

# Strange metals and black holes

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National Science Camp  
(Vijyoshi) - 2017

Indian Institute of Science, Bengaluru  
December 9, 2017

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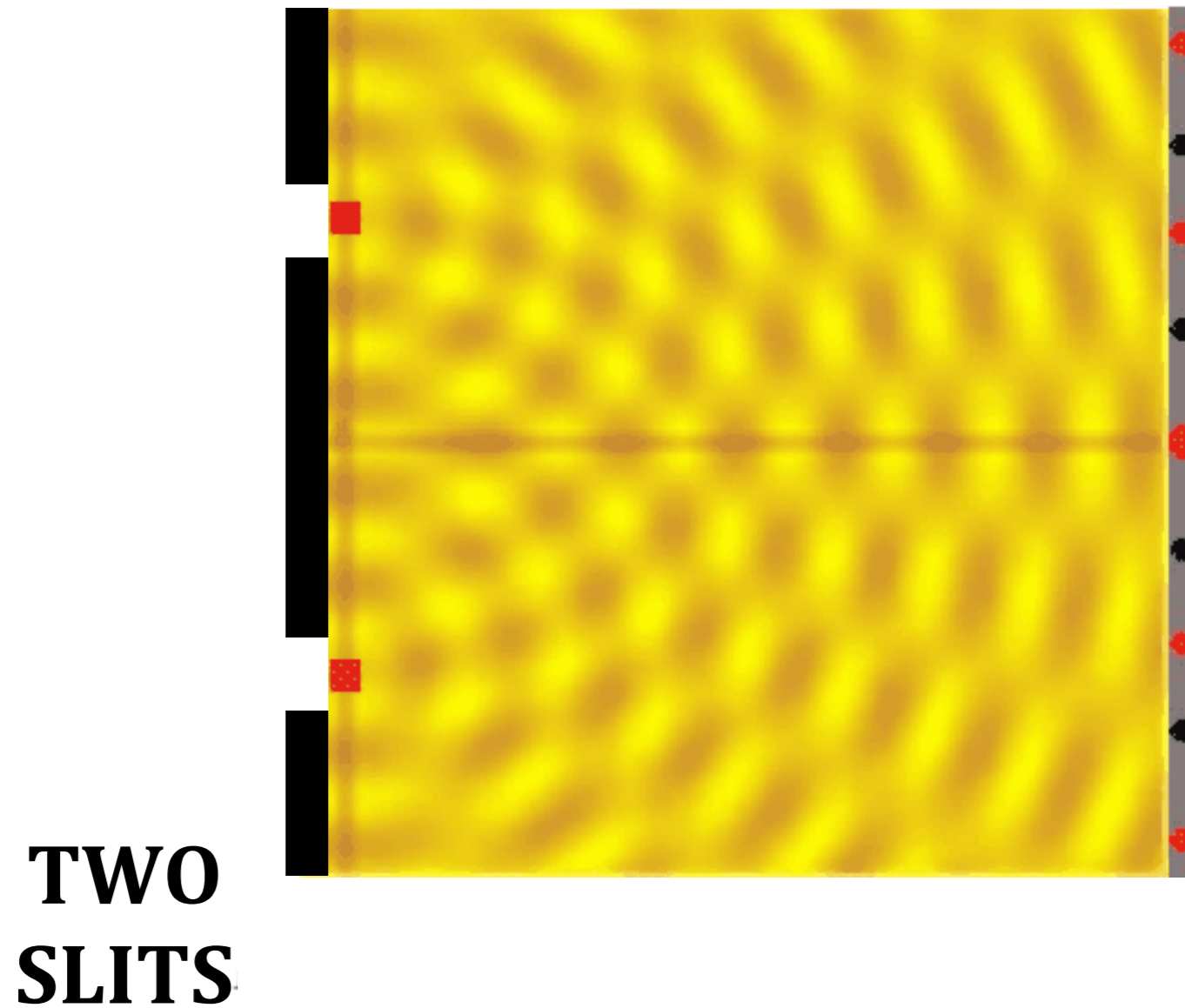
Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)



# Quantum entanglement

# Principles of Quantum Mechanics: I. Quantum Superposition

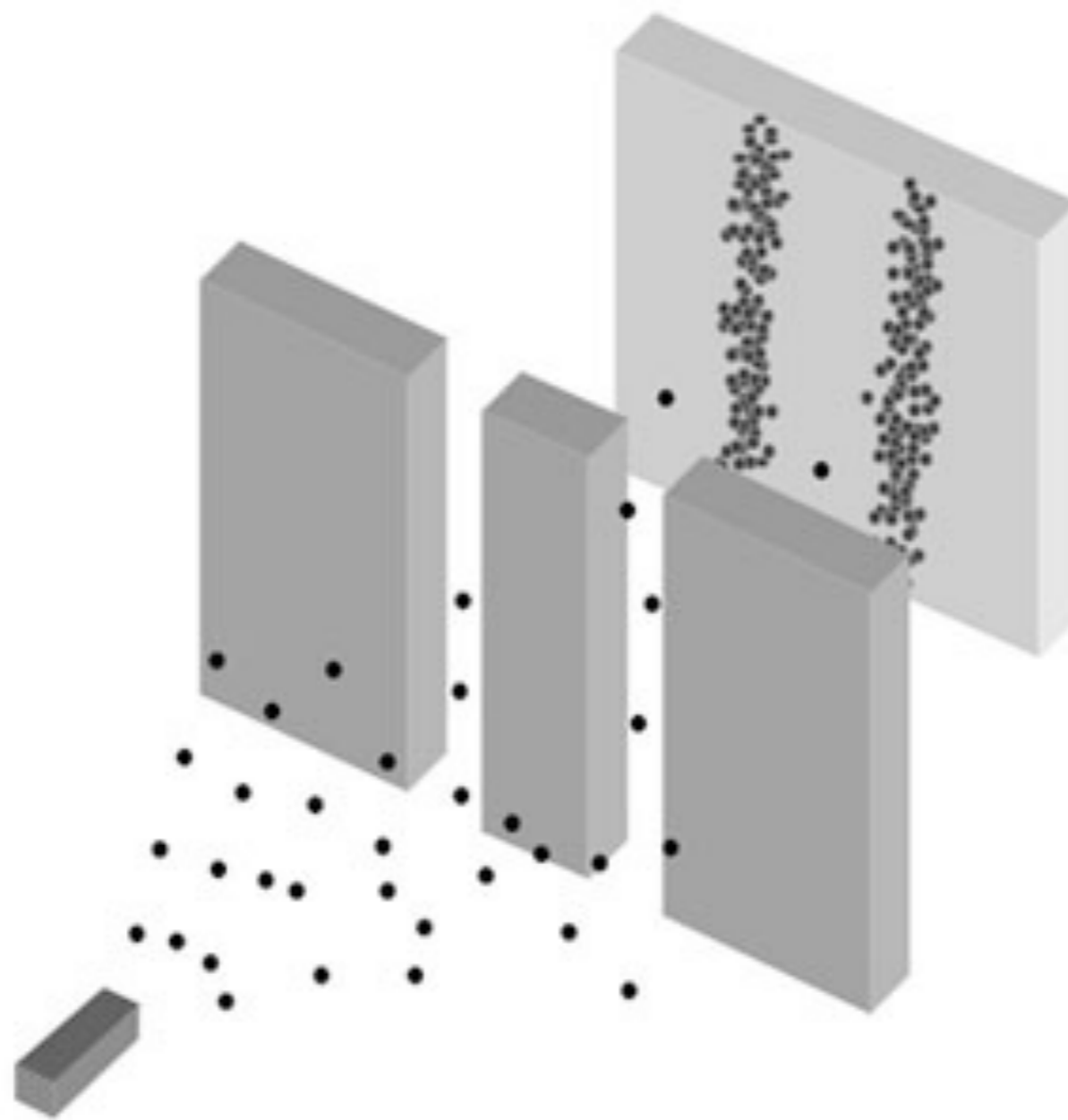
## The double slit experiment



Interference of water waves

# Principles of Quantum Mechanics: I. Quantum Superposition

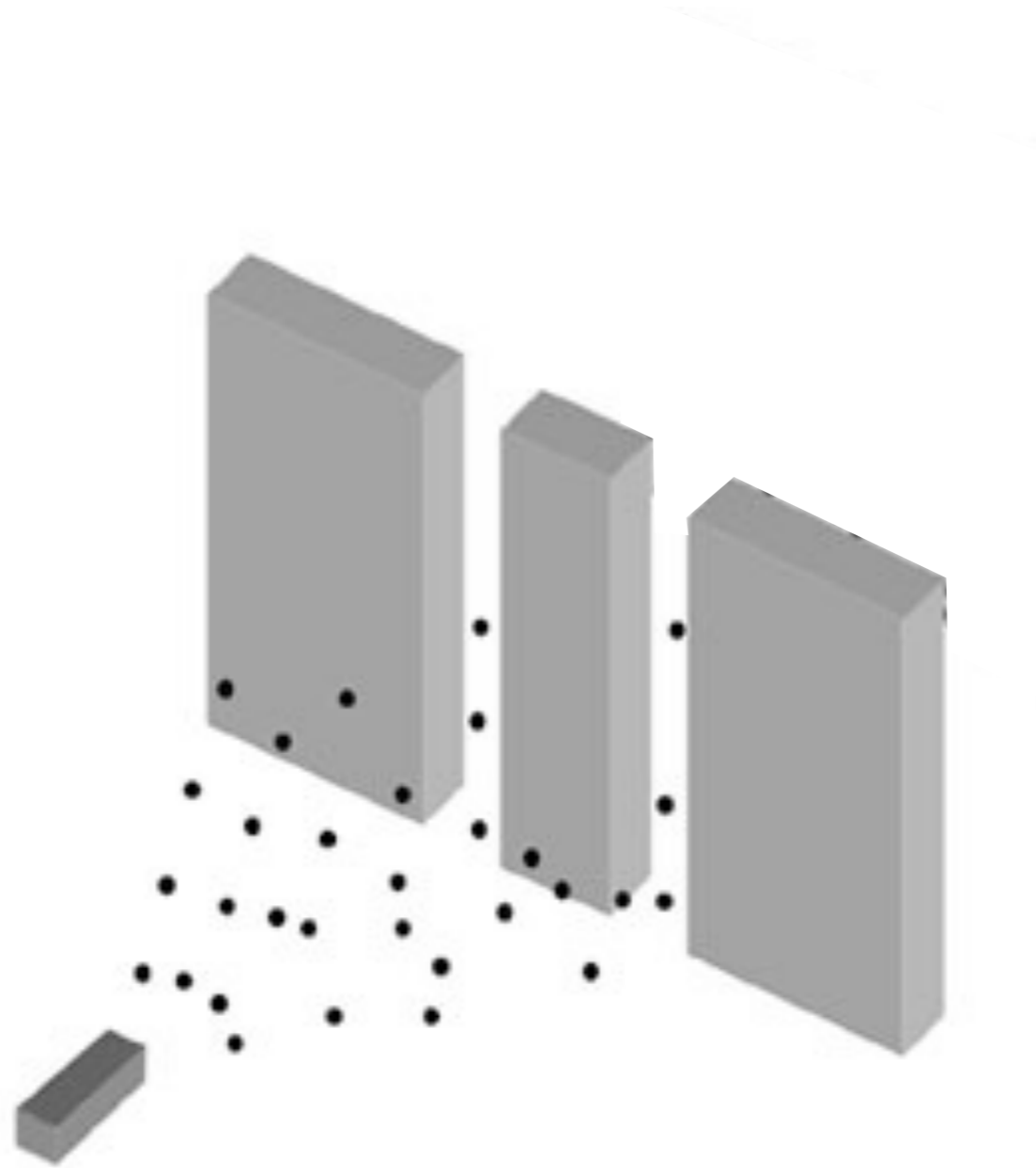
## The double slit experiment



**Bullets**

# Principles of Quantum Mechanics: I. Quantum Superposition

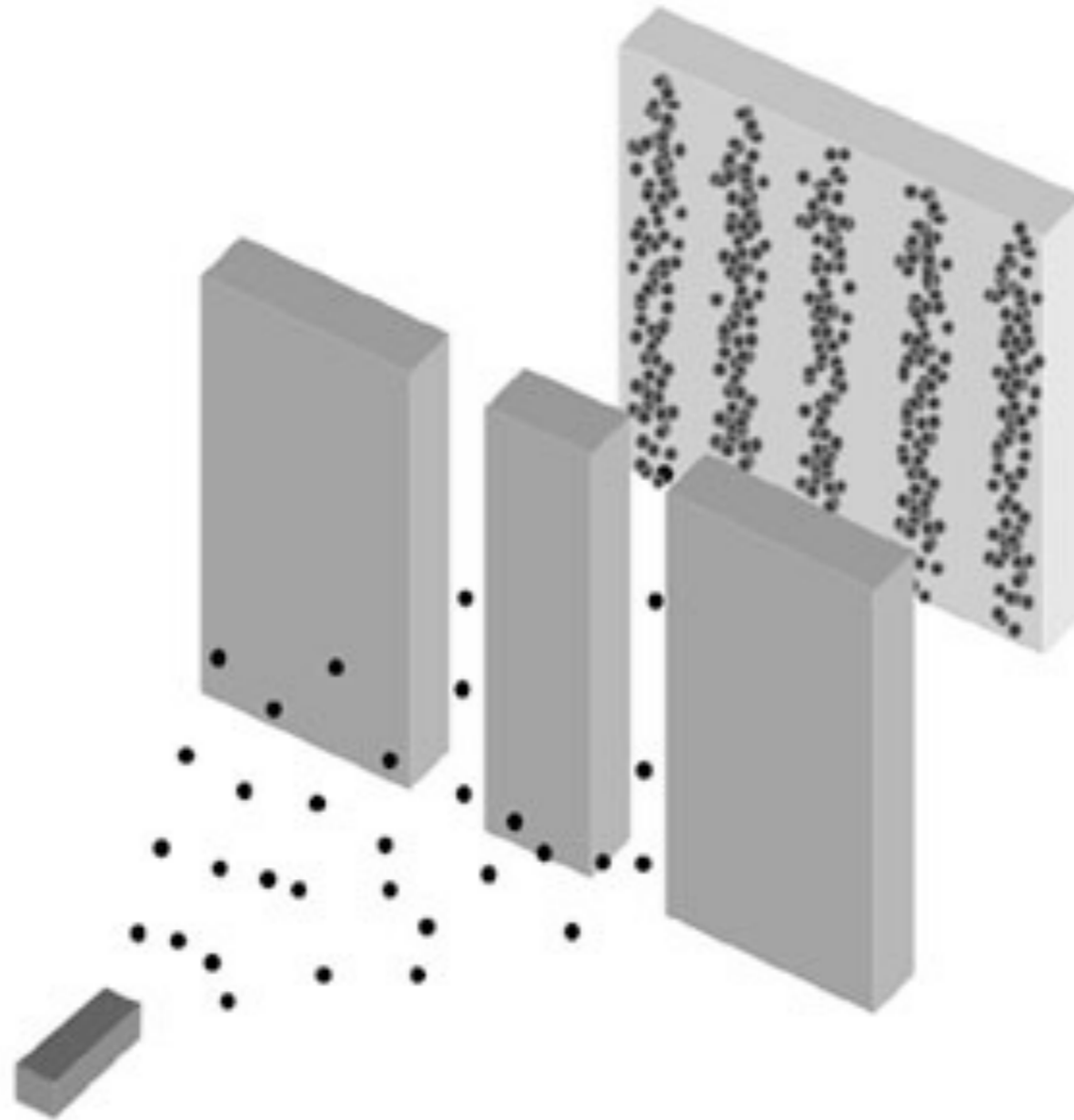
## The double slit experiment



Send electrons through the slits

# Principles of Quantum Mechanics: I. Quantum Superposition

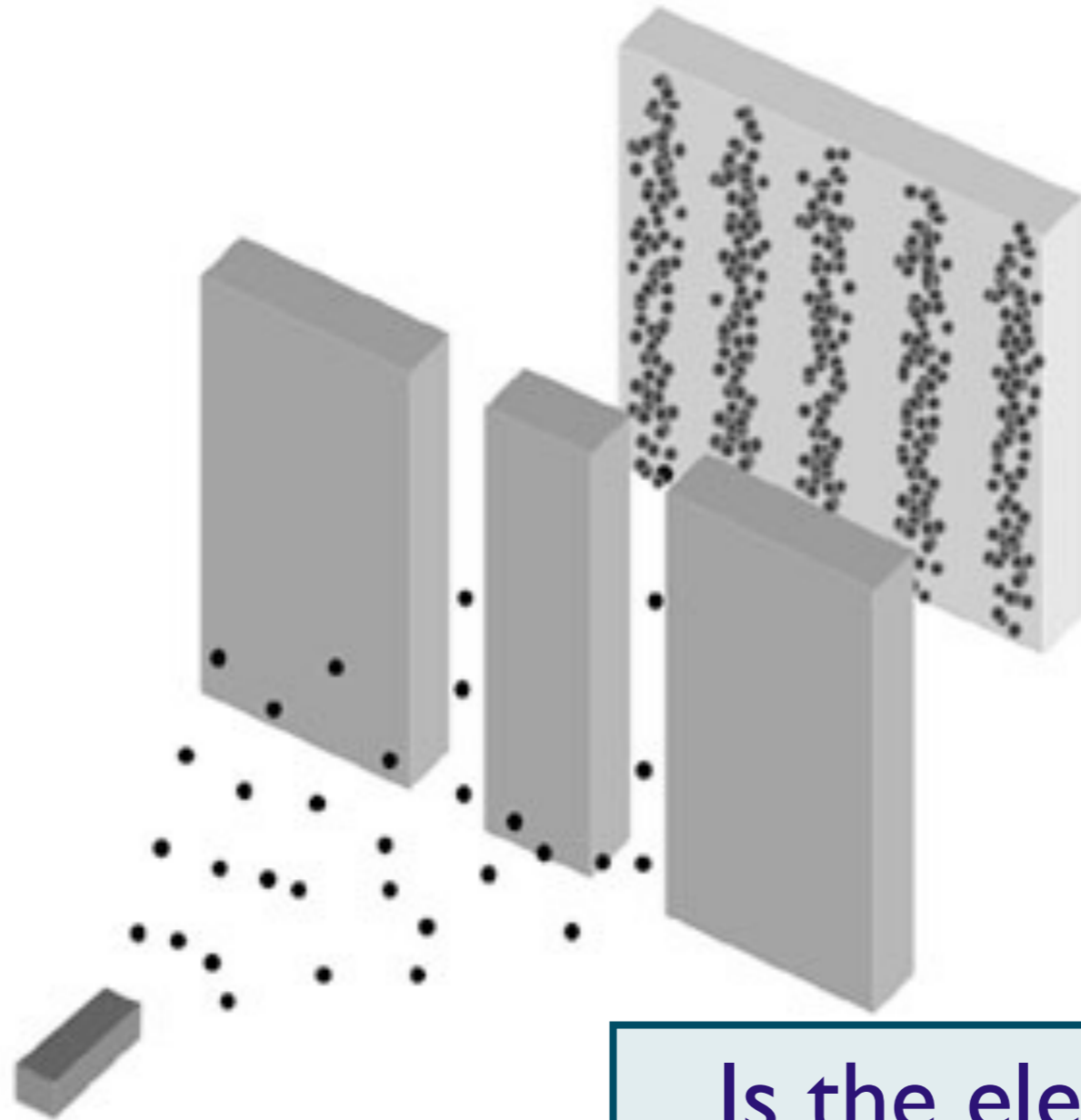
## The double slit experiment



Interference of electrons

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment

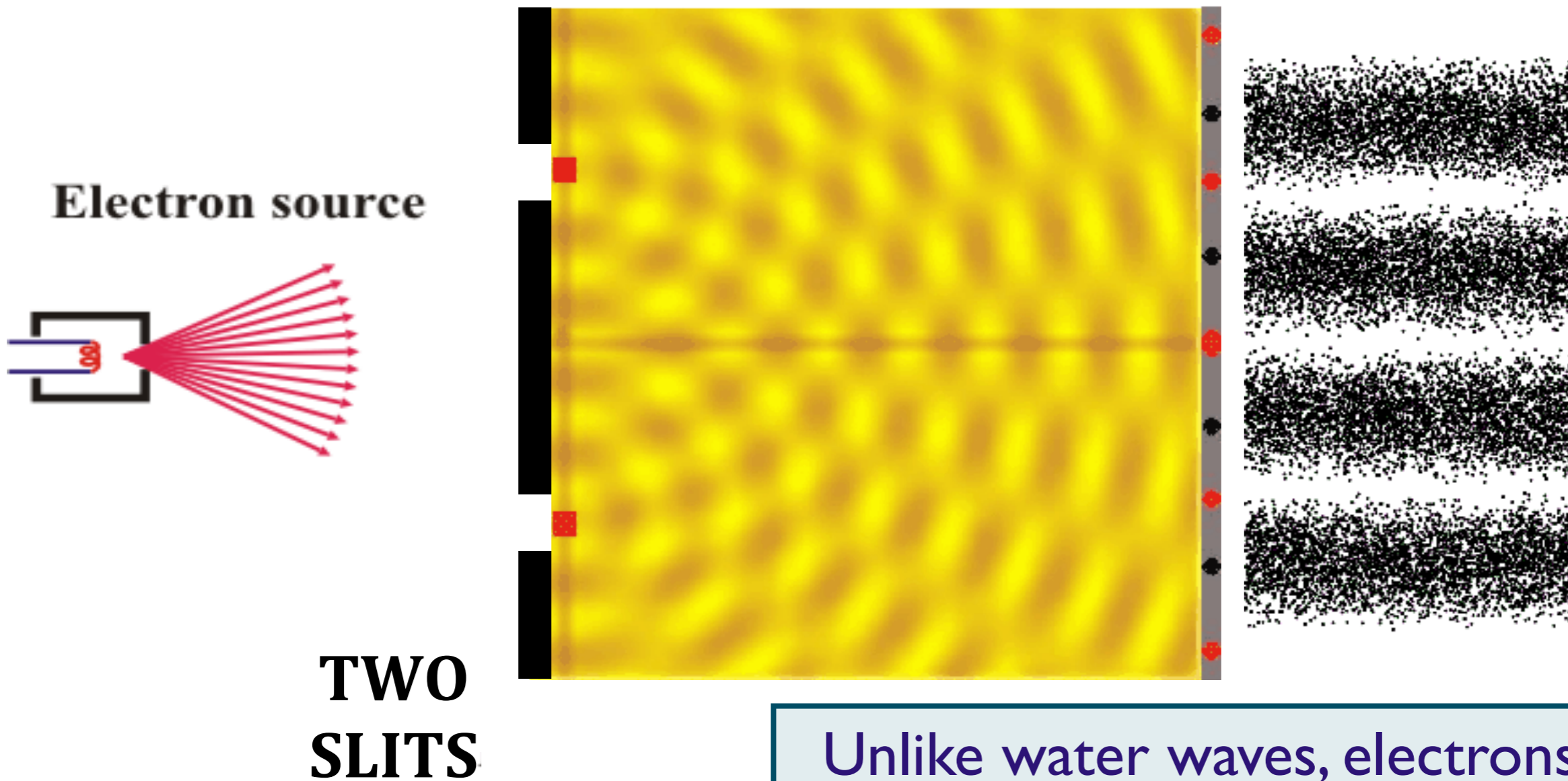


Is the electron a wave ?

