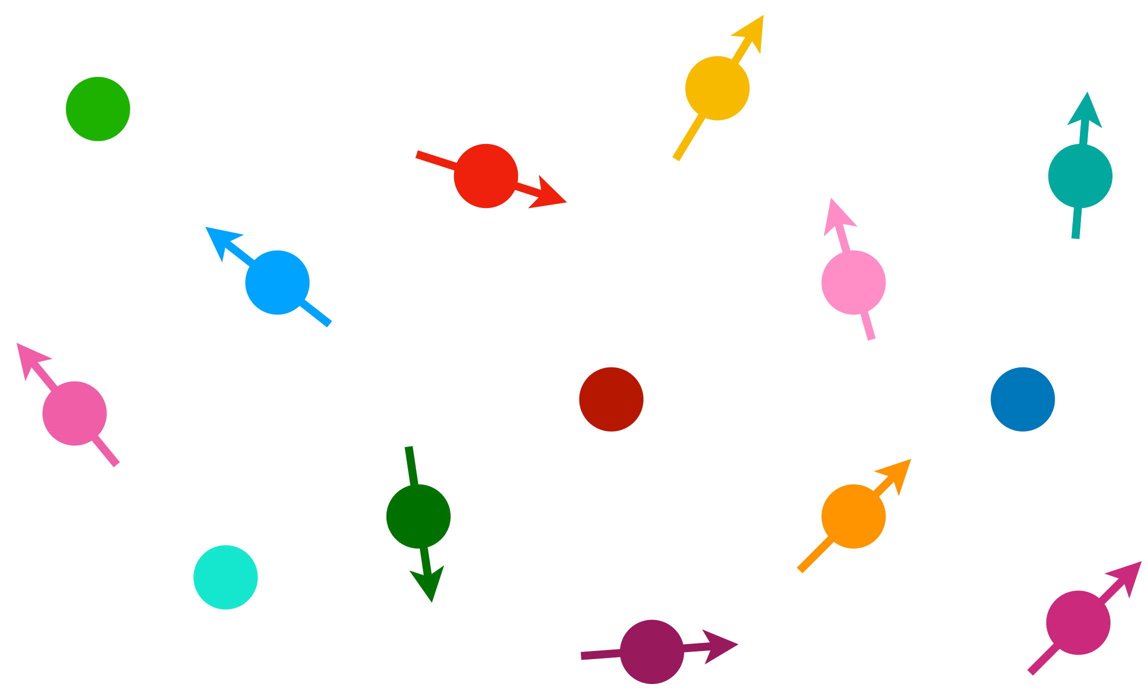

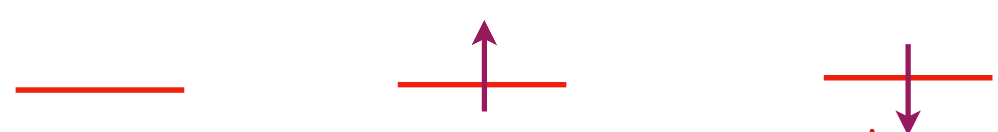

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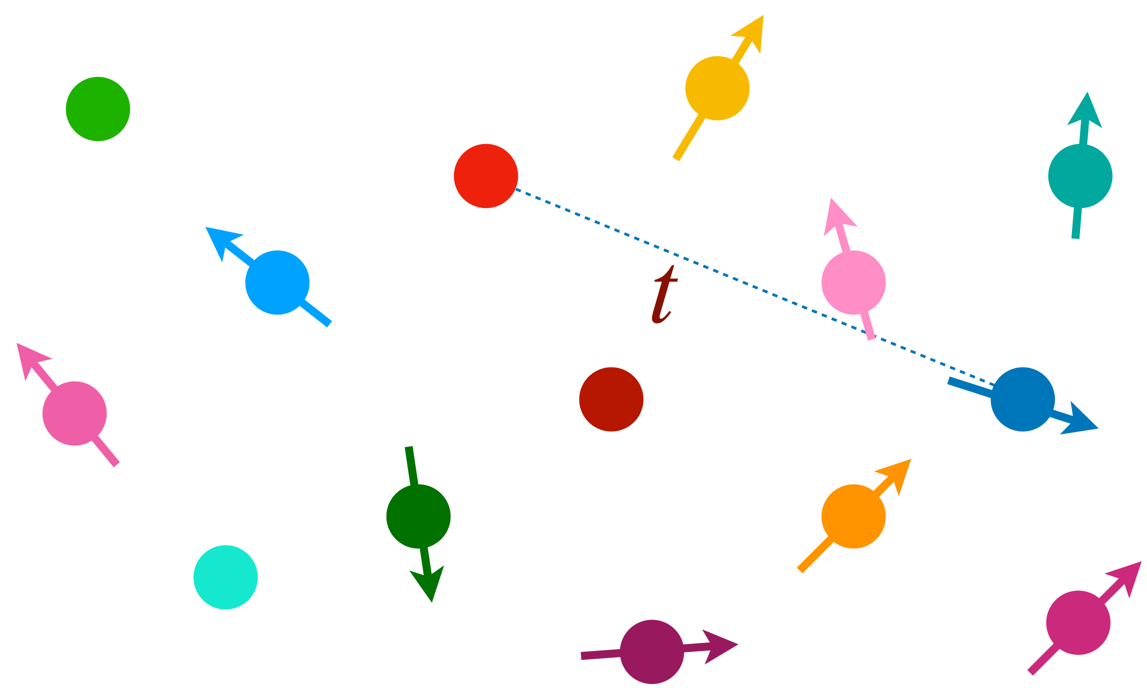

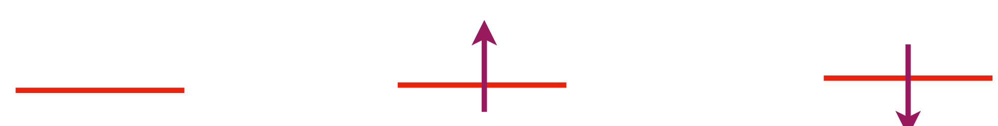

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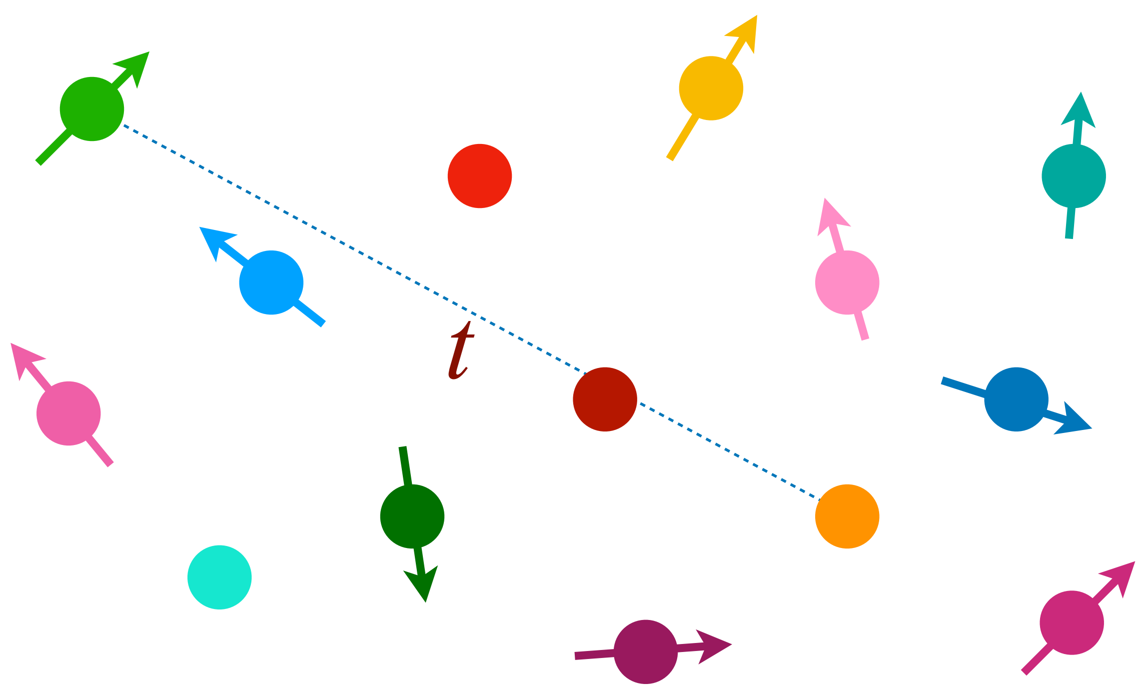

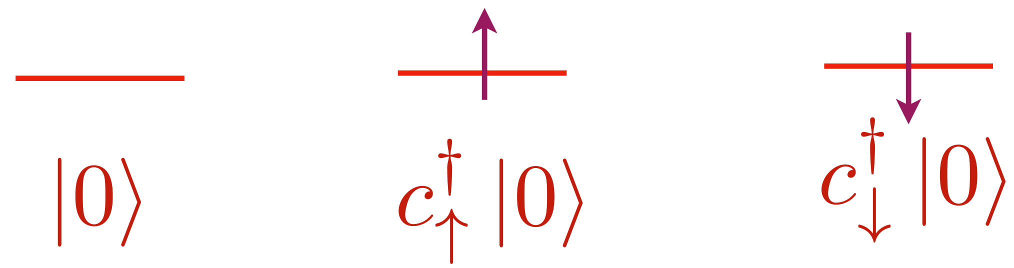

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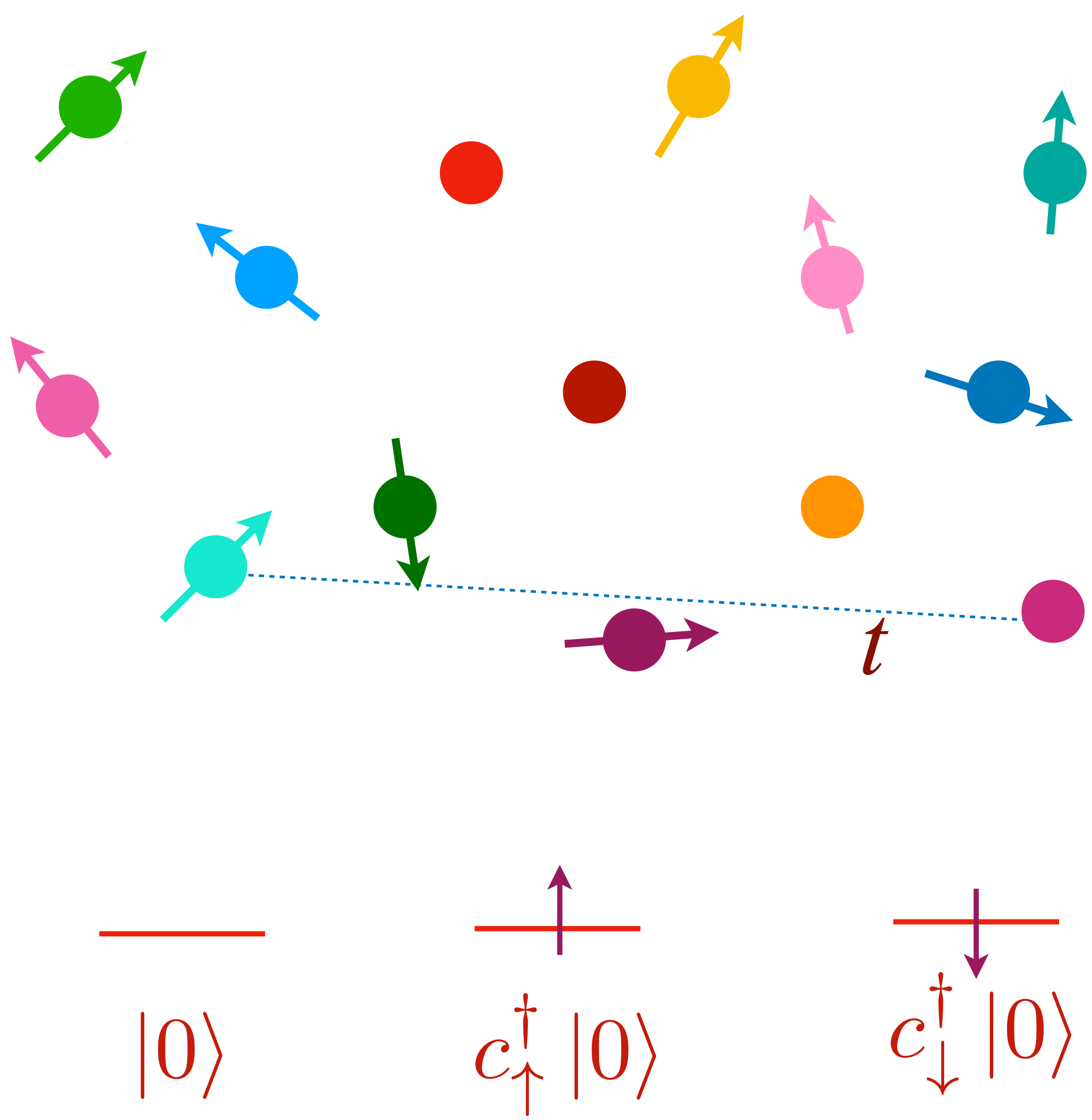