Interference of electrons

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment



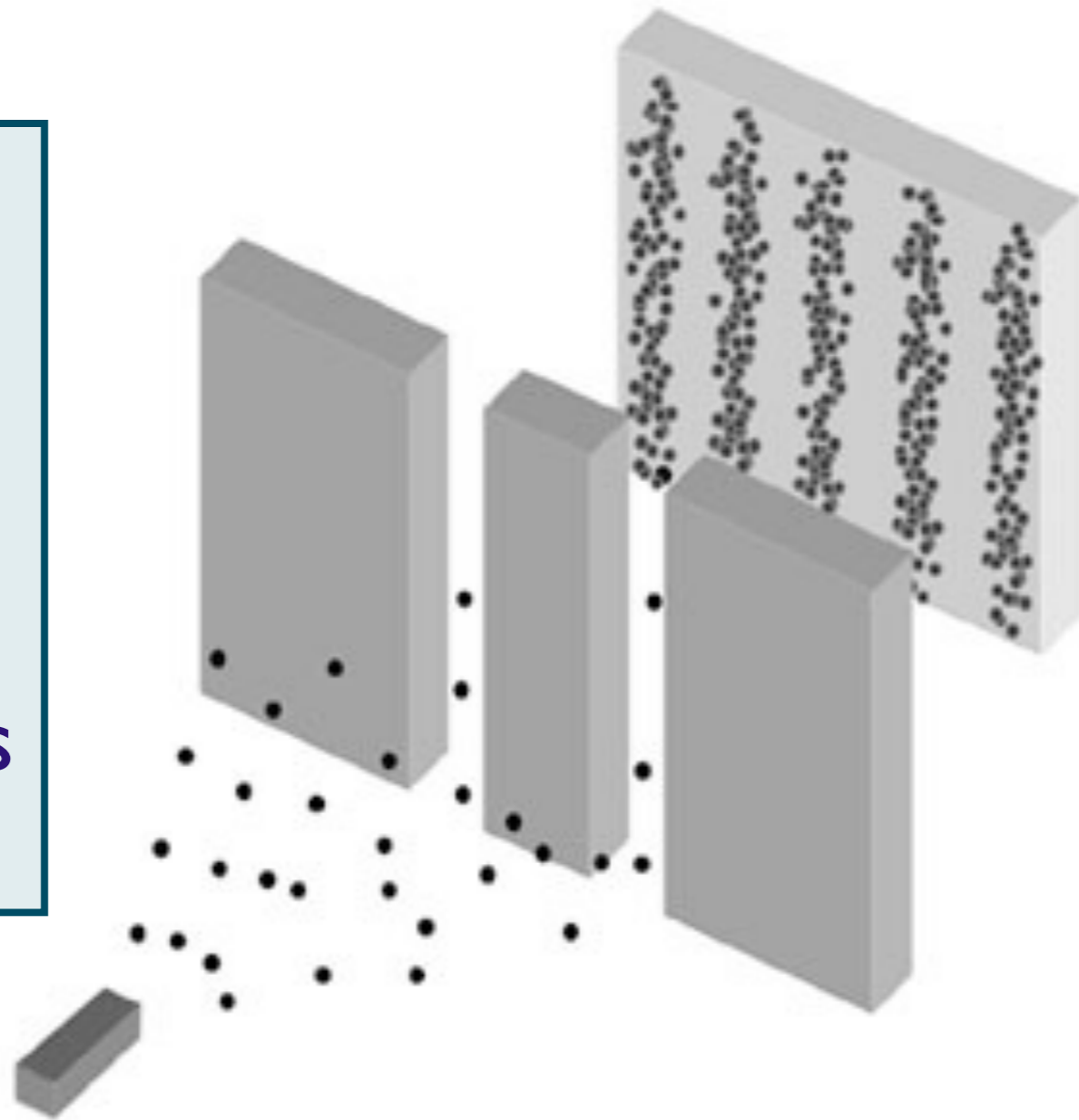
Unlike water waves, electrons arrive one-by-one (so is it like a particle ?)

## Interference of electrons

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment

But if it is like a particle, which slit does each electron pass through ?

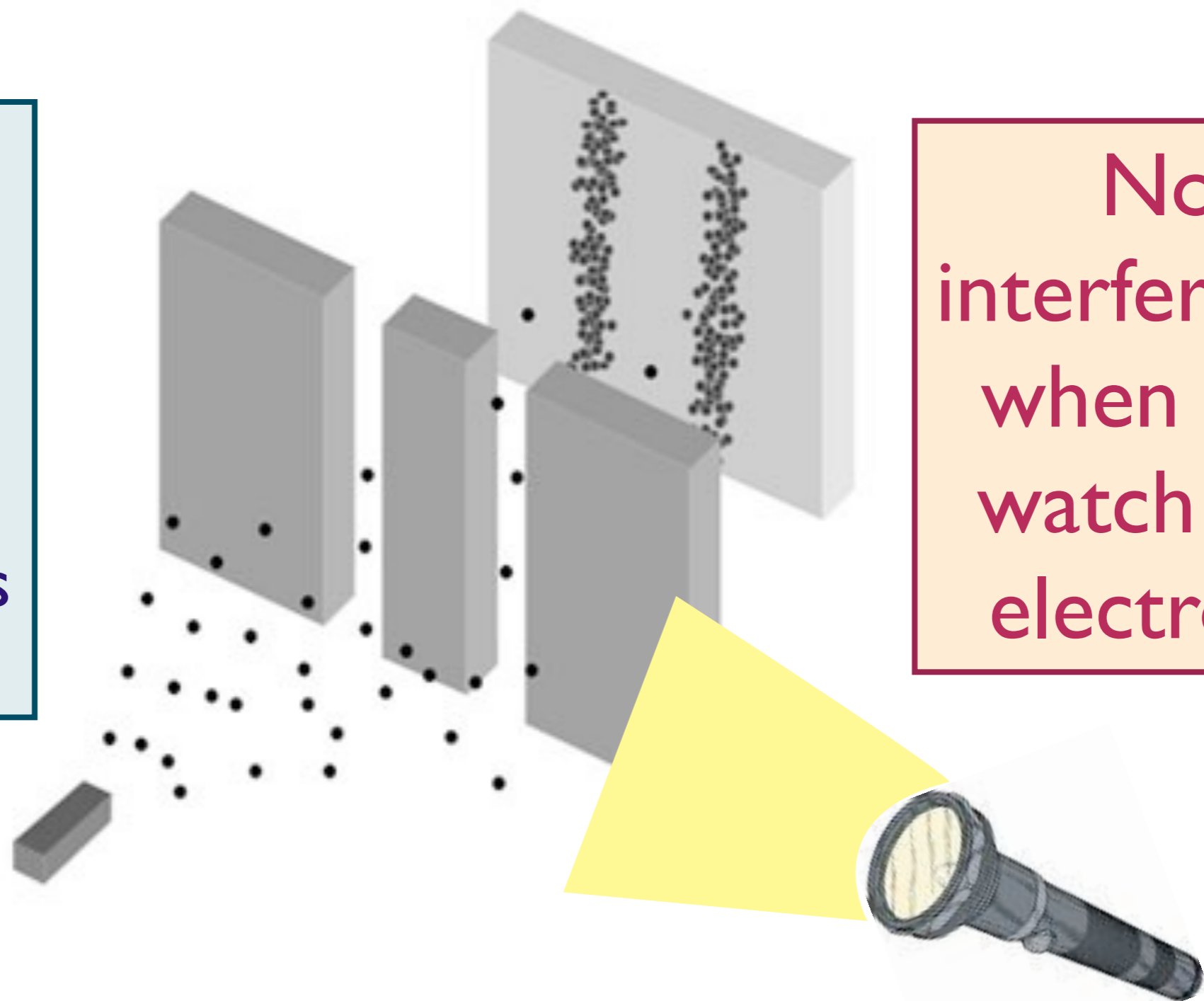


Interference of electrons

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment

But if it is like a particle, which slit does each electron pass through ?



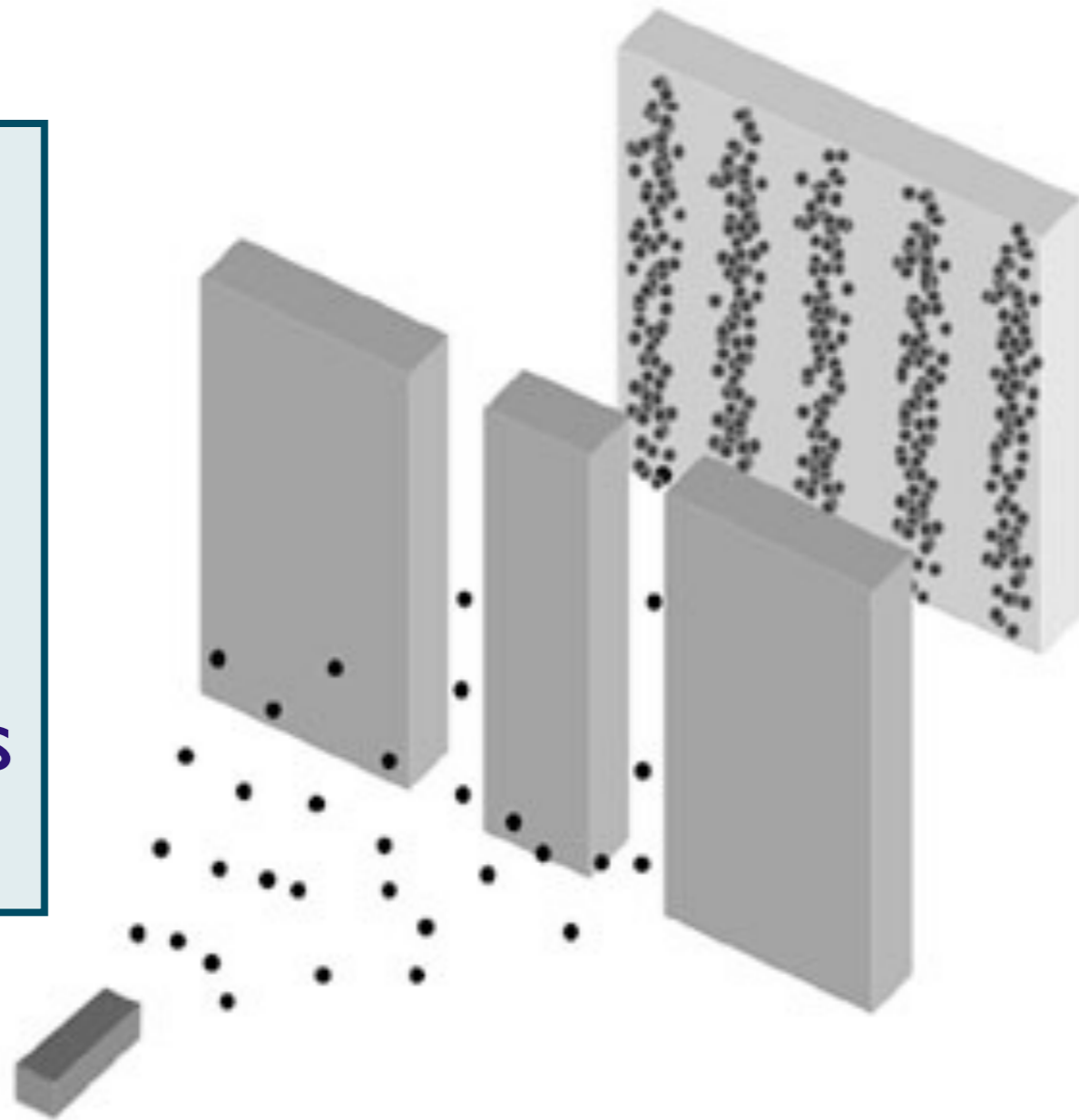
No interference when you watch the electrons

Interference of electrons

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment

But if it is like a particle, which slit does each electron pass through ?

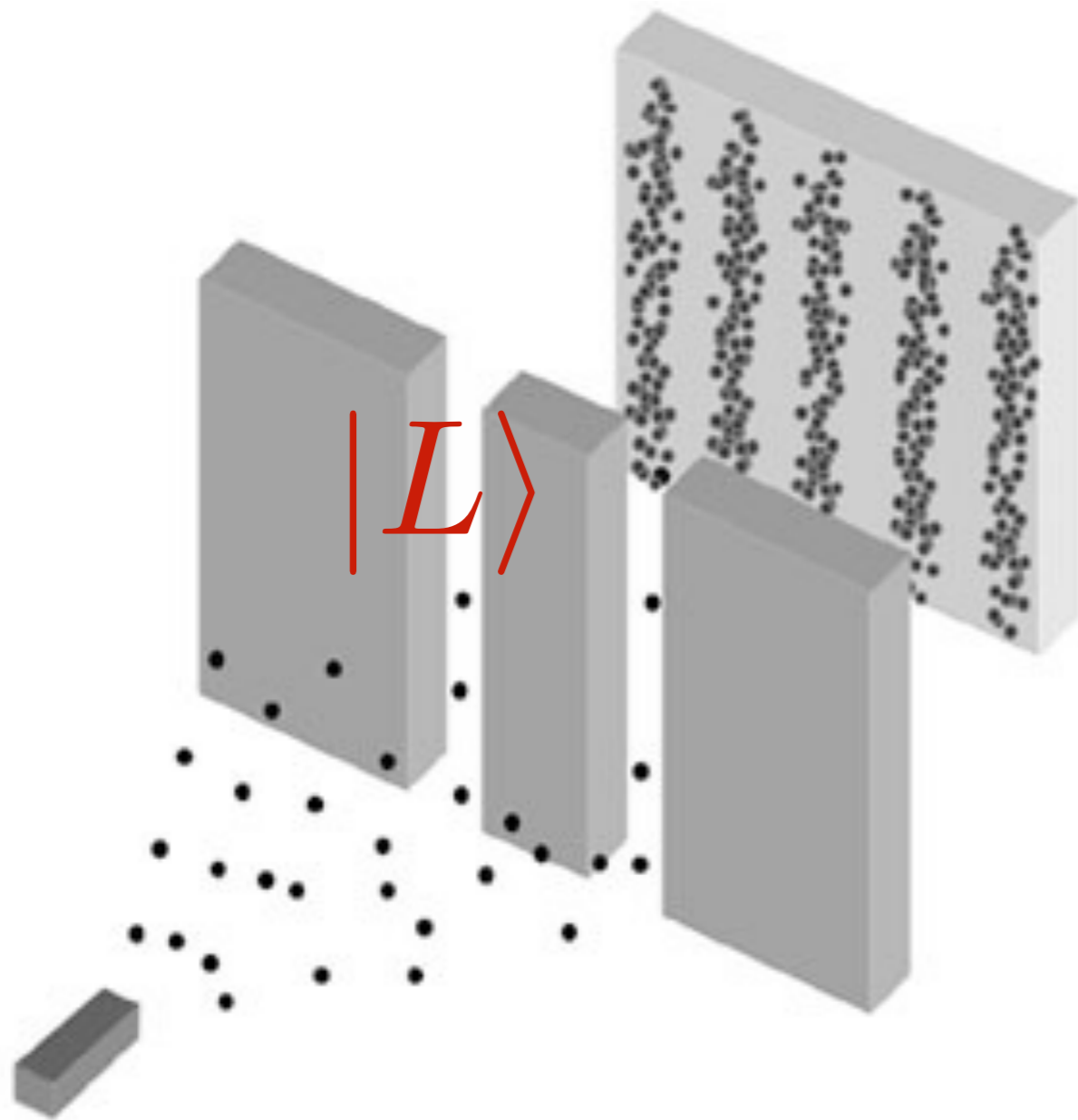


Each electron passes through both slits !

Interference of electrons

# Principles of Quantum Mechanics: I. Quantum Superposition

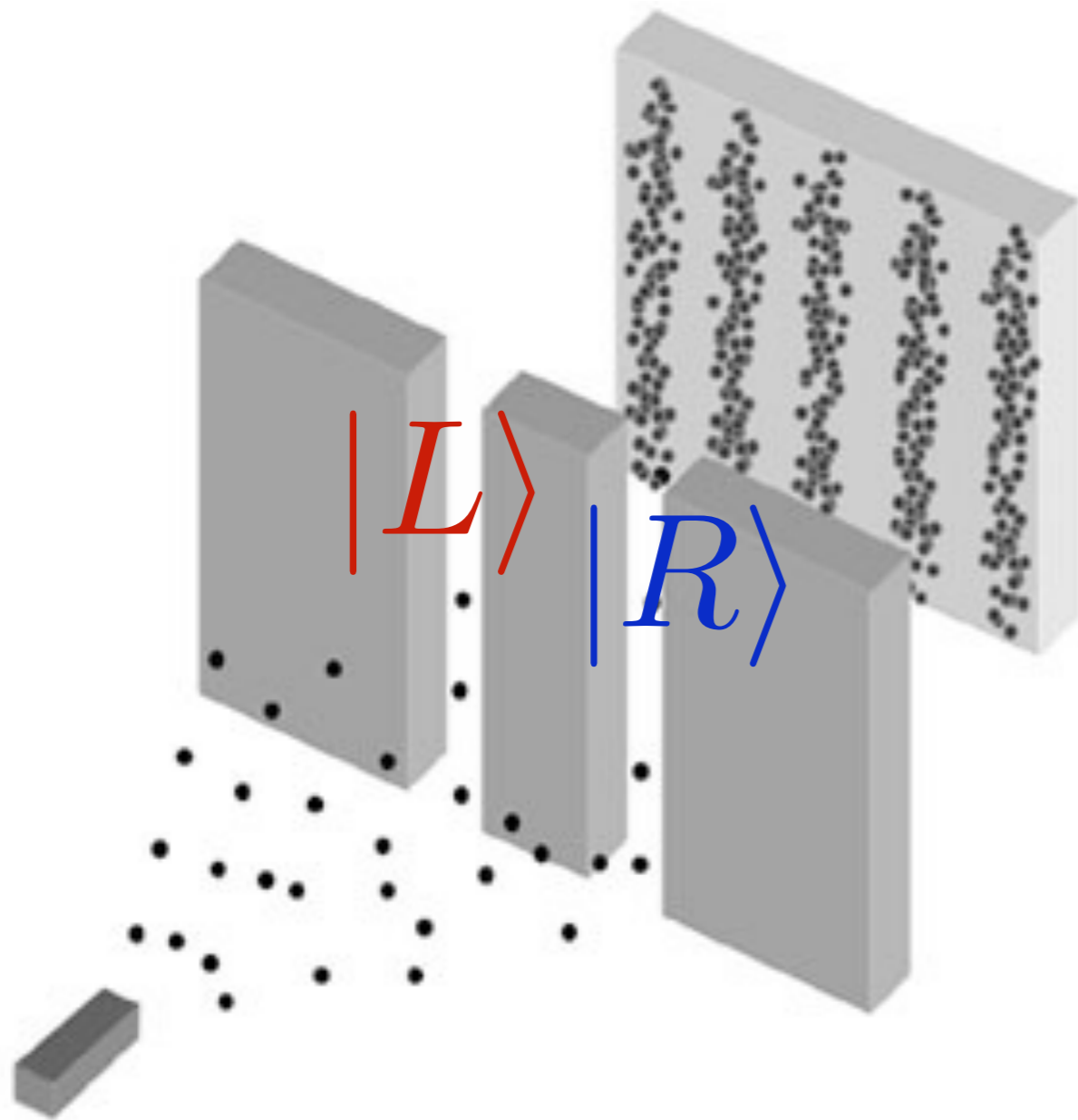
## The double slit experiment



Let  $|L\rangle$  represent the state with the electron in the left slit

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment

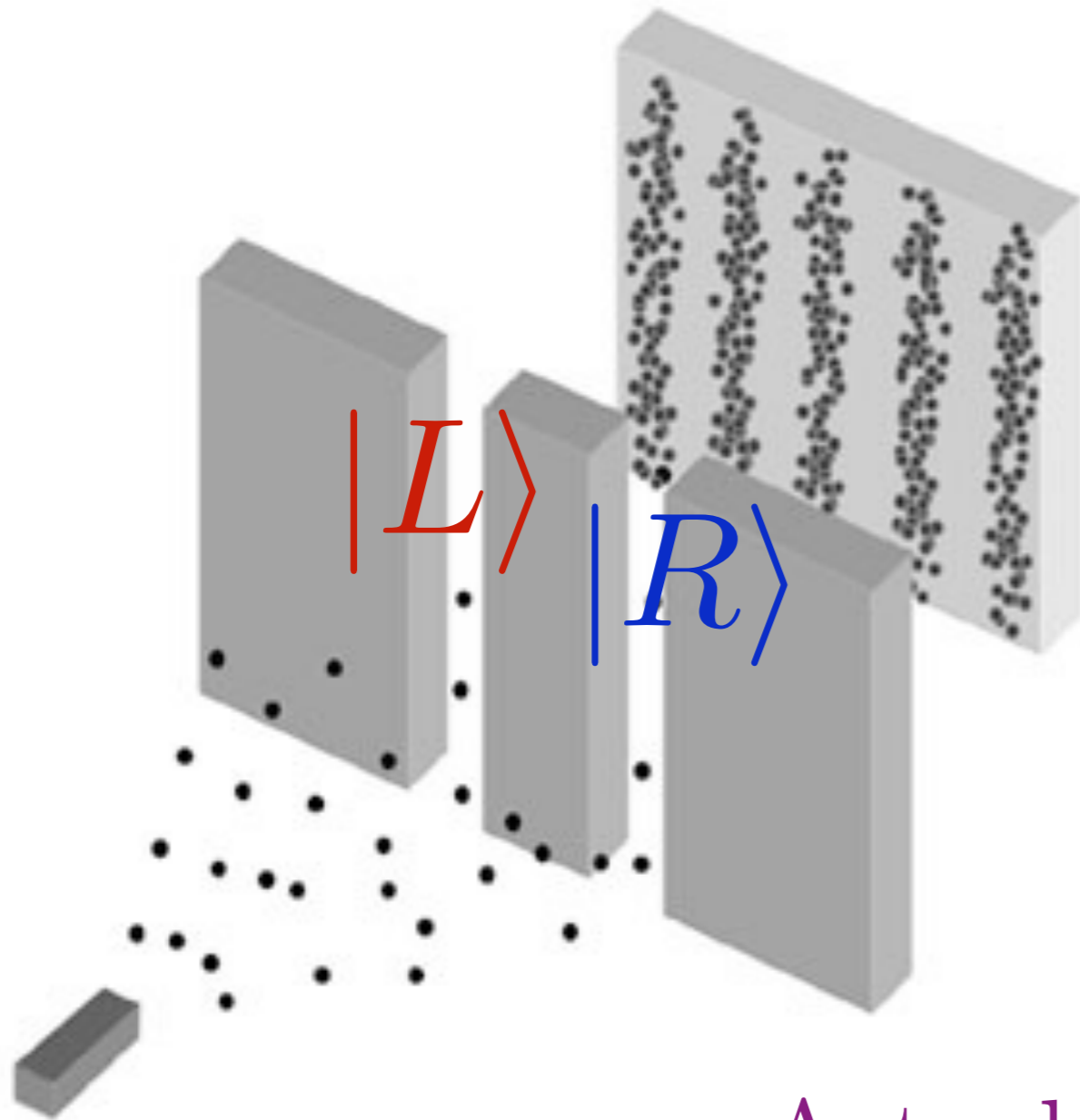


Let  $|L\rangle$  represent the state with the electron in the left slit

And  $|R\rangle$  represents the state with the electron in the right slit

# Principles of Quantum Mechanics: I. Quantum Superposition

## The double slit experiment



Let  $|L\rangle$  represent the state with the electron in the left slit

And  $|R\rangle$  represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

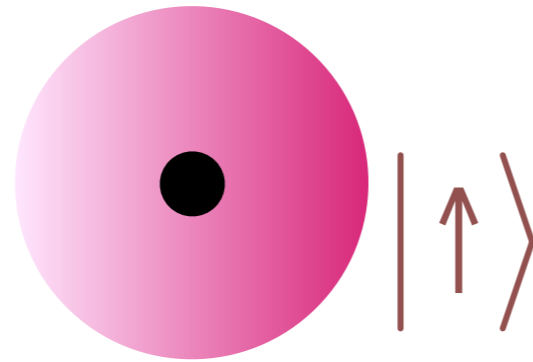
## Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition  
with more than one particle

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Hydrogen atom:

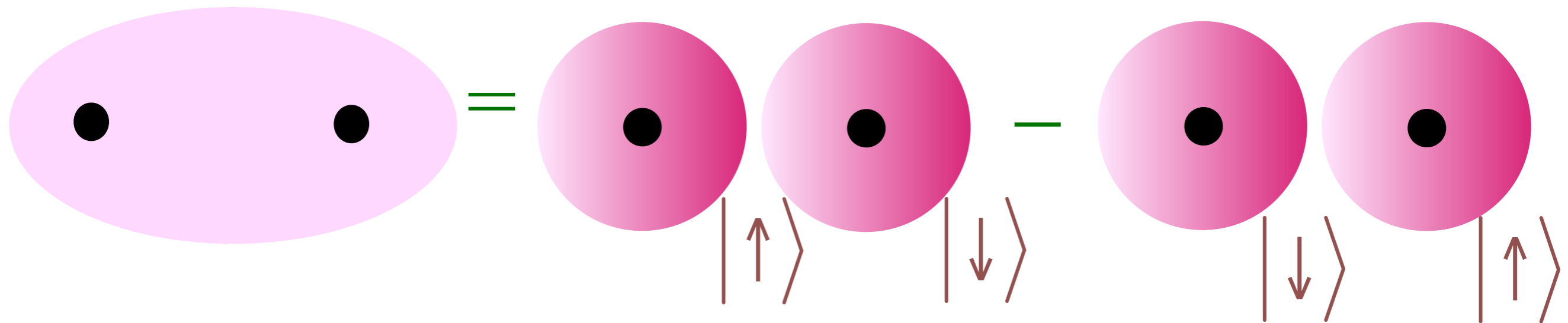


# Principles of Quantum Mechanics: II. Quantum Entanglement

## Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom: 

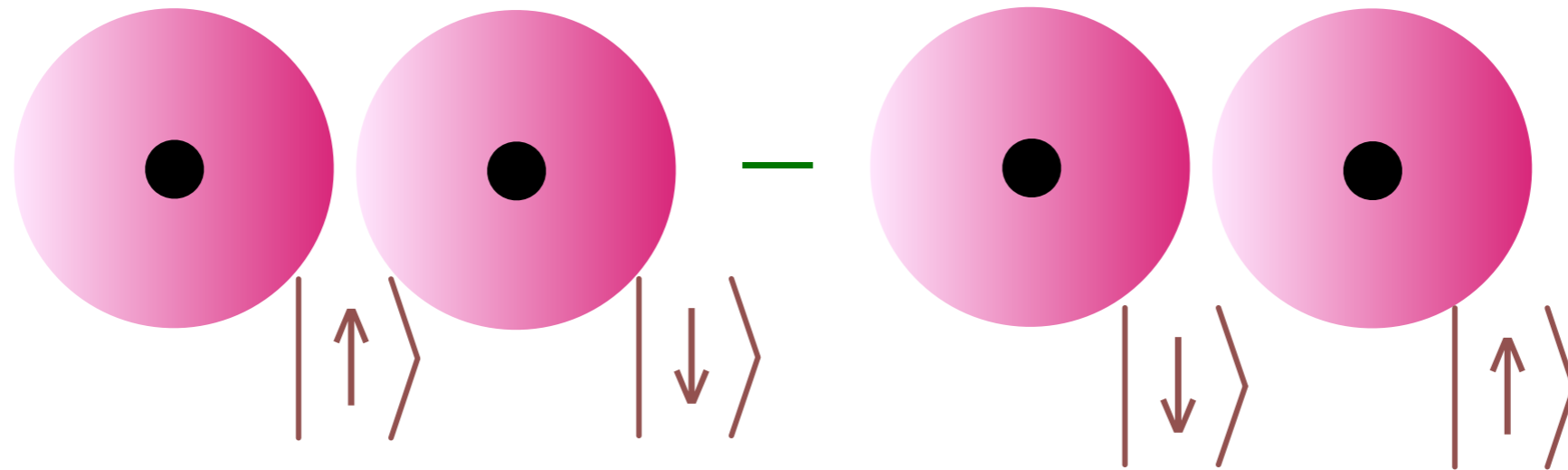
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

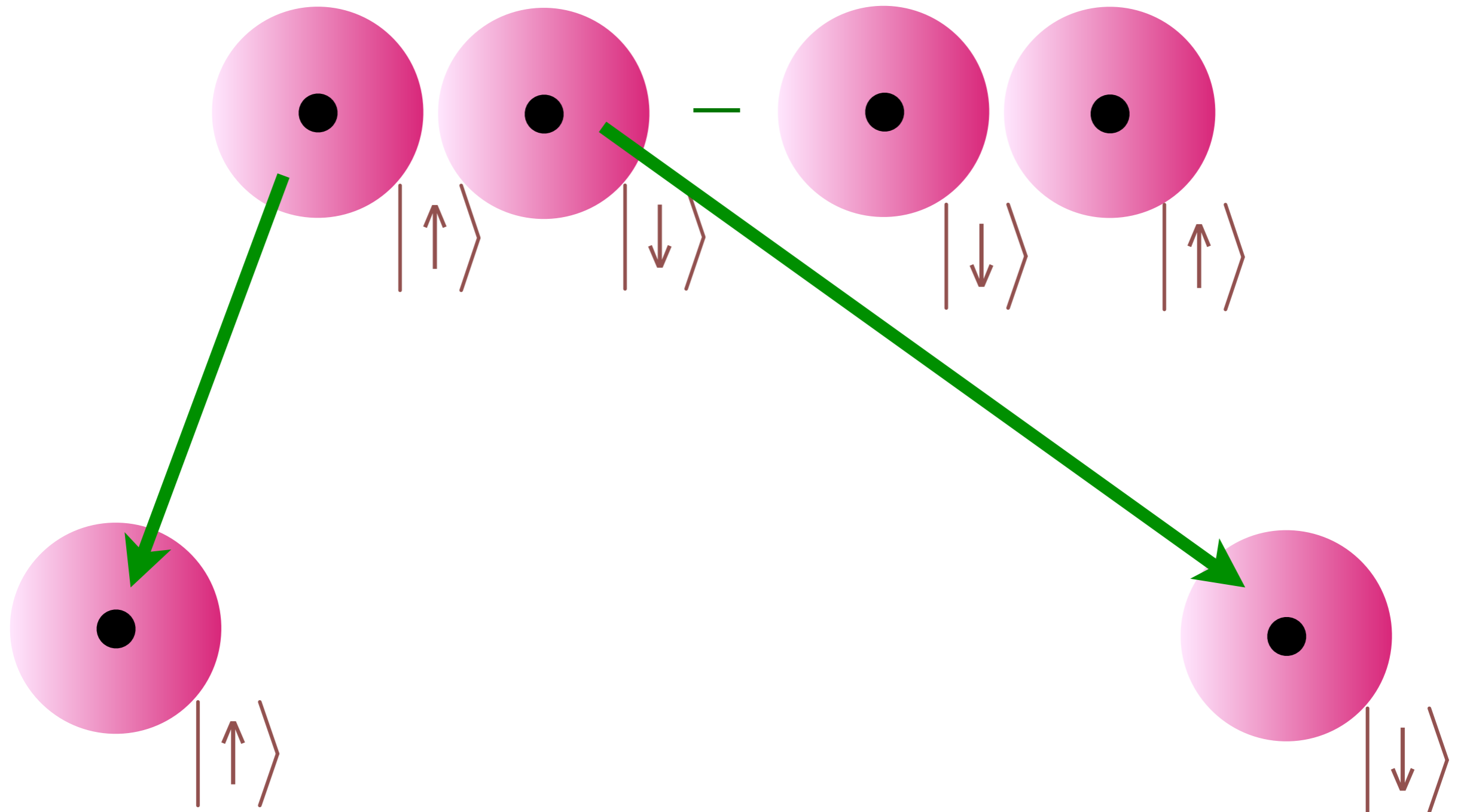
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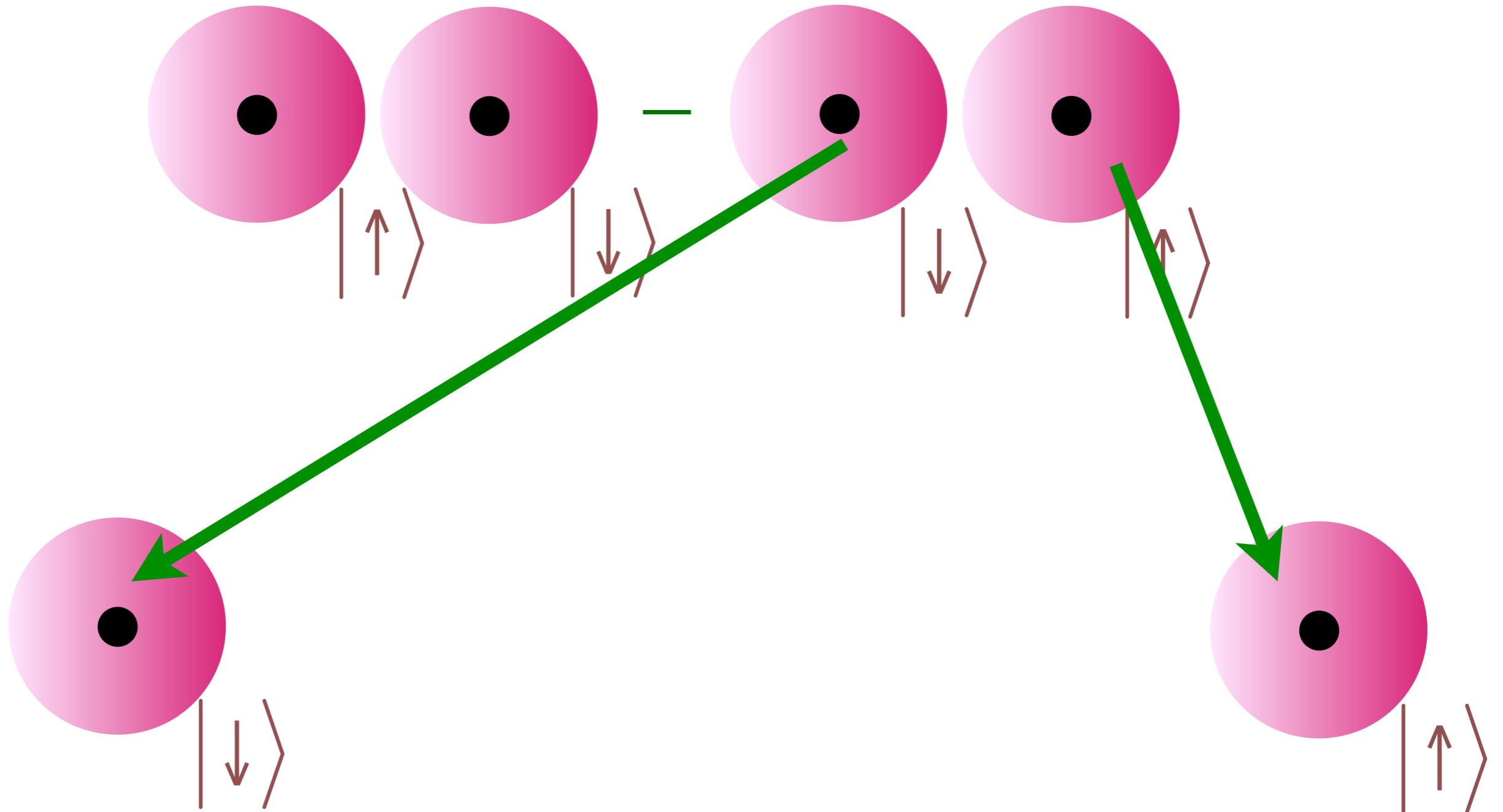
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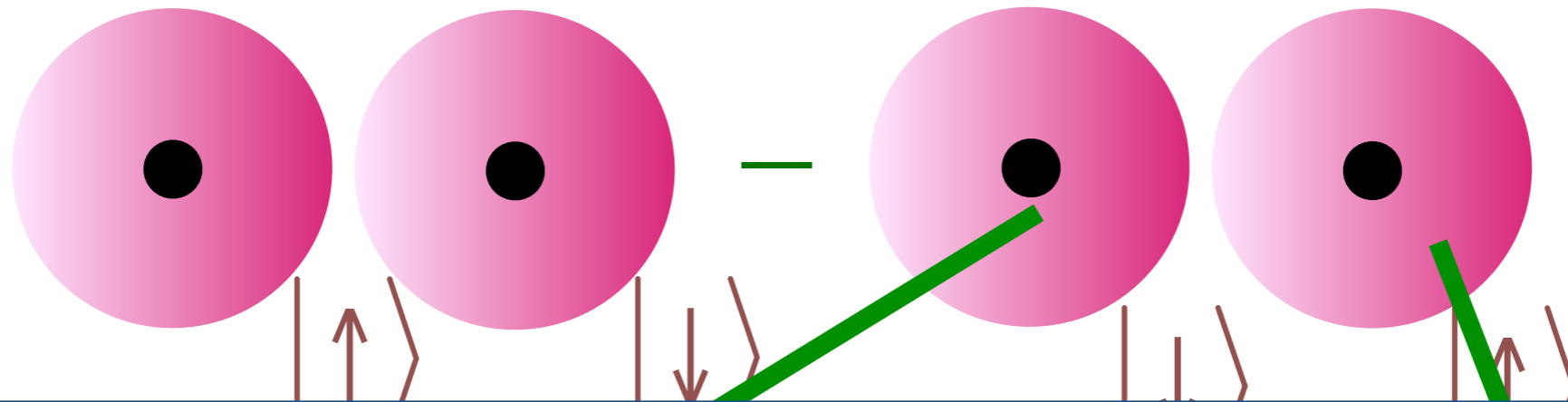
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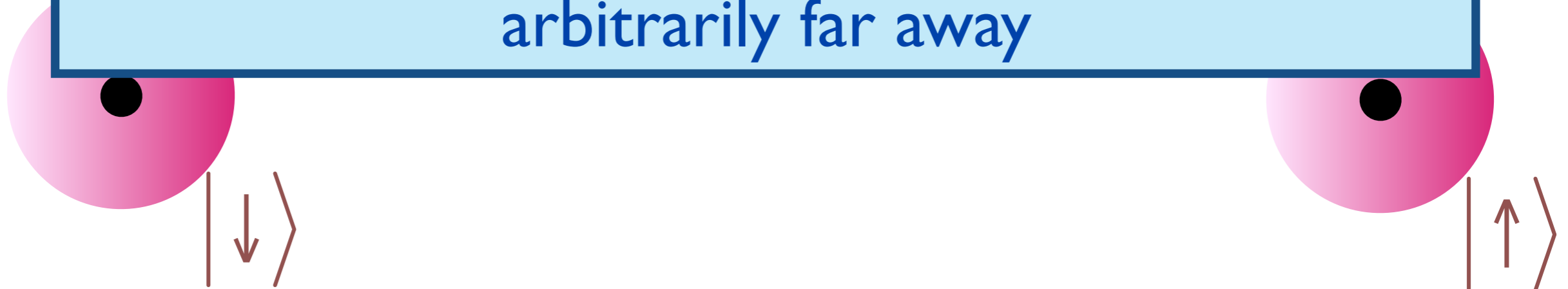


## Principles of Quantum Mechanics: II. Quantum Entanglement

### Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen “paradox” (1935):  
Measurement of one particle instantaneously  
determines the state of the other particle  
arbitrarily far away



# Quantum entanglement

**Quantum  
entanglement**

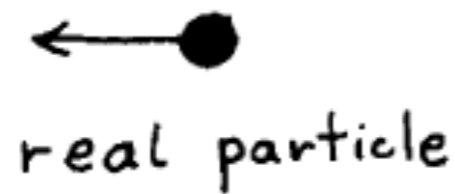
**Strange  
metals**

# Ordinary metals



Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

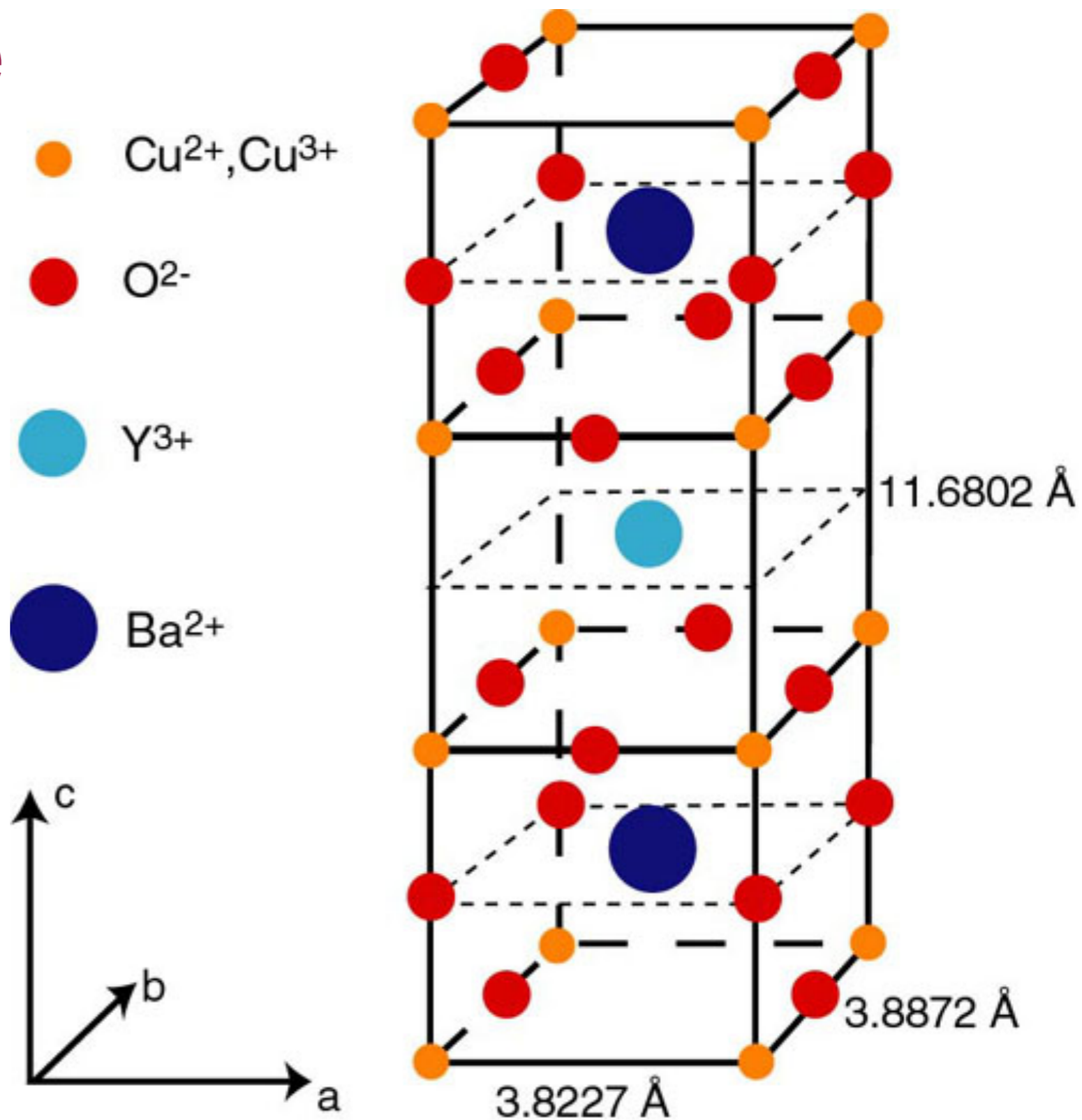
*Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.*

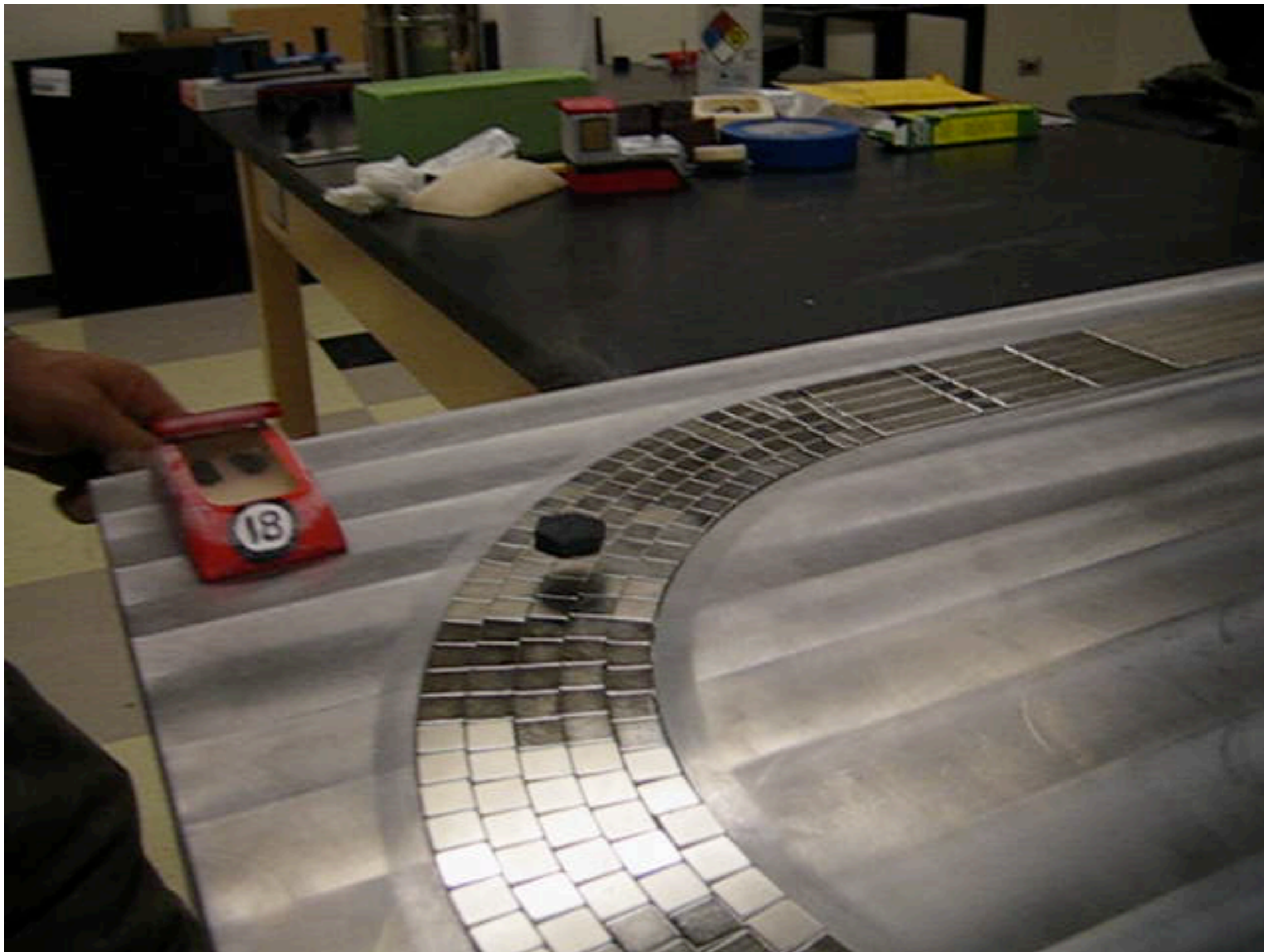


*Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.*

*Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time.*

# High temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

Quantum matter without quasiparticles

# Strange metal

Entangled electrons lead to “strange” temperature dependence of resistivity and other properties