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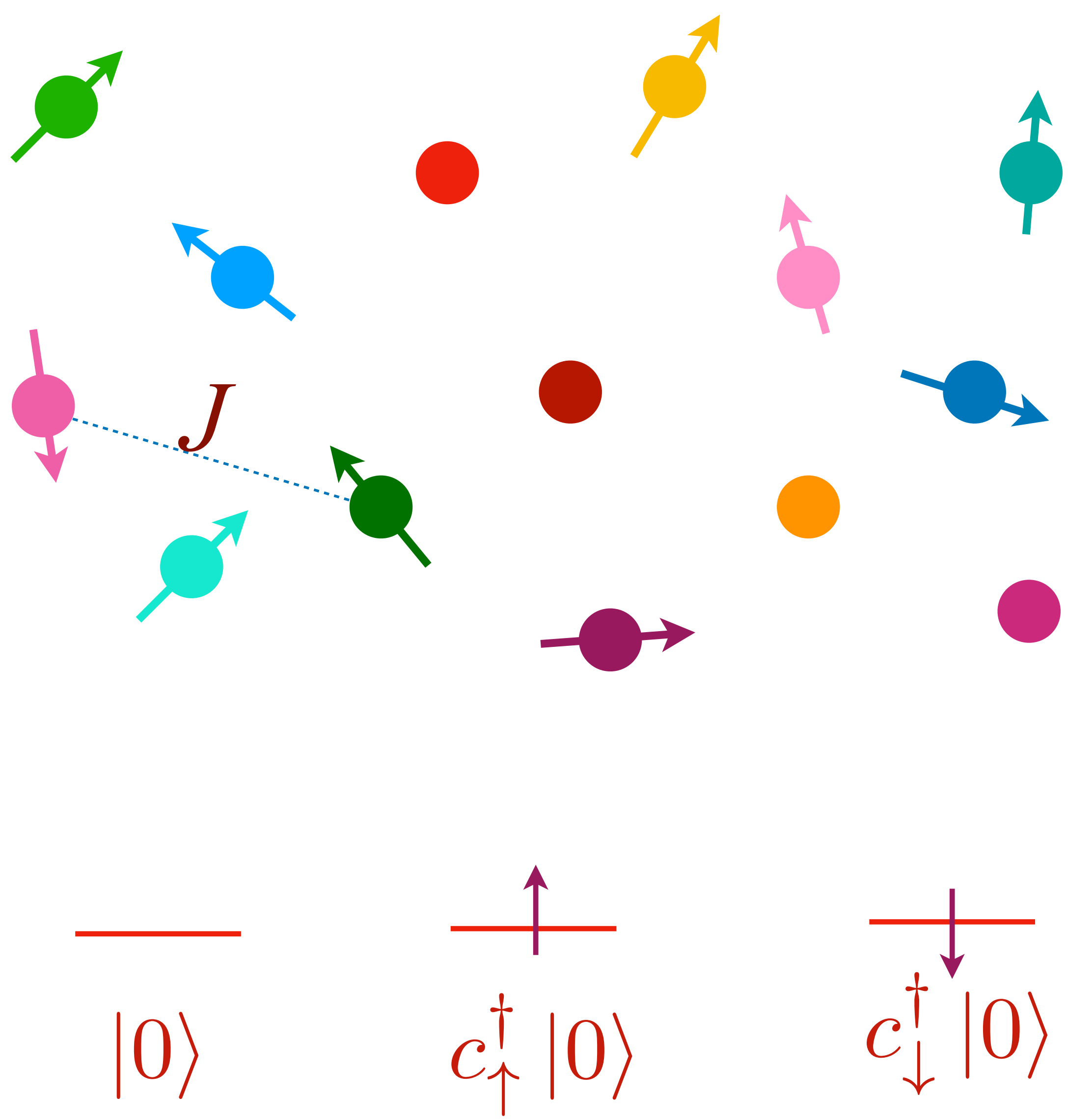

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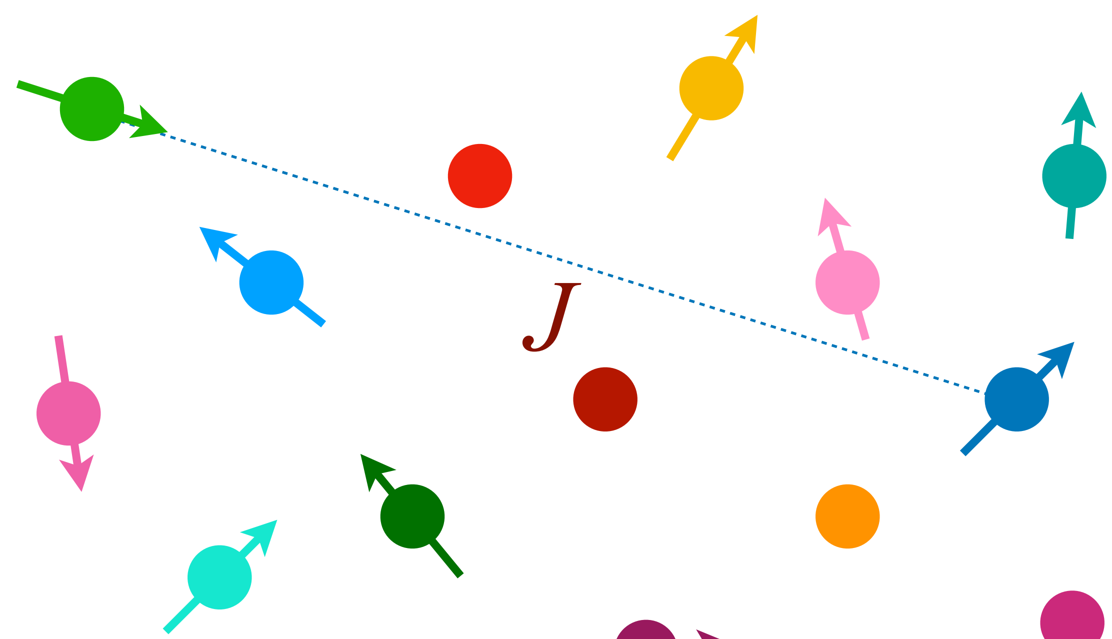