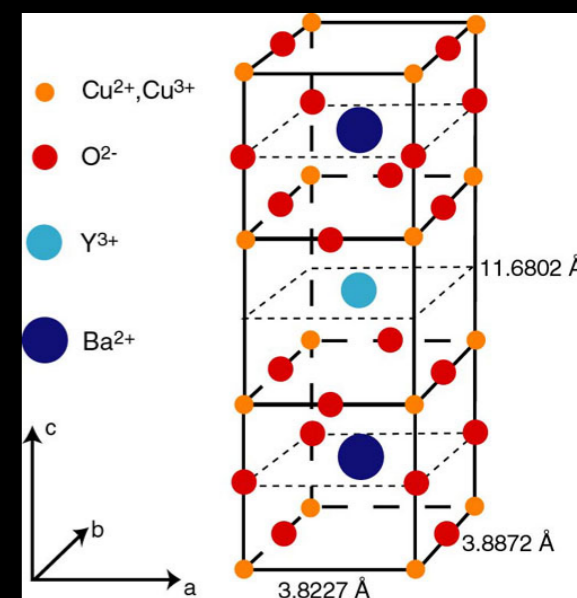
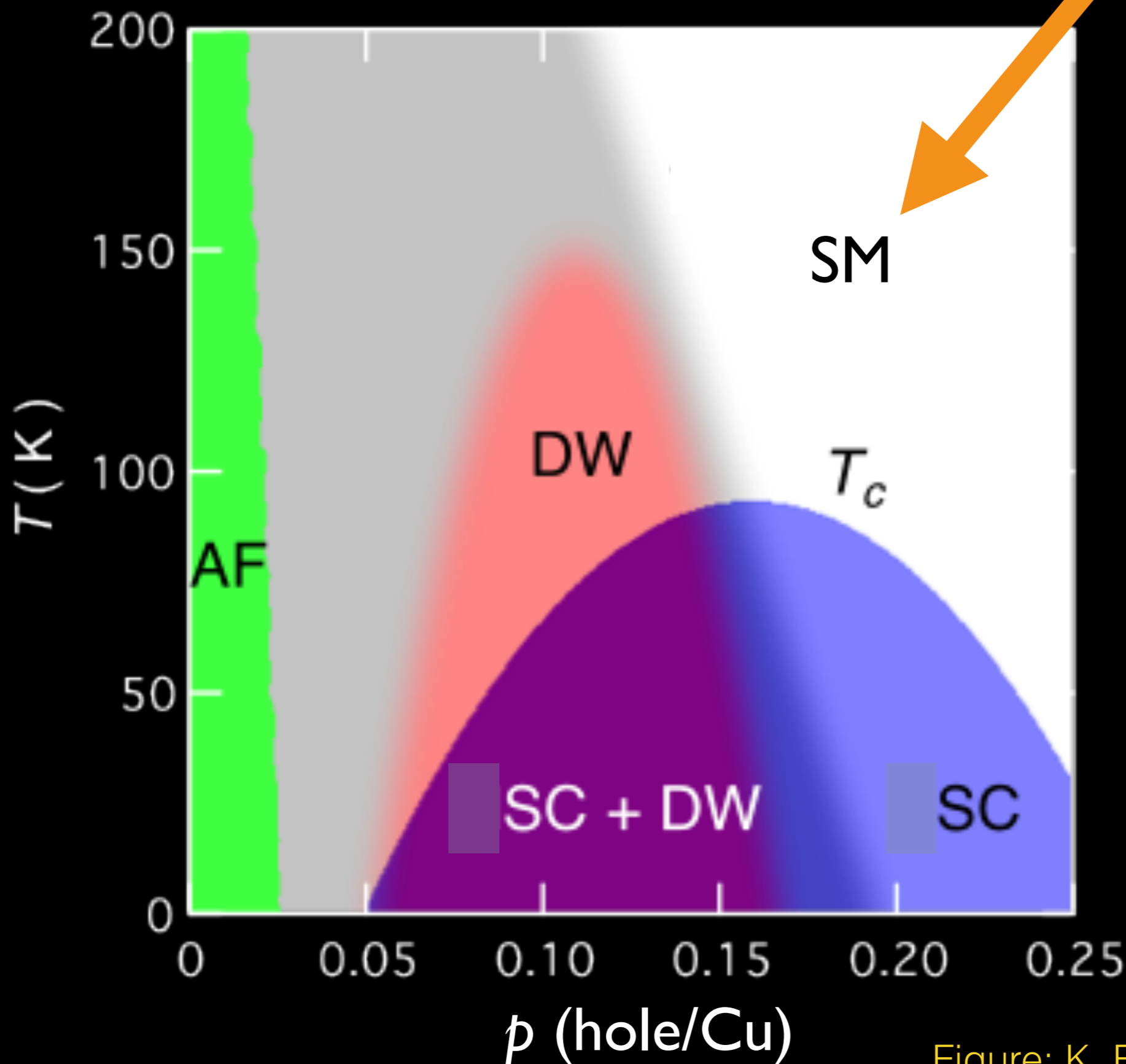


Figure: K. Fujita and J. C. Seamus Davis



“Strange”,

“Bad”,



or “Incoherent”,

metal has a resistivity,  $\rho$ , which obeys

$$\rho \sim T,$$

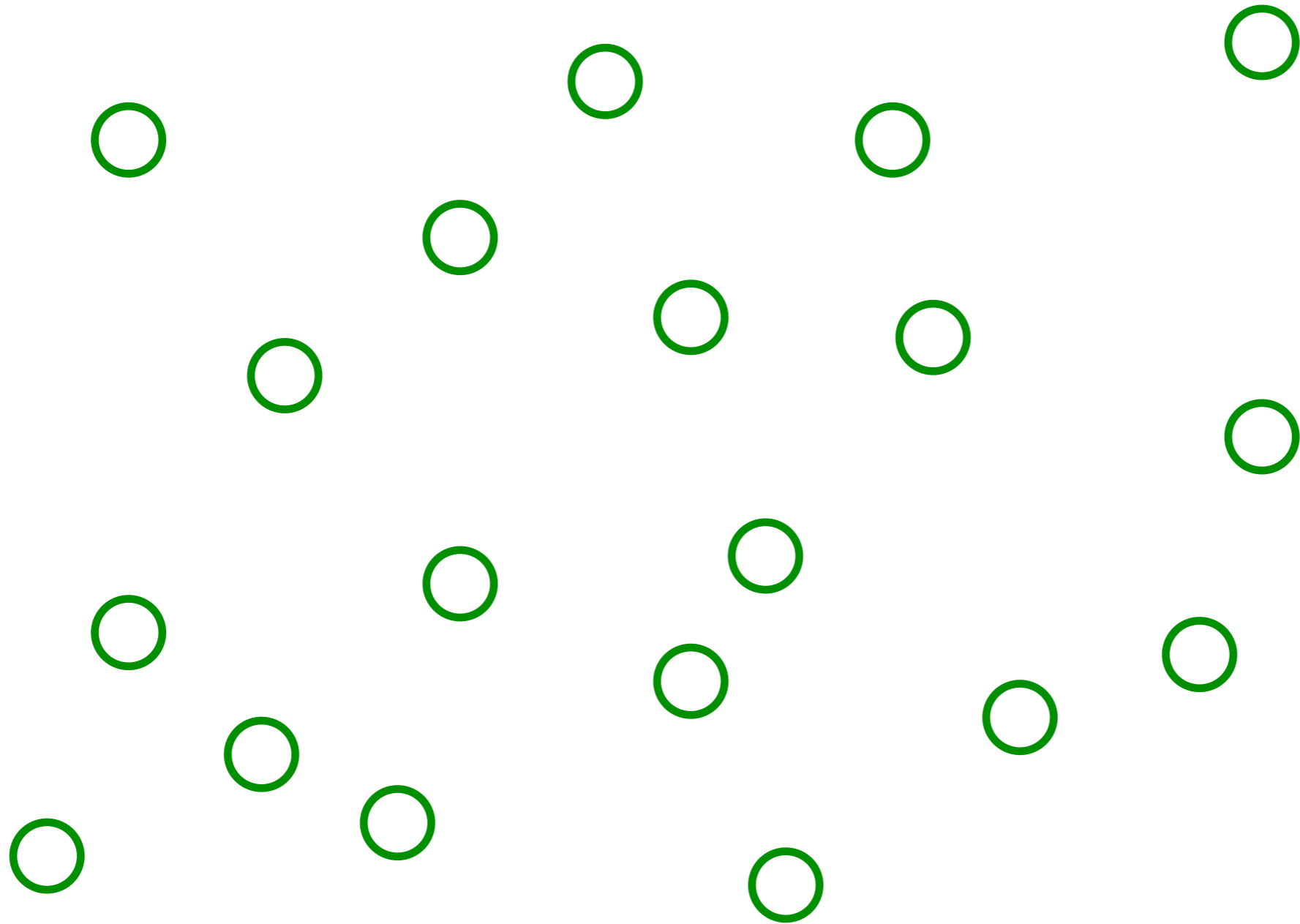
and

in some cases  $\rho \gg h/e^2$

(in two dimensions),

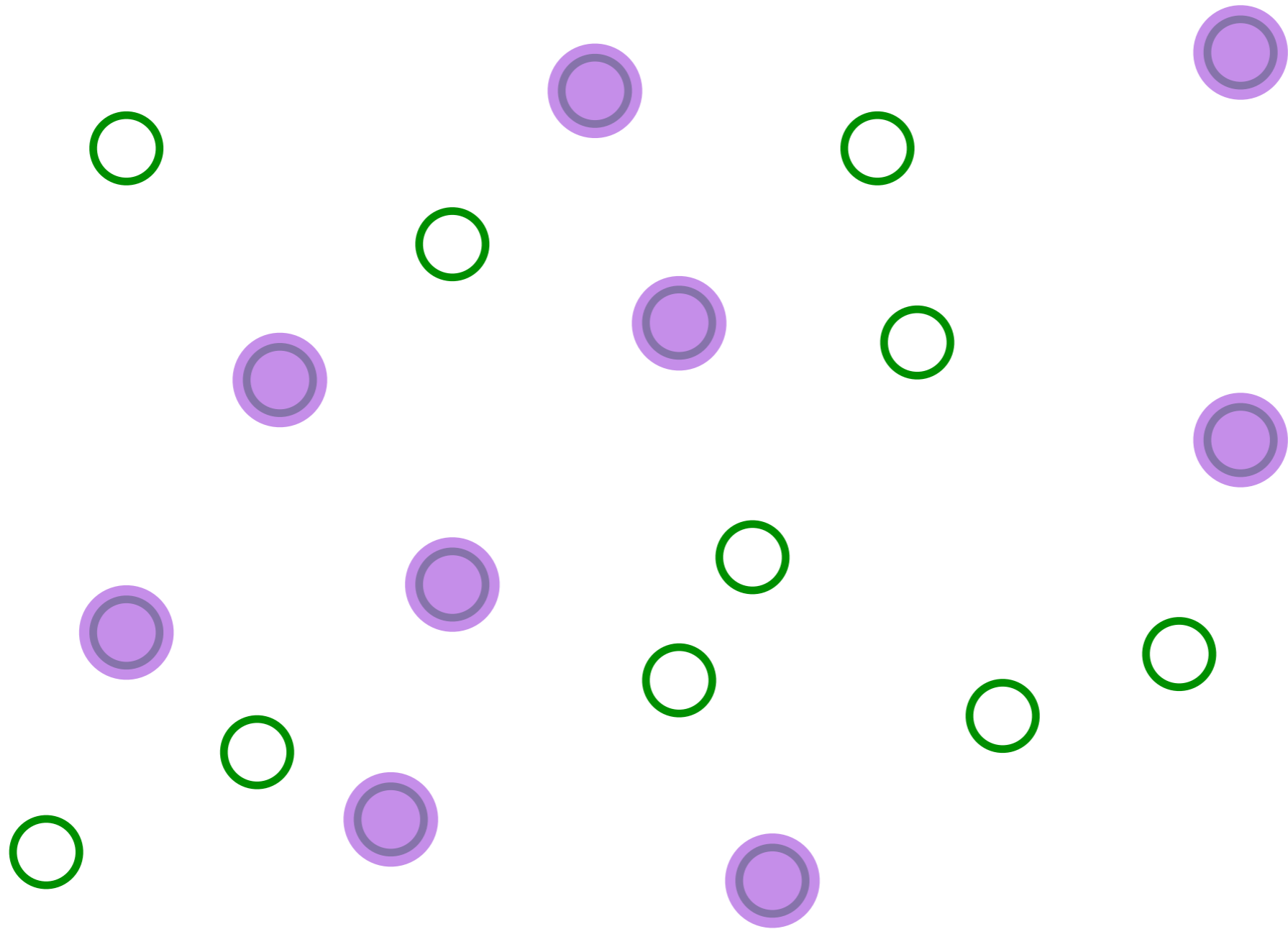
where  $h/e^2$  is the quantum unit of resistance.