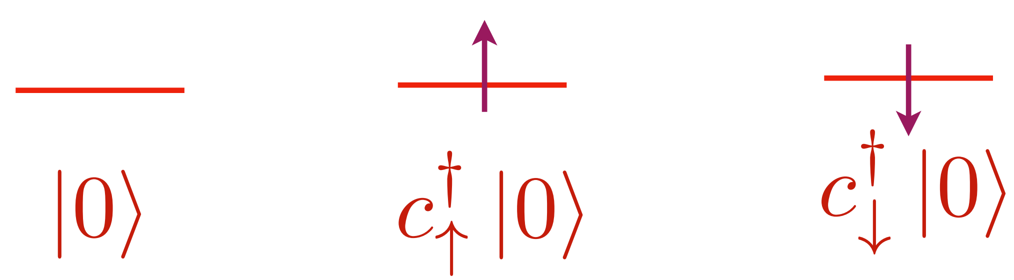

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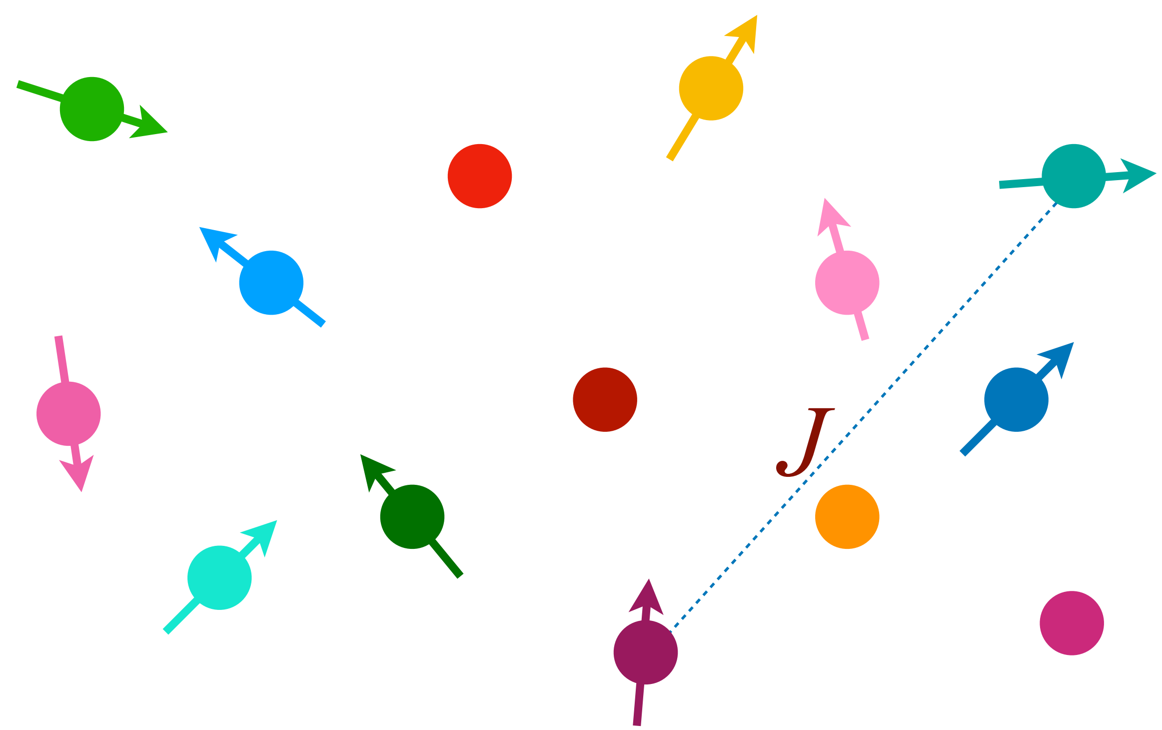
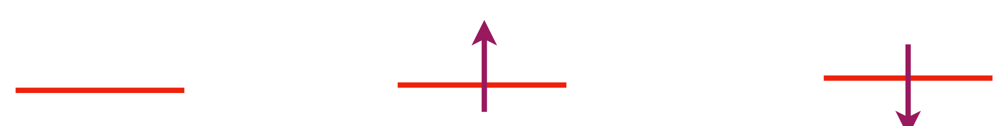
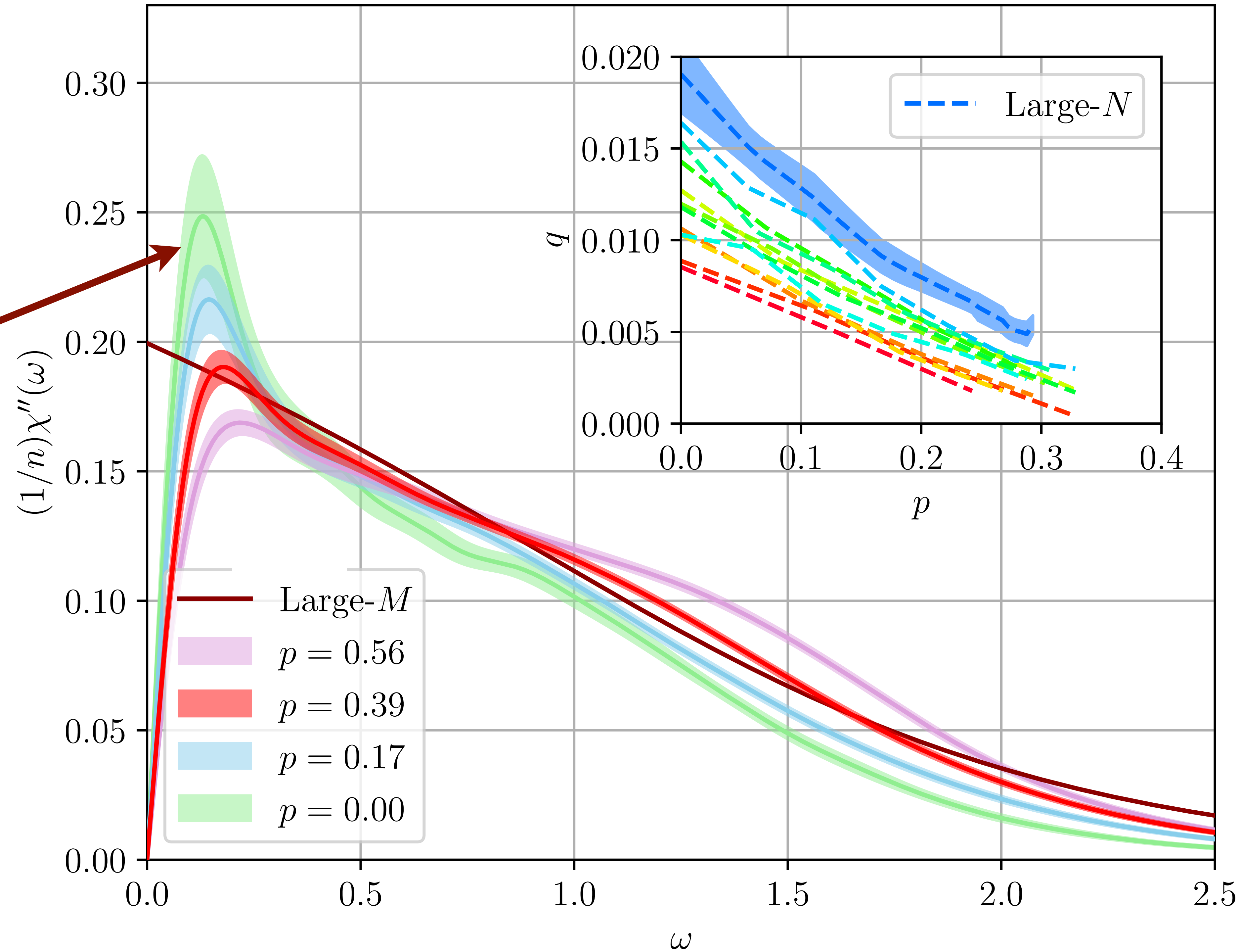




$|0\rangle$ $\quad c_{\uparrow}^{\dagger}|0\rangle$ $\quad c_{\downarrow}^{\dagger}|0\rangle$

Dynamic spin susceptibility

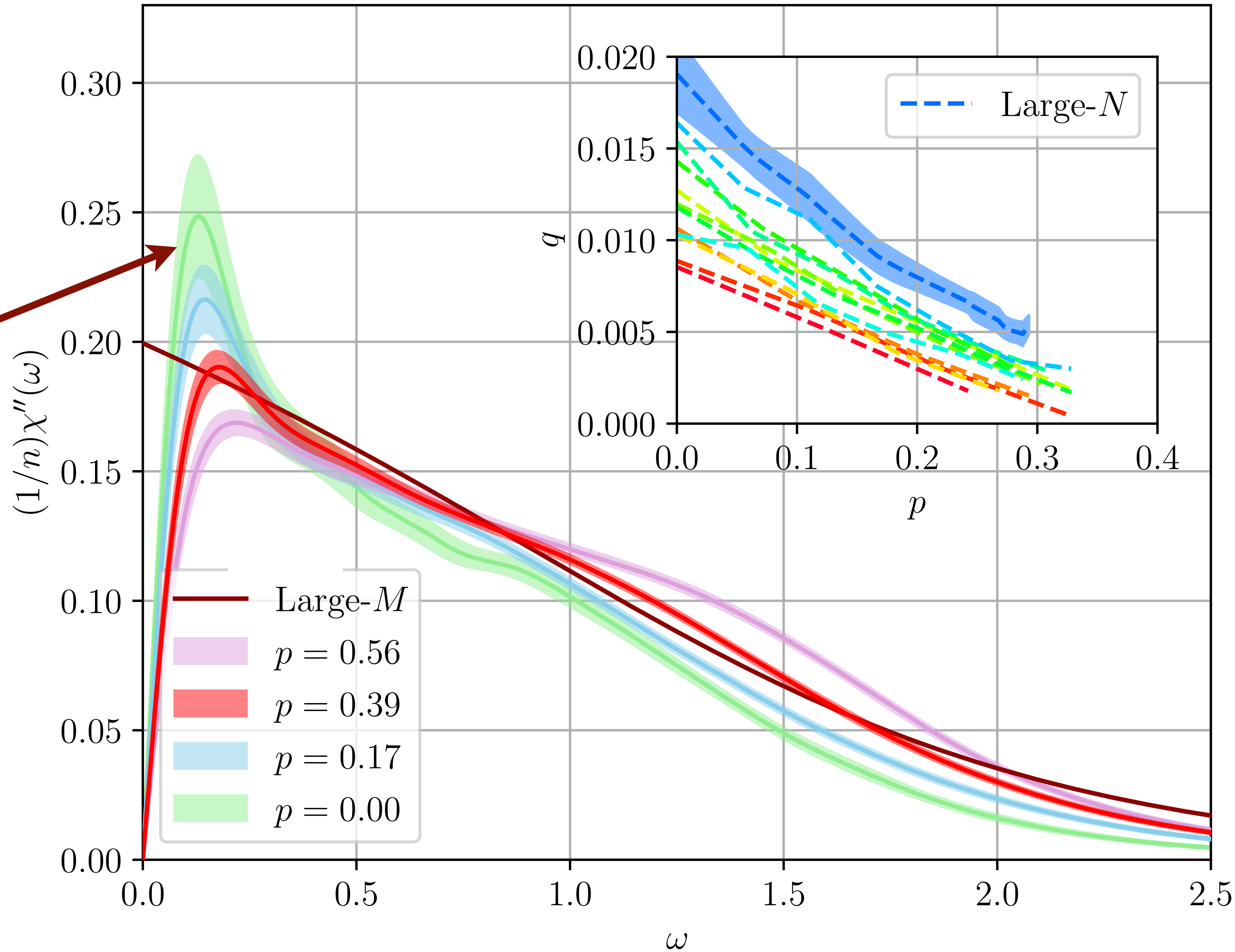
Spin glass order



Evidence for a quantum critical point at $p = p_c \approx 0.3$.
Spin glass order q non-zero for $p < p_c$

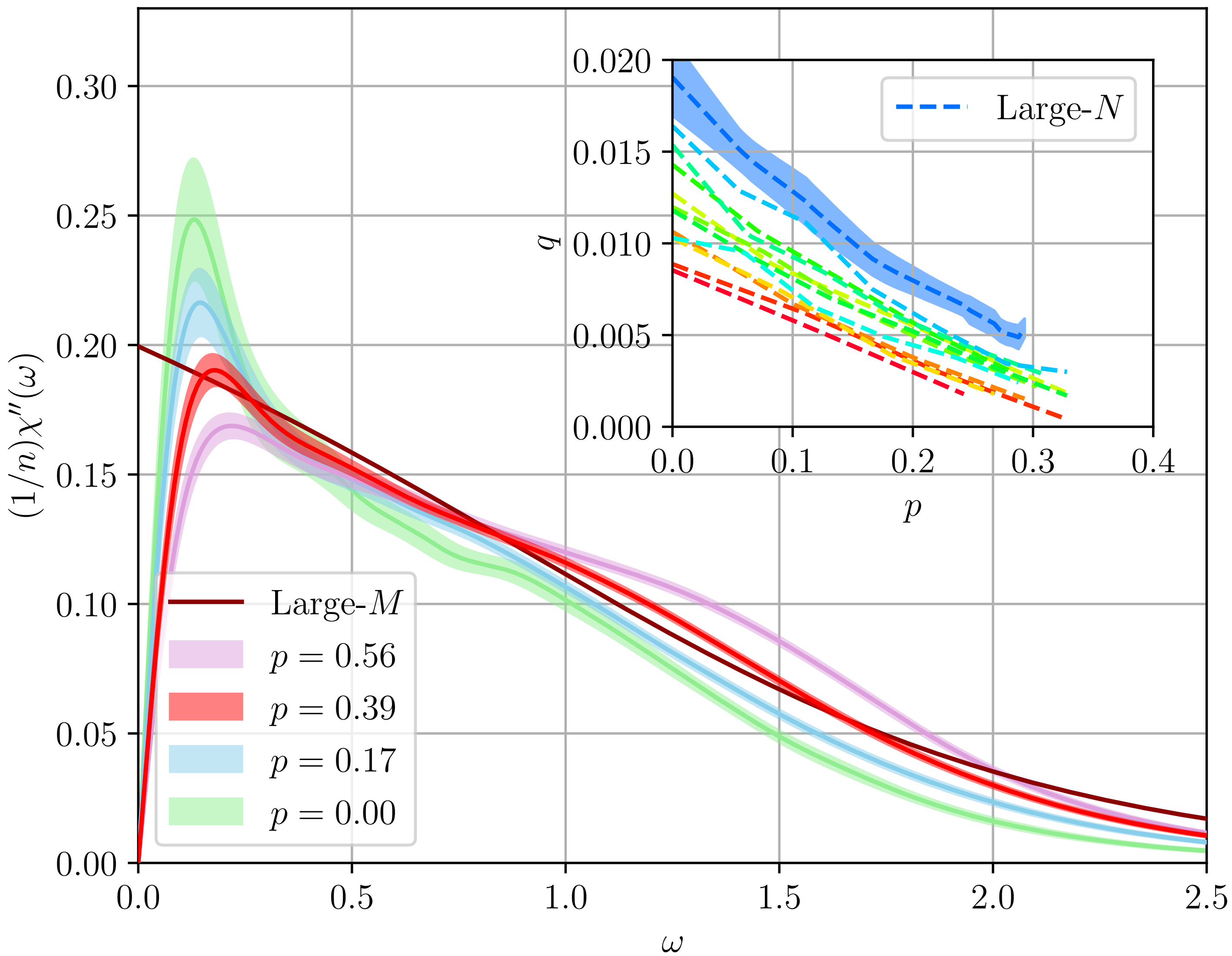
Dynamic spin susceptibility

Spin glass order



Critical spin susceptibility matches the large M SU(M) SYK model.
$\chi''(\omega) \sim \text{sgn}(\omega)\,[1 - \mathcal{C}\gamma|\omega| + \ldots]$ has the 'marginal' $\text{sgn}(\omega)$ form, with a linear ω correction.
Shown is the numerical solution of SYK equations (SY, PRL 1993), after rescaling J.