# The Sachdev-Ye-Kitaev (SYK) model



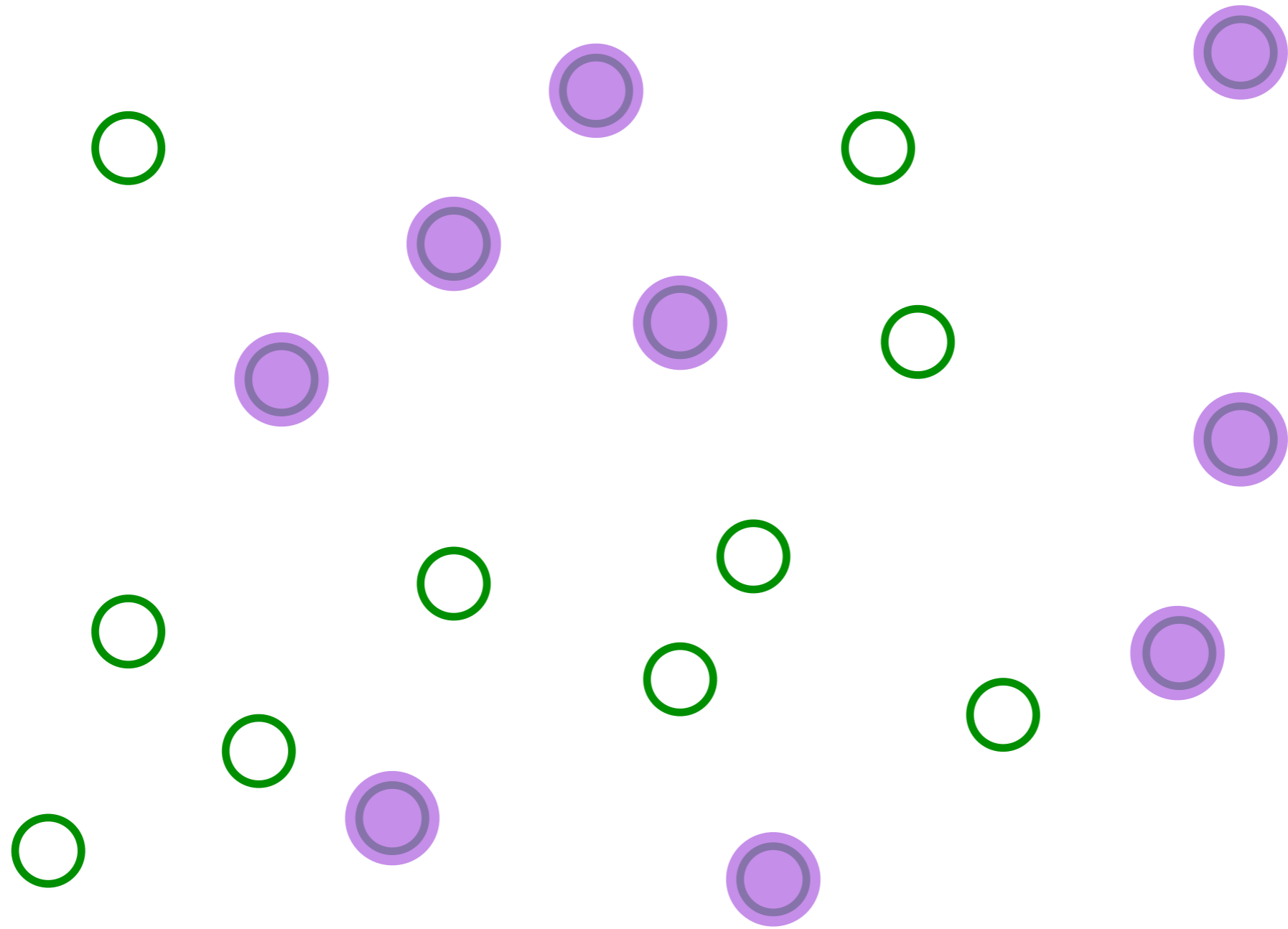
Pick a set of random positions

# The Sachdev-Ye-Kitaev (SYK) model



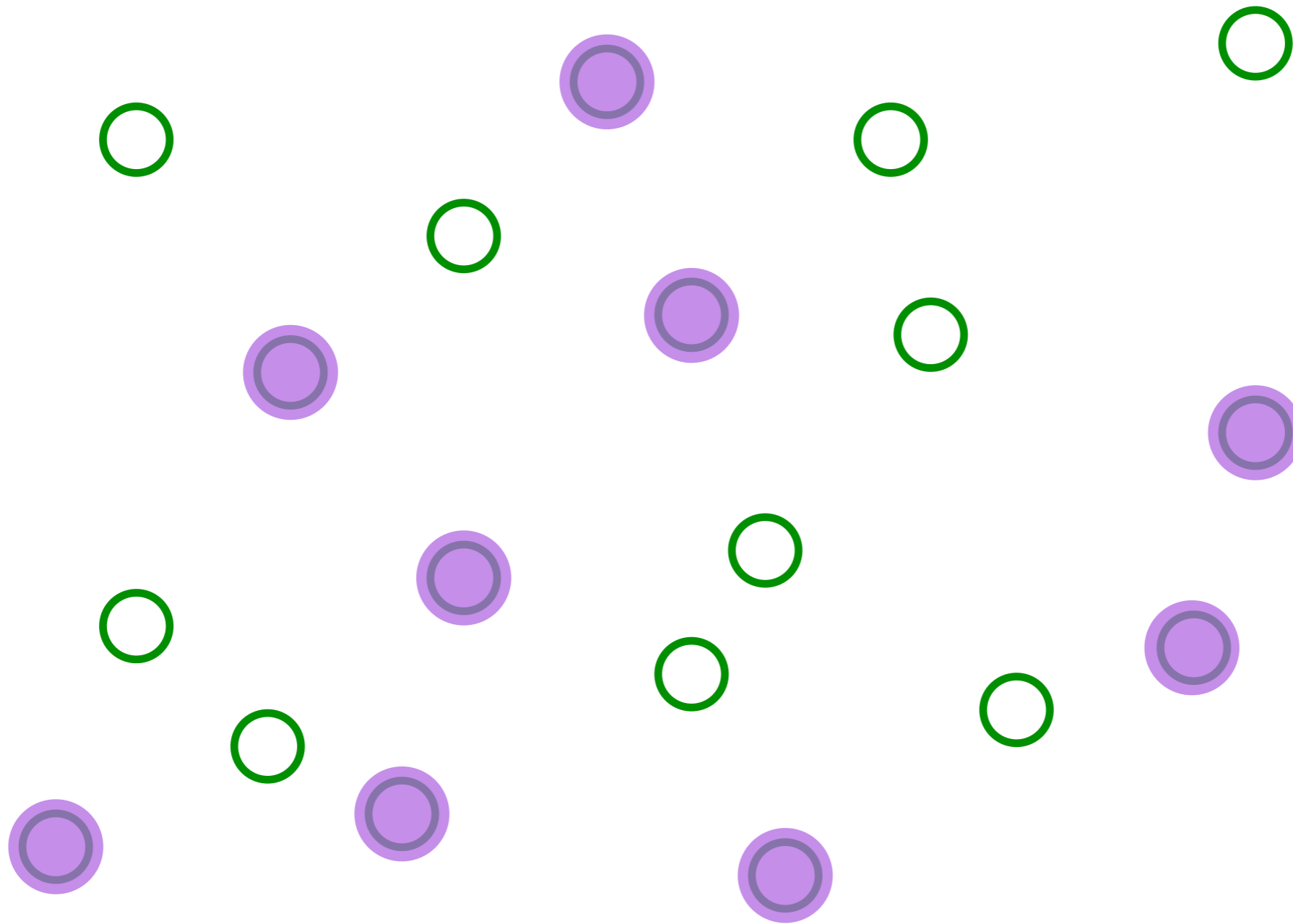
Place electrons randomly on some sites

# The Sachdev-Ye-Kitaev (SYK) model



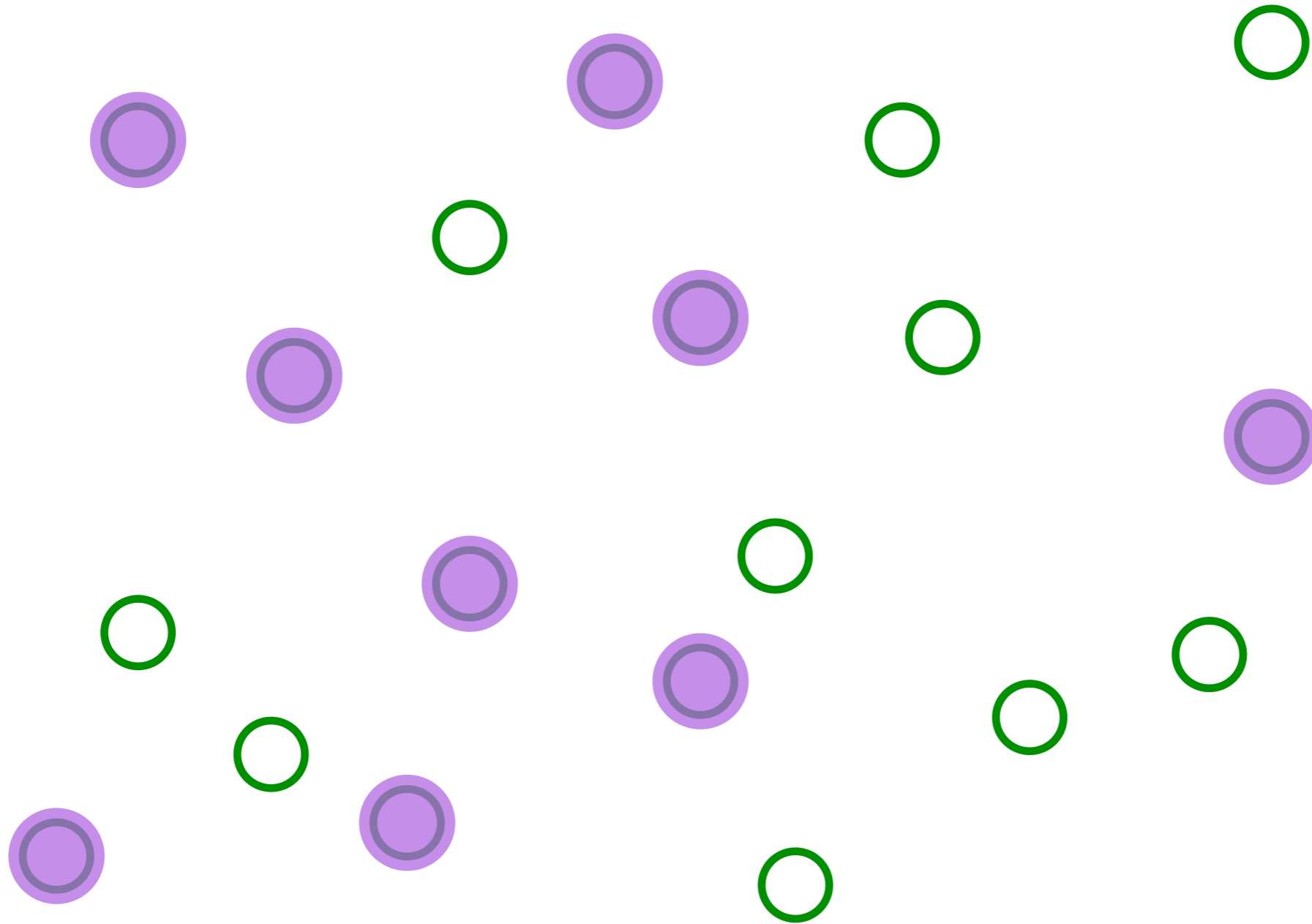
Entangle electrons pairwise randomly

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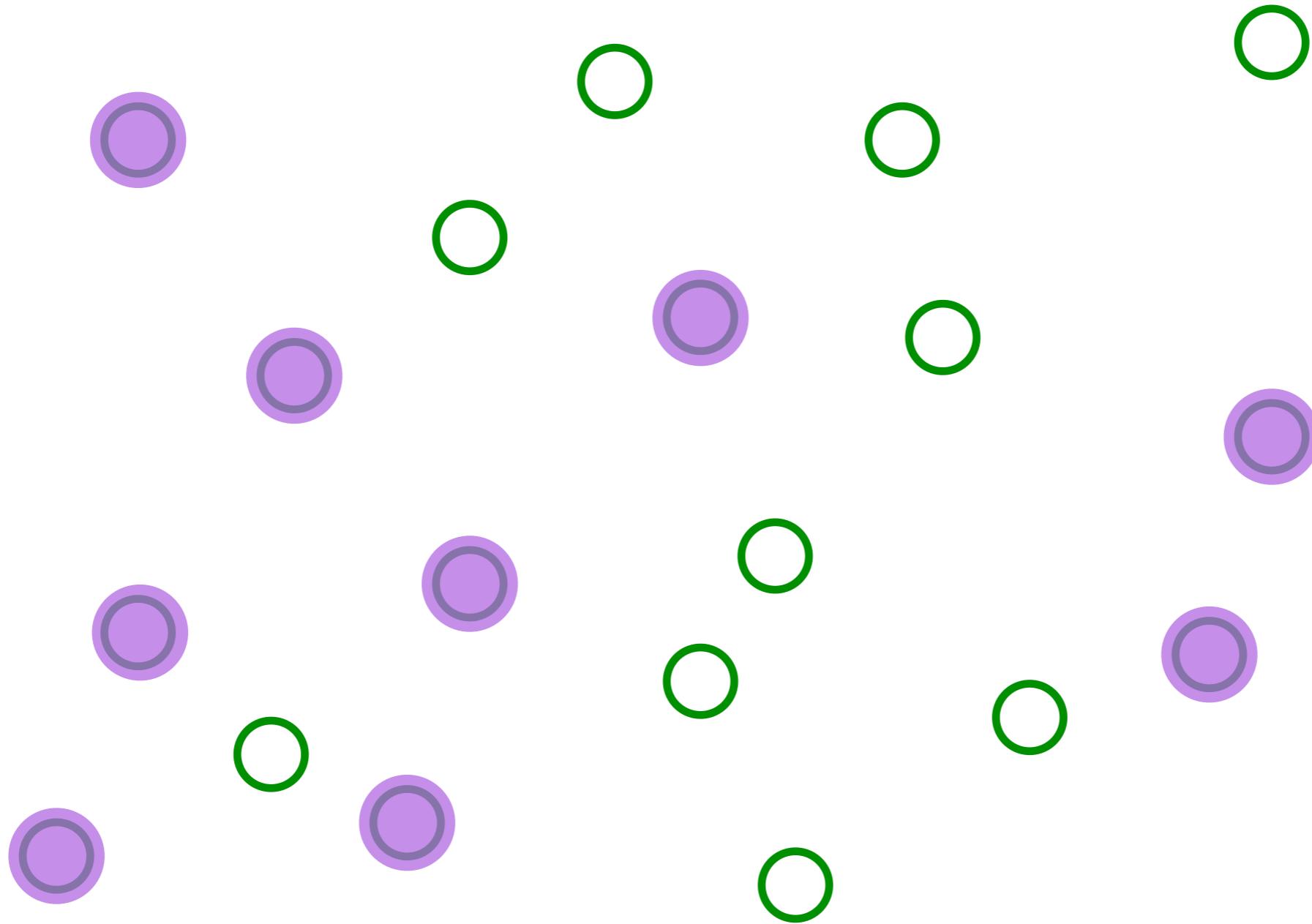
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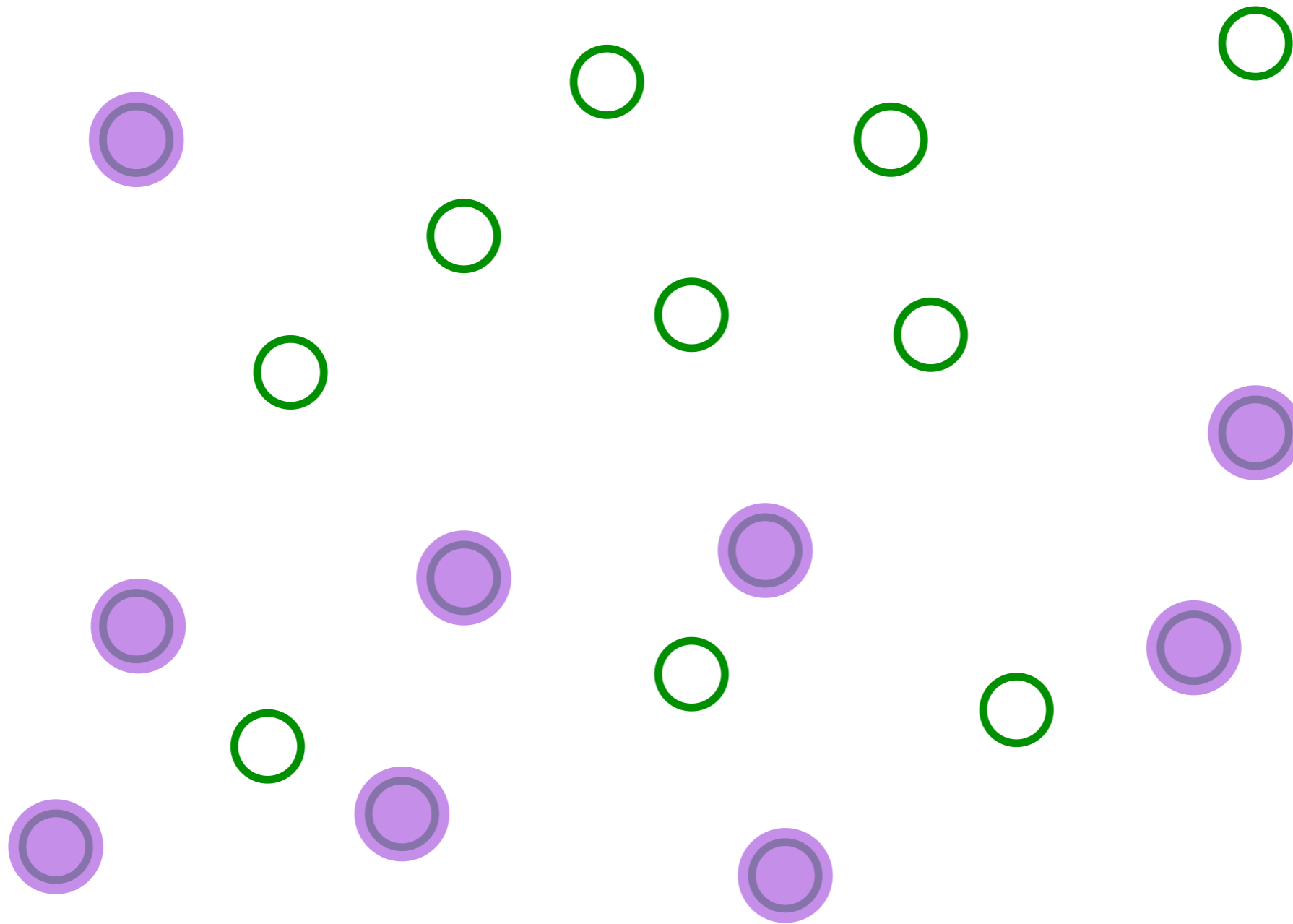
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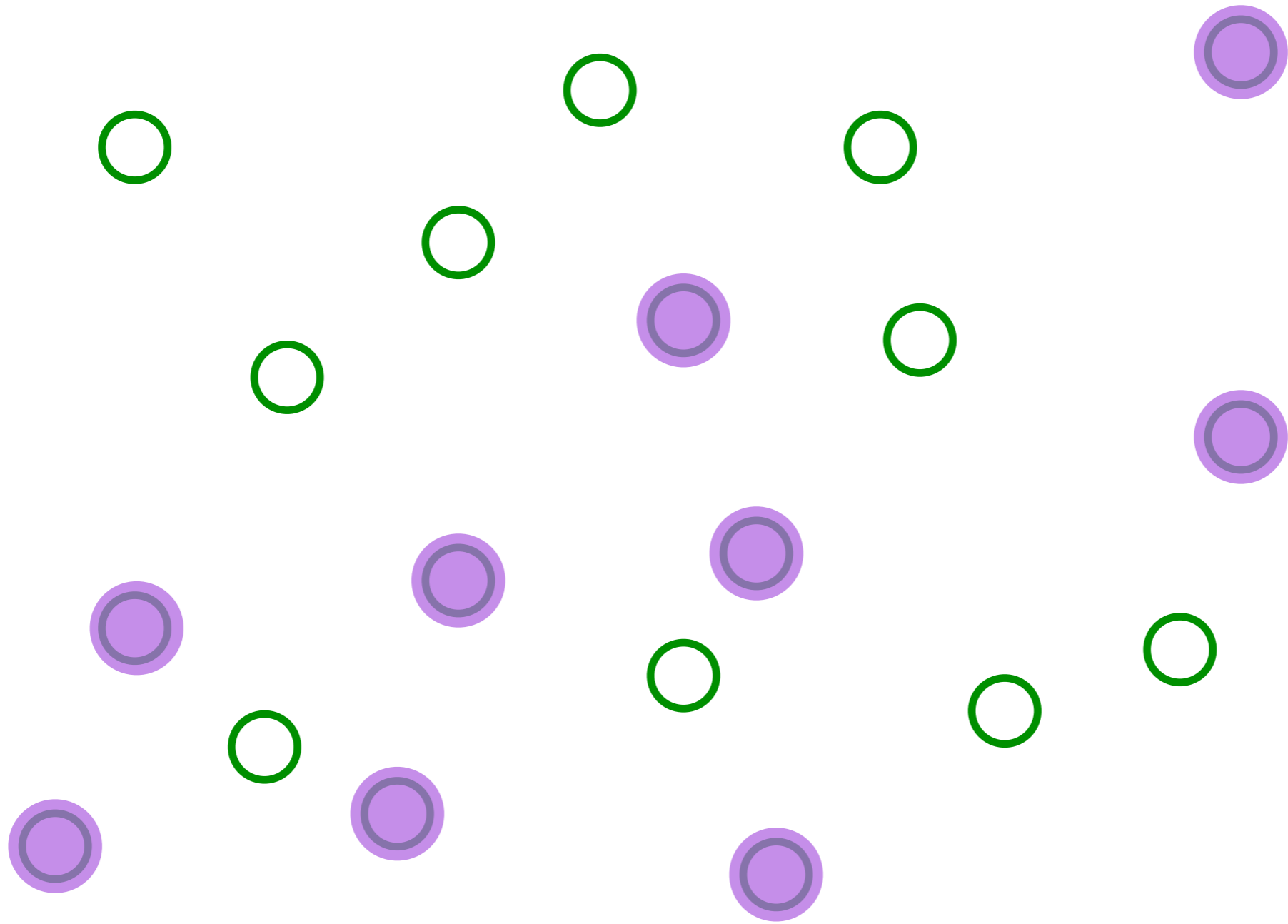
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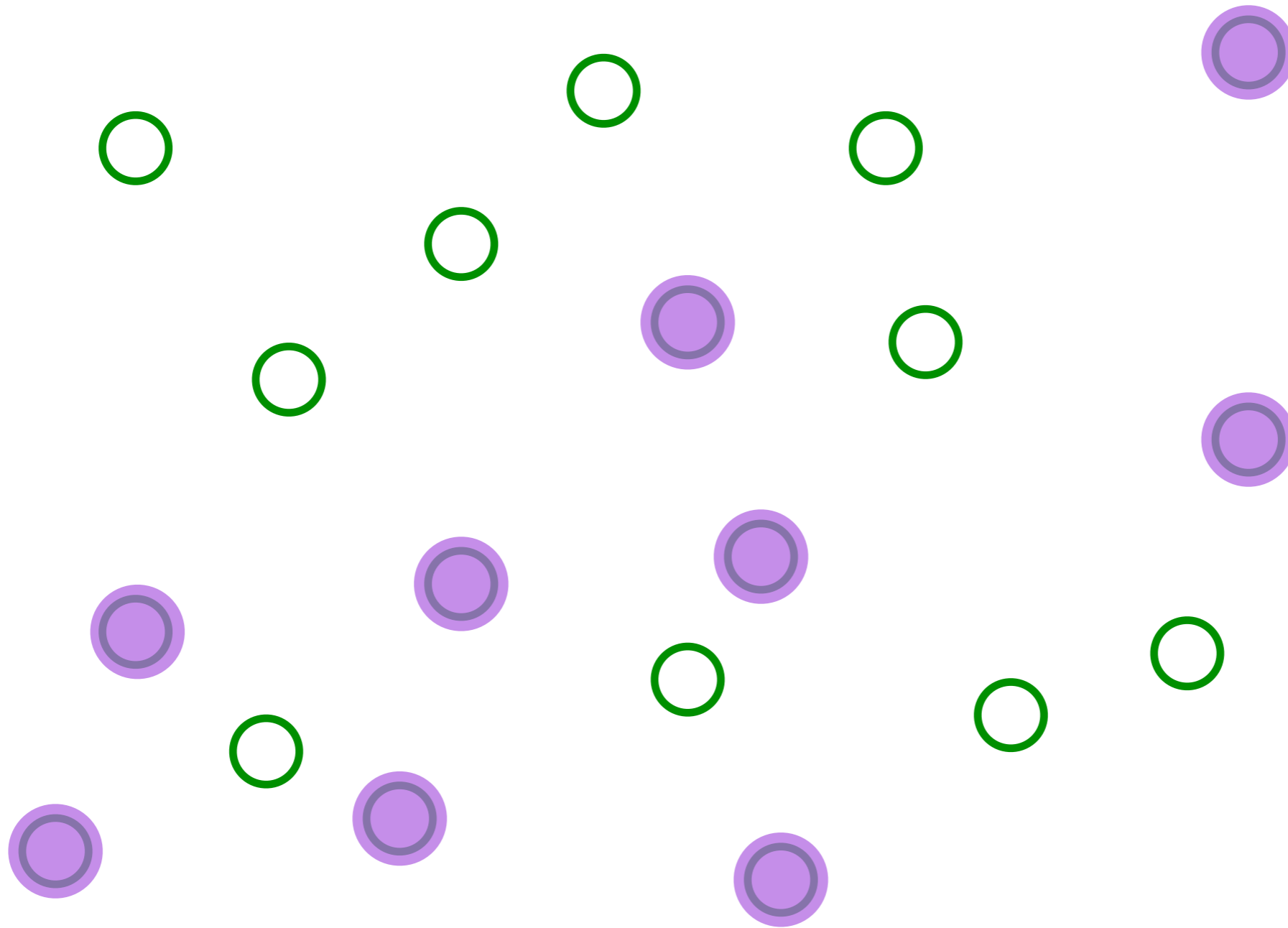
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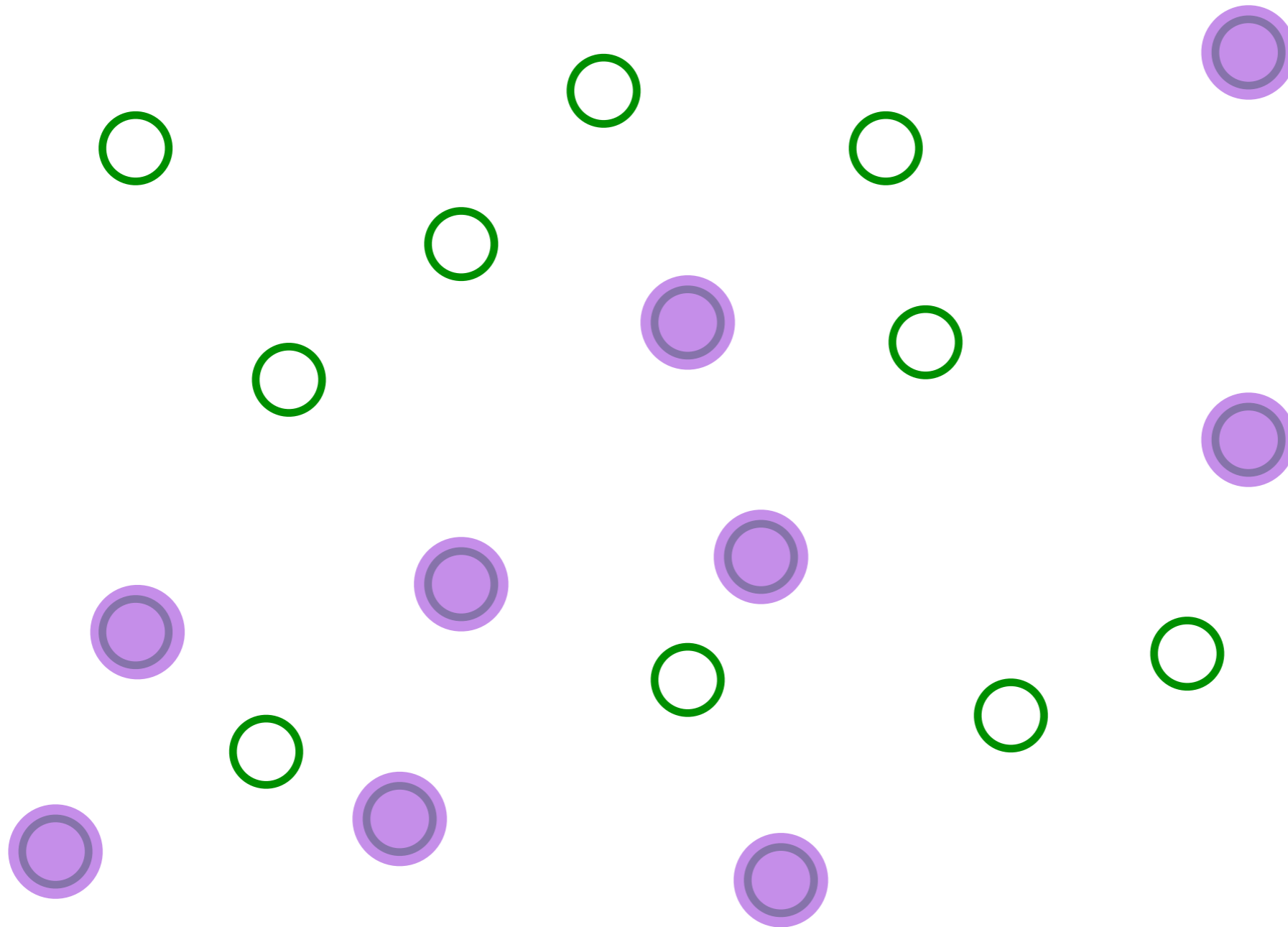
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# The Sachdev-Ye-Kitaev (SYK) model



The SYK model has “nothing but entanglement”

# The Sachdev-Ye-Kitaev (SYK) model

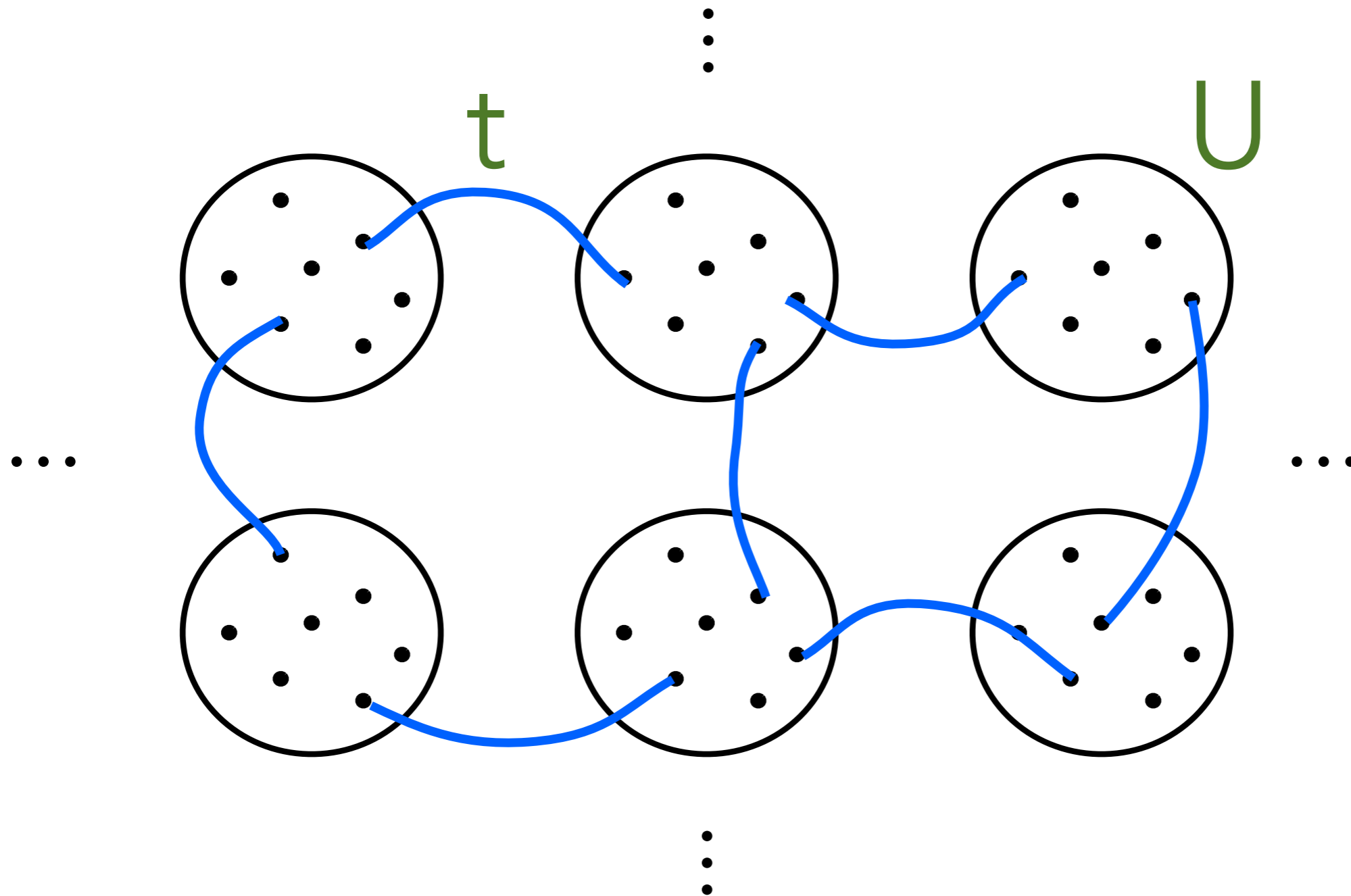


This describes both a strange metal and a black hole!