Dynamic spin susceptibility



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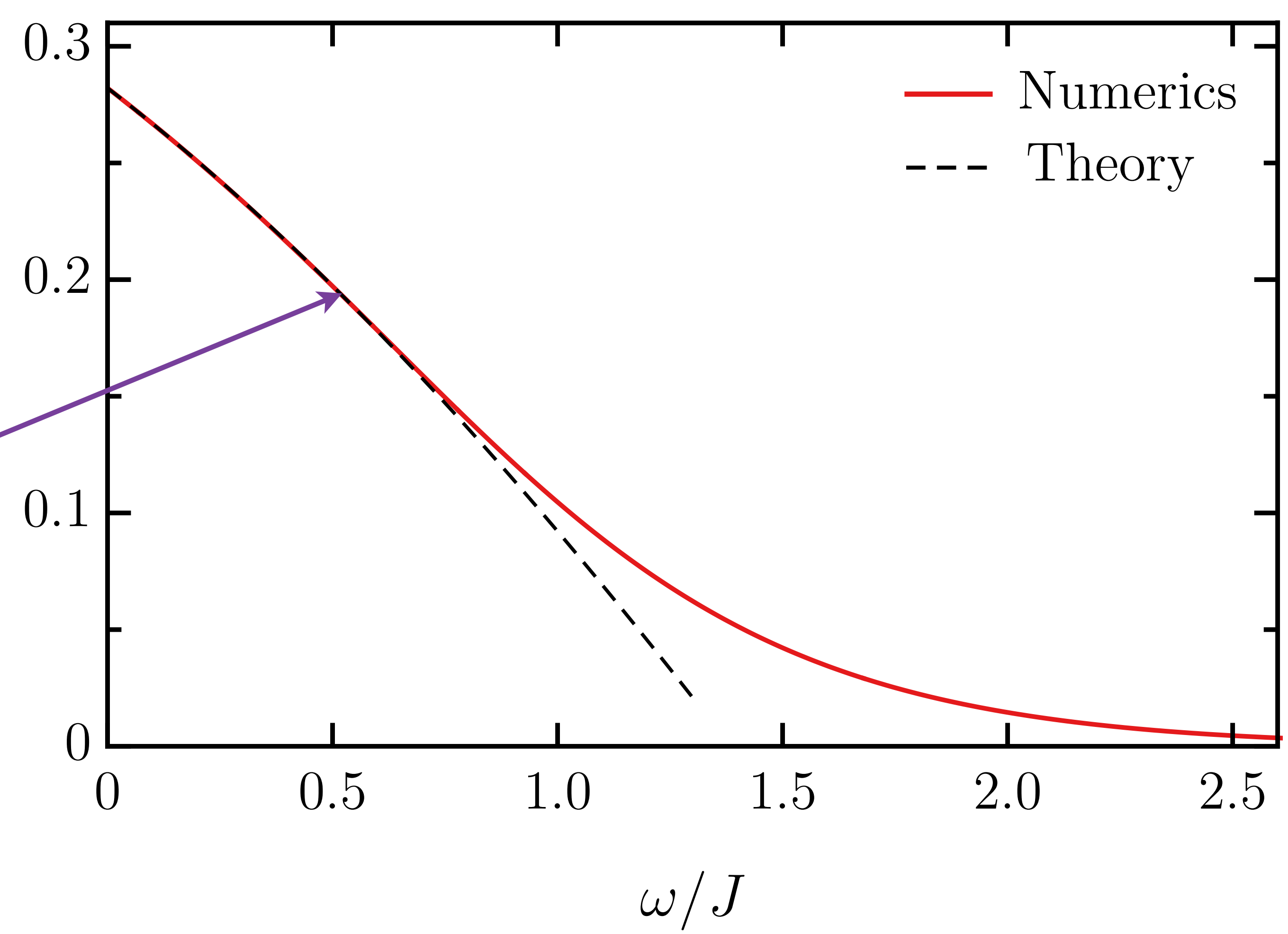
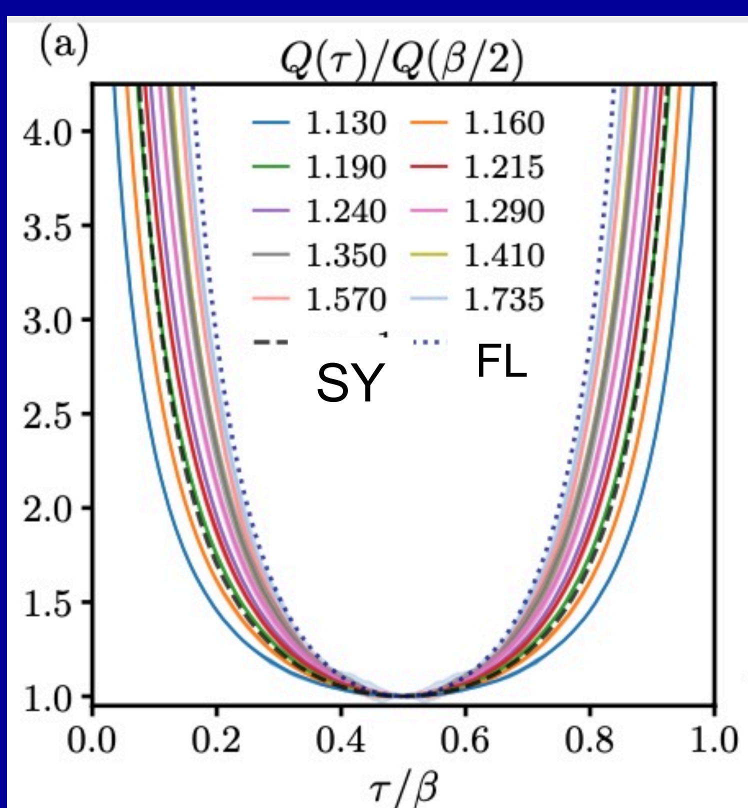
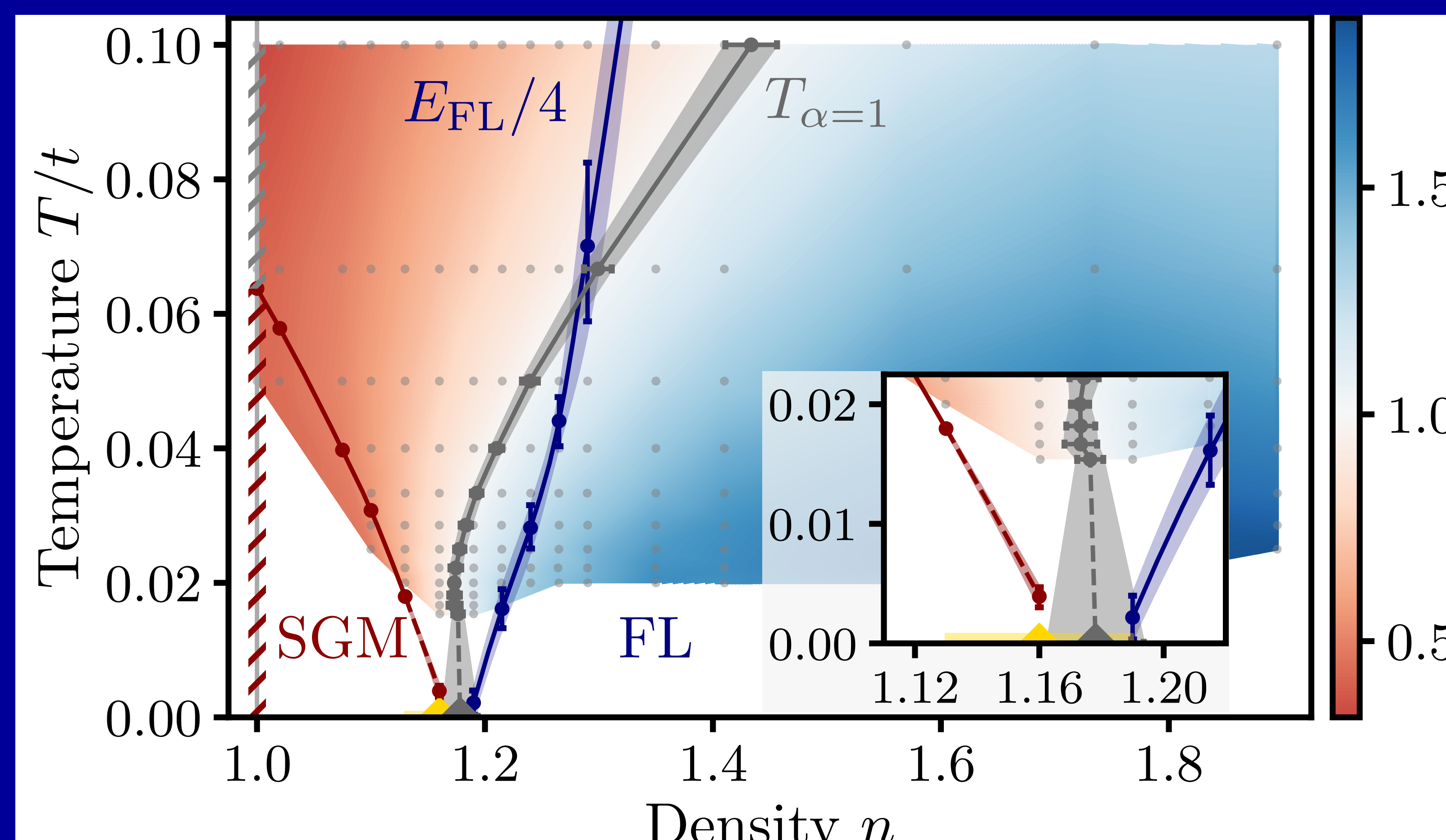