# A strongly correlated metal built from Sachdev-Ye-Kitaev models

Xue-Yang Song, Chao-Ming Jian, and L. Balents, arXiv:1705.00117

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)



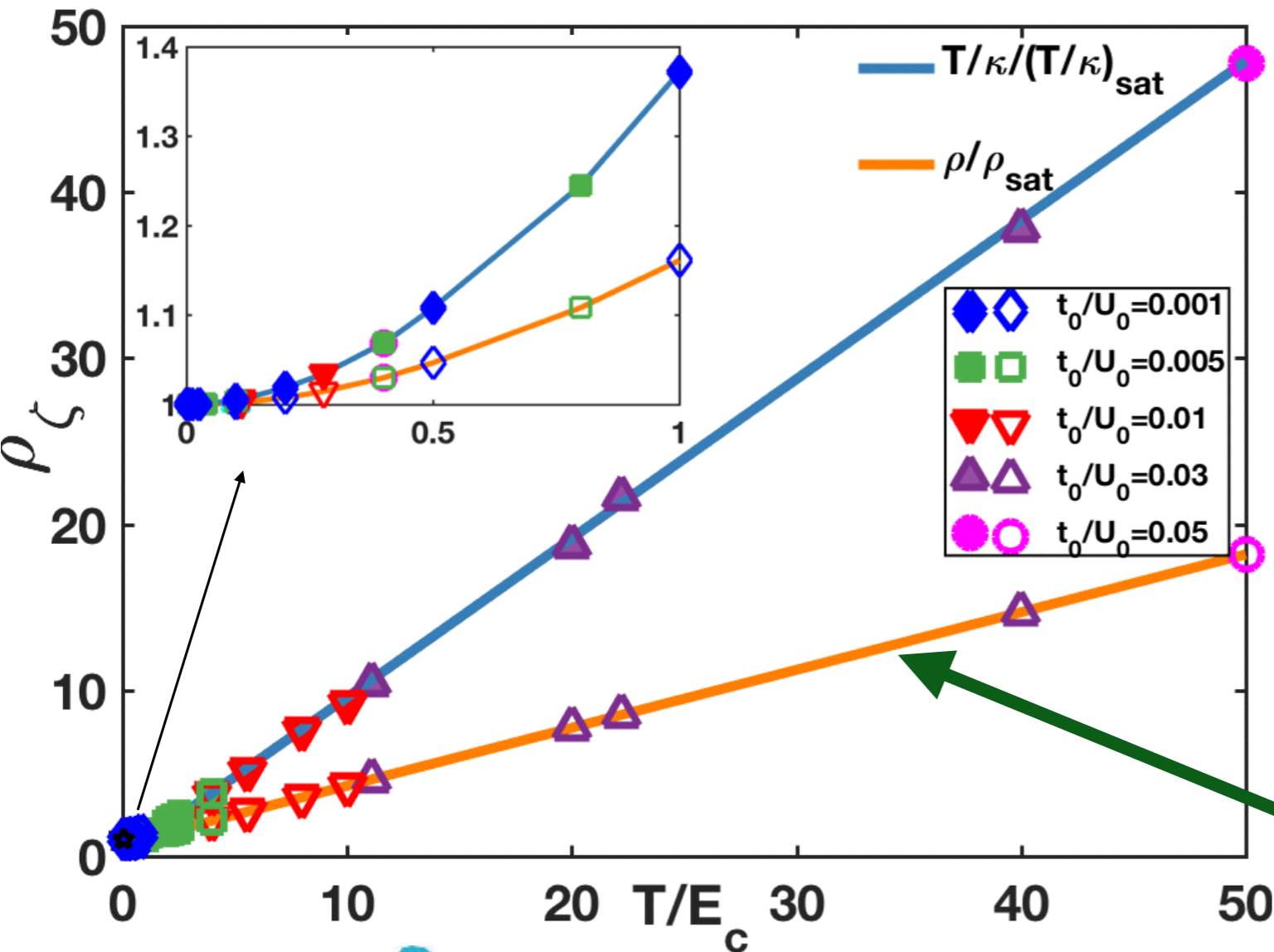
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Low 'coherence' scale

$$E_c \sim \frac{t^2}{U}$$



For  $E_c < T < U$ , the resistivity

$$\rho \sim \frac{h}{e^2} \left( \frac{T}{E_c} \right).$$



# Quantum matter without quasiparticles

*The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.*

# Quantum matter without quasiparticles

*The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.*

Thermal equilibration into a chaotic quantum state happens very rapidly in systems without quasiparticle excitations: it happens in a

shortest possible time of order

$$\frac{\hbar}{k_B T}$$

(SS 1999, Maldacena, Shenker, Stanford 2015)

**Quantum  
entanglement**

**Strange  
metals**

**Quantum  
entanglement**

**Black  
holes**

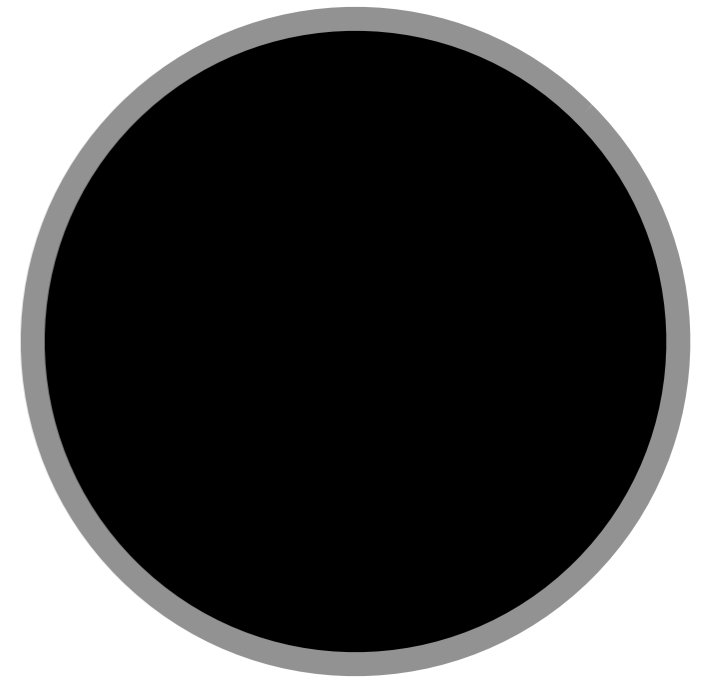
**Strange  
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# Black Holes

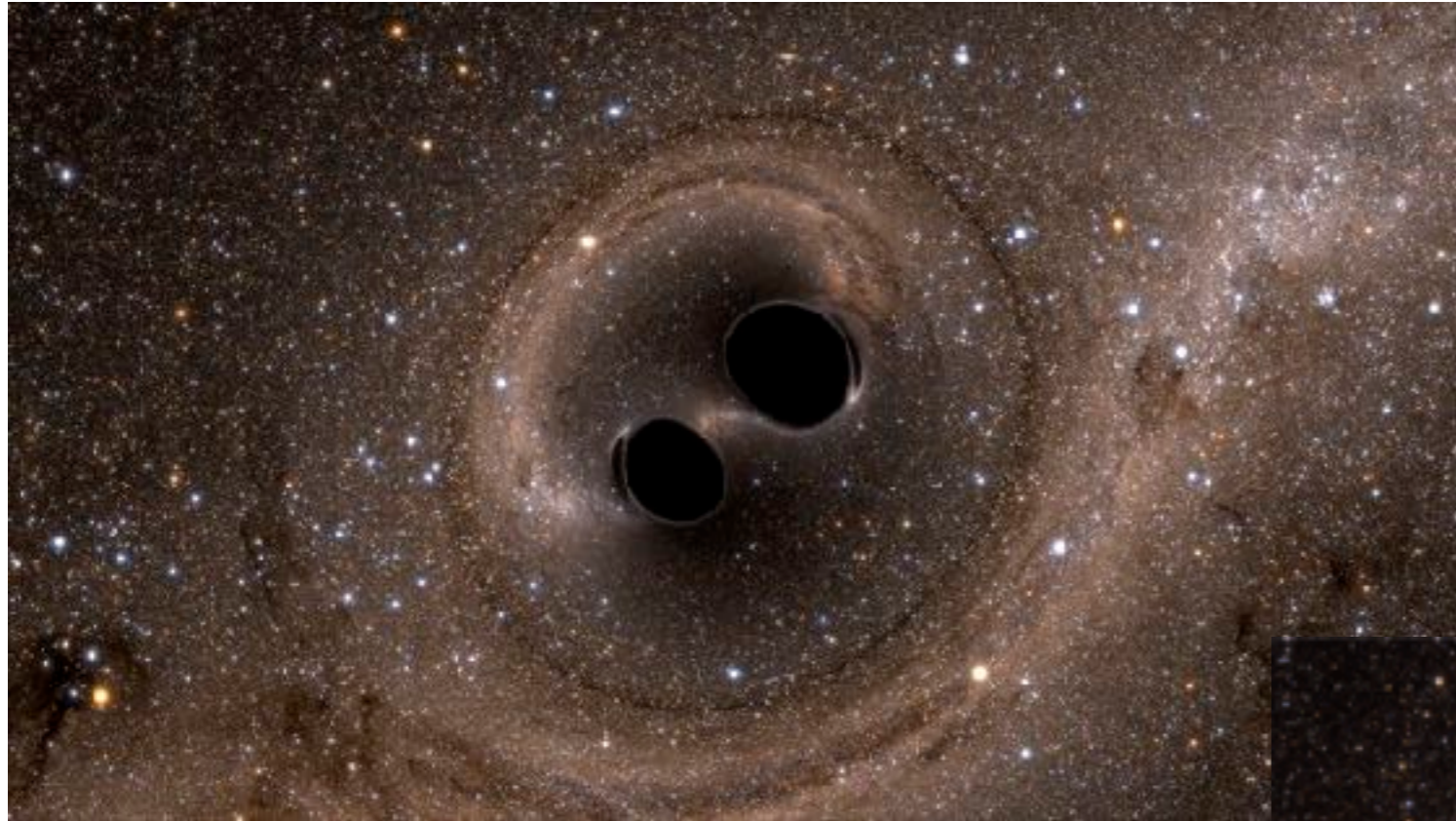
Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

Horizon radius  $R = \frac{2GM}{c^2}$

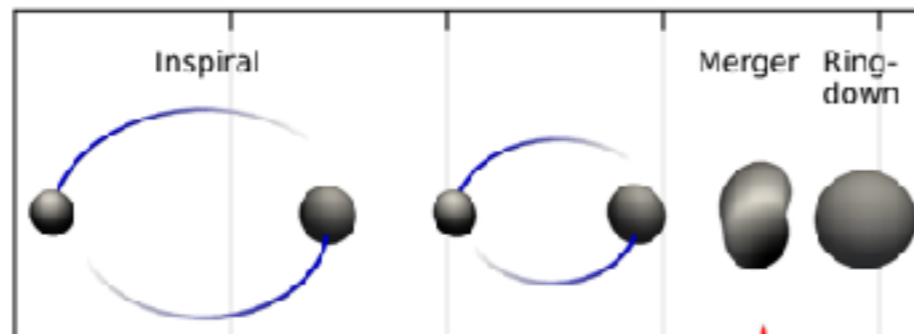
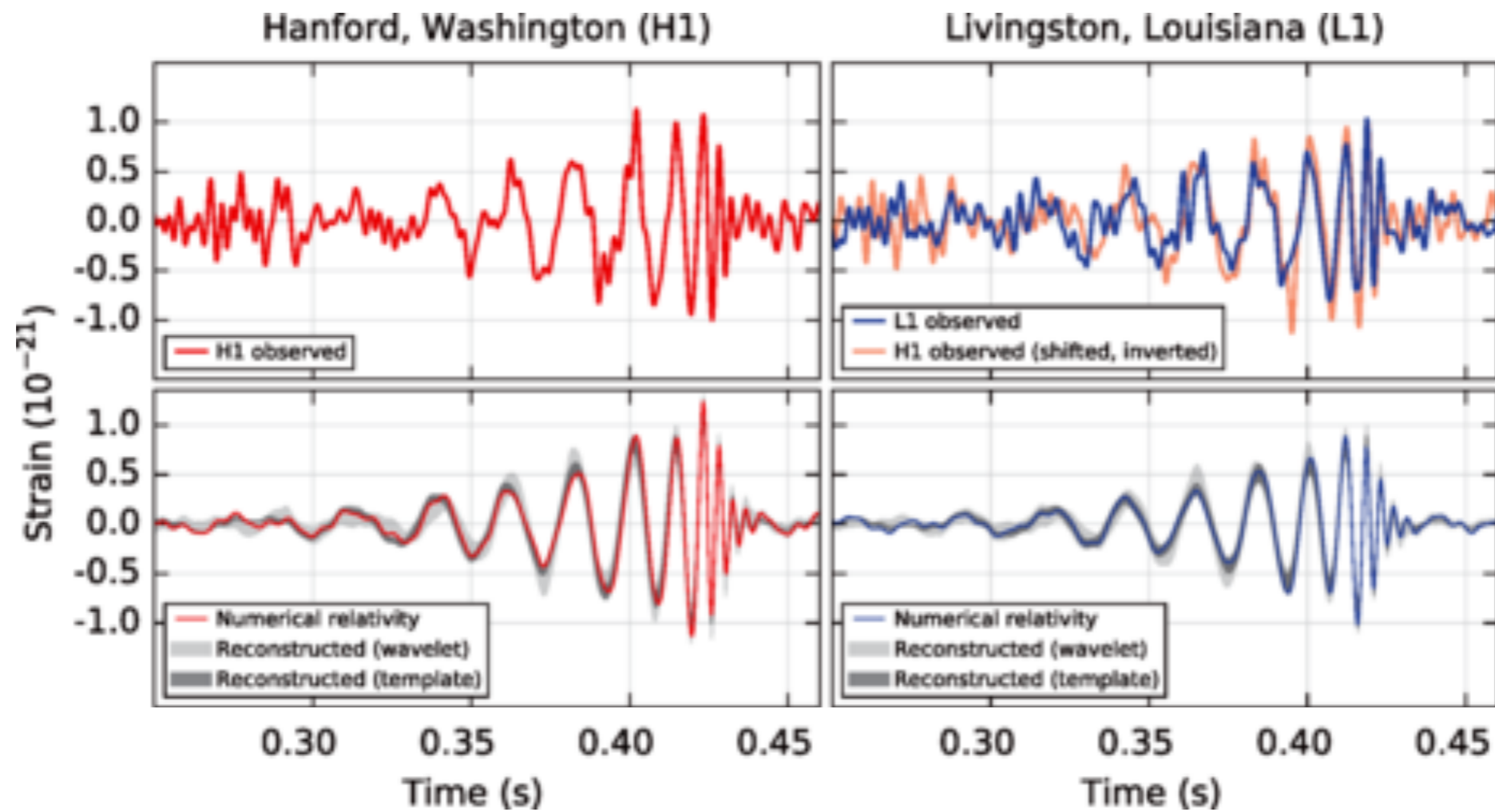


On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



0.1 seconds later !





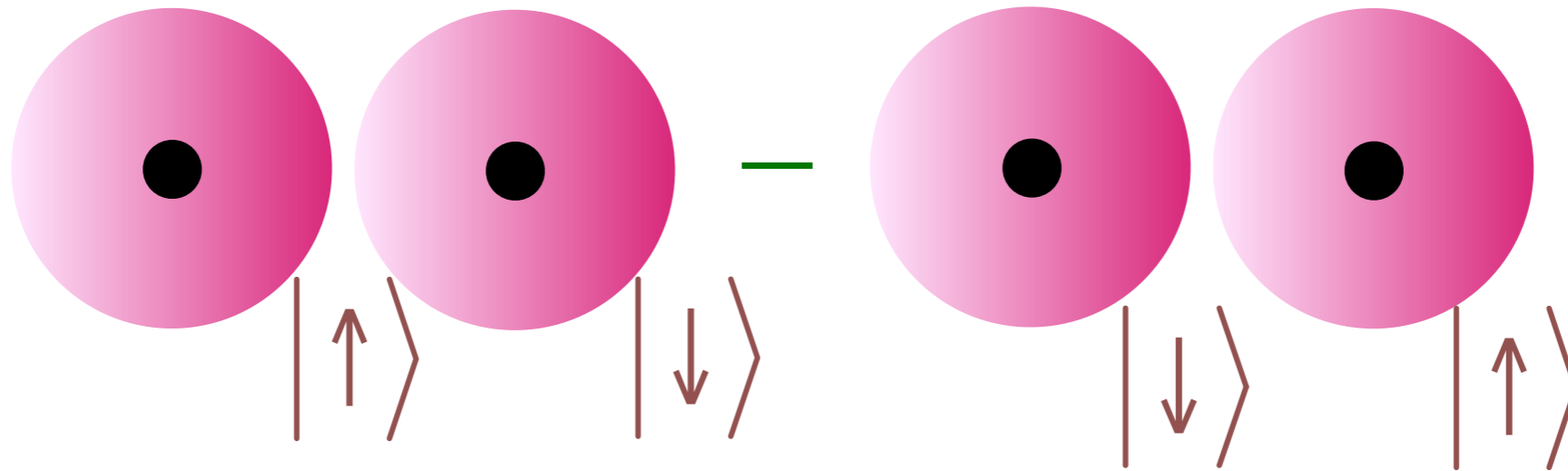
**LIGO**  
**September 14, 2015**

- The ring-down is predicted by General Relativity to happen in a time  $\frac{8\pi GM}{c^3} \sim 8$  milliseconds.

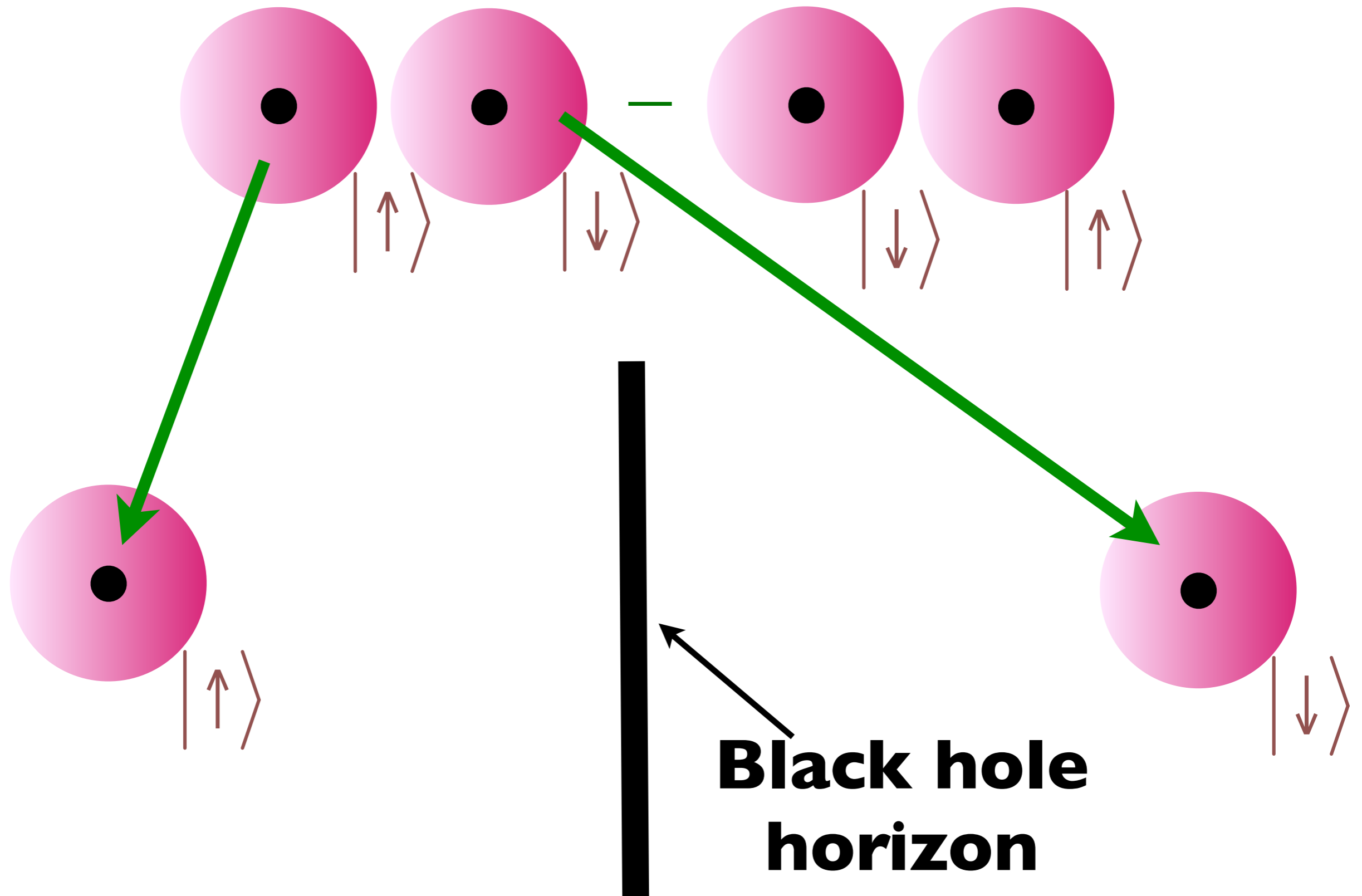
# Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

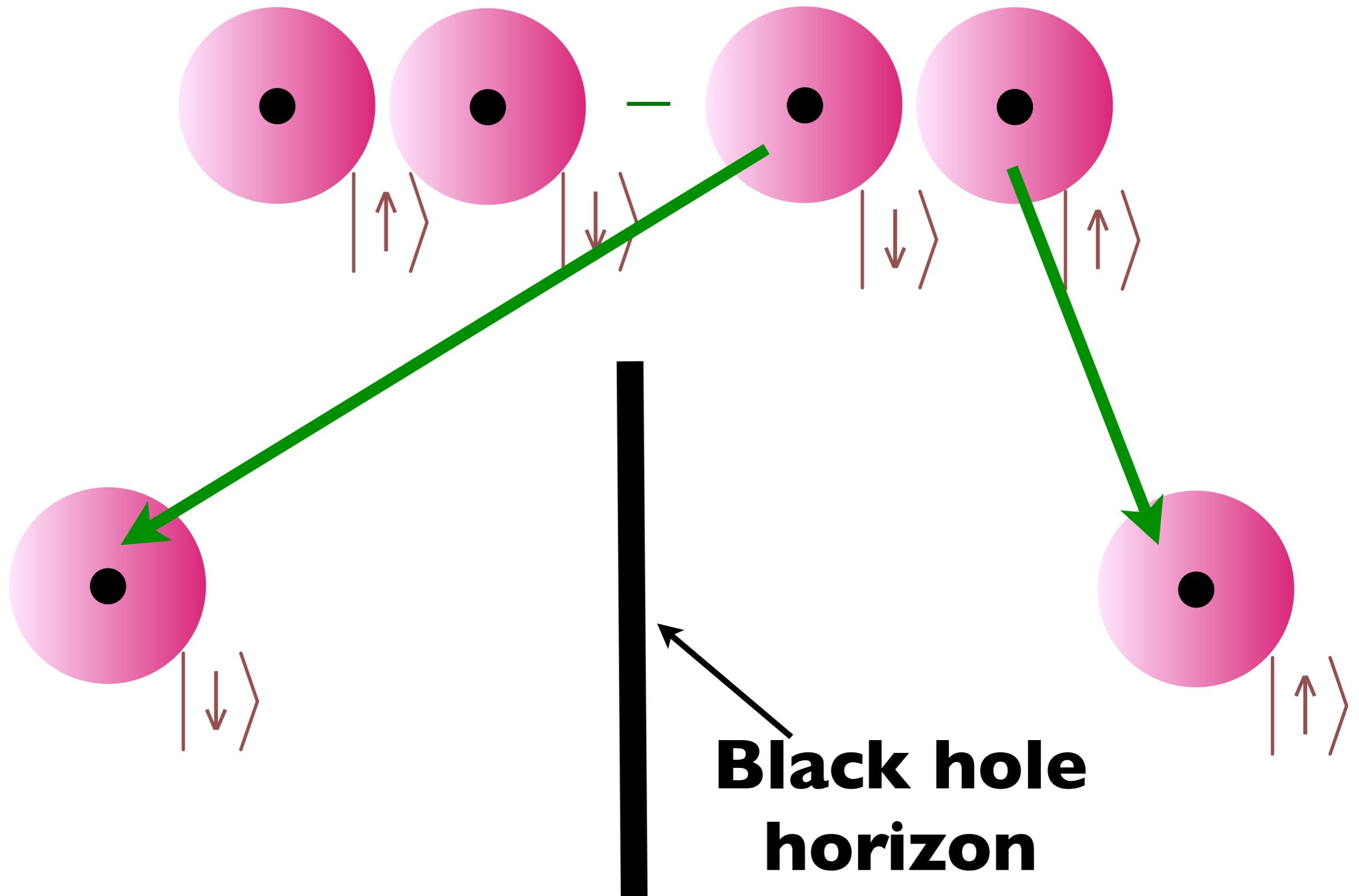
# Quantum Entanglement across a black hole horizon