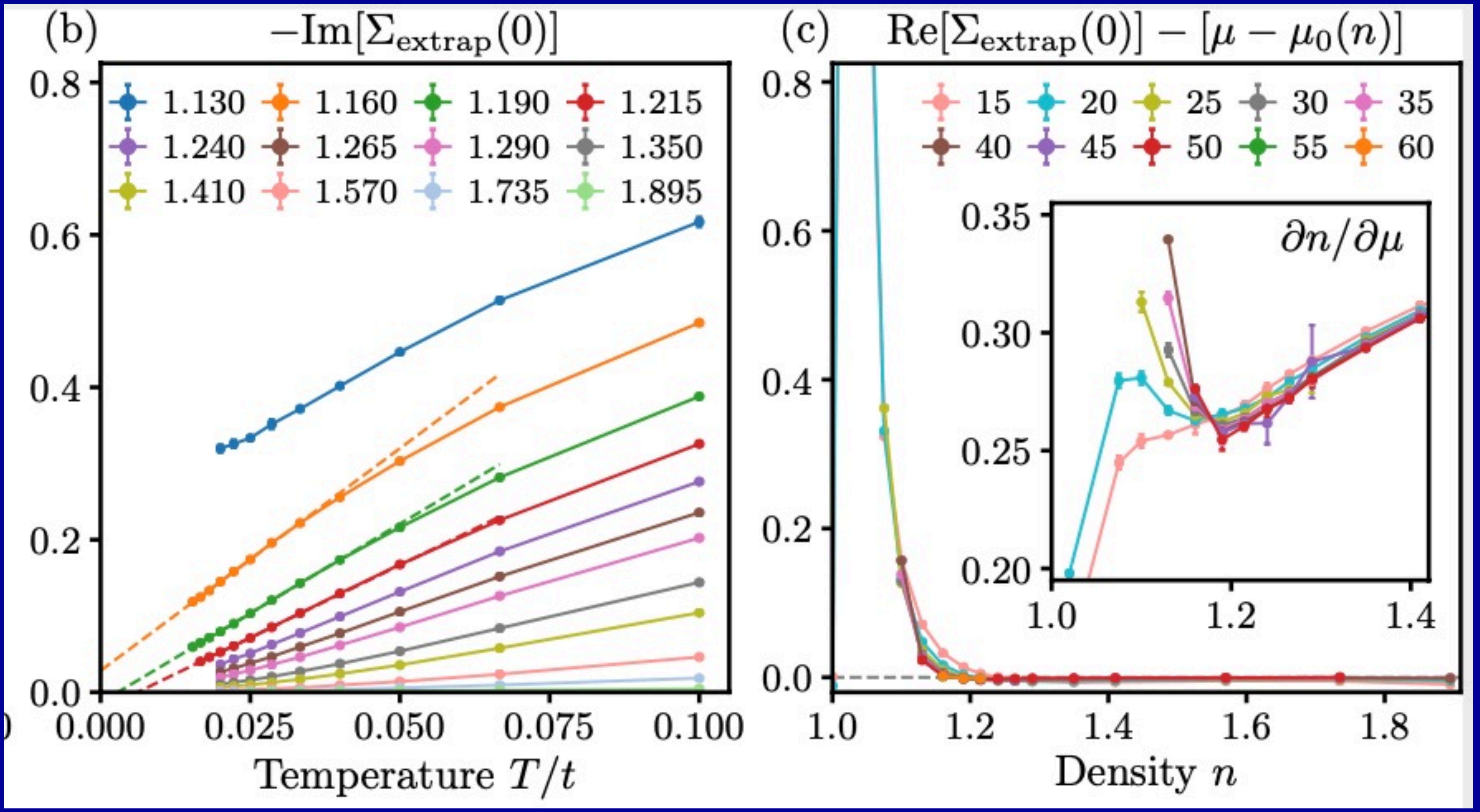
Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Doping the SU(2) t-J-U SY Model : An Intringuing Quantum Critical Point Solving the EDMFT Equations

P.T. Dumitrescu, N. Wentzell, A. Georges,
O. Parcollet, arXiv:2103.08607







Scattering rate
Linear in T
at QCP

$$\frac{1}{\tau^*} \approx 1.4 \frac{k_B T}{\hbar}$$

Luttinger volume of FS
Breaks down at QCP

P.T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

Time reparameterization soft mode and linear-T resistivity

The connection between the random t-J model and SYK criticality can be made explicit in a model in which the SU(2) spin symmetry is replaced by SU($M \to \infty$) spin symmetry. Computing the resistivity in $\rho(T)$ in the this limit via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + 8\alpha_G \frac{T}{J} + \ldots\right) .$$

The α_G term arises from the contribution of the boundary graviton!

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)
Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