# Quantum Entanglement across a black hole horizon

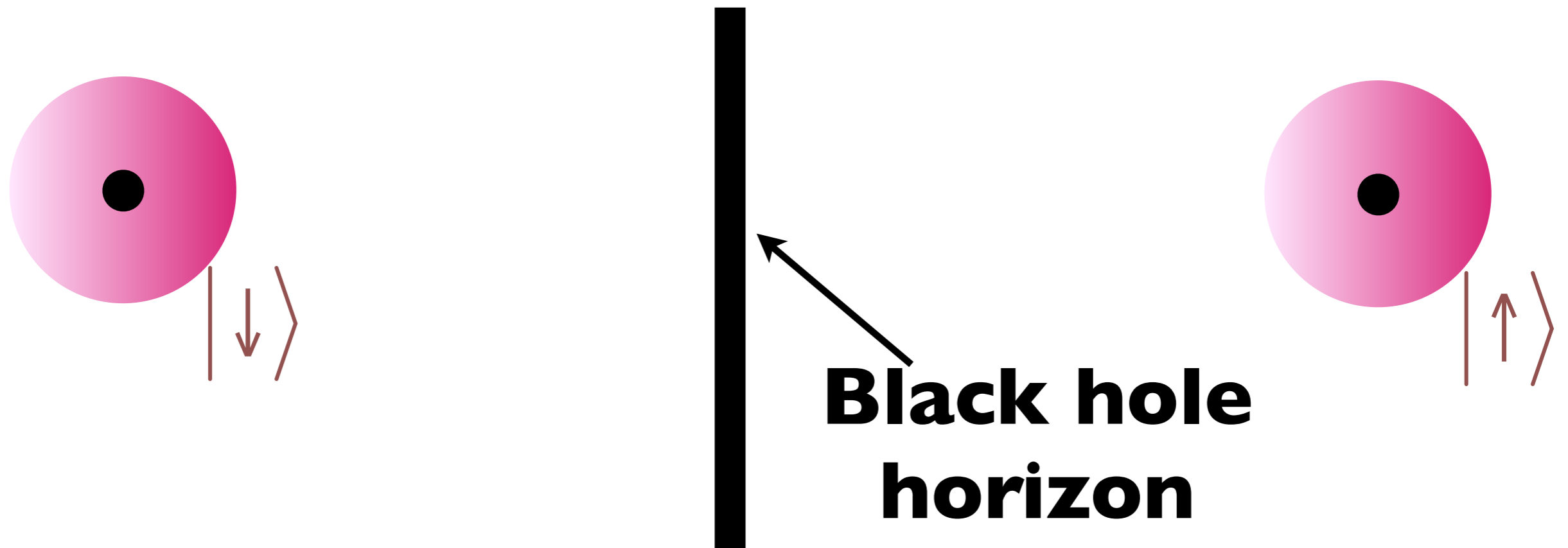


# Quantum Entanglement across a black hole horizon



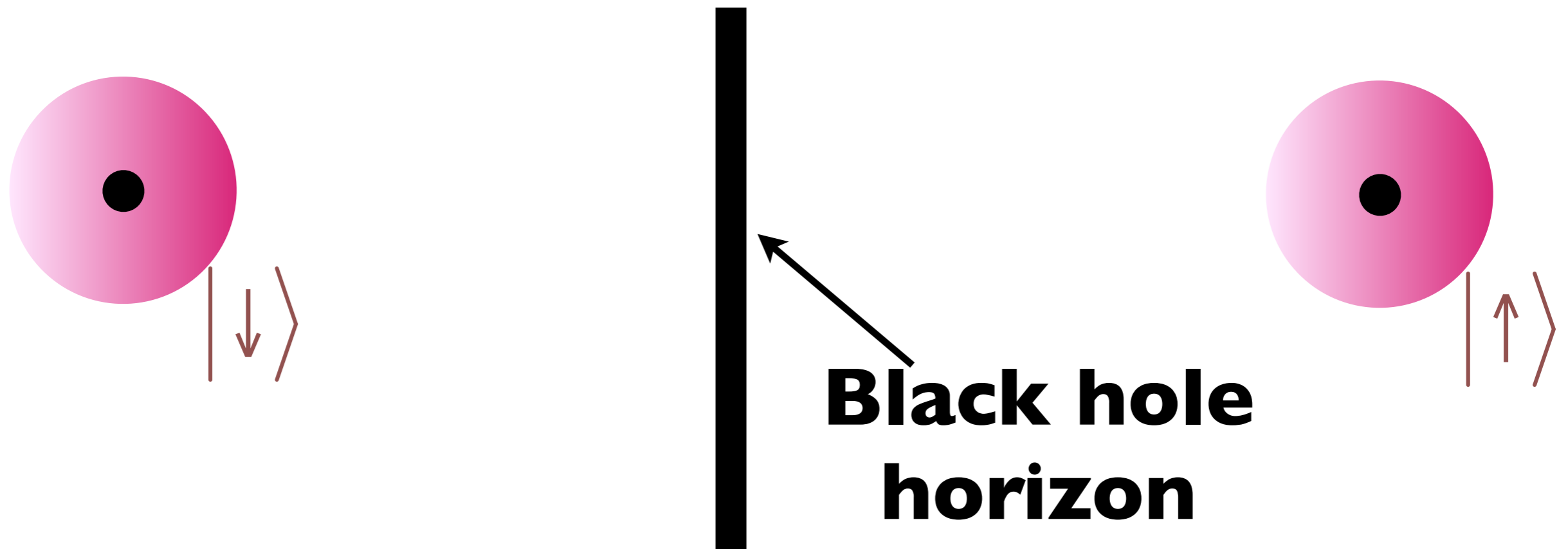
# Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole



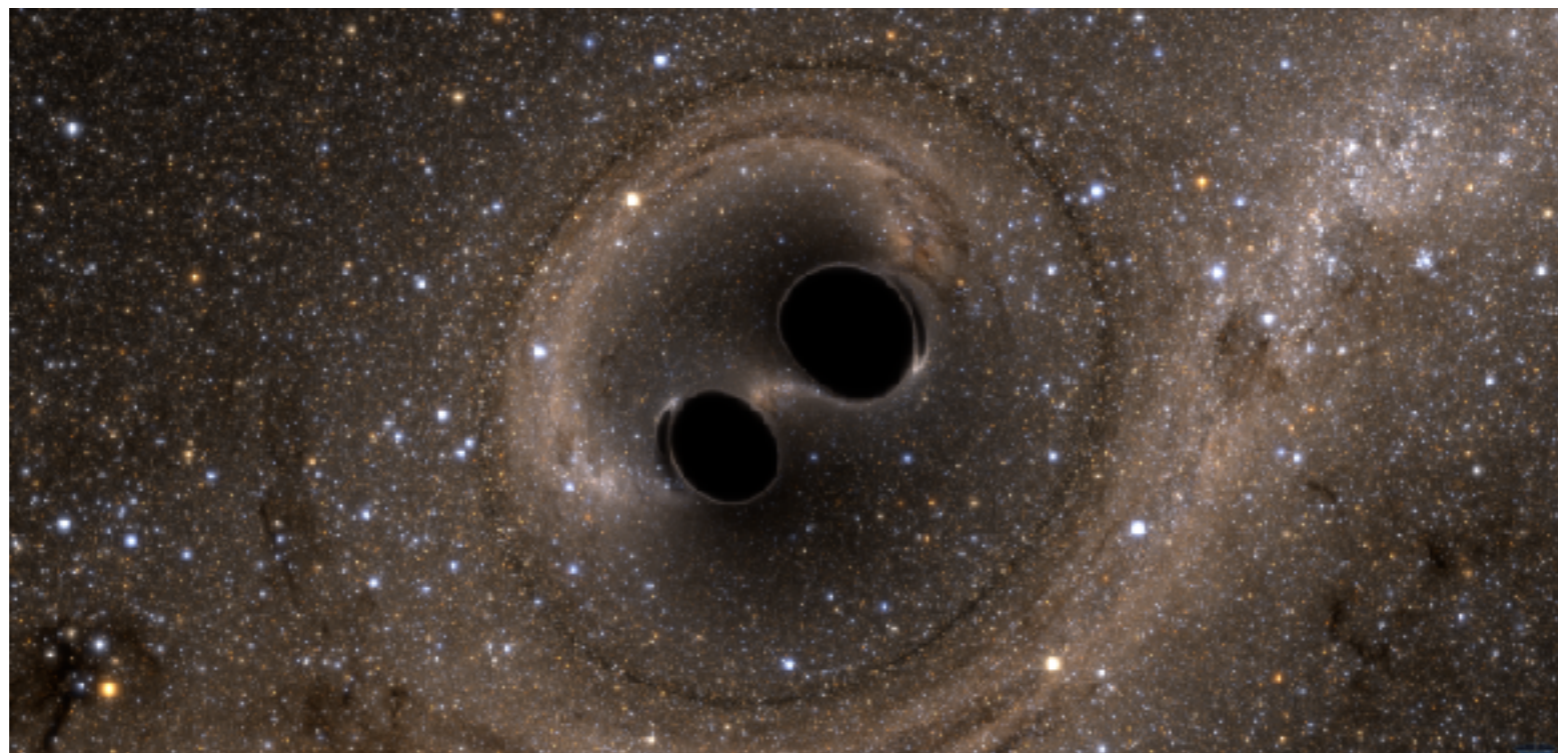
# Quantum Entanglement across a black hole horizon

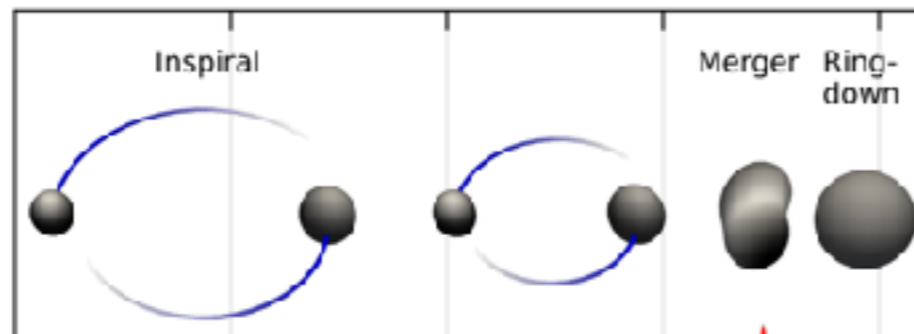
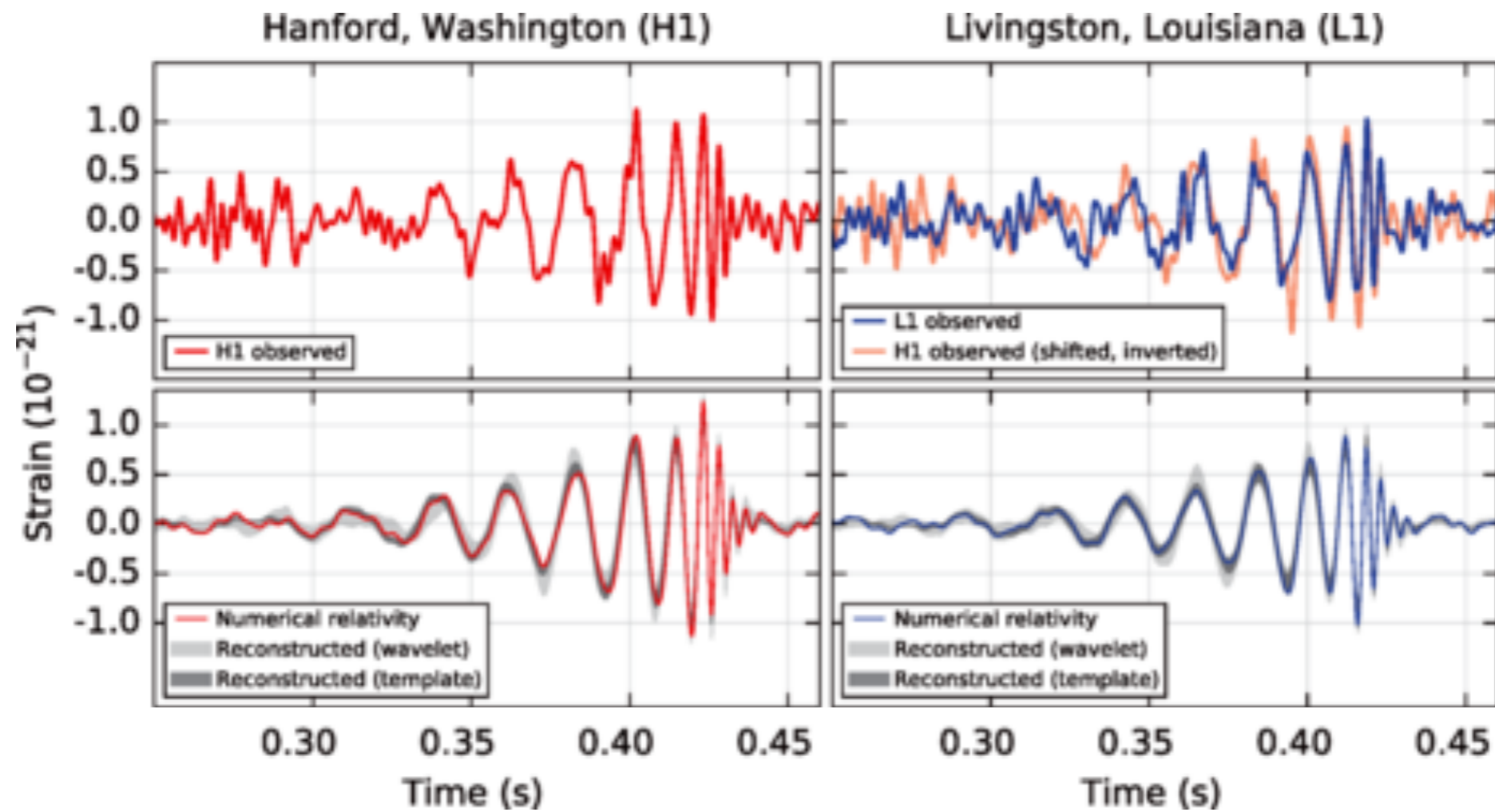
Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)



# Black holes

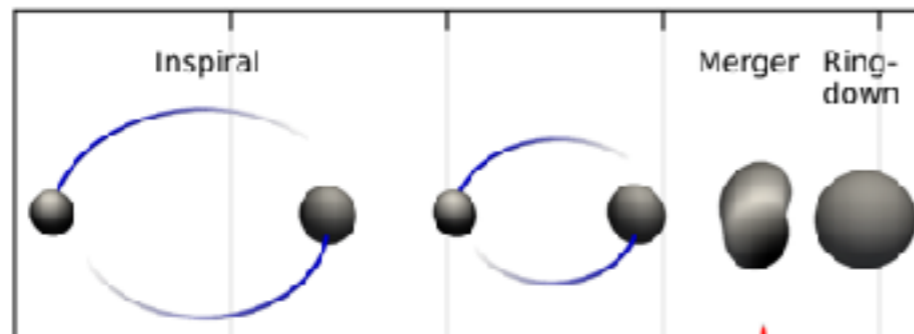
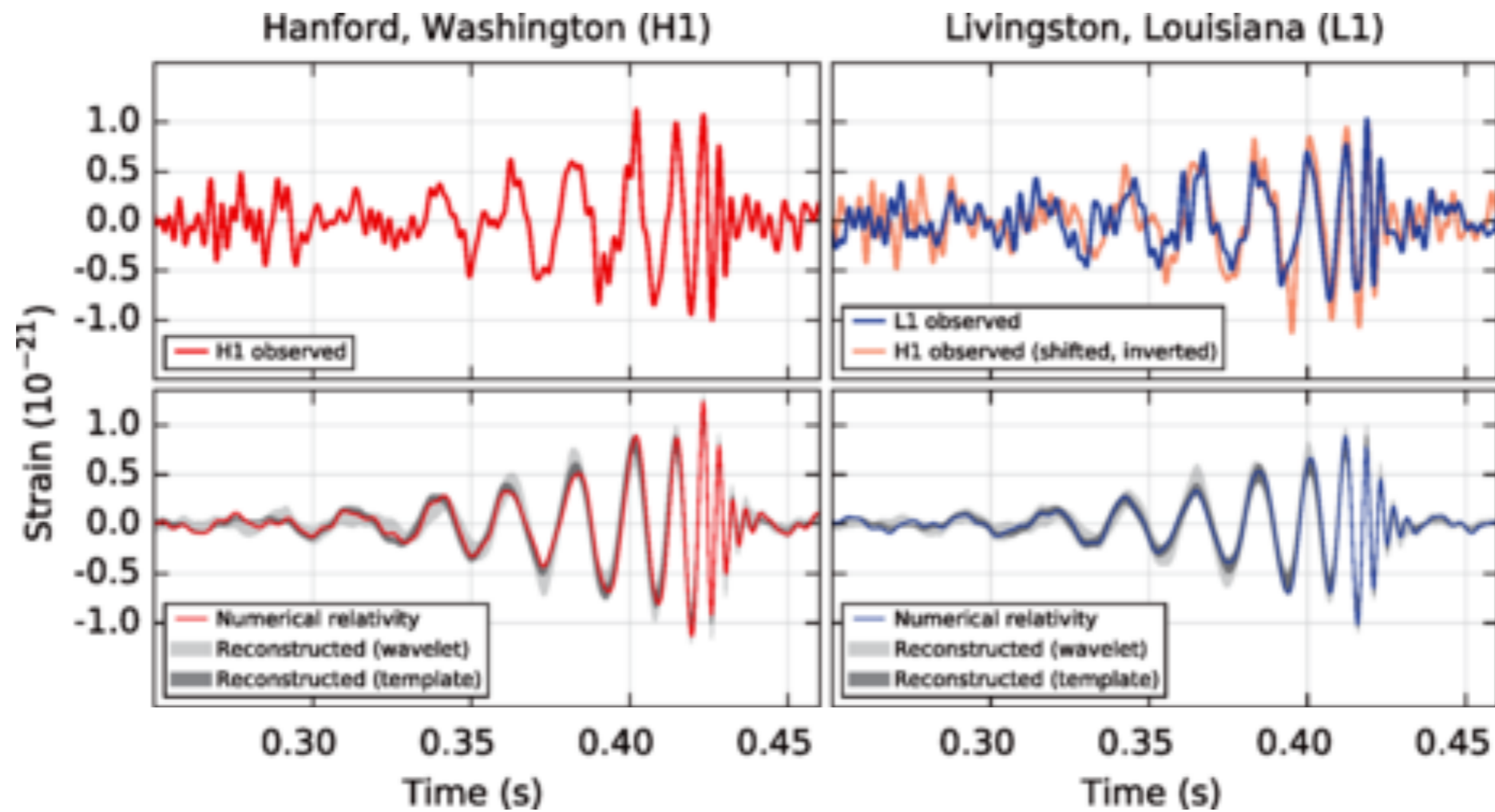
- Black holes have an entropy and a temperature,  $T_H = \hbar c^3 / (8\pi G M k_B)$ .
- The entropy is proportional to their surface area.





**LIGO**  
**September 14, 2015**

- The ring-down is predicted by General Relativity to happen in a time  $\frac{8\pi GM}{c^3} \sim 8$  milliseconds.



**LIGO**  
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- The ring-down is predicted by General Relativity to happen in a time  $\frac{8\pi GM}{c^3} \sim 8$  milliseconds. Curiously this happens to equal  $\frac{\hbar}{k_B T_H}$ ; so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

# Black holes

- Black holes have an entropy and a temperature,  $T_H = \hbar c^3 / (8\pi G M k_B)$ .
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time  $\sim \hbar / (k_B T_H)$ .



**Quantum  
entanglement**

**Black  
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**Strange  
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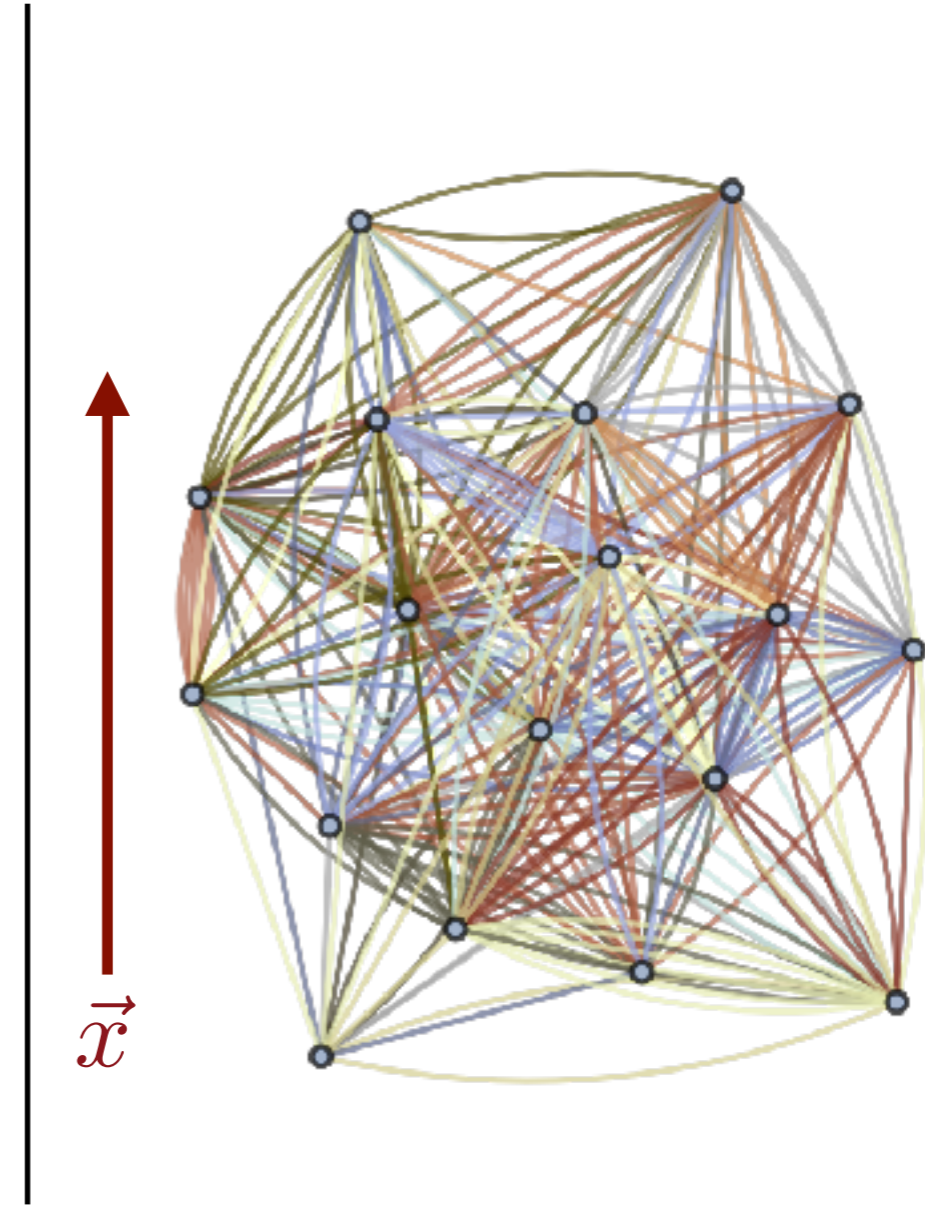
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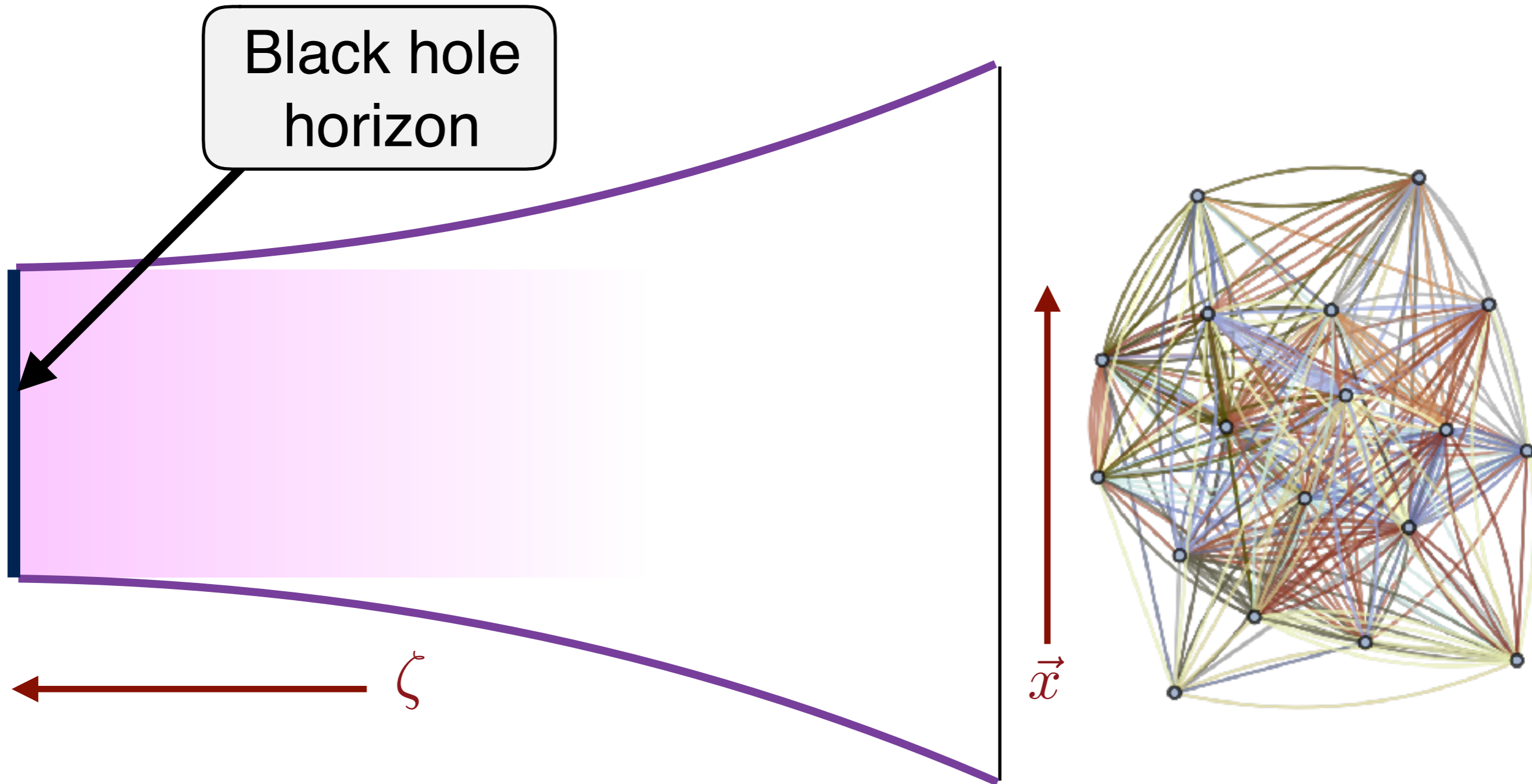
**Strange  
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**The SYK model is both a  
strange metal and a black hole!**

# SYK and black holes

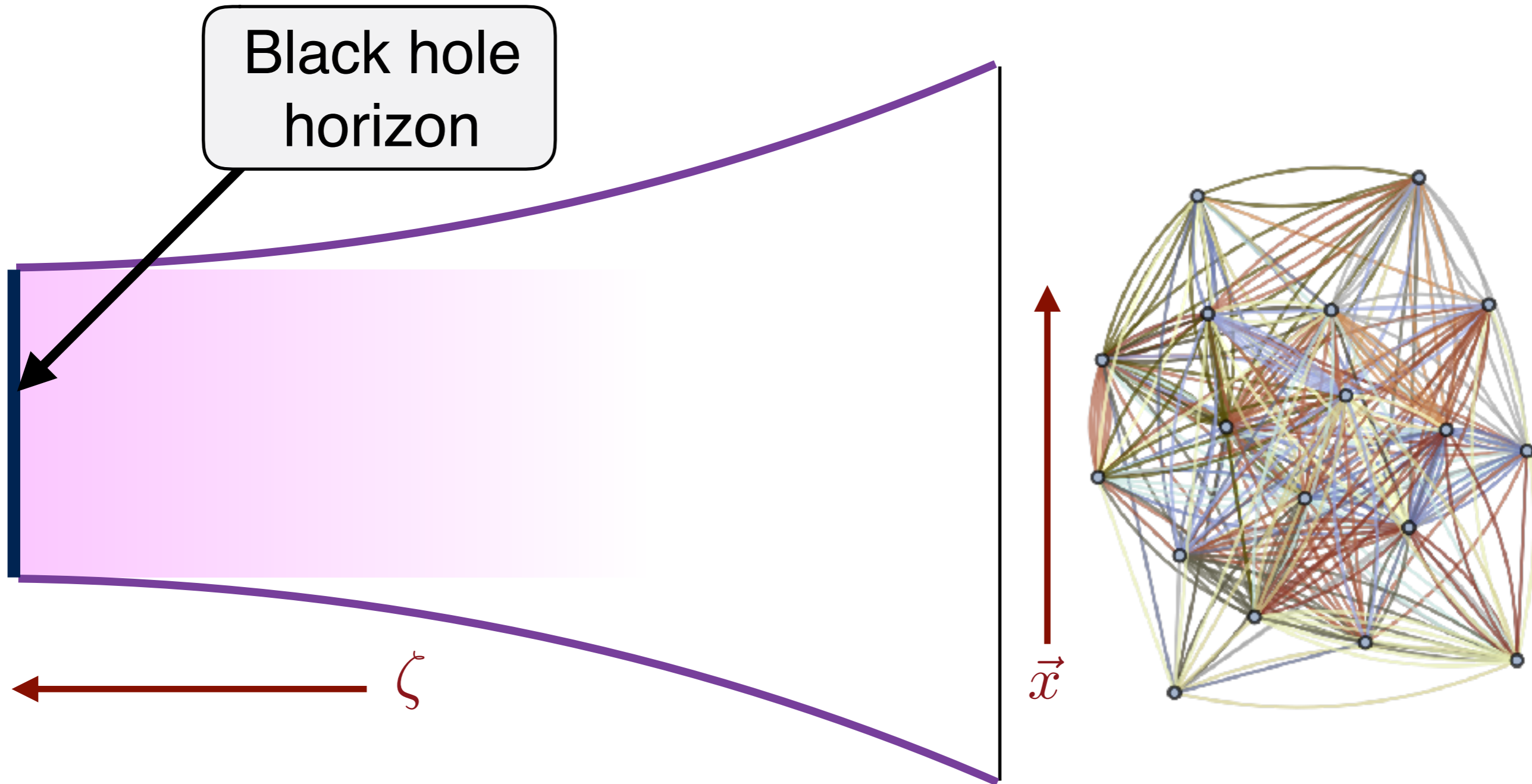


# SYK and black holes



The SYK model has “dual” description in which an extra spatial dimension,  $\zeta$ , emerges. The curvature of this “emergent” spacetime is described by Einstein’s theory of general relativity

# SYK and black holes

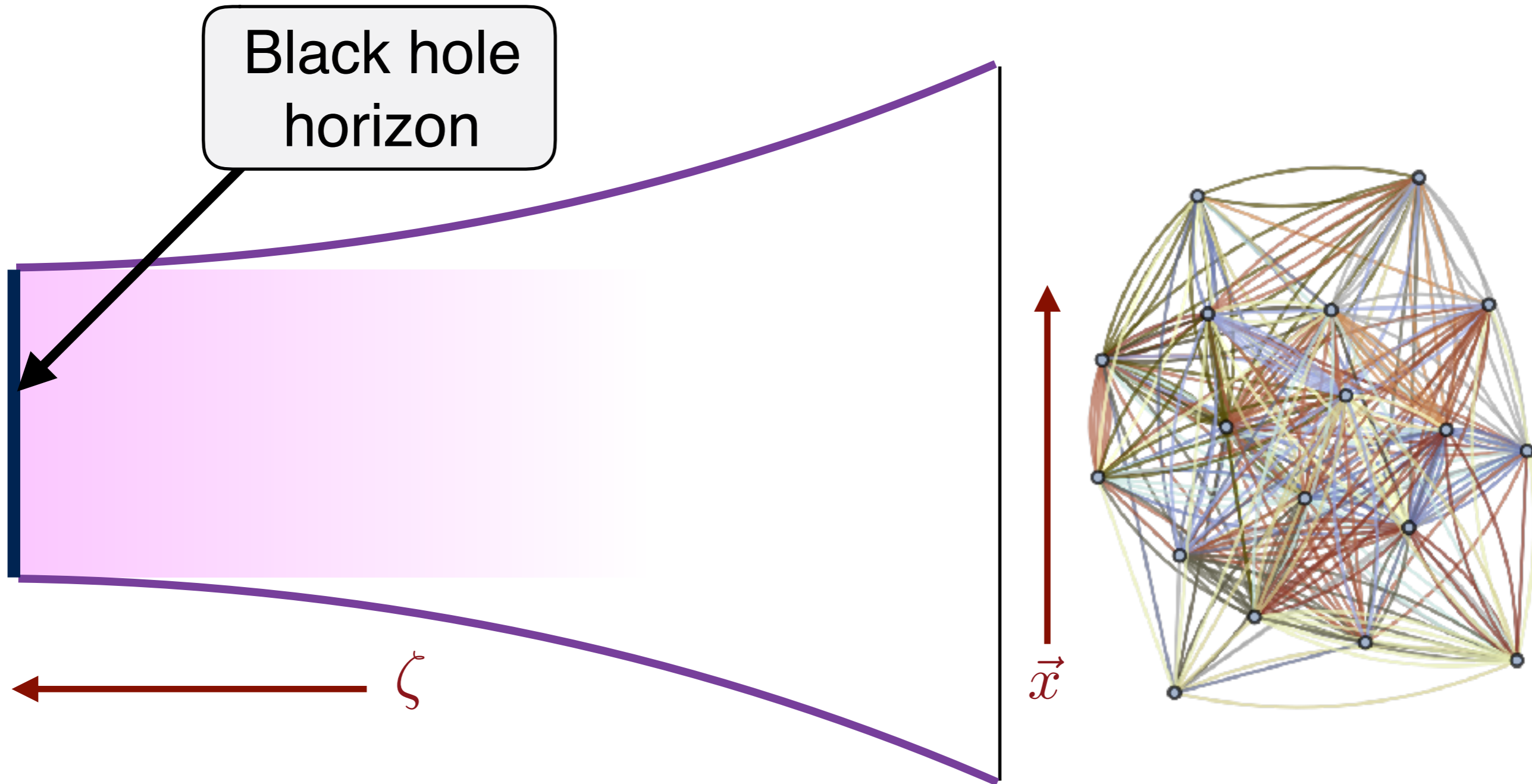


There is a black hole in the emergent spacetime at the same temperature,  $T_H$ , as the SYK model.

The duality explains:

(i) why they have a common thermalization time  $\hbar/(k_B T_H)$ .

# SYK and black holes

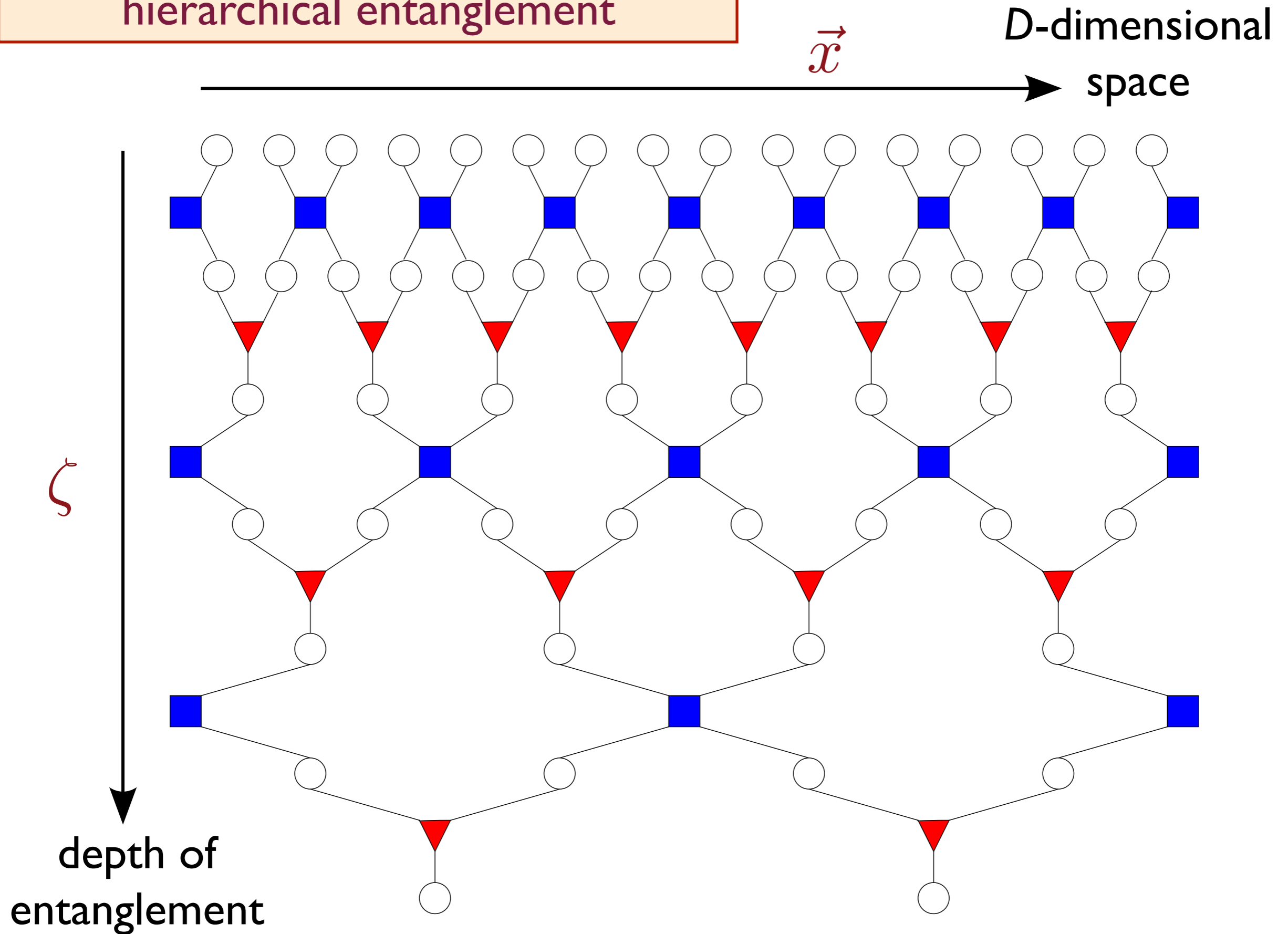


There is a black hole in the emergent spacetime at the same temperature,  $T_H$ , as the SYK model.

The duality explains:

(*ii*) why the black hole entropy is proportional to its surface area.

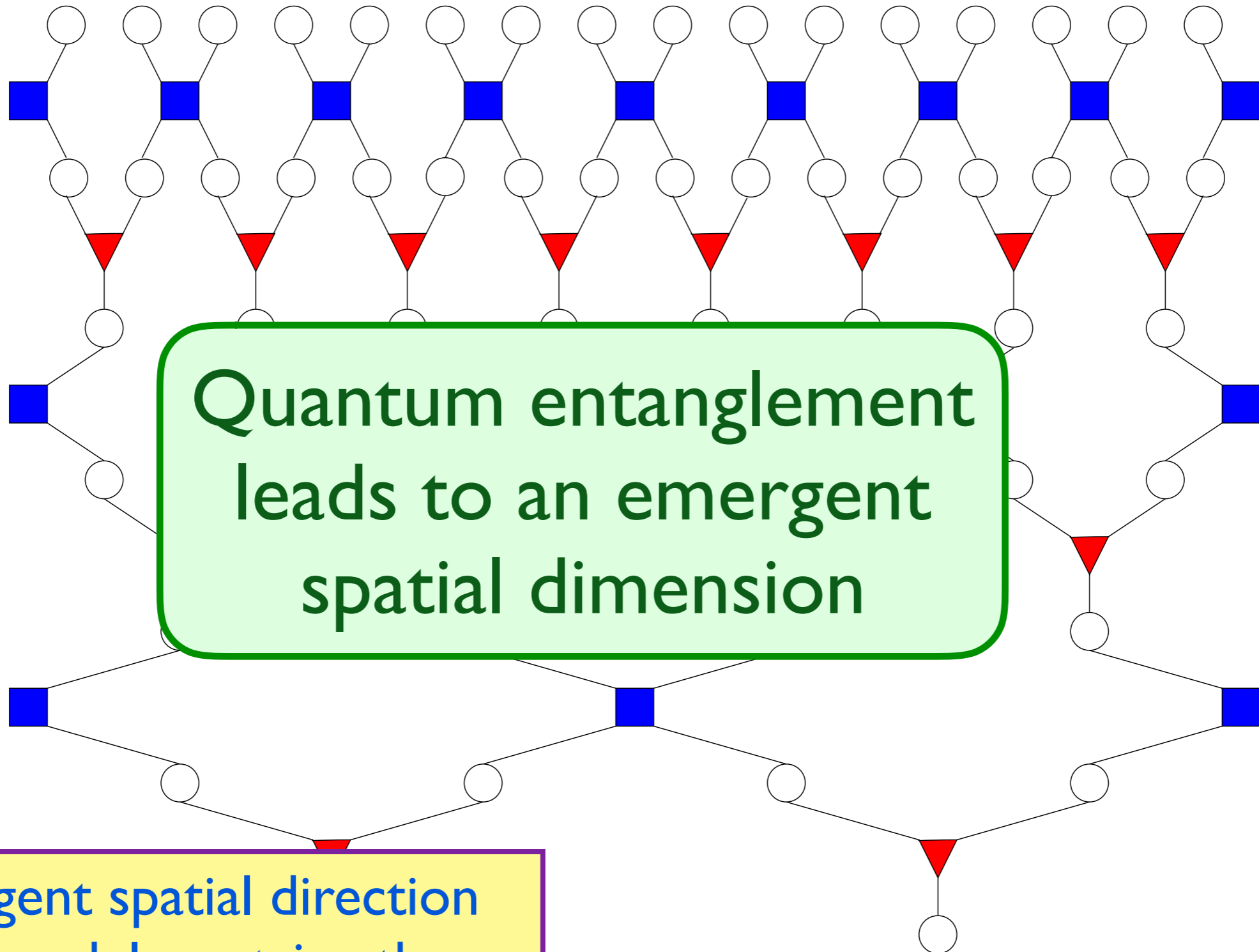
# Tensor network of hierarchical entanglement



Tensor network of hierarchical entanglement

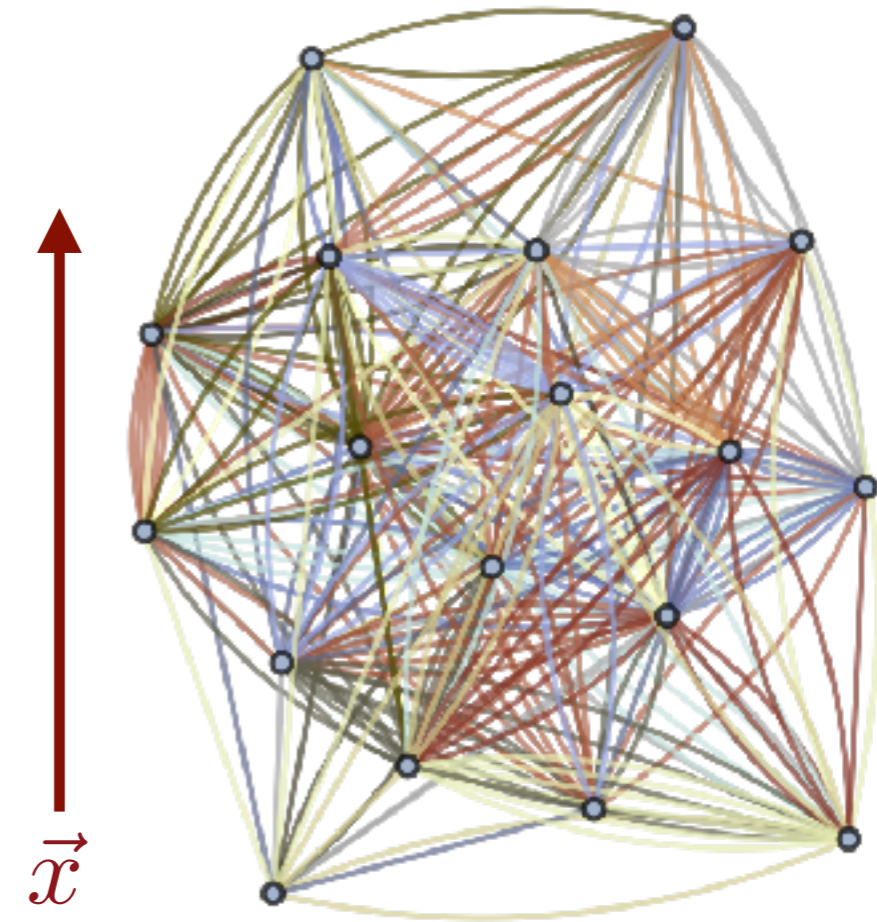
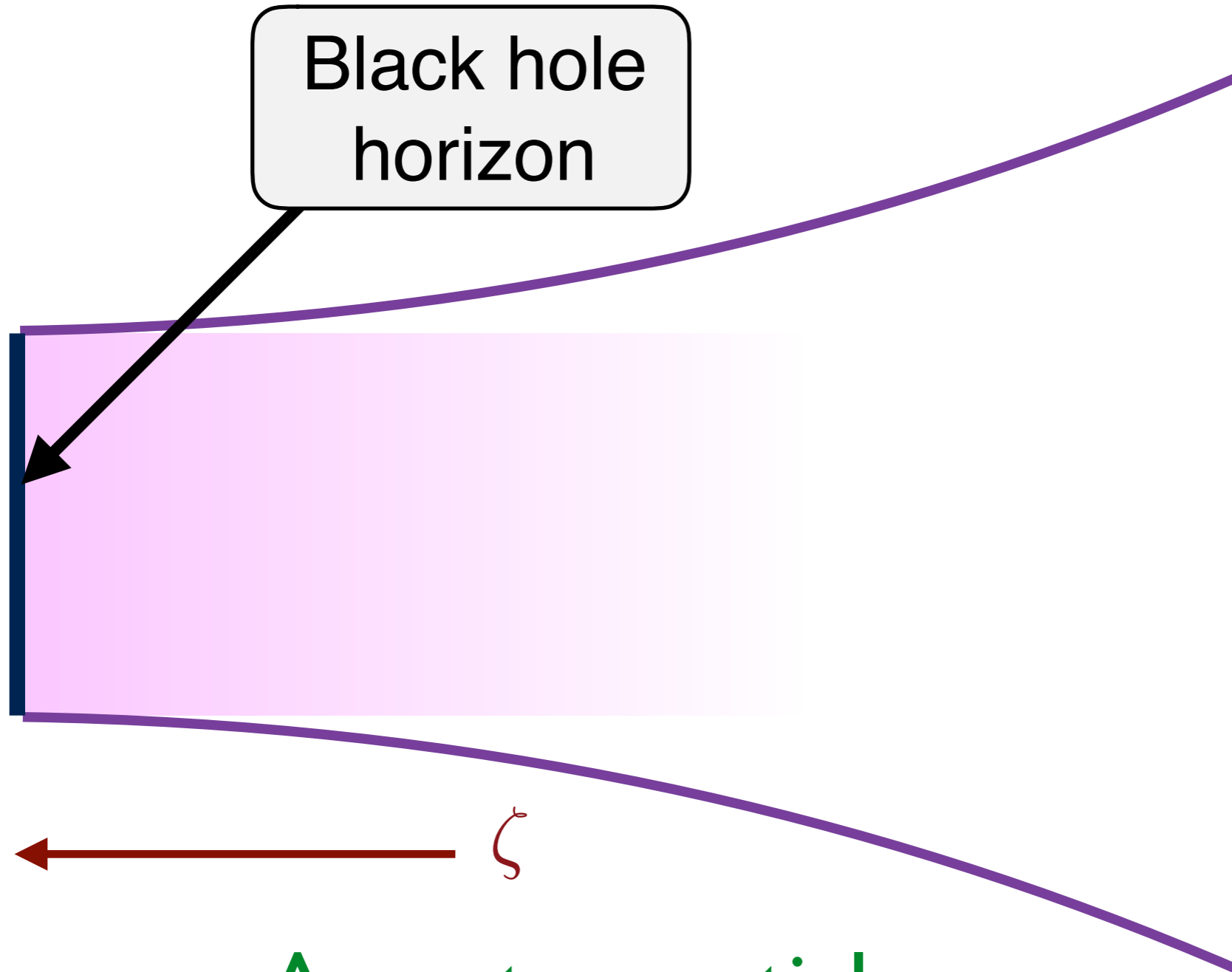
$\vec{x}$

D-dimensional space



Emergent spatial direction of SYK model or string theory

# SYK and black holes



An extra spatial dimension emerges from quantum entanglement!

**Quantum  
entanglement**

**Black  
holes**

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## Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states.
- Thermalization and many-body chaos in the shortest possible time of order  $\hbar/(k_B T)$ .

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- Thermalization and many-body chaos in the shortest possible time of order  $\hbar/(k_B T)$ .
- These are also characteristics of black holes in quantum gravity.