1. Random matrix and SYK models

2. Time reparameterization soft mode

3. Random t-J model

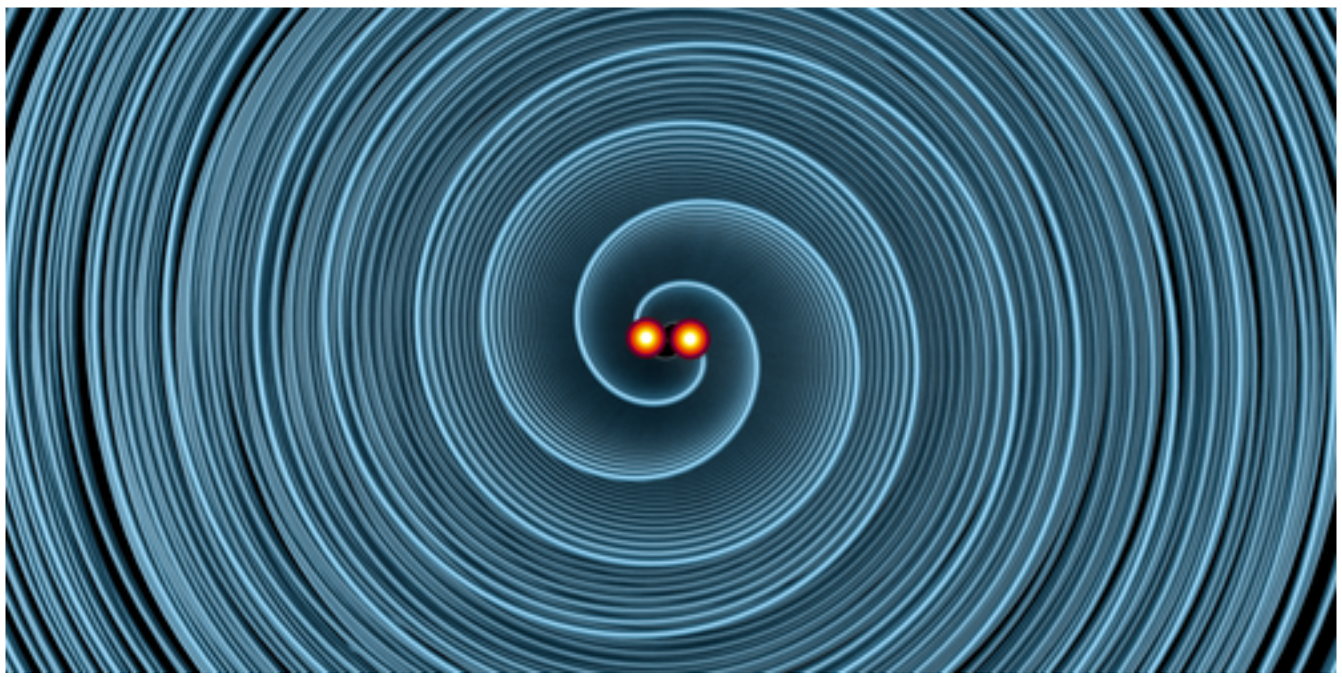
4. Charged black holes

Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

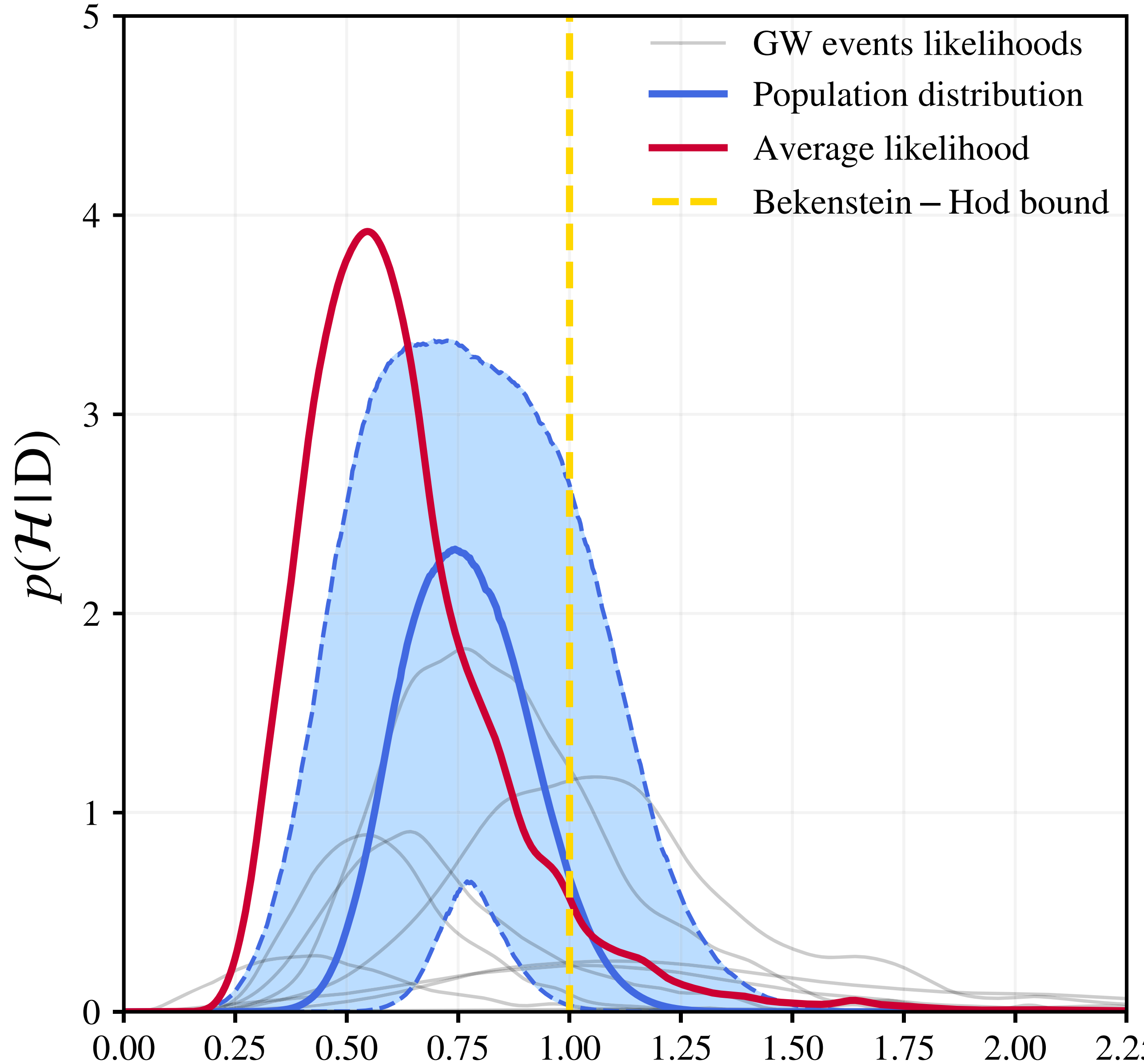
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Thermodynamics of quantum black holes:

$$\int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\, \mathcal{S}^{(d+1)}_{\text{Einstein gravity}}[g_{\mu\nu}]\right)$$

Metric of spacetime

Quantum gravity: a summation over
all possible configurations of spacetime,
each weighted by a factor which is the exponential of
(the 'action' of Einstein gravity)/(Planck's constant)

Thermodynamics of quantum black holes:

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Metric of spacetime

In general, this summation is not well defined, because to the uncontrollably large number of spacetime configurations.

Thermodynamics of quantum black holes:

$$\int \mathcal{D} g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\, \mathcal{S}^{(d+1)}_{\text{Einstein gravity}}[g_{\mu\nu}] \right)$$

$$= \exp\left(S_{BH}\right) \times \left(\quad \text{....????....} \quad \right)$$

Metric of spacetime

Gibbons, Hawking (1977)

$$S_{BH} = \frac{Ac^3}{4G\hbar}$$

A is the area of the black hole horizon.
Interpretation: Black hole entropy is entanglement entropy across the horizon.

Holography and duality

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$$\left. \text{in } d-1 \text{ spatial dimensions } \textit{without} \text{ gravity}\right)$$

Metric of spacetime

Black holes are represented as a `*hologram*' by a quantum many-body system in one lower dimension.

Duality: a `*change of variables*' between the many-particle configurations and the metric of spacetime

Susskind, Maldacena…..

Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

$\vec{x}$

ζ

Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

SYK model and charged black holes

Thermodynamics of charged quantum black holes

$$\int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\rm Einstein-Maxwell\ theory}[g_{\mu\nu}]\right) \;\; T \to 0,$$

$$\approx \int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\rm Gravity\ of\ AdS_2\ and\ boundary}[g_{\mu\nu}]\right)$$

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The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016) ; Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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$$S(T \to 0, \mathcal{Q}) = S_{BH} - \frac{3}{4} \ln \left(\frac{\hbar c^5}{G T^2} \right)$$

$$S_{BH} = \frac{A c^3}{4 G \hbar} \left(1 + \frac{2 (\pi A)^{1/2} T}{\hbar c} \right)$$

A is the area of the charged black hole horizon at $T = 0$, $\mathcal{Q}$ is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016) ; Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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  - $-3/2\ln(1/T)$ correction to Bekenstein-Hawking entropy of low T charged black holes in Einstein gravity.